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# On Portfolio Choice with Savoring and Disappointment

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We revisit the model proposed by Gollier and Muermann [Gollier C, Muermann A (2010) Optimal choice and beliefs with ex ante savoring and ex post disappointment. *Management Sci.* 56(8):1272–1284; hereafter, GM]. In the GM model, for a given lottery, agents form anticipated expected payoffs and the set of possible anticipations is assumed to be exogenously fixed. We propose sets of possible anticipations that are endogenously determined. This permits us to compare and evaluate in a consistent manner lotteries with different supports and to revisit the portfolio choice problem. We obtain new conclusions and interesting insights. Our extended model can rationalize a variety of empirically observed puzzles such as a positive demand for assets with negative expected returns, preference for skewed returns, and underdiversification of portfolios.

**Keywords:** endogenous beliefs; anticipatory feelings; disappointment; optimism; portfolio choice; skewness; underdiversification

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## 1. Introduction

Gollier and Muermann (2010) (hereafter, GM) propose a structural model of subjective belief formation in which beliefs solve a trade-off between ex ante savoring and ex post disappointment. Models of subjective beliefs (with possible cognitive dissonance) go back to Akerlof and Dickens (1982). The GM model is in line with this literature and more precisely builds on the optimal beliefs approach introduced by Brunnermeier and Parker (2005) and Brunnermeier et al. (2007), in which the agents form beliefs endogenously and derive ex ante felicity from expectations of future pleasures. With such an approach, optimal beliefs balance the benefits of higher expectations against the costs of worse decision making and are necessarily biased toward optimism. The GM model also builds on the disappointment theory, introduced by Bell (1985), Loomes and Sugden (1986), and Gul (1991), for which the felicity associated to a given uncertain outcome increases with the difference between the realization and the expectation. In the GM model, agents form an anticipated expected payoff, and optimal beliefs realize the best trade-off between ex ante

savoring and ex post disappointment: high expectations lead to more ex ante savoring at the cost of being disappointed ex post while low expectations lead to the benefits of elation ex post at the cost of less savoring ex ante. Depending on the relative weight of the ex ante and ex post criteria, the optimal belief might be optimistic or pessimistic, leading to a realistic framework to model decision making and to think about endogenous heterogeneous beliefs.

In this paper, we revisit the GM model. In GM the set of possible anticipations is exogenously fixed, whereas we propose relating the set of possible anticipations to the lottery characteristics. The main differences are the following. First, in our setting, as in Brunnermeier and Parker (2005, p. 1096), “in order to believe that something is possible, it must be possible”: feasible subjective probability distributions are assumed to be absolutely continuous with respect to the objective one. In our model, the only possible anticipated expected payoff is the sure payoff when there is no uncertainty—if I am sure to get 100, then I can only believe that I will get 100. In the case of a lottery yielding 0 or 100 with equal probabilities, an agent can believe that he will win 0, that he will win

100, or that he will win *on average* any value between 0 and 100, reflecting a subjective belief that 0 will occur with some probability  $p$  and 100 with  $1 - p$ . However, he cannot believe that he will win some value outside  $[0, 100]$ . Second, the welfare level of a given lottery does not depend on the set of 0 probability possible outcomes that can be added to the lottery support. Third, and as a consequence, as far as the portfolio choice problem is concerned, the set of possible anticipated expected payoffs is not constrained by exogenous bounds as in GM, but it depends on the level of investment in the risky asset, which seems more natural, because this level modifies the support of the possible payoffs.

Our extended model leads to new conclusions and interesting insights, which shed light on a variety of puzzles in decision theory and in portfolio choice literature. First, it appears that the preference function is not necessarily compatible with first- and second-degree stochastic dominance. The intuition behind this result is that an increase in risk may enlarge the support and enable the agent to form an optimal anticipated expected payoff that is more favorable in terms of the savoring and disappointment trade-off and thereby may lead to a higher welfare. We provide and discuss an additional condition on the preference function to restore compatibility with first-degree stochastic dominance: the weight on savoring must be large enough with respect to the weight on disappointment. This condition is consistent with Gneezy et al. (2006), who explain that pure disappointment models permit violations of first-order stochastic dominance (FSD).

Second, and as a consequence, it may be optimal to invest in a risky asset with an expected excess return equal to zero. In our revisited model, risk taking may be optimal even if the expected payoff is negative. The rationale is that investing in the risky asset enables the individual to have a larger range of possible anticipated expected payoffs and possibly a higher welfare.

Third, the agents exhibit a preference for skewed returns as in Brunnermeier et al. (2007): a positive demand for a skewed asset enables the agent to savor more for a given level of risk than the opposite demand. The last two results may explain the popularity of lottery games (Thaler and Ziemba 1988) despite their negative expected returns and the underperformance of lottery-type stocks (Kumar 2009, Bali et al. 2011) (i.e., gambling enables agents to dream). This taste for lottery-type stocks and extreme values is also a possible explanation for portfolio underdiversification (Mitton and Vorkink 2007).

Fourth, the allocation in the risky asset may increase with the weight on savoring, i.e., with the intensity of anticipatory feelings, while in GM, the constant

bounds assumption had the implication that the more the agent savors the less risk he takes. GM showed that a larger weight on savoring increases risk aversion and hence reduces the allocation in the risky asset. In our revisited model, we also have a support effect that may outweigh the effect of the increase in risk aversion.

Finally, we argue that our revisited model provides a suitable framework to think of simultaneous demand for insurance and lotteries, a puzzle pointed out by Friedman and Savage (1948). Consistent with the Lopes (1987) theory of hope and fear and the Shefrin and Statman (2000) behavioral portfolio theory, our model can explain the coexistence of insurance and lottery demand with the fear of disappointment and the desire to savor.

In §2 we present the model, and then in §3 we analyze its properties. Stylized examples to illustrate the differences between the GM model and our model are provided in Appendix A, and proofs are provided in Appendix B.

## 2. The Model

We first present our model, which is directly derived from GM, and then we analyze its relevance and detail the differences with the original model.

### 2.1. Our Decision Criterion and Its Application to Portfolio Choice

The agent faces a risky payoff  $\tilde{c}$ , described by its (objective) probability distribution  $Q$  over the real line. The agent can extract, at date 0, satisfaction from anticipatory feelings. As in Brunnermeier and Parker (2005), the agent can choose a subjective probability distribution in the set  $\mathcal{P}$  of all probability distributions that are absolutely continuous with respect to  $Q$ . The agent then enjoys at date 0 the subjectively expected future utility of the risky payoff  $\tilde{c}$ . This satisfaction from anticipatory feelings comes at the cost of experiencing, at date 1, disappointment. Disappointment is measured with respect to a reference point  $y$  that we will call the *anticipated expected payoff*. For a given realization  $c$  of  $\tilde{c}$ , the agent enjoys at date 1 the satisfaction  $U(c, y)$ , where  $U$  is a bidimensional utility function increasing and concave in its first argument, i.e., such that  $U_c > 0$  and  $U_{cc} < 0$ , and decreasing in the second argument, i.e., such that  $U_y < 0$ , to reflect disappointment. The higher the anticipated expected payoff, the higher the ex post disappointment.<sup>1</sup> The intertemporal welfare of the agent for a given choice of belief  $P$  in  $\mathcal{P}$  is a weighted sum of his ex ante and ex post

