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An Experimental Investigation of Auctions and Bargaining in Procurement

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In reverse auctions, buyers often retain the right to bargain further concessions from the winners. The optimal form of such procurement is an English auction followed by an auctioneer's option to engage in ultimatum bargaining with the winners. We study behavior and performance in this procurement format using a laboratory experiment. Sellers closely follow the equilibrium strategy of exiting the auction at their costs and then accepting strictly profitable offers. Buyers generally exercise their option to bargain according to their equilibrium strategy, but their take-it-or-leave-it offers vary positively with auction prices when they should be invariant. We explain this deviation by modeling buyers' subjective posteriors regarding the winners' costs as distortions of the Bayesian posteriors, calculated using a formulation similar to a commonly used probability weighting function. We further test the robustness of the experimental results and the subjective posterior explanation with three additional experimental treatments.

Data, as supplemental material, are available at <http://dx.doi.org/10.1287/mnsc.2013.1880>.

Keywords: auction; bargaining; experiment; subjective posterior

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1. Introduction

Auctions that set a benchmark price and award exclusive rights to negotiate final purchase terms are commonly used in many areas of commerce such as procurement in supply chains (Elmaghraby 2007, Tunca and Wu 2009), in corporate mergers and acquisitions (Hege et al. 2009), and even in professional sports when teams exchange the rights to players.¹ This practice is particularly common in government procurement. For example, we learned in interviews with the procurement center of the Hunan Province in China that in 2012 they attempted to procure more than 9,000 orthopedic-related objects, with over 12,000 unique suppliers participating in the process. Whenever there were at least three qualified suppliers, they conducted a reverse auction. If the auction price did not meet

their basis price, then they further negotiated with the auction-winning supplier. Sometimes bargaining resulted in an improved price and other times in no purchase at all. This case exhibits two key characteristics of effective auction-bargaining practice: further negotiations are often chosen, and sometimes a trade does not occur even when there are potential benefits to both parties.

There is a strong theoretical basis for using the auction-bargaining mechanism. A seminal article by Bulow and Klemperer (1996) shows that conducting a reverse (forward) English auction with the auctioneer retaining the right to make a take-it-or-leave-it offer to the auction winner implements the optimal mechanism (Myerson 1981) to purchase (sell) an object. On the other hand, they also establish that the auctioneer's expected welfare increases by forgoing this auction-bargaining mechanism and conducting an English auction with an additional serious bidder. Unfortunately, identifying and validating additional serious bidders is often cost prohibitive and difficult, particularly in the procurement setting (Wan and Beil 2009, Wan et al. 2012). Consequently, we feel evaluating behavior and performance in Bulow and Klemperer's auction-bargaining mechanism within a procurement setting is an important task.

¹ For example, the Nippon Professional Baseball League and Korean Baseball Organization have a posting system that allows a player to ask his current team to conduct an auction granting a period of exclusive negotiating rights to a Major League Baseball team; about one player per year leaves the Nippon League through this process. An even more relevant practice, in which no compensation occurs in the absence of a final contract, is the trading of players between teams in the National Basketball Association or National Football League conditional upon the player and new team agreeing to a contract extension.

The Nash equilibrium implementing the optimal mechanism, despite having a simple structure, relies on some behaviorally questionable assumptions. A seller's strategy is to exit the auction when her cost exceeds the auction price and accept any subsequent profitable ultimatum offer. This opposes what is observed in ultimatum game experiments where responders commonly reject profitable offers that are less than equitable, even when experienced (Cooper and Dutcher 2011), or there is incomplete information about the amount of potential gains from exchange (Croson 1996, Harstad and Nagel 2004).² In our experiment, sellers generally exit the auction at their costs, and they rarely reject profitable take-it-or-leave-it offers.

The buyer's equilibrium strategy is characterized by a price threshold that is conditional upon his willingness to pay. He accepts an auction outcome when the price is below his threshold; otherwise, he makes a take-it-or-leave-it offer equal to the threshold. This strategy has the uncanny flavor of an ex post reserve price, and that is not a coincidence. Both the auction-bargaining mechanism and the English auction with an ex ante reserve price implement the optimal direct mechanism. As a consequence, the buyer's threshold in the auction-bargaining mechanism is the optimal reserve price in the English auction, and therefore it is independent of the realized auction price. But this appears paradoxical; the buyer only commits to a threshold after the auction, and hence gaining additional information about the winning seller's cost. We show that the price invariance of the buyer's threshold crucially relies on the buyer forming correct Bayesian posterior beliefs regarding the winner's cost.

This result must seem paradoxical to the subjects as well. Although the buyer's strategy accurately predicts when auction outcomes are rejected in favor of further bargaining, subsequent take-it-or-leave-it offers have a strong positive relationship with the auction price—contradicting the prediction of price invariance. This results in reducing the buyer's surplus by approximately 7.5%. We also find substantial individual heterogeneity in this auction price sensitivity.

We find an explanation for this behavior by generalizing how a buyer formulates his posterior belief regarding the auction winner's cost conditional upon the auction price. We propose the subjective posterior is formed by applying the two-parameter Prelec (1998) probability-weighting function to transform the Bayesian posterior. This allows the subjective posterior to flexibly reflect differing types of perceived affiliation between the auction price and winner's cost. Structural estimates of this model demonstrate its ability to capture both the observed wide varying pattern of

buyer behavior and the general property of the positive relationship between auction price and bargaining offer. We further show that simply allowing for risk aversion does not lead to any relationship between auction price and bargaining offer, and anticipated regret only leads to a negative relationship. Thus, of these three alternatives, our subjective posterior model is the only plausible explanation.

We also conduct additional treatments to challenge the robustness our explanation and findings. First, we enrich the feedback that auction-losing sellers receive by informing them of the actions taken in the negotiation phase. However, this does not give impetus to any increased adherence to their equilibrium strategy in the auction phase, nor does it inspire any rejections of profitable offers. Second, we challenge the distorted posterior explanation with the alternative hypothesis that buyers do not hold accurate beliefs about the seller's strategies. In response, we run a treatment where buyers play against computerized sellers automated to follow the equilibrium strategy. We also inform buyers about the nature of these sellers. The buyer's offers are still sensitive to the prices, and the estimated posteriors, on average, exhibit stronger affiliation between the winner's and loser's costs.

Finally, we explore whether distorted "subjective posteriors" are an easily corrected bias. We augment the computerized seller treatment by giving buyers a calculator that provides detailed information on the probability of acceptance for alternative offers and expected payoffs. We even obligate them to inspect this information for any take-it-or-leave-it offer they make. The results are startling in that buyers do not make offers more in line with the Bayesian benchmark, but rather they show a greater willingness to bargain and make offers reflecting even more distorted posterior beliefs. This has practical implications for management practice; distorted subject posteriors cannot be corrected by simply providing expert decision support systems.

2. Basic Theory

We present an alternative derivation of the Nash equilibrium of the auction-bargaining mechanism that elucidates that how the buyer dynamically processes information influences his strategy. Consider a buyer who desires an indivisible object.³ His value for this object is the random variable v with the absolutely continuous distribution function $H(v)$ on the interval $[\underline{v}, \bar{v}]$, $\underline{v} > 0$, and associated probability distribution function $h(v)$. Only the buyer knows his realized valuation. There are N possible sellers, indexed by i . Upon selling an object, seller i incurs a unit cost of c_i . Each

² An exception is found in Salmon and Wilson (2008), which we discuss in §2.

³ Note that both Bulow and Klemperer (1996) and Myerson (1981) consider the strategically equivalent case of an individual selling an indivisible unit of a good to N possible buyers.

seller's unit cost is an independent random variable with the common absolutely continuous distribution function $F(c)$ on the interval $[0, \bar{c}]$ and associated probability distribution function $f(c)$. We also assume that $c + F(c)/f(c)$ is strictly increasing on the support of F . Only a seller knows her realized cost, which she learns prior to any strategic interaction. The variables $H(v)$ and $F(c)$ are also independent, so a seller's realized cost reveals no additional information about the buyer's value.

Here, are the specific rules of the auction-bargaining mechanism. First, the sellers compete in a reverse English clock auction in which the price starts at \bar{c} , and all N sellers are in the auction. Price falls with time, and a seller can irreversibly exit the auction at any time. The auction closes, setting the auction price p , when either $N - 1$ sellers have exited or the price reaches zero. Ties in either being the $N - 1$ th seller to exit or winning at a price of zero are settled randomly. Any seller who does not win the auction receives a payoff of zero. The buyer is informed of the auction price and chooses to either accept the auction price, resulting in payoffs of $v - p$ for the buyer and $p - c_i$ for the winner, or make a take-it-or-leave-it offer o to the winner. In the case of the latter, the winning seller either accepts the offer, resulting in payoffs of $v - o$ and $o - c_i$, or rejects the offer, and all parties receive a zero payoff.

A seller's strategy has two parts: a function that maps from possible unit costs to auction exit prices and a function that maps from possible counteroffer information sets to reject and accept decisions. The buyer's strategy is a function that maps from possible value and auction price pairs to possible counteroffers joined with accepting the auction outcome. We show the strategy profile in which each seller exits the auction at her unit cost and accepts any profitable take-it-or-leave-it offer, and the buyer accepts all auction prices below some threshold level—conditional on v —and otherwise makes an optimal counteroffer is a Nash equilibrium.

