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To Commit or Not to Commit: Revisiting Quantity vs. Price Competition in a Differentiated Industry

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Previous studies have shown that quantity commitment by all firms in an industry mitigates price competition. This paper shows that when firms have a choice, asymmetric outcomes can arise, with some firms choosing to commit to a quantity and other firms choosing not to. To study the commitment decision, we analyze a multistage game within a duopoly of differentiated firms à la Hotelling. In the first stage of the game, firms choose whether or not to commit to a quantity. In the second stage, each firm that chose to commit sets a quantity, which represents an upper bound on how much it can sell to consumers. In the third stage, both firms set prices strategically, regardless of whether or not they committed. In the final stage, demand is allocated as consumers maximize their utilities. Firm(s) that chose not to commit to a quantity in the first stage can fulfill any quantity that consumers demand at the equilibrium prices. We find that if product differentiation is sufficiently *low*, *both* firms choose to commit to a quantity in equilibrium. This *symmetric* equilibrium allows both firms to avoid the intense price competition associated with low product differentiation. If the level of differentiation is sufficiently *high*, the equilibrium in quantity commitment is *asymmetric* such that one firm chooses to commit and its competitor chooses not to commit. Under this equilibrium, a pricing equilibrium in pure strategies does not exist, only a pricing equilibrium in mixed strategies does, and the equilibrium prices can result in a committed firm not clearing the entire quantity it chose earlier.

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1. Introduction

Although many previous studies have clearly shown that quantity commitment mitigates price competition, the decision to commit or not to commit to a quantity is still not completely understood. For instance, the seminal work of Cournot, later refined by Kreps and Scheinkman (1983), showed that equilibrium prices and profits are higher when competing firms commit to a quantity compared with when they do not commit. Neither study, however, accounted for the role of product differentiation. More importantly, both studies assumed that all firms commit to a quantity and do not consider whether or not firms would actually choose to do so.

Singh and Vives (1984) provided a partial answer by developing a differentiated duopoly model that explored the commitment decision by allowing firms to first choose from a "price contract" or a "quantity contract." They showed that when the goods are imperfect substitutes, both firms will commit to a quantity by choosing the quantity contract. Their quantity contract, however, operationalizes quantity commitment as follows: if a firm chooses the quantity contract, it does not set a price afterward. Rather, it commits to selling that entire quantity at the price that will clear it regardless of what the competitor does. It remains unclear, however, whether or not the equilibrium proposed by Singh and Vives will hold if we allow firms that commit to a quantity to *set prices strategically* at a subsequent stage.

In this paper, we propose a more general modeling approach that allows differentiated firms to choose to commit or not to commit, allows firms that commit to set their quantities, and allows all firms, regardless of whether or not they committed, to strategically set prices. When the level of product differentiation is sufficiently low, our model yields results that are consistent with the aforementioned studies. Nonetheless, when the level of product differentiation is sufficiently high, our model yields some novel insights, and its results depart sharply from those of previous studies.

In particular, we find that if product differentiation is sufficiently *low*, both firms choose to commit to a specific production quantity in equilibrium. This *symmetric* equilibrium allows both firms to avoid the intense price competition that is associated with low product differentiation. If the level of differentiation is sufficiently *high*, the subgame perfect Nash equilibrium (SPNE) is *asymmetric* such that one firm chooses to commit and its competitor chooses not to commit. This asymmetric equilibrium in commitment is our most important and least intuitive result. Under this equilibrium, a pricing equilibrium in pure strategies does not exist, only a pricing equilibrium in mixed strategies does, and the equilibrium prices can result in a committed firm not clearing the entire quantity it chose earlier.

We derive results by analyzing a multistage game within a duopoly of differentiated firms à la Hotelling. In the first stage of the game, firms choose whether or not to commit to a quantity. In the second stage, each firm that chose to commit sets a quantity that represents an upper bound on how much it can sell to consumers. In the third stage, both firms set prices strategically, regardless of whether or not they committed. In the final stage, demand is allocated as consumers maximize their utilities. Firm(s) that chose not to commit to a quantity in the first stage can fulfill any quantity that consumers demand at the equilibrium prices.

The following points explain the intuition behind the SPNE in commitment/noncommitment decisions (we refer to the firms as i and j, and market size is normalized to 1):

- If both firms choose not to commit, then price competition is most intense leading to low profits. For this reason, a firm facing a noncommitted rival will prefer to commit to soften price competition. In other words, when facing a noncommitted rival, a firm achieves a higher profit by committing than by not committing. Because commitment is the best response to noncommitment, at least one firm commits in equilibrium.
- If firm i chooses to commit, its choice of quantity, q_i, will depend on whether or not its rival, j, is committed or noncommitted. If firm j is also committed, then both firms choose complementary quantities, i.e., $q_i + q_j = 1$, which eliminates incentives to compete on price. If, on the other hand, firm j is noncommitted, its demand is unrestricted, and thus it has an incentive to compete on price. That incentive is high (low) when product differentiation is low (high). As such, firm i sets a high quantity when product differentiation is low to claim a larger turf to counter its rival's high incentive to compete on price (and vice versa).
- Whether or not firm j chooses to commit is therefore determined by the quantity chosen by firm i when facing a noncommitted rival. Specifically, if firm i commits, firm j will prefer not to commit only if it knows that firm i will respond to such a choice with a sufficiently low quantity. This is why we find that the SPNE is symmetric (such that both firms commit) when the level of product differentiation is low and asymmetric (such that one firm commits and its rival does not) when the level of differentiation is high.

In practice, firms can employ a variety of mechanisms to commit to a quantity. For instance, plant capacity limits the quantity a manufacturing firm can produce and ultimately sell; warehouse and shelf capacities limit the number of items a firm can stock. Likewise, long replenishment lead time may cause a seasonal goods seller to become quantity constrained if its initial inventory cannot be further replenished. Conversely, firms can also choose *not to commit* to a quantity. The primary lever for eliminating quantity commitment is simply to match supply with demand. This can be achieved through investing in large capacities; by shortening lead times; or by building flexible, just-in-time production systems. A well-known example is that of Dell, whose strategy is to initiate production only when demand from consumers is received (Guo and Iyer 2013).

A firm's decision of whether or not to commit to a quantity is strategically important as it subsequently affects the competitive nature of an industry. This is especially relevant as more firms have the option of adopting flexible manufacturing regimes and/or fast supply chains that can circumvent quantity commitment.

The rest of this paper is organized as follows: §2 discusses previous research, §3 introduces the general setup of our model, §4 contains the bulk of the analysis under various commitment combinations, and §5 discusses the SPNE in commitment choices. Section 6 combines commitment and quantity decision in a single stage and discusses the role of observable commitment on limiting possible SPNEs. Section 7 concludes.

2. Related Research

The Cournot model is perhaps the earliest and most famous model of quantity commitment. In a Cournot game, all firms commit to a quantity by assumption. They set quantities strategically but have no say in setting prices. Rather, each firm sells the entire quantity it chose at the price that will clear such quantity. This lack of price control was the main point behind Bertrand's critique of the Cournot model.

Kreps and Scheinkman (1983) echoed Bertrand's critique and explained that the Cournot model is equivalent to assuming the presence of an auctioneer who auctions off the entire quantities that firms chose. They addressed this issue by building a model in which firms play a two-stage game, where they choose a quantity in the first (production) stage and set prices in the second stage. They found that this two-stage setup yields the same outcome as that of the Cournot model. Although this was certainly an important step in understanding quantity commitment, two important issues still were not well understood. The first issue was the firms' incentive to commit to a quantity—both Cournot and Kreps and Scheinkman (1983) assumed that all competing firms will commit. The second issue is how product differentiation may affect the decision of whether or not to commit and how the potential commitment affects pricing.

The model we analyze in this paper reduces to the Kreps and Scheinkman (1983) model if we assume that both firms commit to a quantity (i.e., remove the first stage of the game) and set the level of product differentiation[1] to zero. The model also reduces to the Singh and Vives (1984) model if we remove the pricing stage for firms that commit to a quantity. When product differentiation is low, our results mirror those of Kreps and Scheinkman (1983) in that both firms commit to a quantity and choose the noncompetitive prices that will simply clear the quantities they chose. They also mirror the results of Singh and Vives (1984) in that both firms choose to commit to a quantity.

If the level of differentiation is sufficiently high, however, our results depart sharply from those of Kreps and Scheinkman (1983) and Singh and Vives (1984). In contrast to both studies, we show that the SPNE is *asymmetric* such that one firm chooses to commit and its competitor chooses not to commit. In contrast to the former, we show that setting prices strategically does not necessarily yield a Cournot outcome, as equilibrium prices can result in a committed firm not clearing the entire quantity it chose earlier.

Others have explored how quantity commitment can relax price competition. An extreme example of this is given by Dixit (1980), who showed that quantity commitment on the part of an incumbent firm can make it hard for a competitor to enter a market. Afterward, the strategic value of quantity commitment has been widely studied, though almost exclusively in markets for nondifferentiated goods. For instance, Brock and Scheinkman (1985), Lambson (1994), Compte et al. (2002), Davidson and Deneckere (1990), and Benoit and Krishna (1987) studied the role of limited capacities in a repeated game of price competition. In some of these earlier papers, asymmetric capacity (commitment) decisions arise in equilibrium. This asymmetry is specifically driven by assuming sequential moves where a Stackelberg leader makes its capacity decision before its rivals do. By contrast, this paper shows that an asymmetric equilibrium can arise when firms move simultaneously.

[1] Operationalized as the Hotelling transportation cost.

When only one firm commits to the quantity, Daughety (1990) found that a Stackelberg leader will commit to a greater quantity than it would in a simultaneous-move game. A reverse result is reported in Shulman (2014), who constructed a model in which one seller commits to a lesser quantity, even though its competitors are not only unconstrained but also sell greater quantities than if no quantity commitment were made. Deneckere and Kovenock (1992) relied on capacity constraints to provide a model where, in equilibrium, the dominant firm chooses to be the price leader. Allen et al. (2000) showed that capacity precommitment may act as a barrier to entry when price competition takes place post entry. Lepore (2008) introduced incomplete information in the Kreps and Scheinkman (1983) model. He analyzed a two-stage game with capacity precommitment followed by price competition where firms have incomplete information about their rival's marginal cost. Lepore's model has a Cournot outcome if and only if the lowest possible marginal cost is sufficiently high relative to the expected marginal cost.

