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We investigate the role of options (contingent claims) in a buyer-supplier system. Specifically using a two-period model with correlated demand, we illustrate how options provide flexibility to a buyer to respond to market changes in the second period. We also study the implications of such arrangements between a buyer and a supplier for coordination of the channel. We show that, in general, channel coordination can be achieved only if we allow the exercise price to be piecewise linear. We develop sufficient conditions on the cost parameters such that linear prices coordinate the channel. We derive the appropriate prices for channel coordination which, however, violate the individual rationality constraint for the supplier. Contrary to popular belief (based on simpler models) we show that credit for returns offered by the supplier does not always coordinate the channel and alleviate the individual rationality constraint. Credit for returns are useful only on a subset of the feasibility region under which channel coordination is achievable with linear prices. Finally, we demonstrate (numerically) the benefits of options in improving channel performance and evaluate the magnitude of loss due to lack of coordination.
(*Supply Contracts; Real Options; Coordination; Flexibility*)

1. Introduction

In recent years, it has become widely recognized that to compete effectively firms need to develop the capability to respond quickly to changing market needs. For example, consider a simple single-supplier, single-buyer system operating in a market with short product life cycles and highly unpredictable demands. In such situations flexibility for a buyer implies the ability to get additional products in response to changes in market demands during the short life cycle of the product. While this flexibility benefits the buyer, it may come at some cost to the supplier.[1]

[1]The additional cost is incurred due to some inflexibility that a supplier faces; e.g., long production lead times or long procurement lead times for some component(s) that the supplier uses in her production process. Consequently, to provide the flexibility to a buyer, she may carry additional inventory of raw material or finished goods, and/or employ expediting or out-source production, all of which entail additional costs.

Therefore, a supplier will usually provide limited flexibility or may offer prices based on the level of flexibility desired by the buyer. We illustrate some practices from three different industries, viz. toys, apparel, and electronics.

Consider the following situation recently faced by the toy maker Mattel, Inc. (Kravetz 1999):

> Mattel was hurt last year by inventory cutbacks at Toys ''R'' Us, and officials are also eager to avoid a repeat of the 1998 Thanksgiving weekend. Mattel had expected to ship a lot of merchandise after the weekend, but retailers, wary of excess inventory, stopped ordering from Mattel. That led the company to report a $500 million sales shortfall in the last weeks of the year. . . . For the crucial holiday selling season this year, Mattel said it will require retailers to place their full orders before Thanksgiving. And, for the first time, the company will no longer take reorders in December, Ms. Barad said. This will enable Mattel to tailor production more closely to demand and avoid building inventory for orders that don't come.

Effectively, Mattel is deciding not to provide any

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flexibility to its retailers to respond to in-season changes in market demand. While this strategy limits the downside risk for Mattel, the upside potential is also limited. The basic premise of this paper is that such measures may hurt overall channel performance. While providing flexibility is costly (e.g., inventory carrying for Mattel), mechanisms exist to mitigate this risk and create a win-win situation for both the manufacturer and the retailer.

The apparel industry is plagued by the increasing cost of markdowns (Frazier 1986, Hammond 1990, Fisher et al. 1994) running into billions of dollars. A primary reason for this phenomenon is that retailers must place firm, SKU-specific orders well in advance of the beginning of the selling season (Nuttle et al. 1991) despite demonstrable advantages to in-season replenishment (Whalen 1993, King and Hunter 1996, Hunter et al. 1996, Pinnow and King 1997). These studies estimate that the savings to the retailer are large enough that he/she can afford to pay an additional 30–50% to a supplier that provides in-season replenishments. In-season replenishments are problematic due to long procurement/production lead times for manufacturers when compared to the length of the selling season faced by a retailer.[2] A manufacturer can make in-season replenishment possible either by reducing the lead time or by carrying inventories. The latter strategy of carrying (raw-material or finished goods) inventories, however, comes with risks of over-stocking. The industry has undertaken several efforts to reduce the lead time; yet, apparently these have not been sufficient to facilitate in-season replenishments as documented by the studies mentioned earlier.

In the apparel catalog industry, contracts similar to ours have been used to provide flexibility to a buyer for in-season replenishment. Under a *backup agreement* (Eppen and Iyer 1997), a buyer commits to a total order quantity for the season. A prespecified fraction of the total quantity is delivered initially. The buyer may purchase additional units up to the remaining commitment at a later date. He pays a penalty for any committed units not purchased. Such contracts are used, for example, by Anne Klein, Finity, DKNY, and Liz Claiborne with the catalog company Catco (Eppen and Iyer 1997).

Finally, in the electronics industry, flexibility for reorders is provided under arrangements known as *quantity-flexibility contracts.* Under such an arrangement, the buyer first provides a forecast of future orders to the supplier. In subsequent periods, he is allowed to place actual orders that are within prespecified limits of the original forecasts; in addition, he may be allowed to update the future forecasts. Such contracts are used, for example, by IBM Printer Division (Bassok et al. 1997), Sun Microsystems (Farlow et al. 1995), Solectron, Hewlett Packard, etc. (Tsay and Lovejoy 1999). In the semiconductor industry, under agreements called *pay-to-delay* capacity reservation, allocation and reservation for wafer fabrication capacity is offered by a supplier in return for a fixed, up-front payment (Brown and Lee 1997). The buyer could then place orders at a later date and use the up-front payment towards actual procurement costs. A large portion of the allocation is usually ''take-or-pay'' capacity—capacity for which the manufacturer will have to pay the full wafer production price even if he does not need the wafers. Brown and Lee (1997) state that according to a recent survey by the Fabless Semiconductor Association, 30% of capacity reservation arrangements are pure take or pay. Recently, however, options for capacity have been offered by the Taiwanese Semiconductor Manufacturing Company (Chang 1996), a semiconductor fabrication foundry company.

In a supply chain context, in addition to the need for a provision of flexibility, the ability to ''coordinate'' decisions between various links in the chain becomes critical (Hammond 1992). When the buyer and the supplier optimize their respective objective functions, ''double marginalization'' (Tirole 1990) adversely impacts channel performance. Several policies have

[2]Lovejoy (1999) states that this lead time consists of the long production lead time of the fabric (estimated to be larger than 10 weeks) and the transportation lead time from off-shore production facilities (4 weeks). He argues that the apparel production itself is relatively fast and takes between two and seven days. The length of the selling season is between 20–24 weeks. Even if we assume that new orders will be placed only four periods into the season, these orders will arrive only towards the end of the season, and will not be of great help.

been proposed for channel coordination, notable among which are nonlinear prices (e.g., a two-part tariff, quantity discounts) and return policies.

In this paper we provide a generic framework for the study of the role of options in a buyer-supplier system for short life cycle products. We show that backup agreements, two-period, quantity-flexibility contracts and pay-to-delay arrangements are but special cases. We then illustrate how options provide flexibility to a buyer to respond to market changes and study the implications of such arrangements between a buyer and a supplier for coordination of the system. In developing a model for the supply chain, we focus particularly on the apparel industry.

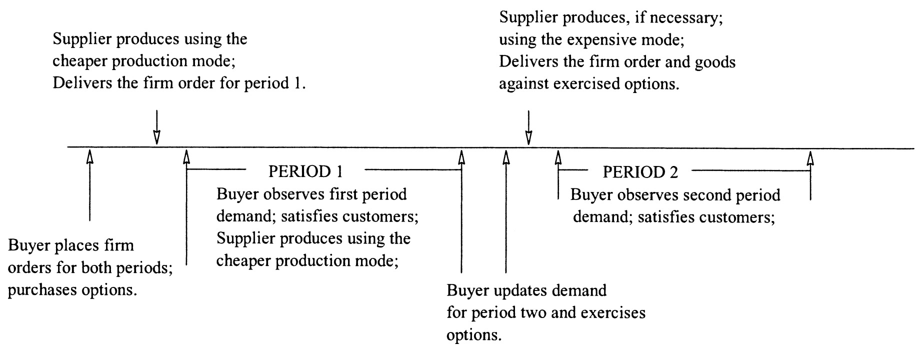
Consider a single-buyer, single-supplier system. The buyer sells a single product to consumers at a fixed market price that is exogenously specified. The product life cycle is short; we refer to this short life cycle as a season and assume it to be of fixed duration. We divide the season into two periods of possibly unequal lengths with correlated demands. The buyer updates the second-period demand and places an additional order after observing and satisfying actual first-period demand.[3] Before the beginning of the horizon, the buyer makes three decisions. He places *firm* orders for goods to be delivered at the beginning of Periods 1 and 2. In addition, he purchases *options* from the supplier that give him an opportunity to order additional units of the product (up to the number of options purchased) before the start of Period 2 but after observing demand in Period 1. The total procurement costs consist of the following components: (i) unit *wholesale prices* for goods procured against firm orders in each period, (ii) an *option price* for every unit of option purchased, and (iii) an *exercise price* for options exercised to obtain additional products in the second period. Excess demand is backordered in the first period and is considered lost sales in the second period. The buyer incurs the standard holding and penalty costs and earns revenues for the products sold for the two periods.

To produce the goods for the buyer, the supplier procures components/raw materials from its upstream supplier that require a long lead time such that there is no opportunity to reorder these during the season. Furthermore, the raw material is costly, so the supplier is unwilling to take the risk of carrying it unless the buyer is willing to share this risk. We believe this accurately reflects the situation in the apparel industry described earlier, where the supplier is the apparel manufacturer and the buyer is the retailer. For the firm orders that a buyer places, the supplier can purchase the exact quantities of raw material needed. There is some uncertainty, however, regarding the number of additional units of the goods a buyer may need in the second period. To be able to provide flexibility to the buyer to order additional goods, the supplier needs to procure sufficient raw material in advance with the associated risks. To mitigate this risk, she offers options to the buyer at a price. The supplier makes a commitment to produce products for the second period, up to the number of options purchased by the buyer. The supplier has two production opportunities. She can produce at a regular (cheaper) cost before and during the first period. In addition, at an extra cost, she can also produce before the beginning of the second period, but *after* observing the number of options exercised by the buyer. The supplier needs to determine the optimal wholesale, option, and exercise prices, as well as determine the optimal production quantities for both periods. A time line of buyer-supplier decisions is illustrated in Figure 1.

In analyzing this basic buyer-supplier model, we consider various channel structures. Primarily we compare a decentralized system in which the buyer and supplier are considered independent profit maximizers to a centrally controlled channel. We then ask: Can *channel coordination* be achieved? Channel coordination is achieved when the supplier and the buyer make decisions (independently) in such a way that the joint profits are maximized. In addition, to compare the performance of the systems with options, we use a benchmark decentralized model in which no options are offered.

[3] Fisher and Raman (1996) have documented the value of updating the forecast after initial sales (e.g., 20% into the season) are observed. Eppen and Iyer (1997) also state that the demand update occurs fairly early in the season, thus justifying the unequal lengths for the two periods.

Figure 1 A Time Line of Buyer-Supplier Decisions



In highlighting our contributions, it is useful to first understand what has been accomplished in the literature. Observe that the two key elements of our model include (i) options as an instrument of flexibility and (ii) incentives for coordination in such environments. Several researchers have considered options or option-like arrangements for flexibility provided through a supply contract. Most of this research, however, takes a single-decision maker approach and derives the optimal policy for the buyer under a given contract. For example, in a two-period problem with correlated demands, Eppen and Iyer (1997) study *backup agreements* and Brown and Lee (1997) study *pay-to-delay* capacity reservation contracts. In a multiperiod setting with uncorrelated demands Bassok and Anupindi (1998) and Tsay and Lovejoy (1999) study *quantity-flexibility contracts.*

Some of the key insights developed by these papers include (i) flexibility-required increases with the coefficient of variation (CV) of demand (Bassok and Anupindi 1998, Tsay and Lovejoy 1999) and (ii) the value of flexibility (through backup or pay-to-delay contracts), which is based more on the ability to learn from early demand than on CV of demand (Eppen and Iyer 1997, Brown and Lee 1997). Some questions remain. For example, do these relationships hold up in equilibrium? When are such contracts efficient (i.e., coordinate the channel)? We show that, in equilibrium, the value of flexibility (through option contracts in our case) depends both on the CV of demand and the correlation coefficient. In fact, there is a ''complementarity'' relationship between the two. That is, the increase in the value of flexibility with correlation increases with the CV of demand. Furthermore, we develop sufficient conditions under which backup agreements are efficient.

The second key element of our study includes incentives for channel coordination. There is a large body of literature that studies incentive contracts for channel coordination under stochastic demand; see for example, Pasternack (1985), Donohue (1996), Ernst and Cohen (1992), Moses and Seshadri (2000), Tsay (1999), Narayanan and Raman (1997), Kouvelis and Gutierrez (1997), Iyer and Bergen (1997), Cachon and Lariviere (1997), and Lee and Whang (1999). Clearly, channel coordination is always achieved when the buyer is able to internalize the costs of the supplier. This happens, for example, if the supplier ''sells the firm'' to the buyer, implemented by a supplier charging marginal costs. There are two issues with this approach. First, marginal cost pricing may not be linear. So if a firm is restricted to linear pricing, can channel coordination still be achieved? We develop sufficient conditions under which this is possible. Second, marginal cost pricing is not *individually rational* for the supplier. Hence extant research has focused on individually rational instruments for coordination; for example, return policies and nonlinear prices. This paper enhances our understanding of such instruments for coordination, on which we now briefly elaborate.

The role of return policies (also called buy-backs) to achieve channel coordination was established by Pasternack (1985) for single-period models under the assumption of identical salvage values for both the manufacturer and the buyer and *frictionless* implementation of return policies.[4] Donohue (1996) extends this to a two-period setting that includes options. In her model the supplier must preposition some portion of component inventory before demand information is known, entailing a risk that is shared via options. Our model differs from hers in several ways. While she limits her model to goods purchased entirely using options, we allow both committed orders and options. Moreover, in her model, while information may be revealed in two stages, demand is realized only once, and hence the model itself remains that of a single period, like in Pasternack (1985). Thus these models do not accurately capture the phenomenon of in-season replenishments. In contrast, in our setting early information comes from actual observed demand leading to a two-period model. Furthermore, her analysis is restricted to the case of perfect correlation, while we present results for general correlation structures. Finally, like Pasternack (1985) she assumes identical salvage values for the buyer and the supplier and a frictionless returns process, while we make no such assumptions.

Not surprisingly, the key findings of Pasternack (1985) and Donohue (1996) are similar; namely, (i) return policies coordinate the channel and (ii) credit for returns allows an arbitrary spread of the channel profits between the buyer and the supplier (Donohue illustrates this under perfect correlation of demand). Several questions remain. What happens when their cost assumptions do not hold? Can return policies still coordinate the channel? Can return policies achieve an arbitrary split of channel profits under more general conditions? Our results show that return policies do not always coordinate the channel. We develop cost conditions under which coordination is achieved (which includes the assumptions made by Pasternack and Donohue). When the channel is coordinated, however, credit for returns does achieve an arbitrary split of channel profits for *any* correlation thus generalizing the result of Donohue.

Finally, using an extensive numerical study, we attempt to characterize environments, providing managerial insights under which (i) benefits of options and coordination are large, (ii) options alone capture most of these benefits, and (iii) incentives for coordination are desirable as well.

[4]A frictionless returns process implies that there is no cost to either party to execute the returns process.

The rest of the paper is organized as follows. In §2 we first present a general model encompassing the various flexibility arrangements discussed earlier as special cases. We also describe the various channel structures and the model assumptions here. Subsequently in §3 we develop expressions for the expected profit of the buyer and the supplier for various channel structures. In §4 we discuss the issue of channel coordination under linear prices. In §5, we numerically compare the performance of various channel structures. We conclude in §6. Appendix A summarizes the notations used. The expressions for profit functions appear in Appendices B and C. Proofs of all results appear in Appendix D.

2. A General Model

We consider a single-buyer, single-supplier system with two periods and correlated demands. The buyer makes three ordering decisions at the beginning of the horizon. First, he orders Q_i units at a unit *wholesale price* of c_i to be delivered at the beginning of period $i \in \{1, 2\}$; we refer to the Q_i as firm orders.[5] In addition, he purchases M options at a unit *option price* of c_o. In Period 2, he may choose to exercise $m \leq M$ options at a per unit *exercise price* of c_e. We assume that one option gives the buyer a right to purchase one unit of good. The buyer sells goods to the consumer at a unit price of r. Let h_i^b and p_i be the unit holding and shortage penalty cost for period $i \in \{1, 2\}$. The holding costs for the two periods are permitted to differ to account for possible disparity in period lengths. The shortage costs differ reflecting the difference between a loss of goodwill due to backlogging of excess demand in the first period and the cost associated with not satisfying a customer at all (lost sale) in the second period. Any

[5]Observe that the second firm order Q_2 does play an important role. Even though he decides Q_2 at the beginning of the horizon, the buyer saves holding cost on finished goods by asking for delivery later during the season. This allows the supplier to better manage her production capacity. In addition, using Q_2 she may increase the average number of products sold using the cheaper mode of production.

leftover finished goods at the end of the second period may be salvaged at a per unit price of $\tilde{v}_f^b$. We define the *effective salvage value* of finished goods for the buyer to be $v_f^b = \tilde{v}_f^b - h_2^b$. Demand in period $i \in \{1, 2\}$, denoted by D_i, is assumed to be normally distributed with mean μ_i and standard deviation σ_i with conditional density and distribution functions of $f_{Di}(\cdot)$ and $F_{D_i}(\cdot)$, respectively. Demands in the two periods are correlated with a correlation coefficient of ρ. Throughout the paper we will use uppercase D_i to denote the demand random variable and lowercase d_i to denote a realization of the appropriate demand random variable. The standard normal distribution function is denoted by $\Phi(\cdot)$.

The supplier must purchase sufficient raw material to produce the maximal quantity possibly requested (that is, $Q_1 + Q_2 + M$, the sum of the committed orders and the options purchased).[6] The per-unit procurement cost of raw material is c_r. As discussed earlier, the supplier has two modes of production. She will produce the required quantity (Q_1) for the first period using the cheaper production mode at a unit labor cost of c_L. In addition, she may choose to produce the firm requirements for the second period (Q_2) and any extra production that she desires (in anticipation of high demand in the second period) using the cheaper production mode (before or during the first period) and thus incur a per-unit holding cost of h^s. Let X_L be the total units produced using the cheaper production mode. In the second period, after the buyer exercises options, the supplier may undertake additional production using an expensive production mode. We assume that additional production is expensive due to increased labor costs (for example, due to overtime costs and/or disruptions of other schedule work); specifically the supplier incurs a total labor cost of $c_L(1 + \gamma)$ where γ is an exogenous parameter. We assume that the capacity in both production modes is sufficient to produce any quantity up to the number of options purchased by the buyer.[7]

[6]We implicitly assume that the raw material has a long acquisition lead time such that it needs to be processed before the start of the season. The supplier can, however, acquire any amount of raw material.

[7]That is, the bottleneck in the supply chain is raw material/component procurement. Once the buyer purchases options, the maximum production quantity of apparel is known to the manufacturer, who can then plan appropriate capacity.

Any raw material left over after supplying the goods needed against options exercised by the buyer is immediately salvaged for v_r^s per unit. Hence, the supplier does not hold any raw material inventory in the second period. We do not explicitly account for raw material holding cost in the first period. Whether or not the supplier incurs this cost depends on how the raw material is delivered. If raw material is staged for delivery just in time for the two production opportunities, then the supplier does not incur any holding cost in the first period. Alternately, all raw material may be delivered to the supplier in the first period. In such cases, any first-period, raw material holding cost can be incorporated into the expedited second-period production cost and salvage value of the raw material. Any leftover finished goods are also immediately salvaged (at the beginning of the second period, after satisfying exercised options) for v_f^s per unit. Since capacity is unconstrained, the supplier will be able to produce exactly what the buyer requests, and hence experiences no shortage cost.

2.1. Special Cases

We now demonstrate that the various flexibility contracts presented in §1 are special cases of our general model.

Backup Agreement. In a model of a backup agreement presented in Eppen and Iyer (1997), at the beginning of the season the buyer makes a firm commitment to purchase Q units during the season. At the first period she purchases $Q_1 = Q(1 - \beta)$ units, at the price c per unit. At the second period the buyer may purchase up to $Q\beta$ units at the price c per unit. If the buyer decides to purchase m units in the second period, where $m < Q\beta$, then she pays a penalty, b per unit, for the remaining $Q\beta - m$ units not purchased. The correspondence between our general model parameters and a backup agreement is shown in Table 1.

