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Consumer Heterogeneity, Product Quality, and Distribution Channels

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This paper shows that the effect of different distribution channel structures on product quality depends on the type of consumer heterogeneity and its distribution in a market. When consumer heterogeneity is uniformly distributed either vertically on willingness to pay or horizontally on transaction costs, a manufacturer may provide the same or lower product quality in a decentralized channel than in a centralized channel. In contrast, when consumer heterogeneity follows a more general distribution on willingness to pay, under certain conditions, the manufacturer may provide higher product quality in a decentralized channel than in a centralized channel. Decentralization also may lead to a higher product quality if consumer heterogeneity is uniformly distributed both vertically and horizontally, but not if consumer heterogeneity is uniformly distributed vertically on each of two product-quality attributes. Additionally, competition at the retail level may amplify these findings.

Key words: product quality; consumer heterogeneity; consumer transaction cost; distribution channel; retail-level competition; game theory
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1. Introduction

In principle, a manufacturer can design its distribution channel to sell its products either directly (i.e., in a centralized channel) or indirectly through an independent retailer (i.e., in a decentralized channel). Yet, as a practical matter, channel design often is not a free variable when it comes to rounding out the marketing mix with product design and pricing decisions. For example, in some industries, product design and pricing is constrained by a fixed channel design because one or more aspects of distribution might be regulated by law. Consider the wine industry in the United States. State shipping laws effectively prohibit some wineries that may sell directly to nearby consumers from selling directly to consumers residing in distant states.[1] Likewise, as Xu (2009) alludes, different regulations within different national boundaries might restrict automobile manufacturers from selling directly to consumers in one market (as is the case in the United States, for example) while allowing them to sell directly in a different market (such as in Japan, for example). In industries such as these where laws either directly or indirectly regulate distribution, manufacturers commonly sell different products through different distribution channels. For example, extrapolating the examples above, different winemakers in the United States often produce different quality wines (as measured by grape crops) to sell through different channel structures (Heron 2010) and automobile manufacturers likewise sell different quality products in different countries depending on the market conditions and channel structure available (Buzzavo and Volpato 2001, Xu 2009).

Even when channel structure is regulated neither directly nor indirectly by law, product design and pricing might be constrained by a fixed channel design because rapid innovation or short product life cycles render channel redesign relatively inflexible as a responsive variable. In such cases, again it is common to find different-quality products matched to different channel designs. Interestingly, however, some manufacturers sell their relatively lower-quality products through their centralized channels, whereas some manufacturers sell their relatively higher-quality products through their centralized channels. For example, Land's End and Levi Strauss opt to sell their relatively lower-quality products through their factory outlet stores while reserving their higher-quality

[1] See, for example, http://wineinstitute.org/ for a summary and overview on how different state shipping laws impose restrictions on distribution, thus rendering certain channel design options infeasible for a given winemaker looking to sell its product in different markets.

products for distribution through retail stores (*Consumer Reports* 1998), and an increasing number of personal computer manufacturers likewise have begun to introduce exclusive products characterized by higher quality to sell through certain independent retailers (Lawton 2007). In contrast, Eureka Forbes Ltd. sells its premium brand vacuum cleaners directly while selling its base models through retail channels (Sridharan et al. 2012). Moreover, it also is commonplace to find different quality products associated with different channel structures when two manufacturers sell substitutable products in a common market. Such examples are prevalent across many industries including but not limited to electronics (Dell versus IBM), fashion (Hermes versus AK Anne Klein), apparel (L. L. Bean versus The North Face), and cosmetics (Avon versus Bare Minerals).[2]

These examples of manufacturers selling different quality products in different channel structures help underscore the observation that neither a centralized nor a decentralized channel structure necessarily implies a higher-quality product. Rather, as the above illustrations suggest, channel structure on the one hand, and product design and pricing on the other hand, represent two sides of a matching set. Accordingly, the goal of this paper is to develop a modeling framework to answer the following twofold question: What is the right product for a given channel, and what are key drivers of that determination?

To that end, we develop a game-theoretic model to study the optimal-quality decisions for a manufacturer when it sells its product either in a centralized or a decentralized channel, considering that consumers are either heterogeneous on their willingness to pay for product quality, heterogeneous on their transaction costs, or heterogeneous on both. We adopt this modeling framework, which is defined fundamentally by the notion of consumer heterogeneity, to remain consistent not only with the related academic literature, but also with predominately observed consumer market phenomena. In this context, we say that consumers are heterogeneous on their willingness to pay if they value product quality differently, everything else being equal; and we say they are heterogeneous on their transaction costs if they incur different travel distances, waiting lines, haggling experiences, learning curves, or the like, everything else being equal.[3] Given that, we first model a market where consumers are defined to be heterogeneous on one dimension, either vertically on their willingness to pay or horizontally on their transaction costs. In this one-dimensional heterogeneity model, we use the case of a uniform consumer distribution to obtain baseline results because that is the most prevalent case in related marketing and economics modeling. We then compare and contrast these baseline results to two controlled variations. In one variation, we continue to require consumers to be heterogeneous on only one dimension (either willingness to pay or transaction costs), but we consider a general consumer distribution. In the second variation, we again require consumers to be uniformly distributed along a given dimension of heterogeneity, but we allow them to be heterogeneous on two dimensions, either willingness to pay and transaction costs or willingness to pay for two product-quality attributes. We then synthesize the results from these different variations to develop deeper insights into how the effect of channel structure on product quality is influenced by the type of consumer heterogeneity and its distribution. Finally, we examine the robustness of our results by exploring the extent to which they extend to the case of retail-level competition.

Our analysis yields interesting results. We find that when consumers are heterogeneous only vertically on their willingness to pay and follow a uniform distribution, the manufacturer in a decentralized channel offers the same or lower product quality than in a centralized channel. However, when consumers remain heterogeneous only vertically on their willingness to pay, but follow a general distribution, then depending on the shape of that distribution, the manufacturer could offer higher product quality in a decentralized channel than in a centralized channel. Intuitively, channel decentralization leads to demand recession from double-marginalization, and that demand recession can hurt the manufacturer when consumers follow a general distribution if a disproportionately large volume of consumers with low willingness to pay exit the market. In contrast, when consumers are heterogeneous only horizontally on their transaction costs, the manufacturer's choice of product quality is independent of distribution channel structure, regardless of the distribution of consumer transaction costs. We therefore conclude that vertical heterogeneity is a necessary, but not sufficient, market ingredient for the manufacturer to opt for a higher product quality in a decentralized channel as compared to a centralized channel.

Similarly, we find that when consumers follow a uniform distribution and are heterogeneous vertically on their willingness to pay for two product-quality attributes, the manufacturer in a decentralized channel offers the same or lower product quality than in

[2] In these comparable sets of examples of manufacturers selling substitutable products characterized by different quality levels, the first manufacturer predominately distributes its product directly whereas the second manufacturer predominately distributes its product indirectly.

[3] More broadly construed, consumer transaction costs serve as a proxy for consumer heterogeneity on a horizontal preference such as taste.

a centralized channel. Yet, when consumers follow a uniform distribution and remain heterogeneous vertically on their willingness to pay for one aggregated set of product-quality attributes (which, throughout this paper, we refer to as *product quality*) but are heterogeneous horizontally on their transaction costs, the manufacturer in the decentralized channel again may provide higher product quality than in a centralized channel. Intuitively, channel decentralization leads to demand recession from double-marginalization. That demand recession can hurt the manufacturer when transaction costs coexist with vertical heterogeneity because desirable consumers with high willingness to pay for product quality will exit the market if their transaction costs are disproportionately high. As a result, similar to the case in which consumers are heterogeneous only on their willingness to pay, but follow a general distribution, a manufacturer in a decentralized channel may optimally increase the product quality to make its product more attractive to the density of consumers who highly value the product quality, but with high transaction costs. We therefore conclude that it takes some form of distribution distortion among consumers to combine with vertical heterogeneity for the manufacturer to opt for a higher product quality in a decentralized channel as compared to a centralized channel.

Finally, we find that our results indicating when a manufacturer may increase product quality in a decentralized channel as compared to a centralized channel, depending on the effective population density of consumers, are amplified if retail-level competition is incorporated into the vertical × horizontal heterogeneity model with uniform distribution. Yet it is eliminated if retail-level competition is incorporated into the one-dimensional vertical heterogeneity model with general distribution. To explain this, note that, in the vertical × horizontal heterogeneity model, retail competition mitigates, but does not eliminate, double-marginalization. As a result, the manufacturer in the corresponding decentralized channel may still have the incentive to provide higher product quality because there still exists the benefit of increasing product quality to attract those consumers with high transaction costs if they also have high willingness to pay for product quality. Moreover, under these circumstances, lower retail prices create additional incentive for the manufacturer to increase its product quality because its channel partners, the competing retailers, will be less able to absorb the benefit of the enhanced quality as opposed to a situation without such competition. In contrast, in the vertical heterogeneity models that do not include horizontal heterogeneity as a market ingredient, retail competition *eliminates* double-marginalization, regardless of the dimensionality of the vertical heterogeneity or the distribution that consumers follow. As a result, the manufacturer in the corresponding decentralized channel lacks any incentive to provide higher product quality. We therefore conclude that although the driving mechanisms underlying the one-dimensional vertical heterogeneity model with general distribution and the vertical × horizontal heterogeneity model are similar, they are not equivalent.