<sup>1</sup> As expressed by Caplin and Leahy (2001, p. 75), “Have you ever felt disappointed about an outcome without having experienced prior feelings of hopefulness?”

satisfactions and given by  $W(P, \tilde{c}) = kE^P[U(\tilde{c}, y)] + E^Q[U(\tilde{c}, y)]$ , where  $k$  measures the intensity of anticipatory feelings. The anticipated expected payoff  $y$  is defined as the (subjective) certainty equivalent of the risky payoff; i.e.,  $U(y, y) = E^P[U(\tilde{c}, y)]$ . We assume that the function  $v(y) \equiv U(y, y)$  is increasing in  $y$  to reflect the fact that receiving a higher payoff in line with expectations increases the agent's utility.<sup>2</sup> Since  $U(y, y) = E^P[U(\tilde{c}, y)]$ , it also means that increasing the anticipated expected payoff raises at date 0 the satisfaction extracted from anticipatory feelings. Since  $W(P, c) = (k+1)U(c, c)$  for a deterministic  $c$ , the condition on  $v$  is also a monotonicity condition on the welfare function over the set of sure payoffs, which is natural.

The agent's optimization problem ( $\mathcal{O}_y$ ) consists of selecting a subjective belief  $P$  in  $\mathcal{P}$  to maximize his welfare  $W(P, \tilde{c})$ . Letting  $c_{\inf(Q)}$  and  $c_{\sup(Q)}$  denote the essential infimum and essential supremum of  $\tilde{c}$  under  $Q$  makes it easy to see that the agent's optimization problem ( $\mathcal{O}_y$ ) is equivalent to the following optimization problem ( $\mathcal{O}_y$ ):

$$\max_{c_{\inf(Q)} \leq y \leq c_{\sup(Q)}} E^Q[F(\tilde{c}, y)], \quad (1)$$

where  $F(c, y) = kU(y, y) + U(c, y)$ . The agent is then endowed with a decision criterion that associates with every risky payoff  $\tilde{c}$  a welfare level  $W(\tilde{c}) \equiv \max_{c_{\inf(Q)} \leq y \leq c_{\sup(Q)}} E^Q[F(\tilde{c}, y)]$ , which corresponds to the optimal trade-off between ex ante savoring and ex post disappointment.<sup>3</sup>

Note that the optimization problem ( $\mathcal{O}_y$ ) is also consistent with the (subjective) expected value of the risky payoff as the reference point instead of the certainty equivalent. Indeed, the optimization problem  $\max_{P \in \mathcal{P}} \{kU(E^P[\tilde{c}], E^P[\tilde{c}]) + E^Q[U(\tilde{c}, E^P[\tilde{c}])]\}$  is equivalent to the optimization problem ( $\mathcal{O}_y$ ).<sup>4</sup> This means that our model is consistent with models of disappointment that adopt the certainty equivalent as reference point, as in Gul (1991), as well as with models that adopt the expected payoff as the reference point, as in Bell (1985) and Loomes and Sugden (1986).

Let us now consider the standard portfolio choice problem with such a decision criterion. The agent has some initial wealth  $z$  at date 0 that can be invested in a riskless asset with a return between date 0

and date 1 that is normalized to one or in a risky asset with an excess return that is described by a random variable  $\tilde{x}$ , with probability distribution  $Q$ . When the agent invests a level  $\alpha$  of his wealth in the risky asset, he faces the risky payoff  $\tilde{c}_\alpha = z + \alpha\tilde{x}$  and, by (1), his intertemporal welfare is given by  $W(\tilde{c}_\alpha) = \max_{c_{\inf(Q)} \leq y \leq c_{\sup(Q)}} E^Q[F(\tilde{c}_\alpha, y)]$ . The agent's portfolio choice problem then consists of choosing the level  $\alpha^*$  of wealth invested in the risky asset to maximize his intertemporal welfare, i.e., such that  $\alpha^* = \arg \max_\alpha W(\tilde{c}_\alpha)$ .

In the remainder of this paper, and as in GM, we make the regularity assumption that the function  $F(c, y)$  is concave in  $y$ . The following first-order condition is then necessary and sufficient to determine the optimal anticipated expected payoff  $y^*$ :

$$E^Q[F_y(\tilde{c}, y)] = kv'(y) \\ + E^Q[U_y(\tilde{c}, y)] \begin{cases} \leq 0 & \text{if } y^* = c_{\inf(Q)}, \\ = 0 & \text{if } y^* \in (c_{\inf(Q)}, c_{\sup(Q)}), \\ \geq 0 & \text{if } y^* = c_{\sup(Q)}. \end{cases} \quad (2)$$

We shall repeatedly consider the additive habit formation specification developed by Constantinides (1990),  $U(c, y) = u(c - \eta y)$ , for an increasing and concave function  $u$  and a positive scalar  $\eta < 1$ . It is easy to verify that this bidimensional function satisfies all the above regularity assumptions.

## 2.2. Our Model vs. the GM Model

Let us be clear about the distinction between the seminal GM model and our extended model and about the relevance of our modifications. GM fix a finite set of possible payoffs  $C = \{c_1 < c_2 < \dots < c_S\}$  and provide a decision criterion for the set  $\mathcal{S}_C$  of simple lotteries with support in  $C$ . A lottery  $Q$  in  $\mathcal{S}_C$  is described by a vector of probabilities  $(q_1, q_2, \dots, q_S)$  with  $q_i \geq 0$  and  $\sum_{i=1}^S q_i = 1$ . For any lottery  $Q$  in  $\mathcal{S}_C$ , the agent's welfare  $W(Q)$  is given by  $W(Q) = \max_{c_1 \leq y \leq c_S} \{kU(y, y) + \sum_{s=1}^S q_s U(c_s, y)\}$ . The welfare level of  $Q$  does not depend on its support but depends on  $C$  (through  $c_1$  and  $c_S$ ). Note the difference with our decision criterion where the bounds are given by  $c_{\inf\{i; q_i > 0\}}$  and  $c_{\sup\{i; q_i > 0\}}$ . In fact, the agent in the GM model can choose a subjective probability that is singular with respect to the objective one, whereas the agent in our model is constrained to choose a probability that is absolutely continuous with respect to the objective one. As a result, when there is no uncertainty, the only possible (and optimal) anticipated expected payoff is equal to the sure payoff in our model. We think that this feature is reasonable because if there is no uncertainty then there is nothing to dream or to be disappointed about. In the GM model, the optimal anticipated expected payoff is also equal to the sure payoff only if  $F_y(x, x) = 0$  for all  $x$ ,

<sup>2</sup> One prefers to consume \$6,000 in line with expectations rather than \$5,000 in line with expectations.

<sup>3</sup> Note that we would obtain analogous results if we considered the more general optimization problem  $\max_{c_{\inf(Q)} \leq y \leq c_{\sup(Q)}} \{kv(y) + E^Q[U(\tilde{c}, y)]\}$  for a general increasing function  $v$  (i.e., not necessarily of the form  $v(y) = U(y, y)$ ) such that  $F(c, y) = kv(y) + U(c, y)$  is concave in  $y$ . In particular, this permits us to consider different date 0 and date 1 utility functions.

