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Improving Store Liquidation

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Store liquidation is the time-constrained divestment of retail outlets through an in-store sale of inventory. The retail industry depends extensively on store liquidation, both to allow managers of going concerns to divest stores in efforts to enhance performance and as a means for investors to recover capital from failed ventures. Retailers sell billions of dollars of inventory annually during store liquidations. This paper introduces the store liquidation problem to the literature and presents a technique for optimizing key store liquidation decisions, including markdowns, inventory transfers, and the timing of store closings. We propose a heuristic for solving the store liquidation problem and evaluate the performance of this method. Through applications, we show that our approach could improve net recovery on cost (i.e., the profit obtained during a liquidation stated as a percentage of the cost value of liquidated inventory) by two to five percentage points in the cases we examined. Further, we discuss ways in which current practice in store liquidation differs from the decisions identified by our method, and we trace the consequences of these differences.

Keywords: retailing; pricing and revenue management; inventory theory and control; OM practice; OM–finance interface
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1. Introduction

Store liquidation, defined as the time-constrained divestment of retail stores through an in-store sale of inventory, is a critical aspect of the retail industry for both going concerns and bankrupt firms. Retailers close thousands of stores each year, liquidating billions of dollars of inventory in the process. For instance, Sears Holdings (Lahart 2011) and Barnes and Noble (Trachtenberg 2013) have announced plans to close hundreds of stores. Circuit City and Linens 'n Things held $1.5 billion and $795.4 million, respectively, of inventory prior to their bankruptcies, which ended in liquidation. Given the size of these liquidations, even a small improvement in net recovery on cost, i.e., the profit obtained during a liquidation stated as a percentage of the cost value of inventories liquidated, can yield substantial profits. For example, a 1% improvement in Circuit City's net recovery on cost would have amounted to approximately $15 million of additional profit.

Bankruptcy and, thus, liquidation are common in retailing. For the United States alone, S&P Capital IQ records 2,013 retailer bankruptcy announcements over the decade beginning in 2000 (http://www.spcapitaliq.com, accessed July 2012). Gaur et al. (2014) find that 3.4% of all public retailers during the past 20 years were liquidated in bankruptcy. Store liquidations operated by asset disposition firms (also referred to as liquidators) like Gordon Brothers Group (GBG) and Hilco Merchant Resources help stakeholders recoup funds from failed firms and allow investors to shunt capital to other ventures. GBG alone liquidated over $2 billion of inventory measured at retail value during 2011.

Store liquidation is also a valuable tool for going concerns, enabling retailers to redeploy resources by liquidating subsets of their stores. Store liquidation allows firms to generate cash from poorly performing stores and chains. For example, Home Depot shut down its EXPO chain in 2009. Barnes and Noble is in the process of closing 200 stores (Trachtenberg 2013). Store liquidation also allows managers to free resources to implement a change in strategy, as in Best Buy's liquidation of 50 big box stores to fund smaller stores selling mobile devices (Bustillo 2012). Other going concerns that rely on liquidators to close stores include Dick's Sporting Goods, Forever 21, JCPenney, Rite Aid, and Saks Fifth Avenue (Hilco Merchant Resources 2015).

The store liquidation problem differs significantly from prior problems studied in the literature. First, unlike in the problem of liquidating seasonal or perishable products (Lazear 1986), retailers must close, i.e., stop operating, stores in the store liquidation problem. The decision of when to close a store and, if needed, move merchandise to another store is an integral part of the store liquidation problem. The decision of which stores to open on a particular day adds a number of binary decision variables to the optimization problem and is a function of demand levels, store operating costs, and inter-store transfer times and costs. Ignoring store closings leads to suboptimal decisions and can substantially impact profit.

Second, consumer demand behaves differently during a store liquidation than at a store under normal operation. Consumers often perceive store liquidations as opportunities for a great bargain. Consumers may also view store liquidations as the last chance to shop at a favorite store or to purchase a specific brand. Extant demand forecasting models for seasonal sales do not incorporate these sources of demand uncertainty. Predicting ex ante how consumers will react to a store liquidation is challenging, and consumers' reactions can differ substantially from one store in a liquidation to another. As the liquidation progresses, the level of demand uncertainty goes down. Consequently, in identifying optimal liquidation approaches, one needs to explicitly incorporate demand uncertainty and forecast updating.

Third, liquidations involve "quirks": special features and constraints that characterize each liquidation event and often individual stores within the same liquidation. For example, lease agreements with stores' landlords may impose limits on when particular stores can be shut down. Moreover, the lack of loading and storage facilities at certain stores may limit inventory transfers. A method for optimizing store liquidation decisions should be flexible enough to accommodate these unique features associated with store liquidation.

In this paper we introduce a method for improving the efficiency of store liquidations, i.e., for increasing the net orderly liquidation value of retail stores (Foley et al. 2012), with a focus on liquidations conducted by asset disposition firms. The method comprises a dynamic program that informs markdown, inventory transfer, and store closing decisions as well as a demand forecasting model. We provide a heuristic for approximating the dynamic program, and we discuss techniques for estimating parameters of the heuristic. We use numerical experiments with parameters informed by empirical data to evaluate the performance of the heuristic, finding that the heuristic decisions attain over 97% of the optimal profit on average for the cases we study.

Furthermore, we present the results of several applications, comprising field experiments and simulations. These applications allow us to assess the benefit of using our method, and we find a two to five percentage point increase in net recovery on cost in comparison to current practice. Moreover, these applications allow us to evaluate extant theoretical results in our novel setting. We discuss several managerial insights drawn from the applications, including systematic deviations between prices determined by managers and prices determined by our model, the benefit of store closing decisions based on reading early consumer demand, the value of inventory transfers, and the importance of inventory transfer lead time. GBG served as the test site for this research.

This paper is organized as follows. Section 2 discusses the process of store liquidation. Section 3 surveys related literature. Section 4 presents the store liquidation dynamic program. Section 5 introduces our solution methodology, comprising a heuristic and forecasting model. Section 6 presents numerical experiments that assess the performance of the heuristic solution relative to the optimal. Section 7 discusses the results of using our method in practice as well as insights garnered through applications. Section 8 presents concluding remarks and future research opportunities.

2. The Process of Store Liquidation

During a store liquidation, a set of retail stores sells inventory, typically at an increasing discount, over a finite time period. The stores cease operations by the end of the liquidation. The length of a liquidation is limited by law for both bankrupt firms and going concerns. The majority of U.S. states constrain the length of all liquidation, distressed inventory, and going-out-of-business sales to protect consumers from firms that might perpetually use liquidation as a marketing tool (Foley et al. 2012). For example, Ohio law constrains liquidations to 90 days, whereas Massachusetts law limits going-out-of-business sales to 60 days. Many other jurisdictions impose similar restrictions.

From the retail asset disposition firm's perspective, the first step of any liquidation is "getting the deal." In the case of a bankruptcy liquidation, the liquidator receives information on store characteristics, inventory, and historical performance from the bankrupt retailer. Typical data include store location and square footage, store-level inventory in terms of cost and retail value, count, and age, as well as store revenues for the prior year. The liquidator must then file a bid with the bankruptcy court for the right to liquidate the bankrupt firm's inventory within the retailer's extant retail outlets. The bidding process transpires quickly and is often limited to less than a week. In the case of a going concern liquidation, the liquidator receives similar information and must engage in a sales process involving the estimation of net liquidation proceeds and the negotiation of fees to earn the right to liquidate.

