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MANAGEMENT SCIENCE
Vol. 63, No. 3, March 2017, pp. 729–748
ISSN 0025-1909 (print), ISSN 1526-5501 (online)

Temporal Profiles of Instant Utility During Anticipation, Event, and Recall

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Received: September 5, 2014
Revised: March 16, 2015
Accepted: August 15, 2015
Published Online in Articles in Advance: April 26, 2016

https://doi.org/10.1287/mnsc.2015.2362

Abstract. We propose the anticipation-event-recall (AER) model. Set in a continuous time frame, the AER model formally links the three components of total utility (i.e., utility from anticipation, event utility, and utility from recall). The AER model predicts the temporal profiles of instant utility experienced before, during, and after a given event. Total utility is calculated as the integral of instant utility. The model builds on the psychological elements of conceptual consumption, adaptation, and time distance. By virtue of its rich formulation, the AER model produces a wide set of insights and testable predictions, including the U shape of instant utility during anticipation and the optimal duration of anticipation for a given event. Using both real and hypothetical events, we provide empirical evidence in support of the main implications of the AER model.

History: Accepted by James Smith, decision analysis.
Funding: This research was first developed at IESE Business School, Spain, and it was partially supported by the Spanish Ministerio de Economía y Competitividad [Grant PSI2013-41909-P].
Supplemental Material: Data are available at https://doi.org/10.1287/mnsc.2015.2362.

Keywords: instant utility • anticipation • recall • time distance • magnitude effect

1. Introduction

> *If for example you come at four o'clock in the afternoon, I shall start feeling happy at three o'clock. As the time passes, I shall feel happier and happier. At four o'clock, I shall become agitated and start worrying; I shall discover the price of happiness. But if you come at just any time, I shall never know when I should prepare my heart to greet you . . . . One must observe the proper rites.*
>
> —Antoine de Saint-Exupéry, *The Little Prince*

The Little Prince exhorts the fox to let him know the exact arrival time of her visit because he does not want to miss the anticipatory feelings of happiness and excitement prior to the upcoming meeting. Indeed, there are many events whose duration is very short relative to the duration of anticipation and recall. Examples include a stopover to admire a beautiful building or natural wonder, a visit by a distant friend or relative, a brief romantic encounter, a short but painful medical procedure, or a run-in with a celebrity. According to Lazarus (1966), certain forms of physical pain, such as pinpricks, do not produce measurable psychological-stress reactions beyond those produced by the mere anticipation of them. Two studies examining travelers' experiences revealed that, regardless of the type of trip, vacationers were happier in the period leading up to their vacation than during the vacation (Nawijn et al. 2010, Mitchell et al. 1997). In such cases, the integral of the utility experienced during the occurrence of the event may be small compared with the total utility derived from the event, that is, considering the utility derived before (anticipation), during (occurrence), and after (recall) the event.

Bentham (1789) was among the first to recognize that anticipation is an important source of pleasure and pain. Subsequently, Jevons (1905) distinguished among anticipation of future events, sensation of present events, and memory of past events. More recently, Kahneman et al. (1997, p. 376) argued that "total utility is a normative concept . . . constructed from temporal profiles of instant utility."

Existing research, however, has not proposed a comprehensive model of instant utility during anticipation, occurrence, and recall of an event. In this paper, we propose the anticipation-event-recall (AER) model that formally links the three components of total utility on a continuous-time interval that includes the anticipation, occurrence, and recollection of the event. The AER model is based on well-established psychological elements such as conceptual consumption, adaptation, and time distance. Given a small set of general inputs (i.e., the magnitude and duration of the event, and the duration of anticipation and recall), the AER model produces the temporal profile of instant utility associated with an event. The integral of instant utility over time produces the total utility associated with the event. The AER model leads to numerous predictions, such as the U shape of instant utility during anticipation and the optimal duration of anticipation.

AER is primarily a descriptive model of instant utility (a.k.a. moment-by-moment utility or experienced utility). The model is also predictive of choices, but only to the extent that individuals accurately predict future total utility and use such criteria to guide their decisions. In the framework of Kahneman et al. (1997), Read (2007), and Morewedge (2016), where a rational decision maker maximizes the time integral of instant utility, our model provides prescriptions for someone willing to "engineer" his or her own happiness.

1.1. Conceptual Consumption, Adaptation, and Time Distance

The process model we propose entails three key psychological elements. The first element is *conceptual consumption* (Ariely and Norton 2009), defined as psychological consumption that is temporally dissociated from physical consumption. Conceptual consumption arises in reaction to mental discrete images of decision outcomes (Damasio 1994). For example, when anticipating an upcoming event, individuals conceptually consume images of the event prior to its physical occurrence. The ability to generate such mental simulations is a fundamental capacity of the human mind (Gilbert and Wilson 2007). Depending on the valence of the event, conceptual consumption produces "savoring" or "dread" during anticipation (Loewenstein 1987, Golub et al. 2009). Similarly, for recall, contemplation of the past through memory produces pleasure or pain in the present (Elster and Loewenstein 1992). People recall salient instants of pleasure or pain and tend to neglect the duration of the event (Kahneman et al. 1993, Fredrickson and Kahneman 1993, Fredrickson 2000). Consistent with this research, we posit that mental images of future events (Elster and Loewenstein 1992) or "snapshots" of the event experienced in the past (Fredrickson and Kahneman 1993) determine the intensity of conceptual consumption before and after the physical occurrence of the event.

The second psychological element is *adaptation* before and during the event. Adaptation, which is understood as a decreased response to a repeated stimuli, has been part of the tool set of psychologists for a long time (Helson 1964). In formal utility models, adaptation is often described by means of a reference point that approaches the consumption rate (Constantinides 1990, Wathieu 1997). We presume that adaptation begins with anticipation—in other words, both conceptual consumption before the event and physical consumption during the event produce adaptation. Anticipating an event increases the level of expectations against which future outcomes will be valued (Kahneman and Miller 1986, Olson et al. 1996). Formally, anticipation modifies the reference point. This is consistent with the finding that overly optimistic expectations can potentially lower a given event's utility by setting high counterfactuals (Shepperd and McNulty 2002). Indeed, extensive research demonstrates that unmet anticipatory expectations produce disappointment in a variety of settings, including romantic dates (Norton et al. 2007), athletic competitions (Medvec et al. 1995), promotions in the workplace (Harvey and Martinko 2009), academic tests (Shepperd and McNulty 2002), and hotel services (Boulding et al. 1993). People typically use a recollection of similar events, which occurred in the past and are stored in their memory, to form their expectations of upcoming events (Weber et al. 2007, Stewart et al. 2006) and to set the reference point against which future outcomes will be measured (Anderson and Milson 1989).

The third psychological element of the AER model is *time distance* to the event. Time distance modulates instant utility during anticipation and recall by means of a discount factor. The discount factor depends on the time distance to and from the event. Discounting captures the notions of decreasing impatience for anticipation (Loewenstein and Prelec 1993, Frederick et al. 2002) and of transience for recall (Ebbinghaus 1913, Wixted 2004). We also incorporate magnitude effects in discounting: the smaller the magnitude of the event, the smaller the discount factor (Thaler 1981, Frederick et al. 2002). This feature captures the "peak" element of the peak-end rule (Kahneman et al. 1993, Fredrickson 2000). Time distance, together with conceptual consumption, is consistent with construal-level theory, which proposes that individuals form abstract mental constructs of distal objects and realities and derive pleasure or pain from these thoughts (Trope and Liberman 2010).

We seek to contribute to the literature by modeling the combined effect of these three psychological elements (i.e., conceptual consumption, adaptation, and time distance) in a unique and comprehensive formulation. As detailed in the description of the model, we capture these elements using a parametric specification (see Table 1).

1.2. Review of Anticipatory Utility Models

Several formal models of anticipation have been proposed (Loewenstein 1987, Brunnermeier and Parker

Table 1. Parameters of the AER Model

Function	Psychological element	Parameters
Value function, v	Loss aversion	$\lambda \geq 1$
Adaptation, $dr_t/dt = \alpha(c_t - r_t)$	Speed of adaptation	$0 \leq \alpha < 1$
Discount rates, $\rho_a = \rho_0/\lvert v(c)\rvert^{\mu}$	Base discount	$\rho_0 > 0$
and $\rho_r = \rho_a e^{\alpha\mu\Delta_a}$	Magnitude effect	$\mu \geq 0$
Discount factor, $e^{-(\tau^{\delta})}$	Diminishing sensitivity to τ	$0 < \delta \leq 1$

2005, Gollier and Muermann 2010, Kőszegi and Rabin 2009, Caplin and Leahy 2001). These models are typically set in discrete time, provide little detail about the process by which future outcomes and events affect current utility, and do not consider the influence of anticipation on event and recall utility. The AER model aims to fill these gaps.

In the seminal paper by Loewenstein (1987), individuals derive utility from anticipation, and such utility is proportional to the total utility that will be obtained during the event. We enrich Loewenstein's formulation by modeling the dynamic interactions between anticipation and event utility and by adding the component of utility from memory to the overall model.

In Brunnermeier and Parker (2005) and Gollier and Muermann (2010), individuals derive utility from being optimistic and choose beliefs that balance positive anticipatory feelings with regret from poor decision making resulting from incorrect beliefs. In our model, individuals may choose to be optimistic by conceptually consuming a positive future scenario. But such thoughts do not alter their beliefs (that is, their subjective probability over the possible scenarios that may occur). Thus, individuals can simultaneously entertain optimistic images and hold correct beliefs.

In Kőszegi and Rabin (2009), individuals derive consumption utility during the event as well as gain/loss utility before the event. Gain/loss utility is driven by changes in expectations. For an event that is certain to occur, individuals experience a boost of utility when such an event enters their calendar, a second boost of utility when they consume, and no utility in between. Our model allows for a richer dynamic of utility, with consumers gradually adapting to the future event and progressively savoring the initial surprise over time.

Finally, Caplin and Leahy (2001) propose a modification of expected utility whereby utility is obtained from psychological states rather than physical outcomes. Psychological states may depend on the current and future physical outcomes and produce anticipatory feelings such as anxiety. Their model, however, does not incorporate the possibility that anticipatory feelings may affect the utility of the event. By contrast, the AER model possesses the realistic feature that anticipation influences event utility.

To keep things simple, we focus on conditions of certainty and on single event cases (e.g., an upcoming dinner at a nice restaurant that is expected to occur with certainty). Set in continuous time, our model predicts the temporal profile of instant utility. The continuous time frame forces us to specify, for instance, what determines the intensity of anticipation at every moment in time. It also yields an exact optimal duration of anticipation.

By virtue of its richer formulation, the AER model produces a wide set of insights and testable implications. Table 3 (reported in the conclusions) provides a summary list of nine predictions of the AER model. For example, consistent with Breznitz (1984) (as well as our experimental results reported in Sections 3.1 and 3.2), the resulting profile of instant utility during anticipation is U-shaped. Moreover, we find that increasing anticipation makes an event less surprising and leads to a decrease in the total utility experienced during and after the event. Thus, there is such a thing as the optimal duration of anticipation (see the experiment reported in Section 6.1). In fact, we identify conditions under which a surprise event (i.e., zero anticipation) is optimal. We also investigate how to optimally anticipate negative events. Finally, the model provides insight into optimal hedonic editing and deceptive postponement.

2. The Anticipation-Event-Recall Model

2.1. The General Model

Let t be a real number denoting calendar time. Four moments in time are particularly relevant: the moment when the event starts to be anticipated, t_0; the moment when the event begins, t_b; the moment when the event ends, t_e; and the moment when the recall of the event ends, t_1. Naturally, $t_0 \le t_b \le t_e \le t_1$. Thus, the event is anticipated during $[t_0, t_b)$, it takes place during $[t_b, t_e)$, and it is recalled during $[t_e, t_1)$. Let $\Delta_a = t_b - t_0$ be the duration of anticipation, let $\Delta_e = t_e - t_b$ be the duration of the event, and let $\Delta_r = t_1 - t_e$ be the duration of recall. Unless stated otherwise, we conveniently set $t_1 = \Delta_r = \infty$.

Events, such as a concert or a minor surgery, can influence utility first through anticipation, then through the unfolding of the experience, and finally through memory (Elster and Loewenstein 1992). The consumer's intake is modeled by means of a consumption rate, $c_t\colon [t_0, t_1) \to \mathbb{R}$. The rate of consumption during the event is determined by the characteristics of the event and is an exogenous input of the AER model. The value of c_t, $t_b \le t < t_e$, is a function of the objective attributes of the event (quantity, quality, etc.). For pleasurable events (e.g., a dinner out), the consumption rate is positive; for painful events (e.g., a surgical procedure), it is negative. For example, consider an individual making a reservation at a high-quality restaurant for the following week. Because the restaurant is high scale (high-quality menu, wine, and ambiance), the consumption rate during the event will be higher (e.g., approximately 80 on an imaginary 100-point scale) than if the reservation had been for a fast-food restaurant (e.g., about 30 out of 100).

The rate of consumption during anticipation and recall, c_t, $t \in [t_0, t_b) \cup [t_e, t_1)$, is interpreted as a rate of *conceptual* consumption (Ariely and Norton 2009). We assume that conceptual consumption before and after the event is composed of samples of snapshots of c_t during the event. In other words, conceptual consumption must take values that are realistically possible. Formally, the level of conceptual consumption at any point in time during anticipation and recall is a decision variable constrained to take values in $C = \{c_t \in \mathbb{R}\colon t_b \le t < t_e\}$. For simplicity, we will assume that consumption is constant throughout the event. In the earlier restaurant example, we have that $c_t = 80$ during the dinner; hence $C = \{80\}$, and the rate of conceptual consumption during anticipation and recall will be 80 as well.

There is a reference point, r_t, $t \ge t_0$, associated with the consumption rate. Given c_t, $t \ge t_0$, the reference point adapts to c_t. Because c_t is a deterministic exogenous variable (during the event) or a deterministic choice variable (during anticipation and recall), the reference point at every moment in time is a deterministic value.

