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Managing Disruption Risk: The Interplay Between Operations and Insurance

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Disruptive events that halt production can have severe business consequences if not appropriately managed. Business interruption (BI) insurance offers firms a financial mechanism for managing their exposure to disruption risk. Firms can also avail of operational measures to manage the risk. In this paper, we explore the relationship between BI insurance and operational measures. We model a manufacturing firm that can purchase BI insurance, invest in inventory, and avail of emergency sourcing. Allowing the insurance premium to depend on the firm's insurance and operational decisions, we characterize the optimal insurance deductible and coverage limit as well as the optimal inventory level. We prove that insurance and operational measures are not always substitutes, and we establish conditions under which they can be complements; that is, insurance can increase the marginal value of inventory and can increase the overall value of emergency sourcing. We also find that the value of insurance is higher for those firms less able to absorb financially significant disruptions. As disruptions become longer but rarer, the value of emergency sourcing increases, and the value of inventory and the value of insurance increase before eventually decreasing.

Key words: disruptions; inventory; insurance
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1. Introduction

The purpose of business interruption (BI) insurance is to protect against losses incurred when a firm cannot operate normally because of a disruption to one of its facilities. Firms can also purchase contingent BI insurance to protect against losses resulting from a disruption to a supplier's facility. Ericsson received an insurance settlement in the region of $200 million resulting from the Philips plant failure in 2000 that disrupted the supply of chips used in its mobile phones (Norrman and Jansson 2004). However, Ericsson lost substantial market share to Nokia whose operational measures, including emergency sourcing, successfully mitigated the impact of the Philips plant failure (Latour 2001). Philips itself received an insurance settlement of approximately 39 million ($35 million at the 2001 exchange rate) due to lost income and physical damage (Philips 2001).

BI insurance is an important line of business in the commercial insurance industry:

> The largest single source of insurance payout in the [September 11] terrorist attacks was not the property claims, but business interruption insurance. In today's dollars, business interruption payouts made up $12.1 billion, or 33 percent of the $35.6 billion in total insured losses.... Business interruption claims from Katrina are expected to cost about half of the $20.8 billion in commercial losses. (Mowbray 2006, p. 1)

There must be three elements in place for BI insurance to take effect. First, the disruption must be directly caused by property damage (PD) that results from a peril covered by the firm's PD insurance policy—fire being the most common (but not only) covered peril. A firm must have a PD insurance policy in order to purchase BI insurance. Second, coverage is provided for a limited time. In U.S. polices, coverage is provided until the facility achieves technical operational readiness, i.e., the point in time at which the facility "should be repaired, rebuilt, or replaced with reasonable speed and similar quality" (Torpey et al. 2004, p. 29).[1] Third, the business must suffer an actual loss because of the disruption.

[1] In UK policies, coverage is provided until commercial operating readiness or the maximum indemnity period is reached, whichever is first. Commercial operating readiness "is when the company is once again able to produce its normal financial results" (Swiss Re 2004, p. 19). This may differ from technical operational readiness if, for example, demand after a disruption is temporarily reduced

Although a BI insurance policy can have a multitude of clauses describing what is and what is not covered, the three crucial elements of the policy are the premium, coverage limit, and deductible. The premium is the price the firm pays the insurer to obtain the insurance coverage. The coverage limit specifies the maximum the insurer will pay the firm in the event of a loss. The deductible specifies the amount of the loss to be absorbed by the firm. The deductible can be a monetary or time deductible. In a monetary deductible, the policy specifies a monetary value and the firm absorbs this amount of any BI related loss. With a time deductible, the policy specifies a duration such that the firm absorbs any loss incurred during that initial duration of a disruption. Before the BI contract is priced and agreed, insurers (or, more precisely, the lead underwriter) are given extensive access to a firm's facilities to assess the risks and potential losses. With fire being the primary risk, particular attention is paid to auditing a facility's fire protection systems and processes. Insurers also collect industry level data on losses and facility reconstruction times by region to help assess risks and to develop broad pricing guidelines that are then tailored to a particular firm's case.

In addition to or in lieu of insurance, the firm may stockpile inventory and/or avail of an emergency supplier. Although insurance is an essential part of risk management in many companies, insurance purchase decisions and operations decisions are often made independently, partly because of the lack of an integrated risk management approach within the organization but also partly because of a limited understanding of the interplay between operations and insurance. In this paper, we explore the use of BI insurance and operational measures for managing disruption risk. We characterize the optimal inventory, deductible, and coverage limit when the insurance price is endogenous, i.e., it depends on the firm's operational and insurance decisions. Following the literature on premium pricing, our pricing function reflects the insurer's expected loss, an expense load, and a risk load. Although intuition might suggest that insurance and operational measures would be substitutes, we show that they can sometimes be complements. That is, insurance may induce the firm to invest in more inventory. We also examine the value of insurance and the value of the two operational measures. We find that the value of insurance is significant for firms that are less able to absorb the financial consequences of disruptions, and that as disruptions become longer but rarer, the value of emergency sourcing increases, and the value of inventory and the value of insurance increase before eventually decreasing.

The rest of this paper is organized as follows. Section 2 discusses the relevant literature. Section 3 describes the model. Profit expressions and the optimal inventory and insurance decisions are developed in §4. The interplay between insurance and operational measures is explored in §5. Risk loading in the premium is examined further in §6. The value of insurance and operational strategies is explored in §7. Conclusions and directions for future research are presented in §8.

2. Literature

The operations literature has typically modeled disruptions in one of two manners: infinite horizon models or single-period models. In the infinite horizon models, a facility alternates between up and down phases with the status of the facility being known at the time of production, e.g., Tomlin (2006) and references therein. These papers, e.g., Tomlin (2006) and Parlar and Perry (1996), often adopt a long-run average objective. Single-period models have been used to explore disruption management for products with short life cycles and long lead times. Production is uncertain and typically modeled as a Bernoulli random variable in which production either succeeds in full or completely fails (e.g., Babich 2006, Chaturvedi and Martínez-de-Albéniz 2011).

The literature has investigated a variety of operational strategies for managing disruption risk: inventory (e.g., Tomlin 2006, Schmitt et al. 2010); emergency sourcing (e.g., Chopra et al. 2007, Yang et al. 2009); dual sourcing (e.g., Parlar and Perry 1996, Dada et al. 2007, Babich et al. 2007); demand management (Tomlin 2009); and process improvement (Wang et al. 2010, Bakshi and Kleindorfer 2009). Financial mechanisms for managing supply risk, e.g., supplier subsidies to reduce the likelihood of bankruptcy-induced disruptions, have been considered by Swinney and Netessine (2009) and Babich (2010). Certain papers use the term "insurance" as an analogy when referring to operational measures for managing disruptions (Lodree and Taskin 2008) or to a contractual arrangement under which one supply chain party offers financial compensation to another party (Sorg et al. 2001, Lin et al. 2010). BI insurance has not been explored in the disruption literature to the best of our knowledge. Addressing that gap is the primary objective of this paper.

The academic insurance literature has extensively studied major issues in the insurance industry, including (but not limited to) risk assessment, the insurance purchase and claim decisions, and the design of the optimal insurance policy (von Lanzenauer and

because the firm has yet to win back customers who switched to a competitor or because the economy was impacted by the disruptive event, e.g., Hurricane Katrina. We focus on the U.S. BI insurance format in this paper.

Wright 1991, Dionne 2000). The insurance premium is a key component of an insurance policy, and it is determined by the expected insurance payout and the application of one of various premium calculation principles to reflect the insurer's administrative costs, profits, and risks (see Goovaerts et al. 1984).

Adverse selection and moral hazard, stemming from asymmetric information and/or a lack of enforceability of precautionary action between the insured and the insurer, are two important issues that insurers must consider when designing insurance contracts (Rothschild and Stiglitz 1976, Wilson 1977, Winter 2000, Dionne et al. 2000, and references therein). However, "neither adverse selection nor moral hazard appears to be a major concern with respect to natural hazard risks" (Grossi and Kunreuther 2005a, p. 36). Adverse selection and moral hazard are also somewhat mitigated with respect to fire hazards in BI insurance because the insurance company, via facility inspection, information system auditing, etc., has access to much of the relevant information when underwriting the policy and processing the claim. In this paper, we take the perspective of the insured company and explore the relationship between operational and insurance measures in the absence of adverse selection and moral hazard.

The commercial importance of BI insurance is reflected by practitioner books dedicated to the topic, e.g., Cloughton (1999) and Torpey et al. (2004), that give in-depth coverage of the accounting and legal considerations. Although many of the frameworks developed for the general insurance product also apply to BI insurance, BI insurance, despite its commercial importance, has received little attention in the academic insurance literature; Kahler (1932) gives an introduction to BI insurance, and Zajdenweber (1996) empirically investigates the distribution of total annual BI claims in France. Recent catastrophic events, e.g., the September 11 terrorist attack and Hurricane Katrina, have led to wide discussions among insurance industry, academicians, and government agencies on whether and how to offer insurance (BI being an important component) to cover extreme risk events (e.g., Kunreuther 2002, Kunreuther and Michel-Kerjan 2004, Grossi and Kunreuther 2005b).

There is a small number of papers at the interface of operations and general insurance; for example, Verter and Erkut (1997) incorporates the cost of liability insurance in a hazardous-material vehicle routing optimization problem. However, the interface of operations and BI insurance seems relatively unexplored.

3. Model

The firm has three disruption strategies at its disposal: inventory, emergency sourcing, and BI insurance. We describe the operational and insurance elements and then summarize the firm's problem.

3.1. Operational Elements

The operational aspects of the model closely resemble the disruption model of Tomlin (2006), with the primary differences being that (i) we allow disruptions to destroy inventory, (ii) we assume lost sales rather than back orders, and (iii) the emergency supplier is immediately available and has infinite capacity.

The firm sells a single product over an infinite (discrete-time) horizon. It operates a single facility that is subject to disruptions. The facility is either operational (up) or nonoperational (down) in a period. When the facility is up it can produce any quantity, but it can produce nothing when down. Disruptive events occur only when the facility is up and cause the facility to remain down for an uncertain duration. We assume that all disruptive events are covered by insurance and that there are two types of disruptive events: one (denoted S) in which all the inventory survives and another (denoted F) in which all the inventory fails, i.e., is destroyed by the event. We assume a constant probability $\theta_Y > 0$ of each disruption type $Y \in \{S, F\}$. When down, the probability of recovery (i.e., facility coming back up at the end of the period) depends only on the disruption type $Y \in \{S, F\}$ and is denoted by $\lambda_Y > 0$. As noted in the introduction, the insurer has detailed access to the firm when assessing the risk and potential losses. We assume, therefore, that the insurer and the firm both know the disruption and recovery probabilities.

Production (including any raw material procurement) incurs a variable cost of v per unit and has a lead time of zero. The firm receives a revenue of $r > v$ per unit sold. Demand in each period is deterministic, and without loss of generality, set equal to 1. Unfilled demand is lost, and the firm incurs an intangible goodwill cost of g per unit lost sale.

We assume the firm operates a base-stock policy in which the ending inventory is brought to a level B in any period that the facility is up. The firm incurs intangible inventory costs (e.g., opportunity, etc.) in each period based on the current inventory level at a cost of h per unit. If the firm adopts emergency sourcing as a strategy, it pays a reservation fee b every period to ensure supplier availability. During a disruption, the firm can purchase units from this supplier at a variable cost of e, with $v < e \leq r$.

The model and analysis can be extended to allow for insurable and noninsurable disruptive events, tangible inventory costs (e.g., physical storage, etc.), and a fixed facility cost per period. The main findings are the same, and we suppress these attributes for ease of presentation.

