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Rationing Capacity in Advance Selling to Signal Quality

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We consider a seller who can sell her product over two periods, advance and spot. The seller has private information about the product quality, which is unknown to customers in advance and publicly revealed in spot. The question we consider is whether the seller has an incentive to signal quality in advance and, if so, how she can convey a credible signal of product quality. We characterize the seller's signaling strategy and find that rationing of capacity in the advance period is an effective tool of signaling product quality. We find that the high-quality seller can distinguish herself by allocating less capacity than the low-quality seller in the advance period. We show that this signaling mechanism exists whenever advance selling would be optimal for both the high-quality and low-quality sellers if quality were known by the consumers. Interestingly, the seller's ability to ration (rationing flexibility) sometimes disadvantages the seller; this effect is independent of product quality.

Keywords: advance selling; signaling quality; capacity rationing
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1. Introduction

Advance selling is used in service industries, including travel and entertainment, as well as in many retail areas, including toys, books, electronics, and media products. With advance selling, sellers offer customers an opportunity to purchase a product or service prior to the consumption time. This can benefit the seller because advance selling may increase total sales and profit. It can also benefit the customers: through buying in advance, customers can usually get guaranteed availability at discounted prices. However, because consumers buy the product before it is available, they are often uncertain about the quality of the product as well as about their own valuation of the product at the time of consumption.

Consider advance selling of French wine, a practice known as *en primeur* ("wine futures" in French), which has been practiced in Bordeaux for centuries as well as in other areas including Burgundy, Tuscany, and Rioja. Typically, 6 to 12 months after the previous year's grape harvest, chateaux and vineyards offer customers opportunities to buy the new vintage. At the time of the *en primeur* sales, the wine is not yet "finished" and is still in barrels. As is typical for advance selling, the payment is due at the time of *en primeur* sales, but the delivery occurs after the wine is finished and bottled, usually 12–18 months after the *en primeur* sales. Interestingly, wineries may sell all or just a portion of their wines through the *en primeur* system, and both wineries and negotiants usually reserve some wine for later sales (Bishop and Hayward 2012). Similar practices are offered by online and brick-and-mortar wine sellers in retail settings.

During *en primeur*, buyers are uncertain about the quality of the wine. Like many other experience goods (Su 2009, Shulman et al. 2009), wines can truly be evaluated through tasting the actual products, which are unavailable during the advance sales. Although tasting of young wines is sometimes offered and some information about the wine—such as the condition of harvested grapes, the total cases made, and how many cases are available for the *en primeur* sales—is available, the wines are six months old, tannic, and still in the stage of malolactic fermentation. They often smell and taste quite unpleasant (at *en primeur*) and can be very different from what they will be in one year or so when the production process is complete.

In contrast, chateaux, as the producers of the wines, have much more information about the grapes used for wine as well as about the entire wine-making processes (Hadj Ali and Nauges 2006, Dubois and Nauges 2010). For example, they have firsthand information on all of the important determinants of wine quality, such as climate, soil, and viticultural and enological practices

(Jackson 2000). Furthermore, they know how representative the sample barrels are in terms of overall quality.

Such information asymmetry is difficult to resolve via information sharing or contracts. The seller (winery or chateau) has private information about what has happened so far to the wine, but with many factors (climate, soil, viticultural practices, etc.) playing a role, no single attribute can precisely determine the quality of wine. Even if the seller is willing to share the information with the consumers during the advance sales (e.g., by showing the soil samples from the vineyards[1]), it is hard for consumers to integrate individual pieces of information to correctly predict the quality. Furthermore, it is also hard to verify the information (e.g., if the grapes used to make the wine indeed grew in the fields from where the soil samples were taken).

The information asymmetry about wine quality is gradually reduced over time and eventually resolved after the wines are bottled and released to the market. Customers can evaluate the quality of the finished wines by reading experts' wine reviews, attending wine tasting events (e.g., Auffrey 2008), ordering trial packs from online retailers (e.g., theorganicwinecompany.com[2]), and tasting wines at social events by chance.

Another example where the seller has significantly more information than the buyers about the products sold in advance is event ticket sales. The advance ticket sales of the Ultra Music Festival in Miami (a festival for electronic music featuring some of the top DJs in the world) and the Bonnaroo Music Festival in Manchester, Tennessee, both take place many months before the complete lineup and schedule are announced.[3] Although the seller has private information about DJs headlining the 2014 Ultra Music Festival (http://www.ultramusicfestival.com), the complete lineup and schedules were not announced until many months after advance sales (most likely January or February of 2014). Similar situations with asymmetric quality information include advance sales of new music albums, games, and electronics before their official release.

When advance selling is offered, the sellers can choose to sell either all or a portion of the products. We observe both practices. For example, some premier designer handbags (so-called "it" bags in Kuksov and Wang 2013) are sold out during preorder and will never arrive at the store. In the *en primeur* market, chateaux release a proportion, ranging from 20% to 90%, of their total production, thus intentionally limiting the wine availability in the advance market. According to the *New York Times* (Prial 1989), the price-setting Bordeaux chateaux sell their wines in stages, or "tranches." In the first stage, they usually release about 20% of their total production at the opening price. This practice is well described by many wine merchants and wine experts.[4] For the 2013 Burning Man Festival, exactly 3,000 of 58,000 tickets were allocated for the advance holiday sales that took place on December 20, 2012 (Associated Press 2013). Limiting preorders of new products is also a common practice in selling electronics. Two of the best-known examples are Microsoft's Xbox 360, released in 2005 (Harford 2005) and Sony PlayStation 3, released in 2006 (Sinclair 2006). In both cases, retailers such as GameStop accepted limited orders or limited the number of units that one consumer could order.

A number of reasons can explain the seller's practice of rationing capacity. Limiting the advance sales may create hype and increase demand for new products (*Retailing Today* 2000, Dye 2000, Brown 2001) or it may simply reflect capacity shortage. In this paper, we show that another explanation for the seller's rationing is to signal product quality.

Offering advance sales when an asymmetry in quality information exists can work for or against the seller. On one hand, it is possible for a seller of low-quality products to hide the inferior quality in advance and to boost sales by locking in many customers who would not have made the purchase were quality known. On the other hand, a seller who cannot prove quality may need to give a considerable price discount to induce customers to buy early, because customers have the option to delay purchase until quality is fully revealed. Given these two opposite drivers, it is not clear whether and when the seller should offer advance selling and, if advance sales are to be made, whether it is possible to convey some of the information about the product quality to buyers through the terms of advance selling (such as price or limited quantities offered for sale). Some key questions we like to address are as follow. How does asymmetric quality information affect the seller's profit, and how much can the seller gain from offering advance selling? Can the seller of high-quality products credibly signal product quality? When is it beneficial to signal the quality level? We study these questions in this paper and, in particular, we examine the role of *capacity rationing* as a signal of quality. We show that a seller can use capacity rationing (i.e., limiting supply in the advance period and choosing to satisfy a portion of advance demand), in conjunction

[1] We thank an anonymous referee for suggesting this.

[2] http://store.theorganicwinecompany.com (accessed May 9, 2014).

[3] The advance tickets for the 2014 Ultra Music Festival (March 28–30, 2014) were already sold out by May 2013 (http://www.ultramusicfestival.com/news; accessed May 9, 2014).

[4] Decanter. "How to buy en primeur." http://www.decanter.com/wine-learning/wine-advice/basics/495393/how-to-buy-en-primeur (accessed May 9, 2014). The Rare Wine Co Newsletter, May 8, 2001, http://www.rarewineco.com (accessed July 18, 2012).

with the corresponding prices, to convey information about the product quality. We also show that, as long as both types of seller offer advance selling when the quality information is known by consumers, there exists an equilibrium in which the seller of high-quality products allocates less capacity in advance than the seller of low-quality products to distinguish the higher quality of the product (i.e., to signal product type).

1.1. Literature Review.

Our work is closely related to the literature on signaling quality. Several different forms of signals of quality have been examined in existing literature, including advertising (Kihlstrom and Riordan 1984, Milgrom and Roberts 1986), pricing (Bagwell and Riordan 1991), warranties (Lutz 1989), money-back guarantee (Moorthy and Srinivasan 1995), umbrella branding (Wernerfelt 1988), and scarcity (Stock and Balachander 2005). Our paper contributes to the literature by showing that capacity rationing in advance selling can be an effective signal of quality.

Among the signaling literature, Stock and Balachander (2005) come closest to our paper. They consider scarcity as a signal of quality and show that a seller who has sufficient capacity to meet all demand may intentionally dispose of some capacity to create scarcity for uninformed customers ("followers"). Thus, a high-quality seller signals quality by making product scarce for followers and charging full-information price for all customers. This strategy is optimal under two conditions: informed customers make purchases first (before followers) and price is constant over two periods. Although both models use sales quantity as a signal of quality, our model is substantially different. First, we assume that quality uncertainty exists before the product is released (e.g., wine *en primeur* or presale of a video game) and is resolved at the product release (e.g., when wines are released to consumers or the game hits the store). Stock and Balachander (2005) assume the opposite: quality is perfectly known in advance, but only to advance customers ("innovators"). Second, in our setting, the seller can dynamically change price over time, and the advance customers can strategically choose when to buy, i.e., whether to buy in advance with imperfect quality information or wait until information is publicly revealed in spot. Such strategic customer behavior is supported by many empirical observations (Su 2007, Aviv and Pazgal 2008) and is not captured in Stock and Balachander (2005). Third, while their model assumes that total capacity can be adjusted, all of the capacity must be available in advance and the seller cannot limit the quantity sold in advance. In contrast, we consider allocation of capacity between the advance (before the product is available) and spot (after it is available) periods.

Our paper is also related to the advance-selling literature, especially papers considering consumers' uncertain valuations (e.g., Xie and Shugan 2001, Gallego and Sahin 2010, Prasad et al. 2010, Chu and Zhang 2011, Yu et al. 2015). All of the papers in this stream, except Chu and Zhang (2011), assume that all information about the product is publicly available, i.e., sellers do not have any private information. In contrast, our paper considers both the customer's uncertain valuation and the seller's private information about quality. The impact of asymmetric quality information on the seller's strategy and profit from advance selling is, in fact, our focus. Among the above papers, the one by Chu and Zhang (2011) is the only work that considers the seller's private information about quality. In their model the seller decides the amount of quality information to release in advance. The paper shows how this decision affects customers' valuation of the product. In contrast, the seller in our model cannot and does not directly control the information released in advance. Quality information can only be *inferred* through the seller's selling strategies (e.g., pricing and capacity rationing).

There are other papers that examine capacity rationing, but not in the context of advance selling. Among these, the paper by Liu and van Ryzin (2008) shows that capacity rationing can induce risk-averse customers to buy early at the regular price instead of waiting for a clearance price. Zhang and Cooper (2008) evaluate the benefit of rationing with both fixed and flexible pricing. Gilbert and Klemperer (2000) find that rationing is preferred to market-clearing price when customers incur seller-specific sunk cost. These papers, however, all ignore the signaling effect of rationing.

The rest of this paper is organized as follows. We define the problem and equilibrium concept in §2 and provide some preliminary results in §3. In §4, the seller's optimal (equilibrium) strategy when there is no quality uncertainty (full-information case) is presented. Sections 5 and 6 are the main thrusts of the paper. In §5, we present the equilibrium strategy and outcome when quality is uncertain and the seller has the option of rationing capacity. In §6, we evaluate the value of rationing and characterize the conditions under which the seller prefers signaling through rationing. We discuss several extensions of our model and conclude the paper in §7. All proofs are presented in the appendices.