The carrier of utility is given by the difference between the consumption rate, c_t, and the reference point, r_t, by means of a value function $v(c_t - r_t)$ (Kahneman and Miller 1986, Kőszegi and Rabin 2006).[1] We label the difference $c_t - r_t$ the *effective consumption* (Figure 1). Continuing with our restaurant example, in the week prior to the dinner, the individual may savor the upcoming event by having thoughts of a tasty entrée in a nice setting. Engaging in such conceptual consumption will progressively elevate the reference point for the upcoming dinner toward the specific level of conceptual consumption (e.g., from 0 to 80 in the case of this high-scale dinner). Thus, when the dinner finally occurs, the effective consumption rate will be determined by the level of conceptual consumption minus the reference point developed during anticipation.

Finally, we consider time distance and discounting. Psychological time distance is defined as calendar distance multiplied by a discount rate. Let $\rho_a, \rho_r > 0$ be the *discount rates* for anticipation and recall. Given discount rates, the *psychological time distance*, τ_t, to and from the event is given by

$$\tau_t = \begin{cases} \rho_a(t_b - t) & t \in [t_0, t_b), \\ 0 & t \in [t_b, t_e), \\ \rho_r(t - t_e) & t \in [t_e, t_1). \end{cases}$$

Discounting is a decreasing function of psychological distance, which we can write as $f(\tau) = e^{-\pi(\tau)}$, where $\pi\colon [0,\infty] \to [0,\infty]$ is a *psychological distance function* (Baucells and Heukamp 2012).[2] To keep things simple, we do not incorporate discounting as a function of calendar time. This implies that the decision maker is indifferent to changes in t_0, provided Δ_a, Δ_e, and Δ_r are maintained.

With these three elements in mind, we are ready to define the AER model.

Definition 1. Given the level of actual and conceptual consumption, the reference point, and psychological distance, *instant utility* in the AER model is given by

$$u(t) = v(c_t - r_t) f(\tau_t), \quad t \in [t_0, t_1);$$

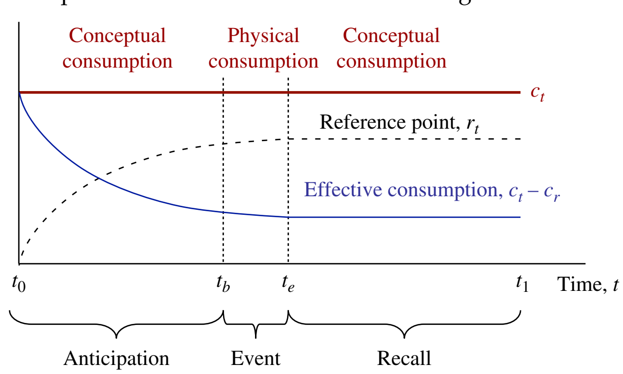
and *total utility* of anticipation, of the event, and of recall is given by

$$U = \int_{t_0}^{t_1} u(t)\,dt = \underbrace{\int_{t_0}^{t_b} u(t)\,dt}_{U^A} + \underbrace{\int_{t_b}^{t_e} u(t)\,dt}_{U^E} + \underbrace{\int_{t_e}^{t_1} u(t)\,dt}_{U^R}.$$

We interpret $u(t) = 0$ as a neutral state and $u(t) > 0$ ($u(t) < 0$) as instants during which the individual is in a positive (negative) state. Considering the absolute value of instant utility, we call $|u(t)|$ the instant (dis)utility at time t and $|U|$ the total (dis)utility.

The proposed model is a single-event model and does not account for divided attention among multiple events. The AER model could be the base for an extended multievent model. Such extension should specify how attention switches between competing events.

Figure 1. (Color online) When Conceptual Consumption Is Constant, Effective Consumption Is Decreasing During Anticipation and Event and Constant During Recall



2.2. Assumptions

To produce a relatively tractable model and derive insights, we make five specific assumptions, which we later discuss in more detail.

Assumption A1 (Constant Consumption Rate). *We set $c_t = c$, $t_b \le t < t_e$, where $c \in \mathbb{R}$ is the consumption level. We call the absolute value of c, $|c|$, the magnitude of the event.*

Assumption A2 (Reference Point). *Let $\alpha \ge 0$ be the speed of adaptation. Initially, we set $r_0 = 0$. We are given c_t, $t_0 \le t < t_1$. During anticipation and the event, the reference point adapts to c_t according to $dr_t/dt = \alpha(c_t - r_t)$, $t_0 \le t < t_e$. During recall, the reference point decays to some value between 0 and r_e. For tractability reasons, we make the simplifying assumption that the reference point stays constant at r_e, or $r_t = r_{t_e}$, $t_e \le t < t_1$.*[3]

Assumption A3 (Value Function). *We set* $v(c) = c \cdot 1_{\{c\geq 0\}} + \lambda c \cdot 1_{\{c<0\}}$, *where* $\lambda \geq 1$ *is the parameter of loss aversion.*

Assumption A4 (Discount Rates). *Let* $\rho_0 > 0$ *be the base discount, and* $\mu \geq 0$ *the parameter of magnitude effect. If* $\mu = 0$, *then we set* $\rho_a = \rho_r = \rho_0$; *if* $\mu > 0$, *then*

$$\rho_a = \frac{\rho_0}{\max_{t\in[t_0,t)} |v(c_t - r_t)|^{\mu}}, \quad \text{and} \tag{1}$$

$$\rho_r = \frac{\rho_0}{\max_{t\in[t_b,t)} |v(c_t - r_t)|^{\mu}}. \tag{2}$$

Assumption A5 (Discount Factor). *We assume that* $\pi(\tau)$ *is a concave function; i.e.,* $f(\tau)$ *is substationary. We consider two specific forms, both involving the sensitivity to time distance parameter,* $\delta \in (0,1]$.

(1) *Power sensitivity*: $\pi(\tau) = \tau^{\delta}$, $\tau \geq 0$.

(2) *Quasi-linear sensitivity*: $\pi(0) = 0$ and $\pi(\tau) = (1-\delta) + \delta\tau$, $\tau > 0$.

The associated discount factors are $f(\tau) = e^{-\tau^{\delta}}$, $\tau \geq 0$ and $f(\tau) = e^{\delta-1}e^{-\delta\tau}$, $\tau > 0$, respectively. Both specifications have exponential discounting as a special case when $\delta = 1$.

Assumptions A1–A5 result in a parametric model with three external decision variables, (c, Δ_a, Δ_e), and five internal parameters (see Table 1). Each parameter captures a distinct psychological element. The different elements can be activated at will. For example, setting $\alpha = 0$ turns off adaptation, setting $\lambda = 1$ eliminates loss aversion, setting $\delta = 1$ produces exponential discounting, and setting $\mu = 0$ eliminates the magnitude effect in discounting.

2.3. Discussion of the Assumptions

In the following section, we discuss each of the assumptions formulated in the previous paragraphs.

Assumption A1: The rate of consumption during the event is assumed to be constant, as in Loewenstein (1987). This automatically implies that $C = \{c\}$, and therefore the level of conceptual consumption during anticipation and recall is $c_t = c$. In this simple setup, the anticipation of the event matches the reality of the event, and so does the recall of it. Hence, the complex problem of choosing the levels of conceptual consumption is made trivial, which allows us to focus our analysis on other aspects of the model.

Assumption A2: The gradual adaptation of the reference point to the consumption level is standard in modeling habit formation (Wathieu 1997, Rozen 2010). Our model is the first, to our knowledge, to consider a gradual process of adaptation before the event. During recall, we keep r_t constant for reasons of tractability. This modeling choice, however, is consistent with the "end" part of the peak-end rule (Kahneman et al. 1993). Assumptions A1 and A2 yield the convenient expression:

$$c_t - r_t = c\, e^{-\alpha(t-t_0)} \cdot 1_{\{t\in[t_0,t_e)\}} + c\, e^{-\alpha(t_e-t_0)} \cdot 1_{\{t\in[t_e,t_1)\}}. \tag{3}$$

Thus, as soon as an upcoming positive event starts to be anticipated, the effective consumption decays exponentially with the passage of time until t_e, and it remains constant thereafter. Adaptation lowers subsequent event and recall utility.

Adaptation during anticipation is very plausible. Suppose a positive event is unexpectedly cancelled at some time after t_0 but before t_b. Because the reference point has increased in the meantime, the AER model predicts that the decision maker will experience disappointment; the intensity of these negative feelings will be stronger the closer to the event the cancellation occurs. Conversely, a cancelled negative event will produce relief, and the intensity of the relief will increase with the length of time one has been dreading the negative event. These predictions are consistent with prior findings on the dynamics of anticipation. Hoch and Loewenstein (1991) argue that learning that a future positive (negative) event is suddenly cancelled induces disappointment (relief). Chen and Rao (2005) confirm that people experience disappointment following the cancellation of a positive event (*dashed hope*) and relief upon cancellation of a negative event (*false alarm*). For auctions, Heyman et al. (2004) provide evidence of *quasi-endowment*: bidders develop partial ownership for objects during an auction, even though they are not the owners yet. Once the bidders gain "mental possession" of the item, not owning the item is perceived as a loss. Indeed, marketers stimulate anticipation and quasi-endowment by advertising vivid product images and simulating ownership through product sampling.

Assumption A3: All of our results generalize to the case of *v power*; that is, $v(c) = c^{\beta^+} \cdot 1_{\{c\geq 0\}} - \lambda|c|^{\beta^-} \cdot 1_{\{c<0\}}$ (Tversky and Kahneman 1992). Because no additional insights are obtained, we set $\beta^+ = \beta^- = 1$.

Assumption A4: Empirical measurements of discount rates consistently show that larger amounts are discounted less than smaller amounts (Thaler 1981, Frederick et al. 2002). The AER model captures magnitude effects by letting the discount rates be a decreasing function of $|c|$. Specifically, the denominator of the discount rates, ρ_a and ρ_r, depends on the "peak" value of $|v(c_t - r_t)|$ before and after the event begins, respectively. For recall, this is consistent with the peak part of the peak-end rule (Kahneman et al. 1993, Fredrickson 2000), according to which recall of experiences is greatly influenced by the peak moments (either good or bad) that stand out regardless of how long the experience lasted. Although ρ_a and ρ_r could depend on t, A1 and A2 ensure that the denominator takes its maximum at $t = t_0$ and $t = t_b$, respectively, yielding

$$\rho_a = \frac{\rho_0}{|v(c)|^{\mu}} \quad \text{and} \quad \rho_r = \frac{\rho_0}{|v(c)|^{\mu} e^{-\alpha\mu\Delta_a}}. \tag{4}$$

Henceforth, we often use the replacement $\rho_r = \rho_a e^{\alpha\mu\Delta_a}$. In other words, anticipation produces adaptation, makes the event less surprising, increases the rate of memory decay, and makes the event less memorable. Caruso et al. (2008) observe that people experience a "wrinkle in time," whereby future events are valued more than equivalent events in the equidistant past. According to AER, the wrinkle in time critically depends on the amount of anticipation. Note also that, because of loss aversion, the discount rate for a negative event will be smaller than the discount rate for an equivalent positive event by a factor of λ^{μ}. This is consistent with the prevalent finding that gains are discounted at a higher rate than losses (Frederick et al. (2002).

Assumption A5: Intuitively, a concave π captures diminishing sensitivity to time distance. Because $\pi(\tau)$ is concave, the discount factor decays rapidly near $\tau = 0$, and the decay rate slows down when τ is large (recall that τ is the distance to the event). This is consistent with patterns of decreasing impatience observed before the event, as well as with patterns of transience in recall (Loewenstein and Prelec 1993, Frederick et al. 2002, Ebbinghaus 1913). The power form was proposed by Ebert and Prelec (2007), and the quasi-linear sensitivity is a version of the quasi-hyperbolic discounting function (Laibson 1997) that includes magnitude effects.

3. The Shape of Temporal Profiles of Instant Utility

Henceforth, assume that A1–A5 hold. Coupling (3) with A3 yields $v(c_t - r_t) = v(c)\,e^{-\alpha(t-t_0)} \cdot 1_{\{t\in[t_0,t_e)\}} + v(c)\,e^{-\alpha(t_e-t_0)} \cdot 1_{\{t\in[t_e,t_1)\}}$, and the profile of instant utility in the time interval $[t_0, t_1)$ is given by

$$u(t) = v(c)\,e^{-\alpha(t-t_0)} \cdot f(\rho_a(t_b - t))\cdot 1_{\{t\in[t_0,t_b)\}} + v(c)\,e^{-\alpha(t-t_0)} \cdot 1_{\{t\in[t_b,t_e)\}} + v(c)\,e^{-\alpha(t_e-t_0)} \cdot f(\rho_r(t-t_e))\cdot 1_{\{t\in[t_e,t_1)\}},$$

where ρ_a and ρ_r are as given in (4).

Clearly, for all t, the sign of instant utility has the same valence as the sign of c; i.e., positive events induce a positive profile, and negative events induce a negative profile. In terms of comparative statics, we note that the entire profile of instant (dis)utility increases as we increase the magnitude of the event, $|c|$; decrease the speed of adaptation, α; or decrease the base discount rate, ρ_0. All these observations are intuitive, especially if we keep in mind that ρ_a and ρ_r, as given in (4), are decreasing functions of $|c|$, and that f is a decreasing function.

