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Salesforce Compensation with Inventory Considerations

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We study a scenario in which a firm designs the compensation contract for a salesperson who exerts unobservable effort to increase the level of uncertain demand and, jointly, the firm also decides the inventory level to be stocked. We use a newsvendor-type model in which actual sales depend on the realized demand but are limited by the inventory available, and unfulfilled demand cannot be observed. In this setup, under the optimal contract, the agent is paid a bonus for meeting a sales quota. Our key result is that it may be optimal for the firm to stock more than the first-best inventory level, because this enables the firm to obtain a more precise indicator of the salesperson's effort. The possibility of stockouts due to limited inventory also leads to several counterintuitive results, including the following: (i) relative to when stockouts are not considered, it may be optimal for the firm to pay a higher bonus even though limited inventory constrains sales; (ii) as inventory becomes more expensive, thereby forcing the firm to lower its inventory, the firm may nevertheless pay the agent a higher bonus; and (iii) if there is a lower probability that the agent's effort exertion leads to high demand, rather than lowering inventory due to the lower sales potential, the firm may increase inventory.

Key words: salesforce compensation; inventory; marketing–operations interface

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1. Introduction

Firms employ salespeople to manage and increase product sales, and use incentive contracts to align the interests of the salesforce with those of the firm. In 2006, firms in the United States spent \$800 billion on salesforce compensation, which was three times the amount spent on advertising (Zoltners et al. 2008). There is an extensive literature that has significantly enhanced our understanding of the issues involved in designing salesforce compensation contracts (Basu et al. 1985, Bhardwaj 2001, Chung et al. 2013, Felli and Villas-Boas 2000, Godes 2004, Hauser et al. 1994, Jain 2012, Jerath et al. 2010, Joseph and Kalwani 1998, Kishore et al. 2013, Misra and Nair 2011, Prendergast 1999, Raju and Srinivasan 1996, Simester and Zhang 2010, Steenburgh 2008). This literature has typically assumed that there are no availability constraints, i.e., whereas demand is uncertain, realized demand is always fulfilled. However, as any manager will attest, this is not true in many practical situations because demand fulfillment is restricted by product availability determined by the inventory in stock at the time sales are realized.

Consider the example of a company selling office products, such as stationery and furniture, which are purchased from manufacturers and stocked in a

local warehouse. Sales agents have the job of increasing demand for the stocked products using samples and catalogs, and are compensated based on sales achieved. Ideally, the generated demand should match the inventory stocked because if demand is below inventory level, the firm incurs a cost for the leftover inventory, and if demand is above inventory level, the salesperson's effort is wasted. It is typically not possible to keep track of demand that was, or could have been, realized but was not fulfilled due to lack of inventory, especially if customers choose not to order or to postpone their purchase rather than back-order the product.¹ As another example, consider a typical car dealership. There is an inventory of cars available at the dealership, and there is high pressure on the sales agents to clear this inventory. On the other hand, generating more demand than the available inventory would lead to unmet demand, and the effort of the salesperson would be wasted. The compensation plan for the sales agents often comprises of a sales target and a bonus is paid if the agent meets the sales target. In the presence of inventory considerations, the inventory level is often an important determinant of the sales target. For instance, in car

¹ We thank Serguei Netessine for suggesting this example.

dealerships, salespeople are often given the explicit target of clearing the inventory in the monthly or quarterly cycle if they want to avail a bonus.

In this paper, we study the salesforce compensation problem in the presence of inventory considerations. We consider a newsvendor-like scenario in which a firm has to match supply with uncertain demand. The firm employs a salesperson whose job is to increase the level of demand through his sales efforts. The firm has to decide the compensation plan of the salesperson and, based on how much demand-enhancing effort the firm will be able to induce from the salesperson, it has to decide the inventory to be stocked. The firm cannot observe the salesperson's actions and demand is uncertain, so that a moral hazard problem arises, which the firm must solve using an outcome-based incentive contract. Therefore, our setting is similar to a standard salesforce compensation problem, but with the critical difference that the firm also has to consider the inventory aspect. The presence of inventory constraints not only leads to sales being limited by the inventory stocked, it also leads to the issue of imperfect observability of demand; i.e., if realized demand happens to be above the inventory level, the firm cannot track lost demand and, therefore, cannot determine the actual level of realized demand. This is also known as "demand censoring" (see Braden and Freimer 1991, Besbes and Muharremoglu 2013; however, these papers do not consider agency issues). If the salesperson works hard to increase the probability of high demand, and it turns out to be above the inventory level, the firm cannot determine how much above the inventory level the realized demand was. This cripples the firm in terms of how well it can reward the salesperson, because for any demand realization above the inventory level, the firm effectively has to pay the salesperson what he would be paid if the realized demand had just met the inventory level. This reduces the salesperson's motivation to work hard to increase demand.

We study the salesforce compensation problem under inventory considerations and find that the interplay between the two provides novel insights for both the salesforce compensation aspect and the inventory management aspect. We find that the firm always chooses a contract to reward the salesperson for achieving the highest observable demand outcome; i.e., the firm uses a quota-bonus contract.² This, however, leads to an inefficiency when the optimal

inventory level is below the highest possible demand outcome, because the highest possible demand outcome is not observable in this case. Therefore, a key result we find is that, to improve the observability of realized demand and obtain a better signal of the salesperson's effort, the firm can have the incentive to overstock inventory. This, however, is beneficial only for nonextreme, i.e., medium, inventory costs—in the case of high inventory cost, the benefit from increased salesperson effort due to increased demand observability is not enough to offset the extra costs due to overstocking inventory, whereas in the case of low inventory cost, the firm already stocks large enough to allow sufficient demand observability.

In addition, we find that to motivate the salesperson to work hard in the situation where high demand outcomes are censored due to limited inventory, the firm can have the incentive to use a higher-powered incentive contract; i.e., the firm gives a larger bonus to the agent. This provides the counterintuitive insight that as inventory cost increases and the firm stocks lower inventory, it pays the salesperson a higher bonus even though it must reduce the sales threshold needed for that bonus. Another counterintuitive insight we obtain is that, if there is a lower probability that the agent's effort exertion leads to high demand, under certain conditions, the firm stocks a higher inventory level.

