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## Manufacturing & Service Operations Management

Publication details, including instructions for authors and subscription information: <a href="http://pubsonline.informs.org">http://pubsonline.informs.org</a>

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#### To cite this article:

Albert Ha, Xiaoyang Long, Javad Nasiry (2016) Quality in Supply Chain Encroachment. Manufacturing & Service Operations Management 18(2):280-298. <a href="http://dx.doi.org/10.1287/msom.2015.0562">http://dx.doi.org/10.1287/msom.2015.0562</a>

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Vol. 18, No. 2, Spring 2016, pp. 280–298 ISSN 1523-4614 (print) | ISSN 1526-5498 (online)



http://dx.doi.org/10.1287/msom.2015.0562 © 2016 INFORMS

# Quality in Supply Chain Encroachment

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We study a supply chain with manufacturer encroachment in which product quality is endogenous and customers have heterogeneous preferences for quality. It is known that, when quality is exogenous, encroachment could make the retailer better off. Yet, when quality is endogenous and the manufacturer has enough flexibility in adjusting quality, we find that encroachment always makes the retailer worse off in a large variety of scenarios. We also establish that, while a higher manufacturer's cost of quality hurts the retailer in absence of encroachment, it could benefit the retailer with encroachment. In addition, we show that a manufacturer offering differentiated products through two channels prefers to sell its high-quality product through the direct channel. Contrary to conventional wisdom, quality differentiation does not always benefit either manufacturer or retailer. Our results may explain why, despite extant theoretical predictions, retailers almost always resent encroachment. These findings also suggest that firms must be cautious when adopting quality differentiation as a strategy to ease channel conflict caused by encroachment.

*Keywords*: encroachment; quality differentiation; dual-channel supply chain *History*: Received: January 7, 2015; accepted: September 1, 2015. Published online in *Articles in Advance* December 7, 2015.

### 1. Introduction

Encroachment occurs when the upstream manufacturer in a supply chain sells directly to consumers, thereby competing with its retail partners in the downstream market. Given the recent advances in e-commerce, encroachment is now a viable strategy for manufacturers to increase profit. If ill managed, however, encroachment could severely damage the relationship between supply chain partners (*Wall Street Journal* 1995). For example, when Bass Ale launched a home delivery program, the supplier's top distributer retaliated by pulling all Bass products off its shelves (Bucklin et al. 1997). To reassure retail partners of Hewlett-Packard's commitment to them, the company's CEO promised that "direct selling behavior will not be tolerated" (Kunert 2013).

To mitigate channel conflict, a manufacturer—rather than forgoing the direct channel entirely—may offer different products through different channels (Bucklin et al. 1997). Many manufacturers implement this strategy via vertical quality differentiation. Eureka Forbes Ltd., which produces vacuum cleaners, sells its premium brand directly and its base models through retailer channels (Sridharan et al. 2012). Kendall-Jackson uses its online channel to sell high-end wines, such as Artisans and Estates, that are usually of higher quality than those available in retail stores (Bucklin et al. 1997). Dell and Toshiba have developed exclusive high-end personal computers, which are generally of higher quality than their regular product offerings, for

retail partners like Best Buy and Circuit City (Lawton 2007). In the apparel industry, retailers increasingly demand exclusive products from designers who also create items for their own boutique stores (Holmes 2011). Clothing companies such as Ralph Lauren, the Armani Group, Hugo Boss, and Baravade have all developed exclusive lines for department stores or their own stores.

In the literature, the encroachment problem is usually studied under the assumption that quality is exogenous and uniform across channels (e.g., Chiang et al. 2003; Arya et al. 2007; Cai 2010; Li et al. 2014, and 2015). It is known that a lower wholesale price increases the retailer channel's profit by creating a higher demand, but it allocates a smaller portion of that channel profit to the manufacturer. While assuming that the encroaching manufacturer's selling cost is higher than that of the retailer, Arya et al. (2007) show that encroachment induces the manufacturer to lower the wholesale price in order to stimulate demand in the retailer channel, even though this comes at the expense of extracting a smaller portion of channel profit. This wholesale price effect is more pronounced when the manufacturer's cost disadvantage is greater. Because of competition, encroachment reduces the retailer channel's profit; however, it gives the retailer a larger share of that profit because of the wholesale price effect. When the manufacturer's cost disadvantage is large, encroachment could lead to a win-win outcome because the wholesale price effect dominates the competition effect for the retailer. This is true even when the products



sold in the two channels are perfectly substitutable. Yet, when the cost disadvantage is small, encroachment hurts the retailer and a win-lose outcome occurs.

Encroachment involves significant manufacturer investments in establishing the direct channel and adopting retail practices. Hence, it is a longer term decision than the quality decision, which is made more frequently (for instance, in each product life cycle). It is therefore possible that quality is endogenous when a manufacturer decides whether or not to encroach. In practice, many manufacturers have adopted quality differentiation as a strategy to mitigate channel conflict due to encroachment. To the best of our knowledge, neither the role of quality in encroachment nor the issue of quality differentiation in a dual-channel setting has been explored in the literature. We hope to fill these gaps by addressing the following research questions: (1) How would quality being endogenous affect the manufacturer's encroachment decision and its impact on firm profits? (2) How should an encroaching manufacturer differentiate in terms of quality? (3) How would quality differentiation affect firm profits?

We consider a supply chain with a manufacturer (she) selling to a retailer (he). If the manufacturer encroaches, she has a selling cost disadvantage and engages in Cournot competition with the retailer. Our model extends Arya et al. (2007) by allowing the consumers to have heterogeneous preferences for quality, which is an endogenous decision made by the manufacturer. In the basic model, we assume that the quality cost function is quadratic and the manufacturer has the flexibility of choosing any quality level. We consider two cases depending on whether or not she can quality differentiate across the two channels.

Suppose the manufacturer cannot quality differentiate. When quality is endogenous, besides the competition and wholesale price effects that exist in Arya et al. (2007), encroachment has a quality distortion effect: it induces the manufacturer to distort quality to shift demand to the retailer channel. Similar to the case of exogenous quality, wholesale price effect here means that encroachment induces the manufacturer to adjust the wholesale price to stimulate the retailer channel's demand at the expense of extracting a smaller portion of channel profit, but the wholesale price is not always lower than that without encroachment because quality may change too. When the manufacturer can distort quality, she relies less on adjusting the wholesale price to influence the retailer's demand and sets it more aggressively for channel profit extraction. Therefore, the wholesale price effect becomes weaker. When the manufacturer's selling cost disadvantage is small, the negative competition effect dominates and makes the retailer worse off. When the manufacturer's selling cost disadvantage is large, the negative competition and quality distortion effects together again make the retailer worse off. Therefore, when quality is endogenous, encroachment always hurts the retailer in the basic model. This result is robust and holds regardless of whether firms make quantity decisions sequentially or simultaneously, whether the manufacturer can segment the market through only the retailer, and whether the cost of quality is fixed or variable.

It is optimal for the manufacturer to encroach when either she has a small cost disadvantage or her cost of quality is low. Moreover, if there is encroachment then the retailer is better off when either the manufacturer has a larger selling cost disadvantage or her cost of quality is higher. The effect of the selling cost disadvantage is intuitive. As the manufacturer's cost of quality increases, she must rely more on adjusting the wholesale price to stimulate the retailer's demand and the wholesale price effect becomes stronger. This hinders the manufacturer's ability in extracting channel profit and makes it more costly for her to encroach; hence, she prefers not to do so unless her cost of quality is low. A stronger wholesale price effect benefits the retailer, which explains why he is better off if the manufacturer's cost of quality is higher. This setting contrasts with that of no encroachment, where the retailer is always worse off if the manufacturer's cost of quality increases.

Suppose the manufacturer *can* quality differentiate. If it is optimal to do so, the manufacturer always prefers to sell the high-quality product (which has a higher profit margin) directly because in that way she captures all the revenue of the high-quality product. A higher level of quality differentiation enables the manufacturer to achieve a higher supply chain profit via market segmentation, but it also diminishes her cost disadvantages and shifts demand from the retailer channel to the direct channel. In determining the level of quality differentiation, the manufacturer faces a trade-off between the benefit of market segmentation and the cost of selling less through the more efficient retailer channel. As the manufacturer's selling cost disadvantage increases, she lowers the level of quality differentiation to shift demand to the retailer channel. When her selling cost disadvantage is large enough, the channel inefficiency effect dominates and it is optimal for her *not* to quality differentiate at all. When the manufacturer's cost of quality increases, market segmentation via quality differentiation becomes more expensive and it is optimal for her not to quality differentiate.

Now consider the effect of quality differentiation on the retailer. Suppose the manufacturer's selling cost disadvantage is small. Without quality differentiation, the manufacturer uses the wholesale price to nullify her selling cost disadvantage and captures most of the demand. Quality differentiation benefits the



retailer because it allows him to increase his demand by focusing on the low-end market segment. Suppose the manufacturer's selling cost disadvantage is large. Quality differentiation now hurts the retailer because his demand is lowered because of the quality disadvantage. Our results imply that quality differentiation eases channel conflict caused by encroachment (though not eliminating it entirely because encroachment still hurts the retailer) when the manufacturer's selling cost disadvantage is small, but exacerbates it otherwise.

By having quality differentiation as another lever, the manufacturer relies less on adjusting the wholesale price to influence the retailer's demand and sets it more aggressively for channel profit extraction. As a result, the wholesale price effect is weakened and, as before, encroachment always hurts the retailer.

Finally, we consider an extension where the manufacturer can choose only quality levels that are not lower than a base level. If the manufacturer's cost of quality is low, the retailer always loses from encroachment. Otherwise, similar to Arya et al. (2007), encroachment could lead to a win-win outcome when the manufacturer's selling cost disadvantage is neither too small nor too large. Based on numerical examples, we show that a win-win outcome is more unlikely to occur when either the base quality level or the base unit production cost (i.e., the unit cost of producing the base quality level) is low.

Our results altogether demonstrate that the retailer always loses from encroachment if the manufacturer has enough flexibility in changing quality (i.e., cost of quality, base quality level, or base unit production cost is low). These are settings where the product quality can vary greatly and where the basic product function as well as the associated production cost are low. We believe that this applies to many industries. The apparel industry is a particularly suitable example because the production cost can be extremely low, whereas the quality range could be very wide. For instance, the cost of making a polo shirt can be from as low as \$1 or \$2 to over \$29 (Binkley 2012). Another example is the consumer electronics industry. The unit production cost of a smartphone has fallen so much that a fully functional one costs just about \$43 (Sparkes 2014, Fagge 2014). Lenovo, which sells smartphones through both retail and online channels, offers phones that vary widely in quality and price (a budget model at less than \$100 to a high-end model at about \$560). Other consumer electronic products (e.g., tablets and blue-ray players) and small home appliances (e.g., hair dryers, electric heaters, and vacuum cleaners) also display similar characteristics.

### 2. Literature Review

Our paper relates to the literature on dual-channel supply chains. Chiang et al. (2003) find that, when

consumers do not strongly prefer the direct channel over the retailer channel, the manufacturer's threat of launching its own direct channel could lead to gains for both channels. Tsay and Agrawal (2004) show that the introduction of a direct channel can benefit both manufacturer and retailer, provided the former is not too inefficient in stimulating demand through sales effort. Cattani et al. (2006) demonstrate that the manufacturer could use pricing strategies (e.g., promising not to undercut the retailer) as a means of mitigating competition and increasing benefits for both firms, especially when the direct channel is costly. Dumrongsiri et al. (2008) find that the manufacturer may be better off establishing a direct channel when demand variability is low. Finally, Arya et al. (2007) show that if an encroaching manufacturer has a selling cost disadvantage relative to the retailer, then the manufacturer has an incentive to lower the wholesale price to maintain the retailer channel's demand; if the selling cost disadvantage is large enough, the downstream retailer would also benefit from encroachment—a win-win outcome. Similarly, Cai (2010) finds that a win-win outcome arises when the retailer channel has a sufficient advantage in base demand or operational cost over the direct channel. Li et al. (2014 and 2015) extend the Arya et al. model to incorporate asymmetric information and show that, when the retailer has private information about demand, encroachment could lead to win-win, win-lose, lose-win, or lose-lose outcomes. Our work differs from these papers in two aspects. First, we consider product quality as an endogenous decision and show that it leads to qualitatively different results. Second, these papers assume that channel differentiation is exogenous. For instance, consumers may have a given preference for one channel over the other, or a channel may be associated with higher costs. We consider endogenous channel differentiation; that is, the manufacturer can choose to differentiate the channels by varying the product quality offered through each channel.