3.2. Insurance Elements

To focus our attention on BI insurance, we assume the firm has PD insurance that fully covers any property loss. BI insurance only covers tangible disruption-related losses. Intangible costs such as goodwill are not relevant to the BI loss calculation. Let $\pi^d(B, n)$ denote the tangible net income during a disruption of n periods including the first period back up, and let $\pi^u(B, n)$ denote the tangible net income that the firm would have earned during that same duration if there had not been a disruption. Expressions for $\pi^d(B, n)$ and $\pi^u(B, n)$ are developed in §4.1. The interruption loss is defined as $IL(B, n) = \pi^u(B, n) - \pi^d(B, n)$, and the BI policy reimburses the firm $R(B, D, L, n) = \min\{[IL(B, n) - D]^+, L\}$, where D is the (monetary) deductible and L the coverage limit. We normalize the firm's claim preparation cost to zero and assume that claims are processed and reimbursed in full in the first period after a disruption ends.[2] The firm chooses the deductible $D \geq 0$ and the coverage limit $L \leq \bar{L}$. The upper bound $\bar{L}$ on the coverage choice reflects the maximum level of coverage that the insurer is willing to provide and is exogenous to the model.

The insurance policy specifies a premium to be paid every period. The premium $p(B, D, L)$ is a function of the firm's base-stock level B, deductible D, and limit L. The insurer can verify the firm's historic inventory levels in the event of a claim by auditing the firm's enterprise resource planning systems. In effect, then, B is observable to all parties and so they can contract on B when setting the premium. We adopt the insurance pricing approach used by Kuzak and Larsen (2005, p. 100) for insurance products that cover catastrophic events:

$$\textit{premium} = \textit{average annual loss} + \textit{expense load} + \textit{risk load}.$$

The *average annual loss* (*AAL*) is the long-run average reimbursement in our model as it is framed in periods rather than years. Let $\bar{R}(B, D, L)$ denote the long-run average reimbursement. The *expense load* "reflects the administrative costs involved in insurance contracts" (Kuzak and Larsen 2005, p. 101), and the insurance literature often models these administrative costs as being proportional to the insurer's average reimbursement payment (Kahler 1979, Schlesinger 2006) or as a constant (Kliger and Levikson 1998). As in Kaplow (1992), we combine these approaches and model the expense load as $m_1 + m_2\bar{R}(B, D, L)$, where m_1 is the constant load and m_2 the proportional load. The *risk load* "is determined by the uncertainty surrounding the AAL… [and it] reflects the insurer's concern with the survival constraint and the need for additional capital" (Kuzak and Larsen 2005, p. 100). In effect, the intent of the risk load is to compensate the insurer for the fact that it faces some costs that are convex increasing in the reimbursement (which makes it risk averse). Rather than assuming a specific risk measure, we explicitly model the insurer as incurring a cost $\psi(R)$ that is a positive piecewise-linear convex increasing function of the reimbursement $R = \min\{[IL(B, n) - D]^+, L\}$. The risk load is then given by the long-run average value of $\psi(R)$, denoted as $\bar{\Psi}(B, D, L)$. Gathering together the average loss, expense load, and risk load, the per-period premium is given by

$$p(B, D, L) = \bar{R}(B, D, L) + m_1 + m_2\bar{R}(B, D, L) + \bar{\Psi}(B, D, L). \quad (1)$$

3.3. Finance-Related Costs

External financing is more expensive than internal financing for a variety of reasons, including information and incentive issues as well as transaction costs (Froot et al. 1993, Kaplan and Zingales 1997, and references therein). Therefore, a shortfall in cash can be costly because the firm either incurs the cost of external financing or underinvests in positive net present value projects (Myers and Majluf 1984). If net income is volatile, then financial risk management can benefit the firm by reducing the need for external financing or mitigating the underinvestment problem (Froot et al. 1993). Theoretical models in finance typically assume that the (extra or deadweight) cost of external financing is convex increasing in the amount raised (Froot et al. 1993, Kaplan and Zingales 1997), although some papers assume a linear relationship (Kim et al. 1998). Altinkiliç and Hansen (2000) and Hennessy and Whited (2007) have empirically estimated elements of external financing costs and both found that there is a positive fixed term, i.e., a cost component that is independent of the amount raised. Altinkiliç and Hansen (2000) found evidence of convexity, whereas Hennessy and Whited (2007) found evidence of a linear term but limited support for a convex term.

Disruptions reduce a firm's net income, and this reduction can create additional costs related to external financing and/or underinvestment. These additional costs are not reimbursed by BI insurance. We capture this finance-related cost issue by introducing a penalty based on the firm's net income loss over the disruption. Let x denote the loss in net income (factoring in any insurance reimbursement) during a disruption as compared to the case of no disruption. This loss is given by $x = IL(B, n) - R(B, D, L, n)$,

[2] Because businesses are more complex than reflected in this model, in practice there can be substantial disagreement between the firm and the insurer as to the actual business loss and what is covered by the policy. The claims process can be arduous and may result in delayed and partial settlements.

with $R(B, D, L, n) = 0$ if the firm does not have insurance. The firm incurs a disruption penalty (in the first period back up) of the form

$$K(x) = \begin{cases} k_1 + k_2(x) & \text{if } x > T, \\ 0 & \text{otherwise,} \end{cases} \quad (2)$$

where $k_1 \geq 0$ and $k_2(x) \geq 0$ is a positive piecewise linear convex increasing function. The fixed k_1 term reflects the fixed cost associated with external financing. The larger the net income loss, the larger the potential funding required, and the $k_2(x)$ function reflects the variable external-financing costs (or underinvestment opportunity cost). The threshold $T \geq 0$ allows us to capture the fact that a firm may require external financing (or be forced to forgo investments) only if its net income loss is significant. We make the natural assumption that $k_2(T) = 0$ but all results are easily adapted to the case of $k_2(T) > 0$.

Because disruptions lead to net income volatility and this volatility creates additional finance-related costs to the firm, one might anticipate that financial risk management, i.e., insurance in this case, to be beneficial. The disruption penalty, which penalizes net income volatility, is *the* motive for BI insurance in our model. Hoyt and Khang (2000) posited and found empirical support for underinvestment as a motive for corporate property insurance. Our model provides theoretical support for the corporate demand for BI insurance based on the general notion of costly external financing.

3.4. Summary of the Firm's Problem

The firm chooses a base-stock $B \geq 0$, whether or not to have an emergency supplier, and whether or not to purchase BI insurance. If the firm chooses to purchase BI insurance, it chooses a deductible $D \geq 0$ and a coverage limit $L \leq \bar{L}$. As in the disruption models of Tomlin (2006) and Parlar and Perry (1996) (among others) and some insurance models (e.g., Rubenstein and Yaari 1980, Dionne 1983), we adopt a long-run average criterion; a reasonable choice given the frequency of inventory replenishment. The firm's objective is to maximize its long-run average profit that includes the disruption penalty.

We adopt the following conventions throughout this paper. The term "profit" refers to the long-run average profit and "insurance" refers to BI insurance unless otherwise stated. The terms increasing, decreasing, larger than, and smaller than are used in the weak sense. Also, $[x]^+ = \max\{x, 0\}$ and $\lfloor x \rfloor$ is the largest integer less than or equal to x. The first and second differences of a function $f(x)$ with respect to an integer variable x are denoted by $\nabla f(x)$ and $\nabla^2 f(x)$. A list of notation can be found in Appendix A.

4. Analysis

Let $\Pi(B)$ denote the profit without insurance for a given base-stock level B, and let $\hat{\Pi}(B, D, L)$ denote the profit with insurance for a given base-stock level B, deductible D, and coverage limit L. We will first develop expressions for $\Pi(B)$ and $\hat{\Pi}(B, D, L)$, and then determine the optimal insurance (deductible and limit) and inventory decisions. We focus on the inventory strategy until §4.3 when emergency sourcing is also allowed.

4.1. Profit Expressions

The model gives rise to a Markov chain with the following states: (DN, n, Y) in which the facility has been down for $n \geq 1$ periods because of a Y-type disruption; (UP, n, Y) in which the facility is back up for the first period after being down n periods because of a Y-type disruption; and $(UP, 0)$ in which the facility is up and was up in the previous period. Because the disruption and recovery probabilities are strictly positive, the Markov chain is irreducible and the states are positive recurrent; thus, the steady-state probabilities $\rho(\cdot)$ exist. They are given by

$$\rho(DN, n, Y) = \rho(UP)\theta_Y(1-\lambda_Y)^{n-1}, \quad n \geq 1, Y \in \{S, F\}, \quad (3)$$

$$\rho(UP, n, Y) = \rho(UP)\theta_Y(1-\lambda_Y)^{n-1}\lambda_Y, \quad n \geq 1, Y \in \{S, F\}, \quad (4)$$

$$\rho(UP, 0) = \rho(UP)(1 - \theta_S - \theta_F), \quad (5)$$

where $\rho(UP) = (1 + \theta_S/\lambda_S + \theta_F/\lambda_F)^{-1}$ is the steady-state probability of being up. The steady-state probabilities are functions of the disruption and recovery probabilities only, and hence are independent of the adoption of the operations strategies (inventory and emergency sourcing) and insurance.

Before presenting the profit expressions with and without insurance, we develop the interruption-loss function as it is a central element in the analysis. Recall that the interruption loss is defined as $IL(B, n) = \pi^u(B, n) - \pi^d(B, n)$, where $\pi^d(B, n)$ is the tangible net income during a disruption of n periods including the first period back up and $\pi^u(B, n)$ is the tangible net income that the firm would have earned over the same $n+1$ periods had there not been a disruption. Now $\pi^u(B, n) = (n+1)(r - v)$, but $\pi^d(B, n)$ depends on the disruption type. During an S-type disruption the firm earns a revenue of $r \min\{n, B\}$. In the first period back up the firm earns r and incurs the variable cost $v(\min\{n, B\} + 1)$ to obtain the base-stock level B. Thus, $\pi_S^d(B, n) = (r - v)(\min\{n, B\} + 1)$. During an F-type disruption the firm earns no revenue as there is no inventory. In the first period back up the firm earns r and incurs the variable cost $v(B + 1)$ to obtain the base-stock level B but PD insurance covers the cost vB for the destroyed inventory because inventory is treated as property. Thus, $\pi_F^d(B,n)=r-v$. Using $IL(B,n)=\pi^u(B,n)-\pi^d(B,n)$, we have

$$IL_S(B,n)=(r-v)[n-B]^+, \tag{6}$$

$$IL_F(B,n)=(r-v)n, \tag{7}$$

where we have made explicit the dependence on the disruption type.

Proposition 1. *Without insurance, the profit for a given base-stock B is*

$$\Pi(B)=\Pi_N(B)-\sum_{n=B+\lfloor T/(r-v)\rfloor+1}^{\infty}\big(k_1+k_2((n-B)(r-v))\big)\rho(UP,n,S)-\sum_{n=\lfloor T/(r-v)\rfloor+1}^{\infty}\big(k_1+k_2(n(r-v))\big)\rho(UP,n,F), \tag{8}$$

where $\Pi_N(B)$, the profit when $K(x)\equiv 0$, is given by

$$\Pi_N(B)=r-(r+g)\cdot\Big(\sum_{n=1}^{\infty}\rho(DN,n,F)+\sum_{n=B+1}^{\infty}\rho(DN,n,S)\Big)-v\Big(\rho(UP)+\sum_{n=1}^{B}n\rho(UP,n,S)+B\sum_{n=B+1}^{\infty}\rho(UP,n,S)\Big)-h\Big(B\rho(UP)+\sum_{n=1}^{B}(B-n)\rho(DN,n,S)\Big). \tag{9}$$

Proof. All proofs are contained in Appendix B. □

The no-penalty profit $\Pi_N(B)$ has three components: (i) the revenue adjusted for the fact that no revenue is earned (and a goodwill cost incurred) when inventory runs out during a disruption, (ii) the variable cost of production (when up) and inventory replenishment post disruption (PD insurance covers this expense when the inventory is destroyed), and (iii) the inventory holding cost. The profit $\Pi(B)$ is lower because of the penalty function $K(\cdot)$. Absent insurance, the penalty is incurred when a Y-type disruption results in a tangible interruption loss $IL(n)$ that exceeds the threshold T. Using the interruption-loss expressions (6) and (7), the penalty is incurred if and only if the disruption length exceeds $B+\lfloor T/(r-v)\rfloor$ for an S-type disruption and $\lfloor T/(r-v)\rfloor$ for an F-type disruption.

