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Turn-and-Earn Incentives with a Product Line

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When manufacturers do not have sufficient capacity to meet demand and cannot increase prices, they have to determine other methods to allocate goods among retailers. A common allocation mechanism is based on a retailer's sales history: a retailer that has ordered larger quantities in the past should get a greater allocation than a retailer that has historically ordered smaller quantities. This mechanism, known as a turn-and-earn allocation rule, is commonly used in many industries such as automobiles, microprocessors, video game consoles, etc. The existing literature has considered the effect of turn-and-earn allocation rules when a manufacturer sells a single product. However, when we consider a product line, it is not clear whether the manufacturer is better off basing its allocation on the sales history of the entire product line or solely on the sales history of the product in short supply. In particular, a shortage of one product can lead retailers and consumers to move toward other products in the line. This, in turn, can have an effect on the manufacturer's optimal allocation mechanism. We examine this issue by developing a model of a supplier selling two substitutable goods through two retailers. Within this setup, we introduce a general turn-and-earn allocation rule that allows the entire sales history to influence allocation levels. Counter to previous work, we show that certain turn-and-earn rules not only help the manufacturer but can also help the retailer and increase total supply chain profits.

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1. Introduction

In the spring of 2008, demand for the Toyota Prius far exceeded the available supply—at one point in California there were only 400 Priuses to share among 65 dealers (Hybrid Cars 2008). As it turns out, these types of shortages are not uncommon in the auto market, e.g., the 1998 GMC Suburban, 1998 Volkswagen Beetle, 2000 Honda Odyssey, and 2006 Toyota Prius were all in short supply. More recently, in anticipation of short supply for its redesigned 2014 Corvette Stingray, General Motors announced in April 2013 that less than a third of its dealers would receive an allocation of Corvettes in the fall (Colias 2013). In instances of product shortage, dealers can respond by raising retail price, resorting to a wait list, or shifting consumers to a model that is in stock. Consumers can decide whether to pay a higher price, wait, or buy a substitute product. If manufacturers cannot increase wholesale prices in response to the shortages, they must decide how to allocate the products in short supply across the dealer network. In Toyota's case, how should it decide on the allocation scheme for the Prius, i.e., which dealers get more units of the Prius and which ones get fewer? And should other cars in the line that are not experiencing any shortages influence the allocation scheme for the Prius? In this paper, we focus on sales of product lines and examine a variety of allocation schemes to understand their effect on the supply chain as well as on the manufacturer and dealers.

Having a shortage of a particular product is not an infrequent occurrence in many markets. What is unique about the problem we are posing is that a manufacturer may have plenty of supply for some of its products and a short supply for others. For example, the Nintendo Wii was in short supply long after it was first introduced in November 2006, but there was plenty of supply of the Nintendo Gamecube (Taub 2007); the iPhone 4 was in short supply but the iPhone 3GS was available. In both these instances, the manufacturer needs to consider not only how to allocate the product in short supply but also the possibility that some consumers may shift their demand to another product. For example, some consumers can opt to buy a Toyota Corolla instead of a Prius. In this paper, we show that if the manufacturer fails to incorporate the interplay among items in its product line when choosing an allocation scheme, it can result in lower profits.

In most cases, a market price is an effective and efficient way to match supply and demand. For example, an increase in price during periods of high demand ensures that the market clears. However, in many business situations, there is a fair degree of price stickiness and prices do not change in the short run (Fishman 1992). For example, once car manufacturers announce their wholesale prices to their dealers, these prices are fixed for several months, regardless of what happens in the market place. This inability to vary prices has an important effect when demand exceeds supply and there is a shortage. Because prices do not adjust, businesses need another mechanism to dole out the scarce product. As a result, manufacturers turn to allocation mechanisms that rely on specific rules, or algorithms, to divide the scarce product among the buyers. Note that although the allocation rule is necessary to convert an infeasible set of demands into a feasible set of capacity assignments, it also has the potential to force the downstream players into a game in which they compete for scarce capacity (Lariviere 2011).

Although allocation mechanisms have been used in many markets, we focus our example on the automobile industry, in which a popular allocation mechanism has been through a "turn-and-earn" rule (Lawrence 1996). Essentially, this scheme works by tying the current allocation to the previous sales rate. In other words, if the dealer "turns" over more vehicles in one period, then it "earns" the right to get a higher allocation in the subsequent period. The beauty of this allocation method is that it is straightforward, observable to both parties, and verifiable should conflicts arise.[1] Interestingly, Ford recently announced a turn-and-earn incentive on the F-150 pickup truck: Any dealer that met aggressive sales targets on F-150 sales would be eligible to potentially win an allocation of a special Shelby edition of the Ford Mustang (Wilson 2007). Although turn-and-earn "moves the metal" for the manufacturer, it has been criticized by some dealers because it has a tendency to create long-lasting asymmetries that put smaller dealers at a disadvantage. For example, Lu and Lariviere (2012) find that sales leadership can be an equilibrium outcome that perpetuates an asymmetry that benefits larger dealers.

However, is turn-and-earn the optimal policy from the manufacturer's perspective? Cachon and Lariviere (1999a) address this question by developing a very interesting model that compares a turn-and-earn allocation mechanism with a straightforward mechanism in which the available capacity is split equally among dealers. In the case of a manufacturer selling a single product via two retailers, they show that a turn-and-earn allocation mechanism increases manufacturer profits but decreases profits for the retailers and for the overall supply chain. From the manufacturer's perspective, a turn-and-earn allocation is effective because it gives each retailer an incentive to order extra units during periods when supply is plentiful in the hope that it will get a higher allocation when there is a future shortage. However, there are two important questions that the previous research has not addressed: If consumers can substitute among items in the product line, then how does turn-and-earn affect profitability? Furthermore, when we allow the manufacturer to choose the optimal wholesale price, how does that affect the optimal allocation mechanism? We address both these questions in this paper.

Product shortages have led to the use of allocation schemes in a wide range of industries such as computers (Zarley and Damore 1996), pharmaceuticals (Hwang and Valeriano 1992), and packaged goods (Harrington 1997). Furthermore, shortages can occur at various levels of the supply chain; e.g., a shortage of Intel microprocessors affects PC manufacturers as well as their downstream retailers (McWilliams 2000), whereas a shortage of the Toyota Prius affects dealers directly. In such instances, allocation becomes a central issue as downstream players jockey for greater units of the scarce good. Researchers have studied the role of capacity allocation in a variety of one-period (e.g., Lee et al. 1997) as well as multiperiod settings (e.g., Cachon and Lariviere 1999a). Lee et al. (1997) show that when the manufacturer allocates insufficient capacity relative to retailers' orders, then strategic retailers inflate their orders and further exacerbate the problem, leading to the bullwhip effect. If retailers hedge against a potential shortage by placing multiple orders with multiple suppliers, then potentially we can see an artificial bubble in demand (Goncalves 2002). If suppliers try to control downstream strategic ordering behavior, then Cachon and Lariviere (1999b) show that no truth-inducing allocation mechanism can maximize retailer profits; furthermore, implementing such a mechanism can lower profits for the entire supply chain. Deshpande and Schwartz (2005) generalize this model by considering both pricing and allocation mechanisms. In some instances, the manufacturer can use scarce inventory to shift consumers to an entirely different distribution channel (Geng and Mallik 2007). More recently, Chen et al. (2013) introduce the idea of a lexicographic allocation mechanism when two retailers compete for a limited supply from a manufacturer. A lexicographic allocation mechanism essentially orders retailers by importance. Interestingly, they find that in terms of supplier's profits, the lexicographic mechanism is better than a standard proportional allocation mechanism.

In this paper, we model a two-period world in which a single manufacturer sells a product line consisting of two products. These products are sold through two retailers that are spatially separated

[1] For an overview of capacity allocation, especially the use of turn-and-earn in the automobile industry, see Lariviere (2011).

and do not compete for each other's customers. The degree of substitutability between the products is captured by a parameter that ranges from zero substitutability to perfect substitutability. Demand for one product is always in a steady state, and the manufacturer always has sufficient capacity of it to supply the quantities demanded by its retailers. On the other hand, demand for the other product in any period can be in either a low or a high state, with specified probabilities. When demand is low, the manufacturer has enough capacity to satisfy the retailers' orders, but if demand is high, there is a shortage of capacity and the manufacturer cannot fully satisfy retailers' order quantities.

Given the potential shortage of a product, we explore a variety of turn-and-earn allocation mechanisms that can be used by the manufacturer. In particular, the manufacturer can choose a turn-and-earn allocation based on the sales history of the entire product line, the sales history of the product in potentially short supply, or the sales history of the product with adequate supply. If the manufacturer does not use turn-and-earn, it simply allocates goods through a fixed allocation, where each dealer gets half of the available supply.

Our results provide interesting insights into the optimal allocation rule. Consistent with Cachon and Lariviere (1999a), we find that a fixed allocation rule can be no better than any of the turn-and-earn allocation rules. However, the optimal choice of wholesale price is crucial in determining which form of turn-and-earn to use with a product line. In particular, we find that in some instances, if the manufacturer implements turn-and-earn based on the sales history of the short-supply product only, then it has lower profits than it would achieve if it used an allocation rule based on the sales history of the product line. This occurs because of the retailer's ability to substitute one product for another. Further, although earlier research finds that a turn-and-earn allocation leads to the largest number of units sold, we find that this need not be the case if the manufacturer sells a product line instead of a single product. In this case, we observe that the total number of units ordered under the turn-and-earn allocation rule can be lower than the number of units ordered under a fixed allocation rule. We also find that contrary to the previous research, the retailers do not always earn lower profits when a manufacturer uses a turn-and-earn allocation rule. We demonstrate that in the presence of product line, both the retailers and the manufacturer can prefer a turn-and-earn rule as opposed to a fixed allocation rule.

The remainder of this paper is organized as follows. In §2, we lay out the model and introduce a generalized turn-and-earn allocation rule for the product line. In §3, we solve the model and present some basic results that describe the set of potentially optimal allocation rules. In §4, we compare the allocation mechanisms from both manufacturers' and retailers' perspectives. In §5, we explore the consequences of relaxing some of our assumptions. In §6, we conclude the paper.

