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The MSOM Society Student Paper Competition: Extended Abstracts of 1999 Winners

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Each year the Manufacturing and Service Operations Management (MSOM) Society of INFORMS conducts a Student Paper Competition. This year, in what the Editor-in-Chief and Senior Editors of the *M&SOM* journal hope will become an annual event, we are publishing extended abstracts of the 1999 winners.

The Chairperson of the 1999 competition was Anant Iyer, i2 Technologies. The judges were Ron Askin (University of Arizona), Frank Ciarallo (University of Arizona), Vibhu Kalyan (i2 Technologies), Janny Leung (City University of Hong Kong), Sarah Patterson (Duke University) and Sridhar Tayur (Carnegie Mellon University).

The 1999 winners were chosen after two rounds of judging. The first round yielded the five papers whose abstracts follow. During the second round each judge examined all five and then voted to determine the winners. All the winners received a certificate recognizing their accomplishments. In addition, the MSOM Society provided a cash award of $400 to the first-prize winner, and $200 to the second-prize winner (shared between the joint winners).

The winners and their faculty mentors are:

First Prize
L. Beril Toktay, INSEAD
Faculty Mentor: Lawrence Wein, MIT
"Analysis of a Production-Inventory System with Stationary Demand and Dynamic Forecast Updates"

Joint Second Prize
Prashant C. Fuloria, Stanford University
Faculty Mentor: Stefanos A. Zenios, Stanford University
"Incentive Efficiency in a Health-Care Delivery System"
and
Itir Z. Karaesmen, Columbia University
Faculty Advisor: Garrett J. van Ryzin, Columbia University
"Overbooking with Substitutable Inventory Classes"

Joint Honorable Mentions
Apurva Jain, University of Washington
Faculty Mentor: Ananth. Iyer, Purdue University
"The Logistics Impact of a Mixture of Order-Streams in a Manufacturer-Retailer System"

James T. Treharne, U.S. Army TRADOC Analysis Center
Faculty Mentor: Charles R. Sox, Auburn University
"Adaptive Inventory Control for Nonstationary Demand and Partial Information"

Extended abstracts of these papers follow.

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Analysis of a Production-Inventory System with Stationary Demand and Dynamic Forecast Updates

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Consider a capacitated manufacturing stage that produces a single product in a make-to-stock fashion. Customer demand for the product evolves according to a discrete-time stationary stochastic process. Updated demand forecasts over a finite forecast horizon become available in each period for use in production decisions. The goal of this paper is to analyze the interrelation between correlated demand, capacitated production, forecast error, and safety stock in such a forecasting-production-inventory setting.

We assume that the forecasting and production-inventory control activities are decentralized: In each period, forecasts of demand over the forecast horizon are generated by a process (e.g., time-series modeling, expert judgement, advance order information) unknown to the production manager, who only observes the output of this process in the form of a sequence of forecast-update vectors. The production manager's goal is to minimize the total expected steady-state inventory holding and backorder cost. To achieve this, he requires a characterization of the inputs to the production stage, namely, of the forecast update and demand processes. The Martingale Model of Forecast Evolution (MMFE) developed independently by Graves et al. (1986, 1998) and Heath and Jackson (1994) provides this modeling capability. The MMFE posits that under relatively mild assumptions (see Heath and Jackson for details), the sequence of forecast-update vectors can be modeled as iid mean zero multivariate normal random variables (with covariance matrix Σ). To characterize an existing forecast-update process satisfying the MMFE assumptions, it is sufficient to estimate the elements of the covariance matrix Σ using historical forecast-update data.