Quantity Flexibility (QF) Contract. Consider a two-period version of the commonly used quantity flexibility contract (Bassok and Anupindi 1998, Tsay and Lovejoy 1999). The wholesale price is c for both periods. The quantities committed for the two periods are $\tilde{Q}_1$ and $\tilde{Q}_2$. Let the second-period upward and downward flexibility be α_u and α_d, respectively. That

Table 1 Special Cases

General Model	Q_1	Q_2	M	c_1	c_2	c_o	c_e
Backup Agreement	$Q(1-\beta)$	0	$Q\beta$	c	—	b	$c-b$
QF Contract	$\tilde{Q}_1$	$(1-\alpha_d)\tilde{Q}_2$	$(\alpha_d+\alpha_u)\tilde{Q}_2$	c	c	0	c
Pay-to-Delay (Single-Period Demand)	—	y	$z-y$	—	c_f	c_o	c_e
Pay-to-Delay (Two-Period Demand)	y	—	$z-y$	c_f	—	c_o	c_e

is, in the second period the maximum quantity that can be purchased is $\tilde{Q}_2(1+\alpha_u)$, and the minimum quantity that needs to be purchased is $\tilde{Q}_2(1-\alpha_d)$ at the unit price of c. The correspondence between the general model parameters and a QF contract is shown in Table 1.

Pay-to-Delay Capacity Reservation. Under this form of capacity reservation analyzed by Brown and Lee (1997), a buyer makes a total reservation z of which he is obligated to purchase at least $y < z$ units (called take or pay). He pays a unit cost c_f for the take-or-pay capacity and a unit option cost of c_o for $z-y$ units. Additional units up to a maximum $z-y$ can be bought at an extra unit cost of c_e.[8] As discussed earlier, their analysis is limited to a single-period model in which information about demand is revealed in two stages. In our general model this corresponds to the second-period problem where the first-period demand realization is treated as information about demand. In that case the correspondence between the general model and pay-to-delay reservation (labeled as "Single-Period Demand") is as shown in Table 1. Alternately, one could apply the pay-to-delay capacity reservation agreement to allow for two periods of actual demand. The correspondence follows analogously as shown in Table 1.

Thus for any given backup agreement and simple QF contract there exists a corresponding option contract with restrictions on the number of options and the option/exercise price. Thus the performance of the channel and the supplier will be no worse with the more general options contract we propose. Our model specialized to a capacity investment setting with single-period demand realization is equivalent to a pay-to-delay arrangement presented in Brown and Lee (1997).[9] We now proceed with the analysis of the general model proposed in this paper.

2.2. Channel Structures and Assumptions

We compare three channel structures: In the *Centralized System* (*CS*), we assume that there is one decision maker who controls the channel and, hence, makes decisions to optimize total system profits. Observe that under this scenario, the wholesale, option, and exercise prices play no role in determining the system profits. The only appropriate decision variables are production decisions for the two periods under the two production modes, the shipment quantities, and the quantity of raw material to be ordered at the beginning of the season. The optimal solution to the CS is called the *first-best* solution. In the *Decentralized System* (*DS*) system, the buyer and supplier play a Stackelberg game where the supplier is the leader and the buyer the follower. This is reasonable whenever the supplier has more power in the channel than the buyer.[10] In this situation, for a given set of prices—wholesale, option, and exercise—announced by the supplier, the buyer places orders in the two periods which maximize his expected profits. These orders act as implicit demand functions for the supplier, who then determines the optimal prices and appropriate production quantities that maximize her profits. A *Decentralized System with No Options* (*DSNO*) is similar to the DS but without options. Since there are

[8]Brown and Lee (1997) allow for additional unreserved units to be purchased at a unit cost of $c_p > c_e + c_o$. In contrast, our model does not allow for this additional source of supply.

[9]Our analysis, with minor modifications, can be used to study capacity reservation issues as in Brown and Lee (1997).

[10]The alternate situation of the buyer being the leader is also possible and is an area of future research.

no options, the buyer cannot order any additional quantity in the second period after observing demand; he places firm orders for the two periods at the beginning of Period 1.

The CS is used as a benchmark case to investigate if there exists an appropriate decentralization mechanism (through prices and/or quantities) such that the DS will achieve the first-best solution. In addition, we use the DSNO as a benchmark case to determine if both the supplier and the buyer increase their profits from the use of options, and numerically quantify the benefits of using options. Finally, we make the following assumptions:

(1) **Cost Assumptions.** We assume that the raw material, labor and salvage costs exhibit the following relationships:

ASSUMPTION C1. $v_f^s \leq c_r + c_L + h^s$, *and* $v_r^s \leq c_r$.

ASSUMPTION C2. $v_f^b \leq c_r + c_L + h^s$.

ASSUMPTION C3. $h^s \leq c_L \gamma$.

ASSUMPTION C4. $h^s < h_1^b$.

Assumptions C1 and C2 ensure that the system profits in the CS are finite. Assumption C1, in addition, ensures that the supplier's profits in the DS and DSNO are finite. Assumption C3 says that it is never more expensive to produce a unit in the cheaper mode and incur holding costs than to produce it in the expedited mode. Finally, Assumption C4 states that the buyer's first-period holding cost is larger than that of the supplier. Assumptions C3 and C4 are reasonable and allow us to limit the analysis to a small number of permutations of the cost parameters.

(2) **Correlation Structure.** We will assume that $\rho < 1$. This allows us to focus on a two-period problem. If $\rho = 1$, we effectively have a single-period problem, and it can be shown that channel coordination is always achieved (Barnes-Schuster et al. 2002).

(3) **Information Structure.** Demand distributions are common knowledge. Since the supplier's profits are affected by the buyer's order quantities and the number of options exercised, she needs to know the distribution of demands in both periods to be able to set optimal prices.

(4) **Delivery Commitment.**

- The supplier is obligated to supply goods against options purchased. This obligation is part of the contract because the buyer, by purchasing options, acquires the right to purchase a certain amount of goods. This assumption is also made by other researchers; e.g., Eppen and Iyer (1997), Donohue (1996), and Brown and Lee (1997).
- All goods against exercised options are delivered immediately.

(5) **Salvage Location.** We initially assume that in the CS any excess finished goods at the supplier and buyer locations will be salvaged at their respective locations regardless of the salvage values. In §4 when we discuss return policies, we allow the CS to salvage in the most favorable location. This allows us to make a fair comparison of CS and DS in each of the cases—with and without return policies.

3. Analysis

To analyze the various systems, we first need to develop expressions for the expected profits in each system. For clarity of exposition, we first derive the individual profit functions of the supplier and the buyer in the DS and then aggregate appropriately to write the profit function for the CS.

3.1. Buyer's Profit Function in the DS

The buyer's ordering problem can be set up as a two-stage stochastic dynamic program. That is, first we calculate the optimal policy for exercising options given an initial inventory (on-hand + on-order), $I_2 = Q_1 + Q_2 - d_1$, at the beginning of the second period. We then substitute the resulting function into the two-period total profit function. Let $(\cdot)^+$ denote the positive part of $(\cdot)$.

Buyer's Order Policy for Period 2. We first develop the buyer's last-period conditional expected profit function,

$$\begin{aligned}\Pi_2^b(m, I_2) &= E_{D_2}[r \min(D_2, (I_2 + m)^+) \\ &\quad + r \min((d_1 - Q_1)^+, Q_2 + m) - c_e m \\ &\quad - p_2(D_2 - I_2 - m)^+ + v_f^b(I_2 + m - D_2)^+]. \qquad (1)\end{aligned}$$

The first term is the buyer's revenue from current sales; the second term is revenue from first-period backorders satisfied in Period 2, and the third term is the cost to exercise m options. The fourth and fifth terms are the shortage and effective salvage costs, respectively. The buyer then solves

$$\max_{0 \le m \le M} \Pi_2^b(m, I_2).$$

This is equivalent to a capacitated inventory model whose optimal solution is known to be a *modified base-stock policy* (Federgruen and Zipkin 1986). That is, the buyer will order up to the optimal base-stock level for the uncapacitated problem, provided the amount ordered does not exceed M. The optimal base-stock level, Z, is given by

$$F_{D_2|d_1}(Z) = \frac{p_2 + r - c_e}{p_2 + r - v_f^b}, \tag{2}$$

where $F_{D_i|d_1}$ is the conditional cumulative distribution function of Period 2 demand. Thus the optimal number of options to exercise, m^*, is:

$$m^* = \min\{\max\{Z - I_2, O\}, M\}.$$

We can develop an explicit expression for Z. Observe that the conditional mean and standard deviation of a normally distributed and correlated random variable are as follows:

$$\mu_{D_2|d_1} = \mu_2 + \rho\frac{\sigma_2}{\sigma_1}(d_1 - \mu_1) \quad \text{and}$$

$$\sigma_{D_2|d_1} = \sigma_2\sqrt{1 - \rho^2}.$$

Define k_m to be such that $\Phi(k_m) = (p_2 + r - c_e)/(p_2 + r - v_f^b)$. Then, $Z = \mu_{D_2|d_1} + k_m\sigma_{D_2|d_1}$. Let

$$\delta^d = \mu_2 - \rho\frac{\sigma_2}{\sigma_1}\mu_1 + k_m\sigma_2\sqrt{1 - \rho^2} \quad \text{and}$$

$$\eta = 1 + \rho\frac{\sigma_2}{\sigma_1}. \tag{3}$$

Noting that $I_2 = Q_1 + Q_2 - d_1$ and substituting for Z, m^* can be written as a function of the first-period demand outcome, d_1, and the order quantities, Q_1, Q_2, and M giving

$$m^* = \begin{cases} G(\delta^d, Q_1 + Q_2) - G(\delta^d, Q_1 + Q_2 + M) \\ \quad \text{if } c_e > v_f^b \\ M \quad \text{if } c_e \le v_f^b \end{cases} \tag{4}$$

where

$$G(x, y) = \max\{x + \eta d_1 - y, 0\}. \tag{5}$$

Buyer's Order Policy for Period 1. At the beginning of Period 1 the buyer needs to decide on the order quantities Q_1, Q_2, and the number of options, M, to purchase. Recall that any demand not satisfied in the first period will be back-ordered and satisfied in the second period. The first-period expected profit function is:

$$\begin{aligned}\Pi_1^b&(Q_1, Q_2, M)\\ &= E_{D_1}[r \min(D_1, Q_1) - c_oM - c_1Q_1 - c_2Q_2\\ &\qquad - p_1(D_1 - Q_1)^+ - h_1^b(Q_1 - D_1)^+].\end{aligned} \tag{6}$$

The first term is the total revenue the buyer receives. The second term is the cost to purchase M options and the third and fourth terms represent the purchase cost for firm orders. The last two terms are the shortage and holding costs. The total expected profit is then given by $\Pi^b(Q_1, Q_2, M) = \Pi_1^b(Q_1, Q_2, M) + E_{I_2}\Pi_2^b(m^*, I_2)$. Define a vector $\mathbf{X} = (X_1, X_2, X_3) = (Q_1, Q_1 + Q_2, Q_1 + Q_2 + M)$. The nonnegativity constraints on Q_i imply $0 \le X_1 \le X_2 \le X_3$. Substituting for Q_1, Q_2, and M in $\Pi^b(Q_1, Q_2, M)$ allows us to write the total expected profits of the buyer as a separable function in X_1, X_2, and X_3 as follows:

$$\Pi_{\text{DS}}^b(X_1, X_2, X_3) = J_1(X_1) + J_2(X_2) + J_3(X_3) \tag{7}$$

where $0 \le X_1 \le X_2 \le X_3$, and $J_i(X_i)$ for $i = 1, 2, 3$ are as given in Appendix A.[11]

It is then straightforward to establish joint concavity of $\Pi_{\text{DS}}^b(X_1, X_2, X_3)$ in X_i, $i = 1, 2, 3$. Joint concavity with a linear constraint ($X_1 \le X_2 \le X_3$) implies that there will be a unique maximum.

3.2. Supplier's Profit Function in the DS

We now turn to the supplier's decisions. The supplier needs to have enough raw materials on hand to produce and deliver a maximum of X_3 units of goods. She produces X_L units using the cheaper mode of

[11]Details of the derivation of (7) are available from the authors.

production and the remainder, if necessary, using the expensive mode of production. From Assumption C3 it is clear that $X_2 \leq X_L \leq X_3$. The expected profit function of the supplier is

$$\begin{aligned}
&\Pi^s_{\mathrm{DS}}(X_L, c_1, c_2, c_o, c_e) \\
&\quad = E_{D_1}[c_1X_1 + c_2(X_2 - X_1) + c_o(X_3 - X_2) + c_em^* \\
&\qquad - h^s(X_L - X_1) \\
&\qquad + v^s_r(X_3 - X_2 - \max(X_L - X_2, m^*)) \\
&\qquad + v^s_f(X_L - X_2 - m^*)^+ - c_rX_3 - c_LX_L \\
&\qquad - c_L(1 + \gamma)(m^* - (X_L - X_2))^+].
\end{aligned}$$

The first and second terms represent the revenues received from firm orders for the first and second periods, respectively. The third term is the revenue received from the sale of options, and the fourth term is the revenue received from options exercised. The fifth term is the cost of holding leftover finished goods at the end of the first period. The sixth and seventh terms are the revenues received from salvaging raw material and finished goods, respectively. The eighth term is the cost of procuring the required raw material. The ninth term is the total labor cost in the cheaper production mode and the last term is the labor cost in the expensive production mode to produce the number of options exercised by the buyer.

The supplier then solves the following problem:

$$\max_{X_L, c_1, c_2, c_o, c_e} \Pi^s_{\mathrm{DS}}(X_L, c_1, c_2, c_o, c_e) \quad \text{s.t.} \quad X_2 \leq X_L \leq X_3.$$

Later we explore the issue of channel coordination by setting appropriate transfer prices; this would still involve solving for optimal X_L for a given set of prices. It is easy to show that $\Pi^s_{\mathrm{DS}}(X_L, c_1, c_2, c_o, c_e)$ is concave in X_L for a given c_1, c_2, c_o, and c_e. We close this subsection with the optimal policy structure for X_L, the total production quantity using the cheaper mode. The derivation is analogous to that of m^* and is hence omitted.

Proposition 1. *Let $\hat{X}^*_L$ be the optimal solution of $\Pi^s_{\mathrm{DS}}(X_L, c_1, c_2, c_o, c_e)$ for a given c_1, c_2, c_o, and c_e when X_L is unconstrained. Define $k_o = \Phi^{-1}[(c_L\gamma - h^s)/(v^s_r + c_L(1 + \gamma) - v^s_f)]$. Then, $\hat{X}^*_L = \delta^d + \eta(\mu_1 + \sigma_1k_o)$, where δ^d and η are given by (3). The optimal (constrained) production amount, X^*_L, for the supplier to produce using the cheaper production mode is $X^*_L = \max(X_2, \min(X_3, \hat{X}^*_L))$.*

3.3. Joint Profit Function in the CS

We now derive the joint profit function in the CS. Note that c_1, c_2, c_o, and c_e play no role in the expression for the joint profits because they merely affect transfer payments between the two parties. As a result, the only decisions that affect system profit are decisions concerning order and production quantities. Furthermore, the second-period firm order (Q_2) used in the DS plays no role in the CS. The amount shipped to the buyer's location (to satisfy demand) in the second period is a function of the total production (using both modes) and the number of options exercised.

Let X^c_1, X^c_L, and X^c_3 represent the respective quantities in the CS corresponding to X_1, X_L, and X_3 in the DS. In addition, let $I^c_2 = X^c_1 - d_1$ be the on-hand inventory at the beginning of the second period in the CS. The expected profit function for the last period, $\Pi^c_2(m_c, I^c_2)$, follows by appropriately combining the second-period profit function of the buyer, $\Pi^b_2(m, I_2)$, and the second-period cash flows for the supplier in the DS. The main difference is that while $\Pi^b_2(m, I_2)$ was independent of the supplier's production decision given by X_L, $\Pi^c_2(m_c, I^c_2)$ will depend on X^c_L. As before, we are concerned here about the number of goods the supplier will produce using the expensive production mode, which depends on the relationship between the salvage value of finished goods (either at the supplier's or buyer's location) and the marginal cost of expensive production. We have two cases to consider:

Case (i). $v^s_f \leq v^s_r + c_L(1 + \gamma)$, and $v^b_f \leq v^s_r + c_L(1 + \gamma)$. In this case, the CS will not convert all raw material to finished goods. We then write the profit function as a sum of two sets of terms as follows:

$$\begin{aligned}
&\Pi^c_2(m_c, I^c_2) \\
&\quad = E_{D_2|d_1}\{[r \min(D_2, (I^c_2 + m_c)^+) \\
&\qquad + r \min((d_1 - X^c_1)^+, m_c) - p_2(D_2 - I^c_2 - m_c)^+ \\
&\qquad + v^b_f(I^c_2 + m_c - D_2)^+] \qquad (8a)
\end{aligned}$$

$$+ [-c_L(1+\gamma)(m_c - (X_L^c - X_1^c))^+ + v_r^s(X_3^c - X_1^c - \max(m_c, (X_L^c - X_1^c))) + v_f^s(X_L^c - X_1^c - m_c)^+]\}. \quad (8b)$$

The first set of terms given by (8a) is analogous to $\Pi_2^b(m, I_2)$ given by (1), and the second set of terms given by (8b) is analogous to the second-period cash flows of the supplier in the DS.

Case (ii). $v_f^s > v_r^s + c_L(1+\gamma)$ or $v_f^b > v_r^s + c_L(1+\gamma)$. In this case the system can turn a profit on every unit of raw material converted to finished goods. Furthermore, because $h^s \le \gamma c_L$ (Assumption C3), all raw material will be converted to finished goods using the cheaper production mode, and hence $X_L^c = X_3^c$. Thus the cash flows in the second period at the supplier's location represented by (8b) reduce to $v_f^s(X_3^c - X_1^c - m_c)$.

The decision maker, in either case, optimizes $\Pi_2^c(m_c, I_2^c)$ for the number of options, m_c, to exercise such that $m_c \le X_3^c - X_1^c$. The optimal number of options will depend on the various cost conditions and the correlation coefficient as stated in the following lemma.

LEMMA 1. *The optimal number of options to exercise in the CS, m_c^*, depends on cost conditions as follows:*

Conditions	m_c^*
$v_f^s \le v_f^b$ *and* $v_r^s + c_L(1+\gamma) > v_f^b$	$X_L^c - X_1^c + G(\delta_2^c, X_L^c) - G(\delta_2^c, X_3^c)$
$v_f^s \le v_f^b$ *and* $v_r^s + c_L(1+\gamma) \le v_f^b$	$X_3^c - X_1^c$
$v_f^s > v_f^b$ *and* $v_r^s + c_L(1+\gamma) \le v_f^s$	$G(\delta_1^c, X_1^c) - G(\delta_1^c, X_3^c)$
$v_f^s > v_f^b$ *and* $v_r^s + c_L(1+\gamma) > v_f^s$	$G(\delta_1^c, X_1^c) - G(\delta_1^c, X_L^c) + G(\delta_2^c, X_L^c) - G(\delta_2^c, X_3^c)$

where $\delta = \mu_2 - \rho(\sigma_2/\sigma_1)\mu_1$, *and* $\delta_i^c = \delta + k_{m_i}^c\sigma_2\sqrt{1-\rho^2}$ *for* $i \in \{1, 2\}$ *with*

$$\Phi(k_{m_1}^c) = \frac{p_2 + r - v_f^s}{p_2 + r - v_f^b} \quad \text{and}$$

$$\Phi(k_{m_2}^c) = \frac{p_2 + r - v_r^s - c_L(1+\gamma)}{p_2 + r - v_f^b},$$

and $G(x, y)$ is given by (5).