Our work also relates to the literature on a firm's quality decision in the presence of heterogeneous consumers. Moorthy (1988) studies firms' vertical positioning strategies in a duopoly with variable quality cost. Chambers et al. (2006, p. 1885) provide a review of the extensive work in economics and marketing on quality-based competition; these authors observe that "the defining characteristic of quality is that the marketplace consists of individuals who all agree that a higher level is always preferable to a lower level." We consider vertical differentiation with either variable or fixed quality cost in a framework similar to that used in Motta (1993).

There are a number of papers addressing the effect of channel decentralization on product quality. Jeuland and Shugan (1983) and Economides (1999) were the first to establish that quality should be lower (or the



same) in decentralized channels. Xu (2009) extends this result to show that the reverse may hold depending on the marginal revenue function. Shi et al. (2013) examine the case of vertical *and* horizontal consumer heterogeneity. They find that the effect of channel decentralization on product quality is sensitive to the distribution of consumer preferences. Our work differs from these papers in that, instead of comparing centralized versus decentralized channels, we examine the case where both channels exist simultaneously for one manufacturer and show that product quality under encroachment could be either higher or lower depending on the selling cost in the direct channel.

### 3. Model and Benchmark Analysis

In this section, we outline the basic model and investigate a benchmark case with no encroachment. We study encroachment in §§4 and 5 and compare the results with the benchmark case.

#### 3.1. Basic Model

Consider a market, with size normalized to 1, that consists of consumers with heterogenous preferences for quality. A manufacturer (she) sells a product through a retailer (he) but may also establish her own channel to sell directly to consumers.

A consumer's surplus from purchasing the product with quality u > 0 and price p is  $U(u, p; \theta) = \theta u - p$ , where  $\theta$  represents the consumer's sensitivity to quality. We model consumer heterogeneity by assuming that  $\theta$  is uniformly distributed on [0, 1] (Ronnen 1991, Lehmann-Grube 1997). If there is only one product of quality u in the market, then a consumer with sensitivity  $\theta$  is indifferent between buying and not buying when  $\theta u - p = 0$ . Therefore, all customers with  $\theta \ge p/u$  would buy the product. Hence, demand is q = 1 - p/u, which yields the inverse demand function p = u(1 - q).

We assume that the manufacturer's unit cost for making a product with quality level u is  $ku^2$ , where k > 0 captures the manufacturer's cost of quality. This functional form implies that the manufacturer can choose any quality level, and it has been extensively used in the literature (e.g., Moorthy and Png 1992). In §6.4, we consider a more general cost function under which the manufacturer has less flexibility in choosing the quality level. Similarly to Arya et al. (2007), we assume that the manufacturer incurs a selling cost  $c \ge 0$  for every unit of product sold through the direct channel and normalize the retailer's selling cost to 0. This direct selling disadvantage could be due to inexperience, lack of consumer knowledge, or the additional cost of e-commerce (The Economist 2013, Kapner 2014). Finally, we assume that consumers can accurately perceive product quality.

The timeline is as follows: (i) the manufacturer decides on the quality level(s) of the product and the wholesale price w; (ii) after observing the quality level(s) and w, the retailer decides on his order quantity  $q_R$ ; (iii) if a direct channel exists, then the manufacturer decides on the quantity  $q_M$  that she will sell directly. This decision sequence entails three models: no encroachment, encroachment with uniform quality, and encroachment with quality differentiation. Without encroachment, the manufacturer sells the product of quality u through the retailer channel only. If a direct channel exists and only one type of product is sold (encroachment with uniform quality) then, after receiving the retailer's order, the manufacturer decides on the quantity she wants to sell directly. Both firms face the same market-clearing price. We assume that quantity decisions are made sequentially because the manufacturer can observe the retailer's order decision, whereas the manufacturer's own quantity decision is usually unknown to the retailer. In other words, the manufacturer cannot credibly commit to not changing her quantity after receiving the retailer's order. (In §4.2, we study the case in which the manufacturer and the retailer make simultaneous quantity decisions.) Finally, the manufacturer can decide to quality differentiate by offering product of quality u through the direct channel and product of quality tu, t > 0, through the retailer (encroachment with quality differentiation).

The model of encroachment with uniform quality corresponds to the case where it is not practical for the manufacturer to offer a variety of products in the supply chain—for instance, owing to the high product development cost or high production cost associated with a small volume. This model also serves as a base case for understanding the role of quality in encroachment when quality differentiation across channels is not an important issue.

### 3.2. Benchmark: No Encroachment

Here we establish a benchmark case with no direct channel. The problem is solved by backward induction. Given wholesale price w and quality u, the retailer chooses order quantity  $q_R$  to maximize his profit,  $\Pi_R^N(q_R, w, u) = (u(1 - q_R) - w)q_R$ , from which it follows that  $q_R^N(w, u) = \frac{1}{2} - w/(2u)$ ; N stands for no encroachment. Anticipating the retailer's order decision, the manufacturer solves

$$\max_{w, u} (w - ku^2) q_R^N(w, u) = \max_{w, u} (w - ku^2) \left(\frac{1}{2} - \frac{w}{2u}\right);$$

this expression yields

$$w^{N}(u) = \frac{ku^{2}}{2} + \frac{u}{2},\tag{1}$$

and hence  $q_R^N(u) = \frac{1}{4} - ku/4$ . The manufacturer's and the retailer's profits as a function of quality u are, respectively,

$$\Pi_M^N(u) = \frac{u(1-ku)^2}{8} \quad \text{and} \quad \Pi_R^N(u) = \frac{u(1-ku)^2}{16}.$$
 (2)



Finally, the manufacturer determines the optimal quality by maximizing  $\Pi_M^N(u)$ , which yields  $u^N = 1/(3k)$ ,  $w^N = 2/(9k)$ , and  $q_R^N = \frac{1}{6}$ . The equilibrium profits under no encroachment are

$$\Pi_M^N = \frac{1}{54k} \quad \text{and} \quad \Pi_R^N = \frac{1}{108k}.$$

Without a direct channel, the two firms' interests are perfectly aligned for the quality decision: according to (2), the quality level  $u^N$  that maximizes the manufacturer's profit  $\Pi_R^N(u)$  also maximizes the retailer's profit  $\Pi_R^N(u)$ .

### 4. Encroachment with Uniform Quality

In this section, we consider the case where the manufacturer sells directly to consumers and there is no quality differentiation across channels.

### 4.1. Sequential Quantity Decisions

Following the timeline in the basic model (§3.1), we assume that the retailer determines his order quantity before the manufacturer determines her selling quantity.

In the last stage, the manufacturer's optimization problem (given u, w, and  $q_R$ ) is

$$\max_{q_{M}} \{ (w - ku^{2})q_{R} + (u - uq_{M} - uq_{R} - c - ku^{2})q_{M} \},$$

which gives her best response quantity  $q_M^U(q_R, w, u) = (\frac{1}{2} - q_R/2 - c/(2u) - ku/2)^+$ . Anticipating this, the retailer determines his order quantity by maximizing his own profit:

$$\max_{q_R} (u(1 - q_R - q_M^U(q_R, w, u)) - w)q_R$$

which, assuming  $q_R \leq 1-c/u-ku$ , simplifies into  $\max_{q_R} \left\{ -\frac{1}{2}uq_R^2 + \left(\frac{1}{2}u + \frac{1}{2}ku^2 + \frac{1}{2}c - w\right)q_R \right\}$ . Solving this optimization problem and substituting the retailer's optimal quantity into the manufacturer's best response quantity  $q_M^U(q_R, w, u)$  yields the following solution for the quantity competition subgame when w and u are given:

$$q_R^U(w, u) = \frac{1}{2} - \frac{w}{u} + \frac{ku}{2} + \frac{c}{2u} \quad \text{and}$$

$$q_M^U(w, u) = \frac{1}{4} + \frac{w}{2u} - \frac{3ku}{4} - \frac{3c}{4u}.$$
(3)

In the first stage, by anticipating the equilibrium quantities  $q_R^U(w, u)$  and  $q_M^U(w, u)$ , the manufacturer solves the following optimization problem:

$$\begin{aligned} \max_{w,u} & \big\{ (w - ku^2) q_R^U(w,u) \\ & + (u - u q_M^U(w,u) - u q_R^U(w,u) - c - ku^2) q_M^U(w,u) \big\}. \end{aligned}$$

It is straightforward to show that, for a given u, the optimal wholesale price is

$$w^{U}(u) = \frac{ku^2}{2} + \frac{u}{2} - \frac{c}{6} \tag{4}$$

with the corresponding quantities

$$q_R^U(w^U(u), u) = \frac{2c}{3u}$$
 and 
$$q_M^U(w^U(u), u) = -\frac{ku}{2} - \frac{5c}{6u} + \frac{1}{2}.$$

The profit functions then become

$$\Pi_{M}^{U}(u) = \frac{k^{2}u^{3}}{4} + \frac{kcu}{2} + \frac{7c^{2}}{12u} - \frac{ku^{2}}{2} + \frac{u}{4} - \frac{c}{2} \quad \text{and} \quad (5)$$

$$\Pi_{R}^{U}(u) = \frac{2c^{2}}{9u}.$$

Comparing the retailer's profit under encroachment  $\Pi_R^U(u)$  in (5) with the benchmark profit under no encroachment  $\Pi_R^N(u)$  in (2), we see that the retailer is better off with encroachment if and only if

$$\frac{3(u-ku^2)}{4\sqrt{2}} < c < \frac{3(u-ku^2)}{5},\tag{6}$$

where the left inequality follows from  $\Pi_R^U(u) - \Pi_R^N(u) > 0$  and the right inequality is the constraint for encroachment to occur:  $q_M^U(u) = -ku/2 - 5c/(6u) + \frac{1}{2} > 0$ . Condition 6 is exactly the same as that in Proposition 2 of Arya et al. (2007) with  $a = u - ku^2$ . In their model, the demand function is a - bq and the unit production cost is normalized to zero. When u is exogenously given, our model is the same as theirs with  $a = u - ku^2$  and b = u. Similar to the wholesale price effect identified by Arya et al. (2007), from (1) and (4),  $w^N(u) - w^U(u) = c/6 \ge 0$  and therefore  $w^N(u) - w^U(u)$  is increasing in c.

When quality is endogenous, the above analysis no longer applies because the manufacturer's quality decision depends on whether there is a direct channel or not. The following proposition characterizes the manufacturer's quality decision under encroachment and shows that, unlike the case of exogenous quality, a win-win outcome is no longer possible (all proofs are in the appendix).

PROPOSITION 1. (i) There exists a threshold  $\tilde{c}$  such that the manufacturer encroaches if and only if  $c < \tilde{c}$ .

- (ii) Under encroachment, (a) the product quality  $u^U$  is first increasing and then decreasing in c, and (b) there exists a threshold  $c^u$  such that  $u^U \ge u^N$ , if  $c \le c^u$  and  $u^U < u^N$  otherwise.
- (iii) When encroachment happens, the manufacturer always wins,  $\Pi_M^U \Pi_M^N > 0$ , and the retailer always loses,  $\Pi_R^U \Pi_R^N < 0$ .



Here, as in the rest of the paper, "encroachment" means that the manufacturer actually sells through her direct channel ( $q_M^U > 0$ ). In the appendix, we consider the case of  $q_M^U = 0$  (i.e., no sales through the direct channel) when we characterize the threshold value  $\tilde{c}$ . Note that this latter case differs from the benchmark case, in which no direct channel exists.