2. The Model

We consider a risk-neutral seller offering a product to risk-neutral customers over two periods, advance period and spot period. While the seller knows the quality of the product in advance, the quality is not observable by customers until the spot period. The seller decides the price and quantity to offer for the advance period and then later the price for the spot period. We assume that customers are strategic and choose whether and when to buy the product. In what follows, we describe the seller's and the customer's problems, and then define the sequence of the events.

2.1. The Seller

The seller's product can be of either high (H) or low (L) quality (with $H > L$). We assume that the seller's total capacity is T and the marginal production cost is c, both of which are common knowledge.[5] It should be noted that our analysis can be extended to the case where high-quality products are more costly to produce (all of our results continue to hold). We follow the signaling literature (see a comprehensive review in Sobel 2007 or Kirmani and Rao 2000) and assume that the quality of the seller's product is exogenously given and cannot be chosen by the seller.[6]

To isolate the effect of rationing as a signal, we assume that the seller's capacity is exogenously determined and that the seller does not have freedom to change the capacity. There are a number of examples supporting this. For instance, in wine sales, only the grapes harvested from a certain lot can become *Premier Cru*, thus the total capacity cannot be changed freely by the seller. In event ticket selling, exactly 58,000 tickets are available for the 2013 Burning Man Festival, because the number of attendants is regulated by Pershing County, Nevada, where the festival takes place. Similarly, the total number of tickets for the 2013 Ultra Music Festival requires approval from Miami–Dade County.

2.2. The Customers

Customers are strategic and risk-neutral: they choose the option that maximizes their expected utility when facing multiple purchasing opportunities. If customers do not buy the product, their reservation utility is zero. Otherwise, a customer's net utility is $U = t + \alpha - p$, where p is the price of the product, $t \in \{H, L\}$ represents the product quality (H or L), and α is the customer's private valuation that captures the heterogeneity in individual customers' willingness to pay. In particular, α corresponds to the combined effect of all idiosyncratic factors, such as individual preferences about the flavor of the wines or customers' moods at the times of consumption (Hauser and Wernerfelt 1990). We assume that the sum of quality and individual valuation, $t + \alpha$, is nonnegative for all realizations. We note that all derived results also hold for a multiplicative utility function (i.e., $U = \alpha t - p$).

In the advance period, customers are uncertain about both quality t and valuation α, both of which are resolved in the spot period. For example, in the *en primeur* market, customers are uncertain not only about wine quality but also about their individual valuation of consumption, which depends on their mood. Similarly, customers prepaying for festivals are unsure about both the quality of the event (which depends on many factors, such as which DJs will perform) and their personal states (e.g., mood, health, scheduling conflicts) on the event date.

Specifically, let q be the probability that the product quality is high (H), a common prior belief for all customers in the advance period. While individual valuations are different for different customers in spot, in advance they all follow the same prior distribution with cdf $G(\cdot)$ and pdf $g(\cdot)$. This approach is taken by several papers on advance selling, including Xie and Shugan (2001), Gallego and Sahin (2010), Prasad et al. (2010), and Nasiry and Popescu (2012). Throughout the paper, we impose the following assumptions on the distribution function $G(\cdot)$:

(1) $G(\cdot)$ is twice continuously differentiable and has a compact support $[\underline{\alpha}, \bar{\alpha}]$.

(2) $g(\cdot) = G'(\cdot) > 0$ on $(\underline{\alpha}, \bar{\alpha})$ and is log concave.

(3) For any $k \in [0, 1]$, $(G(x)\bar{G}(x)/g(x))' + \bar{G}(x) - k$ crosses zero at most once and from above.[7]

These assumptions are not very restrictive and cover many distributions and their truncated versions, including uniform, exponential, logistic, normal, extreme value, power, Weibull, beta, gamma, and χ (χ^2) with most parameter values. Note that condition (2) implies that the distribution has an increasing failure rate (IFR) and that the tail distribution $\bar{G}(\cdot) = 1 - G(\cdot)$ has an inverse on $[0, 1]$, which we denote by $(\bar{G})^{-1}(\cdot)$.

Following the standard approach in the advance-selling literature (see Xie and Shugan 2001, Gallego and Sahin 2010, Yu et al. 2015, to name a few), we use the fluid model, wherein the proportion of customers with an individual valuation less than or equal to x is $G(x)$.

2.3. The Game

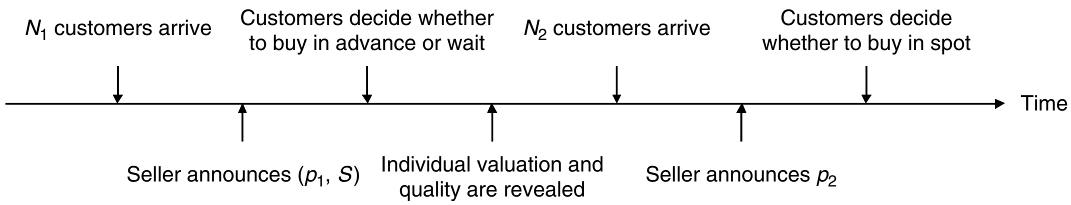
The sequence of events is illustrated in Figure 1. A cohort of N_1 consumers who are uncertain about both product quality, t, and individual valuation, α_i, arrive in the advance period. The seller decides whether to

[5] We follow Stock and Balachander (2005) and consider the case when the seller's marginal cost is independent of quality: high-quality and low-quality sellers have the same constant marginal cost of production. Walton (1986) and Srinivasan and Lovejoy (1997) show that high-quality products are not necessarily more costly to produce. For instance, the prices of some wines are largely distanced from their production costs (Wine Anorak. "Why does wine cost what it does?" http://www.wineanorak.com/whydoeswinecost.htm; accessed May 9, 2014).

[6] If the product quality is a function of the seller's efforts, then a moral hazard problem may arise, especially when the effort decision is made after advance sales. To focus on the signaling problem that occurs in the advance selling, we do not consider the case of endogenous effort and ensuing moral hazard.

[7] A function $a(\cdot)$ crosses zero at most once and from above, if and only if $a(x_0) < 0$ implies $a(x) < 0$ for all $x > x_0$.

Figure 1 **Sequence of Events**



offer advance selling and, if so, announces the advance price, p_1, and the rationing decision, S, which is the maximum the seller is willing to sell in advance.

Upon observing the seller's offer, customers update their belief about the product quality. Let $b(p_1, S)$ denote customers' posterior belief about the probability that the seller offers high-quality products. Based on this updated belief, customers decide whether to buy in advance or to wait and postpone the purchasing decision until the spot period.

In the spot period, another cohort of N_2 customers arrive, and the seller sells the remaining quantity at price p_2. At this time, the product is available for consumption, and the customers know both the product quality and their own individual valuations. All the remaining customers, including those who have not bought in advance and those who arrived in spot, decide whether to buy in spot. At the end of the spot period, consumption takes place and there is no salvage value for any unsold capacity. Note that having two separate streams of arrivals, N_1 and N_2, allows us to capture situations in which not every customer is aware of the product release in the advance period. For example, while some consumers may become aware of the release of a new game and place a preorder, others (who may have the same willingness to pay) may not be aware of the product release until the game actually hits the shelf; see Prasad et al. (2010) and Stock and Balanchander (2005) for more examples and discussion. Furthermore, all of our results hold for the case $N_2 = 0$: a single stream of buyers arriving in the advance period.

In our model, the seller's rationing decision is observable to the customers. As reported in trade articles (e.g., Prial 1989), chateaux release a portion of wines for sales during *en primeur*. Some vineyards announce the total quantity and the amount they sell in advance up front. For instance, Midsummer Cellar, a Californian winery located in St. Helena, announced that, of approximately 300 cases of their 2010 Canon Creek Cabernet Sauvignon, only 100 cases (1,200 bottles) would be available during the advance period.[8] Similarly, the organizers of the 2013 Burning Man Festival announced at the beginning of a selling season how many tickets would be available at each of the sales dates and committed to this specific plan (e.g., 3,000 in December 2012, 40,000 in February 2013, and 1,000 in August 2013). The holiday sales took place on December 20, 2012, and, in fact, all 3,000 allocated tickets were sold in a matter of hours and the press repeatedly reported the sales of the tickets (Associated Press 2013, Chase 2013, Marcus 2013). The commitment to the quantity sold in advance can be verified—if in dispute, a judge can order ex post to audit sales data to verify that the announced quantity was indeed sold. The credibility of commitment is tightly linked to the reputation of the seller as well. Su and Zhang (2009, p. 717) note, "when stocking quantities and service levels are verifiable ex post, the seller may be averse to misrepresentation due to reputation concerns."

We assume that the seller determines and announces the spot price at the beginning of the spot period. If the seller can commit to a spot price during advance selling, then the spot price itself may be used as a signal of quality. Such signaling role of price has been studied in a static model (one-shot sales) by Bagwell and Riordan (1991). However, in some situations it is difficult to commit to a certain spot price. For example, the spot price of a wine (often 1–3 years after the en primeur) is influenced by the seller's production process, which has not yet taken place.

2.4. Equilibrium Concepts

Our signaling game is a sequential game with incomplete information, and the equilibrium concept we employ is perfect Bayesian equilibrium (PBE) (Fudenberg and Tirole 1991, Katok et al. 2014). Typically, two classes of PBE, separating and pooling equilibria, exist for a signaling game. In a separating equilibrium, a high-quality seller can successfully distinguish himself from a low-quality seller during the advance sales by choosing a strategy that the low-quality seller does not have an incentive to mimic. Consequently, customers can perfectly infer the seller's type. In contrast, in a pooling equilibrium, the high-quality seller cannot economically differentiate himself, and both types of sellers adopt the same strategy in advance. Resultantly, customers cannot infer any information about the quality of the seller in equilibrium.

In our game, the seller has two potential tools to signal quality to customers in the advance period: price

[8] http://www.midsummercellars.com/Futures.htm (accessed May 9, 2014).

and capacity ration. Hence, we define a separating equilibrium as an equilibrium in which either only one of the two types of sellers offer advance selling, or both types sell in advance but differ in their advance prices or/and capacity rations. On the other hand, we define a pooling equilibrium as one in which either both types of sellers only sell in spot, or both types sell in advance and use the same advance price and ration. To avoid trivialities, we focus on the set of *participating* equilibria where, if an advance offer is made in equilibrium, the customers' response in equilibrium is to accept the offer and buy in advance. If a seller makes an advance offer that is rejected by customers in equilibrium, then it is equivalent to the seller not offering any advance selling. The concept of "participating equilibrium" is also used in economics and finance literature, e.g., Janssen et al. (2005) and Easley and O'Hara (2009).

In addition, to limit the number of equilibria in a signaling game, we apply the *intuitive criterion* of Cho and Kreps (1987) for customers' beliefs on off-equilibrium paths. The intuitive criterion requires that, if only one type of seller benefits from following an off-equilibrium strategy compared with following the equilibrium strategy, then observing the off-equilibrium strategy enables the customers to correctly identify the seller's type. In addition to the intuitive criterion, we also impose Pareto dominance: if multiple equilibria exist, we will focus on the equilibrium that Pareto-dominates all other ones from the seller's point of view, i.e., the equilibrium where both types of the seller obtain (weakly) higher profits than they do in any other equilibrium. Such an equilibrium is a *focal equilibrium* and is supported by evidence from behavioral experiments (Schelling 1960). Also, because in our model the seller moves first, the seller can always choose the equilibrium most appealing to himself and expects customers to foresee his choice. If there are multiple equilibria and both types of sellers are indifferent in choosing any of them, we will focus on the equilibrium at which the advance sales are the largest (lexicographically largest, similar to Federgruen and Heching 1999).

In the next section, we formally formulate the problem and provide preliminary results. In the following sections, we consider a baseline case when advance customers know the true quality (full-information setting), and then we proceed to the focus of the paper: when advance customers are uncertain about quality (asymmetric-information setting).

