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A Generalized Norton–Bass Model for Multigeneration Diffusion

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The Norton–Bass (NB) model is often credited as the pioneering multigeneration diffusion model in marketing. However, as acknowledged by the authors, when counting the number of adopters who substitute an old product generation with a new generation, the NB model does not differentiate those who have already adopted the old generation from those who have not. In this study, we develop a generalized Norton–Bass (GNB) model that separates the two different types of substitutions. The GNB model provides closed-form expressions for both the number of units in use and the adoption rate, and offers greater flexibility in parameter estimation, forecasting, and revenue projection. An appealing aspect of the GNB model is that it uses exactly the same set of parameters as the NB model and is mathematically consistent with the later. Empirical results show that the GNB model delivers better overall performance than previous models both in terms of model fit and forecasting performance. The analyses also show that differentiating leapfrogging and switching adoptions based on the GNB model can help gain additional insights into the process of multigeneration diffusion. Furthermore, we demonstrate that the GNB model can incorporate the effect of marketing mix variables on the speed of diffusion for all product generations.

Key words: Norton–Bass model; multigeneration diffusion; leapfrogging; switching
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1. Introduction

Technological advances fuel the development of new products and services. Examples are abundant. Decades ago, black-and-white TV was replaced by color TV, which is now ceding market share to high-definition TV (HDTV). Even within the HDTV category, newer models are continuing to emerge, with the most recent variety 3D-capable, because of even more recent technologies. In the cellular phone market, the earliest generation was equipped with only basic calling features; the following generation was enhanced to include cameras, media players, etc., whereas the newest generation, called smart phones, allows users to surf the Web, check email, and run more sophisticated applications. The same phenomenon also exists in the software market, where vendors keep releasing new versions to meet users' ever-increasing appetite for functionalities and take advantage of improvements in hardware technologies. The Microsoft Windows and Office lines of products are two well-known examples, with new versions typically introduced every few years.

The diffusion of successive product generations has been well studied in the prior literature. Most of the existing multigeneration diffusion models are inspired by the seminal Bass model (Bass 1969). Among them, the model proposed by Norton and Bass (1987) (NB model for short) is often credited as the pioneering work in describing multigeneration diffusion. The NB model assumes that each generation has its own market potential and market penetration process, and adopters of earlier generations can shift to newer generations. After the Norton and Bass (1987) model, several other notable multigeneration diffusion models were proposed. Speece and MacLachlan (1995) extended the NB model to incorporate the influence of pricing and tested it with multigenerational data for fluid milk packaging technologies. Mahajan and Muller (1996) developed a model that captures the number of systems in use for each generation and used it to study the optimal market entry timing for successive generations. Jun and Park (1999) combined the diffusion effects and choice effects and proposed two integrated models: the Type I model distinguishes first-purchase demand and upgrade demand, whereas the Type II model does not. Kim et al. (2000) proposed a dynamic market growth model that captures not only the diffusion of multiple generations within the same product category, but also the complementarity and competition presented by related product categories. Danaher et al. (2001) developed a two-generation model that includes both first-time sales and periodic renewals. By selecting appropriate adoption time distributions, their model can also incorporate the impact of market mix variables. More recently, Jiang (2010) proposed a simple two-generation model to analyze the optimal free offer policy for successive software versions.

Despite the progresses made in the last two decades, a review of the literature reveals that the NB model remains the most tested and extended multigeneration diffusion model to date. We believe that the desirable mathematical properties (e.g., offering closed-form expressions, parsimonious, and continuous-time based) of the model plays a key role behind its popularity. However, the NB model is not applicable to all business scenarios. This is primarily because when counting the number of adopters who substitute an old generation with a new generation, the NB model does not differentiate those who have already adopted an earlier generation and those who have not. In their study, Norton and Bass (1987, p. 1074) do acknowledge the existence of the two different types of substitutions, but admit that their model does not differentiate them. Without such differentiation, the NB model cannot be used to estimate the number of cross-generation repeat purchases, nor can it help forecast future demand or project revenue for certain business scenarios (e.g., when revenue is generated through both product sale and after-sale service). In this study, we propose a generalized Norton–Bass (GNB) model that overcomes this limitation while retaining the desirable mathematical properties of the NB model. As we will demonstrate later, the proposed model offers greater flexibility in parameter estimation, forecasting, and revenue projection for a wider range of scenarios.

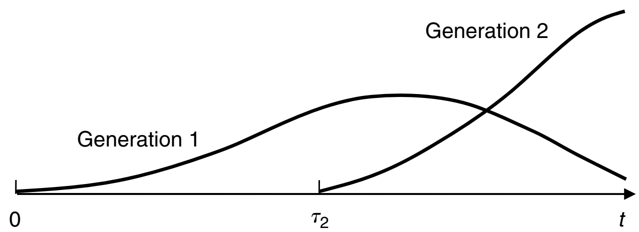
The remainder of this paper is organized as follows. In §2, we briefly review the Norton–Bass model. The detailed derivation of the GNB model is presented in §3. Empirical analyses of the GNB model and competing models are reported in §4. In §5, the GNB model is further extended to incorporate the effect of marketing mix variables. Finally, we discuss the managerial implications of this study and some future research directions in §6.

2. Review of the Norton–Bass Model

The NB model can be illustrated using two product generations. As shown in Figure 1, generation 1 (G1) and generation 2 (G2) are introduced at time 0 and τ_2, respectively. Based on the NB model, the sales rates of the two generations can be represented by two equations:

$$\begin{aligned} S_1(t) &= m_1F_1(t) - m_1F_1(t)F_2(t-\tau_2) \\ &= m_1F_1(t)[1-F_2(t-\tau_2)], \end{aligned} \tag{1}$$

Figure 1 Diffusion Curves for Two Successive Product Generations



$$\begin{aligned} S_2(t) &= m_2F_2(t-\tau_2) + m_1F_1(t)F_2(t-\tau_2) \\ &= F_2(t-\tau_2)[m_2+m_1F_1(t)]. \end{aligned} \tag{2}$$

In Equations (1) and (2), m_1 represents the market potential for generation 1, and m_2 is the market potential unique to generation 2. According to Norton and Bass (1987), all potential adopters of generation 1 are also possible adopters of generation 2. In addition, $F_G(t)$ in the NB model takes the following form:

$$F_G(t) = \begin{cases} 0 & t < 0, \\ \dfrac{1-e^{-(p_G+q_G)t}}{(q_G/p_G)e^{-(p_G+q_G)t}+1} & t \geq 0, \end{cases} \tag{3}$$

where p_G and q_G are the *coefficient of innovation* and *coefficient of imitation*, respectively, for generation G. In this study, we interpret $F_G(t)$ as representing the *diffusion of adoption* concerning generation G, and $S_G(t)$ as representing the number of units in use for generation G.[1]

