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Production Planning Under Yield and Demand Uncertainty with Yield-Dependent Cost and Price

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This paper studies production planning with random yield and demand. It is a departure from previous studies of random yield in that it defines the sale price and the purchasing cost as exogenous and increasing with decreasing yield. While this behavior can be observed in various industries (e.g., citrus), the paper focuses on the olive oil industry as its application. Production of olive oil is a challenging business as olives grow every other year; thus, a risky investment is involved. A new practice among olive oil producers involves leasing farm space from farmers to grow olives. When the yield of olives is low (because of weather, disease, etc.), the oil producer gets a second chance to buy olives from other farmers at a unit cost varying with the yield. In this case, the sale price of olive oil increases in the market place because of the reduced supply. When the yield is high, the company uses some of its olives for olive oil production and some are salvaged. After olives are pressed and olive oil is produced, the company experiences an uncertain demand. The paper makes four contributions: First, it is shown that the objective function is concave in the amount of farm space leased, so that the first-order conditions provide the globally optimal solution. Second, it illustrates how the total production of olive oil changes with the yield. Third, it proves that the optimal amount of farm space leased decreases under the presence of a second (and reliable) source of supply. Finally, unlike traditional yield papers, the fourth result shows that increased yield variance does *not* necessarily increase the optimal amount of farm space to be leased when there is a second chance to obtain supplies.

Key words: production planning; yield and demand uncertainty; stochastic programming; yield-dependent price; yield-dependent cost; olive oil production
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1. Introduction

This paper investigates production planning decisions under yield and demand uncertainty. The modeling approach of this study differs from traditional random yield papers because it captures a unique perspective on how the sale price and some of the costs are affected by the realized yield. More specifically, the sale price and the purchasing cost, while being exogenous, are inversely impacted with the realized yield because they both increase with decreasing yield. This relationship can be observed in various industries such as the production of olive oil, orange juice, timber, and wood, etc. Therefore, the findings of this paper extend to a broad range of applications.

We focus on olive oil production because it presents several challenging decisions involving yield and demand uncertainty. Unlike many agricultural products (e.g., fruits and vegetables), the growing and selling seasons of olives are longer. Olive trees are unique because they bear fruit every other year, and are therefore a riskier investment than other crops. At the end of a growing season, farmers collect the olive crop (in late November and early December) and sell them to olive oil producers. In less than 48 hours, they are pressed to obtain olive oil. At this point, the purchasing cost of olives changes depending on the crop yield in the region. Weather conditions, diseases, insects, etc. can influence the yield. As the yield varies, so does the purchasing cost of the olives. The purchasing cost is not the only parameter influenced by such variations. The sale price of the final product, olive oil, is also affected. In Turkey, for example, the vast majority of olives for olive oil are grown in a small geographic area known as Edremit Bay. Because the growers are in close proximity to each other, they share a similar yield uncertainty. A low yield for one implies a low yield for others. With over

70% of Turkey's naturally pressed olive oil coming from the same region, the yield has an impact on the entire country. For example, when the yield was low in 1997 (by 40%) and 1999 (by 30%) compared to 2001, the average sale price was 68% and 45% higher, respectively.[1] Therefore, a low yield increases the sale price of olive oil.

A recent practice growing in popularity is for olive oil producers to lease farm space from growers—the lease is usually measured by the number of olive trees. While the lease is for 2 years (growing season), the leased trees are mature and at least 10 years old. This method enables the oil producing company to have better control over the growth of olives and the quality of the crop. The oil producer incurs a growing cost of olives (which includes pruning, stem cutting, fertilization, weed control, and insect and disease management) in the leased farm space. The oil producer has access to credits and financial instruments at better rates than the farmers. Due to such financial efficiencies and economies of scale (most farmers do not have a large amount of land), the cost of growing olives for the oil producer is typically less than it is for the farmers. While the oil producer incurs a lower cost by growing their own olives as opposed to purchasing them from farmers, the producer has to deal with the risk of yield uncertainty. However, the producer can mitigate this risk through the option of having a second chance to buy extra olives from other growers after the crop yield is observed. It should be noted here that the oil producer cannot diversify the yield risk by leasing land from various farmers in the same region—they all have similar yield.[2] However, leasing is intended to reduce the future purchasing costs of olives, especially when the yield is low. Therefore, the question for the company is what makes the leasing option more profitable than the traditional practice of producing olive oil with solely purchased olives. This paper identifies the conditions that lead to a profitable leasing alternative.[3]

In this study a two-stage decision-making process is considered for a representative olive oil producer. The growing season of olives is the first stage and the selling of olive oil is the second stage. Each stage is approximately two years in length. In Stage 1, the oil producer decides on the size of the olive farm to be leased. After the crop is collected and the yield is observed, the producer decides how much of the realized yield should be used for the production of olive oil and, if necessary, how much to purchase from other farmers. If the crop yield from the leased farm is low, then the producer firm has to decide whether it should purchase more olives from other farmers, and if so, the amount. If the realized yield is high, however, the oil producer has to choose the amount of olives to be pressed for olive oil production, and the rest of the yield is salvaged at a low return. The objective in the second stage is to maximize expected profit under demand uncertainty. We also need to emphasize that olives not used for oil are not packaged for sale. This is because there are two kinds of olives: one is for olive oil (the seed is significantly larger, and its flesh is rich in oil) and the other is for packaged (e.g., canned) olives that have a different texture, taste, and appearance. Therefore, olives that are not pressed for oil can be sold only at salvage value without creating a secondary market. After olive oil is produced, the demand is observed and revenues are collected. The price of olive oil is determined in the overall market, depending on the yield. However, because all olive oil producers operate in nearly identical supply conditions, an individual producer is assumed to face an estimated demand as its market share remains constant regardless of the total supply and demand in the overall market. Therefore, the price received by an olive oil producer is assumed to be inversely related to the observed olive yield.

[1] This data was provided by the Ayvalik Chamber of Commerce, Turkey.

[2] In general, geographic diversification requires transporting olives from a different region. Because olives need to be pressed in less than 48 hours after collection, complex logistical operations are necessary and costs and the quality risk increase. Thus, geographic diversification requires locating a factory in every growing region, and increases capital expenditures. Furthermore, both the kind and quality of olives grown in other parts of Turkey do not match those of Edremit Bay, so the oil producers are not interested in leasing land in other regions of the country.

[3] One can argue that the oil producer should purchase the land (rather than leasing) for its long-term benefits. There are three reasons why this does not occur. The first one is cultural: People who own olive trees perceive the land as their most valuable asset (a guarantee for the future). The leasing also allows them to work different jobs. The second reason is the higher land price because of the proximity to a shoreline. The third reason is a subsidy provided by the World Bank that pays farmers between 17¢ and 21¢ per tree to keep the ownership of their land.

Unlike manufacturing products, olive oil producers do not carry inventory from one selling season to the other. The decision to not hold olive oil inventory is primarily based on maintaining the reputation of the quality of the oil. The process of combining aged oil with new oil is referred to as "blending." Although blending is practiced by bottling companies in some countries, it presents a high risk for oil producers because the aged olive oil naturally carries a higher acidic content than the newly pressed one. Salvaging unsold olive oil (with no carrying of inventory) forces the oil producer to solve this problem every season. As a result, the formulation considered here assumes a single period (two years). Although the salvage revenue (from leftover olive oil) can also change with the realized yield, an exogenous and constant value is assumed in the formulation. However, the main conclusions would remain under a variable price scenario.

The model incorporates an inverse relationship between the realized yield, the purchasing cost of olives, and the sale price of olive oil, but it makes no assumption regarding the form of this relationship. Furthermore, no assumption is made regarding the distribution of random yield and demand. Thus, the results hold under quite general conditions.

The paper makes four contributions. First, it shows that the objective function, maximizing expected profit, is concave in the amount of farm space leased, so that a global optimal solution can be obtained from the first-order conditions. The optimal solution depends on the yield distribution and how the sale price and the purchasing cost change with respect to (w.r.t.) the yield. Second, it offers a detailed discussion on how the total amount of olive oil production changes with the yield. The conditions on various kinds of supply behavior (increasing or decreasing) provide managerial insight. It also demonstrates the implications of having a second source of supply. Third, it proves that the optimal amount of farm space to be leased decreases under the presence of a second chance. It is commonly reported in the random yield literature that higher yield variance increases the optimal amount of initial production. In contrast, this study shows that a higher yield variance does *not* necessarily lead to an increased amount of leased farm space when the oil producer can purchase olives from other farmers (even at a higher cost). The second chance allows the producer to obtain more supplies with a smaller amount of farm space leased. The paper also presents an empirical application to substantiate these theoretical findings.