3. Model Setup

We consider a duopoly industry of two competing firms that sell differentiated goods to heterogeneous consumers. We begin by describing consumers and firms; then we describe the decision sequence.

3.1. Consumers

Consumers are heterogeneous in their product preferences à la Hotelling. They are indexed by their location, $x \sim \mathrm{U}(0,1)$, along a line of unit length. The utility a consumer located at x gets from buying product j at a price p_j is given by

$$U_j(x) = 1 - ts_j(x) - p_j, \tag{1}$$

where $s_j(x)$ denotes the Hotelling distance between the consumer's location, x, and product j's.

3.2. Firms

The two competing firms/products will be indexed by a and b; firm a is located at the extreme left ($x=0$), and firm b is located at the extreme right ($x=1$) of the Hotelling line. The Hotelling distances in Equation (1) therefore are $s_a = x$ and $s_b = 1-x$. Both firms are risk neutral and incur zero marginal production cost. We assume zero marginal cost to simplify the exposition and identify the effect of strategic considerations (as opposed to purely cost concerns) in quantity commitment decisions. The online appendix (available as supplemental material at http://dx.doi.org/10.1287/mnsc.2015.2213) explains how allowing positive marginal costs does not change the main message of the results of the paper.

Before firms sell their products, they choose whether or not to commit to a quantity. If firm $j \in \{a, b\}$ chooses to commit, it needs to set its commitment quantity, q_j, with the stipulation that it cannot sell more than q_j. This quantity represents either the firm's production capacity or the quantity it has to preorder prior to selling (see §3.4 regarding the interpretation of q_j). On the other hand, if a firm chooses not to commit, it can fulfill any demand that arises after both firms set their prices.

After making their commitment and quantity decisions, firms set prices *strategically*. This applies to *both* firms, regardless of whether or not they decided to commit. This is an *important difference* between our model and the models of Cournot and Singh and Vives (1984), both of which assume that when a firm commits to a quantity, it chooses whatever price clears such quantity regardless of what the competitor does.

We use $d_j(p_i, p_j)$ to denote the demand that firm j achieves at prices p_i and p_j, where $i, j \in \{a, b\}$: $i \neq j$. If firm j chooses to commit to quantity q_j, it sets a price that achieves the maximum in the following problem:

$$\begin{aligned} &\max_{p_j} \quad \pi_j \equiv p_j d_j(p_i, p_j) \\ &\text{subject to} \quad d_j(p_i, p_j) \leq q_j. \end{aligned} \tag{2}$$

If firm j chooses not to commit to any quantity, then it simply sets a price, p_j, to achieve

$$\max_{p_j} \pi_j \equiv p_j d_j(p_i, p_j). \tag{3}$$

3.3. Additional Assumptions

Our analysis makes use of three additional assumptions:

1. *Product differentiation:* We assume that the level of product differentiation, t, is not too high. Specifically, $0 \leq t \leq \frac{2}{3}$, which ensures that the firms compete for the marginal consumer and do not act as local monopolists.
2. *Efficient rationing:* As explained earlier, a quantity commitment imposes a constraint on the number of consumers a firm can serve. Under certain pricing conditions, this may lead to product shortages, which occur precisely when the number of consumers who prefer the firm's product exceeds the quantity to which it had committed earlier. Whenever such a case arises, we assume that inventory is cleared by those consumers who achieve the highest utility for it (this rule is efficient in the sense that it maximizes consumer surplus; see, for instance, Kreps and Scheinkman 1983).
3. *Tie-breaking quantity decision:* If a firm chooses to commit to a quantity and can choose multiple order quantities that yield the same expected profit, we assume that it will choose the smallest quantity. This assumption is necessitated by the zero marginal cost normalization and will *only* be active in a situation when a committed firm faces another committed firm (see §4.3). (Note that, if marginal cost were set to an arbitrarily small amount, this assumption would not be required. For additional discussion, see the online appendix.)

3.4. Game Sequence

What follows is a summary of the sequence of stages in the game between the two firms:

Stage 1. Each firm chooses whether or not to commit to a quantity.

Stage 2. If firm j chooses to commit, it sets the quantity/capacity, q_j.

Stage 3. The selling stage starts and firms choose prices to solve (2) and/or (3).

Stage 4. Each consumer chooses whether or not to buy and which product version to buy.

The sequence above describes a multistage game of perfect information. We solve for the SPNE by backward induction. In the above sequence, notice that the separation of Stages 1 and 2 implies *observable commitment*, where a firm that chooses to commit first observes its rival's commitment decision before it chooses quantity. Alternatively, Stages 1 and 2 could be combined into a single stage implying *nonobservable commitment*, where a firm that chooses to commit has to choose quantity *before* it observes its rival's commitment decision. To the best of our knowledge, Singh and Vives (1984) is the only previous research study that endogenized the decision of whether or not to commit. In that paper, commitment is *observable*, which is consistent with the game sequence presented above. Singh and Vives operationalized the quantity commitment decision as a choice between a quantity contract and a price contract; after firms observe the rival's choice, they set quantities and/or prices.

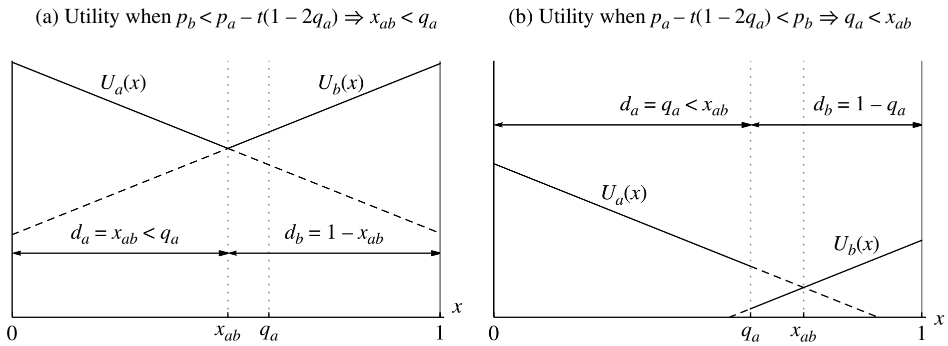
We proceed with the analysis while assuming observable commitment, keeping Stages 1 and 2 separate. In §6, we discuss different industry scenarios in which commitment can be either observable or not. We also investigate the effect of combining Stages 1 and 2 on the final SPNE. As will be seen, the SPNE that arises under observable commitment is also an SPNE under nonobservable commitment. However, an additional SPNE can arise when commitment is nonobservable.[2]

4. Subgame Analyses

We begin the analysis by examining the subgames of Stages 2–4 *conditional* on the quantity commitment decision of Stage 1. Then, in §5, we explore the Stage 1 decision of whether "to commit" or "not to commit."

[2] We thank two anonymous reviewers for their thoughtful comments on this issue.

Figure 1 **Consumers' Utility Under N-C**



The quantity commitment choices in the first stage yield three possible outcomes (subgames):

N-N: Neither firm commits to quantity.

N-C: Only one firm commits to quantity and the other firm does not.

C-C: Both firms commit.

We analyze cases (subgames) N-N, N-C, and C-C in succession.

4.1. Symmetric Noncommitment (N-N)

First, suppose that neither firm commits to a quantity in Stage 1, and define $x_{ab} \equiv (t + p_b - p_a)/(2t)$ as the location of the consumer who is indifferent between buying products a and b at respective prices of p_a and p_b. Since neither firm is committed to a quantity, no demand constraints exist, and the firms' demands are $d_a = x_{ab}$ and $d_b = 1 - x_{ab}$. Simultaneously solving the first-order conditions (FOCs) of the firms' objective functions as given by problem (3) yields the following equilibrium price and profits:[3]

$$p^*_{n-n} = t \quad \text{and} \quad \pi^*_{n-n} = \frac{t}{2}. \qquad (4)$$

4.2. Asymmetric Choices (N-C)

Next, suppose that only one firm commits to quantity in Stage 1. Without loss of generality, suppose that firm a commits to a quantity q_a and firm b makes no quantity commitment. Then the total demand that firm a can serve, $d_a(p_a, p_b)$, becomes constrained by the quantity q_a (see 2), whereas the total demand that firm b can serve, $d_b(p_b, p_a)$, remains unconstrained (see (3)).

We proceed with the analysis of the N-C case by first solving for the equilibrium prices of Stage 3. Then we solve for firm a's equilibrium quantity of Stage 2.

4.2.1. Equilibrium Prices. As before, $x_{ab} = (t + p_b - p_a)/2t$ denotes the location of the consumer who is indifferent between buying product a or product b. Firm a, however, can sell at most q_a units, which means that the farthest consumer firm a can serve is located at $x = q_a$. Now, if we were to apply the classical Cournot model or the quantity contract of Singh and Vives (1984), firm a would set a price $p_a = p_b + t(1 - 2q_a)$, which implies $x_{ab} = q_a$. In other words, no matter how firm b prices, firm a would simply price so as to *equate* its demand with the quantity q_a. This, however, may not necessarily be optimal because, as will be seen, equilibrium prices can result in either $x_{ab} < q_a$ or $q_a < x_{ab}$.

The situation $x_{ab} < q_a$ arises when firm b's price is low enough compared with that of firm a. Specifically, if $p_b < p_a - t(1 - 2q_a)$, firm a will be unable to sell its entire inventory because $x_{ab} < q_a$ (see consumers' utility in Figure 1(a)).

By contrast, the situation $q_a < x_{ab}$ arises when p_b is high enough compared with p_a. Then the number of consumers who prefer to purchase from firm a exceeds firm a's available inventory (see consumers' utility in Figure 1(b)). In this case, the *efficient rationing* assumption explained in §3.3 implies that consumers located in $[0, q_a]$ will purchase from firm a, consumers located in $(q_a, x_{\text{ab}}]$ would prefer to purchase from firm a but will be forced to purchase from firm b because of firm a's inventory shortage, and consumers located in $(x_{\text{ab}}, 1]$ will purchase from firm b.

