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Supply Chain Contract Design Under Financial Constraints and Bankruptcy Costs

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We study contract design and coordination of a supply chain with one supplier and one retailer, both of which are capital constrained and in need of short-term financing for their operations. Competitively priced bank loans are available, and the failure of loan repayment leads to bankruptcy, where default costs may include variable (proportional to the firm's sales) and fixed costs. Without default costs, it is known that simple contracts (e.g., revenue-sharing, buyback, and quantity discount) can coordinate and allocate profits arbitrarily in the chain. With only variable default costs, buyback contracts remain coordinating and equivalent to revenue-sharing contracts but are Pareto dominated by revenue-sharing contracts when fixed default costs are present. Thus, for general bankruptcy costs, contracts without buyback terms are of most interest. Quantity discount contracts fail to coordinate the supply chain, since a necessary condition for coordination is to proportionally reallocate debt obligations within the channel. With only variable default costs and with high fixed default costs exhibiting substantial economies-of-scale, revenue-sharing contracts with working capital coordination continue to coordinate the chain. Unexpectedly, for fixed default costs with small economies-of-scale effects, the two-firm system under a revenue-sharing contract with working capital coordination might have higher expected profit than the one-firm system. Our results provide support for the use of revenue-sharing contracts with working capital coordination for decentralized management of supply chains when there are bankruptcy risks and default costs.

Keywords: newsvendor; supply contract; supply chain coordination; bankruptcy/default costs; working capital management

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1. Introduction

It is not uncommon in a supply chain that either the supplier or the retailer is the subsidiary of the other or that they are both subsidiaries of the same parent company. For example, Target Brands and Target Stores are subsidiaries of Target Corporation, Visteon was a previous subsidiary of Ford (that spun off in 2000), Motorola Mobility is a subsidiary of Google, Nokia Mobile is a subsidiary of Microsoft, and so on. Often the parent company manages regular intracompany transfers of funds among itself and subsidiaries to reallocate working capital for various purposes, e.g., risk concerns, tax reduction, profitability, etc.¹

In this paper, we argue that such practice is strategically advantageous from a supply chain coordination perspective in the presence of bankruptcy risks and costs.

In fact, the transfer of funds between supply chain partners is observed beyond just subsidiary-parent style supply chains. When Visteon faced financial distress in 2005, Visteon issued to Ford warrants to purchase \$25 million shares at an exercise price of \$6.90 per share, and Ford provided \$400 million to Visteon for restructuring its businesses, as stated in Visteon's Form 10-K in March 2006. (A parallel story is recounted for another pair of supply chain partners, Delphi and General Motors.) Although Visteon was no longer a subsidiary of Ford at that time, Ford

¹ As stated at <http://us-subsidiary.claytonmckerverve.com/en/international-cash-management.htm>: "Centralized cash management systems offer more efficiently handled cash and produce a greater rate of return on cash investments. Under a centralized system, each subsidiary only worries and forecasts cash demands for their own subsidiary. The parent company then controls and distributes cash around the organization to meet required working capital needs, or maximize investment returns." The Statoil ASA

2011 annual report on Form 20-F states on page 247: "Normally the parent company, Statoil ASA, incurs debt and then extends loans or equity to wholly owned subsidiaries to fund capital requirements within the group." And "When partially owned subsidiaries or investments in associates and jointly controlled entities are financed, it is Statoil's policy to finance according to ownership share and on equal terms with the other owners."

helped with the funds since Visteon was recognized by Ford as a strategic supplier.

In contrast, when the video retailer Blockbuster faced financial distress in the mid- to late-2000s, its suppliers, prominent Hollywood studios, rather than providing financing help, cut off their trade credit provisions to Blockbuster. As stated in Blockbuster's Form 10-Q in August 2009, "Given our liquidity limitations and uncertainty surrounding our ability to finance our obligations... If the studios further tighten their credit terms or if studios eliminate their provision of credit to us altogether, this could result in up-front cash commitments that we may be unable to sustain on a long-term basis. ..." Despite the revenue-sharing contracts between Blockbuster and the studios to better align inventory risks and operational costs of the supply chain parties, we clearly observed a coordination failure in the presence of a financial distress event in this chain. Our paper argues that active working capital coordination among supply chain parties is needed, in addition to supply chain contracts, to effectively handle bankruptcy risks and optimize supply chain performance.

In this paper, we study contract design and coordination in a one-supplier-one-retailer supply chain in the presence of bankruptcy risks and costs due to capital constraints of the firms. Hereafter, and for ease of exposition, the supplier and retailer are referred to as "supply chain parties" ("party" singular). Based on the number of bankruptcy-exposed firms in the supply chain and with or without coordinated working capital management, we classify supply chains into three types: two firms without coordinated working capital management, two firms with coordinated working capital management, and one firm with centralized control of all decisions including working capital management. The first two cases are referred to as a "two-firm system," and the third case is referred to as a "one-firm system." In the two-firm system, the parties manage financial and material flows via a supply contract. The supply chain literature is particularly concerned about "coordinating" contracts that achieve "first best" for the two-firm system; i.e., the total profit of the two-firm system equals that of the one-firm system.

A classic example of a coordinating supply chain contract is the revenue-sharing contract. Blockbuster and major studios signed such contracts in 1998. Before that, studios sold videotapes via a traditional wholesale price contract to Blockbuster and other small retailers for \$65 per tape, with retailers collecting \$3 per rental. With the revenue-sharing contract, Blockbuster paid studios 40% of its rental revenue in exchange for a reduction in the unit price from \$65 to \$8. The contract helped Blockbuster to gain market share, while the studios ensured high levels of

availability of their videos to customers. The revenue-sharing contract effectively aligned the interests of supply chain parties in this instance, and the overall industry's profitability increased by 10% to 20% (Mortimer 2007).

In the existing literature, supply chain contracts are studied without concerns of cash flow constraints and threats of bankruptcy. However, such concerns are prevalent in real supply chains. The previously mentioned Visteon situation is typical for the automotive industry, where undercapitalized suppliers often face bankruptcy risks due to continuous price pressures from manufacturers in a volatile demand environment (for more detail, see Swinney and Netessine 2009). In a recent presentation,² the VP of Global Supply Chain for B/E Aerospace stated that "...bankruptcy risk of my 2,500 suppliers is my major risk management concern..." From the current literature, we know (see Kouvelis and Zhao 2012) that the bankruptcy risk itself does not change the nature of the coordinating contracts, as long as any needed financing for the existing capital constraints is provided through competitively priced bank loans. However, there are no studies to address coordination issues in the presence of costly defaults.

We address these issues in this paper by assuming that our supply chain is operated under a general contract (i.e., it subsumes class of quantity discount, buy-back, and revenue-sharing contracts) in the presence of capital market imperfections such as significant default costs. Both retailer and supplier are capital constrained and may need to borrow short-term bank loans to finance their operational requirements. Failure of a party to repay the loan will lead to bankruptcy; i.e., if the realized retail sales are low, the supplier and retailer might not be able to repay the loans and become insolvent.

We assume that banks operate in highly competitive financial markets, and all bank loans are competitively pricing risks. To account for the relevant default risks and related costs, the banks charge a premium above the risk-free interest rate. In the event of a default, the banks have the right to seize all remaining sales receipts, but only after covering relevant bankruptcy costs (also referred to as "default costs"). The default costs may include, but are not limited to, fixed costs and variable costs proportional to the residual value of a firm's assets. These two types of default costs are discussed in detail in corporate finance literature (see Ang et al. 1982, Tirole 2006), and in the recent operations literature (see Kouvelis and Zhao 2011).

² Industry seminar in the Fall 2013 at Boeing Center on Technology, Information, and Manufacturing, Olin Business School, Washington University in St. Louis.

To the best of our knowledge, our paper is the first to study supply contract design with considerations of default costs and working capital management. Unlike the corporate finance literature, which mainly takes a single-firm perspective, we study the strategic interactions of supply chain parties. Our research question is how the structure of default costs affects the supply contract design and performance, and whether or not expected default loss may be allocated among supply chain parties in a way that aligns their incentives and achieves the “first best” profits.

Our study contributes to the literature in two ways. First, we show how the structure of default costs (fixed, variable, etc.) interacts with contract terms and affects the supply chain performance. With only variable default costs, both revenue-sharing and buyback contracts with working capital coordination (aligning loan sizes to be proportional to revenue shares of the parties) continue to coordinate the chain. Quantity discount contracts fail to do so, since the retailer has to borrow a loan larger than that in the one-firm system. With fixed default costs, however, a buyback contract induces less profits than does a revenue-sharing contract.

The existence of fixed default costs complicates the problem and leads to interesting results. Although the one-firm system is preferred with substantial economies-of-scale effects in the fixed default costs, it is possible for the two-firm system with coordinated working capital management to outperform the one-firm system with small such effects. The two-firm system takes advantage of the flexibility to better allocate loan responsibilities to each party according to its default costs, thus allowing the party with smaller default costs to borrow more.

Our second contribution is to point out the importance of, and offer effective ways to execute, coordinated working capital management in a two-firm system. Our proposed revenue-sharing contracts through appropriate adjustment of wholesale prices achieve the needed alignment in loan sizes of the two firms. With only variable default costs or with high fixed default costs exhibiting substantial economies-of-scale, these contracts achieve the “first best” in supply chain performance. More strikingly, with fixed default costs of the one-firm system higher than a threshold, such contracts may even induce better profits than the one-firm system.

The paper is organized as follows. Section 2 briefly reviews the related literature. In §3, we introduce default costs and profit models of supply chain parties. In §4, we show contracts with buyback terms are Pareto dominated by contracts without such terms. Section 5 examines the case of only variable default costs and shows that revenue-sharing contracts might coordinate the chain, but quantity discount contracts

fail to do so. Section 6 studies the case of fixed default costs and shows the two-firm system under well-designed revenue-sharing contracts may outperform the one-firm system. Finally, §7 summarizes our results and future research suggestions.

2. Literature Review

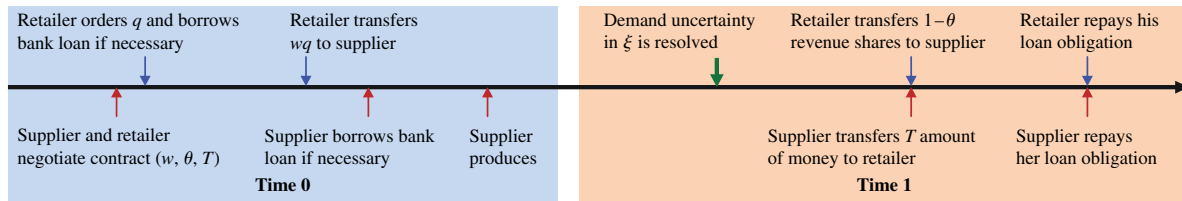
Our work fits in the broad area of the interfaces of operations and financial decisions, which has recently received substantial interest (representative references include Babich and Sobel 2004 and Ding et al. 2007). We are particularly interested in the impact of cash flow constraints on inventory decisions and supply chain coordination. Until the last few years, most of the related literature either ignored the impact of short-term financing issues on operational decisions (see Cachon 2003, Cachon and Lariviere 2005, Pasternack 1985, and Moorthy 1987 for more references) or considered financing issues in the absence of default costs (see Buzacott and Zhang 2004, Zhou and Groenevelt 2007, and Kouvelis and Zhao 2012 for more references).