We model the production stage as a discrete-time continuous-state single-item make-to-stock queue driven by forecast updates that are modeled via the MMFE. In the finite-horizon, deterministic-capacity setting, we show that the optimal policy is a nonstationary modified base-stock policy with respect to forecast-corrected inventory (current inventory level minus the aggregate demand forecast over the forecast horizon). Based on this result, we propose an order-release rule that maintains a constant base-stock level with respect to forecast-corrected inventory: The number of orders released to the production stage in each period equals the aggregate demand forecast update over the forecast horizon. The production quantity is then given by the smaller of the current order queue length and the production capacity, which is assumed to be random in order to model yield uncertainty. Under the proposed policy, the production-inventory control problem in the steady-state random-capacity setting reduces to finding the cost-minimizing base-stock level with respect to forecast-corrected inventory.

We use an approximate analysis based on heavy-traffic and random-walk theory to characterize the optimal forecast-corrected base-stock level. We then consider the asymptotic case where the backorder cost is

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much larger than the holding cost to obtain an approximate closed-form expression for this quantity. Discrete-event simulation shows that the derived expression is accurate under high utilization and is robust with respect to the backorder-to-holding cost ratio.

Our analytical results provide a basis for investigating the impact of demand and forecast characteristics on the performance of a capacitated production-inventory system driven by forecast updates. We observe that the optimal base-stock level increases with utilization level, demand uncertainty, capacity uncertainty and the variability of the total forecast error over the forecast horizon. Compared to iid demand, positively (negatively) correlated demand necessitates more (less) safety stock.

Different forecasting schemes corresponding to the same demand process and satisfying the MMFE assumptions may differ in the variance of total forecast error over the forecast horizon, or alternatively, in the amount of unresolved demand variability over the forecast horizon. We define a higher quality forecasting scheme to be one with a lower level of unresolved demand variability. Our analysis reveals that higher quality forecasting schemes allow the production-inventory system to carry less safety stock. To see the nature of this effect, we compare the optimal base-stock level in the setting described above with a benchmark case where no advance information is used; that is, where the number of order releases in each period equals the demand realized in the same period. The difference in the optimal base-stock levels under these two scenarios is the product of two factors: the total expected excess capacity over the forecast horizon and the fraction of the total system (demand and production) variability over the forecast horizon that has already been resolved. This result highlights that safety stock and excess capacity are interchangeable resources in a make-to-stock system driven by forecast updates: The system's reliance on safety stock (as opposed to excess capacity) to cope with unresolved system variability decreases with the quality of the forecast. The value of advance information vanishes with increasing system utilization; in this regime, the only resource to counter unresolved variability is safety stock.

We investigate the impact of forecast bias and forecast model misspecification (by the exogenous forecaster) on system performance and determine that they can result in high cost suboptimality. Our results suggest that operational decisions regarding the usage of available information, such as using updated forecast information to determine releases and aggregating forecasts, can improve system performance, but the relative cost improvement thus obtained is low when compared to the improvement achieved by correct forecast model specification. We conclude that the main value-added step in forecasting is the specification step, which is arguably not carried out thoroughly in practice. It would thus be of value to design inventory-control policies that are robust with respect to forecast model misspecification.

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Incentive Efficiency in a Health-Care Delivery System

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Healthcare delivery systems are becoming increasingly decentralized, with medical decisions delegated by the purchasers of health services to noncooperative specialized providers. The performance of a decentralized health-care delivery system is affected by the payment scheme or *reimbursement mechanism* used by the purchaser of a health service to reimburse the provider of that service. This happens because the delegation of medical decision making creates financial-incentive problems which are amplified when the decisions that are desirable to the purchaser are expensive for the provider (Kuttner 1999). Consequently, an important research problem is to design reimbursement mechanisms that will alleviate financial-incentive issues, lead to a more efficient distribution of the purchaser's funds, and generate better health outcomes.