The following result characterizes the subgame equilibrium prices conditional on all possible values of q_a such that $\max\{0, 1 - 1/(2t)\} \le q_a \le 1$.[4] The expressions for $\underline{q}$, $\bar{q}$, $\hat{p}_a$, $\bar{p}_b$, $\underline{p}_b$, and ρ are given in Table 1.

Proposition 1. *Suppose firm a chooses to commit to a quantity q_a such that* $\max\{0, 1 - 1/(2t)\} \le q_a \le 1$, *and firm b chooses not to commit. Then the subgame pricing equilibrium is as follows:*

1. *If firm a commits to a low quantity such that* $\max\{0, 1 - 1/(2t)\} \le q_a \le \underline{q}$, *the equilibrium prices are* $p^*_a = 1 - t\,q_a$ *and* $p^*_b = \bar{p}_b = 1 - t(1 - q_a)$. *The firms do not directly compete on price, and surplus is fully extracted from the marginal consumer at* $x = q_a$.

[3] This is the standard Hotelling result (e.g., see Tirole 1988, p. 279).

[4] See Appendix B for the formal expressions of the demand functions across the entire pricing strategy space and the firms' best response functions. There, in Corollary B.1, we also show that firm a will never set $q_a < 1 - 1/(2t)$. Thus, we establish that $\max\{0, 1 - 1/(2t)\} \le q_a$ for all $0 \le t \le \frac{2}{3}$.

Table 1 Expressions and Notation

Expression	Description
$\underline{q} \equiv \max\left\{0, \frac{3t-1}{3t}\right\}$	If q_a falls below this quantity, firm b will not compete on price and only go after the remaining $(1-q_a)$ consumers.
$\bar{q} \equiv \frac{\sqrt{1-2t^2}-(1-2t)}{2t}$	If q_a exceeds this quantity, firm b will compete on price and go after more than $(1-q_a)$ consumers.
$\hat{p}_a \equiv 2\sqrt{2t(1-q_a)(1-t(1-q_a))}-t$	If $p_a = \hat{p}_a$, firm b will be indifferent between competing and not competing on price.
$\bar{p}_b \equiv 1-t(1-q_a)$	The optimal *noncompetitive* price for firm b to capture only $(1-q_a)$ consumers.
$\underline{p}_b \equiv \frac{1}{2}(\hat{p}_a+t)$	The optimal *competitive* price for firm b to capture more than $(1-q_a)$ consumers.
$\rho \equiv \frac{3\sqrt{2t(1-q_a)(1-t(1-q_a))}-3t}{3\sqrt{2t(1-q_a)(1-t(1-q_a))}+t(2q_a-3)}$	Equilibrium mixing probability for firm b.

2. *If firm a commits to an intermediate quantity such that $\underline{q} < q_a < \bar{q}$, no pure strategy equilibrium in prices exists. Nonetheless, a mixed strategy equilibrium exists where firm a plays $p_a^* = \hat{p}_a$ with probability 1 and firm b plays $p_b^* = \bar{p}_b$ with probability ρ and $p_b^* = \underline{p}_b$ with probability $(1-\rho)$.*

3. *If firm a commits to a large quantity such that $\bar{q} \leq q_a \leq 1$, the equilibrium prices are $p_a^* = p_b^* = t$. The firms compete on price, and the marginal consumer at $x = \frac{1}{2}$ is left with positive surplus.*

Because of the quantity commitment, firm a cannot serve more than q_a consumers. Thus, regardless of the price set by its competitor, firm b can always capture the remaining $(1-q_a)$ at a high price of $\bar{p}_b$. Firm b can also choose to set a lower competitive price to capture more than $(1-q_a)$ consumers.

Part (1) of Proposition 1 considers the case when firm a commits to a low quantity: $q_a \leq \underline{q}$. Such a low quantity removes any incentive for firm b to compete for more than $(1-q_a)$ consumers, as it can garner a higher profit by choosing the high price, $\bar{p}_b$, and capturing exactly $(1-q_a)$ consumers. In this case, both firms set high prices to extract full surplus from the marginal consumer at $x = q_a$.

Part (3) of Proposition 1 considers the opposite extreme, where firm a commits to a large quantity $\bar{q} \leq q_a \leq 1$. Such a large quantity renders the quantity commitment irrelevant as the profit firm b garners by choosing the high price, $\bar{p}_b$, becomes too low. In this case, the pricing equilibrium is fully competitive as if no quantity commitment had occurred.[5]

Now, consider part (2) of Proposition 1, where firm a commits to an intermediate quantity $\underline{q} < q_a < \bar{q}$. Under such conditions, a pure strategy equilibrium in prices does not exist. The intuition behind this result is as follows: when q_a is intermediate, the profits firm b captures by setting a high price are close to the profits it captures by setting a competitive price to serve more than $(1-q_a)$ consumers. Therefore, firm b's optimal price will ultimately depend on the price set by its competitor. Specifically, if firm a decides on a sufficiently low price, $p_a < \hat{p}_a$, firm b will not want to compete on price and will set a high price of $\bar{p}_b$. Nonetheless, if firm b chooses such a high price, firm a will not have an incentive to set its price below $\hat{p}_a$. Conversely, if firm a decides on a sufficiently high price (above $\hat{p}_a$), firm b will prefer to set a competitive price to capture more than $(1-q_a)$ consumers. Nonetheless, if firm b chooses such a competitive price, firm a will have an incentive to set its price below $\hat{p}_a$.[6]

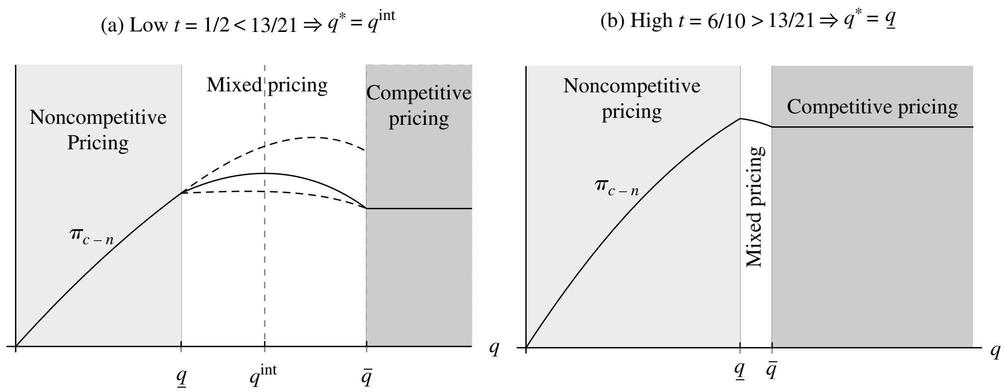
Under the mixed strategy described in part (2) of Proposition 1, firm a always plays $\hat{p}_a$, which makes firm b indifferent between setting a competitive price and setting a high price. Firm b randomizes between playing the high price of $\bar{p}_b$ and playing the competitive price of $\underline{p}_b$. If firm b plays $\underline{p}_b$, consumer utilities are as shown in Figure 1(a). Firm b captures more than $(1-q_a)$ consumers, and firm a does not sell its entire inventory. If firm b plays $\bar{p}_b$, consumer utilities are as shown in Figure 1(b). It captures exactly $(1-q_a)$ consumers, and firm a sells its entire inventory.

4.2.2. Equilibrium Quantity. Having solved for equilibrium prices conditional on all possible quantity levels, we now turn our attention to firm a's optimal quantity decision in Stage 2 (firm b chose not to commit, so it does not set a quantity). Not surprisingly, firm a will never choose any quantity above $\bar{q}$ because this will result in intense price competition. Instead, firm a will either set a low quantity, $\underline{q}$, which guarantees that firm b does not compete on price, or it will set an intermediate quantity, which will result in firm b randomizing between setting a competitive price and setting a

[5] As will be seen later, firm a will never choose such a high quantity in Stage 2.

[6] See the online appendix for a formal exposition of the firms' bestresponse functions.

Figure 2 **Committed Firm's Profits Under N-C at Different Levels of t**



noncompetitive one. The choice will ultimately depend on the level of product differentiation as follows.

Proposition 2. *Let q^* denote the optimal order quantity of firm a. For all $0 \le t \le 13/21$, there exists an interior optimal order quantity, $\underline{q} < q^{\text{int}} < \bar{q}$, such that*

1. *if the competing firms are not sufficiently differentiated, i.e., if $0 \le t < 13/21$, firm a sets $q^* = q^{\text{int}}$, and the pricing equilibrium is in mixed strategies as specified by Proposition* 1.2; *and*
2. *if the competing firms are sufficiently differentiated, i.e.,* $13/21 \le t \le 2/3$, *firm a sets $q^* = \underline{q}$, and the pricing equilibrium is in pure strategies as specified by Proposition* 1.1.

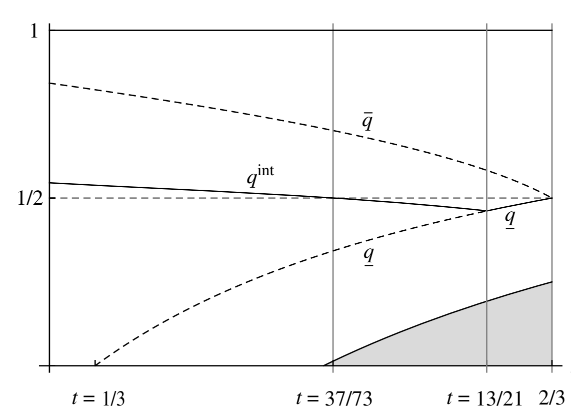
For a given level of product differentiation, t, the quantity $\underline{q}(t) \equiv \max\{0, (3t-1)/(3t)\}$ represents the maximum quantity to which firm a can commit without triggering any incentive on the part of firm b to compete on price. (As discussed above, if firm a commits to any $q_a \le \underline{q}$, firm b will find it optimal to set a high price so as to capture $1 - q_a$ consumers. Consequently, firm a will never order $q_a < \underline{q}$ since by setting $q_a = \underline{q}$, firm a achieves a higher profit while guaranteeing noncompetitive pricing on the part of firm b.) A closer look at the expression for $\underline{q}$ readily reveals that for all $t \le 1/3$, the incentive to compete is so high that firm b will compete on price for *any* positive quantity to which firm a commits. For all $t > 1/3$, $\underline{q} > 0$ is increasing in t, implying that as the level of product differentiation increases, firm a can commit to a higher quantity without evoking a competitive pricing response from its rival. Figure 3 illustrates the relationship between t and $\underline{q}$ graphically.