<sup>4</sup> This does not mean that it is possible to replace  $y$  with the subjective expected value of the risky payoff in the initial problem of GM.



or if the set of possible payoffs is reduced to a singleton, namely, the sure payoff. For the additive habit specification,  $F_y(x, x) = 0$  for all  $x$  is satisfied only if  $k = \eta/(1 - \eta)$ . More generally, in our setting, the anticipated expected payoff belongs to the (convex hull of the) support of the objective lottery.

Note that in the case when the support of the objective distribution of the lottery  $Q$  under consideration coincides with the set  $C$  of possible payoffs in GM,  $c_1 = c_{\inf\{i; q_i > 0\}}$  and  $c_S = c_{\sup\{i; q_i > 0\}}$ , and the welfare level of  $Q$  in GM coincides with its welfare level in our extended model. However, the GM model only permits us to compare lotteries with the same support. Our decision criterion can then be seen as an extension of the GM decision criterion to lotteries with different supports. In the case when the set  $C$  in GM and the support of the objective distribution do not coincide, the model presented here is not an extension but a modification of GM because it does not lead to the same welfare levels.

As far as the portfolio choice problem is concerned, GM impose exogenous bounds  $y_{\inf}$  and  $y_{\sup}$  on anticipated expected payoffs, and these bounds are the same for all payoffs  $\tilde{c}_\alpha = z + \alpha\tilde{x}$ , independent of  $\alpha$ . In our model, if an agent does not invest in the risky asset ( $\alpha = 0$ ), the only possible (and optimal) anticipated expected payoff is equal to the sure payoff  $z$  ( $y^*(0) = z$ ) (i.e., the individual cannot extract anticipatory feelings without investing in the risky asset). In GM, the agent can choose any anticipated expected payoff in  $[y_{\inf}, y_{\sup}]$ , even though he is sure to get  $z$  (i.e., the individual can savor a high anticipated expected payoff even if he does not invest in the risky asset and is hence sure to keep the same wealth  $z$ ). More generally, in our model, the level of investment modifies the range of possible realizations of possible anticipated expected payoffs.

### 3. Results and Predictions

Our extended model leads to new conclusions and interesting insights.

#### 3.1. Comparative Statics

##### 3.1.1. Optimal Anticipated Expected Payoffs.

First, it is easy to show, exactly as in GM, that an increase in the intensity of anticipatory feelings weakly increases the optimal anticipated expected payoff; i.e.,  $\partial y^*/\partial k \geq 0$ . As intuition suggests, when the intensity of anticipatory feelings increases, the agent can get more benefit from his dreams, and he biases his beliefs toward more optimism.

Most results in GM about the impact of stochastic dominance on the optimal anticipated expected payoff are no longer valid in our setting without additional assumptions. Detailed stylized counterexamples can be found in Appendix A, but the main idea is the following: In our extended model, modifying

the support of the objective distribution changes the range of the possible anticipated expected payoffs and may authorize anticipated expected payoffs that are more favorable in terms of the savoring and disappointment trade-off. The only result that remains valid is the following.

**PROPOSITION 1.** *If  $U_y$  is increasing in the payoff  $c$ , then any FSD dominated shift in the probability distribution  $Q$  weakly reduces the optimal anticipated expected payoff  $y^*$ .*

The condition  $U_{yc} > 0$  means that the agent is disappointment averse. Note that for the habit formation specification  $U(c, y) = u(c - \eta y)$ , we always have  $U_{yc} > 0$ .

**3.1.2. Welfare.** GM show that any second-order stochastic dominance (SSD) dominated shift (and in particular, any FSD dominated shift) in the probability distribution  $Q$  weakly reduces the agent's intertemporal welfare (Proposition 5). In our setting, in the absence of an additional condition, the impact of an FSD dominated shift on welfare is ambiguous.<sup>5</sup> As just seen, modifying the support of the objective distribution may authorize more favorable trade-offs between savoring and disappointment and then lead to higher welfare.

The simplest forms of FSD dominated shifts are given by the shift from the binary lottery  $L = (\{x_1, x_2\}, (\pi, 1 - \pi))$  with  $x_1 < x_2$  to the sure payoff  $x_1$  or by the shift from the sure payoff  $x_2$  to the lottery  $L$ . Gneezy et al. (2006) define the internality axiom for decision models as the fact that for any binary lottery, these two simple shifts reduce welfare. Equivalently, this axiom imposes the restriction that for any binary lottery, the welfare level associated to the lottery ranges between the welfare level of its lowest and highest outcomes. Note that, as expressed by Gneezy et al. (2006), disappointment models permit violations of the internality requirement. This means that even with this simplest form of FSD, an additional condition is needed for our decision criterion. The following result shows that the internality requirement is equivalent to the condition  $F_y(x, x) \geq 0$  for all  $x$ . Moreover, it also shows that this condition guarantees that our decision criterion is consistent with FSD shifts.

**PROPOSITION 2.** *The three following conditions are equivalent:*

1. *The decision criterion  $W$  satisfies the internality requirement.*
2. *For all  $x$ ,  $F_y(x, x) \geq 0$ .*
3. *Any FSD dominated shift in the probability distribution  $Q$  weakly reduces the agent's intertemporal welfare.*

<sup>5</sup> An example of an FSD dominated shift leading to a decrease in welfare can be found in Appendix A (Example 4).

**Table 1** Allais Paradox

Lottery $\{0, 1, 5\}$ $Q = (q_1, q_2, q_3)$	$A_1$ (0, 1, 0)	$A_2$ (0.01, 0.89, 0.1)	$B_1$ (0.9, 0, 0.1)	$B_2$ (0.89, 0.11, 0)
Preference	$A_1 > A_2$		$B_1 > B_2$	
$k = 0.75$				
$y^*(Q)$	1	0.7744	0	0
$W(Q)$	-0.1728	-0.1790	-0.5502	-0.5513
$k = 2$				
$y^*(Q)$	1	1.0404	0.1989	0.2002
$W(Q)$	-0.2963	-0.3117	-0.9126	-0.9132

*Notes.* This table presents for  $k$  equal to 0.75 and 2 the choice of an agent endowed with a utility function  $U(c, y) = -(1 + c - y/2)^{-3}/3$  between the lotteries considered in the Allais paradox. The different lotteries yield 0 with probability  $q_1$ , 1 with probability  $q_2$ , and 5 with probability  $q_3$ . They differ by the values of  $(q_1, q_2, q_3)$ . The Allais paradox is explained when  $A_1 > A_2$  and  $B_1 > B_2$ . This is the case when  $k$  remains sufficiently close to  $\eta/(1 - \eta)$ , which is equal to 1 in this example.

