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Add-on Pricing by Asymmetric Firms

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This paper uses an analytical model to examine the consequences of add-on pricing when firms are both horizontally and vertically differentiated and there is a segment of boundedly rational consumers who are unaware of the add-on fees at the time of initial purchase. We find that consumers who know the add-on fees can be penalized—and increasingly so—by the existence of boundedly rational consumers. Our consideration of quality asymmetries on base goods and add-ons, plus the inclusion of boundedly rational consumers, leads to several novel findings regarding firm profits. When quality asymmetry is on base goods only and with boundedly rational consumers, add-on pricing can diminish profit for a qualitatively superior firm and increase profit for an inferior firm (i.e., a lose–win result), compared to when add-on pricing is prohibited or infeasible. When quality asymmetries exist on both base goods and add-ons and without boundedly rational consumers, the opposite win–lose result prevails. When quality asymmetries exist on both base goods and add-ons and with boundedly rational consumers, the result can be win–win, win–lose, or lose–win, depending on the magnitude of quality differentiation on add-ons.

Key words: game theory; add-on pricing; vertical differentiation; bounded rationality
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1. Introduction

Consumers are often faced with fees for add-ons not included in the price of a base product (or service). For example, hotel guests, after room rates are already paid, often have to pay additional charges for services such as Internet access, safe access, parking, a fitness center, and pool use.[1] Banks charge fees for consumers who opt to cancel a check, use a foreign ATM, or use a debit card.[2] Air travelers frequently find themselves paying for pets, snacks, checked luggage, and even blankets.[3] Cell phone service providers charge subscribers to send text messages and download data (see Sullivan 2007, p. 108).

The central focus of this paper is determining the implications of competing firms charging separately for the add-ons. Whereas the previous add-on pricing literature assumes quality symmetry between the horizontally differentiated firms, we account for the fact that competing firms may actually differ in quality. As a consequence, the implication of add-on pricing may be different for a superior firm than for an inferior firm. We also allow for asymmetry in the quality of the add-on to determine whether add-on pricing has contrasting implications depending on whether there is quality asymmetry for the base products (e.g., differently rated hotels), the add-ons (e.g., fitness centers of varying quality), or both.

In addition to quality asymmetry, this paper uniquely incorporates market realities regarding consumer segmentation. Specifically, there is often heterogeneity in the willingness to pay for the add-on and heterogeneity in the anticipation of the add-on fee. To illustrate heterogeneity in willingness to pay for the add-on, consider that some consumers may need to park a car or use Internet services, whereas a consumer without a car (computer) does not value parking (Internet service). With regard to heterogeneity in anticipating the add-on, we allow for some consumers to be knowledgeable about the add-on price yet still purchase the add-on, whereas other consumers are boundedly rational in that they do not take the add-on price into consideration at the time of initial purchase (nor do they form rational expectations).

Two reasons explain the existence of boundedly rational consumers in add-on pricing. First, consumers are sometimes not informed of add-on fees when purchasing a base product—a recurring theme

[1] See http://www.independenttraveler.com/resources/article.cfm?AID=816&category=7 (accessed August 27, 2012) for a long list of hotel fees.

[2] See U.S. Government Accountability Office (2008) for a list of bank fees.

[3] See http://www.kayak.com/airline-fees (accessed August 27, 2012) for an overview of airline fees.

in the add-on pricing literature that is attributed to the lack of advertisement of add-on fees and even deliberate information hiding (Verboven 1999, Ellison 2005, Sullivan 2007, Gabaix and Laibson 2006). Second, although many consumers may know these add-on fees at the time of their initial purchase decision, there is strong anecdotal evidence that many other consumers are surprised by the charges for services they thought were included in the posted price. Internet chat rooms are filled with complaints from hotel consumers who were faced with fees they had not anticipated. Sullivan (2007) describes consumers who were surprised to learn how much it costs to send each text message. Our consideration of boundedly rational consumers is also consistent with the increasing attention on bounded rationality in the broader business research context.[4]

This paper addresses three questions.

1. *Who will gain from the advent of add-on pricing* (*the inferior firm*, *the superior firm*, *both firms*, *or neither firm*)?

2. *What happens to prices and consumer surplus when more consumers are boundedly rational and do not anticipate the add-on prices*?

3. *Who will charge for add-ons* (*the inferior firm*, *superior firm*, *or both*)?

The results of the research can be used to determine the implications of a newly realized ability to charge separately for an add-on previously included in the posted price (either through innovative thinking or technological advances). They also help show the implications of government intervention intended to limit the ability to charge for an add-on (for instance, President Obama has taken aim at bank fees (Mui 2011) and senator Charles Schumer has proposed prohibiting certain airline fees (Gormley 2010)). The answers to these questions are not straightforward because they each depend critically on the mix of consumer segments and/or the nature of asymmetry between firms.

In the popular press, add-on pricing is often viewed as a means for firms to boost profit via locking consumers into decisions that are more costly than anticipated. There is ample evidence that consumers make a purchase with the inaccurate expectation that the add-on is included in the posted price. After such a purchase, as one traveler opined, "It's like the people are at the mercy of the [companies]" (Belko 2010, p. C-1). Although the intuition would suggest that add-on pricing benefits firms, a well-known result in the unadvertised prices and add-on pricing literatures is that charging for add-ons results in equivalent profit as an all-inclusive price. We aim to resolve this discrepancy by showing when the profit irrelevance result will hold and when firms will benefit from add-on pricing. We also aim to identify conditions for when add-on pricing will diminish a firm's profitability.

Intuitively, the effect on prices of firms serving a greater number of boundedly rational consumers is unclear. On the one hand, the higher add-on revenue from exploiting boundedly rational consumers incentivizes firms to charge lower posted base prices to attract customers. With the add-on price capped by the customer's valuation on the add-on, the influx of boundedly rational consumers increases competitive intensity in base prices, thereby lowering the total price and thus increasing surplus. This logic is consistent with the predictions of the add-on literature (e.g., Gabaix and Laibson 2006). On the other hand, boundedly rational consumers expect a lower total price (i.e., base price plus add-on fee) than observed by consumers who are knowledgeable about add-on fees and thus may have a higher willingness to pay. The higher willingness to pay may increase total prices and hurt all consumers.[5] This latter effect has not been found in the current add-on pricing literature. We aim to resolve these two viewpoints and identify when total prices will (and will not) increase in the number of boundedly rational consumers.

In the previous literature, firms are symmetric in their add-on prices. In reality, however, firms vary in the add-on prices charged (Dickler 2011). Our model proposes asymmetry in firm quality as a driver of variance in add-on prices. The question becomes whether the superior firm or inferior firm is more likely to charge add-on prices. Because add-on prices are a form of price discrimination (Ellison 2005), applying predictions from another form of price discrimination, such as rebates, would suggest the inferior firm is more likely to price discriminate than the superior firm (Dogan et al. 2010). We examine whether this finding holds true for add-on pricing.

2. Literature Review

A well-known result in the unadvertised prices and add-on pricing literatures is that charging for add-ons results in equivalent profit as when add-on pricing is prohibited or infeasible. Any gains from the unadvertised price are competed away through the posted prices. Lal and Matutes (1994) modeled a retailer's use of loss-leader products coupled with unadvertised prices of other products that are not discounted.[6] Verboven (1999) looked at add-on pricing

[4] We offer a brief survey of business research on bounded rationality in the next section.

[5] We thank the associate editor for nicely summarizing this logic.

[6] In two related papers, Rao and Syam (2001) address the problem of why unadvertised prices may be discounted, and Janssen and Non (2009) identify why the advertised price may be higher than the unadvertised prices.

for products in which the price of the base product (i.e., the car) is advertised and the firms can choose whether to advertise the prices of premium upgrade add-ons. Gabaix and Laibson (2006) produced the profit-irrelevancy result of add-on prices with a segment of boundedly rational consumers who inaccurately believe the add-on is included in the posted price. Our paper differs from the above papers in that we show add-on pricing can affect firm profits.

Ellison (2005) identified how add-on pricing can actually lead to improved profit for the firms if consumers who are more sensitive to interfirm price differences are less likely to purchase costly add-ons. The correlation between consumer price sensitivity and add-on valuation creates an adverse selection problem that softens competition in base prices. Our paper complements Ellison's (2005) in that we find alternative venues in which add-on pricing can influence firm profits. Furthermore, we show that add-on pricing can have opposite profit implications for two competing firms. Ellison and Wolitzky (2012) extended the sequential search model of Stahl (1989) and showed that concealing prices—which increases search costs—reduces the fractions of consumers who search and thus benefits firms.[7] Ellison and Ellison (2009) provided empirical support for the explanations of Ellison (2005) and Ellison and Wolitzky (2012).[8] Kuksov and Xie (2010) found that concealing benefits in the form of offering frills after purchase can benefit a monopolist in a dynamic game by positively influencing word of mouth.

Because add-on pricing is a form of price discrimination, our paper relates to a body of research on targeted pricing. Shaffer and Zhang (1995) found that being able to charge different prices to loyal customers and consumers open to switching between firms intensifies competition and will result in a prisoner's dilemma that reduces profit relative to uniform pricing. Chen et al. (2001) found that price discrimination between loyal customers and switchers can benefit both firms if a proportion of loyal customers are misclassified by the firms as switchers (and vice versa). The ability to price discriminate between old and new customers is shown to diminish profits for competing firms (Villas-Boas 1999) and a durable-good monopolist (Villas-Boas 2004). Shaffer and Zhang (2002) allowed for asymmetric firms and found that the firm with the greater number of loyal customers will be better off when both firms use one-to-one promotions than when neither firm uses one-to-one promotions. Corts (1998) modeled vertically differentiated firms and showed that price discrimination between customers who value quality and those who do not value quality will result in lower profit for both the high-quality firm and the low-quality firm. Chen (2008) modeled firms of differentiated quality and found that the inferior firm earns less profit when price discrimination based on consumer purchase history is feasible than when uniform pricing is adopted. To the best of our knowledge, our paper is the first to demonstrate that price discrimination, in the form of add-on pricing, can improve profit for the inferior firm while *reducing profit for the superior firm*. Also, we are the first to develop a single model that uniquely identifies conditions defining when price discrimination will lead to three distinct outcomes (a win–win, win–lose, and lose–win) for the superior firm and inferior firm, respectively.

Establishing conditions that define when charging separately for add-ons leads to win–win, win–lose, and lose–win outcomes is also new to the bundling literature. There are a variety of papers that theoretically and empirically identify various reasons to offer bundles rather than price each element separately (e.g., Adams and Yellen 1976, Schmalensee 1984, McAfee et al. 1989, Fang and Norman 2005, Geng et al. 2005). Specifically, Whinston (1990), Chen (1997), Nalebuff (2004), and Ghosh and Balachander (2007) analyzed bundling in an oligopolistic setup. Whinston (1990) showed that a firm can leverage its monopolistic power in one market into another oligopolistic market via product tying. Chen (1997), from a different research angle of ex ante homogeneous firms, showed that bundling can serve as a product differentiation tool. Our research also differs from existing works in that the bundling literature treats all prices as common knowledge, whereas our paper allows for the possibility that add-on fees are not observable to at least some consumers. Such unobservability can result in a high magnitude of add-on fees—a signature dynamic in the add-on literature—that is absent from bundling research.

Gabaix and Laibson (2006) were among the first in the add-on literature to allow for boundedly rational consumers. As noted by Spiegler (2006), consumers may find it difficult to think and account for all possible dimensions (e.g., all possible services that may or may not be included in the posted price) and thus resort to simplifying heuristics (e.g., posted price) to save cognitive resources. The concept of bounded rationality has been around for decades (Simon 1955), though relatively recently it has gained more acceptance in economics (Rubinstein 1998). Conlisk (1996) reviewed a large body of evidence supporting bounds on rationality and provided a discussion

[7] The add-on pricing literature assumes two products: base and add-on products. In contrast, Ellison and Wolitzky (2012) only consider a single product in a search model.

[8] Another stream of work extends search models to study the scenario where a product is not free yet the information service about this product, such as product review by shoppers, is free (Wu et al. 2004, Shin 2007).

of when and why they should be incorporated in economic models. In our setting, there is a large body of anecdotal evidence supporting bounded rationality in terms of expectations surrounding hidden add-on fees. As described by Conlisk (1996), human cognition is a scarce resource, and thus a consumer's inability to accurately forecast the price for each possible add-on is consistent with a process by which cognitive resources are allocated elsewhere.