Part (1) of Proposition 2 considers the case where the level of product differentiation, t, is low, which implies a small $\underline{q}$. This in turn limits the profits firm a achieves from committing to $\underline{q}$ and guaranteeing a noncompetitive pricing outcome. In other words, when t is small, firm a will be better off setting $\underline{q} < q_a$. The relationship between the profits of committed firm, a, and the commitment quantity when t is low is seen graphically in Figure 2(a).

Note how the (expected) profits exhibit an inverse U shape between $\underline{q}$ and $\bar{q}$ and achieve a unique maximum at q^{int}. To understand why, recall from Proposition 1.2 that any quantity within this range will result in mixed pricing where firm b plays a high (noncompetitive) price, $\bar{p}_b$, with probability $\rho(q_a)$ and a low (competitive) price, $\underline{p}_b$, with probability $(1 - \rho(q_a))$. In the former case, the corresponding firm a's profits are represented by the upper dashed curve in Figure 2(a); in the latter case, they are represented by the lower dashed curve. The solid curve represents expected profits. The probability $\rho(q_a)$ is strictly decreasing in q_a (see Corollary B.2 in the appendix). That is, an increase in q_a makes firm b more (less) likely to play the competitive low (noncompetitive, high) price.

As q_a increases beyond $\underline{q}$, two forces affecting firm a's (expected) profit are in play: (1) Firm a achieves higher profits whenever firm b plays the high price, $\bar{p}_b$. (2) The probability of firm b playing the high price, $\rho(q_a)$, decreases. Jointly, these two effects result in the inverse

Figure 3 **Optimal Order Quantity Under Asymmetric Commitment**



Notes. Above, $\max\{0, 1 - 1/(2t)\} \le q_a \le \underline{q}$ yields a noncompetitive pricing equilibrium, $\underline{q} < q_a < \bar{q}$ yields a mixed strategy pricing equilibrium, and $\bar{q} \le q_a \le 1$ yields a fully competitive pricing equilibrium. The shaded region represents $0 \le q < 1 - 1/(2t)$, which is ruled out in Appendix B.

U shape as follows: whenever $\underline{q} < q_a < q^{\text{int}}$, firm a's expected profits increase in q_a because the first effect dominates the second. By contrast, whenever $q^{\text{int}} < q_a < \bar{q}$, firm a's expected profits decrease in q_a because the second effect dominates the first.

Part (2) of Proposition 2 considers the case where the level of product differentiation, t, is high, implying that $\underline{q}$ is high. That is, firm a can choose a sufficiently high quantity without triggering a competitive pricing response on the part of firm b, and its profits are maximized at the corner solution of $q_a = \underline{q}$. In mathematical terms, $13/21 < t \leq 2/3 \Rightarrow q^{\text{int}} < \underline{q} \Rightarrow q^* = \underline{q}$. The relationship between the profits of committed firm, a, and the commitment quantity when t is high, as seen graphically in Figure 2(b).

4.3. Symmetric Commitment (C-C)

We conclude the subgame analyses by discussing the case where both firms choose to commit to quantities. The same set of steps that led to Propositions 1 and 2 now lead to Proposition 3. (Since the results here are somewhat simpler than those of the N-C case, we report equilibrium prices and quantities in a single proposition.)

Proposition 3. *If both firms choose to commit to quantities, then there exists a continuum of equilibria in quantity choices such that firm a commits to a quantity* $q_a^{c-c} \in [1-\bar{q}_{c-c}, \bar{q}_{c-c}]$ *and sets a price of* $p_a = 1 - tq_a^{c-c}$; *firm b commits to a quantity of* $q_b^{c-c} = 1 - q_a^{c-c}$ *and sets a price of* $p_b = 1 - t(1 - q_a^{c-c})$. *The firms do not compete on price, and surplus is fully extracted from the marginal consumer located at* $x = q_a^{c-c}$.

To understand the intuition behind Proposition 3, first suppose that firm a sets a quantity of $q_a \in [1-\bar{q}_{c-c}, \bar{q}_{c-c}]$. Firm b can choose one of two responses:[7]

1. A noncompetitive response of setting a complementary quantity $q_b = (1 - q_a)$, which removes all incentives to compete on price, leaving each firm to sell exactly the quantity it orders at the highest possible price. This is the equilibrium described in Proposition 3, which yields firm b a profit of $\pi_{c-c} = (1-q_a)(1-t(1-q_a))$.
2. A competitive response of setting a higher quantity $q_b > (1-q_a)$ and competing with firm a for the marginal consumer. A mixed strategy equilibrium in prices will arise similar to that of the asymmetric (N-C) case discussed in §4.2. The highest profit firm b can capture in this case is that of a noncommitted firm when facing a committed rival, $\pi_{n-c} = (1-q_a)(1-t(1-q_a))$.

Note that the profits associated with both responses are equal.[8] However, the competitive response involves setting a higher quantity and can thus be ruled out by the tie-breaking assumption (see §3.3). If we set marginal cost to an arbitrarily small amount, the tie-breaking assumption would *not* be required as the profits associated with ordering a higher quantity would be automatically lower. (See the online appendix for a discussion of positive marginal cost.) In other words, the tie-breaking assumption rules out a committed firm mimicking a noncommitted one by setting a very high quantity in the absence of marginal cost.

Next, suppose firm a chooses a high quantity that is outside the range prescribed by Proposition 3 (i.e., $\bar{q}_{c-c} < q_a$). Such a high quantity renders firm a's quantity commitment irrelevant as it will be acting as a *noncommitted* firm. Firm b's best response would be to act as a committed firm facing a noncommitted rival by setting $q_b = q^*$ (see Proposition 2). In such a case, firm a's profits would be exactly the same as if it had ordered the complement $q_a = 1 - q_b$, as prescribed by Proposition 3. Consequently, $\bar{q}_{c-c} < q_a$ is ruled out by the tie-breaking assumption.

Now, within the continuum of equilibria in quantity choices, only one is symmetric where both firms order the same quantity and set the same price, respectively, given by

$$q_{c-c}^* = \frac{1}{2}, \quad \text{and} \quad p_{c-c}^* = 1 - \frac{t}{2}. \tag{5}$$

For ease of exposition, we will restrict attention to this symmetric equilibrium. Note that any other equilibrium will not qualitatively change the results of the final decision of whether or not to commit. Keep in mind that under any other (C-C) equilibrium, one firm achieves higher profits while the other achieves less. This will make the latter more inclined to choose noncommitment.

5. SPNE: To Commit or Not to Commit

So far, we have completed the analysis of all possible subgame equilibria for Stages 2–4, conditional on the commitment choice of Stage 1. To fully characterize the SPNE, we now compare the subgame equilibrium profits that result from all possible commitment combinations. Let π_{n-n}^* (π_{c-c}^*) denote both firms' profits under the symmetric N-N (C-C) combination, and let π_{n-c}^* (π_{c-n}^*) denote the noncommitted (committed) firm's profits under the asymmetric N-C combination. The commitment choice game can now be represented in normal form, as shown in Table 2.

Our analysis reveals that $\pi_{c-n}^* > \pi_{n-n}^*$ always, which rules out N-N as an equilibrium combination. This is not surprising, since if neither firm commits to an order quantity, price competition is most intense. What remains is to compare C-C profits to N-C profits.

[7] Firm b will never order less than $(1-q_a)$, as it is suboptimal to leave any consumers unserved whenever $t \leq 2/3$.

[8] For details regarding the N-C mixed pricing subgame equilibrium, see the discussion that follows Proposition 1.

Table 2 Commitment Choice Game in Normal Form

	Not to commit	To commit
Not to commit	$(\pi^*_{n-n}, \pi^*_{n-n})$	$(\pi^*_{n-c}, \pi^*_{c-n})$
To commit	$(\pi^*_{c-n}, \pi^*_{n-c})$	$(\pi^*_{c-c}, \pi^*_{c-c})$

Since $\pi^*_{c-n} > \pi^*_{n-n}$ always, whenever $\pi^*_{n-c} > \pi^*_{c-c}$, the equilibrium commitment combination is the asymmetric N-C; whenever $\pi^*_{c-c} > \pi^*_{n-c}$, the equilibrium regime combination is the symmetric C-C.

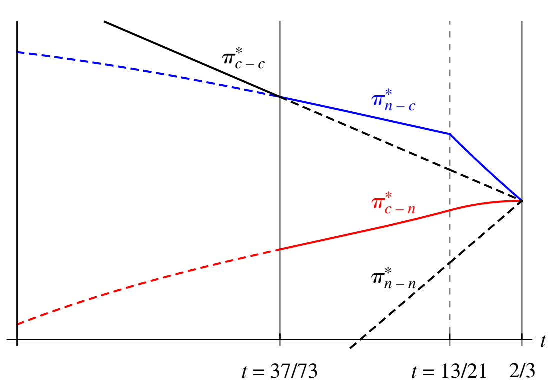
Proposition 4. *The quantity commitment decision SPNE depends on the level of product differentiation, t, as follows*:

1. *If the level of product differentiation is sufficiently low, i.e., $0 \le t < 37/73$, then $\pi^*_{c-c} > \pi^*_{n-c}$, and the SPNE is symmetric such that both firms choose to commit to a quantity.*
2. *If the level of product differentiation is sufficiently high, i.e., $37/73 < t \le 2/3$, then $\pi^*_{n-c} > \pi^*_{c-c}$, and the SPNE is asymmetric such that one firm chooses to commit to a quantity and the other chooses not to commit.*

The profits under the three possible commitment combinations are shown in Figure 4, which graphically illustrates the results of Proposition 4. When product differentiation is low, i.e., $0 \le t < 37/73$, price competition is fierce, and firms avoid it by choosing to commit to quantities. When product differentiation is high, i.e., $37/73 < t \le 2/3$, price competition is less fierce, and one firm will find it optimal not to commit, knowing that its competitor will commit in order to avoid full price competition.