In the public sector, the most important example of the problem analyzed in this research is the relationship between Medicare and Medicare providers. Medicare, the largest purchaser of health services in the United States, provides coverage for outpatient and inpatient services to more than 37 million Americans. The services covered by Medicare are provided by a vast network of private health plans, hospitals, physicians, and other specialists, which are reimbursed based on a complicated cost-reimbursement formula approved by the U.S. Congress. However, there is strong evidence that this formula is plagued by incentive problems because it does not hold the providers accountable for the outcomes of their services (Farley et al. 1996). In fact, the Balanced Budget Act of 1997 authorized Medicare to consider alternative reimbursement mechanisms that represent more efficient dissemination of Medicare's funds (U.S. Congress 1997). In that spirit, our research identifies reimbursement mechanisms that alleviate some of these incentive problems and generate improved clinical outcomes.

Another example is provided by private health insurance corporations which purchase health services for their members. Consider, for instance, Aetna US Healthcare, the largest private purchaser of health services in the United States. Aetna provides health coverage for more than 13.7 million members by contracting with over 330,000 providers, including primary-care physicians, hospitals, and other specialists. Recognizing that the providers are the main determinants of patient outcomes, and that these outcomes can be compromised by poorly designed financial arrangements, Aetna's CEO made the following statement in his 1998 letter to the shareholders: ". . . the best health outcomes are achieved through strong relationships and accountability among physicians, patients and health plans" (Huber 1998). Our research identifies mechanisms that increase the accountability of the health-care providers and lead to these "best" health outcomes.

The principal-agent paradigm from the economics literature is relevant to the analysis of the purchaser-provider relationship in a decentralized health-care delivery system (Zweifel and Breyer 1997). Using this paradigm, we consider a situation in which a purchaser of health services for a patient population (a principal whom we refer to as "her") purchases these services from an independent noncooperative provider (an agent whom we refer to as "him"). We assume that the patient population is homogenous; however, multiple patient classes can be introduced without any difficulty. New patients arrive and join the population obeying an exogenous Poisson process. All patients are assumed to suffer from a chronic disease, and hence leave the population only through death. While alive, each patient requires periodic outpatient services which are administered by the agent. The intensity of each outpatient service is assumed to be specified by the agent and allowed to vary dynamically from treatment to treatment. The agent incurs an associated cost for each treatment which is assumed to be strictly increasing and convex in the intensity. The principal cannot directly monitor the delivered intensity, but she does observe clinically relevant patient outcomes, such as mortality and demand for inpatient services, the occurrence of which is influenced by the agent's medical decisions. Specifically, the probabilities of undesirable patient outcomes are related to the treatment intensity received through a proportional-hazards model (Cox and Oakes 1984), with the probabilities decreasing in the treatment intensity. The principal, who receives a reward per patient per unit time and also incurs the inpatient costs, wishes to design a reimbursement mechanism that rewards the agent based on observed outcomes and motivates him to make decisions that maximize her expected net rewards.

As is customary in principal-agent models, we also assume that the principal is risk-neutral and that she is the party with the power to determine the terms of the reimbursement mechanism. By contrast, the agent is assumed to be a "small" health-care provider, and is thus risk-averse. Following Fudenberg et al. (1990), he is also assumed to have access to banking; this assumption is made in order to focus attention on incentive problems that arise due to the delegation of medical decisions from the principal to the agent. Additional incentive problems arise when the agent does not have access to banking, because then the agent relies on the principal to maintain a smooth flow of cash. Plambeck and Zenios (1999) show that the banking assumption leads to an analytically-tractable dynamic principal-agent model which can be solved using a two-step dynamic programming procedure.

Our analysis involves placing a very reasonable linearity restriction on the allowed class of reimbursement mechanisms. Given this restriction, we show that the optimal financial-incentive scheme has several desirable properties that hold promise for practical applications: It is both history and state independent and can also be characterized by a small number of parameters. The optimal incentive scheme has two components: a gold-standard, which represents the treatment intensity desirable to the principal, and a reimbursement mechanism, which motivates the agent to adopt this standard. The agent's total reimbursement in this incentive scheme is linked to patient outcomes: It rewards him for outcomes that are consistent with the gold standard and penalizes him for adverse outcomes that indicate deviations from this standard. Significantly, in this scheme, the gold standard is worse than the standard adopted if the provider and purchaser were a monolithic cooperative team. This indicates that the quality of the delivered clinical services depends on the organizational structure of the healthcare delivery system and on the incentive mechanisms that are in place.

