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Multidimensional Ellsberg

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The classical Ellsberg experiment presents individuals with a choice problem in which the probability of winning a prize is unknown (uncertain). In this paper, we study how individuals make choices between gambles in which the uncertainty is in different dimensions: the winning probability, the amount of the prize, the payment date, and the combinations thereof. Although the decision-theoretic models accommodate a rich variety of behaviors, we present experimental evidence that points at systematic behavioral patterns: (i) no uncertainty is preferred to uncertainty on any single dimension and to uncertainty on multiple dimensions, and (ii) "correlated" uncertainty on multiple dimensions is preferred to uncertainty on any single dimension.

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1. Introduction

When making decisions under uncertainty, we are often confronted with incomplete objective information, or uncertainty, on multiple aspects of the decision problem. The amount of monetary earnings may be uncertain; the likelihood of the possible earnings may be uncertain; the date at which payments will be made may be uncertain; etc. Our willingness to take on uncertainty may depend not only on which dimensions are uncertain, but also on the extent to which our decision can affect the levels of uncertainty in the different dimensions. The goal of this paper is to gain some understanding of how individuals approach this form of "multidimensional" uncertainty.[1]

The starting point of modern analysis of decision making under uncertainty is the seminal thought experiment of Ellsberg. In one of its variants, participants face an urn with 60 poker chips: 20 of these chips are black, and each of the remaining 40 chips is one of the "uncertain" colors—red or green. Participants are asked to guess the color of a randomly drawn chip. A participant who guesses correctly wins a prize of \$20 that is paid immediately.

Two features of this classic experiment are particularly relevant for us. First, the gamble involves uncertainty in only one dimension, namely, the likelihood of winning; other dimensions, such as the prize amount and the payment date, are known. Second, a participant can remove all uncertainty by betting on black—the option with no uncertainty in any dimension.

To explore a more general setup with uncertainty in multiple dimensions, consider the following variations on the classical Ellsberg experiment. Participants face the same urn as above, and as before only a single chip will be drawn, and the participants are asked to select a color. In one variation, a participant is paid only if a black chip is drawn, but he is paid a number of dollars equal to the number of chips in the urn of his chosen color: if X is the number of chips in the urn that have the color chosen by the participant, he wins \$$X$ if a black chip is drawn. In this case the probability of winning is not uncertain, because the lottery is paid only if a black chip is drawn, but the amount won is. In another variation, the participant is again asked to choose a color, and is paid if a chip of *that* color is extracted, a number of dollars equal to the number of chips of that color in the urn. In this variation, there is a sense in which the uncertainty is on "two dimensions": not only the likelihood of winning, but also the amount won. In yet another variation, if the participant guesses correctly, he wins \$$X$, but is paid X days from the date of the experiment. Here

[1] In line with the literature, we use the term "uncertainty" to refer to a situation in which some objective parameters of the decision problem, such as the likelihood of different outcomes or the monetary value of outcomes, are not objectively specified. (Some papers refer to this as "ambiguity.") We use the term "risk" to refer to situations in which all the relevant parameters of the decision problem are specified.

we have added uncertainty on a "third dimension": how soon the prize is paid.

The above variations of the classical Ellsberg experiment allow us to answer three classes of questions. A first set of questions is concerned with the correlation of uncertainty attitudes across dimensions. Are participants uncertainty averse also when prizes, and not probabilities, are uncertain? Or when dates are uncertain? Moreover, is uncertainty aversion a stable feature "across dimensions"? That is, if a participant is uncertainty averse in one dimension, is he more likely to be uncertainty averse in another?

A second set of questions is concerned with situations in which multiple dimensions can be uncertain at the same time and uncertain variables across different dimensions may be correlated. How does an individual's willingness to accept uncertainty in one dimension depend on the presence of uncertainty in other dimensions? For example, suppose an individual could decide only whether the probability of winning will be uncertain. How is her decision affected by whether the amount of winning is uncertain as well? How is it affected by whether the date of payment is uncertain? And if it is affected, does the agent choose to opt for the more uncertain option in which the various dimensions are positively correlated, or not? These questions have a particular relevance because in real life we often face options that involve uncertainties in many dimensions at the same time, some of which are fixed and cannot be completely removed.

The third and last set of questions explores whether participants prefer options with no uncertainty to options in which two or more dimensions are uncertain. Are participants still uncertainty averse when facing multidimensional uncertainty? Is the proportion of participants who are averse to "multidimensional" uncertainty higher or lower than the proportion who are averse to each form of "single-dimensional" uncertainty?

This paper addresses these questions with an experimental design that adopts the above variation of the canonical Ellsberg framework. One feature of this framework is that all uncertainty is generated by a single urn—specifically, by the distribution of colors in a three-color Ellsberg urn.[2] This is important for our analysis because any difference in behavior cannot be due to differences in beliefs.

[2] In our experiment, the outcome of each bet depends either on the color of the chip extracted from the urn or on the number of red or green balls in it. Although conceptually these may be seen as different sources, they are clearly directly, and objectively, linked. This is in contrast to other experiments in which there are two separate urns either with no objective relations or that are explicitly independent, as in Eichberger et al. (2015) and Epstein and Halevy (2014), discussed in §1.2.

Although there are intuitive differences between the various forms of uncertainty above, these differences are less apparent in the typical formalization of these bets in economics, in which all uncertainty is simply described as an unknown probability distribution on a state space, with possibly a multidimensional outcome space (prize, dates, etc.). In other words, in the typical formalization, uncertainty appears only over probabilities, and we can naturally express our experiment using this language as well. However, we find it useful to organize the data using the terminology of "different dimensions of uncertainty," and we will describe the behavior of participants in terms of how their decisions change as a function of the dimensions over which there is no objective information. We think this is the clearest and most transparent way to organize the data, which is possibly also close to how subjects perceive it in our experiment.

1.1. Overview of the Results

Our main findings are as follows:

1. *The majority of participants are averse to having uncertainty in only one dimension* (*single-dimensional uncertainty*), *regardless of what that dimension is* (*probability, prize, time*). The proportions, however, vary across the different dimensions. Furthermore, the sets of participants who are uncertainty averse in the various dimensions are, with some degree of error, nested: the largest set includes participants who are averse only to uncertain prizes (82%); participants who are averse to uncertain probabilities tend to be a subset of it (76%); and the set of participants who are averse to uncertain dates is then a subset of both (52%).

2. *When one dimension is fixed and uncertain in all available options and participants can choose whether to have uncertainty in another dimension, the majority of participants prefer uncertainty in both dimensions. In particular, they tend to choose the option for which the uncertainties in both dimensions are perfectly correlated.* In other words, participants who prefer no uncertainty when that option is available tend to prefer the gamble with the "*most exposure*" to the uncertain variable, when (at least) one of the gamble's dimensions is uncertain and fixed in all the available options. To illustrate, consider first the standard Ellsberg gamble in which a participant is paid \$20 at the end of the experiment if he correctly guesses the color of the chip to be drawn. About 76% of participants bet on black, and only 12% bet on green. However, when the prize amount is fixed at \$$g$ (the number of green chips in the urn), then only 33% bet on black, whereas 55% bet on green. Similarly, consider the gambles that pay \$20 if a black chip is drawn, but the payment is made in x days, where x is the number of chips of the color chosen by the subject. About 52% of participants

choose black, and only 14% choose green, in line with standard uncertainty aversion. In contrast, when the participant wins $\$r$ (the number of red chips) only if the color red comes up, only 19% of participants choose black, whereas 58% choose green (the option where the prize amount is perfectly correlated with how quickly the payment is received).[3]

3. *When comparing options with uncertainty in multiple dimensions against options with no uncertainty, the majority of participants prefer the option with no uncertainty*. However, the proportion of participants who prefer no uncertainty to uncertainty in multiple dimensions tends to be smaller than the proportion preferring no uncertainty to uncertainty in a single dimension. For instance, 82% of participants are averse to uncertainty only in prizes, 76% are averse to uncertainty only in probability, but only 67% prefer the risky option to an option in which both prize and probabilities are uncertain and perfectly correlated (see Table 1 and Table B.7 in Online Appendix B).

Our findings suggest that individuals tend to exhibit one particular pattern of behavior: aversion to single-dimensional uncertainty, (milder) aversion to multidimensional uncertainty, and a preference for multidimensional uncertainty over single-dimensional uncertainty. Put differently, when subjects have the option to remove all uncertainty, the majority opt for that option. When uncertainty cannot be completely removed, the majority of subjects prefer perfectly correlated uncertainty on several dimensions to having only one uncertain dimension.

As a robustness check, we also run an additional experiment in a very different participant pool, Amazon's Mechanical Turk (MT). We find very similar patterns (see §4).

In §5 we investigate how the results above relate to existing models of choice under uncertainty. We address two questions: Are existing models of decision making under uncertainty consistent with *all* of our main findings? And, conversely, can our results be used to *refine* the parameters of these models?

Our key observation is that leading existing models are compatible with the main patterns exhibited in our experiment, but also with their opposite: even assuming uncertainty aversion in the classical Ellsberg experiment, these models make little to no predictions over the choices in our experiment. Let us illustrate this in the context of the well-known max-min expected utility (MMEU) of Gilboa and Schmeidler (1989). (In §5 we show that the same holds for the recursive nonexpected utility model of Segal (1987) and Segal (1990).) According to this model, a decision maker considers a *set* of priors Π and evaluates each act by computing its expected utility using the most pessimistic prior. For simplicity, let us consider a risk-neutral participant (the analysis easily generalizes). We will argue that depending on the set Π, many patterns of preferences between options with no uncertainty, single-dimensional uncertainty, "double negatively correlated uncertainty," and "double positively correlated uncertainty" are possible, as long as no uncertainty is ranked above single-dimensional uncertainty as in the typical Ellsberg experiment. Consider for example the set that includes the belief "the urn has only x red chips" and the symmetric counterpart, "the urn has only x green chips," where $x < 20$. This set would lead subjects to prefer both single-dimensional uncertainty and double negatively correlated uncertainty to double positively correlated uncertainty—but the opposite is true in our data. Whereas the priors above are degenerate, and hence have zero variance, an alternative plausible set of beliefs might consist of priors with some variance, for example, "with probability 0.6 there are 40 red (green) chips, and with probability 0.4 there are 40 green (red) chips." Using beliefs of this kind, double positively correlated uncertainty becomes much better than any other option, including the one with no uncertainty. This follows from observing that the uncertainty affects both the winning probability and the winning prize, which are multiplied by each other, generating a convexity that makes the expected value of these options very high if the variance of the prior is high enough—because convexity of the evaluation induces a high expected value when the variance is high. *However, a strong preference for these options is not consistent with our data:* more than 65% prefer no uncertainty to double positively correlated uncertainty.