The condition  $F_y(x, x) \geq 0$  for all  $x$  is a condition on the relative weights of savoring and disappointment. It amounts to assuming that when the anticipated expected payoff and the payoff are in line, the decrease in ex ante utility induced by a decrease in the anticipated expected payoff—due to lower anticipatory feelings—is greater than the increase in ex post utility—due to lower disappointment. A slight decrease<sup>6</sup> in the anticipated expected payoff then induces a decrease in intertemporal welfare. As explained above, because pure models of disappointment violate the internality requirement, our condition ensures that the weight on savoring is high enough compared to the weight on disappointment to induce the agent to bias his beliefs upward when the anticipated expected payoff and the actual payoff are in line. For the habit formation specification  $U(c, y) = u(c - \eta y)$ , the additional condition  $F_y(x, x) \geq 0$  for all  $x$  is satisfied if and only if  $k \geq \eta/(1 - \eta)$ .

Finally, we show in Table 1 that for some specifications, our model, like the GM model, can help explain the Allais paradox.

### 3.2. Positive Demand for Assets with Negative Expected Return

The following proposition shows that the agent may take nonzero positions on zero-mean risk assets in contrast to Proposition 8 in GM and in contrast to the standard expected utility model. As previously mentioned, the intuition is that in our setting, the presence of risk permits a larger range of possible anticipated expected payoffs and hence possibly higher

savoring or less disappointment compensating for risk aversion.

**PROPOSITION 3.** Let  $\tilde{x}$  be a bounded, nonzero, zero-mean risk and let  $z$  denote the agent's initial wealth. If  $F_y(z, z) \neq 0$ , then the optimal investment  $\alpha^*$  in the risky asset  $\tilde{x}$  is nonzero.

This proposition shows that there are zero-mean risks for which the optimal demand is positive (even if it means changing  $\tilde{x}$  into  $-\tilde{x}$ ). Slight perturbations of  $\tilde{x}$  or  $-\tilde{x}$  would then permit us to construct negative mean risks for which the optimal demand is positive. Note that state lotteries typically have a negative average payoff. In our framework, the positive demand for such lotteries is rationalized by the savoring of favorable future prospects. Given the equivalence between portfolio choice and insurance demand problems, Proposition 3 also shows that full insurance is not optimal for actuarially fair insurance when  $F_y(z, z) \neq 0$ , which may help to explain the annuities puzzle.<sup>7</sup> More generally, the proposition implies that risky prospects might be desirable. This explains why there is no systematic effect of SSD shifts on welfare in our setting (see Example 3, Appendix A).

For the habit formation specification  $U(c, y) = u(c - \eta y)$ , we have  $F_y(z, z) \neq 0$  for all  $z$  if and only if  $k \neq \eta/(1 - \eta)$ . Under this assumption, Proposition 3 applies for all possible initial wealth levels, and the agent might then invest in a risky asset with a negative expected return. For example, for  $k > \eta/(1 - \eta)$  and  $E^Q[\tilde{x}] < 0$ , we can see, using the proof of Proposition 3, that if shortsales are not allowed, the optimal investment level  $\alpha^*$  is positive as soon as  $x_{\text{sup}} > -(E^Q[\tilde{x}])/(k(1 - \eta) - \eta)$  or, in other words, as soon as the expected loss is moderate relative to the maximum possible gain. This is typically the case with state lotteries for which the expected gain is negative, shortsales are not allowed, and the maximum possible gain is high. Note that the focus on the maximum possible gain is consistent with Cook and Clotfelter (1993), who document that per capita lottery sales increase with the population base—indeed a higher possible jackpot makes higher dreams possible.

It is also interesting to note that under the condition  $F_y(x, x) \geq 0$ , SSD dominated shifts are undesirable when they do not affect the maximum possible dream.

**PROPOSITION 4.** Assume that  $F_y(x, x) \geq 0$  for all  $x$ . An SSD dominated shift in the probability distribution  $Q$ , which does not modify the maximum possible payoff, weakly reduces the agent's intertemporal welfare.

<sup>6</sup> This local property is also satisfied at the global level, and slight decreases might be replaced by general decreases. Indeed, since  $F$  is concave in  $y$ , the condition  $F_y(x, x) \geq 0$  for all  $x$  is equivalent to the fact that the function  $y \mapsto F(x, y)$  is nondecreasing on  $\{y \leq x\}$ .

<sup>7</sup> See, for instance, Benartzi et al. (2011) for a review on this subject.

This result might help to explain simultaneous demand for insurance and lotteries: an agent who holds a lottery ticket and faces some risk that does not affect the lottery jackpot would be interested because of a risk reduction caused by insurance.

### 3.3. Underdiversification

An interesting corollary of Proposition 3 is the possible preference for underdiversified portfolios. Indeed, let us consider a financial market with several assets and with a zero idiosyncratic risk. A perfectly diversified portfolio would then be nonrisky. Let us normalize its return to zero. Proposition 3 implies that when facing the perfectly diversified portfolio and any other underdiversified portfolio with zero average return, an agent with a total wealth  $z$ , and such that  $F_y(z, z) \neq 0$ , would choose to invest a nonzero fraction of his wealth in the underdiversified portfolio leading to an underdiversified overall portfolio, while a classic expected utility agent would choose to invest his whole wealth in the perfectly diversified portfolio.

Underdiversified portfolio holdings of individual investors have been documented, for instance, by Mitton and Vorkink (2007) and Goetzmann and Kumar (2008); they find that underdiversified portfolio holdings are concentrated in stocks with high idiosyncratic volatility and high skewness, i.e., stocks with maximum upside potential. This is consistent with our model, which predicts that agents underdiversify to savor the upside potential.

### 3.4. Binary Risk and Preference for Skewed Returns

In this section, we assume that  $U(c, y) = \ln(c - \eta y)$  and that  $\tilde{x}$  is a binary risk. The next proposition solves the portfolio choice problem for general zero-mean binary risks and shows a preference for skewed returns. This is consistent with, e.g., Mitton and Vorkink (2007, p. 1255), who find that “investors sacrifice mean variance efficiency for higher skewness exposure.”

**PROPOSITION 5.** *Let  $z$  denote the agent's initial wealth. Suppose that  $U(c, y) = \ln(c - \eta y)$  for  $0 < \eta < 1$  and that the excess return of the risky asset has a zero mean and yields  $x_{\sup} > 0$  with probability  $\pi$  and  $x_{\inf} < 0$  with probability  $1 - \pi$ . For  $\pi \leq \frac{1}{2}$  (respectively,  $\pi \geq \frac{1}{2}$ ), the optimal investment level is given by*

$$\alpha^* \equiv \alpha_1 = \frac{k(1 - \eta) - \eta}{(k + 1)(\eta x_{\sup} - x_{\inf})} z,$$

respectively,

$$\alpha^* \equiv \alpha_2 = -\frac{k(1 - \eta) - \eta}{(k + 1)(x_{\sup} - \eta x_{\inf})} z,$$