To grasp the intuition behind the equilibrium commitment choices, suppose firm a chooses to commit. If firm b also chooses to commit (see §4.3), both firms will set their order quantities to $\frac{1}{2}$ and achieve a profit of $\pi^*_{c-c} = \frac{1}{4}(2-t)$. Alternatively, if firm b chooses not to commit (see §4.2), firm a will set its order quantity to q^*, and the resulting price equilibrium will be in

Figure 4 Profits Under Different Regime Combinations



Note. SPNE profits are denoted by solid lines, and off-equilibrium profits are denoted by dashed lines.

mixed strategies, in which case firm b achieves its guaranteed profit from covering $(1-q^*)$ consumers, $\pi^*_{n-c} = (1-q^*)(1-t(1-q^*))$. Whether or not firm b chooses to commit, therefore, is determined by the order quantity firm a will choose when facing a noncommitted competitor.

Thus, if firm a commits, firm b will prefer not to commit only if it knows that firm a will respond to such a choice with a sufficiently low order quantity. Specifically, if $q^* < \frac{1}{2}$, firm b prefers not to commit, and vice versa (for details, see the proof of Proposition 4). The intuition is that the lower the quantity firm a orders, the higher the profits firm b can make as it captures the profits of covering the complement of firm a's quantity, $(1-q^*)$, while extracting full surplus from the farthest consumer.

Recall from §4.2 that the committed firm will order $q^* = q^{\text{int}}$, which is seen graphically in Figure 3. Note how q^* decreases in t as the committed firm, firm a, orders fewer units to fend off its competitor, firm b. Therefore the logic behind firm b's regime choice can be summarized as follows: when t is low (high), firm a will respond to firm b's noncommitment with a large (small) quantity; hence, firm b prefers to commit (not to commit).[9]

In summary, when the level of differentiation is sufficiently low ($0 \le t < 37/73$), both firms choose to commit because such a combination completely removes any price competition. If either firm switches to noncommitment, the competitor will respond by ordering a high quantity, rendering the switch unprofitable. When the level of differentiation is high ($37/73 < t \le 2/3$), one firm will find noncommitment optimal, knowing that its competitor will respond with a low enough order quantity making the switch profitable. The other firm, however, does not switch to noncommitment as this will lead to more intense price competition, which in turn results in lower profits for both firms.

6. The Role of Observable Commitment

This section investigates the issue of observable versus nonobservable commitment raised in §3.4. The following examples illustrate two practical scenarios where Stages 1 and 2 are combined (nonobservable commitment) or separate (observable commitment).

Nonobservable commitment: A firm commits to a quantity by way of building a plant with a specific production capacity, thus limiting the quantity of goods it can ultimately sell. For such a firm, Stages 1 and 2 can be combined into a single stage because building the plant and deciding its capacity can be viewed as a single decision.

[9] This can be expressed mathematically as $1/3 < t < 37/73 \Rightarrow 1/2 < q^* \Rightarrow \pi^*_{n-c} < \pi^*_{c-c}$, and $37/73 < t < 2/3 \Rightarrow q^* < 1/2 \Rightarrow \pi^*_{c-c} < \pi^*_{n-c}$.

Observable commitment: A firm selling a seasonal good commits to a quantity by way of offshore contract manufacturing with long lead times. (If lead times are longer than the length of the selling season, then long lead-time production implies commitment.) In Stage 1, the firm makes a strategic choice to rely on a long lead-time production—a decision that requires that the firm locate suitable offshore suppliers—and in Stage 2, the firm sets the quantity. Stages 1 and 2 are separate if locating suitable contract manufacturers requires time. A practical example of this situation is the garment industry (Ferdows et al. 2004, Caro and Martínez-de-Albéniz 2010). A firm such as Gap Inc., in Stage 1, decides to rely on a network of offshore contract manufacturers with lead times that exceed the length of the selling season. This is a decision to commit, which does not automatically entail setting quantities. After establishing such a network, Gap sets a quantity every season by placing orders with its contract manufacturers. Conversely, Zara, in Stage 1, decides to rely on short lead-time production via close-to-market, flexible suppliers. The short lead time allows for fast replenishment during the selling stage and therefore represents a decision not to commit to a quantity.

So far, the SPNE presented in Proposition 4 was derived assuming observable commitment, i.e., keeping Stages 1 and 2 separate. The SPNE derived under observable commitment is also an SPNE when commitment is nonobservable. However, an additional SPNE may arise. Observable commitment can therefore be seen as a coordinating mechanism because it limits the number of equilibria that can arise.

Corollary 1. *If commitment is nonobservable, i.e., Stages* 1 *and* 2 *are combined, then for all* $t \in [0, \frac{2}{3}]$, *there are two possible types of SPNE*:

1. *asymmetric SPNE such that one firm chooses to commit to a quantity and the other chooses not to commit, and*
2. *symmetric SPNE such that both firms choose to commit to a quantity.*

Proof of Corollary 1. The reason behind the existence of two possible types of equilibria when commitment is unobservable can best be summarized in two points:

(i) The best response to a noncommitted rival is to commit and set a quantity at q^*, which rules out symmetric noncommitment (N-N) as an equilibrium and establishes that at least one firm will choose to commit to quantity.

(ii) Whenever a firm commits and sets a quantity *simultaneously*, its rival will be indifferent between commitment and noncommitment. Taken together with (i), this establishes both asymmetric commitment (N-C) and symmetric commitment (C-C) as equilibria.

For (i), suppose firm b chose not to commit and consider firm a's optimal response. If firm a chose not to commit, a fully competitive pricing subgame would follow yielding profits of π^*_{n-n} (see Equation (4)). Alternatively, if firm a chose to commit, it would optimally set quantity at q^* (see Proposition 2), and a mixed pricing subgame equilibrium would follow (see Propositions 1 and 2), yielding profits of π^*_{c-n}. Section 5 establishes that $\pi^*_{n-n} < \pi^*_{c-n}$ (for details, see the proof of Proposition 4). Hence, *the optimal response to a noncommitted rival is to commit and set a quantity to* q^*. This rules out symmetric noncommitment (N-N) as an equilibrium and establishes that at least one firm will choose to commit to quantity.

For (ii), suppose firm a chose to commit to some quantity q_a within the bounds given in Proposition 3 and consider firm b's optimal response. If firm b chooses not to commit, a mixed pricing subgame equilibrium will follow (see Propositions 1 and 2), yielding profits of $\pi_{n-c}(q_a) = (1-q_a)(1-t(1-q_a))$.[10] Alternatively, if firm b chooses to commit, it will optimally set quantity at $(1-q_a)$, and a noncompetitive pricing subgame equilibrium will follow (see Proposition 3), yielding profits of $\pi_{c-c}(q_a) = (1-q_a)(1-t(1-q_a))$. Note that the profits are equal, which establishes that if firm a commits to a given quantity q_a, firm b is indifferent between committing to $(1-q_a)$ and not committing to a quantity. □

This shows that the results under observable commitment are robust under nonobservable commitment in that the SPNE derived under observable commitment is also an SPNE under nonobservable commitment. We also find that an additional SPNE is also possible at every level of product differentiation. Observable commitment can therefore be seen as a coordinating mechanism because it limits the number of equilibria that can arise.[11] Industry profits are higher under C-C than under N-C. When t is low, observable commitment rules out N-C and coordinates the industry to higher combined profits under C-C. Conversely, when t is high, observable commitment rules out C-C and coordinates the industry to lower combined profits under N-C.

7. Conclusion

As discussed in §1, there exist multiple mechanisms that allow firms to either commit to quantity or to circumvent quantity commitment. The decision of

[10] Recall that the mixed pricing equilibrium under N-C results in the noncommitted firm becoming indifferent between setting a high price that captures exactly $(1-q_a)$ consumers and setting a low competitive price that captures more than $(1-q_a)$ consumers.

[11] We are grateful to an anonymous reviewer for suggesting that the model should be rich enough to capture the implications of observable commitment.

whether or not to commit to a quantity is certainly of strategic importance because it affects the competitive nature of an industry. In some industries, we observe asymmetric commitment choices. In the computer industry, for instance, Dell circumvents quantity commitment through its flexible, just-in-time manufacturing regime, whereas most of its competitors do not (Guo and Iyer 2013). In the garment industry, Zara circumvents quantity commitment through slashing lead time via a fast supply chain, whereas most of its competitors (e.g., Gap) do not (e.g., Ferdows et al. 2004, Caro and Martínez-de-Albéniz 2010). (For the most recent developments on quantity commitment and lead-time management—not just in the garment industry—see a survey by den Bosshe et al. 2014, highlighting fast supply chains that can help firms circumvent quantity commitment.)

The objective of this paper is to investigate the incentives of differentiated firms to commit to a specific quantity or to circumvent such commitment. To achieve that objective, we take a more general approach than that taken in previous studies. We analyze a multistage game within a duopoly of differentiated firms à la Hotelling. In the first stage of the game, firms choose whether or not to commit to a quantity; in the second stage, any firm that chose to commit sets its quantity; in the third stage, both firms set prices strategically, regardless of whether or not they committed; and in the final stage, demand is allocated as consumers maximize their utilities.

We find that if product differentiation is sufficiently low, *both* competing firms choose to commit to a specific production quantity in equilibrium. This *symmetric* equilibrium allows both firms to avoid the intense price competition associated with low product differentiation. If the level of differentiation is sufficiently high, the SPNE is *asymmetric* such that one firm chooses to commit while its competitor chooses not to commit. This asymmetric equilibrium in commitment is perhaps our most important and least intuitive result. Under this equilibrium, a pricing equilibrium in pure strategies does not exist, only a pricing equilibrium in mixed strategies does, and the equilibrium prices can result in a committed firm not clearing the entire quantity it chose earlier.