The first two conditions in Lemma 1 consider the case when the salvage value of finished goods is larger at the buyer's location. In such situations the salvage of a unit of leftover finished goods will fetch v_f^b. Therefore we need to focus on whether the marginal cost of expensive production is larger or smaller than v_f^b giving the first two conditions. The last two conditions in Lemma 1 consider the case when the salvage value of finished goods is larger at the supplier's location. Then the marginal cost of expensive production needs to be compared only with v_f^s, giving the last two conditions. The derivation of m_c^* for each of these conditions is similar to the derivation of the optimal number of options that a buyer has to exercise as discussed in the DS and is hence omitted. The first period expected profit of the CS is written as

$$\Pi_1^c(X_1^c, X_2^c, X_3^c) = E_{D_1}[r\min(D_1, X_1^c) - c_r X_3^c - c_L X_L^c - p_1(D_1 - X_1^c)^+ - h_1^b(X_1^c - D_1)^+ - h^s(X_L^c - X_1^c)],$$

where the first term is the revenue received, the second term is the total purchase cost of raw material, the third term is the total labor cost of production in the cheaper mode, and the fourth term is the backorder penalty cost of excess demand in the first period. The last two terms represent the holding cost of finished goods leftover at the two locations. Since $h^s < h_1^b$ (Assumption C4), not all of the first period production is transferred to the buyer's location.

The total expected system profit can then be written as:

$$\Pi_{CS}(X_1^c, X_L^c, X_3^c) = \Pi_1^c(X_1^c, X_2^c, X_3^c) + E_{I_2}[\Pi_2^c(m_c, I_2^c)] = J_1^c(X_1^c) + J_L^c(X_L^c) + J_3^c(X_3^c),$$

where $0 \le X_1^c \le X_L^c \le X_3^c$ and $J_i^c(X_i^c)$, $i = \{1, L, 3\}$ are given in Appendix A for the first condition given in Lemma 1; expressions for other conditions can be derived analogously. Finally, it is straightforward to establish joint concavity of $\Pi_{CS}(X_1^c, X_L^c, X_3^c)$ in X_i^c for $i \in \{1, L, 3\}$.

4. Channel Coordination

Consider the optimal profits, Π^*_{CS}, in the CS; this is the *first-best* solution. We say that *channel coordination* is achieved if there exists a mechanism (prices and/or quantities) such that the joint (supplier + buyer) profit in the DS is Π^*_{CS}. Channel coordination is always achieved when the buyer is able to internalize the costs of the supplier. This happens, for example, if the supplier ''sells the firm'' to the buyer, implemented by a supplier charging marginal costs. In this section, we first explore if the DS can achieve the first-best solution under linear pricing schemes. We first demonstrate that, in general, channel coordination cannot be achieved with linear prices. We develop sufficient conditions under which linear prices lead to the first-best solution. These prices, however, are not *individually rational* for the supplier. We then show that use of return policies by the supplier allow her to satisfy the individual rationality constraint and extract all of the channel profits. Return policies, however, are applicable only for a subset of the feasible region under which channel coordination is achievable with linear prices. Thus the two main problems with linear prices (with/without credit for returns)—the inability to always coordinate the channel and violation of the individual rationality constraint even when channel is coordinated—remain. It is possible to overcome these using nonlinear prices and is discussed in a longer version of this paper (Barnes-Schuster et al. 2002).

4.1. Linear Prices

In our model thus far the buyer makes the following transactions with the supplier: purchases up to X_1 units at the unit wholesale price, c_1, purchases up to $X_2 - X_1$ units at the unit wholesale price, c_2, purchases M options at c_o per unit and exercises m^* options for a unit price of c_e. While the prices in the DS are restricted to be linear, the marginal costs for various decisions in the CS may not be linear. For example, the marginal cost of providing goods against exercised options in the CS is not constant. This is because the supplier produces $X_L - X_1$ units using the cheaper production mode and the rest, if necessary, using the expensive production mode. In the DS, however, the buyer is offered a constant unit exercise price, c_e, regardless of the number of options exercised. Thus, in general, the number of options exercised in the CS and the DS will differ. This appears to pose the main difficulty in achieving channel coordination. We develop sufficient conditions that (i) effectively ensure that the number of options exercised in the two systems are identical and (ii) equate other production/order decisions. Clearly, the sufficient conditions will depend on the cost/salvage parameters of the model and the correlation coefficient. It then only remains to find the right wholesale, option, and exercise prices.

PROPOSITION 2. *Assume* $\rho < 1$. *Then*

(a) *for the conditions given in the following table channel coordination is achieved at the corresponding prices:*

	Condition	Wholesale Price (c_1)	Wholesale Price (c_2)	Option Price (c_o)	Exercise Price (c_e)
(a.1)	$v_f^s \leq v_f^b$ *and* $v_r^s + c_L + h^s \leq v_f^b$	$c_r + c_L$	$c_1 + h^s$	$c_2 - c_e$	$\leq v_f^b$
(a.2)	$v_f^s \leq v_f^b$ *and* $v_r^s + c_L + h^s > v_f^b$	$c_r + c_L$	$c_1 + h^s$	$c_r - v_r^s$	$v_r^s + c_L(1 + \gamma)$
(a.3)	$v_f^s > v_f^b$ *and* $v_r^s + c_L + h^s \leq v_f^s$	$c_r + c_L$	$c_1 + h^s$	$c_2 - c_e$	v_f^s

(b) *If* $v_f^s > v_f^b$, $v_r^s + c_L + h^s > v_f^s$, *then channel coordination may not be achieved using linear wholesale, option, and exercise prices.*

Intuitively the conditions in Proposition 2 can be interpreted using the following terms:

Access to Salvage Markets is represented by the salvage value of finished goods that a player can get. In (a.1) and (a.2) $v_f^s \leq v_f^b$, implying that the supplier at best has the *same* access to salvage markets as the buyer but no more. On the other hand, in (a.3) and (b) $v_f^s > v_f^b$, implying that the supplier has *strictly better* access to salvage markets.

Conditional Marginal Channel Risk of Advanced Production. The risk of advanced production (using the cheaper mode) is that it may not be needed after all (i.e., in the second period). If this event occurs, then the good has to be salvaged at either the buyer's or the supplier's location (v_f^b or v_f^s) while incurring the

	Condition	Wholesale Price (c_1)	Wholesale Price (c_2)	Option Price (c_o)	Exercise Price (c_e)
(a.1)	$v_f^s = v_f^b$, $h_2^b = t_{bs} = 0$, $v_r^s + c_L + h^s \le v_f^b$	$(p_2 + r - h^s)(1 - Y) + (c_r + c_L)Y$	$c_1 + h^s$	$c_2 - c_e$	$\le c_b$
(a.2)	$v_f^s = v_f^b$, $h_2^b = t_{bs} = 0$, $v_r^s + c_L + h^s > v_f^b$	$(p_2 + r - h^s)(1 - Y) + (c_r + c_L)Y$	$c_1 + h^s$	$(c_r - v_r^s)Y$	$(p_2 + r)(1 - Y) + (v_r^s + c_L(1 + \gamma))Y$
(a.3)	$v_f^s > v_f^b$, $v_r^s + c_L + h^s \le v_f^s$	$(c_b - v_f^s + t_{bs}\mathcal{Z}(X_1^*) + (c_r + c_L)Y + (v_f^s - h^s)(1 - Y)$	$(p_2 + r - h^s)(1 - Y) + (c_r + c_L)Y + h^s$	$c_2 - c_e$	$(p_2 + r)(1 - Y) + v_f^s Y$

labor cost (c_L) and holding cost (h^s). If $v_f^s \le v_f^b$, as in (a.1) and (a.2), then the maximum surplus/deficit generated in the channel is $v_f^b - c_L - h^s$. Alternately, if $v_f^s > v_f^b$, as in (a.3) and (b), then the maximum surplus/deficit generated in the channel is $v_f^s - c_L - h^s$. Conditional on the unit not being needed, the value of not producing ahead is the salvage value of the raw material (v_r^s); observe that the raw material cost is sunk at this time. Combining the maximum surplus generated due to advanced production with the value of not producing at all, we get the second part of the conditions (a.1)–(a.3) and (b). Specifically, the conditional marginal channel risk of advanced production is zero in (a.1) and (a.3) and positive in (a.2) and (b).

Recall that the primary difficulty in achieving channel coordination is the possibility that the marginal cost of goods supplied against exercised options may be nonlinear. We now illustrate that when conditions (a.1)–(a.3) hold, this is not the case, and hence we achieve channel coordination.

First consider (a.1) and (a.3). For these conditions, because the conditional marginal channel risk of advanced production is *zero,* all raw material is converted to finished goods in the CS. Furthermore in (a.1) and (a.3) it is profitable to salvage all the leftover goods at the buyer's and the supplier's location, respectively. Now consider the DS. For (a.1) the supplier could produce ahead all of the goods. To ensure that the buyer will purchase all of these goods (less what was purchased in the first period), she merely needs to set an exercise price at or below the buyer's salvage value. In (a.3), because the supplier has the access advantage, it is profitable for the channel to salvage any leftover goods (over and above the number of options exercised) at the supplier location as in the CS. So she sets the exercise price at $c_e = v_f^s > v_f^b$. In fact we have that for (a.1) and (a.3), $X_1^* = X_2^* = X_1^{c*}$ and $X_L^* = X_L^{c*} = X_3^* = X_3^{c*}$.

Next consider (a.2) and (b). Because the conditional marginal channel risk of advanced production is *positive,* the supplier will not convert all of the raw material into finished goods using the cheaper production mode. However, she will produce more than what is required in the first period. Under (a.2), however, the buyer has the salvage advantage. In the CS, any leftover goods produced using the cheaper production mode will always be salvaged at the buyer's location. To ensure that this happens in the DS, the supplier produces exactly the total committed quantity using the cheaper production mode. In fact $X_1^* = X_1^{c*}$, $X_2^* = X_L^* = X_L^{c*}$, and $X_3^* = X_3^{c*}$. This ensures that the marginal cost of production of goods against exercised options in the DS will be linear. Under (b), however, the supplier has the access advantage. So in the CS any leftover goods that were produced using the cheaper production mode will be salvaged at the supplier's location. To ensure this in the DS, the supplier's production quantity in the cheaper production mode will be different from the total committed quantity. This difference, however, implies that the marginal cost of goods available to meet exercised options will be nonlinear. Hence, given the restriction on linear prices, channel coordination cannot be achieved.

Thus we conclude that channel coordination is not achieved using linear prices whenever the supplier has better access to salvage markets and the conditional marginal risk of advanced production is positive.

Individual Rationality. While channel coordination implies that the first-best solution is achieved in the DS, it is important to know its impact on the expected profits of the buyer and the supplier, individually.

PROPOSITION 3. *When channel coordination is achieved under the conditions mentioned in Proposition 2, the supplier makes zero profits.*

The prices derived in cases (a.1)–(a.3) of Proposition 2 effectively imply that the supplier ''sells the firm'' to the buyer because she supplies all goods at marginal cost, leaving zero profits for herself. This means that the set of linear (marginal cost) prices that achieve channel coordination may not be *individually rational* for the supplier; that is, she may be unwilling to participate to achieve coordination.

Comparison with Other Contracts. It is useful to compare the pricing scheme derived in Proposition 2 to some of the existing contracts discussed in §2. In particular, we consider the backup agreement. First observe (see Table 1) that the structure of the backup agreement is such that $Q_1 > 0$, $Q_2 = 0$, and $M > 0$ and $c_1 = c_o + c_e$. Recall the optimal decisions for conditions (a.1) and (a.3). Notice that they are structurally similar to the quantity decisions in a backup agreement. Furthermore, the pricing structure for cases (a.1) and (a.3) is such that $c_o + c_e = c_2 = c_1 + h^s > c_1$. Thus while the decisions are similar to that in a backup agreement, the pricing is not unless $h^s = 0$. Recall that the conditional marginal channel risk of advanced production is zero for both (a.1) and (a.3) and, hence, the supplier converts all material into finished goods. While Eppen and Iyer (1997) did not have an explicit model of the supplier, they assumed that the supplier will always produce ahead. *Thus Proposition 2 suggests that complete, advanced production, and a backup agreement structure, is an equilibrium.* The pricing, however, is not individually rational for the supplier. In the next subsections we show how return policies can be used to mitigate this problem.

4.2. Return Policies

Return policies are used in many industries (Padmanabhan and Png 1995). Let c_b refer to the return price the supplier pays the buyer for one unit of finished good at the end of the season, and let t_{bs} refer to the cost the supplier incurs (e.g., transportation costs) to get that good shipped back from the buyer. Clearly, returns are profitable for the system only when the salvage value for the supplier (minus any costs for return) is no less than that of the buyer.[12] Recall that the salvage value for the supplier is v_f^s, and the salvage value for the buyer is $\tilde{v}_f^b = v_f^b + h_2^b$. Thus we require that $v_f^s - t_{bs} \geq v_f^b + h_2^b$. Furthermore, the supplier will choose c_b, such that it is attractive for the buyer to actually return the leftover goods; that is $c_b > v_f^b + h_2^b$. The following proposition gives conditions and corresponding prices under which channel coordination is achieved with return policies.

PROPOSITION 4. *Assume* $\rho < 1$.

(a) *Let*

$$Y = \frac{p_2 + r - (c_b - h_2^b)}{p_2 + r - (v_f^s - t_{bs} - h_2^b)}, \quad \text{and}$$

$$Z(X) = \Phi(k_{m_1}^c)\left[1 - F_{D_1}\left(\frac{X - \delta_1^c}{\eta}\right)\right] + \int_0^{(x-\delta_1^c)/\eta} F_{D_2|d_1}(X - d_1)\, dF_{D_1}(d_1).$$

If $v_f^s - t_{bs} \geq v_f^b + h_2^b$ and the conditions in the table at top of page hold, then for any $c_b > v_b^f + h_2^b$ channel coordination is achieved at the corresponding prices.

(b) *If $v_f^s > v_f^b$, $v_r^s + c_L + h^s > v_f^s$, then channel coordination may not be achieved using linear wholesale, option, and exercise prices, and credit for returns.*

Observe that there is a close parallel between conditions in Propositions 2 and 4. Specifically, conditions (a.1) and (a.2) in Proposition 4 are subsets of conditions (a.1) and (a.2) in Proposition 2. Recall that in the CS, it is profitable to use returns only when $v_f^s - t_{bs} \geq v_f^b + h_2^b$. Combining this with $v_f^s \leq v_f^b$, as in (a.1) and (a.2) in Proposition 2, we see that returns will be used by the CS only when $v_f^s = v_f^b$ and $h_2^b = t_{bs} = 0$, which gives conditions (a.1) and (a.2) of Prop-

[12]Any significant costs of return incurred by the buyer can always be folded into h_2^b.

osition 4. Furthermore, conditions (a.3) and (b) in both the propositions are identical.

The term Y in Proposition 4 *represents the ratio of incremental profits of the buyer of selling in the retail market over returning the goods to incremental profits of the channel of selling in the retail market over returning the goods to salvage at the supplier location.* For conditions (a.1) and (a.2) in Proposition 4, $Y \leq 1$. In (a.1) and (a.2) of Proposition 4, $Y = 1$ whenever $c_b = v_f^s$. The buyer is then indifferent between returning goods and salvaging it himself. Furthermore, since $v_f^s = v_f^b$, there is no disadvantage due to the location of salvage. With $Y = 1$, the supplier resorts to marginal cost pricing (as in (a.1) and (a.2) of Proposition 2) and makes zero profits. At the other extreme when $Y = 0$, the supplier extracts all the profits in the channel by charging a wholesale price $c_1 = p_2 + r - h^s$, which is the "effective" margin a supplier will earn if she sells in the retail market directly. For intermediate situations ($Y < 1$), we can consider the wholesale price c_1 to be a weighted average of "effective retail margin" a supplier earns ($p_2 + r - h^s$) and the marginal cost of advanced production ($c_r + c_L$). The supplier raises the wholesale price above the marginal cost to compensate for the loss she incurs in accepting returned goods (as $Y < 1$ implies $c_b > v_f^s - t_{bs}$).

The term $Z(X_1^{c*})$ *represents the probability of not running out of goods in the second period or, alternatively, the probability of having to salvage goods at the end of the horizon.* In Proposition 5 we will show that $Y < 1$ is required for the supplier to make positive profits. Now for condition (a.3), $Y < 1$ only when $c_b > v_f^s - t_{bs}$. This implies that the supplier incurs a loss for every unit that is returned by the buyer. Consequently, she increases the first-period wholesale price (c_1) by the expected marginal loss due to returned goods, denoted by $(c_b - v_f^s + t_{bs}) Z(X_1^{c*})$ and the second-period wholesale price by a fraction of the effective retail margin ($p_2 + r - h^s$). When returns are not allowed, we set $Y = 1$ and $Z(X_1^{c*}) = 0$ to get the prices for (a.3) in Proposition 2. Condition (a.3) allows for $Y > 1$. In this case, however, while the channel is coordinated, the supplier's profit is negative.

Naturally, return policies give some flexibility in setting prices when credit for returns is chosen such that $Y < 1$. In fact, as stated in Proposition 5, this flexibility enables the supplier to extract all of the channel profits.

Proposition 5. *When channel coordination is achieved under the conditions mentioned in Proposition 4:*

(1) The supplier's profits increase monotonically with c_b;

(2) the supplier's profits are positive whenever $c_b > v_f^s - t_{bs}$ (i.e., $Y < 1$); and

(3) she extracts all of the channel profits.

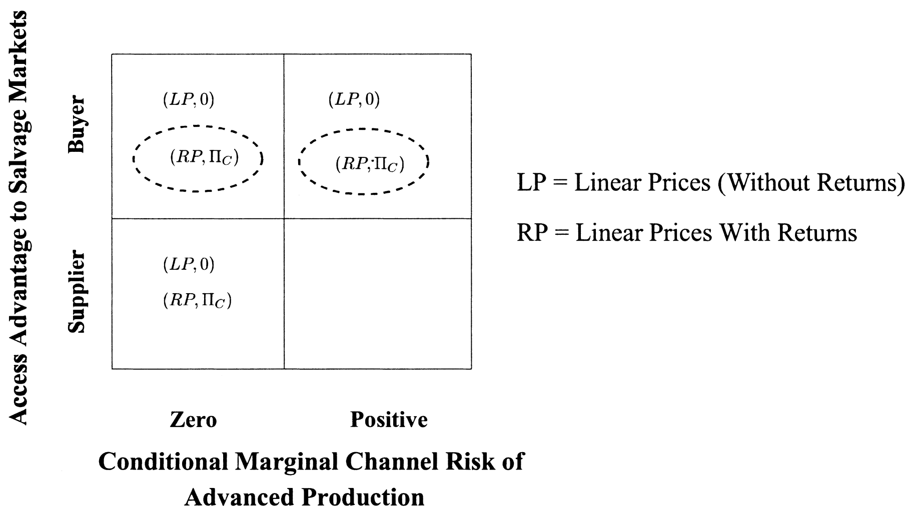
Thus return policies allow a supplier to make positive profits whenever channel coordination is achieved as outlined in Proposition 4. In fact, for every c_b there is a corresponding c_1 (see Proposition 4) and c_1 increases with c_b. This allows the supplier to extract any portion of the total channel profits.

Pasternack (1985) and Donohue (1996) have considered return policies to simultaneously achieve channel coordination and allocate the channel profits between the buyer and the supplier. There are three ways we enhance our understanding of the role of return policies:

- Pasternack and Donohue both assume *equal access to salvage* ($v_f^b = v_f^s$) and *frictionless return* ($h_2^b = t_{bs} = 0$). Conditions (a.1) and (a.2) in Proposition 4 correspond to these assumptions. Thus Proposition 4 suggests that any friction (h_2^b, $t_{bs} > 0$) in the models proposed by Pasternack (1985) or Donohue (1996) may prevent arbitrary allocation of profits between channel members using credit for returns.[13] Thus returns policies are not always useful in alleviating the individual rationality problem arising from marginal cost pricing.
- We develop conditions (e.g., (a.3)) under which, even in the presence of friction, coordination and arbitrary allocation of profits in the channel can be achieved.
- Furthermore, Pasternack (1985) illustrates that credit for returns (c_b) can be used to allocate the chan-

[13]Based on Proposition 2, we can claim that channel coordination in the presence of friction can still be achieved by marginal cost pricing. Under such pricing, returns are never used and the supplier makes zero profits.

Figure 2 Summary of Results on Channel Coordination



nel profits between the two parties in a single-period problem; Donohue (1996) illustrates the same under perfect demand correlation across periods. Proposition 5 generalizes these results for a two-period problem with arbitrary correlation.