3. Formulation and Preliminary Results

Following backward induction, we first examine the seller's decision in the spot period, when quality information is fully revealed.

3.1. Spot Period

Consider a subgame where the seller sold S units in the advance period. Because the seller's total capacity is T and the number of advance customers is N_1, clearly $S \in [0, \min(T, N_1)]$, the remaining capacity is $T-S$, and there are N_1+N_2-S customers in the spot period. Because the customers know the product quality and their individual valuations, a customer with valuation α will buy the product of quality t, $t=H, L$, in spot if and only if her utility is nonnegative, i.e., $U=t+\alpha-p_2\geq 0$ or $\alpha \geq p_2 - t$. Hence, the number of customers who want to buy the product in spot is $(N_1+N_2-S)\bar{G}(p_2-t)$.

Because the sales quantity in the spot period is constrained by both the seller's remaining capacity and the spot demand, for given remaining capacity, $T-S$, the type t seller chooses a spot price p_2 to maximize the expected spot profit $\pi_{2t}(p_2, S)$.

$$\pi_{2t}(p_2, S) = p_2 \min\{T-S, (N_1+N_2-S)\bar{G}(p_2-t)\}. \quad (1)$$

Let the optimal spot price be $p^*_{2t}(S)$ and the corresponding spot profit $\pi^*_{2t}(S)=\pi_{2t}(p^*_{2t}(S), S)$. The following lemma characterizes $p^*_{2t}(S)$:

Lemma 1.

$$p^*_{2t}(S) = \max(p^U_{2t}, p^B_{2t}(S)) = \begin{cases} p^U_{2t} & \textit{if } \dfrac{T-S}{N_1+N_2-S} \geq \bar{G}(p^U_{2t}-t), \\ p^B_{2t}(S) & \textit{otherwise}, \end{cases} \quad (2)$$

where p^U_{2t} maximizes the unconstrained profit $p_2\bar{G}(p_2-t)$ and $p^B_{2t}(S)$ is the market-clearing price. Specifically,

$$p^U_{2t} \begin{cases} \in (t+\underline{\alpha}, t+\bar{\alpha}) \textit{ and is a solution to} \\ \quad p^U_{2t} = \dfrac{\bar{G}(p^U_{2t}-t)}{g(p^U_{2t}-t)} \qquad \textit{if } t < \bar{t} = \dfrac{1}{g(\underline{\alpha})} - \underline{\alpha}, \\ = t+\underline{\alpha} \qquad \textit{if } t \geq \bar{t}, \end{cases} \quad (3)$$

and $\bar{G}(p^B_{2t}(S)-t) = \min(1, (T-S)/(N_1+N_2-S))$.

Lemma 1 shows that product quality, valuation uncertainty, and the seller's remaining capacity all play a role in determining the optimal spot price. When the product quality is high enough to dominate valuation uncertainty (i.e., $t > \bar{t}$) and the seller has sufficient capacity, the seller finds it optimal to sell to all remaining customers by setting the spot price to $t+\underline{\alpha}$. With lower quality ($t \leq \bar{t}$), the seller with sufficient capacity will charge an interior spot price. Thus, even when the seller has ample capacity, the optimal price changes in product quality. The threshold quality, $\bar{t}$, will play a role both in full information and asymmetric information cases.

On the other hand, when the seller's capacity is tight so that spot demand at p^U_{2t} exceeds the remaining capacity $T-S$, it is optimal to charge the capacity clearing price, $p_{2t}^B(S)$. Clearly, when the spot price is raised from p_{2t}^U to $p_{2t}^B(S)$, the profit is increased while the sales remain equal to the remaining capacity. This also means that no shortage can take place in the spot period, which is summarized in the following corollary:

Corollary 1. *For*

$$S \in [0, \min(T, N_1)],$$
$$T - S \geq (N_1 + N_2 - S)\bar{G}(p_{2t}^*(S) - t).$$

The fact that shortage in supply never occurs in spot will affect the customers' purchasing decision in advance, and, consequently, the seller's decision.

3.2. Advance Period

If the seller decides not to sell in advance, setting advance ration S to zero, all N_1 customers must wait until the spot period and the seller's total profit over the two periods is simply $\pi_{2t}^*(0)$. If, however, the seller offers advance selling with advance price p_1 and *positive* capacity ration S, advance customers update their belief about the probability of high quality to $b(p_1, S)$. Based on this belief, customers choose to buy in advance or wait and delay the purchasing decision to spot, by comparing the expected utilities. When purchasing in advance, a customer's expected utility is $U_A(p_1, S) = \mathrm{E}_{\alpha,t}[t+\alpha-p_1]$. If she decides to wait, she will buy only if the revealed quality and individual valuation are high enough. From Lemma 1 and Corollary 1, the expected utility from waiting until spot is $U_D(p_1, S) = \mathrm{E}_{\alpha,t}[\max(t+\alpha-p_{2t}^*(S), 0)]$. Because $U_D \geq 0$, buying in the advance period is optimal for the customer if and only if $U_A(p_1, S) \geq U_D(p_1, S)$, which is equivalent to

$$p_1 \leq \mathrm{E}_{\alpha,t}[\min(p_{2t}^*(S), t+\alpha)]. \tag{4}$$

Note that the right-hand side of Equation (4) provides the customer's maximum willingness to pay in advance. Intuitively, a customer would not pay any advance price higher than the expectation of the minimum of spot price and total product value (the sum of quality and individual valuation), since the customer always has the option of waiting until spot.

Although customers will eventually realize valuations at the time of consumption, their decision in the advance period is based on expected utility. Because customers share the same *ex ante* distribution about individual valuation, α, and the same updated belief about quality, t, they have the same maximum willingness to pay, as expressed in Equation (4). Consequently, for given price p_1 and ration $S>0$, either all or none of them buy in advance, resulting in the seller selling out all of the S units or making no sales at all. Thus, for any ration $S>0$, the seller can choose a price leading to the sales of all S units.

After explicitly including the posterior belief $b(p_1, S)$ in Equation (4) we have

$$p_1 \leq b(p_1, S)p_{1H}^*(S) + (1-b(p_1, S))p_{1L}^*(S), \tag{5}$$

where $p_{1t}^*(S) = \mathrm{E}_\alpha[\min(p_{2t}^*(S), t+\alpha)]$ represents advance customers' maximum willingness to pay when they believe the seller is of type t.

Let $\pi_t^a(p_1, S, b)$ denote the expected total profit for a type t seller who sets an advance price p_1 and rations capacity S to advance customers, who believe the seller to be of high-quality type with probability $b = b(p_1, S)$. Superscript a stands for *asymmetric information*. If the condition in Equation (5) is satisfied, all customers choose to buy in advance and the seller sells all S units. Thus, we have

$$\pi_t^a(p_1, S, b) = p_1 S + \pi_{2t}^*(S). \tag{6}$$

For p_1 not satisfying Equation (5), none of the customers buys in advance and the seller's total profit $\pi_t^a(p_1, S, b) = \pi_{2t}^*(0)$. Thus, in correspondence to a set of customers' beliefs $\{b(p_1, S)\}$, the type t seller chooses the price-ration pair (p_1, S) to maximize his expected total profit $\pi_t^a(p_1, S, b)$.

Recall that we consider only participating equilibria, wherein the seller's advance offer induces all customers to buy in advance, i.e., Equation (5) is satisfied. Because $b(p_1, S) \in [0, 1]$, customers would never buy at any price higher than $p_{1H}^*(S)$ and yet would always buy at a price lower than $p_{1L}^*(S)$. For the remainder of the paper, when identifying participating equilibria, it will suffice to consider only *feasible* strategies: $S=0$ (when p_1 is irrelevant), or $0 < S \leq \min(T, N_1)$ and $p_1 \in [p_{1L}^*(S), p_{1H}^*(S)]$.

Our objective is to evaluate the impact of asymmetric information about quality. To do this, we first analyze a benchmark case in which customers know the true quality of the product in advance.

4. Base Case: Full Information About Quality

When customers know the true quality t in advance, for given advance ration S, their maximum willingness to pay is simply $p_{1t}^*(S)$. Thus, $p_{1t}^*(S)$ is exactly the advance price the seller will quote: any lower price is strictly dominated and any higher price is rejected by customers. With the optimal spot and advance prices characterized as functions of S, the seller chooses a capacity ration S to maximize her total expected profit, $\pi_t^f(S)$:

$$\max_{S \in [0, \min(T, N_1)]} \left\{ \pi_t^f(S) = p_{1t}^*(S)S + \pi_{2t}^*(S) \right\}. \tag{7}$$

Denote the optimal ration in the full-information case by S_t^f and the corresponding optimal advance price by p_{1t}^f, where superscript f stands for the *full information* case. The following theorem characterizes when it is optimal to use advance selling. Later we describe price p_{1t}^f and ration S_{1t}^f for each type.

Theorem 1. (i) *There exist two critical numbers, T_1 and T_D, $0 \le T_1 \le T_D \le N_1 + N_2$, such that*

- *if $T \le T_1$, then $S_t^f = 0$* [no advance selling];
- *if $T \in (T_1, T_D)$, then $0 < S_t^f < \min(T, N_1)$* [limited advance selling];
- *if $T_D \le T < N_1 + N_2$ or $T \ge N_1 + N_2$ and $t < \bar{t}$, then $S_t^f = \min(T, N_1)$* [full advance selling]; *and*
- *if $T \ge N_1 + N_2$ and $t \ge \bar{t}$, then S_t^f is any value between zero and N_1* [advance selling and spot-only selling are equivalent].

(ii) *T_1 and T_D are independent of quality level.*

In the case when advance selling and spot-only selling are equivalent (the last bullet in part (i)), we assume that the seller offers advance selling and ration $S = N_1$ because the seller can accrue the revenue early.

Advance selling allows the seller to take advantage of customers' uncertainty about their individual valuations. If accepting a discount in advance, customers are willing to buy before their valuations are revealed. This discount increases the sales as well as the profit. Note in Theorem 1 that the total capacity plays a significant role: The seller prefers to offer advance selling (or at least has no preference between spot-only or advance selling) when the capacity is large. On the other hand, the seller rations capacity in advance ($S < \min(T, N_1)$) when capacity is at intermediate level ($T \in (T_1, T_D)$). To see why, note that the seller would not be able to sell all of his capacity if he does not sell in advance. On the other hand, in the case of full advance selling, $S = \min(T, N_1)$, a significant portion of customers would buy at a (possibly heavily) discounted advance price. Offering some quantity in advance not only increases the sales quantity but also raises the spot price. The product quality affects the seller's policy, but only when the seller has large capacity ($T \ge N_1 + N_2$). If the product quality is very high ($t \ge \bar{t}$), it is optimal for the seller to set the price low enough so that all customers want to buy and the seller becomes indifferent in selling the product between the two periods.

The obvious next questions are which of the two sellers—high-type or low-type—quotes a higher price and which sells a bigger quantity in advance. The following theorem answers these questions.

Theorem 2. *Consider the full-information case.*

(i) *The two types of sellers always ration the same amount of capacity in advance, i.e., $S_H^f = S_L^f$.*

(ii) *When both types sell in advance ($S_H^f > 0$), the high-type seller charges a strictly higher advance price, $p_{1H}^f > p_{1L}^f$.*

As expected, the high-type seller charges a strictly higher price in advance. This is intuitive because customers who are aware in advance of quality are willing to pay more for high quality. However, the two types of sellers always ration the same amount of capacity in advance. This is because the difference in quality is perfectly captured by the price difference. Thus, in the full-information setting, quality difference is only reflected in price. We shall see that the result is drastically different when customers are uncertain about the quality.