We now turn our attention to the case in which the firm has insurance. In the first up-period after an n-period Y-type disruption, the firm receives a reimbursement of $R_Y(B,D,L,n)=\min\{[IL_Y(B,n)-D]^+,L\}$, $Y\in\{S,F\}$. Using (6) and (7), the long-run average reimbursement is $\bar R(B,D,L)=\bar R_S(B,D,L)+\bar R_F(D,L)$, where

$$\bar R_S(B,D,L)=\sum_{n=B+\lfloor D/(r-v)\rfloor+1}^{B+\lfloor (D+L)/(r-v)\rfloor}\big((n-B)(r-v)-D\big)\rho(UP,n,S)+L\sum_{n=B+\lfloor (D+L)/(r-v)\rfloor+1}^{\infty}\rho(UP,n,S), \tag{10}$$

$$\bar R_F(D,L)=\sum_{n=\lfloor D/(r-v)\rfloor+1}^{\lfloor (D+L)/(r-v)\rfloor}\big(n(r-v)-D\big)\rho(UP,n,F)+L\sum_{n=\lfloor (D+L)/(r-v)\rfloor+1}^{\infty}\rho(UP,n,F). \tag{11}$$

Proposition 2. *With insurance, the profit for a given base-stock B, deductible D and limit L is*

$$\widehat\Pi(B,D,L)=\Pi(B)-\big(m_1+m_2\bar R(B,D,L)+\bar\Psi(B,D,L)\big)+\Delta\bar K(B,D,L), \tag{12}$$

where $\Pi(B)$ is the profit without insurance, and

$$\Delta\bar K(B,D,L)=\sum_{n=B+\lfloor T/(r-v)\rfloor+1}^{B+\lfloor (T+L)/(r-v)\rfloor}\big(k_1+k_2((n-B)(r-v))\big)\rho(UP,n,S)+\sum_{n=B+\lfloor (T+L)/(r-v)\rfloor+1}^{\infty}\big(k_2((n-B)(r-v))-k_2((n-B)(r-v)-L)\big)\rho(UP,n,S)+\sum_{n=B+\lfloor T/(r-v)\rfloor+1}^{\lfloor (T+L)/(r-v)\rfloor}\big(k_1+k_2(n(r-v))\big)\rho(UP,n,F)+\sum_{n=\lfloor (T+L)/(r-v)\rfloor+1}^{\infty}\big(k_2(n(r-v))-k_2(n(r-v)-L)\big)\cdot\rho(UP,n,F) \tag{13}$$

for $D\le T$, and

$$\Delta\bar K(B,D,L)=\sum_{n=B+\lfloor D/(r-v)\rfloor}^{B+\lfloor (D+L)/(r-v)\rfloor}\big(k_2((n-B)(r-v))-k_2(D)\big)\rho(UP,n,S)+\sum_{n=B+\lfloor (D+L)/(r-v)\rfloor+1}^{\infty}\big(k_2((n-B)(r-v))-k_2((n-B)(r-v)-L)\big)\rho(UP,n,S)$$

$$+\sum_{n=\lfloor D/(r-v)\rfloor}^{\lfloor (D+L)/(r-v)\rfloor}\big(k_2(n(r-v))-k_2(D)\big)\rho(UP,n,F)$$
$$+\sum_{n=\lfloor (D+L)/(r-v)\rfloor+1}^{\infty}\big(k_2(n(r-v))-k_2(n(r-v)-L)\big)\cdot\rho(UP,n,F) \quad (14)$$

for $D > T$.

Define the *value of insurance* (at a given B, D, and L) as the profit with insurance less the profit without insurance. Observe from (12) that the value of insurance comprises two terms. The first term $m_1 + m_2\bar{R}(B,D,L) + \bar{\Psi}(B,D,L)$ is the premium less the average reimbursement; see (1). We call this the *net premium*. The second term $\Delta\bar{K}(B,D,L)$ is the additional benefit of insurance above and beyond the average reimbursement. We call this the *net benefit of insurance*. The value of insurance is the net benefit less the net premium. Insurance will be purchased (at a given B, D, and L) only if it provides a positive value.

Insurance provides a net benefit to the firm because the reimbursement reduces the firm's overall net income loss during a disruption (and thus reducing the need for external financing, for example). This benefit materializes itself through the disruption penalty function $K(x)$ because $x = IL(B,n) - R(B,D,L,n)$, with $R(B,D,L,n) = 0$ if the firm does not have insurance. When $D \le T$ insurance provides a net benefit by reducing the set of states in which the disruption penalty is incurred (penalty avoidance) and reducing the size of the penalty in any given state (penalty reduction). The penalty avoidance is reflected in the first and third terms in (13) as the penalty is incurred only if the disruption exhausts the coverage limit L, i.e. it lasts longer than $B + \lfloor (T+L)/(r-v)\rfloor$ periods if inventory succeeds and longer than $\lfloor (T+L)/(r-v)\rfloor$ periods if inventory fails. When $D > T$, i.e., the deductible exceeds the penalty threshold, insurance does not change the set of states in which the penalty is incurred and so the penalty avoidance benefit disappears but the penalty reduction benefit persists.

Before examining the firm's optimal deductible, limit, and inventory choices, we make the following observation: For any B and L, the net benefit $\Delta\bar{K}(B,D,L)$ does not depend on the deductible D when $D \le T$; see (13). The net premium, however, decreases in D because the average reimbursement $\bar{R}(B,D,L)$ decreases in D and the risk load $\bar{\Psi}(B,D,L)$ also decreases in D (because all reimbursements, not just the average, decrease in D). Using (12), the profit therefore increases in D for $D \le T$. It then follows that the optimal deductible for any given B and L is greater than or equal to T. We can therefore restrict attention to $D \ge T$.

For expositional ease, we assume that the penalty threshold T is an integer multiple of $r - v$, i.e., $T = n_T(r-v)$ with n_T integer. We also assume that any breakpoints in the $\psi(\cdot)$ and $k_2(\cdot)$ piecewise-linear functions occur only at integer multiples of $r - v$. It then follows from Proposition 2 and expressions (6) and (7) that there is an optimal deductible and limit that are both integer multiples of $r - v$ (because the profit is linear between these integer multiples). Writing $D = n_D(r-v)$ and $L = n_L(r-v)$, where n_D and n_L are integers, we can therefore express the firm's problem as determining the optimal n_D, n_L, and base-stock B. For simplicity, we refer to n_D and n_L as the deductible and limit, respectively, with the understanding that the actual deductible and limit are given by $D = n_D(r-v)$ and $L = n_L(r-v)$. Using the observation in the preceding paragraph, in all that follows we restrict attention to $n_D \ge n_T$ without loss of generality.

4.2. Optimal Insurance and Inventory Decisions

We now examine the optimal insurance decisions (deductible and coverage limit) and the optimal inventory level. We focus on the case in which there is no risk load in the premium function (1), i.e., $\bar{\Psi}(B,D,L) \equiv 0$. This is relaxed in §6. Absent the risk measure, the premium equals the average reimbursement plus the expense load, i.e., $p(B,D,L) = \bar{R}(B,D,L) + m_1 + m_2\bar{R}(B,D,L)$. This special case reflects a competitive insurance market with risk-neutral insurers; such a model is common in the general insurance literature, e.g., Kahler (1979), Schlesinger (1983), Kahane and Kroll (1985), Eeckhoudt et al. (1988), and Kaplow (1992).

Using Proposition 2, the steady-state probability expressions (3)–(5), and tailoring expressions (10), (11), (13), and (14) to the case of T, D, and L being integer multiples of $r - v$, the profit with insurance can be written (after some algebraic manipulation) as

$$\begin{aligned}\widehat{\Pi}(B,n_D,n_L) &= \Pi(B) - m_1 + \rho(UP)\\ &\quad\cdot\begin{pmatrix}\theta_S(1-\lambda_S)^{B+n_D}(1-(1-\lambda_S)^{n_L})a_S(n_D)\\ +\theta_F(1-\lambda_F)^{n_D}(1-(1-\lambda_F)^{n_L})a_F(n_D)\end{pmatrix},\end{aligned} \quad (15)$$

where

$$\begin{aligned}a_Y(n_D) = k_1 I(n_D) + \sum_{n=0}^{\infty}\big(&k_2((n_T+n+1)(r-v))\\ &-k_2((n_T+n)(r-v)) - m_2(r-v)\big)(1-\lambda_Y)^n\end{aligned} \quad (16)$$

for $Y \in \{S, F\}$, and $I(n_D)$ is an indicator function such that $I(n_T) = 1$ and $I(n_D) = 0$ for $n_D > n_T$.[3] Recall that

[3] When introducing the penalty function, we made the assumption that $k_2(T) = 0$. All results in the paper extend to the case of $k_2(T) > 0$ with one transformation: the $a_Y(n_T)$ function becomes $a_Y(n_T) = k_1 + k_2((n_T+1)(r-v)) - m_2(r-v) + \sum_{n=1}^{\infty}(1-\lambda_Y)^n \cdot (k_2((n_T+n+1)(r-v)) - k_2((n_T+n)(r-v)) - m_2(r-v))$.

we can restrict attention to $n_D \geq n_T$ without loss of generality because, as discussed above, the optimal deductible is at least T.

Ignoring the constant premium term m_1 for the moment, the value of insurance is given by the third term in (15). Observe that the value is separable by the disruption type. Let us first consider an S-type disruption. The $a_S(n_D)$ term captures both the penalty-reduction advantage and the net-premium disadvantage of insurance; the indicator function in (16) reflects the fact that insurance does not protect the firm from incurring the fixed penalty k_1 if the deductible exceeds the penalty threshold. This insurance value, which may be positive or negative, is scaled by the firm's limit choice through the $1-(1-\lambda_S)^{n_L}$ term. It is also scaled by $(1-\lambda_S)^{B+n_D}$ because the firm receives an insurance payout only if the disruption length exceeds $B+n_D$, i.e., it has used up its inventory and its net income loss has exceeded the deductible. A similar structure holds for the value of insurance attributable to F-type disruptions, but in this case an insurance payout occurs only if the disruption length exceeds n_D because there is no inventory to delay losses.

Continuing to ignore the constant premium term m_1, the value of insurance attributable to Y-type disruptions is positive if and only if $a_Y(n_D)$ is positive; recall that $a_Y(n_D)$ captures the tension between penalty reduction and net-premium payment. If both $a_S(n_D)$ and $a_F(n_D)$ are negative, then the firm will not purchase insurance. If at least one of these two terms is positive, then the firm will purchase insurance if the overall value of insurance, i.e., the sum of the second and third terms in (15), is positive. Finally, we note that (15) does not imply that the value of insurance is increasing in $\rho(UP)$; recall from above that $\rho(UP)=(1+\theta_S/\lambda_S+\theta_F/\lambda_F)^{-1}$, and so an increase in $\rho(UP)$ must be accompanied by a decrease in at least one of the θ_Y or an increase in at least one of the λ_Y.

With profit expression (15) in hand, we now turn our attention to determining the optimal deductible and limit for any given base-stock B. Let $n_D^*(B, n_L)$ denote the optimal deductible for any given B and n_L.

Theorem 1. *Let $\hat{n}$ be the smallest $n \geq 0$ such that $k_2((n+1)(r-v))-k_2((n)(r-v)) \geq m_2(r-v)$. For any B and n_L,* (i) *if $\hat{n} \leq n_T$, then $n_D^*(B, n_L)=n_T$;* (ii) *if $\hat{n} > n_T$, then $n_D^*(B, n_L)=n_T$ if*

$$\begin{aligned}&\theta_S(1-\lambda_S)^{B+n_T}\big(1-(1-\lambda_S)^{n_L}\big)\big(a_S(n_T)-(1-\lambda_S)^{\hat{n}-n_T}a_S(\hat{n})\big)\\&\quad+\theta_F(1-\lambda_S)^{n_T}\big(1-(1-\lambda_F)^{n_L}\big)\\&\quad\cdot\big(a_F(n_T)-(1-\lambda_F)^{\hat{n}-n_T}a_F(\hat{n})\big)\geq 0,\end{aligned}$$

and $n_D^*(B, n_L)=\hat{n}$ otherwise.