The problem of random yield and demand has received considerable attention in the literature. An extensive review of production and inventory problems under yield uncertainty can be found in Yano and Lee (1995). Gerchak et al. (1988), Gerchak (1992), Henig and Gerchak (1990), Henig and Levin (1992), and Shih (1980) consider models with production capabilities in single and multiple periods. Gerchak et al. (1988) and Henig and Gerchak (1990) find that the optimal production quantity does not depend on the yield distribution in a periodic review inventory problem. The study by Henig and Levin (1992) determines the optimal order quantity with the choice of the vendor and the quantity to be delivered to customers. Hsu and Bassok (1999) find the amount of optimal production with the availability of downward substitution. Bollapragada and Morton (1999) provide efficient myopic heuristics for periodic review inventory problems. Random yield and demand are also considered in assembly lines (see Gerchak et al. 1994; Gurnani et al. 1996, 2000) and in N-stage serial systems (see Lee and Yano 1988). Additional literature on random yield and demand can be found in papers that study multiple lot sizing in make-to-order systems. An extensive review of these problems is provided in Grosfeld-Nir and Gerchak (forthcoming). The model developed in this study differs from these studies by incorporating a relationship between the realized yield, purchasing cost, and sale price. When the yield is observed, the second stage of the model becomes a newsboy problem with an uncertain demand (Hadley and Whitin 1963). The newsboy problem also presents a different feature by incorporating a secondary source (i.e., the option to purchase more olives from other farmers). The second opportunity to obtain olives resembles the setting in Jones et al. (2001). In their paper, the hybrid seed corn producer gets a second chance for production in a different region of the world and experiences yield uncertainty. Our problem differs from Jones et al. (2001) in two ways: (1) the sale price and the purchasing cost are functions of the realized yield and

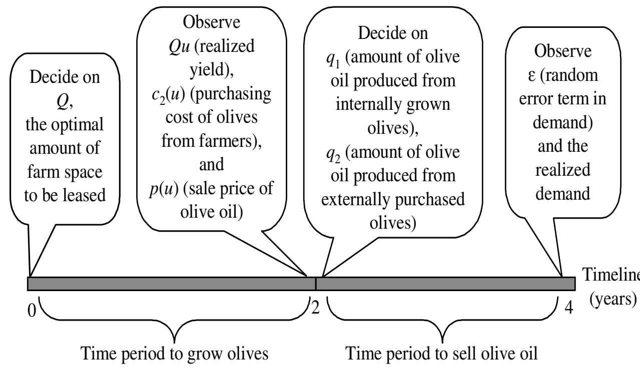
(2) when the olive oil producer purchases olives from other farmers, she does not experience another yield uncertainty. Finally, the demand and its distribution are affected by a variable price scheme in the present formulation.

The paper is organized in the following order: §2 details the description of the problem and presents the model for production planning under yield and demand uncertainty with the presence of yield-dependent cost and price. In §3, a structural analysis explores analytical results for both the growing stage and the sale stage of the problem. Section 4 presents an empirical application by defining a specific relationship that describes how the purchasing cost of olives and the sale price of olive oil change with the realized yield. Conclusions and managerial insights are presented in §5 along with future research directions.

2. The Problem Definition and the Model

This section describes the details of the problem and presents a mathematical model that features random yield and demand with yield-dependent cost and price. Because olive trees are productive once every two years, an olive oil producer's timeline can be divided into two stages as shown in Figure 1. The first two years comprise the growing season and correspond to the first stage of the model. In this stage, the producer decides the number of olive trees to be leased from farmers. At the end of the growing season (Stage 1), the producer collects olives and observes the realized yield. Stage 2 marks the time when the producer faces several decisions. If the yield is low, the producer processes all of the realized yield for olive oil production. If the yield is insufficient to produce the most profitable amount of olive oil, the producer may purchase more olives from other farmers at a relatively higher cost than its own. As a result, the producer determines the total amount of olives to be pressed for olive oil—including the internally grown and purchased olives. However, if the yield is high, then the producer does not purchase olives, and perhaps does not even process its entire yield. Instead, the producer can sell some of the olives at a salvage value. Therefore, at the beginning of the second stage, the producer decides on the amount of olive oil to be

Figure 1 **The Natural Sequence of Events on a Timeline**



produced and sold in the market. During this two-year period, the producer faces an uncertain demand for his product.

We first present a two-stage stochastic programming model for the olive oil production planning problem under yield and demand uncertainty. We use the following notation in our formulation:

u: a random variable representing yield as the fraction of the amount produced, Q (Qu is the number of olives realized after leasing Q units of echelon farm space, e.g., olive trees).

$g(u)$: probability density function (pdf) of random yield parameter, u, on a support $[0, B]$ where $0 < B \leq 1$.

c_1: unit cost of leasing trees (echelon land for each unit of olive oil) for growing olives (includes maintaining the land and collecting olives at the end of the growing season).

$c_2(u)$: unit cost of purchasing (echelon) olives (from other olive growers) after olives are collected; $c_2(u)$ is continuous and is a decreasing function of u, and $c_2(u = B) > c_1$.

c_p: unit cost of processing olives to produce olive oil.

h_1: unit revenue of salvaging (echelon) olives without processing for olive oil production, $h_1 < c_1$.

h_2: unit revenue of salvaging olive oil at the end of the selling season, $h_1 < h_2 < c_2(u = B)$; $h_2 < h_1 + c_p$.

b: unit penalty cost for unsatisfied demand of olive oil.

$p(u)$: unit sale price of olive oil; $p(u)$ is continuous and is a decreasing function of u; $p(u) > c_2(u)$ for all u values, and $p(u = B) > c_1 + c_p > h_2$.

$D(p(u))$: demand function when the price of olive oil is $p(u)$.

Suppose that the firm faces a demand that is a function of the price of olive oil, $p(u)$, which in turn is a decreasing function of the realized yield. We use the following expression:

$$D(p(u)) = K - \beta p(u) + \varepsilon \tag{1}$$

where K is the expected demand for olive oil if the price were set to zero ($K > 0$), β is the rate that demand for olive oil decreases per unit increase in price ($\beta > 0$), and ε is the random error term. It is assumed that ε is independent of the price level and represents the noise around expected demand when price is set to $p(u)$. We assume that ε is distributed according to $f(\varepsilon)$, the pdf of the random error term, with a mean of zero on a support $[-A_1, A_2]$ where A_1 and $A_2 > 0$. $F(\varepsilon)$ is defined as the cumulative density function (cdf) of the random error term and assumed to be continuous, invertible, and twice-differentiable. We also assume that $K - \beta p(u) - A_1 > 0$, ensuring that the demand for olive oil is always positive even when the yield is zero and the sale price of olive oil in the market is set to its highest level. It should be noted here that the expected demand for a given yield u decreases with increasing prices, i.e., $E_\varepsilon[D(p(u)) \mid u] = K - \beta p(u)$. Thus, the demand variance does not change w.r.t. the sale price for a given u. Furthermore, for the same u, the demand coefficient of variation increases in the price.[4] Also note that no assumption has been made regarding the mean and variance of the overall demand function.

Stage 1 Decision Variables

Q: amount of (echelon) olive trees (for the target amount of olive oil) that the oil producer leases in order to grow olives.

Stage 2 Decision Variables

q_1: amount of olive oil produced from olives grown by the producer (equivalently, this is equal to the echelon amount of internally grown olives pressed for olive oil), $q_1 \le Qu$.

q_2: amount of olive oil produced from the olives purchased from other growers (equivalently, this variable can be interpreted as the echelon amount of olives that need to be purchased from other farmers).

It should be noted here that while Q represents the number of olive trees leased for growing olives, q_1 and q_2 are defined as the amount of olive oil produced from internally grown and purchased olives, respectively. q_1 and q_2 are also equal to the echelon amount of olives that will be converted to olive oil.[5] We next present the model which uses the above notation.

The Model

The second-stage problem maximizes the expected revenues from the sale of olive oil under demand uncertainty. Given the realized yield of Qu units of olives, this stage determines the optimal amount of olive oil to be produced from internally grown olives (q_1) and from purchased olives (q_2). The sum $(q_1 + q_2)$ gives the total amount of olive oil production. It should be noted that second stage revenue depends on the random demand level. The expected second-stage return function, $E_\varepsilon[\Psi(q_1, q_2 \mid Q, u)]$, can be written as follows:

$$\begin{aligned} E_\varepsilon&[\Psi(q_1, q_2 \mid Q, u)] \\ &= -c_2(u)q_2 - c_p(q_1+q_2) + h_1(Qu - q_1)^+ \\ &\quad + \int_{-A_1}^{(q_1+q_2)-[K-\beta p(u)]} \big[p(u)[K - \beta p(u) + \varepsilon] + h_2((q_1+q_2) \\ &\qquad - [K - \beta p(u) + \varepsilon])\big] f(\varepsilon)\, d\varepsilon \\ &\quad + \int_{(q_1+q_2)-[K-\beta p(u)]}^{A_2} \big[p(u)(q_1+q_2) - b([K - \beta p(u) + \varepsilon] \\ &\qquad - (q_1+q_2))\big] f(\varepsilon)\, d\varepsilon. \end{aligned} \tag{2}$$

The first term in the right-hand side (RHS), $c_2(u)q_2$, is the purchasing cost of echelon olives from other growers for the production of q_2 units of olive oil. The second term, $c_p(q_1 + q_2)$, is the processing cost of olives

[4] Alternatively, we could use a functional form such as $D(p(u)) = Kp(u)^{-\beta}\varepsilon$, $K > 0$ and $\beta > 1$, where demand is multiplied by an error term. In this case, while the variance of demand would vary with price, the demand coefficient of variation would remain constant and independent of the price. A more general form of the demand function is $D(p(u), \varepsilon)$ where $\partial D(p(u), \varepsilon)/\partial p(u) < 0$ and $\partial D(p(u), \varepsilon)/\partial \varepsilon > 0$. Although not presented here, the analytical results (e.g., the concavity of the objective function in the amount of farm space leased) do not change when this general demand function is used.

