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Coordination of Price Promotions in Complementary Categories

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In this paper, I investigate the outcome of a price competition between two firms, each producing two complementary products. Specifically, I study each firm's decision to coordinate price promotions of its products. Consumers are divided into loyals, who purchase both products from their preferred firm, and heterogeneous switchers, who choose between four possible bundles or buy a product in a single category. The switchers are willing to pay some price premium in order to purchase two complementary products that share the same brand name and are produced by the same firm, because they believe that these products are a better match than two complementary products with different brand names. I find that each firm predominantly promotes its complementary products together. This finding is correlationally supported by data in the shampoo and conditioner and in the cake mix and cake frosting categories.

Key words: price promotions; complementary products; heterogeneous consumers
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1. Introduction

When a firm produces goods in several related categories, it has to take into account cross-category price effects in order to develop an optimal pricing strategy. Several recent empirical studies contributed to developing the framework for estimating these cross-category effects at the category level (Chintagunta and Haldar 1998, Manchanda et al. 1999, Russell and Petersen 2000, Duvvuri et al. 2007, Song and Chintagunta 2007, Niraj et al. 2008, Sriram et al. 2010) and at the brand level (Wedel and Zhang 2004, Song and Chintagunta 2006, Mehta 2007, Ma et al. 2012). Using estimated cross-category elasticities, it is possible to find the single optimal price for each product (Song and Chintagunta 2006) or to compute the total effect of a price promotion on profits (Manchanda et al. 1999, Duvvuri et al. 2007, Niraj et al. 2008). However, there exists little theoretical work that analyzes firms' optimal strategies involving price promotions in related categories. The current paper fills this gap by introducing a theoretical model that predicts that complementary products that share the same brand name and are produced by the same firm go on sale at the same time. This prediction finds correlational empirical support in two pairs of complementary categories: shampoo and conditioner and cake mix and cake frosting.

In my model, there are two firms, each selling products in two complementary categories. Following Varian (1980) and Narasimhan (1988), I model price promotions (sales) as mixed strategies in price competition between these firms. There are two types of consumers: loyals and switchers. The loyal consumers buy both products from their preferred firm. The switchers mix and match between the brands offered by the two firms or buy a product in only one category. They choose the single product or the bundle that offers them the highest utility. Simester (1997) studied a similar model, but assumed that the switchers purchase both products from the same firm. I allow the switchers to purchase two complementary products from different firms, but assume that they are willing to pay a price premium in order to purchase products that share the same brand name.[1] This assumption reflects a consumer belief that the complementary products carrying the same brand name are a better match. Often, firms themselves contribute to this consumer perception by placing product advertisements on the labels of their complementary products. For example, Pantene Pro-V shampoo and conditioner bottle labels encourage "For best results, try other products in our collection." The last step in the recipe on the back of a Betty Crocker cake mix box is "Frost with Betty Crocker Creamy Deluxe or Whipped Frosting," and the writing on the lid of a can of BEHR paint states "For Lifetime Guarantee, always use BEHR PRIMERS."

I further assume that the switchers have heterogeneous tastes for the different single products and

[1] Although the source of this consumer behavior is the complementarity in consumption, the firms, when setting their prices, care about the purchase complementarity. In this paper, I assume that the consumption complementarity leads to the purchase complementarity.

bundles, and I model their demand as logit—the popular demand specification in the empirical literature. Sinitsyn (2008a) showed that for such demands, the support of the mixed-strategy Nash equilibrium in prices consists of a finite number of points. Subsequent research in the area of price promotions with heterogeneous consumers (Sinitsyn 2008b, 2009) outlined the techniques for studying these equilibria in various models of price competition in single categories. Here, I extend these methods to allow for competition in multiple categories. I compute the equilibria for various values of the parameters of the demand function and find that although the firms sometimes use the price pairs in which one product goes on sale whereas the complementary product does not, these occurrences are rare. Most probability is assigned to the price pairs in which both products are discounted. This is the central result of the theoretical section—price promotions of complementary products are synchronized.

Next, I examine supermarket price data on the brands in three pairs of complementary categories. In two of the three pairs—shampoo and conditioner and cake mix and cake frosting—the firms use the same brand names for their products in complementary categories, and I expect that consumers gain additional utility from purchasing the complementary products sharing the same brand name. Correlationally consistent with the theory, I find that in these category pairs, the firms often put their complementary products on sale together, and I reject the hypothesis that their promotions are independent. In another complementary products pair, detergent and fabric softener, the top two firms, Procter & Gamble and Unilever, use different brand names for the products in different categories. Hence, it is likely that in the absence of matching brand names, consumers do not gain extra utility from purchasing the products produced by the same firm. Correspondingly, I find that there is almost no coordination of sales of detergents and fabric softeners—there is evidence to reject the hypothesis of independent price promotions in only 1 out of the 14 possible brand pairs present in the data.

2. Evidence of Brand Matching Preferences

In this section, I provide some empirical and experimental evidence for the assumption that the switching consumers are willing to pay a price premium in order to purchase two complementary products that share the same brand name and are produced by the same firm. I model this preference by explicitly adding the premium to the consumer's utility for a bundle with matching brand names. This approach was recently used by Ma et al. (2012), who estimated a multicategory brand-choice model for cake mixes and cake frostings in which they included coefficients that captured the brand-level complementarity between brands in different categories. They found that two out of three brands (Betty Crocker and Duncan Hines) had the strongest brand-level complementarity with their umbrella brand in the complementary category. For the third brand, Pillsbury, Ma et al. (2012) found a large positive correlation between the error terms for cake mix and frosting, which means that if a household makes a purchase of Pillsbury cake mix for some unobserved reason, it is also more likely to buy Pillsbury frosting. Simonin and Ruth (1995) conducted an experiment in which subjects evaluated bundles consisting of a tube of toothpaste and a complementary dental hygiene product, either sharing the toothpaste's brand or not. Their results suggest that the subjects placed a premium on within-brand bundles. Mulhern and Leone (1991) and Walters (1991) estimated cross-category effects for cake mix and cake frosting categories and found "stronger complementary relationships for items that have the same brand name" (Mulhern and Leone 1991, p. 72).

I chose to supplement the evidence provided by the above studies by running an experiment with the undergraduate students at an East Coast university. Sixty-one subjects answered a questionnaire with three types of questions. The questions were designed to evaluate their preferences for matching between two brands of cake mix and cake frosting: Betty Crocker and Duncan Hines.

Endowment questions presented a promotion of the type "Buy one package of cake mix, get Duncan Hines frosting free," then offered the choice of, in this example, Betty Crocker or Duncan Hines cake mix at an equal price. Each subject was asked four endowment questions, corresponding to each of the four products. Any consumer preferences in which brand preference in each category does not vary in response to the brand chosen in the complementary category would manifest as a 50% matching rate. However, the participants chose matching bundles 71% of the time, with the rates of matching varying from 62% to 80% among the four products—each greater than 50%, significant at the 0.05 level. Additionally, classifying the respondents according to the type of preferences revealed, I find that "matchers" are the modal group, comprising 42% of respondents, and that 27% of the respondents are loyal to one of the two brands. These data provide evidence validating the assumption of coexistence of loyal consumers and switchers, which is crucial to the model presented in this paper.

Valuation questions asked subjects to provide a valuation on a one to nine scale for each of the four individual products, and each of the four possible cake mix and cake frosting bundles. Using these valuations, I conducted a conjoint analysis, estimating an individual's valuation for each bundle as a function of component product valuations and bundle-specific fixed effects. I find that the coefficient for matching brands is positive and significant at the 0.001 level. Additionally, by comparing the estimated value of the "matching brand" coefficient, 0.59, to the mean bundle valuation of 5.53, I find that the additional utility consumers derive from brand-level complementarity is substantial.

Discrete-choice questions presented subjects with a series of purchase scenarios in which each of the four products was listed at either regular price or sale price, and asked them to select the cake mix and cake frosting they would purchase given those prices. I modeled their decision process as multinomial logit, consistent with the empirical literature and with the theoretical model presented in this paper. Regressing the respondents' bundle choices on the price of each bundle and the components of the bundle, I find that the coefficient for matching brands is positive and significant at the 0.001 level, as in the conjoint analysis. It is likely, however, that the presence of the brand-loyal consumers who choose a same-brand bundle with little or no price elasticity upwardly biases the estimated coefficient for brand complementarity. Therefore, I conducted a series of robustness checks: excluding the loyals identified using discrete-choice responses, excluding the loyals or including only the matchers identified using endowment responses, and adding the component valuations as regressors. The finding of a positive and significant coefficient for brand complementarity is robust to all of these modifications. In addition, modeling the consumers' decision process using a bivariate probit model yields qualitatively similar results, which withstand all of the same robustness checks. The discrete-choice data, therefore, provide reliable evidence of consumer preference for matching brands across complementary categories, consistent with the results obtained from the conjoint and endowment choice analyses.[2]

3. Simple Model

Before presenting my main model that relies on numerical solutions, in this section, I present a simple, analytically tractable model that outlines the basic mechanism of price promotions in complementary categories. Assume that there are two firms, each producing two complementary products. The set of consumers has measure one, and each consumer purchases both complementary products. The firms can either charge a regular price normalized to 1 or a sale price $p < 1$ for each product. Thus, there are four possible price pairs available to the firms: $(p; p)$, $(p; 1)$, $(1; p)$, and $(1; 1)$. The firms are symmetric, so if the prices are identical in both markets, each firm gets 0.5 of the consumers in each market. The following variables describe the additional portion of the market a firm can gain if it runs a price promotion.

I begin by considering the case where, in one category, a firm charges the sale price p whereas its rival charges 1, and the prices of the two firms are identical within the other category. The firm running a promotion in the first category gets s_P consumers in this category (the subscript P denotes the primary effect), where $0.5 < s_P \leq 1$. When only one firm offers a discount in one category and the prices are identical in the other category, there is a spillover effect of the promotion on the purchase behavior in the other category. Some of the s_P consumers who purchased the product on sale in the first category would like to match it with the same brand in the second category, even though the prices in that category are identical. Therefore, a firm running a sale in the first category will also gain additional s_D consumers in the second category (the subscript D denotes the secondary effect). Note that s_P measures the *total* amount of consumers purchasing the product on sale ($s_P > 0.5$) whereas s_D measures the *additional* (over 0.5) amount of consumers gained in the second category. Restricting s_D to be less than $s_P - 0.5$ guarantees that the secondary effect is not larger than the primary effect, allowing for the presence of some consumers who do not care about matching the brands.

For simplicity, I also assume that if a firm puts both products on sale, $(p; p)$, whereas the rival charges regular prices, $(1; 1)$, then the firm that offers promotions gets s_P in both categories. Also, if each firm promotes one product in different categories, there are no spillover effects—each firm gets s_P consumers in the category in which it offers a promotion and $1 - s_P$ in the category in which its rival offers a promotion. Table 1 summarizes the outcomes and serves as a payoff matrix for the 4×4 game between the two firms.