Liquidators may execute a sale for a fixed fee or on an equity basis. In the equity case, typically associated with bankruptcy liquidations, the asset disposition firm pays up front for the right to liquidate

Figure 1 **Liquidation Multipliers Across Retail Segments**

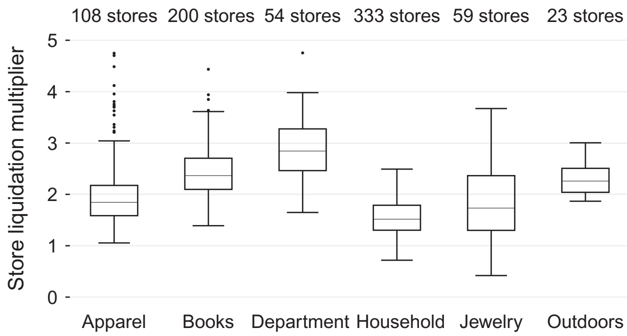
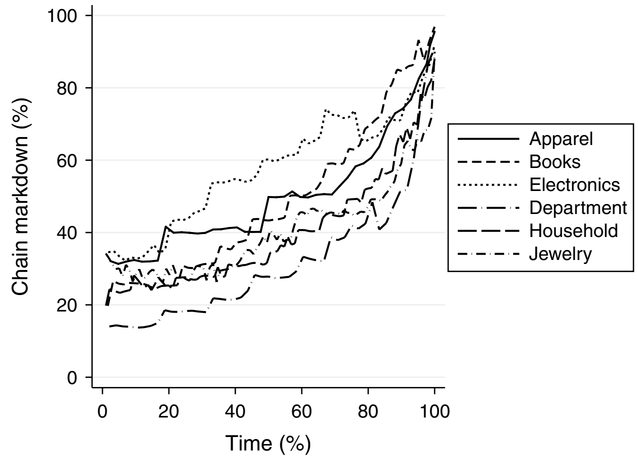


Figure 2 **Realized Markdowns for Retailers in Six Segments**



the inventory and may share profits with the retailer. For instance, during the final liquidation of Borders Group in 2011, liquidators paid Borders' estate 72% of the audited cost value of inventory present at the outset of liquidation and shared all operating profit above the initial payment with the retailer (Checkler 2011). Given the speed of store liquidation, even a small improvement in net recovery on cost leads to a substantial increase in return on investment.

After securing a deal, the liquidator quickly begins to execute the sale, often within days. Once the liquidation commences, store-level supervisors work to increase profitability through sales and marketing efforts (e.g., placing signs with price changes displayed prominently), inventory placement (to provide a pleasant shopping environment throughout a sale, liquidators collapse a store by moving inventory toward the front of the store and cordoning off the back), and expense management (e.g., controlling inventory shrinkage). To obtain managers with liquidation experience, liquidators often recruit their supervisors from firms subject to prior going-out-of-business sales.

Prior exposure to liquidation is important to managers because stores in liquidation behave differently than stores in normal operation. To illustrate this, we construct liquidation multipliers, i.e., the ratio of revenue earned during the liquidation of a store to the revenue generated by that store over the same period during the prior year. Figure 1 presents box plots of liquidation multipliers from store liquidations in the apparel, book, department store, household goods, jewelry, and outdoor goods segments. Each box plot represents a single chain. Most stores see a significant increase in revenue because of liquidation, although there is substantial variation in performance within a given liquidation. The median store in both the book and household furniture liquidations more than doubled its revenue rate. Moreover, this revenue perspective understates the physical volume of product sold, since liquidation discounts exceed normal operating discounts on average.

In current practice, central managers at liquidators plan a common markdown schedule across all stores. Managers choose this markdown schedule based on prior experience with a given retail segment. Broadly, the markdowns start relatively low—usually around 25%—and increase to approximately 85% over the course of a sale. Store-level managers then set prices, typically at the category level, designed to achieve the centrally chosen target. Figure 2 plots the realized markdown levels over time across six retail chains in different segments. Liquidators continue to operate each store until its inventory is sold through or until it turns unprofitable, i.e., when operating costs exceed revenues. In the vast majority of cases, liquidators sell through a store's inventory in place; when there is leftover inventory, the liquidator sells it at a large discount to a jobber. As will be discussed in §7, our method departs from this practice in several ways.

Managers operating liquidations track inventory at the cost level rather than at the item level. This is because of the coarseness of the information an asset disposition firm receives from a retailer and the need to execute a liquidation quickly. We formulate the optimization and forecasting models proposed herein accordingly.

We have identified three key operational levers for central managers to improve profitability during store liquidations: markdowns, inventory transfers, and store closings. Markdowns allow managers to affect the volume and timing of demand as well as the revenue realized from the sale. For instance, at the outset of a sale, liquidators tend to use smaller discounts to maximize the revenue from high-demand goods. Inventory transfers allow managers to move inventory to stores that are more attractive from a demand or cost perspective but incur additional costs. Finally, store closings, used in conjunction with inventory transfers and markdowns, reduce operating costs by shuttering certain locations in advance of the overall time limit.

We introduce a dynamic program that selects the markdowns, transfers, and store closings that maximize the profitability of a store liquidation subject to random demands, where profitability is equal to revenues less operating expenses such as payroll and shipping costs. This model departs from prior works by explicitly incorporating store operating expenses. Thus, the model may use inventory transfers, markdowns, and store closings to mitigate operating costs. In §6, we show that ignoring store closing decisions substantially impairs profitability. We now provide a brief overview of related research before turning to our formulation of the store liquidation problem.

3. Related Literature

This work extends the literature on retailing perishable and seasonal goods, which seeks to optimize pricing, inventory, and other decisions for merchandise in liquidation. Often, research focuses on markdown optimization. For example, Gallego and van Ryzin (1994) examine optimal prices during the liquidation of a fixed stock of product subject to price-dependent demand. See Bitran and Caldentey (2003) and Elmaghraby and Keskinocak (2003) for reviews. Other research focuses on inventory: Fisher and Raman (1996) study preseason and replenishment inventory decisions when selling a seasonal good, and Caro and Gallien (2007) examine assortment decisions at a fast-fashion retailer. To the best of our knowledge, this paper is the first to introduce the decisions and practical considerations that separate store liquidation from product liquidation.

This paper augments research on formal tools for seasonal and perishable goods retailing. As noted by Bitran et al. (1998), most research in this area focuses on strategies rather than formal tools. Exceptions include Smith and Achabal (1998), which introduces pricing and inventory policies for seasonal goods when demand is a function of price, inventory level, and season. The authors discuss the application of these policies at multiple retail chains. Mantrala and Rao (2001) present a decision-support system used by retail buyers making order quantity and markdown decisions for fashion goods. Heching et al. (2002) compare actual pricing decisions at an apparel retailer to prices determined by a variety of models. Caro and Gallien (2010) detail work with a fast-fashion retailer on a novel process for distributing a fixed amount of inventory across the retailer's stores. This process incorporates both a demand forecasting model and an inventory optimization model. Caro and Gallien (2012) introduce a markdown optimization model that has been implemented by a fast-fashion retailer. Formal tools for related problems include methods for retail price experiments (Gaur and Fisher 2005) and fashion product testing (Fisher and Rajaram 2000).

Bitran et al. (1998) propose a related model that maximizes revenue from the liquidation of a single product across a retail chain. A central planner sets a single price across all stores, and inventory may be transferred among stores. The authors test the efficacy of their model using a simulation based on data from a Chilean retail chain. The store liquidation problem treated herein extends their research in several respects, such as the inclusion of store operating costs and the ability to close stores, prices that vary across stores, and nonzero lead times for inventory transfers.

4. Store Liquidation Model

This section introduces a dynamic program for optimizing the profitability of a store liquidation, where profitability is defined as revenues less operating costs. The decision variables are markdowns, inventory transfers, and store closings. Let S be the set of stores to be liquidated. Define T to be the ordered set of periods over which the stores are liquidated (typically days or weeks, depending on the application). Let M be the set of potential markdowns (e.g., inventory may only be sold at a 5%, 10%, … discount because of signage limitations). Let $\tau_{sr}(x)$ be the cost of transferring inventory of retail value x from store $s \in S$ to store $r \in S$. Let l_{sr} be the transfer time in periods for shipments from store s to store r, where transfer time includes not only transportation but also the time required to package and reshelve the inventory.