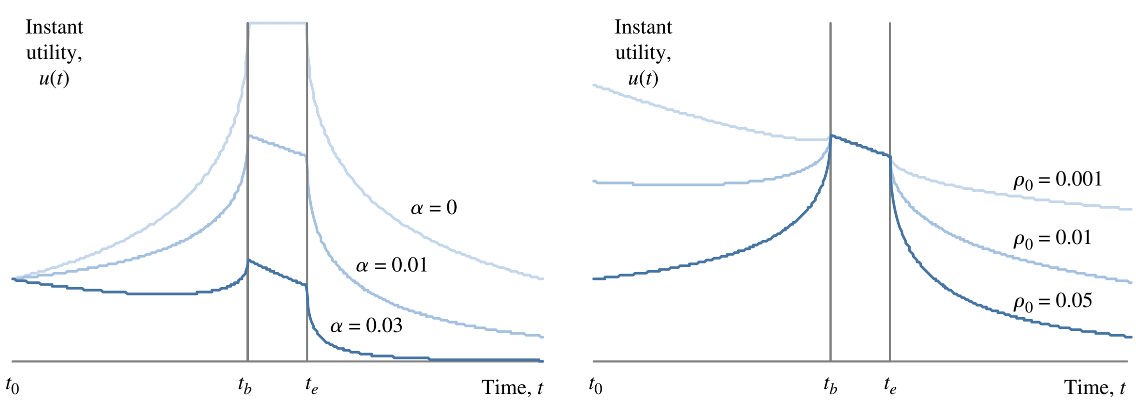
As a function of the passage of time, instant (dis)-utility, $|u(t)|$, is decreasing in t during the event because of adaptation. It is also decreasing in t during recall because of increased temporal distance (see Figure 2). During anticipation, however, two opposite forces determine the shape of the temporal profile of instant utility. On the one hand, temporal distance shrinks with the passage of time. On the other hand, adaptation decreases effective consumption with the passage of time. The net result is that the profile of instant (dis)utility during anticipation is unimodal.

Proposition 1. *Assume that A1–A5 hold. Instant (dis)-utility is unimodal during anticipation; i.e., there exists a $t_m \in [t_0, t_b]$ such that $|u(t)|$ is decreasing on $[t_0, t_m]$ and increasing on $[t_m, t_b]$.*

All proofs are contained in Appendix B.

If the function π is strictly concave, then instant (dis)utility during anticipation is strictly unimodal.[4] For example, under A5(1) we find that instant

Figure 2. (Color online) The Temporal Profiles of Instant Utility, $u(t)$, for Different Levels of Adaptation α (Left) and Base Discount ρ_0 (Right)



Note. Base case parameters: $c = 1$, $\Delta_a = 40$, $\Delta_e = 10$, $\alpha = 0.01$, $\pi(\tau) = \tau^{0.5}$, $\rho_0 = 0.05$, and $\mu = 2$.

(dis)utility during anticipation takes its minimum at time

$$t_m = \max\left\{t_0, t_b - \frac{1}{\rho_a}\left(\frac{\delta\rho_a}{\alpha}\right)^{1/(1-\delta)}\right\}.$$

Jevons' (1905) intuitions about anticipation[5] as well as previous theoretical models of anticipation (Loewenstein 1987) predict only the increasing portion of the U shape. The AER model allows for individuals to get very excited when they first learn about an upcoming event, such as a concert or a holiday. The excitement then decays, but it rekindles when the event draws near.

Consistent with Proposition 1, Breznitz (1984) suggests that once an individual is fully aware of an upcoming threat, the time path of anxiety tends to be U-shaped. There is intense fear when an individual is first informed of an upcoming threat. Such fear then diminishes before rising sharply in anticipation of the impact closer to the event. To provide further evidence that the temporal profile of instant utility during anticipation is unimodal and generally U-shaped, we conducted two experiments.

3.1. Study 1A: The Predicted Profile of Instant Utility Before a Hypothetical Event

We recruited a total of 282 paid respondents online (43% female, mean age 34) through Amazon Mechanical Turk. All participants were asked to "imagine a friend just told you that he/she bought you a ticket for the concert of your favorite band as a present. The concert is happening in 3 months. Think about how you would feel about the concert throughout the time before the event. Take a look at the five patterns in the following page describing your excitement regarding the concert prior to the event date."

Five graphs of excitement patterns (see companion experimental materials available online) were displayed in randomized order: (a) *stable* throughout time; (b) *increasing* throughout time as the event draws nearer; (c) *decreasing* throughout time as the event draws nearer; (d) [*U shape*] high when you first learn about the event, it then slightly decreases, and it increases again as the event is about to happen; and (e) [*inverted U shape*] low when you first learn about the event, it then slightly increases, and it decreases again as the event is about to happen.

Subsequently, participants were asked, "In your opinion, which of the five graphs better describes the pattern of your excitement regarding the concert prior to the event date (pick one)?"

Most participants selected the U-shaped pattern [61%, $\chi^2 = 355.2$, $p < 0.001$]; some an increasing pattern [27%]; and very few a stable pattern [6%], an inverted U shape [4%], or a decreasing pattern [2%]. Thus, the notion that anticipation might be U-shaped or unimodal, as predicted by Proposition 1, resonates with most people.

3.2. Study 1B: The Reported Instant Utility Before a Real Event

We recruited 373 participants (38% female, mean age = 32) who had signed up for an upcoming Tough Mudder event through the Tough Mudder mailing list. Tough Mudder is a series of obstacle course competitions billed as "probably the toughest one day event on the planet." We asked participants, "On a scale from 1 to 7, how excited are you today about your upcoming event?" We also asked participants when they started thinking about the event (our t_0), the date of their next Tough Mudder challenge from a list of 42 upcoming races (our t_b), and other demographic questions (see Appendix A for details). All responses were collected on July 16, 2014 (our t).

For each individual, we calculate $\theta = (t - t_0)/(t_b - t_0) \in [0, 1]$, which is a comparable measure across participants of their relative location on the anticipation time interval. For example, an individual who signed up for a race taking place 30 days after the survey's date and who started thinking about the race 60 days ago will have $\theta = 60/(60 + 30) = 0.66$.

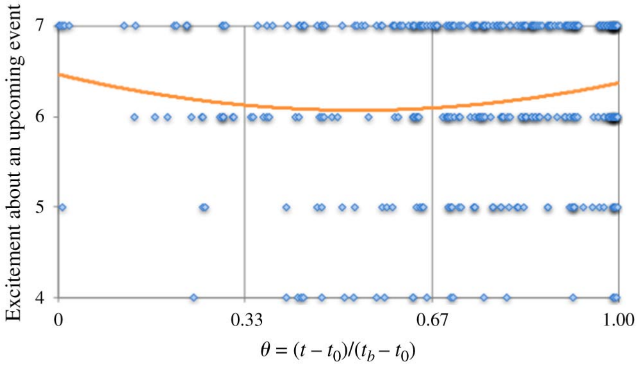
The mean level of excitement was 6.2. We predict the level of excitement using ordered probit. Our main independent variables are θ and θ^2. The latter will allow us to detect the curvature of the anticipation pattern. As control variables, we include *Time since sign-up* (the time distance between the day of the survey (July 16, 2014) and the day participants signed up for the race), the number of races run in the past (zero for about half the sample) (*Number of races before*), and *Gender*. If θ is a relative measure, the time since sign-up is an absolute measure of time distance. Table 2 shows the regression results of the ordered probit (the results using ordinary least squares (OLS) are very similar and yield the same conclusions).

The critical test for the U-shaped curve is whether the regression line exhibits positive curvature. Indeed, the coefficient $b_{\theta^2} = 2.03$ is positive ($p = 0.026$). To see the relevance of this coefficient, consider the total effect of θ on excitement, $b_\theta\theta + b_{\theta^2}\theta^2$. A U-shaped pattern

Table 2. Regression Using Ordered Probit of the Current Level of Excitement for Participants of an Upcoming Tough Mudder Event ($N = 373$)

Independent variable	Coefficient	Std. dev.	p-value
θ	−1.79	1.15	0.119
θ^2	**2.03**	0.91	**0.026**
Time since sign-up (days)	**−0.0021**	0.00085	**0.010**
Number of races before	**0.16**	0.062	**0.008**
Gender (1 = male)	0.054	0.12	0.662

Figure 3. (Color online) Excitement About an Upcoming Event as a Function of θ ($N = 373$)



Notes. The solid line is a quadratic fit. Four answers below 4 are not shown.

would exhibit negative slope at $\theta = 0$ and positive slope at $\theta = 1$. The slope is equal to $b_\theta + 2b_{\theta^2}\theta$, which is indeed negative at $\theta = 0$, $b_\theta = -1.79$ [$p = 0.119$] and positive at $\theta = 1$, $b_\theta + 2b_{\theta^2} = 2.3$ (in OLS, the F-test $b_\theta + 2b_{\theta^2} = 0$ yields $F(1, 367) = 8.8$ and $p = 0.003$). More importantly, the difference between the slope at $\theta = 1$ and at $\theta = 0$, given by $2b_{\theta^2} = 4.06$, is positive and statistically different from zero ($p = 0.026$). The variable *Time since sign-up* has a negative and significant effect and is consistent with the proposed U-shaped pattern ($b = -0.0021, p = 0.010$). Participating in previous Tough Mudder challenges produces a mild but significant increase in excitement ($b = 0.16, p = 0.008$).[6] Finally, we observe no gender effects.

Figure 3 shows the reported excitement level as a function of θ and a quadratic fit. Note that for θ away from the extremes (e.g., the interval $[1/3, 2/3]$), some participants are lukewarm and report a level of excitement less than 6. By contrast, for $\theta \in [0, 1/3)$ (i.e., participants started thinking about the event recently) or $\theta \in (2/3, 1]$ (i.e., the event is near), the large majority of respondents report a level of excitement of 6 or 7. Specifically, the fraction of participants reporting a level of excitement of 6 or more in the first, second, and third segments is 88%, 70%, and 81%, respectively; a level of excitement of 4 or less is 3%, 16%, and 4%, respectively.[7]

4. Total Utility

In this section, our focus will be on the effect of c on total utility, U. In Sections 5 and 6, we will then consider the effect of Δ_e and Δ_a on U, respectively. To obtain total utility, we integrate instant utility over time. Recall that $v(c) = c \cdot 1_{\{c \geq 0\}} + \lambda c \cdot 1_{\{c<0\}}$.

Proposition 2. *Let $\Sigma = \int_0^{\rho_r(t_1 - t_e)} f(\tau)\,d\tau$ be the coefficient of recall.*[8] *Under A1–A5, if $c \geq 0$, then total utility is given by*

$$U = v(c)e^{-\alpha\Delta_a}\left[\frac{1}{\rho_a}\int_0^{\rho_a\Delta_a} e^{(\alpha/\rho_a)\tau} f(\tau)\,d\tau + \frac{1}{\alpha}(1 - e^{-\alpha\Delta_e}) + \frac{\Sigma}{\rho_a}e^{-\alpha\mu\Delta_a}e^{-\alpha\Delta_e}\right]. \quad (5)$$

If the discount rates ρ_a and ρ_r are independent of c (i.e., $\mu = 0$), then the term in brackets does not depend on c. This implies that total utility of consumption is proportional to $v(c)$. In other words, the rest of complexities—adaptation, discounting, duration of the anticipation and of the event—modify the value function by means of a constant of proportionality.

4.1. Anticipation, Event, and Recall

Assume $\mu > 0$ and $\Delta_a > 0$. If the discount rates depend on c, then the details of the event must be considered when calculating the functional relationship between U and c. Note that increasing $|c|$ has a double effect: it increases $|v(c)|$ and lowers the discount rates for anticipation and recall. Whereas $|U|^E$ remains proportional to $|v(c)|$ (elasticity equal to 1), $|U|^A$ and $|U|^R$ might be convex in $|c|$ (elasticity greater than 1).

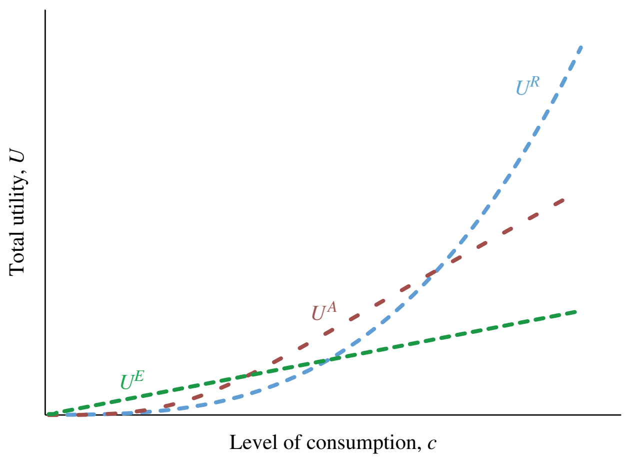
Proposition 3. *Assume that A1–A5 hold. The elasticity of $|U|^A$, $|U|^E$, and $|U|^R$ with respect to $|c|$ is given by $1 + \mu\psi$, 1, and $1 + \mu$, respectively, where $g_\tau = e^{(\alpha/\rho_a)\tau} f(\tau)$ and*

$$\psi = \frac{\int_0^{\rho_a\Delta_a} \tau\pi'(\tau) g_\tau\,d\tau}{\int_0^{\rho_a\Delta_a} g_\tau\,d\tau}.$$

The function ψ is bell-shaped. Hence, as a function of $|c|$, we observe that $|U|^A$ is close to linear for small values of $|c|$, convex for intermediate values of $|c|$, and again approaches linearity for large values of $|c|$.[9] By contrast, $|U|^E$ is proportional to $|c|$. Finally, because $1 + \mu > 1$, $|U|^R$ is a convex function of $|c|$ (large events might be more than twice as memorable as events half the size).

For small $|c|$, the discount rates for anticipation and recall are very high, leading to $|U| \approx |U|^E$. As $|c|$ increases, the discount rate for anticipation, ρ_a, decreases, and utility of anticipation takes a more prominent role and $|U| \approx |U|^E + |U|^A$. As $|c|$ further increases, ρ_r decreases and recall becomes more prominent (if T is large, the term $|U|^R$ becomes the largest). In Figure 4, we illustrate the effect of the level of consumption on the three components of total utility. For small experiences (e.g., eating ice cream), most utility will be event utility. For intermediate experiences (e.g., a weekend outing), anticipation will play a key role. For large experiences (e.g., a honeymoon), the model predicts that most of the utility will be derived from recall.