I search for the mixed-strategy equilibrium in this game that puts positive weights θ, β, η, and λ on strategies $(p; p)$, $(p; 1)$, $(1; p)$, and $(1; 1)$. I also find the range of parameters of the model for which the promotions are correlated. To check for the correlation of promotions, I use the following method. For each firm, the promotion of the first product occurs

[2] Although none of the experimental or empirical analyses can characterize consumer reasoning underlying brand-level complementarity, some anecdotal evidence can be found in online forums where visitors discuss their reasons for matching their brands of shampoo and conditioner. Some examples of individuals' reasoning are as follows: "I usually match…but I think it's just because I like things to be uniform," and "I prefer mine to match, as well. I feel like it must work best as a pair, instead of using different brands and types." From the forum comments, it seems that consumers prefer to match the brands of shampoo and conditioner either because of the belief that they work better together or because of the desire to have a uniform bundle.

Table 1 Payoff Matrix for the Simple Model of Price Promotions

	$(p; p)$	$(p; 1)$	$(1; p)$	$(1; 1)$
$(p; p)$	p; p	$ps_P + p(0.5 + s_D)$; $p(0.5 - s_D) + (1 - s_P)$	$ps_P + p(0.5 + s_D)$; $p(0.5 - s_D) + (1 - s_P)$	$2ps_P$; $2(1 - s_P)$
$(p; 1)$	$p(0.5 - s_D) + (1 - s_P)$; $ps_P + p(0.5 + s_D)$	$0.5p + 0.5$; $0.5p + 0.5$	$ps_P + (1 - s_P)$; $ps_P + (1 - s_P)$	$ps_P + (0.5 + s_D)$; $(1 - s_P) + (0.5 - s_D)$
$(1; p)$	$p(0.5 - s_D) + (1 - s_P)$; $ps_P + p(0.5 + s_D)$	$ps_P + (1 - s_P)$; $ps_P + (1 - s_P)$	$0.5p + 0.5$; $0.5p + 0.5$	$ps_P + (0.5 + s_D)$; $(1 - s_P) + (0.5 - s_D)$
$(1; 1)$	$2(1 - s_P)$; $2ps_P$	$(1 - s_P) + (0.5 - s_D)$; $ps_P + (0.5 + s_D)$	$(1 - s_P) + (0.5 - s_D)$; $ps_P + (0.5 + s_D)$	1; 1

with probability $\theta + \beta$. The promotion of the second product occurs with probability $\theta + \eta$. If the promotion decisions were independent in these categories, the joint promotions of the two complementary products would occur with probability $(\theta + \beta)(\theta + \eta)$. The actual probability of a joint promotion is θ. Thus, if $\theta > (\theta + \beta)(\theta + \eta)$, then the promotions are positively correlated. The following proposition describes the conditions under which this happens.[3]

Proposition 1. *There exists a mixed-strategy Nash equilibrium in which the promotions are positively correlated iff $s_P < 1/(1 + p^2)$ and $s_D > (1 - s_P(1 + p^2))/p$.*

If the first condition of Proposition 1 holds, that is, if $s_P < 1/(1 + p^2)$, then $s_P(1 + p^2) < 1$. Thus, the second condition of Proposition 1 requires s_D to be a positive number. This means that price promotions are positively correlated only if there is a cross-category spillover effect of a promotion. As s_P approaches $1/(1 + p^2)$ from below, $(1 - s_P(1 + p^2))/p$ (the lower bound on s_D) decreases and converges to zero. Then, for s_P sufficiently close to $1/(1 + p^2) > 0.5$, there always exists s_D that satisfies the assumption that the secondary effect is smaller than the primary effect ($s_D < s_P - 0.5$). Thus, for any discount price p, there always exist parameters s_P and s_D such that the promotions are positively correlated.

The following proposition shows that the promotions become more coordinated when cross-category effect s_D increases.

Proposition 2. *The degree of correlation between promotions of complementary products $(\theta/((\theta + \beta)(\theta + \eta)))$ increases with the strength of cross-category effect s_D.*

The intuition for this result is as follows. Assume that both firms use the same mixed strategy and, therefore, are indifferent between using all four price pairs, including $(p; p)$ and $(1; 1)$. Holding everything else constant, an increase in s_D increases the profitability of a price promotion, because more consumers are captured in the complementary category. The expected profit from using $(p; p)$ increases and the expected profit from using $(1; 1)$ decreases. To restore the mixed-strategy equilibrium, the payoff from $(p; p)$ has to decrease and/or the payoff from $(1; 1)$ has to increase. The expected payoff from using $(p; p)$ increases with s_D because using $(p; p)$ against the rival who uses $(p; 1)$ or $(1; p)$ becomes more profitable (the payoff from using $(p; p)$ against the rival who uses $(p; p)$ or $(1; 1)$ does not change). Therefore, if the rival decreases the probability of using $(p; 1)$ and $(1; p)$, the expected profit from using $(p; p)$ will go down. Similarly, the expected payoff from using $(1; 1)$ decreases with s_D because using $(1; 1)$ against the rival who uses $(p; 1)$ or $(1; p)$ becomes less profitable (the payoff from using $(1; 1)$ against the rival who uses $(p; p)$ or $(1; 1)$ does not change). Thus, if the rival decreases the probability of using $(p; 1)$ and $(1; p)$, the expected profit from using $(1; 1)$ will go up. Hence, the mixed-strategy equilibrium is restored if some probability is shifted from $(p; 1)$ and $(1; p)$ toward $(p; p)$ and $(1; 1)$. Such a shift causes the promotions of complementary products to become more correlated.

The advantages of the model presented above are that it is simple, has a closed-form solution, and does not rely on functional form assumptions about consumer demand. A major disadvantage of this model, however, is that the sale price p is exogenous. In reality, the firms would respond to changes in s_D not only by shifting the probabilities, but also by adjusting the promotion depth. In addition, in this model the firms are restricted to using the same sale price for the single-category discounts and for joint discounts, although, in reality, the firms might choose to use different discounts, depending on whether they run promotions in one category or in both categories. To make the sale price endogenous, I have to specify how consumers react to price changes, that is, I have to specify the demand function. Even very simple demand specifications render the model analytically intractable. Because I have to proceed by using numerical solutions, I choose to work with the logit demand specification that is often used in the empirical literature to estimate demand for complementary products.

[3] All proofs are in Appendix A.

4. Full Model

There are two complementary product categories. Within each category, there are two brands produced by firms A and B. The firms have constant marginal cost c and compete in prices. The highest price the firms can charge for any of their products is the reservation price r—the maximum price any consumer is willing to pay. The rescaling of prices $p \to (p-c)/(r-c)$ normalizes the marginal cost to 0 and the reservation price to 1.

The set of consumers has measure 1. The consumers are divided into loyals and switchers. Each firm has a share α of loyal consumers—these consumers buy both complementary products from their preferred firm, provided that the price of each of the products is less than or equal to 1. The remaining $1-2\alpha$ consumers are the switchers. They can either buy one product or buy a bundle. The switchers then have heterogeneous tastes for the four single products—$A0$, $B0$, $0A$, and $0B$ (0 denotes a no-purchase in the corresponding category)—and for the four possible bundles—AA, AB, BA, and BB. The switching consumer s has the following utility from purchasing a bundle ij (either i or j can be 0):

$$U_{sij} = \delta_{1,i} + \delta_{2,j} - p_{1,i} - p_{2,j} + d_c \cdot I(i, j \neq 0) + d_b \cdot I(i = j) + \varepsilon_{sij}, \tag{1}$$

where $\delta_{k,i}$ is the base utility of product i in category k, $p_{k,f}$ $(k = 1, 2;\ f = A, B)$ is firm f's price for its product in category k $(p_{k,0} = 0)$, d_c is the additional utility the consumers gain from purchasing products in complementary categories, d_b is the additional utility the consumers gain from purchasing the same brands in complementary categories, and ε_{sij} are independently and identically Gumbel distributed with scale parameter μ (μ is the degree of consumer heterogeneity).[4] Including an additional term d_c in the utility function to capture the effect of complementarity at the category level is a standard specification in the empirical literature (Russell and Petersen 2000, Song and Chintagunta 2006, Gentzkow 2007, Niraj et al. 2008, Sriram et al. 2010). In addition to having category-level complementarity d_c, I also allow for complementarity at the brand level by introducing d_b in the utility function. The justification for this assumption is presented in §2 of the paper.

The setup of the full model is similar to the bundling model of Matutes and Regibeau (1992). In their model, the firms are able to bundle their products, that is, to offer a discount to the consumers who purchase both products from one firm. The effect of bundling is equivalent to having d_b in (1), except that the firms can choose a bundle discount, whereas, in my model, d_b is exogenous.[5] Matutes and Regibeau (1992) do not have loyal consumers; thus, the equilibrium in their model is in pure strategies. The presence of loyal consumers in my model leads to mixed strategies and allows for analysis of price promotions.

The utility from (1) gives rise to the following logit choice probability of purchasing a bundle ij:

$$P_{ij} = \big(\exp((\delta_{1,i} + \delta_{2,j} - p_{1,i} - p_{2,j} + d_c \cdot I(i, j \neq 0) + d_b \cdot I(i = j) + \varepsilon_{sij})/\mu)\big) \cdot \Bigg(\sum_{\substack{f_1, f_2 \in \{A, B, 0\} \\ f_1 \neq 0 \vee f_2 \neq 0}} \exp\big((\delta_{1,f_1} + \delta_{2,f_2} - p_{1,f_1} - p_{2,f_2} + d_c \cdot I(f_1, f_2 \neq 0) + d_b \cdot I(f_1 = f_2))/\mu\big) \Bigg)^{-1}. \tag{2}$$

Then, the profit function of firm i consists of profit from the loyals, $\alpha(p_{1,i} + p_{2,i})$; profit from the switchers who bought both products from firm i, $(1-2\alpha) \cdot (p_{1,i} + p_{2,i})P_{ii}$; and profit from the switchers who bought only one of the products from firm i, $(1-2\alpha)p_{1,i}(P_{ij} + P_{i0})$ and $(1-2\alpha)p_{2,i}(P_{ji} + P_{0i})$.

To find a pure-strategy equilibrium, it is necessary to differentiate the profit functions of both firms with respect to the two prices each firm sets and then solve the resulting system of four equations with four unknowns. However, as I show in the following section, when consumer heterogeneity μ is low enough, a pure-strategy equilibrium does not exist, so it is necessary to search for the mixed-strategy equilibria.

The demand functions and the profit functions are analytic. Sinitsyn (2008a) showed that for price competition with analytic demands, the support of the mixed-strategy equilibrium is finite.[6] This means that the equilibrium strategy of firm i involves charging price vectors $\{p^n_{1,i}, p^n_{2,i}\}_{n=1}^{N_i}$ with corresponding

[4] The random term ε_{sij} reflects the unobserved consumer preferences. A larger variance of ε_{sij} implies more dispersed consumer tastes and, hence, a greater consumer heterogeneity. The scale parameter of the Gumbel distribution, μ, is proportional to the variance; thus, μ is also a measure of consumer heterogeneity.

[5] To keep the analysis simple, I do not let the firms offer bundle discounts, but the model can easily be extended to allow for them. Balachander et al. (2010) analyze a model in which they allow for both price promotions and bundle discounts in an unrelated pair of categories.