3. The GNB Model

When a new product generation is introduced, potential adopters may not rush to adopt it. Some potential adopters may still purchase an older generation, possibly because they do not know of the availability of the new generation, or because they perceive the new generation as unproven or lacking product support. Gradually, the newer generation will become better known, and better product support will become available, thus making the new generation more appealing to potential adopters. Consequently, the proportion of potential adopters who are willing to skip previous generation(s) and directly adopt the new generation will increase over time. Similar to Danaher et al. (2001), we call the behavior of potential adopters skipping previous generation(s) and directly adopting a newer generation as

[1] Our interpretation of $S_G(t)$ is the same as Jun and Park's (1999). There is another interpretation that the NB model represents a repeat-purchase framework (e.g., Mahajan and Muller 1996), where $S_G(t)$ equals the size of the installed base times the frequency of purchases by an average user. The two interpretations do not contradict each other, because the first one is mainly applicable to durable products, whereas the second is applicable to nondurable products.

leapfrogging. In addition to leapfroggers, some existing adopters of the immediate previous generation may also be willing to purchase the new generation, if they perceive the improvements in the new generation as worth the investment. We call this behavior *switching*. Similar to leapfrogging, the proportion of switching adoptions also increases with time. The primary difference between leapfrogging and switching is that the former skips generation(s), whereas the latter does not. Therefore, this differentiation allows us to count cross-generation repeat purchases by the same adopters.

As discussed earlier, the NB model does not differentiate leapfrogging and switching. In this section, we generalize the NB model by separating the two types of behaviors. Whenever applicable, we keep the same notations of the NB model. We first consider a two-generation case, as shown in Figure 1. Product G1 is introduced at time 0, and product G2 at time $\tau_2 \geq 0$. Before the introduction of G2, the diffusion of G1 follows the Bass (1969) model. After G2 becomes available, a fraction of the potential adopters of G1, who would have adopted G1 if it were the only generation available, will leapfrog to G2 instead. As explained earlier, we expect the proportion of leapfrogging adoptions (hereafter referred to as the *leapfrogging multiplier*) to increase with time, as a direct result of the diffusion of adoption regarding G2, represented by $F_2(t-\tau_2)$. This is similar in spirit to the leapfrogging multipliers adopted by Danaher et al. (2001) and Jun and Park (1999). In all three models, the probability of leapfrogging from G1 to G2 is time varying and particularly influenced by the diffusion rate of G2. Further, all three models are parsimonious in that the rate of leapfrogging is a function of the same parameters used to define the basic diffusion rate for each generation.[2]

Given that the leapfrogging multiplier between G1 and G2 is $F_2(t-\tau_2)$, the number of leapfrogging adoptions during a small time interval $[t-\varepsilon, t]$ can be expressed as

$$m_1[F_1(t)-F_1(t-\varepsilon)]F_2(t-\tau_2).$$

Hence, the instantaneous rate of leapfrogging at time t, denoted by $u_2(t)$, equals

$$u_2(t) = \lim_{\varepsilon\to 0}\frac{m_1[F_1(t)-F_1(t-\varepsilon)]F_2(t-\tau_2)}{\varepsilon} = m_1 f_1(t)F_2(t-\tau_2), \quad t \geq \tau_2, \tag{4}$$

where $f_1(t)$ is the derivative of $F_1(t)$ and represents the diffusion rate of G1 at time t. Hence, the cumulative number of leapfrogging adoptions from G1 to G2 by time t is

$$U_2(t) = \int_{\tau_2}^{t} u_2(\theta)\,d\theta = m_1\int_{\tau_2}^{t} f_1(\theta)F_2(\theta-\tau_2)\,d\theta. \tag{5}$$

Along with leapfrogging, switching also starts to occur after time τ_2. We propose the following functional form to represent the rate of switching at time t:

$$w_2(t) = m_1F_1(t)f_2(t-\tau_2), \quad t \geq \tau_2; \tag{6}$$

its cumulative form is

$$W_2(t) = \int_{\tau_2}^{t} w_2(\theta)\,d\theta = m_1\int_{\tau_2}^{t} F_1(\theta)f_2(\theta-\tau_2)\,d\theta. \tag{7}$$

Equation (6) can be easily explained for $t=\tau_2$, when the rate of switching is

$$w_2(\tau_2) = m_1F_1(\tau_2)f_2(0). \tag{8}$$

In Equation (8), $m_1F_1(\tau_2)$ represents the number of existing adopters of G1 at time τ_2, and $f_2(0)$ is the instantaneous diffusion rate for G2 upon its introduction. Hence, all existing adopters of G1 are immediately treated as potential adopters of G2 as soon as G2 enters the market. Interpreting (6) is less straightforward for $t > \tau_2$ because, as a result of leapfrogging, the cumulative number of adopters of G1 is $m_1F_1(t) - U_2(t)$ instead of $m_1F_1(t)$. Upon further analyses, we find that the proposed functional form in (6) is still quite reasonable because, once the quality of the new generation is known, the probability of an existing adopter purchasing an update is expected to be higher than $f_2(t-\tau_2)$. Note that $f_2(t-\tau_2)$ represents the diffusion rate of G2 across those potential adopters who are unique to G2 (represented by m_2). For a typical product line, the diffusion rate of G2 is faster across those who have already adopted G1 than across those G2-unique potential adopters, primarily because the existing adopters are more likely to pay attention to G2 and appreciate its value, and may also have higher affordability and be more innovative. As an evidence, an article from Apple Insider (Hughes 2010) reports that 77% of the new iPhone 4s are sold to existing iPhone users. A similar phenomenon is observed in the software market. For instance, for various reasons, those who have not purchased an earlier version of Microsoft Office are less likely to purchase a new version than those who have used the product before. Therefore, given that the diffusion rate is $f_2(t-\tau_2)$ across those G2-unique potential adopters, we assume that the rate is $((m_1F_1(t))/(m_1F_1(t)-U_2(t)))f_2(t-\tau_2)$ for those who have already adopted G1. We refer to this rate as the *switching multiplier*. With this multiplier, we have

$$w_2(t) = [m_1F_1(t)-U_2(t)]\cdot\frac{m_1F_1(t)}{m_1F_1(t)-U_2(t)}f_2(t-\tau_2) = m_1F_1(t)f_2(t-\tau_2).$$

[2] This is quite reasonable because the same underlying factors (e.g., characteristics of the population, product, and the market) that govern the basic diffusion rate should also influence the speed of leapfrogging and switching.

