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$\mathbf{E}$xchanges sell both trading services and price information. We study how the joint pricing of these products affects price discovery and the distribution of gains from trade in an asset market. A wider dissemination of price information reduces pricing errors and the transfer from liquidity traders to speculators. This effect reduces the fee that speculators are willing to pay for trading. Therefore, to raise its revenue from trading, a for-profit exchange optimally charges a high fee for price information so that only a fraction of speculators buy this information. As a result, price discovery is not as efficient as it would be with free price information. This problem is less severe if the exchange must compensate liquidity traders for a fraction of their losses.

Key words: sale of market data; transparency; price discovery
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1. Introduction

No member of this board, nor any partner of a member, shall hereafter give the prices of any kind of Stock, Exchange or Specie, to any printer for publication [. . .]. (Constitution of the NYSE Board, 1817; quoted in Mulherin et al. 1991, p. 597)

Black box traders, direct market access traders and algorithmic traders are all in a race to beat the other guy. And the best way to do that is to get their hands on the market data first. (Chapman 2009)

An essential function of securities markets is to discover asset values (see Baumol 1965, Bresnahan et al. 1992, O'Hara 2003). This function is critical for an efficient allocation of capital in the economy, because better price discovery in the stock market translates into better capital allocation decisions (see, e.g., Subrahmanyam and Titman 1999, Bond et al. 2012). For this reason, regulators and academics often see the maximization of price discovery as one important goal. For instance, O'Hara (1997, p. 270) writes, "How well and how quickly a market aggregates and impounds information into the price must surely be a fundamental goal of market design [. . .] If market prices reflect true asset values more quickly and accurately then presumably the allocation of capital can better reflect its best uses." Market design depends in large part on the decisions of stock exchanges, which are now for-profit firms. Exchanges' income derives from trading revenues and increasingly from the sale

of information on prices.[1] In this paper, we show that the efficiency of price discovery is, among other factors, determined by the fee charged by exchanges for price information.[2]

Exchanges often supply information at various speeds, charging a higher fee to traders who receive price information more quickly.[3] In the past, trading took place on the floor of stock or derivatives exchanges. In this case, floor brokers had, thanks to their physical presence on the floor, a much faster access to price information than off-floor traders (see Easley et al. 2013a). Today, trading is increasingly electronic, which greatly accelerates the speed at

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[1] Major exchanges (e.g., NYSE-Euronext, NASDAQ, London Stock Exchange, Chicago Mercantile Exchange) are for profit (see Aggarwal and Dahiya 2006 for a survey of exchanges' governances around the world). A report of the Aite Group estimates that market data revenue accounts for 19% of revenue on average for NYSE-Euronext, NASDAQ, Deutsche Börse, and the Tokyo Stock Exchange. Other revenues stem from the sale of listing services.

[2] We do not model how price discovery affects investment because this link is already formalized in several papers (sometimes in the CARA-Gaussian rational expectations framework used here; see, e.g., Subrahmanyam and Titman 1999).

[3] Information on past trades is generally available for free only after some delay (e.g., 20 minutes on the NYSE, 15 minutes on NASDAQ and Euronext). See http://finance.yahoo.com/exchanges for the delays after which information on transaction prices from major stock exchanges is freely released on http://www.yahoo.com. Real-time price information is not free, and given the speed at which trading takes place in electronic securities markets, information delayed by even one second is already stale.

which price information can be delivered to market participants. Yet there are still fast and slow traders in terms of access to price and trade data (see Hasbrouck and Saar 2011). Indeed, some proprietary trading firms buy direct access to trading platforms' data feeds, and other market participants obtain the same information with a delay, at a lower cost.[4] Trading firms buying direct access to streaming prices enjoy, in cyberspace, the informational advantage that floor traders used to have in physical space.

Do exchanges have sufficient incentives to price their real-time data feeds so that price discovery is maximized, or is there reason for regulatory intervention in this market? Should regulators allow fast and slow tracks for access to price information? How does differential access to price information affect the distribution of gains from trades among traders? Economic analyses of these questions are scarce despite their importance in recent regulatory debates (see, e.g., SEC 2010).

To fill this gap, we consider the market for a security featuring risk-averse speculators and liquidity traders. We interpret speculators as proprietary trading firms (the sell side) specialized in liquidity provision. They post a schedule of prices (limit orders) at which they are willing to absorb liquidity traders' orders. There are two types of speculators: the "insiders," who observe prices in real time (i.e., they know all prices up to the last transaction before submitting their orders), and the "outsiders," who observe past prices with a lag (latency). Because transaction prices contain information on the asset payoff, insiders have an informational advantage over outsiders. The market structures in which speculators are either all insiders or all outsiders result in a level playing field since, in either case, speculators have access to price information at the same speed. Otherwise, some traders (the insiders) receive price information faster than others.

The market is organized by a for-profit exchange that charges a fee for real-time price information and a trading fee (a fixed participation fee) to speculators. In equilibrium, the fraction of insiders is inversely related to the fee for real-time price information. We show that a decrease in this fee is associated with smaller pricing errors (the average squared deviation between the payoff of the security and the transaction price). Indeed, speculators bet more aggressively against pricing errors when they have information on past prices, because they face less uncertainty on the final payoff of their position. Accordingly, price discovery is improved (pricing errors are reduced) when more speculators are insiders, that is, when the fee for price information is low enough.

A reduction in this fee, however, lowers the expected profit of the exchange in two ways— (i) directly (common in models of information sales): the exchange earns less from the sale of information per insider; and (ii) indirectly (specific to the information sale problem studied in our paper): the fee that speculators are willing to pay for trading is smaller because pricing errors are smaller. In particular, we show that speculators' welfare is always smaller when they are all insiders than when they are all outsiders. Accordingly, the exchange optimally restricts access to price information: it sets a high fee for price information so that either no speculator purchases price information or only a fraction of speculators buy this information. In the second case, the market features fast and slow traders in terms of access to price information, very much as is observed in today's markets. In either case, price discovery is not maximal.

This policy makes the market more illiquid because it reduces competition among insiders, which increases the adverse price impact of liquidity traders' orders and therefore their trading losses. Hence, by charging a high fee for price information, an exchange risks losing liquidity traders unless it reduces the cost of trading for these investors. We account for this effect by considering the possibility that the exchange compensates liquidity traders for a fraction of their trading losses. We show numerically that as this fraction increases, the exchange lowers its price for information, which works to improve price discovery and market liquidity.

To sum up, limited access to information on past prices softens competition among speculators. Hence, it increases speculators' average welfare at the expense of liquidity traders and price discovery. In the past, exchanges were owned by proprietary trading firms, and they strove to restrict the dissemination of information on prices (see the opening quotation in the introduction). This was indeed in the best interest of their members, according to the model. In today's markets, exchanges are for profit and regulation does not allow them to disclose transaction prices selectively (e.g., only to their members).[5] However, by

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[4] This delay can be very small, but it is still sufficient to give an informational advantage to traders with direct access to market data. For instance, U.S. trading platforms must also transmit their data to plan processors (the Consolidated Tape Association and Consolidated Quote Association), which consolidate the data and distribute them to the public (the proceeds are then redistributed among contributors). Because this process takes time, market participants with direct access to the trading platforms' data feeds can obtain market data faster than participants who obtain the data from plan sponsors (for a discussion, see SEC 2010, §IV.B.2).

[5] In the United States, stock exchanges must make their data available since 1975, according to the so-called Quote Rule. See Mulherin et al. (1991) for a historical account of how exchanges established their property rights over market data.

setting a high fee for price information, they can still soften competition among trading firms and recover a fraction of their rents by charging participation fees. Our model suggests that this practice harms price discovery and liquidity traders. It therefore offers a rationale for regulating the sale of price information by exchanges.

Our framework is closely related to Easley et al. (2013b), and some of our findings are similar. For instance, as in Easley et al. (2013b), liquidity traders' losses (or speculators' welfare) are minimal when price information is free in our model. Our paper differs from their paper in three important ways, however. First, our focus is different: Easley et al. (2013b) show that the cost of capital is minimal when all investors have price information, whereas we show that price discovery is maximal in this case. Both results imply that a greater dissemination of price information should affect firms' real decisions but for different reasons: a lower cost of capital enables firms to invest more, whereas a more informationally efficient market enables firms to better identify positive net present value projects (Bond et al. 2012).

Second, we do not model information sales in the same way: In Easley et al. (2013b), the seller of price information does not derive revenues from trading fees, whereas it does in our model. As shown in §7.3, and discussed further in §7.4, this difference matters. Depending on the extent to which the exchange internalizes liquidity traders' losses, allowing the exchange to sell price information reduces or increases the price of price information relative to the price charged by a pure information seller. Thus, the way in which price information is sold affects welfare and price discovery. Moreover, in our setting, it can be optimal for the exchange to abstain from selling price information, although this is never the case when the exchange only derives revenues from information.

Last, we show that there exist cases in which speculators' average welfare is maximal for an interior fraction of insiders, that is, when there is differential access to price information among speculators. This result, which plays an important role for the analysis of the exchange's optimal selling policy, is not derived by Easley et al. (2013b).

2. Related Literature

Our paper is also related to the literature on financial market transparency (see, e.g., Biais 1993, Madhavan 1995, Pagano and Roëll 1996, Bloomfield and O'Hara 1999, Boehmer et al. 2005, Rindi 2008) and the literature on information sales (e.g., Admati and Pfleiderer 1986, 1987, 1990; Fishman and Hagerthy 1995; Veldkamp 2006; Cespa 2008; Lee 2009, Garcia and Sangiorgi 2011).

The literature on transparency mainly focuses on pretrade transparency: the information on current quotes and orders (e.g., information on posted limit orders). In contrast, posttrade transparency (the swift dissemination of information on the terms of trades) has not received much attention, although it is frequently discussed in regulatory debates. One issue is whether exchanges have natural incentives to be posttrade transparent. We contribute to this question by showing that an exchange always optimally restricts access to posttrade information when its revenues primarily derive from the sell side (proprietary trading firms). It may sometimes even be optimal for the exchange to provide no posttrade information at all, which provides an explanation for the existence of market structures with high posttrade opacity (e.g., over-the-counter markets).

We depart from the literature on information sales because an exchange is not a pure information seller (as in, e.g., Admati and Pfleiderer 1986); it also obtains revenues from the sale of trading services. As shown in §7.3, this feature creates systematic differences between the optimal pricing of information by an exchange and that of a pure information seller. Furthermore, the literature on information sales has shown that it is sometimes optimal for the seller to add noise to its information. Exchanges do not add noise to their price reports, maybe because this would be considered illegal. Instead, they have the possibility to delay the moment at which they disclose posttrade information for free, an aspect not analyzed in the literature on information sales. In §7.3, we show that a pure information seller never releases delayed information for free (unless obliged by regulation), whereas an exchange sometimes finds it optimal to do so.[6]

In reality, proprietary trading firms buy information on two distinct types of prices: (i) quotes in the market at a given point in time (the entire limit order book for a security) and (ii) prices at which transactions actually took place (including the most recent transaction price). Both types of data are useful to reduce uncertainty on execution prices for market orders ("execution risk"). This motivation for buying price information is analyzed by Boulatov and Dierker (2007). It is not present in our model: as speculators submit limit orders, they face no uncertainty on the price at which they trade a given quantity. Yet, as we show here, speculators value information on prices because prices help them better forecast future

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[6] In the literature on information sales, the information seller must commit to provide information truthfully because his information is in general not verifiable. This commitment problem does not arise for transaction prices, since these prices are verifiable. This is another reason why the sale of price information is different from other types of information sale.

price changes. This is another major reason that proprietary trading firms (e.g., high-frequency traders) want to get super fast access to prices in real time.[7] Thus, our approach is complementary to Boulatov and Dierker (2007).