[6] A function is analytic if it has a Taylor series about each point x that converges to this function in an open neighborhood of x. The exponential function is analytic. The sums, products, compositions, and reciprocals of analytic functions are analytic; therefore, P_{ij} from (2) is analytic. Theorem 1 from Sinitsyn (2008a) states that if the strategy space is compact and the demand functions are analytic and are positive at zero for any price of the competitor, then the support of the price distribution in any mixed-strategy Nash equilibrium has a finite number of points. Although the theorem is stated for the single-dimensional case, it also applies to multiple dimensions.

probabilities $\{\gamma_i^n\}_{n=1}^{N_i}$, where N_i is the number of price vectors firm i uses. The system of equations, the solution to which is an equilibrium price distribution with finite support, is standard. It includes the first-order conditions—the profit function must be maximized at each point of the support. In addition, the profits at all the points of the support of the price distribution have to be equal to each other. It is impossible to solve this system analytically, thus, I solve it numerically. Sinitsyn (2008b, 2009) provides insights on the solution procedure for the case when price has one dimension. These methods naturally extend to the current case with two-dimensional price. In the next section, I analyze the equilibrium strategies of the firms.

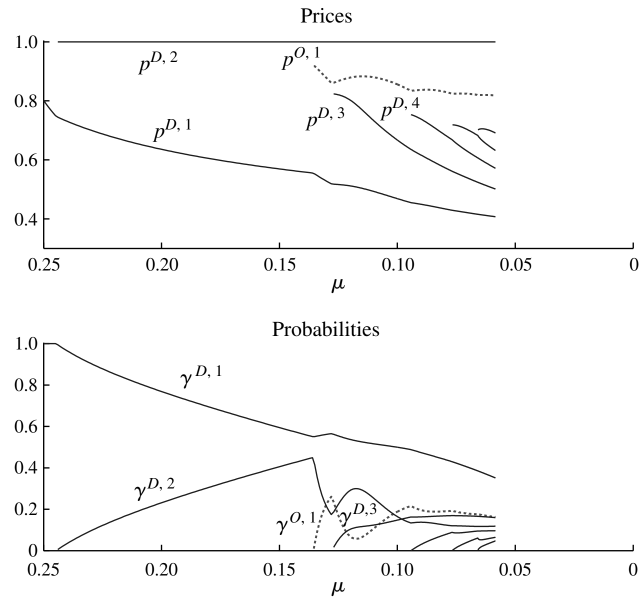
5. Equilibrium Strategies

The firms' equilibrium strategies depend on the parameters of the demand function—percentage of loyal consumers α, consumer heterogeneity μ, category-level complementarity d_c, brand-level complementarity d_b, and base utility levels $\delta_{k,i}$. To illustrate the structure of the equilibria in this model, I fix $\alpha = 0.25$, $d_c = d_b = 0.4$, $\delta_{k,i} = 0$ ($k \in \{1, 2\}$, $i \in \{A, B, 0\}$)[7] and examine how the firms' strategies change as consumer heterogeneity declines.

When μ is high, there exists a pure-strategy equilibrium. For large μ, the consumer preferences are quite dispersed. Therefore, if a firm decreases its price, it gains relatively few additional consumers. Hence, the demands are inelastic. With inelastic demands, price competition is relatively soft, and both firms charge the reservation price for both of their products. As μ decreases, the incentives to undercut the rival become larger, and in a pure-strategy equilibrium, both firms start charging prices that are smaller than the reservation price. The prices for two complementary products are the same, so I label both of them $p^{D,1}$, where D stands for diagonal, because these prices lie on the diagonal of the price support square.

Figure 1 shows how prices respond to further decrease in μ. The diagonal prices are represented in this figure by the solid lines. As consumers become less heterogeneous, $p^{D,1}$ decreases until it becomes so low that the firms would prefer to deviate by charging the reservation prices and serving mainly their loyal consumers. Of course, charging the reservation prices with certainty cannot be an equilibrium either, because the firms will have incentives

Figure 1 **Equilibrium Prices and Probabilities for** $\alpha = 0.25$, $d_c = 0.4$, **and** $d_b = 0.4$



to undercut. At this point, the pure-strategy equilibrium does not exist anymore, but the equilibrium is restored in mixed strategies, with both firms charging two sets of prices: $(p^{D,1}; p^{D,1})$ with probability $\gamma^{D,1}$ and $(p^{D,2}; p^{D,2}) = (1; 1)$ with remaining probability $\gamma^{D,2} = 1 - \gamma^{D,1}$.

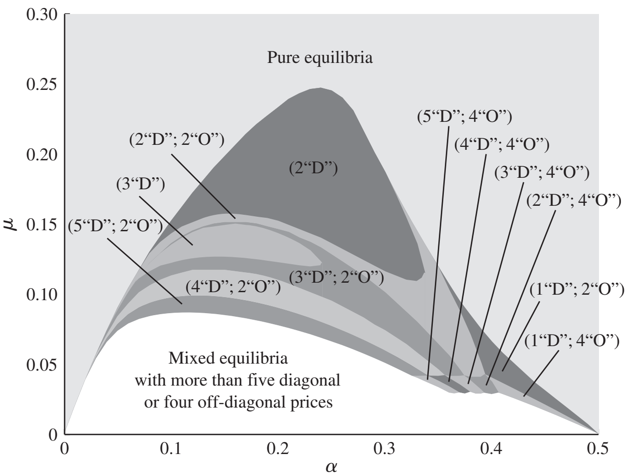
As μ keeps decreasing, $\gamma^{D,2}$—the probability of charging the reservation prices—increases, thus increasing the incentives to undercut them. When μ reaches 0.135, two new prices appear in the equilibrium. The firms find it profitable to undercut (1; 1), but along only one dimension of the price support square, that is, either charging $(1; p^{O,1})$ or $(p^{O,1}; 1)$. Here, O stands for off-diagonal, because these prices lie off the diagonal of the price support square. In Figure 1, the off-diagonal prices are represented by the dotted lines. In the symmetric mixed-strategy equilibrium, the firms will charge both $(1; p^{O,1})$ and $(p^{O,1}; 1)$ with equal probability. I label the *total* probability of charging these price pairs as $\gamma^{O,1}$ (so, $\gamma^{O,1}/2$ is the probability of charging one of these price pairs).

As μ declines from 0.135, the probability of charging the off-diagonal prices increases until it reaches 0.2635 at $\mu = 0.1277$. At this point, it becomes profitable to undercut $(1; p^{O,1})$ or $(p^{O,1}; 1)$ with a diagonal price that I label $(p^{D,3}; p^{D,3})$. The support of the firms' optimal mixed strategies now consists of the reservation prices $(p^{D,2}; p^{D,2}) = (1; 1)$, two price pairs with identical discounts—$(p^{D,1}; p^{D,1})$ and $(p^{D,3}, p^{D,3})$—and two price pairs in which only one product is discounted—$(1; p^{O,1})$ and $(p^{O,1}; 1)$.

Figure 1 shows the rest of the pricing strategies as μ decreases until 0.0584. Figure B.1 in Appendix B gives an alternative graphic presentation of these

[7] These parameters were chosen so that the percentage of joint purchase in the two categories matches the empirical estimates. For the smallest value of consumer heterogeneity at which the firms charge reservation prices in both categories, the percentage of joint purchases that is predicted by the model is 22%. This is in line with the empirical estimates in the categories I consider in this paper, which range from 14% to 34%.

Figure 2 **Regions with Mixed-Strategy Nash Equilibria for $d_c = 0.4$ and $d_b = 0.4$**



strategies using bubble plots. Some patterns emerge that are also present for other parameters of the demand function. First, as μ keeps decreasing, more and more diagonal prices are added to the mixed-strategy equilibrium. Second, the off-diagonal prices are not always present in the mixed-strategy equilibrium, and when they are a part of the equilibrium, the probability of charging them is low. For Figure 1, the highest value for the total off-diagonal probability, $\gamma^{O,1}$, is 0.2635, at the point right before $(p^{D,3}; p^{D,3})$ appears.

The equilibrium pricing strategies are similar for other values of α. Figure 2 shows the regions with different mixed-strategy equilibria for the different values of α and μ. Each region is characterized by the number of diagonal ("D") and off-diagonal ("O") prices the firms use. The equilibria with up to five diagonal prices or four off-diagonal prices were computed. I conjecture that the remaining equilibria contain more than five diagonal prices or four off-diagonal prices. Although a theoretical proof of this conjecture that the number of prices generally increases with a decrease in consumer heterogeneity does not exist, previous work on single category price promotions with heterogeneous consumers (Sinitsyn 2008b, 2009) and multiple robustness checks in this paper indicate that, indeed, this is the case. Figure 1 suggests that no significant new insights can be gained from examining the equilibria with a larger number of prices. Also, four different levels of promotional depths are likely to be at the upper limit of the managerially relevant choice.

Figure 2 shows that for all values of α, when consumer heterogeneity is high, there exists a pure-strategy equilibrium. For smaller values of μ, the equilibria are in mixed strategies and the number of diagonal prices increases as μ declines. When they are present, the number of off-diagonal prices is usually two, with four appearing only for a small subset of the parameters and, in particular, for large values of α. The estimates in the literature (Villas-Boas 1995, Huang et al. 2006) suggest that a brand's loyalty share is usually below 10%.[8] These estimates were made for a single category, which means that the percentage of consumers loyal to the same brand in two categories is likely to be even smaller. Therefore, more relevant predictions of the model are the ones for these smaller values of α.

The main variable of interest is not the number of off-diagonal prices, but their frequency. For each value of α, it is possible to find the maximum value of the total probability of off-diagonal prices, $\gamma^{O_{\mathrm{MAX}}}$, over the studied range of μ. It turns out that this probability increases with α. Whereas for larger values of α, $\gamma^{O_{\mathrm{MAX}}}$ can be significant (for example, it is 0.4129 for $\alpha = 0.35$), for the empirically relevant range of α, $\gamma^{O_{\mathrm{MAX}}}$ is small (for example, it is 0.0267 for $\alpha = 0.05$ and 0.0893 for $\alpha = 0.1$). It is impossible to compute all equilibria for any fixed α, because for small levels of consumer heterogeneity, the number of prices and probabilities the firms use and, correspondingly, the number of equations in the system to solve, become too large. Therefore, the computed $\gamma^{O_{\mathrm{MAX}}}$ covers only the shaded regions shown in Figure 2. It is unlikely, however, that there are higher values for off-diagonal probabilities in the unexplored unshaded region. For any nonlarge fixed α, as consumer heterogeneity decreases and the firms start using off-diagonal prices, the maximum value of the total off-diagonal probability occurs on the lower bound of the region (2"D"; 2"O"), just before the third diagonal price is added to the equilibrium. As μ decreases further, although the total probability of off-diagonal prices has both increasing and decreasing regions, the general trend is downward.

The intuition for why a large probability cannot be placed on off-diagonal prices in a mixed-strategy equilibrium is illustrated by the following example. Consider the extreme case, in which each firm charges two off-diagonal prices, $(0.5; 1)$ and $(1; 0.5)$, with equal probability. If, as in the preceding analysis, $d_b = 0.4$, then when the firms happen to charge different price pairs, the consumers will purchase a mixed bundle.[9] Consumers buy the discounted products

[8] Villas-Boas (1995) shows that the percentage of loyal consumers depends on the brand and ranges from 0.3% to 10.4% for coffee and from 0.4% to 2.2% for saltine crackers. Huang et al. (2006) estimate the *total* percentage of loyal consumers in the refrigerated orange juice and frozen orange juice categories to be 12.2% and 29.5%, respectively. Because there are more than three brands in each category, the average percentage of loyal consumers per brand is less than 10%.

