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The Rich Domain of Risk

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We report on two experiments challenging the common assumption that events with objective probabilities constitute a unique source of uncertainty. We find that, similar to the domain of ambiguity, the domain of risk is rich in the sense that behavior is systematically different when subjects face risky bets based on simple or more complex events. Furthermore, we find a tight association between attitudes toward complex risky bets and attitudes toward both ambiguity and compound lotteries. These results raise questions about the characterization of ambiguity aversion and the modeling of decisions under uncertainty.

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> The question then is not "Are there uncertainties that are not risks?," as posed by Ellsberg, but "Are there risks that are not risks?" (Smith 1969, p. 329)

1. Introduction

The literature on uncertainty generally distinguishes decisions made under risk (with objective probabilities) from decisions made under ambiguity (with unknown probabilities).[1] Recently, Abdellaoui et al. (2011) (ABPW hereafter) showed that the domain of ambiguity is rich, in the sense that a decision maker may have different attitudes toward different sources of ambiguity. As is usual, however, events with objective probabilities are assumed to constitute a unique source. This assumption is important because ambiguity aversion is typically measured in contrast with attitude toward risk.

In this paper, we conduct two within-subject experiments to test the common assumption of a unique source of risk. The first is an urn experiment that replicates ABPW's Ellsberg experiment under risk and ambiguity. The second is a dice experiment with simple and compound risks. In both experiments, we add a new risky treatment with events whose objective probabilities are arguably more difficult to calculate.

We find that the domain of risk is rich in the sense that subjects have systematically different attitudes toward risks depending on whether the events are simple or more complex. Furthermore, we find that attitudes toward complex risk are tightly related to attitudes toward ambiguity and compound lotteries.

These results have implications for the characterization and the modeling of ambiguity attitudes. In particular, ambiguity attitudes cannot be unequivocally characterized in contrast to attitudes toward risk. Furthermore, most models of choices under risk and ambiguity are incomplete because they fail to explain attitudes toward complex risk. In contrast, our results are consistent with a modified version of ABPW's "source method," under which the many kinds of uncertainties are differentiated by their degree of subjective complexity.

2. The Source Method—The Binary Case

ABPW's source method combines Chew and Sagi's (2008) concept of probabilistic sophistication within sources with Tversky and Kahneman's (1992) (cumulative) prospect theory.[2] In the binary case, a decision maker (DM) faces a bet $\bar{x}_E\underline{x}$, i.e., win $\underline{x}$ when event E realizes and $\underline{x} \leq \bar{x}$ otherwise. A source S is loosely defined as "a group of events that is generated by a common mechanism of uncertainty" (ABPW, p. 696). Formally, ABPW assume that sources are algebras of events. A source is said to be "uniform" if probabilistic sophistication holds within the source. When S is uniform, ABPW write the DM's utility:

$$U(\bar{x}_E\underline{x}) = w_S(p)u(\bar{x}) + (1 - w_S(p))u(\underline{x}), \quad (1)$$

[1] In this paper, the term uncertainty captures both risk and ambiguity.

[2] Because we only consider binary prospects, note that prospect theory (Kahneman and Tversky 1979) and cumulative prospect theory (Tversky and Kahneman 1992) coincide.

where p is the subjective probability of E and w_S is a probability weighting function, called "source function," associated with S. ABPW assume that the utility function u is the same regardless of the source S.

In the case of risk, E has an objective probability P, and $p = P$. ABPW assume that every source of risk has the same source function, and they denote it $w(P)$. In contrast, w_S can differ depending on the source of ambiguity S. Hence, the DM has the same attitude toward every source of risk, but he may have different attitudes toward different sources of ambiguity. The difference between w and w_S characterizes the DM's ambiguity attitude toward S.

3. Experiment 1: The Urn Experiment

3.1. The Design

The within-subject experiment consists of three treatments, the two treatments in ABPW's Ellsberg experiment and a new treatment. Subjects face a series of binary bets $\bar{x}_E\underline{x}$ for which they report a certainty equivalent using ABPW's computerized iterative choice list method. Details on the bets as well as screen shots of the experiment and instructions are provided in Appendix A.

In the first risky (known) treatment (denoted by K), the bet is settled by drawing a ball from a transparent urn containing eight balls of different colors. Elementary events are thus equally likely with probability 1/8. In treatment K, the bets are based on simple events (e.g., "the ball is red").

In the ambiguous treatment (denoted U), the bet is settled by drawing a ball from an opaque urn containing eight balls. The balls' possible colors are the same as in treatment K, but the composition of the opaque urn is unknown. The bets in treatment U involve the same events as in treatment K.

For the new risky treatment (denoted K_2), there are two transparent urns each containing eight balls of different colors (as in treatment K). The bet is settled by the simultaneous draw of two balls, one from each urn. Elementary events (i.e., a pair of colored balls) are equally likely (as in treatment K), but with probability 1/64. The bets in treatment K_2 are based on what may be considered more complex events than in treatment K (e.g., "the two balls are of different colors").[3] To simplify, we will often refer to the risky bets in treatments K and K_2 as "simple" and "complex," respectively.[4]

Following ABPW, the order of the treatments and bets is fixed; every subject faces the same sequence of 13, 19, and 19 bets corresponding to treatments K, K_2, and U, respectively. The bets in treatments K and U are the same as in ABPW: the first bets have events that combine from one to seven colors while $\{\underline{x}, \bar{x}\}$ is fixed at $\{0, 25\}$; the last six bets are based on a four-color event while $\underline{x}$ and $\bar{x}$ vary from 0 to 25. Subjects faced no time limit and did not have access to calculators. At the beginning of the experiment, each subject was told that one of his choices would be randomly selected for payment.

The implementation of the experiment is similar to ABPW with four notable exceptions: (i) bets are settled by having the subject draw ball(s) from physical urn(s); (ii) the experiment was not conducted individually but in sessions with 17 to 21 subjects; (iii) a show-up fee of €5 was paid to every subject; (iv) the 77 subjects in our experiment were university students in Toulouse (France), not students at elite graduate engineering schools.

3.2. Raw Results

Figure 1 shows that the average certainty equivalents (divided by 25) for the bets 25_p0 are close to the diagonal for any P in treatment K. In contrast, they are below the diagonal for $P > 1/4$ in treatments U and K_2.[5] As shown in Table F.1 in Appendix F, a series of Wilcoxon signed-rank tests confirms statistically (at the 5% level) that, for any $P > 1/4$, simple risky bets in treatment K are valued differently than corresponding bets in treatments U and K_2. In contrast, the valuation of ambiguous and complex risky bets cannot be differentiated at standard significance levels for any P. These raw results suggest that (i) risky bets are valued differently depending on whether they are based on simple or more complex events, and (ii) the value of an ambiguous bet relative to a risky bet depends on the complexity of the events on which the risky bet is based.

3.3. Structural Econometric Approach

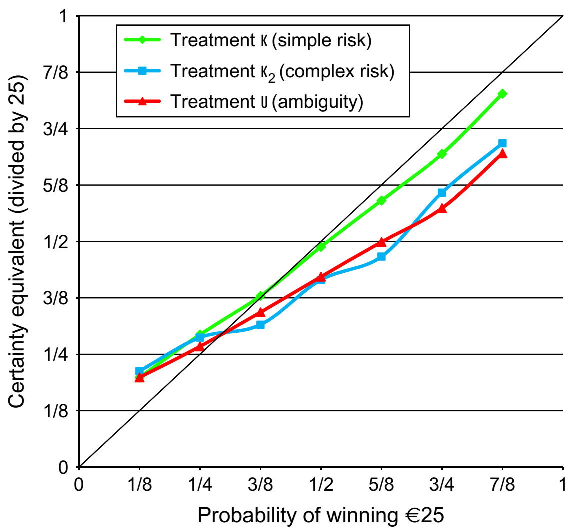
We now estimate the subjects' source and utility functions. Following ABPW, we assume that subject i in treatment $t \in \{\mathrm{K}, \mathrm{K}_2, \mathrm{U}\}$ has a power utility function $u_{it}(x) = x^{r_{it}}$ and a source function $w_{it}(p) = \exp(-\beta_{it}[-\ln p]^{\alpha_{it}})$, where $\alpha_{it} > 0$ and $\beta_{it} > 0$

[3] The psychology literature has identified several factors that can bias probability judgment, such as conjunctive or disjunctive events (Bar-Hillel 1973), negative frames (Tversky and Kahneman 1981), or the use of frequencies and ratios (Gigerenzer 1991, Pacini and Epstein 1999). When designing treatment K_2, we took special care to avoid such factors so as not to generate any systematic bias compared to treatment K.

[4] Observe that treatment K_2 and the experiment in Epstein and Halevy (2014) have similar designs. The key difference is that the composition of the two urns is known to the subjects in K_2, whereas it is unknown in Epstein and Halevy (2014).

[5] As shown in Appendix B, we fail to reject the hypothesis of a uniform source in treatments U and K_2 at any usual significance level. Thus, we follow ABPW and assume that subjects behave as if elementary events are uniformly distributed in each treatment.

Figure 1 (Color online) Average Certainty Equivalents in Experiment 1



(Prelec 1998). To estimate the structural model, however, we reparametrize the source function into $w_{it}(p) = \exp(\ln(b_{it})[\ln(p)/\ln(b_{it})]^{a_{it}})$, where $\alpha_{it} = a_{it}$ and $\beta_{it} = (-\ln b_{it})^{1-a_{it}}$. We argue that this reparametrization offers two advantages: First, it is easier to visualize the shape of the source function because a_{it} and b_{it} each capture a simple property of w_{it}: b_{it} captures where w_{it} crosses the diagonal (i.e., $w_{it}(b_{it}) = b_{it}$), and a_{it} is the slope at this fixed point (i.e., $w'_{it}(b_{it}) = a_{it}$). Second, it is easier to characterize how w_{it} deviates from its benchmark under expected utility (i.e., the diagonal), which obtains when $a_{it} = 1$.[6] Finally, observe that, similar to ABPW, a_{it} can be interpreted as a likelihood sensitivity index and b_{it} as a pessimism (optimism) index when w_{it} is (inverse) S shaped, that is, when $a_{it} > 1$ ($a_{it} < 1$).