To represent inventory transfers, let x_{srt} be the retail value of inventory that leaves store s during period t for delivery to store r. We assume that transfers arrive and depart prior to demand. Let $\mathbf{x}_{st} = (x_{s1t}, \ldots, x_{s|S|t})'$, and let $\mathbf{X}_t = (\mathbf{x}_{1t}, \ldots, \mathbf{x}_{|S|t})'$ be the matrix of transfers leaving during period t, where $\mathbf{X}_t \in \Lambda_t$, the set of feasible transfers at time t. Restrictions such as carrier capability, inventory type, and store facilities may limit transfers. Besides transfers, the remaining decisions are markdowns and store closings. Store s sells its inventory at markdown $m_{st} \in M$ during period t. Let $\mathbf{m}_t = (m_{1t}, \ldots, m_{|S|t})'$. Define b_{st} to be a binary variable that takes the value 1 if store $s \in S$ is open during period $t \in T$ and 0 otherwise. We assume that store s incurs operating expenses, such as occupancy and payroll expenses, equal to $o_{st}(b_{st})$ during period t. Let $\mathbf{b}_t = (b_{1t}, \ldots, b_{|S|t})'$.

Store and in-transit inventory characterize the state of the system. Let z_{st} be the value of inventory available for sale or transfer at store s in period t. Let $\mathbf{z}_t = (\mathbf{z}_{1t}, \ldots, \mathbf{z}_{|S|t})'$. To track scheduled transfers, let $\chi_{st} = \sum_{r \in S} x_{r,s,t-l_{rs}}$, i.e., the amount of inventory arriving at store s at time t. Let $\mathscr{X}_t$ be a matrix containing the amount of inventory scheduled, as of time t, to arrive at each store in each time period from $t+1$ to the final period, i.e., $(\mathscr{X}_t)_{ij} = \chi_{ij}$ for all $i \in S$ and $j \in \{t+1, \ldots, |T|\}$. Then the state at time t is given by $(\mathbf{z}_t, \mathscr{X}_t)$.

Let Y_{st} be the set of possible demands at store s during period t, where demands are measured at the retail value of merchandise. Let Ψ_t be the Cartesian product of Y_{st} over all $s \in S$, and let $\mathbf{y}_t = (y_{1t}, \ldots, y_{st})'$ be a vector specifying a demand for each store at time t. Define $F_t(\mathbf{y}_t \mid \mathbf{m}_t, \mathbf{z}_t)$ as the joint distribution of demand across all stores for $\mathbf{y}_t \in \Psi_t$. This formulation accommodates warehouses, which are treated as stores with no demand. We assume a relationship between demand and inventory to capture a number of important dynamics, including customers' expectations about future service level (Cachon and Swinney 2009), the broken assortment effect (Smith and Achabal 1998, Caro and Gallien 2012), and the promotional role of inventory (Balakrishnan et al. 2008).

Using the above definitions, the current period cost for all stores during period t is $C_t(\mathbf{b}_t, \mathbf{z}_t, \mathbf{X}_t) = \sum_{s\in S}[\sum_{r\in S} \tau_{sr}(x_{srt}) + o_{st}(b_{st})]$. The revenue across all stores is a function of the store status indicator, markdown, demand, and inventory available for sale: $R_t(\mathbf{y}_t, \mathbf{b}_t, \mathbf{m}_t, \mathbf{X}_t, \mathbf{z}_t) = \sum_{s\in S} b_{st}(1 - m_{st}) \min[y_{st}, z_{st} - \sum_{r\in S} x_{srt}]$.

The optimal profit during period $|T|$, the final period, is expected revenue less cost:

$$L_{|T|}(\mathbf{z}_{|T|}, \mathscr{X}_{|T|}) = \max_{\mathbf{b}_{|T|}, \mathbf{m}_{|T|}, \mathbf{X}_{|T|}} \left\{ \int_{\Psi_{|T|}} R_{|T|}(\mathbf{y}_{|T|}, \mathbf{b}_{|T|}, \mathbf{m}_{|T|}, \mathbf{X}_{|T|}, \mathbf{z}_{|T|}) - C_{|T|}(\mathbf{b}_{|T|}, \mathbf{z}_{|T|}, \mathbf{X}_{|T|}) \right\}$$

for $\mathbf{b}_{|T|} \in \mathbb{B}^{|S|}$, $\mathbf{m}_{|T|} \in M^{|S|}$, and $\mathbf{X}_{|T|} \in \Lambda_{|T|}$. Given the profit function for period $|T|$, the profit function for all other periods is as follows for $\mathbf{b}_t \in \mathbb{B}^{|S|}$, $\mathbf{m}_t \in M^{|S|}$, and $\mathbf{X}_t \in \Lambda_t$.

$$L_t(\mathbf{z}_t, \mathscr{X}_t) = \max_{\mathbf{b}_t, \mathbf{m}_t, \mathbf{X}_t} \left\{ \int_{\Psi_t} [R_t(\mathbf{y}_t, \mathbf{b}_t, \mathbf{m}_t, \mathbf{X}_t, \mathbf{z}_t) + L_{t+1}(\mathbf{z}_{t+1}(\mathbf{y}_t, \mathbf{b}_t, \mathbf{X}_t, \mathscr{X}_t), \mathscr{X}_{t+1})]\, dF_t(\mathbf{y}_t \mid \mathbf{m}_t, \mathbf{z}_t) - C_t(\mathbf{b}_t, \mathbf{z}_t, \mathbf{X}_t) \right\},$$

where $\mathbf{z}_{t+1}(\mathbf{y}_t, \mathbf{b}_t, \mathbf{X}_t, \mathscr{X}_t)$ is a vector for all s representing the inventory available for sale or transfer at store s in period $t+1$, i.e., $z_{s,t+1}(y_{st}, b_{st}, \mathbf{x}_{st}) = \max(z_{st} - \sum_{r\in S} x_{srt} - b_{st} y_{st}, 0) + \chi_{s,t+1}$. It is straightforward to adjust the problem to meet the requirements of a particular liquidation, although this may increase the number of state variables. For example, if markdowns must be nondecreasing, we would add the constraints $m_{st} \geq m_{s,t-1}$ for all s and $t > 1$.

The dimension of the state space, which, in the general case, reflects the potential current and in-transit inventories of each store, grows rapidly with problem size. For example, suppose that inventories are restricted to nonnegative integers and that transfer times equal one period. Then there are $\binom{|S|+\sum_{s\in S} z_{s1}}{|S|}$ possible inventory arrangements at the outset of each period, assuming the existence of prices at which all inventory sells. For \$1,000 of inventory and 10 stores, this yields on the order of 10^{23} suboptimizations in each period. To address this issue, we introduce a heuristic that has been tested in practice by a major asset disposition firm.

5. Solution Methodology

In this section we present a solution methodology for use by an asset disposition firm during a store liquidation. This method comprises a heuristic for the store liquidation dynamic program as well as a demand forecasting model. We begin with the modified optimization model.

The modified model reduces the complexity of the store liquidation problem by solving it in the absence of recourse, i.e., by committing to all decisions at the outset of the planning horizon, an approach used in related research (Gallego and van Ryzin 1994, Bitran et al. 1998). Even with this simplification, the combination of pricing, transfer, and closing decisions yields a nonlinear mixed-integer program that does not reduce to a readily solved form (see, e.g., Grossmann 2002) and that does not easily incorporate the idiosyncratic constraints imposed by instances of the store liquidation problem. Thus, we decompose the problem into a subproblem and a master problem. The subproblem is a mixed-integer program with bilinear constraints that optimizes profit over inventory transfers and store closings subject to a fixed markdown schedule. The master problem searches for the markdown schedule that maximizes the objective value of the subproblem. As discussed in §§6 and 7, this heuristic performs well relative to both the optimal and actual managerial decisions when solved on a rolling horizon basis throughout a store liquidation.

5.1. Heuristic Subproblem: Inventory Transfers and Store Closings

The subproblem requires a number of simplifying assumptions for tractability. We highlight the most significant assumptions in this paragraph and discuss others as they are introduced. First, the subproblem assumes there is a single opportunity to transfer inventory. As noted in the seasonal sales context (Bitran et al. 1998), this assumption is in keeping with practice. Central managers operating store liquidations are reluctant to attempt multiple rounds of transfers because of the challenge of coordinating carriers and store managers. Therefore, managers prefer to execute transfers in a single batch. Although our formulation assumes transfers depart during the first period, representing the case in which the model is solved immediately prior to making transfer decisions, the heuristic is readily modified to treat transfers in an arbitrary period. Hence, managers can solve the heuristic for different transfer periods to determine when to execute transfers. Second, the heuristic assumes that the transfer time between all stores is a constant. Since transportation itself requires only a day or two, the time required to pick product from shelves, pack it in boxes, unpack the boxes, and reshelve product is the primary component of transfer time and is similar across stores. These assumptions reduce the number of decision variables and constraints required to model transfers: this formulation uses $|S|(|S|-1)$ transfer variables as opposed to the $|S|(|S|-1)(|T|-1)$ variables required to model transfers with varying lead times that depart on an arbitrary period. Finally, the heuristic assumes the relationship between demand and inventory can be modeled using a specific functional form that is discussed below.

