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We focus on a logistics system where inventory is held at three levels: the customers, the retail store, and the warehouse. Retail customer segments are heterogeneous and differ in their reservation prices for product as well as their holding costs. They purchase product from a retail store managed by a retailer. The retailer chooses a retail pricing scheme to maximize his expected profit given a model of customer temporal response to retail pricing. This retailer is supplied product from a warehouse managed by a manufacturer.

The manufacturer is responsible for maintaining inventory level at the warehouse and providing 100% service level for retailer orders. The manufacturer uses all available information to generate an inventory policy that maximizes expected profit subject to the service-level requirement. We evaluate the manufacturer's optimal expected profit under two possible schemes: (1) no information regarding the timing of retail promotion plans, and (2) full information regarding the timing of retail promotion plans.

We show: (1) as the predictability of the sales impact of a promotion decreases, it may be optimal for the retailer to eliminate retail promotions; (2) increased stockpiling tendency of customers increases retailer profits and decreases manufacturer profits; and (3) retail-promotion information sharing can make retail promotions change from being less profitable than no promotions to being more profitable than no promotions for the manufacturer. We show the impact of fitting the model to a grocery store data set that provided data regarding retail sales (and associated prices) of canned tomato soup over two years. We also explore managerial insights suggested by the model.
(*Information Sharing; Retail Promotions; Customer Heterogeneity; Inventory Models*)

1. Introduction

Recently there has been considerable interest in the area of grocery supply chains (Saporito 1994). Various industry groups (FMI 1993) project substantial savings that can be generated by coordinating grocery supply chains. Many studies question the profitability of trade promotions (Buzzell et al. 1990) and suggest flat manufacturer prices (called EDLPP or Every Day Low Purchaser Price) as a profitable strategy. Other studies have examined flat retailer prices (called EDLP or Every Day Low Price at the store) as part of the solution. A few marketing studies have suggested that retail promotions might be a more profitable strategy (Hoch et al. 1994 and Bell et al. 1996) than flat retailer prices.

We consider a retailer who faces a promotion-sensitive retail customer environment. Retail customer segments are heterogeneous and differ in their reservation prices for product as well as their holding costs. We consider a system with *two* different customer

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segments. We model retail sales as a random variable whose level, conditioned on a customer-stockpiling model along with associated forecasted error, follows a lognormal distribution.[1]

Given this retailer sales model, the retailer chooses a promotion plan that maximizes expected profits over time. The retailer also chooses a corresponding inventory policy to support this promotion plan. Retail store sales each period then generate retail orders for the warehouse.

These retail orders are filled by a warehouse that is managed by a manufacturer. The manufacturer is responsible for maintaining the warehouse inventory level and choosing an inventory policy that maximizes its expected profit through time. The manufacturer's inventory policy must provide a 100% service level for retail orders. We evaluate the manufacturer expected profit under two possible schemes: (1) no information regarding the timing of retail promotion plans; and (2) full information regarding the timing of retail promotion plans.

We show: (1) as the predictability of the sales impact of promotions decreases, it may be optimal for the retailer to eliminate retail promotions; (2) increased stockpiling tendency of customers in response to promotions increases retailer profits and decreases manufacturer profits; and (3) retail-promotion information sharing may make retail promotions change from being less profitable than no promotions to being more profitable than no promotions for the manufacturer. Proofs for theorems and lemmas are provided in the appendix unless otherwise stated.

2. Retailer Model Description

The retailer sets a price each period. In the absence of promotions, the price is set at the regular price of p_h. The retailer has to choose a promotion price level p_l and declare that price with an associated frequency.

[1]Our model is motivated by a data set of sales of canned tomato soup from 60 stores over a two-year period examined by Iyer and Ye (1997). They used a statistical procedure to fit a multiple customer segment model to the data. Results show a high R^2 level, over 75% for over 80% of the stores. The study suggested that a multiple customer-stockpiling model provides a reasonable statistical model when fit to real data sets. Figure 1 shows an example data set, the fitted model, and associated R^2.

We first describe a model of retail customer segments and their purchase response to a retail price level. We then develop an optimal retailer inventory policy given a price. Finally, we develop an optimal retail-promotion plan that maximizes expected retailer profit over time.

2.1. The Retail Customer Model

We consider a retail environment populated by two customer segments, 1 and l. A description of the two segments follows:

1. In the absence of any forecast error, segment 1 purchases a quantity c_h every period. The regular retail price paid by segment 1 is p_h. The distribution of sales to segment 1 each period is unaffected by price decreases. Given forecast error, we model sales to this segment as $c_h\, e^{\epsilon}$, where ϵ follows a Normal distribution with mean 0 and variance σ^2. Thus, sales to segment 1 each period follow a lognormal distribution with mean $c_h e^{\sigma^2/2}$ and variance $c_h^2 e^{\sigma^2}(e^{\sigma^2} - 1)$.

2. Segment l purchases when prices in a period are below r_l (where $r_l < p_h$). If this segment consumes in a period, it consumes c_l. The holding cost per unit per unit time for segment l is h_l. In the absence of forecast error, if the price in a period is $p \leq r_l$, the quantity purchased by this segment is $((r_l - p)c_l/h_l)$. This purchase quantity maximizes the consumer's surplus (i.e., the difference between the reservation price r_l and the sum of purchase and holding costs) each period.

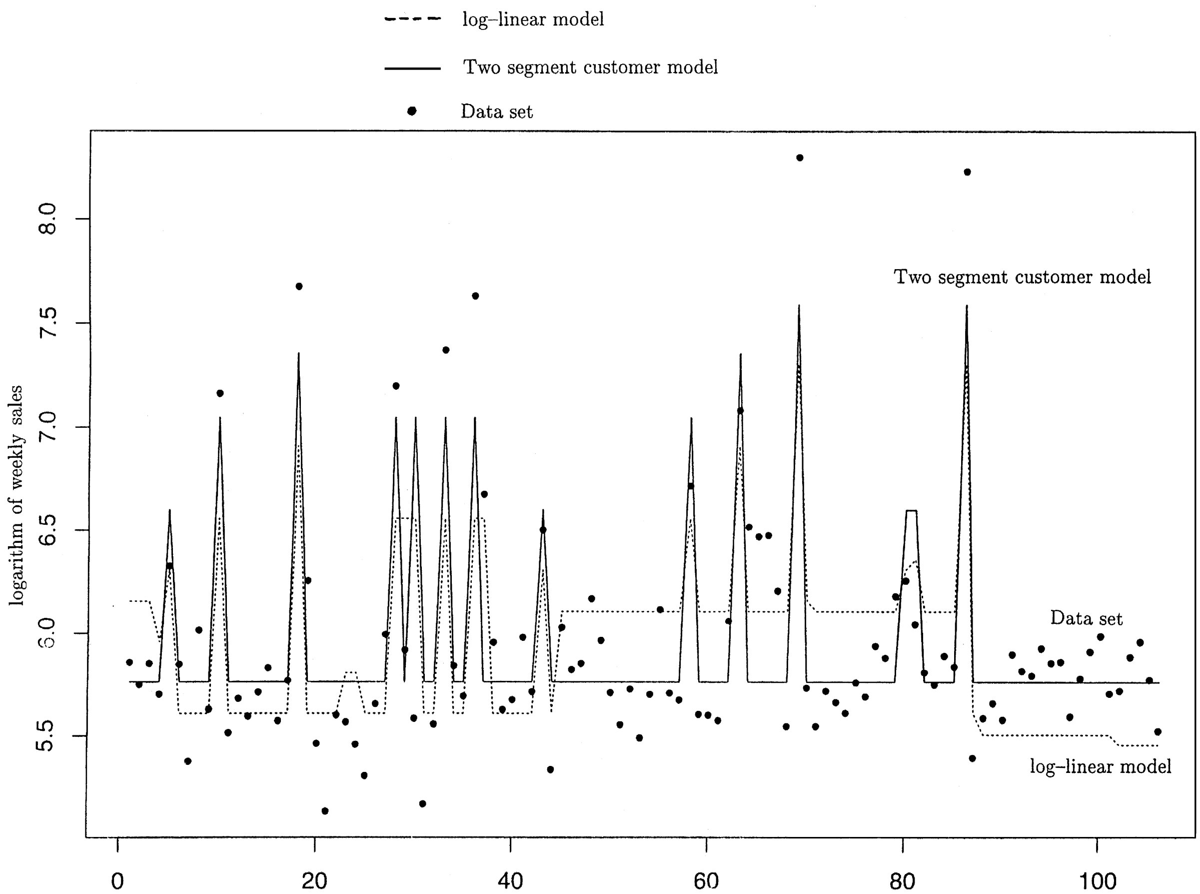
In the presence of forecast error, sales to segment l are modeled as a random variable $((r_l - p)c_l/h_l)\, e^{\epsilon}$ where ϵ follows a Normal distribution[2] with mean 0 and variance σ^2. Thus sales to the segment l customers during periods when the price $P_e \leq r_l$ follow a lognormal distribution with mean $((r_l - p)c_l/h_l)\, e^{\sigma^2/2}$ and variance $(((r_l - p)c_l/h_l))^2\, e^{\sigma^2}\, (e^{\sigma^2} - 1)$.