The key results are summarized in Figure 2. On the vertical axis we consider the ''access advantage to salvage markets,'' and on the horizontal axis we consider the ''conditional marginal channel risk of advanced production.'' This results in four quadrants. In each quadrant, we specify a tuple. The first element of this tuple denotes the pricing scheme that achieves channel coordination and the second denotes the profits of the supplier under the pricing scheme. The dotted ovals suggest that the pricing scheme is only valid for a subset of the cost region—corresponding to frictionless returns and equal salvage access—defining the quadrant. As we can see, linear prices do not always coordinate the channel. In Barnes-Schuster et al. (2002), we illustrate how nonlinear prices (e.g., piecewise linear exercise prices/quantity discount schemes) can be used to coordinate the channel regardless of the cost parameters.

5. Computational Study

In this section, we explore the performance of the DSNO and the DS under linear prices and compare it to the performance of the CS. Due to the complexity of the domain being analyzed, we are unable to completely characterize the structural properties of the solutions and hence cannot compare the DSNO and the DS with linear prices and the CS analytically. Therefore we resort to a numerical study. This paper has proposed two sets of actions: (i) *options* as an instrument for increasing flexibility and (ii) *incentives* for channel coordination. Therefore, in the numerical study, we attempt to characterize environments, providing managerial insights, under which: (i) neither action is warranted (i.e., benefits of options and incentives for coordination are minimal), (ii) options alone are sufficient (benefits of options and coordination are large but options capture most of the benefits), and (iii) both options and incentives for coordination are useful.

We first divide the market into two broad categories: *fashion goods* and *basic goods.*[14] Fashion goods are characterized (the numbers in brackets indicate the values chosen by our numerical study) by high levels of coefficient of variation (CV) of demand ($\sigma/\mu \in \{0.33, 0.5\}$), medium to high correlation ($\rho \in \{0.5, 0.75, 0.9\}$), high margin ($r = 8.0$), and low salvage values for finished goods ($v_f^s \in \{2, 0.8\}$, $v_f^b \in \{1, -0.2\}$) and

[14] Fisher (1997) has used the classification of *innovative* and *functional* goods, respectively.

raw material ($v_r^s = 1.5$). Basic goods, on the other hand, are characterized by low levels of demand CV ($\sigma/\mu \in \{0.125, 0.25\}$), low correlation ($\rho \in \{0.0, 0.05, 0.1\}$), low margin ($r = 5.0$), and high salvage values for finished goods ($v_f^s = 4.0$, $v_f^b = 3.0$) and raw material ($v_r^s = 3.0$). The margins are computed based on the base marginal production cost consisting of the cost of raw material ($c_r = 3$) and the cost of labor ($c_L = 1$). Thus $r = 8$ corresponds to a maximum margin of 100% whereas $r = 5.0$ corresponds to a maximum margin of 25%.[15] Similarly, salvage values are computed as a fraction of the base marginal production cost of $c_r + c_L = 4$. Thus, fashion goods are characterized by low salvage values assumed to be 20 and 50% of production cost,[16] and basic goods are characterized by high salvage values assumed to be 100% of production cost. Finally, we selected penalty costs p_1 and p_2 from {0.25, 2.0, 19.5} to achieve low (65–68%), medium (82–87%), and high (97–98%) levels of fill-rates for the DS.

For the purpose of our numerical study, we will assume that the supplier uses a single wholesale price for both periods, i.e., $c_1 = c_2$. This simplifies the computations for the supplier greatly. We believe that the nature of the results obtained will remain unaffected if we allow for two wholesale prices. Thus the decision variables for the supplier in the DS include the prices—(single) wholesale, option and exercise—and the production quantities. The buyer's decision variables are the order quantities for both periods and the number of options to exercise in the second period. Let Π^*_{DS} and Π^*_{DSNO} be the sum of the optimal profits of the buyer and the supplier in the DS and the DSNO, respectively. In both the DS and DSNO, the supplier optimizes her expected profit subject to a constraint that the buyer's expected profit be no less than his reservation profit which, without loss of generality, is assumed to be zero. In our computational studies for both the DSNO and the DS, we observe that the prices charged by the supplier are such that the individual rationality constraint on the buyer is often binding. The first-best solution, however, is not achieved; that is, $\Pi^*_{\mathrm{DS}} < \Pi^*_{\mathrm{CS}}$.

The value of options for the channel in the DS, denoted by VO, is given by the percentage difference between Π^*_{DS} and Π^*_{DSNO}. The potential gain that could accrue from channel coordination in a system that already uses options, denoted by VC, is captured by the percentage increase from Π^*_{DS} to Π^*_{CS}. Finally, the joint effect of options and coordination, denoted by VOC, is captured by the difference between the expected profits of the CS and the DSNO. That is,

$$\mathrm{VO} = \frac{\Pi^*_{\mathrm{DS}} - \Pi^*_{\mathrm{DSNO}}}{\Pi^*_{\mathrm{DSNO}}} \times 100 \quad \text{and}$$

$$\mathrm{VC} = \frac{\Pi^*_{\mathrm{CS}} - \Pi^*_{\mathrm{DS}}}{\Pi^*_{\mathrm{DS}}} \times 100;$$

$$\mathrm{VOC} = \frac{\Pi^*_{\mathrm{CS}} - \Pi^*_{\mathrm{DSNO}}}{\Pi^*_{\mathrm{DSNO}}} \times 100.$$

In addition, we compute the ratio VO/VOC (reported as a percentage) as a fraction of the total possible improvement in the system (VOC) captured solely by options (VO). The three measures along with the ratio allow us to quantify the objectives of this numerical study mentioned earlier. High values of VO suggest that options are valuable. In addition, high values of VC suggest that achieving coordination is important. A high VO/VOC ratio will indicate that options capture most of the benefits, whereas a low ratio will imply that we may need to aim for coordination as well.

We perform four studies as described below. We ran a total of more than 200 scenarios across these four studies. In the interest of space, we report a representative subset of the results. The discussion, however, relies on the larger experimental setup. For each of these studies, we attempt to give an explanation for the behavior of VO, VC, and VOC based on a set of operational actions. It is hard, however, to similarly explain the change in the ratio VO/VOC (why it increases or decreases) because monotonicity of VO and VOC do not guarantee a certain directional change in their ratio.

Before we discuss the various studies in detail, we discuss a hypothesis that correlates the metric VC

[15]The actual retail margins are lower due to double-marginalization effect.

[16]For the buyer it is effective salvage that is salvage less holding cost.

Table 2(a) Demand and Salvage Risk for Fashion Goods

Demand Correlation	Demand CV = 0.33						Demand CV = 0.5					
	Π_{DSNO}	Q_{DS}/Q_{CS}	VO	VOC	VO/VOC	VC	Π_{DSNO}	Q_{DS}/Q_{CS}	VO	VOC	VO/VOC	VC
Medium Salvage ($v_f^s = 2.0$, $v_f^b = 1.0$)												
0.5	1150.40	82.61%	1.81	13.19	13.71	11.18	895.03	80.95%	2.43	15.09	16.10	12.36
0.75	1126.29	85.60%	6.57	16.51	39.81	9.32	864.59	83.91%	9.24	20.89	44.24	10.66
0.9	1112.50	89.30%	13.15	20.95	62.79	6.89	846.60	90.08%	19.32	29.24	66.07	8.31
Low Salvage ($v_f^s = 0.8$, $v_f^b = -0.2$)												
0.5	1078.64	84.39%	3.00	13.25	22.61	9.96	785.05	83.61%	4.55	15.79	28.79	10.76
0.75	1051.15	88.32%	9.61	18.78	51.14	8.37	747.90	86.89%	15.89	26.88	59.13	9.48
0.9	1035.40	89.89%	18.46	25.79	71.59	6.18	726.52	91.70%	31.57	41.35	76.36	7.43

with the quantities purchased in the DS and the CS. This, coupled with the pricing power of the supplier as a Stackelberg leader, will be useful in explaining the behavior of VC in several of the scenarios later. Let Q_{DS} and Q_{CS} be the total quantity of raw material purchased in DS and CS, respectively. Our hypothesis is that the ratio Q_{DS}/Q_{CS} is a good proxy for the metric VC. That is, as Q_{DS}/Q_{CS} increases, VC decreases. The basic intuition is that double-marginalization reduces the quantity purchased in the DS leading to lower profits. If Q_{DS} is closer to Q_{CS}, then the double-marginalization effect is lower and the need for coordination will also be lower leading to a lower VC. This is certainly true for a single-period newsvendor problem. In applying this argument to our model, we are ignoring the effect of allocation of quantities across the two periods. We observe that the effect of total quantities usually dominates the allocation effect. Furthermore, based on classical single-period newsvendor analysis, we will use a critical fractile as a measure of the quantities. Specifically, for the DS it is $\beta_{DS} = (r + p - c)/(r + p + h - v)$, and for the CS it is $\beta_{CS} = [r + p - (c_r + c_L)]/(r + p + h - v)$, where r represents the unit revenue, p is the unit penalty cost, c is the unit wholesale price in DS, h represents the holding cost, v represents the salvage value, and $(c_r + c_L)$ represents the production cost. While the quantities in our model are not so neatly captured by a critical fractile, they nevertheless serve as useful approximations. These critical fractiles will be useful in comparative statics to explain the behaviors of Q_{DS} and Q_{CS}, and hence that of VC. Our explanations are based on conjectures formed from the extensive numerical studies. Formal verification of these conjectures are potential areas for future research.

5.1. Effect of Demand Risk

Here we evaluate the value of options and coordination for both types of goods with changes in demand CV and correlation under equal access to salvage markets. The results are summarized in Table 2. We assume the following values for other parameters: $\gamma = 0.5$, $h^s = 0.4$, and $p_1 = p_2 = 2.0$.

Effect of Demand Variability. In Table 2, the results for demand CV appear in adjacent sets of columns. Comparing the metrics across these columns, we observe that *VO and VOC increase with demand CV.* In general increasing CV of demand increases the probability of mismatch between supply and demand in both periods. The use of options, however, allows the buyer to compensate for this mismatch using exercised options in two ways: (a) excess inventory/demand at the end of the first period is corrected and (b) the potential second-period mismatch is minimized by demand updating. We therefore observe that VO increases with demand CV. A similar phenomenon in the CS explains the behavior of VOC.

Next, we observe from Table 2 that VC increases with CV of demand. That is, coordination is more valuable when demand variability is higher. Recall our discussion on Q_{DS}, Q_{CS}, and VC. For a given wholesale price in the DS, $\beta_{DS} < \beta_{CS}$. This implies that Q_{DS} increases at a rate slower than Q_{CS} and hence the ratio Q_{DS}/Q_{CS} is decreasing with CV of demand for

Table 2(b) Demand Risk for Basic Goods

Demand Correlation	Demand CV = 0.125						Demand CV = 0.25					
	Π_{DSNO}	Q_{DS}/Q_{CS}	VO	VOC	VO/VOC	VC	Π_{DSNO}	Q_{DS}/Q_{CS}	VO	VOC	VO/VOC	VC
0	249.39	89.51%	0.06	1.94	3.32	1.88	162.61	83.12%	0.16	3.59	4.55	3.42
0.05	248.49	88.13%	0.21	2.21	9.71	2.00	160.85	80.86%	0.61	4.43	13.80	3.80
0.1	247.60	86.80%	0.44	2.56	17.08	2.11	159.12	78.66%	1.29	5.50	23.46	4.16

fixed prices. Consequently coordination will be more valuable with increasing demand CV for fixed prices. The buyer's profit, however, reduces with CV of demand. To get the buyer to buy more products, the supplier decreases the wholesale and option prices. The decreasing prices reduce double marginalization increasing the ratio Q_{DS}/Q_{CS}. Thus, CV of demand decreases the ratio whereas the supplier's pricing power increases it. How do these opposing forces play out? Whenever the critical fractiles in DS and CS are such that $\beta_{DS} \ll \beta_{CS}$ (e.g., low-medium penalty costs or low salvage values), changes in CV of demand dominate the supplier's pricing power effect. Consequently, we observe that the ratio Q_{DS}/Q_{CS} is decreasing with CV leading to an increasing VC as in Table 2(a). When both the critical fractiles are high and close to each other (e.g., due to high penalty costs), we observe (results not reported here) that often the supplier's wholesale pricing power reverses the direction of change in the ratio Q_{DS}/Q_{CS} and VC decreases with CV. To summarize, *for fashion goods VC increases with CV of demand for low to moderate service levels, whereas it decreases with CV of demand for high service levels. For basic goods,* since the value of options are low, the supplier's pricing power (which affects both the wholesale and option prices) is dominated by the effect of CV of demand at fixed prices. Consequently (see Table 2(b)), we observe that *VC increases with CV of demand.*

Effect of Demand Correlation. Higher correlation implies that the first-period demand realization is more informative about the second-period demand. Options allow the channel to exploit this information by shifting procurement from (inflexible) commitments to (flexible) options. Thus *VO and VOC increase with demand correlation.*

Demand correlation impacts operational decisions in two ways. In the first period before any decisions are made, total demand variability across the two periods increases with correlation. This affects the committed quantities. After the first period, the second-period demand variability decreases with correlation. Based on the earlier discussion on the effect of demand variability on VC, we then conclude that there are two opposing forces acting on VC due to correlation. In the beginning, VC increases as total demand variability across the two periods increases with correlation, whereas for the second period VC decreases as the remaining demand variability decreases. The overall effect on VC with respect to correlation will depend on which of these effects dominate. *For fashion goods,* options partially mitigate the first effect such that *VC is decreasing with correlation. For basic goods,* however, there is little value of options and *VC increases with correlation.*

Finally, we also observe (Table 2(a)) that the increase in both VO and VOC with correlation, increases with the CV of demand. This is a *complementarity* result implying that *more information is more valuable when demand variability is higher; provided we have a mechanism to respond to it.*

5.2. Effect of Salvage Risk

We say that salvage risk is high when salvage value is low and vice versa. Comparing the sets of results for the two levels of salvage values in Table 2(a), we observe that *both VO and VOC increase as salvage value decreases.* Intuitively, when salvage value decreases, both the supplier and the buyer, respectively, under-produce or under-order to avoid the risk of salvage. Flexibility provided through options allows the players to mitigate this risk and increase the channel profits.

There are two forces that impact VC. First, in the absence of any pricing effects and using the analogy

Table 3 Serviceability for Fashion and Basic Goods

Penalty Cost	Fashion Goods						Basic Goods					
	Π_{DSNO}	Π_{DS}	VO	VOC	VO/VOC	VC	Π_{DSNO}	Π_{DS}	VO	VOC	VO/VOC	VC
Low	822.93	870.89	5.83	42.08	13.85	34.26	218.42	218.45	0.02	9.94	0.16	9.92
Medium	864.59	944.49	9.24	20.89	44.24	10.66	162.61	162.87	0.16	3.59	4.55	3.42
High	432.15	602.11	39.33	42.31	92.96	2.14	23.17	26.48	14.26	28.81	49.51	12.73

from a single-period model, we expect that VC will increase with salvage value. This is due to the fact that an increase in salvage value impacts β_{CS} more than β_{DS}. This implies that Q_{CS} will increase at a rate faster that Q_{DS}, decreasing the ratio Q_{DS}/Q_{CS} and increasing VC. The second effect, viz., the pricing power that the supplier exercises in the channel, works as follows. As profits of the buyer increase with salvage value, the supplier attempts to extract away some of these higher profits by increasing the prices. This, however, increases the double-marginalization effect, increasing VC further. Thus the two effects reinforce each other and we observe that *VC increases in salvage value.*

5.3. Effect of Serviceability

We test the effect of serviceability (measured as fill-rate) for both types of goods by varying the penalty costs. Observe that service levels are proportional to penalty costs. The results are shown in Table 3. The values of other parameters for fashion goods are $h^s = 0.4$, CV $= 0.5$, $\gamma = 0.5$, $v_f^s = 2$, $v_f^b = 1$, and $\rho = 0.75$; similarly the values of other parameters for basic goods are $h^s = 0.4$, CV $= 0.25$, $\gamma = 0.5$, $v_f^s = 4$, $v_f^b = 3$, and $\rho = 0$. We make the following observations.

We observe that *VO increases with serviceability.* As penalty costs increase, the need to more accurately match supply and demand increases. The buyer, with proper incentives from the supplier, achieves this by buying more options and postponing more of his orders from commitments to exercised options, which increases VO.

Next we observe that both VOC and VC are convex—decreasing and eventually increasing—in penalty costs. Our hypothesis is based on the following three effects that impact profits in both decentralized systems[17]—DSNO and DS:

- Under the first effect, when (wholesale/option) prices are unchanged, the buyer's profit decreases and the supplier's profit increases with the penalty cost. The total system profits in DSNO/DS may then decrease or increase with the penalty costs.
- The second effect, viz., the pricing power that the supplier exercises in the channel, works as follows. In general, as the penalty cost increases, the buyer tends to buy more goods to satisfy a higher service level. A higher penalty cost, on the other hand, allows the supplier to increase prices without significantly affecting the service level. The increasing prices lead to a larger double-marginalization effect, decreasing profits in DSNO and DS. This happens until the buyer's individual rationality constraint[18] (IRC) is binding.
- The last effect is also related to the pricing power of the supplier but when the IRC is binding. As the penalty cost increases and the supplier increases the wholesale price (second effect), the buyer's profit will ultimately turn negative. To satisfy the IRC, however, the supplier then has to decrease the wholesale price. This decreases the double marginalization effect putting upward pressure on profits in DSNO and DS.

Regardless of whether the first effect leads to an increase or decrease in system profits, in combination with the second effect, we observe that the total system profits always decrease with penalty costs. When the IRC kicks in, the behavior of profits depends on a combination of the first and third effect. In this region we observe that initially the third effect dominates, increasing profits. Ultimately, the first effect dominates and profits start decreasing again. Thus, we expect the profits in DSNO and DS to exhibit a convex-concave behavior. This is indeed our observation in our larger numerical study. In Table 3 for

[17]We find it easier to explain VC based on the behavior of profits rather than quantities as before.

[18]He needs to make at least nonzero profit.

Table 4 Effect of Supplier Flexibility

γ	Profits in DSNO	Total Committed Qty. (X_2)	Number of Options M	Advanced Production	Q_{DS}/Q_{CS}	VO	VOC	VO/VOC	VC
2.0	432.15	183	590	590	95.63%	18.11	21.06	85.99	2.50
1.5	432.15	183	604	540	96.40%	19.08	23.41	81.52	3.63
1.0	432.15	183	612	454	96.40%	24.37	28.57	85.30	3.38
0.5	432.15	183	622	239	96.59%	39.33	42.31	92.96	2.14
0.4	432.15	183	634	0	97.80%	46.57	49.28	94.51	1.85

Table 5 Summary—Value of Options and Coordination

	VO	VOC	VO/VOC	VC
Demand CV (↑)	↑	↑	↑	↕
Demand Correlation (↑)	↑	↑	↑	↕
Salvage Risk (↑)	↑	↑	↑	↓
Serviceability (↑)	↑	↓↑	↑	↓↑
Flexibility (↑)	↑	↑	↓↑	↑↓

A sequence of two arrows signifies that the change reverses direction; e.g., ↑ followed by a ↓ implies concavity. A bidirectional arrow (↕) signifies that the change, while monotone, could be either direction.

fashion goods, the IRC is not binding for low penalty but subsequently is giving rise to a concave behavior. In the same table for basic goods the IRC is always binding. Consequently, we see the decreasing part of the profit curve as the first effect dominates the third.

Observe that profits in CS always decrease with penalty costs. *For fashion goods* profits in the DSNO and DS are concave in the region and we see that *VOC and VC are convex.* For basic goods, profits in DSNO and DS are decreasing in the region (the IRC is always binding). Initially (when penalty cost is low), however, the rate of decrease in profits of DSNO and DS is less than that of CS due to the third effect identified above. This leads to decreasing VOC and VC. Ultimately, the first effect dominates and profits in DSNO and DS decline faster than in the CS leading to an increasing VOC and VC. The net result is that even *for basic goods, VOC and VC are convex.*

5.4. Effect of Supplier Flexibility

To study the effect of flexibility available to a supplier on the value of options and coordination for fashion goods, we varied the incremental cost of expedited production. The results are presented in Table 4. The values of other parameters are $h^s = 0.4$, CV $= 0.5$, $v_f^s = 2$, $v_f^b = 1$, $\rho = 0.75$, $p_1 = p_2 = 19.5$. We make the general observation that both *VO and VOC increase with supplier flexibility* (decreasing γ). In the DS, as the supplier's cost of expedited production decreases, she reduces the option price (values not shown here), thus incentivizing the buyer to buy more options. The buyer buys more options (as seen from the column labeled M of the table) and the supplier produces less in advance (see the column labeled advanced production) improving the match between supply and demand. This increases VO. The same effects are present in the CS and hence VOC too increases.

