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Win-Win Capacity Allocation Contracts in Coproduction and Codistribution Alliances

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Abstract. In some strategic alliances, a firm shares its manufacturing capacity with another, and the latter shares its distribution capacity with the former. Even though such *bidirectional alliances* have become more common, they remain challenging to manage because of the frequent disputes over capacity allocation, especially when demand is uncertain. In this paper, we investigate whether there exists a contractual mechanism that can mitigate the extent of these disputes while improving the profits of all participating firms. We consider two types of bidirectional contracts, namely, the *ex post transfer payment contract* and the *ex ante capacity reservation contract*. By modeling the capacity allocation and the bidirectional contract design as a noncooperative game between two firms with noncompeting product lines, we show that, relative to a situation with no contract, either contract can improve the alliance's total profit in equilibrium. In terms of distribution of the total surplus, we find that capacity reservation contracts always make both firms better off, whereas ex post transfer payment contracts may make one firm worse off. Hence, capacity reservation contracts are more likely to be implemented in practice in such bidirectional alliances.

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1. Introduction

To sustain profitable growth in consumer goods markets (or durable goods markets), many firms have attempted to expand their product portfolios (or enter new markets) while reducing their operating costs. Yet, achieving both objectives can be quite challenging for two reasons. First, the high degree of demand uncertainty associated with new product categories (or new markets) makes it difficult for firms to assess how much capacity they need to satisfy their future demand. Second, the lumpy nature of capacity investments in those industries may constrain firms to overinvest in capacity.[1] Together, these two factors often result in capacity underutilization.

When facing capacity underutilization, it may be more economical for a firm to use its facility to make products for other (preferably noncompeting) firms that seek to outsource some of their operations. Among the various types of outsourcing arrangements, a new form of alliance has recently emerged, in which one firm shares its production capacity with another firm, and the latter shares its distribution capacity with the former. We offer a few examples. First, Nestlé USA and Ocean Spray have formed a long-term strategic operations alliance to increase their manufacturing and supply chain efficiency in the United States. Under this alliance, Nestlé has transitioned the manufacturing of its Juicy Juice and Libby's Kern Nectars to Ocean Spray facilities. In addition, the two companies have pooled purchases of ingredients, bottles, and packaging, and have shared warehouse and distribution costs for their juices. As one Ocean Spray shareholder put it, "they have the strength of (distribution), and we have the bottling" (Melo 2002). Second, Fiat partnered with Tata Motors to produce both Fiat and Tata vehicles in one of Fiat's plants in India. Under that partnership, Fiat produced its own cars and Tata's cars, and Tata Motors shared its dealership network and managed the marketing and distribution of Fiat's cars in India (Purkayashta and Samad 2013). Similar comanufacturing and codistribution alliances have been adopted by Panasonic and Bosch (Hoshi 2014) and Panasonic and Gorenje (Groznik 2013), and have been under consideration by A.B. InBev and Carlsberg (Cunningham 2010).

These alliances are unique in several respects. First, they are *bidirectional*, in contrast to the standard, unidirectional outsourcing arrangements. Second, they involve *complementary resources*, in contrast to many other horizontal alliances or joint ventures, which involve substitute resources. Consequently, the partners in such alliances are on an equal footing: they both need each other and they both have a say in how capacity is allocated.

Based on our discussions with supply chain managers of various companies that engage in bidirectional alliances, we learned that, even though firms recognize the benefits of sharing capacities, they find themselves spending a lot of time negotiating over capacity allocation. They also felt that, in some cases, the negotiations ended up with solutions that were driven by politics and/or emotions. For instance, some Nestlé's executives believed they may not always get enough capacity from Ocean Spray, and that the negotiated production plan is sometimes suboptimal (Wang 2014). Similarly, the Fiat-Tata alliance stumbled when Fiat blamed Tata's dealers for not putting in enough efforts to sell its cars, e.g., by excluding Fiat's Petra from display in the dealerships (Purkayashta and Samad 2013). Their dissatisfactions have motivated us to examine the following research questions: Are such disputes unavoidable? Is it possible to structure a bidirectional alliance so as to achieve more efficient capacity allocation while making both parties better off?

In this paper, we investigate how to improve the efficiency of the capacity allocation decisions in a bidirectional alliance that involves the sharing of two complementary resources (e.g., production and distribution capacity). We propose that contractually specifying certain capacity allocation rules in advance may help avoid disputes later on. The intent is to improve the alliance's long-term stability by improving the efficiency of the capacity allocation decisions, which are made repeatedly, e.g., every year. Specifically, we consider two types of contracts: (1) *ex post transfer payments* and (2) *ex ante capacity reservation*. Whereas ex post transfer payments facilitate the firms' negotiation process for capacity transfers *after* demand is realized, ex ante capacity reservations take place *before* demand is realized.

We model the capacity allocation and the bidirectional contract design as a game between two firms with noncompeting product lines. Consistent with our observation that, in practice, firms engage in an alliance to maximize their individual payoffs and independently decide on how capacity must be allocated, we adopt a *noncooperative* game-theoretic framework. We consider a multistage game, in which firms first set the contractual terms noncooperatively, then fulfill the terms of the contract, and finally allocate the remaining capacity noncooperatively, in the context of a noncooperative unanimity game (see Harsanyi 1982b).

By analyzing two separate contracting games, we obtain the following results. We first show that structuring the alliance with either contract always improves supply chain efficiency. Specifically, we find that ex post transfer payment contracts improve the efficiency of the noncooperative capacity allocation process by better aligning the firms' profit margins, whereas ex ante capacity reservation contracts reduce the need for those ex post negotiations by allocating capacity ex ante. Although both contracts improve efficiency, using the two contracts in combination may not always be desirable. This is because structuring the noncooperative capacity allocation process with an ex post transfer payment contract dilutes the incentives to reserve capacity ex ante.

In addition, we show that the ex ante capacity reservation contracts lead to very attractive outcomes. First, they are Pareto improving, whereas ex post transfer payment contracts may not be so. Second, we find numerically that they often achieve high levels of efficiency and we analytically characterize that, under mild conditions on the demands, full efficiency is achieved. This suggests that ex ante capacity reservation contracts, commonly used in outsourcing arrangements, can also be win-win arrangements in bidirectional alliances that involve complementary resources. Therefore, as two firms evolve from a unidirectional to a bidirectional alliance, they can simply extend their existing unidirectional capacity reservation contracts to accommodate the bidirectional nature of their alliance.

2. Literature Review

We first review the related economics literature on horizontal alliances and justify our choice of noncooperative game-theoretic framework. We then review the related contracting literature in supply chains and position our contribution with respect to that literature.

2.1. Horizontal Alliances

Alliances have been studied from different perspectives with different research questions. For instance, transaction cost economics investigate when an alliance structure dominates vertical/horizontal integration and markets (Williamson 1991) and cooperative game theory studies the stability of alliances with many firms (see, e.g., Yin 2010, Huang et al. 2016). In this paper, we adopt a noncooperative game-theoretic framework to investigate whether contractual mechanisms can improve the efficiency of capacity allocation decisions in bidirectional alliances.

A large body of the empirical literature on strategic alliances indeed indicates that the long-term stability of alliances relies on the participating firms' individual incentives. For instance, Parkhe (1993, p. 794) notes that "mutual cooperation, although desirable, is not automatic; the self-interest orientation of each party can lead to actions that are individually rational yet produce a collectively suboptimal outcome," drawing a parallel between alliances and the prisoner's dilemma (Axelrod and Hamilton 1981). Similarly, Kogut (1989, p. 185) argues that joint ventures are characterized, among others, by "competitive rivalry over ... control of the operations management of the assets." Consistent with that literature, we adopt a noncooperative game-theoretic approach to assess which contractual mechanisms can improve the alliance's efficiency when the participating firms compete over control of the operations management of complementary assets. In short, we investigate whether contracts can improve coordination, in the same spirit as the empirical findings of Luo (2002), who report that, in international joint ventures, contracts and cooperation are strategic complements.

In particular, we adopt a noncooperative game-theoretic framework to model how firms allocate capacity ex post. Specifically, we assume that firms play a simultaneous-move noncooperative unanimity game (Harsanyi 1982b), according to which each firm proposes a production plan. If the production plans coincide, the production plan is implemented; otherwise, firms get zero profit. Since this game has many possible equilibria in case of capacity rationing, we adopt the payoff- and risk-dominant equilibrium selection rule proposed by Harsanyi and Selten (Harsanyi 1982a).

In general, these noncooperative negotiations will lead to inefficient capacity allocations, consistent with what we have observed in practice (Wang 2014). We note, however, that in other settings, negotiations may not necessarily be inefficient. In particular, in the property rights literature (Grossman and Hart 1986, Hart and Moore 1990), the ex post negotiation process is often assumed to lead to an efficient outcome, but inefficiency is created by ex ante decisions, such as investments in product innovation or production capacity (Plambeck and Taylor 2005, Plambeck and Taylor 2007). Yet, even within that literature, there is a recent stream of work that acknowledges that "the emphasis on noncontractible ex ante investments seems overplayed" and that "it is essential to depart from a world in which Coasian renegotiation always leads to ex post efficiency" (Hart and Moore 2008, pp. 2–3). In particular, Baker et al. (2008, p. 153), based on what they have learned from numerous discussions with practitioners, assume that, consistent with our noncooperative framework, "whoever holds the decision right ex ante will make the decision that is in her best interest ex post; because decisions are not contractible, no Coasian bargaining can occur to achieve ex post efficiency."

2.2. Coordinating Capacity Allocation in Supply Chains

Most of the literature on supply chain contracting has been in the context of vertical supply chains; see Cachon (2003) for a review. In particular, capacity reservation contracts have been used as a way to lower production costs by reserving capacity early, provided that the early demand signals are of high quality (Brown and Lee 2003). When capacity commitments are verifiable, this type of contract is known to generally coordinate the production decisions in the supply chain, similar to backup agreements and quantity-flexibility contracts (Barnes-Schuster et al. 2002). However, when capacity commitments are not verifiable, the downstream party has incentives to inflate its demand forecast to secure supply (Cachon and Lariviere 2001), thereby creating a "bullwhip effect" (Lee et al. 1997). Similar order inflation pattern arises in distribution chains involving multiple retailers and one capacity-constrained manufacturer (see, e.g., Cachon and Lariviere 1999, Cho and Tang 2014) and among multiple product managers requesting the same manufacturing capacity (Karabuk and Wu 2005). Although our newsvendor modeling setup is similar to that literature, our focus is different given that we consider horizontal alliances and not vertical transactions. Information asymmetry, leading to order inflation, is indeed more prevalent in vertical arm's length transactions than in horizontal strategic alliances. Accordingly, we assume that firms share their demand information truthfully and instead investigate which contractual mechanisms result in win-win decentralized capacity allocation decisions. Hence, our approach is complementary to that literature, and we leave it for future research to design contractual mechanisms that induce truthful demand revelation and coordinate decentralized decision making in horizontal alliances.

In comparison to vertical supply chains, horizontal alliances have been relatively understudied. In a seminal paper, Van Mieghem (1999) studies different outsourcing arrangements under demand uncertainty. For these unidirectional relationships, he shows that simple transfer price contracts fully coordinate capacity allocation decisions (but not capacity investment decisions, which we consider sunk here). Similarly, Wu et al. (2013) show that capacity reservation contracts are also coordinating such unidirectional outsourcing arrangements. However, we find that these two coordinating contracts, which enable one party to extract all surplus, are not enforceable in the bidirectional outsourcing arrangements considered here. This is because, in bidirectional arrangements with complementary resources, both firms must agree on how capacity is allocated, in contrast to unidirectional agreements, in which capacity allocation is decided by only one firm.

Among the different forms of horizontal alliances, inventory transshipment games have perhaps received the most attention from the research community; see Anupindi et al. (2001), Rudi et al. (2001), Granot and Sošić (2003), Hu et al. (2007), and Huang and Sošić (2011) among others. This stream of research has studied how to design a contract that not only coordinates the ex post inventory transshipment so that no coalition of retailers has incentives to leave the grand coalition, but also induces the retailers to order their first-best inventories before demand is realized. In those games, the contract is typically imposed to the retailers by a central authority. In contrast, we consider here a setting where the contract design decision is decentralized, and therefore investigate whether the capacity allocation contracts are self-enforcing.

More recently, other forms of horizontal alliances have been studied, such as joint ventures (Çetinkaya et al. 2014, Levi et al. 2014) and airline code-sharing alliances (Hu et al. 2013, Adler and Hanany 2016). In general, those papers do not consider capacity rationing, which lies at the core of our problem, either because it is assumed that demand can always be met by changing prices ex post (Çetinkaya et al. 2014, Adler and Hanany 2016), or because the alliance has only one product (Levi et al. 2014), or because capacity allocation happens ex ante (Hu et al. 2013). In contrast, we focus on the ex post capacity allocation decisions.

3. Model

We consider two firms that enter a bidirectional alliance by sharing two complementary resources (e.g., manufacturing and distribution), each of which is owned by one firm only. For instance in the Nestlé-Ocean Spray alliance, Ocean Spray is in charge of the bottling for both firms' fruit drinks, whereas Nestlé shares its procurement and distribution processes with Ocean Spray (Melo 2002). In the Fiat-Tata alliance, Fiat owns the manufacturing plant that produces cars for both firms, whereas Tata's dealership network distributes and promotes both firms' cars (Purkayashta and Samad 2013). The intent of the alliance is that the participating firms can utilize their respective capacity more effectively when the demands for their products are uncertain.[2] Hence, the focus of this strategic alliance is on operational efficiencies.

We consider a situation where the firms' products are not competing, or at least, are not direct competitors. For instance, in the Nestlé-Ocean Spray alliance, Nestlé's Juicy Juices are mostly targeted toward children, whereas Ocean Spray's juices, which typically contain sour cranberries, are mostly targeted toward adults (Wang 2014). Similarly, in the Fiat-Tata alliance, Fiat's cars were generally perceived as more prestigious than Tata's middle-class cars (Purkayashta and Samad 2013). We assume that each firm manages only one product line so as to focus on the economic impact of different capacity allocation contracts on both firms.

Moreover, we assume that the retail prices are set exogenously by the market. In other words, the strategic intent of the bidirectional alliance is to reduce each firm's operating cost, but not to increase selling prices because of less competition, similar to outsourcing agreements (e.g., Van Mieghem 1999) and unlike mergers (e.g., Cho 2014). Let p_i denote the "unit gross profit margin" of product i offered by firm i, where $i = 1, 2$.[3] Without loss of generality, we assume that $p_1 \geq p_2$.