Once again, our switching multiplier is similar to those adopted by Danaher et al. (2001) and Jun and Park (1999)—in all three models, the time-varying switching multiplier is a function of the same parameters used to define the basic adoption rate for each generation, and is particularly influenced by the adoption rate of the new generation. More importantly, all three models assume that the switching multiplier is larger than the probability that a G2-unique potential adopter will adopt the new generation. In addition, all three models are supported by empirical data (Danaher et al. 2001, Jun and Park 1999; §4 of this study).

Because of leapfrogging after time τ_2, the noncumulative adoption rate for G1 at time t, denoted by $y_1(t)$, takes the following form:

$$y_1(t) = \begin{cases} m_1 f_1(t), & t < \tau_2, \\ m_1 f_1(t) - u_2(t) \\ \quad = m_1 f_1(t)[1 - F_2(t-\tau_2)], & t \geq \tau_2. \end{cases} \tag{9}$$

The cumulative number of adoptions of G1 by time t is

$$Y_1(t) = \begin{cases} m_1 F_1(t), & t < \tau_2, \\ m_1 F_1(t) - U_2(t) \\ \quad = m_1 F_1(t) - m_1 \int_{\tau_2}^{t} f_1(\theta) F_2(\theta - \tau_2)\, d\theta, & t \geq \tau_2. \end{cases} \tag{10}$$

Taking into consideration the potential adopters that are unique to G2 (counted in m_2), as well as leapfrogging and switching adoptions (by those counted in m_1), we obtain the adoption rate for G2:

$$\begin{aligned} y_2(t) &= m_2 f_2(t-\tau_2) + u_2(t) + w_2(t) \\ &= [m_2 + m_1 F_1(t)] f_2(t-\tau_2) \\ &\quad + m_1 f_1(t) F_2(t-\tau_2), \qquad t \geq \tau_2. \end{aligned} \tag{11}$$

Hence, the cumulative number of adopters of G2 is

$$\begin{aligned} Y_2(t) &= m_2 F_2(t-\tau_2) + U_2(t) + W_2(t) \\ &= [m_2 + m_1 F_1(t)] F_2(t-\tau_2), \quad t \geq \tau_2. \end{aligned} \tag{12}$$

It is worth noting that the cumulative number of leapfrogging and switching adoptions by time t is

$$U_2(t) + W_2(t) = m_1 F_1(t) F_2(t-\tau_2), \tag{13}$$

which is the same as the substitution term in Equations (1) and (2) of the NB model.

Based on the above results, we also obtain the number of units in use for each generation. The number of units in use of G1 equals the cumulative number of adoptions of G1 minus the number of adopters who have switched to G2, i.e.,

$$S_1(t) = Y_1(t) - W_2(t) = m_1 F_1(t)[1 - F_2(t-\tau_2)]. \tag{14}$$

On the other hand, because G2 is the latest generation, the number of units in use is the same as the cumulative number of adoptions of G2, i.e.,

$$S_2(t) = Y_2(t) = [m_2 + m_1 F_1(t)] F_2(t-\tau_2). \tag{15}$$

Note that Equations (14) and (15) match Equations (1) and (2) of the NB model. Therefore, the proposed model is mathematically consistent with the NB model for the two-generation scenario.[3]

3.1. N-Generation Scenario

We now extend the GNB model to an N-generation scenario. For a generation i (Gi) introduced at time τ_i, we denote its adoption rate and its cumulative number of adoptions at time t, assuming it is the last generation available, by $\tilde{y}_i(t)$ and $\tilde{Y}_i(t)$, respectively. Based on the results for two and three generations, we have

$$\begin{cases} \tilde{y}_1(t) = m_1 f_1(t), & t \geq 0, \\ \tilde{y}_i(t) = [m_i + \tilde{Y}_{i-1}(t)] f_i(t-\tau_i) \\ \qquad\quad + \tilde{y}_{i-1}(t) F_i(t-\tau_i), & t \geq \tau_i,\ i \geq 2, \end{cases} \tag{16}$$

$$\begin{cases} \tilde{Y}_1(t) = m_1 F_1(t), & t \geq 0, \\ \tilde{Y}_i(t) = [m_i + \tilde{Y}_{i-1}(t)] F_i(t-\tau_i), & t \geq \tau_i,\ i \geq 2, \end{cases} \tag{17}$$

where m_i is the market potential unique to generation i. After generation $i+1$ is introduced at time τ_{i+1}, the rate of leapfrogging from generation i to generation $i+1$ equals

$$u_{i+1}(t) = \tilde{y}_i(t) F_{i+1}(t-\tau_{i+1}), \quad t \geq \tau_{i+1}, \tag{18}$$

and the cumulative number of such leapfrogging adoptions is

$$U_{i+1}(t) = \int_{\tau_{i+1}}^{t} u_{i+1}(\theta)\, d\theta. \tag{19}$$

Similarly, the rate of switching from generation i to generation $i+1$ equals

$$w_{i+1}(t) = \tilde{Y}_i(t) f_{i+1}(t-\tau_{i+1}), \quad t \geq \tau_{i+1}, \tag{20}$$

and the cumulative number of such switching adoptions is

$$W_{i+1}(t) = \int_{\tau_{i+1}}^{t} w_{i+1}(\theta)\, d\theta. \tag{21}$$

[3] Following a similar logic, we can extend the above model to the three-generation scenario. The detailed derivation can be found from the e-companion to this paper (available at http://ssrn.com/abstract=2046847) or can be obtained from the authors.

Because of leapfrogging, the adoption rate for generation i $(1 < i < N)$ is reduced after time τ_{i+1}. Hence,

$$y_i(t) = \begin{cases} \tilde{y}_i(t), & \tau_i \le t < \tau_{i+1}, \\ \tilde{y}_i(t) - u_{i+1}(t), & t \ge \tau_{i+1}, \end{cases} \tag{22}$$

and its cumulative form is

$$Y_i(t) = \begin{cases} \tilde{Y}_i(t), & \tau_i \le t < \tau_{i+1}, \\ \tilde{Y}_i(t) - U_{i+1}(t), & t \ge \tau_{i+1}. \end{cases} \tag{23}$$

The adoption rate and the cumulative number of adoptions for the last generation, generation N, take the following functional forms:

$$y_N(t) = \tilde{y}_N(t), \quad t \ge \tau_N, \tag{24}$$

$$Y_N(t) = \tilde{Y}_N(t), \quad t \ge \tau_N. \tag{25}$$

Based on the above derivations, the number of units-in-use for each generation can also be obtained:

$$\begin{cases} S_i(t) = Y_i(t) - W_{i+1}(t) \\ \qquad = \tilde{Y}_i(t)[1 - F_{i+1}(t - \tau_{i+1})], \quad i < N, \\ S_N(t) = Y_N(t) = \tilde{Y}_N(t). \end{cases} \tag{26}$$

Equations (16) through (26) jointly constitute the *generalized Norton–Bass model*. It can be verified that the expressions shown in (26) are exactly the same as the NB model for all generations. This serves as a theoretical verification that although they are not separately computed, both leapfrogging and switching adoptions are implicitly included in the NB model.