We will assume that this sales level will be observed if the promotion prices are offered at intervals of time following a negative-binomial distribution with parameters[3] 2 and $q(p)$. Thus a promotional price p $(\leq r_l)$ is offered by the retailer with a mean time between

[2]We discuss generalizations to this model in § 5

[3]This means that there are no two successive periods with promotions. The probability density function of the number of periods (x) between successive promotions is $f(x) = \binom{x-1}{1}\, q(p_l)^2 (1 - q(p_l))^{x-2}$ $x = 2, 3, \ldots.$

Figure 1 A data set showing the logarithm of weekly sales of canned tomato soup in a store over two years. Also shown is a fitted log-linear model as well as a model with parameters fitted to a two-segment model. The associated R^2 for the log-linear model was 52%, the R^2 for the fitted two-segment model was 68%.



promotions of $2/(q(p))$ and a variance of $(2(1 - q(p))/q(p)^2)$. We will model $q(p) = 2/((r_l - p)/h_l)$. The average consumption level each period for segment l customers implied by a price $p_l < r_l$ and frequency of purchases of $q(p_l)$ is c_l. Thus, once a decision is made to sell to the low holding-cost customers, p_l only affects the timing of purchases, not the long-run average consumption. The average time between successive promotions matches the average time for the low-reservation-price customers to consume the quantity purchased at p_l.

We will assume that the parameters satisfy the inequality $h_l \leq (c_h(p_h - r_l)/4c_l)$. We will see later that for the optimal value of p_l (see Lemma 1) $(r_l - p_l)/h_l \geq 2$.

The total sales in a period is thus the sum of the sales to these two segments. Our model is motivated by data regarding sales of canned tomato soup over a two-year period. We fit the model to the data and identified model parameters that maximized the likelihood function.[4] Figure 1 shows (1) the logarithm of sales over time in the original data set, (2) the logarithm of sales over time implied by the fitted parameters for the proposed two-segment model, and (3) the logarithm of sales over time implied by the fitted parameters for a

[4]Details such as the fitting procedure are presented in a paper by Iyer and Ye (1997). We thus omit details regarding parameter fitting in this paper.

log-linear model.[5] The R^2 of the fitted parameters for the two-segment model was 68%. This contrasts with an R^2 of 52% when a log-linear model was fitted to the data. Thus Figure 1 suggests that the two-segment model provides a reasonable statistical description of sales in a promotional retail environment. Also, the residual variance implied by the model is used as an estimate of the variance of the model fit with the underlying data (the parameter σ).

We provide a brief summary of related models in the literature. Blattberg et al. (1981) present a model that assumes equal reservation prices for both segments and no forecast error. Jeuland and Narasimhan (1985) present a model that has the same structure as that presented in this section but assumes no residual variance. Assuncao and Meyer (1993) present a model where the distribution of future prices is stochastic, consumer purchase quantity and consumption decisions are based on an expectation of future prices and the link between quantity consumed, and consumer utility is a known function. They focus on the impact of high consumer inventories on consumer consumption. Note that we incorporate the impact of model forecast error, a feature that allows us to quantify the impact of model predictability on buffer inventory levels. We also build detailed models of the cost implications of promotion information sharing between the manufacturer and the retailer. Many authors have identified other reasons for temporal price variation. For example, in Lazear (1986), pricing over time is used to identify reservation prices in the presence of ex ante uncertainty regarding customer reservation prices. Varian (1980) models temporal price variation as a mechanism to exploit the difference between informed customers (who are aware of prices charged by all stores and shop at the cheapest price store each period) and uninformed customers. Clearly, alternative models of the link between temporal prices and sales and its fit to data could be used to model the link between price variation and its impact on demand. We leave such explorations to future research.

2.2. Retailer Cost and Revenue Parameters

The retailer purchases product from the manufacturer at a wholesale price of w per unit. Holding costs at the retailer are h per unit per unit time. These holding costs are charged for leftover retail inventory at the store before any returns to the warehouse. The retailer sells the product at the price of p_h during nonpromotion periods and at a chosen price of p_l during promotion periods. Goodwill costs associated with lost retail sales (when demand during a period exceeds retail inventory level) are charged at π per unit short. The retailer has to choose a retail inventory level at the start of the period (before observing demand) and can return any unwanted product to the manufacturer at the end of the period. Holding costs are charged for returned goods before they are shipped back to the warehouse. In the model we do not include any other return charges. However, such charges can be included. Lead time for the manufacturer to fill a retail order is negligible at the start of the period (the only time when deliveries are made by the manufacturer).[6]

2.3. The Retailer Model

The retailer's objective is to choose a promotion scheme to maximize its expected profits over time. We will first derive the retailer expected profit for a given value of p_l. This will enable us to generate the optimal value of retailer promotion price p_l^*.

Lemma 1 shows the optimal retailer inventory level (a Newsboy model) and associated expected retailer profit for a given declared price p in a period.

LEMMA 1. *The optimal retailer expected profit for each period t with a retail price of p, given a demand of $d_t(p)e^{\epsilon}$ where $\epsilon \sim N(0, \sigma^2)$, is attained at an inventory level of $e^{\sigma Z(p)}d_t(p)$, where $\Phi(Z(p)) = (p + \pi - w)/(p + \pi + h - w)$ and $\Phi(\cdot)$ is the cumulative density function of a standard normal random variable with mean 0 and variance 1 and $d_t(p) = (c_h + ((r_l - p)c_l/h_l))$ if $p \le r_l$ and c_h otherwise. The associated expected profit is*

$$G(p) = d_t(p)\{(p - c)V_1 - hV_2 - \pi V_3\},$$

where $V_1 = e^{\sigma^2/2}\Phi(Z(p) - \sigma) + e^{Z(p)\sigma}(1 - \Phi(Z(p)))$, $V_2 = e^{Z(p)\sigma}\Phi(Z(p)) - e^{\sigma^2/2}\Phi(Z(p) - \sigma)$, $V_3 = e^{\sigma^2/2}(1 - \Phi(Z(p) - \sigma)) - e^{Z(p)\sigma}(1 - \Phi(Z(p)))$.

Thus, during regular nonpromotion periods when the retail price is p_h, the retail inventory level is

[5] In the log-linear model we express sales as the $Ae^{-(b_1p+b_2)}$ where A, b_1, and b_2 are fitted to the data.

[6] We discuss extensions in § 5.

$I(p_h) = e^{Z(p_h)\sigma} c_h$. In the absence of any promotions, the retailer expected profit per unit time is $G(p_h)$. During promotion periods with a price of p_l, the associated retail inventory level is $I(p_l) = e^{Z(p_l)\sigma} (c_h + ((r_l - p_l)c_l/h_l))$. Note that given this expression, the inventory level is just the sum of the optimal inventory levels if each segment were considered in isolation. This is a property of the lognormal distribution. The associated expected profit is $G(p_l)$. Note that the promotion price of p_l is declared at the retail store with the average interval between promotions of $E(q(p_l)) = 2/(q(p_l))$. This implies that the average sales per period to segment l is c_l in the absence of forecast error. Thus, the expected retailer profit per unit time is (following the renewal reward theorem) as follows:

$$E_{ret}(p_l) = G(p_h) + \frac{G(p_l) - G(p_h)}{E(q(p_l))}.$$

Lemma 2 provides the optimal promotion price p_l^*.

Lemma 2. *If we set* $Z(p_l) = Z(p_h)$,[7] *the optimal price level* $p_l^* = r_l - \sqrt{(c_h h_l (p_h - r_l))/c_l}$ *if* $G(p_l^*) \geq G(p_h)$. *Otherwise, if* $G(p_l^*) < G(p_h)$, *it is optimal to have no retail promotions and set retail price each period to* p_h.