[5] There is a maximum amount of olives that a tree can produce (about 200 kilograms). Similarly, there is a conversion rate for the amount of olives needed per liter of olive oil—this ratio is typically five kilograms of olives per liter of olive oil. These conversion rates are omitted in the discussion for simplicity. Instead, the cost parameters are adjusted to reflect the role of these parameters.

to press a total of (q_1+q_2) units of olive oil. The term $h_1(Qu-q_1)^+$ is the salvage revenue obtained from leftover olives that are not used for oil production. When the demand is less than or equal to the total amount of olive oil production, i.e., $K-\beta p(u)+\varepsilon \le q_1+q_2$, the second-stage return function, $\Psi(q_1, q_2 \mid Q, u)$, includes the revenue from selling $D(p(u))$ units at price $p(u)$. In this case, the unsold olive oil, $q_1+q_2-D(p(u))$, is salvaged at a unit value of h_2. Otherwise, when the realized demand exceeds the total amount of olive oil produced (i.e., $K-\beta p(u)+\varepsilon > q_1+q_2$), the second-stage return function includes the revenue from selling only q_1+q_2 units of olive oil at price $p(u)$. In this case, each unit of unsatisfied demand, $D(p(u))-(q_1+q_2)$, incurs a penalty cost of b. The resulting second-stage optimization model can then be expressed as

$$PA(Q,u) = \max_{(q_1,q_2)} \quad E_\varepsilon[\Psi(q_1,q_2 \mid Q,u)] \tag{3}$$

$$\text{s.t.} \quad q_1 \le Qu$$

$$q_1, q_2 \ge 0 \tag{4}$$

where Q units are planned in the first stage and u% of it is realized. Note that $h_1(Qu-q_1)^+$ in (2) can be replaced with $h_1(Qu-q_1)$ because constraint (4) ensures that the amount of olive oil produced from internally grown olives cannot exceed the realized yield in the first stage.

Stage 1 maximizes the expected profit from leasing Q units of olive trees, $E[\Pi(Q)]$, which is equal to the expected value of the second-stage profit over yield uncertainty, $E_u[PA(Q,u)]$, less the cost of growing olives in the leased farm space, c_1Q.

$$\max_{Q} \quad E[\Pi(Q)] = -c_1Q + E_u[PA(Q,u)] \tag{5}$$

$$\text{s.t.} \quad Q \ge 0.$$

3. The Analysis

The purpose in this section is to show that the objective function in Stage 1, $E[\Pi(Q)]$, is a concave function of the amount of farm space leased, Q. To accomplish this, we first derive structural results for the optimal policy in the second stage and use it to construct a similar analysis for the first stage.

3.1. The Structural Analysis of the Second Stage

This section derives the optimal policies for the amount of olive oil production (both from internally grown and purchased olives) and the conditions that lead to them. The proofs of all the propositions stated below can be found in the Appendix.

Proposition 1. *For any realized yield, u, and the first-stage decision Q, the maximand of $E_\varepsilon[\Psi(q_1,q_2 \mid Q,u)]$ defined in* (3) *is concave in q_1 and q_2.*

Next, two special cases are considered. The first case corresponds to the traditional practice of olive oil producers when they do not lease farm space. In this case, Q is equal to zero; therefore q_1 is also zero, and the producer needs to purchase olives from other farmers to produce q_2 units of olive oil. These olives are purchased at a cost of $c_2(u)$ for every q_2 units of olive oil, and thus are still subject to a cost varying with the realized yield. In the second case, the producer is not allowed to purchase olives from other farmers. The analysis of these two cases provides insight into both the structural results and managerial decisions.

3.1.1. Case 1: Traditional Practice. The next proposition establishes the optimal quantity of purchased olives for any realized yield parameter, u, in the traditional practice, where the company does not lease any farm space (thus, $Q=0$).

Proposition 2. *For any realized yield, u, and first-stage decision $Q=0$, the optimal quantity to be purchased for producing olive oil is equal to*

$$q_2^* = [K-\beta p(u)] + F^{-1}\left(\frac{p(u)+b-c_2(u)-c_p}{p(u)+b-h_2}\right). \tag{6}$$

From this point on, we use the following expressions in the notation.

$$\tau_1(u) = \frac{p(u)+b-c_2(u)-c_p}{p(u)+b-h_2} \quad \text{where} \quad 0<\tau_1(u)<1,$$

$$s_1(u) = F^{-1}\left(\frac{p(u)+b-c_2(u)-c_p}{p(u)+b-h_2}\right), \quad \text{and}$$

$$TS_1(u) = [K-\beta p(u)] + F^{-1}\left(\frac{p(u)+b-c_2(u)-c_p}{p(u)+b-h_2}\right).$$

While the fraction $\tau_1(u)$ corresponds to the critical fractile in the newsboy problem, $s_1(u)$ resembles the amount of safety stock and $TS_1(u)$ represents the "target amount of olive oil production" (e.g., order-up-to quantity). The following lemma establishes the conditions in which $s_1(u)$ is a strictly increasing (decreasing) function of u.

Lemma 3. (a) $s_1(u)$ *is a strictly increasing function of* u *when*

(i) $p'(u) \geq c_2'(u)$, or (7)

(ii) $p'(u) < c_2'(u)$ and $\tau_1(u) > \dfrac{p'(u) - c_2'(u)}{p'(u)}$, (8)

for all values of u; (b) $s_1(u)$ *is a strictly decreasing function of* u *when*

$$p'(u) < c_2'(u) \quad \text{and} \quad \tau_1(u) < \frac{p'(u) - c_2'(u)}{p'(u)} \tag{9}$$

for all values of u.

The conditions stated above provide insight about the behavior of $s_1(u)$. If the profit margin from purchasing olives increases with the yield, then $s_1(u)$ increases in u. This occurs when the reduction in price is less than that of the purchasing cost. In this case, the producer expects to have its worst profit margins when the yield is at its lowest value and vice versa. When the profit margin decreases with the yield, however, $s_1(u)$ can still be increasing in u as long as the newsboy fractile, $\tau_1(u)$, is greater than the percentage change in the profit margin w.r.t. the change in price. Note that the RHS of conditions (8) and (9) are the same and can be interpreted as the percentage of change in profit margin through purchasing of olives w.r.t. the change in price. While the left-hand side in (8) is an increasing function of u, the RHS can be either increasing or decreasing, depending on how the price and the purchasing cost change w.r.t. the yield. For example, when a linear relationship is assumed between the price, the purchasing cost, and the yield, then the RHS becomes a constant. As long as $\tau_1(u)$ is greater than the RHS (a constant) at the lowest yield value, $u = 0$, $s_1(u)$ strictly increases in the yield. Alternatively, if $\tau_1(u)$ is less than the RHS, even at the highest yield value, $u = B$, then $s_1(u)$ is strictly decreasing in the yield. The following lemma states the behavior of the target amount of olive oil production w.r.t. the yield, namely under decreasing returns w.r.t. the yield, the target amount of olive oil production is a strictly increasing function of u.

Lemma 4. (a) $TS_1(u)$ *is a strictly increasing function of* u *when*

(i) $p'(u) \geq c_2'(u)$, *or*

(ii) $p'(u) < c_2'(u)$ *and*

$$\tau_1(u) > \frac{p'(u) - c_2'(u)}{p'(u)} + \frac{p(u) + b - h_2}{p'(u) cv_1'(u)} \tag{10}$$

for all values of u; (b) $TS_1(u)$ *is a strictly decreasing function of* u *when*

$$p'(u) < c_2'(u) \quad \text{and}$$
$$\tau_1(u) < \frac{p'(u) - c_2'(u)}{p'(u)} + \frac{p(u) + b - h_2}{p'(u) cv_1'(u)} \tag{11}$$

for all values of u, *where* $cv_1'(u) = [\partial F^{-1}(\tau_1(u)) / \partial \tau_1(u)] / (-\beta p'(u))$.

To conclude that $TS_1(u)$ is an increasing function, $s_1(u)$ may not necessarily be increasing. $TS_1(u)$ can be increasing even when the increase in expected demand is greater than or equal to the decrease in safety stock, $s_1(u)$. This explains why a wider range of $\tau_1(u)$ ensures an increasing behavior for $TS_1(u)$ than for $s_1(u)$.

3.1.2. Case 2: No Purchasing of Olives From Other Farmers. We show that when $q_2 = 0$, the optimal amount of olive oil production depends on the realized yield.

Proposition 5. *For any realized yield*, Qu, *the optimal amount of olive oil produced from internally grown olives is equal to*

$$q_1^* = \begin{cases} Qu & \text{when} \quad Qu \leq [K - \beta p(u)] + F^{-1}\left(\dfrac{p(u)+b-h_1-c_p}{p(u)+b-h_2}\right) \\ [K - \beta p(u)] + F^{-1}\left(\dfrac{p(u)+b-h_1-c_p}{p(u)+b-h_2}\right) & \text{when} \quad Qu > [K - \beta p(u)] + F^{-1}\left(\dfrac{p(u)+b-h_1-c_p}{p(u)+b-h_2}\right). \end{cases} \tag{12}$$

To simplify, we use the expressions that will follow the notation below:

$$\tau_2(u) = \frac{p(u) + b - h_1 - c_p}{p(u) + b - h_2} \quad \text{where } 0 < \tau_2(u) < 1,$$

$$s_2(u) = F^{-1}\left(\frac{p(u) + b - h_1 - c_p}{p(u) + b - h_2}\right), \quad \text{and}$$

$$TS_2(u) = [K - \beta p(u)] + F^{-1}\left(\frac{p(u) + b - h_1 - c_p}{p(u) + b - h_2}\right).$$

Again, $\tau_2(u)$ corresponds to the newsboy fractile, $s_2(u)$ resembles the safety stock, and $TS_2(u)$ represents the target amount of olive oil production.