A number of recent researchers have also recognized the existence and importance of bounded rationality in various business contexts. Spann and Tellis (2006) showed in the context of name-your-own-price auctions that consumers often deviate from rational decision making. Ofek et al. (2007) analyzed how boundedly rational consumers derive information from their past decisions in making subsequent decisions. Ho and Zhang (2008) demonstrated bounded rationality in manufacturer-channel relationships regarding the influence of how the fixed fee is framed. Su (2008) studied the impact of bounded rationality of decision makers, rather than consumers, on newsvendor models. Jain (2009) examined how myopic consumers behave in the presence of goals. Kuksov and Villas-Boas (2010) recognized that processing information about attributes of each alternative may be costly and thus avoided altogether. Chen et al. (2010) studied how some consumers' limited memory of pricing affects market competition. Thomadsen and Bhardwaj (2011) allowed players in a strategic game to be forgetful. Therefore, the current research contributes to the growing body of business research examining how boundedly rational decision makers affect equilibrium outcomes. Although bounded rationality can manifest itself in many forms, we consider the specific bounded rationality where consumers inaccurately expect the add-on to be available for free.

3. The Model

3.1. Firms

There are two firms, 1 and 2, that are horizontally differentiated in that they are located at $x_1 = 0$ and $x_2 = 1$ on a linear city. We also allow for asymmetry in the quality of each firm's offerings, which we will specify shortly when we discuss consumer reservation utilities. As in Liu and Serfes (2005), we assume the cost of developing a certain quality to be a fixed cost (i.e., the cost of developing a product or building a brand name), which has no effect on the marginal cost of production (see also Motta 1993, Ikeda and Toshimitsu 2010).[9] Each firm offers a *base good* with marginal cost c_b. Each firm also offers an *add-on*, with marginal cost c_a, to consumers who purchase the firm's base good. For example, a base good can be a hotel room, checking account, wireless plan, or flight seat, and the associated add-on can be pool access, overdraft protection, data usage, or pillow usage, respectively.

Firm j ($j = 1, 2$) can charge a posted price p_{jb} for the base good and then charge a separate fee p_{ja} for the add-on. Posted prices are easily accessible by consumers at negligible costs, such as through popular Internet-based price comparison sites like Expedia.com, MyRatePlan.com, and Google Product Search. As a result, consumers have full information over posted prices. In contrast, p_{ja} is not easily discoverable by consumers. For example, on popular price comparison sites such as Expedia.com, hotels post their room rates yet often keep silent on their Internet access, pool usage, or parking fees, and airlines show their ticket prices yet not their charges for check-in baggage, pet fees, or Internet access. Although some consumers may track down this information, others may not know to do so. Outside of the travel industry, marketing letters from credit card companies highlight APRs, yet keep details on how default rates are triggered in fine print.[10] Wireless companies like Verizon and AT&T clearly announce their access fees for voice plans, but per-unit text messaging rates are in the fine print.

3.2. Consumers

Consumer i's utility from buying the base product from firm j is equal to $v_{jb} - t|x_j - \theta_i| - p_{jb}$, where transportation cost t is a positive constant, consumer locations θ_i are uniformly distributed along the linear city with a mass normalized to one, and the reservation value for the base good from firm j, v_{jb}, depends on the quality of the firm. Without loss of generality, let firm 1 be the qualitatively superior firm with quality normalized to 1 (i.e., $v_{1b} = v_b$). Firm 2 is weakly inferior with quality of the base good $q \le 1$ (i.e., $v_{2b} = qv_b$). Similarly to Villas-Boas (1999) and Shin and Sudhir (2010), we assume that qv_b is large enough to ensure full market coverage.[11] Consumer i's utility from buying the add-on from firm j is equal to $v_{jia} - p_{ja}$, where

[9] Alternatively, asymmetric quality could be modeled as having asymmetric marginal costs of production (Moorthy 1988, Kuksov and Lin 2010). Incorporating this assumption would complicate the model, yet logic suggests the current findings are preserved when the difference in costs is sufficiently less than the difference in qualities. The basis for this logic is that our results hold when the difference in costs is zero, and thus hold for local deviations.

[10] One controversial yet popular trigger is called "universal default," where a credit card's default rate is triggered if a consumer defaults on credit cards issued by any other financial institute.

[11] In §4.4 we provide the explicit condition for full market coverage in equilibrium.

Table 1 **Description of Consumer Segments**

	α fraction $v_{ia} = v_a > 0$	$(1-\alpha)$ fraction $v_{ia} = 0$
β fraction think $p_{ja} = 0$ before purchase	**Boundedly Rational Segment** ($\alpha\beta$ fraction) *Base purchase behavior*: Buys product that maximizes $v_{jb} - t\lvert\theta_i - x_j\rvert - p_{jb} + v_{ja}$ *Add-on purchase behavior*: Buys add-on of purchased base product if $v_{ja} \geq p_{ja}$	**Base Segment** ($(1-\alpha)$ fraction) *Base purchase behavior*: Buys product that maximizes $v_{jb} - t\lvert\theta_i - x_j\rvert - p_{jb}$ *Add-on purchase behavior*: Does not buy add-on
$(1-\beta)$ fraction know value of p_{ja} before purchase	**Knowledgeable Segment** ($\alpha(1-\beta)$ fraction) *Base purchase behavior*: Buys product that maximizes $v_{jb} - t\lvert\theta_i - x_j\rvert - p_{jb} + (v_{ja} - p_{ja})^+$ *Add-on purchase behavior*: Buys add-on of purchased base product if $v_{ja} \geq p_{ja}$	

$v_{1ia} = v_{ia}$ and $v_{2ia} = gv_{ia}$ with firm 1's quality of the add-on normalized to 1 and firm 2's quality of the add-on equal to $g \leq 1$.

We allow for consumer heterogeneity in willingness to pay for the add-on service as well as in bounded rationality.[12] As such we have three segments that we describe and provide justification for.

3.2.1. Base Consumers. We allow a proportion $(1-\alpha)$ of consumers to not derive utility from the add-on (i.e., $v_{ia} = 0$). Consumers may be in the base segment because they do not value the add-on or because they have found reasonable substitutes from the outside market. Examples include a wireless customer who does not want to text message, a checking account holder who balances the books and does not require overdraft protection or cancel-check services, and a flyer who packs light for a trip or carries their own pillow and thus doesn't check a bag or pay for use of a pillow. In ancillary analyses in Appendix B, we confirm our results when an additional segment exists who will consume the add-on unless it carries a charge.

3.2.2. Boundedly Rational Consumers. Whereas base consumers have zero value for the add-on, all remaining consumers have a positive value for the add-on (i.e., $v_{ia} = v_a > 0$). We assume $gv_a > c_a$, i.e., the reservation value of the add-on for either firm is higher than the marginal cost of the add-on. As in Gabaix and Laibson (2006), we allow for consumers to both value the add-on and inaccurately believe that the add-on is provided for free. Let β represent the proportion of nonbase consumers who have this form of bounded rationality (i.e., consumers who believe $p_{ja} = 0$). In other words, the proportion of the entire consumer population who fall in this segment is equal to $\alpha\beta$. As described by Gabaix and Laibson (2006), these consumers incompletely analyze the future game tree and do not anticipate the add-on fee at the time of base purchase. There are several reasons why consumers may be boundedly rational in this sense. It has been documented that even experienced travelers are surprised to learn that there are charges for services that had at a previous time been free (Stoller 2009). Alternatively, consumers may not have enough knowledge about the product to know what services will or will not be included in the price. Consistent with Conlisk (1996), boundedly rational consumers do not allocate cognitive resources toward finding the actual add-on fee or toward forming accurate expectations.

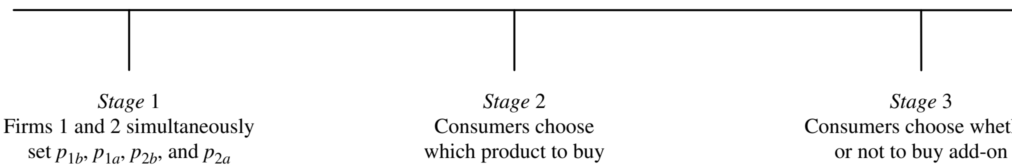
3.2.3. Knowledgeable Consumers. The remaining $\alpha(1-\beta)$ consumers have $v_{ia} = v_a > 0$ and know the add-on fees. Their knowledge of add-on fees may come from their prior experience or search. As such, knowledgeable consumers explicitly account for the add-on fee in their base purchase decision. See Table 1 for a description of consumer segments and Table 2 for notations.

Table 2 **Parameters and Decision Variables**

Symbol	Definition
c_b	Marginal cost of providing base product
c_a	Marginal cost of providing add-on
x_j	Location of product j
p_{jb}	Posted price for base offering from firm j
p_{ja}	Price for add-on from firm j
v_b	Consumer reservation utility of consuming the superior firm's base product
v_{ia}	Consumer i's utility from consuming the superior firm's add-on
q	Quality of firm 2's base good
g	Quality of firm 2's add-on
t	Transportation cost for consumers
θ_i	Consumer i's ideal taste parameter
α	Proportion of consumers who value the add-on (i.e., $v_{ia} = v_a > 0$)
β	Proportion of consumers who believe add-on is free

[12] In Appendix C, we examine a model in which consumers know that they will be charged add-on fees, but do not know what those are before committing to purchase the base product.

Figure 1 Sequence of Events



3.3. Timing of the Model

The sequence of the game is as follows. In Stage 1, firms simultaneously choose their posted and add-on prices. In Stage 2, consumers choose from which firm to buy. In Stage 3, all consumers become aware of the add-on fee and only nonbase consumers purchase the add-on if the add-on fee is less than or equal to the utility it provides. Consumers in Stage 3 are locked in to their Stage 2 purchase and cannot switch firms. Figure 1 depicts the sequence of events.

4. Results

We analyze the model using backward induction. We have three different consumer segments to examine: $(1-\alpha)$ base consumers, $\alpha\beta$ boundedly rational consumers, and $\alpha(1-\beta)$ knowledgeable consumers. The decision rules for these segments are described in Table 1. For the add-on to be purchased by consumers in Stage 3, it must be that $p_{1a} \le v_a$ and $p_{2a} \le g v_a$. Otherwise, no consumer will consume the add-on.

To our knowledge this is the first research on add-on pricing that considers all of the aforementioned three segments of consumers. It turns out, as the results will demonstrate, that insights new to the literature arise when both firms serve *all three* segments of consumers in equilibrium. We therefore will focus our attention on the scenario where all three segments of consumers purchase in equilibrium in the body of the paper, and we will provide the corresponding regularity conditions.

To derive the quantity of consumers who purchase each base product and each add-on in Stages 2 and 3, we identify the marginal consumer in each segment who is indifferent between purchasing product 1 and purchasing product 2. Given firm prices (p_{jb}, p_{ja}) for $j = 1, 2$, the marginal consumer in the knowledgeable segment satisfies

$$\begin{aligned} &v_b + v_a - t|0-\theta^k| - p_{1b} - p_{1a} \\ &\quad = q v_b + g v_a - t|1-\theta^k| - p_{2b} - p_{2a}, \end{aligned}$$

and thus she is located at $\theta^k = 1/2 + ((1-q)v_b + (1-g)v_a)/(2t) + (p_{2b} + p_{2a} - p_{1b} - p_{1a})/(2t)$. All consumers in the knowledgeable segment with $\theta < \theta^k$ ($\theta > \theta^k$) will purchase both the base product and the add-on service from firm 1 (firm 2).

Similarly the locations of the marginal base consumer and the marginal boundedly rational consumer are $\theta^b = 1/2 + ((1-q)v_b + p_{2b} - p_{1b})/(2t)$ and $\theta^{br} = 1/2 + ((1-q)v_b + (1-g)v_a + p_{2b} - p_{1b})/(2t)$, respectively. Demand of each firm's base product, γ_{jb}, and add-on, γ_{ja}, are then

$$\begin{aligned} \gamma_{jb} = 1/2 - \eta(j)[&\alpha(1-\beta)(p_{1a}-p_{2a}) + (p_{1b}-p_{2b}) \\ &- \alpha(1-g)v_a - (1-q)v_b]/(2t), \end{aligned} \tag{1}$$

$$\begin{aligned} \gamma_{ja} = \alpha/2 - \eta(j)\alpha[&(1-\beta)(p_{1a}-p_{2a}) + (p_{1b}-p_{2b}) \\ &- (1-g)v_a - (1-q)v_b]/(2t), \end{aligned} \tag{2}$$

where $\eta(j) = 1$ if $j = 1$, and $\eta(j) = -1$ if $j = 2$. Firm profits are

$$\pi_j = (p_{jb} - c_b)\gamma_{jb} + (p_{ja} - c_a)\gamma_{ja}. \tag{3}$$

In this model with three consumer segments and two firms that are vertically differentiated along two dimensions as well as horizontally differentiated, the Stage 1 pricing analysis is relatively involved. Similarly to Shin and Sudhir (2010), we proceed by first analyzing three special cases of this general model. Examining the special cases first has two benefits. First, it allows us to isolate the impacts of having three distinct consumer segments served by the firms (in contrast to the previous literature examining only two consumer segments), asymmetry in quality of the base products, and asymmetry in quality of the add-ons. Second, it allows the main insights to take center stage rather than the technical complexities inherent to the general model. We then conclude our results section by discussing the general case that produces unique interactions between the model features.