Last, our paper contributes to the research agenda described by Cantillon and Yin (2011). They call for using a combination of industrial organization and finance to understand market structures as the "microstructure of exchanges is often part of their business models" (Cantillon and Yin 2011, p. 335). This is precisely our approach: the posttrade transparency of the market (the fraction of insiders) and the efficiency of price discovery are ultimately determined by the optimal pricing decisions of the exchange in our model.

3. Model

We consider the market for a risky asset with payoff $v \sim N(0, \tau_v^{-1})$. Trades take place at dates $1, 2, \ldots, N$ between two types of traders: a continuum of speculators (indexed by $i \in [0, 1]$) and liquidity traders. Traders leave the market at the end of each trading round and are replaced by a new cohort of traders. We denote by e_n the net trade from liquidity traders at date n. A positive (negative) value of e_n means that liquidity traders are net sellers (buyers) of the security. The variable, e_n, is normally distributed with mean $\bar{e}$ and variance τ_e^{-1}. We set $\bar{e} = 0$, that is, the asset is in zero net aggregate supply. This assumption simplifies some derivations and is not key for our main findings, as shown in §A of the online appendix (available at http://www.cass.city.ac.uk/__data/assets/pdf_file/0003/171075/internet-appendix-MS-February-2013.pdf).

Each speculator i at date n observes a private signal s_{in} about the payoff of the security:

$$s_{in} = v + \epsilon_{in}, \qquad (1)$$

where $\epsilon_{in} \sim N(0, \tau_{\epsilon_n}^{-1})$. The precision of private signals τ_{ϵ_n} is the same for all speculators in any period n but can change across periods. "Fresh" information is available at date n if the precision of speculators' signals at this date is strictly positive, $\tau_{\epsilon_n} > 0$. Error terms ϵ_{in} are independent across speculators, across periods, and from v and e_n. Moreover, they cancel out on average (i.e., $\int_0^1 s_{in} \, di = v$, almost surely).

[7] For instance, in describing the activity of one major high-frequency trading firm on Chi-X (the third largest European trading platform), Jovanovic and Menkveld (2011, p. 38) write that this firm "is particularly well positioned to quickly do the statistics and infer a security's change in fundamental value by tracking price series that are correlated with it, e.g., the index level, same industry stocks, foreign exchange rate, etc." In the same vein, Brogaard et al. (2012) show that high-frequency traders exploit the information about subsequent returns contained in posted limit orders.

We denote by p_n the clearing price at date n and by p^n the record of all transaction prices up to date n (the "ticker"):

$$p^n = \{p_t\}_{t=0}^n, \quad \text{with } p_0 = E[v] = 0. \qquad (2)$$

Speculators differ in their speed of access to ticker information. Speculators with type I (the *insiders*) observe the ticker in real time, whereas speculators with type O (the *outsiders*) observe the ticker with a lag equal to $l \geq 2$ periods. That is, insiders arriving at date n observe p^{n-1} before submitting their orders, and outsiders arriving at date n observe p^{n-l^*}, where $l^* = \min\{n, l\}$. That is,

$$p^{n-l^*} = \begin{cases} \{p_0, p_1, p_2, \ldots, p_{n-l}\} & \text{if } n > l, \\ p_0 & \text{if } n \leq l. \end{cases} \qquad (3)$$

We refer to p^n as the "real-time ticker" and to p^{n-l^*} as the "lagged ticker." The "delayed ticker" is the set of prices unobserved by outsiders (i.e., $p^n \backslash p^{n-l^*}$). The proportion of insiders at date n is denoted by μ. When $0 < \mu < 1$, some speculators (the insiders) have access to ticker information faster than other speculators. In the first period, the distinction between insiders and outsiders is irrelevant, since there are no prior transactions. We refer to l as the *latency in information dissemination* and to μ as the *scope of information dissemination (in real time)*. Figure 1 shows the timing of the model.

Each speculator has a CARA utility function with risk tolerance γ. Thus, if speculator i holds x_{in} shares of the risky security at date n, her expected utility is

$$E[U(\pi_{in}) \mid s_{in}, \Omega_n^k] = E[-\exp\{-\gamma^{-1}\pi_{in}\} \mid s_{in}, \Omega_n^k], \qquad (4)$$

where $\pi_{in} = (v - p_n)x_{in}$ and Ω_n^k is the price information available at date n to a speculator with type $k \in \{I, O\}$.

As usual in a rational expectations model, the clearing price in each period aggregates speculators' private signals and therefore constitutes an additional signal about the asset payoff. As speculators submit price-contingent demand functions, they all act as if they were observing the contemporaneous clearing price (whether they have information on past transaction prices or not). Thus, in period $n \geq 2$, we have $\Omega_n^I = \{p^n\}$ and $\Omega_n^O = \{p^{n-l^*}, p_n\}$. We denote the demand function of an insider by $x_n^I(s_{in}, p^n)$ and that of an outsider by $x_n^O(s_{in}, p^{n-l^*}, p_n)$. The clearing price in period n, p_n, is such that speculators' aggregate demand is equal to the asset supply, e_n, in this period; that is,

$$\int_0^\mu x_n^I(s_{in}, p^n) \, di + \int_\mu^1 x_n^O(s_{in}, p^{n-l^*}, p_n) \, di = e_n \quad \forall n. \qquad (5)$$

The structure of the model in each period is similar to other rational expectations model (e.g., Hellwig 1980).

Figure 1   The Timeline
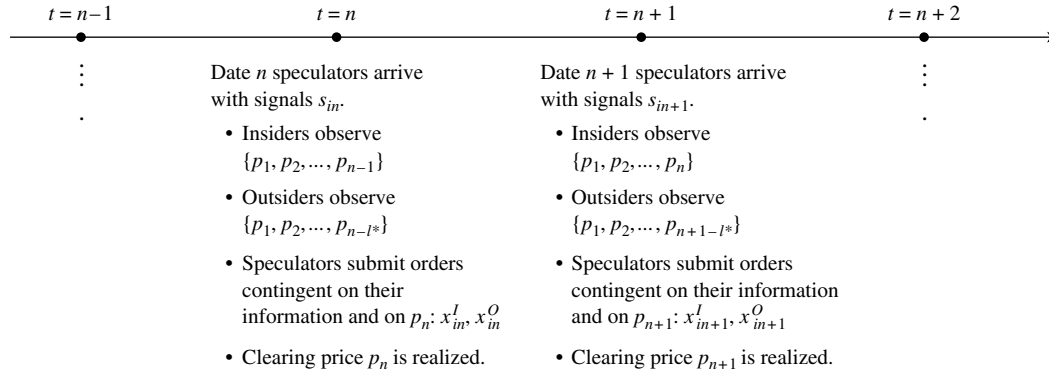
Multiperiod rational expectations models usually assume that all investors have information on past prices; i.e., $\mu = 1$ (see, e.g., Grundy and McNichols 1989, Cespa and Vives 2012). We consider the more general case, in which $0 \leq \mu \leq 1$, so that some speculators have faster access to information on past prices than others. The first step in our analysis is to study the equilibrium of the security market in each period (next section). We then analyze the effect of varying μ on price discovery and speculators' welfare. Finally, we endogenize the scope of information dissemination, μ, by introducing a market for price information.

Investors (including proprietary trading firms) buy information about past transaction prices because they do not participate in all transactions and therefore do not automatically know the terms of prior transactions. To capture this in the simplest way, we assume that speculators stay in the market for only one period. A more general model could also feature speculators who participate in all trading rounds ("recurrent speculators") in addition to "episodic" speculators, who stay for one trading round. Only episodic speculators need information on the price history (recurrent speculators know this history because they participate in all trades). We have studied this more general case when $n = 2$ (see §B in the online appendix). The model becomes significantly more difficult to analyze, but it does not deliver additional insights relative to the case with only episodic speculators.

4. Equilibrium Prices with Differential Access to Price Information

We refer to $\tau_n(\mu, l) \stackrel{\text{def}}{=} (\text{Var}[v \mid p^n])^{-1}$ as the informativeness of the real-time ticker at date n and to $\hat{\tau}_n(\mu, l) \stackrel{\text{def}}{=} (\text{Var}[v \mid p^{n-l^*}, p_n])^{-1}$—the precision of outsiders' forecasts conditional on their price information at date n—as the informativeness of the "truncated" ticker. Lemma 1 provides a characterization of

the unique linear rational expectations equilibrium in each period.

Lemma 1. *In any period n, there is a unique linear rational expectations equilibrium. In this equilibrium, the price is given by*

$$p_n = A_n v - \sum_{j=0}^{l^*-1} B_{n,j} e_{n-j} + D_n E[v \mid p^{n-l^*}], \qquad (6)$$

where A_n, $\{B_{n,j}\}_{j=0}^{l^*-1}$, D_n are positive constants characterized in the proof of the lemma. Moreover, speculators' demand functions are given by

$$x_n^I(s_{in}, p^n) = \gamma(\tau_n + \tau_{\epsilon_n})(E[v \mid s_{in}, p^n] - p_n), \qquad (7)$$

$$x_n^O(s_{in}, p^{n-l^*}, p_n) = \gamma(\hat{\tau}_n + \tau_{\epsilon_n})(E[v \mid s_{in}, p^{n-l^*}, p_n] - p_n), \quad (8)$$

where $\tau_n + \tau_{\epsilon_n} \equiv \text{Var}[v \mid p^n, s_{in}]^{-1}$ and $\hat{\tau}_n + \tau_{\epsilon_n} \equiv \text{Var}[v \mid p_n, p^{n-l^*}, s_{in}]^{-1}$.

To interpret the expression for the equilibrium price, consider the case in which $l = 2$ (the same interpretation applies for $l > 2$). In this case, Equation (6) becomes

$$p_n = A_n v - B_{n,0} e_n - B_{n,1} e_{n-1} + D_n E[v \mid p^{n-2}]$$
$$\text{for } n \geq 2. \qquad (9)$$

We now contrast two particular cases. In the first case, speculators do not receive fresh information at dates $n-1$ and n, whereas in the second case, fresh information is available at date $n-1$ but not at date n. For the discussion, we define $z_n \stackrel{\text{def}}{=} a_n v - e_n$ and $a_n \stackrel{\text{def}}{=} \gamma \tau_{\epsilon_n}$.

Case 1: No fresh information is available at dates $n-1$ and n ($\tau_{\epsilon_{n-1}} = \tau_{\epsilon_n} = 0$, for $n \geq 3$). In this case, $A_n = 0$, $B_{n,0} = (\gamma \tau_{n-2})^{-1}$, $B_{n,1} = 0$, and $D_n = 1$ (see the expressions for these coefficients in the appendix). Thus, the equilibrium price at date n is

$$p_n = E[v \mid p^{n-2}] - (\gamma \tau_{n-2})^{-1} e_n. \qquad (10)$$

Because speculators entering the market at dates n and $n-1$ do not possess fresh information, the

clearing price at date n cannot reflect information above and beyond that contained in the lagged ticker, p^{n-2}. Thus, the clearing price is equal to the expected value of the security conditional on the lagged ticker adjusted by a risk premium (the compensation required by speculators to absorb liquidity traders' net supply).

