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Scott Webster, Z. Kevin Weng,

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A Risk-free Perishable Item Returns Policy

Scott Webster • Z. Kevin Weng

Syracuse University, School of Management, Syracuse, New York 13244-2130 stwebste@syr.edu University of Wisconsin–Madison, School of Business, Madison, Wisconsin 53706 kweng@bus.wisc.edu

A returns policy, which specifies a schedule of rebates from manufacturer to retailer for product left over at the end of the selling season, encourages larger order quantities and can increase manufacturer profit. One downside from a manufacturer's perspective is the possibility of very low profit due to high rebate expense when demand is lower than expected. We take the viewpoint of a manufacturer selling a short life-cycle product to a single risk-neutral retailer and describe returns policies that, when compared to no returns, satisfy two conditions: (1) the retailer's expected profit is increased and (2) the manufacturer's profit is at least as large as when no returns are allowed. We call such a returns policy *risk-free*. (Supply Chain; Contacts; Returns Policies)

1. Introduction

Consider a manufacturer-retailer supply chain for a short life-cycle product where the retailer places a one-time order and the manufacturer produces and delivers the order to the retailer prior to the selling season. Before approaching the retailer, the manufacturer determines her selling price and a policy, if any, for returns. A returns policy defines the rebate to the retailer for unsold units at the end of the selling season.

Research on returns policies and other tactics for managing supply chains is increasing in importance. An environment characterized by very short product life cycles and high market volatility is becoming more common in a number of industries. Consider the case of Compaq Computer Corp. (Burrows, Armstrong, and McWilliams 1997):

Compaq also is playing hardball with retailers. It pushes store owners to estimate sales precisely so that low-end PCs can be made in one batch. If retailers underestimate, they probably won't get more product. And Compaq won't take back PCs that have been sitting on store shelves unless they are faulty.

Compaq's policy does a good job controlling risk; the profit level is certain once the order is placed. Compaq might consider offering a rebate on unsold items, which generally will result in a larger order (see, e.g., Donohue 1996), but it carries the risk of a large payout if the market softens. The issue of returns policy risk is one reason why some companies are hesitant to employ such policies (see Kandel 1996 or Padmanabhan and Png 1995 for a discussion of this and other reasons).

In this paper, we take the viewpoint of a manufacturer considering whether to offer a returns policy to single risk-neutral retailer, frame the decision as a tradeoff between risk and return, and investigate whether there are returns policies that transcend the risk-return tradeoff regardless of the manufacturer's attitude towards risk. In particular, we are interested in returns policies that, when compared to the alternative of no allowance for returns, satisfy the following conditions: (1) the retailer's expected profit is increased and (2) the manufacturer's profit is at least as large as when no returns are allowed. We call such a returns policy *risk free*.

The main limiting assumptions of our analysis are: First, we focus on the relationship between a manufacturer and a *single* retailer. Since most manufacturers sell to multiple retailers, it is important to note that any

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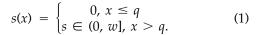
returns policy implemented in the United States is susceptible to claims under the Robinson-Patman Act unless it is applied consistently for all retailers. As a consequence, the degree to which returns policies can increase expected profit while controlling risk is generally more limited than what we will see in this paper (See §3). Second, we focus on the manufacturer's perspective, and in particular, manufacturer risk. We ignore the impact of a returns policy on retailer risk by assuming that the retailer is risk-neutral. Third, we assume that retail price is exogenous. The introduction of a returns policy may influence the retail price, which, in turn, may influence the probability distribution of end-customer demand. Fourth, from a practical standpoint, a manufacturer's ability to design a risk-free returns policy that increases manufacturer and retailer profits depends on the accuracy with which the manufacturer can estimate the quantity the retailer would order without a returns policy. We will discuss the managerial implications of the policy and its limitations further in §3.

2. Model and Analysis

We model the manufacturer-retailer short life-cycle product supply chain as follows (where possible we use the notation in Emmons and Gilbert 1998). Prior to the selling season, the retailer orders Q units from the manufacturer. The manufacturer incurs a quantity independent unit cost c to produce and deliver the order to the retailer. The manufacturer's unit selling price is w > c and the retailer's unit selling price is r > w. Let D denote a nonnegative continuous random variable for end-customer demand during the selling season; f(x) and F(x) are the PDF and the CDF of D, respectively. We assume that the value of any unsold unit at the end of the season is known, and to simplify notation without loss of generality, we let this value be zero. The actual responsibility for salvaging leftover units may reside with either the manufacturer or the retailer depending upon which alternative makes more economic sense.