Moorthy (1987) is one of the early works discussing supply chain coordination and shows that all unit quantity discount contracts can coordinate the channel. Pasternack (1985) studies the coordinating nature of buyback contracts. More recently, Cachon and Lariviere (2005) analyze revenue-sharing contracts and establish the equivalence between a buyback and a revenue-sharing contract. Cachon (2003) provides a comprehensive review of supply chain coordination in a supplier selling to a newsvendor retailer setting.

For a capital constrained retailer without default costs, Buzacott and Zhang (2004) discuss asset-based financing and analyze how a strategically behaving bank decides the interest rate of the loan to the retailer. However, the authors do not consider coordination issues in their work. Kouvelis and Zhao (2012) study trade credit contracts for a capital-constrained retailer, with the retailer obtaining financing from the bank and/or the supplier. Analyzing a Stackelberg leading supplier selling to a newsvendor setting, the paper considers bankruptcy risks but assumes costless default.

Yang and Birge (2013) study how the retailer, facing a default cost proportional to sales revenue, decides an optimal portfolio of financing resources, including bank credits and trade credits from the supplier. Kouvelis and Zhao (2011) study the retailer's ordering problem under a wholesale price-only contract and in the presence of general default costs, including costs proportional to sales revenue and to the pledged collateral to the loan, and fixed administrative costs. However, in both Yang and Birge (2013) and Kouvelis

Figure 1 (Color online) The Sequence of Events



and Zhao (2011), the supplier is a profit-maximizing Stackelberg leader in the supply chain, and in neither paper is there a discussion of supply chain coordination issues under default costs. Our current paper explicitly focuses on the contract design and coordination issues in the presence of bankruptcy risks and costs for both the retailer and supplier, and it brings to focus working capital management considerations among supply chain parties.

3. Model Description, Assumptions, and Notation

In the two-firm system the retailer (he) orders a single product from the supplier (she) before the sales season to satisfy future uncertain demand. The sequence of events is as follows (see Figure 1):

1. Before the sales season (time 0), the supplier and retailer engage in negotiation for the parameterized contract (w, θ, T) , where w is the wholesale price, θ is the retailer's revenue share, and T is the fixed amount of money the supplier transfers to the retailer at the end of the sales season. Upon contract agreement, the retailer places an order of quantity q and transfers the amount wq to the supplier. Then, the supplier produces the order for the retailer. If necessary, the supplier and retailer independently borrow competitively priced bank loans to finance their production and ordering decisions, respectively.

2. At the end of the sales season (time 1), the retailer transfers $1 - \theta$ shares of sales revenue to the supplier, and the supplier transfers T amount of money to the retailer. Next, the supplier and retailer repay their debt obligations to the banks. Any party who defaults has to declare bankruptcy, and the banks gain control of the defaulting party's remaining wealth after covering bankruptcy proceedings and all other relevant default costs.

REMARK 1. Here we assume contractually agreed transfer payments between supply chain parties occur before bankruptcy proceedings. In fact, it is not an innocuous assumption. Debt holders are considered to be more senior claimants to the bankrupt firm's assets than the suppliers. Thus, suppliers often stop shipping goods to the firm in anticipation of bankruptcy; e.g., many suppliers stopped shipments

during K-Mart's bankruptcy (see Yang and Birge 2013 for more references).

However, the law is not always followed, and junior claimants (i.e., suppliers) might be paid before senior claimants (i.e., banks) because of the implicit relational benefits between the retailers and suppliers. That is, most bankruptcies result in reorganization, and the firms continue to be in business after reorganization (e.g., as described in Altman 1984, p. 1071, "...after the firm has declared bankruptcy and is attempting to operate and manage a return to financial health"), and it is important to maintain good relationships with key supply chain partners in the long term. Finally, loans from the banks might have longer duration than the planning horizon. Therefore, transactions between the retailer and the supplier are settled before loan payments are due.

Let random variable ξ be the demand on $[0, \infty)$. The probability density function is $f(\cdot)$, cumulative distribution function (CDF) is $F(\cdot)$, and complementary CDF is $\bar{F}(\cdot) = 1 - F(\cdot)$. Let F be differentiable and increasing and $F(0) = 0$. The failure rate is $z(\cdot) = f(\cdot)/\bar{F}(\cdot)$ on $[0, \infty)$. We restrict our attention to distributions of an increasing failure rate (IFR); i.e., $z(\xi_2) \geq z(\xi_1)$ for $0 \leq \xi_1 < \xi_2 < \infty$.

Let the banks' interest rate on a loan offered be R . Exogenous parameters are r_f , the risk-free interest rate in the period from time 0 to 1; p , the retail price at time 1; and c , the unit production cost at time 0. We ignore the salvage values of unsold items and goodwill loss for unmet demand. To avoid trivial cases, let $p > c(1 + r_f)$. For ease of exposition, we assume the supplier's and retailer's opportunity cost of capital is r_f , normalize the retail price $p = 1$,³ and let the risk-free rate $r_f = 0$.

If necessary, we use subscripts r , s , c , and d to denote the retailer, supplier, one-firm system, and two-firm system, respectively. Major notation and assumptions are listed in Tables 1 and 2, respectively. Note that assumption A_5 eliminates moral hazard possibilities in loan repayment; i.e., supply chain parties purposefully hide money against loan or contractual obligations. Assumptions A_5 and A_2 eliminate the

³ Let $\xi' = p\xi$, $q' = pq$, $w' = w/p$, and $c' = c/p$. Our model with ξ' , q' , w' , c' , and $p = 1$ is the same as the original model.

Table 1 Notation

c	Supplier's production cost before the sales season, $c(1 + r_f) < p = 1$ (and for $r_f = 0$, $c < p$)
ξ	Random variable, the future uncertain demand
y	Working capital before the sales season
q	Retailer's order quantity before the sales season
R	Banks' interest rate determined after competitively pricing the risks associated with the loan
k	Default threshold, i.e., the minimal realized demand for a party to fully repay its loan obligation
w	Contract parameter, the supplier's wholesale price
θ	Contract parameter, the retailer's revenue share from the sales
T	Contract parameter, fixed money transfer from the supplier to retailer after demand realization
π	The expected profit of a firm
δ	The repayment from a firm to the banks before default costs at time 1 (a random variable)
Δ	The banks' expected repayment from a firm, i.e., the expectation of δ minus default costs
B	The default cost of a party as a function of demand realization and contract parameters
L	The loan amount of a party as a function of ordering quantity and contract parameters
C	The expected default loss of a party as a function of contract parameters

possibility that the supplier with limited cash on hand fails to fulfill the buyback obligation in the case of low demand realization. She has to borrow an adequate bank loan at time 0; as under the full information assumption, the retailer will agree to their transaction only if the supplier borrows adequately to cover her obligations.

3.1. A General Contract (w, θ, T)

We consider a supply chain of a supplier and a retailer facing uncertain demand under a general contract $GC(w, \theta, T)$ (for notational simplicity, hereafter, we simply refer to it as contract (w, θ, T)), where $0 \leq w \leq 1$ is the wholesale price, $0 \leq \theta \leq 1$ is the retailer's revenue share, and $T \geq 0$ is a fixed money transfer from the supplier to the retailer at time 1. (We briefly discuss in §4 the case $T \leq 0$, the fixed money transfer from the retailer to the supplier.) Let the retailer's ordering quantity be $q \geq 0$. In terms of assumed product flows in the contract, at time 0 there

Table 2 Assumptions

A_1	The banks, retailer, supplier, and one-firm system are risk neutral.
A_2	Full information, i.e., the working capital of both the retailer and supplier, and demand information are common knowledge to the three parties.
A_3	All bank loans are competitively priced, and the banks are assumed to face no default risk.
A_4	Both the supplier and retailer have long-term capital structure that is solely equity financed (for exposition convenience).
A_5	There are no moral hazard issues; i.e., supply chain parties have no ex ante intention to break the contractual agreement, and will ex post repay their loan obligations to the extent possible.

is a flow of q units from the supplier to the retailer, and at time 1 the remaining inventory of $(q - \xi)^+$ units can be returned to the supplier. Compensation for returned units is captured via adjusting T , as we will further explain. Although θ is assumed to be independent of q , both w and T can be functions of q . However, for ease of exposition, we generally omit their dependence on q .

Our general contract represents the three well-studied types of coordinating contracts in the literature, by appropriately assigning w , θ , and T values and time 1 returned items. If $w(q)$ is a properly specified function, $\theta = 1$ and $T = 0$, we get a quantity discount contract. If $w = c\theta$ and $T = 0$, we get a revenue-sharing contract. We next show the general contract can be reduced to a buyback contract $BC(w, b)$, where the wholesale price is w , the buyback price is b , and all remaining inventory, $(q - \xi)^+$, is returned. In $BC(w, b)$, at time 0, the product flow is q units from the supplier to the retailer, and cash flow is wq from the retailer to the supplier. At time 1, the product flow is $(q - \xi)^+$ from the retailer to the supplier, and cash flow is $b(q - \xi)^+$ from the supplier to the retailer.

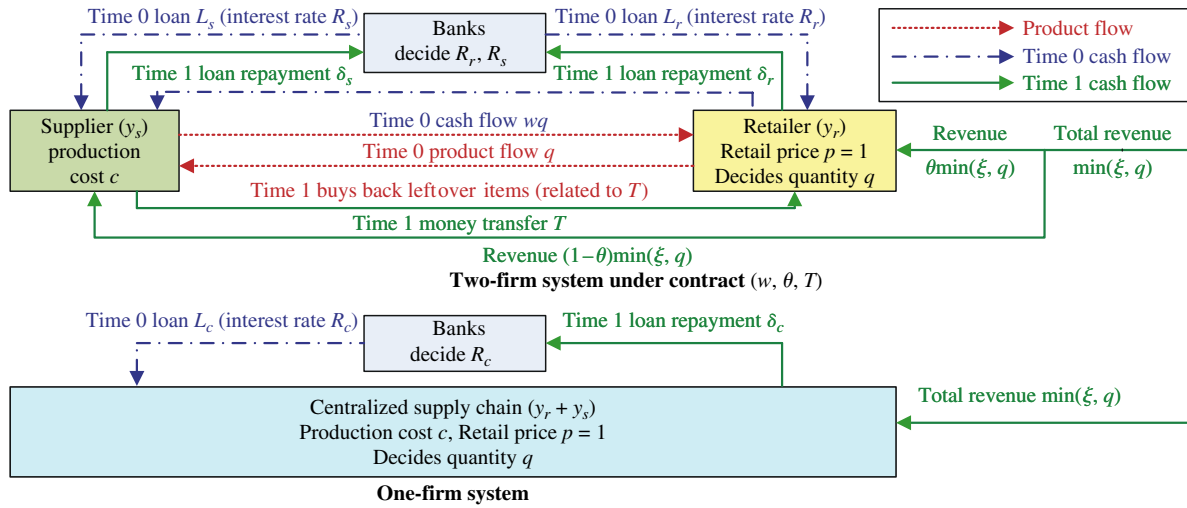
For our contract (w, θ, T) , the time 0 product flow q and cash flow wq match those of the buyback contract $BC(w, b)$. Let $\theta = 1 - b$ and $T = (1 - \theta)q = bq$. Note that the time 1 net cash flow from the supplier to the retailer is $T - (1 - \theta) \min(\xi, q) = bq - b \min\{\xi, q\} = b(q - \xi)^+$; i.e., the supplier buys back the retailer's remaining inventory $(q - \xi)^+$ at a unit price b . At time 1, both contracts have the same product flows. Then, the general contract $(w, 1 - b, bq)$ induces the same product flows and cash flows as does the buyback contract $BC(w, b)$; i.e., they are equivalent.⁴

In our contract (w, θ, T) , the supplier-to-retailer money transfer $T(q)$, with $T(0) = 0$, is assumed to be an increasing and concave function of q (i.e., $T'(q) \geq 0$ and $T''(q) \leq 0$). That is, the supplier does not want the money transfer to increase too fast in q to avoid the placement of unrealistically large quantities by the retailer. This assumption is satisfied by the buyback contract where $T = (1 - \theta)q$.