The discussion above highlights that there are two opposing forces at play when subjects evaluate options with positively correlated uncertainty on multiple dimensions. On the one hand, standard uncertainty aversion pushes against them. On the other hand, their expected value is higher the higher the variance of the priors used, even for "pessimistic" priors. Which of these forces is greater? We see no a priori reason to expect one force to be greater than another. One of the goals of this paper is to shed light on this question and actually test quantitatively the trade-off between these two forces.

We can therefore use the results of our paper to suggest possible *refinements* of the sets of priors. The flexibility of these models, or lack of predictive power, is desirable if there is substantial heterogeneity of behavior in the population. If, however, there exists some systematic, representative behavior that individuals are more likely to exhibit, then we would instead like to have models that capture this pattern—models

[3] All the changes described are significant at the 1% level.

with a higher predictive power. Identifying such regularities is then a first necessary step to understanding which additional restrictions should be imposed on existing models.[4] In §5, we show that for the MMEU model, the set of priors that are compatible with *all* the patterns we document must contain priors that are not only "pessimistic," as in any MMEU model that can explain the choices from Ellsberg urns, but also must satisfy some precise conditions on the relation between variance and expectation: Pessimistic priors must have a relatively high variance, ruling out the example above with degenerate priors, but they must also be "pessimistic" enough, ruling out the other example above. (Precise conditions are discussed in §5.) We interpret this condition as follows: Even though subjects are pessimistic in their evaluations, they are also aware of being pessimistic and that they might be wrong in being so, and therefore use priors with a relatively high variance—as opposed to priors that are pessimistic and have low variance, in which case they acted as if they were sure that they should be pessimistic. On the other hand, their pessimism is still strong enough that they rank no uncertainty above anything else. We refer to this behavior as *skeptical pessimism*.[5]

Our findings may have potential implications for decision making under uncertainty in applications. In most concrete economic settings, a decision maker cannot remove all uncertainty from the choice problems he faces. For example, even if the decision maker could choose a safe investment that guarantees a certain interest, the inflation rate may be uncertain, there may be uncertainty regarding the decision maker's need for liquidity (hence, the time of payment), etc. Our results suggest that in many circumstances, decision makers may be more likely to prefer uncertain prospects in which more dimensions are uncertain and correlated.

1.2. Outline and Related Literature

In the remainder of this section we discuss the related literature. Section 2 describes the experimental design. Section 3 presents the results of the lab experiment. Section 4 compares the lab results to the results of an online experimental treatment conducted via Amazon's Mechanical Turk. Section 5 relates our results to existing theoretical models. Concluding remarks are given in §6. The online appendix (available as supplemental material at http://dx.doi.org/10.1287/mnsc.2015.2240) contains supplementary analysis, the additional data not directly discussed in the main body of this paper, as well as the instructions and the screenshots of the experiment.

Our paper contributes to the literature on uncertainty attitudes over outcomes and delay. Ho et al. (2002) investigated the effect of framing a decision problem as a gain or a loss on the tendency to choose an uncertain option. Subjects were presented with hypothetical managerial cases in which they had to choose between two projects, one with incomplete (uncertain) and one with complete (risky) information on its internal rate of return. In the "outcome" treatment, the information was the actual rate of return, whereas in the "probability" treatment, the information was the probability that the rate was below/above some threshold. In both treatments, the question was framed once as a loss and once as a gain. They found that in the gain frame, subjects were more likely to choose the risky project in the "outcome" treatment than in the "probability" one. No significant differences were observed in the loss frame. This study had very different goals from ours, and there are several important differences in the design. Among them, in their experiment, the risky options differed across the two treatments, and the source of uncertainty was not specified to be the same; hence, participants' set of priors could be different across treatments. By contrast, in our study the uncertainty depends only on the composition of a *single* urn, and the risky option is always the same; this is essential for our analysis.

Attitudes toward uncertainty in delay are studied by Weber and Tan (2012), who used an analogue of the Ellsberg three-color experiment reformulated as a hypothetical choice between two delivery services, one with a known delivery time and another with an uncertain delivery time. Their results suggest that the majority of participants exhibited the Ellsberg pattern of choice, but the percentage was lower compared to that observed in the original Ellsberg urn design. Once again, as opposed to our experiment, the two decision problems—the original Ellsberg and the delivery-service analogue—are very different from each other, and participants may have a distinct set of beliefs over the two, whereas in our case we can directly compare attitudes toward uncertainty in different dimensions since they are all represented as choices between bets over the same urn.

Epstein and Halevy (2014) study choice between bets on the colors of two balls, each drawn from different urns. Subjects are given identical information about the two urns, but no information about

[4] To better illustrate, it is helpful to make the analogy to the experimental literature on repeated games. The various folk theorems establish that "almost anything" can be supported in equilibrium. But do individuals who actually engage in indefinite repeated interaction exhibit such heterogeneity of behavior? This question has given rise to a growing experimental literature that, like our work, investigates whether actual behavior tends to systematically select one of the predicted behaviors. These findings have in turns led to the development of various refinements meant to increase the predictive power of the theories.

[5] We thank Paolo Ghirardato for suggesting this term.

their composition or their relation. This allows them to study the attitude toward the uncertainty coming from the correlation between the two urns. They find that subjects are averse to this form of uncertainty, and that this aversion is associated with, but distinct from, classical uncertainty aversion as measured in the typical Ellsberg experiment.

The closest study to ours is the recent independent work of Eichberger et al. (2015). They study whether individuals who are faced with two unrelated and independent sources of uncertainty treat these two sources as independent of each other. They use a natural extension of the classical Ellsberg two-urn experiment in which a participant wins prize $x \in \{a, b\}$ if he guesses correctly, and $y \in \{a, b\}/\{x\}$ otherwise. They investigate whether the behavior of participants change if they do not know whether $x = a$ or $x = b$, where this uncertainty is determined independently of the composition of the urn. The authors find that significantly fewer subjects choose the risky urn when the prize is uncertain. This is consistent with our finding that fewer subjects choose the risky gamble in the presence of gambles with multiple uncertain dimensions (see §3.4). Although both this study and ours incorporate uncertainty in probability and prize, the two papers ask different questions and employ completely different designs. In particular, their experiment exploits explicitly distinct sources of uncertainty for prize and probabilities—also with the goal of testing whether subject consider them as independent. By contrast, as we mention above, in our analysis all uncertainty comes from a single urn.

2. Experimental Design

The lab experiment was carried out in the Social Science Experimental Laboratory (SSEL) at the California Institute of Technology during the month of February 2012. Participants were undergraduate students at Caltech, recruited from a pool of volunteer participants maintained by the SSEL. There were a total of 97 participants in four sessions, each lasting about 30 minutes. Each participant took part in only one session.

All sessions used the following procedure. The participants read instructions that were printed on paper, which they could refer to at any point in the experiment (a copy of the instructions appears in Online Appendix D). The experimenter stood in front of the participants and showed them an opaque cloth bag. Participants were told that the bag contained 60 colored poker chips. Of these, 20 were black, whereas each of the remaining 40 chips could have one of two colors: red or green. Let r denote the number of red chips in the bag, and let g denote the number of green chips. The participants knew neither the values of r and g nor how these values were determined. They were told that they could inspect the contents of the bag at the end of the experiment. At the end of the experiment, a single chip was drawn from the bag, and the participants' task was to make choices between groups of two or three gambles that depended on the composition of the bag and on the color of the chip extracted.

A gamble assigns to each color that could be extracted an amount of dollars m and a time of payment t. For simplicity of exposition, in what follows we shall use the following notation. Given the (uncertain) composition of the bag, the probabilities of extracting a black, red, and green chip are, respectively, 20/60, $r/60$, and $g/60$. We then denote a gamble by a triplet $(p, \$m, t)$, by which we understand the gamble that pays $\$m$ in t days with probability p, and \$0 otherwise.[6] Any of the three dimensions of a gamble can depend on the composition of the bag: for example, $(r/60, \$g, r \text{ days})$ denotes the gamble that pays $\$g$ in r days with probability $r/60$, i.e., if a red chip is extracted.

Participants made their choices on a computer using a custom-designed interface.[7] The decision tasks were divided into seven screens. Five of the screens displayed nine gambles arranged in three rows of three gambles; two screens had only six gambles arranged in two rows of three gambles. On each screen, for each row/column, a participant was asked to choose his most preferred gamble in the row/column. For screens with nine gambles, a participant was also asked to choose his most preferred gamble on the main diagonal (from top left to bottom right).[8] Figure 1 displays a screenshot of one of the seven screens. (Online Appendix F contains the remaining six screenshots.)

The choice problems in these screens can be categorized as follows.

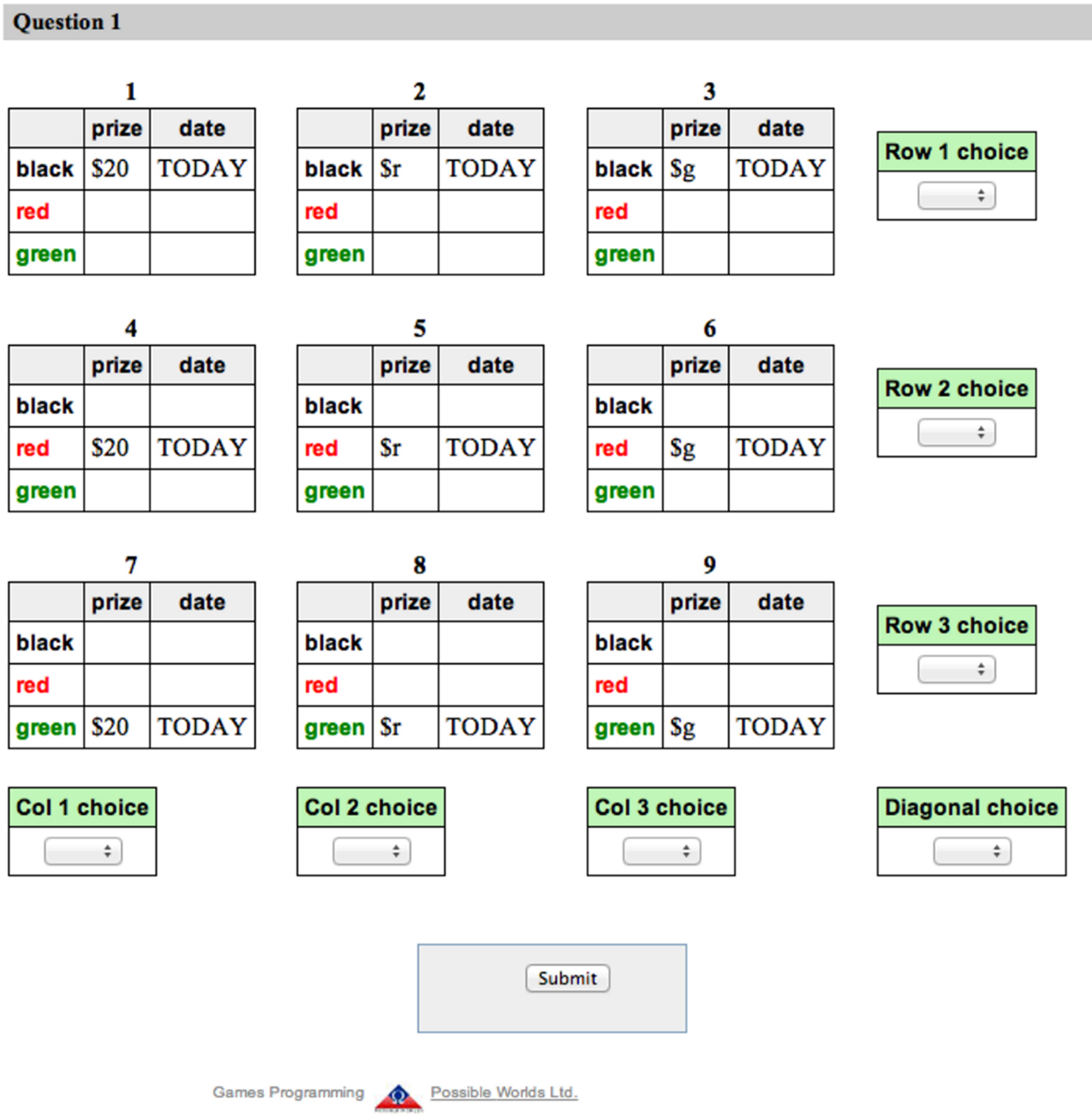
1. *Attitude toward "uncertainty in probability."* The participant chooses the probability of winning from $\{20/60, r/60, g/60\}$ for gambles in which the prize amount is fixed at x dollars and the date of payment is fixed at t days away, where x had values in $(20, r, g)$, and t had values in $(0, 20, r, g)$ (0 days means that they were paid at the end of the experiment).

