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Production Smoothing and the Bullwhip Effect

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The bullwhip effect and production smoothing appear antithetical because their empirical tests oppose one another: production variability exceeding sales variability for bullwhip, and vice versa for smoothing. But this is a false dichotomy. We distinguish between the phenomena with a new production smoothing measure, which estimates how much more variable production would be absent production volatility costs. We apply our metric to an automotive manufacturing sample comprising 162 car models and find 75% smooth production by at least 5%, despite the fact that 99% exhibit the bullwhip effect. Indeed, we estimate both a strong bullwhip (on average, production is 220% as variable as sales) and robust smoothing (on average, production would be 22% more variable without deliberate stabilization). We find firms smooth both production variability and production uncertainty. We measure production smoothing with a structural econometric production scheduling model, based on the generalized order-up-to policy.

Keywords: production smoothing; bullwhip effect; demand signal processing; generalized order-up-to policy; martingale model of forecast evolution

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1. Introduction

Production smoothing is production stabilization intended to mitigate the tolls of production variability: overtime fees and strained toolsets on the one hand, idle capacity on the other. Firms smooth production by using safety stocks to insulate production schedules from demand volatility. This topic interests both economists (Blinder and Maccini 1991, Ramey and West 1999) and operations researchers (Holt et al. 1960, Klein 1961, Graves et al. 1986, Aviv 2007). Researchers have tested for smoothing by measuring if production is less variable than sales. But this test fails to identify the phenomenon since “production is more variable than sales in all major sectors and in most industries” (Blinder and Maccini 1991, p. 80). Indeed, production variability exceeds sales variability in 99% of our auto manufacturing sample—do auto manufacturers disregard production stability 99% of the time? Unlikely.

Muddling the traditional production smoothing test is another supply chain phenomenon: the bullwhip effect. Bullwhip is the amplification of demand fluctuations propagating up a supply chain (Lee et al. 1997). A constellation of factors underpin bullwhip, e.g., order batching, pipeline inventory discounting, and production cost shocks. These all increase the variance of production relative to sales, biasing the traditional production smoothing measure—eclipsing it entirely in most cases. Accordingly, we develop a new production smoothing metric, which estimates how much

more variable production would be if firms did not moderate it. By benchmarking production to production, rather than to demand, the measure eliminates bullwhip bias.

Measuring production smoothing in a sample of 162 car models produced by 20 auto manufacturers, we find sizable smoothing: auto production would be 22% more variable without deliberate production stabilization. And this smoothing exists amid a robust bullwhip effect: auto production is 220% as variable as auto sales. Coexistence of these phenomena upends our sense of bullwhip vis-à-vis smoothing. Until now the supply chain literature has framed bullwhip as counter to production smoothing, reporting the former when production variability exceeds sales variability and the latter when sales variability exceeds production variability (e.g., Lee et al. 1997, Cachon et al. 2007, Bray and Mendelson 2012). But the phenomena are operationally distinct and *not* mutually exclusive: smoothing pertains to production cost convexities compelling producers to stabilize production (Blinder and Maccini 1991), while bullwhip pertains to supply chain logistics amplifying demand shocks (Chen and Lee 2012). Our measurements led us astray: we considered the phenomena two sides of the same coin because they shared a common metric. Giving production smoothing its own yardstick frees us to embrace both.

To calculate production smoothing, we must anticipate how an auto manufacturer would produce if

it disregarded production stability. To conduct this counterfactual analysis, we use structural estimation (Lucas 1976, Reiss and Wolak 2007, Keane 2010); i.e., we (i) describe the data generating process with a production scheduling model, (ii) estimate the model's primitives with a sample of production schedules, and (iii) use these primitive estimates to simulate how the manufacturers would have produced in the absence of production volatility cost. Our structural model incorporates the *martingale model of forecast evolution* (MMFE) and the *generalized order-up-to policy* (GOUTP) (Hausman 1969; Graves et al. 1986, 1998; Chen and Lee 2009). A firm optimizes over the *impulse response functions* (IRFs) that govern its *demand signal processing* (DSP), its transformation of demand forecast revisions into production forecast revisions. We show that when a firm faces linear-quadratic costs, each set of model primitives has a unique DSP signature; we reverse engineer this DSP pattern to estimate the firm's marginal costs.

Our production scheduling model relates to those of Graves et al. (1986, 1998), Balakrishnan et al. (2004), Aviv (2007), and Chen and Lee (2009). We (i) broaden the objective function of Chen and Lee (2009), (ii) extend the production processes of Graves et al. (1986) and Balakrishnan et al. (2004), and (iii) refine the demand processes of Graves et al. (1998) and Aviv (2007). Each of these theoretical generalizations furthers our empirical agenda.

We estimate our model with auto manufacturing data. OM researchers have studied the auto industry for decades: Bresnahan and Ramey (1992) and Hall (2000) estimated production cost functions; Ramey and Vine (2006) and Copeland and Hall (2011) measured the effect of demand shocks on production schedules; Gopal et al. (2013) and Shah et al. (2013) related product launches and recalls to plant utilization levels; Cachon and Olivares (2010) and Cachon et al. (2012a) looked into dealership inventory levels; Guajardo et al. (2012) investigated warranties; and Moreno and Terwiesch (2011) studied production flexibility.

2. Model

Underpinning our empirical production scheduling model is Graves et al. (1998) and Chen and Lee's (2009) GOUTP, "an elegant model of...production and inventory planning" (Aviv 2007, p. 778). DSP, the translation of demand forecast revisions into production forecast revisions, drives the model dynamics (Lee et al. 1997).

2.1. Martingale Model of Forecast Evolution

We model the firm's information structure with the martingale model of forecast evolution (MMFE) (Hausman 1969, Graves et al. 1986, Heath and

Jackson 1994, Oh and Özer 2013). The MMFE describes the conditional expectations of a covariance-stationary, discrete stochastic process, $\{x_t\}_{-\infty}^{\infty}$. Let $\mathbb{E}_s(x_t)$ denote a forecaster's conditional expectation of x_t at time $s \leq t$. The forecaster observes x_t in period t , so $\mathbb{E}_t(x_t) = x_t$. And the forecaster has perfect memory, so its forecasts follow a martingale:

$$\mathbb{E}_r(x_t) = \mathbb{E}_r[\mathbb{E}_s(x_t) | \mathbb{E}_s(x_t)] = \mathbb{E}_r[\mathbb{E}_s(x_t)], \quad \text{for } r \leq s \leq t.$$

This is the MMFE's namesake property.

We suppose that the forecaster has an H -period-long forecast horizon, beyond which its forecasts are constant: $\mathbb{E}_t(x_{t+l}) = \mu$, for $l \geq H$. With this, we decompose x_t into a sum of forecast revisions:

$$\begin{aligned} x_t &= [\mathbb{E}_t(x_t) - \mathbb{E}_{t-1}(x_t)] + [\mathbb{E}_{t-1}(x_t) - \mathbb{E}_{t-2}(x_t)] + \dots \\ &\quad + [\mathbb{E}_{t-H+1}(x_t) - \mathbb{E}_{t-H}(x_t)] + \mathbb{E}_{t-H}(x_t) \\ &= \mu + \sum_{l=0}^{H-1} e'_l \epsilon_{t-l}, \end{aligned}$$

where e_l is a unit vector with a one in its $(l+1)$ th position and $\epsilon_t = \mathbb{E}_t[x_t, \dots, x_{t+H-1}]' - \mathbb{E}_{t-1}[x_t, \dots, x_{t+H-1}]'$ is a "signal vector" housing the forecast revisions the firm makes in period t . Henceforth, we use "signal" and "forecast revision" interchangeably and let e_l 's length be context specific.

The forecast's martingale property implies signal vectors ϵ_s and ϵ_t are uncorrelated, for $s < t$:

$$\begin{aligned} \mathbb{E}_s(\epsilon_s \epsilon'_t) &= \epsilon'_s \mathbb{E}_s[\mathbb{E}_t[x_t, \dots, x_{t+H-1}] - \mathbb{E}_{t-1}[x_t, \dots, x_{t+H-1}]] \\ &= \epsilon'_s [\mathbb{E}_s[x_t, \dots, x_{t+H-1}] - \mathbb{E}_s[x_t, \dots, x_{t+H-1}]] \\ &= 0. \end{aligned}$$

However, ϵ_t has general covariance matrix Σ , the trace of which denotes x_t 's variance:

$$\begin{aligned} \mathbb{T}(\Sigma) &= \sum_{l=0}^{H-1} e'_l \mathbb{E}(\epsilon_{t-l} \epsilon'_{t-l}) e_l = \sum_{l=0}^{H-1} \mathbb{V}(e'_l \epsilon_{t-l}) \\ &= \mathbb{V}\left(\sum_{l=0}^{H-1} e'_l \epsilon_{t-l}\right) = \mathbb{V}(x_t). \end{aligned}$$

2.2. Production Logistics

An auto manufacturer faces exogenous demand for a single car model. In each period, the firm starts producing a new batch of cars, taking ϕ months to finish each batch. The firm sources inputs quickly and backlogs demand when it stocks out of finished goods: the supply chain is decoupled (Kahn 1987, Gavirneni et al. 1999, Lee et al. 2000, Chen and Lee 2009). The firm has an H -period-long forecast horizon, rolling forecasts for the next H demands. Its retail price is fixed because "automakers only modestly respond with changes in price when faced with a demand

shock to a particular vehicle. Instead, demand shocks are almost entirely absorbed by changes in sales and production" (Copeland and Hall 2011, p. 233).

Two exogenous processes drive the firm's inventory dynamics:

$$d_t = \mu + \sum_{l=0}^{H-1} e'_l \epsilon_{t-l} \quad \text{and} \quad (1)$$

$$c_t = \mu^c + \sum_{l=0}^{H-1} e'_l \epsilon_{t-l}^c. \quad (2)$$

Equation (1) expresses the period t demand, d_t , as an MMFE with mean μ and signal vector $\epsilon_t = \mathbb{E}_t[d_t, \dots, d_{t+H-1}]' - \mathbb{E}_{t-1}[d_t, \dots, d_{t+H-1}]'$. Equation (2) expresses the period t marginal production cost, c_t , as an MMFE with mean μ^c and signal vector $\epsilon_t^c = \mathbb{E}_t[c_t, \dots, c_{t+H-1}]' - \mathbb{E}_{t-1}[c_t, \dots, c_{t+H-1}]'$. Cost shocks c_t reflect metal prices (Hall and Rust 2000), weather conditions (Cachon et al. 2012a, b), available work shifts (Copeland and Hall 2011), and labor relations (Bresnahan and Ramey 1992). Signal vectors ϵ_t and ϵ_t^c have general covariance matrices $\Sigma = \mathbb{E}(\epsilon_t \epsilon_t')$ and $\Sigma^c = \mathbb{E}(\epsilon_t^c \epsilon_t^{c'})$, but are uncorrelated with one another: $\mathbb{E}(\epsilon_t \epsilon_t^{c'}) = 0$.

