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Fragility, stress, and market returns ☆,☆☆

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ABSTRACT

We propose a novel risk measure that relates to subsequent negative conditional stock market returns. Our risk measure considers both the fragility and stress of the market. Fragility is measured by the Fragility Index developed by Berger and Pukthuanthong (2012) and market stress is based on several economic variables. Results show that incorporating both market stress and fragility improves the information content of a risk measure. Our risk measure relates to poor subsequent monthly market returns. We show the risk measure contains predictive information in a purely ex-ante specification.

1. Introduction

Given the recent financial environment, as well as the impact that crashes may have on investor wealth, financial crises and risk have been the focus of significant research. Recent research identifies many variables that may predict an increasing likelihood of a market downturn, or of negative joint co-exceedances across markets (e.g., Markwat et al., 2009; Kumar et al., 2003; Christiansen and Ranaldo, 2009; Kritzman et al., 2011, Berger and Pukthuanthong, 2012). However, the extant literature largely focuses on conditional probabilities, and does not consider first moments. Therefore, it is unclear if the existing risk measures relate to conditional mean returns. We develop a new measure of market risk, and investigate the relation between risk and subsequent returns. A key innovation is that our novel risk measure incorporates information regarding market fragility, as well as market stress, and consequently offers a contribution relative to extant measures. Conditional mean market returns for months following the risky state are negative, and significantly lower than mean returns conditional on the stable state.

It is well known that second moments of financial data are persistent (cf., Poterba and Summers, 1986). Extending the literature on volatility, recent research considers the probability of financial crashes, thereby focusing on the most extreme periods of volatility. For example, Markwat et al. (2009) document a domino effect in which an initial local or regional shock increases the likelihood of a subsequent global shock. Kumar et al. (2003) identify economic variables that predict an increasing probability of currency devaluation. Christiansen and Ranaldo (2009) find that closer economic linkages following EU entry increase the probability of a joint co-exceedance across nations. Kritzman et al. (2011) create their Absorption Ratio and find that significant stock downturns are frequently preceded by spikes in this measure. Berger and Pukthuanthong (2012) study joint co-exceedances across nations. They find high levels of their Fragility Index indicate significantly higher probabilities of market crashes across many nations.

The literature above indicates that information variables exist that may be associated with an increasing likelihood of a market downturn. However, excluding Kritzman et al. (2011), these studies focus on second moment estimation or on the conditional probability of an event occurring. Variables that predict an increasing probability of a shock do not necessarily imply poor subsequent average returns. Periods in which risk is high, or risk sources are concentrated, may lead to especially strong returns in the event

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of a positive innovation in an underlying factor. For example, Kritzman et al. (2011) note that in many instances strong stock performance follows spikes in the Absorption Ratio. Similarly, Berger and Pukthuanthong (2012) show that their Fragility Index also predicts joint co-exceedances in the right tail of the distribution, indicating a greater likelihood of strong simultaneous performance across multiple markets. From this, it is not clear that the risk measures present in the literature relate to subsequent mean returns. Consequently, we focus the current study on the relation between risk and subsequent conditional returns.

Our risk measure considers both fragility and market stress. Fragility can be considered as the susceptibility of the system to a shock. A fragile system can be expected to 'break' in the event of a shock occurring. For example, Kritzman et al. (2011) discuss that high levels of their Absorption Ratio indicate fragility, which may be a necessary, but not sufficient, criteria for a sharp downturn. Similarly, Berger and Pukthuanthong (2012) argue that their measure captures periods in which a shock would be most damaging, if it were to occur. Or alternatively, they argue that the impact of a given shock would be the greatest during periods of high market fragility. A key point is that fragility alone will not necessarily lead to a crash. In this context, the occurrence of a crash would depend on the system being fragile, as well as a shock occurring. We combine a fragility measure similar to that of Berger and Pukthuanthong (2012), with a number of economic variables that identify periods of market stress, which can be considered as turbulent periods within the market in which shocks may be more likely to occur. Arguably, the intersection of increasing fragility, indicative of a system that is susceptible to a shock, with increasing market stress, indicative of a likely shock, will precede many market downturns.

We find strong results for our novel risk measure, indicating increases in the risk measure precede a flight to quality dynamic in which prices adjust following innovations in risk. These results indicate that the intersection of fragility and market stress strongly relates to subsequent poor conditional mean returns. In isolation, we show that neither fragility nor market stress alone contains the same information as our combined risk measure. As examples of our results, we use lagged market volatility as the primary indicator of market stress. Considering mean returns, our measure identifies 131 months within our sample as risky based on an increase in both fragility and stress during the previous month. The average excess US market return during the subsequent months is −0.62%. The average excess return during the remaining 445 months is significantly higher, and equal to +0.74%. Results further indicate that minimum daily returns are larger in magnitude, and large daily losses occur with a greater frequency during months that follow the risky state. We find similar results across additional measures of market stress. For example, using VIX as an indicator of stress, average excess returns during month t are −0.78% and +0.83% conditional on increasing and decreasing values of our risk measure during month $t-1$, respectively. Finally, our regression results reveal the importance of both stress and fragility. Specifically, the combination of stress and fragility decreases conditional mean returns, as well as the lower percentiles of the return distribution. For example, conditional on the fragile state, a one standard deviation increase in stress decreases the conditional 10th percentile of monthly returns from −4.09% to −6.71%, and this change is much larger in magnitude when compared to the impact of stress in isolation (stress without fragility). Finally, results indicate that absent fragility, stress may lead to additional upside in returns. For example, if the market is not in the fragile state, a one standard deviation increase in stress increases the conditional 90th percentile of monthly returns from 5.47% to 6.83%.

Taken in total, the results present a novel measure of risk, in which increases in both stress and fragility precede market downturns in the short-run. In addition, there is evidence that long-run returns relate positively to increases in stress and fragility.[1] Therefore, our primary interpretation of the results is that increases in risk lead to price adjustments (and consequently negative returns) as these innovations in risk are priced in the market. However, the results may also be consistent with a form of neglected risk. In particular, Gennaioli et al. (2012) present a standard model of innovation, but assume that investors ignore certain unlikely risks. Their model builds on the 'local thinking' of Gennaioli and Shleifer (2010), and emphasizes that an agent may not consider all possible outcomes for a risky asset, but rather only the most likely outcomes, while the least likely outcomes are neglected. Considering that our quantile regression results suggest the strongest impact of fragility and stress on returns manifests in the lower quantiles of the distribution, the high stress and high fragility state would be a good candidate for the form of unlikely and neglected risk discussed above. This idea may also relate to crash risk. For example, Kim et al. (2011) find high levels of crash risk relate to corporate tax avoidance. Baron and Xiong (2014) find that credit expansion predicts increased crash risk, as well as lower mean returns over the subsequent one to eight quarters, suggesting that this form of risk may also be neglected. Therefore, an alternative interpretation of our results would be that the results represent an example of an unlikely outcome that is neglected under local thinking, and consequently not priced in the market. We leave the issue of potential pricing of fragility risk for future research, and focus the current study on the short-run relation between innovations in risk and market returns.

The paper proceeds as follows. In Section 2, we discuss the intuition behind the novel risk measure, and highlight the importance of both stress and fragility. Sections 3 and 4 present our primary results relating conditional monthly returns, as well as measures of large daily losses, to our identified risk states. Section 5 considers the relation between multiple stress variables, and assesses the information content of the risk measure across multiple stress variables. Finally, Section 6 concludes.

2. Data and risk measures

In this section we describe the risk measure and necessary data. We focus on the monthly excess US market return as our test portfolio. In general, our analyses relate risk to subsequent market returns. In contrast to the short-term predictive power shown by Berger and Pukthuanthong (2012), we consider a longer window of monthly returns. Applying a longer window may provide a more relevant measure for policy makers. We further extend Berger and Pukthuanthong (2012) by combining stress and fragility and considering conditional mean returns. Fragility represents the susceptibility of the market to a shock, and stress captures the possibility of shocks occurring.

Considering stress and fragility, we focus on *increases* in both measures, as opposed to *levels* throughout our analysis. First, across a lengthy sample, absolute levels of risk may vary, and relative risk may be more valuable. Specifically, an absolute level of fragility, or of a stress variable, may indicate varying levels of risk at different points within our sample (cf., Kritzman et al., 2011). Second, our analysis attempts to relate risk in month $t-1$ to negative mean returns during month t. This requires some level of mispricing. Therefore, we expect that general levels of risk may be priced accurately, but innovations in risk may precede market downturns. Consequently, we focus our analysis on changes in

[1] In unreported results, we regress calendar year excess market returns on changes in fragility during the previous calendar year and find positive and significant results. For example, regressing calendar year returns in year t on relative fragility measured from July through December, compared to January through June in year $t-1$ leads to a positive and significant coefficient, with an R^2 of 7.9%.

risk; however, within Table 1 we also consider levels of short-term fragility, and show that these levels also relate to subsequent returns.

