Management Science

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This paper studies the design of group-buying mechanisms in a two-period game where cohorts of consumers arrive at a deal and make sign-up decisions sequentially. A firm can adopt either a sequential mechanism where the firm discloses to second-period arrivals the number of sign-ups accumulated in the first period, or a simultaneous mechanism where the firm does not post the number of first-period sign-ups and hence each cohort of consumers faces uncertainty about another cohort's size and valuations when making sign-up decisions. Our analysis shows that, compared with the simultaneous mechanism, the sequential mechanism leads to higher deal success rates and larger expected consumer surpluses. This result holds for a multiperiod extension and when the firm offers a price discount schedule with multiple breakpoints. Finally, when the firm can manage the sequence of arrivals, it should inform the smaller cohort of consumers first.

1. Introduction

A group-buying mechanism is a scheme designed to help coordinate a group of interested buyers so that they can reach their common purchase goals. In a typical group-buying mechanism, no transaction will take place unless the total number of committed purchases exceeds a specified threshold within a certain time period. Various forms of group-buying mechanisms have been observed on a wide range of occasions and for a long time. One version is street performer protocol (SPP), an agreement between an artist and a group of potential users. The artist does not start the creative work until the potential funders have pledged a required amount of support. Renowned musicians like Beethoven and Mozart used such arrangements to ensure that enough tickets were sold for their concerts. The recent emergence of social media has popularized these funding concepts and generated similar concepts like threshold pledge systems and crowd funding. Websites like Kickstarter.com allow artists, museums, and entrepreneurs to post project proposals and seek funding from interested donors. In another version of the group-buying mechanism, a number of buyers pool their purchases, often through the facilitation of a third party, to obtain a quantity discount from sellers. Online group-buying websites first appeared in the late 1990s, as part of the wave of innovative online market-based mechanisms. Usually the consumers had to make the purchase

commitment through escrow payment systems. Most of the representative group-buying websites that became popular in the late 1990s, including Mercata, Mobshop, and Letsbuyit, either ceased operating or changed their business models a few years later (Kauffman and Wang 2002). Interestingly, despite the failure of these pioneering group-buying sites, a decade later another generation of social buying websites like Groupon and LivingSocial emerged. Led by the market leader, Groupon, these newcomers typically offer "a deal a day" tailored to each local market (Wortham 2009). The market enthusiasm for online group buying peaked when Groupon declined a $6 billion offer from Google (Weiss 2010).

This paper investigates the information management strategy for group-buying mechanisms, specifically, whether or not the sponsor should ask participants to make decisions without knowing the decisions of others. We take the perspective of third-party group-buying platforms like Groupon and Kickstarter and investigate the impact of alternative information management mechanisms on deal success rates. Knowing how to improve the success rates is important because group-buying firms typically earn revenues from successful listings only and not all group-buying deals succeed. For example, from the launch of Kickstarter.com in April 2009 to March 2011, a total of 20,371 projects were proposed. Among them, 7,496 projects attracted enough funds, leading to a success rate of 43% (blog.kickstarter.com).

Zhang and Liu (2012), who studied requests for microloans listed in Prosper.com, reported a success rate of 12.3% among 49,693 listings. Without careful analysis, the firm's decision does not appear straightforward because of the uncertainty about the number of consumer arrivals and their individual valuations. Looking forward, it can be beneficial to post the number of sign-ups if a large cohort of consumers with high individual valuations turn out in the early stage, but it can be detrimental if the first cohort of consumers turn out to be small and have low individual valuations.

To investigate the influence of information management mechanisms on deal success rates, we develop a two-period model where two cohorts of consumers arrive at the deal sequentially. The two-period model is a stylized capture of the fact that earlier arrivals are faced with more uncertainty in the deal's success rate than later arrivals. The firm being studied chooses between a "sequential mechanism" where the firm posts the number of sign-ups at the end of the first period, and a "simultaneous mechanism" where the firm does not post the first-period outcome. Somewhat surprisingly, our analysis shows that the deal's success rate is always higher under the sequential mechanism. To see the reason for this result requires a backward-inductive reasoning, starting with the second period and then moving back to the first period. A sequential mechanism increases the ex ante expected sign-up rates of the second cohort of consumers by eliminating the uncertainty facing them. The increased expected sign-up rates of the second cohort enhance the confidence of the first cohort of consumers, thereby increasing the ex ante expected sign-up rates of the first cohort. This result underscores the importance of modeling and investigating the dynamics of sign-up behavior under group-buying mechanisms. The result also offers a potential explanation for why firms like Groupon and Kickstarter display the updated number of sign-ups along with the minimum number required to unlock the deals.

When implementing the information management mechanism, a firm may control the sequence of consumer arrivals by, for example, contacting different groups of consumers separately. Since the sequential mechnism always leads to a higher success rate over the simultaneous mechanism, it is better off for the firm to have cohorts of consumers arrive sequentially rather than simultaneously. Interestingly, our analysis also finds that the firm should first inform the cohort with a potentially smaller number of consumers, and then the potentially larger cohort of consumers. The reason for this result can again be seen from the backward-inductive reasoning described earlier. When the firm is uncertain about the size of each

cohort, it is better to inform the potentially larger cohort of consumers later because the confidence of consumers arriving earlier can be boosted by a subsequent stronger cohort who does not suffer discounting in belief due to information asymmetry.

To further understand the dominance of sequential mechanisms over simultaneous mechanisms, we extend our model in a number of directions. First, we extend our model to one with more than two periods, where the threshold of group buying is equal to the number of arrivals. Such a model allows a sequential mechanism to have different reporting frequencies. We find that the mechanism with the highest reporting frequency, that is, updating after each period, leads to the highest success rate. Because all of the consumers have to commit to purchases for the deal to be on, the intuition behind the backward induction described earlier still applies when we trace along the sole sign-up trajectory that leads to the success. Second, we extend the single-level scheme to a multilevel pricing schedule where the participants can receive a greater benefit when the total number of sign-ups reaches a higher level of threshold. Our analysis shows that, all else being equal, the sequential mechanism still always yields higher success rates than the simultaneous mechanism. Third, when the firm faces a capacity constraint, the expected number of sign-ups may be lower under the sequential mechanism than that under the simultaneous mechanism. This is because the capacity constraint introduces negative externality as the chance to receive service decreases when more people have signed up to the deal. Knowing that a large number of people have already signed up can simply depress the expected values of a group-buying deal.

1.1. Literature Review
Our paper is related to research on private provision of public goods. Like the group-buying deals, provision of public goods requires successful coordination among the private contributors. In the literature on private provision of public goods, most papers examine the simultaneous mechanism (e.g., Palfrey and Rosenthal 1984, Bagnoli and Lipman 1989, Tabarrok 1998). Work on sequential mechanisms includes Varian (1994), Gächter et al. (2010), and Romano and Yildirim (2001). Varian (1994) shows that a simultaneous decision-making mechanism leads to more supplied public goods than a sequential mechanism where the outcome of early decisions is revealed to those participants who make decisions later. In Varian (1994), one player can free ride on another's contributions under the sequential mechanism. This result was further supported by evidence from lab experiments (Gächter et al. 2010). In contrast, Romano and Yildirim (2001) show that a sequential

mechanism could perform better than a simultaneous mechanism with more general utility functions that include psychological values associated with the warm-glow or snob appeal effect. In their model, an individual may derive a positive value from donating more money than others to support causes. Such psychological values mitigate the incentive to free ride, which is the key driving force in Varian (1994). Our results are similar to Romano and Yildirim (2001), but with different reasons in a different context. Unlike the research on public goods, in our paper free riding on contributions does not exist. A consumer cannot benefit from the deal without signing up with it. Since group-buying arrangements reimburse consumers when a deal is not on, this type of mechanisms belongs to assurance contracts for discrete public goods (Bagnoli and Lipman 1989, Tabarrok 1998). Our paper contributes to this literature by examining the assurance contracts in a dynamic model and, more specifically, analyzing how the sequential order of sign-up decisions can affect the success rate.

Our paper is also related to the small but growing theoretical literature on group-buying mechanisms. In the presence of demand uncertainty, the group-buying mechanism is shown to outperform posted pricing under demand heterogeneity, economies of scale (Anand and Aron 2003), and risk-seeking sellers (Chen et al. 2007). Chen et al. (2010) compare the uniform group-buying price with nonuniform-price group-buying mechanisms. Jing and Xie (2011) explore the role of group buying in facilitating consumer social interaction and show that group buying can dominate other related promotional schemes. Edelman et al. (2010) examine how the vendors can potentially use group buying as a mechanism for price discrimination and advertising. Wu et al. (2013) empirically identify two threshold effects in online group-buying diffusion, induced by the minimum sign-up quantity of a deal. Unlike these papers, we assume the firm has adopted the group-buying mechanism and focus on the deal's success rate under distinct information disclosure schemes.

Our model and analysis bear some similarities with the literature on technology adoption and network externality (e.g., Farrell and Saloner 1985; Katz and Shapiro 1985, 1986). Research on network externality studies the markets such as telecommunications and computer softwares where a user's product valuation increases with the number of other users who adopt the same product. The uncertainty on other users' adoptions and hence the uncertainty on the relative value of new technologies may create inertia against the adoption of new technologies (Farrell and Saloner 1985). Positive network effect exists in our setting because a deal is more likely to be on when more consumers sign up. Thus, our paper contributes

to the network externality literature by examining the unique form of externality effect in the group-buying context. A group-buying deal is on if and only if a minimum number of consumers sign on. Moreover, a group-buying deal can consist of multiple levels with the deal offering a greater value when the number of sign-ups reaching a higher level of threshold. Finally, network externality effect can also be negative when a group-buying deal is offered by a service firm constrained by some service capacity. In this case, it is more difficult for a consumer to secure a space when a larger number of people are interested in signing on to the deal.

2. A Two-Person Model
In this section we develop a two-person and two-period model to study the firm's design of a group-buying mechanism and subsequent consumer responses. In practice, firms may use variations of group-buying formats depending on the specific products or services being promoted. For instance, Kickstarter sets the total amount of demanded dollar commitment as the threshold, and the project sign-up horizon typically can last for several weeks. We base our description of the model on group-buying websites like Groupon; nevertheless, the results also apply to settings like Kickstarter where the individual sign-up amount is not binary.

2.1. An Illustrating Example
We start with a numerical example to illustrate our core results and the driving forces behind. Consider a firm selling a group-buying deal to two consumers. The firm earns a positive profit whenever the deal is on. If *both* consumers sign up to the deal, the deal will be on and each consumer will receive a surplus s (which is the difference between the deal value and deal price). Otherwise, the deal is off and each consumer receives zero surplus. Each consumer has private information regarding her surplus s; that is, the consumer knows the value of her surplus s, but the firm and another consumer only know the probability distribution of s in the market. The distribution follows a four-point discrete distribution as follows:

$$S = \begin{cases} s^H = \$18 & \text{with prob. } \frac{1}{4}, \\ s^M = \$7 & \text{with prob. } \frac{1}{4}, \\ s^L = \$5 & \text{with prob. } \frac{1}{4}, \\ s^0 = \$0 & \text{with prob. } \frac{1}{4}. \end{cases}$$

Finally, a consumer incurs an opportunity cost $c = \$4$ when signing up to a deal. When a consumer faces uncertainty in the deal's success, we assume that the consumer uses the other consumer's sign-up likelihood as the belief in making sign-up decisions. This

approach is consistent with the equilibrium concepts in multiplayer sign-up games that we analyze in §§3 and 4.