In accordance with the GOUTP, we suppose production satisfies all demand within H periods and follows a linear time-invariant function of observed signals, ϵ_{t-l} and ϵ_{t-l}^c :

$$p_t = \mu + \sum_{l=0}^{H-1} e'_l \epsilon_{t-l}^p, \quad (3)$$

where

$$\epsilon_t^p = A \epsilon_t + A^c \epsilon_t^c, \quad (4)$$

$$\iota' A^c = 0, \quad \iota' A = \iota', \quad \text{and} \quad \iota = \sum_{l=0}^{H-1} e_l. \quad (5)$$

Equation (3) expresses the period t production start, p_t , as an MMFE with mean μ and signal vector $\epsilon_t^p = \mathbb{E}_t[p_t, \dots, p_{t+H-1}]' - \mathbb{E}_{t-1}[p_t, \dots, p_{t+H-1}]'$.

Equation (4) expresses these production signals as a linear combination of demand and cost signals. Specifically, the firm maps ϵ_t and ϵ_t^c into ϵ_t^p with $H \times H$ matrices A and A^c , respectively. Matrix A characterizes DSP: the m th element of A routes the $(n-1)$ -period-ahead demand forecast revision, $e'_n \epsilon_t$, into the $(m-1)$ -period-ahead production forecast revision, $e'_m \epsilon_t^p$. And matrix A^c characterizes cost signal processing (CSP): the m th element of A^c routes the $(n-1)$ -period-ahead cost forecast revisions, $e'_n \epsilon_t^c$, into the $(m-1)$ -period-ahead production forecast revisions, $e'_m \epsilon_t^p$. The columns of A and A^c house the IRFs that characterize the production policy: the n th column of A is the IRF that maps $(n-1)$ -period-information-lead-time demand signals into production quantities and the n th column of A^c is the IRF

that maps $(n-1)$ -period-information-lead-time cost signals into production quantities. Accordingly, we call A the DSP IRF matrix and A^c the CSP IRF matrix.

Equation (5) provides market-clearing constraints: The $\iota' A^c = 0$ constraint makes the CSP IRFs integrate to zero, which makes aggregate workloads insensitive to cost shocks, and the $\iota' A = \iota'$ constraint makes the DSP IRFs integrate to one, which makes production satisfy demand within $H + \phi - 1$ periods.

Since the firm fulfills all demand within $H + \phi - 1$ periods, its forecast of inventories $H + \phi$ periods hence remains constant: $\mathbb{E}_t(i_{t+H+\phi}) = \mu^i$ (Graves et al. 1998). With this, we express the firm's finished goods inventories in terms of the difference between its cumulative production starts, delayed ϕ periods, and its cumulative sales:

$$\begin{aligned} i_t &= \mathbb{E}_{t-H-\phi}(i_t) + [i_t - \mathbb{E}_{t-H-\phi}(i_t)] \\ &= \mu^i + \left[\sum_{l=0}^{H+\phi-1} (p_{t-\phi-l} - d_{t-l}) - \mathbb{E}_{t-H-\phi} \left(\sum_{l=0}^{H+\phi-1} (p_{t-\phi-l} - d_{t-l}) \right) \right] \\ &= \mu^i + \sum_{l=0}^{H+\phi-1} \sum_{i=0}^l e'_i (D_\phi \epsilon_{t-l}^p - I_\phi \epsilon_{t-l}) \\ &= \mu^i + \sum_{l=0}^{H+\phi-1} \left(\sum_{i=0}^l e'_i \right) (D_\phi \epsilon_{t-l}^p - I_\phi \epsilon_{t-l}) \\ &= \mu^i + \sum_{l=0}^{H+\phi-1} e'_l \epsilon_t^i, \end{aligned} \quad (6)$$

where $\epsilon_t^i = C_\phi (D_\phi \epsilon_{t-l}^p - I_\phi \epsilon_{t-l})$.

Equation (6) expresses the end-of-period- t finished goods inventory level, i_t , as an MMFE with mean μ^i and signal vector

$$\epsilon_t^i = \mathbb{E}_t[i_t, \dots, i_{t+H+\phi-1}]' - \mathbb{E}_{t-1}[i_t, \dots, i_{t+H+\phi-1}]'.$$

We define this inventory signal vector with three operators:

- D_x is a delay-by- x operator, an $(H+x) \times H$ matrix with ones in the x th subdiagonal and zeros elsewhere. For example, $D_2[v_1, v_2, \dots, v_H]' = [0, 0, v_1, v_2, \dots, v_H]'$. Multiplying the production signals by D_ϕ accounts for the ϕ period production lead-time delay.

- I_x is a lengthen-by- x operator, an $(H+x) \times H$ -dimensional version of the identity matrix. For example, $I_2[v_1, v_2, \dots, v_H]' = [v_1, v_2, \dots, v_H, 0, 0]'$. Multiplying the demand signals by I_ϕ gives them the same dimension as the delayed production signals.

- C_x is a cumulative sum operator, an $(H+x) \times (H+x)$ lower triangular matrix of ones. For example, $C_0[v_1, v_2, \dots, v_H]' = [v_1, v_1 + v_2, \dots, v_1 + \dots + v_H]'$.

Multiplying the production and demand signals by C_ϕ makes inventory depend on the cumulative sum of production and demand.

2.3. Objective Function

The firm faces linear-quadratic costs (Holt et al. 1960, Blinder and Maccini 1991). In period t , it incurs baseline production cost $C_t^1 = c_t p_t$, inventory overage-underage cost $C_t^2 = i_t^2$, production capacity overage-underage cost $C_t^3 = \alpha p_t^2$, and production input overage-underage cost $C_t^4 = \sum_{l=1}^h \beta_l (p_t - \mathbb{E}_{t-l}(p_t))^2$, for $h < H - 1$. For example, suppose producing a car requires (i) line capacity, which the firm builds well in advance, (ii) labor, which the firm schedules one month ahead, and (iii) raw materials, which the firm orders two months ahead. In this case, α parameterizes the line capacity cost, which is convex in production p_t , β_1 parameterizes the labor cost, which is convex in one-month forecast error $p_t - \mathbb{E}_{t-1}(p_t)$, and β_2 parameterizes the raw material cost, which is convex in two-month forecast error $p_t - \mathbb{E}_{t-2}(p_t)$.

The firm's expected operating cost is $\mathbb{E}(C_t^1 + C_t^2 + C_t^3 + C_t^4) = \mathbb{E}(c_t p_t) + \mathbb{V}(i_t) + \alpha \mathbb{V}(p_t) + \sum_{l=1}^h \beta_l \mathbb{V}(p_t - \mathbb{E}_{t-l}(p_t))$. The firm chooses the IRF matrices that minimize this expected cost, subject to the market-clearing constraints of (5):

$$\begin{aligned} \min_{A, A^c} \quad & \mathbb{E}(c_t p_t) + \mathbb{V}(i_t) + \alpha \mathbb{V}(p_t) \\ & + \sum_{l=1}^h \beta_l \mathbb{V}(p_t - \mathbb{E}_{t-l}(p_t)), \\ \text{s.t.} \quad & \iota' A = \iota' \quad \text{and} \quad \iota' A^c = 0. \end{aligned} \quad (7)$$

Relative to the marginal cost of inventory variability, parameter $\alpha \geq 0$ denotes the marginal cost of production variability and $\beta_l \geq 0$ denotes the marginal cost of “lead- l production uncertainty,” the mean square error of the l -period-ahead production forecast, $\mathbb{V}(p_t - \mathbb{E}_{t-l}(p_t))$. In other words, α parameterizes the firm's aversion to overall production volatility and β_l parameterizes its aversion to production volatility that resolves in the last l periods. Aviv (2007, p. 780) calls $\sum_{l=1}^h \beta_l \mathbb{V}(p_t - \mathbb{E}_{t-l}(p_t))$ in expression (7) an “adherence to production plans [metric], a measure commonly used in the industry” to measure production uncertainty.

2.4. Optimal Policy

The following proposition characterizes the optimal IRF matrices in terms of ϕ , α , and $\beta = [\beta_1, \dots, \beta_h]$.

PROPOSITION 1. *The firm sets $A = A(\alpha, \beta, \phi)$ and $A^c = A^c(\alpha, \beta, \phi)$, where*

$$\begin{aligned} A(\alpha, \beta, \phi) = & J + K(K'(D'_\phi C'_\phi C_\phi D_\phi + \alpha I + L_\beta)K)^{-1} \\ & \cdot (K'D'_\phi C'_\phi C_\phi (I_\phi - D_\phi J) - \alpha K'J), \end{aligned}$$

$$A^c(\alpha, \beta, \phi) = -K(K'(D'_\phi C'_\phi C_\phi D_\phi + \alpha I + L_\beta)K)^{-1}K',$$

$$J = e_{H-1}\iota',$$

$$K = (I - J)I'_{-1}, \quad \text{and}$$

$$L_\beta = \sum_{l=1}^h \sum_{i=0}^{l-1} \beta_l e_i e_i'.$$

Figure 1 depicts Proposition 1's optimal DSP IRF matrix under various parameter values. A plot's n th curve depicts the $n - 1$ th column of $A(\alpha, \beta, \phi)$, the IRF that determines how demand signals with n -period information lead times translate into production quantities:

- When $\phi = 0$ and α and β are small, A mirrors an identity matrix and production mirrors demand. In this case, the firm acts like a cross-dock facility with stable inventories.

- When α is large, the firm disperses mass throughout A 's columns, which spreads demand shocks across the production horizon. This is *signal pooling*: rather than an individual demand signal, each production quantity responds to a weighted sum of all demand signals. Signal pooling attenuates production variability.