For our primary market stress variable, we use the standard deviation of daily excess market returns. Given volatility persistence, we would expect increasing volatility in month $t-1$ to be followed by additional stress in the subsequent month. Aggregating daily volatility to measure longer-window volatility is related to the realized volatility approach of Andersen et al. (2001) and Andersen et al. (2003). A further advantage of this specification is the lengthy sample for which the necessary data are available. We calculate changes in market volatility, $\Delta\sigma_{mkt,t-1}$, as the difference between the standard deviation of daily excess returns in month $t-1$ and the standard deviation over months $t-12$ through $t-1$. In this way, the variable represents changes in volatility from a short run relative to long run window, with positive values indicating periods of increasing volatility. In Section 5 we consider relationships across multiple stress measures, and show that our primary results are robust to alternative measures of stress.

To measure fragility, we first calculate the US market factor as the first principal component formed from the correlation matrix of excess returns to 49 industry portfolios of US firms, with firms assigned to industry based on four-digit SIC codes.[2] To capture increases in fragility, we regress daily excess returns for each industry on the first PC during month $t-1$, as well as during months $t-12$ through $t-1$. We subtract the average coefficient on the first PC across each industry over the $t-12$ through $t-1$ sample from the average loading from the month $t-1$ sample, and denote this variable ΔFI_{t-1}. From our approach, ΔFI_{t-1} represents the relative level of fragility in month $t-1$, in context of fragility over the previous year. Consequently, positive values of the measure indicate average exposure to the US market factor (first PC) is increasing, which we take to be periods in which the system is becoming fragile. Hereafter, we refer to this relative difference variable as 'fragility' or 'FI.'

We now discuss a number of considerations with respect to the construction of fragility. First, throughout we present two approaches to estimating the first PC within our study. One approach uses the full-sample correlation matrix formed from daily excess returns to extract the eigenvectors. In this approach, the eigenvectors used to construct the first PC will be static. However, the first PC, given by daily returns to the industry portfolios and the relative weights from the eigenvectors, will vary, and our risk measure will consider how loadings on this first PC change through time. As a second approach, we compliment the full sample results with rolling-window PC estimation. In particular, we show that our main results are robust to estimating the first PC with a rolling three-year window. Considering the two approaches in total, the full sample approach will isolate the impact of changing loadings on the first PC, holding the PC weights constant. These results will reveal if time-variation in loadings relates to subsequent returns. The rolling-window approach will reveal if the risk measure is implementable on a purely ex-ante basis.

As an additional consideration, we focus our fragility measure on loadings on the market factor, rather than goodness of fit from the regression model. To explain, Berger and Pukthuanthong (2012) develop a measure of international fragility from the integration measure of Pukthuanthong and Roll (2009). Pukthuanthong and Roll develop a measure of market integration by regressing country market returns on 10 principal components (PCs) formed from seventeen prominent developed markets. In this setting, the R-square from the regression is a measure of integration, and the loading on the first PC represents exposure to a world market factor. It is important to differentiate between integration and risk. Pukthuanthong and Roll (2009) discuss that the R-square measure of integration in the PC setting is not necessarily equivalent to factor exposures in the context of a risk measure. Berger and Pukthuanthong (2012) discuss this point as well. Within the context of a risk measure, the factor loadings or exposures seem relevant, while integration does not necessarily imply risk.[3] Finally, our focus on the market factor, measured by the first PC, allows us to identify measures of stress that correspond to this factor. Alternative measures of fragility that utilize multiple PCs may not necessarily be able to identify the factors underlying the additional PCs, and consequently would not be able to identify related measures of stress.

For both the stress and fragility variables, for ease of interpretation, we normalize the variables to mean of 0, and standard deviation of 1. Both stress and fragility exhibit high levels of autocorrelation. The first order autocorrelations of the changes variables, ΔFI_{t-1} and $\Delta\sigma_{mkt,t-1}$, are 0.254 and 0.553, respectively. First order autocorrelations for the levels of fragility and stress are 0.507 and 0.695, respectively. Our sample consists of the daily return data beginning July 1963. We utilize up to 12 month rolling windows to estimate fragility, and require a minimum of six months of data. Given the six month minimum requirement, we have sufficient data to estimate FI beginning December 1963. We then match this initial observation of FI, the relative difference between the average exposure to the US factor in December 1963 and the average exposure from July 1963 through December 1963, to market returns in January 1964 for the first month of our return sample. We initially present plots of stress and fragility through time. Fig. 1 plots the changes variables, ΔFI_{t-1} and $\Delta\sigma_{mkt,t-1}$, and Fig. 2 plots the levels variables. In Figs. 1 and 2, gray shading identifies periods in which both stress and fragility are increasing, or both are above their full sample average, respectively.

3. Risk and conditional returns

We assess the information content of the risk measure, and consider conditional mean returns across risk states. Panel A of Table 1 presents conditional returns during month t following increases and decreases in fragility ($\Delta FI_{t-1} > 0$ and $\Delta FI_{t-1} < 0$), increases and decreases in stress ($\Delta\sigma_{mkt,t-1} > 0$ and $\Delta\sigma_{mkt,t-1} < 0$), as well as joint increases in stress and fragility ($\Delta FI_{t-1} > 0$ and $\Delta\sigma_{mkt,t-1} > 0$) compared to the condition in which either stress or fragility decreases ($\Delta FI_{t-1} < 0$ or $\Delta\sigma_{mkt,t-1} < 0$). All risk states are identified during month $t-1$, and related to returns during month t. Panel B of Table 1 similarly presents returns during month t conditional on risk and fragility in month $t-1$, but in this Panel, fragility is identified using three-year rolling window PC estimation. This analysis will confirm if fragility and stress contain predictive content in a purely ex-ante fashion. Within Panel C, we present returns during month t conditional on risk partitions also identified during month t. This analysis will have implications for the interpretation of our primary results. Finally, within Panel D, we identify risk states based on levels of fragility, FI_{t-1}, estimated during

[2] We obtain monthly US excess market returns, monthly US risk-free rates, daily US excess market returns, daily industry returns, and daily risk free rates from Kenneth French's data library.

[3] For intuition, consider the simple setting of standard market risk and a portfolio consisting of 95% cash and a 5% allocation to the market portfolio. In the regression setting, the R-square would be 100% as the market portfolio return would explain the entirety of variation in the portfolio return, but the factor loading would be small. The factor loading would best capture the risk of the portfolio, as any loss within the market would correspond to a loss in the investment portfolio of 1/20th the magnitude.

Table 1
Risk and conditional excess returns.

Market condition	Mean	*t*-statistic	P25	P10	σ	Skewness	*N*
Panel A: Risk states and subsequent market returns							
Full sample	0.430	–	−2.210	−5.120	4.557	−0.530	576
$\Delta FI_{t-1} > 0$	0.073	1.97	−2.425	−5.760	4.730	−0.808	300
$\Delta FI_{t-1} < 0$	0.817	(0.024)	−1.885	−4.420	4.336	−0.098	276
$\Delta\sigma_{mkt,t-1} > 0$	0.252	0.79	−2.680	−6.725	5.285	−0.660	250
$\Delta\sigma_{mkt,t-1} < 0$	0.566	(0.214)	−2.100	−3.930	3.911	−0.155	326
$\Delta FI_{t-1} > 0$ and $\Delta\sigma_{mkt,t-1} > 0$	−0.615	2.59	−4.160	−7.790	5.554	−0.865	131
$\Delta FI_{t-1} < 0$ or $\Delta\sigma_{mkt,t-1} < 0$	0.737	(0.005)	−2.030	−4.120	4.176	−0.130	445
Panel B: Returns with three-year rolling window PC estimation							
$\Delta FI_{t-1} > 0$ and $\Delta\sigma_{mkt,t-1} > 0$	−0.353	1.95	−2.880	−7.160	5.340	−0.841	126
$\Delta FI_{t-1} < 0$ or $\Delta\sigma_{mkt,t-1} < 0$	0.649	(0.026)	−2.100	−4.425	4.293	−0.282	450
Panel C: Risk and contemporaneous returns							
$\Delta FI_{t-1} > 0$	0.021	2.27	−2.620	−5.530	4.673	−0.839	300
$\Delta FI_{t-1} < 0$	0.879	(0.012)	−1.790	−4.620	4.392	−0.105	276
$\Delta\sigma_{mkt,t-1} > 0$	−0.592	4.58	−3.610	−7.525	5.411	−0.368	250
$\Delta\sigma_{mkt,t-1} < 0$	1.217	(0.000)	−1.310	−3.520	3.589	−0.116	326
$\Delta FI_{t-1} > 0$ and $\Delta\sigma_{mkt,t-1} > 0$	−1.374	4.53	−4.090	−7.930	5.493	−0.688	131
$\Delta FI_{t-1} < 0$ or $\Delta\sigma_{mkt,t-1} < 0$	0.964	(0.000)	−1.630	−4.260	4.101	−0.137	445
Panel D: Risk levels and subsequent market returns							
$FI_{t-1} > 0$	0.723	1.56	−1.740	−4.620	4.525	−0.627	289
$FI_{t-1} < 0$	0.134	(0.06)	−2.710	−5.690	4.577	−0.440	287
$FI_{t-1} > 0$ and $\Delta\sigma_{mkt,t-1} > 0$	0.602	1.58	−2.045	−4.370	4.368	−0.526	452
$FI_{t-1} < 0$ or $\Delta\sigma_{mkt,t-1} < 0$	−0.198	(0.057)	−3.550	−7.050	5.157	−0.440	124

The table presents summary statistics for excess monthly market returns during month t conditional on market conditions specified in the initial column. Fragility, ΔFI_{t-1}, represents the difference between the average loading on the first PC during month $t-1$, relative to the average loading from months $t-12$ through $t-1$. In Panel B the first PC is estimated with a rolling three-year window. Excluding this panel, the first PC is estimated from the full sample. Stress, $\Delta\sigma_{mkt,t-1}$, represents the difference between the standard deviation of daily excess market returns calculated with observations during month $t-1$, and months $t-12$ through $t-1$. Panel C presents conditional returns during month t with market conditions identified in month t. All remaining panels match market returns during month t with market conditions identified in month $t-1$. Finally, Panel D presents statistics based on levels of fragility, FI_{t-1}, measured as the average coefficient on the first PC taken across industries from month $t-1$. All risk variables are standardized to mean of 0, standard deviation of 1, such that a positive value of FI_{t-1} in Panel D indicates the fragility level is above the full-sample average.