2. Model

In this section, we detail the assumptions about the manufacturer, the retailers, consumer demand for the products, and the product-allocation mechanisms available to the manufacturer. The basic assumptions of our analysis mirror Cachon and Lariviere (1999a). We consider a two-period world in which a single manufacturer sells a product line consisting of two products (A and B). These products are partial substitutes, so each product has an influence on the other's demand. For example, consumers may view the Toyota Prius and Toyota Corolla as being similar, thus raising the possibility that if the Prius is not available or if it is too expensive, a consumer can instead buy a Corolla. The manufacturer has a per-period production capacity K_A for product A and K_B for product B. These capacities are fixed and cannot be altered over the two-period horizon. These assumptions are consistent with manufacturing in capital-intensive industries in which capacity tends to be fixed in the short run.

The manufacturer sells its products through two retailers, R_1 and R_2. Each retailer is a local monopolist and does not compete with the other retailer for customers. This would be the case where they are spatially separated (e.g., located in different states such as Ohio and Utah) or if they are assigned exclusive territories (e.g., pharmaceutical representatives). Each retailer has the option to purchase products A and B at wholesale prices w_A and w_B, respectively. These wholesale prices are fixed and do not vary over the two periods of analysis. In each period, the retailers choose the optimal quantity to order based on the wholesale prices and the level of demand. Each retailer's marginal costs are constant and are set to zero without further loss of generality.

2.1. Consumer Demand

Consumer demand for products A and B is represented by the following demand functions in each period:

Period 1	Period 2	
$p_A = \alpha - q_A - sq_B$	$p_A = \sigma - q_A - sq_B,$	(1)
$p_B = \alpha - q_B - sq_A$	$p_B = \alpha - q_B - sq_A,$	(2)

where p_i and q_i are the price and quantity of product i $(i = A, B)$ and $0 \leq s \leq 1$ is a substitutability parameter. If $s = 0$, there is no substitutability and no competition between the two products. On the other hand, if $s=1$, then the two products are perfect substitutes. For example, one can imagine that a Toyota Sienna and a Toyota Corolla will have a low degree of substitutability, whereas the degree of substitutability between Toyota Corolla and Toyota Prius will be higher.

Following Cachon and Lariviere (1999a), there are two demand states—"high" and "low." In the high demand state, the intercept is $\sigma=1$, and in the low state $\sigma=\alpha<1$. Thus, while product B's demand is always in the steady low state, demand for product A is low in period 1, but in period 2, it can be high with probability ϕ or low with probability $(1-\phi)$. As in Cachon and Lariviere (1999a), we assume that capacity K_A is enough to satisfy the optimal order quantities in the low-demand state, but not enough to satisfy the optimal order quantity if demand for product A is high.[2] On the other hand, we assume there is plenty of supply available for product B, which allows us to explore whether it is optimal for the manufacturer to tie sales of product B to the turn-and-earn policy for product A. We note that this assumption of sufficient capacity for product B is not necessary for the results to hold.

For product A, the assumption that demand is low in period 1 but potentially high in period 2 is consistent with demand patterns in several industries, such as automobiles, wine and Broadway shows. Within the automobile industry, demand for SUVs may be low today because of the high price of gasoline but may increase later if the price of gasoline decreases. On the other hand, both wine and Broadway shows are characterized by a fixed capacity—wines of a particular vintage have a fixed supply and the number of seats in the theater is fixed. Within the context of the model, both these industries can have products that face a low demand state in period 1, but if they are "discovered" by critics, the products can end up in a high demand state in period 2. For example, the 2003 Fisher Chardonnay had plenty of availability until it received 94 points from the *Wine Spectator*, after which point demand increased and the wine became hard to find. A similar surge in demand (i.e., a high demand state) can also be seen whenever a reputable source such as *Consumer Reports* gives a seal of approval or a high rating to a product. Finally, this assumption makes the comparison with Cachon and Lariviere (1999a) more straightforward.

2.2. Allocation Rules

The need for an allocation rule arises only if demand is greater than the manufacturer's capacity to supply the product. In particular, this occurs when capacity K_A is enough to fulfill the retailers' orders if demand is low, but not if demand is high.[3] Yet there is always sufficient capacity for product B. Therefore, at the beginning of the game, in addition to announcing the wholesale prices, the manufacturer also has to specify an allocation mechanism or rule that will be used to divide the available units between the two retailers. Importantly, the manufacturer cannot force a retailer to accept more goods than it has ordered, and the retailer is required to pay for all the units it orders.

In our setting, the simplest method of allocating products to the retailers is a *fixed rule*. In this case, each retailer's guaranteed allocation is simply a fixed proportion of capacity. With symmetric retailers, this amounts to a guaranteed allocation of $K_A/2$ such that each retailer can order up to but not exceed $K_A/2$ units. An alternate method of allocating products comes from a *turn-and-earn allocation rule*, where each retailer's guaranteed allocation depends on its prior sales history. Thus, whichever retailer sold more units in the past is identified as the sales leader and gets a correspondingly higher guaranteed allocation in the future. As we show below, the turn-and-earn allocation rule can take a variety of forms.

We now consider a turn-and-earn rule in which each retailer's allocation rule is based on its sales history of the entire product line, i.e., on the sales history of products A *and* B. Let δ^r denote the weighted difference in retailers' first-period sales:

$$\delta^r = \theta_A(q_A^r - q_A^{3-r}) + \theta_B(q_B^r - q_B^{3-r}), \tag{3}$$

where q_i^r is the quantity of product i ($i=A,B$) ordered in the first period by retailer R_r ($r=1,2$) and $0\le\theta_i\le 1$ are the product line turn-and-earn parameters that capture the extent to which the allocation of product A depends on the prior history of product i. Retailer R_r is defined as a sales leader if $\delta^r>0$ and is considered a sales laggard otherwise. Under a turn-and-earn allocation, the sales leader R_r is guaranteed the first δ^r units of capacity for product A, and the remaining capacity of $(K_A-\delta^r)$ is split equally between the retailers. Thus, the sales leader's guaranteed allocation of product A is $\delta^r+(K_A-\delta^r)/2$, and the sales laggard's guaranteed allocation is $(K_A-\delta^r)/2$. By substituting Equation (3) for δ^r above, we can express each retailer's maximum guaranteed allocation, G_A^r, as follows:

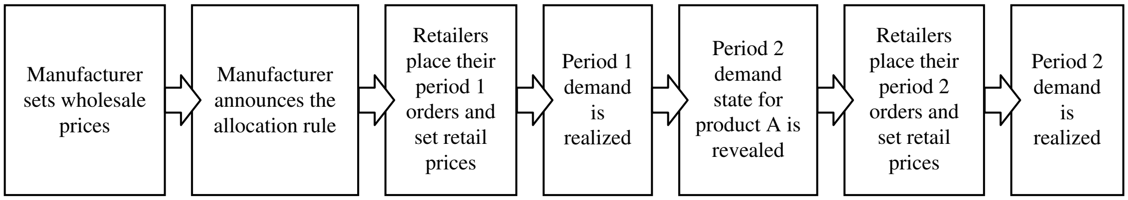
$$G_A^r = \frac{K_A}{2} + \theta_A\frac{q_A^r - q_A^{3-r}}{2} + \theta_B\frac{q_B^r - q_B^{3-r}}{2}. \tag{4}$$

Note that if $\theta_A=\theta_B=0$, the guaranteed allocation is equivalent to a fixed allocation rule, $G_A^r=K_A/2$.

[2] In §5.2, we consider the consequences of endogenizing capacity.

[3] Specifically, when $K_A > \alpha(2-\phi)/((1+s)(4-\phi)) = K_{\min}^{\mathrm{FX}}$ and $K_A < (4-\alpha(2(1+s)-\phi)-\phi)/(4(1-s^2)) = K^{\mathrm{NS}}$.

Figure 1 **Sequence of Events**



On the other hand, when θ_A, $\theta_B > 0$, then it is clear that if one retailer orders more units than the other, it guarantees itself a higher allocation in the future.

2.3. Sequence of Events

At the beginning of the game, the manufacturer chooses the turn-and-earn parameters θ_A and θ_B and the wholesale prices w_A and w_B for each product. Neither the wholesale prices nor the allocation rule changes over the two-period horizon. Subsequently, at the beginning of each period, the retailers observe the demand state for each product and then submit their orders simultaneously. If the sum of the orders for a particular product is lower than the capacity, the manufacturer fulfills those orders. If capacity is insufficient, the manufacturer uses all its capacity by allocating units based on the mechanism announced at the start of the game. Once retailers receive their orders, they choose an optimal price at which to sell their units. We assume that holding costs are sufficiently high so retailers do not hold any inventory. Furthermore, there is a zero salvage value for any unsold units, so the retailers have an incentive to sell all the units they have ordered. Figure 1 depicts the sequence of events in this model.

3. Analysis

In solving this model, we use the concept of subgame perfection and use backward induction to solve for the optimal order quantities. Thus, we first analyze the subgame in period 2 and then solve the problem in period 1.

3.1. Period 2 Analysis

In period 2, each retailer maximizes its profits by choosing the optimal quantities of each good to order. Because this is the last period, the retailers, regardless of the allocation rule, do not have an incentive to sell more than the quantity that maximizes their second-period profit. If demand for product A is high, retailer R_r orders and sells:

$$q_A^{rH} = \min\left\{G_A^r, \frac{1-w_A-s(\alpha-w_B)}{2(1-s^2)}\right\}, \qquad q_B^{rH} = \frac{\alpha-w_B}{2} - sq_A^{rH}, \tag{5}$$

where G_A^r is a guaranteed allocation for retailer R_r as defined by Equation (4). In this case, if the manufacturer adopts a turn-and-earn allocation policy with $\theta_i > 0$, then one retailer can potentially gain an advantage over the other by ordering more than the other retailer in the first period. In this case, it is clear that by ordering more in period 1, when the demand is low, the retailer can potentially get a better allocation in period 2 if demand turns out to be high and the manufacturer has insufficient capacity to meet all its orders.