- When β_l is large, the firm diverts mass away from A 's first l rows, which attenuates production schedule changes made with less than l periods notice. This is *signal delaying*: the firm postpones its response to demand shocks to stabilize short-run production schedules. Signal delaying attenuates production uncertainty.

The following section develops an algorithm to reverse engineer α , β , and ϕ from the measured degree of signal pooling and signal delaying.

3. Identification and Estimation

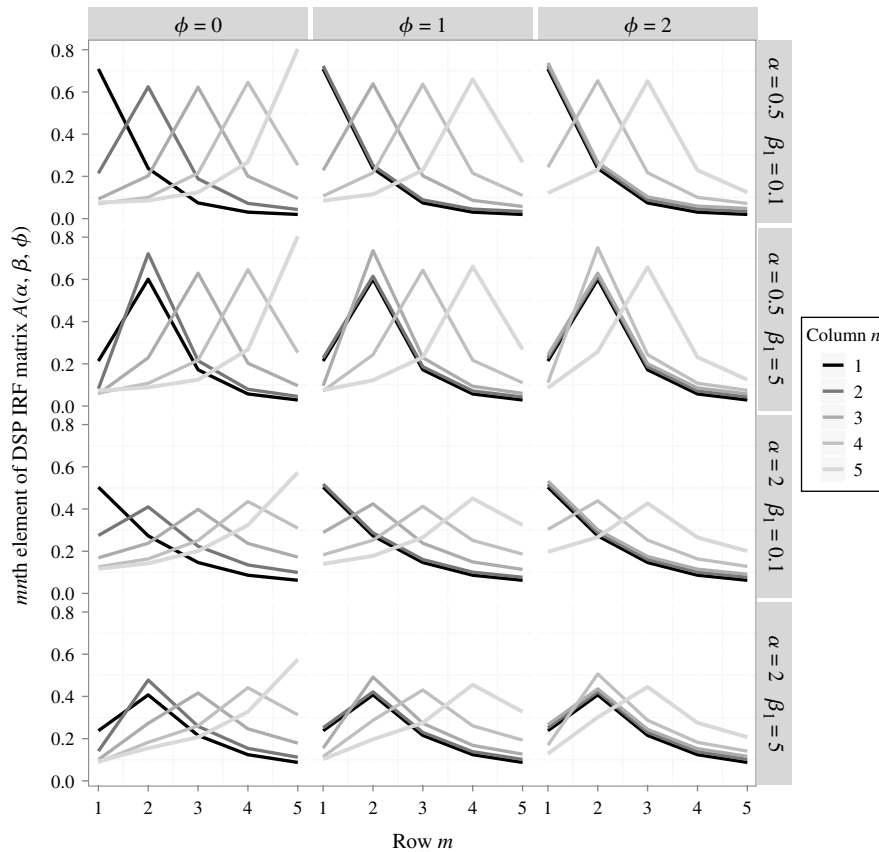
We now develop our theoretical model into an empirical model. We treat ϵ_t^p as a dependent variable, ϵ_t as an independent variable, and $e_t = A^c \epsilon_t^c$ as statistical error. Section 3.1 presents our basic identification conditions and estimators, and §3.2 refines these results.

3.1. Basic Specification

We present three propositions: The first defines an inverse mapping from optimal production policies to model primitives, the second uses this inverse mapping to establish a set of identifying moment conditions, and the third uses these identifying moment conditions to create generalized method of moment (GMM) estimators of our model primitives.

The following proposition establishes that $A(\alpha, \beta, \phi)$ is one-to-one—that each set of primitives has a unique DSP signature.

Figure 1 Optimal Demand Signal Processing Impulse Response Functions



PROPOSITION 2. When $H > \phi + 1$, there exists inverse mapping $A^{-1}(\cdot)$ that satisfies $A^{-1}(A(\alpha, \beta, \phi)) = \{\alpha, \beta, \phi\}$.

We characterize A^{-1} in the appendix. The function enables us to empirically identify a firm's primitives from its DSP IRFs. The identification argument is straightforward: production variability relative to inventory variability identifies α , lead- l production uncertainty relative to inventory variability identifies β_l , and the correlation between demand and lagged production identifies ϕ , since the firm tries to match production starts in period $t - \phi$ with demand in period t .

Proposition 2 enables us to derive α , β and ϕ from A ; Proposition 3, in turn, enables us to derive A from ϵ_t and ϵ_t^p .

PROPOSITION 3. Matrix A is empirically identified in a sample of demand signals ϵ_t , production signals ϵ_t^p , and instrumental variables ξ_t , if

$$\text{Rank}[E(\epsilon_t \xi_t')] = H \quad \text{and} \quad (8)$$

$$E(e_t \xi_t') = 0. \quad (9)$$

Instrumental variables ξ_t enable us to determine the causal effect of independent variables ϵ_t on dependent variables ϵ_t^p . Condition (8) is a classic instrumen-

tal variable inclusion restriction, ensuring the instruments have enough linearly independent variation to characterize the demand signals (Cameron and Trivedi 2005). And condition (9) is a classic instrumental variable exclusion restriction, ensuring the instruments do not correlate with the error terms.

With Proposition 3's moment conditions, we define estimators of α , β and ϕ in terms of matrices $E = [\epsilon_1, \dots, \epsilon_T]'$, $E^p = [\epsilon_1^p, \dots, \epsilon_T^p]'$, and $Z = [\xi_1, \dots, \xi_T]'$:

PROPOSITION 4. If (8) and (9) hold, then the following estimators are consistent:

$$\{\hat{\alpha}, \hat{\beta}, \hat{\phi}\} = A^{-1}(\hat{A}), \quad \text{where } \hat{A} = E^p Z Z' E (E' Z Z' E)^{-1}.$$

Note \hat{A} is a GMM estimator corresponding to (9)'s moment conditions (Cameron and Trivedi 2005).

3.2. Refined Specification

Proposition 4's estimators have two drawbacks: (i) its empirical requirements grow quickly with forecast horizon H , and (ii) it needlessly sacrifices H^2 degrees of freedom by pre-estimating A . We now refine our identification conditions and estimators to address these shortcomings.

Define $\underline{\epsilon}_t = I_{H-H} \epsilon_t$ as the first H elements of ϵ_t , $\underline{\epsilon}_t^p = I_{H-H} \epsilon_t^p$ as the first H elements of ϵ_t^p , $\underline{A}(\alpha, \beta, \phi) = I_{H-H} A(\alpha, \beta, \phi) I_{H-H}'$ as the top-left $H \times H$ submatrix of A , and $\underline{e}_t = \underline{\epsilon}_t^p - \underline{A}(\alpha, \beta, \phi) \underline{\epsilon}_t$ as a vector of statistical

errors, where \underline{H} satisfies $1 + \max(\phi, h) < \underline{H} \leq H$. The following proposition uses these variables to define identification conditions whose data requirements do not grow with H :

PROPOSITION 5. *Primitives α , β , and ϕ are empirically identified in a sample of truncated demand signals $\underline{\epsilon}_t$, truncated production signals $\underline{\epsilon}_t^p$, and instrumental variables ξ_t , if*

$$\text{Rank}[\mathbb{E}(\underline{\epsilon}_t \xi_t')] = \underline{H} \quad \text{and} \quad (10)$$

$$\mathbb{E}(\underline{\epsilon}_t \xi_t') = 0. \quad (11)$$

Conditions (10) and (11) are analogous to (8) and (9)'s inclusion and exclusion restrictions, but they only pertain to signals with information lead times shorter than \underline{H} . Thus, Proposition 5 establishes that the transformation of short-information-lead-timed demand signals into short-information-lead-timed production signals identifies our model primitives.

With Proposition 5's moment conditions, we define estimators of α , β and ϕ in terms of matrices $\underline{E} = [\underline{\epsilon}_1, \dots, \underline{\epsilon}_T]'$, $\underline{E}^p = [\underline{\epsilon}_1^p, \dots, \underline{\epsilon}_T^p]'$, and $Z = [\xi_1, \dots, \xi_T]'$:

PROPOSITION 6. *If (10) and (11) hold, then the following estimators are consistent:*

$$\{\hat{\alpha}, \hat{\beta}, \hat{\phi}\} = \arg \min_{\alpha, \beta, \phi} \text{vec}[(\underline{E}^{p'} - \underline{A}(\alpha, \beta, \phi) \underline{E}') Z'] \cdot W \text{vec}[(\underline{E}^{p'} - \underline{A}(\alpha, \beta, \phi) \underline{E}') Z],$$

where vec is matrix vectorization and W is a GMM weighting matrix.

4. Data

Hall (2000, p. 684) explains that

...most production decisions for automobile assembly plants are made at the monthly frequency. Once a month, there is a capacity planning meeting in which production schedules are set. At this meeting managers are presented with last month's sales and inventory numbers and a sales forecast. The managers must then set and revise their production schedule.

We estimate these monthly production schedules with monthly Wards Auto InfoBank data (WardsAuto Group 2014), which provide physical-unit sales and inventory levels of all cars produced in North America from 1985 to 2013.

First, we construct each car model's demand and production series. We use sales as a proxy for demand, and we calculate production by summing sales and the change in inventory. We begin a car model's time series when sales first exceed 1,000 cars and end it when sales last exceed 1,000 cars. We detrend each sales and production series, dividing them by their LOWESS regression fitted values

Table 1 Sample Overview

	Number of Distinct		
	Cars	Periods	Obs.
American			
Chrysler	12	339	2,906
Ford	23	339	5,521
GM	46	339	9,550
Total	81	339	17,977
Asian			
Honda	8	339	1,840
Hyundai	4	291	919
Isuzu	2	238	382
Kia Motors	2	152	299
Mazda	5	292	1,090
Mitsubishi	5	326	1,119
Nissan	9	339	2,128
Subaru	4	287	928
Toyota	19	339	4,139
Total	58	339	12,844
European			
Audi	2	210	419
BMW	4	339	1,168
Daimler	6	339	1,636
Jaguar	2	336	408
Porsche	1	305	305
Saab	2	308	308
VW	5	339	1,197
Volvo	1	145	145
Total	23	339	5,586
Total	162	339	36,407

(Cameron and Trivedi 2005). Our sample comprises the 162 time series that are at least 144 months long. Table 1 provides summary statistics.