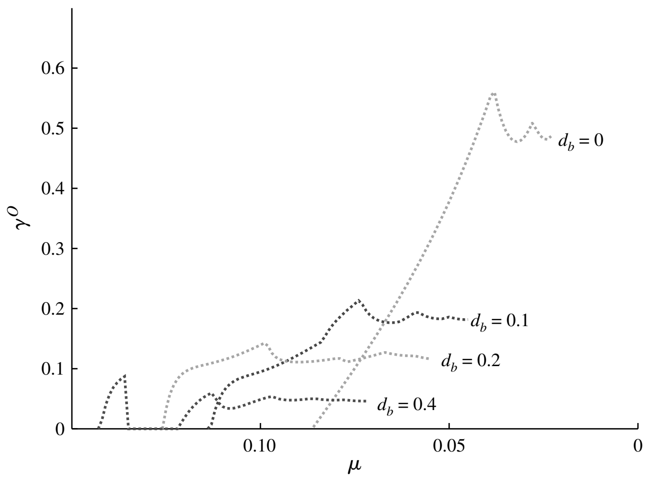
[9] For simplicity, I consider only the consumers who buy in both categories and assume that consumer heterogeneity is negligible.

from the different firms because the premium they are willing to pay for getting both products from the same firm, 0.4, is not large enough to overcome the price difference of 0.5 (1 for a mixed bundle versus 1.5 for a bundle from the same firm). Then the firms have a profitable deviation to a diagonal price pair (0.75; 0.75). This incentive to deviate is explained entirely by the switchers' demands, because the price paid by the loyals remains 1.5. When both firms use the mixed strategy involving (0.5; 1) and (1; 0.5), there is a 50% probability that they charge the same price pair, and each firm gets half the switchers, who pay 1.5. There is also a 50% probability that they charge the opposite price pairs, and each firm gets all the switchers who buy only their discounted product, paying 0.5. When one firm charges (0.75; 0.75) instead, the switchers no longer want to buy a mixed bundle: the mixed bundle will cost 1.25, the matching bundles still cost 1.5, and consumers are willing to pay 0.4 to get the matching bundle. Thus, regardless of whether the rival charges (1; 0.5) or (0.5; 1), the deviating firm will get half the switchers, who pay 1.5. Because half of the switchers paying 1.5 bring more revenue than all of the switchers paying 0.5, charging (0.75; 0.75) brings more profit than charging (1; 0.5) and (0.5; 1) with equal probabilities.[10] More broadly, when the firm charges off-diagonal prices such that the switchers purchase a mixed bundle, the firm sells only the discounted product to all of them. If the firm deviates to charging a diagonal price pair while keeping the price of the bundle constant, then it will get half of the switchers, each paying more than double of the original discounted price, thereby increasing the firm's profits.

The analysis of the simple model in §3 reveals that the firms coordinate their promotions more when the strength of cross-category effect increases (Proposition 2). This result is conserved in the full model. Figure 3 shows the effect of μ on total off-diagonal probability, γ^O, for different values of brand-level complementarity d_b ($\alpha = 0.1$, $d_c = 0.4$, $\delta_{k,i} = 0$). Equilibria with up to eight diagonal prices or six off-diagonal prices were computed for this figure.

[10] This example can be extended to show why off-diagonal prices might be charged with small probability. If, in addition to charging (0.5; 1) and (1; 0.5), both firms charge (0.6; 0.6) with a large probability, the deviation to (0.75; 0.75) is unprofitable for some values of consumer heterogeneity. This happens because a sizable portion of the switchers will buy a mixed bundle with prices (0.5; 0.6) over the cheapest matching bundle, (0.6; 0.6), but very few will buy a mixed bundle with prices (0.75; 0.6). Therefore, although charging (0.75; 0.75) is more profitable than (0.5; 1) against the rival charging (0.5; 1) or (1; 0.5), charging (0.75; 0.75) is less profitable than (0.5; 1) against the rival charging (0.6; 0.6). If the probability of the rival charging (0.6; 0.6) is large, the deviation to (0.75; 0.75) is unprofitable, and the off-diagonal prices can exist.

Figure 3 **Total Off-Diagonal Probability for Different Values of d_b** ($\alpha = 0.1$, $d_c = 0.4$, $\delta_{k,i} = 0$)



Some common trends are present for all values of d_b. As μ decreases, off-diagonal prices start to appear and the total probability of charging them increases. This probability increases until reaching its maximum, $\gamma^{O_{\text{MAX}}}$, and then it fluctuates with small deviations around some lower level. As d_b increases, $\gamma^{O_{\text{MAX}}}$ decreases. For example, $\gamma^{O_{\text{MAX}}} = 0.56$ for $d_b = 0$. Then $\gamma^{O_{\text{MAX}}}$ decreases to 0.2133, 0.1425, and 0.0893 for d_b equal to 0.1, 0.2, and 0.4, correspondingly.[11] In addition to the total off-diagonal probability, the number of off-diagonal prices in relation to diagonal prices is also smaller for the larger values of d_b. For example, for $d_b = 0.4$, the last computed equilibrium has eight diagonal and two off-diagonal prices, whereas for $d_b = 0$, the last computed equilibrium has two diagonal and six off-diagonal prices. Therefore, similar to the predictions of the simple model, in the full model, an increase in the cross-category brand-level complementarity increases the coordination of price promotions.

In summary, this theoretical model predicts that although the firms sometimes place only one of the complementary products on sale, these occasions are rare. Most of the time, the complementary products are promoted together. The frequency of copromotions is greater for higher values of brand-level complementarity and for smaller levels of consumer loyalty.

6. Extensions

In this section, I show that the predictions of the base model from the previous section also hold for more realistic settings. This analysis is motivated by the industry structure in the cake mix and cake frosting

[11] The same trend is present for the other values of α. For example, for $\alpha = 0.05$, $\gamma^{O_{\text{MAX}}}$ decreases from 0.1368 to 0.0915, 0.0569 and 0.0267 as d_b increases from 0 to 0.1, 0.2, and 0.4.

categories—one of the complementary category pairs studied in §7. These categories exhibit an asymmetric response to a promotion in the other category (Manchanda et al. 1999); the production processes of cake mixes and cake frostings are different, likely leading to different costs; the regular prices of cake mix and cake frosting differ; and there are three major producers. I deal with each of these issues in turn.

6.1. Asymmetric Promotional Response

In Equation (1), I assume that both complementary products enter the consumers' utility function symmetrically. Thus, the effect of a price promotion in the first category on the brands in the second category is the same as the effect of an identical price promotion in the second category on the brands in the first category. However, it has been established in the empirical literature that cross-category price effects can be asymmetric within pairs of categories (Manchanda et al. 1999, Russell and Petersen 2000, Mehta 2007, Niraj et al. 2008). This literature shows the presence of the asymmetry at the category level. Recent work by Song and Chintagunta (2006) demonstrated that this asymmetry also exists at the brand level. I also model the asymmetry in the promotional response at the brand level by modifying the utility function from Equation (1) in the following way:

$$U_{sij} = \delta_{1,i} + \delta_{2,j} - tp_{1,i} - p_{2,j} + d_c \cdot I(i, j \neq 0) + d_b \cdot I(i = j) + \varepsilon_{sij}. \quad (3)$$

Here, t measures the relative importance of the first category product's price in the consumers' utility function. If t is greater than 1, then the impact of a change in the price of firm i's first category product on the probability of purchase of its secondary category product is greater than the impact of the change in the price of firm i's second category product on the probability of purchase of its first category product. Using the terminology from Manchanda et al. (1999), category 1 is the primary category and category 2 is the secondary category.

To illustrate the equilibrium strategies for this case, I follow the procedure used in §5. I fix $\alpha = 0.1$, $d_c = d_b = 0.4$, $t = 1.5$, $\delta_{k,i} = 0$ ($k \in \{1, 2\}$, $i \in \{A, B, 0\}$) and examine what happens to the firms' strategies as consumer heterogeneity μ declines.[12] As expected, because the products enter the firms' profit functions asymmetrically, even if the firms discount their complementary products together, the sizes of these discounts will be different. It is, thus, unfeasible to construct an informative analog of Figure 1 for this asymmetric case. A convenient way to represent the firms' strategies is to use the bubble graphs, which are shown in Figure B.2 in Appendix B.

For high levels of consumer heterogeneity, both firms charge the reservation price for both complementary products. As μ decreases and demand becomes more elastic, both firms start discounting their primary category brands, each charging prices $(p^{O,1}; 1)$. Eventually, the firms have enough incentive to undercut the price along the second dimension. In equilibrium, the firms now charge $(\mathbf{p^{D,1}})$, where $\mathbf{p^{D,1}}$ is a two-dimensional vector with both prices less than 1.[13] A decrease in μ leads to lower prices in both categories until the prices become low enough that the firms would prefer to deviate to $(1; 1)$. At this point, the pure-strategy equilibrium no longer exists, and the firms use mixed strategies, charging either $(1; 1)$ or $(\mathbf{p^{D,1}})$ (Figure B.2(a)).

A further decrease in μ causes the firms to undercut $(1; 1)$ by dropping the price of the secondary product. That is, the firms start charging $(1; p^{O,1})$ instead of $(1; 1)$, which results in the equilibrium with price pairs $(\mathbf{p^{D,1}})$ and $(1; p^{O,1})$ (Figure B.2(b)). As μ decreases further and these sale prices decrease, the firms again find a profitable deviation to $(1; 1)$, which leads to an emergence of an equilibrium with three prices (Figure B.2(c)). In this equilibrium, the firms charge the reservation price for both products, put both complementary products on sale at the same time, or promote only the secondary product.

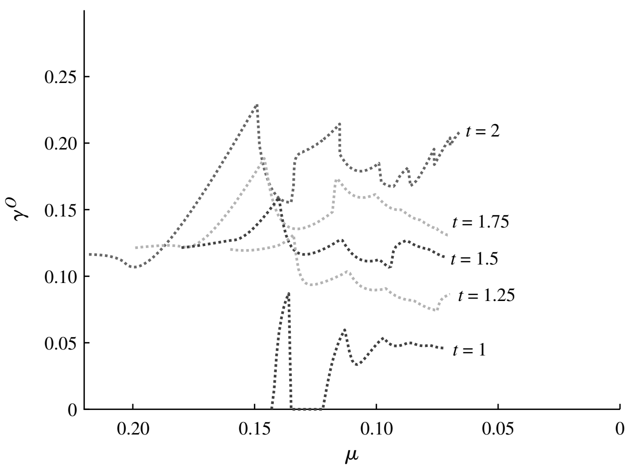
As μ keeps decreasing, the firms add more price pairs in which both products are promoted. In Figure B.2(d), the firms use two diagonal prices, and in Figure B.2(e) they use three diagonal prices. As μ decreases further, more and more prices appear in the equilibria (Figure B.2(f)). These new price pairs constitute either promotions in both categories, $(\mathbf{p^{D,4}})$, $(\mathbf{p^{D,5}})$, $(\mathbf{p^{D,6}})$, or promotions in only the secondary category, $(1; p^{O,2})$.