with

$$y(\alpha_1) = \frac{k + \pi}{(k + 1)(\pi + \eta(1 - \pi))} z,$$

respectively,

$$y(\alpha_2) = \frac{k + 1 - \pi}{(k + 1)(1 - \pi + \eta\pi)} z.$$

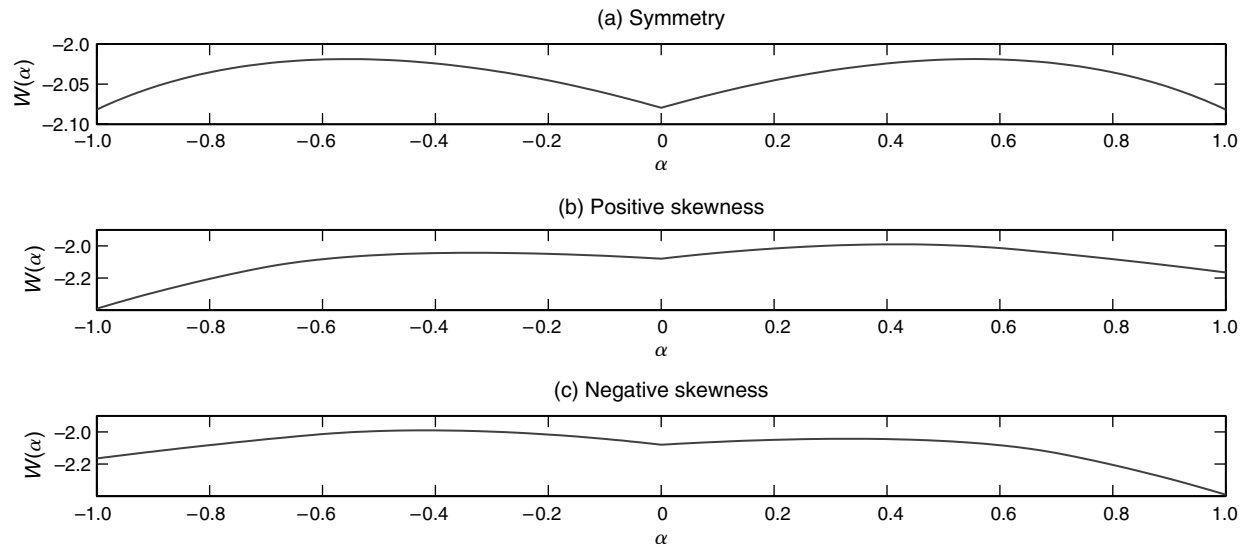
In particular, it is optimal not to invest in the zero-mean return portfolio, i.e.,  $\alpha^* = 0$ , if and only if  $k = \eta/(1 - \eta)$ ; in this case,  $y^* = z$ . In the general case, the optimal investment is nonzero. Note that we only need to consider one of the two cases,  $\pi \leq \frac{1}{2}$  or  $\pi \geq \frac{1}{2}$ , because they are symmetric.

The case  $\pi \leq \frac{1}{2}$  corresponds to a positively skewed distribution of payoffs and hence to a positively skewed range of values for the anticipated expected payoff. When the intensity of anticipatory feelings is high enough relative to the intensity of disappointment ( $k > \eta/(1 - \eta)$ ), then the positive skewness enables the agent to dream. To savor these high possible anticipated expected payoffs at date 0, the agent has a positive optimal demand  $\alpha^* = \alpha_1 = (k(1 - \eta) - \eta)/((k + 1)(\eta x_{\sup} - x_{\inf}))z > 0$  and an optimistic optimal subjective belief  $y^* = y(\alpha_1) = (c_{\alpha_1})_{\sup} = z + \alpha_1 x_{\sup} > z$ . When the intensity of disappointment is high enough relative to the intensity of anticipatory feelings ( $\eta/(1 - \eta) > k$ ), then the negative skewness of the random variable  $-\tilde{x}$  enables the agent to profit from elation. The agent has then a negative demand of  $\tilde{x}$  (or equivalently a positive demand of the negatively skewed risky payoff  $-\tilde{x}$ ) with  $\alpha^* = \alpha_1 = (k(1 - \eta) - \eta)/((k + 1)(\eta x_{\sup} - x_{\inf}))z < 0$  and a pessimistic optimal belief  $y^* = y(\alpha_1) = z + \alpha_1 x_{\sup} < z$  to benefit from elation at date 1. We retrieve the fact that, depending on the relative intensity of anticipatory feelings and disappointment, the agent's optimal belief can be pessimistic or optimistic.

Moreover, it is easy to see that  $\partial \alpha_1 / \partial k > 0$  and  $\partial \alpha_1 / \partial \eta < 0$ , which means that the optimal investment in a positively skewed asset increases with  $k$  and decreases with  $\eta$ . As intuition suggests, a higher intensity of anticipatory feelings, which as seen in §3.1 is associated with more optimism, leads to a higher position in a positively skewed risky asset, and a higher intensity of disappointment reduces the level of investment in the positively skewed risky asset. Here again, the implications of our model differ from those of the GM model, because GM find that, for the additive habit specification with  $u$  DARA, the optimal investment in the risky asset decreases with  $k$  (Proposition 9.1 of GM) and that the optimal investment in the risky asset decreases with (respectively, increases with, is independent of)  $\eta$  iif relative risk aversion is larger than (smaller than, equal to) 1 (Proposition 9.2 of GM).

Figure 1 illustrates Proposition 5 for  $k > \eta/(1 - \eta)$ , i.e., when the intensity of anticipatory feelings is high enough relative to the intensity of disappointment. The top graph represents the welfare  $W(\alpha)$  as a function of the investment in a symmetric binary risk

Figure 1 Skewness and Welfare



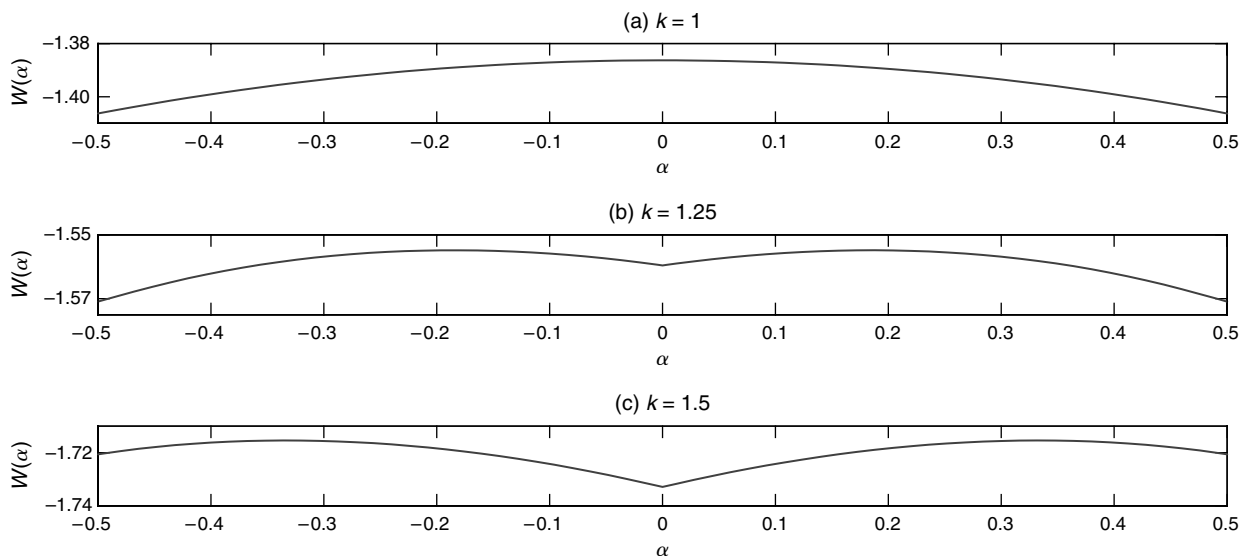
**Notes.** This figure presents the welfare  $W(\alpha)$  as a function of the investment level  $\alpha$  for a symmetric (a), positively skewed (b), and negatively skewed (c) binary risk. The utility function is given by  $U(c, y) = \ln(c - y/2)$  and  $k = 2$ . Initial wealth is given by  $z = 1$ . (a) For  $x_{\text{sup}} = -x_{\text{inf}} = 0.2$  (and  $\pi = 1/2$ ), the risk is symmetric and there are two optimal investment levels  $\alpha_1 = -\alpha_2 = 0.56$ . (b) If we maintain  $x_{\text{inf}} = -0.2$  and take  $x_{\text{sup}}$  equal to 0.4 (with  $\pi = 1/3$  to keep a zero mean), the optimal investment level is positive and given by  $\alpha_1 = 0.42$ . (c) If we maintain  $x_{\text{sup}} = 0.2$  and take  $x_{\text{inf}}$  equal to  $-0.4$  (with  $\pi = 2/3$  to keep a zero mean), the optimal investment level is negative and given by  $\alpha_2 = -0.42$ .