If demand for product A is low, each retailer orders and sells

$$q_A^L = \frac{\alpha-w_A-s(\alpha-w_B)}{2(1-s^2)}, \qquad q_B^L = \frac{\alpha-w_B-s(\alpha-w_A)}{2(1-s^2)}. \tag{6}$$

In this case, there is no shortage and hence no need for an allocation rule.

3.2. Period 1 Analysis

In period 1, each retailer maximizes its expected profits over the two-period horizon. Note that retailer R_r's expected second-period profit is

$$\pi_2^r = (1-\phi)\begin{bmatrix}(\alpha-q_A^L-sq_B^L-w_A)q_A^L \\ +(\alpha-q_B^L-sq_A^L-w_B)q_B^L\end{bmatrix} + \phi\begin{bmatrix}(1-q_A^{rH}-sq_B^{rH}-w_A)q_A^{rH} \\ +(\alpha-q_B^{rH}-sq_A^{rH}-w_B)q_B^{rH}\end{bmatrix}. \tag{7}$$

As noted earlier, in period 2 the probability of scarcity of capacity for product A is ϕ. Thus, with probability $(1-\phi)$ there is no scarcity and each retailer receives its optimal allocation of product A, q_A^L. The latter case is reflected in the first part of Equation (7), when the retailer receives its optimal allocation for both products. When scarcity occurs, the second-period order quantities depend on the first-period order quantities q_A^r and q_B^r. These order quantities affect the maximum guaranteed allocation G_A^r which comes into the second part of Equation (7) through order quantities q_j^{rH}.

Given the expected profits in period 2, in the first period, each retailer R_r maximizes the overall profit, given below by choosing the order quantities q_A^r and q_B^r:

$$\pi^r = (\alpha - q_A^r - sq_B^r - w_A)q_A^r + (\alpha - q_B^r - sq_A^r - w_B)q_B^r + \pi_2^r. \quad (8)$$

By solving the first-order conditions and using the fact that retailers are symmetric, we obtain the first-period optimal order quantities for each retailer:

$$q_A^* = \begin{cases} q_A^L + \dfrac{\phi(\theta_A - s\theta_B)(1-K_A(1-s^2)-w_A-s(\alpha-w_B))}{4(1-s^2)} \\ \quad \text{if } \dfrac{K_A}{2} \le \dfrac{(1-w_A-s(\alpha-w_B))}{2(1-s^2)}, \\ \dfrac{\alpha - w_A - s(\alpha-w_B)}{2(1-s^2)} \quad \text{otherwise;} \end{cases}$$
$$q_B^* = \begin{cases} q_B^L + \dfrac{\phi(\theta_B - s\theta_A)(1-K_A(1-s^2)-w_A-s(\alpha-w_B))}{4(1-s^2)} \\ \quad \text{if } \dfrac{K_A}{2} \le \dfrac{(1-w_A-s(\alpha-w_B))}{2(1-s^2)} \ge 0, \\ \dfrac{\alpha - w_B - s(\alpha-w_A)}{2(1-s^2)} \quad \text{otherwise.} \end{cases} \quad (9)$$

Given the retailers' optimal order quantities in period 1, we can now solve the manufacturer's optimization problem of choosing the optimal allocation rule and wholesale prices. First, the manufacturer maximizes profits over the two-period horizon by choosing the optimal allocation mechanism, which amounts to choosing the optimal θ_A and θ_B:

$$\max_{\theta_A,\,\theta_B} \Pi_M = 2w_A[q_A^* + (1-\phi)q_A^L + \phi q_A^{rH}] + 2w_B[q_B^* + (1-\phi)q_B^L + \phi q_B^{rH}]. \quad (10)$$

Substituting expressions (7) and (9) into the manufacturer's profit function (10) yields the following expression for the manufacturer's profit over two periods:

$$\Pi_M = \begin{pmatrix} [\phi(\theta_A(w_A - sw_B) + \theta_B(w_B - sw_A)) \\ \cdot(1-K_A(1-s^2)-w\alpha - s(\alpha-w_B))]\cdot(4(1-s^2))^{-1} \\ -[\phi(w_A-sw_B)(\alpha(1-s)-(w_A-sw_B) \\ -K_A(1-s^2))]\cdot(2(1-s^2))^{-1} \\ -\dfrac{(w_A^2-2sw_Aw_B+w_B^2-(1-s)(w_A+w_B)\alpha)}{(1-s^2)} \end{pmatrix}, \quad \text{if } \frac{K_A}{2} \le \frac{(1-w_A-s(\alpha-w_B))}{2(1-s^2)};$$
$$= [\phi(w_A - sw_B)(1-\alpha) - 2(w_A^2 - 2sw_Aw_B + w_B^2 - (1-s)(w_A+w_B)\alpha)]\cdot(2(1-s^2))^{-1}, \quad \text{otherwise.}$$

Note that because the profit function is linear in both θ_A and θ_B, the solutions to the profit maximization problem with respect to θ_i will always be at a corner. That is, the manufacturer would choose either $\theta_i = 0$ or $\theta_i = 1$. Thus, in equilibrium, there can be only four specific allocation rules:

1. *Fixed Allocation* (FX). In this case $\theta_A = 0$ and $\theta_B = 0$ and the guaranteed allocation for retailer R_r is given by $G_A^r(\text{FX}) = K_A/2$.
2. *Single Product A Allocation* (SPA). In this case $\theta_A = 1$ and $\theta_B = 0$, and the allocation is based on the prior sales history of product A such that the guaranteed allocation for retailer R_r is given by $G_A^r(\text{SPA}) = K_A/2 + (q_A^r - q_A^{2-r})/2$. This is equivalent to the turn-and-earn rule developed in Cachon and Lariviere (1999a).
3. *Single Product B Allocation* (SPB). In this case $\theta_A = 0$ and $\theta_B = 1$, and the allocation is based on the prior sales history of product B such that the guaranteed allocation for retailer R_r is given by $G_A^r(\text{SPB}) = K_A/2 + (q_B^r - q_B^{2-r})/2$.
4. *Product Line Allocation* (PL). In this case $\theta_A = 1$ and $\theta_B = 1$, and allocation is based on the prior sales history of both products A and B such that the guaranteed allocation for retailer R_r is given by $G_A^r(\text{PL}) = K_A/2 + (q_A^r - q_A^{2-r})/2 + (q_B^r - q_B^{2-r})/2$.

The final step in solving the manufacturer's profit maximization problem is to choose the optimal wholesale price for each of the four allocation rules. Table 1 in the electronic companion (available at http://business.rice.edu/uploadedFiles/Faculty_and_Research/Academic_Areas/Marketing/T-a-E_in_a_PL-Tables.pdf) summarizes optimal wholesale prices and order quantities along with their corresponding profits. In the next section, we identify the necessary conditions for a specific allocation mechanism to be optimal and analyze the effect of allocation rules on the manufacturer's, retailers', and supply chain profits.

4. Results

In this section, we consider the manufacturer's optimal allocation rule and highlight its effect on profits. Subsequently, we discuss the implications for the retailer and the supply chain.

4.1. Optimal Allocation Rule

As a preliminary step in determining the optimal rule, we have the following lemma:

LEMMA 1. *The SPB turn-and-earn allocation mechanism is always weakly dominated by one of the other three allocation mechanisms and hence is never optimal for the manufacturer.*

Lemma 1 shows that the manufacturer cannot benefit solely from using the product in plentiful supply as an inducement toward a better allocation of the scarce product. As a result, we turn our attention to the other three mechanisms: FX, SPA, and PL.

It is straightforward to see that an allocation mechanism kicks in only when capacity is lower than the demand. From the manufacturer's perspective, the optimal allocation mechanism will vary depending on the capacity, demand, and potential shortage in the market. To understand why this is the case, consider the simplest case, where there is sufficient capacity for all demand states. This leads to the following lemma:

Lemma 2. *If capacity is sufficiently high, $K_A > K^{NS} = (4-\alpha(2(1+s)-\phi)-\phi)/(4(1-s^2))$, there is no capacity shortage in any demand state; hence, the choice of allocation rule does not matter: fixed and both turn-and-earn (SPA and PL) rules lead to the same profits, order quantities, and prices for the manufacturer.*

Therefore, if capacity is sufficiently plentiful, the allocation rule does not matter and the manufacturer can choose the optimal wholesale prices to charge without any capacity constraints. It is important to note that the capacity needed for product A before it can be considered plentiful also depends on the other parameters. In particular, from the expression for K^{NS} in Lemma 2, we see that it is affected by α, the level of demand in the low state, by ϕ, the probability of the high state, and by s, the degree of demand substitutability between products A and B. It is straightforward to see that as α increases, K^{NS} decreases. This occurs because as α increases, there is less of a difference between the base demand in the low and high states; hence, the maximum capacity needed for product A decreases. Similarly, as ϕ increases, it makes the high demand state more likely; i.e., it is more likely that there will be a difference in demand between the two products, hence K^{NS} decreases.

Finally, it is important to note that because the products can weakly substitute for each other in terms of consumer demand, they can substitute for each other in terms of capacity, which can then affect the optimal levels of capacity. However, this substitutability in terms of capacities is different from flexibility of capacity. Substitution of capacities arises from demand substitution but also involves a complex effect through optimal retail and wholesale prices. Therefore, the effect of s on maximum capacity is less straightforward. In particular, K^{NS} tends to decrease with s for lower levels of s, but increases with s for higher levels of s. Intuitively, this suggests that capacities can substitute for each other when the level of demand substitutability is above a threshold.

