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Equilibrium Innovation Ecosystems: The Dark Side of Collaborating with Complementors

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We provide a rationale for the recent burst in the amount of collaborative activities among firms selling complementary products, highlighting factors that may result in a lower profitability for such firms overall. To this end, we examine a game-theoretic model in which firms can collaborate with producers of complementary goods to enhance the quality of the systems formed by their components. Collaboration makes it cheaper to enhance such quality, so building innovation ecosystems results in firms investing more than if collaboration were impossible. In markets reaching saturation, firms are trapped in a prisoner's dilemma: the greater investments create more value, but this does not translate into greater value capture because the value created relative to competitors does not change. We also examine the (dis)advantages for a firm of having open or closed interfaces for the component it sells when the environment is competitive as well as how this is related to the endogenous emergence of two-sided platforms.

Keywords: systems competition; complementary products; interoperability; coopetition; exclusivity; endogenous two-sided platform
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1. Introduction

The recent decades have witnessed a shift in the competitive paradigm in high-tech industries that is driven to a large extent by the increasing importance of product complementarity. Indeed, cooperation among firms selling complementary products is playing a prominent role in industries such as consumer electronics, semiconductors, and telecommunications. More generally, hardware–software industries have exhibited a surge in the extent of cooperation among producers of complementary goods with the aim of improving the interoperability of their respective products (see, e.g., Moore 1996, Gawer and Cusumano 2002, Adner 2006, Gawer and Henderson 2007, Adner and Kapoor 2010).[1] Building such *innovation ecosystems* (Adner 2006) with the producers of complementary goods seems to be the key competitive weapon in most high-tech industries, in which the notion of competition has been displaced by that of *coopetition* (Brandenburger and Nalebuff 1996). A noteworthy feature of collaboration with complementors (i.e., firms selling products that complement each other) is that it is not unusual for firms to collaborate with several complementors that sell substitutes of each other.[2]

A natural question that arises in these settings is whether such extensive collaboration is desirable from the standpoints of firms and consumers. Intuitively, one would be tempted to think that collaboration in improving the interoperability of complementary products is jointly optimal for the firms involved, and in fact for society as a whole. The purpose of this paper is to show that this intuition may be valid for society, but not for the firms involved, which may be trapped in a prisoner's dilemma. Collaboration may then result in equilibria in which firms are worse off than when they do not collaborate with complementors. We show that this may hold regardless of whether collaboration ties are exclusive or not.

[1] The interoperability between components refers to their coherence to work together with each other as a sole system. This is largely related to the absence of conflicts arising from incompatibility issues.

[2] To give concrete examples, mobile phone manufacturer Nokia allied first with Intel to develop the MeeGo operating system for smartphones, and later signed an agreement with Microsoft to support the Windows Phone operating system (Nokia was finally acquired by Microsoft in 2013). In addition, the Intel Architecture Lab was formed to foster investment in components complementary to Intel's microprocessors by firms that many times competed against each other.

To formally analyze these issues, we consider a game played by two firms X_1 and X_2 that sell components that (perfectly) complement those sold by firms Y_1 and Y_2 (both of which are also engaged in the game). In this mix-and-match setting (Matutes and Regibeau 1988, Economides 1989), there are four systems contemplated by consumers when they make their purchase decisions: X_1Y_1, X_1Y_2, X_2Y_1, and X_2Y_2. The game consists of three stages. In the first stage, each firm decides whether to form a (pairwise) collaboration link with each of its possible complementors (collaboration among firms selling substitute components of a system is not allowed). In the second stage, each firm decides how much to invest in improving the interoperability of its component with each of its complementors. It is assumed that a firm that has formed a collaboration link with a complementor faces lower costs when enhancing interoperability with such complementor. In the third and final stage, each firm decides independently on the price of its component, given past interoperability investments of all the firms involved in the game.

We find in this setting that the (unique) equilibrium collaboration network involves each firm forming (pairwise) collaboration links with its two complementors. If collaboration ties can be formed only in an exclusive manner, then exactly the same forces (subject to the exclusivity restriction) imply that in equilibrium each firm forms a collaboration link with just one of its complementors. In both the exclusive and nonexclusive settings, equilibria exhibit all firms collaborating with at least one complementor, which seems to accord well with the empirical evidence on innovation ecosystems.

These equilibrium outcomes seem quite intuitive, but it is worth noting that intuition may conceal the effect of several forces working at the same time. Thus, two complementors that form a new collaboration link between them benefit from cost synergies and increase their investments in enhancing the interoperability with each other. This effect conforms to the intuition that one may have on the impact of a new collaboration link. However, two firms that form a new collaboration tie with each other must also bear in mind that the other firms will react strategically. Thus, the complementors not involved in the new collaboration tie reduce their investments in the firms that form such a new tie. Even though this harms the two firms that start collaborating with each other, such firms benefit a lot from the firms not involved in the new collaboration tie also reducing their investments in each other. Hence, the strategic effect of collaboration turns out to be positive, and hence reinforces the effect of the cost synergy that arises when two firms start collaborating. Factoring all the incentives, it holds that it is always desirable to form a new collaboration tie with a complementor with which a firm does not have one. This rat race ends when no more ties are possible, and hence each firm collaborates with as many complementors as it can.

Although a firm would benefit from its competitor committing to not collaborate with any complementor, it holds that all the firms would be better off if each could make such a commitment. Hence, the equilibrium outcome exhibits the features of a prisoner's dilemma even though all firms are more productive in enhancing interoperability between complementary components. Being more productive, each firm invests more in interoperability than in the absence of any collaboration among complementors. The greater investment leads to higher investment costs, incurred with the aim of vertically differentiating the systems in which a firm participates. Because all other firms act in the same way, firms boost investments but do not manage to vertically differentiate any system. When market expansion effects are weak, it follows that firms attain roughly the same profit in the product market as in the absence of collaboration. The growth in investment costs without substantially greater product market profits explains why the equilibrium outcome is jointly suboptimal for firms. As for consumers, all of them benefit in equilibrium from the better functionality of every system relative to when firms do not collaborate. This explains why collaboration arising as an equilibrium outcome enhances social welfare relative to the situation in which firms do not collaborate with complementors.

Our central finding that endogenous network formation leads firms into a "Bertrand supertrap" (Cabral and Villas-Boas 2005) is shown to be quite robust to a number of extensions (such as adding an extra producer of one of the components). Our analysis also shows that enough differentiation between systems allows firms to avoid the prisoner's dilemma when market expansion effects are significant. Perhaps more importantly, firms do not face such a trap when collaboration between two complementors (in the form of information sharing) is essential for their components to properly work. In these instances in which interoperability between components can be unilaterally refused, the jointly optimal profits may well be reached in equilibrium. However, this outcome is characterized by some but not full collaboration. The reason is that one of the firms is left out of the market because its competitor has signed exclusive contracts with its complementors, so equilibria in these cases are highly asymmetric in both outcomes and payoffs.

Our paper contributes to the management literature on innovation ecosystems (e.g., Gawer and Cusumano 2002, Yoffie and Kwak 2006, Adner 2012) by covering settings in which a firm is confronted by rivals that can develop such ecosystems on an equal footing. Our model allows us to examine standard prescriptions for firms that seek to stimulate system-specific investments. An instance of one such a prescription is that firms open interfaces for their components (see, e.g., Gawer and Cusumano 2014). This recommendation may be sound if a component producer enjoys a dominant position, but our analysis shows that it might lead to overinvestment if there is intense competition among systems. We also argue that pursuing an initially closed specification for interfaces, accompanied by exclusive contracting to open them up, may well result in highly asymmetric outcomes in which a firm dominates the market. That incompatibility results in multiplicity of equilibria with possibly extreme outcomes is well known in other settings relevant for high-tech industries, as when network effects are present (see, e.g., Casadesus-Masanell and Ruiz-Aliseda 2009). It is worth remarking that our results do not rely on there being network effects, though.

Our main finding that research and development (R&D) collaboration among complementors may make firms worse off is in stark contrast with the result that R&D collaboration among producers of substitute goods may be desirable both for firms and society, as shown by D'Aspremont and Jacquemin (1988) and Kamien et al. (1992). These papers do not consider whether a firm has incentives to collaborate with other firms, a limitation overcome by Bloch (1995) using a coalitions approach, and by Goyal and Moraga-González (2001) using a bilateral link formation approach.[3] Both of these papers show that excessive collaboration may arise in equilibrium. Although we also contend that collaboration may be jointly suboptimal for firms, it is worth noting that the results in Bloch (1995) and Goyal and Moraga-González (2001) are derived for substitute goods, not for complementary goods, as is our focus.

Our paper also contributes to the literature analyzing strategic competition when there exists at least one complementor whose pricing activities interact with those of two firms selling components that constitute substitutes for each other. This literature was pioneered by Economides and Salop (1992) as an extension of early work by Cournot (1838), who analyzed the effect of a merger of two monopolists that produce complementary goods. Significant papers following this literature stream are those by Casadesus-Masanell et al. (2008) and Hermalin and Katz (2013).[4]

Our work is most closely related to that of Hermalin and Katz (2013). They show that a commoditized component may have an implicit platform value, which could be exploited by the producers of this component by having complementors sign an exclusive contract. Exclusive contracting allows the commoditized component to endogenously emerge as a (two-sided) platform, something that would be impossible if contract exclusivity were banned. Each producer of the commoditized component is then willing to incur the cost of entering the market, since they anticipate making some profit as a two-sided platform. This more competitive outcome would not arise if exclusivity were banned, so they conclude that exclusive contracting need not lower (consumer) welfare. Our paper differs from theirs in two ways. It differs in the objective (showing that collaboration in enhancing interoperability can be jointly suboptimal for firms) and in the means (since we consider endogenous network formation even if components cannot be made incompatible with one another). Finally, we note that, unlike in our paper, Hermalin and Katz (2013) do not find equilibrium market foreclosure when one of the components is a commodity. Our results, based on sufficiently differentiated components, complement rather than substitute theirs.

2. The Baseline Model

We define a system as a pair of perfectly complementary goods consumed in a proportion of one to one (e.g., hardware and software). The two perfect complements are called components X and Y. Component X is produced at no cost by two firms, X_1 and X_2, whereas component Y is costlessly produced by two firms, Y_1 and Y_2.[5] Hence, there are $n = 4$ systems: X_1Y_1, X_1Y_2, X_2Y_1, and X_2Y_2. System X_iY_j $(i, j = 1, 2)$ can be bought by any consumer at price $p_{i,j} = p_{X_i} + p_{Y_j}$, where p_{X_i} and p_{Y_j} respectively denote the prices at which components X_i and Y_j are sold. With some slight abuse of notation, we will often write p_{ij} instead of $p_{i,j}$ for system X_iY_j. Also, firms X_1 and X_2 are typically referred to as the complementors of Y_1 and Y_2, and vice versa.