Let s(x) denote the rebate schedule as a function of the xth unit ordered by the retailer, i.e., the rebate on the xth unit ordered if it is left over is $\int_{x-1}^{x} s(t)dt$. In this paper, we will limit consideration to rebate schedules of the following simple form:

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The literature on the design of manufacturer-retailer contracts to improve channel performance is extensive (see Tsay, Nahmias, and Agrawal 1999 for a review). However, there is relatively little work on returns policies contracts, in part because of a tendency in the literature to favor settings where demand is deterministic (Lariviere 1999). Early work on returns policies can be traced to Pasternack (1985). He analyzes a class of returns policies where q is defined in terms of the maximum percent of an order that can be returned, i.e., $q = (1 - \rho)Q$ where ρ is the fraction of an order that can be returned for rebate s. Pasternack observes that expected system profit is suboptimal if s = 0, and he shows that there is an infinite set of returns policies (ρ , s) with $\rho \in (0, 1]$ and $s \in (0, w]$ that maximize expected system profit, i.e., achieve channel coordination. One disadvantage of returns policies with ρ < 1 relative to ρ = 1 is that the channel coordinating value of s depends upon the CDF, as well as r, w, and c. If $\rho = 1$, then the channel coordinating value of s depends solely on r, w, and c.

Emmons and Gilbert (1998) build on the work of Pasternack (1985) by analyzing a *partial credit*/*full returns* policy (i.e., s < w, $\rho = 1$) with the generalization that the probability distribution of end-customer demand depends on the retail price r. Consequently, a retailer seeking to maximize his expected profit must decide on a value for r and Q, rather than just Q. Given uniformly distributed demand, the authors show that a rebate on leftover product increases the expected profit of the manufacturer and the retailer whenever w exceeds a threshold value.

A number of researchers (e.g., Marvel and Peck 1995, Kandel 1996, Padmanabhan and Png 1997, Lariviere 1999) have analyzed the form of an optimal full returns policy from the manufacturer's perspective when the manufacturer is free to set w and s. Lariviere (1999) also presents comparative statics of a *full credit/partial returns* policy (i.e., s = w, $\rho < 1$). In general, results show that a returns policy can be a useful tactic for increasing manufacturer profit and, with some cases, increasing retailer profit as well.

All of the returns policy models from the literature

assume that the manufacturer and the retailer are riskneutral. Consideration of attitude towards risk and its influence on decision making presents some modeling challenges (Tsay, Nahmias, and Agrawal 1999; see Jia, Dyer, and Butler 1999 for a discussion of opposing views on appropriate risk preference models). However, there has been some analysis of risk-averse retailer behavior in a perishable item setting when no returns are allowed (e.g., Horowitz 1970, Baron 1973, and Winkler 1974, Lau Britney Sankarasubramanian and Kumaraswamy 1983, Li, and Lau 1990, Eeckhoudt, Gollier, Schlesinger 1995). One general conclusion from this research stream is that a risk-averse retailer will order less than a risk-neutral retailer. Other, more specific, conclusions depend upon specific models. Eeckhoudt, Gollier, Schlesinger (1995), for example, model a riskaverse retailer with a concave utility function and find that behavior can depend on the specific utility function and the specific probability distribution of demand (e.g., order quantity may increase or decrease with changes in demand variance or cost parameters).

Assuming that the retailer is risk-neutral, it follows from standard newsvendor analysis that the retailer order quantity when no returns are allowed is

$$Q_1^* = F^{-1}((r - w)/r) \tag{2}$$

and the deterministic manufacturer profit is

$$\pi_M^* = (w - c)Q_1^*. (3)$$

We let Q_2^* denote the order quantity that maximizes expected retailer profit when a returns policy is offered. The following property gives a necessary and sufficient condition for a risk-free returns policy (the proof is straightforward and therefore omitted).