5. Asymmetric Information About Quality

We now study the case when customers in the advance period are not sure about the product quality. We examine whether the seller benefits by signaling product quality through the terms of advance selling (i.e., price and quantity) and, if the seller does benefit, how a high-type seller differentiates from a low-type seller through her quantity and price decision. We show that, unlike the full-information case, quality cannot be differentiated by price alone. Instead, the quality uncertainty forces the seller offering high-quality products to distort the rationing level and adjust price accordingly. We first characterize the properties of a separating equilibrium, where the terms of advance selling perfectly communicate the quality information to customers. We then examine pooling equilibria, where advance selling is uninformative about the quality.

In preparation for equilibrium analysis, it is useful to examine the seller's profit in a special case, where all consumers believe that the product quality is high, i.e., $b(p_1, S) = 1$. We will show that the "single-crossing property" (Athey 2001) holds in this case, which will be used later for the general setting. When $b(p_1, S) = 1$, customers believe that the true quality is high and will accept any feasible advance price, $p_1 \le p_{1H}^*(S)$. Hence, the type t seller's total profit is

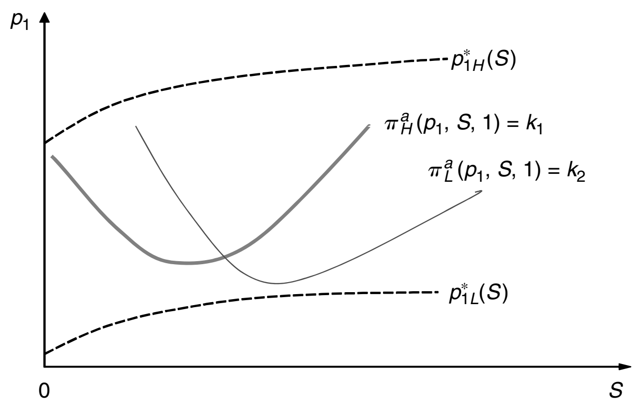
$$\pi_t^a(p_1, S, 1) = p_1 S + \pi_{2t}^*(S), \tag{8}$$

where superscript a stands for asymmetric information. The following lemma shows that iso-profit curves of the two types of seller satisfy the single-crossing property.

Lemma 2. *Consider $S > 0$ and $b = 1$. Iso-profit curves for two types of sellers (low and high) cross at most once. When they cross, the high-type seller's iso-profit curve crosses the low-type seller's from below, i.e., the high-type seller's iso-profit curve is below the low type seller's on the left of the crossing point and is above on the right.*

Figure 2 illustrates Lemma 2. A point (p_1, S) represents the seller's strategy—his advance price and rationed quantity, respectively. The two increasing curves, $p_{1L}^*(S)$ and $p_{1H}^*(S)$, represent the lower and

Figure 2 **Single Crossing of Iso-Profit Curves for $b = 1$ (k_1, k_2 Are Constants)**



upper bounds of the feasible price for a given rationing quantity, $S > 0$. Inside the feasible region, we show iso-profit curves, one for high-type seller and one for low-type seller. For each type of seller, the total profit remains constant on the corresponding iso-profit curve. This "single-crossing property" is important in our analysis. It is used to show that the high-type seller will decrease his rationing quantity to signal his quality during advance selling (Theorem 3). It is also used to show that certain pooling equilibria will be ruled out by the intuitive criterion (Theorem 4).

5.1. Separating Equilibrium

In a separating equilibrium, the seller's quality will be fully revealed in the advance period because low- and high-type sellers use different strategies. In the context of our game, a separating equilibrium must follow one of the following cases: either (1) only one type of seller offers advance selling, or (2) both do, but they are different in price, in quantity rationed, or in both. Denote type t seller's equilibrium strategy pair by (p^a_{1t}, S^a_t), where $t = H$ or L. In any separating equilibrium, customers can infer the true quality: $b(p^a_{1H}, S^a_H) = 1$ and $b(p^a_{1L}, S^a_L) = 0$.

Following the standard argument, it is straightforward to show that, in any separating equilibrium, the strategy used by the low-type seller is always the same as in the full-information case, i.e., $p^a_{1L} = p^f_{1L}$ and $S^a_L = S^f_L$. This can be shown by following the standard arguments as in, e.g., Lutz (1989) and Sobel (2007): because the low-type seller is always perfectly discerned in a separating equilibrium, the above strategy will yield the highest payoff for her. On the other hand, the problem that the high-type seller needs to solve is as follows:

$$(p^a_{1H}, S^a_H) = \arg\max_{p_1, S} \{\pi^a_H(p_1, S, 1) = p_1 S + \pi^*_{2H}(S)\}$$

subject to

$$S = 0, \quad \text{or } \{S \in (0, \min(T, N_1)] \text{ and } p_1 \in [p^*_{1L}(S), p^*_{1H}(S)]\}, \tag{9}$$

$$\pi^a_L(p_1, S, 1) \le \pi^a_L(p^f_{1L}, S^f_L, 0), \tag{10}$$

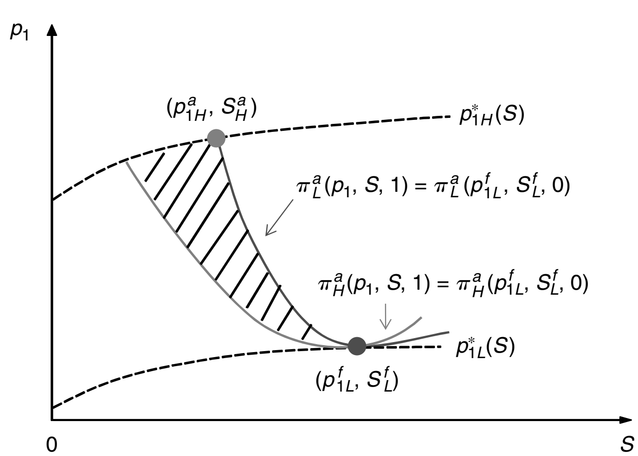
$$\pi^a_H(p_1, S, 1) \ge \pi^a_H(p^f_{1L}, S^f_L, 0), \tag{11}$$

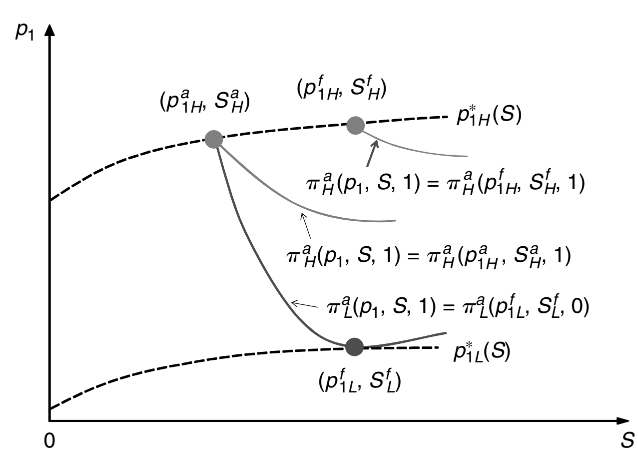
$$(p_1, S) \ne (p^f_{1L}, S^f_L). \tag{12}$$

In the above formulation, the high-type seller maximizes his own profit, subject to the following four constraints: first, the feasibility condition defined in §3.2; second and third, the low type prefers to be perceived as a low type rather than to imitate the high type's strategy, and the high type does not have an incentive to imitate the low type (incentive compatible); and fourth, the high type's strategy is not identical to the low type's.

Figure 3 (left) shows two iso-profit curves where the profit of each seller equals the profit when both sellers use the low type's equilibrium strategy. Clearly these

Figure 3 **A Separating Equilibrium**





two curves contain (and thus cross at) the strategy point (p_{1L}^f, S_L^f). Note that the seller's profit will increase if she manages to sell all the S units at a higher advance price p_1. Hence, the points above a given iso-profit curve correspond to higher profits. By the single-crossing property (Lemma 2) and the incentive-compatibility constraints, only the points inside the shaded area are feasible strategies for the high-type seller. From the graph, any feasible point satisfies $S < S_L^f$; that is, to differentiate himself from the low-quality seller, the high-quality seller should ration strictly lower capacity S during the advance sales. The next theorem formally proves this result and shows that this separating equilibrium exists if and only if both sellers are going to sell in the advance period in the full-information case. Figure 3 (right) illustrates the strategies of the low-type and high-type sellers in the full-information case, and in the separating equilibrium described above, as well as the corresponding iso-profit curves. In this separating equilibrium, the high-type seller sets a lower advance price and sells fewer units during advance sales, compared to what he would do in the full-information case. The following theorem formally proves this result.

Theorem 3. (i) *A separating equilibrium exists if and only if $T > T_1$, i.e., if and only if the capacity is sufficiently large so that advance selling is optimal for both types in the full-information case.*

(ii) *In a separating equilibrium, the low-type seller follows her full-information strategy (p_{1L}^f, S_L^f).*

(iii) *In a separating equilibrium, the following characterize the H-type seller's rationing*:

(a) *The high-type seller rations strictly less than the low-type seller, $S_H^a < S_L^a$.*

(b) *The high-type seller rations strictly less than he would in the full-information case, $S_H^a < S_H^f$.*

(c) *When the high-type seller offers advance selling ($S_H^a > 0$), his price is lower than the price he would charge in the full-information case, $p_{1H}^a \leq p_{1H}^f$. This price, however, is the same as the price that he would charge when the same quantity S_H^a is sold in the full-information case, i.e., $p_{1H}^a = p_{1H}^*(S_H^a)$.*

(d) *For a given low quality L, the high-type seller's equilibrium ration S_H^a is nonincreasing in his quality H.*

Part (i) of Theorem 3 implies that a separating equilibrium arises whenever capacity is not too limited ($T > T_1$) so that both sellers offer advance selling in the full-information case (see Theorem 1). Note that no additional condition is required for the existence of a separating equilibrium. It implies that, under fairly general conditions, the high-type seller can differentiate from the low-type seller with the terms of advance selling, (p_1, S). On the other hand, if the capacity is tight ($T \leq T_1$), this separating equilibrium breaks apart because neither type wants to sell in advance in the full-information case.

Parts (ii) and (iii) of Theorem 3 further characterize this separating equilibrium. The information asymmetry only affects the high-type seller because the low-type seller follows her strategy in the full-information case. The high-type seller needs to distort his strategy from the full-information levels and differentiate himself from the low-type seller. What is interesting is how the high-type seller accomplishes it. If both types of sellers offered the same quantity during the advance sales, the high-type seller's advance price would be higher and, consequently, low-type seller could increase her profit by matching the high-type seller's advance price, without any consequence on the low-type seller's profits in spot. To signal his type, the high-type seller needs to change the ration for advance sales. Part (iii(a)) of Theorem 3 shows that the high-type seller will decrease the quantity available during the advance sales to an extent that cannot be economically mimicked by low-type seller. Note that for a given advance price, reducing the capacity ration will decrease both types' advance profits by the same amount. However, in the spot period when the quality information is revealed, the high-type seller can charge a higher spot price than a low-type seller. Consequently, decreasing the advance sales hurts the low-type seller more than the high-type seller. Part (iii(b)) of Theorem 3 immediately follows from Theorem 2, $S_L^f = S_H^f$, and part (ii) of Theorem 3, $S_L^a = S_L^f$. Reducing the ration signals the product quality in the following way. Customers, upon observing that only a small portion of the total capacity is offered in advance, infer that the seller is very confident about her quality and reserves a lot to sell in the spot period. In contrast, a large ration in advance will be associated with low quality, because the low-type seller expects a weak spot market and has a strong incentive to sell a lot in advance. Part (iii(d)) of Theorem 3 reinforces this intuition. As the quality difference increases, the high-type seller will reserve more capacity to sell in the spot market.