asset. Because the risk is symmetric, there are two symmetric possible values for the optimal portfolio  $\alpha^*$  yielding the same welfare. Note that the welfare function is not globally concave in  $\alpha$ . When the return is positively skewed (second graph), the welfare still has two local maxima but only the positive one is a global maximum. The positive demand for the risky asset yields higher welfare because the maximum return  $x_{\text{sup}}$  is higher (in absolute value) than the minimum

return  $x_{\text{inf}}$ . Therefore, a positive demand for the asset enables the agent to savor more for a given level of risk than the opposite demand. The third graph represents the symmetric situation with negatively skewed returns.

Figure 2 represents  $W(\alpha)$  and illustrates the impact of  $k$  in a symmetric returns framework. For  $k = 1$  (which corresponds in the example to  $\eta/(1 - \eta)$ ), the optimal demand is zero. When  $k$  increases, zero

Figure 2 Intensity of Anticipatory Feelings and Welfare



**Notes.** This figure presents the welfare  $W(\alpha)$  as a function of the investment level  $\alpha$  for three different values of the intensity of anticipatory feelings  $k$  and in the case of a zero-mean symmetric binary risk ( $x_{\text{sup}} = 0.2$  and  $x_{\text{inf}} = -0.2$ ). The utility function is given by  $U(c, y) = \ln(c - y/2)$ . Initial wealth is given by  $z = 1$ . For  $k = 1$ , the optimal demand is zero. When  $k$  increases, zero becomes a local minimum and the two (symmetric) maxima move away from zero.



becomes a local minimum of the welfare function, and the two symmetric maxima go away from zero.

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### Appendix A. Stylized Counterexamples for Comparative Statics Results

The following examples illustrate the differences between our model and the GM model in terms of comparative statics. Their stylized feature permits to clearly highlight the differences.

**EXAMPLE 1 (FSD AND THE OPTIMAL ANTICIPATED EXPECTED PAYOFF).** A utility function such that  $U_{cy} < 0$  for which there is an FSD dominated shift that decreases the optimal anticipated expected payoff  $y^*$ : Let  $U$  be defined by  $U(c, y) = c - \eta y - \frac{1}{2}\beta(c + \eta y)^2$  on  $[0, 1] \times [0, 1]$  with  $\beta = \frac{4}{19}$  and  $\eta = \frac{1}{2}$ . We take  $k = 2$ ,  $Q^1(\{\frac{1}{2}\}) = Q^1(\{1\}) = \frac{1}{2}$  and  $Q^2(\{\frac{1}{2}\}) = \frac{1}{2}$ . We have  $Q^1 \succ_{\text{FSD}} Q^2$  and  $y^2 = y^*(Q^2) = \frac{1}{2} - \frac{1}{38} < \frac{1}{2} = y^*(Q^1) = y^1$ .

**EXAMPLE 2 (INCREASES IN RISK AND THE OPTIMAL ANTICIPATED EXPECTED PAYOFF).**

(2a) A utility function such that  $U_{ccy} < 0$  and for which there is an increase in risk in the sense of Rothschild and Stiglitz (1970) that increases the optimal anticipated expected payoff  $y^*$ : Let  $U$  be defined by  $U(c, y) = \ln(c - \frac{1}{2}y)$  on  $[\frac{9}{10}, \frac{11}{10}] \times [\frac{9}{10}, \frac{11}{10}]$ . We take  $k = 3$ ,  $Q^1$ , and  $Q^2$  such that  $Q^1(\{1\}) = 1$  and  $Q^2(\{\frac{9}{10}\}) = Q^2(\{\frac{11}{10}\}) = \frac{1}{2}$ . The distribution  $Q^2$  is more risky than  $Q^1$  in the sense of Rothschild and Stiglitz (1970), and we have  $y^2 = y^*(Q^2) = \frac{11}{10} > 1 = y^*(Q^1) = y^1$ .

(2b) A utility function such that  $U_{ccy} > 0$  and for which an increase in risk exists in the sense of Rothschild and Stiglitz (1970) that decreases the optimal anticipated expected payoff  $y^*$ : Let  $U$  be defined by  $U(c, y) = c - \frac{1}{2}y - \frac{1}{4}(c - \frac{1}{2}y)^2 + \frac{1}{86}(c + \frac{1}{2}y)^3$  on  $[\frac{9}{10}, \frac{11}{10}] \times [\frac{9}{10}, \frac{11}{10}]$ . We take  $k = \frac{1}{2}$ ,  $Q^1(\{1\}) = 1$ , and  $Q^2(\{\frac{9}{10}\}) = Q^2(\{\frac{11}{10}\}) = \frac{1}{2}$ . The distribution  $Q^2$  is more risky than  $Q^1$  in the sense of Rothschild and Stiglitz (1970), and we have  $y^2 = y^*(Q^2) = \frac{9}{10} < 1 = y^*(Q^1) = y^1$ .

**EXAMPLE 3 (SSD AND WELFARE).** An SSD dominated shift in the probability distribution  $Q$  that increases the intertemporal welfare: Take the same utility function and the same distributions as in (2a). We check that  $W(Q^1) < W(Q^2)$ .

**EXAMPLE 4 (FSD AND WELFARE).** An FSD dominated shift in the probability distribution  $Q$  that increases intertemporal welfare: Let  $U$  be defined by  $U(c, y) = \ln(c - \frac{1}{2}y)$  on  $[\frac{9}{10}, 1] \times [\frac{9}{10}, 1]$ . We take  $k = \frac{1}{2}$ ,  $Q^1(\{1\}) = 1$ , and  $Q^2(\{\frac{9}{10}\}) = 1 - Q^2(\{1\}) = 0.01$ . We have  $Q^1 \succeq_{\text{FSD}} Q^2$ , and we check that  $W(Q^1) < W(Q^2)$ .