Proposition 6. *$s_2(u)$ is a strictly decreasing function of u.*

The optimal value of q_1^* depends on the realized yield. When the realized yield is high enough, the producer chooses to use only a portion of the yield, which is equal to $TS_2(u)$. The amount of $TS_2(u)$ can be considered as the target amount of olive oil production when there is a sufficiently high yield for any given u.

Lemma 7. (a) *$TS_2(u)$ is a strictly increasing function of u when*

$$\tau_2(u) > 1 + \frac{p(u)+b-h_2}{p'(u)cv_2'(u)} \tag{13}$$

for all values of u; (b) *$TS_2(u)$ is a strictly decreasing function of u when*

$$\tau_2(u) < 1 + \frac{p(u)+b-h_2}{p'(u)cv_2'(u)} \tag{14}$$

for all values of u, where $cv_2'(u) = [\partial F^{-1}(\tau_2(u))/\partial\tau_2(u)]/(-\beta p'(u))$.

It should be noted that the conditions for the increasing and decreasing behavior of $TS_2(u)$ are similar to those developed for $TS_1(u)$. Because

$$\frac{p'(u)-c_2'(u)}{p'(u)} < 1 \quad \text{when} \quad c_2'(u) > p'(u),$$

the RHS of (13) and (14) are greater than the RHS of (10) and (11). Thus, the condition for $TS_2(u)$ to be increasing is greater than that of $TS_1(u)$.

The next proposition compares the optimal olive oil production under the traditional practice (no leasing of farm space) and the case of not purchasing olives.

Proposition 8. *For a given value of u, the optimal amount of olive oil production targeted in the case of no farm space leasing, $q_1^* = TS_2(u)$, is greater than that of the case of not purchasing olives, $q_1^* = TS_2(u)$, i.e., $q_1^* = TS_2(u) > q_1^* = TS_2(u)$.*

Denote the values of yield u that solve $Qu = TS_1(u)$, $Qu = TS_2(u)$ by $u_1(Q)$, and $u_2(Q)$, respectively. The above proposition implies that $u_1(Q) = (1/Q)TS_1(u) < u_2(Q) = (1/Q)TS_1(u)$. Furthermore, one can see that $u_1(Q)$ and $u_2(Q)$ are both decreasing in Q. Finally, the structural results of the second stage can be summarized with the following proposition for the general model (presented in §2).

Proposition 9. *For a given u the optimal values of internally grown and purchased olives used for olive oil production is as follows:*

$$(q_1^*, q_2^*) = \begin{cases} (Qu, TS_1(u) - Qu) \\ \quad \textit{for } u \in R_1 = \{u\colon Qu < TS_1(u)\} \\ (Qu, 0) \\ \quad \textit{for } u \in R_2 = \{u\colon TS_1(u) \le Qu < TS_2(u)\} \\ (TS_2(u), 0) \quad \textit{for } u \in R_3 = \{u\colon Qu \ge TS_2(u).\} \end{cases}$$

Using the optimal policies in each region, the expected second-stage return function for each interval can be written as

$$\begin{aligned}
&E_\varepsilon[\Psi_{R_1}(q_1^*, q_2^* \mid u \in R_1)] \\
&\quad = (p(u) - (c_2(u) + c_p))[K - \beta p(u)] \\
&\qquad - (c_2(u) + c_p - h_2)s_1(u) + c_2(u)Qu \\
&\qquad - (p(u) + b - h_2)\int_{s_1(u)}^{A_2} [\varepsilon - s_1(u)] f(\varepsilon)\,d\varepsilon \\
&E_\varepsilon\big[\Psi_{R_2}(q_1^*, q_2^* \mid u \in R_2)\big] \\
&\quad = (p(u) - h_2)[K - \beta p(u)] \\
&\qquad - (c_p - h_2)Qu - (p(u) + b - h_2) \\
&\qquad \cdot \int_{Qu-[K-\beta p(u)]}^{A_2} \big[\varepsilon - (Qu - [K - \beta p(u)])\big] f(\varepsilon)\,d\varepsilon \\
&E_\varepsilon[\Psi_{R_3}(q_1^*, q_2^* \mid u \in R_3)] \\
&\quad = (p(u) - c_p - h_1)[K - \beta p(u)] \\
&\qquad - (c_p + h_1 - h_2)s_2(u) + h_1 Qu \\
&\qquad - (p(u) + b - h_2)\int_{s_2(u)}^{A_2} [\varepsilon - s_2(u)] f(\varepsilon)\,d\varepsilon.
\end{aligned}$$

In light of this observation, one can see that $Qu = TS_1(u)$ and $Qu = TS_2(u)$ are the two break points where the optimal policy changes. Furthermore, it can be observed that the expected second-stage return function maintains continuity at these two break points.

Lemma 10. *The following always hold*:

(a) $E_\varepsilon[\Psi_{R_1}(q_1^*, q_2^* \mid u \in R_1)]|_{Qu=TS_1(u)}$
$$= E_\varepsilon[\Psi_{R_2}(q_1^*, q_2^* \mid u \in R_2)]|_{Qu=TS_1(u)}, \quad \text{and}$$

(b) $E_\varepsilon[\Psi_{R_2}(q_1^*, q_2^* \mid u \in R_2)]|_{Qu=TS_2(u)}$
$= E_\varepsilon[\Psi_{R_3}(q_1^*, q_2^* \mid u \in R_3)]|_{Qu=TS_2(u)}$.

Proposition 11. *$PA(Q,u)$ is continuous in Qu and has break points at $Qu = TS_1(u)$ and $Qu = TS_2(u)$.*

3.2. Analysis of the First Stage

This section presents the optimality conditions in the first stage. In light of the analysis of the second stage presented in §3.1, a complete expression for $E[\Pi(Q)]$ is derived first. Then it is shown that $E[\Pi(Q)]$ is concave in Q, and thus the optimal Q^* can be found by equating the first-order derivative to zero. The objective function stated in Equation (5) can be written as

$$E[\Pi(Q)] = -c_1 Q + \begin{cases} \int_0^B [E_\varepsilon[\Psi_{R_1}(q_1^*, q_2^* \mid Qu \in R_1)]]g(u)\,du \\ \quad \text{when } u_1(Q) \ge B \\ \int_0^{u_1(Q)} [E_\varepsilon[\Psi_{R_1}(q_1^*, q_2^* \mid Qu \in R_1)]]g(u)\,du \\ \quad + \int_{u_1(Q)}^B [E_\varepsilon[\Psi_{R_2}(q_1^*, q_2^* \mid Qu \in R_2)]]g(u)\,du \\ \quad \text{when } u_1(Q) < B \text{ and } u_2(Q) \ge B \\ \int_0^{u_1(Q)} [E_\varepsilon[\Psi_{R_1}(q_1^*, q_2^* \mid Qu \in R_1)]]g(u)\,du \\ \quad + \int_{u_1(Q)}^{u_2(Q)} [E_\varepsilon[\Psi_{R_2}(q_1^*, q_2^* \mid Qu \in R_2)]]g(u)\,du \\ \quad + \int_{u_2(Q)}^B [E_\varepsilon[\Psi_{R_3}(q_1^*, q_2^* \mid Qu \in R_3)]]g(u)\,du \\ \quad \text{when } u_2(Q) < B. \end{cases} \tag{15}$$

Proposition 12. *$E[\Pi(Q)]$ is continuous and concave in Q.*

The concavity is important for sufficiency of the first-order optimality conditions. Although the optimal amount of farm space to be leased, Q^*, cannot be expressed in a closed-form expression (due to the lack of explicit functional relationships between price, purchasing cost, and yield as well the pdf of yield and demand), it can be calculated by equating the first-order derivative of $E[\Pi(Q)]$ to zero. The condition that makes leasing farm space a profitable investment for olive oil producers relates to the unit cost of leasing and the expected savings from not having to purchase olives, as stated below:

Proposition 13. *The optimal amount of farm space leased is strictly positive, i.e., $Q^* > 0$, when*

$$\int_0^B [uc_2(u)]g(u)\,du - c_1 > 0. \tag{16}$$

The condition in Equation (16) has managerial implications. It shows how much money is saved (in expected value) by *not* having to purchase olives from external farmers (in the second stage) for each unit of investment made in the first stage at a cost of c_1. The term $uc_2(u)$ is the yield times the purchasing cost of olives at this specific value of the yield while $g(u)$ is its associated probability. Thus, a corresponding amount would be saved in the second stage for each unit of investment in the first stage. The integral over $[uc_2(u)]g(u)$ determines the expected amount of savings in the second stage for each unit of investment made in the first stage.

3.3. The Value of First Chance (Leasing) and Second Chance (Purchasing)

This section investigates the value of leasing and having a second chance to obtain olives. A similar analysis is shown in Jones et al. (2001) when a hybrid seed corn producer gets a second chance of production (or purchasing) in the second stage. Our analysis extends their work in two ways: (1) the value depends on the sale price and the purchasing cost that vary with the realized yield and (2) the value of initial investment can be calculated separately due to the fact that an olive oil producer can satisfy its demand exclusively by purchasing olives from farmers without leasing farm space.