Part (iii(c)) of Theorem 3 explains how the high-type seller changes the advance selling price. One may think that the high-type seller should increase the advance selling price as he reduces the advance ration. However, the effect is exactly opposite. As the capacity ration decreases, more capacity is available in spot and spot price decreases. As spot price decreases, customers are willing to pay less in advance. Nevertheless, it should be noted that this price change is simply a consequence of the change in the seller's rationing rather than the seller's deliberate attempt to use price as a signal. To see this, note that the high-type seller offers the same price as he would when he sells the quantity S_H^a in the full-information case, implying that there is no additional price distortion due to information asymmetry.

While price always increases in quality in the full-information case (recall Theorem 2), we find that price may increase or decrease in quality in the asymmetric-information case. Specifically, in the separating equilibrium, we observe in numerical study that the high-type seller's advance price may be either higher or lower than the low-type seller's advance price. Although the result seems counterintuitive at first, it can be explained as follows. First, note that, for a given level of rationing, the high-type seller's advance price is strictly higher than that of the low-type seller (indicating a price premium), because consumers are willing to pay a higher price in advance when they believe that the product is of high quality. However, in the separating equilibrium, the high-type seller rations less capacity in advance than the low-type seller. For a given posterior belief, consumers' willingness to pay in advance decreases when the seller rations a smaller quantity, because more capacity will be available in spot, and consequently the spot price will decrease (indicating cost of rationing signal). Depending on which of the two forces plays a predominant role, the high-type seller's price can be either higher or lower than the low-type seller's price.

It should be noted that the high-type seller uses exactly the opposite strategy of the one he would use in the full-information case. In the full-information case (see §4), price reflects the quality difference and rationing does not: both sellers use the same rationing policy, but a high-type seller charges a higher price than a low-type seller. In the asymmetric-information case, price alone cannot signal quality. The seller uses rationing, combined with the price that corresponds to the ration, to signal quality.

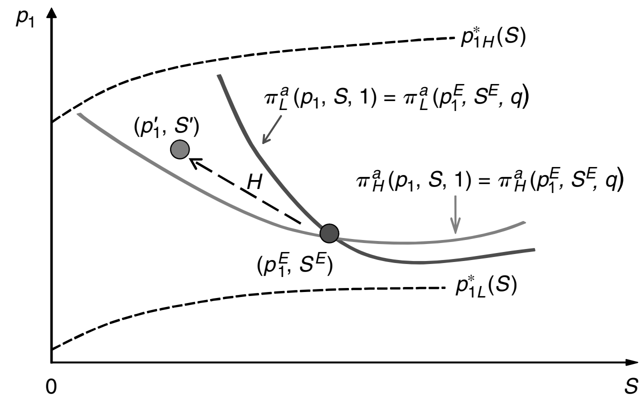
5.2. Pooling Equilibrium

In a pooling equilibrium, both sellers follow the same strategy during the advance sales—i.e., either both types sell in advance with the same price and ration, or neither sells in advance. Thus, the quality information will not be revealed. We show that applying the intuitive criterion eliminates a pooling equilibrium in which both sellers offer advance selling.

Theorem 4. *By the intuitive criterion, a pooling equilibrium wherein both types offer the same quantity during advance sales, $S > 0$, cannot be sustained.*

Figure 4 illustrates why this is the case. Suppose a pooling equilibrium (p_1^E, S^E) exists with $S^E > 0$. Such a strategy (a point in Figure 4) must be in the interior of the feasible region. Customers will not pay any advance price greater than or equal to p_{1H}^* when they believe that the product might be sold by a low-quality seller. On the other hand, any price less than or equal to

Figure 4 Intuitive Criterion Eliminates a Pooling Equilibrium



p_{1L}^* is Pareto-dominated from the sellers' perspectives. From the single-crossing property (Lemma 2), the high-type seller's iso-profit curve must lie below the low-type seller's curve for $S \in (0, S^E)$, and the two curves must cross at (p_1^E, S^E). Hence, if the high-type seller unilaterally reduces the advance ration and deviates to (p_1', S') for some $0 < S' < S^E$, this move will make the high-type seller strictly better off and the low-type strictly worse off. From the intuitive criterion, customers will believe that the seller is the high type. This supports the high-type seller's unilateral deviation and breaks the pooling equilibrium.

Theorem 4 further highlights why rationing can be effectively used to signal quality. As long as advance selling is desirable, the high-type seller would never pool with the low-type seller during advance sales, because he can gain more from revealing his type to customers by unilaterally lowering the capacity ration in advance.

5.3. Structure of Equilibrium

Following Theorems 3 and 4, we further characterize the seller's equilibrium strategies as functions of capacity T and quality levels L and H. Theorem 5 summarizes the result.

Theorem 5. *When quality information is asymmetric in advance, equilibrium strategies are as follows:*

(i) *If $T \le T_1$, neither type offers advance selling.*

(ii) *If $T \in (T_1, T_D)$, both types offer limited advance selling, and the high-type seller rations less in advance than the low-type seller.*

(iii) *If $T \in [T_D, N_1 + N_2)$, both types offer advance selling, and the high-type seller limits advance sales while the low-type seller does not.*

(iv) *If $T \ge N_1 + N_2$, then the equilibrium depends on the values of L and H. Specifically,*

(a) *if $L < \bar{t}$, the high-type seller offers limited advance selling and the low-type seller offers full advance selling;*

(b) *if $L \ge \bar{t}$, the high-type seller only sells in spot while the low-type seller is indifferent between offering advance selling and selling only in spot.*

Theorem 5 implies that the quality uncertainty does not affect the strategy and profit of the low-quality seller. However, the high-type seller has to sacrifice a portion of his profit by limiting the amount sold in advance (and possibly also lowering the price quoted in advance) to differentiate his type during advance sales. One may expect that in such a case, the high-quality seller is less likely to sell in advance. Interestingly, as long as advance selling is strictly preferred by the high type in the full-information case (parts (ii), (iii) and (iv(a)) of Theorem 5), the high-quality seller continues to offer advance selling. Although information asymmetry reduces the profit gain achieved by advance selling, its effect does not distort the seller's strategy enough to abandon advance selling. Reducing rationing quantity is sufficient to differentiate his type from the low-quality seller. The only exception to this rule is the case described by part (iv(b)) of Theorem 5, when the seller has large capacity, $T \geq N_1 + N_2$, and the quality of the low-type product exceeds the threshold, $\underline{t}$. In this situation, the seller has much capacity and finds it optimal to sell to all customers. Hence, in the full-information case, the seller charges the same price in both advance and spot periods (see Theorem 1). However, when the quality is uncertain and the high-type seller offers the same rationing quantity as the low-type seller, the low-type seller can easily mimic the high-type seller and remove the ability to signal. Consequently, the high-type seller is better off selling only in spot.

6. Value of Rationing

Our previous results show that in the presence of quality uncertainty, sellers can signal quality by limiting the amount sold in advance. One interesting question that follows is how big the benefit from such signaling is. In the full-information case, an option to ration capacity never hurts the seller because it gives the seller more choices in allocating capacity: instead of 0 or $\min(T, N_1)$, S could be any quantity $S \in [0, \min(T, N_1)]$. In fact, as shown in part (i) of Theorem 1, rationing makes the seller strictly better off when his capacity is at an intermediate level. The value of capacity rationing, however, becomes less evident when quality is uncertain. This is because, although the ability to ration helps the high-type seller to differentiate from the low-type seller, signaling by reducing the ration in the advance sales will decrease the profit.

To examine whether the seller always benefits from rationing, we first examine the equilibrium outcome when sellers cannot ration their capacity during advance selling. That is, if the seller offers advance selling, he needs to accept all demand up to his total capacity: $S = \min(N_1, T)$. A few observations immediately follow in the no-rationing case. If both sellers set out to offer advance selling, the quantity will be the same, and the low-type seller has an incentive to mimic the high-type seller's price. Consequently, the high-type seller can never differentiate himself by advance selling. Theorem 6 characterizes the equilibrium for the no-rationing case.

Theorem 6. *For given H, let $\delta = H - L$ denote the difference in quality levels. When rationing is not allowed (i.e., $S = \min(T, N_1)$ or 0), there exists a function $t^D(T)$ such that the following holds in equilibrium*:

(i) *If $L > t^D(T)$, neither type sells in advance.*

(ii) *If $L \leq t^D(T) < H$, only the low-type seller offers advance selling.*

(iii) *If $H \leq t^D(T)$, there exists a threshold $\bar{\delta} \geq 0$ and a function $\bar{q}(\delta) \in [0, 1]$ for $\delta > 0$ such that*

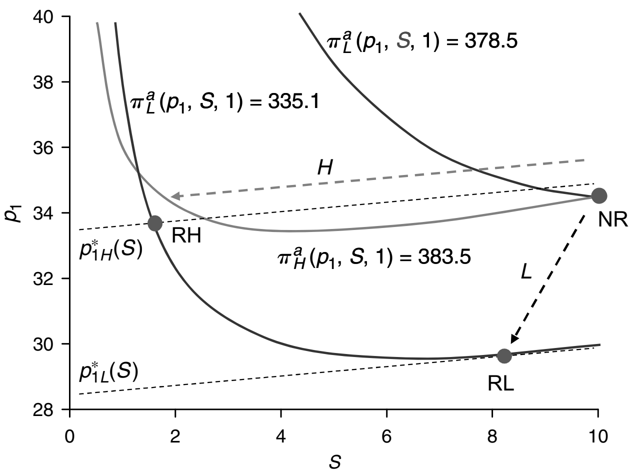
(a) *if $\delta > \bar{\delta}$ and $q < \bar{q}(\delta)$, only the low-type sells in advance;*

(b) *otherwise (i.e., $\delta \leq \bar{\delta}$ or $q \geq \bar{q}(\delta)$), both types sell in advance with a common advance price.*

Part (i) of Theorem 6 implies that, when sellers cannot ration, neither seller offers advance selling if the qualities of both types are sufficiently high ($L > t^D(T)$). When this happens, neither seller benefits from advance selling at a discounted price. When the quality of the low-type product falls below the threshold, $t^D(T)$, at least one of the two sellers offers advance selling in equilibrium. The low-type seller has an incentive to sell as much as possible in advance before the quality information is revealed. The high-type seller chooses to sell in advance only if customers strongly believe that the product quality is high (i.e., the probability that the seller is a high type), $q \geq \bar{q}(\delta)$, or the quality difference is sufficiently small, $\delta \leq \bar{\delta}$. In such cases, a pooling equilibrium where both types charge a sufficiently high price in advance emerges, and both sellers benefit from selling in advance (compared to selling only in spot).

We now compare the equilibria in the rationing (Theorem 5) and no-rationing (Theorem 6) cases. We find that having an operational flexibility to ration does not necessarily benefit the seller. In other words, both sellers can be better off when they do not have the ability to ration. To see why this is the case, note that the high-quality seller always uses rationing to signal his quality as long as advance selling is desirable. Thus, in the no-rationing pooling equilibrium, the low-type seller may be able to hide her inferior quality and quote a higher advance price compared to a separating equilibrium that arises in the rationing case. Thus, the low-type seller will be better off without the ability to ration. Interestingly, the inability to ration can benefit the high-type seller, too. Although rationing enables the seller to signal its type, this signaling can be costly, because the high-type seller is forced to reduce the amount sold in advance and to lower both spot and advance prices. The cost associated with

Figure 5 **An Example Where Both Types Are Strictly Worse Off by Capacity Rationing:** $q = 0.9$, $N_1 = 10$, $N_2 = 10$, $\alpha \sim$ **Uniform**$[-5, 5]$, $T = 11$, $H = 35$, $L = 30$



Note. For the labeled points, the corresponding strategy pair and profits for type t seller (p_1, S; $\pi_L^a(p_1, S, 1)$, $\pi_H^a(p_1, S, 1)$), are as follows: NR(34.45, 10; 378.5, 383.5), RL(29.75, 8.39; 335.1, 348.1), RH(33.69777, 1.62; 335.1, 381.9).

signaling can outweigh any profit incurred by the pooling equilibrium in the no-rationing case.