Property 1. A returns policy with $Q_2^* > Q_1^*$ is risk-free if and only if

$$s(Q_2^* - q) \le (w - c)(Q_2^* - Q_1^*).$$
 (4)

Otherwise, the probability that the returns policy results in lower manufacturer profit than when no returns are allowed is

$$F(O_2^* - (w - c)(O_2^* - O_1^*)/s).$$
 (5)

The left-hand side of (4) is the maximum possible rebate whereas the right-hand side of (4) is the preseason

increase in manufacturer profit due to the returns policy. If (4) does not hold, then $Q_2^* - (w - c)(Q_2^* - Q_1^*)/s$ is the realization of demand such that the rebate is equal to the preseason increase in manufacturer profit due to the returns policy. Thus, manufacturer profit is lower with a returns policy than without a returns policy whenever demand is less than the argument of the CDF in (5).

If

$$q = (1 - \rho)Q, \tag{6}$$

then the expected retailer profit is concave in the order quantity, and the unique retailer optimal order quantity satisfies

$$Q_2^* = F^{-1}((r - w)/(r - s) - F((1 - \rho)Q_2^*)(1 - \rho)s/(r - s))$$
 (7)

(Pasternack 1985). We refer to a returns policy with q according to (6) as a *percent rebate policy* because the schedule of allowances is a function of a percentage of the order quantity. Alternatively, the value of q may be independent of Q, in which case we refer to the returns policy as a *quantity rebate policy*. If $\rho = 1$, then (7) reduces to

$$Q_2^* = F^{-1}((r - w)/(r - s)),$$
 (8)

which also holds for a quantity rebate policy if

$$q \le F^{-1}((r - w)/r) = Q_1^*$$
 (9)

and more generally if expected retailer profit increases with order quantity Q_2^* , i.e.,

$$E[\pi_R(Q_1^*)] < E[\pi_R(Q_2^*)].$$
 (10)

A full-returns policy qualifies as both a percent rebate policy and a quantity rebate policy, but how do the two classes of policies compare when q > 0 and $\rho < 1$? A percent rebate policy is arguably simpler in the sense that the value of q adjusts to changes in the demand distribution (e.g., q increases with the optimal retailer order quantity when there is a right-shift in the CDF). On the other hand, the linkage between q and the retailer order quantity comes with a cost that becomes apparent in (4) and (7). For a fixed optimal retailer-order quantity Q_2^* , an increase in q lowers the left-hand side of the risk-free condition (4). But, due to

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Table 1 Returns Policy Performance Statistics: r=\$100, w=\$40, c=\$10, $D\sim U[0,100]$

r	Returns Policy	Q_2^{\star}	$E[\pi_{\scriptscriptstyle M}^{\star}]$	$E[\pi_{\scriptscriptstyle R}^{\star}]$	$E[\pi_{\mathit{M}}^{\star}] + E[\pi_{\mathit{R}}^{\star}]$	$P[\pi_M^* < 1800]$	minimum mfr profit	E[<i>R</i> (<i>D</i>)]
A1	$s = $30, \rho = 1-6^{-1/2} \approx 59\%$	80	\$1600	\$2400	\$4000	60.0%	\$979.80	\$800
B1	s = \$25, q = 60	80	\$2050	\$1950	\$4000	0.0%	\$1900.00	\$350
C1	s = \$25, q = 0	80	\$1600	\$2400	\$4000	56.0%	\$400.00	\$800
A2	$s = $40, \rho = 1-6^{-1/2} \approx 59\%$	90	\$1350	\$2700	\$4050	67.5%	\$569.69	\$1350
B2	s = \$100/3, q = 60	90	\$1950	\$2100	\$4050	63.0%	\$1700.00	\$750
C2	s = \$100/3, q = 0	90	\$1350	\$2700	\$4050	63.0%	-\$300.00	\$1350

the linkage between q and Q, an increase in q must be accompanied by an increase in s in order to keep Q_2^* fixed—a change that offsets some or all of the effect of increasing q. This is in contrast with a quantity rebate policy where an increase in q does not necessarily require an increase in s. The effect of the linkage can also be observed in the marginal cost at equilibrium. At Q_2^* , the marginal expected retailer cost of increasing the order quantity is equal to the marginal expected retailer revenue. If the linkage between q and Q is removed while keeping q fixed, then only the marginal expected retailer cost changes and it is reduced (because q no longer increases with the order quantity). The effect is an increase in the optimal retailer quantity, or if the marginal expected retailer cost is forced to be fixed, then the retailer order quantity is unchanged and the effect is to reduce s. These are the insights that underlie the following characterization of relative performance (the proof is in the Appendix).

