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# Optimizing Delivery Fees for a Network of Distributors

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The third-party logistics industry has grown rapidly in recent years, accounting for \$46 billion of the total \$921 billion in logistics spending in the United States during 1999. This figure is expected to grow by 15 to 20% annually as manufacturing firms increasingly partner with third-party logistics providers to cost effectively distribute their products, while meeting increasingly stringent service expectations of customers. These logistics partnerships have introduced a new set of decision requirements to negotiate compensation for distribution services. Based on a collaborative project with a leading building products manufacturer, this paper describes the development and implementation of a novel linear programming model to decide the delivery fees paid to distributors. The model applies to manufacturer-distributor partnerships where distributors are compensated using fee values that depend on delivery weights and distances. It ensures that the expected compensation, considering stochastic demands, is adequate to cover the aggregate distribution costs for each distributor, and permits imposing various consistency conditions to ensure that fee values are credible. The model proved effective in helping the manufacturer develop a new fee table that generated considerable economic savings and provided more equitable compensation to distributors.

*(Third-Party Logistics; Distribution; Supply Chain; Compensation; Applied Optimization; Linear Programming)*

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## Introduction

The increasing emphasis in recent years on customer orientation—providing the right product at the right price, time, and place—has propagated up the supply chain. Retailers have raised their service expectations, demanding from their suppliers frequent deliveries of mixed loads in small batches. Likewise, distributors expect better delivery service from manufacturers. This trend is especially obvious in contexts where the so-called “big-box” retailers have become dominant. These high-volume retailers impose stringent supply requirements while also exerting pressure to reduce margins. Meeting these needs requires efficient distribution operations and tightly coordinated supply

chains, inducing manufacturers to form strategic partnerships with third-party logistics firms. These arrangements permit manufacturers to exploit the distribution expertise of their logistics partners while focusing on their own core competencies. The new logistics partnerships have created a new set of decision requirements, such as, which supply chain partners to select, who coordinates the logistics operations, and how to compensate partners for their services. Based on a collaborative project with a leading building products manufacturer, this paper develops and applies an optimization model to develop a fair and effective payment schedule for distribution services.

Big-box retailers are national chains of superstores

such as Wal-Mart, Toys R' Us, The Home Depot, and various office supply retail chains that are capturing market share from traditional retailers by offering wide product variety at low cost. These organizations benefit from the scale economies of their vast networks of retail outlets. In many instances, big-box retailers have come to dominate sales channels and are often manufacturers' largest customers. Consider, for instance, the trends over the past two decades in the building products market. In the 1970s, sales to the residential segment of the building products market were made primarily through independent lumberyards, hardware stores, and home-improvement stores that were relatively small (typically, less than 15,000 square feet of floor space). This traditional retailing model would not last however. Since 1980, national chains such as The Home Depot have grown rapidly and gained a dominant share in the do-it-yourself market for building materials. For example, at the end of 1999, The Home Depot (founded in 1978) operated over 900 stores in North America, with annual revenues exceeding \$38 billion. Each of its stores carries 40,000 to 50,000 SKUs, and has an average size of 105,000 square feet. The company expects to more than double its number of stores within the next four years. In 1999, The Home Depot was named America's Most Admired Retailer by *Fortune* magazine for the sixth consecutive year.

Big-box retailers exploit economies of scale via centralized purchasing, using their leverage to negotiate low prices and stringent delivery terms from manufacturers. Some of these retail firms operate their own distribution systems, while others have adopted a decentralized strategy whereby each store directly receives shipments from manufacturers. To be cost-competitive while providing high variety, inventories must be kept low. Avoiding costly stockouts requires that retailers receive frequent and reliable deliveries with short order lead times. Moreover, to reduce overheads, retailers want a single point of contact for ordering and consolidated deliveries of all ordered items.

To cope with these challenging expectations, manufacturers are increasingly partnering with third-party logistics providers who offer distribution expertise and

even specialized information technology that is difficult and expensive to develop in-house (Bradley 1998). The third-party logistics industry has grown rapidly in recent years (Leahy, Murphy, and Poist 1995), increasing its revenues from \$34 billion in 1997 (Fahrenwald 1998) to \$46 billion in 1999. In some instances logistics outsourcing is a by-product of the manufacturing organization's conscious strategy to focus on its core competencies (Bowersox 1990). Large corporations such as Kodak and 3M often heavily use third-party logistics firms, and approximately 70% of the Fortune 500 firms outsource at least some of their logistics activities (Lieb and Maltz 1998). Logistics providers can assume some or all of a firm's materials management and distribution functions, ranging from simple warehousing services to product customization and intercontinental supply chain management activities (Cooke 1998). Their most typical functions are as distributors, providing warehousing and transportation services.

Operating a lean and responsive distribution chain requires tight linkages between manufacturer and distributor(s), both in terms of seamless and timely information flows and close coordination of physical activities and material flows. Because such linkages require trust, mutual understanding, and good communications, manufacturers are entering into long-term alliances with distributors. These partnerships are characterized by mutual interdependence and require significant investments of resources (e.g., to develop interoperable information systems and consistent operating procedures). A manufacturer's ability to offer good customer service depends upon its distributors; conversely, distributors often rely on a single manufacturer for a significant portion of their revenues. Because of these synergistic relationships, firms have a vested interest in maintaining the viability of their distributors by providing adequate compensation for their services.

Designing an effective distribution channel in partnership with independent logistics firms requires careful planning and extensive negotiations. In this context, the broad hierarchy of system design decisions includes:

- deciding the number of distribution centers, and selecting distribution partners;

- defining each distributor's geographic coverage, role, and modes of interaction;
- specifying inventory and transportation service requirements (lead times, service levels); and
- determining appropriate compensation for distributors' services.

This paper considers the last among these decisions, namely, setting payments for delivery services,<sup>1</sup> assuming that distributors have already been selected and their territories assigned.

The delivery compensation issue emerged as a promising context for decision modeling at Armstrong World Industries, a leading producer of flooring and ceiling products that recently established a logistics partnership with a group of independent distributors. Each of Armstrong's distributors is responsible for delivering to all stores in an exclusive region, several hundred miles in diameter. Stores require deliveries at regularly scheduled times, but the quantities they order vary from week to week. Because these quantities are typically much less than a full truckload, distributors use multi-stop delivery routes, originating at the warehouse, to combine deliveries for multiple stores. To compensate distributors for their delivery expenses, Armstrong adopted a fee structure similar to the tariff tables used by the LTL (less-than-truckload) carrier industry. Under this scheme, for each store delivery the distributor receives a fee that depends on the store's distance from the distributor's warehouse and the weight of products delivered. Distances and weights are aggregated into ranges. The fees are determined using a prenegotiated fee table that specifies the amount payable for each distance and weight range. Armstrong selected this compensation scheme because it conforms to industry practice for LTL shipments, is easy to communicate and administer, and is independent of distributors' routing decisions. Moreover, to reduce conflicts in an environment where distributors share information, the company wanted to use a common fee table that applies to all distributors.

Periodically (e.g., each year), the company reviews the fee values in the table, and revises them through

informal negotiations with its distributors. This paper address this periodic fee setting decision problem: to decide, for a given set of distance and weight ranges, the fee that distributors should receive for each distance and weight combination. Because the logistics partnership arrangement provides long-term commitment and exclusive coverage regions to distributors, the manufacturer expects to pay fees that are lower than commercial LTL rates. To engender acceptance of these fee values by distributors, the company must ensure that delivery fee values are credible and will be sufficiently large to cover distributors' costs. Because these costs depend on distributors' routing decisions, matching the payment for each delivery with the cost of that delivery is impossible. Instead, the company wishes to ensure that for each distributor the expected total compensation (e.g., each week), obtained using the fee table, equals or exceeds the distributor's total transportation cost. Because the same fee table applies to all distributors, some distributors might be compensated more than their costs. The company's goal is to reduce such overcompensation.

The operations and logistics literature has not previously discussed this fee-setting decision. This paper contributes to the literature by defining the decision problem, modeling it as a linear optimization problem, and discussing the application of this model to support a real fee-setting decision. We formulate the fee-setting problem as a linear program that is easy to solve and surprisingly versatile. Because the stores' demands vary randomly from week to week, so do the actual fees that distributors receive. Despite the stochastic nature of this problem, the formulation accurately represents the expected total compensation for each distributor without introducing nonlinearities. Moreover, the model can incorporate a variety of fee credibility constraints. It also extends easily to alternative compensation functions that are based on discrete weight and distance ranges.

We implemented and applied the fee-setting model, using data for over 1,500 home-improvement superstores in Armstrong's U.S. big-box supply network, to support the company's annual fee-setting exercise. Using this model, the company developed and introduced a new fee schedule for all the distributors participating in this network. The new schedule not only

<sup>1</sup>We use the terms compensation, payments, and fees interchangeably to denote the monetary amounts that distributors receive for delivering products to customers.



ensures more equitable compensation across distributors but also provides savings to the company. Perhaps more importantly, the model helped logistics managers better understand the trade-offs and considerations in compensation design, and for the first time permitted them to incorporate demand variability in their fee planning. Though motivated by the decision problem facing a specific manufacturer, the model can support compensation negotiations in other environments where manufacturers must determine fee schedules for distribution services.