Figure 4. (Color online) Utility of Anticipation, of the Event, and of Recall as a Function of c



Note. Base case parameters: $\Delta_a = 10$, $\Delta_e = 5$, $\alpha = 0.03$, $\pi(\tau) = \tau$, $\rho_0 = 0.1$, and $\mu = 2$.

4.2. The Preference to Segregate Losses

The shape of U as a function of $|c|$ also has implications for the strategic aggregation or desegregation of gains and losses. Hedonic editing, as conceptualized by Thaler (1985), predicts a preference for segregating gains and aggregating losses. This strategy is based on the curvature properties of the S-shaped value function. If the magnitude effect in discounting is turned off, $\mu = 0$, then the AER model also recommends aggregating losses and segregating gains.

If $\mu > 0$, however, then the AER model is more nuanced and runs against segregating gains and aggregating losses. Indeed, aggregating gains under AER might be optimal because it produces a lasting memorable experience. Conversely, aggregating losses may not be optimal because it induces a large negative experience that will be ingrained in memory for a long time. Because the temporal discount factor for losses is higher (by a factor of λ^μ), negative memories will be more persistent than positive ones. Hence, the AER model supports the persistently observed preference for individuals to segregate losses (Linville and Fischer 1991, Thaler and Johnson 1990).

5. Unique vs. Repeated Experiences

In this section, we focus on the effect of Δ_e on total utility. As can be seen in expression (5), Δ_e has no effect on anticipation. Instant utility during the event is decreasing over time, but we extend the interval of integration, so that event (dis)utility increases with Δ_e. In their experiments on recall utility, Kahneman et al. (1993) demonstrate that the intensity of recall is insensitive to the duration of the event, an effect for which they coin the term "duration neglect." The AER model captures the notion of duration neglect in a very strong sense. The model predicts that extending the duration of the event induces more adaptation and makes the event less memorable.

Proposition 4. *Increasing the duration of the event, Δ_e, has no effect on the (dis)utility of anticipation, $|U|^A$; increases the (dis)utility of the event, $|U|^E$; and strictly decreases the (dis)utility of recall, $|U|^R$. Total (dis)utility, $|U|$, increases with Δ_e if and only if $\alpha\Sigma/\rho_r \le 1$.*

According to Proposition 4, the optimal value of Δ_e, assuming we preserve the integrity of the experience, is either 0 ($\rho_r < \alpha\Sigma$) or infinity ($\rho_r \ge \alpha\Sigma$). In practice, Δ_e can be shortened by never repeating the same exact event. Conversely, Δ_e can be increased by repeating the exact same experience multiple times, as, for example, by dining out regularly in the same restaurant. The AER model predicts a dichotomy between events that are best experienced just once and those that are best experienced multiple times. Factors that contribute to one-time experiences are those that increase the utility of recall: high speed of adaptation (α), high coefficient of recall (Σ, which increases as the sensitivity to time distance, δ, decreases), and low discounting for recall (ρ_r). Because the discounting for recall decreases with the magnitude of the event, the events to be experienced only one time will tend to be the large-magnitude ones.

Do people agree with the notion that extending an experience through repetition might lower the utility of recall? Inspired by Loewenstein's (1987) experimental stimuli, in Study 2 (see Appendix A for details), we asked participants whether they would consider it more memorable to kiss their favorite movie star one time only or once a day for one week (i.e., seven times). The majority of respondents selected "one time" over "seven times" (68%, $\chi^2 = 18.2$, $p < 0.001$), giving it a higher score on a seven-point scale (6.4 versus 5.5, $p < 0.001$). Thus, this prediction of the model resonates with people's predictions about recall utility (which does not necessarily mean these predictions are accurate).

The optimality of avoiding repetition is consistent with Zauberman et al. (2008), who find that when people truly enjoy an experience, they forgo ever repeating it.[10] The authors suggest that such aversion is driven by a desire to protect the memory of the event from future experiences that might not be as pleasurable. The AER model rationalizes this highly psychological process.

Our current conclusion that Δ_e has to be either as short as possible or as long as possible critically hinges on A1. However, the constant consumption rate is not a very realistic assumption. In a more realistic model, C would be a function of Δ_e (a longer experience is more likely to have a wider range of consumption rates and thus a higher peak). In this case, new interactions between Δ_e and the utility of anticipation and recall would emerge, leading to a more complex optimal choice of Δ_e.

6. Duration of Anticipation

In this section, we focus on the effect of Δ_a on total utility. Decision-making research has documented that total utility may increase given more time to savor anticipation (Loewenstein 1987, Nowlis et al. 2004). There might be, however, an optimal duration of anticipation. In an experiment entailing real consumption of chocolate, Chan and Mukhopadhyay (2010) found that participants who had to wait one week before consumption evaluated the chocolate more highly than those who were given the chocolate immediately as well as those who were given it after delays of two and four weeks.

In some cases, decision makers have some discretion over the duration of anticipation. If t_0 is known and fixed, they may vary t_b by choosing the date of the event. If t_b is fixed, they may vary t_0 by choosing the date at which to start planning (e.g., for a holiday trip or deciding how long in advance to release news some about an upcoming event). In what follows, we set the duration of anticipation as a decision variable and seek to find its ideal length.

The effect of Δ_a on instant utility and total utility is threefold. First, a positive *duration effect*: Δ_a increases the interval over which anticipation is experienced. Second, a negative *adaptation effect*: Δ_a reduces utility by a factor $e^{-\alpha\Delta_a}$. Third, a negative *magnitude effect*: Δ_a increases the discount rate for recall, ρ_r, and reduces the utility of recall. Under A1–A5,

$$\frac{\partial U}{\partial \Delta_a} = \underbrace{v(c)f(\rho_a\Delta_a)}_{\text{Duration}} - \underbrace{\alpha U}_{\text{Adaptation}} - \underbrace{\alpha\mu U^R}_{\text{Magnitude}}$$
$$= \underbrace{v(c)f(\rho_a\Delta_a) - \alpha U^A}_{\partial U^A/\partial\Delta_a} - \underbrace{\alpha U^E}_{\partial U^E/\partial\Delta_a} \underbrace{-\alpha(1+\mu)U^R}_{\partial U^R/\partial\Delta_a}.$$

Clearly, both U^E and U^R are decreasing with Δ_a. The effect of Δ_a on U^A is mixed: as the right panel of Figure 5 shows, when Δ_a increases, instant utility lasts longer, but adaptation reduces the average instant utility.

The left panel of Figure 5 illustrates the effect of Δ_a on total utility. In the figure, both U and U^A are unimodal (only U^A is guaranteed to be so in general). We now show that the duration of anticipation that maximizes U is shorter than the duration of anticipation that maximizes U^A.

Proposition 5. *Assume that A1–A5 hold, $\alpha > 0$, and $c > 0$. Let*

$$G(\Delta) = f(0^+) - \int_0^{\rho_a\Delta} e^{(\alpha/\rho_a)\tau} f(\tau)\pi'(\tau)\,d\tau.$$

Utility of anticipation, U^A, is a unimodal function of Δ_a, reaching the peak at $\Delta_A \in (0,\infty)$, the unique solution to $G(\Delta) = 0$. Total utility, U, is maximized at $\Delta_* \in [0, \Delta_A)$. Specifically, if $(\partial U/\partial\Delta_a)|_{\Delta_a=0} \le 0$, then $\Delta_* = 0$; otherwise, $\Delta_* > 0$ solves

$$G(\Delta) = 1 - e^{-\alpha\Delta_e} + e^{-\alpha\Delta_e}\frac{\alpha(1+\mu)\Sigma}{\rho_a}e^{-\alpha\mu\Delta}.$$

Moreover, there is a $\hat{\mu} > 0$ such that if $0 \le \mu < \hat{\mu}$, then Δ_* is unique.

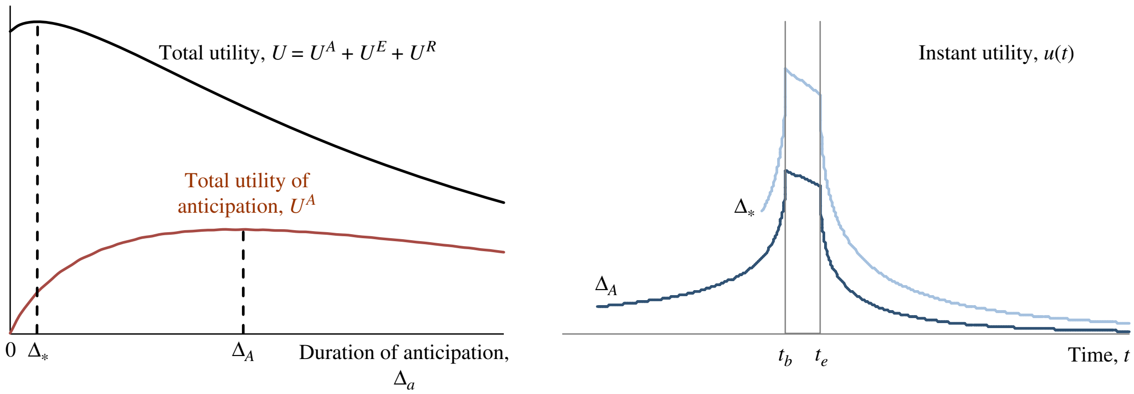
In the proof, we also establish the following comparative statics results. If $\mu = 0$, then both Δ_A and Δ_* are independent of c; but if $\mu > 0$ and $-\tau\pi''_\tau/\pi'(\tau) < 1$ (a mild condition satisfied by A5(1) and A5(2)), then Δ_A is increasing in c, whereas the effect of c on Δ_* is ambiguous.

We conclude this section by providing an analytic solution for the quasi-linear sensitivity case.

Proposition 6. *Assume that A1–A4 and A5(2) hold, $\alpha > 0$, and $c > 0$. If $\delta\rho_a = \alpha$, then $\Delta_A = 1/\alpha$; otherwise,*

$$\Delta_A = \frac{\ln\delta\rho_a - \ln\alpha}{\delta\rho_a - \alpha}.$$

Figure 5. (Color online) Left: Total Utility and Total Utility of Anticipation as a Function of Δ_a; Right: The Temporal Profile of Instant Utility for Two Durations of Anticipation, $\Delta_A = 55$ and $\Delta_* = 7$



Notes. Total utility (left) is the integral of profiles (right) for different values of Δ_a. Base case parameters: $c = 1$, $\Delta_e = 10$, $\alpha = 0.01$, $\pi(\tau) = \tau^{0.5}$, $\rho_0 = 0.1$, and $\mu = 1$.

If $e^{1-\delta}(1-e^{-\alpha\Delta_e})+(\alpha(1+\mu)/(\delta\rho_a))e^{-\alpha\Delta_e}\geq 1$, *then* $\Delta^*=0$; *otherwise,* $\delta\rho_a>\alpha$ *and* $\Delta^*>0$ *is the unique solution of the recursion*

$$\Delta_* = \left(-\ln\left\{1-\frac{\delta\rho_a-\alpha}{\delta\rho_a}\left[1-e^{1-\delta}(1-e^{-\alpha\Delta_e}) - \frac{\alpha(1+\mu)}{\delta\rho_a}e^{-\alpha\Delta_e}e^{-\alpha\mu\Delta_*}\right]\right\}\right)\cdot(\delta\rho_a-\alpha)^{-1}.$$

6.1. Experimental Evidence

For a given event, do people have an intuition about the ideal duration of anticipation? If so, does the ideal duration change with the magnitude of the event? To empirically address these two questions, we provided participants with a randomized list of 11 different positive events (see Study 3 in Appendix A for details). We told participants to assume that all outcomes were certain to occur at the designated time. We also instructed them to ignore organizational issues (e.g., no booking or reservation issues). We then asked respondents to indicate how long in advance they would like to be told about each event.

The results, shown in Appendix A, Table A.1, indicate that participants do have an intuition about the ideal date to begin anticipating an upcoming event and that such a duration is monotonic with the magnitude of the event. For example, most participants said they wanted to start anticipating the "wedding of their best friend" six months ahead of time, the "concert of their favorite band" one month before, or a "dinner in a fancy restaurant" one week prior. To verify that this ideal anticipation time increases with the magnitude of the event, we compare pairs of similar events but with different magnitudes. For example, participants wished to anticipate the wedding of their best friend longer than the wedding of a distant relative (180 versus 54 days, $p<0.001$) and to anticipate dining at a fancy restaurant longer than eating ice cream (7 days versus 1 day, $p<0.001$).

The findings of Study 3 are consistent with the AER model and provide an indirect indication of magnitude effects in discounting.

6.2. Positive Surprises

Proposition 5 shows that, for positive events, it may be optimal to set the duration of anticipation to zero. The necessary and sufficient condition to produce $\Delta^*=0$ is $(\partial U/\partial\Delta_a)|_{\Delta_a=0}\leq 0$.[11] For simplicity, assume the psychological distance function is continuous (i.e., $f(0^+)=1$). Then the combination of parameters that establishes the optimality of positive surprises is[12]

$$\alpha(1+\mu)\Sigma/\rho_a \geq 1. \qquad (6)$$

Recall that for positive events, the discount rate during anticipation is $\rho_a=\rho_0/c^{\mu}$. Factors contributing to a positive surprise are a high speed of adaptation (α), high coefficient of recall (Σ, which increases as the sensitivity to time distance, δ, decreases), low base discount rate (ρ_0), and high magnitude of the event (c).

Novels, soap operas, sports events, and casinos all create value by revealing information over time in a manner that makes the experience more exciting (Ely et al. 2015). Indeed, surprise endings are a common element in many folktales, story jokes, and advertising campaigns. Loewenstein et al. (2001) show that readers find the "repetition–break" plot structure very engaging (this plot structure uses repetition among obviously similar items to create a pattern and then uses a break in this pattern to generate shock or surprise at the end). In our framework, repetition creates adaptation/expectation, and the contrasting element provokes the surprise.