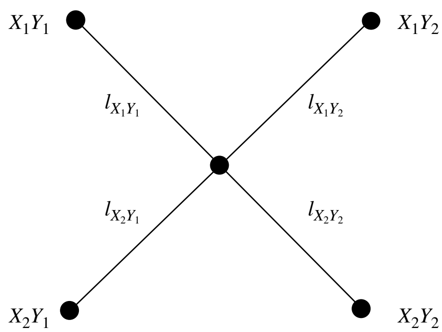
There also exists a unit mass of consumers willing to buy at most one system. System X_iY_j is assumed to create a gross utility of $v_{i,j}$ to any consumer (again, we will typically write v_{ij} instead of $v_{i,j}$). The gross valuation v_{ij} is largely the outcome of choices by firms X_i and Y_j. More specifically, for some given

[3] See Bloch (2005) for a comprehensive survey covering strategic network formation games with R&D activities.

[4] There is a recent literature on (pure and mixed) bundling by firms that produce two perfectly complementary components in competition with firms that produce just one of these components (see, e.g., Denicolò 2000, Choi 2008). This stream of research is not directly related to our work as we do not consider bundling, an issue that certainly deserves a separate analysis beyond the scope of our paper.

[5] That production is costless is without loss of generality if the marginal cost of production is constant.

Figure 1 **The Spokes Model**



scalar $v > 0$, we have that $v_{ij} = v + x_i^j + y_j^i$, where x_i^j is firm X_i's R&D investment in improving the quality of the match with firm Y_j's component, and y_j^i is firm Y_j's R&D investment in improving the quality of the match with firm X_i's component.[6] Thus, the investment variables x_i^j and y_j^i affect the vertical attributes of system X_iY_j. Given their system specificity, they can be viewed as investments in improving the interoperability of components X_i and Y_j, although other interpretations are possible and may be more appealing in other contexts.

Besides (possibly) being vertically differentiated, systems are perceived by consumers as being horizontally differentiated in an exogenous manner. In this respect, we follow Chen and Riordan (2007) in using their "spokes" model of nonlocalized differentiation. Thus, each of the $n = 4$ systems desired by consumers is represented by a point at the origin of a line of length 1/2, a line which is denoted by $l_{X_iY_j}$ for system X_iY_j $(i, j = 1, 2)$. The other end of a line is called its terminal, and it is assumed that the terminals of all lines meet at a point called the center (see Figure 1). All existing consumers are uniformly distributed along the four lines. A consumer who is located on line $l_{X_iY_j}$ at distance $d_{X_iY_j} \in [0, 1/2]$ from system X_iY_j must incur a transportation cost of $td_{X_iY_j}$ when buying X_iY_j, where $t \geq 0$ is a unit transportation cost. The same consumer must incur transportation cost $t(1 - d_{X_iY_j})$ when purchasing any other system (since $l_{X_iY_j} = 1/2$ for all $i, j = 1, 2$). It is assumed that X_iY_j is the preferred system for any consumer on $l_{X_iY_j}$, and any other system has probability $1/(n-1) = 1/3$ of constituting the benchmark against which X_iY_j is to be compared by a consumer on $l_{X_iY_j}$. A system that is not deemed as preferred or as a benchmark for a consumer is assumed to yield no utility to such a consumer. This assumption completes the description of the spokes model.

Given these features of firms and consumers, we study a game that consists of three stages. The first one is the network formation stage, in which firms X_i $(i = 1, 2)$ simultaneously form pairwise collaboration links with firms Y_j $(j = 1, 2)$. We let $g_{ij} = 1$ if a (costless) collaboration link between X_i and Y_j is formed and $g_{ij} = 0$ otherwise. We denote the network by g; that is, $g = \{g_{11}, g_{12}, g_{21}, g_{22}\} \in \{0, 1\}^4$. Note that we allow a firm to form more than one collaboration link with its complementors.

The second one is the value creation stage, in which X_i chooses x_i^j at the same time as Y_j chooses y_j^i $(i, j = 1, 2)$. Given network g and parameter $\gamma \in (0, 1)$, investments of x_i^1 and x_i^2 by X_i result in an R&D cost equal to $C_{X_i}(x_i^1, x_i^2 \mid g) = \sum_{j=1}^{2} \gamma^{g_{ij}} (x_i^j)^2$, whereas investments of y_j^1 and y_j^2 by Y_j result in R&D cost $C_{Y_j}(y_j^1, y_j^2 \mid g) = \sum_{i=1}^{2} \gamma^{g_{ij}} (y_j^i)^2$.[7] Hence, collaboration between X_i and Y_j yields that it is easier/cheaper to enhance the match quality with the component provided by the complementor. This captures, in a simple manner, useful but costless information exchanges that aim at improving the interoperability of system X_iY_j. For this reason, the inverse of parameter γ can be understood as representing the extent of information sharing and its economic relevance: lowering the value of γ represents in our model more exchange of technically useful information among collaborators.

The third and last stage is the value capture stage, in which prices p_{X_i} and p_{Y_j} are set simultaneously in the standard noncooperative manner, and consumers make their purchase decisions given p_{ij} for $i, j = 1, 2$.

The solution concept is the same as in Goyal and Moraga-González (2001). Thus, for each possible g, we will look for subgame perfect Nash equilibria, which will give equilibrium payoffs given g. To solve for the equilibrium network structure in the first stage, we will use the pairwise stability notion proposed by Jackson and Wolinsky (1996). This concept is very weak, and aims at capturing (possibly) complex communication and negotiation activities that would be hard to capture through noncooperative game theory.

Introducing the concept of pairwise stability requires some notation. In particular, $g - g_{ij}$ denotes the network that results from suppressing an existing collaboration link between X_i and Y_j in network g, whereas $g + g_{ij}$ denotes the network that results from adding a new collaboration link between them. Denoting the equilibrium payoffs obtained by X_i and Y_j given network g by $\Pi^*_{X_i}(g)$ and $\Pi^*_{Y_j}(g)$, network g would be pairwise stable if the following conditions held for all $i, j \in \{1, 2\}$: (i) $\Pi^*_{X_i}(g) \geq \Pi^*_{X_i}(g - g_{ij})$ and $\Pi^*_{Y_j}(g) \geq \Pi^*_{Y_j}(g - g_{ij})$, and (ii) $\Pi^*_{X_i}(g + g_{ij}) \geq \Pi^*_{X_i}(g)$

[6] See Goyal et al. (2008) for another setting with relationship-specific actions.

[7] For example, if $g = \{1, 0, 0, 0\}$, then $C_{X_1}(x_1^1, x_1^2 \mid g) = \gamma(x_1^1)^2 + (x_1^2)^2$, $C_{X_2}(x_2^1, x_2^2 \mid g) = (x_2^1)^2 + (x_2^2)^2$, $C_{Y_1}(y_1^1, y_1^2 \mid g) = \gamma(y_1^1)^2 + (y_1^2)^2$ and $C_{Y_2}(y_2^1, y_2^2 \mid g) = (y_2^1)^2 + (y_2^2)^2$.

implies that $\Pi^*_{Y_j}(g+g_{ij}) < \Pi^*_{Y_j}(g)$. The first condition requires that neither X_i nor Y_j have an incentive to unilaterally break an existing collaboration relationship. In turn, the second condition requires that a desire by X_i to form a new collaboration link with Y_j should not be reciprocal. It is worth noting that the results we derive still hold if the network is required to be pairwise Nash stable, that is, if a firm is allowed to unilaterally break more than one collaboration link at a time.[8]

3. Resolution of the Model

3.1. Third Stage

As is standard, we solve the last two stages of the game by working backward. So assume that first-stage and second-stage choices lead to a gross valuation of v_{ij} for system X_iY_j, $i, j = 1, 2$. We first derive the demand functions for each system, and then we find out profits attained by each firm as a function of v_{11}, v_{12}, v_{21}, and v_{22}. It is assumed throughout that v is such that the market is always fully covered and all firms make positive sales.[9]

We proceed to derive the demand function just for system X_1Y_1, the other demands being derived analogously. Consider a consumer who happens to be on either line $l_{X_1Y_1}$ or line $l_{X_1Y_2}$. This occurs either because X_1Y_1 is her preferred system and X_1Y_2 is the benchmark or because X_1Y_2 is her preferred system and X_1Y_1 is the benchmark. The consumer will be indifferent between both systems if her distance $d_{11}^{12} \in [0,1]$ from X_1Y_1 is given by $v_{11} - p_{11} - td_{11}^{12} = v_{12} - p_{12} - t(1-d_{11}^{12})$.[10] It readily follows that

$$d_{11}^{12} = \frac{t + v_{11} - v_{12} + p_{12} - p_{11}}{2t}.$$

Because the measure of consumers between the locations of systems X_1Y_1 and X_1Y_2 is 1/2, the number of consumers who prefer X_1Y_1 over X_1Y_2 given p_{11} and p_{12} must be $d_{11}^{12}/2$. Similarly, the number of consumers who prefer X_1Y_1 over X_2Y_j ($j = 1, 2$) can be shown to be $d_{11}^{2j}/2$, where

$$d_{11}^{2j} = \frac{t + v_{11} - v_{2j} + p_{2j} - p_{11}}{2t}.$$

Conditional on X_1Y_1 being the preferred system or the benchmark one, it holds that X_1Y_2, X_2Y_1 and X_2Y_2 each have probability $1/(n-1) = 1/3$ of being the system with respect to which X_1Y_1 is to be assessed by consumers. It follows that demand for X_1Y_1 is

$$Q_{11} = \frac{2(d_{11}^{12} + d_{11}^{21} + d_{11}^{22})}{n(n-1)}.$$

Simple algebra yields that

$$Q_{11} = \frac{3t + 3v_{11} - v_{12} - v_{21} - v_{22} - 3p_{11} + p_{12} + p_{21} + p_{22}}{12t}.$$

Similar steps lead to the following demand for system X_iY_j ($i, j = 1, 2$):

$$Q_{ij} = (3t + 3v_{i,j} - v_{3-i,j} - v_{i,3-j} - v_{3-i,3-j} - 3p_{i,j} + p_{3-i,j} + p_{i,3-j} + p_{3-i,3-j})/(12t).$$

Recalling that $p_{i,j} = p_{X_i} + p_{Y_j}$ and letting $Q_{X_i} \equiv Q_{i1} + Q_{i2}$ denote X_i's demand, we then have that

$$Q_{X_i}(p_{X_i}, p_{X_{3-i}}) = \frac{3t + v_{i,1} + v_{i,2} - v_{3-i,1} - v_{3-i,2} - 2p_{X_i} + 2p_{X_{3-i}}}{6t}.$$

We have rendered the arguments of Q_{X_i} explicit to highlight that X_i's sales volume does not depend on how any complementary good is priced. Under full market coverage, different prices by Y_1 and Y_2 just affect with which component X_i wishes to be matched, but X_i's demand solely depends on p_{X_i} and $p_{X_{3-i}}$. One can similarly obtain that

$$Q_{Y_j}(p_{Y_j}, p_{Y_{3-j}}) = \frac{3t + v_{1,j} + v_{2,j} - v_{1,3-j} - v_{2,3-j} - 2p_{Y_j} + 2p_{Y_{3-j}}}{6t},$$

where $Q_{Y_j} \equiv Q_{1j} + Q_{2j}$.