This paper is related to two streams of literature. The first stream studies the effects of channel structure and channel coordination on distribution channel relationships (Zusman and Etgar 1981; McGurie and Staelin 1983; Moorthy 1987, 1988b; Jeuland and Shugan 1983, 1988; Lucas 1996; Villas-Boas 1998; Economides 1999; Desai et al. 2004; Raju and Zhang 2005; Luo et al. 2007; Xu 2009; Zhao et al. 2009; Hua et al. 2011; Williams et al. 2011). Among this stream of literature, the following articles are particularly related to this paper because they focus on product design and pricing decisions. Jeuland and Shugan (1983) establish that channel decentralization drives the manufacturer to lower its product quality below the joint maximum level. Similarly, Villas-Boas (1998) shows that channel decentralization leads to lower product quality for low valuation consumers in product line design when consumers are only heterogeneous on their willingness to pay; Economides (1999) shows that two independent vertically related monopolists provide products of lower-quality level than does a sole integrated monopolist when consumers have different willingness to pay. In contrast, Xu (2009) finds that channel decentralization actually could lead to higher product quality depending on the functional specifics of a specialized utility function. However, as a set, these papers do not consider the effects of different types of consumer heterogeneity on a firm's optimal product quality, which is a fundamental focus of our paper so that we can identify and explore key drivers underlying the effects of distribution channels on quality provision.

The second related stream of literature studies the research on product quality in the context of different types of consumer heterogeneities, namely, vertical (e.g., Villas-Boas 1998, Vadenbosch and Weinberg 1995) or two-dimensional (Economides 1989, Neven and Thisse 1990, Desai 2001, Tyagi 2004, Ellison 2005, Hotz and Xiao 2013). Among this stream of literature, the following articles are particularly related to this paper. Desai (2001) studies a monopolist's, as well as two competing firms', product quality and price decisions in a market where consumers, with either high- or low-quality valuations, are continuously heterogeneous on their taste preferences. Economides (1989) and Neven and Thisse (1990) demonstrate max-min differentiation on two competing firms' product decisions when consumers are multidimensionally heterogeneous instead of maximal differentiation when consumers are one-dimensionally heterogeneous. Ellison (2005) and Hotz and Xiao (2013) focus on duopoly competition in a market where consumers are two-dimensionally heterogeneous. Tyagi (2004), which is especially instrumental to our paper because it provides the inspiration to investigate a manufacturer's product-quality decision in different distribution channels when consumers are heterogeneous on two dimensions, studies the effect of two-dimensional consumer heterogeneity on a monopolist's decision making. As a set, the papers that constitute this literature stream do not address the effect of distribution channel structures on firms' optimal-quality decisions, which is a focus of our paper, but they provide the fundamental support and motivation for us to do so.

The remainder of this paper is organized as follows. In §2, we study a manufacturer's product-quality decision in a market when consumers are one-dimensionally heterogeneous, either vertically on their willingness to pay or horizontally on their transaction costs. In §3, we similarly examine a manufacturer's product-quality decision in a market when consumers are two-dimensionally heterogeneous, either vertically on their willingness to pay plus horizontally on their transaction costs, or vertically on their willingness to pay for two product-quality attributes. In §4, we present a unified framework to better pinpoint key drivers behind the results of §§2 and 3. In §5, we discuss the extent to which the main results extend to the case of retail-level competition, and we conclude the paper with §6. Proofs and detailed derivations are included in the appendices.

2. One-Dimensional Consumer Heterogeneity

2.1. Vertical Heterogeneity

Given a manufacturer's unit production cost that is a quadratic function of product quality $c = q^2$ (Moorthy 1988a, Desai 2001, Tyagi 2004),[4] we first consider a market where consumers are vertically heterogeneous with respect to their willingness to pay (θ) for product quality. The mass of consumers in the market is normalized to one. Consumer utility is defined as $u(\theta, q, p) = \theta q - p$, and each consumer with non-negative utility will purchase one unit of product. Next, we examine the effect of distribution channel structure on a manufacturer's product-quality decision first with a uniform consumer distribution, then with a general consumer distribution.

[4] In the online technical appendix (available at http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2166198), we show that the results of this section continue to hold under the more general cost structure $c = C(q)$, where $qC'(q)/C(q) > 0$. Note that this general cost structure includes as a special case $C(q) = bq^2$, where $b > 0$ can be understood as the level of production technology (Desai 2001).

2.1.1. Uniform Distribution. If consumers are uniformly distributed over $\theta \in [a, 1]$, where $0 \le a < 1$, then for given product quality q and retail price p, consumers who are located at $\theta \in [\max\{a, p/q\}, 1]$ will purchase the product. The demand function thus is derived as $D = \min\{(1/(1-a))(1-p/q), 1\}$.

When the manufacturer sells its product directly to end consumers, its profit is given by $\pi_{vc} = (p - q^2)D$. Jointly maximizing π_{vc} over p and q, we therefore obtain the optimal product quality for the centralized channel $q^*_{vc} = 1/3$ if $a \le 2/3$ and $q^*_{vc} = a/2$ if $2/3 < a < 1$. In contrast, when the manufacturer sells its product through an independent retailer to end consumers, the manufacturer first sets the product quality q and wholesale price w, and then the retailer sets the retail price p. The channel members' respective profit functions are given by $\pi_{vm} = (w - q^2)D$ and $\pi_{vr} = (p - w)D$. Solving the problem based on backward induction, we therefore obtain the optimal product quality $q^*_{vd} = 1/3$ if $a \le 5/6$ and $q^*_{vd} = (2a-1)/2$ if $5/6 < a < 1$. Direct comparison thus indicates that $q^*_{vd} \le q^*_{vc}$ if consumers are heterogeneous only vertically on their willingness to pay and follow a uniform distribution. This determination is consistent with literature (e.g., Jeuland and Shugan 1983, Villas-Boas 1998, Economides 1999) and provides a baseline for subsequent analysis to isolate and explore key drivers underlying the effects of distribution channel on product quality. In that regard, the first variation we consider is the case of a general consumer distribution, ceteris paribus.

2.1.2. General Distribution. In this section, we now allow consumers' willingness to pay to follow a general distribution. Accordingly, let $\theta \in [0, 1]$ with cumulative distribution function $F(\theta)$. The demand function is thus derived as $D = \Pr\{\theta q - p \ge 0\} = 1 - F(p/q) = \bar{F}(p/q)$.

In the centralized channel, analogous to §2.1.1, the manufacturer's profit function is given by $\pi_{vc} = (p - q^2)\bar{F}(p/q)$. Accordingly, we maximize the manufacturer's profit π_{vc} jointly over p and q by solving the two first-order conditions $\partial \pi_{vc}/\partial p = \bar{F}(p/q) + \bar{F}'(p/q)(p - q^2)/q = 0$ and $\partial \pi_{vc}/\partial q = -2q\bar{F}(p/q) - (p - q^2)p\bar{F}'(p/q)/q^2 = 0$. Let $s = p/q$ denote the manufacturer's retail price per unit quality, and let $\epsilon(s) = -s\bar{F}'(s)/\bar{F}(s)$ denote the elasticity of demand associated with that price per unit quality. Then $\partial \pi_{vc}/\partial p = 0$ reduces to $q = [\epsilon(s) - 1]s/\epsilon(s)$ and $\partial \pi_{vc}/\partial q = 0$ reduces to $-2q + (s - q)\epsilon(s) = 0$. Given this, let q^*_{vc} be the optimal product quality. Then $q^*_{vc} = \sqrt{p^*_{vc}/2}$.

In the decentralized channel, again analogous to §2.1.1, the manufacturer's and retailer's profit functions are $\pi_{vm} = (w - q^2)\bar{F}(p/q)$ and $\pi_{vr} = (p - w) \cdot \bar{F}(p/q)$, respectively. Following backward induction, we maximize the retailer's profit first. The first-order condition is given by $d\pi_{vr}/dp = \bar{F}(p/q) + \bar{F}'(p/q) \cdot (p - w)/q = 0$ which, analogous to above, reduces to $p[1 - 1/\epsilon(p/q)] = w$. This gives the retailer's best pricing response as an implicit function of (w, q). Accordingly, from the manufacturer's perspective, choosing w can be thought of as implicitly choosing the retail price p through this response function. In other words, knowing the retailer's best response, the manufacturer's profit function can be rewritten as a function of q and p as follows: $\pi_{vm} = [p(1 - 1/\epsilon(p/q)) - q^2]\bar{F}(p/q)$. Given this, the manufacturer implicitly maximizes its profit by jointly solving the two first-order conditions

$$\frac{\partial \pi_{vm}}{\partial p} = \left[1 - \frac{1}{\epsilon(p/q)} + \frac{p\epsilon'(p/q)}{q\epsilon^2(p/q)}\right]\bar{F}\left(\frac{p}{q}\right) + \bar{F}'\left(\frac{p}{q}\right)\frac{p - p/\epsilon(p/q) - q^2}{q} = 0 \quad \text{and}$$

$$\frac{\partial \pi_{vm}}{\partial q} = \left[-\frac{p^2\epsilon'(p/q)}{q^2\epsilon^2(p/q)} - 2q\right]\bar{F}\left(\frac{p}{q}\right) - \bar{F}'\left(\frac{p}{q}\right)\frac{p}{q^2}\left[p - \frac{p}{\epsilon(p/q)} - q^2\right] = 0.$$

Let $X(s) = s(\epsilon(s) - 1)/\epsilon(s)$. Note that, analogous to s in the centralized case above, $X(s)$ denotes the manufacturer's *wholesale* price per unit quality. Thus, $\partial\pi_{vm}/\partial p = 0$ reduces to $q = [1/\epsilon(s) - s\epsilon'(s)/\epsilon^2(s) + \epsilon(s) - 2](s/\epsilon(s))$ and $\partial\pi_{vm}/\partial q = 0$ reduces to $2q = -s^2\epsilon'(s)/\epsilon^2(s) + \epsilon(s)[s - q - s/\epsilon(s)]$. Given this, let q^*_{vd} be the optimal product quality. Then we have the following proposition, the proof of which is in the appendix.