Earlier research has established that if there is a shortage of capacity such that K_A is not enough to fulfill the retailers' orders in the high-demand state, then compared with a fixed allocation rule, a turn-and earn allocation always results in higher profits for the manufacturer. The intuition behind this result is straightforward: compared with the quantity ordered under a fixed allocation rule, under a turn-and-earn allocation mechanism, each retailer has an incentive to order more in the first period, when demand is low. As a result, the total sales, capacity utilization, and manufacturer profits over the two periods are higher under the turn-and-earn allocation rule. This turn-and-earn mechanism works because to "earn" the additional guaranteed allocation in the future, the retailer needs to "turn" a higher quantity in the present. Furthermore, the competition among retailers for potentially scarce capacity in the future compels both of them to order higher quantities in the present. An important assumption underlying this result is that when demand is high, the retailers do not have substitute products that could make up for some of the shortfall. In particular, if the retailer had a product line with one product in plentiful supply, would we still expect the turn-and-earn mechanism to work in the same manner and yield the same result? This leads to the following result:

Proposition 1. *In comparing manufacturer profits across the allocation mechanisms:*

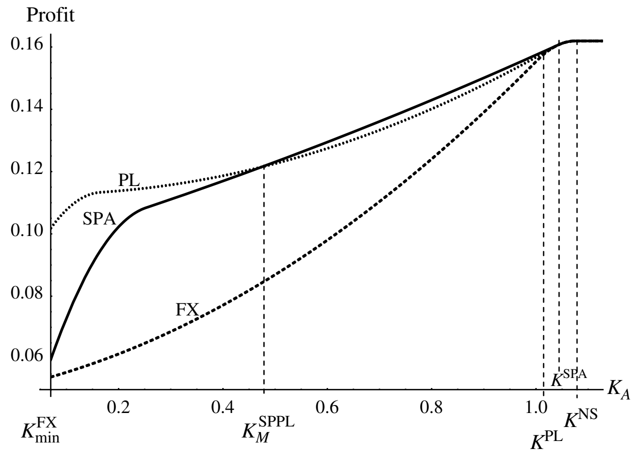
1. *PL turn-and-earn results in highest profits when $\alpha > \alpha^* = 4s\phi/(8-4s(2-\phi)-\phi+s^2\phi)$ or when $\alpha < \alpha^*$ and $K_A < K_M^{SPPL}$.*[4]
2. *SPA turn-and-earn results in highest profits when $\alpha < \alpha^*$ and $K_A \in (K_M^{SPPL}, K^{SPA})$.*
3. *FX profits are always lower than or equal to both SPA and PL profits. Specifically, SP results in strictly higher profits than FX when $K_A < K^{SPA}$, and PL results in strictly higher profits than FX when $K_A < K^{PL}$.*

Proposition 1 establishes that the fixed allocation rule is dominated by both the turn-and-earn allocation rules. However, depending on the parameters, either the SPA or the PL turn-and-earn allocation rule may be optimal. In particular, note that if $\alpha > \alpha^*$, then the PL rule is always optimal. This threshold level α^* is affected by both the level of substitutability and the probability that demand will be in the high state in period 2. Note that when $s = 0$, then $\alpha^* = 0$ and PL is the optimal allocation rule. In other words, even when the products are not substitutes, the manufacturer is always better off basing the allocation rule on the sales history of the entire product line than on the past sales of a single product. In addition, α^* increases with both s and ϕ. This suggests that as either s or ϕ increases, it is less likely that a PL rule will be optimal. Importantly, Proposition 1 also establishes that even if the manufacturer sells a product line, that does not mean that a product line turn-and-earn allocation rule is always preferable to a single product allocation rule.

In Figure 2, we present a simple numerical example to highlight the essential results of Proposition 1.

[4] The closed-form expression of K_M^{SPPL} is derived in the proof of Proposition 1 in the appendix.

Figure 2 Manufacturer's Profit Under Different Allocation Rules ($s = 0.65$, $\alpha = 0.3$, $\phi = 0.75$)



In particular, we plot the manufacturer profits as a function of product A's capacity. Note that at higher capacity levels, the potential shortage of product A in period 2 is less, such that there is less of an opportunity cost for getting a suboptimal allocation. To develop the intuition behind these results, we compare each turn-and-earn allocation rule with the fixed allocation rule.

4.1.1. Comparing FX and SPA. From Table 1 in the electronic companion and Proposition 1, we see that SPA turn-and-earn is more profitable than FX only when the capacity is low enough ($K_A < K^{\mathrm{SPA}} = (8 - \alpha(s(4-\phi) + 2(2-\phi)) - 3\phi)/((1-s^2)(8-\phi))$); if not, both FX and SPA allocation rules result in the same profits. When the retailer sells a single product A, then under an SPA turn-and-earn allocation rule, it increases its purchases of product A in period 1. However, when it sells a product line consisting of products A and B, the higher quantities of product A, $q_A^{\mathrm{FX}} < q_A^{\mathrm{SPA}}$, are coupled with a lower sales quantity of product B, $q_B^{\mathrm{FX}} > q_B^{\mathrm{SPA}}$. The difference in profits between the FX and SPA rules arises because of the interplay between wholesale prices and quantities. With an SPA rule, even though the retailer can partially substitute one product for the other, the manufacturer still benefits from a turn-and-earn allocation because it can choose the wholesale prices optimally.[5] Note that the wholesale price for product B is the same under SPA as it is under FX (see Table 1 in the electronic companion). Because the order quantity of product B is lower under the SPA rule, the manufacturer clearly makes lower profits on sales of product B by switching from FX to SPA. The only way for the manufacturer to make up for the "loss" associated with product B is if it can earn higher profits with the additional unit sales of product A. Notice that the increase in order quantity imposed by the SPA allocation rule, $q_A^{\mathrm{SPA}} - q_A^{\mathrm{FX}}$, is decreasing in capacity K_A. This implies that the smaller the potential shortfall between demand in the high state and the capacity, the smaller is the increase in unit sales imposed by the SPA allocation rule. Intuitively, with the turn-and-earn allocation rule, the larger the available capacity, the smaller is the incentive that retailers have to increase their first-period order quantity. Further, it can be shown that the wholesale price for product A is lower under the SPA allocation rule than under FX when

$$K_A > K^{\mathrm{SPA}}_{w_A} = \frac{(2-\phi)(2(8-\phi-3s\phi)-\alpha(24-\phi-s^2\phi-s(8+6\phi)))}{(1-s^2)(32-4(1+5s)\phi+(1+s)^2\phi^2)},$$
$$\text{and} \quad K^{\mathrm{SPA}}_{w_A} < K^{\mathrm{SPA}}.$$

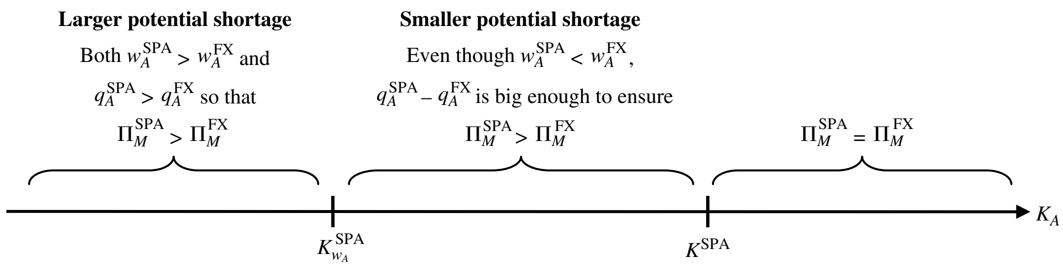
Because a fixed allocation rule does not induce the retailer to order higher quantities of product A in period 1, it should be clear that the supplier's demand function for product A is more elastic with respect to wholesale price w_A under the SPA allocation than under fixed allocation. Consequently, when the potential shortfall between available demand in the high state and the capacity is low (i.e., $K^{\mathrm{SPA}}_{w_A} < K_A < K^{\mathrm{SPA}}$), the wholesale price decreases under the SPA allocation rule and the retailer orders more units of product A and, because of substitution, fewer units of product B. In this case, the potential shortage is still high enough to ensure that the increase in order quantities will be enough to offset both the lower wholesale price and the loss of profit associated with lower sales of product B. Interestingly, when the available capacity is very small, $K_A < K^{\mathrm{SPA}}_{w_A}$, the opportunity cost of the potential shortage is so high that the manufacturer can increase the wholesale price under the SPA rule and still induce higher order quantities of product A. These results are summarized in Figure 3.

4.1.2. Comparing FX and PL Allocation Rules. A similar story plays out when we consider the difference between an FX allocation and the PL turn-and-earn allocation. We see that PL turn-and-earn is more profitable only when the capacity K_A is low enough ($K_A < K^{\mathrm{SPA}}$), which means that the shortage of capacity in the high demand state, or the opportunity cost for the retailer, is large. If the opportunity cost is low, i.e., the capacity shortage is not that large, then the allocation rules are equivalent.

The effect on quantities is similar to the case with SPA, but there is an important difference: Compared with a fixed allocation, a PL turn-and-earn allocation

[5] If wholesale prices were exogenous to the model, then for some wholesale prices (specifically, when $w_B < sw_A$), a turn-and-earn allocation rule based on the sales history of the product in short supply (SPA) leads to lower manufacturer profits because of partial product substitution.

Figure 3 **Comparison of Manufacturer Profits and Wholesale Prices Under SPA and FX Allocation Rules for Different Values of Capacity,** K_A



leads to a higher quantity of product A ordered in period 1; however, the quantity ordered for product B can be higher or lower: $q_A^{\mathrm{PL}} > q_A^{\mathrm{FX}}$ and $q_B^{\mathrm{PL}} <> q_B^{\mathrm{FX}}$. This highlights an important effect of a PL turn-and-earn policy: compared with the FX allocation rule, when

$$K_A < [(2-\phi)(2(24-(9+s)\phi)+\alpha(3s^2\phi-2s(20-\phi) -3(8-5\phi)))]\cdot[(1-s^2)(96-4(15-s)\phi +(6+5s-s^2)\phi^2)]^{-1},$$

the units ordered for both products increase. This occurs because the manufacturer can use sales of both products A and B in period 1 to count toward a more favorable allocation of product A in period 2. In this instance, we do not see the substitution outlined earlier. However, an important difference now is that under certain conditions, the total units ordered in period 1 can be *lower* under a product line turn-and-earn. This is important because it highlights that, contrary to earlier research, a turn-and-earn policy does not always lead to higher order quantities. In particular, if the turn-and-earn policy is based on sales of other items in the line, then it can actually lead to a *decrease* in the total quantities. Interestingly, in spite of this lower level of unit sales, the manufacturer still earns higher profits with a PL allocation rule.