We observe that *VC is concave—increases and then decreases—with flexibility.* Once again there are two effects to consider. First, when prices are kept unchanged, we expect the total quantity in DS (Q_{DS}) to be unaffected by flexibility. The total quantity in the CS (Q_{CS}), however, is increasing. This implies that their ratio is decreasing leading to an increasing VC. The second effect viz., the pricing power of the supplier works as follows. As flexibility increases, her cost of providing goods in expedited mode decreases, which allows her to incent the buyer to buy more options. This increases Q_{DS}, putting upward pressure on the ratio Q_{DS}/Q_{CS} and, consequently, downward pressure on VC. Furthermore, because the buyer has potential to make more profits, the supplier tries to extract some of this additional profit via increased wholesale prices. This increases double marginalization, putting upward pressure on VC. The ultimate behavior of VC is a combination of these effects. Initially, we observe that the supplier does not decrease option prices enough to counteract the other effects and hence VC increases. As flexibility increases, the incentives to buy more options and shift production increases reversing the negative effect of higher wholesale prices leading to a decreasing VC.

General Discussion. Table 5 summarizes the directional changes of various parameters on the different metrics identified. While the patterns of VO, VC, and VOC are important (as we have discussed elaborately), their magnitudes are no less important. In particular, we are interested in knowing situations in which options alone (VO) capture a large fraction of the total value (given by VOC) and situations where coordination may also be necessary. Our analysis provides some insight into this. From the larger numerical study, we observe that for fashion goods VOC varies from 6.71 to 80.52% with a mean of approximately 30%. Similarly, for basic goods VOC varies between 0.95 and 33.52% with an average of about 7%. *Higher values of VOC occur for high values of demand CV and correlation, higher salvage risk (smaller salvage values), higher serviceability, and higher supplier flexibility.* A high VOC indicates that an action (options and/or coordination) will result in a significant increase in channel profits. For basic goods, in general VOC is low except in high serviceability environments (see Table 3); this, however, is explained more by the low value of profits in the system with no options.

To see circumstances when options alone may capture most of the value, we focus on the ratio VO/VOC. Based on our numerical studies, we find that *for fashion goods the level of service ability is the predominant factor that explains whether or not options capture at least 50% of the value. Two other factors that influence a higher value of VO/VOC include correlation and salvage risk.* All of these environments call for a shift in procurement and production strategy from (inflexible) commitments to (flexible) options. When service levels are high (above 97% fill-rate) options capture between 61–98% (with an average of 86%) of the value, regardless of the levels of other parameter values (see for example, Tables 3 and 4). At the other extreme, when service levels are low (average fill-rate of 65%), options never capture more than 41% (average of 18%) of the value. The only exception is when, in addition, both demand correlation and the risk of salvage are high leading to about 44–47% of the value capture. When service levels are moderate (average fill-rate of 85%), options capture between 14–76% of the value, with an average of 44%. For this case, in combination with either a high demand correlation or a medium demand correlation, but a high risk of salvage (see Table 2(a)), we observe that options capture above 50% of the total value.

6. Conclusion

In this paper we investigate the role of options in a buyer-supplier system. Using a two-period model with correlated demand, we demonstrate how options offered by a supplier provide flexibility to a buyer to respond to market changes, thus increasing the profits of both the buyer and the supplier. In addition, we derive sufficient conditions and the corresponding linear transfer prices under which channel coordination is achieved. Unfortunately, at these prices, the supplier makes zero profits. We then illustrate that, for a subset of these conditions, return policies could be used to coordinate the channel and give the supplier positive profits. Finally, using numerical studies, we quantify the value of options and of coordination as a function of demand and salvage risks, serviceability, and flexibility when prices are restricted to be linear. Obviously the joint profits of the (decentralized and centralized) channel always increase with options. Furthermore, this increase in profits increases with demand correlation. In the decentralized system with linear prices the supplier is able to extract all of the channel profits, even though she is unable to coordinate the channel. Thus options are useful instruments for increasing flexibility and channel profits. The risk to the supplier of offering a higher level of flexibility is partly mitigated by the use of options. Thus options can be an effective mechanism to implement in-season replenishments in the channel.

We have also illustrated how return policies, in conjunction with linear prices, can be used to coordinate the channel and allow the supplier to extract the channel profits. These results significantly expand the scope of return policies beyond those studied in the context of ''single-'' period models by Pasternack (1985) and Donohue (1996).

Our model assumes that the length of the first period is exogenous. A buyer updates demand at the end of the first period and exercises options. One could argue that the timing of the demand update should be a decision variable for the buyer. Ideally, a buyer may update the demand density in a dynamic fashion after observing a certain level of sales. An alternate approach could be that the timing of demand update is announced a priori (at the beginning of the horizon), so that the supplier can make some appropriate planning decisions. A buyer then decides on this optimal timing, in addition to the various order quantities. In a preliminary study, we use a discrete time model to study the sensitivity of the timing of updates to the level of correlation, labor costs, etc. Numerical testing shows that the optimal timing of demand updates is usually later in the DS (with linear prices) than in the CS. A complete analysis of this issue is an area of further research.

Finally, we have assumed information symmetry between the buyer and the supplier. Analysis under information asymmetry (regarding costs and/or demand parameters) are also areas of future research.

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Appendix A. Notation

c_i	unit wholesale price for firm orders to be delivered in period i.
c_e	unit exercise price.
c_o	unit option price.
c_L	unit labor cost of production.
c_r	unit cost of raw material.
γ	additional unit labor cost of production in the expedited mode.
X_L	production quantity of the supplier in the cheaper mode of production; that is, at labor cost of c_L per unit.
v_r^s	unit salvage value of raw material for the supplier.
v_f^s	unit salvage value of finished goods for the supplier.
h^s	unit Period 1 holding cost for finished goods at the supplier.
Q_i	order quantity to be delivered at the beginning of period i, $i \in \{1, 2\}$ at a wholesale price of c_i.
M	number of options purchased at the beginning of Period 1 at a unit price of c_o.
m	number of options exercised at the beginning of Period 2 at a price of c_e.
r	unit selling price of finished good to the consumer.
h_i^b	unit holding cost of the buyer in period $i \in \{1, 2\}$.
p_i	unit shortage penalty cost incurred by the buyer in period $i \in \{1, 2\}$.
$\tilde{v}_f^b$	unit salvage value of finished goods for the buyer.
v_f^b	effective salvage value of finished goods for the buyer; equal to $\tilde{v}_f^b - h_2^b$.
D_i	demand in period $i \in \{1, 2\}$ assumed to be normally distributed.
μ_i	mean of D_i.
σ_i	standard deviation of D_i.
ρ	correlation coefficient of D_1 and D_2.
$F_{D_i}(\cdot)$	conditional distribution function of D_i.
$f_{D_i}(\cdot)$	conditional density function of D_i.
$\Phi(\cdot)$	distribution function of standard normal.
c_b	price the supplier gives the buyer for goods returned at the end of the season.
t_{bs}	cost the supplier pays to ship goods back from the buyer at the end of the season.

Appendix B. Expressions for the Profit Functions

- **Profit Function of the Buyer in DS**

$$\Pi_{\mathrm{DS}}^b(X_1, X_2, X_3) = J_1(X_1) + J_2(X_2) + J_3(X_3),$$

where

$$J_1(X_1) = -p_2\mu_2 - \mu_1(p_1 + p_2) - (p_1 + h_1^b)\int_0^{X_1}(X_1 - d_1)\,dF_{D_1}(d_1) + (p_1 + c_2 - c_1)X_1,$$

$$\begin{aligned}
J_2(X_2) &= (p_2 + r - (c_2 - c_o))X_2 + (p_2 + r - c_e)\int_{(X_2-\delta^d)/\eta}^{\infty}(\delta^d + \eta d_1 - X_2)\,dF_{D_1}(d_1)\\
&\quad - (p_2 + r - v_f^b)\int_0^{(X_2-\delta^d)/\eta}\int_0^{X_2-d_1}(X_2 - d_1 - d_2)\,dF_{D_2|d_1}(d_2)\,dF_{D_1}(d_1)\\
&\quad - (p_2 + r - v_f^b)\int_{(X_2-\delta^d)/\eta}^{\infty}\int_0^{\delta^d+(\eta-1)d_1}(\delta^d + (\eta - 1)d_1 - d_2)\,dF_{D_2|d_1}(d_2)\,dF_{D_1}(d_1),
\end{aligned} \tag{9}$$

$$\begin{aligned}
J_3(X_3) &= -c_oX_3 - (p_2 + r - c_e)\int_{(X_3-\delta^d)/\eta}^{\infty}(\delta^d + \eta d_1 - X_3)\,dF_{D_1}(d_1)\\
&\quad + (p_2 + r - v_f^b)\Bigg[\int_{(X_3-\delta^d)/\eta}^{\infty}\int_0^{\delta^d+(\eta-1)d_1}(\delta^d + (\eta - 1)d_1 - d_2)\,dF_{D_2|d_1}(d_2)\,dF_{D_1}(d_1)
\end{aligned}$$

$$- \int_{(X_3-\delta^d)/\eta}^{\infty} \int_0^{X_3-d_1} (X_3 - d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1)\Bigg]. \tag{10}$$

- **Profit Function of the System in CS, $v_f^s \le v_f^b$ and $v_r^s + c_L(1+\gamma) > v_f^b$**

$$\Pi_{CS}(X_1^c, X_L^c, X_3^c) = J_1^c(X_1^c) + J_L^c(X_L^c) + J_3^c(X_3^c),$$

where

$$J_1^c(X_1^c) = -p_2\mu_2 - \mu_1(p_1+p_2) - (p_1+h_1^b)\int_0^{X_1^c} (X_1^c - d_1)\, dF_{D_1}(d_1) + (p_1+h^s)X_1^c, \tag{11}$$

$$\begin{aligned} J_L^c(X_L^c) &= (p_2 + r - v_r^s - c_L - h^s)X_L^c \\ &\quad + (p_2 + r - v_r^s - c_L(1+\gamma)) \int_{(X_L^c-\delta_2^c)/\eta}^{\infty} (\delta_2^c + \eta d_1 - X_L^c)\, dF_{D_1}(d_1) \\ &\quad - (p_2 + r - v_f^b)\Bigg[\int_0^{(X_L^c-\delta_2^c)/\eta} \int_0^{X_L^c-d_1} (X_L^c - d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1) \\ &\quad + \int_{(X_L^c-\delta_2^c)/\eta}^{\infty} \int_0^{\delta_2^c+(\eta-1)d_1} (\delta_2^c + (\eta-1)d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1)\Bigg], \end{aligned} \tag{12}$$

$$\begin{aligned} J_3^c(X_3^c) &= -(c_r - v_r^s)X_3^c - (p_2 + r - v_r^s - c_L(1+\gamma)) \int_{(X_3^c-\delta_2^c)/\eta}^{\infty} (\delta_2^c + \eta d_1 - X_3^c)\, dF_{D_1}(d_1) \\ &\quad + (p_2 + r - v_f^b)\Bigg[\int_{(X_3^c-\delta_2^c)/\eta}^{\infty} \int_0^{\delta_2^c+(\eta-1)d_1} (\delta_2^c + (\eta-1)d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1) \\ &\quad - \int_{(X_3^c-\delta_2^c)/\eta}^{\infty} \int_0^{X_3^c-d_1} (X_3^c - d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1)\Bigg]. \end{aligned} \tag{13}$$

Appendix C. Expressions for the Remaining Profit Functions

- **Profit Function of the System in CS, $v_f^s \le v_f^b$ and $v_r^s + c_L(1+\gamma) \le v_f^b$**

$$\Pi_{CS}(X_1^c, X_L^c, X_3^c) = J_1^c(X_1^c) + J_L^c(X_L^c) + J_3^c(X_3^c),$$

where $X_L^c = X_3^c$, $J_1^c(X_1^c)$ is as in (11) and

$$J_3^c(X_3^c) = (p_2 + r - c_r - c_L - h^s)X_3^c - (p_2 + r - v_f^b)\int_0^{X_3^c} (X_3^c - d_{12})\, dF_{D_1+D_2}(d_{12}). \tag{14}$$

- **Profit Function of the System in CS, $v_f^s > v_f^b$ and $v_r^s + c_L(1+\gamma) \le v_f^s$**

$$\Pi_{CS}(X_1^c, X_L^c, X_3^c) = J_1^c(X_1^c) + J_3^c(X_3^c),$$

where $X_L^c = X_3^c$ and

$$\begin{aligned} J_1^c(X_1^c) &= -p_2\mu_2 - \mu_1(p_1+p_2) - (p_1+h_1^b)\int_0^{X_1^c} (X_1^c - d_1)\, dF_{D_1}(d_1) \\ &\quad + (p_1 + p_2 + r - v_f^s + h^s)X_1^c + (p_2 + r - v_f^s)\int_{(X_1^c-\delta_1^c)/\eta}^{\infty} (\delta_1^c + \eta d_1 - X_1^c)\, dF_{D_1}(d_1) \end{aligned}$$

$$- (p_2 + r - v_f^b)\left[\int_0^{(X_1^c-\delta_1^c)/\eta} \int_0^{X_1^c-d_1} (X_1^c - d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1) + \int_{(X_1^c-\delta_1^c)/\eta}^{\infty} \int_0^{\delta_1^c+(\eta-1)d_1} (\delta_1^c + (\eta - 1)d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1)\right], \quad (15)$$

$$J_3^c(X_3^c) = (v_f^s - c_r - c_L - h^c)X_3^c - (p_2 + r - v_f^s) \int_{(X_3^c-\delta_1^c)/\eta}^{\infty} (\delta_1^c + \eta d_1 - X_3^c)\, dF_{D_1}(d_1)$$

$$- (p_2 + r - v_f^b)\left[\int_{(X_3^c-\delta_1^c)/\eta}^{\infty} \int_0^{X_3^c-d_1} (X_3^c - d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1) - \int_{(X_3^c-\delta_1^c)/\eta}^{\infty} \int_0^{\delta_1^c+(\eta-1)d_1} (\delta_1^c + (\eta - 1)d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1)\right]. \quad (16)$$

- **Profit Function of the System in CS, $v_f^s > v_f^b$ and $v_r^s + c_L(1 + \gamma) > v_f^s$**

$$\Pi_{CS}(X_1^c, X_L^c, X_3^c) = J_1^c(X_1^c) + J_L^c(X_L^c) + J_3^c(X_3^c),$$

where $J_1^c(X_1^c)$ is as in (15), $J_3^c(X_3^c)$ is as in (13), and

$$J_L^c(X_L^c) = (v_f^s - v_r^s - c_L - h^s)X_L^c - (p_2 + r - v_f^s) \int_{(X_L^c-\delta_1^c)/\eta}^{\infty} (\delta_1^c + \eta d_1 - X_L^c)\, dF_{D_1}(d_1)$$

$$+ (p_2 + r - c_L(1 + \gamma) - v_r^s) \int_{(X_L^c-\delta_2^c)/\eta}^{\infty} (\delta_2^c + \eta d_1 - X_L^c)\, dF_{D_1}(d_1)$$

$$- (p_2 + r - v_f^b)\left[\int_{X_L^c-\delta_1^c)/\eta}^{(X_L^c-\delta_2^c)/\eta} \int_0^{X_L^c-d_1} (X_L^c - d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1) - \int_{(X_L^c-\delta_1^c)/\eta}^{\infty} \int_0^{\delta_1^c+(\eta-1)d_1} (\delta_1^c + (\eta - 1)d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1) + \int_{(X_L^c-\delta_2^c)/\eta}^{\infty} \int_0^{\delta_2^c+(\eta-1)d_1} (\delta_2^c + (\eta - 1)d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1)\right]. \quad (17)$$

Appendix D. Proofs

DERIVATION of (7). Let us begin by simplifying (6). Collecting terms and taking expectations gives

$$\Pi_1^b(Q_1, Q_2, M) = E_{D_1}[rD_1 - r(D_1 - Q_1)^+ - c_oM - c_1Q_1 - c_2Q_2 - h_1^b(Q_1 - d_1)^+ - p_1(d_1 - Q_1)^+ + E_{D_2}[\Pi_2^b(m^*|d_1)]]$$

$$= -c_oM - c_1Q_1 - c_2Q_2 - (h_1^b + p_1) \int_0^{Q_1} (Q_1 - d_1)\, dF_{D_1}(d_1) - p_1(\mu_1 - Q_1) + E_{D_1}[\Pi_2^b(m^*|d_1) + rD_1 - r(D_1 - Q_1)^+]. \quad (18)$$

Expanding out $E_{D_1}[\Pi_2^b(m^*|d_1) + rD_1 - r(D_1 - Q_1)^+]$, we get

$$E_{D_1}[\Pi_2^b(m^*|d_1) + rD_1 - r(D_1 - Q_1)^+]$$

$$= \int_0^{\infty} \left[(r - c_e)m^* - p_2(\mu_2 - (\eta - 1)\mu_1 + \eta D_1 - Q_1 - Q_2 - m^*) + r(Q_1 + Q_2) - (p_2 + r - v_f^b) \int_0^{Q_1+Q_2-d_1+m^*} (Q_1 + Q_2 + m^* - d_1 - d_2)\, dF_{D_2|d_1}(d_2)\right] dF_{D_1}(d_1)$$

$$= (p_2 + r - c_e) \int_0^{\infty} m^*\, dF_{D_1}(d_1) - p_2(\mu_1 + \mu_2 - Q_1 - Q_2) + r(Q_1 + Q_2)$$

$$- (p_2 + r - v_f^b)\Bigg(\int_0^{(Q_1+Q_2-\delta^d)/\eta} \int_0^{Q_1+Q_2-d_1} (Q_1 + Q_2 - d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1)$$

$$+ \int_{(Q_1+Q_2-\delta^d)/\eta}^{(Q_1+Q_2+M-\delta^d)/\eta} \int_0^{\delta^d+(\eta-1)d_1} (\delta^d + (\eta - 1)d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1)$$

$$+ \int_{(Q_1+Q_2+M-\delta^d)/\eta}^{\infty} \int_0^{Q_1Q_2+M-d_1} (Q_1 + Q_2 + M - d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1)\Bigg). \quad (19)$$

Substituting, $(X_1, X_2, X_3) = (Q_1, Q_1 + Q_2, Q_1 + Q_2 + M)$, we rewrite $\Pi^b(Q_1, Q_2, M)$ as

$$\Pi^b_{\mathrm{DS}}(X_1, X_2, X_3) = -c_oX_3 + (r - (c_2 - c_o))X_2 - (c_2 - c_1)X_1 - (h_1^b + p_1) \int_0^{X_1} (X_1 - d_1)\, dF_{D_1}(d_1)$$

$$- p_1(\mu_1 - X_1) + E_{D_1}[\Pi_2^b((\delta^d + \eta D_1 - X_2)^+ - (\delta^d + \eta D_1 - X_3)^+)]$$

$$+ rE_{D_1}[\delta^d + \eta D_1 - X_2)^+ - (\delta^d + \eta D_1 - X_3)^+ - (X_2 + m^* - D_1)^+]. \quad (20)$$

where

$$E_{D_1}[\Pi_2^b((\delta^d + \eta D_1 - X_2)^+ - (\delta^d + \eta D_1 - X_3)^+) + rm^* - r(X_2 + m^* - D_1)^+]$$

$$= -p_2(\mu_1 + \mu_2 - X_2) + (p_2 + r - c_e) \int_{(X_2-\delta^d)/\eta}^{\infty} (\delta^d + \eta d_1 - X_2)\, dF_{D_1}(d_1) - (p_2 + r - c_e) \int_{(X_3-\delta^d)/\eta}^{\infty} (\delta^d + \eta d_1 - X_3)\, dF_{D_1}(d_1)$$

$$- (p_2 + r - v_f^b) \int_0^{(X_2-\delta^d)/\eta} \int_0^{X_2-d_1} (X_2 - d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1)$$

$$- (p_2 + r - v_f^b) \int_{(X_2-\delta^d)/\eta}^{(X_3-\delta^d)/\eta} \int_0^{\delta^d+(\eta-1)d_1} (\delta^d + (\eta - 1)d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1)$$

$$- (p_2 + r - v_f^b) \int_{X_3-\delta^d)/\eta}^{\infty} \int_0^{X_3-d_1} (X_3 - d_1 - d_2)\, dF_{D_2|d_1}(d_2) dF_{D_1}(d_1). \quad (21)$$