We modify the notation of §4 as follows. Let z_s be the pretransfer beginning inventory at store s during the first period of T. We modify the operating and transfer cost definitions as follows. Let o_{st} be the cost of operating store s during period t, which is assessed when the store is open. Let τ_{rs} be the cost of moving one retail dollar of inventory from store $r \in S$ to store s. Further, let l be the transportation lead time for all stores. The markdown in store s during period t retains the notation m_{st} but is not a decision variable in the subproblem. Let $\mathbf{m}$ be a matrix containing m_{st} for all $s \in S$ and $t \in T$. Let $\mathcal{M}$ be the set of possible markdown matrices.

We model demand using a set of discrete demand scenarios, where each scenario specifies a base level of demand for each store during each period. As an input to the subproblem, this base level of demand incorporates the effect of markdowns and beginning inventory levels but does not account for the effect of inventory transfers. We discuss the construction of demand scenarios from historical data in §5.3. Let $\mathbf{d}$ be a demand scenario that specifies a demand d_{st} for each store during each period, where demand is measured in retail dollars of inventory. Therefore, $\mathbf{d}$ is a matrix of size $|S| \times |T|$. Let D be the set of possible demand scenarios, and let $\phi(\mathbf{d})$ be a probability measure on D. We index D by $k \in K$, where $\mathbf{d}^k$ refers to a demand scenario, and d_{st}^k refers to the demand for store s during period t for that demand scenario. To model the relationship between demand and inventory, we introduce a parameter, g_s for store s, which is defined as the percentage increase or decrease in posttransfer demand associated with a one-dollar inbound or outbound inventory transfer. This relationship mirrors empirical observations while maintaining tractability. We discuss g_s further in §5.3.

The decision variables represent store closings and inventory transfers. As before, b_{st} is a binary decision variable that takes the value 1 if store s is open during period t and 0 otherwise. The decision variables x_{rs} equal the retail dollars of inventory transferred from store r to store s at the outset of the first time period. Our formulation uses the following auxiliary variables. Let ζ_{kst} be the posttransfer beginning inventory at store s during period t under demand scenario $\mathbf{d}^k$. Let ξ_{kst} be lost sales, i.e., the amount by which demand exceeds supply in store s during period t under demand scenario d^k. Let $\gamma_{st} \geq 0$ be the scaling of demand at store s at time t as a result of inventory transfers. The subproblem is

$$\begin{aligned}
&\Theta(\mathbf{m}, D, \phi(\cdot))\\
&\quad = \max\Big\{\sum_{k\in K}\phi(\mathbf{d}^k)\sum_{s\in S}\sum_{t\in T}[(1-m_{st})(\gamma_{st}d_{st}^k-\xi_{kst})]\\
&\qquad -\sum_{s\in S}\sum_{t\in T}o_{st}b_{st}-\sum_{r\in S}\sum_{s\in S}\tau_{rs}x_{rs}\Big\}\\
\text{s.t.}\quad &\gamma_{st}=1-g_s\sum_{r\in S}x_{sr}\quad \forall s\in S,\ t<l,\\
&\gamma_{st}=1+g_s\Big(\sum_{r\in S}x_{rs}-\sum_{r\in S}x_{sr}\Big)\quad \forall s\in S,\ t\geq l,\\
&\sum_{r\in S}x_{sr}\leq z_s\quad \forall s\in S,\\
&\zeta_{ks1}=z_s-\sum_{r\in S}x_{sr}\quad \forall k\in K,\ s\in S,\\
&\zeta_{ksl}=\zeta_{ks,\,l-1}-\gamma_{st}d_{s,\,l-1}^k+\xi_{ks,\,l-1}+\sum_{r\in S}x_{rs}\ \forall k\in K,\ s\in S,\\
&\zeta_{kst}=\zeta_{ks,\,t-1}-\gamma_{st}d_{s,\,t-1}^k+\xi_{ks,\,t-1}\\
&\qquad\qquad \forall k\in K,\ s\in S,\ t\notin\{1,l\},\\
&\xi_{kst}\geq b_{st}(\gamma_{st}d_{st}^k-\zeta_{kst})+(1-b_{st})\gamma_{st}d_{st}^k\\
&\qquad\qquad \forall k\in K,\ s\in S,\ t\in T;\\
&b_{st}\in\mathbb{B}\quad \forall s\in S,\ t\in T;\qquad x_{rs}\geq 0\quad \forall r\in S, s\in S;\\
&\xi_{kst}\geq 0\quad \forall k\in K,\ s\in S,\ t\in T.
\end{aligned}$$

The objective is expected revenue less store operating costs and transfer costs. The first two constraints specify the effect of transfers on demand, where transfers do not affect demand during the transfer time. The third constraint ensures that a store does not transfer more than its beginning inventory. The fourth constraint calculates first-period inventory as a result of transfers. The fifth constraint calculates inventory in period l as a result of transfers. In the special case $l = 1$ (i.e., no lead time), the fourth and fifth constraints collapse to $\zeta_{ks1} = z_s - \sum_{r\in S} x_{sr} + \sum_{r\in S} x_{rs}$ $\forall\, k \in K, s \in S$.

The sixth constraint links inventory across periods. The seventh constraint calculates unsatisfied demand. The final constraint ensures that closed stores remain closed. (Although this constraint may be omitted, it reflects a common feature of store liquidations.) Thus, the subproblem calculates the optimal inventory transfers and store closings for fixed per-store markdown schedules subject to the aforementioned assumptions. This formulation easily accommodates instance-specific constraints, e.g., if store 1 is not allowed to close before period p because of an agreement with the store's landlord, then we add the constraint $b_{1t} = 1$ $\forall\, t < p$.

5.2. Heuristic Master Problem: Markdowns

To incorporate markdown decisions, we employ a master problem that models the relationship between price and demand. The master problem creates the demand scenarios used by the subproblem, where the scenarios are a function of markdowns and beginning inventory. Section 5.3 discusses the construction of these demand scenarios. The master problem maximizes the objective value of the subproblem over the set of possible markdowns.

To reflect the fact that the master problem generates the subproblem's demand scenarios, we introduce the notation $D(\mathbf{m})$, which is the set of demand scenarios subject to a given markdown schedule, $\mathbf{m}$. The probability measure on this demand set is $\phi(\cdot \mid \mathbf{m})$. Given this notation, the master problem is $\max_{\mathbf{m}} \Theta(\mathbf{m}, D(\mathbf{m}), \phi(\cdot \mid \mathbf{m}))$ for $\mathbf{m} \in \mathcal{M}$.

Restrictions such as increasing markdowns are captured by $\mathcal{M}$, and consumer reactions to markdown schedules are modeled in $D(\mathbf{m})$ and $\phi(\cdot \mid \mathbf{m})$. We implemented this model using MATLAB for the master problem and IBM ILOG CPLEX for the subproblem. In our implementation, we solve the master problem via the following algorithm. Let $\min(\mathbf{m})$ be the smallest markdown used for any store during any period for a given markdown schedule. We order all markdown schedules in $\mathcal{M}$ by $i \in \{0, 1, \ldots, |\mathcal{M}| - 1\}$ such that $\min(\mathbf{m}^i)$ is nondecreasing in i, where $\mathbf{m}^i$ refers to the ith markdown schedule. Let $\bar{\Theta}^i$ be the maximum objective value of the master problem observed when evaluating markdown schedules $0, \ldots, i$. The algorithm comprises the following steps:

1. Set $i = 0$ and $\bar{\Theta}^{i-1} = 0$.
2. If $[1 - \min(\mathbf{m}^i)] \sum_{s\in S} z_s \le \bar{\Theta}^{i-1}$, then stop and return $\bar{\Theta}^{i-1}$.
3. Solve the master problem for $\mathbf{m}^i$. Let Θ^i be the resulting optimal value. Set $\bar{\Theta}^i = \max(\Theta^i, \bar{\Theta}^{i-1})$.
4. If i equals $|\mathcal{M}| - 1$, then stop and return $\bar{\Theta}^i$. Otherwise, increment i and return to Step 2.