Our performance comparison benchmark is the one-firm system: a single bankruptcy-exposing firm controls the total working capital of the channel and serves as a single decision maker on all relevant decisions. The one-firm system is a capital-constrained newsvendor. See Figure 2.

⁴ We ignore the salvage values for notational convenience. In the more general case that parties have different salvage values, our general contract needs to specify the product flows accordingly and explicitly, to simulate an existing contract in the literature. However, the introduction of the salvage value will not change our main insights.

Figure 2 (Color online) Two-Firm System and One-Firm System



3.2. The Expected Profit and Expected Default Loss of the Retailer

The retailer borrows a bank loan $L_r = (wq - y_r)^+$ for his purchase needs at time 0. Using the interest rate R_r as the decision variable to control associated risks, the banks would demand $L_r(1 + R_r)$ as a repayment promise. Let the actual repayment the banks receive from the retailer be δ_r , a random variable depending on demand realization. If the retailer defaults on the loan, the banks incur bankruptcy costs B_r . Thus, the expected amount the banks receive from the retailer is $\Delta_r = \mathbb{E}[\delta_r - B_r \mathbf{1}_{bnkr}]^+$, where the indicator function $\mathbf{1}_{bnkr} = 1$ if the retailer goes bankrupt and 0 otherwise. The banks are perfectly competitive and their rate of return is assumed to be zero. Then, for them to extend the loan of size L_r at rate R_r , the following equilibrium condition must hold:

$$\Delta_r = \mathbb{E}[\delta_r - B_r \mathbf{1}_{bnkr}]^+ = L_r. \quad (1)$$

Note that $\mathbb{E}[\delta_r - (\delta_r - B_r \mathbf{1}_{bnkr})^+]$ is the part of assets that the retailer loses but the banks cannot receive because of the existence of B_r , which we refer to as “expected bankruptcy (default) loss.” Let

$$C_r = \mathbb{E}[\delta_r - (\delta_r - B_r \mathbf{1}_{bnkr})^+] = \mathbb{E}[\delta_r] - \Delta_r. \quad (2)$$

Then, $\mathbb{E}[\delta_r] = C_r + L_r$ from (1). As a result, the retailer’s expected profit is

$$\begin{aligned} \pi_r(q, w, \theta, T) &= \underbrace{\mathbb{E}[\theta \min(\xi, q) + T]}_{\text{Total revenue at time 1}} - \underbrace{(wq - L_r)}_{\text{Net cost at time 0}} \\ &\quad - \underbrace{\mathbb{E}[\delta_r]}_{\text{Repayment to the banks at time 1}} \\ &= \theta \mathbb{E}[\min(\xi, q)] + T - wq - C_r, \end{aligned} \quad (3)$$

where the second equality holds because of $L_r - \mathbb{E}[\delta_r] = -C_r$.

The actual repayment δ_r will be the minimum between what is promised to the banks ($L_r(1 + R_r)$) and the money the retailer has on hand at the repayment time ($\theta \min(\xi, q) + T$), which is fraction of the revenue $\theta \min(\xi, q)$ plus contract payment T from the supplier. That is, $\delta_r = \min(L_r(1 + R_r), \theta \min(\xi, q) + T)$. No loan will be offered if the quantity q is insufficient to repay the loan even assuming infinite demand. Therefore, whenever there exist loan transactions, R_r and q together must satisfy $L_r(1 + R_r) \leq \theta q + T$. As a result, one can simplify the expression for repayment as

$$\delta_r = \min[L_r(1 + R_r), \theta \xi + T].$$

The retailer’s bankruptcy condition is $L_r(1 + R_r) > \theta \xi + T$. Then, his default threshold is

$$k_r = \frac{1}{\theta} [L_r(1 + R_r) - T]^+; \quad (4)$$

i.e., he will go bankrupt if the realized demand is below this threshold. When $k_r = 0$, $\mathbf{1}_{bnkr} \equiv 0$ and the banks will receive $L_r(1 + R_r)$ for sure so that (1) implies that $R_r = 0$ and (2) implies that $C_r = 0$. Thus, (1) becomes trivial and the retailer’s profit is reduced to the situation without cash constraints. Then, we focus on $k_r > 0$ in this section, and we can simplify δ_r into

$$\begin{aligned} \delta_r &= \theta \min(\xi, k_r) + T \quad \text{and} \\ (\delta_r - B_r \mathbf{1}_{bnkr})^+ &= \begin{cases} (\theta \xi + T - B_r)^+ & \text{if } \xi < k_r, \\ \theta k_r + T & \text{otherwise.} \end{cases} \end{aligned}$$

The default cost B_r follows a stylized assumption in corporate finance literature that the default cost is incurred as a proportion of the bankrupt firm’s total residual assets. For a reference to the associated structural modeling work, see the classical debt pricing

models of Leland (1994) and Leland and Toft (1996), and the references therein. Several models in the operations literature follow the proportional default cost assumption, such as Xu and Birge (2004) and Lai et al. (2009).

Note that the above default cost is not related to other factors such as the loan size, the repayment size, the short amount, etc. That is, the more residual assets a bankrupt firm has, the more default cost it will incur, no matter how much the firm borrows or how much is left unpaid.

Some criticism of the proportional to residual assets default costs is pointed out in Elkamhi et al. (2012, p. 70): “Although empirical work tends to report estimates as proportions of bankrupt firm value (or near this value), there may in reality be a mixture of fixed and proportional costs. For example, court and attorney fees are likely to be fairly similar across firms, while asset sale discounts may vary in proportion to firm size. See, for example, Gruber and Warner (1977) and Bris et al. (2006).” The recent work of Kouvelis and Zhao (2011) includes both fixed costs and proportional costs.

In our model, the retailer’s total remaining asset, given default, is $\xi\theta + T$ (we assume that the retailer does not have other assets than the receiving from the current business, for simplicity of exposition). The total remaining asset incurs the total value loss

$$B_r = B(\xi, \theta, \beta_r) \equiv \alpha\xi\theta + \beta_r. \quad (5)$$

That is, the retailer loses $0 \leq \alpha \leq 1$ portion of his share of sales revenue $\xi\theta$ and $\beta_r \geq 0$ bankruptcy administrative fees (e.g., court and attorney fees for bankruptcy proceedings). Note that β_r are independent of the current sales and are referred to as “fixed default costs.” (Here we assume that the fixed money transfer T does not directly lose value.) Note that we intentionally omit the subscript r from the function B , since this function applies to the supplier and one-firm system as well, as long as we use the variables corresponding to those parties. We apply the same logic for the repayment function Δ , bankruptcy threshold function k , and expected default loss function C .

Substituting $(\delta_r - B_r \mathbf{1}_{bnkr})^+$ into (1) yields the retailer’s expected repayment to the banks:

$$\Delta_r = \Delta(k_r, \theta, \beta_r, T) \equiv \int_0^{k_r} [\xi\theta + T - B(\xi, \theta, \beta_r)]^+ dF(\xi) + \int_{k_r}^{\infty} (\theta k_r + T) dF(\xi). \quad (6)$$

Note that Equation (1), i.e., $\Delta(k_r, \theta, \beta_r, T) = L_r$, implicitly defines the retailer’s bankruptcy threshold k_r as a function of θ, β_r, T , and L_r . Then, let $k_r = k(\theta, L_r, \beta_r, T)$. Note that this definition of k_r does not conflict with (4), which is used by the banks to determine the corresponding interest rate.

Substituting δ_r into (2) and applying (1) yield the retailer’s expected default loss:

$$\begin{aligned} C_r &= C(\theta, L_r, \beta_r, T) \equiv \theta \int_0^{k_r} \bar{F}(\xi) d\xi + T - L_r \\ &= \theta \int_0^{k(\theta, L_r, \beta_r, T)} \bar{F}(\xi) d\xi + T - L_r. \end{aligned} \quad (7)$$

Again, we will see in §§3.3 and 3.4 that the functions B , Δ , k , and C can be applied to the supplier and one-firm system as well, as long as we use the variables corresponding to those parties. Therefore, we omit the corresponding subscripts from those functions.

As a summary, the retailer decides an optimal quantity to maximize π_r in (3), subject to (1) through which banks competitively price the loan by choosing a proper interest rate R_r .

3.3. The Expected Profit and Expected Default Loss of the Supplier

We now analyze the supplier’s cash flows in a similar way. Let the supplier’s loan size at time 0 be L_s . The banks would demand $L_s(1 + R_s)$ as a repayment promise, using the interest rate R_s as the decision variable. The supplier’s actual repayment is a random variable δ_s . If the supplier defaults, the banks incur bankruptcy costs B_s so that the expected amount the banks receive is $\Delta_s = \mathbb{E}[\delta_s - B_s \mathbf{1}_{bnkr}]^+$, where $\mathbf{1}_{bnkr} = 1$ if the supplier goes bankrupt and 0 otherwise. For the perfectly competitive banks to extend the loan L_s at rate R_s , the following equilibrium condition must hold:

$$\Delta_s = \mathbb{E}[\delta_s - B_s \mathbf{1}_{bnkr}]^+ = L_s. \quad (8)$$

Note that $\mathbb{E}[\delta_s - (\delta_s - B_s \mathbf{1}_{bnkr})^+]$ is the expected bankruptcy (default) loss that the supplier loses but the banks cannot receive. Let

$$C_s = \mathbb{E}[\delta_s - (\delta_s - B_s \mathbf{1}_{bnkr})^+] = \mathbb{E}[\delta_s] - \Delta_s. \quad (9)$$

Then, $\mathbb{E}[\delta_s] = C_s + L_s$ from (8). As a result, the supplier’s expected profit of the equity holders is

$$\begin{aligned} \pi_s(q, w, \theta, T) &= \underbrace{\mathbb{E}[(1 - \theta) \min(\xi, q) - T]}_{\text{Total revenue from the retailer at time 1}} - \underbrace{[(cq - wq) - L_s]}_{\text{Net cost at time 0}} \\ &\quad - \underbrace{\mathbb{E}[\delta_s]}_{\text{Repayment to the banks at time 1}} \\ &= (1 - \theta) \mathbb{E}[\min(\xi, q)] - T - cq + wq - C_s, \end{aligned} \quad (10)$$

where the second equality holds from $L_s - \mathbb{E}[\delta_s] = -C_s$.

We now discuss the supplier’s loan size L_s . The supplier’s total assets after receiving payment wq from the retailer are $y_s + wq$. Her total liabilities are cq , the production cost, plus the net money transfer $T - (1 - \theta) \min(\xi, q)$ to the retailer. As we have already

explained, all contractually agreed transfer payments between supply chain parties occur before bankruptcy proceedings. To avoid the moral hazard issues (see assumption A_5), the supplier needs T cash on hand, for her buyback responsibility if $\xi = 0$, on top of cq . Therefore, the supplier's loan is $L_s = [T + cq - (wq + y_s)]^+$.