3.3.1. Case 1: Traditional Practice. In this section, we determine the expected profit of an olive oil producer who does not lease farm space. In this case, $Q = 0$ and $q_1 = 0$. However, the purchasing cost of olives and the sale price of olive oil continue to change w.r.t. the realized yield. Proposition 2 implies that $q_2^* = TS_1(u)$ under this scenario. Defining $\Pi_{C1}(Q=0)$ as the profit of the traditional olive oil producer, the expected profit function can be written as follows:

$$E[\Pi_{C1}(Q=0)] = \int_0^B \Big\{(p(u) - c_2(u) - c_p)[K - \beta p(u)] - (c_2(u) + c_p - h_2)s_1(u) - (p(u) + b - h_2) \cdot \int_{s_1(u)}^A [\varepsilon - s_1(u)]f(\varepsilon)\,d\varepsilon\Big\} g(u)\,du.$$

It should be noted that the value of $E[\Pi_{C1}(Q=0)]$ depends on how the sale price of olive oil, $p(u)$, and the purchasing cost of olives, $c_2(u)$, change w.r.t. the crop yield u, as well as the pdf of the yield. We define the value added through leasing farm space as the difference between the expected profit obtained from leasing the optimal amount of farm space, Q^*, and the expected profit of the case when no leasing occurs: i.e., $V_1 = E[\Pi(Q^*)] - E[\Pi_{C1}(Q=0)]$.

3.3.2. Case 2: No Purchasing of Olives from Other Farmers. The value obtained from a second opportunity of purchasing olives from other farmers is analyzed as in Jones et al. (2001). Purchasing olives would be needed when the realized yield is low. In this scenario, q_2 is forced to be zero, and the expected profit is not influenced by the purchasing cost, but it remains to be dependent on how the sale price changes with the yield. Proposition 5 implies that $q_1^* = Qu$ when $Qu \le TS_2(u)$, and $q_1^* = TS_2(u)$ when $Qu > TS_2(u)$. In this case, $TS_2(u)$ serves as the only break point in the expected profit function. Denoting the profit obtained from this case with $\Pi_{C2}(Q)$, the expected profit function can be written as

$$
\begin{aligned}
E[\Pi_{C2}(Q)] &= -c_1 Q \\
&+ \begin{cases}
\displaystyle\int_0^B \Big\{ (p(u)-h_2)[K-\beta p(u)] \\
\quad -(c_p-h_2)Qu-(p(u)+b-h_2) \\
\quad \cdot \displaystyle\int_{Qu-[K-\beta p(u)]}^{A_2} [\varepsilon-(Qu-[K-\beta p(u)])] f(\varepsilon)\,d\varepsilon \Big\} \\
\quad \cdot g(u)\,du \\
\qquad\qquad\qquad\qquad \text{when } u_2(Q)\ge B \\
\displaystyle\int_0^{u_2(Q)} \Big\{ (p(u)-h_2)[K-\beta p(u)] \\
\quad -(c_p-h_2)Qu-(p(u)+b-h_2) \\
\quad \cdot \displaystyle\int_{Qu-[K-\beta p(u)]}^{A_2} [\varepsilon-(Qu-[K-\beta p(u)])] f(\varepsilon)\,d\varepsilon \Big\} \\
\quad \cdot g(u)\,du \\
\displaystyle\int_{u_2(Q)}^B \Big\{ (p(u)-c_p-h_2)[K-\beta p(u)] \\
\quad -(c_p+h_1-h_2)s_2(u)+h_1 Qu-(p(u)+b-h_2) \\
\quad \cdot \displaystyle\int_{s_2(u)}^{A_2} [\varepsilon-s_2(u)] f(\varepsilon)\,d\varepsilon \Big\} g(u)\,du \\
\qquad\qquad\qquad\qquad \text{when } u_2(Q)< B.
\end{cases}
\end{aligned}
$$

The value of having a second chance (purchasing olives from other farmers) is given by $V_2 = E[\Pi(Q^*)] - E[\Pi_{C2}(Q^*_{C2})]$, where Q^* and Q^*_{C2} are the optimal amounts of farm space leased that maximize $E[\Pi(Q)]$ and $E[\Pi_{C2}(Q_{C2})]$, respectively. The following proposition shows that the optimal amount of farm space leased in the original problem is always less than that of Case 2.

Proposition 14. *The optimal amount of farm space leased, Q^*, is always smaller than the optimal amount Q^*_{C2}.*

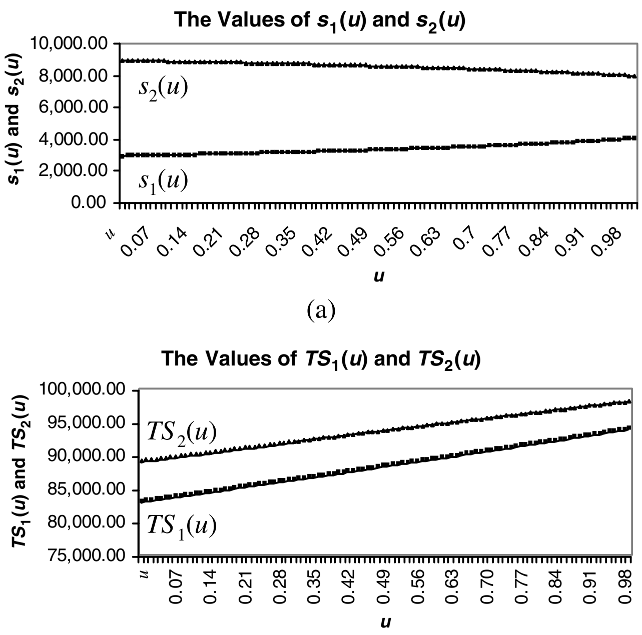
The above result has managerial implications for decision making under yield uncertainty. The flexibility of having a second chance to obtain more supplies can reduce the optimal amount of olive production. Producers who have an alternative source of supply in the second stage do not need to invest in a large amount of production in the first stage. Similarly, if a manufacturer has the option of outsourcing from a supplier after realizing the yield, she should produce a smaller amount of products in the first stage.

4. An Empirical Application

This section presents an empirical application of the above theoretical framework. The key price and cost data were provided by the Ayvalık Chamber of Commerce in Turkey (the largest organization in Edremit Bay that monitors olive oil production). These data were also confirmed by other producers operating in the same region.[6] The following linear relationships were used in the model: $p(u) = 19.86 - 9.93u$; $c_2(u) = 8.22 - 4.11u$; $c_1 = \$2.64/\text{can}$; $c_p = \$3.13/\text{can}$; $h_1 = \$1.97/\text{can}$; $h_2 = \$4.00/\text{can}$; $b = \$5.00/\text{can}$; $K = 100{,}000$; and $\beta = 1{,}000$. Therefore, the demand function is $D(p(u)) = 100{,}000 - 1{,}000p(u) + \varepsilon$ where ε is distributed uniformly between $-10{,}000$ and $+10{,}000$, i.e., $A_1 = A_2 = 10{,}000$. To highlight the impact of yield uncertainty, we begin our analysis with a distribution that has a high variance, where u follows a discrete uniform distribution between 0.01 and 1.00 with increments of 0.01 and a corresponding probability of 0.01. Using these data, Figure 2

[6] A five-liter can of naturally pressed olive oil is used as the final product, and the cost of its echelon olives and farm space are calculated by the following conversion rates: (i) five kilograms of olives are used to obtain a liter of olive oil, and therefore a can corresponds to 25 kilos of olives; (ii) each olive tree provides at most (e.g., perfect yield) 200 kilograms of olives, and thus a can corresponds to 12.5% of the maximum outcome of a tree.

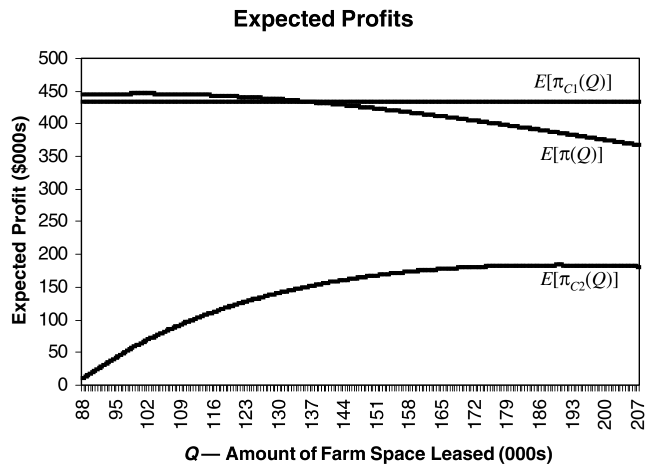
Figure 2 **(a) The Amount of Safety Olive Oil in the Case of Traditional Practice ($s_1(u)$) and No Purchasing of Olives ($s_2(u)$) as a Function of Yield u; (b) The Target Amount of Olive Oil Production in the Cases of Traditional Practice ($TS_1(u)$) and No Purchasing of Olives ($TS_2(u)$) as a Function of Yield u in the Empirical Application**



(a)

(b)

Figure 3 **The Expected Profit Functions of the Original Problem ($E[\Pi(Q)]$) and the Two Special Cases of Traditional Practice ($E[\Pi_{C1}(Q)]$) and No Purchasing of Olives ($E[\Pi_{C2}(Q)]$)**



shows how $s_1(u)$, $s_2(u)$ (in Part a), and $TS_1(u)$ and $TS_2(u)$ (in Part b) vary w.r.t. the yield, u. Proposition 6 implies that $s_2(u)$ is always decreasing in u, while $s_1(u)$ can exhibit either an increasing or a decreasing behavior. In the example here, $s_1(u)$ was increasing in u. Furthermore, the increase in expected demand was higher than the decrease in $s_2(u)$, therefore both $TS_1(u)$ and $TS_2(u)$ exhibited an increasing behavior in u.