The firm faces the following trade-off as it increases its deductible: the penalty avoidance/reduction benefit decreases (a disadvantage) but the net premium also decreases (an advantage). When $n_D > n_T$, insurance does not protect the firm from incurring the fixed penalty k_1, and so the penalty effect is driven solely by $k_2(\cdot)$. In this case, the marginal value of increasing the deductible is driven by $k_2((n_D+1)(r-v)) - k_2((n_D)(r-v)) - m_2(r-v)$, where $m_2(r-v)$ captures the net premium effect. When $k_2((n_D+1)(r-v)) - k_2((n_D)(r-v)) < m_2(r-v)$, then the advantage of a net-premium decrease outweighs the disadvantage of the penalty-reduction decrease; so the firm prefers n_D+1 to n_D until $n_D=\hat{n}$. However, if the firm sets $n_D=n_T$, it gains the additional benefit of reducing the probability of incurring the fixed penalty k_1; so it can be advantageous to set the deductible to n_T even if $\hat{n} > n_T$.

For reasons of space and because it leads to more interesting results, we focus the remainder of this paper on the case in which n_T is the optimal deductible for all B; the necessary and sufficient condition for this is given at the end of the proof of Theorem 1. For later use, we note that the necessary condition implies that (i) $k_1 > 0$ and (ii) $a_S(n_T)$, and $a_F(n_T)$ cannot both be negative; that is the fixed penalty term is strictly positive and insurance adds positive value at a deductible of n_T. As discussed in §3.3, there is empirical support that external financing has a positive fixed cost. Later in the paper, we comment on if and how results change when $\hat{n}$ is the optimal deductible.

We now proceed to characterize the optimal limit $n_L^*(B)$ for any given B. Recall that actual coverage is $n_L(r-v)$ and that $\bar{L}$ is the maximum-allowed actual coverage.

Theorem 2. *Let $n_{\bar{L}} \equiv \lfloor \bar{L}/(r-v)\rfloor$. For any given B, $n_L^*(B)=n_{\bar{L}}$ if $A(B, n_T, n_{\bar{L}}) \geq 0$; otherwise, $n_L^*(B)$ is the minimum (integer) value of n_L such that $A(B, n_T, n_L) < 0$, where*

$$\begin{aligned}A(B, n_T, n_L) = {}&\left(\frac{1-\lambda_S}{1-\lambda_F}\right)^{n_T+n_L}(1-\lambda_S)^B\theta_S\lambda_S a_S(n_T)\\&+\theta_F\lambda_F a_F(n_T).\end{aligned} \tag{17}$$

The firm incurs the additive expense load m_1 regardless of its coverage if it purchases insurance. Therefore, the magnitude of m_1 dictates whether the firm should purchase insurance but not the optimal coverage if it does purchase. Recall from above that the value of insurance attributable to Y-type disruptions is positive if and only if $a_Y(n_T)$ is positive, where $Y \in \{S, F\}$. When $a_S(n_T)$ and $a_F(n_T)$ are both positive, the above theorem implies that the firm should purchase the maximum-allowed level of coverage, i.e., $n_L^*(B)=n_{\bar{L}}$, because all other terms in (17) are positive.

When exactly one of $a_S(n_T)$ and $a_F(n_T)$ is positive, let Y^+ (Y^-) denote the disruption type for which $a_Y(n_T)$ is positive (negative). In this case, the firm prefers a higher coverage if thinking only of Y^+ type disruptions because the penalty reduction benefit dominates net premium attributable to the Y^+ type, but the firm prefers a lower coverage if thinking only of Y^- type disruptions because the associated net premium dominates the penalty reduction. Of course, the firm cannot purchase separate policies for the different disruption types, and in balancing these opposing desires, the firm may find it optimal to select an interior coverage limit, i.e., purchase less coverage than the maximum allowed.

We now turn our attention to the base-stock decision. For any given B, let $\widehat{\Pi}(B)$ denote the profit (with insurance) at the optimal deductible and coverage for that B. We also consider the no-insurance case. Recall that the profit without insurance is denoted $\Pi(B)$.

Theorem 3. *The profit without insurance $\Pi(B)$ is concave in B.*

Theorem 4. *The profit with insurance $\widehat{\Pi}(B)$ is concave in B if $n_L^*(B) = n_{\bar{L}}$ for all B.*

Although we have not been able to analytically establish that $\widehat{\Pi}(B)$ is concave when an interior coverage limit can be optimal, our numerical tests found that $\widehat{\Pi}(B)$ was concave in B when $n_L^*(B) < n_{\bar{L}}$. In §5, we will explore whether and how insurance influences the firm's inventory investment.

4.3. Emergency Sourcing

We now consider the case when the firm uses emergency sourcing in addition to holding inventory. The firm can choose zero inventory if it wishes. Recall that when the firm uses the emergency sourcing strategy, it pays a reservation fee b (every period) to ensure availability and pays e per unit sourced from the emergency supplier. Because $e > v$, the firm only sources from the emergency supplier during a disruption, and then only when it runs out of inventory (because filling demand from inventory is preferable as it reduces the inventory holding cost). Let $\Pi_N^E(B)$ denote the profit without insurance when there is no disruption penalty, i.e., $K(x) \equiv 0$. The development of the $\Pi_N^E(B)$ expression follows in a similar manner to the inventory-only expression $\Pi_N(B)$ (see (9)) except that instead of incurring the goodwill cost g when inventory runs out, the firm incurs the variable emergency sourcing cost e. It also incurs b every period. Accounting for these differences,

$$\Pi_N^E(B) = \Pi_N(B) - b + (r + g - e) \cdot \left[\sum_{n=B+1}^{\infty} \rho(DN, n, S) + \sum_{n=1}^{\infty} \rho(DN, n, F)\right]. \quad (18)$$

Emergency sourcing is used if a disruption lasts long enough to completely deplete the firm's inventory. Expression (18) shows that in this case the firm prevents lost revenue and goodwill, but at a cost e. The firm's per-period tangible loss (after any inventory is exhausted) is $e - v$ as opposed to $r - v$ without emergency sourcing. The tangible loss is thus lower with emergency sourcing as $e \le r$ by assumption.

Assume for expositional convenience that the penalty threshold T is an integer multiple of $e - v$ and that any breakpoints in the $k_2(x)$ function occur only at integer multiples of $e - v$. It is readily shown (proof omitted) that *all* of the above propositions and theorems for the inventory-only strategy apply also to the emergency-sourcing and inventory strategy with a simple transformation: $\Pi_N^E(B)$ replaces $\Pi_N(B)$ in Proposition 2, and e replaces r in expressions (8), (10), (11), (13), (14), and (16). Also, e replaces r in the denominator for n_T, n_D, and n_L, that is, $n_T = T/(e - v)$, $n_D = D/(e - v)$, and $n_L = L/(e - v)$.

5. The Interplay of Operational Measures and Insurance

We now explore the interplay between operational measures and insurance. In particular, we examine whether they are substitutes or complements. We first analyze inventory and insurance and then emergency sourcing and insurance. We continue to focus on the case of no risk load, i.e., $\bar{\Psi}(B, D, L) \equiv 0$ but this is relaxed in §6.

5.1. Inventory and Insurance

We define inventory and insurance to be substitutes (complements) if insurance reduces (increases) the marginal value of inventory.

Definition 1. Inventory and insurance are substitutes (complements) if $\nabla\widehat{\Pi}(B) \le (\ge) \nabla\Pi(B)$ for all B.

Because inventory and insurance both mitigate the impact of disruptions, one might intuitively expect them to be substitutes. The following theorem establishes that they can in fact be complements.

Theorem 5. *Inventory and insurance are substitutes (complements) if and only if $a_S(n_T) \ge (\le) 0$.*

To understand what drives the substitute/complement result, it is important to recognize that insurance and inventory interact in two manners. First, insurance and inventory both serve to help the firm avoid/reduce the disruption penalty. In that regard, insurance reduces the marginal value of inventory; the penalty interaction is a substitution effect. Second, inventory reduces the reimbursement, and hence reduces the net premium paid for insurance, i.e., $m_1 + m_2\bar{R}(B, D, L)$. In that regard, insurance inflates the marginal value of inventory; the net-premium interaction is a complementary effect. These two insurance–inventory interactions, penalty and net premium, are fully captured by the $a_S(n_T)$ expression, which was given by (16). The penalty interaction (substitution effect) is strong when the fixed penalty term k_1 is high and/or the proportional term $k_2(x)$ increases rapidly, both of which drive $a_S(n_T)$ higher. The net-premium interaction (complementary effect) is strong when the proportional premium load m_2 is high, and $a_S(n_T)$ decreases in m_2.

Insurance and inventory are substitutes when the penalty interaction dominates the net-premium interaction, i.e., $a_S(n_T) \geq 0$. In this case, the firm invests in *less* inventory when it has insurance. Insurance and inventory are complements when the penalty interaction dominates the net-premium interaction, i.e., $a_S(n_T) \leq 0$. In this case, the firm invests in *more* inventory when it has insurance. Again, the reason for this complementarity is that inventory drives down the price of insurance to an extent that dominates the penalty interaction effect.

Although not immediately apparent, Theorem 5 implies certain conditions for complementarity that we want to bring to the reader's attention. First, recall that we are focusing on the case in which n_T is the optimal deductible. If there is no fixed disruption penalty, i.e., $k_1 = 0$, or if $a_S(n_T)$ and $a_F(n_T)$ are both negative (which can occur if m_2 is sufficiently high), then the firm prefers the deductible $\hat{n}$, which can be ∞ (effectively no insurance) if m_2 is extremely high. The firm purchases the maximum-allowed coverage limit when $\hat{n}$ is the optimal deductible, and inventory and insurance are substitutes; the proof of these results follow in similar manner to proofs of Theorems 2 and 5 and the fact that $a_S(\hat{n}) \geq 0$ and $a_F(\hat{n}) \geq 0$ by definition of $\hat{n}$. Second, the complementarity condition also implies that $\lambda_F > \lambda_S$; that is, the firm recovers faster (on average) from disruptions that destroy inventory than from ones that do not. This might seem unlikely to hold in practice. For example, recovering from a larger fire that causes more damage (including inventory destruction) would likely take longer than recovering from a smaller fire that caused less damage and did not destroy the inventory. There are reasonable circumstances, however, in which $\lambda_F > \lambda_S$ would hold. Consider a two-step process, such as pharmaceutical production followed by filling/packing, in which the production technology for the first step is difficult to repair/replace but the technology for the second step can be easily repaired/replaced. If the steps are in different zones of the plant and the finished goods inventory is stored in close proximity to the easy-to-repair technology, then a fire that damages the easy-to-repair technology may destroy the inventory but recovery time will be short.

5.2. Emergency Sourcing and Insurance

We now discuss the interplay between emergency sourcing and insurance. Let $\widehat{\Pi}^E$ (Π^E) denote the optimal profit with (without) insurance when there is an emergency source. It is straightforward to show that emergency sourcing and inventory are substitutes using a definition analogous to Definition 1. Different to the inventory level, insurance and emergency sourcing are both binary decisions; the firm either uses the strategy or not. As such, we define insurance and emergency sourcing to be substitutes (complements) if emergency sourcing reduces (increases) the value of insurance.

Definition 2. Emergency sourcing and insurance are substitutes (complements) if $\Delta V \leq 0$ ($\Delta V \geq 0$), where $\Delta V = \widehat{\Pi}^E - \Pi^E - (\widehat{\Pi} - \Pi)$.