To better understand the added flexibility of turn-and-earn, consider the implications of our allocation mechanisms. A fixed allocation simply splits capacity equally among the retailers so there is no strategic advantage one retailer can gain over the other. As a result, in period 1, the retailer simply orders the optimal quantity for that period. On the other hand, a turn-and-earn allocation mechanism induces the retailer to order more units and move away from the "optimal." It is important to remember that in equilibrium, both retailers order additional units; thus, the additional units ordered in period 1 simply ensure that the retailer is not put at a disadvantage. Finally, note that the variance for the manufacturer's profit comes not only from the uncertainty about the high demand state ϕ but also from the demand intercept α. Indeed, as α increases, the difference in demand between high and low states decreases; hence, the potential difference in profits is smaller and there is less potential for a shortage. Furthermore, the cutoff points K^{SP} and K^{PL} are linear and decreasing in α such that an increase in α decreases the region where a turn-and-earn allocation rules brings strictly higher profits than fixed allocation.

4.2. Effect on the Retailers and on the Supply Chain

When the retailer sells a single product, a turn-and-earn allocation increases the manufacturer's profits but decreases the retailer's profits (Cachon and Lariviere 1999a). In other words, the manufacturer prefers turn-and-earn but the retailer prefers fixed allocation, thus leading to an ambiguous effect for the supply chain. Now we consider the impact of a turn-and-earn allocation rule in the presence of product line on retailers' and the supply chain's profits. In looking at allocation mechanisms within the context of a product line, the story changes considerably from a single product case. This leads to the following proposition:

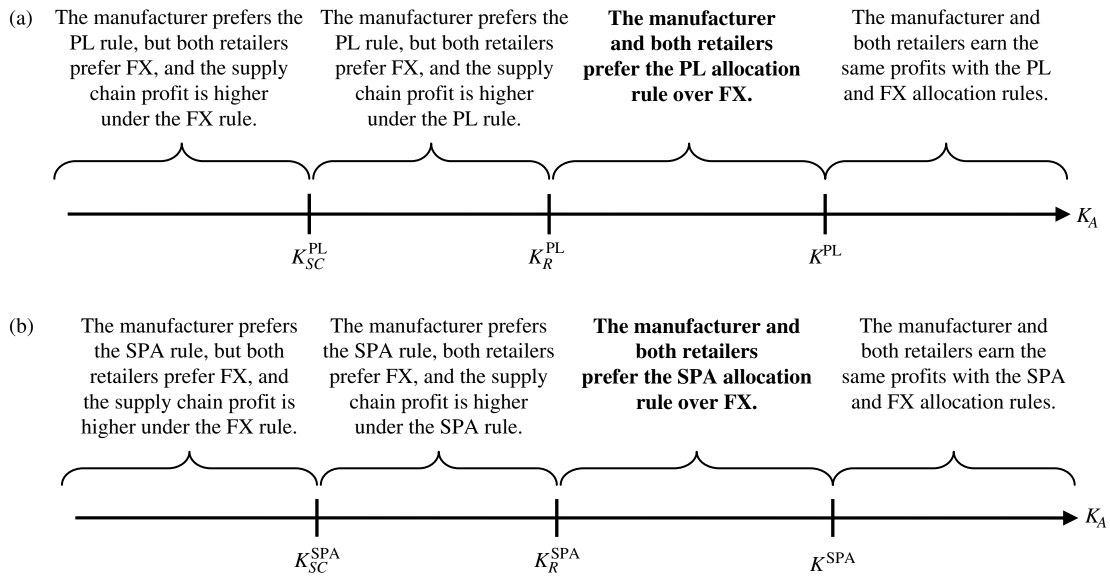
Proposition 2. (a) *Retailers prefer PL over FX allocation when* $K_A > K_R^{\mathrm{PL}}$. *Moreover,* $K_R^{\mathrm{PL}} < K^{\mathrm{PL}}$, *so for* $K_A \in [K_R^{\mathrm{PL}}, K^{\mathrm{PL}}]$, *both retailers and the manufacturer prefer PL over the FX allocation rule.*

(b) *Retailers prefer SPA over FX allocation when* $K_A > K_R^{\mathrm{SPA}}$. *Moreover,* $K_R^{\mathrm{SPA}} < K^{\mathrm{SPA}}$, *so for* $K_A \in [K_R^{\mathrm{SPA}}, K^{\mathrm{SPA}}]$, *both retailers and the manufacturer prefer SPA over the FX allocation rule.*[6]

A single product (SPA) turn-and-earn confers an advantage to a retailer if it orders additional units of product A in period 1. However, by ordering more units of product A, because of the substitution effect, the retailer orders fewer units of product B and thus can, at least partially, compensate for the larger order quantity of product A. This substitution is possible only if the retailer sells a line of products. In a PL turn-and-earn, the strategic advantage to a retailer comes from an ability to order larger quantities of either product. In particular, the retailer orders additional units of product A and, in certain cases, additional units of product B as well. Thus, the retailer has more

[6] Closed-form expressions of K_R^{PL} and K_R^{SPA} are provided in the proof of Proposition 2 in the appendix.

Figure 4 Manufacturer, Retailer, and Supply Chain Preferences for (a) PL vs. FX and (b) SPA vs. FX



flexibility when the allocation mechanism is based on the sales history of the whole product line.

Because of the computational complexity of the expressions, we are unable to compare the retailer's profits under SPA and PL turn-and-earn allocation rules. However, it is important to note that when the retailer sells a product line, for $K_A > \max[K_R^{SPA}, K_R^{PL}]$ one of the turn-and-earn mechanisms is preferable to a fixed allocation rule. In other words, a fixed allocation rule is not a dominant choice for the retailers. However, if the retailer does not sell a product line, then the retailers would always prefer a fixed allocation. It is important to note that Proposition 2 also implies that there are instances where both the manufacturer and the retailers prefer the same allocation rule. This result is in stark contrast to the finding in the earlier research that retailers' and the manufacturer's preferences for an allocation rule are never aligned.

Finally, we note the effect of our allocation rules on the profits of the supply chain. As should be evident from our earlier discussion, when the manufacturer and the retailers prefer the same allocation rule, supply chain profits are highest for that particular allocation mechanism. This leads to the following proposition:

Proposition 3. *When the retailer sells a product line, then the effect on supply chain profits is as follows*:

(a) *The supply chain profit is higher under PL than under FX allocation when $K_A > K_{SC}^{PL}$. Because $K_{SC}^{PL} < K_R^{PL} < K^{PL}$, when $K_A \in (K_{SC}^{PL}, K^{PL})$, the manufacturer and the supply chain as a whole are better off with PL than FX allocation.*

(b) *The supply chain profit is higher under SPA allocation than under FX when $K_A > K_{SC}^{SPA}$. Because $K_{SC}^{SPA} < K_R^{SPA} < K^{SPA}$, when $K_A \in (K_{SC}^{SPA}, K^{SPA})$, the manufacturer and the supply chain as a whole are better off with SPA than FX allocation.*[7]

In summary, Figure 4 illustrates the alignment of preferences for a turn-and-earn versus fixed allocations.

5. Model Extensions

In setting up and analyzing the model, we made several simplifying assumptions. In this section, we briefly explore the implications of relaxing some of the main assumptions. In particular, we look at the following embellishments: allowing wholesale prices to vary over time and endogenizing the choice of capacity.

5.1. Flexible Wholesale Pricing

The need for an allocation mechanism arises when demand is greater than the capacity available to the manufacturer and prices cannot be adjusted to account for the shortage of supply. If capacity were infinitely flexible or if prices could be adjusted, then there would be no need for an explicit allocation mechanism. As we have shown, the manufacturer always prefers some form of a turn-and-earn allocation rule. Yet this is not the case for the retailer, which may prefer a fixed allocation rule.

In many business markets, manufacturer prices are sticky and tend to remain fixed over time, at least in the short run, e.g., automobiles, computers, pharmaceuticals, etc. In spite of these fixed prices, an interesting question to consider is the following: If the manufacturer could vary wholesale prices in each

[7] Closed-form expressions of K_{SC}^{PL} and K_{SC}^{SPA} are provided in the proof of Proposition 3 in the appendix.

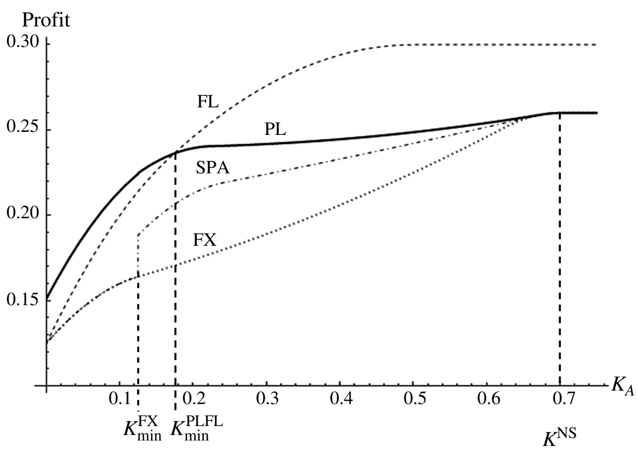
period, would that be more profitable than using a turn-and-earn policy with constant prices over time? Allowing for wholesale prices that can vary over time and clear demand in any state is usually the efficient solution. However, the turn-and-earn rule affords an element that is not available to the flexible price solution. In particular, even though the turn-and-earn rule has fixed prices, its requirements can induce retailers to order additional units. The question is whether these additional sales can compensate for the lack of flexibility in prices.

