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Noncooperative Games for Subcontracting Operations

George L. Vairaktarakis
Department of Operations, Weatherhead School of Management, Case Western Reserve University, Cleveland, Ohio 44106, gxv5@case.edu

Consider a set of manufacturers, all of which can subcontract part of their workload to a third party. For simplicity, we assume that every manufacturer as well as the third party each possess a single production facility. Each manufacturer has to decide the amount of workload to be subcontracted so as to minimize the completion time of his in-house and subcontracted workloads. In an effort to provide good service to all, the third party gives priority to manufacturers whose subcontracted workload is small. This incentive scheme forces manufacturers to compete for position in the third-party processing sequence. We develop pure Nash equilibria schedules under three distinct protocols for production.

Key words: operations strategy; production planning and scheduling; supply chain management; incentives and contracting
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1. Introduction

Industrial and service organizations increasingly use third parties to subcontract part of their operations so that they can deliver goods and services faster to their customers. As noted by Van Mieghem (1999), subcontracting refers to situations where a firm contracts with a supplier part of its workload; outsourcing is the special case where the firm has no means to produce on its own.

As a result of subcontracting, a third party typically serves multiple manufacturers or service providers. In turn, this forces manufacturers to compete for the use of the third-party capacity so as to serve their individual objective. The third party, on the other hand, has her own objective, which may be related to the overall service to the manufacturers, or to the utilization of her manufacturing/service capacity. Conflicting interests among manufacturers and the third party result in a serious capacity allocation problem.

One can think of two ways to resolve the capacity management problem created by subcontracting to a third party: cooperation or competition. The literature on cooperation of operations suggests that a centralized minimum cost solution be used together with incentive payment schemes that make the centralized schedule more profitable than any of the alternatives for every member of the production chain. The concept of competition suggests that the third party announces a priority rule to the manufacturers, who in turn decide independently the amount of work that should be subcontracted so as to maximize their individual benefit.

Cooperation and competition pose new challenges. For cooperation the challenge is to process and analyze large amounts of information collected from many sources in different locations, solve a large problem involving all manufacturers, and find a "fair" incentive payment scheme that causes players to coordinate. Evidently, this is a tall order that only recently has been made possible due to Internet technologies that make the exchange of information fast and inexpensive. Competition avoids many of these challenges but requires the third party to devise a priority rule that will serve her own objective and at the same time render her capacity beneficial to all manufacturers.

Concepts of cooperation and competition have received significant attention since the 1950s, originally in the context of economics in industrial organization and subsequently in operations management. Since the 1980s, these concepts have been responsible for a large body of literature on supply chain management (see, e.g., Cachon 2003) where the primary focus is on coordination of inventories. Concepts of competition in relation to inventory management have also been heavily studied, with emphasis on the cost of the (decentralized) equilibrium solution compared to the centralized optimal solution; see, e.g., Lariviere and Porteus (2001) and Cachon (2003).

Research at the shop floor level is conspicuously scarce even though supply coordination necessitates the coordination of production activities. Cooperative scheduling games were introduced by Curiel et al. (1989), who considered a single machine game with the weighted completion time objective for each player. They showed that the corresponding sequencing game is convex, which implies that a reasonable payment scheme called *the core* of the game is guaranteed to exist; see Shapley (1971). Curiel et al. (2002) presented a survey of sequencing games and considered core strategies. A typical model in this literature assumes a single machine and one job per player. Therefore, the results produced are primarily of theoretical interest. Cai and Vairaktarakis (2012) considered a related model motivated by the coordination of manufacturing operations at Cisco's supply chain. A set of manufacturers subcontract operations to a third party who books her capacity at a price. Knowing these prices, manufacturers book available production days in a first-come, first-book order so as to minimize booking, overtime, and tardiness penalties. Subsequently, the third party creates savings by coordinating operations and devises an allocation scheme so that the coordinated schedule is more profitable to all chain members. Aydinliyim and Vairaktarakis (2010) considered a related model where tardiness penalties are replaced by weighted flow-time costs.

We are not aware of any paper dealing with competition at the shop floor production level except possibly the work of Hain and Mitra (2004), where each manufacturer subcontracts a single job to a third party who is committed to processing jobs in nondecreasing order of processing times (or shortest processing time (SPT) order). In an effort to gain processing priority, each manufacturer has the incentive to quote a shorter than actual processing time for his job, the validity of which cannot be verified by the third party. To resolve this problem, the authors develop a money transfer mechanism based on the job durations announced by the manufacturers. The mechanism is such that every chain member is better off announcing his true processing requirement, thus ensuring the third party of the SPT order on her facility.

In our models, players subcontract part of their workload to improve their overall completion time or *makespan*. As a result, subcontracting is not viewed as a penalty but as a reward. When it is cheaper to subcontract work to a third party rather than processing it in-house, the problem may be viewed as one of *scheduling with rejection*. In our model, the rejection cost corresponds to the opportunity cost forfeited by in-house processing. In the scheduling with rejection literature, a job may be rejected from scheduling on a production system and incur a job-dependent penalty. This notion was introduced by Bartal et al. (2000) and attracted attention; see, e.g., Sengupta (2003), Seiden (2001), Engels et al. (2003), and Hoogeveen et al. (2003).

Other related areas include job processing with controllable processing times and *project crashing*. Here, the makespan (or project completion) can be reduced at a cost. In this literature, the cost of crashing is considered explicitly, but not the resource that enables it. Key contributions in this literature are presented in a survey by Shabtay and Steiner (2007). When different players contend for the same resources, we encounter *agent scheduling* problems with controllable processing times. Agnetis et al. (2004) and Wan et al. (2010) considered problems with two competing agents.

In our models, players compete for scheduling priority at the third-party machine. There is a large body of literature in priority scheduling of queues where multiple customer types compete for service. In this setting, competition via incentives was considered by Mendelson and Whang (1990), who derived a closed-form pricing mechanism for various customer types (corresponding to our players) that is incentive-compatible; namely, the customer arrival rates and the service priorities jointly maximize the expected net present value of the system while being determined on a decentralized basis by the players. Priority pricing in queues was introduced by Kleinrock (1967), where the trade-off between the delay cost and the price paid by a player was chosen so as to optimize a systemwide objective. In a related paper, Edelson and Hildebrand (1975) introduced a no-balking model in which players decide to join the system without observing its current state. They showed that, in this case, revenue maximization by individual players coincides with the optimal for the grand coalition. Devising pricing schemes that induce individual customers to implement system-optimal solutions is a popular theme in this literature; see, e.g., Bell and Stidham (1983), Dewan and Mendelson (1990), and Lederer and Li (1994).