Moreover, individuals can also administer surprises to themselves and benefit from unanticipated positive events. For example, one possibility is by means of instant and unplanned purchases. Although impulse buying behaviors are often considered a sign of low self-control (Baumeister 2002), their high prevalence suggests that they may be optimal in some occasions. Yet another possibility is to rely on others. Surprise gifts are common in many cultures. The AER model shows that surprise gifts can be optimal. As a matter of fact, the "rational" approach of asking the recipient her desires in advance might trigger anticipation and reduce the effect of surprise. In relationships, for example, one may strategically decide when to deliver good news or offer a gift to produce greater surprise. For instance, receiving an engagement ring is often a surprise experience, and the instant utility increases when the recipient is not (yet) expecting it.

Consistent with Proposition 5, shortening the anticipation time may be welfare increasing in some circumstances. Many successful business models are based on shortening the time between planning and execution of consumption experiences. For example, Toyota had a competitive advantage in the 1990s because of its reputation for fast delivery, and similarly today, NikeID is one of the leaders in mass customization processes thanks to the shortened delivery time of individually customized items. Today, the "Amazon Prime Now" app offers free two-hour deliveries in addition to one-hour deliveries for a few dollars.

6.3. Coping with Negative Events

An upcoming negative event induces anxiety. Anticipating the negative experience, however, can help one endure the event and reduce total pain. The literature on coping identifies several ways to respond to upcoming stress (Carver et al. 1989) and examines coping strategies for health-related events (Carver et al. 1993). Our current setup allows us to examine the effect of adaptation on modifying the reference point and reducing total disutility.

Suppose we learn that we need to undergo a surgery. We have certain flexibility regarding the calendar date of the surgery, e.g., any time within the next three months. In the context of the AER model, when should we schedule the surgery? Alternatively, suppose we need to tell a loved one that he or she has to undergo some critical surgery. The earliest available slot for the critical surgery is one month from now. When should we deliver the news—now, in one week, in two weeks, or a few days before the surgery? In both examples, the goal is to decide the optimal amount of anticipation before a negative event.

When the negative event can be postponed far into the future at no cost, the AER model recommends doing so. Intuitively, the anticipatory disutility reaches its highest point at the beginning, near t_0, before adaptation soothes the pain. But if the event occurs in the distant future, then time distance will greatly reduce the initial anticipatory disutility.

Proposition 7. *Assume that A1–A5 hold and that $\tau f(\tau)$ goes to zero as τ goes to infinity. If $\alpha > 0$, then total disutility tends to zero (not necessarily in a monotonic way) as Δ_a goes to infinity. Hence, $\Delta_a = \infty$ minimizes disutility.*

Often, negative events are imposed on us, and we cannot avoid them or postpone them into the far future. Indeed, postponing the event may either be very costly (e.g., delaying a surgery may compromise the medical situation of the patient) or be infeasible (e.g., the surgery cannot be rescheduled). Henceforth, we assume that Δ_a needs be chosen in some bounded interval $[0, \Delta]$, where Δ is the longest possible duration of anticipation. Because disutility may not be monotonic in Δ_a, the optimal duration of anticipation may be shorter than Δ. Let Δ_*^- denote such optimal anticipatory time.

The formulation of Loewenstein (1987) produces two optimal strategies that can be labelled "get over it as soon as possible" ($\Delta_*^- = 0$) or "adapt for as long as possible" ($\Delta_*^- = \Delta$). The AER model admits a third possibility, "some right amount of time to adapt" and prepare for the negative event ($0 < \Delta_*^- < \Delta$). This mathematically interior solution is only possible if the parameter of magnitude effect, μ, is sufficiently large. If μ is large, some anticipation has the positive effect of lowering the discount rates for recall. Figure 6 plots disutility as a function of Δ_a in three representative parameter specifications.

The following results assume that A5(2) holds to ensure the uniqueness of the solution, although the threefold typology of solutions holds for any psychological distance function.

Proposition 8. *Assume that A1–A4 and A5(2) hold, $\alpha > 0$, and $c < 0$; also assume that the duration of adaptation can be set to any value between 0 (get over it as soon as possible) and some $\Delta > 0$ (delay for as much as possible). The optimal duration of anticipation is as follows:*

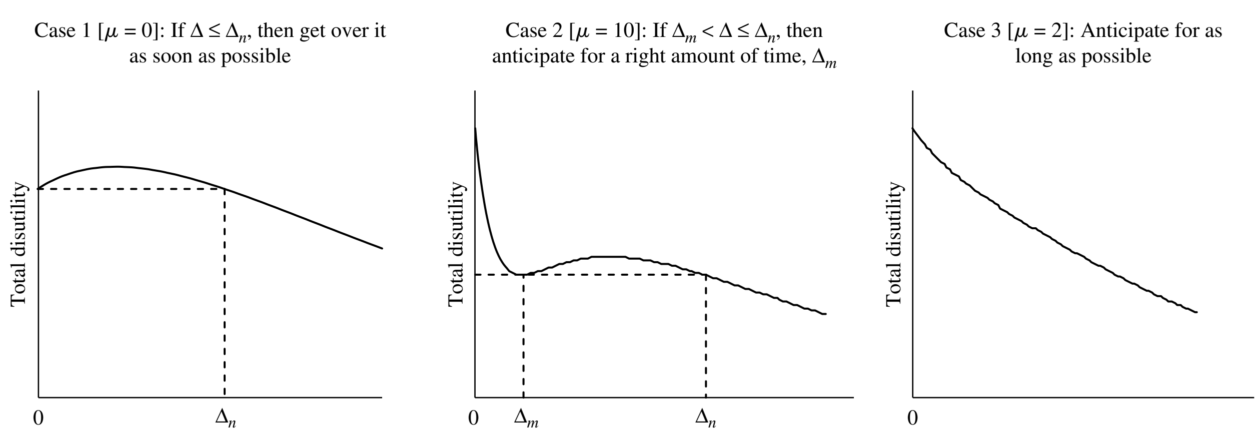
(1) *If $(\partial |U| / \partial \Delta_a)|_{\Delta_a=0} > 0$, then there is a unique $\Delta_n > 0$ solving $U_0 = U_{\Delta_n}$. If $\Delta \le \Delta_n$, then $\Delta_*^- = 0$; otherwise, $\Delta_*^- = \Delta$.*

(2) *If $(\partial |U| / \partial \Delta_a)|_{\Delta_a=0} < 0$ and μ is sufficiently large (at least $\sqrt{1/4 + e^{\alpha\Delta_e}(\delta\rho_a/\alpha)^2} - 1/2$), then $|U|$ has a unique local (but not global) minimum at $\Delta_m > 0$ and a unique $\Delta_n > \Delta_m$ solving $U_{\Delta_m} = U_{\Delta_n}$. If $\Delta \le \Delta_m$, then $\Delta_*^- = \Delta$; if $\Delta_m < \Delta \le \Delta_n$, then $\Delta_*^- = \Delta_m$; and if $\Delta > \Delta_n$, then $\Delta_*^- = \Delta$.*

(3) *If none of the above holds, then $\Delta_*^- = \Delta$.*

Note that $\Delta_*^- = 0$ only in case 1. Facilitators of this "get over it as soon as possible" strategy are similar to those producing an optimal positive duration of anticipation: low adaptation (α), small magnitude of the event (c), low coefficient of recall (Σ), and high base discount (ρ_0). The AER model predicts that people will prefer to quickly experience negative events of small magnitude, such as Loewenstein's (1987) mild electroshock, but prefer more time to anticipate and adapt

Figure 6. Disutility for a Negative Event as a Function of the Duration of Anticipation and for Three Values of μ



Notes. In the left graph, the disutility at $\Delta_a = \Delta_n$ is the same as the disutility at $\Delta_a = 0$. In the center graph, the disutility at $\Delta_a = \Delta_n$ is the same as the disutility at $\Delta_a = \Delta_m$, and Δ_m is the unique local minimum. Base case parameters: $c = -0.5$, $\Delta_e = 5$, $\lambda = 2$, $\alpha = 0.01$, $\pi(\tau) = \tau$, and $\rho_0 = 0.02$.

to larger negative events such as surgery, provided the postponement has a bearable cost.

6.4. Deceptive Postponement

Models of utility of anticipation face the problem of reverse time consistency or *deceptive postponement* (Loewenstein 1987): upon reaching t_b, the individual may gain utility by postponing t_b to a later date.[13] From a rational viewpoint, the strategy is dubious, as it requires the self-deception of not knowing in advance that t_b will be moved. In many practical circumstances, the scheduling of events, such as concerts, are out of the decision maker's control. By contrast, other events, such as an outing, are at the decision maker's discretion and can be postponed. It is conceivable, therefore, that individuals may use commitment mechanisms to avoid deceptive postponement. Ways to ensure that the event happens at $t = t_b$ include buying tickets in advance, or rejecting cancelation options.

Still, in the absence of frictions, the AER model also exhibits a tendency to postpone events at t_b. When the date of the event is postponed, the discount factor immediately adjusts downward because of the updated time distance to the event. But the instant utility obtained between t_0 and the original date, t_b, is not affected by this readjustment of the discount factor. Thus, the sudden postponement of t_b produces additional utility.

In the AER model, the marginal benefit of such postponement is given by

$$\left.\frac{\partial U}{\partial \Delta_a}\right|_{\Delta_a = t_b - t_0} = e^{-\alpha\Delta_a} \left.\frac{\partial U}{\partial \Delta_a}\right|_{\Delta_a = 0}. \tag{7}$$

Hence, the condition that ensures that anticipation is optimal, $(\partial U/\partial \Delta_a)|_{\Delta_a=0} > 0$, also implies that engaging in deceptive postponement is optimal. Note, however, that the net marginal benefit is proportional to $e^{-\alpha\Delta_a}$, which decreases with the total time of anticipation. If $\alpha\Delta_a$ is large and there is some cost to postponement, then postponement is not advantageous. The problem of deceptive postponement is more acute in Loewenstein's (1987) model, where the gain to deceptive postponement does not decay over time.

The AER model supports the notion that delaying a gratification may not be costly. This is consistent with Baumeister and Tierney (2011), who argue that one of the few psychological strategies to exercise self-control without depleting willpower is to *delay* (rather than *deny*) immediate gratification.

7. Conclusions

In this paper, we propose the anticipation-event-recall model that formally links the three components of total utility (i.e., utility from anticipation, event utility, and recall utility) in a comprehensive formulation. By virtue of its continuous time setting, the AER model produces the temporal profile of moment-by-moment utility throughout the entire event timeline. The AER model entails several unique modeling features capturing the psychological elements of conceptual consumption, adaptation during anticipation, and magnitude effects in discounting.

Although the goal of the AER model is primarily descriptive, its implications entail some prescriptive value for rational individuals seeking to maximize total utility, as well as for firms seeking to maximize customers' experience. As summarized in Table 3, the model provides several insights, some of them new (e.g., U shape of instant utility during anticipation, the trade-off between anticipation and total utility) and some of them consistent with common intuition or previous research findings (e.g., duration neglect). In three sets of experiments with real and hypothetical events, we find empirical support for the U shape of anticipatory utility, the preference for unique (versus repeated) experiences, and the optimal duration of anticipation.

The AER model predicts a trade-off between anticipation and memory: the longer the duration of anticipation, the more adaptation, the lower the surprise,

Table 3. Summary of Predictions from the AER Model

Prediction	Result	Supporting evidence
The profile of instant utility during anticipation is U-shaped.	Proposition 1	Studies 1A and 1B
Small events tend to produce more event utility, medium events more anticipatory utility, and large events more memory utility.	Proposition 3	
Segregating losses reduces bad memories and might be optimal.	Proposition 3	Linville and Fischer (1991)
Increasing the duration of the event makes it less memorable, and experiencing an event only once may be optimal.	Proposition 4	Study 2 Zauberman et al. (2008)
Instant utility decreases with the duration of anticipation, and there is an ideal duration of anticipation.	Proposition 5	Study 3 Chan and Mukhopadhyay (2010)
There is a trade-off between anticipation and total utility.	Proposition 6	
Surprises (i.e., no anticipation) might be optimal.	Equation (6)	Loewenstein et al. (2001)
There is a threefold strategy to cope with negative events.	Propositions 7 and 8	
Delaying (rather than denying) a gratification may not be costly.	Equation (7)	Baumeister and Tierney (2011)

Note. References to the literature are illustrative and by no means exhaustive.

and the lower the recall utility. Besides shortening anticipation, individuals could mitigate the effect of adaptation by ensuring that the experience differs from what is expected. For example, maintaining some ambiguity about an upcoming event (e.g., avoiding detailed information by not reviewing Web images or reading book guides) can lead to a positive surprise.

Adding elements of surprise can also increase the satisfaction derived from services and consumption events (Karmarkar and Karmarkar 2014). For example, reviewers often compliment haute cuisine by saying that customers can "expect the unexpected." The AER model suggests that the benefits of such strategies may reside in creating surprise even after prolonged anticipation. For negative events, the opposite seems advisable. The more detailed knowledge and vivid imagery individuals have about the upcoming reality, the more they might find the actual event to be not as bad.

Our assumptions strike a balance between realism, tractability, and fruitfulness of insights. We do not claim that our approach is unique, and future research may explore the large Pareto frontier generated by these three attributes.

We have set up our exploration in conditions of certainty and for flat events. If the event can take multiple potential levels and/or there is uncertainty about how good or bad the upcoming event will be, then conceptual consumption can take values in some nontrivial range C, creating many interesting possibilities. For one, the optimal level of conceptual consumption may vary during the anticipatory time interval. When conceptual consumption during the time of anticipation is set as a decision variable taking values on $C = [0, \bar{c}]$, preliminary numerical results suggest that it might be optimal to set conceptual consumption very highly at first (e.g., we may imagine that a vacation will be extraordinary three months prior to the departure date) and then lower the level of conceptual consumption as the event draws nearer, so that the event can still generate a final pleasant surprise.