Consider the optimality of the buyer's strategy conditional upon those of the sellers. The buyer's conditional payoff function is

$$\pi(o | v, p) = \max\{v - p, \max(v - o)G(o | p)\}; \quad (1)$$

$G(o | p)$ is the buyer's subjective probability distribution of the auction winner's cost, which need be not derived according to Bayes' rule, but $o + G(o | p)/g(o | p)$ must be strictly increasing. The first-order condition for an interior maximum of the second argument of (1) implies

$$v - o^* = \frac{G(o^* | p)}{g(o^* | p)}. \quad (2)$$

We provide an economic interpretation of Equation (2) from the theory of a price-setting monopolist,

a direct analog to the classic interpretations of optimal forward auctions with monopoly theory (Bulow and Roberts 1989, Bulow and Klemperer 1996). In this case, the buyer is a monopsonist, the probability of purchase is analogous to market quantity q , and the offer o is analogous to market price set by the monopsonist. The buyer's marginal and average values (of additional probability of successfully purchasing) are the same constant v . Now the market supply function is the buyer's subjective posterior distribution of the winner's cost; i.e., $q = G(o | p)$. And the marginal cost (of additional probability of successfully purchasing) is $MC(G(o | p)) = o + G(o | p)/g(o | p)$. If there is an optimal interior quantity $G(o^* | p)$, we know it is found where $MR = MC$, which is simply Equation (2).

The relationship between v , market supply, and marginal cost is depicted in the left-hand graph of Figure 1. In the right-hand side of this figure, we depict an interior solution and two types of corner solutions. In one corner solution, marginal cost never exceeds v , in which case $o^* = G^{-1}(1 | p_1)$. In the other corner solution, the buyer believes with probability zero that the winner's cost is below his value (see $G(o | p_2)$), and thus setting $o^* = v$ is an optimal offer.⁴ In all three cases, the buyer accepts the auction outcome if $v - p \geq (v - o^*)G(o^* | p)$.

Thinking of $G(o | p)$ as a state-dependent supply function, where the state is the realized auction price, the optimal state pricing rule is found by rewriting Equation (2) as

$$o^* = \frac{v}{1 + 1/E_{G,o}(o^*)}, \quad \text{where} \quad E_{G,o}(o) = \frac{o}{G(o | p)}g(o | p). \quad (3)$$

In other words, $E_{G,o}(o)$ is the price elasticity of the supply function, and Equation (3) is the standard monopsony inverse elasticity pricing rule.

When $G(o | p)$ is calculated according to Bayes' rule, this elasticity function is the same across all states, and o^* is state invariant for interior solutions. To see this, first consider the following proposition.

PROPOSITION 1. *Let c_i , $i = 1, \dots, N$, be independent realizations from the distribution F , with ordering $c_1 \leq c_2 \leq \dots \leq c_N$. Then the conditional distribution of c_1 , given c_2 , is the same distribution as F truncated at c_2 .*

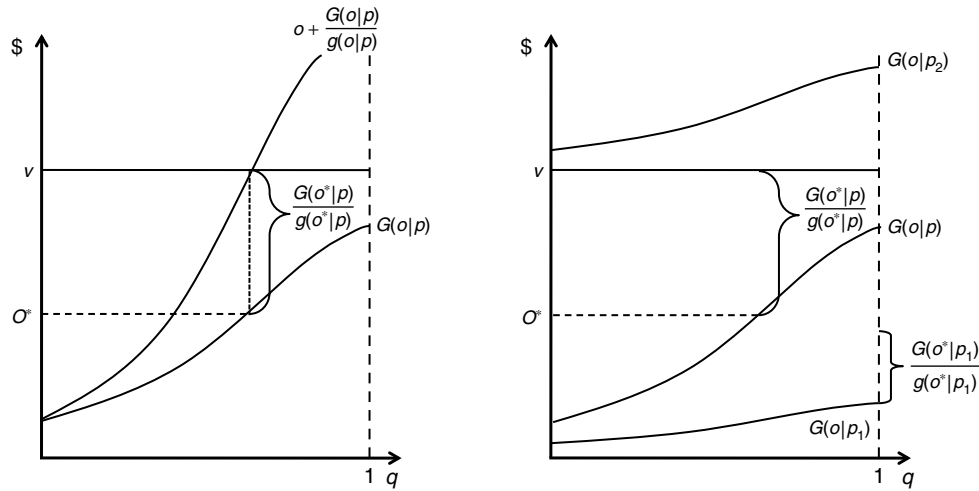
PROOF. This is Theorem 2.7 of David (1981).

In equilibrium, the auction price equals the realized second-lowest cost, and according to this proposition, the Bayesian posterior is

$$G(o | p) = \frac{F(o)}{F(p)}. \quad (4)$$

⁴ In this case the optimal offer is set valued; $o^* = [0, G^{-1}(0 | p_2)]$.

Figure 1 Optimal Offer



Note. The left-hand graph illustrates an interior solution. The right-hand graph illustrates an interior solution for auction price p , a corner solution for p_1 where marginal cost never crosses v and $o^* = G^{-1}(1 | p_1)$, and another corner solution for p_2 where $G^{-1}(0 | p_2) > v$ and $o^* = v$.

Thus, the state-dependent supply functions have a multiplicative relationship. For example, for two different auction outcomes, p' and p'' , their Bayesian posteriors are related as follows: $G(o | p') = (F(p'')/F(p')) \cdot G(o | p'')$. One of the properties of an elasticity measure is that it is invariant to multiplicative scaling of the relationship. Accordingly, one can show that $E_{G,o}(o) = E_{F,o}(o)$ and the optimal monopsony pricing rule of Equation (3) is invariant of the price; i.e.,

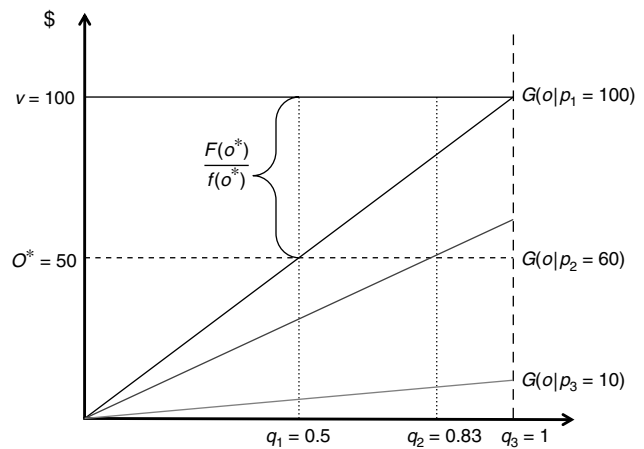
$$o^* = \frac{v}{1 + 1/E_{F,o}(o^*)}. \quad (5)$$

In summary, the Bayesian posterior is simply a scalar proportion of the prior. Likewise, the elasticity of the posterior does not change, and neither does the optimal offer, as indicated by Equation (5). However, the probability that the optimal offer is accepted by the auction winner does change and is inversely proportional to the prior evaluated at the auction price, as indicated by Equation (4).

In brief, the seller's strategy is an optimal response given the buyer's strategy because she can never strictly increase her payoff by accepting an offer above her cost or by rejecting one above her cost. Likewise, exiting the auction at her cost is optimal by standard arguments (for example, see Krishna 2009, p. 15) for exiting at cost in typical private cost English auctions.

Consider the following example, which is also the setting we use in our experiment. The distribution of the buyer's value $H(v)$ is the uniform distribution on $[50, 150]$. There are two sellers, and $F(c)$ is the uniform distribution on $[0, 100]$. Notice that $F(c)$ is a linear function, so its elasticity is always 1, as is the elasticity of any posterior. For a realized value v , by Equation (3), $o^* = v/2$. So whereas the optimal threshold does not depend on price, the postauction

Figure 2 The Optimal Offer for $v = 100$ and $F(c) = c/100$



Notes. For $p > 50$, the optimal counteroffer is 50 and does not vary with the price, but the corresponding probability of purchasing, $G(50 | p)$, does. For $p \leq 50$, the buyer accepts the auction outcome.

probability of a purchase when engaging in take-it-or-leave-it bargaining does. Suppose $v = 100$ and consider three scenarios: $p_1 = 100$, $p_2 = 60$, and $p_3 = 10$. Figure 2 depicts that the optimal offer is the same in the first two scenarios, but the probability of purchase differs, and the optimal decision is to accept the auction in the last scenario.

3. Experimental Design and Hypotheses

3.1. Experimental Design

Our experiment consists of the following session flow. First, we recruit 18 subjects to participate in a two-hour experimental session. We randomly designate 6 subjects as buyers and 12 subjects as sellers; these designations

are fixed for the session. The session consists of 2 practice periods and 30 rounds for which subjects receive compensation based on their decisions. Every period we randomly form six trios consisting of two sellers and one buyer. Participants are informed of the random rematching protocol and that all costs and values are redrawn each period.

Each trio plays the previous example of the auction-bargaining mechanism, and we induce common knowledge by publicly reading and displaying instructions at the start of the experiment.⁵ Each period starts with each buyer and seller learning his or her respective value and cost. The two sellers in a trio participate in a descending English clock auction, without knowing the buyer's realized value. The initial auction price is \$100 and decreases by \$1 every 0.7 second. When a seller exits the auction, the auction concludes with a winner and price determination as indicated previously.⁶ Next, the buyer is informed of his auction price and presented the choice to either accept the auction outcome or make a take-it-or-leave-it offer to the auction-winning seller. If he accepts the auction outcome, all trio members are informed of their payoffs, and the period concludes. If the buyer instead engages in bargaining, the auction winner is presented with the counteroffer and decides whether to accept or reject. In either case, payoffs are reported, and the period ends.