The next section introduces the specific problem context that motivated our model. Section 2 discusses the requirements and challenges in setting delivery fees, and describes the ingredients for a model to support this decision. Section 3 presents the linear programming formulation for the fee-setting problem, and discusses some variants and extensions. Section 4 covers our implementation and application of this model, and §5 concludes by identifying opportunities for further work.

## 1. Problem Context

During its 130-year history, Armstrong World Industries, Inc. has evolved from its roots as a small cork manufacturer to its current position as a global producer of floor and ceiling products. In 1999, Armstrong's annual revenues of \$3.44 billion stemmed from strong sales in both the residential and commercial segments, with residential sales to professional builders and do-it-yourself consumers accounting for 48% of its total U.S. sales revenues. Over time the characteristics of the residential segment have changed substantially, and Armstrong has adapted to the changing needs of this market.

Historically, Armstrong's organizational structure was heavily product focused; floor and ceiling products were manufactured and sold by separate divisions via independent supply chains. This separation even extended to the retail level, where many of Armstrong's traditional retailers specialized in either ceiling or flooring products. The emergence of big-box retailers in the home improvement industry placed new demands on Armstrong's divisional structure. Rather than negotiate with two different sales departments,

place separate orders on the two divisions, and receive separate deliveries, the big-box retailers insisted on a single point of contact in the organization, and required consolidated deliveries of both ceiling and floor products. They also required:

- frequent scheduled deliveries (e.g., once or twice a week);
- short order lead times;
- near-perfect order and item fill rate; and,
- unconstrained order quantities, from a few pieces to complete pallets.

To meet these needs, Armstrong initiated internal organizational changes and also decided to redesign its distribution network. A new logistics coordination unit in the corporate office was dedicated solely to servicing the big-box retail accounts—negotiating sales, processing orders, and coordinating deliveries from plants and warehouses to customer locations. Armstrong also identified a select set of independent distributors to supply the big-box customers and assigned each distributor an exclusive geographic region. Armstrong's choices in designing this new big-box distribution network were driven by several strategic and tactical considerations. The company decided to limit the number of distributors in this network in order to reduce coordination costs, focus its managerial efforts on developing close working relationships with distributors, and exploit economies of scale in transporting products from its plants. The company deliberately chose to assign each customer location to a single distributor. This single-sourcing strategy not only reduces contention among distributors, but more importantly, permits the company to clearly assign the responsibility for supplying each location to one distributor, a critical requirement to meet the stringent expectations of big-box customers.

The benefits to the big-box retailers of these changes in the organization and distribution channel are highlighted in Armstrong's 1997 annual report:

*[Armstrong's] "total category management" approach entails differentiated programs for "big box" retailers, providing products and services that drive value for customers such as The Home Depot and Lowe's. Working with them, we determine their needs and identify the right products from among Armstrong's flooring, ceiling and grid, installation and insulation offerings. We advise on the proper product mix and merchandising, provide in-store training,*

and structure the ordering and billing processes to fit each retailer's needs.

In 1997, Armstrong was honored as Supplier of the Year by Lowe's, (a large big-box chain with approximately 500 stores) from among Lowe's more than 2,200 suppliers.

The new big-box distribution network operates using a new division of responsibilities and incentives. For instance, for the big-box channel Armstrong directly markets the products, sets the pricing, and handles receivables. Hence, in the new system, distributors' primary functions are stocking products and transporting them, as needed, to customer locations. In contrast, distributors in the traditional network were responsible for selling to retailers, and arranging or verifying customer credit. To ensure efficient distribution, Armstrong works closely with its distributors to help improve operations, and promotes the sharing of best practices among distributors.

Figure 1 shows a schematic of the material and information flows in Armstrong's new distribution network for big-box retailers. The big-box customers (their warehouses and over 1,500 retail stores) directly convey their orders to Armstrong's logistics coordination unit. Stores can place orders up to 24 hours prior to their scheduled delivery time. Armstrong then forwards each order to the appropriate regional distributor. Distributors maintain warehouses (typically, just one warehouse each) and are responsible for managing their inventory. Upon receiving retailers' orders,

distributors retrieve the items from inventory, load their trucks, and deliver to various stores and other retailer locations at scheduled times. Being independent businesses, distributors plan and manage their own distribution resources (e.g., trucks and drivers). For instance, distributors decide what delivery routes to follow and how to sequence the deliveries, based upon customers' delivery time-window preferences and anticipated delivery volumes.

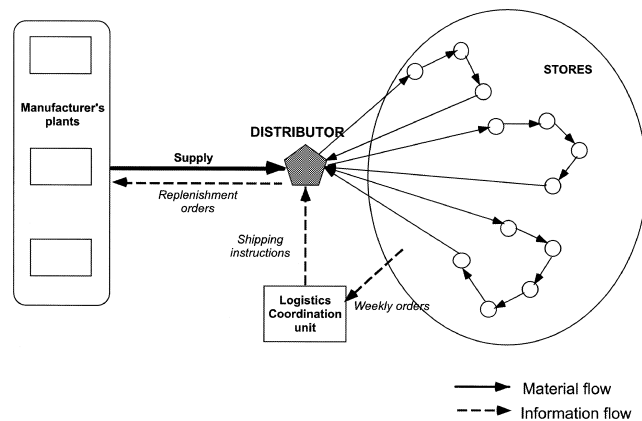
To compensate distributors for warehousing and distribution services, Armstrong implemented a new payment mechanism consisting of two parts—one to cover warehousing (storage, inventory, overhead) costs, and the other to cover the costs of delivering orders to customer locations. This paper focuses on the latter component, specifically, to determine the fees that Armstrong should pay distributors for warehouse-to-store deliveries. In the next section, we motivate and develop the ingredients of this decision problem, illustrating the challenges and considerations in deriving a common fee table for a network of distributors serving different regions.

## 2. Problem Definition

The payment scheme to compensate distributors for their delivery services must be activity-driven, fair, easy to administer, and uniformly applicable to all distributors. Fairness includes ensuring solvency of all distributors. Uniformity refers to using the same compensation scheme for all distributors, based on common activity metrics and compensation rates. This requirement, important for logistics partnerships such as the one Armstrong has established, helps avoid conflicts among distributors (e.g., perceptions that certain distributors have more favorable terms). To promote efficiency and provide proper incentives, compensation must be activity-driven, i.e., payments must be linked to services provided rather than simply resources consumed. To meet these various compensation objectives, Armstrong adopted a payment scheme analogous to the tariff structure used by the LTL carrier industry. We next motivate this approach and explain its ingredients.

Tariff structures implemented by common carriers provide a natural starting point to decide an appropriate delivery compensation scheme for distributors.

**Figure 1** Schematic of Material and Information Flow in Big-Box Supply Chain



Commercial freight rates are based upon freight characteristics of the product (e.g., size, weight, special handling requirements) and shipping distances (Harmatuck 1990, 1991, Swenseth and Godfrey 1996). They are typically specified in the form of tables showing the freight charge for various weight classes and distance zones. LTL rate structures vary across companies and regions, and can be quite complex (Baker 1991, Bohman 1995a, 1995b, Smith and Hilton 1985). Of course, published common-carrier rates typically pertain to individual shipments, and therefore may not be appropriate within the context of long-term distribution partnerships, which reduce uncertainty and provide scale economies to distributors. Nevertheless, common-carrier rates can provide guidelines for setting compensation. For instance, their structure confirms that delivery distance and the volume or weight of products delivered are the primary metrics for distribution activity, and so are suitable bases for compensation.<sup>2</sup>

Armstrong's activity-based compensation system, consistent with commercial rate structures, pays distributors on a per-delivery basis using rates specified in a *fee table*. The fee paid for delivery to a particular store<sup>3</sup> depends on the direct driving distance from the warehouse to that store, the so-called *fee distance*, and the actual total *weight* of products delivered. Fee distances and weights are aggregated into distance and weight *ranges*; the fee table specifies the delivery fee for each pair of distance and weight ranges. Table 1 shows an illustrative fee table. For each delivery, the distributor receives the fee value specified in the corresponding cell of the fee table, independent of how distributors utilize and route their trucks to deliver stores' orders. The number of ranges and the width of each distance and weight range in the fee table define the table *structure*; the fee values populating the corresponding cells define a particular fee *schedule*. The fee values are reviewed and revised once or twice a year, and applied uniformly across all distributors.

<sup>2</sup>Because all flooring and ceiling products fall under the same freight class as defined for commercial transportation, we do not need differential rates for different products, but might need some adjustments for product densities.