4.1. Symmetric Firms and Boundedly Rational Consumers

The first special case is when the two firms are symmetric, i.e., $q = 1$ and $g = 1$. We impose the following assumption regarding the level of horizontal differentiation relative to the add-on value to ensure that firms choose to serve all three consumer segments:

Assumption 1. $t > v_a - c_a$.

Applying $q = 1$ and $g = 1$ to Equations (1) and (2), in this special case we have $\gamma_{jb} = 1/2 - \eta(j)[\alpha(1-\beta) \cdot (p_{1a} - p_{2a}) + (p_{1b} - p_{2b})]/(2t)$ and $\gamma_{ja} = \alpha/2 - \eta(j) \cdot \alpha[(1-\beta)(p_{1a} - p_{2a}) + (p_{1b} - p_{2b})]/(2t)$. We can then derive the equilibrium posted prices and add-on fees from the first-order conditions of profit expressions in (3). It turns out there are two mutually exclusive cases.

Lemma 1. *Given symmetric firms, i.e., $q = 1$ and $g = 1$:*

(i) *if $\beta/((1-\alpha)(1-\beta)) < (v_a - c_a)/t$, the unique equilibrium is an inner equilibrium, in which firms charge posted prices $p_{1b} = p_{2b} = c_b + t - (\alpha\beta/((1-\alpha)(1-\beta)))t$ and add-on fees $p_{1a} = p_{2a} = c_a + (\beta/((1-\alpha)(1-\beta)))t$;*

(ii) *if $\beta/((1-\alpha)(1-\beta)) \geq (v_a - c_a)/t$, the unique equilibrium is a corner equilibrium, in which firms charge posted prices $p_{1b} = p_{2b} = c_b + t - \alpha(v_a - c_a)$ and add-on fees $p_{1a} = p_{2a} = v_a$.*

See Appendix A for all proofs. In contrast to the Gabaix and Laibson (2006) prediction that firms will charge the maximum possible add-on price, Lemma 1 shows this result is conditional on the relative magnitude of two ratios $\beta/((1-\alpha)(1-\beta))$ and $(v_a - c_a)/t$. Although part (ii) of Lemma 1 is consistent with Gabaix and Laibson's (2006) shrouded prices equilibrium, part (i) indicates when the equilibrium add-on price may be less than the consumer valuation of the add-on.[13] The following Proposition highlights a key insight gained from considering knowledgeable consumers who buy the add-on.

Proposition 1. *Given symmetric firms, the total price for buying the base product and add-on is increasing in β if and only if $\beta/((1-\alpha)(1-\beta)) < (v_a - c_a)/t$. As such, the consumer welfare for knowledgeable consumers and boundedly rational consumers is decreasing in the number of boundedly rational consumers if and only if $\beta/((1-\alpha)(1-\beta)) < (v_a - c_a)/t$.*

Proposition 1 identifies when knowledgeable consumers who value the add-on are penalized—and increasingly so—by the existence of boundedly rational consumers. To our knowledge this result is new to the add-on pricing literature and contrasts with the results of Gabaix and Laibson (2006, p. 517), who found that consumers who account for the add-on during the base purchase earn greater surplus with more boundedly rational consumers.[14] Proposition 1 also shows that a boundedly rational consumer is worse off when there are a greater number of peers who are also boundedly rational. Gabaix and Laibson (2006) found the opposite result when prices are hidden.

The intuition behind the contrasting results lies in how the sizes of the consumer segments affect posted prices and add-on fees. When a greater number of consumers will purchase the add-on, firms engage in increasingly intensified competition over posted prices to lock in additional profit on the add-on. This effect is present in our model and that of Gabaix and Laibson (2006). In the Gabaix and Laibson (2006) paper, all consumers who buy the add-on when its price is hidden are boundedly rational and the add-on prices are set at their maximum (further increasing them will trigger consumers to stop buying the add-on). Therefore, the only effect of increasing the number of boundedly rational consumers is to lower posted prices, which in turn means lower total prices. However, when there is a sufficient number of knowledgeable consumers and base consumers (i.e., the condition in part (i) of Lemma 1 holds), add-on prices are not set at their maximum. As such, in this parameter range there is a second effect of increasing the number of boundedly rational consumers. In this scenario, firms soften their competition over the add-on—and thus increase their add-on fees—to take advantage of locked-in boundedly rational consumers. The increased add-on fees dominate the reduced posted prices, resulting in higher total prices that knowledgeable and boundedly rational consumers pay.

As the intuition suggests, Proposition 1 is driven by the fact that the add-on price is set below its maximum level. We refer to this case as the inner equilibrium. The new result in Proposition 1 regarding total price and consumer welfare requires participation in the marketplace by each of the three consumer segments. To see this, the condition from Proposition 1 can be rewritten as follows:

$$\begin{aligned}&([\text{number of boundedly rational consumers}])\\&\quad\cdot([\text{number of base consumers}]\\&\qquad\cdot[\text{number of knowledgeable consumers}])^{-1}\\&\qquad< \frac{(v_a - c_a)}{t}.\end{aligned} \tag{4}$$

Equation (4) highlights that the equilibrium is the inner equilibrium when there is a sufficient number of *both* base consumers and knowledgeable consumers. Notice that the condition holds trivially when there are no boundedly rational consumers, and the condition never holds when all consumers who buy the add-on are boundedly rational (i.e., there are no knowledgeable consumers). Intuitively, the inclusion of enough base consumers dampens the firms' incentive to slash posted prices in an attempt to exploit boundedly rational consumers. Furthermore,

[13] Note that combined with Assumption 1, the inner equilibrium holds for $\beta/[(1-\alpha)(1-\beta)] < (v_a - c_a)/t < 1$. This inequality is true for any $v_a - c_a$ and t that satisfy Assumption 1 if β is small enough and $\alpha < 1$.

[14] Gabaix and Laibson (2006) consider both a shrouded prices equilibrium in which firms hide the add-on fee from boundedly rational consumers and an unshrouded prices equilibrium in which some boundedly rational consumers know about the add-on fee. In their shrouded prices equilibrium, the surplus of consumers who know about the fee increases in the number of myopic consumers; under unshrouded prices equilibrium, this surplus is a constant that is lower than the surplus under the shrouded prices equilibrium.

when there are enough knowledgeable consumers, firms will not set total prices too high, because otherwise they will drive away knowledgeable consumers. The above two dynamics together drive down the add-on price. Hereafter we refer to (4) as the "inner-equilibrium condition" for ease of exposition. Its essence that there should be enough numbers of both base and knowledgeable consumers to ensure inner add-on fees will hold across the more general cases, though the condition will be relatively more involved.

When (4) does not hold, Lemma 1 shows that firms will charge the maximum possible add-on fees, under which total price is independent of β. The equilibrium prices are analogous to the shrouded prices equilibrium in Gabaix and Laibson (2006) and thus do not add new insight.[15] For the rest of this paper, we focus attention on the more novel case in which equilibrium add-on fees are not corner solutions.

The inclusion of consumers who know about the add-on price before base purchase yet still buy the add-on has a notable impact on the welfare of the consumer segments. However, this model feature has no effect on the profit irrelevancy of add-on prices result. Firm profit in either case of Lemma 1 is equal to $t/2$. When add-on pricing is prohibited or infeasible (i.e., $p_{1a} = p_{2a} = 0$), the equilibrium prices are $p_1^{\mathrm{NA}} = p_2^{\mathrm{NA}} = t + c_b + \alpha c_a$, and profit is also $t/2$.[16] In the following subsection, we demonstrate the profit implications of add-on pricing when there is asymmetry between firms in the quality of the base good.

4.2. Asymmetric Base Good Quality and Boundedly Rational Consumers

Our second special case adds quality differentiation on base goods, i.e., $q < 1$. For example, hotels can differ in their star ratings. For ease of exposition, we frequently refer to firm 1 with base good quality 1 as the superior firm and firm 2 with base good quality $q < 1$ as the inferior firm. To isolate the consequences of quality differentiation over base goods, in this special case we consider symmetric quality of the add-on, which is represented by $g = 1$. Add-ons such as Internet access, check cancellation, and text messaging likely fall in this category because their add-on quality is likely irrespective of the quality of the base offering. We impose the following assumption to ensure that each firm will serve all three segments:

Assumption 2. $t > v_a - c_a + (1-q)v_b/3$.

After applying $g = 1$ to Equations (1) and (2), we can then derive the equilibrium posted prices and add-on fees from the first-order conditions of profit expressions in (3). Similar to Lemma 1, we again get mutually exclusive cases and present the conditions leading to the inner equilibrium below.

Lemma 2. *Consider symmetric add-on quality, i.e., $g = 1$. If*

$$\frac{\beta}{(1-\alpha)(1-\beta)} < \frac{1}{t}\left[v_a - c_a - \frac{\beta}{9(1-\beta)-\alpha(9-9\beta+2\beta^2)}(1-q)v_b\right],$$

the firms charge the following posted prices and add-on fees:

$$p_{1b} = c_b + t - \frac{\alpha\beta}{(1-\alpha)(1-\beta)}t + \frac{3(1-\beta)-\alpha(3-2\beta)}{9(1-\beta)-\alpha(9-9\beta+2\beta^2)}(1-q)v_b,$$

$$p_{2b} = c_b + t - \frac{\alpha\beta}{(1-\alpha)(1-\beta)}t - \frac{3(1-\beta)-\alpha(3-2\beta)}{9(1-\beta)-\alpha(9-9\beta+2\beta^2)}(1-q)v_b,$$

$$p_{1a} = c_a + \frac{\beta}{(1-\alpha)(1-\beta)}t + \frac{\beta}{9(1-\beta)-\alpha(9-9\beta+2\beta^2)}(1-q)v_b,$$

$$p_{2a} = c_a + \frac{\beta}{(1-\alpha)(1-\beta)}t - \frac{\beta}{9(1-\beta)-\alpha(9-9\beta+2\beta^2)}(1-q)v_b.$$

The above parametric condition ensuring $p_{ja} < v_a$ in equilibrium for each firm j is analogous to the inner-equilibrium condition in the symmetric case, yet more involved because of asymmetric base qualities.[17] This condition, combined with Assumption 2, ensures that the indifferent consumers in each consumer segment are located in the interior. Also note that full market coverage again requires a high enough qv_b.[18] We again limit our discussion to the more novel case where both firms charge inner add-on fees. Notice that the inner-equilibrium condition holds trivially as the

[15] Classifying $\alpha(1-\alpha)$ as the number of consumers who buy (do not buy) the add-on, the prices in part (ii) of Lemma 1 are equivalent to the prices in Equations (3) and (4) of Gabaix and Laibson (2006).

[16] The equilibrium in the absence of add-on pricing follows straightforwardly from setting $p_{1a} = p_{2a} = 0$ in (3) and solving for the posted prices that simultaneously satisfy each firm's first-order conditions.

[17] Note that firm 1's (firm 2's) prices are increasing (decreasing) in v_b because, as we show in the proof of Lemma 2, conditions $9(1-\beta)-\alpha(9-9\beta+2\beta^2) > 0$ and $3(1-\beta)-\alpha(3-2\beta) > 0$ hold under the inner equilibrium.

[18] We provide proof that it is feasible to have all regularity conditions for the inner equilibrium to hold simultaneously when we analyze the general model in §4.4.

number of boundedly rational consumers approaches zero (because the left-hand side of the inequality approaches zero and the right-hand side is strictly positive by fact that $v_a > c_a$). As such, both the inner-equilibrium condition and Assumption 2 (which does not depend on β) can hold simultaneously.