We add to a thin but growing literature on jointly modeling incentive issues and inventory considerations. Chen (2000, 2005) focuses on designing contracts for sales agents such that inventory can be managed more effectively by smoothing demand and eliciting more information about market conditions. Plambeck and Zenios (2003) develop a principal-agent model for a make-to-stock production system, in which they establish an optimal incentive contract for a production manager. Saghaian and Chao (2011) develop a model with moral hazard and adverse selection to determine a commission-based contract for a salesperson in a dynamic inventory setting, where the contract is renegotiated in every time period. Jerath et al. (2010) study how the marketing and operations departments in a firm can be coordinated using incentive contracts. More generally, we add to the broader literature on the marketing and operations interface (Balasubramanian and Bhardwaj 2004, Cachon 2003, Desai et al. 2007, Eliashberg and Steinberg 1987, Ho and Tang 2004, Iyer et al. 2007, Netessine and Taylor 2007, Pasternack 1985, Shulman et al. 2009).

Our work is related to that of Chu and Lai (2013), who study a similar problem of salesforce compensation under limited inventory. As in their paper, we find that the firm is biased toward stocking more

² We note that a quota-bonus contract can be optimal even without limited inventory (Laffont and Martimort 2001). However, the subtle difference is that without limited inventory, the highest *observable* demand outcome and the highest *possible* demand outcome are the same, whereas with limited inventory, the highest observable demand outcome (for which the bonus is awarded) may be lower than the highest possible demand outcome.

inventory than first best.³ However, in the framework used by Chu and Lai (2013), the support of the distribution of outcomes may depend on the amount of effort exerted by the agent. Therefore, in their framework, if the agent does not exert the optimal postulated effort level (i.e., he “shirks”), there is a nonzero probability that the outcome reveals that the agent shirked, and he can be punished strongly in this situation. In other words, the firm can always achieve the first-best outcome by using sufficiently large off-equilibrium punishments (as per the “Mirrlees argument”; see Laffont and Martimort 2001, p. 163). Such contracts, which Chu and Lai (2013) rule out by assumption, may nevertheless be possible in reality under some circumstances. For instance, if the outcome reveals that effort was suboptimal, then the firm may be able to punish the agent by firing him; otherwise he will be paid according to a particular compensation schedule (Harris and Raviv 1979).⁴ In our framework, the support of the sales distribution is independent of effort, so the firm is never able to learn the agent’s effort level with certainty, and the Mirrlees argument does not apply. Consequently, in our framework, the firm can never achieve the first-best outcome.

The remainder of this paper is organized as follows. In §2, we present our modeling framework. In §3, we analyze two benchmark scenarios that consider agency considerations and inventory considerations one at a time. In §4, we analyze the full scenario with both agency and inventory considerations. In §5, we analyze the impact of agency and inventory considerations by comparing the results for the full scenario to those for the benchmark scenarios. In §6, we conclude.

2. Model

We consider a risk-neutral firm selling one product. The firm employs a salesperson whose job is to

exert effort to increase the demand for the product. Demand is uncertain and the realization of demand, denoted by ξ , can be H (high), M (medium), or L (low), where $H > M > L > 0$. The salesperson’s effort level, denoted by e , can be either e_H (high effort) or e_L (low effort); i.e., he either “works” or “shirks.” The salesperson’s effort influences the distribution of the demand in the following way. For $s \in \{H, M, L\}$,

$$\Pr(\xi = s | e) = \begin{cases} p_s & \text{if } e = e_H, \\ q_s & \text{if } e = e_L, \end{cases} \quad \text{where} \\ \sum_{s \in \{H, M, L\}} p_s = 1 \text{ and } \sum_{s \in \{H, M, L\}} q_s = 1.$$

Following the principal-agent literature, we assume that these probabilities satisfy the monotone likelihood ratio property (MLRP); i.e.,

$$\frac{p_L}{q_L} < \frac{p_M}{q_M} < \frac{p_H}{q_H}. \quad (1)$$

The MLRP assumption, which is commonly made in the principal-agent literature (see Laffont and Martimort 2001, p. 164), implies that a higher-demand outcome is a more reliable indicator that the salesperson exerted effort e_H rather than effort e_L . In addition, the MLRP assumption (along with the fact that $\sum_{s \in \{H, M, L\}} p_s = 1$ and $\sum_{s \in \{H, M, L\}} q_s = 1$) implies that $p_H > q_H$ and $p_L < q_L$. This implies that when the salesperson exerts the high effort level rather than the low effort level, the high-demand outcome will be more likely to occur, and the low-demand outcome will be less likely to occur. Note that p_M could be higher or lower than q_M . We also assume $p_L > 0$, which ensures that the support of outcomes is invariant with effort.

Models with discrete outcomes and/or discrete effort levels have been used frequently in agency theory research in marketing and economics (e.g., Felli and Villas-Boas 2000, Fudenberg and Tirole 1990, Godes 2004, Simester and Zhang 2010). As a stylized example of a scenario where discretization of outcomes and effort may be appropriate for modeling purposes, consider a pharmaceutical salesperson on a detailing visit to a physician. He may simply drop off reading materials with the physician (low effort), or may spend a significant amount of time and effort explaining the effectiveness of the drug to the physician through advanced materials and details of clinical trial results (high effort). Based on the detailing visit, the physician may choose to prescribe the drug to none or a minimal number of her patients (low demand), a significant fraction of her patients (medium demand), or all her patients (high demand). More effort by the salesperson is expected to lead to more prescriptions by the physician.⁵

³ The primary result in Chu and Lai (2013) is that a quota-based bonus contract is optimal for the firm, and under different conditions, this contract can or cannot achieve the first best. Park (1995), Kim (1997), and Oyer (2000) also show quota-bonus contracts to be optimal and discuss when they can or cannot achieve the first best, though these papers do not consider inventory. Furthermore, in Chu and Lai (2013), when the firm cannot achieve the first best, it may stock inventory to maintain a service level higher than in the first best. This overlaps with our results, with the difference that in our framework the first best is never achievable, and the firm maintains a higher service level for medium inventory cost.

⁴ As a more concrete example, the firm may know that if a salesperson exerts optimal effort, then he should be able to generate, say, at least 10 sales leads per month. Then, if less than 10 sales leads are generated, the firm can be certain that the salesperson must have shirked and he can be fired, whereas if 10 or more sales leads are generated, then the salesperson is paid according to the agreed-upon compensation schedule.