We conducted eight sessions at the Finance and Economics Experimental Laboratory at Xiamen University.⁷ This gives us a total of 144 subjects (48 buyers and 96 sellers), each with 30 observations. Subjects, on average, earned RMB 70 for their participation. We recruited subjects, all of whom were undergraduate and graduate students enrolled in Xiamen University, through the Online Recruitment System for Economic Experiments (ORSEE) system (Greiner 2004), and none had previous experience in this study. The experimental software was programmed in Z-tree (Fischbacher 2007).

3.2. Hypotheses

Our ex ante hypotheses consists of one regarding economic performance and three regarding buyer and seller behavior. The main result of the Bulow and Klemperer (1996) study is that whereas the auction-bargaining mechanism offers the auctioneer more value than a simple N -bidder English auction, it offers the auctioneer less value than adding another serious bidder⁸ and conducting an $N + 1$ bidder English auction.

⁵ Instructions are available upon request from the authors.

⁶ We never observe a case where both sellers do not exit and the auction closes at a price of zero.

⁷ This facility is designed for the purpose of conducting economic experiments and has privacy carrels, a private payment and sign-in area, and a separate monitor room from which the experimenter conducts the experiment.

⁸ In our design, sometimes a seller fails to satisfy the serious bidder criteria $c_i \leq v$. However, our design does satisfy the more lax sufficient

HYPOTHESIS 1. *Buyer profit is greater than the expected profit in a two-bidder English auction and less than the expected profit in a three-bidder English auction.*⁹

Next, the seller's Nash equilibrium strategy provides two additional hypotheses. This first is about the seller's behavior in the auction phase of the auction-bargaining mechanism.

HYPOTHESIS 2. *Sellers exit the auction at their realized costs.*

We have a strong prior for confirming this hypothesis due to previous experimental results on independent private value forward (Coppinger et al. 1980) and private cost reverse (Shachat and Wei 2012) English auctions that show close adherence to theoretical predictions regarding expected price and bidders' strategy. However, there is uncertainty regarding how the bargaining phase plays out in the experiment and how this change in feedback affects the saliency of the seller's optimal action in the auction. For example, a rational seller may "overstay" in the auction if she has a "joy of winning" the auction component in her utility function or if she believes with probability one there will be a counteroffer and an opportunity to reject it. Alternatively, she may fail to make a rational choice because of difficulty in perceiving the optimal strategy, as often is found in the sealed-bid second-price auctions.

Our second hypothesis about sellers concerns behavior when a seller confronts a take-it-or-leave-it offer.

HYPOTHESIS 3. *Sellers do not reject profitable offers and reject nonprofitable ones.*

This hypothesis derived from sequential rationality has a stronger alternative hypothesis than it would appear at first glance. The bargaining phase of the game is strategically equivalent to an ultimatum game in which the buyer and seller only respectively know the upper and lower ends of the "pie" interval to be shared. The very large literature on ultimatum game experiments, starting with Guth et al. (1982), has shown that a nonnegligible proportion of responders reject minimally profitable offers. In particular, this still holds true in studies that introduce asymmetric information about the pie size (Croson 1996, Huck 1999, Harstad and Nagel 2004). However, a glaring counterexample are the results of Salmon and Wilson (2008), who study a two-unit experimental English auction where the first unit is sold to the auction winner and a take-it-or-leave-it offer is made to the last exiting bidder for the

condition that the losing supplier's expected cost is no more than the buyer's value. See Bulow and Klemperer (1996) p. 185, Footnote 15 for more detail.

⁹ In this paper, we use the theoretical predictions of the two- and three-bidder English auction benchmarks to test this hypothesis rather than run separate control treatments.

second unit. In this study, approximately only 4% of profitable offers were rejected. Our study is similar in that we imbed the ultimatum game as an after stage to the auction to determine possible responder participation. However, our setting differs because we have two-sided incomplete information, and the responder does not have incentives to misrepresent her type (cost).

Our final hypothesis, developed in the previous section, regards the buyer's behavior.

HYPOTHESIS 4. *Buyers follow their optimal strategy according to Bayesian posteriors.*

4. Results

Our experimental data set consists of 1,440 plays of the auction-bargaining mechanism, each including the losing seller's exit price and the buyer's decision of whether to bargain. Buyers chose to bargain 963 times, and the winning seller rejected the take-it-or-leave-it offer 280 of these times. We start by comparing observed economic performance with theoretical benchmarks for buyer profit, seller profit, and welfare-improving trades. Then we examine the extent to which subjects follow their Nash equilibrium strategies.

4.1. Market Performance

We start by presenting various summary statistics of our auction bargaining with two-seller experiments and the corresponding theoretical benchmarks from English auctions with two and three sellers. Table 1 presents these statistics and theoretical benchmarks. Note that when calculating the theoretical benchmarks of the English auctions for expected buyer and seller profits and potential social surplus, $\max\{0, v - \min\{c_i\}\}$, we use the realizations of the buyers' values and sellers' costs rather than the distributions they are drawn from.

A first observation is that the main theorem of Bulow and Klemperer (1996) holds.

RESULT 1. Using the auction-bargaining mechanism, buyer profit is greater than the expected profit in a

Table 1 Realized Economic Performance vs. Theoretical Benchmarks

Performance measure	AB experiment	Theoretical prediction		
		AB equilibrium	EA 2 bidders	EA 3 bidders
Average buyer profit	40.66	43.92	36.83	51.64
Standard deviation	32.54			
Average seller profit	21.91	16.84	25.23	21.11
Standard deviation	20.92			
% periods with trade	80.56	79.58	95.14 ^a	97.57 ^a

Note. AB, auction-bargaining mechanism; EA, English auction.

^aFor the English auctions, the reported value is the percentage of auctions for which $v \geq \min\{c_i\}$.

Table 2 Hypothesis Tests Comparing Observed Profit and Theoretical Benchmarks

Player role	Null hypothesis	Alternative hypothesis	<i>t</i> -statistic ^a	<i>p</i> -value
Buyer	AB = AB equilibrium	AB ≠ AB equilibrium	−3.80	0.00
	AB = EA 2 bidders	AB > EA 2 bidders	4.47	0.00
	AB = EA 3 bidders	AB < EA 3 bidders	−12.78	0.00
Auction	AB = AB equilibrium	AB ≠ AB equilibrium	9.18	0.00
Winning	AB = EA 2 bidders	AB > EA 2 bidders	−6.02	0.00
Seller	AB = EA 3 bidders	AB > EA 3 bidders	1.45	0.93

Note. AB, auction-bargaining mechanism; EA, English auction.

^aThe *t*-statistic is calculated as $(\bar{\mu} - \mu_0)/\hat{\sigma}_{\bar{\mu}}/\sqrt{n}$, where $\bar{\mu}$ is the sample average, μ_0 is the theoretical prediction, the number of observations is $n = 1,440$, and $\hat{\sigma}_{\bar{\mu}}$ is the sample standard deviation. The test statistic has a Student's *t*-distribution with $n - 1$ degrees of freedom.

two-bidder English auction and less than the expected profit in a three-bidder English auction. Hypothesis 1 is confirmed by the *t*-tests of the third and fourth rows of Table 2.

Although the qualitative prediction regarding the buyer's welfare is confirmed, Table 2 shows that we reject the theoretical predictions of average buyer and seller profits. Either the buyers or sellers are not uniformly following their respective equilibrium strategies. We show it is the buyers, not the sellers, who are deviating.

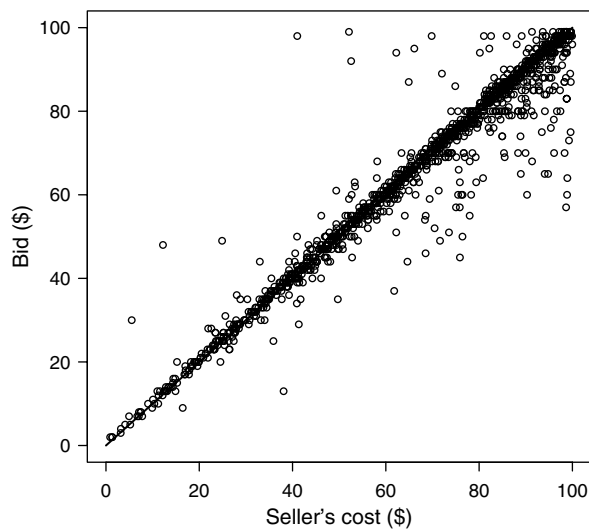
4.2. Seller Behavior

Do sellers exit the auction when the price reaches their cost? Consider Figure 3, which plots the 1,440 exit prices for the auction-losing sellers versus their respective costs. There is a clear concentration of observations along the 45° line. To provide quantitative evidence that sellers exit at prices equal to costs, we conduct an ordinary least squares (OLS) regression, finding an intercept of 3 and a slope coefficient of 0.95 (with an adjusted R^2 statistic of 0.94). If we suppress the intercept term, then the slope coefficient is 0.99. These results are consistent with those found previously in English auctions in both forward (Coppinger et al. 1980) and reverse (Shachat and Wei 2012) contexts. Admittedly, there is some evidence that the potential bargaining phase leads to some overstaying in the auction for high costs, as seen in Figure 3, and noisy adherence to the equilibrium strategy. For example, the percentage of absolute deviations of bids from costs less than \$1, \$2, and \$3 are 56.5%, 75.0%, and 82.6%, respectively. With these minor caveats, we state our next result.

RESULT 2. Sellers tend to exit the auction at their realized costs, confirming Hypothesis 2.