[6] Each gamble was presented to the subjects as a 3×2 table, where each row corresponded to a color and the two columns represented the payment–date pair (see Figure 1). Moreover, since the term "gamble" may have a negative connotation, we used the term "lottery" in the experiment.

[7] This interface was programmed by Possible Worlds Ltd. and run on a Web browser.

[8] Hence, five screens consisted of seven choice problems (with three alternatives each), whereas two screens had five choice problems (three with two alternatives, and two with three alternatives).

Figure 1 (Color online) Screenshot of the Typical Screen



2. *Attitude toward "uncertainty in prizes."* The participant chooses the dollar amount of winnings from $\{20, r, g\}$ when the probability of winning is fixed at p and the date of payment is fixed at t days away, where p had values in $(20/60, r/60, g/60)$, and t had values in $(0, 20, r, g)$.

3. *Attitude toward "uncertainty in timing."* The participant chooses whether payments will be made in 20 days, r days, or g days, when the amount of winning is fixed at x dollars and the probability of payment is fixed at p, where x had values in $(20, r, g)$, and p had values in $(20/60, r/60, g/60)$.

4. *Attitude toward uncertainty in "two dimensions."* Each dimension may be viewed as having a value measured in units appropriate to that dimension: each unit in the probability dimension is $1/60$, each unit in the prize dimension is \$1, and each unit in the time dimension is one day. The participant chooses $x \in \{20, r, g\}$ such that the values in two dimensions are both x units, whereas the value of the third dimension is fixed and known (and zero when it is the date).

5. *Attitude toward uncertainty in "all three dimensions."* The participant chooses $x \in \{20, r, g\}$ such that the value of each dimension is x units.

In addition, in the last two screens, subjects were asked similar questions to measure their risk aversion.[9] All participants started with the screen in which all payments were made at the end of the experiment (depicted in Figure 1). This screen contained the standard Ellsberg question. Following this screen,

[9] These questions were very similar to the ones above except that they involved only bets on risky events. One of the screens tested whether subjects were risk loving or not, whereas the other screen tested whether subjects were risk averse or not.

each participant was assigned to a random sequence of the six remaining screens.[10] All participants completed a total of 45 choice problems. At the end of the experiment, for each participant the program randomly selected one of the 45 choice problem, and then the participants witnessed the experimenter draw a chip from the bag and show the content of the bag. This determined the amount won by the participants given the selected question and their choice in that question.[11]

All participants received a show-up fee of $5 at the end of the experiment. Any participant who won an additional amount to be paid with no delay also received this payment at the end of the experiment. Participants who earned additional amounts to be paid at a later date were given the choice between three methods of delivery: personally picking up a payment in cash from a staff member, receiving a check by mail, or being paid through PayPal (see the instructions in Online Appendix D for more detail).[12] Of note, a large majority of participants chose to pick up their payment in person from a staff member.

There are many possible ways of displaying the choice problems we are interested in. Our choice of display is motivated by the following considerations. First, we wanted to minimize inconsistencies due to errors or carelessness. If participants were presented with just a vertical list of choice problems, they would be more likely to miss inconsistencies in their answers. Our display gives participants a better opportunity to internalize the relationship between different choice problems that have common elements. Similarly, we wanted to minimize the possibility that the effect of uncertainty in a fixed dimension (in the sense that the participant's decision cannot remove this uncertainty) on choice could be due to mistake or inattention. The matrix display emphasizes the multidimensionality of the problem, thus increasing the likelihood that participants would understand that they can remove uncertainty in some dimensions but not in others.

In all the screens that involved uncertain bets, the first option (leftmost or top) corresponded to no uncertainty in the relevant dimension (i.e., betting on black), the middle option corresponded to betting on red, and the last option corresponded to betting on green. Since we are interested in the question of whether individuals who are uncertainty averse in probabilities are willing to take on uncertainty in other dimensions, we wanted to make it difficult to switch away from the no-uncertainty decision. Participants who are inattentive and just want to finish the task are more likely to just click on the first option, because options are chosen from a drop-down menu. Thus, by keeping no uncertainty as the first option, we minimize the risk of misinterpreting such inattentive participants as being uncertainty seeking.

Fixing the positions of the red and green options allows us to check whether participants are sensitive to the positive and negative correlations across the uncertain dimensions net of order effects. For example, consider the problem of guessing what color chip will be drawn when a correct guess pays $\$r$. As explained later, we will interpret a guess of red as choosing "more exposure to uncertainty" and a guess of green as choosing to "hedge against uncertainty." If we allow the two colors to appear in a different order, it is hard to disentangle whether this is due to order effects. If, instead, we fix the order and we compare with the identical question in which the prize is fixed at $\$g$ and find the same results, then we know it cannot be due to order effects.[13]

3. Results

Our analysis of the data proceeds as follows. We start with a basic analysis of the consistency of choices. Next, we analyze participants' attitudes toward uncertainty when only a single dimension (prize, date, or probability) is uncertain. We then turn to analyze how participants react to the contemporaneous presence

[10] Only after a participant completed making all choices in a screen could he press a button to take him to the next screen. Once a participant left a screen, he could not return to it.

[11] The use of a random incentive mechanism, which pays for only one randomly chosen answer, is standard in the literature. Some recent papers (Azrieli et al. 2014, Bade 2014, Baillon et al. 2014) studied whether they are incentive compatible in the presence of uncertainty. The concern is that in this case subjects may have an incentive to hedge between answers. Azrieli et al. (2014) show that the mechanism is incentive compatible if subjects never choose dominated gambles. Although our procedure does not allow us to test for this property, we partially follow the recommendation of Baillon et al. (2014), who show that sufficient conditions for the random mechanism to be incentive compatible under uncertainty aversion (with mild assumptions) are that (1) the randomization about the question to be paid is carried out before the state of nature is resolved (which is what we do), and (2) both the question and the state are realized before subjects make their decisions (our procedure, like virtually all papers in the literature, does not satisfy this part).

[12] Note that in our experiment, whenever a participant had the option to choose an uncertain date of payment, the non-uncertain date was a known delay of 20 days; that is, all options involved some delay. Hence, it is not crucial for the purpose of this experiment to equate the transaction cost of payment today versus payment in the future. Since participants may differ in their ranking of the various payment methods, we allowed participants to choose their preferred method (rather than impose one of the three). Thus, if we normalize the transaction cost of the most preferred method, then each participant is facing this normalized cost.

[13] Alternatively, one could randomize the order and then check whether the location of the question had a statistically significant effect on choice. The disadvantage of this approach is that there would be fewer observations for each choice, because a choice would involve not only the color chosen, but also the location of the color in the display.

of multiple forms of uncertainty. First, we *fix* one dimension to be uncertain and investigate whether the attitude toward uncertainty in another dimension is affected. Second, we study the agent's preferences between options with no uncertainty and options with different forms of uncertainty at the same time. Overall, in this section we discuss only the relevant portion of our data. Online Appendix B contains the answers to all other questions not directly reported here.

3.1. Preliminaries

We begin by discussing some basic properties of our data. In terms of risk attitudes, we label as "weakly risk-loving" participants who chose a gamble that pays \$40 in 20 days if a black chip is drawn (and \$0 otherwise) over a gamble that pays \$20 in 20 days if a red or green chip is drawn (and \$0 otherwise). A participant was labeled "strictly risk averse" if he chose a gamble that paid \$13 in 20 days no matter which chip was drawn over a gamble that paid \$40 in 20 days if a black chip was drawn (and \$0 otherwise). About 68% of all lab participants were *not* weakly risk loving, and about 44% were strictly risk averse. Thus, about a quarter of the participants could be labeled as risk neutral or very mildly risk averse.[14]

Because the experiment is designed to be entirely symmetric with respect to the two uncertain colors, red and green, with the exception of the order in which options are presented (one color always appears before the other), we should expect participants to treat these colors symmetrically. This seems to be the case in our data. First, bets on either color are not significantly different. Second, the behavior when one dimension is fixed and depends on the number of red chips is essentially identical to the behavior when it depends on the number of green chips.[15]

We also investigated to what extent participants gave consistent answers. One form of consistency is transitivity in the answers. We find that about 64% of participants gave transitive answers across all questions in the seven screens they faced.[16] A second form of consistency concerns answers to identical questions. One of the questions in the experiment was asked three times, and 71% of the participants gave consistent answers. This consistency is much stronger in aggregate: essentially identical proportions apply to the three questions. Finally, we verified that most participants gave the same answers to questions that differed only in the color in which the uncertainty was fixed.[17]

In what follows we report our results for the entire participant pool. Results are qualitatively and in most cases also quantitatively very similar if we restrict attention to only transitive participants.