We also examine a numerical application of our model and analytical results; the interested reader is referred to Fuloria and Zenios (1999). This application is motivated by Medicare's ESRD (End-Stage Renal Disease) program, a major entitlement program in which patients requiring dialysis therapy (usually an outpatient service) are eligible for Medicare coverage (Zenios and Fuloria 1998). Medicare typically reimburses dialysis providers using a traditional fee-for-service (FFS) mechanism and has recently started experimenting with a capitated plan, commonly known as the HMO (Health Maintenance Organization) plan, in which dialysis providers receive a capitated annual payment per patient and are responsible for both patient dialysis services and any additional inpatient hospital services. We use our purchaser-provider model to numerically evaluate these two reimbursement mechanisms and compare their performance with that of our analytically derived optimal reimbursement mechanism. Our finding that the HMO plan is a substantial improvement over the FFS mechanism provides a validation for Medicare's recent experimentation with the former. However, our proposed optimal reimbursement mechanism performs even better, resulting in a patient life-expectancy improvement of more than 9% over the HMO plan, and more than 39% over the FFS mechanism, for the same Medicare expenditure per patient.

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Overbooking with Substitutable Inventory Classes

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The idea of yield management, in general, is to improve revenues by more effectively managing service capacity. Overbooking (i.e., accepting more reservations than one has physical capacity to serve, as a hedge against cancellations and no-shows) is one of the oldest and, from a revenue standpoint, one of the most important yield management tactics. Despite limited attention in the literature, there are many examples of real-world operations that involve overbooking multiple, substitutable inventory classes. At most airlines, for example, overbooked business and economy class passengers are upgraded to the first class cabin on oversold flights. Similarly, hotel customers are sometimes upgraded to luxury rooms; mid-size cars substituted for compact cars, etc., depending on demand and available capacity. A less obvious, but important example occurs in airlines that have frequent departures on the same route, in which case later departures can be used to service the overflow from earlier, overbooked flights. Because substitution may decrease (or increase) the quality of service, it has an important impact on customers. At the same time, substitution is a potentially valuable option to enhance the effectiveness of overbooking. Therefore, it is important to understand how to balance its potential benefits and costs.

In this paper, we consider a reservation/overbooking control problem in which management can accept or deny requests for n different reservation classes. Demands for reservation classes can be satisfied using any one of m different classes of inventory. The cost (out-of-pocket and/or good-will cost) of assigning demand to an inventory class depends on the particular reservation-inventory class pairing. There are two periods in our model of the booking process: (1) a reservation period, in which reservations are accepted, and (2) a service period, in which demand is realized and assigned to the m inventory classes. In the first period, reservations are accepted given only probabilistic knowledge about cancellations. In the second period, cancellations are realized and surviving customers are assigned to the various inventory classes in order to maximize the net benefit of assignments (e.g. minimize downgrading penalties). We use this two-period formulation to find joint overbooking levels for n reservation classes that maximize the revenues net of overbooking or substitution penalties.

The overbooking levels of different inventory classes are related in a natural way. In order to show that, we first analyze the properties of the deterministic (ex post) service period allocation problem, the most important of which are that its value is *submodular* and *component-wise concave* with respect to the realized demand levels. This means that an increase in the realized demand of one class reduces the marginal benefit of additional demand in any other class. Second, we extend these properties to the expected net revenue function of the overbooking problem in the reservation period, using stochastic-convexity ideas. Specifically, we show that the expected net revenue function is component-wise concave in each booking level and that the marginal benefit of accepting an additional booking in any class is nonincreasing in the level of bookings for all other reservation classes. In practical terms, this means that optimal booking levels for a class will decline as the number of reservations on hand for the other classes increases. This property is natural, since, because of substitution, reservations indirectly compete for the same capacity. It also describes qualitatively how reservation levels of one class affect the optimal overbooking levels of other classes.