We consider here a situation where demand is uncertain at the time of contracting, either because of changing consumer tastes (as is the case of Nestlé's and Ocean Spray's juices) or because the firms are entering a new market (as is the case for Fiat's cars in India).[4] We denote by D_1 and D_2 the demands for products 1 and 2, which, unless otherwise specified, may be correlated. Let us denote by $F_i(x)$ the marginal cumulative distribution function of D_i, i.e., $F_i(x) = \mathbb{P}[D_i \leq x]$, assumed to be continuously differentiable with $F_i(0) = 0$, by $f_i(x)$ its density, i.e., $f_i(x) = F_i'(x)$, and by $\bar{F}_i(x) = 1 - F_i(x)$ its complementary distribution function. Let also $\mathbb{E}[\cdot]$ denote the expectation operator. Throughout the paper, we denote $[x]^+$ as $\max\{x, 0\}$.

We assume that, at the time of contracting, capacity investments are fixed (and sunk), and we therefore focus on contracting for capacity allocation, which occurs after demand is realized. For instance, Nestlé and Ocean Spray typically negotiate their production plan once a year, but their investments in production facilities are multiyear projects (Wang 2014). Let c_i be the capacity installed by firm i. For tractability, we assume that the relative resource usage per product is constant, which, after rescaling the capacities and profit margins, is equivalent to assuming that each product requires one unit of resource, a reasonable assumption if the products are similar. For instance, Nestlé and Ocean Spray's juices have standard-size bottles and Fiat and Tata produced both sedans and hatchbacks through the alliance. We also assume that each firm owns only one type of resource and has access to the other type of resource only through the alliance. For instance, Fiat India appears to have had very few dealers before the alliance (Purkayashta and Samad 2013) and hence, through its alliance with Tata, Fiat can gain access to a large dealership network.[5]

Each firm may have a "guaranteed base demand" b_i, i.e., $\mathbb{P}[D_i < b_i] = 0$, and, because of strategic business objectives, which lie outside of the scope of our model, they may be required to satisfy their base demand before sharing any capacity. Accordingly, the negotiation over capacity allocation pertains to the "leftover capacity;" i.e., to $c_i - b_1 - b_2$. Potential disputes over capacity allocation arise only when there is capacity rationing, i.e., when $D_1 + D_2 > c_i$. Our model is therefore relevant mostly to alliances for which the share of leftover capacity of the two complementary processes, after fulfilling the base demands, is significant and for which the probability of capacity rationing is significant. To simplify our exposition and without loss of generality, we set $b_1 = b_2 = 0$ throughout this paper so that the leftover capacity of each firm is denoted by c_i and the residual demand is denoted by D_i.

Throughout our analysis, we assume that firms make noncooperative contracting decisions to maximize their expected profits. We also assume that firms truthfully share information about their respective profit margins, demands, and capacity.[6]

4. Two Benchmarks

In this section, we present two benchmarks to guide our discussion on bidirectional contracting. We first characterize the "first-best" capacity allocation, i.e., the capacity allocation that maximizes the alliance's total profit. Although the first-best capacity allocation may not be attainable, it constitutes a natural goal for both firms to strive toward. We then characterize the noncooperative capacity allocation without contract, upon which bidirectional contracts should improve. These two benchmarks enable us to motivate the design of bidirectional contracts as a mechanism to move from the noncooperative capacity allocation toward the first-best capacity allocation.

4.1. First-Best (FB) Capacity Allocation

Given that each product requires exactly one unit of each resource and given that each firm has (leftover) capacity c_i, the capacity allocation that maximizes the alliance's total profit (under a centralized control system) solves the following problem for any realized demand $\mathbf{D}$:

$$\begin{aligned} \mathbf{x}^{FB}(\mathbf{D}) = \arg\max_{\mathbf{x}} \; & p_1 x_1 + p_2 x_2 \\ \text{s.t.} \quad & x_1 + x_2 \le c_i \quad \forall\, i = 1,2, \\ & 0 \le x_i \le D_i \quad \forall\, i = 1,2. \end{aligned} \tag{1}$$

Under our assumption that each product requires one unit of each resource, one of the constraints turns out to be redundant, and the optimal production plan can be expressed in terms of the bottleneck capacity $c \doteq \min\{c_1, c_2\}$.

Because $p_1 \ge p_2$, the optimal capacity allocation possesses the following property: product 1 has priority access to both firms' capacity; and product 2 has access to the residual capacity (if any).[7] Therefore, the optimal production plan, referred to as the first-best (*FB*) capacity allocation, can be expressed as

$$\begin{aligned} & x_1^{FB}(\mathbf{D}) = \min\{D_1, c\}, \quad \text{and} \\ & x_2^{FB}(\mathbf{D}) = \min\{D_2, c - x_1^{FB}(\mathbf{D})\}. \end{aligned} \tag{2}$$

Also, the ex ante total expected optimal profit under the FB capacity allocation is equal to $p_1 \mathbb{E}[x_1^{FB}(\mathbf{D})] + p_2 \mathbb{E}[x_2^{FB}(\mathbf{D})]$. Because the FB capacity allocation yields the highest total expected profit for the alliance, it will be qualified as *efficient*.

4.2. Noncooperative (NC) Capacity Allocation Without Contracts

Although the FB production plan yields the highest total expected profit for the alliance, it will typically not be implementable in a decentralized system. In a bidirectional alliance with complementary resources, each firm needs the other firm's resource; as a result, both firms will have a say on how capacity should be allocated. In contrast, in unidirectional outsourcing agreements, capacity allocation is decided by one firm only, i.e., the subcontractor (Van Mieghem 1999).

Following Harsanyi (1982b), we model this noncooperative capacity allocation process as a simultaneous-move noncooperative unanimity game, in which each firm i proposes a feasible, nonwasteful production plan. If the firms' production plans are identical, the jointly agreed production plan is implemented; otherwise, the negotiations break down and firms get zero profit. Formally, denoting by $x_{i,j}$ the proposed capacity allocation made by firm i for product j, an equilibrium $((\bar{x}_{1,1}, \bar{x}_{1,2}), (\bar{x}_{2,1}, \bar{x}_{2,2}))$ in the noncooperative unanimity game satisfies, for $i = 1,2$,

$$\begin{aligned} (\bar{x}_{i,1}, \bar{x}_{i,2}) = \arg\max \; & p_i x_{i,i} \mathbb{1}_{[(x_{i,1}, x_{i,2}) = (\bar{x}_{j,1}, \bar{x}_{j,2})]} \\ \text{s.t.} \quad & x_{i,1} + x_{i,2} \le c_k, \quad k = 1,2 \\ & 0 \le x_{i,l} \le D_l, \quad l = 1,2, \end{aligned} \tag{3}$$

in which the indicator function $\mathbb{1}_{[x=y]}$ is equal to one if $x = y$ and zero otherwise.[8]

Similar to the FB capacity allocation optimization problem (1), one capacity constraint turns out to be redundant in that formulation. Hence, the NC solution can be expressed in terms of the bottleneck capacity $c = \min\{c_1, c_2\}$. Although firms negotiate over the capacity allocation of both resources, we have observed in practice that their disputes tend to be anchored on one particular resource, such as the bottling capacity in the Nestlé and Ocean Spray alliance (Wang 2014) and the showroom capacity in the Fiat-Tata alliance (Purkayashta and Samad 2013), consistent with our mathematical formulation.

When $D_1 + D_2 > c$, there exists an infinite number of equilibrium points to this unanimity game. To select the equilibrium, we adopt the payoff- and risk-dominance selection rule proposed by Harsanyi and Selten (Harsanyi 1982a). Harsanyi (1982b) shows that the payoff- and risk-dominant solution to this noncooperative game solves the following optimization problem:

$$\begin{aligned} \mathbf{x}^{NC}(\mathbf{D}) = \arg\max_{\mathbf{x}} \; & \left[(p_1 x_1) \cdot (p_2 x_2)\right] \\ \text{s.t.} \quad & x_1 + x_2 \le c \\ & 0 \le x_i \le D_i \quad \forall\, i = 1,2. \end{aligned} \tag{4}$$

In the same spirit as Nash (1950), we say that a capacity allocation is *fair* if there exists no transfer of resource that would increase the profit of one firm by a greater percentage than it would decrease the profit of the other firm (Bertsimas et al. 2011). In our setting, the next proposition shows that this results in a symmetric NC capacity allocation; that is, both firms have equal access to capacity irrespective of their profit margins.

Proposition 1. *Without upfront contract, the equilibrium capacity allocation in the noncooperative unanimity game (3) with the payoff- and risk-dominant equilibrium selection rule (4) is inefficient, but fair. Specifically, the noncooperative capacity allocation without contract $(x_1^{NC}(\mathbf{D}), x_2^{NC}(\mathbf{D}))$ is equal to*

$$(x_1^{NC}(\mathbf{D}), x_2^{NC}(\mathbf{D})) = \begin{cases} (D_1, D_2), & \text{if } D_1 + D_2 \le c; \\ (c - D_2, D_2), & \text{if } D_1 + D_2 \ge c, D_2 \le c/2; \\ (D_1, c - D_1), & \text{if } D_1 + D_2 \ge c, D_1 \le c/2; \\ (c/2, c/2), & \text{if } D_1 \ge c/2, D_2 \ge c/2. \end{cases} \quad (5)$$

The proof of this proposition, as well as all other proofs, appear in the online appendix.

Although fair, the NC capacity allocation is inefficient because it allocates capacity suboptimally in case of capacity rationing, i.e., when $D_1 + D_2 > c$, and when the demand for product 1 is large, i.e., when $D_1 > c/2$. As a result, the alliance's total expected profit generated under the noncooperative capacity allocation is lower than that attained with the FB production plan, i.e., $p_1\mathbb{E}[x_1^{NC}(\mathbf{D})] + p_2\mathbb{E}[x_2^{NC}(\mathbf{D})] \le p_1\mathbb{E}[x_1^{FB}(\mathbf{D})] + p_2\mathbb{E}[x_2^{FB}(\mathbf{D})]$.[9]

4.3. Bidirectional Contracts for Improving Supply Chain Efficiency

In this section, we motivate the design of bidirectional contracts as a mechanism to move from the NC capacity allocation toward the FB production plan. Figure 1 displays the feasible set of capacity allocations (in gray) for a given demand realization $\mathbf{D}$ with $c \ge D_1 \ge c/2$ and $c \ge D_2 \ge c/2$, and depicts the first-best production plan ($\mathbf{x}^{FB}$), which gives product 1 priority access to the capacity, and the noncooperative capacity allocation ($\mathbf{x}^{NC}$), which is fair, but inefficient.

In the following section, we investigate the following questions:

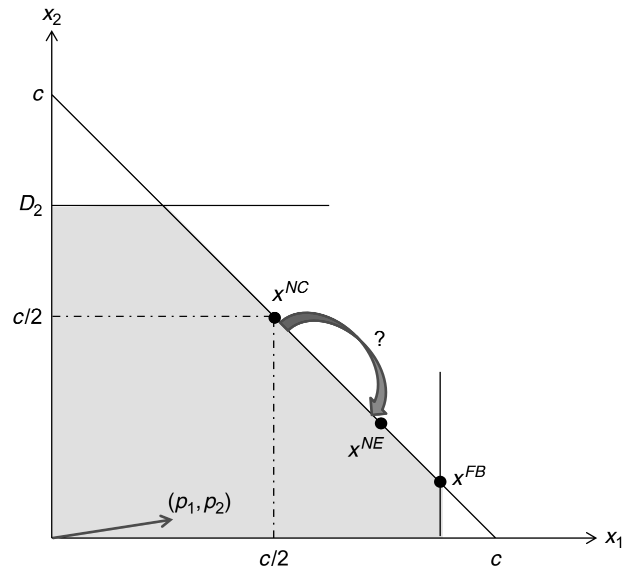
• Can a simple, practical bidirectional contract improve the capacity allocation efficiency upon the noncooperative case with no contract?

• If a bidirectional contract can improve the efficiency of capacity allocation, can it do so in a way that increases both firms' profits (i.e., that is Pareto improving)?

To be enforceable, such bidirectional contracts would need to be agreed upon by both firms in equilibrium. Accordingly, we will refer to such contracts with the superscript *NE*, for Nash equilibrium.

Motivated by practice, we consider the following two bidirectional contracts that are commonly used for managing unidirectional contract manufacturing (e.g., Brown and Lee 2003):

Figure 1. First-Best Capacity Allocation (x^{FB}), Noncooperative Capacity Allocation (x^{NC}), and Equilibrium Capacity Allocation Under a Bidirectional Contract (x^{NE}) When $c \ge D_1 \ge c/2$ and $c \ge D_2 \ge c/2$

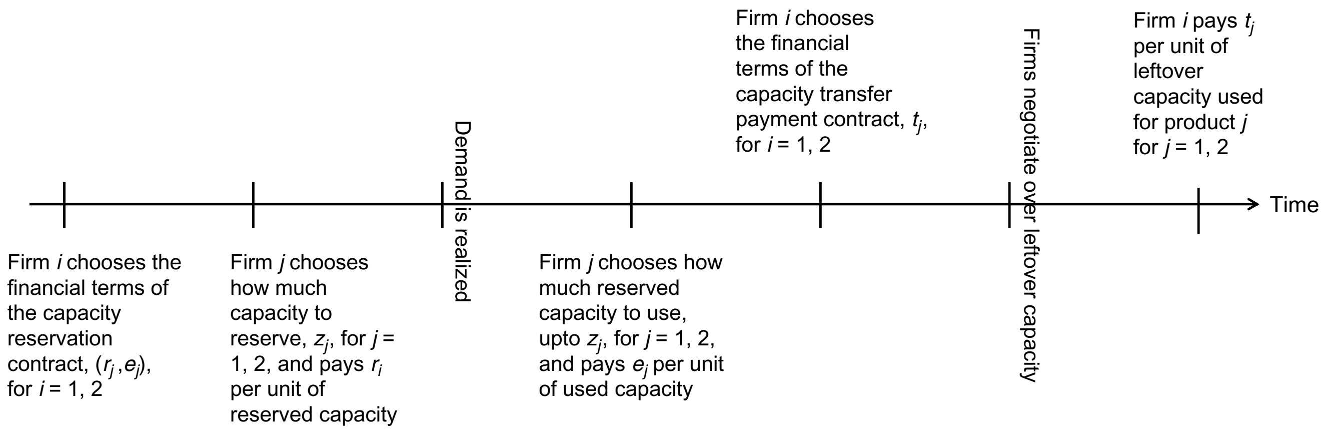


1. *Ex post transfer payments*. Instead of freely transferring capacity, the parties could set transfer prices to govern those transactions. Specifically, this contract, if offered by firm i, sets a price t_j for each unit of firm i's capacity used by product j. This contract is offered ex post, i.e., *after* demand realization.[10] To guarantee firm j's participation, we require that $t_j \le p_j$.

Because of the bidirectional nature of the alliance, we assume that each firm has the right to offer such a contract to the other firm. If both firms choose to offer such contracts, we assume that the financial terms (i.e., the capacity transfer prices) are set simultaneously and noncooperatively so that (t_1, t_2) is a Nash equilibrium. The sequence of events associated with this contractual mechanism are depicted in Figure 2, above the timeline.