The substitute/complement question can be explored either in the absence of inventory (all strategies use zero inventory) or in its presence (i.e., at the appropriate optimal base-stock levels). We focus on the case of zero inventory. Industries in which inventory is very expensive or very perishable will not use inventory to manage disruption risk, and the zero-inventory case reflects such industries. Also, by focusing on the zero-inventory case, we can highlight the interplay between emergency sourcing and insurance. The analytical findings reported below were numerically observed to also hold when a positive inventory (set optimally) is allowed.

Both the penalty interaction and the net-premium interaction exist between emergency sourcing and insurance. Whereas the net-premium interaction is necessary for the complementarity between inventory and insurance, the complementarity between emergency sourcing and insurance can be *solely* driven by the penalty interaction. To highlight this distinction, we remove the proportional expense load in the insurance premium and the convex variable cost in the disruption penalty, respectively, by letting $m_2 = 0$ and $k_2(\cdot) = 0$. For simplicity we let $\lambda_Y = \lambda$, $Y \in \{S, F\}$. By Theorem 5 (and $a_S(n_T) > 0$) inventory and insurance are substitutes in this setting. The following theorem states that emergency sourcing and insurance are complements if the recovery probability λ is low and the emergency cost e is high.

Theorem 6. *Assume that $m_2 = 0$, $k_2(\cdot) = 0$, $\lambda_Y = \lambda$ for $Y \in \{S, F\}$, and $B = 0$. If $\lambda < 1 - (T/(T + \bar{L}))^{(r-v)/\bar{L}}$, then there exists a threshold $\tilde{e}(\lambda) < r$ such that insurance and emergency sourcing are substitutes (complements) for $e \leq \tilde{e}(\lambda)$ ($e > \tilde{e}(\lambda)$). If $\lambda > 1 - (T/(T + \bar{L}))^{(r-v)/\bar{L}}$, then insurance and emergency sourcing are substitutes.*

Because emergency sourcing reduces the per-period net income loss from $r - v$ to $e - v$, (a) it takes a longer disruption (as compared to no emergency sourcing) to exceed the penalty threshold T, and (b) it allows the coverage limit $\bar{L}$ to absorb longer disruptions as compared to no emergency sourcing. Whereas effect (a) decreases the value of insurance (substitute), effect (b) increases the value of insurance (complement). Effect (a) dominates, resulting in insurance and emergency sourcing being substitutes, unless the recovery probability is low and the variable emergency sourcing cost is high in which case the long disruption duration strengthens effect (b) and the high emergency cost weakens effect (a), and insurance and emergency sourcing are complements.

6. Risk Loading

The profit expression in Proposition 2 was developed allowing for a positive risk load in the premium, but we assumed a zero risk load, i.e., $\bar{\Psi}(B, D, L) \equiv 0$, when characterizing the optimal decisions in §§4.2 and 4.3 and when analyzing whether operational measures and insurance were substitutes or complements. In this section, we establish that our insights about the interplay of insurance and inventory are robust, that is, both complementarity and substitution persist in the presence of risk loading. We focus on the case of no-emergency sourcing, but as in §4.3, the following analysis and results extend easily to the case of emergency sourcing using a simple transformation whereby e replaces r in the relevant expressions.

Recall that the risk load $\bar{\Psi}(B, D, L)$ is the average value of $\psi(R)$, where $\psi(R)$ is a positive piecewise-linear convex increasing function of the reimbursement $R = \min\{[IL(B, n) - D]^+, L\}$. Using the interruption loss expressions (6) and (7), and assuming that any breakpoints of $\psi(R)$ occur only at integer multiples of $r - v$, we can write

$$\begin{aligned}
\bar{\Psi}(B, n_D, n_L) &= \sum_{n=B+n_D+1}^{B+n_D+n_L} \rho(UP, n, S)\psi((n - B - n_D)(r - v)) \\
&\quad + \psi(n_L(r - v)) \sum_{n=B+n_D+n_L+1}^{\infty} \rho(UP, n, S) \\
&\quad + \sum_{n=n_D+1}^{n_D+n_L} \rho(UP, n, F)\psi((n - n_D)(r - v)) \\
&\quad + \psi(n_L(r - v)) \sum_{n=n_D+n_L+1}^{\infty} \rho(UP, n, S),
\end{aligned}$$

where (as before) we frame the deductible and limit in terms of integer multiples of $r - v$. Applying the steady-state probability expressions (3)–(5), it can be shown that $\bar{\Psi}(B, n_D, n_L) = \bar{\Psi}_S(B, n_D, n_L) + \bar{\Psi}_F(n_D, n_L)$, where

$$\bar{\Psi}_S(B, n_D, n_L) = \rho(UP)\theta_S\lambda_S(1 - \lambda_S)^{B+n_D}\phi_S(n_L),$$
$$\bar{\Psi}_F(n_D, n_L) = \rho(UP)\theta_F\lambda_F(1 - \lambda_F)^{n_D}\phi_F(n_L),$$

and

$$\phi_Y(n_L) = \sum_{n=1}^{n_L} \psi(n(r - v))(1 - \lambda_Y)^{n-1} + \psi(n_L(r - v))\frac{(1 - \lambda_Y)^{n_L}}{\lambda_Y}$$

for $Y \in \{S, F\}$. Generalizing the profit expression (15) to allow for a positive risk load in the premium, we obtain

$$\begin{aligned}
&\hat{\Pi}(B, n_D, n_L) \\
&\quad = \Pi(B) - m_1 + \rho(UP) \\
&\qquad \cdot \begin{pmatrix} \theta_S(1-\lambda_S)^{B+n_D}((1-(1-\lambda_S)^{n_L})a_S(n_D) - \phi_S(n_L)) \\ +\theta_F(1-\lambda_F)^{n_D}((1-(1-\lambda_F)^{n_L})a_F(n_D) - \phi_F(n_L)) \end{pmatrix},
\end{aligned} \tag{19}$$

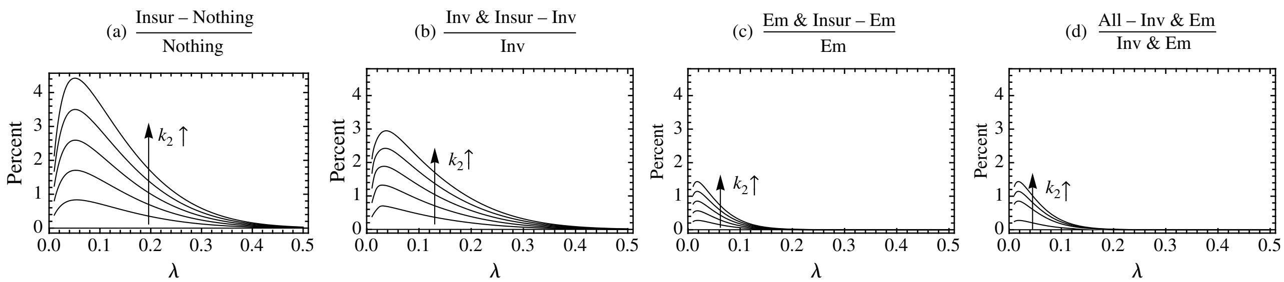
where $a_Y(n_D)$ was given in (16). Comparing (15) and (19) (or more precisely the first differences of these expressions) it follows that the risk load in the premium weakly increases (decreases) the firm's optimal deductible (limit). The intuition is that a higher deductible (lower limit) reduces reimbursements and this reduces the risk load, which in turn reduces the net premium. The optimal deductible remains at n_T, however, if the fixed penalty term k_1 is sufficiently large.

Focusing on the case where the optimal deductible is n_T with and without risk loading, we examine how risk loading influences our earlier result that insurance and inventory can be complements. Now, risk loading influences the value of insurance only through its impact on the net premium. It does not influence the net benefit of insurance, i.e., the penalty reduction. Therefore, risk loading has no influence on the penalty interaction of insurance and inventory. Risk loading, however, amplifies the net-premium interaction; that is, the net premium decreases faster in B when there is a risk load because the risk load $\bar{\Psi}(B, D, L)$ decreases in B. Intuitively, then, one might expect risk loading to make it more likely that insurance and inventory be complements because it is the net-premium interaction that drives complementarity. This is verified in the following theorem.

Theorem 7. *With a risk load in the premium, inventory and insurance are complements if $a_S(n_T) \le 0$.*

Without a risk load, $a_S(n_T) \le 0$ was the necessary and sufficient condition for complementarity; see Theorem 5. With a risk load, $a_S(n_T) \le 0$ is a sufficient but not necessary condition because, as discussed above, risk loading amplifies the net premium interaction. We note that a sufficient condition for inventory and insurance to be substitutes (in the presence of risk loading) is given at the end of the proof of Theorem 7.

Figure 1 Value of Insurance as k_2 Increases from 1 to 5



Note. The flat line at bottom represents the case of $k_1 = k_2 = 0$.

7. Value of Strategies

We now explore how model parameters influence the attractiveness of the disruption strategies. Because the directional effect of strategy-specific parameters (inventory costs, emergency sourcing costs, and the insurance markups) are obvious, we focus our attention on two key drivers that influence the performance of all three strategies: the disruption profile and the disruption penalty. The disruption profile refers to the concept that, for a given percentage uptime, disruptions can range from short but frequent to long but rare. The consequences of financially significant disruptions, as captured by the penalty parameters k_1 and $k_2(\cdot)$, may differ across firms.

For simplicity we make two assumptions, both can be easily relaxed. First, we assume that the recovery probability does not depend on the disruption type. The disruption profile is then captured by the recovery probability λ, and the expected disruption length is given by $1/\lambda$. Holding the steady-state probability of being up constant, disruptions become shorter and more frequent as λ increases. Second, we assume an affine penalty function $K(x) = k_1 + k_2 x$ if $x > T$, zero otherwise. The magnitude of the disruption penalty can be adjusted by scaling k_1 and k_2.

7.1. Value of Insurance

The value of insurance at a given B, D, and L, defined in §4.1, is the increase in profit gained by adding insurance to an existing strategy. In this section, we examine how the model parameters influence the value of insurance when B, D, and L are set to their optimal values for a given strategy. In Figure 1, we plot the *relative* value of (adding) insurance to four strategies ((a) nothing, (b) inventory, (c) emergency sourcing, and (d) inventory and emergency sourcing) as a function of the disruption profile, measured by λ, and the proportional disruption penalty k_2, with the steady-state probability of being up 0.99.[4] We omit the analogous plot for k_1 as a similar pattern holds. Two observations are apparent in Figure 1(a); the relative value of insurance (i) increases in the penalty k_2, and (ii) increases and then decreases in λ. Both of these observations can be proven analytically for the value of insurance (proofs omitted). Observation (i) is not surprising but observation (ii) merits discussion.

If the penalty threshold is positive, then disruptions must be sufficiently long to trigger the disruption penalty. As such, one might expect that insurance would become more valuable as disruptions become longer on average, i.e., as λ decreases. This intuition is correct when the disruption probability θ is held constant in which case the percentage uptime decreases. In that case, the likelihood of incurring the penalty increases as λ decreases. However, when the percentage uptime is held constant instead (as in Figure 1(a)), disruptions also become rarer as the average length increases. As disruptions become longer and rarer, i.e., moving from right to left on the x-axis, the relative value of insurance increases to a peak but then decreases, suggesting insurance is not an effective risk management tool when disruptions are very rare. This can be proven for the value of insurance (proof omitted). The reason for this behavior is that the likelihood of incurring the disruption penalty is not monotonic in λ. As disruptions become very long but very rare the overall probability of incurring the penalty decreases.