Finally, we develop a simulation-based optimization method for determining joint overbooking levels. Our algorithm is based on stochastic gradients of the expected revenue function. We apply our method to an example from the airline industry to determine what effect the substitution option has on revenue performance and service levels. Numerical results show that our procedure can increase revenues, even when compared to the ad hoc use of substitution. It appears that accounting for substitution options when one is setting overbooking levels has a small but significant impact on overall revenue and service performance. This insight should stimulate practitioners to reexamine their overbooking practices for adjacent flights, multicabin flights, and other yield management applications where substitution options are prevalent and widely used in an ad hoc fashion, with an eye toward more precisely setting joint booking levels.

The Logistics Impact of a Mixture of Order-Streams in a Manufacturer-Retailer System

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The grocery industry has seen a number of significant logistics trends recently—ECR (Efficient Consumer Response) and CPR (Continuous Product Replenishment) are notable initiatives (see Kurt Salmon Associates 1993). As a part of these initiatives, changes have been implemented at two levels. The first level is manufacturer-to-retailers: Various manufacturers offer both traditional trade-promotion prices as well as value-pricing or *Every Day Low Purchaser's Prices* (EDLPP). The goal of EDLPP prices is to decrease retailers' incentives for forward-buying, which increases the variability in the retailers' order streams. Data from industry show, however, that not all retailers agree to switch to the new pricing policy. Campbell Soup estimates that about 16% of its retailers are on CPR (see the Harvard case Campbell Soup Company, Clark (1994)). This generates a mixture or order streams of different variability. The second level is retailers-to-customers: Retail formats show a diversity of pricing policies. Lal and Rao (1997) identify Albertson's, Food Lion, and Hannaford as *every day low prices* (EDLP) supermarkets and Vons, Safeway and Kroger as *Hi-Lo* (offering retail promotions) supermarkets. In this case too, the demand received by the manufacturer is a mixture of the retailer warehouse order streams of different variability.

For a manufacturer that takes the supply-chain view for reducing variability, the above discussion highlights the need to devise variability reduction programs that encourage retailers to adopt the low-variability behavior based on their individual incentives. We analyze a model that captures the essential features of this situation: the coexistence of two retailers' demand processes with different variability, the ability to change their mix, and to track the two retailers' inventory costs.

The two retailer warehouses, Retailer M (i.e., EDLP pricing policy) and Retailer H (i.e., Hi-Lo pricing policy) carry inventory to satisfy their respective stores' demands, and order replenishments from a manufacturer. Unmet stores' demands are backordered at the warehouses. Store orders are for a single unit (for example, truck loads) and are assumed to be instantaneously communicated to the manufacturer at no cost. Each retailer's inventory is managed using an (S-1,S) policy. The manufacturer produces on a make-to-order basis. We model the demand process at Retailer M as a Poisson process with intensity λ and the demand process at Retailer H as a renewal process where the interarrival times follow a Hyperexponential distribution of degree 2 (H_2) with balanced means. This choice not only provides us with a reasonable approximation to the spikes of a promotional demand pattern, but also, because of its relative simplicity, allows us to develop analytical insights. Define: ν = intensity of the demand process at Retailer H, and hcv = coefficient of variation of the demand process at Retailer H. The manufacturer is modeled as a single-server, first-come-first-served queue with exponential service rate μ. The order-arrival process at the manufacturer is the superposition of these two processes. Hence, the queuing system at the manufacturer is $H_2 + M/M/1$. Define: $\rho = \lambda/\mu$ = Retailer M's load; $\sigma = \nu/\mu$ = Retailer H's load; $\tau = \rho + \sigma < 1$, total load on the manufacturer's queuing system; and f = Retailer M's load as a fraction of the total load $= \rho/\tau$. For Retailer j (j = H,M), we use L_j to denote the lead-time random variable for its orders. Given holding cost and penalty cost values, let EC_j^* denote the optimal expected base-stock inventory cost at Retailer j.