By managing created expectations, individuals may find it optimal to set negative levels of anticipation to leave room for pleasant outcomes (Shepperd and McNulty 2002). The well-documented strategy of defensive pessimism involves setting unrealistically low expectations for success (Norem and Cantor 1986, Martin et al. 2001). Similarly, firms may find that moving customers from a negative to a positive state through a surprising service recovery (e.g., an airline announcing that a forecasted delay on departure time has been fixed) helps customers realize a higher satisfaction than if the negative incident had never occurred (Chen and Rao 2005, Karmarkar and Karmarkar 2014). The hubris and catharsis structure of Greek tragedies (as well as many modern-day movies) follows a pattern of final recovery of the protagonist after near defeat. An analogous kind of high excitement follows from last-minute recoveries and victories in sports events.

The extension of the AER model to conditions of uncertainty, together with the assumption that conceptual consumption is driven by images of upcoming events, would naturally capture the observation that people react more to the *possibility* of good or bad outcomes rather than to the *probability* of those good or bad outcomes (Kahneman and Tversky 1979).

The assumption of a single initiating time, t_0, is obviously a simplification. In many circumstances, it is hard to identify one single point in time when anticipation begins. Rather, the uncertainty about the occurrence of the event, or one's probability of attendance, may unfold over time, producing spikes of anticipation or disappointment. We are confident that the current framework can be modified in order to produce a prediction of how adaptation, discounting, and instant utility unfold in such cases.

The model still has room for more psychological realism. For example, research suggests that recall of past experiences might be driven by prior beliefs and distorted positive images of reality (Mitchell et al. 1997, Xu and Schwarz 2009, Ross 1989). The process of anticipation and forecasting is also subject to a variety of biases such as people's reliance on highly available but unrepresentative memories of the past (Morewedge et al. 2005, Ungemach et al. 2009). Indeed, future research could expand the AER model to capture such psychological processes and lead to predictions of instant utility that can improve the match with robust empirical findings.

In conclusion, the anticipation-event-recall model is a step toward providing a more articulated, yet tractable, model of total event utility that captures the psychological elements of adaptation, time distance, and conceptual consumption.

Acknowledgments

The authors are grateful to the Tough Mudder Company for the chance to run an experiment with its customers. The authors thank participants at the Society for Judgment and Decision Making, Foundations of Utility and Risk, INFORMS, and the International Federation of Operation Research Societies conferences for useful feedback, as well as the feedback they received at seminar presentations at Duke University, the University of Mannheim, Harvard University, and University Pompeu Fabra. The opening quotation was suggested by Miguel Lobo.

Appendix A. Experiments

A.1. Study 1B: The Reported Instant Utility Before a Real Event

Objective. Testing whether instant utility before an upcoming real event is U-shaped.

Method. We recruited 373 participants (38% female, mean age = 32) who signed up for an upcoming Tough Mudder event through the Tough Mudder mailing list. Each event consists of a 7- to 12-mile trail run over uneven, hilly, and wet ground followed by 17–20 sets of military-style obstacles. The event is designed to be "more convivial than marathons and triathlons, but more grueling than shorter runs or novelty events" (Branch 2010). Respondents, after agreeing to complete an online survey, were first asked to select their Tough Mudder challenge from a list of the upcoming events (42 races happening in the next 12 months, as of July 16, 2014).

Participants were asked, "When did you first start thinking about this event? Indicate the number of days. For example, if you started thinking about it a week ago, write 7 days; if you started thinking about it 1 month ago, write 30 days; if you started thinking about it 3 months ago, write 90 days, etc." We use this answer as a proxy for t_0, the time when anticipation begins. Next, participants were asked to rate their excitement about the event: "On a scale from 1 to 7, how excited are you today about your upcoming event?" The scale ranged from 1 ("not excited at all") to 7 ("extremely excited"). Finally, participants indicated when they had signed up for the challenge and whether they had participated in a Tough Mudder race before ("yes/no") and, if yes, in how many races. The survey ended with a series of general demographic questions (e.g., gender, age).

***Results*.** Results are reported in the main text.

A.2. Study 2: Preference for Unique, Nonrepeated Experiences

***Objective*.** Testing whether the memorability of experiences, such as kissing one's favorite movie star, is higher when the event occurs once versus multiple times.

***Method*.** Participants were 148 individuals in the Boston area (52% female, mean age = 22) who participated in a series of unrelated lab studies for monetary compensation. We first asked participants to imagine they were given the chance to kiss their favorite movie star. We then asked them, "Would it be more memorable if you kissed your favorite movie star only once (i.e., one time) *or* once a day per one week (i.e., seven times)?" Subsequently, participants were asked to rate on a scale from 1 to 7 the memorability of each of the two kisses experiences. The scale ranged from 1 ("not memorable at all") to 7 ("extremely memorable"). Finally, we collected a series of demographic variables (gender and age).

***Results*.** Sixty-eight percent of respondents selected the kiss one time as the most memorable experience between the two ($\chi^2 = 18.2$, $p < 0.001$). Moreover, participants rated kissing the movie star only once as significantly more memorable than kissing the movie star once a day per one week (6.4 versus 5.5, $t = 6.6$, $p < 0.001$). In conclusion, results from this study confirm that, ceteris paribus, the memorability of a unique experience is higher when the event happens only once rather than multiple times.

A.3. Study 3: The Optimal Duration of Anticipation

***Objective*.** Testing whether individuals have an ideal date to begin anticipating an upcoming event and whether such an ideal date depends on the magnitude of the event.

Table A.1. Answers to "How Long in Advance Would You Ideally Like to Be Told About Each of the Following Events?" (Response Time Scale: 1 = One Year, 2 = Nine Months, 3 = Six Months, 4 = Three Months, 5 = One Month, 6 = Two Weeks, 7 = One Week, 8 = The Day Prior; $N = 155$)

	Ideal anticipation	
Upcoming event	Avg. response (Std. dev.)	Days (interpolated)
Wedding of your best friend	3.1 (1.6)	180
Two-week vacation	4.3 (1.3)	60
Wedding of a distant relative	4.4 (1.5)	54
Concert of your favorite band	5.4 (1.3)	24
A weekend vacation	5.8 (1.1)	18
One day at a relax spa	6.7 (1.0)	9
Dinner at a fancy restaurant	7.0 (0.8)	7
Receiving a relaxing massage	7.2 (0.9)	6
Going to the cinema	7.5 (0.7)	4
Movie at home on DVD	7.7 (0.6)	2
Eating ice cream	7.9 (0.3)	1

***Method*.** Participants were 155 individuals in the Boston area (55% female, mean age = 22) who participated in a series of unrelated lab studies for monetary compensation. We provided respondents with a list of positive events and asked them to "imagine you can decide when to be told about each event. In other words, you can decide for how long you will be anticipating the event." We asked participants to ignore potential complications that might arise in the future: "There are no other issues or constraints and the event will happen on the anticipated day (e.g., events will not be sold out, there are no booking or reservation issues, some other obligation will not get on its way)." The list included the 11 events listed in Table A.1.

The order of events was randomized. Next, participants were asked, "How long in advance would you ideally like to be told about each of the following events?" Responses were measured on the following 1–8 time scale: (1) one year, (2) nine months, (3) six months, (4) three months, (5) one month, (6) two weeks, (7) one week, and (8) the day prior.

***Results*.** We calculated average ratings and, using linear interpolation, the equivalent time in days. As seen in Table A.1, participants indicated "wedding of your best friend" as the event that they wanted to start anticipating at the earliest date (3.1, equivalent to six months prior), followed by "two-week vacation" (4.4, three months). The events that participants wanted to anticipate for the shortest period of time were "movie at home on DVD" (7.7, the day prior) and "eating ice cream" (7.9, the day prior). Furthermore, the comparisons between events of similar nature, but different magnitude, revealed that participants clearly preferred a longer period of anticipation for events of larger magnitude. Ratings of ideal anticipation time were analyzed using one-way repeated-measures analysis of variance. Participants expressed a preference for anticipating earlier the wedding of their best friend rather than the wedding of a distant relative (3.1 versus 4.4, $t = 9.3$, $p < 0.001$), a two-week vacation rather than a weekend vacation (4.4 versus 5.8, $t = 16.5$, $p < 0.001$), and dining at a restaurant rather than eating ice cream (7.0 versus 7.9, $t = 14.7$, $p < 0.001$).

Presumably, the longer time needed to make arrangements for a larger event (e.g., wedding dress purchase, lodging for a vacation) may lead to desire for a longer period of anticipation. Yet our results persist even for events requiring similar preparations (e.g., weddings of relatives or best friends).

Appendix B. Proofs

Proof of Proposition 1. For $t \in [t_0, t_b)$, the derivative of $|u(t)|$ with respect to t is given by

$$|u'(t)| = |v(c)| e^{-\alpha\Delta_a} e^{\alpha(t_b - t)} f[\pi'(\tau_t)\rho_a - \alpha].$$

Because $|v(c)| e^{-\alpha\Delta_a} e^{\alpha(t_b-t)} f > 0$, instant (dis)utility increases with t iff $\pi'(\tau_t)\rho_a \geq \alpha$. By concavity, π' decreases with $\tau_t = \rho_a(t_b - t)$. Hence, π' increases with t, and so does $\pi'(\tau_t)\rho_a - \alpha$. This implies that $u'(t)$ can change signs at most once. If $\pi'(0^+)\rho_a < \alpha$, then let $t_m = t_b$. Otherwise, let $t_m = \inf\{t \in [t_0, t_b)\colon \pi'(\tau_t)\rho_a \geq \alpha\}$. It follows that $|u'(t)| < 0$ if $t < t_m$, and $|u'(t)| \geq 0$ if $t \geq t_m$. $\square$

Proof of Proposition 2. We integrate $u(t)$ over the three relevant time intervals. Recall that

$$\begin{aligned} u(t) = v(c)[&e^{-\alpha(t-t_0)} \cdot f(\rho_a(t_b - t)) \cdot 1_{\{t\in[t_0,t_b)\}} + e^{-\alpha(t-t_0)} \\ &\cdot 1_{\{t\in[t_b,t_e)\}} + e^{-\alpha(t_e-t_0)} \cdot f(\rho_r(t-t_e)) \cdot 1_{\{t\in[t_e,t_1)\}}]. \end{aligned}$$

For anticipatory utility, the change of variable $\tau = \rho_a(t_b - t)$ yields

$$\begin{aligned} U^A &= v(c) \int_{t_0}^{t_b} e^{-\alpha(t-t_0)} f(\rho_a(t_b - t))\, dt \\ &= v(c) e^{-\alpha\Delta_a} \frac{1}{\rho_a} \int_0^{\rho_a\Delta_a} e^{\alpha/\rho_a \tau} f(\tau)\, d\tau. \end{aligned}$$

For event utility, we directly obtain

$$\begin{aligned} U^E &= v(c) \int_{t_b}^{t_e} e^{-\alpha(t-t_0)}\, dt \\ &= v(c) e^{-\alpha\Delta_a} \frac{1 - e^{-\alpha\Delta_e}}{\alpha}. \end{aligned}$$

For recall utility, remember that $\Sigma = \int_0^{\rho_r(t_1-t_e)} f(\tau)\, d\tau$. The change of variable $\tau = \rho_r(t - t_e)$ yields

$$\begin{aligned} U^R &= v(c) e^{-\alpha(t_e-t_0)} \int_{t_e}^{t_1} f(\rho_r(t - t_e))\, dt \\ &= v(c) e^{-\alpha\Delta_a} e^{-\alpha\Delta_e} \frac{1}{\rho_r} \int_0^{\rho_r(t_1-t_e)} f(\tau)\, d\tau. \end{aligned}$$

Replacing $\int_0^{\rho_r(t_1-t_e)} f(\tau)\, d\tau = \Sigma$ and $\rho_r = \rho_a e^{\alpha\mu\Delta_a}$ yields the proposed expression for U^R. Adding U^A, U^E, and U^R produces the desired expression for U.

For future use, we apply integration by parts and $f'(\tau) = -f(\tau)\pi'(\tau)$ to obtain the alternative expression

$$\begin{aligned} U^A = \frac{v(c)}{\alpha} \Bigg[&f(\rho_a\Delta_a) - f(0^+) e^{-\alpha\Delta_a} \\ &+ e^{-\alpha\Delta_a} \int_0^{\rho_a\Delta_a} \underbrace{e^{(\alpha/\rho_a)\tau} f(\tau)}_{g_\tau} \pi'(\tau)\, d\tau \Bigg]. \quad \square \end{aligned} \tag{B.1}$$

Proof of Proposition 3. For $* \in \{A, E, R\}$, let $k_* = |U|^*/|v(c)|$. In this proof, all derivatives are taken with respect to $|c|$. If v is piecewise linear, then $|c||U|'^*/|U|^* = 1 + |c|k_*'/k_*$. It remains to show that $k_A' = \mu\psi k_A/|c|$, $k_E' = 0$, and $k_R' = \mu k_R/|c|$.

Differentiating (B.1) with respect to $|c|$ (and noting that the term $f\pi'\Delta_a\rho_a'$ cancels) yields

$$k_A' = \frac{\mu}{|c|} \frac{e^{-\alpha\Delta_a}}{\rho_a} \int_0^{\rho_a\Delta_a} \tau\pi'(\tau) g_\tau\, d\tau = \mu\psi k_A/|c| > 0.$$

Because k_E does not depend on c, we have that $k_E' = 0$.

For recall, we let $t_1 = \infty$. Then, Σ does not depend on c, and k_R is inversely proportional to ρ_r—hence directly proportional to $|c|^\mu$—and $k_R' = \mu k_R/|c|$.