3.2. Comparison with the NB Model and Other Multigeneration Diffusion Models

The proposed GNB model includes exactly the same set of model parameters as the NB model and is mathematically consistent with the later. However, because the GNB model distinguishes leapfrogging from switching, it offers some important advantages over the NB model:

- The GNB model can fit both units-in-use data and sales data,[4] whereas the NB model can only fit units-in-use data. The GNB model can predict both units in use and the adoption rate, whereas the NB model can estimate only units in use. The GNB model can estimate repeat purchases across generations, whereas the NB model cannot.
- The GNB model can be used to project revenue for more scenarios than the NB model. Specifically, depending on the type of products/services, a firm can generate its revenue through one-time product sale (e.g., dynamic random-access memory (DRAM), TV sets), continuous service (e.g., software license, cable TV), or both product sale and continuous service (e.g., printers and inks, cellular phones, and cellular network service). For durable goods, the NB model can be used to project revenue only if revenue is generated through continuous services, whereas the GNB model can project revenue for all three scenarios.
- As we will show in §5, the GNB model can capture the influence of marketing mix variables on the diffusion of each generation, whereas the NB model does not consider such influence.

There are several existing multigeneration models (Mahajan and Muller 1996, Jun and Park 1999, Danaher et al. 2001, Jiang 2010) that also separate leapfrogging from switching. The GNB model compares favorably to other models on multiple aspects. It is the only model that can be fit to both units-in-use and sales data and provides closed-form expressions for both the number of units in use and the adoption rate. It is also the only model that is mathematically consistent with the NB model, which is an important advantage because the NB model remains the most applied and extended multigeneration diffusion model to date. Similar to the Type I model proposed by Jun and Park (1999), the GNB model provide formulations for any number of generations. In addition, as a continuous-time-based model, the GNB model is more flexible than discrete-time models. For instance, it can capture the diffusion dynamics within a time period, and parameter values can be estimated even if available data are not for consecutive time periods. Furthermore, the GNB model uses time-varying leapfrogging and switching multipliers as advocated by Danaher et al. (2001). Our empirical analyses also indicate that models using time-varying multipliers tend to deliver better fit to data. Last, similar to the models of Jun and Park (1999) and Danaher et al. (2001), the GNB model can incorporate the effect of marketing mix variables, which is an important advantage over the models that do not include marketing mix variables.

4. Empirical Analysis

In this section, we empirically evaluate the performance of the GNB model in relation to existing multigeneration diffusion models. In addition, we show that the GNB model can help us gain a better understanding of the process of multigeneration diffusion.

4.1. Performance Comparison

Because the GNB model can fit both sales data and units-in-use data, we compare it separately with two different groups of models. The first group includes the models proposed by Mahajan and Muller (1996)

[4] If repeat purchases are insignificant, sales data can be used to approximate the adoption rate.

Table 1 Parameter Estimates of GNB for U.S. Cellular Subscribers

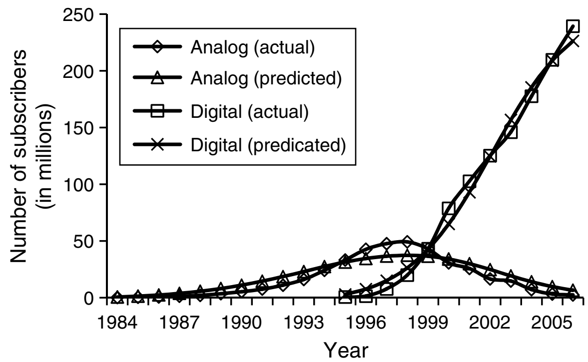
Parameter	Estimate	Approx. std. err.	Approx. Pr > |t|
p	0.00943	0.00435	0.0426
q_1	0.337	0.0788	0.0004
q_2	0.477	0.1252	0.0011
m_1	5.03×10^7	6.32×10^6	<0.0001
m_2	21.1×10^7	30.6×10^6	<0.0001

and Danaher et al. (2001), and Jun and Park's (1999) Type I model; these three models can fit only units-in-use data. The second group includes Jun and Park's (1999) Type II model and the model by Jiang (2010); both are suitable only for sales data. Similar to Danaher et al. (2001), the SAS PROC MODEL procedure with the full information maximum likelihood method is used to estimate the parameters for all tested models. Consistent with more recent studies (e.g., Islam and Meade 1997, Danaher et al. 2001), whenever applicable, we let the coefficient of innovation (p_i) remain constant and the coefficient of imitation (q_i) differ across generations.

4.1.1. Units-In-Use Data. We adopt a data set from the World Telecommunication/ICT Indicators Database, which includes the numbers of analog (G1) and digital (G2) cellular phone subscribers in the United States each year. The observations for analog subscriptions range from 1984 to 2006, and those for digital subscriptions range from 1995 to 2006. We first fit the GNB model to this data set. The parameter estimates are summarized in Table 1. All estimates are significant at the 95% level. The adjusted R^2 values for the two generations are 0.8564 and 0.9936, respectively. The actual and predicated numbers of subscribers are shown in Figure 2. We can see that the GNB model fits the data quite well.

The U.S. cellular subscription data are also used to test the three other models that are suitable for units-in-use data. Among the three, Jun and Park's (1999) Type I model and the model of Danaher et al. (2001) both offer good fit to the data. The model by Mahajan and Muller (1996), on the other hand, does not converge when the necessary positive-value constraint is imposed on the leapfrogging/switching multiplier. Therefore, we leave out Mahajan and Muller's (1996) model from further comparison. The sums of square errors (SSEs) for the three compared models, which

Figure 2 Actual and Predicated U.S. Cellular Subscriptions Based on GNB



share the same number of parameters, are summarized in columns 2–4 of Table 2. As shown in the table, the GNB model provides a better fit than Jun and Park's (1999) Type I model for both analog and digital data. In another comparison, although the Danaher et al. (2001) model fits analog subscriptions better than the GNB model, the GNB model fairs better for digital subscriptions. In terms of the overall model fit, the GNB model is the best among the three models.