Turning our attention to seller behavior in the bargaining phase, Figure 4 plots the 963 take-it-or-leave-it offers versus the winning seller's cost. The 45° line

Figure 3 Auction Exit Price vs. Realized Cost



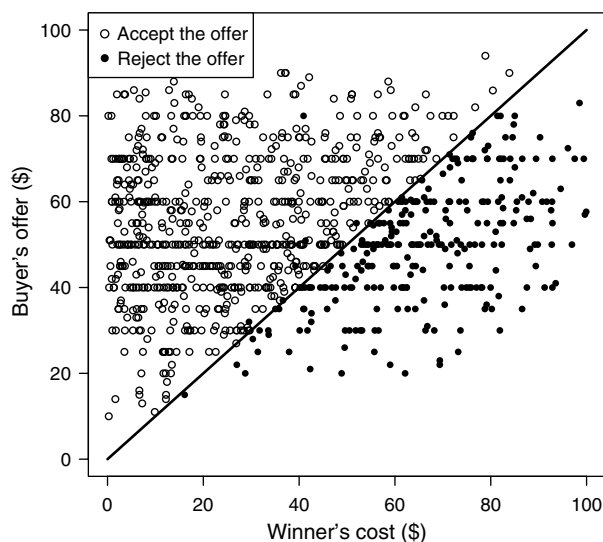
separates profitable and nonprofitable offers. First note that sellers reject all 274 nonprofitable offers, but only 6 of the 689 offers that exceed the seller's cost. Clearly, the types of reciprocal behavior and other-regarding preferences found pervasive in ultimatum bargaining experiments are not a factor here. These results also demonstrate the robustness of the findings in Salmon and Wilson (2008).

RESULT 3. Sellers rarely reject profitable offers and always reject nonprofitable ones, confirming Hypothesis 3.

4.3. Buyer Behavior

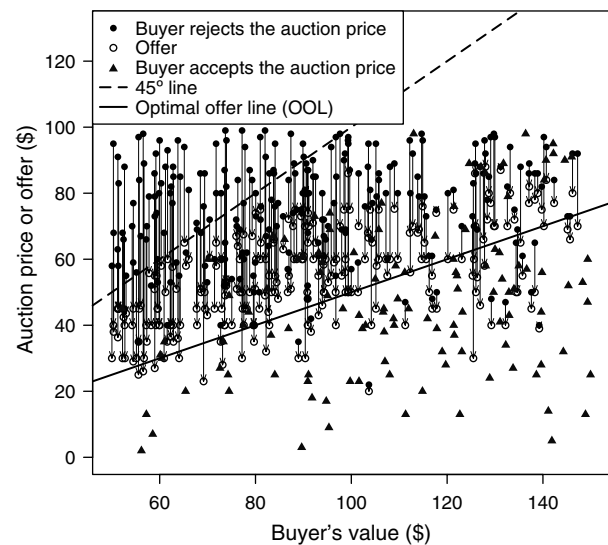
Our assessment of how the buyers' choices agree with their equilibrium strategies starts with a visualization

Figure 4 Plot of Take-It-or-Leave-It Offers vs. the Auction-Winning Seller's Cost



Note. Open circles mark accepted offers, and solid circles mark rejected offers.

Figure 5 Buyer Choices Conditional on Realized Value: Accepted Auction Prices and Reject Auction Prices with Counteroffer



Notes. Solid triangles are accepted auction prices. Solid circles are rejected auction prices, and the connected open circles are the subsequent counteroffers. The upper dashed line is the 45° line indicating where auction prices equal buyer values. The solid line is the theoretical optimal offer line. In equilibrium, we should see all prices above this line rejected with counteroffers on the line and all prices below the line accepted.

of the buyers' decisions in Figure 5. In this figure the x axis is the buyer's value and the y axis is the value of the auction price and take-it-or-leave-it offer. A triangle marks an accepted auction price. A closed circle marks a rejected auction price, and the connected open circle is the corresponding take-it-or-leave-it offer. The upper line flowing from the bottom left to the top right is where auction price (and counteroffer) equals the buyer value. The lower line is where auction price (and counteroffer) equals one-half the buyer value, i.e., the theoretical prediction. We refer to this as the optimal offer line (OOL). If buyers adhere perfectly to the equilibrium strategy, all auction prices below the OOL are accepted and those above are rejected and countered with offers on the OOL. Note that we only plot a randomly selected one-fourth of the data to avoid overcluttering.

The figure suggests mixed evidence regarding how theory does track the data. Buyers reject 82% of the auction prices above the OOL and accept 74% of those below the OOL, roughly matching the theoretical predictions. However, we can recognize that there are three types of behavior that deviate from the theoretical prediction. First, we observe that when some high auction prices are rejected, the subsequent counteroffers are greater than the theoretical optimal offer. Second, some auction prices above the OOL are accepted. Third, some counteroffers are below the OOL, with some associated with rejected auction prices below the OOL. We explore whether these deviations are from the

Table 3 Maximum Likelihood Estimation of the Linear Tobit Model for the Pooled Data

Variable	Estimate	Standard error ^a
Constant	−0.24	3.23
Price	0.35***	0.05
Value	0.34***	0.02
σ_ϵ	12.68***	0.73
Log-likelihood	Pooled	−4,070
Log-likelihood	Individual	−3,444

^aStandard errors are clustered by session.

***Significant at the 1% level.

unbiased noisy adoption of equilibrium strategies or reflect alternative structural behavior.

A natural way to test the veracity of the buyer's equilibrium strategy, modulo unbiased error terms, is to nest it in the following Tobit regression model. We assume that a buyer's optimal offer is a linear function of his value and the auction price plus some normally distributed error term. We observe his optimal offer whenever it is less than the auction price; otherwise, it is censored at the auction price. Formally, the Tobit model is

$$o_{it} = \begin{cases} \alpha + \beta p_{it} + \gamma v_{it} + \epsilon_{it} & \text{if } \alpha + \beta p_{it} + \gamma v_{it} + \epsilon_{it} < p_{it}, \\ p_{it} & \text{if } \alpha + \beta p_{it} + \gamma v_{it} + \epsilon_{it} \geq p_{it}, \end{cases} \quad (6)$$

where ϵ_{it} is a normally distributed error term, $N(0, \sigma_\epsilon^2)$.

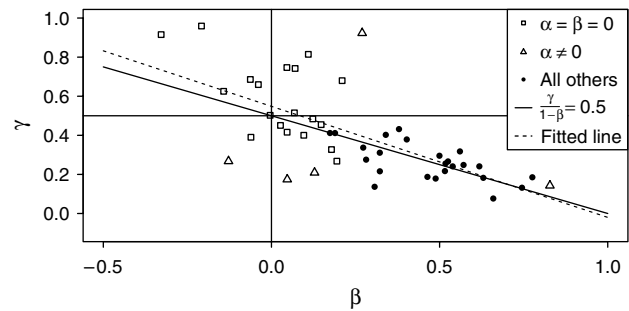
If buyers follow the optimal strategy, given in Equation (1), then we should find $\alpha = \beta = 0$ and $\gamma = 0.5$. We report the maximum likelihood estimates of Equation (6) in Table 3.¹⁰

The Tobit regression results in Table 3 reveal that the theory does a remarkably good job of predicting when a buyer chooses to bargain but fails to capture important aspects of what determines the size of take-it-or-leave-it offers. First, we expect a buyer to bargain when $(\alpha + \gamma v)/(1 - \beta) < p$. We find that α is not significant, and then by substituting parameter estimates for γ and β , we estimate that the condition for choosing to bargain is $(0.34/(1 - 0.35))v = 0.52v < p$, almost exactly what the theory predicts. However, inconsistent with the theoretical prediction is the significant positive relationship price has on the take-it-or-leave-it offer amount. We seek to explain this systematic deviation from the theory.

First, we identify significant individual heterogeneity in the buyers' sensitivities to price. We estimate an individual linear Tobit model for each buyer and report

¹⁰ Out of concern that buyers use one rule to determine when to bargain and another to determine the counteroffer, we attempt to estimate a sample selection model. However, the correlation coefficient ρ always converges to one, leading to a singular information matrix. We interpret this as strong evidence that the selection and offer models are the same.

Figure 6 Individual Estimated Price and Value Coefficients from the Linear Tobit Model

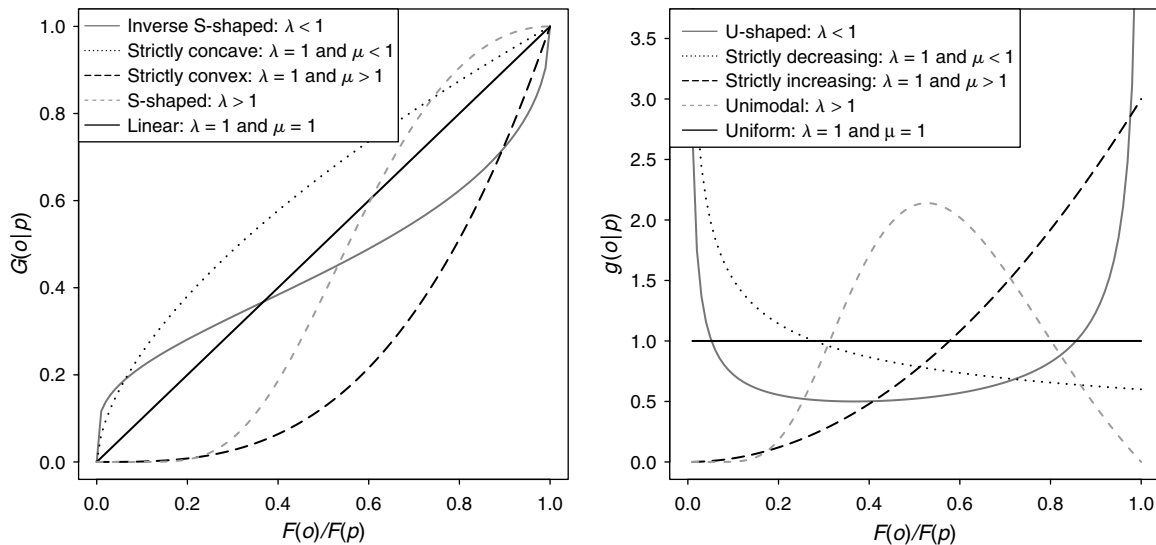


the sum of individual log-likelihood values in the last row of Table 3. A likelihood ratio test soundly rejects the pooled model in favor of the individual model. In Figure 6 we present scatterplots of the estimated individual coefficients for price and value. We classify three types of individual estimates: open squares mark the cases that we cannot reject that α and β are jointly zero via an F -test, open triangles mark the estimated coefficients for the five subjects whose estimated α is significantly different from zero, and closed circles mark the estimated coefficients for all others.

