Management Science

Managing Retention in Service Relationships

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In a repeat business context, past experiences with a service provider affect customers' decisions to renew their contract. How should a strategic firm manage customized service over time to maximize the long-term value from each customer relationship? We propose a dynamic model that relies on behavioral theories and empirical evidence to capture the effect of past service experiences on service quality expectations, customer satisfaction, and retention. Although firms can benefit from managing service expectations at the beginning of a relationship, we find that varying service in the long run is not optimal. Behavioral regularities explain the structure of optimal service policies and limit the value of responsive service. Loss aversion expands the range of optimal constant policies; however, if satisfying experiences are more salient, then firms should constantly vary service levels. Loyal or high-margin customers need not warrant better service; those who anchor less on past service experiences do—provided that retention is improved by better past experiences. The effect of customer memory on service levels is determined by whether habituation or rather goodwill drives defection decisions.

1. Introduction

The growth of the service sector has brought about a paradigm shift from managing transactions to managing customer relationships, with an increasing focus on retention as a driver of profitability (Gupta and Zeithaml 2006). In both business-to-consumer (B2C) and business-to-business (B2B) sectors, evidence suggests that a customer's assessment of the value of the relationship and subsequent repatronage decisions are critically influenced by the dynamics of service experiences with the firm (Bolton et al. 2006). Hence, the question for firms is how service can be managed over time to capture the most value from each customer relationship.

DeVine and Gilson (2010) pose the problem as follows: "Every year, companies have thousands, even millions, of interactions with human beings— also known as customers. Their perceptions of an interaction...are influenced by...its sequence of painful and pleasurable experiences. Companies care deeply about the quality of those interactions.... Yet the application of behavioral science to service operations seems spotty at best." We respond to this need by proposing a behavioral model for managing customized service over time to maximize the expected long-term profit from a firm's customer base. In each period, the provider decides how much

effort, or service, to invest in the customer relationship. Improving service is costly, but it increases a customer's immediate and overall satisfaction, her perception of service quality (or goodwill), and hence the probability that she will renew the contract. The distinctive premise of our model is that customers' *memory* of past service experiences affects repurchase decisions. We rely on empirical evidence and behavioral decision theories—such as adaptive expectations and prospect theory—to model realistic effects of service experiences on customers' service quality expectations and renewal decisions. In a context where firms can manage service as a retention driver, we ask two broad questions: (1) When does it make more sense for firms to vary service than to maintain it at a constant level? (2) Which aspects of consumer behavior are critical in determining the firm's decisions— in particular, how do consumer memory and renewal decision processes affect the optimal service policy and customer lifetime value (CLV)?

Customer retention is a primary concern in mature markets, where it delivers substantially higher returns on investment than acquisition (Bolton and Tarasi 2006). In such markets, our focus is on managing retention via responsive service in contractual settings where departures are observable (as when, e.g., the customer cancels a contract or fails to renew a

subscription).[1] We define responsive service broadly as the noncontractible level of effort that the firm expends on retaining an individual customer; this includes sales-force effort, number of contact hours, response time, value-added services, direct targeted emails, promotions, and so forth.[2]

Evidence from a variety of industries has shown that service effort can be used to influence retention. In high-technology markets, for example, Bolton et al. (2006) find that the sequence of service experiences—as measured by average engineer work-minutes per contract—is a managerially actionable service operations measure that can predict renewal rates of support service contracts. DeVine and Gilson (2010) describe how an insurance provider managed a schedule of targeted phone calls from nurses to individual patients to improve client satisfaction and retention. Toward that same end, Survival Chic, a lifestyle membership, deploys targeted emails with weekly information and access to special deals and events, as does DBS, a retail bank. Television stations and Internet publishers control service quality, and hence advertiser retention, by airing additional ads (make-goods) for client campaigns (Araman and Popescu 2010). Customer churn is a major concern for telecom and Internet providers, who use promotion, call center effort (Sun and Li 2011), and, more recently, differentiated service quality (at the IP-address level, thanks to Cisco's "service control" technology) to retain subscribers.

As firms increase their capabilities to manage targeted services, our goal as researchers is understanding when and how they should do so proactively, to fuel retention and maximize CLV. Despite existing capabilities, firms are often loath to adjust service, for fear of failing to meet customers' expectations (Entel 2007). In seeking to establish whether such fears are justified, we develop a stylized dynamic model and derive structural results that explain how behavioral processes and characteristics (e.g., customer loyalty, memory, expectations) affect firm policies and profits.

A key insight from our research is that *behavioral asymmetries*—in customer memory or decision processes—*determine the long-run structure of service policies*. Even though managing service quality

expectations benefits the firm at the beginning of a relationship, we show that varying service is not optimal in the long run. The reason is not that customers dislike variability per se, but rather that they are (weakly) loss averse—that is, they anchor more on negative than positive experiences. This fundamental aspect of consumer behavior is consistent with prospect theory and empirical evidence (Tversky and Kahneman 1991, Bolton et al. 2006). We show that loss aversion reduces CLV and leads to a range of optimal constant policies, but if the asymmetry is reversed, i.e., consumers are delight seeking, then the service policy oscillates. In other words, *loss aversion supports the prevalence of constant service policies*. It can also explain why a firm may be impelled to consistently deliver the service expected by loss-averse customers.

Absent behavioral asymmetries, prior expectations do not bias long-run service levels; however, those levels will be affected by the extent and mechanism through which expectations influence customers' decisions to defect. Indeed, we shall demonstrate that the weight customers place on past experiences has systematic effects on service levels and customer value. The direction of these memory effects is determined by whether goodwill or habituation is a stronger driver of defection—a testable dichotomy. Specifically, anchoring on past experiences results in lower service levels, assuming that a better service history reduces churn; this assumption is consistent with goodwill models (Nerlove and Arrow 1962) and ample empirical evidence (Zeithaml 2000). However, the effect is reversed if clients who are accustomed to good service become more difficult to satisfy and retain, as hypothesized by habituation models (Baucells and Sarin 2010).

Finally, as a by-product of endogenizing the effect of service quality on repurchase decisions, our results also explain why higher-CLV customers need not warrant better service. Neither do higher-margin customers or those who are more loyal—such relationships are typically unimodal, unlike conventional wisdom may suggest (see, e.g., Gupta and Zeithaml 2006).

With the aim of providing structural, analytical insights for this complex problem, we develop in §3 a parsimonious model of optimizing service dynamics in response to realistic customer behavior. Section 4 characterizes the optimal long-run and transient policy for exponentially smooth memories. General adaptation and behavioral asymmetries, such as loss aversion, are studied in §5. Section 6 presents sensitivity insights. Section 7 presents an alternative decision model where retention is not driven by goodwill, but instead by habituation. Our results are summarized in §8. All proofs are given in the appendix, which also includes extensions that reinforce our main insights and illustrate the versatility of our framework.

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[1] Examples include subscription and membership services, insurance, banking, and utilities contracts in B2C contexts, as well as maintenance, support, and advertising contracts in B2B. By contrast, in noncontractual settings (e.g., retail or airline travel) the focus is on customer share as opposed to defections (see Fader and Hardie 2009).

[2] We focus on managing service because it is typically a more effective driver of customer retention than are prices, which are often fixed for a variety of reasons (see, e.g., Liu et al. 2007, Pfeifer and Ovchinnikov 2011). Our results can be extended to manage prices, instead of service levels.

2. Related Literature

Our work draws on established behavioral decision theories and dynamic optimization techniques to address a fundamental question that interfaces the literatures on service operations and marketing: how to proactively manage service effort over time when doing so affects retention.

In the service operations literature, several authors endogenize the effect of past service experiences on demand. One class of papers studies static service policies in oligopolies where consumers do not know each firm's predetermined service quality but learn about it from experience, either adaptively (Gaur and Park 2007) or in a Bayesian fashion (Gans 2003). In contrast, our customers form adaptive expectations as the firm strategically varies service. Hall and Porteus (2000) dynamically optimize service over a finite horizon in a duopoly, where market shares depend on current "service failures," determined by each firm's investment in capacity. The customers in their setting are purely reactive in their switching behavior; unlike ours, they have no memory of past experiences. A similar assumption is made in Olsen and Parker (2008), who study the optimality of base-stock policies when customers defect upon facing a stockout but can be reacquired via advertising. These papers focus on aggregate market size dynamics between competitive firms. In contrast, we capture boundedly rational individual behavior in a contractual (e.g., subscription) context where customers defect for good, and competition is implicit in their choice to do so.

Our work belongs to a growing behavioral literature on dynamic models where demand evolves adaptively as firms strategically vary prices (Popescu and Wu 2007), capacity (Liu and van Ryzin 2011), or quality decisions (Caulkins et al. 2006). Liu et al. (2007) study the allocation of an exhaustible resource (sales-force effort) over a fixed customer lifetime when immediate satisfaction relative to prior service experiences (i.e., disconfirmation) affects short-term profit but not retention. In parallel work, Adelman and Mersereau (2013) use approximation techniques to allocate capacity among clients whose stochastic demands depend on goodwill derived from past fill rates. Retention considerations are absent from these models, as customers never defect.

Managing customer relationships is a fundamental concern in the marketing field, which largely focuses on customer classification and associated metrics, such as CLV (Bolton and Tarasi 2006). Few papers consider optimal investment decisions toward maximizing CLV. Ho et al. (2006) derive static service policies when customer purchases follow a memoryless (Poisson) process whose rate depends on a customer's immediate satisfaction; the probability p that a customer is satisfied in a given period is controlled by the firm. Pfeifer and Ovchinnikov (2011) optimize static spending on acquisition and retention when only a limited number of customers can be served; we do not consider capacity constraints. These papers, unlike ours, do not capture dynamic effects of past firm policies on customer behavior. Exceptions are Lewis (2005) and Sun and Li (2011), who estimate such dynamics empirically and then numerically optimize prices and call-center allocation decisions, respectively. In contrast, we rely on behavioral decision theories (rather than data) to set up a stylized, nonparametric model, and then we derive structural insights analytically.

In sum, a combination of features distinguishes our work from the preceding literature. First, we specifically model customer defection as an observable and controllable probabilistic construct in contractual settings. In doing so, we draw on behavioral theories—such as prospect theory and adaptive expectations—to capture realistic memory effects of past service experiences on customer defections decisions. Finally, we derive structural analytical insights and go beyond steady-state analysis to investigate the endogenous dynamics of service policies and their sensitivity to behavioral factors such as consumer memory and loss aversion. The model we propose thus addresses a dual need to "incorporate findings from psychology and marketing into OM" [operations management] models of service management (Bitran et al. 2008, p. 80) and to develop "dynamic models [of CLV]…to reflect the evolution of customer preferences and behaviors over time, so that the path-dependent nature of organizational decisions is explicitly recognized" (Bolton and Tarasi 2006, pp. 17–18; see also Rust and Chung 2006, p. 575).

3. The Model

A profit-maximizing firm decides in each period t what level of noncontractible service (or retention effort) $x_t^i \in [0, 1]$ to offer each active customer $i \in \{1, \dots, N\}$.[3] Upon receiving service level x_t^i, a customer may decide to terminate the relationship (e.g., by canceling or not renewing the contract), and this observable decision depends on her history of service experiences $X_t^i = (x_0^i, \dots, x_t^i)$, as well as other customer- or contract-specific factors and uncertain events. A customer who defects is "lost for good," an assumption typical of contractual settings, in contrast with noncontractual settings where "always-a-share" models are the norm (Fader and Hardie 2009).

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[3] Measuring service in percentage terms captures a variety of service decisions, including the binary (service, no service) models typically used in the literature; there, x represents the fraction of time that (good) service is offered within a given time span, such as the fraction of mailers sent to a customer over a contractual period.

By dynamically managing service levels for each customer, x_t^i, the firm's objective is to maximize the expected long-term value from its customer base. This value is given by

$$\lim_{T \to \infty} \sum_{t=0}^{T} \beta^t \sum_{i=1}^{N} \mathbb{P}(\text{customer } i \text{ is active at time } t \mid X_{t-1}^i)$$
$$\cdot \, \pi_i(x_t^i; X_{t-1}^i), \tag{1}$$

where $\beta \in (0, 1)$ is the firm's discount factor and π_i its short-term profit. Motivated by the examples in §1, the cost of responsive service is assumed to be separable across customers—a common (albeit strong) assumption, as Pfeifer and Ovchinnikov (2011) explain. Absent pooling effects, problem (1) is separable at the customer level; hence we can omit the customer index i from notation and focus on managing individual relationships—that is, maximizing the CLV from each customer.

3.1. Customer Behavior

This section establishes a general parsimonious model for the customer's decision to quit the relationship at the end of period t, as a function of her service history X_t. Specifically, we posit that, upon receiving service level x_t, an active customer revisits the firm in the next period with probability $R(X_t) = R(x_t, s_t)$, where the "memory" construct s_t summarizes her overall evaluation of past service experiences X_{t-1}. We next describe stylized models of consumer memory and decision processes, which motivate our customer retention model.