Figure 5 illustrates in more details how both sellers can be strictly better off in the no-rationing equilibrium. According to Theorem 6, an outcome where both sellers offer the full advance selling at the same price (point NR) is a pooling equilibrium when rationing is disallowed. Suppose now that both sellers can ration their capacity in advance. First note that from Theorem 4, no pooling equilibrium where both types sell in advance can be sustained, thus NR cannot be an equilibrium and a deviation must occur. In this situation, the low-type seller will follow the full-information strategy (point RL) (see Theorem 3). Consider the iso-profit curve for the low-type seller that runs through RL. Any point above this curve cannot be chosen by the high-type seller, because it can be mimicked profitably by the low-type seller (this is formally written in constraint (10)). To prevent the low-type seller from mimicking, the high-type seller needs to lower rationing to a point where the low-type seller is indifferent between mimicking and following her full-information strategy. However, doing so will also lower both the advance and spot prices and erode the seller's profit (point RH). At this point, both sellers earn strictly lower profits than in the no-rationing situation. In short, the high-type seller is worse off because signaling costs too much and the low-type seller is worse off as it cannot pool with the high-type seller. Hence, both types of seller would prefer not to have the ability to ration.

An immediate question is why both sellers cannot follow a pooling equilibrium when they have the flexibility to ration. The answer is that, similar to the prisoner's dilemma, the no-rationing equilibrium, while achieving the Pareto-dominant outcome, cannot be enforced once rationing becomes feasible. The high-type seller, induced by short-term increase of profit, cannot resist the temptation to deviate to reduce rationing in advance. This incentive of rationing, however, triggers a downward spiral, leading to an outcome where both sellers lose.

It should be noted that the phenomenon that pooling is better for both sellers occurs only when customers strongly believe that the product quality is high (i.e., q is high). In fact, it can be shown that, *ceteris paribus*, the value of rationing decreases in the prior belief, q. As q increases, customers are more optimistic about high quality and are willing to pay a higher price. On the other hand, as Theorem 5 illustrates, the rationing equilibrium is independent of the prior belief q. Hence, as q increases, the increase in the pooling price makes the no-rationing pooling equilibrium more appealing and thus the rationing option becomes less desirable. Our result shows that although capacity rationing can signal quality, it is sometimes costly and makes both sellers worse off compared to the no-rationing case. As the prior belief of high quality decreases, the value of rationing increases.

7. Conclusion

As advance selling has been rapidly adopted in practice, academic research has examined a number of reasons why firms should offer advance selling. These reasons include consumer's risk aversion (Png 1989), advance demand information (Tang et al. 2004, Li and Zhang 2013), and consumer's valuation uncertainty (Xie and Shugan 2001, Gallego and Sahin 2010). To the best of our knowledge, this paper is the first one that describes and analyzes the role of advance selling as a signal of product quality. We show that, when consumers do not have perfect information about product quality, the seller can signal the product quality by rationing the capacity available during advance sales.

We show that the consumer's uncertainty about product quality always worsens the profit of a seller offering a high-quality product, because his product cannot be fully appreciated during advance sales. Pricing alone cannot be a signal for quality, because the low-quality seller can easily mimic the high-quality seller. In order to differentiate herself from the low-quality seller, the high-quality seller sacrifices some profit by reducing the capacity ration during advance sales from the rationing level that the same seller would choose if the quality were known to consumers. Although it may seem optimal for the high-quality seller to bypass advance selling and sell only after the quality information is released, selling a portion of the capacity in advance can still increase the high-quality seller's profit except in two extreme cases. The first case is when the total capacity is very tight. In this case, the seller can clear the capacity at a very high spot price, and neither seller wants to sell in advance. Thus, the presence of the low-quality seller does not reduce the high-type seller's profit. The second case is when the seller has a large capacity and would like to sell to all customers by pricing the product very low. Instead of sending a costly signal, the high-type seller sells only in the spot period while the low-type seller is indifferent between selling in advance and selling only in spot. In all other cases, the high-quality seller uses reduced ration as a primary signal device. Interestingly, as long as both types of sellers offer advance selling in the full-information case, then the high-quality seller can use rationing to distinguish himself. Our finding on rationing capacity to signal quality is consistent with several examples in practice. One such examples is the premium French wine's advance (*en primeur*) market, when chateaux intentionally limit the availability of wine sold *en primeur* to convey high quality (Prial 1989, Stimpfig 2012).

Although rationing can be a very effective tool for signaling and for increasing profits, we show that the seller does not always benefit from the ability to ration. When compared to the case where rationing is not allowed, rationing flexibility can make both high-quality and low-quality types of sellers strictly worse off. This happens when customers are optimistic about the product's high quality and the seller's capacity is not too tight. Although rationing enables the seller to signal his type, reducing the amount that he sells in advance (and consequently lowering both spot and advance prices) can outweigh any profit from simply pooling with the low-type seller (which is the outcome when rationing is not allowed). Because consumers are optimistic about product quality, the high-type seller does not lose too much by pooling with the low-type seller.

Our model and results can be extended in a number of directions. First, our results are robust in two variations of the model. We show that all major results and insights carry through when the marginal cost of a high-quality product (c_H) is different from that of a low-quality seller (c_L). All results also hold when the utility of a consumer is a multiplicative function ($U = \alpha t - p$) instead of an additive function ($U = \alpha + t - p$). The proofs for these two variations are provided in the online supplementary document to this paper (available at http://www.bm.ust.hk/isom/staff/manyu.html).

Second, it is interesting to compare the rationing signal with advertising, another signaling tool that has been studied extensively in the literature (Kihlstrom and Riordan 1984, Milgrom and Roberts 1986, Bagwell and Ramey 1988, Stock and Balachander 2005). Consider the case where advertising is a pure dissipative cost that the seller incurs to her customers for signaling. We can show that such uninformative advertising alone cannot signal quality in our setting. This implies that in our setting signaling by rationing and pricing can be more efficient than by advertising. This is because advertising is a pure cost to the seller, whereas rationing helps the seller to partially offset the signaling cost by adjustment of both the availability and price in the spot period. These proofs are also are available in the online supplementary document.

Third, although we have assumed exogenous capacity in our model, it is interesting to consider the variation wherein the seller can limit not only the supply in advance but also the total supply in the two periods.[9] In this case, the seller can use three signals—total capacity, advance price, and advance rationing level—to convey quality. In particular, for a given advance price and rationing level, the high-type seller has an incentive to choose a higher total capacity level than a low-type seller has, because the high-type seller expects larger sales in the spot period when the quality is revealed. The high-type seller now has a choice of signaling quality through a higher total capacity level, or a lower advance capacity ration, or both. We conjecture that, with the high-type seller having more degrees of freedom to differentiate her quality, mimicking the behavior of the high-type seller is more difficult for the low-type seller, and signaling becomes less costly for the high-type seller. However, the analysis involves a multidimensional signaling problem over two different periods, which we leave to future research.

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Appendix A. Proof of Lemma 1 and Corollary 1

The proof, directly adapted from Yu et al. (2015), is provided in the online supplementary document.

Appendix B. Proof of Theorem 1

(i) The proof, directly adapted from Yu et al. (2015), is provided in the online supplementary document.

(ii) First recall the definitions of T_1 and T_D in Yu et al. (2015): for $T < N_1 + N_2$ and $S \in [0, \min(T, N_1)]$, let $f_t^B(S) = \mathrm{E}[\min(t+\alpha, p_{2t}^B(S))]S + p_{2t}^B(S)(T-S)$; T_1 is the largest T such that $f_t^B(S)$ is maximized at $S = 0$ and T^D is the smallest T such that $f_t^B(S)$ is maximized at $S = \min(T, N_1)$.

To show that T_1 and T_D are independent of the quality level t, it then suffices to show that $df_t^B(S)/dS$ is independent of t. To this end, note that by Lemma 1,

[9] We thank an anonymous referee for suggesting this variation.

$p_{2t}^B(S) = t + (\bar{G})^{-1}((T-S)/(N_1+N_2-S))$. Substituting this in the expression of $f_t^B(S)$, we have $f_t^B(S) = \mathrm{E}[\min(\alpha, (\bar{G})^{-1}((T-S)/(N_1+N_2-S)))]S + (\bar{G})^{-1}((T-S)/(N_1+N_2-S))(T-S)+tT$, which depends on t only through the term tT. Clearly, $df_t^B(S)/dS$ is independent of t; hence, so are T_1 and T_D.

Appendix C. Proof of Theorem 2

The proof uses the following lemma. The proof of all the lemmas are available in the online supplementary document.

Lemma C.1. $p_{2H}^*(S) \ge p_{2L}^*(S)$, $p_{1H}^*(S) > p_{1L}^*(S)$.

By Lemma C.1, to show $p_{1H}^f > p_{1L}^f$, it suffices to show $S_H^f = S_L^f$. Meanwhile, by Theorem 1, to show $S_H^f = S_L^f$, it suffices to show that S_t^f is independent of t for $T \in (T_1, T_D)$. By Yu et al. (2015), for $T \in (T_1, T_D)$, S_t^f satisfies $df_t^B(S)/dS = 0$, where $f_t^B(S)$ is as defined in the proof of Theorem 1(ii). Since $df_t^B(S)/dS$ is independent of t (shown in the proof of Theorem 1(ii)), so is S_t^f for $T \in (T_1, T_D)$.

Appendix D. Proof of Lemma 2

The proof uses the following lemma:

Lemma D.1. *$\pi_{2H}^*(S) - \pi_{2L}^*(S)$ strictly decreases in S.*

For $b = 1$, $S > 0$, and constants k_1 and k_2, let the iso-profit curves of the high-type seller for profit k_1 and of the low-type seller for profit k_2 be $\{(S, p_1)\colon \pi_H^a(p_1, S, 1) = k_1, (p_1, S)$ is feasible$\}$ and $\{(S, p_1)\colon \pi_L^a(p_1, S, 1) = k_2, (p_1, S)$ is feasible$\}$, respectively. We prove by contradiction that (i) they cross at most once and (ii) when they do cross, the former crosses the latter from below.

(i) Suppose that the two curves cross at two *distinct* points (p_1', S') and (p_1'', S'') with $0 < S'' \le S'$. By definition of the iso-profit curve, $\pi_H^a(p_1', S', 1) = \pi_H^a(p_1'', S'', 1) = k_1$ and $\pi_L^a(p_1', S', 1) = \pi_L^a(p_1'', S'', 1) = k_2$. From the expression of $\pi_t^a(p_1, S, 1)$ in Equation (8), we get

$$p_1'S' + \pi_{2H}^*(S') = p_1''S'' + \pi_{2H}^*(S'') = k_1, \tag{D1}$$

$$p_1'S' + \pi_{2L}^*(S') = p_1''S'' + \pi_{2L}^*(S'') = k_2. \tag{D2}$$

Subtracting Equation (D2) from Equation (D1), we get $\pi_{2H}^*(S') - \pi_{2L}^*(S') = \pi_{2H}^*(S'') - \pi_{2L}^*(S'')$. Since $\pi_{2H}^*(S) - \pi_{2L}^*(S)$ strictly decreases in S (Lemma D.1), we immediately have $S' = S''$. This result, together with Equation (D1) and the fact $S' > 0$, further implies $p_1' = p_1''$. This, however, contradicts the assumption that (p_1', S') and (p_1'', S'') are two distinct points.