⁵ We thank Christophe Van den Bulte for suggesting this example.

We assume that the salesperson is effort averse: when the salesperson chooses the effort level e_H , he incurs an effort cost of $\psi > 0$; when the salesperson chooses the effort level e_L , the corresponding effort cost is normalized to zero. To ensure that the firm will want to motivate high effort from the salesperson, we assume that ψ is small enough; we present the exact conditions when we present our analysis. We assume that the salesperson is risk neutral but has limited liability; i.e., his salary must be nonnegative under any outcome of demand.⁶ Limited liability is a distinct feature of salesforce contracts in the industry, and this assumption has also been widely made in previous literature (Kim 1997, Oyer 2000, Park 1995, Sappington 1983, Simester and Zhang 2010). Limited liability on the part of the salesperson models his tendency to avoid downside risks. We normalize the salesperson's reservation utility (due to, say, his outside options) to zero without loss of generality.

The firm stocks an inventory level denoted by Q . We model the cost associated with ordering inventory Q as cQ , where $c > 0$ is the per-unit marginal cost. We assume that every unit sold generates a revenue of $r > c$. The sales after demand realization cannot exceed the inventory level. One can think of this as being a product with a long-enough lead time such that if realized demand exceeds available inventory, then new units cannot be ordered fast enough to meet the extra demand. If there is a mismatch between realized demand and inventory, then, if demand is lower than inventory, the surplus inventory is wasted, whereas if demand is higher than inventory, the firm has to turn away the unserved customers. We assume that when realized demand is higher than inventory, the firm cannot observe the actual number of unmet demand units (which is the phenomenon of “demand censoring”). Therefore, due to limited inventory, the observability of realized demand is imperfect.

The timing of the game is as follows. First, the firm jointly decides the inventory level, Q , that it will stock and the compensation contract of the salesperson. Then the salesperson accepts or rejects the contract. If he accepts, he exerts effort e , which is unob-

servable to the firm. Following this, demand and sales are realized, and the salesperson is paid.

To rule out the trivial case in which the firm would choose $Q < M$ (in which case the firm does not need to motivate any effort from the salesperson, and the incentive problem is moot), we define the following:

$$C = (p_H + p_M)r - \frac{\psi}{(M - L)(1 - (q_H + q_M)/(p_H + p_M))}$$

and we make the assumption that $c < C$, i.e., the marginal cost is not too high. This is a sufficient assumption to ensure that the firm will not choose an inventory level strictly lower than M (in both the Benchmark 2 scenario and in the full scenario).

3. Benchmark Scenarios

In this section, we establish two benchmarks. The first benchmark is a scenario in which the firm employs a salesperson to enhance demand and cannot observe his effort, but we assume away any inventory considerations. This is the scenario that the existing literature has typically considered. The second benchmark is the first-best scenario with a salesperson and with inventory considerations. That is, the firm employs a salesperson to enhance demand, and it faces the newsvendor problem of limited supply; however, the firm can perfectly observe the salesperson's effort so that there is no moral hazard problem.

3.1. Benchmark 1: Incentive Contract Without Inventory Considerations

In this scenario, moral hazard is present, but inventory considerations are absent. That is, the effort of the salesperson is unobservable, and the firm has to use an incentive contract, but the firm can perfectly match supply with realized demand. Because of MLRP, it suffices to consider the following compensation scheme: the salesperson is paid $S_{H,b1} > 0$ if the outcome is H , and is paid zero if the outcome is M or L (see the appendix for details). The subscript $b1$ stands for “Benchmark 1.” This compensation scheme always satisfies the limited liability constraint. The firm's contracting problem can be written as

$$\begin{aligned} \max_{S_{H,b1} > 0} \quad & p_H[(r - c)H - S_{H,b1}] + p_M(r - c)M + p_L(r - c)L \\ \text{s.t.} \quad & p_H S_{H,b1} - \psi \geq q_H S_{H,b1} - 0, \end{aligned} \quad (2)$$

where (2) is the incentive compatibility constraint that ensures that the salesperson prefers to choose the effort level e_H rather than e_L . The participation constraint, given by $p_H S_{H,b1} - \psi \geq 0$, is implied by (2), as follows:

$$p_H S_{H,b1} - \psi = \underbrace{p_H S_{H,b1} - \psi - q_H S_{H,b1}}_{\geq 0} + \underbrace{q_H S_{H,b1}}_{\geq 0} \geq 0.$$

⁶ We note here that in Chu and Lai (2013) the support of the demand distribution may depend on salesperson effort, so they need the limited liability assumption off the equilibrium path as well; otherwise, the firm can *always* achieve the first-best outcome. To see this, note that in their model, a low sales outcome can definitively reveal that the agent exerted low effort. The “Mirrlees argument” then suggests that because there is a positive probability that the firm can with certainty discover that the agent chose a low effort level, the firm can impose an extreme penalty that prevents the agent from shirking. Such concerns do not arise in our model because the support of the demand distribution is independent of effort, and the firm can *never* detect shirking by the agent. Therefore, we do not need the limited liability assumption off the equilibrium path.

Solving the above program gives the following optimal payment if high demand is realized:

$$S_{H,b1}^* = \frac{\psi}{p_H - q_H}. \quad (3)$$

The expected payment to the salesperson in this scenario is

$$ES_{b1}^* = \frac{\psi}{1 - q_H/p_H}, \quad (4)$$

and the expected profit of the firm is

$$(r - c)(p_H H + p_M M + p_L L) - \frac{\psi}{1 - q_H/p_H}. \quad (5)$$

The following condition on ψ is sufficient to ensure that the firm wants to motivate high effort from the salesperson (see the appendix for details):

$$\psi \leq (1 - q_H/p_H)(r - c) \cdot ((p_H - q_H)H + (p_M - q_M)M + (p_L - q_L)L). \quad (6)$$

We now formally state the key result for this scenario as follows.

RESULT 1. *The firm pays the salesperson a reward of $\psi/(p_H - q_H)$ if the realization of demand is H , and zero otherwise.*

We note that the above outcome is clearly not the first-best outcome for the firm. In the first-best outcome, the firm would pay the salesperson exactly ψ to induce high effort, whereas in this outcome the expected amount that the firm pays to the salesperson is $\psi/(1 - q_H/p_H)$, which is greater than ψ . This reduces the firm's expected profit compared to the first-best expected profit.