We focus now on the profit implications of add-on pricing relative to the situation in which add-on pricing is prohibited or infeasible and reserve a discussion of consumer welfare to the presentation of the general model's results. When add-on pricing is prohibited or infeasible, the equilibrium prices are $p_1^{\mathrm{NA}} = t + c_b + \alpha c_a + [(1-q)v_b]/3$ and $p_2^{\mathrm{NA}} = t + c_b + \alpha c_a - [(1-q)v_b]/3$. The equilibrium profit for the superior firm is $\pi_1^{\mathrm{NA}} = (3t + (1-q)v_b)^2/(18t)$, and the equilibrium profit for the inferior firm is $\pi_2^{\mathrm{NA}} = (3t - (1-q)v_b)^2/(18t)$. When add-on pricing is feasible and the parametric conditions lead to the inner equilibrium from Lemma 2, the equilibrium profits are

$$\pi_1^* = \frac{t}{2} + \frac{3(1-\beta) - \alpha(3 - 3\beta + \beta^2)}{9(1-\beta) - \alpha(9 - 9\beta + 2\beta^2)}(1-q)v_b + \frac{(1-\alpha)(1-\beta)}{2(9(1-\beta) - \alpha(9 - 9\beta + 2\beta^2))t}(1-q)^2 v_b^2$$

and

$$\pi_2^* = \frac{t}{2} - \frac{3(1-\beta) - \alpha(3 - 3\beta + \beta^2)}{9(1-\beta) - \alpha(9 - 9\beta + 2\beta^2)}(1-q)v_b + \frac{(1-\alpha)(1-\beta)}{2(9(1-\beta) - \alpha(9 - 9\beta + 2\beta^2))t}(1-q)^2 v_b^2.$$

When all consumers know the add-on prices, the add-on prices are set to marginal cost, and add-on pricing does not affect profit for either firm (i.e., when $\beta = 0$, the above π_j^* is equal to π_j^{NA} for $j = 1, 2$). This result, which holds even for firms who are asymmetric in quality of the base good (i.e., $q < 1$), is consistent with the bulk of add-on pricing research (see Lal and Matutes 1994, Verboven 1999, Gabaix and Laibson 2006). However, the existence of boundedly rational consumers in the context of asymmetric base qualities has unique profit implications.

Proposition 2. *Given the presence of boundedly rational consumers and horizontally differentiated firms that are asymmetric in the quality of the base product but symmetric in the quality of the add-on (i.e., $\beta > 0$, $q < 1$, $g = 1$), add-on pricing by both firms will diminish profit for the superior firm and increase profit for the inferior firm when the conditions lead to the inner equilibrium.*

Proposition 2 is the first finding in the add-on pricing literature to identify opposing effects of add-on pricing on profitability for a superior firm and for an inferior firm. It also demonstrates how add-on pricing with boundedly rational consumers differs from other forms of price discrimination. The finding that the superior firm earns less profit when both firms practice price discrimination via add-on pricing is counter to the finding of Shaffer and Zhang (2002) that the firm with larger loyal following earns greater profit with one-to-one promotions. The finding that the inferior firm can earn greater profit when both firms practice price discrimination via add-on pricing is counter to Chen (2008), who found that the inferior firm is always worse off with price discrimination based on purchase histories. It also contrasts with the finding that price discrimination between consumers who value quality and consumers who do not value quality will lead to lower profits for both the high- and low-quality firms (Corts 1998).

To understand this contrasting result, first consider the fact that $g = 1$ implies the reservation value differential between firms is the same for each type of consumer. Specifically, base consumers who do not buy the add-on experience the same reservation value differential, absent transportation cost, $(1-q)v_b$ as both knowledgeable and boundedly rational consumers because $v_b + v_a - (qv_b + v_a) = (1-q)v_b$. In the absence of the boundedly rational consumer segment, both firms will earn all profit on the base consumers and give the add-on at marginal cost.[19] However, when the boundedly rational consumer segment exists, both firms have an incentive to boost revenue by charging higher add-on fees, which does not affect the base purchase decision by the boundedly rational consumers.[20] As a result, there is greater competitive intensity in posted prices that will attract consumers who will generate further income via add-on purchase. This diminishes profitability on the base consumers and increases profitability on the nonbase consumers. However, the superior firm has a larger number of base consumers to which this diminished profit margin applies. For the superior firm, the loss in profit margin on base consumers outweighs the profit gain from selling the add-on above cost. The opposite is true for the inferior firm. As a consequence, the superior firm loses profit and the inferior firm gains profit with the advent of add-on pricing.

Having identified the implications of a boundedly rational consumer segment and quality asymmetry in the base offering, we next allow for asymmetry in the quality of the add-on. To isolate the effects of add-on asymmetry, in the next subsection we assume all consumers know the add-on price.

[19] This follows straightforwardly from simplifying the prices of Lemma 2 with $\beta = 0$.

[20] Following the proof of Lemma 2, we show that $p_{2a} > c_a$ in the inner equilibrium.

4.3. Asymmetry in Quality of Add-on and Base Good, and No Boundedly Rational Consumers

Asymmetry in the quality of add-on, for instance, the pool at the Best Western relative to the pool at the Hilton, is captured by $g < 1$. The case in which all consumers know the add-on price is represented by $\beta = 0$. After applying $\beta = 0$ to Equations (1) and (2), we can derive the equilibrium posted prices and add-on fees from the first-order conditions of profit expressions in (3). Different from Lemmas 1 and 2, in absence of boundedly rational consumers, the equilibrium always satisfies the inner-equilibrium condition where neither firm charges its maximum add-on fee.

Lemma 3. *In the absence of the boundedly rational consumer segment, i.e., $\beta = 0$, firms charge the following posted prices and add-on fees*:

$$p_{1b} = c_b + t + \tfrac{1}{3}(1-q)v_b, \quad p_{2b} = c_b + t - \tfrac{1}{3}(1-q)v_b,$$
$$p_{1a} = c_a + \tfrac{1}{3}(1-g)v_a, \quad p_{2a} = c_a - \tfrac{1}{3}(1-g)v_a.$$

Proposition 3 below follows directly from an analysis of these prices.

Proposition 3. *Given all consumers know the add-on prices and the horizontally differentiated firms are asymmetric in the quality of the base and the add-on, the inferior firm will not charge separately for the add-on if there is significant asymmetry in the add-on quality (i.e., $g < 1 - 3c_a/v_a$). The superior firm will charge separately for the add-on.*

Proposition 3 shows that the inferior firm may choose not to price discriminate via add-on pricing while the superior firm does price-discriminate via add-on pricing. The result is in contrast to research on rebates which finds that there is no pure strategy equilibrium in which the superior firm price discriminates (via rebates) and the inferior firm does not price discriminate (see Dogan et al. 2010, p. 70). The rebate result is driven by the redemption cost and heterogeneity in price sensitivity, features absent in our model of add-on pricing. Proposition 3 is driven by the increased reservation value differential between firms for consumers who purchase the add-on. The inferior firm chooses a posted price that reflects its quality position on the base good and gives the add-on away to attract its desired number of nonbase consumers for whom the quality differential is more pronounced.

We focus now on the profit implications of add-on pricing relative to the situation in which add-on pricing was prohibited or infeasible. The no-add-on pricing equilibrium is derived and presented in the proof of the following Proposition. Below we compare profit from each case.

Proposition 4. *Given all consumers know the add-on prices and the horizontally differentiated firms are asymmetric in the quality of the base and the add-on (i.e., $\beta = 0$, $q < 1$, $g < 1$), add-on prices charged by both firms will increase profit for the superior firm and diminish profit for the inferior firm.*

Proposition 4 nicely isolates the impact of add-on quality differentiation on the profitability of add-on pricing. In fact, asymmetry in the quality of the add-on has the opposite effects on profitability than the boundedly rational consumers (recall Proposition 2). A notable difference between the cases is that consumers who value the add-on have a higher reservation value differential between firms than the base consumers (i.e., $(1-q)v_a + (1-g)v_b > (1-q)v_a$). Shaffer and Zhang (2002, p. 1153) similarly found that one-to-one promotions can benefit the firm with the larger loyal following and provide a nice explanation of the intuition that also applies here. In contrast to the Shaffer and Zhang (2002) result that requires a sufficient level of vertical differentiation, the add-on pricing result occurs even for minute differences in vertical quality.[21] The reason for this difference is that the increased price competition effect of Shaffer and Zhang (1995, 2002) is absent in a model of add-on pricing with all rational consumers (see Lal and Matutes 1994, Verboven 1999). Therefore, the only effect of add-on pricing with asymmetry in add-on quality is the market-share effect in which the superior firm gains share from being able to price separately to consumers for whom the quality differential between firms is less pronounced (i.e., the base consumers) at the expense of the inferior firm.

Proposition 4 therefore makes two contributions to the literature. It is the first result in the add-on pricing literature to identify a condition for which the superior firm will gain from add-on pricing while the inferior firm suffers. Although the result is similar to the one-to-one promotions result of Shaffer and Zhang (2002), the contrast with Proposition 2 demonstrates that applying the one-to-one promotion predictions to add-on pricing is not a straightforward exercise. Second, relative to the price discrimination literature, it demonstrates that significant differentiation between firms is not necessary for this result to occur.

4.4. General Case: Asymmetric Base and Add-on Qualities and Boundedly Rational Consumers

We next consider the general model by relaxing the assumptions on β, g, and q. Whereas each of the asymmetric firm special cases predict that add-on pricing will improve profit for one firm and diminish

[21] It should be noted that the magnitude of the positive profit effect is increasing in the differentiation.

profit for the other firm, the general case shows an interaction effect of allowing $\beta > 0$ and $g < 1$.

The following assumption ensures that each firm will serve all three segments of consumers:

Assumption 3. $t > v_a - c_a + (2/3)(1-g)v_a + (1/3)(1-q)v_b$.

Similar to Lemmas 1 and 2, we get mutually exclusive cases and list the inner equilibrium below.

Lemma 4. *If*

$$\frac{\beta}{(1-\alpha)(1-\beta)} < \min\left\{ v_a - c_a - \frac{3-\alpha(3-\beta)}{9(1-\beta)-\alpha(9-9\beta+2\beta^2)}(1-g)v_a - \frac{\beta}{9(1-\beta)-\alpha(9-9\beta+2\beta^2)}(1-q)v_b, \; g v_a - c_a + \frac{3-\alpha(3-\beta)}{9(1-\beta)-\alpha(9-9\beta+2\beta^2)}(1-g)v_a + \frac{\beta}{9(1-\beta)-\alpha(9-9\beta+2\beta^2)}(1-q)v_b \right\} \cdot \frac{1}{t},$$

firms charge the following posted prices and add-on fees constituting the inner equilibrium:

$$p_{1b} = c_b + t - \frac{\alpha\beta}{(1-\alpha)(1-\beta)}t + \frac{3(1-\beta)-\alpha(3-2\beta)}{9(1-\beta)-\alpha(9-9\beta+2\beta^2)}(1-q)v_b - \frac{\alpha\beta}{9(1-\beta)-\alpha(9-9\beta+2\beta^2)}(1-g)v_a,$$

$$p_{2b} = c_b + t - \frac{\alpha\beta}{(1-\alpha)(1-\beta)}t - \frac{3(1-\beta)-\alpha(3-2\beta)}{9(1-\beta)-\alpha(9-9\beta+2\beta^2)}(1-q)v_b + \frac{\alpha\beta}{9(1-\beta)-\alpha(9-9\beta+2\beta^2)}(1-g)v_a,$$

$$p_{1a} = c_a + \frac{\beta}{(1-\alpha)(1-\beta)}t + \frac{\beta}{9(1-\beta)-\alpha(9-9\beta+2\beta^2)}(1-q)v_b + \frac{3-\alpha(3-\beta)}{9(1-\beta)-\alpha(9-9\beta+2\beta^2)}(1-g)v_a,$$

$$p_{2a} = c_a + \frac{\beta}{(1-\alpha)(1-\beta)}t - \frac{\beta}{9(1-\beta)-\alpha(9-9\beta+2\beta^2)}(1-q)v_b - \frac{3-\alpha(3-\beta)}{9(1-\beta)-\alpha(9-9\beta+2\beta^2)}(1-g)v_a.$$

The inner-equilibrium condition in Lemma 4 is analogous to the ones in Lemmas 1 and 2, yet its right-hand side is more complicated because of quality asymmetries on both base good and add-on. Yet again, the inner-equilibrium condition implies that $\beta \neq 1$ and is satisfied for lower values of β. Also note that full market coverage requires qv_b to be sufficiently large.[22] In the proof of Lemma 4, we show the inner equilibrium is feasible: Assumption 3, the inner-equilibrium condition, and the full-market-coverage condition can hold simultaneously. We also verify all marginal consumers are located in the interior.