Note that $E_{ret}(r_l - p_l) = G(r_l - h_l) \leq E_{ret}(p_l^*)$. Thus, we need not explicitly consider the case where we permit everyday low prices to the low reservation price segment. Also, given the relation between the parameters in § 2.1, i.e., $h_l \leq (c_h(p_h - r_l))/4c_l$, thus $p_l^* \leq r_l - 2h_l$, $q(p_l^*) \leq 1$, and $(r_l - p_l^*)/h_l \geq 2$. We will refer to the optimal expected retailer profit given that the retail promotion price is set at p_l^* as E_{ret}^*, i.e.,

$$E_{ret}^* = E_{ret}(p_l^*) = G(p_h) + \frac{G(p_l^*) - G(p_h)}{E(q(p_l^*))}.$$

2.4. Retailer Model Results

Our objective in this section is to identify the effect of changes in σ and h_l on the optimal retailer expected profit and the optimal retail promotion price p_l^* as well as the quantity sold under promotion and the frequency of promotions.

Lemma 3 summarizes the effect of σ and h_l on the retailer's optimal expected profit E_{ret}^*.

Lemma 3.

(3a) $$\frac{dE_{ret}^*}{dh_l} = -c_h V_1 - \frac{V_1 \sqrt{c_h c_l (p_h - r_l)}}{\sqrt{h_l}}.$$

(3b) *There exists a value of* $h^* = f(p, \pi, w)$ *such that for all* $h \geq h^*$, $(dG(p)/d\sigma) \leq 0$.

(3c) *If* $h \geq h^* = f(p_h, \pi, w)$, $(dE_{ret}^*/d\sigma) \leq 0$.

Note that we take derivatives and simplify for Lemmas 3a and 3b.

Lemma 4 shows the impact of h_l on the retailer optimal promotion price p_l^*.

Lemma 4. $$\frac{dp_l^*}{dh_l} = -\sqrt{\frac{c_h(p_h - r_l)}{4c_l h_l}}.$$

The proof is obtained by taking the derivative of p_l^* with respect to h_l and simplifying.

Lemma 5 shows the impact of h_l and r_l on the quantity sold during promotions at the retailer optimal promotion price level p_l^*. The expected quantity sold during promotions is $Q^* = V_1 \{c_h + (r_l - p_l^*)c_l/h_l\}$, where V_1 is defined in Lemma 1. This quantity sold during promotion periods is affected by the average time between promotions (which is equal to $(r_l - p_l^*)/h_l$).

Lemma 5. $$\frac{dQ^*}{dh_l} = -V_1 \frac{\sqrt{c_h c_l (p_h - r_l)}}{2h_l^{3/2}}.$$

The proof is obtained by taking the derivative of Q^* with respect to h_l and simplifying.

2.5. Model Implications

The model suggests that, holding all other parameters constant, increasing h_l (i.e., decreasing the stockpiling tendency of segment l) decreases retailer profits. This is indicated by the fact that dE_{ret}^*/dh_l is negative in Lemma 3a. We interpret this effect by noting that Lemma 4 shows dp_l^*/dh_l is negative, and Lemma 5 shows that the expected quantity sold during promotions (Q^*) also decreases as h_l increases (i.e., dQ^*/dh_l is negative).

We also note that if retailer holding cost is larger than the threshold value h^* in Lemmas 3b and 3c, then

[7] This implies that we maintain the service level during promotion and nonpromotion periods. The retailer's profit-maximizing decisions are single period Newsboy models for both promotion and nonpromotion periods. Note that it is optimal to *decrease* the service level during promotion periods because of the lower retail margin during promotion periods.

retailer optimal expected profits decrease as the model predictability decreases; i.e., as σ increases (i.e., $dE^*_{ret}/d\sigma$ is negative). Note also that a higher σ results in an increase in retailer safety stocks to compensate for the higher demand uncertainty. The holding cost of this increased safety stock decreases retailer expected profits. Also, for a high enough value of σ, it may be optimal to eliminate all promotions and maintain the retail price at p_h for all periods because $G(p_h)$ exceeds $G(p_l)$. This also suggests that if retail holding costs are significant, then at high levels of sales unpredictability, the retailer might maximize profit by eliminating retail promotions.

3. The Manufacturer Model

We now explore the impact on the manufacturer. Orders for the warehouse are generated by the retailer—their size and timing depend on the retailer's promotion plans. The manufacturer is responsible for managing the warehouse inventory. The manufacturer chooses the warehouse inventory policy to maximize its expected profit over time. Our analysis assumes that the manufacturer is obligated to provide a 100% service level to the retailer.[8] We first consider the impact of the retailer's orders on the manufacturer expected profits under two schemes.

1. *No promotion-timing information sharing:* Under this scheme, the manufacturer knows the promotion price, the regular price, and their associated maximum inventory levels. The manufacturer is *not* informed of the specific periods when the retailer plans to promote.

2. *Full promotion information sharing:* Under this scheme, the retailer informs the manufacturer in advance regarding the specific periods when retail promotions will occur.

3.1. Manufacturer Cost and Revenue Parameters

The manufacturer incurs a cost of c per unit to make the product and deliver it to the warehouse. Wholesale price paid by the retailer to the manufacturer is w per unit. The retailer is offered full flexibility to return unsold product to the warehouse.[9]

Manufacturer warehouse holding costs are h_m per unit per period. These holding costs are charged at the end of the period. All deliveries are received at the start of the period. Also, the manufacturer orders to the plant to replenish the warehouse face a lead time of L. We also assume that the manufacturer has the ability to cancel part of any orders that are in the pipeline but not received at the warehouse at no cost.[10]

3.2. No Promotion-Timing Information-Sharing System

By assumption, the manufacturer knows the retailer's order-up-to levels during promotion and nonpromotion periods. However, in the absence of information regarding the precise timing of retail promotions, the inventory position at the warehouse must be maintained at $LI(p_l)$ to support a 100% retail service level.

We will first derive the expected outstanding warehouse order size. This will enable us to derive the average on-hand physical inventory and the associated expected profit for the manufacturer.

LEMMA 6. *The expected warehouse order size under a no promotion-timing information-sharing system is*

$$E[Order] = E(d_h) + \frac{E(d_l) - E(d_h)}{E(q(p_l))},$$

where

$$E(d_h) = V_1 c_h \text{ and } E(d_l) = V_1\left(c_h + \frac{(r_l - p_l)c_l}{h_l}\right),$$

$$V_1 = e^{\sigma^2/2}\Phi(Z - \sigma) + e^{Z\sigma}(1 - \Phi(Z)).$$

THEOREM 1. *The manufacturer's maximum expected profit under a no promotion-timing information-sharing system (when $Z_{p_l} \sim Z_{p_h}$) is*

$$((w - c) * E[Sales]) - h_m L(I(p_l) - E[Order])),$$

where

$$E[Sales] = E[Order] = E(d_h) + \frac{E(d_l) - E(d_h)}{E(q(p_l)},$$

$$E(d_h) = V_1 c_h,$$

and

[8] We discuss the impact of this assumption in § 5.

[9] Note that the retailer returns product to the warehouse only if demand during a promotion period is $< I(p_l) - I(p_h)$. Also the returns are combined with deliveries and are charged holding costs at the end of the period. We discuss extensions in § 5.

[10] The expressions developed permit us to include costs for order cancellations.

$$E(d_l) = V_1 \left(c_h + \frac{(r_l - p_l)c_l}{h_l}\right),$$

$$I(p_l) = \left(c_h + \frac{(r_l - p_l)c_l}{h_l}\right) e^{Z\sigma},$$

$$V_1 = e^{\sigma^2/2}\Phi(Z - \sigma) + e^{Z\sigma}(1 - \Phi(Z)),$$

$$\Phi(Z) = \frac{p_h + \pi - w}{p_h + \pi + h - w}.$$

We will refer to the optimal expected manufacturer profit under no promotion-timing information-sharing (with the retailer promotion price p_l^* and its associated optimal frequency) as $E_{mfr}^{no\text{-}info}$.

Lemma 7 summarizes the effect of σ and h_l on the manufacturer's optimal expected profit $E_{mfr}^{no\text{-}info}$.

LEMMA 7.

(7A) $$\frac{dE_{mfr}^{no\text{-}info}}{dh_l} = \frac{h_m L e^{Z\sigma} \sqrt{c_h c_l}(p_h - r_l)}{2h_l^{3/2}}.$$

(7B) *There exists an* $h_m^* = g(c_h, c_l, r_l, c, w, p_h, h_l, L, Z)$ *(with* Z *defined as* $\Phi(Z) = (p_h + \pi - w)/(p_h + \pi + h - w)$*) such that if* $h_m \geq h_m^*$*, then* $dE_{mfr}^{no\text{-}info}/d\sigma \leq 0$.