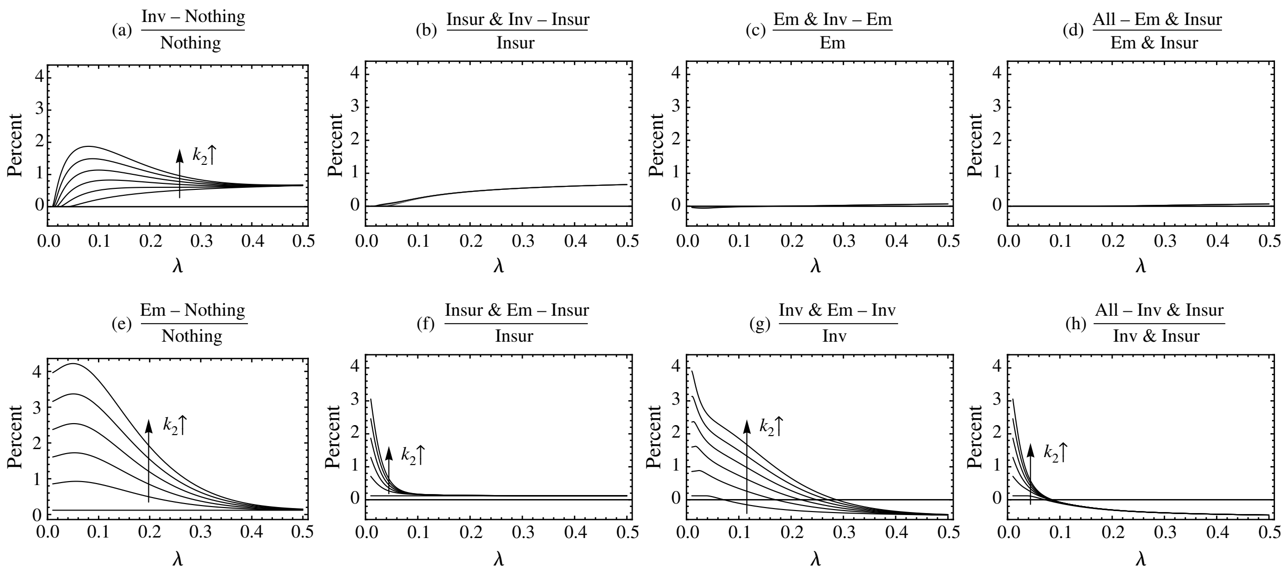
Similar behaviors with regard to k_2 and λ are observed in Figures 1(b)–1(d), but the value of insurance is significantly diminished by the presence of an operational strategy.

7.2. Value of Operational Measures

We now explore the value of inventory and emergency sourcing. Again, the value of a strategy is measured as the increase in profit gained by adding

[4] The following revenue and costs were used for all figures in this section: $r = 1$, $v = 0.1$, $h = 0.0002691$, $g = 0.5$, $b = 0.01$, and $e = 0.4$. With periods being interpreted as weeks, the holding cost represents an annual holding cost rate of 15%. The penalty threshold $T = 10(r - v)$, i.e., the penalty is incurred if the interruption loss exceeds 10 weeks of variable net income, and the fixed penalty cost $k_1 = 1$. Inventory fails in 50% of disruptions. The insurance markups are $m_1 = 0$ and $m_2 = 0.1$ and the risk load is zero. The maximum coverage limit $\bar{L} = 52(r - v)$.

Figure 2 Value of Operational Measures as k_2 Increases from 1 to 5; (a)–(d) Illustrate the Value of Inventory, and (e)–(h) Illustrate the Value of Emergency Sourcing



Note. The flat line at bottom represents the case of $k_1 = k_2 = 0$.

that strategy to an existing strategy (or combination of strategies) with B, D, and L set to their appropriate optimal values. Unlike insurance, inventory and emergency sourcing provide benefits above and beyond penalty avoidance/reduction because they mitigate revenue/goodwill loss during a disruption; see the no-penalty profit expressions (9) and (18) for both strategies.

In Figures 2(a)–2(d), we plot the *relative* value of adding inventory to four strategies: (a) nothing, (b) insurance, (c) emergency sourcing, and (d) emergency sourcing and insurance. In Figures 2(e)–2(h), we plot the *relative* value of adding emergency sourcing to four strategies: (e) nothing, (f) insurance, (g) inventory, and (h) inventory and insurance. The inventory strategy never has a negative value because the base-stock level can be set to zero. Emergency sourcing, however, can have a negative value as the firm pays a reservation fee b to add it to its portfolio.

The penalty mitigation benefit of inventory (or emergency sourcing) is high when it is the only disruption strategy employed by the firm, especially when disruptions are moderately long but rare (see Figures 2(a) and 2(e)). When disruptions become very long and rare, inventory is an expensive means to avoid/reduce the penalty, whereas emergency sourcing remains effective. As disruptions become short but frequent the value of the two operational measures is mainly driven by their revenue/goodwill mitigation abilities (as the penalty is not often incurred because disruptions are short on average). The presence of insurance strategy diminishes the value of operational measures when disruptions are long but rare (i.e., when insurance is effective at mitigating the penalty), but does not affect their values when disruptions are short but frequent (i.e., when the main concern is revenue/goodwill loss, which insurance cannot influence) (see Figures 2(b) and 2(f)). Because of substitution, the presence of one operational measure nullifies the effect of the other operational measure (see Figures 2(c), 2(d), 2(g), 2(h); values are lower than those in Figures 2(a) and 2(e), respectively). We note that similar intuition holds when the disruption probability θ is held constant (in which case the percentage uptime decreases as λ decreases), except that the value of operational measures increases as λ decreases.

With regard to the robustness of these observations, we note that patterns similar to those in Figure 2 were observed for other values of the inventory holding cost (h), emergency sourcing cost (e), and insurance markup (m_2). (In Figures 2(a) and 2(e), the levels that the value of inventory and the value of emergency sourcing converge to as λ increases are influenced by the relative magnitudes of inventory holding cost (h) and emergency sourcing cost (e).)

8. Conclusions

In this research we have examined BI insurance, inventory, and emergency sourcing as strategies for managing a firm's disruption risk. Adopting an endogenous insurance pricing model in which the price depends on the firm's choices, we have characterized the optimal insurance and operational decisions, i.e., deductible, coverage limit, and inventory.

We established that, contrary to what one might expect, insurance and operational measures are not always substitutes. They are complements when the net-premium interaction between operations and insurance dominates the penalty-reduction interaction. The interplay between operations and insurance has managerial implications: risk management decisions regarding insurance and operational levers are sometimes made independently of each other by distinct organizational units. This research highlights the importance of linking these decisions, a compelling reason for an integrated risk management approach within organizations. We also found the value of insurance is much higher for firms less able to cope with financially significant disruptions. Insurance companies may find their insurance products more appealing to those customers and might consider this when pricing the insurance premium. Comparing the value of the three strategies, the value of insurance is most significant when disruptions are modestly long and rare; when disruptions are longer but rarer, the value of emergency sourcing remains high while the value of insurance and inventory decrease.

Although our model focused on an internal plant, the results would carry over to a supplier disruption (with BI insurance now being augmented by "contingent" BI insurance) but with a caveat. Our model does not reflect any strategic interactions or misaligned incentives between the party investing in protection (insurance, inventory, and emergency sourcing) and the party that owns and operates the plant. Whether interactions and incentive issues would alter the paper's findings is a subject for future research.

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Appendix A. List of Notation

Primitive Variables–Operational Elements

r, v, h, g, b, e: respectively, revenue per unit, variable production cost per unit, intangible inventory holding cost per unit, goodwill cost per unit lost sale, reservation fee paid to the emergency supplier per period, emergency sourcing cost per unit;

B: inventory level at the end of a period;

Y: disruption type, $Y \in \{S, F\}$, where S/F represents disruptions in which inventory survives/fails;

θ_Y,: probability that a Y-type disruption occurs in an up period;

λ_Y: probability of recovery at the end a period when the facility is down because of a Y-type disruption;

(DN, n, Y): the state representing the facility has been down for $n \geq 1$ periods (including the current period) because of a Y-type disruption;

(UP, n, Y): the state representing the facility is back up for the first period after being down n periods because of a Y-type disruption;

$(UP, 0)$: the state representing the facility is up and was up in the previous period;

$\rho(s)$: the steady-state probability of state s;

IL_Y: interruption loss because of a Y-type disruption;

$K(x)$: penalty incurred when x, the net-income loss during a disruption, exceeds a threshold T, $K(x) = 0$ if $x \leq T$, $K(x) = k_1 + k_2(x)$ otherwise;

E: superscript denoting the presence of emergency sourcing.

Primitive Variables—Insurance Elements

D: insurance deductible;

L: insurance coverage limit;

$\bar{L}$: maximum coverage limit that the insurer is willing to provide;

$p(B, D, L)$: insurance premium, a function of B, D, and L;

$R(B, D, L)$: insurance reimbursement paid in the first-period back up from a disruption;

$\bar{R}(B, D, L)$: long-run average reimbursement;

m_1: constant expense load in the premium calculation;

m_2: proportional expense load in the premium calculation;

$\psi(R)$: positive piecewise linear convex increasing function of the reimbursement;

$\bar{\Psi}(B, D, L)$: risk load, average value of ψ;

$\hat{}$: accent denoting the presence of insurance.

Intermediate Variables

$\Delta\bar{K}$: the additional benefit of insurance above and beyond the average reimbursement;

$a_Y(n_D)$: scaled difference of the disruption penalty and the net insurance premium (excluding the fixed premium m_1);

$n_D, n_L, n_{\bar{L}}, n_T$: respectively, representing D, L, $\bar{L}$, T as integer multiples of $r - v$.

Appendix B. Proofs

Proof of Proposition 1. Let $w(s)$ denote the profit in state s and $\rho(s)$ the steady-state probability. Applying Theorem 3.3.3 in Tjims (2003), the profit is given by $\Pi(B) = \sum_{s\in S} \rho(s)w(s)$, where S is the collection of all states. We first develop the expression for the profit $\Pi_N(B)$ when there is no penalty function, i.e., when $K(x) \equiv 0$. Accounting for the revenue and various costs, we obtain $w(UP, 0) = r - v - hB$, $w(UP, n, S) = r - v(\min\{n, B\} + 1) - hB$, $w(UP, n, F) = r - v - hB$, $w(DN, n, S) = r - h(B - n)$ if $n \leq B$ and $w(DN, n, S) = -g$ otherwise, and finally $w(DN, n, F) = -g$. (To understand why $w(UP, n, F)$ does include a vB term, recall that PD insurance reimburses destroyed inventory in an F-type disruption and we assume the firm has PD insurance.) Substituting these $w(s)$ expressions into $\Pi(B) = \sum_{s\in S} \rho(s)w(s)$ and rearranging terms, we obtain (9). We now consider the case of a positive penalty function $K(x)$, where x is the interruption loss because there are no BI reimbursements. The profit differs from the no-penalty case only in that the penalty $K(x)$, given by (2), is incurred in the first up-period after a disruption. Using (2), $K(x)$ can be expressed in terms of B and n as $K(B,n)=k_1+k_2(IL(B,n))$ if $IL(B,n)>T$ and $K(B,n)=0$ otherwise, where the interruption loss functions (for S and F type disruptions) are given by (6) and (7). It follows from these that $IL_S(B,n)>T$ if and only if (iff) $n>B+T/(r-v)$ and $IL_F(B,n)>T$ iff $n>T/(r-v)$. Proof of (8) then follows. □