The firm chooses from two alternative mechanisms: the simultaneous mechanism under which each consumer signs up separately without knowing the decision of the other, or the sequential mechanism under which one consumer makes a decision first and the decision is revealed to the second consumer. Before analyzing each mechanism, it is useful to provide a benchmark case where two consumers know the values of each other's surplus. Such a full-information case leads to the first-best solution, under which the deal's success rate is equal to $\frac{3}{4} \cdot \frac{3}{4} = \frac{9}{16}$ because the deal will be on unless at least one consumer has a zero surplus. In the first-best solution, whenever both consumers can gain from the deal (i.e., the positive surplus s is above the opportunity cost c), the deal will be on. In other words, two consumers can perfectly coordinate on signing up for the deal.

Under the simultaneous mechanism, when consumers make the sign-up decision, each needs to estimate the sign-up likelihood of the other consumer. Since two consumers are identical, we denote such a belief by q. A consumer will sign up if her expected surplus is no less than the opportunity cost, i.e., $q \cdot s \geq c$. Then the likelihood for each consumer to sign up is $P(S \geq c/q)$, and the likelihood for deal success is $P(S \geq c/q) \cdot P(S \geq c/q)$, where P denotes probability. In the equilibrium, the consumer's sign-up likelihood needs to be consistent with the consumer's original estimate, i.e., $P(S \geq c/q) = q$. This consistency condition yields to the equilibrium sign-up likelihood $q^* = \frac{1}{4}$. To verify it, given $q^* = \frac{1}{4}$, one can see that $q^* \cdot s^H - c = \frac{1}{4} \cdot \$18 - \$4 > 0$ but $q^* \cdot s^M - c = \frac{1}{4} \cdot \$7 - \$4 < 0$ and $q^* \cdot s^i - c < 0$ for $i = L, 0$, because s^L and s^0 are even smaller than s^M. Thus, a consumer will sign up to the deal if and only if the consumer's surplus turns out to be s^H with probability $\frac{1}{4}$. The likelihood that both consumers sign up to make the deal succeed is thus $\frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$. One can further verify that $q^* = \frac{1}{4}$ is the unique consistent estimate under the simultaneous mechanism.[1]

Under the sequential mechanism, one consumer moves before the other. We denote the first mover's estimate of the second consumer's sign-up likelihood by Q. The first mover will sign up if and only if this consumer's expected surplus is no less than the opportunity cost. This implies the first mover's likelihood of signing up as $P(S \geq c/Q)$. Next, knowing that the first mover signs up, the second mover

will sign up as long as her surplus is no less than the opportunity cost, i.e., $s \geq c$, with the probability $P(S \geq c)$. Given the sequential nature of the process, the deal success rate is the product of the likelihood that the first mover signs up and the conditional probability that the second mover signs up given the first mover has already signed up, i.e., $P(S \geq c/Q) \cdot P(S \geq c)$. Under the equilibrium, the first mover's estimate should be consistent with the second consumer's behavior, i.e., $Q^* = P(S \geq c) = 3/4$. With a belief of $Q^* = \frac{3}{4}$ as the second consumer's sign-up probability, the first mover will sign on to the deal when her surplus turns out to be s^H or s^M with a sign-up probability being $\frac{1}{2}$ because $Q^* \cdot s^H - c = \frac{3}{4} \cdot \$18 - \$4 > 0$ and $Q^* \cdot s^M - c = \frac{3}{4} \cdot \$7 - \$4 > 0$ but $Q^* \cdot s^i - c < 0$ for $i = L, 0$. Overall the probability that both consumers will sign up is $\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$.

We now compare the success rates under alternative mechanisms. The above results indicate that the sequential mechanism yields a higher expected success rate than the simultaneous mechanism. First, under the sequential mechanism, given that the first mover has signed up to the deal, the second mover's sign-up probability is $\frac{3}{4}$. This is higher than individual consumer's sign-up probability $\frac{1}{4}$ in the simultaneous mechanism. Clearly the second mover under the sequential mechanism benefits from the certainty. Second, under the sequential mechanism, the first mover's sign-up probability is $\frac{1}{2}$, which is higher than either consumer's sign-up probability $\frac{1}{4}$ in the simultaneous mechanism. Thus, the sequential mechanism, by eliminating the uncertainty facing the second mover, also boosts the confidence of the first mover. Intuitively, the superiority of the sequential mechanism stems from the need for consumers to coordinate their sign-up decisions. Consumers' sign-up decisions are *strategic complements* with a common goal of benefiting from the deal. In the first-best solution under full information, two consumers can perfectly coordinate. Both the firm and two consumers can benefit from the deal as long as neither consumer has a zero surplus. Under the sequential mechanism, the second mover faces certainty as in the full-information case, but the first mover still faces uncertainty regarding the second mover's surplus. As a result, coordination fails when the first mover has $S = s^L = \$5$. Finally, under the simultaneous mechanism, both consumers face the uncertainty that clearly exacerbates the coordination. In this case, even when both consumers have $S = s^M = \$7$, the coordination fails and the deal will be off. Thus, the sequential mechanism provides a second-best solution that leads to a better coordination than the simultaneous mechanism by reducing uncertainty for both consumers. Next we propose analytical models to investigate the differences between simultaneous and sequential group-buying mechanisms. We conduct

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[1] Note that zero is always a consistent belief under the simultaneous mechanism. It is self-fulfilling that the deal will fail if both consumers are convinced that the other consumer will not sign up. However, this result is trivial.

formal equilibrium analysis with belief perturbations. We also study multiple directions of model extensions to identify the potential boundary conditions for our results.[2]

2.2. Model Setup

We consider a market where a firm (either a producer or a broker) uses the group-buying format to promote a product or service to consumers. At the beginning of the first period, the firm posts its group-buying deal, which is characterized by three elements: group-buying price w, minimum number of buyers N required, and a time horizon of two periods for signing up. When firms such as Groupon use group-buying mechanisms for promotion, the firms also post a regular price p that consumers would otherwise have to pay in the market. We assume that the threshold number N and the group-buying price w are exogenously determined. The threshold may be the minimum number at which the product or service provider can enjoy economies of scale and offer the discount. Naturally the group-buying price should be at a discount to the regular price, i.e., $w < p$. Alternatively, a firm may replace the deal price with the discount level in its announcement. For example, a group-buying deal may offer a 50% discount for a lunch buffet at a popular Japanese sushi restaurant, regularly priced at \$20 per person, on condition that the number of buyers surpasses 150 within one day. This is equivalent to a deal price of \$10.

We consider a two-person model as an efficient way to capture the sequential nature of consumer arrivals and the interdependence between purchase decisions of early and late arrivals. The longer the sign-up horizon, the more time discounting the early arrival takes into account in calculating the expected surplus at the end of the horizon. In practice the length of the time horizon can vary from one day to several months. For example, most deals offered by Groupon expire within 24 hours. In the two-person model, we assume that one consumer arrives in the first period, and the other in the second period. Each consumer demands and may purchase up to one unit of the product or service. We limit the threshold N to be 2 in this section, but generalize it in the extensions. The deal succeeds if and only if both consumers sign up for the deal, and they receive the product or service at price w. Otherwise, the deal is off and no transaction takes place. The individual product valuation for consumer i is denoted by V_i, which is drawn from a cumulative distribution function (CDF) $F_i(\cdot)$.

A consumer decides to sign on to the group-buying deal if and only if she expects a discounted utility from the deal at least as high as that of not signing on to the deal. To sign on to the deal is a commitment to purchase if the deal is on. In making the decision, a consumer takes into consideration her own valuation for the deal as well as her expectation about the success rate of the group-buying deal. In the two-person case, one consumer uses the other consumer's sign-up likelihood as the belief in making sign-up decisions, as conditional on signing on to the deal by herself, the deal's success is a direct consequence of the other consumer's sign-up decision. We assume that consumers form rational expectations and focus on the pure strategy equilibria of the game. The use of the rational expectations hypotheses is standard in the literature (Frydman 1982, Jerath et al. 2010). Specifically, we denote by $H_i(q_{-i})$ the likelihood of an individual consumer i's signing on to a group-buying deal, where q_{-i} is consumer $-i$'s sign-up likelihood that consumer i expects. This notation emphasizes that the likelihood of signing up depends on the other consumer's sign-up likelihood, but suppresses its dependence on the characteristics of the deal. We assume the following relation between $H_i(q_{-i})$ and q_{-i}.

ASSUMPTION 1 (SIGN-UP LIKELIHOOD). *For all i and $q_{-i} \in [0, 1]$, we assume*
   (i) $H_i(q_{-i} = 0) = 0$;
   (ii) $H_i(q_{-i})$ is nondecreasing in q_{-i}.

Assumption 1(i) states that if a consumer expects with certainty that the other consumer will not sign up, then she expects zero benefits from signing on to the deal and will definitely not sign up. Assumption 1(ii) indicates that the higher the likelihood that a consumer expects the other consumer to sign up, the higher the probability that the consumer will sign up.

If both consumers can fully communicate with each other before making sign-up decisions, the first-best solution of the deal's success rate can be obtained at $H_1(1) \cdot H_2(1)$. However, a full-information structure is uncommon in reality where typical consumers do not reveal their own private information to strangers in online group-buying settings. Instead, the firm considers two alternative group-buying mechanisms: a sequential mechanism and a simultaneous mechanism. The firm's goal is to achieve a higher success rate. Higher success rates lead to higher expected profits when the firm receives a fixed lump-sum profit for each successful deal and/or a variable profit for each committed purchase given the deal is successful. Given any group-buying mechanism, the game consumers play is in nature a *coordination game*. To achieve a higher success rate, the firm aims at achieving a better coordination between consumers. We interpret the success rate in the following sense.

DEFINITION 1 (SUCCESS RATE). The success rate measures the ex ante likelihood that a group-buying deal is successful before the consumers arrive. The higher

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[2] We thank an anonymous reviewer for offering a similar example to illustrate the key insights on coordination.

the success rate, the higher the expected payoffs for the firm and the higher the expected individual and total surpluses for consumers.

Henceforth it suffices to compare the success rates under the two different mechanisms. The distinction between these two mechanisms is created by the firm's decision on whether to reveal information to the consumer who arrives in the second period; specifically, the second consumer can see the decision of the consumer who arrives in the first period under a sequential mechanism, but not under a simultaneous mechanism. When a firm adopts the simultaneous mechanism, although the second consumer arrives and makes the decision later than the first consumer, she does not gain any information advantage by arriving late. Not disclosing the decision of the first consumer makes the process equivalent to one where consumers simultaneously make decisions under appropriate time discounting adjustment. Thus, we define the simultaneous and sequential mechanisms from an information perspective, not based on the sequence of arrivals. Our approach follows the tradition of Varian (1994). The distinction between simultaneous and sequential mechanisms in this paper is similar in nature to that between sealed and sequential auctions. We assume away observational learning where consumers may draw quality inferences from observation of peer choices, though the sign-up pattern under the sequential mechanism is similar to information cascades (Zhang 2010).

2.3. Micromodeling of Consumer Decisions

To gain granularity on exactly how consumers' purchase decisions may depend on the success rate, we define and examine two distinct types of products or services that fit into our framework: necessity goods and luxury goods. We define necessity goods as those products considered to be essential for a certain consumption purpose. More specifically, if a consumer does not buy from the group-buying deal, she will buy the product elsewhere, for example, from a local store. We define luxury goods as those products or services that consumers do not have to consume. Examples of such luxury goods might be a dinner at an expensive restaurant, an hour of pampering at a fancy spa, or a donation to a music project proposed by a remote museum. Note that we do not follow the standard income-elasticity approach to define necessity versus luxury goods. Our definition is based on whether or not the no consumption is a viable option. We define these two categories for the purpose of illustration, and our results are by no means limited to these special cases.