Case 2: *Fresh information is available at date $n-1$ but not at date n ($\tau_{\epsilon_n} = 0$ but $\tau_{\epsilon_{n-1}} > 0$).* In this case, the transaction price at date $n-1$ contains new information on the asset payoff ($A_{n-1} > 0$). Specifically, we show in the proof of Lemma 1 (see the appendix) that the observation of the price at date $n-1$ conveys a signal $z_{n-1} = a_{n-1}v - e_{n-1}$ on the asset payoff. Moreover, the equilibrium price at date n can be written as follows:

$$p_n = E[v \mid p^{n-2}] + A_n a_{n-1}^{-1}(z_{n-1} - E[z_{n-1} \mid p^{n-2}]) - B_{n,0}e_n. \tag{11}$$

If $\mu = 0$, we have $A_n = 0$ and the expression for the equilibrium price at date n is identical to its formulation in Case 1 (Equation (10)). Indeed, in this case, no speculator observes the last transaction price. Thus, the information contained in this price (z_{n-1}) cannot transpire into the price at date n.

In contrast, if $\mu > 0$, some speculators at date n observe the last transaction price and trade on this information. Thus, the information contained in the price at date $n-1$ "percolates" into the price at date n and the latter is informative ($A_n > 0$), even though there is no fresh information at date n. Specifically, Equation (11) shows that an outsider can extract a signal $\hat{z}_n$, from the clearing price at date n:

$$\hat{z}_n = \alpha_1 z_{n-1} - \alpha_0 e_n = \alpha_1 z_{n-1} + \alpha_0 z_n, \tag{12}$$

with $\alpha_0 \stackrel{\text{def}}{=} A_n^{-1} B_{n,0}$ and $\alpha_1 \stackrel{\text{def}}{=} a_{n-1}^{-1}$. This signal does not perfectly reveal insiders' information (z_{n-1}) because it also depends on the supply of the risky security at date n (e_n). Thus, at date n, outsiders obtain information ($\hat{z}_n$) from the clearing price, but this information is not as precise as insiders' information (since $\alpha_0 > 0$). For this reason, being an insider is valuable in our set-up.

When no fresh information is available at dates $\{n-1, \ldots, n-l^*+1\}$ (as in Case 1), observing the delayed ticker is useless and there is no difference between insiders and outsiders. Thus, to focus on the interesting case, we assume from now on that, at any date n, there is at least one date $j \in \{n-1, \ldots, n-l^*+1\}$ at which fresh information is available (i.e., $\tau_{\epsilon_j} > 0$). This restriction does not exclude the possibility that no fresh information is available at date n (i.e., $\tau_{\epsilon_n} = 0$, as in Case 2).

In general, the price at date n contains information on the asset payoff (i.e., $A_n > 0$) because

(i) speculators' demand depends on their private signals (when $\tau_{\epsilon_n} > 0$) and (ii) insiders' demand depends on the signals $\{z_{n-j}\}_{j=1}^{l^*-1}$ that they extract from the prices yet unobserved by outsiders at date n (as in Case 2). For outsiders, the clearing price at date n conveys the following signal (see the proof of Lemma 1):

$$\hat{z}_n = \sum_{j=0}^{l^*-1} \alpha_j z_{n-j}, \tag{13}$$

where the coefficient α_j is positive, for $j = 0, 1, \ldots, l^*-1$. Intuitively, the signal $\hat{z}_n$ provides a less precise estimate of the asset payoff than the set of signals $\{z_{n-j}\}_{j=0}^{l^*-1}$ since $\hat{z}_n$ is a linear combination of the signals in this set. Hence, the current clearing price is not a sufficient statistic for the entire price history, as the latter enables insiders to recover the signals $\{z_{n-j}\}_{j=1}^{l^*-1}$. For this reason, observing past prices has value even though speculators can condition their demand on the contemporaneous clearing price.[8] We analyze the determinants of this value in §7.

5. Price Discovery and the Scope of Information Dissemination

We now study the effect of the scope of information dissemination (μ) and of latency (l) on price discovery. We first consider how these variables affect (i) the informativeness of the "truncated ticker," $\hat{\tau}_n(\mu, l) = (\text{Var}[v \mid p^{n-l^*}, p_n])^{-1}$; and (ii) the informativeness of the real-time ticker, $\tau_n(\mu, l) = (\text{Var}[v \mid p^n])^{-1}$. The first measure of price informativeness takes outsiders' viewpoint since it measures the residual uncertainty on the asset payoff conditional on the prices that outsiders observe. The second measure takes insiders' viewpoint.

Let $\psi_n^m(\mu, l) \stackrel{\text{def}}{=} (\text{Var}[\hat{z}_n \mid v])^{-1}$. Proposition 1 shows that ψ_n^m is the contribution of the nth clearing price to the informativeness of the truncated ticker. For this reason, we refer to ψ_n^m as the informativeness of the nth clearing price for outsiders.

PROPOSITION 1. *At any date $n \geq 2$, the informativeness of the truncated ticker and the real-time ticker are as follows:*

1. *The informativeness of the truncated ticker, $\hat{\tau}_n$, is*

$$\hat{\tau}_n(\mu, l) = \tau_{n-l^*} + \psi_n^m(\mu, l), \tag{14}$$

where $\hat{\tau}_n(\mu, l)$ increases in the scope of information dissemination (μ), (weakly) decreases in latency (l), and is

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[8] In Brown and Jennings (1989) and Grundy and McNichols (1989), clearing prices are not a sufficient statistic for past prices as well. In contrast, Brennan and Cao (1996) and Vives (1995) develop multiperiod models of trading in which the clearing price in each period is a sufficient statistic for the entire price history. In this case, observing past prices has no informational value.

strictly smaller than the informativeness of the real-time ticker, τ_n.

2. *The informativeness of the real-time ticker, τ_n, is independent of latency and the scope of information dissemination. It is given by*

$$\tau_n(\mu, l) = \tau_v + \tau_e \sum_{t=1}^{n} a_t^2, \quad \text{with } a_t = \gamma \tau_{\epsilon_t}. \quad (15)$$

To save space, the proof of this result and the proofs of all subsequent results are shown in the online appendix.

As explained previously, the nth clearing price is informative about the signals $\{z_{n-j}\}_{j=1}^{l^*-1}$ obtained by insiders from the delayed ticker (the prices yet unobserved by outsiders). For this reason, the precision of an outsider's forecast at date n is greater than if he could not condition his forecast on the contemporaneous clearing price ($\hat{\tau}_n > \tau_{n-l^*}$). Yet an insider's forecast is more precise than an outsider's forecast ($\hat{\tau}_n < \tau_n$) because the clearing price at date n is not a sufficient statistic for the delayed ticker.

As the proportion of insiders increases, their demand (and therefore their signals) weighs more on the clearing price realized in each period. Hence, the informativeness of the truncated ticker increases in μ. In addition, the informativeness of the truncated ticker decreases with latency because a higher latency implies that outsiders (i) have access to a shorter and therefore less informative price history and (ii) are uninformed about a greater number of transaction prices. Because they have only one signal (the current clearing price) about the information contained in these prices, their inference is less precise.

In contrast, the informativeness of the real-time ticker, p^n, does not depend on μ (second part of the proposition). Actually, in equilibrium, a speculator's demand can be written as

$$x_n^k(s_{in}, \Omega_n^k) = (\gamma \tau_{\epsilon_n}) s_{in} - \varphi_n^k(\Omega_n^k), \quad (16)$$

where φ_n^k is a linear function of the prices observed by a speculator with type $k \in \{I, O\}$. Thus, the sensitivity of speculators' demand ($\gamma \tau_{\epsilon_n}$) to their private signals (s_{in}) is identical for outsiders and insiders. Accordingly, the sensitivity of each clearing price to fresh information—and therefore the informativeness of the entire price history—does not depend on the proportion of insiders.

Price discovery is more efficient when transaction prices are closer to an asset's fundamental value. Hence, we measure the efficiency of price discovery by the mean squared deviation between the payoff of the security and the clearing price (the average "pricing error" at date n).[9] Because the average order

Figure 2 **Variance of the Pricing Error Var[$v - p_n$] as a Function of Latency**
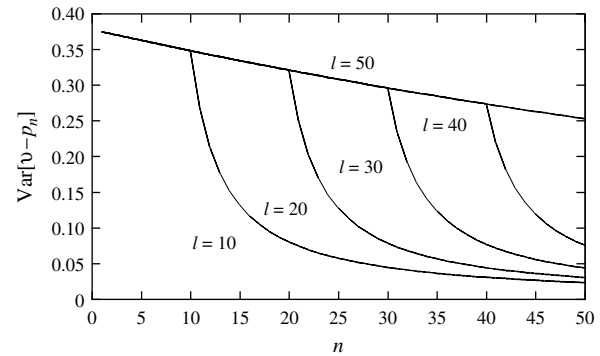
Note. Parameter values: $\tau_v = 2$, $\tau_e = 1$, $\tau_{\epsilon_n} = 1$, for $n = 1, 2, \ldots, N$, $\gamma = 1$, $\mu = 0.01$, $N = 50$, and $l \in \{10, 20, 30, 40, 50\}$.

imbalance, $\bar{e}$, from liquidity traders is zero, the average pricing error at date n is equal to Var[$v - p_n$].[10]

PROPOSITION 2. *At any date $n \geq 2$, Var[$v - p_n$] and therefore the average pricing error decreases with μ, the proportion of insiders at date n.*

When the fraction of insiders increases, the clearing price in each period becomes closer to their forecast. This effect improves price discovery since insiders' forecast of the asset value is closer to the true value of the asset than outsiders' forecast (Proposition 1).

We have not been able to study analytically the effect of an increase in latency on the average pricing error. However, extensive numerical simulations indicate that an increase in latency has a positive impact on the average pricing error at each date $n \geq 2$, as illustrated in Figure 2 (compare, e.g., the pricing error for $l = 10$ and $l = 20$), where we assume that fresh information arrives at each date ($\tau_{\epsilon_n} > 0$, $\forall n$). This information is reflected into subsequent prices through trades by insiders and outsiders. For this reason, the pricing error decreases over time (i.e., n). Interestingly, Figure 2 shows that the speed at which the pricing error decays with n increases sharply when outsiders start obtaining information on past prices, that is, when $l < n$. Intuitively, in this case, the information contained in past prices is better reflected in current prices because all speculators (insiders and outsiders) trade on this information. This effect dramatically accelerates the speed of learning about the asset payoff, compared to the case in which outsiders trade in the "dark" ($n \leq l$).

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[9] This measure of market efficiency is often used in experimental studies where the asset payoff is known to the econometrician (e.g., see Bloomfield and O'Hara 1999).

[10] Indeed, $E[(v - p_n)^2] = E[v - p_n]^2 + \text{Var}[v - p_n]$. Using Equation (6) and the Law of Iterated Expectations, we deduce that $E[v - p_n] = 0$ when $\bar{e} = 0$. We show in §A of the online appendix that Proposition 2 holds even when $\bar{e} \neq 0$.