Under the source method, the indifference of subject i between bet j and his elicited certainty equivalent CE_{ijt} implies

$$(CE_{ijt})^{r_{it}} = \exp(\ln(b_{it})[\ln(P_j)/\ln(b_{it})]^{a_{it}}) \cdot [(\bar{x}_j)^{r_{it}} - (\underline{x}_j)^{r_{it}}] + (\underline{x}_j)^{r_{it}}. \quad (2)$$

As explained in Appendix C, we prefer to adopt a different econometric approach than ABPW. Using (2), we estimate the structural parameters $\{r_{it}, a_{it}, b_{it}\}$ jointly by nonlinear least squares with all the data collected for subject i in treatment t. To test for differences across treatments, we define the following:

$$\begin{aligned} r_{it} &= 1.0 + r_{i\mathrm{K}} + r_{i\mathrm{K}_2}\mathrm{K}_2 + r_{i\mathrm{U}}(\mathrm{K}_2 + \mathrm{U}), \\ a_{it} &= 1.0 + a_{i\mathrm{K}} + a_{i\mathrm{K}_2}\mathrm{K}_2 + a_{i\mathrm{U}}(\mathrm{K}_2 + \mathrm{U}), \quad (3) \\ b_{it} &= 0.5 + b_{i\mathrm{K}} + b_{i\mathrm{K}_2}\mathrm{K}_2 + b_{i\mathrm{U}}(\mathrm{K}_2 + \mathrm{U}), \end{aligned}$$

[6] When $a_{it} = 1$, b_{it} is irrelevant and can be set to an arbitrary value, e.g., $b_{it} = 0.5$.

where K_2 (U) is a dummy variable equal to 1 when $t = \mathrm{K}_2$ ($t = \mathrm{U}$).

This specification of the parameters allows us to conduct several tests. First, observe in (3) that the default values for r_{it}, a_{it}, and b_{it} are 1, 1, and 0.5, respectively. Thus, a subject i in treatment K has a linear utility function when $r_{i\mathrm{K}} = 0$ and he satisfies expected utility when $a_{i\mathrm{K}} = 0$ (since his source function is the diagonal in that case). Finally, the source function of subject i crosses the diagonal at 0.5 in treatment K when $b_{i\mathrm{K}} = 0$.

Furthermore, the specification in (3) allows us to compare treatment K with treatment U, and treatment U with treatment K_2. In particular, we can test ABPW's assumption that an agent has the same utility regardless of the source of uncertainty. Indeed, $r_{i\mathrm{U}}$ captures the possible difference in u_{it} when subject i faces ambiguous instead of simple risky bets, whereas $r_{i\mathrm{K}_2}$ captures the difference in u_{it} when he faces complex risky instead of ambiguous bets.

To assess the robustness of the results, we estimate both a fully heterogeneous model as in ABPW (i.e., $\{r_{it}, a_{it}, b_{it}\}$ differs across subjects) and a homogeneous model in which $\{r_{it}, a_{it}, b_{it}\} = \{r_t, a_t, b_t\}$. Each approach has advantages and drawbacks. Estimates from the homogeneous model are severely constrained but easy to interpret. Estimates from the heterogeneous model are unrestricted across subjects but rely on small samples (e.g., $\{r_{i\mathrm{K}}, a_{i\mathrm{K}}, b_{i\mathrm{K}}\}$ is estimated with 13 observations). Finally, to account for the small sample size, the standard deviations and p-values are calculated by nonparametric bootstrap.

3.4. Results from the Structural Estimation

The estimation results reported in panel (a) of Table 1 indicate that the subjects' utility functions in treatment K are nearly linear on average (as in ABPW), but highly heterogeneous across subjects. Indeed, r_{K} is not significantly different from 0 in the homogeneous model, and the average estimated $r_{i\mathrm{K}}$ is close to 0 in the heterogeneous model. Furthermore, the large standard deviation of the estimated $r_{i\mathrm{K}}$ (0.496) indicates important differences in utility across subjects. In fact, we can reject the linearity of u_i for about half of the subjects in treatment K. The parameter r_{U} (r_{K_2}) is insignificant in the homogeneous model, and $r_{i\mathrm{U}}$ ($r_{i\mathrm{K}_2}$) is significant for only 14% (9%) of the subjects in the heterogeneous model. Thus, a subject generally exhibits the same utility in treatments K, K_2, and U. Similarly, ABPW found the same u_i in treatments K and U.

We now turn to the estimation of the source functions in Table 1. The parameters a_{K} and b_{K} are close to −0.2 and 0 in the homogeneous model. Thus, similar to ABPW, the likelihood sensitivity and optimism indexes are around 0.8 and 0.5 in treatment K. The

Table 1 Structural Estimation of Utility and Source Functions

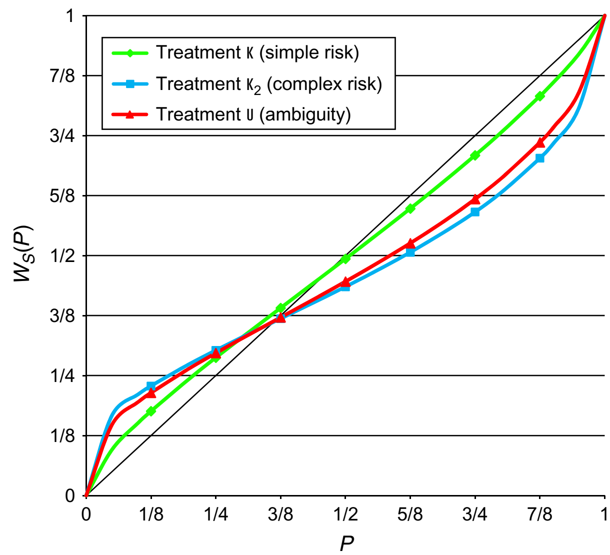
	Homogeneous	Heterogeneous: Average across subjects	Heterogeneous: Std across subjects	Heterogeneous: % of subjects with significant parameter at 5%
		(a) Experiment 1 (urn)		
r_K	−0.012 (0.025)	0.081	0.496	49.4
r_U	−0.031 (0.039)	−0.056	0.534	14.3
r_{K_2}	0.013 (0.045)	0.009	0.481	9.1
a_K	−0.211*** (0.019)	−0.181	0.208	84.4
a_U	−0.205*** (0.057)	−0.231	0.388	57.1
a_{K_2}	−0.069 (0.065)	−0.066	0.254	14.3
b_K	−0.009 (0.035)	−0.038	0.336	46.8
b_U	−0.084** (0.042)	−0.096	0.351	31.2
b_{K_2}	−0.022 (0.039)	−0.004	0.356	16.9
		(b) Experiment 2 (dice)		
r_K	−0.181** (0.082)	−0.090	0.643	58.1
r_C	0.056 (0.088)	0.008	0.495	9.3
r_{K_2}	−0.099 (0.113)	−0.023	0.501	7.0
a_K	−0.217** (0.086)	−0.275	0.280	90.7
a_C	−0.184*** (0.043)	−0.209	0.461	48.8
a_{K_2}	−0.083** (0.036)	−0.105	0.374	39.5
b_K	−0.031 (0.098)	0.036	0.467	58.1
b_C	−0.029 (0.106)	−0.043	0.228	16.3
b_{K_2}	−0.004 (0.047)	0.003	0.224	4.7

Notes. Estimates' standard errors and test statistics' distributions are estimated by bootstrap (size = 10,000) to account for the finiteness of the sample. As a comparison, $\{r_K, a_K, b_K\} = \{0.05, -0.15, 0.04\}$ and $\{r_U, a_U, b_U\} = \{0.04, -0.21, -0.20\}$ in ABPW.

*** and ** represent parameters significant at the 1% and 5% levels, respectively.

estimation of the heterogeneous model confirms that we cannot reject the hypothesis that the source function crosses the diagonal at 1/2 (i.e., $b_{iK} = 0.5$) for the majority of the subjects in treatment K. Furthermore, expected utility is rejected for 84% of the subjects in treatment K because their source function is significantly different from the diagonal (i.e., $a_{iK} = 1$).

Figure 2 (Color online) Source Functions in Experiment 1



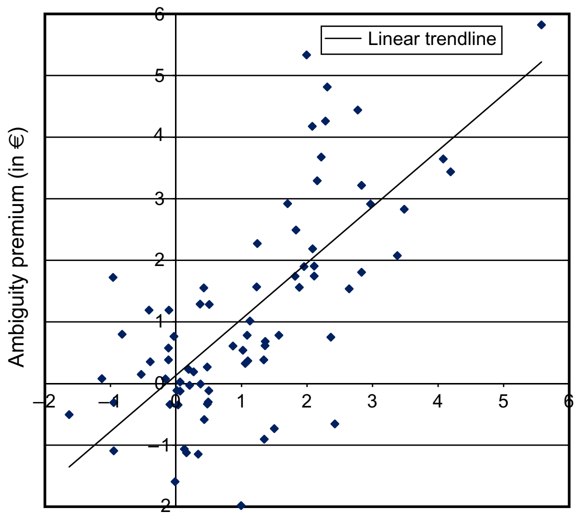
Note. Based on average estimated parameters from the heterogeneous model (see Table 1).

With respect to treatment effects, a_U and b_U are both significantly lower than 0 in the homogeneous model, thereby indicating that the optimism and likelihood sensitivity indexes are significantly larger in treatment K than in treatment U. The heterogeneous model supports this result because we can reject the equality of the likelihood sensitivity index (optimism index) in treatments K and U for 57% (31%) of the subjects. In contrast, a_{K_2} and b_{K_2} are negative but insignificant in the homogeneous model and insignificant for most subjects in the heterogeneous model. Thus, as seen in Figure 2, the optimism and likelihood sensitivity indexes in treatment K_2 are significantly lower than in treatment K, but statistically indistinguishable from those in treatment U.

To sum up, we find that subjects have (i) similar utility functions across treatments, (ii) different source functions for simple and complex risky bets, and (iii) similar source functions for ambiguous and complex risky bets.