Several noteworthy patterns emerge from Figure B.2. First, for a large range of consumer heterogeneity μ, the lowest sale price of the primary product is below 0. Because marginal cost is normalized to 0, this means that the product is sold at a price below cost. This is consistent with a loss leader pricing strategy. In general, the promotion depth is larger for the

[12] It can be shown that for the utility function (3), the cross elasticity of demand of the second category product with respect to the price of the first category product is t times the cross elasticity of demand of the first category product with respect to the price of the second category product. This ratio of these elasticities was estimated to be close to 1.5 in Manchanda et al. (1999). Therefore, I chose $t = 1.5$ for the main analysis of the firms' strategies in this subsection.

[13] With some abuse of notation, I keep using the superscript D to indicate a price pair in which both products are on sale, even though, because the size of the discounts is different in different categories, this price no longer lies on the diagonal of the price support square.

Figure 4 **Total Off-Diagonal Probability for Different Values of** t ($\alpha = 0.1$, $d_b = d_c = 0.4$, $\delta_{k,i} = 0$)



primary product. Second, for a large subset of price pairs (for example, for ($\mathbf{p}^{\mathbf{D},1}$), ($\mathbf{p}^{\mathbf{D},2}$), ($\mathbf{p}^{\mathbf{D},3}$), ($\mathbf{p}^{\mathbf{D},4}$), ($\mathbf{p}^{\mathbf{D},5}$) in Figure B.2(f)), the discount depth of the secondary product is almost the same for each price pair, whereas the discount depth of the primary product varies. Taken together, these two observations lead to the conclusion that the firms use deeper promotions and more varied discount levels for their primary product. Also, although there are price pairs in which only the secondary product is promoted, there are no price pairs in which only the primary product is promoted.

Finally, consistent with the findings in the symmetric case, most of the probability is put on the price pairs, in which both products are sold at the reservation price or promoted together. Figure 4 shows the effect of μ on total off-diagonal probability, γ^O, for different values of cross-category asymmetry in promotional response t ($\alpha = 0.1$, $d_b = d_c = 0.4$, $\delta_{k,i} = 0$). Equilibria with up to eight diagonal prices were computed for this figure.

Figure 4 shows that the total off-diagonal probability is greater for the larger values of t. However, this probability is still relatively small, reaching a maximum value of 0.2294 for $t = 2$. Thus, for the case of asymmetric promotional response, the model also predicts that the complementary products should be promoted together.

6.2. Different Regular Prices and Costs

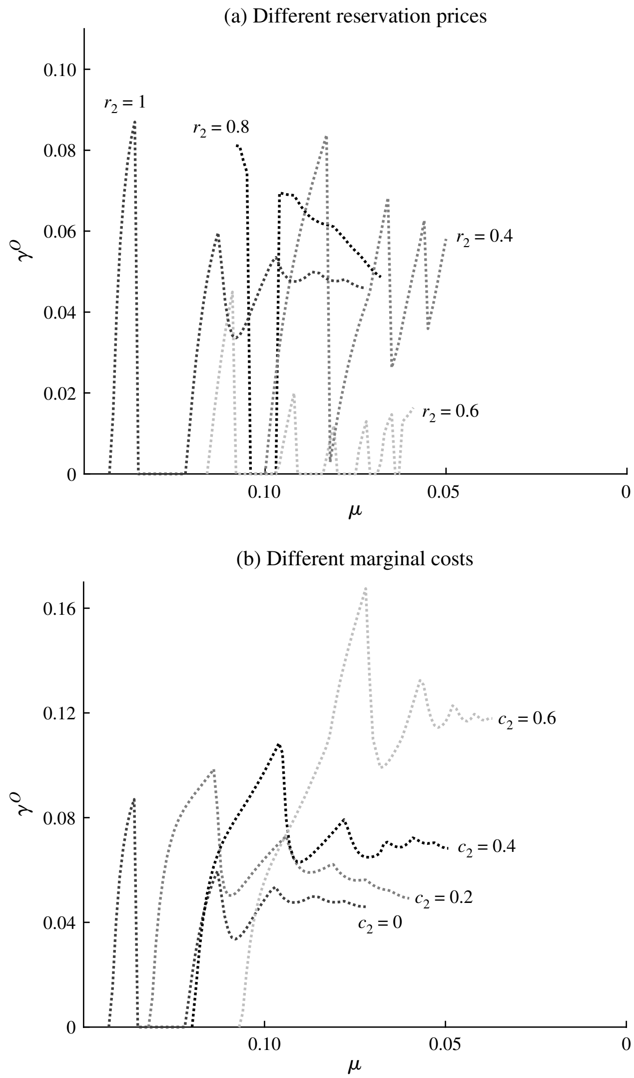
One of the assumptions made in §4 is that the two complementary products have identical reservation prices. In the subsequent analysis, I relax this restriction and consider the case of different reservation prices. The consumers' reservation prices for the complementary products are now $r_1 = 1$ and $r_2 \leq 1$, where r_1 is still normalized to 1. Figure B.3 in Appendix B illustrates the equilibrium strategies for different values of μ, first when the difference in reservation prices is large ($r_2 = 0.6$; left column), and second when the difference in reservation prices is small ($r_2 = 0.8$; right column).

Both cases are characterized by an increase in the number of diagonal prices as μ decreases. In each case, single-category promotions exist for only one category, but which category is promoted on its own depends on r_2. The off-diagonal price $(1; p^{O,1})$ with only the second category product on sale is present only for the high values of r_2. This happens because the firms typically use reservation prices $(1; r_2)$ in their mixed strategies, and it is not profitable to undercut this price pair in the second category if the reservation price is already small. The off-diagonal price $(p^{O,1}; r_2)$ with only the first category product on sale is present only for the small values of r_2. This happens because as μ decreases and new diagonal prices ($\mathbf{p}^{\mathbf{Di}}$) appear in the equilibrium, if p^{Di} is smaller than r_2, then the limit on the reservation price is not binding. However, if p^{Di} is greater than r_2, then the firm cannot charge p^{Di} in the second category. Therefore, the new price pair that is added to the equilibrium is (p^{Di}, r_2), in which the first category product is on promotion and the second category product is sold at the reservation price.

When the off-diagonal prices are present, the probability of charging them is once again small. Figure 5(a) shows the effect of μ on total off-diagonal probability, γ^O, for different values of reservation price r_2 ($\alpha = 0.1$, $d_b = d_c = 0.4$, $\delta_{k,i} = 0$). The dependence of total off-diagonal probability on r_2 is U-shaped. Its maximum value is 0.0812 for $r_2 = 0.8$; then it decreases to 0.0198 for $r_2 = 0.7$ before increasing to 0.0450 for $r_2 = 0.6$ and 0.0837 for $r_2 = 0.4$. With all of these probabilities less than 10%, the model predicts that the complementary products should be promoted together for the case of asymmetric reservation prices.

Another assumption made in §4 was that the two complementary products have identical marginal costs. To examine the effect of different marginal costs on the firms' strategies, I normalize the reservation price of both products to 1, normalize the marginal cost of the first category product to zero, and let the marginal cost of the second category product, c_2, be greater than zero. The equilibrium strategies (not presented in the paper, but available from the author) respond to the changes in μ as follows: as μ decreases, more price pairs with both products on sale are added to the equilibrium. In all of these price pairs, the first product (with the smaller cost) is promoted more deeply than the second product. There is also an increase in the number of price pairs in which only the first product is on promotion. Figure 5(b) shows that the probability of charging these off-diagonal prices is higher for the larger values of c_2. However,

Figure 5 **Total Off-Diagonal Probability for Different Values of r_2 and c_2 ($\alpha = 0.1$, $d_b = d_c = 0.4$, $\delta_{k,i} = 0$)**



even for $c_2 = 0.6$, the maximum off-diagonal probability is only 0.1675. The number of off-diagonal prices is also higher for the larger values of c_2—for the equilibrium with eight diagonal prices, there are five off-diagonal prices for $c_2 = 0.6$, two off-diagonal prices for $c_2 = 0.4$, and one off-diagonal price for $c_2 = 0.2$.

6.3. Three Firms and Three Complementary Categories

For some of the product categories that I consider later in the paper (shampoos, conditioners, cake mixes, cake frostings), there are three major producers. This subsection examines whether the predictions of the two-firm model hold for the three-firm model. Equation (1) is easily extended to the case of three firms. I consider only symmetric equilibria.[14] The structure of these equilibria and the responses of prices and probabilities to a decrease in μ are very similar to the ones described in §5 for the case of two firms. In the case of three firms, for the identical parameter values ($\alpha = 0.1$, $d_c = d_b = 0.4$, $\delta_{k,i} = 0$), off-diagonal prices first appear for smaller values of μ ($\mu = 0.107$ for three firms versus $\mu = 0.142$ for two firms). In addition, the maximum total off-diagonal probability is lower for the case of three firms—it is equal to 0.0233, whereas for the case of two firms it was 0.0893.

Sometimes consumers consider purchases in three complementary categories. For example, they can purchase cake mix, cake frosting, and toppings, or shampoo, conditioner, and body wash. The model is easily extended to allow for purchases in three complementary categories by specifying that consumers obtain an additional premium if they buy products from the same firm in all three categories. Analysis of the firms' optimal strategies for this case reveals that the firms will use all possible variations of price triplets: only one product on sale, two products on sale, all three products on sale, and all three products sold at the reservation price. The probability of using the first two types of these triplets is small, however—most of the time, the firms either charge the reservation price for the three products or promote all of them together. For example, for the same basic parameters of the demand function that were used in previous sections ($\alpha = 0.1$, $d_c = d_b = 0.4$, $\delta_{k,i} = 0$), if the consumers get an additional utility of 0.2 for buying in all three categories and another 0.4 for matching the brands of all three complementary products, the maximum total off-diagonal probability is 0.1013.

6.4. Summary

The main prediction of the base model is that price promotions in only one category, while possible, are rare. This prediction is conserved in the various modifications of the base model, which are presented in this section. These extensions generate a number of other testable predictions of the theory, which are summarized in Table 2.

The probability of putting only one product on sale is smaller when the consumer loyalty is smaller, when the brand-level complementarity is larger, when the asymmetry in promotional response or the difference in marginal costs is smaller, when the difference in reservation prices for the two products is intermediate, or when there are three firms competing instead of two.

In addition, for the case of asymmetric promotional response, the firms use deeper promotions and more

[14] Baye et al. (1992) show that the Varian model of sales with more than two firms has a continuum of asymmetric equilibria. However, they also show that the unique subgame-perfect equilibrium of a metagame in which both firms and consumers are players is symmetric. For simplicity, I consider only symmetric equilibria.

Table 2 **Impact of Parameters on the Total Off-Diagonal Probability**

An increase in	γ^0
Consumer loyalty	Increases the total off-diagonal probability
Brand-level complementarity	Decreases the total off-diagonal probability
Asymmetry in promotional response	Increases the total off-diagonal probability
Difference in reservation prices	First decreases and then increases the total off-diagonal probability
Difference in marginal costs	Increases the total off-diagonal probability

varied discount depths for their primary product. For some parameters of the demand function, this might include selling the primary product at a price below the marginal cost, making the primary product a loss leader. For the case of different reservation prices, when both products are on sale, their nominal sale prices are very similar, which means that the product with the larger reservation price is discounted more in percentage terms. For the case of different marginal costs, when both products are on sale, the product with the smaller cost is promoted more deeply.