Second, we estimate demand and production forecasts $\mathbb{E}_t(d_{t+l})$ and $\mathbb{E}_t(p_{t+l})$ for $l \leq \underline{H}$. We derive our forecast estimates from the following forecast variables:

- From Wards Auto InfoBank, we get monthly sales and inventory levels at the model, firm, and industry levels, and sales and inventory levels squared at the model level.
- From the website of the Office of Highway Policy Information (Federal Highway Administration 2013), we get monthly aggregate vehicle miles traveled in the United States.
- From the website of the Bureau of Labor Statistics (2013), we get the monthly producer price index of the primary metal manufacturing industry.
- From the website of the Conference Board (Consumer Confidence Survey 2014), we get the monthly consumer confidence index.
- From the website of the U.S. Energy Information Administration (2014), we get the monthly average New York Harbor conventional gasoline regular spot price.

We store the 12 forecast variables that resolve in period t in vector x_t . We suppose $\mathbb{E}_t(d_{t+l})$ and $\mathbb{E}_t(p_{t+l})$ are linear in variables $\{x_t, \dots, x_{t+l-\underline{H}}\}$ and seasonal

dummies s_t . Accordingly, we define forecast estimates $\hat{\epsilon}_t(d_{t+l})$ and $\hat{\epsilon}_t(p_{t+l})$ as the projections of d_{t+l} and p_{t+l} on $\{s_t, x_t, \dots, x_{t+l-H}\}$.

Third, we difference these forecast estimates to obtain our truncated MMFE signal estimates:

$$\begin{aligned}\hat{\epsilon}_t &= [\hat{\epsilon}_t(d_t), \dots, \hat{\epsilon}_t(d_{t+H-1})]' \\ &\quad - [\hat{\epsilon}_{t-1}(d_t), \dots, \hat{\epsilon}_{t-1}(d_{t+H-1})]' \quad \text{and} \\ \hat{\epsilon}_t^p &= [\hat{\epsilon}_t(p_t), \dots, \hat{\epsilon}_t(p_{t+H-1})]' \\ &\quad - [\hat{\epsilon}_{t-1}(p_t), \dots, \hat{\epsilon}_{t-1}(p_{t+H-1})]'.\end{aligned}$$

Fourth, we derive the instrumental variables. To satisfy (8), the instruments must correlate with ϵ_t 's demand forecast revisions. Since these forecast revisions won't correlate with anything forecastable prior to period t (because of the MMFE's martingale property), the natural choices of instruments are innovations in the demand forecast variables, $x_t - \mathbb{E}_{t-1}(x_t)$. To satisfy (9), however, we must exclude the forecast variables that may correlate with cost shocks c_t . Accordingly, we use instruments $\xi_t = z_t - \mathbb{E}_{t-1}(z_t)$, where $z_t \subset x_t$ are a subset of forecast variables that we assume are uncorrelated with production costs. We include in z_t the demand levels, the consumer confidence index, and the aggregate vehicle miles traveled; we exclude from z_t the inventory levels, which incorporate production quantities, the metal manufacturers' producer price index, which drives the cost of goods sold, and the price of gasoline, which correlates with power costs. On average, ξ_t explains 48% of $e'_4 \hat{\epsilon}_t$, 55% of $e'_3 \hat{\epsilon}_t$, 53% of $e'_2 \hat{\epsilon}_t$, 67% of $e'_1 \hat{\epsilon}_t$, and 100% of $e'_0 \hat{\epsilon}_t$ (since $e'_0 \hat{\epsilon}_t$ is an instrument).

Finally, we specify forecast horizons \underline{H} and H . Since z_t has six elements, (8) holds for $\underline{H} \leq 6$; we set $\underline{H} = 5$ for an extra degree of freedom. We set $H = 24$ (two years).

5. Parameter Estimates

We estimate each car model's parameters separately, with Proposition 6's estimators. Substituting estimates $\hat{\epsilon}_t$ and $\hat{\epsilon}_t^p$ for signals ϵ_t and ϵ_t^p makes our GMM standard errors inconsistent (see Newey 1984), so we compute standard errors with the block bootstrap, which is valid with sequential estimators (Berkowitz and Kilian 2000, Hardle et al. 2003).

5.1. Cost Parameters

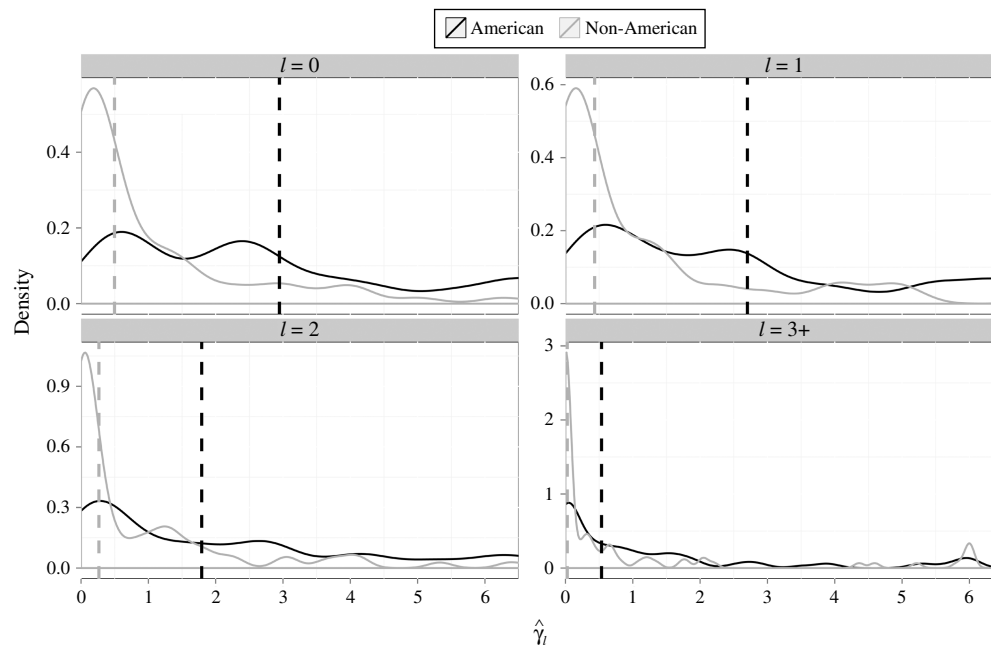
Table 2 presents $\hat{\alpha}$ and $\hat{\beta}_l$, which respectively estimate the production variability and lead- l production uncertainty marginal costs, relative to the inventory variability marginal cost. We find aversions to both production variability and uncertainty, with significantly positive mean and median $\hat{\alpha}$ and $\hat{\beta}$. Overall, 95% of our sample exhibits some aversion to production instability, with positive $\hat{\alpha}$, $\hat{\beta}_1$, $\hat{\beta}_2$, or $\hat{\beta}_3$.

Table 2 Cost Parameter Estimates

	Mean				Median			
	$\hat{\alpha}$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\alpha}$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$
American								
Chrysler	0.45 (0.29)	0.63 (0.37)	1.10 (0.42)	0.71 (0.35)	0.00 (0.21)	0.00 (0.04)	0.50 (0.31)	0.24 (0.20)
Ford	0.95 (0.29)	0.11 (0.26)	1.15 (0.28)	1.03 (0.26)	0.41 (0.13)	0.00 (0.06)	0.36 (0.15)	0.74 (0.26)
GM	1.66 (0.20)	0.47 (0.26)	1.71 (0.27)	2.10 (0.20)	0.88 (0.19)	0.00 (0.11)	0.99 (0.39)	0.85 (0.21)
Total	1.28 (0.13)	0.39 (0.16)	1.46 (0.19)	1.59 (0.15)	0.54 (0.09)	0.00 (0.02)	0.62 (0.23)	0.68 (0.16)
Asian								
Honda	2.55 (0.69)	0.00 (0.46)	1.33 (0.74)	1.62 (0.53)	1.16 (0.63)	0.00 (0.13)	0.42 (0.50)	0.71 (0.48)
Hyundai	0.27 (0.22)	0.04 (0.27)	0.28 (0.33)	0.16 (0.28)	0.13 (0.11)	0.00 (0.01)	0.25 (0.14)	0.00 (0.08)
Isuzu	1.02 (0.79)	0.00 (0.71)	0.33 (0.81)	0.13 (0.55)	1.02 (0.79)	0.00 (0.71)	0.33 (0.81)	0.13 (0.55)
Kia Motors	0.00 (0.04)	0.00 (0.01)	0.00 (0.11)	0.00 (0.05)	0.00 (0.04)	0.00 (0.01)	0.00 (0.11)	0.00 (0.05)
Mazda	1.74 (0.53)	0.00 (0.09)	0.13 (0.24)	0.53 (0.28)	0.62 (0.15)	0.00 (0.07)	0.04 (0.19)	0.82 (0.20)
Mitsubishi	1.24 (0.37)	0.00 (0.30)	0.12 (0.29)	0.72 (0.32)	0.01 (0.00)	0.00 (0.01)	0.02 (0.03)	0.04 (0.03)
Nissan	1.57 (0.55)	0.85 (0.61)	2.09 (0.52)	1.50 (0.58)	0.66 (0.60)	0.18 (0.48)	1.54 (0.67)	0.97 (0.64)
Subaru	0.73 (0.47)	0.00 (0.37)	1.22 (0.69)	0.69 (0.45)	0.36 (0.27)	0.00 (0.00)	0.76 (0.28)	0.58 (0.31)
Toyota	0.46 (0.22)	0.39 (0.24)	0.64 (0.30)	0.53 (0.25)	0.06 (0.07)	0.02 (0.05)	0.13 (0.14)	0.06 (0.07)
Total	1.11 (0.16)	0.26 (0.18)	0.85 (0.19)	0.80 (0.15)	0.22 (0.04)	0.00 (0.00)	0.14 (0.06)	0.20 (0.05)
European								
Audi	0.00 (0.47)	0.11 (0.99)	0.00 (0.64)	0.55 (0.66)	0.00 (0.47)	0.11 (0.99)	0.00 (0.64)	0.55 (0.66)
BMW	0.00 (0.08)	0.00 (0.01)	0.15 (0.09)	0.00 (0.19)	0.00 (0.01)	0.00 (0.00)	0.13 (0.08)	0.00 (0.04)
Daimler	0.05 (0.15)	0.02 (0.20)	0.00 (0.03)	0.28 (0.16)	0.00 (0.02)	0.00 (0.01)	0.00 (0.00)	0.01 (0.04)
Jaguar	0.01 (0.09)	0.00 (0.62)	0.27 (0.76)	0.17 (0.21)	0.01 (0.09)	0.00 (0.62)	0.27 (0.76)	0.17 (0.21)
Porsche	0.00 (0.35)	0.00 (0.12)	0.06 (0.06)	0.00 (0.02)	0.00 (0.35)	0.00 (0.12)	0.06 (0.06)	0.00 (0.02)
Saab	3.00 (1.14)	0.00 (1.63)	0.31 (1.33)	2.71 (1.09)	3.00 (1.14)	0.00 (1.63)	0.31 (1.33)	2.71 (1.09)
VW	0.50 (0.57)	1.34 (0.72)	0.98 (0.70)	0.87 (0.50)	0.29 (0.42)	0.20 (0.49)	0.00 (0.63)	0.10 (0.48)
Volvo	0.02 (0.14)	0.00 (1.10)	0.29 (1.96)	0.07 (1.52)	0.02 (0.14)	0.00 (1.10)	0.29 (1.96)	0.07 (1.52)
Total	0.38 (0.16)	0.31 (0.21)	0.30 (0.25)	0.56 (0.17)	0.00 (0.01)	0.00 (0.00)	0.00 (0.04)	0.01 (0.03)
Total	1.09 (0.09)	0.33 (0.12)	1.08 (0.13)	1.16 (0.10)	0.28 (0.05)	0.00 (0.00)	0.27 (0.08)	0.37 (0.07)