EXAMPLE 1 (NECESSITY GOODS). Consumers sign up for group-buying deals of necessity goods to obtain

the desired product at a bargain price, but take the risk that the deal may not be successful and they will have to go elsewhere to buy the same product at a regular price and with a delay. For example, a consumer who shops around for a camcorder before a family vacation may want to sign up for a group-buying bargain if time permits, making a trade-off between buying from a local store with the ability to enjoy it right away, versus waiting for the bargain to be realized at a later time with a possibility that the deal may not be successful. In such a case, when a consumer arrives at a group-buying deal, she will sign up if and only if the present value of the expected surplus resulting from signing on with the group-buying deal is no less than the present value of buying the same product immediately through another channel. The expected surplus from signing up for a group-buying deal has two components: if the deal turns out to be successful, the consumer enjoys the product at the bargain price w; otherwise, the consumer purchases the product elsewhere at a later time after the sign-up period ends. For any consumer i holding a belief of the other consumer's sign-up likelihood being q_{-i} and an individual product valuation v_i, the mathematical form of this trade-off statement is to sign on if and only if $v_i \geq w$ and $[q_{-i}(v_i - w) + (1 - q_{-i})(v_i - p)]\rho_i \geq (v_i - p)$, where ρ_i is a discounting factor used by consumer i in calculating the value of signing on to the deal. The discounting factor can incorporate time discounting, since the outcome of a success resolves at a latter time. From the above condition, we can derive the range of consumer i's valuation within which she will sign up for the group-buying deal as follows: $w \leq v_i \leq p + (\rho_i/(1 - \rho_i))(p - w)q_{-i}$. If consumer i has a product valuation higher than $p + (\rho_i/(1 - \rho_i)) \cdot (p - w)q_{-i}$, then the consumer will resort to an immediate access to the product through the alternative channel instead of going after the group-buying deal. Let consumer i have the valuation v_i drawn from a continuous distribution with CDF $F_i(\cdot)$, then we can derive the probability of consumer i signing up for the group-buying deal as follows: $H_i(q_{-i}) = F_i[p + (\rho_i/(1 - \rho_i))(p - w)q_{-i}] - F_i(w)$, $q_{-i} \in (0, 1]$, which is nondecreasing in q_{-i}. We see that the consumer belief plays an important role in sign-up decisions.

From the seller's perspective, $H_i(q_{-i})$ is the sign-up likelihood for an individual consumer i. The market for necessity goods tends to be very competitive because retailers as alternative purchase channels may have already enjoyed economies of scale. As a result, the benefit of group-buying deals over purchasing from a retailer as measured by price discount $p - w$ is likely to be limited, and therefore the individual sign-up likelihood can be low. In §2.8, we show that the expected low individual sign-up likelihood

may reduce other consumers' confidence in the deal's success, eventually leading to a low success rate of group buying for necessity goods.

EXAMPLE 2 (LUXURY GOODS). Consumers sign up for group-buying deals of luxury goods to obtain more valuable ways to spend their discretionary income. Specifically, we assume that any consumer i will buy luxury goods if and only if the present value of the expected surplus from signing on to a group-buying deal is not less than a predetermined threshold a_i, which is the surplus of her no-purchase option. In practice, the luxury goods sold through group-buying sites are usually the try-it-for-the-first-time products or services. The expected surplus of signing up has two components: if the deal turns out to be successful, the consumer enjoys the product at price w; otherwise, the consumer takes the no-purchase option at a later time after the deal turns off. The sign-up decision for consumer i for this type of product can be characterized as $[q_{-i}(v_i - w) + (1 - q_{-i})a_i]\rho_i \geq a_i$. Similar to the case of necessity goods, ρ_i is a broadly defined discounting factor for consumer i used in calculating the value of signing on to the deal. In the case of buying luxury goods through group-buying discounts, the regular price of this type of product or service is usually higher than the valuation of consumers. In other words, for the target consumer i, we have $v_i - p < a_i$, which indicates that the consumer would not purchase the luxury goods at the regular price. We can then derive the range of the consumer's valuation of signing up for the group-buying deal as $a_i[1 - (1 - q_{-i})\rho_i]/(q_{-i} \cdot \rho_i) + w \leq v_i < p + a_i$. Given the distribution of valuations, $F_i(\cdot)$, for consumer i, we can specify the consumer i's sign-up likelihood function for luxury goods as $H_i(q_{-i}) = F_i(p + a_i) - F_i(a_i[1 - (1 - q_{-i})\rho_i]/(q_{-i} \cdot \rho_i) + w)$, $q_{-i} \in (0, 1]$, which is nondecreasing in q_{-i}. Note that $a_i[1 - (1 - q_{-i})\rho_i] \geq 0$ is the loss of surplus from signing up to a deal that turns out to be off at the end. The larger such a loss, the less likely the consumer will sign on to the deal.

2.4. Equilibrium Analysis Under Simultaneous Mechanism

When the firm adopts the simultaneous mechanism, neither consumer is informed of the sign-up decision of the other. As a result, each consumer bases her sign-up decision on her belief in the valuation of the other one. The game is a Bayesian game in the Harsanyi sense (Harsanyi 1968) where "types" are defined by valuations. Specifically, the realized type of consumer i is defined by its realized valuation v_i, which is private information known to the consumer herself but not to the other consumer. Similarly, we denote by V_i the corresponding random variable of

consumer i's types, whose distribution is public information. For any consumer i, $s_i(v_i) \in \{0, 1\}$ denotes the decision rule that consumer i takes to decide whether or not to sign up, given her realized type being v_i. Given consumer i's sign-up decision $s_i(v_i)$ and the other consumer $-i$'s sign-up decision $s_{-i}(v_{-i})$, the surplus of consumer i is denoted as $\pi_i(s_i(v_i), s_{-i}(v_{-i}))$.

DEFINITION 2 (BAYESIAN EQUILIBRIUM). The Bayesian equilibrium strategy $\{s_i^*(v_i)\}$ for the simultaneous game is defined by the best-response strategy played by each consumer i, $s_i^*(v_i) \in \arg\max_{s_i(v_i) \in \{0, 1\}} E_{V_{-i}}\{\pi_i(s_i(v_i), s_{-i}^*(V_{-i}))\}$ for all v_i.

The existence of Bayesian equilibria follows standard arguments (Tabarrok 1998). When consumers make sign-up decisions under the simultaneous mechanism, the consumer who arrives in the second period faces the same uncertainty in the sign-up probability of the other consumer as the one arrives in the first period. The solution scheme for this type of game is a threshold strategy: there exists a valuation range for consumer i such that the consumer signs up if and only if her valuation falls into such a range. At the beginning of the game, the ex ante belief of the firm on the success rate under a Bayesian equilibrium strategy $\{s_i^*(v_i)\}$ can be characterized by $q^* = P(\sum_{i=1}^2 s_i^*(V_i) = 2)$, which is the success rate at equilibrium. Since the revenue of a group-buying site depends on the success of deals, from this point on we compare different mechanisms by the seller's expected deal success rate.

Denote q_i the belief of consumer i's sign-up likelihood held by consumer $-i$. As a result, consumer 1 signs up with probability $H_1(q_2)$, and consumer 2 signs up with probability $H_2(q_1)$. Equilibrium is characterized by a pair of beliefs (q_1^*, q_2^*) that satisfies the following conditions:

$$H_1(q_2^*) = q_1^*, \quad H_2(q_1^*) = q_2^*.$$

Or equivalently, q_i^* is given by

$$q = H_i(H_{-i}(q)), \quad i = 1, 2.$$

The above equations characterize equilibria in the sense that any Bayesian equilibrium results in a pair of beliefs (q_1^*, q_2^*) that satisfies the equations, and any pair of beliefs (q_1^*, q_2^*) that satisfies the equations corresponds to a Bayesian equilibrium where consumer i behaves as if consumer $-i$ signs up for the deal with probability q_{-i}^*. The existence of such a pair of beliefs (q_1^*, q_2^*), and equivalently, the existence of a Bayesian equilibrium in pure strategies, is a direct consequence of Tarski's fixed point theorem, since $H_i(H_{-i}(q))$ is nondecreasing in $q \in [0, 1]$, $i = 1, 2$.

Following the preceding discussion, the success rate q^* at equilibrium can be characterized by

$$q^* = P(X_1(q_2^*) + X_2(q_1^*) = 2) = H_1(q_2^*) \cdot H_2(q_1^*), \quad (1)$$

where $X_i(q)$ is a Bernoulli random variable with success probability $H_i(q)$, $i = 1, 2$.

Note that q_i^* is given by $q = H_i(H_{-i}(q))$, $i = 1, 2$, so there may exist multiple equilibria. Similar to Jackson and Yariv (2007), we categorize equilibria into two types, stable equilibria and tipping points, depending on their sensitivity to minor perturbation in belief. Next we provide a formal definition and include an illustration in Figure 1, where (A_1, A_2) is a tipping point, and (B_1, B_2) and (B_1', B_2') are stable equilibria.[3]

DEFINITION 3 (STABLE EQUILIBRIUM AND TIPPING POINT). A pair of beliefs (q_1^*, q_2^*) constitutes a stable equilibrium (tipping point) if for all $i = 1, 2$, there exists $\epsilon' > 0$ such that for all $\epsilon \in (0, \epsilon')$, when consumer i expects that the other consumer makes her decision with the belief $q_i^* - \epsilon$, consumer i's sign-up likelihood will be higher (lower) than $q_i^* - \epsilon$; when consumer i expects that the other consumer makes her decision with the belief $q_i^* + \epsilon$, consumer i's sign-up likelihood will be lower (higher) than $q^* + \epsilon$.

2.5. Equilibrium Analysis Under Sequential Mechanism

Under the sequential mechanism, at the beginning of the second period the firm posts the decision of the consumer who arrives in the first period. Because two consumers make decisions sequentially, the first consumer needs to make a prediction on the sign-up probability of the second consumer. The sequential game we analyze follows the concept of rational expectations (RE) equilibrium.

DEFINITION 4 (RE EQUILIBRIUM). For any realization v_i of the valuation for consumer i who moves first, an RE equilibrium $q_{-i}^*(v_i)$ in the sequential game satisfies: (i) Consumer i plays an optimal strategy of whether to sign up $S_i^*(v_i) \in \{0, 1\}$, given belief q_{-i} about the sign-up likelihood of consumer $-i$. (ii) Given the decision $S_i^*(v_i)$ from consumer i and any realization v_{-i} of the valuation of consumer $-i$. consumer $-i$ plays a best-response strategy $S_{-i}^*(v_{-i}; S_i^*(v_i)) \in \{0, 1\}$. (iii) The belief is consistent with the sign-up likelihood of consumer $-i$: $q_{-i} = q_{-i}^*(v_i) = P(S_{-i}^*(V_{-i}; S_i^*(v_i)) = 1)$.

Suppose consumer i arrives in the first period and consumer $-i$ arrives in the second period. When the

second consumer makes her decision, there is no more uncertainty about the future. Given the decision of the first consumer, the optimal strategy for the second consumer can be characterized by a valuation range: to sign up if and only if (a) the first consumer signs up and (b) her own valuation falls into the range for signing up with the likelihood of the first consumer's sign-up being 1. After solving the best response from the second consumer, we move backward to the first consumer. Suppose consumer i expects that consumer $-i$ will sign up with probability q_{-i}. Then consumer i signs up with probability $H_i(q_{-i})$ in the first period. At equilibrium q_{-i}^* is consistent with consumer $-i$'s sign-up likelihood, i.e., $q_{-i}^* = H_{-i}(1)$.