<sup>3</sup>For convenience, we will henceforth refer to all locations that distributors serve as stores.

**Table 1** Sample Fee Table with Three Weight Ranges and Three Distance Ranges

Weight	Distance		
	0–99 miles	100–299 miles	300–500 miles
0–999 pounds	\$150	\$200	300
1000–4999 pounds	\$250	\$300	\$450
5000–15000 pounds	\$350	\$500	\$800

This per-delivery compensation scheme, using a common fee table, offers many advantages in the context of logistics partnerships. Distributors can readily relate to this scheme since its structure conforms to common industry practice. The scheme is also easy to administer and flexible. Changes in costs due to inflation, new store assignments, and product mix variations can be accommodated by periodically adjusting the fee values, and these adjustments can be easily communicated to distributors. Note that the compensation structure remains relatively stable, i.e., the number and specification of the distance and weight ranges does not change as frequently as fee values. Over the long term basing compensation on a consistent structure alleviates difficulties associated with renegotiating distributor contracts.

The delivery fees paid to distributors represent costs to the firm. The firm's objective is to minimize its outlays, yet the delivery fee values must be credible and ensure that each distributor receives adequate total compensation to cover delivery costs in each period. Designing a single fee table that meets these conditions is a challenging task, particularly since distributors differ in their service profiles, i.e., in the number, spatial density, and demand patterns of the stores they serve. Next, we discuss the cost coverage and credibility requirements in greater detail.

## 2.1. Cost Coverage and Fee Credibility Requirements

**Adequate and Equitable Cost Coverage.** For distributors to accept a set of fee values, they must be assured that the fees they receive are sufficient to cover their distribution costs and include an appropriate profit

margin.<sup>4</sup> The manufacturer cannot guarantee a precise match between compensation and cost, even for a single distributor, due to uncontrollable variability in store demands and because distributors select their own routes, which determine their costs. Distributors can aggregate deliveries to multiple stores into a single truck that follows a multistop route. Thus, actual transportation costs depend on the distributors' truck packing and routing decisions, which in turn depend on the stores' relative locations and demand distributions. Moreover, since stores' order quantities vary stochastically from week to week, compensation also varies. Because of these difficulties in matching compensation with cost at the detailed activity level, Armstrong specified its cost coverage requirement in aggregate terms as follows: the fee values must be such that, for each distributor the *expected total* compensation per period (e.g., week or month) over all deliveries must equal or exceed that distributor's (projected) total distribution cost per period. Note that this approach requires obtaining reliable estimates of total costs for all distributors. These costs might be estimated via activity-based costing, benchmarking, or using historical data. In § 2.3 and later in § 4.3, we discuss how Armstrong independently obtained reliable cost estimates using a vehicle routing model.

**Fee Table Credibility.** Delivery fees are part of the overall cooperative agreement with distributors. To ensure acceptance by distributors, the fee values must be "credible," i.e., they should broadly conform to conventional pricing policies in the logistics industry. For instance, fees must increase with distance and weight, and should not differ markedly from commercial rates. Moreover, fees should not change abruptly from one range to the next; otherwise, they might create dysfunctional incentives for distributors. Managers might also desire fee values that reflect scale economies in distribution costs, e.g., incremental fees per mile or per pound must decrease with increasing distance or weight.

## 2.2. Challenges and Approaches for Fee-Setting

Given the structure of the fee table, the fee-setting task consists of selecting credible fee values for all distance

and weight ranges that ensure adequate cost coverage for every distributor. In thinking about how to determine these fee values, two natural approaches come to mind—tariff-based or cost-based.

The *tariff-based* method uses commercial freight rates to guide fee value choices. This approach presents several challenges. First, as we noted earlier, commercial rates vary by carrier and region, whereas Armstrong seeks a single nationwide fee schedule. Second, even within a region, transportation companies may not use the same weight and distance ranges, or provide a format from which comparable rates can be easily obtained. One possibility is to obtain quotes for several representative origin-destination pairs to determine the range of rates within each cell of fee table. Even if the values in each cell are restricted to be within these limits, selecting a specific value for each cell is a combinatorial problem. Finally, we expect that the fees a manufacturer pays its distributors will be lower (on average) than commercial rates because distributors operate under long-term contracts and have assured business in their respective exclusive service regions.

The *cost-based* approach estimates the cost of making each store delivery, and expresses this cost as a function of fee distance and weight. This function, which might be derived from statistical analysis, could then be used to decide the fee value in each cell of the table. The main drawback of this method is in determining the cost of each delivery. A substantial portion of the transportation costs incurred by distributors is fixed—consisting of depreciation (or leasing costs) for trucks and other distribution assets, salaries for regular drivers, etc. Moreover, the variable transportation costs depend on the driving distances and times which, in turn, depend on the choice of delivery routes. Therefore, to determine the full cost for a delivery the fixed costs must be allocated to routes, and then the cost of each route to deliveries. Depending on the bases used to allocate the costs, the per-delivery costs can vary widely even for a single distributor. Thus, this approach can narrow the range of feasible fee values in each cell, but does not identify specific values.

Both the tariff-based method and the cost-based approach have another disadvantage—neither method incorporates the aggregate cost coverage requirement for each distributor. Quite likely, fee values based

<sup>4</sup>Henceforth, we will assume that "cost" includes the required profit margin, return on investment, etc.



solely on tariffs will overcompensate all distributors, while the cost-based approach might undercompensate some distributors. Consequently, manual adjustments would typically be required to ensure that expected total compensation per week equals or exceeds the projected total distribution cost for each distributor. The adjustment process might sequentially modify the fee values in various cells of the fee table, or simultaneously scale all values by a common factor. But, the complex linkages among these fee values, created by the cost coverage and fee credibility requirements, greatly complicate this process. Also, increasing the fee value in one cell of the fee table does not provide the same benefit to all distributors since their delivery profiles (i.e., store distances and delivery weight distributions) vary widely. Computing the incremental amount each distributor receives per unit increase in a particular fee value, is itself nontrivial because of probabilistic demand. Thus, deciding fee values heuristically and manually can be arduous and time-consuming, and will likely produce a table that results in wide variations in distributors' surpluses and high total cost for the firm.

### 2.3. Prior Fee-Setting Exercise

When Armstrong established its new distribution system for big-box retailers, it decided a set of initial fee values using a variant of the cost-based approach combined with managerial judgment. First, a detailed activity-based costing analysis was performed to estimate each distributor's transportation costs. Using forecasts of average weekly demand (in total pounds) for each retail location, a deterministic routing algorithm identified a set of optimal delivery routes,<sup>5</sup> taking into account truck capacities (with a safety margin to account for randomness in delivery quantities). The total cost of the trucks and drivers needed to operate these routes was then allocated to each *route* based on the duration of that route. In turn, this route cost was allocated to *deliveries* within that route in proportion to the fee distances of the respective stores. Using the distribution of fee distances and delivery volumes as a guide, a set of weight and distance ranges for the fee

table was selected. The average allocated cost per-delivery over all the deliveries in each cell of the fee table served as the initial fee value for that cell. The fee values were then adjusted via an iterative manual process to ensure adequate cost coverage for each distributor. To determine if fees covered costs, the total weekly payments to each distributor were estimated by assuming deterministic store demands. Certain cell values were then adjusted to overcome deficits or to better match common carrier rates. This manual approach to setting fees resulted in uneven cost coverage (and larger than necessary total fee payments) and some anomalous fee values.

By exploiting the special structure of the problem, we model the compensation design problem as a linear program. The model minimizes the expected total fees that the manufacturer pays, while ensuring that fee values are credible and each distributor's expected compensation equals or exceeds its costs. Based on user-specified parameters characterizing the credibility requirements, the linear program generates an optimal fee table that meets all requirements. Next, we describe the principles underlying the linear programming model, and present the problem formulation before discussing our implementation and results.

## 3. Model Formulation

As we noted earlier, the distance and weight ranges of the fee table do not change as often as fee values. This paper focuses on the more frequent fee-setting decision. Given the distance and weight ranges that define the cells of the fee table, the fee-setting problem requires selecting a delivery fee value for each pair of distance and weight ranges. As we discussed previously, the chosen values must be credible, and must ensure cost coverage for every distributor. We treat the fee credibility and distributor cost coverage requirements as constraints, while the objective is to minimize the total expected overpayment to distributors or; equivalently, minimize the total fees paid by the manufacturer to all distributors.