7. Empirical Evidence

In this section, I show that the pattern of real world promotions of complementary products is correlationally consistent with the theoretical predictions of my model. The data that I use were collected by ACNielsen in a large metropolitan area in the United States between January 1993 and March 1995 for a total of 124 weeks. I examine the prices from the two largest supermarket chains, which together total 39 stores.

The three pairs of complementary categories chosen for this study are shampoo and conditioner, cake mix and cake frosting, and detergent and fabric softener. On an intuitive level, these three pairs exhibit different strengths of brand-level complementarity. In the detergent and fabric softener categories, the two leading manufacturers (Procter & Gamble (P&G) and Unilever) use different names in the complementary categories (for example, P&G produces detergents Tide and Cheer and fabric softeners Bounce and Downy, whereas Unilever produces detergents Surf and Wisk and fabric softener Snuggle). It is likely that in the absence of matching brand names, consumers do not gain extra utility from purchasing the complementary products that are produced by the same firm ($d_b = 0$). This intuition is consistent with the estimates of the cross-price elasticities between detergents and fabric softeners computed by Song and Chintagunta (2006). All cross-price elasticities between a firm's product in one category and its own and its rival's brands in the complementary category are almost identical. For example, the effect of powder Tide's price on the demand for liquid Downy, both of which are produced by P&G, is exactly the same as its effect on the demand for liquid Snuggle, which is produced by Unilever: −0.112 (Table 5 in Song and Chintagunta 2006). Therefore, in the absence of brand-level complementarity, the theoretical model predicts that the coordination of price promotions of complementary products within the detergent and fabric softener categories should be very small or nonexistent.

On the contrary, in the shampoo and conditioner and cake mix and cake frosting categories, the firms use the same brand names for their complementary products. It is likely, then, that in these categories, consumers are willing to pay a substantial premium in order to match the brand names of complementary products. Therefore, the theoretical model predicts that the promotions of complementary products within these categories should be positively correlated.

Because of space limitations, I present a full analysis of the cake mix and cake frosting categories and briefly summarize the findings for other category pairs. The cake mix and cake frosting categories are dominated by three brands—Betty Crocker (BC), Duncan Hines (DH), and Pillsbury (PB)—which jointly account for over 90% of the market. BC has around 50% market share, whereas the remaining 40% is split almost evenly between DH and PB. At the time when these data were collected, each of the brands was owned by a separate company: General Mills, P&G, and Grand Metropolitan, respectively. Each of the brands has multiple flavors, but the prices and sizes within each brand are the same. For these brands, I have data only for the first 82 weeks of the sample. Table 3 provides summary statistics.

The number of weeks during which only one of the complementary products was on sale is relatively high—27% for BC, 13% for DH, and 37% for PB. These values are so high partly because of the different frequency of sales of cake mixes and cake frostings. For example, in the second chain, DH cake mix was on sale for 26 weeks whereas DH frosting was on sale for 17 weeks. However, these two products were on sale together for 16 weeks. Thus, although it is evident that the price promotions of these two brands were coordinated (if DH frosting was promoted, that promotion was almost always accompanied by a promotion on DH cake mix), there were still 11 weeks when only one of these products was on sale. Now I formally test the hypothesis that the price promotions of cake mixes and cake frostings are independent.

Assume that out of W weeks there were S_1 weeks when a firm puts its product in the first category on sale and S_2 weeks when it puts its complementary product in the second category on sale. Then, the probability of a sale of the first product is S_1/W

Table 3 **Average Regular and Sale Prices for Cake Mixes and Cake Frostings**

	First chain			Second chain		
	Avg. regular	Avg. sale price		Avg. regular	Avg. sale price	
	Price	in $	in %	Price	in $	in %
BC cake	$1.47	$1.23 (0.18)	84% (12)	$1.47	$1.01 (0.18)	69% (12)
DH cake	$1.2 (0.05)	$0.95 (0.11)	79% (9)	$1.25 (0.09)	$0.96 (0.16)	77% (13)
PB cake	$1.34 (0.04)	$1.02 (0.25)	76% (19)	$1.35 (0.04)	$0.88 (0.19)	65% (14)
BC frosting	$1.83	$1.70 (0.16)	93% (9)	$1.83	$1.52 (0.16)	83% (9)
DH frosting	$1.57 (0.08)	$1.38 (0.13)	88% (8)	$1.58 (0.06)	$1.44 (0.05)	91% (3)
PB frosting	$1.76 (0.11)	$1.64 (0.09)	93% (5)	$1.79 (0.13)	$1.68 (0.04)	94% (2)

Notes. Standard deviations are in parentheses. Because of the changes in pricing regimes, there is more than one regular price for some of the products.

and the probability of a sale of the second product is S_2/W. If the sales in these two categories were independent, the probability of a joint sale would be $(S_1/W)(S_2/W)$. Therefore, under the null hypothesis of independent sales, the number of weeks with joint sales is distributed binomially with parameters W and $(S_1/W)(S_2/W)$. In Table 4, I present the values of the cumulative distribution function (CDF) of this binomial distribution at the actual number of weeks both products were on sale together. High values of this CDF indicate that the hypothesis of independent sales can be rejected and that the firm puts its two complementary products on sale together.

The hypothesis of independent sales can be rejected at the 1% level for four out of six brand pairs and at the 5% level for five out of six brand pairs. This suggests that brand managers of cake mixes and cake frostings do not view these product lines separately, but rather coordinate pricing decisions. It is interesting to note that in the second chain, Duncan Hines and Pillsbury—the two smaller firms—have a high concurrence of their cross-brand cross-category promotions (DH cake and PB frosting and PB cake and DH frosting) and a high concurrence of their cross-brand within-category promotions (DH cake and PB cake and DH frosting and PB frosting).

Prior research has identified cake mix as a primary category and cake frosting as a secondary category (Manchanda et al. 1999). This means that discounts on cake mixes have a larger impact on the sales of cake frosting than discounts on cake frostings have on the sales of cake mixes. The theoretical model of asymmetric promotional response (§6.1) predicts that the product in the primary category (cake mix) should have more varied discounts depths as well as deeper discounts. This prediction is largely supported by the data—the number of different discount prices Pillsbury used for cake mixes is 10 (in chain 1) and 8 (in chain 2), whereas for cake frostings it is 5 (in chain 1) and 6 (in chain 2). The corresponding numbers for Betty Crocker are 9 and 6 versus 3 and 6, and for Duncan Hines, 8 and 6 versus 7 and 4. With respect to the depth of the discounts, the largest discounts offered on cake mixes for Pillsbury were 50% and 62%, whereas its largest discounts on cake frostings were 16% and 8%. The corresponding numbers for Betty Crocker are 50% and 66% versus 50% and 32%, and for Duncan Hines, 34% and 36% versus 28% and 13%.

Table 4 **Joint Sales of Cake Mixes and Cake Frostings**

	Weeks on sale	Weeks on sale together with					
		BC cake	DH cake	PB cake	BC frosting	DH frosting	PB frosting
First chain							
BC cake	39	—	0.7013 (15)	0.5725 (23)	**0.9995 (30)**	0.4079 (11)	0.5801 (8)
DH cake	29		—	0.5671 (17)	0.3261 (11)	**0.9999 (22)**	0.7477 (7)
PB cake	48			—	0.1482 (17)	0.6516 (16)	**0.8823 (13)**
BC frosting	37				—	0.2469 (9)	0.2098 (5)
DH frosting	26					—	0.8295 (7)
PB frosting	17						—
Second chain							
BC cake	48	—	0.4305 (14)	0.6287 (20)	**0.9712 (25)**	0.2073 (7)	0.5907 (22)
DH cake	26		—	0.9858 (17)	0.8907 (13)	**0.9999 (16)**	0.9600 (17)
PB cake	33			—	0.7403 (14)	0.9819 (12)	**0.9953 (24)**
BC frosting	31				—	0.8079 (8)	0.5722 (14)
DH frosting	17					—	0.9207 (11)
PB frosting	37						—

Notes. The number in each cell shows the CDF of a binomial distribution of independent joint sales at the number of weeks with joint sales. The number of weeks with joint sales is in parentheses. The joint sales of the products produced by the same manufacturer are in bold.

In the shampoo and conditioner categories, the prices that the firms charged for their complementary products were almost always identical. In fact, the value of the CDF of the binomial distribution under the hypothesis of independent sales is 0.9999 for the three leading brands that I studied: Pantene (produced by P&G), Suave (at the time produced by Helene Curtis), and White Rain (at the time produced by Gillette). The hypothesis of independent sales is rejected in favor of the conclusion that the price promotions of shampoos and conditioners happened simultaneously, consistent with the theoretical predictions. Moreover, the coordination of price promotions was stronger for the shampoo and conditioner categories than it was for the cake mix and cake frosting categories. The theoretical model predicts that this should happen if brand-level complementarity is stronger for the brands in the shampoo and conditioner categories than it is for the brands in the cake mix and cake frosting categories. Although I do not have a measure of the brand-level complementarity in these categories, it is plausible that consumers place a larger premium on matching the brands of shampoos and conditioners based on the notion of uniformity in the bundle. The shampoo and conditioner bottles sharing the same brand name have a very similar design and are typically located next to each other on the shelf. The cake mixes and cake frostings of the same brand, on the other hand, have a less uniform design (cake mixes come in boxes while cake frostings come in cans) and are not typically placed next to each other (all brands of cake mixes are located together and all brands of cake frostings are located together).

For the final category pair, detergents and fabric softeners, the complementary brands produced by the same firm have different names, and thus we expect that there is little or no coordination of price promotions. Indeed, the hypothesis that the sales are independent can be rejected for only 1 out of 14 possible brand pairs at the 5% level, and for 3 out of 14 possible brand pairs at the 10% level. In summary, the empirical evidence from three pairs of complementary categories is correlationally consistent with the predictions of the theoretical model. In the category pairs for which we expect a large degree of brand-level complementarity, the firms promoted their complementary products together, whereas in the category pair in which brand-level complementarity is likely to be absent, there was almost no evidence of joint promotions.

8. Concluding Remarks

This paper presents a theoretical model of price promotions of complementary products. I assume that consumers are willing to pay a price premium for purchasing complementary products that share the same brand name and are produced by the same firm, and I find that each firm predominantly promotes its complementary products together. The supermarket price data in the shampoo and conditioner and cake mix and cake frosting categories is correlationally consistent with the predictions of the theoretical model—in these categories, the firms usually put their complementary products on sale at the same time.

There are, however, alternative explanations for the observed simultaneity of price promotions of complementary products. If a firm wants to increase the perception of complementarity of its products, simultaneous promotions encourage joint purchase and, hence, joint consumption of its products, leading to the desired result. On the demand side, consumers might respond more strongly to a joint promotion than to two separate single-product promotions. For example, Chintagunta and Haldar (1998) found some synergies to promoting pasta and pasta sauce jointly—a price promotion in both categories led to a larger increase in purchase probability of both products than the combined effect of two separate price promotions of the same depth. Similarly, on the cost side, there likely exist economies of scope in promotions—it is cheaper to include both products in one promotion rather than run two separate promotions.

However, the reasoning of these alternative explanations does not require that the complementary products share the same brand name, and should thus be equally applicable to the coordination of price promotions in detergent and fabric softener categories. Because the promotions of detergents and fabric softeners are not coordinated, I regard these alternatives as less likely explanations of the observed simultaneity of price promotions of complements than the proposed model.