6. Investors' Welfare and the Dissemination of Price Information

The effects of the dissemination of price information on welfare are often discussed in regulatory debates. To study these effects, in this section, we analyze how a change in the scope of information dissemination, μ, affects the welfare of the different groups of traders in the model. This analysis also lays the ground for the next section, in which we study the optimal pricing policy for the sale of price information by the exchange.

We first analyze how a change in the fraction of insiders affects speculators' expected utilities. In particular, we compare speculators' welfare in three market structures: (i) the market in which *all* speculators are outsiders ($\mu = 0$), (ii) the market in which *all* speculators are insiders ($\mu = 1$), and (iii) a two-tiered market in which only a *fraction* of speculators observe prices in real time ($0 < \mu < 1$).

To facilitate the exposition, we measure speculators' welfare by the certainty equivalent of their ex ante expected utility (as in Dow and Rahi 2003). Findings are identical if we work directly with speculators' expected utilities. By definition, the certainty equivalent is the maximal fee that a speculator is willing to pay to participate to the market. We denote this fee by $C_n^k(\mu, l)$ for a speculator with type k entering the market at date n and we call it the *speculator's payoff*.

Speculators' payoffs can be written as follows:[11]

$$C_n^k(\mu, l) = \frac{\gamma}{2} \ln\left(\frac{\mathrm{Var}[v - p_n]}{\mathrm{Var}[v \mid s_{in}, \Omega_{in}^k]}\right)$$
$$= \frac{\gamma}{2} \ln(1 + \gamma^{-1} \mathrm{Cov}[x_{in}^k, v - p_n]). \quad (17)$$

Thus, a speculator's payoff increases in the covariance between the true return on the security ($v - p_n$) and its position in the security (x_n^k). This covariance is positive (speculators tend to buy the asset when it is undervalued and sell it otherwise) and higher when pricing errors are larger.[12] Because the precision of insiders' forecasts is higher, they are more likely to buy the asset when it is undervalued ($p_n < v$) and sell it when it is overvalued ($p_n > v$). As a result,

the covariance between their position and the return on their position ($v - p_n$) is higher and they obtain a higher expected payoff than outsiders, as shown in Proposition 3.

PROPOSITION 3. *At any date n, other things being equal, an insider's ex ante expected utility is strictly greater than an outsider's expected utility. Specifically,*

$$C_n^I(\mu, l) - C_n^O(\mu, l) = \frac{\gamma}{2} \ln\left(\frac{\tau_{\epsilon_n} + \tau_n(\mu, l)}{\tau_{\epsilon_n} + \hat{\tau}_n(\mu, l)}\right) > 0. \quad (18)$$

When μ increases, some speculators shift from being outsiders to being insiders. If the increase in μ is small, those who switch are always better off (Proposition 3). Speculators who do not shift groups—we call them "incumbent insiders" and "remaining outsiders"—are always worse off, however, as shown in the next proposition.

PROPOSITION 4. *At any date $n \geq 2$, the welfare of incumbent insiders and remaining outsiders declines when the proportion of insiders increases.*

Hence, acquisition of ticker information by one speculator exerts a *negative externality* on other speculators. The reason is that it reduces pricing errors (Proposition 2) and thereby speculators' expected profits, since the latter take positions against these errors. Figure 3 illustrates Propositions 3 and 4 for specific parameter values and $n = l = 2$.

All speculators have equal access to price information when either $\mu = 0$ or $\mu = 1$. However, Proposition 5 shows that they prefer the market structure in which price information is delayed for all speculators ($\mu = 0$) to the market structure in which price information is available in real time to all speculators ($\mu = 1$).

PROPOSITION 5. *At any date $n \geq 2$, speculators' welfare is higher when $\mu = 0$ than when $\mu = 1$; that is, $C_n^I(1, l) < C_n^O(0, l)$.*

In this proposition, we are not comparing the expected utilities of a *given group* of speculators for two different values of μ as in Proposition 4. Instead, we compare the expected utility of speculators when they are all insiders ($\mu = 1$) with their expected utility when they are all outsiders ($\mu = 0$). All speculators trade more aggressively against deviations between their forecast of the final payoff and the price of the asset when $\mu = 1$ than when $\mu = 0$ because their forecast is more precise in the former case.[13] As a result,

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[11] The derivation for investors' certainty equivalent in the CARA-Gaussian framework is standard (see, e.g., Admati and Pfleiderer 1987). When the average demand of liquidity traders is different from zero, $\bar{e} \neq 0$, the expression for speculators' payoffs is more complex, which precludes a formal proof for Proposition 5. However, we have checked (analytically for $n = 2$ and numerically otherwise) that this proposition still holds when $\bar{e} \neq 0$ (see §A in the online appendix).

[12] Take the extreme case in which the pricing error vanishes ($p_n = v$). Then, the covariance between a speculator's position and the true return on the security would be zero.

[13] To see this, observe that the sensitivity of speculators' demand to the difference between their forecast of the asset and its price is $\gamma(\tau_n + \tau_{\epsilon_n})$ when $\mu = 1$ and $\gamma(\hat{\tau}_n + \tau_{\epsilon_n})$ when $\mu = 0$. Since $\tau_n(1, l) > \hat{\tau}_n(0, l)$ (Proposition 1), speculators trade against deviations between the price and their forecast more forcefully when $\mu = 1$ than when $\mu = 0$.

Figure 3 **Speculators' Welfare Is Higher in a Fully Opaque Market**
($\mu = 0$) **Than in a Fully Transparent One** ($\mu = 1$):
$C_2^O(0, 2) > C_2^I(1, 2)$
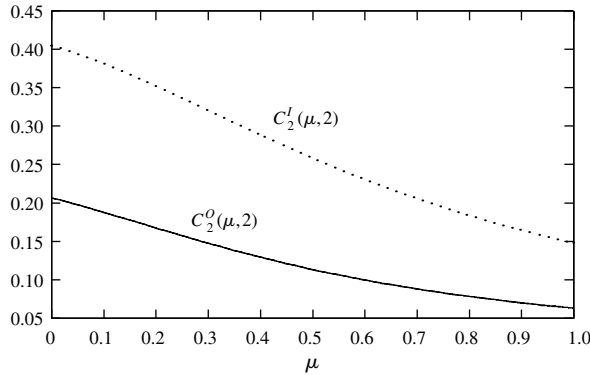
Note. Parameters' values $\tau_v = 2$, $\tau_\theta = 1$, $\tau_{\epsilon_1} = 1$, $\tau_{\epsilon_2} = 0.05$, $\gamma = 1$ and $n = N = 2$.

the clearing price in each period better reflects the information contained in speculators' positions (x_{in}^I) when $\mu = 1$, which in turn implies that the covariance between each speculator's position and the future return, $v - p_n$, is smaller when $\mu = 1$ than when $\mu = 0$. Consequently, speculators' payoffs are smaller in the former case than in the latter case (see Equation (17)).

Thus, speculators prefer posttrade opacity to full posttrade transparency because opacity is a way to soften competition among speculators. As an illustration, consider Figure 3. Speculators have a payoff equal to $C_2^O(0, 2) = 0.206$ when they are all outsiders and a payoff equal to $C_2^I(1, 2) = 0.147$ when they are all insiders.

Now we compare the market structure in which information is delayed for all speculators ($\mu = 0$) with a two-tiered market structure in which only some investors have access to information in real time ($\mu > 0$). To this end, let $\bar{\mu}_n$ be the fraction of insiders at date n such that $C_n^I(\bar{\mu}_n, l) = C_n^O(0, l)$. That is, when $\mu = \bar{\mu}_n$, insiders have exactly the same payoff in a two-tiered market structure with $\mu = \bar{\mu}_n$ or a market structure in which information is delayed for all speculators. Proposition 3 implies that $C_n^I(0, l) > C_n^O(0, l)$, whereas Proposition 5 implies that $C_n^I(1, l) < C_n^O(0, l)$. As $C_n^I(\mu, l)$ decreases with μ, $\bar{\mu}_n$ is strictly greater than zero and strictly smaller than one. Moreover,

$$C_n^I(\mu, l) < C_n^O(0, l) \quad \text{if } \bar{\mu}_n < \mu \leq 1, \tag{19}$$

$$C_n^I(\mu, l) > C_n^O(0, l) \quad \text{if } 0 < \mu < \bar{\mu}_n. \tag{20}$$

We deduce the following proposition.

Proposition 6. 1. *If $\bar{\mu}_n < \mu \leq 1$, then at date n, insiders and outsiders would be better off if information was delayed for all speculators.*

2. *If $\mu < \bar{\mu}_n$, then at date n, if information were delayed for all speculators, insiders would be worse off and outsiders would be better off.*

As an illustration, consider Figure 3 again. In this example, $\bar{\mu}_2 = 0.696$. Moreover, if information is delayed for all speculators, their payoff is equal to $C_2^O(0, 2) = 0.206$. Now suppose that μ switches from zero to 10%. The speculators who become insiders are better off, since their payoff becomes $C_2^I(0.1, 2) = 0.381$, whereas those who remain outsiders are worse off, since their payoff becomes $C_2^O(0.1, 2) = 0.187$.

Let $W_n(\mu, l) \overset{\text{def}}{=} \mu C_n^I(\mu, l) + (1 - \mu)C_n^O(\mu, l)$ be the average payoff of speculators at date n, when the fraction of insiders is μ, and let $\mu_n^{\max}$ be the value of μ that maximizes $W_n(\mu, l)$. Proposition 6 implies that $W_n(0, l) > W_n(1, l)$. Thus, $\mu_n^{\max}$ is always strictly less than one. However, it is not necessarily equal to zero. For instance, for the parameter values considered in Figure 3, $W_2(0, 2) = 0.206$ and $W_2(0.1, 2) = 0.207$. In this case, the aggregate increase in the payoff of speculators who become insiders when μ increases from zero to 10% more than offsets the welfare loss of speculators who remain outsiders.

This happens because insiders obtain a higher expected profit than outsiders.[14] Thus, speculators switching from outsiders to insiders work to increase the expected profit on speculators' average position ($\mu x_n^I + (1 - \mu)x_n^O$), at the expense of liquidity traders since speculators' net position is the opposite of liquidity traders' net trade. However, as the fraction of insiders increases, the average pricing error decreases, which lowers the expected profit of all speculators. This countervailing effect always dominates for μ large enough, but it may not for small values of μ, which explains why $\mu_n^{\max} > 0$ for some parameter values. As a result, a two-tier market structure in which some speculators access price information faster than others, can sometimes be Pareto superior from the speculators' viewpoint, provided a fraction of the gains from insiders is used to compensate outsiders for their losses, relative to the case in which $\mu = 0$.

In any case, an increase in speculators' expected profits is obtained at the expense of liquidity traders. Because an increase in μ reduces all speculators' expected payoffs, the expected trading loss for liquidity traders, $E[(v - p_n)e_n]$, should be minimal (in absolute value) when all speculators have access to price information.[15] This is indeed the case, as shown by the next proposition.

Proposition 7. *Liquidity traders' expected trading loss, $|E[(p_n - v)e_n]|$, is minimal for $\mu = 1$.*

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[14] Indeed, a speculator's expected profit is $E[x_{in}^k(v - p_n)] = \text{Cov}[x_{in}^k, v - p_n]$, where the equality follows from the fact that $E[x_{in}^k] = 0$. We have $E[(v - p_n)x_{in}^I] > E[(v - p_n)x_{in}^O]$ since $\text{Cov}[x_{in}^I, v - p_n] > \text{Cov}[x_{in}^O, v - p_n]$ (a consequence of Proposition 3).