Below we discuss the implications of this general model on firm pricing and profits sequentially. An examination of the prices reveals that firm 1's total price always increases in β under conditions such that the equilibrium is an inner equilibrium. Firm 2's total price in these conditions, however, increases in β if and only if t is above a threshold:

$$t > \frac{(1-\alpha)(1-\beta)^2}{(9(1-\beta)-\alpha(9-9\beta+2\beta^2))^2} \cdot [(9-\alpha(3-2\beta)^2)(1-q)v_b + 3(9-\alpha(9-4\beta))(1-g)v_a]. \tag{5}$$

Therefore, when (5) holds, the findings of Proposition 1 are replicated in the general model.[23]

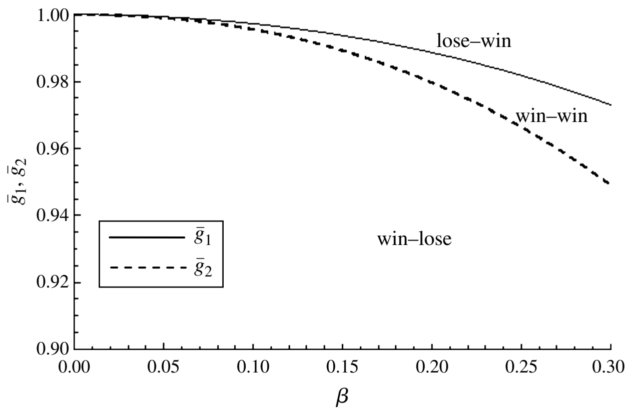
Nevertheless, when t is below the threshold in Equation (5), the effect of β on firm 2's total price is negative. This is because the inferior firm experiences an additional dynamic than discussed earlier in §4.1. Because the superior firm is increasing its total price to get a higher margin out of the boundedly rational consumers (while experiencing a greater market share due to its quality superiority), the inferior firm has an incentive to actually cut its total price because the marginal increment in market share is now more significant. The second dynamic dominates the first dynamic when t is small enough: when t is small, consumers are more price sensitive, and thus the inferior firm can be more effective in acquiring market share by lowering price.

To determine the profit implications of add-on pricing in this general model, we compare the add-on pricing equilibrium with a model in which add-on pricing is prohibited or infeasible. Proposition 5 summarizes the findings.

[22] Full market coverage requires $\min\{((1+q)v_b)/2 - c_b - ((3-3\alpha-3\beta+\alpha\beta)/(2(1-\alpha)(1-\beta)))t, ((1+q)v_b+(1+g)v_a)/2 - c_b - c_a - ((3-\beta)/(2(1-\beta)))t\} \geq 0$.

[23] Analysis of the impact of β on total prices is included at the end of the proof for Lemma 4. To see that the condition in Equation (5) can be satisfied at the inner equilibrium, observe that the right-hand side of the inequality is bounded as β approaches zero. Therefore, the inner-equilibrium condition and the equality of Equation (5) can be satisfied for low values of β.

Figure 2 Comparisons of Profits with Add-on Pricing to No-Add-on Pricing



Proposition 5. *With the presence of boundedly rational consumers and asymmetry in the quality of the base good and the add-on, add-on pricing improves combined firm profit when conditions lead to the inner equilibrium. Furthermore, there exists $\bar{g}_2 < \bar{g}_1 < 1$ such that,*

(i) (*lose–win*) *if the level of asymmetry in add-on quality is low, i.e., $g > \bar{g}_1$, add-on pricing by both firms will diminish profit for the superior firm and increase profit for the inferior firm;*

(ii) (*win–win*) *if the level of asymmetry in add-on quality is medium, i.e., $\bar{g}_1 > g > \bar{g}_2$, add-on pricing by both firms will increase profits for both firms;*

(iii) (*win–lose*) *if the level of asymmetry in add-on quality is high, i.e., $g < \bar{g}_2$, add-on pricing by both firms will increase profit for the superior firm and diminish profit for the inferior firm.*

Notice that both $\bar{g}_1$ and $\bar{g}_2$ converge from below to 1 when β converges to 0. Thus, given a small enough β, each of the three cases of Proposition 5 (lose–win, win–win, and win–lose) can exist for some according values of g. When β is large, however, the existence of all three cases is not guaranteed: for instance, the win–lose case may not exist with large β (i.e., $\bar{g}_2$ may fall out of the feasible parameter set).

We use Figure 2 to illustrate the findings in Proposition 5. Figure 2 shows lose–win, win–win, and win–lose regions in the $\beta - g$ plane for parameter values $\alpha = 0.25$, $t = 1$, $q = 0.9$, $v_a = 0.9$, $v_b = 2$, $c_a = 0.1$, and $c_b = 0.1$. We have verified that Assumption 3, the condition for the inner equilibrium, and the condition for full market coverage are satisfied for the full range presented. Part (i) of Proposition 5 (illustrated by the lose–win region in Figure 2) is consistent with Proposition 2, and part (iii) of Proposition 5 (illustrated by the win–lose region in Figure 2) is consistent with Proposition 4. Part (ii) of Proposition 5 shows a unique win–win result of the general model: when the quality differentiation on the add-on is neither too small nor too large and a segment of consumers is boundedly rational regarding add-on prices, it is possible for both firms to benefit from add-on pricing (compared to when add-on pricing is prohibited or infeasible).

The remaining question to be answered is why the combination of model features in the general case uniquely leads to a win–win, whereas the special cases lead to profit irrelevance, lose–win, or win–lose. As summarized in Table 3, there are three generalizations considered ($q < 1, \beta > 0, g < 1$), and a combination of at least two of the three is necessary for the profit-irrelevance result to be overturned. For each of the subcases that overturn the profit-irrelevance result, total industry profit is greater with add-on pricing than without. The parameters then determine the allocation between the two firms of this gain. Ceteris paribus, the superior (inferior) firm is worse (better) off with add-on pricing in the presence of boundedly rational consumers (see Proposition 2). Ceteris paribus, the superior (inferior) firm is better (worse) off with add-on pricing if the add-on quality is also superior (see Proposition 4). The combination of these features implies both positive and negative effects of add-on pricing for each firm. It is possible, then, that the positive effect outweighs the negative effect for *both* firms, and each enjoys a share of the industry's gain.

Table 3 Profit Implications of Boundedly Rational Consumers, Base Quality Asymmetry, and Add-on Quality Asymmetry

No boundedly rational consumers ($\beta = 0$)		Boundedly rational consumers ($\beta > 0$)	
	Profit implications of add-on pricing		Profit implications of add-on pricing
Symmetric firms ($\beta = 0, g = 1, q = 1$)	Profit irrelevance	Symmetric firms ($\beta > 0, g = 1, q = 1$)	Profit irrelevance
Asymmetric base goods ($\beta = 0, g = 1, q < 1$)	Profit irrelevance	Asymmetric base goods ($\beta > 0, g = 1, q < 1$)	lose–win
Asymmetric base goods, asymmetric add-ons ($\beta = 0, g < 1, q < 1$)	win–lose	Asymmetric base goods, asymmetric add-ons ($\beta > 0, g < 1, q < 1$)	win–win, lose–win, or win–lose

5. Discussion

This paper extends prior work on add-on pricing in two dimensions. First, it allows for asymmetry in the quality of the base good and in the quality of the add-on. Second, it accommodates three segments of consumers: base consumers who do not derive utility from the add-on, knowledgeable consumers who value the add-on and know the add-on fee, and boundedly rational consumers who value the add-on yet do not take the add-on price into consideration at the time of initial purchase. Each of these model considerations has substantive implications and reverses predictions from prior research.

Specifically, previous research in add-on pricing predicts that consumers who account for the add-on price in their base purchase decision will enjoy greater surplus when there are more boundedly rational consumers in the marketplace. However, we show that this is not the case when there are enough base and knowledgeable consumers who the firms serve. In this case, an increasing number of boundedly rational consumers results in lower base prices and higher add-on fees. We find the add-on fee increase dominates the base price reduction for both firms when the quality differential between the firms is not too large. Consequently, a greater number of boundedly rational consumers can lead to increased total prices and lower consumer surplus for the boundedly rational consumers and the knowledgeable consumers.

Our consideration of quality asymmetry (along with the inclusion of boundedly rational consumers) leads to new findings regarding firm profits. Three specific features are discussed: quality asymmetry in the base good, quality asymmetry in the add-on, and the presence of boundedly rational consumers. Strikingly, the standard profit-irrelevancy result of add-on pricing is robust to any one of these three features, but is challenged in the presence of two or more of the features. To the best of our knowledge, we are the first to demonstrate how a form of price discrimination can lead to diminished profit for the superior firm and enhanced profit for the inferior firm. This occurs when there is asymmetry only in the quality of the base good and there exist boundedly rational consumers. This finding is preserved when there is a moderate level of asymmetry in the quality of the add-on.

We also find when the superior firm will gain from add-on pricing and the inferior will suffer from add-on pricing. This occurs when there is asymmetry in both the quality of the add-on and the quality of the base offering. It is preserved with a moderate level of boundedly rational consumers. Although the finding is new to the add-on pricing literature, it is consistent with the price discrimination literature on one-to-one promotions. However, the analogous result in the price discrimination literature requires a high level of vertical differentiation relative to horizontal differentiation, whereas our result occurs for even the smallest differences in vertical quality.

In contrast to the one-to-one promotions research, we also establish when add-on pricing will increase profit for both the superior firm and the inferior firm. The presence of boundedly rational consumers combined with sufficient asymmetry in both the add-on and base good will lead both the inferior firm and the superior firm to gain from add-on pricing. Whereas previous add-on pricing research has found a win–win associated with add-on pricing only when consumers who value the add-on are less price sensitive than other consumers, our result occurs even without this. Thus, we identify new conditions for when add-on pricing can increase each firm's profit.

This research can be extended in several directions. First, the current model does not allow firms to commit to a pricing policy—e.g., credibly committing to free add-on pricing. Such a commitment would be interesting to study. Second, we have assumed that consumers are homogeneous in their appreciation of quality. An interesting direction for future research is to study the effect of heterogeneity in quality preferences and allow the heterogeneity to be correlated with the willingness to pay for the add-on. Third, an avenue for future research is to nest the current model in a customer lifetime value framework where customers are repeat buyers. Such a model could allow boundedly rational consumers to punish firms that charge add-on prices by switching to competitors. The add-on prices by each firm, and the feasibility of equilibrium add-on pricing in general, would depend on the firm's discount rate on future payoffs, the size of each consumer segment, and the manner in which consumer beliefs are based on purchase history. Such a dynamic model would be interesting to consider, and not considering it is a limitation of this research.[24]

In summary, our paper is the first to examine add-on pricing with three segments of consumers and with asymmetric firms. The joint consideration of these features leads to unique predictions. As such, the insights gained from the model contribute to the theories of both add-on pricing and price discrimination.

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Appendix A. Proof of Lemmas and Propositions

Proof of Lemma 1. First consider the case where $\beta/[(1-\alpha)(1-\beta)] < (v_a - c_a)/t$—which we refer to as the "inner-equilibrium condition"—holds. Plug $q=1$ and $g=1$ into Equations (1) and (2), then solve the following first-order conditions simultaneously: $\partial\pi_j/\partial p_{jb}=0$ and $\partial\pi_j/\partial p_{ja}=0$, $j=1,2$. We get posted prices $p_{1b}=p_{2b}=c_b+t-[\alpha\beta/((1-\alpha)(1-\beta))]t$ and add-on fees $p_{1a}=p_{2a}=c_a+[\beta/((1-\alpha)(1-\beta))]t$. For these prices to constitute an equilibrium, we need all following four conditions to hold: (1) $p_{ja}<v_a$ for any j, (2) second-order conditions hold at the equilibrium prices, (3) no firm will deviate by abandoning knowledgeable consumers, and (4) full market coverage. Notice that (1) is apparently true under the inner-equilibrium condition, and (4) holds if v_b is sufficiently large (as we assumed). We will prove that (2) and (3) hold in the proof of the general case (Lemma 4) and thus they also hold in this specific case.

Next consider the case where the $\beta/[(1-\alpha)(1-\beta)] \geq (v_a-c_a)/t$. The above inner equilibrium cannot hold now because the add-on fee would surpass v_a (implying no consumer would purchase the add-on). To show that parametric conditions can lead to the corner equilibrium (as specified in part (ii) of this Lemma), we need the following two conditions to hold at equilibrium prices: (1) $\partial\pi_j/\partial p_{jb}=0$ and (2) $\partial\pi_j/\partial p_{ja}>0$ for $j=1,2$. Without loss of generality, we only consider $j=1$. Conditional on $p_{2b}=c_b+t-\alpha\cdot(v_a-c_a)$ and $p_{2a}=v_a$, we have

$$\begin{aligned}\pi_1 &= [\alpha(p_{1a}-c_a)(2t-p_{1b}+c_b-\alpha(v_a-c_a)+(1-\beta)(v_a-p_{1a}))\\ &\quad +(p_{1b}-c_b)(2t-p_{1b}+c_b-\alpha(p_{1a}-c_a)-\alpha\beta(v_a-p_{1a}))]/(2t).\end{aligned}$$

(1) It is straightforward to verify that $\partial\pi_1/\partial p_{1b}=0$ at $p_{1b}=c_b+t-\alpha(v_a-c_a)$ and $p_{1a}=v_a$.