If the supplier defaults, then $L_s > 0$ so that $L_s + wq + y_s - cq - T = 0$. The supplier's remaining asset is $wq + y_s + L_s - cq - T + (1 - \theta) \min(\xi, q) = (1 - \theta) \min(\xi, q)$, where $wq + y_s + L_s - cq$ is the remaining cash the supplier has at time 0 (after borrowing L_s and investing cq), and $T - (1 - \theta) \min(\xi, q)$ is the net transfer amount from the supplier to the retailer.

The actual repayment δ_s made by the supplier to the banks will be the minimum between what is promised, $L_s(1 + R_s)$, and her remaining asset, $(1 - \theta) \min(\xi, q)$. No loan will be offered if the quantity q is insufficient to repay the loan even assuming infinite demand; i.e., whenever there exist loan transactions, R_s and q must satisfy $L_s(1 + R_s) \leq (1 - \theta)q$. Then,

$$\begin{aligned}\delta_s &= \min(L_s(1 + R_s), (1 - \theta) \min(\xi, q)) \\ &= \min[L_s(1 + R_s), (1 - \theta)\xi].\end{aligned}$$

The supplier's bankruptcy condition is $L_s(1 + R_s) > (1 - \theta)\xi$. Then, her default threshold is

$$k_s = \frac{1}{1 - \theta} L_s(1 + R_s); \quad (11)$$

i.e., she will go bankrupt if the realized demand is below k_s . When $k_s = 0$, $\mathbf{1}_{bnkr} \equiv 0$ and the banks will receive $L_s(1 + R_s)$ for sure. From (8), $R_s = 0$, and from (9), $C_s = 0$. Thus, the banks' equation (8) becomes trivial and the supplier's profit is reduced to the situation without cash constraints. Then, we focus on $k_s > 0$ in this section. In this case, we can simplify δ_s into

$$\begin{aligned}\delta_s &= (1 - \theta) \min(\xi, k_s) \quad \text{and} \\ (\delta_s - B_s \mathbf{1}_{bnkr})^+ &= \begin{cases} ((1 - \theta)\xi - B_s)^+ & \text{if } \xi < k_s, \\ (1 - \theta)k_s & \text{otherwise.} \end{cases}\end{aligned}$$

As we have established, the supplier's remaining asset, given default, is $(1 - \theta) \min(\xi, q) = (1 - \theta)\xi$ (recall we require $\xi < q$ when default happens) from the current business (again, we assume the supplier does not have other assets than the current business, for ease of exposition). Let $B_s = \alpha\xi(1 - \theta) + \beta_s$, where α is the percentage loss of her share of sales revenue and β_s is the bankruptcy administrative cost, the fixed default cost. It is reasonable for the supplier to have the same α as the retailer, because she shares revenue with the retailer and is subject to the same percentage loss. Note that

$$B_s = \alpha\xi(1 - \theta) + \beta_s = B(\xi, 1 - \theta, \beta_s),$$

where the function B is defined in (5), with corresponding variables for the supplier.

Substituting $(\delta_s - B_s \mathbf{1}_{bnkr})^+$ into (8) yields the supplier's expected repayment to the banks:

$$\begin{aligned}\Delta_s &= \int_0^{k_s} [\xi(1 - \theta) - B(\xi, 1 - \theta, \beta_s)]^+ dF(\xi) \\ &\quad + \int_{k_s}^{\infty} (1 - \theta)k_s dF(\xi) = \Delta(k_s, 1 - \theta, \beta_s, 0),\end{aligned} \quad (12)$$

where the function Δ is defined in (6). Note that Equation (8), i.e., $\Delta(k_s, 1 - \theta, \beta_s, 0) = L_s$, implicitly defines k_s as a function of $1 - \theta$, β_s , 0, and L_s . Since defined through the same function Δ , this function is just the function k defined in §3.2 for the retailer; i.e., $k_s = k(1 - \theta, L_s, \beta_s, 0)$. Substituting δ_s into (9) and applying (8) yields the supplier's expected default loss:

$$\begin{aligned}C_s &= (1 - \theta) \int_0^{k_s} \bar{F}(\xi) d\xi - L_s \\ &= (1 - \theta) \int_0^{k(1 - \theta, L_s, \beta_s, 0)} \bar{F}(\xi) d\xi - L_s \\ &= C(1 - \theta, L_s, \beta_s, 0),\end{aligned} \quad (13)$$

where the function C is defined in (7).

3.4. The Expected Profit and Expected Default Loss of the One-Firm System

The one-firm system has the channel working capital $y_r + y_s$. At time 0, the one-firm system needs to borrow $L_c = (cq - y_r - y_s)^+$ for the production requirement, with the banks' interest rate R_c . The actual repayment is a random variable δ_c , and if the one-firm system defaults, the banks incur bankruptcy costs B_c . Then, the expected amount the banks receive is $\Delta_c = \mathbb{E}[\delta_c - B_c \mathbf{1}_{bnkr}]^+$, where $\mathbf{1}_{bnkr} = 1$ if the one-firm system goes bankrupt and 0 otherwise. For competitive banks to extend the loan of size L_c at rate R_c , the following equilibrium condition must hold:

$$\Delta_c = \mathbb{E}[\delta_c - B_c \mathbf{1}_{bnkr}]^+ = L_c. \quad (14)$$

Note that $\mathbb{E}[\delta_c - (\delta_c - B_c \mathbf{1}_{bnkr})^+]$ is the expected bankruptcy (default) loss that the one-firm system loses but the banks cannot receive. Let

$$C_c = \mathbb{E}[\delta_c - (\delta_c - B_c \mathbf{1}_{bnkr})^+] = \mathbb{E}[\delta_c] - \Delta_c. \quad (15)$$

Then, $\mathbb{E}[\delta_c] = C_c + L_c$ from (14). As a result, the one-firm system's expected profit is

$$\begin{aligned}\pi_c(q, c, 1, 0) &= \underbrace{\mathbb{E}[\min(\xi, q)]}_{\text{Total revenue at time 1}} - \underbrace{(cq - L_c)}_{\text{Net cost at time 0}} \\ &\quad - \underbrace{\mathbb{E}[\delta_c]}_{\text{Repayment to the banks at time 1}} \\ &= \mathbb{E}[\min(\xi, q)] - cq - C_c,\end{aligned} \quad (16)$$

where the second equality holds because of $L_c - \mathbb{E}[\delta_c] = -C_c$.

The actual repayment δ_c is the minimum between $L_c(1 + R_c)$, the amount owed to banks, and $\min(\xi, q)$, cash on hand at the repayment time. No loan will be offered if the quantity q is insufficient to repay the loan even assuming infinite demand. Thus, whenever there exist loan transactions, R_c and q together must satisfy $L_c(1 + R_c) \leq q$, and we can simplify the expression for repayment as

$$\delta_c = \min(L_c(1 + R_c), \xi).$$

The one-firm system's bankruptcy condition is $L_c(1 + R_c) > \xi$. Then, the default threshold is

$$k_c = L_c(1 + R_c). \quad (17)$$

When $k_c = 0$, $\mathbf{1}_{bnkr} \equiv 0$ and the banks will receive $L_c(1 + R_c)$ for sure. From (14), $R_c = 0$, and from (15), $C_c = 0$. Thus, (14) becomes trivial and the one-firm system's profit is reduced to the case without cash constraints. Then, we focus on $k_c > 0$ in this section. We have

$$\begin{aligned} \delta_c &= \min(\xi, k_c) \quad \text{and} \\ (\delta_c - B_c \mathbf{1}_{bnkr})^+ &= \begin{cases} (\xi - B_c)^+ & \text{if } \xi < k_c, \\ k_c & \text{otherwise.} \end{cases} \end{aligned}$$

Let the default cost of the one-firm system be $B_c = \alpha\xi + \beta_c$, where α is the percentage loss of the sales revenue (the same α as the supplier and retailer) and β_c is the fixed default cost. In §3.5, we illustrate that the definition of the one-firm system is not always obvious, and thus we consider the range of $\min\{\beta_r, \beta_s\} \leq \beta_c \leq \beta_r + \beta_s$. Note that

$$B_c = \alpha\xi + \beta_c = B(\xi, 1, \beta_c),$$

where the function B is defined in (5).

Substituting $(\delta_c - B_c \mathbf{1}_{bnkr})^+$ into (14) yields the expected repayment to the banks,

$$\begin{aligned} \Delta_c &= \int_0^{k_c} [\xi - B(\xi, 1, \beta_c)]^+ dF(\xi) \\ &\quad + \int_{k_c}^{\infty} k_c dF(\xi) = \Delta(k_c, 1, \beta_c, 0), \end{aligned} \quad (18)$$

where the function Δ is defined in (6). Note that Equation (14), i.e., $\Delta(k_c, 1, \beta_c, 0) = L_c$, implicitly defines k_c as a function of $1, \beta_c, 0$, and L_c . Since defined through the same function Δ , this function is just the function k defined in §3.2 for the retailer; i.e., $k_c = k(1, L_c, \beta_c, 0)$.

Substituting δ_c into (15) and applying (14) yields the expected default loss,

$$\begin{aligned} C_c &= \int_0^{k_c} \bar{F}(\xi) d\xi - L_c = \int_0^{k(1, L_c, \beta_c, 0)} \bar{F}(\xi) d\xi - L_c \\ &= C(1, L_c, \beta_c, 0), \end{aligned} \quad (19)$$

where the function C is defined in (7).

3.5. New Features in Supply Chain Contract Design and Coordination

The operations literature proposes various contracts that achieve “first best” for the supply chain while allowing different degrees of profit sharing between the retailer and supplier. However, factors such as capital constraints and default costs are generally ignored. We address such concerns in this study. Note that by competitively pricing loans, banks do not claim any supply chain profits.

Under our revenue-sharing contracts, a party's profit will be from either the down payment or the shares of sales revenue, both of which will increase the bankruptcy risks of the other party. Therefore, which is better—the one-firm system or the two-firm system—is nontrivial, and this research can shed light on it. From Equations (3), (10), and (16), the total profits of the two-firm system and one-firm system are given in (20) and (21), respectively:

$$\begin{aligned} \pi_r(q, w, \theta, T) + \pi_s(q, w, \theta, T) &= \underbrace{\mathbb{E}[\min(\xi, q)] - cq}_{\text{Newsvendor profits}} \\ &\quad - \underbrace{[C(\theta, L_r(q), \beta_r, T) + C(1 - \theta, L_s(q), \beta_s, 0)]}_{\text{Expected default loss}}, \end{aligned} \quad (20)$$

$$\pi_c(q, c, 1, 0) = \underbrace{\mathbb{E}[\min(\xi, q)] - cq}_{\text{Newsvendor profits}} - \underbrace{C(1, L_c(q), \beta_c, 0)}_{\text{Expected default loss}}. \quad (21)$$

From (20) and (21), without default costs and thus the expected default loss function $C(\cdot, \cdot, \cdot, \cdot) \equiv 0$, the two-firm system under coordinating contracts performs identically to the one-firm system. However, when default costs, particularly fixed ones, exist, $C(\cdot, \cdot, \cdot, \cdot)$ affects the relative values of the two-firm system and one-firm system.

From (21), if β_c is small enough ($\beta_c \leq \min\{\beta_r, \beta_s\}$, implied by Proposition 6 in §6.2.2), then the one-firm system is guaranteed to be better than the two-firm system. In fact, $\beta_c = \min\{\beta_r, \beta_s\}$ corresponds to the case that the single decision maker controlling the supplier and retailer together lets the party with smaller fixed default costs borrow all channel loans, has that party be exposed to all bankruptcy risks, and collects all the sales revenues. The other party is secured from bankruptcy and shares the channel profits only if the bankruptcy-exposing party survives.