Analytical Results

Our analysis of the queuing system at the manufacturer, H_2 + M/M/1, follows that of Kuczura (1973) and extends it. The property that is exploited in this analysis is that between two consecutive Retailer H order arrivals, the system is Markovian. The analysis leads to the explicit characterization of the lead-time distributions and base-stock expected inventory cost for the two retailers. We show that L_H has a generalized hyperexponential distribution, that L_M has a hyperexponential distribution, and that they are both different-weighted mixtures of the same two exponential distributions. We prove the following comparative results:

THEOREM 2.1. *The lead time experienced by Retailer H is more dispersive than the lead-time experienced by Retailer M in the sense* $L_H \geq_{Disp} L_M$.

THEOREM 2.2. *If the demand intensity* ν *for Retailer H and the demand intensity* λ *for Retailer M are ordered as* $\nu \geq \lambda$, *then their optimal expected base-stock inventory costs are ordered as* $EC_H^* \geq EC_M^*$.

Managerial Implications

Our analysis suggests two alternative approaches for a manufacturer interested in proposing a variability reduction program to Retailer H:

Centralized Approach. In this approach the manufacturer changes policies so that at each incremental step of the variability reduction program there is a small reduction in the variability of all the high-variability demand—we interpret this as a decrease in *hcv* and as a reduction in the depth of discounts and unpredictability in the Hi-Lo retail promotion-pricing policy. Based on our numerical study, we draw the following inferences: (1) As *hcv* decreases, the optimal expected cost for a high-variability retailer decreases. However, the optimal expected cost for a low-variability retailer also decreases. If the implementation of lower *hcv* retail environments requires sharing of information between the two types of retailers, this model provides a rationale for such information sharing. (2) As the optimal service level for products increases, the benefits of decreasing *hcv* increase. This suggests that for implementing variability reduction programs, manufacturers should first target products that have a high service level requirement. (3) The reduction in optimal expected cost decreases as *hcv* decreases. This diminishing returns to variability reduction efforts suggests a possible reason why centralized approaches to variability reduction program implementation may not result in full participation by the retailers.

Decentralized Approach. In this approach the manufacturer changes policies so that at each incremental step of the variability reduction program a small fraction of the high-variability demand is completely converted to the low-variability demand—we interpret this as an increase in $f(=\rho/\tau)$ and as shifting a small fraction of demand from Hi-Lo promotion pricing policy to EDLP nonpromotion pricing policy. Based on our numerical study, we draw the following inferences: (1) The benefit to a small fraction of high-variability demand from completely converting to the low-variability demand is small for low f and increases as f increases. This suggests greater reluctance for retailers to convert to nonpromotion policy initially than when there are already a large number of retailers under the nonpromotion policy, and therefore suggests greater manufacturer effort required to promote the variability reduction program in the beginning. (2) The higher the variability of the small fraction of demand that is shifted to the low-variability non-promotion policy, the higher is the total benefit to the system. This suggests that for implementing the variability reduction program the manufacturer should first target a retailer with high *hcv* (perhaps due to being in a promotion sensitive environment). (3) The rate at which system-wide and individual retailer costs decrease is maintained as f increases. Thus, such a decentralized approach does not seem to suffer from the diminishing returns to scale effect, as does the centralized approach.

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Adaptive Inventory Control for Nonstationary Demand and Partial Information

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We examine an inventory-control problem in which the demand process is partially observed and nonstationary. The demand process is partially observed because the distribution of demand in a given period is not known with certainty, and it is nonstationary because the distribution may randomly change over time.