(2) $\partial\pi_j/\partial p_{ja}=\alpha(t\beta-(1-\alpha)(1-\beta)(v_a-c_a))/(2t)$, which is positive if the inner-equilibrium condition does not hold. Thus, the constraint that $p_{1a}\leq v_a$ for the add-on fee to be relevant is binding. Q.E.D.

Proof of Lemmas 2–4. The flow of this proof is analogous to that of Lemma 1, albeit more involved because of quality asymmetry. We prove that the general case (see Lemma 4) and the findings of Lemmas 2 and 3 arise from the appropriate simplifications.

Suppose the inner equilibrium exists (we will verify its regularity conditions shortly). Given profit functions in (3), we solve first-order conditions and get equilibrium prices as presented in Lemma 4. These prices simplified at $g=1$ yield the prices in Lemma 2, whereas simplification of $\beta=1$ yields the prices in Lemma 3. For these prices to constitute an inner equilibrium, we need all of the following conditions to hold: (1) $p_{1a}<v_a$ and $p_{2a}<gv_a$, (2) second-order conditions hold, (3) no firm deviates by abandoning knowledgeable consumers, (4) full market coverage, and (5) marginal consumers between 0 and 1. We will also show that all regularity conditions can hold simultaneously.

From (1) we get the inner-equilibrium condition that guarantees $p_{2a}<gv_a$, $p_{1a}<v_a$ in equilibrium:

$$\begin{aligned}&\frac{\beta}{(1-\alpha)(1-\beta)}\\ &\quad<\min\left\{v_a-c_a-\frac{(3-\alpha(3-\beta))(1-g)}{9(1-\beta)-\alpha(3-\beta)(3-2\beta)}v_a\right.\\ &\qquad-\frac{\beta(1-q)}{9(1-\beta)-\alpha(3-\beta)(3-2\beta)}v_b,\\ &\qquad gv_a-c_a+\frac{(3-\alpha(3-\beta))(1-g)}{9(1-\beta)-\alpha(3-\beta)(3-2\beta)}v_a\\ &\qquad\left.+\frac{\beta(1-q)}{9(1-\beta)-\alpha(3-\beta)(3-2\beta)}v_b\right\}\cdot\frac{1}{t}.\end{aligned}$$

The inner-equilibrium condition in Lemma 2 is derived from setting $g=1$ in the above inequality. At the prices in Lemma 3, $p_{1a}<v_a$ and $p_{2a}<gv_a$ by fact that add-on value is greater than its cost: $gv_a>c_a$.

In the subsequent analysis we will frequently use the fact that $\beta/((1-\alpha)(1-\beta))<1$, which directly follows the inner-equilibrium condition and Assumption 3. We will also frequently use the expression $9(1-\beta)-\alpha(3-\beta)(3-2\beta)$, or, equivalently, $9(1-\beta)-\alpha(9-9\beta+2\beta^2)$, which is positive because it equals $9(1-\alpha)(1-\beta)-2\alpha\beta^2>9\beta-2\alpha\beta^2>0$.

For (2) to be true, we need the Hessian determinant $\partial^2\pi_j/\partial p_{jb}\partial p_{jb}\cdot\partial^2\pi_j/\partial p_{ja}\partial p_{ja}-\partial^2\pi_j/\partial p_{jb}\partial p_{ja}\cdot\partial^2\pi_j/\partial p_{ja}\partial p_{jb}>0$, i.e., $\alpha\beta^2<4(1-\alpha)(1-\beta)$. This inequality always holds because $\beta/((1-\alpha)(1-\beta))<1$.

We prove (3) by contradiction. Assume that firm 2 prefers to deviate from this equilibrium by abandoning knowledgeable consumers (and thus will charge the maximum possible add-on fee). Given p_{1b} and p_{1a} as specified in this lemma and $p_{2a}=v_a$, by maximizing firm 2's profit over base and boundedly rational consumers combined (i.e., $\pi_2=(p_{1b}-c_b)(1-\theta^b+1-\theta^{br})+(v_a-c_a)(1-\theta^{br})$), we get the following posted price for firm 2:

$$\begin{aligned}p_{2b}&=c_b+\frac{1-\beta-\alpha(1-\beta/2)}{(1-\alpha)(1-\beta)}t\\ &\quad-\frac{3(1-\beta)-\alpha(3-7\beta/2+\beta^2)}{9(1-\beta)-\alpha(9-9\beta+2\beta^2)}(1-q)v_b\\ &\quad-\frac{\alpha\beta}{2(1-\alpha(1-\beta))}(v_a-c_a)\\ &\quad-\frac{\alpha\beta(10-9\beta-2\alpha(5-(5-\beta)\beta))}{2(1-\alpha(1-\beta))(9(1-\beta)-\alpha(3-\beta)(3-2\beta))}(1-g)v_a.\end{aligned}$$

A contradiction arises if, given the above prices, the knowledgeable consumer indifferent between the two firms is located within the linear city. We have $\theta^k<1$ when plugging the above prices into $\theta^k=1/2+((1-q)v_b+(1-g)v_a)/(2t)+(p_{2b}+p_{2a}-p_{1b}-p_{1a})/(2t)$ (thus the contradiction) if and only if

$$\begin{aligned}t&>\frac{(1-\alpha)(1-\beta)}{1-\alpha(1-\beta)}(v_a-c_a)\\ &\quad+\frac{(1-\alpha)(1-\beta)(6-8\beta-\alpha(2-\beta)(3-2\beta))}{(2-\alpha(2-\beta))(9(1-\beta)-\alpha(3-\beta)(3-2\beta))}(1-q)v_b\\ &\quad+\big[(1-\alpha)(1-\beta)\big[6(2-3\beta)-\alpha(2-\beta)(12-13\beta)\\ &\quad+2\alpha^2(6-\beta(10-(6-\beta)\beta))\big]\big]\end{aligned}$$

$$\cdot [(2-\alpha(2-\beta))(1-\alpha(1-\beta))$$
$$\cdot (9(1-\beta)-\alpha(3-\beta)(3-2\beta))]^{-1}(1-g)v_a.$$

Each of the three items on the right-hand side are decreasing functions of β. The first term is apparently decreasing in β. The second, $(6-8\beta-\alpha(2-\beta)(3-2\beta))/(9(1-\beta)-\alpha(3-\beta)\cdot(3-2\beta))$, or equivalently, $1-(3(1-\alpha)-\beta+2\alpha\beta)/(9(1-\beta)-\alpha(3-\beta)(3-2\beta))$, is decreasing in β because

$$\partial\left(\frac{3(1-\alpha)-\beta+2\alpha\beta}{9(1-\beta)-\alpha(3-\beta)(3-2\beta)}\right)\Big/\partial\beta = \frac{9(1-\alpha)(2-\alpha)+2\alpha\beta((6-\beta)\cdot(1-\alpha)+\alpha\beta)}{(9(1-\beta)-\alpha(3-\beta)(3-2\beta))^2} > 0.$$

We next show that the third item is also decreasing in β. Specifically, because $(1-\alpha)(1-\beta)/(2-\alpha(2-\beta))$ is decreasing in β, we only need to show that

$$\frac{6(2-3\beta)-\alpha(2-\beta)(12-13\beta)+2\alpha^2(6-\beta(10-(6-\beta)\beta))}{((1-\alpha(1-\beta))(9(1-\beta)-\alpha(3-\beta)(3-2\beta)))}$$

or

$$1+\frac{1-\alpha}{1-\alpha(1-\beta)}-\frac{6-\alpha(6-\beta)}{9(1-\beta)-\alpha(3-\beta)(3-2\beta)}$$

is decreasing in β. For this we only need to show that $(6-\alpha(6-\beta))/(9(1-\beta)-\alpha(3-\beta)(3-2\beta))$ is increasing in β. Defining $y(\alpha,\beta)=54+\alpha(-99+24\beta+\alpha(45+2(-12+\beta)\beta))$, we have

$$\partial\left(\frac{6-\alpha(6-\beta)}{9(1-\beta)-\alpha(3-\beta)(3-2\beta)}\right)\Big/\partial\beta = y(\alpha,\beta)/(9(1-\beta)-\alpha(3-\beta)(3-2\beta))^2.$$

Therefore, we only need to show that $y(\alpha,\beta)>0$. It is straightforward to verify that $\partial y(\alpha,\beta)/\partial\alpha<0$; therefore, $y(\alpha,\beta)>y(1,\beta)=2\beta^2>0$.

Therefore, if the above inequality holds at $\beta=0$, it will hold for any positive β. At $\beta=0$, the above inequality simplifies to $t>v_a-c_a+(2/3)(1-g)v_a+(1/3)(1-q)v_b$, which is our Assumption 3. Thus the contradiction. For $g=1$, the inequality simplifies to Assumption 2. The analysis of firm 1 abandoning knowledgeable consumers is analogous and omitted.

For (4), we now derive the full-market-coverage condition. The market is fully covered in equilibrium if the marginal consumer in each segment who is indifferent between purchasing product 1 and 2 derives a nonnegative utility, i.e., the following three inequalities hold: $v_b-p_{1b}-\theta^b t>0$, $v_b+v_a-p_{1b}-\theta^{br}t>0$, and $v_b+v_a-p_{1b}-p_{1a}-\theta^k t>0$. Plugging equilibrium prices into these inequalities (and afterward it is straightforward that the second inequality will hold if the first one holds), we can simplify the first and the third inequalities to the following, respectively: $u_{\theta b}\triangleq((1+q)v_b)/2-c_b-((3-3\alpha-3\beta+\alpha\beta)/(2(1-\alpha)(1-\beta)))t\geq 0$ and $u_{\theta k}\triangleq[(1+q)v_b+(1+g)v_a]/2-c_b-c_a-[(3-\beta)/(2(1-\beta))]t\geq 0$. Therefore, full market coverage is true if $\min\{u_{\theta b},u_{\theta k}\}\geq 0$—we refer to this as the full-market-coverage condition.

For (5), we simplify the location of the marginal consumer indifferent between firm 1 and firm 2 at the equilibrium prices for each segment:

$$\begin{aligned}
\theta^b &= \frac{1}{2}+\frac{\alpha\beta}{t(9(1-\beta)-\alpha(3-\beta)(3-2\beta))}(1-g)v_a\\
&\quad+\frac{(1-\beta)(3-\alpha(3-2\beta))}{2t(9(1-\beta)-\alpha(9-9\beta+2\beta^2))}(1-q)v_b,\\
\theta^{br} &= \frac{1}{2}+\frac{(1-\beta)(9-\alpha(9-2\beta))}{2t(9(1-\beta)-\alpha(3-\beta)(3-2\beta))}(1-g)v_a\\
&\quad+\frac{(1-\beta)(3-\alpha(3-2\beta))}{2t(9(1-\beta)-\alpha(3-\beta)(3-2\beta))}(1-q)v_b,\\
\theta^k &= \frac{1}{2}+\frac{3-9\beta-\alpha(3-\beta(9-2\beta))}{2t(9(1-\beta)-\alpha(3-\beta)(3-2\beta))}(1-g)v_a\\
&\quad+\frac{3-5\beta-\alpha(1-\beta)(3-2\beta)}{2t(9(1-\beta)-\alpha(3-\beta)(3-2\beta))}(1-q)v_b.
\end{aligned}$$

It can be shown that these values are each between 0 and 1 given Assumption 3 and the inner-equilibrium condition. We describe the steps to prove this here, with detailed algebra available from the authors. Notice that $\theta^{br}-\theta^b=(1-g)v_a/(2t)\geq 0$ and $\theta^{br}-\theta^k=((1-g)(3+\alpha(-3+\beta))v_a+(1-q)\beta v_b)/(t(9(1-\beta)-\alpha(3-\beta)(3-2\beta)))\geq 0$. Thus if $\theta^{br}<1$, then $\max\{\theta^b,\theta^k\}<1$. Let $t'\equiv v_a-c_a+(2/3)(1-g)v_a+(1/3)(1-q)v_b+\varepsilon$, where $\varepsilon>0$ to satisfy Assumption 3. Simplify the inner-equilibrium condition at $t=t'$. Rearrange this condition to be written as an expression $\mathrm{A}>0$. Simplify $2t(1-\theta^{br})$ at $t=t'$. Call this expression BR. If $\mathrm{BR}-\mathrm{A}>0$, then $(1-\theta^{br})>0$. Collecting terms on v_a and v_b, the intercept is positive, the coefficient on v_a is positive based on prior proofs, and the coefficient on v_b is positive (the denominator is positive by prior proofs, and the numerator is decreasing in α and positive at $\alpha=1$).