The main results of the paper were derived while assuming that commitment is observable; i.e., a committed firm first observes its rival's commitment decision and then sets a quantity. We extend the analysis to investigate the robustness of the results when commitment is nonobservable, i.e., when a committed firm sets a quantity before observing its rival's commitment decision. We find the results to be robust in that the SPNE derived under observable commitment is also an SPNE under nonobservable commitment. We also find that an additional SPNE is also possible at every level of product differentiation. Observable commitment can therefore be seen as a coordinating mechanism as it limits the number of equilibria that can arise.

Our results are consistent with those of previous studies only when the level of product differentiation is sufficiently low. Specifically, both firms choose to commit to a quantity as reported by Singh and Vives (1984), and choosing a quantity then setting prices at a subsequent stage yields Cournot outcomes as reported by Kreps and Scheinkman (1983). Nonetheless, our results depart sharply from those studies when the level of product differentiation is sufficiently high. Unlike Singh and Vives, we show that when firms that commit to a quantity then set prices strategically, the equilibrium outcome is *asymmetric* such that one firm commits and the other does not. To the best of our knowledge, no previous study has shown that product differentiation can explain such asymmetry. Unlike Kreps and Scheinkman, we show that under this asymmetric equilibrium, quantity commitment followed by price competition does *not* yield Cournot outcomes.

Our findings highlight the effect of the managerial decision of whether or not to commit to a quantity on the competitive nature of an industry. This is especially relevant as more firms decide to adopt flexible manufacturing regimes and/or fast supply chains that can circumvent quantity commitment.

On the research front, we hope that this paper invites others to theoretically and empirically investigate the topic further. For example, we have abstracted away from demand uncertainty. We did so to better understand the effect of product differentiation on commitment decisions by isolating it. How the presence of demand uncertainty affects quantity commitment decisions is a potential question for theoretical research. We also assumed that firms operate with full information, which allows each firm to infer its competitor's optimal commitment and quantity choices. This raises another potential research question: How would the equilibrium change if firms possess some private information that prevents competitors from exactly inferring their quantity decisions? On the empirical side, our model makes different predictions of equilibrium commitment outcomes for different levels of product differentiation, which can be empirically tested across different industries. Also, our model can provide a new framework for empirical structural models that investigate competition within asymmetric industries where some firms commit to a quantity whereas others do not.

Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/mnsc.2015.2213.

Appendix A

Proof of Proposition 1. See Appendix B. □

Proof of Proposition 2. In Stage 2 of the game, firm a's objective function is given by

$$\max_{q_a} \pi_{c-n} = \begin{cases} \pi^{*\text{touch}}_{c-n} = (1-tq_a)q_a & \text{if } 0 \le q_a \le \underline{q}, \\ \pi^{*\text{mix}}_{c-n} = \dfrac{q_a(B-4t\sqrt{A})}{3\sqrt{A}-(3-2q_a)t} & \text{if } \underline{q} < q_a < \bar{q}, \\ \pi^{*\text{comp}}_{c-n} = \dfrac{t}{2} & \text{if } \bar{q} \le q_a \le 1, \end{cases} \tag{A1}$$

where $A = 2(1-q_a)t(1-t(1-q_a))$, $B = t(8(1-q_a) - t(7-16q_a+8q_a^2))$, and $0 \le t \le \frac{2}{3}$. The expressions for $\underline{q}$ and $\bar{q}$ are given in Table 1; $\pi^{*\text{touch}}_{c-n}$, $\pi^{*\text{mix}}_{c-n}$, and $\pi^{*\text{comp}}_{c-n}$ are profits that correspond to a noncompetitive pricing equilibrium that arises from setting a low quantity, q_a; profits that correspond to a mixed pricing strategy equilibrium that arises from setting an intermediate quantity, q_a; and profits that correspond to the competitive price that arises from setting a high quantity, q_a, respectively. (For a derivation of π_{c-n}, see Appendix B.) Also, let

$$q^* \equiv \arg\max_{q_a}\{\pi_{c-n}(q_a)\colon 0 \le q_a \le 1\}.$$

First, if $\pi_{c-n} = \pi^{*\text{comp}}_{c-n} = t/2$, then $q^{*\text{comp}} = \bar{q}$. To see this, note that $t/2$ is not a function of q_a. Using the tie-breaking condition (see §3.3),

$$q^{*\text{comp}} = \arg\max_{q_a}\left\{\frac{t}{2}\colon \bar{q} \le q_a \le 1\right\} = \bar{q}.$$

Second, if $\pi_{c-n} = \pi^{*\text{touch}}_{c-n} = (1-tq_a)q_a$, then $q^{*\text{touch}} = \underline{q}$. To see this, note that

$$q^{*\text{touch}} = \arg\max_{q_a}\{(1-tq_a)q_a\colon 0 \le q_a \le \underline{q}\} = \min\left\{\frac{1}{2t}, \underline{q}\right\}.$$

Hence, for all $0 \le t \le \frac{2}{3}$, we have $\underline{q} \le q^* \le \bar{q}$.

Now, if $13/21 \le t \le \frac{2}{3}$, then $\pi^{*\text{mix}}_{c-n} \ge \pi^{*\text{comp}}_{c-n} = t/2$ and $\partial\pi^{*\text{mix}}_{c-n}/\partial q_a < 0$ (see Lemma 7 in the online appendix). Therefore if $13/21 \le t \le \frac{2}{3}$, then $q^* = \underline{q}$, which proves part (2) of Proposition 2.

If $0 \le t < 13/21$, then $\pi^{*\text{mix}}_{c-n}$ is strictly concave in q_a and achieves a unique maximum at $q_a = q^{\text{int}}$ (see Lemma 1 in the online appendix). Moreover, if

- $q_a = \underline{q}$, then $\pi^{*\text{mix}}_{c-n} = \pi^{*\text{touch}}_{c-n}$ and $\partial\pi^{*\text{mix}}_{c-n}/\partial q_a > 0$ (see Lemma 6 in the online appendix), implying that $\underline{q} < q^* = q^{\text{int}}$ whenever $0 \le t < 13/21$; and
- $q_a = \bar{q}$, then $\pi^{*\text{mix}}_{c-n} = \pi^{*\text{comp}}_{c-n}$ and $\partial\pi^{*\text{mix}}_{c-n}/\partial q_a < 0$ (see Lemma 6 in the online appendix), implying that $q^* = q^{\text{int}} < \bar{q}$ whenever $0 \le t < 13/21$.

Therefore, if $0 \le t < 13/21$, then $\underline{q} < q^* = q^{\text{int}} < \bar{q}$, which proves part (1) of Proposition 2. □

Proof of Proposition 3. To prove that the strategies dictated by Proposition 3 are an equilibrium, we start by focusing on the symmetric equilibrium defined by Equations (5) and show that if both firms follow them, neither will have an incentive to deviate. First, suppose that firm b deviates in the second stage of the game and orders some $q_{\text{low}} < q^*_{c-c} = \frac{1}{2}$. As such, the combined inventory of both firms is below one, and some consumers will be left unserved. The deviating firm's optimal price and profits are $(1-tq_{\text{low}})q_{\text{low}}$ and $(1-tq_{\text{low}})q_{\text{low}} < \pi^*_{c-c}$, respectively.

Next, suppose that firm b deviates in the second stage of the game and orders some $q_{\text{high}} > q^*_{c-c}$. Similar to the N-C combination, the pricing equilibrium will be in mixed strategies, where the deviating firm prices as the noncommitted firm and the nondeviating firm prices as the committed firm. From the results in §4.2 that the deviating firm will randomize its prices, it will always achieve a profit at the pricing stage of $(1-tq^*_{c-c})(1-q^*_{c-c})$, which is the same profit it achieves by not deviating. The same logic applies to all equilibria in the continuum. The boundaries of the continuum, $[1-\bar{q}_{c-c}, \bar{q}_{c-c}]$, are determined by $\bar{q}_{c-c}$, which solves the following equation:

$$\pi_{c-c}(q_i = 1-\bar{q}_{c-c}) = (1-\bar{q}_{c-c})(1-t(1-\bar{q}_{c-c})) = \pi_{c-n}(q_i = q^*), \quad i = a, b,$$

where q^* is as defined in Proposition 2. □

Proof of Proposition 4. First, we rule out N-N as an SPNE by showing that $\pi^*_{c-n} \ge \pi^*_{n-n}$ always. From the proof of Proposition 2, if $0 \le t \le 13/21$, then $\pi^*_{c-n} = \pi^{*\text{mix}}_{c-n}$. Moreover, from the definition of q^*, $\pi^{*\text{mix}}_{c-n}(q = \frac{1}{2}) \le \pi^{*\text{mix}}_{c-n}(q = q^*)$. However, as will be seen, $\pi^{*\text{mix}}_{c-n}(q = \frac{1}{2}) \ge \pi^*_{n-n} = t/2$, which implies that $\pi^{*\text{mix}}_{c-n}(q = q^*) \ge \pi^*_{n-n} = t/2$. To see that $\pi^{*\text{mix}}_{c-n}(q = \frac{1}{2}) \ge t/2$, note that

$$\pi^{*\text{mix}}_{c-n}\left(q = \frac{1}{2}\right) - \frac{t}{2} = \frac{t(2t - 7\sqrt{2}\sqrt{(2-t)t} + 8)}{6\sqrt{2}\sqrt{(2-t)t} - 8t} \ge 0 \quad \forall\, t \in \left[0, \frac{13}{21}\right].$$

If $13/21 \le t \le 2/3$, then $\pi^*_{c-n} = \pi^{*\text{touch}}_{c-n}$ and $q^* = \underline{q}$. Moreover, $\pi^{*\text{touch}}_{c-n}(q = \underline{q}) \ge \pi^*_{n-n} = t/2$. To see that $\pi^{*\text{touch}}_{c-n}(q = \underline{q}) \ge t/2$, note that

$$\pi^{*\text{touch}}_{c-n}(q = \underline{q}) - \frac{t}{2} = \frac{-27t^2 + 30t - 8}{18t} \ge 0 \quad \forall\, t \in \left[\frac{13}{21}, \frac{2}{3}\right].$$

Therefore $\pi^*_{c-n} \ge \pi^*_{n-n}$ for all $t \in [0, \frac{2}{3}]$.