### 3.1. Notation and Parameters

Let  $i = 1, \dots, I$  and  $j = 1, \dots, J$  respectively, denote the indices of the given weight and distance ranges in increasing order of weight and distance. The pair  $(i, j)$

<sup>5</sup>These routes were developed solely for cost estimation purposes; distributors select their own daily delivery routes.

represents the cell of the fee table corresponding to weight range  $i$  and distance range  $j$ . Let  $w_i$  denote the given lower limit (in pounds) for weight range  $i$ , and  $d_j$  the lower limit (in miles) for distance range  $j$ . With our range indexing scheme,  $0 = w_1 \leq w_2 \leq \dots \leq w_I < w_{I+1} = \infty$ , and  $0 = d_1 \leq d_2 \leq \dots \leq d_J < d_{J+1} = \infty$ . Thus, if a distributor makes a delivery of  $u$  pounds to a store located  $l$  miles away from the warehouse, and if  $w_i \leq u < w_{i+1}$  and  $d_j \leq l < d_{j+1}$ , the distributor receives the fee corresponding to cell  $(i, j)$  of the fee table for this delivery.

Let  $M$  and  $K$  denote the number of stores and distributors in the system. Stores are indexed from  $m = 1$  to  $M$ , and distributors from  $k = 1$  to  $K$ . For convenience, we define a store as any location that receives at most one delivery per time period. If an actual store is scheduled to receive more than one delivery per period, we consider multiple "stores" corresponding to that location so that each new store receives no more than one delivery per period. In our actual application the vast majority of actual stores (over 90%) receive one delivery per week, and so our planning period is one week. A few locations receive two deliveries per week, while others receive one delivery every two weeks. We are given the following data on stores and distributors:

(i) the list of stores  $M(k) \subseteq \{1, \dots, M\}$  served by each distributor  $k$ ;

(ii) for each store  $m$ , the *fee distance*  $l_m$ , defined as the distance to drive directly to store  $m$  from its assigned distributor's warehouse (without stopping at other stores);

(iii) the frequency  $\lambda_m$  of deliveries to each store  $m$ , i.e., the number of scheduled deliveries per time period to store  $m$ . For example, a store that receives one delivery every two weeks has  $\lambda_m = 1/2$ . By our previous definition of a store,  $\lambda_m \leq 1$ ;

(iv) the cumulative distribution  $\phi_m(\cdot)$  of demand (in pounds per delivery) for each store  $m$ ; and,

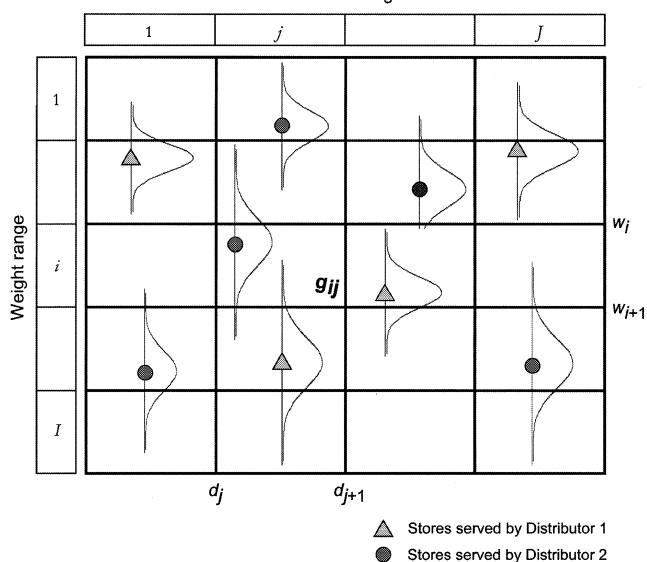
(v) the anticipated total distribution cost per period  $C_k$  for each distributor  $k$ .

In logistics partnerships, distributors might be willing to share detailed operational and financial data about their operations, permitting the manufacturer to obtain reliable estimates of the distributor costs  $C_k$ . Alternatively, as outlined in §2.3, the manufacturer can

independently estimate these costs by modeling each distributor's operations using a vehicle routing algorithm.

Figure 2 shows a schematic representation of store locations and demand, overlaid on a fee table. The circles and triangles in this figure represent the "coordinates"—fee distance and mean delivery weight—of individual stores served by two different distributors. The demand distribution around these coordinates shows that deliveries made (and hence fees paid) to the same store can span multiple cells of the fee table. We note here that because of Armstrong's single sourcing strategy, the customer locations  $M(k)$  covered by each distributor  $k$  in its big-box network are mutually exclusive. However, the model applies even with multiple sourcing, i.e., when more than one distributor can supply the same store, as long as the relative product volumes supplied by each distributor are reasonably stable and predictable. To model this situation we can either adjust the store's delivery frequency parameters for each distributor or introduce dummy stores corresponding to each original multisourced store. In the latter case each dummy store is served by a single distributor, and its demand (and delivery frequency) reflects the volume of products delivered by the corresponding distributor. Note that, with multiple

Figure 2 Fee Table with Relative Delivery Locations and Demands



sourcing, the distribution cost estimates  $C_k$  must take into account the delivery-sharing arrangements.

Each store  $m$ 's fee distance  $l_m$  determines the distance range index  $j(m) = \max \{j: d_j \leq l_m\}$  to use for calculating fees for deliveries to that store. Let  $\beta_{im} = \{\phi_m(w_{i+1}) - \phi_m(w_i)\}$  denote the probability that a delivery to store  $m$  will fall in weight range  $i$ . Then,  $\lambda_m \beta_{im}$  is the expected number of deliveries per period to store  $m$  in weight range  $i$ . Hence, distributor  $k$ 's expected number of deliveries per period,  $a_{ijk}$ , in weight range  $i$  and distance range  $j$  is the sum of  $\lambda_m \beta_{im}$ , over all stores  $m$  that are served by distributor  $k$  and belong to distance range  $j$ , i.e.,

$$a_{ijk} = \sum_{m \in M(k); j(m)=j} \lambda_m \beta_{im} \text{ for all } i = 1, \dots, I, \\ j = 1, \dots, J, \text{ and } k = 1, \dots, K.$$

These parameters constitute the coefficients of the decision variables in the cost coverage constraints and the objective function of the linear programming model.

In addition to the data on distributors and stores, the user also specifies various parameters characterizing the constraints that enforce the fee credibility requirements. These constraints, referred to as *consistency* conditions, fall into two classes: *bounds* on fee values, and *shape* constraints. For each cell  $(i, j)$  (or for selected cells) of the fee table, the user might specify minimum and maximum permissible fee values  $\underline{p}_{ij}$  and  $\bar{p}_{ij}$ , with respective default values of 0 and  $\infty$ . These bounds might be determined, for instance, based on common carrier freight rates. The shape constraints, governing how the fee values change as a function of distance and weight, relate the fee values across cells in the fee table. As we noted earlier, managers prefer not to have large, abrupt fee increases from one range to the next; but, at the same time, fee values in adjacent cells must be noticeably different. The model can accommodate these requirements by imposing upper and lower limits on the fee increments from one cell to the next along both the distance and weight dimensions. With unequal range widths managers might find it more convenient to express these conditions in terms of incremental fees per pound or per mile, i.e., in terms of *rates* of change in fees. Accordingly, we define  $q_i$  and  $\bar{q}_i$  as the minimum and maximum fee increment per pound in going from weight range  $i$  to weight range

$i + 1$ . Similarly, let  $r_j$  and  $\bar{r}_j$  represent the minimum and maximum fee increment per mile between distance ranges  $j$  and  $j + 1$ . A more stringent version of these shape conditions might require *concavity* of the fee values with respect to both distance and weight, i.e., the incremental fee per additional mile (pound) must decrease as the distance (weight) increases, consistent with economies of scale in distribution costs. Our formulation incorporates these conditions as well.

### 3.2. Mathematical Formulation

The natural decision variables for the fee setting optimization model are the fee values (in \$ per delivery), denoted as  $g_{ij}$ , for each cell  $(i, j)$  of the fee table. To reduce the number of constraints (and to later facilitate modeling a distance-sensitive fee structure using the same formulation), we consider a different but equivalent set of decision variables. For  $i = 1, \dots, I$ , let  $f_i$  denote the fee value in cell  $(i, 1)$ , and let  $s_{ij}$ , for  $j = 1, \dots, J - 1$  be the incremental fee per unit distance (\$ per mile) for weight range  $i$  and distance range  $j$ . In terms of these alternate decision variables, the fee  $g_{ij}$  that a distributor receives for a delivery that falls in cell  $(i, j)$  of the fee table is:

$$g_{i1} = f_i \quad \text{for } i = 1, \dots, I, \text{ and} \\ g_{ij} = f_i + \sum_{j'=1}^{j-1} s_{ij'}(d_{j'+1} - d_{j'}) = g_{i,j-1} + \\ s_{i,j-1}(d_j - d_{j-1}) \quad \text{for } i = 1, \dots, I, \text{ and } j = 2, \dots, J.$$

Using the decision variables  $f_i$  and  $s_{ij}$ , the fee setting problem has the following linear programming formulation [FSLP]:

$$\text{[FSLP] Minimize } \sum_{k=1}^K \sum_{i=1}^I \left\{ f_i \left( \sum_{j=1}^J a_{ijk} \right) + \sum_{j=1}^{J-1} s_{ij}(d_{j+1} - d_j) \left( \sum_{j'=j+1}^J a_{ij'k} \right) \right\} \quad (1)$$

subject to:

*Distributor cost coverage:*

$$\sum_{i=1}^I \left\{ f_i \left( \sum_{j=1}^J a_{ijk} \right) + \sum_{j=1}^{J-1} s_{ij}(d_{j+1} - d_j) \left( \sum_{j'=j+1}^J a_{ij'k} \right) \right\} \\ \geq C_k \text{ for } k = 1, \dots, K, \quad (2)$$

Monotonicity with weight:

$$f_{i+1} \geq f_i \quad \text{for } i = 1, \dots, I-1, \quad (3a)$$

$$\begin{aligned} f_{i+1} + \sum_{j'=1}^{j-1} s_{i+1,j'}(d_{j'+1} - d_{j'}) \\ \geq f_i + \sum_{j'=1}^{J-1} s_{i,j'}(d_{j'+1} - d_{j'}) \\ \text{for } i = 1, \dots, I-1, j = 2, \dots, J, \end{aligned} \quad (3b)$$

Upper & lower bounds on fee values:

$$\underline{p}_{i1} \leq f_i \leq \bar{p}_{i1} \quad \text{for } i = 1, \dots, I, \quad (4a)$$

$$\begin{aligned} \underline{p}_{ij} \leq f_i + \sum_{j'=1}^{j-1} s_{ij'}(d_{j'+1} - d_{j'}) \leq \bar{p}_{ij} \\ \text{for } i = 1, \dots, I, j = 2, \dots, J, \end{aligned} \quad (4b)$$

Upper & lower bounds on incremental fee per pound:

$$\begin{aligned} \underline{q}_i(w_{i+1} - w_1) \leq f_{i+1} - f_i \leq \bar{q}_i(w_{i+1} - w_i) \\ \text{for } i = 1, \dots, I-1, \end{aligned} \quad (5a)$$

$$\begin{aligned} \underline{q}_i(w_{i+1} - w_i) \leq \left\{ f_{i+1} + \sum_{j'=1}^{J-1} s_{i+1,j'}(d_{j'+1} - d_{j'}) \right\} \\ - \left\{ f_i + \sum_{j'=1}^{J-1} s_{ij'}(d_{j'+1} - d_{j'}) \right\} \leq \bar{q}_i(w_{i+1} - w_i) \\ \text{for } i = 1, \dots, I-1, j = 2, \dots, J, \end{aligned} \quad (5b)$$

Upper & lower bounds on incremental fee per mile:

$$\underline{r}_j \leq s_{ij} \leq \bar{r}_j \quad \text{for } i = 1, \dots, I, j = 1, \dots, J-1, \quad (6)$$

Concavity with weight:

$$\begin{aligned} (w_i - w_{i-1})(f_{i+1} - f_i) \leq (w_{i+1} - w_i)(f_i - f_{i-1}) \\ \text{for } i = 2, \dots, I-1, \end{aligned} \quad (7a)$$

$$\begin{aligned} (w_i - w_{i-1}) \left\{ f_{i+1} + \sum_{j'=1}^{J-1} s_{i+1,j'}(d_{j'+1} - d_{j'}) \right\} \\ - \left\{ f_i + \sum_{j'=1}^{J-1} s_{ij'}(d_{j'+1} - d_{j'}) \right\} \\ \leq (w_{i+1} - w_i) \left\{ f_i + \sum_{j'=1}^{J-1} s_{i,j'}(d_{j'+1} - d_{j'}) \right\} \\ - \left\{ f_{i-1} + \sum_{j'=1}^{J-1} s_{i-1,j'}(d_{j'+1} - d_{j'}) \right\} \\ \text{for } i = 2, \dots, I-1, j = 2, \dots, J, \end{aligned} \quad (7b)$$

Concavity with distance:

$$\begin{aligned} s_{i,j+1} \leq s_{ij} \quad \text{for } i = 1, \dots, I, \\ j = 1, \dots, J-1, \text{ and} \end{aligned} \quad (8)$$

Non-negativity:

$$f_i \geq 0, s_{ij} \geq 0 \quad \text{for } i = 1, \dots, I, j = 1, \dots, J-1. \quad (9)$$

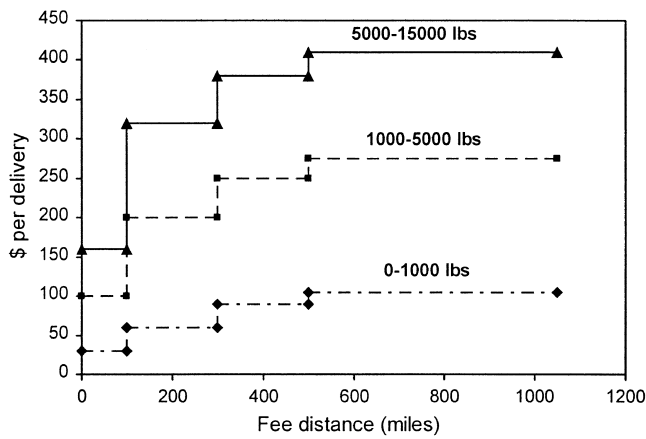
The objective function (1) minimizes the expected total fees paid each period to all distributors. The distributor cost coverage constraints (2) ensure that the expected total fees paid to each distributor exceeds that distributor's anticipated distribution cost per period. The weight monotonicity constraints (3) state that for any distance range, the fee values should not decrease as the delivery weight increases. The nonnegativity constraints (9) on the  $s_{ij}$  variables enforce distance monotonicity by ensuring that fees do not decrease with distance. Constraints (4) through (8) represent the consistency requirements. Constraints (4), (5), and (6) impose upper and lower bounds on the fee values and their increments, while constraints (7) and (8) are the concavity conditions with respect to weight and distance. These latter constraints specify that, for each distance (weight) range the rate of change in fees should not increase as the weight (distance) increases.

### 3.3. Model Variants and Extensions

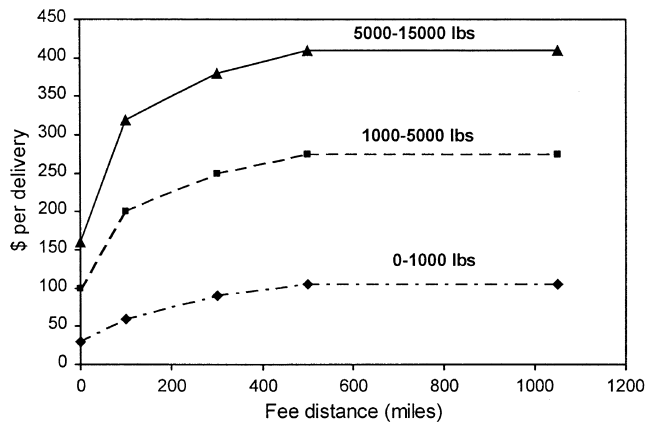
**Distance-Sensitive and Weight-Sensitive Compensation Functions.** The fee-setting linear program [FSLP] can also be used to model *distance-sensitive* and/or *weight-sensitive* compensation functions. Recall that, for each distance and weight range, we currently assume that the fee table specifies a single fixed fee value for all deliveries corresponding to that cell. This approach corresponds to a *stepwise* compensation function since the graph of fees versus distance (or weight) increases in steps at the endpoints of each range (see Figure 3a). Alternatively, we might consider a piecewise linear compensation function that distinguishes between deliveries even within the same cell based on the exact (continuous) weight and/or distance values. To illustrate this approach, consider two stores located 125 miles and 290 miles from their respective distributors. For the fee ranges shown in Table 1, both stores belong to the same distance range. If their shipment sizes also fall in the same range, the stepwise function



Figure 3 Graphical Representation of Compensation Functions



(3a) Step Function Fee Structure



(3b) Distance-Sensitive Fee Structure

would provide the same delivery fee for both stores (e.g., \$200 for the 1,000 to 4,999 pound range in Table 1). For a distance-sensitive function (see Figure 3b), on the other hand, we specify an additional per-mile fee for each cell that permits fee variations based upon exact distances. Thus, deliveries to the store that is 125 miles away would be assessed the base fee for the 100-299 mile distance range, plus 25 times the incremental fee.

By interpreting the variables  $s_{ij}$  as the incremental fee for each cell, adding a new variable  $s_{ij}$  corresponding to the last distance range  $J$ , and appropriately changing the coefficients in the cost coverage constraints (2) and objective function (1), we see that the linear program [FSLP] easily models this distance-sensitive compensation function. In particular, for each

store  $m$ , let  $\mu_{im} = \lambda_m \beta_{im} (l_m - d_{j(m)})$  where, as before,  $j(m)$  is the index of the distance range corresponding to store  $m$ . Define

$$b_{ijk} = \sum_{m \in M(k): j(m)=j} \mu_{im}$$

for all  $i = 1, \dots, I, j = 1, \dots, J$ , and  $k = 1, \dots, K$ .