Finally, we observe that as $|c|$ increases, ψ goes to 0 and the elasticity of $|U|^A$ goes to 1. The result holds automatically for $\mu = 0$; otherwise, we use $\tau\pi'(\tau)$ increasing and π concave to establish that

$$\begin{aligned} 0 \leq \psi &= \frac{\int_0^{\rho_a\Delta_a} \tau\pi'(\tau) g_\tau\, d\tau}{\int_0^{\rho_a\Delta_a} g_\tau\, d\tau} \leq \frac{\rho_a\Delta_a\pi'(\rho_a\Delta_a) \int_0^{\rho_a\Delta_a} g_\tau\, d\tau}{\int_0^{\rho_a\Delta_a} g_\tau\, d\tau} \\ &= \rho_a\Delta_a\pi'(\rho_a\Delta_a) \leq \pi(\rho_a\Delta_a) - \pi(0^+). \end{aligned}$$

If $\mu > 0$ and $|c| \to \infty$, then $\rho_a \to 0$, $\pi(\rho_a\Delta_a) \to \pi(0^+)$, and $\psi \to 0$. $\square$

Proof of Proposition 4. Differentiating total utility with respect to Δ_e yields

$$\begin{aligned} \frac{\partial U}{\partial \Delta_e} &= \underbrace{0}_{\partial U^A/\partial\Delta_e} + \underbrace{v(c)e^{-\alpha(\Delta_a+\Delta_e)}}_{\partial U^E/\partial\Delta_e} - \underbrace{v(c)e^{-\alpha(\Delta_a+\Delta_e)} \frac{\alpha\Sigma}{\rho_r}}_{\partial U^R/\partial\Delta_e} \\ &= v(c)e^{-\alpha(\Delta_a+\Delta_e)} \left[1 - \frac{\alpha\Sigma}{\rho_r}\right]. \end{aligned}$$

Hence, total utility is increasing in Δ_e if and only if $\alpha\Sigma \leq \rho_r$. Note also that, because $\rho_r \geq \rho_a$, if $\alpha\Sigma \geq \rho_r$, then $\alpha\Sigma \geq \rho_a$, (6) holds, and $(\partial U/\partial\Delta_a)|_{\Delta_a=0} \leq 0$. $\square$

Proof of Proposition 5. Let $g_\tau = e^{(\alpha/\rho_a)\tau} f(\tau)$, and let $G(\Delta_a) = f(0^+) - \int_0^{\rho_a\Delta_a} g_\tau \pi'(\tau)\, d\tau$. We differentiate each component of U with respect to Δ_a to obtain (for U^R, we assume $t_1 = \infty$)

$$\begin{aligned} \frac{\partial U^A}{\partial \Delta_a} &= v(c)e^{-\alpha\Delta_a} G(\Delta_a), \\ \frac{\partial U^E}{\partial \Delta_a} &= -v(c)e^{-\alpha\Delta_a}(1 - e^{-\alpha\Delta_e}), \quad \text{and} \\ \frac{\partial U^R}{\partial \Delta_a} &= -v(c)e^{-\alpha(1+\mu)\Delta_a} \frac{\alpha(1+\mu)\Sigma}{\rho_a} e^{-\alpha\Delta_e}. \end{aligned}$$

To maximize U^A, the first-order condition is $G(\Delta_a) = 0$. The solution is strictly positive ($G(0) = f(0^+) > 0$), finite ($\lim_{\Delta_a\to\infty} G(\Delta_a) = f(0^+) - \int_0^\infty g_\tau\pi'(\tau)\, d\tau < f(0^+) - \int_0^\infty f'(\tau)\, d\tau < 0$), and unique ($G'(\Delta_a) = -\rho_a e^{\alpha\Delta_a} f(\rho_a\Delta_a)\pi'(\rho_a\Delta_a) < 0$).

To maximize U, the first-order condition is

$$G(\Delta_a) - (1 - e^{-\alpha\Delta_e}) - e^{-\alpha\Delta_e} \frac{\alpha(1+\mu)\Sigma}{\rho_a} e^{-\alpha\mu\Delta_a} = 0. \tag{B.2}$$

Note that if $\Delta_a \geq \Delta_A$, then $G(\Delta_a) \leq 0$, and (B.2) is strictly negative.

Next, we now show that if μ is not large, then U is unimodal in Δ_a (switching from increasing to decreasing at most once), and the optimal anticipatory time is unique. It suffices to show that the term inside the bracket in (B.2) is strictly decreasing in Δ_a, which, by differentiating and arranging, is the case if and only if

$$\mu(1+\mu) < \left(\frac{\rho_a}{\alpha}\right)^2 \frac{f(\rho_a\Delta_a)\pi'(\rho_a\Delta_a)}{e^{-\alpha(1+\mu)\Delta_a}e^{-\alpha\Delta_e}\Sigma}. \tag{B.3}$$

If $\mu = 0$ (i.e., $\rho_a = \rho_0$), then the right-hand side is bounded from below by $(\rho_0/\alpha)^2(f(\rho_0\Delta_A)\pi'(\rho_0\Delta_A)/e^{-\alpha\Delta_e}\Sigma) > 0$ for all $\Delta_a \in [0, \Delta_A]$. By continuity, there is a value of $\hat{\mu} > 0$ for which (B.3) holds for all $\Delta_a \in [0, \Delta_A]$ and all $\mu \in [0, \hat{\mu})$. We conclude that if $\mu \in [0, \hat{\mu})$, then U is unimodal on $\Delta_a \in [0, \Delta_A]$, and $\Delta_* \in [0, \Delta_A)$ is unique.

Note that (B.2) is a decreasing function to which we subtract an exponential decay. The typical shape of $U(\Delta_a)$ is threefold. If $(\partial U/\partial\Delta_a)|_{\Delta_a=0} > 0$, then U is unimodal, and $\Delta_* > 0$. If $(\partial U/\partial\Delta_a)|_{\Delta_a=0} \le 0$, then U can be either decreasing throughout or decreasing first, then increasing, and finally decreasing. In either of these two cases, we show that $U(\Delta_a) < U(0)$, $\Delta_a > 0$ to conclude that $\Delta_a = 0$ is a global maximum.

Step 1. The claim holds for $\Delta_e = 0$. Note that

$$\begin{aligned} &U(0)|_{\Delta_e=0} - U(\Delta_a)|_{\Delta_e=0} \\ &\quad= \frac{\Sigma}{\rho_a}(1-e^{-\alpha(1+\mu)\Delta_a}) - \frac{e^{-\alpha\Delta_a}}{\rho_a}\int_0^{\rho_a\Delta_a} g_\tau\, d\tau > 0. \end{aligned} \tag{B.4}$$

Consider a parameter change where α is replaced by $\hat{\alpha}$, $\alpha(1+\mu)$ is kept constant by setting $\hat{\mu} = (\alpha/\hat{\alpha})(1+\mu) - 1$, and ρ_a is also constant by setting $\hat{\rho}_0 = \rho_0(v(c)^{\hat{\mu}}/v(c)^{\mu})$. Without loss of generality, we set $v(c) = 1$. Then $U(\Delta_a)|_{\Delta_e=0}$ becomes

$$\hat{U}(\Delta_a, \hat{\alpha}) = \frac{1}{\rho_a}\int_0^{\rho_a\Delta_a} e^{-\hat{\alpha}(\Delta_a - \tau/\rho_a)} f(\tau)\, d\tau + \frac{\Sigma}{\rho_a} e^{-\alpha(1+\mu)\Delta_a}.$$

By construction, if $\hat{\alpha} = \alpha$, then $\hat{U}(\Delta_a, \alpha) = U(\Delta_a)|_{\Delta_e=0}$. For all Δ_a, $\hat{U}(\Delta_a, \hat{\alpha})$ increases as $\hat{\alpha}$ decreases, and strictly so for $\Delta_a > 0$. Thus,

$$\hat{U}(\Delta_a, 0) = \frac{1}{\rho_a}\int_0^{\rho_a\Delta_a} f(\tau)\, d\tau + \frac{\Sigma}{\rho_a} e^{-\alpha(1+\mu)\Delta_a} \ge U(\Delta_a)|_{\Delta_e=0}.$$

That $(\partial U/\partial\Delta_a)|_{\Delta_a=0} \le 0$ implies $\alpha(1+\mu)\Sigma/\rho_a \ge f(0^+)$. The latter implies that $\hat{U}(\Delta_a, 0)$ is unimodal, decreasing first and then increasing as $\Delta_a \to \infty$. To see this, the first-order condition $\partial\hat{U}(\Delta_a, 0)/\partial\Delta_a = 0$ becomes $\alpha(1+\mu)\Delta_a - \pi(\rho_a\Delta_a) = \ln(\alpha(1+\mu)\Sigma) - \ln(\rho_a)$. Because π is concave, there is at most one strictly positive solution, a local minimum. Hence, the function takes its maximum either at 0 or at ∞, which in this case is immaterial because $\hat{U}(0,0) = \hat{U}(\infty, 0) = \Sigma/\rho_a$. Hence, for $\Delta_a \in (0,\infty)$, $U(\Delta_a)|_{\Delta_e=0} = \hat{U}(\Delta_a, \alpha) < \hat{U}(\Delta_a, 0) < \Sigma/\rho_a = U(0)$.

Step 2. The claim holds for $\Delta_e > 0$. Using (B.4), we obtain

$$\begin{aligned} &U(0) - U(\Delta_a) \\ &\quad= U(0) - U(0)|_{\Delta_e=0} + U(0)|_{\Delta_e=0} - U(\Delta_a)|_{\Delta_e=0} \\ &\qquad + U(\Delta_a)|_{\Delta_e=0} - U(\Delta_a) \\ &\quad= \frac{1-e^{-\alpha\Delta_e}}{\alpha} + \frac{\Sigma}{\rho_a}e^{-\alpha\Delta_e} - \frac{\Sigma}{\rho_a} + \frac{\Sigma}{\rho_a}(1-e^{-\alpha(1+\mu)\Delta_a}) \\ &\qquad - \frac{e^{-\alpha\Delta_a}}{\rho_a}\int_0^{\rho_a\Delta_a} g_\tau\, d\tau + \frac{e^{-\alpha\Delta_a}}{\rho_a}\int_0^{\rho_a\Delta_a} g_\tau\, d\tau + \frac{\Sigma}{\rho_a}e^{-\alpha(1+\mu)\Delta_a} \\ &\qquad - \frac{e^{-\alpha\Delta_a}}{\rho_a}\int_0^{\rho_a\Delta_a} g_\tau\, d\tau - e^{-\alpha\Delta_a}\frac{1-e^{-\alpha\Delta_e}}{\alpha} - \frac{\Sigma}{\rho_a}e^{-\alpha(1+\mu)\Delta_a}e^{-\alpha\Delta_e} \\ &\quad= \frac{1-e^{-\alpha\Delta_a}}{\alpha}(1-e^{-\alpha\Delta_e}) + \frac{\Sigma}{\rho_a}(1-e^{-\alpha(1+\mu)\Delta_a})e^{-\alpha\Delta_e} \\ &\qquad - \frac{e^{-\alpha\Delta_a}}{\rho_a}\int_0^{\rho_a\Delta_a} g_\tau\, d\tau \\ &\quad> (1-e^{-\alpha\Delta_e})\left[\frac{1-e^{-\alpha\Delta_a}}{\alpha} - \frac{e^{-\alpha\Delta_a}}{\rho_a}\int_0^{\rho_a\Delta_a} g_\tau\, d\tau\right] \\ &\quad= \frac{1-e^{-\alpha\Delta_e}}{\rho_a}\left[\int_0^{\rho_a\Delta_a} e^{-\alpha(\tau/\rho_a)}\, d\tau - \int_0^{\rho_a\Delta_a} e^{-\alpha(\Delta_a-\tau/\rho_a)} f(\tau)\, d\tau\right]. \end{aligned}$$

To see that the term in brackets is ≥ 0, apply the change of variable $\tau' = \rho_a\Delta_a - \tau$ to $\int_0^{\rho_a\Delta_a} e^{-\alpha(\Delta_a-(\tau/\rho_a))} f(\tau)\, d\tau$ and use $f(\tau) \le 1$ to obtain

$$\begin{aligned} \int_0^{\rho_a\Delta_a} e^{-\alpha(\Delta_a-\tau/\rho_a)} f(\tau)\, d\tau &= \int_0^{\rho_a\Delta_a} e^{-\alpha(\tau'/\rho_a)} f(\rho_a\Delta_a - \tau')\, d\tau' \\ &\le \int_0^{\rho_a\Delta_a} e^{-\alpha(\tau'/\rho_a)}\, d\tau'. \end{aligned}$$

Finally, we show that Δ_A decreases with α and ρ_0 and increases with c. By the implicit function theorem, and knowing that $\partial G/\partial\Delta_a < 0$, it suffices to show that $\partial G/\partial\alpha < 0$, $\partial G/\partial\rho_0 < 0$, and $\partial G/\partial c \ge 0$, respectively. Let $\tau_A = \rho_a\Delta_A$. Note that $g'_\tau = (\alpha/\rho_a)g_\tau - g_\tau\pi'(\tau)$. Using $f(0^+) = \int_0^{\tau_A} g_{\tau_A}\pi'(\tau)\, d\tau$ and $g(0^+) = f(0^+)$, we conclude that $g_{\tau_A} = \int_0^{\tau_A} g'_\tau\, d\tau = (\alpha/\rho_a)\int_0^{\tau_A} g_\tau\, d\tau$. Note also that if $-\tau\pi''/\pi' < 1$, then $(\pi'\tau)' = \tau\pi'' + \pi' > 0$ and $\pi'\tau$ is strictly increasing. Thus,

$$\begin{aligned} \frac{\partial G}{\partial\alpha} &= -\frac{1}{\rho_a}\int_0^{\tau_A} \tau^2 g_\tau\pi'(\tau)\, d\tau < 0, \\ \frac{\partial G}{\partial\rho_0} &= -\frac{1}{\rho_0}\left[g_{\tau_A}\pi'(\tau_A)\tau_A + \frac{\alpha}{\rho_a}\int_0^{\tau_A} g_\tau\pi'(\tau)\tau\, d\tau\right] \\ &= -\frac{\alpha}{\rho_0\rho_a}\left[\int_0^{\tau_A} g_\tau\pi'(\tau_A)\tau_A\, d\tau - \int_0^{\tau_A} g_\tau\pi'(\tau)\tau\, d\tau\right] < 0, \quad \text{and} \\ \frac{\partial G}{\partial c} &= \frac{\mu}{c}\left[g_{\tau_A}\pi'(\tau_A)\tau_A - \frac{\alpha}{\rho_a}\int_0^{\tau_A} g_\tau\pi'(\tau)\tau\, d\tau\right] \\ &= \frac{\mu}{c}\frac{\alpha}{\rho_a}\left[\int_0^{\tau_A} g_\tau\pi'(\tau_A)\tau_A\, d\tau - \int_0^{\tau_A} g_\tau\pi'(\tau)\tau\, d\tau\right] \ge 0. \quad \square \end{aligned}$$