In another body of literature related to our models, games are studied where players are represented by retailers competing for the scarce resource of a supplier (corresponding to the third party in this paper). Each retailer privately observes his demand and orders stock from the supplier who decides her capacity level. A one-period model with identical retailers was studied by Lee et al. (2000), who found that allocating the supplier capacity in proportion to the retailer orders may lead retailers to inflate their orders. In a related paper, Cachon and Lariviere (1999) studied the effect of publicly known allocation mechanisms on the members of the chain. They showed that a broad class of mechanisms is prone to order inflation, causing the supplier to choose a higher level of capacity compared to a truth-inducing mechanism, yet much less than the prevailing distribution of orders. They showed that there does not exist a truth-inducing mechanism that maximizes total retailer profits, and in many cases true orders can lower profits for the supplier, the chain, and even the retailers.

In the present paper, we apply the Nash equilibrium concept (see Nash 1951) to study competition in the context of subcontracting operations. A schedule is said to be a *pure Nash equilibrium* or (for brevity) a *Nash schedule* when the strategy of each player is optimal given the strategies of the other players. In the next section we describe our model as well as the objectives of the third party and the players.

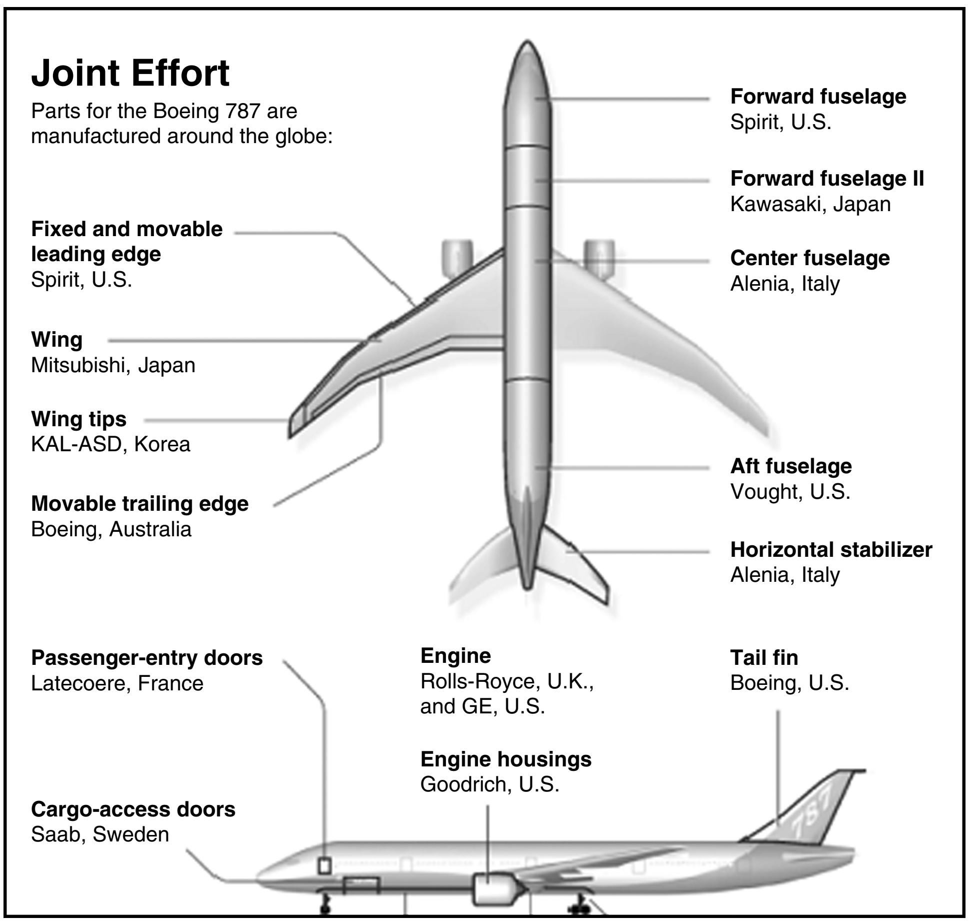
2. Model Description

Consider a set M of manufacturers (also referred to as *players*) each having known total workload P_i: $i \in M$. Let N_i denote the job set of player $i \in M$, $p^i_{\max}$ the processing time of a longest job in N_i, and p_{ij} the processing time of job $j \in N_i$. Player i has to determine the amount $x_i \leq P_i$ of work to be subcontracted to machine F owned by the third party P. The remaining amount $P_i - x_i$ will be processed in-house on machine M_i owned by player $i \in M$. The amount x_i: $i \in M$ is referred to as the *strategy* of player i. Player i's objective is to minimize his maximum completion time of the in-house and subcontracted portions of his workload, henceforth referred to as the *makespan* of player i. Evidently, the players in M want to compete for resource F which forces P to announce the rules of engagement. Throughout this paper we assume that P's objective is to maximize the amount $\sum_{i \in M} x_i$ subcontracted.

A production chain that closely reflects our modeling approach is Boeing's production of the 787 Dreamliner. Boeing authorized a team of parts suppliers to design and build major sections of the 787 Dreamliner, a portion of which is depicted in Figure 1. Lunsford (2007) reported in the *Wall Street Journal* that Boeing's suppliers for the 787 Dreamliner have had a working relationship with the company for over 30 years and were "handpicked." The plan called for suppliers to ship mostly completed fuselage sections, already fitted with wiring and other systems, to Boeing facilities near Seattle so that they could be put together in as few as three days, making timely delivery from suppliers extremely important. Prior to the Dreamliner project, this production mode was never employed before at Boeing, which normally kept large planes in the final assembly area for a month.

To deal with time pressures, many of Boeing's suppliers subcontracted part of their workload to even smaller companies instead of using their own

Figure 1 Manufacturing Sites for Boeing 787 Parts



Source. Lunsford (2007). Copyright Dow Jones & Company, Inc. Reprinted with permission.

engineers. In an article titled "Boeing Scrambles to Repair Problems with New Plane," Lunsford (2007) reported that some of the smaller subcontractors ended up overloading themselves with work from multiple 787 suppliers.