3.3. Model Implications

Lemma 7A shows that as the holding cost of the promotion-sensitive customers (h_l) increases, the manufacturer's optimal expected profit increases (i.e., $dE_{mfr}^{no\text{-}info}/dh_l$ is positive). We observed earlier (from Lemma 3A) that as h_l increases, the retail expected quantity sold during promotions decreases and retail promotion frequency increases. Also the associated manufacturer inventory level required to support promotions ($I(p_l^*)$) decreases as h_l increases. The manufacturer thus faces lower expected holding costs to support the retailer, thereby increasing manufacturer profit. Note that this effect of h_l on the manufacturer is the reverse of the effect of h_l on the retailer.

Lemma 7B shows that for a sufficiently large manufacturer holding cost h_m, the manufacturer's optimal expected profit decreases with σ (i.e., $dE_{mfr}^{no\text{-}info}/d\sigma$ is negative for $h_m \geq h_m^*$). As σ increases, safety stocks required to support the (less predictable) demand increase, i.e., manufacturer inventory level required to support promotions ($I(p_l^*)$) increases as does $I(p_h)$. Thus, if the manufacturer holding costs are significant, this increased inventory level will reduce manufacturer profits.

3.4. A Manufacturer Model with Full Promotion Information Sharing

We now focus on a scheme where the retailer provides the manufacturer with advance knowledge (i.e., more than L periods in advance) of the specific periods when promotion will occur at the retail level. Such information enables the manufacturer to adjust orders (L periods in advance) to enable a 100% service level to be offered. We will focus on the effect of this information on the manufacturer's expected profit. We will examine a manufacturer inventory policy that we term a *modified policy*.

Under the *modified policy*, the manufacturer inventory position is maintained at a minimum of $(I(p_l) - I(p_h)) + LI(p_h)$ for all periods. If there are $k \geq 1$ promotions scheduled during the next L periods, inventory position is $k(I(p_l) - I(p_h)) + LI(p_h)$. This modified policy guarantees that the inventory position (including retailer returns) at the warehouse is always below the base-stock level. Thus this policy: (1) provides tractability of the model, (2) permits us to generate a lower bound on the effect of information sharing, and (3) permits the retailer the ability to add an unplanned promotion within L periods. The main role of the promotion information sharing is to synchronize manufacturer inventory with retailer promotion plans. This synchronization of manufacturer inventory policy with retail promotion plans saves holding costs for the manufacturer and thus improves manufacturer profits for a given level of retail promotions.

LEMMA 8. *The expected inventory position across time under a full promotion information-sharing system is*

$$E[InvPosition] = LI(p_h) + (I(p_l) - I(p_h)) \left\{\frac{Lq(p_l)}{2} + b(0, L, q(p_l)) + 0.5 * b(1, L, q(p_l))\right\},$$

where $b(i, j, q)$ *refers to the binomial probability that there are exactly* i *successes in* j *trials with a probability of success of a trial of* q*. In this expression,* $I(p_l) = (c_h + ((r_l - p_l)c_l/h_l))e^{Z\sigma}$, $I(p_h) = e^{Z\sigma}c_h$, $\Phi(Z) = (p_h + \pi - w)/(p_h + \pi + h - w)$

LEMMA 9. *The optimal expected order size under a full promotion information-sharing system is*

$$E[Order] = E(d_h) + \frac{E(d_l) - E(d_h)}{E(q(p_l))},$$

where

$$E(d_h) = V_1 c_h \text{ and } E(d_l) = V_1 \left(c_h + \frac{(r_l - p_l)c_l}{h_l}\right),$$

$$V_1 = e^{\sigma^2/2}\Phi(Z - \sigma) + e^{Z\sigma}(1 - \Phi(Z)).$$

Note that the expected order size under full promotion information-sharing is the same as the expected order size under no promotion-timing information sharing (i.e., Lemma 9 and Lemma 6 generate the same expected order size). This is because the expected order size reflects the expected retail sales, which are the same for the two cases. However, under full promotion information sharing, the timing of manufacturer orders (to satisfy orders placed by the retailer to cover demand during retail promotion periods) can be adjusted so that their deliveries at the warehouse synchronize with shipments to fill retail orders. Under no promotion-timing information sharing, these manufacturer orders (to cover retail promotion-related demand) would be placed as replenishment orders immediately following a retail promotion. Thus the timing of order deliveries at the warehouse and hence the higher moments of the manufacturer orders differ across the two cases.

THEOREM 2. *The expected manufacturer profit under a modified policy under full promotion information sharing with the retailer is*

$$(w - c)E[Sales] - h_m(E[InvPosition] - (L \times E[Order])),$$

where

$$E[Sales] = E[Order] = E(d_h) + \frac{E(d_l) - E(d_h)}{E(q(p_l))},$$

$$E(d_h) = V_1 c_h,\ E(d_l) = V_1 \left(c_h + \frac{(r_l - p_l)c_l}{h_l}\right),$$

$$V_1 = e^{\sigma^2/2}\Phi(Z - \sigma) + e^{Z\sigma}(1 - \Phi(Z)),$$

and

$$E[InvPosition]$$

is defined in Lemma 8.

We now show the impact of changes in h_l and σ on the manufacturer's optimal expected profit using the modified policy. We denote the manufacturer's optimal expected profit under full information sharing using the modified policy as E^{info}_{mfr}.

LEMMA 10.

$$\text{(10A)}\ \frac{dE^{info}_{mfr}}{dh_l} = -h_m \frac{dE[Pipeline]}{dq}\frac{dq}{dh_l} = h_m c_l e^{Z\sigma} \left\{\frac{2L(1 - q)^{L-1}}{q} + \frac{2(1 - q)^L}{q^2} + L(L - 1)(1 - q)^{L-2}\right\} \sqrt{\frac{c_l}{c_h(p_h - r_l)}}\frac{1}{\sqrt{h_l}},$$

where

$$q = 2\sqrt{\frac{c_l h_l}{c_h(p_h - r_l)}}.$$

(10B) *There exists a value of $h^*_m = m(c_h, c_l, r_l, c, w, p_h, h_l, L, Z)$ (with Z defined as $\Phi(Z) = (p_h + \pi - w)/(p_h + \pi + h - w)$) such that for $h_m \geq h^*_m$, $dE^{info}_{mfr}/d\sigma \leq 0$.*

$$\text{(10C)}\ \frac{dE^{info}_{mfr}}{dh_l} - \frac{dE^{no\text{-}info}_{mfr}}{dh_l} = K_{14}\frac{1}{h_l^{3/2}},$$

where

$$K_{14} = \frac{h_m e^{Z\sigma}\sqrt{c_h c_l(p_h - r_l)}}{2}\{(1 - q)^L + Lq(1 - q)^{L-1} + \frac{L(L - 1)}{2}(1 - q)^{L-2}q^2 - L\}$$

and

$$q = 2\sqrt{\frac{c_l h_l}{c_h(p_h - r_l)}}.$$

3.5. Model Implications

Lemma 10A suggests that as h_l increases, the manufacturer's optimal expected profit increases (i.e., dE^{info}_{mfr}/dh_l is positive). This is a consequence of the decrease in the quantity sold during retail promotions and the increase in the promotion frequency with increasing h_l. Thus the manufacturer inventory levels decrease, thereby increasing manufacturer profits. Note again that the effect of increasing h_l on manufacturer profits is the reverse of the effect on the retailer.

As σ increases, retail safety stocks increase, thus increasing manufacturer inventory levels and associated holding costs, i.e., $dE_{mfr}^{info}/d\sigma$ is negative for $h_m \geq h_m^*$. The increased manufacturer holding costs decrease manufacturer expected profits.

The benefit of information sharing decreases as h_l increases. This is shown by the fact that (from Lemma 10c) $(dE_{mfr}^{info}/dh_l) - (dE_{mfr}^{no\text{-}info}/dh_l) \leq 0$. As h_l increases, manufacturer expected profits increase as manufacturer inventory levels decrease. Since the benefit of information sharing is to decrease the manufacturer inventory costs, we should expect this benefit to decrease as h_l increases.