3.2. Attitude Toward Uncertainty in Each Dimension

We now turn to investigate how participants approach uncertainty when only a single dimension of the gamble is uncertain. In what follows, we say that a participant is averse to uncertainty in a choice problem if he chooses the gamble with the most objective information: for example, in the classical Ellsberg experiment a participant is classified as uncertainty averse if they choose to bet on a black ball.[18] Table 1 summarizes the aggregate results.[19] The next observation follows.

Observation 1. The majority of participants are averse to uncertainty in each dimension. However,

[14] The two screens that presented the risk-attitude question also tested whether participants changed their decisions if the date of payment was uncertain (see screenshots in Online Appendix F). We found no significant change in the answers.

[15] The only exception is a difference between red and green for choosing the date when everything else is fixed: when choosing between $(20/60, \$20, 20)$, $(20/60, \$20, r)$, and $(20/60, \$20, g)$, over twice as many participants choose the second compared to the third option (33 versus 14). We see no obvious explanation for this asymmetry, which is confined to this specific question.

[16] For a given screen, there are 3^7 ways to answer (there cannot be violations of transitivity in the two screens testing for risk attitudes). Of these, 1,176 (53.77%) are intransitive. We should emphasize that a violation of transitivity in this context need not represent a violation of "rationality," but could simply be due the fact that participants are indifferent, say, between betting on red or green, but break indifferences in different ways depending on the questions.

[17] We also checked whether the distribution of responses for each of question in a matrix is independent of the position that the matrix appears in (2nd, ..., 7th) by running 38 Fisher exact tests. None of these tests were significant at the 1% level, and only three (out of the 38) were significant at the 5% level.

[18] Assuming symmetric priors on the distribution of red and green, a subject is uncertainty averse if and only if she prefers to bet on black. If beliefs are not symmetric, then any uncertainty-loving subject should choose either red or green, and we can classify those who choose black as uncertainty averse. On the other hand, a subject could be uncertainty averse but choose to bet on red or green if she has asymmetric priors. Thus, the results that follow could be seen as a lower bound of the number of uncertainty-averse subjects. Notice, however, that if beliefs are asymmetric, they should be asymmetric for all questions, and such asymmetry should play a role in the other questions as well.

[19] The question in which participants are asked their attitude toward uncertain dates, i.e., to choose between $(20/60, \$20, 20)$, $(20/60, \$20, r)$, and $(20/60, \$20, g)$, was actually asked three times during the lab experiment. Table 1 contains the average answers. Answers are very consistent: the fraction of participants choosing $(20/60, \$20, 20)$ ranges between 51% and 55%, the fraction choosing $(20/60, \$20, r)$ between 33% and 36%, and the fraction choosing $(20/60, \$20, g)$ between 12% and 16%. By analyzing individual questions, 71% of participants exhibit the same attitude in all three questions (the remaining 29% of participants might have given different answers because they were indifferent and broke the indifference in a different way in one of the three questions).

Table 1 Attitude Toward Unidimensional Uncertainty

		(1)	(2)	(3)	(4)	(5)
The color (x) chosen by the participant	Probability	$x/60$	20/60	$x/60$	20/60	20/60
	Amount	\$20	\$$x$	\$20	\$$x$	\$20
	Date	0 days	0 days	20 days	20 days	x days
Black[a] (%)		76	82	73	75	52
Red (%)		11	9	13	15	34
Green (%)		12	8	13	9	14

Notes. We can conduct tests of similarity of the answers by testing whether the probability of choosing black versus another color is statistically different in each column. A test of proportions returns that column (5) is statistically different form all others ($p < 0.001$ for comparisons); other columns are not (columns (1) and (2), $p = 0.38$; columns (1) and (3), $p = 0.62$; columns (1) and (4), $p = 0.86$; columns (2) and (3), $p = 0.17$; columns (2) and (4), $p = 0.30$; columns (3) and (4), $p = 0.75$).
[a]The non-uncertain option.

the proportions of uncertainty-averse participants vary across dimensions: whereas we find no significant difference in the proportion of subjects averse to uncertainty in probability and prizes, the proportion of subjects averse to uncertainty in the time dimension is significantly lower.

Observation 1 is based only on aggregate data. This raises the question of whether, at an individual level, the set of individuals who are averse to single-dimensional uncertainty expands as we change the uncertain dimension from dates to probability to prize. This is approximately true: between 89% and 92% of participants who are uncertainty averse over dates are also uncertainty averse over prizes, and between 84% and 86% are also uncertainty averse over probabilities;[20] about 91% (67/74) of participants who are uncertainty averse over probabilities are also uncertainty averse over prizes. This suggests that, with some approximation, we can partition our data set into four groups: those participants who are not uncertainty averse to any dimensions, those participants who are uncertainty averse only over prizes, those who are averse to uncertainty in prizes and probabilities, and those who are averse to all ambiguities. The last group contains at most 52% of the pool.

Notice that our data are split 50–50 between participants who are averse to uncertainty (only) in the date to those who are not. One possible explanation for this is that there is heterogeneity in the agent's patience and beliefs on the composition of the bag. Under the assumption that participants' preferences are additively separable over time with a standard exponential discount factor δ, in §5 we show that the following is true: For a given prior belief on the number of red chips r in the bag, there exists a threshold discount factor δ^* such that the risky gamble is preferred to a gamble in which only the payment date is unknown if and only if the discount factor is at least δ^* (where the value of the threshold δ^* depends on the prior belief); that is, even allowing subjects to hold "pessimistic" beliefs, if they are impatient enough, then they will prefer the uncertain gamble. Intuitively, this is due to the fact that exponential discounting is convex, inducing a preference for variance. Therefore, our data suggest that about half of our subjects are impatient enough to prefer the gamble with an uncertain payment date.[21]

Finally, note that the relatively high proportion of aversion to uncertainty (only) in prizes may be due not to a higher aversion to uncertainty, but to risk aversion. To see why, note that a decision maker who is uncertainty *neutral* and follows expected utility would still dislike uncertainty over prizes just by risk aversion.[22] However, this does not seem to be the main driving force in our data: if we redo Table 1 for risk-averse participants, we obtain very similar proportions. Redoing it for risk-loving participants yields Table 2, where the proportion of participants who are averse to uncertainty in probability remains the same, but the proportion of participants who are averse to uncertainty in prizes is still high but decreases, in line with the intuition that risk loving increases the attractiveness of the uncertain options in this case.[23]

3.3. "Separability" in the Dimensions of Uncertainty

We now turn to analyze whether the dimensions of uncertainty are "separable" in the sense that the

[20] In the case of aversion toward uncertain dates, we have a range of answers because, as we mentioned above, the question over uncertain dates was asked three times. More precisely, the fractions of subjects who are uncertainty averse over prizes among those who are uncertainty averse over dates are 43/49, 45/49, and 47/53; the fractions of subjects who are uncertainty averse over probabilities among those who are uncertainty averse over dates are: 42/49, 41/49, and 45/53.

[21] Note that even though the time horizon in our experiment is at most 40 days, past experiments have shown that lab participants tend to exhibit abnormally low discount factors (see Camerer 1995). Hence, there is no a priori reason to expect most participants in the lab experiments to necessarily exhibit aversion to uncertainty in the payment date.

[22] We also checked whether winning for sure affects the aversion to uncertainty on the prize (i.e., we asked participants to choose between a sure prize of \$20, a sure prize of \$$r$, and a sure prize of \$$g$ and compared this with their choice in the case in which the prize is paid only if the black ball is extracted). The results are virtually identical ($p = 0.980$ with a chi-square test).

[23] In line with this discussion, chi-square tests on whether subjects choose black with the same frequency depending on whether they are risk averse or not yield the following: columns (2) and (4) are different (p-values, 0.009 and 0.001); columns (1), (3), and (5) are not (p-values, 0.74, 0.88, and 0.77, respectively).

Table 2 Attitude Toward Unidimensional Uncertainty for Risk-Loving Participants

		(1)	(2)	(3)	(4)	(5)
The color (X) chosen by the participant	Probability	$x/60$	20/60	$x/60$	20/60	20/60
	Amount	\$20	\$$x$	\$20	\$$x$	\$20
	Date	0 days	0 days	20 days	20 days	x days
Black[a] (%)		74	68	74	55	48
Red (%)		10	19	10	32	33
Green (%)		16	13	16	13	18

[a]The non-uncertain option.

presence of a (fixed) uncertainty in one dimension affects the uncertainty attitude of the agent in another dimension. To illustrate, let us compare two choice situations in which the agent chooses the color to bet on: (1) the standard Ellsberg question, in which the agent chooses between $(20/60, \$20, 0)$, $(r/60, \$20, 0)$, and $(g/60, \$20, 0)$, and (2) the identical question in which the prize is fixed at $\$r$ instead of \$20, i.e., the choice between $(20/60, \$r, 0)$, $(r/60, \$r, 0)$, and $(g/60, \$r, 0)$. Will the decision maker choose the same color in the two decision problems? If he does, we say that he exhibits a "separable" attitude toward uncertainty in probability.

We do not consider this separability either a priori obvious or normatively desirable. For example, an agent who chooses to bet on black in the first (Ellsberg) problem might decide to bet on green in the second one because he is already "exposed" to the number of red chips via the uncertainty in prizes, and may prefer to "hedge," or "reduce his exposure," by betting on green: if there are few red chips, he might win a small amount, but at least he wins with a high probability.[24] Alternatively, this individual might instead decide to bet on red to "increase his exposure": if there are many red chips, he wins a large sum with a large probability, whereas if there are few red chips, he wins a small amount with a small probability. In particular, we say that an individual "switches to more exposure" if he bets on black when the prize is \$20 and the date is known, but he bets on red (green) in a decision problem where the prize is $\$r$ ($\$g$) and the date is known.

The notions of separability, hedging, and more exposure can naturally be generalized to all of our questions. We will follow this terminology in describing the results. Notice, however, that when the date is known, choosing more exposure means choosing a gamble where the prize and probability both depend positively on the *same* color; by contrast, when the date is uncertain, choosing more exposure means choosing the option for which the date depends on a *different* color than the prize or the winning probability—because earlier dates are better, and thus the agent wants a smaller value for the date.[25]

Table 3 displays aggregate choices in decision problems with uncertainty in probability, prize, and date, both when all other dimensions are known and when they are fixed and uncertain. (The notes of the table include statistical tests of the differences between the answers for the various questions in each column.) Table 4 contains the data at an individual level: for each (relevant) pair of questions, we look at the proportion of subjects who chose each color in each question, generating a 3×3 matrix, where the main diagonal displays the fraction of participants who chose the same color in both choice problems.[26]

We start from aggregate data, which reveal a clear pattern of choice.