We modify the basic model by assuming that wholesale prices vary across both periods. In particular, at the beginning of each period, the manufacturer observes the demand state and chooses the wholesale prices for each product. Retailers choose the optimal number of units to order based on the market demand and the wholesale price. Because flexible pricing leads to an optimal wholesale price that clears demand, there is no need for an allocation mechanism. The rest of the game is solved in the manner outlined earlier. This leads to the following:

Proposition 4. *The manufacturer earns higher profits with flexible pricing than it does with either fixed or SPA turn-and-earn allocation rules. However, there exists K^{PLFL} such that when capacity is $K_A \in (0, K^{\mathrm{PLFL}})$ a PL turn-and-earn allocation rule leads to higher profits for the manufacturer than flexible pricing.*

Figure 5 illustrates Proposition 4. Interestingly, flexible prices are always preferred to fixed and SPA allocation rules. To understand why the fixed allocation rule dominates, note that flexible pricing can do no worse than fixed prices, because flexible prices can always replicate the fixed prices. When it comes to the SPA rule, the manufacturer gains some advantage by selling additional units of the product that may potentially be in short supply; however, the sales of

Figure 5 **Manufacturer's Profit Under Different Allocation Rules Compared to Flexible Pricing ($s = 0.5$, $\alpha = 0.5$, $\phi = 0.8$)**



the other product go down, so in comparison it is better to adjust wholesale prices for the product in potentially short supply and not worry about allocation.

The more interesting result of our analysis is that a PL turn-and-earn rule can induce sufficiently higher sales to make it a more profitable policy than the "efficient" flexible pricing policy. As detailed in the proof of Proposition 4, PL does better than flexible pricing only when there is not enough product to meet demand in the low state (very tight capacity, $K_A < K^{\mathrm{FX}}_{\min}$) or in certain ranges of moderately tight capacity ($K_A \in (K^{\mathrm{FX}}_{\min}, K^{\mathrm{PL}}_{\min})$); i.e., the capacity is insufficient to meet the additional orders induced by PL turn-and-earn. In these instances of tight capacity, an allocation rule based solely on the sales of product in short supply is not going to make a difference compared with fixed allocation, because an SPA allocation rule cannot induce higher order quantities. In contrast, because the PL allocation rule also takes into account the sales history of product B, retailers can order more of product B, to secure a more favorable future allocation of product A. It turns out this benefit from the PL rule is especially salient when capacity is sufficiently constrained; when it is not, flexible pricing is the best option for the manufacturer.

5.2. Endogenous Capacity

An important assumption of our analysis is that the choice of capacity is exogenous. Although this simplifying assumption helps us in terms of making the model tractable, it is clear that given the uncertain nature of demand, the manufacturer's optimal choice of capacity could also be affected. Below, we explore the implications of allowing the manufacturer to choose capacity for the product that may potentially be in short supply. In particular, the question we address is whether a fixed allocation can do worse than turn-and-earn when capacities are chosen optimally. A complete analysis with endogenous capacity is not possible, but our approach highlights some important results and shows that the main results of our analysis still hold.

To simplify the case, we consider only the capacity choice for product A, which may be in short supply. Recall that the manufacturer always has sufficient capacity for product B. The game is played as outlined in §3; the only change is that with each allocation rule, there is a corresponding optimal choice of capacity. This is highlighted in the following lemma.

Lemma 3. (1) *Under fixed allocation, one of the following three capacity choices maximizes the manufacturer's profits*: $(0, K^{\mathrm{FX}}_{\min}, K^{\mathrm{NS}})$.

(2) *Under SPA allocation, one of the following four capacity choices maximizes the manufacturer's profits*: $(0, K^{\mathrm{FX}}_{\min}, K^{\mathrm{NS}}, K^{\mathrm{SPA}}_{\min})$.

(3) *Under PL allocation, one of the following four capacity choices maximizes the manufacturer's profits*: $(0, K^{\mathrm{FX}}_{\min}, K^{\mathrm{NS}}, K^{\mathrm{PL}}_{\min})$.[8]

From Lemma 3, we see that each allocation rule has a capacity strategy and that the first three capacity choices are identical for the three allocation rules. The first three choices correspond to build no capacity; build enough to cover orders in the low demand state; and build enough to cover orders in the high demand state. The fourth capacity choice in SPA and PL allocations corresponds to building just enough capacity to account for the increased period 1 demand that comes from the particular allocation rule.

To get a sense of how optimal capacity affects profits, note that the first three capacity levels lead to the same level of profits for the FX and SPA allocation rules. This is to be expected, because at these capacity levels, both a fixed rule and an SP rule will lead to the same level of sales of product A. On the other hand, at these capacity levels, the PL rule results in the highest profits when capacity is just enough to cover demand in the low state. This occurs because the PL rule leads to higher sales of product B, as the retailers can use turn-and-earn based on the sales history of the entire product line.

Earlier research has shown that when capacity is fixed, then the reason turn-and-earn works is that it encourages the retailers to order higher quantities in the low demand state (Cachon and Lariviere 1999a). When we expand the product offerings to include substitutable products, the retailer has another degree of freedom to manage potential shortages. However, the ability to substitute products also implies that the capacity required to meet the potentially high demand for product A can also be lower. As a result, if capacities can be chosen optimally, then the SPA or PL turn-and-earn rule should always do better than the fixed allocation rule.

6. Conclusion

When manufacturers have products in short supply, then basing allocations on sales histories through turn-and-earn can be an attractive allocation mechanism. In particular, not only is the sales history readily observable and verifiable, the allocation policy is easy to understand and implement. Indeed, allocation rules are used extensively in many industries such as automobiles, microprocessors, pharmaceuticals, video game consoles, etc. In this paper, we delve deeper into the allocation mechanism by focusing on the product line issues associated with a turn-and-earn policy.

The interaction between a product line and a turn-and-earn allocation mechanism raises an interesting problem for manufacturers: Should the allocation rule depend solely on the sales history of the product that is in short supply, or should it depend on the prior sales of all the items in the line? The answer to this question is not readily apparent. The seminal work in this area is a model developed in Cachon and Lariviere (1999a), in which they showed that a turn-and-earn policy is always more profitable than a fixed allocation policy. However, that model looked at the case of a manufacturer that produces a single product and did not model the manufacturer's choice of optimal wholesale prices. Thus, it does not address the issue of a manufacturer selling a product line. Our paper extends this prior work and develops some new insights.

We develop a general allocation rule that encompasses a fixed allocation as well as a continuum of turn-and-earn mechanisms. We show that there are three main allocation rules: fixed, single product, and product line. A fixed rule splits capacity equally among the retailers; a single product turn-and-earn allocation rule is based solely on the product in potentially short supply; and a product line turn-and-earn is based on the sales history of the entire product line. Consistent with earlier research, we find that a turn-and-earn rule is always the optimal choice for the manufacturer. However, we show that the presence of a product line affects the form of the optimal turn-and-earn rule, i.e., whether it should be based on a single product or on a product line. In addition, when the manufacturer sells a product line, the effect on order quantities is more complicated. In particular, compared with a fixed allocation mechanism, a single product turn-and-earn policy leads to an increase in sales of the product that can potentially be in short supply, but it leads to a decrease in sales of the product in plentiful supply. Hence, the optimal choice of wholesale pricing becomes crucial for turn-and-earn allocation to provide higher profits than fixed allocation. If the manufacturer uses a product line turn-and-earn allocation mechanism, then we can see a decrease in total orders for both products. In summary, we show that depending on the capacity levels, either a single product or a product line turn-and-earn allocation rule is optimal from the manufacturer's point of view.

In terms of the supply chain, previous research showed that a turn-and-earn mechanism resulted in higher profits for the manufacturer but lower profits for the retailer, thus leading to an ambiguous effect on the profitability of the supply chain. We find that things change considerably when we consider a product line that gives the retailer more degrees of freedom. Now, we see that there are instances where

[8] The expressions for these values are defined in Table 1 of the electronic companion.

both the retailers and the manufacturer prefer the same turn-and-earn allocation rule. As a result, compared with the fixed allocation mechanism, the supply chain can be better off with a turn-and-earn allocation mechanism. These are important results in that they establish that turn-and-earn mechanisms can align the incentives of both parties in the supply chain.

Although the model in this paper addresses some of the issues of using a turn-and-earn allocation mechanism with a product line, we see several important extensions for future research. In particular, it is not unusual for the demand states to be correlated across the product line. If this is the case, how does it affect the optimal turn-and-earn mechanism? Two further embellishments that are important are allowing for competing retailers and analyzing the full game when they can hold inventory and trade it among themselves. We see these as important areas for future research.

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Appendix. Proofs

Proof of Lemma 1. (1) As seen from Table 1 in the electronic companion, $\Pi_M^{\mathrm{SPB}} = \Pi_M^{\mathrm{FX}}$ for $K \geq K^{\mathrm{SPB}}$.

(2) Now consider the interval $K \in (K_{\min}^{\mathrm{FX}}, K^{\mathrm{SPB}})$. The difference in profits $\Pi_M^{\mathrm{SPB}} - \Pi_M^{\mathrm{FX}}$ is quadratic in K with positive quadratic coefficient. Denote the roots of the quadratic equation $\Pi_M^{\mathrm{SPB}} - \Pi_M^{\mathrm{FX}} = 0$ as K_1 and K_2, such that $K_1 < K_2$. It can be demonstrated that $K_2 > K^{\mathrm{NS}} > K^{\mathrm{SPB}}$ for all feasible parameter values. Hence, SPB is dominated by FX for all $K \in (K_1 K^{\mathrm{SPB}})$, where

$$\begin{aligned} K_1 = {}& [(2-\phi)(-2\phi(4+8s-\phi+s^2\phi)+\alpha(-32+12\phi-\phi^2 \\ & +s^3\phi^2+s^2\phi(4+\phi)+s(32+8\phi-\phi^2)))] \\ & \cdot[(1-s^2)\phi(s^2\phi^2-16s(4-\phi)-(4-\phi)^2)]^{-1} \\ & -\sqrt{2}[((2-\phi)(64-16(2+s)\phi-(1-s^2)\phi^2) \\ & \cdot(\alpha(4-2s(2-\phi)-\phi+s^2\phi)-2s\phi)^2)]^{1/2} \\ & \cdot[(1-s^2)\phi(s^2\phi^2-16s(4-\phi)-(4-\phi)^2)]^{-1}. \end{aligned}$$

(3) One can show that at $K = K_{\min}^{\mathrm{FX}}$, $\Pi_M^{\mathrm{SPB}} < \Pi_M^{\mathrm{PL}}$ and $\partial\Pi_M^{\mathrm{PL}}/\partial K > \partial\Pi_M^{\mathrm{SPB}}/\partial K$. Hence, SPB is dominated by PL for all $K \in (K_{\min}^{\mathrm{FX}} K_1)$. □

Proof of Lemma 2. The statement of the lemma follows directly from the expressions for the manufacturer's optimal profits in Table 1 of the electronic companion. □

Proof of Proposition 1. (1) *FX vs. SPA profits.*

(i) For $K > K^{\mathrm{SP}}$, $\Pi_M^{\mathrm{FX}} = \Pi_M^{\mathrm{SPA}}$ (see Table 1 in the electronic companion).