As the direct selling cost c increases, the manufacturer would like to shift more sales to the retailer channel. Proposition 1 states that she can do so by first increasing u when c is small, and then decreasing it when c is large. By setting c = 0 in (5), we can show that the optimal quality level for the centralized system (i.e., the manufacturer owns the retailer channel) is 1/(3k), which is the same as  $u^N$ . Proposition 1 states that the manufacturer distorts quality upward ( $u^U \ge u^N$ ) when *c* is small and downward ( $u^U < u^N$ ) when *c* is large. These results can be explained as follows. In (3), the third and fourth terms of  $q_R^U(w, u)$  are positive whereas those of  $q_M^U(w, u)$  are negative. This shows that given w and u, the manufacturer has competitive disadvantages because of her production cost  $ku^2$  and selling cost c. A larger u increases her production cost and makes this disadvantage more significant (ku in the third term becomes larger). However, a larger *u* improves the market profitability and makes the selling cost disadvantage less significant (c/u in the fourth term becomes smaller). These effects are reversed when u becomes smaller. When c is small, the production cost disadvantage effect dominates the selling cost disadvantage effect, and a larger u shifts demand from the direct channel to the retailer channel. When *c* is large, however, the selling cost disadvantage effect dominates and a smaller u shifts demand from the direct channel to the retailer channel.

By distorting quality, the manufacturer can rely less on the wholesale price to influence the retailer's demand and uses it more aggressively to extract profit from the retailer channel. It is intuitive that the retailer is now more unlikely to benefit from encroachment. Proposition 1, however, shows a stronger result: the retailer no longer wins. This can be explained as follows. Encroachment has three effects on the retailer: a competition effect (a portion of the retailer channel's profit is lost to the direct channel), a wholesale price effect (the retailer receives a larger portion of the channel profit because the manufacturer adjusts wholesale price to boost retailer channel's demand), and a quality distortion effect (quality is distorted to boost retailer channel's demand at the expense of a lower channel profit). The first two effects are similar to those in Arya et al. (2007), except wholesale price does not always go down because encroachment could induce a higher quality level. When c is small, the negative competition effect dominates the other two effects. When c is large, the negative quality distortion effect is strong and, together with the negative competition effect, makes the retailer lose from encroachment. Figure 1 illustrates the results and shows that quality distortion due to encroachment can be as large as 16%.

Our result that encroachment always hurts the retailer depends critically on the assumed quality cost function. Obviously it is valid only if the assumed cost function is realistic (i.e., the marginal quality cost is increasing in the current quality level and the manufacturer can choose any quality level). Nevertheless it is consistent with what we observe in practice where retailers almost always resent encroachment (see the examples of Bass Ale and Hewlett-Packard in §1). In §6.4, we consider a more general quality cost function to further investigate the conditions under which the result holds.

COROLLARY 1. The threshold  $\tilde{c}$  is decreasing in k. Given c > 0, the retailer's profit under encroachment is increasing in k.

Without encroachment, the retailer's profit decreases in k because it becomes more expensive for the supply chain to use quality to increase consumers' willingness to pay (see §3.2). With encroachment, when k is close to 0, market profitability is very high because of inexpensive quality and the manufacturer's selling disadvantage becomes insignificant. Therefore the retailer's demand is very small when *k* is close to 0. As *k* increases, it benefits the retailer in two ways. First, his demand increases because his selling cost advantage becomes more significant. Second, the manufacturer relies less on distorting quality and more on adjusting the wholesale price to stimulate the retailer's demand. Hence, the wholesale price effect becomes more pronounced, which benefits the retailer. When the wholesale price effect is stronger, encroachment is more costly to the manufacturer; hence, the threshold for encroachment  $\tilde{c}$  is decreasing in k. Figure 2 shows how k affects firms' profits. Unsurprisingly, the manufacturer's profit is decreasing in k.

#### 4.2. Simultaneous Quantity Decisions

In the basic model with uniform quality, we assumed the firms make quantity decisions sequentially. Here we examine the case where they make their decisions simultaneously; in other words, the retailer no longer has a first-mover advantage.

The following proposition characterizes the conditions under which encroachment occurs as well as its effects on the manufacturer and retailer profits.

PROPOSITION 2. (i) When both firms choose quantities simultaneously, there exists a threshold  $\tilde{c}'$  such that the manufacturer encroaches if and only if  $c < \tilde{c}'$ . Under encroachement, there exists  $c_L$  such that the retailer always loses  $(\Pi_R^S < \Pi_R^N)$ , whereas the manufacturer loses  $(\Pi_M^S < \Pi_M^N)$  if and only if  $c_L < c < \tilde{c}'$ . (ii) The manufacturer is more likely to encroach under simultaneous than under sequential quantity decisions:  $\tilde{c} < \tilde{c}'$ .

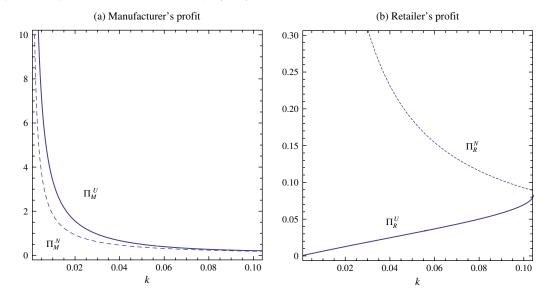


(a) Wholesale price and quality level (b) Manufacturer's profit (c) Retailer's profit 0.010  $\Pi_R^N$ 0.35 0.008 0.030 0.30 0.006 0.25 0.025 0.004 0.20 0.002 0.020 0.15 0.02 0.04 0.10 0.06 0.08 0.04 0.02 0.04 0.02 0.10 0.06 0.08 0.10 0.06 0.08

c

Figure 1 (Color online) Firms' Decisions and Profits vs. Manufacturer's Selling Cost Disadvantage  $c\ (k=1)$ 

Figure 2 (Color online) Firms' Profits vs. Cost of Quality k (c = 1)



Proposition 2 states that encroachment may hurt the manufacturer under simultaneous quantity decisions. So when it is optimal for the manufacturer to encroach, she could be better off if encroachment were not an option (i.e., if the direct channel did not exist). Figure 3 illustrates this result. The lose-lose outcome arises because the simultaneous game intensifies competition in the downstream market, which hurts both the retailer and the manufacturer. Arya et al. (2007) find that, when quality is exogenous, encroachment with simultaneous quantity decisions yields win-lose, lose-win, or loselose outcome. In comparison, our results show that endogenizing quality eliminates the possibility of a retailer benefitting from encroachment. With or without endogenous quality, the manufacturer is more likely to encroach under simultaneous quantity decisions (i.e.,  $\tilde{c} < \tilde{c}'$ ) because the retailer no longer has the first-mover advantage. For the rest of this paper, we maintain the assumption of sequential quantity decisions because in

most industries, it is more likely that the manufacturer can revise her production quantity after receiving the retailer's order.

c

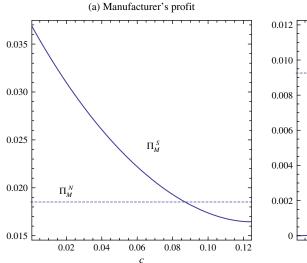
### 5. Encroachment with Quality Differentiation

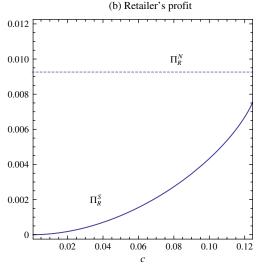
In this section, we study the case where the manufacturer can encroach by offering products of different quality levels through the two channels. This is motivated by the conventional wisdom that channel conflict can be eased when different products are offered in different channels.

Suppose the manufacturer distributes a product of quality u through the direct channel and a product of quality tu through the retailer channel. Because the demand functions are different depending on which channel carries the higher-quality product, we distinguish two cases:  $0 < t \le 1$  (high-quality encroachment)



Figure 3 (Color online) Comparison of Firms' Profits With and Without Encroachment Under Simultaneous Quantity Decisions (k = 1)





and  $t \ge 1$  (low-quality encroachment). If  $0 < t \le 1$  then consumers for whom  $\theta \ge (p_M - p_R)/(u - tu)$  choose quality u and those for whom  $p_R/(tu) \le \theta < (p_M - p_R)/(u - tu)$  choose quality tu, where  $p_M$  and  $p_R$  correspond to the market-clearing prices of (respectively) the directand retailer-channel products. The market is thus segmented, and the respective inverse demand functions are  $p_M = u(1 - q_M - tq_R)$  and  $p_R = tu(1 - q_M - q_R)$ . The demand functions when  $t \ge 1$  can be derived similarly.

We first establish that if it is optimal for the manufacturer to offer two quality-differentiated products, she prefers to offer the high-quality product through the direct channel. This is consistent with the Eureka Forbes example in §1.

Proposition 3. Under encroachment, the manufacturer never offers a product of strictly lower quality through the direct channel.

Because the high-quality product caters to consumers with greater willingness to pay for quality, the total surplus that could be extracted is larger, which leads to a higher margin for the higher quality product (Moorthy 1988). Then, because the manufacturer receives only a part of the retailer channel's revenue, she prefers to distribute the high-quality product through the direct channel while selling the low-quality product through the retailer channel. But is quality differentiation always optimal? Without a direct channel, it is known that quality differentiation benefits a monopoly (Mussa and Rosen 1978, Moorthy 1984) or two competing firms (Moorthy 1988, Motta 1993) because it enables firms to segment the market and extract more consumer surplus. Yet, we can show that, under certain conditions, a uniform quality policy is preferred by the encroaching manufacturer.

Table 1 Manufacturer's Equilibrium Decisions

c	t*	u*	$q_M^*$
$0 \leq c < c_1$	$t^* = \frac{6}{5} - \frac{2}{5}\sqrt{4 - \frac{5c}{ku^{*2}}} < 1$	$u^* > \sqrt{\frac{4c}{3k}}$	$q_M^* > 0$
$c_1 < c \leq c_2$	$t^* = 1$	$u^* < \sqrt{\frac{4c}{3k}}$	$q_M^* > 0$
$C > C_2$	N/A	N/A	$q_M^*=0$

Proposition 4. (i) There exist two thresholds  $c_1 < c_2$  that characterize the manufacturer's equilibrium decisions as given in Table 1.

(ii) When encroachment happens, the manufacturer always wins  $(\Pi_M^* > \Pi_M^N)$  and the retailer always loses  $(\Pi_R^* < \Pi_R^N)$ .

When c is relatively small (i.e.,  $c < c_1$ ), the manufacturer sells a lower-quality product through the retailer. However, when the selling disadvantage is sufficiently large (i.e.,  $c_1 < c \le c_2$ ), the encroaching manufacturer is better off by not quality differentiating at all. To see why, we first look at the effect of quality differentiation on the retailer channel's demand. In the quantity competition subgame, given w, u, and t, where t < 1, the equilibrium quantities are

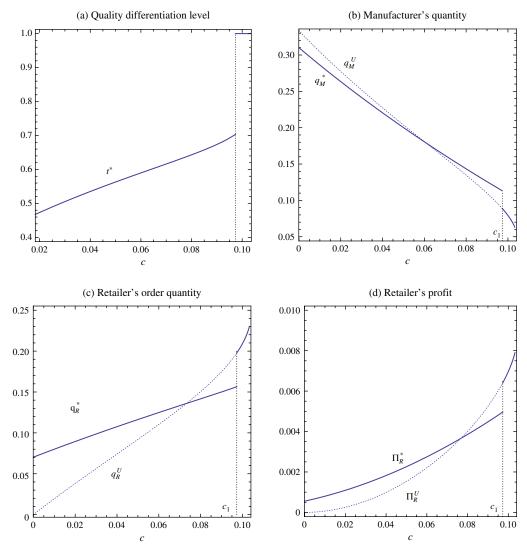
$$q_{R}(w, u, t) = \frac{1}{2(2-t)} - \frac{w}{t(2-t)u} + \frac{ku}{2(2-t)} + \frac{c}{2(2-t)u},$$

$$q_{M}(w, u, t) = \frac{4-3t}{4(2-t)} + \frac{w}{2(2-t)u} - \frac{(4-t)ku}{4(2-t)} - \frac{(4-t)c}{4(2-t)u}.$$
(7)

Similar to the case of no quality differentiation, given w, u, and t, the manufacturer has competitive



Figure 4 (Color online) Firms' Decisions and Retailer's Profit vs. Manufacturer's Selling Cost Disadvantage c (k = 1)



disadvantages because of her production cost  $ku^2$  and selling cost c. (In (7), the third and fourth terms of  $q_R(w, u, t)$  are positive, whereas those of  $q_M(w, u, t)$  are negative.) Increasing the level of quality differentiation (i.e., lowering t) has two effects. On the one hand, it allows the manufacturer to increase consumers' willingness to pay and achieve a higher supply chain profit via market segmentation. On the other hand, it diminishes her production and selling cost disadvantages and shifts demand from the more efficient retailer channel to the less efficient direct channel (the magnitude of each of the third and fourth terms of  $q_R(w, u, t)$  and  $q_M(w, u, t)$  is increasing in t). In determining the level of quality differentiation, the manufacturer must trade off the benefit of market segmentation against the cost of selling less through the more efficient retailer channel. As *c* increases, the manufacturer increases t to get it closer to 1 (i.e., lowers the level of quality differentiation) in order to shift demand to the retailer channel. When c is large enough ( $c > c_1$ ), she prefers not to quality differentiate at all. Figures 4(a) to 4(c) illustrate these results.