[15] As in Easley et al. (2013b) or Leland (1992), we assume that the expected trading loss of liquidity traders is a proxy for their gains from trade.

In our model, in a given period, the sensitivity of the clearing price to liquidity traders' supply is given by $B_{n,0}$ (see Equation (6)). This sensitivity is often used as a measure of market illiquidity (see, e.g., Kyle 1985) and determines liquidity traders' expected trading losses. Using Equation (6), we have

$$E[(p_n - v)e_n] = -B_{n,0}\tau_e^{-1}. \qquad (21)$$

It follows from Proposition 7 that, as for price discovery, liquidity is maximal when $\mu = 1$.

To sum up, sell-side traders (proprietary trading firms specialized in liquidity provision) will oppose a too-wide distribution of real-time price information, since $\mu_n^{\max} < 1$. In contrast, liquidity traders prefer a market structure in which this information is widely available, since price impact costs are minimized when all speculators are insiders. These implications are consistent with the fact that changes in the scope of access to real-time price information are very often controversial. Another implication is that enforcing a situation in which the fraction of insiders is less than one is difficult, since speculators who can switch from being outsiders to being insiders always enjoy an increase in their payoff. Hence, speculators will individually seek to obtain posttrade information, even though collectively speculators would be better off with no access ($\mu_n^{\max} = 0$) or moderate access ($\mu_n^{\max} > 0$) to price information.

7. Optimal Sale of Price Information by for Profit Exchanges

As explained in the introduction, in today's markets exchanges cannot directly control who is an insider and who is not. Rather, price information is sold, and the number of insiders in the market is ultimately equal to the number of speculators who choose to buy price information. This number, of course, is ultimately determined by the fee for price information set by exchanges. We now analyze the determination of this fee and the resulting equilibrium value for the fraction of insiders.

In each period, the price at which speculators can observe the real-time ticker is set by a for-profit exchange. In reality, exchanges obtain revenues from both the sale of information *and* from trading fees. Accordingly, we assume that in each period, the exchange charges two fees: a fee for real-time ticker information, ϕ_n, and a market access fee, F_n (a "membership fee").[16] All speculators pay the access fee and

only insiders pay the fee for real-time information. In §7.1, we consider the case in which the exchange does not account for liquidity traders' losses in setting its fees. We then relax this assumption in §7.2.

7.1. Baseline Case: Free Participation for Liquidity Traders

At the beginning of each period, before receiving their private signals, speculators decide (i) whether to participate to the market and (ii) whether to purchase ticker information. Let $\bar{\phi}_n(\mu, l)$ be the *maximum* fee that a speculator is willing to pay for observing the real-time ticker at date n. This fee is

$$\bar{\phi}_n(\mu, l) = C_n^I(\mu, l) - C_n^O(\mu, l). \qquad (22)$$

We call it the *value of the ticker*. Using Equation (18), we obtain that

$$\begin{aligned}\bar{\phi}_n(\mu, l) &= \frac{\gamma}{2}\ln\left(\frac{\tau_{\epsilon_n} + \tau_n(\mu, l)}{\tau_{\epsilon_n} + \hat{\tau}_n(\mu, l)}\right) \\ &= \frac{\gamma}{2}\ln\left(1 + \frac{\tau_n(\mu, l) - \hat{\tau}_n(\mu, l)}{\tau_{\epsilon_n} + \hat{\tau}_n(\mu, l)}\right) > 0. \quad (23)\end{aligned}$$

The value of the ticker increases with the difference between the informativeness of the real-time ticker and the informativeness of the truncated ticker. Proposition 1 implies that this difference is reduced when the proportion of insiders increases or when latency is reduced. Thus, we obtain the following result.

PROPOSITION 8. *For a fixed latency, the value of the real-time ticker at any date $n \geq 2$ decreases with the proportion of insiders. Moreover, for a fixed proportion of insiders, the value of the real-time ticker weakly increases with the latency in information dissemination, l. More precisely,*

$$\bar{\phi}_n(\mu, l) < \bar{\phi}_n(\mu, l+1) \quad \text{for } n > l,$$
$$\bar{\phi}_n(\mu, l) = \bar{\phi}_n(\mu, l+1) \quad \text{for } n \leq l.$$

At date n, a speculator buys ticker information if the price of the ticker is strictly less than the value of the ticker ($\phi_n < \bar{\phi}_n(\mu, l)$). She does not buy information if $\phi_n > \bar{\phi}_n(\mu, l)$. Finally, she is indifferent between buying ticker information or not if $\phi_n = \bar{\phi}_n(\mu, l)$. Thus, the equilibrium proportion of insiders, $\mu^e(\phi_n, l)$, is

$$\mu^e(\phi_n, l) = \begin{cases} 1 & \text{if } \phi_n \leq \bar{\phi}_n(1, l), \\ \mu & \text{if } \phi_n = \bar{\phi}_n(\mu, l), \\ 0 & \text{if } \phi_n \geq \bar{\phi}_n(0, l). \end{cases} \qquad (24)$$

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[16] In reality, participation fees usually have both a fixed component (e.g., exchanges charge annual or monthly membership fees) and a variable component (members pay a fee per share traded). See http://www.nasdaqtrader.com/Trader.aspx?id=PriceListTrading2#membership (accessed April 24, 2013) for an

example of these fees on NASDAQ. In our model, the membership fee is sufficient for the exchange to extract all surplus from speculators. Hence, to simplify the analysis, we assume that the trading fee per share is zero (the analysis is much less tractable with a fee per share).

Figure 4   Value of the Ticker for Two Levels of Latency when $n > l$
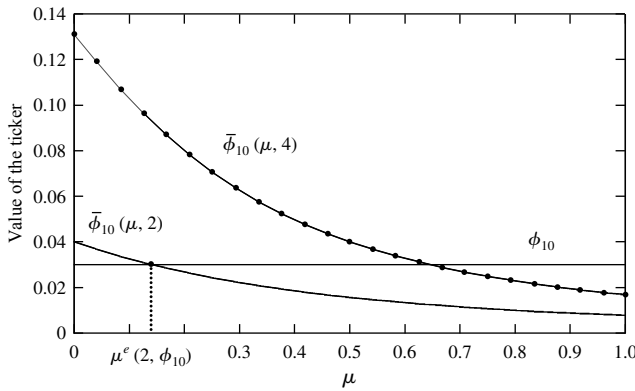
Note. Parameters' values: $\tau_v = 2$, $\tau_e = 1$, $\gamma = 1$, $l \in \{2, 4\}$, and $\tau_{\epsilon_n} = 1$, for $n = 1, 2, \ldots, 50$.

Figure 4 shows how the equilibrium proportion of insiders is determined. As the value of the ticker declines with μ (Proposition 8), for each price of the ticker there is a unique equilibrium proportion of insiders $\mu^e(\phi_n, l)$. Moreover, the equilibrium proportion of insiders decreases with the price of ticker information (to see this, consider an upward shift in ϕ_{10} in Figure 4).

In each period, the for-profit exchange chooses its tariff (ϕ_n, F_n) to maximize its expected profit:[17]

$$\Pi_n(\mu^e(\phi_n, l), l) = \mu^e(\phi_n, l)\phi_n + F_n.$$

As the exchange is a monopolist, it optimally chooses its tariff to extract all gains from trade from speculators. Hence,

$$F_n = C_n^O(\mu^e(\phi_n, l), l)$$

(outsiders' net payoff is zero),   (25)

$$\phi_n + F_n = C_n^I(\mu^e(\phi_n, l), l)$$

(insiders' net payoff is zero).   (26)

The access fee, F_n, is determined by the fee for real-time ticker Information, since this fee determines the equilibrium proportion of insiders. Hence, ultimately, ϕ_n is the only decision variable of the for-profit exchange. Equations (25) and (26) imply

$$\phi_n = C_n^I(\mu^e(\phi_n, l), l) - C_n^O(\mu^e(\phi_n, l), l) = \bar{\phi}_n(\mu^e(\phi_n, l), l),$$

[17] The choice of a tariff in a given period does not influence the exchange's expected profits in subsequent periods because the speculators' payoffs in each period only depend on the fraction of insiders in this period and investors' signals are exogenous. Hence, the tariffs that maximize the exchange's per period expected profits also maximize the total expected profit of the exchange over all periods. This will not hold if investors' signals are endogenous and $n > 2$. Indeed, in this case the precision of signals acquired in a given period will affect the informativeness of the price in this period for investors arriving at subsequent dates. In §7.4, we endogenize the precision of investors' signals, but we focus on the case $n = 2$ only.

where the last equality follows from the definition of $\bar{\phi}_n(\mu^e(\phi_n, l), l)$. Thus, the objective function of the for-profit exchange is

$$\max_{\phi_n} \Pi_n(\mu^e(\phi_n, l), l) = \mu^e(\phi_n, l)\bar{\phi}_n(\mu^e(\phi_n, l), l)$$
$$+ C_n^O(\mu^e(\phi_n, l), l).   (27)$$

The solution to this optimization problem can be found by solving

$$\max_{\mu} \Pi_n(\mu, l) = \mu\bar{\phi}_n(\mu, l) + C_n^O(\mu, l),   (28)$$

because there is a one-to-one relationship between the equilibrium fraction of insiders and the fee charged for ticker information. Consequently, if μ_n^* is the solution of (28), then $\phi_n^* = \bar{\phi}_n(\mu_n^*, l)$ solves (27).

The for-profit exchange faces the following trade-off. On the one hand, by increasing the fraction of insiders, the exchange gets a larger revenue from the sale of information ($\mu\bar{\phi}_n(\mu, l)$). However, it must lower both the price for ticker information (since $\partial\bar{\phi}_n(\mu, l)/\partial\mu < 0$) and the access fee, since speculators' gain from market participation decreases with the proportion of insiders ($\partial C_n^O(\mu, l)/\partial\mu < 0$). Using the definition of $\bar{\phi}_n(\mu, l)$, we rewrite (28) as

$$\max_{\mu} \Pi_n(\mu, l) = \mu C_n^I(\mu, l) + (1 - \mu)C_n^O(\mu, l)$$
$$= W_n(\mu, l).   (29)$$

Hence, the exchange's expected profit is equal to the average payoff of all speculators. This is intuitive since the exchange's fees are set to extract all speculators' surplus.