(ii) On the interval $K \in [K_{\min}^{\mathrm{SPA}}, K^{\mathrm{SP}}]$ the difference in the manufacturer's profits $(\Pi_M^{\mathrm{FX}} - \Pi_M^{\mathrm{SPA}})$ is quadratic in terms of capacity K with positive quadratic coefficient. The roots of equation $\Pi_M^{\mathrm{FX}} - \Pi_M^{\mathrm{SPA}} = 0$ are

$$\begin{aligned} K_1 = {}& -\frac{(\alpha(4-s(4-\phi))-\phi)(2-\phi)}{(1-s^2)(6-\phi)\phi} \\ & -\frac{\sqrt{2}(\phi+\alpha(2-2s-\phi))\sqrt{8-6\phi+\phi^2}}{(1-s^2)(6-\phi)\phi}, \quad \text{and} \\ K_2 = {}& -\frac{(\alpha(4-s(4-\phi))-\phi)(2-\phi)}{(1-s^2)(6-\phi)\phi} \\ & +\frac{\sqrt{2}(\phi+\alpha(2-2s-\phi))\sqrt{8-6\phi+\phi^2}}{(1-s^2)(6-\phi)\phi}. \end{aligned}$$

It is easy to check that $K_1 < 0$ and $K_2 > K^{\mathrm{FX}}$; hence, $\Pi_M^{\mathrm{FX}} < \Pi_M^{\mathrm{SPA}}$ for all $K \in [K_{\min}^{\mathrm{SPA}}, K^{\mathrm{SP}}]$.

(iii) On the interval $K \in [K_{\min}^{\mathrm{FX}}, K_{\min}^{\mathrm{SPA}}]$, both profits are monotonically increasing. Furthermore, $\Pi_M^{\mathrm{FX}} < \Pi_M^{\mathrm{SPA}}$ at $K = K_{\min}^{\mathrm{SPA}}$ and $\Pi_M^{\mathrm{FX}} = \Pi_M^{\mathrm{SPA}}$ for $K \leq K_{\min}^{\mathrm{FX}}$; hence, $\Pi_M^{\mathrm{FX}} \leq \Pi_M^{\mathrm{SPA}}$ for all K.

(2) *FX vs. PL profits.*

(i) For $K > K^{\mathrm{PL}}$, $\Pi_M^{\mathrm{FX}} = \Pi_M^{\mathrm{PL}}$.

(ii) On the interval $K \in [K_{\min}^{\mathrm{PL}}, K^{\mathrm{PL}}]$, the difference in profits $(\Pi_M^{\mathrm{FX}} - \Pi_M^{\mathrm{PL}})$ is quadratic in K with a positive quadratic coefficient. Denote the roots of quadratic equation $\Pi_M^{\mathrm{FX}} - \Pi_M^{\mathrm{PL}} = 0$ as K_1 and K_2, such that $K_1 < K_2$. It is easy to check that $K_1 < 0$ and $K_2 > K^{\mathrm{FX}}$; hence, $\Pi_M^{\mathrm{FX}} < \Pi_M^{\mathrm{PL}}$ for all $K \in [K_{\min}^{\mathrm{PL}}, K^{\mathrm{PL}}]$.

(iii) On the interval $K \in [K_{\min}^{\mathrm{FX}}, K_{\min}^{\mathrm{PL}}]$, both profits are monotonically increasing. Furthermore, $\Pi_M^{\mathrm{FX}} < \Pi_M^{\mathrm{PL}}$ at $K = K_{\min}^{\mathrm{PL}}$, and $\Pi_M^{\mathrm{FX}} < \Pi_M^{\mathrm{SPA}}$ for $K = K_{\min}^{\mathrm{FX}}$; hence, $\Pi_M^{\mathrm{FX}} < \Pi_M^{\mathrm{SPA}}$ for all $K \in [K_{\min}^{\mathrm{FX}}, K_{\min}^{\mathrm{PL}}]$.

(iv) On the interval $K \in [0, K_{\min}^{\mathrm{FX}}]$, both profits are monotonically increasing. Furthermore, $\Pi_M^{\mathrm{FX}} < \Pi_M^{\mathrm{PL}}$ at $K = 0$ and $\Pi_M^{\mathrm{FX}} < \Pi_M^{\mathrm{SPA}}$ for $K = K_{\min}^{\mathrm{FX}}$; hence, $\Pi_M^{\mathrm{FX}} \leq \Pi_M^{\mathrm{SPA}}$ for all K.

(3) *SPA vs. PL profits.*

(i) For $K > \max\{K^{\mathrm{PL}}, K^{\mathrm{SPA}}\}$, $\Pi_M^{\mathrm{SPA}} = \Pi_M^{\mathrm{PL}}$.

(ii) When $\alpha \geq \alpha^* = (4s\phi)/(8-4s(2-\phi)-\phi+s^2\phi)$, $K^{\mathrm{PL}} > K^{\mathrm{SPA}}$. Then for $K \in [K^{\mathrm{SPA}}, K^{\mathrm{PL}}]$, $\Pi_M^{\mathrm{PL}} > \Pi_M^{\mathrm{SP}} = \Pi_M^{\mathrm{FX}}$. Further, for $K < K^{\mathrm{SPA}}$, the difference in profits $(\Pi_M^{\mathrm{PL}} - \Pi_M^{\mathrm{SPA}})$ is quadratic in K with a positive quadratic coefficient. Denote the roots of the quadratic equation $\Pi_M^{\mathrm{PL}} - \Pi_M^{\mathrm{SPA}} = 0$ as K_1 and K_2, such that $K_1 < K_2$. It is easy to demonstrate that $K_2 > K^{\mathrm{SPA}}$ always, and $K_1 < K_{\min}^{\mathrm{FX}}$ for $\alpha \geq \alpha^*$. Taking into account that $\Pi_M^{\mathrm{PL}} > \Pi_M^{\mathrm{SP}} = \Pi_M^{\mathrm{FX}}$ for $< K_{\min}^{\mathrm{FX}}$, PL is the optimal allocation rule for the manufacturer when $\alpha \geq \alpha^*$.

(iii) When $\alpha < \alpha^*$, PL results in greater profits than SPA only if $K < K_1$. Here,

$$\begin{aligned} K_1 = {}& [256(1-s)\alpha+64(1-(2-s)\alpha)\phi \\ & -4(1+s)(8-7\alpha-s(4-(2+s)\alpha))\phi^2 \\ & +(1-s^2)(3-(2+s)\alpha)\phi^3] \\ & \cdot[(1-s^2)\phi(16s(8-\phi)-(8-\phi)^2-s^2\phi^2)]^{-1} \\ & -[2\phi(16s(8-\phi)+(8-\phi)^2-s^2\phi^2) \\ & \cdot\sqrt{(4-\phi)(64-(1+s)\phi(16+(1-s)\phi))} \\ & \cdot(\alpha(8-\phi-s(8-(4+s)\phi))-4s\phi)]\cdot(1-s^2)^{-1} \end{aligned}$$

Denote $K_M^{\mathrm{SPPL}} = K_1$.

(iv) Further, $\partial K_M^{\mathrm{SPPL}}/\partial s$ is linear in α, and negative with $\alpha=0$ and $\alpha=\alpha^*$. Hence, $K_M^{\mathrm{SPPL}}$is decreasing in s. $\square$

Proof of Proposition 2. *Comparing PL with FX profits.*

1. For $K>K^{\mathrm{PL}}$, $\Pi_R^{\mathrm{FX}}=\Pi_R^{\mathrm{PL}}$.
2. On the interval $K\in[K_{\min}^{\mathrm{PL}},K^{\mathrm{PL}}]$, consider the difference in profits $\Pi_R^{\mathrm{PL}}-\Pi_R^{\mathrm{FX}}$, which is quadratic in K with negative quadratic coefficient. Denote K_1 and K_2 the roots of quadratic equation $\Pi_R^{\mathrm{PL}}-\Pi_R^{\mathrm{FX}}=0$ such that $K_1<K_2$. It can be demonstrated that $K_2>K^{\mathrm{NS}}>K^{\mathrm{PL}}$. Hence, $\Pi_R^{\mathrm{PL}}>\Pi_R^{\mathrm{FX}}$ for all feasible $K_1<K$, where

$$\begin{aligned}K_1=\;&\{(2-\phi)[64\phi(96-4(13+5s)\phi+(5+6s+s^2)\phi^2)\\&+\alpha[4{,}096-1{,}024(3+5s)\phi+128(19+10s+7s^2)\phi^2\\&-32(9+11s+3s^2+s^3)\phi^3-(1-s^2)^2\phi^4]]\\&-\sqrt{2}\{[(2-\phi)[4\alpha\phi(512+128(8-5s)\phi\\&-16(53+5s-4s^2)\phi^2+4(42+25s-s^2)\phi^3+(1+s)^2\phi^4)\\&+\alpha^2[4{,}096+4{,}096(1-2s)\phi-64(107-56s-23s^2)\phi^2\\&+32(91+15s-17s^2-s^3)\phi^3\\&-(421+290s-16s^2-10s^3+3s^4)\phi^4-2(1+s)^2\phi^5]\\&+2\phi^2(128+64(6-s)\phi-(132+60s-8s^2)\phi^2-(1+s)^2\phi^3)]]^{1/2}\}\}\\&\cdot[(1-s^2)\phi(10{,}240-512(19+3s)\phi+32(59+58s-s^2)\phi^2\\&-8(11-s)(1+s)^2\phi^3+(1-s)(1+s)^2\phi^4)]^{-1}.\end{aligned}$$

Denote $K_1=K_R^{\mathrm{PL}}$.

