Management Science

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John Morgan, Felix Várdy,

MANAGEMENT SCIENCE
Vol. 59, No. 6, June 2013, pp. 1344–1353
ISSN 0025-1909 (print) | ISSN 1526-5501 (online)

http://dx.doi.org/10.1287/mnsc.1120.1639

The Fragility of Commitment

John Morgan
Haas School of Business and Department of Economics, University of California, Berkeley,
Berkeley, California 94720, morgan@haas.berkeley.edu

Felix Várdy
Haas School of Business, University of California, Berkeley, Berkeley, California 94720; and
International Monetary Fund, Washington, DC 20431, fvardy@haas.berkeley.edu

We show that the value of commitment is fragile in many standard games. When the follower faces a small cost to observe the leader's action, equilibrium payoffs are identical to the case where the leader's actions are unobservable. Applications of our result include standard Stackelberg–Cournot and differentiated product Bertrand games, as well as forms of indirect commitment, highlighted in Bulow et al. [Bulow J, Geanakoplos J, Klemperer P (1985) Multimarket oligopoly: Strategic substitutes and strategic complements. *J. Political Econom.* 93:488–511]. Weakening full rationality in favor of boundedly rational solution concepts such as quantal-response equilibrium restores the value of commitment.

Key words: Cournot; Bertrand; Stackelberg; observation cost; value of commitment; first-mover advantage; second-mover advantage; costly leader game
History: Received June 9, 2011; accepted August 15, 2012, by Peter Wakker, decision analysis. Published online in *Articles in Advance* January 8, 2013.

1. Introduction

An important lesson to emerge from game theory is the value of commitment. By tying its hands, a firm can influence the competitive response of its rival and gain a strategic advantage. Although this idea takes many forms, a simple illustration is the transformation of a simultaneous Cournot game into a sequential Stackelberg game. Here, the leader gains an advantage by committing to overproduce, which triggers scaled-back production by its rival.

This paper amends the standard model by adding the arguably realistic feature that observing the leader's action is costly, although the cost may be trivial. The follower can still observe the leader's action, but must pay for this information. Of course, the follower is free not to pay, in which case no information is revealed. Our main result shows that when strategically sophisticated players meet in such settings, the value of commitment vanishes completely, even if the cost of observation is arbitrarily small.

The intuition for the result proceeds in two steps: first, the result must hold if we limit attention to pure-strategy subgame-perfect equilibria; second, all subgame-perfect equilibria are, in fact, in pure strategies. The value of commitment is lost in all pure-strategy equilibria because the follower optimally chooses to *never* observe the leader's action. The reason is that, in a pure-strategy equilibrium, the leader takes a certain action with probability 1. Before observing, the follower holds some beliefs about which action the leader has taken and, in equilibrium, these beliefs are correct. Therefore, the follower can perfectly predict the leader's action. Thus, it is never rational for the follower to pay any amount, no matter how small, to observe and merely confirm his (correct) beliefs. This effectively reduces the game to a game with simultaneous moves, thereby destroying the value of commitment in pure-strategy equilibria.

The absence of mixed-strategy equilibria follows from the well-behavedness of the players' optimization problems. If the leader's optimization problem is strictly concave given subgame-perfect equilibrium beliefs about the follower's strategy, then the leader always has a *unique* best reply, given his beliefs. In that case, mixing is not optimal and the leader must be playing a pure strategy. Strict concavity of the follower's problem given his beliefs about—or observation of—the action of the leader leads to an analogous conclusion. Therefore, all equilibria are in pure strategies.

One might conjecture that our result is the consequence of a deliberate strategy by the follower to undermine the leader's value of commitment. After all, in a Stackelberg game, the follower is disadvantaged by the leader's ability to commit. Therefore, when the option of not observing presents itself, it may seem unsurprising that the follower chooses it. However, the result also holds for games where the follower *gains* from the leader's commitment, as in differentiated product Bertrand competition. Here, the problem is that, although the follower would like to convince the leader that he will pay to observe, in equilibrium, this promise is not credible. Similarly, our result also extends to indirect commitment, i.e., to situations where the value of commitment stems from actions in a related market, in the manner of Bulow et al. (1985).