PROPOSITION 9. *At any date $n \geq 2$ and for all values of the parameters, the for-profit exchange chooses its tariff so that the proportion of speculators buying ticker information maximizes speculators' average payoff. That is, $\mu_n^* = \mu_n^{\max}$ and $\phi_n^* = \bar{\phi}_n(\mu_n^{\max}, l)$. Thus, the exchange never sells price information to all speculators ($\mu_n^* < 1$) and sometimes finds it optimal not to sell price information at all ($\mu_n^* = 0$).*

Figure 5 illustrates Proposition 9 by plotting the exchange's profit (Π_2) as a function of the proportion of insiders, for specific parameter values ($\tau_v = 2$, and $\tau_e = \gamma = \tau_{\epsilon_1} = 1$). For these parameters, when $\tau_{\epsilon_2} = 0.05$, the exchange's expected profit peaks when $\mu_2^* = \mu_2^{\max} \simeq 6\%$, in which case $\bar{\phi}(\mu_2^*, 2) \simeq 0.2$; when τ_{ϵ_2} increases to 1, then the exchange chooses or $\mu_2^* = \mu_2^{\max} = 0$, setting $\bar{\phi}(\mu_2^*, 2) \simeq 0.11$. In the first case, a two-tier market structure with fast and slow traders optimally strikes a balance between two opposite goals: (i) maximize speculators' willingness to pay for trading, which requires restricting access to price information; and (ii) make profits on the sale of information, which requires selling information to at least

Figure 5   Optimal Choice of μ by the Exchange

(a)

$\Pi_2(\mu, 2)$

(b)

$\Pi_2(\mu, 2)$
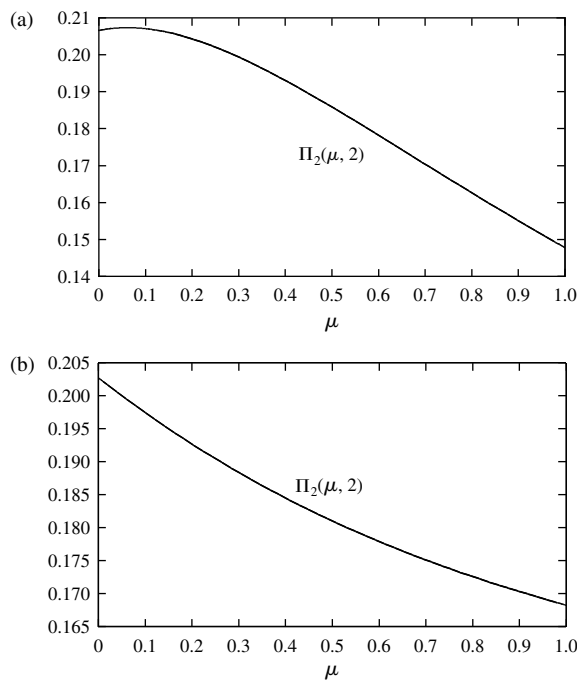
Notes. Parameters' values: $\tau_v = 2$, $\tau_e = \gamma = \tau_{\epsilon_1} = 1$. In panel (a), $\tau_{\epsilon_2} = 0.05$; in panel (b), $\tau_{\epsilon_2} = 1$.

some speculators. In the second case, the exchange optimally chooses not to sell price information at all to maximize the access fee.

We have assumed that the exchange is a monopolist, so it leaves no surplus to speculators. Obviously, the same findings are obtained if the exchange can only recover a fixed fraction $\delta > 0$ of the payoffs earned by insiders and outsiders. In this case, its expected profit is $\Pi_n(\mu, l) = \delta W_n(\mu, l)$ and is therefore maximized for the same value of μ. It is reasonable to assume that exchanges exert market power in pricing their own price information ($\delta > 0$). Indeed, many studies, starting with Hasbrouck (1995), have gauged the contribution of each trading venue for a stock to price discovery for this stock. These studies typically find that each trading venue for a stock has an "information share" higher than zero, which means that the transaction price on one trading venue is not a sufficient statistic for prices on all trading venues. Hence, the signals contained by real-time prices from different trading platforms are not perfect substitutes, even when they are for the same asset. This suggests that competition between exchanges in the market for information is a form of monopolistic competition. Analyzing this competition in detail raises complex modeling issues that are beyond the scope of this paper.[18]

[18] There is no standard model yet of imperfect competition among sellers in the literature on information sale in securities markets,

7.2.   Extension: Subsidized Participation for Liquidity Traders

The previous section emphasizes one mechanism: By restricting access to price information, an exchange raises speculators' expected trading profits and can accordingly charge them higher trading fees. As a result, liquidity traders obtain smaller expected profits since speculators' profits are obtained at the expense of liquidity traders. If exchanges' pricing policies harm liquidity traders too much, the latter may trade less on the exchange.

To account for this possibility while keeping the model simple, we assume now that the exchange must compensate liquidity traders for a fraction β of their trading losses. Specifically, the exchange subsidizes liquidity traders for an amount $L_n(\mu) = \beta E[(p_n - v)e_n]$ with $\beta \in [0, 1]$.[19] We interpret this subsidy as the rebate that the exchange should concede to liquidity traders to prevent them from migrating to another market.[20] Insofar as traders' types (e.g., retail investors versus proprietary trading firms) are informative on trading motivations, exchanges can charge different prices for clienteles with different trading motives.[21] For instance, they could charge lower fees for brokers who mainly channel orders from retail investors, as these orders are more likely to come from liquidity traders.[22]

The case $\beta = 0$ corresponds to the case analyzed in the previous section (liquidity traders are captive), whereas the case $\beta = 1$ corresponds to the case in which the exchange must fully compensate liquidity traders for their trading losses. A higher β captures (in reduced form) the effect of increased competition for order flow from other exchanges.

In this case, the expected profit becomes

$$\max_{\mu} \Pi_n(\mu, l, \beta) = W_n(\mu, l) + \beta L_n(\mu, l). \quad (30)$$

in particular when sellers sell different signals. To our knowledge, Simonov (1999) and Lee (2009) are the only two models in which two sellers compete for the sale of two different signals. Veldkamp (2006) considers a case with multiple sellers, but they all have the same signal.

[19] Boulatov and Dierker (2007) use a similar approach.

[20] This rebate must be interpreted as a discount given to liquidity traders relative to a fixed baseline fee, which is implicitly normalized to zero in our analysis.

[21] Linnainmaa and Saar (2012) show that brokers' identities are informative about their clientele type (uninformed versus informed). In particular, they show that orders from brokers who mainly channel orders from retail investors are perceived as uninformative by market participants.

[22] This is, in fact, what some exchanges do. For instance, some options exchanges (e.g., the AMEX or the CBOE) charge little or nothing to execute retail customer orders (they actually pay for retail customer order flow) but charge "marketing fees" to market makers who execute retail customer orders (see, e.g., Battalio et al. 2011).

Table 1   Effect of an Increase in β on the Fraction of Insiders, the Price of the Ticker, the Welfare of Informed Speculators, and Liquidity Traders' Expected Losses

	β			
	0	0.31	0.6	1
μ_2^*	0	0.60	1	1
$\phi_2(\mu_2^*, 2)$	0.14	0.04	0.03	0.03
$W_2(\mu_2^*, 2)$	0.25	0.21	0.20	0.20
$L_2(\mu_2^*, 2)$	0.67	0.53	0.50	0.50

Note. Parameter values: $\tau_v = \tau_{\epsilon_n} = \tau_u = \gamma = 1$.

Table 2   Effect of an Increase in β on the Fraction of Insiders, the Price of the Ticker, the Welfare of Informed Speculators, Liquidity Traders' Expected Losses, and the Variance of the Pricing Error for the Exchange and the Pure Seller Case

	Exchange				
	β				
	0	0.31	0.6	1	Pure seller
μ_2^*	0	0.60	1	1	0.04
$\phi_2(\mu_2^*, 2)$	0.14	0.04	0.03	0.03	0.66
$W_2(\mu_2^*, 2)$	0.25	0.21	0.20	0.20	0.21
$L_2(\mu_2^*, 2)$	0.67	0.53	0.50	0.50	0.53
$\mathrm{Var}[v - p_2]$	0.50	0.40	0.38	0.38	0.39

Note. Parameter values: $\tau_v = \tau_{\epsilon_n} = \tau_u = \gamma = 1$.

We have not been able to study analytically the effect of β on the optimal fraction of insiders, μ_n^*, chosen by the exchange. However, in line with intuition, numerical simulations show that as β increases from zero to one, the exchange optimally reduces its fee for price information so that it disseminates price information to a wider number of speculators. In this way, the exchange intensifies competition among speculators, which reduces trading costs for liquidity traders. As an example, Table 1 shows the evolution of the optimal fee charged by the exchange, the resulting fraction of insiders, the average welfare of speculators, and liquidity traders' trading losses when β increases from 0 to 1.

In recent years, competition among stock exchanges has increased with the emergence of new trading platforms (such as BATS in the United States or Chi-X in Europe) that are trading shares of stocks listed on incumbent markets such as NYSE-Euronext and NASDAQ. As explained in the previous section, this competition does not per se reduce exchanges' market power with respect to their price information, because transaction prices from different platforms are not perfect substitutes. Yet it could force exchanges to reduce their fees for price information since the coexistence of multiple trading platforms makes liquidity traders less captive, which should induce exchanges to care more about liquidity traders' losses (an increase in β in our model) and cut their fees for price information.

7.3.   Pure Information Sellers vs. Exchanges

The literature on information sales in securities markets (e.g., the pioneering paper by Admati and Pleiderer 1986) has focused on *pure* information sellers, that is, sellers who derive revenues only from the sale of information.[23] In this section, we compare the optimal selling of price information by a pure information seller and an exchange.

The pure information seller faces the same problem as the exchange, except that by definition it cannot charge an access fee. Hence, at date n, the pure information seller optimally chooses a fee $\bar{\phi}_n(\mu^{**}(l), l)$ where $\mu^{**}(l)$ maximizes $\Pi_n^{\text{pure}}(\mu, l) = \mu \bar{\phi}_n(\mu, l)$ with respect to μ. In contrast to the exchange, the pure seller does not internalize the effect of disseminating price information on speculators' willingness to pay for trading since it cannot charge speculators for market participation. Consequently, when β is small enough, the exchange charges a higher fee for price information than a pure information seller because the exchange internalizes the negative effect of disseminating price information on speculators' revenues but the pure information seller does not. As shown in Figure 5, the exchange may even find it optimal not to sell price information at all ($\mu_n^* = 0$), whereas this is clearly never optimal for a pure information seller. In contrast, when β is high enough, the exchange wants to disseminate price information more widely than a pure information seller to minimize trading costs for liquidity traders. Accordingly, the fee that the pure information seller charges is at least as high as that chosen by the exchange. Table 2 illustrates this point by comparing, for different values of β, the optimal fee for price information charged by the exchange and a pure information seller and the fractions of speculators buying price information in each case. The table also reports the size of pricing errors ($\mathrm{Var}[v - p_n]$), speculators' average payoffs, and liquidity traders' losses.

As shown by Admati and Pleiderer (1986), an information seller can optimally add noise to the information it sells to increase profit. This is not a possibility for exchanges, since, in reality, they must report a price "as is."[24] However, exchanges have

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[23] An exception is the case in which the information seller can also obtain profits by trading on his information (e.g., Admati and Pleiderer 1988). This case, however, is not relevant for exchanges, since they do not trade securities for their own account.

[24] This precludes the possibility of adding personalized error terms to the information sold by the exchange (what Admati and Pleiderer 1986 call personalized signals).

another tool at their disposal, not analyzed in the literature on information sales: they can delay the moment at which price information becomes available for free, i.e., increase the latency in information dissemination, l. Using the envelope theorem, it is immediate that the optimal latency for the exchange maximizes $\Pi_n(\mu^*(l), l)$ where $\mu^*(l)$ solves (30).