Next the original problem was solved along with the two special cases presented in §3. As shown in Figure 3, the expected profit of the original problem was maximized for $Q^* = 100{,}941$ with an optimum expected profit of $E[\Pi(Q)] = \$446{,}137.61$. When there was no leasing of farm space ($Q = 0$), the expected profit obtained by solving the newsboy problem for each realization of u (and later taking the expectation) was found as $E[\Pi_{C1}(Q)] = \$434{,}421.26$. In this particular application, the option of leasing farm space increased the expected profit of the producer by $V_1 = \$11{,}716.35$, which corresponds to a 2.70% increase. The value of the second chance (purchasing olives from other farmers) is clearly seen in Figure 3. When this flexibility was eliminated ($q_2 = 0$), the producer had to lease the optimal amount $Q^* = 189{,}985$, and the expected profit would reduce to $E[\Pi_{C2}(Q)] = \$183{,}924.40$. Therefore, the value of the second chance, V_2, is equal to \$262,213.21, which increases the expected profit by 142.57%. This expected profit is significantly lower than that of the original problem, which is caused primarily by the losses that would occur when the realized yield was low. Under this scenario, the producer cannot purchase olives externally to eliminate the loss equalling the sum of the opportunity cost of not selling the product $p(u) - c_p - c_2(u)$ and the cost of unsatisfied demand, b.

While in traditional yield uncertainty studies the optimal production quantity increases with higher yield variance, an opposite reaction can occur in the particular problem studied here—i.e., the optimal amount of farm space leased can decrease with increased yield variance. This observation can be best exemplified when the yield is set to a point distribution $u = 0.505$ with a probability of 1 (and no variation). In this case the optimal amount to be leased would become $Q^* = 183{,}976$, and the corresponding profit was \$516,665.40. Thus, the optimal amount of farm space to be leased is significantly higher than the optimal solution when the yield variance is higher, and an opposite impact of yield uncertainty is observed. A comparison of the optimal solutions of the no-variance and higher yield variance scenarios validates the results commonly reported in the traditional random yield literature. It should be observed here that when the oil producer leases $Q^* = 183{,}976$ olive trees under the no-variance case, she does not purchase olives from other farmers. In this case, the realized yield would be $Qu = 92{,}907$, which exceeds $TS_1(u) = 88{,}497$, and the producer would not need to purchase more olives. Therefore, we can compare the optimal solution of the no-variance case with that of Case 2 (no purchasing of olives from other farmers) with higher yield variance. The optimal solution recommends leasing $Q^*_{C2} = 189{,}985$ trees, which is higher than the no-variance solution ($Q^* = 183{,}976$). This validates the result that higher yield variance leads to an increase in the optimal production quantity under the traditional yield uncertainty setting. Furthermore, the expected profit under higher yield variance is significantly lower than the no-variance scenario.

One of the key factors that requires the attention of an implementing manager is the relationship between the price of olive oil, the purchasing cost of olives, and the olive yield; namely, $p(u)$ and $c_2(u)$. While it is impossible to know the exact functional forms of $p(u)$ and $c_2(u)$, the producers do not have sufficient data to accurately predict them either. This is due to the fact that olive trees produce fruit every other year making their environment unique from those in repetitive manufacturing cases (where frequent experiments can be conducted to construct such definitions). At this juncture, the model presented here benefits producers by signaling the importance of understanding the relationship between the olive oil price, the purchasing cost of olives, and the yield. Linearly decreasing functions of the yield for $p(u)$ and $c_2(u)$ are defined in the above empirical application. The decision to use linearly decreasing functions is made for two reasons: The first is the above-mentioned lack of data to accurately determine the form of $p(u)$ and $c_2(u)$. The second is the surprising consensus among managers that the profit margin is lower when the yield is high and is higher when the yield is low. This observation is captured by linearly decreasing functions in the yield for $p(u)$ and $c_2(u)$. It should be noted here that the benefits of using this approach would increase when oil producers collect sufficient data to define these relationships more precisely. In different production environments the profit margin can have an opposite reaction to yield than the olive oil market. The approach presented in this paper is general enough to accommodate a wide range of functions that reflect various market conditions.

5. Conclusions and Managerial Insights

This paper presents a model to be used in production planning under yield and demand uncertainty where the sale price and the purchasing cost are dependent on the realized yield. The model is a two-stage stochastic program with recourse, and finds a focused application in the olive oil industry. While a traditional olive oil producer purchases olives from farmers and only experiences the demand uncertainty when producing olive oil, a recent practice of leasing farm space to increase profits introduces an additional yield uncertainty. The model determines the optimal amount of farm space to be leased in the first stage, the amount of olives to be purchased from other growers, and the total amount of olive oil to be produced in the second stage. While the amount of farm space to be leased can be solved optimally, the paper determines the conditions that lead to the optimal choices of olive oil production. Furthermore, it addresses the value of a first chance (leasing farm space) and a second chance (purchasing olives from other farmers) of obtaining olives for oil production. The theoretical findings are tested in an empirical application with real data obtained from a firm.

Our paper shows that the optimal amount of farm space leased by the oil producer decreases under the presence of a second source of supply. While traditional yield uncertainty papers commonly conclude that the initial production investment increases with higher yield variation, this paper shows that the optimal amount of farm space leased by the olive oil producer may decrease. This is primarily due to the second chance of obtaining olives from other farmers when the realized yield is low. This result is also useful for manufacturers. Under the presence of an alternative (reliable) supplier for the same product, manufacturers can utilize this supplier in the cases of lower yield. When practiced, this can reduce their production lot sizes as well.

This paper is the first of its kind in terms of the yield-dependent price and purchasing cost definitions in the area of random yield and demand. Using this definition, it can be extended to other areas of application as well as different environments through future research.

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Appendix

Proof of Proposition 1. We first show that $E_\varepsilon[\Psi(q_1, q_2 \mid Q, u)]$ is concave in q_1 and q_2. The proof uses the first- and second-order derivatives w.r.t. q_1 and q_2.

$$\begin{aligned}\frac{\partial E_\varepsilon[\Psi(q_1,q_2\mid Q,u)]}{\partial q_1} &= (p(u)+b-c_p-h_1)-(p(u)+b-h_2)\\&\quad\cdot F((q_1+q_2)-[K-\beta p(u)]),\\ \frac{\partial^2 E_\varepsilon[\Psi(q_1,q_2\mid Q,u)]}{\partial q_1^2} &= -(p(u)+b-h_2)\\&\quad\cdot f((q_1+q_2)-[K-\beta p(u)])<0,\end{aligned}$$

because $p(u=B)>h_2$ and $p(u)$ is increasing with decreasing values of u, $p(u)>h_2$ for all u values. Similarly,

$$\begin{aligned}\frac{\partial E_\varepsilon[\Psi(q_1,q_2\mid Q,u)]}{\partial q_2} &= (p(u)+b-c_2(u)-c_p)-(p(u)+b-h_2)\\&\quad\cdot F((q_1+q_2)-[K-\beta p(u)]),\\ \frac{\partial^2 E_\varepsilon[\Psi(q_1,q_2\mid Q,u)]}{\partial q_2^2} &= -(p(u)+b-h_2)\\&\quad\cdot f((q_1+q_2)-[K-\beta p(u)])<0,\\ \frac{\partial^2 E_\varepsilon[\Psi(q_1,q_2\mid Q,u)]}{\partial q_1\partial q_2} &= -(p(u)+b-h_2)\\&\quad\cdot f((q_1+q_2)-[K-\beta p(u)])<0.\end{aligned}$$

Furthermore, the Hessian is negative definite. Therefore, $E_\varepsilon[\Psi(q_1, q_2 \mid Q, u)]$ is concave in q_1 and q_2. As a result, the maximand of $E_\varepsilon[\Psi(q_1, q_2 \mid Q, u)]$ for any given Q and u is also concave in q_1 and q_2.

Proof of Proposition 2. First, we write down the expected second-stage return function when $q_1=0$.

$$\begin{aligned}&E_\varepsilon[\Psi(q_1=0, q_2\mid Q,u)]\\&\quad=-c_2(u)q_2-c_pq_2+\int_{-A_1}^{q_2-[K-\beta p(u)]}[p(u)[K-\beta p(u)+\varepsilon]\\&\qquad+h_2(q_2-[K-\beta p(u)+\varepsilon])]f(\varepsilon)\,d\varepsilon\\&\qquad+\int_{q_2-[K-\beta p(u)]}^{A_2}[p(u)q_2-b([K-\beta p(u)+\varepsilon]-(q_2))]f(\varepsilon)\,d\varepsilon.\end{aligned}$$

Because of concavity, the optimal q_2^* can be obtained by equating the first-order derivative to zero.

$$\begin{aligned}\frac{\partial E_\varepsilon[\Psi(q_1=0,q_2\mid Q,u)]}{\partial q_2} &= (p(u)+b-c_2(u)-c_p)-(p(u)+b-h_2)\\&\quad\cdot F(q_2-[K-\beta p(u)])=0\\ F(q_2-[K-\beta p(u)]) &= \frac{p(u)+b-c_2(u)-c_p}{p(u)+b-h_2}\\ q_2^* &= [K-\beta p(u)]\\&\quad+F^{-1}\left(\frac{p(u)+b-c_2(u)-c_p}{p(u)+b-h_2}\right).\end{aligned}$$