Our model closely captures the Dreamliner project at Boeing. In our model, Boeing's suppliers form the set M of "manufacturers," and a single small company represents P. In our model, completion times—not cost—are the objective. Indeed, consider the business environment in 2004 following the economic downturn. Overcapacity plagued companies who downsized quickly and were less than eager to make capital investments in a lagging economy. According to an article published by Wayne (2007) in the *Wall Street Journal*, Boeing's delivery delay of the Dreamliner followed a three-month delay in the plane's flight-test program, caused in part by a worldwide shortage of fasteners that hold together the plane's fuselage, wing, and tail sections. This is indicative that time is of primary concern, and that the third-party capacity is scarce, similar to our model.

Rather than time, one may view cost or penalty clauses as a viable way for Boeing to enforce deadlines. This is not workable in Boeing's case for multiple reasons. First, Boeing was not aware of the smaller companies represented by P. Second, Boeing handpicks its suppliers and helps them develop using development programs rather than imposing penalties. A supplier cannot join Boeing's production chain unless the firm conforms to high quality standards and has the specific knowledge and technology to produce jet components. In the event of delays, Boeing works with its suppliers toward problem resolution. According to a report by Lunsford and Michaels (2008) titled "More Delays Plague Boeing's Dreamliner," Boeing sent armies of its own employees to help suppliers prepare to turn out as many as seven of the wide-body jets each month. In some cases, Boeing—not the suppliers—was responsible for the delays. For example, Boeing was three to eight months late in giving the suppliers final specifications for structures and systems. Rather than a contractual penalty structure, many suppliers agreed not to be paid until planes were delivered.

Without any oversight structure in place, it was impossible for Boeing to manage the impact of a third party on the production chain. Unfortunately, Boeing had to announce delays twice in 2007. With orders at risk, the need to deliver airplanes "as early as possible" became paramount. As with the case of the fasteners producer, in this paper we take the view of a third party, P, who wants to maximize her own utilization with little oversight or direct control of the broader production chain.

We study our model under three different production protocols: overlapping, preemption, and nonpreemption. Overlapping allows processing parts of a job of player i simultaneously on M_i and F. Preemption allows processing part of a job of player i on M_i and the rest on F, however, not simultaneously. Nonpreemption stipulates that preemption is not possible for any job. Suppose, for example, that N_i includes a job j that requires five units of processing. Overlapping production allows simultaneous processing of j on M_i and F. For example, j could be partially processed on M_i during the time interval $[5, 8]$ and on F during $[7, 9]$. Such processing would not be allowed under the preemptive protocol. Instead, j could be scheduled during the interval $[5, 7]$ on M_i and $[8, 11]$ on F. Finally, in nonpreemptive production, job j should be scheduled entirely on F or entirely on M_i.

Let C^i denote the makespan of player $i \in M$ in schedule S, and let x_i be the amount of work subcontracted by manufacturer $i \in M$ to the third party P. Possible objectives for the third party include (i) minimizing the overall service time to manufacturers (expressed by $\sum_{i\in M} C^i$) or (ii) maximizing the total workload subcontracted by manufacturers in M (expressed by $\sum_{i\in M} x_i$). As an example, objective (i) would be appropriate if the third party is a flexible manufacturing center of a firm serving various internal departments. In this paper, however, we assume that the third party looks out for her own interest as in objective (ii).

Consider an arbitrary order $[1], [2], \ldots, [|M|]$ of manufacturers. If P announces that the work subcontracted by player [1] will be processed first, then [1] will contribute as much of his workload as is beneficial without regard to other manufacturers. Similar will be the strategy of the next few manufacturers who will occupy all the early processing capacity of F. Consequently, subsequent manufacturers cannot benefit from F and will not subcontract any of their workload to F. In this case the total workload subcontracted to P is expected to be smaller, and $\sum_{i\in M} C^i$ is expected to suffer. Therefore, P would like to announce priority rules to manufacturers so that they compete for her capacity more productively. As we show in the following two sections, when overlapping or preemption is allowed, equilibrium schedules exist where each manufacturer $i \in M$ attains his makespan on M_i, rather than on F. Then,

$$\sum_{i\in M} C^i = \sum_{i\in M} (P_i - x_i),$$

and hence minimizing the service objective $\sum_{i\in M} C^i$ is consistent with maximizing the total workload $\sum_{i\in M} x_i$ subcontracted to F. A somewhat similar result holds for nonpreemptive schedules as well. Therefore, with appropriate incentives to manufacturers, the third party P can achieve both superior service as well as maximum utilization of her capacity.

An important assumption of the models considered in this paper is that all players are of equal importance to P, and none of them receives priority treatment. The case where some players have special contractual agreements (e.g., long-term just-in-time contracts) with P was studied by Aydinliyim and Vairaktarakis (2008).

Player strategies depend on the information and production protocols, which in turn affect the rule-making process of P. We consider four different information protocols:

(IP1) Value $|M|$ is disclosed to all manufacturers.

(IP2) Values P_i, $i \in M$, are disclosed to all manufacturers.

(IP3) Values P_i and $p^i_{\max}$, $i \in M$, are disclosed to all manufacturers.

(IP4) Complete job-processing profiles $\{p_{ij}\colon j \in N_i\}$, $i \in M$, are disclosed to all manufacturers.

In all of the above information protocols, it is assumed that the information shared is common knowledge. For example, in IP2, player i knows P_j, and player j knows that player i has this information, for every $i, j \in M$. Evidently, IP4 corresponds to complete information. When overlapping is allowed, IP2 is equivalent to complete information because jobs can be preempted and/or processed simultaneously on F and M_i ($i \in M$) irrespective of their exact processing requirements. When preemption is allowed but overlapping is not, then IP3 is equivalent to complete information because $p^i_{\max}$ is a lower bound on the makespan for player $i \in M$, and, as will be shown shortly, the remaining workload $P_i - p^i_{\max}$ can always be processed without overlapping. Hence, detailed job-processing information does not provide additional preemption opportunities. On the other hand, detailed job information is useful in nonpreemptive schedules.

In the next three sections we consider competition under complete information for the three production protocols, respectively. We conclude with some observations in §6.