3.2. Benchmark 2: First-Best With Inventory Considerations

In this scenario, inventory considerations are present, but moral hazard is absent. That is, the firm has to make the inventory decision, and sales are limited by this inventory, but the firm does not have to determine an incentive contract for the salesperson, and the salesperson only has to be compensated for his effort. This is therefore the first-best scenario with inventory, in which the firm can observe the salesperson's effort level and can simply pay a fixed amount to cover the salesperson's effort cost. Since the firm only has to make the inventory decision, its problem can be written as

$$\max_{M \leq Q_{b2} \leq H} \{p_H r \min\{Q_{b2}, H\} + p_M r \min\{Q_{b2}, M\} + p_L r \min\{Q_{b2}, L\} - cQ_{b2} - \psi\},$$

where the subscript $b2$ stands for "Benchmark 2." Note that the first-best inventory level must be either

H or M . To see this, suppose that the firm chooses an inventory level $Q_{b2} \in (M, H)$. Then the firm's expected profit is $(p_H r - c)Q_{b2} + p_M r M + p_L r L - \psi$. If $p_H r - c < 0$, this profit is dominated by choosing $Q_{b2} = M$, and if $p_H r - c \geq 0$, it is (weakly) dominated by choosing $Q_{b2} = H$. We obtain the first-best inventory level as follows:

$$Q^{FB} = Q_{b2}^* = \begin{cases} H & \text{if } c < \bar{c}, \\ M & \text{if } \bar{c} \leq c < C, \end{cases} \quad \text{where } \bar{c} = p_H r.$$

The firm's expected profit is

$$\pi^{FB} = \pi_{b2}^* = \begin{cases} r(p_H H + p_M M + p_L L) - cH - \psi & \text{if } c < \bar{c}, \\ r((p_H + p_M)M + p_L L) - cM - \psi & \text{if } \bar{c} \leq c < C. \end{cases} \quad (7)$$

The following condition on ψ is sufficient to ensure that the firm wants to motivate high effort from the salesperson (see the appendix for details):

$$\psi \leq r((p_H + p_M - q_H - q_M)M + (p_L - q_L)L) - c(H - L). \quad (8)$$

We now formally state the key result for this scenario as follows.

RESULT 2. *If $c < \bar{c}$, the firm stocks inventory equal to H , and if $\bar{c} \leq c < C$, the firm stocks inventory equal to M . The firm directs the salesperson to exert high effort and pays him exactly ψ to reimburse his effort cost.*

We see that, in the first-best scenario, the firm stocks high inventory ($Q^{FB} = H$) when the marginal inventory cost is low, and reduces the inventory stocked ($Q^{FB} = M$) if the marginal inventory cost is high enough ($c \geq \bar{c}$). In addition, the profit of the firm is decreasing in the inventory cost. These results conform to standard intuition.

4. Full Scenario: Incentive Contract With Inventory Considerations

After establishing the benchmark scenarios, we now analyze the scenario of focal interest in which both moral hazard and inventory considerations are present simultaneously. In this scenario, the firm cannot observe the salesperson's effort level. In addition, the firm also faces the newsvendor problem of sales being limited by the inventory stocked. In the remainder of this paper, we refer to this scenario as the full scenario.

To derive the optimal incentive contract, we first analyze the firm's problem conditioning on the inventory level, and then combine the cases to determine the optimal inventory level and compensation scheme.

Case a. If the firm chooses $Q = H$, then the firm can always observe the actual demand realization. The optimal contract is as follows: the salesperson is paid a reward of $S_\Omega^* = \psi/(p_H - q_H)$ if the quantity sold is H (where Ω denotes the highest possible sales outcome, which is H in this case), and 0 otherwise. The firm's expected profit is

$$\begin{aligned}\pi_{Q=H}^* &= p_H r H + p_M r M + p_L r L - c H - p_H \frac{\psi}{p_H - q_H} \\ &= p_H r H + p_M r M + p_L r L - c H - \frac{\psi}{1 - q_H/p_H}.\end{aligned}$$

Case b. If the firm chooses $Q = M$, then when the quantity sold turns out to be M , due to demand censoring, the firm cannot tell whether the actual realization is H or M . The optimal contract is as follows: the salesperson is paid a reward of $S_\Omega^* = \psi/((p_H + p_M) - (q_H + q_M))$ if the quantity sold is M (where Ω denotes the highest possible sales outcome, which is M in this case), and 0 otherwise. (Note that $p_L < q_L$ implies that $p_H + p_M > q_H + q_M$.) The firm's expected profit is

$$\begin{aligned}\pi_{Q=M}^* &= (p_H + p_M) r M + p_L r L - c M \\ &\quad - (p_H + p_M) \frac{\psi}{(p_H + p_M) - (q_H + q_M)} \\ &= (p_H + p_M) r M + p_L r L - c M \\ &\quad - \frac{\psi}{1 - (q_H + q_M)/(p_H + p_M)}.\end{aligned}$$

Case c. If the firm chooses $M < Q < H$, then this would be (weakly) dominated by Cases a and b. Below we show the reason for this. In this case, the firm can perfectly infer the realized demand from the selling quantity, and the expected optimal payment to the salesperson is $\psi/(1 - q_H/p_H)$. The firm's expected profit is

$$\pi_{M < Q < H}^* = (p_H r - c)Q + r(p_M M + p_L L) - \frac{\psi}{1 - q_H/p_H}.$$

Note that it must be the case that $p_H r - c \geq 0$; otherwise, the firm can increase its expected profit by lowering its inventory position to an arbitrary quantity $Q' = Q - \epsilon$, where $0 < \epsilon < Q - M$. But $p_H r - c \geq 0$ means that the firm can be (weakly) better off by ordering H units of inventory instead, because the firm's profit is weakly increasing in Q , and when the firm orders H units of inventory, the expected payment to the salesperson remains the same.

The justification for the argument that the salesperson should be given a reward only for the highest output outcome in each of Cases a–c is provided in the appendix.