Finally, collecting terms and restating, we get our result. □

PROOF OF CONCAVITY OF $\Pi^b_{\mathrm{DS}}(X_1, X_2, X_3)$. We begin by taking the first partials of $J_1(X_1)$, $J_2(X_2)$, and $J_3(X_3)$ (given in Appendix A) with respect to X_1, X_2, and X_3.

$$\frac{\partial J_1(X_1)}{\partial X_1} = -(h_1^b + p_1)F_{D_1}(X_1) + p_1 + c_2 - c_1,$$

$$\frac{\partial J_2(X_2)}{\partial X_2} = p_2 + r - (c_2 - c_o) - (p_2 + r - c_e) \int_{(X_2-\delta^d)/\eta}^{\infty} dF_{D_1}(d_1) - (p_2 + r - v_f^b) \int_0^{(X_2-\delta^d)/\eta} \int_0^{X_2-d_1} dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1)$$

$$+ \frac{p_2 + r - v_f^b}{\eta}\Bigg(\int_0^{X_2-(X_2-\delta^d)/\eta} \left(X_2 - \frac{X_2 - \delta^d}{\eta} - d_2\right) dF_{D_2|(X_2-\delta^d)/\eta}(d_2) f_{D_1}\left(\frac{X_2 - \delta^d}{\eta}\right)$$

$$- \int_0^{\delta^d+(\eta-1)[(X_2-\delta^d)/\eta]} \left(\delta^d + (\eta - 1)\frac{X_2 - \delta^d}{\eta} - d_2\right) dF_{D_2|(X_2-\delta^d)/\eta}(d_2) f_{D_1}\left(\frac{X_2 - \delta^d}{\eta}\right)\Bigg)$$

$$= c_o + c_e - c_2 + (p_2 + r - c_e)F_{D_1}\left(\frac{X_2 - \delta^d}{\eta}\right) - (p_2 + r - v_f^b) \int_0^{(X_2-\delta^d)/\eta} F_{D_2|d_1}(X_2 - d_1)\, dF_{D_1}(d_1),$$

$$
\begin{aligned}
\frac{\partial J_3(X_3)}{\partial X_3} = &- c_o + (p_2 + r - c_e) \int_{(X_3-\delta^d)/\eta}^{\infty} dF_{D_1}(d_1) \\
&- \frac{p_2 + r - v_f^b}{\eta} \int_0^{\delta^d + (\eta-1)[(X_3-\delta^d)/\eta]} \left(\delta^d + (\eta - 1)\frac{X_3 - \delta^d}{\eta} - d_2\right) dF_{D_2|(X_3-\delta^d)/\eta}(d_2) f_{D_1}\left(\frac{X_3 - \delta^d}{\eta}\right) \\
&+ \frac{p_2 + r - v_f^b}{\eta} \int_0^{X_3 - [(X_3-\delta^d)/\eta]} \left(X_3 - \frac{X_3 - \delta^d}{\eta} - d_2\right) dF_{D_2|(X_3-\delta^d)/\eta}(d_2) f_{D_1}\left(\frac{X_3 - \delta^d}{\eta}\right) \\
&- (p_2 + r - v_f^b) \int_{(X_3-\delta^d)/\eta}^{\infty} \int_0^{X_3 - d_1} dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1) \\
= &\ p_2 + r - c_o - c_e - (p_2 + r - c_e) F_{D_1}\left(\frac{X_3 - \delta^d}{\eta}\right) - (p_2 + r - v_f^b) \int_{(X_3-\delta^d)/\eta}^{\infty} F_{D_2|d_1}(X_3 - d_1)\, dF_{D_1}(d_1).
\end{aligned}
$$

The function Π_{DS}^b will be strictly concave if the Hessian matrix of $-J_1(X_1) - J_2(X_2) - J_3(X_3)$ is positive definite. Due to the earlier transformations, all cross-partials will be zero. We need only to calculate the three second partials. For the derivations listed below, the following identity is needed.

$$
F_{D_2|(X_2-\delta^d)/\eta}\left(X_2 - \frac{X_2 - \delta^d}{\eta}\right) = \Phi\left(\frac{X_2 - \dfrac{X_2 - \delta^d}{\eta} - \mu_2 - \rho\dfrac{\sigma_2}{\sigma_1}\left(\dfrac{X_2 - \delta^d}{\eta} - \mu_1\right)}{\sigma_2\sqrt{1 - \rho^2}}\right) = \Phi(k_m). \tag{22}
$$

Using this the second partials can be calculated as follows:

$$
\frac{\partial^2 J_1(X_1)}{\partial X_1^2} = -(p_1 + h_1^b) f_{D_1}(X_1), \tag{23}
$$

$$
\begin{aligned}
\frac{\partial^2 J_2(X_2)}{\partial X_2^2} = &\ \frac{1}{\eta}(p_2 + r - c_e) f_{D_1}\left(\frac{X_2 - \delta^d}{\eta}\right) - \frac{1}{\eta}(p_2 + r - v_f^b) f_{D_1}\left(\frac{X_2 - \delta^d}{\eta}\right) F_{D_2|(X_2-\delta^d)/\eta}\left(X_2 - \frac{X_2 - \delta^d}{\eta}\right) \\
&- (p_2 + r - v_f^b) \int_0^{(X_2-\delta^d)/\eta} f_{D_2|d_1}(X_2 - d_1)\, dF_{D_1}(d_1) \\
= &-(p_2 + r - v_f^b \int_0^{(X_2-\delta^d)/\eta} f_{D_2|d_1}(X_2 - d_1)\, dF_{D_1}(d_1),
\end{aligned} \tag{24}
$$

$$
\begin{aligned}
\frac{\partial^2 J_3(X_3)}{\partial X_3^2} = &-\frac{1}{\eta}(p_2 + r - c_e) f_{D_1}\left(\frac{X_3 - \delta^d}{\eta}\right) + \frac{1}{\eta}(p_2 + r - v_f^b) F_{D_2|(X_3-\delta^d)/\eta}\left(X_3 - \frac{X_3 - \delta^d}{\eta}\right) f_{D_1}\left(\frac{X_3 - \delta^d}{\eta}\right) \\
&- (p_2 + r - v_f^b) \int_{(X_3-\delta^d)/\eta}^{X_3} f_{D_2|d_1}(X_3 - d_1)\, dF_{D_1}(d_1) \\
= &-(p_2 + r - v_f^b \int_{(X_3-\delta^d)/\eta}^{X_3} f_{D_2|d_1}(X_3 - d_1)\, dF_{D_1}(d_1).
\end{aligned} \tag{25}
$$

Clearly, all of the second partials are negative, and the result follows. □

PROOF OF PROPOSITION 1. In writing the expression for $\Pi_{\mathrm{DS}}^s(\cdot)$, we have implicitly assumed that the supplier will never produce more than what the buyer asks for using the expensive mode of production. This is certainly true whenever $v_f^s \le v_r^s + c_L(1 + \gamma)$. Alternately, if $v_f^s > v_r^s + c_L(1 + \gamma)$, then the supplier is motivated to turn all raw material to finished goods. However, if this is the case, then the supplier will simply set $X_L = X_3$ and will never produce anything using the more expensive mode of production. Both of these cases are captured in the profit function. Therefore, to find the optimal production level, we need to differentiate Π_{DS}^s with respect to X_L and equate this to zero.

$$\frac{\partial \Pi_{DS}^{s}}{\partial X_L} = v_f^s - v_r^s - c_L - h^s - [v_f^s - v_r^s - c_L(1-\gamma)]\left[1 - F_{D_1}\left(\frac{X_L - \delta^d}{\eta}\right)\right]$$

$$= c_L\gamma - h^s + [v_f^s - v_r^s - c_L(1-\gamma)]F_{D_1}\left(\frac{X_L - \delta^d}{\eta}\right),$$

$$\frac{\partial^2 \Pi_{DS}^{s}}{\partial X_L^2} = \frac{1}{\eta}[v_f^s - v_r^s - c_L(1-\gamma)]f_{D_1}\left(\frac{X_L - \delta^d}{\eta}\right).$$

Notice that the second partial will be negative for $v_f^s - v_r^s - c_L(1-\gamma) < 0$ and positive otherwise. Hence,

$$F_{D_1}\left(\frac{\hat{X}_L^* - \delta^d}{\eta}\right) = \frac{c_L\gamma - h^s}{v_r^s + c_L(1-\gamma) - v_f^s} \equiv \Phi(k_o)$$

so long as $v_f^s < v_r^s + c_L(1-\gamma)$. Otherwise, $k_o = \infty$, implying $X_L^* = X_3$. Hence, $X_L^* = \max(X_2, \min(X_3, \hat{X}_L^*))$. □

Proof of Lemma 1. We will show this for the most general case, which corresponds to the fourth condition of the lemma. The proofs for other cases are quite similar and are hence omitted. Consider when $v_f^s > v_f^b$ and $v_r^s + c_L(1+\gamma) > v_f^s$. In this case, the expected profit function for the last period corresponds to (8a) and (8b), which can be simplified as

$$\Pi_2^c(m_c, I_2^c) = (p_2 - v_f^s)X_1^c + (v_f^s - v_r^s)X_L^c + v_r^s X_3^c + (p_2 + r - v_f^s)m_c + p_2(\mu_{D_2|d_1} + d_1)$$

$$+ r(X_1^c - d_1)^+ + (v_f^s - v_r^s - c_L(1+\gamma))(X_1^c + m_c - X_L^c)^+ - (p_2 + r - v_f^b)\int_0^{X_1^c + m_c - d_1}(X_1^c + m_c - d_1 - d_2)\,dF_{D_2|d_1}(d_2).$$

Let $\mathbf{1}_x = 1$ if x is true and 0 otherwise. Taking partials with respect to m_c, we get

$$\frac{\partial \Pi_2^c(m_c, I_2^c)}{\partial m_c} = p_2 + r - v_f^s + (v_f^s - v_r^s - c_L(1+\gamma))\mathbf{1}_{\mathbf{m_c > X_L^c - X_1^c}} - (p_2 + r - v_f^b)F_{D_2|d_1}(X_1^c + m_c - d_1),$$

$$\frac{\partial^2 \Pi_2^c(m_c, I_2^c)}{\partial m_c^2} = -(p_2 + r - v_f^b)f_{D_2|d_1}(X_1^c + m_c - d_1).$$

Hence, it is easy to see that $\Pi_2^c(m_c, I_2^c)$ is concave and has a maximum where the first partial is equated to zero. That is,

$$F_{D_2|d_1}(X_1^c + m_c - d_1) = \frac{p_2 + r - v_f^s + (v_f^s - v_r^s - c_L(1+\gamma))\mathbf{1}_{\mathbf{m_c > X_L^c - X_1^c}}}{p_2 + r - v_f^b}.$$

This implies that the optimal value of m_c depends on how much was produced ahead of time, as well as the initial demand. Define $k_{m_i}^c$ as follows:

$$\Phi(k_{m_1}^c) = \frac{p_2 + r - v_f^s}{p_2 + r - v_f^b} \quad \text{and} \quad \Phi(k_{m_2}^c) = \frac{p_2 + r - v_r^s - c_L(1+\gamma)}{p_2 + r - v_f^b}.$$

Hence, $k_{m_i}^c$ corresponds to the optimal m_c should that value not exceed $X_L^c - X_1^c$. That is, $m_c^* = d_1 + \mu_{D_2|d_1} - X_1^c + \sigma_{D_2|d_1}k_{m_1}^c = \eta d_1 + \delta_1^c - X_1^c$ should this not exceed $X_L - X_1$. This will happen whenever $d_1 \le (X_L^c - \delta_1^c)/\eta$. Because the buyer cannot send units back, however, should $F_{D_2|d_1}(X_1^c - d_1) < (p_2 + r - v_f^s)/(p_2 + r - v_f^b)$ (that is if $d_1 \le (X_1^c - \delta_1^c)/\eta$), then $m_c^* = 0$.

In a similar fashion, whenever $d_1 \ge (X_L^c - \delta_2^c)/\eta$, then $m_c^* = d_1 + \mu_{D_2|d_1} - X_1^c + \sigma_{D_2|d_1}k_{m_2}^c = \eta d_1 + \delta_2^c - X_1^c$. However, as the buyer cannot acquire more total units than X_3^c from the supplier, if $F_{D_2|d_1}(X_3^c - d_1) > [p_2 + r - v_r^s - c_L(1+\gamma)]/(p_2 + r - v_f^b)$ (that is if $d_1 \ge (X_3^c - \delta_2^c)/\eta$), then $m_c^* = X_3^c - X_1^c$.

Therefore, only demands such that $(X_L^c - \delta_1^c)/\eta < d_1 < (X_L^c - \delta_2^c)/\eta$ are unaccounted for. Such a demand outcome implies that $F_{D_2|d_1}(X_L^c - d_1) > (p_2 + r - v_f^s)/(p_2 + r - v_f^b)$, but $F_{D_2|d_1}(X_L^c - d_1) < [p_2 + r - v_r^s - c_L(1+\gamma)]/(p_2 + r - v_f^b)$. Hence, we set m_c^* equal to the boundary value, $X_L^c - X_1^c$.

Putting these scenarios together into one equation yields $G(\delta_1^c, X_1^c) - G(\delta_1^c, X_L^c) + G(\delta_2^c, X_L^c) - G(\delta_2^c, X_3^c)$. □

Proof of $\Pi_{CS}(X_1^c, X_L^c, X_3^c)$. The proof is similar to proof of concavity of $\Pi_{DS}^b(X_1, X_2, X_3)$ and hence omitted. □

Proof of Proposition 2.

Part a.1. When v_f^b exceeds both v_f^s and $v_r^s + c_L + h^s$, then the CS will convert all of the raw material purchased into finished goods using the cheaper production mode and then salvage all leftover finished goods at v_f^b. That is, $X_L^{c*} = X_3^{c*}$ and $m_c^* = X_3^{c*} - X_1^{c*} = X_L^{c*} - X_1^{c*}$. For $\rho < 1$, this means that the profit function in the CS will be

$$\Pi_{CS}(X_1^c, X_L^c, X_3^c) = J_1^c(X_1^c) + J_L^c(X_L^c), \tag{26}$$

where $J_1^c(X_1^c)$ is as in (11) and

$$J_L^c(X_L^c) = (p_2 + r - c_r - c_L - h^s)X_L^c - (p_2 + r - v_f^b)\int_0^{X_L^c} (X_L^c - d_{12})\, dF_{D_{12}}(d_{12}), \tag{27}$$

where $D_{12} = D_1 + D_2$ with a cdf of $F_{D_{12}}(\cdot)$.

From (9) and (11) we have that $X_1^* = X_1^{c*}$ if $c_2 - c_1 = h^s$. If $c_e \le v_f^b$, then $m^* = X_3^* - X_2^*$ in the DS. This means that

$$\Pi_{DS}^b(X_1, X_2, X_3) = J_1(X_1) + X_2(c_e + c_o - c_2) + X_3(p_2 + r - c_o - c_e) - (p_2 + r - v_f^b)\int_0^{X_3} (X_3 - d_{12})\, dF_{D_{12}}(d_{12}). \tag{28}$$

Setting $c_2 = c_o + c_e$ assures us that the buyer will buy as few firm orders as possible in the second period, so $X_2^* = X_1^*$ (an option good costs the same as a firm order good). If we set $c_o + c_e = c_r + c_L + h^s$, then $X_L^{c*} = X_3^*$, and $m_c^* = m^*$. Clearly, $X_3^* = X_3^{c*}$. Substituting these values of prices and decisions gives a supplier's profit function of

$$\Pi_{DS}^s(X_L, c_r + c_L, c_r + c_L + h^s, c_r + c_L + h^s - c_e, c_e) = X_L(c_L\gamma - h^s) - X_3(c_L\gamma - h^s). \tag{29}$$

Because $h^s \le c_L\gamma$ (Assumption C3), the supplier will set $X_L = X_3$ and make zero profits. But $X_3^* = X_L^{c*}$; hence $X_L^* = X_L^{c*}$. Therefore, the decisions and profits of DS and CS are the same and we have channel coordination.

Part a.2. If $v_f^s \le v_f^b$, then in the CS all finished goods inventory will be salvaged at v_f^b. In addition, the first condition of Lemma 1 also holds and $m_c^* = X_L^c - X_1^c + G(\delta_2^c, X_L^c) - G(\delta_2^c, X_3^c)$. This means that the minimum number of goods that will be sent to the buyer in the second period is $X_L^c - X_1^c$. Furthermore, because $v_r^s + c_L + h^s > v_f^b$, $X_L^c < X_3^c$. The profit function for the CS is given by $\Pi_{CS}(X_1^c, X_L^c, X_3^c) = J_1^c(X_1^c) + J_L^c(X_L^c) + J_3^c(X_3^c)$, where $J_1^c(X_1^c)$, $J_L^c(X_L^c)$, and $J_3^c(X_3^c)$ are given by (11)–(13).

For channel coordination, we want the DS to mimic the CS by setting appropriate values of c_1, c_2, c_o, and c_e. From (11) we have that $X_1^* = X_1^{c*}$ if $c_2 - c_1 = h^s$. With $c_2 - c_o = c_L + v_r^s + h^s$ and $c_e = v_r^s + c_L(1 + \gamma)$ we get that $X_2^* = X_L^{c*}$. In addition, $c_e = v_r^s + c_L(1 + \gamma)$ implies that $\delta^d = \delta_2^c$, and so $m_c^* = m^* + X_L^{c*} - X_1^{c*} = m^* + X_2^* - X_1^*$. By substituting the value of c_e into (10) along with $c_o = c_r - v_r^s$ and comparing with (13), we get that $J_3(X_3) = J_3^c(X_3^c)$ giving $X_3^* = X_3^{c*}$.

Finally, we need to ensure that the supplier in the DS will also choose the CS's optimal production quantity as her production quantity; that is, $X_L^* = X_L^{c*}$. From Proposition 1, we know that, for the DS, $X_L^* = \max(X_2, \min(X_3, \hat{X}_L^*))$. For the transfer prices set as above, we have,

$$X_L^* = \max(X_2^*, \min(X_3^*, \hat{X}_L^*)) = \max(X_L^{c*}, \min(X_3^{c*}, \hat{X}_L^*)), \tag{30}$$

where $\hat{X}_L^*$ satisfies

$$\frac{\partial \Pi_{DS}^s}{\partial X_L} = c_L\gamma - h^s + (v_f^s - v_r^s - c_L(1 + \gamma))F_{D_1}\left(\frac{\hat{X}_L^* - \delta_2^c}{\eta}\right) = 0.$$

Now taking the derivative of (12) we get,

$$\begin{aligned}
\frac{\partial J_L^c(X_L^c)}{\partial X_L^c} &= p_2 + r - v_r^s - c_L - h^s - (p_2 + r - v_r^s - c_L(1 + \gamma))\left(1 - F_{D_1}\left(\frac{X_L^c - \delta_2^c}{\eta}\right)\right) \\
&\quad - (p_2 + r - v_f^b)\int_0^{(X_L^c - \delta_2^c)/\eta} F_{D_2|d_1}(X_L^c - d_1)\, dF_{D_1}(d_1) \\
&\ge c_L\gamma - h^s + (p_2 + r - v_r^s - c_L(1 + \gamma))F_{D_1}\left(\frac{X_L^c - \delta_2^c}{\eta}\right) - (p_2 + r - v_f^b)\int_0^{(X_L^c - \delta_2^c)/\eta} dF_{D_1}(d_1) \\
&= c_L\gamma - h^s + (v_f^b - v_r^s - c_L(1 + \gamma))F_{D_1}\left(\frac{X_L^c - \delta_2^c}{\eta}\right) \ge c_L\gamma - h^s + (v_f^s - v_r^s - c_L(1 + \gamma))F_{D_1}\left(\frac{X_L^c - \delta_2^c}{\eta}\right) \\
&= \left.\frac{\partial \Pi_{DS}^s}{\partial X_L}\right|_{X_L = X_L^c}.
\end{aligned}$$

Then,

$$\left.\frac{\partial J_L^c(X_L^c)}{\partial X_L^c}\right|_{X_L^c = X_L^{c*}} = 0 \ge \left.\frac{\partial \Pi_{DS}^s}{\partial X_L}\right|_{X_L = X_L^{c*}}.$$

Hence, $X_L^{c*} \ge \hat{X}_L^*$. Because $X_3^{c*} \ge X_L^{c*}$, it follows, by substituting into (30), that $X_L^* = X_L^{c*}$. Thus we have shown all decisions in the CS and the DS are identical and we get channel coordination.