Firm X_i ($i = 1, 2$) chooses p_{X_i} to maximize $\pi_{X_i}(p_{X_i}, p_{X_{3-i}}) \equiv p_{X_i} Q_{X_i}(p_{X_i}, p_{X_{3-i}})$. Using the strict concavity of profit functions, one can solve the system that consists of X_1 and X_2's first-order conditions so as to find out the following equilibrium prices: $p^*_{X_i} = (9t + v_{i,1} + v_{i,2} - v_{3-i,1} - v_{3-i,2})/6$, $i = 1, 2$. Similarly, one can show that $p^*_{Y_j} = (9t + v_{1,j} + v_{2,j} - v_{1,3-j} - v_{2,3-j})/6$, $j = 1, 2$. It then holds that the sales of system X_iY_j are $Q^*_{ij} = (9t + 5v_{i,j} + v_{3-i,3-j} - 3v_{i,3-j} - 3v_{3-i,j})/(36t)$. Letting $\pi^*_{X_i} \equiv p^*_{X_i} \sum_{j=1}^{2} Q^*_{ij}$ and

[8] See Jackson (2008, pp. 156, 371–376) for a thorough discussion of the virtues and limitations of pairwise stability as a solution concept.

[9] Parameter v should be large enough so that the market be covered. However, it can not be too large, otherwise a firm may have a unilateral incentive to charge a much higher price than in equilibrium, thus destroying the same equilibrium existence. The details are technically complex and can be found in the online appendix (available as supplemental material at http://dx.doi.org/10.1287/mnsc.2014.2140). (Essentially, γ should not differ too much from 1.)

[10] To compute d_{11}^{12} for such an indifferent consumer, suppose that she is located on $l_{X_1Y_1}$. Then d_{11}^{12} is her distance from system X_1Y_1. If she is instead located on $l_{X_1Y_2}$, then d_{11}^{12} is one minus her distance from X_1Y_2 (since she has to "travel" along line $l_{X_1Y_1}$ to get to system X_1Y_1's location, and such a line has a measure of one-half, as does $l_{X_1Y_2}$).

$\pi^*_{Y_j} \equiv p^*_{Y_j} \sum_{i=1}^{2} Q^*_{ij}$ respectively denote the overall profits made by X_i and Y_j $(i, j = 1, 2)$, and recalling that $v_{ij} = v + x_i^j + y_j^i$, we can write them as a function of second-stage choices:

$$\pi^*_{X_i} = \frac{(9t + x_i^1 + x_i^2 - x_{3-i}^1 - x_{3-i}^2 + y_1^i + y_2^i - y_1^{3-i} - y_2^{3-i})^2}{108t}$$

and

$$\pi^*_{Y_j} = \frac{(9t + x_1^j + x_2^j - x_1^{3-j} - x_2^{3-j} + y_j^1 + y_j^2 - y_{3-j}^1 - y_{3-j}^2)^2}{108t}.$$

The following is worth noting for $i, j = 1, 2$:

Remark 1. It holds that

$$\frac{\partial^2 \pi^*_{X_i}}{\partial x_i^j \partial x_i^{3-j}} = \frac{\partial^2 \pi^*_{X_i}}{\partial x_i^j \partial y_j^i} = \frac{\partial^2 \pi^*_{X_i}}{\partial x_i^j \partial y_{3-j}^i} > 0$$

and

$$\frac{\partial^2 \pi^*_{X_i}}{\partial x_i^j \partial x_{3-i}^j} = \frac{\partial^2 \pi^*_{X_i}}{\partial x_i^j \partial x_{3-i}^{3-j}} = \frac{\partial^2 \pi^*_{X_i}}{\partial x_i^j \partial y_j^{3-i}} = \frac{\partial^2 \pi^*_{X_i}}{\partial x_i^j \partial y_{3-j}^{3-i}} < 0.$$

Similarly,

$$\frac{\partial^2 \pi^*_{Y_j}}{\partial y_j^i \partial y_j^{3-i}} = \frac{\partial^2 \pi^*_{Y_j}}{\partial y_j^i \partial x_i^j} = \frac{\partial^2 \pi^*_{Y_j}}{\partial y_j^i \partial x_{3-i}^j} > 0$$

and

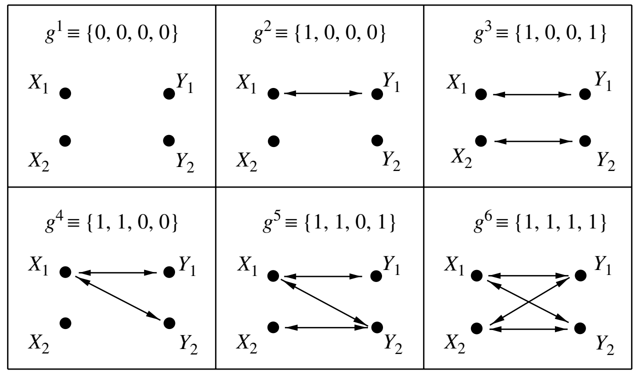
$$\frac{\partial^2 \pi^*_{Y_j}}{\partial y_j^i \partial y_{3-j}^i} = \frac{\partial^2 \pi^*_{Y_j}}{\partial y_j^i \partial y_{3-j}^{3-i}} = \frac{\partial^2 \pi^*_{Y_j}}{\partial y_j^i \partial x_i^{3-j}} = \frac{\partial^2 \pi^*_{Y_j}}{\partial y_j^i \partial x_{3-i}^{3-j}} < 0.$$

Remark 1 will be heavily used in what follows, so it is worthwhile expressing in words what it means. Essentially, a firm's incentive to invest in enhancing the match quality with any one of its complementors intensifies as there is more investment in any of the systems in which the firm participates. This incentive is also strengthened as there is less investment in any of the systems in which it does not participate. To better understand these incentives to perform R&D in the second stage, let us suppose that v_{11} increases for some unspecified reason. Then firm X_1 foresees charging a higher price for its component X_1. This in turn induces firm X_1 to stimulate more sales of X_1Y_1 and X_1Y_2 by doing R&D, so that it can better exploit the higher price it is led to charge owing to the larger v_{11}. The opposite happens when either of v_{21} and v_{22} increases, hence the result.

3.2. Second and First Stages

We now consider the investment subgames for each of the possible network structures arising from the first stage. Up to a relabeling of firms, there are six network structures that should be considered (see Figure 2): $g^1 \equiv \{0, 0, 0, 0\}$, $g^2 \equiv \{1, 0, 0, 0\}$, $g^3 \equiv \{1, 0, 0, 1\}$, $g^4 \equiv \{1, 1, 0, 0\}$, $g^5 \equiv \{1, 1, 0, 1\}$, and $g^6 \equiv \{1, 1, 1, 1\}$.

Figure 2 **Network Structures**



Besides characterizing equilibrium play for each, we also show which one emerges as the unique (pairwise) stable network, thus providing a complete resolution of the network formation game. We start by analyzing networks structures when no firm can have more than one collaboration link (i.e., $g \in \{g^1, g^2, g^3\}$), which may be due to exclusivity, for instance.[11] We then consider network architectures when firms can have more than one collaboration tie (i.e., $g \in \{g^4, g^5, g^6\}$).

At this point, it is useful to define the following functions:

$$\begin{aligned} &\Pi_{X_i}(x_i^1, x_i^2, x_{3-i}^1, x_{3-i}^2, y_j^1, y_j^2, y_{3-j}^1, y_{3-j}^2 \mid g) \\ &\quad \equiv \pi^*_{X_i} - C_{X_i}(x_i^1, x_i^2 \mid g), \\ &\Pi_{Y_j}(y_j^1, y_j^2, y_{3-j}^1, y_{3-j}^2, x_i^1, x_i^2, x_{3-i}^1, x_{3-i}^2 \mid g) \\ &\quad \equiv \pi^*_{Y_j} - C_{Y_j}(y_j^1, y_j^2 \mid g). \end{aligned}$$

Given network architecture g, firm X_i $(i = 1, 2)$ chooses $x_i^1 \geq 0$ and $x_i^2 \geq 0$ to maximize Π_{X_i}, whereas firm Y_j $(j = 1, 2)$ chooses $y_j^1 \geq 0$ and $y_j^2 \geq 0$ to maximize Π_{Y_j}, where we have suppressed the arguments of the functions to avoid clutter. We also recall that all second-stage choices are made simultaneously. Throughout, we make the assumption that

$$t > \hat{t} \equiv (3 - 2\gamma + \sqrt{4\gamma^2 - 6\gamma + 3})/(54\gamma) > 0$$

to ensure that, regardless of the network considered, all firms are willing to sink the costs of investing in creating value in any of the systems in which each participates. For low values of t, competition among

[11] The formation of exclusive collaboration links may be due to explicit or implicit contracting requirements, or to highly competitive conditions that preclude several complementors from being willing to collaborate with the same firm.

systems is very intense, and not all of them can be active.[12]

3.2.1. Network Structures Under Exclusivity. When collaboration is exclusive in the sense that no firm can establish more than one link with complementors, the unique stable network is $g = g^3$. As we formally show below (see Proposition 1), neither $g = g^1$ nor $g = g^2$ is stable because any unlinked firm would do better by establishing a collaboration tie with an unlinked complementor. For instance, both X_1 and Y_1 would benefit from forming a collaboration tie if the network were $g = g^1$.