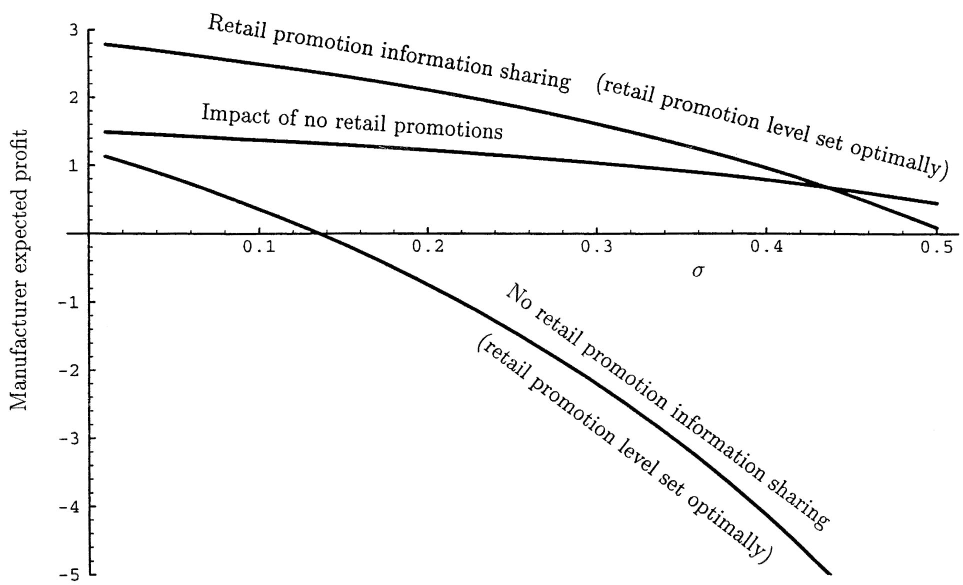
Figure 2 shows the manufacturer's optimal expected profit under full information sharing and under no promotion-timing information sharing. The figure also shows the manufacturer's optimal expected profits if there were no retail promotions. Note that full information sharing makes the manufacturer profits higher when there are retail promotions than when there are no retail promotions for a range of σ values. This is in contrast to the situation when there is no promotion-timing information sharing, where the manufacturer prefers a no promotion retail system than one where there are retail promotions. Thus, information sharing can make retail promotions change from being less profitable than no promotions to being more profitable than no promotions for the manufacturer.

4. Managerial Implications

In this section, we will discuss some managerial implications of the model and its analysis. We will thus link model implications to observed products and demand characteristics in grocery stores.

It has been suggested (Neslin et al. 1995) that if one compares analgesics to toilet paper one should expect a lower h_l for analgesics. Thus our model would suggest that grocery-store promotion prices for analgesics should be higher than for toilet paper. However, one would also expect the quantity sold during promotions

Figure 2 **Manufacturer expected profit under no promotion information sharing and promotion information sharing vs. σ. Results for the optimally chosen retailer promotion p_l^* for $h_l = 0.005$. The figure also shows the manufacturer profits for the no retail promotion case. Note that for the range of parameters (shown below) the manufacturer prefers the no retail promotion case to the case with retailer promotions when there is no promotion information sharing. Information sharing causes the manufacturer to prefer retail promotions to no retail promotions.**



Notes. $p_h = 0.5$, $c_h = 10$, $r_l = 0.35$, $c_l = 10$, $\pi = 5$, $c = 0.25$, $h = 0.01$, $w = 0.1$, $h_m = 0.01$, $L = 4$.

for analgesics to be higher than that for toilet paper, and thus one would expect promotions for analgesics to be less frequent. Consequently, the manufacturer's benefit from full promotion information sharing should be expected to be greater for analgesics than for toilet paper.

We hypothesize that manufacturer profit under full promotion information sharing would increase as the customer stockpiling tendency increases (i.e., h_l decreases) and as the unpredictability (σ) of the retail environment increases. This suggests that manufacturers of analgesics would be more interested in retail alliances than toilet paper manufacturers because they would see a greater benefit from such information sharing. Similarly, retail stores in more competitive environments (with high σ) would see manufacturers with a greater interest in information sharing than retail stores in less competitive and more predictable environments.

Studies (Lee 1994) have shown that in most retail environments, EDLP (Every Day Low Price) merely refers to a lower frequency of promotions rather than no retail promotions. Our model suggests a number of situations that might result in EDLP retail environments. In an environment with a high level of customer stockpiling (i.e., low h_l), the optimal frequency of promotions would be low, and thus such situations would have price levels in a majority of the periods close to p_h. Our model also suggests that when the sales impact of retail promotions is more difficult to predict, the profitability of retail promotions decreases (i.e., σ increases), thus making EDLP at the retailer a more profitable alternative than maintaining retail promotions. Thus both low h_l, i.e., high promotion sensitivity by the promotion-sensitive customers, or high σ, i.e., low predictive power of the impact of promotions, would suggest that the retailer adopt an EDLP environment.

5. Model Assumptions and Their Impact on Conclusions

In this section we address two issues: (1) the impact of changes in model assumptions on model structure, and (2) the potential impact of changing model assumptions on the conclusions generated by the model.

1. Impact of Alternate Choice of Uncertainty Distributions. An extension of the model presented in this paper is a multiplicative model of the type $g(p)f(\epsilon)$ (see Zabel 1970) where $f(\epsilon)$ refers to the probability density function of demand uncertainty ϵ and $g(p)$ is a deterministic relation between price p and demand quantity $g(p)$. Given this model of demand, for a given price p, the optimal (expected profit maximizing) inventory is $g(p)F^{-1}$ (ser) where *ser* is the optimal service level during that period,[11] and $F(\bullet)$ is the cumulative probability function of the random-demand component. The associated expected profit is $g(p)EP(f(\epsilon))$, where $EP(f(\epsilon))$ refers to the expected profit involving only the marginal cost and revenue parameters and parameters of the demand uncertainty distribution (similar to Lemma 1).

In our analysis, $g(p)$ is described in § 2.1, and the demand uncertainty $f(\epsilon)$ is described by a lognormal distribution. If we choose distributions other than a normal or lognormal distribution to model demand uncertainty $f(\epsilon)$, it will have the following impact on model structure: (1) the fitting of model parameters should use the maximum likelihood function—this corresponds to minimizing squared error for the lognormal distribution but not for other distributions, and (2) the identification of the optimal inventory and expected profit for a given choice of parameters is less explicit for nonnormal distributions.

Note that if $g(p)$ is maintained as in § 2.1 and $f(\epsilon)$ is changed, we still expect conclusions with respect to the impact of h_l to be maintained. The magnitude of the impact of σ on expected costs will depend on the specific distribution chosen.

2. Impact of Returnable Inventory. Our model assumes that after a retail price promotion with low demand (i.e., if $d_l < I(p_l) - I(p_h)$), the retailer pays holding costs, returns $I(p_l) - I(p_h) - d_l$, and thus starts with retail inventory at the optimal (nonpromotion on-hand inventory $I(p_h)$) inventory the next period. This model was influenced by practice for the chain we observed, in which retail promotions were very rarely continued if the sales response was poor.

If the retailer could not return inventory, then the retailer expected profit following a promotion would

[11] As defined in Lemma 1.

be dependent on the realized customer sales response as well as a model of the impact of such a response on future period demands. This would require the inventory and pricing decisions to be dependent on the sample path of observed demands. Explicitly modeling the impact of no return policy would increase model complexity and detract from the observed system reality. We thus leave such extensions for future research.

We expect, however, that changing these model assumptions will not change the model conclusions. This conjecture is suggested by the following model feature. Suppose the model had no demand uncertainty (i.e., $\sigma = 0$). Note that we would still get the same results regarding the optimal value of p_l, the impact of h_l on expected profits, and the impact of information sharing. Also the retailer would never have any leftover inventory to return to the manufacturer. We therefore conjecture that the basic results would remain unchanged.

3. Impact of Manufacturer Order Cancellation. Our model assumes that when the retailer returns inventory to the manufacturer after a retail promotion that did not generate sufficient sales (i.e., if $d_l < I(p_l) - I(p_h)$), then the manufacturer adjusts orders already placed (partial cancellation) so as to maintain the optimal inventory level at the warehouse. This permits the manufacturer to avoid committing inventory at the warehouse that is not required. In the absence of this ability, the manufacturer inventory levels following a promotion would have to be adjusted to be the nonoptimal level of base stock. Such a model feature creates base-stock levels that are sample path dependent. We leave such extensions to future work.