Proof of Lemma 3. We first prove Part (a) of the lemma. $\partial s_1(u)/\partial u = [\partial F^{-1}(\tau_1(u))/\partial\tau_1(u)][\partial\tau_1(u)/\partial u]$. Because $F^{-1}(\cdot)$ is an increasing function, $\partial F^{-1}(\tau_1(u))/\partial\tau_1(u)>0$ and the sign of $\partial s_1(u)/\partial u$ is the same as that of $\partial\tau_1(u)/\partial u$.

$$\begin{aligned}\frac{\partial\tau_1(u)}{\partial u} &= \frac{(p'(u)-c_2'(u))(p(u)+b-h_2)-p'(u)(p(u)+b-c_2(u)-c_p)}{(p(u)+b-h_2)^2}\\ &= \frac{(p'(u)-c_2'(u))-p'(u)\tau_1(u)}{(p(u)+b-h_2)}.\end{aligned}$$

Because $p(u)+b-h_2>0$ for all u, the numerator determines the sign of $\partial\tau_1(u)/\partial u$. Note that $p'(u)<0$ and $c_2'(u)<0$. Since $-p'(u)\tau_1(u)>0$, the positivity of $p'(u)-c_2'(u)$ suffices for $\partial\tau_1(u)/\partial u$ to be positive. $p'(u)-c_2'(u)$ is positive when $p'(u)\ge c_2'(u)$. This proves the first condition of Part (a). However, when $p'(u)<c_2'(u)$, we need $(p'(u)-c_2'(u))-p'(u)\tau_1(u)>0$. This is satisfied when $\tau_1(u)>[p'(u)-c_2'(u)]/p'(u)$. Part (b) of the lemma can be proven by enforcing the first-order derivative of $\tau_1(u)$ w.r.t. u to be negative for all values of u. This can only happen when $p'(u)<c_2'(u)$ and $(p'(u)-c_2'(u))-p'(u)\tau_1(u)<0$. The latter is satisfied when $\tau_1(u)<[p'(u)-c_2'(u)]/p'(u)$ for all u.

Proof of Lemma 4. Taking the first-order derivative yields $\partial TS_1(u)/\partial u = -\beta p'(u)+\partial s_1(u)/\partial u$. The previous lemma implies that $s_1(u)$ is strictly increasing in u when conditions (7) and (8) hold. Under (7), because $-\beta p'(u)>0$ and $\partial s_1(u)/\partial u>0$, $\partial TS_1(u)/\partial u>0$. Condition (ii) allows more possibility for the sign of $\partial TS_1(u)/\partial u$ to be positive. For $\partial TS_1(u)/\partial u$ to be positive, we need

$-\beta p'(u) + \partial s_1(u)/\partial u > 0$. Because $F^{-1}(\cdot)$ is an increasing function, $\partial F^{-1}(\tau_1(u))/\partial\tau_1(u) > 0$ and the sign of $\partial s_1(u)/\partial u$ is the same with that of $\partial\tau_1(u)/\partial u$. This means we need $-\beta p'(u) + [\partial F^{-1}(\tau_1(u))/\partial\tau_1(u)][\partial\tau_1(u)/\partial u] > 0$. Because $p'(u) - c_2'(u) < 0$, $-p'(u)\tau_1(u) > 0$,

$$1 + cv_1'(u)\left[\frac{(p'(u) - c_2'(u)) - p'(u)\tau_1(u)}{p(u)+b-h_2}\right] > 0$$

$$\tau_1(u) > \frac{p'(u) - c_2'(u)}{p'(u)} + \frac{p(u)+b-h_2}{p'(u)cv_1'(u)}.$$

Part (b) of the lemma can be proven by changing the sign under the same conditions.

Proof of Proposition 5. Proposition 1 implies that

$$E_\varepsilon[\Psi(q_1, q_2 = 0 \mid Q, u)]$$

is concave in q_1. When constraint set (4) is ignored, the optimal amount of olive oil production from internally grown olives can be found by equating the first-order derivative of $E_\varepsilon[\Psi(q_1, q_2 = 0 \mid Q, u)]$ to zero.

$$\frac{\partial E_\varepsilon[\Psi(q_1, q_2 = 0 \mid Q, u)]}{\partial q_1} = (p(u)+b-c_p-h_1) - (p(u)+b-h_2) \cdot F(q_1 - [K - \beta p(u)]) = 0.$$

$F(q_1 - [K - \beta p(u)]) = (p(u)+b-h_1-c_p)/(p(u)+b-h_2)$ and $q_1^* = [K - \beta p(u)] + F^{-1}((p(u)+b-h_1-c_p)/(p(u)+b-h_2))$. Due to concavity, when (4) is included, the optimal q_1^* becomes

$$q_1^* = \begin{cases} Qu & \text{when} \quad Qu \le [K-\beta p(u)] + F^{-1}\left(\dfrac{p(u)+b-h_1-c_p}{p(u)+b-h_2}\right) \\ [K-\beta p(u)] + F^{-1}\left(\dfrac{p(u)+b-h_1-c_p}{(p(u)+b-h_2)}\right) & \\ & \text{when} \quad Qu > [K-\beta p(u)] + F^{-1}\left(\dfrac{p(u)+b-h_1-c_p}{(p(u)+b-h_2)}\right). \end{cases}$$

Proof of Proposition 6. $\partial s_2(u)/\partial u = [\partial F^{-1}(\tau_2(u))/\partial\tau_2(u)] \cdot [\partial\tau_2(u)/\partial u]$, and because $\partial F^{-1}(\tau_2(u))/\partial\tau_2(u)$ is always positive, it is sufficient to show that $\partial\tau_2(u)/\partial u$ is negative for all u.

$$\frac{\partial\tau_2(u)}{\partial u} = \frac{p'(u)(p(u)+b-h_2) - p'(u)(p(u)+b-h_1-c_p)}{(p(u)+b-h_2)^2} = \frac{p'(u)(h_1+c_p-h_2)}{(p(u)+b-h_2)^2} < 0,$$

because $h_1 + c_p - h_2 > 0$ by definition, $p'(u) < 0$, and $p(u) + b - h_2 > 0$.

Proof of Lemma 7. Because $s_2(u)$ is a strictly decreasing function of u, for $\partial TS_2(u)/\partial u$ to be positive (Part a), we need $-\beta p'(u) > \partial F^{-1}(\tau_2(u))/\partial\tau_2(u)|\partial\tau_2(u)/\partial u|$. This implies $(p(u)+b-h_2)/cv_2'(u) > |p'(u)(1-\tau_2(u))|$ and is satisfied when $(p(u)+b-h_2)/(|p'(u)|cv_2'(u)) > 1 - \tau_2(u)$. Thus $\tau_2(u) > 1 + (p(u)+b-h_2)/(p'(u)cv_2'(u))$ for all values of u. Part (b) can be shown by changing the sign of the inequality.

Proof of Proposition 8. When $Qu > TS_2(u)$, $q_1^* = TS_2(u)$. For the same realization of yield fraction, u, $q_2^* = TS_1(u)$. The difference between the target optimals of Case 1 and Case 2 is

$$q_1^* - q_2^* = F^{-1}\left(\frac{p(u)+b-h_1-c_p}{p(u)+b-h_2}\right) - F^{-1}\left(\frac{p(u)+b-c_2(u)-c_p}{p(u)+b-h_2}\right).$$

$h_1 < c_1$ and $c_1 < c_2(u = B)$ for all values of u, and $(p(u)+b-h_1-c_p) > (p(u)+b-c_2(u)-c_p)$. Because

$$\frac{p(u)+b-h_1-c_p}{p(u)+b-h_2} > \frac{p(u)+b-c_2(u)-c_p}{p(u)+b-h_2},$$

we have

$$F^{-1}\left(\frac{p(u)+b-h_1-c_p}{p(u)+b-h_2}\right) > F^{-1}\left(\frac{p(u)+b-c_2(u)-c_p}{p(u)+b-h_2}\right).$$

Therefore,

$$q_1^* - q_2^* = F^{-1}\left(\frac{p(u)+b-h_1-c_p}{p(u)+b-h_2}\right) - F^{-1}\left(\frac{p(u)+b-c_2(u)-c_p}{p(u)+b-h_2}\right) > 0.$$

Proof of Proposition 9. We first show the proof when $Qu < TS_1(u)$. In this interval of Qu values, each unit of q_2 costs the company $c_2(u)$ units to purchase in the second stage, while the internally grown olives do not cost extra. The comparison of the first-order derivatives w.r.t. q_1 and q_2 shows that the expected second-stage return function increases more by utilizing the internally grown olives. Therefore, the company gives priority to increase q_1 before starting to purchase q_2. Proposition 5 implies that q_1 can be increased up to $TS_2(u)$. However, because Qu is less than $TS_1(u)$, which is less than $TS_2(u)$ as shown in Proposition 8, q_1 can increase only up to Qu. Therefore, the optimal amount of internally grown olives used for olive oil production, q_1^*, is equal to Qu. When this is the case, we can rewrite the expected second-stage return function as follows:

$$\begin{aligned} E_\varepsilon[\Psi(q_1 = Qu, q_2 \mid Q, u)] &= -c_2(u)q_2 - c_p(Qu+q_2) \\ &\quad + \int_{-A_1}^{(Qu+q_2)-[K-\beta p(u)]} \big[p(u)[K-\beta p(u)+\varepsilon] + h_2((Qu+q_2) - [K-\beta p(u)+\varepsilon])\big] f(\varepsilon)\, d\varepsilon \\ &\quad + \int_{(Qu+q_2)-[K-\beta p(u)]}^{A_2} \big[p(u)(Qu+q_2) - b([K-\beta p(u)+\varepsilon] - (Qu+q_2))\big] f(\varepsilon)\, d\varepsilon. \end{aligned}$$

The first-order derivative is sufficient to locate the optimal q_2:

$$\frac{\partial E_\varepsilon[\Psi(q_1=Qu, q_2 \mid Q, u)]}{\partial q_2} = (p(u)+b-c_2(u)-c_p)-(p(u)+b-h_2) \cdot F((Qu+q_2)-[K-\beta p(u)]).$$

Because $F((Qu+q_2)-[K-\beta p(u)]) = (p(u)+b-c_2(u)-c_p)/(p(u)+b-h_2)$, $q_2^* = TS_1(u)-Qu$. This completes the proof for $Qu < TS_1(u)$. In the next two intervals, the optimal values of q_1^* directly follow from Proposition 5.