These scatterplots of individual estimated pairs β and γ demonstrate that individuals generally agree on when to bargain but differ in how strongly counteroffers depend on the auction price. We provide a vertical reference line at $\beta = 0$ and a horizontal reference line at $\gamma = 0.5$. The line with a slope of negative one-half passing through $(0, 0.5)$ represents the pairs of parameter values corresponding to the rule of negotiating when $p > 0.5v$. Movements toward the bottom right along this line, away from $(0, 0.5)$, indicate counteroffers with an increasing positive relationship with auction prices. We separate individuals into those whose offers depend on price, identifying them with solid circles, and those whose offers do not, identifying them with open squares. The distribution of the estimated parameter pairs is clustered around the theoretical line, as suggested by the close agreement between this and the fitted line, for rejection of the auction price, but it spans a large range of price sensitivity of counteroffers. A natural question is whether these dependencies of take-it-or-leave-it offers on auction prices are consistent with less restrictive assumptions on subject preferences or formation of beliefs.

5. Alternative Models

In this section we explain buyer behavior by supposing posterior beliefs regarding the auction winner's cost are distortions of the Bayes' rule-determined posteriors. We parameterize these distortions using a well-known probability weighting function that allows an interpretation of subjects perceiving various kinds of affiliation

Figure 7 Alternative Shapes for Transformed Posterior Distribution Function $G(o|p)$ and Density Function $g(o|p)$ 

between the lowest and second-lowest realized costs. We also show that the observed patterns of when buyers choose to bargain and the positive relationship between take-it-or-leave-it offers and auction prices cannot be explained by models solely assuming risk aversion or anticipated regret, models that have previous success explaining empirical auction behavior.

5.1. A Model of Transformed Bayesian Posteriors

We showed previously that the invariance of the buyer's optimal offer depends on the use of Bayes' rule to formulate the posterior distribution of the auction-winning seller's cost. We now generalize from Bayesian updating by using a two-parameter transformation of Bayesian posteriors—namely, the buyer's subjective posterior of the winner seller's cost is

$$G(o|p) = \begin{cases} 0 & \text{if } o = 0, \\ e^{-\mu(-\ln(F(o)/F(p)))^\lambda} & \text{if } 0 < o \leq p, \end{cases} \quad (7)$$

where μ and $\lambda > 0$.

Readers may recognize that Equation (7) is the Prelec (1998) form of a probability weighting function. Probability weighting is a component of prospect theory (Kahneman and Tversky 1979) capturing the empirical regularity that an individual's valuation of a risky prospect is more sensitive to changes in probabilities close to zero and one than at more central quantiles. This common pattern of valuation results in an inverted S-shaped probability weighting function. Since we are modeling distorted posteriors, we adopt the Prelec form, which allows various forms of biases. Using our uniform prior, for example, the unbiased transformation of the Bayesian posterior is the linear $G(o|p)$ presented in Figure 7. Now consider the short-dashed S-shaped $G(o|p)$ in the same figure. In this case, the bias reflects

a single-peaked posterior probability density function (PDF) with a modal belief that the winner's cost is about 50% of the auction price, essentially perceiving a nonexistent affiliation between the auction price and the winner's cost. A stronger bias toward a positive affiliation in costs is found in the convex-shaped $G(o|p)$, whereas a concave $G(o|p)$ suggests a bias in favor of low costs.¹¹ In this case, the subjective posterior puts increasing probability mass nearer the auction price. Let us finally consider the inverted S-shaped $G(o|p)$, which implies increasing posterior beliefs on costs close to zero and the price. This suggests a rather unusual U-shaped posterior PDF.

Now suppose a buyer formulates his posterior distribution of the auction winner's cost according to Equation (7). The next proposition describes when there is a unique optimal offer and under what conditions an interior optimal offer is strictly increasing in price.

PROPOSITION 2. Assuming that $F(c)$ is the uniform distribution and $G(o|p)$ is calculated according to Equation (7),

(i) if $\lambda \geq 1$ and $f'(o) \leq 0$, then $o + G(o|p)/g(o|p)$ is strictly increasing and there is a unique o^* ;

(ii) if $\lambda > 1$, then an interior optimal offer o^* is strictly increasing in the auction price p .

PROOF. See the appendix.

A buyer's Nash equilibrium strategy, when following our two-parameter subjective posterior model, leads

¹¹ When $\lambda = 1$, this model is behaviorally indistinguishable from a model of a buyer whose expected utility function is $u(x) = x^{1/\mu}$. One can establish this by making the preference-invariant transformation $Z((v-o)(F(o)/F(p))^\mu) = (v-o)^{1/\mu}(F(o)/F(p))$, where $Z(y) = y^{1/\mu}$. We refer the reader to Goeree et al. (2002, p. 265) for further discussions.

Table 4 Maximum Likelihood Estimation of the Nonlinear Tobit Model with the Two-Parameter Subjective Posterior Model

Parameter	Estimate	Standard error
μ	2.39***	0.002
λ	1.35***	0.002
σ_ϵ	11.31***	0.248
Log-likelihood	Pooled	−4,070 ^a
Log-likelihood	Individual	−3,613

^aThis is not a typo. This is the same value for the maximized likelihood as we find for the linear Tobit model. However, the linear Tobit model has one more parameter than the nonlinear Tobit model.

***Significant at the 1% level.

directly to the following nonlinear Tobit regression model:

$$o_{it} = \begin{cases} o_{it}^*(\mu, \lambda | p_{it}, v_{it}) + \epsilon_{it} & \text{if } o_{it}^*(\mu, \lambda | p_{it}, v_{it}) + \epsilon_{it} < p_{it}, \\ p_{it} & \text{if } o_{it}^*(\mu, \lambda | p_{it}, v_{it}) + \epsilon_{it} \geq p_{it}, \end{cases}$$

where ϵ_{it} is a normally distributed error term, $N(0, \sigma_\epsilon^2)$, and $o_{it}^*(\mu, \lambda | p_{it}, v_{it})$ comes from the first-order condition $v - o^* = G(o^* | p) / g(o^* | p)$.

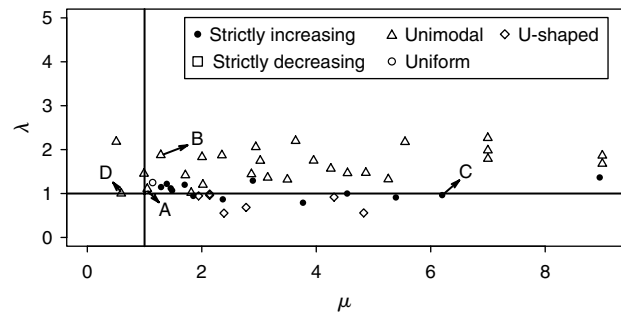
The likelihood function of the nonlinear Tobit model is

$$\begin{aligned} \max_{\mu, \lambda} L(\mu, \lambda, \sigma^2 | o, v, p) \\ = \prod_{i=1}^{48} \prod_{t=1}^{30} \prod_{p_{it} \leq o_{it}} \Pr(o_{it} \geq p_{it}) \\ \cdot \prod_{p_{it} > o_{it}} f(o_{it} | o_{it} < p_{it}) \Pr(o_{it} < p_{it}). \end{aligned}$$

Table 4 reports the maximum likelihood estimation results of this nonlinear Tobit model. The estimates of $\mu = 2.39$ and $\lambda = 1.35$ imply that $G(o | p)$ is S-shaped and there is a positive relationship between the offers and auction prices.¹² A Wald test rejects the Bayesian model, $\mu = \lambda = 1$, in favor of this subjective prior model with a p -value of less than 0.001. Once again there is significant heterogeneity across individuals, as a full fixed coefficient model cannot be rejected with a likelihood ratio test (see the last row of Table 4 for the log-likelihood value of the fixed coefficient regression).

The fixed coefficient model exhibits a diversity of behavioral rules that we now connect to the individual estimates of the structural parameters μ and λ . First, we present in Figure 8 a scatterplot of each buyer's joint estimate of (μ_i, λ_i) . We classify each buyer's

Figure 8 Individual Estimates of Parameters μ and λ from the Subjective Posterior Model



subjective posterior PDF according to the following joint hypothesis tests:¹³

1. Uniform—we fail to reject $\mu = \lambda = 1$.
2. Unimodal—we reject $\lambda = 1$ in favor of $\lambda > 1$.
3. U-shaped—we reject $\lambda = 1$ in favor of $\lambda < 1$.
4. Strictly increasing—we fail to reject $\lambda = 1$, and we reject $\mu = 1$ in favor of $\mu > 1$.
5. Strictly decreasing—we fail to reject $\lambda = 1$, and we reject $\mu = 1$ in favor of $\mu < 1$.