Comparing the firm's expected profit for Cases a and b in the preceding analysis, we can determine the firm's optimal inventory level as follows:

$$Q^* = \begin{cases} H & \text{if } c < \hat{c}, \\ M & \text{if } \hat{c} \leq c < C, \end{cases} \quad \text{where } \hat{c} = p_H r + \frac{\psi}{H - M} \cdot \left(\frac{1}{1 - (q_H + q_M)/(p_H + p_M)} - \frac{1}{1 - q_H/p_H} \right).$$

The following lemma compares \hat{c} and \bar{c} .

LEMMA 1. $\hat{c} > \bar{c}$.

PROOF. The MLRP assumption implies that $q_H/p_H < q_M/p_M$. Therefore, we have

$$\begin{aligned}& \frac{1}{1 - (q_H + q_M)/(p_H + p_M)} - \frac{1}{1 - q_H/p_H} \\ &= \frac{1}{(1 - (q_H + q_M)/(p_H + p_M))(1 - q_H/p_H)} \\ &\quad \cdot \frac{q_M/p_M - q_H/p_H}{1 + p_H/p_M} > 0,\end{aligned}$$

which implies that $\hat{c} > p_H r = \bar{c}$. Q.E.D.

The reward for achieving the highest possible sales outcome, which also clears the inventory stocked, is

$$S_\Omega^* = \begin{cases} \frac{\psi}{p_H - q_H} & \text{if } c < \hat{c}, \\ \frac{\psi}{(p_H + p_M) - (q_H + q_M)} & \text{if } \hat{c} \leq c < C, \end{cases} \quad (9)$$

and the expected payment to the salesperson is, therefore,

$$S^* = \begin{cases} \frac{\psi}{1 - q_H/p_H} & \text{if } c < \hat{c}, \\ \frac{\psi}{1 - (q_H + q_M)/(p_H + p_M)} & \text{if } \hat{c} \leq c < C. \end{cases} \quad (10)$$

The firm's expected profit is

$$\pi^* = \begin{cases} r(p_H H + p_M M + p_L L) - c H - \frac{\psi}{1 - q_H/p_H} & \text{if } c < \hat{c}, \\ r((p_H + p_M)M + p_L L) - c M - \frac{\psi}{1 - (q_H + q_M)/(p_H + p_M)} & \text{if } \hat{c} \leq c < C. \end{cases} \quad (11)$$

The following condition on ψ is sufficient to ensure that the firm wants to motivate high effort from the salesperson:

$$\begin{aligned}\psi &\leq \left(1 - \frac{q_H + q_M}{p_H + p_M}\right) (r((p_H + p_M) - q_H - q_M)M \\ &\quad + (p_L - q_L)L) - c(H - L). \quad (12)\end{aligned}$$

The analysis to obtain (12) is on the lines of the analysis to obtain (8).

We summarize the above solution in the following proposition.

PROPOSITION 1. *If $c < \hat{c}$, the firm stocks inventory equal to H and pays the salesperson a reward of $\psi/(p_H - q_H)$ to clear this inventory (with an expected reward of $\psi/(1 - q_H/p_H)$), and if $\hat{c} \leq c < C$, the firm stocks inventory equal to M and pays the salesperson a reward of $\psi/((p_H + p_M) - (q_H + q_M))$ to clear this inventory (with an expected reward of $\psi/(1 - (q_H + q_M)/(p_H + p_M))$). As inventory cost increases, the firm can have the incentive to pay a higher bonus for a lower sales threshold.*

The above solution has the following characteristics. First, similar to the first best in Benchmark 2, the firm stocks high inventory ($Q^* = H$) when the marginal inventory cost is low, and reduces the inventory stocked ($Q^* = M$) if the marginal inventory cost is large enough ($\hat{c} \leq c < C$).

Second, the reward for achieving the highest possible sales outcome and clearing the inventory is $\psi/(p_H - q_H)$ for $c < \hat{c}$, and it is $\psi/((p_H + p_M) - (q_H + q_M))$ for $\hat{c} \leq c < C$. The latter can be higher or lower than the former. If $p_M \geq q_M$, then the reward $\psi/((p_H + p_M) - (q_H + q_M))$, which is paid when M is stocked, is less than the reward $\psi/(p_H - q_H)$, which is paid when H is stocked, whereas if $p_M < q_M$, then the reverse holds. Intuitively, this is because if inventory equal to M is cleared, a large p_M makes it easy for the firm to determine whether the agent exerted high effort or not, so that the firm has to pay a smaller reward to induce high effort, and vice versa. This gives the following result—the firm can have the incentive to pay the salesperson a higher reward for a lower sales threshold.

Third, the expected reward to the salesperson is $\psi/(1 - q_H/p_H)$ for $c < \hat{c}$, and it is $\psi/(1 - (q_H + q_M)/(p_H + p_M))$ for $\hat{c} \leq c < C$. Under the MLRP assumption, $\psi/(1 - (q_H + q_M)/(p_H + p_M)) > \psi/(1 - q_H/p_H)$. This gives another counterintuitive result along the lines of the one obtained above—the firm pays the agent a higher expected reward when it stocks a lower inventory level, compared to the expected reward when it stocks a higher inventory level. (The subtle differences are that the current result is for expected reward and the reward is always higher.) The insight is that, when c is large, the firm stocks lower inventory (equal to M), which implies that the demand realization is not fully observable. (If the inventory is cleared, the firm cannot determine whether realized demand was M or H .) However, to motivate high effort from the salesperson, the firm has to pay him a reward for clearing this inventory. In the ensuing outcome, the agent exerts high effort and is paid the reward for a larger set of demand realizations, which

leads to a high expected reward. For smaller c , on the other hand, the firm stocks an inventory level of H and pays the salesperson a reward only if the realized demand is H and all the inventory is cleared. In other words, the reward is paid for a smaller set of demand realizations, and its expected value is therefore lower.

5. Comparing Across Scenarios

We now proceed to obtain insights from the analyses in §§3 and 4 by comparing the outcomes in the full scenario to the outcomes in the two benchmark scenarios. First, we compare the reward that the agent is given in the full scenario to the reward in the scenario without inventory (Benchmark 1). In other words, assuming that agency issues are present, we analyze the impact of incorporating inventory. We obtain the following result.