This algorithm stops when the optimal profit observed while evaluating markdown schedules $0, 1, \ldots, i-1$ is greater than or equal to the revenue earned from selling all initial inventory at the minimum discount of markdown schedule i. Since the minimum discount of all remaining markdown schedules is at least $\min(\mathbf{m}^i)$, none of these schedules can yield higher profits than those already observed. Since low initial markdowns are typically optimal in practice, this stopping condition can rule out a substantial portion of feasible markdowns.

Although $|\mathcal{M}|$ may be large, several organizational constraints reduce its size. For example, liquidators generally use nondecreasing markdown schedules, and managers often set minimum initial markdowns as well as specific markdowns for given periods for advertising purposes. Further, central managers often prefer to use the same markdown schedule for all stores operated by a given store manager. If necessary for computational purposes, managers can employ coarser markdowns and longer time periods. The problem may also be simplified in other ways. In many store liquidations, there are constraints on the decision types available to the liquidator. The case studies in §7 illustrate how such restrictions arise in practice.

5.3. Demand Forecasting

Asset disposition firms can readily obtain the majority of the parameters for both the dynamic program and the modified problem, including store operating costs, beginning inventories, and transportation costs. However, the distribution of demand is often difficult to determine, since stores in liquidation perform differently than stores in normal operation. In this section, we develop and evaluate a demand forecasting model for store liquidation. To test this forecasting model, we use historical data provided by GBG comprising 43 retail store liquidations since 2006. These sales involve 2,516 stores and approximately \$7.5 billion of inventory at retail value.

Our forecasting model differs from prior research for several reasons. Most importantly, demand during store liquidation differs substantially from demand during normal operations. During the first week of liquidation, the average store in our data generates 2.49 times as much revenue as it did in the same week the year prior, despite offering an average markdown of 21%, which is similar to the average markdown offered by many retailers during normal operations. Thus, change in price alone does not fully predict store liquidation demand. Further, as will be discussed, demand can vary substantially across stores during a store liquidation. To account for this source of demand uncertainty, our forecasting method employs information about how consumers have responded to prior liquidation events as well as information about a store's local economic environment. Second, as discussed in §2, liquidators manage inventory by value rather than by count during a store liquidation, and this demand model is phrased accordingly. Third, liquidators do not receive the data, such as a store's item-level prices, inventory, and sales prior to liquidation, necessary to implement the demand forecasting models suggested in prior research on seasonal sales.

During a store liquidation, managers track sales performance in terms of multipliers, which reflect the revenue lift caused by the store liquidation. Let $Revenue_{st}$ be the liquidation revenue of store s during period t. Let $\textit{Last-Year Revenue}_{st}$ be the revenue of store s during normal operations one year prior to period t. Then the multiplier for period t at store s is the ratio of $Revenue_{st}$ to $\textit{Last-Year Revenue}_{st}$. Multipliers are defined over arbitrary time periods, e.g., the first week of a liquidation or the entire liquidation.

To illustrate the challenge of forecasting demand during a store liquidation, consider the variation in store performance exhibited in Figure 1. During the liquidation of the chain of 59 jewelry stores, some stores performed roughly the same as they did in the prior year, whereas many other stores more than tripled their prior year's revenue. Thus, demand at stores in liquidation can vary substantially within the same chain.

We construct the demand forecasting model as a function of store characteristics as well as current economic conditions. The store-level variables are inventory, price (i.e., $1 - \text{markdown}$), store square footage, last-year sales, liquidation duration (i.e., the maximum remaining duration of the liquidation), and retail segment (e.g., apparel store or bookstore). The inventory variable captures the broken assortment effect: when inventory is low, remaining items tend to be less desirable than those already sold (Smith et al. 1994, Smith and Achabal 1998). When fitting the model after a liquidation has begun, we include the store's sales during the liquidation.

To account for local economic conditions, which help explain differences in performance within a single chain, we employ data on the demographics of a store's neighborhood, which is determined by the store's zip code, as well as a store random effect. The United States Census Bureau provides the demographic data at the zip code level. Data from the 2000 census include median household income, number of households, average household size, and number of houses for sale. Data from the 2007 economic census include the number of local business establishments as well as the average payroll of these businesses. Prior research has identified wealth, represented by household income, as a key factor affecting demand among strategic consumers (Su 2007).

To capture the state of the broader economy, the model uses the Thomson Reuters/University of Michigan consumer sentiment index, which is released on a monthly basis. Consumer confidence can affect buying decisions, for example, by causing consumers to become more or less patient with regard to markdowns (Su 2007). Other measures, such as the economic activity data released by the U.S. Bureau of Economic Analysis, yield similar insights about the relationship between liquidation demand and broader economic conditions but are released too infrequently for forecasting in practice. When estimating the forecasting model, we assign to a store the value of the consumer sentiment index that was released in the month prior to the month in which the store's liquidation commenced. This ensures data availability when forecasting upcoming store liquidations.

We selected the following form for the forecasting model based on both input from managers and performance during the cross-validation procedures discussed below:

$$\begin{aligned}\ln(\mathit{Revenue}_{st}) = {}& \beta_1 \ln(\mathit{Beginning\ Inventory}_{st}) \\ & + \beta_2 \ln(\mathit{Price}_{st}) + \beta_3 \ln(\mathit{Square\ Footage}_s) \\ & + \beta_4 \ln(\mathit{Liquidation\ Duration}_{st}) \\ & + \beta_5 \ln(\mathit{Last\text{-}Year\ Revenue}_{st}) \\ & + \boldsymbol{\delta}'(\mathbf{Local\ Economic\ Variables}_s) \\ & + \nu \ln(\mathit{Consumer\ Sentiment}_s) \\ & + \mathit{Store\ Random\ Effect}_s \\ & + \mathit{Segment\ Fixed\ Effect}_s + \epsilon_{st}. \qquad (1)\end{aligned}$$

This functional form assumes a constant elasticity of demand relative to price, a common assumption (Smith et al. 1994). When fitting the model during period t of a liquidation, when sales from prior periods of the liquidation are available, we add a term to reflect obtained sales information, $\beta_6 \ln(\mathit{Current\ Revenue}_{st})$, where $\mathit{Current\ Revenue}_{st}$ is the sum of revenue at store s over periods 1 through $t-1$. Incorporating the sales of other stores within a chain—e.g., by including overall chain sales through period t—did not improve fit. Further, coefficients estimated assuming that errors are independent across time outperformed estimates assuming autoregressive errors in the cross-validation procedures.

The heuristic subproblem requires an approximation of the relationship between demand and inventory. We let β_1 represent the percent change in demand that results from a percent change in inventory. Then, for store s, this relationship is modeled by $g_s = \beta_1 / z_s$, where z_s is the store's pretransfer beginning inventory.

To create demand scenarios for the subproblem, we use the forecasting model to generate a revenue forecast, which we scale by price, yielding demand in terms of retail dollars. We employ the discretized normal distribution with the point estimate of demand as the mean. We evaluated two methods for constructing the variance of demand. First, we used the variance of the demand forecast from the model. Second, we multiplied the demand forecast by the empirical coefficient of variation of sales calculated from historical data. Both methods produce similar results. For larger problems, sampling the demand scenarios simplifies computation of the proposed heuristic. Following Caro and Gallien (2012), we tested for heteroscedasticity and implemented the error smearing factor of Duan (1983), which was close to 1 in most cases.