Proof of Proposition 6. Recall that $\pi(\tau) = (1-\delta) + \delta\tau$, $\delta \in (0,1]$. Let $\Lambda = \delta\rho_a - \alpha$. Henceforth, if $\Lambda = 0$, then we replace $(1-e^{-\Lambda\Delta_a})/\Lambda$ for its limit, Δ_a. We obtain $f(0^+) = e^{\delta-1}$, $\Sigma = e^{\delta-1}/\delta$, $\pi'(\tau) = \delta$,

$$g_\tau\pi'(\tau) = \delta\, g_\tau = \delta\, e^{\delta-1}e^{(\alpha/\rho_a-\delta)\tau}, \quad \text{and}$$
$$\int_0^{\rho_a\Delta_a} g_\tau\pi'(\tau)\, d\tau = \delta\int_0^{\rho_a\Delta_a} g_\tau\, d\tau = \delta e^{\delta-1}\rho_a\frac{1-e^{-\Lambda\Delta_a}}{\Lambda}.$$

Solving for $\int_0^{\rho_a\Delta_a} g_\tau\pi'(\tau)\, d\tau = f(0^+)$ yields $\Delta_A = 1/(\delta\rho_a)$ if $\delta\rho_a = \alpha$; otherwise, $\Delta_A = (\ln\delta\rho_a - \ln\alpha)/(\delta\rho_a - \alpha)$.

Recall that Δ_* either is 0 or is the $\Delta_a \in (0, \Delta_A)$ that sets $\partial|U|/\partial\Delta_a = 0$. The sign of this derivative is given by $f(0^+) - \int_0^{\rho_a\Delta_a} g_\tau\pi'(\tau)\, d\tau - (1-e^{-\alpha\Delta_e}) - e^{-\alpha\Delta_e}(\alpha(1+\mu)\Sigma/\rho_r)$ or, in our case, by

$$1 - \delta\rho_a\frac{1-e^{-\Lambda\Delta_a}}{\Lambda} - (1-e^{-\alpha\Delta_e})e^{1-\delta} - e^{-\alpha\Delta_e}\frac{\alpha}{\delta\rho_a}(1+\mu)e^{-\alpha\mu\Delta_a}.$$

If $\mu = 0$, then only the second term depends on Δ_a, this expression is decreasing in Δ_a, and there is a unique local and global maximum. Henceforth, assume that $\mu > 0$.

(i) $\Lambda=0$. The extremum solves $1-\delta\rho_a\Delta_a=(1-e^{-\alpha\Delta_e})e^{1-\delta}+e^{-\alpha\Delta_e}(1+\mu)e^{-\alpha\mu\Delta_a}$ (we use $\alpha=\delta\rho_a$). As a function of Δ_a, the left-hand side is a line with negative slope, whereas the right-hand side is a decaying exponential. At $\Delta_a=0$, the line is greater than or equal to the decaying exponential (because the right-hand side is a convex combinations of $e^{1-\delta}\geq 1$ and $1+\mu>1$), which implies that $|U|$ is strictly decreasing, and $\Delta_*=0$ is a local maximum. If the line is less than or equal to the exponential for all Δ_a, then $\Delta_*=0$. If the line intersects the exponential, it does so twice. Then, $|U|$ is strictly decreasing until the first intersection point and strictly increasing until the second intersection point. It follows that Δ_* is either 0 or the second intersection point.

If $\Lambda\neq 0$, then let

$$A=1-\frac{\Lambda}{\delta\rho_a}[1-e^{1-\delta}(1-e^{-\alpha\Delta_e})],\quad\text{and}$$
$$B=\Lambda\frac{\alpha(1+\mu)}{(\delta\rho_a)^2}e^{-\alpha\Delta_e}.$$

The extremum solves $e^{-\Lambda\Delta_a}=A+Be^{-\alpha\mu\Delta_a}$. We consider two cases.

(ii) $\Lambda>0$. Then, $0<A<1$ and $B>0$. Let $C=\alpha\mu/\Lambda>0$. Then, $x_*=e^{-\Lambda\Delta_*}$ is a fixed point of

$$H(x)=A+Bx^C,\ x\in[0,1].$$

We have three subcases.

(a) *Case* $A+B<1$. Because $H(0)=A>0$, $H(1)=A+B<1$, and H is either concave ($0<C\leq 1$) or convex ($C>1$), we have that $H(x)$ has one unique fixed point, x_*, which can be found recursively. (Dis)utility increases up to $\Delta_*=-\ln x_*/\Lambda$ and then decreases.

(b) *Case* $A+B=1$. Here, $x_*=1$ is the only fixed point, and $\Delta_*=0$.

(c) *Case* $A+B>1$. If $C\leq 1$, then $H(x)$ is concave, there is no fixed point, and $\Delta_*=0$. If $C>1$, then H is convex, and $H(x)=x$ has at most two solutions. The shape of $|U|$ is as described in the case of $\Lambda=0$.

(iii) $\Lambda<0$. Then, $A>1$, $B<0$, and

$$A+B=1-\frac{\alpha-\delta\rho_a}{\delta\rho_a}\left[e^{1-\delta}(1-e^{-\alpha\Delta_e})+\frac{\alpha(1+\mu)}{\delta\rho_a}e^{-\alpha\Delta_e}-1\right]<1.$$

Let $C=\alpha\mu/(\alpha-\delta\rho_a)>0$. Then, $x_*=e^{-\Lambda\Delta_*}$ is a fixed point of $H(x)=A+B/x^C$, $x\in[1,A]$. Because H is concave, $H(x)=x$ may have at most two solutions, and the shape of $|U|$ is as described for $\Lambda=0$. □

Proof of Proposition 7. In view of (5), both $|U|^E$ and $|U|^R$ tend to 0 as Δ_a increases. It remains to show that $|U|^A$ also goes to 0. Note that

$$\begin{aligned}|U|^A&=\frac{|v(c)|}{\rho_a}\int_0^{\rho_a\Delta_a}e^{-\alpha(\Delta_a-\tau/\rho_a)-\pi(\tau)}\,d\tau\\&\leq\frac{|v(c)|}{\rho_a}\left[\int_0^{\rho_a\Delta_a/2}e^{-\alpha(\Delta_a/2)}\,d\tau+\int_{\rho_a\Delta_a/2}^{\rho_a\Delta_a}e^{-\pi(\rho_a\Delta_a/2)}\,d\tau\right]\\&\leq\frac{|v(c)|}{\rho_a}\left[e^{-\alpha(\Delta_a/2)}\frac{\rho_a\Delta_a}{2}+e^{-\pi(\rho_a\Delta_a/2)}\frac{\rho_a\Delta_a}{2}\right].\end{aligned}$$

By assumption, $\tau e^{-\pi(\tau)}\to 0$ as $\tau\to\infty$. Hence, the last term goes to 0 as $\Delta_a\to\infty$. □

Proof of Proposition 8. Consider the sign of the derivative $(\partial|U|/\partial\Delta_a)|_{\Delta_a=0}>0$. In view of Proposition 6, if $(\partial|U|/\partial\Delta_a)|_{\Delta_a=0}>0$, then $\Lambda>0$, and $|U|$ is unimodal. Solution 1 then follows.

If $(\partial|U|/\partial\Delta_a)|_{\Delta_a=0}=0$, then $|U|$ decreases, and solution 3 follows.

In view of Proposition 6, if $(\partial|U|/\partial\Delta_a)|_{\Delta_a=0}<0$, then we may have zero or two extrema. Solution 2 corresponds to the case of two extrema, where disutility decreases until reaching the first fixed point, increases until reaching the second fixed point, and decreases thereafter. Solution 3 corresponds to the case of zero extremum, and disutility (weakly) decreases with Δ_a. □

Endnotes

[1] A *value function*, v: $\mathbb{R}\to\mathbb{R}$, is any strictly increasing function with $v(0)=0$ (Kahneman and Tversky 1979). It is a ratio-scale function—that is, unique up to multiplication by a positive scalar.

[2] Note that π is a strictly increasing function with $\pi(0)=0$ and $\pi(\infty)=\infty$. Of course, this implies that $f(0)=1$ and $f(\infty)=0$. The case of $\pi(\tau)=\tau$ corresponds to exponential discounting.

[3] Otherwise, an expression for total utility is not available, even if we let r_t decay at a constant rate. We acknowledge, however, that letting r_t decay during recall is more realistic and should be used in numerical applications.

[4] To find t_m, we solve for $u'(t)=0$, $t_0\leq t<t_b$, where $u(t)=v(c)e^{-\alpha(t-t_0)}e^{-\pi(\rho_a(t_b-t))}$. This produces $\rho_a\pi'(\rho_a(t_b-t))=\alpha$. If π' is decreasing and $\pi'(0^+)>\alpha/\rho_a$, then there is a unique solution $t_m<t_b$; otherwise, $t_m=t_b$ (recall that zero distance is at $t=t_b$).

[5] Jevons (1905, p. 64) wrote, "The nearer the date fixed for leaving home approaches, the greater does the intensity of anticipal pleasure become: at first, when the holiday is still many weeks ahead, the intensity increases slowly; then, as time grows closer, it increases faster and faster, until it culminates on the eve of departure."

[6] Note that the AER model predicts decreasing excitement with repetition as a result of adaptation. Granted that each Tough Mudder race entails several elements of novelty (different location, obstacles, and trails) and adaptation to the event will begin afresh in many of its dimensions, selection bias may also partially account for the observed increase in excitement. Novices may be anxious. Indeed, some will have a negative experience and be less likely to participate again. After each race, there may be some additional attrition. Accordingly, the average excitement in the sample may increase with previous races, and the variability around excitement may decrease with the number of races. Indeed, the standard deviation around the mean level of excitement for novices is $\sigma=1.108$, whereas for repeaters, it is $\sigma=0.865$ ($F=4.56$, $p=0.033$, using Levene's homoscedasticity test).

[7] We created a 3×4 table of θ (three segments of one-third each) times excitement (7, 6, 5, and ≤ 4). We clearly can reject the hypothesis that excitement is independent of θ ($\chi^2=17.6$, p-value $=0.007$). As a further check, we examined the list of races and looked at location and seasonality of each event. The observed U-shaped pattern does not seem to reflect a spurious combination of interesting races in the immediate and distant future and more boring races in the moderate future.

[8] Assume $t_1=\infty$. Under Assumption A5(1) we have that $\Sigma=\Gamma(1/\delta+1)=(1/\delta)!$, whereas Assumption A5(2) yields $\Sigma=e^{\delta-1}/\delta$.

[9] To see this, observe that $|U|^A$ is bounded from below by $|c|^{1+\mu}e^{-\alpha\Delta_a}\Sigma/\rho_0$ and bounded from above by $|c|f(0^+)\Delta_a$. For $|c|$ small, $\partial|U|^A/\partial|c|\to e^{-\alpha\Delta_a}\Sigma/\rho_0$, and for $|c|$ large, $\partial|U|^A/\partial|c|\to f(0^+)\Delta_a$. More formally, if $\tau\pi'(\tau)$ is increasing, then ψ will be bell-shaped, taking value 0 at $|c|=0$, increasing with $|c|$ up to a point, and then decreasing back to 0 as $|c|$ tends to infinity.

[10] In one study, participants in one condition were asked to recall a special evening out; in the other condition, they were asked to recall a typical evening out. Naturally, special evenings were rated more highly than typical ones. But when the researchers then asked participants which experience they would want to repeat, participants were more likely to want to repeat the typical evening than the special evening, even though they had just rated this experience as providing less utility.

[11] This result is not trivial. For some parameter values, U initially decreases with Δ_a, then it reaches a local minimum, then it increases to a local maximum, and finally it decreases to zero. In the proof of Proposition 5, we show that when $\Delta_a = 0$ is a local maximum, then it is necessarily a global maximum.

[12] In view of (B.2), we have that $(\partial U/\partial \Delta_a)|_{\Delta_a=0} \leq 0$ iff $\alpha(1+\mu)e^{-\alpha\Delta_e} \cdot \Sigma/\rho_a + 1 - e^{-\alpha\Delta_e} \geq f(0^+)$, which becomes (6) if $f(0^+) = 1$.

[13] Issues of dynamic consistency with respect to one's actions are commonplace in models of anticipation (Caplin and Leahy 2001, Kőszegi and Rabin 2006). Basically, if the value of certain state variable today (e.g., the current reference point) depends on what individual i thought yesterday that he or she would do today, as it does in the AER model, then rational expectations require that i be consistent and carry out the anticipated plan. The requirement creates a circularity in the model that needs to be resolved by means of a Nash equilibrium between the "multiple selves" involved in the model. In single-individual contexts, such an equilibrium is called a *personal equilibrium*. A personal equilibrium often takes the form of a precommitment.

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