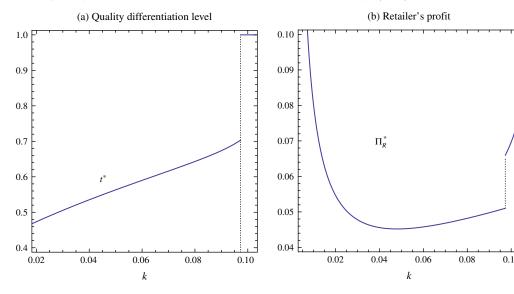
As the manufacturer's selling disadvantage c increases, her incentive to encroach decreases. There is a threshold  $c_2$  such that, if  $c > c_2$ , then the manufacturer does not sell anything through the direct channel.

Under encroachment, when the manufacturer can adjust the level of quality differentiation, she relies less on adjusting the wholesale price to shift demand to the retailer channel and sets it more aggressively for channel profit extraction. Because the wholesale price effect is subdued, similar to the case of uniform quality, encroachment always hurts the retailer.

Figure 4(d) shows that under encroachment, quality differentiation benefits the retailer when c is small but hurts him otherwise. This finding contradicts the conventional wisdom, which holds that the retailer is better off if the encroaching manufacturer sells a different product. Suppose c is small. Without quality differentiation, the manufacturer uses the wholesale



Figure 5 (Color online) Quality Differentiation Decision and Retailer's Profit vs. Cost of Quality k (c=1)



price to nullify his cost disadvantage and captures most of the sales ( $q_R^U$  is close to zero). Quality differentiation benefits the retailer because it allows him to increase his sales ( $q_R^* > q_R^U$ ) by focusing on the low-end market segment. Now suppose c is large. Quality differentiation hurts the retailer because his sales are lowered ( $q_R^* < q_R^U$ ) because of the quality disadvantage.

In practice, a manufacturer may have to let a powerful retailer sell the high-quality product (i.e., t > 1). In this case, our numerical analysis shows that, as c increases, the manufacturer decreases t to get it closer to 1 (i.e., lowers the level of quality differentiation) in order to shift demand to the retailer channel. It remains true that encroachment always hurts the retailer. However, under encroachment, quality differentiation always benefits the retailer.

Finally, we examine how k, the manufacturer's cost of quality, affects her decisions and the retailer's profit.

COROLLARY 2. Given c > 0, as k increases (i) the manufacturer becomes less likely to encroach or quality differentiate (i.e., both  $c_1$  and  $c_2$  decrease); and (ii) the retailer's profit first decreases and then increases.

Recall from Corollary 1 that, absent quality differentiation, a higher k makes it more costly for the manufacturer to encroach and the retailer's profit is increasing in k. It follows that the threshold for encroachment  $c_2$  is decreasing in k. From Figure 5, as k increases, the level of differentiation decreases because it becomes more expensive to quality differentiate. Therefore, the threshold for no quality differentiation  $c_1$  is decreasing in k as well. Unlike the case of no quality differentiation, the retailer can now focus on the low-end market segment and obtain a positive demand even when k is close to 0. As k increases, it has two opposing effects on the retailer's profit. On

the one hand, market profitability is lower because of the higher cost of quality and this hurts the retailer. On the other hand, it decreases quality differentiation, which benefits the retailer. When k is small, the negative effect of lower market profitability dominates the positive effect of lower quality differentiation, and the reverse is true when k is large. Therefore, under quality differentiation the retailer's profit first decreases and then increases in k. Figure 5 illustrates the result.

#### 6. Extensions

We now consider several variations of the basic model to show that our main insights are robust.

### 6.1. Segmentation Through the Retailer

Instead of selling two different products through two different channels, the manufacturer might sell both products through only the retailer channel. In this section we develop a model that accounts for this possibility and, by comparing it with the model in §5, show that the win-lose outcome of encroachment is robust.

Suppose that there is no direct channel and that the manufacturer sells two products of different quality levels through the retailer. Given  $w_1$ ,  $w_2$ ,  $u_1$ , and  $u_2$  (without loss of generality, we let  $u_1 > u_2$ ), the retailer chooses  $q_1$  and  $q_2$  to solve

$$\max_{q_1, q_2} \{ (u_2(1 - q_2 - q_1) - w_2)q_2 + (u_1 - u_1q_1 - u_2q_2 - w_1)q_1 \},$$

which yields

$$q_1(u_1, u_2, w_1, w_2) = \frac{u_1 - u_2 - w_1 + w_2}{2(u_1 - u_2)} \quad \text{and} \quad q_2(u_1, u_2, w_1, w_2) = \frac{u_2 w_1 - u_1 w_2}{2u_2(u_1 - u_2)}.$$



The manufacturer's profit maximization problem is then

$$\begin{aligned} \max_{w_1,\,w_2,\,u_1,\,u_2} &\{(w_1-ku_1^2)q_1(u_1,\,u_2,\,w_1,\,w_2) \\ &+(w_2-ku_2^2)q_2(u_1,\,u_2,\,w_1,\,w_2)\}. \end{aligned}$$

It is straightforward to show that the optimal solution to the manufacturer's problem is

$$\begin{split} u_1^{N2} &= \frac{2}{5k}, \quad u_2^{N2} = \frac{1}{5k}, \quad w_1^{N2} = \frac{7}{25k}, \\ w_2^{N2} &= \frac{3}{25k}, \quad \text{and} \quad q_1^{N2} = q_2^{N2} = \frac{1}{10}. \end{split}$$

Furthermore,  $\Pi_M^{N2}=1/(50k)$  and  $\Pi_R^{N2}=1/(100k)$ . The manufacturer clearly should prefer to sell two products of different quality levels  $(u_1^{N2}\neq u_2^{N2})$  through the retailer to benefit from market segmentation  $(\Pi_M^{N2}=1/(50k)>\Pi_M^N=1/(54k))$ . The retailer also benefits from this strategy  $(\Pi_R^{N2}=1/(100k)>\Pi_R^N=1/(108k))$ .

Without a direct channel, the manufacturer always prefers selling two quality-differentiated products instead of one product to achieve market segmentation. If the retailer has the power to dictate the product assortment decision and chooses not to carry the full product line (e.g., under different consumer preferences for quality; see Villas-Boas 1998), it seems logical for the manufacturer to counterbalance this power by encroaching and selling some products through its direct channel to improve market segmentation. Proposition 4, however, suggests that this is not necessarily true because the manufacturer has an incentive to lower the level of quality differentiation in order to shift demand to the more efficient retailer channel.

By comparing with results from the model in §5, we obtain our next proposition.

Proposition 5. When encroachment happens, the manufacturer always wins  $(\Pi_M^* - \Pi_M^{N2} > 0)$ , and the retailer always loses  $(\Pi_R^* - \Pi_R^{N2} < 0)$ .

Similar to Proposition 4, encroachment hurts the retailer. This is not surprising because under encroachment, the retailer now loses the high-quality product to the direct channel and the negative competition effect of encroachment becomes more significant (when compared with the case in Proposition 4).

### 6.2. Encroachment with Fixed Cost of Quality

In some industries, the cost of quality is predominantly driven by a fixed cost that does not depend on production volume. For example, there is usually a high fixed cost associated with research and development in the high-tech industry and with hiring high-profile designers in the apparel industry. In this section, we consider encroachment in a setting where there is a fixed cost associated with quality.

As in §5, the manufacturer can choose to quality differentiate between the two channels with one product of quality u offered directly to consumers and another of quality tu through the retailer. The fixed cost of quality is then given by  $\max(ku^2, kt^2u^2)$ . This corresponds to the case of a high-quality product being convertible to a low-quality one by reducing some features, which in some industries is a common way of achieving quality differentiation.

PROPOSITION 6. Under encroachment, (i) quality differentiation is not optimal (i.e., t = 1) and (ii) the manufacturer always wins and the retailer always loses.

The intuition is that the cost of improving the quality of the lower quality product is zero and consumers all prefer higher quality to lower quality. The manufacturer is therefore incentivized to increase the quality of the lower quality product, resulting in no quality differentiation. Finally, Proposition 6(ii) shows that encroachment always hurts the retailer, which is consistent with the case where quality cost is variable.

### 6.3. No Quality Commitment for Direct Channel

Here we consider the "fast design" case where the manufacturer cannot commit to a particular level of product quality in the direct channel. Lack of commitment could be due to design lead times short enough that the manufacturer can adjust the product's design quality for the direct channel after receiving the retailer's order. Because of this design flexibility, the manufacturer cannot credibly commit to the quality of the direct channel product before the retailer places an order. The timeline for this model is as follows: (i) the manufacturer announces the product quality  $u_R$  and the wholesale price w; (ii) the retailer chooses the order quantity  $q_R$ ; (iii) the manufacturer determines the product quality  $u_M$  and selling quantity  $q_M$ .

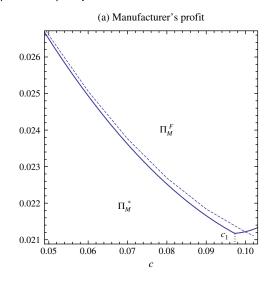
Proposition 7. If the encroaching manufacturer cannot commit to a level of quality in the direct-channel product, then quality differentiation across channels is always optimal.

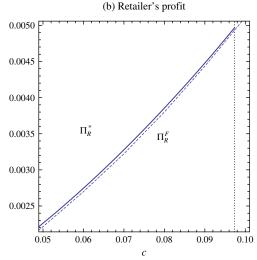
Proposition 7 provides an interesting contrast to Proposition 4. Recall from our discussion of Proposition 4 that, in deciding whether or not to quality differentiate, the manufacturer faces a trade-off between the benefit of market segmentation and the cost of selling less through the more efficient retailer channel. Without commitment, the product quality in the direct channel can no longer affect the retailer's order directly and the manufacturer always prefers to quality differentiate and thereby benefit from market segmentation.

Because we are unable to characterize analytically the optimal quality levels, we investigate the effect of commitment on firms' profits numerically by computing the optimal encroachment solution for a range of values of c in  $[0, c_2)$ . The upper bound  $c_2$  is chosen



Figure 6 (Color online) Comparison of Firms' Profits With and Without Quality Commitment Under Encroachment (k = 1)





such that we can compare the results with those under commitment. Figure 6 illustrates the results in this scenario. We find that, as in §5, encroachment with the high-quality product  $(u_M > u_R)$  is always optimal for the manufacturer. In addition, no quality commitment in the direct channel benefits the manufacturer when her cost disadvantage *c* is small but hurts her otherwise  $(\Pi_M^F > \Pi_M^*)$  if c < 0.101 in Figure 6(a)). This result can be explained as follows. No quality commitment allows the manufacturer to alleviate the retailer's first-mover advantage in the quantity decision because she can use quality differentiation afterward to influence competition. However, the manufacturer can no longer use the committed quality to affect the retailer's order quantity directly, and the result is insufficient use of the more efficient retailer channel. When *c* is small, the manufacturer makes most of her sales through the direct channel and so the competition effect dominates the channel inefficiency effect. However, the situation is reversed when c is large. Our results imply that, if the manufacturer suffers from a high selling cost disadvantage, then she may be better served by committing (if possible) to a strategy of no differentiation rather than maintaining design flexibility. Finally, we discover numerically that the retailer is worse off when there is no quality commitment ( $\Pi_R^F < \Pi_R^*$  in Figure 6(b)). The reason is that the retailer's first-mover advantage in the quantity decision is reduced owing to the manufacturer's flexibility in choosing quality differentiation afterward.