Part a.3. If $v_f^s \geq v_r^s + c_L + h^s$, the supplier will always produce all goods in the cheaper mode of production in both the CS and the DS. That is, $X_L^* = X_3^*$ and $X_L^{c*} = X_3^{c*}$. Moreover, $c_o + c_e = c_2$ and $c_e > v_f^b$ imply that $[\partial J_2(X_2)]/\partial X_2 < 0$ for all X_2 as shown below:

$$\begin{aligned}
\frac{\partial J_2(X_2)}{\partial X_2)} &= (p_2 + r - c_e)F_{D_1}\left(\frac{X_2 - \delta^d}{\eta}\right) - (p_2 + r - v_f^b)\int_0^{(X_2-\delta^d)/\eta} F_{D_2|d_1}(X_2 - d_1)\, dF_{D_1}(d_1) \\
&< (p_2 + r - c_e)F_{D_1}\left(\frac{X_2 - \delta^d}{\eta}\right) - (p_2 + r - v_f^b)\int_0^{(X_2-\delta^d)/\eta} F_{D_2|d_1}\left(X_2 - \frac{X_2 - \delta^d}{\eta}\right) dF_{D_1}(d_1) \\
&= (p_2 + r - c_e)F_{D_1}\left(\frac{X_2 - \delta^d}{\eta}\right) - (p_2 + r - v_f^b)\int_0^{(X_2-\delta^d)/\eta} \Phi(k_m)\, dF_{D_1}(d_1) \\
&= (p_2 + r - c_e)F_{D_1}\left(\frac{X_2 - \delta^d}{\eta}\right) - (p_2 + r - v_f^b)\Phi(k_m)F_{D_1}\left(\frac{X_2 - \delta^d}{\eta}\right) \\
&= 0.
\end{aligned} \tag{31}$$

This implies that $X_2^* = X_1^*$. Substituting into the buyer's profit function and setting $c_1 - c_o = v_f^s - h^s$ and $c_e = v_f^s$ we get $J_1(X_1) + J_2(X_1) = J_1^c(X_1)$, where $J_1^c(X_1)$ is as in (15). Therefore, $X_1^* = X_1^{c*}$. Because $X_L^{c*} = X_3^{c*}$, m_c^* is only a function of δ_1^c. Finally, $c_e = v_f^s$ implies that $\delta_1^c = \delta^d$, hence $m^* = m_c^*$.

Now, we only need to show that $X_3^* = X_3^{c*}$. Recall that $X_L^{c*} = X_3^{c*}$. Substituting into the profit function for the CS, we get

$$\begin{aligned}
J_3^c(X_3^c) &= J_L^c(X_3^c) + J_3^c(X_3^c) \\
&= -(c_r + h^s - v_f^s + c_L)X_3 - (p_2 + r - v_f^s)\int_{(X_3^c-\delta_1^c)/\eta}^{\infty} (\delta_1^c + \eta d_1 - X_3^c)\, dF_{D_1}(d_1) \\
&\quad - (p_2 + r - v_f^b)\left(\int_{(X_3^c-\delta_1^c)/\eta}^{\infty}\int_0^{X_3^c-d_1} X_3^c - d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1) - \int_{(X_3^c-\delta_1^c)/\eta}^{\infty}\int_0^{\delta_1^c+(\eta-1)d_1} (\delta_1^c + (\eta - 1)d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1)\right).
\end{aligned} \tag{32}$$

Because $v_f^b \leq v_f^s$, δ_1^c is finite. Setting $c_o = c_r + c_L + h^s - v_f^s$ with $c_e = v_f^s$ we get $J_3(X_3) = J_3^c(X_3)$, and $X_3^* = X_3^{c*}$. Hence, all of the order quantities will be the same, and we have channel coordination.

Part b. Consider the scenario where $X_1^{c*} < X_L^{c*} < X_3^{c*}$. Recall that $v_f^s > v_f^b$ and $v_r^s + c_L + h^s > v_f^s$. This implies that $\delta_1^c > \delta_2^c$ and the optimal number of options exercised, $m_c^* = G(\delta_1^c, X_1^c) - G(\delta_1^c, X_L^c) + G(\delta_2^c, X_L^c) - G(\delta_2^c, X_3^c)$. We can rewrite m_c^* as follows:

$$m_c^* = \begin{cases} 0, & \text{for } d_1 \leq (X_1^c - \delta_1^c)/\eta, \\ \delta_1^c + \eta d_1 - X_1^c, & \text{for } (X_1^c - \delta_1^c)/\eta < d_1 \leq (X_L^c - \delta_1^c)/\eta, \\ X_L^c - X_1^c, & \text{for } (X_L^c - \delta_1^c)/\eta < d_1 \leq (X_L^c - \delta_2^c)/\eta, \\ \delta_2^c + \eta d_1 - X_1^c, & \text{for } (X_L^c - \delta_2^c)/\eta < d_1 \leq (X_3^c - \delta_2^c)/\eta, \\ X_3^c - X_1^c, & \text{for } (X_3^c - \delta_2^d)/\eta < d_1. \end{cases}$$

Thus the number of options exercised depends on the first-period demand being in one of the five distinct regions. The regions are distinct because $\delta_1^c > \delta_2^c$ and $X_1^{c*} < X_L^{c*} < X_3^{c*}$. In the DS, however, the optimal number of options exercised depends on first-period demand being in at most one of three regions; that is,

$$m^* = \begin{cases} 0, & \text{for } d_1 \leq (X_2 - \delta^d)/\eta, \\ \delta^d + \eta d_1 - X_2, & \text{for } (X_2 - \delta^d)/\eta < d_1 \leq (X_3 - \delta^d)/\eta, \\ X_3 - X_2, & \text{for } (X_3 - \delta^d)/\eta < d_1. \end{cases}$$

Because δ^d, given by (3), is a function of c_e through k_m, the supplier may control the setting of region boundaries. There are, however, only two such boundaries available to influence in the DS. Hence, there is no way to make $m^* + X_2^* - X_1^* = m_c^*$ for every outcome d_1. Therefore, the DS cannot imitate the actions of the CS along every sample path, and so there will be no channel coordination. □

Proof of Proposition 3. Part a.1: Shown in the Proof of Proposition 2, Part a.1.

Part a.2. The prices are $c_1 = c_r + c_L$, $c_2 = c_r + c_L + h^s$, $c_o = c_r - v_r^s$, and $c_e = v_r^s + c_L(1 + \gamma)$, and the resulting decisions are $X_1^* = X_1^{c*}$, $X_2^* = X_L^* = X_L^{c*}$ and $X_3^* = X_3^{c*}$. The marginal cost of supplying up to X_1 units of goods is $c_r + c_L$ for which the supplier gets c_1 per unit. The marginal cost of supplying up to $X_2 - X_1$ units of goods is $c_r + c_L + h^s$ for which the supplier gets c_2 per unit. Furthermore,

$X_2^* = X_L^*$ implies that if any options are exercised in the DS, the supplier has to produce them in the expedited mode. Thus marginal cost of supplying goods against options is the higher labor cost $c_L(1 + \gamma)$ plus the salvage value of the raw material v_r^s. Observe that at the beginning of second period when options are exercised, raw material has already been bought and hence the cost of raw material is essentially a sunk cost; all that matters is its salvage value. By setting $c_e = v_r^s + c_L(1 + \gamma)$, the supplier supplies goods to the buyer against exercised options at marginal cost. Finally, the marginal cost of purchasing additional raw material is $c_r - v_r^s$, which is also the price at which the supplier sells options to the buyer. Therefore, all transactions between the supplier and the buyer occur at marginal cost to the supplier and hence she makes zero profits.

Part a.3. The prices are $c_1 = c_r + c_L$, $c_2 = c_r + c_L + h^s$, $c_o = c_r + c_L + h^s - v_f^s$, and $c_e = v_f^s$, and the resulting decisions are $X_1^* = X_1^{c*} = X_2^*$, $X_L^* = X_L^{c*} = X_3^* = X_3^{c*}$. The marginal cost of supplying up to X_1 units of goods is $c_r + c_L$ for which the supplier gets c_1 per unit. The marginal cost of supplying up to $X_2 - X_1$ units of goods is $c_r + c_L + h^s$ for which the supplier gets c_2 per unit. Furthermore, $X_3^* = X_L^*$ implies that if any options are exercised in the DS, the supplier has already produced them in the preliminary mode. By setting $c_o + c_e = c_r + c_L + h^s$ the supplier supplies goods to the buyer against exercised options at marginal cost. Finally, the marginal cost of making an unused good is $c_r + c_L + h^s - v_f^s$, which is also the price at which the supplier sells options to the buyer. Therefore, all transactions between the supplier and the buyer occur at marginal cost to the supplier and hence she makes zero profits. □

Proof of Proposition 4. Let us consider a scenario where the supplier also offers to buy back any remaining finished goods the buyer may have, for a price c_b. However, it will cost t_{bs} per unit to have the goods returned. This is a charge the supplier incurs, though she may pass that on to the buyer through an appropriate c_b.

The buyer's expected profit function will remain the same as before, except that now $\max(v_f^b, c_b - h_2^b)$ will replace v_f^b everywhere it appears. That is, we get

$$\Pi_{\mathrm{DS}}^b(X_1, X_2, X_3) = J_1(X_1) + J_2(X_2) + J_3(X_3),$$

where

$$J_1(X_1) = -p_2\mu_2 - \mu_1(p_1 + p_2) - (p_1 + h_1^b)\int_0^{X_1}(X_1 - d_1)\,dF_{D_1}(d_1) + (p_1 + c_2 - c_1)X_1,$$

$$\begin{aligned} J_2(X_2) = {} & (p_2 + r - (c_2 - c_o))X_2 + (p_2 + r - c_e)\int_{(X_2-\delta^d)/\eta}^{\infty}(\delta^d + \eta d_1 - X_2)\,dF_{D_1}(d_1) \\ & - (p_2 + r - \max(v_f^b, c_b - h_2^b))\int_0^{(X_2-\delta^d)/\eta}\int_0^{X_2-d_1}(X_2 - d_1 - d_2)\,dF_{D_2|d_1}(d_2)\,dF_{D_1}(d_1) \\ & - (p_2 + r - \max(v_f^b, c_b - h_2^b))\int_{(X_2-\delta^d)/\eta}^{\infty}\int_0^{\delta^d+(\eta-1)d_1}(\delta^d + (\eta - 1)d_1 - d_2)\,dF_{D_2|d_1}(d_2)\,dF_{D_1}(d_1), \end{aligned}$$

$$\begin{aligned} J_3(X_3) = {} & -c_oX_3 - (p_2 + r - c_e)\int_{(X_3-\delta^d)/\eta}^{\infty}(\delta^d + \eta d_1 - X_3)\,dF_{D_1}(d_1) \\ & + (p_2 + r - \max(v_f^b, c_b - h_2^b))\int_{(X_3-\delta^d)/\eta}^{\infty}\int_0^{\delta^d+(\eta-1)d_1}(\delta^d + (\eta - 1)d_1 - d_2)\,dF_{D_2|d_1}(d_2)\,dF_{D_1}(d_1) \\ & - (p_2 + r - \max(v_f^b, c_b - h_2^b))\int_{(X_3-\delta^d)/\eta}^{\infty}\int_0^{X_3-d_1}(X_3 - d_1 - d_2)\,dF_{D_2|d_1}(d_2)\,dF_{D_1}(d_1). \end{aligned}$$

The supplier's profit function will now also need to incorporate the added salvage of finished goods that the buyer may return. That is, if $v_f^b < c_b - h_2^b$, then the supplier will also need to salvage $X_2 + m^* - d_1 - d_2$. For each of these units, she will get $v_f^s - c_b - t_{bs}$. Therefore, when this is the case, we can write the supplier's profit function as

$$\begin{aligned} & \Pi_{\mathrm{DS}}^s(X_L, c_1, c_2, c_o, c_e) \\ & = X_1(h^s - (c_2 - c_1)) + X_2(c_2 - c_o - v_f^s) - X_L(v_r^s - v_f^s + c_L + h^s) + X_3(c_o + v_r^s - c_r) + (c_e - v_f^s)\int_{(X_2-\delta^d)/\eta}^{\infty}(\delta^d - X_2 + \eta d_1)\,dF_{D_1}(d_1) \\ & \quad + (v_f^s - v_r^s - c_L(1 + \gamma))\int_{(X_L-\delta^d)/\eta}^{\infty}(\delta^d - X_L + \eta d_1)\,dF_{D_1}(d_1) + (v_r^s + c_L(1 + \gamma) - c_e)\int_{(X_3-\delta^d)/\eta}^{\infty}(\delta^d - X_3 + \eta d_1)\,dF_{D_1}(d_1) \end{aligned}$$

$$+ (v_f^s - t_{bs} - c_b) \int_0^{(X_2-\delta^d)/\eta} \int_0^{X_2-d_1} (X_2 - d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1)$$

$$+ (v_f^s - t_{bs} - c_b) \int_{(X_2-\delta^d)/\eta}^{\infty} \int_0^{\delta^d+(\eta-1)d_1} (\delta^d + (\eta - 1)d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1)$$

$$- (v_f^s - t_{bs} - c_b) \int_{(X_3-\delta^d)/\eta}^{\infty} \int_0^{\delta^d+(\eta-1)d_1} (\delta^d + (\eta - 1)d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1)$$

$$+ (v_f^s - t_{bs} - c_b) \int_{(X_3-\delta^d)/\eta}^{\infty} \int_0^{X_3-d_1} (X_3 - d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1).$$

When $v_f^b > c_b - h_2^b$, then the last four terms will be omitted from the expression, and the supplier's profit will be the same as the nonbuy-back case. Note that the supplier's unconstrained production decision, $\hat{X}_L^*$, remains unaffected by whether or not the buyer will return goods.

In the centralized system, the buy-back price is another transfer price and as such will not matter for the centralized system. However, under a buy-back-type arrangement, it now becomes possible for the finished goods to be salvaged wherever it is more lucrative. Hence, if costs are such that $v_f^s - t_{bs} > \tilde{v}_f^b = v_f^b + h_2^b$ (i.e., $v_f^s - t_{bs} - h_2^b > v_f^b$), then the goods will be shipped back to the supplier for salvage. Otherwise, they will be salvaged at the buyer. In any case, $\bar{v} = \max(v_f^b, v_f^s - t_{bs} - h_2^b)$ will replace v_f^b in all CS profit equations.

Clearly, when $v_f^s - t_{bs} - h_2^b < v_f^b$, the CS will choose to salvage at the buyer. Hence, there is no change in the decisions made and general channel coordination is not possible. However, when $v_f^s - t_{bs} - h_2^b \geq v_f^b$, the CS is either indifferent between salvage locations (=) or prefers to salvage at the supplier (>), in which cases there might now be a new opportunity to channel coordinate. Therefore, let us consider this case.

Part a.1. Because $v_f^s - h_2^b - t_{bs} \geq v_f^b$ and $v_f^s \leq v_f^b$, it must be true that $h_2^b = t_{bs} = 0$ and $v_f^s = v_f^b$. When v_f^b equals v_f^s and exceeds $v_r^s + c_L + h^s$, then the CS will convert all of the raw material purchased into finished goods using the cheaper production mode and then salvage all leftover finished goods. It is indifferent between salvage locations, however. That is, $X_L^{c*} = X_3^{c*}$ and $m_c^* = X_3^{c*} - X_1^{c*} = X_L^{c*} - X_1^{c*}$. This means that the profit function in the CS will be

$$\Pi_{CS}(X_1^c, X_L^c, X_3^c) = J_1^c(X_1^c) + J_L^c(X_L^c) \tag{33}$$

where $J_1^c(X_1^c)$ is as in (11) and

$$J_L^c(X_L^c) = (p_2 + r - c_r - c_L - h^s)X_L^c - (p_2 + r - v_f^s) \int_0^{X_L^c} (X_L^c - d_{12})\, dF_{D_{12}}(d_{12}), \tag{34}$$

where $D_{12} = D_1 + D_2$ with a cdf of $F_{D_{12}}(\cdot)$.

From (9) and (11) we have that $X_1^* = X_1^{c*}$ if $c_2 - c_1 = h^s$. If the supplier is to take advantage of the equal salvage access, she must convince the buyer to return goods. That is, she must set $c_b > v_f^b$. Then, if $c_e \leq c_b$, we have $m^* = X_3^* - X_2^*$ in the DS. Hence, the supplier will turn all raw material into finished goods using the cheaper production mode, and so $X_L^* = X_3^*$. This also means that

$$\Pi_{DS}^b(X_1, X_2, X_3) = J_1(X_1) + X_2(c_e + c_o - c_2) + X_3(p_2 + r - c_o - c_e) - (p_2 + r - c_b) \int_0^{X_3} (X_3 - d_{12})\, dF_{D_{12}}(d_{12}). \tag{35}$$

Setting $c_2 = c_o + c_e$ assures us that the buyer will buy as few firm orders as possible in the second period, and so $X_2^* = X_1^*$ (an option good costs the same as a firm order good). Equating first-order conditions for X_3 and X_3^c gives us

$$\frac{p_2 + r - c_2}{p_2 + r - c_b} = \frac{p_2 + r - c_r - c_L - h^s}{p_2 + r - v_f^s}.$$

Hence $c_1 = p_2 + r - h^s - (p_2 + r - c_r - c_L - h^s)(p_2 + r - c_b)/(p_2 + r - v_f^s)$ will assure that $X_3^* = X_3^{c*}$, and we will coordinate the channel.

Part a.2. Because $v_f^s - h_2^b - t_{bs} \geq v_f^b$ and $v_f^s \leq v_f^b$, it must be true that $h_2^b = t_{bs} = 0$ and $v_f^s = v_f^b$, so all finished goods inventory will be salvaged at v_f^b. In addition, the first condition of Lemma 1 also holds and $m_c^* = X_L^c - X_1^c + G(\delta_2^c, X_L^c) - G(\delta_2^c, X_3^c)$. This means that the minimum number of goods that will be sent to the buyer in the second period is $X_L^c - X_1^c$. Furthermore, because $v_r^s + c_L + h^s > v_f^b$, $X_L^c < X_3^c$. The profit function for the CS is given by $\Pi_{CS}(X_1^c, X_L^c, X_3^c) = J_1^c(X_1^c) + J_L^c(X_L^c) + J_3^c(X_3^c)$ where $J_1^c(X_1^c)$, $J_L^c(X_L^c)$, and $J_3^c(X_3^c)$ are given by (11)–(13).

For channel coordination, we want the DS to mimic the CS by setting appropriate values of c_1, c_2, c_b, c_o, and c_e. To get the buyer to ship goods back, the supplier needs to make sure that $c_b > v_f^b$. From (11) we have that $X_1^* = X_1^{c*}$ if $c_2 - c_1 = h^s$. To get $m^* + X_2 - X_1 = m_c^*$ we also need $\delta^d = \delta_2^c$ and $X_2^* = X_L^{c*}$. To get identical deltas, we must have

$$\frac{p_2 + r - c_e}{p_2 + r - c_b} = \frac{p_2 + r - v_r^s - c_L(1 + \gamma)}{p_2 + r - v_f^s}.$$

First-order conditions for X_2 and X_L^c also indicate that we must have

$$\frac{p_2 + r - c_2 + c_o}{p_2 + r - c_b} = \frac{p_2 + r - v_r^s - c_L - h^s}{p_2 + r - v_f^s}$$

to have $X_2 = X_L^c$.