Figure 2 depicts the medians (with vertical dashed lines) and probability density functions (PDFs) of $\hat{\gamma}_l = \hat{\alpha} + \sum_{i=l+1}^H \hat{\beta}_i$, our estimates of the marginal cost of the variance of production signals with l -period information lead times. First, the American firms are particularly averse to production schedule revisions—the median American $\hat{\gamma}_l$ is more than five times the median non-American $\hat{\gamma}_l$, for $l = 0, 1, 2$, and $3+$. This finding points to Detroit's sluggishness. Second,

Figure 2 Production Volatility Cost Estimate Medians and PDFs by Information Lead Time



surprising production fluctuations cost more than predictable ones: the mean $\hat{\gamma}_1$ is statistically larger than the mean $\hat{\gamma}_2$, which is statistically larger than the mean $\hat{\gamma}_3$. For example, the median American manufacturer deems last-minute production schedule changes to be three times as costly as inventory fluctuations, but deems three-month-out production schedule changes to be only half as costly as inventory fluctuations.

5.2. Lead Times

Table 3 tabulates the discrete PDFs of our production lead-time estimates, $\hat{\phi}$ (we cap $\hat{\phi}$ to two months since producing a car should take less than 60 days). Most $\hat{\phi}$ are zero months, which supports the Lieberman et al. (1995, p. 9) finding that production lead times only “sometimes exceed one month.” At 0.52 months, the average Asian lead time is statistically smaller than the average non-Asian lead time, at 0.80 months. This finding highlights the Asian firms’ proclivity for JIT manufacturing. Indeed, of the 13 companies that

have at least three car models in our sample, Toyota—the inventor of the Toyota Production System—has the shortest production lead time, at 0.11 months; Toyota produces statistically faster than the average firm.

5.3. DSP Matrices

We estimate matrices \hat{A} , $\hat{\Sigma} = \mathbb{E}(\epsilon_t \epsilon_t')$, and $\hat{\Sigma}^e = \mathbb{E}(\epsilon_t^e \epsilon_t^{e'})$ with $\hat{A} = \hat{A}(\hat{\alpha}, \hat{\beta}, \hat{\phi})$, $\hat{\Sigma} = T^{-1}E'E$, and $\hat{\Sigma}^e = T^{-1}(E^{p'} - \hat{A}E')(E^{p'} - \hat{A}E')'$. Figure 3 plots these estimates, normalizing $\mathbb{T}(\hat{\Sigma})$ to one. The \hat{A} estimates demonstrate substantial production policy heterogeneity: our data take advantage of our model’s flexibility. The mostly positive $\hat{\alpha}$ give \hat{A} their dispersion; the mostly positive $\hat{\beta}$ give \hat{A} their asymmetry; and the mostly zero $\hat{\phi}$ give \hat{A} their diagonal ridge. The $\hat{\Sigma}$ estimates confirm that demand signals are positively correlated and more volatile with shorter information lead times. The large $\hat{\Sigma}^e$ estimates indicate that the error terms are influential, which limits our R^2 values: on average, $e_t' \hat{A} \hat{\epsilon}_t$ account for 38%, 40%, 41%, 36%, and 36% of $e_t' \hat{\epsilon}_t^p$, for $l = 0, 1, 2, 3$, and 4.

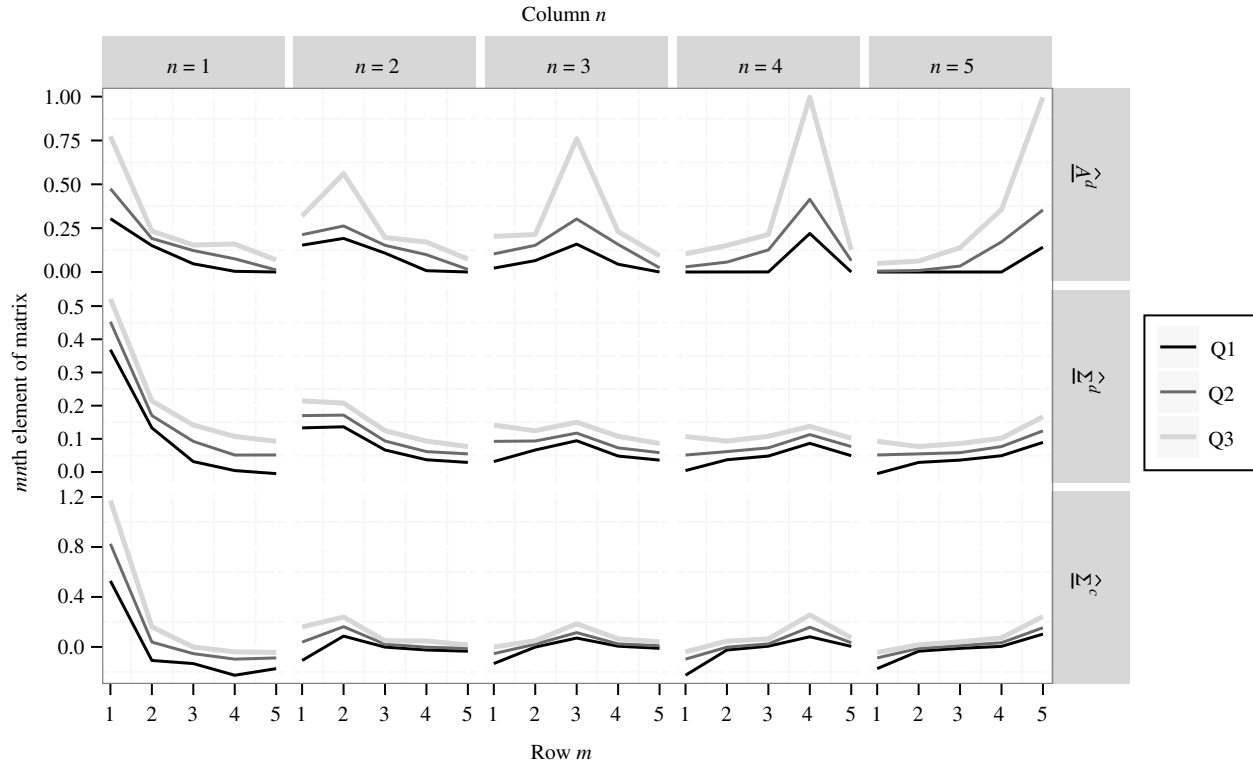
Table 3 Lead-Time Estimate PDFs

	$\hat{\phi} = 0$	$\hat{\phi} = 1$	$\hat{\phi} = 2$
American	0.49 (0.05)	0.30 (0.04)	0.21 (0.04)
Asian	0.64 (0.06)	0.21 (0.05)	0.16 (0.04)
European	0.57 (0.08)	0.17 (0.08)	0.26 (0.08)
Total	0.56 (0.04)	0.25 (0.03)	0.20 (0.03)

6. Application

We now use our model to disentangle production smoothing from the bullwhip effect. Production smoothing is production stabilization intended to reduce production volatility costs. Microeconomists predicted “[i]f marginal production costs are increasing

Figure 3 Quantiles of the DSP and CSP Matrix Estimates



and sales vary over time, a cost-minimizing strategy that equates marginal costs across time periods will smooth production relative to sales” (Blinder and Maccini 1991, p. 78). The bullwhip effect, on the other hand, is the tendency for demand shocks to amplify, like the crack of a whip, as they wend their way up a supply chain (Lee et al. 1997). Although distinct, these phenomena have shared a common measure—the ratio of production variability to sales variability (Cachon et al. 2007, Bray and Mendelson 2012, Shan et al. 2013). Mapping these two concepts to a single measure has forced them to be antithetical: either a firm exhibited the bullwhip effect, with production more volatile than sales, or production smoothing, with sales more volatile than production. We have therefore implicitly defined production smoothing to be the anti-bullwhip. Empirically, this framing has enabled the bullwhip effect to steamroll production smoothing; e.g., production is more variable than demand in 160 of the 162 car models in our sample.

We reconcile the false dichotomy between production smoothing and the bullwhip effect with a new production smoothing measure. Rather than benchmark production variability to sales variability—an apples-to-oranges comparison, because of the bullwhip—we benchmark it to the production variability in the hypothetical scenario in which firms have no incentive to smooth. That is, we measure a firm’s smoothing with the ratio of what its production variability actually is to what it would be if

it were indifferent to production stability. We simulate the stability-indifferent counterfactual by setting α and β to zero, since these parameters compel the firm to stabilize production:

$$\widehat{\text{PS}} = \frac{\hat{V}_{\alpha=\hat{\alpha}, \beta=\hat{\beta}}(p_t)}{\hat{V}_{\alpha=0, \beta=0}(p_t)} = \frac{\mathbb{T}[A(\hat{\alpha}, \hat{\beta}, \hat{\phi})\hat{\Sigma}A(\hat{\alpha}, \hat{\beta}, \hat{\phi})' + \hat{\Sigma}^e]}{\mathbb{T}[A(0, 0, \hat{\phi})\hat{\Sigma}A(0, 0, \hat{\phi})' + \hat{\Sigma}^e]}. \quad (12)$$

Equation (12)’s smoothing estimate relies on two simplifications. First, it supposes the e_t statistical errors don’t change with α or β . Second, it pertains to $\underline{p}_t = p_t - \mathbb{E}_{t-\underline{H}}(p_t)$ rather than p_t , because $\hat{\Sigma}_t^p$ doesn’t capture the production fluctuations firms can anticipate $\underline{H} = 5$ months early. (Removing $\mathbb{E}_{t-\underline{H}}(p_t)$ sacrifices about a tenth of p_t ’s variation.)