As far as the results are concerned, note that if σ were 0, the manufacturer would never receive retailer returns and thus never have to adjust orders placed. Note that we would still get the same results regarding the optimal value of p_l, the impact of h_l on expected profits, and the impact of information sharing. We therefore conjecture that the basic results would remain unchanged.

4. Markovian Retailer Promotion Periods. We set the promotion periods to be Markovian in the model with an interval between promotions having an expected value equal to the average time for the low reservation price segments to consume their purchases. Thus the expected time between promotions matches the expected time for the segment to complete consumption, thereby making the system consistent regarding expectations.

A more elaborate model wherein the customers build in expectations regarding promotions based on the observed sample path of promotions could be set up—we leave such variants as extensions of the model. We remark that adding such detail represents a tradeoff between model complexity and its impact on the residual error σ. We expect a model that maintains the customer stockpiling tendencies as part of the model structure to generate conclusions (regarding the impact of h_l, σ, etc.) similar to the current model.

5. Impact of Lead Time for Delivery from the Warehouse. We assume that orders from the warehouse are shipped to retail stores instantaneously. For the retail store we used as an example, the warehouse was located in the same city, and orders were shipped to stores immediately on receipt at the warehouse. Thus orders were delivered fairly quickly to the store as compared to the manufacturer lead time to get product from the plant to the warehouse.

Clearly, adding a nonzero lead time (say l) for warehouse delivery to the retail store would increase retail safety stock and increase the retail-store inventory position. We will have to modify the model and permit the retailer to have information regarding promotions l periods in advance so as to schedule deliveries.

We do not expect the results implied by the model (regarding the impact of h_l and σ) to change because the longer retail lead time merely increases retailer safety stocks.

6. Impact of the 100% Manufacturer Service-Level Constraint. Given that the order size to the manufacturer during a period is no more than the retailer base stock during a period, a 100% service level can be provided by the manufacturer at a finite cost. Also, this manufacturer service level permits the retailer to place orders knowing that deliveries at time t would match orders placed at the end of $t - 1$. In the absence of a 100% manufacturer service level, a rational retailer ordering policy would take into account uncertainties in the delivered quantity. This would affect the orders received by the manufacturer. Such a system would include a game theoretic component to choose manufacturer and retailer equilibrium inventory policies.

To maintain focus on the basic idea, we constrain the manufacturer service level to be 100%. We note, however, that if σ were equal to 0, it is optimal to provide a 100% service level, and the results from this article would hold. We conjecture that the results in this article will continue to hold if we include an equilibrium model of manufacturer and retailer inventory choices. We leave explorations of the impact of deviations from the 100% service level to future research.

7. Impact of Multiple Retail Stores. Our model focused on one retail store location. We discuss two possible situations that would affect the modeling of multiple stores: (1) all retail stores offer identical retail prices to the customer, and (2) all retail stores set retail prices independently.

We first discuss the case where all stores offer identical retail prices. Such a scheme is applicable in grocery chains that use zone pricing whereby all stores in a zone offer identical retail prices to customers. Under such a scheme, the regular price p_h, the promotion price p_l^*, and its associated timing would be identical for all stores. However, the store customer characteristics, i.e., the parameters r_l, h_l, c_l, c_h, σ, could vary by store location. If the entire chain (total across all retail stores) orders and sales were treated as one large store, the results of our current model apply. The retailer profits, now measured across all locations ($i = 1, 2, \ldots, N_s$, where N_s refers to the number of stores), would be $\Sigma_{i=1}^{N_s} E^*_{ret,i}(p_l)$ (where $E^*_{ret,i}(p_l)$ refers to the expected profit in store location i associated with a promotion price of p_l). The retailer chooses p_l^* to maximize the total expected retail profit across all stores. The manufacturer faces the combined orders across all stores. The increase in the number of stores would generate some risk-pooling benefits and thus decrease manufacturer safety stock under full promotion information sharing. However, the rest of the analysis follows as in the current model.

If retail prices are set independently in each retail location, then the demand faced by the manufacturer would depend on store inventory levels and store promotion plans. The impact of the manufacturer constraint to provide 100% service level suggests that the inventory position under no promotion-timing information sharing is just the sum of the order-up-to levels at promotion periods adjusted for manufacturer lead time. The impact of full promotion information sharing requires the modeling of the timing of individual store retail promotions. We leave the computation of the exact optimal manufacturer expected cost and other details for future research.

6. Conclusions and Discussion

We examined the impact of customer heterogeneity on manufacturer and retailer expected profits in a two-level promotional retail environment. We showed the impact of the extent of predictive power of promotions on the retailer and manufacturer expected profits and retail promotion levels. Finally, we have examined the impact of customer stockpiling characteristics on manufacturer and retailer expected profits.

Our results show the value of information sharing between retailer and manufacturer in a promotional retail environment. Information sharing mitigates some of the costs imposed on the manufacturer by the high inventories required to support promotions. Inclusion of the model forecast error enables us to include the inventory costs associated with the extent of model fit to realized sales in the expected profits for the retailer and manufacturer. Our results show that as the customer stockpiling tendency for a given promotion level increases, the retailer profits increase while manufacturer profits decrease. As the forecast error associated with retail sales estimates increase, promotion information sharing may make retail promotions move from being less profitable than no retail promotions to being more profitable (than no retail promotions) for the manufacturer.

The models suggest managerial implications for grocery logistics decision making. The models suggest testable hypotheses that deserve further empirical analysis. An important next step would be to evaluate the impact of statistical fit of the proposed models to empirical datasets across categories.

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Appendix

Note. Throughout this Appendix, we will use $\Phi(\bullet)$ and $\phi(\bullet)$ to refer to the cumulative density function and the probability density function of the standard normal distribution.

LEMMA 1. *The optimal retailer expected profit for each period t with a retail price of p, given a demand of $d_t(p)e^{\epsilon}$ where $\epsilon \sim N(0, \sigma^2)$, is attained at an inventory level of $e^{Z(p)\sigma}d_t(p)$, where $\Phi(Z(p)) = (p + \pi - w)/(p + \pi + h - w)$, $\Phi(\bullet)$ is the cumulative density function of a standard normal random variable with mean 0 and variance 1 and $d_t(p) = (c_h + ((r_l - p_l)c_l/h_l))$ if $p \le r_l$ and c_h otherwise. The associated expected profit is*

$$G(p) = d_t(p)\{(p - c)V_1 - hV_2 - \pi V_3\},$$

where

$$V_1 = e^{\sigma^2/2}\Phi(Z(p) - \sigma)) + e^{Z(p)\sigma}(1 - \Phi(Z(p))),$$

$$V_2 = e^{Z(p)\sigma}\Phi(Z(p)) - e^{\sigma^2/2}\Phi(Z(p) - \sigma),$$

$$V_3 = e^{\sigma^2/2}(1 - \Phi(Z(p) - \sigma)) - e^{Z(p)\sigma}(1 - \Phi(Z(p))).$$

PROOF. Following the Newsboy model, the expected profit associated with an inventory level of Q is $(p - c)V_1 - hV_2 - \pi V_3$, where $V_1 = \int_0^Q \xi f(\xi)d\xi + \int_Q^\infty Qf(\xi)d\xi$, $V_2 = \int_0^Q (Q - \xi)f(\xi)d\xi$, and $V_3 = \int_Q^\infty (\xi - Q)f(\xi)d\xi$. In these integrals, $f(\xi)$ refers to the probability density function of a lognormal distribution. Setting $Q = e^{Z(p)\sigma}d_t(p)$ for some Z and differentiating with respect to Z, we get the optimal value of Q as $Q = d_t(p)e^{Z^*\sigma}$, where Z^* is defined as $\Phi(Z^*) = (p + \pi - w)/(p + \pi + h - w)$. Substituting this value for Q and simplifying, we get the result. □

LEMMA 2. *Under the condition that we set $Z(p_l) = Z(p_h)$, the optimal price level $p_l^* = r_l - \sqrt{(c_h h_l(p_h - r_l))/c_l}$ if $G(p_l^*) \ge G(p_h)$. Otherwise it is optimal to have no promotions and set the price each period to p_h.*

PROOF. Given the condition that $Z(p_l) = Z(p_h)$, the average retailer profit per unit time (that is affected by p_l) is obtained as

$$\frac{c_h(p_l - p_h)V_1 + \left\{\dfrac{(r_l - p_l)}{h_l} c_l((p_l - c)V_1 - hV_2 - \pi V_3)\right\}}{\left(\dfrac{r_l - p_l}{h_l}\right)}.$$

If we take the derivative with respect to p_l and simplify, we get p_l^* as $r_l - \sqrt{(c_h h_l(p_h - r_l))/c_l}$. Clearly this promotion level will be chosen only if $G(p_l^*) \ge G(p_h)$. Otherwise it is optimal to have no promotions and set price each period to p_h. □

LEMMA 3B. *There exists a value of $h^* = f(p, \pi, c)$ such that for all $h \ge h^*$, $(dG(p)/d\sigma) \le 0$.*

PROOF. Let Z^* be the largest value such that the condition below holds:

$$\frac{\Phi\left(Z - \dfrac{1}{Z}\right) - Z\phi\left(Z - \dfrac{1}{Z}\right)}{\Phi(Z)} \le \frac{\pi}{p + \pi - w}.$$

Define $h^* = \dfrac{(p + \pi - w)(1 - \Phi(Z^*))}{\Phi(Z^*)}$.