Observation 2. Compared to a choice problem that includes a gamble with no uncertainty (and where two of the three dimensions are fixed and certain), making one of the dimensions uncertain (but fixed) leads to a significant change in behavior, most of which is in the direction of more exposure.

To illustrate this observation, note that 76% of the participants exhibit the standard Ellsberg pattern of betting on black when the prize is \$20 and the payment date is today. Only 11% choose to bet on red in this choice problem. However, when the prize is changed to $\$r$ (the number of red chips), more than half of the participants who chose to bet on black (i.e., the risky gamble) in the original choice problem now switch to bet on red—thus, more exposure. Only 28% of the participants bet on black both when the prize is \$20 and when it is $\$r$ (i.e., they continue to choose the gamble with the least uncertainty).

A similar effect is observed when *two* dimensions are made uncertain (see, for example, the bottom

[24] To see why a participant may decide to "hedge," assume he has max-min expected utility preferences (see §5) with linear utility and only two priors: probability 1 on $r = 1$, and probability 1 on $r = 39$. If he chooses either $(r/60, r, 0)$ or $(g/60, g, 0)$, his expected utility is 1/60. If, however, he chooses $(r/60, g, 0)$ or $(g/60, r, 0)$, his expected utility is 39/60.

[25] For example, if the winning probability is fixed at 20/60 and prize is fixed at $\$r$, then the participant opts for more exposure if he chooses to be paid in g days: with the latter bet, when r is high, the prize is high and is paid soon (g is small), and when r is low, the prize is low and it is delayed.

[26] As mentioned above, the choice between $(20/60, \$20, 20)$, $(20/60, \$20, r)$, and $(20/60, \$20, g)$ was asked three times during the experiment, in different screens. Answers were very consistent. Table 3 contains the average answers. Table 4, instead, uses the answers subjects gave in one of the screens (Screen 2, the earliest one), since if we averaged them the probabilities would not sum to 100%. Identical tables constructed using the answers given in the other two screens (3 and 5) can be found in Table A.6 in Online Appendix A. Results are essentially the same.

Table 3 Effect of a Fixed Uncertain Dimension (Aggregate)

Choosing prob.		Choosing prize		Choosing date	
(20/60, \$20, 0)	76%	(20/60, \$20, 0)	83%	(20/60, \$20, 20)	52%
(r/60, \$20, 0)	11%	(20/60, \$$r$, 0)	9%	(20/60, \$20, r)	34%
(g/60, \$20, 0)	12%	(20/60, \$$g$, 0)	8%	(20/60, \$20, g)	14%
(20/60, \$$r$, 0)	31%	(r/60, \$20, 0)	36%	(r/60, \$20, 20)	29%
(r/60, \$$r$, 0)	52%	(r/60, \$$r$, 0)	52%	(r/60, \$20, r)	15%
(g/60, \$$r$, 0)	18%	(r/60, \$$g$, 0)	12%	(r/60, \$20, g)	56%
(20/60, \$20, 20)	73%	(20/60, \$20, 20)	75%	(20/60, \$$r$, 20)	30%
(r/60, \$20, 20)	13%	(20/60, \$$r$, 20)	15%	(20/60, \$$r$, r)	22%
(g/60, \$20, 20)	13%	(20/60, \$$g$, 20)	9%	(20/60, \$$r$, g)	48%
(20/60, \$20, r)	49%	(20/60, \$20, r)	63%	(r/60, \$$r$, 20)	19%
(r/60, \$20, r)	21%	(20/60, \$$r$, r)	18%	(r/60, \$$r$, r)	24%
(g/60, \$20, r)	30%	(20/60, \$$g$, r)	20%	(r/60, \$$r$, g)	58%

Notes. We can perform a chi-square test of the difference of the answers, testing whether, for each column, the answer to the first question is different from that of the second one, and the third is different from the fourth. (As noted above, the first question in the last column was asked three times. For these tests, we use the first time it was asked. Almost identical results hold using the other times.) All differences are statistically significant at the 1% level, except for the following: for the second column (choosing price), in the case of the third and fourth questions we have a $p = 0.092$, and for the third column (choosing date), the third and fourth questions are not different ($p = 0.178$).

Table 4 Effect of a Fixed Uncertain Dimension (Individual)

		(20/60, 20, 0)	(r/60, 20, 0)	(g/60, 20, 0)	
Choose PROB.–	(20/60, r, 0)	0.28	0.02	0.01	0.31
Effect of fixed	(r/60, r, 0)	0.40	0.05	0.06	0.52
ambiguous PRIZE	(g/60, r, 0)	0.08	0.04	0.05	0.18
		0.76	0.11	0.12	1
		(20/60, 20, 20)	(r/60, 20, 20)	(g/60, 20, 20)	
Choose PROB.–	(20/60, 20, r)	0.45	0.02	0.02	0.49
Effect of fixed	(r/60, 20, r)	0.08	0.08	0.04	0.21
ambiguous DATE	(g/60, 20, r)	0.20	0.03	0.07	0.30
		0.73	0.13	0.13	1
		(20/60, 20, 0)	(20/60, r, 0)	(20/60, g, 0)	
Choose PRIZE–	(r/60, 20, 0)	0.36	0.00	0.00	0.36
Effect of fixed	(r/60, r, 0)	0.41	0.06	0.04	0.52
ambiguous PROB.	(r/60, g, 0)	0.05	0.03	0.04	0.12
		0.82	0.09	0.08	1
		(20/60, 20, 20)	(20/60, r, 20)	(20/60, g, 20)	
Choose PRIZE–	(20/60, 20, r)	0.62	0.01	0.00	0.63
Effect of fixed	(20/60, r, r)	0.08	0.07	0.02	0.18
ambiguous DATE	(20/60, g, r)	0.05	0.07	0.07	0.20
		0.75	0.15	0.09	1
		(20/60, 20, 20)	(20/60, 20, r)	(20/60, 20, g)	
Choose DATE–	(r/60, 20, 20)	0.24	0.03	0.02	0.29
Effect of fixed	(r/60, 20, r)	0.03	0.09	0.03	0.15
ambiguous PROB.[a]	(r/60, 20, g)	0.24	0.21	0.11	0.56
		0.51	0.33	0.16	1
		(20/60, 20, 20)	(20/60, 20, r)	(20/60, 20, g)	
Choose DATE–	(20/60, r, 20)	0.27	0.02	0.01	0.30
Effect of fixed	(20/60, r, r)	0.07	0.12	0.02	0.22
ambiguous PRIZE[a]	(20/60, r, g)	0.16	0.19	0.13	0.48
		0.51	0.33	0.16	1
		(20/60, 20, 20)	(20/60, 20, r)	(20/60, 20, g)	
Choose DATE–	(r/60, r, 20)	0.15	0.01	0.02	0.19
Effect of fixed	(r/60, r, r)	0.08	0.11	0.04	0.24
Ambiguous PRIZE	(r/60, r, g)	0.27	0.21	0.10	0.58
and ambiguous PROB.[a]		0.51	0.33	0.16	1

[a]The columns' choice problem was asked three times during the experiment. The table contains the answers given when the question was asked in the earliest screen, Screen 2. Identical tables with the answers to the same choice problem only from Screens 3 and 5 can be found in Table A.6 in Online Appendix A. Results are essentially the same.

Table 5 Multivariate Logit Regressions in Which the Dependent Variable Is the Log Ratio of Choosing the Uncertain Color Red Relative to the Non-uncertain Color Black When the Participants Can Affect the Uncertainty of One Dimension Only, Controlling for Whether Other Fixed Dimensions Are Uncertain or Not

Choosing probability	Coeff. (std. err.)	RRR	Choosing prize	Coeff. (std. err.)	RRR	Choosing date	Coeff. (std. err.)	RRR
Constant	−1.7987 (0.2773)		Constant	−1.7987 (0.2472)		Constant	−0.4308** (0.1855)	
Amb. prize red	2.3095*** (0.3486)	10.069	*Amb. prob. red*	2.1440*** (0.2876)	8.534	*Amb. prob. red*	−0.1943 (0.2690)	0.824
Amb. prize green	0.8179** (0.3519)	2.266	*Amb. prob. green*	0.4790 (0.3434)	1.276	*Amb. prob. green*	1.0708*** (0.2115)	2.918
Amb. date red	0.9232*** (0.2843)	2.517	*Amb. date red*	0.1779 (0.2311)	2.241	*Amb. prize red*	0.1599 (0.2278)	1.173
Amb. date green	1.5039 (0.2550)	4.499	*Amb. date green*	0.7777*** (0.1828)	1.842	*Amb. prize green*	0.8812*** (0.1730)	2.414
						Amb. prob. and prize red	0.6759** (0.3080)	1.966
						Amb. prob. and prize green	1.14173*** (0.2659)	4.126
N	582		*N*	873		*N*	1,164	
Number of subjects	97		Number of subjects	97		Number of subjects	87	
Log pseudolikelihood	−532.2324		Log pseudolikelihood	−715.4545		Log pseudolikelihood	−1,166.459	

Notes. Errors are clustered by subject. The constant denotes the benchmark case in which the fixed dimensions are not uncertain. RRR denotes the relative risk ratio. *Amb.* denotes ambiguity.
**Significant at the 5% level; ***significant at the 1%level.

matrix in Table 4). About half of the participants choose a payment date of 20 days over r and g days when a prize of \$20 is awarded if a black chip is drawn. When the prize and winning probabilities are changed to \$$r$ and $r/60$, about half of the participants who chose a date of 20 now choose a date of g days, thereby choosing a gamble in which the prize, probability, and how soon the payment is made are all uncertain and perfectly correlated. In fact, close to 60% of all participants choose this triple-uncertainty gamble.

The only case in which making one of the fixed dimensions uncertain has a small effect is when subject decide on the prize—\$20, \$$r$, or \$$g$—and the date turns uncertain. In this case, 75% of participants choose a prize of \$20 when the payment date is 20 days away, whereas 63% choose it when it is r days away.[27] Indeed, according to Table 4, making the date uncertain appears to have the smallest impact on choices relative to making the prize or the probability uncertain.

While Table 3 only shows us the effects of fixing one dimension to be uncertain and equal to the number of red chips, in the experiment we also ask the corresponding questions in which the fixed uncertain dimension is equal to the number of green chips. Results are remarkably similar, confirming the robustness of these findings. (For brevity, this table appears in Online Appendix A, Table A.1.) In fact, nearly every participant who switches from betting on black, when one of the fixed dimensions is positively correlated with the number of red chips, also tends to switch *in the same direction* (i.e., toward more exposure or hedging) when the same fixed dimension is positively correlated with the number of greens. (This is illustrated in Table A.2 in Online Appendix A.)