To understand why firms X_1 and Y_1 would be better off, note that, relative to $g = g^1$, several incentives arise for such firms if $g = g^2$. On the one hand, the investment cost synergy leads them to increase x_1^1 and y_1^1, which in turn creates pressure toward increasing x_1^2 and y_1^2 in light of Remark 1. On the other hand, the negative impact of higher x_1^2 on firm X_2's marginal payoff is completely offset by the positive impact of higher y_1^2 (by Remark 1). Taking this into consideration as well as the fact that system X_1Y_1 is stronger, it then follows from Remark 1 that X_2 prefers to lower x_2^1, and it does so in such a way that $x_2^1 + y_1^2$ does not vary with respect to the level under $g = g^1$. In an analogous fashion, total investment in system X_1Y_2 does not vary because Y_2 lowers y_2^1 in a way that offsets the increase in x_1^2. The strength of systems X_1Y_2 and X_2Y_1 is then unaffected, but firms X_2 and Y_2 end up respectively decreasing x_2^2 and y_2^2 because system X_1Y_1 becomes stronger. It is interesting to observe that firm X_2 reduces x_2^1 and x_2^2 by the same amount (and analogously for Y_2 with y_2^1 and y_2^2). The point is that it equally benefits from investing in the match with Y_1 or Y_2, so both x_2^1 and x_2^2 must be reduced by the same amount because the strict convexity of R&D costs implies that it is more efficient to spread R&D effort over two complementors rather than concentrating it on just one.

In short, g^1 is not a stable network because firms X_1 and Y_1 mutually benefit from forming a link because of both the cost synergy fostered by their collaboration and the positive strategic effect of such collaboration.[13] Even though firms X_1 and Y_1 benefit from the fact that X_2 and Y_2 reduce their investment in each other, firms X_2 and Y_2 exploit the incentive that X_1 and Y_1 have to invest more in systems X_1Y_2 and X_2Y_1 by cutting down their respective investments in such systems, slightly harming X_1 and Y_1. Overall, the strategic reaction of firms X_2 and Y_2 benefits firms X_1 and Y_1, thus reinforcing the positive direct effect of the cost synergies exploited by X_1 and Y_1.

The explanation of why $g = g^2$ cannot constitute an equilibrium network is somewhat similar, so it is omitted for the sake of brevity. Again, collaboration by firms X_2 and Y_2 results in a positive strategic effect that reinforces the cost synergies that arise because of their collaboration, and hence both mutually benefit from forming a link with each other.

Even though the unique stable network that arises when collaboration is exclusive is $g = g^3$, it holds that each firm would be better off if collaboration were not possible. Gross profits are the same under $g = g^1$ and $g = g^3$ because firms do not change their pricing and end up with the same sales. (Of course, systems X_1Y_1 and X_2Y_2 are bought more under $g = g^3$, but this is at the expense of X_1Y_2 and X_2Y_1.) However, total investment costs are greater under $g = g^3$ than $g = g^1$ (more precisely, $(1+\gamma)/(144\gamma)$ versus 1/72), which explains why a firm's payoff decreases when going from $g = g^1$ to $g = g^3$. Although firms are worse off, consumers are much better off under $g = g^3$, and as a result social welfare increases. In particular, we have the following result.

Proposition 1. *Let $t > \hat{t}$, and suppose that collaborating with a complementor precludes a firm from collaborating with the complementor's competitor. Then, it holds that*:

(i) *The unique* (*up to a relabeling of the X-firms*) *equilibrium network is $g^* = \{1, 0, 0, 1\}$.*

(ii) *In equilibrium, firm X_1 invests $x_1^1(g^*) = 1/(12\gamma)$ in improving the match quality with complementor Y_1, whereas it invests $x_1^2(g^*) = 1/12$ in improving the match quality with complementor Y_2. In turn, firm Y_1 invests $y_1^1(g^*) = 1/(12\gamma)$ in improving the match quality with complementor X_1, whereas it invests $y_1^2(g^*) = 1/12$ in improving the match quality with complementor X_2. In addition, each firm earns a payoff of $(108t\gamma - 1 - \gamma)/(144\gamma)$.*

(iii) *The equilibrium network $g^* = \{1, 0, 0, 1\}$ results in a payoff for each firm smaller than that achieved when $g = \{0, 0, 0, 0\}$, even though $g^* = \{1, 0, 0, 1\}$ is socially preferred over $g = \{0, 0, 0, 0\}$.*

[12] For instance, only one system can be active when systems exhibit no horizontal differentiation at all. If forming a collaboration link involves incurring some arbitrarily small cost, then it can be shown (proof available upon request) that $g = g^2$ is the unique stable network (up to a relabeling of firms), so the network that emerges in equilibrium is asymmetric even if firms are ex ante identical.

[13] We compute the direct (profit) effect of collaboration between firms X_1 and Y_1 through the following thought experiment. Upon collaborating, both of these firms react to the change in their investment costs, taking into account the reactions of each other in an optimal manner, but keeping the investments of X_2 and Y_2 as in $g = g^1$ (i.e., X_2 and Y_2 do not react to the change in the network architecture). This yields some profit for X_1 and Y_1, which, after respectively subtracting $\Pi^*_{X_1}(g^1)$ and $\Pi^*_{Y_1}(g^1)$, gives the direct effect of collaboration. The difference between $\Pi^*_{X_1}(g^2) - \Pi^*_{X_1}(g^1)$ and the direct effect for X_1 then gives the strategic (profit) effect of collaboration for this firm, that is, how its profits change because of the reaction of X_2 and Y_2 to the collaboration between X_1 and Y_1. One can compute the strategic (profit) effect of collaboration for Y_1 in an analogous manner.

Proof. See the appendix. □

Given that the social welfare comparison is entirely driven by the increase in consumer surplus, it is worthwhile explaining why it happens. Note first that, under $g = g^3$, the investment in enhancing the match quality with a complementor with which a firm does not collaborate is the same as under $g = g^1$: $x_1^2(g) = x_2^1(g) = y_1^2(g) = y_2^1(g) = 1/12$ for $g \in \{g^1, g^3\}$. However, the investment in enhancing the match quality with a complementor with which a firm does collaborate increases relative to $g = g^1$: $x_1^1(g^3) = x_2^2(g^3) = y_1^1(g^3) = y_2^2(g^3) = 1/(12\gamma) > x_1^1(g^1) = x_2^2(g^1) = y_1^1(g^1) = y_2^2(g^1) = 1/12$. As component prices are the same under $g = g^1$ and $g = g^3$, it follows that some systems are more appealing in their vertical attributes when $g = g^3$, and hence are bought more than when $g = g^1$. However, *no* consumer is worse off under $g = g^3$ than under $g = g^1$. Those who were already consuming one of the enhanced systems are obviously better off. In turn, the new consumers attracted by any one of the enhanced systems experience greater transportation costs, but still prefer purchasing one of such systems. This revealed preference argument shows that they are better off under $g = g^3$. Finally, those consumers who were already buying one of the systems whose quality is not enhanced derive the same utility under $g = g^3$ than $g = g^1$, since prices and total investments in these systems do not change.

3.2.2. Network Structures Under Nonexclusivity. We now deal with networks in which at least one firm has more than one collaboration link. As before, in equilibrium, firms try to collaborate with as many complementors as possible, which rules out $g = g^3$ as a plausible equilibrium outcome. The underlying economic forces parallel those behind Proposition 1, with collaboration between two firms inducing positive direct and strategic effects. Thus, start from a network architecture in which X_i and Y_j $(i, j = 1, 2)$ are not linked. If these firms form a new collaboration tie, then system X_iY_j becomes stronger and $X_{3-i}Y_{3-j}$ becomes weaker, whereas neither system X_iY_{3-j} nor system $X_{3-i}Y_j$ end up having a change in their overall investment levels. The outcome of these economic forces leads to the following result.

Proposition 2. *Let $t > \hat{t}$, and suppose that collaborating with a complementor does not preclude a firm from collaborating with the complementor's competitor. Then, it holds that*:

(i) *The unique equilibrium network structure is the complete network $g^{**} = \{1, 1, 1, 1\}$.*

(ii) *In equilibrium, firm X_i invests $x_i^j(g^{**}) = 1/(12\gamma)$ in improving the match quality with complementor Y_j, whereas firm Y_j invests $y_j^i(g^{**}) = 1/(12\gamma)$ in improving match quality with complementor X_i $(i, j = 1, 2)$. Each firm earns a payoff of $(54t\gamma - 1)/(72\gamma)$.*

(iii) *The equilibrium network $g^{**} = \{1, 1, 1, 1\}$ results in a payoff for each firm smaller than that achieved when $g = \{0, 0, 0, 0\}$, even though $g^{**} = \{1, 1, 1, 1\}$ is socially preferred over $g = \{0, 0, 0, 0\}$.*

Proof. See the appendix. □

Hence, collaboration with complementors results in a prisoner's dilemma. Firms engage in a futile fight to vertically differentiate the systems in which they participate by collaborating with as many complementors as possible and by boosting investments accordingly. The larger investments increase investment costs, and the greater investment costs end up being only a wasteful rent dissipation, since nothing is gained in return, *despite the (possibly substantial) downward shift in cost functions*.[14] So it holds that the greater value creation does not translate into greater value capture because the value created relative to competitors does not change. However, all consumers greatly benefit from the greater functionality of every existing system, which explains why society would be worse off without collaborative activities involving information sharing among complementors.

To understand what drives the prisoner's dilemma result, note that the empty network's payoff structure (i.e., $\Pi^*_{X_i}(g^1) = \Pi^*_{Y_j}(g^1) = (54t - 1)/72$) corresponds to that of the complete network for $\gamma = 1$. Hence, we can parametrize the comparison between g^1 and g^6 as a reduction in the value of γ starting from $\gamma = 1$. This way of looking at the comparison between these two networks is useful because it will allow us to use the power of the envelope theorem. Thus, the fact that $\Pi^*_{X_i}(g^6) = \Pi^*_{Y_j}(g^6) = (54t\gamma - 1)/(72\gamma)$ is increasing in γ implies that lowering γ from one to any (admissible) level below one would reduce a firm's profit. In words, collaboration is harmful for firms relative to the situation in which none collaborate.

To get a more precise understanding of this result, consider what happens when all firms simultaneously start collaborating with their complementors (i.e., when γ marginally decreases in $g = g^6$ starting from $\gamma = 1$). The greater investments of each complementor in the systems in which it participates do not affect a firm's profit because these effects cancel out: *the existence of complementors is irrelevant for the result that firms prefer the empty network over the complete one*. In particular, an envelope argument shows that the result is simply driven by the positive direct effect on a firm's profit of having lower investment costs and the negative strategic effect of having the competitor

[14] Thus, if firms invested as much as under $g = g^1$, their profits would indeed augment because the cost function shifts downward. But precisely this downward shift in the cost function of a firm leads all of them to futilely invest more, thus dissipating the potential gains from the change in the cost function.

invest more in all the systems in which it participates. The latter effect dominates the former for any admissible value of $\gamma < 1$, which explains why any firm is better off when no collaboration at all takes place. In addition, the strategic effect becomes more important than the direct effect as γ decreases. As a result, a firm's preference for the empty network over the complete one is accentuated as γ is lowered, that is, as information sharing among complementors makes it cheaper to enhance the quality of the match with a complementor. Also, using Propositions 1 and 2 yields that firms are better off under exclusivity than under nonexclusivity.