Proposition 1. *If consumers are heterogeneous only vertically on their willingness to pay, but consumers follow general distribution $F(\theta)$, then channel decentralization leads to higher product quality (i.e., $q^*_{vd} > q^*_{vc}$) if $\bar{F}(s)$ is such that* (a) $\epsilon'(s) > 0$ *and* (b) $\bar{F}(s)/\bar{F}(X(s))$ *is increasing in s.*

Proposition 1, in effect, establishes sufficient conditions on demand that, if satisfied, lead the manufacturer to optimally choose higher product quality under a decentralized channel as compared to under a centralized channel.[5] Of these conditions, the first ($\epsilon'(s) > 0$) is standard in the sense that increasing elasticity is a typical property of demand functions prevalent in economics and marketing. For insight into the second condition, recall that s represents the price per unit quality that the manufacturer charges (its consumers) in a centralized channel, whereas $X(s)$ represents the analogous price per unit quality that the manufacturer charges (its retailer) in a decentralized channel. Accordingly, condition (b) indicates that, for the manufacturer to increase product quality under decentralization, it suffices for the consumer market to be such that the ratio of the manufacturer's demand under consumer pricing (per unit quality) to the manufacturer's demand under wholesale pricing (per unit quality) be increasing. In other words, when the market is such that a per-quality-unit increase in wholesale price has smaller relative (negative) impact on demand than the equivalent per-quality-unit increase in consumer price, the manufacturer will respond by increasing its product quality.

Given Proposition 1, note that, consistent with the baseline comparison of §2.1.1, the uniform distribution does not meet the stated conditions for achieving $q^*_{vd} > q^*_{vc}$. In particular, although a uniform distribution implies that $\epsilon(s) = s/(1-s)$, thus satisfying condition (a), $\bar{F}(s)/\bar{F}(X(s)) = (1-s)/(2(1-s)) = 1/2$ which does not satisfy condition (b). Nevertheless, as the following corollary illustrates, the conditions of Proposition 1 can be satisfied with even simple distributions.[6]

Corollary 1. *If consumers follow the Power distribution, $F(\theta) = \theta^k$, then $q^*_{vd} > q^*_{vc}$ when $k < 1$.*

Intuitively, the driving force behind Proposition 1 and Corollary 1 is as follows: When the market has a relatively higher concentration of lower willingness-to-pay consumers as compared to a relatively lower concentration of higher willingness-to-pay consumers, which is the case for a Power distribution when $k < 1$, double-marginalization leaves a disproportionately large volume of consumers unserved because they have relatively low willingness to pay. Hence, the manufacturer's optimal response is to provide higher product quality to retain this large volume of consumers. As an illustration of this intuition, consider the apparel industry vis-à-vis the high fashion industry. Consistent with Proposition 1 and the discussion that follows it, for apparel products like jeans, the market for which can be characterized by a relatively higher concentration of lower willingness-to-pay consumers, it is typical to find higher-quality products sold in independently owned department stores as compared to those sold in manufacturer-owned outlet stores. In contrast, for high fashion products, the market for which can

[5] Note that, although this result is consistent with Xu (2009), it differs at a basic level. Whereas Xu (2009) formulates a special type of consumer utility function, $u(Q, p) = \beta(Q) - \alpha(Q)p + \epsilon$, we adopt the traditional consumer utility function, $u(\theta, q, p) = \theta q - p$ that is most prevalent in the marketing literature.

[6] Note that, to keep this section consistent with the others, here we restrict θ to be such that $\theta \in [0, 1]$. However, if this restriction were relaxed so that θ were characterized simply as $\theta \sim F(\theta)$, then another simple example that satisfies the conditions of Proposition 2 is the Weibull distribution, $F(\theta) = 1 - e^{\lambda\theta^k}$, with shape parameter $k < 1$.

be characterized by a relatively higher concentration of higher willingness-to-pay consumers, it is often the case that comparatively higher-quality products (like those manufactured by Hermes, for example) are sold in manufacturer-owned boutique shops whereas comparatively lower-quality products (like those manufactured by AK Anne Klein, for example) are sold in independently owned department stores.

Given that the results of this section indicate that the manufacturer potentially has the incentive to increase its product quality under channel decentralization, depending on consumer density in the market, one may ask whether this incentive will remain if consumers are heterogeneous only horizontally along transaction costs in lieu of only vertically along willingness to pay. To answer this question, we next analyze the horizontal heterogeneity case.

2.2. Horizontal Heterogeneity

We consider a market where consumers are not vertically heterogeneous with respect to their willingness to pay, but instead are horizontally heterogeneous with respect to their transaction costs. Specifically, consumer utility is defined by $u(q, p, t, x) = q - p - tx$, where $t > 0$ is the unit transaction cost and x follows a general distribution over a Hotelling line $x \in [0, 1]$ with cumulative density function $G(x)$ (Hotelling 1929).[7] The store is located at $x = 0$. Given that a consumer with nonnegative utility will purchase one unit of product, the demand function is derived as $D = \Pr\{q - p - tx \geq 0\} = G((q-p)/t)$. Accordingly, let q^*_{hc} and q^*_{hd} be the optimal product quality in the centralized and decentralized channels, respectively. Then the following proposition, the proof of which is in the appendix, summarizes the comparison between these two optimal qualities.

Proposition 2. *If consumers are heterogeneous only horizontally on their transaction costs, then regardless of the distribution that consumers follow, channel structure does not affect product quality. Specifically,* $q^*_{hc} = q^*_{hd} = 1/2$.

Proposition 2 indicates that horizontal heterogeneity, taken alone, does not influence the effect that channel structure has on the product-quality decision. Regardless of the distribution of consumers, product quality is independent of channel structure when consumers are horizontally heterogeneous, but not vertically heterogeneous. This result is in contrast to Proposition 1, which indicates that product quality does depend on channel structure when consumers are vertically heterogeneous, but not horizontally heterogeneous, depending on the distribution of consumers. One way to explain this is as follows: When consumers vary on their willingness to pay, the manufacturer's action of increasing product quality affects consumers differently. Thus, the manufacturer can effectively use this decision to exploit the market opportunity of consumer density generated from a general distribution. If, instead, all consumers have the same willingness to pay, as is the case when consumers are heterogeneous only horizontally on their transaction costs, the manufacturer's action of increasing product quality affects all the consumers the same. Hence, without vertical heterogeneity, the manufacturer lacks the incentive to provide higher product quality under decentralization regardless of the distribution of consumers.

Taken together, Propositions 1 and 2 indicate that vertical heterogeneity is necessary for the manufacturer to choose higher product quality under decentralization. Yet, as §2.1.1 establishes, vertical heterogeneity alone is not sufficient to yield that choice. Rather, to establish $q^*_{vd} > q^*_{vc}$, vertical heterogeneity needs to be combined, for example, with a general distribution of consumers' willingness to pay. This therefore begs the question: If consumers are vertically heterogeneous, is this general distribution not only sufficient, but also necessary to establish that $q^*_{vd} > q^*_{vc}$? To explore that question, we next consider two variants of the case in which consumers are two-dimensionally heterogeneous, but follow a uniform distribution along each dimension of heterogeneity.

3. Two-Dimensional Consumer Heterogeneity

In this section, we consider the variation to the baseline model of §2.1.1 that mirrors the variation from §2.1.2 by now requiring that consumers follow a uniform distribution along a given dimension of heterogeneity, but allowing for consumers to be heterogeneous along two dimensions. In particular, we first consider the case in which one of the two dimensions is defined by horizontal heterogeneity (§3.1). We then consider the case in which neither of the two dimensions is defined by horizontal heterogeneity (§3.2).

3.1. Vertical × Horizontal

When consumers are two-dimensionally heterogeneous with one vertical dimension and one horizontal dimension, their heterogeneity is captured by a square model that has been well established in literature (e.g., Neven and Thisse 1990, Tyagi 2004). In this square model, the vertical dimension captures consumers' heterogeneity on willingness to pay ($\theta \in [0, 1]$), and

[7] The term tx can be understood as other than consumer transaction costs. For example, it can be consumers' disutility when the product's horizontal feature is different from their ideal preferences. In this case, t can be explained as the weight consumers have on their disutility from horizontal mismatch. Because this weight t is relative, it can vary and serve as a parameter.

the horizontal dimension captures consumers' heterogeneity on transaction costs ($x \in [0, 1]$). A unit mass of consumers is uniformly distributed over the unit square, and the retailer is located at $x = 0$.

Consistent with §§2.1 and 2.2 and following the related literature involving two-dimensional consumer heterogeneity (Desai 2001, Tyagi 2004, Ellison 2005, Hotz and Xiao 2013), we define consumer utility as follows. Given product quality q and retail price p, for a consumer located at (θ, x), utility from buying the product is given by $u(\theta, x, q, t, p) = \theta q - p - tx$. Accordingly, consumers located on the line $s(x) = (p + tx)/q$ are marginal consumers who are indifferent between purchasing the product and not purchasing. Note that, from a consumer's perspective, $s(x)$ represents the total cost per unit quality that a consumer with transaction cost x incurs, including transaction costs, to obtain the product. Thus, all consumers with willingness to pay greater than this total cost will purchase the product. Accordingly, given product quality q and retail price p, the manufacturer's demand is therefore defined as $D = 1 - p/q - t/(2q)$ if $q - p \geq t$ and $D = (q - p)^2/(2qt)$ if $q - p < t$.[8]

In the centralized channel, we maximize the manufacturer's profit $\pi_{vhc} = (p - q^2)D$ jointly over p and q to establish the associated optimal product quality as $q^*_{vhc} = (1 + \sqrt{1 + 6t})/6$ if $0 < t \leq 4/25$ and $q^*_{vhc} = 2/5$ if $t > 4/25$.[9] In the decentralized channel, the manufacturer sets the wholesale price w and product quality q to maximize its profit $\pi_{vhm} = (w - q^2)D$, knowing that the retailer responds optimally by setting the retail price p to maximize its profit $\pi_{vhr} = (p - w)D$. Accordingly, we obtain the following optimal product quality (see the proof of Proposition 3 in the appendix for derivation details): $q^*_{vhd} = (1 + \sqrt{1 + 6t})/6$ if $0 < t \leq 3/32$, $q^*_{vhd} = \sqrt{3t/2}$ if $3/32 < t \leq 8/75$, and $q^*_{vhd} = 2/5$ if $t > 8/75$. By directly comparing q^*_{vhd} with q^*_{vhc}, we therefore have the following result.