#### 6.4. Generalized Cost Function

Suppose the unit production cost is b if  $u \le u_0$  and  $b+k(u-u_0)^2$  if  $u>u_0$ , where  $u_0\ge 0$  and  $b\ge 0$ . Thus the manufacturer cannot choose a quality level that is lower than a base level  $u_0$ ,, which can be produced at a base unit production cost b. She can improve quality

by an increment  $u_1 = u - u_0$  with an additional unit cost  $ku_1^2$ . A larger b means a lower margin for the manufacturer to invest in quality. Therefore the degree of flexibility for the manufacturer to change quality is lower when either  $u_0$  or b is larger. This cost function captures the model in Arya et al. (2007) with  $u_0 > 0$ , b = 0, and  $k \to \infty$ , and our basic model with  $u_0 = b = 0$ , and k > 0.

PROPOSITION 8. There exists a threshold  $\bar{k} > 0$  such that (i) if  $k \leq \bar{k}$ , then the retailer always loses from encroachment; (ii) if  $k > \bar{k}$ , then there exist  $\underline{c}$  and  $\bar{c}$  such that the retailer wins from encroachment when  $\underline{c} < c < \bar{c}$ .

When quality is endogenous, the manufacturer relies less on adjusting the wholesale price to shift demand to the retailer channel and sets it more aggressively for channel profit extraction. That is, the wholesale price effect becomes weaker. A large k, however, means that the manufacturer cannot afford to change quality as much as she wants, and so the wholesale price effect becomes stronger. When the manufacturer's flexibility in changing quality is restricted by  $u_0$  and b, Proposition 8 shows that the retailer can win from encroachment if k is large, which is reminiscent of Proposition 2 in Arya et al. (2007). If k is small, however, similar to our basic model, the retailer always loses. Figure 7 illustrates these results and shows that *k* increases if  $u_0$  becomes smaller. Moreover, the region where the retailer wins from encroachment shrinks as k decreases. Similarly, Figure 8 shows that k increases if b becomes smaller. For the generalized cost function, we can show that  $u^N$  is still the same as the optimal quality level of the centralized system. Figure 9 shows that when k = 50 and b = 0, quality is distorted by as much as 16% for the case of  $u_0 = 0$ , which is much larger than the maximum distortion of 1% for the case of  $u_0 = 0.3$ . This illustrates that when  $u_0$  is larger, the



Figure 7 (Color online) Effect of  $u_0$  on the Region Where Retailer Wins from Encroachment (b=0) (a)  $u_0=1$  (b)  $u_0=0.1$ 

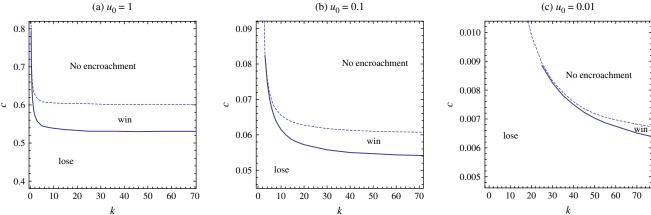
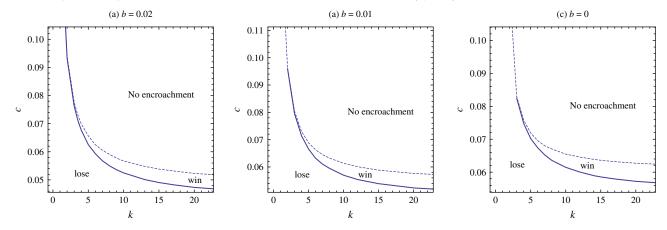


Figure 8 (Color online) Effect of b on the Region where Retailer Wins from Encroachment ( $u_0 = 0.1$ )



manufacturer prefers to rely less on quality distortion to influence retailer channel's demand.

We also consider nonquadratic quality cost functions where the unit cost function is  $ku^x$  and x > 1. As x increases, it is increasingly cheap to produce quality up to a certain level (u = 1) and increasingly expensive to produce beyond this quality level. This becomes similar to the case of  $u_0 > 0$  and large k in the generalized cost model, because the manufacturer cannot afford to invest in quality beyond a cheap base level, regardless of supply chain structure. Therefore, for large x, the win–win outcome may emerge again.

### 7. Conclusion

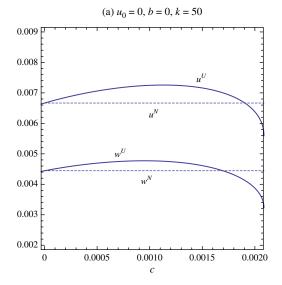
In this paper we study the manufacturer encroachment problem by allowing the quality decision to be endogenous, and we obtain novel results that are qualitatively different from those in the existing literature. We identify the manufacturer's selling cost disadvantage and her cost of quality as two key performance drivers and characterize how they affect the firms' decisions and their profits.

There are several limitations to our study. First, we consider quantity competition even though price

competition may be more appropriate in some industrial settings. Arya et al. (2007) show that, when compared with quantity competition, price competition reduces the manufacturer's incentive to encroach and increases the retailer's gain when encroachment happens. It would be interesting to see how these results might change when quality is endogenous. Second, we assume that consumers are equally accepting of direct and retailer channels—although in practice it is plausible that some consumers prefer to shop at the retailer, whereas others prefer the manufacturer's store. In that case, the products from the two channels are imperfect substitutes. Third, the manufacturer could offer products that are horizontally (rather than vertically) differentiated for the two channels. For example, a clothing company could decide to sell a trendy line through its online store while offering a classic line through department stores. Finally, although we find the manufacturer should distribute the higher quality product directly, in real life she may not do so because of the retailer's bargaining power, horizontal differentiation strategies, or exogenous channel differences. We leave these issues for future research.



Figure 9 (Color online) Manufacturer's Quality and Wholesale Price Decisions



### Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/msom.2015.0562.

### Acknowledgments

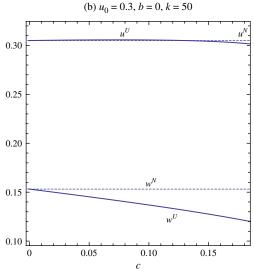
The authors thank Editor-in-Chief Christopher Tang, the associate editor, and three anonymous reviewers for their constructive comments and suggestions. Financial support was provided by Wei Lun Foundation and the Research Grants Council of Hong Kong [Project HKUST16502414].

### Appendix. Proofs

Proof of Proposition 1(i). Recall that the manufacturer's optimal quantity decision in the last stage is  $q_M^U(q_R,w,u)=(\frac{1}{2}-q_R/2-c/(2u)-ku/2)^+$ . We distinguish three cases:  $(C_1)$  if  $\frac{1}{2}-q_R/2-c/(2u)-ku/2>0$ , then  $q_M^U>0$  and the manufacturer encroaches;  $(C_2)$  if  $\frac{1}{2}-q_R/2-c/(2u)-ku/2=0$ , then  $q_M^U=0$  and the retailer chooses  $q_R$  in order to prohibit the manufacturer from selling directly;  $(C_3)$  if  $\frac{1}{2}-q_R/2-c/(2u)-ku/2<0$ , then  $q_M^U=0$  and the existence of the direct channel does not affect the firms' decisions. We use backward induction to derive the manufacturer's optimal profit, given u, for each of the three cases:

$$\Pi_{M}(u) = \begin{cases} \Pi_{M}^{U}(u) \equiv \frac{k^{2}u^{3}}{4} + \frac{kcu}{2} + \frac{7c^{2}}{12u} - \frac{ku^{2}}{2} + \frac{u}{4} - \frac{c}{2} \\ & \text{if } c \leq \frac{3u(1-ku)}{5}; \\ \Pi_{M}^{UZ}(u) \equiv \left(\frac{1}{2}ku^{2} - \frac{1}{2}u + \frac{3}{2}c\right)\left(1 - \frac{c}{u} - ku\right) \\ & \text{if } \frac{3u(1-ku)}{5} \leq c \leq \frac{5u(1-ku)}{6}; \\ \Pi_{M}^{N}(u) = \frac{u(1-ku)^{2}}{8} & \text{if } c \geq \frac{5u(1-ku)}{6}. \end{cases}$$

We solve each of the three constrained maximization problems (denoted by  $C_1$ ,  $C_2$ , and  $C_3$ ) and then compare to find the conditions for encroachment—that is, when the solution to  $C_1$  is the global maximum.



*Problem*  $C_1$ : We rewrite the constraint as

$$c \le \frac{3u(1-ku)}{5} \iff 3ku^2 - 3u + 5c \le 0$$

$$\iff \underline{u}_1 = \frac{3 - \sqrt{3(3-20ck)}}{6k} \le u \le \frac{3 + \sqrt{3(3-20ck)}}{6k} = \overline{u}_1. (8)$$

The interior critical points solve the first-order condition (FOC)  $d\Pi_{\rm M}^{\rm U}(u)/du=3k^2u^2/4-ku+\frac{1}{4}+kc/2-7c^2/(12u^2)=0$  or, equivalently,

$$9k^2u^4 - 12ku^3 + (3 + 6kc)u^2 - 7c^2 = 0. (9)$$

The derivative of the left-hand side (LHS) of Equation (9) with respect to u is  $6u(6k^2u^2 - 6ku + 2kc + 1)$ .

For c>1/(8k), by (8), we have  $6k^2u^2-6ku+2kc+1<6k^2u^2-6ku+10kc\leq 0$ ; hence  $d\Pi_M^U(u)/du$  is strictly decreasing in  $[\underline{u}_1,\bar{u}_1]$ . Since  $d\Pi_M^U(\underline{u}_1)/du=2ck(12ck+\sqrt{3(3-20ck)}-3)/(\sqrt{3(3-20ck)}-3)^2<0$ , it follows that  $d\Pi_M^U(u)/du<0$ ,  $\forall\,u\in[\underline{u}_1,\bar{u}_1]$ . Therefore,  $\Pi_M^U(u)$  is strictly decreasing in  $[\underline{u}_1,\bar{u}_1]$  and the maximizer of  $C_1$  is  $u=\underline{u}_1$ .

For  $c \leq 1/(8k)$ , the expression  $6k^2u^2 - 6ku + 2kc + 1 = 0$  has two roots,  $u_1 < u_2$ , in the interval  $[\underline{u}_1, \overline{u}_1]$ . Hence,  $d\Pi_M^U(u)/du$  is increasing in  $[\underline{u}_1, u_1)$ , decreasing in  $[u_1, u_2]$ , and increasing in  $(u_2, \overline{u}_1]$ . It follows that if an interior maximizer for  $\Pi_M^U(u)$  exists then it must be unique. Denote this unique interior maximizer by  $u^U$ . Now observe that  $d\Pi_M^U(\underline{u}_1)/du > 0$  if and only if (iff)  $c \leq 1/(12k)$  and that  $d\Pi_M^U(\overline{u}_1)/du = -(2ck(\sqrt{9-60ck}+3-12ck))/(\sqrt{9-60ck}+3)^2 < 0$ . So on the one hand, if  $c \leq 1/(12k)$  then the interior solution  $u^U$  is the optimal solution to  $C_1$ . On the other hand, if c > 1/(12k) and  $d\Pi_M(u_1)/du \geq 0$ —that is, if

$$c \leq \frac{1}{48k} \left( 5 - \sqrt{1 + 6\sqrt[3]{14}} \right.$$

$$\left. + \sqrt{\frac{2(1 + 3\sqrt[3]{14} - 18 \ 14^{2/3} + 55\sqrt{1 + 6\sqrt[3]{14}})}{1 + 6\sqrt[3]{14}}} \right) \approx 0.104309/k$$

—then the manufacturer's optimal profit is  $\max\{\Pi_M^U(u^U), \Pi_M^U(\underline{u}_1)\}$ . If c > 0.104309/k, then  $d\Pi_M^U(u)/du < 0$  for all  $u \in [\underline{u}_1, \bar{u}_1]$  and so the optimal solution is  $u = \underline{u}_1$ .