Naturally, one may wonder whether the difference in attitudes toward simple and complex probabilities could be explained by random calculation errors. Under this hypothesis, subjective probabilities are random variables whose distributions capture calculation errors. Consistent with the psychology literature (Kahneman et al. 1982), the calculation errors can be assumed to generate an inverse-S-shaped judgment bias in the sense that, on average, the agent overestimates low probabilities and underestimates high probabilities. In that case, the differences in behavior we observed between treatments K and K_2 could be explained by the fact that subjects make larger calculations errors in K_2. We estimate in Appendix D such a structural model that accounts for calculation

Figure 3 (Color online) Average Ambiguity and Complex Risk Premia per Subject in Experiment 1



errors and find that although subjects make larger calculation errors for complex risky bets, there is still a significant difference between the source functions in treatment K and K_2.

3.5. Attitudes Toward Ambiguity and Complex Risk

Is there a link between attitudes toward ambiguity and attitudes toward complex risk? To address this question, we follow Abdellaoui et al. (2015) and define for each subject i and each $P = 1/8, \ldots, 7/8$ the ambiguity and "complex risk" premium as $CE_{ipK} - CE_{ipU}$ and $CE_{ipK} - CE_{ipK_2}$, respectively. These individual premia (averaged across $P = 1/8, \ldots, 7/8$) are plotted in Figure 3. As indicated by the positive trend line, there is a strong positive correlation ($\rho = 0.714$) between a subject's ambiguity and complex risk premia.[7] Furthermore, the relative concentration of subjects around the origin in Figure 3 suggests that neutrality to ambiguity and neutrality to complex risky bets are associated (as shown statistically in Appendix E). Thus, we find a tight link between attitudes toward ambiguity and attitudes toward complex risk.

4. Experiment 2: The Dice Experiment

We conduct a second experiment to confirm that the domain of risk is rich and to study the link between attitudes toward compound and complex risks.

To assess the robustness of the results, the second experiment differs from the first in several dimensions: Uncertainty is generated with dice, the acts are generated with a quadratic scoring rule (as in Andersen et al. 2009), the experiment is conducted with pen and paper, the subject pool comes from a developing country, and incentives are substantially higher.

4.1. The Design

The within-subject experiment also consists of three treatments. In each treatment of Experiment 2, subjects predict the probability of 10 risky events. Each of the events describes the outcome of the roll of two 10-sided dice (one black, one red) showing numbers from zero to nine. For comparison, the 10 events have the same objective probabilities in each treatment (3, 5, 15, 25, 35, 45, 61, 70, 80, and 90%). The events and the experimental instructions are provided in Appendix G.

In treatment K, the red (black) die determines the first (second) digit of a number between 01 and 100 (represented by both dice showing 0). Elementary events thus follow a uniform distribution. The events in treatment K have objective probabilities that are simple to calculate (e.g., the 25% probability event is described as "the number is between 1 (included) and 25 (included)").[8]

In treatment K_2, the two dice are added to form a number between 0 and 18. Elementary events thus follow a triangular-shaped distribution. The events in treatment K_2 have objective probabilities that are arguably more difficult to calculate than those in treatment K (e.g., the 25% probability event is described as "the sum is between 2 (included) and 6 (included)"). Note, however, that the same mechanism (i.e., the roll of two 10-sided dice) generates uncertainty in both treatments. Only the construction of the events differs.

In treatment C, the subjects face compound lotteries, i.e., lotteries whose prizes are other lotteries. Specifically, subjects have to make a single prediction for two possible events. They are told that uncertainty is resolved in two steps at the end of the experiment, first by tossing a coin to identify one of the two events, and then by rolling the dice to determine whether that event occurs. The events in treatment C are similar to those in treatment K, i.e., the roll of the dice produces a number between 1 and 100. For instance, the compound lottery with a reduced probability of 25% is described as, "If the coin lands on **heads**, then the event is 'the number is between 82 (included) and 89 (included)'; otherwise, if the coin

[7] As shown in Figure F.2 in Appendix F, this positive relationship holds for each $P = 1/8, \ldots, 7/8$. It also holds when the ambiguity and complex risk premia are calculated with respect to expected value (computed with P) instead of simple risk ($\rho = 0.698$), and when we compare the source functions $w_{iU}(P)$ and $w_{iK_2}(P)$ for $P = 1/8, \ldots, 7/8$ ($\rho = 0.659$).

[8] The data collected in treatment K have been used by Armantier and Treich (2013).

lands on **tails**, the event is 'the number is between 25 (included) and 66 (included).'"

The subjects are rewarded for the accuracy of their predictions according to a quadratic scoring rule: when a subject predicts a probability q, he receives $1-(1-q)^2$ when the event occurs and $(1-q^2)$ when the event does not occur. At the beginning of the experiment, the subjects were told that only one of the 30 events would be randomly selected for actual payment. As in Experiment 1, the order of the events and treatments ($\mathtt{K}$, $\mathtt{K}_2$, and $\mathtt{C}$) are fixed.

Experiment 2 took place in Ouagadougou, the capital of Burkina Faso. The subjects were recruited by a local recruiting firm (Opty-RH) by placing fliers around the city. To be eligible, subjects had to be at least 18 years old and be current or former university students. Two sessions with 21 and 22 subjects were conducted, each taking around 90 minutes to complete. Subjects were familiar with probabilities. In particular, 65% reported having taken a college-level course in probability or statistics. Subjects earned 3,000 CFA francs on average, which corresponds to a three-day wage for a university graduate.

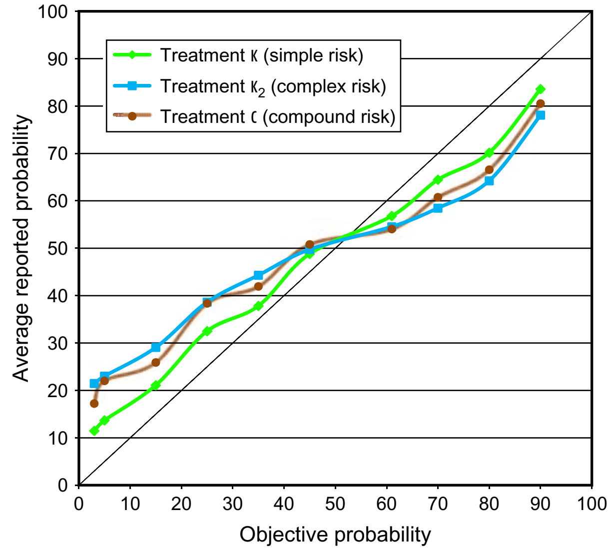
4.2. Experimental Results

It is well known that the quadratic scoring rule is incentive compatible only when subjects maximize expected payoffs (Winkler and Murphy 1970). Under model (1), the relationship between objective and reported probabilities is expected to be inverse S shaped (Offerman et al. 2009, Armantier and Treich 2013). Nevertheless, if the source of risk is unique, then a subject's should report the same probabilities for the three treatments. As in Experiment 1, we find evidence against this hypothesis. Indeed, Figure 4 displays systematic differences: Treatment $\mathtt{K}$ (with simple events) generates the smallest biases for virtually all objective probabilities (i.e., reported probabilities are consistently closest to the diagonal), whereas treatment $\mathtt{K}_2$ (with complex events) generates the largest biases. This ranking across treatments is confirmed statistically by nonparametric Friedman tests (see Table F.2 in Appendix F).

As in Experiment 1, we estimate the structural parameters $\{r_{it}, a_{it}, b_{it}\}$ characterizing subject i's utility and source functions in treatment $t \in \{\mathtt{K}, \mathtt{K}_2, \mathtt{C}\}$.[9] The results are presented in panel (b) of Table 1. Similar to Experiment 1, we cannot reject the hypothesis that the subjects have the same utility function across treatments. In contrast to Experiment 1 however, the subjects' average utility function is concave (which may

[9] The parameters are estimated by nonlinear least squares by comparing for every event j subject i's reported probability $\hat{P}_{ijt}$ with $P_j^*(r_{it}, a_{it}, b_{it})$ the optimal report under a quadratic scoring rule by an agent with utility and source functions characterized by $\{r_{it}, a_{it}, b_{it}\}$.

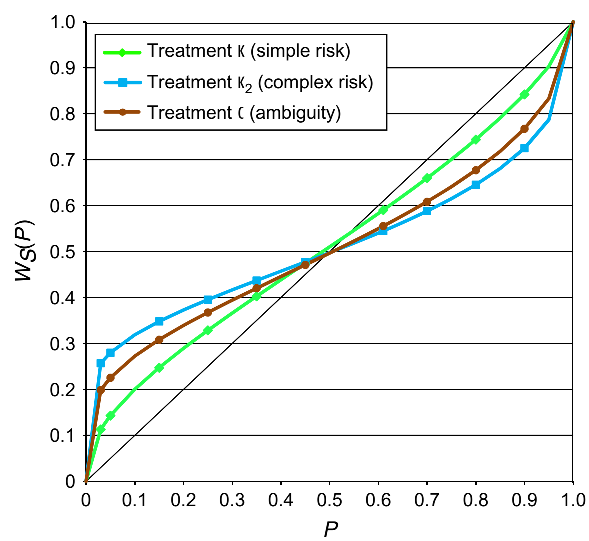
Figure 4 (Color online) Reported Probabilities in Experiment 2



be due to the higher earnings at stakes). With respect to the source functions, we find $b_{\mathtt{C}}$ and $b_{\mathtt{K}_2}$ to be negative but insignificant, thereby revealing no statistical difference in the optimism index across treatments. In contrast, $a_{\mathtt{C}}$ and $a_{\mathtt{K}_2}$ are both significantly smaller than 0 in the homogeneous model. Thus, as shown in Figure 5, the curvature of the source function is more pronounced for compound than simple events, and the most pronounced for complex risky events.