Let us consider four subjects with differently shaped subjective posterior PDFs and compare their behaviors in the experiment. First, consider Buyer A, whose parameter estimates are marked “A” in Figure 8 and are close to the Bayesian type at (1, 1). Buyer A's estimated $g(o | p)$ and his choice data are presented in the first row of plots of Figure 9. Here, we see that his subjective posterior transformation is nearly one-to-one with the Bayesian posterior, and correspondingly, his experimental choices of when to bargain and consequent take-it-or-leave-it offers closely agree with the theory. Buyer B, in contrast, has a unimodal subjective PDF consistent with a disproportionately high perception that the auction winner's cost is between 20% and 80% of the auction price. This leads Buyer B to reject every auction outcome and make aggressive counteroffers. Buyer C's posterior reflects a different belief of affiliation; the strictly increasing posterior PDF reflects a belief that the winner's cost is very likely close to the auction price.¹⁴ Consequently, the buyer seldom rejects the auction outcome and demands very small price reductions in the few cases he does. On the other end of the optimism spectrum is Buyer D, whose strictly decreasing posterior PDF exhibits a strong negative bias on the winner's cost and leads to a high rejection rate and very aggressive counteroffers.

¹² Finding probability weighting functions that are not inverted S-shaped is common in strategic decision tasks; for example, bidders in first-price sealed-bid auctions have been estimated to have convex-shaped functions (Goeree et al. 2002) and S-shaped functions in normal form games (Goeree et al. 2003).

¹³ We choose a test size of 0.05 in each case.

¹⁴ The reader may notice that we estimate seven buyers having inverted S-shaped $G(o | p)$; however, in each of these cases, the function is essentially convex, with only a slight overweighing of a small interval of low probabilities.

Figure 9 Four Different Estimated $g(o|p)$ and the Corresponding Buyer Subject's Behavior

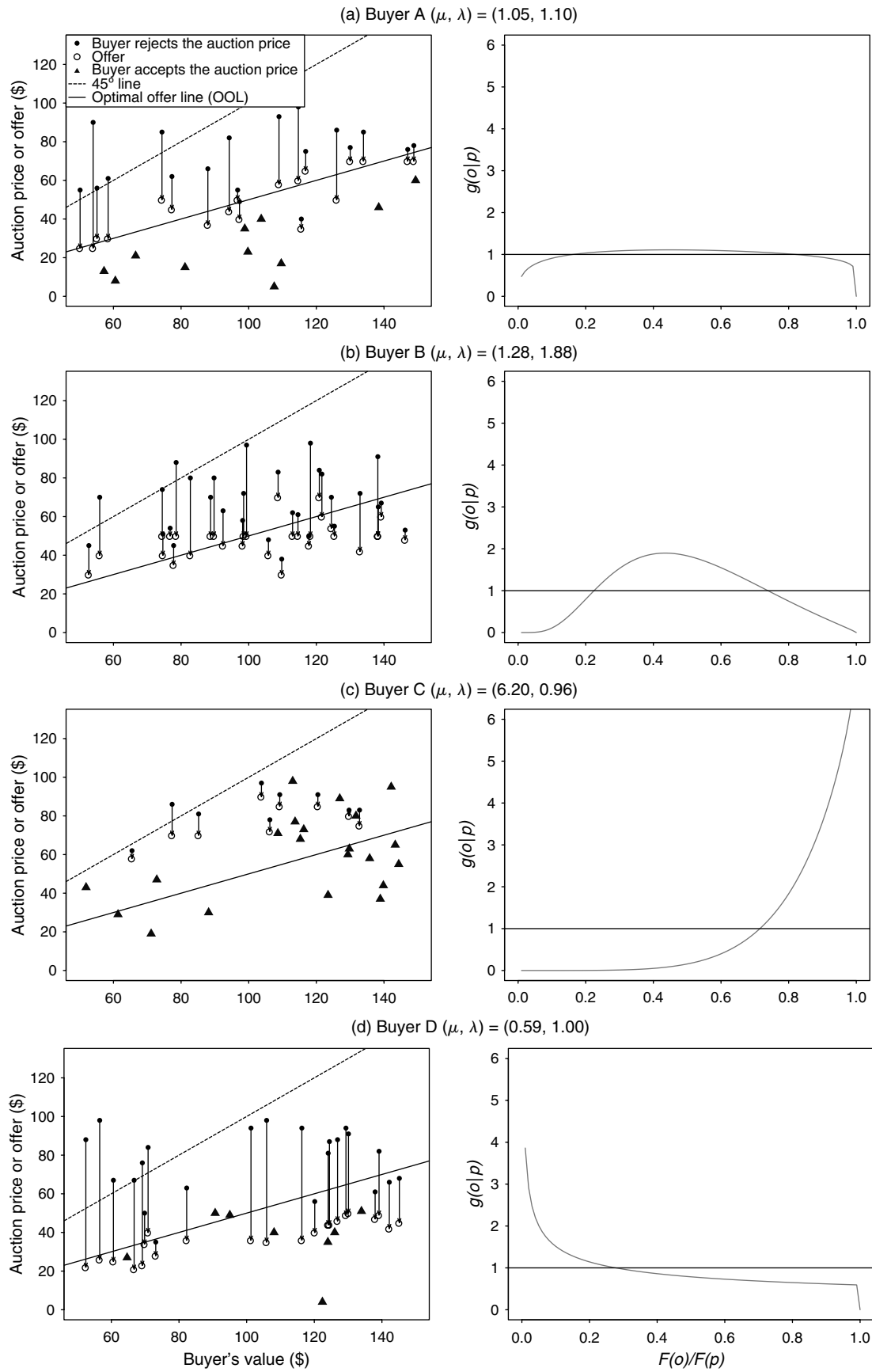
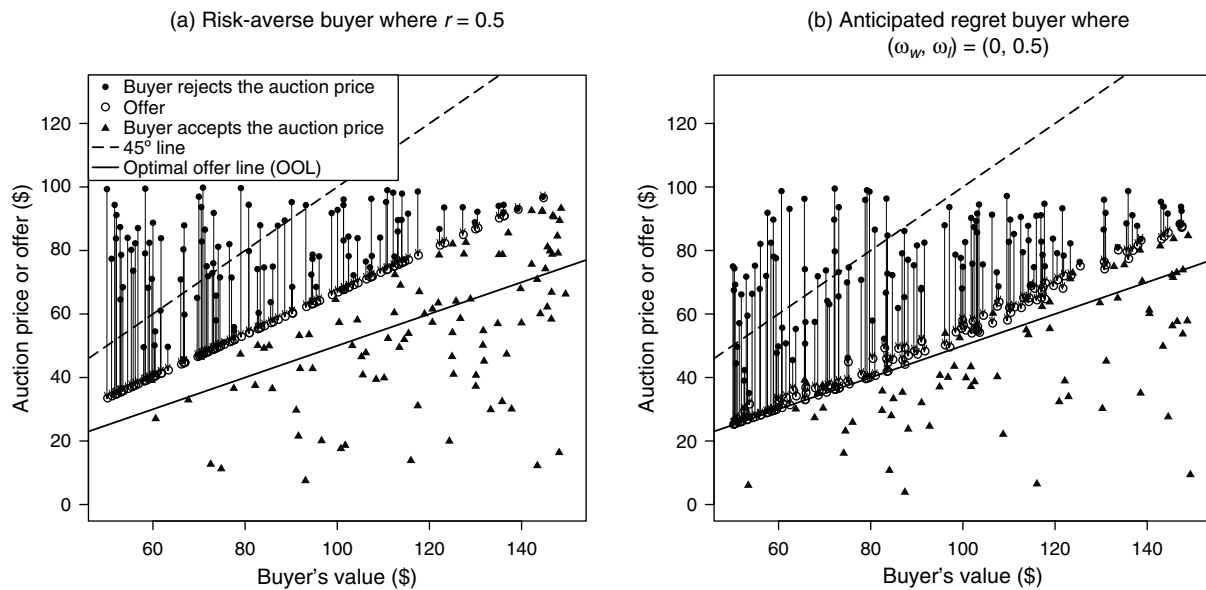


Figure 10 Buyer Behavior Under Risk Aversion and Anticipated Regret: Some Hypothetical Decisions for Randomly Selected Scenarios



5.2. Risk Aversion and Anticipated Regret: Two (Non)Explanations

Two behavioral models successfully used to explain bidder deviations from Nash equilibrium strategies are risk aversion (Cox et al. 1982, 1988) and anticipated regret (Engelbrecht-Wiggans and Katok 2007, 2009; Filiz-Ozbay and Ozbay 2007). However, neither model can explain the positive relationship between auction prices and take-it-or-leave-it offers. Risk aversion can only influence the location of the optimal offer line, whereas in the case of anticipated loss regret, there is no impact on the buyer's strategy when the auction price exceeds his value, and there is a negative relationship between take-it-or-leave-it offers and auction prices when his value exceeds the auction price.

First consider the case where the buyer is strictly risk averse but forms Bayesian posteriors. The following proposition shows that for any v , the optimal take-it-or-leave-it offer exceeds the risk-neutral offer and does not vary with price.

PROPOSITION 3. Assume the buyer's expected utility function satisfies $u(0) = 0$, $u'(x) > 0$, and $u''(x) < 0$. Then

- the optimal offer o^* is greater than the optimal offer for the risk-neutral case;
- for an interior optimal offer o^* , $\partial o^* / \partial p = 0$.