From the seller's perspective, the success rate Q_i^* at equilibrium can be characterized by

$$Q_i^* = P(X_i(q_{-i}^*) + X_{-i}(1) = 2) = H_i(q_{-i}^*) \cdot H_{-i}(1), \quad (2)$$

where $X_i(q)$, $X_{-i}(1)$ are Bernoulli random variables with success probability $H_i(q)$ and $H_{-i}(1)$, respectively. It is noteworthy that the equilibrium under the sequential mechanism is guaranteed to be unique, and hence it is stable.

## 2.6. Mechanism Design: Simultaneous or Sequential?

Given the potential presence of multiple equilibria under the simultaneous mechanism, how can one compare the success rates under simultaneous and sequential mechanisms? In this paper, we adopt the approach proposed in Jackson and Yariv (2007) because it allows us to compare the set of equilibria regardless of equilibrium multiplicity. First, we formalize our criteria for comparisons of each consumer's belief regarding the other consumer's sign-up likelihood by the following definition.

DEFINITION 5 (HIGHER BELIEFS). For each individual consumer, one scenario or mechanism generates a higher belief than another if, for any belief at a stable equilibrium of the latter, there exists a higher belief at a stable equilibrium of the former, and for any belief at a tipping point of the latter there is a lower belief at a tipping point of the former or no lower tipping points of the former at all.

We illustrate the comparison criteria defined above in Figure 1. In the figure, given all others the same, the solid line and the dashed line correspond to the right-hand side of the equilibrium characterization equations under the simultaneous mechanism, and the sequential mechanism, respectively. Equilibria are the intersections of these lines with the diagonals. Figure 1(a) illustrates the belief regarding consumer 2's sign-up likelihood held by consumer 1 who moves first under the sequential mechanism, and

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[3] In Figure 1, $(0, 0)$ is also an equilibrium under the simultaneous mechanism. It is self-fulfilling that the deal will fail if everyone is convinced that the other consumer will not sign up. However, this result is trivial and thus is not examined in this paper.

Figure 1   **Comparison Between Simultaneous and Sequential Mechanisms in the Two-Person Model**

(a) The first consumer

(b) The second consumer
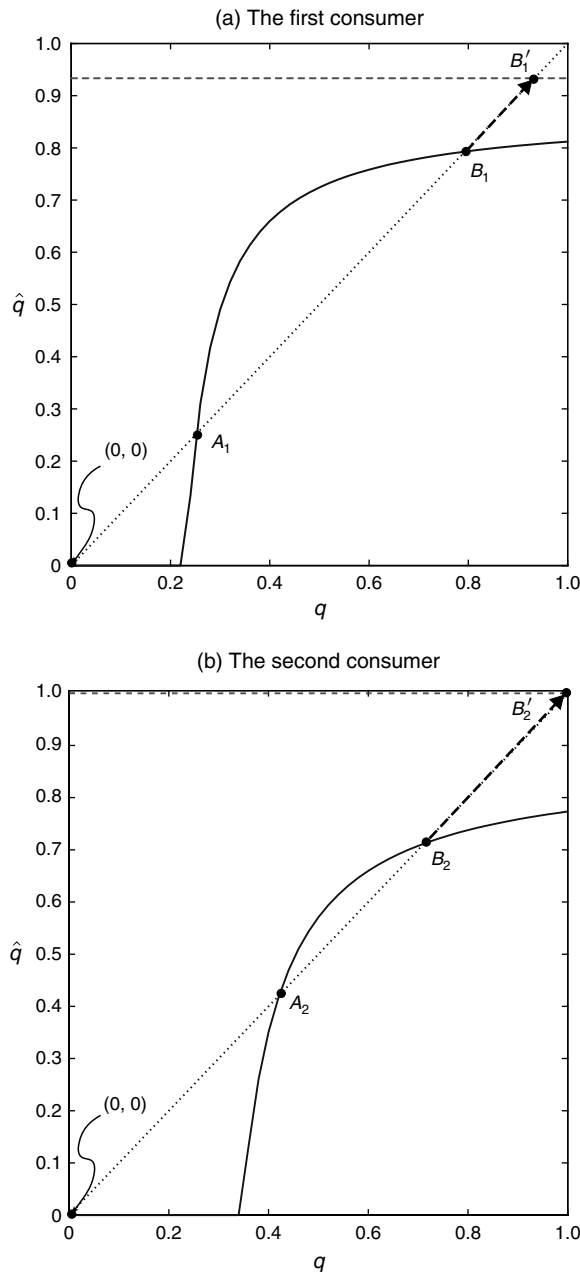
Notes. The parameters are specified as follows: V_i, $i = 1, 2$, follows a uniform distribution with support [15, 30], and $p = 30$, $w = 10$, $\rho_1 = 0.4$, $\rho_2 = 0.5$, and $a = 3$. In this numerical example, (0, 0) is a trivial equilibrium, (A_1, A_2) is a tipping point, and (B_1, B_2) is a stable equilibrium under the simultaneous mechanism; (B_1', B_2') is a stable equilibrium under the sequential mechanism.

Figure 1(b) illustrates the belief of consumer 2. The figure shows that the simultaneous mechanism yields a lower belief B_1 (B_2) at the stable equilibrium than the stable equilibrium B_1' (B_2') under the sequential mechanism for consumer 1 (consumer 2). Moreover, there exists a tipping point (A_1, A_2) under the simultaneous

mechanism, while there is no tipping point under the sequential mechanism.

The reason why the sequential mechanism yields higher beliefs for both consumers is twofold. First, with no tipping point, it is more likely for consumers' expectations to move upward to the beliefs at the stable equilibrium. Second, when the stable equilibrium is reached, the beliefs regarding the other consumer's sign-up likelihood under the sequential mechanism is higher than that under the simultaneous mechanism for both consumers. If both consumers have higher expectations at equilibrium, then each individual is more likely to sign up, and thus the deal is more likely to succeed. Consequently, we can compare the deal's success rate by comparing the belief held by each individual consumer.

Definition 6 (Higher Success Rates). One scenario or mechanism generates a higher success rate than another if the belief of each consumer in the former is higher than the belief of the consumer in the latter in the sense of Definition 5.

To formally compare the belief held by each individual under alternative mechanisms, consider $f(q)$ and $g(q)$ as two functions parameterized by $q \in [0, 1]$. Suppose $f(q) \geq g(q)$ for any $q \in [0, 1]$. That is, $f(q)$ is always higher than $g(q)$ pointwisely for any belief $q \in [0, 1]$. As illustrated in Figure 1(a), for any left-to-right crossing point (e.g., point B_1) of curve $g(q)$ with the 45 degree line, there always exists a higher crossing point (e.g., point B_1') of curve $f(q)$; for any right-to-left crossing point (e.g., point A_1) of curve $g(q)$, there exists no lower crossing point of curve $f(q)$. Consequently, if $f(q) \geq g(q)$ for all $q \in [0, 1]$, we can conclude that mechanism with equilibrium characterization $f(q) = q$ yields higher belief than mechanism with equilibrium characterization $g(q) = q$.

Recall that when consumer i arrives first and consumer $-i$ arrives later, the belief held by consumer $-i$, q_i^*, under the simultaneous mechanism is given by $q = H_i(H_{-i}(q))$, and the belief held by consumer $-i$ under the sequential mechanism is simply 1. As $1 \geq H_i(H_{-i}(q))$ for any $q \in [0, 1]$, the sequential mechanism yields a higher belief than the simultaneous mechanism for consumer i. Similarly, for the first mover consumer i, as $H_{-i}(1) \geq H_{-i}(H_i(q))$ for any $q \in [0, 1]$, the sequential mechanism yields higher belief than the simultaneous mechanism for consumer i as well. Given the preceding discussions, we have the following proposition.

Proposition 1 (Mechanism Comparison for the Two-Person Game). *Given all others the same, the sequential mechanism always yields higher success rates than the simultaneous mechanism.*

The driving force behind this result is that in the simultaneous mechanism, each consumer faces uncertainty about the other consumer when making

decisions. However, in the sequential mechanism, the second consumer decides after the uncertainty about the first consumer has been resolved. Moreover, in anticipation that the second consumer will sign up for the deal without discounting belief in the sign-up likelihood, the first consumer's confidence about the second consumer's sign-up likelihood is consequently boosted. This result may explain why the prevalent practice of online group buying nowadays is sequential mechanisms.[4]

2.7. Endogenous Sequencing

So far we have assumed that the sequence of arrivals of two consumers is exogenously determined. However, in some cases group-buying firms can manage the sequence of arrivals of different cohorts by, for example, contacting one cohort ahead of another. In such cases, a natural question is which sequence of arrivals will yield the higher success rate? To investigate this problem, we add a stage 0 before our current two-person game. In stage 0, two consumers simultaneously decide between making the sign-up decisions first or second. If both consumers decide to move first or move later, then the simultaneous mechanism applies. If one decides to move first and the other decides to move second, then the sequential mechanism applies and the game plays out according to the determined sequence. When making these decisions, the uncertainties regarding consumer valuations and preferences have not been resolved yet. Finally, we assume that the individual consumer's sign-up likelihood function $H_i(q_{-i})$ is independent of decision time. In other words, the time discounting does not play a significant role as in today's a-deal-a-day practice.

We summarize the payoff matrix at stage 0 in Table 1. Given the nondecreasing one-to-one correspondence between the success rate and the payoffs facing consumers and the firm, we can use the success rate as a proxy for the payoff and compare success rates in the sense of Definition 6. As an immediate result of Proposition 1, we can identify the equilibria of stage 0 game.

PROPOSITION 2 (THE TWO-PERSON GAME). *There exist two equilibria (Q_1^*, Q_1^*) and (Q_2^*, Q_2^*), both of which correspond to sequential mechanisms.*

[4] Dominance of the sequential mechanism over the simultaneous mechanism can be strengthened if the late consumer can learn about the value of deal from the sign-up decision of the first person. In such a case, if the first person does not sign on, then the deal is off and any information disclosure from the firm will not affect the outcome. However, if the first person signs up, under the sequential mechanism, the second person may increase the perceived value of deal and thus the group-buying deal is more likely to succeed. Because such a positive learning effect does not exist under the simultaneous mechanism, overall the benefit of the sequential mechanism will be even stronger.

Table 1   The Payoff Matrix

	Consumer 2 selects leader	Consumer 2 selects follower
Consumer 1 selects leader	(q^*, q^*)	(Q_1^*, Q_1^*)
Consumer 1 selects follower	(Q_2^*, Q_2^*)	(q^*, q^*)

If the firm can manage the sequence of arrivals, the firm will compare Q_1^* and Q_2^*, and investigate the conditions under which one of the equilibria leads to higher success rates. Recall that $Q_i^* = H_i(H_{-i}(1)) \cdot H_{-i}(1)$, $i = 1, 2$. Following Assumption 1, we identify the following condition under which success rates are higher when consumer $-i$ moves first than the other way around. (See the proof in Appendix B.)