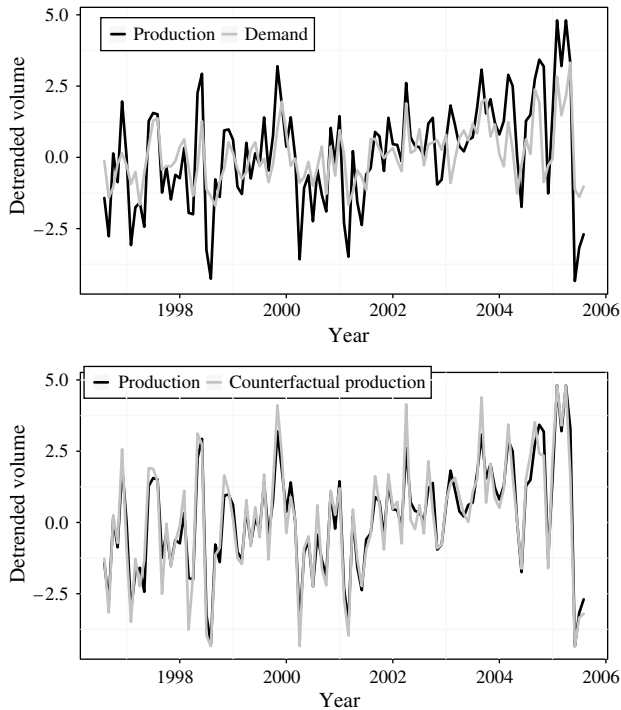
We measure the bullwhip effect with the traditional variance amplification ratio:

$$\widehat{\text{BW}} = \hat{V}(\underline{p}_t)/\hat{V}(\underline{d}_t) = \frac{\mathbb{T}[A(\hat{\alpha}, \hat{\beta}, \hat{\phi})\hat{\Sigma}A(\hat{\alpha}, \hat{\beta}, \hat{\phi})' + \hat{\Sigma}^e]}{\mathbb{T}[\hat{\Sigma}]}$$

Note, to match our smoothing metric, we measure the bullwhip in terms of $\underline{p}_t = p_t - \mathbb{E}_{t-\underline{H}}(p_t)$ and $\underline{d}_t = d_t - \mathbb{E}_{t-\underline{H}}(d_t)$, rather than p_t and d_t (the results are similar either way).

With our new smoothing measure, we no longer must pit the bullwhip effect against production

Figure 4 Sales, Production, and Stability-Indifferent Counterfactual Production of the Dodge Neon



smoothing. For example, the Dodge Neon exhibits both the bullwhip effect and production smoothing in Figure 4. The figure plots the Neon's demand, production, and counterfactual production in the $\alpha = \beta = 0$ scenario. (We estimate the counterfactual production with $\sum_{i=0}^{H-1} e'_i \underline{A}(0, 0, \hat{\phi}) \hat{\varepsilon}_t + \hat{\varepsilon}_t$, where $\hat{\varepsilon}_t = \hat{\varepsilon}_t^p - \underline{A}(\hat{\alpha}, \hat{\beta}, \hat{\phi}) \hat{\varepsilon}_t$.) The figure's top panel depicts the bullwhip effect, with production 290% more variable than demand. The bottom panel depicts production smoothing, with production 14% less variable than it would be in the stability-indifferent counterfactual scenario.

Extending these results to the rest of our sample, Figure 5 plots the joint and marginal distributions of our bullwhip and smoothing estimates. We find both phenomena: 99% of our bullwhip estimates exceed

one (on average, production is 240% as variable as demand), and 95% of our production smoothing estimates fall short of one (on average, production is only 81% as variable as it would be without explicit stabilization). The median American manufacturer smooths significantly more than the median non-American manufacturer and thus has a significantly smaller bullwhip—intuitively, Americans are better suited to smoothing, since being less lean gives them more stabilizing inventories.

So far, we have only considered production variability, but firms also attenuate production uncertainty. We measure the smoothing of lead- l production uncertainty by comparing its values in the current and stability-indifferent scenarios:

$$\begin{aligned} \widehat{PS}_l &= \hat{V}_{\alpha=0, \beta=0}(p_t - E_{t-l}(p_t)) / \hat{V}_{\alpha=\hat{\alpha}, \beta=\hat{\beta}}(p_t - E_{t-l}(p_t)) \\ &= \frac{\sum_{i=0}^{l-1} e'_i [\underline{A}(0, 0, \hat{\phi}) \hat{\Sigma} \underline{A}(0, 0, \hat{\phi})' + \hat{\Sigma}^e] e_i}{\sum_{i=0}^{l-1} e'_i [\underline{A}(\hat{\alpha}, \hat{\beta}, \hat{\phi}) \hat{\Sigma} \underline{A}(\hat{\alpha}, \hat{\beta}, \hat{\phi})' + \hat{\Sigma}^e] e_i}. \end{aligned}$$

We likewise define the lead- l bullwhip effect as the amplification of lead- l uncertainty, from demand to production (Bray and Mendelson 2012):

$$\begin{aligned} \widehat{BW}_l &= \hat{V}(p_t - E_{t-l}(p_t)) / \hat{V}(d_t - E_{t-l}(d_t)) \\ &= \frac{\sum_{i=0}^{l-1} e'_i [\underline{A}(\hat{\alpha}, \hat{\beta}, \hat{\phi}) \hat{\Sigma} \underline{A}(\hat{\alpha}, \hat{\beta}, \hat{\phi})' + \hat{\Sigma}^e] e_i}{\sum_{i=0}^{l-1} e'_i \hat{\Sigma}^e e_i}. \end{aligned}$$

Table 4 tabulates \widehat{BW}_l and \widehat{PS}_l . The \widehat{BW}_l estimates, significantly greater than one, indicate that uncertainty increases from demand to production, and the \widehat{PS}_l estimates, significantly less than one, indicate that uncertainty increases from actual production to stability-indifferent counterfactual production. The median lead-1 bullwhip is significantly larger than the median lead-5 bullwhip, which confirms the finding of Bray and Mendelson (2012) that firms amplify last-minute surprise more than predictable fluctuations: the bullwhip thrives in a time crunch.

Figure 5 Production Smoothing and Bullwhip Effect Estimates

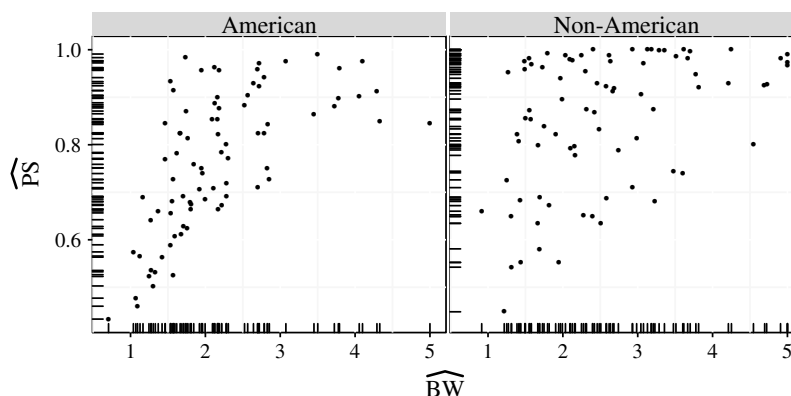


Table 4 Quartiles of the Lead-/ Production Smoothing and Bullwhip Effect Estimates

	$l = 1$	$l = 2$	$l = 3$	$l = 4$	$l = 5$
\widehat{BW}_l					
Q1	1.74 (0.06)	1.63 (0.06)	1.61 (0.05)	1.65 (0.06)	1.63 (0.06)
Q2	2.51 (0.10)	2.18 (0.10)	2.12 (0.09)	2.18 (0.09)	2.16 (0.06)
Q3	3.42 (0.17)	3.05 (0.13)	2.98 (0.11)	3.05 (0.12)	2.85 (0.12)
\widehat{PS}_l					
Q1	0.56 (0.03)	0.59 (0.03)	0.62 (0.02)	0.67 (0.02)	0.69 (0.02)
Q2	0.75 (0.02)	0.76 (0.02)	0.79 (0.02)	0.84 (0.01)	0.84 (0.01)
Q3	0.88 (0.02)	0.90 (0.01)	0.92 (0.01)	0.94 (0.01)	0.95 (0.01)

Nevertheless, firms attenuate surprising production fluctuations more aggressively; e.g., the median firm smooths lead-1 uncertainty 56% more than lead-5 uncertainty. Since unexpected production schedule changes cost more, firms attenuate them more. The firms smooth lead-1 uncertainty with signal delaying, postponing their response to surprising demand shocks to freeze short-term production plans, and they smooth lead-5 uncertainty with signal pooling, averaging out demand fluctuations across the production horizon (see §2.4).

7. Conclusion

In this paper, we do three things. First, we create a new production smoothing measure: rather than benchmark production variability to sales variability—a comparison bullwhip corrupts—we benchmark production variability to what it would be without production stabilization. Whereas the traditional measure suggests 1% of our sample smooths production, our new measure suggests 75% smooths production by at least 5%. Further, we find production smoothing and the bullwhip effect can coexist—indeed, they do so in the majority of our sample. Thus, the bullwhip effect is more than merely the opposite of production smoothing: it has distinct operational underpinnings and ramifications.

Second, we advance the notion of smoothing production uncertainty. Not all production fluctuations are equal; some are more surprising and hence more costly. We measure production uncertainty with the Aviv (2007, p. 780) “adherence to production plans” metric, which weighs production fluctuations by their information lead time. We find that firms smooth production uncertainty, attenuating surprising production fluctuations (those they anticipate with less than two months’ notice) significantly more than predictable production fluctuations (those they anticipate with more than four months’ notice).