When these conditions exist, optimal policies are very difficult to calculate. Managers often will estimate the unknown demand distribution, and then use some type of suboptimal certainty equivalent control (CEC) policy that assumes the estimate is exact. We present results that demonstrate that there exist practical suboptimal control policies to solve realistic instances of this problem without requiring the assumption that the estimate is exact. Furthermore, in most cases these control policies achieve much better performance, that is, lower average cost, than the CEC policies commonly used in practice.

We examine a finite-horizon problem with positive lead time, full backordering, and linear holding and backorder costs. However, the model can be extended to allow a positive fixed order cost and observed or unobserved lost sales. Also, many of the results hold for convex costs. We model the demand process as a composite-state, partially-observed Markov decision process. Partially-observed Markov decision processes have been well studied in the past, but with few applications in inventory and production control. We assume that the demand in any given period arises from one of a finite collection of N known candidate distributions. The demand state process $\{d_t\}$ is not known with certainty and is referred to as the *core process*. However, the decision maker may not have complete certainty about which of the distributions generates the demand in a given period. A known, discrete a priori distribution, π_t, is assumed for $\{d_t\}$ that is used, along with subsequent observations, to compute a *posterior* distribution. We define the demand observations as w_t. The vector π_t characterizes the current belief of the distribution of d_t given all prior observations (demands) of the information process $\{w_t\}$. Furthermore, the distribution may randomly change from one period to the next. Any pattern of transitions from one distribution to another can be modeled. So, the system is modeled as a finite-state Markov chain, $d_t \in \{1, \ldots, N\}$, governed by the transition matrix P. If $d_t = j$, the probability distribution for demand in t is r_j, *i.e.*, $P[w_t = k \mid d_t = j] = r_{jk}$ for $k = 0, 1, \ldots, M$. Therefore, the problem can be viewed as a Markov decision process on the state space (u_t, π_t), where u_t is the inventory position.

The control process begins with an information vector, I_t, which includes a prior distribution, π_t, for period t, and then selects an order quantity, a_t. For positive lead time, an order placed l periods in the past is received. The demand, w_t, then occurs, and the inventory costs for period t are incurred. The time index advances to $t + 1$, and the core process advances from d_t to d_{t+1} according to the transition matrix P. The new prior distribution π_{t+1} is computed in a Bayesian fashion using π_t, w_t and P.

The problem can be solved by dynamic programming. We formulate the problem in terms of the *inventory position,* which is the net inventory level (on-hand less backorders) plus inventory on-order at the beginning of t, $u_t = x_t + \Sigma_{n=1}^{l} a_{t-n}$. The inventory position follows the transition equation $u_{t+1} = u_t + a_t - w_t$, and if we redefine $S_t = u_t + a_t$, then the dynamic-programming recursion is:

$$J_t(u_t, \pi_t) = -cu_t + \min_{S_t \geq u_t} \{cS_t + G_t(S_t \mid \pi_t, l) + E_{w_t \mid \pi_t}[J_{t+1}(S_t - w_t, T(\pi_t \mid w_t))]\}, \; t = 0, \ldots, T;$$

where c is unit cost, $G_t(\cdot)$ is the expected single period cost, and $T(\cdot)$ computes the posterior distribution.

We show that $J_t(u_t, \pi_t)$ is a convex function of u_t for all π_t. This result demonstrates that a state-dependent base-stock policy is optimal for this problem. Although the optimal policy has a relatively simple structure, it is computationally difficult to compute the policy because the stock level is state-dependent. Therefore, we describe several possible suboptimal control policies that can be computed much more quickly.