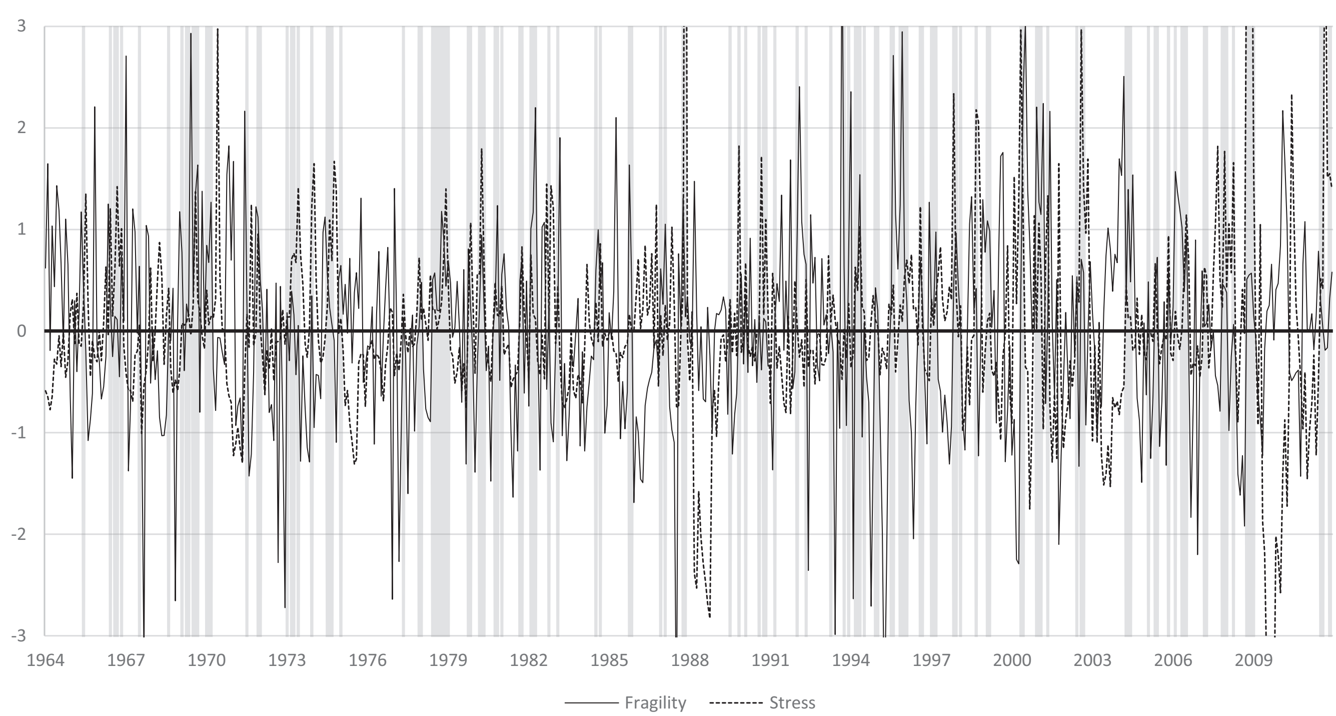

Fig. 1. This figure plots changes in fragility, ΔFI_{t-1}, and changes in stress, $\Delta\sigma_{mkt,t-1}$, through time. To calculate ΔFI_{t-1} the average loading on the first PC of each industry portfolio during months $t-12$ through $t-1$ is subtracted from the average loading during month $t-1$. Changes in stress, $\Delta\sigma_{mkt,t-1}$, is similarly calculated as the difference in standard deviation of daily excess market returns during months $t-12$ through $t-1$ from month $t-1$. Both variables are standardized to mean of zero and standard deviation of one. Vertical gray bars indicate months in which both fragility and stress are increasing.

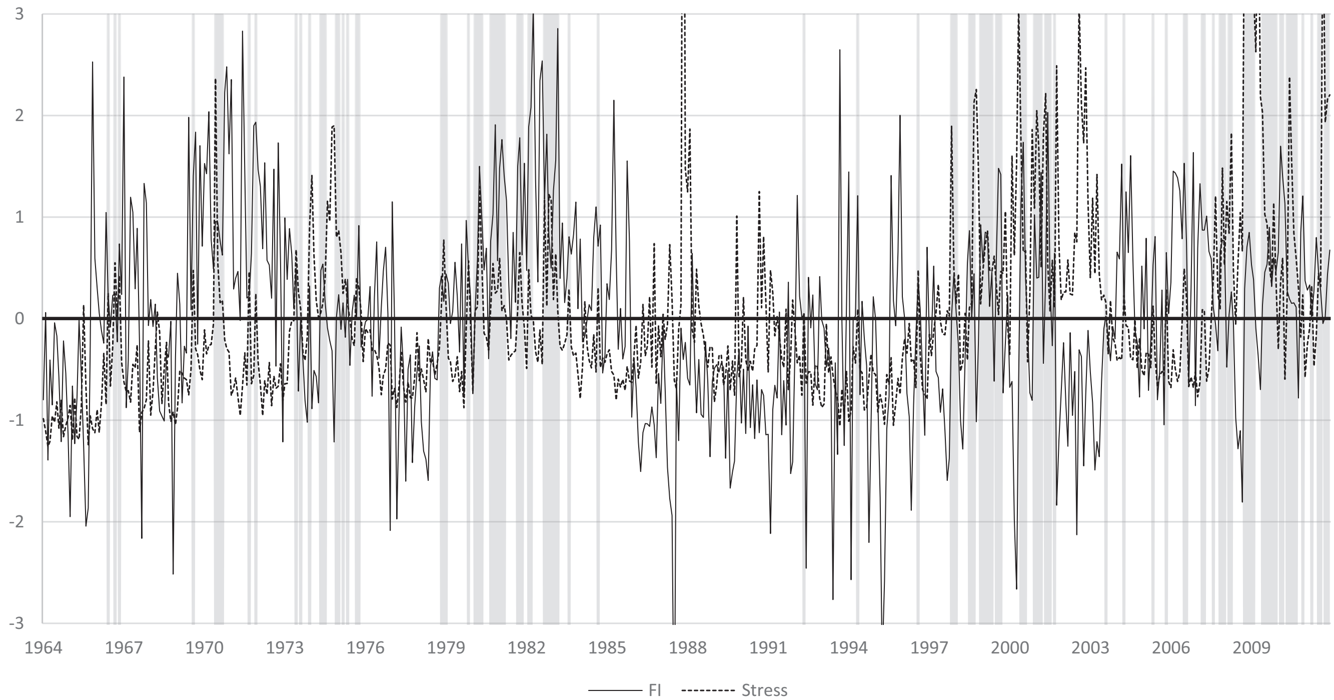

Fig. 2. This figure plots levels of fragility, FI_{t-1}, and levels of stress, $\sigma_{mkt,t-1}$, through time. FI_{t-1} represents the average loading of each industry portfolio on the first PC during month $t-1$. Stress is similarly calculated as the in standard deviation of daily excess market returns during month $t-1$. Both variables are standardized to mean zero and standard deviation of one. Vertical gray bars indicate months in which both fragility and stress are positive, indicating values above the full-sample average.

month $t-1$, opposed to changes in fragility. As all stress and fragility variables are standardized to mean of 0, positive levels of fragility indicate months in which the risk level is above the full sample average. Within all Panels, we report t-statistics, and associated p-values in parentheses, under the null hypothesis that the return in the risky state is not less than the return in the low risk state.

Considering fragility and returns within Panel A of Table 1, we observe lower, but non-negative, mean excess returns during month t following an increase in fragility during month $t-1$. Specifically, following an increase in fragility, the mean, standard deviation, and skewness of returns are 0.07%, 4.73%, and -0.81, respectively. These values compare to statistics of 0.82%, 4.34% and -0.10, indicating substantial decreases in returns and skewness within the increasing risk state, and a marginal increase in volatility. The t-statistic indicates a significant difference in mean returns across conditions. Considering the stress variable, $\Delta\sigma_{mkt,t-1}$, we find no evidence that an increase in stress in isolation significantly impacts subsequent mean returns. Specifically, the t-statistic reveals insignificant differences in mean excess returns across levels of the stress variable. However, the dramatic difference in volatility, 5.29% relative to 3.91%, indicates the stress measure does capture turbulent periods within the market. The final two rows within Panel A reveal the primary results for our risk measure. Following an increase in both stress and fragility, the conditional mean return is equal to -0.62%. This value is 135 basis points below the conditional return following a decrease in either stress or fragility, and the difference across returns is highly significant. Returns following the high risk state also exhibit high levels of volatility, and large in magnitude skewness, compared to the comparable values following a decrease in either stress or fragility. The 10th percentile of the empirical distribution is -7.79% following increases in both stress and fragility, compared to a value of -4.12% following the neutral condition. Finally, within Panel B, using the ex-ante PC specification, the conditional return following the risky condition of -0.35% is 100 basis points below the counterpart conditional return of 0.65%. The difference across returns is significant at the 5% level, and confirms that the risk measure identifies risk states in an ex-ante specification.