Third, we develop a new empirical approach: gleaning operational insights from demand signal

processing. We show that the transformation of demand into production empirically identifies manufacturing costs. Our model-cum-estimators give empiricists a flexible means to invoke inventory theory: managers could use our framework to conduct operational counterfactuals, such as estimating the value of shortening production lead times or improving demand forecasts. A large Midwest manufacturer plans to use our empirical DSP model to determine whether their idle upstream capacity stems from overly optimistic demand forecasts.

Acknowledgments

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Appendix. Proposition Proofs

PROOF OF PROPOSITION 1. First, we show that $H \times H$ matrix X satisfies $\iota'X = 0$ if and only if $X = K\underline{X}$, for some $(H-1) \times H$ matrix \underline{X} . Each column of K sums to zero, which means $\iota'K = 0$, which means $X = K\underline{X}$ implies $\iota'X = \iota'K\underline{X} = 0$. Matrix J takes the form $[0, \iota']$, which means $\iota'X = 0$ implies $JX = 0$. Thus if $\iota'X = 0$, then let $\underline{X} = I_{-1}X$, and $K\underline{X} = KI_{-1}X = (I - J)X = X$.

Second, we show that $H \times H$ matrix X satisfies $\iota'X = \iota'$ if and only if $X = J + K\underline{X}$, for some $(H-1) \times H$ matrix \underline{X} . Each column of J sums to one, which means $\iota'J = \iota'$. Each column of $I - J$ sums to zero, which means $\iota'(I - J) = 0$. Thus if $X = J + K\underline{X}$, then $\iota'X = \iota'(J + K\underline{X}) = \iota'J + \iota'K\underline{X} = \iota' + 0\underline{X} = \iota'$. Matrix J takes the form $[0, \iota']$, which means $\iota'X = \iota'$ implies $JX = J$. Thus, if $\iota'X = \iota'$, then let $\underline{X} = I_{-1}X$, and $J + K\underline{X} = J + ((I - J)I_{-1})I_{-1}X = J + (I - J)X = JX + (I - J)X = X$.

Third, we use these results to remove the market clearing constraints from the firm’s objective. Doing so yields

$$\begin{aligned}
& \min_{\underline{A}, \underline{A}^c} \underbrace{\mathbb{T}(K\underline{A}^c \Sigma^c)}_{\mathbb{E}(c_t p_t)} \\
& + \underbrace{\mathbb{T}(C_\phi(D_\phi(J + K\underline{A}) - I_\phi)\Sigma((J' + \underline{A}'K')D'_\phi - I'_\phi)C'_\phi + C_\phi D_\phi K\underline{A}^c \Sigma^c \underline{A}^{c'} K' D'_\phi C'_\phi)}_{\mathbb{V}(i_t)} \\
& + \underbrace{\mathbb{T}(\alpha(J + K\underline{A})\Sigma(J' + \underline{A}'K') + \alpha K\underline{A}^c \Sigma^c \underline{A}^{c'} K')}_{\alpha \mathbb{V}(p_t)} \\
& + \underbrace{\mathbb{T}(L_\beta(J + K\underline{A})\Sigma(J' + \underline{A}'K') + L_\beta K\underline{A}^c \Sigma^c \underline{A}^{c'} K')}_{\sum_{l=1}^h \beta_l \mathbb{V}(p_t - \mathbb{E}_{t-l}(p_t))}.
\end{aligned}$$

Fourth, we differentiate this objective with respect to \underline{A} . We do so with two matrix calculus identities: $\partial \mathbb{T}(AXB)/\partial X = A'B'$ and $\partial \mathbb{T}(AXBX'A')/\partial X = A'AX(B + B')$ (Petersen and Pedersen 2008):

$$\begin{aligned}
& \frac{\partial}{\partial \underline{A}} [\mathbb{T}(K\underline{A}^c \Sigma^c) + \mathbb{T}(C_\phi(D_\phi(J + K\underline{A}) - I_\phi)\Sigma((J' + \underline{A}'K')D'_\phi - I'_\phi)C'_\phi \\
& + C_\phi D_\phi K\underline{A}^c \Sigma^c \underline{A}^{c'} K' D'_\phi C'_\phi)
\end{aligned}$$

$$\begin{aligned}
 & + \mathbb{T}(\alpha(J + KA)\Sigma(J' + A'K') + \alpha KA^c \Sigma^c A'^c K') \\
 & + \mathbb{T}(L_\beta(J + KA)\Sigma(J' + A'K') + L_\beta KA^c \Sigma^c A'^c K')) = 0 \\
 \Rightarrow & \frac{\partial}{\partial A} [\mathbb{T}(C_\phi D_\phi KA \Sigma A' K' D_\phi' C_\phi' + 2C_\phi D_\phi KA \Sigma(J' D_\phi' - I_\phi) C_\phi') \\
 & + \mathbb{T}(\alpha KA \Sigma A' K' + 2\alpha KA \Sigma J') \\
 & + \mathbb{T}(L_\beta^{1/2} KA \Sigma A' K' L_\beta^{1/2'} + 2L_\beta^{1/2} KA \Sigma J' L_\beta^{1/2'})] = 0 \\
 \Rightarrow & K' D_\phi' C_\phi' C_\phi D_\phi KA \Sigma + K' D_\phi' C_\phi' C_\phi (D_\phi J - I_\phi) \Sigma \\
 & + \alpha K' K A \Sigma + \alpha K' J \Sigma + K' L_\beta KA \Sigma + K' L_\beta J \Sigma = 0 \\
 \Rightarrow & A = (K' (D_\phi' C_\phi' C_\phi D_\phi + \alpha I + L_\beta) K)^{-1} \\
 & \cdot (K' D_\phi' C_\phi' C_\phi (I_\phi - D_\phi J) - \alpha K' J - K' L_\beta J) \\
 \Rightarrow & A = (K' (D_\phi' C_\phi' C_\phi D_\phi + \alpha I + L_\beta) K)^{-1} \\
 & \cdot (K' D_\phi' C_\phi' C_\phi (I_\phi - D_\phi J) - \alpha K' J) \\
 \Rightarrow & A = J + K (K' (D_\phi' C_\phi' C_\phi D_\phi + \alpha I + L_\beta) K)^{-1} \\
 & \cdot (K' D_\phi' C_\phi' C_\phi (I_\phi - D_\phi J) - \alpha K' J).
 \end{aligned}$$

Differentiating the firm's objective with respect to A^c likewise yields A^c . \square

PROOF OF PROPOSITION 2. Let $A^{-1}(A) = \{\alpha(A), \beta(A), \phi(A)\}$. First we derive $\phi(A)$ with three identities:

1. $K' D_\phi' C_\phi' C_\phi D_\phi K + K' L_\beta K$ is symmetric and positive definite.
2. $(D_\phi' C_\phi' C_\phi (I_\phi - D_\phi J) - L_\beta J) e_{\phi+1} = (D_\phi' C_\phi' C_\phi (I_\phi - D_\phi J) - L_\beta J) e_\phi - e_0$.
3. $(D_\phi' C_\phi' C_\phi (I_\phi - D_\phi J) - L_\beta J) e_l = (D_\phi' C_\phi' C_\phi (I_\phi - D_\phi J) - L_\beta J) e_{l+1}$, for $l < \phi$.

The first identity implies $K(K' D_\phi' C_\phi' C_\phi D_\phi K + K' L_\beta K)^{-1} K'$ is symmetric and positive definite, which implies $e_0' K(K' D_\phi' C_\phi' C_\phi D_\phi K + K' L_\beta K)^{-1} K' e_0$ is nonzero. The second identity yields

$$\begin{aligned}
 e_0' A e_{\phi+1} &= e_0' [J + K(K' D_\phi' C_\phi' C_\phi D_\phi K + K' L_\beta K)^{-1} \\
 & \cdot K' (D_\phi' C_\phi' C_\phi (I_\phi - D_\phi J) - L_\beta J)] e_{\phi+1} \\
 &= e_0' K(K' D_\phi' C_\phi' C_\phi D_\phi K + K' L_\beta K)^{-1} \\
 & \cdot K' (D_\phi' C_\phi' C_\phi (I_\phi - D_\phi J) - L_\beta J) e_{\phi+1} \\
 &= e_0' K(K' D_\phi' C_\phi' C_\phi D_\phi K + K' L_\beta K)^{-1} \\
 & \cdot K' [(D_\phi' C_\phi' C_\phi (I_\phi - D_\phi J) - L_\beta J) e_\phi - e_0] \\
 &\neq e_0' K(K' D_\phi' C_\phi' C_\phi D_\phi K + K' L_\beta K)^{-1} \\
 & \cdot K' (D_\phi' C_\phi' C_\phi (I_\phi - D_\phi J) - L_\beta J) e_\phi \\
 &= e_0' [J + K(K' D_\phi' C_\phi' C_\phi D_\phi K + K' L_\beta K)^{-1} \\
 & \cdot K' (D_\phi' C_\phi' C_\phi (I_\phi - D_\phi J) - L_\beta J)] e_\phi \\
 &= e_0' A e_\phi.
 \end{aligned}$$

The third identity implies that $e_0' A e_l = e_0' A e_{l+1}$, for $l < \phi$. Putting this together, we get $\phi(A) = \min(\{l: e_0' A e_l \neq e_0' A e_{l+1}\})$.