Finally, we amend the model to allow for limited strategic sophistication. Here, our main result is that less strategic sophistication can restore the value of commitment. Specifically, if the leader is prone to making mistakes when choosing his actions, as in a quantal-response equilibrium, then the value of commitment is restored, provided that observation costs are small relative to the degree of the leader's bounded rationality. Similarly, when the follower incurs cognitive processing costs for formulating beliefs about the leader's action, paying to observe can economize on these costs. If observation costs are small relative to computation costs, again, the value of commitment is restored.

The contribution of our paper is twofold. First, by "unpacking" the usual leader–follower setup and endogenizing the observation decision of the follower, our results highlight a key implicit assumption in the standard analysis. The conventional recipe for commitment has two key ingredients: (1) irreversibility and (2) observability (see, e.g., Dixit and Skeath 2004). However, as we show, it is not enough that the leader's action *can* be observed. For commitment to have value, his action *must* be observed. But if observing is at all costly, this will generally not happen in equilibrium, because it is not incentive compatible. In other words, unless the follower is somehow committed to observing the leader's action, he will not do so. As a result, the value of commitment breaks down.

The second contribution is to identify a novel and practically relevant source of commitment value in business environments, namely, limited strategic sophistication. "Humanizing" the players by allowing for mistakes or cognitive processing costs makes paying to observe optimal and thereby restores the value of commitment. The necessary bounds on strategic sophistication vary with the cost of observation. When observation is cheap, small amounts of bounded rationality suffice. When observation is expensive, more generous amounts of bounded rationality are required.

To summarize, the value of commitment is most fragile when parties are strategically sophisticated and most robust under bounded rationality. Hence, the value of commitment is unusual in that it works better in practice than in (neoclassical) theory.

This paper proceeds as follows: The remainder of this section places our findings in the context of the extant literature. In §2, we show that the value of commitment is destroyed in Stackelberg–Cournot and differentiated product Bertrand games when observing the leader's action is costly. In §3, we offer a general model illustrating the broad applicability of the idea. Section 4 extends this result. Using the multimarket oligopoly model of Bulow et al. (1985), we show that *indirect* commitment is likewise destroyed by small observations costs. In §5, we relax the full rationality assumption in favor of quantal-response equilibrium and computational costs. Under both specifications of bounded rationality, the value of commitment is restored when observation costs are sufficiently low. Section 6 concludes. In Appendix A we show that, in the Stackelberg–Cournot model with costly observation, the fragility of commitment carries over to an arbitrary number of followers. Appendix B contains proofs relegated from the main text.

Related Literature. Worries about the fragility of commitment date back to the seminal paper by Bagwell (1995). He points out that, when the follower receives a noisy signal about the leader's action rather than observing it precisely, the value of commitment is destroyed in any pure-strategy equilibrium. Van Damme and Hurkens (1997) show that when mixed-strategy equilibria are admitted, this conclusion is reversed: there always exists a mixed-strategy equilibrium that preserves the value of commitment as the signal noise vanishes.

Várdy (2004) considers the same issue, but endogenizes the follower's observation decision in the same fashion as we do. His main findings parallel those in noisy observation games—commitment is destroyed in pure strategies and preserved in mixed strategies. Morgan and Várdy (2004) present experimental evidence that, in practice, the value of commitment survives when observation costs are sufficiently small.

Presumably for tractability reasons, this earlier literature assumed that the action space was discrete. However, Morgan and Várdy (2007) point out that, at least in the setting of contests, this assumption is not innocuous. In particular, they find that the value of commitment is destroyed in *all* equilibria when observation is costly and the action space is continuous. Of course, the contest setting is rather special—best-response functions are nonmonotonic and contests are ill-suited to analyze market competition among firms. Our contribution is to substantially generalize this earlier result. In particular, we show that the value of commitment in many standard models of imperfect competition suffers from a fragility problem when observation is endogenized. Perhaps, more importantly, we highlight how forms of bounded rationality create new and realistic avenues for restoring the value of commitment in business settings.