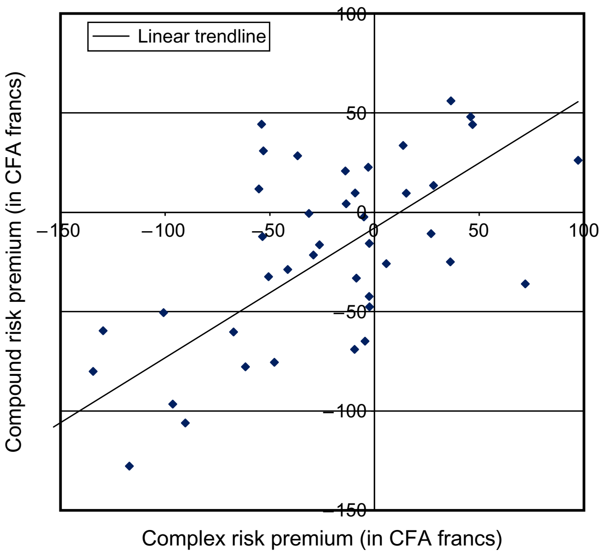
Finally, we calculate the complex (compound) risk premium as $\pi_{ip\mathtt{K}} - \pi_{ip\mathtt{K}_2}$ $(\pi_{ip\mathtt{K}} - \pi_{ip\mathtt{C}})$, where π_{ipt} is the expected payoff corresponding to subject i prediction for the event with probability P in treatment $t \in \{\mathtt{K}, \mathtt{K}_2, \mathtt{C}\}$. Figure 6, where the individual premia

Figure 5 (Color online) Source Functions in Experiment 2



Note. Based on average estimated parameters from the heterogeneous model (see Table 1).

Figure 6 (Color online) Average Compound and Complex Risk Premia per Subject in Experiment 2



averaged across all P are plotted, reveals a strong positive correlation ($\rho = 0.688$) between complex and compound risk premia. Furthermore, consistent with Halevy (2007), we find a tight association between neutrality to complex risk and the reduction of compound lotteries (see Appendix E).

To sum up, Experiment 2 provides further evidence that the domain of risk is rich in the sense that subjects have different attitudes toward risks based on simple, complex, or compound events. Furthermore, attitudes toward complex and compound risks are found to be highly correlated.

5. Discussion

5.1. Summary

We conducted two experiments to test the common assumption of a unique source of risk. We found evidence against this hypothesis, as subjects displayed significantly different attitudes when facing risks based on simple events and risks based on more complex events. Furthermore, these differences appeared to be systematic and not driven by calculation errors. Thus, we found that, similar to the domain of ambiguity, the domain of risk is rich. We also identified a tight link between attitudes toward complex risky bets and attitudes toward ambiguity and compound risk. In particular, subjects were essentially neutral to ambiguity and compound risk when those were measured in contrast to complex risk. Finally, our experiment shows that complexity attitudes can be empirically relevant, as they affected behavior as much as ambiguity and compound risk attitudes. We now discuss possible implications of these results.

5.2. Complexity Attitudes

Our results reveal a rich pattern of attitudes toward complex risk. Indeed, we find less optimism and less likelihood sensitivity under complex risk, but only the second effect is statistically significant in both experiments. Furthermore, the complex risk premia we calculated reveal complexity aversion for high probabilities and complexity loving for low probabilities, which is the same pattern generally observed for ambiguity attitudes (see Trautmann and van de Kuilen 2015). Our experiment, however, leaves a number of questions unanswered regarding the determinants of complex risk attitudes. In particular, how would people behave if they were told the objective probability of a complex risk? Do people react to complex risk because of the cost of calculating complex probabilities or because of the complex mechanism with which risk is generated? Would attitudes toward complex risk be less pronounced if evaluated in isolation of simple risk, similar to Fox and Tversky's (1995) comparative ignorance hypothesis for ambiguity attitudes?

5.3. Characterization of Ambiguity Aversion

Finding that the domain of risk is rich raises questions about the characterization of ambiguity attitudes in theory and in practice. In particular, an agent's attitude toward an ambiguous source S is defined under ABPW's source method as the difference between the source function for S and the source function for risk. Thus, ambiguity attitudes cannot be characterized uniquely if there are many sources of risk. Moreover, the central "uncertainty aversion" axiom of Gilboa and Schmeidler (1989) maxmin model implicitly relies on the assumption of a unique source of risk. In practice, ambiguity aversion is almost exclusively measured in Ellsberg-like experiments by comparing attitudes toward known and unknown probabilities (Trautmann and van de Kuilen 2015). Experimental measures of ambiguity aversion are thus contingent on the source of risk considered.

5.4. Modeling of Ambiguity Aversion

A variety of models have been proposed to capture attitudes toward both risk and ambiguity. Whereas the class of multiple prior models considers that beliefs cannot be represented by a unique distribution (Wald 1950, Gilboa and Schmeidler 1989), the now popular multistage approach induces ambiguity aversion through a failure to reduce compound lotteries (Segal 1987, Klibanoff et al. 2005, Seo 2009). The latter approach recently found support in experiments showing an association between the inability to reduce compound objective probabilities and ambiguity aversion (Halevy 2007, Abdellaoui et al. 2015). Our results suggest that these ambiguity (aversion) models are incomplete because they fail to capture attitudes toward complex risk. Furthermore, the tight association between attitudes toward complex risk (with no obvious interpretation as compound lotteries) and attitudes toward both ambiguity and compound risk suggests that ambiguity and compound risk attitudes may be special cases of complexity attitudes.

5.5. Implication for the Source Method

The source method has been found to outperform alternative theories to rationalize behavior under risk and ambiguity (Kothiyal et al. 2014). Nevertheless, one may question the practical relevance of the source method. Indeed, because it is context dependent, the source method has an infinite number of degrees of freedom (i.e., a different source function for each source of uncertainty). As a result, the source method does not lend itself to out of sample predictions: knowing an agent's attitudes toward one source does not provide guidance as to the attitudes of that agent toward a different source. However, the high correlation we found between choices under ambiguity and compound and complex risks suggests that an agent's behavior is connected across sources of uncertainty. Thus, predictions in general environments may be possible if the theory can be generalized to capture this connection. A possible approach to do so may be to model how complexity affects choices under uncertainty, as discussed next.

5.6. Accounting for Complexity

Our results suggest that a comprehensive model of choices under uncertainty should account for attitudes toward complexity. To do so, one may follow one of the many bounded rationality approaches proposed in various fields of economics.[10] Alternatively, one may consider ABPW's source method under the assumption that the whole domain of uncertainty is rich, regardless of whether probabilities are known or not. Consistent with our results, the source function could then be interpreted as reflecting the subjective degree of "complexity" of the source. This descriptive approach may offer several advantages: it is rooted in decision theory, it is based on an empirically validated model (prospect theory), and changes in complexity attitudes may be studied by varying the shape of the source function.

Acknowledgments

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Appendix A. The Design of Experiment 1

The description of the events in each treatment of Experiment 1 is shown in Figure A.1. The instructions are shown in Figure A.2. The subject went through two examples illustrating the three phases of the computerized iterative choice list method, as shown in Figures A.3–A.5. In Phase 1, the range of payments was divided into 5 categories. In Phase 2, the range of payments corresponding to the choices in Phase 1 was subdivided into 10 categories. In the confirmation phase, the subject saw the choices she made in Phases 1 and 2. She could then confirm or change her choices. See ABPW for further details on the design.

Appendix B. Uniform Source Test

Following ABPW, we verify that the sources in treatments $\mathtt{U}$ and $\mathtt{K}_2$ are uniform; that is, we test whether the bet 25_E0 is valued equally when event E is expressed using different colors. For instance, as explained by ABPW, if the source in treatment $\mathtt{U}$ is uniform, then a subject should report the same certainty equivalent for the events "the ball is blue" and "the ball is red," thereby indicating that he perceives the two events as equiprobable. Using a nonparametric Friedman test, we fail to reject the hypothesis of a uniform source in treatments $\mathtt{U}$ and $\mathtt{K}_2$ at any usual significance level. Specifically, in treatment $\mathtt{U}$, the p-values are 0.51, 0.37, and 0.66 for the events with one, two and three colors, respectively, whereas in treatment $\mathtt{K}_2$, the p-values are 0.39, 0.45, and 0.28 for the events with probabilities 1/8, 2/8, and 3/8, respectively.

Appendix C. ABPW Empirical Approach

ABPW's econometric approach to test whether the source function under risk differs from the source function under ambiguity consists of three steps.

In Step 1, r_{it} and an auxiliary parameters δ_{it} are estimated for subject i in treatment t by nonlinear least squares using the certainty equivalents CE_{ijt} elicited for the six bets j in which $\underline{x}_j$ and $\bar{x}_j$ vary from 0 to 25 while P_j is fixed at 1/2. The equality used to implement the nonlinear least squares estimation is based on the cumulative prospect theory value function with $\delta_{it} = w_{it}(1/2)$:

$$(CE_{ijt})^{r_{it}} = \delta_{it}[(\bar{x}_j)^{r_{it}} - (\underline{x}_j)^{r_{it}}] + (\underline{x}_j)^{r_{it}}.$$

In Step 2, using the parameter $\hat{r}_{it}$ estimated in Step 1, the parameters $\{\alpha_{it}, \beta_{it}\}$ from the source function proposed by Prelec (1998) are estimated by nonlinear least squares

[10] For example, macroeconomics (Sims 2006), game theory (Gale and Sabourian 2005), industrial organization (Ellison and Ellison 2009), or finance (Caballero and Simsek 2013).

Figure A.1 **The Design of Experiment 1**

Description of the events in each treatment of Experiment 1		
	Treatment *K* [and *U*]	Treatment K_2
	An urn contains the following 8 balls: ● ● ● ● ● ● ● ● We draw one ball at random from the urn. [Treatment *U*: Identical except that the number of balls of each color in the urn is unknown]	There are 2 urns. In each urn there are the following 8 balls: ● ● ● ● ● ● ● ● We draw one ball at random from each of the 2 urns. The order in which the balls are drawn does not matter. Only the color of the balls drawn matters. We denote with brackets the color of the balls drawn. For instance, {●,●} means that we drew one blue ball and one red ball.
Probability	**The subject wins if**	**The subject wins if**
1/8	The ball drawn is ●.	One ball is ● or ●, the other ball is ● or ●. For instance, you win if we draw {●,●}, you lose if we draw {●,●} or {● ,● }.
2/8	The ball drawn is ● or ●.	There is no ● ball, no ● ball, no ● ball, no ● ball. For instance, you win if we draw {● ,● }, you lose if we draw {●,●}.
3/8	The ball drawn is ● or ● or ●.	One ball is ● or ●, the other ball is neither ● nor ●. For instance, you win if we draw {●, ●}, you lose if we draw {●,●} or {● ,●}.
4/8	The ball drawn is ● or ● or ● or ●.	One ball is ● or ● or ● or ●, the other ball is ● or ● or ● or ●. For instance, you win if we draw {●, ● }, you lose if we draw {●, ●}.
5/8	The ball drawn is ● or ● or ● or ● or ●.	There is no ● ball and no ● ball, or the 2 balls are ● and/or ●. For instance, you win if we draw {●, ●} or {●, ●} or {● ,●}, you lose if we draw {●, ●}.
6/8	The ball drawn is ● or ● or ● or ● or ● or ●.	There is no ● ball, and the 2 balls are not both ●. For instance, you win if we draw {●,●} or {●,●}, you lose if we draw {●,● } or {●,●}.
7/8	The ball drawn is ● or ● or ● or ● or ● or ● or ●.	The 2 balls drawn are not of the same color. For instance, you win if we draw {●, ●}, you lose if we draw {●, ●}.