Although the profit-sharing mechanism described above seems to be ideal, we claim that there are various situations for the mechanism to fail in practice. First, if the two supply chain parties are independent firms but not parent-subsidiary or subsidiary-subsidiary, then the mechanism might fail because of the serious moral hazard issue for the secured party to

share net profits with the bankruptcy-exposing party.⁵ Second, even for the parent-subsidiary or subsidiary-subsidiary supply chains, given that the single decision maker wants one firm to take all bankruptcy risks, choosing which firm to assign the risks to might be a strategic decision and depend on many factors beyond just fixed default costs. For example, the parent company might want the bankruptcy risks on the subsidiary's side but not on its side, even if the parent company has smaller fixed default costs. Also, tax rates of the two firms might affect the risk assignment decision as well (we leave the study for tax rates in the future research).

Finally, if the one-firm system is merged with the two-firm system and all resources are combined, we typically have $\min\{\beta_r, \beta_s\} \leq \beta_c \leq \beta_r + \beta_s$, depending on how the two firms are merged. Consider two scenarios:

1. The supplier and retailer are in different countries. If the merged single firm declares bankruptcy, it is handled by the court of the retailer's country and by the court of the supplier's country (see LoPucki 1999). In this case, $\beta_c = \beta_r + \beta_s$.

2. The retailer and supplier are in the same country and located near each other. Then, $\beta_c < \beta_r + \beta_s$, since one attorney team might handle the retailer and supplier together and save some attorney fees. Other scenarios might also exist so that the merged single firm might gain benefits by having only one legal entity. In general, we have $\min\{\beta_r, \beta_s\} \leq \beta_c \leq \beta_r + \beta_s$.

The above discussion implies that there is no obvious way to define a β_c that is appropriate for all situations. Therefore, in this work, we consider all the $\beta_c \in [\min\{\beta_r, \beta_s\}, \beta_r + \beta_s]$.

⁵ The moral hazard issues of profit-sharing contracts between two independent agents arise when one agent cannot fully monitor all of the relevant expenses so that the other agent has opportunities to manipulate the report of the profit. We illustrate this through the profit-sharing contracts used as employment contracts, where the independent agents are Hollywood studios and their actors, but the same idea applies to the supply chain of two independent firms. As Weinstein (1998, pp. 67–68) points out: "Litigation about employment contracts in Hollywood is widely reported. These suits are usually brought by people who had contracted for a share of the 'net profits' from a movie. After the movie is, arguably, successful, the individual discovers that the 'net profits' are small and perhaps zero. The common perception is that the studios use strange and arcane accounting practices to eliminate any profit." Moreover, "A contrast is often drawn between those who have little bargaining power—such as Art Buchwald—and sign contracts with 'net-profit' shares and big stars—such as Tom Hanks—who are able to sign for shares of the 'gross.' The latter are believed to be unaffected by studio chicanery. Indeed, the fact that some major stars get a percentage of the gross is considered one of the reasons the 'net profits' are reduced" (p. 68). In our supply chain, since both parties have some bargaining powers, we follow the trend and focus on revenue-sharing contracts instead of profit-sharing contracts.

In our study, as in the traditional literature, we show that the one-firm system dominates the two-firm system when fixed default costs do not exist (§5) and also when they exist but β_c is less than a threshold between $\min\{\beta_r, \beta_s\}$ and $\beta_r + \beta_s$ (§6.2). We say coordination is achieved if the contract used in the two-firm system results in decisions that generate the optimal profits of the one-firm system. However, §6.2 also shows that the two-firm system may outperform the one-firm system when β_c is larger than the threshold. For such cases, we are no longer searching for "coordinating" contracts in the above sense, but for contracts that result in a set of the two-firm system's optimal actions that have Nash equilibrium; i.e., no supply chain party has a profitable unilateral deviation from these optimal actions (see Cachon 2003, p. 230).

In our next analysis, we work with a generalized form of the default cost function defined in (5),

$$B(\xi, \theta, \beta) = g(\xi)\theta + \beta, \quad (22)$$

where $g(\xi) \geq 0$ with $g(0) = 0$ and $0 \leq g'(\xi) \leq 1$. We assert that a contract is more efficient (or has a larger efficiency) than another if it can generate larger expected profits in the two-firm system. Unless stated explicitly, we use comparatives in a weak sense throughout the paper.

4. Contracts (w, θ, T) with Nonzero Transfer Payment T

4.1. Positive Fixed Transfer Payment $T > 0$ (Buyback Contracts)

We first discuss the general contracts with a fixed transfer payment $T > 0$ from the supplier to the retailer at time 1, which can be interpreted as a buyback-related term. The buyback term has two effects on the retailer.

1. To honor the buyback, the supplier charges a larger wholesale price than the production cost c , and thus the retailer has to borrow a larger loan principal than under the one-firm system, which leads to the overborrowing effect.

2. The buyback amount can be used as the collateral at time 1 to secure the retailer's loan. This is referred to as the collateral effect. However, similar to the sales revenue, this buyback collateral may lose some of its value in the case of default, which influences the retailer's expected default loss and expected profit.

Recall that both w and T can be functions of q . Without loss of generality, let $w(q) = \omega(q) + \gamma$, where $\gamma \geq 0$ is a constant independent of q . Let $q_d(w(q_d), \theta, T(q_d)) = \arg \max_{q \geq 0} \pi_r(q | w(q), \theta, T(q))$ be the retailer's optimal ordering quantity under a contract $(w(q), \theta, T(q))$. Then, we have the following proposition.

PROPOSITION 1. For IFR demand distributions and a contract $(w(q), \theta, T(q))$ where $T(q)$ satisfies $0 \leq qT'(q) \leq T(q)$, there exists a counterpart revenue-sharing contract $(\hat{w}(q) - T(q)/q, \theta, 0)$ with $\hat{w}(q) = \omega(q) + \hat{\gamma}$ and $\hat{\gamma} \geq \gamma$, i.e., the same or increased (shifted by a positive constant) wholesale price, so that we have the following cases.

1. The retailer's optimal ordering quantity under the revenue-sharing contract $(\hat{w}(q) - T(q)/q, \theta, 0)$ is the same as that under the contract $(w(q), \theta, T(q))$, i.e., $q_d(\hat{w}(q_d) - T(q_d)/q_d, \theta, 0) = q_d(w(q_d), \theta, T(q_d))$.
2. The contract $(w(q), \theta, T(q))$ is (weakly) Pareto dominated by the contract $(\hat{w}(q) - T(q)/q, \theta, 0)$. That is, both the retailer and supplier are (weakly) better-off under the contract $(\hat{w}(q) - T(q)/q, \theta, 0)$; i.e.,

$$\pi_r\left(q_d, \hat{w}(q_d) - \frac{T(q_d)}{q_d}, \theta, 0\right) \geq \pi_r(q_d, w(q_d), \theta, T(q_d)),$$

$$\pi_s\left(q_d, \hat{w}(q_d) - \frac{T(q_d)}{q_d}, \theta, 0\right) \geq \pi_s(q_d, w(q_d), \theta, T(q_d)).$$

For the case of $\beta_r = 0$, we have $\hat{\gamma} = \gamma$ and $\hat{w}(q) = w(q)$. Furthermore, the equalities in the two inequalities above hold, and the two contracts are equivalent to each other.

For the contract $(w(q), \theta, T(q))$, let us first consider the contract $(w(q) - T(q)/q, \theta, 0)$. We can show that whenever the retailer under the contract $(w(q) - T(q)/q, \theta, 0)$ orders the optimal quantity under $(w(q), \theta, T(q))$, the retailer will be better off and the supplier's expected profits are the same under the two contracts. The driving factor for $(w(q), \theta, T(q))$ to be less efficient is that $T(q)$ may lose value in the retailer's bankruptcy. When comparing the buyback contract $(w(q), \theta, T(q))$ to the contract $(w(q) - T(q)/q, \theta, 0)$, the retailer pays an extra $T(q)$ at time 0. Then, at time 1, the supplier pays back the $T(q)$ to the retailer when buying the retailer's leftover inventory. We refer to it as the "overcharge-then-payback" mechanism of the buyback contract $(w(q), \theta, T(q))$. From (6) and (22), if $\beta_r = 0$, then $T(q)$ is separable from other terms in Δ_r . From (1), there will be no difference whether $T(q)$ happens at time 0 to reduce L_r or at time 1 to reduce Δ_r . That is, this mechanism is still efficient and the two contracts are equivalent to each other (Cachon and Lariviere 2005).

However, if $\beta_r > 0$, this mechanism may cause profit loss, since part of the buyback collateral may have to cover the retailer's fixed default cost. That is, the reduced collateral amount at time 1 does not offset the increased loan obligation at time 0 anymore. Then, the banks charge a higher rate and the retailer ends up with smaller expected profit under the original buyback contract $(w(q), \theta, T(q))$ than the revenue-sharing contract $(w(q) - T(q)/q, \theta, 0)$.

The above observations hold under the assumption that the retailer orders the same quantities under

$(w(q), \theta, T(q))$ and $(w(q) - T(q)/q, \theta, 0)$. It is, however, possible that the retailer may order a different quantity under $(w(q) - T(q)/q, \theta, 0)$ than under $(w(q), \theta, T(q))$. As case 1 of Proposition 1 shows, to ensure that the retailer orders the same quantity under $(\hat{w}(q) - T(q)/q, \theta, 0)$ as under $(w(q), \theta, T(q))$, the supplier might have to appropriately increase the wholesale price from $w(q) = \omega(q) + \gamma$ to $\hat{w}(q) = \omega(q) + \hat{\gamma}$, where $\hat{\gamma} \geq \gamma$. We refer to this appropriately adjusted wholesale price revenue-sharing contract $(\hat{w}(q) - T(q)/q, \theta, 0)$ as the counterpart revenue-sharing contract of the buyback contract $(w(q), \theta, T(q))$. Note that, if $\beta_r = 0$, then $\hat{\gamma} = \gamma$ and $(w(q) - T(q)/q, \theta, 0)$ is equivalent to $(w(q), \theta, T(q))$.

Case 2 of Proposition 1 shows that both the retailer and supplier will receive (weakly) better profits under $(\hat{w}(q) - T(q)/q, \theta, 0)$ than those under the contract $(w(q), \theta, T(q))$. As a result, we conclude that $(w(q), \theta, T(q))$ is (weakly) Pareto dominated by, and less efficient than, the counterpart revenue-sharing contract $(\hat{w}(q) - T(q)/q, \theta, 0)$.⁶

4.2. Negative Fixed Transfer Payment $T < 0$ (Partial Down Payment)

We now discuss what happens if the retailer only pays part of wq as down payment at time 0. In this case, the supplier will require the retailer to pay the remaining amount at time 1, i.e., $T \leq 0$. Then, the smaller down payment at time 0 forces the supplier to borrow a larger loan, and the retailer's remaining payment at time 1 serves as a collateral to secure the supplier's loan. The retailer also needs to carefully plan and borrow against his time 1 payment obligations. Under our full information assumption A_2 , if the retailer does not borrow enough, the supplier will know it at time 0 and demand from the retailer to do so before engaging in any transactions. Our observations are parallel to those made for $T > 0$ in Proposition 1, but now differentiating on cases for $\beta_s > 0$ or $\beta_s = 0$. The following lemma summarizes our results.

LEMMA 1. Let $T(q) < 0$. Consider the contracts $(w(q), \theta, T(q))$ and $(w(q) - T(q)/q, \theta, 0)$. We have the following cases.