Problem C<sub>2</sub>: The constraint  $3u(1-ku)/5 \le c \le 5u(1-ku)/6$  can be written as  $\underline{u}_2 \equiv (5-\sqrt{5(5-24ck)})/(10k) \le u \le \underline{u}_1$  or  $\bar{u}_1 \le u \le (5+\sqrt{5(5-24ck)})/(10k) \equiv \bar{u}_2$ , where  $\underline{u}_1$  and  $\bar{u}_1$  are as in (8). The interior critical points solve the FOC  $d\Pi_M^{IJZ}(u)/du = \frac{1}{2}(-1-4ck+3c^2/u^2+4ku-3k^2u^2) = 0$  or, equivalently,

$$-3k^2u^4 + 4ku^3 + 3c^2 - (4ck+1)u^2 = 0. (10)$$

The derivative of the LHS of (10) is  $2u(-6k^2u^2 + 6ku - (4ck + 1))$ .

For c < 1/(6k) we have  $-6k^2u^2 + 6ku - (4ck + 1) < -6k^2u^2 + 6ku - 10ck \le 0$  (because  $3u(1 - ku)/5 \le c$ ), which implies that  $d\Pi_M^{UZ}(u)/du$  is decreasing in each of the two intervals  $[\underline{u}_2, \underline{u}_1]$  and  $[\bar{u}_1, \bar{u}_2]$ . Since

$$\frac{d\Pi_{M}^{UZ}(\bar{u}_{1})}{du} = -\frac{2ck(\sqrt{9 - 60ck} + 3 - 12ck)}{(\sqrt{9 - 60ck} + 3)^{2}} < 0,$$

it follows that  $d\Pi_M^{UZ}(u)/du < 0$  in  $[\bar{u}_1,\bar{u}_2]$ . Hence  $\Pi_M^{UZ}(u)$  is decreasing in  $[\bar{u}_1,\bar{u}_2]$  and the maximum in the interval  $[\bar{u}_1,\bar{u}_2]$  is obtained at  $\bar{u}_1$ . If  $c \leq 1/(12k)$  then

$$\frac{d\Pi_M^{\rm UZ}(\underline{u}_1)}{du} = \frac{2ck(12ck + \sqrt{3(3-20ck)} - 3)}{(\sqrt{3(3-20ck)} - 3)^2} \ge 0,$$

so  $\Pi_M^{UZ}(u)$  is increasing in  $[\underline{u}_2, \underline{u}_1]$ . In this case the solution to  $C_2$  is  $\max\{\Pi_M^{UZ}(\underline{u}_1), \Pi_M^{UZ}(\bar{u}_1)\}$ . If  $1/(12k) \le c \le 1/(6k)$ , then

$$\frac{d\Pi_{M}^{\text{LIZ}}(\underline{u}_{1})}{du} < 0 \quad \text{and}$$

$$\frac{d\Pi_{M}^{\text{LIZ}}(\underline{u}_{2})}{du} = \frac{2ck(87ck - 20 + 4\sqrt{5}\sqrt{5 - 24ck})}{(\sqrt{5}\sqrt{5 - 24ck} - 5)^{2}} > 0;$$

hence there exists a unique interior maximizer  $u^Z \in [\underline{u}_2, \underline{u}_1]$  and so the optimal solution to  $C_2$  is  $\max\{\Pi_M^{UZ}(u^Z), \Pi_M^{UZ}(\bar{u}_1)\}$ .

For c > 1/(6k), since the solution to  $C_1$  is the corner solution  $u = \underline{u}_1$  and since  $\Pi_M(u)$  is continuous, we need not solve  $C_2$  to realize that its solution weakly dominates  $C_1$ .

Problem  $C_3$ : The constraint is  $u \le \underline{u}_2$  or  $\bar{u}_2 \le u < 1/k$ .<sup>1</sup> Because  $\Pi_M^N(u) = u(1-ku)^2/8$  is increasing in (0,1/(3k)) and decreasing in (1/(3k),1/k), if c < 5/(27k) (i.e.,  $\underline{u}_2 < 1/(3k) < \bar{u}_2$ ) then the solution to problem  $C_3$  is either  $u = \underline{u}_2$  or  $u = \bar{u}_2$ . If  $c \ge 5/(27k)$ , the optimal solution is  $u^N = 1/(3k)$ .

Comparison: Given that  $\Pi_M(u)$  is continuous, the preceding analysis implies that if  $c \leq 1/(12k)$  then the global optimum is  $\Pi_M^U(u^U)$ . If c > 0.104309/k then the global optimum is the solution to  $C_2$  or  $C_3$ . If 1/(12k) < c < 0.104309/k, we shall demonstrate a unique threshold  $\tilde{c} \in (1/(12k), 0.104309/k)$  such that  $\Pi_M^U(u^U) > \Pi_M^{UZ}(u^Z)$  if and only if  $c < \tilde{c}$ .

First we show that  $\Pi_M^U = \Pi_M^U(u^U)$  is convex in c. By the envelope theorem,  $d\Pi_M^U(u^U)/dc = \partial \Pi_M^U(u^U)/\partial c$ . Since  $u^U$  solves  $\partial \Pi_M^U(u)/\partial u = 0$ , it follows that

$$\frac{du^{U}}{dc} = -\frac{\partial^{2}\Pi_{M}^{U}(u^{U})}{\partial u \partial c} / \frac{\partial^{2}\Pi_{M}^{U}(u^{U})}{\partial u^{2}}$$

(implicit function theorem). Therefore,

$$\frac{d^2\Pi_M^U}{dc^2} = \frac{\partial^2\Pi_M^U(u^U)}{\partial c^2} + \frac{\partial^2\Pi_M^U(u^U)}{\partial u\partial c} \cdot \frac{du^U}{dc}$$

$$\begin{split} &= \frac{\partial^2 \Pi_M^U(u^U)}{\partial c^2} + \frac{\partial^2 \Pi_M^U(u^U)}{\partial u \partial c} \left[ -\frac{\partial^2 \Pi_M^U(u^U)}{\partial u \partial c} \middle/ \frac{\partial^2 \Pi_M^U(u^U)}{\partial u^2} \right] \\ &= \frac{\partial^2 \Pi_M^U(u^U)}{\partial c^2} - \left( \frac{\partial^2 \Pi_M^U(u^U)}{\partial u \partial c} \right)^2 \middle/ \frac{\partial^2 \Pi_M^U(u^U)}{\partial u^2} \\ &= \frac{7}{6u^U} - \left( \frac{\partial^2 \Pi_M^U(u^U)}{\partial u \partial c} \right)^2 \middle/ \frac{\partial^2 \Pi_M^U(u^U)}{\partial u^2} > 0, \end{split}$$

where the last inequality follows because  $\partial^2 \Pi_M^{II}(u^{II})/\partial u^2 < 0$  (by definition of  $u^{II}$ ). Next, we show that  $\Pi_M^{UZ} = \Pi_M^{UZ}(u^Z)$  is concave in c:

$$\begin{split} \frac{d^2\Pi_M^{UZ}}{dc^2} &= \frac{\partial^2\Pi_M^{UZ}(u^Z)}{\partial c^2} - \left(\frac{\partial^2\Pi_M^{UZ}(u^Z)}{\partial u\partial c}\right)^2 \bigg/ \frac{\partial^2\Pi_M^{UZ}(u^Z)}{\partial u^2} \\ &= -\frac{3}{u^Z} - \left(\frac{3c}{(u^Z)^2} - 2k\right)^2 \bigg/ \left(2k - \frac{3c^2}{(u^Z)^3} - 3k^2u^Z\right) \\ &= \frac{ku^Z(12c - 6u^Z + 5k(u^Z)^2)}{2k(u^Z)^3 - 3c^2 - 3k^2(u^Z)^4} < 0. \end{split}$$

The inequality follows because the denominator is negative and the numerator is positive. The denominator is negative because at  $u^z$ ,

$$\frac{\partial^2 \Pi_M^{UZ}(u)}{\partial u^2} = 2k - \frac{3c^2}{u^3} - 3k^2u = \frac{2ku^3 - 3c^2 - 3k^2u^4}{u^3} < 0;$$

the numerator is positive because  $u^Z < \underline{u}_1 = (6k)^{-1} \cdot (3 - \sqrt{3(3 - 20ck)})$ , which implies that  $12c - 6u^Z + 5k(u^Z)^2 > 0$ .

Hence  $d^2\Pi_M^{II}/dc^2 - d^2\Pi_M^{IIZ}/dc^2 > 0$ , which means that  $\Pi_M^{II} - \Pi_M^{IIZ}$  is convex in c. We observe that  $\Pi_M^{II} - \Pi_M^{IIZ} > 0$  at c = 1/(12k), but that  $\Pi_M^{II} - \Pi_M^{IIZ} < 0$  at c = 0.104309/k. It follows that there is a unique threshold  $\tilde{c}$  at which  $\Pi_M^{II} = \Pi_M^{IIZ}$ . We find  $\tilde{c} \approx 0.1019/k$ , completing the proof.  $\square$ 

Proof of Proposition 1(II). (a) We prove that  $u^U$  is increasing in  $c \in [0, (4-\sqrt{7})/(24k))$  and decreasing in  $c \in [(4-\sqrt{7})/(24k), \tilde{c})$ . By the implicit function theorem,  $du^U/dc = -(\partial^2\Pi_M^U(u^U)/\partial u\partial c)/(\partial^2\Pi_M^U(u^U)/\partial u^2)$ . Because  $\partial^2\Pi_M^U(u^U)/\partial u^2 < 0$ , we have  $du^U/dc \geq 0$  iff  $\partial^2\Pi_M^U(u^U)/\partial u\partial c = k/2 - 7c/(6u^{U^2}) \geq 0$  or, equivalently, iff  $u^U \geq \sqrt{7c/(3k)}$ . When c < 1/(12k), recall from the proof of Proposition 1(i) that  $\Pi_M^U(u)$  is increasing in  $[\underline{u}_1, u^U)$  and decreasing in  $(\underline{u}^U, \bar{u}_1]$ . Therefore,  $u^U > \sqrt{7c/(3k)} \in (\underline{u}_1, \bar{u}_1)$  iff  $d\Pi_M^U(\sqrt{7c/(3k)})/du = \frac{1}{4} + 2ck - \sqrt{7ck/3} > 0$  or, equivalently, iff  $c < (4 - \sqrt{7})/(24k)$ . If  $1/(12k) < c < \tilde{c}$  then  $\Pi_M^U(u)$  is decreasing in  $[\underline{u}_1, u_1)$ , increasing in  $[u_1, u^U)$ , and decreasing in  $(\underline{u}^U, \bar{u}]$ ; again if  $c \leq (4 - \sqrt{7})/(24k)$  then we have  $u^U \geq \sqrt{7c/(3k)}$ , otherwise,

$$\frac{d\Pi_{M}^{U}(\sqrt{7c/(3k)})}{du} < 0$$
 and  $\sqrt{\frac{7c}{3k}} > u_{1} = \frac{3 - \sqrt{3}\sqrt{1 - 4ck}}{6k}$ 

from which it follows that  $u^U < \sqrt{7c/(3k)}$ . (b) Observe that at c=0 we have  $u^U=u^N=1/(3k)$  but at c=0.1/k we have  $u^U=0.313981/k < u^N$ . Because  $u^U$  is increasing in  $c \in [0, (4-\sqrt{7})/(24k))$  and decreasing in  $c \in [(4-\sqrt{7})/(24k), \tilde{c})$ , a unique threshold  $c^u$  exists.  $\square$ 

Proof of Proposition 1(iii). The proof of Proposition 1(i) shows that  $\Pi^U_M$  is convex in  $c \in [0, \tilde{c})$ . Consider c = 2/(21k), at which  $u^U = 1/(3k)$ . Substituting shows that  $d\Pi^U_M/dc|_{c=2/(21k)} = 0$  and  $\Pi^U_M|_{c=2/(21k)} > \Pi^N_M$ . Therefore, when the manufacturer encroaches  $(c < \tilde{c})$ ,  $\Pi^U_M$  is first decreasing and then



<sup>&</sup>lt;sup>1</sup> The condition u < 1/k is necessary because  $q_R^N(u) = \frac{1}{4}(1 - ku) > 0$ .

increasing in c, and we have  $\Pi_M^U > \Pi_M^N$ . We next show that  $\Pi_R^U(u^U) < \Pi_R^N(u^U)$ , or  $c < (3u^U(1-ku^U))/(4\sqrt{2})$ . Substituting  $7c^2 = 9k^2(u^U)^4 - 12k(u^U)^3 + 3(1+2kc)(u^U)^2$  (see (9)) into the second-order condition (SOC)  $\partial^2\Pi_M(u^U)/\partial u^2 = (3k^2u^U)/2 - k + 7c^2/(6(u^U)^3) < 0$  yields  $3u^U(ku^U-1) + 1/(2k) + c < 0$ , which implies that  $3u^U(ku^U-1) + 4\sqrt{2}c < 0$  because, under encroachment,  $c < \tilde{c} < 1/(2k(4\sqrt{2}-1))$ . Since  $\Pi_R^N(u^U) \le \Pi_R^N$  by definition, it follows that  $\Pi_R^U < \Pi_R^N$ . This completes the proof.  $\square$ 

Proof of Corollary 1. See the e-companion (available as supplemental material at http://dx.doi.org/10.1287/msom.2015.0562).  $\Box$ 

Proof of Proposition 2. (i) The proof is similar to that of Proposition 1 and so is omitted for brevity. (ii) The manufacturer encroaches iff  $c < \tilde{c}' \approx 0.122/k$ ; since  $\tilde{c} \approx 0.1019/k$  (Proposition 1),  $\tilde{c} < \tilde{c}'$ .  $\square$ 

To prove Propositions 3 and 4, we first establish the following useful result.