3. Overlapping Allowed

Throughout this section we use x^O_i to denote the strategy of player $i \in M$ when overlapping is allowed. As discussed in the previous section, the third party would like to process the workload of manufacturers in M in a predetermined order of P_i values, but such an announcement would result in a bipartition of the manufacturers: those that subcontract large amounts, and those that are essentially precluded from using F, resulting in inferior overall service (as measured by the sum of all makespans) and reduced total subcontracted workload. In this section we assume the following:

- overlapping is allowed for jobs;
- values P_i, $i \in M$, are disclosed to all manufacturers as in IP2; and
- sequencing on F will be done according to the *incentive rule* when *overlap* is allowed (abbreviated as IRO), defined next.

Definition 1 (IRO). If $x^O_i \le x^O_k$, then player i precedes k on F (this is the SPT order of subcontracted workloads); break ties with smaller P_i first and then arbitrarily.

For all players who have the same total workload P_i and strategy x^O_i, in the above definition we choose an arbitrary priority sequence on F to maintain transitivity among the players. Incentive IRO gives priority to a player who subcontracts to P a small amount of workload. This rule presents each manufacturer i with a dilemma: if he subcontracts a small amount x^O_i, his remaining workload $P_i - x^O_i$ is likely large, and his makespan will probably be attained on M_i. If x^O_i is large, then it is likely to be preceded by several other manufacturers, thus delaying the completion of his workload on F, forfeiting potential benefits from utilizing the capacity of F more effectively. The following lemma provides an upper bound on the amount a player can be reasonably expected to subcontract. All proofs are included in the online supplement (available at http://dx.doi.org/10.1287/msom.1120.0410).

Lemma 1. *With any Nash schedule, we have $x^O_i \le P_i/2$ for $i \in M$.*

The following result suggests that, when every player looks out only for himself, a necessary condition for any Nash schedule is that players are sequenced at the third-party machine in nondecreasing order of P_i values. Moreover, the later a player appears in the sequence, the greater the amount of workload he subcontracts to P. The second part of Theorem 1 suggests that the makespan of player i is attained on M_i for every $i \in M$.

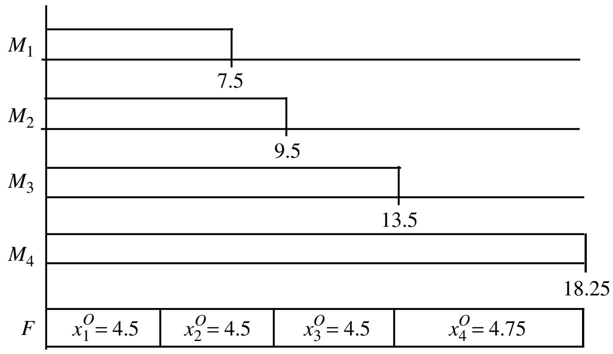
Theorem 1. *Reindex manufacturers so that $P_1, P_2, \ldots, P_{|M|}$ is a nondecreasing order of player workloads. Then, every Nash schedule is such that $x^O_1 \le x^O_2 \le \cdots \le x^O_{|M|}$ and*

$$\sum_{k=1}^{i} x^O_k \le P_i - x^O_i \quad \textit{for every } i \in M.$$

Theorem 2. *There exists a pure Nash equilibrium where players are reindexed so that $P_1, P_2, \ldots, P_{|M|}$ is a nondecreasing order of P_i values, and the player strategies are*

$$x^O_k = \min_{i \ge k} \frac{P_i - x^O_1 - \cdots - x^O_{k-1}}{i + 2 - k} \quad \textit{for } k = 1, 2, \ldots, |M|.$$

Figure 2 **An Example Nash Schedule**



In the above theorem we do not claim that there exists a unique Nash schedule because of instances with $P_i = P_j$ for players $i \neq j$. However, whatever nondecreasing P_i order is chosen, the strategies of Theorem 2 yield a unique pure Nash equilibrium for that order. When there are no ties among $P_1, P_2, \ldots, P_{|M|}$, the Nash schedule is unique.

Example 1. To illustrate the above results, consider the following problem instance with $|M| = 4$ and $P_1 = 12$, $P_2 = 14$, $P_3 = 18$, and $P_4 = 23$. As we saw in Theorem 1, the four players will be sequenced on F in the order 1, 2, 3, 4, and the subcontracted workload of player 1 is $x_1^O = \min\{12/2, 14/3, 18/4, 23/5\} = 4.5$. If player 1 did not hedge against other manufacturers and was processed first, he would prefer to subcontract 12/2 units for a resulting makespan $C^1 = 6$. However, the minimizer in the expression for x_1^O is attained at player 3 (because $18/4 = 4.5$), indicating that, unless $x_1^O = 4.5$, player 3 will be processed ahead of player 1 on F, resulting in an inferior makespan for player 1; see Figure 2. Indeed, suppose player 1 subcontracted to P more than 4.5 units of work, say 5. Then, players 2 and 3 could subcontract 4.7 units, and the completion times of players 2 and 3 would improve because $4.7 < 5$, and according to IRO, both players would precede player 1 on F.

Similarly, $x_2^O = \min\{(14-4.5)/2, (18-4.5)/3, (23-4.5)/4\} = 4.5$, $x_3^O = 4.5$, and $x_4^O = 4.75$. Note that, given that players 1 and 2 subcontract the maximum possible according to IRO, it is not beneficial for player 3 to subcontract more than 4.5 because when $x_1^O = x_2^O = x_3^O = 4.5$, makespan $C^3 = 13.5$ is attained on both F and M_3. Finally, $\sum_i x_i^O = 18.25$ and $\sum_i C^i = \max\{7.5, 4.5\} + \max\{9.5, 9\} + \max\{13.5, 13.5\} + \max\{18.25, 18.25\} = 48.75 = \sum_i P_i - \sum_i x_i^O$; the latter equality is due to the fact that player i attains C^i on M_i for $i \in M$ (see Theorem 1).

4. Preemption Allowed

Recall that N_i is the set of jobs that belong to manufacturer $i \in M$, $p_{\max}^i$ is the processing time requirement of a job in N_i with largest processing time, and $P_i = \sum_{j \in N_i} p_{ij}$ is the total workload of manufacturer $i \in M$. When preemption is allowed but overlapping is not, each manufacturer will generally follow a different strategy, say x_i^P: $i \in M$. In this section we assume the following:

- preemption is allowed for jobs but overlapping is not;
- values P_i, $p_{\max}^i$: $i \in M$ are disclosed to all manufacturers as in IP3; and
- sequencing on F will be done according to the *incentive rule* when *preemption* is allowed (abbreviated as IRP), defined next.