Then, modeling the distance-sensitive function entails replacing the distributor cost coverage constraints (2) in formulation [FSLP] with the following constraints:

$$\sum_{i=1}^I \left\{ f_i \left( \sum_{j=1}^J a_{ijk} \right) + \sum_{j=1}^J s_{ij} b_{ijk} + \sum_{j=1}^{J-1} s_{ij} (d_{j+1} - d_j) \left( \sum_{j'=j+1}^J a_{ij'k} \right) \right\} \geq C_k \quad \text{for } k = 1, \dots, K. \quad (2a)$$

The new objective function is:

$$\text{Minimize } \sum_{k=1}^K \sum_{i=1}^I \left\{ f_i \left( \sum_{j=1}^J a_{ijk} \right) + \sum_{j=1}^J s_{ij} b_{ijk} + \sum_{j=1}^{J-1} s_{ij} (d_{j+1} - d_j) \left( \sum_{j'=j+1}^J a_{ij'k} \right) \right\}. \quad (1a)$$

We could similarly consider *weight-sensitive* or *distance and weight-sensitive* functions. However, because delivery weights are stochastic, the computations necessary to determine the constraint coefficients (representing partial expectations of cell fees) of the linear programming model become more complex.

**Auxiliary Objectives.** The model [FSLP] uses the *minisum* objective of minimizing the manufacturer's expected *total* payments per period for distribution services. If, for a given problem instance, the model has alternative optimal solutions, then the company might wish to select a solution that minimizes the maximum surplus (expected payment minus cost) over all distributors. This *minimax* objective reflects the company's goal of providing equitable payments to distributors. To identify the minimax solution among alternative optima, the model changes as follows.

Suppose we have solved the original [FSLP] model. Let  $Z^*$  denote its optimal objective function value. We construct an *auxiliary* model [AFSLP] by:

(i) adding to [FSLP] a constraint specifying that the total fee payments for all distributors must be less or equal to  $Z^*$ , i.e.,

$$\sum_{k=1}^K \sum_{i=1}^I \left\{ f_i \left( \sum_{j=1}^J a_{ijk} \right) + \sum_{j=1}^{J-1} s_{ij}(d_{j+1} - d_j) \left( \sum_{j'=j+1}^J a_{ij'k} \right) \right\} \leq Z^*; \quad (10)$$

(ii) introducing nonnegative surplus variables,  $x_k$ , for  $k = 1, \dots, K$ , in the distributor cost coverage constraints (2), i.e., replacing constraints (2) with

$$\sum_{i=1}^I \left\{ f_i \left( \sum_{j=1}^J a_{ijk} \right) + \sum_{j=1}^{J-1} s_{ij}(d_{j+1} - d_j) \left( \sum_{j'=j+1}^J a_{ij'k} \right) \right\} - x_k = C_k \quad \text{for } k = 1, \dots, K, \quad (2a)$$

and adding the nonnegativity restrictions

$$x_k \geq 0 \quad \text{for } k = 1, \dots, K; \quad (11)$$

(iii) introducing a new variable  $X$  denoting the *maximum surplus* over all distributors, and relating this variable to the distributor surplus variables  $x_k$  by adding the constraints

$$X \geq x_k \quad \text{for } k = 1, \dots, K; \text{ and} \quad (12)$$

(iv) replacing the original objective function (1) with

$$\text{Minimize } X. \quad (13)$$

The optimal solution to this auxiliary linear program [AFSLP] selects the minimax solution from among the optimal solutions to [FSLP]. Note that, without constraint (10), the model [AFSLP] models the minimax version of the original problem, namely to find the set of fee values that minimize the maximum surplus among all distributors.

## 4. Implementation and Application

As with most modeling efforts (e.g., Urban [1974]), our development and implementation of the fee-setting model followed an iterative process—starting with a basic model, testing it using actual data, and enhancing the model based on user feedback. The process of modeling the fee-setting problem and representing it formally as a mathematical program proved valuable to the organization, and demonstrated the potential economic benefits of fee optimization. The modeling process also served to clarify the constraints on fee selection, and highlighted to managers the complex

linkages between the total fees paid and various problem parameters such as the range specifications, demand variability, and store-to-distributor assignments.

### 4.1. The CODES System

We implemented the delivery fee-setting model in Microsoft Excel, using Excel's built-in linear programming solver to optimize the embedded linear program. The system, referred to as CODES (for COnpensation DESign Support), can run in evaluation mode or optimization mode. In the optimization mode, it solves the fee-setting linear program to generate an optimal fee table and permits sensitivity analysis. Users can decide which among the consistency conditions (4) through (8) to include in the model. The system also provides an option to optimize fee values for a distance-sensitive fee structure instead of step function fees. In the evaluation mode, CODES accepts any user-proposed fee table, determines the total expected weekly compensation for each distributor, and computes the deficit or surplus for each distributor.

For a problem with, say, 8 weight ranges, 14 distance ranges, and around 25 distributors, the linear programming formulation (with all consistency conditions) contains more than 100 variables and over 500 constraints. We were able to solve these problem instances on a personal computer in only a few minutes of execution time. Calculating the probabilities needed to determine the  $a_{ijk}$  coefficients in the formulation required more time than actually solving the linear program. For example, a problem with 2,000 stores and 8 weight ranges requires 16,000 probability calculations.

### 4.2. Using the Fee-Setting Model

The fee-setting model is a valuable tool to support the periodic fee-change negotiations. As an optimizer and evaluator it can propose an initial fee schedule via optimization, and later evaluate any manual adjustments made to reflect other fee-setting considerations. More importantly, the system can help assess the impact of various changes in the distribution system, and even serve as a diagnostic tool for prioritizing operations-improvement initiatives. We elaborate on these capabilities below.

*Evaluating the impact of changes in the distribution system:* Distribution systems are dynamic. New stores open, existing stores close, and improvements in logistics

management alter distributors' costs. Also, the locations and store assignments of distributors can change, and store demand patterns might shift due to new products or changing demographics. Each of these changes affects distributors' compensation and costs. Consequently, over time, a given fee schedule might lead to under- or over-compensation for certain distributors, and possibly larger than necessary total payments by the manufacturer. Because changes in the distribution system require only simple modifications to the input data, the model can quickly evaluate the financial impact of these changes as they arise.

*Adjusting the fee table:*

For minor changes in the environment, management might wish to adjust the existing table rather than introduce a completely revised (and optimized) fee table. CODES facilitates such manual fee adjustments. For instance, if the fee table does not provide adequate cost coverage for a distributor, the coefficients  $a_{ijk}$  in the cost coverage constraints provide a convenient metric to prioritize the cells that might require fee adjustment. Suppose the table does not fully cover distributor  $l$ 's cost. Because  $a_{ijl}$  represents the increase in distributor  $l$ 's revenue per unit increase in the fee value for cell  $(i, j)$ , the cells with high values of  $a_{ijl}$  might appear to be attractive candidates for increasing fee values. However, if these cells also have large coefficients  $a_{ijk}$  corresponding to another distributor  $k$ , then increasing these fee values also increases overpayments to that distributor. Hence, to cover distributor  $l$ 's cost while limiting the overall cost to the manufacturer, an effective heuristic strategy might be to increase fees in those cells that have high values of the ratio  $a_{ijl}/\sum_k a_{ijk}$ .

*Diagnosing cost bottlenecks and identifying improvement opportunities:*

Although CODES is primarily aimed at fee-table evaluation and optimization, it also provides diagnostic capabilities. For instance, it can identify distributors who are "influential" from a fee-setting perspective, i.e., distributors whose cost coverage requirement forces the model to select high fee values. In the linear programming solution, the cost coverage constraints for such distributors have consistently high shadow prices over many fee table scenarios. Reassigning the stores served by these distributors, helping them improve

operations, or refining their cost estimates might permit fee revisions that significantly reduce the manufacturer's overall fee expenses. Similarly, suppose stores can be persuaded to reduce the variability in their ordering patterns, permitting distributors to reduce their costs, (e.g., by decreasing the needed safety capacity in trucks). The model helps assess the benefits of such variability reduction by determining the expected total compensation for various uncertainty levels.

*Changing the compensation function and fee structure:*

The model permits managers to quickly optimize and evaluate alternative compensation functions, such as a distance-sensitive function. It can also evaluate the impact of changing the number and width of the weight and distance ranges. Recall that the linear programming model assumes that the ranges are prespecified. However, by applying the model repeatedly with different range specifications, we can empirically determine a set of weight and distance ranges that reduce the total fees paid (while still satisfying all constraints).

In summary, the system can support a variety of decisions that go beyond setting delivery fees. Its optimization and evaluation capabilities, combined with sensitivity analysis outputs, provide a powerful platform for problem diagnosis and fee structure refinement. Furthermore, it provides an implicit method to benchmark and compare distributors' delivery costs.