PROOF. See the appendix.

Consider an example in which a buyer in our experiment has the expected utility function $u(x) = x^r$, characterized by the constant coefficient of relative risk aversion $1 - r$. For a strictly positive r , the buyer's Nash equilibrium strategy is to accept all auction prices below the threshold $o^*(v; r)$ and to counteroffer this

threshold otherwise. It is straightforward to show that $o^*(v; r) = v/(1 + r)$. In Figure 10, the left-hand side plot shows the hypothetical behavior of a buyer for whom $r = 0.5$. Clearly, the offers do not depend on the price, and the OOL is steeper than in the risk-neutral case.

Now we show that in a model of anticipated regret, there is no relationship between the optimal offer and auction price when this price exceeds the buyer's value, and the relationship is negative when the auction price is below the buyer's value. In an anticipated regret model, the decision maker's utility function includes disutility for the failure to capture all ex post realized potential gains by ex ante decisions. For buyers in our experiment, both loss and win regrets are possible. A loss regret occurs when the buyer opts to bargain rather than accept a profitable auction price and then his take-it-or-leave-it offer is rejected. A win regret occurs when an offer is accepted and the buyer realizes a lower offer could have been accepted. In this case the magnitude of the win regret is unknown, unlike the case of loss regret. Here, we follow the suggestion of Davis et al. (2011) and set the win regret equal to the accepted offer less the expected winner seller's cost conditional on this bargaining outcome, namely, $o/2$.¹⁵

Let us consider an explicit model in which win and loss regrets result in decrements to utility by

¹⁵ We proceed assuming there is no winner's regret for accepting the auction outcome. Doing so only influences the decision of whether to bargain, not the actual amount of the optimal offer conditional on bargaining.

proportional penalties ω_w and ω_l , respectively. In this case we can express the expected utility of an offer o as

$$E[u(o | v, p; \omega_w, \omega_l)] = \begin{cases} \left(v - o \left(1 + \frac{\omega_w}{2}\right)\right) \frac{o}{p} & \text{if } p \geq v, \\ \left(v - o \left(1 + \frac{\omega_w}{2}\right)\right) \frac{o}{p} - \omega_l(v - p) \left(1 - \frac{o}{p}\right) & \text{if } p < v. \end{cases} \quad (8)$$

Conditional upon bargaining, the optimal take-it-or-leave-it offer is

$$o^*(v, p; \omega_w, \omega_l) = \begin{cases} \frac{v}{2(1 + \omega_w/2)} & \text{if } p \geq v, \\ \frac{v + \omega_l(v - p)}{2(1 + \omega_w/2)} & \text{if } p < v. \end{cases} \quad (9)$$

From Equation (9), when the auction price exceeds the buyer's value, his optimal offer does not depend on the auction price. When the price is below his value, then there exists a *negative* relationship, which is the opposite of what we observe. When deciding whether to bargain, the buyer evaluates whether $v - p$ exceeds his expected utility (8) evaluated at his optimal offer given in Equation (9). We present the hypothetical decisions of a buyer for whom $\omega_w = 0$ and $\omega_l = 0.5$ on the right-hand side of Figure 10.

6. Experimental Treatment Testing Robustness

After reporting our main experimental results and subsequent ex post behavioral explanations, it is natural to question the robustness of these results to the information subjects are given and alternative explanations of buyer behavior. We report on three additional experimental treatments to address these concerns. First, we examine whether sellers learn to adhere more closely to the equilibrium strategy in the auction phase or learn to be more “imperfect” in the bargaining phase when given more extensive feedback on the procurement outcome. Second, we test whether our estimated transformed subjective posteriors are actually nonequilibrium beliefs by having buyers play against automated equilibrium playing sellers—and informing buyers of this. Finally, we investigate whether distorted posteriors can be corrected by providing buyers with an expert decision support system that is based on the true probabilities an offer will be accepted.

6.1. Effects of Enriching Sellers' Feedback

Procurement processes in practice generally provide a richer set of feedback to auction-losing sellers than we do. The Nash equilibrium does not change if we

inform the auction loser of the subsequent actions taken in the bargaining phase; however, this additional information could affect seller behavior through more rapid learning or invoking social preference norms. Accordingly, we conduct two sessions using the same pairings, values, and costs from two sessions of our original treatment, but now we inform the auction-losing seller about the buyers' decision to bargain, the amount of any take-it-or-leave-it offer, and the winning seller's response. We call this the enriched feedback (EF) treatment.

We find no significant differences in the sellers' auction or bargaining behaviors. We pool the auction data from the first two sessions of the original treatment with the two EF sessions and then estimate the bid function component of the seller's strategy. Table 5 reports the OLS estimates in panel A. First, we note that an F -tests fails to reject the absence of treatment effects in both the slope and constant terms. Furthermore, the slope, as in the original treatment, is very close to one. Given the lack of a treatment effect, we find that with enriched feedback sellers still do not exactly bid their cost. Panel B of Table 5 shows a slight increase in the number of absolute bid deviations of less than one dollar as well as a slight increase in the number of deviations greater than three dollars. With respect to the bargaining phase, there is essentially no treatment effect. Sellers rejected only 3 of 188 profitable offers in the EF treatment and 0 of 169 profitable offers in the corresponding two sessions of the original treatment.

Table 5 Comparison of Sellers' Behavior in the Auction Phase for the Original and Enriched Feedback Treatments

Panel A: OLS estimation of the sellers' auction bid functions			
Variable	Estimate	Standard error	
Constant	2.79***	0.79	
d_o^a	−1.19	1.13	
$Cost$	0.97***	0.01	
$d_o \times Cost$	0.01	0.02	
Adjusted R^2	0.95		
H_0 : Coefficients of d_o and $d_o \times Cost$ are jointly zero	$F\text{-stat.} = 2.63^b$	$p\text{-value} = 0.07$	
Panel B: Comparison of cumulative absolute bid deviations from realized costs			
Treatment	$ b - c \leq 1$ (%)	$ b - c \leq 2$ (%)	$ b - c \leq 3$ (%)
Enriched feedback	66.39	74.17	78.61
Original—first 2 sessions	60.83	78.89	87.50
Original—all 8 sessions	56.53	74.93	82.64

^a d_o is the dummy variable for the original treatment

^bThe distribution of the test statistic is $F_{2,716}(x)$.

***Significant at the 1% level.

6.2. Robustness of the Transformed Bayesian Posterior Model

An alternative to our transformed posterior model is that subjects' posteriors are Bayesian, but their beliefs regarding the sellers' strategies are incorrect.¹⁶ To test this alternative hypothesis, we conduct the automated seller (AS) treatment. In this treatment, all subjects are buyers who face computerized sellers following their Nash equilibrium strategy. We inform buyers that sellers are following this strategy.¹⁷ We conduct three sessions, each with 16 subjects, of this treatment. If buyers deviate from their equilibrium strategy in the original treatment because of disequilibrium beliefs, then buyer behavior should conform to theoretical predictions. As we show shortly, they do not.

In our final treatment, we test whether transformed posteriors are a simple-to-correct bias. In the automated seller with calculator (ASC) treatment, we augment the AS treatment by providing buyers with a calculator both when deciding whether to bargain and when choosing the offer size. When the buyer enters a potential offer, the calculator returns six items: the profit if the buyer accepts the auction outcome, probability the offer will be accepted, profit if the offer is accepted, probability the offer will be rejected, profit if the offer is rejected (always zero), and expected profit from making the offer. The calculator also has a history window showing all previously evaluated offers of the current period and corresponding output. Furthermore, before a buyer can submit a take-it-or-leave-it offer, he is required to successfully enter the offer in the calculator. This ensures the buyer has seen the correct probability of his offer being accepted and the expected payoff. Despite providing—and in fact obligating buyers to use—an expert decision support system, the positive relationship between price and offers strengthens.

We first show how the two automated seller treatments alter the relationship between auction prices and buyers' offers. Table 6 presents the linear Tobit estimates of the buyers' offers from the pooled original, AS, and ASC treatments. Although a likelihood ratio test rejects this pooled regression in favor of a full fixed coefficient model, we still believe this is an informative starting point for discussions. When switching from human (original) to automated sellers, the sensitivity of offers with respect to price and value does not change. When we provide buyers with calculators, the results are striking in that buyers' offers have an even stronger positive relationship with the auction prices.

In both the AS and ASC treatments, buyers exhibit subject posterior density functions that are unimodal

¹⁶ For an discussion on the extensive use of rational models with disequilibrium beliefs, please see Crawford et al. (2013).

¹⁷ We use the same sequences of realized costs and values as the buyers faced in our original treatment.

Table 6 Linear Tobit Estimates for the Original, AS, and ASC Treatments: Pooled Data

Variable	Estimate ^a	Standard error ^a
Constant	−0.45	3.10
d_{AS}	−5.41*	3.16
d_{ASC}	−4.49	3.31
Price	0.35***	0.04
$d_{AS} \times Price$	0.03	0.04
$d_{ASC} \times Price$	0.09***	0.05
Value	0.34***	0.01
$d_{AS} \times Value$	−0.03**	0.01
$d_{ASC} \times Value$	−0.06***	0.02
σ_ϵ	12.34***	0.26
Log-likelihood pooled	−13,451	
Log-likelihood individual	−11,518	
Likelihood ratio test stat.	3,864	$p\text{-value} = 0.00$

Notes. The null of the specification test is the pooled model, and the alternative is the individual model. The test statistic is $LR = -2(LL \text{ Pooled} - LL \text{ Individual})$ and is distributed χ^2 with 566 degrees of freedom.