PROPOSITION 3 (DISCOUNTING AND ENDOGENOUS SEQUENCING). *If $H_i(1) = H_{-i}(1)$ and $H_i(q) \leq H_{-i}(q)$ for all $q \in [0, 1)$, then the scenario where consumer $-i$ acts as the leader has higher success rates than the scenario where consumer i acts as the leader.*

The condition $H_i(1) = H_{-i}(1)$ says that if the other consumer is expected for sure to sign up, both consumers share the same likelihood of signing up. The condition $H_i(q) \leq H_{-i}(q)$, $q \in [0, 1)$ indicates that given everything else being identical, consumer $-i$ is less averse to the possibility that the other consumer may not sign up. In the context of necessity goods (Example 1) and luxury goods (Example 2), one sufficient condition is that the two consumers have the same valuation distribution, i.e., $V_i =_{st} V_{-i}$, but consumer i discounts time to a larger extent than consumer $-i$, i.e., $\rho_i \leq \rho_{-i}$. The proposition then predicts that the consumer who discounts less is preferred by all to move first, given that all the other characters are identical between two consumers. In other words, if consumer valuations have the identical distribution, the weaker consumer in the sense of discounting more is preferred by all as the information free riders in the sequential coordination game. This is because if one of the two consumers has the chance to decide later when uncertainty regarding the other is resolved, it is mutually beneficial to have the consumer who discounts more to enjoy such a luxury to minimize the chances of both walking away from the deal due to uncertainty discounting.

2.8. Comparative Statics for Sequential Mechanism

In keeping with the prevalent adoption of sequential mechanisms in the current group-buying sites, we have shown that the sequential mechanism always yields higher success rates than the simultaneous mechanism. But how does the success rate vary across different types of products? Which products

are more suitable for the group-buying selling format? To explore these issues, we conduct comparative statics analysis on the success rate under the sequential mechanism. To derive specific insights, we resort to the necessity and luxury goods defined earlier in Examples 1 and 2. Next we discuss the results, leaving formal statements and proofs in Appendix A.

Our analysis shows that, first, the success rate is higher if the products are of less immediate need. Luxury goods are more suitable for group buying because consumers are more tolerant of waiting for the goods not considered to be necessities. A shorter deal duration can also help boost the success rate because the potential payoff from the deal is less discounted from waiting. This may partially explain why we observe lower success rates in the earlier wave of group-buying websites a decade ago, whose deals usually lasted for one to two months (Tang 2008). Because the consumers were reluctant to sign on due to a long waiting period, the success rate would remain low in spite of the long duration for signing up. In contrast, today's successful firms like Groupon usually offer deals that expire within 24 hours. Second, a group-buying deal is more likely to succeed when the deal offers a deeper discount. For both types of goods, the number of consumers who may potentially sign up to a deal increases with the size of the deal discount. In general, the discount $p - w$ tends to be small for necessity goods such as digital cameras because the retail market is already very competitive. It is hard for group-buying websites to offer a lower price than what is available at retail giants like Walmart or Best Buy that have already enjoyed buying power and economies of scale. In contrast, luxury local services like spa and leisure activities are more differentiated and associated with low variable costs. As a result, the discounts offered by group-buying sites tend to be larger. The typical discount level offered by current group-buying firms, like Groupon and LivingSocial, is 50% off regular prices for service products. Combining the above results, we conclude that necessity goods are less suitable than luxury goods for the group-buying format. These findings may explain the stark contrast between the fate of the first-generation group-buying sites and the enormous success enjoyed by their counterparts in the current generation. This view is echoed in a recent speech by Groupon's CEO Andrew Mason:

> I thought about why collective buying sites had failed in the past. ...Mercata was from the dot-com era. The more people bought, the lower the price would go. The trouble was it took a week to get enough people to drive the price down. They might buy a camera, but they'd have to wait a week.   (Cutler 2010)

3. A Multiperiod Extension

So far we have used a two-person model to demonstrate that a sequential mechanism leads to higher expected success rates than a simultaneous mechanism. In reality a group-buying deal faces a large number of potential consumers arriving at the deal in sequence. To implement a sequential mechanism, the firm could periodically post the cumulative number of sign-ups, with the reporting frequency ranging from never updating to updating after every period. In this section, we extend our base model to a multiperiod model and show that it is optimal for the firm to update as frequently as possible.

Consider an N-period model where, similar to the two-person game, exactly one consumer will arrive during each period and the preset threshold is equal to N; that is, everyone has to sign up for the deal to be on. The firm can choose to release the cumulative number of sign-ups after every τ periods, where $\tau \geq 1$. In one extreme case, the firm can update the cumulative number of sign-ups after every period, i.e., $\tau = 1$. In another extreme case, the firm follows the simultaneous mechanism where $\tau = N$.

We let V_i, $i = N, N - 1, \ldots, 1$, denote the valuation of the $(N + 1 - i)$th arrival, which is drawn from a given CDF $F_i(\cdot)$. We let $H_i(q)$ denote the likelihood of the $(N + 1 - i)$th arrival to sign up for the deal as a function of her belief q on the likelihood that all remaining consumers sign up. Whereas the valuation of each consumer is private information, sign-up likelihood functions are public information. Under the information updating scheme with frequency τ, every τ consumer can be viewed as a cohort because these consumers, although arriving and making decisions sequentially, do not know the valuations or sign-up decisions of the other consumers within the same cohort. Consequently, within each cohort, to any individual, there exists uncertainty on the valuations of the other consumers, and the consumers behave as if the firm implements the simultaneous mechanism within each cohort. Effectively the N people are divided into a number of cohorts, indexed by $j = 1, 2, \ldots, \lceil N/\tau \rceil$, where $\lceil x \rceil$ is the smallest integer greater than or equal to x. Given that consumers in all previous cohorts sign up, let q_j^τ denote the deal's success likelihood before j more cohorts arrive, namely, the likelihood that all consumers in the remaining cohorts sign up. Again given that consumers in all previous cohorts sign up, denote $q_{j,i}^\tau$, the belief of deal's success rate held by consumer i, $i = (j - 1)\tau + 1, \ldots, \min\{j\tau, N\}$ within cohort $\lceil N/\tau + 1 - j \rceil$. Note that $q_{j,i}^\tau$ is derived from the sign-up likelihoods of consumers within the same cohort, and those consumers from later cohorts. Since the threshold is equal to the number of arrivals, all consumers need to sign up for the deal to be on. Hence, conditional on that

all consumers in previous cohorts sign up, the deal's success likelihood before j more cohorts arrive can be characterized by

$$q_j^\tau = \prod_{i=(j-1)\tau+1}^{\min\{j\tau, N\}} H_i(q_{j,i}^\tau) \cdot q_{j-1}^\tau, \quad j = 2, \ldots, \lceil N/\tau \rceil, \quad (3)$$

$$q_1^\tau = \begin{cases} \prod_{i=1}^\tau H_i(q_{1,i}^\tau), & \tau \geq 2; \\ H_1(1), & \tau = 1. \end{cases} \quad (4)$$

The belief held by each consumer at equilibrium should be self-enforced by other consumers' behavior. That is, $q_{j,i}^\tau = q_{j-1}^\tau \cdot \prod_{l=(j-1)\tau+1, l\neq i}^{\min\{j\tau, N\}} H_l(q_{j,l}^\tau)$, $j \geq 2$, $i = (j-1)\tau+1, \ldots, \min\{j\tau, N\}$, and $q_{1,i}^\tau = \prod_{l=1, l\neq i}^\tau H_l(q_{1,l}^\tau)$, $i = 1, \ldots, \tau$. For the firm, the ex ante success rate of a group-buying deal with threshold N before the consumers arrive is $q_{\lceil N/\tau \rceil}^\tau$, which can be computed recursively by Equation (3) with the initialization step (4). We can prove the following result by induction.

PROPOSITION 4 (PERIODIC UPDATING). *In the multi-period model, given all others being equal, the sequential mechanism with $\tau = 1$ always yields higher success rates than any periodic updating mechanism with a lower frequency.*

The above proposition indicates that in the multi-period model, the firm should implement a sequential mechanism that updates the cumulative number of sign-ups after each period. This result is consistent with observations in practice.

4. A Two-Cohort Extension

To further explore the impact of information management on group-buying deal success rates, in this section we extend the two-person model to a two-cohort model with multiple consumers within each cohort. In particular, there are two cohorts of consumers, indexed by $i = 1, 2$. At the beginning of the game, the firm knows the probability distribution of the size of each cohort, but does not know the exact number. Consumers within each cohort may have different valuations for the firm's product or service. Although the firm does not know an individual consumer's valuation, the firm knows the distribution of valuations among consumers within each cohort. We denote the number of consumers of cohort i by a discrete random variable M_i, with support $\{1, 2, \ldots, m_i\}$. The individual product valuation for consumers in cohort i, denoted by V_i, is drawn from a given CDF $F_i(\cdot)$. These two cohorts of consumers can differ in both size and individual valuations.

The distributions for the two cohorts' sizes and valuations of consumers in each cohort are public information, known to the firm as well as to all consumers. We assume that consumers know exactly how much

they value the product and that their valuations are not influenced by other people's sign-up decisions. We further assume that when one cohort of consumers arrive at the deal and need to make sign-up decisions, these consumers find out the size of the cohort and its members' valuations. Such information becomes available to all consumers within the cohort, but not to the other cohort. This assumption on intra-cohort communication can be restrictive, but substantially simplifies our analysis. The model with periodic information disclosure in §3 can be viewed as a scenario where consumers within a cohort do not share private information.

Recall that in the two-person case, each consumer forms a belief in the likelihood of the other consumer signing up for the deal when making the sign-up decision. In the case of the two cohorts, consumers form beliefs on the number of sign-ups from the other cohort.[5] We represent such a consumer belief within cohort i by a discrete probability distribution, denoted by $\mathcal{Q}_{-i}$. Let $H_i(\mathcal{Q}_{-i})$, $i = 1, 2$, denote the individual sign-up likelihood of cohort i given the individual consumer's belief being $\mathcal{Q}_{-i}$. Because it is more convenient to work with tail distributions, we let $q_{i,l} \equiv P(\mathcal{Q}_i \geq l)$, $l = 1, \ldots, m_i$, and for convention $q_{i,m_i+1} \equiv 0$. The belief variable $\mathcal{Q}_i$ can be characterized by its tail distribution, namely, the vector $\vec{q}_i = (q_{i,1}, q_{i,2}, \ldots, q_{i,m_i})^T$. (We interchangeably use the random variable $\mathcal{Q}$ and its tail distribution vector $\vec{q}$ when referring to a belief distribution.) We generalize Assumption 1 to the following assumption.

ASSUMPTION 2. *The individual sign-up likelihood $H_i(\mathcal{Q}_{-i})$, $i = 1, 2$, is nondecreasing in $\mathcal{Q}_{-i}$, in the sense of the first-order stochastic dominance (Shaked and Shanthikumar 2007).*

4.1. Simultaneous vs. Sequential Mechanism

4.1.1. Simultaneous Mechanism. Under the simultaneous mechanism, consumers in each cohort base their sign-up decisions on the realized size and valuations of their own cohort, as well as their beliefs in the size and valuations of the other cohort. Thus, cohort sizes and valuations define the "types" for the Bayesian game that has been studied. Similar to the argument in the two-person game, equilibrium is

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[5] For donors at Kickstarter.com, we can define the expected individual donation amount $H(\mathcal{Q})$ as a function of the other donor's donation amount $\mathcal{Q}$, according to the distribution of potential donation amounts and the characteristics of the proposed project. As long as Assumption 2 is satisfied for $H(\mathcal{Q})$, all results in the subsequent sections apply except for those specifically related to the Groupon context, for example, the coexistence of the discount price and regular price for a deal.

characterized by a pair of belief distributions $(\mathcal{Q}_1^*, \mathcal{Q}_2^*)$ that satisfies the following conditions:

$$P\left(\sum_{k=1}^{M_1} X_{1,k}(\mathcal{Q}_2^*) \geq l\right) = q_{1,l}^*, \quad l = 1, \ldots, m_1, \quad (5)$$

$$P\left(\sum_{k=1}^{M_2} X_{2,k}(\mathcal{Q}_1^*) \geq l\right) = q_{2,l}^*, \quad l = 1, \ldots, m_2. \quad (6)$$

The existence of such a pair of belief distributions is a direct consequence of the multidimensional version of Tarski's fixed point theorem. The success rate q^* at equilibrium can be characterized by

$$q^* = P\left(\sum_{k=1}^{M_1} X_{1k}(\mathcal{Q}_2^*) + \sum_{k=1}^{M_2} X_{2k}(\mathcal{Q}_1^*) \geq N\right), \quad (7)$$

where $X_{ik}(\mathcal{Q}_{-i})$, $i = 1, 2$, all k, are independent, and $X_{ik}(\mathcal{Q}_{-i})$, all k, $i = 1, 2$, are identically distributed Bernoulli random variables with success probability $H_i(\mathcal{Q}_{-i})$. We use the same notation $X_{ik}(\mathcal{Q}_{-i})$ for individual sign-up random variables throughout the rest of the paper.