Proposition 3. *If consumers are heterogeneous both vertically on their willingness to pay and horizontally on their transaction costs, but follow a uniform distribution along each dimension of heterogeneity, then channel decentralization leads to higher product quality* (*i.e.*, $q^*_{vhd} > q^*_{vhc}$) *if* $3/32 < t < 4/25$.

[8] When the consumer with the highest willingness to pay for product quality and the largest transaction cost $(\theta, x) = (1, 1)$ buys a product, $u(\theta, x, q, t, p) = q \times 1 - p - t \times 1 \geq 0$, which means $q - p \geq t$. In this case, $D = 1 - p/q - t/(2q)$. When the consumer with the highest willingness to pay for product quality and the largest transaction cost $(\theta, x) = (1, 1)$ does not buy a product, $u(\theta, x, q, t, p) = q \times 1 - p - t \times 1 < 0$, which means $q - p < t$. In this case, $D = (q - p)^2/(2qt)$.

[9] Note that for $0 < t \leq 4/25$, the problem in the centralized channel has been solved in Tyagi (2004).

Proposition 3 shows that a manufacturer may produce higher product quality in a decentralized channel as compared to a centralized channel even when consumer heterogeneity follows a uniform distribution, if horizontal heterogeneity is combined with vertical heterogeneity. This happens when the unit transaction cost is intermediate ($3/32 < t < 4/25$). The intuition behind Proposition 3 is as follows: When the unit transaction cost is intermediate ($3/32 < t < 4/25$), the manufacturer in the centralized channel can optimally decide its quality and price such that not only do all consumers with the highest willingness to pay ($\theta = 1$) remain in the market, but also some consumers with the largest transaction cost ($x = 1$) remain as well, if their willingness to pay is sufficiently high. However, in a decentralized channel, double-marginalization leads to a higher retail price; thus, it is more difficult to retain the same consumers that the centralized channel retains in the market. As a result, some consumers with large transaction costs exit the market even if they have relatively high willingness to pay. Because those consumers with a high willingness to pay are profitable to the manufacturer, the manufacturer has the incentive to retain those consumers in the decentralized channel. To do so, the manufacturer has two options. The first is to decrease its wholesale price, and the second is to increase its product quality. The first option, lowering the wholesale price, would benefit all consumers the same way, including those consumers who otherwise would buy the product anyway. Consequently, a lower wholesale price would reduce the surplus the manufacturer can extract from each consumer it retains. In contrast, the second option, increasing product quality, is efficient for targeting the consumers with a high willingness to pay because they value quality increases more than do consumers with a low willingness to pay. Therefore, a decentralized channel induces the manufacturer to increase its product quality rather than to lower its wholesale price to retain those consumers with a high willingness to pay.

Consider the personal computer industry as an illustration. In that industry, consumers can be vertically heterogeneous on willingness to pay in the sense that different consumers may value product quality differently based on their individual needs, and consumers can be heterogeneous on transaction costs in the sense that consumers may have different learning curves. With personal computers, individual learning curves can be considerable but typically not excessive. Thus, consistent with Proposition 3 and the discussion that follows it, it makes sense that manufacturers that mainly sell indirectly through decentralized channels (like IBM, for example) produce products with comparatively higher quality, while manufacturers that mainly sell directly through centralized channels (like Dell, for example) produce products with comparatively lower quality. Similarly, in the apparel industry, consumers can be vertically heterogeneous in the sense that their willingness to pay for fabric quality could differ, and they can be horizontally heterogeneous in the sense that their preferences for styles and colors could differ. Thus, to the extent that consumers' unit disutility from style and color mismatches is relatively intermediate as compared to their willingness to pay for product quality, Proposition 3 would help explain the relatively higher-quality apparel that typically is found in department stores as compared to outlet stores.

3.2. Vertical × Vertical

When consumers are two-dimensionally heterogeneous, but neither dimension is defined by horizontal heterogeneity, their heterogeneity can be captured by a weighted average of their willingness to pay for each of two product-quality attributes. In this case, consumer utility can be defined as $u(\theta_1, \theta_2, q_1, q_2, p) = m\theta_1 q_1 + (1-m)\theta_2 q_2 - p$, where $0 < m \le 1/2$, q_1 and q_2 are measures of two vertical attributes of the product, p is the retail price, and both θ_1 and θ_2 follow a uniform distribution along $[0, 1]$. Given this, to keep the comparisons analogous to previous sections, we follow Economides (1999) by defining product quality as $q = \min\{q_1, q_2\}$.[10] Accordingly, the optimal design of the product-quality attributes is to set $q_1 = q_2 = q$. Thus, we can rewrite the consumer utility function as $u(\theta_1, \theta_2, q_1, q_2, p) = Vq - p$, where $V \equiv m\theta_1 + (1-m)\theta_2$ follows a trapezoidal distribution.[11]

Let $F(v)$ denote the cumulative distribution of V and let $s = p/q$ denote the price per unit quality as in §2. Then, given that $F(v)$ is characterized by a trapezoidal distribution, the demand function is derived as

$$\bar{F}(s) = \begin{cases} 1 - \dfrac{s^2}{2m(1-m)}, & \text{if } 0 \le s \le m; \\[2ex] 1 - \dfrac{2s-m}{2(1-m)}, & \text{if } m < s \le 1-m; \\[2ex] \dfrac{(1-s)^2}{2m(1-m)}, & \text{if } 1-m < s \le 1. \end{cases} \tag{1}$$

[10] This definition is directly applicable if a product with multiple functional attributes is such that the failure of any given product attribute directly leads to the failure of the product as a whole. Nevertheless, the specifics that follow in this section remain valid for any definition of product quality that would result in $q_1 = q_2$ as the corresponding optimal design of the product-quality attributes. For cases of more general (asymmetric) mappings of product-quality attributes, we point to the applicability of Proposition 5 in §4.

[11] Note that this distribution reduces to a triangular distribution for the special case in which the weights of the product attributes are equal (i.e., $m = 1/2$).

Given (1), in the centralized channel we maximize the manufacturer's profit $\pi_{vvc} = q(s-q)\bar{F}(s)$ jointly over q and s to obtain the associated optimal quality $q^*_{vvc} = (2-m)/6$ (see the online technical appendix for details). In the decentralized channel, using backward induction, we first maximize the retailer's profit $\pi_{vvr} = q(s - w/q)\bar{F}(s)$ with respect to s to obtain the retailer's optimal response function $s(q, w)$, and then maximize the manufacturer's profit $\pi_{vvm} = q(w/q - q)\bar{F}(s(q, w))$ jointly over w and q to obtain the associated optimal quality (see the online technical appendix for details) $q^*_{vvd} = (2-m)/6$ if $0 < m \le 2/7$, $q^*_{vvd} = (2-3m)/4$ if $2/7 < m \le 1/3$, and $q^*_{vvd} = 1/4$ if $1/3 < m \le 1/2$. Direct comparison of q^*_{vvc} and q^*_{vvd} then leads to the following result.

Proposition 4. *If consumers are heterogeneous vertically on their willingness to pay for two product-quality attributes and follow a uniform distribution on each of these two dimensions, then channel decentralization leads to the same or lower-product quality, i.e.,* $q^*_{vvd} \le q^*_{vvc}$.

Proposition 4 indicates that two-dimensional heterogeneity, in and of itself, is not sufficient for the manufacturer to optimally choose higher quality under a decentralized channel, as compared to under a centralized channel, if consumers follow a uniform distribution along each dimension. Thus, qualitatively, Proposition 4 hones Proposition 3 similar to the way that Proposition 2 hones Proposition 1. In particular, Proposition 1 establishes the possibility that decentralization could lead to higher product quality if consumers follow a general distribution as opposed to a uniform distribution, but then Proposition 2 helps clarify that vertical heterogeneity, and not horizontal heterogeneity, is necessary for that conclusion to be true. Similarly, if consumers are vertically heterogeneous, Proposition 3 establishes the possibility that decentralization could lead to higher product quality even if consumers follow a uniform distribution, if they also happen to be heterogeneous on a second dimension. However, Proposition 4 then clarifies that the added dimension needs to be horizontal heterogeneity and not vertical heterogeneity for that conclusion to be true. Thus Propositions 1–4 combine to establish that a decentralized channel can provide higher product quality than a centralized channel, but for that to occur, two market ingredients are necessary. Specifically, as the first market ingredient, consumers need to be heterogeneous vertically on their willingness to pay and, as the second market ingredient, consumers need either to follow a general (i.e., nonuniform) distribution along that vertical dimension of heterogeneity or they need to be horizontally heterogeneous as a complement to their vertical heterogeneity.

To better understand the combination of market forces that leads to higher product quality in a decentralized channel, it is useful to probe the qualitative similarities between Propositions 1 and 3. Like Proposition 3, Proposition 1 illustrates how product quality can be used as a lever to mitigate double-marginalization. Unlike in Proposition 3, however, in Proposition 1 that lever arises not because consumers' vertical heterogeneity is complemented by horizontal heterogeneity, but rather because consumers' vertical heterogeneity is spread unevenly along its spectrum. Nonetheless, a common thread to Propositions 1 and 3 is that an asymmetry of consumers precipitates different market coverage mappings when viewed through the respective lenses of decentralization and centralization. Such asymmetry can occur in different ways, however. For example, in the case of two-dimensional heterogeneity on willingness to pay and transaction costs, such asymmetry effectively occurs as a result of double-marginalization because consumers with high willingness to pay otherwise would leave the market if their transaction costs are disproportionately high. In the case of one-dimensional heterogeneity on willingness to pay, similar asymmetry effectively occurs as a result of double-marginalization because a disproportionately high volume of consumers with low willingness to pay would otherwise leave the market. It is with this intuition in mind that we present a unified framework next, in §4, to synthesize and characterize the key drivers behind matching the right product to a given channel structure.