3.1.1. Customer Memory Process and Service Quality. Consistent with empirical findings (e.g., Boulding et al. 1993), we assume that customers adaptively form an overall evaluation of the service history X_t, represented by a one-dimensional recursive *service quality* construct:

$$s_{t+1} = \Lambda(x_t, s_t) = \hat{\Lambda}(x_t - s_t, s_t). \tag{2}$$

Although assuming a single sufficient statistic for consumer memory is restrictive, models such as (2) capture critical memory features, including recency and interaction effects, as well as nonlinearities and biases in perception. They also capture differences in overall evaluation, depending on customers' immediate satisfaction, as measured by the *disconfirmation* gap, $x_t - s_t$, between service and expectations. In particular, we allow asymmetries in service quality perception, notably *loss aversion*—i.e., that dissatisfying experiences $(x_t < s_t)$ are more salient than satisfying ones $(x_t > s_t)$—as motivated by prospect theory (Tversky and Kahneman 1991). Specifically, we study

a loss-averse adaptation model similar to Gaur and Park (2007):

$$s_{t+1} = \Lambda(x_t, s_t) = \begin{cases} s_t + (1 - \lambda)(x_t - s_t) & \text{if } x_t \geq s_t, \\ s_t + \rho(1 - \lambda)(x_t - s_t) & \text{if } x_t < s_t, \end{cases} \tag{3}$$

where $\rho > 1$ captures loss aversion. Exponential smoothing, $s_{t+1} = \lambda s_t + (1 - \lambda)x_t$, obtains for $\rho = 1$. The memory parameter $\lambda \in [0, 1)$ is the weight that the customer puts on prior experiences. Customers with lower λ focus more on recent experiences; in the limit, if $\lambda = 0$, then loss-neutral $(\rho = 1)$ customers are memoryless; i.e., their decisions are not affected by service history. This exponential smoothing model captures the essence of theoretical models of belief formation (Hogarth and Einhorn 1992), service quality expectations (Cronin and Taylor 1992), and economic goodwill (Nerlove and Arrow 1962). Such models have been tested empirically in a service context (Boulding et al. 1993, Bolton et al. 2006) and used extensively to capture memory effects in operational models (e.g., Caulkins et al. 2006, Liu et al. 2007, Adelman and Mersereau 2013). In addition to being theoretically and empirically supported, we will show that model (3), simple as it is, delivers robust insights.

3.1.2. Customer Renewal Decision Process. A random utility model, albeit not necessary for our setup, is typically used to capture renewal decisions (e.g., Lewis 2005, Bolton et al. 2006); thus, at time t, the customer's utility from the relationship with the firm is given by

$$\tilde{u}_t = u_t(X_t; I) + \tilde{\varepsilon}_t = u(x_t, s_t; I) + \tilde{\varepsilon}_t. \tag{4}$$

Here, u_t is a deterministic value function of the service history X_t. The information vector of parameters, I, captures individual characteristics (e.g., loyalty, memory, initial expectations) as well as factors that are contract, firm, or industry specific (e.g., price P, switching costs). Additional random factors and events that affect the customer's decision are summarized in the random component $\tilde{\varepsilon}_t$, assumed independent and identically distributed (i.i.d.) with a general distribution. The probability that an active customer with service history X_{t-1}, summarized by s_t, renews after receiving service level x_t is then $R(x_t, s_t; I) = R_t(X_t; I) = \mathbb{P}(\tilde{u}_t \geq 0) = \mathbb{P}(\tilde{\varepsilon} \geq -u(x_t, s_t; I))$.[4] Competing hypotheses exist on whether past experiences, summarized by s_t, have a positive or negative effect on a customer's probability to return. These motivate

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[4] Such models can be estimated from longitudinal data and can be extended to account for customer inertia and choice among competing offers. In this case $R_t(X_t; I) = \mathbb{P}(\tilde{u}_t \geq \tau + \tilde{v}_t) = \mathbb{P}(\tilde{\varepsilon} - \tilde{\epsilon} \geq v - u(x_t, s_t; I) + \tau)$, where $v_t = v + \tilde{\epsilon}_t$ is the i.i.d. utility from the competitive offer and τ is a parameter reflecting consumer inertia.

us to study two alternative consumer decision models (goodwill versus habituation) typically used in the literature.

Goodwill models of retention assume that service quality is the main driver of customer's decision utility—that is, $u_t(X_t) = u(\Lambda(x_t, s_t)) = u(s_{t+1})$, and so the repurchase probability $R(x_t, s_t) = R(s_{t+1})$ is increasing in s_{t+1}. The implication, widely supported in the literature (see Zeithaml 2000), is that *customers who have had better cumulative experiences with the firm are less likely to defect*. This assumption is consistent with goodwill models (Nerlove and Arrow 1962) as well as the adaptive expectations framework (Cronin and Taylor 1992), under which previous service experiences create endogenous "will" expectations that evidence a positive effect on retention (Boulding et al. 1993).[5]

Habituation models, by contrast, posit a reference-dependent decision utility, $u_t(X_t) = u(x_t - s_t)$, where the disconfirmation gap $x_t - s_t$ drives renewal decisions. Thus, the repurchase probability $R(x_t, s_t) = R(x_t - s_t)$ increases with $x_t - s_t$. Such models imply that *customers who have received better service quality in the past become more difficult to satisfy and retain*, consistent with habituation and satisfaction treadmill effects (Brickman and Campbell 1971, Baucells and Sarin 2010).

In sum, the probability that an active customer renews after being served at time t is either $R(s_{t+1})$ for goodwill models or $R(x_t - s_t)$ for habituation models. Retention functions $R(\cdot)$ commonly estimated in the literature include logit (Rust et al. 2004), exponential (Berger and Nasr 1998), and double-exponential functions (Bolton et al. 2006). We make no parametric or shape assumption on the function R (or on the underlying utility u) other than that it is continuously increasing in its argument. In other words, "happier" customers—i.e., those with higher goodwill or higher immediate satisfaction (disconfirmation), depending on the hypothesized model—are more likely to renew.

We start our analysis by focusing on the goodwill hypothesis, given its wide empirical support, and then extend the results to habituation models in §7, where we also unify the goodwill and habituation hypotheses under a general reference-dependent model $R(x_t, s_t)$.

3.2. The Firm's Dynamic Optimization Problem
The maximum expected long-term value (or CLV) from an active customer—given her history of service

experiences X_{t-1} summarized by the state s_t—can be obtained recursively by solving the following dynamic program:

$$V(s_t) = \max_{\{x_t\}} \Bigg\{ \pi(x_t; s_t)$$
$$+ \sum_{l=1}^{\infty} \beta^l \left(\prod_{k=0}^{l-1} R(x_{t+k}, s_{t+k}) \right) \pi(x_{t+l}; s_{t+l}) \Bigg\}$$
$$= \max_{x_t \in [0, 1]} \{ \pi(x_t; s_t) + \beta R(x_t, s_t) V(s_{t+1}) \}. \quad (5)$$

Here, for all t, $s_{t+1} = \Lambda(x_t, s_t)$, and s_0 denotes consumer's initial expectation, or goodwill stock.[6] This is a discounted stochastic shortest-path model with bounded profit per stage and state-dependent transition probability $R(x_t, s_t)$. Thus, $V(\cdot)$ is bounded and uniquely defined by the Bellman equation (5), which admits an optimal stationary policy $x^*(\cdot)$ (Bertsekas and Shreve 1996).[7]

The main trade-off in (5) is between the short-term cost of providing high service and the long-term benefit of increasing retention and future revenue streams. To isolate the effect of past service experiences on *retention*, which is our distinctive focus, our base model ignores volume effects; that is, we suppose $\pi(x_t; s_t) \equiv \pi(x_t)$, assumed to be strictly concave. This assumption is consistent with need-based services (e.g., maintenance, insurance, utilities) and all-inclusive subscriptions (e.g., broadband, lifestyle memberships); for example, $\pi(x) = P - c(x)$, where P is the subscription price and $c(x)$ is the convex cost of service. Our insights extend after accounting for the positive effect of past service experiences on current customer spending, visit frequency, and referrals, as well as for Bayesian learning about customer characteristics I (see Appendix A).

4. Goodwill Models Under Exponential Smoothing
The next three sections study model (5) under the goodwill hypothesis $R(x, s) = R(\Lambda(x, s))$, starting here with an exponentially smoothed memory (3) with $\rho = 1$, i.e., $\Lambda(x, s) = \lambda x + (1 - \lambda)s$:

$$V(s) = \max_{x \in [0, 1]} \{ \pi(x) + \beta R(\lambda s + (1 - \lambda)x)$$
$$\cdot V(\lambda s + (1 - \lambda)x) \}. \quad (6)$$

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[5] These authors distinguish between *will* and *should* expectations, and their respective effects on retention; the former are based on the customer's previous experiences with the firm and typically dominate the latter, which are based on exogenous factors, such as industry standards, competitive offers, or the firm's advertising. Our focus is on service expectations that are based on prior experiences with the firm (as summarized by s_t) and hence on *will* expectations.

[6] For an existing customer, s_0 summarizes the service quality delivered so far and can be estimated from longitudinal data; for new customers, s_0 represents an initial stock of goodwill, as for example, from the firm's reputation. In marketing terms, s_0 reflects will expectations (Boulding et al. 1993).

[7] When multiple solutions exist, we define $x^*(\cdot)$ as the largest maximizer of (5). Our results extend without discounting $\beta = 1$, if $R(\lambda) < 1$ in which case the policy $x \equiv 0$ is proper.

We seek to understand whether pathwise, conditional on the customer renewing, optimal service levels converge in the long run. If so, they must converge to a steady state. We say that s^{**} is a *steady state* if the firm has no incentive to deviate from this service level as long as the customer is active—i.e., if s^{**} is a fixed point of the optimal service policy $x^*(s^{**}) = s^{**}$. In other words, if the customer is accustomed to service quality s^{**}, then it is optimal for the firm to offer her $x \equiv s^{**}$ in each period until the customer defects; this occurs with probability one if the policy is proper.

4.1. Steady State

To characterize a steady state of problem (6), we first define the "memoryless" CLV from offering constant service x in each period to a memoryless ($\lambda = 0$) customer:

$$\Pi(x) = \sum_{t=0}^{\infty} \beta^t R^t(x) \pi(x) = \frac{\pi(x)}{1 - \beta R(x)} = \pi(x) L(x). \quad (7)$$

For customers who do not remember past service, setting $\lambda = 0$ in (6) confirms that past service does not affect their optimal value: $V(s; \lambda = 0) \equiv V = \max\{\pi(x) + \beta R(x) V\} = \max_x \Pi(x)$. It is therefore optimal to offer these customers a static (memoryless) service level $x_t \equiv \arg\max_{x \in [0,1]} \Pi(x) = \bar{s}$. This exceeds the myopic service $\underline{s} = \arg\max_{x \in [0,1]} \pi(x) \leq \bar{s}$; $\underline{s}$ is unique because π is strictly concave. We assume that Π is strictly quasi-concave on $[0,1]$, and so is the following function:[8]

$$W(x) = \lambda \pi(x) + (1 - \lambda) \Pi(x). \quad (8)$$

Our main result in this section shows that, in steady state, a firm that endogenizes the effects of consumer memory on retention (by solving the dynamic program (6)) behaves *as if* it was maximizing the simple static objective W, i.e., the memory (λ)-weighted average of short-term profit π and memoryless lifetime value Π. In particular, for firms to be compelled to maintain 100% long-run service levels (as exemplified in Jones and Sasser 1995), W must increase on $[0,1]$. Similarly, W must be decreasing for a firm to stop investing in a customer ($s^{**} = 0$) in the long run; this may be because she is too costly to maintain or she dwells too heavily on past experiences.

PROPOSITION 1. (i) *Customer value $V(s)$ is increasing, and $\Pi(s) \leq V(s) \leq \pi(\underline{s})/(1 - \beta)$, for $s \in [0,1]$. (ii) Problem (6) admits a unique steady state s^{**}, which maximizes* $W(s)$ *over $s \in [0,1]$. Furthermore, $\underline{s} \leq s^{**} \leq \bar{s}$ and s^{**} is decreasing in λ and increasing in β.*

Proposition 1 is useful for relating the firm's long-run service policy and profit to customer- and contract-specific factors; we do so in §6. A firm with a short-term outlook puts less weight β on future cash flows (e.g., $\beta = 0$ for a fully myopic firm) and thus provides lower long-run service because it focuses on (short-term) cost savings. In particular, the steady-state service level exceeds the myopic benchmark: $\underline{s} \leq s^{**}$, but it is lower than the static service level offered to memoryless customers $\bar{s} \geq s^{**}$. By leveraging memory effects, firms offer *lower service* levels as consumers anchor (more) on past experiences; i.e., $s^{**}(\lambda)$ decreases in λ. This result is driven by the fact that past experiences affect retention, and it persists as long as this effect is positive (see §7).

4.2. Global Stability

The previous subsection characterized the steady-state service level but did not indicate how the firm might reach it over time. The following result establishes global stability of the unique steady state determined by Proposition 1, and it also characterizes the structure of the transient policy. Define the optimal service quality (or state) policy as $s^*(s) = \lambda s + (1 - \lambda) x^*(s)$, where $x^*(\cdot)$ solves (6). All convergence results are stated pathwise, conditional on the customer being active.

PROPOSITION 2. *The optimal service quality policy $s^*(\cdot)$ is increasing. For any initial expectations s_0, the corresponding optimal path $\{s_t^*\}$ converges monotonically to the unique steady state s^{**} characterized by Proposition 1, and the optimal service path $\{x_t^*\}$ also converges to s^{**}.*

Proposition 2 shows that all service policies converge in the long run to the unique steady state s^{**}, regardless of customers' prior expectations s_0. The optimal way of managing service induces a monotone service quality path, as illustrated in Figure 1(a). If customers have relatively low initial perceptions of service quality s_0, then our model prescribes gradually increasing their expectations to s^{**}; the opposite holds if customers' initial expectations are too high.[9] Initially, the firm may be imperfectly informed about customer expectations s_0; this may affect monotonicity but not stability results (see Appendix A). Whereas the service quality policy s^* is increasing, the optimal service policy x^* need not be, as illustrated in Figure 1(b); §6 explains when this is the case.