Proof of Proposition 2. The profit $w(s)$ in each state s is the same as in the no-insurance case except that (i) the firm pays an insurance premium $p(B,D,L)$ every period, (ii) it receives a reimbursement of $R_Y(B,D,L,n)=\min\{[IL_Y(B,n)-D]^+,L\}$, $Y\in\{S,F\}$ in the first period up after a disruption, and (iii) this reimbursement reduces the firm's net income loss over a Y-type disruption ($Y\in\{S,F\}$), and hence the penalty function in the first period back up applies to $IL_Y(B,n)-R_Y(B,D,L,n)$ rather than $IL_Y(B,n)$ as is the case with no BI insurance. Let $\Delta\bar{K}(B,D,L)$ denote the reduction in the long-run average penalty cost achieved by having insurance (i.e., the average value of (iii)). We then have $\widehat{\Pi}(B,D,L)=\Pi(B)-p(B,D,L)+\bar{R}(B,D,L)+\Delta\bar{K}(B,D,L)$, where $\bar{R}(B,D,L)$ is the average reimbursement. Using (1) to substitute for $p(B,D,L)$, we obtain (12), the profit expression in the proposition statement. It remains to prove the $\Delta\bar{K}(B,D,L)$ expressions (13) $[D\le T]$ and (14) $[D>T]$, where $\Delta\bar{K}(B,D,L)$ is the difference in the average penalty incurred with and without insurance. Substituting the interruption loss functions (6) and (7) into $R_Y(B,D,L,n)=\min\{[IL_Y(B,n)-D]^+,L\}$, $Y\in\{S,F\}$ yields

$$R_S(B,D,L,n)=\begin{cases}0 & n<B+\dfrac{D}{r-v},\\ (n-B)(r-v)-D & B+\dfrac{D}{r-v}\le n<B+\dfrac{D+L}{r-v},\\ L & n\ge B+\dfrac{D+L}{r-v};\end{cases}\tag{B1}$$

$$R_F(B,D,L,n)=\begin{cases}0 & n<\dfrac{D}{r-v},\\ n(r-v)-D & \dfrac{D}{r-v}\le n<\dfrac{D+L}{r-v},\\ L & n\ge\dfrac{D+L}{r-v}.\end{cases}\tag{B2}$$

In the first period back up after a Y-type disruption, the firm incurs a penalty $K(IL_Y(B,n))$ without insurance but $K(IL_Y(B,n)-R_Y(B,D,L,n))$ with insurance, where $K(x)$ is given by (2) and $R_Y(B,D,L,n)$, $Y\in\{S,F\}$, are given by (B1) and (B2). The $\Delta\bar{K}(B,D,L)$ expressions (13) $[D\le T]$ and (14) $[D>T]$ then follow by taking the difference in the average value of $K(x)$ with and without insurance. □

Proof of Theorem 1. By definition, $\hat{n}$ is the smallest $n\ge 0$ such that $k_2((n+1)(r-v))-k_2((n)(r-v))\ge m_2(r-v)$. We first establish that $\hat{n}$ is well defined. By assumption, $k_2(x)$ is convex increasing in x. Therefore, $k_2((n+1)(r-v))-k_2(n(r-v))$ is increasing in n. Thus, either a unique finite $\hat{n}$ exists or $\hat{n}=\infty$ if $k_2((n+1)(r-v))-k_2((n)(r-v))\ge m_2(r-v)$ for all n. We have already established (see §4.1) that an optimal deductible satisfies $n_D\ge n_T$. Consider $n_D>n_T$. The first difference of the profit expression (15) with respect to (w.r.t.) n_D is

$$\begin{aligned}\nabla_{n_D}\widehat{\Pi}(B,n_D,n_L) &= -\rho(UP)(k_2((n_D+1)(r-v))-k_2((n_D)(r-v))-m_2(r-v))\\ &\quad\cdot(\theta_S(1-(1-\lambda_S)^{n_L})(1-\lambda_S)^{B+n_D}\\ &\quad+\theta_F(1-(1-\lambda_F)^{n_L})(1-\lambda_F)^{n_D})\end{aligned}$$

when $n_D>n_T$. All terms are positive except for $(k_2((n_D+1)(r-v))-k_2((n_D)(r-v))-m_2(r-v))$, which can be positive or negative. Therefore (because of the negative sign in front of $\rho(UP)$), $\nabla_{n_D}\widehat{\Pi}(B,n_D,n_L)>0$ iff $k_2((n_D+1)\cdot(r-v))-k_2((n_D)(r-v))<m_2(r-v)$, i.e., iff $n<\hat{n}$. Therefore, $\widehat{\Pi}(B,n_D,n_L)$ is quasiconcave in n_D for $n_D>n_T$, with the profit decreasing in n_D iff $n\ge\hat{n}$. Therefore, the global optimal deductible is either $\hat{n}$ or n_T. To determine which is optimal we need to examine $\nabla_{n_D}\widehat{\Pi}(B,n_D,n_L)$ at $n_D=n_T$. Accounting for the discontinuity in the $a(n_D)$ function moving from n_T to n_T+1 (due to the k_1 term appearing in (16) only when $n_D=n_T$), we obtain

$$\begin{aligned}\nabla_{n_D}\widehat{\Pi}(B,n_D,n_L) &= -\rho(UP)(k_1+k_2((n_T+1)(r-v))-m_2(r-v))\\ &\quad\cdot(\theta_S(1-(1-\lambda_S)^{n_L})(1-\lambda_S)^{B+n_D}\\ &\quad+\theta_F(1-(1-\lambda_F)^{n_L})(1-\lambda_F)^{n_D}).\end{aligned}$$

All terms are positive except for $(k_1+k_2((n_T+1)(r-v))-m_2(r-v))$, which can be positive or negative. Therefore (because of the negative sign in front of $\rho(UP)$), at $n_D=n_T$, $\nabla_{n_D}\widehat{\Pi}(B,n_D,n_L)>0$ iff $k_1+k_2((n_T+1)(r-v))-m_2(r-v)<0$, or equivalently iff $k_1+k_2((n_T)(r-v))+k_2((n_T+1)(r-v))-k_2((n_T)(r-v))<m_2(r-v)$. If $\hat{n}\le n_T$, then $k_2((n_T+1)(r-v))-k_2((n_T)(r-v))>m_2(r-v)$ and so $\nabla_{n_D}\widehat{\Pi}(B,n_D,n_L)<0$ at $n_D=n_T$ because $k_1\ge 0$ and $k_2(\cdot)\ge 0$. Therefore, $n_D^*(B,n_L)=n_T$ if $\hat{n}\le n_T$, which proves (i). (With the convention in the paper being $k_2(x)=0$ for $x<T$, $\hat{n}$ cannot be strictly less than n_T. However, if we allow $k_2(x)>0$ for $x<T$, then $\hat{n}$ can be strictly less, and the theorem statement and proof holds in this more general case.) Next consider $\hat{n}>n_T$. Using (15), $\widehat{\Pi}(B,n_T,n_L)\ge\widehat{\Pi}(B,n_T+1,n_L)$ iff

$$\begin{aligned}&\theta_S(1-\lambda_S)^{B+n_T}(1-(1-\lambda_S)^{n_L})a_S(n_T)\\ &\quad+\theta_F(1-\lambda_F)^{n_T}(1-(1-\lambda_F)^{n_L})a_F(n_T)\\ &\ge\theta_S(1-\lambda_S)^{B+\hat{n}}(1-(1-\lambda_S)^{n_L})a_S(\hat{n})\\ &\quad+\theta_F(1-\lambda_F)^{\hat{n}}(1-(1-\lambda_F)^{n_L})a_F(\hat{n}).\end{aligned}$$

Rearranging terms proves (ii).[5] □

[5] We note (proof omitted) that n_T is the optimal deductible for all B iff at least on one of the following four conditions holds: (i) $\hat{n}\le n_T$; (ii) $\hat{n}=\infty$; (iii) $q_S(n_T,\hat{n})\ge 0$ and $q_F(n_T,\hat{n})\ge 0$, where $q_Y(n_T,\hat{n})=a_Y(n_T)-(1-\lambda_Y)^{\hat{n}-n_T}a_Y(\hat{n})$; or (iv) $q_S(n_T,\hat{n})<0$ and $q_F(n_T,\hat{n})\ge 0$, and $W(0,n_T,n_L^0)\ge W(0,\hat{n},\hat{n},n_{\bar{L}})$, where $W(B,n_D,n_L)=\theta_S(1-\lambda_S)^{B+n_D}(1-(1-\lambda_S)^{n_L})a_S(n_D)+\theta_F(1-\lambda_F)^{n_D}(1-(1-\lambda_F)^{n_L})a_F(n_D)$, and where n_L^0 denotes the minimum value of n_L such that $A(0,n_T,n_L)\le 0$ and $n_{\bar{L}}$ and $A(\cdot)$ are defined in Theorem 2. When $k_2(x)$ is linear in x, then (i) or (ii) must hold and so n_T is the optimal deductible for all B in that special case.

Proof of Theorem 2. At $n_D = n_T$, the first difference of the profit expression (15) w.r.t. n_L is $\nabla_{n_L}\widehat{\Pi}(B, n_T, n_L) = \rho(UP)\theta_S\lambda_S(1-\lambda_S)^{n_L+n_T+B}a_S(n_T) + \rho(UP)\theta_F\lambda_F(1-\lambda_F)^{n_L+n_T}a_F(n_T)$. Now, $\nabla_{n_L}\widehat{\Pi}(B, n_T, n_L) < 0$ iff $A(B, n_T, n_L) < 0$, where $A(B, n_L)$ is given by (17) in the theorem statement. We consider four different cases corresponding to different combinations of $a_S(n_T)$ and $a_F(n_T)$ being positive and negative. (i) $a_S(n_T) \geq 0$ and $a_F(n_T) \geq 0$: In this case $A(B, n_T, n_L) \geq 0$ for all n_L and so $n_L^*(B, n_D) = n_{\bar{L}}$, i.e., the maximum feasible value. (ii) $a_S(n_T) > 0$ and $a_F(n_T) \leq 0$: We must have $\lambda_S > \lambda_F$ in this case (proof omitted). Thus, $((1-\lambda_S)/(1-\lambda_F))^{n_T+n_L}$ decreases in n_L. Now $a_S(n_T) > 0$ and so $A(B, n_T, n_L)$ decreases in n_L. Thus, if $A(B, n_T, n_L) < 0$ for some n_L then $A(B, n_T, n_L) < 0$ for all larger n_L. Therefore, $\widehat{\Pi}(B, n_T, n_L)$ is quasiconcave in n_L, and so the first order condition (subject to boundaries 0 and $n_{\bar{L}}$) is sufficient for optimality. That is, $n_L^*(B)$ is given by the smallest n_L such that $A(B, n_T, n_L) < 0$ and $n_L^*(B, n_T) = n_{\bar{L}}$ if $A(B, n_T, n_{\bar{L}}) \geq 0$. (iii) $a_S(n_T) \leq 0$ and $a_F(n_T) > 0$: We must have $\lambda_S < \lambda_F$ in this case (proof omitted). Thus, $((1-\lambda_S)/(1-\lambda_F))^{n_T+n_L}$ increases in n_L. Now $a_S(n_T) \leq 0$. Therefore, $A(B, n_L)$ decreases in n_L. Proof then follows identically to (iii). (iv) $a_S(n_T) < 0$ and $a_F(n_T) < 0$: In this case, $A(B, n_T, n_L) < 0$ for all n_L and so $n_L^*(B, n_D) = 0$. We note, however, that n_T would never be the optimal deductible in this case as $\hat{n}$ is (weakly) better. We present the case for completeness however. □

Proof of Theorem 3. Recall that $T = n_T(r - v)$. Using Theorem 1 and taking the second difference of (8) w.r.t. B, we obtain $\nabla_B^2\Pi(B) = \nabla_B^2\Pi_N(B) - \rho(UP)\theta_S(\lambda_S^2)\cdot(1-\lambda_S)^{B+n_T}d_S(n_T)$, where

$$d_S(n_T) = k_1 + \sum_{n=0}^{\infty}(1-\lambda_S)^n \cdot\big(k_2((n_T+n+1)(r-v)) - k_2((n_T+n)(r-v))\big). \quad \text{(B3)}$$

Noting that $d_S(n_T) \geq 0$ because $k_2(x)$ is increasing in x, we then have $\nabla_B^2\Pi(B) \leq \nabla_B^2\Pi_N(B)$. Now, $\nabla^2\Pi_N(B) = (v - r - g)\rho(UP, B+1, S) - h\rho(DN, B+1, S) \leq 0$ because $h \geq 0$, $g \geq 0$ and $r > v$ by assumption. Therefore, $\nabla_B^2\Pi(B) \leq 0$. □