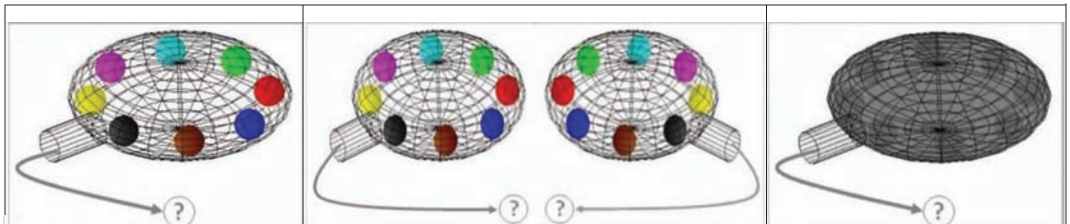
Figure A.2 **Instructions (Screen Shots Translated from French)**

Hello and thanks for your participation in this experiment!

During this experiment you will see several series of questions. In these questions, you will see different urns. Each urn contains 8 balls.
The balls can be of the following colors :

■ ■ ■ ■ ■ ■ ■ ■

The urns you will face are as follows :



In this urn, we know that there is one ball of each color. One ball is drawn at random from this urn.	**There are 2 urns. In each urn, we know that there is one ball of each color. One ball is drawn at random from each of the 2 urns. The order in which the balls are drawn does not matter. Only the color of the balls drawn matters. For instance, {■,■} indicates that a red ball and a blue ball have been drawn.**	**In this urn, the proportion of balls of each color is not known. One ball is drawn at random from this urn.**

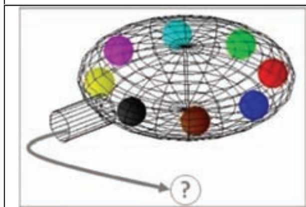
In each question you have to choose between two options:
- Option 1 indicates with which urn(s) and under which conditions you can win an amount between 0 and 25 €.
- Option 2 is a list of sure amounts.
For each question you have to choose between Option 1 and each of the amounts listed under Option 2.

Start the Examples

Figure A.3 **Phase 1**

Question 1

Which of these two options do you prefer?



Earn 25 € if
Earn 0 € if or or or or or or

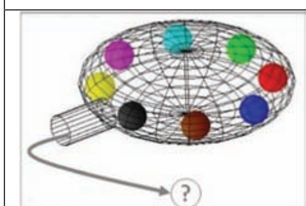
Option 1 *Play the lottery below*	1	2	Option 2 *Receive the amount below for sure*
			0 €
			5 €
			10 €
			15 €
			20 €
			25 €

Continue

Figure A.4 **Phase 2**

Question 1

Which of these two options do you prefer?



Earn 25 € if
Earn 0 € if or or or or or or

Option 1 *Play the lottery below*	1	2	Option 2 *Receive the amount below for sure*
			20 €
			20.5 €
			21 €
			21.5 €
			22 €
			22.5 €
			23 €
			23.5 €
			24 €
			24.5 €
			25 €

Back Continue

using the remaining 7 or 13 certainty equivalents elicited for the bets in which P_j varies from 1/8 to 7/8 while $\{\underline{x}_j, \bar{x}_j\}$ remains fixed at $\{0, 25\}$. The equality used to implement the nonlinear least squares estimation is based on subject i's indifference under cumulative prospect theory between a prospect j and his elicited certainty equivalent:

$$(CE_{ijt})^{\hat{r}_{it}} = \exp(-\beta_{it}[-\ln P_j]^{\alpha_{it}})[(\bar{x}_j)^{\hat{r}_{it}} - (\underline{x}_j)^{\hat{r}_{it}}] + (\underline{x}_j)^{\hat{r}_{it}}.$$

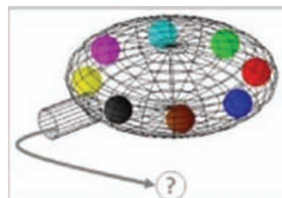
In Step 3, the estimated parameters $\{\hat{\alpha}_{it}, \hat{\beta}_{it}\}$ are used to calculate subject i's estimated source function value $\hat{w}_{it}(j/8) = \exp(-\hat{\beta}_{it}[-\ln(j/8)]^{\hat{\alpha}_{it}})$ for $j = 1, \ldots, 7$. To test for treatment effects for each $j = 1, \ldots, 7$, the distributions of $\hat{w}_{it}(j/8)$ are compared across treatments using a t test. Additionally, a likelihood sensitivity index and a pessimism index are calculated for each subject and compared across treatments.

We believe ABPW's empirical approach may be improved in three ways. First, the t tests conducted in Step 3 to compare the distributions of $\hat{w}_{it}(j/8)$ across treatments are valid if one treats the $\hat{w}_{it}(j/8)$ as (recoded) data, but they are not valid if one treats the $\hat{w}_{it}(j/8)$ as econometric estimates, i.e., random variables whose standard deviations depend on the sampling error from the estimation of $\{\hat{r}_{it}, \hat{\alpha}_{it}, \hat{\beta}_{it}\}$ in Steps 1

Figure A.5 **Confirmation Phase**

Question 1

Which of these two options do you prefer?



Earn 25 € if
Earn 0 € if or or or or or or

Option 1 *Play the lottery below*	1	2	Option 2 *Receive the amount below for sure*
			0 €
			0.5 €
			1 €
			1.5 €
			2 €
			2.5 €
			3 €
			3.5 €
			4 €
			4.5 €
			5 €
			5.5 €
			6 €
			6.5 €
			7 €
			7.5 €
			8 €
			8.5 €
			9 €
			9.5 €
			10 €
			10.5 €
			11 €
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			22 €
			22.5 €
			23 €
			23.5 €
			24 €
			24.5 €
			25 €

Back Validate

and 2. Consistent with the econometric literature, our empirical approach treats $\{\hat{r}_{it}, \hat{\alpha}_{it}, \hat{\beta}_{it}\}$ as econometric estimates.[11] Second, ABPW estimate four parameters $\{\hat{\delta}_{it}, \hat{r}_{it}, \hat{\alpha}_{it}, \hat{\beta}_{it}\}$ when only three are necessary: $\{\hat{r}_{it}, \hat{\alpha}_{it}, \hat{\beta}_{it}\}$. Given the small sample size with which the parameters are estimated (13 or 19 observations, depending on the treatment), adding an auxiliary parameter reduces the efficiency of the estimates. In our empirical approach, we therefore estimate only the

[11] As explained in §3.3, we also estimate $\{\hat{\alpha}_{it}, \hat{\beta}_{it}\}$, but report the reparametrization $\{\hat{a}_{it}, \hat{b}_{it}\}$ ($\hat{\alpha}_{it} = \hat{a}_{it}$ and $\hat{\beta}_{it} = (-\ln \hat{b}_{it})^{1-\hat{a}_{it}}$) to facilitate interpretation.

parameters of interest, $\{\hat{r}_{it}, \hat{\alpha}_{it}, \hat{\beta}_{it}\}$. Third, by segmenting the data between Step 1 and Step 2, some information is ignored when estimating the parameters. Indeed, $\{\hat{\delta}_{it}, \hat{r}_{it}\}$ are estimated with 6 observations, while $\{\hat{\alpha}_{it}, \hat{\beta}_{it}\}$ are estimated with the remaining 7 or 13 observations (depending on the treatment). Instead, we estimate $\{\hat{r}_{it}, \hat{\alpha}_{it}, \hat{\beta}_{it}\}$ jointly with all the 13 or 19 observations collected for subject i in treatment t. In general, such a joint estimation should provide more precise estimates. Finally, note that, otherwise, our estimation method is identical to that of ABPW with respect to the data used, the structural and parametric assumptions, and the identifying condition (Equation (2)).

Appendix D. Structural Model with Calculation Errors

Did subjects report different certainty equivalents in treatments $\mathtt{K}$ and $\mathtt{K}_2$ because of errors they made when calculating objective probabilities? To address this question, we relax the assumption that p_{ijt}, the subjective probability of subject i for prospect j in treatment t, is equal to the objective probability P_j. Instead, p_{ijt} is now assumed to be a random variable, in which case the difference $p_{ijt} - P_j$ can be interpreted as a random calculation error. Following the psychology literature (e.g., Kahneman et al. 1982), we can assume that $E[p_{ijt}] > P_j$ when P_j is close to 0, and $E[p_{ijt}] < P_j$ when P_j is close to 1, so that the subject has an inverse-S-shaped judgment bias (i.e., on average, the subject overestimates low probabilities and underestimates high probabilities). Then, the differences in behavior we observed across treatments could be explained by differences in calculation errors.