1. The retailer's optimal ordering quantity under the revenue-sharing contract $(w(q) - T(q)/q, \theta, 0)$ is the same as that under the contract $(w(q), \theta, T(q))$, i.e., $q_d(w(q_d) - T(q_d)/q_d, \theta, 0) = q_d(w(q_d), \theta, T(q_d))$.
2. The contract $(w(q), \theta, T(q))$ is (weakly) Pareto dominated by the contract $(w(q) - T(q)/q, \theta, 0)$. More specifically, the retailer gets the same optimal expected profit under both contracts, and the supplier gets (weakly)

⁶ Note that concave functions $T(q)$ are a subset of functions satisfying $0 \leq qT'(q) \leq T(q)$.

better optimal expected profit under the contract $(w(q) - T(q)/q, \theta, 0)$; i.e.,

$$\begin{aligned} \pi_r\left(q_d, w(q_d) - \frac{T(q_d)}{q_d}, \theta, 0\right) &= \pi_r(q_d, w(q_d), \theta, T(q_d)), \\ \pi_s\left(q_d, w(q_d) - \frac{T(q_d)}{q_d}, \theta, 0\right) &\geq \pi_s(q_d, w(q_d), \theta, T(q_d)). \end{aligned} \quad (23)$$

If $\beta_s = 0$, the equality in inequality (23) holds and the two contracts are equivalent to each other.

Note that the wholesale purchase payment wq is split between an upfront and a remnant payment. The remnant payment is paid at time 1 under the $T(q) < 0$ buyback contract but is paid at time 0 under the revenue-sharing contract. Under our assumptions of full information and that all contractual agreements are executed before bankruptcy, it is guaranteed that the retailer makes an equal amount payment. Then, case 1 of Lemma 1 concludes that the retailer's optimal ordering quantities are the same under the buyback contract and its counterpart revenue-sharing contract.

By the same logic, case 2 of Lemma 1 concludes that the retailer's optimal expected profits are the same under the two contracts. However, in the case of the buyback contract, the supplier may have to borrow more at time 0 and use as a collateral payment to that loan the amount to be received at time 1. In the case of fixed default cost $\beta_s > 0$, this collateral might be subject to losing some of its value and might not be covering some of the overborrowed amount at time 0. As a result, if $\beta_s > 0$, then the supplier receives smaller than expected profits, and the contract $(w(q), \theta, T(q))$ is Pareto dominated by $(w(q) - T(q)/q, \theta, 0)$. Otherwise, the supplier receives the same expected profits under the two contracts, and the two contracts are equivalent to each other.

Combining Proposition 1 for $T \geq 0$ and Lemma 1 for $T \leq 0$, we get the results below.

LEMMA 2. For contracts $(w(q), \theta, T(q))$ with a fixed money transfer $T(q) \neq 0$ at time 1 and $T'(q) \geq 0$ and $T''(q) \leq 0$ if $T(q) \geq 0$, there exists a counterpart revenue-sharing contract $(\hat{w}(q) - T(q)/q, \theta, 0)$ that induces the same retailer's ordering quantity as $(w(q), \theta, T(q))$, and both parties in the two-firm system are (weakly) better off under $(\hat{w}(q) - T(q)/q, \theta, 0)$ than under $(w(q), \theta, T(q))$.

Our analysis in §4 clearly shows that for any general contract $(w(q), \theta, T(q))$ with $T(q) > 0$ or $T(q) < 0$, we can find an equivalent or Pareto dominant counterpart revenue-sharing contract (with $T(q) = 0$), because the fixed default cost will create friction on the buyback term. However, we observe that buyback contracts are used in practice. Note that revenue-sharing contracts might incur friction in practice as well, e.g., suppliers expending monitoring costs to verify the stated

revenue of the retailer. Our full information assumption A_2 (demand realization is then common knowledge) implicitly assumes that the revenue-monitoring mechanism is costless to build, or all associated investments for it are sunk costs and there are no currently incurred variable operating costs linked to such activities. Thus, it is plausible that buyback contracts might in practice prove to be efficient in channels with substantial revenue-monitoring costs.

From Lemma 2, in the search for Pareto dominant coordinating contracts in §§5 and 6, we do not consider fixed transfer payment; i.e., $T = 0$. Therein, for ease of exposition, we drop the term T in all relevant functions. We want to point out that our two parameter contracts are more general than the revenue-sharing contracts in the literature, because the wholesale price might be a function of the retailer's ordering quantity.

5. Coordinating Contracts for the Case of Only Variable Default Costs

We study the case wherein the retailer, supplier, and one-firm system do not have fixed bankruptcy costs at all, i.e., $\beta_r = \beta_s = \beta_c = 0$. As mentioned earlier, we only need to study contracts with $T = 0$. For simplicity, the contract $(w, \theta, 0)$ going forward is denoted as (w, θ) . Let q_c be the optimal production quantity of the one-firm system and $L_c(q_c) > 0$. Then, we make the following proposition.

PROPOSITION 2. For IFR demand distributions and for any $q \geq 0$, we have

$$C(\theta, L_r(q), 0) + C(1 - \theta, L_s(q), 0) \geq C(1, L_c(q), 0), \quad (24)$$

where the equality holds if and only if

$$\frac{L_r(q)}{\theta} = \frac{L_s(q)}{1 - \theta} = \frac{L_c(q)}{1}. \quad (25)$$

Condition (25) is a necessary, and also sufficient if $L_c(q_c) > 0$, condition to coordinate the chain. That is, let $(\bar{w}(q), \theta)$ be the contract according to (25) and $q_d(w(q_d), \theta) = \arg \max_{q \geq 0} \pi_r(q | w(q), \theta)$ be the retailer's optimal quantity under a given contract $(w(q), \theta)$ in the two-firm system. Then,

$$\begin{cases} q_d(\bar{w}(q_d), \theta) = q_c, \\ \pi_r(q_d(w(q_d), \theta), w(q_d), \theta) + \pi_s(q_d(w(q_d), \theta), w(q_d), \theta) \\ \leq \pi_c(q_c, c, 1), \quad \text{for all } w(q) \geq 0, \\ \pi_r(q_d(\bar{w}(q_d), \theta), \bar{w}(q_d), \theta) + \pi_s(q_d(\bar{w}(q_d), \theta), \bar{w}(q_d), \theta) \\ = \pi_c(q_c, c, 1), \\ \pi_r(q_d(\bar{w}(q_d), \theta), \bar{w}(q_d), \theta) = \lambda \pi_c(q_c, c, 1), \end{cases}$$

where $\lambda = \frac{\theta[\pi_c(q_c, c, 1) + y_r + y_s] - y_r}{\pi_c(q_c, c, 1)}$.

As a result, $(\bar{w}(q), \theta)$ is the optimal contract among the contracts $(w(q), \theta)$ with $w(q) \geq 0$ and $0 \leq \theta \leq 1$, and

it coordinates the chain with arbitrary profit allocation between the supplier and retailer.

The case of supply chains without any financial constraint, i.e., $L_r(q) = L_s(q) = L_c(q) = 0$, is a special case of Proposition 2. In the case that $L_c(q_c) > 0$, i.e., the one-firm system has bankruptcy risks, under condition (25), the retailer's and supplier's profits are proportional to the one-firm system's profit (except for a constant term). Then, the retailer's optimal ordering quantity is also q_c . Furthermore, since both the expected default losses of the retailer and the supplier are proportional to the expected default loss of the one-firm system, the expected profit of the two-firm system with the retailer ordering q_c equals that of the one-firm system. Therefore, the revenue-sharing contract $(\bar{w}(q), \theta)$ achieves coordination. Note that the retailer's portion of channel profits is $-(y_r/\pi_c(q_c, c, 1)) \leq \lambda \leq (\pi_c(q_c, c, 1) + y_s)/\pi_c(q_c, c, 1)$. If we want $0 \leq \lambda \leq 1$, we require $y_r/(\pi_c(q_c, c, 1) + y_r + y_s) \leq \theta \leq (\pi_c(q_c, c, 1) + y_r)/(\pi_c(q_c, c, 1) + y_r + y_s)$.

From Proposition 2, it is clear that (25) is the necessary condition for supply chain coordination. In other words, the only possible way for a contract to coordinate is to allocate the channel's total debt to each party proportional to his/her revenue share. From condition (25), $(\bar{w}(q), \theta) = (c\theta - (\theta(y_r + y_s) - y_r)/q, \theta)$, which coordinates the supply chain for IFR demand distributions, from Proposition 2. Thus, for a retailer with small y_r but large revenue share θ , i.e., $y_r < \theta(y_r + y_s)$, the wholesale price the retailer receives is smaller than $c\theta$, and vice versa. However, we can equivalently interpret the proposed revenue-sharing contractual mechanism as the wholesale price always being $c\theta$, but the working capital positions of the two supply chain parties are adjusted to be proportional to the party's revenue share. Effectively, it ensures implementation of the necessary condition (25).

Note that condition (25) implies $L_r + L_s = L_c$. We have shown that, for a given revenue share, a party's expected default losses C are increasing and convex in the loan amount (see the proof of Proposition 2 in the online appendix, available as supplemental material at <http://dx.doi.org/10.1287/mnsc.2015.2248>). As a result, if $L_r(q) + L_s(q) > L_c(q)$ for any $q \geq 0$ with $L_c(q) > 0$, then the strict inequality in (24) holds, and the optimal expected profit the two-firm system can possibly achieve will be strictly smaller than the optimal profit of the one-firm system. We refer to this coordination failure case as due to overborrowing effects, and it explains the coordination failure of quantity discount contracts in bankruptcy cost settings. We have the following lemma.

LEMMA 3. *For IFR demand distributions, in the case that the one-firm system has default risks, i.e., $cq_c > y_r + y_s$, any quantity discount contract with $w > c$ fails to coordinate.*

Under quantity discount contracts, since $w > c$, only the retailer borrows; i.e., $L_s(q) = 0$ for any $q \geq 0$. If $L_c(q) = cq - y_r - y_s > 0$, then we have $L_r(q) = (wq - y_r)^+ = wq - y_r > L_c(q) > 0$ and thus $L_r(q) + L_s(q) > L_c(q)$. Because of the larger loan size, the two-firm system incurs larger expected default loss and gets smaller expected profit, and quantity discount contracts fail to coordinate.

REMARK 2. Our model does not include any fixed production cost K . In fact, including such fixed cost does not change our results and insights in Proposition 2. Note that $L_c = cq - y_r - y_s + K$, $L_r = wq - y_r$, and $L_s = cq - wq - y_s + K$. Then, $L_r = \theta L_c \Leftrightarrow wq - y_r = c\theta q - \theta(y_r + y_s) + \theta K \Leftrightarrow w = c\theta - (\theta(y_r + y_s - K) - y_r)/q$ (thus, we can verify that $L_s = (1 - \theta)L_c$). As a result, the working capital adjustment for the retailer is $\theta(y_r + y_s - K) - y_r$. A similar argument applies for other kinds of costs, as long as such costs can be appropriately shared among supply chain parties (so that a party's loan size is proportional to its revenue share).

REMARK 3. Since $\beta_r = 0$ in this section, from Proposition 1, the contract $(c\theta - (\theta(y_r + y_s) - y_r)/q, \theta)$ is the counterpart revenue-sharing contract of the buyback contract $(c\theta - (\theta(y_r + y_s) - y_r)/q + (1 - \theta), \theta, (1 - \theta)q)$, and both contracts coordinate the supply chain. Note that here $T = (1 - \theta)q$; i.e., the supplier buys back all of the retailer's leftover inventory at unit price $1 - \theta$ (see §3.1).