Panel C of Table 1 presents contemporaneous market returns across risk states. Our previous results within Panels A and B of Table 1 document a strong relation between increasing risk in month $t-1$ and returns in month t. Therefore, documenting negative returns in month $t-1$ as well as month t is potentially indicative of an explanation in which prices initially underreact to the observed increase in risk. Within Panel C, we find significant differences in returns all conditions considered. Returns during months in which fragility decreases are 0.88%, and this value is 86 basis points greater than returns in months in which fragility increases. Similarly, returns are 1.22% and -0.59% during months in which volatility decreases or increases, respectively. The difference in returns across states appears the largest in months in which both stress and fragility increase, relative to months in which either measure decreases, with conditional returns of 0.96% and -1.37%, respectively.

Overall, considering the relation between risk and conditional returns in both months $t-1$ and t that are presented in Panels A through C of Table 1, we find that poor returns occur during months in which both fragility and stress increase, and increases in these risk measures further precede subsequent negative returns. The poor contemporaneous returns conditional on increases in stress and fragility are similar to the poor returns during months in which volatility increases, but larger in magnitude. This suggests that fragility may amplify the role of stress, and the negative conditional returns are evidence of a flight to quality dynamic. Finally, within Panel D of Table 1, we consider analogous results based on levels, not increases, in fragility. We now find that high levels of fragility also relate to subsequent poor returns. Considering levels of fragility in isolation, the difference in returns following the high and low risk states is 59 basis points, and marginally significant. Similarly, combining levels of fragility with increases in stress reveals conditional subsequent returns of 0.602

Table 2
Risk and conditional daily excess returns.

Market condition	Mean	P10	σ	Skewness	N
Panel A: Conditional minimum daily excess returns					
$\Delta FI_{t-1} > 0$ and $\Delta\sigma_{mkt,t-1} > 0$	−1.999	−3.290	1.942	−4.733	131
$\Delta FI_{t-1} < 0$ or $\Delta\sigma_{mkt,t-1} < 0$	−1.579	−2.710	1.011	−2.323	445
Panel B: Conditional minimum daily excess returns with rolling window PC					
$\Delta FI_{t-1} > 0$ and $\Delta\sigma_{mkt,t-1} > 0$	−1.970	−3.290	1.931	−4.923	126
$\Delta FI_{t-1} < 0$ or $\Delta\sigma_{mkt,t-1} < 0$	−1.592	−2.705	1.036	−2.408	450

Market condition	$r_{mkt,t} < 0\%$	$r_{mkt,t} < -1\%$	$r_{mkt,t} < -2\%$	$r_{mkt,t} < -3\%$
Panel C: Average proportion of daily returns below threshold				
$\Delta FI_{t-1} > 0$ and $\Delta\sigma_{mkt,t-1} > 0$	0.482	0.142	0.040	0.015
$\Delta FI_{t-1} < 0$ or $\Delta\sigma_{mkt,t-1} < 0$	0.464	0.095	0.020	0.005
Panel D: Average proportion of returns below threshold with rolling-window PC				
$\Delta FI_{t-1} > 0$ and $\Delta\sigma_{mkt,t-1} > 0$	0.486	0.134	0.038	0.014
$\Delta FI_{t-1} < 0$ or $\Delta\sigma_{mkt,t-1} < 0$	0.463	0.097	0.021	0.006

The table presents statistics covering daily excess returns during month t, conditional on risk states identified at the end of month $t-1$. Panels A and B present results across risk conditions for the minimum daily excess return during month t, utilizing the full-sample and rolling window PCs, respectively. P10 represents the tenth percentile. Panels C and D present results for the average proportion of daily returns below the given threshold during month t across risk conditions, for the full-sample and rolling window PC approach, respectively. Return thresholds are identified in the column headings in Panels C and D.

and −0.198, following the safe and risky conditions, respectively. Again, this difference is marginally significant. In general, this analysis indicates that levels of fragility may also be useful in identifying periods of risk.

Our primary results consider conditional returns measured through the end of month t, relative to risk states identified at the end of month $t-1$. However, markets may crash during the beginning of the month, and partially recover through the end of the month. Therefore, the conditional monthly returns may present a conservative picture of the risk investors face following risk increases.[4] To investigate within-month dynamics, we consider the occurrence of poor daily returns during month t, based on the risk condition from the previous month, and present results within Table 2. Specifically, Panels A and B of Table 2 present results for the conditional within-month minimum daily excess return across risk states. This analysis will detail if the worst daily returns are more extreme following the risky state. Panels C and D present results for the proportion of daily excess returns during month t that fall below several thresholds. This analysis will reveal if daily negative returns, as well as large daily losses occur with a greater frequency following the risky state.

Considering minimum daily excess returns within Panels A and B of Table 2, we find the worst daily returns within months that follow the risky state are larger in magnitude relative to the comparable values following the stable state. Specifically, within Panel A, the average minimum daily return following the risky state is −2.00% with skewness of −4.73. These values compare to −1.58% and −2.32 following the safe state. Similarly, Panel B utilizes the ex-ante PC approach, and finds a comparable difference (−1.97% relative to −1.59%). Finally, within Panels C and D, we find that large in magnitude daily losses occur with a greater frequency in months that follow the risky state. To explain, the occurrence of negative daily returns, $r_{mkt,t} < 0\%$, is relatively similar across risky states. For example, Panel C details that negative returns occur on 48% of the days and 46% of the days that follow the risky and safe condition, respectively. Panel D, based on the rolling-window PC approach reveals comparable values (49% and 46%). However, focusing on the large daily losses, the differences across conditions are dramatic. In Panel C, the proportion of daily losses of more than 1%, 2%, and 3% occur with a proportion of 0.095, 0.020, and 0.005, respectively, following the safe condition. These proportions increase to 0.142, 0.040 and 0.015 following the risky condition. The results indicate that returns below 2% and 3% are two times, and three times, more likely, following the risky state.

We now study the performance of a dynamic investment strategy based on our risk measure. This analysis will detail the magnitude of the impact of our risk measure on returns, and provide direct measurement of economic significance. We consider 'fully-invested' as a benchmark strategy. This strategy invests 100% in the US market portfolio. We compare this benchmark to results from multiple 'dynamic strategies.' First, $Dynamic_{full\text{-}sample}$, is based on the full sample principal component extraction, and allocates 100% to the US market portfolio for month t, unless both fragility and market stress are measured as increasing during month $t-1$, indicative of the increasing risk condition previously defined. In the event both fragility and stress increase during month $t-1$, the dynamic strategy reallocates completely to risk free securities for investment during month t.[5] Second, $Dynamic_{short}$, is constructed similarly to the previous strategy, but establishes a short position in the market, rather than a position in treasuries, following any month in which both stress and fragility increase. Finally, $Dynamic_{rolling\text{-}window}$ allocates between the market and treasuries, with risk state identification based on the rolling window principal component approach. This will confirm if the results of the dynamic strategies are robust to a purely ex-ante specification. To compare investment strategies we report mean excess returns, standard deviation of monthly excess returns, and Sharpe Ratios for each strategy within Table 3. We conduct Ledoit and Wolf (2008) tests for differences in Sharpe Ratios. A similar test was first proposed by Jobson and Korkie (1981) and then revised by Memmel (2003). The Ledoit and Wolf (2008) test is based on a studentized time series bootstrap approach to address heavy-tails and time-series characteristics of the data.

Table 3 details strong superior performance of the dynamic strategy relative to the fully-invested strategy. This indicates that our risk measure identifies periods of high risk with low returns. The strategies that allocate between the market and treasuries, $Dynamic_{full\text{-}sample}$ and $Dynamic_{rolling\text{-}window}$ reveal modest increases in

[4] We thank an anonymous referee for this insight.

[5] We assume the allocation decision for month t is made at the end of month $t-1$, and based only on the variables at this point. Future research may consider a strategy in which the investor pulls out of the market at the onset of a high risk state, and only reallocates to the market following a drawdown.

Table 3
Dynamic strategy performance.