Again, the optimal policy of an exchange and a pure information seller with respect to l do not necessarily coincide. A pure information seller chooses l to maximize $\mu_n^{**}(l)\bar{\phi}_n(\mu_n^{**}(l), l)$. The value of information increases with l (Proposition 8) for a fixed fraction of insiders. Accordingly, the pure information seller always chooses the highest possible latency, $l = N$.[25] In contrast, when l increases, the exchange faces a trade-off: on the one hand, a higher latency increases the value of price information, and on the other hand it reduces the fee that speculators are willing to pay for trading and aggravates liquidity traders' losses, forcing the exchange to grant them a greater discount (if $\beta > 0$). Accordingly, there exist parameter values for which the exchange optimally sets a latency smaller than the highest possible latency (that is, less than N).

Figure 6 provides a numerical example of this situation (the complexity of the optimization problem for the exchange precludes an analytic solution for the choice of its latency by the exchange). In the figure, we assume that there are $N = 10$ trading rounds and plot the expected profits of the exchange (plain line) and the pure information seller (dotted line) in the last trading round, for three different latencies: $l \in \{2, 3, 10\}$. As expected, the pure information seller's expected profit is maximal when $l = N = 10$. In contrast, the exchange's expected profit is maximal when latency is shorter and equal to $l = 3$. This choice strikes a balance between the revenues from the sale of price information (which are maximal for $l = 10$) and the revenues from market participation, which tends to be higher when l is smaller.

7.4. Implications

Does the sale of price information by exchanges promote price discovery? The previous sections show that the answer to this question depends on the extent to which exchanges need to compensate liquidity traders for trading costs, that is, β. If β is small (e.g., because liquidity traders are captive), the answer

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[25] Suppose that this is not true. Then there is a date n at which the pure information seller optimally sets $l < n$. Its expected profit is then $\mu^{**}(l)\bar{\phi}_n(\mu_n^{**}(l), l)$. Now consider the following alternative policy for the pure information seller. It sets $l = n$ and a price information fee equal to $\bar{\phi}_n(\mu_n^{**}(l), n)$. With this policy, the pure information seller's expected profit is $\mu^{**}(l)\bar{\phi}_n(\mu_n^{**}(l), n)$, which is strictly higher than $\mu^{**}(l)\bar{\phi}_n(\mu_n^{**}(l), l)$ since $\bar{\phi}_n(\mu_n^{**}(l), n) > \bar{\phi}_n(\mu_n^{**}(l), l)$ (Proposition 8). This is a contradiction.

Figure 6 **Optimal Choice of μ by the Exchange and the Pure Information Seller, for Different Values of l, and for $\tau_v = 2$, $\tau_u = \gamma = \tau_{\epsilon_n} = 1$, and $\beta = 0.7$**
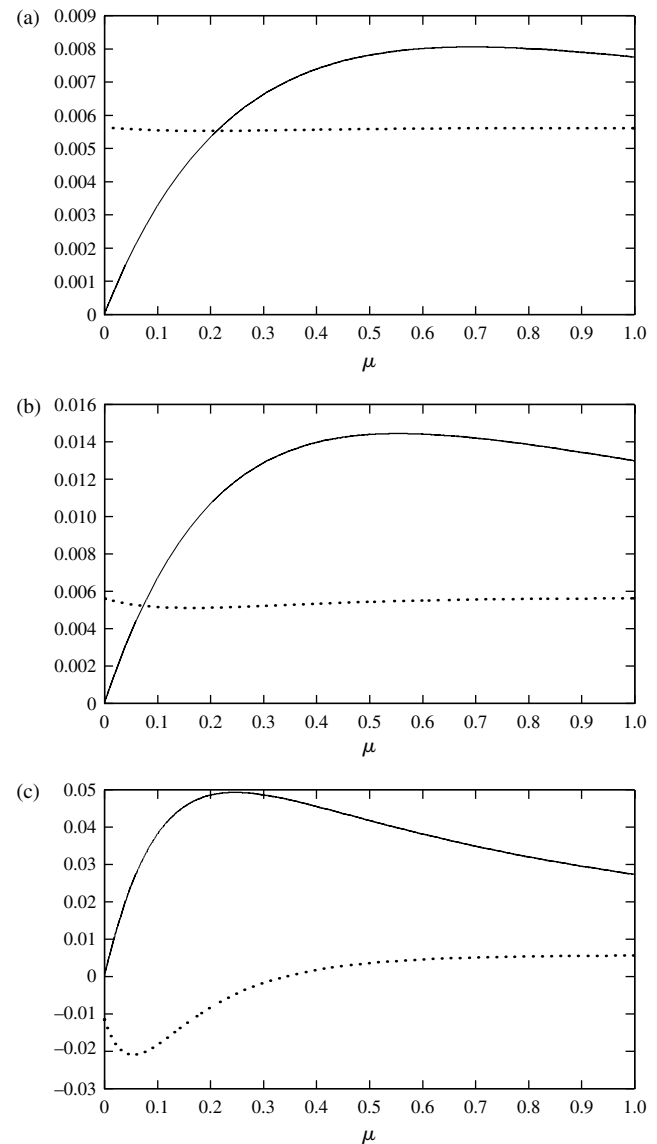
Notes. The continuous and dotted lines represent, respectively, the profit of the pure information seller and of the exchange at date 10. In panel (a), we set $l = 2$; in panel (b), we set $l = 3$; and in panel (c), we set $l = 10$. As the comparison of the three panels shows, the pure data vendor obtains the highest profit in panel (c) for $l = 10$ (setting $\mu^* \approx 0.25$, and obtaining a corresponding profit of about 0.05), whereas the exchange would choose $l = 3$ in panel (b) (setting $\mu^* = 1$ and obtaining a profit of about 0.0059).

is clearly negative, since the exchange always optimally restricts access to price information by charging a high fee for this information. As the efficiency of price discovery increases with the fraction of insiders, the exchange pricing policy harms price discovery. If β is high, the answer is less clear cut. Actually, the exchange optimally charges a smaller fee for price information than a pure information seller would and therefore disseminates more price information.

The sale of price information by the exchange therefore results in more efficient price discovery than if price information were sold by a pure information seller. Yet it may still be the case that only a fraction of speculators buy information, leaving room for an improvement in price discovery by lowering the fee for price information (see, e.g., the case $\beta = 0.31$ in Table 2).

Accordingly, if regulators' objective is to maximize the efficiency of price discovery, they should consider capping the fee for price information to make sure that a large fraction of investors chooses to buy this information. One drawback, however, is that as price information becomes more widely available, speculators might have less incentive to acquire information from other sources. A decrease in the fee for price information could then negatively affect price discovery, instead of strengthening it. To study this point, we have considered an extension of the model in which, as in Verrecchia (1982), speculators can choose the precision τ_{ϵ_n} of their fresh information, s_{in}, at a cost $c(\tau_{\epsilon_n})$ where $c(\cdot)$ is quadratic. For brevity, we present this extension in details in §C of the online appendix, and for tractability we only consider the case $n = 2$. We find that when the fraction of insiders, μ, increases, speculators choose a private signal of lower precision in equilibrium. Hence, in line with intuition, speculators substitute fresh information with price information. However, despite this substitution effect, we show through numerical simulations that pricing errors still decrease with μ. Hence, price discovery remains maximal when $\mu = 1$, that is, when the fee for price information is low.

Of course, regulators should also care about welfare. On this front, a decrease in the fee for price information has ambiguous effects because, as shown in §6, there is no Pareto-dominant level for μ: an increase in μ harms incumbent insiders and remaining outsiders while helping those new insiders (Proposition 4). Moreover, speculators always prefer a fully opaque market ($\mu = 0$) to a fully transparent market ($\mu = 1$), whereas liquidity traders have exactly opposite views. If regulators primarily care about liquidity traders, the recommendation, however, is nonambiguous, because, as for price discovery, liquidity traders' losses are minimized when $\mu = 1$. As the fee charged by an exchange or a pure information seller may be too high to obtain this outcome, regulators may again want to cap the fee for price information.

One way to test the model is to study empirically the effects of variations in the fee for price information. The model implies that a reduction in this fee should improve price discovery and liquidity by intensifying competition among liquidity providers. Testing this prediction is challenging, as the fee for price information is endogenous. Hence, empiricists need to find adequate instruments for variations in these fees.

8. Conclusions

Exchanges play an important role by providing venues for risk sharing and price discovery. However, they are not per se interested in maximizing the efficiency of price discovery or risk sharing. Rather, they make their decisions to maximize their profits, which derive in large part from the sale of trading services and of information on prices. In this paper, we show that there is a conflict between the efficiency of price discovery and profit maximization by exchanges.

This conflict arises because selling real-time price information to more speculators (proprietary trading firms specialized in the provision of liquidity) enhances price discovery, but it reduces speculators' expected trading profits and therefore the fee (e.g., membership fee) that they are willing to pay for trading. Hence, a for-profit exchange faces a trade-off between its expected trading revenue from speculators and its expected revenue from the sale of price information. When the exchange does not need to compensate liquidity traders for their losses, it always resolves this trade-off by choosing a high fee for price information, so that no speculator or only a fraction of all speculators buys access to price information in real time. This pricing policy is detrimental to price discovery, since price discovery is most efficient when all speculators observe prices in real time. It also harms liquidity traders, since their losses are minimal when all speculators observe price information in real time. Overall, these findings suggest that there is ground for regulating the sale of price information by exchanges.

Appendix

PROOF OF LEMMA 1. *Step* 1: *Informational content of equilibrium prices*. In a symmetric linear equilibrium, speculators' demand functions in period $n \geq 1$ can be written as follows:

$$x_n^I(s_{in}, p^n) = a_n^I s_{in} - \varphi_n^I(p^n), \qquad (31)$$

$$x_n^O(s_{in}, p^{n-2}, p_n) = a_n^O s_{in} - \varphi_n^O(p^{n-l^*}, p_n), \qquad (32)$$

where $\varphi_n^k(\cdot)$ is a linear function of the clearing price at date n and the past prices observed by a speculator with type $k \in \{I, O\}$ ($\varphi_1^I(\cdot) = \varphi_1^O(\cdot)$, since price information is identical for insiders and outsiders at date 1). In any period n, the clearing condition is $\int_0^\mu x_{in}^I \, di + \int_\mu^1 x_{in}^O \, di = e_n$. Thus, using Equations (31) and (32), we deduce that at date n

$$a_n v - \mu \varphi_n^I(p^n) - (1-\mu)\varphi_n^O(p^{n-l^*}, p_n) = e_n \quad \forall n \geq 2, \qquad (33)$$

with $a_n \stackrel{\text{def}}{=} \mu a_n^I + (1-\mu)a_n^O$. We deduce that p^n is observationally equivalent to $z^n = \{z_1, z_2, \ldots, z_n\}$ with $z_n = a_n v - e_n$.

Step 2: *Equilibrium in period* n.