To show $\min\{\theta^b,\theta^{br},\theta^k\}>0$, notice that $\theta^b=1/2$ at $q=1$, $g=1$ and is decreasing in g and q. Thus $\theta^b>0$, and hence $\theta^{br}>0$. To prove $\theta^k>0$, simplify $2t\theta^k$ at $t=t'$ and label it expression K. If $\mathrm{K}-\mathrm{A}>0$ then $\theta^k>0$. Collecting terms on v_a and v_b, the intercept is positive. To show that the coefficient on v_a is positive, notice that the denominator is positive and the numerator is decreasing in α and positive at $\alpha=1$. To show the coefficient on v_a is positive, notice that the denominator is always positive, and the minimum of the numerator is at the constraint boundary (i.e., $(1-\alpha)=\beta/(1-\beta)$) and is positive at this point.

We next show that all regularity conditions for the inner equilibrium in this general model are feasible to hold simultaneously: the full-market-coverage condition; Assumption 3; and the inner-equilibrium condition. We simplify this feasibility proof using the following fact: all sides of all the above conditions are continuous functions of q. Therefore, if all three conditions can hold *strictly* at $q=1$, it must be true that they will also hold when q is close enough to one. Applying $q=1$ to the above three regularity conditions, they are simplified to

$$\min\left\{v_b-c_b-\frac{3-3\alpha-3\beta+\alpha\beta}{2(1-\alpha)(1-\beta)}t,\right.$$
$$\left.v_b-c_b+\frac{(1+g)v_a}{2}-c_a-\frac{3-\beta}{2(1-\beta)}t\right\}\geq 0,$$
$$t>v_a-c_a+\frac{2}{3}(1-g)v_a,$$

$$\frac{\beta}{(1-\alpha)(1-\beta)}<\min\left\{v_a-c_a-\frac{(3-\alpha(3-\beta))(1-g)}{9(1-\beta)-\alpha(3-\beta)(3-2\beta)}v_a,\right.$$
$$\left. gv_a-c_a+\frac{(3-\alpha(3-\beta))(1-g)}{9(1-\beta)-\alpha(3-\beta)(3-2\beta)}v_a\right\}\cdot\frac{1}{t}.$$

Only the first condition involves v_b. When $q=1$ and for any given values of all other model parameters, this first condition holds if v_b is large enough. When $q=1$, the second condition holds if v_a is sufficiently small relative to t. For the third condition, notice that as β goes to zero, the left-hand side goes to zero, and the right-hand side is strictly positive (by fact that $gv_a>c_a$). Therefore, when $q=1$, and given values of all model parameters except for β, this third condition holds if β is low enough.

Given that all sides of all three regularity conditions for this general model are continuous functions of q, we can now conclude that all regularity conditions for the existence of the inner equilibrium will hold simultaneously when (i) q is large enough, (ii) v_b is sufficiently large relative to t, (iii) v_a is sufficiently small relative to t, and (iv) β is low enough.

Finally we show how each firm's total price changes in β:

$$\frac{\partial(p_{1b}+p_{1a})}{\partial\beta}=\frac{t}{(1-\beta)^2}+\big[(1-\alpha)\big[3(9-\alpha(9-4\beta))(1-g)v_a+(9-\alpha(3-2\beta)^2)(1-q)v_b\big]\big]\cdot[(9(1-\beta)-\alpha(9-9\beta+2\beta^2))^2]^{-1}>0,$$

$$\frac{\partial(p_{2b}+p_{2a})}{\partial\beta}=\frac{t}{(1-\beta)^2}-\big[(1-\alpha)\big[3(1-g)(9-\alpha(9-4\beta))v_a+(1-q)(9-\alpha(3-2\beta)^2)v_b\big]\big]\cdot[(9(1-\beta)-\alpha(9-9\beta+2\beta^2))^2]^{-1}.$$

This last expression is positive if and only if condition (5) in the main text holds. Q.E.D.

Proof of Two Claims Following the Discussion of Lemma 2. The first claim is that firm 1's (firm 2's) prices are increasing (decreasing) in v_b. We already know $9(1-\beta)-\alpha(9-9\beta+2\beta^2)>0$. From $\beta/((1-\alpha)(1-\beta))<1$ we have $3(1-\beta)-\alpha(3-2\beta)=3(1-\alpha)(1-\beta)-\alpha\beta>3\beta-\alpha\beta>0$. Hence the claim is true.

The second claim is that at $g=1$, add-on prices exceed add-on cost. This is apparent for p_{1a}. To show $p_{2a}>c_a$, we only need to show that

$$\frac{t}{(1-\alpha)(1-\beta)}>\frac{(1-q)v_b}{9(1-\beta)-\alpha(9-9\beta+2\beta^2)}.$$

Because $t>v_a-c_a+(1-q)v_b/3>(1-q)v_b/3$, we only need to show that $3(1-\alpha)(1-\beta)<9(1-\beta)-\alpha(9-9\beta+2\beta^2)$, or $6(1-\alpha)(1-\beta)-2\alpha\beta^2>0$. The last inequality is true from $\beta/((1-\alpha)(1-\beta))<1$. Q.E.D.

Proof of Proposition 2. From the equilibrium profit expressions of §4.2, the firms' profit differentials between when add-on pricing is feasible and when add-on pricing is prohibited or infeasible are

$$\pi_1-\pi_1^{\mathrm{NA}}=-\frac{\alpha\beta^2(1-q)v_b(3t-(1-q)v_b)}{9t(9(1-\beta)+\alpha(3-\beta)(3-2\beta))}<0\quad\text{and}$$
$$\pi_2-\pi_2^{\mathrm{NA}}=\frac{\alpha\beta^2(1-q)v_b(3t+(1-q)v_b)}{9t(9(1-\beta)-\alpha(3-\beta)(3-2\beta))}>0.$$

It should be noted that the pricing equilibrium without add-on fees results in $q_{2b}^{\mathrm{NA}}=(3t-(1-q)v_b)/(6t)$. Thus, $3t-(1-q)v_b>0$ is a necessary condition for both firms to sell to consumers when add-on pricing is prohibited or infeasible. Q.E.D.

Proof of Proposition 3. Firm 2's add-on price from Lemma 3 is $p_{2a}=c_a-(1-g)v_a/3>0$ if and only if $g>1-3c_a/v_a$. Firm 1's add-on is $p_{1a}=c_a+(1-g)v_a/3>0$ for any $v_a>0$. Q.E.D.

Proof of Proposition 4. In this special case, $\beta=0$. As a benchmark, we first derive firm prices and profits when add-on pricing is prohibited or infeasible. Given zero add-on fees, by maximizing profits in Equation (3) using the first-order conditions, we get prices as follows:

$$p_1^{\mathrm{NA}}=t+c_b+\alpha c_a+(\alpha(1-g)v_a+(1-q)v_b)/3\quad\text{and}$$
$$p_2^{\mathrm{NA}}=t+c_b+\alpha c_a-(\alpha(1-g)v_a+(1-q)v_b)/3.$$

Consequently, when add-on pricing is infeasible, equilibrium profits for the superior firm and inferior firm, respectively, are $\pi_1^{\mathrm{NA}}=[\alpha^2(1-g)^2v_a^2+(3t+(1-q)v_b)^2-\alpha(9(1-\alpha)\cdot c_a-2(3t+(1-q)v_b))(1-g)v_a]/(18t)$ and $\pi_2^{\mathrm{NA}}=[\alpha^2(1-g)^2\cdot v_a^2+(3t-(1-q)v_b)^2+\alpha(9(1-\alpha)c_a-2(3t-(1-q)v_b))\cdot(1-g)v_a]/(18t)$.

Now consider the inner equilibrium under add-on pricing from Lemma 3. Firm profits are

$$\pi_1=[(1-g)^2\alpha v_a^2+2(1-g)\alpha v_a(3t+(1-q)v_b)+(3t+(1-q)v_b)^2]/(18t)\quad\text{and}$$
$$\pi_2=[(1-g)^2\alpha v_a^2-2(1-g)\alpha v_a(3t-(1-q)v_b)+(3t-(1-q)v_b)^2]/(18t).$$

Therefore, the firms' profit differentials between when add-on pricing is feasible and when add-on pricing is prohibited or infeasible are

$$\pi_1-\pi_1^{\mathrm{NA}}=[(1-g)(1-\alpha)\alpha v_a(9c_a+(1-g)v_a)]/(18t)>0\quad\text{and}$$
$$\pi_2-\pi_2^{\mathrm{NA}}=-[(1-g)(1-\alpha)\alpha v_a(9c_a-(1-g)v_a)]/(18t)<0.$$

The last inequality holds whenever the inferior firm charges for the add-on (i.e., $g>1-3c_a/v_a$). Q.E.D.

Proof of Proposition 5. We first derive equilibrium profits when add-on pricing is infeasible. Setting $p_{2a}=p_{1a}=0$ and maximizing profits in Equation (3) using the first-order conditions, prices are as follows:

$$p_1^{\mathrm{NA}}=t+c_b+\alpha c_a+(\alpha(1-g)v_a+(1-q)v_b)/3\quad\text{and}$$
$$p_2^{\mathrm{NA}}=t+c_b+\alpha c_a-(\alpha(1-g)v_a+(1-q)v_b)/3.$$

Consequently, when add-on pricing is prohibited or infeasible, equilibrium profit for the superior firm is $\pi_1^{\mathrm{NA}}=[\alpha^2(1-g)^2v_a^2+(3t+(1-q)v_b)^2-\alpha(9(1-\alpha)c_a-2(3t+(1-q)v_b))(1-g)v_a]/(18t)$, and equilibrium profit for the inferior firm is $\pi_2^{\mathrm{NA}}=[\alpha^2(1-g)^2v_a^2+(3t-(1-q)v_b)^2+\alpha(9(1-\alpha)c_a-2(3t-(1-q)v_b))(1-g)v_a]/(18t)$.

Now consider the inner equilibrium under add-on pricing. From Equation (3) and equilibrium prices as shown in Lemma 4, we can derive firm profits under add-on pricing as follows:

$$\begin{aligned}\pi_1 = {}& \frac{t}{2} + \frac{(1-g)^2(1-\alpha)\alpha v_a^2}{2t(9(1-\beta)-\alpha(9-9\beta+2\beta^2))}\\ &+ \frac{(1-q)(3(1-\beta)-\alpha(3-3\beta+\beta^2))v_b}{9(1-\beta)-\alpha(9-9\beta+2\beta^2)}\\ &+ \frac{(1-q)^2(1-\alpha)(1-\beta)v_b^2}{2t(9(1-\beta)-\alpha(9-9\beta+2\beta^2))}\\ &+ v_a\bigg(\frac{(1-g)\alpha(6-5\beta-\beta^2-\alpha(6-5\beta+\beta^2))}{2(1-\beta)(9(1-\beta)-\alpha(9-9\beta+2\beta^2))}\\ &\qquad + \frac{(1-g)(1-q)(1-\alpha)\alpha(2-\beta)v_b}{2t(9(1-\beta)-\alpha(9-9\beta+2\beta^2))}\bigg),\end{aligned}$$

$$\begin{aligned}\pi_2 = {}& \frac{t}{2} + \frac{(1-g)^2(1-\alpha)\alpha v_a^2}{2t(9(1-\beta)-\alpha(9-9\beta+2\beta^2))}\\ &- \frac{(1-q)(3(1-\beta)-\alpha(3-3\beta+\beta^2))v_b}{9(1-\beta)-\alpha(9-9\beta+2\beta^2)}\\ &+ \frac{(1-q)^2(1-\alpha)(1-\beta)v_b^2}{2t(9(1-\beta)-\alpha(9-9\beta+2\beta^2))}\\ &+ v_a\bigg(-\frac{(1-g)\alpha(6-5\beta-\beta^2-\alpha(6-5\beta+\beta^2))}{2(1-\beta)(9(1-\beta)-\alpha(9-9\beta+2\beta^2))}\\ &\qquad + \frac{(1-g)(1-q)(1-\alpha)\alpha(2-\beta)v_b}{2t(9(1-\beta)-\alpha(9-9\beta+2\beta^2))}\bigg).\end{aligned}$$