Strategy	Mean	σ	Sharpe Ratio (%)	Transactions	*p*-value
Fully-invested	0.430	4.557	9.430	–	–
Dynamic$_{full\text{-}sample}$	0.569	3.683	15.462	73	(0.000)
Dynamic$_{short}$	0.709	4.522	15.686	73	(0.067)
Dynamic$_{rolling\text{-}window}$	0.507	3.803	13.331	76	(0.054)

This table reports summary performance measures. We report mean excess returns, standard deviation of monthly excess returns, and the Sharpe Ratio for a strategy, *Fully-invested*, that is 100% allocated to the market portfolio throughout the sample period. We report comparable statistics from multiple dynamic strategies. *Dynamic$_{full\text{-}sample}$*, and *Dynamic$_{rolling\text{-}window}$* are based on the full-sample PC and a 36-month rolling window PC, respectively. These strategies allocate 100% to the risk free rate for any month t, that follows month $t-1$ in which both fragility, ΔFI_{t-1}, and stress, $\Delta\sigma_{mkt,t-1}$, increase. In all other months, these dynamic strategies allocate 100% to the market portfolio. *Dynamic$_{short}$* is a strategy that takes a short position in the market portfolio following the high-risk state, and invests fully in the market otherwise. The transactions column reports the number of round trip transactions associated with each dynamic strategy. The *p*-value column reports *p*-values from an *F*-test for equal variances, and a studentized time series bootstrap test of equal Sharpe ratios developed by Ledoit and Wolf (2008).

mean returns, with benefits largely accruing through risk reduction when compared to the fully-invested benchmark. For example, the full-sample and rolling-window strategies offer improvements in mean returns of 14 and eight basis points, respectively. However, these dynamic strategies decrease monthly volatility by 87 and 75 basis points. Overall, these strategies increase the Sharpe Ratio from 9.43% to 15.46% and 13.33%, for *Dynamic$_{full-sample}$* and *Dynamic$_{rolling-window}$*, respectively with results significant at the 10% level or greater. Finally, *Dynamic$_{short}$* offers substantial improvements in mean returns with little change in volatility. This strategy increases monthly mean returns from 0.43% to 0.71%, with a four basis point decrease in volatility. Overall, this strategy increases the Sharpe Ratio to 15.69%, with the difference being significant at the 10% level. To further detail the economic significance of the dynamic strategies, we consider total holding period returns for the *Dynamic$_{short}$* strategy, relative to fully-invested. Across the full-sample, geometric average raw monthly returns for the two strategies are 1.04% and 0.94%, respectively. These values correspond to annual returns of 13.27% and 11.84%. As the *Dynamic$_{short}$* strategy has a comparable standard deviation, relative to the fully-invested strategy, the increase in annual returns reveals the additional benefit for an investor, without bearing additional risk.

4. Regression analysis

In this section, we perform regressions with excess market returns as the dependent variable. We use stress and fragility as explanatory variables, and also create interaction terms between stress and fragility. Specifically, $\Delta\sigma_{mkt,t-1}I(\Delta FI^{+}_{t-1})$ takes the value of the stress variable for all months $t-1$ in which fragility is increasing, and takes the value of zero otherwise. Similarly, $\Delta FI_{t-1}I(\Delta\sigma^{+}_{mkt,t-1})$ represents fragility for any month in which stress is increasing, and zero otherwise. This analysis reveals how the impact of stress may vary conditional on increasing or decreasing fragility. Finally, we denote the product of stress and fragility as $\Delta FI_{t-1} * \Delta\sigma_{mkt,t-1}$, and the product conditional on both variables increasing as $\Delta FI^{+}_{t-1} * \Delta\sigma^{+}_{mkt,t-1}$. We present estimation results within Table 4. Importantly, for models that include any of the interaction terms, we present a Chi-squared test from the null hypothesis that the sum of the coefficient on the primary regressor and the interaction term is equal to zero. For example, considering the model presented in the second row of the table, the Chi-squared test is based on the sum of the $\Delta\sigma_{mkt,t-1}$ and $\Delta\sigma_{mkt,t-1}I(\Delta FI_{t-1,+})$ terms. These tests will indicate if, conditional on increasing fragility, the net impact of market stress significantly impacts future returns or if, conditional on increasing stress, the net impact of fragility significantly impacts future returns. Panel A presents results based on full-sample PC estimation. Panel B presents results in the case in which the first PC is estimated a rolling three-year window.

Regression results in Table 4 indicate the importance of the interaction between stress and fragility. In Panel A, the initial

Table 4
Primary regressions.

Intercept	$\Delta\sigma_{mkt,t-1}$	ΔFI_{t-1}	$\Delta\sigma_{mkt,t-1}I(\Delta FI_{t-1,+})$	$\Delta FI_{t-1}I(\Delta\sigma_{mkt,t-1,+})$	$\Delta FI_{t-1}I(\Delta\sigma^{+}_{mkt,t-1})$	χ^2
0.430	−0.503	−0.299				–
(0.026)	(0.030)	(0.085)				
0.429	0.264	−0.325	−1.124			11.54
(0.025)	(0.481)	(0.062)	(0.013)			(0.001)
0.423	−0.495	−0.093		−0.621		4.02
(0.029)	(0.031)	(0.619)		(0.117)		(0.045)
0.781	0.241				−3.897	−3.897
(0.001)	(0.576)				(0.047)	(0.047)
0.692		−0.048			−2.851	7.63
(0.000)		(0.796)			(0.010)	(0.006)
0.430	−0.513	−0.244				–
(0.026)	(0.028)	(0.160)				
0.427	0.028	−0.262	−0.787			8.08
(0.026)	(0.943)	(0.132)	(0.108)			(0.005)
0.414	−0.501	−0.068		−0.563		2.93
(0.033)	(0.029)	(0.726)		(0.183)		(0.087)
0.632	−0.054				−2.061	3.32
(0.006)	(0.882)				(0.145)	(0.069)
0.653		0.036			−2.297	5.27
(0.001)		(0.854)			(0.029)	(0.022)

The table presents regression results. Each model includes an intercept term, and the regressors identified in the column headings. Table entries represent coefficient estimates, with associated *p*-values. Regressors include changes in market volatility, $\Delta\sigma_{mkt,t-1}$, and changes in fragility, ΔFI_{t-1}. Variables $\Delta\sigma_{mkt,t-1}I(\Delta FI_{t-1,+})$ and $\Delta FI_{t-1}I(\Delta\sigma_{mkt,t-1,+})$ represent changes in volatility conditional on increasing fragility, and changes in fragility conditional on increasing volatility, respectively. Finally, $\Delta FI_{t-1} * \Delta\sigma_{mkt,t-1}$ represents the product of changes in fragility and changes in volatility, and $\Delta FI^{+}_{t-1} * \Delta\sigma^{+}_{mkt,t-1}$ represents this product conditional on both variables being positive. Panels A and B present results when fragility is estimated using full-sample, and a three-year rolling window estimation, respectively. The χ^2 presents the results from the null hypothesis that the sum of the coefficients for the primary regressor and interaction term is equal to 0. *P*-values are based on Newey West standard errors.

model reports parameter estimates of −0.50 (significant at the 5% level) and −0.30 (significant at the 10% level) for stress and fragility, respectively. These results reveal a negative relation between both stress and fragility and subsequent returns. The second model includes stress, fragility, and the $\Delta\sigma_{mkt,t-1}I(\Delta FI^{+}_{t-1})$ interaction parameter representing stress conditional on increasing fragility. In this specification, we find a negative and marginally significant estimate of −0.33 for the fragility term. Interestingly, the point estimate for $\Delta\sigma_{mkt,t-1}$, equal to 0.26, is insignificant, but the coefficient estimate of −1.12 for the interaction between changes in stress conditional on increasing fragility is negative, large in magnitude, and significant at the 5% level. The Chi-squared statistic of 11.54 strongly rejects that the sum of the stress and the interaction term is equal to zero. Taken together, this indicates that a negative relation between stress and returns exists only conditional on increasing fragility. Considering the third model in Panel A, we find a negative and significant parameter estimate of −0.50 for the stress variable. The fragility parameter estimate is insignificant, but the Chi-squared test rejects the null hypothesis that the sum of the fragility and interaction parameters is equal to zero. Similar to the previous model, this indicates that conditional on increasing stress, fragility relates negatively to subsequent returns. From the parameter estimates in Table 4, we find a substantial impact of the interaction between stress and fragility, conditional on positive realizations of each variable. For example, for the models that include stress, $\Delta\sigma_{mkt,t-1}$, or fragility, ΔFI_{t-1}, coefficient estimates on the product of positive stress and fragility are −3.90 and −2.85, respectively (presented in the fourth and fifth rows of Panel A).

Given the normalized, mean zero and unit standard deviation stress and fragility variables, the coefficient estimates in Table 4 provide a direct measure of the economic significance of the results. For example, considering the third model in Panel A, the coefficient estimate of −0.09 on the primary fragility regressor, ΔFI_{t-1}, indicates a one standard deviation increase in fragility relates to a point estimate of a nine basis point decrease in subsequent conditional excess market returns. However, the sum of the fragility and interaction terms (−0.09 − 0.62 = −0.71) indicates that, conditional on increasing stress, a one standard deviation increase in fragility relates to a 71 basis point decrease in subsequent conditional returns. Results from the fourth model in Panel A that includes the product of stress and fragility conditional on positive realizations of both variables indicate a substantial impact on subsequent returns. If both variables concurrently increase by one standard deviation, the forecasted return decreases by 366 basis points to −2.88% (=0.78% + 0.24%*1–3.90%*1*1). Conditioning on the monthly observations in which both stress and fragility increase, the means of these variables are 0.76 and 0.61. As stress and fragility increase to these conditional means, the forecasted return decreases 163 basis points to −0.85% (=0.78% + 0.24%*0.76–3.90%*0.76*0.61).