Property 2.

- (i) For any percent rebate policy, there exists a quantity rebate policy with higher expected manufacturer profit.
- (ii) For any percent rebate policy with s < w c, there exists a quantity rebate policy with higher expected manufacturer profit and higher minimum manufacturer profit.
- (iii) For some r, w, c, and F(x), there are no risk-free percent rebate policies whereas there always exists a risk-free quantity rebate policy.

Before discussing managerial implications, we illustrate alternative policies through a simple numerical example. Suppose that the retail unit margin is \$60, the manufacturer unit margin is \$30, and units left over at the end of the season can be salvaged for \$10 less than the unit manufacturing and delivery cost, or r = \$100,

w=\$40, and c=\$10. If end-customer demand is a continuous uniform random variable with a range of [0,100], then without a returns policy, the order quantity, manufacturer profit, and expected retailer profit are $Q_1^* = F^{-1}((r-w)/r) = 60$, $\pi_M^* = 100 (w-c) (r-w)/r = \1800 , $\mathrm{E}[\pi_R^*] = 50(r-w)^2/r = \1800 . Table 1 reports returns policy performance statistics. In the table, $\mathrm{P}[\pi_M^* < 1800]$ is the probability that manufacturer profit with a returns policy is less than manufacturer profit without a returns policy. The seventh column reports the minimum possible manufacturer profit and the last column reports the expected rebate to the retailer. The detailed formulas and calculations are omitted, but interested readers can obtain them by contacting the authors.

Policies A1 through C1 induce the retailer to increase his order from 60 to 80 for a corresponding 11% increase in expected system profit. The policies differ in their allocation of expected profit among manufacturer and retailer, and the level of manufacturer risk. Policy A1 limits returns to about 59% of the order, policy B1 limits returns to Q-60, and policy C1 offers a rebate on unlimited returns. Comparing policies A1 and C1, we see that policy A1 results in a slightly greater chance that manufacturer profit will be less than \$1800—the deterministic manufacturer profit when no returns policy is offered—but assures a substantially higher minimum manufacturer profit. Compared to A1 and C1, the quantity rebate policy B1 results in a larger expected manufacturer profit and is risk-free.

Policies A2 through C2 are the same as policies A1 through C1 except the rebate is increased to a level necessary to achieve channel coordination. The differences between policies A2 and C2 are similar to what

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was observed for policies A1 and C1. Comparing policy B2 to policy B1, the rebate is increased from \$25.00 to \$33.33, which is greater than the manufacturer margin of \$30.00 and consequently the policy is no longer risk-free. Furthermore, expected manufacturer profit decreases from \$2050 to \$1950 while expected system profit increases. One way to improve the performance of policy B2 (from the perspective of the manufacturer) is to increase q. If q is increased from 60 to 66, for example, $Q_2^* = 90$, $E[\pi_M^*] = 2076 , $E[\pi_R^*] = 1974 , $P[\pi_M^* < 1800] = 0\%$, E[R(D)] = \$624, and manufacturer profit will be no less than \$1900. Increasing q benefits the manufacturer to the detriment of the retailer while leaving expected system profit unchanged. However, the degree of increase is limited to the point where $E[\pi_R^*] = $1800 (q \approx 73 \text{ in this case})$; once q goes beyond this critical value, a retailer interested in maximizing his expected profit will ignore the returns policy and order Q_1^* .

3. Discussion

A percent rebate policy specifies a rebate schedule as a function of percentages of the order quantity. A quantity rebate policy specifies a rebate schedule as a function of absolute units, and except for the case of q=0, is new to the literature. The elimination of the linkage between order quantity and q provides additional flexibility that explains why: (1) there always exists a quantity rebate policy with higher expected manufacturer profit than any percent rebate policy, and (2) there always exists a risk-free quantity rebate policy while a risk-free percent rebate policy may not exist.