Since the results above may be driven by heterogeneity of preferences types (i.e., there may be particular distributions of subjects with different attitudes to uncertainty that would be consistent with the aggregate proportions above), we now turn to analyze behavior at an individual level. This appears in Table 4. If we focus on an agent who has chosen a particular color to affect the uncertainty in one dimension when the other dimensions are known, we can analyze which color he would tend to choose when one other dimension is fixed and uncertain. From the table it is clear how participants change the color to bet on in the direction of *more exposure*.

We conclude our analysis by estimating the effect of introducing fixed uncertain dimensions using multinomial logistic regressions (with errors clustered by subjects), to account for the possibility of random errors. Table 5 displays the estimation results for each of the three dimensions when participants could affect the uncertainty in only a single dimension. (Table A.3 in Online Appendix A displays estimation results of linear regressions.)[28]

[27] Using a chi-square test to test whether they are a different distribution returns a p-value of 0.092. See the caption of Table 3 for more statistical tests on the difference between the answers.

[28] These regressions estimate the log ratio of choosing the uncertain color red relative to the non-uncertain color black. The constant

To better understand these estimates, consider the case in which the participant chooses red when the only dimension he can affect is the winning probability. The constant term in this case is -1.8, which is the log of the ratio of participants who choose red when all fixed dimensions are objectively known (0.12) to the ratio of participants who choose black in this case (0.75). When the prize is changed from \$20 to $\$r$, the distribution of choices changes to 31% on black and 52% on red. This corresponds to a log ratio of 0.51, which is 2.31 times higher than the constant. This is the coefficient of the variable *Amb. prize red*, which is positive and significant (at 1%), reflecting the fact that the log odds ratio of choosing red has gone up when the winning probability was changed from 20/60 to $r/60$. These findings support our conclusions from the above analysis of individual behavior. Tables A.4 and A.5 in Online Appendix A display the regression analysis for choosing the color green, which is essentially identical, and for the remaining cases that were covered by Table 3 above. These regressions also support our conclusions above on the propensity to choose more exposure.

Finally, we should emphasize that we can interpret bets as having more or less exposure to uncertainty in a way that is independent of the participant's beliefs only because in our experiment all uncertainty—whether it is on probability, prize, date or any combination of these dimensions—is determined by the *same random variable*: the number of red (green) chips in the urn. To understand the importance of this, consider an alternative environment where each dimension of the gamble is determined by a different random variable. For example, suppose there were two separate urns with 60 chips each, both of which contain 20 black chips, but with possibly different distributions of red and green. Now consider a bet that pays if a given color is drawn from one urn, but the monetary prize (to be paid immediately) equals the number of red chips in the second urn. In this example, whether betting on red on the first urn means more or less exposure to uncertainty depends on the participant's beliefs regarding the *joint* distribution over the composition of the two urns (e.g., a participant may believe that the number of red chips in one urn equals the number of green chips in the other, or that the two urns are identical).[29]

term estimates this log ratio in the benchmark case where the fixed dimensions are not uncertain (e.g., \$20 and 20 days when participant chooses probability). Table 5 reports both the coefficients and the relative risk ratio. (The relative risk ratio is the exponential of the regression coefficient; it is often interpreted as representing how the probability of choosing the dependent variable relative to zero changes if we increase the regressor by one unit.)

[29] This alternative environment is similar to the situation studied in Eichberger et al. (2015).

3.4. Attitude Toward Compound Uncertainty

We now turn to analyze the case in which participants face the choice between no uncertainty and gambles having uncertainty in more than one dimension. Although our analysis in the previous section shows that participants might prefer to have multiple uncertain dimensions to only one uncertain dimension, we have not yet discussed how gambles with multiple uncertain dimensions compare with gambles that have no uncertain dimensions. Table 6 displays this comparison, leading to the following observation.

Observation 3. The majority of participants choose the risky gamble over gambles in which multiple dimensions are uncertain.

Observation 3 above shows that, even though we have seen that a large fraction of participants are attracted to an uncertain gamble where the prize is perfectly correlated with the winning probability, participants still prefer options with no uncertainty. Consider, for example, the gamble that pays $\$r$ today if a red chip comes up. About 52% of participants choose this gamble when they can only affect the *winning probability* when the prize and date are fixed at $\$r$ and zero days, respectively. Exactly the same percentage chooses this gamble when participants can only affect the *prize* while the winning probability and date are fixed at $r/60$ and zero days, respectively. However, the proportion choosing $(r/60, \$r, 0)$ drops to 18% when the risky gamble $(20/60, \$20, 0)$ is also available.

Even if the majority of participants prefer the option with no uncertainty, if we compare Table 6 with Table 1, we observe that the fraction of participants who choose the no-uncertainty gamble $(20/60, \$20, 20)$ drops from 75% to 64% when $(20/60, \$r, 20)$ and $(20/60, \$g, 20)$ are replaced with $(r/60, \$r, 20)$ and $(g/60, \$g, 20)$. Instead, when the time dimension becomes also uncertain—i.e., when $(r/60, \$r, 20)$ and $(g/60, \$g, 20)$ are replaced with $(r/60, \$r, r)$ and $(g/60, \$g, g)$—the percentage of participants who opt for no uncertainty rises from 64% to almost 80%.

These behavioral patterns are also confirmed at the individual level.

Table 6 Choice Between No Uncertainty and Multidimensional Uncertainty

Ambiguity in prob and prize		*Ambiguity in prize and date*	
$(20/60, \$20, 20)$	64%	$(20/60, \$20, 20)$	77%
$(r/60, \$r, 20)$	21%	$(20/60, \$r, r)$	15%
$(g/60, \$g, 20)$	15%	$(20/60, \$g, g)$	7%
Ambiguity in prob and date		*Ambiguity in prob, prize, and date*	
$(20/60, \$20, 20)$	71%	$(20/60, \$20, 20)$	78%
$(r/60, \$20, r)$	19%	$(r/60, \$r, r)$	14%
$(g/60, \$20, g)$	10%	$(g/60, \$g, g)$	7%

Table 7 **Effect on Choosing No Uncertainty of Allowing Uncertainty in Multiple Dimensions**

Choose prob.	Coefficient	$P > z$	Choose prize	Coefficient	$P > z$	Choose date	Coefficient	$P > z$
Prob and prize	-0.445^{**} (0.198)	0.025	*Prize and prob*	-0.627^{***} (0.159)	0.000	*Prob and prize*	0.840^{***} (0.247)	0.001
Prob and date	-0.183 (0.240)	0.445	*Prob and date*	0.020 (0.133)	0.879	*Prob and date*	1.224^{***} (0.230)	0.000
Constant	1.085^{***} (0.206)	0.000	Constant	1.266^{***} (0.213)	0.000	Constant	0.062 (0.173)	0.721
N	485		*N*	679		*N*	679	
No. of subjects	97		No. of subjects	97		No. of subjects	97	
Log pseudol.	-292.9691		Log pseudol.	-379.69574		Log pseudol.	-428.39992	

Note. Each column is a multinomial logit with errors clustered by subject. Pseudol. denotes the psuedolikelihood.
**Significant at the 5% level; ***significant at the 1% level.

Observation 4. Consider the set of participants who (i) choose no uncertainty when they can affect the uncertainty in only a single dimension, while the remaining dimensions are objectively known, but (ii) choose an uncertain gamble with more exposure when one of the fixed objective dimensions becomes uncertain. Most of these participants prefer no uncertainty to an uncertain gamble with more exposure.

In particular, the only situation in which multiple uncertainty with more exposure is available alongside no uncertainty is when there is uncertainty in probability and prize. When choosing prizes, there are 40 participants (out of 97) who "switch" from the known color to the more uncertain one with more exposure when the probability becomes fixed at $r/60$. Of these, however, 28 choose no uncertainty to multiple uncertainty with more exposure when both are available. (Similarly, 39 switch when the probability is fixed at $g/60$, and 29 of them choose no uncertainty over multiple uncertainty.) When choosing probabilities, there are 40 participants who switch from the known color to the uncertain one with more exposure when the prize becomes fixed at $\$r$. Of these, 31 choose no uncertainty over multiple uncertainty with more exposure when both are available. (Similarly, 39 switch when the prize is fixed at $\$g$, and 29 of them choose no uncertainty over multiple uncertainty.)

To take a systematic account of possible errors, and to better understand the impact of allowing for uncertainty on multiple dimensions, we ran a series of multinomial regressions, with errors clustered by subject (linear regressions return essentially identical results). The results are summarized in Table 7.[30] Consistent with our observations above, relative to the case in which participants choose either the winning probability or the prize amount (while the remaining dimensions are known), allowing for uncertainty in both dimensions significantly lowers the fraction who choose no uncertainty, whereas allowing for uncertainty in the date has no significant effect. In contrast, compared to the case in which participants choose only the date, allowing for uncertainty in prize or probability significantly increases the fraction who choose no uncertainty.

4. A Robustness Treatment

A variant of the lab experiment was also conducted on Amazon.com's Mechanical Turk platform (see https://www.mturk.com). We view these data as complementary to our Caltech lab findings and as evidence on the robustness of these findings. Although MT is a nonstandard source of data in experimental economics, several recent studies have shown that these data can be quite reliable (see, e.g., Berinsky et al. 2012, Horton et al. 2011, Paolacci et al. 2010). A total of 355 participants participated in the MT treatment,[31] which used the procedure used in the Caltech lab with the following modifications. First, participants read the instructions online and were asked to imagine an opaque bag with colored chips. Second, all choice problems were hypothetical, and

[30] To understand these results, consider the regression that takes choice over probabilities as the benchmark (the top part of Table 7). This regression asks: does allowing participants to choose uncertainty in prizes and dates change their tendency of choosing the non-uncertain option? When participants choose the winning probability (from 20/60, $r/60$, and $g/60$), while the prize and date are known, about 75% of them (on average) choose no uncertainty (this percentage corresponds to the constant term in the linear regression, whereas the constant in the multinomial regression is the log of the odds ratio, $\log(0.748/(1-0.748)) = 1.08$). However, when the prize can also be chosen to be uncertain, only 65% choose no uncertainty. Hence, 9.3% fewer subjects choose the non-uncertain option when they can also control the uncertainty in prizes. This is the coefficient of *Prob and prize* in the regression. The log odds ratio here is $\log(0.655/(1-0.655)) = 0.63$, which is 0.45 less than before. This difference of -0.45 gives the logit coefficient of *Prob and prize*.

[31] The experiment was conducted in January 2012, and participation was restricted by location (United States only) and acceptance rate (above 95%). Participants were allowed to participate only once by not allowing multiple logins with the same email address. In general, existing studies find that duplicate observations are not a major concern in MT (Berinsky et al. 2012, pp. 365–366).

participants were paid a flat fee of $1 (participation fees on MT are usually very low and range between $0.25 and $2). Third, because MT participants were unlikely to have the patience and attention to answer too many (repetitive) questions, they were presented with only two screens of problems: the first screen (with payment today) and a second, randomly selected screen.