Results in Panel B of Table 4, utilizing the ex-ante principal component specification, confirm the primary results. That is, in the models that include the interaction term, the primary regressor is insignificant, but the sum of the primary regressor and the interaction term is negative and significant for both stress and fragility. In the model that includes the stress interaction term, which is measured as stress conditional on increasing fragility, the point estimate on the stress variable is 0.03 and insignificant. However, the sum of the stress variable and the interaction term is −0.76, which is large in magnitude and significant as indicated by the Chi-square statistic. Similarly, with fragility, the primary regressor is insignificant, but the sum of fragility and the interaction term (−0.07 − 0.56 = −0.63) is significant at the 10% level.

To complete the regression analysis, we perform quantile regressions to assess the ability of our risk measure to relate to the downside of the conditional distribution of equity returns. This analysis addresses the issue that the extant literature emphasizes how shocks can impact the economy non-linearly. Our previous results have focused on the impact of risk on the conditional mean. This analysis will further consider the impact of risk across the full distribution of returns. We would expect the impact of our risk measure to manifest within the left-tail of the distribution. Within this analysis, to illustrate the relation between risk states and subsequent returns, as well as to highlight the importance the interaction between stress and fragility, we separately regress monthly excess market returns during month t on stress, $\Delta\sigma_{mkt,t-1}$, stress conditional on increasing fragility, $\Delta\sigma_{mkt,t-1}I(\Delta FI^{+}_{t-1})$, and a new variable $\Delta\sigma_{mkt,t-1}I(\Delta FI^{-}_{t-1})$ representing stress conditional on decreasing fragility. Similarly, considering fragility, we present results from regressing subsequent market returns on fragility, ΔFI_{t-1}, as well as fragility conditional on increasing and decreasing stress, $\Delta FI_{t-1}I(\Delta\sigma^{+}_{mkt,t-1})$ and $\Delta FI_{t-1}I(\Delta\sigma^{-}_{mkt,t-1})$, respectively.

We present quantile regression results for stress and fragility in Fig. 3, respectively. Specifically, we estimate coefficients for each percentile of the return distribution between the tenth and 90th percentiles. We then plot the coefficient estimates for the given percentile, represented as the solid line in Fig. 3, as well as the 90% confidence intervals for the coefficient estimates, represented by the shaded area. If the 90% confidence interval excludes 0, we may conclude that the parameter estimate for the given variable is significant at the given percentile of the return distribution.

We initially consider the estimates of the coefficient on stress across the quantiles of the excess market return distribution presented in the first Panel of Fig. 3. We tend to find a moderate and negative relation between stress and the left-hand side of the return distribution. Specifically, every coefficient estimate below the 35th quantile is significant at the 10% level or greater, and ranges from a high of −0.46 for the 33rd quantile, to a low of −0.93 for the 14th quantile. As a reminder, with the standardized stress variables, the coefficients provide direct measure of the economic impact. Considering the examples above, a one standard deviation increase in stress would decrease the conditional 33rd and 14th percentiles by 46 and 93 basis points, respectively. We find no relation between stress and the right-hand side of the return distribution, as there is no coefficient estimate that is significant for the 50th percentile or above.

Considering the coefficient estimate on, $\Delta\sigma_{mkt,t-1}I(\Delta FI^{+}_{t-1})$, the interaction between changes in stress and the indicator variable for increasing fragility, we find evidence of a large in magnitude negative relation between stress and the left hand side of the return distribution, conditional on increasing fragility. Specifically, in the second Panel of Fig. 3, all coefficient estimates for the 68th percentile or below are significant at the 10% level or greater. Further, the estimates appear larger in magnitude relative to the previous panel (stress without the interaction between increasing fragility). For example, the stress coefficient estimate for the 10th percentile is −1.81 conditional on increasing fragility, which compares to an estimate of −0.79 from the previous panel. Finally, considering the interaction between changes in stress and *decreasing* fragility, $\Delta\sigma_{mkt,t-1}I(\Delta FI^{-}_{t-1})$, we find evidence that stress without fragility may lead to especially strong subsequent returns. Coefficient estimates are positive and significant at the 10% level or greater for the 53rd percentile and above. For example, parameter estimates are 0.81 and 1.36 for the 75th and 90th percentiles, respectively. These values indicate that, conditional on decreasing fragility, a one standard deviation increase in stress increases these conditional percentiles by 81 and 136 basis points. Summarizing, Fig. 3 illustrates the importance of a risk measure that considers both stress and fragility. Stress in isolation relates to moderate decreases in the left tail of the distribution. However, stress

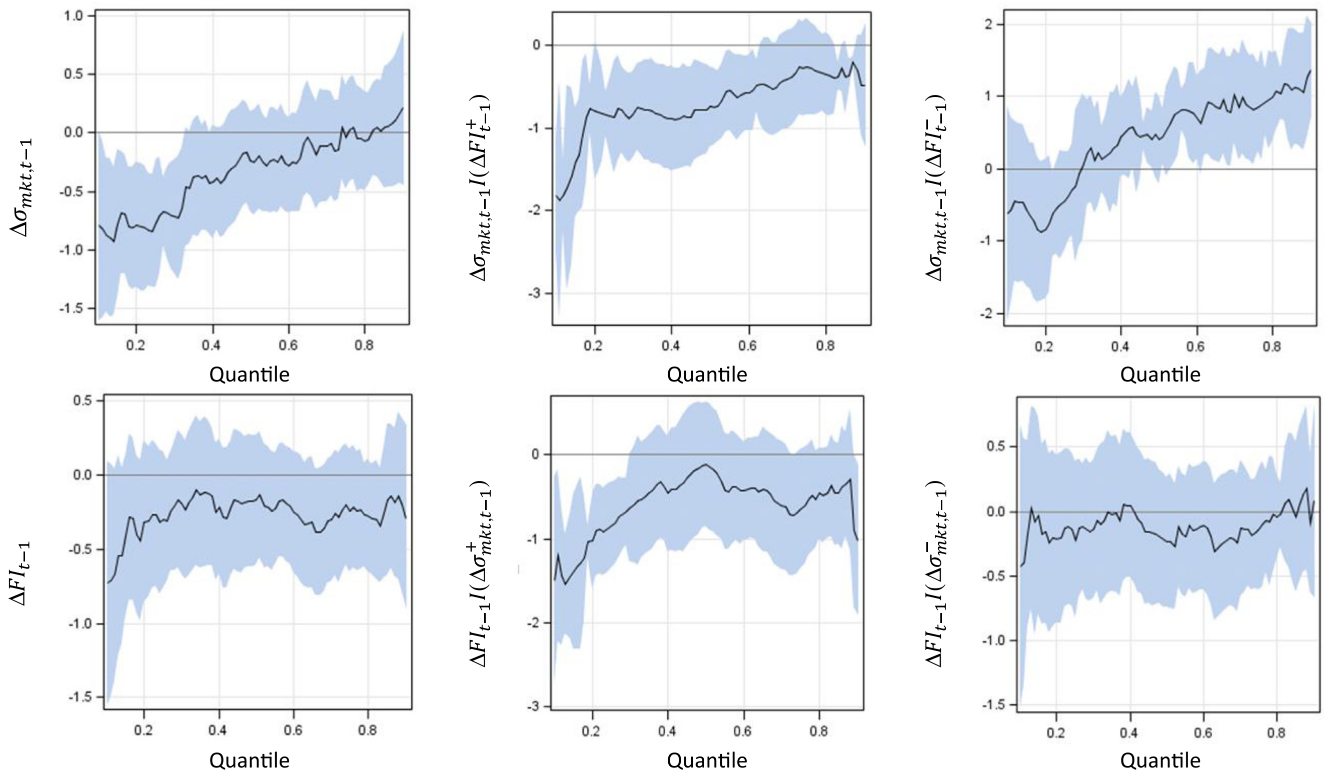

Fig. 3. The figure plots coefficient estimates from quantile regressions. Quantiles are estimated from 10th to the 90th percentile in increments of 1%. The solid line represents the estimate, and the shaded area represents the 90% confidence interval. $\Delta\sigma_{mkt,t-1}$ represents market volatility, $\Delta\sigma_{mkt,t-1}I(\Delta \mathrm{FI}^{+}_{t-1})$ represents market volatility conditional on fragility being positive, and $\Delta\sigma_{mkt,t-1}I(\Delta \mathrm{FI}^{-}_{t-1})$ represents market volatility conditional on fragility being negative. $\Delta \mathrm{FI}_{t-1}$, $\Delta \mathrm{FI}_{t-1}I(\Delta\sigma^{+}_{mkt,t-1})$, and $\Delta \mathrm{FI}_{t-1}I(\Delta\sigma^{-}_{mkt,t-1})$ represent fragility, fragility conditional on stress increasing, and fragility conditional on stress decreasing, respectively.

conditional on increasing (decreasing) fragility leads to large in magnitude decreases (increases) across the left (right) hand side of the distribution.