Proposition 3. *Let $t > \hat{t}$. Then equilibrium payoffs decrease as γ decreases. Furthermore, the equilibrium network under formation of nonexclusive collaboration ties results in a payoff for each firm smaller than that achieved under exclusive collaboration ties.*

4. Robustness of the Baseline Model

The purpose of this section is to check the robustness of our findings. Before proceeding, we would like to discuss how our framework is related to the notion of "Bertrand supertraps" introduced by Cabral and Villas-Boas (2005). In our setting, firms end up falling into a Bertrand supertrap because of the collaboration links they endogenously form. Thus, what is new in our setup is that the change in the parameter exhibiting Bertrand supertrap features is endogenously triggered. Technically speaking, ours is not a comparative statics exercise such as the ones they consider. However, we have just showed that the result in part (iii) of Proposition 2 can be interpreted as comparative statics on γ. In light of Cabral and Villas-Boas (2005, Proposition 1), we are therefore inclined to believe that our central result is quite general (given symmetry and full market coverage) regardless of our functional form assumptions. Even though their results (based on strategic complementarities) may seem not directly applicable to our case, note that complementors' strategic reactions owing to a shift from $g = g^1$ to $g = g^6$ have no impact on a firm's payoff.[15] This neutral impact of complementors' strategic reactions on payoffs was not obvious a priori and turns out to be critical in showing that we can safely use the conceptual apparatus they develop. In fact, the profit neutrality of complementors' reactions to changes in γ can be shown to hold even when one departs from their assumption of roughly symmetric firms.

[15] Recall when shifting from $g = g^1$ to $g = g^6$ that the positive effect on X_i's payoff of larger y_1^i and y_2^i cancels out with the negative effect of equally larger y_1^{3-i} and y_2^{3-i}. So the strategic substitutabilities that complementors bring in are entirely offset by the strategic complementarities they bring in too.

Besides the findings by Cabral and Villas-Boas (2005) reinforcing the robustness of our results, there are many other changes in the baseline model with no substantial effects. Our results that any firm collaborates with as many complementors as possible and that this leads to a prisoner's dilemma still hold in different scenarios. In the first place, they hold if firm X_1 is initially stronger than X_2 along some dimension such as production cost or quality: in such a case, the effect of the prisoner's dilemma is more intense for firm X_2 than for X_1. In the second place, they continue to hold if the R&D investments by producers of X are more effective than those undertaken by their complementors (namely, if $v_{ij} = v + x_i^j + \phi y_j^i$ for some parameter $\phi \in (0, 1)$); when this occurs, the effect of the prisoner's dilemma is more intense for the producers of X because their complementors free ride on their investment efforts. In the third place, our results persist if there is one more producer of Y; in such a case, the effect of the prisoner's dilemma is more intense for the firms operating in the component market that happens to be more concentrated, namely, X. This is an intuitive result insofar as strategic effects are more intense the more concentrated a market is. In the last place, our results hold if collaboration between any two complementors implies that each cares to some extent about the other's payoff when making investment choices (but not when setting prices); in such a case, collaboration is socially undesirable, though. For the sake of space constraints, all the proofs can be found in the online appendix. (See Mantovani and Ruiz-Aliseda 2011 for the last extension.)

5. Escaping from the Prisoner's Dilemma

In light of the robustness of our findings, it is important to understand why firms may not be trapped in a prisoner's dilemma, since it will give us a platform for drawing managerial implications in the next section. The two aspects that constitute the object of this section are the existence of market expansion effects and the possibility that each firm can unilaterally refuse compatibility with any complementor's component.

5.1. Market Expansion Effects

A substantive limitation of the baseline spokes model has to do with the absence of market expansion effects implied by the full market coverage assumption, an assumption that may overemphasize competitive pressure. In markets not yet saturated, one may be inclined to think that results might differ from those derived in the previous sections.

To show the impact of market expansion effects, we depart from the spokes model by considering a model with a single/representative consumer whose utility function is

$$U = m + \sum_{i=1}^{2}\sum_{j=1}^{2} v_{ij}Q_{ij} - \frac{1}{2}\left(\sum_{i=1}^{2}\sum_{j=1}^{2} Q_{ij}^2\right) - \theta\left(\prod_{j=1}^{2}\left(\sum_{i=1}^{2} Q_{ij}\right) + \sum_{j=1}^{2}\left(\prod_{i=1}^{2} Q_{ij}\right)\right),$$

where m is the consumption of a *numéraire* good, and $\theta \in (0,1)$ indicates the inverse of the degree of horizontal differentiation between any two systems. Given a wealth $W > 0$ for the consumer, maximizing her utility subject to the budget constraint results in the following demand functions:

$$Q_{ij} = \left[(1+2\theta)(v_{i,j} - p_{i,j}) - \theta(v_{i,3-j} + v_{3-i,j} + v_{3-i,3-j} - p_{i,3-j} - p_{3-i,j} - p_{3-i,3-j})\right]/((1-\theta)(1+3\theta)) \quad (i,j=1,2).$$

The third stage now displays both strategic complementarities (with respect to the competitor's price) and strategic substitutabilities (with respect to the complementors' prices). Solving for the Nash equilibrium, third-stage profits for $i, j = 1, 2$ equal

$$\pi^*_{X_i} = \frac{(1+\theta)^2}{32(3-\theta)^2(1-\theta)(1+2\theta)^2(1+3\theta)} \times \left[(2 + x_i^1 + x_i^2 + y_1^i + y_2^i)(5+10\theta-7\theta^2) - (2 + x_{3-i}^1 + x_{3-i}^2 + y_1^{3-i} + y_2^{3-i})(1+6\theta+\theta^2)\right]^2$$

and

$$\pi^*_{Y_j} = \frac{(1+\theta)^2}{32(3-\theta)^2(1-\theta)(1+2\theta)^2(1+3\theta)} \times \left[(2 + x_1^j + x_2^j + y_j^1 + y_j^2)(5+10\theta-7\theta^2) - (2 + x_1^{3-j} + x_2^{3-j} + y_{3-j}^1 + y_{3-j}^2)(1+6\theta+\theta^2)\right]^2,$$

where we have let $v = 1$ to simplify matters.[16] It is easy to demonstrate that a symmetric increase of investment levels by all firms will increase third-stage profits, unlike the spokes model, although the increase in profits will be smaller the higher θ is.

The full resolution of this model (see the online appendix) is extremely lengthy and algebraically complex, but one can show that all firms choose to form a collaboration tie with their complementors for all admissible values of θ and γ. As in the spokes model, several forces are simultaneously at work. For example, if no firms collaborate with one another and X_1 and Y_1 form a new link, the cost synergy leads them to increase x_1^1 and y_1^1, which creates pressure toward increasing x_1^2 and y_1^2. These effects cancel out for X_2 and Y_2, so the strengthening of X_1Y_1 induces them to respectively lower x_2^1 and y_2^1. However, differently from the spokes model, the increases in x_1^2 and y_1^2 are respectively much stronger than the falls in y_2^1 and x_2^1, so $x_1^2 + y_2^1$ and $x_2^1 + y_1^2$ end up increasing. In fact, even though X_1Y_1 is stronger, it holds that both systems X_1Y_2 and X_2Y_1 are favorably affected by the new collaboration link. Facing stronger competing systems, X_2Y_2 is obviously weaker, and hence X_2 and Y_2 end up reducing x_2^2 and y_2^2, respectively. As in the spokes model, X_2 finds it optimal to reduce x_2^1 and x_2^2 by the same amount (and analogously for Y_2 with y_2^1 and y_2^2). In short, even though the mechanism differs somewhat from that in the spokes model, the result is the same: the new collaboration tie has a positive direct effect, and it also elicits an overall positive strategic reaction from the firms not involved in the tie.

Once we know that the complete network is the only stable one, intuition about how market expansion effects may change results with respect to the spokes model becomes more transparent. With this objective in mind, note that we can again parametrize the comparison between the complete network and the empty network through changes in γ. Starting from a value of $\gamma = 1$ (which captures the empty network), lowering γ has a positive direct effect on a firm's payoff because of the cost synergies. Moreover, it also allows the firm to enjoy the beneficial effects deriving from complementors' strategic reactions: unlike the spokes model, these effects do *not* cancel out now and, in fact, become more important as the intensity of systems competition (measured through θ) diminishes.[17] As in the spokes model, though, the firm's competitor also invests more, which gives rise to a negative strategic effect whose strength diminishes as θ falls and systems become more horizontally differentiated. As a result, when θ is relatively low, the competitor's strategic effect induced by lowering γ is very weak, and

[16] This assumption is without loss of generality, since v can be shown to simply scale up second-stage equilibrium profits. Hence, v does not affect the comparison across different network structures.

[17] To show this, focus for instance on firm X_i, and, taking into account that $\partial y_j^i/\partial\gamma = \partial y_j^{3-i}/\partial\gamma > 0$, note that $(\partial\pi^*_{X_i}/\partial y_j^i)(\partial y_j^i/\partial\gamma) + (\partial\pi^*_{X_i}/\partial y_j^{3-i})(\partial y_j^{3-i}/\partial\gamma)$ evaluated at the equilibrium investment levels for $g = g^6$ ($x_i^j(g^6) = y_j^i(g^6) = (5+\theta[15+\theta(3-7\theta)])/(2\{2\gamma(3-\theta)^2[1+\theta(5+6\theta)] - (1+\theta)[5+\theta(10-7\theta)]\})$ for all $i, j = 1, 2$) is equal to

$$\frac{\partial y_j^i}{\partial\gamma}\left(\frac{2(1-\theta^2)(2x_i^j(g^6)+1)}{(3-\theta)^2(1+3\theta)}\right) = \frac{\partial y_j^i}{\partial\gamma}\left(\frac{4\gamma(1-\theta^2)(1+2\theta)}{2\gamma(3-\theta)^2(1+\theta(5+6\theta)) - (1+\theta)(5+\theta(10-7\theta))}\right) > 0.$$

This shows that X_i benefits from the complementors' greater R&D investments, which contrasts with the profit neutrality of complementors' reactions to changes in γ in the baseline model (and in the extensions).

such an effect cannot dominate the positive effects of smaller γ. However, when θ is relatively high, the competitor's strategic effect is very strong and dominates the positive effects, as in the spokes model. Investing in enhancing match quality is costly but hardly enhances third-stage profits when the environment is rather competitive.