Overall, the MT data appear to be very similar to the Caltech data.[32] Table 8 shows the results when participants can affect the uncertainty of only a *single* dimension. (All the tables in this section focus on the case in which the additional uncertainty is perfectly correlated with the number of red chips. Similar tables for correlation with green appear in Table C.1 in Online Appendix C.)

Table 8 shows that the main patterns exhibited in the lab are present in the MT data set as well. There are, however, some differences. First, the proportion of MT participants who chose no uncertainty is weakly higher for each of the dimensions.[33] In addition, the effect of introducing a fixed uncertain dimension on the choice of uncertain gambles is milder compared with the Caltech data: relative to the Caltech participants, there is a lower fraction of participants who chose the uncertain gamble with more exposure.[34] Coherently, the estimates of multinomial regressions (for the effect of making a fixed dimension uncertain) are qualitatively similar to the estimates obtained from the lab data, but the magnitude of the effect is smaller. (Table C.2 in Online Appendix C displays the estimation results for the MT data.) The distributions of choices in the lab and MT data are even more similar when participants have the option between no uncertainty and uncertainty in multiple dimensions. (See Table C.3 in Online Appendix C.) We also estimated a logit model to study how agents compare options with multiple ambiguities and options with no ambiguities. The estimation results are again very similar to those obtained for the Caltech data (see Table C.4 in Online Appendix C).[35]

Table 8 **Choice Distributions of MT When Participants Can Affect the Uncertainty of Only a Single Dimension**

Choosing prob.		Choosing prize		Choosing date	
$(20/60, \$20, 0)$	74%	$(20/60, \$20, 0)$	90%	$(20/60, \$20, 20)$	72%
$(r/60, \$20, 0)$	17%	$(20/60, \$r, 0)$	9%	$(20/60, \$20, r)$	21%
$(g/60, \$20, 0)$	10%	$(20/60, \$g, 0)$	1%	$(20/60, \$20, g)$	7%
$(20/60, \$r, 0)$	54%	$(r/60, \$20, 0)$	69%	$(r/60, \$20, 20)$	55%
$(r/60, \$r, 0)$	34%	$(r/60, \$r, 0)$	22%	$(r/60, \$20, r)$	27%
$(g/60, \$r, 0)$	12%	$(r/60, \$g, 0)$	8%	$(r/60, \$20, g)$	18%
$(20/60, \$20, 20)$	73%	$(20/60, \$20, 20)$	86%	$(20/60, \$r, 20)$	47%
$(r/60, \$20, 20)$	18%	$(20/60, \$r, 20)$	8%	$(20/60, \$r, r)$	32%
$(g/60, \$20, 20)$	9%	$(20/60, \$g, 20)$	6%	$(20/60, \$r, g)$	21%
$(20/60, \$r, 20)$	48%	$(20/60, \$20, r)$	74%	$(r/60, \$r, 20)$	48%
$(r/60, \$r, 20)$	26%	$(20/60, \$r, r)$	20%	$(r/60, \$r, r)$	35%
$(g/60, \$r, 20)$	26%	$(20/60, \$g, r)$	6%	$(r/60, \$r, g)$	17%

5. Relating the Data to Theory

In this section we address the following questions: Is the modal behavior of our participants compatible with existing models of uncertainty aversion? If so, is it compatible with some of the most typical parametric specifications of these models, and, in general, can our results be used to refine these parametric specifications? We will show that most models have little to no predictive power in this case, i.e., they can accommodate almost all possible rankings of the gambles considered in our experiment—not only those exhibited by the majority of our participants, but also very different ones. Thus, at least in theory, there is no a priori reason to expect the emergence of a systematic behavior (from two very different pools). On the other hand, we argue that the behavior exhibited by our subjects does not appear to be "obvious" from the point of view of the models. For example, their behavior is not compatible with some of the most "immediate" parametric specifications. We argue, instead, that the systematic behavior observed in our experiment can be used to impose specific and novel restrictions on these models and thus on attitudes toward uncertainty.

5.1. Uncertainty in Prize and Probability

We begin with uncertainty on the prize and/or probability. We discuss uncertainty in the date separately,

[32] MT participants were also fairly consistent in their answers: 271 out of 355 (76%) gave transitive answers in the two screens they faced. To compare this percentage with that of the lab participants, we need to account for the fact that they had more opportunities to violate transitivity because they faced seven screens instead of two. We therefore considered the answers that lab participants gave on the first two screens they saw (the "today" screen was always the first and the second screen was randomly generated). We find that 84 out of 97 Caltech subjects (86.6%) were transitive on their first two screens.

[33] In contrast, the fraction of risk-loving participants is much higher in the MT sample: 61% compared with 32% in the Caltech data. This relatively high proportion may be due to the fact that the MT questionnaire was hypothetical.

[34] For example, when only the prize can be chosen ($20, $r, or $g) and the winning probability is changed from 20/60 to r/60, the fraction of participants who choose no uncertainty drops from 82% to 36% in Caltech, whereas in MT the drop is from 90% to 69%.

[35] There are only a few notable exceptions. First, relative to the case in which participants can only affect uncertainty in *probability* (*payment date*), there is *no* significant effect on the choice of no uncertainty when participants can also affect the uncertainty in the *prize* (*winning probability*). Second, relative to the case in which participants can only affect uncertainty in the *prize amount*, there is a *significant negative* effect on the choice of no uncertainty when participants can also affect the uncertainty in the *winning probability*.

because the intuition is different for that case. We start our analysis by focusing a well-known model of decision making under uncertainty: max-min expected utility of Gilboa and Schmeidler (1989). In the MMEU model, a decision maker has a set of priors Π and a Bernoulli utility function over prizes u. Let $h(x) = (p(x), m(x), 0)$ be a gamble in which the probability and prize are functions of $x \in \{r, g\}$. According to this model, the value V of a gamble g is the minimal expected utility from the gamble, where the minimum is taken over all the priors in Π; that is, assuming $u(0) = 0$,

$$V(h(x)) = \min_{\pi \in \Pi} \sum_{x=0}^{40} \pi(x) \cdot p(x) \cdot u(m(x)).$$

We impose the standard consistency assumption that any prior must assign probability 1/3 to a black chip being extracted, and that, conditional on there being r red chips in the urn, the probability of drawing a black, red, or green chip is $1/3$, $r/60$, $(40-r)/60$, respectively. This implies that the prior belief of a participant is defined, effectively, on the number of red chips in the urn (or, equivalently, the number of green chips). We also assume the decision maker treats the colors red and green symmetrically:[36] for any prior $\pi \in \Pi$ there exists a prior $\pi' \in \Pi$ such that $\pi'(40-r) = \pi(r)$ for $r = 0, 1, \ldots, 40$. As long as Π is not a singleton, this model predicts the standard Ellsberg aversion to uncertainty in probability. For almost identical reasons, if the decision maker is not risk loving, then he would prefer no uncertainty to uncertainty in prize.

Note, however, that unless we impose further restrictions on Π, then *any* ranking of $(20/60, 20, t)$, $(x/60, x, t)$, and $(20/60, x, t)$ that respects aversion to single-dimensional uncertainty (i.e., $(20/60, 20, t) \succ (x/60, 20, t)$ and $(20/60, 20, t) \succ (20/60, x, t)$) is compatible with MMEU for some Π and some (weakly) concave u. Intuitively, this is due to the fact that there are two opposing forces at play when evaluating $(x/60, x, t)$: first, the more pessimistic the priors in Π, the less the agent will like this option; but second, the higher the variance of these priors, the *more* he will like it. The latter point follows from the observation that in computing the expected utility of this option even with a pessimistic prior, the utility and the probability are multiplied by each other, which generates a convexity that renders $(x/60, x, t)$ attractive if the variance of the prior is high enough (and the prior is not too pessimistic). For example, with linear u and a prior π such that $\pi(0) = \pi(39) = \frac{1}{2}$, a prior with very high variance, the expected utility of $(x/60, x, t)$ is higher than that of $(x/60, 20, t)$, $(20/60, x, t)$, or even $(20/60, 20, t)$. Depending on the expected value and on the variance of each prior in Π, it is easy to see how we could generate any ranking between the relevant options.[37]

Since the MMEU model makes no predictions on the behavior subjects should exhibit in our experiment, we now turn to analyze whether the modal behavior we observe is compatible with the MMEU model assuming some common parametric specifications, and if we can use our results to *refine* the set of priors Π.[38] In particular, we will be looking for sets of priors that are compatible with (1) preferring no uncertainty to both single and double uncertainty, and (2) preferring double correlated uncertainty to single uncertainty and to hedging. First of all, it is easy to see how the modal behavior in our experiment is *not* compatible with the casual "classroom" example of MMEU, in which subjects include in their set Π a degenerate prior according to which there are only $r < 20$ red balls ($\pi(r) = 1$) in the urn: any such prior would lead the agent to prefer single uncertainty to double correlated uncertainty. Intuitively, to guarantee the latter, all priors must have a higher variance.

This suggests that we could use the finding of our experiment to *refine* the set of priors in Π.

Observation 5. A risk-neutral MMEU subject with set of priors Π exhibits the modal ranking of our experiment if and only if

$$20^2 \geq \min_{\pi \in \Pi} (\mathrm{VAR}_\pi + \mathbb{E}_\pi[x]^2) \geq \min_{\pi \in \Pi} 20 \cdot \mathbb{E}_\pi[x]. \tag{1}$$

Observation 5 shows that the MMEU model generates the modal ranking in our experiment only if the priors in Π have three characteristics:

1. They are pessimistic, as $\min_{\pi \in \Pi} \mathbb{E}_\pi[x] < 20$.
2. They have a nontrivial variance, as $\min_{\pi \in \Pi} (\mathrm{VAR}_\pi + \mathbb{E}_\pi[x]^2) \geq \min_{\pi \in \Pi} 20 \cdot \mathbb{E}_\pi[x]$. Notice that this implies $\min_{\pi \in \Pi} \mathbb{E}_\pi[x^2] \geq \min_{\pi \in \Pi} \mathbb{E}_\pi[20 \cdot x]$, which means that there cannot be any $\pi \in \Pi$ whose support is entirely in $\{0, \ldots, 19\}$.
3. Their pessimism must be high enough, or, conversely, their variance cannot be too high, as $20^2 \geq \min_{\pi \in \Pi} (\mathrm{VAR}_\pi + \mathbb{E}_\pi[x]^2)$.

[36] This is assumed just for simplicity of notation, but it is not necessary for the discussion that follows. (The extension to the general case is trivial.)