4. Unified Framework

In this section,[12] we summarize and unify the results and insights from §§2 and 3 to better understand the root drivers behind the determination of the right product for a given channel structure, especially those that lead a decentralized channel to provide a higher-quality product than a centralized channel. Toward that determination, we begin by comparing and contrasting the two-dimensional vertical × vertical heterogeneity model from §3.2 with its one-dimensional vertical counterpart from §2.1. Recall from §3.2 that although the consumer utility function is defined by consumers following a uniform distribution on each of two dimensions of willingness to pay for different product-quality attributes, that utility function can be reduced to one in which consumers follow a trapezoidal (i.e., nonuniform) distribution on a single dimension of willingness to pay for one aggregated set of product-quality attributes by optimizing over the two product attributes (q_1, q_2). As such, the resulting reduced utility function from the vertical × vertical case in §3.2 can be viewedas a special case of that from the one-dimensional vertical heterogeneity model in §2.1.2. Accordingly, Proposition 1 from §2.1.2 can be applied directly to the trapezoidal distribution defined by (1) to determine whether it is sufficient to lead the manufacturer to optimally choose higher product quality under a decentralized channel compared to under a centralized channel. Given that, notice from (1) that although the trapezoidal distribution satisfies condition (a) of Proposition 1, it does not satisfy condition (b). Thus, it makes sense that Proposition 4 prescribes $q^*_{vvd} \leq q^*_{vvc}$ for the vertical × vertical case.

By the same token, the two-dimensional vertical × horizontal model from §3.1 can be mapped, in principle, to an equivalent one-dimensional heterogeneity model in which consumers follow a general distribution on willingness to pay. By leveraging this notion more broadly, we point to the utility function as the common thread that synthesizes and extrapolates the models of the previous sections. In particular, we note that each of the different models of heterogeneity studied previously can be defined in general by the following unified utility function: $u = V(q) \times q - p$, where $V(q) = \Theta - t\chi/q$ can be interpreted as consumers' willingness to pay. Note that in this construct, $V(q)$ is a compound random variable defined as such because either Θ or χ or both are random variables that represent, respectively, consumers' vertical heterogeneity, horizontal heterogeneity, or both. Therefore, the models and results of §§2 and 3 can be synthesized and cataloged by the distributional specifications of Θ and χ, as summarized in Table 1.

Consistent with the discussions following Propositions 1–4, Table 1 underscores the observation that vertical heterogeneity is a necessary market component if the manufacturer is to consider providing a higher-quality product in a decentralized channel as compared to in a centralized channel. However, given that necessary component, Table 1 further illustrates that the consumer heterogeneity forces required to drive the manufacturer to match a higher-quality product with channel decentralization appear to be twofold: First, a nonuniform distribution of consumers' willingness to pay is required. Second, nonuniform distribution either needs to satisfy the shape characteristics specified by Proposition 1 or it needs to be a strictly increasing function of q. In other words, given that consumers are vertically heterogeneous and consumer utility can be defined as $u = V(q) \times q - p$, the answer to whether channel decentralization can lead to higher product quality appears to depend on two fundamental elements, namely, whether the willingness-to-pay distribution has the "right" shape (as defined in this context by Proposition 1) and, if not, then whether the manufacturer can "reshape" that distribution (as defined in this context by $V'(q) > 0$).

[12] We owe special thanks to an anonymous referee for comments that spurred this line of thinking.

Table 1 **Cataloging of Heterogeneity Models and Results**

Section ref.	(Vertical) distribution of Θ	(Horizontal) distribution of χ	Resulting distribution of $V(q) = \Theta - t\chi/q$	Note: Does distribution of $V(q)$ satisfy Proposition 1(b)?	Note: Is $V'(q) > 0$?	Is $q_d^* > q_c^*$ possible?
2.1.1	Uniform	None	Uniform	No	No	No
3.2	Not uniform	None	Not uniform	No	No	No
2.1.2	Not uniform	None	Not uniform	Yes	No	Yes
3.1	Uniform	Uniform	Not uniform	No	Yes	Yes
2.2	None	Not uniform	Not uniform			No

Given these qualitative observations, we conclude this section with Proposition 5, which generalizes Table 1 and the results of §§2 and 3 from which it derives. To frame that generalization, let $u = V(q) \times q - p$ denote consumer utility, where $V(q)$ is a random variable that is defined to be nonincreasing in q. Moreover, let $\bar{F}_q(s)$ denote the cumulative distribution function of $V(q)$ for a given q so that $\bar{F}_q(s)$ represents consumer demand, and define associated demand elasticity measures as follows: $\epsilon_s(s,q) = -(s/\bar{F}_q(s))\partial\bar{F}_q(s)/\partial s \geq 0$ and $\epsilon_q(s,q) = (q/\bar{F}_q(s))\partial\bar{F}_q(s)/\partial q \geq 0$. Consistent with Proposition 1(a), we assume that $\partial\epsilon_s(s,q)/\partial s \geq 0 \geq \partial\epsilon_s(s,q)/\partial q$.

Finally, let $\pi_{gc}(q) = q[s_c(q) - q]\bar{F}_q(s_c(q))$ denote the manufacturer's profit function in the centralized channel given that $s_c(q)$ specifies the centralized manufacturer's optimal price per unit quality for a given q; and let $\pi_{gm}(q) = q[X_d(q) - q]\bar{F}_q(s_d(q))$ analogously denote the manufacturer's profit function in the decentralized channel given that $X_d(q) = s_d(q) \cdot (1 - 1/\epsilon_s(s_d(q),q))$ specifies the decentralized manufacturer's optimal wholesale price per unit quality for a given q, and that $s_d(q)$ specifies the decentralized retailer's optimal retail price per unit quality for a given q set in response to $X(q)$. Then the following is true (proof provided in the appendix):

Proposition 5. *If either $\pi_{gc}(q)$ or $\pi_{gm}(q)$ or both are unimodal functions of q, then $q_d^* > q_c^*$ if $T_d(q) > T_c(q)$, and $q_d^* \leq q_c^*$ if $T_d(q) \leq T_c(q)$, where*

$$T_c(q) = \frac{s_c(q)[1 + \epsilon_q(s_c(q), q)]}{2 + \epsilon_q(s_c(q), q)},$$

$$T_d(q) = \frac{X_d(q)[1 + \epsilon_q(s_d(q), q)]}{2 + \epsilon_q(s_d(q), q) - N(q)}, \quad \textit{and}$$

$$N(q) = \left[\frac{s}{\epsilon_s(s,q)} \frac{\partial\epsilon_s(s,q)}{\partial q}\right]_{s_d(q)}.$$

To demonstrate how Proposition 5 applies in general, consider the specifics from Corollary 1 to Proposition 1 as a special case. If $F_q(s) = s^k$, where $k < 1$, then $\epsilon_s(s,q) = ks^k/(1-s^k)$ and $\epsilon_q(s,q) = 0$. Moreover, $\partial\epsilon_s(s,q)/\partial q = 0$. Accordingly, $s_c(q) = \arg\max_s\{q(s-q) \cdot (1-s^k)\}$ and $s_d(q) = \arg\max_s\{q(s(1-1/\epsilon_s(s,q)) - q) \cdot (1-s^k)\}$ together imply that $X_d(q) = s_d(q)(1 - 1/\epsilon_s(s_d(q),q)) > s_c(q)$. Therefore, $T_c(q) = s_c(q)/2 < X_d(q)/2 = T_d(q)$. Moreover, $\pi_{gc}(q) = q(s_c(q) - q) \cdot (1 - s_c(q)^k)$ is a unimodal function of q. Thus, Proposition 5 applies directly, and Corollary 1 follows accordingly. In a similar vein, Propositions 1, 3, and 4 can be viewed as special cases of Proposition 5.

5. The Effect of Retail-Level Competition

In this section, we test the robustness of the results indicating when channel decentralization leads to higher product quality by extending our analysis to the case of retail-level competition. Specifically, we first extend our vertical × horizontal model from §3.1. Then we extend our one-dimensional model with a general distribution from §2.1.2. We conclude this section with a comparative discussion accordingly.

To make a direct comparison to the decentralized channel with two competing retailers, we define the centralized channel such that the manufacturer directly sells to consumers through two stores, located at $x = 0$ and $x = 1$. The manufacturer's associated demand is $D = 1 - p/q - t/(4q)$ if $q - p \geq t/2$ and $D = (q-p)^2/(qt)$ if $q - p < t/2$. Correspondingly, the manufacturer's profit function is given by $\pi^c = (p - q^2)D$. Maximizing π^c jointly over p and q therefore leads to the associated optimal quality (see the online technical appendix for derivation details) $q^{c*} = (1 + \sqrt{1+3t})/6$ if $0 < t \leq 8/25$ and $q^{c*} = 2/5$ if $t > 8/25$. By comparing q^{c*} to its single-store analog given by q_{vhc}^*, note that the addition of a second store to the centralized channel puts pressure on the manufacturer to decrease its optimal product quality when consumers have relatively lower transaction costs, but has no effect on optimal product when consumers have relatively higher transaction costs. Specifically, note that if we define $\Delta_c = q_{vhc}^* - q^{c*}$ as the amount of product-quality decrease that results from adding a second store to the centralized channel of the vertical × horizontal heterogeneity model of §3.1, then $\Delta_c > 0$ for $0 < t < 8/25$ but $\Delta_c = 0$ when $t \geq 8/25$.