To compare the three models' forecasting performance, we reestimate the model parameters using the subscription data from 1984 to 2000; the new parameter values are then used to predict the numbers of subscribers for the period 2001–2006. The predicated values and the actual values for the six time periods are compared and the mean absolute derivations (MADs) are summarized in the last three columns of Table 2. The results show that the GNB model outperforms Jun and Park's (1999) Type I model in the predictions of both analog and digital subscriptions. Compared to the Danaher et al. (2001) model, the GNB model delivers a better prediction for digital subscriptions, but not as good a prediction for analog subscriptions. In terms of the overall forecasting accuracy, the GNB model performs slightly better than the Danaher et al. (2001) model; both models outperform Jun and Park's (1999) Type I model by a large margin. Based on the model fit and forecasting performance, we conclude that the GNB model delivers the best overall performance among the models that are suitable for units-in-use data.

Table 2 Comparison of Model Fit and Forecasting Performance for U.S. Cellular Subscribers

	Model fit (SSE)			Six-period-ahead forecast (MAD)		
	Analog	Digital	Overall	Analog	Digital	Overall
GNB	8.42×10^{14}	7.68×10^{14}	16.1×10^{14}	6.19×10^6	29.2×10^6	17.7×10^6
Jun and Park (1999) Type I	17.7×10^{14}	10.7×10^{14}	28.5×10^{14}	9.38×10^6	50.1×10^6	29.7×10^6
Danaher et al. (2001)	1.49×10^{14}	26.7×10^{14}	28.2×10^{14}	5.93×10^6	29.8×10^6	17.9×10^6

Table 3 Parameter Estimates of GNB for DRAM Shipments

Parameter	Estimate	Approx. std. err.	Approx. Pr > $\|t\|$
p	0.00162	0.000048	<0.0001
q_1	0.258	0.00342	<0.0001
q_2	0.194	0.00570	<0.0001
q_3	0.312	0.00537	<0.0001
m_1	3.16×10^5	3.97×10^3	<0.0001
m_2	13.4×10^5	50.6×10^3	<0.0001
m_3	20.2×10^5	179×10^3	<0.0001

4.1.2. Sales Data. For tests on the rate of adoptions, we adopt three generations of quarterly DRAM shipment data from 1974 to 1984.[5] There are 44 data points for G1 (4K), 32 data points for G2 (16K), and 15 data points for G3 (64K). We first fit the GNB model to this data set. All parameter estimates are again statistically significant, as shown in Table 3. The adjusted R^2 values for the three generations are 0.9853, 0.9707, and 0.999, respectively. From Figure 3, we can see that the sales predicated by the GNB model closely match the actual sales for all three generations.

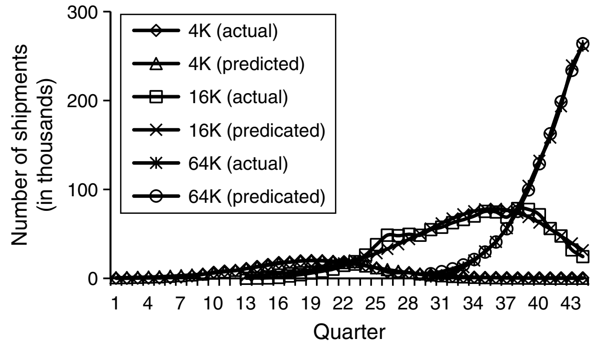
Using the DRAM shipment data, we next compare the GNB model with two other models that are also suitable for sales data, i.e., Jun and Park's (1999) Type II model and Jiang's (2010) two-generation model. These three competing models also have the same number of parameters. Because Jun and Park's (1999) Type II model includes formulations for N generations, we use all three DRAM generations in the comparison. The results for model fit and six-period-ahead forecasting are summarized in Table 4. We can see that the GNB model performs better than Jun and Park's (1999) Type II model on all measures of model fit and forecasting performance.

Because Jiang's (2010) model provides formulations only for the two-generation scenario, it cannot be tested using all three generations of the DRAM data. Therefore, we compare the GNB model and Jiang's (2010) model first using data for G1 and G2 (4K and 16K), and then using data for G2 and G3 (16K and 64K).[6] When G2 and G3 are used, we are only able to conduct two-period-ahead forecasting because Jiang's (2010) model fails to produce a reasonable fit with three or more periods of data held out. Table 5 summarizes the model fit and forecasting performance based on data for G1 and G2, and Table 6 shows the comparison based on data for G2 and G3. Regarding model fit, we can see that the GNB model shows a significantly better fit than Jiang's (2010) model in both tables. Regarding the forecasting performance, except for the 4K DRAM shown in Table 5, the GNB model also delivers more accurate predictions. With both generations considered, the forecasting accuracy of the GNB model is strictly better than that of the Jiang's (2010) model. From the results shown in Tables 4–6, it is evident that the GNB model performs the best among the competing models.

[5] We thank Portia I. Bass for providing us the data. We approached John A. Norton but were not able to obtain the original data used by Norton and Bass (1987).

[6] We have to impose a bound on the leapfrogging multiplier of Jiang's model in order to obtain realistic estimates and reasonable model fit.

Figure 3 Actual and Predicated DRAM Sales Based on the GNB Model



4.2. Identifying Leapfrogging and Switching Adoptions

We now demonstrate how the GNB model can help us develop a more detailed picture of multigeneration diffusion. We again use both the U.S. cellular subscription data and the DRAM shipment data to illustrate the benefits of the GNB model.

4.2.1. U.S. Cellular Subscription. Based on the parameter values shown in Table 1, we calculate the number of leapfrogging and switching adoptions between analog and digital phones, as shown in Figure 4. We can see that the number of switching adoptions first increases and then decreases after the peak is reached, and so does the number of leapfrogging adoptions. As is also evident from the figure, switching dominates leapfrogging in this case. In fact, the number of switching adoptions from 1995 to 2006 is approximately 13 times the number of leapfrogging adoptions during the same period.