2. Cournot and Bertrand Competition

To illustrate the main point of the paper, we begin by analyzing the effect of endogenous and costly observation on the value of commitment in two workhorse models of imperfect competition: Stackelberg–Cournot quantity competition and differentiated product Bertrand price competition.

2.1. Stackelberg–Cournot Games

Perhaps the earliest application of the value of commitment is due to von Stackelberg (1934). Von Stackelberg observed that when two otherwise identical firms engage in quantity competition, it is to the advantage of either of the firms to commit to its quantity ahead of the other. Upon observing the inflated quantity chosen by the leader, the follower optimally reduces its quantity choice, which increases the leader's profits.

A simple illustration of this idea is as follows: There are two firms, $i = 1, 2$, each of which chooses a quantity x_i. They face a linear inverse demand curve $P = 1 - x_1 - x_2$ and have zero production costs. When the firms choose quantities simultaneously, the unique Nash equilibrium is for each to produce $x_i = 1/3$, thereby earning profits $\pi_i = 1/9$. If, however, firm 1 gets to choose its quantity first and this quantity is observed by firm 2, then firm 1 selects $x_1 = 1/2$. Firm 2 replies with $x_2 = 1/4$, and firm 1 gains a first-mover advantage by virtue of its ability to commit to a larger quantity ahead of its rival. Indeed, in this case, firm 1 earns $\pi_1 = 1/8$, whereas firm 2 earns only $\pi_2 = 1/16$.

Now consider a variation of the sequential game. As before, firm 1 selects its quantity first. Following this, the decision by firm 2 to observe firm 1's choice is endogenous. Firm 2 can pay a cost $\varepsilon \geq 0$ and perfectly observe firms 1's quantity, or decline this option and observe nothing prior to making its own quantity choice. First, suppose that the observation cost is $\varepsilon = 0$. In that case, observation is endogenous, but there are no "frictions" associated with firm 2's decision to observe. Although firm 2 might prefer not to observe so as to negate firm 1's first-mover advantage, this is not credible. To see this, notice that any mixed strategy in which firm 2 sometimes chooses not to observe firm 1's choice is weakly dominated by the pure strategy where firm 2 observes with probability 1. Therefore, in any subgame-perfect equilibrium in undominated strategies, firm 2 must observe firm 1's choice with certainty. Hence, the unique subgame-perfect equilibrium in undominated strategies induces the Stackelberg outcome.

A similar argument can be made more generally: endogenizing the observation decision in a frictionless setting does nothing to undermine the value of commitment. Thus, for the remainder of the paper, we focus on the more realistic situation where observation costs are strictly positive. In that case, the situation changes dramatically, as the following proposition shows.

Proposition 1. *In the Stackelberg model with observation costs, there is no first-mover advantage. That is, the Cournot outcome obtains in the unique subgame-perfect equilibrium.*

Why is the value of commitment destroyed? When observation is costly, the only way for firm 1 to derive any advantage from commitment is to induce firm 2 to observe its quantity choice at least some of the time. However, for that to happen, firm 2 must derive value from this observation. If firm 1 plays a pure strategy, in equilibrium, its quantity will be perfectly anticipated by firm 2, and there is no point in observing. Thus, firm 1 must randomize its choice to derive value from commitment. However, randomization runs afoul of the fact that, in standard Stackelberg–Cournot games, firm 1's problem is strictly concave. Put differently, firm 1 cannot credibly implement a randomization strategy because, regardless of its beliefs about firm 2's strategy, its optimization problem always has a *unique* global maximizer. We may therefore conclude that, in any equilibrium, firm 1 does not randomize and firm 2 does not observe. The game becomes, in effect, simultaneous, and there is no value of commitment.

2.2. Differentiated Product Bertrand Games

One might conjecture that the fragility of the value of commitment in the Stackelberg–Cournot game stems from the fact that firm 1's commitment makes firm 2 worse off. Indeed, given the opportunity, firm 2 would prefer to move simultaneously or commit to never observe firm 1's quantity. Our next application shows that this intuition is wrong.