4.1.2. Sequential Mechanism. Suppose cohort i arrives in the first period and cohort $-i$ arrives in the second period. Under the sequential mechanism, the equilibrium behavior can be solved by backward induction. When individual consumers of cohort $-i$ make decisions, there is no more uncertainty about the decisions made by consumers of cohort i. Given the number of sign-ups from cohort i revealed as n, the subgame equilibrium strategy in the second period depends on the number of potential subscribers in cohort $-i$, that is, those consumers who will sign up with a belief $\vec{q}_i = \vec{e}$, where $\vec{e}$ is a vector with all entries equal to 1. Specifically, if the potential subscribers are no fewer than $N - n$, then these consumers will sign up and the belief for the deal to be on will be fulfilled. Otherwise, nobody in the second cohort will sign up and the deal is off. After solving the subgame equilibrium in the second period, we move backward to the first period when cohort i makes sign-up decisions. At equilibrium, the belief $\mathcal{Q}_{-i}$ held by cohort i should be self-enforced. That is, any equilibrium $\mathcal{Q}_{-i}^*$ satisfies the condition

$$P(\mathcal{Q}_{-i}^* = l) = P\left(\sum_{k=1}^{M_{-i}} X_{-ik}(\vec{e}) = l\right). \quad (8)$$

From the seller's perspective, the success rate Q_i^* at equilibrium can be characterized by

$$Q_i^* = P\left(\sum_{k=1}^{M_i} X_{ik}(\mathcal{Q}_{-i}^*) + \sum_{k=1}^{M_{-i}} X_{-ik}(\vec{e}) \geq N\right). \quad (9)$$

4.1.3. Mechanism Comparison. In the two-cohort case, the belief held by consumers of cohort i is characterized by a discrete random variable $\mathcal{Q}_{-i}$. Consequently, we need to extend the pointwise dominance used for comparing mechanisms in the two-person case to the first-order stochastic dominance. Denote $(\mathcal{Q}_{1,se}^*, \mathcal{Q}_{2,se}^*)$ and $(\mathcal{Q}_{1,sm}^*, \mathcal{Q}_{2,sm}^*)$ the pairs of equilibrium beliefs held by consumers under the sequential mechanism and the simultaneous mechanism, respectively. First, it is easy to show that the number of sign-ups from cohort $-i$ under the sequential mechanism always stochastically dominates the number of sign-ups under the simultaneous mechanism, i.e., $\sum_{k=1}^{M_{-i}} X_{-ik}(\vec{e}) \geq_{st} \sum_{k=1}^{M_{-i}} X_{-ik}(\mathcal{Q}_{i,sm}^*)$. The inequality is due to Assumption 2 and the fact that the belief of the deal being on with certainty is the most promising belief distribution that consumers can possibly have. As $\mathcal{Q}_{-i,se}^* = \sum_{k=1}^{M_{-i}} X_{-ik}(\vec{e}) \geq_{st} \sum_{k=1}^{M_{-i}} X_{-ik}(\mathcal{Q}_{i,sm}^*) = \mathcal{Q}_{-i,sm}^*$, we have $\sum_{k=1}^{M_i} X_{ik}(\mathcal{Q}_{-i,se}^*) \geq_{st} \sum_{k=1}^{M_i} X_{ik}(\mathcal{Q}_{-i,sm}^*)$ again by Assumption 2. Consequently, we obtain that the total number of sign-ups from the two cohorts under the sequential mechanism is greater than the total number of sign-ups under the simultaneous mechanism in the sense of first-order stochastic dominance, i.e., for all $i = 1, 2$, $\sum_{k=1}^{M_i} X_{ik}(\mathcal{Q}_{-i,se}^*) + \sum_{k=1}^{M_{-i}} X_{-ik}(\vec{e}) \geq_{st} \sum_{k=1}^{M_1} X_{1k}(\mathcal{Q}_{2,sm}^*) + \sum_{k=1}^{M_2} X_{2k}(\mathcal{Q}_{1,sm}^*)$, and thus the sequential mechanism leads to higher success rates. Therefore, the sequential mechanism should generate higher expected profit for the firm and yield higher expected individual and total consumer surpluses. We summarize the above discussions in the next proposition.

PROPOSITION 5 (MECHANISM COMPARISON FOR TWO COHORTS). *Given all others the same, compared with the simultaneous mechanism, the sequential mechanism yields higher success rates.*

4.1.4. Endogenous Sequencing. Similar to the two-person game, we can investigate the impact of endogenous sequencing by adding a stage 0 before our current two-period game. In stage 0, the firm decides whether two cohorts should arrive simultaneously or sequentially and if sequentially, which cohort should arrive first. In practice, group-buying firms can predict the purchase likelihood and cohort size of their members for any given product by tracking their purchase histories.

At stage 0, the firm and cohorts face the payoff matrix exactly the same as the one shown in Table 1. As a result, in stage 0 of the two-stage game, there exist two equilibria, both of which correspond to sequential mechanisms. Comparing these two equilibria, we identify the conditions under which the equilibrium where cohort $-i$ should move before cohort i. We summarize the condition in the following proposition and relegate the proof to Appendix B.

PROPOSITION 6 (HETEROGENEOUS COHORT SIZE AND DISCOUNTING). *If $M_i \geq_{st} M_{-i}$, $H_i(\vec{e}) = H_{-i}(\vec{e})$ and $H_i(\mathcal{Q}) \leq H_{-i}(\mathcal{Q})$ for all belief distribution $\mathcal{Q}$, then the equilibrium where cohort $-i$ acts as the leader Pareto dominates the other equilibrium where cohort i acts as the leader.*

Recall that in the two-person game, the consumer who discounts time to a less extent should be chosen to move first. Proposition 6 shows that a similar result still holds here. Moreover, the condition $M_i \geq_{st} M_{-i}$ indicates that cohort i is the larger cohort in the probabilistic sense. Thus, given all others the same, the firm should choose the cohort that is potentially smaller to move first. This result is somewhat surprising because one may want to have the potentially larger cohort to move first to assure the smaller cohort. However, our result indicates that the firm prefers to have the larger cohort to be the information free riders in the second period. Such an arrangement can eventually benefit both cohorts by boosting the confidence of the earlier cohort, who is in anticipation of potentially larger later cohort.

4.2. Multilevel Price Schedule
Our main analysis investigates group-buying deals with a single threshold. In practice, a group-buying deal can have multiple levels of thresholds, and a higher threshold associated with a deeper discount. Such multilevel price schedules were commonly offered by the first-generation group-buying websites (Kauffman and Wang 2002). In this section we extend our analysis of simultaneous versus sequential mechanisms to the group-buying deals with multilevel price schedules. Without a careful analysis, it is unclear if the benefit of sequential mechanisms on deal success rates will remain when a deal consists of multiple levels of thresholds. Overall, the analysis of multilevel deals generates additional insights on how the mechanism choice may affect the likelihood of achieving different levels of thresholds.

Consider a J-level price schedule where the level of discounts depends on the total number of cumulative sign-ups X. Specifically, the group-buying price $w(X) = w_j$ if $N_j \leq X < N_{j+1}$, $j = 1, \ldots, J$, where $N_j \in \mathbb{N}$ for all $j = 1, \ldots, J$, $N_{J+1} = \infty$ and $w_1 > w_2 > \cdots > w_J$. Similar to the single-threshold case, consumers form beliefs on the number of sign-ups from the other cohort while making decisions. However, a deal's success rate needs to be extended to a vector with J elements, which characterizes the likelihood of the cumulative number of sign-ups exceeding each level of the discount schedule N_j, $j = 1, \ldots, J$.

4.2.1. Simultaneous Mechanism. Similar to the single-threshold case, the pair of belief distributions at equilibrium $(\mathcal{Q}_1^*, \mathcal{Q}_2^*)$ satisfies Equations (5) and (6). Under the simultaneous mechanism, the likelihood of the cumulative number of sign-ups exceeding the discount schedule N_j can be characterized by

$$q_j^* = P\left(\sum_{k=1}^{M_1} X_{1k}(\mathcal{Q}_2^*) + \sum_{k=1}^{M_2} X_{2k}(\mathcal{Q}_1^*) \geq N_j\right), \quad j = 1, \ldots, J.$$

4.2.2. Sequential Mechanism. Suppose cohort i arrives in the first period and cohort $-i$ arrives in the second period. The equilibrium belief $\mathcal{Q}_{-i}^*$ held by cohort i should satisfy the same condition as that in the single-threshold model, which is given by Equation (8). The likelihood of the cumulative number of sign-ups exceeding the discount schedule N_j, Q_{ij}^*, can be characterized by

$$Q_{ij}^* = P\left(\sum_{k=1}^{M_i} X_{ik}(\mathcal{Q}_{-i}^*) + \sum_{k=1}^{M_{-i}} X_{-ik}(\vec{e}) \geq N_j\right), \quad j = 1, \ldots, J.$$

4.2.3. Mechanism Comparison. Invoking the same proof as that in §4.1, we can show that the sequential mechanism results in a higher likelihood of the cumulative number of sign-ups exceeding each threshold level N_j, $j = 1, \ldots, J$. That is, when a firm offers a multilevel group-buying deal, a sequential mechanism not only leads to a higher likelihood of exceeding the lowest threshold and unlocking the deal but also increases the chance to achieve a higher level of threshold. As a result, the sequential mechanism leads to larger expected consumer surpluses. We summarize the result in the proposition below.

PROPOSITION 7 (MECHANISM COMPARISON FOR MULTILEVEL PRICE SCHEDULE). *If the deal is in the form of a multilevel price schedule, the sequential mechanism always yields a higher likelihood of reaching any threshold level than the simultaneous mechanism.*

4.3. Capacity Limit
Group-buying deals are often offered by service firms with capacity constraints. Examples of such service firms include restaurants, salons, and art schools. Because these businesses earn much higher profit margins from regular customers who do not use group-buying deals, they allocate only the excess capacities for group-buying customers. As a result, when a large number of consumers are anticipated to sign up to a deal that has a capacity limit, the expected value of the deal for each consumer may decrease because of the possibility of being rationed. To incorporate such a negative externality effect into our model, we consider a group-buying deal with a capacity limit C and a minimum threshold N. If the total number of sign-ups at the end of the second period is greater than C, we assume that C units of services will be randomly allocated to the subscribers with equal probability.