Definition 2 (IRP). The manufacturer workloads x_i^P will be processed in quasi-SPT order, i.e., player i precedes k on F if $x_i^P \leq x_k^P$ (break ties with smaller P_i, then arbitrarily), unless $x_i^P = P_i - p_{\max}^i \leq x_k^P$ and $P_i > P_k$.

All players subcontracting the same amount x are ordered in nondecreasing order of P_i values, and in case several x, P_i pairs are identical, choosing an arbitrary subsequence avoids transitivity issues.

Incentive rule IRP coincides with IRO when the workload P_i of every $i \in M$ consists of many very small jobs. However, if N_i contains a very large job that requires $p_{\max}^i$ time and the remaining workload $P_i - p_{\max}^i$ is small, then player i will process at least $p_{\max}^i$ units of work on M_i and at most $P_i - p_{\max}^i$ on F, as we will prove shortly. We refer to $P_i - p_{\max}^i$ as the *disposable* workload of player i because it is the maximum possible amount that player i is reasonably expected to subcontract to P. When $P_i - p_{\max}^i$ is small and this amount is subcontracted, delaying player i according to IRP does not hurt him (as we will see below), allows other players to subcontract more workload than they otherwise would, and provides better overall service by letting P utilize the SPT rule with respect to the P_i values rather than the SPT rule with respect to the x_i^P values. As for the players, in choosing their strategy they face the same dilemma as in the case when overlapping is allowed.

Lemma 2. *If a pure Nash Schedule exists, then there exists one where*
(a) $x_i^P \leq \min\{P_i - p_{\max}^i, P_i/2\}$; *and*
(b) $x_{[1]}^P + \cdots + x_{[i]}^P \leq P_{[i]} - x_{[i]}^P$ *for every* $i \in M$, *where* $[i]$ *denotes the ith player processed on* F.

Note that it may be optimal for player i to subcontract $x_i^P > \min\{P_i - p_{\max}^i, P_i/2\}$. For example, suppose N_1 has two jobs requiring 1 and 10 time units of processing, respectively, and all other manufacturers have a single job each requiring 100 units. Then player 1 can subcontract 10 units of work (instead of 1), and all other manufacturers would process their job in-house. In this example, $x_1^P > P_1/2$ and $P_1 - x_1^P < x_1^P$. In contrast, the proof of Lemma 2(b) shows that Nash schedules that satisfy (a) necessarily satisfy (b).

The main finding in this section is that there exists a Nash equilibrium in which IRP processes players in SPT order of their P_i values, as was the case in Theorem 1. In Theorem 1 this ordering coincides with the SPT order of workloads. In this section we show that it coincides with the quasi-SPT order defined in the beginning of the section. The reason for the quasi-SPT order when overlapping is not allowed is due to the longest job of each manufacturer. In light of Lemma 2, the workload subcontracted by player i should not exceed $P_i - p^i_{\max}$. For a very long job $p^i_{\max}$, the amount $P_i - p^i_{\max}$ is small, thus commanding an early position in the SPT sequence of manufacturer workloads even though P_i is large. In this case, if P used the SPT order to process manufacturer workloads, player i would be scheduled early even though machine M_i completes the remaining workload much later, and would unnecessarily delay all subsequent players and force them to subcontract less of their workload to P. This action would result in inferior service to all manufacturers (as measured by the sum of all completion times) and would decrease the total amount subcontracted to P. Incentive IRP avoids both of these undesirable effects.

Theorem 3. *If a Nash schedule exists, there is one where manufacturers are processed on F in nondecreasing order of P_i values, $i \in M$.*

Theorem 3 suggests an ordering of manufacturers on F. The following theorem suggests an ordering of their associated workloads.

Theorem 4. *Suppose a Nash schedule exists in which manufacturers are processed in nondecreasing order of P_i values, and reindex manufacturers so that*

$$\min\{P_1/2,\, P_1 - p^1_{\max}\} \le \min\{P_2/2,\, P_2 - p^2_{\max}\} \le \cdots \le \min\{P_{|M|}/2,\, P_{|M|} - p^{|M|}_{\max}\}.$$

Then, the subcontracted workloads satisfy $x^P_1 \le x^P_2 \le \cdots \le x^P_{|M|}$.

The previous two theorems suggest an ordering of manufacturers and workloads on F but not the amount of workload to be contributed by each manufacturer. This is computed in the next theorem. But first, we give a key property of pure equilibrium strategies.

Lemma 3. *If $P_k - p^k_{\max} < x^O_k$, then $x^P_k = P_k - p^k_{\max}$ is an equilibrium strategy for player k in a Nash schedule, if one exists. Otherwise, $x^P_k \ge x^O_k$.*

Theorem 5. *Reindex the manufacturers in nondecreasing order of P_i values. When preemption is allowed, the strategies $x^P_k = x^P_k(|M|)$, $k \in M$ obtained by the recurrence relations*

$$\Gamma(0) = \{k \in M \mid P_k - p^k_{\max} < x^O_k\},$$

$$x^P_{[k]}(r) = \min\left\{ \min_{\substack{k\le i\le |M|\\ i\notin\Gamma(r-1)}} \left\{ \left(P_{[i]} - x^P_{[1]}(r) - \cdots - x^P_{[k-1]}(r) - \sum_{\substack{j<k\\ j\in\Gamma(r-1)}} (P_j - p^j_{\max}) \right) \Big/ (i+2-k-n_{k,i}(r-1)) \right\},\ P_{[k]} - p^{[k]}_{\max} \right\}, \quad k \in M, \quad \text{and} \tag{1}$$

$$\Gamma(r) = \{k \in M \mid x^P_k(r) = P_k - p^k_{\max}\} \quad \text{for } r = 1, 2, \ldots, |M|$$

are in equilibrium, where $n_{k,i}(r-1)$ is the number of players among $k, \ldots, i$ in $\Gamma(r-1)$.

Two observations are in order. First, the recurrence relations in Theorem 5 require $\mathcal{O}(|M|^2)$ effort including updating the $n_{k,i}(r)$ values in each iteration. Second, given a nondecreasing order of P_i values, the strategies produced by the recurrence relations of Theorem 5 yield a unique pure Nash equilibrium. Tie breaks and the implicit requirements of Lemma 2 indicate that alternative Nash equilibria can exist.