[37] To give concrete examples, assuming a linear u, we obtain $(20/60, 20, t) \succ (x/60, x, t)$ if Π is the closed convex hull of the set $\{\pi_1, \pi_2\}$, where $\pi_1(10) = 1$ and $\pi_2(30) = 1$. The opposite ranking, $(x/60, x, t) \succ (20/60, 20, t)$, could be obtained if Π is the closed convex hull of the set $\{\pi_3, \pi_4\}$, where $\pi_3(0) = \pi_3(39) = \frac{1}{2}$ and $\pi_4(1) = \pi_4(40) = \frac{1}{2}$.

[38] In what follows we focus on linear u. (This is not inconsistent with our findings on risk attitudes; see §3.1.) It is straightforward to derive similar conditions for generic u.

Whereas the first condition is fundamental to capture uncertainty aversion in the MMEU model, the second and the third ones are novel, as they pertain to the *variance* of the priors in Π. In particular, such variance cannot be too low, to guarantee that the second conditions holds, ruling out priors with support that is "entirely" pessimistic, i.e., in $\{0,\dots,19\}$;[39] but it cannot be too high, to guarantee that the third condition holds as well.

One possible interpretation for condition (1) is the following. Although subjects can act as if they are pessimistic about the number of red or green balls—they have a prior with a low expected value—they cannot act as if they are *sure* about this pessimistic valuation: they should incorporate in this prior the awareness that they are being pessimistic, and hence incorporate some variance in the priors. In particular, they cannot incorporate beliefs that put support only in the "pessimistic" side. On the other hand, for how much variance they incorporate, their pessimism must still be strong enough to lead them to prefer no uncertainty to double uncertainty, which takes place only if the variance is not too high; that is, even if they do recognize that incorporating some variance could improve the valuation of some options, they are still not ready to prefer them to the options with no uncertainty. We refer to this behavior as *skeptical pessimism*.

The above observations are not specific only to MMEU; similar implications hold for other models as well. Consider first the recursive nonexpected utility model of Segal (1987, 1990). This model, which is very different from the MMEU model, received recent empirical support (Halevy 2007) and was used recently to explain some "paradoxes" that challenge many of the existing models of decision making under uncertainty (see Machina 2009, 2014; Baillon et al. 2011; Dillenberger and Segal 2012).

To simplify the exposition, we impose some restrictions. (We will show that a large variety of ranking can be obtained even in this specific case.) First, we assume that the agent applies the rank-dependent utility functional of Yaari (1987) to first- and second-stage lotteries, with a linear utility u and a *convex*, i.e., pessimistic, weighting function. Second, we restrict attention to simple, symmetric beliefs of the following form. The belief on r has only two elements in the support: either $20-k$ or $20+k$, with equal probability, where k is an integer between 0 and 20. To describe the corresponding functional, let h_i be a gamble in which there is probability p_i of winning m_i (to be paid today), and zero otherwise. Let h_1 and h_2 be two gambles with $m_1 \geq m_2$, and let h be a gamble in which there is a 50% chance of playing h_1 and a 50% chance of playing h_2. According to this model, there exists a utility function u, which we assume to be linear, and a probability weighting function f, which we assume to be convex and satisfying $f(0)=0$ and $f(1)=1$, such that the value V assigned h is, assuming $u(0)=0$,

$$V(h)=u(m_1)\cdot f(p_1)\cdot f(\tfrac{1}{2})+u(m_2)\cdot f(p_2)\cdot[1-f(\tfrac{1}{2})].$$

It is easy to see that, as long as f is not linear but convex, this generates the typical Ellsberg ranking of $(20/60,20,t)\succ(x/60,20,t)$. For gambles with more than two outcomes, Segal (1987) gives conditions on f that are sufficient to exhibit this aversion to uncertain probability. One of the examples he gives for f is p^{α}, where $\alpha>1$. It is easy to verify that for many pairs (α,k), the induced behavior is *not* consistent with our data. For instance, if $\alpha=2$ there is no k between 0 and 20 that works. On the other hand, $k=4$ and $\alpha=1.2$ does work. Again, the patterns documented in our paper could help us restrict the space of parameters for this model.

Consider now the smooth ambiguity model of Klibanoff et al. (2005), where the agent has a subjective prior μ over priors over the states of the world, as well as a transformation ϕ, and evaluates acts by $\mathbb{E}_{\mu}\,\phi(\mathbb{E}_{\pi}\,u\circ f)$. The concavity of ϕ leads to the standard Ellsberg paradox. For this model to be compatible with our data, the prior over priors μ must put enough weight on priors that have a relatively high variance and that are not entirely optimistic or pessimistic—otherwise single uncertainty would be better than double correlated uncertainty. At the same time, if μ puts a high weight only on priors with a relatively high variance, then the agent must be also highly ambiguity averse—ϕ must be very concave—otherwise no uncertainty would be ranked below double correlated uncertainty.[40]

Finally, consider the Choquet expected utility (CEU) model of Schmeidler (1989), where agents evaluate acts using a possibly nonadditive capacity v. The model is compatible with the standard Ellsberg paradox if the capacity is convex. As was the case for the models discussed above, for CEU to be compatible with the modal behavior in our experiments,

[39] For instance, any degenerate prior according to which there are only $r<20$ red balls ($\pi(r)=1$) is not compatible with condition (1).

[40] The intuition is similar to the one illustrated for MMEU. For example, a μ that admits in its support only two degenerate priors—thus with zero variance—that assign probability 1 to 19 or 21 red balls, respectively, is not compatible with our data: it would imply that single uncertainty is better than double correlated uncertainty. On the other hand, once μ gives high weight to priors with a high variance, ϕ must be sufficiently concave. To see why, consider μ' that assigns equal probability to π_1, which assigns probabilities 0.9 to 0 and 0.1 to 40 red balls, and π_2, which assigns probabilities 0.1 to 0 and 0.9 to 40 red balls. For the model to be compatible with our data, the ambiguity aversion should be extremely high (ϕ must be *extremely* concave: it would work with $\phi(x)=\ln(\ln(x))$, but not with $\phi(x)=\ln(x)$); otherwise, the agent would rank double correlated uncertainty above no uncertainty.

the capacity must have a "high enough variance" to guarantee that double correlated uncertainty is ranked above single uncertainty; however, if the variance is also very high, then the capacity must also be "very convex"—high ambiguity aversion—to guarantee that double correlated uncertainty is ranked below no uncertainty.[41]

5.2. Uncertainty in the Payment Date

We now turn to discuss the theoretical predictions regarding attitudes toward uncertainty in the payment date. Consider the choice between $(20/60, 20, 20)$ and $(20/60, 20, r)$ (a similar argument can be made for a gamble where the date is g days away). Assume the participants' preferences are additively separable over time with a standard exponential discount factor $\delta \in (0, 1)$. Let us now compare the expected utilities of these bets, using a prior $\pi = (\pi_0, \ldots, \pi_{40})$ on the number of red chips. (This prior π could, for example, be the most pessimistic one in a MMEU representation.) It is easy to see that a participant prefers the risky gamble if and only if

$$\delta^{20} \geq \sum_{k=0}^{40} \pi_k \cdot \delta^k. \tag{2}$$

This means, for example, that if π has an expectation equal to 20, then the agent will always prefer the uncertain bet; we would in fact have $\sum_{k=0}^{40} \pi_k \cdot \delta^k > \delta^{20}$, because δ^k is a convex function. (In fact, exponential discounting and expected utility imply that subjects prefer risky payment time to certain payment time; see Dejarnette et al. (2015).) On the other hand, if the expectation of π is low enough, i.e., if the agent has a sufficiently pessimistic set of priors, then the agent will prefer the risky outcome. In general, the more pessimistic the priors in the set, the more subjects will like the fixed date; whereas the lower δ, i.e., the more impatient they are, the more they will like the uncertain one.

To summarize this section, the following conclusions can be drawn from the decision-theoretic models we considered. The models do generate predictions regarding single-dimensional uncertainty: a decision maker who is averse to uncertainty only in the probability dimension (i) would also be averse to uncertainty only in the prize dimension if he is risk averse, and (ii) would also be averse to uncertainty only in the time dimension if he is sufficiently patient. However, even under reasonable restrictions on the prior beliefs, the models have *no* prediction regarding attitudes toward multidimensional uncertainty. In particular, they allow a decision maker who is averse to single-dimensional uncertainty to prefer uncertainty in more dimensions. Our experimental results could then be used to refine the parameters used in the models.

[41] Consider the following examples. For simplicity, we restrict attention to capacities v such that $v(\{40, \ldots, 20+x\}) = \alpha$, $v(\{40, \ldots, 20-x\}) = (1-\alpha)$, $v(\{0, \ldots, 20-x\}) = \alpha$, $v(\{0, \ldots, 20+x\}) = (1-\alpha)$ for some $20 \geq x \geq 0$ and $\alpha \in [0, 0.5]$. (Conceptually, these are symmetric capacities that put all weight on two states, $20+x$ and $20-x$, and assign to each a "weight" α or $(1-\alpha)$.) Notice that the higher x is, the higher the "variance" is; and the smaller α is, the more ambiguity averse these capacities are. First, when $x = 1$ and $\alpha = \frac{1}{3}$, the model is compatible with the standard Ellsberg paradox, but not with our data: the "variance" is too low, and single uncertainty is ranked above double correlated uncertainty. It would instead work with $x = 8$ and $\alpha = \frac{1}{3}$. When $x = 18$, the variance is very high, and the model is compatible with our data with $\alpha = \frac{1}{8}$, but not $\alpha = \frac{1}{3}$.

6. Concluding Remarks

The goal of this paper was to explore how individuals approach uncertainty when the uncertainty may be in different or in multiple dimensions. Although most decision problems in real life involve uncertainty in multiple dimensions, essentially all models in decision making under uncertainty are agnostic to the distinction between these dimensions and, in particular, provide no guidance as to what behavior we should expect when individuals choose between gambles that involve uncertainty in more than one dimension. This paper explores the question of whether there is some systematic, representative behavior that the majority of individuals exhibit.

We address this question by conducting an experiment on two very different participant pools: the lab and Mechanical Turk. The majority of participants in both participant pools exhibit the same systematic behavior: (i) they prefer no uncertainty to uncertainty on any single dimension and to uncertainty on multiple dimensions, and (ii) they prefer "correlated" uncertainty on multiple dimensions to uncertainty on any single dimension. In other words, when subjects have the option to remove *all* uncertainty, the majority opt for that option. However, when uncertainty cannot be completely removed, the majority prefer options with (perfectly correlated) uncertainty on several dimensions to those with only one uncertain dimension.

Our results suggest that in economic settings, where some uncertainty is always present, decision makers may be more likely to choose uncertain prospects with "more exposure to uncertainty" (in the sense that more dimensions are uncertain and correlated).

Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/mnsc.2015.2240.

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