To assess the accuracy of the forecasting method, we cross-validate the model by estimating the coefficients on data from 42 of the 43 store liquidations and then forecasting revenues for the omitted store liquidation. We examine the accuracy of revenues forecast over two periods: the first week of a liquidation and the entire liquidation, where the forecast for the entire liquidation incorporates sales realized during the first week of the liquidation. The coefficients of determination for these two cross-validations are 73% and 89%, respectively. We also tested an alternate method of cross-validation by omitting randomly selected stores. Specifically, we randomly selected 59 stores to omit (the average number of stores per liquidation event in our database is 58.51). We then fit the model using the remaining stores, forecast for the omitted stores, and recorded the resulting coefficient of determination. We repeated this procedure by dropping different selections of 59 stores until the 99% confidence interval for the mean coefficient of variation was within 1% of the running average of coefficient of variation. This resulted in an average coefficient of variation of 76% for first-week revenue and 91% for overall revenue.

These results demonstrate that sales during the first week of a liquidation convey considerable information about sales throughout a liquidation, even though first-week liquidation revenues are only 12.23% of overall revenues on average. Researchers have observed this pattern in other retail contexts (Fisher et al. 1994, Fisher and Rajaram 2000). Although the store-level forecasts benefit greatly from a small amount of realized sales data, the preliquidation forecasts are still useful.

Figure 3 **Actual Chain Multipliers vs. Expert and Statistical Forecast Chain Multipliers**

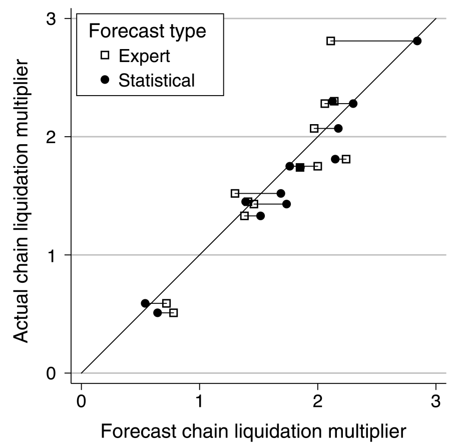


Figure 3 compares the accuracy of liquidation multiplier forecasts generated via this model to expert forecasts for 13 retail chain liquidations recorded by GBG (historical forecasts for the other liquidations in our database were not available). We aggregate the forecasts to the chain level by taking the ratio of forecast chain liquidation revenue to chain last-year revenue. Across the sample of 13 liquidations, the forecasting model realizes a 37% reduction in mean absolute deviation from actual multipliers in comparison to the expert forecasts. Specifically, the mean absolute deviation of the expert forecasts is 0.21 in comparison to 0.13 for the model forecasts.

Although we focus on employing these forecasts to inform operational decisions, we note that our research partner also used the forecasting model during the preliquidation bidding and sales processes. As part of these processes, managers estimate both the revenue and operating costs of a potential store liquidation to determine a forecast operating profit. During our research, the revenue forecast by the model influenced the bids the liquidator filed for several liquidations by predicting a substantially higher or lower revenue than managers had initially forecast.

6. Numerical Experiments

In this section we assess the performance of the heuristic proposed in §5 relative to the optimal expected profit of the dynamic program introduced in §4. These experiments employ parameters derived from the historical liquidation data introduced in §5.3. Because of computational limitations, we restrict our attention to smaller instances of the store liquidation problem, an approach used in related research (Gallego and van Ryzin 1994, Caro and Gallien 2012). We discuss the performance of the heuristic during large-scale applications in §7.

We conduct numerical experiments for a problem with two stores and three liquidation periods.

This problem size allows us to evaluate the heuristic while maintaining reasonable computation times. We assume that inventory can be transferred in all periods with a single period transfer time. In keeping with practice, we restrict markdowns to the set $\{0\%, 5\%, \ldots, 95\%\}$ and assume nondecreasing markdowns. The parameters for the numerical experiments are initial store inventory, store operating costs, transfer costs, and the distribution of demand.

To construct the experimental parameter space, we begin by fixing the retail value of the inventory at one store, Store A, at \$100. We examine the historical liquidation data and find that, within a given liquidation, the maximum ratio of the highest store inventory to the lowest store inventory is approximately 10. Thus, we vary the inventory of the other store, Store B, from \$100 to \$1,000 in increments of \$50. To simulate the range of store operating expenses observed in our sample, we vary store operating costs per period from 5% to 15% of beginning inventory in increments of 5% for each store separately (i.e., each period incurs 20 days of store operating costs in the case of a 60-day liquidation). To model transfer costs, we use managerial estimates ranging from \$0.01 to \$0.10 per dollar of inventory transferred, in increments of \$0.01.

We use the coefficients from the demand forecasting model, Equation (1), fit on the entire historical database. We simulate different levels of demand by varying last-year revenue, an input to the forecasting model, so that the ratio of inventory to last-year sales varies from 0.5 to 5.5, a range that captures 94% of all empirically observed cases, in increments of 0.5. Forecast demands are increasing in last-year revenue. We employ the mean values observed in the historical database for the remaining variables that do not change across our numerical experiments (store square footage, liquidation duration, consumer sentiment, and the local economic variables). We assume that demand is normally distributed, and, to capture the random component of demand, we calculate the coefficient of variation of daily store liquidation revenue for all stores in the historical database, which yields a range of 0.24 to 1.41. Thus, we employ coefficients of variation from 0.2 to 1.5 in increments of 0.1. In total, our parameter space yields 263,340 experiments.

When solving the full dynamic program, we assume that inventory levels and transfers are integral. (We tested an alternate assumption using increments of \$0.01 for a random subset of the parameter combinations and found that the results did not materially change.) For both the dynamic program and the heuristic, we discretize the demand distribution to five points: percentiles 10, 30, 50, 70, and 90, where each occurs with equal probability. We tested using up to 10 points, which yielded similar results to the five-point distribution.

To simulate the performance of the heuristic, we use the following steps for each parameter combination. First, we solve the heuristic for the first period. Then we draw random demands and repeatedly simulate the second and third periods assuming the heuristic is solved at the outset of each of these periods. For each parameter combination's simulation, we terminate the simulation when the 95% confidence interval of liquidation profit is within 1% of liquidation profit.

On average, the heuristic achieves 97.34% of the optimal profit, with a minimum of 85.61% and a maximum of 99.98%. We observed several trends in the performance of the heuristic. First, as the coefficient of variation of demand grows, the performance of the heuristic decreases. When the coefficient of variation of demand is on $[0.2, 0.4]$, the heuristic achieves 98.21% of the optimal profit, and when the coefficient of variation of demand is on $[1.3, 1.5]$, the heuristic achieves 96.15% of the optimal profit. For intermediate values of the coefficient of variation, on $[0.7, 1]$, the heuristic attains 97.47% of the optimal profit. As increased variability of demand leads to a wider variety of outcomes in later periods, the heuristic's assumption of no recourse becomes more onerous.

Heuristic performance also varies with demand. When demand is high (i.e., the ratio of inventory to last-year sales is 0.5 or 1), the heuristic achieves 97.12% of the optimal profit on average. When demand is low (i.e., the aforementioned ratio is 5 or 5.5), the heuristic achieves 97.48% of the optimal profit. This may be partially explained by the fact that the heuristic does not incorporate inventory transfers in late periods through recourse. When demand is high relative to the historical liquidations, late inventory transfers are more likely to be beneficial, since stores have more capacity to sell transferred inventory. When both demand and demand uncertainty are high, the heuristic performs worst relative to the dynamic program. Specifically, when the ratio of inventory to last year sales is 0.5 or 1, and when the coefficient of variation of demand is on $[1.3, 1.5]$, the heuristic achieves 95.84% of the optimal profit on average, and the worst-case performance of 85.61% occurs under these conditions.

The heuristic is not materially affected by the initial imbalance of inventory between the two stores. When Store B's inventory is between \$100 and \$200, the heuristic achieves 97.36% of the optimal; when Store B's inventory is between \$900 and \$1,000, the heuristic achieves 97.33% of the optimal. Finally, the performance of the heuristic is not substantially affected by transportation costs. This is because of the magnitude of the transportation costs and the fact that the heuristic's initial decisions closely approximate the optimal: the average absolute deviation between the heuristic's first-period transfers and the optimal first-period transfers is 4.29% of the optimal transfers.