One drawback of a quantity rebate policy, with respect to avoiding risk, is that it relies on the manufacturer's ability to estimate the quantity the retailer would order without a returns policy (i.e., Q_1^*). One pragmatic implementation is to select $s \in (0, w - c]$ and let q to be an estimate of what the retailer would order without a returns policy. The policy is risk-free as long as $q \ge Q_1^*$, and if in addition the retailer orders more than q, the expected profits of both parties are higher than would be the case without the returns policy. More generally, this pragmatic implementation results in higher expected manufacturer profit with no risk whenever the policy results in a larger order than

would be placed otherwise, a condition that may hold in some more complicated models.

We considered a single manufacturer-retailer model. One reason for the nearly exclusive focus on full returns policies ($\rho = 1$) over partial returns policies ($\rho <$ 1) in the literature is because the channel coordinating value of s depends only on r, w, and c, and not the CDF. Thus, a manufacturer selling to two retailers with the same r but different CDFs could offer the same channel coordinating contract (s, w) to both retailers and capture the same fraction of total channel profits. As Lariviere (1999) notes, the advantage of distributional independence may be overvalued in the literature because, at least when interpreted literally, it makes little economic sense; i.e., a manufacturer will likely be hesitant to enter into a transaction without an estimate of her resulting profit. Furthermore, the distributional dependence afforded by a partial returns policy could be an advantage when designing a menu of optimal self-selecting contracts (Lariviere 1999). For example, consider two retailers with identical cost structures (i.e., r and minimum level of expected profit to stock the product) and each with one of two possible CDFs. If the manufacturer knew which CDF applied to which retailer, she could offer two individual contracts that would maximize system profit while providing each retailer with their minimum expected profit requirement. Since a full returns contract is independent of the CDF, it is impossible to induce the retailers to self-select the optimal contract (from the manufacturer's perspective) when faced with a menu of choices. The distributional dependence of a percent rebate policy with ρ < 1 at least allows the possibility of designing optimal self-selecting contracts.

Quantity rebate policies occupy a middle ground between the extremes of complete distributional independence of a full returns policy and the tight linkage between the CDF and values of channel coordinating parameters given any $\rho < 1$ (cf., (7)). For example, consider two retailers with identical cost structures and known, but different, CDFs. While it is impossible to offer a single channel coordinating percent rebate policy with $\rho < 1$, there are many channel coordinating quantity rebate policies (e.g., any full returns channel coordinating contract but with $q \in (0, \min Q_1^*]$ where $\min Q_1^*$ denotes the minimum Q_1^* value among the two

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retailers). Similarly, while it is impossible to offer a menu of optimal self-selecting full returns policies, the distributional dependence of a quantity rebate policy raises the possibility of *optimal* self-selection. Furthermore, due to the risk-controlling characteristics of quantity rebate policies, the term "optimal" in italics may refer to the traditional objective of maximizing expected profit, or alternatively, an objective incorporating both risk and return. We leave this as a possible issue for future research.

4. Conclusion

While returns policies can increase the "upside potential" of manufacturer profit by encouraging retailers to order more, they may also introduce "downside exposure" through high rebate costs when demand is lower than expected. The work presented in this paper is motivated by the question of whether there are returns policies that lead to increased expected manufacturer and retailer profit without any, or at least minimal, downside exposure for the manufacturer. With a trend towards shorter product life-cycles, the issue of leftover product is becoming more pronounced and there is a need for additional research on structuring terms and conditions of sale to increase profits and to allocate profit and risk in a manner that benefits all parties.¹

Appendix

PROOF OF PROPERTY 2. For a percent rebate policy, the expected retailer profit is

$$E[\pi_R(Q)] = (r - w)Q - (r - s) \int_0^Q F(y)dy - s \int_0^{(1-\rho)Q} F(y)dy.$$
 (A1)

The first order condition is $dE[\pi_R(Q)]/dQ = r - w - (r - s)F(Q) - s(1 - \rho)F((1 - \rho)Q) = 0$, which has unique stationary point Q_2^* satisfying

$$Q_2^* = F^{-1}((r - w)/(r - s) - F((1 - \rho)Q_2^*)(1 - \rho)s/(r - s)).$$
 (A2)