We also estimate the rates of initial adoptions over the same time period, which are shown in Figure 5. Based on the estimation, the initial adoptions of analog and digital phones reached their peaks around 1993 and 2002, respectively. Therefore, by the time digital phones were introduced in 1995, the majority of the consumers who are interested in analog phones had already adopted the phone. This explains why the number of leapfrogging adoptions is significantly smaller than the number of switching adoptions from 1995 to 2006. Note that what we conclude from Figure 5 cannot be inferred from the units-in-use data

Table 4 Comparison of Model Fit and Forecasting Performance for DRAM Shipments (G1–G3)

	Model fit (SSE)				Six-period-ahead forecast (MAD)			
	4K	16K	64K	Overall	4K	16K	64K	Overall
GNB	3.17×10^7	106×10^7	18.1×10^7	127×10^7	4.04×10^2	87.0×10^2	95.4×10^2	62.1×10^2
Jun and Park (1999) Type II	98.0×10^7	911×10^7	34.3×10^7	$1{,}040 \times 10^7$	15.2×10^2	216×10^2	$1{,}060 \times 10^2$	432×10^2

Table 5 Comparison of Model Fit and Forecasting Performance for DRAM Shipments (G1–G2)

	Model fit (SSE)			Six-period-ahead forecast (MAD)		
	4K	16K	Overall	4K	16K	Overall
GNB	2.91×10^7	128×10^7	131×10^7	4.13×10^2	83.4×10^2	43.8×10^2
Jiang (2010)	31.8×10^7	438×10^7	470×10^7	1.99×10^2	133×10^2	67.5×10^2

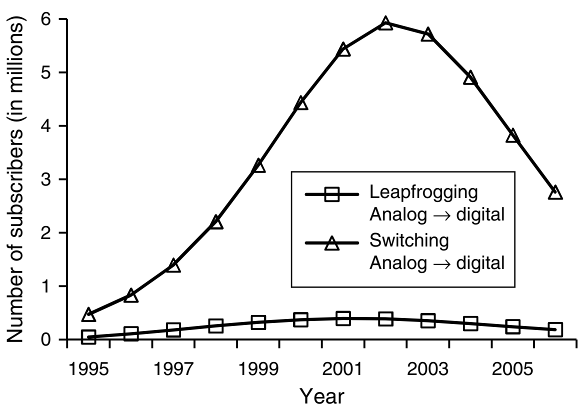
Table 6 Comparison of Model Fit and Forecasting Performance for DRAM Shipments (G2–G3)

	Model fit (SSE)			Two-period-ahead forecast (MAD)		
	16K	64K	Overall	16K	64K	Overall
GNB	11.0×10^8	1.79×10^8	12.8×10^8	7.28×10^3	27.4×10^3	17.3×10^3
Jiang (2010)	61.0×10^8	152×10^8	213×10^8	63.2×10^3	75.4×10^3	69.3×10^3

or from Figure 2. This demonstrates the benefits of examining the units-in-use curve and the initial sales curve separately based on the GNB model.

4.2.2. DRAM Shipments. Using the parameter estimates from Table 3, we calculate the rates of leapfrogging and switching across the three DRAM generations; the results are shown in Figure 6. Note that leapfrogging and switching exist not only from 4K to 16K and from 16K to 64K, but also from 4K directly to 64K. In this example, the rates of leapfrogging between 4K and 64K are much lower than the rates of leapfrogging and switching between consecutive generations during the same period, mainly because m_1 is significantly smaller than m_2. From the figure, we also observe that the rates of leapfrogging and switching from 4K to 16K first increase and then decrease after the peak is reached.

Figure 4 Predicated Leapfrogging and Switching Adoptions Between Cellular Phone Generations



The GNB model also helps us analyze the composition of the adopters at any time. For instance, our results show that out of the adoptions of 64K DRAM between Quarter 30 and Quarter 44, 60% were by adopters only interested in 64K, 33% were switching adoptions from 16K to 64K, and the rest were from other leapfrogging and switching adoptions from 4K or 16K to 64K. Here, the 64K-unique adopters account for the largest share, because m_3 is significantly larger than m_1 and m_2 (see Table 3). This is probably because the computer market expanded during that time period and adopters had strong preference for larger computer memory. This illustration also highlights the additional insights that can be gained based on the GNB model.

Similar to prior studies, because of the limitation of the available data, we are not able to directly compare the estimated rates of leapfrogging and switching

Figure 5 Predicated Cellular Phone Adoption Rates

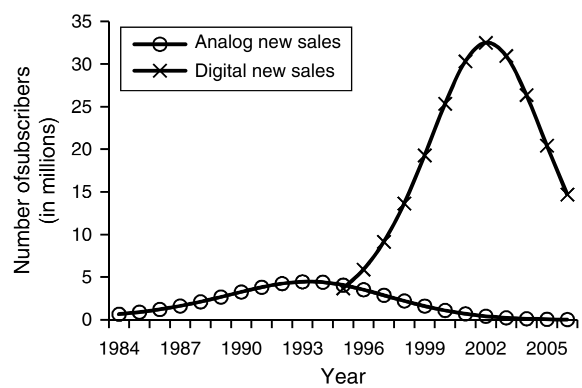
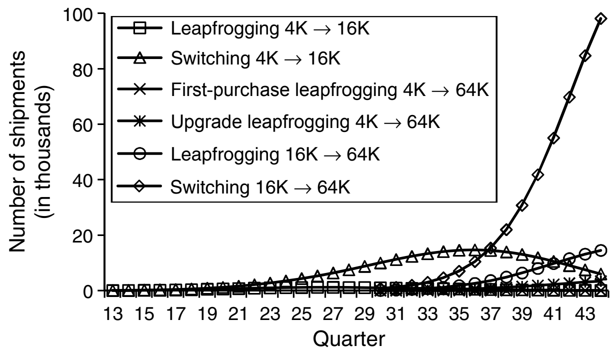


Figure 6 **Predicated Leapfrogging and Switching Adoptions Across DRAM Generations**



against the true rates. Lacking an adequate real data set, we resort to simulation to generate appropriate data sets, which are then used to evaluate the GNB model's performance regarding the separation of leapfrogging and switching adoptions. The simulation is based on the parameter values estimated from the U.S. cellular subscription data. The results show that the rates of leapfrogging and switching projected by the GNB model are better than those produced by competing models (i.e., Jun and Park's (1999) Type I model and the Danaher et al. (2001) model).[7]

5. Incorporating Marketing Mix Variables

It is well documented that marketing mix variables (e.g., price, advertising) can influence the diffusion of a single-generation product (e.g., Jain and Rao 1990, Bass et al. 1994). Several existing multigeneration models have also incorporated marketing mix variables into their formulations (e.g., Speece and MacLachlan 1995, Jun and Park 1999, Danaher et al. 2001). In this section, we explore how the GNB model can be further extended to capture the effect of marketing mix variables.