The main result with market expansion effects is hence the following: the prisoner's dilemma arises if and only if rents accruing from investments in better match quality are competed away because systems are not differentiated enough.

Proposition 4. *Suppose that parameters are such that investment levels, sales, and profits are all nonnegative in equilibrium. Then, it holds that*:

(i) *The unique equilibrium network structure is the complete network, namely,* $g^{**} = \{1, 1, 1, 1\}$, *which is always socially preferred over* $g = \{0, 0, 0, 0\}$.

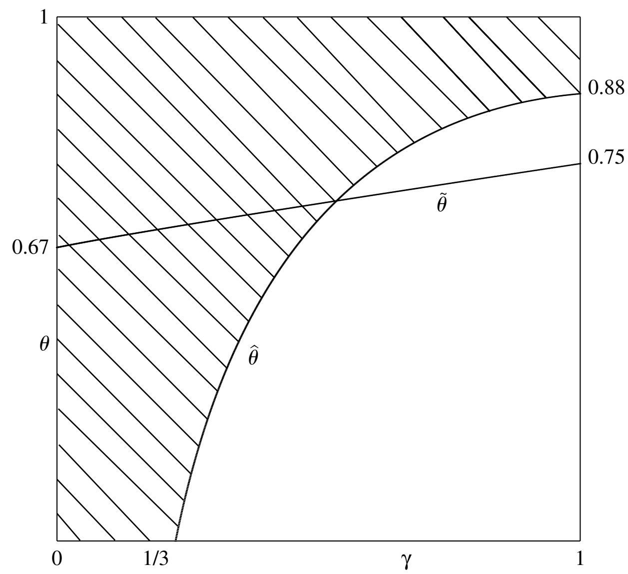
(ii) *The payoff achieved by a firm under* $g^{**} = \{1, 1, 1, 1\}$ *is lower than that achieved under* $g = \{0, 0, 0, 0\}$ *if in addition it holds that* $\theta > 3/4$.

The proposition (part (ii)) is not stated in the more general form to convey the message more easily. The point is that the value of γ may have an influence on how far θ needs to be from one for the prisoner's dilemma to disappear. Graphically, Figure 3 shows the region in the (γ, θ)-space where the prisoner's dilemma arises (see the region above the $\tilde{\theta}$-curve); outside this region (i.e., for θ low enough), firms manage to achieve their jointly preferred outcome through full collaboration. Notice that there exists an equilibrium with all systems active regardless of the network configuration only outside the shaded area in the figure (i.e., for $\theta < \hat{\theta}$, the equivalent to $t > \hat{t}$ in the spokes model).

5.2. Incompatibility and Market Foreclosure

To conclude with our formal analyses, we note that there are instances in which the lack of collaboration between two complementors results in a system being unavailable. Our aim now is to model (pairwise) collaboration among complementors as sharing of information that is essential for them to properly work (e.g., application program interfaces in hardware–software industries). To this end, we proceed to modifying our baseline spokes model by assuming that the network formation stage is followed by the price competition stage (so there is no investment stage). It also holds that $g_{ij} = v$ if firms X_i and Y_j collaborate with each other, and $g_{ij} = 0$ otherwise. The idea is that system X_iY_j is not available for consumers if X_i and Y_j do not make their complements compatible by exchanging essential information. Contrary to what was assumed before, interoperability is then a

Figure 3 **Prisoner's Dilemma**



joint decision, not a unilateral one. The proofs of this section can be found in the online appendix.

We shall also assume for simplicity that there is always a unit mass of consumers who have preferences over available systems and that a consumer's transportation cost does not depend on how many systems are available. If there is just one system active in the market, we will assume that it is located at the end of a Hotelling segment. If there is an extra system, then it will be located at the other end of this segment. In case of either three or four systems, we will use the appropriate variant of the spokes model employed earlier. Because we want to compare the various situations depending on how many systems are active, we will assume throughout that $v \in [15t/4, 35t/8]$ (a necessary and sufficient condition if the market is to be covered for any network structure).

It is laborious but relatively straightforward to prove that the equilibrium profits for firms in each network structure are as follows: $\pi^*_{X_i}(g^1) = \pi^*_{Y_j}(g^1) = 0$ for $i, j = 1, 2$; $\pi^*_{X_1}(g^2) = \pi^*_{Y_1}(g^2) = (v-t)/2$ and $\pi^*_{X_2}(g^2) = \pi^*_{Y_2}(g^2) = 0$; $\pi^*_{X_i}(g^3) = \pi^*_{Y_j}(g^3) = t/2$; $\pi^*_{X_1}(g^4) = v - 3t/2$, $\pi^*_{X_2}(g^4) = 0$ and $\pi^*_{Y_1}(g^4) = \pi^*_{Y_2}(g^4) = t$; $\pi^*_{X_1}(g^5) = \pi^*_{Y_2}(g^5) = 49t/48$ and $\pi^*_{X_2}(g^5) = \pi^*_{Y_1}(g^5) = 25t/48$; and $\pi^*_{X_i}(g^6) = \pi^*_{Y_j}(g^6) = 3t/4$ for $i, j = 1, 2$.

Although we omit the details, it is easy to demonstrate that, given this payoff structure, the unique pairwise stable network is the complete one, which again is not jointly optimal (since $g = g^4$ maximizes industry profits).[18] Even though this result reinforces our previous one, we have now solid motives to

[18] The same holds if one looks at coalition-proof Nash equilibria, as Jeon and Menicucci (2011) have recently done, instead of looking at pairwise stable equilibria.

depart from pairwise stability as the equilibrium notion so as to allow for explicit transfers between complementors. There are at least two reasons why such transfers may be more prominent when a firm can unilaterally refuse compatibility. First, in most occasions it is easier to contract on interoperability than on investments in enhancing it when it is impossible to refuse it. Second, compatibility can radically change the structure of the components' markets because a firm can be partly or fully excluded, and it seems desirable to consider the incentives of a firm to exclude another one.

Modeling equilibrium network formation under multilateral bargaining is not an easy task (suffering from an ad hoc character), so we will content ourselves with the following sequential game for the negotiations stage. In the first round of play, one of the four firms is randomly chosen, where each firm has the same probability of being chosen. Such firm then makes a simultaneous take-it-or-leave-it offer to each of its complementors, where an offer can be contingent upon demanding exclusivity or not. Once complementors have accepted/refused the offer, the game proceeds without the firm that has made the offer and without any complementor that may have accepted an offer demanding exclusivity. In the second round of play, any one of the firms that remain active in the game has equal probability of being selected to make a take-it-or-leave-it offer to any complementor left, where again an offer can be contingent upon demanding exclusivity or not. Once complementors have accepted/refused the offer, the game proceeds without the firms that have made the offer and without any firm that may have accepted any such offer demanding exclusivity. In the third round, the game ends if there are no complementors with which any of the remaining firms can bargain; otherwise, any of the two remaining firms is chosen with probability one-half to make a take-it-or-leave-it offer to the other firm that remains, and the game ends once this offer is accepted or rejected.

In this multilateral bargaining game, all parties have some bargaining power, with subsequent threat points endogenously depending on past negotiation outcomes. Intuitively, getting a complementor to sign an exclusive contract in the first round should strike a balance between two aspects: the compensation for the bargaining power possibly given up by the complementor in potential future negotiations and the provision of some safeguard against adverse positions in future bargaining situations.[19] In turn, getting a complementor to sign a nonexclusive contract should allow the firm making the first-round offer to demand a pretty high transfer because the future bargaining position of the complementor is enhanced regardless of whether it makes an offer in the future or actually receives it. However, offering nonexclusive contracts weakens the strategic positioning of the firm making the first-round offer when competing in the product market.

In what follows, we will look at subgame perfect equilibria of the game just described. We will show that the greater surplus that can be extracted through nonexclusive contracts relative to exclusive contracts is smaller than the profit lost in the marketplace because of the more intense competition. This will explain why excluding a rival from the market arises as the equilibrium outcome of the bargaining game. To show this result, let us suppose that firm X_i is chosen to make the offers in the first round. In particular, let us examine what happens if it makes exclusive offers to both Y_j and Y_{3-j}:

- If both accept an offer demanding exclusivity, it holds that $\Pi_{X_i} = v - 3t/2$ and $\Pi_{Y_j}^{++} = \Pi_{Y_{3-j}}^{++} = t$ (the payoffs corresponding to network structure $g = g^4$).
- If Y_j, say, refuses (but Y_{3-j} accepts), it will make a take-it-or-leave-it offer to X_{3-i} with probability one-half, but it will receive a take-it-or-leave-it offer from X_{3-i} with complementary probability. Their collaboration is efficient for both, so it follows that X_{3-i} and Y_j will collaborate regardless of which firm makes the offer. With probability one-half, Y_j will get all the surplus created by their collaboration, which is equal to $\pi^*_{X_{3-i}}(g^3) + \pi^*_{Y_j}(g^3) = t$, but Y_j will get nothing with complementary probability. So firm Y_j's expected profit equals $\Pi_{Y_j}^{+-} = t/2$. Similarly, firm Y_{3-j}'s expected profit if it is the only firm to refuse firm X_i's offer equals $\Pi_{Y_{3-j}}^{+-} = t/2$.
- Firm X_i then chooses to demand a transfer T (which may be negative) such that its total payoff $v - 3t/2 + 2T$ is maximized subject to the constraint that each of the complementors finds it optimal to accept the exclusive contract given that the other one is expected to accept (i.e., $\Pi_{Y_j}^{++} - T \geq \Pi_{Y_j}^{+-}$).[20] It then

[19] As we shall see, the latter of these two effects dominates when both complementors are offered an exclusive contract. In the case in which one, and only one, is offered an exclusive contract, the "insurance" aspect of the problem will again be the one that determines why the firm receiving the exclusive contract offer must pay to sign such a contract. We remark that the aspect of compensating for bargaining power lost is crucial for results to be robust. If firm X_1 were chosen to make the first-round offer and Y_j and Y_{3-j} could never make take-it-or-leave-it offers in the second round of play, only the "insurance" aspect would play a role, and X_i could appropriate the maximal industry profit by offering an exclusive contract to each complementor in which X_i demands a transfer that fully extracts rents and on top of that excludes X_{3-i}.

[20] To avoid uninteresting equilibria, we implicitly assume throughout that there are no coordination problems among complementors when deciding on whether or not to accept an offer, and hence firm X_i can induce the outcome it desires.

follows that the optimal exclusive contract involves requiring a transfer of $t/2$ from each complementor, and firm X_i earns a total payoff equal to $\Pi^*_{X_i} = v - t/2$. The payoffs for firms X_{3-i}, Y_j, and Y_{3-j} are $\Pi^*_{X_{3-i}} = 0$ and $\Pi^*_{Y_j} = \Pi^*_{Y_{3-j}} = t/2$.