We also conducted numerical experiments to assess the relative importance of the three types of decisions. Specifically, we modified the full dynamic program in three ways. To assess the value of differential pricing, we required both stores to use the same price. To evaluate the importance of inventory transfers, we constrained the transfer variables to be zero. To assess the value of the store closing decisions, we solved the model with store operating costs set to zero and then added store operating costs for periods in which a store's inventory was positive. Therefore, rather than assuming that stores must remain open when store closing decisions are not incorporated, we assume that the decision maker optimizes without accounting for the cost of operating the stores and then closes stores that run out of inventory.

The single-price restriction yielded an average of 88.79% of the optimal profit with a minimum of 71.41%. Without transfers, the model achieved an average of 92.13% of the optimal profit with a minimum of 77.94%. Finally, for the case without store operating costs, the model achieved 94.47% of the optimal profit on average with a minimum of 75.93%. Therefore, for the scenarios we studied, restricting the ability to price differentially across stores decreases performance more than restrictions on the other two decisions. Intuitively, when one price must be chosen for all stores, decision makers are less able to respond to high or low levels of inventory and demand during a liquidation. This reduces the value of the other two decisions, since, e.g., inventory transferred from a store with high inventory to a store with low inventory will be sold at the same markdown schedule regardless of where it is located.

To assess the performance that would result from treating a store liquidation as a product liquidation, we restricted the model to a single markdown, allowed inventory transfers, and omitted store operating costs. These restrictions yield an average profit of 85.51% of the optimal profit with a minimum of 65.52%. Therefore, incorporating the additional store liquidation decisions increases performance substantially.

7. Case Studies

In this section we discuss the application of our methods to recent store liquidations. These case studies comprise field experiments and, in cases where we are limited by our research site, simulations. We evaluate the monetary benefit of applying our methods to augment current liquidation practice (see §2 for an overview of current practice), assess extant theoretical results in our novel setting, and provide general insights for practitioners operating store liquidations.

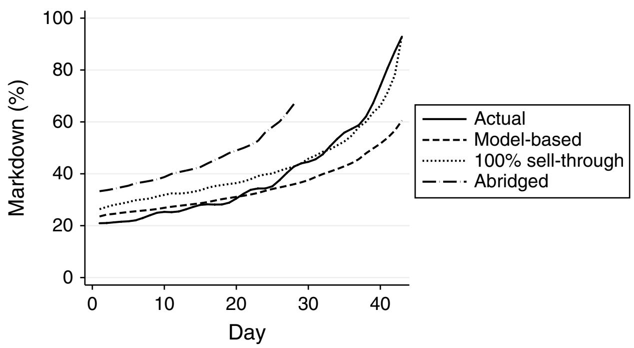
7.1. Comparing Managers' Pricing Decisions with Model Recommendations

Our first case study examines store liquidation decisions in a context without the option to close stores early or transfer inventory across stores. This case allows us to examine managers' pricing decisions in isolation of the inventory transfer and store closing decisions. We find consistent with prior research that, toward the end of a liquidation, managers use deeper markdowns than those recommended by our model. The finding is reversed at the beginning of the store liquidation when managers favor smaller markdowns than those recommended by our model. We hypothesize and show supportive evidence for the claim that the difference between managers' and model decisions can be traced at least in part to managers' desire to sell-through all inventory at the store.

For this case study, we examine the liquidation of stores owned by a branded apparel manufacturer that primarily sold its merchandise through department stores. The company's 84 stores contributed a small portion of overall revenues (less than 5%) and were located near department stores that also sold the company's products. The apparel manufacturer's managers elected not to allow inventory transfers during liquidation, since transferring large amounts of inventory into any of their own stores risked harming their relationships with nearby department stores. Further, because of the terms of the company's leases, managers could not close stores early.

Our model recommends lower markdowns than the ones managers use toward the end of the liquidation, and it also recommends higher markdowns than what managers use toward the beginning of the liquidation. We fit the forecasting model in Equation (1) to the historical database introduced in §5.3, omitting data from the company's stores. This generates an estimate of store-level conditional demand distributions. We use these demand distributions to determine the heuristic's pricing decisions, i.e., the heuristic conservatively does not use any information generated during the actual liquidation. We then estimate new demand distributions using realized sales during the liquidation of the company's stores, where actual sales in retail dollars is the mean of demand and the observed variance in sales in retail dollars across days is the variance of demand. To assess the revenue realized by the heuristic decisions, we simulate the liquidation using the latter demand distributions. We ran the simulation until the 99% confidence

Figure 4 Model-Based Markdown Schedule for a Retail Chain



interval of average revenue was within 1% of average revenue. On average, the model decisions generated a 4.1% increase over the actual revenue. Incorporating operating costs, this improvement yields a 5.0 percentage-point increase in net recovery on the cost value of inventory. Whereas managers sold through all inventory, the model left an average of 8% of inventory unsold.

Figure 4 graphs the actual managers' decisions against the heuristic's average decisions at the chain level, which are labeled as model-based markdowns. This figure illustrates several patterns that we have seen across the cases we studied. First, as Figure 4 illustrates, the model generally recommends lower markdowns than managers use toward the end of a liquidation. This result is consistent with findings in the product liquidation context (Bitran et al. 1998, Heching et al. 2002, Caro and Gallien 2012) and in behavioral experiments (Bearden et al. 2008). Second, our formal method prescribes larger markdowns at the outset of a liquidation than liquidators use in practice. For seasonal products, Bitran et al. (1998) argue that high initial prices (i.e., low markdowns) result when managers attempt to reach their profitability targets quickly.

We explored the differences between managers' choices and model recommendations in our field research and hypothesized that some differences were caused by managers' desire to sell through all inventory in a store. Managers were reluctant to have unsold inventory at the end of a liquidation, since clients might perceive unsold inventory as a failure to maximize the profitability of a liquidation. Consistent with this hypothesis, our model, when constrained to sell through all inventory, generates prices similar to those that managers used, as demonstrated by the 100% sell-through markdowns in Figure 4. When unconstrained, the model decisions lead to more unsold inventory than managerial decisions, which contrasts with the finding of higher sell-through for model decisions in the third clearance pricing implementation of Smith and Achabal (1998).

7.2. Reading Early Consumer Demand and Reacting with Markdowns and Closings

This case study examines the benefits of taking deeper markdowns and closing some stores early based on reading early consumer demand. The study also illustrates the benefit of varying prices at different stores. As in product liquidation (Fisher and Raman 1996), consumer demand can be difficult to predict accurately prior to store liquidation. Moreover, as we have shown, consumer reaction to store liquidation can differ substantially among stores in a retail chain, and early consumer demand is useful for predicting demand later during a liquidation. Consequently, it can be beneficial to read early sales and to close some stores earlier, and take steeper markdowns, than planned. This improves profitability by controlling store operating expenses.

In this case, the method was applied during the liquidation of discount department stores. The liquidation was limited to two months, and nine of the chain's 40 stores were subject to a lease constraint that rent would have to be paid through the 60th day of the liquidation if the stores were occupied on the 46th day (the 46th day was the first day of a month, and the stores' leases had "in for one, in for 15" clauses). Critically, landlords had to be notified two weeks in advance of a store's closing. Initially, managers planned to operate these nine stores through the 60th day of the liquidation. Two weeks into the liquidation, the heuristic indicated that it would be better to close the nine stores in advance of the 46th day. Thus, the model provided enough lead time for all landlords to be notified ahead of the deadline.

To estimate the benefits of the early store closings, we solved our heuristic with the constraint that the nine stores stay open until the end of the liquidation, which resulted in a 3.1% decrease in net recovery on cost. Another way to assess the value of closing these stores is through realized revenues: for the last two weeks that the nine stores operated, operating costs were 88% of actual revenues. Since revenues often decrease substantially toward the end of a liquidation—in the historical data, the average store's revenue during the final two weeks is 21% of its revenue for the prior two weeks—continuing to operate the stores likely would have been unprofitable.