### 4.3. Application to Armstrong's Distribution Network

Prior to using CODES, the logistics organization at Armstrong developed a  $5 \times 4$  fee table, with five weight ranges and four distance ranges, using a manual and iterative procedure as discussed earlier. Distributors were compensated based on this table for the first two years of the new distribution system's operation. Subsequent increases in distribution costs were accommodated using a standard "inflation" factor applied to all fee values in the table. But, as the number of stores grew and distributors' service regions were reconfigured, Armstrong needed to revise the fee table. CODES provided the capability to quickly evaluate and optimize fee values; most importantly, it incorporates demand variability whereas previous evaluations were deterministic.

To test the system, we first obtained data on all of Armstrong's U.S. distributors and big-box customer locations (including over 1,500 stores), as of early 1998. For each store, the expected weekly demand was estimated from past data. Rather than specify detailed empirical demand distributions, we assumed that store demands followed normal distributions (truncated at zero), and the ratio of standard deviation to mean demand, i.e., the coefficient of variation (COV), is the same for all stores.<sup>6</sup> This assumption reduced the model's input data needs, and was validated by computing the actual past COV values for select stores and noting that they did not differ markedly across stores.

To determine distribution cost, Armstrong made the conservative assumption that distributors dedicate resources exclusively to deliver the company's products even though distributors might derive scale economies by sharing resources (e.g., trucks, drivers) with their other lines of businesses. Ignoring these scale economies helped ensure fairness and uniformity across distributors. As we noted in § 2.3, Armstrong had previously applied a deterministic vehicle routing algorithm to estimate each distributor's costs. Based on the distributor's warehouse location, assigned stores, estimated mean store demands, and effective capacity of each truck, the algorithm groups stores into routes and sequences the deliveries on each route to minimize total travel distance. Using commonly accepted cost metrics for owning and operating vehicles, the company computed the cost of each route generated by the algorithm. Adding these costs provided an initial estimate of each distributor's total cost. Moreover, as part of the partnership agreement, distributors are required to provide detailed financial reports to Armstrong. This data provided yet another means to validate the cost figures.

Based upon the demand and cost data provided by Armstrong, we evaluated the expected compensation for each distributor based on the existing  $5 \times 4$  fee table (referred to as the *original* table). Using this table some distributors received large surpluses whereas others had deficits because distributor costs had increased and their coverage regions had changed since

the table's introduction. Moreover, the table's design did not consider stochastic variations in delivery weights. To generate a proper benchmark for comparing with our subsequent optimized tables, we applied a scaling procedure that increases all the fee values in the original table by the same proportion to ensure that expected compensation covers projected costs for every distributor. A 12% inflation factor was the minimum necessary to achieve this goal. We refer to the scaled version of the original table as the *scaled* table. Scaling is a convenient strategy that is consistent with previous fee table revisions and is readily understood by distributors. Although the scaled table ensured adequate compensation for every distributor, the resulting *distributor surplus %*, defined as the expected \$ surplus per week divided by projected cost for each distributor, varied widely—ranging from 0% to 59.2%. Some distributor surplus is inevitable when a uniform table is applied to heterogeneous distribution operations, but wide variation in surpluses is undesirable. Optimizing the fee values minimizes total surplus, thus reducing undue overcompensation.

For the first optimization run, we used the same distance and weight ranges specified for the original  $5 \times 4$  table structure, and did not impose any consistency conditions other than monotonicity. The fee values obtained using this first optimization run were not deemed credible by Armstrong management. For instance, the optimal solution selected the same fee values for several contiguous cells in the table. Imposing concavity constraints provided a more credible table, but increased the total fees considerably. By replacing the concavity constraints with less stringent consistency requirements (such as lower limits on fee values and upper and lower bounds on fee increments), the model generated fee values that distributors could agree to. Instead of explicitly specifying lower limits on the fee values for every cell of the table, we specified minimum required fees for only the four corner cells in the table (i.e., for the first and last weight and distance ranges). Lower bounds for intermediate cells are obtained by interpolating these "anchor" values. For fee functions that reflect economies of distance and weight, this method is justified since the straight line connecting the lower bounds at the extreme ranges provides valid lower bounds for the intermediate

<sup>6</sup>The system can accommodate more general (including empirical) demand distributions, and also handle store or region-specific demand variation.



ranges. As expected, the new optimized  $5 \times 4$  table ensured greater parity in surplus, reducing the maximum distributor surplus to under 28%. Relative to using the scaled table, this optimized  $5 \times 4$  table reduced total fees by more than one million dollars per year. Interestingly, the total fees payable using the optimized table were even lower than the total payments using the original (unscaled) table which did not fully cover costs for some distributors.

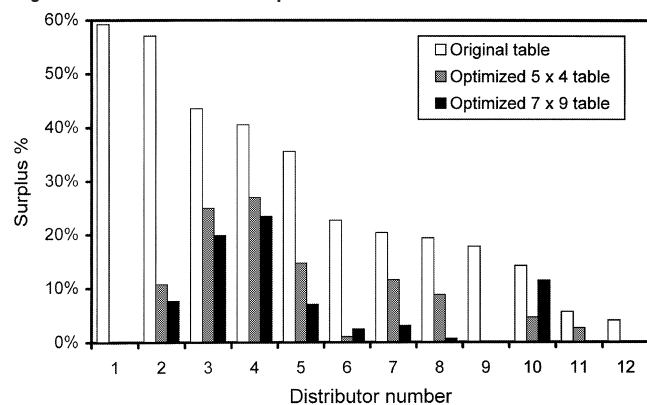
We also experimented with alternative specifications of the distance and weight ranges in an attempt to further improve the performance of the fee table by changing the *structure* of the table (i.e., the number of ranges and their widths). We generated and tested approximately ten candidate fee table structures ranging in size from  $5 \times 4$  through  $8 \times 14$  by making intuitive choices regarding distance and weight intervals, based on an examination of the distributions of store distances and delivery weights. Increasing the number of distance and weight ranges allows more degrees of freedom, but does not necessarily reduce the total fees paid due to the consistency constraints. Our tests indicated that increasing the table size beyond the existing  $5 \times 4$  limits yielded only modest economic benefits. After experimenting with various candidate table structures, the company chose a  $7 \times 9$  table. For this table, managers iteratively changed the parameters of the fee consistency conditions (e.g., the fee anchors and the upper and lower limits on fee increments) and applied the model to obtain fee values that distributors would find acceptable. The final optimized  $7 \times 9$  table yielded an optimal expected cost 2.9% lower than the cost of the optimized  $5 \times 4$  table. Thus, while the cost savings gained from attempts to improve upon the existing  $5 \times 4$  fee structure were quite modest (2.9%), significant cost savings were realized by optimizing the fee values within the existing  $5 \times 4$  fee structure (12.6%).

The problem instances corresponding to the  $5 \times 4$  and  $7 \times 9$  table structures had unique optimal solutions. So, the auxiliary optimization discussed in Section 3.3 was not needed. Two factors might explain the uniqueness of the optimal solution. First, because we consider stochastic demands, the coefficient matrix  $a_{ijk}$  is dense, i.e., most if not all cells of the fee table have some associated deliveries. Consequently, changing

any fee value impacts both cost coverage and the objective function. Second, the consistency conditions relate the fee value in each cell to fee values in adjacent cells, thus limiting the degrees of freedom needed for alternate optima.

Figure 4 compares the surplus percentage for a sample set of 12 distributors using the scaled version of the prior fee table, the optimized  $5 \times 4$  table, and the optimized  $7 \times 9$  table. As the figure shows, the optimized tables distribute surplus much more equitably than the scaled table. In particular, with the scaled table, two distributors receive over 50% surplus, and 10 out of 12 distributors receive more than 10% surplus. With the optimized  $7 \times 9$  table, the maximum surplus is less than 25%, and only three out of 12 distributors gain more than 10% surplus. Finally, we note that removing the fee consistency conditions (all the upper and lower limits on fee increments, except monotonicity and fee anchors) for the  $7 \times 9$  table reduced total expected payments to all distributors by approximately 3%. This figure represents the expected cost to the company for ensuring that fees are credible and will be accepted by distributors. The total payments further decreases by 1% when we ignore the fee anchors. The remaining gap between total payments and the sum of the distributors' costs can be attributed to the company's policy of requiring a uniform fee schedule (without consistency requirements) for all distributors. Surprisingly, this gap turned out to be only 0.75% of the total expected payment to distributors.

Figure 4 Distributors' Surplus for Three Different Fee Tables\*



\*Distributor surplus % is defined as each distributor's (expected compensation minus projected cost) as a percentage of projected cost.