6. Supply Chain Contracts for the Cases with Fixed Default Costs

6.1. Fixed Default Costs: No Economies-of-Scale

We now study the case that firms face fixed default costs without economies-of-scale effects; i.e., $\beta_c = \beta_r + \beta_s$. Then, $B(\xi, \theta, \beta_r) = g(\xi)\theta + \beta_r$ for the retailer, $B(\xi, 1 - \theta, \beta_s) = g(\xi)(1 - \theta) + \beta_s$ for the supplier, and $B(\xi, 1, \beta_c) = g(\xi) + \beta_c$ for the one-firm system. Note that $B(\xi, 1, \beta_c) = B(\xi, \theta, \beta_r) + B(\xi, 1 - \theta, \beta_s)$, i.e., no economies-of-scale effects for the total default costs as well.

Unexpectedly, for this case of fixed default costs, the two-firm system might have smaller expected default losses and thus induce larger expected profit than the one-firm system. This is formally established in the following proposition. Recall that q_c is the optimal production quantity of the one-firm system and recall that we denote a contract $(w(q), \theta, 0)$ by $(w(q), \theta)$ for simplicity.

PROPOSITION 3. *For the case that $\beta_c = \beta_r + \beta_s$, we have the following results.*

1. *For $q \geq 0$ with $L_c(q) > 0$, there might exist combinations of revenue shares and corresponding loans for the two*

parties, $(\theta, L_r(q), \beta_r)$ and $(1 - \theta, L_s(q), \beta_s)$, with $L_r(q) + L_s(q) = L_c(q)$ so that

$$\begin{aligned} & C(\theta, L_r(q), \beta_r) + C(1 - \theta, L_s(q), \beta_s) \\ & < C(1, L_c(q), \beta_c); \end{aligned} \quad (26)$$

2. There might exist contracts $(\tilde{w}(q), \theta)$ so that

$$\begin{aligned} & \pi_r(q_d(\tilde{w}(q_d), \theta), \tilde{w}(q_d), \theta) + \pi_s(q_d(\tilde{w}(q_d), \theta), \tilde{w}(q_d), \theta) \\ & > \pi_c(q_c, c, 1), \end{aligned}$$

where $q_d(\tilde{w}(q_d), \theta) = \arg \max_{q \geq 0} \pi_r(q | \tilde{w}(q), \theta)$ is the retailer's optimal ordering quantity in the two-firm system. Thus, the two-firm system may end up with larger expected profit than the one-firm system, especially if $q_d(\tilde{w}(q_d), \theta) = \arg \max_{q \geq 0} \pi_r(q | \tilde{w}(q), \theta) + \pi_s(q | \tilde{w}(q), \theta)$; i.e., $(\tilde{w}(q), \theta)$ induces the retailer to order the optimal quantity of the two-firm system.

Note that by substituting $B(\xi, \theta, \beta_r) = g(\xi)\theta + \beta_r$ into Equation (6) and $B(\xi, 1 - \theta, \beta_s) = g(\xi)(1 - \theta) + \beta_s$ into Equation (12), we observe that the fixed default costs β_r and β_s cannot be perfectly aligned with revenue shares θ and $1 - \theta$ (except for specific value $\theta = \beta_r/(\beta_r + \beta_s)$) and with corresponding sizes of the retailer and supplier. In fact, such misalignment may benefit the two-firm system for nonconvex expected default loss functions C .

In other words, in the two-firm system, we have the flexibility to better allocate loan responsibilities to each party according to its default cost, e.g., to allow the party with smaller default costs to borrow more. The loan flexibility might be easier to implement in parent-subsidiary or subsidiary-subsidiary (of the same parent company) supply chains through a well-designed contract with coordinated working capital. However, other chains can implement such schemes through third parties (potentially banks) agreeing to serve as working capital coordinators.

According to case 1 of Proposition 3, for an arbitrary q with $L_c(q) > 0$, if the retailer independently orders the optimal quantity of the one-firm system, (26) implies that it is possible for the revenue-sharing contract that induces such a retailer's order to result in larger total expected profit of the two-firm system than that of the one-firm system. Of course, if the contract induces the retailer to order the quantity that maximizes the total expected profit of the two-firm system, the performance of the two-firm system can be even better, as case 2 of Proposition 3 implies. (For details on the existence of such contracts, see §EC.1 in the online appendix.)

We further study our modified revenue-sharing contract $(\tilde{w}(q), \theta) = (c\theta - (\theta(y_r + y_s) - y_r)/q, \theta)$, which implements the loan alignment condition (25) of Proposition 2. Although such contracts may not be the optimal contracts that maximize the total profits

of the two-firm system, the two-firm system may still outperform the one-firm system. (See Figure EC.6 of §EC.1 in the online appendix for a graphical interpretation of such a case.) We have the following proposition for two-firm systems operated under the revenue-sharing contracts $(\tilde{w}(q), \theta)$.

PROPOSITION 4. Consider IFR demand distributions with $z''(\cdot) \geq 0$; i.e., $z(\cdot)$ is increasing and convex. Let $g''(\cdot) \geq 0$. Under the contract $(\tilde{w}(q), \theta)$, there exist $\bar{q}(\theta, \beta_r, \beta_s) \leq \bar{\bar{q}}(\theta, \beta_r, \beta_s)$ so that we have the following cases.

1. For $q \leq \bar{q}(\theta, \beta_r, \beta_s)$, we have $C(\theta, L_r(q), \beta_r) + C(1 - \theta, L_s(q), \beta_s) \leq C(1, L_c(q), \beta_c)$.
2. For $q \geq \bar{\bar{q}}(\theta, \beta_r, \beta_s)$, we have $C(\theta, L_r(q), \beta_r) + C(1 - \theta, L_s(q), \beta_s) \geq C(1, L_c(q), \beta_c)$.
3. If $q_c \leq \bar{q}(\theta, \beta_r, \beta_s)$, the contract $(\tilde{w}(q), \theta)$ might induce

$$\begin{aligned} & \pi_r(q_d(\tilde{w}(q_d), \theta), \tilde{w}(q_d), \theta) + \pi_s(q_d(\tilde{w}(q_d), \theta), \tilde{w}(q_d), \theta) \\ & > \pi_c(q_c, c, 1), \end{aligned}$$

where $q_d(\tilde{w}(q_d), \theta) = \arg \max_{q \geq 0} \pi_r(q | \tilde{w}(q), \theta)$ is the retailer's optimal ordering quantity. Thus, the two-firm system under $(\tilde{w}(q), \theta)$ might end up with larger expected profit than the one-firm system, especially if $q_d(\tilde{w}(q_d), \theta) = \arg \max_{q \geq 0} \pi_r(q | \tilde{w}(q), \theta) + \pi_s(q | \tilde{w}(q), \theta)$; i.e., the contract $(\tilde{w}(q), \theta)$ induces the retailer to order the optimal quantity of the two-firm system.

The definitions of the two thresholds $\bar{q}(\theta, \beta_r, \beta_s) \leq \bar{\bar{q}}(\theta, \beta_r, \beta_s)$ are in the proof of Proposition 4 in the online appendix. According to the proposition, the two-firm system has smaller expected default loss for $q \leq \bar{q}(\theta, \beta_r, \beta_s)$, and larger expected default loss for $q \geq \bar{\bar{q}}(\theta, \beta_r, \beta_s)$, than the one-firm system. Note that in case 3 of Proposition 4, if $C(\theta, L_r(q_c), \beta_r) + C(1 - \theta, L_s(q_c), \beta_s) < C(1, L_c(q_c), \beta_c)$, then even if the retailer's quantity that maximizes his own profit, q_d , is not precisely the quantity that maximizes the total profit of the two-firm system, but the two quantities are close enough, the conclusion that the two-firm system outperforms the one-firm system still holds.

The stated conditions in the proposition hold for many settings. The increasing and convex failure rate $z''(\cdot) \geq 0$ characterizes many commonly used distributions; e.g., uniform, exponential, power, Weibull with the shape parameter greater than or equal to two, and truncated normal meet this needed requirement (see Kouvelis and Zhao 2012, Zhou and Groenevelt 2007).

Proposition 4 sharpens the result of Proposition 3 by clarifying that working capital coordination as described in condition (25) is not sufficient to optimized two-firm system profits, especially when $q_c \geq \bar{q}(\theta, \beta_r, \beta_s)$. Although we correct the misalignment of revenue shares and loan sizes of supply chain parties through the use of the modified revenue-sharing

contract $(\bar{w}(q), \theta)$, the misalignment between fixed default costs β_r and β_s and revenue shares θ and $1 - \theta$ is still there. Particularly, from Equations (6) and (12), the misalignment increases in k_r and k_s , which increase in q . Cases 1 and 2 of Proposition 4 indicate that the above misalignment benefits the two-firm system when q is small and hurts it when q becomes large.

6.2. Fixed Default Costs With Economies-of-Scale

We now extend our discussion from §6.1 to the more general case $\min\{\beta_r, \beta_s\} \leq \beta_c \leq \beta_r + \beta_s$. We refer to this property of default costs as exhibiting “economies-of-scale effects.”

6.2.1. The Special Case of Fixed Default Costs:

$\beta_r = \beta_s = \beta_c = \beta > 0$. We start from the simple case of $\beta_r = \beta_s = \beta_c = \beta > 0$. Then, $B(\xi, \theta, \beta) = g(\xi)\theta + \beta$ for the retailer, $B(\xi, 1 - \theta, \beta) = g(\xi)(1 - \theta) + \beta$ for the supplier, and $B(\xi, 1, \beta) = g(\xi) + \beta$ for the one-firm system. We have $B(\xi, \theta, \beta) + B(\xi, 1 - \theta, \beta) > B(\xi, 1, \beta)$; i.e., the two-firm system has a larger default cost for each demand realization. That is, the economies-of-scale effects apply to total default costs.

Recall that q_c is the optimal production quantity of the one-firm system and let $k_c^o = k(1, L_c, \beta) |_{q=q_c}$ be the corresponding default threshold. We focus on the case that $L_c(q_c) > 0$ (or alternatively, $k_c^o > 0$); i.e., the one-firm system has bankruptcy risk.

PROPOSITION 5. For IFR demand distributions and under our general contract $(w(q), \theta)$, we have the following cases.

1. For $q \geq 0$, the two-firm system has larger expected default loss than the one-firm system; i.e.,

$$C(\theta, L_r(q), \beta) + C(1 - \theta, L_s(q), \beta) \geq C(1, L_c(q), \beta). \quad (27)$$

2. Let $q_d(w(q_d), \theta) = \arg \max_{q \geq 0} \pi_r(q | w(q), \theta)$ be the retailer's optimal quantity under $(w(q), \theta)$. Then,

$$\begin{aligned} \pi_r(q_d(w(q_d), \theta), w(q_d), \theta) + \pi_s(q_d(w(q_d), \theta), w(q_d), \theta) \\ \leq \pi_c(q_c, c, 1), \quad \text{for all } w(q) \geq 0; \end{aligned} \quad (28)$$

i.e., the optimal expected profit of the two-firm system has as an upper bound the optimal profit of the one-firm system. More specifically,

i. If $0 < \beta < k_c^o - g(k_c^o)$, the strict inequality in (28) holds for all $w(q) \geq 0$, and coordination fails;

ii. If $\beta \geq k_c^o - g(k_c^o)$, then for $(\bar{w}(q), \theta) = (c\theta - (\theta(y_r + y_s) - y_r)/q, \theta)$ that implements condition (25),

$$\begin{cases} q_d(\bar{w}(q_d), \theta) = q_c, \\ \pi_r(q_d(\bar{w}(q_d), \theta), \bar{w}(q_d), \theta) + \pi_s(q_d(\bar{w}(q_d), \theta), \bar{w}(q_d), \theta) \\ = \pi_c(q_c, c, 1), \\ \pi_r(q_d(\bar{w}(q_d), \theta), \bar{w}(q_d), \theta) = \lambda \pi_c(q_c, c, 1), \end{cases} \quad \text{where } \lambda = \frac{\theta[\pi_c(q_c, c, 1) + y_r + y_s] - y_r}{\pi_c(q_c, c, 1)}.$$

As a result, $(\bar{w}(q), \theta)$ is the optimal contract and coordinates the chain with arbitrary profit allocation between the supplier and retailer.