Proof of Lemma 10. Substituting $Qu = TS_1(u)$ in $E_\varepsilon[\Psi_{R_1}(q_1^*, q_2^* \mid u \in R_1)]$ and $E_\varepsilon[\Psi_{R_2}(q_1^*, q_2^* \mid u \in R_2)]$, and $Qu = TS_2(u)$ in $E_\varepsilon[\Psi_{R_2}(q_1^*, q_2^* \mid u \in R_2)]$ and $E_\varepsilon[\Psi_{R_3}(q_1^*, q_2^* \mid u \in R_3)]$ provides the proof for Parts (a) and (b), respectively.

Proof of Proposition 11. Proof follows Lemma 10.

Proof of Proposition 12. We first show that for any given u, the second-stage return function $PA(Q,u)$ is concave in Q. We already know from Lemma 10 that for any given u, $PA(Q,u)$ is continuous in Q.

$$\frac{\partial E_\varepsilon[\Psi_{R1}(q_1^*, q_2^* \mid Qu \in R_1)]}{\partial Q} = uc_2(u), \quad \text{when } u_1(Q) \geq B$$

$$\frac{\partial E_\varepsilon[\Psi_{R2}(q_1^*, q_2^* \mid Qu \in R_2)]}{\partial Q} = (p(u)+b-c_p)u - u(p(u)+b-h_2) \cdot F(Qu-[K-\beta p(u)]), \quad \text{when } u_1(Q) < B \text{ and } u_2(Q) \geq B$$

$$\frac{\partial E_\varepsilon[\Psi_{R3}(q_1^*, q_2^* \mid Qu \in R_3)]}{\partial Q} = uh_1, \quad \text{when } u_2(Q) < B.$$

It should be observed that $F(Qu-[K-\beta p(u)])$ increases with increasing values of Q, i.e.,

$$\frac{\partial F(Qu-[K-\beta p(u)])}{\partial Q} > 0,$$

and thus

$$\frac{\partial^2 E_\varepsilon[\Psi_{R_2}(q_1^*, q_2^* \mid u \in R_2)]}{\partial Q^2} < 0.$$

Furthermore, $\partial^2 E_\varepsilon[\Psi_{R_1}(q_1^*, q_2^* \mid u \in R_1)]/\partial Q^2$ and $\partial^2 E_\varepsilon[\Psi_{R_3}(q_1^*, q_2^* \mid u \in R_3)]/\partial Q^2$ are both equal to zero. Therefore, the second derivative of $PA(Q,u)$ is less than or equal to zero for any given value of u. Thus, $PA(Q,u)$ is concave in Q for any given value of u.

Equation (15) presents the objective function $E[\Pi(Q)]$ as the integral of concave functions plus a linear term $-c_1Q$. Because integrals of concave functions are also concave (see p. 65 of Boyd and Vandenberghe 2002), $E[\Pi(Q)]$ is concave in Q.

Proof of Proposition 13. For small values of Q close to zero, we know that the objective function is $E[\Pi(Q)] = -c_1Q + \int_0^B [E_\varepsilon[\Psi_{R_1}(q_1^*, q_2^* \mid u \in R_1)]]g(u)\,du$. When this is the case, while the first-order derivative is $\partial E[\Pi(Q)]/\partial Q = -c_1 + \int_0^B [uc_2(u)]g(u)\,du$, the second-order derivative is zero. Therefore, if $\int_0^B [uc_2(u)]g(u)\,du - c_1 > 0$, due to linearity, Q would be increased up to the first point when $u_1(Q) = B$. Due to concavity, this is at least the highest point of $E[\Pi(Q)]$, even if it is not increasing beyond this point. Therefore, $Q=0$ cannot be optimal, and thus $Q^* > 0$.

Proof of Proposition 14. Consider the optimal amount of farm space leased in the case that purchasing olives is not allowed (Case 2), Q_{C2}^*. Because of concavity, the first-order derivative of $E[\Pi_{C2}(Q)]$ determines this optimal amount. Thus,

$$\left.\frac{\partial E[\Pi_{C2}(Q)]}{\partial Q}\right|_{Q=Q_{C2}^*} = -c_1 + h_1\bar{u} + \int_0^{u_2(Q)} \big[(p(u)+b-c_p-h_1)u - u(p(u)+b-h_2) \cdot F(Qu-[K-\beta p(u)])\big]g(u)\,du = 0,$$

where $\bar{u}$ represents the expected value (mean) of the yield random variable. Proposition 5 implies that $F(TS_2(u)-[K-\beta p(u)]) = (p(u)+b-c_p-h_1)/(p(u)+b-h_2)$, and substituting $p(u)+b-c_p-h_1 = (p(u)+b-h_2)F(TS_2(u)-[K-\beta p(u)])$ into above expression we obtain

$$\left.\frac{\partial E[\Pi_{C2}(Q)]}{\partial Q}\right|_{Q=Q_{C2}^*} = -c_1 + h_1\bar{u} + \int_0^{u_2(Q)} \big[u(p(u)+b-h_2)\{F(TS_2(u)-[K-\beta p(u)]) - F(Qu-[K-\beta p(u)])\}\big]g(u)\,du = 0.$$

Note that $c_1 > h_1$, and because $0 < \bar{u} < 1$, $-c_1 + h_1\bar{u} < 0$. We also know that $u(p(u)+b-h_2) > 0$ for all u values. Then, to equate the above expression to zero, we need the integral term to be positive and equal to $c_1 - h_1\bar{u}$. Also note that Q_{C2}^* is such that at $u = u_2(Q)$ we have $Qu = TS_2(u)$. Observe that Q_{C2}^* is such that in the region $0 \leq u < u_2(Q_{C2}^*)$, $TS_2(u) > Q_{C2}^*u$. For example, when $u=0$, $TS_2(u=0) > Q_{C2}^*u = 0$. Indeed, the difference of $TS_2(u) - Qu > 0$ is decreasing with increasing values of $u=0$ to $u=u_2(Q)$. We next calculate the first-order derivative of the original problem at the point when $Q = Q_{C2}^*$, and show that the sign is negative.

$$\left.\frac{\partial E[\Pi(Q)]}{\partial Q}\right|_{Q=Q_{C2}^*} = -c_1 + h_1\bar{u} + \int_0^{u_1(Q)} [uc_2(u) - uh_1]g(u)\,du + \int_{u_1(Q)}^{u_2(Q)} \big[(p(u)+b-c_p-h_1)u - u(p(u)+b-h_2) \cdot F(Qu-[K-\beta p(u)])\big]g(u)\,du.$$

By adding and subtracting $\int_0^{u_1(Q)}[(p(u)+b-c_p-h_1)u-u(p(u)+b-h_2)F(Qu-[K-\beta p(u)])]g(u)\,du$ we obtain

$$\left.\frac{\partial E[\Pi(Q)]}{\partial Q}\right|_{Q=Q^*_{C2}} = \left.\frac{\partial E[\Pi_{C2}(Q)]}{\partial Q}\right|_{Q=Q^*_{C2}} - \int_0^{u_1(Q)}\big[(p(u)+b-c_p-c_2(u))u-u(p(u)+b-h_2) \cdot F(Qu-[K-\beta p(u)])\big]g(u)\,du.$$

Proposition 2 implies that $F(TS_1(u)-[K-\beta p(u)])=(p(u)+b-c_p-c_2(u))/(p(u)+b-h_2)$, so we can write the above expression as

$$\left.\frac{\partial E[\Pi(Q)]}{\partial Q}\right|_{Q=Q^*_{C2}} = -\int_0^{u_1(Q)}\big[u(p(u)+b-h_2)(F(TS_1(u)-[K-\beta p(u)]) - F(Qu-[K-\beta p(u)]))\big]g(u)\,du.$$

Similarly, note that $TS_1(u)=Qu$ at $u=u_1(Q)$, $TS_1(u=0)>Qu=0$ when $u=0$. Thus, for the values of u in between zero and $u_1(Q)$, we get $TS_1(u)>Qu$. Because $u(p(u)+b-h_2)>0$ for all u values, $[u(p(u)+b-h_2)(F(TS_1(u)-[K-\beta p(u)])-F(Qu-[K-\beta p(u)]))]>0$ and

$$\left.\frac{\partial E[\Pi(Q)]}{\partial Q}\right|_{Q=Q^*_{C2}} = -\int_0^{u_1(Q)}\big[u(p(u)+b-h_2)(F(TS_1(u)-[K-\beta p(u)]) - F(Qu-[K-\beta p(u)]))\big]g(u)\,du<0.$$

Since the first-order derivative is negative at $Q=Q^*_{C2}$ in the original problem and the objective function is concave, the optimal amount of farm space leased, Q^*, is less than Q^*_{C2}.

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