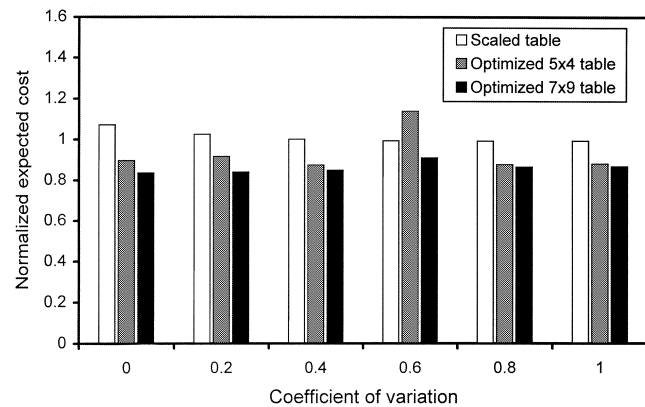
To summarize, the model generated a fee table that provides more equitable compensation to all distributors at lower total fee expenses. Armstrong has recently implemented this table. Also, the diagnostic capabilities of the model, as noted in Section 4.2 (specifically, the ability to identify improvement opportunities and influential constraints), proved useful to management. Our optimization exercise led to re-examination and refinement of the projected costs for certain distributors, resulting in further improvements to the solution.

A major benefit of the fee-setting model is its ability to incorporate demand variability, as specified by the coefficient of variation (COV) for store deliveries. Fee value optimization must be performed for a specific coefficient of variation, but the optimized table (or any other user-specified table) can then be tested at other levels of variability. Note that the total expected fees and the compensation that distributors receive might increase or decrease as the COV increases. The specific behavior depends not only on the demand distributions but also the table structure, the dispersion of stores in the cells, and the fee values. Analysis of past data suggested that the COV in stores' order weights was around 0.4. Thus, when optimizing both the  $5 \times 4$  and  $7 \times 9$  fee tables, we used this value of 0.4 as the COV for store demands. To study the behavior of expected compensation if order variability changes, we evaluated the optimized tables and the scaled table for five other values of COV.

Figure 5 compares the total expected compensation, normalized with respect to the total compensation using the scaled table when COV is 0.4, for each of the three tables and for COV values ranging from 0 to 1. Surprisingly, the expected compensation for the scaled table actually *increases* as COV decreases.

This outcome ran counter to our expectation that reducing variability would reduce total payments. We note, however, that this behavior depends on the distributors' service profiles, and need not hold for other distribution networks. The results in Figure 5 show that the optimized  $7 \times 9$  table is robust—it provides relatively constant total compensation for all COV values, and is always superior to the scaled table. The optimized  $5 \times 4$  table fares nearly as well at all variability levels but one; for a coefficient of variation of

**Figure 5** Comparison of Three Fee Tables for Different Demand Variability Levels



0.6 its expected cost increases sharply. Thus, the system permits evaluating candidate fee tables for various parameter settings and variability levels, a capability that was not previously available.

## 5. Conclusions

More demanding customer delivery expectations, combined with the emergence of capable third-party logistics providers present manufacturers with both new challenges and opportunities for managing their supply chains. To respond to the needs of important customers, such as big-box retailers, companies often enter into long-term partnerships with logistics providers, permitting increased focus on core competencies while still allowing active participation in supply chain planning and oversight. One issue raised by outsourcing distribution functions concerns how distributors should be compensated for their services. Compensation policies could range from paying a fixed periodic amount (e.g., a predetermined monthly payment) to negotiating payments for each individual service transaction, with most viable contracts being likely to fall somewhere between these two extremes. Fee tables, with agreed upon rates for various distance and weight ranges, provide an appropriate method for compensating independent distributors. This approach, similar to the pricing scheme used by LTL carriers, permits activity-based compensation and is easy

to administer. In the context of multidistributor partnerships such as Armstrong's, using a uniform fee table for all distributors promotes fairness and helps avoid conflicts.

Without a structured optimization approach, manually selecting acceptable fee values for all weight-distance combinations is difficult and might not be effective. The fee-setting model developed in this paper incorporates the primary concerns of both manufacturer and distributors by minimizing expected total expenses for the manufacturer, while ensuring that distributors are adequately compensated. The linear programming modeling framework is very flexible, and even incorporates demand variability. The model provided Armstrong with the ability to develop a new fee table that provides significant annual savings while enabling equitable compensation to distributors.

Two ingredients of the fee-setting model—cost estimation and uniformity—might appear to limit its applicability. The model requires reliable estimates of distributor's costs; Armstrong's approach of modeling and costing each distributor's logistics operations using standard underlying resource costs illustrates one way to generate such cost estimates. Uniformity, obtained by using a common fee table for all distributors, will inevitably result in overcompensation for some distributors. Nevertheless, Armstrong chose this approach to minimize conflicts and simplify implementation. Alternatively, we might consider developing here, a customized fee table that guarantees cost coverage for each distributor; the linear programming model applies to this situation as well. Interestingly, as our analysis of Armstrong's distribution data demonstrates, the benefits of using distributor-specific tables might be relatively small. The optimized table suggested by the model is only 4.75% more expensive than the sum of the distributors' costs (which is also the lowest possible amount that the company can expect to pay using distributor-specific tables). Of this gap, 4% stems from the fee credibility constraints (including fee anchors). In view of these observations, employing a single fee table for all distributors appears to be a reasonable and practical compensation strategy that is applicable to other third-party logistics partnerships. For such contexts the linear programming model provides a principled methodology to decide

acceptable fee values. More generally, the model applies to problem contexts in which random outcomes can be mapped onto a multidimensional grid, and the value associated with each cell in the grid must be chosen to optimize the total expected value over all stochastic outcomes.

## Future Directions

This work motivates future research in two broad directions: modeling and methodological development for optimization problems related to the fee table-based compensation approach, and broader examination of compensation schemes for third-party logistics providers. We next briefly discuss both these directions.

This paper has focused on supporting the periodic fee-setting decision for a given set of weight and distance ranges, a decision that occurs more frequently than adjusting the structure of the fee table. We can extend this modeling framework to encompass the longer-term decisions associated with defining the table structure (i.e., the specification of distance and weight ranges). However, this extension, which we refer to as *range selection*, increases the problem complexity significantly. Even for a table of fixed size, the problem of optimizing the width of each row and column becomes a large-scale mixed integer program. For example, suppose we require five distance ranges spanning 0–300 miles, and assume for simplicity that the weight ranges are prespecified. The model must decide how to partition the 300-mile range into five contiguous distance segments, and also, simultaneously determine the fee value for each pair of distance and weight ranges. The problem is complex because the distance range containing each store delivery is no longer a parameter, but depends on the choice of range limits. With far more decision variables and constraints than [FSLP], and with many variables restricted to be binary the range selection problem is difficult to solve optimally. Developing tailored solution techniques to find probably near-optimal solutions (e.g., using decomposition techniques) for this problem is a promising avenue for investigation. It might also be interesting to consider extensions to the model that consider tables of higher dimensionality.

Delivery planning with stochastic demands and scheduled deliveries presents another promising research opportunity. Because the costs for delivery resources (vehicles, personnel) are largely fixed, distributors must simultaneously decide resource capacities and their deployment (e.g., vehicle routing and delivery scheduling). Moreover, since demands are stochastic, they must take into account overflow options when the actual demand on a route exceeds the capacity of the truck assigned to that route. This complex stochastic optimization problem has interesting embedded subproblems (related the stochastic bin packing problem). Because this model explicitly considers demand variability, it can also serve as a means to refine the distributor cost estimates in the fee-setting model, particularly to study the impact of different levels of demand variability.

The generic issue of *designing compensation schemes* for third-party delivery service providers presents several interesting empirical and theoretical questions. Appropriate compensation designs will depend on the nature of manufacturer-distributor relationships and operational details of service provision. Armstrong chose to use a uniform fee table with a stepwise compensation function, but other contexts might require consideration of alternative approaches. For instance, even within the framework of using fee tables, we discussed distance and/or weight-sensitive compensation functions that provide different fees for different weights and distances within the same range. More generally, rather than assume a discrete set of weight and distance ranges, the fee for any given weight or distance might be specified via a single function, determined using curve fitting or calculus of variations techniques. For situations where the manufacturer helps decide (perhaps jointly with distributors) actual delivery routes, a route-based or resource-based compensation method might be feasible. Finally, instead of using a common scheme, one might consider negotiating distributor specific (either lump sum or activity-based) fees. In evaluating the suitability of these different approaches for a given context, we must consider several factors including their ease of use, credibility, incentive effects, and ability to match compensation with costs. Practitioners would benefit from

empirical studies or theoretical investigations (e.g., incentive compatibility issues) that provide insights on industry circumstances under which each approach is appropriate.

The compensation scheme used to pay distributors also has implications for *supply chain configuration and design*. For instance, when evaluating a particular configuration—choice of distributors, their warehouse locations, etc.—the chosen compensation scheme will determine the cost of that configuration. With the fee table-based approach (for a given set of fee values) the manufacturer's cost for distributing products to the last echelon of the chain (from warehouse to customers) is a simple additive function of the warehouse-to-customer distances and customer demands. On the other hand, for a route-based compensation scheme, the supply chain configuration model must simultaneously decide delivery routes.

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