There are several different ways to capture the influence of marketing mix variables on the speed of diffusion; interested readers are referred to Bass et al. (2000) for further details. After some comparison, we decided to adopt the multiplicative factors of the generalized Bass model (GBM) (Bass et al. 1994), mainly because the GNB model has been empirically tested and demonstrates a number of important advantages, such as preserving the closed-form solution of the Bass (1969) model. In the GBM, the *current marketing effort* and *cumulative marketing effort* for a product at time t are denoted by $x(t)$ and $X(t)$, respectively, with

$$X(t) = \int_0^t x(\theta)\,d\theta.$$

[7] The details can be found from the e-companion or can be obtained from the authors.

With the marketing effort considered, the cumulative and noncumulative market penetration (as a fraction of the market potential) can be represented, respectively by

$$F(t) = \frac{1 - e^{-(p+q)X(t)}}{(q/p)e^{-(p+q)X(t)} + 1} \quad \text{and}$$

$$f(t) = \frac{(p+q)^2}{p} x(t) \frac{e^{-(p+q)X(t)}}{[(q/p)e^{-(p+q)X(t)} + 1]^2}.$$

We next demonstrate that the multiplicative factors for the single-generation GBM can be incorporated into the GNB model. Because market efforts are not expected to be the same across generations, we denote current and cumulative marketing efforts for generation G by $x_G(t)$ and $X_G(t)$, respectively. Analogous to the GBM, incorporating the effect of marketing efforts into Equation (3) yields

$$\mathbf{F_G(t)} = \frac{1 - e^{-(p_G+q_G)X_G(t)}}{(q_G/p_G)e^{-(p_G+q_G)X_G(t)} + 1}. \tag{27}$$

From (27), we obtain the noncumulative rate of diffusion for generation G:

$$\mathbf{f_G}(t) = \frac{(p_G+q_G)^2}{p_G} x_G(t) \frac{e^{-(p_G+q_G)X_G(t)}}{[(q_G/p_G)e^{-(p_G+q_G)X_G(t)} + 1]^2}. \tag{28}$$

Note that in (27) and (28), the notations in the left-hand side are displayed in bold. This is to differentiate them from the results derived without marketing mix variables. The same rule is followed in the remaining discussion, where the bold notations represent quantities derived with the marketing mix variables considered.

Taking into consideration the effect of marketing mix variables, we next examine the diffusion dynamics for a two-generation scenario. Before the introduction of G2, the diffusion of G1 follows the GBM, i.e.,

$$\begin{aligned}\mathbf{Y_1}(t) &= m_1 \mathbf{F_1}(t)\\ &= m_1 \frac{1 - e^{-(p_1+q_1)X_1(t)}}{(q_1/p_1)e^{-(p_1+q_1)X_1(t)} + 1}, \quad t < \tau_2.\end{aligned} \tag{29}$$

After G2 is introduced at time τ_2, some potential adopters of G1 will leapfrog to G2. We assume that the leapfrogging multiplier under this scenario is $\mathbf{F_2}(t - \tau_2)$, because the cumulative market effort for G2 is expected to influence the rate of leapfrogging to G2. Similar to Equation (4), the rate of leapfrogging between G1 and G2 equals

$$\begin{aligned}\mathbf{u_2}(t) &= \lim_{\varepsilon\to 0} \frac{m_1[\mathbf{F_1}(t) - \mathbf{F_1}(t-\varepsilon)]\mathbf{F_2}(t-\tau_2)}{\varepsilon} = m_1 \mathbf{f_1}(t)\mathbf{F_2}(t-\tau_2)\\ &= m_1 \frac{(p_1+q_1)^2}{p_1} x_1(t) \frac{e^{-(p_1+q_1)X_1(t)}}{[(q_1/p_1)e^{-(p_1+q_1)X_1(t)} + 1]^2}\\ &\quad \cdot \frac{1 - e^{-(p_2+q_2)X_2(t-\tau_2)}}{(q_2/p_2)e^{-(p_2+q_2)X_2(t-\tau_2)} + 1}, \quad t \geq \tau_2.\end{aligned} \tag{30}$$

Regarding the rate of switching between G1 and G2, similar to the scenario without marketing mix variables, we assume that the switching multiplier is $((m_1\mathbf{F_1}(t))/(m_1\mathbf{F_1}(t)-\mathbf{U_2}(t)))\mathbf{f_2}(t-\tau_2)$. Hence, the rate of switching between G1 and G2 is

$$\begin{aligned}\mathbf{w_2}(t) &= [m_1\mathbf{F_1}(t)-\mathbf{U_2}(t)]\cdot\frac{m_1\mathbf{F_1}(t)}{m_1\mathbf{F_1}(t)-\mathbf{U_2}(t)}\mathbf{f_2}(t-\tau_2)\\ &= m_1\mathbf{F_1}(t)\mathbf{f_2}(t-\tau_2) = m_1\frac{1-e^{-(p_1+q_1)X_1(t)}}{(q_1/p_1)e^{-(p_1+q_1)X_1(t)}+1}\\ &\quad\cdot\frac{(p_2+q_2)^2}{p_2}x_2(t-\tau_2)\\ &\quad\cdot\frac{e^{-(p_2+q_2)X_2(t-\tau_2)}}{[(q_2/p_2)e^{-(p_2+q_2)X_2(t-\tau_2)}+1]^2},\quad t\ge\tau_2. \end{aligned}\tag{31}$$

Analogous to the derivation shown in §3, once the rates of leapfrogging and switching are determined, the rates of adoptions for G1 and G2 can be easily obtained:

$$\mathbf{y_1}(t) = m_1\mathbf{f_1}(t)[1-\mathbf{F_2}(t-\tau_2)],\quad t\ge\tau_2,\tag{32}$$

$$\begin{aligned}\mathbf{y_2}(t) &= [m_2+m_1\mathbf{F_1}(t)]\mathbf{f_2}(t-\tau_2)\\ &\quad+m_1\mathbf{f_1}(t)\mathbf{F_2}(t-\tau_2),\quad t\ge\tau_2.\end{aligned}\tag{33}$$

The numbers of units in use for the two generations are

$$\mathbf{S_1}(t) = m_1\mathbf{F_1}(t)[1-\mathbf{F_2}(t-\tau_2)],\tag{34}$$

$$\mathbf{S_2}(t) = [m_2+m_1\mathbf{F_1}(t)]\mathbf{F_2}(t-\tau_2).\tag{35}$$

From (32)–(35), we can see that with the marketing mix variables considered, the GNB model still retains the closed-form expressions for both the rate of adoptions and the number of units in use for each generation.

Similarly, by substituting $F_G(t)$ and $f_G(t)$ with $\mathbf{F_G}(t)$ and $\mathbf{f_G}(t)$, the effect of marketing mix variables can be incorporated into the N-generation GNB model. Once again, all closed-form expressions are preserved after this generalization.