Next we define $\alpha(A)$ and $\beta(A)$ implicitly. Let $w_l = e_l' A e_0$. Scalars w_l do not depend on Σ , Σ^c , or ϕ , so we can set $\Sigma = I$, $\Sigma^c = 0$, and $\phi = 0$. At the optimum, the benefit of shifting a marginal unit of mass from w_{l-1} to w_l is zero, a

fact that yields the following difference equations (Graves et al. 1998):

$$\left(\alpha + \sum_{i=l+1}^h \beta_i \right) (w_l - w_{l-1}) - \beta_l w_{l-1} + 1 - \sum_{i=0}^{l-1} w_i = 0,$$

$$l \in \{1, \dots, h\},$$

$$\alpha(w_{h+1} - w_h) + 1 - \sum_{i=0}^h w_i = 0.$$

These $h+1$ linear equations implicitly define $\alpha(A)$ and $\beta(A)$. \square

PROOFS OF PROPOSITIONS 3–6. First, estimator \hat{A} is consistent (note, condition (8) enables us to invert $\mathbb{E}(\epsilon_t \xi_t') \mathbb{E}(\xi_t \epsilon_t')$ and condition (9) enables us to drop $\mathbb{E}(e_t \xi_t')$):

$$\begin{aligned}
 \lim_{T \rightarrow \infty} \hat{A} &= \lim_{T \rightarrow \infty} E^{p'} Z Z' E (E' Z Z' E)^{-1} \\
 &= \mathbb{E}(\epsilon_t^p \xi_t') \mathbb{E}(\xi_t \epsilon_t') (\mathbb{E}(\epsilon_t \xi_t') \mathbb{E}(\xi_t \epsilon_t'))^{-1} \\
 &= \mathbb{E}((A \epsilon_t + e_t) \xi_t') \mathbb{E}(\xi_t \epsilon_t') (\mathbb{E}(\epsilon_t \xi_t') \mathbb{E}(\xi_t \epsilon_t'))^{-1} \\
 &= A \mathbb{E}(\epsilon_t \xi_t') \mathbb{E}(\xi_t \epsilon_t') (\mathbb{E}(\epsilon_t \xi_t') \mathbb{E}(\xi_t \epsilon_t'))^{-1} \\
 &= A.
 \end{aligned}$$

Since \hat{A} converges to A , $A^{-1}(\hat{A})$ must converge to $\{\alpha, \beta, \phi\}$, which proves Propositions 3 and 4. Second, since Proposition 2's inverse mapping only references elements in A 's top-left $H \times H$ submatrix, function $\underline{A}(\alpha, \beta, \phi)$ is also one-to-one, which implies (11)'s moment conditions are consistent at the true α, β , and ϕ values only:

$$\begin{aligned}
 \lim_{T \rightarrow \infty} T^{-1} (E^{p'} - \underline{A}(\hat{\alpha}, \hat{\beta}, \hat{\phi}) E') Z \\
 &= \lim_{T \rightarrow \infty} T^{-1} \sum_{t=1}^T (\epsilon_t^p - \underline{A}(\hat{\alpha}, \hat{\beta}, \hat{\phi}) \epsilon_t) \xi_t' \\
 &= \lim_{T \rightarrow \infty} T^{-1} \sum_{t=1}^T ((\underline{A}(\alpha, \beta, \phi) - \underline{A}(\hat{\alpha}, \hat{\beta}, \hat{\phi})) \epsilon_t + e_t) \xi_t' \\
 &= (\underline{A}(\alpha, \beta, \phi) - \underline{A}(\hat{\alpha}, \hat{\beta}, \hat{\phi})) \mathbb{E}(\epsilon_t \xi_t') \\
 &= 0
 \end{aligned}$$

if and only if $\{\hat{\alpha}, \hat{\beta}, \hat{\phi}\} = \{\alpha, \beta, \phi\}$.

This proves Propositions 5 and 6. \square

References

- Aviv Y (2007) On the benefits of collaborative forecasting partnerships between retailers and manufacturers. *Management Sci.* 53(5):777–794.
- Balakrishnan A, Geunes J, Pangburn MS (2004) Coordinating supply chains by controlling upstream variability propagation. *Manufacturing Service Oper. Management* 6(2):163–183.
- Berkowitz J, Kilian L (2000) Recent developments in bootstrapping time series. *Econometric Rev.* 19(1):1–48.
- Blinder AS, Maccini LJ (1991) Taking stock: A critical assessment of recent research on inventories. *J. Econom. Perspect.* 5(1):73–96.
- Bray RL, Mendelson H (2012) Information transmission and the bullwhip effect: An empirical investigation. *Management Sci.* 58(5):860–875.
- Bresnahan TF, Ramey VA (1992) Output fluctuations at the plant level. *Quart. J. Econom.* 109(3):593–624.

- Bureau of Labor Statistics (2013) Producer Price Index Industry Data. Government Printing Office, Washington, DC. Accessed January 31, 2015, <http://download.bls.gov/pub/time.series/pc/pc.data.17.PrimaryMetal>.
- Cachon GP, Olivares M (2010) Drivers of finished-goods inventory in the U.S. automobile industry. *Management Sci.* 56(1):202–216.
- Cachon GP, Gallino S, Olivares M (2012a) Does inventory increase sales? The billboard and scarcity effects in U.S. automobile dealerships. Working paper, University of Pennsylvania, Philadelphia.
- Cachon GP, Gallino S, Olivares M (2012b) Severe weather and automobile assembly productivity. Columbia Business School Research Paper 12/37, Columbia University, New York.
- Cachon GP, Randall T, Schmidt GM (2007) In search of the bullwhip effect. *Manufacturing Service Oper. Management* 9(4):457–479.
- Cameron AC, Trivedi PK (2005) *Microeconometrics: Methods and Applications* (Cambridge University Press, Cambridge, UK).
- Chen L, Lee HL (2009) Information sharing and order variability control under a generalized demand model. *Management Sci.* 55(5):781–797.
- Chen L, Lee HL (2012) Bullwhip effect measurement and its implications. *Oper. Res.* 60(4):771–784.
- Consumer Confidence Survey (2014) Consumer Confidence Index. The Conference Board, New York. Accessed January 31, 2015, <https://www.conference-board.org/data/consumerdata.cfm>.
- Copeland A, Hall G (2011) The response of prices, sales, and output to temporary changes in demand. *J. Appl. Econometrics* 26(2):232–269.
- Federal Highway Administration (2013) Historical monthly VMT report. Office of Highway Policy Information, Bureau of Public Roads, Washington, DC. Accessed January 31, 2015, http://www.fhwa.dot.gov/policyinformation/travel_monitoring/historicvmt.cfm.
- Gavirneni S, Kapuscinski R, Tayur S (1999) Value of information in capacitated supply chains. *Management Sci.* 45(1):16–24.
- Gopal A, Goyal M, Netessine S, Reindorp M (2013) The impact of new product introduction on plant productivity in the North American automotive industry. *Management Sci.* 1909(25):1–20.
- Graves SC, Kletter DB, Hetzel WB (1998) A dynamic model for requirements planning with application to supply chain optimization. *Oper. Res.* 46(3):S35–S49.
- Graves SC, Meal HC, Dasu S, Qui Y (1986) Two-stage production planning in a dynamic environment. Axsater S, Schneeweiss C, Silver E, eds. *Multi-Stage Production Planning and Inventory Control* (Springer, Berlin), 9–43.
- Guajardo JA, Cohen MA, Netessine S (2012) Service competition and product quality in the U.S. automobile industry. Working paper, Wharton Faculty and Research, University of Pennsylvania, Philadelphia.
- Hall GJ (2000) Non-convex costs and capital utilization: A study of production scheduling at automobile assembly plants. *J. Monetary Econom.* 45:681–716.
- Hall G, Rust J (2000) An empirical model of inventory investment by durable commodity intermediaries. *Carnegie-Rochester Conf. Ser. Public Policy* 52:171–214.
- Hardle W, Horowitz J, Kreiss J-P (2003) Bootstrap methods for time series. *Internat. Statist. Rev.* 71(2):435–459.
- Hausman WH (1969) Sequential decision problems: A model to exploit existing forecasters. *Management Sci.* 16(2):93–111.
- Heath DC, Jackson PL (1994) Modeling the evolution of demand forecasts ITH application to safety stock analysis in production/distribution systems. *IIE Trans.* 26(3):17–30.
- Holt CC, Modigliani F, Muth JF, Simon HA (1960) *Planning Production, Inventories, and Work Forces* (Prentice-Hall, Englewood Cliffs, NJ).
- Kahn JA (1987) Inventories and the volatility of production. *Amer. Econom. Rev.* 77(4):667–679.
- Keane MP (2010) Structural vs. atheoretic approaches to econometrics. *J. Econometrics* 156(1):3–20.
- Klein M (1961) On production smoothing. *Management Sci.* 7(3):286–293.
- Lee HL, Padmanabhan V, Whang S (1997) Information distortion in a supply chain: The bullwhip effect. *Management Sci.* 43(4):546–558.
- Lee HL, So KC, Tang CS (2000) The value of information sharing in a two-level supply chain. *Management Sci.* 46(5):626–643.
- Lieberman MB, Demeester L, Rivas R (1995) Inventory reduction in the Japanese automotive sector, 1965–1991. Working paper, University of California, Los Angeles, Los Angeles.
- Lucas RE (1976) Econometric policy evaluation: A critique. *Carnegie-Rochester Conf. Series Public Policy* 1:19–46.
- Moreno A, Terwiesch C (2011) Pricing and production flexibility: An empirical analysis of the U.S. automotive industry. Working paper, Northwestern University, Evanston, IL.
- Newey WK (1984) A method of moments interpretation of sequential estimators. *Econom. Lett.* 14(2–3):201–206.
- Oh S, Özer Ö (2013) Mechanism design for capacity planning under dynamic evolutions of asymmetric demand forecasts. *Management Sci.* 59(4):987–1007.
- Petersen KB, Pedersen MS (2008) *The Matrix Cookbook* (Technical University of Denmark, Kongens Lyngby, Denmark).
- Ramey VA, Vine DJ (2006) Declining volatility in the US automobile industry. *Amer. Econom. Rev.* 96(5):1876–1889.
- Ramey VA, West KD (1999) Inventories. Taylor JB, Woodford, M, eds. *Handbook of Macroeconomics*, Vol. 1, Part B, Chap. 13 (Elsevier B.V., Amsterdam), 863–923.
- Reiss PC, Wolak FA (2007) Structural econometric modeling: Rationales and examples from industrial organization. Heckman JJ, Leamer EE, eds. *Handbook of Econometrics*, Vol. 6, Part A (Elsevier B.V., Amsterdam), 4277–4415.
- Shah R, Ball G, Netessine S (2013) Plant operations and product recalls in the automotive industry: An empirical investigation. INSEAD Working Paper 2013/116/TOM, INSEAD, Fontainebleau, France.
- Shan J, Yang S, Zhang J (2013) An empirical study of the bullwhip effect in China. *Production Oper. Management* 23(4):537–551.
- U.S. Energy Information Administration (2014) Spot prices: Petroleum and other liquids. U.S. Department of Energy, Washington, DC. Accessed January 31, 2015, http://www.eia.gov/dnav/pet/pet_pri_spt_s1_d.htm.
- WardsAuto Group (2014) Wards Auto InfoBank. Penton Media, Chicago, IL. Accessed January 31, 2015, <http://wardsauto.com/>.