2. *Ex ante capacity reservation*. Instead of postponing the capacity allocation decision until demand is realized, firms could reserve some capacity ex ante, i.e., *before* demand realization, and decide, once demand is realized, how much of that reserved capacity they want to utilize (Wu et al. 2013). Specifically, under this contract, firm i specifies two parameters before demand realization: (i) a reservation price r_j for each unit of firm i's capacity that firm j can "reserve" for its product, and (ii) an exercise price e_j for each unit of reserved capacity firm j can "exercise" for firm i to make its product. To guarantee firm j's participation, we require that $r_j \le p_j$ and $e_j \le p_j$. Given a contract (r_j, e_j) specified by firm i, firm j needs to decide on the amount of capacity z_j that it wants to reserve before demand is realized, where $z_j \le c$. Because of the bidirectional nature of the contract, each firm thus needs to make

Figure 2. Timeline of Events with Ex Post Transfer Payment Contracts (Top) and Ex Ante Capacity Reservation Contracts (Bottom)



Notes. The events depicted above the timeline are specific to the ex post transfer payment contract, whereas those depicted below the timeline are specific to the ex ante capacity reservation contract. The events cutting across the timeline apply to both contracts.

three decisions before demand realization. Specifically, firm i needs to decide on the contract (r_j, e_j) for firm j to consider, as well as how much capacity to reserve from firm j for its own product, z_i, based on the contact (r_i, e_i) specified by firm j.

Because of the bidirectional nature of the alliance, we assume that each firm has the right to offer such a contract to the other firm. If both firms choose to offer such contracts, we assume that the financial terms (i.e., the capacity reservation and exercise prices) are set simultaneously and noncooperatively so that $((r_1, e_1), (r_2, e_2))$ is a Nash equilibrium. Then, once those financial terms have been set, firms are assumed to choose simultaneously and noncooperatively how much capacity to reserve, with the constraint that $z_1 + z_2 \le c$, i.e., (z_1, z_2) must be a Nash equilibrium. Finally, if there is some leftover capacity and unfulfilled demand after both firms have exercised their reserved capacity, the remaining capacity is allocated through negotiation in the context of a noncooperative unanimity game, similar to §4.2. The sequence of events associated with this contractual mechanism are depicted in Figure 2, below the timeline.

In terms of notation, we adopt the convention that the subscripts of contractual decisions refer to the *product* index to which those decisions are related, as summarized in Table 1.

Table 1. Contractual Decisions

Contract	Ex ante capacity reservation		Ex post transfer payments	
Decision maker	Product i	Product j	Product i	Product j
Firm i	z_i	r_j, e_j	∅	t_j
Firm j	r_i, e_i	z_j	t_i	∅

In principle, these two bidirectional contracts can be used in combination. Specifically, the firms could reserve capacity ex ante, then, after demand is realized, choose whether to exercise their option of using the reserved capacity. If, after that, there is still some leftover capacity and unfulfilled demand, the firms could then offer each other an ex post transfer payment contract and then negotiate over the remaining capacity. See Figure 2 for a complete timeline of events when the two contracts are used in combination. In that case, the contracting game consists of four "nested" Nash equilibria: to determine the financial terms of the capacity reservation contracts, to choose how much capacity to reserve, to determine the financial terms of the ex post transfer payment contracts, and to allocate the residual capacity. We first study separately the ex post transfer payment contract in §5 and the ex ante capacity reservation contract in §6. We then study the combined contract in §7.1 and show that combining these contract features may not necessarily result in more efficient decisions.

5. Ex Post Transfer Payment Contracts

In this section, we analyze the ex post transfer payment contracts. Since each contract specifies a capacity transfer price t_i, the capacity allocation decisions will depend on $\mathbf{t} = (t_1, t_2)$. As is customary, we solve the game backward. First, we identify the noncooperative capacity allocation $\mathbf{x}^{NC}(\mathbf{D}, \mathbf{t})$ for a given $\mathbf{t}$, which is determined in the context of the aforementioned noncooperative unanimity game (Harsanyi 1982b). We then characterize the equilibrium transfer prices (t_1^{NE}, t_2^{NE}), when firms anticipate the NC capacity allocation plan. We finally analyze the equilibrium profits and compare them to the FB profits and the profits obtained without contract. To simplify the exposition, we present here only one formal statement, characterizing the efficiency of ex post transfer payment contracts, and only outline the argument underlying the other results, for which the formal statements appear in the online appendix.

5.1. Noncooperative Capacity Allocation $\mathbf{x}^{NC}(\mathbf{D},\mathbf{t})$

For any given transfer payment contract $\mathbf{t}$ and any feasible capacity allocation $\mathbf{x}$, the ex post profit of firm i can be expressed as $(p_i-t_i)x_i+t_jx_j$. Similar to the case with no contract, described in §4.2, we model the noncooperative capacity allocation game as a noncooperative unanimity game (Harsanyi 1982b), similar to (3), with firm i's profit function now equal to $(p_i-t_i)x_{i,i}+t_jx_{i,j}$ if firm i's proposed production plan $(x_{i,1},x_{i,2})$ is equal to firm j's proposed production plan $(x_{j,1},x_{j,2})$, and zero otherwise. Similar to the case with no contract, there exists an infinite number of equilibrium solutions when $D_1+D_2>c$, and we select the following payoff- and risk-dominant equilibrium solution $\mathbf{x}^{NC}(\mathbf{D},\mathbf{t})$:

$$\begin{aligned}&\mathbf{x}^{NC}(\mathbf{D},\mathbf{t})\\&=\arg\max_{\mathbf{x}}\left[((p_1-t_1)x_1+t_2x_2)\cdot((p_2-t_2)x_2+t_1x_1)\right]\\&\text{s.t.}\quad x_1+x_2\le c,\\&\qquad\quad 0\le x_i\le D_i,\quad \forall\, i=1,2.\end{aligned}\tag{6}$$

Depending on the transfer prices, the noncooperative capacity allocation $\mathbf{x}^{NC}(\mathbf{D},\mathbf{t})$ can be efficient or not. We consider two cases, depending on whether $p_1\ge t_1+t_2\ge p_2$ or not. When $p_1\ge t_1+t_2\ge p_2$, firm 1's profit margin on product 1 (p_1-t_1) is greater than or equal to its profit margin on product 2 (t_2) and similarly, firm 2's profit margin on product 1 (t_1) is greater than or equal to its profit margin on product 2 (p_2-t_2). Consequently, both firms agree that product 1 should have priority access to their respective capacities, and the FB production plan (2) is implemented through the noncooperative capacity allocation, i.e., $\mathbf{x}^{NC}(\mathbf{D},\mathbf{t})=\mathbf{x}^{FB}(\mathbf{D})$ for any $\mathbf{D}$.[11]

In contrast, when either $t_1+t_2\ge p_1$ or $t_1+t_2\le p_2$, the NC capacity allocation is in general inefficient. Formally, we show in the online appendix that, when $t_1+t_2\ge p_1$ or $t_1+t_2\le p_2$, the NC capacity allocation is given by

$$\begin{aligned}&x_1^{NC}(\mathbf{D},\mathbf{t})=\min\{c,D_1,\max\{0,c-D_2,c\cdot\chi(\mathbf{t})\}\}\quad\text{and}\\&x_2^{NC}(\mathbf{D},\mathbf{t})=\min\{c,D_2,\max\{0,c-D_1,c\cdot(1-\chi(\mathbf{t}))\}\},\\&\qquad\text{where }\chi(\mathbf{t})=\frac{1}{2}\left(\frac{p_2-t_2}{p_2-t_1-t_2}-\frac{t_2}{p_1-t_1-t_2}\right).\end{aligned}\tag{7}$$

Therefore when either $t_1+t_2\ge p_1$ or $t_1+t_2\le p_2$, $\mathbf{x}^{NC}(\mathbf{D},\mathbf{t})$ may be different from the first-best production plan $\mathbf{x}^{FB}(\mathbf{D})$. In particular, when $t_1=t_2=0$ (such that $t_1+t_2\le p_2$) or when $t_i=p_i$ for $i=1,2$ (such that $t_1+t_2\ge p_1$), $\chi(\mathbf{0})=\chi(\mathbf{p})=1/2$ and the NC capacity allocation is the same as the plan without contract as given in (5), i.e., $\mathbf{x}^{NC}(\mathbf{D},\mathbf{0})=\mathbf{x}^{NC}(\mathbf{D})$, which is inefficient.

Although the noncooperative capacity allocation with the ex post transfer payment contract $\mathbf{x}^{NC}(\mathbf{D},\mathbf{t})$ may be inefficient (when either $t_1+t_2\ge p_1$ or $t_1+t_2\le p_2$), it may still improve upon the plan with no contract $\mathbf{x}^{NC}(\mathbf{D})$. Because the efficiency of a capacity allocation hinges upon the priority access given to product 1, $\mathbf{x}^{NC}(\mathbf{D},\mathbf{t})$ is more efficient than $\mathbf{x}^{NC}(\mathbf{D})$ if it allocates more capacity to product 1, i.e., if $x_1^{NC}(\mathbf{D},\mathbf{t})\ge x_1^{NC}(\mathbf{D})=x_1^{NC}(\mathbf{D},\mathbf{0})$ for all $\mathbf{D}$. This happens when $\chi(\mathbf{t})\ge\chi(\mathbf{0})$, i.e., when $t_1(p_1-t_1)\ge t_2(p_2-t_2)$. By noting that t_1 is the capacity transfer payment firm 2 receives from making product 1 and that (p_1-t_1) is the net margin firm 1 receives from making its own product, we conclude that both firms agree to allocate more capacity to product 1 if the product of their respective margins on that product, $t_1\cdot(p_1-t_1)$, is higher.

We next characterize the equilibrium transfer prices, which will determine if, in equilibrium, the noncooperative capacity allocation is efficient (if $p_1\ge t_1+t_2\ge p_2$) or not (if either $t_1+t_2\ge p_1$ or $t_1+t_2\le p_2$), and in the latter case, if it is nevertheless more efficient than the capacity allocation with no contract.

5.2. Equilibrium Transfer Payments (t_1^{NE},t_2^{NE})

Under the transfer payment contract, the equilibrium transfer payments are chosen simultaneously and noncooperatively. In equilibrium, they should satisfy the following conditions:

$$\begin{aligned}&t_j^{NE}(\mathbf{D})=\arg\max_{0\le t_j\le p_j}\Pi_i(t_j;t_i^{NE}(\mathbf{D}),\mathbf{D})\ \ \forall\, j=1,2,\\&\quad\text{where }\Pi_i(t_j;t_i,\mathbf{D})=(p_i-t_i)\cdot x_i^{NC}(\mathbf{D},\mathbf{t})+t_j\cdot x_j^{NC}(\mathbf{D},\mathbf{t}),\end{aligned}\tag{8}$$

in which $\mathbf{x}^{NC}(\mathbf{D},\mathbf{t})$ is given by (7) if $t_1+t_2\ge p_1$ or $t_1+t_2\le p_2$ and $\mathbf{x}^{NC}(\mathbf{D},\mathbf{t})=\mathbf{x}^{FB}(\mathbf{D})$ otherwise.

Upon examining (8), it can be shown that firm 1 will attempt to capture all product 2's profit margin and set $t_2^{NE}(\mathbf{D})=p_2$, irrespective of firm 2's decision and of the demand realization. That is, setting $t_2^{NE}(\mathbf{D})=p_2$ is a dominant strategy for firm 1.

In contrast, it may happen, in equilibrium, that $t_1^{NE}(\mathbf{D})<p_1$, i.e., that firm 2 leaves some profit margin to firm 1. When $t_1^{NE}(\mathbf{D})=p_1$, firm 1 earns p_2x_2 and firm 2 earns p_1x_1. In that case, the equilibrium capacity allocation is the same as the allocation with no contract, since $\mathbf{x}^{NC}(\mathbf{D},\mathbf{p})=\mathbf{x}^{NC}(\mathbf{D})$. As we discussed above, this outcome is quite inefficient since product 1 receives only its "fair" share of capacity despite its higher profit margin. Accordingly, it may be in firm 2's best interest to slightly give up on product 1's profit margin by setting $t_1^{NE}(\mathbf{D})<p_1$ so as to induce the negotiations to evolve toward a more efficient outcome, by allocating a greater share of capacity to product 1. In that

case, what firm 2 loses on product 1's profit margin is recaptured through greater sales. Formally, we show in the online appendix that the equilibrium transfer payments $(t_1^{NE}(\mathbf{D}), t_2^{NE}(\mathbf{D}))$ solving (8) can be expressed as

$$(t_1^{NE}(\mathbf{D}), t_2^{NE}(\mathbf{D})) = \begin{cases} (p_1, p_2), & \text{if } D_1 \le \dfrac{c}{2} \text{ or } \\ & p_1(c-D_2) > (p_1-p_2)\min\{c, D_1\} + p_2\dfrac{c}{2}; \\ \left(p_1 - \dfrac{p_2}{2}, p_2\right), & \text{if } D_1 \ge c \text{ and } D_2 \ge \dfrac{p_2}{p_1}\dfrac{c}{2}; \\ \left(p_1 - p_2 + \dfrac{p_2}{2}\dfrac{c}{D_1}, p_2\right), & \text{if } c > D_1 > \dfrac{c}{2} \text{ and } \\ & p_1(c-D_2) \le (p_1-p_2)D_1 + p_2\dfrac{c}{2}. \end{cases} \tag{9}$$

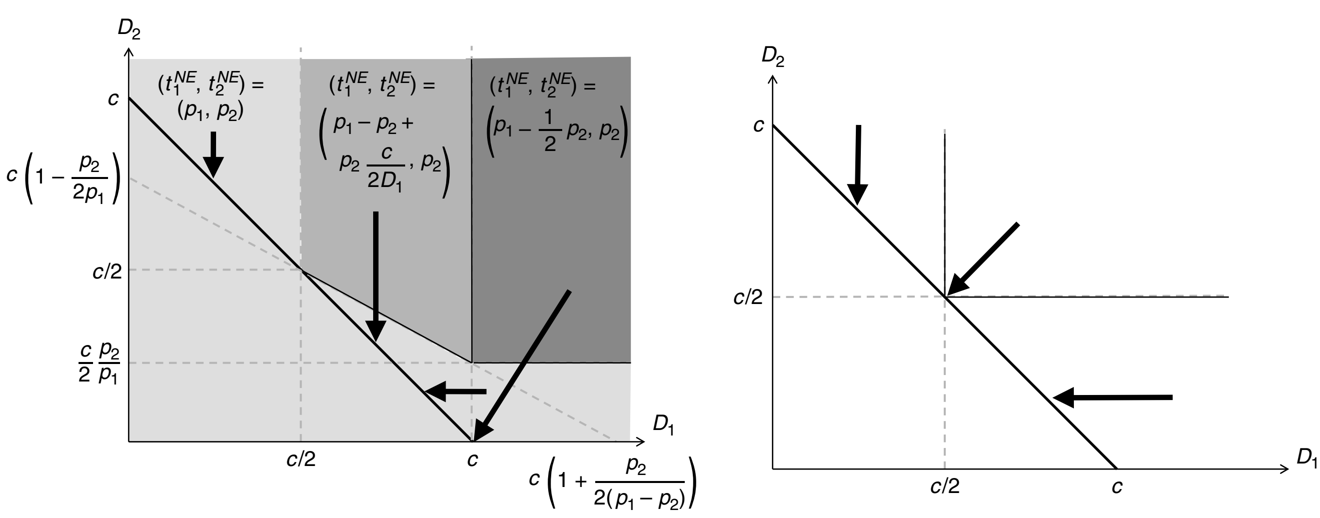
In general, this equilibrium is unique everywhere except when a product's equilibrium capacity allocation is zero or at the boundaries between the three regions defined in (9). In the former case, all equilibria are payoff invariant,[12] whereas in the latter case, they have measure zero.[13] Accordingly, we assume without loss of generality that, in case of multiple equilibria, the selected equilibrium is given by (9).