**Lemma** 1. Suppose u is given and that encroachment happens. Then the following statements hold.

- (i) Assume that the manufacturer offers a lower-quality product through the retailer. If  $c \le 3ku^2/4$  then  $t(u) = \frac{6}{5} \frac{2}{5}\sqrt{4 5c/(ku^2)}$ ; otherwise, t(u) = 1.
- (ii) Assume that the manufacturer offers a higher-quality product through the retailer. If  $c \le (3u 6ku^2)/4$  then

$$t(u) = \frac{2 + 5ku + \sqrt{4 - 48kc + 8ku - 23k^2u^2}}{12ku};$$

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$$\frac{3u - 6ku^2}{4} < c < \frac{4 + 8ku - 23k^2u^2}{48k} \quad and \quad u \le \frac{2}{7k},$$

then t(u) is either 1 or  $(2+5ku+\sqrt{4-48kc+8ku-23k^2u^2})/(12ku)$ . Otherwise, t(u)=1.

(iii) For every c < 1/(12k), there is a unique threshold  $\tilde{u}(c)$  such that t(u) > 1 if  $\sqrt{4c/(3k)} < u < \tilde{u}(c)$  and t(u) < 1 if  $\tilde{u}(c) \le u < (3 + \sqrt{3(3 - 32kc)})/(12k)$ .

PROOF OF LEMMA 1(I). The manufacturer first solves  $\max_{w} \Pi_{M}(w, t, u)$ ; this yields  $w(t, u) = tu/2 + kt^{2}(7 - 4t)u^{2}/(2(8 - 5t)) - ct^{2}/(2(8 - 5t))$ . Then the manufacturer maximizes

$$\begin{split} \Pi_M(t,u) &= \left(4k^2u^3t^3 - (8k^2u^3 + 8kcu)t^2 - \left(k^2u^3 + 2kcu + \frac{c^2}{u}\right)t \right. \\ &+ 8k^2u^3 + 16kcu + \frac{8c^2}{u}\right) \cdot (4(8-5t))^{-1} \\ &- \frac{2ku^2 - u + 2c}{4}, \end{split}$$

under the constraints

$$q_M(t, u) > 0 \implies c < \frac{(8 - 5t)u}{8 - 3t} - \frac{(-2t^2 - 3t + 8)ku^2}{8 - 3t}$$
 (11)

and t < 1. Now, given u, There are three roots to

$$\frac{\partial \Pi_M(t,u)}{\partial t} = \frac{-2u(t-c/(ku^2)-1)(5k^2u^2t^2-12k^2u^2t+4k^2u^2+4kc)}{(8-5t)^2} = 0:$$

$$t_1 = \frac{6}{5} - \frac{2}{5}\sqrt{4 - \frac{5c}{ku^2}}, \quad t_2 = \frac{6}{5} + \frac{2}{5}\sqrt{4 - \frac{5c}{ku^2}},$$
  
and  $t_3 = 1 + \frac{c}{ku^2}.$ 

Obviously,  $t_2 > 1$  and  $t_3 > 1$ . If  $t_1 > 1$  then  $\Pi_M(t, u)$  is increasing in  $t \in [0, 1]$ ; in contrast, if  $t_1 \le 1$  then  $\Pi_M(t, u)$  is increasing in  $t \in [0, t_1)$  and decreasing in  $t \in [t_1, t_1]$ . Therefore, with encroachment the manufacturer chooses  $t(u) = \min\{\frac{6}{5} - \frac{2}{5}\sqrt{4 - 5c/(ku^2)}, 1\}$ .  $\square$ 

Proof of Lemma 1(II). See the e-companion.  $\Box$ 

PROOF OF LEMMA 1(III). Combining the first two parts of Lemma 1 yields an interval of u for which both t>1 and t<1 dominate t=1. In particular, combine  $c\leq (3u-6ku^2)/4$  and  $c\leq 3ku^2/4$  to get  $\sqrt{4c/(3k)}< u<(3+\sqrt{3(3-32kc)})/(12k)$ ; the interval exists iff c<1/(12k). It remains to determine whether t<1 or t>1 is the optimal strategy. Given u and assuming that encroachment happens, denote the manufacturer's profit function under t<1 as  $\Pi_M^H(u)$  and under t>1 as  $\Pi_M^L(u)$ , where the H and L denote high- and low-quality encroachment. Note that

$$\begin{split} \frac{d}{du}\Pi_{M}(t(u),u) &= \frac{\partial}{\partial u}\Pi_{M}(t(u),u) + \frac{\partial}{\partial t}\Pi_{M}(t(u),u)\frac{dt(u)}{du} \\ &= \frac{\partial}{\partial u}\Pi_{M}(t(u),u), \end{split}$$

since  $(\partial/\partial t)\Pi_M(t(u), u) = 0$  by definition of t(u). We prove the following result.

Claim 1.  $d\Pi_M^H(u)/du > d\Pi_M^L(u)/du$ ,  $\forall u \in [\sqrt{4c/(3k)}, (3 + \sqrt{3(3 - 32kc)})/(12k)]$ .

Proof of Claim 1. See the e-companion.  $\Box$ 

Because  $\Pi_{M}^{H}(u) = \Pi_{M}^{U}(u) < \Pi_{M}^{L}(u)$  at  $u = \sqrt{4c/(3k)}$  and  $\Pi_{M}^{H}(u) > \Pi_{M}^{U}(u) = \Pi_{M}^{L}(u)$  at  $u = (3 + \sqrt{3(3 - 32kc)})/(12k)$ , by Claim 1 there is a unique  $\tilde{u} \in [\sqrt{4c/(3k)}, (3 + \sqrt{3(3 - 32kc)})/(12k)]$  at which  $\Pi_{M}^{H}(u)$  and  $\Pi_{M}^{L}(u)$  cross. This completes the proof of Lemma 1(iii).  $\square$ 

Proof of Proposition 3. By Lemma 1(ii), for any (c, u) such that

$$\left(c > \frac{4 + 8ku - 23k^2u^2}{48k}, u \le \frac{2}{7k}\right) \text{ or } \left(c > \frac{3u(1 - 2ku)}{4}, u > \frac{2}{7k}\right),$$

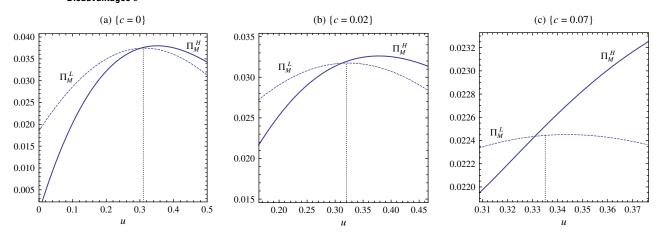
encroachment with uniform quality (t=1) dominates low-quality encroachment (t>1). For the remaining region (which is equivalent to  $u\in [\underline{u},\bar{u}]$  and c<9/(92k), where  $\underline{u}=\max\{0,(4-2\sqrt{3(9-92kc)})/(23k)\}$  and where  $\bar{u}=(\sqrt{3(3-32kc)}+3)/(12k)$  if  $c\le9/(98k)$  or  $\bar{u}=(4+2\sqrt{3(9-92kc)})/(23k)$  if 9/(98k)< c<9/(92k)), we proceed to show that low-quality encroachment (t>1) is always dominated by high-quality encroachment  $(t\le1)$ . Toward that end, we first establish that, given c, there exists at most one interior maximizer for  $\Pi^L_M(u)$  in  $[\underline{u},\bar{u}]$ ; we then show that this unique interior maximum is always dominated by  $\max_{u\in [\underline{u},\bar{u}]}\Pi^H_M(u)$ .

Step 1 (unique interior maximizer for  $\Pi_M^L(u)$  in  $[\underline{u}, \bar{u}]$ ). By analysing the profit function, we find that  $d\Pi_M^L(u)/du$  has at most two critical points (see the e-companion). Since  $d\Pi_M^L(\underline{u})/du \le 0$  and since  $d\Pi_M^L(\bar{u})/du \le 0$  if  $c \le 15/(196k)$ , it follows that no interior maximizer exists for  $c \ge 15/(196k)$  and that, for c < 15/(196k), the interior maximizer is unique.

Step 2 (interior maximizer of  $\Pi_M^L(u)$  is never optimal). By Step 1, if  $c \ge 15/(196k)$  then the low-quality encroachment



Figure A.1 (Color online) Manufacturer Profits vs. Quality Under Low- and High-Quality Encroachment—Three Representative Selling Cost Disadvantages c



problem has no interior maximizer in  $[\underline{u}, \overline{u}]$  and so encroachment under t > 1 is suboptimal. If c < 15/(196k), then  $\underline{u} = 0$ and  $\bar{u} = (3 + \sqrt{3(3 - 32kc)})/(12k)$ ; denote the unique interior maximizer of  $\Pi_M^L$  by  $u^L$ . We shall demonstrate that  $\Pi_M^L(u^L)$  <  $\Pi_M^H(u^L)$ ; the proof proceeds as follows. For any  $c \leq 15/(196k)$ , we identify a value  $\hat{u} \in (u, \bar{u})$  such that (1)  $d\Pi_M^L(\hat{u})/du > 0$ and (2)  $\Pi_M^H(\hat{u}) > \Pi_M^L(\hat{u})$ . Since  $d\Pi_M^L(\bar{u})/du < 0$  (see Step 1) and since  $u^L$  is unique, it follows that condition (1) guarantees  $u^L > \hat{u}$ . Condition (2) implies that  $\hat{u} > \tilde{u}$  because  $\tilde{u}$ is the unique point at which  $\Pi_M^H(u)$  and  $\Pi_M^L(u)$  cross (see Lemma 1(iii)). Hence it follows from these two conditions that  $u^L > \tilde{u}$ , which implies  $\Pi_M^H(u^L) > \Pi_M^L(u^L)$ . It is easy to verify that  $\hat{u} = 0.31/k$  satisfies the two conditions for  $c \le$ 0.016/k. For  $0.016/k < c \le 0.038/k$ , we find  $\hat{u} = 0.32/k$  satisfies the two conditions; for  $0.038/k < c \le 0.063/k$ ,  $\hat{u} = 0.329/k$ satisfies the two conditions; for  $0.063/k < c \le 15/(196k)$ ,  $\hat{u} = 0.335/k$  satisfies the two conditions. Therefore, the interio<u>r</u> maximum of  $\Pi_M^L$  is dominated by  $\max\{\Pi_M^H(u) \mid u \in$  $[\sqrt{4c/(3k)}, (3+\sqrt{3(3-32kc)})/(12k)]$  for c < 15/(196k). For a graphical illustration see Figure A.1, in which the vertical dotted line represents the position of  $\hat{u}$  and the domain is  $u \in (\sqrt{4c/3}, (3+\sqrt{3(3-32c)})/12)$ . We still need to verify that  $\max\{\Pi_M^H(u) \mid u \in [\sqrt{4c/(3k)}, (3+\sqrt{3(3-32kc)})/(12k)]\}$  is feasible (i.e., that constraint (11) is satisfied) for c < 15/(196k). The inclusion  $u \in [\sqrt{4c/(3k)}, (3+\sqrt{3(3-32kc)})/(12k)]$  is equivalent to  $c \le 3ku^2/4$  and  $c \le (3u - 6ku^2)/4$ , from which we obtain  $c < (3u - 3ku^2)/5$ . To show (11) it suffices to observe

$$\frac{(8-5t)u}{8-3t} - \frac{(-2t^2 - 3t + 8)ku^2}{8-3t} - \frac{3u - 3ku^2}{5}$$
$$= \frac{2(1-t)u(8 - (5t + 8)ku)}{5(8-3t)} > 0$$

(because t < 1 and u < 1/(2k)). This completes the proof.