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[8] The assumption on W only needs to hold on $[\underline{s}, \bar{s}]$, thus ensuring that the interior steady state is unique. This holds, e.g., if Π is concave or if $\pi'(x)/L'(x)$ is strictly decreasing in $x \in [\underline{s}, \bar{s}]$. The latter assumption holds for subscription services $\pi(x) = P - c(x)$, under common parametric retention models used in the literature, such as logistic $R(x) = 1/(1 + \exp(-\alpha x))$ or exponential $R(x) = 1 - \exp(-\alpha x)$ models, and for power cost $c(x) = x^{1+\theta}$ with $\theta \geq \alpha$.

[9] If, for strategic reasons, a firm commits to not decrease service quality, one can show that any $s_0 \geq s^{**}$ is a steady state; for $s_0 < s^{**}$, the policy remains as prescribed in Proposition 2.

Figure 1   Optimal Service and State Paths (a), Optimal Policies (b), and Customer Lifetime Value (c)
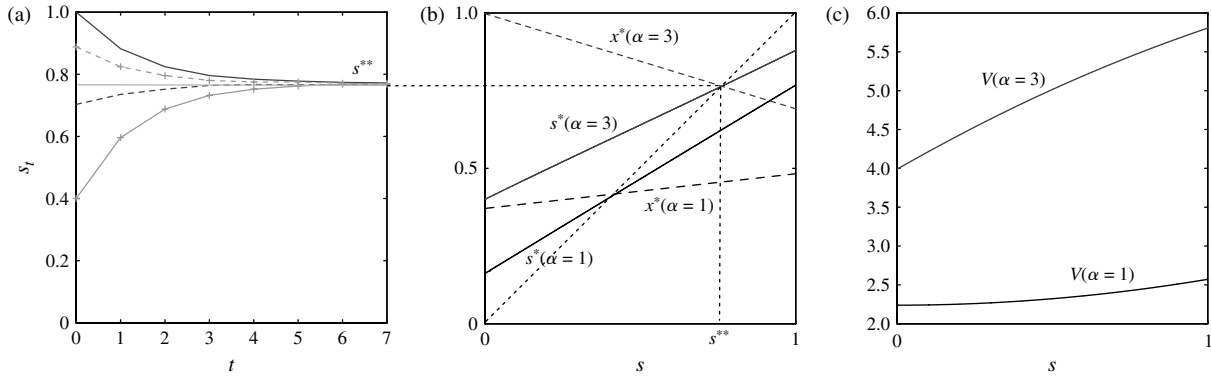
Note. $\pi(x, P) = Pd(x, P) - c(x)$, $d(x, P) = 0.5x - P + 2$, $c(x) = x^2$, $R(s) = 1/(1 + \exp(\tau - \alpha s + P))$, $\tau = 1$, $\lambda = 0.6$, $\beta = 0.94$, and $P = 1$.

5.   Goodwill Models with Asymmetric and Nonlinear Memory

In this section we investigate, along lines that are consistent with prospect theory and empirical evidence, the effect of behavioral asymmetries on adaptation processes. We show that these asymmetries have important structural effects on the firm's policy: (i) loss aversion leads to a range of steady states, which depend on initial expectations; and (ii) no steady state exists if the asymmetry is reversed, i.e., if consumers are gain seeking. These insights hold under general adaptation models.

5.1.   Behavioral Asymmetries: Loss Aversion

Prospect theory postulates that decision makers code new information as gains or losses relative to a status quo, a principle that applies to both decision and experienced (or remembered) utility (Kahneman et al. 1997). Further, a negative change from the status quo (perceived loss) has a larger effect on value than a positive change (perceived gain) of the same magnitude. Such "loss aversion" has vast empirical support in the service context—not only in B2C but also in B2B markets (Bolton et al. 2006).

The Bellman equation for asymmetric (kinked) adaptation processes (3) is

$$V^K(s_t) = \max_{x_t \in [0, 1]} \{\pi(x_t) + \beta R(s_{t+1}) V^K(s_{t+1})\} \quad (9)$$

s.t. $s_{t+1} = s_t + (1 - \lambda)(x_t - s_t)^+ + \rho(1 - \lambda)(x_t - s_t)^-$.   (10)

Denote by V^G and V^L the values of the smooth problems (6) corresponding to $\lambda_G = \lambda$ and $\lambda_L = 1 - \rho(1 - \lambda)$, respectively, and let $x_G^*(\cdot)$ and $x_L^*(\cdot)$ be their optimal policies. Loss aversion, $\rho > 1$—or equivalently, $\lambda_L < \lambda_G$—implies that the respective steady states satisfy $s_L^{**} = s_L^{**}(\rho) = s^{**}(\lambda_L) > s^{**}(\lambda_G) = s_G^{**}$ (see Proposition 1).

PROPOSITION 3.   *Assume that customers are loss averse, $\rho > 1$. (i) Then problem (9) admits a range of steady states*

$[s_G^{**}, s_L^{**}]$; *that is, starting from any $s_0 \in [s_G^{**}, s_L^{**}]$, a constant service path $x_t^{K*} \equiv s_0$ is optimal, and $V^K(s_0) = \Pi(s_0)$. (ii) For $s_0 > s_L^{**}$, the state path $\{s_t^{K*}\}$ decreases to s_L^{**}, following the optimal policy for the smooth V^L model $x^{K*}(s_0) = x_L^*(s_0)$, and $V^K(s_0) = V^L(s_0)$. Similarly, for $s_0 < s_G^{**}$, $\{s_t^{K*}\}$ increases to s_G^{**}, $x^{K*}(s_0) = x_G^*(s_0)$, and $V^K(s_0) = V^G(s_0)$. In particular, the optimal service path $\{x_t^{K*}\}$ converges to the same steady state as the corresponding state path.*

Adding loss aversion to the model leads to a range of steady states $[s_G^{**}, s_L^{**}]$ (see Figure 2(a)). This range becomes wider (upward) the more loss averse consumers are; indeed, $s_L^{**} = s_L^{**}(\rho)$ increases with the loss aversion coefficient $\rho \geq 1$, with $s_L^{**}(1) = s_G^{**}$, by Proposition 1. In other words, loss aversion increases the prevalence of constant service policies and improves service quality. Intuitively, firms have less leverage to improve perception when customers anchor more strongly on negative experiences.[10]

Unlike the smooth adaptation case ($\rho = 1$) studied in Proposition 2, when $\rho > 1$, the transient policy converges to a steady state that does depend on initial customer expectations. The higher these expectations, the higher the customer's value and the service she gets in the long run. In particular, for a range of initial expectations $s_0 \in [s_G^{**}, s_L^{**}]$, the firm will find it too risky to manage expectations by varying service over time, and that is because of customers' loss aversion. For the same reason, firms become more cautious not to disappoint high-expectation (henceforth, "high-CLV") customers and hence maintain their service quality above $s_L^{**}(\rho)$. In sum, loss aversion can justify why, even in the long run, some firms feel compelled to deliver service levels that customers were trained to expect; such practice would be suboptimal, if not for the behavioral asymmetry.

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[10] In the extreme case when $\lambda_L = 0$, $\lambda_G = 1$, we have $s_{t+1} = \min\{s_t, x_t\} = \min\{X_t\}$; i.e., customers anchor on the lowest service experience. In this case, every service level $s \in [\underline{s}, \bar{s}]$ is a steady state.

Figure 2 Optimal Service Quality Paths for (a) Loss-Averse ($\rho = 2$) Consumers and (b) Gain-Seeking ($\rho = 0.5$) Consumers
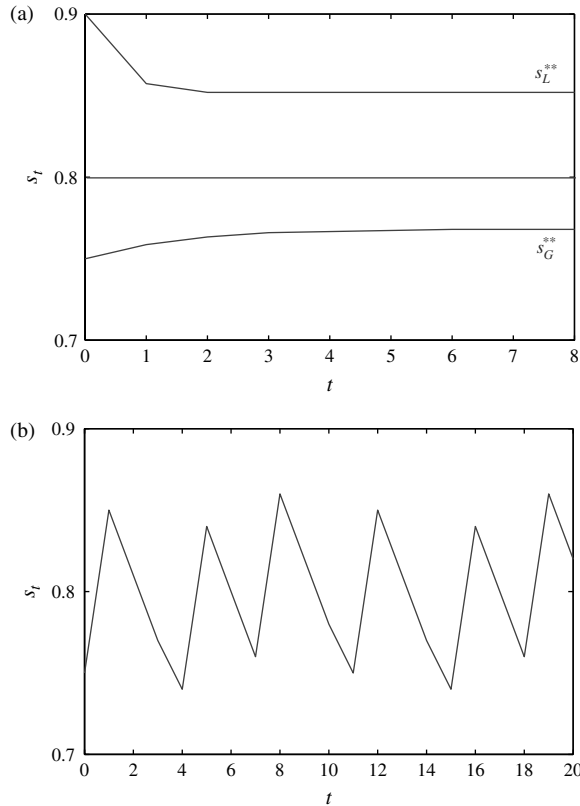
Note. $\pi(x, P) = Pd(x, P) - c(x)$, $d(x) = 0.5x - P + 2$, $c(x) = x^2$, $R(s) = 1/(1 + \exp(-3s + P))$, $P = 1$, $\lambda = 0.6$, and $\beta = 0.94$.

5.2. Delight-Seeking Consumers

For completeness, we briefly consider the case where customers are *gain* (or *delight*) *seeking*; i.e., $\rho < 1$. This means that service experiences above expectations (positive disconfirmation) are more salient than those below expectations (negative disconfirmation). Bolton et al. (2000) find evidence that members of loyalty rewards programs tend to discount or overlook negative service experiences. In this case, we find it interesting that no interior steady state exists; that is to say, any optimal service path oscillates (see Figure 2(b)). Under a high–low policy, the firm benefits in the long run by manipulating delight-seeking customers' expectations. Here, the benefits to the firm are attributable to the positive net effect of first increasing service and then decreasing it.

PROPOSITION 4. *If $\rho < 1$, then problem (9) admits no interior steady state.*

Together, Propositions 3 and 4 suggest that behavioral asymmetries explain the long-run structure of service policies, i.e., whether or not firms should vary service in the long run. As illustrated in Figure 2, constant long-run policies prevail if, and only if, consumers are (weakly) loss averse. This behavioral

regularity offers a reason why firms heed to customers' service expectations.

5.3. Robustness: General Memory Models

We argue in this section that our main insights so far, under a (piecewise) linear memory model (3), extend under a general memory process $s_{t+1} = \Lambda(x_t, s_t)$ such that $\Lambda(x, s)$ is increasing in x and in s, with $\Lambda(s, s) = s$. That both current and previous service levels have a positive effect on (the overall perception of) service quality is consistent with empirical evidence (Boulding et al. 1993). The Bellman equation corresponding to this problem is

$$V(s) = \max_{x \in [0,1]} \{\pi(x) + \beta R(\Lambda(x, s))V(\Lambda(x, s))\}. \quad (11)$$

Let $\lambda(s) = \Lambda_2(s, s) = 1 - \Lambda_1(s, s) \in [0, 1]$, the partial derivative of $\Lambda(x, s)$ evaluated at $x = s$; for exponential smoothing, we have $\lambda(s) \equiv \lambda$. We show in the appendix that our main insights extend under this general adaptation process by replacing λ with $\lambda(s)$, as follows:

i. Proposition 1 extends to model (11), where s^{**} solves $\lambda(s)\pi'(s) + (1 - \lambda(s))\Pi'(s) = 0$. It is interesting that $s^{**} \leq \bar{s}$ depends on the adaptation process Λ only via $1 - \lambda(s) = \Lambda_1(s, s)$, the marginal effect of current service on service quality, when service is aligned with expectations.

ii. A range of steady states persists under loss aversion; i.e., Proposition 3(i) extends. To capture (weak) loss aversion, we set Λ as the minimum of two smooth functions that cross at $x = s$ (cf. Popescu and Wu 2007, p. 421). Formally, $\Lambda(x, s) = \min\{\Lambda^L(x, s), \Lambda^G(x, s)\}$, with $(\Lambda^L(x, s) - \Lambda^G(x, s))(x - s) \geq 0$, and $1 - \lambda^L(s) = \Lambda_1^L(s, s) \geq \Lambda_1^G(s, s) = 1 - \lambda^G(s)$; i.e., a marginal decrease in service away from the status quo $x = s$ has a steeper effect on memory than a marginal increase.

iii. Transient results can be derived analytically if $\pi(\Lambda^{-1}(s_t, s_{t+1}))$ is supermodular—in particular, if π is sufficiently concave or if $\Lambda(x, s)$ is supermodular and concave in x.[11] These conditions are sufficient, but they are not necessary for global stability. Although an analytical proof is difficult, our numerical experiments suggest that optimal service paths still converge to the steady state even when these conditions are violated, extending the insights in Propositions 2 and 3(ii).

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[11] The assumptions on Λ mean that the marginal effect of current service on overall service quality diminishes, and it is higher for customers who had better past experiences. These conditions are satisfied, for example, if $\Lambda(x, s) = \lambda(x)s + (1 - \lambda(x))x$, where $\lambda(x)$ is increasing with bounded curvature, $\lambda'(x) \geq |\lambda''(x)|/2$ (as with, e.g., $\lambda(x) = x$, e^{x-1}, and $1 - e^{-x}$). In this model, lower service is more salient in memory—that is, the lesser the current experience, the more it weighs on service quality.

6. Sensitivity Insights

We further illustrate how service levels and customer lifetime value are affected by behavioral characteristics (such as memory, loss aversion, or loyalty) and by prices and switching costs. By endogenizing the effect of service quality on defection, our model reveals systematic effects of customer memory on service and value. It also explains why loyal or high margin customers need not fetch better service and why firms may experience increasing marginal returns to service quality.