To get $X_3^* = X_3^{c*}$, first-order conditions tell us that we need

$$\frac{c_o}{p_2 + r - c_b} = \frac{c_r - v_r^s}{p_2 + r - v_f^s}.$$

Finally, we need to ensure that the supplier in the DS will also choose the CS's optimal production quantity as her production quantity; that is, $X_L^* = X_L^{c*}$. This is the exact same situation as Proposition 2, Part a.2, and from that proof we know that $X_L^{c*} \geq \hat{X}_L^*$. Because $X_3^{c*} \geq X_L^{c*}$, it follows that $X_L^* = X_L^{c*}$. Thus we have shown all decisions in the CS and the DS, which are identical, and we get channel coordination.

Part a.3. Here, $v_f^s - h_2^b - t_{bs} \geq v_f^b$ and $v_f^s > v_f^b$ imply that $v_f^s - h_2^b - t_{bs} > v_f^b$. Now, if $v_f^s \geq v_r^s + c_L + h^s$, the supplier will always produce all goods in the cheaper mode of production in both the CS and the DS. That is, $X_L^* = X_3^*$ and $X_L^{c*} = X_3^{c*}$. Then, the profit function for the CS appears as in (15)–(16). For channel coordination, we want the DS to mimic the CS by setting appropriate values of c_1, c_2, c_b, c_o, and c_e. To get the buyer to ship goods back, the supplier needs to make sure that $c_b - h_2^b > v_f^b$.

To get $m^* + X_2 - X_1 = m_c^*$, we need to have $\delta^d = \delta_1^c$, and this will happen when

$$\frac{p_2 + r - c_e}{p_2 + r - c_b + h_2^b} = \frac{p_2 + r - v_r^s - c_L(1 + \gamma)}{p_2 + r - v_f^s + h_2^b + t_{bs}}.$$

To get $X_3 = X_3^c$, we need to consider the first-order condition of (16). This indicates that we need

$$\frac{c_o}{p_2 + r - c_b + h_2^b} = \frac{c_r + c_L + h^s - v_f^s}{p_2 + r - v_f^s + h_2^b + t_{bs}}.$$

To get $X_1 = X_1^c$, we first need to have $X_1 = X_2$. This will certainly happen when $c_o + c_e = c_2$. Define $\mathcal{Z}(X)$ as follows.

$$\mathcal{Z}(X) = \Phi(k_{m_1}^c)\left(1 - F_{D_1}\left(\frac{X - \delta_1^c}{\eta}\right)\right) + \int_0^{(X-\delta_1^c)/\eta} F_{D_2|d_1}(X - d_1)\, dF_{D_1}(d_1).$$

Then, the first-order conditions of (15) and the profit function for the DS, give us

$$p_1 + p_2 + r - v_f^s + h^s - (h_1^b + p_1)F_{D_1}(X_1^c) - (p_1 + r - v_f^s + t_{bs} + h_2^b)\mathcal{Z}(X_1^c) = 0 \quad \text{and}$$

$$p_1 + p_2 + r - c_1 + c_o - (h_1^b + p_1)F_{D_1}(X_1) - (p_1 + r - c_b + h_2^b)\mathcal{Z}(X_1) = 0.$$

Hence, to get $X_1^* = X_1^{c*}$, we need $c_1 = c_o + v_f^s - h^s + \mathcal{Z}(X_1^{c*})(c_b - v_f^s + h^s)$.

Therefore, all of the order quantities will be the same, and we have channel coordination.

Part b. Same as the Proof of Proposition 2, Part b. □

Proof of Proposition 5. We wish to show that the supplier can operate in an individually rational manner and still coordinate the supply chain by using a return policy.

Part a.1. In this case, $m^* = X_2^* - X_3^*$, $X_1^* = X_2^*$ and $X_L^* = X_3^*$, we can write the supplier's profit function as follows.

$$\begin{aligned}\Pi_{\mathrm{DS}}^s(X_L^*, c_1, c_2, c_o, c_e) &= c_1X_1^* + c_2(X_2^* - X_1^*) + c_o(X_3^* - X_2^*) + c_e(X_3^* - X_2^*) - h^s(X_L^* - X_1^*)\\ &\quad - c_rX_3^* - c_LX_L^* + (v_f^s - c_b)\int_0^{X_3^*}(X_3^* - d_{12})\, dF_{D_1+D_2}(d_{12})\\ &= c_1X_1^* + (c_o + c_e)(X_3^* - X_1^*) - h^s(X_3^* - X_1^*) - (c_r + c_L)X_3^* + (v_f^s - c_b)\int_0^{X_3^*}(X_3^* - d_{12})\, dF_{D_1+D_2}(d_{12}). \qquad (36)\end{aligned}$$

Then, noting that $c_e + c_o = c_2 = c_1 + h^s$ and $c_1 = (p_2 + r - h^s)(1 - Y) + (c_r + c_L)Y$ in the pricing scheme and that $v_f^s = v_f^b$, we can further simplify and get

$$\begin{aligned}\Pi^s_{\mathrm{DS}}(X^*_L, c_1, c_2, c_o, c_e) &= (c_1 - c_r - c_L)X^*_3 + (v^s_f - c_b)\int_0^{X^*_3}(X^*_3 - d_{12})\, dF_{D_1+D_2}(d_{12})\\ &= (1 - Y)\left[(p_2 + r - h^s - c_r - c_L)X^*_3 - (p_2 + r - v^b_f)\int_0^{X^*_3}(X^*_3 - d_{12})\, dF_{D_1+D_2}(d_{12})\right]\\ &= (1 - Y)J^c_L(X^*_3). \qquad (37)\end{aligned}$$

However, $X^*_3 = X^*_L = X^{c*}_L$ with our current prices. Moreover, because $c_b > v^b_f + h^b_2$, we know that $Y \le 1$. Therefore, the supplier will make nonnegative profits if $J^c_L(X^{c*}_L) \ge 0$. First note that the first-order conditions of $J^c_L(X^c_L)$ indicate that $F_{D_1+D_2}(X^{c*}_L) = (p_2 + r - c_r - c_L - h^s)/(p_2 + r - v^s_f)$. Substituting this into $J^c_L(X^c_L)$ gives us

$$\begin{aligned}J^c_L(X^c_L) &= (p_2 + r - c_r - c_L - h^s)X^c_L - (p_2 + r - v^s_f)\int_0^{X^c_L}(X^c_L - d_{12})\, dF_{D_1+D_2}(d_{12})\\ &= (p_2 + r - c_r - c_L - h^s)X^c_L - (p_2 + r - v^s_f)X^c_L F_{D_1+D_2}(X^c_L) + (p_2 + r - v^s_f)\int_0^{X^c_L} d_{12}\, dF_{D_1+D_2}(d_{12})\\ &= (p_2 + r - v^s_f)\int_0^{X^c_L} d_{12}\, dF_{D_1+D_2}(d_{12})\\ &\ge 0.\end{aligned}$$

Therefore, the supplier makes $(1 - Y)J^c_L(X^{c*}_L)$. Moreover, the larger she sets c_b, the more she makes, up to the system profits less the buyer's reservation profit.

Part a.2. In this case, we have that $X_2 = X_L = X^*_L$. This means that the supplier's profit function can be written as

$$\begin{aligned}\Pi^s_{\mathrm{DS}}(X_L, c_1, c_2, c_o, c_e) &= X_1(h^s - (c_2 - c_1)) + X_L(c_2 - c_o - v^s_r - c_L - h^s) + X_3(c_o + v^s_r - c_r)\\ &+ (c_e - v^s_r - c_L(1 + \lambda))\int_{(X_L-\delta^d)/\eta}^{\infty}(\delta^d - X_L + \eta d_1)\, dF_{D_1}(d_1)\\ &+ (v^s_r + c_L(1 + \gamma) - c_e)\int_{(X_3-\delta^d)/\eta}^{\infty}(\delta^d - X_3 + \eta d_1)\, dF_{D_1}(d_1)\\ &+ (v^s_f - c_b)\int_0^{(X_L-\delta^d)/\eta}\int_0^{X_L-d_1}(X_L - d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1)\\ &+ (v^s_f - c_b)\int_{(X_L-\delta^d)/\eta}^{\infty}\int_0^{\delta^d+(\eta-1)d_1}(\delta^d + (\eta - 1)d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1)\\ &- (v^s_f - c_b)\int_{(X_3-\delta^d)/\eta}^{\infty}\int_0^{\delta^d+(\eta-1)d_1}(\delta^d + (\eta - 1)d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1)\\ &+ (v^s_f - c_b)\int_{(X_3-\delta^d)/\eta}^{\infty}\int_0^{X_3-d_1}(X_3 - d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1).\end{aligned}$$

Substituting in for the wholesale, option, and exercise prices, we get that $c_2 - c_o - v^s_r - c_L - h^s = (1 - Y)\ (p_2 + r - h^s - c_L - v^s_r)$, $c_o + v^s_r - c_r = (1 - Y)(v^s_r - c_r)$, and $c_e - v^s_r - c_L(1 + \gamma) = (1 - Y)(p_2 + r - v^s_r - c_L(1 + \gamma))$. This means that we can rewrite the supplier's profit function as

$$
\begin{aligned}
\Pi^s_{\mathrm{DS}}(X_L, c_1, c_2, c_o, c_e) &= X_L(1-Y)(p_2 + r - h^s - c_L - v^s_r) + X_3(1-Y)(v^s_r - c_r)\\
&\quad + (1-Y)(p_2 + r - v^s_r - c_L(1+\gamma)) \int_{(X_L-\delta^d)/\eta}^{\infty} (\delta^d - X_L + \eta d_1)\, dF_{D_1}(d_1)\\
&\quad - (1-Y)(p_2 + r - v^s_r - c_L(1+\gamma)) \int_{(X_3-\delta^d)/\eta}^{\infty} (\delta^d - X_3 + \eta d_1)\, dF_{D_1}(d_1)\\
&\quad + (1-Y)(p_2 + r - v^b_f) \int_0^{(X_L-\delta^d)/\eta} \int_0^{X_L-d_1} (X_L - d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1)\\
&\quad + (1-Y)(p_2 + r - v^b_f) \int_{(X_L-\delta^d)/\eta}^{\infty} \int_0^{\delta^d+(\eta-1)d_1} (\delta^d + (\eta-1)d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1)\\
&\quad - (1-Y)(p_2 + r - v^b_f) \int_{(X_3-\delta^d)/\eta}^{\infty} \int_0^{\delta^d+(\eta-1)d_1} (\delta^d + (\eta-1)d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1)\\
&\quad + (1-Y)(p_2 + r - v^b_f) \int_{(X_3-\delta^d)/\eta}^{\infty} \int_0^{X_3-d_1} (X_3 - d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1)\\
&= (1-Y)[J^c_L(X_L) + J^c_3(X_3)]
\end{aligned}
$$

where $J^c_L(\cdot)$ and $J^c_3(\cdot)$ are as in (12) and (13). Because $X^*_L = X^{c*}_L$ and $X^*_3 = X^{c*}_3$, we only need to show that $J^c_L(X^{c*}_L) + J^c_3(X^{c*}_3) \geq 0$ for the supplier to have nonnegative profits. Using the first-order condition of (12) we can rewrite $J^c_L(X^{c*}_L)$ as follows:

$$
\begin{aligned}
J^c_L(X^{c*}_L) &= (p_2 + r - v^s_r - c_L - h^s)X^{c*}_L - (p_2 + r - v^s_r - c_L(1+\gamma))\left(1 - F_{D_1}\left(\frac{X^{c*}_L - \delta^c_2}{\eta}\right)\right)X^{c*}_L\\
&\quad + (p_2 + r - v^s_r - c_L(1+\gamma)) \int_{(X^{c*}_L-\delta^c_2)/\eta}^{\infty} (\delta^c_2 + \eta d_1)\, dF_{D_1}(d_1)\\
&\quad - (p_2 + r - v^b_f)\Bigg[\int_0^{(X^{c*}_L-\delta^c_2)/\eta} X^{c*} F_{D_2|d_1}(X^c_L - d_1)\, dF_{D_1}(d_1) - \int_0^{(X^{c*}_L-\delta^c_2)/\eta} \int_0^{X^c_L-d_1} (d_1 + d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1)\\
&\qquad + \int_{(X^c_L-\delta^c_2)/\eta}^{\infty} \int_0^{\delta^c_2+(\eta-1)d_1} (\delta^c_2 + (\eta-1)d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1)\Bigg]\\
&= \left.\frac{\partial J^c_L(X^c_L)}{\partial X^c_L}\right|_{X^{c*}_L} + (p_2 + r - v^s_r - c_L(1+\gamma)) \int_{(X^{c*}_L-\delta^c_2)/\eta}^{\infty} (\delta^c_2 + \eta d_1)\, dF_{D_1}(d_1)\\
&\quad + (p_2 + r - v^b_f)\Bigg[\int_0^{(X^{c*}_L-\delta^c_2)/\eta} \int_0^{X^c_L-d_1} (d_1 + d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1) - \int_{(X^c_L-\delta^c_2)/\eta}^{\infty} \int_0^{\delta^c_2+(\eta-1)d_1} (\delta^c_2 + (\eta-1)d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1)\Bigg].
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
J^c_3(X^{c*}_3) &= \left.\frac{\partial J^c_3(X^c_3)}{\partial X^c_3}\right|_{X^{c*}_3} - (p_2 + r - v^s_r - c_L(1+\gamma)) \int_{(X^{c*}_3-\delta^c_2)/\eta}^{\infty} (\delta^c_2 + \eta d_1)\, dF_{D_1}(d_1)\\
&\quad + (p_2 + r - v^b_f)\Bigg[\int_{(X^{c*}_3-\delta^c_2)/\eta}^{\infty} \int_0^{X^c_3-d_1} (d_1 + d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1)\\
&\qquad + \int_{(X^c_3-\delta^c_2)/\eta}^{\infty} \int_0^{\delta^c_2+(\eta-1)d_1} (\delta^c_2 + (\eta-1)d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1)\Bigg].
\end{aligned}
$$

Therefore, we get

$$J_L^c(X_L^{c*}) + J_3^c(X_3^{c*}) = (p_2 + r - v_r^s - c_L(1+\gamma)) \int_{(X_L^{c*}-\delta_2^c)/\eta}^{(X_3^{c*}-\delta_2^c)/\eta} (\delta_2^c + \eta d_1)\, dF_{D_1}(d_1)$$

$$+ (p_2 + r - v_f^b)\Bigg[\int_{(X_3^{c*}-\delta_2^c)/\eta}^{\infty} \int_0^{X_3^c - d_1} (d_1 + d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1) + \int_0^{(X_L^{c*}-\delta_2^c)/\eta} \int_0^{X_L^c - d_1} (d_1 + d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1)$$

$$- \int_{(X_L^c-\delta_2^c)/\eta}^{(X_3^c-\delta_2^c)/\eta} (\delta_2^c + \eta d_1) F_{D_2|d_1}(\delta_2^c + (\eta-1)d_1)\, dF_{D_1}(d_1) - \int_{(X_L^c-\delta_2^c)/\eta}^{(X_3^c-\delta_2^c)/\eta} \int_0^{\delta_2^c + (\eta-1)d_1} (d_1 + d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1)\Bigg].$$

However, because $p_2 + r - v_r^s - c_L(1+\gamma) \geq p_2 + r - v_f^b$ and $F_{D_2|d_1}(\delta_2^c + (\eta-1)d_1) \leq 1$, we have that the sum is greater than or equal to zero, and hence the supplier makes nonnegative profits.

Part a.3. In this case, $X_L^* = X_3^* = X_L^{c*} = X_3^{c*}$ and $X_1^* = X_2^*$, and we get

$$\Pi_{\mathrm{DS}}^s(X_L, c_1, c_2, c_o, c_e) = X_1(h^s + c_1 - c_o - v_f^s) - X_3(c_r + c_L + h^s - c_o - v_f^s)$$

$$+ (c_e - v_f^s) \int_{(X_1-\delta^d)/\eta}^{\infty} (\delta^d - X_1 + \eta d_1)\, dF_{D_1}(d_1) + (v_f^s - c_e) \int_{(X_3-\delta^d)/\eta}^{\infty} (\delta^d - X_3 + \eta d_1)\, dF_{D_1}(d_1)$$

$$+ (v_f^s - t_{bs} - c_b) \int^{(X_L-\delta^d)/\eta} \int_0^{X_2-d_1} (X_2 - d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1)$$

$$+ (v_f^s - t_{bs} - c_b) \int_{(X_2-\delta^d)/\eta}^{\infty} \int_0^{\delta^d + (\eta-1)d_1} (\delta^d + (\eta-1)d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1)$$

$$- (v_f^s - t_{bs} - c_b) \int_{(X_3-\delta^d)/\eta}^{\infty} \int_0^{\delta^d + (\eta-1)d_1} (\delta^d + (\eta-1)d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1)$$

$$+ (v_f^s - t_{bs} - c_b) \int_{(X_3-\delta^d)/\eta}^{\infty} \int_0^{X_3-d_1} (X_3 - d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1).$$

Using the optimal prices, we get that $h^s + c_1 - c_o - v_f^s = (c_b - v_f^s + t_{bs})\mathcal{Z}(X_1^{c*}) = (1 - Y)(p_2 + r + h_2^s + t_{bs} - v_f^s)\mathcal{Z}(X_1^{c*})$, $c_r + c_L + h^s - c_o - v_f^s = (1 - Y)(c_r + c_L + h^s - f_v^s)$ and $c_e - v_f^s = (1 - Y)(p_2 + r - v_f^s)$. Substituting in gives us

$$\Pi_{\mathrm{DS}}^s = X_1(1-Y)(p_2 + r + h_2^s + t_{bs} - v_f^s)\mathcal{Z}(X_1^{c*}) - X_3(1-Y)(c_r + c_L + h^s - f_v^s)$$

$$+ (1-Y)(p_2 + r - v_f^s) \int_{(X_1-\delta^d)/\eta}^{\infty} (\delta^d - X_1 + \eta d_1)\, dF_{D_1}(d_1)$$

$$- (1-Y)(p_2 + r - v_f^s) \int_{(X_3-\delta^d)/\eta}^{\infty} (\delta^d - X_3 + \eta d_1)\, dF_{D_1}(d_1)$$

$$+ (1-Y)(p_2 + r + h_2^b + t_{bs} - v_f^s) \int_0^{(X_1-\delta^d)/\eta} \int_0^{X_1-d_1} (X_1 - d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1)$$

$$+ (1-Y)(p_2 + r + h_2^b + t_{bs} - v_f^b) \int_{(X_1-\delta^d)/\eta}^{\infty} \int_0^{\delta^d + (\eta-1)d_1} (\delta^d + (\eta-1)d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1)$$

$$- (1-Y)(p_2 + r + h_2^b + t_{bs} - v_f^b) \int_{(X_3-\delta^d)/\eta}^{\infty} \int_0^{\delta^d + (\eta-1)d_1} (\delta d + (\eta-1)d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1)$$

$$+ (1-Y)(p_2 + r + h_2^b + t_{bs} - v_f^b) \int_{(X_3-\delta^d)/\eta}^{\infty} \int_0^{X_3-d_1} (X_3 - d_1 - d_2)\, dF_{D_2|d_1}(d_2)\, dF_{D_1}(d_1)$$

$$= (1-Y)\left[J_1^c(X_1^{c*}) + \mu_2 p_2 + \mu_1(p_1 + p_2) - (h_1^b + p_1)\int_0^{X_1^{c*}} d_1 dF_{D_1}(d_1)\right] + (1-Y)J_3^c(X_3)$$

$$= (1 - Y)[J_1^c(X_1^{c^*}) + J_3^c(X_3) + \mu_2 p_2 + \mu_1(p_1 + p_2) - (h_1^b + p_1)(\mu_1 F_{D_1}(X_1^{c^*}) - \sigma_1^2 f_{D_1}(X_1^{c^*}))]$$

$$= (1 - Y)[J_1^c(X_1^{c^*}) + J_3^c(X_3) + (\mu_1 + \mu_2)p_2 + \mu_1(p_1 - (h_1^b + p_1)F_{D_1}(X_1^{c^*})) + (h_1^b + p_1)\sigma_1^2 f_{D_1}(X_1^{c^*}))].$$

Because $F_{D_1}(X_1^{c^*}) \le p_1/(p_1 + h_1^b)$, we have that the supplier will make nonnegative profits. □

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