It is possible that economies of scope in promotions are stronger for brands that share the same brand name. In addition, the incentives for the coordination of promotions in the two categories might be driven by similar inventory cycles or observations of common demand shocks that affect both categories. It might be easier to recognize these effects and implement a joint management strategy in the shampoo and conditioner and cake mix and cake frosting categories, where the complementary brands share the same name.[15]

Although I cannot rule out these alternative explanations, I have investigated whether sharing a brand

[15] It is also possible that, historically, the products in detergent and fabric softener categories were set up under differing brand names, leading to several strong brand management teams pursuing separate objectives without coordination.

name in multiple categories drives the coordination of promotions. I examined the prices of products that have the same brand name as the products studied in §7, but belong to other, noncomplementary categories. I found only three categories where these products are present, and I considered each firm's leading product in these categories. Those products are Betty Crocker Dunk-a-Roos 6 oz (cookies), Pillsbury Frozen Microwaveable Buttermilk Pancakes 15.2 oz, and Pillsbury Hungry Jack Maple Syrup 24 oz. Using the same procedure to test for the independence of the price promotions as in §7, I computed the values of the CDF for the joint sales of each firm's noncomplementary products. For BC cake mix and BC Dunk-a-Roos, they are 0.8457 (first chain) and 0.8260 (second chain). For BC frosting and BC Dunk-a-Roos, they are 0.6346 and 0.2801. For PB cake mix and PB pancakes, they are 0.9289 and 0.6234. For PB frosting and PB pancakes, they are 0.6578 and 0.5271. For PB cake mix and PB syrup, they are 0.8912 and 0.1778. For PB frosting and PB syrup, they are 0.8621 and 0.4433. The hypothesis that the promotions of two products with the same brand name in unrelated categories are independent can be rejected for only one brand pair out of twelve at the 10% level.[16] Simply sharing a brand name thus appears to be insufficient to cause coordinated promotions.

This paper has several limitations, some of which could be addressed in future work. First, I model the competition between manufacturers and do not consider their interactions with retailers. Intuitively, the qualitative predictions should not change if I add to the model nonstrategic retailers who set their prices by charging a certain markup over the wholesale price (the markups can be different for the regular and the sale prices). Including one or more strategic retailers would be a useful extension of the current model. Second, I provided only correlational empirical evidence in support of the main prediction of the theoretical model. Further work in this direction involves empirical testing of other implications of the model that are summarized in Table 2. Third, because the full model does not have closed-form solutions, the theoretical predictions are based on numerical computations for a finite set of parameters.

[16] Although Pillsbury pancakes and syrup are also complements, their promotions are not coordinated—the values of the CDF for the joint sales are 0.8297 and 0.2621. This finding does not necessarily contradict my model, because the industry structure in these categories is different from the ones I examined in this paper. The markets for frozen waffles/pancakes and syrups are dominated by the firms that produce in only one category. In addition to Pillsbury, only Aunt Jemima produces in both categories, but the shares of both companies are small: Pillsbury has 1.9% in the frozen waffles/pancakes market and 3% in the syrup market, whereas Aunt Jemima has 12.8% and 10.4% shares in the respective markets.

Another avenue for future research could involve a reexamination of the role of umbrella branding in light of the findings that complementary products sharing the same brand name are generally promoted together. Finally, a natural complement to the current paper would be the analysis of coordination of price promotions by a firm producing substitute products. Combining such an analysis with the results of the current paper would further advance our understanding of price promotion strategies for firms producing multiple products in each of several related categories.

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Appendix A

Proof of Proposition 1. Assume that the second firm chooses its four strategies with probabilities θ, β, η, and $\lambda = 1-\theta-\beta-\eta$. Then the profit of the first firm, if it chooses $(p;p)$, is

$$\theta(p)+\beta(ps_P+p(0.5+s_D))+\eta(ps_P+p(0.5+s_D)) \\ +(1-\theta-\beta-\eta)(2ps_P).$$

The profit from choosing $(p;1)$ is

$$\theta(p(0.5-s_D)+1-s_P)+\beta(0.5p+0.5)+\eta(ps_P+1-s_P) \\ +(1-\theta-\beta-\eta)(ps_P+(0.5+s_D)).$$

The profit from choosing $(1;p)$ is

$$\theta(p(0.5-s_D)+1-s_P)+\beta(ps_P+1-s_P)+\eta(0.5p+0.5) \\ +(1-\theta-\beta-\eta)(ps_P+(0.5+s_D)),$$

and, finally, the profit from choosing $(1;1)$ is

$$\theta(2(1-s_P))+\beta(1-s_P+(0.5-s_D))+\eta(1-s_P+(0.5-s_D)) \\ +(1-\theta-\beta-\eta)(1).$$

Equating all four profits and solving for the probabilities, we get $\theta=(s_D+ps_D+p^2s_P-1+s_P)/(s_D(1+p)^2)$, $\beta=\eta=(1-s_P-p^2s_P)/(s_D(1+p)^2)$, and $\lambda=1-\theta-\beta-\eta=(ps_D+p^2s_D+p^2s_P-1+s_P)/(s_D(1+p)^2)$.

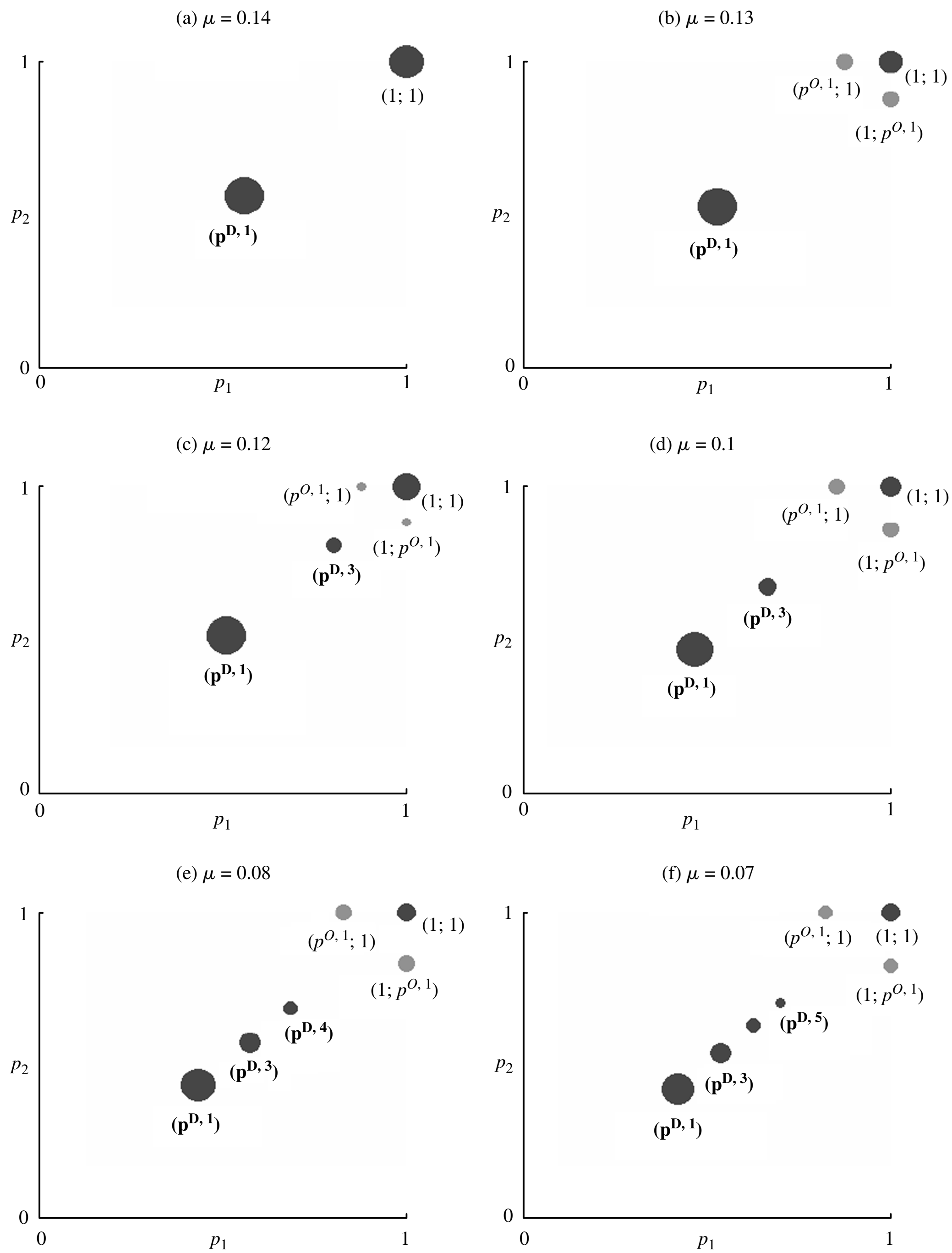
Now, we need to check that all probabilities are greater than zero. $\theta>\lambda$ (since $p<1$), so it is left to check that $\beta>0$ and $\lambda>0$. $\beta>0$ iff $1-s_P-p^2s_P>0$ or $s_P<1/(1+p^2)$. $\lambda>0$ iff $ps_D+p^2s_D+p^2s_P-1+s_P>0$ or $s_P(1+p^2)>1-ps_D-p^2s_D$ or $s_D>(1-s_P(1+p^2))/(p(1+p))$. In addition, the sales are positively correlated iff $\theta > (\theta+\beta)(\theta+\eta)$. Using formulas for θ, β, and η obtained above and noting that $\theta+\beta=\theta+\eta=1/(1+p)$, we get $(s_D+ps_D+p^2s_P-1+s_P)/(s_D(1+p)^2) > (1/(1+p))^2$. Solving for s_P, we get $s_P > (1-ps_D)/(1+p^2)$, which is equivalent to $s_D > (1-s_P(1+p^2))/p$. □

Proof of Proposition 2. From the formulas for the probabilities given in the proof of Proposition 1, $\theta/((\theta+\beta)(\theta+\eta)) = (s_D+ps_D+p^2s_P-1+s_P)(1+p)^2/(s_D(1+p)^2) = 1+p-(1-s_P-p^2s_P)/s_D$, which increases with s_D since $1-s_P-p^2s_P>0$ for $s_P<1/(1+p^2)$. □