Next, we show that if $0 \le t < 37/73$, then $\pi^*_{n-c} < \pi^*_{c-c}$. Conversely, if $37/73 \le t \le 2/3$, then $\pi^*_{c-c} < \pi^*_{n-c}$. Suppose firm a commits. Firm b will prefer not to commit (N-C) only if it knows that firm a will respond to such a choice with a sufficiently low order quantity. Specifically, if $q^* < \frac{1}{2}$, firm b prefers not to commit, and vice versa. To see this, note that if firm b does not commit in Stage 1, its payoff will be (as a function of q^*, which is firm a's quantity):

$$\pi_{n-c} = (1-q^*)(1-t(1-q^*)).$$

If both firms commit in Stage 1, firm b's payoff will be

$$\pi_{c-c} = \frac{1}{2}\left(1 - \frac{t}{2}\right).$$

For all $0 \le q^* \le \frac{1}{2}$ and $0 \le t \le \frac{2}{3}$, we have $\pi_{c-c} \le \pi_{n-c}$. Conversely, for all $\frac{1}{2} \le q^* \le 1$ and $0 \le t \le \frac{2}{3}$, we have $\pi_{n-c} \le \pi_{c-c}$.

Now, Lemma 5 in the online appendix asserts that if $0 \le t < 37/73$, then $q^* > \frac{1}{2}$. Conversely, if $37/73 < t \le 13/21$, then $q^* < 1/2$. The proof of Proposition 2 is that if $13/21 \le t \le 2/3$, then $q^* = \underline{q} < 1/2$. Taken together, $q^* > 1/2$ whenever $0 \le t < 37/73$ and $q^* < 1/2$ whenever $37/73 < t \le 2/3$. □

Appendix B. Best Response Pricing Functions Under Asymmetric Commitment (N-C)

This appendix presents the analysis of the firms' best response pricing functions when firm a commits to a quantity q_a and firm b does not commit. First, we derive the firms' best response pricing functions conditional on firm a's quantity choice, $q_a \in [\max\{0, 1-1/(2t)\}, 1]$. Then in Corollary B.1, we rule out any quantity choice $q_a < 1-1/(2t)$.

Let (p_a, p_b) be a pair of prices chosen by firms a and b. Any feasible pair (p_a, p_b) will belong to one of four possible regions (subsets of the pricing space $p_a \times p_b$), which are formally defined in (B1) and are seen graphically in Figure B.1:

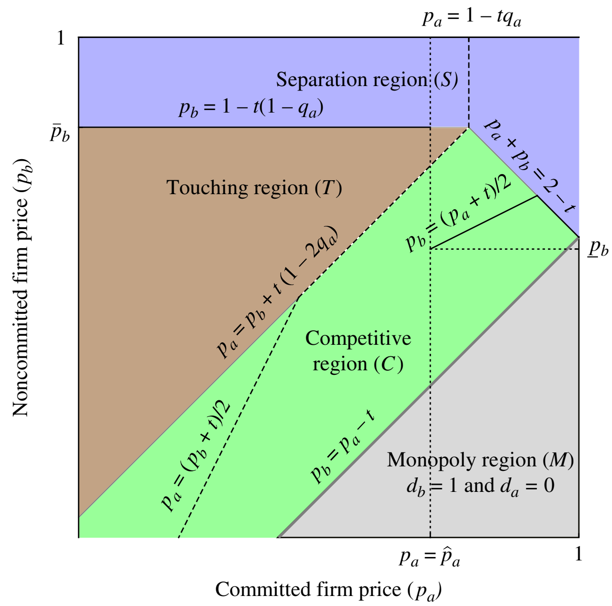
$$\begin{aligned}
&M \equiv \{(p_a, p_b)\colon\ t \le p_a \le 1, 0 \le p_b \le p_a - t\},\\
&C \equiv \{(p_a, p_b)\colon\ 0 \le p_a \le 1, \max\{p_a - t, 0\}\\
&\qquad < p_b \le \min\{p_a - t(1-2q_a), 2 - t - p_a\}\},\\
&T \equiv \{(p_a, p_b)\colon\ 0 \le p_a \le 1 - tq_a,\\
&\qquad p_a - t(1-2q_a) < p_b \le 1 - t(1-q_a)\}, \quad \text{and}\\
&S \equiv \{(p_a, p_b)\colon\ 0 \le p_a \le 1,\\
&\qquad \max\{1 - t(1-q_a), 2 - t - p_a\} < p_b \le 1\}.
\end{aligned} \tag{B1}$$

In the monopoly region, M, firm b's prices are so low that firm a achieves zero demand. In the competitive region, C, both firms price so that $x_{ab} < q_a$, where $x_{ab} = (t + p_b - p_a)/(2t)$ is the location of the consumer who is indifferent between buying product a or product b. This implies that in the region C, firm a does not sell its entire inventory (see consumers' utility in Figure 1(a)). In the touching region, T, p_a is low relative to p_b, which results in $q_a < x_{ab}$ (see consumers' utility in Figure 1(b)). In the separation region, S, firms price such that product b's utility for the consumer located at $x = q_a$ or both products' utility for the consumer located at $x = x_{ab}$ falls below zero. As a result, some consumers in the middle of the line do not buy. The corresponding demands under N-C are therefore as follows:

$$d_a = \begin{cases} 0 & \text{if } (p_a, p_b) \in M,\\ (t - p_a + p_b)/(2t) & \text{if } (p_a, p_b) \in C,\\ q_a & \text{if } (p_a, p_b) \in T,\\ (1-p_a)/t & \text{otherwise;} \end{cases}$$
$$d_b = \begin{cases} 1 & \text{if } (p_a, p_b) \in M,\\ 1 - (t + p_b - p_a)/(2t) & \text{if } (p_a, p_b) \in C,\\ 1 - q_a & \text{if } (p_a, p_b) \in T,\\ (1-p_b)/t & \text{otherwise.} \end{cases} \tag{B2}$$

Substituting the demand functions into Equations (2) and (3) yields the objective functions of firms a and b under N-C. Lemma B.1 below asserts the nonexistence of a pure strategy equilibrium.

Figure B.1 **Best Pricing Response Functions Under Asymmetric Commitment**



Note. $v = 1$ and $c = 0$.

Lemma B.1. *Suppose firm a chooses to commit to a quantity $q_a \in (\underline{q}, \bar{q})$ and firm b chooses not to commit. Then the best response pricing functions of the two firms do not intersect and a pure strategy equilibrium in pricing does not exist.*

Proof of Lemma B.1. The firms' best response function can either intersect inside the competitive region (C) or at the intersection of the boundaries between the competitive (C), separation (S), and touching regions (T) (see Figure B.1). A competitive equilibrium may take place at the intersection of $p_a = \frac{1}{2}(p_b + t)$ and $p_b = \frac{1}{2}(p_a + t)$; i.e., at $(p_a, p_b) = (t, t)$. This intersection point will lie on the firm's best response functions only if $\hat{p}_a \le t$. Note that $\hat{p}_a \le t$ if and only if $\bar{q} \le q_a$. (See Table 1 for expressions.) The noncompetitive equilibrium may occur at $(p_a, p_b) = (1 - tq_a, 1 - t(1-q_a))$. This point will lie on the firm's best response functions only if $1 - t(1-q_a) \le \hat{p}_a$. Also note that $1 - t(1-q_a) \le \hat{p}_a$ if and only if $q_a \le \underline{q}$. Therefore, $\underline{q} < q_a < \bar{q} \Leftrightarrow t < \hat{p}_a < 1 - t(1-q_a)$, and the best response pricing functions do not intersect. □

A good way to understand the pricing equilibrium is to consider the firms' best response functions. The best response pricing function of firm a is represented by the thick dashed line in Figure 1 and that of firm b is represented by the thick solid line. First, consider firm b's best response and its optimal pricing strategy within the competitive and touching regions. If firm b chooses a price within the competitive region, its profit is given by $p_b(t + p_a - p_b)/(2t)$. The FOC yields an optimal competitive price and profit given by

$$p_b^{\text{comp}} = \frac{1}{2}(p_a + t), \quad \text{and} \quad \pi_b^{\text{comp}} = \frac{(p_a + t)^2}{8t}. \tag{B3}$$

If firm b chooses a price within the touching region, its profit is given by $p_b(t + p_a - p_b)/(2t)$. The FOC then yields an optimal competitive price and profit given by $p_b(1-q_a)$, which yields a corner optimal price and profit given by

$$p_b^{\text{touch}} = 1 - t(1-q_a), \quad \text{and} \quad \pi_b^{\text{touch}} = (1-t(1-q_a))(1-q_a). \tag{B4}$$

A comparison between the profits given in Equations (B3) and (B4) reveals that $\pi_b^{\text{comp}} < \pi_b^{\text{touch}} \Leftrightarrow p_a < \hat{p}_a$. This is why we observe the discontinuity in firm b's best response function around $p_a = \hat{p}_a$. Note also that if p_a is too large, $\frac{1}{3}(4-3t) < p_a$, the interior competitive price p_b^{comp} exceeds the boundary between the competitive and separation regions. The best pricing response function of the firm b can be written as

$$p_b^{*n-c} = \begin{cases} p_b^{\text{touch}} = 1 - t(1-q_a) & \text{if } 0 \le p_a < \hat{p}_a, \\ p_b^{\text{comp}} = \frac{1}{2}(p_a + t) & \text{if } \hat{p}_a \le p_a < \frac{1}{3}(4-3t), \\ 2 - p_a - t & \text{otherwise.} \end{cases} \tag{B5}$$