Proof of Theorem 4. By theorem assumption, $n_L^*(B) = n_{\bar{L}}$ for all B. Taking the second difference of the profit expression (15) w.r.t. B (at $n_D = n_T$ and $n_L = n_{\bar{L}}$), we obtain

$$\nabla_B^2\widehat{\Pi}(B, n_T, n_{\bar{L}}) = \nabla_B^2\Pi_N(B) + \rho(UP)\theta_S(\lambda_S)^2(1-\lambda_S)^{B+n_T} \cdot\big((1-(1-\lambda_S)^{n_{\bar{L}}})a_S(n_T) - d_S(n_T)\big),$$

where $a_S(n_T)$ is given by (16) and $d_S(n_T)$ by (B3). Now $d_S(n_T)$ is positive but $a_S(n_T)$ can be positive or negative. However, because $a_S(n_T) \leq d_S(n_T)$ we must have $(1-(1-\lambda_S)^{n_{\bar{L}}})a_S(n_T) - d_S(n_T) \leq 0$ in either case. Now $\nabla_B^2\Pi(B) \leq 0$ from Theorem 3. Therefore, $\nabla_B\widehat{\Pi}(B, n_T, n_{\bar{L}}) \leq 0$. □

Proof of Theorem 5. By Definition 1, inventory and insurance are substitutes (complements) if $\nabla\widehat{\Pi}(B) \leq (\geq)\cdot \nabla\Pi(B)$. Writing out the first difference explicitly, inventory and insurance are substitutes (complements) if $\widehat{\Pi}(B+1, n_L^*(B+1)) - \widehat{\Pi}(B, n_L^*(B)) \leq (\geq)\Pi(B+1) - \Pi(B)$, where we explicitly note the dependence of the optimal limit on B. (Note the optimal deductible is n_T for all B.) For use below, we note that the cross-partial (difference, to be precise) of $\Pi(B, n_L)$ w.r.t. B and n_L is $\nabla_{B,n_L}^2\widehat{\Pi}(B, n_D, n_L) = -\rho(UP)\theta_S(\lambda_S)^2(1-\lambda_S)^{n_L+B+n_D}a_S(n_D)$.

Case 1: $a_S(n_D) \geq 0$. From above, the cross-partial of $\Pi(B, n_L)$ is negative, that is, $\nabla_B\widehat{\Pi}(B, n_L)$ is decreasing in n_L. Therefore,

$$\begin{aligned}
&a_S(n_D) \geq 0\\
&\quad\Rightarrow \nabla_B\widehat{\Pi}(B, n_L) \leq \nabla_B\widehat{\Pi}(B, 0)\\
&\quad\Rightarrow \widehat{\Pi}(B+1, n_L) - \widehat{\Pi}(B, n_L) \leq \widehat{\Pi}(B+1, 0) - \widehat{\Pi}(B, 0)\\
&\quad\Rightarrow \widehat{\Pi}(B+1, n_L) - \widehat{\Pi}(B, n_L) \leq \Pi(B+1) - \Pi(B)\\
&\quad\Rightarrow \widehat{\Pi}(B+1, n_L^*(B+1)) - \widehat{\Pi}(B, n_L^*(B+1)) \leq \Pi(B+1) - \Pi(B)\\
&\quad\Rightarrow \widehat{\Pi}(B+1, n_L^*(B+1)) - \widehat{\Pi}(B, n_L^*(B)) \leq \Pi(B+1) - \Pi(B),
\end{aligned}$$

where we have used the fact that $\widehat{\Pi}(B, 0) = \Pi(B)$ in going from the second to the third line above, and we used $\widehat{\Pi}(B, n_L^*(B+1)) \leq \widehat{\Pi}(B, n_L^*(B))$ in going from the second last to the last line. Therefore, $a_S(n_D) \geq 0$ implies inventory and insurance are substitutes.

Case 2: $a_S(n_D) \leq 0$. From above, the cross-partial of $\Pi(B, n_L)$ is positive, that is, $\nabla_B\widehat{\Pi}(B, n_L)$ is increasing in n_L. Therefore,

$$\begin{aligned}
&a_S(n_D) \leq 0\\
&\quad\Rightarrow \nabla_B\widehat{\Pi}(B, n_L) \geq \nabla_B\widehat{\Pi}(B, 0)\\
&\quad\Rightarrow \widehat{\Pi}(B+1, n_L) - \widehat{\Pi}(B, n_L) \geq \widehat{\Pi}(B+1, 0) - \widehat{\Pi}(B, 0)\\
&\quad\Rightarrow \widehat{\Pi}(B+1, n_L) - \widehat{\Pi}(B, n_L) \geq \Pi(B+1) - \Pi(B)\\
&\quad\Rightarrow \widehat{\Pi}(B+1, n_L^*(B)) - \widehat{\Pi}(B, n_L^*(B)) \geq \Pi(B+1) - \Pi(B)\\
&\quad\Rightarrow \widehat{\Pi}(B+1, n_L^*(B+1)) - \widehat{\Pi}(B, n_L^*(B)) \geq \Pi(B+1) - \Pi(B),
\end{aligned}$$

where we have used the fact that $\widehat{\Pi}(B, 0) = \Pi(B)$ in going from the second to the third line above, and we used $\widehat{\Pi}(B+1, n_L^*(B+1)) \geq \widehat{\Pi}(B+1, n_L^*(B))$ in going from the second last to the last line. Therefore, $a_S(n_D) \leq 0$ implies inventory and insurance are complements. □

Proof of Theorem 6. Using Definition 2, insurance and emergency sourcing are substitutes (complements) if $\Delta V \doteq \widehat{\Pi}^E(0) - \Pi^E(0) - (\widehat{\Pi}(0) - \Pi(0)) < 0\ (> 0)$. The development of the long-run average profit expressions follows in a similar manner to the inventory-only expressions developed in §4.1, except that instead of incurring the goodwill cost g when inventory runs out, the firm incurs the variable emergency sourcing cost e. It also incurs b every period. Accounting for these differences, the firm's profit without insurance for a given base-stock B is

$$\Pi^E(B) = \Pi_N^E(B) - \sum_{n=B+\lfloor T/(e-v)\rfloor+1}^{\infty}\big(k_1 + k_2(n-B)(e-v)\big)\rho(UP, n, S) - \sum_{n=\lfloor T/(e-v)\rfloor+1}^{\infty}\big(k_1 + k_2(n(e-v))\big)\rho(UP, n, F),$$

where $\Pi_N^E(B)$ is given in (18); the firm's profit without insurance for a given base-stock B is

$$\widehat{\Pi}(B,D,L) = \Pi(B) - m_1 + \rho(UP) \cdot \begin{pmatrix} \theta_S(1-\lambda_S)^B((1-\lambda_S)^{\lfloor D/(e-v)\rfloor} - (1-\lambda_S)^{\lfloor (D+L)/(e-v)\rfloor})a_S^E(n_D^E) \\ +\theta_F((1-\lambda_F)^{\lfloor D/(e-v)\rfloor} - (1-\lambda_F)^{\lfloor (D+L)/(e-v)\rfloor})a_F^E(n_D^E), \end{pmatrix}, \tag{B4}$$

where

$$a_Y^E(n_D^E) = k_1 I^E(n_D^E) + \sum_{n=0}^{\infty}\big(k_2\big((n_T^E+n+1)(e-v)\big) - k_2\big((n_T^E+n)(e-v)\big) - m_2(e-v)\big)(1-\lambda_Y)^n$$

for $Y\in\{S,F\}$, and $I^E(n_D^E)$ is an indicator function such that $I^E(n_T^E)=1$ and $I^E(n_D^E)=0$ for $n_D^E > n_T^E$, and $n_T^E = \lfloor T/(e-v)\rfloor$. Under the assumption that $k_2(\cdot) = m_2 = 0$, the optimal deductible and coverage limit are $D^* = T$ and $L^* = \bar{L}$ for both inventory only and emergency sourcing. For $B=0$ and $\lambda_Y = \lambda$, by (15) and (B4),

$$\Delta V(e,\lambda) = \rho(UP)(\theta_S+\theta_F)\big(\big((1-\lambda)^{T/(e-v)} - (1-\lambda)^{(T+\bar{L})/(e-v)}\big) - \big((1-\lambda)^{T/(r-v)} - (1-\lambda)^{(T+\bar{L})/(r-v)}\big).$$

Taking the partial derivative of ΔV with respect to e,

$$\frac{\partial \Delta V}{\partial e} = \rho(UP)(\theta_{IS}+\theta_{IF})(1-\lambda)^{T/(e-v)}\frac{\ln(1-\lambda)}{(e-v)^2}\chi(e,\lambda),$$

where $\chi(e,\lambda) \doteq (-T+(1-\lambda)^{\bar{L}/(e-v)}(T+\bar{L}))$. Because $\ln(1-\lambda) < 0$, the sign of $\partial\Delta V/\partial e$ is determined by the sign of $\chi(e,\lambda)$. It is straightforward to show that $\chi(e,\lambda)$ is an increasing function of e but a decreasing function of λ, and establish the following: For $\lambda < 1-(T(T+\bar{L}))^{(r-v)/\bar{L}}$, there exists $\hat{e}(\lambda)$ such that $\chi(e,\lambda) < (>)0$ for $e \le \hat{e}(\lambda)(e > \hat{e}(\lambda))$; for $\lambda \ge 1-(T/(T+\bar{L}))^{(r-v)/\bar{L}}$, $\chi(e,\lambda) < 0$. This implies that for $\lambda < 1-(T/(T+\bar{L}))^{(r-v)/\bar{L}}$, $\partial\Delta V/\partial e > (<)0$ for $e \le \hat{e}(\lambda)(e > \hat{e}(\lambda))$; for $\lambda \ge 1-(T/(T+\bar{L}))^{(r-v)/\bar{L}}$, $\partial\Delta V/\partial e > 0$. Because $\Delta V(v,\lambda) < 0$ and $\Delta V(r,\lambda) = 0$, it follows that if $\lambda < 1-(T/(T+\bar{L}))^{(r-v)/\bar{L}}$ then $\Delta V(e,\lambda)$ increases in e for $e < \hat{e}(\lambda)$ and decreases in e for $e > \hat{e}(\lambda)$. There exists $\tilde{e}(\lambda)$ such that $\Delta V(e,\lambda) \le (>)0$ for $e \le \tilde{e}(\lambda)(e > \tilde{e}(\lambda))$; if $\lambda \ge 1-(T/(T+\bar{L}))^{(r-v)/\bar{L}}$ then $\Delta V(e,\lambda) < 0$. □

Proof of Theorem 7. Using (19), the cross-partial (difference, to be precise) of $\Pi(B,n_L)$ w.r.t. B and n_L is

$$\nabla^2_{B,n_L}\widehat{\Pi}(B,n_T,n_L) = -\rho(UP)\theta_S(\lambda_S)^2(1-\lambda_S)^{n_L+B+n_T} \cdot \left(a_S(n_T) - \frac{\psi((n_L+1)(r-v)) - \psi(n_L(r-v))}{\lambda_F}\right).$$

Now $\psi(\cdot)$ is an increasing function, and so $\psi((n_L+1)(r-v)) - \psi(n_L(r-v)) \le 0$ for all n_L. Therefore, the cross-partial of $\Pi(B,n_L)$ is positive if $a_S(n_T) \le 0$ (a sufficient but not necessary condition). Proof of complementarity then follows in identical manner to Case 2 in proof of Theorem 5.[6] □

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[6] By assumption, $\psi(\cdot)$ is a convex function. Therefore, $(\psi((n_L+1)\cdot(r-v)) - \psi(n_L(r-v)))/\lambda_F$ is increasing in n_L, and so the cross-partial of $\Pi(B,n_L)$ is negative for all feasible n_L if $a_S(n_T) \ge (\psi((n_{\bar{L}})\cdot(r-v)) - \psi((n_{\bar{L}}-1)(r-v)))/\lambda_F$. If this condition holds, then inventory and insurance are substitutes (following in identical manner to Case 1 in proof of Theorem 5).

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