Example 2. Consider the instance of Example 1 with the additional information that $p^1_{\max} = 5$, $p^2_{\max} = 11$, $p^3_{\max} = 7$, and $p^4_{\max} = 8$. Recall that the nondecreasing order of P_i values is 1, 2, 3, 4. Then, $\Gamma(0) = \{2\}$ because $P_2 - p^2_{\max} = 3 < x^O_2 = 4.5$. Hence, according to Lemma 3, the strategy for player 2 is $x^P_2 = 3$. Knowing $\Gamma(0)$, we obtain $x^P_1(1) = \min\{12/2, (18-3)/3, (23-3)/4\} = 5$—in the latter two fractions $n_{1,3}(0) = 1$ and $n_{1,4}(0) = 1$ because player 2 (who subcontracts $P_2 - p^2_{\max}$) is the only one in $\Gamma(0)$ between players 1 and 3 and between 1 and 4. Note that the minimand for player 1 is attained by both players 3 and 4, with the terms $(18-3)/3$ and $(23-3)/4$, respectively. For player 2 we have $x^P_2(1) = 3$ because $P_2 - p^2_{\max} = 3 < x^O_2 = 4.5$. The inside min operator in (1) is greater than $P_2 - p^2_{\max} = 3$, and hence $x^P_2(r)$ will remain the same for every $1 < r \le |M|$. Also, $x^P_3(1) = \min\{(18-5-3)/2, (23-5-3)/3\} = 5$, and $x^P_4(1) = (23-5-3-5)/2 = 5$. In this strategy scenario, no player other than player 2 subcontracts amount equal to $P_k - p^k_{\max}$; sets $\Gamma(1), \Gamma(2)$, and $\Gamma(3)$ equal $\Gamma(0)$; and hence the remaining three iterations of the recurrence relation $x^P_i(r)$ will not change the $x^P_i(1)$ values. Therefore, $(x^P_1, x^P_2, x^P_3, x^P_4) = (5, 3, 5, 5)$, $\sum_i x^P_i = 18$, and $\sum_i C^i = \max\{5, 7\} + \max\{8, 11\} + \max\{13, 13\} + \max\{18, 18\} = 49 = \sum_i (P_i - x^P_i)$; the latter is due to the fact that all the players attain their makespan on M_k, $k \in M$ (see Lemma 2(b)).

It is not difficult to verify that $x^P = (5, 3, 5, 5)$ is an equilibrium strategy profile, i.e., no player may profitably deviate from x^P given the strategies of other players. For example, if player 1 were to subcontract more than $x_1^P = 5$ units of work, say 6, he would be processed after players 3 and 4 on F, thus suffering makespan losses. Hence, $x_1^P(1) = 6$ is not an equilibrium strategy for player 1; instead, $x_1^P(1) = 5$ is the best strategy he can hope for. Similarly, player 4 could subcontract more than five units if this were beneficial, but this is not the case because the cumulative workload subcontracted by players 1, 2, and 3 is 13, and $P_4 = 23$. Therefore, five units is the most that player 4 could subcontract profitably.

Observe how solution x^P differs from $(x_1^O, x_2^O, x_3^O, x_4^O) = (4.5, 4.5, 4.5, 4.75)$ obtained in Example 1: knowing that player 2 cannot subcontract more than three units does not let player 1 take all the benefits (i.e., $x_2^O - x_2^P = 4.5 - 3 = 1.5$ units) because he still has to hedge his processing order on F. Instead, IRP forces players to distribute the 1.5 units among them almost equitably. In particular, players 1 and 3 subcontract 0.5 units more, whereas player 4 (who in Example 1 subcontracted 4.75 units—0.25 more than players 1 and 3) subcontracts only 0.25 units more. In effect, IRP helped players to equitably utilize F.

5. The Nonpreemptive Problem

In this section we consider Nash equilibria when jobs must be processed without interruption. When preemption is not allowed, the player strategies (denoted by x_i^N) are significantly different from those in previous sections. In her effort to motivate better utilization of F, the third party needs an appropriate incentive rule. For this, we need the following operator:

Definition 3. Operator $f_i(w)$ is the maximum workload of any subset $A_i \subseteq N_i$ that does not exceed w.

Operator $f_i(w)$ is equivalent to a knapsack problem with a knapsack of size w, where jobs are the items, and job $j \in N_i$ has size p_{ij} and value p_{ij}. In the rest of this section we assume the following:

- preemption is not allowed for jobs;
- complete job profiles $\{p_{ij}: j \in N_i\}_{i \in M}$ are disclosed to all manufacturers as in IP4; and
- sequencing on F will be done according to the *incentive rule* for the *nonpreemptive* problem (abbreviated by IRN), as defined next.

Definition 4 (IRN). Manufacturer workloads x_i^N will be processed in nondecreasing order of w_i values; where $f_i(w_i) = x_i^N$, $i \in M$; break ties with smaller P_i first, then arbitrarily.

Breaking ties with smaller P_i values is done to avoid transitivity problems among the players who have the same total workload. IRN suggests that the jobs subcontracted by each player i are those that fit in a knapsack of size $w_i \geq w_k$ for $k < i$, i.e., a knapsack that is at least as big as the size of any of the knapsacks used by his predecessors on F. IRN works similarly to IRO: players subcontracting from bigger knapsacks are punished by being processed later on F. Therefore, even though early manufacturers would like to subcontract as much as possible, they are forcibly restrained so as to allow subsequent manufacturers the opportunity to subcontract from even bigger knapsacks. And because the workload of later manufacturers is larger, it is likely that $\sum_{i \in M} x_i^N$ is larger, thus increasing the total workload subcontracted to P.