## 6.1. Customer Memory Has Systematic Effects on Service and CLV

In the long run, customers who adapt faster require higher service ($s^{**}(\lambda)$ decreases in λ by Proposition 1) and hence require higher maintenance cost—but does this mean they are less profitable overall? The next result shows that this need not be the case: the effect of memory λ on a customer's value depends on how her initial expectations s_0 fare relative to the steady-state service quality, as illustrated in Figure 3(b). Among customers who had relatively good prior experiences with the firm, those who adapt more slowly (high λ) or are less loss averse (lower ρ) provide better return on investment because they yield higher value V and demand lower long-run service s^{**}. Among customers

who expect low service quality, those with shorter memory (lower λ) are more profitable overall—even as they require higher service in the long run—because their expectations are easier to change.

PROPOSITION 5. (i) *Customer memory λ has a positive marginal effect on value, i.e., $(d/d\lambda)V^K(s;\lambda,\rho) \geq 0$, if $s > s_L^{**}(\lambda,\rho)$, and a negative effect if $s < s_G^{**}(\lambda)$. (ii) Loss aversion has a negative marginal effect on customer value $(d/d\rho)V^K(s;\lambda,\rho) \leq 0$, if $s > s_L^{**}(\lambda;\rho)$, and no effect otherwise.*

The effects of consumer memory on firm policies and profits have been mostly ignored in the literature (e.g., Ho et al. 2006, Bitran et al. 2008), or studied in interaction with other operational concerns, which preclude a direct comparison with our findings. For example, memory effects interact with inventory competition in Gaur and Park (2007), whose consumers learn adaptively (from shopping experiences) about firms' true service quality. They find that asymmetry in perception, i.e., loss aversion ($\rho > 1$), leads to higher service and lower profits (consistent with Propositions 3 and 5(ii)), but smooth memory λ has no such effect (unlike Proposition 1). In an average-cost, multiclient model with capacity constraints, Adelman and Mersereau (2013) find a

Figure 3 The Effect of Memory (λ), Loyalty (α) and Price (P) on Long-Run Service and Value
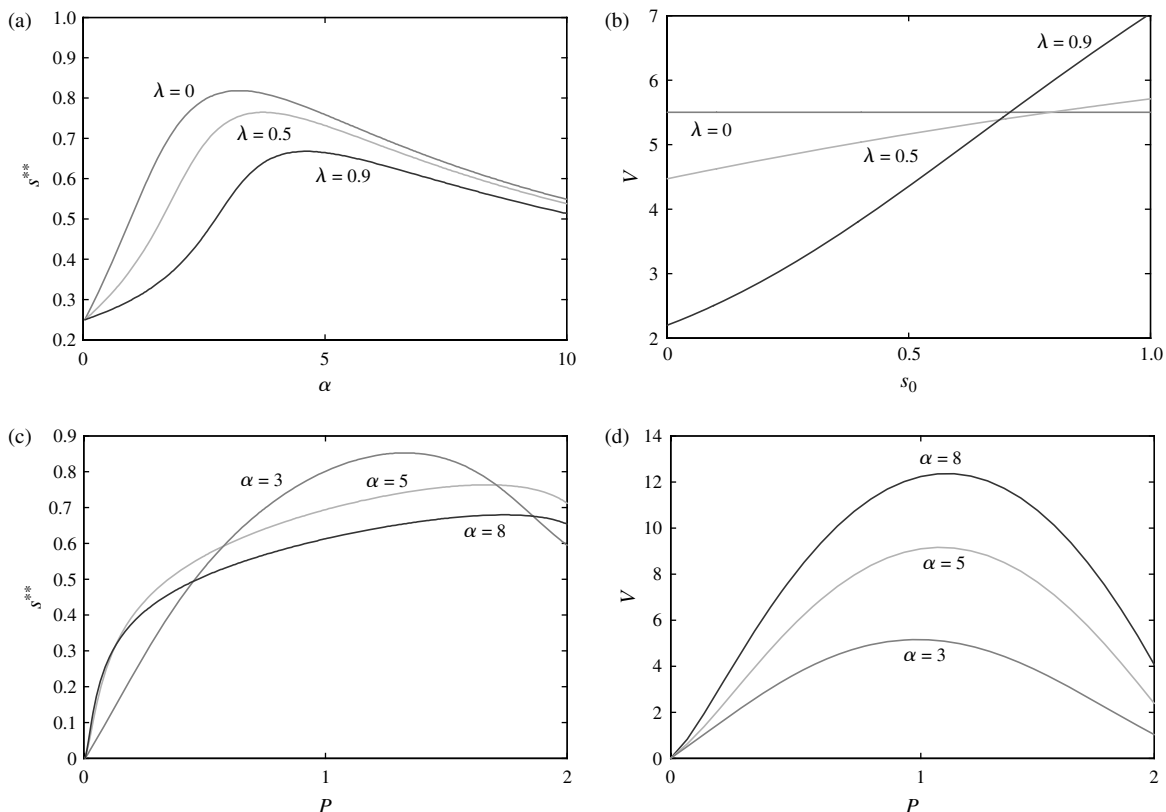
Note. Same model as in Figure 1, with $R(s) = 1/(1 + \exp(\tau - \alpha s + P))$; default $\beta = 0.94$, $\tau = 1$, $P = 1$, $\alpha = 3$, $\lambda = 0.5$, and $s_0 = 0.5$.

generally ambiguous effect of exponentially smoothed customer memory on profits, but this effect vanishes when purchase volumes, and the implicit evolution of goodwill, are deterministic (as they are in our model). We further briefly illustrate biases that can arise from ignoring customer memory in our context.

6.1.1. Miscalibration. A firm that ignores the effect of customer memory and adaptation processes on retention will offer suboptimal service levels $\hat{s}$, either too high or too low, depending on how this miscalibrated firm might estimate retention rates. Indeed, this firm has no incentive to vary service, and it forms a static (biased) estimate of the renewal probability $\hat{R}(x)$ from offering a constant service x in each period.[12] Assuming that $\hat{R}(x)$ does capture a consumer's *long-run* behavior accurately, we have $\hat{R} \equiv R$, the probability that, at service level x, a customer accustomed to this service quality ($s = x$) renews. This miscalibrated firm will then deliver in each period the static service level $\hat{s} = \bar{s} = \arg\max \Pi \geq s^{**}$. In other words, a firm that ignores memory effects (but correctly estimates steady-state retention rates) will offer higher than optimal service levels. By contrast, a firm that completely ignores the effect of service quality on defection will assume that $\hat{R}$ is constant and deliver a low, myopic service $\hat{s} = \underline{s} \leq s^{**}$.

6.2. Marginal Returns to Service Quality May Increase

The marginal returns to service quality depend on the shape of the retention probability R; this depends on customer loyalty and switching costs—in particular, if customers have low switching costs, as evidenced in more competitive industries, then R can be convex (Jones and Sasser 1995).[13] By leveraging the memory effect, firms can then not only increase CLV but also extract increasing marginal returns to service quality (absent memory, $V(s; \lambda = 0) \equiv \Pi(\bar{s})$ is constant). With increasing (or even constant) marginal effects of service quality on retention, firms will offer higher transient service to customers with better service quality perceptions; these yield higher expected returns—despite being more expensive to serve—because they are easier to retain. Convex CLV has been evidenced in Ho et al. (2006), in a model

where immediate satisfaction drives memoryless purchase rates, and costs are not too steep. Our result is independent of costs but is driven instead by the effect of customer *memory* on her probability to defect (which does not figure in their paper).

PROPOSITION 6. *If $R(s)$ is convex, then $V(s)$ is also convex, and the optimal service policy $x^*(s)$ is increasing. These relationships are ambiguous if $R(s)$ is concave; see Figures* 1(b) *and* 1(c).

Contrasting existing predictions (Jones and Sasser 1995), increasing marginal returns can persist in our model even as service quality has a diminishing marginal effect on retention (i.e., for concave R; see Figure 1(c)). Intuition suggests that the shape of the value function depends on the degree of concavity of R, in particular, V is concave for sufficiently concave R. This intuition is confirmed in numerical experiments for parametric logit, exponential, and power specifications of R (not reported here).[14]

6.3. Loyalty, Switching Costs, and Price Have Unimodal Effects on Service

As a result of endogenizing the effect of service quality on retention, other retention drivers such as price and customer loyalty need not systematically determine who should get better service. These nonmonotonic effects, illustrated numerically in Figure 3 based on the smooth model (6), extend to the loss-averse case via Proposition 3 and persist in the absence of service dynamics (i.e., for $\bar{s}$ and Π) as long as service quality, price, and loyalty moderate retention rates R.

6.3.1. Loyalty and Switching Costs. We capture loyalty with a parameter α that increases retention $R(x; \alpha)$, and hence profit $V(s; \alpha)$. Thus, all else equal, loyal customers are more valuable; but do they also get better service? Using the characterization in Proposition 1, Figure 3(a) shows that α typically has a nonmonotonic effect on steady-state service $s^{**}(\alpha)$. We illustrate this for a parametric logit model where α has a multiplicative effect on service quality s; the effect of an additive-switching cost τ is similar. Beyond a critical loyalty or switching cost level, the firm will treat customers as a captured audience and reduce its costly investment in retention.[15]

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[12] The *true* renewal probability is nonstationary, $R(\lambda^t s_0 + (1 - \lambda^t)x)$. A miscalibrated firm might correctly estimate this probability at $t = 1$, $\hat{R}(x) = R(\lambda s_0 + (1 - \lambda)x)$, and then use this estimate (wrongly) for all t. Depending on s_0, $\hat{s}$ can then be higher or lower than s^{**}. We thank one of the referees for suggesting this point.

[13] Jones and Sasser (1995) cite Xerox, which found that increasing overall satisfaction ratings from 4 to 5 (on a 5-point scale) for its office products customers increased their retention by a factor of 6—significantly more than for lower-rating customers.

[14] Characterizing the relationship analytically is complicated by the multiplicative interaction effects in the value to go (6), which make preservation of concavity difficult. A sufficient condition for $V(s)$ to be concave (respectively, convex) is the concavity (convexity) of $(RV)(s)$; see the proof of Proposition 6. Yet this property is not preserved in general because RV need not be concave even when both R and V are (increasing and) concave. This fact significantly complicates the analysis of our problem.

[15] By moderating R's degree of concavity, α also links to the level of competition in the market (Jones and Sasser 1995), suggesting

6.3.2. Contract Price. Practical considerations such as price rigidity, competitive forces, and branding concerns suggest that prices may not be optimized in response to noncontractible service levels in a contractual setting such as ours. In this case, an exogenous change in prices will have a unimodal effect on steady-state service $s^{**}(P)$, even if short-term profit $\pi(\cdot; P)$ is unimodal, or increasing in P, as with subscription services (i.e., $\pi(x) = P - c(x)$). As Figure 3(c) illustrates, higher-margin customers command better service—up to a certain point, when the pattern is reversed and the relationship becomes more transactional. When prices are too high, customers are more likely to defect because they are left with less surplus ($u(\cdot; P)$, and hence $R(\cdot; P)$ is decreasing in P; see (4)), and better service cannot indefinitely make up in retention what is lost by extracting such high rents. This trade-off is also reflected in the unimodal relationship between price and customer value V, illustrated in Figure 3(d).

Although in this analysis the contractual price P was exogenously fixed, the patterns in Figures 3(a) and 3(b) persist when the pricing decision is endogenized, i.e., when $P = \arg\max_p \Pi(s^{**}(p); p)$ is set optimally with respect to steady-state profits; plots are omitted for brevity.[16]

7. Alternative Decision Models: Habituation vs. Goodwill

The results so far relied on the premise that customers who had better past experiences with the firm are more likely to renew. This assumption, consistent with goodwill models and will expectations, has the widest empirical support in the literature (Boulding et al. 1993, Zeithaml 2000). In this section we consider an alternative hypothesis—namely, that customers who have received better service are *less* likely to renew, given a service level x. Intuitively, they become harder to please (and easier to disappoint) as they grow accustomed to better service, owing to the satisfaction treadmill effect (Brickman and Campbell 1971) and consistent with *habituation* models (Baucells and Sarin 2010). We further propose a general model that

that—from the customers' point of view—there is an optimal degree of competition that results in the highest service level. Our result is partially consistent with Hall and Porteus (2000), who find that loyalty decreases long-run service levels in an oligopoly. Our model captures competitive effects implicitly in a customer's choice to defect; strategic interaction remains to be researched.

[16] Our focus is on managing service, but our framework can be adapted to dynamically manage any type of retention effort, including prices; x_t then captures the discount from an endogenous, adaptive reference price. In our model, unlike the dynamic pricing literature (e.g., Lewis 2005, Popescu and Wu 2007), these reference prices affect defection probabilities.

unifies the two hypotheses and enriches our framework to capture the dual effects of habituation and goodwill on retention.

7.1. Habituation Hypothesis: Disconfirmation Drives Defection

This section considers a model where retention R is driven by disconfirmation, $d = x - s$, i.e., the perceived gain or loss relative to expectations, as evidenced, e.g., in Bolton and Drew (1991). All else equal, a customer A who is used to better service quality than B, $s_t^A > s_t^B$, is more likely to be dissatisfied by a service experience x_t; i.e., $x_t - s_t^A < x_t - s_t^B$. For this reason, this model suggests that A is *less* likely to renew than B, even as A received overall better service quality, $s_{t+1}^A > s_{t+1}^B$. Disconfirmation drives consumer purchases in Liu et al. (2007) and Ho et al. (2006).