Hence, in every demand scenario, it appears from (9) that $t_2^{NE}(\mathbf{D}) = p_2$ and $t_1^{NE}(\mathbf{D}) > p_1 - p_2$. Therefore, the condition $p_1 \ge t_1 + t_2 \ge p_2$ will not happen in equilibrium, and the resulting capacity allocation will not be efficient since firms will not uniformly agree that product 1 should have priority access to their capacities. In particular, by substituting the equilibrium transfer prices (9) into (7), we obtain the following noncooperative capacity allocation under the equilibrium transfer price contract:

$$(x_1^{NC}(\mathbf{D}, \mathbf{t}^{NE}(\mathbf{D})), x_2^{NC}(\mathbf{D}, \mathbf{t}^{NE}(\mathbf{D}))) = \begin{cases} (D_1, D_2), & \text{if } D_1 + D_2 \le c; \\ (D_1, c - D_1), & \text{if } D_1 + D_2 \ge c, D_1 < c, \text{ and } \\ & p_1(c-D_2) \le (p_1-p_2)D_1 + p_2 c/2; \\ (c, 0), & \text{if } D_1 \ge c, D_2 \ge (p_2/p_1)(c/2); \\ (c - D_2, D_2), & \text{if } D_1 + D_2 \ge c \text{ and } \\ & p_1(c-D_2) > (p_1-p_2)D_1 + p_2 c/2 \\ & \text{or if } D_1 \ge c, D_2 < (p_2/p_1)(c/2). \end{cases} \tag{10}$$

Figure 3 compares the noncooperative capacity allocations with and without ex post transfer payment contracts, i.e., (10) and (5), respectively, as a function of the demand realizations. From the figure, it appears that the noncooperative capacity allocation under the equilibrium ex post transfer payment contract is more efficient than the plan without an upfront contract. To investigate this further, let us first consider the case when $c > D_1 \ge c/2$ and $D_2 \ge c/2$. The noncooperative capacity allocation without contract (5) yields $(x_1^{NC}(\mathbf{D}), x_2^{NC}(\mathbf{D})) = (c/2, c/2)$, which is inefficient. Because this case falls into the second demand scenario in (10), the noncooperative capacity allocation under the equilibrium transfer payment contract yields $(x_1^{NC}(\mathbf{D}, \mathbf{t}^{NE}(\mathbf{D})), x_2^{NC}(\mathbf{D}, \mathbf{t}^{NE}(\mathbf{D}))) = (D_1, c - D_1)$, which is the FB production plan (2) for the case when $c > D_1 \ge c/2$ and $D_2 \ge c/2$. Hence, by

Figure 3. Noncooperative Capacity Allocations with An Ex Post Transfer Payment Contract (Left) and Without It (Right)



Notes. The figure illustrates the noncooperative capacity allocations (10) and (5), respectively, with an ex post transfer price contract (left) and without contract (right). In both cases, the capacity allocation depends on whether demand falls into one out of four regions, delimited in the figure by solid lines (including the capacity constraint). For each of those regions, one or two arrows depict the equilibrium capacity allocation corresponding to a representative demand vector from that region; specifically, the demand vector lies at the origin of the arrow and the equilibrium plan lies at the destination of the arrow. In the left figure, the different colors of regions are associated with different capacity transfer prices (t_1^{NE}, t_2^{NE}).

giving product 1 full priority to the capacity, the capacity allocation under the equilibrium transfer payment contract maximizes the alliance's total profit. Next, consider the case when $D_2 \le c/2$ and $D_1 + D_2 \ge c$. The noncooperative capacity allocation without contract (5) yields $(x_1^{NC}(\mathbf{D}), x_2^{NC}(\mathbf{D})) = (c - D_2, D_2)$, which is inefficient. However, under the equilibrium transfer payment contract, this inefficient capacity allocation $(x_1^{NC}(\mathbf{D}, \mathbf{t}^{NE}(\mathbf{D})), x_2^{NC}(\mathbf{D}, \mathbf{t}^{NE}(\mathbf{D}))) = (c - D_2, D_2)$ arises only under more restrictive conditions on demands. (To see this graphically, compare the regions that result in a horizontal arrow as depicted in Figure 3 (left and right figures).) Therefore, even though the noncooperative capacity allocation under the equilibrium transfer payment contract is not fully efficient, it is more efficient than that without contract.

5.3. Equilibrium Profits

It follows from the discussion above that, relative to the case without contract as presented in §4.2, the ex post transfer payment contract improves the total profit of the alliance, even though it may not achieve full efficiency. More formally, by denoting $\mathbb{E}[\Pi_i(t_j^{NE}(\mathbf{D}); t_i^{NE}(\mathbf{D}), \mathbf{D})]$ as firm i's ex ante expected profit in equilibrium under the ex post transfer payment contract, we can conclude that

$$\begin{aligned} &p_1\mathbb{E}[x_1^{FB}(\mathbf{D})] + p_2\mathbb{E}[x_2^{FB}(\mathbf{D})] \\ &\quad \ge \mathbb{E}[\Pi_1(t_2^{NE}(\mathbf{D}); t_1^{NE}(\mathbf{D}), \mathbf{D})] + \mathbb{E}[\Pi_2(t_1^{NE}(\mathbf{D}); t_2^{NE}(\mathbf{D}), \mathbf{D})] \\ &\quad \ge p_1\mathbb{E}[x_1^{NC}(\mathbf{D})] + p_2\mathbb{E}[x_2^{NC}(\mathbf{D})]. \end{aligned}$$

Although the alliance's total profit increases under the ex post transfer payment contract, a firm may end up with a lower expected profit with the transfer payment contract than without. That is, the ex post transfer payment contract may not make both firms better off, i.e., it may not be Pareto improving. To elaborate, consider the case where there is no capacity rationing, i.e., $D_1 + D_2 \le c$. Under the noncooperative capacity allocation without contract (5), firm 1 earns $p_1 D_1$ and firm 2 earns $p_2 D_2$. However, under the ex post transfer payment contract, the equilibrium transfer prices as given in (9) are $(t_1^{NE}(\mathbf{D}), t_2^{NE}(\mathbf{D})) = (p_1, p_2)$ (see Figure 3, left). Accordingly, firm 1 earns $p_2 D_2$ and firm 2 earns $p_1 D_1$. Therefore, unless $p_1 D_1 = p_2 D_2$, one firm thus ends up with a lower profit with the contract whereas the other firm obtains higher profit. In general, demand may be random, and there could be capacity rationing; but provided that the probability that demand falls into the region where $(t_1^{NE}(\mathbf{D}), t_2^{NE}(\mathbf{D})) = (p_1, p_2)$ (light gray region in Figure 3, left), is significant, the contract will not be Pareto improving.

The following proposition summarizes the above results and discussion in a formal manner.

Proposition 2. *In equilibrium, the ex post transfer payment contract results in a more efficient capacity allocation than the noncooperative capacity allocation without contract as given in (5), even though full efficiency is in general not achieved. However, the ex post transfer payment contract may not be Pareto improving in the sense that one firm may earn less under the capacity allocation associated with the ex post transfer payment contract than under the noncooperative capacity allocation without contract.*

To recap the intuition behind this result, the ex post transfer payment contract improves the efficiency of the noncooperative capacity allocation process, relative to a situation with no contract, because it better aligns the firms' profit margins and therefore ensures that product 1 is given more often priority access to capacity. It may not, however, be Pareto improving because, in some demand scenarios, firms will attempt to extract all profit margin from the other firm's product, making firm i earn all profit on product j, and vice versa.

Because the ex post transfer payment contract may not be Pareto improving, it may be challenging to implement in practice. That is, firms may find it difficult to agree on adopting this contract since one of them may end up with a smaller profit. Although it is possible for a firm to offer a lump-sum side payment as a way to share the gain with the other firm who earns a lower profit, lump-sum payments are generally difficult to negotiate in practice, especially when demands are uncertain. This motivates us to examine next whether the ex ante capacity reservation contract is Pareto improving and is more efficient than without contract.

6. Ex Ante Capacity Reservation Contracts

We now analyze the ex ante capacity reservation contract. An ex ante capacity reservation contract offered by firm j specifies a reservation price (r_i) and an exercise price (e_i), based on which firm i would choose how much capacity to reserve (z_i). As in the previous section, we solve the corresponding game via backward induction. We first characterize the noncooperative capacity allocation $\mathbf{x}^{NC}(\mathbf{D}, \mathbf{z})$ after demand $\mathbf{D}$ is realized and after firms have exercised their reserved capacity $\mathbf{z}$. We then study how much capacity (z_i) firms will reserve ahead of time, before demand is realized; we will however do so not for any capacity reservation price (r_i) and capacity exercise price (e_i), but only for those that are offered in equilibrium, thereby studying these three decisions simultaneously. We then characterize the equilibrium profits and finally establish some mild conditions under which full efficiency is achieved under the ex ante capacity reservation contract. Similar to the previous section, we formally present only the efficiency characterization of the contracts and, to simplify the exposition, only outline here the other results, which are formally stated in the online appendix.

6.1. Noncooperative Capacity Allocation $\mathbf{x}^{NC}(\mathbf{D},\mathbf{z})$

Let us first determine how capacity is allocated after demand is realized and after both firms exercise their reserved capacity. Essentially, after demand is realized, firm i will request firm j to allocate $\min\{D_i, z_i\}$ units of capacity to product i, $i = 1, 2$. Hence, the residual demand for firm i is equal to $[D_i - z_i]^+$, for $i = 1, 2$, and the remaining capacity for each firm is equal to $c - \min\{D_1, z_1\} - \min\{D_2, z_2\}$. As in the case with no contract described in §4.2, we model the capacity negotiation game as a noncooperative unanimity game (Harsanyi 1982b), similar to (3) with the demand D_i replaced by the residual demands $[D_i - z_i]^+$, for $i = 1, 2$, and the capacity c replaced with the residual capacity $c - \min\{D_1, z_1\} - \min\{D_2, z_2\}$. When $D_1 + D_2 > c$, there exists an infinite number of equilibria to the unanimity game, and similar to §4.2, we select the payoff- and risk-dominant equilibrium, which solves

$$\begin{aligned}\mathbf{x}^{NC}&(\mathbf{D},\mathbf{z})\\ &= \arg\max_{\mathbf{x}} \left[(p_1 x_1)\cdot(p_2 x_2)\right]\\ &\quad \text{s.t.} \quad x_1 + x_2 \le c - \min\{D_1, z_1\} - \min\{D_2, z_2\}\\ &\qquad\qquad 0 \le x_i \le [D_i - z_i]^+ \quad \forall\, i = 1, 2.\end{aligned} \tag{11}$$

Because this problem has the same structure as the noncooperative capacity allocation with no contract as defined in (4), its solution, $\mathbf{x}^{NC}(\mathbf{D},\mathbf{z})$, is of the same nature as $\mathbf{x}^{NC}(\mathbf{D})$, stated in (5), with D_i replaced with the residual demand $[D_i - z_i]^+$, for $i = 1, 2$ and the capacity c replaced with the "residual capacity" $c(\mathbf{D},\mathbf{z}) \doteq c - \min\{D_1, z_1\} - \min\{D_2, z_2\}$. Specifically, it is easy to check that the corresponding noncooperative capacity allocation under the capacity reservation contract $\mathbf{x}^{NC}(\mathbf{D},\mathbf{z})$ can be expressed as

$$\begin{aligned}&(x_1^{NC}(\mathbf{D},\mathbf{z}), x_2^{NC}(\mathbf{D},\mathbf{z}))\\ &= \begin{cases} ([D_1 - z_1]^+, [D_2 - z_2]^+), \\ \qquad \text{if } [D_1 - z_1]^+ + [D_2 - z_2]^+ \le c(\mathbf{D},\mathbf{z}); \\ (c(\mathbf{D},\mathbf{z}) - [D_2 - z_2]^+, [D_2 - z_2]^+), \\ \qquad \text{if } [D_1 - z_1]^+ + [D_2 - z_2]^+ \ge c(\mathbf{D},\mathbf{z}), \\ \qquad [D_2 - z_2]^+ \le 0.5\cdot c(\mathbf{D},\mathbf{z}); \\ ([D_1 - z_1]^+, c(\mathbf{D},\mathbf{z}) - [D_1 - z_1]^+), \\ \qquad \text{if } [D_1 - z_1]^+ + [D_2 - z_2]^+ \ge c(\mathbf{D},\mathbf{z}), \\ \qquad [D_1 - z_1]^+ \le 0.5\cdot c(\mathbf{D},\mathbf{z}); \\ (0.5\cdot c(\mathbf{D},\mathbf{z}), 0.5\cdot c(\mathbf{D},\mathbf{z})), \\ \qquad \text{if } [D_1 - z_1]^+ \ge 0.5\cdot c(\mathbf{D},\mathbf{z}), \\ \qquad [D_2 - z_2]^+ \ge 0.5\cdot c(\mathbf{D},\mathbf{z}). \end{cases}\end{aligned} \tag{12}$$

Observe from (12) that, when neither firm reserves any capacity, i.e., $z_1 = z_2 = 0$, the NC capacity allocation with an ex ante capacity reservation contract is the same as the one with no contract, i.e., $\mathbf{x}^{NC}(\mathbf{D},\mathbf{0}) = \mathbf{x}^{NC}(\mathbf{D})$. Hence, similar to Proposition 1, the noncooperative capacity allocation under an ex ante capacity reservation contract is inefficient, but fair.