To illustrate, let us assume that p_{ijt} follows a normal distribution around P_j truncated on $[0, 1]$, $N_{[0,1]}(P_j, \sigma_t^2)$, where σ_t^2 captures the magnitude of the calculation errors across subjects in treatment t. Thus, $\sigma_{\mathtt{K}_2} > \sigma_{\mathtt{K}}$ implies that $E[p_{ij\mathtt{K}_2}] > E[p_{ij\mathtt{K}}] > P_j$ when $P_j < 1/2$ (respectively, $E[p_{ij\mathtt{K}_2}] < E[p_{ij\mathtt{K}}] < P_j$ when $P_j > 1/2$), in which case subjects in treatment in $\mathtt{K}_2$ choose, on average, higher (respectively, lower) certainty equivalents than subjects in treatment $\mathtt{K}$. In other words, differences in σ_t could explain (at least in part) why subjects generally select different certainty equivalents across treatments.[12]

Under the source method, the indifference of subject i between prospect j and his elicited certainty equivalent implies

$$(CE_{ijt})^{r_{it}} = \exp(\ln(b_{it})[\ln p_{ijt}/\ln(b_{it})]^{a_{it}})[(\bar{x}_j)^{r_{it}} - (\underline{x}_j)^{r_{it}}] + (\underline{x}_j)^{r_{it}}$$

$$\Rightarrow \quad p_{ijt} = \exp\left\{[\ln(b_{it})]^{1-1/a_{it}}\left[\ln\left(\frac{(CE_{ijt})^{r_{it}} - (\underline{x}_j)^{r_{it}}}{(\bar{x}_j)^{r_{it}} - (\underline{x}_j)^{r_{it}}}\right)\right]^{1/a_{it}}\right\}.$$

The structural parameters (i.e., $\{r_{it}, a_{it}, b_{it}, \sigma_t\}$ when we assume that $p_{ijt} \sim N_{[0,1]}(P_j, \sigma_t^2)$) can then be estimated by maximum likelihood. Similar to the other parameters, we write $\sigma_t = \sigma_{\mathtt{K}} + \sigma_{\mathtt{K}_2}\mathtt{K}_2 + \sigma_{\mathtt{U}}(\mathtt{K}_2 + \mathtt{U})$.

As shown in Table F.3 in Appendix F, $\sigma_{\mathtt{U}}$ and $\sigma_{\mathtt{K}_2}$ are both positive and significant. Thus, subjects facing an ambiguous bet appear to make larger calculation errors than when they face a risky bet with simple events. Furthermore, subjects make the largest calculation errors when they face risky bets with complex events (which are arguably more difficult to evaluate than bets with simple events).

These differences in calculation errors, however, are not sufficient to explain the differences in certainty equivalents across treatments. Indeed, comparing the results in Table F.1 with those in panel (a) of Table 1 shows that the sign, magnitude, and significance of the other parameters remain essentially unchanged when we account for calculation errors. In other words, although we find differences in calculation errors for simple, complex, and ambiguous bets, we still find that a subject's source function is statistically different for risky bets based on simple events and for risky bets based on more complex events.

[12] Although similar in spirit, this model with calculation errors is different from random utility models (e.g., Blavatskyy 2007) in which an error is added to the expected utility. In particular, it is easy to show that such models do not necessarily produce the pattern just described, i.e., higher (lower) certainty equivalents on average when $P_j < 1/2$ ($P_j > 1/2$).

Appendix E. Neutrality to Complex Risk, Compound Risk, and Ambiguity

To explore the link between neutrality to ambiguity and neutrality to complex risky bets in the urn experiment (Experiment 1), we calculate for each subject the average (across $P = 1/8, \ldots, 7/8$) absolute value of the ambiguity and complex risk premia defined as $CE_{ip\mathtt{K}} - CE_{ip\mathtt{U}}$ and $CE_{ip\mathtt{K}} - CE_{ip\mathtt{K}_2}$, respectively, where CE_{ipt} is the certainty equivalent subject i provided for the event with probability P in treatment $t \in \{\mathtt{K}, \mathtt{U}, \mathtt{C}\}$. If a subject is perfectly neutral to ambiguity (complex risk), then his average absolute ambiguity (complex risk) premium is 0. As shown in Figure 3, no subject was perfectly neutral to ambiguity or complex risk for every $P = 1/8, \ldots, 7/8$. If we define a subject with an average absolute premium lower than €1 as "nearly neutral," then, out of the 77 subjects in Experiment 1, 10 are nearly neutral to both ambiguity and complex risk, 4 are nearly neutral to ambiguity only, and 3 are nearly neutral to complex risk only. Thus, 77% of the subjects with similar attitudes toward simple and complex risky bets are ambiguity nearly neutral. A Fisher exact test confirms (p-value $= 2.2\ 10^{-7}$) that near neutrality to ambiguity and near neutrality to complex risk can be considered tightly associated.

To explore the link between neutrality to compound risk and neutrality to complex risky bets in the dice experiment (Experiment 2), we calculate for each subject the average absolute value of the complex and compound risk premia defined as $\pi_{ip\mathtt{K}} - \pi_{ip\mathtt{K}_2}$ and $\pi_{ip\mathtt{K}} - \pi_{ip\mathtt{C}}$, respectively, where π_{ipt} is the expected payoff corresponding to subject i's prediction for the event with probability P in treatment $t \in \{\mathtt{K}, \mathtt{K}_2, \mathtt{C}\}$. If we define near neutrality as having an average absolute premium lower than 75 CFA francs (1/40th of the 3,000 CFA francs earned on average), then out of the 43 subjects in Experiment 2, 8 are nearly neutral to both compound and complex risks, 3 are nearly neutral to compound risk only, and 2 are nearly neutral to complex risk only.[13] Thus, 80% of the subjects with similar attitudes toward simple and complex risks are nearly neutral to compound risk. A Fisher exact test confirms (p-value $= 4.4\ 10^{-5}$) that near neutrality to compound and complex risks can be considered tightly associated.

[13] Alternative definitions of near neutrality yield similar results.

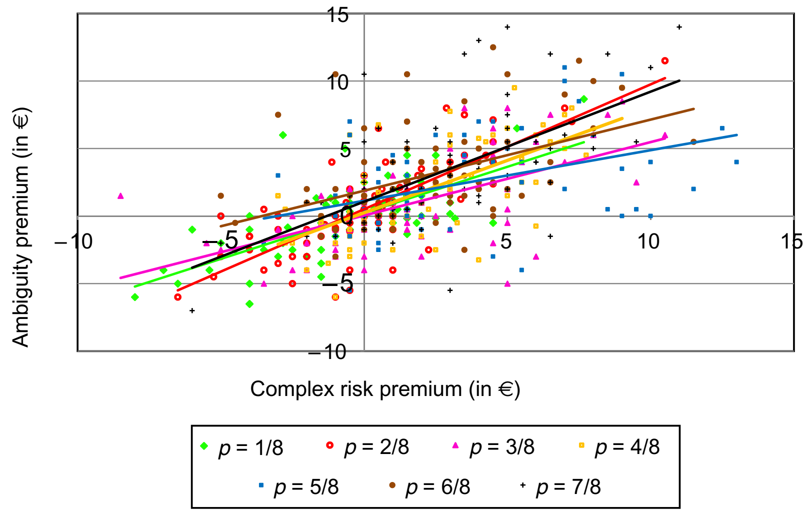
Appendix F. Additional Figures and Tables

Table F.1 Comparison of Certainty Equivalents Across Treatments in Experiment 1

Treatments	$P=1/8$	$P=2/8$	$P=3/8$	$P=4/8$	$P=5/8$	$P=6/8$	$P=7/8$
K vs. U	$Z=0.229$ $P=0.819$	$Z=1.362$ $P=0.173$	$Z=1.873$ $P=0.061$	$Z=3.578^{**}$ $P=3.5\text{E}{-4}^{**}$	$Z=5.127^{**}$ $P=2.9\text{E}{-7}^{**}$	$Z=6.399^{**}$ $P=1.6\text{E}{-10}^{**}$	$Z=5.881^{**}$ $P=4.1\text{E}{-9}^{**}$
K vs. K_2	$Z=-0.577$ $P=0.564$	$Z=0.249$ $P=0.803$	$Z=3.615^{**}$ $P=3.0\text{E}{-4}^{**}$	$Z=4.542^{**}$ $P=5.6\text{E}{-6}^{**}$	$Z=6.362^{**}$ $P=2.0\text{E}{-10}^{**}$	$Z=5.410^{**}$ $P=6.3\text{E}{-8}^{**}$	$Z=5.969^{**}$ $P=2.4\text{E}{-9}^{**}$
U vs. K_2	$Z=1.181$ $P=0.238$	$Z=1.842$ $P=0.065$	$Z=-1.884$ $P=0.059$	$Z=-0.582$ $P=0.560$	$Z=-1.200$ $P=0.230$	$Z=1.697$ $P=0.090$	$Z=1.013$ $P=0.311$

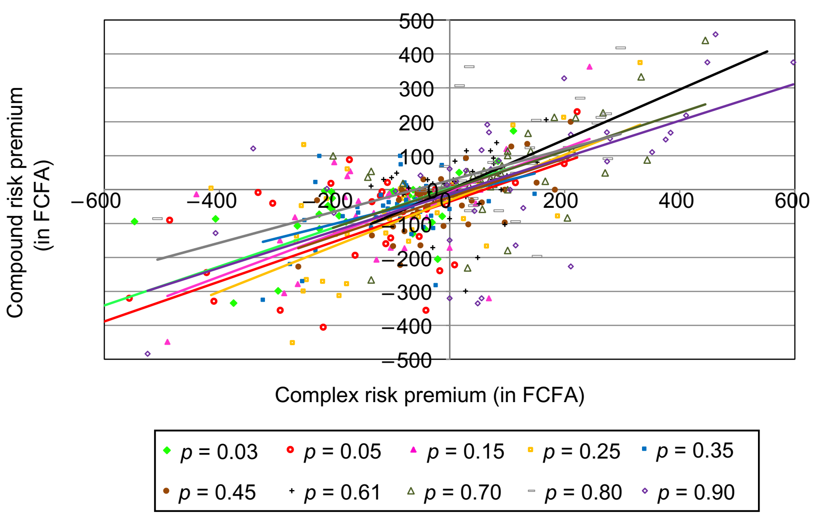
Notes. Wilcoxon sign-ranked tests are conducted to compare subjects' certainty equivalents across treatments. Under the null hypothesis, the distributions of subjects' certainty equivalents are the same across treatments.
**Indicates tests significant at the 5% level.

Figure F.1 Ambiguity and Complex Risk Premia in Experiment 1



Notes. Each dot represents a subject. Each color represents a probability $P=1/8,\ldots,7/8$. Each line represents the linear trend line for the corresponding color/probability. The figure shows that the positive relationship between ambiguity and complex risk premia holds for each $P=1/8,\ldots,7/8$.

Figure F.2 Compound and Complex Risk Premia in Experiment 2



Notes. Each dot represents a subject. Each color represents a probability $P=3\%,\ldots,90\%$. Each line represents the linear trend line for the corresponding color/probability. The figure shows that the positive relationship between compound and complex risk premia holds for each $P=3\%,\ldots,90\%$.