Note that

$$\frac{dG(p)}{d\sigma} \le c_h e^{\sigma^2/2}(p + \pi - w)\sigma \left\{\frac{\sigma\Phi(Z - \sigma) - \phi(Z - \sigma)}{\sigma\Phi(Z)} - \frac{\pi}{p + \pi - w}\right\}$$

$$\le c_h e^{\sigma^2/2}(p + \pi - w)\sigma \left\{\frac{\Phi\left(Z - \dfrac{1}{Z}\right) - Z\phi\left(Z - \dfrac{1}{Z}\right)}{\Phi(Z)} - \frac{\pi}{p + \pi - w}\right\}$$

Verify that for $h = h^*$ the right-hand side is equal to 0. Hence the result. □

LEMMA 3C. *If $h \ge h^* = f(p_h, \pi, c)$, $\dfrac{dE^*_{ret}}{d\sigma} \le 0$.*

PROOF. We will first show that there exists an h^* such that $G(p_l^*)$ with the value of Z chosen such that $\Phi(Z) = (p_h + \pi - w)/(p_h + \pi + h - w)$ decreases with σ increasing. Set $G(p_l^*) = G^*(p_l^*) - h'V_2$. In this expression, $G^*(p_l^*) = (c_h + ((r_l - p_l^*)c_l/h_l))\,\{(p_l^* - w)V_1 - h_1V_2 - \pi V_3\}$, where $h_1 = ((p_l^* + \pi - w)(1 - \Phi(Z))/\Phi(Z))$. Note that for this value of h_1, $G^*(p_l^*)$ will provide the optimal expected profit. Also, note that given the definition of Z, $h' \ge 0$.

We will first show that $(dG^*(p_l^*)/d\sigma) \le 0$. Observe that if we repeat the steps in Lemma 3B with p replaced by p_l^*, we get a Z_l^* and an associated h_l^*. Verify that $h_1 \ge h_l^*$ (because $Z_l^* \ge Z$). Thus $(dG^*(p_l^*)/d\sigma) \le 0$ for $h \ge h_1$.

We will now show that $(dV_2/d\sigma) \ge 0$. Note that it can be verified that $(d/dZ)(dV_2/d\sigma) \ge 0$. Also, $(dV_2/d\sigma)|z = 0 = (1/\sqrt{2\pi})\,\{1 - (\sigma\Phi(-\sigma)/\phi(-\sigma))\}$. This can be verified to be ≥ 0 for all $\sigma \ge 0$. Hence $(dV_2/d\sigma) \ge 0$. Thus $(dG(p_l^*)/d\sigma) \le 0$ for $h \ge h_1$.

Note that $(dE^*_{ret}/d\sigma) = (dG(p_h)/d\sigma)\,(1 - K_7) + (dG(p_l^*)/d\sigma)\,K_7$ where $K_7 = 1/((r_l - p_l)/h_l)$. Thus for $h \ge h^*$, we have $(dE^*_{ret}/d\sigma) \le 0$. Hence the result. □

LEMMA 6. *The expected warehouse order size under a no promotion-timing information-sharing system is*

$$E[Order] = E(d_h) + \frac{E(d_l) - E(d_h)}{E(q(p_l))},$$

where

$$E(d_h) = V_1c_h$$

and

$$E(d_l) = V_1\left(c_h + \frac{(r_l - p_l)c_l}{h_l}\right),$$

$$V_1 = e^{\sigma^2/2}\Phi(Z - \sigma) + e^{Z\sigma}(1 - \Phi(Z)).$$

PROOF. Consider a renewal process that starts immediately following a pair of successive promotions that are $\geq L$ periods apart, includes all the successive promotions that are $< L$ periods apart, and includes the first pair of promotions that are $\geq L$ apart. Let the promotion points be labeled $S_1, S_2, \ldots, S_{M-1}, S_M$. Thus, the gap $S_M - S_{M-1}$ is greater than L, while all other $S_j - S_{j-1}$ is less than L for $1 \leq j \leq M - 1$. Note that the orders delivered during this period are as follows:

During promotion intervals k where $d_l^k < (I(p_l) - I(p_h))$ (d_l^k refers to the retail sales during the promotion period S_k, d_h^k refers to retail sales during nonpromotion period k), the delivery at period $S_k + L$ is $d_l^k + d_h^{S_k - 1}$ and at period $S_k + L + 1$ is 0. During promotion intervals k where $d_l^k \geq (I(p_l) - I(p_h))$, the delivery at period $S_k + L$ is $(I(p_l) - I(p_h)) + d_h^{S_k - 1}$ and at period $S_k + L + 1$ is $d_l^k - (I(p_l) - I(p_h))$. All other periods j get deliveries of $d_h^{(j-L)}$. The average quantity delivered during this period is $E[Quantity] = \{E(d_h)(E(y_1) - 1) + E(d_l)\}E(n) + E(d_h)(E(y_2) - 1) + E(d_l)$. The length of this renewal cycle is $E[Length] = E(y_1)E(n) + E(y_2)$, where $E(y_1)$ is the expected number of periods between successive promotions given that the gap is $< L$, $E(y_2)$ is the expected number of periods between successive promotions given that the gap is $\geq L$, $E(n)$ is the expected number of successive cycles with the gap between promotions in each of the cycles $< L$.

Note that following the renewal reward theorem, the ratio

$$\frac{E(y_1)E(n) + E(y_2)}{E(n) + 1} = E(q(p_l)).$$

Thus,

$$\frac{E[Quantity]}{E[Length]} = E(d_h) + \frac{E(d_l) - E(d_h)}{E(q(p_l))}.$$

Hence the result. □

THEOREM 1. *The manufacturer's maximum expected profit under a no promotion-timing information-sharing system (when $Z_{p_l} = Z_{p_h}$) is*

$$E_{mfr}^{no\text{-}info} = ((w - c) * E[Sales]) - h_m L(I(p_l) - E[Order])),$$

where

$$E[Sales] = E[Order] = E(d_h) + \frac{E(d_l) - E(d_h)}{E(q(p_l)},$$

$$E(d_h) = V_1 c_h,$$

and

$$E(d_l) = V_1\left(c_h + \frac{(r_l - p_l)c_l}{h_l}\right),$$

$$I(p_l) = \left(c_h + \frac{(r_l - p_l)c_l}{h_l}\right) e^{Z\sigma},$$

$$V_1 = e^{\sigma^2/2}\Phi(Z - \sigma) + e^{Z\sigma}(1 - \Phi(Z)),$$

$$\Phi(Z) = \frac{p_h + \pi - w}{p_h + \pi + h - w}.$$

PROOF. To provide a 100% service level in the absence of promotion information from the retailer, the manufacturer inventory policy is an order-up-to policy with the base-stock level set as $LI(p_l)$. The average physical inventory level at the warehouse is = Order Up-to Level − ($L * E[Order]$), where L is the manufacturer lead time and $E[Order]$ is the expected order size from Lemma 6. The average retail sales (that trigger orders to the warehouse) is equal to $E[Order]$. Hence the result. □

LEMMA 7C. *There exists an $h_m^* = g(c_h, c_l, r_l, c, w, p_h, h_l, L, Z)$ (with Z defined as $\Phi(Z) = (p_h + \pi - w)/(p_h + \pi + h - w)$) such that if $h_m \geq h_m^*$, then $(dE_{mfr}^{no\text{-}info}/d\sigma) \leq 0$.*

PROOF. Identify the smallest value of σ^* such that $(\Phi(Z - \sigma)/\phi(Z - \sigma)) = (\sigma/1 - \sigma(Z - \sigma))$. The value of h_m^* is obtained by setting $\sigma = \sigma^*$ in the expression $((w - c)/L)\ \{1/(K_{33} - 1)\}$ with

$$K_{33} = \frac{\left(c_h + \sqrt{\dfrac{c_h c_l(p_h - r_l)}{h_l}}\right)}{c_h + c_l}$$

$$\left\{\frac{Ze^{Z\sigma}}{\Phi(Z - \sigma)\sigma e^{\sigma^2/2} - e^{\sigma^2/2}\phi(Z - \sigma) + (1 - \Phi(Z))Ze^{Z\sigma}}\right\}.$$

Furthermore,

$$K_{33} \geq \frac{1}{\sigma^* \dfrac{\phi(Z)}{Z}\dfrac{\Phi(Z - \sigma^*)}{\phi(Z - \sigma^*)} - \phi(Z) + (1 - \Phi(Z))} = K_{33}^*$$

with σ^* as defined earlier.