In the decentralized channel with two competing retailers located at $x = 0$ and $x = 1$, the retail prices are p_1 and p_2. Accordingly, the retailers' demand functions are:

$$(D_1, D_2) = \begin{cases} \left(\dfrac{(4q-3p_1-p_2-t)(t-p_1+p_2)}{8qt}, \right. \\ \quad \left. \dfrac{(4q-3p_2-p_1-t)(t+p_1-p_2)}{8qt}\right), \\ \qquad\qquad \text{if } p_1+p_2 \le 2q-t; \\ \left(\dfrac{(q-p_1)^2}{2qt}, \dfrac{(q-p_2)^2}{2qt}\right), \quad \text{if } p_1+p_2 > 2q-t. \end{cases} \tag{2}$$

The channel members' profit functions are given by $\pi^m = (w - q^2)(D_1 + D_2)$, $\pi^{r1} = (p_1 - w)D_1$ and $\pi^{r2} = (p_2 - w)D_2$. Thus, using backward induction, we first maximize the retailers' profits simultaneously for given w and q, and then maximize the manufacturer's profit over w and q to get the resulting optimal product quality q^{d*}. (Derivation details are provided in the online technical appendix.) By then comparing q^{d*} to its single-retailer analog given by q^*_{vhd}, we find that the addition of a second retailer to the decentralized channel has a qualitatively similar effect as adding a second store to its centralized counterpart, i.e., it again puts downward pressure on the manufacturer to decrease its optimal product quality when consumers have relatively lower transaction costs, but has no effect on optimal product quality when consumers have relatively higher transaction costs. However, the reach of that pressure is different than in the centralized channel. Specifically, if we define $\Delta_d = q^*_{vhd} - q^{d*}$ as the amount of quality decrease that results from adding a second retailer to the decentralized channel of the vertical × horizontal heterogeneity model from §3.1, then, in contrast to its centralized analog, $\Delta_d > 0$ for $0 < t < 16/75$ and $\Delta_d = 0$ when $t \ge 16/75$. Accordingly, by comparing Δ_d to Δ_c determined above, we can assess the extent to which retail-level competition influences the effect of channel structure on optimal product quality. The following proposition summarizes that comparison.

Proposition 6. *If consumers are heterogeneous both vertically on their willingness to pay and horizontally on their transaction costs and consumers follow a uniform distribution along each dimension of heterogeneity, then*

(a) *channel decentralization with two competing retailers leads to higher product quality (i.e., $q^{d*} > q^{c*}$) if $0 < t < 8/25$, and*

(b) *retail competition in the decentralized channel leads to lower product quality (i.e., $q^{d*} < q^*_{vhd}$) if $0 < t < 16/75$ and has no effect on product quality (i.e., $q^{d*} = q^*_{vhd}$) if $t \ge 16/75$.*

Proposition 6 not only highlights how retail competition puts pressure on the decentralized manufacturer to decrease its product quality, but it also demonstrates that Proposition 3, which indicates that decentralization can lead to higher product quality in the single-retailer case, extends to the analogous case of two competing retailers. As noted, adding a second sales outlet could decrease the manufacturer's optimal product quality regardless of whether the distribution channel is centralized or decentralized, depending on consumers' transaction costs. However, the second sales outlet puts greater pressure on the manufacturer to lower its product quality in the centralized channel than in the decentralized channel, and it does so over a larger range of transaction costs, resulting in $q^{d*} > q^{c*}$ as per Proposition 6(a). Intuitively, this makes sense because, when the unit consumer transaction cost t is low, competition between the two retailers is especially intense. Although this intense competition lowers the retail price that consumers are charged, it also provides additional incentive for the manufacturer to increase its product quality because the more intense the competition, the lower the retail price, and thus the smaller the benefit that transfers to the retailers when product quality is increased. Given, then, that the manufacturer can benefit more from the increase of product quality, the incentive exists to do just that.

Note that when $t = 0$, the analysis here converges to the retail-competition case of the one-dimensional model in which consumers follow a uniform distribution on their willingness to pay. Note also that, from the derivation of this section in the online technical appendix, the two retailers' optimal retail prices when $t = 0$ are $p_1^* = p_2^* = w$. This is explained by the extreme competition that results from having two stores selling the same product at the same location. In this special case, the manufacturer's correspondingly optimal product quality under centralization and decentralization is $q^{c*} = q^{d*} = 1/3$, which, from §2.1.1, is the same as in the analogous model with only one retailer. Moreover, note that if $t = 0$, it remains true that $q^{d*} = q^{c*}$ even if consumers follow a general distribution on their willingness to pay. This is because double-marginalization is eliminated by the two retailers' head-to-head competition, and without double-marginalization the manufacturer's decision is the same in both distribution channels.[13] Therefore, although Proposition 6 extends the result of Proposition 3 to the case of retail-level competition when consumers are both vertically and horizontally heterogeneous, the analog of Proposition 6 does not likewise extend the result of Proposition 1 to the case of retail-level competition when consumers are one-dimensionally (vertically) heterogeneous with a general distribution. This implies that retailer competition

[13] This argument is also valid for extending the vertical × vertical model from §3.2 to the analogous case of retail-level competition.

alone is not a sufficient incentive for the manufacturer to provide higher product quality in the decentralized channel as compared to the centralized channel.

Proposition 6, together with the discussion of the special case when $t = 0$, confirm that the driving forces behind Propositions 1 and 3 are similar, but not equivalent. Whereas downstream competition amplifies the manufacturer's incentive to provide higher product quality along channel decentralization when consumers are uniformly heterogeneous, both vertically and horizontally (Proposition 3), downstream competition actually *eliminates* the manufacturer's incentive to provide higher product quality under channel decentralization when consumers are only vertically heterogeneous, but follow a general distribution.[14] To summarize, both vertical heterogeneity and double-marginalization are necessary, but not sufficient, to create the incentive for the manufacturer to increase product quality under channel decentralization. Given these two necessary market ingredients, either a general distribution of consumers or a willingness-to-pay function that strictly increases in product quality is sufficient, although neither is necessary, to yield a product-quality increase under decentralization.

6. Conclusion

In this paper, we study a manufacturer's optimal-quality decision in different channel structures with different types of consumer heterogeneity and different consumer distributions to isolate, explore, and synthesize key drivers underlying the effects of distribution channels on quality provision. We develop several interesting results with important managerial implications for marketing practice.

First, when consumers are heterogeneous only vertically on willingness to pay and follow a uniform distribution, channel decentralization leads to same or lower product quality. However, when consumers follow a general distribution on their willingness to pay, the manufacturer has incentive to provide higher product quality in the decentralized channel.

Second, when consumers are heterogeneous only horizontally on their transaction costs, channel decentralization leads to same product quality regardless of consumer distribution. This implies that a general distribution alone is not sufficient, and that consumers' vertical heterogeneity is necessary, for the manufacturer to provide higher product quality under channel decentralization. Practically, marketing managers need to identify the type of consumer heterogeneity before making product-quality decisions in different distribution channels.

Third, when consumers follow a uniform distribution and are heterogeneous vertically on their willingness to pay for two product-quality attributes, the manufacturer in a decentralized channel offers the same or lower product quality than in a centralized channel. Yet, when consumers follow a uniform distribution and are heterogeneous both vertically on their willingness to pay and horizontally on their transaction costs, the manufacturer again has incentive to provide higher product quality in the decentralized channel as compared to the centralized channel. This, when extrapolated and combined with the previous two points, implies that a consumer willingness-to-pay function that strictly increases with product quality is sufficient, but not necessary, to provide the manufacturer with incentive to increase product quality under channel decentralization.

Fourth, we supplement our findings by analyzing the case of downstream retailers' competition. Interestingly, when consumers are both vertically and horizontally heterogeneous, competition amplifies the incentive for the manufacturer to increase product quality under decentralization. Yet, in contrast, when consumers are one-dimensionally heterogeneous, competition eliminates the manufacturer's incentive to increase product quality under channel decentralization. This suggests that double-marginalization is another necessary incentive for the manufacturer to consider increasing product quality under decentralization. Moreover, it implies that the driving forces behind the model of vertical heterogeneity with consumer general distribution and the model of vertical $\times$ horizontal heterogeneity are similar, but they are not equivalent.

We acknowledge some limitations of this paper. We test the findings considering downstream retailers' competition. Future research can study competition at the upstream level or at both levels, when consumers differ both horizontally and vertically. In addition, it is interesting to study the case in which retailers may provide value-added services that complement the manufacturer's product. We hope this research can inspire more interest in this area.

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Appendix

Proof of Proposition 1. We first prove that if $\epsilon'(s) > 0$, then $\pi_{vc}(q) = \pi_{vc}(s_{vc}(q), q)$ is unimodal in q, where

[14] Note that when consumers are only heterogeneous on horizontal dimension, retail-level competition cannot create incentive for the manufacturer to offer higher product quality along channel decentralization. This further confirms that horizontal heterogeneity alone is neither necessary nor sufficient for a quality-enhancing effect to occur.