The profit attained by firm X_i, $\Pi^*_{X_i} = v - t/2$, can be shown to exceed the joint profit under the network $g \neq g^4$ that maximizes total profits, namely, $g = g^5$. This total profit of $\sum_{i=1}^{2} \pi^*_{X_i}(g^5) + \sum_{j=1}^{2} \pi^*_{Y_j}(g^5) = 37t/12$ provides an upper bound on rent acquisition by X_i if it does not pursue network architecture $g = g^4$, so the assumption that $v \in [15t/4, 35t/8]$ yields that $\Pi^*_{X_i} > 37t/12$. Therefore, X_i should offer an exclusive contract to both Y_j and Y_{3-j} if there exists an equilibrium for this bargaining game for the parameter values considered, something that can be always shown to hold (see the online appendix).

As a result, we have that the payoffs for firms X_i, X_{3-i}, Y_j, and Y_{3-j} are $\Pi^*_{X_i} = v - t/2$, $\Pi^*_{X_{3-i}} = 0$, and $\Pi^*_{Y_j} = \Pi^*_{Y_{3-j}} = t/2$ whenever firm X_i is chosen to make the first offer. At the beginning of the game, the assumption that each firm has the same probability of being chosen to make such an offer implies that any firm expects to make the following payoff: $\Pi_{X_i} = \Pi_{X_{3-i}} = \Pi_{Y_j} = \Pi_{Y_{3-j}} = (2v + t)/8$. Regardless of which firm makes the first offer, the outcome is that one of them always excludes its rival from the market by obtaining exclusivity from any available complementor. Such a dominant firm is also compensated by any of the complementors from which it demands exclusivity (so the dominant firm ends up operating a two-sided platform). The point is that the dominant firm anticipates that any complementor refusing its offer would be left in a difficult bilateral bargaining position given that its component would remain incompatible with that of the firm whose offer was refused. Note that the result we are to state now contrasts with the no equilibrium foreclosure findings of Hermalin and Katz (2013) when one of the components is a commodity, an environment that we do not contemplate.

Proposition 5. *When firms can unilaterally refuse interoperability with any complementor's component, any equilibrium of the bargaining game results in a firm getting its two complementors to sign exclusive contracts. Such a firm earns $v - t/2$, whereas its competitor, excluded from the market, earns 0. Complementors earn $t/2$. In equilibrium, the network structure that emerges, $g = g^4$, is the jointly optimal network.*

6. Managerial Implications and Concluding Remarks

Essentially all the literature on innovation ecosystems has entirely focused on the "coopetitive" relationship of a single firm with its (potential) complementors (e.g., Gawer and Cusumano 2002, Yoffie and Kwak 2006, Adner 2012). The insights offered are very useful whenever this single firm has a strong dominant position, as when switching costs or network effects are present (e.g., Microsoft's Windows), or as happens with technological pioneers or disruptive entrants (e.g., when the iPhone was introduced). Our goal in this paper was to extend these well-developed frameworks to situations in which the focal firm has lost (or simply does not have) such a market dominance or lead-time advantage. In reality, firms that interact with complementors often face competition from producers of substitute goods. Early industry examples are given by the VCR and computer industries, whereas a more recent one is given by the smartphone industry, in which Samsung has quickly caught up with Apple.

In these more competitive scenarios, the main message of this paper is that a firm should have an integral view of its innovation efforts when building the ecosystem for its product(s). In particular, the firm should be aware that the firm's competitors may be driven by the same incentives. A firm will strive to become a "platform leader" (as defined by Gawer and Cusumano 2002), but its competitors will certainly aim at platform leadership as well.[21] Such competition for leadership can well be very destructive, as we have tried to argue.[22] In our case, all firms have "platform potential" (Gawer and Cusumano 2002), and they destroy rents in their quest to realize it: no firm emerges as a platform and the extensive collaboration they foster ends up backfiring. As far as we are aware, this is a novel aspect to the discourse on the formation of innovation ecosystems.

Our baseline model and its extensions give a lens through which to assess some managerial indications that might prove useful in situations close to monopoly, but that might actually backfire under competition. For instance, Gawer and Cusumano (2014, p. 429) draw on the case of Intel to offer some prescriptions for becoming a platform leader. Among other things, the addition of connectors and interfaces so that other firms can build on the firm's component is one of the critical issues they advocate for.

[21] In our setting, each of the component providers has the potential to become a platform leader, but none of them has been exogenously given this role. In the baseline model, none realizes such potential, unlike the in model in which component providers can unilaterally refuse compatibility and sign (possibly exclusive) contracts with complementors. In such cases, a system's component may endogenously become a (two-sided) platform that makes money on both complementors and consumers.

[22] Adner (2012) examines the contrast between ecosystem leadership strategies and ecosystem followership strategies. Whereas his focus is on initial choices and coordination, our focus is on equilibrium profits and welfare.

A related one is that this be followed by the sharing of the intellectual property of these connectors to reduce complementors' costs to connect to the platform and thus incentivize complementary innovation. Our baseline model, based on the satisfaction of the former premise, shows that the latter prescription may be detrimental if rivals end up doing the same.

As the results in §5 suggest, an initially closed specification for interfaces, accompanied by exclusive contracting to open them up, may lead to the endogenous emergence of a platform product that may well gather all complementors (and their investments). Accounting for the (in)compatibility of components, an endogenous feature that was treated as exogenous in our framework, may be one way out of the prisoner's dilemma. Indeed, many of the current collaboration activities have to do with firms agreeing to make their components compatible with each other. However, a firm's ability to refuse compatibility with a complementary component opens the door to other issues such as multiplicity of (highly) asymmetric equilibria even if firms are ex ante identical. These insights reinforce the message from the two-sided platform competition literature that, contrary to compatibility, incompatibility may well result in market dominance, but that multiple equilibria exist when platforms are incompatible.[23] Our paper suggests that this is a robust feature even if there are no network effects. Although quite challenging, it would be interesting to extend our work to endogenize the degree of interface openness for each component before collaboration links are formed.

The smartphone industry following Apple's entry offers a good example of a gradual increase in the degree of connectedness between leading smartphone producers and mobile network operators. This has fostered extensive collaboration and, as a result, greater investments with the aim of differentiating the various systems (both vertically and horizontally). Thus, Apple's iPhone was initially operational in the United States exclusively on AT&T's wireless network, but the fourth generation was already available with many other carriers—e.g., the largest one, Verizon—that also commercialized (some variant of) the Samsung Galaxy S, as did AT&T. Interestingly, operators have been quite eager to differentiate systems from a horizontal point of view, which may be another way to escape the prisoner's dilemma in light of §5's results (even if formally analyzing the incentive to differentiate is an aspect beyond the scope of the current paper). For instance, the Verizon iPhone 4 offered unlimited data and personal hotspot tethering (in addition to a bigger and more reliable network), but AT&T offered a wider international roaming service or allowed users to browse the Web while simultaneously talking on their iPhone 4. Similarly, AT&T's Samsung Captivate is somewhat different from Verizon's Samsung Fascinate, even though the differences are probably not as pronounced as with the iPhone because Samsung has been less willing to customize its hardware.[24] In any case, with the market reaching saturation in this context of extensive collaboration all across the board, our results suggest that profitability should eventually diminish.[25]

We conclude this paper by mentioning that our prisoner's dilemma results suggest that firms might have a strategic incentive to integrate with complementors (even leaving foreclosure aside).[26] Cisco has pursued this practice extensively for decades for a number of reasons, and it might be worthwhile inspecting it more closely in light of our findings.

Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/mnsc.2014.2140.

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[23] See Casadesus-Masanell and Ruiz-Aliseda (2009).

[24] In fact, the Samsung experience suggests that there might be a trade-off between the extent of horizontal differentiation of the systems in which it participates and the commitment not to engage in system-specific enhancements whose rents may be competed away.

[25] This seems to be happening already (see *Agence France-Presse News* 2014), but it is an empirical matter to separate out the direct effect of market saturation from the indirect effect that was the object of analysis of our baseline model.

[26] Economides and Salop (1992) perform a related analysis of the strategic effect of unilateral integration by X_1 and Y_1 on product market competition given that X_2 and Y_2 remain unintegrated. One could therefore build on their work and ours to address this question, which is suggested by Yoffie and Kwak (2006) for other reasons.

Appendix

Proof of Proposition 1. We first consider equilibrium payoffs under network $g = g^1 \equiv \{0,0,0,0\}$. It is easy to show that the unique equilibrium is symmetric, and it is characterized by each firm investing $x_i^j(g^1) = y_j^i(g^1) = 1/12$ $(i,j = 1,2)$ in trying to (unilaterally) improve the match with each complementary component. Equilibrium profits for each firm under $g = g^1$ are

$$\Pi^*_{X_i}(g^1) = \Pi^*_{Y_j}(g^1) = (54t - 1)/72,$$

which are positive since $t > \hat{t} > 1/54$ (recall that $\hat{t} \equiv (3 - 2\gamma + \sqrt{4\gamma^2 - 6\gamma + 3})/(54\gamma)$).

We now turn to the case in which there is just one collaboration link, i.e., $g = g^2 \equiv \{1,0,0,0\}$. It holds that the unique equilibrium is characterized by the following investments in match quality: $x_1^1(g^2) = y_1^1(g^2) = (27t-1)/6(54t\gamma - 1 - \gamma)$, $x_1^2(g^2) = y_1^2(g^2) = \gamma(27t-1)/6(54t\gamma - 1 - \gamma)$, and $x_2^1(g^2) = x_2^2(g^2) = y_2^1(g^2) = y_2^2(g^2) = (27t\gamma - 1)/6(54t\gamma - 1 - \gamma)$. Equilibrium profits under $g = g^2$ are then

$$\Pi^*_{X_1}(g^2) = \Pi^*_{Y_1}(g^2) = \frac{\gamma(108t\gamma - 1 - \gamma)(27t-1)^2}{36(54t\gamma - 1 - \gamma)^2}$$

and

$$\Pi^*_{X_2}(g^2) = \Pi^*_{Y_2}(g^2) = \frac{(54t-1)(27t\gamma - 1)^2}{18(54t\gamma - 1 - \gamma)^2}.$$

All profits are positive for $t > \hat{t}$, as can be easily demonstrated. Such parametric assumption also ensures that equilibrium quantities of each system are positive, as required by the full market coverage assumption we have made. Noting that $g^2 = g^1 + g_{11}$, it holds that $\Pi^*_{X_1}(g^2) = \Pi^*_{Y_1}(g^2) > \Pi^*_{Y_1}(g^1) = \Pi^*_{X_1}(g^1)$ for $t > \hat{t}$, and firms X_1 and Y_1 would mutually benefit from forming a link with each other, so $g = g^1$ cannot be a stable network.