Considering fragility and the distribution of returns in the lower panels of Fig. 3, we again find the importance of the intersection of stress and fragility. Specifically, fragility in isolation, as well as fragility conditional on decreasing stress leads to largely insignificant parameter estimates. However, conditioning changes in fragility on increasing stress, we find a negative and significant relation for many quantiles in the left hand side of the distribution. For example, if stress is positive, a one standard deviation increase in fragility decreases the conditional 25th percentile by 84 basis points from −2.21% to −3.05%, and decreases the conditional tenth percentile by 150 basis points from −5.29% to −6.79%.

5. Alternative market stress measures

Within this section we consider additional measures of market stress. This analysis will confirm the robustness of the risk measure across multiple stress measures. These stress variables include VIX, the default spread, which is calculated as the difference between AAA and BAA corporate bond index yields and captures aggregate default likelihood perceptions, and the term spread, measured as the difference between the yield from 10-year Treasury Bonds and the 3-month Treasury Bills. Increases in the term spread may represent stress (cf., Kritzman and Li (2010)). Specifically, our stress measures consider the relative change in month $t-1$ relative to the previous 12 months. A sudden increase in the term structure relative to recent averages may be indicative of a rapidly shifting macroeconomic environment. The second and third measures are commonly used proxies for economic conditions. They have a low correlation (0.17), thus reflecting different aspects of market conditions.[6] Our fourth measure of stress is the Aruoba–Diebold–Scotti (ADS) business conditions index provided by the Philadelphia Federal Reserve Bank. This index provides a measure of business conditions on a high frequency basis (Aruoba and Diebold, 2010). For consistency with our previous measures of stress, we multiply the raw ADS index by negative one, such that high levels of the variable denote high levels of stress, and negative values indicate periods of improving conditions. We hypothesize that high levels of this transformed variable, which represent deteriorating business conditions, will relate to stress in the financial market as the economic news is incorporated into prices. Finally, we use the Kansas City Financial Stress Index (KCFSI), capturing financial stress based on multiple financial market variables. This variable is detailed in Hakkio and Keeton (2009).

Similar to the market volatility variable previously discussed, we calculate ΔVIX_{t-1}, ΔDS_{t-1}, $\Delta Term_{t-1}$, and ΔADS_{t-1} as the average of daily values of VIX, the default spread, the term spread, and the ADS measure, respectively, during month $t-1$, less the counterpart long run daily average over months $t-12$ through $t-1$. The KCFSI is provided monthly, and therefore we compare the value during month $t-1$, relative to the average value across months $t-12$ through $t-1$. Hereafter, we use the name of the underlying stress measure (eg. KCFSI) to specifically refer to the relative difference variables (eg. $\Delta KCFSI_{t-1}$) that we have described above. We use the previously described fragility measure, and the stress variables described above to identify our risky state. Specifically, a month t will be identified as the high risk state if both stress and fragility increase in month $t-1$. Conversely, a month t

[6] See, for example, Fama and French (1989), Chen et al. (1986), Brandt (1999) and Chalmers et al. (2011) for discussion of these proxies.

Table 5
Correlations of fragility and stress.

	$Stress_{t-1}$	$\Delta\sigma_{mkt,t-1}$	ΔVIX_{t-1}	ΔDS_{t-1}	$\Delta Term_{t-1}$	ΔADS_{t-1}	$\Delta KCFSI_{t-1}$
Panel A: Principal component loadings							
PC_1	–	0.437	0.472	0.380	−0.031	0.440	0.498
PC_2	–	−0.068	−0.016	−0.079	0.976	0.190	0.028
PC_3	–	−0.524	−0.375	0.707	−0.042	0.288	0.018
Panel B: Correlations							
ΔFI_{t-1}	−0.117	−0.026	−0.085	−0.088	0.051	−0.006	−0.095
	(0.061)	(0.538)	(0.139)	(0.125)	(0.337)	(0.883)	(0.128)
$I(\Delta FI_{t-1} > 0)$	0.025	0.000	−0.085	−0.087	0.065	0.025	−0.091
	(0.557)	(0.998)	(0.137)	(0.129)	(0.220)	(0.557)	(0.146)
$Stress_{t-1}$	–	0.846	0.913	0.853	−0.060	0.735	0.964
		(0.000)	(0.000)	(0.000)	(0.338)	(0.000)	(0.000)
$\Delta\sigma_{mkt,t-1}$	–	–	0.832	0.462	0.030	0.243	0.769
			(0.000)	(0.000)	(0.576)	(0.000)	(0.000)
ΔVIX_{t-1}	–	–	–	0.598	0.081	0.384	0.862
				(0.000)	(0.157)	(0.000)	(0.000)
ΔDS_{t-1}	–	–	–	–	0.091	0.593	0.840
					(0.113)	(0.000)	(0.000)
$\Delta Term_{t-1}$	–	–	–	–	–	−0.087	−0.040
						(0.101)	(0.518)
ΔADS_{t-1}	–	–	–	–	–	–	0.634
							(0.000)

The table presents correlations of the stress and fragility measures. The fragility index, ΔFI_{t-1}, and market volatility, $\Delta\sigma_{mkt,t-1}$, are as described in Table 1. ΔVIX_{t-1}, ΔDS_{t-1}, $\Delta Term_{t-1}$, and ΔADS_{t-1} represent the average of daily observations during month $t-1$ of VIX, the Default Spread, calculated as the yield from BAA corporate bonds minus the yield to AAA corporate bonds, the Term Spread, calculated as the yield from 10-year Treasury Bonds less the yield from 3-month Treasury Bills, and the Aruoba–Diebold–Scotti Business Conditions Index, calculated as the index multiplied by negative one, respectively, less the comparable average from daily observations during months $t-12$ through $t-1$. $\Delta KCFSI_{t-1}$ represents the value of the Kansas City Financial Stress Index during month $t-1$, less the average monthly value from months $t-12$ through $t-1$. $Stress_{t-1}$, represents the first principal component of the six stress variables. Eigenvectors from the first three principal components are reported in the first three rows. Panel B reports p-values for the given correlation in parentheses.

may be considered as being in the robust or stable state if either stress, fragility, or both, decrease during month $t-1$.

We initially consider the relation across the various stress measures. Specifically, we conduct principal component analysis to extract common variation in the multiple stress measures. We denote the first principal component of the various stress measures as $Stress_{t-1}$. We report the principal component loadings, as well as cross-correlations of the stress measures in Table 5.

The principal component and correlation analysis in Table 5 reveals interesting dynamics regarding the stress measures. Considering the principal component analysis reported in Panel A, we find the first PC loads positively on volatility, VIX, the default spread, ADS, and KCFSI. The second PC has a very large in magnitude loading on the terms structure, with relatively marginal loadings on the remaining variables. A benefit of our risk measure is the consideration of both stress and fragility. A potential concern could be that the variables capture similar periods and thus are redundant. However, the correlation analysis presented in Panel B reveals that this is not the case. We observe small in magnitude, and insignificant correlations between fragility and the stress variables in every possible case. The largest in magnitude correlation between fragility and an individual stress measure is −0.100 with KCFSI. The correlation between fragility and the principal component stress measure is −0.117. This supports the use of our risk measure, suggesting stress and fragility are separate phenomena. The insignificant statistic between FI and VIX or FI and volatility also indicate that FI is not simply mechanically capturing volatility. For completeness, we report correlations across our measures of market stress. We would expect strong relations between several of our measures of stress. For example, the correlation of 0.832 between market volatility and VIX is not surprising. Interestingly, many of our stress measures do appear to capture separate phenomenon. For example, each cross-correlation between the term structure and remaining stress variables is 0.1 or smaller. The default spread variable exhibits moderate correlations of 0.462 and 0.598 with market volatility and VIX, respectively. As a composite variable, KCFSI exhibits correlations above 0.7 with all stress variables excluding the term structure. Consistently, Giglio et al. (2012) show, besides the recent financial crisis, the correlations of these stress variables are low and each measure represents different information at different periods. Given the relative moderate correlations across many stress variables, we continue our study analyzing risk with these various measures.

We now consider average market excess returns conditional on both fragility and the alternative measures of market stress. We report these results within Table 6. Column headings of Table 6 indicate the risk state in question. The initial data column in Table 6 reports statistics of conditional market returns during month t from the high risk state in which both fragility and stress increase during month $t-1$, $\Delta FI_{t-1} > 0$ and $VAR_{t-1} > 0$. The second data column reports statistics from the low risk state in which either fragility or stress decreases. For completeness, the third through fifth data columns present results further partitioning the low risk state on the specific decrease in either stress, fragility, or both. Data columns two through five all report heteroskedastic consistent t-statistics comparing that state's mean return, to the mean return in the high risk state. We present results for all stress variables considered, as well as $Stress_{t-1}$, the first principal component across the measures. We have hypothesized that poor returns follow increases in stress and fragility; therefore, the reported p-values are results from the directional hypothesis that mean returns following the high risk state are lower than the counterparts in any of the low risk states.

Within Table 6, and considering the high risk state identified during month $t-1$, we find negative point estimates for average excess returns during month t, for all stress variables excluding the term structure. To illustrate, the conditional return during month t is −0.40% following months in which the first component of the stress measures, $Stress_{t-1}$, and fragility both increase. This value is 119 basis points below the conditional return of 0.79%

Table 6
Double-partition conditional market excess returns.