$s_{vc}(q) \equiv \arg\max_s \pi_{vc}(s, q)$. Note that $s_{vc}(q)$ is the value of s that satisfies $d\pi_{vc}(s, q)/ds = 0$.

$$\frac{d\pi_{vc}(q)}{dq} = \frac{\partial \pi_{vc}(s, q)}{\partial q}\bigg|_{s=s_{vc}(q)} + \frac{\partial \pi_{vc}(s, q)}{\partial s}\bigg|_{s=s_{vc}(q)} \frac{ds_{vc}(q)}{dq}$$
$$= \frac{\partial \pi_{vc}(s, q)}{\partial q}\bigg|_{s=s_{vc}(q)} = \bar{F}(s_{vc}(q))[s_{vc}(q) - 2q].$$

Note that $\partial \pi_{vc}(s, q)/\partial s = \bar{F}(s) + \bar{F}'(s)(s - q)$. Therefore, $s_{vc}(q)$ satisfies $s_{vc}(q) = q\epsilon(s_{vc}(q))/(\epsilon(s_{vc}(q)) - 1)$. Thus, $d\pi_{vc}(q)/dq = \bar{F}(s_{vc}(q))q[1/(\epsilon(s_{vc}(q)) - 1) - 1]$. This implies that $\operatorname{sign}(d\pi_{vc}(q)/dq) = \operatorname{sign}[1/(\epsilon(s_{vc}(q)) - 1) - 1]$. Let $L(q) = 1/(\epsilon(s_{vc}(q)) - 1) - 1$, then $L'(q) = -(\epsilon'(s_{vc})(ds_{vc}/dq))/(\epsilon(s_{vc}) - 1)^2$. From $s_{vc}(q) = q\epsilon(s_{vc}(q)/\epsilon(s_{vc}(q)) - 1)$, take total derivative: $(ds_{vc}(q)/dq)[q/s + \epsilon'(s)s/\epsilon(s)^2]|_{s=s_{vc}(q)} = 1$. Therefore, $ds_{vc}(q)/dq > 0$ because $\epsilon'(s) > 0$, by assumption. So, from $L'(q)$ and $L(q)$, note that $L(q) = 0 \Rightarrow L'(q) < 0$. This implies that $L(q) = 0$ is unique. Moreover, this implies that when $L(q)$ crosses 0, it goes from (+) to (−). Therefore, $\pi_{vc}(q)$ is unimodal in q.

Next, we prove that if $(d/ds)[\ln(\bar{F}(s)/\bar{F}(X(s)))] > 0$, then $s_{vc}(q) < X(s_{vd}(q))$.

Recall that $X(s) = s(\epsilon(s) - 1)/\epsilon(s)$. Let $A(s) = 1 - (sX'(s)/X(s))/\epsilon(s)$ and $B(s) = 1 - 1/\epsilon(s)$. Then $A(s) = B(s) - sB'(s)/(B(s)\epsilon(s))$. Note that

$$\frac{d}{ds}\left[\ln \frac{\bar{F}(s)}{\bar{F}(X(s))}\right] > 0$$
$$\Rightarrow \frac{d}{ds}[\ln(\bar{F}(X(s))) - \ln(\bar{F}(s))] < 0$$
$$\Rightarrow \frac{d \ln \bar{F}(X(s))}{ds} < \frac{d \ln \bar{F}(s)}{ds}$$
$$\Rightarrow \frac{\bar{F}'(X(s))X'(s)}{\bar{F}(X(s))} < \frac{\bar{F}'(s)}{\bar{F}(s)}$$
$$\Rightarrow X'(s) > \frac{\bar{F}'(s)\bar{F}(X(s))}{\bar{F}(s)\bar{F}'(X(s))}$$
$$\Rightarrow X'(s) > \frac{B(s)\epsilon(s)}{\epsilon(sB(s))}$$
$$\Rightarrow sB'(s) + B(s) > \frac{B(s)\epsilon(s)}{\epsilon(sB(s))}$$
$$\Rightarrow \frac{sB'(s)}{B(s)} > \frac{\epsilon(s)}{\epsilon(sB(s))} - 1$$
$$\Rightarrow \frac{sB'(s)}{\epsilon(s)B(s)} > B(s) - B(sB(s))$$
$$\Rightarrow B(s) - \frac{sB'(s)}{B(s)\epsilon(s)} < B(sB(s))$$
$$\Rightarrow A(s)X(s) < X(s)B(X(s))$$
$$\Rightarrow X(s_{vd}(q))\left[1 - \frac{sX'(s)/X(s)}{\epsilon(s)}\right]\bigg|_{s=s_{vd}(q)} < X(X(s_{vd}(q))).$$

Note, from the first-order condition in the decentralized channel

$$\frac{\partial \pi_{vm}}{\partial s} = q\bar{F}'(s_{vd})\left\{X(s_{vd}(q))\left[1 - \frac{sX'(s)/X(s)}{\epsilon(s)}\right]\bigg|_{s=s_{vd}(q)} - q\right\} = 0$$
$$\Rightarrow X(s_{vd}(q))\left[1 - \frac{sX'(s)/X(s)}{\epsilon(s)}\right]\bigg|_{s=s_{vd}(q)} = q$$

for any q. Note also that from the first-order condition in the centralized channel $\partial \pi_{vc}/\partial p = 0$ reduces to $s_{vc}(q) \cdot (\epsilon(s_{vc}(q)) - 1)/\epsilon(s_{vc}(q)) = X(s_{vc}(q)) = q$ for any q. So,

$$X(s_{vd}(q))\left[1 - \frac{sX'(s)/X(s)}{\epsilon(s)}\bigg|_{s=s_{vd}(q)}\right] < X(X(s_{vd}(q)))$$

implies that $X(s_{vc}(q)) < X(X(s_{vd}(q)))$. Therefore, $s_{vc}(q) < X(s_{vd}(q))$ because $X(s)$ is increasing in s.

Finally, we prove that $q_{vd}^* > q_{vc}^*$.

By definition, $q_{vd}^* = \arg\max \pi_{vd}(q)$ where $\pi_{vd}(q) = \pi_{vd}(s_{vd}(q), q)$. From the first-order condition

$$\frac{d\pi_{vd}(q)}{dq} = \frac{\partial \pi_{vd}(s, q)}{\partial q}\bigg|_{s=s_{vd}(q_{vd})}$$
$$= \bar{F}(s_{vd}(q_{vd}))\left[\frac{\epsilon(s_{vd}(q_{vd})) - 1}{\epsilon(s_{vd}(q_{vd}))} s_{vd}(q_{vd}) - 2q_{vd}\right] = 0,$$

we have

$$2q_{vd} = \frac{\epsilon(s_{vd}(q_{vd})) - 1}{\epsilon(s_{vd}(q_{vd}))} s_{vd}(q_{vd}) = X(s_{vd}(q_{vd})).$$

Given that $s_{vc}(q) < X(s_{vd}(q))$ is valid,

$$\frac{d\pi_{vc}(q)}{dq}\bigg|_{q=q_{vd}} = \bar{F}(s_{vc}(q_{vd}))[s_{vc}(q_{vd}) - 2q_{vd}]$$
$$= \bar{F}(s_{vc}(q_{vd}))[s_{vc}(q_{vd}) - X(s_{vd}(q_{vd}))] < 0.$$

Because $\pi_{vc}(q)$ is unimodal, $(d\pi_{vc}(q)/dq)|_{q=q_{vd}} < 0 \Rightarrow q_{vc}^* < q_{vd}^*$. Q.E.D.

Proof of Corollary 1. For Power distribution defined by $F(s) = s^k$ $(k < 1)$, $\bar{F}(s) = 1 - s^k$, and $\epsilon(s) = ks^k/(1 - s^k)$, $\epsilon'(s) = k^2 s^{k-1}/(1 - s^k)^2 > 0$; thus, condition (a) in Proposition 1 is satisfied.

Let $B(s) = 1 - 1/\epsilon(s)$. Then $B'(s) = \epsilon'(s)/\epsilon(s)^2 = k/(s(1 - s^k) \cdot \epsilon(s))$. Then $\bar{F}(X(s)) = 1 - s^k B(s)^k$.

$$\frac{\bar{F}(s)}{\bar{F}(X(s))} = \frac{k}{\epsilon(1 - B(s)^k) + k}$$
$$\Rightarrow \ln \frac{\bar{F}(s)}{\bar{F}(X(s))} = \ln k - \ln(\epsilon(1 - B(s)^k) + k)$$
$$\Rightarrow \frac{d}{ds} \ln \frac{\bar{F}(s)}{\bar{F}(X(s))} = -\frac{(1 - B'(s) - k\epsilon(s)B(s)^{k-1}B'(s))}{\epsilon(1 - B(s)^k) + k}$$
$$= \frac{\epsilon'(s)}{\epsilon(s)[k + \epsilon(s)(1 - B(s)^k)]}\left[kB(s)^{k-1} - \frac{1 - B(s)^k}{1 - B(s)}\right]$$
$$= \frac{\epsilon'(s)}{k + \epsilon(s)(1 - B(s)^k)}[kB(s)^{k-1} - kB(s)^k + B(s)^k - 1]$$
$$= \frac{\epsilon'(s)}{k + \epsilon(s)(1 - B(s)^k)}[B(s)^{k-1}[k(1 - B(s)) + B(s)] - 1]$$
$$\Rightarrow \frac{d}{ds} \ln \frac{\bar{F}(s)}{\bar{F}(X(s))} > 0$$

when $k < 1$. Therefore, condition (b) in Proposition 1 is satisfied. Q.E.D.

Proof of Proposition 2. In the centralized channel, the manufacturer's profit is given by $\pi_{hc} = (p - q^2)G((q - p)/t)$.

We therefore maximize its profit by solving the first-order conditions $\partial\pi_{hc}/\partial p = G((q-p)/t) - ((p-q^2)/t)G' \cdot ((q-p)/t) = 0$ and $\partial\pi_{hc}/\partial q = -2qG((q-p)/t) + ((p-q^2)/t) \cdot G'((q-p)/t) = 0$. By replacing $((p-q^2)/t)G'((q-p)/t)$ with $G((q-p)/t)$ in $\partial\pi_{hc}/\partial q = 0$, we get $2q^*_{hc} = 1$. Therefore, the optimal quality is $q^*_{hc} = 1/2$.