Table F.2 Comparison of Reported Probabilities Across Treatments in Experiment 2

Objective probability (%)	Rank sum for each treatment K	K_2	C	Friedman statistic	*p*-value
3	53.5	108	96.5	40.26	1.81E–9**
5	58	103	97	28.43	6.71E–7**
15	60	103	96	25.02	3.68E–6**
25	67	98	94	13.49	0.001**
35	59	113	87	35.57	1.89E–8**
45	70	95	94	10.92	0.004**
61	102	78	79	9.86	0.007**
70	109	66	84	23.45	8.10E–6**
80	110	60	89	32.19	1.02E–7**
90	105	70	84	15.99	1.74E–3**

Notes. Friedman tests are conducted to compare a subject's reported probabilities across treatments. Under the null hypothesis, the distributions of a subject's reported probabilities are the same across treatments.
**Indicates tests significant at the 5% level.

Table F.3 Structural Estimation of Utility and Source Functions: Models with Calibration Errors

	Homogeneous	Heterogeneous: Average across subjects	Std across subjects	% of subjects with significant parameter at 5% (%)
r_K	−0.039 (0.050)	0.095	0.452	51.9
r_U	−0.030 (0.071)	−0.018	0.413	11.7
r_{K_2}	0.023 (0.095)	0.006	0.596	10.4
a_K	−0.183*** (0.043)	0.002	0.218	80.5
a_U	−0.201*** (0.053)	−0.242	0.413	51.9
a_{K_2}	−0.054 (0.058)	−0.068	0.377	18.2
b_K	0.012 (0.032)	−0.022	0.385	48.1
b_U	−0.113** (0.049)	−0.085	0.423	33.8
b_{K_2}	−0.047 (0.041)	−0.044	0.417	14.3
σ_K	0.098*** (0.003)	0.060	0.033	—
σ_U	0.039*** (0.008)	0.027	0.045	37.7
σ_{K_2}	0.018** (0.009)	0.026	0.053	27.3

Notes. Estimates' standard errors and test statistics' distributions are estimated by bootstrap (size=10,000) to account for the finiteness of the sample.
*** and ** represent parameters significant at the 1% and 5% levels, respectively.

Appendix G. Instructions for Experiment 2 (Translated from French)

Your Identification Code: ——

You are about to take part in an experiment aimed at better understanding decisions made under uncertainty. In the experiment you will earn an amount of money. This amount of money will be paid to you at the end of the experiment, outside the lab, in private, and in cash. The amount of money you will earn may be larger if:

1. You read the instructions below carefully;
2. You follow these instructions precisely;
3. You make thoughtful decisions during the experiment.

If you have any questions while we read the instructions or during the experiment, then call us *by raising your hand*. Any form of communication between participants is absolutely forbidden. If you do not follow this rule, then we will have to exclude you from the experiment without any payment.

The Task

You will be given 30 different "events," divided into three series of 10. Each of these events describes the possible outcome produced by the roll of two dice. One of the die is red, the other die is black. Each die has 10 sides numbered from 0 to 9. Each die is fair, which means that any of the 10 sides has an equal chance to come up when the die is rolled. Consider now two examples of events we could give you:

- Event 1: *The red die equals 5 and the black die equals 3.*
- Event 2: *The red die produces a number strictly greater than the black die.*

As explained below, 1 out of the 30 events will be randomly selected for payment at the end of the experiment. We will then roll the two dice once to determine whether the event occurs or whether the event does not occur. For instance, if Event 1 above is randomly selected for payment, then we will say that Event 1 occurs when the outcome of the roll of the two dice is such that the red die produces a 5 and the black die produces a 3. For any other number produced by either the black or the red die, we will say that Event 1 did not occur. Likewise, if Event 2 is randomly selected for payment, then we will say that Event 2 occurs when the outcome of the roll of the 2 dice is such that the red die produces a number strictly greater than the black die. Otherwise, we will say that Event 2 did not occur.

Your Choices

For each of the 30 events, you will be asked to make a choice. One of these choices will determine the amount of money you will earn both when the event randomly selected for payment occurs and when it does not occur. Each of your choices consists in selecting a number between 1 and 149 in the table we gave you separately. We will now explain how your choice for the event randomly selected for payment affects the amount of money you will earn.

If you look at the table, you can see that there are two amounts associated with each of the 149 possible choice numbers. The first is the amount of money you receive if the event occurs. The second is the amount of money you receive if the event does not occur. For instance, you can see in the table that the amounts associated with the choice number "1" are 53 and 4,000. This means that the amount of money you earn will be 53 CFA francs if the event occurs or 4,000 CFA francs if the event does not occur. As you can see, when the choice number increases from 1 to 149, the amounts in the first columns increase, whereas the amounts in the second column decrease. For instance, the amounts associated with the choice number "90" are 3,360 and 2,560 CFA francs. In other words, if you choose the number "90" instead of the number "1," then you will earn more if the event occurs (3,360 CFA francs instead of 53), but you will earn less if the event does not occur (2,560 CFA francs instead of 4,000). Note also that the highest choice numbers (those closer to 149) produce the largest amounts of money when the event occurs, but the smallest amounts of money when the event does not occur. For instance, the choice number "140" produces 3,982 CFA francs if the event occurs, but only 516 CFA francs if the event does not occur.

For each of the 30 events, you are free to select any choice number you want. Note that there is no correct or incorrect choice. The choice numbers selected may differ from one individual to the next. In general, however, you may find it profitable to choose a higher choice number when you think the chances that the event will occur are higher. Indeed, as we just explained, such a choice number will produce a larger amount if the event occurs. Conversely, you may find it profitable to choose a smaller number when you think the chances that the event will occur are lower.

Your Payment

The amount of money you receive today will be determined in three steps. In a first step, we will randomly select one of the 30 events for payment. In a second step, we will roll the two dice once to determine whether the event selected for payment occurs or does not occur. Finally, in a third step, we will look at the choice number you chose for the event selected for payment to determine the amount of money you will receive.

We will proceed as follows to select one of the 30 events for payment. At the beginning of the experiment, we will ask you to write your identification code on a piece of paper that you will then fold. Your identification code is located on the top right-hand corner on the first page of the instructions. At the end of the experiment, we will draw at random one of the pieces of paper. The person whose identification number has been drawn will randomly choose 1 out of 30 numbered tokens from a bag. The number written on the token selected indicates the event that will be considered for the payment of each person in the room.

We will then draw at random a second piece of paper. The person whose identification code has been drawn will roll the two dice once to determine whether the event selected occurs or not. This single roll will be used to determine the payment of each person in the room.

If you do not wish to be one of the persons rolling the dice or drawing the token, then simply leave your piece of paper blank. Just fold it without writing your identification code.

Comprehension Test

Understanding the instructions well is important if you want to improve your chances to earn a larger amount of money during the experiment. To make sure you understand the instructions well, we will now conduct a quick test without monetary consequences. Imagine first that Event 1 (*the red die equals* 5 *and the black die equals* 3) has been selected for payment. In addition, imagine that an individual selected the choice number **98** for this event, whereas a different individual selected the choice number **139**. Please, write in the table below the amount of money each of these two individuals would receive if the roll of the dice produces the following outcomes:

	Payment to the individual with	
Outcome produced by the roll of the 2 dice	A choice number of **98**	A choice number of **139**
The red die equals 6 and the black die equals 4	——CFA francs	——CFA francs
The red die equals 5 and the black die equals 4	——CFA francs	——CFA francs
The red die equals 5 and the black die equals 3	——CFA francs	——CFA francs

Imagine now that Event 2 (*the red die produces a number strictly greater than the black die*) has been selected for payment. In addition, imagine that an individual selected the choice number **6** for this event, whereas a different individual selected the choice number **71.** Please write in the table below the amount of money each of these two individuals would receive if the roll of the dice produce the following outcomes:

	Payment to the individual with	
Outcome produced by the roll of the 2 dice	A choice number of **6**	A choice number of **71**
The red die equals 3 and the black die equals 9	——CFA francs	——CFA francs
The red die equals 5 and the black die equals 2	——CFA francs	——CFA francs
The red die equals 0 and the black die equals 5	——CFA francs	——CFA francs

Please do not hesitate to raise your hand now if the instructions we just read were not perfectly clear. Once the experiment starts you can still call us to answer any question *by raising your hand*.

Note that the amount of money you will receive today may be larger or smaller depending on your choices *and* on the outcome produced by the roll of the two dice. By accepting to participate in the experiment, you accept the consequences associated with your choices and with the roll of the dice. If you do not wish to participate in the experiment, you are free to leave now, in which case you will receive a flat fee of 500 CFA francs.

Series 1
For the first series of 10 events, we will consider that the red die determines the first digit (meaning 0, 10, 20, 30, 40, 50, 60, 70, 80, or 90) and the black die determines the second digit (meaning 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9) of a number between 1 and 100 (both dice equal to zero corresponds to the number 100). As a result, every number between 1 and 100 has an equal chance to come out from the roll of the two dice.

Event	Description	Your choice number
1	*"The number is between* 1 (*included*) *and* 25 (*included*)*"*	
2	*"The number is between* 62 (*included*) *and* 66 (*included*)*"*	
3	*"The number is between* 16 (*included*) *and* 76 (*included*)*"*	
4	*"The number is between* 3 (*included*) *and* 92 (*included*)*"*	
5	*"The number is between* 52 (*included*) *and* 96 (*included*)*"*	
6	*"The number is between* 9 (*included*) *and* 88 (*included*)*"*	
7	*"The number is between* 44 (*included*) *and* 58 (*included*)*"*	
8	*"The number is between* 23 (*included*) *and* 25 (*included*)*"*	
9	*"The number is between* 37 (*included*) *and* 71 (*included*)*"*	
10	*"The number is between* 28 (*included*) *and* 97 (*included*)*"*	

Series 2
For the next series of 10 events, we will sum the outcome of the red die to the outcome of the black die. Since each die can only produce a number between 0 and 9, the sum obtained can only be a number between 0 and 18. Observe that some of these sums (for instance, 0) can only be obtained from a unique combination of the two dice, whereas other sums (for instance, 6) can be obtained from multiple combinations of the two dice. As a result, some of the 19 possible sums have more chances to come out than other sums.