We first verify that the combined firm profits are always higher when add-on pricing is feasible than profits, when add-on pricing is prohibited or infeasible:

$$\begin{aligned}&\pi_1+\pi_2-\pi_1^{\mathrm{NA}}-\pi_2^{\mathrm{NA}}\\ &\quad = (\alpha[(1-g)^2(3-\alpha(3-\beta))(3-\alpha(3-2\beta))v_a^2\\ &\qquad + (1-g)(1-q)\beta(9-9\alpha+4\alpha\beta)v_a v_b + 2(1-q)^2\beta^2 v_b^2])\\ &\qquad \cdot [9t(9(1-\beta)-\alpha(3-\beta)(3-2\beta))]^{-1} > 0.\end{aligned}$$

Now consider firm 1's profit differential between add-on pricing being feasible and infeasible:

$$\begin{aligned}&\pi_1-\pi_1^{\mathrm{NA}}\\ &\quad = \frac{\alpha}{18t(9(1-\beta)-\alpha(3-\beta)(3-2\beta))}\\ &\qquad \cdot \bigg\{(1-g)^2(3-\alpha(3-\beta))(3-\alpha(3-2\beta))v_a^2\\ &\qquad\quad - 2(1-q)\beta^2 v_b(3t-(1-q)v_b) + \frac{(1-g)v_a}{1-\beta}\\ &\qquad\quad \cdot [3\beta(21(1-\beta)-\alpha(3-\beta)(7-4\beta))t\\ &\qquad\qquad + (1-\beta)[9(1-\alpha)(9(1-\beta)-\alpha(3-\beta)(3-2\beta))c_a\\ &\qquad\qquad + (1-q)\beta(9-9\alpha+4\alpha\beta)v_b]]\bigg\}.\end{aligned}$$

From Proposition 2, $\pi_1-\pi_1^{\mathrm{NA}}<0$ at $g=1$. We next show that $\pi_1-\pi_1^{\mathrm{NA}}$ is a decreasing function of g:

$$\begin{aligned}&\frac{\partial(\pi_1-\pi_1^{\mathrm{NA}})}{\partial g}\\ &\quad = \frac{\alpha v_a}{18t(9(1-\beta)-\alpha(3-\beta)(3-2\beta))}\\ &\qquad \cdot \{-9(1-\alpha)(9(1-\beta)-\alpha(3-\beta)(3-2\beta))c_a\\ &\qquad\quad - 3t\beta[21(1-\beta)-\alpha(3-\beta)(7-4\beta)]/(1-\beta)\\ &\qquad\quad - 2(1-g)(3-\alpha(3-\beta))(3-3\alpha+2\alpha\beta)v_a\\ &\qquad\quad - (1-q)\beta(9-9\alpha+4\alpha\beta)v_b\}.\end{aligned}$$

On the right side, all four components within the curly brackets are negative given $\beta/((1-\alpha)(1-\beta))<1$. Therefore, $\partial(\pi_1-\pi_1^{\mathrm{NA}})/\partial g<0$.

Let $\bar{g}_1$ denote the feasible solution of g to $\pi_1-\pi_1^{\mathrm{NA}}=0$. Because $\pi_1-\pi_1^{\mathrm{NA}}=0$ is a quadratic equation in g, it has no more than two roots. One root must be larger than 1 (and thus infeasible) because $\partial^2(\pi_1-\pi_1^{\mathrm{NA}})/\partial g^2>0$ for any g and $\pi_1-\pi_1^{\mathrm{NA}}<0$ at $g=1$. Therefore, $\bar{g}_1$ is unique. Let $\bar{g}_2$ denote the solution of g to $\pi_1-\pi_1^{\mathrm{NA}}=\pi_1+\pi_2-\pi_1^{\mathrm{NA}}-\pi_2^{\mathrm{NA}}$. Similarly $\bar{g}_2$ is also unique. Furthermore, because $\pi_1+\pi_2-\pi_1^{\mathrm{NA}}-\pi_2^{\mathrm{NA}}>0$, $\bar{g}_1<\bar{g}_2$ is impossible (as otherwise some $\bar{g}_1<g<\bar{g}_2$ will satisfy both $\pi_1-\pi_1^{\mathrm{NA}}<0$ and $\pi_2-\pi_2^{\mathrm{NA}}<0$). Therefore, under inner equilibrium,

- if $g>\bar{g}_1$, it is a lose–win scenario, i.e., the superior firm loses from add-on pricing, whereas the inferior firm gains from add-on pricing;
- if $\bar{g}_1>g>\bar{g}_2$, it is a win–win scenario;
- if $g<\bar{g}_2$, it is a win–lose scenario. Q.E.D.

Appendix B. Add-on Usage Dependent on Price

In the main text we assume that base consumers never use the add-on regardless of whether it is free or not. In practice, however, there might be scenarios where consumers who do not need a product or service asks for one if it is offered for free. In this appendix we show that our results are robust to such consumer behavior. Consider a fraction, ρ, of the base consumers who will consume the add-on if it is provided for free and will not consume the add-on if $p_a>0$.[25] Thus, we now have four consumer segments: knowledgeable consumers, boundedly rational consumers, base consumers who never use add-on regardless of whether it is free or not, and base consumers who will use add-on only if it is free.

Firm j's profit can now be written as $\pi_j=\gamma_{jb}(p_{jb}-c_b)+\gamma_{ja}(p_{ja}-c_a)-\Phi\rho(\gamma_{jb}-\gamma_{ja})c_a$, where $\Phi=1$ if $p_{ja}=0$, and $\Phi=0$ otherwise. Notice the term $\Phi\rho(\gamma_{jb}-\gamma_{ja})c_a$ captures the fact that a fraction ρ of the consumers who do not buy the add-on (which is equal to $\gamma_{jb}-\gamma_{ja}$ in the previously derived expressions) will use the add-on if and only if $p_{ja}=0$. Without add-on pricing, each firm's optimization problem $\max_{p_{jb}}\pi_j|_{p_{ja}=0}$ is concave ($\partial^2\pi_j/\partial p_{jb}^2=-1/t<0$), and simultaneously

[25] This incorporates the so-called "penny gap" where "in most cases, just a penny—a seemingly inconsequential price—can stop the vast majority of consumers in their tracks" due to the imposed cost of thinking (Anderson 2009, p. 59).

solving the first-order conditions leads to equilibrium prices of $p_1^{NA} = t + c_b + (\alpha + \rho - \alpha\rho)c_a + (\alpha(1-g)v_a + (1-q)v_b)/3$ and $p_2^{NA} = t + c_b + (\alpha + \rho - \alpha\rho)c_a - (\alpha(1-g)\cdot v_a + (1-q)v_b)/3$. Equilibrium profit for the superior firm is $\pi_1^{NA} = [\alpha^2(1-g)^2v_a^2 + (3t + (1-q)v_b)^2 - \alpha(9(1-\alpha)(1-\rho)\cdot c_a - 2(3t + (1-q)v_b))(1-g)v_a]/(18t)$, and the equilibrium profit for the inferior firm is

$$\pi_2^{NA} = [\alpha^2(1-g)^2v_a^2 + (3t - (1-q)v_b)^2 + \alpha(9(1-\alpha)(1-\rho)c_a - 2(3t - (1-q)v_b))(1-g)v_a]/(18t).$$

We then have $\partial\pi_1^{NA}/\partial\rho = \alpha(1-g)(1-\alpha)c_av_a/(2t) > 0$ and $\partial\pi_2^{NA}/\partial\rho = -\alpha(1-g)(1-\alpha)c_av_a/(2t) < 0$. These derivatives are zero if $g = 1$; hence Propositions 1 and 2 are trivially preserved. In the general case, the effect of price-dependent add-on usage is to expand the region for which the superior firm suffers from add-on pricing and to expand the region for which the inferior firm gains from add-on pricing. For $g < 1$, $\beta = 0$, $\pi_1 - \pi_1^{NA} = [(1-g)(1-\alpha)\alpha v_a(9(1-\rho)c_a + (1-g)v_a)]/(18t) > 0$, and $\pi_2 - \pi_2^{NA} = [(1-g)(1-\alpha)\cdot \alpha v_a(9(1-\rho)c_a - (1-g)v_a)]/(18t)$. Thus, the profit improvement for firm 1 in Proposition 4 is preserved, but the profit loss for firm 2 is confirmed if and only if $\rho < 1-(1-g)v_a/c_a$. The potential for a win–win when the boundedly rational segment does not exist is the only insight unique to price-dependent add-on usage.

Appendix C. Consumers Know They Will Be Charged for an Add-on But Do Not Know the Fee

In this appendix, we consider the case in which consumers are rational and know they will be charged for the add-on, but do not know the add-on price before committing to purchase the base product. As such, there are two consumer segments: $(1-\alpha)$ consumers for whom $v_{ia} = 0$ and α consumers for whom $v_{ia} = v_a > 0$.

Claim 1. *Consumers will expect that after they have committed to a firm by buying its base product, the add-on price will be at the maximal level.*

Proof of Claim 1. For any consumer belief, firms optimally charge their maximum possible add-on fees (i.e., v_a for firm 1 and gv_a for firm 2). If firms charge above this price, no consumers will buy the add-on in stage 3. If firms charge below this price, then the add-on margin will be lower without an effect on sales. Therefore, the only rational belief is that the add-on fees will be at their maximum. Q.E.D.

Claim 2. *Profit can be expressed as* $\pi_1 = (p_{1b} + \alpha v_a - \alpha c_a - c_b)(t - p_{1b} + p_{2b} + (1-q)v_b)/(2t)$ *and* $\pi_2 = (p_{2b} + \alpha g v_a - \alpha c_a - c_b)(t - p_{2b} + p_{1b} - (1-q)v_b)/(2t)$ *for the superior and inferior firms, respectively.*

Claim 2 follows from the analysis of marginal base consumer and the marginal consumer who values the add-on and rationally expects $p_{1a} = v_a$ and $p_{2a} = gv_a$.

The first-order conditions are uniquely satisfied at $p_{1b}^* = t + \alpha c_a + c_b - (2\alpha v_a + g\alpha v_a - (1-q)v_b)/3$ and $p_{2b}^* = t + \alpha c_a + c_b - (\alpha v_a + 2g\alpha v_a + (1-q)v_b)/3$, leading to equilibrium profits of $\pi_1^* = (3t + \alpha(1-g)v_a + (1-q)v_b)^2/(18t)$ and $\pi_2^* = (3t - \alpha(1-g)v_a - (1-q)v_b)^2/(18t)$ provided differentiation between firms is such that both firms compete for both consumer segments.

In comparison, when add-on pricing is infeasible, the superior and inferior firms maximize $\pi_1 = [(p_{1b} - c_b)\cdot (t - p_{1b} + p_{2b} + (1-g)\alpha v_a + (1-q)v_b) - \alpha c_a(t - p_{1b} + p_{2b} + (1-g)v_a + (1-q)v_b)]/(2t)$ and $\pi_2 = [(p_{2b} - c_b)(t + p_{1b} - p_{2b} - (1-g)\alpha v_a - (1-q)v_b) - \alpha c_a(t + p_{1b} - p_{2b} - (1-g)\cdot v_a - (1-q)v_b)]/(2t)$. The first-order conditions are uniquely satisfied at $p_{1b}^{NA} = t + \alpha c_a + c_b + (\alpha(1-g)v_a + (1-q)v_b)/3$, and $p_{2b}^{NA} = t + \alpha c_a + c_b - (\alpha(1-g)v_a + (1-q)v_b)/3$, which results in profits equal to $\pi_1^{NA} = (3t + (4-g)\alpha v_a + (1-q)v_b)(3t - 2(1-g)\alpha v_a + (1-q)v_b)/(18t)$ and $\pi_2^{NA} = (3t + 2(1-g)\alpha v_a - (1-q)v_b)(3t - (1-4g)\alpha v_a - (1-q)v_b)/(18t)$.

The differences in profit from add-on pricing relative to when add-on pricing is prohibited are $\pi_1^* - \pi_1^{NA} = (1-g)\cdot (1-\alpha)\alpha c_a v_a/(2t) \geq 0$ and $\pi_2^* - \pi_2^{NA} = -(1-g)(1-\alpha)\cdot \alpha c_a v_a/(2t) \leq 0$. Thus, the profit irrelevancy of add-on pricing result holds unless both $\alpha < 1$ and $g < 1$. If base consumers exist *and* there is asymmetry in the add-on quality, then add-on pricing represents a win–lose situation for the superior and inferior firms, respectively. Industry profits are unaffected by add-on pricing (i.e., $\pi_1^* - \pi_1^{NA} + \pi_2^* - \pi_2^{NA} = 0$).

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