	$\Delta FI_{t-1} > 0$ and $VAR_{t-1} > 0$	$\Delta FI_{t-1} < 0$ or $VAR_{t-1} < 0$	$\Delta FI_{t-1} < 0$ and $VAR_{t-1} < 0$	$\Delta FI_{t-1} < 0$ and $VAR_{t-1} > 0$	$\Delta FI_{t-1} > 0$ and $VAR_{t-1} < 0$
Panel A: $\Delta VAR_{t-1} = Stress_{t-1}$					
Mean	−0.401	0.788	1.120	1.272	0.267
t-statistic		1.55	1.74	1.66	0.85
		(0.061)	(0.043)	(0.050)	(0.200)
σ	5.422	4.238	3.776	5.569	3.369
Skewness	−1.315	−0.225	0.047	−0.415	−0.385
N	58	199	52	59	88
Panel B: $\Delta VAR_{t-1} = \Delta\sigma_{mkt,t-1}$					
Mean	−0.615	0.737	0.524	1.205	0.606
t-statistic		2.59	1.98	2.78	2.15
		(0.005)	(0.024)	(0.003)	(0.016)
σ	5.554	4.176	3.924	4.817	3.910
Skewness	−0.865	−0.130	−0.079	−0.194	−0.227
N	131	445	157	119	169
Panel C: $\Delta VAR_{t-1} = \Delta VIX_{t-1}$					
Mean	−0.784	0.830	0.338	1.829	0.607
t-statistic		2.14	1.34	2.69	1.74
		(0.017)	(0.092)	(0.004)	(0.042)
σ	5.547	4.403	4.042	5.293	4.030
Skewness	−1.759	−0.394	−0.319	−0.455	−0.703
N	62	244	75	61	108
Panel D: $\Delta VAR_{t-1} = \Delta DS_{t-1}$					
Mean	−0.458	0.837	1.132	0.870	0.585
t-statistic		2.01	2.14	1.53	1.45
		(0.023)	(0.017)	(0.064)	(0.075)
σ	5.113	4.502	4.046	5.327	4.225
Skewness	−1.073	−0.799	−0.080	−0.323	−1.948
N	79	227	71	65	91
Panel E: $\Delta VAR_{t-1} = \Delta TERM_{t-1}$					
Mean	0.183	0.807	0.751	1.077	0.605
t-statistic		1.06	0.79	1.27	0.61
		(0.144)	(0.214)	(0.102)	(0.271)
σ	5.195	4.368	4.569	4.317	4.251
Skewness	−1.315	−0.413	−0.194	−0.251	−0.857
N	99	255	85	83	87
Panel F: $\Delta VAR_{t-1} = \Delta ADS_{t-1}$					
Mean	−0.473	0.783	0.604	1.028	0.714
t-statistic		2.79	2.05	2.68	2.22
		(0.003)	(0.021)	(0.004)	(0.014)
σ	5.089	4.286	4.007	4.643	4.199
Skewness	−0.542	−0.433	−0.228	−0.052	−1.188
N	162	414	137	139	138
Panel G: $\Delta VAR_{t-1} = \Delta KCFSI_{t-1}$					
Mean	−0.330	0.795	1.065	1.334	0.253
t-statistic		1.57	1.63	1.74	0.78
		(0.058)	(0.053)	(0.043)	(0.219)
σ	5.159	4.310	4.192	5.352	3.520
Skewness	−1.351	−0.281	0.023	−0.494	−0.578
N	63	194	55	56	83

The table presents mean excess market returns during month *t*, conditional on levels of fragility and market stress variables both from month $t-1$. All conditioning variables are described in Table 4. Within Panels A through *F*, column headings indicate the level of each conditioning variable, and panel headings define the generic variable ΔVAR. The *t-stat* presents the heteroskedastic consistent t-statistic, and associated *p*-value, from the one-sided test of the null hypothesis that the mean excess return in the initial column is not less than the mean excess return in the given column.

following any month in which either stress or fragility decrease, and the *t*-statistic of 1.55 reveals this difference is significant at the 10% level. In addition, four of the remaining six stress variables reveal significant differences across risk states at the 5% level or greater, and the KCFSI stress variable reveals a difference that is significant at the 10% level. Across all seven stress measures, the average conditional return during month *t* is −0.41%, and this value is 121 basis points below the average conditional return of 0.80% following the stable state. Summarizing, excess market returns conditional on our high risk state appear negative, and are significantly lower than excess market returns across all remaining observations. Further, the higher returns following the low risk state appear to also offer smaller standard deviations.

Within Table 6 the further partition of the low risk state presented in the final three columns reveals interesting dynamics. First, the state characterized by decreasing fragility and decreasing stress may be indicative of stable markets. To explain, in this state we observe positive, but small in magnitude mean returns. For example, Panel C details that following a decrease in fragility and a decrease in VIX, $\Delta FI_{t-1} < 0$ and $VAR_{t-1} < 0$, mean excess returns are 0.34%, while mean returns are 1.83% and 0.61% in the alternative partitions of the low risk state. Further, standard deviations in this state are on average 100 basis points below their comparable values in the high risk state across all seven stress variables. Results suggest that in this state risk is diffuse as evidenced by decreasing fragility, and measures of market stress indicate a

Table 7
Conditional mean and minimum returns with rolling window PC estimation.

	$Stress_{t-1}$	$\Delta\sigma_{mkt,t-1}$	ΔVIX_{t-1}	ΔDS_{t-1}	$\Delta Term_{t-1}$	ΔADS_{t-1}	$\Delta KCFSI_{t-1}$
Panel A: Conditional mean excess market return during month t							
$\Delta FI_{t-1} > 0$ and $VAR_{t-1} > 0$	−0.240	−0.353	−0.787	−0.144	−0.053	−0.345	−0.125
$\Delta FI_{t-1} < 0$ or $VAR_{t-1} < 0$	0.731	0.649	0.805	0.717	0.884	0.685	0.695
t-statistic	1.30	1.95	1.98	1.31	1.56	2.24	1.11
	(0.097)	(0.026)	(0.024)	(0.095)	(0.060)	(0.013)	(0.134)
Panel B: Average minimum daily excess market return during month t							
$\Delta FI_{t-1} > 0$ and $VAR_{t-1} > 0$	−2.489	−1.970	−2.714	−2.308	−2.215	−1.766	−2.492
$\Delta FI_{t-1} < 0$ or $VAR_{t-1} < 0$	−1.849	−1.592	−1.832	−1.897	−1.784	−1.645	−1.851

The table presents conditional excess market returns across risk states. Fragility, ΔFI_{t-1} is estimated using 36-month rolling-windows to form the first principal component. Stress variables, VAR_{t-1}, are identified in the column headings. Entries in Panel A represent the conditional mean excess market return during month *t*, following the risk state identified in the initial column. The *t*-statistic row presents results from a one-sided test under the null that the mean return following the risky state is not less than the mean return otherwise. Entries in Panel B represent the average minimum daily excess market return during month *t* across risk conditions.

tranquil period. Therefore stability in returns is observed. Second, the state characterized by increasing stress and decreasing fragility may indicate large in magnitude positive returns. For example, mean excess returns are 1.21% and 1.83% conditional on fragility decreasing, but market volatility or VIX increasing, respectively. These results suggest that, absent fragility, the market may benefit from the upside associated with turbulence. That is, in this state, risk is diffuse, but shocks may occur, offering strong upside potential in this robust state.

Finally, we consider the robustness of our conditional return analysis across multiple stress measures in an ex-ante basis. Specifically, within Table 7, we report conditional excess monthly returns and average minimum daily returns during month *t*, relative to risk states identified in month $t-1$. Panel A contains results for monthly excess returns, while Panel B contains results for the minimum daily returns.

The results in Table 7 illustrate the predictive content of the risk measure, as both stress and fragility are specified in an ex-ante fashion in this analysis. Specifically, point estimates of conditional returns are negative for all seven stress measures. Returns following the high risk state are significantly lower than returns in the stable state at the 10% level for six of the seven stress measures, with results based on three of the measures significant at the 5% level. Across all seven stress measures, the conditional monthly return following the risky state is −0.29%, and this is 103 basis points below the average conditional return following the stable state. Results in Panel B similarly confirm that the risk measures contains predictive information regarding large daily market declines across multiple stress measures and in an ex-ante specification. The average minimum daily return following the high risk state is significantly lower than the comparable value following the stable state for six of the seven stress measures. Averaging across all stress measures, the statistic of −2.28% for the average minimum daily return following the risky state is 50 basis points below the average following the stable state.

6. Conclusions

Following the financial crisis from 2007 to 2009, many studies have proposed measures that predict the probability of crashes or extreme events. Indicators of an increased likelihood do not necessarily imply lower average stock returns. In fact, the literature has documented incidences of strong returns during high risk periods. In this study, we propose a risk measure that specifically predicts stock downturns on a monthly basis. The risk measure relies on capturing both stress and fragility.

We show that increases in our risk measure predict poor monthly market returns, as well as large in magnitude minimum monthly daily returns. The results are robust across multiple stress specifications. Considering contemporaneous and subsequent returns, as well as our focus on changes in risk measures, our results are most consistent with a rational adjustment process, in which prices adjust following innovations in risk. The relatively long frequency of our data (monthly, opposed to daily) may provide additional benefits for investors and policy makers.

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