Insiders. An insider's demand function in period n, $x_n^I(s_{in}, p^n)$, maximizes

$$E\left[-\exp\left\{ -\frac{(v-p_n)x_{in}^I}{\gamma} \right\} \,\bigg|\, s_{in}, p^n \right].$$

We deduce that

$$x_n^I(s_{in}, p^n) = \gamma \frac{E[v-p_n \mid s_{in}, p^n]}{\text{Var}[v-p_n \mid s_{in}, p^n]} = \gamma \frac{E[v \mid s_{in}, p^n] - p_n}{\text{Var}[v \mid s_{in}, p^n]}.$$

Since p^n is observationally equivalent to z^n, we deduce (using well-known properties of normal random variables)

$$E[v \mid s_{in}, p^n] = E[v \mid s_{in}, z^n]$$
$$= (\tau_n(\mu, l) + \tau_{\epsilon_n})^{-1}(\tau_n(\mu, l)E[v \mid z^n] + \tau_{\epsilon_n}s_{in}),$$
$$\text{Var}[v \mid s_{in}, p^n] = \text{Var}[v \mid s_{in}, z^n] = (\tau_n(\mu, l) + \tau_{\epsilon_n})^{-1},$$

where

$$\tau_n(\mu, l) \stackrel{\text{def}}{=} (\text{Var}[v \mid p^n])^{-1} = (\text{Var}[v \mid z^n])^{-1} = \tau_v + \tau_e \sum_{t=1}^n a_t^2, \qquad (34)$$

and

$$E[v \mid s_{in}, p^n] = \frac{\tau_e \sum_{t=1}^n a_t z_t + \tau_{\epsilon_n}s_{in}}{\tau_n(\mu, l) + \tau_{\epsilon_n}}.$$

Thus,

$$x_n^I(s_{in}, p^n) = \gamma(\tau_n + \tau_{\epsilon_n})(E[v \mid s_{in}, p^n] - p_n)$$
$$= a_n^I(s_{in} - p_n) + \gamma \tau_n (E[v \mid p^n] - p_n), \qquad (35)$$

where $a_n^I = \gamma \tau_{\epsilon_n}$.

Outsiders. An outsider's demand function in period n, $x_n^O(s_{in}, p^{n-l^*}, p_n)$, maximizes

$$E\left[-\exp\left\{ -\frac{(v-p_n)x_{in}^O}{\gamma} \right\} \,\bigg|\, s_{in}, p^{n-l^*}, p_n \right].$$

We deduce that

$$x_n^O(s_{in}, p^{n-l^*}, p_n) = \gamma \frac{E[v-p_n \mid s_{in}, p^{n-l^*}, p_n]}{\text{Var}[v-p_n \mid s_{in}, p^{n-l^*}, p_n]}$$
$$= \gamma \frac{E[v \mid s_{in}, p^{n-l^*}, p_n] - p_n}{\text{Var}[v-p_n \mid s_{in}, p^{n-l^*}, p_n]}.$$

In equilibrium, outsiders correctly anticipate that the clearing price at each date is given by

$$p_n = A_n v - \sum_{j=0}^{l^*-1} B_{n,j} e_{n-j} + D_n E[v \mid p^{n-l^*}] \quad \text{for } n \geq 1. \qquad (36)$$

When $A_n \neq 0$, the price at date n is informative about v. This happens iff (i) $\mu > 0$ and $\tau_{\epsilon_{n-j}} > 0$ for some $j \in \{1, \ldots, l^*-1\}$ or (ii) $\tau_{\epsilon_n} > 0$. In this case, let $\hat{z}_n$ be the signal on v that an outsider can obtain from the equilibrium price p_n, given that he observes p^{n-l^*}. Using Equation (36), we obtain that

$$\hat{z}_n = \frac{p_n - D_n E[v \mid p^{n-l^*}]}{A_n} = v - \sum_{j=0}^{l^*-1} \frac{B_{n,j}}{A_n} e_{n-j} \qquad (37)$$

Thus, $\{s_{in}, p^{n-l^*}, p_n\}$ is observationally equivalent to $\{s_{in}, p^{n-l^*}, \hat{z}_n\}$. Moreover, Equation (37) implies that

$$\hat{z}_n \mid v \sim N\left(v, A_n^{-2}\left(\sum_{j=0}^{l^*-1} B_{n,j}^2 \right)\tau_e^{-1} \right).$$

Let $\Gamma \stackrel{\text{def}}{=} A_n^2 (\sum_{j=0}^{l^*-1} B_{n,j}^2)^{-1}$. Using well-known properties of normal random variables, we obtain $E[v \mid s_{in}, p^{n-l^*}, p_n] = (\hat{\tau}_n(\mu, l) + \tau_{\epsilon_n})^{-1}(\hat{\tau}_n(\mu, l)E[v \mid p^{n-l^*}, p_n] + \tau_{\epsilon_n}s_{in})$, and $\text{Var}[v \mid s_{in}, p^{n-l^*}, p_n] = (\hat{\tau}_n(\mu, l) + \tau_{\epsilon_n})^{-1}$, where

$$\hat{\tau}_n(\mu, l) \stackrel{\text{def}}{=} (\text{Var}[v \mid p^{n-l^*}, p_n])^{-1} = (\text{Var}[v \mid z^{n-l^*}, \hat{z}_n])^{-1}$$
$$= \tau_{n-l^*}(\mu, l) + \Gamma \tau_e. \qquad (38)$$

When $A_n = 0$ (which happens when (i) $\mu = 0$ or $\tau_{\epsilon_{n-j}} = 0$ for all $j \in \{1, \ldots, l^*-1\}$, *and* (ii) $\tau_{\epsilon_n} = 0$), the price at date n does not provide information about v. This is as if the precision of the signal inferred from the price at date n by insiders were nil, that is, $\Gamma = 0$. This is indeed what we obtain for Γ when $A_n = 0$.

Thus, for all values of μ and τ_{ϵ_n}, we have

$$x_n^O(s_{in}, p^{n-l^*}, p_n)$$
$$= \gamma(\hat{\tau}_n(\mu, l) + \tau_{\epsilon_n})(E[v \mid s_{in}, p^{n-l^*}, p_n] - p_n)$$
$$= a_n^O(s_{in} - p_n) + \gamma\hat{\tau}_n(\mu, l)(E[v \mid p^{n-l^*}, p_n] - p_n), \qquad (39)$$

with $a_n^O = a_n^I = \gamma\tau_{\epsilon_n}$. In the rest of the proofs, we sometimes omit arguments μ and l in $\hat{\tau}_n(\mu, l)$ and $\tau_n(\mu, l)$ for brevity.

Clearing price in period n. The clearing condition in period n imposes $\int_0^\mu x_{in}^I \, di + \int_\mu^1 x_{in}^O \, di = e_n$. Using Equations (35) and (39), we solve for the equilibrium price and obtain

$$p_n = \frac{1}{K_n}(z_n + \mu\gamma\tau_n E[v \mid p^n] + (1-\mu)\gamma\hat{\tau}_n E[v \mid p^{n-l^*}, p_n]), \qquad (40)$$

where

$$K_n = a_n + \gamma(\mu\tau_n + (1-\mu)\hat{\tau}_n), \qquad (41)$$

with $a_n = \mu a_n^I + (1-\mu)a_n^O = \gamma\tau_{\epsilon_n}$. Observe that

$$E[v \mid p^{n-l^*}, p_n] = E[v \mid p^{n-l^*}, \hat{z}_n] = \hat{\tau}_n^{-1}(\tau_{n-l^*}E[v \mid p^{n-l^*}] + \Gamma\tau_e\hat{z}_n)$$

and

$$E[v \mid p^n] = E[v \mid p^{n-l^*}, \{z_{n-j}\}_{j=0}^{l^*-1}]$$
$$= \tau_n^{-1}\left(\tau_{n-l^*}E[v \mid p^{n-l^*}] + \tau_e \sum_{j=0}^{l^*-1} a_{n-j}z_{n-j} \right).$$

Substituting $E[v \mid p^{n-l^*}, p_n]$ and $E[v \mid p^n]$ by these expressions in Equation (40), we can express p_n as a function of v, $\{e_{n-j}\}_{j=0}^{l^*-1}$, and $E[v \mid p^{n-l^*}]$. In equilibrium, the coefficients on these variables must be identical to those in Equation (36). This condition imposes

$$A_n = \frac{(a_n + \mu\gamma\tau_e \sum_{t=n-(l^*-1)}^{n} a_t^2) + (1-\mu)\gamma\Gamma\tau_e}{K_n}, \quad (42)$$

$$B_{n,0} = \frac{1 + \mu\gamma a_n\tau_e + (1-\mu)\gamma\tau_e\Gamma B_{n,0}A_n^{-1}}{K_n}, \quad (43)$$

$$B_{n,j} = \frac{\mu\gamma a_{n-j}\tau_e + (1-\mu)\gamma\tau_e\Gamma B_{n,j}A_n^{-1}}{K_n},$$

$$\forall j \in \{1, \ldots, l^*-1\}, \quad (44)$$

$$D_n = \frac{\gamma\tau_{n-l^*}}{K_n}. \quad (45)$$

Equations (42)–(44) form a system with l^*+1 unknowns: A_n and $\{B_{n,j}\}_{j=0}^{j=l^*-1}$. Solving this system of equations, we obtain

$$A_n = \frac{a_n + \mu\gamma(\tau_n - \tau_{n-l^*})}{K_n}$$
$$\cdot \left(1 + \frac{(1-\mu)\gamma\tau_e(a_n + \mu\gamma(\tau_n - \tau_{n-l^*}))}{(1+\mu\gamma a_n\tau_e)^2 + \sum_{j=1}^{l^*-1}(\mu\gamma a_{n-j}\tau_e)^2}\right), \quad (46)$$

$$B_{n,0} = \frac{A_n(1 + \mu\gamma a_n\tau_e)}{a_n + \mu\gamma(\tau_n - \tau_{n-l^*})}, \quad (47)$$

$$B_{n,j} = \frac{A_n(\mu\gamma a_{n-j}\tau_e)}{a_n + \mu\gamma(\tau_n - \tau_{n-l^*})} \quad \text{for } 1 \le j \le l^*-1, \quad (48)$$

$$D_n = \frac{\gamma\tau_{n-l^*}}{K_n}. \quad (49)$$

Observe that A_n is positive. Moreover, it is equal to zero iff (i) $\mu = 0$ or $\tau_{\epsilon_{n-j}} = 0$ for all $j \in \{1, \ldots, l^*-1\}$, and (ii) $\tau_{\epsilon_n} = 0$, as conjectured. Last, we deduce from these expressions and Equation (38) that

$$\hat{\tau}_n(\mu, l) = \tau_{n-l^*} + \frac{(a_n + \mu\gamma(\tau_n - \tau_{n-l^*}))^2}{(1+\mu\gamma a_n\tau_e)^2 + \sum_{j=1}^{l^*-1}(\mu\gamma a_{n-j}\tau_e)^2}\tau_e. \quad (50)$$

This achieves the characterization of the equilibrium in each period in closed-form.

REMARK. Remember that when $A_n > 0$, the observation of the nth transaction price at date n conveys the following signal to outsiders: $\hat{z}_n = v - \sum_{j=0}^{l^*-1}(B_{n,j}/A_n)e_{n-j}$. Using Equations (43), (46), and (48), we deduce that $\hat{z}_n = \sum_{j=0}^{l^*-1}\alpha_j z_{n-j}$, with

$$\alpha_0 = (1 + \mu\gamma\tau_e)\left(a_n(1 + \mu\gamma\tau_e) + \mu\gamma\tau_e\left(\sum_{j=1}^{l^*-1} a_{n-j}\right)\right)^{-1}$$

and

$$\alpha_j = (\mu\gamma\tau_e)\left(a_n(1 + \mu\gamma\tau_e) + \mu\gamma\tau_e\left(\sum_{j=1}^{l^*-1} a_{n-j}\right)\right)^{-1}. \quad \square$$

The proofs of the remaining results of the paper are available in the online appendix.

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