6.2. Equilibrium Capacity Reservation $\mathbf{z}^{NE}(\mathbf{r},\mathbf{e})$

We next characterize how much capacity each firm will reserve in equilibrium, before demand is realized, with any given contract $(\mathbf{r},\mathbf{e})$, anticipating the noncooperative capacity allocation for the remaining capacity $\mathbf{x}^{NC}(\mathbf{D},\mathbf{z})$. When firms choose their reserved capacity simultaneously and noncooperatively, the equilibrium reserved capacities solve

$$z_i^{NE}(\mathbf{r},\mathbf{e}) = \arg\max_{z_i \le c - z_j^{NE}(\mathbf{r},\mathbf{e})} \Pi_i(z_i; z_j^{NE}(\mathbf{r},\mathbf{e}), \mathbf{r}, \mathbf{e}) \quad \forall\, i = 1, 2, \tag{13}$$

where firm i's (ex ante) expected profit associated with any given contract $(\mathbf{r},\mathbf{e})$ is equal to

$$\begin{aligned}\Pi_i(z_i; z_j, \mathbf{r}, \mathbf{e}) = (p_i - e_i)\mathbb{E}[\min\{D_i, z_i\}] + p_i \mathbb{E}[x_i^{NC}(\mathbf{D},\mathbf{z})]\\ + e_j \mathbb{E}[\min\{D_j, z_j\}] - r_i z_i + r_j z_j.\end{aligned} \tag{14}$$

In that profit function, the first term corresponds to the sales generated with the reserved capacity. For each unit sold, firm i generates a gross profit p_i, but must pay firm j the capacity exercise price e_i. The second term corresponds to the sales generated from the units that are allocated noncooperatively through the unanimity game (12), after firms have exercised their capacity reservation contract. For each of those units, firm i generates p_i units of gross profit since there are no transfer payments associated with this ex post capacity allocation process, unlike with the ex post transfer payment contract (discussed in §5). The third term corresponds to the revenue firm i makes when firm j exercises its reserved capacity. The fourth term corresponds to the payment firm i must make to firm j to reserve capacity, and the last term corresponds to the payment firm i receives from firm j to reserve capacity.

Similar to the ex post transfer payment contract presented in §5, the ex ante capacity reservation contract can, in principle, achieve the FB capacity allocation. Specifically, when $z_1 = c$ and $z_2 = 0$, product 1 has priority access to the firms' capacities and the FB capacity allocation is attained. However, we shall show that this particular capacity reservation with $z_1 = c$ and $z_2 = 0$ may not always arise in equilibrium. Instead of characterizing the equilibrium choices of reserved capacity $\mathbf{z}^{NE}(\mathbf{r},\mathbf{e})$ for any ex ante capacity reservation contract $(\mathbf{r},\mathbf{e})$, it is actually more convenient for us to characterize $\mathbf{z}^{NE}(\mathbf{r}^{NE},\mathbf{e}^{NE})$; i.e., the equilibrium choices of reserved capacity for the equilibrium contract $(\mathbf{r}^{NE},\mathbf{e}^{NE})$. This motivates us to characterize the equilibrium capacity reservation contract $(\mathbf{r}^{NE},\mathbf{e}^{NE})$ next.

6.3. Equilibrium Capacity Reservation Contract $(\mathbf{r}^{NE}, \mathbf{e}^{NE})$

Anticipating the capacities that each firm i will reserve in equilibrium $z_i^{NE}(\mathbf{r}, \mathbf{e})$, which satisfy (13), for $i = 1, 2$, the equilibrium capacity reservation contract $(\mathbf{r}^{NE}, \mathbf{e}^{NE})$ solves

$$(r_j^{NE}, e_j^{NE}) = \arg\max_{r_j, e_j} \Pi_i(r_j, e_j; r_i^{NE}, e_i^{NE}) \quad \forall\, i = 1, 2, \quad (15)$$

in which firm i's (ex ante) profit is equal to

$$\Pi_i(r_j, e_j; r_i, e_i) = \Pi_i(z_i^{NE}(\mathbf{r}, \mathbf{e}); z_j^{NE}(\mathbf{r}, \mathbf{e}), \mathbf{r}, \mathbf{e}), \quad (16)$$

and where the profit function $\Pi_i(z_i^{NE}(\mathbf{r}, \mathbf{e}); z_j^{NE}(\mathbf{r}, \mathbf{e}), \mathbf{r}, \mathbf{e})$ can be obtained from (14).

Upon examining (15), it can be shown that, regardless of firm 2's decision, firm 1 will attempt to extract all surplus from firm 2 by setting r_2 as large as possible. That is, $r_2^{NE} = p_2$. As a result, firm 2 never reserves capacity in equilibrium, i.e., $z_2^{NE}(\mathbf{r}^{NE}, \mathbf{e}^{NE}) = 0$.

This outcome is due to a misalignment between the firms' equal bargaining powers, which results in an even allocation of capacity ex post, and their different profit margins, which make capacity more valuable to firm 1. To understand this tension, let us examine the total quantity that firm 2 will receive, i.e., $\min\{D_2, z_2\} + x_2^{NC}(\mathbf{D}, \mathbf{z})$. We first argue that, unless $[D_1 - z_1]^+ \geq 0.5 \cdot c(\mathbf{D}, \mathbf{z})$ and $[D_2 - z_2]^+ \geq 0.5 \cdot c(\mathbf{D}, \mathbf{z})$, firm 2 will receive the same total quantity regardless of its reservation decision z_2. To establish this result, we successively consider the first three demand scenarios in (12). First, let us consider the case when there is enough capacity to satisfy both demands, i.e., when $[D_1 - z_1]^+ + [D_2 - z_2]^+ \leq c(\mathbf{D}, \mathbf{z})$, or equivalently, when $D_1 + D_2 \leq c$. In this case, reserving capacity has obviously no effect on the quantities produced for any firm i since there is no capacity rationing. The second and third demand scenarios are symmetric and we consider them jointly. When there is capacity rationing (i.e., $D_1 + D_2 \geq c$), and all demand for product i is satisfied in the negotiated plan (i.e., when $[D_i - z_i]^+ \leq 0.5 \cdot c(\mathbf{D}, \mathbf{z})$), then reserving capacity for product i has obviously no effect on the quantities produced for firm i; and therefore, reserving capacity for product i does not affect the quantity produced for firm j either.

It remains to consider the fourth demand scenario in (12) that has $[D_i - z_i]^+ \geq 0.5 \cdot c(\mathbf{D}, \mathbf{z})$ for both $i = 1, 2$, or equivalently, when $D_i \geq (c + z_i - z_j)/2$ for $i = 1, 2$ and $j \neq i$. We now argue that, under this demand scenario, firm 2 will not reserve capacity from firm 1. In this fourth demand scenario, (12) reveals that each firm is allocated half of the residual capacity; i.e., $0.5 \cdot c(\mathbf{D}, \mathbf{z})$. Accordingly, each unit of capacity in that demand scenario is worth $p_1/2$ to firm 1 and $p_2/2$ to firm 2. Hence, firm 2 will reserve capacity if the total expected cost for reserving and exercising capacity is smaller than its ex post valuation, i.e., if $r_2 + e_2\mathbb{P}[D_2 \geq z_2] \leq (p_2/2)\mathbb{P}[D_1 \geq (c + z_1 - z_2)/2, D_2 \geq (c + z_2 - z_1)/2]$. Conversely, firm 1 will let firm 2 reserve capacity if the total price it expects to receive from firm 2 is larger than its ex post valuation, i.e., if $r_2 + e_2 \mathbb{P}[D_2 \geq z_2] \geq (p_1/2)\mathbb{P}[D_1 \geq (c + z_1 - z_2)/2, D_2 \geq (c + z_2 - z_1)/2]$. When $p_1 > p_2$, the parties cannot agree, and firm 2 does not reserve capacity in equilibrium.[14]

By contrast, firm 1 will in general reserve capacity in equilibrium, i.e., $z_1^{NE}(\mathbf{r}^{NE}, \mathbf{e}^{NE}) \geq 0$, since its ex post valuation of capacity, $(p_1/2)\mathbb{P}[D_1 \geq (c + z_1 - z_2)/2, D_2 \geq (c + z_2 - z_1)/2]$, is larger than firm 2's ex post valuation of capacity, $(p_2/2)\mathbb{P}[D_1 \geq (c + z_1 - z_2)/2,\ D_2 \geq (c + z_2 - z_1)/2]$, provided of course that $\mathbb{P}[D_1 \geq (c + z_1 - z_2)/2, D_2 \geq (c + z_2 - z_1)/2] > 0$. If the latter condition does not hold, i.e., if $\mathbb{P}[D_1 \geq (c + z_1)/2, D_2 \geq (c - z_1)/2] = 0$ given that $z_2^{NE}(\mathbf{r}^{NE}, \mathbf{e}^{NE}) = 0$, then firm 1 will receive no benefit from reserving capacity and, unless $r_1 = e_1 = 0$, it will be tempted to reserve as little capacity as possible. In particular, if $\mathbb{P}[D_1 \geq (c + z_1)/2, D_2 \geq (c - z_1)/2] = 0$ for all $z_1 \geq 0$, it is optimal for firm 1 not to reserve any capacity since it anticipates to receive all capacity it needs to fulfill its demand through the ex post noncooperative capacity allocation process.

Moreover, in equilibrium, when $z_1^{NE}(\mathbf{r}^{NE}, \mathbf{e}^{NE}) > 0$, it can be shown that firm 2 will let firm 1 freely reserve capacity and will charge only when it exercises its reserved capacity, i.e., firm 2 sets $r_1^{NE} = 0$ and $e_1^{NE} > 0$. Intuitively, capacity reservation introduces an additional risk for firm 1, in case product 1's demand turns out be smaller than the reserved capacity. This overage risk lowers firm 1's incentives to reserve capacity, and therefore negatively affects the alliance's total profit, since the FB capacity allocation is achieved when firm 1 reserves all capacity. Fortunately, this risk is totally avoidable by making the payments ex post, i.e., contingent on the exercise of reserved capacity, which is why firm 2 only charges firm 1 through the exercise price e_1.

Note that, even though the reservation price is zero, i.e., $r_1^{NE} = 0$, firm 1 may not want to reserve all capacity in equilibrium because it must pay e_1 per unit of capacity, whereas the negotiated capacity comes at no cost. As a result, firm 1 will need to trade off the benefit of having a guaranteed access to capacity with its additional cost e_1.

In summary, the bidirectional capacity reservation contract mimics the unidirectional capacity reservation contract in equilibrium: only firm 1 will reserve capacity in equilibrium. Moreover, it can be shown that its structure is similar to the coordinating contracts for unidirectional alliances, although its parameters need to be adjusted to reflect the fact that firms anticipate that the residual capacity that has not been reserved ex ante will be allocated noncooperatively ex post. This suggests that, as the relationship between two firms evolves from a unidirectional to bidirectional, they can simply extend the existing contracts for their unidirectional alliances to coordinate the new alliance structure.

6.4. Equilibrium Profits

Based on our characterization of the equilibrium capacity reservation contract $(\mathbf{r}^{NE}, \mathbf{e}^{NE})$, we now argue that, relative to the case without contract as presented in §4.2, the capacity reservation contract improves the profits of both firms in equilibrium. To see why this is the case, notice that firm 1 can always choose to reserve zero units of capacity (i.e., by setting $z_1 = 0$), and so, if it does reserve capacity, it must gain from it. Similarly, firm 2 can always choose to price out capacity by setting $r_1 = p_1$ so that firm 1 does not reserve capacity; hence, if firm 2 prices capacity such that firm 1 opts to reserve some, it must gain from it. As we argued above, firm 2 will set in equilibrium r_1 below p_1 and firm 1 will reserve capacity in equilibrium. Therefore, the alliance's total profit increases under the ex ante capacity reservation contract in equilibrium; that is,

$$\begin{aligned} p_1\mathbb{E}[x_1^{FB}(\mathbf{D})] + p_2\mathbb{E}[x_2^{FB}(\mathbf{D})] \\ \geq \Pi_1(r_2^{NE}, e_2^{NE}; r_1^{NE}, e_1^{NE}) + \Pi_2(r_1^{NE}, e_1^{NE}; r_2^{NE}, e_2^{NE}) \\ \geq p_1\mathbb{E}[x_1^{NC}(\mathbf{D})] + p_2\mathbb{E}[x_2^{NC}(\mathbf{D})]. \end{aligned}$$

The following proposition summarizes the above results and discussion.

Proposition 3. *In equilibrium, under the ex ante capacity reservation contract, firm 1 will set $r_2^{NE} = p_2$ so that firm 2 does not reserve capacity $z_2^{NE}(\mathbf{r}^{NE}, \mathbf{e}^{NE}) = 0$. However, firm 2 will set $r_1^{NE} = 0$ so that firm 1 will reserve capacity $z_1^{NE}(\mathbf{r}^{NE}, \mathbf{e}^{NE}) > 0$, except when $\mathbb{P}[D_1 \geq (c+z)/2, D_2 \geq (c-z)/2] = 0$ for all z. Moreover, the ex ante capacity reservation contract will result in a more efficient capacity allocation than the noncooperative capacity allocation without contract (5). Furthermore, both firms earn more under the ex ante capacity reservation contract than under the noncooperative capacity allocation without contract given in (5). In other words, the ex ante capacity reservation contract is Pareto improving.*

To recap the intuition behind this result, ex ante capacity reservation contracts are more efficient than no contract because they give a chance to firm 1 to reserve capacity for its product, thereby reducing the need to have recourse to inefficient ex post negotiations. In addition, it is Pareto improving because firm 1 benefits from having capacity reserved for its product and firm 2 is able to capture some of that surplus through the exercise price. Since both firms gain from adopting an ex ante capacity reservation contract, unlike the ex post transfer payment contract, it will be easier to implement in practice.

6.5. Efficiency of the Ex ante Capacity Reservation Contract

Knowing that the capacity reservation contract is Pareto improving, we now examine the efficiency of that contract. Because the FB capacity allocation plan is achieved when product 1 has priority access to the joint resources, the capacity reservation contract is fully efficient when $z_1^{NE}(\mathbf{r}^{NE}, \mathbf{e}^{NE}) = c$. The next proposition identifies a sufficient condition on the demand distributions under which this outcome emerges in equilibrium.

Proposition 4. *Suppose that D_1 and D_2 are independent, with $\bar{F}_1(c) > 0$, and that $h_2((c-z)/2) - h_1((c+z)/2) + 2h_1(z) \geq 0$ for all $0 \leq z \leq c$, where $h(\cdot)$ is the hazard rate function (i.e., $h(x) \doteq f(x)/\bar{F}(x)$). Then, firm 1 reserves all available capacity, i.e., $z_1^{NE}(\mathbf{r}^{NE}, \mathbf{e}^{NE}) = c$, and the ex ante capacity reservation contract achieves the first-best capacity allocation, i.e., $\mathbf{x}^{NC}(\mathbf{D}, \mathbf{z}^{NE}(\mathbf{r}^{NE}, \mathbf{e}^{NE})) = \mathbf{x}^{FB}(\mathbf{D})$ for all $\mathbf{D}$.*

The condition stated in Proposition 4 is only sufficient. In particular, independence of the demands is mostly assumed to simplify the expression in terms of hazard rate, but is not needed in general. To interpret the condition stated in Proposition 4, we next consider three examples. The first two examples are cases where the sufficient condition holds, whereas the third example shows that the capacity reservation contract can be inefficient when the sufficient condition does not hold.