The GNB model has the flexibility to take any appropriate functional form to represent the effect of marketing mix variables on multigeneration diffusion. For instance, based on the original study by Bass et al. (1994), we can adopt the following to represent the effect of pricing:

$$X_G(t) = t+\beta_G\,\mathrm{Ln}\left(\frac{v_G(t)}{v_G(0)}\right),\tag{36}$$

$$x_G(t) = 1+\beta_G\frac{v'_G(t)}{v_G(t)},\tag{37}$$

where $v_G(t)$ and $v'_G(t)$ represent the absolute price and the rate of change in price, respectively, for generation G at time t, and β_G reflects the sensitivity to the change in price for generation G.

The functional forms shown in (36) and (37) are not the only options available. For instance, based on the models of Danaher et al. (2001) and Jun and Park (1999), two other functional forms also could be used to represent the effect of price in the GNB model. The first one is copied from the adoption time cumulative distribution function used by Danaher et al. (2001):

$$X_G(t) = \sum_{j=\tau_{G+1}}^{t}\exp[\beta_G v_G(j)].\tag{38}$$

The second one is similar to the form used by Jun and Park (1999), but is revised to maintain the mathematical properties of the GNB model:

$$X_G(t) = t+\beta_G v_G(t).\tag{39}$$

The data sets used in our empirical analysis do not include information on marketing mix variables; hence we are not able to assess the performance of the GNB model with marketing mix variables. If such a data set were available, we could test different functional forms and select the one that leads to the best model fit or forecasting performance. This flexibility is another important advantage of the GNB model.

6. Discussions and Future Research Directions

The development and marketing of successive product generations are very critical to many industries. To develop effective product development and marketing strategies, it is important that firms understand the diffusion dynamics across multiple generations. Two of the most important aspects of multigeneration diffusion are leapfrogging and switching. When multiple generations coexist in the market, newer generations can cannibalize the sale of older generations because of leapfrogging. Switching, on the other hand, not only increases cross-generation repeat purchases, but also helps speed up the diffusion of newer generations, because the probability of switching by existing adopters is expected to be higher than the probability of adoption by first-time adopters. The GNB model developed in this study generalizes the well-known Norton and Bass (1987) model to capture leapfrogging and switching, thus enabling it to estimate both the number of units in use and the adoption rate for each generation. Although existing multigeneration models also provide some of the capabilities of the GNB model, the GNB model not only offers greater flexibility and additional capabilities, but also delivers better overall performance both in terms of model fit and forecasting performance.

As demonstrated in this article, the GNB model is the only model that is mathematically consistent with the NB model, which is an important advantage because the NB model remains the most applied and extended multigeneration diffusion model to date. Furthermore, the GNB model can incorporate the effects of marketing mix variables while still retaining the closed-form expressions. Therefore, the GNB model is well suited to help firms make important decisions regarding the planning and marketing of multigeneration products.

The GNB model could be extended or applied in future research. For instance, the single-generation generalized Bass model has been used to study optimal pricing and advertising polices for single-generation products (Krishnan et al. 1999, Krishnan and Jain 2006). Similarly, based on the GNB model, the total profit for a multigeneraiton product line can be formulated as a function of some marketing mix variables. Based on such a formulation, one could derive the best pricing or advertising polices for multigeneration products. Furthermore, because it can help project future revenue regardless of whether the source of revenue is product sale, continuous service, or both, the GNB model also can help determine the optimal market entry timing for future generations (Wilson and Norton 1989, Mahajan and Muller 1996).

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References

Bass FM (1969) A new product growth model for consumer durables. *Management Sci.* 15(5):215–227.

Bass FM, Jain DC, Krishnan T (2000) Modeling the marketing-mix influence in new-product diffusion. Mahajan V, Muller E, Wind Y, eds. *New-Product Diffusion Models* (Kluwer Academic, Boston), 99–122.

Bass FM, Krishnan TV, Jain DC (1994) Why the bass model fits without decision variables. *Marketing Sci.* 13(3):203–223.

Danaher PJ, Hardie BGS, Putsis WP (2001) Marketing-mix variables and the diffusion of successive generations of a technological innovation. *J. Marketing Res.* 38(4):501–514.

Hughes N (2010) Apple's recurring revenue stream: 77% of iPhone 4 sales were upgrades. *Apple Insider* (June 25), http://www.appleinsider.com/articles/10/06/25/apples_recurring_revenue_stream_77_of_iphone_4_sales_were_upgrades.html.

Islam T, Meade N (1997) The diffusion of successive generations of a technology: A more general model. *Tech. Forecasting Soc. Change* 56(1):49–60.

Jain DC, Rao RC (1990) Effect of price on the demand for durables: modeling, estimation, and findings. *J. Bus. Econom. Statist.* 8(2):163–170.

Jiang Z (2010) How to give away software with successive versions. *Decision Support Systems* 49(4):430–441.

Jun DB, Park YS (1999) A choice-based diffusion model for multiple generations of products. *Tech. Forecasting Soc. Change* 61(1):45–58.

Kim N, Chang DR, Shocker AD (2000) Modeling intercategory and generational dynamics for a growing information technology industry. *Management Sci.* 46(4):496–512.

Krishnan TV, Jain DC (2006) Optimal dynamic advertising policy for new products. *Management Sci.* 52(12):1957–1969.

Krishnan TV, Bass FM, Jain DC (1999) Optimal pricing strategy for new products. *Management Sci.* 45(12):1650–1663.

Mahajan V, Muller E (1996) Timing, diffusion, and substitution of successive generations of technological innovations: The IBM mainframe case. *Tech. Forecasting Soc. Change* 51(2):109–132.

Norton JA, Bass FM (1987) A diffusion theory model of adoption and substitution for successive generations of high-technology products. *Management Sci.* 33(9):1068–1086.

Speece MW, MacLachlan DL (1995) Application of a multi-generation diffusion model to milk container technology. *Tech. Forecasting Soc. Change* 49(3):281–295.

Wilson LO, Norton JA (1989) Optimal entry timing for a product line extension. *Marketing Sci.* 8(1):1–17.

CORRECTION

In this article, "A Generalized Norton–Bass Model for Multigeneration Diffusion" by Zhengrui Jiang and Dipak C. Jain (first published in *Articles in Advance* May 18, 2012, *Management Science*, DOI:10.1287/mnsc.1120.1529), "first-time" has been corrected to read as "potential" throughout §3, and Figure 6 has been updated in §4.