PROPOSITION 2. (i) *The expected payment to the salesperson is weakly higher in the full scenario than in the no-inventory scenario.*

(ii) *The reward for achieving the highest sales outcome is weakly higher in the full scenario than in the no-inventory scenario if $p_M < q_M$, and weakly lower if $p_M \geq q_M$.*

The expected payments to the salesperson in the full scenario with inventory and in the scenario without inventory are given by (10) and (4), respectively. Comparing the two, we see that the expected payment is the same in both scenarios for small c , but it is larger in the full scenario for $\hat{c} \leq c < C$. The insight for this is similar to that discussed in the previous section—when c is large, the firm stocks inventory equal to M , which implies that the firm cannot distinguish between medium and high demand realizations. Therefore, the salesperson is paid a reward for a larger set of demand realizations with probability $p_M + p_H$, which increases the expected reward.

Furthermore, the rewards for achieving the highest sales outcomes in the full scenario with inventory and in the scenario without inventory are given by (9) and (3), respectively. The rewards are the same for $c < \hat{c}$. However, for $\hat{c} \leq c < C$, the reward in the scenario with inventory can be larger or smaller, depending on whether $p_M < q_M$ or $p_M \geq q_M$. The intuition is again similar to that discussed in the previous section—the firm stocks lower inventory for large c , which inhibits the observability of demand. The reward is paid for clearing the inventory, and if p_M is small, it is more difficult for the firm to distinguish whether high effort was exerted or not, thus making a larger reward necessary to motivate effort. Taken together, these results show that the presence of inventory constraints can lead to a higher-powered incentive contract. In other words, when inventory availability is an issue, not only does the firm have to contend with the fact that it might lose some sales due to stockouts, it has to

pay the agent higher bonus for achieving the sales threshold. Both of these reduce the firm's profit.

Next, we compare the inventory decisions in the full scenario and the first-best scenario (Benchmark 2). In other words, assuming that inventory considerations are present, we analyze the impact of incorporating agency issues. This gives us the following key result.

PROPOSITION 3. *The optimal inventory level in the full scenario is weakly higher than the optimal inventory level in the first-best scenario without agency considerations.*

In the first-best scenario, the firm stocks H units of inventory for $c < \bar{c}$, and stocks M units otherwise. With agency considerations, the firm stocks H units of inventory for $c < \hat{c}$, and stocks M units otherwise. Since $\hat{c} > \bar{c}$ by Lemma 1, in the latter scenario, the firm overstocks inventory compared to the first-best scenario for $\bar{c} \leq c < \hat{c}$. The intuition behind this result is as follows. When the firm cannot observe the effort of the salesperson, it has to rely on the outcome generated by the salesperson's effort, i.e., the realized demand, to determine whether the salesperson exerted high effort. However, as a result of the demand censoring problem due to limited inventory, the firm cannot always fully observe the realized demand; i.e., it gets a distorted signal of the salesperson's effort. Therefore, to be able to obtain a better signal of the salesperson's effort, the firm stocks more inventory.

In other words, if for $\bar{c} \leq c < \hat{c}$ the firm stocked M units, as in the first-best scenario, and if all M units were sold, the firm would not be able to determine whether realized demand was M or H . However, by stocking H units, the firm can make this determination. Given this, the salesperson has the incentive to exert high effort, otherwise he will not be paid if M units are sold. Stocking inventory above the first-best level hurts the firm's profit, but being able to compensate the salesperson more efficiently offsets this disadvantage. However, if the inventory cost becomes too high, specifically, above \hat{c} , then the firm does not find it profitable any more to gain better observability of realized demand by overstocking inventory. Note also that, for small inventory cost, it is optimal to stock H in both scenarios. Therefore, the result on overstocking inventory holds for nonextreme values of inventory cost; i.e., it holds for medium inventory cost and does not hold when inventory cost is too high or too low. A natural implication of stocking higher inventory is that the stockout probability is lower; i.e., with agency considerations, the stockout probability is lower. In summary, agency considerations and inventory considerations interact to produce the following insight—because limited inventory distorts the signal that the firm obtains of the salesperson's unobservable effort, the firm, in turn, distorts

the inventory level to make the signal of effort more informative.

Next, we study the impact of the parameter p_H , which denotes the probability of high demand under high effort.

PROPOSITION 4. *As p_H decreases, i.e., under a lower probability that the agent's effort exertion leads to high demand, the expected payment to the salesperson in the full scenario increases (weakly) faster than in the no-inventory scenario.*

For $\hat{c} \leq c < C$, the expected payments in the full scenario and the no-inventory Benchmark 1 scenario are different, and this is the case of interest. Consider the no-inventory scenario. Decreasing p_H implies that, after demand realization, it is more difficult for the firm to determine whether the salesperson exerted high effort. Therefore, the firm has to offer a higher-powered contract to the salesperson to motivate high effort. In the presence of inventory, i.e., in the full scenario, for $\hat{c} \leq c < C$, the firm stocks an inventory level of M , which leads to demand censoring; i.e., the reward has to be offered for a larger set of demand realizations. Intuitively speaking, this further compounds the issue of determining whether high effort is exerted or not as p_H decreases, which explains the higher expected payment.

We also obtain the following proposition by comparing expressions in the full scenario with those in the first-best scenario, i.e., the Benchmark 2 scenario. For the discussion below, we define the following:

$$\tilde{\psi} = \frac{r(H - M)}{(q_H + q_M)/(p_H + p_M - q_H - q_M)^2 - q_H/(p_H - q_H)^2}.$$

PROPOSITION 5. *If $\psi \geq \tilde{\psi}$, as p_H decreases, i.e., under a lower probability that the agent's effort exertion leads to high demand, the firm stocks a higher inventory level in the full scenario, which does not happen in the first-best scenario without agency considerations.*

In the first-best scenario, i.e., the Benchmark 2 scenario, decreasing p_H implies that the firm has less incentive to stock a high inventory level, which we see by the fact that \bar{c} decreases as p_H decreases. In other words, as p_H decreases, high inventory is stocked for a smaller range of inventory cost. However, in the full scenario, as p_H decreases, \hat{c} can decrease or increase. To understand the reason behind this, note that there are two effects as p_H decreases. The first effect is the one that operates in the first-best scenario as discussed above—decreasing p_H implies decreasing probability of high demand, and the firm has an incentive to reduce \hat{c} , i.e., stock lower inventory. The second effect is that, as p_H decreases, it is more difficult for the firm to determine whether the agent exerted high effort; therefore, it becomes

increasingly valuable for the firm to stock more inventory to improve its observability of high demand outcomes, i.e., \hat{c} increases. If $\psi \geq \tilde{\psi}$, i.e., if the agent's effort cost is large enough, the second effect dominates the first effect.⁷ This is because with a large effort cost, it is more difficult for the firm to motivate effort from the salesperson, and so the firm stocks more inventory to improve its observability of the high-demand outcome to obtain a better signal of the agent's effort. When $\psi < \tilde{\psi}$, i.e., the agent's effort cost is small, it is easier to motivate the agent's effort, and the benefit from overstocking inventory is lesser. In this case, the first effect dominates, and as p_H decreases, the firm stocks lower inventory. This discussion shows that agency considerations can impact inventory decisions in counterintuitive ways such that results can reverse compared to the case in which agency considerations are absent.