(ii) To show that the high-type seller's curve crosses the low-type seller's from below, we will prove that the high-type seller's curve is below the low-type seller's on the left of the crossing point. The result for the other side of the crossing point can be shown in a same way.

Suppose that the two curves cross at point (p_1', S') with $S' > 0$ and that the high-type seller's curve is above the low-type seller's on the left of the crossing point. Hence, there must exist two points (p_{1H}'', S'') and (p_{1L}'', S'') on the high-and low-type seller's curves, respectively, such that $0 < S'' < S'$ and $p_{1H}'' > p_{1L}''$. By definition of the iso-profit curve and Equation (8), we have

$$p_1'S' + \pi_{2H}^*(S') = p_{1H}''S'' + \pi_{2H}^*(S'') = k_1, \tag{D3}$$

$$p_1'S' + \pi_{2L}^*(S') = p_{1L}''S'' + \pi_{2L}^*(S'') = k_2. \tag{D4}$$

Subtracting Equation (D4) from Equation (D3), we get $(p_{1H}'' - p_{1L}'')S'' + \pi_{2H}^*(S'') - \pi_{2L}^*(S'') = \pi_{2H}^*(S') - \pi_{2L}^*(S')$. Since $p_{1H}'' > p_{1L}''$ and $S'' > 0$, we have $\pi_{2H}^*(S'') - \pi_{2L}^*(S'') < \pi_{2H}^*(S') - \pi_{2L}^*(S')$. This, however, contradicts the fact $S'' < S'$ and Lemma D.1.

Appendix E. Proof of Theorem 3

(i) First note that by Theorem 1, $S_L^f > 0$ if and only if $T > T_1$. Hence, it suffices to show that a separating equilibrium exists if and only if $S_L^f > 0$.

($\Rightarrow$) Prove by contradiction. Suppose $S_L^f = 0$ and a separating equilibrium exists. First note that Equations (10) and (11) jointly imply $\pi_{2H}^*(S_H^a) - \pi_{2L}^*(S_H^a) \ge \pi_{2H}^*(S_L^f) - \pi_{2L}^*(S_L^f)$, which further implies $S_H^a \le S_L^f$ by Lemma D.1. Since $S_L^f = 0$ and $S_H^a \ge 0$ (constraint (9)), we immediately have $S_H^a = 0$. In other words, both types of sellers sell only in spot in the separating equilibrium. This, however, contradicts the definition of a separating equilibrium.

($\Leftarrow$) It suffices to show that when $S_L^f > 0$, there exists a solution for the high-quality seller's problem (Equations (9)–(12)). To this end, define a function of S for $S \in [0, \min(T, N_1)]$:

$$\begin{aligned} M(S) &= \pi_L^a(p_{1H}^*(S), S, 1) - \pi_L^a(p_{1L}^f, S_L^f, 0) \\ &= p_{1H}^*(S)S + \pi_{2L}^*(S) - [p_{1L}^f S_L^f + \pi_{2L}^*(S_L^f)]. \end{aligned}$$

Clearly, $M(S)$ is continuous in S. Also recall the following facts: $p_{1L}^f = p_{1L}^*(S_L^f)$, S_L^f maximizes $\pi_L^a(p_{1L}^*(S), S, 0)$, and $p_{1H}^*(S) > p_{1L}^*(S)$ (Lemma C.1). These facts imply the values of $M(S)$ at two boundary points, 0 and S_L^f:

$$\begin{aligned} M(0) &= \pi_{2L}^*(0) - \pi_L^a(p_{1L}^*(S_L^f), S_L^f, 0) \le 0, \\ M(S_L^f) &= (p_{1H}^*(S_L^f) - p_{1L}^f)S_L^f = (p_{1H}^*(S_L^f) - p_{1L}^*(S_L^f))S_L^f > 0. \end{aligned} \tag{E1}$$

Hence, there exists a point $S \in [0, S_L^f)$ satisfying $M(S) = 0$. Let $\underline{S} = \min\{S \in [0, S_L^f)\colon M(S) = 0\}$. Below we show that $(p_{1H}^*(\underline{S}), \underline{S})$ satisfies all the constraints in the high-type seller's problem:

- Constraint (9): By definition of $\underline{S}$, $0 \le \underline{S} < S_L^f \le \min(T, N_1)$. Also, when $\underline{S} > 0$, clearly $p_1 = p_{1H}^*(\underline{S})$ satisfies $p_1 \in [p_{1L}^*(\underline{S}), p_{1H}^*(\underline{S})]$.
- Constraint (10): By definition of $\underline{S}$, $\pi_L^a(p_{1H}^*(\underline{S}), \underline{S}, 1) = \pi_L^a(p_{1L}^f, S_L^f, 0)$.
- Constraint (11): By Lemma D.1 and the fact $\underline{S} < S_L^f$, $\pi_H^a(p_{1H}^*(\underline{S}), \underline{S}, 1) - \pi_H^a(p_{1L}^f, S_L^f, 0) > \pi_L^a(p_{1H}^*(\underline{S}), \underline{S}, 1) - \pi_L^a(p_{1L}^f, S_L^f, 0) = 0$.
- Constraint (12): By definition of $\underline{S}$, $\underline{S} < S_L^f$. Hence, $(p_{1H}^*(\underline{S}), \underline{S}) \ne (p_{1L}^f, S_L^f)$.

(ii) First note that if L type sells in advance in the separating equilibrium (i.e., $S_L^a > 0$), then his advance price must be equal to $p_{1L}^*(S_L^a)$. That is because any price lower than $p_{1L}^*(S_L^a)$ is strictly dominated and any price higher will not be accepted by advance customers, as L type is perfectly discerned in the separating equilibrium. If, however, L type does not sell in advance (i.e., $S_L^a = 0$), his profit in equilibrium is simply $\pi_{2L}^*(0)$. In both cases, the low-type seller's profit in the separating equilibrium is $p_{1L}^*(S_L^a)S_L^a + \pi_{2L}^*(S_L^a)$. Furthermore, since S_L^f maximizes $p_{1L}^*(S)S + \pi_{2L}^*(S)$ for $S \in [0, \min(T, N_1)]$ (Equation (7)), the *highest* profit that the low-type seller can make in the equilibrium is $p_{1L}^*(S_L^f)S_L^f + \pi_{2L}^*(S_L^f)$.

On the other hand, if L type follows his full-information strategy $(p_{1L}^*(S_L^f), S_L^f)$ and $S_L^f > 0$, customers will always buy in advance regardless of their posterior belief b, because their maximum willingness-to-pay is at least $p_{1L}^*(S_L^f)$ (Equation (5)). Consequently, the *lowest* profit that the low-quality type can guarantee to make in a separating equilibrium is also $p_{1L}^*(S_L^f)S_L^f + \pi_{2L}^*(S_L^f)$.

From the two facts above, L type always follows his full-information strategy in a separating equilibrium, i.e., $p_{1L}^a = p_{1L}^*(S_L^f) = p_{1L}^f$ and $S_L^a = S_L^f$.

(iii) We first show the first half of (iii(c)): if $S_H^a > 0$, $p_{1H}^a = p_{1H}^*(S_H^a)$, and then use it to prove the other results.

(iii(c)) *if* $S_H^a > 0$, $p_{1H}^a = p_{1H}^*(S_H^a)$: Prove by contradiction. Suppose $S_H^a > 0$ and $p_{1H}^a \neq p_{1H}^*(S_H^a)$. By constraint (9), we immediately have $p_{1H}^a < p_{1H}^*(S_H^a)$. To reach a contradiction, it suffices to show that, compared to (p_{1H}^a, S_H^a), the feasible strategy $(p_{1H}^*(\underline{S}), \underline{S})$ identified in part (i) strictly improves H type's profit.

To this end, first note that by definition of $\underline{S}$ and the fact $M(0) \le 0$, we have $M(S) \le 0$ for all $S \le \underline{S}$. Meanwhile, since $p_{1H}^a < p_{1H}^*(S_H^a)$, constraint (10) must be binding with (p_{1H}^a, S_H^a), otherwise the high-quality seller can strictly increase the total profit by slightly raising p_{1H}^a while fixing S_H^a. The binding constraint (10) and $p_{1H}^a < p_{1H}^*(S_H^a)$ jointly imply $\pi_L^a(p_{1H}^*(S_H^a), S_H^a, 1) > \pi_L^a(p_{1H}^a, S_H^a, 1) = \pi_L^a(p_{1L}^f, S_L^f, 0)$, i.e., $M(S_H^a) > 0$. All of these results jointly imply $S_H^a > \underline{S}$.

Next we prove that $(p_{1H}^*(\underline{S}), \underline{S})$ dominates (p_{1H}^a, S_H^a). By definition of $\underline{S}$, the binding constraint (10), Lemma D.1, and the fact $S_H^a > \underline{S}$, we have

$$
\begin{aligned}
&\pi_H^a(p_{1H}^*(\underline{S}), \underline{S}, 1) - \pi_H^a(p_{1H}^a, S_H^a, 1)\\
&\quad = \pi_L^a(p_{1H}^*(\underline{S}), \underline{S}, 1) - \pi_{2L}^*(\underline{S}) + \pi_{2H}^*(\underline{S})\\
&\qquad - [\pi_L^a(p_{1H}^a, S_H^a, 1) - \pi_{2L}^*(S_H^a) + \pi_{2H}^*(S_H^a)]\\
&\quad = \pi_L^a(p_{1L}^f, S_L^f, 0) - \pi_{2L}^*(\underline{S}) + \pi_{2H}^*(\underline{S})\\
&\qquad - [\pi_L^a(p_{1L}^f, S_L^f, 0) - \pi_{2L}^*(S_H^a) + \pi_{2H}^*(S_H^a)]\\
&\quad = \pi_{2H}^*(\underline{S}) - \pi_{2L}^*(\underline{S}) - [\pi_{2H}^*(S_H^a) - \pi_{2L}^*(S_H^a)] > 0.
\end{aligned}
$$

(iii(a)) Prove by contradiction: Suppose $S_H^a \ge S_L^a$. Since $S_L^a = S_L^f$ by part (ii), we have $S_H^a \ge S_L^f$. Meanwhile, as shown in part (i), $S_H^a \le S_L^f$. These facts jointly imply $S_H^a = S_L^f$. Also note that in an equilibrium, $S_L^f > 0$ by part (i). Hence, $p_{1H}^a = p_{1H}^*(S_H^a) = p_{1H}^*(S_L^f)$ by part (iii(c)). However, from Equation (E1), $(p_{1H}^*(S_L^f), S_L^f)$ violates constraint (10) and hence cannot be H type's equilibrium strategy.

(iii(b)) Follows immediately from parts (ii) and (iii(a)), as well as Theorem 2.

(iii(c)) *if* $S_H^a > 0$, $p_{1H}^a \le p_{1H}^f$: $p_{1H}^a = p_{1H}^*(S_H^a)$ by part (iii(c)) and $p_{1H}^f = p_{1H}^*(S_H^f)$ by definition of p_{1H}^f. The result then follows from the facts that $p_{1H}^*(S)$ is nondecreasing in S and $S_H^a < S_H^f$ (shown above).