In the decentralized channel, the channel members' profit functions are $\pi_{hm} = (w-q^2)G((q-p)/t)$ and $\pi_{hr} = (p-w) \cdot G((q-p)/t)$. Following backward induction, we first maximize the retailer's profit by solving $d\pi_{hr}/dp = G((q-p)/t) - ((p-w)/t)G'((q-p)/t) = 0$, and get the retailer's optimal response as a function of (w, q). Accordingly, we write the wholesale price as an implicit function of the retail price $w = p - (t/G'((q-p)/t))G((q-p)/t)$. Given this, we therefore maximize the manufacturer's profit $\pi_{hm} = [p - (t/G'((q-p)/t))G((q-p)/t) - q^2]G((q-p)/t)$ over q and p. The corresponding first-order conditions are $\partial\pi_{hm}/\partial p = [2 - GG''/(G')^2]G - ((p - tG/G' - q^2)/t)G' = 0$ and $\partial\pi_{hm}/\partial q = [-1 + GG''/G'^2 - 2q]G + ((p - tG/G' - q^2)/t)G' = 0$. By replacing $((p - tG/G' - q^2)/t)G'$ with $[2 - GG''/(G')^2]G$ in $\partial\pi_{hm}/\partial q = 0$, we get $2q^*_{hd} = 1$. Therefore, the optimal quality is $q^*_{hd} = 1/2 = q^*_{hc}$. Q.E.D.

Proof of Proposition 3. To compare the optimal product quality in different distribution channels, we first solve the profit-maximization problem in each distribution channel. In a centralized channel, maximizing the manufacturer's profit $\pi_{vhc} = (p-q^2) \times D$, where $D = 1 - p/q - t/(2q)$ if $q - p \geq t$ and $D = (q-p)^2/(2qt)$ if $q - p < t$, over p and q, we obtain the optimal quality in the centralized channel

$$q^*_{vhc} = \begin{cases} \dfrac{1+\sqrt{1+6t}}{6}, & \text{if } 0 < t \leq \dfrac{4}{25}; \\[2mm] \dfrac{2}{5}, & \text{if } t > \dfrac{4}{25}. \end{cases} \tag{3}$$

In a decentralized channel, the manufacturer's profit is $\pi_{vhm} = (w-q^2) \times D$ and the retailer's profit is $\pi_{vhr} = (p-w) \times D$, where $D = 1 - p/q - t/(2q)$ if $q - p \geq t$ and $D = (q-p)^2/(2qt)$ if $q - p < t$. We solve the game based on backward induction and maximize the retailer's profit π_{vhr} with respect to the retail price p. The retailer's best responses are $p(w,q) = (2q + 2w - t)/4$ if $q - w \geq 3t/2$ and $p(w,q) = (q+2w)/3$ if $q - w < 3t/2$.

Given the retailer's best responses, we maximize the manufacturer's profit π_{vhm} over w and q, and we obtain the optimal product quality in the decentralized channel

$$q^*_{vhd} = \begin{cases} \dfrac{1+\sqrt{1+6t}}{6}, & \text{if } 0 < t \leq \dfrac{3}{32}; \\[2mm] \sqrt{\dfrac{3t}{2}}, & \text{if } \dfrac{3}{32} < t < \dfrac{8}{75}; \\[2mm] \dfrac{2}{5}, & \text{if } t \geq \dfrac{8}{75}. \end{cases} \tag{4}$$

When $t \in (3/32, 8/75)$, $q^*_{vhd} = \sqrt{3t/2} > (1+\sqrt{1+6t})/6 = q^*_{vhc}$. When $t \in [8/75, 4/25)$, $q^*_{vhd} = 2/5 > (1+\sqrt{1+6t})/6 = q^*_{vhc}$. Q.E.D.

Proof of Proposition 5. Let $\pi_{gc}(s,q) = q(s-q)\bar{F}_q(s)$ denote the manufacturer's profit in the centralized channel for a given quality q and price per unit quality s. Then, by definition, $s_c(q) = \arg\max_s \pi_{gc}(s,q)$. Therefore, $s_c(q)$ satisfies

$$0 = \frac{\partial\pi_{gc}(s,q)}{\partial s} = \frac{q\bar{F}_q(s)}{s}[s - (s-q)\epsilon_s(s,q)],$$

which implies that $s_c(q)(1 - 1/\epsilon_s(s_c(q),q)) = q$. Accordingly, q^*_c, the manufacturer's optimal product quality in the centralized channel is $q^*_c = \arg\max_q \pi_{gc}(q) = \arg\max_q \pi_{gc}(s_c(q),q)$. Given this, note that

$$\begin{aligned}\frac{d\pi_{gc}(q)}{dq} &= \frac{\partial\pi_{gc}(s,q)}{\partial q}\bigg|_{s_c(q)} = \bar{F}_q(s_c(q))[s - 2q + (s-q)\epsilon_q(s,q)]|_{s_c(q)} \\ &= \bar{F}_q(s_c(q))[2 + \epsilon_q(s_c(q),q)][T_c(q) - q],\end{aligned}$$

where $T_c(q) = s_c(q)(1+\epsilon_q(s_c(q),q))/(2+\epsilon_q(s_c(q),q))$. Therefore, $q^*_c = T_c(q^*_c)$.

Next, let $\pi_{gr}(s,q,w) = q(s - w/q)\bar{F}_q(s)$ and $\pi_{gm}(s,q,w) = q(w/q - q)\bar{F}_q(s)$ denote the retailer's and the manufacturer's profit, respectively, in the decentralized channel for a given quality q, price per unit quality s, and wholesale price w. Then, the retailer's optimal response for a given q and w solves $d\pi_{gr}(s,q,w)/ds = 0$, which implies that $w/q = s(1 - 1/\epsilon_s(s,q))$. Substituting this for w/q in $\pi_{gm}(s,q,w)$ allows the decentralized channel manufacturer's profit to be written as a function of the two variables s and q as follows: $\pi_{gm}(s,q) = q[s(1 - 1/\epsilon_s(s,q)) - q]\bar{F}_q(s)$. Given this, the manufacturer implicitly maximizes its profit by jointly maximizing $\pi_{gm}(s,q)$ over s and q. Toward that end, note that, by definition, $s_d(q) = \arg\max_s \pi_{gm}(s,q)$. Thus, $s_d(q)$ satisfies $0 = \partial\pi_{gm}(s,q)/\partial s = (q\bar{F}_q(s)/s)[s(1 - 1/\epsilon_s(s,q)) + (s^2/\epsilon_s(s,q)^2)(\partial\epsilon_s(s,q)/\partial s) - s(\epsilon_s(s,q) - 1) + q\epsilon_s(s,q)]$, which implies that $q = X_d(q)(1 - 1/\epsilon_s(s_d(q),q)) - [(s^2/\epsilon_s(s,q)^3)(\partial\epsilon_s(s,q)/\partial s)]_{s_d(q)}$, where $X_d(q) \equiv s_d(q)[1 - 1/\epsilon_s(s_d(q),q)]$. Accordingly, q^*_d, the manufacturer's optimal product quality in the decentralized channel, is $q^*_d = \arg\max_q \pi_{gm}(q) = \arg\max_q \pi_{gm}(s_d(q),q)$. Given this, note that

$$\begin{aligned}\frac{d\pi_{gm}(q)}{dq} &= \frac{\partial\pi_{gm}(s,q)}{\partial q}\bigg|_{s_d(q)} \\ &= \bar{F}_q(s_d(q))\left[s\left(1 - \frac{1}{\epsilon_s(s,q)}\right)(1+\epsilon_q(s,q)) \right. \\ &\qquad \left. -q\left(2 + \epsilon_q(s,q) - \frac{s}{\epsilon_s(s,q)^2}\frac{\partial\epsilon_s(s,q)}{\partial q}\right)\right]_{s_d(q)} \\ &= \bar{F}_q(s_d(q))[2 + \epsilon_q(s_d(q),q) - N(q)][T_d(q) - q],\end{aligned}$$

where $N(q) = [(s/\epsilon_s(s,q)^2)(\partial\epsilon_s(s,q)/\partial q)]_{s_d(q)} > 0$ and $T_d(q) \equiv X_d(q)[(1+\epsilon_q(s_d(q),q))/(2+\epsilon_q(s_d(q),q) - N(q))]$. Therefore, $q^*_d = T_d(q^*_d)$.

Given that $q^*_c = T_c(q^*_c)$ and $q^*_d = T_d(q^*_d)$, note that $\text{sign}[d\pi_{gc}(q)/dq]_{q^*_d} = T_c(q^*_d) - q^*_d = T_c(q^*_d) - T_d(q^*_d)$. Hence, if $\pi_{gc}(q)$ is a unimodal function of q, then

$$T_c(q) < T_d(q) \;\Rightarrow\; \left[\frac{d\pi_{gc}(q)}{dq}\right]_{q^*_d} < 0 \;\Rightarrow\; q^*_d > q^*_c,$$

and

$$T_c(q) \geq T_d(q) \;\Rightarrow\; \left[\frac{d\pi_{gc}(q)}{dq}\right]_{q^*_d} \geq 0 \;\Rightarrow\; q^*_d \leq q^*_c.$$

Similarly, note that $\text{sign}[d\pi_{gm}(q)/dq]_{q_c^*} = T_d(q_c^*) - q_c^* = T_d(q_c^*) - T_c(q_c^*)$. Hence, if $\pi_{gm}(q)$ is a unimodal function of q, then $T_c(q) < T_d(q) \Rightarrow [d\pi_{gm}(q)/dq]_{q_c^*} > 0 \Rightarrow q_d^* > q_c^*$, and

$$T_c(q) \geq T_d(q) \quad \Rightarrow \quad \left[\frac{d\pi_{gm}(q)}{dq}\right]_{q_c^*} \leq 0 \quad \Rightarrow \quad q_d^* \leq q_c^*. \quad \text{Q.E.D.}$$

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