Common business practice for most inventory-control systems is to use some type of certainty equivalence control. In the CEC approach, some function of the information vector, I_t, is used to generate an estimate, $\hat{d}_t$, of d_t. Assuming that $d_t = \hat{d}_t$, a complete information problem is solved to determine the initial base-stock level S_0. In practice, this process is repeated at the beginning of each period using the newly updated information vector. The estimate $\hat{d}_t$ is computed using some type of forecasting model, then, depending on the problem, either a stationary or nonstationary demand model is used to compute S_0. This approach ignores the uncertainty in the estimate $\hat{d}_t$, and does not account for the use of feedback about d_{t+n} in future periods when calculating the current solution.

We suggest the use of other practical and more cost effective suboptimal policies than the CEC policies. These suboptimal procedures include open-loop feedback control (OLFC) and limited look-ahead control (LLAC). Both of these policies account for more of the inherent uncertainty in the demand process and, therefore, often outperform the CEC policies.

Open-loop strategies do not use feedback to update estimates of π_t for $t > 0$. A decision is made at time zero for an initial order-up-to level under the assumption that no feedback will be used in the future to calculate new stock levels. However, in practice, an open-loop control problem may be solved each period with the previous demand observation used as feedback to update the current prior. This type of control has been referred to in the literature as Open-Loop Feedback Control. This approach takes advantage of both open- and closed-loop control. The cost-to-go equation assumes that feedback will not be used in future periods (open loop) and thereby greatly reduces the computational requirements. However, because π_t is updated using the prior observations (closed loop), it does incorporate the information available at t to characterize the distribution of w_t. Another way of describing this approach is that an open-loop control is implemented on a "rolling-horizon" basis.

Limited look-ahead control policies are suboptimal control policies that optimize the dynamic problem for only a limited amount of time into the future, say L periods. If $L = 0$, we call it a myopic policy. Computational requirements increase exponentially as the look-ahead period increases.

We test the OLFC and LLAC control policies over a wide range of problem instances and compare them with an optimal policy and three CEC policies. We specify three discrete distributions that form the set of possible distributions. We examine the effects among policies for various parameter variations including seven transition matrices, three lead times, and three critical values (by varying penalty costs). We also examine cases that vary the confidence that a specific distribution is in effect. We do so with a function δ which measures the dispersion of the prior distribution π_t from a discrete uniform distribution on $\{1, \ldots, N\}$. As δ approaches 1, there is a high confidence that a specific distribution is in effect. Similarly, as δ approaches 0, there is a low confidence that a specific distribution is in effect. We also consider the effects of *ergodicity* for a transition matrix. This quantity is a measure of how quickly the Markov process converges to its stationary distribution.

The most significant result is that the full consideration of the uncertainty of the demand distribution, even over a limited horizon, leads to substantial improvements over the CEC policies. Even the myopic policy performs substantially better than the CEC policies. The look-ahead, myopic, and OLFC control policies perform much better than the CEC policies. Most importantly, the CEC policies rarely outperform the look-ahead or OLFC policies. Because CEC or similar policies are often used in practice, these results suggest that even a very simple policy, such as the myopic policy, that fully considers the demand uncertainty, will perform significantly better than a CEC policy. The computational-time performance for the OLFC, myopic, and look-ahead policies are comparable to the CEC policies. There is certainly a tradeoff between computational time and solution quality. However, the solution quality of these policies is significantly improved without a significant increase in computational time compared to the CEC policies. We have demonstrated that a policy that uses feedback and fully accounts for the demand uncertainty in this problem can substantially outperform a certainty equivalent policy. By incorporating more of the actual uncertainty of the demand process even over a limited horizon into a suboptimal policy can provide dramatic improvements in the solution of this problem. In many cases even a myopic policy will provide significantly better results. Additionally, open-loop feedback control policies also will often perform exceptionally well when implemented in a "rolling horizon" fashion. Look-ahead policies should also be tractable for many realistic problems. The most effective policies consider the uncertainty of the demand distribution through an a priori distribution. This prior can be estimated through many different means and some future research can focus on efficient ways to obtain these priors as well as evaluate the sensitivity of the solution to the accuracy of the prior.

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