In the case with a capacity limit, the characterizations of equilibria and deals' success rates are

similar to our basic two-cohort model. That is, the success rate q^* under the simultaneous mechanism is given by $q^* = P(\sum_{k=1}^{M_1} X_{1k}(\mathbb{Q}_2^*) + \sum_{k=1}^{M_2} X_{2k}(\mathbb{Q}_1^*) \geq N)$, where the pair of equilibrium belief $(\mathbb{Q}_1^*, \mathbb{Q}_2^*)$ satisfies Equations (5) and (6). When cohort i arrives in the first period and cohort $-i$ arrives in the second period, the success rate Q_i^* under the sequential mechanism is characterized by $Q_i^* = P(\sum_{k=1}^{M_i} X_{ik}(\mathbb{Q}_{-i}^*) + \sum_{k=1}^{M_{-i}} X_{-ik}(\bar{e}) \geq N)$, where $\mathbb{Q}_{-i}^*$ is given by Equation (8).

However, Assumption 2 no longer holds when a capacity limit is imposed to the deal. With a capacity limit, there are two opposing forces driving the sign-up decisions. One is the positive network externality due to the need to coordinate to meet the minimum threshold, and the other is a negative network externality due to the capacity limit. When a consumer makes the sign-up decision, she bases her decision not only on the probability of the deal being on but also on the chance of eventually receiving the service. Consequently, for any individual consumer, knowing that a larger number of people in the other cohort will sign up to the deal does not always increase her intention to sign up.

Under the sequential mechanism, the relative strength of these two forces depends on the number of sign-ups in the first cohort. Specifically, if only a small number of consumers sign up in the first period, the positive network externality dominates. In this case, since rationing is unlikely to occur, consumers in the second cohort do not need to discount their perceived values for the deal. However, if a large number of consumers have signed up for the deal in the first period, then the negative externality effect due to the capacity limit will be much stronger, depressing the expectations of the second cohort. Compared with the simultaneous mechanism, the sequential mechanism amplifies both types of network effects because consumers of the second cohort observe the decisions of the first cohort and adjust their expected valuations accordingly.

We use the numerical results in Figure 2 to illustrate the intuitions regarding the capacity limit. Our numerical example concerns a group-buying deal for luxury goods as specified in Example 2. We consider three potential consumers, two of them arriving in the first period, and one arriving in the second period. Both the deal threshold and capacity limit are equal to 2, i.e., $N = C = 2$. We plot the expected number of sign-ups and the deal success rate in Figures 2(a) and 2(b), respectively. Given all other parameters the same, the negative externality effect increases when the surplus of the nonpurchase option a decreases. Figure 2(b) shows that the success rate under the sequential mechanism still dominates that under the simultaneous mechanism. However, as shown in Figure 2(a), when a is sufficiently small, the expected

Figure 2 Comparisons of Expected Numbers of Sign-Ups and Success Rates with Various Surpluses of Nonpurchase Option a
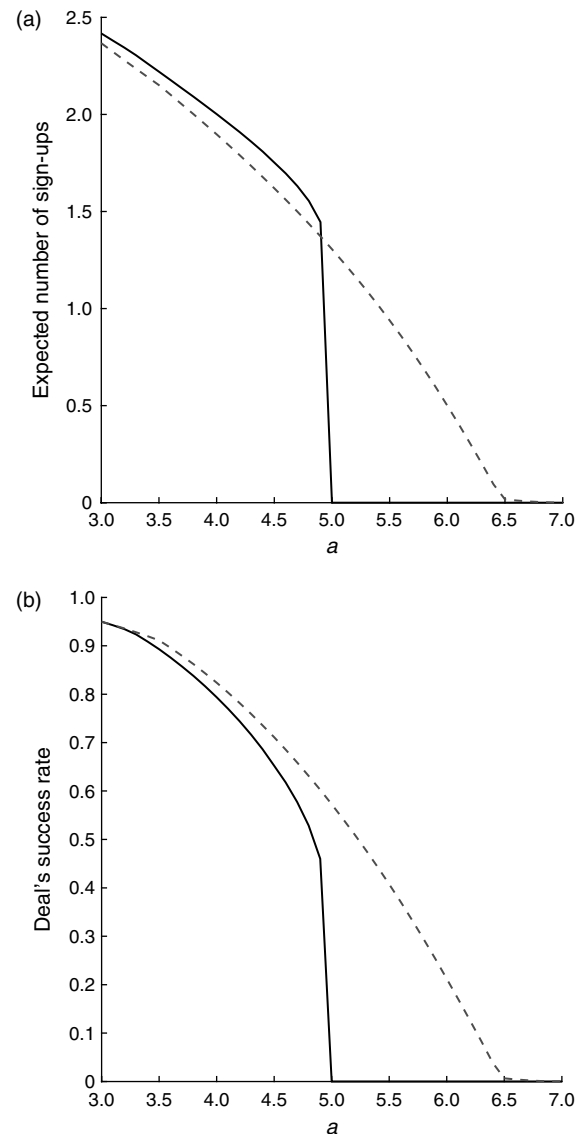
Notes. The solid and dashed lines correspond to simultaneous and sequential mechanisms, respectively. The parameters are specified as follows: V_i, $i = 1, 2$, follow uniform distributions with support $[15, 30]$, and $p = 30$, $w = 10$, $N = 2$, $C = 2$, $\rho_1 = 0.4$, and $\rho_2 = 0.7$, and the surpluses of nonpurchase option a vary from 3 to 7.

number of sign-ups under the simultaneous mechanism can exceed that under the sequential mechanism. Thus, the sequential mechanism no longer dominates the simultaneous mechanism in terms of the number of sign-ups. In essence, our model without capacity limits is a pure coordination game with positive externality effect only. Capacity constraints introduce negative externalities through competition among consumers that reduce the information benefit associated with the sequential mechanism. The extent of competition among consumers and the resulting negative externality effect is stronger with a higher sign-up likelihood from each individual consumer.

5. Conclusion

This paper studies optimal group-buying mechanisms in a two-period game where cohorts of consumers arrive at the deal sequentially. The two cohorts cannot fully communicate with each other about their sizes and consumer valuations. Instead, the firms use group-buying mechanisms to coordinate. In particular, we examine the success rate of a group-buying deal under two alternative mechanisms: a sequential mechanism and a simultaneous mechanism. Our analysis shows that, all other things being equal, a sequential mechanism dominates a simultaneous mechanism. Interestingly, posting the cumulative number of sign-ups from the first period can reduce uncertainty and thus increase the expected sign-ups among the second cohort of consumers. The increased second-period sign-ups can in turn improve the confidence among the first cohort of consumers and lead to a higher expected number of sign-ups in the first period, thus further increasing the deal's success rate. This backward-inductive perspective, starting from the second period and going back to the first period, is crucial to understanding the intuition behind our result. The driving force behind our result is that consumers essentially play a coordination game, and the sequential mechanism, with information revealed by one cohort to the other, allows for a better coordination.

This result is consistent with the observed dominance of sequential mechanisms in practice. Broadly speaking, the superiority of the sequential mechanism can also be implemented by exploiting communication among consumers who are adjacent in a social network. Furthermore, our analysis provides useful guidance on how a group-buying firm may properly arrange the sequence of consumer arrivals to increase the success rate and the expected number of sign-ups. Having the smaller cohort come first and the larger cohort come later, or having the cohort who discounts time to a less extent come first and the cohort who discounts more come later, can increase the deal's success rate.

We enhance our key insights through a number of model extensions. First, when a group-buying deal requires the entire set of consumers to participate in a multiperiod model, we find that it is optimal for the firm to update the sign-up numbers as frequently as possible. Second, when the firm offers a multilevel group-buying deal that hands out greater benefits for reaching higher levels of thresholds, the sequential mechanism not only leads to a higher deal success rate, but also increases the chance to reach higher levels of thresholds. Third, when the group-buying firm faces a capacity limit, negative externality may arise because the limited capacity has to be rationed among the consumers who have signed up. As a result, the sequential mechanism, which may amplify the negative externality effect, could lead to a lower number of sign-ups when the firm faces a stringent capacity constraint.

Appendix A. Comparative Statics for Sequential Mechanism in a Two-Person Model

Consider two sets of signing up probability functions $H = (H_i)$ and $\tilde{H} = (\tilde{H}_i)$, which are sustained by two distinct group-buying deals. Suppose that $\tilde{H}_i(q) \geq H_i(q)$ for all $q \in [0, 1]$ and $i = 1, 2$, then we have $\tilde{Q}_i^* = \tilde{H}_i(\tilde{H}_{-i}(1))\tilde{H}_{-i}(1) \geq H_i(H_{-i}(1))H_{-i}(1) = Q_i^*$, $i = 1, 2$. This implies that one can compare the success rates of two group-buying deals by simply comparing the individual sign-up probability. We summarize the results in the following corollaries.

COROLLARY 1 (TIME DISCOUNTING). *Consider either necessity goods or luxury goods. A deal $\tilde{H}$ yields higher success rates than another deal H under the sequential mechanism, if, ceteris paribus, the deal $\tilde{H}$ has higher values of the discount rate (less discounting) than those of the deal H for each cohort, i.e., $\tilde{\rho}_i \geq \rho_i$, $i = 1, 2$.*

PROOF OF COROLLARY 1. By Assumption 1(i), it is sufficient to show that $\tilde{H}_i(q) \geq H_i(q)$, $i = 1, 2$ for all $q \in (0, 1]$. Recall that, for the necessity goods, $H_i(q)$ is in the form of $H_i(q) = F_i(p + (\rho_i/(1-\rho_i))(p-w)q) - F_i(w)$ for $q \in (0, 1]$. If $\tilde{\rho}_i \geq \rho_i$, ceteris paribus, then $\tilde{H}_i(q) \geq H_i(q)$ for $q \in (0, 1]$, since the function $F_i(p + (\rho_i/(1-\rho_i))(p-w)q)$ is increasing in ρ_i. For the luxury goods, $H_i(q)$ is in the form of $H_i(q) = F_i(p + a_i) - F_i(a_i[1 - (1-q)\rho_i]/(q \cdot \rho_i) + w)$ for $q \in (0, 1]$. If $\tilde{\rho}_i \geq \rho_i$, ceteris paribus, then $\tilde{H}_i(q) \geq H_i(q)$ for $q \in (0, 1]$, since the function $-F(a_i[1 - (1-q)\rho_i]/(q \cdot \rho_i) + w)$ is increasing in ρ_i. □

COROLLARY 2 (PRICE DISCOUNT). *Consider either necessity goods or luxury goods. A deal $\tilde{H}$ yields higher success rates than another deal H under the sequential mechanism, if, ceteris paribus, the deal $\tilde{H}$ has a higher regular price and a lower group-buying price than those of the deal H, i.e., $\tilde{p} \geq p$ and $\tilde{w} \leq w$.*

PROOF OF COROLLARY 2. By Assumption 1(i), it is sufficient to show that $\tilde{H}_i(q) \geq H_i(q)$, $i = 1, 2$ for all $q \in (0, 1]$. Recall that, for the necessity goods, $H_i(q)$ is in the form of $H_i(q) = F_i(p + (\rho_i/(1-\rho_i))(p-w)q) - F_i(w)$ for $q \in (0, 1]$. If $\tilde{p} \geq p$ and $\tilde{w} \leq w$, ceteris paribus, then

$[\tilde{w}, \tilde{p} + (\rho_i/(1 - \rho_i))(\tilde{p} - \tilde{w})q] \supseteq [w, p + (\rho_i/(1 - \rho_i))(p - w)q]$. Consequently, $\tilde{H}_i(q) \geq H_i(q)$ for all $q \in (0, 1]$. For the luxury goods, $H_i(q)$ is in the form of $H_i(q) = F_i(p + a_i) - F_i(a_i[1 - (1 - q)\rho_i]/(q \cdot \rho_i) + w)$ for $q \in (0, 1]$. If $\tilde{p} \geq p$ and $\tilde{w} \leq w$, ceteris paribus, then $[a_i[1 - (1 - q)\rho_i]/(q \cdot \rho_i) + \tilde{w}, \tilde{p} + a_i] \supseteq [a_i[1 - (1 - q)\rho_i]/(q \cdot \rho_i) + w, p + a_i]$. Consequently, $\tilde{H}_i(q) \geq H_i(q)$ for all $q \in (0, 1]$. $\square$