Event	Description	Your choice number
11	*"The sum is between* 0 (*included*) *and* 4 (*included*)*"*	
12	*"The sum is between* 2 (*included*) *and* 10 (*included*)*"*	
13	*"The sum is equal to* 16*"*	
14	*"The sum is between* 4 (*included*) *and* 14 (*included*)*"*	
15	*"The sum is between* 5 (*included*) *and* 13 (*included*)*"*	
16	*"The sum is between* 0 (*included*) *and* 14 (*included*)*"*	
17	*"The sum is between* 10 (*included*) *and* 18 (*included*)*"*	
18	*"The sum is equal to* 4*"*	
19	*"The sum is between* 11 (*included*) *and* 17 (*included*)*"*	
20	*"The sum is between* 2 (*included*) *and* 6 (*included*)*"*	

Series 3
The last series of 10 events is similar to the first series. The red die determines the first digit and the black die determines the second digit of a number between 1 and 100. The difference with the first series is that when you select your choice number, you are not facing one, but two possible events. For instance, the first of the two possible events could be *"the number is between* 1 (*included*) *and* 25 (*included*)*"* and the second of the two possible events could be *"the number is between* 55 (*included*) *and* 59 (*included*)*."* You are asked to select a single choice number without knowing which of the two possible events will be used to determine your payment. It is only at the end of the experiment that we will toss a coin to identify which of the two possible events will be used for payment. If the coin lands on **heads**, then your payment will be determined using the first event. If the coin lands on **tails**, then your payment will be determined using the second event. As with Series 1, we will then roll the two dice to determine whether the event identified by the coin toss occurs or not. Here is an example:

♦ If the coin lands on **heads**, then the event is *"the number is between* 1 (*included*) *and* 25 (*included*)*."*
♦ **Or**, if the coin lands on **tails**, then the event is *"the number is between* 55 (*included*) *and* 59 (*included*)*."*

You must select a unique choice number before you know which of the possible two events will be used for payment. Imagine for instance that an individual selects the choice number 70. We have to distinguish between different two situations to determine how much the individual will be paid:

♦ **Either** the coin tossed at the end of the experiment lands on **heads**. In this case, the event used for payment is *"the number is between* 1 (*included*) *and* 25 (*included*)*."* Then, the event occurs if the two dice produce a number that is indeed between 1 (included) and 25 (included), and the individual in our example is paid 2,862 CFA francs. On the other hand, if the two dice produce a number that is not between 1 (included) and 25 (included), then the event does not occur, and the individual in our example is paid 3,129 CFA francs.

♦ **Or**, the coin tossed at the end of the experiment lands on **tails.** In this case, the event identified is *"the number is between* 55 (*included*) *and* 59 (*included*)*."* Then, the event occurs if the two dice produce a number that is indeed between 55 (included) and 59 (included), and the individual in our example is paid 2,862 CFA francs. On the other hand, if the two dice produce a number that is between 55 (included) and 59 (included), then the event does not occur, and the individual in our example is paid 3,129 CFA francs.

To summarize, there are only two cases under which the event occurs: (1) The coin lands on **heads** and the two dice produce a number between 1 (included) and 25 (included), **or** (2) the coin lands on **tails** and the two dice produce a number between 55 (included) and 59 (included). In all other cases, the event does not occur. Thus, when you select your choice number, you might want to imagine the different cases under which the event occurs and does not occur.

If these explanations are not sufficiently clear, please call us by *raising your hand*. We will then come to your desk to answer any questions you may have. We would like to remind you that it is important for you to understand the instructions well so that you can make the decisions that suit you the best.

Event	Description	Your choice number
21	♦ If the coin lands on **heads**, then the event is: *"the number is between* 48 (*included*) *and* 82 (*included*)." ♦ Or, if the coin lands on **tails**, then the event is: *"the number is between* 14 (*included*) *and* 48 (*included*)."	
22	♦ If the coin lands on **heads**, then the event is: *"the number is between* 21 (*included*) *and* 35 (*included*)." ♦ Or, if the coin lands on **tails**, then the event is: *"the number is between* 30 (*included*) *and* 44 (*included*)."	
23	♦ If the coin lands on **heads**, then the event is: *"the number is between* 25 (*included*) *and* 89 (*included*)." ♦ Or, if the coin lands on **tails**, then the event is: *"the number is between* 2 (*included*) *and* 96 (*included*)."	
24	♦ If the coin lands on **heads**, then the event is: *"the number is between* 66 (*included*) *and* 97 (*included*)." ♦ Or, if the coin lands on **tails**, then the event is: *"the number is between* 13 (*included*) *and* 70 (*included*)."	
25	♦ If the coin lands on **heads**, then the event is: *"the number is between* 56 (*included*) *and* 58 (*included*)." ♦ Or, if the coin lands on **tails**, then the event is: *"the number is between* 78 (*included*) *and* 80 (*included*)."	
26	♦ If the coin lands on **heads**, then the event is: *"the number is between* 82 (*included*) *and* 89 (*included*)." ♦ Or, if the coin lands on **tails**, then the event is: *"the number is between* 25 (*included*) *and* 66 (*included*)."	
27	♦ If the coin lands on **heads**, then the event is: *"the number is between* 7 (*included*) *and* 88 (*included*)." ♦ Or, if the coin lands on **tails**, then the event is: *"the number is between* 3 (*included*) *and* 100 (*included*)."	
28	♦ If the coin lands on **heads**, then the event is: *"the number is equal to* 12." ♦ Or, if the coin lands on **tails**, then the event is: *"the number is between* 49 (*included*) *and* 57 (*included*)."	
29	♦ If the coin lands on **heads**, then the event is: *"the number is between* 26 (*included*) *and* 86 (*included*)." ♦ Or, if the coin lands on **tails**, then the event is: *"the number is between* 14 (*included*) *and* 74 (*included*)."	
30	♦ If the coin lands on **heads**, then the event is: *"the number is between* 1 (*included*) *and* 83 (*included*)." ♦ Or, if the coin lands on **tails**, then the event is: *"the number is between* 36 (*included*) *and* 91 (*included*)."	

References

Abdellaoui M, Klibanoff P, Placido L (2015) Experiments on compound risk in relation to simple risk and to ambiguity. *Management Sci.* 61:1306–1322.
Abdellaoui M, Baillon A, Placido L, Wakker PP (2011) The rich domain of uncertainty: Source functions and their experimental implementation. *Amer. Econom. Rev.* 101:695–723.
Andersen S, Fountain J, Harrison G, Rutström E (2009) Estimating aversion to uncertainty. Mimeo, Copenhagen Business School, Copenhagen.
Armantier O, Treich N (2013) Eliciting beliefs: Proper scoring rules, incentives, stakes and hedging. *Eur. Econom. Rev.* 62:17–40.
Bar-Hillel M (1973) On the subjective probability of compound events. *Organ. Behav. Human Performance* 9:396–406.
Blavatskyy P (2007) Stochastic expected utility theory. *J. Risk Uncertainty* 34:259–286.
Caballero R, Simsek A (2013) Fire sales in a model of complexity. *J. Finance* 68:2549–2587.
Chew S, Sagi J (2008) Small worlds: Modeling attitudes toward sources of uncertainty. *J. Econom. Theory* 139:1–24.
Ellison G, Ellison S (2009) Search, obfuscation, and price elasticities on the internet. *Econometrica* 77:427–452.
Epstein L, Halevy Y (2014) No two experiments are identical. Mimeo, Boston University, Boston.
Fox CR, Tversky A (1995) Ambiguity aversion and comparative ignorance. *Quart. J. Econom.* 110:585–603.
Gale D, Sabourian H (2005) Complexity and competition. *Econometrica* 73:739–769.
Gigerenzer G (1991) How to make cognitive illusions disappear: Beyond heuristics and biases. *Eur. Rev. Soc. Psych.* 2:83–115.
Gilboa I, Schmeidler D (1989) Maxmin expected utility with nonunique prior. *J. Math. Econom.* 18:141–153.
Halevy Y (2007) Ellsberg revisited: An experimental study. *Econometrica* 75:503–536.
Kahneman D, Tversky A (1979) Prospect theory: An analysis of decision under risk. *Econometrica* 47:263–291.
Kahneman D, Slovic P, Tversky A (1982) *Judgment Under Uncertainty: Heuristics and Biases* (Cambridge University Press, New York).
Klibanoff P, Marinacci M, Mukerji S (2005) A smooth model of decision making under ambiguity. *Econometrica* 73:1849–1892.
Kothiyal A, Spinu V, Wakker PP (2014) An experimental test of prospect theory for predicting choice under ambiguity. *J. Risk Uncertainty* 48:1–17.
Offerman T, Sonnemans J, van de Kuilen G, Wakker PP (2009) A truth-serum for non-Bayesians: Correcting proper scoring rules for risk attitudes. *Rev. Econom. Stud.* 76(4):1461–1489.
Pacini R, Epstein S (1999) The relation of rational and experiential information processing styles to personality, basic beliefs, and the ratio-bias phenomenon. *J. Personality Soc. Psych.* 76: 972–987.
Prelec D (1998) The probability weighting function. *Econometrica* 66:497–527.
Segal U (1987) The Ellsberg paradox and risk aversion: An anticipated utility approach. *Internat. Econom. Rev.* 28:175–202.
Seo K (2009) Ambiguity and second order belief. *Econometrica* 77:1575–1605.
Sims C (2006) Rational inattention: Beyond the linear-quadratic case. *Amer. Econom. Rev.* 96:158–163.
Smith VL (1969) Measuring nonmonetary utilities in uncertain choices: The Ellsberg urn. *Quart. J. Econom.* 83:324–329.
Trautmann S, van de Kuilen G (2015) Ambiguity attitudes. Keren G, Wu G, eds. *The Wiley Blackwell Handbook of Judgment and Decision Making* (John Wiley & Sons, Chichester, UK).
Tversky A, Kahneman D (1981) The framing of decisions and the psychology of choice. *Science* 21:453–458.
Tversky A, Kahneman D (1992) Advances in prospect theory: Cumulative representation of uncertainty. *J. Risk Uncertainty* 5:297–323.
Wald A (1950) *Statistical Decision Functions* (Wiley, New York).
Winkler R, Murphy A (1970) Nonlinear utility and the probability score. *J. Appl. Meteorology* 9:143–148.