To illustrate how markdowns change as the number of days a store is open decreases, we return to the case discussed in §7.1. Figure 4 includes the model-based markdown schedule when that liquidation is shortened by 15 days, referred to as the abridged markdowns. The model-based markdowns at the outset and the end of the abridged liquidation are larger than those for the unabridged liquidation. Under the simulation procedure described in §7.1, the abridged liquidation generates 13% less revenue than the full liquidation and 9.4% less profit. Since profit is a function of overall liquidation duration, liquidators would like to have the freedom to operate sales over longer time frames. However, regulators, when setting limits on liquidation duration, seek to maximize social welfare, which depends upon consumers, retailers, liquidators, and investors.

The value of reading consumer demand and reacting with changes in the supply chain has been identified previously in the literature (see, e.g., Hammond 1990, Fisher and Raman 1996). To our knowledge, the benefit of responding to early sales data with store closings and steeper markdowns has not been considered in prior literature. Our analysis also shows the benefit of allowing prices to differ from one store to another. In the absence of store-level pricing, the stores that closed early would have employed the same markdown schedule as the other stores, and the liquidator would not have benefited as much from closing stores early.

7.3. Benefits of Transferring Inventory Among Stores Based on Early Sales Data

Liquidators can transfer inventory from one store to another based on consumer demand at various locations. Demand during liquidation is frequently higher than expected at certain locations and lower than expected at others. Under those circumstances, the former store is likely to run out of inventory faster than the latter, and—depending on transfer costs and time—the liquidator may benefit from transferring inventory from the latter to the former. Hence, inventory transfers play a role in store liquidation similar to the role that inventory pooling and delayed differentiation have been shown to play in other contexts (Eppen and Schrage 1981, Lee and Tang 1997).

This study occurred during the liquidation of a major retail chain involving hundreds of stores holding over $500 million of inventory measured at retail value. GBG liquidated the chain as a joint venture with other liquidators. Because of the reporting structure established for the joint venture, GBG did not have direct control over markdowns and closings. However, GBG managers were able to transfer inventory among stores.

Three weeks into the liquidation, we solved the heuristic using the latest available information (i.e., including early liquidation sales) to determine inventory transfers. This yielded a schedule of roughly $20 million of inventory to be transferred among 119 stores, with 56 stores receiving inventory and 63 stores donating inventory. The transfers commenced during the fourth week of liquidation.

To estimate the impact of a transfer on the sales of any given store, we compare the posttransfer revenues of stores that received or donated inventory, referred to as transfer stores, to the revenues of a peer group of stores that were not affected by transfers. For each transfer store, the comparison group is the group of 10 nontransfer stores that had the closest revenues (measured by absolute deviation) to those of the transfer store during the pretransfer period. The results do not change materially when stores are matched on inventory instead of sales. Only the date on which inventory leaves a donor store is recorded in our data, so the posttransfer period includes the transfer time. For each transfer store, we calculate the difference between the store's revenues and the average revenue of the comparison group in both the pre- and posttransfer periods. The transfer effect is then the difference in posttransfer revenues less the difference in pretransfer revenues.

Figure 5 Example Inventory Transfer

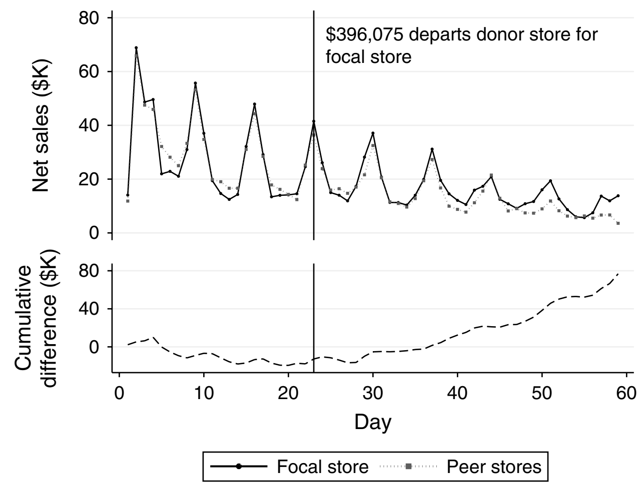
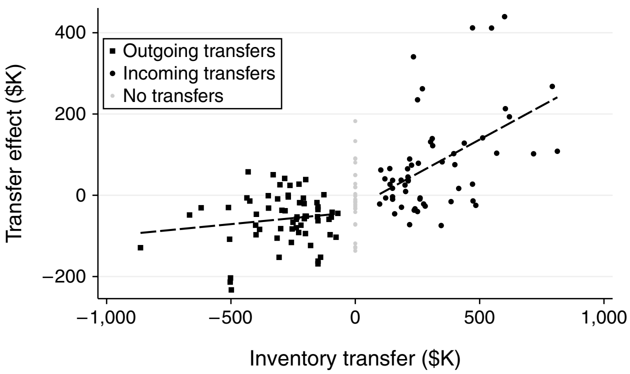


Figure 5 illustrates one inventory transfer. The focal store received $396,075 of inventory at retail value. The top panel plots the focal store's revenue as well as the average revenue of the comparison group. The bottom panel charts the cumulative difference (focal store minus comparison group) between the two time series. The vertical line indicates the day when inventory left the donor store. Up to the transfer day, the focal store generated revenues similar to those of the comparison stores: the cumulative difference is −$17,281. At the end of the sale, the cumulative difference is $76,833. Hence, the transfer effect is $94,114 for the store depicted.

Applying this method for calculating the transfer effect to all transfers provides an estimate of the overall impact of the transfers. Figure 6 plots the transfer effect for each store against the amount of inventory transferred in or out of the store, where negative transfers are outgoing. The lines of best fit illustrate that the benefits of the inbound transfers outweighed the costs of removing inventory from the donor stores. The t values for the slopes are 4.12 for

Figure 6 **The Effect of Chain-Wide Inventory Transfers**



the inbound transfers and 1.14 for the outbound transfers. Including the operating expenses of the transfers, we estimate that the profit increase was 2.5% of overall revenue and that the net recovery on cost increased 2.0 percentage points. This application, to the best of our knowledge, is the first to demonstrate the potential benefit of inventory transfers in retail store and product liquidation practice.

Our analysis also documents the significant impact the time required to transfer product between stores has on the value of inventory transfers. Intuitively, longer transfer times keep inventory off store shelves longer and cause transferred inventory to arrive when markdowns are higher at receiving stores. Managers estimated that each of the transfers took around 20 days to execute. According to our model, reducing the transfer time from 20 days to 15 days would increase the benefits associated with the transfers from 2.5% of overall revenue to 3.1%. Further reducing the transfer time to 10 days would lead to a profit increase of 3.4% of overall revenue.

8. Conclusion

Store liquidation is important for firms and investors, affecting retailer performance, how retailers are financed, and how investors are compensated (Foley et al. 2012). Further, store liquidation is fundamental to innovation in the retail sector, since extracting value from defunct stores and firms is a critical step in the process of creative destruction (Schumpeter 2008). In this paper we introduce methods for increasing the efficiency of store liquidations.

Although the literature has addressed markdowns and inventory transfers in the context of liquidating seasonal or fashion goods, the store liquidation problem differs significantly, structurally and managerially. First, during a store liquidation, managers must make not only pricing and inventory decisions but also store closing decisions. Omitting the store closing decisions leads to a substantial decrease in profitability. Second, customers behave differently during a store liquidation than they do during normal operations, which makes demand difficult to predict. Third, each store liquidation may involve idiosyncratic constraints, which are often caused by legal and lease requirements. Finally, asset disposition firms manage inventory at the dollar rather than unit level.

Store liquidation presents numerous research opportunities related to the liquidator's operating problem and to the broader asset-based lending cycle. Future research could identify optimal policies for the decisions studied herein and incorporate features such as strategic consumers. Research could identify when it is beneficial to incur the costs of managing inventory at the item rather than the dollar level. These costs include the cost of data collection and processing as well as menu costs, e.g., because of modifying item price tags. In addition, store liquidation is an important aspect of retailers' and investors' inventory, investment, and financial contracting decisions. Studying store liquidation outcomes could yield a better understanding of how asset liquidation values affect ex ante decisions, such as whether to use inventory secured debt. Finally, there are a number of legal factors that affect store liquidation, including limits on the length of both the bankruptcy process and liquidation sales as well as the structure of bankruptcy auctions. Future research could study these policy decisions to assess their impact on consumers, firms, and investors.

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