Loss aversion in a customer's decision (as opposed to memory) processes suggests that a dissatisfying experience has a stronger impact on retention than a satisfying one of the same magnitude. Formally, this translates into an asymmetric repurchase probability:

$$R_d(x - s)$$
$$= \begin{cases} R(\rho(x - s)) & \text{if } x < s \text{ (perceived loss)}, \\ R(x - s) & \text{if } x \geq s \text{ (perceived gain)}, \end{cases} \quad (12)$$

where R is an increasing function. We use the same parameter ρ as in §5.1 to captures loss aversion in decision processes ($\rho > 1$). The Bellman equation for the disconfirmation model is

$$V_d(s) = \max_{x \in [0, 1]} \{\pi(x) + \beta R_d(x - s) V_d(\lambda s + (1 - \lambda)x)\}. \quad (13)$$

PROPOSITION 7. (i) $V_d(s)$ *is decreasing in* s. (ii) *If* $\rho \geq 1$, *then problem* (13) *admits a range of steady states* $[s_d^{**}(1), s_d^{**}(\rho)]$, *where* $s_d^{**}(\rho) \geq \bar{s}$ *solves* $(1 - \lambda \beta R(0)) \cdot \pi'(s) + \beta \rho R'(0)\pi(s) = 0$. *Moreover* $s_d^{**}(\rho)$ *is increasing in* λ *and* ρ *and decreasing in* β. (iii) *If, moreover, R is concave, then the optimal service quality path decreases to* $s_d^{**}(\rho)$ *if* $s_0 > s_d^{**}(\rho)$, *and it increases to* $s_d^{**}(1)$ *if* $s_0 < s_d^{**}(1)$. (iv) *If* $\rho < 1$, *then no steady state exists.*

When disconfirmation drives retention, customers who received better treatment demand higher service, because of the satisfaction treadmill effect, and this makes them less profitable in the long run (V is decreasing). We obtain a reversal of the insights from Proposition 1—in particular, the steady-state service s_d^{**} decreases with the firm's discount factor β, and it increases the more customers anchor on past experiences (higher λ). The effect of memory λ on customer value V reverses relative to Proposition 5, whereas the effects of loss aversion, loyalty, and margin evidenced in §6 persist under this disconfirmation model.

We find it interesting that, although memory effects reverse, the structural results concerning global

stability and the effects of behavioral asymmetries on long-run policies are preserved under this conceptually different disconfirmation model. Loss aversion, this time in decision processes, hurts profits and leads again to a (higher) range of steady states. It also explains why, even in the long run, firms may anchor on customers' service expectations. This is because the (cost and retention) benefits from lowering these expectations are outweighed by the risk of losing a dissatisfied customer if she is loss averse in her renewal decisions. This risk is higher for high-expectation customers, so firms will serve them better in the long run even though they are less profitable—because they are loss averse.

7.2. General Reference-Dependent Decision Model

The results in the previous section illustrated how the insights in Proposition 1, notably the effect of customer memory on service, can be reversed if disconfirmation, as opposed to service quality, drives consumer repurchase decisions. In this section, we unify these opposing insights under a general retention model $R(x, s)$—in particular, $R(x, s) = R(\lambda s + (1 - \lambda)x)$ for the goodwill model (6) and $R(x, s) = R(x - s)$ for the habituation model (13). This model corresponds to a general reference-dependent utility $u(x, s) = v(d, s)$, $d = x - s$ underlying consumer renewal decisions (4) and resulting in renewal probability $R(x, s) = \mathbb{P}(u(x, s) + \tilde{\varepsilon} \geq 0)$. Using for simplicity the same notation as in §4, the memoryless service $\bar{s}$ maximizes $\Pi(x) = \pi(x)/(1 - \beta R(x, x))$, and the Bellman equation is

$$V(s) = \max_{x \in [0, 1]} \left\{ \pi(x) + \beta R(x, s) V(\lambda s + (1 - \lambda)x) \right\}. \quad (14)$$

PROPOSITION 8. (a) *If $R(x, s)$ is increasing in s, then customer value $V(s)$, in (14) is increasing in s and the steady state s^{**} decreases with customer memory λ—in particular, $s^{**} \leq \bar{s}$. (b) If $R(x, s)$ is decreasing in s, these relationships are reversed; i.e., $V(s)$ is decreasing in s and s^{**} increases with λ—in particular, $s^{**} \geq \bar{s}$.*

Proposition 8 explains the dichotomy between the results of Propositions 1 and 7: part (a) applies for the goodwill model in §4, because $R(\lambda s + (1 - \lambda)x)$ is increasing in s, and part (b) encompasses the disconfirmation model in §7.1, because $R(x - s)$ is decreasing in s. In general, the effect of customer memory on service appears to be driven by whether past service experiences (summarized by s) have a positive or negative effect on retention—that is, whether the renewal probability $R(x, s)$ is increasing or decreasing in s. In other words, what matters is whether service quality s or disconfirmation $d = x - s$ is a stronger driver of renewal decisions $R(x, s) = R(d + s, s)$. Indeed, monotonicity of $R(x, s)$ in s is determined by the sign

of $R_2(x, s) = (\partial/\partial s)R(x, s) = (\partial/\partial s - \partial/\partial d)R(d + s, s)$. Global monotonicity of $R(x, s)$ is sufficient but not necessary to explain the effect of memory λ on steady-state service. As we show in the appendix, this effect is fully characterized by the sign of $R_2(s^{**}, s^{**})$ alone—that is, whether locally, *at steady state*, customer renewal decisions are more sensitive to disconfirmation or to service quality.

8. Conclusions

We relied on behavioral theories to develop a dynamic programming model that uses responsive service strategies to manage the lifetime value of customers whose defection decisions anchor on past service experiences. In this context, we showed that varying service to manage expectations is only beneficial at the beginning of a relationship; in the long run, service policies converge toward a steady-state service level from which it is suboptimal to deviate until the customer defects. We discover that behavioral asymmetries, notably loss aversion in customer memory or decision processes, explain this structure of optimal long-run policies. Specifically, the more customers are averse to disappointing service experiences, or the more salient such experiences are in their memory, the wider is the range of optimal constant service policies offered by the firm and the lower its profits. If, however, satisfying experiences are more salient, then firms should constantly vary service. Loss aversion thus offers one reason why—despite targeted service control capabilities—firms may have no better choice but to deliver service levels that customers expect.

Beyond this direct effect of customer expectations on long-run service levels, the weight and process through which these expectations translate into defection decisions has systematic effects on policies and CLV. As long as better service history has positive (goodwill) effects on retention, the firm offers lower service in the long run to less adaptive customers, i.e., those who anchor more on past experiences. These customers are more profitable than those who focus on recent experiences, but only if they have received better treatment in the past. The memory effects reverse, however, if habituation (disconfirmation) rather than goodwill (service quality) drives defection decisions. These results motivate the importance of testing this dichotomy empirically.

The endogenous effect of service quality on customer retention explains why higher-margin customers are not always more valuable and need not receive better service; this is because they are more likely to defect. Similarly, customers who are more loyal or who have higher switching costs, albeit more valuable, need not receive better service if they

Table 1 Effects of Margin and Consumer Behavior on Policy and Value

Characteristics (I)	Long-run service (steady state s^{**})	Customer value (V)
Memory (λ)[a]	Decreasing	Decreasing for high s_0; increasing for low s_0
Loss aversion ($\rho > 1$)	Expands range up (oscillates if $\rho < 1$)	Decreasing
Initial expectations (s_0)	No effect ($\rho = 1$) (increasing if $\rho > 1$)	Increasing[a]
Loyalty (α)	Inverse U-shape[b]	Increasing
Price (P)	Inverse U-shape[b]	Inverse U-shape[b]

[a]These effects are reversed when disconfirmation drives retention.
[b]These effects are evidenced numerically; all others are proved analytically.

are inherently too sticky. We find unimodal effects of retention drivers such as prices and loyalty on long-run service; these factors influence the marginal returns to service quality, which may increase in our model. Table 1 summarizes the effect of consumer characteristics on service and CLV.

This paper is a first step toward capturing, in an analytical framework, the behavioral effects of adaptive service expectations on customer retention and profitability in contractual relationships. We have therefore strived for parsimony in developing the simplest stylized model capable of transmitting robust structural insights from this framework. Ample opportunities exist for future research to extend this model and address its limitations from an operational, marketing, or economic perspective—for example, by incorporating strategic interactions, customer acquisition, shared resources, or richer operational structures. For expository purposes, we cast our model and results in the context of managing service relationships. However, our setup can be translated to other contexts, including dynamic pricing, employee retention, and effort and quality management.

Appendix A. General Profit Models and Parameter Uncertainty

Our stylized model has focused on customer retention and ignored the positive effect of past service experiences on customer spending, purchase frequency, and acquisition.

This section demonstrates that incorporating these effects, as well as the firm's uncertainty about customer characteristics, does not change our structural insights based on model (6).

A.1. Volume Effects

Suppose that better past experiences increase purchase volume, so $\pi(x)$ is replaced by $\pi(x, s)$, an increasing function of s. For example, Bolton et al. (2006) show that prior service experiences affect the purchase of additional support contracts. Let $\pi_2(s, s) = (\partial/\partial s)\pi(x, s)|_{x=s}$.

PROPOSITION 9. *Proposition 1 extends under volume effects, with the steady state $s^{**} \in [\underline{s}, \bar{s}]$ solving $W'(s) = (1 - \lambda)\pi_2(s, s)$—in particular, $s^{**}(\lambda) \le \bar{s}$ is decreasing in λ.*

We find it interesting that no additional assumption is needed for steady-state results to extend under volume effects; in particular, the effect of loss aversion carries on as well. Propositions 2–5 also extend, for example, if short-term profit $\pi(x, s)$ is supermodular and concave in x (see Appendix B). Supermodularity of $\pi(x, s)$ means that customers who had better experiences with the firm are more sensitive to changes in service, or that the marginal cost of serving them is lower; this holds, for example, if cost and customer spending are separable $\pi(x, s) = r(s) - c(x)$. These assumptions are sufficient to extend (using similar proof techniques) transient results and global stability, but they are not necessary for the result to hold; in particular, a weaker sufficient condition is $(1 - \lambda)\pi_{12} \ge \lambda\pi_{11}$.

A.2. Customer Visits

Our stylized model (6) assumes that customers periodically visit the firm and decide whether or not to renew. However, our main insights are not changed by uncertainty in visits and renewal timing decisions nor by the positive effects of service quality on the frequency of these events.[17]

Endogenous Visit Frequency. Suppose that, in each period, a customer who has received better quality of service s has a higher probability $\nu(s)$ of visiting the firm. We then obtain $\bar{V}_\nu(s) = \nu(s)[\max_x\{\pi(x, s) + \beta R(\Lambda(x, s))\bar{V}_\nu(\Lambda(x, s))\}] + \beta(1 - \nu(s))R(s)\bar{V}_\nu(s)$. Denoting $R_\nu(s) = \nu(s)/(1 - \beta R(s)(1 - \nu(s)))$, we obtain that $V_\nu(s) = \bar{V}_\nu(s)/R_\nu(s)$ solves model (6) with R replaced by $F \cdot R_\nu$, which is increasing in s. All our results so far extend in this case.

Viscous Demand. In contractual settings such as insurance, utilities, and subscriptions, the customer visits the firm regularly but only occasionally—that is, with probability $p(s)$—considers whether or not to defect. A boundedly rational attention budget can explain such demand viscosity: "The consumer rethinks such decisions from time to time, regularly or at some random intervals, perhaps triggered by some events" (Radner 2003, p. 190). The resulting model is $V_p(s) = \max_x\{\pi(x, s) + \beta[p(S)R(S)V_p(S) + (1 - p(S))V_p(S)]\}$, where $S = \Lambda(x, s)$. This model has the same structure as model (6), with R replaced by the increasing function $1 - p(s)(1 - R(s)) \ge R(s)$.

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[17] Our results extend also to situations where contract renewal decisions are scheduled over a wider horizon (e.g., year), during which the customer makes multiple visits to the firm (e.g., weekly/monthly).

A.3. Customer Acquisition

As stated in §1, our model is limited to maximizing profit from an existing customer base, and it ignores the acquisition of new customers; this focus is justified in mature markets (Bolton et al. 2006, Gupta and Zeithaml 2006). In particular, our result that customer memory leads firms to offer lower long-run service levels (Proposition 1) did not account for the possibility that these low service levels might erode the ability to acquire new customers. We briefly argue here that the memory effect persists in (proportional) growth markets, when service quality affects the firm's ability to acquire new customers and these customers' expectations. Indeed, suppose that service quality s increases not only the firm's retention rate $R(s)$ but also the acquisition rate $A(s)$ of identical customers. This is consistent with a viral marketing context where, in each period, with probability $A(s)$, an existing customer may bring in a new customer with the same service quality expectations s, reflecting "word of mouth." The Bellman equation for a fluid model with customer base N is $V_A(N, s) = \max_x \{N\pi(x) + \beta V_A(R(S)N + A(N, S), S)\}$, where $S = \Lambda(x, s)$, and $A(N, S) = NA(S)$ is the number of new customers acquired. So, parallel to the argument in Appendix 3, $V_A(N, s) = NV_A(s)$, where $V_A(s) = \max_x \{\pi(x) + \beta(R(S) + A(S))V_A(S)\}$ is the acquisition-adjusted CLV, and we recover our original model with $R + A$ instead of R. The memory effect persists as V_A is bounded if $\beta(R + A) < 1$; otherwise, $s^{**} = \bar{s} = 1$.[18] Few papers consider dynamic models for managing both retention and acquisition. Ovchinnikov et al. (2012) do so for two customer classes under capacity constraints; the effect of consumer memory in their context remains to be explored.