Example 1 (DFR Distribution for D_1). When D_1 follows a decreasing failure rate (DFR) distribution, its hazard rate, $h_1(x)$, is decreasing in x (Barlow and Proschan 1965). DFR distributions include the exponential distribution, the Weibull distribution with shape parameter between 0 and 1, the gamma distribution with shape parameter between 0 and 1, or any mixture of those. Hence, when D_1 has a DFR distribution, it is easy to check that $h_1(z) \geq h_1((c+z)/2)$ because the reserved capacity $z \leq c$. Therefore, $h_2((c-z)/2) - h_1((c+z)/2) + 2h_1(z) \geq 0$ for all $0 \leq z \leq c$ regardless of the distribution of D_2, provided that it is independent of D_1.

The next example shows that the sufficient condition may also hold when D_1 is not DFR.

Example 2 (Uniform Distribution for D_1 with Long Tail). When D_1 follows a uniform distribution between l_1 and $u_1 > c$, its hazard rate is equal to $h(x) = 1/(u_1 - x)$. By noting that $2h_1(z) \geq h((c+z)/2)$ if and only if $u_1 \geq c$, we can conclude that $h_2((c-z)/2) - h_1((c+z)/2) + 2h_1(z) \geq 0$ for all $0 \leq z \leq c$ regardless of the distribution of D_2, provided that it is independent of D_1.

Together, these two examples suggest that the sufficient condition will be satisfied when the demand for product 1 is stochastically large. In particular, DFR distributions are such that $\bar{F}(x)$ decays at a slower rate than the exponential distribution, and therefore have a heavy tail.[15]

Alternatively, the demand for product 1 could be so small that $F_1(c/2) = 1$ so that $D_1 \leq c/2$, in which case, the FB capacity allocation is achieved through the noncooperative capacity allocation process (3) with the payoff- and risk-dominant equilibrium selection rule, and there is no need for capacity reservation.

For the intermediate case, i.e., when D_1 is neither too small nor too large, the ex post noncooperative capacity allocation is inefficient, but it does not provide enough incentives for firm 1 to reserve all capacity. In those cases, the capacity reservation contract will lead to suboptimal capacity allocations, as the following counterexample illustrates.

Example 3 (A Counterexample). Consider the case when D_1 follows a discrete uniform distribution between 0 and 10 and D_2 has a discrete uniform distribution between 0 and 20, when $p_1 = 7$, $p_2 = 4$, $c = 13$, the equilibrium prices are $(r_1^{NE}, r_2^{NE}) = (0, 4)$ and $(e_1^{NE}, e_2^{NE}) = (0.819, 0)$, leading to $z_2^{NE}(\mathbf{r}^{NE}, \mathbf{e}^{NE}) = 0$ and $z_1^{NE}(\mathbf{r}^{NE}, \mathbf{e}^{NE}) = 5$, and generating a total profit for the alliance of 58.9697, which is less than the FB total profit of 59.1905.

Fortunately, these suboptimal outcomes occur only over a limited range of demand distributions. Moreover, the loss of optimality associated with these cases is, in general, small. Indeed, it is precisely when the noncooperative capacity allocation is relatively efficient that firm 1 has limited incentives to reserve capacity and would therefore reserve less than the full capacity. Our numerical analysis in §7.2 confirms this intuition.

7. Contract Comparison

In this section, we compare the ex post transfer payment contract and the ex ante capacity reservation contract, first analytically, by combining them, and then numerically.

7.1. Combining Ex Post Capacity Transfers with Ex Ante Capacity Reservation

Given that both the ex post transfer payment contract and the ex ante capacity reservation contract improve the alliance's total profit (relative to the case of no upfront contract), we now investigate whether it is worthwhile to combine them. As we show next, this combined contract may not necessarily improve upon the simple ex ante capacity reservation contract in equilibrium; hence, more options may not be more desirable.

The timeline of the combined contract is depicted in Figure 2. Similar to the previous sections, our analysis proceeds by backward induction. We first analyze the noncooperative capacity allocation with transfer payments, after demand is realized and the reserved capacities have been exercised. We then study the optimal choices of reserved capacity for the equilibrium capacity reservation contract. We finally discuss the efficiency of such a combined contract.

7.1.1. Ex Post Capacity Allocation and Transfer Payment.

To begin, let us first consider the last decision that both firms need to make after demand is realized, i.e., the ex post noncooperative capacity allocation via a noncooperative unanimity game. At that point, both firms exercise their reserved capacity: firm i will request firm j to produce $\min\{D_i, z_i\}$ for firm i, $i = 1, 2$ under the ex post transfer contract. By noting that the residual demand for firm i is equal to $[D_i - z_i]^+$, for $i = 1, 2$ and the remaining capacity for each firm is equal to $c - \min\{D_1, z_1\} - \min\{D_2, z_2\}$, both firms will enter a negotiation under the ex post transfer contract $(\mathbf{t})$ in the context of a noncooperative unanimity game (Harsanyi 1982b). The game can be formulated in a similar fashion to (3) with firm i's profit equal to $(p_i - t_i)x_{i,i} + t_j x_{i,j}$ if firm i's proposed capacity allocation $(x_{i,1}, x_{i,2})$ is equal to firm j's proposed capacity allocation $(x_{j,1}, x_{j,2})$, and zero otherwise; with the residual capacity $c - \min\{D_1, z_1\} - \min\{D_2, z_2\}$; and with the residual demands $[D_i - z_i]^+$, for $i = 1, 2$. Similar to (3), there exists an infinite number of equilibria when $D_1 + D_2 > c$, and we adopt the payoff- and risk-dominant equilibrium selection rule proposed by Harsanyi and Selten (Harsanyi 1982a), which solves

$$\begin{aligned} &\mathbf{x}^{NC}(\mathbf{D}, \mathbf{z}, \mathbf{t}) \\ &= \arg\max_{\mathbf{x}} \left[((p_1 - t_1)x_1 + t_2 x_2) \cdot ((p_2 - t_2)x_2 + t_1 x_1)\right] \\ &\qquad \text{s.t.} \quad x_1 + x_2 \le c - \min\{D_1, z_1\} - \min\{D_2, z_2\} \\ &\qquad\qquad 0 \le x_i \le [D_i - z_i]^+ \quad \forall\, i = 1, 2. \end{aligned} \tag{17}$$

Denoting by $\Pi_i(t_j; t_i, \mathbf{D}, \mathbf{z}) = (p_i - t_i)x_i^{NC}(\mathbf{D}, \mathbf{z}, \mathbf{t}) + t_j x_j^{NC}(\mathbf{D}, \mathbf{z}, \mathbf{t})$ the ex post profit for firm i, for $i = 1, 2$, the equilibrium transfer payments $(\mathbf{t}^{NE})$ should satisfy the following Nash equilibrium conditions:

$$t_j^{NE}(\mathbf{D}, \mathbf{z}) = \arg\max_{0 \le t_j \le p_j} \Pi_i(t_j; t_i^{NE}(\mathbf{D}, \mathbf{z}), \mathbf{D}, \mathbf{z}) \quad \forall\, j = 1, 2. \tag{18}$$

Similar to (9), we obtain that there exists a transfer price equilibrium given by

$$\begin{aligned} &(t_1^{NE}(\mathbf{D}, \mathbf{z}), t_2^{NE}(\mathbf{D}, \mathbf{z})) \\ &= \begin{cases} (p_1, p_2), \quad \text{if } [D_1 - z_1]^+ \le 0.5 \cdot c(\mathbf{D}, \mathbf{z}) \\ \qquad\qquad \text{or if } p_1(c(\mathbf{D}, \mathbf{z}) - [D_2 - z_2]^+) \\ \qquad\qquad > (p_1 - p_2)\min\{c(\mathbf{D}, \mathbf{z}), [D_1 - z_1]^+\} \\ \qquad\qquad\quad + p_2 \cdot 0.5 \cdot c(\mathbf{D}, \mathbf{z}); \\ \left(p_1 - \dfrac{p_2}{2}, p_2\right), \quad \text{if } [D_1 - z_1]^+ \ge c(\mathbf{D}, \mathbf{z}) \text{ and} \\ \qquad\qquad p_1[D_2 - z_2]^+ \ge p_2 \cdot 0.5 \cdot c(\mathbf{D}, \mathbf{z}); \\ \left(p_1 - p_2 + p_2 \dfrac{0.5 \cdot c(\mathbf{D}, \mathbf{z})}{[D_1 - z_1]^+}, p_2\right), \\ \quad \text{if } c(\mathbf{D}, \mathbf{z}) > [D_1 - z_1]^+ > 0.5 \cdot c(\mathbf{D}, \mathbf{z}) \\ \quad \text{and if } p_1(c(\mathbf{D}, \mathbf{z}) - [D_2 - z_2]^+) \\ \quad\quad \le (p_1 - p_2)[D_1 - z_1]^+ + p_2 \cdot 0.5 \cdot c(\mathbf{D}, \mathbf{z}), \end{cases} \end{aligned} \tag{19}$$

in which $c(\mathbf{D},\mathbf{z}) = c - \min\{D_1, z_1\} - \min\{D_2, z_2\}$. This equilibrium is unique as long as both products receive some positive capacity allocation (and is otherwise payoff invariant) or as long as demand does not fall exactly at the boundaries of one of these three cases (which is an event that has measure zero). Similar to our analysis of ex post transfer payment contracts in §5, we assume that, in case of multiple equilibria, equilibrium (19) is selected.

7.1.2. Ex Ante Capacity Reservation. With the equilibrium transfer prices $\mathbf{t}^{NE}(\mathbf{D},\mathbf{z})$, firm i's (ex ante) expected profit associated with any capacity reservation contract and reserved capacity $(\mathbf{r},\mathbf{e},\mathbf{z})$ is equal to

$$\begin{aligned}\Pi_i(z_i; z_j, \mathbf{r}, \mathbf{e}) = {} & (p_i - e_i)\mathbb{E}[\min\{D_i, z_i\}] \\ & + \mathbb{E}[(p_i - t_i^{NE}(\mathbf{D},\mathbf{z}))x_i^{NC}(\mathbf{D},\mathbf{z},\mathbf{t}^{NE}(\mathbf{D},\mathbf{z}))] \\ & + \mathbb{E}[t_j^{NE}(\mathbf{D},\mathbf{z})x_j^{NC}(\mathbf{D},\mathbf{z},\mathbf{t}^{NE}(\mathbf{D},\mathbf{z}))] \\ & + e_j\mathbb{E}[\min\{D_j, z_j\}] - r_i z_i + r_j z_j. \qquad (20)\end{aligned}$$

In this case, the equilibrium capacity reservations solve

$$z_i^{NE}(\mathbf{r},\mathbf{e}) = \operatorname*{arg\,max}_{0 \le z_i \le c - z_j^{NE}(\mathbf{r},\mathbf{e})} \Pi_i(z_i; z_j^{NE}(\mathbf{r},\mathbf{e}), \mathbf{r}, \mathbf{e}) \quad \forall\, i = 1,2. \qquad (21)$$

If there exists an equilibrium in the capacity reservation game, firm i's profit is equal to

$$\Pi_i(r_j, e_j; r_i, e_i) = \Pi_i(z_i^{NE}(\mathbf{r},\mathbf{e}); z_j^{NE}(\mathbf{r},\mathbf{e}), \mathbf{r}, \mathbf{e}), \qquad (22)$$

and the equilibrium capacity reservation and execution prices solve

$$(r_j^{NE}, e_j^{NE}) = \operatorname*{arg\,max}_{r_j, e_j} \Pi_i(r_j, e_j; r_i^{NE}, e_i^{NE}) \quad \forall\, i = 1,2.$$

Most of the results characterizing the capacity reservation contract without ex post contract (§6) carry over to the case where the negotiation process is structured with an ex post transfer payment contract. Specifically, in equilibrium, $z_2^{NE}(\mathbf{r}^{NE}, \mathbf{e}^{NE}) = 0$. Moreover, $z_1^{NE}(\mathbf{r}^{NE}, \mathbf{e}^{NE}) > 0$ unless the noncooperative capacity allocation with the ex post transfer payment contract guarantees enough capacity allocation to fulfill product 1's demand. This will happen under milder conditions than under Proposition 3's requirement that $\mathbb{P}[D_1 \ge (c+z)/2, D_2 \ge (c-z)/2] = 0$ for all z, given that the noncooperative capacity allocation with the ex post transfer payment contract is more efficient than that without contract, by Proposition 2 (see Figure 3), thereby providing less incentives to firm 1 to reserve capacity. In addition, in equilibrium, $r_1^{NE} = 0$ since firm 1's incentives to reserve capacity are the highest when it faces no risk of overage.

7.1.3. The Combined Contract in Equilibrium. Because the ex post transfer payment contract yields a more efficient capacity allocation than without contract, and because the ex ante capacity reservation contract makes both firms better off (since firm 1 finds it beneficial to reserve nonzero capacity and firm 2 finds it beneficial to induce firm 1 to reserve capacity), the total profit of the alliance under this combined contract is larger than without contract. It is however not Pareto improving, since the ex post transfer payment contract is not Pareto improving.

Proposition 5. *The combined ex ante capacity reservation and ex post transfer payment contract has a similar structure, in equilibrium, to the equilibrium structure of the single ex ante capacity reservation contract, even though the actual values of the contractual terms may differ. Overall, the combined ex ante capacity reservation and ex post transfer payment contract results in a more efficient capacity allocation than the noncooperative capacity allocation without contract (5), but it may not be Pareto improving.*

7.1.4. Efficiency of the Combined Contract. How attractive is the combination of the two contract features? We show in the online appendix that it is always beneficial to add the ex ante capacity reservation feature to an ex post transfer payment contract. This is because, if firm 1 reserves capacity in equilibrium, then both firms must find it profitable to do so. The following example illustrates that point.

Example 4 (Adding Ex Ante Capacity Reservation Is Always Beneficial). Consider the case when D_1 and D_2 are independent negative binomial distributions, truncated over $[0,20]$, with parameters $(0.25, 15)$ and $(0.75, 5/3)$, respectively. When $p_1 = 7$, $p_2 = 4$, $c = 13$, the ex post transfer payment contract generates a total surplus of 86.6871. However, by adding the ex ante capacity reservation feature to the ex post transfer payment contract, the alliance's total profit is increased to 88.5928.

We next show that it may be detrimental to add an ex post transfer payment feature to an ex ante capacity reservation contract. The next example makes that point.

Example 5 (Adding Ex Post Transfer Payment Can Be Hurtful). Consider the same parameters as in Example 4. The ex ante capacity reservation contract generates a total surplus of 90.5499, which is equal to the FB total profit. However, by adding the ex post transfer payment feature to the ex ante capacity reservation contract, the alliance's total profit is reduced to 88.5928.

The next proposition summarizes the results illustrated by the two examples above.