Appendix B

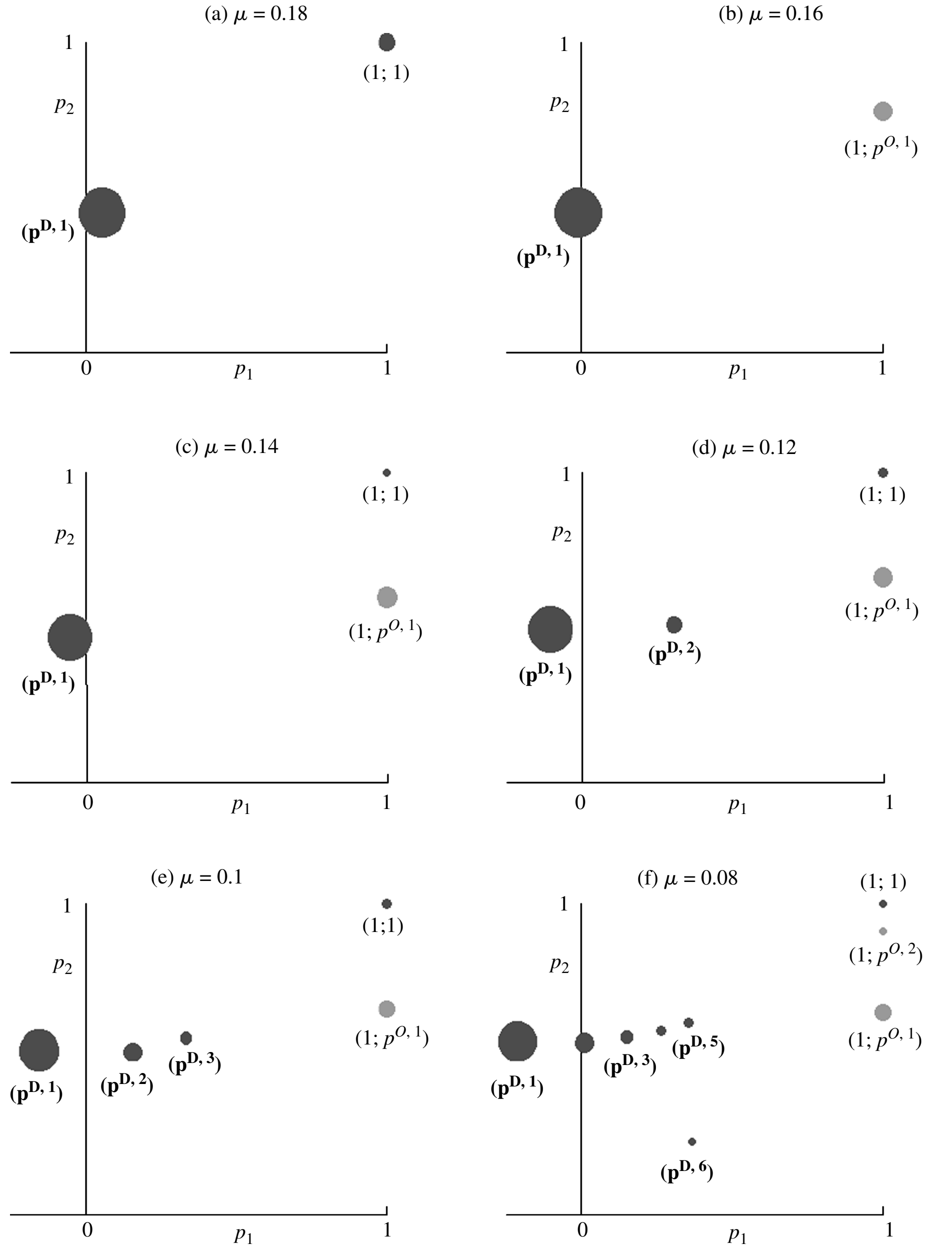
Figure B.1 shows an alternative bubble plot presentation of the equilibrium pricing strategies that were presented in Figure 1. Figure B.2 shows the equilibrium pricing strategies for the case of asymmetric promotional response. Figure B.3 shows the equilibrium pricing strategies for the case of different regular prices.

Figure B.1 Equilibrium Strategies for $\alpha=0.25$, $d_c=0.4$, and $d_b=0.4$



Note. Size of bubble denotes probability of charging a price pair.

Figure B.2 **Equilibrium Strategies for** $\alpha = 0.1$, $d_c = 0.4$, $d_b = 0.4$, **and** $t = 1.5$



Note. Size of bubble denotes probability of charging a price pair.

Figure B.3 **Equilibrium Strategies for** $\alpha = 0.1$, $d_c = 0.4$, $d_b = 0.4$, **and Two Values for** r_2, 0.6 **and** 0.8

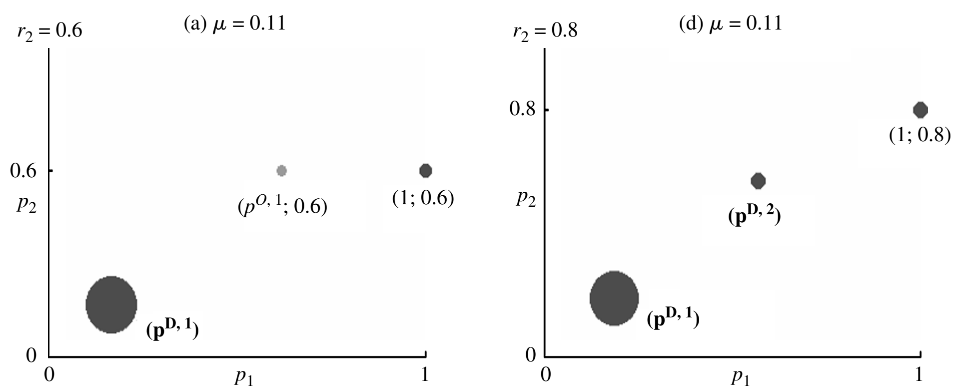
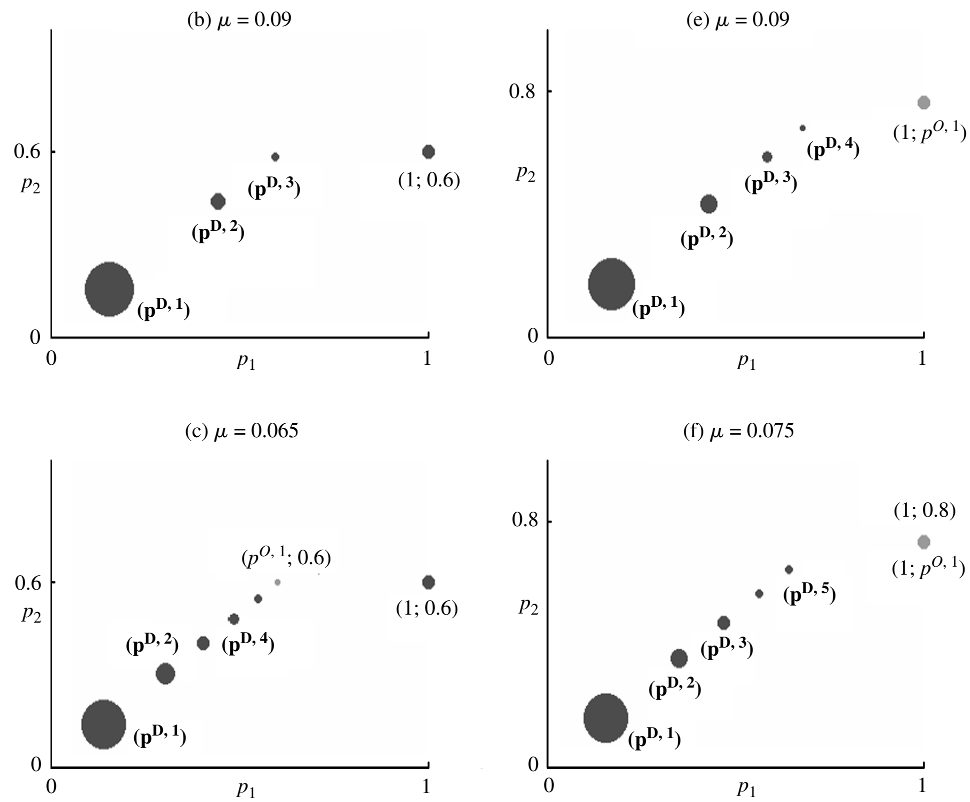


Figure B.3 (Continued)



Note. Size of bubble denotes probability of charging a price pair.

References

Balachander S, Ghosh B, Stock A (2010) Why bundle discounts can be a profitable alternative to competing on price promotions. *Marketing Sci.* 29(4):624–638.

Baye MR, Kovenock D, de Vries CG (1992) It takes two to tango: Equilibria in a model of sales. *Games Econom. Behav.* 4(4):493–510.

Chintagunta PK, Haldar S (1998) Investigating purchase timing behavior in two related product categories. *J. Marketing Res.* 35(1):43–53.

Duvvuri SD, Ansari A, Gupta S (2007) Consumers' price sensitivities across complementary categories. *Management Sci.* 53(12):1933–1945.

Gentzkow M (2007) Valuing new goods in a model with complementarity: Online newspapers. *Amer. Econom. Rev.* 97(3):713–744.

Huang R, Perloff JM, Villas-Boas SB (2006) Effects of sales on brand loyalty. *J. Agricultural Food Indust. Organ.* 4(1):Article 5.

Ma Y, Seetharaman PB, Narasimhan C (2012) Modeling dependencies in brand choice outcomes across complementary categories. *J. Retailing* 88(1):47–62.

Manchanda P, Ansari A, Gupta S (1999) The "shopping basket": A model for multicategory purchase incidence decisions. *Marketing Sci.* 18(2):95–114.

Matutes C, Regibeau P (1992) Compatibility and bundling of complementary goods in duopoly. *J. Indust. Econom.* 40(1):37–54.

Mehta N (2007) Investigating consumers' purchase incidence and brand choice decisions across multiple product categories: A theoretical and empirical analysis. *Marketing Sci.* 26(2):196–217.

Mulhern FJ, Leone RP (1991) Implicit price bundling of retail products: A multiproduct approach to maximizing store profitability. *J. Marketing* 55(4):63–76.

Narasimhan C (1988) Competitive promotional strategies. *J. Bus.* 61(4):427–449.

Niraj R, Padmanabhan V, Seetharaman PB (2008) A cross-category model of households' incidence and quantity decisions. *Marketing Sci.* 27(2):225–235.

Russell GJ, Petersen A (2000) Analysis of cross category dependence in market basket selection. *J. Retailing* 76(3):367–392.

Simester D (1997) Note. optimal promotion strategies: A demand-sided characterization. *Management Sci.* 43(2):251–256.

Simonin BL, Ruth JA (1995) Bundling as a strategy for new product introduction: Effects on consumers' reservation prices for the bundle, the new product, and its tie-in. *J. Bus. Res.* 33(3):219–230.

Sinitsyn M (2008a) Characterization of the support of the mixed strategy price equilibria in oligopolies with heterogeneous consumers. *Econom. Lett.* 99(2):242–245.

Sinitsyn M (2008b) Technical note: Price promotions in asymmetric duopolies with heterogeneous consumers. *Management Sci.* 54(12):2081–2087.

Sinitsyn M (2009) Price dispersion in duopolies with heterogeneous consumers. *Internat. J. Indust. Organ.* 27(2):197–205.

Song I, Chintagunta PK (2006) Measuring cross-category price effects with aggregate store data. *Management Sci.* 52(10):1594–1609.

Song I, Chintagunta PK (2007) A discrete-continuous model for multicategory purchase behavior of households. *J. Marketing Res.* 44(4):595–612.

Sriram S, Chintagunta PK, Agarwal MK (2010) Investigating consumer purchase behavior in related technology product categories. *Marketing Sci.* 29(2):291–314.

Varian HR (1980) A model of sales. *Amer. Econom. Rev.* 70(4):651–659.

Villas-Boas JM (1995) Models of competitive price promotions: Some empirical evidence from the coffee and saltine crackers markets. *J. Econom. Management Strategy* 4(1):85–107.

Walters RG (1991) Assessing the impact of retail price promotions on product substitution, complementary purchase, and interstore sales displacement. *J. Marketing* 55(2):17–28.

Wedel M, Zhang J (2004) Analyzing brand competition across subcategories. *J. Marketing Res.* 41(4):448–456.