Next, to examine the “bottom line” impact of moral hazard and demand censoring on the firm, we compare the firm's expected profit in the first-best scenario (Benchmark 2 scenario) and the full scenario. Let $\Delta\pi$ denote this profit gap. We have from (7) and (11) that

$$\Delta\pi = \pi^{FB} - \pi^* = \begin{cases} \psi \frac{q_H}{p_H - q_H} & \text{if } c < \bar{c}, \\ (c - \bar{c})(H - M) + \psi \frac{q_H}{p_H - q_H} & \text{if } \bar{c} \leq c < \hat{c}, \\ \psi \frac{q_H + q_M}{(p_H + p_M) - (q_H + q_M)} & \text{if } \hat{c} \leq c < C. \end{cases} \quad (13)$$

We obtain the following proposition.

PROPOSITION 6. *The gap between the first-best profit and the optimal profit in the full scenario*

- (i) *increases (weakly) as the cost of inventory increases; and*
- (ii) *increases (weakly) under a lower probability that the agent's effort exertion leads to high demand.*

For part (i) of the proposition, we know from previous results that as the cost of inventory increases, the inventory ordered (weakly) decreases, which exacerbates the demand censoring problem. When $c < \bar{c}$ or $\hat{c} \leq c < C$, the profit gap is simply the rent that the salesperson gains due to moral hazard. However,

when $\bar{c} \leq c < \hat{c}$, the profit gap comes from a combination of two effects: first, the firm's expected revenue less its inventory cost is lower than in the first-best scenario, due to overstocking inventory; second, the salesperson gains a lower rent than he would have gained if the firm chose the same inventory level as in the first-best scenario. The first effect contributes negatively to the firm's profit, whereas the second effect contributes positively. When $\bar{c} \leq c < \hat{c}$, the second effect dominates and the firm overstocks to improve its observability of the salesperson's effort so that it can reduce its expected payout. For $\hat{c} \leq c < C$, the firm has the option to overstock inventory but, because the inventory is expensive, the additional benefit of increased demand observability is not sufficiently large; therefore, the firm does not overstock inventory. For $c < \bar{c}$, the firm already stocks inventory at the highest possible level. For part (ii) of the proposition, under a lower probability that the agent's effort exertion leads to high demand, i.e., p_H decreases, it is more difficult for the firm to ascertain the effort level of the salesperson. This implies that the agency problem is more severe and, therefore, the profit gap from the first best increases.

6. Conclusions

In many practical situations, demand for products is uncertain, and firms stock limited inventory by balancing revenues and costs. Sales of a product are, therefore, limited by the inventory stocked. The fact that sales are limited by inventory can be expected to play an important role in determining the compensation contract of a salesperson whose job is to increase the expected level of demand. Existing literature on salesperson compensation, however, has typically ignored inventory issues. In this paper, we take a step forward in this direction.

Limited inventory leads to imperfect observability of realized demand; i.e., it leads to the firm obtaining a distorted signal of the salesperson's effort, which increases the moral hazard problem. We find that, to reduce this inefficiency, the firm can increase the inventory level stocked so that a larger range of demand realizations is observable. In addition, because high demand realizations will be censored, the firm can increase the salesperson's reward to induce him to work harder. We also find that, as the likelihood that effort exertion by the agent leads to high demand reduces, under certain conditions, the firm stocks more inventory to obtain a better indicator of the salesperson's effort through improved demand observability.

We use a three-point discrete demand distribution for our analysis. However, we show that our results hold qualitatively for any discrete distribution with

⁷ Equation (12) puts an upper bound on ψ , whereas $\psi \geq \tilde{\psi}$, needed for Proposition 5, puts a lower bound on ψ . These constraints are mutually satisfiable, for instance, for the values $\{r, c, H, M, L, p_H, p_M, p_L, q_H, q_M, q_L\} = \{100, 10, 100, 75, 5, 0.77, 0.15, 0.08, 0.08, 0.49, 0.43\}$.

three or more levels, conditional on MLRP and certain other regularity conditions being satisfied. (Details of the analysis are available from the authors upon request.) We also assume two effort levels for the agent in our model. However, since our main results on stocking excess inventory are driven primarily by unobservability of high-demand outcomes due to limited inventory, we expect them to continue to hold in a model with multiple effort levels for the agent. Finally, in our analysis, we assume away inventory overage and underage costs. However, our results will not change qualitatively if these costs are explicitly incorporated.⁸

Our study provides useful managerial insights. We find that agency considerations motivate stocking higher inventory, and inventory considerations lead to higher-powered incentive contracts. Therefore, managers should be cognizant of both the salesforce compensation aspect and the inventory aspect when making decisions about either aspect. Moreover, the distortion in the observability of demand due to limited inventory reduces the profit of the firm—in this context, the expression for the profit gap (derived in (13)) has the potentially useful managerial interpretation that it gives the maximum investment that the firm may incur to alleviate the moral hazard problem in the presence of inventory considerations, e.g., by setting up improved monitoring systems.