(iii(d)) By parts (iii(a)) through (iii(c)), the high-type seller's problem is equivalent to $p_{1H}^a = p_{1H}^*(S_H^a)$ if $S_H^a > 0$ and

$$S_H^a = \arg\max_{S \in [0, S_H^f)} \; \pi_H^a(p_{1H}^*(S), S, 1) = p_{1H}^*(S)S + \pi_{2H}^*(S)$$

subject to

$$\pi_L^a(p_{1H}^*(S), S, 1) \le \pi_L^a(p_{1L}^f, S_L^f, 0), \tag{E2}$$

$$\pi_H^a(p_{1H}^*(S), S, 1) \ge \pi_H^a(p_{1L}^f, S_L^f, 0). \tag{E3}$$

Note that $\pi_t^a(p_{1t}^*(S), S, 1) = \pi_t^f(S)$ for $t = H$ or L. Based on the property of $\pi_t^f(S)$ characterized in Yu et al. (2015), below we prove by considering three cases:

- $T_1 < T < N_1 + N_2$: By Yu et al. (2015), both $\pi_H^a(p_{1H}^*(S), S, 1)$ and $\pi_L^a(p_{1L}^*(S), S, 1)$ strictly increase in $S \in [0, S_H^f)$. It is easy to show that $(p_{1H}^*(S) - p_{1L}^*(S))S$ is nondecreasing in S, hence $\pi_L^a(p_{1H}^*(S), S, 1) = \pi_L^a(p_{1L}^*(S), S, 1) + (p_{1H}^*(S) - p_{1L}^*(S))S$ strictly increases in $S \in [0, S_H^f)$. As a result, constraint (E2) must be binding when $S = S_H^a$. Fixing L and S, we have $\pi_L^a(p_{1H}^*(S), S, 1)$ strictly increases in $H > L$, while $\pi_L^a(p_{1L}^f, S_L^f, 0)$ remains constant. Hence, S_H^a is nonincreasing in $H > L$. (It is also easy to see $S_H^a > 0$, as the inequality in constraint (E2) is strict when $S = 0$. We will use this result in a later proof.)
- $T \ge N_1 + N_2$ and $L \ge \bar{t}$: By Yu et al. (2015), both $\pi_H^a(p_{1H}^*(S), S, 1)$ and $\pi_L^a(p_{1L}^*(S), S, 1)$ are independent of S for $S \in [0, \min(T, N_1)]$. Meanwhile, since $p_{1H}^*(S) > p_{1L}^*(S)$, constraint (E2) is satisfied only when $S = 0$. It is easy to show that $S = 0$ also satisfies constraint (E3). Hence, $S_H^a = 0$ for all $H > L$.
- $T \ge N_1 + N_2$ and $L < \bar{t}$: By Yu et al. (2015), $\pi_L^a(p_{1L}^*(S), S, 1)$ strictly increases in $S \in [0, S_H^f)$. If $H < \bar{t}$, then the same monotonicity applies to $\pi_H^a(p_{1H}^*(S), S, 1)$. Following the same logic as in the first bullet, we can show that S_H^a is nonincreasing in $H > L$ (and $S_H^a \in (0, S_H^f)$). If, however, $H \ge \bar{t}$, then $\pi_H^a(p_{1H}^*(S), S, 1)$ is independent of S. Since $\pi_L^a(p_{1H}^*(S), S, 1)$ strictly increases in S, there exists an interval $[0, \bar{S}]$ for some $\bar{S} \in (0, S_H^f)$ such that constraint (E2) is binding at $S = \bar{S}$ and the high-type seller is indifferent in choosing any $S \in [0, \bar{S}]$. Hence, $S_H^a = \bar{S}$ as it leads to the lexicographically largest equilibrium, and it decreases in $H > L$ since $\pi_L^a(p_{1H}^*(S), S, 1)$ strictly increases in H.

Appendix F. Proof of Theorem 4

Suppose a pooling equilibrium exists in which both types of sellers offer advance selling at (p_1^E, S^E). Per definition, $S^E > 0$ and customers' posterior belief $b(p_1^E, S^E)$ is the same as the prior belief q. Furthermore, since customers would buy in advance in the equilibrium (ref. participating equilibrium), we have $p_1^E \le q p_{1H}^*(S^E) + (1-q) p_{1L}^*(S^E)$, further implying $p_1^E < p_{1H}^*(S^E)$ by Lemma C.1. To show that the intuitive criterion always eliminates such a pooling equilibrium, it suffices to show that there always exists a strategy pair (p_1', S') such that the high-quality seller strictly prefers choosing (p_1', S') and being perceived as a high-quality seller, rather than pooling at (p_1^E, S^E), while the low-quality seller has the opposite preference.

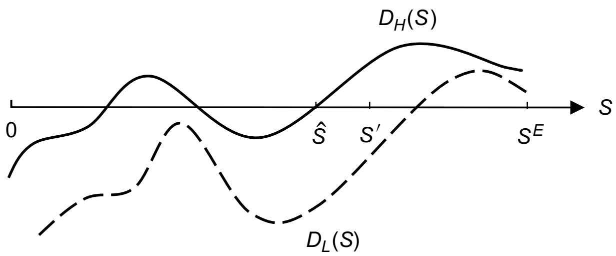
To this end, we first define two functions of S for $S \in [0, \min(T, N_1)]$:

$$
\begin{aligned}
D_H(S) &= \pi_H^a(p_{1H}^*(S), S, 1) - \pi_H^a(p_1^E, S^E, q)\\
&= p_{1H}^*(S)S + \pi_{2H}^*(S) - [p_1^E S^E + \pi_{2H}^*(S^E)],\\
D_L(S) &= \pi_L^a(p_{1H}^*(S), S, 1) - \pi_L^a(p_1^E, S^E, q)\\
&= p_{1H}^*(S)S + \pi_{2L}^*(S) - [p_1^E S^E + \pi_{2L}^*(S^E)].
\end{aligned}
$$

The two functions are illustrated in Figure F.1.

Clearly both $D_H(S)$ and $D_L(S)$ are continuous in S. Furthermore, note that $D_H(S^E) = (p_{1H}^*(S^E) - p_1^E)S^E > 0$ since $p_1^E < p_{1H}^*(S^E)$ and $S^E > 0$, and that $D_H(0) \le 0$ since otherwise the high-quality seller would strictly prefer selling

Figure F.1 Illustration of $D_H(S)$ and $D_L(S)$



only in spot to selling in advance with (p_1^E, S^E). Hence, there exists at least a point $S \in [0, S^E)$ satisfying $D_H(S) = 0$. Let $\hat{S} = \max\{S\colon S \in [0, S^E), D_H(S) = 0\}$. Clearly, $D_H(S) > 0$ for all $S \in (\bar{S}, S^E]$. Meanwhile, by Lemma D.1, for $S < S^E$, $D_H(S) - D_L(S) = \pi_{2H}^*(S) - \pi_{2L}^*(S) - (\pi_{2H}^*(S^E) - \pi_{2L}^*(S^E)) > 0$, implying $D_L(\hat{S}) < D_H(\hat{S}) = 0$. Since $D_L(S)$ is continuous in S, there must exist a S' in the right neighborhood of $\hat{S}$ such that $D_H(S') > 0$ and $D_L(S') < 0$. Let $p_1' = p_{1H}^*(S')$. Clearly, (p_1', S') satisfies $\pi_H^a(p_1', S', 1) > \pi_H^a(p_1^E, S^E, q)$ and $\pi_L^a(p_1', S', 1) < \pi_L^a(p_1^E, S^E, q)$.

Appendix G. Proof of Theorem 5

By Theorem 4, only two classes of equilibria are possible: separating equilibrium and pooling equilibrium wherein neither type sells in advance. Below we prove by considering the following four cases:

- (i) $T \le T_1$: By Theorem 3(i), there does not exist a separating equilibrium. Furthermore, by Theorem 1, neither type sells in advance in the full-information case. Hence, in the asymmetric-information case, neither type would deviate from selling only in spot.
- (ii) and (iii) $T \in (T_1, N_1 + N_2)$: First note that both types selling only in spot cannot be sustained as an equilibrium, because otherwise L type always has an incentive to deviate to selling in advance with his full-information strategy. On the other hand, by Theorem 3(i) through 3(ii(b)), a separating equilibrium always exists with $S_L^a = S_L^f$ and $S_H^a < S_L^a$. Furthermore, by Theorem 1, both types strictly prefer advance selling to selling only in spot in the full-information case. Thus, $S_H^a = 0$ is always dominated by some $S_H^a > 0$. Hence, by Theorem 1, $S_H^a \in (0, \min(T, N_1))$ and if $T \in (T_1, T_D)$, $S_L^a \in (0, \min(T, N_1))$ and if $T \in [T_D, N_1 + N_2)$, $S_L^a = \min(T, N_1)$.
- (iv(a)) $T \ge N_1 + N_2$ and $L < \bar{t}$: Similar to the second bullet, only a separating equilibrium exists. By Theorems 1 and 3, $S_L^a = \min(T, N_1)$ and the low-quality type of seller strictly prefers advance selling to selling only in spot. Meanwhile, by the proof of Theorem 3(iii(d)), $S_H^a \in (0, S_H^f) \subset (0, \min(T, N_1))$.
- (iv(b)) $T \ge N_1 + N_2$ and $L \ge \bar{t}$: As shown in the proof of Theorem 3(iii(d)), $S_H^a = 0$. Meanwhile, by Theorem 1, the low-quality type is indifferent between (full or limited) advance selling and selling only in spot. Hence, both kinds of equilibria can be sustained: either a separating equilibrium wherein H type sells only in spot and L type offers (full or limited) advance selling, or a pooling equilibrium wherein neither type sells in advance.

Appendix H. Proof of Theorem 6

The proof uses the following three lemmas, where Lemma H.1 shows the seller's optimal no-rationing strategy in the full-information case, and Lemmas H.2 and H.3 characterize the separating equilibrium and pooling equilibrium for the no-rationing model, respectively.

Lemma H.1. *When capacity rationing is not allowed and customers in advance are perfectly informed of quality, there exists a function $t^D(T)$ for $T > 0$ such that if $t \le t^D(T)$, the seller should offer full advance selling; otherwise, the seller should sell only in spot.*

Lemma H.2. *When capacity rationing is not allowed, in any separating equilibrium, L type offers full advance selling at price $p_1^L(\min(T, N_1))$, while H type sells only in spot.*

Lemma H.3. (i) *In the focal pooling equilibrium where both types of sellers sell in advance, the equilibrium price is* $p_1^E = qp_{1H}^*(\min(T, N_1)) + (1-q)p_{1L}^*(\min(T, N_1))$. (ii) *The focal pooling equilibrium is sustained only if* $H \le t^D(T)$. (iii) *When* $H \le t^D(T)$, *there exist a threshold* $\bar{\delta} \ge 0$ *and a function* $\bar{q}(\delta) \in [0, 1]$ *for* $\delta > 0$ *such that the focal pooling equilibrium is sustained if either* $\delta \le \bar{\delta}$ *or* $q \ge \bar{q}(\delta)$.

The proof of Theorem 6 is naturally divided into the following three cases:

- $L > t^D(T)$: By Lemma H.1, neither type sells in advance in the full-information case. Hence, in the asymmetric-information case, neither type would deviate from selling only in spot.
- $L \le t^D(T) < H$: By Lemma H.1, only L type sells in advance in the full-information case. Hence, in the asymmetric-information case, H type does not have incentive to sell in advance, as the maximum profit he can get from selling in advance is $\pi_H^f(\min(T, N_1))$, which does not exceed $\pi_H^f(0)$ by Lemma H.1. Similarly, L type does not have an incentive to sell only in spot, as $\pi_L^f(\min(T, N_1)) \ge \pi_L^f(0)$ by Lemma H.1.
- $H \le t^D(T)$: By Lemma H.1, both types sell in advance in the full–information case. In the asymmetric-information case, both types selling in advance occurs only in a pooling equilibrium (by Lemma H.2), and the focal pooling equilibrium is sustained if either $\delta \le \bar{\delta}$ or $q \ge \bar{q}(\delta)$ (by Lemma H.3). If, however, $\delta > \bar{\delta}$ and $q < \bar{q}(\delta)$, pooling with L type in advance makes H type worse off compared to selling only in spot. Meanwhile, L type would rather sell in advance alone than sell only in spot (by Lemma H.1). Hence, a separating equilibrium is sustained.

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