From $d^2E[\pi_R(Q)]/dQ^2 = -rf(Q) - s(1-\rho)^2f((1-\rho)Q) < 0$, we see

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that expected retailer profit is strictly concave, and Q_2^* is the optimal retailer order quantity. For a quantity rebate policy, the expected retailer profit is

 $E[\pi_R(Q)] =$

$$\begin{cases} (r-w)Q-r\int_0^Q F(y)dy, Q \leq q \\ (r-w)Q-(r-s)\int_0^Q F(y)dy-s\int_0^q F(y)dy, Q > q. \end{cases} \tag{A3}$$

Both of the expressions in the brackets are strictly concave, and the two stationary points are

$$Q_1^* = F^{-1}((r - w)/r), \tag{A4}$$

$$Q_2^* = F^{-1}((r - w)/(r - s)).$$
 (A5)

Thus, Q_2^* is optimal if $E[\pi_R(Q_2^*)] \ge E[\pi_R(Q_1^*)]$ and Q_1^* is optimal if $E[\pi_R(Q_2^*)] \le E[\pi_R(Q_1^*)]$.

Property 2i follows from two observations. First, for a percent rebate policy with any given ρ , the optimal expected retailer profit is strictly increasing in s (i.e., expected retailer profit is strictly higher with the returns policy than without the returns policy). Second, for any given s in a quantity rebate policy, the value of q can be increased to the point where $\mathrm{E}[\pi_R(Q_2^*)] = \mathrm{E}[\pi_R(Q_1^*)] + \epsilon$ for ϵ a positive value arbitrarily close to zero. Thus, for any given percent rebate policy, there exists a quantity rebate policy with the same optimal retailer order quantity and total expected system profit, and where all of the increase in expected system profit (or less of the decrease) due to the returns policy goes to the manufacturer.

For Property 2ii, note that a retailer considering a quantity rebate policy will order Q_2^* if $q \leq Q_1^*$ (a sufficient, but not a necessary condition). Thus, for a given s, the optimal retailer order quantity is the same for returns policies with $\rho = 1$ (equivalent to q = 0) and q = 0Q₁*. However, expected manufacturer profit and the minimum manufacturer profit are higher at $q = Q_1^*$. This shows that compared to a full returns policy, there always exists a quantity rebate policy with higher expected manufacturer profit and higher minimum manufacturer profit. Now consider a percent rebate policy with s < w $c, \rho < 1$, and optimal retailer order quantity Q_2^* . Define a corresponding quantity rebate policy with $q = (1 - \rho)Q_2^*$ and the same rebate s. Given this quantity rebate policy, the retailer is able to increase his expected profit by increasing his order to the quantity defined in (A5) (due to benefit of fixed q instead of q increasing with the order size). Since s < w - c, the larger retailer order results in higher expected manufacturer profit and higher minimum manufacturer profit compared to the percent rebate policy. This shows that for any percent rebate policy with s < w - c and $\rho < 1$, there exists a quantity rebate policy that results in higher expected retailer profit, higher expected manufacturer profit, and higher minimum manufacturer profit.

Property 2iii can be verified from two observations. First, compared to no returns allowed, a quantity rebate policy with $q = Q_1^*$ and $s \in (0, w - c)$ is risk-free and results in higher expected manufacturer profit; i.e., there always exists a risk-free quantity rebate policy. Second, a simple example shows that there does not always



exist a risk-free percent rebate policy. Suppose that *D* is a continuous random variable with range [0, 100]. Then F(Q) = Q/100 and F((1 + Q)) = Q/100 $(-\rho)Q = (1-\rho)F(Q)$, which when substituted into (2) and (7) yield $Q_2^* = 100(r - w)/[r - s(2\rho - \rho^2)].$ $Q_1^* = 100(r - w)/r$ and From Property 1, a percent rebate policy is not risk-free if $s\rho Q_2^*/(Q_2^*-Q_1^*)>w-c$, which can be rewritten as $r(2-\rho)>w$ − *c*. Since $\rho \in (0, 1]$, it follows that there is no risk-free percent rebate policy if r > 2(w - c).

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