Suppose, for example, that the possible subcontracting strategies of player $[i]$ (i.e., the player processed ith on F) are 0, or 10 or 15, and that his total workload is 35. This means that player $[i]$ can split his job set into two groups: one to be processed on $M_{[i]}$ and another on F. Note that, in this example, player $[i]$ cannot subcontract 12 units because there is no bipartition of jobs into $23 + 12 = 35$ units. Suppose $x_{[i]}^N = 10$. Then, for the purpose of processing priority on F, rule IRN allows for $w_{[i]} \in \{10, 11, 12, 13, 14\}$ because $f_{[i]}(w_{[i]}) = 10$. Suppose $w_{[i]} = 13$. Then, if player $[i-1]$ has (for example) a total workload $P_{[i-1]} = 30$ units that can be partitioned in two groups of 12 and 18 units, then $[i-1]$ could subcontract 12 units of work and be processed ahead of $[i]$ because for $w_{[i-1]} = 12 < w_{[i]} = 13$ we have $f_{[i-1]}(12) = 12$. In this example, IRN allows for $w_{[i-1]} \in \{12, 13, 14, 15, 16, 17\}$ and $w_{[i]} \in \{10, 11, 12, 13, 14\}$. Choosing $w_{[i-1]} = 12$ and $w_{[i]} = 13$ allows player $[i-1]$ to subcontract 12 units of work and be processed before $[i]$, who subcontracts just 10 units. This feature of IRN is similar to IRP and allows deviation from the SPT order of subcontracted workloads. Contrary to IRO and IRP, which operate based on actual workloads, IRN operates based on knapsack sizes.

As we saw, there are mutliple knapsack sizes w_i for which $f_i(w_i) = x_i^N$ for $i \in M$. Hence, an issue arises of what sizes are actually used for IRN purposes. Clearly, player $i \in M$ would like to use the smallest possible size that preserves his equilibrium order on F to prevent other players (those with large P_k values) being processed ahead of i. Therefore, we assume that the third party P finds an ordering so that it can assign to the player processed ith on F the knapsack size

$$w_{[i]} = \max_{k \leq i} x_{[k]}^N, \quad \text{for } i \in M.$$

Clearly, according to IRN, this knapsack size guarantees player $[i]$ the ith position in the SPT order of F while subcontracting $x_{[i]}^N$.

It is easy to observe that IRN reduces to IRO (IRP) in the case that overlapping (preemption) is allowed because $f_i(w) = w = x_i^O$ ($f_i(w) = w = x_i^P$). Regarding the size w, note that for all w such that $\min\{P_i - p^i_{\max}, P_i/2\} \le w \le P_i/2$, we have $f_i(w) = \min\{P_i - p^i_{\max}, P_i/2\}$. The equilibrium properties of IRN are different than those for IRO and IRP. We start with the following observations:

Lemma 4. *Let $x_1^N, x_2^N, \ldots, x_{|M|}^N$ be the player strategies in a Nash schedule S, if one exists. Then,*

(a) $x^N_{[1]} + \cdots + x^N_{[i]} - (P_{[i]} - x^N_{[i]}) < \min_{j\in A_i} p_{ij}$, *where $A_i \subseteq N_i$, denotes the subset of jobs subcontracted to P by player i, $i \in M$; and*

(b) $x_i^N \le \min\{f_i(P_i/2), P_i - p^i_{\max}\}$, $\forall i \in M$.

Lemma 4(a) states that if the makespan of player i is attained on F, then the subcontracted workload cannot complete $\min_{j\in A_i} p_{ij}$ or more time units after the completion time of M_i. Therefore, for some player(s) the makespan may be attained on F, and hence $C^i > P_i - x_i^N$. Hence, when preemption is not allowed, it may be that

$$\sum_i C^i \ge \sum_i P_i - \sum_i x_i^N.$$

This is in contrast with Theorem 1 and Lemma 2(b): when overlaps or preemption is allowed, the makespan of every manufacturer is attained on his processor. The following result highlights an equilibrium order induced by IRN.

Theorem 6. *If Nash schedules exist, then there is one where manufacturers are ordered in nondecreasing order of P_i values.*

In light of Theorem 6, suppose $P_1 \le P_2 \le \cdots \le P_{|M|}$ is an SPT order of P_i values. An equilibrium strategy for player 1 that satisfies IRN is given by the following mathematical program:

$$(IP_1) \quad x_1^N = \min_{w\ge 0} \max\{P_1 - f_1(w), f_1(w)\}$$
$$\text{s.t. } \max\left\{P_i - f_i(w), \sum_{j=1}^{i} f_j(w)\right\} \tag{2}$$
$$\le \max\{P_i - f_i(w_i^-), f_i(w_i^-)\} \quad \text{for } i > 1,$$

where w_i^- is the largest strategy of player i that does not exceed $f_i(w)$ (i.e., $f_i(w_i^-) < f_i(w)$; define $w_i^- = 0$ if $w = 0$). Constraint (2) tests whether the players $i = 2, \ldots, |M|$ are better off subcontracting from a smaller knapsack size than w, thus forcing player 1 to subcontract less. After finding x_1^N, we know that players $i > 1$ can profitably draw from knapsacks of size $w_1 = x_1^N$, and therefore they seek to potentially improve upon this strategy. And because $P_i \ge P_1$ for $i > 1$, by Theorem 6, players $2, \ldots, |M|$ do not need to consider preceding player 1 on F. This observation motivates the following two operators:

$$g_{ki}(w) = \max\left\{P_i - f_i(w), \sum_{j=1}^{k-1} x_j^N + \sum_{j=k}^{i} f_j(w)\right\} \quad \text{and}$$

$$h_{ki}(w) = \max\left\{P_i - f_i(w), \sum_{j=1}^{k-1} x_j^N + f_i(w)\right\}$$

$$\text{for } 1 \le k \le i \le |M|.$$

Then, (IP_1) can be stated as $\min_{w\ge 0} g_{11}(w)$ s.t. $g_{1i}(w) \le h_{1i}(w_i^-)$ for $i > 1$. Iteratively, x_k^N can be found by solving the mathematical program (IP_k) for $k = 1, 2, \ldots, |M|$:

$$(IP_k) \quad x_k^N = \min_{w \ge w_{k-1}} g_{kk}(w)$$
$$\text{s.t. } g_{ki}(w) \le h_{ki}(w_i^-) \quad \text{for } i > k.$$

A few observations are in order. Define the set of profitable strategies for player $i \in M$ as follows:

$$X_i = \left\{x \ge 0\colon x \le \min\left\{\frac{P_i}{2}, P_i - p^i_{\max}\right\} \text{ and } f_i(x) = x\right\}, \tag{3}$$

and let $w_r = \max_{j\le r} x_j^N$ for $r = 1, \ldots, |M|$; $w_0 = 0$. Then, in IP_k, we find knapsack size w_k for which players $i > k$ profitably subcontract $f_i(w_k)$. Therefore, in IP_{k+1} we can use $w_i^- = w_k$ so as to benchmark the new strategy $f_i(w_{k+1})$ against the proven strategy $f_i(w_k)$. Therefore, program IP_k can be restated as

$$(IP_k) \quad x_k^N = \min_{\substack{x\in X_k \\ f_k(x)\ge f_k(w_{k-1})}} g_{kk}(x)$$
$$\text{s.t. } g_{ki}(\max\{x, w_{k-1}\}) \le h_{ki}(w_{k-1}) \quad \text{for } i > k. \tag{4}$$

We summarize our findings in the following theorem.