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Appendix: Additional Analyses

Analysis for §3.1

Analysis to show that reward should be given only for the highest output outcome. If the firm only pays a positive reward for the highest output outcome, then the expected payment is $\psi/(1 - q_H/p_H)$. Suppose that the firm uses a different incentive contract where the salesperson is paid $t_s \geq 0$ for $s \in \{H, M, L\}$, and at least one of $\{t_M, t_L\}$ is positive. The incentive compatibility constraint

$$t_H p_H + t_M p_M + t_L p_L - \psi \geq t_H q_H + t_M q_M + t_L q_L$$

⁸ Since we conduct an expected case analysis, as long as the distribution of demand conditional on effort is known, to incorporate inventory underage costs of the nature of “loss of goodwill” costs from unavailability of the product, we do not need to assume that excess demand should be observable. However, our model cannot deal with backordering of excess units (and associated costs) because realized excess units are assumed to be unobservable.

gives $t_H \geq (\psi - \epsilon_s - \epsilon_s)/(p_H - q_H)$, where $\epsilon_s = t_s(p_s - q_s)$ for $s \in \{M, L\}$. Therefore, the firm’s total expected payment to the salesperson is

$$\begin{aligned} \sum_{s \in \{H, M, L\}} p_s t_s &\geq p_H \frac{\psi - \epsilon_M - \epsilon_L}{p_H - q_H} + p_M \frac{\epsilon_M}{p_M - q_M} + p_L \frac{\epsilon_L}{p_L - q_L} \\ &= (\psi - \epsilon_M - \epsilon_L) \frac{1}{1 - q_H/p_H} + \epsilon_M \frac{1}{1 - q_M/p_M} \\ &\quad + \epsilon_L \frac{1}{1 - q_L/p_L} \\ &> (\psi - \epsilon_M - \epsilon_L) \frac{1}{1 - q_H/p_H} + \epsilon_M \frac{1}{1 - q_H/p_H} \\ &\quad + \epsilon_L \frac{1}{1 - q_H/p_H} \quad (\text{by MLRP}) \\ &= \frac{\psi}{1 - q_H/p_H}. \end{aligned}$$

Analysis to obtain (6). For the firm to motivate higher effort, the firm’s expected profit when the induced effort level is e_H should be higher than the firm’s expected profit when the induced effort level is e_L ; i.e., the following condition should hold (the left-hand side and the right-hand side correspond to the firm’s expected profits when the induced effort levels are e_H and e_L , respectively):

$$\begin{aligned} (r - c)(p_H H + p_M M + p_L L) - \frac{\psi}{1 - q_H/p_H} \\ \geq (r - c)(q_H H + q_M M + q_L L). \end{aligned}$$

This directly implies (6).

Analysis for §3.2

Analysis to obtain (8). Let Q_{b2}^* be the optimal inventory level if the salesperson exerts e_L . It is straightforward to see that $Q_{b2}^{**} < Q_{b2}^*$. To make sure that the firm has the incentive to induce e_H rather than e_L , we need to have

$$\begin{aligned} r \sum_{s \in \{H, M, L\}} p_s \min\{Q_{b2}^*, s\} - c Q_{b2}^* - \psi \\ \geq r \sum_{s \in \{H, M, L\}} q_s \min\{Q_{b2}^{**}, s\} - c Q_{b2}^{**}, \end{aligned}$$

which gives

$$\psi \leq r \sum_{s \in \{H, M, L\}} (p_s \min\{Q_{b2}^*, s\} - q_s \min\{Q_{b2}^{**}, s\}) - c(Q_{b2}^* - Q_{b2}^{**}).$$

We now derive a lower bound of the right-hand side of the above inequality:

$$\begin{aligned} r \sum_{s \in \{H, M, L\}} (p_s \min\{Q_{b2}^*, s\} - q_s \min\{Q_{b2}^{**}, s\}) - c(Q_{b2}^* - Q_{b2}^{**}) \\ > r \sum_{s \in \{H, M, L\}} (p_s - q_s) \underbrace{\min\{Q_{b2}^*, s\}}_{Q_{b2}^{**} < Q_{b2}^*} - c \underbrace{(H - L)}_{L \leq Q_{b2}^{**}, Q_{b2}^* \leq H} \\ \geq r \sum_{s \in \{H, M, L\}} (p_s - q_s) \underbrace{\min\{M, s\}}_{Q_{b2}^* \geq M} - c(H - L). \end{aligned}$$

Therefore, a sufficient condition is that $\psi \leq r((p_H + p_M - q_H - q_M)M + (p_L - q_L)L) - c(H - L)$.

Analysis for §4

Analysis to show that reward should be given only for the highest output outcome. If the firm chooses $Q > M$, then the proof is the same as in §3.1. If the firm chooses $Q = M$, then due to demand censoring, the sales can be either M or L ; the firm can either (i) choose to pay a positive reward only when the sales are M , or (ii) pay a positive reward in both cases. By following the former strategy, the firm's expected payment to the salesperson is $\psi \cdot 1/(1 - (q_H + q_M)/(p_H + p_M))$. We shall prove that the second strategy is dominated by the first one. Let $\hat{t}_M, \hat{t}_L > 0$ denote the firm's payment to the salesperson when the sales are M and L , respectively. The incentive compatibility constraint

$$(p_H + p_M)\hat{t}_M + p_L\hat{t}_L - \psi \geq (q_H + q_M)\hat{t}_M + q_L\hat{t}_L$$

gives $\hat{t}_M \geq (\psi - \hat{\epsilon}_L)/((p_H + p_M) - (q_H + q_M))$, where $\hat{\epsilon}_L = \hat{t}_L(p_L - q_L)$. Therefore, the firm's expected payment to the salesperson is

$$\begin{aligned} & (p_H + p_M)\hat{t}_M + p_L\hat{t}_L \\ & \geq (p_H + p_M) \frac{\psi - \hat{\epsilon}_L}{(p_H + p_M) - (q_H + q_M)} + p_L \frac{\hat{\epsilon}_L}{p_L - q_L} \\ & = (\psi - \hat{\epsilon}_L) \frac{1}{1 - (q_H + q_M)/(p_H + p_M)} + \hat{\epsilon}_L \frac{1}{1 - q_L/p_L} \\ & > (\psi - \hat{\epsilon}_L) \frac{1}{1 - (q_H + q_M)/(p_H + p_M)} \\ & \quad + \hat{\epsilon}_L \frac{1}{1 - (q_H + q_M)/(p_H + p_M)} \\ & = \psi \frac{1}{1 - (q_H + q_M)/(p_H + p_M)}. \end{aligned}$$

The last inequality is true because MLRP gives $q_H/p_H < q_M/p_M < q_L/p_L$ and, therefore, $(q_H + q_M)/(p_H + p_M) < q_L/p_L$.

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