A.4. Parameter Uncertainty

In this section, we argue that our main insights extend when we acknowledge the firm's potential uncertainty about customer characteristics. Suppose the firm knows the distribution $\tilde{\theta}$ of customer types, with support $[\theta_L, \theta_H]$. For example, $\tilde{\theta}$ may capture the uncertainty about loyalty $\tilde{\alpha}$ or about initial expectations $\tilde{s}_0$. The firm's objective, given $\tilde{\theta}$ and residual information I, is

$$\tilde{V}(\tilde{\theta}, I) = \max_{X = \{x_t\}} \mathbb{E}_{\tilde{\theta}}[V(X; \tilde{\theta}, I)]. \tag{A1}$$

A recursive expression of this imperfect observation Borel model requires, for a given policy X, the probability that the consumer renews at stage t given that she has renewed in all previous stages:

$$\mathbb{P}(\tilde{u}_{t+1}(s_{t+1}; \tilde{\theta}) \geq 0 \mid \tilde{u}_i(s_i; \tilde{\theta}) \geq 0, i = 1, \ldots, t)$$
$$= \frac{\mathbb{E}_{\tilde{\theta}}\left[\prod_{i=1}^{t+1} R(s_i; \tilde{\theta})\right]}{\mathbb{E}_{\tilde{\theta}}\left[\prod_{i=1}^{t} R(s_i; \tilde{\theta})\right]} \triangleq \mathbb{E}_{\tilde{\theta}_t}[R(s_{t+1}; \tilde{\theta}_t)].$$

The distribution $\tilde{\theta}_t = \tilde{\theta}_t(X)$ of active types at time t follows the Bayesian update: $\tilde{\theta}_{t+1} \sim (\tilde{\theta}_t \mid \text{customer renews at time } t)$ with $\tilde{\theta}_0 = \tilde{\theta}$. The bounded value (A1) of this imperfect

observation model solves the following Bellman equation (Bertsekas and Shreve 1996, Chap. 10):

$$\tilde{V}(s_t, \tilde{\theta}_t) = \max\{\pi(x_t) + \beta\mathbb{E}_{\tilde{\theta}_t}[R(s_{t+1}; \tilde{\theta}_t)]\tilde{V}(s_{t+1}, \tilde{\theta}_{t+1})\}, \tag{A2}$$

where $s_{t+1} = \lambda s_t + (1 - \lambda)x_t$ and $\forall k \in [\theta_L, \theta_H]$, $\mathbb{P}(\tilde{\theta}_{t+1} \geq k) = (\mathbb{E}[R(s_{t+1}; \tilde{\theta}_t) \mid \tilde{\theta}_t \geq k]/\mathbb{E}[R(s_{t+1}; \tilde{\theta}_t)])\mathbb{P}(\tilde{\theta}_t \geq k)$. Suppose that higher types are more likely to renew, i.e., R increases in θ; then, as they do so, the firm updates its beliefs $\tilde{\theta}_t$ upward, in a stochastic sense. This sorting argument confirms the numerical findings in Sun and Li (2011) and the empirical evidence of increasing retention rates, as illustrated in Fader and Hardie (2009) for the case of $R(s_t; \theta) \equiv \theta$. We extend their setup to capture the effect of service dynamics on retention and derive implications for the optimization problem of the firm.

Proposition 10. *Suppose that R is strictly increasing in θ. For any distribution $\tilde{\theta}$ with support $[\theta_L, \theta_H]$ and any feasible solution $X = \{x_t\}$ to problem (A2) we have $\tilde{\theta}_t(X) \preceq_{FSD} \tilde{\theta}_{t+1}(X)$ and pointwise $\tilde{\theta}_t(X) \to \tilde{1}_{\theta_H}$, a point mass at θ_H. Moreover, if the optimal policy $X^* = \{x_t^*\}$ of (A2) converges as the customer renews, then it must converge to $s^{**}(\theta_H)$. In particular, for $\tilde{\theta} = \tilde{s}_0$, this is the same steady state s^{**} determined in Proposition 1, and it does not depend on $\tilde{s}_0$.*

Problem (A2) is generally intractable owing to the dimensionality of the state space. For two-point distributions, $\tilde{\theta} = [\theta_H, p; \theta_L, 1 - p]$, (A2) is amenable to a two-dimensional state dynamic program with $\tilde{\theta}_t$ replaced by p_t and updated as $p_{t+1} = (p_t R(s_{t+1}; \theta_H))/(p_t R(s_{t+1}; \theta_H) + (1 - p_t)R(s_{t+1}; \theta_L))$. We shall use this formulation in numerical experiments to illustrate the effect of uncertainty about customer expectations and loyalty (or switching costs) on the firm's policy and profit and to confirm global stability.[19] We report representative results in Figure A.1 and Table A.1; these insights appear to be robust under the habituation hypothesis (the disconfirmation model; results not reported here).

When customer's loyalty $\tilde{\theta} = \tilde{\alpha}$ is uncertain, the firm initially treats the customer as transactional (low type), but, as she renews, the posterior probability of her being a high type increases, and thus the firm gradually adopts the policy for loyal types (see Figure A.1(b)). Ignoring uncertainty in loyalty (i.e., assuming $\alpha = \mathbb{E}\tilde{\alpha}$) may lead the firm to offer long-run service levels that are either too high or too low because of the unimodal service-loyalty relationship; see §6.3.1.[20] In contrast, uncertainty about customer expectations $\tilde{\theta} = \tilde{s}_0$, does not affect long-run service levels, as Figure A.1(a) shows. In sum, our numerical results confirm the insights from Proposition 10: with imperfect information, as long as customers renew, the service trajectory converges to the long-run service $s^{**}(\alpha_H)$ delivered under full information to most loyal types; this does not depend on (the distribution of) initial expectations.

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[18] Our fluid results extend to the stochastic model, as well as when disconfirmation drives retention $R(x - s)$ and acquisition $A(x - s)$. Other criteria (e.g., average cost) can be more appropriate to handle acquisitions in a growing market.

[19] Stability is difficult to prove analytically because (i) the state paths need not be monotone (see Figure 3) and (ii) the value function need not be concave, so the optimal policy need not be unique. Thus, conventional policy convergence and turnpike results do not apply.

[20] Indeed, Figure A.1(b) shows that $\tilde{s}^{**}(\tilde{\alpha} = [1, \frac{1}{3}; 4, \frac{2}{3}]) = s^{**}(4) < s^{**}(3) < s^{**}(7) = \tilde{s}^{**}(\tilde{\alpha} = [1, \frac{2}{3}; 7, \frac{1}{3}])$.

Figure A.1 **Perception (State) Paths for the Optimal Policy Under Full Information (Dashed Line) vs. Uncertain (a) Initial Expectations $\tilde{s}_0$ (Full Lines: s_t^H and s_t^L for $\alpha = 3$) and (b) Loyalty $\tilde{\alpha}$ (Full Lines: s_t for $s_0 = 0.7$)**
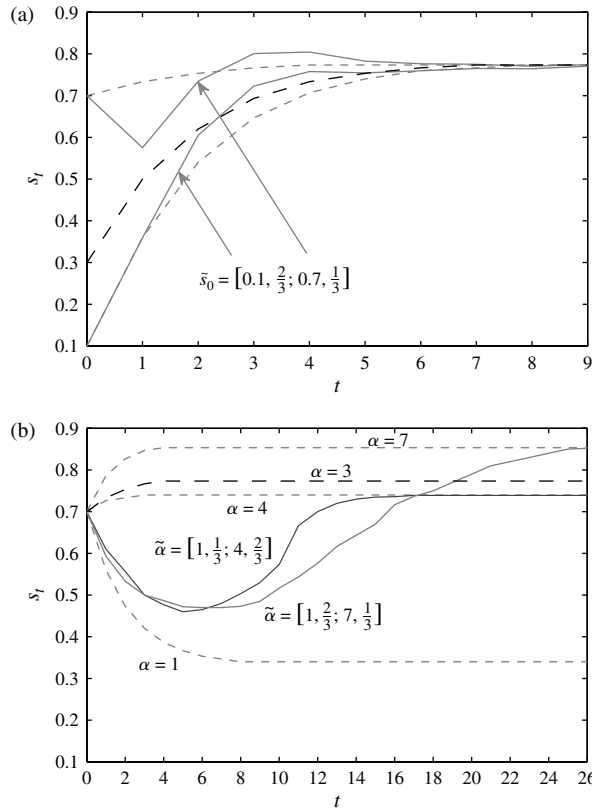
Note. Same model as in Figure 1, with $R(s) = 1/(1 + \exp(-\alpha s + P))$, $P = 1$, $\lambda = 0.6$, and $\beta = 0.94$.

Unlike with full information, customer perceptions need not evolve monotonically as the firm learns about the customer. The speed of convergence depends on the prior distribution $\tilde{\theta}_0$, and it is faster for $\tilde{s}_0$ than $\tilde{\alpha}$ because of the decaying memory effect of initial expectations on retention.

These differences suggest that it is more important to acquire information about customer loyalty and switching costs, rather than initial expectations. Table A.1 confirms this intuition by reporting, along three dimensions, the value of information regarding customer types: (1) the effect of uncertainty in $\tilde{\theta}$ on customer value (i.e., the percentage

Table A.1 **Value of Information About s_0 and α**

Uncertainty measure		Initial expectations $\tilde{\theta} = \tilde{s}_0$ (%)	Loyalty $\tilde{\theta} = \tilde{\alpha}$ (%)
Price of uncertainty	$\dfrac{V(\mathbb{E}\,\tilde{\theta}) - \tilde{V}}{V(\mathbb{E}\,\tilde{\theta})}$	2	15
Cost of ignoring uncertainty	$\dfrac{\tilde{V}(X^*(\mathbb{E}\,\tilde{\theta}); \tilde{\theta}) - \tilde{V}}{\tilde{V}}$	< 1	3
Value of screening	$\dfrac{\mathbb{E}[V(\tilde{\theta})] - \tilde{V}}{\tilde{V}}$	1	118

Note. Parameters as in Figure A.1; $\tilde{s}_0 = [0.1, \frac{2}{3}; 0.7, \frac{1}{3}]$ and $\tilde{\alpha} = [1, \frac{2}{3}; 7, \frac{1}{3}]$.

gap from certainty equivalence), (2) the cost of assuming that the customer is of average type $\theta = \mathbb{E}\,\tilde{\theta}$ and delivering the optimal service policy $X^*(\theta) = \arg\max V(\theta)$ for the corresponding full-information θ model, and (3) the value of knowing and targeting individual customer types $\mathbb{E}[V(\tilde{\theta})]$. As the second row suggests, there is limited cost to ignoring uncertainty about initial expectations (or loyalty) and using instead the full-information model corresponding to average expectations. This insight, confirmed throughout our numerical experiments, supports our main focus on the full-information model.

Appendix B. Proofs

PROOF OF PROPOSITION 1. (i) Monotonicity of the value function holds because $R(\lambda s + (1 - \lambda)x)$ is increasing in s. Monotonicity is preserved by induction for the corresponding finite-horizon model and then at the limit for our infinite-horizon formulation. The firm can extract at least $\Pi(s)$ from the customer by maintaining the service quality at current expectations (i.e., by offering $\{x_t \equiv s\}$), so $\Pi(s) \leq V(s)$. An upper bound on customer value is obtained if the customer never defects and the firm offers the short-term profit maximizing service; in this case, $V(s) \leq \sum_{t=0}^{\infty} \beta^t \max_s \pi(s) = \pi(\underline{s})/(1 - \beta)$. Because V and R are increasing, so is the profit to go in the Bellman equation (6). Because π is concave, its maximizer $\underline{s} \leq x^*(s)$ for all s; in particular, we obtain the following.

REMARK 1. $\pi'(x^*(s)) \leq 0$.

(ii) Define $\bar{\pi}(s, S) = \pi((S - \lambda s)/(1 - \lambda))$. In terms of the variable $s_{t+1} = \lambda s_t + (1 - \lambda)x_t$, problem (6) becomes

$$V(s_t) = \max_{s_{t+1} \in \mathbf{s}(s_t)} \{\bar{\pi}(s_t, s_{t+1}) + \beta R(s_{t+1})V(s_{t+1})\}. \quad (B1)$$

Here, $\mathbf{s}(s_t) = [\lambda s_t, \lambda s_t + (1 - \lambda)]$ is the feasible set of next-period service quality s_{t+1} associated with the constraint $x_t \in [0, 1]$. We will use this alternative formulation for technical convenience and because it facilitates extensions.