The intuition behind b's best response function is as follows. When the competitor's price is too low, $0 \le p_a < \hat{p}_a$, firm b prefers not to compete and chooses to serve only those consumers that firm a cannot serve because of its inventory constraint $(1-q_a)$. It chooses the touching price, p_b^{touch}, which extracts full surplus from the consumer located at $x = q_a$. Under this scenario, some consumers who prefer to buy product a because of its low price end up buying product b since the former runs out. Whenever $\hat{p}_a \le p_a < \frac{1}{3}(4-3t)$, firm b switches to the competitive price, p_b^{comp}, and both firms compete for the marginal consumer located at $x = x_{ab}$. Under this scenario, each consumer buys the product she prefers, and a's inventory is not exhausted. Whenever, p_a is too high (i.e., $\frac{1}{3}(4-3t) \le p_a$), firm b prices at the border of separation and competitive regions such that all surplus is extracted form the indifferent consumer located at $x = x_{ab}$, and a's inventory is not exhausted.[12]

Following similar steps, we can write the best pricing response function of firm a as

$$p_a^{*c-n} = \begin{cases} \frac{1}{2}(p_b + t) & \text{if } 0 \le p_b < t(4q_a - 1), \\ p_b + t(1-2q_a) & \text{if } t(4q_a-1) \le p_b < 1 - t(1-q_a), \\ 1 - tq_a & \text{otherwise.} \end{cases} \tag{B6}$$

The best response functions in Equations (B5) and (B6) highlight the asymmetry in pricing competition under N-C. Recall that when faced with a low price by its competitor, the noncommitted firm, b, chooses not to compete and prices high (p_b^{touch}) to serve only those consumers that the committed firm cannot reach. By contrast, when the committed firm, a, faces a low price from its competitor, it has to choose a competitive price, $\frac{1}{2}(p_b + t)$, as firm b does not face any demand constraints and can drive it off the market. This scenario is illustrated by the upper prong of Equation (B6).

[12] Economides (1984) refers to this as *touching*. His model, however, does not consider quantity commitment. In our model, touching refers to the scenario under which the committed firm exhausts its entire inventory and the noncommitted firm serves the remaining consumers.

Whenever $t(4q_a - 1) \le p_b < 1 - t(1-q_a)$, the competitive price of firm a will not satisfy its demand constraint as more consumers will prefer a to b. Hence, firm a switches to pricing at the boundary between the competitive and touching regions and chooses $p_a = p_b + t(1-2q_a)$. This price equates $x_{ab} = q_a$ and results in firm a exhausting its inventory. Whenever, p_b is too high $(1 - t(1-q_a) \le p_b)$, firm a sets $p_a = 1 - tq_a$ to exhaust its entire inventory while extracting all surplus from the consumer located at $x = q_a$.

As Lemma B.1 states, the best response functions do not intersect (see Figure B.1) if $\underline{q} \le q_a \le \bar{q}$. If firm a commits to a low order quantity $(q_a < \underline{q})$, firm b will not have an incentive to fully compete, and the best response functions will intersect such that a unique pure strategy equilibrium will exist. If firm a commits to a large order quantity $(\bar{q} < q_a)$, firm b will always have an incentive to fully compete, and the unique pure strategy equilibrium will be similar to the N-N case. We are now ready to prove Proposition 1.

Proof of Proposition 1. For part (2), suppose $\underline{q} < q_a < \bar{q}$. If a plays $p_a = \hat{p}_a$ with probability 1, b will be indifferent between touching and competing, per Equation (B5). Substituting $\hat{p}_a$ for p_a in the top and middle prongs of Equation (B5) yields $\bar{p}_b$ and $\underline{p}_b$, respectively. Either price allows b to capture a profit of $\pi_{n-c} = (1-t(1-q_a))(1-q_a)$. Consider now firm a's optimal price when b randomizes between $\bar{p}_b$ and $\underline{p}_b$ with probabilities ρ and $(1-\rho)$, respectively. Here, a sets its price to solve $\max_{p_a} p_a(\rho q_a + (1-\rho)(t + \underline{p}_b - p_a)/(2t))$. The FOC yields the optimal price of

$$p_{c-n}^* = \frac{1}{2}\left(\sqrt{2(1-q_a)t(1-t(1-q_a))} + t + \frac{2\rho t q_a}{1-\rho}\right).$$

Solving for the probability ρ that makes $p_{c-n}^* = \hat{p}_a$ yields

$$\rho = \frac{3\sqrt{2t(1-q_a)(1-t(1-q_a))} - 3t}{3\sqrt{2t(1-q_a)(1-t(1-q_a))} + t(2q_a - 3)} \quad \Leftrightarrow \quad p_{c-n}^* = \hat{p}_a.$$

For part (1), whenever $\max\{0, 1-1/(2t)\} \le q_a \le \underline{q}$, the best pricing response functions will intersect at $p_a^{*\text{touch}} = 1 - tq_a$ and $p_b^{*\text{touch}} = 1 - t(1-q_a)$ (at the corner of regions T, C, and S). For part (3), whenever $\bar{q} < q_a \le 1$, the best response pricing functions will intersect within the competitive region at $p_a^{*\text{comp}} = p_b^{*\text{comp}} = t$ (the intersection point of $p_a^{\text{comp}} = \frac{1}{2}(p_b + t)$ and $p_b^{\text{comp}} = \frac{1}{2}(p_a + t)$).

We can now write the subgame equilibrium profits of the pricing Stage 3 as follows:

$$\pi_{c-n} = \begin{cases} \pi_{c-n}^{*\text{touch}} = (1 - tq_a)q_a & \text{if } 0 \le q_a \le \underline{q}, \\ \pi_{c-n}^{*\text{mix}} = \dfrac{q_a(B - 4t\sqrt{A})}{3\sqrt{A} - (3-2q_a)t} & \text{if } \underline{q} < q_a < \bar{q}, \\ \pi_{c-n}^{*\text{comp}} = \dfrac{t}{2} & \text{if } \bar{q} \le q \le 1, \end{cases} \tag{B7}$$

where $A = 2(1-q_a)t(1-t(1-q_a))$, $B = t(8(1-q_a) - t(7 - 16q_a + 8q_a{}^2))$, and $0 \le t \le \frac{2}{3}$. □

Finally, we show that under N-C, the committed firm, a, will never set its quantity too low such that $0 \le q_a < 1 - 1/(2t)$ (note that $0 < 1 - 1/(2t)$ only if $1/2 < t$).

Corollary B.1. *Suppose firm a chooses to commit to a quantity, q_a, and firm b chooses not to commit. Firm a will never choose to set its quantity such that $0 \le q_a < 1 - 1/(2t)$.*

Proof of Corollary B.1. The result is established by showing that for all $q_a \in [0, 1-1/(2t))$, firm a's equilibrium profit in the pricing subgame is strictly increasing in q_a. This happens because the noncommitted firm, b, will set its price to capture less than $(1-q_a)$ and leave some consumers unserved. In other words, it will set $p_b > 1 - t(1-q_a)$ and act as a local monopolist, and the pricing equilibrium will occur in the separation region S. In this case, firm b's profits are given by $p_b(1-(1-p_b)/t)$ (see the bottom prong of firm b's demand in Equation (B2)). The FOC yields an interior solution of $p_b^* = \frac{1}{2}$, which results in a demand of $d_b^* = 1/(2t) < 1 - q_a$, $\forall\, q_a \in [0, 1-1/(2t))$. This can be summed up mathematically as follows: $0 \le q_a < 1-1/(2t) \Rightarrow p_b^* = \frac{1}{2} > 1 - t(1-q_a) \Rightarrow d_b^* = 1/(2t) < (1-q_a)$. Since firm b acts as a local monopolist, firm a will set its price at $p_a^* = 1 - tq_a$ to capture q_a consumers and extract full surplus from the consumer located at $x = q_a$. The profit firm a achieves in this case is $(1-tq_a)q_a$, which is strictly increasing in q_a $\forall\, q_a \in [0, 1-1/(2t))$. Therefore, firm a will never set a quantity below $1-1/(2t)$. □

Corollary B.2.

$$\frac{\partial \rho(q)}{\partial q} = \frac{\partial}{\partial q}\left(\frac{3\sqrt{2t(1-q)(1-t(1-q))} - 3t}{3\sqrt{2t(1-q)(1-t(1-q))} + t(2q-3)}\right) < 0 \quad \textit{for all } (q,t) \in \mathscr{H},$$

where

$$\mathscr{H} = \left\{(q,t)\colon 0 \le t \le \frac{13}{21}, \max\left\{0, \frac{3t-1}{3t}\right\} \le q \le \frac{\sqrt{9-18t^2}+6t-3}{6t}\right\}.$$

Proof of Corollary B.2. Since

$$\frac{\partial \rho(q)}{\partial q} = \left(3\sqrt{2}t^2\left(\sqrt{2(1-q)t(1-(1-q)t)} - (2-q(1-2t)-2t)\right)\right) \cdot \left(\sqrt{(1-q)t((q-1)t+1)} \cdot \left((2q-3)t + 3\sqrt{2}\sqrt{(1-q)t((q-1)t+1)}\right)^2\right)^{-1},$$

the sign of $\partial\rho(q)/\partial q$ will be determined by the sign of the numerator, which is negative on $\mathscr{H}$. To see that, note that

$$\begin{aligned} &\sqrt{2(1-q)t(1-(1-q)t)} - (2-q(1-2t)-2t) \le 0 \\ &\Leftrightarrow\ 2(1-q)t(1-(1-q)t) \le (2-q(1-2t)-2t)^2 \\ &\Leftrightarrow\ q^2(6t^2-4t+1) - 2q(6t^2-7t+2) + (6t^2-10t+4) \ge 0. \end{aligned}$$

At $q=0$, $q^2(6t^2-4t+1) - 2q(6t^2-7t+2) + (6t^2-10t+4) = (6t^2-10t+4) > 0$ for all $0 \le t \le \frac{2}{3}$. Moreover, for all $0 \le t \le \frac{2}{3}$, $\nexists\, q \in \mathbb{R}$ for which

$$q^2(6t^2-4t+1) - 2q(6t^2-7t+2) + (6t^2-10t+4) \le 0.$$

This is because the left side of the above inequality *equals* zero for values of q given by

$$q_{1,2} = \frac{(3t-2)(2t-1) + \sqrt{t(3t-2)}}{6t^2-4t+1},$$

which are real only for values of $t > \frac{2}{3}$. □

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