Comparing SPA with FX profits.

1. For $K>K^{\mathrm{SPA}}$, $\Pi_R^{\mathrm{FX}}=\Pi_R^{\mathrm{SPA}}$.
2. On the interval $K\in[K_{\min}^{\mathrm{SPA}},K^{\mathrm{SPA}}]$, the difference in profits $(\Pi_R^{\mathrm{SPA}}-\Pi_R^{\mathrm{FX}})$ is quadratic in K. The quadratic coefficient is negative when $\phi<2(3-\sqrt{7})$ and positive otherwise. The roots of the quadratic equation are as follows:

$$\begin{aligned}K_1=\;&[(2-\phi)(\phi(40-28\phi+3\phi^2)-\alpha(s(32+24\phi-12\phi^2+\phi^3)\\&-2(16-8\phi+8\phi^2-\phi^3)))]\\&\cdot[(1-s^2)\phi(48-80\phi+18\phi^2-\phi^3)]^{-1}\\&-\frac{2(4-\phi)(\phi+\alpha(2-2s-\phi))\sqrt{16+16\phi-22\phi^2+5\phi^3}}{(1-s^2)\phi(48-80\phi+18\phi^2-\phi^3)}\\K_2=\;&[(2-\phi)(\phi(40-28\phi+3\phi^2)-\alpha(s(32+24\phi-12\phi^2+\phi^3)\\&-2(16-8\phi+8\phi^2-\phi^3)))]\\&\cdot[(1-s^2)\phi(48-80\phi+18\phi^2-\phi^3)]^{-1}\\&+\frac{2(4-\phi)(\phi+\alpha(2-2s-\phi))\sqrt{16+16\phi-22\phi^2+5\phi^3}}{(1-s^2)\phi(48-80\phi+18\phi^2-\phi^3)}.\end{aligned}$$

If $\phi<2(3-\sqrt{7})$ then $K_1<K_2$ and $K_2>K^{\mathrm{NS}}>K^{\mathrm{SPA}}$. Hence, $\Pi_R^{\mathrm{SPA}}>\Pi_R^{\mathrm{FX}}$ for all feasible $K_A>K_1$. If $\phi>2(3-\sqrt{7})$ then $K_1>K_2$ and $K_2<0$. Hence, $\Pi_R^{\mathrm{SPA}}>\Pi_R^{\mathrm{FX}}$ for all feasible $K>K_1$.

Thus, we have demonstrated that $\Pi_R^{\mathrm{SPA}}>\Pi_R^{\mathrm{FX}}$ for all feasible $K>K_1$. Denote $K_1=K_R^{\mathrm{SPA}}$.

Finally, via straightforward comparison one can see that $K_R^{\mathrm{PL}}<K_M^{\mathrm{PL}}$ and $K_R^{\mathrm{SPA}}<K_M^{\mathrm{SPA}}$. $\square$

Proof of Proposition 3. The supply chain profit for allocation rule j is $\Pi_{SC}^j=\Pi_M^j+2\Pi_R^j$.

Comparing PL with FX profits.

1. For $K>K^{\mathrm{PL}}$, $\Pi_{SC}^{\mathrm{FX}}=\Pi_{SC}^{\mathrm{PL}}$.
2. On the interval $K\in[K_{\min}^{\mathrm{PL}},K^{\mathrm{PL}}]$ the difference in profits $(\Pi_{SC}^{\mathrm{PL}}-\Pi_{SC}^{\mathrm{FX}})$ is quadratic in K with negative quadratic coefficient. Denote K_1 and K_2 as the roots of quadratic equation $\Pi_{SC}^{\mathrm{PL}}-\Pi_{SC}^{\mathrm{FX}}=0$. It can be demonstrated that $K_2>K^{\mathrm{NS}}>K^{\mathrm{PL}}$. Hence, $\Pi_{SC}^{\mathrm{PL}}>\Pi_{SC}^{\mathrm{FX}}$ for all feasible $K_1<K_A$, where

$$\begin{aligned}K_1=\;&\{-64+16(769+s)\phi-(12{,}287+2{,}048s+s^2)\phi^2\\&+512(7+3s)\phi^3-8\phi^4(33+s^2)(1+s)\\&+4(1-s)(1+s)^2\phi^5\\&-\alpha(2-\phi)[4{,}096-2{,}176\phi^2+272\phi^3-4s^3(4-\phi)\phi^3\\&-\phi^4+3s^4\phi^4-2s^2\phi^2(192+8\phi+\phi^2)\\&+4s\phi(512-128\phi+68\phi^2-\phi^3)]\\&+(1-s^2)(64-16\phi-16s\phi-\phi^2+s^2\phi^2)\sqrt{2(2-\phi)}\\&\cdot\{2\phi^2[128+320\phi-4(47-s)\phi^2-(1+s)^2\phi^3]\\&+4\alpha\phi[512-128(-5+4s)\phi+16(-59+11s+2s^2)\phi^2\\&+2(127+17s-13s^2+s^3)\phi^3+(1+s)^2\phi^4]\\&+\alpha^2[4{,}096-5{,}824\phi^2+3{,}808\phi^3\\&-2s^3(16-17\phi)\phi^3-685\phi^4-3s^4\phi^4-2\phi^5\\&+2s^2\phi^2(736-528\phi+76\phi^2-\phi^3)\\&-2s\phi(4{,}096-3{,}328\phi+464\phi^2+93\phi^3+2\phi^4)]\}^{1/2}\}\\&\cdot\{(1-s^2)\phi[14{,}336-512(21+5s)\phi+32(53+54s+s^2)\phi^2\\&-8(7-s)(1+s)^2\phi^3+3(1-s)(1+s)^2\phi^4]\}^{-1}.\end{aligned}$$

Denote $K_1=K^{\mathrm{SCPL}}$. By straightforward comparison it can be demonstrated that $K_{SC}^{\mathrm{PL}}<K_M^{\mathrm{PL}}$. Hence, $(K_{SC}^{\mathrm{PL}},K_M^{\mathrm{PL}})$ is a nonempty set.

Comparing SPA with FX profits.

1. For $K>K^{\mathrm{SPA}}$, $\Pi_{SC}^{\mathrm{FX}}=\Pi_{SC}^{\mathrm{SPA}}$ (see Table 1 in the electronic companion).
2. On the interval $K\in[K_{\min}^{\mathrm{SPA}},K^{\mathrm{SPA}}]$, consider the difference in profits $\Pi_{SC}^{\mathrm{SPA}}-\Pi_{SC}^{\mathrm{FX}}$, which is quadratic in K with negative quadratic coefficient. Denote K_3 and K_4 as the roots of quadratic equation $\Pi_{SC}^{\mathrm{SPA}}-\Pi_{SC}^{\mathrm{FX}}=0$, such that $K_3<K_4$. It can be demonstrated that $K_4>K^{\mathrm{NS}}>K^{\mathrm{SP}}$. Hence, $\Pi_{SC}^{\mathrm{SPA}}>\Pi_{SC}^{\mathrm{FX}}$ for all feasible $K_3<K_A$, where

$$\begin{aligned}K_3=\;&\{(2-\phi)[\phi(56-32\phi+3\phi^2)-\alpha(s(-32+56\phi-16\phi^2+\phi^3)\\&+2(16-8\phi^2+\phi^3))]\}\\&\cdot[(1-s^2)\phi(144-120\phi+22\phi^2-\phi^3)]^{-1}\\&-\frac{2(4-\phi)\sqrt{16+8\phi-22\phi^2+7\phi^3}(\phi+\alpha(2-2s-\phi))}{(1-s^2)\phi(144-120\phi+22\phi^2-\phi^3)}.\end{aligned}$$

Denote $K_3=K_{SC}^{\mathrm{SPA}}$. It can be demonstrated by direct comparison that $K_{SC}^{\mathrm{SPA}}<K_M^{\mathrm{SPA}}$. This means that $(K_{SC}^{\mathrm{SPA}},K_M^{\mathrm{SPA}})$ is always a nonempty set. $\square$

Proof of Lemma 3. Let c denote the cost of capacity K. Then the manufacturer earns profits of $\Pi_M^j(c)=-cK+\Pi_M^j$ with allocation rule $j\in\{\mathrm{FX},\mathrm{SPA},\mathrm{PL}\}$, where Π_M^j is given in Table 1 of the electronic companion. One can see that for each allocation rule Π_M^j is a piecewise function defined over a corresponding set of intervals. On each of these intervals

$\Pi^j_M(c)$ is monotone and nondecreasing in K. Hence, the value of K that maximizes $\Pi^j_M(c)$ can only be located at the end of one of those intervals. □

Proof of Proposition 4. From the profit expressions provided in Table 1 of the electronic companion one can see that $\Pi^{FL}_M < \Pi^{PL}_M$ with $K \to 0$ and that $\Pi^{FL}_M > \Pi^{PL}_M$ at $K = K^{PL}_{min}$. Further, Π^{FL}_M is monotone and strictly increasing for $K \in (0, K^{PL}_{min})$, and Π^{PL}_M is monotone and strictly increasing on each of the intervals $(0, K^{FX}_{min})$ and $[K^{FX}_{min}, K^{PL}_{min})$. Hence, there exists only one K that solves the equation $\Pi^{FL}_M = \Pi^{PL}_M$ on $K \in (0, K^{PL}_{min})$. Denote it K^{PLFL}. □

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