### Appendix B. Proofs

**PROOF OF PROPOSITION 1.** Let  $Q^1 \succeq_{\text{FSD}} Q^2$ . For  $i = 1, 2$ ,  $y^i$ ,  $c_{\text{inf}}^{Q^i}$ , and  $c_{\text{sup}}^{Q^i}$  denote the optimal anticipated expected payoff, the essential infimum, and the essential supremum under  $Q^i$ , respectively. Since  $U_{cy} > 0$ , we have  $F_{cy} > 0$  and then  $E^{Q^1}[F_y(\tilde{c}, y^1)] \geq E^{Q^2}[F_y(\tilde{c}, y^1)]$ . Furthermore, FSD shifts the support to lower payoffs, that is,  $c_{\text{sup}}^{Q^1} \geq c_{\text{sup}}^{Q^2}$  and  $c_{\text{inf}}^{Q^1} \geq c_{\text{inf}}^{Q^2}$ . The domain over which  $E^{Q^2}[F(\tilde{c}, y)]$  is maximized intersects with  $(-\infty, y^1]$ . If  $E^{Q^1}[F_y(\tilde{c}, y^1)] \leq 0$ , then  $E^{Q^2}[F_y(\tilde{c}, y^1)] \leq 0$ , and since  $F$  is concave in  $y$ , we have  $y^2 \leq y^1$ . If  $E^{Q^1}[F_y(\tilde{c}, y^1)] > 0$ , then  $y^1$  corresponds to the highest possible payoff under  $Q^1$  and we necessarily have  $y^2 \leq y^1$ .  $\square$

**PROOF OF PROPOSITION 2.** (2)  $\Rightarrow$  (1): Consider the lottery  $L = (\{x_1, x_2\}, (\pi, 1 - \pi))$  with  $x_1 < x_2$  and denote by  $y^*$  the optimal anticipated expected payoff of the lottery. We have  $W(x_1) = F(x_1, x_1) \leq \pi F(x_1, x_1) + (1 - \pi)F(x_2, x_1) \leq \pi F(x_1, y^*) + (1 - \pi)F(x_2, y^*) = W(L)$ , where the first inequality is because  $F_c > 0$  and the second inequality comes from the optimality of  $y^*$ . The inequality  $W(x_1) \leq W(L)$  is then always satisfied.

Since  $F_{yy} < 0$ ,  $F_y(x_2, x_2) \geq 0$  implies that  $F_y(x_2, x) \geq F_y(x_2, x_2) \geq 0$  for  $x \leq x_2$  and then  $F(x_2, x) \leq F(x_2, x_2)$  for all  $x \leq x_2$ . Thus, we have  $W(L) = \pi F(x_1, y^*) + (1 - \pi)F(x_2, y^*) \leq F(x_2, y^*) \leq F(x_2, x_2) = W(x_2)$ , where the first inequality follows from  $F_c > 0$ .

(1)  $\Rightarrow$  (2): Assume that there exist  $x_2$  and  $y < x_2$  with  $F(x_2, y) > F(x_2, x_2)$ . Let  $x_1 < y$  and consider the lottery  $l = (\{x_1, x_2\}, (\pi, 1 - \pi))$  with an optimal anticipated expected payoff denoted by  $y^l$ . We have  $W(l) = \pi F(x_1, y^l) + (1 - \pi)F(x_2, y^l)$ , and hence by optimality,  $W(l) \geq \pi F(x_1, y) + (1 - \pi)F(x_2, y)$ . Choosing a small enough  $\pi$ , we have  $W(l) > F(x_2, x_2) = W(x_2)$ , which leads to a contradiction. For all  $x_2$ , we then have  $F(x_2, y) \leq F(x_2, x_2)$  for all  $y \leq x_2$ , and hence,  $F_y(x_2, x_2) \geq 0$ .

(2)  $\Rightarrow$  (3): Let  $Q^1 \succeq_{\text{FSD}} Q^2$  and let  $y^1$  and  $y^2$  denote the optimal anticipated expected payoffs respectively associated with  $Q^1$  and  $Q^2$ . Since  $F_c > 0$ , we have  $W(Q^2) = E^{Q^2}[F(\tilde{c}, y^2)] \leq E^{Q^1}[F(\tilde{c}, y^2)]$ . If  $y^2 \geq c_{\text{inf}}^{Q^1}$ , then  $y^2 \in [c_{\text{inf}}^{Q^1}, c_{\text{sup}}^{Q^1}]$  (see the proof of Proposition 1), and by optimality, we have  $E^{Q^1}[F(\tilde{c}, y^2)] \leq E^{Q^1}[F(\tilde{c}, y^1)] = W(Q^1)$ . If  $y^2 < c_{\text{inf}}^{Q^1}$ , we have, for all  $c$  in the support of  $Q^1$ ,  $F(c, y^2) \leq F(c, c_{\text{inf}}^{Q^1})$  since  $F_y(x, x) \geq 0$  for all  $x$  implies that  $F(c, y)$  is increasing in  $y$  for  $y \leq c$  (see above). Therefore,  $E^{Q^1}[F(\tilde{c}, y^2)] \leq E^{Q^1}[F(\tilde{c}, c_{\text{inf}}^{Q^1})] \leq E^{Q^1}[F(\tilde{c}, y^1)] = W(Q^1)$ , where the last inequality is due to the optimality of  $y^1$ .

(3)  $\Rightarrow$  (1): Immediate.  $\square$

**PROOF OF PROPOSITION 3.** Assume that  $F_y(z, z) > 0$ . If  $\alpha > 0$  and is sufficiently small,  $y^*(\alpha)$  is sufficiently close to  $z$  to have  $kv'(y^*(\alpha)) + E^Q[U_y(\tilde{c}_\alpha, y^*(\alpha))] > 0$ . This implies that  $y^*(\alpha) = (c_\alpha)_{\text{sup}}$ . Hence, for a  $\alpha > 0$  that is sufficiently small,  $W_\alpha(\alpha) = E[\tilde{x}U_c(\tilde{c}_\alpha, y^*(\alpha)) + x_{\text{sup}}F_y(\tilde{c}_\alpha, y^*(\alpha))]$  and  $\lim_{\alpha \rightarrow 0^+} W_\alpha(\alpha) = x_{\text{sup}}F_y(z, z) > 0$ . We prove similarly that  $\lim_{\alpha \rightarrow 0^-} W_\alpha(\alpha) = x_{\text{inf}}F_y(z, z) < 0$  and  $\alpha = 0$  is a local minimum for  $W(\alpha)$ . The optimal investment is then nonzero. The case  $F_y(z, z) < 0$  is treated similarly.  $\square$

PROOF OF PROPOSITION 4. Let  $Q^1 \succeq_{SSD} Q^2$  with  $c_{sup}^{Q^1} = c_{sup}^{Q^2}$ , and let  $y^1$  and  $y^2$  denote the optimal anticipated expected payoffs respectively associated with  $Q^1$  and  $Q^2$ . Since  $F$  is concave in  $c$ , we have  $W(Q^2) = E^{Q^2}[F(\tilde{c}, y^2)] \leq E^{Q^1}[F(\tilde{c}, y^2)]$ . If  $y^2 \geq c_{inf}^{Q^1}$ , then  $y^2 \in [c_{inf}^{Q^1}, c_{sup}^{Q^1}]$  (since by assumption  $c_{sup}^{Q^1} = c_{sup}^{Q^2}$ ), and by optimality, we have  $E^{Q^1}[F(\tilde{c}, y^2)] \leq E^{Q^1}[F(\tilde{c}, y^1)] = W(Q^1)$ . If  $y^2 < c_{inf}^{Q^1}$ , we have, for all  $c$  in the support of  $Q^1$ ,  $F(c, y^2) \leq F(c, c_{inf}^{Q^1})$  since  $F_y(x, x) \geq 0$  for all  $x$  implies that  $F(c, y)$  is increasing in  $y$  for  $y \leq c$ . Therefore,  $E^{Q^1}[F(\tilde{c}, y^2)] \leq E^{Q^1}[F(\tilde{c}, c_{inf}^{Q^1})] \leq E^{Q^1}[F(\tilde{c}, y^1)] = W(Q^1)$ , where the last inequality is due to the optimality of  $y^1$ .  $\square$

PROOF OF PROPOSITION 5. Let us first study the concavity of  $W$  and let us assume  $\alpha > 0$  (the  $\alpha < 0$  case is treated similarly).