^aStandard errors are clustered by session.

*Significant at the 10% level; **significant at the 5% level; ***significant at the 1% level.

Table 7 Nonlinear Tobit Estimates of the Subjective Posterior Model for the Original, AS, and ASC Treatments: Pooled Data

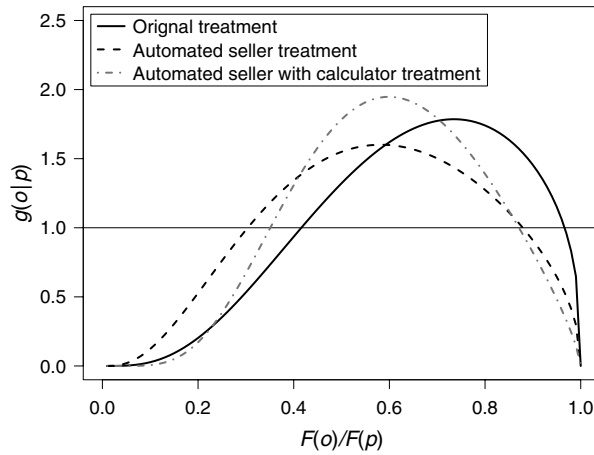
Parameter	Original treatment	AS treatment	ASC treatment
μ	2.39	1.68	2.24
λ	1.35	1.46	1.71
σ_ϵ	11.31	11.80	11.35
Log-likelihood pooled	−4,070	−4,769	−4,592
Log-likelihood individual	−3,613	−4,359	−4,012

Notes. All coefficients are significant at the 1% level. Likelihood ratio tests reject the pooled model in favor of the individual model at a 1% level of significance for all three models.

and whose modes occur at lower quantiles. Table 7 presents the nonlinear Tobit estimates of the subjective posterior model for the original, AS, and ASC treatments. The three estimated subjective posterior densities are plotted in Figure 11. For the AS and ASC treatments, the estimated posteriors reflect modes occurring at lower quantiles and lower density in the top three deciles than for the original treatment posterior. This reflects behaviorally in a greater willingness to bargain and more aggressive offers because the perceived affiliation between the winner's cost and the auction price is stronger, and the expected winner's cost is a smaller percentage of the price.

7. Concluding Remarks

In this study, we examine behavior in reverse English auctions, followed by a buyer option to engage in ultimatum bargaining. As Bulow and Klemperer (1996) showed, a Nash equilibrium of this game implements the optimal mechanism for procurement. We find strong support for this equilibrium modulo buyer's ultimatum

Figure 11 Estimated Subjective Posteriors for the Original, AS, and ASC Treatments

offers having a positive relationship with auction prices. This turns out to have *economic* significance because the average buyer's surplus in our experiment was about 7.5% less than would have been earned with optimal offers. For the procurement official using reverse auctions, our results provide justification for the practice of engaging in postauction negotiations when qualifying additional suppliers is not feasible. This justification is twofold: expected buyer surplus increases and it appears sellers do not harbor strong social utility concerns when being forced to negotiate after an auction. However, the behavioral issue of decision makers failing to use the information revealed in the auction in an optimal Bayesian manner leads to unrealized potential benefits.

The bad news for procurement organizations is that our results reveal that the suboptimal processing of auction information is difficult to correct. As we show, simply providing accurate decision support systems does not affect surplus-improving behavior. This suggests that either the subjective posterior model is a preference primitive and organizations need to consider how to construct mechanisms to account for this or it is a deep-seated bias that requires more intensive training to correct.

Supplemental Material

Supplemental material to this paper is available at <http://dx.doi.org/10.1287/mnsc.2013.1880>.

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Appendix

PROPOSITION 2. Assuming that $F(c)$ is the uniform distribution on the interval $[0, \bar{c}]$ and $G(o|p)$ is calculated according to Equation (7), then

- (i) if $\lambda \geq 1$ and $f'(o) \leq 0$, then $o + G(o|p)/g(o|p)$ is strictly increasing and there is a unique o^* ;
- (ii) if $\lambda > 1$, then an interior optimal offer o^* is strictly increasing in the auction price p .

PROOF. Start by noting the following

$$g(o|p) = f(o) \frac{\mu\lambda}{F(o)} \left(-\ln \frac{F(o)}{F(p)} \right)^{\lambda-1} G(o|p),$$

and

$$\begin{aligned} g'(o|p) &= f'(o) \frac{\mu\lambda}{F(o)} \left(-\ln \frac{F(o)}{F(p)} \right)^{\lambda-1} G(o|p) \\ &\quad - f^2(o) \frac{\mu\lambda}{F^2(o)} \left(-\ln \frac{F(o)}{F(p)} \right)^{\lambda-1} G(o|p) \\ &\quad - (\lambda-1) f^2(o) \frac{\mu\lambda}{F^2(o)} \left(-\ln \frac{F(o)}{F(p)} \right)^{\lambda-2} G(o|p) \\ &\quad + f^2(o) \frac{\mu^2 \lambda^2}{F^2(o)} \left(-\ln \frac{F(o)}{F(p)} \right)^{2\lambda-2} G(o|p). \end{aligned}$$

Now the derivative we are interested in is

$$\begin{aligned} \frac{d(o + G(o|p)/g(o|p))}{do} &= 1 + \frac{g^2(o|p) - G(o|p)g'(o|p)}{g^2(o|p)} \\ &= 2 - \frac{G(o|p)g'(o|p)}{g^2(o|p)}. \end{aligned}$$

Simplifying we get

$$\begin{aligned} \frac{d(o + G(o|p)/g(o|p))}{do} &= 1 + \frac{1}{\mu\lambda(-\ln(F(o)/F(p)))^{\lambda-1}} + \frac{\lambda-1}{\mu\lambda(-\ln(F(o)/F(p)))^{\lambda}} \\ &\quad - \frac{f'(o)}{\mu\lambda f^2(o)(1/F(o))(-\ln(F(o)/F(p)))^{\lambda-1}}. \end{aligned} \quad (10)$$

By inspection of Equation (10) we can that our assumption that $\lambda \geq 1$ and $f'(o) \leq 0$ ensures this expression is positive.

Next, when we substitute for $G(o|P)$ and $g(o|P)$ into Equation (2), the first-order condition for an interior optimal take-it-or-leave-it offer, we have

$$\mu\lambda \frac{p}{o^*} \left(-\ln \left(\frac{o^*}{p} \right) \right)^{\lambda-1} = \frac{p}{v-o^*}.$$

Taking the natural logarithm of both sides,

$$\ln(\mu\lambda) - \ln(o^*) + (\lambda-1) \ln \left(-\ln \frac{o^*}{p} \right) + \ln(v-o^*) = 0.$$

We differentiate with respect to p at the optimal solution to obtain the following:

$$\frac{\lambda-1}{\ln(p)-\ln(o^*)} \left(\frac{1}{p} - \frac{1}{o^*} \frac{\partial o^*}{\partial p} \right) - \frac{1}{o^*} \frac{\partial o^*}{\partial p} - \frac{1}{v-o^*} \frac{\partial o^*}{\partial p} = 0. \quad (11)$$

Rearranging terms, we obtain

$$\frac{\partial o^*}{\partial p} = \frac{(\lambda - 1)(v - o^*)o^*}{(v \ln(p/o^*) + (\lambda - 1)(v - o^*))p}. \quad (12)$$

Obviously, the optimal offer should be smaller than the value and larger than the auction price. Hence, when $\lambda > 1$, $\partial o^*/\partial p$ is strictly positive, and the optimal offer is strictly increasing in the auction price. Also note that when $\lambda = 1$, there is no relationship between optimal offer o^* and auction price p —not surprising as the model becomes observationally equivalent to assuming the buyer is risk averse/loving. Finally, when $\lambda < 1$, $\partial o^*/\partial p$ is ambiguous.

PROPOSITION 3. Assume the buyer's expected utility function satisfies $u(0) = 0$, $u'(x) > 0$, and $u''(x) < 0$, then

- (i) the optimal offer o^* is greater than the optimal offer for the risk-neutral case;
- (ii) for an interior optimal offer o^* , $\partial o^*/\partial p = 0$.

PROOF. The first-order condition for maximization can be expressed as

$$u(v - o^*) \frac{f(o^*)}{F(p)} - u'(v - o^*) \frac{F(o^*)}{F(p)} = 0.$$

Rearranging terms yields

$$\frac{u(v - o^*)}{u'(v - o^*)} = \frac{F(o^*)}{f(o^*)}. \quad (13)$$

Let $z(x) = (u(v - x))/(u'(v - x))$. Then

$$z'(x) = \frac{u(v - x)u''(v - x)}{u'(v - x)^2} - 1.$$

By the strict concavity of $u(x)$, $z'(x) < -1$.

When $o^* = 0$, the left-hand side of (13) is strictly positive and the right-hand side is zero. Furthermore, because the left-hand side is strictly decreasing and the right-hand side increasing, they will intersect at most one time on the domain $[0, p]$. Since the slope of the left-hand side is less than -1 it is decreasing faster than in the risk-neutral case. Also, the utility at $o^* = v$ is zero for both the risk-averse and risk-neutral cases, so $u(v - o^*) < v - o^*$ for $o^* \in [0, u]$. Therefore, it will intersect the right-hand side at a higher offer level than in the risk-neutral case. Thus, the optimal offer is higher than in the risk-neutral case.

Finally, the independence of the optimal offer from the auction price is clear from the first-order condition expressed as in (13).

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