Thus

$$\frac{dE_{mfr}^{no\text{-}info}}{d\sigma} = L(I_{p_l} - E(Order))\left\{\frac{w - c}{L\{K_{33} - 1\}} - h_m\right\}$$

$$\leq L(I_{p_l} - E(Order))\left\{\frac{(w - c)}{L}\left\{\frac{1}{K_{33} - 1}\right\} - h_m\right\}$$

$$\leq 0$$

for all $h_m \geq h_m^*$, by definition of h_m^*. Hence the result. □

LEMMA 8. *The expected inventory position across time under a full promotion information-sharing system is: $E[InvPosition] = LI(p_h) + (I(p_l) - I(p_h))\ \{(Lq(p_l)/2) + b(0, L, q(p_l)) + 0.5 * b(1, L, q(p_l))\}$ where $b(i, j, q)$ refers to the binomial probability that there are exactly i successes in j trials with a probability of success of a trial of q. In this expression, $I(p_l) = (c_h + ((r_l - p_l)c_l/h_l))e^{Z\sigma}$, $I(p_h) = e^{Z\sigma} c_h$, $\Phi(Z) = (p_h + \pi - w)/(p_h + \pi + h - w)$.*

PROOF. We will focus on a *modified policy* under which the manufacturer places orders to maintain an inventory position. If there are k promotions scheduled during the next L periods and $k \geq 1$, then inventory position is maintained as $k\ (I(p_l) - I\ (p_h)) + L\ I(p_h)$. If there is no promotion scheduled in the next L periods, then inventory position is maintained as $(I(p_l) - I(p_h)) + L\ I\ (p_h)$. Consider the probability of j successes in L consecutive trials with a probability $q(p_l)$ of a success. We will consider $j = 1, 2, \ldots, L$. These j successes correspond to exactly $j/2$ promotions if j is even. Verify that the expected increase in the inventory position given j successes in L consecutive periods is $(j/2)(I(p_l) - I(p_h))$. In addition, we also place an order when there are no promotions in the next L periods. This has a probability of occurrence of $b(0, L, q(p_l)) + 0.5 * b(1, L, q(p_l))$. Thus the expected inventory position is

$$\left\{\left(\sum_{j=1}^{L} \frac{j}{2}\, b(j, L, q(p_l))\right) + b(0, L, q(p_l)) + 0.5 * b(1, L, q(p_l))\right\}$$

$$(I(p_l) - I(p_h)) + LI(p_h) =$$

$$\left\{\frac{Lq(p_l)}{2} + b(0, L, q(p_l)) + 0.5 * b(1, L, q(p_l))\right\}$$

$$(I(p_l) - I(p_h)) + LI(p_h). \qquad \square$$

Lemma 9. *The expected order size under a full promotion information-sharing system is*

$$E[Order] = E(d_h) + \frac{E(d_l) - E(d_h)}{E(q(p_l))},$$

where

$$E(d_h) = V_1 c_h,$$

$$E(d_l) = V_1\left(c_h + \frac{(r_l - p_l)c_l}{h_l}\right),$$

and

$$V_1 = e^{\sigma^2/2}\Phi(Z - \sigma) + e^{Z\sigma}(1 - \Phi(Z)).$$

Proof. Under a promotion information-sharing system, the manufacturer schedules warehouse deliveries to synchronize with the retail promotion periods subject to the condition in the modified policy. Thus we will show that the average order size is the same as in Lemma 6 with no promotion information sharing; however, the timing of the deliveries is different under a full promotion information-sharing system. Define the renewal process as in the proof in Lemma 6. The main difference for a promotion information-sharing system arises because, following the promotion period S_k, deliveries to support promotions are scheduled to arrive at $S_k + L$ if $S_{k+1} > S_k + L$ and at S_{k+1} if $S_{k+1} \le S_k + L$. Thus, if $d_l^k < I(p_l) - I(p_h)$ (d_l^k refers to the retail sales during the promotion period S_k; d_h^k is the retail sales during nonpromotion period k), the deliveries at period S_{k+1} are $d_l^k + d_h^{S_{k+1}-L}$ and at period $S_k + L + 1$ are 0. During promotion intervals k where $d_l^k \ge (I(p_l) - I(p_h))$, the delivery at period S_{k+1} is $(I(p_l) - I(p_h)) + d_h^{S_{k+1}-L}$ and at period $S_k + L + 1$ is $d_l^k - (I(p_l) - I(p_h))$. All other periods j get deliveries of $d_h^{(j-L)}$. The remaining steps in the proof follow that for Lemma 6 with the identical results for $E[Order]$. $\square$

Theorem 2. *The optimal expected manufacturer profit under a modified policy under full promotion information sharing with the retailer is*

$$(w - c)E[Sales] - h_m(E[InvPosition] - (L \times E[Order])),$$

where

$$E[Sales] = E[Order] = E(d_h) + \frac{E(d_l) - E(d_h)}{E(q(p_l))},$$

$$E(d_h) = V_1 c_h,$$

and

$$E(d_l) = V_1\left(c_h + \frac{(r_l - p_l)c_l}{h_l}\right)$$

and $E[InvPosition]$ *is defined in Lemma 8.*

Proof. By definition, the manufacturer expected profit consists of the average sales times the gross margin of $w - c$, less the average holding cost of inventory at the warehouse. Given the expected inventory position generated in Lemma 8, we have the result. $\square$

Lemma 10b. *There exists a value of* $h_m^* = m(c_h, c_l, r_l, c, w, p_h, h_l, L, Z)$ *(with* Z *defined as* $\Phi(Z) = (p_h + \pi - w)/(p_h + \pi + h - w)$*) such that for* $h_m \ge h_m^*$, $(dE_{mfr}^{info}/d\sigma) \le 0$.

Proof. Identify the smallest value of σ^* such that

$$\frac{\Phi(Z - \sigma)}{\phi(Z - \sigma)} = \frac{\sigma}{1 - \sigma(Z - \sigma)}.$$

Define K_{23} defined as

$$\frac{K_{13}}{c_h + c_l}\left\{\frac{Ze^{Z\sigma}}{\Phi(Z - \sigma)\sigma e^{\sigma^2/2} - e^{\sigma^2/2}\,\phi(Z - \sigma) + (1 - \Phi(Z))Ze^{Z\sigma}}\right\}$$

and

$$K_{13} = \left\{c_h + c_l\left\{1 + \frac{2(1 - q)^L}{Lq} + (1 - q)^{L-1}\right\}\right\}$$

and

$$q = 2\sqrt{\frac{c_l h_l}{c_h(p_h - r_l)}}.$$

The value of h_m^* is obtained by setting $\sigma = \sigma^*$ in the expression $((w - c)/L)\,\{1/(K_{23} - 1)\}$. Also

$$K_{23} \ge \frac{K_{13}}{c_h + c_l}\,\frac{1}{\sigma^* \dfrac{\phi(Z)}{Z}\dfrac{\Phi(Z - \sigma^*)}{\phi(Z - \sigma^*)} - \phi(Z) + (1 - \Phi(Z))} = K_{23}^*$$

with σ^* as defined earlier.

Thus,

$$\frac{dE_{mfr}^{info}}{d\sigma} = (E(InvPosition) - L \times E(Order))\left\{\frac{w - c}{L\{K_{23} - 1\}} - h_m\right\}$$

$$\le (E(InvPosition) - L \times E(Order))$$

$$\left\{\frac{(w - c)}{L}\left\{\frac{1}{K_{23}^* - 1}\right\} - h_m\right\} \le 0$$

for all $h_m \ge h_m^*$, by definition of h_m^*. Hence the result. $\square$

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