We interpret the case $0 < \beta < k_c^o - g(k_c^o)$ as “small fixed default costs.” Then, there is a $0 < \bar{\xi} < k_c^o$ so that $\beta = \bar{\xi} - g(\bar{\xi})$. For $\bar{\xi} < \xi \leq k_c^o$, we have $\beta < \xi - g(\xi)$ and $B(\xi, 1, \beta) = g(\xi) + \beta < \xi$. For such demand realizations, the one-firm system can retain some of the remaining assets' value after bankruptcy. Since the default costs of each party in the two-firm system are strictly larger for demand realizations leading to bankruptcy, the two-firm system will have smaller total expected profit than the one-firm system. That is, economies-of-scale in default costs allow the one-firm system to reduce the expected default loss and increase the expected profits by combining the loans and receiving all sales revenues. Then, for small fixed default costs, our contracts cannot coordinate the two-firm system, even with coordinated working capital management.

We interpret the case $\beta \geq k_c^o - g(k_c^o)$ as “large fixed default costs.” Given $0 \leq g'(\xi) \leq 1$, we have $\beta \geq \xi - g(\xi)$ for $0 < \xi < k_c^o$. Then, $B(\xi, 1, \beta) = g(\xi) + \beta \geq \xi$ and the one-firm system in the case of bankruptcy loses all the remaining value of assets because of default costs. Both parties in the two-firm system fare equally badly in bankruptcy as a result of the economies-of-scale property of the default costs. However, because the one-firm system cannot leverage its economies-of-scale by borrowing larger loans and collecting all the sales revenue, the two-firm system can still achieve the best profits of the one-firm system under the proposed revenue-sharing contract.

From Proposition 5, default costs with economies-of-scale do not automatically lead to coordination failure. For small fixed default costs with economies-of-scale, the one-firm system is more effective in handling default costs, and our proposed revenue-sharing contract (w, θ) fails to coordinate the two-firm system. However, high fixed default costs cannot be absorbed by the one-firm system, and in that case, the one-firm system and the two-firm system are equally ineffective in handling default. Then, the supply chain can still be coordinated with the proposed revenue-sharing contract $(\bar{w}(q), \theta)$ and with working capital coordination.

6.2.2. General Cases of Fixed Default Costs: $\min\{\beta_r, \beta_s\} \leq \beta_c \leq \beta_r + \beta_s$. We now study the more general cases $\min\{\beta_r, \beta_s\} \leq \beta_c \leq \beta_r + \beta_s$. We have the following result.

PROPOSITION 6. For the fixed default costs such that $\min\{\beta_r, \beta_s\} \leq \beta_c \leq \beta_r + \beta_s$,

1. There exists a threshold $\min\{\beta_r, \beta_s\} \leq \bar{\beta}_c \leq \beta_r + \beta_s$ so that the one-firm system is guaranteed to have larger expected profits than the two-firm system if

$\min\{\beta_r, \beta_s\} \leq \beta_c \leq \bar{\beta}_c$, and the two-firm system might outperform the one-firm system if $\beta_c < \beta_r + \beta_s$;

2. If $\beta_c \geq k_c^o - g(k_c^o)$, $\beta_r \geq \theta(k_c^o - g(k_c^o))$ and $\beta_s \geq (1 - \theta)(k_c^o - g(k_c^o))$, then for $(\bar{w}(q), \theta) = (c\theta - (\theta(y_r + y_s) - y_r)/q, \theta)$ that implements the condition (25),

$$\begin{cases} q_d(\bar{w}(q_d), \theta) = q_c, \\ \pi_r(q_d(\bar{w}(q_d), \theta), \bar{w}(q_d), \theta) + \pi_s(q_d(\bar{w}(q_d), \theta), \bar{w}(q_d), \theta) \\ = \pi_c(q_c, c, 1), \\ \pi_r(q_d(\bar{w}(q_d), \theta), \bar{w}(q_d), \theta) = \lambda \pi_c(q_c, c, 1), \end{cases}$$

$$\text{where } \lambda = \frac{\theta[\pi_c(q_c, c, 1) + y_r + y_s] - y_r}{\pi_c(q_c, c, 1)}.$$

Thus, the two-firm system under the contract $(\bar{w}(q), \theta)$ has the same expected profit as the one-firm system. Moreover, such contracts are the optimal contracts if $\min\{\beta_r, \beta_s\} \leq \beta_c \leq \bar{\beta}_c$.

It can be shown that a party's expected profit decreases in the fixed default cost (see the proof of Proposition 3 in the online appendix). Consequently, when the fixed default costs exhibit strong economies-of-scale, i.e., $\min\{\beta_r, \beta_s\} \leq \beta_c \leq \bar{\beta}_c$ where $\bar{\beta}_c$ is a threshold, the one-firm system is guaranteed to end up with larger expected profits than the two-firm system. However, if the economies-of-scale in the fixed default costs are not that strong, i.e., $\bar{\beta}_c < \beta_c \leq \beta_r + \beta_s$, then the two-firm system might outperform the one-firm system. This is case 1 of Proposition 6.

In case 2, for $\beta_c \geq k_c^o - g(k_c^o)$, $\beta_r \geq \theta(k_c^o - g(k_c^o))$, and $\beta_s \geq (1 - \theta)(k_c^o - g(k_c^o))$, the retailer and supplier lose all remaining assets in the case of bankruptcy under the contract $(\bar{w}(q), \theta) = (c\theta - (\theta(y_r + y_s) - y_r)/q, \theta)$. The same applies to the one-firm system. Thus, the one-firm system does not gain any benefits by combining the loans, and it has the same expected profit as the two-firm system under the proposed revenue-sharing contract. However, we still need the condition $\min\{\beta_r, \beta_s\} \leq \beta_c \leq \bar{\beta}_c$ to guarantee that such contracts are the optimal contracts. That is, if $\beta_c > \bar{\beta}_c$, the two-firm system may outperform the one-firm system, and then $(\bar{w}(q), \theta)$ may not be the optimal contract that maximizes the expected profit of the two-firm system.

Note that Proposition 3 now becomes a special case of Proposition 6 with $\beta_c = \beta_r + \beta_s$. Also, Proposition 5 can be viewed as a special case of Proposition 6. For $\beta_r = \beta_s = \beta_c = \beta$, we have $\beta = \min\{\beta_r, \beta_s\} \leq \bar{\beta}_c \leq 2\beta = \beta_r + \beta_s$ and $\beta_c = \beta \leq \bar{\beta}_c$. Also, $\beta_c \geq k_c^o - g(k_c^o)$, $\beta_r \geq \theta(k_c^o - g(k_c^o))$ and $\beta_s \geq (1 - \theta)(k_c^o - g(k_c^o))$ are reduced to $\beta \geq k_c^o - g(k_c^o)$. Then, Proposition 6 establishes the results of Proposition 5; i.e., the one-firm system always has larger expected profits, and the modified revenue sharing contract $(\bar{w}(q), \theta)$ can coordinate the supply chain.

7. Conclusions and Future Research

We study contract design and coordination in a one-supplier-one-retailer supply chain facing uncertain demand. Both the supplier and retailer may be capital constrained and need to borrow competitively priced bank loans for their operational requirements. Failure of loan repayment will lead to bankruptcy. Without default costs, it is known that simple contracts, e.g., revenue sharing, buyback, and quantity discount, can coordinate and arbitrarily allocate profits in our supply chain. However, supply chains with leveraged firms need to address realistic concerns of bankruptcy risks and costly default, and in such cases the contract design and coordination become challenging. Our study brings to light such concerns and analyzes the performance of a general contract (w, θ, T) . By assigning proper values to the parameters, the general contract can be reduced to the traditional revenue sharing, buyback, and quantity discount contracts. Thus, we are able to inform the performance of these widely used contracts in leveraged and default exposed supply chains, while addressing relevant supply chain coordination issues.

First, we discuss contracts with a fixed money transfer T from the supplier to the retailer ($T > 0$), or in the reverse direction ($T < 0$), at time 1. Without fixed default costs, the buyback amount T will not lose value and cause contract efficiency loss. However, with fixed default costs, since the buyback amount has to be used for covering fixed default costs in the case of low demand realization, the contract is Pareto dominated by the counterpart revenue-sharing contract without any buyback term. Thus, we proceed in our analysis with revenue-sharing contracts that have no buyback terms and offer insights for Pareto dominant coordinating contracts for general default costs.

In our further analysis of revenue-sharing contracts, we first study the case that there are only variable default costs (i.e., proportional to sales revenue). In this case, the necessary condition to coordinate is to align the loan amounts each supply chain party borrows with their revenue shares, or what we refer to as "coordinated working capital management." Such working capital coordination is not possible for a quantity discount contract, and thus it fails to coordinate in the presence of default costs. The large wholesale price charged by the supplier necessitates the retailer borrowing a larger loan than the corresponding loan amount in the one-firm system (overborrowing effect), which then increases the expected default loss of the two-firm system.

For fixed default costs with small economies-of-scale (i.e., the fixed default costs of the one-firm system are larger than a threshold), we establish that it is possible for the two-firm system to achieve (strictly)

larger expected profits than the one-firm system. The two-firm system uses its flexibility to better allocate the loan responsibilities among supply chain parties according to their default costs (e.g., the party with lower default costs borrows more). This can happen if a contract induces the retailer to order the quantity that maximizes the total expected profit of the two-firm system, or even if the retailer independently orders the same quantity as does the one-firm system.

Then, we analyze the case that fixed default costs exhibit substantial economies-of-scale effects (i.e., the fixed default costs of the one-firm system are smaller than the threshold). With high fixed default cost (relative to the shares of sales revenue) for each party, both supply chain parties and the one-firm system lose all of their residual assets in the case of default, which eliminates any opportunity to leverage the economies-of-scale on default costs in the one-firm system. Then, our proposed revenue-sharing contract can still coordinate the chain. However, for small fixed default costs, both parties and the one-firm system may retain some remaining asset value after covering the default costs, and the economies-of-scale property of the default costs favors the one-firm system. Then, revenue-sharing contracts fail to coordinate.

In summary, our results illustrate the importance and necessity of coordinating initial cash positions and/or loan amounts among supply chain parties. Frequently, our proposed revenue-sharing contracts with adjusted wholesale price can be used to effectively achieve the desired working capital coordination.

For supply chain parties with small economies-of-scale effects in the fixed default costs, it might be challenging to find the optimal contracts for the two-firm system. In such situations, how to align the loan responsibilities and revenue shares with fixed default costs for optimal channel performance remains a challenging research question.

Supplemental Material

Supplemental material to this paper is available at <http://dx.doi.org/10.1287/mnsc.2015.2248>.

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