Proposition 6. *Although it is always beneficial to add the ex ante capacity reservation feature to an ex post transfer payment contract, the opposite may not be true. That is, less efficient capacity allocations may occur under the combined contract than under the single ex ante capacity reservation contract.*

Accordingly, combining the two contractual features may not necessarily improve efficiency, even if each of these features individually improves efficiency. This outcome is the result of the following tension. On the one hand, the noncooperative capacity allocation is more efficient with an ex post transfer payment contract than that without a contract (Proposition 2). Because firm 1, when deciding how much capacity to reserve, balances the costs of reserving capacity with the costs of leaving the capacity allocation decision up for ex post negotiation, this improvement in efficiency of the noncooperative capacity allocation process should *reduce* firm 1's incentives to reserve capacity. On the other hand, firm 1 is not able to capture all these efficiency gains. In particular, for many demand realizations, firm 2 sets $t_1^{NE}(\mathbf{D},\mathbf{z}) = p_1$ (Figure 3, left), which precludes firm 1 from appropriating any such efficiency gains. The prospect of not capturing any profit margin from product 1 in the ex post capacity negotiations should *increase* firm 1's incentives to reserve capacity ex ante, so as to preserve its margins. As a result, allowing for ex post transfer payments in addition to ex ante capacity reservation can result in either more or less reserved capacity.

To summarize, adding features may not necessarily lead to better outcomes because it may dilute incentives. Moreover, the combined contract is not Pareto improving, in contrast to the capacity reservation contract. These two results therefore suggest that the capacity reservation contract alone may be a more robust choice in practice.

7.2. Numerical Analysis

In this section, we numerically compare the ex post transfer payment contract, the ex ante capacity reservation contract, and the combined contract analyzed in the previous section.

In our simulations, we assume that $p_1 = 7$ and $p_2 = 4$ and that both demands are independent discrete uniforms. We consider 14 different problem instances, taking c equal to 8 or 13, and taking the largest value of D_1 or D_2 as either 5, 10, or 20. In all problem instances, the lower values of D_1 and D_2 are set to zero, consistent with our interpretation that D_1 and D_2 represent the residual demands after fulfilling the base demands. The problem instances are identified by a capital letter (A–N); see the first four columns of Table 2.

For each problem instance, the fifth column in Table 2 evaluates $\mathbb{P}[D_1 + D_2 > c, D_1 > c/2]$, which indicates the extent to which having no upfront contract is inefficient, since, in the noncooperative capacity allocation, suboptimal outcomes occur only in those cases, as Figure 3 (right) illustrates. (Another indicator of inefficiency of the situation with no contract is of course the ratio of profit margins, p_1/p_2, which we consider fixed here.) As the table indicates, the problem instances cover a wide spectrum of values for that probability. In particular, we expect that the gain from structuring the alliance with a contract will be stronger in instances B–D, H–K, and M than in instances E–G and N; in fact, the noncooperative capacity allocation process governed by the unanimity game never results in suboptimal outcomes in instances E and G, so adopting a contract will not be necessary in such situations.

For the ex post transfer payment contract and the combined contract, we take the ex post transfer prices given by (9) and (19). Based on Propositions 3 and 4, we take, in the ex ante capacity reservation contract and the combined contract, $r_2^{NE} = 4 = p_2$ and $e_2^{NE} = 0$ (so as to have $z_2^{NE}(\mathbf{r}^{NE}, \mathbf{e}^{NE}) = 0$), and take r_1^{NE} either equal to 0 or to p_1 (in case $z_1^{NE}(\mathbf{r}^{NE}, \mathbf{e}^{NE}) = 0$). We vary e_1 from 0 to $p_1 = 7$ by increments of 0.007, and for each value of (r_1, e_1), identify the best response $z_1^{NE}(\mathbf{r}, \mathbf{e})$ by enumeration, varying z_1 from 0 to 10. (Because demand

Table 2. Comparison of the Capacity Reservation Contract Parameters

					Ex ante capacity reservation		Combined	
Instance	c	D_1	D_2	$\mathbb{P}[D_1+D_2>c, D_1>c/2]$	(r_1,r_2,e_1,e_2)	(z_1,z_2)	(r_1,r_2,e_1,e_2)	(z_1,z_2)
A	13	$U[0,10]$	$U[0,10]$	18%	$(0, 0.553, 4, 0)$	$(5,0)$	$(0, 6.699, 4, 0)$	$(13,0)$
B	13	$U[0,10]$	$U[0,20]$	27%	$(0, 0.819, 4, 0)$	$(5,0)$	$(0, 6.566, 4, 0)$	$(13,0)$
C	13	$U[0,20]$	$U[0,10]$	55%	$(0, 1.687, 4, 0)$	$(13,0)$	$(0, 6.293, 4, 0)$	$(3,0)$
D	13	$U[0,20]$	$U[0,20]$	60%	$(0, 2.163, 4, 0)$	$(13,0)$	$(0, 5.964, 4, 0)$	$(3,0)$
E	13	$U[0,5]$	$U[0,10]$	0%	$(7, 0, 4, 0)$	$(0,0)$	$(7, 0, 4, 0)$	$(0,0)$
F	13	$U[0,10]$	$U[0,5]$	5%	$(0, 0.091, 4, 0)$	$(7,0)$	$(0, 6.979, 4, 0)$	$(2,0)$
G	13	$U[0,5]$	$U[0,5]$	0%	$(7, 0, 4, 0)$	$(0,0)$	$(7, 0, 4, 0)$	$(0,0)$
H	8	$U[0,10]$	$U[0,10]$	46%	$(0, 1.806, 4, 0)$	$(8,0)$	$(0, 6.342, 4, 0)$	$(1,0)$
I	8	$U[0,10]$	$U[0,20]$	50%	$(0, 2.100, 4, 0)$	$(8,0)$	$(0, 6.083, 4, 0)$	$(1,0)$
J	8	$U[0,20]$	$U[0,10]$	72%	$(0, 2.352, 4, 0)$	$(8,0)$	$(0, 6.216, 4, 0)$	$(1,0)$
K	8	$U[0,20]$	$U[0,20]$	74%	$(0, 2.695, 4, 0)$	$(8,0)$	$(0, 5.824, 4, 0)$	$(1,0)$
L	8	$U[0,5]$	$U[0,10]$	11%	$(0, 0.4900, 4, 0)$	$(2,0)$	$(7, 0, 4, 0)$	$(0,0)$
M	8	$U[0,10]$	$U[0,5]$	39%	$(0, 1.295, 4, 0)$	$(8,0)$	$(0, 6.797, 4, 0)$	$(1,0)$
N	8	$U[0,5]$	$U[0,5]$	6%	$(0, 0.259, 4, 0)$	$(2,0)$	$(7, 0, 4, 0)$	$(0,0)$

Table 3. Comparison of the Equilibrium Expected Profits Under Each Contract

Instance	c	D_1	D_2	Base case (Π_1^b, Π_2^b)	FB Π^{FB}	Ex post capacity transfers (Π_1^t, Π_2^t)	Ex ante capacity reservation (Π_1^r, Π_2^r)	Combined (Π_1^b, Π_2^b)
A	13	$U[0,10]$	$U[0,10]$	(32.5702, 18.6116)	52.2231	(18.7107, 33.4628)	(32.5841, 19.4644)	(18.7281, 33.4950)
B	13	$U[0,10]$	$U[0,20]$	(31.3030, 26.3030)	59.1905	(26.355, 32.8095)	(31.5067, 27.4630)	(26.3605, 32.8300)
C	13	$U[0,20]$	$U[0,10]$	(46.0303, 17.8874)	70.1905	(18.6320, 50.7922)	(46.0460, 21.1445)	(18.6463, 51.1676)
D	13	$U[0,20]$	$U[0,20]$	(41.8889, 23.9365)	73.8730	(24.3265, 49.1451)	(41.9207, 31.9523)	(24.3267, 49.3490)
E	13	$U[0,5]$	$U[0,10]$	(17.5000, 19.7576)	37.2576	(19.7576, 17.5000)	(17.5000, 19.7576)	(19.7576, 17.5000)
F	13	$U[0,10]$	$U[0,5]$	(34.5758, 10.0000)	44.7576	(10.0909, 34.5758)	(34.5946, 10.1629)	(10.0969, 34.6152)
G	13	$U[0,5]$	$U[0,5]$	(17.5000, 10.0000)	27.5000	(10.0000, 17.5000)	(17.5000, 10.0000)	(10.0000, 17.5000)
H	8	$U[0,10]$	$U[0,10]$	(24.5289, 14.0165)	42.2149	(14.1488, 27.7934)	(24.5535, 17.6614)	(14.1519, 27.9638)
I	8	$U[0,10]$	$U[0,20]$	(23.1515, 16.6926)	44.1039	(16.7619, 27.1991)	(23.1636, 20.9403)	(16.7644, 27.2876)
J	8	$U[0,20]$	$U[0,10]$	(29.2121, 13.2294)	48.7792	(13.4719, 34.7749)	(29.216, 19.5632)	(13.4739, 35.1235)
K	8	$U[0,20]$	$U[0,20]$	(27.0476, 15.4558)	49.7687	(15.5828, 33.9070)	(27.0600, 22.7087)	(15.5871, 34.0863)
L	8	$U[0,5]$	$U[0,10]$	(16.7576, 15.3939)	32.4697	(15.3939, 17.0758)	(16.7650, 15.7047)	(15.3939, 17.0758)
M	8	$U[0,10]$	$U[0,5]$	(26.9394, 9.5758)	39.1515	(9.8182, 28.8330)	(26.9691, 12.1824)	(9.8209, 29.1488)
N	8	$U[0,5]$	$U[0,5]$	(17.1111, 9.7778)	27.0556	(9.7778, 17.2778)	(17.1115, 9.9441)	(9.7778, 17.2778)

is discrete, the equilibrium reserved capacity will be an integer; because $\Pi_1(z; 0, \mathbf{r}, \mathbf{e})$ is in general not quasi-concave, all possible solutions need to be considered.) We then set (r_1^{NE}, e_1^{NE}) as the largest pair that maximizes $\Pi_2(r_1, e_1; r_2, e_2)$.

Table 2 reports the equilibrium capacity reservation contractual terms, for both the ex ante capacity reservation contract and the "combined" contract. We find that, except for instances A and B, the combined contract leads to larger reservation prices r_1 and lower reserved capacities z_1 than the single capacity reservation contract.

Table 3 reports the following equilibrium expected profits:

• The equilibrium profit of each firm under the non-cooperative capacity allocation without contract (i.e., under the base case ("b")) is given by

$$(\Pi_1^b, \Pi_2^b) = (p_1 \mathbb{E}[x_1^{NC}(\mathbf{D})], p_2 \mathbb{E}[x_2^{NC}(\mathbf{D})]).$$

• The equilibrium profit of the alliance under the first-best ("FB") solution that maximizes the alliance's total profit is given by

$$\Pi^{FB} = p_1 \mathbb{E}[x_1^{FB}(\mathbf{D})] + p_2 \mathbb{E}[x_2^{FB}(\mathbf{D})].$$

• The equilibrium profit of each firm under the ex post transfer ("t") payment contract is given by

$$(\Pi_1^t, \Pi_2^t) = (\mathbb{E}[\Pi_1(t_2^{NE}(\mathbf{D}); t_1^{NE}(\mathbf{D}), \mathbf{D})], \mathbb{E}[\Pi_2(t_1^{NE}(\mathbf{D}); t_2^{NE}(\mathbf{D}), \mathbf{D})]).$$

• The equilibrium profit of each firm under the ex ante capacity reservation ("r") contract is given by

$$(\Pi_1^r, \Pi_2^r) = (\Pi_1(r_2^{NE}, e_2^{NE}; r_1^{NE}, e_1^{NE}), \Pi_2(r_1^{NE}, e_1^{NE}; r_2^{NE}, e_2^{NE})),$$

where $\Pi_i(r_i, e_i; r_j, e_j)$ is given by (14).

• The equilibrium profit of each firm under the combined ("c") contract is given by

$$(\Pi_1^c, \Pi_2^c) = (\Pi_1(r_2^{NE}, e_2^{NE}; r_1^{NE}, e_1^{NE}), \Pi_2(r_1^{NE}, e_1^{NE}; r_2^{NE}, e_2^{NE})),$$

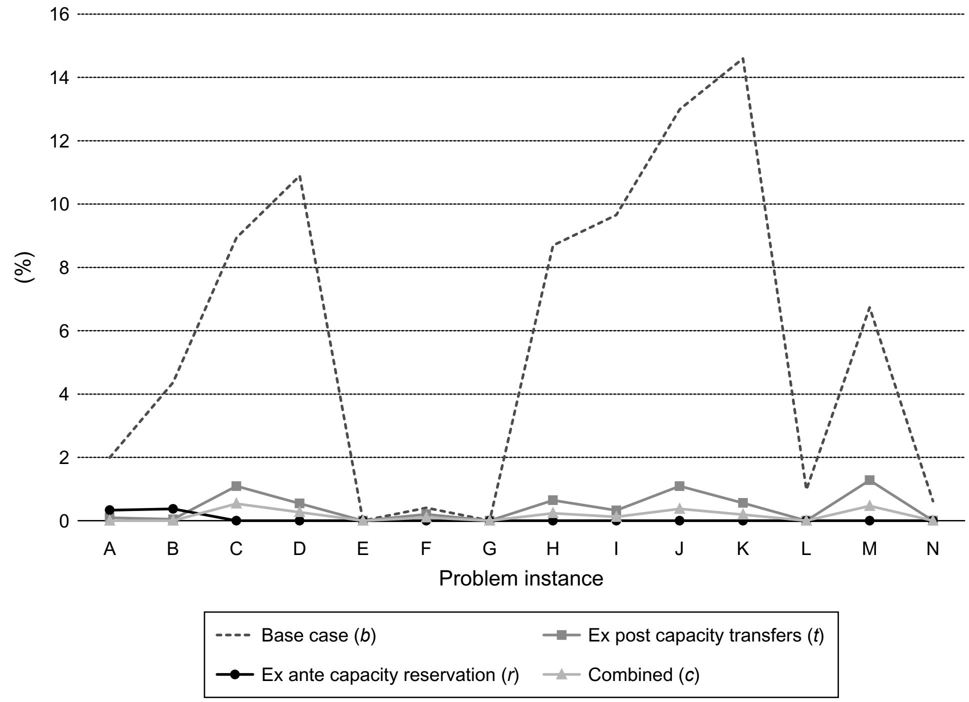
where $\Pi_i(r_i, e_i; r_j, e_j)$ is given by (20).