COROLLARY 3 (NONPURCHASE OPTION). *Consider the luxury goods. A deal $\tilde{H}$ yields higher success rates than another deal H under the sequential mechanism, if, ceteris paribus, the deal $\tilde{H}$ has a higher sum of the regular price and nonpurchase value, and a lower nonpurchase value than those of the deal H for each cohort, i.e., $\tilde{p} + \tilde{a}_i \geq p + a_i$ and $\tilde{a}_i \leq a_i$, $i = 1, 2$.*

PROOF OF COROLLARY 3. By Assumption 1(i), it is sufficient to show that $\tilde{H}_i(q) \geq H_i(q)$, $i = 1, 2$ for all $q \in (0, 1]$. Recall that, for the luxury goods, $H_i(q)$ is in the form of $H_i(q) = F_i(p + a_i) - F_i((a_i[1 - (1 - q)\rho_i])/(q \cdot \rho_i) + w)$ for $q \in (0, 1]$. If $\tilde{p} + \tilde{a}_i \geq p + a_i$ and $\tilde{a}_i \leq a_i$, ceteris paribus, then $[\tilde{a}_i[1 - (1 - q)\rho_i]/(q \cdot \rho_i) + w, \tilde{p} + \tilde{a}_i] \supseteq [a_i[1 - (1 - q)\rho_i]/(q \cdot \rho_i) + w, p + a_i]$. Consequently, $\tilde{H}_i(q) \geq H_i(q)$ for all $q \in (0, 1]$. $\square$

Appendix B. Proofs

Proposition 3 is a special case of Proposition 6, where M_i, $i = 1, 2$, are both equal to 1, and the predetermined threshold N is 2. As a result, we include proof for Proposition 6, but not for Proposition 3 to avoid repetition.

PROOF OF PROPOSITION 4. For convenience, we focus on the largest stable equilibrium; nevertheless, the result holds generally in the sense of Definition 6, because what we will verify is essentially the pointwise dominance needed for the equilibrium comparison. We start by showing the dominance of the sequential mechanism with $\tau = 1$ over the simultaneous mechanism (i.e., $\tau = N$) in the multiperiod model, which is summarized in the following corollary. This result will be repeatedly invoked in the rest of the proof. If the firm adopts the sequential mechanism with $\tau = 1$, Equations (3) and (4) reduce to $q_i^1 = H_i(q_{i-1}^1)q_{i-1}^1$, for $i = 2, \ldots, N$, and $q_1^1 = H_1(1)$. When the firm adopts the simultaneous mechanism (i.e., $\tau = N$), the ex ante success rate is characterized by $q_1^N = \prod_{i=1}^{N} H_i(q_{1,i}^N)$, for $N \geq 2$, and $q_1^1 = H_1(1)$, where $q_{1,i}^N = \prod_{l=1, l \neq i}^{N} H_l(q_{1,l}^N)$, $i = 1, \ldots, N$.

COROLLARY 4. *In the multiperiod model, given all others being equal, $q_N^1 \geq q_1^N$.*

PROOF OF COROLLARY 4. First, we make the following observation: $q_{1,i}^N \geq q_{1,i}^{N+1}$, $i = 1, \ldots, N$. That is, for any consumer i, her belief regarding the likelihood that the rest will sign up for deals is nonincreasing in the market size N. This is intuitive in the sense that if the $(N + 1)$th consumer signs up the deal with certainty, then the uncertainty facing the other consumers is the same as the case when the market size is N. However, generally, the likelihood that consumer $N + 1$ signs up for the deal is less than 1, so the other consumers' beliefs regarding the deal's success likelihood at their stage of decision making given that all consumers in previous cohorts have signed up, will decrease with the introduction of consumer $N + 1$. Next we prove by induction on N that $q_N^1 \geq q_1^N$. It is obvious for $N = 1$. If $N = 2$, the multiperiod model is

reduced to our basic two-person model, and thus we have $q_2^1 \geq q_1^2$. Suppose we have $q_N^1 \geq q_1^N$ for some $N \geq 2$, we want to show that $q_{N+1}^1 \geq q_1^{N+1}$. By the recursive definitions, we have $q_1^{N+1} = \prod_{i=1}^{N+1} H_i(q_{1,i}^{N+1}) = q_{1,N+1}^{N+1} H_{N+1}(q_{1,N+1}^{N+1})$ and $q_{N+1}^1 = H_{N+1}(q_N^1)q_N^1 \geq H_{N+1}(q_1^N)q_1^N$, where the last inequality is given by the induction result in the previous step. To this end, all we need to show is that $q_1^N \geq q_{1,N+1}^{N+1}$. From the definitions, we have

$$q_{1,N+1}^{N+1} = \prod_{i=1}^{N} H_i(q_{1,i}^{N+1}), \quad \text{and}$$

$$q_{1,i}^{N+1} = H_{N+1}(q_{1,N+1}^{N+1}) \cdot \prod_{l=1, l \neq i}^{N} H_l(q_{1,l}^{N+1}), \quad i = 1, \ldots, N,$$

$$q_1^N = \prod_{i=1}^{N} H_i(q_{1,i}^N), \quad \text{and}$$

$$q_{1,i}^N = \prod_{l=1, l \neq i}^{N} H_l(q_{1,l}^N), \quad i = 1, \ldots, N.$$

As $q_{1,i}^{N+1} \leq q_{1,i}^N$, $i = 1, 2, \ldots, N$, we have $q_{1,N+1}^{N+1} \leq q_1^N$, and thus we obtain the announced result. $\square$

For any τ, we prove by induction on j that $q_{\min\{j\tau, N\}}^1 \geq q_j^\tau$ for $j = 1, \ldots, \lceil N/\tau \rceil$. The desired result can be obtained when $j = \lceil N/\tau \rceil$. If $j = 1$, the last cohort under the information release scheme with frequency τ behaves exactly the same as under the simultaneous mechanism. Consequently, the dominance of $q_{\min\{\tau, N\}}^1$ over q_1^τ is a direct consequence of Corollary 4. Suppose we have $q_{\min\{j\tau, N\}}^1 \geq q_j^\tau$ for some $j \geq 1$, we want to show that $q_{\min\{(j+1)\tau, N\}}^1 \geq q_{j+1}^\tau$. By Equation (3), $q_l^1 = \prod_{i=j\tau+1}^{l} H_i(q_{i-1}^1) \cdot q_{j\tau}^1$, $l = j\tau + 1, \ldots, \min\{(j+1) \cdot \tau, N\}$ and $q_{j+1}^\tau = \prod_{i=j\tau+1}^{\min\{(j+1)\tau, N\}} H_i(q_{j+1,i}^\tau) \cdot q_j^\tau$, where $q_{j+1,i}^\tau = q_j^\tau \cdot \prod_{l=j\tau+1, l \neq i}^{\min\{(j+1)\tau, N\}} H_l(q_{j+1,l}^\tau)$. Following the proof of Corollary 4, we can show, by induction on each individual consumer $l = 1, \ldots, \tau$ within the $(\lceil N/\tau \rceil + 1 - (j + 1))$th cohort, that $q_{\min\{(j+1)\tau, N\}}^1 \geq q_{j+1}^\tau$. $\square$

PROOF OF PROPOSITION 6. It is easy to verify that the stipulations of Proposition 6 provide a sufficient condition for the following conditions: $M_i \geq_{st} M_{-i}$, and for all $@$, $H_i(\bar{e})[1 - H_i(@)] \geq H_{-i}(\bar{e})[1 - H_{-i}(@)]$ as inequality (H1), $[1 - H_i(\bar{e})]H_i(@) \leq [1 - H_{-i}(\bar{e})]H_{-i}(@)$ as inequality (H2). Hence, it suffices to prove the desired result under these more general conditions. Recall that $X_{ik}(@)$ is a Bernoulli random variable with success probability $H_i(@)$ and $X_{ik}(\bar{e})$ is a Bernoulli random variable with success probability $H_i(\bar{e})$. Hence,

$$X_{ik}(\bar{e}) - X_{ik}(@) =_{st} \begin{cases} 1 & \text{with prob. } H_i(\bar{e})[1 - H_i(@)], \\ -1 & \text{with prob. } [1 - H_i(\bar{e})]H_i(@), \\ 0 & \text{otherwise.} \end{cases}$$

Given inequality (H1), we know that $X_{ik}(\bar{e}) - X_{ik}(@)$ has a larger mass in value of 1 than $X_{-ik}(\bar{e}) - X_{-ik}(@)$. By inequality (H2) alone, $X_{ik}(\bar{e}) - X_{ik}(@)$ has a smaller mass in value of -1 than $X_{-ik}(\bar{e}) - X_{-ik}(@)$; equivalently, $X_{ik}(\bar{e}) - X_{ik}(@)$ has a larger mass in values of 0 and 1 combined than $X_{-ik}(\bar{e}) - X_{-ik}(@)$. Hence, under conditions (H1) and (H2), we have $X_{ik}(\bar{e}) - X_{ik}(@) \geq_{st} X_{-ik}(\bar{e}) - X_{-ik}(@)$. Furthermore, if

$M_i \geq_{st} M_{-i}$, by Shaked and Shanthikumar (2007) Theorem 1.A.4., we have $\sum_{k=1}^{M_i}[X_{ik}(\vec{e}) - X_{ik}(\mathcal{Q})] \geq_{st} \sum_{k=1}^{M_{-i}}[X_{-ik}(\vec{e}) - X_{-ik}(\mathcal{Q})]$, which is equivalent to $\sum_{k=1}^{M_i} X_{ik}(\mathcal{Q}) + \sum_{k=1}^{M_{-i}} X_{-ik}(\vec{e}) \leq_{st} \sum_{k=1}^{M_{-i}} X_{-ik}(\mathcal{Q}) + \sum_{k=1}^{M_i} X_{ik}(\vec{e})$.

Recall that the deal's success rates under the two cases are given by $Q_i^* = P(\sum_{k=1}^{M_i} X_{ik}(\mathcal{Q}_{-i}^*) + \sum_{k=1}^{M_{-i}} X_{-ik}(\vec{e}) \geq N)$ and $Q_{-i}^* = P(\sum_{k=1}^{M_{-i}} X_{-ik}(\mathcal{Q}_i^*) + \sum_{k=1}^{M_i} X_{ik}(\vec{e}) \geq N)$, respectively. Given $H_i(\vec{e}) = H_{-i}(\vec{e})$ and $M_i \geq_{st} M_{-i}$, we know that $\mathcal{Q}_i^* = \sum_{k=1}^{M_i} X_{ik}(\vec{e}) \geq_{st} \sum_{k=1}^{M_{-i}} X_{-ik}(\vec{e}) = \mathcal{Q}_{-i}^*$. Combining the above results, we have $\sum_{k=1}^{M_i} X_{ik}(\mathcal{Q}_{-i}^*) + \sum_{k=1}^{M_{-i}} X_{-ik}(\vec{e}) \leq_{st} \sum_{k=1}^{M_i} X_{ik}(\mathcal{Q}_i^*) + \sum_{k=1}^{M_{-i}} X_{-ik}(\vec{e}) \leq_{st} \sum_{k=1}^{M_{-i}} X_{-ik}(\mathcal{Q}_i^*) + \sum_{k=1}^{M_i} X_{ik}(\vec{e})$, and the desired result follows. □

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