Theorem 7. *Let $1, 2, \ldots, |M|$ be an SPT order of P_i values. Then, the strategies $\{x_k^N\}_{k\in M}$ produced by IP_k, $k \in M$, yield a pure strategy equilibrium in $\mathcal{O}(\max_i |N_i||M|P^2_{|M|})$ time.*

The presentation of an iterative algorithm NPN (for nonpreemptive Nash) together with its complexity are included in the online companion. It is interesting to observe that the Nash equilibrium produced by NPN is not unique even if the SPT order of the players is fixed. This is because there may be more than one optimal strategy for some player(s) i. Hence, the choice x_i^N greatly affects x_k^N for $k > i$, resulting in different equilibria. In algorithm NPN (presented in the online companion) we assume that each player i chooses his smallest strategy among the optimal ones because even though he is indifferent among them, subsequent players can potentially subcontract more.

Example 3. Consider the instance used in Examples 1 and 2 when the processing time profiles are $\{p_{1j}\} = \{5, 4, 2, 1\}$, $\{p_{2j}\} = \{11, 2, 1\}$, $\{p_{3j}\} = \{7, 6, 5\}$, and $\{p_{4j}\} = \{8, 8, 7\}$. The players will again be processed in the order 1, 2, 3, 4 on F, but the workload of each must come from a knapsack no larger than that of any of his successors. The set X_1 of bipartitions for player 1 where one part includes at least half of the total workload is $X_1 = \{6, 5, 4, 3, 2, 1, 0\}$ (e.g., he can subcontract a workload of $6 = 4+2$ or 5 due to the job of length 5, etc.). If player 1 subcontracted six units of processing in a Nash schedule, then player 2 would have to pick jobs from a knapsack of size at least 6. Note that $X_3 = \{7, 6, 5, 0\}$, and hence player 3 would have to subcontract six or more units of workload. But then, his completion time on F would be $6+3+6 = 15$, whereas the remaining workload on M_3 would be $7+5 = 12$, and hence $C^3 = 15$. Strategy $x_3 = 6$ is not optimal for player 3 (given strategies $f_1(6)$ and $f_2(6)$ for players 1 and 2) because he can subcontract five instead of six units and attain makespan $\max\{6+3+5, 7+6\} = 14 < 15$. Hence, $x_3^N = 6$ is not an equilibrium strategy for player 3. On the other hand, $x_1^N = 5$ yields $x_2^N = 3$ (because $X_2 = \{3, 2, 1, 0\}$), $x_3^N = 5$, $x_4^N = 7$ (because $X_4 = \{8, 7, 0\}$), and $\max\{23-x, 5+3+5+x\}$ is minimized for $x = 7$. Evidently, $\sum_i x_i^N = 20$, and $\sum_i C^i = \max\{7, 5\} + \max\{11, 8\} + \max\{13, 13\} + \max\{16, 20\} = 51$. In this example, $\sum_i P_i - \sum_i x_i^N = 47 \neq \sum_i C^i$, which shows that when preemption is not allowed, it is possible to have

$$\sum_i C^i > \sum_i P_i - \sum_i x_i^N$$

because some player(s) attain their makespan on F, and hence $C^i > P_i - x_i^N$.

6. Conclusions

At this point it is instructive to look back at the example presented in §2 and examine how it may explain the Boeing experience with the Dreamliner project. Our analysis indicates that a third party whose main objective is to maximize her utilization tends to equalize her availability across all manufacturers by giving priority to those who subcontract smaller workloads. Then, not only do large manufacturers not have exclusive use of P, but they are scheduled late on F, thus experiencing maximum risk of completing late the subcontracted amount of their workload. In terms of Boeing, consider a supplier i who is given a relatively large and *critical* activity for the Dreamliner. He is eager to avoid delays by subcontracting amount x_i of his workload to P. Scheduled near the end of the sequence on F, he is exposed to the greatest possible risk of finishing later than anticipated because he is preceded by several other manufacturers on F. Possible delays on F accumulate against the timely delivery of his workload x_i, whereas possible gains do not; he still has to complete the in-house portion $P_i - x_i$ of his workload. Evidently, for the Dreamliner to be delivered on time, hundreds of Boeing suppliers had to be "lucky" with their third parties and finish on time. In retrospect, it is clear that the only way around this problem was for Boeing to gain oversight of not just its suppliers, but also their third parties. In the absence of any such centralized control, these results were predictable and unsupportive of Boeing's objective toward timely delivery of the Dreamliner.

In this paper we considered a competitive model of subcontracting operations to a third party so as to best take advantage of its capacity. In the first part of this paper we developed pure equilibrium strategies for three production protocols with complete information sharing. Subsequently, for the overlapping and preemptive protocols, we observed that optimal performance can be achieved without the burden of centralized control if P chooses an appropriate incentive rule and the manufacturers subcontract so as to maximize their own makespan. A number of future research directions are possible beyond the countless variations in production configurations, player objectives (including cost- rather than time-based objectives), and incentive rules.

In this paper, we considered the case of perfect information. Applications abound where information is imperfect. These issues point to fruitful research directions on truth-inducing mechanisms for games with imperfect information. Note that under complete information sharing, it is unclear whether players can benefit by revealing false P_i values. If player i inflates his true P_i value, rules IRO, IRP, and IRN process player i later on F, thus risking unnecessary loses in makespan. If player i deflates his true P_i value, then he is processed earlier on F but is required to subcontract a smaller amount. This would result in a greater amount processed on M_i, again resulting in makespan loses. Intuitively, conveying false information exposes players to significant risk.

Under incomplete information, players may respond with strategies that hedge against the worst possible outcome. Alternatively, distributional assumptions for possible outcomes within a Bayesian framework may be used. Another obvious research direction is the analysis of the centralized problem presented in this paper and the study of cooperative games that help players achieve the results of coordination by means of transfer payment schemes instead of rules imposed by the third party.

Electronic Companion

An electronic companion to this paper is available as part of the online version at http://dx.doi.org/10.1287/msom.1120.0410.

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