We first focus on interior steady states, which are given by the following Euler equation:

$$\frac{\partial}{\partial s_{t+1}} \{\bar{\pi}(s_t, s_{t+1}) + \beta R(s_{t+1})(\bar{\pi}(s_{t+1}, s_{t+2}) +$$
$$\beta R(s_{t+2})\Pi(s_{t+2}))\}_{s_t = s_{t+1} = s_{t+2} = s} = 0, \quad \text{or} \quad (B2)$$
$$\bar{\pi}_2(s, s) + \beta R'(s)\Pi(s) + \beta R(s)\bar{\pi}_1(s, s) = 0.$$

Differentiating $\Pi(s) = \bar{\pi}(s, s)/(1 - \beta R(s))$ gives $(1 - \beta R(s)) \cdot \Pi'(s) = \bar{\pi}_1(s, s) + \bar{\pi}_2(s, s) + \beta R'(s)\Pi(s)$, which allows to write the Euler equation (B2) as

$$\Pi'(s) = \bar{\pi}_1(s, s). \quad (B3)$$

This is equivalent to $W'(s) = \lambda \pi'(s) + (1 - \lambda)\Pi'(s) = 0$, because $\bar{\pi}_1(s, s) = -(\lambda/(1 - \lambda))\pi'(s)$. Thus, an interior steady state maximizes W, and the strict quasi-concavity of W ensures uniqueness. Furthermore, s^{**} must lie between the maximizers $\underline{s}$ and $\bar{s}$ of the unimodal functions π and Π, because it maximizes their weighted average W. To show that $s^{**}(\lambda)$ is decreasing in λ, we use the envelope theorem to derive $W_{s\lambda}(s^{**}) = \pi'(s^{**}) - \Pi'(s^{**}) = (1/(1 - \lambda))\pi'(s^{**}) \leq 0$,

as argued previously. A similar argument shows that $s^{**}(\beta)$ is increasing in β.

So far we have focused on interior steady states. The next lemma shows that problem (6) cannot admit a boundary steady state unless W, and in particular Π, is monotone. In this case there is no interior steady state, and the boundary steady state maximizes W on $[0, 1]$, as claimed.

Lemma 1. (i) *If* $s^{**} = 0$ *is a steady state of problem (6), then* $W'(0) \le 0$ *and* Π *is decreasing.* (ii) *If* $s^{**} = 1$ *is a steady state of problem (6), then* $W'(1) \ge 0$ *and* Π *is increasing.*

Proof of Lemma 1. We prove part (i); (ii) follows similarly. If 0 is a steady state, then for all x,

$$\Pi(0) = V(0) \ge \pi(x) + \beta R((1-\lambda)x)V((1-\lambda)x) \quad (B4)$$

$$\ge \pi(x) + \beta R((1-\lambda)x)\Pi((1-\lambda)x)$$

$$= \pi(x) - \pi((1-\lambda)x) + \Pi((1-\lambda)x). \quad (B5)$$

Rearranging terms and dividing by x, we can write this as $(1-\lambda)(\Pi((1-\lambda)x) - \Pi(0)/(1-\lambda)x) + \lambda((\pi(x) - \pi((1-\lambda)x))/(\lambda x)) \le 0$. Taking limits when $x \downarrow 0$ gives the desired result. $\square$

In sum, we have showed that, if problem (6) admits a steady state, then this is unique. If neither 0 or 1 are steady states, then $s^*(0) > 0$ and $s^*(1) < 1$; so, by continuity, there must exist $x \in [0, 1]$ so that $s^*(x) = x$. We conclude that, indeed, there does exist a steady state for problem (6), and it is the unique maximizer of W on $[0, 1]$.

Proof of Proposition 2. Since $\pi(x)$ is concave, it follows that $\bar\pi(s_t, s_{t+1})$ is supermodular and so is the right-hand side of the Bellman equation (B1). Moreover, the feasible sets $\mathbf{s}(s_t)$ are ascending in s_t; that is, for any $s_t \le s_t'$, $r \in \mathbf{s}(s_t)$, and $r' \in \mathbf{r}(s_t')$, we have $\min(r, r') \in \mathbf{s}(s_t)$ and $\max(r, r') \in \mathbf{s}(s_t')$. Therefore, by Topkis's theorem (Topkis 1998, Theorem 2.8.2), the policy function $s^*(\cdot)$ is increasing on $[0, 1]$. This implies that the state path $\{s_t^*\}$ is monotonic (by induction); because the feasible set $\mathbf{s}(\cdot)$ is compact, $\{s_t^*\}$ must converge to a steady state, which, by Proposition 1, equals s^{**}.[21] $\square$

Proof of Proposition 3. We transform problem (9) to obtain

$$V^K(s_t) = \max_{s_{t+1} \in \mathbf{s}(s_t)} \left\{ \bar\pi(s_t, s_{t+1}) + \beta R(s_{t+1})V^K(s_{t+1}) \right\}, \quad (B6)$$

where the state transition s_t satisfies (3). This can also be written as $s_{t+1} = \min\{s_{t+1}^G, s_{t+1}^L\}$, where $s_{t+1}^j = s_t + (1-\lambda_j)(x_t - s_t)$, $j \in \{G, L\}$. Indeed, when $x_t \ge s_t$, we have $s_{t+1} = s_{t+1}^G \le s_{t+1}^L$ (because $1 - \lambda_L > 1 - \lambda_G$) and when $x_t \le s_t$, we have $s_{t+1} = s_{t+1}^L \le s_{t+1}^G$.

We use (P_κ) to denote the smooth problem (6) with $\lambda = \lambda_\kappa = \kappa\lambda_G + (1-\kappa)\lambda_L$, so $s_{t+1}^\kappa = \kappa s_{t+1}^G + (1-\kappa)s_{t+1}^L$. We also define (P_G) and (P_L) as (P_κ) when $\kappa = 0$ and $\kappa = 1$, respectively. Let $V^i(s_t)$ denote the value function and s_i^{**} the steady state of the corresponding problems (P_i), $i \in \{G, L, \kappa\}$. In particular, $s_{\kappa=0}^{**} = s_L^{**}$ and $s_{\kappa=1}^{**} = s_G^{**}$.

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[21] In particular, because the service quality paths converge monotonically to s^{**}, which maximizes the unimodal function W, it follows that W is a Lyapunov function for our problem.

Lemma 2. (i) $V^K(s) \le V^\kappa(s)$ *for all* s. (ii) *If* s_κ^{**} *is a steady state for* (P_κ), *then it is also a steady state for problem (B6).*

Proof. (i) The claim follows from $s_{t+1}^\kappa = \kappa s_{t+1}^G + (1-\kappa)s_{t+1}^L \ge \min\{s_{t+1}^G, s_{t+1}^L\} = s_{t+1}^K$ by using induction on the corresponding finite-horizon problems and then value iteration. (ii) Starting from s_κ^{**}, a constant state path is optimal for (P_κ). This constant path is feasible for our kinked problem (B6) and achieves the same value $V^K(s_\kappa^{**}) = V^\kappa(s_\kappa^{**}) = \Pi(s_\kappa^{**})$. Therefore, by Lemma 2(a), the constant path s_κ^{**} must be optimal for (B6), and so s_κ^{**} is also a steady state for this problem. $\square$

By Proposition 2, if we start from $s_0 > s_{\kappa=0}^{**} = s_L^{**}$, then the optimal service quality path in (P_L) decreases to s_L^{**}. Because $x_t^* \le s_t^*, \forall t$, this path is feasible for problem (B6) and yields the same value as in problem (P_L). But since the latter is an upper bound, $V^L(s) \ge V^K(s)$ for all s, the optimal path for (P_L) must be optimal for (B6) as well. The case $s_0 < s_{\kappa=1}^{**} = s_G^{**}$ is analogous.

By Proposition 1, s_κ^{**} solves $W'(s; \lambda_\kappa) = 0$ when $\lambda_\kappa = \kappa\lambda_G + (1-\kappa)\lambda_L$. For $\kappa = 0$ (respectively, $\kappa = 1$), the solution to $W'(s; \lambda_\kappa) = 0$ is s_L^{**} (respectively, s_G^{**}). Continuity of $W'(s; \lambda)$ ensures that, for all $s \in [s_G^{**}, s_L^{**}]$, there exists a $\kappa \in [0, 1]$ such that $W'(s; \lambda_\kappa) = 0$; in other words, s is a steady state for (P_κ). By Lemma 2, it is also steady state for problem (B6) and hence for problem (9). $\square$

Proof of Proposition 4. Suppose that the interior steady state exists and is equal to x. It follows that, starting from $s_0 = x$, any deviation from the constant service path $\{x_t \equiv x, \forall t\}$ is not profitable. Consider the two deviations: $X^+(e) = \{x_1 = x + e, x_2 = x - d, x_t = x, t \ge 3\}$ and $X^-(e) = \{x_1 = x - e, x_2 = x + f, x_t = x, t \ge 3\}$; here, $d = (\lambda_L(1-\lambda_G)/(1-\lambda_L))e$ and $f = (\lambda_G(1-\lambda_L)/(1-\lambda_G))e$. It follows that $s_2 = x$ for each path, and their respective profits are

$$V(X^+(e)) = \pi(x + e) + \beta R(\lambda_G x + (1-\lambda_G)(x + e))$$
$$\cdot (\pi(x - d) + \beta R(x)\Pi(x)), \quad (B7)$$

$$V(X^-(e)) = \pi(x - e) + \beta R(\lambda_L x + (1-\lambda_L)(x - e))$$
$$\cdot (\pi(x + f) + \beta R(x)\Pi(x)). \quad (B8)$$

Suboptimality of any deviation from the constant path $\{x\}$ implies that the derivative of $V(X^+(e))$ and $V(X^-(e))$ with respect to e, evaluated at $e = 0$, should be negative. Since $\pi'(x) \le 0$ at a steady state, it follows that

$$\left[\frac{d}{de}V(X^+(e)) + \frac{d}{de}V(X^-(e)) \right]\Big|_{e=0} = \beta(\lambda_L - \lambda_G)$$
$$\cdot \left(R'(x)\Pi(x) - \frac{1 - \lambda_L\lambda_G}{(1-\lambda_G)(1-\lambda_L)}R(x)\pi'(x) \right) \le 0.$$

This expression contradicts $\lambda_L > \lambda_G$, so an interior steady state does not exist. Boundary steady states are ruled out in the same way as in Proposition 1. $\square$

Proof of the Results in §5.3. Part (i) follows from the Euler equation; $s^{**} \in [\underline{s}, \bar{s}]$ by the same argument as in the proof of Proposition 1. Part (ii) follows the same logic as the proof of Proposition 3—in particular, the result of Lemma 2 extends by defining the upper bound problem

$V^\kappa \geq V$ based on the smooth adaptation process $\Lambda_\kappa = \kappa \Lambda^G + (1-\kappa)\Lambda^L \geq \Lambda$ and showing that any optimal constant policy (i.e., steady state) of V^κ is also a steady state for V. (iii) Let $\chi = \Lambda^{-1}$ denote the inverse function of $\Lambda(x, s)$; thus, $\chi(\Lambda(x, s), s) = x$. The Bellman equation is

$$V(s_t) = \max_{s_{t+1} \in \mathbf{s}(s_t)} \{\pi(\chi(s_{t+1}, s_t)) + \beta R(s_{t+1})V(s_{t+1})\}, \quad (B9)$$

where $\mathbf{s}(\cdot)$ represents the corresponding feasible set. Monotonicity of Λ ensures that $\pi'(x_t) \leq 0$ at an optimal solution (i.e., Remark 1 extends), so we can focus on the interval where π is decreasing. Note that $\Lambda(x_t, s_t)$ is increasing in s_t, and so the feasible sets $\mathbf{s}(s_t)$ are ascending. Much as in the proof of Proposition 2, supermodularity of $Q(s_{t+1}, s_t) = \pi(\chi(s_{t+1}, s_t))$ implies monotonicity of the policy function $s^*(\cdot)$, which is a sufficient, albeit not necessary, condition for global stability:

$$Q_{12}(s_t, s_{t+1}) = \pi''(\chi(s_{t+1}, s_t))\chi_1(s_{t+1}, s_t)\chi_2(s_{t+1}, s_t)$$
$$+ \pi'(\chi(s_{t+1}, s_t))\chi_{12}(s_{t+1}, s_t), \quad (B10)$$

where derivatives are denoted by corresponding subscripts. Differentiating $\chi(\Lambda(s_{t+1}, s_t), s_t) = s_{t+1}$ with respect to s_{t+1} and s_t, separately and then together, yields $\chi_1(s_{t+1}, s_t) = 1/\Lambda_1(x_t, s_t) > 0$, $\chi_2(s_{t+1}, s_t) = -\Lambda_2(x_t, s_t)/\Lambda_1(x_t, s_t) \leq 0$, and $\chi_{12}(s_{t+1}, s_t) = (\Lambda_2\Lambda_{11} - \Lambda_{12}\Lambda_1)/[\Lambda_2]^3 \leq 0$, respectively. Since π is decreasing and concave on the relevant domain, it follows that $Q_{12} \geq 0$. The rest of the proof resembles that of Proposition 2. $\square$

PROOF OF PROPOSITION 5. We show (i) for $\rho = 1$; the rest follows by Proposition 3. For a given s_0, consider the optimal path $s_{t+1} = s^*(s_t)$ for all t. By the envelope theorem, for all t, we have

$$\frac{\partial}{\partial \lambda}V(s_t; \lambda) = \frac{s_{t+1} - s_t}{(1-\lambda)^2}\pi'\left(\frac{s_{t+1} - \lambda s_t}{1-\lambda}\right)$$
$$+ \beta R(s_{t+1})\frac{\partial}{\partial \lambda}V(s_{t+1}; \lambda). \quad (B11)$$

In particular, at $s_t = s_{t+1} = s^{**}(\lambda)$, this gives

$$\frac{\partial}{\partial \lambda}V(s; \lambda)\bigg|_{s=s^{**}} = \beta R(s)\frac{\partial}{\partial \lambda}V(s; \lambda)\bigg|_{s=s^{**}} = 0.$$

Suppose $s_0 \geq s^{**}(\lambda)$; the other case is proved similarly. Proposition 2 shows that the optimal service quality path is decreasing; $s_{t+1} \leq s_t$ for all t. Because $\pi'(x_t) \leq 0$ on an optimal path by Remark 1, the first term on the right-hand side of (B11) is positive. Thus, for any $t > 0$, we have

$$\frac{\partial}{\partial \lambda}V(s_0; \lambda) \geq \beta R(s_1)\frac{\partial}{\partial \lambda}V(s_1; \lambda)$$
$$\geq \beta^2 R(s_1)R(s_2)\frac{\partial}{\partial \lambda}V(s_2; \lambda) \geq \cdots \quad (B12)$$
$$\geq \lim_{t\to\infty}\beta^t\left(\prod_{i=1}^{t}R(s_i)\right)\frac{\partial}{\partial \lambda}V(s_t; \lambda) = 0. \quad (B13)$$

The last derivative is bounded as $t \to \infty$ because $s_t \to s^{**}(\lambda)$ (Proposition 2). $\square$

PROOF OF PROPOSITION 6. The result holds because convexity is preserved by maximization and limits. Indeed,

if $R(\cdot)$ is convex, then it follows by induction that the corresponding finite-horizon value function is also convex. By value iteration, we can take limits to obtain that the infinite-horizon value function $V(s)$ is convex; therefore, $V_R(s) = R(s)V(s)$ is convex.