If  $(c_\alpha)_{inf} < y(\alpha) < (c_\alpha)_{sup}$  (Regime 1), then by the implicit function theorem, we have  $y'(\alpha) = -E[\tilde{x}U_{cy}]/(kv'' + E[U_{cc}])$  and  $W_{\alpha\alpha}(\alpha) = E[\tilde{x}^2 U_{cc}] + y'(\alpha)E[\tilde{x}U_{cy}]$ , where all functions are taken at  $y = y(\alpha)$  and  $\tilde{c}_\alpha$ .  $W_{\alpha\alpha}(\alpha)$  is negative if

$$\frac{kv''E[\tilde{x}^2 u''] + \eta^2(E[u'']E[\tilde{x}^2 u''] - E[\tilde{x}u'']^2)}{kv'' + \eta^2 E[u'']} < 0,$$

where the derivatives of  $v$  (respectively,  $u$ ) are taken at  $(1 - \eta)y(\alpha)$  (respectively,  $\tilde{c}_\alpha - \eta y(\alpha)$ ), and this inequality is satisfied because of the concavity of  $u$  and  $v$  and the Cauchy-Schwarz inequality.

When  $y(\alpha) = (c_\alpha)_{sup}$  (Regime 2),  $W_{\alpha\alpha}(\alpha)$  is given by  $W_{\alpha\alpha}(\alpha) = x_{sup}^2[kv'' + E U_{yy}] + 2x_{sup}E[\tilde{x}U_{cy}] + E[\tilde{x}^2 U_{cc}] < 0$ , where all functions are taken at  $y = (c_\alpha)_{sup}$  and  $c = z + \alpha\tilde{x}$ . The concavity condition is then given by

$$k(1 - \eta)^2 u''((1 - \eta)(c_\alpha)_{sup})x_{sup}^2 + E(\tilde{x} - \eta x_{sup})^2 u''(z(1 - \eta) + \alpha(\tilde{x} - \eta x_{sup})) < 0,$$

which is automatically satisfied by concavity of  $u$ . The same applies for  $y(\alpha) = (c_\alpha)_{inf}$  (Regime 3).

Finally, note that  $y(\alpha)$  and  $E^Q[F_y(\tilde{c}_\alpha, y(\alpha))]$  are continuous. This means that when we switch from Regime 2 to Regime 1 (or from Regime 3 to Regime 1), and conversely, at some  $\hat{\alpha} > 0$ , we have  $E^Q[F_y(\tilde{c}_\alpha, y(\hat{\alpha}))] = 0$  and  $W'(\hat{\alpha}^-) = W'(\hat{\alpha}^+) = E[U_c(\tilde{c}_\alpha, y(\hat{\alpha}))]$ . Thus,  $W_\alpha(\alpha)$  is continuous at  $\hat{\alpha}$ , and since  $W$  is concave at the left and at the right of  $\hat{\alpha}$ , it is concave on a neighborhood of  $\hat{\alpha}$ . The unique remaining cases correspond to switches from Regime 2 to Regime 3 and conversely. Since  $y(\alpha)$  is continuous, such a switch can only occur for  $\alpha = 0$ . In conclusion,  $W$  is concave on  $\mathbb{R}_-$  and  $\mathbb{R}_+$  but might not be concave at 0.

Let us then consider separately the two following problems:  $\max_{\alpha \geq 0, (c_\alpha)_{inf} \leq y \leq (c_\alpha)_{sup}} \{k \ln((1 - \eta)y) + E[\ln(\tilde{c}_\alpha - \eta y)]\}$  and  $\max_{\alpha \leq 0, (c_\alpha)_{sup} \leq y \leq (c_\alpha)_{inf}} \{k \ln((1 - \eta)y) + E[\ln(\tilde{c}_\alpha - \eta y)]\}$ .

Let us start with the first one, i.e.,  $\alpha \geq 0$ . The objective function is concave in  $(\alpha, y)$  and the domain  $\{(\alpha, y): \alpha \geq 0, (c_\alpha)_{inf} \leq y \leq (c_\alpha)_{sup}\}$  is convex. The first-order necessary and sufficient conditions for an interior solution are then given by  $k/y - \eta E[1/(\tilde{c}_\alpha - \eta y)] = 0$ , and  $E[\tilde{x}/(\tilde{c}_\alpha - \eta y)] = 0$ . Deriving  $y$  from the first equation and replacing it in the second equation, we obtain  $\alpha = 0$ , which is only optimal if  $k = \eta/(1 - \eta)$ . Otherwise, there is no interior solution. The same applies for  $\alpha \leq 0$ .

This means that the solutions of (2) are necessarily such that  $y^*(\alpha^*) = z + \alpha^* x_{sup}$  or  $y(\alpha^*) = z + \alpha^* x_{inf}$ . It suffices then to solve the two following problems:

$$\max_{\alpha} \{k \ln((1 - \eta)(z + \alpha\tilde{x})) + E[\ln((1 - \eta)z + \alpha(\tilde{x} - \eta\tilde{x}))]\},$$

with  $\tilde{x} = x_{sup}$  or  $\tilde{x} = x_{inf}$ , and to compare their values to determine  $\alpha^*$ . We obtain

$$\alpha_1 = \frac{k(1 - \eta) - \eta}{(k + 1)(\eta x_{sup} - x_{inf})} z, \quad \alpha_2 = \frac{\eta - k(1 - \eta)}{(k + 1)(x_{sup} - \eta x_{inf})} z, \quad \text{and}$$

$$W(\alpha_0) - W(\alpha_1) = (k + \pi) \ln \left( \frac{(k + \pi)(1 - \eta)}{\pi(1 - \eta) + \eta} \right) - (k + 1 - \pi) \ln \left( \frac{(k + 1 - \pi)(1 - \eta)}{1 - \pi(1 - \eta)} \right) = \Delta(\pi).$$

We check that  $\Delta(\pi)$  is decreasing on  $[0, 1]$  with  $\Delta(1/2) = 0$ . Consequently,  $\alpha^* = \alpha_1$  for  $\pi \leq \frac{1}{2}$  and  $\alpha^* = \alpha_2$  for  $\pi \geq \frac{1}{2}$ .  $\square$

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