Recall that in the standard differentiated product Bertrand setting, both firms prefer to move sequentially rather than simultaneously. Thus, if it could commit, firm 2 would prefer to *always* observe the action taken by firm 1. However, this is not credible. As we now show, despite the fact that both firms would benefit, the value of commitment is still destroyed when observation is costly.

Suppose that firms face linear demand curves $q_i = 1 - x_i + x_j$, where x_i is now interpreted as the price chosen by firm i. We then have the following result.

Proposition 2. *In the differentiated product Bertrand model with observation costs, commitment has no value, and there is no second-mover advantage.*

The point is that because firm 1 cannot credibly commit to randomize its price (though it would like to), firm 2 cannot commit to observe firm 1's move. The end result is that the game collapses to what is, essentially, a simultaneous move game.

3. A Generalization

Next, we generalize the examples given above to describe a class of sequential games where the value of commitment vanishes in the presence of observation costs. For future reference, we refer to this game, along with Assumptions 1 and 2 below, as "the general model with observation costs."

Consider the following sequential-move game. First, player 1 takes an action $x_1 \in X_1 = [\underline{x}_1, \bar{x}_1]$. Then, player 2 gets to observe player 1's action if and only if he pays an observation cost $\varepsilon > 0$. Finally, player 2 takes an action $x_2 \in X_2 = [\underline{x}_2, \bar{x}_2]$, and payoffs are realized. The payoff Π_1 to player 1 only depends on the pair of actions x_1, x_2. The payoff Π_2 to player 2 also depends on whether he has observed player 1's action. That is, $\Pi_1 = \pi_1(x_1, x_2)$ and $\Pi_2 = \pi_2(x_2, x_1) - I\varepsilon$, where I is an indicator function that is equal to 1 if player 2 chose to observe and 0 otherwise.

We make the following (fairly standard) assumptions about the profit functions $\pi_i(x_i, x_j)$, $i \in \{1, 2\}$ and $j \neq i$:

Assumption 1. *Profit function $\pi_i(x_i, x_j)$ is continuous, twice differentiable, and strictly concave in $x_i \in X_i$.*

Assumption 1 is a usual one for obtaining "well-behaved" payoff functions for both players. In particular, if $x_i(x_j)$ denotes the best response of player i to action x_j, then Assumption 1 implies that $x_i(x_j)$ is a continuous function. As a consequence of the structure of the best-response functions and the application of Brouwer's fixed-point theorem, we have the following fact:

Fact 1. *There exists a pure-strategy Nash equilibrium in the simultaneous move game.*

For analyzing subgame-perfect equilibria in sequential games, a slightly stronger assumption is often invoked. We shall do so here.

Assumption 2. *Profit function $\pi_1(x_1, x_2(x_1))$ is strictly concave in $x_1 \in X_1$.*

Assumption 2 implies that in the sequential game where player 2 observes 1's choice, player 1 has a unique best action. Together with Assumption 1, this implies the following:

Fact 2. *The sequential-move game without endogenous observation admits a unique subgame-perfect equilibrium.*

Notice that the examples previously studied satisfy these assumptions, as do many standard applications in the industrial organization literature. We now establish the following result.

Proposition 3. *In the general model with observation costs, all subgame-perfect equilibria of the sequential game are payoff equivalent to a Nash equilibrium of the simultaneous game. In other words, commitment has no value.*

To see why the proposition holds, notice that, regardless of player 2's decision to observe and subsequent play, firm 1's problem remains strictly concave. Thus, player 1's best response to any strategy by player 2 is to play a pure strategy. This implies that, in equilibrium, player 2 can perfectly anticipate player 1's choice; hence, there is no reason to pay to observe it. Absent observation by player 2, the game is in effect simultaneous, destroying any value of commitment.

Discrete Strategy Spaces. One might suspect that the argument above crucially relies on the fact that strategy spaces are *continuous*. This would severely limit the applicability of the results because, in most applications, the continuum is merely an analytically convenient way of representing the idea that the strategy grid is sufficiently fine. For instance, there clearly is some discreteness in the number of units a firm can produce. Similarly, there is a smallest unit of account governing a firm's pricing.