Figure 4 depicts the suboptimality gaps achieved by the base case (no contract) and each contract, i.e., $1 - (\Pi_1^\phi + \Pi_2^\phi)/\Pi^{FB}$, in which $\phi \in \{b, t, r, c\}$ depending on whether we consider the base case, the ex post transfer payment, the ex ante capacity reservation, or the combined contract. As illustrated in the figure, structuring the alliance with a contract dramatically improves its efficiency. Although the potential loss from using no contract may appear moderate, ranging from 15% (instance K) to 0% (instances E and G), we note that alliances are typically short-lived (e.g., a few years) and bad outcomes may not have the chance to be compensated with good outcomes. Accordingly, firms may care not only about their expected profits, but also about their worst-case profits. In that spirit, we evaluated the maximum optimality gaps and found that, with no contract, they could be as large as 21.4% in instances C–D, H–K, and M and 12.8% in instances A and B. Hence, even if the average optimality gaps appear moderate, the maximum optimality gaps can be quite severe. In that respect, structuring the alliance with a contract can help reduce the magnitude of a bad outcome. In particular, we noticed that the maximum loss of efficiency across all problem instances was reduced to 10.7% with ex post transfer payment contracts and to 3.7% with ex ante capacity reservation contracts.

From Table 3, we make the following observations. First, consistent with Propositions 2 and 5, Table 3 shows that setting ex post transfer payments may not be Pareto improving. In fact, in all simulated instances, firm 1 is always worse off with either the ex post transfer payment contract or the combined contract, compared to the base case with no contract. Moreover, the profit drop can be quite substantial, sometimes greater than 50%. Hence, firm 1 will in general be reluctant to allowing for ex post transfer payments.

Second, consistent with Proposition 3, we observe from Table 3 that the ex ante capacity reservation contract improves the profits of both firms, relative to the

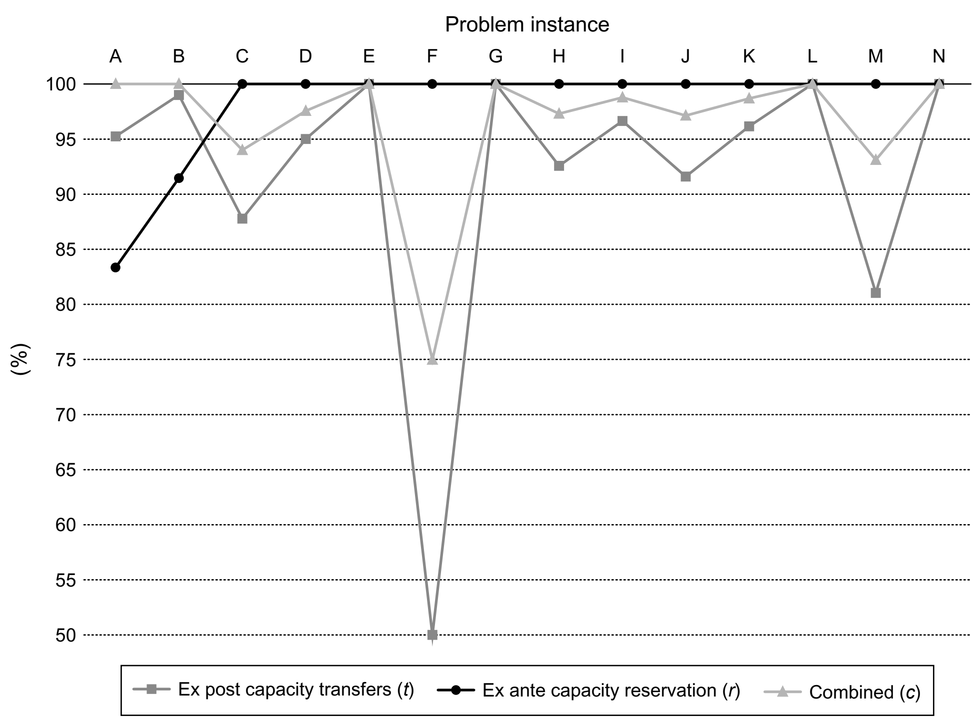
Figure 4. Suboptimality Gaps, Relative to the First-Best Scenario, Achieved by the Base Case (No Contract) and Each Contract



base case with no contract. Moreover, the improvement is strict unless the FB outcome is achieved with no contract, as in instances E and G. Comparing the profits obtained under the combined contract to those obtained under the single ex post transfer payment contract reveals that adding the ex ante capacity reservation feature improves both firms' profits, but the improvement tends to be marginal. Hence, if the base case constitutes a natural reference point, most of the benefit is already achieved with an ex ante capacity reservation contract alone.

To gain more insight into the relative performance of each contract, Figure 5 displays the relative potential gain, measured as the total profit difference between

Figure 5. Percentage of the Potential Gain, from Base Case to First Best, Achieved by Each Contract



the first best and the base case, attained by each contract, i.e.,

$$\frac{(\Pi_1^\phi+\Pi_2^\phi)-(\Pi_1^b+\Pi_2^b)}{\Pi^{FB}-(\Pi_1^b+\Pi_2^b)},$$

in which $\phi \in \{t, r, c\}$ depending on whether the contract under consideration is the ex post transfer payment, the ex ante capacity reservation, or the combined contract.[16]

Consistent with Proposition 6, we observe that (i) adding the capacity reservation feature to an ex post transfer payment contract always results in greater efficiency, but that (ii) adding the ex post transfer payment feature to an ex ante capacity reservation contract may not result in greater efficiency.

In fact, the simple ex ante capacity reservation contract dominates the other two contract types in all problem instances except instances A and B. Moreover, it attains 100% of the potential gain in instances C–N.

Consistent with our discussion of Proposition 3, it is when the demand for product 1 is neither too small (so that firm 1 would not reserve capacity since it would be guaranteed to receive all capacity it needs through the ex post noncooperative capacity allocation process) nor too large (so that firm 1 would have large incentives to reserve capacity and avoid inefficient ex post capacity allocations) that firm 1 reserves too little capacity and that that decision has an impact on the alliance's total profits. Nevertheless, even in those cases, the ex ante capacity reservation contract achieves more than 80% of the potential gain, from the base case to the first-best solution. Moreover, the optimality gap, $1-(\Pi_1^r+\Pi_2^r)/\Pi^{FB}$, turns out to be no larger than 0.4%, so the remaining inefficiency is truly marginal.

What explains this good performance of the ex ante capacity reservation contract? As we discussed in §4.2, the noncooperative capacity allocation achieved through negotiations is inefficient. With an ex ante capacity reservation contract, there is a large chance that all the demand for product 1 will be fulfilled from the capacity firm 1 reserved ex ante and therefore that ex post negotiations over capacity allocation may not be needed. Hence, capacity reservation contracts help reduce the share of capacity that will be subject to ex post negotiation, whereas ex post transfer price contracts only improve the efficiency of those negotiations.

8. Conclusion

In this paper, we investigate how to contractually structure a bidirectional alliance that involves the sharing of two complementary resources in the presence of demand uncertainty when firms have noncompeting product lines. We consider two practical contracts, namely, the ex post transfer payment contract and the ex ante capacity reservation contract, and a combination of these two contracts. Our key findings can be summarized as follows:

- Structuring the alliance with either contract (or a combination thereof) significantly increases the alliance's total profit over a situation where capacity is allocated noncooperatively without contract.
- Under either contract, the firm that has the most profitable product line will attempt to extract all surplus from the other firm's product line. The converse is not true, however, because the firms realize that greater total surplus will be achieved if they both have incentives to allocate more capacity to the most profitable product.
- The equilibrium "bidirectional" capacity reservation contract degenerates into a "unidirectional" capacity reservation contract: in equilibrium, only the most profitable firm reserves capacity.
- The ex ante capacity reservation contract tends to generate more profit for the alliance than the ex post transfer payment contract, especially when the demand for the most profitable product has a long tail, or, alternatively, when that demand is so small that, through the ex post negotiation process, capacity will always be allocated to the most profitable product. This is because ex ante capacity reservation contracts reduce the need for such negotiations, whereas ex post transfer payment contracts only improve their efficiency.
- It is always beneficial to enhance an ex post transfer payment contract to allow for ex ante capacity reservation. In contrast, it may not be beneficial to enhance an ex ante capacity reservation contract to allow for ex post transfer payments.
- The ex ante capacity reservation contract always increases the profits of both firms. In contrast, the ex post transfer payment contract may increase the profit of only one firm.

Overall, the above results have two practical implications:

1. As some firms are transforming their unidirectional alliance into bidirectional alliances, they can extend their existing ex ante capacity reservation contracts to coordinate the new form of bidirectional alliance.
2. In particular, ex ante capacity reservation contracts, which are common in unidirectional alliances (Brown and Lee 2003), work very well in bidirectional alliances for the following reasons: (i) they often achieve full efficiency; (ii) when they do not, the suboptimality gap is marginal; (iii) they are Pareto improving, i.e., their adoption leads to an increase in the profits of both firms.

Future research could consider alternate contracts and/or settings where our modeling assumptions do not apply, such as settings where the products are direct competitors (provided that the alliance is approved by antitrust agencies), settings where the technology lifecycles are so short that capacity investments must be made in each selling season, or settings where firms may not have full information about their respective demand and cost structure. Given the recent development of bidirectional alliances, addressing those questions would be worthwhile.

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Endnotes

[1] In those industries, capacity cannot typically be sold in bits and pieces. For example, in many production facilities with automated assembly lines, the production capacity cannot be freely reduced because of requisite equipment.

[2] In practice, firms may control more than capacity (e.g., production capacity, showroom space, number of dealers). For instance, they may also be able to shape their or their partner's demand through discretionary effort (e.g., sales effort, product quality). We will focus on the capacity lever here and will leave it for future research to incorporate discretionary efforts into our model.

[3] Because of economies of scale in procurement and production, the gross profit margins may be higher under the alliance than if the firms had operated independently. Yet, we assume that the unit procurement and production costs remain constant within the alliance's range of production volume, similar to Van Mieghem (1999), Cho (2014), and Cho and Wang (2017).

[4] It indeed took several years for both Fiat and Tata to figure out Indian customers' tastes; see Mishra and Surendar (2010).

[5] In certain situations, however, a firm may have access to the other firm's resource through an outside option, albeit at a higher cost. For instance, Nestlé has access to bottling capacity outside its alliance with Ocean Spray (Wang 2014). Because of its higher cost, the firm would use that outside option in last recourse, either when it receives less capacity than what it needs or when the negotiations for capacity allocation break down. Although such an outside option can easily be incorporated into the model, it makes the notation cumbersome without changing the insights and we shall therefore assume, for simplicity, that such an outside option either does not exist or yields zero profit.

[6] Although firms are forbidden by antitrust agencies to share information on their respective profit margins, they are usually able to infer it, especially for such staple products as juices. Also, based on our discussions with practitioners, it appeared that demand forecast sharing is usually truthful in such strategic horizontal alliances (Wang 2014).

[7] Recall that, since base demands b_1 and b_2 have been filled before any capacity negotiation, the actual capacity allocated to product 2 is at least b_2.

[8] Suppose firm i has access to resource j outside the alliance, in unlimited capacity, but only earns a profit margin $v_i < p_i$ with that outside option. In that case, firm i's payoff is equal to $p_i x_i + v_i \min\{[D_i - x_i]^+, [c_i - x_1 - x_2]^+\}$ when the negotiations succeed and equal to $v_i \min\{D_i, c_i\}$ if they break down. According to Binmore et al. (1986), this breakdown profit should be modeled as a participation constraint, i.e., $p_i x_i + v_i \min\{[D_i - x_i]^+, [c_i - x_1 - x_2]^+\} \geq v_i \min\{D_i, c_i\}$ and does not affect the disagreement (status quo) point, which is assumed here to be zero. Although such outside options can easily be incorporated, it unnecessarily complicates the notations without changing the fundamental insights, and we shall henceforth assume that $v_i = 0$ for $i = 1, 2$.

[9] Although an analysis of the contracts under demand information asymmetry is beyond the scope of this paper, we note that, in this unanimity game, the firms would truthfully reveal their demands if they were private information. That is, neither firm would have an incentive to inflate or deflate its demand before negotiating over capacity allocation, since firm i's ex post profit $p_i \min\{D_i, x_i^{NC}(\hat{D}_i, D_j)\}$ is maximized when $\hat{D}_j = D_i$.

[10] Alternatively, the transfer payment contracts could be offered ex ante, i.e., *before* demand is realized. However, it can be shown that firm 1 would have an incentive to change its contract after demand is realized, i.e., that the ex ante contract would be renegotiated.

[11] Similarly, in unidirectional alliances, price-only contracts can coordinate the subcontractor's capacity allocation decisions as long as its profit margin on product 1 exceeds that on product 2 (Van Mieghem 1999). Moreover, this FB capacity allocation can be sustained in equilibrium in a unidirectional outsourcing agreement: Taking advantage of its complete control of the production capacity, the subcontractor could indeed offer a take-it-or-leave-it contract with $t_i = p_i$ so as to capture the buyer's total surplus and implement the FB production plan. By contrast, in bidirectional alliances, the FB capacity allocation will in general not be sustained in equilibrium, as we will discuss in the next section, since both firms have control over capacity and are therefore both entitled to make take-it-or-leave-it offers.

[12] In particular, when $D_1 \geq c$ and $D_2 \geq (p_2/p_1)(c/2)$, it turns out that $x_2^{NC}(\mathbf{D}, \mathbf{t}) = 0$ when $\mathbf{t}$ is given by (9). Because product 2 is allocated zero capacity in equilibrium, firm 1 could in principle charge any transfer price, and there may exist other equilibria with $t_2^{NE}(\mathbf{D}) < p_2$. Nevertheless, firm 1's profit on product 2, $t_2^{NE}(\mathbf{D}) \cdot x_2^{NC}(\mathbf{D}, \mathbf{t})$, is equal to zero in every such equilibrium, and therefore all equilibria are payoff equivalent.

[13] For instance, when $D_1 \geq c$ and $D_2 = (p_2/p_2) \cdot (c/2)$, both $\mathbf{t} = \mathbf{p}$ and $\mathbf{t} = (p_1 - p_2/2, p_2)$ are equilibria. However, because demand is assumed to have a continuous distribution, $\mathbb{P}[D_1 \geq c, D_2 = (p_2/p_2) \cdot (c/2)] = 0$.

[14] Although firm 2 may reserve capacity in equilibrium when $p_1 = p_2$ if $r_2 + e_2 \mathbb{P}[D_2 \geq z_2] = (p_1/2)\mathbb{P}[D_1 \geq (c + z_1 - z_2)/2, D_2 \geq (c + z_2 - z_1)/2]$, neither firm 1 nor firm 2 makes money (in expectation) from this operation, and we assume that capacity reservation does not arise in this case, similar to when $p_1 > p_2$.

[15] Recall that D_1 represents the residual demand after satisfying the base demand b_1. This condition thus suggests that there is a lot of upward uncertainty, beyond the guaranteed demand of b_1. Since product 1 is the most profitable, this condition suggests that there is a large potential upside for the most profitable product. In the case of the Fiat-Tata alliance, sales of Fiat's cars, which were more premium, were very variable from year to year; for instance they grew by more than 200% in 2009-2010 to decline the following year (Purkayashta and Samad 2013).

[16] When the FB solution is achieved without contract, i.e., when $x_i^{FB}(\mathbf{D}) = x_i^{NC}(\mathbf{D})$ for $i = 1, 2$, as in instances E and G, we adopt the convention that each contract achieves 100% of the gain.

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