However, network $g = g^2 \equiv \{1,0,0,0\}$ cannot arise in equilibrium either. To show this, consider $g = g^3 \equiv \{1,0,0,1\}$. When $t > \hat{t}$, the unique equilibrium is symmetric, being characterized by $x_1^1(g^3) = x_2^2(g^3) = y_1^1(g^3) = y_2^2(g^3) = 1/(12\gamma)$ and $x_1^2(g^3) = x_2^1(g^3) = y_1^2(g^3) = y_2^1(g^3) = 1/12$. Equilibrium profits under $g = g^3$ are

$$\Pi^*_{X_i}(g^3) = \Pi^*_{Y_j}(g^3) = (108t\gamma - 1 - \gamma)/(144\gamma),$$

which are positive for $t > \hat{t}$. Noting that $g^3 = g^2 + g_{22}$, it holds that $\Pi^*_{X_2}(g^3) = \Pi^*_{Y_2}(g^3) > \Pi^*_{Y_2}(g^2) = \Pi^*_{X_2}(g^2)$ for $t > \hat{t}$, and firms X_2 and Y_2 would mutually benefit from forming a link with each other, so $g = g^2$ cannot be a stable network.

The unique stable network that arises when collaboration is exclusive is $g = g^3$. It remains to show that profits decrease but social welfare increases relative to the empty network. Both for $g = g^1$ and $g = g^3$, it holds that $p^*_{X_1} = p^*_{X_2} = p^*_{Y_1} = p^*_{Y_2} = 3t/2$, so $p^*_{11} = p^*_{12} = p^*_{21} = p^*_{21} = 3t$. In addition, the number of consumers purchasing system X_iY_j $(i,j = 1,2)$ under $g = g^1$ is $Q^*_{ij}(g^1) = 1/4$. However, under $g = g^3$, the number of consumers purchasing the four systems is $Q^*_{11}(g^3) = Q^*_{22}(g^3) = \frac{1}{4} + (1-\gamma)/(36t\gamma)$ and $Q^*_{12}(g^3) = Q^*_{21}(g^3) = \frac{1}{4} - (1-\gamma)/(36t\gamma)$. Taking into account that $l_{X_iY_j}$ $(i,j = 1,2)$ has a length of 1/2 and that that there exists a unit mass of consumers uniformly spread all over the four existing lines, the aggregate consumer surplus under $g = g^1$ is

$$CS(g^1) = 4\left[\frac{1}{2}\left(v + \frac{1}{12} + \frac{1}{12} - 3t - t\int_0^{1/2} z\,dz\right)\right],$$

while the aggregate consumer surplus under $g = g^3$ is

$$\begin{aligned} CS(g^3) &= 2\left[\frac{1}{2}\left(v + \frac{1}{12\gamma} + \frac{1}{12\gamma} - 3t - t\int_0^{1/2+(1-\gamma)/(18t\gamma)} z\,dz\right)\right] \\ &\quad + 2\left[\frac{1}{2}\left(v + \frac{1}{12} + \frac{1}{12} - 3t - t\int_0^{1/2-(1-\gamma)/(18t\gamma)} z\,dz\right)\right]. \end{aligned}$$

The assumption that $t > \hat{t}$ yields $t > 1/(27\gamma)$, so $54t\gamma > 27t\gamma > 1 > \gamma$ implies that

$$CS(g^3) - CS(g^1) = (1-\gamma)(54t\gamma + \gamma - 1)/(324t\gamma^2) > 0$$

and

$$\begin{aligned} &\sum_{i=1}^{2}[\Pi^*_{X_i}(g^3) - \Pi^*_{X_i}(g^1)] + \sum_{j=1}^{2}[\Pi^*_{Y_j}(g^3) - \Pi^*_{Y_j}(g^1)] \\ &\quad = -(1-\gamma)/(36\gamma) < 0, \end{aligned}$$

so it follows from $45t\gamma > 27t\gamma > 1$ that social surplus increases by $((1-\gamma)(45t\gamma + \gamma - 1))/(324t\gamma^2) > 0$ when going from $g = g^1$ to $g = g^3$ even though firms are worse off. □

Proof of Proposition 2. Assume that $t > \hat{t}$, which ensures the strict concavity of payoffs and the nonnegativity of equilibrium profits, investment levels, and quantities sold in the following networks.

First consider $g = g^4 \equiv \{1,1,0,0\}$. It is straightforward to prove that $x_1^1(g^4) = x_1^2(g^4) = (54t\gamma + 1 - 3\gamma)/(12\gamma(54t\gamma - 1 - \gamma))$, $x_2^1(g^4) = x_2^2(g^4) = (54t\gamma - 3 + \gamma)/(12\gamma(54t\gamma - 1 - \gamma))$, $y_1^2(g^4) = y_2^2(g^4) = \frac{1}{12}$, and $y_1^1(g^4) = y_2^1(g^4) = 1/12\gamma$. Equilibrium profits are

$$\Pi^*_{X_1}(g^4) = \frac{(54t\gamma - 1)(54t\gamma + 1 - 3\gamma)^2}{72\gamma(54t\gamma - 1 - \gamma)^2},$$

$$\Pi^*_{X_2}(g^4) = \frac{(54t\gamma - 1)(54t\gamma - 3 + \gamma)^2}{72\gamma(54t\gamma - 1 - \gamma)^2},$$

and

$$\Pi^*_{Y_j}(g^4) = \frac{108t\gamma - 1 - \gamma}{144\gamma}, \quad j = 1, 2.$$

We analyze now the cases in which $g = g^5 \equiv \{1,1,0,1\}$. Solving for an equilibrium yields $x_1^1(g^5) = x_1^2(g^5) = y_1^1(g^5) = y_2^2(g^5) = y_2^2(g^5) = (27t - 1)/6(54t\gamma - 1 - \gamma)$, $x_2^1(g^5) = y_1^2(g^5) = (27t\gamma - 1)/6(54t\gamma - 1 - \gamma)$, and $x_2^2(g^5) = y_2^1(g^5) = (27t\gamma - 1)/(6\gamma(54t\gamma - 1 - \gamma))$. As for profits, they are

$$\Pi^*_{X_1}(g^5) = \Pi^*_{Y_2}(g^5) = \frac{\gamma(54t\gamma - 1)(27t - 1)^2}{18(54t\gamma - 1 - \gamma)^2}$$

and

$$\Pi^*_{X_2}(g^5) = \Pi^*_{Y_1}(g^5) = \frac{(108t\gamma - 1 - \gamma)(27t\gamma - 1)^2}{36\gamma(54t\gamma - 1 - \gamma)^2}.$$

Noting that $g^5 = g^3 + g_{12}$, it holds that $\Pi^*_{X_1}(g^5) = \Pi^*_{Y_2}(g^5) > \Pi^*_{Y_2}(g^3) = \Pi^*_{X_1}(g^3)$ for $t > \hat{t}$, and it follows that firms X_1 and Y_2 would mutually benefit from forming a link with each other, so $g = g^3$ cannot be a stable network. Network $g = g^4$ can also be discarded as an equilibrium outcome. To show this, note that $g^5 = g^4 + g_{22}$, so the fact that $\Pi^*_{X_2}(g^5) > \Pi^*_{X_2}(g^4)$ and $\Pi^*_{Y_2}(g^5) > \Pi^*_{Y_2}(g^4)$ for $t > \hat{t}$ implies that firms X_2 and Y_2 would mutually benefit from forming a link with each other.

We deal now with the final network that needs to be considered, namely, $g = g^6 \equiv \{1, 1, 1, 1\}$. The unique equilibrium is symmetric and involves the following investment levels: $x_i^j(g^6) = y_j^i(g^6) = 1/(12\gamma)$ $(i, j = 1, 2)$. Equilibrium profits under $g = g^6$ are

$$\Pi^*_{X_i}(g^6) = \Pi^*_{Y_j}(g^6) = (54t\gamma - 1)/(72\gamma),$$

which are positive for $t > 1/(54\gamma)$.

We can rule out $g = g^5$ as an equilibrium network. Noting that $g^6 = g^5 + g_{21}$, it holds that $\Pi^*_{X_2}(g^6) > \Pi^*_{X_2}(g^5)$ and $\Pi^*_{Y_1}(g^6) > \Pi^*_{Y_1}(g^5)$ for $t > \hat{t}$. Firms X_2 and Y_1 would mutually benefit from forming a link with each other, and hence $g = g^5$ cannot be a stable network. The fact that $\Pi^*_{X_i}(g^6) > \Pi^*_{X_i}(g^5)$ for $i = 1, 2$ and $\Pi^*_{Y_j}(g^6) > \Pi^*_{Y_j}(g^5)$ for $j = 1, 2$ implies that $g = g^6$ is the equilibrium network for $t > \hat{t}$, which proves parts (i) and (ii).

To examine the efficiency properties of the equilibrium network in the absence of exclusivity constraints and thus prove (iii), note that it holds both for $g = g^1$ and $g = g^6$ that $p^*_{X_1} = p^*_{X_2} = p^*_{Y_1} = p^*_{Y_2} = 3t/2$, so $p^*_{11} = p^*_{12} = p^*_{21} = p^*_{21} = 3t$ and $Q^*_{ij} = 1/4$ for $i, j = 1, 2$. Therefore, the aggregate consumer surplus under $g = g^1$ is

$$CS(g^1) = 4\left[\frac{1}{2}\left(v + \frac{1}{12} + \frac{1}{12} - 3t - t\int_0^{1/2} z\,dz\right)\right],$$

whereas the aggregate consumer surplus under $g = g^6$ is

$$CS(g^6) = 4\left[\frac{1}{2}\left(v + \frac{1}{12\gamma} + \frac{1}{12\gamma} - 3t - t\int_0^{1/2} z\,dz\right)\right].$$

It then holds that $CS(g^6) - CS(g^1) = (1 - \gamma)/(3\gamma) > 0$. Because

$$\sum_{i=1}^{2}[\Pi^*_{X_i}(g^6) - \Pi^*_{X_i}(g^1)] + \sum_{j=1}^{2}[\Pi^*_{Y_j}(g^6) - \Pi^*_{Y_j}(g^1)]$$
$$= -(1-\gamma)/(18\gamma) < 0,$$

it follows that social welfare increases by $5(1-\gamma)/(18\gamma) > 0$ when going from $g = g^1$ to $g = g^6$ even though firms are worse off than when collaboration is forbidden. □

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