PROOF OF PROPOSITION 4(I). According to Proposition 3, low-quality encroachment is suboptimal. We first establish the existence of the threshold  $c_1$  by solving the high-quality encroachment problem and comparing the optimal profit with  $\Pi_M^U$ . We complete the proof by showing that  $q_M^* = 0$  if and only if  $c \ge c_2$ .

Step 1 (high-quality versus uniform-quality encroachment). If c < 1/(12k) then by Lemma 1 and Proposition 3, high-quality encroachment dominates uniform-quality encroachment provided  $u^U > (3-\sqrt{3(3-32kc)})/(12k)$ , which holds because  $u^U > (3-\sqrt{3(3-20kc)})/(6k)$  by (8). It remains to compare high- and uniform-quality encroachment for  $c \ge 1/(12k)$ . The high-quality encroachment problem is  $\max_u \Pi_M^H(u)$  subject to  $u \ge \sqrt{4c/(3k)}$  (see Lemma 1). The optimal quality in the uniform-quality encroachment problem is as given in Proposition 1. Observe that at  $u = \sqrt{4c/(3k)}$  the profit function is continuous:  $\Pi_M^H(u) = \Pi_M^U(u)$ . We focus on interior solutions and ignore the  $q_M > 0$  constraint except to show feasibility. If the interior maximizer does not exist or is not the optimal solution, then  $q_M = 0$  (as will be discussed in Step 2).

Denote the interior maximizer of  $\Pi_M^H(u)$  (if it exists) by  $u^H$ . Then  $u^H$  must satisfy the second-order condition

$$\frac{d^2\Pi_M^H(u)}{du^2} < 0. ag{12}$$

We simplify the expression in (12) by imposing the transformation  $x = \sqrt{4 - 5c/(ku^2)}$ , which is admissible because there is a one-to-one correspondence between  $u \in [\sqrt{4c/(3k)}, \infty)$  and  $x \in [\frac{1}{2}, 2)$ . Equation (12) then becomes

$$kc(x^5 - 8x^3 + 192x^2 - 341x + 768) - 50x\sqrt{5kc(4 - x^2)} < 0$$
 or 
$$\sqrt{kc} < \frac{50x\sqrt{5(4 - x^2)}}{(x^5 - 8x^3 + 192x^2 - 341x + 768)}.$$

We can show that any (u,c) satisfying (12) also satisfies the encroachment constraint by performing the same transformation  $x = \sqrt{4-5c/(ku^2)}$  in (11) (note  $t = \frac{6}{5} - 2x/5$ ); we thus have the equivalent constraint  $\sqrt{kc} < (5(x+1)\sqrt{5(4-x^2)})/(3(21+17x-5x^2-x^3))$ . It is straightforward to verify that

$$\frac{50x\sqrt{5(4-x^2)}}{x^5-8x^3+192x^2-341x+768} < \frac{5(x+1)\sqrt{5(4-x^2)}}{3(21+17x-5x^2-x^3)}$$

for all  $x \in [\frac{1}{2}, 2)$ , so all interior maxima for  $\Pi_M^H(u)$  are feasible. Next we consider

$$\begin{split} \frac{d\Pi_{M}^{H}}{dc} &= \frac{\partial \Pi_{M}^{H}(t(u^{H}), u^{H})}{\partial t} \left( \frac{\partial t(u^{H})}{\partial u} \frac{du^{H}}{dc} + \frac{\partial t(u^{H})}{\partial c} \right) \\ &+ \frac{\partial \Pi_{M}^{H}(t(u^{H}), u^{H})}{\partial u} \frac{du^{H}}{dc} + \frac{\partial \Pi_{M}^{H}(t(u^{H}), u^{H})}{\partial c} \end{split}$$



$$= \frac{\partial \Pi_M^H(t(u^H), u^H)}{\partial c} \quad \text{(because } \frac{\partial \Pi_M^H(t(u^H), u^H)}{\partial t} = 0$$
 
$$\text{and } \frac{\partial \Pi_M^H(t(u^H), u^H)}{\partial u} = 0\text{)}$$
 
$$= \frac{c(8-t) - u^H(4t^2ku^H + t(ku^H - 5) - 8ku^H + 8)}{2(8-5t)ku^H}.$$

Substituting  $x = \sqrt{4 - 5c/(ku^2)}$  and  $t = \frac{6}{5} - 2x/5$  into  $d\Pi_M^H/dc < 0$  yields the equivalent condition  $\sqrt{kc} < (5\sqrt{5(4-x^2)})/(-x^2-24x+81)$ . Again we have

$$\frac{50x\sqrt{5(4-x^2)}}{x^5 - 8x^3 + 192x^2 - 341x + 768} < \frac{5\sqrt{5(4-x^2)}}{-x^2 - 24x + 81}$$

for all  $x \in [\frac{1}{2}, 2)$ , so the SOC implies that  $d\Pi_M^H/dc < 0$ . Therefore, the interior maximum profit under high-quality encroachment  $\Pi_M^H$  is decreasing in c.

Recall that  $\Pi_M^U$  is U-shaped in c,  $d\Pi_M^U/dc|_{c=2/(21k)}=0$  (see the proof of Proposition 1). We shall compare  $\Pi_M^H$  and  $\Pi_M^U$  for  $c \ge 1/(12k)$  in two cases.

Case 1.  $1/(12k) \le c < 2/(21k)$ . Here both  $\Pi_M^H$  and  $\Pi_M^U$  are decreasing in c. For  $x = \sqrt{4 - 5c/(ku^2)}$ , solving  $\max_u \Pi_M^H(u)$  subject to  $u \in [\sqrt{4c/(3k)}, \infty)$  is equivalent to solving  $\max_x \Pi_M^H(u(x))$  subject to  $x \in [\frac{1}{2}, 2)$ . To ease the notation, we let  $\Pi_M^H(x) = \Pi_M^H(u(x))$  and use  $x^H$  to denote the optimal solution. We can show that  $\Pi_M^H$  is convex in c as follows:

$$\begin{split} \frac{d^2\Pi_M^H(x^H)}{dc^2} &= \frac{\partial^2\Pi_M^H(x^H)}{\partial c^2} + \frac{\partial^2\Pi_M^H(x^H)}{\partial x\partial c} \cdot \frac{dx}{dc} \\ &= \frac{\partial^2\Pi_M^H(x^H)}{\partial c^2} - \left(\frac{\partial^2\Pi_M^H(x^H)}{\partial x\partial c}\right)^2 \bigg/ \frac{\partial^2\Pi_M^H(x^H)}{\partial x^2} > 0. \end{split}$$

The inequality holds because

$$\frac{\partial^2 \Pi_M^U(x^H)}{\partial c^2} = \frac{3kc(x^2 + 38x + 57)(x - 3)^2 - 25(4 - x^2)}{80\sqrt{5}(4 - x^2)^{3/2}k^{5/2}c^{3/2}} > 0,$$

which follows from  $kc>25(4-x^2)/(3(3-x)^2(57+38x+x^2))$  (the right-hand side is bounded from above by 1/12). Because  $d\Pi_M^U/dc|_{c=1/(12k)}>d\Pi_M^H/dc|_{c=2/(21k)}$ , the convexities of  $\Pi_M^H$  and  $\Pi_M^U$  in c imply that

$$\left. \frac{d\Pi_M^U}{dc} > \frac{d\Pi_M^U}{dc} \right|_{c=1/(12k)} > \left. \frac{d\Pi_M^H}{dc} \right|_{c=2/(21k)} > \frac{d\Pi_M^H}{dc}$$
 for all  $\frac{1}{12k} \le c < \frac{2}{21k}$ .

As a result,  $\Pi_M^U - \Pi_M^H$  is strictly increasing in c. Since  $\Pi_M^U - \Pi_M^H < 0$  at both c = 1/(12k) and c = 2/(21k), it follows that  $\Pi_M^U$  and  $\Pi_M^H$  do not intersect in  $c \in (1/(12k), 2/(21k))$ .

Case 2. c > 2/(21k). Here  $\Pi_M^H$  is strictly decreasing in c but  $\Pi_M^U$  is strictly increasing in c. Since  $\Pi_M^U < \Pi_M^H$  at c = 2/(21k) and  $\Pi_M^U > \Pi_M^H$  at c = 1/(10k), there is a unique threshold  $c_1 \in (2/(21k), 1/(10k))$  at which  $\Pi_M^U < \Pi_M^H$  iff  $c < c_1$ . Numerically, we find  $c_1 \approx 0.0973/k$ .

Finally, it is easy to show that an interior maximizer of  $\Pi_M^H(u)$  exists for  $c < c_1$  because  $d\Pi_M^H(\sqrt{5c/(3k)})/du > 0$  and  $d\Pi_M^H(1/(2k))/du < 0$ .

Step 2 (encroachment versus no encroachment). We prove that  $q_M^* = 0$  if and only if c is larger than a certain threshold.

Claim 2. The manufacturer encroaches if and only if  $c \le c_2 \approx 0.1019/k$ .

Proof of Claim 2. See the e-companion.  $\Box$ 

PROOF OF PROPOSITION 4(II). The manufacturer always wins because  $\Pi_M^* \ge \Pi_M^U > \Pi_M^M$  for all  $c < c_2$  (the latter inequality follows from Proposition 1 and  $\tilde{c} = c_2$ ). Similarly, if  $c_1 \le c < c_2$  then  $\Pi_R^u = \Pi_R^u < \Pi_R^u$ .

If  $c < c_1$  then we show that  $\Pi_R^*$  is increasing in c by substituting  $x^* = \sqrt{4 - 5c/(ku^{*2})}$  into

$$\begin{split} \frac{d\Pi_R^*}{dc} &= \frac{\partial \Pi_R(u^*)}{\partial c} + \frac{\partial \Pi_R(u^*)}{\partial u} \cdot \frac{du^*}{dc} \\ &= \frac{\partial \Pi_R(u^*)}{\partial c} + \frac{\partial \Pi_R(u^*)}{\partial u} \left( -\frac{\partial^2 \Pi_M(u^*)}{\partial u \partial c} \middle/ \frac{\partial^2 \Pi_M(u^*)}{\partial u^2} \right), \end{split}$$

which yields

$$\frac{d\Pi_R^*}{dc} = A(x)[50\sqrt{5}(4x^3 + 3x^2 - 16x - 12)\sqrt{ck/(4 - x^2)} + ck(2x^5 - 9x^4 - 182x^3 + 84x^2 + 72x + 4,185)]$$

for  $A(x) = (k(x-3)^2/(6250x(4-x^2)))/(\partial^2 \Pi_M^*/\partial u^2) < 0$ . Combining the FOC (substitute x into  $d\Pi_M^H/du = 0$ )

$$ck(x^4 + 146x^2 - 384x - 243) + 100\sqrt{5(4 - x^2)ck} - 25(4 - x^2) = 0$$
(13)

with the SOC in (12) yields

$$\sqrt{ck} < \frac{1}{4}\sqrt{\frac{4-x^2}{5}} \left(1 - \frac{500x^2(x^4 + 146x^2 - 384x - 243)}{(x^5 - 8x^3 + 192x^2 - 341x + 768)^2}\right)$$

$$< \frac{-50\sqrt{5/(4-x^2)}(4x^3 + 3x^2 - 16x - 12)}{2x^5 - 9x^4 - 182x^3 + 84x^2 + 72x + 4,185}$$

for all  $x^* \in (\frac{1}{2}, 2)$ , which implies that  $d\Pi_R^*/dc > 0$ . Therefore,  $\Pi_R^*$  is increasing in c. Because  $\Pi_R^* < \Pi_R^N$  at  $c = c_1$ , the retailer always loses.  $\square$ 

Proof of Corollary 2. See the e-companion.  $\Box$ 

Proofs of Propositions 5–8. See the e-companion. □

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