We further show that V_R convex (respectively, concave) is sufficient for the service policy to be increasing (decreasing) and the value function V to be convex (concave), confirming the statement at the end of §6.2. Now, V_R convex (concave) is equivalent to the right-hand side of the Bellman equation, $Q(x, s) = \pi(x) + \beta V_R(\lambda s + (1-\lambda)x)$, being supermodular (submodular). Monotonicity of the optimal policy then follows by Topkis's theorem. Moreover, K convex implies that $Q(x, s)$ is convex in s, so V is convex. On the other hand, the fact that V_R and π are concave implies that $Q(x, s)$ is jointly concave, so V must be concave. But this does not ensure V_R to be concave. $\square$

PROOF OF PROPOSITION 7. (i) For the finite-horizon version of the problem, the result follows by induction because $R_d(x - s)$ is decreasing in s and monotonicity is preserved by maximization. Since monotonicity is preserved by limits, the infinite-horizon result follows by value iteration.

(ii) We characterize the steady state for the smooth version of problem (13), where $R_d(x - s) = R(\rho(x - s))$. The Euler equation states that the following expression, evaluated at $s_t = s_{t+1} = s_{t+2} = s_d^{**}(\rho)$, equals zero:

$$\frac{\partial}{\partial s_{t+1}}\bigg\{\bar{\pi}(s_t, s_{t+1}) + \beta R\left(\frac{\rho(s_{t+1} - s_t)}{1-\lambda}\right)$$
$$\cdot \left(\bar{\pi}(s_{t+1}, s_{t+2}) + \beta F\left(\frac{\rho(s_{t+2} - s_{t+1})}{1-\lambda}\right)\Pi(s_{t+2})\right)\bigg\}. \quad (B14)$$

Equivalently, $\bar{\pi}_2(s, s) + (\rho\beta/(1-\lambda))R'(0)\Pi(s) + \beta R(0) \cdot (\bar{\pi}_1(s, s) - (\rho\beta/(1-\lambda))R'(0)\Pi(s)) = 0$. Because $\Pi(s)(1 - \beta R(0)) = \bar{\pi}(s, s)$, this can be written as $\bar{\pi}_2(s, s) + \beta R(0) \cdot \bar{\pi}_1(s, s) + ((\rho\beta)/(1-\lambda))R'(0)\bar{\pi}(s, s) = 0$. After simplification, this gives the stated result. This implies $s_d^{**} \geq \bar{s} = \underline{s}$ and sensitivity results follow.

(iii) The Bellman equation for the smooth disconfirmation model in (ii) can be written as $V_d(s_t) = \max\{\pi((s_{t+1} - \lambda s_t)/(1-\lambda)) + \beta R(\rho(s_{t+1} - s_t)/(1-\lambda))V_d(s_{t+1})\}$. Concavity of π and R, together with V_d decreasing (from part (i)), implies that the function on the right-hand side of the Bellman equation is supermodular in (s_t, s_{t+1}). Therefore the optimal state policy $s_d^*(\cdot)$ is increasing, and so all transient results follow by the same logic as in Proposition 2.

(iv) The proof for the loss-averse case mimics that of Proposition 3. Defining $R_\kappa(x - s) = R((\kappa + (1-\kappa)\rho)(x - s)) \geq R_d(x - s)$ with $\kappa \in [0, 1]$, the smooth problems based on R_κ provide value function upper bounds $V_\kappa \geq V_d$. By the logic in Lemma 2, their steady states (characterized previously) span the range $[s_d^{**}(1), s_d^{**}(\rho)]$ and are also steady states for our kinked problem. The proof of the gain-seeking case ($\rho < 1$) mimics that of Proposition 4 on the paths: $X^+(e) = \{x_1 = x + e, x_2 = x + e(1-\lambda), x_t = x, t \geq 3\}$ and $X^-(e) = \{x_1 = x - e, x_2 = x - e(1-\lambda), x_t = x, t \geq 3\}$. $\square$

PROOF OF PROPOSITION 8. Monotonicity of the value function follows the same logic as for Propositions 1(i) and 7(i). Writing the model with respect to $S = \lambda s + (1 - \lambda)x$, as

$V(s) = \max_S \{\bar{\pi}(s, S) + \beta \bar{R}(s, S)V(S)\}$, where $\bar{R}(s, S) = R(x, s)$, the corresponding Euler equation is

$$\frac{\partial}{\partial s_{t+1}} \big\{ \bar{\pi}(s_t, s_{t+1}) + \beta \bar{R}(s_t, s_{t+1})(\bar{\pi}(s_{t+1}, s_{t+2}) + \beta \bar{R}(s_{t+1}, s_{t+2})\Pi(s_{t+2})) \big\}_{s_j = s^{**}} = 0.$$

Omitting arguments for simplicity, this equation can be written as $\bar{\pi}_2 + \beta\bar{R}_2\Pi + \beta\bar{R}\bar{\pi}_1 + \beta^2\bar{R}_1\bar{R}\Pi = 0$. After substituting $\beta\bar{R}\Pi = \Pi - \pi$ in the last term, we obtain that s^{**} solves

$$\bar{\pi}_2 + \beta\bar{R}\bar{\pi}_1 + \beta(\bar{R}_1 + \bar{R}_2)\Pi - \beta\bar{R}_1\pi = 0. \tag{B15}$$

Differentiating $\Pi(s) = \bar{\pi}(s, s)/(1 - \beta\bar{R}(s, s))$ gives $(1 - \beta\bar{R}) \cdot \Pi' = \bar{\pi}_1 + \bar{\pi}_2 + \beta(\bar{R}_1 + \bar{R}_2)\Pi$, which allows us to write (B15) as $(1 - \beta\bar{R})(\Pi'(s) - \bar{\pi}_1(s, s)) = \beta\bar{R}_1\pi$, or $\Pi'(s) - \bar{\pi}_1(s, s) = \beta\bar{R}_1\Pi$. Because $\bar{\pi}_1(s, s) = -(\lambda/(1 - \lambda))\pi'(s)$, and $\bar{R}_1 = R_2 - (\lambda/(1 - \lambda))R_1$, we obtain, after some algebra, that the steady state s^{**} satisfies $\Pi'(s)(1 - \lambda\beta R(s, s)) = \beta R_2(s, s)\Pi(s)$. It follows that the sign of $R_2(s^{**}, s^{**})$ determines that of $\Pi'(s^{**})$ and hence also the relationship between s^{**} and $\bar{s} = \arg\max\Pi$, as well as the monotonicity of $s^{**}(\lambda)$. $\square$

PROOF OF PROPOSITION 9. We follow the same steps used to obtain the Euler equation (B3) in the proof of Proposition 1. We obtain $\Pi'(s) = \bar{\pi}_1(s, s) = \pi_2(s, s) - (\lambda/(1 - \lambda))\pi_1(s, s)$, or equivalently, $w(s; \lambda) = \lambda\pi'(s) + (1 - \lambda)\Pi'(s) - (1 - \lambda)\pi_2(s, s) = 0$, where we denote $\pi(s) = \pi(s, s)$. For all $s > \bar{s} \geq \underline{s}$, we have $\pi'(s) < 0$ and $\Pi'(s) < 0$, and so $w(s) < 0$, because $\pi_2 \geq 0$; this shows $s^{**} \leq \bar{s}$. That s^{**} is decreasing in λ follows because $(\partial/\partial\lambda)w(s; \lambda) = \pi'(s) - \Pi'(s) + \pi_2(s, s)$, which evaluated at s^{**} equals $\pi'(s^{**})/(1 - \lambda) \leq 0$, using the Euler equation.

Transient results and global stability (Proposition 2) extend, e.g., if $\bar{\pi}(s, S) = \pi((S - \lambda s)/(1 - \lambda), s)$ is supermodular in (s, S), in particular if $\pi(x, s)$ is supermodular in (x, s) and concave in x. This ensures that the state policy $s^*(\cdot)$ is increasing, a condition that is sufficient but not necessary for global stability (showing that $s^*(\cdot)$ is a contraction mapping would suffice). It is also easy to see that Proposition 3(ii) extends, because Remark 1 does; part (i) holds if $\pi(x, s)$ is convex in s. $\square$

PROOF OF PROPOSITION 10. Let $G_0(k) = \mathbb{P}(\tilde{\theta}_0 \geq k)$, $G_t(k; X) = \mathbb{P}(\tilde{\theta}_t(X) \geq k)$. Because R is increasing in θ, $G_{t+1}(k; X) = (\mathbb{E}[R(s_{t+1}; \tilde{\theta}_t) \mid \tilde{\theta}_t \geq k]/\mathbb{E}[R(s_{t+1}; \tilde{\theta}_t)])G_t(k; X) \geq G_t(k; X)$, i.e., $\tilde{\theta}_t \preceq_{FSD} \tilde{\theta}_{t+1}$ (in fact, $\tilde{\theta}_t \preceq_{LR} \tilde{\theta}_{t+1}$). Thus, for any $k, X, G_t(k; X) \in [0, 1]$ is increasing in t, so it converges: $\lim_{t \to \infty} G_t(k; X) = G^*(k; X)$, i.e., $\tilde{\theta}_t(X) \to \tilde{\theta}^*(X)$ pointwise, where $\mathbb{P}(\tilde{\theta}^*(X) \geq k) = G^*(k; X)$.

For $k < \theta_H$, $G^*(k; X) \geq G_t(k; X) \geq G_0(k) > 0$, and taking limits in Bayes rule, $1 = \lim_t(G_{t+1}(k; X)/G_t(k; X)) = \lim_t(\mathbb{E}[R(s_{t+1}; \tilde{\theta}_t) \mid \tilde{\theta}_t \geq k]/\mathbb{E}[R(s_{t+1}; \tilde{\theta}_t)])$. Any feasible state path $\{s_t\}$ is bounded so it contains a convergent subsequence $s_{t_j} \to s^*$; then $1 = \mathbb{E}[R(s^*; \tilde{\theta}^*) \mid \tilde{\theta}^* \geq k]/\mathbb{E}[R(s^*; \tilde{\theta}^*)]$, $\forall k \in [\theta_L, \theta_H)$; i.e., $(1 - G^*(k; X))(1 - \mathbb{E}[R(s^*; \tilde{\theta}^*) \mid \tilde{\theta}^* < k]$ $\mathbb{E}[R(s^*; \tilde{\theta}^*) \mid \tilde{\theta}^* \geq k]) = 0$. Because R is strictly increasing, the second term is strictly positive, so $G^*(k; X) = 1 \forall k \neq \theta_H$; i.e., $\tilde{\theta}^*(X)$ is a point mass at θ_H for any X. Finally, if the optimal policy $X^* = \{x_t^*\}$ converges (as the customer renews), then so does the corresponding state path; its limit

$(s^{**}, \tilde{\theta}^{**} = \tilde{1}_{\theta_H})$ must be a steady state, so $s^{**} = s^{**}(\theta_H)$ from the Euler equation.

For $\tilde{\theta} = \tilde{s}_0$, we separate the uncertainty from the state by defining the deterministic *delivered* service quality $s_{t+1} = \lambda s_t + (1 - \lambda)x_t$, with $s_0 = 0$. Then $\tilde{s}_{t+1} = \lambda\tilde{s}_t + (1 - \lambda)x_t = s_{t+1} + \lambda^{t+1}\tilde{\theta}_t$, where $\tilde{\theta}_{t+1} \sim (\tilde{\theta}_t \mid \text{customer renews})$. To write a stationary recursion, we use $\tilde{\xi}_t = \lambda^t\tilde{\theta}_t$ instead of $\tilde{\theta}_t$ as a state variable, so $R(\tilde{s}_{t+1}) = R(s_{t+1} + \lambda\tilde{\xi}_t) = R(s_{t+1}; \tilde{\xi}_t)$ is stationary, and we obtain

$$\tilde{V}(s_t, \tilde{\xi}_t) = \max \pi(x_t) + \beta\mathbb{E}_{\tilde{\xi}_t}[R(s_{t+1}; \tilde{\xi}_t)]\tilde{V}(s_{t+1}, \tilde{\xi}_{t+1}), \tag{B16}$$

where $s_{t+1} = \lambda s_t + (1 - \lambda)x_t$ and $\tilde{\xi}_{t+1} = \lambda(\tilde{\xi}_t \mid \text{customer renews})$. The Bellman equation (B16) determines the unique bounded value function $\tilde{V}$ in (A1), and an optimal stationary policy $x^*(s, \tilde{\xi})$ exists and solves (B16) (Bertsekas and Shreve 1996). The result follows like above, using $\tilde{\theta}_t \succeq_{FSD} \tilde{\theta}_{t-1}$. $\square$

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