The next proposition demonstrates that the loss of commitment also holds for a sufficiently fine grid over the set of strategies. In other words, although the continuum assumption is helpful for mathematical tractability, it is not fundamental to the result that costly observation destroys the value of commitment. Define the grid $G_i(m)$ on X_i to be $\{\underline{x}_i, \underline{x}_i + (\bar{x}_i - \underline{x}_i)/m, \underline{x}_i + 2((\bar{x}_i - \underline{x}_i)/m), \ldots, \underline{x}_i + (m-1)((\bar{x}_i - \underline{x}_i)/m), \bar{x}_i\}$, where parameter is m some strictly positive integer. The larger is m, the finer is the grid.

Proposition 4. *Fix $\varepsilon > 0$. In the general model with observation costs and a sufficiently fine discretization of strategy spaces, commitment has no value.*

Formally, fix $\varepsilon > 0$ and constrain players' actions x_i, $i \in \{1, 2\}$, to values on the grid $G_i(m)$. For m sufficiently large, all subgame-perfect equilibria of the sequential game are payoff equivalent to a Nash equilibrium of the simultaneous game.

The key to this result is the order of limits: We fix the cost of observation, ε, and let the grid become arbitrarily fine. In that case, the value of commitment disappears. If, on the other hand, we fix a discrete grid $\{G_1(m), G_2(m)\}$ and let ε go to zero, then, as shown in Várdy (2004), there always exists a (mixed-strategy) equilibrium in which the value of commitment is preserved.

4. Indirect Commitment

In their seminal paper, Bulow et al. (1985; hereafter, BGK) highlight the value of indirect commitment. To see how indirect commitment works, consider a two-period model of a market with experience curves. In period 1, firm 1 acts as a monopolist and chooses its output. In period 2, perhaps because firm 1's patent protection has run out, firms 1 and 2 compete in quantities. Firm 1's marginal cost in period 2 is affected by its "experience" (i.e., output) in period 1. The more it produces in period 1, the lower its marginal cost in the next period. Hence, firm 1 achieves a strategic advantage and associated value of commitment by overproducing in period 1 relative to the case where it is a monopoly in both periods. As BGK show, the same idea holds whenever firms 1 and 2 engage in oligopolistic competition in some market, k, and prior to this, firm 1 commits to some action in a different market, l, that affects its marginal profitability in market k.

For the costly observation version of BGK, one might be tempted to apply Proposition 3 above. However, in BGK, the required conditions do not hold. Specifically, with indirect commitment, player 1 acts in *both* periods. Nevertheless, the same result obtains: when observation is costly, the value of (indirect) commitment disappears.

To see this, consider the following sketch of their model. First, player 1 takes an action $c_1 \in C_1 = [\underline{c}_1, \bar{c}_1]$. Next, players 1 and 2 simultaneously take actions $x_1 \in X_1 = [\underline{x}_1, \bar{x}_1]$ and $x_2 \in X_2 = [\underline{x}_2, \bar{x}_2]$, respectively. Finally, payoffs are realized. The payoff Π_1 to player 1 depends on c_1, x_1, x_2. The payoff Π_2 to player 2 depends on x_1, x_2. Hence, $\Pi_1 = \pi_1(x_1, x_2, c_1)$ and $\Pi_2 = \pi_2(x_2, x_1)$.

Consider two benchmark games: (1) Player 1's choice of c_1 is unobservable to player 2. (2) Player 1's choice of c_1 is observed perfectly by player 2. We shall refer to these benchmarks as the "simultaneous game" and the "sequential game," respectively. Throughout, we restrict attention to interior solutions.

We make two assumptions to ensure that the simultaneous and sequential games are well behaved.

BGK Assumption 1. *Profit function $\pi_1(c_1, x_1, x_2)$ is continuous, twice differentiable and negative definite on $[\underline{c}_1, \bar{c}_1] \times [\underline{x}_1, \bar{x}_1]$. Similarly, $\pi_2(x_2, x_1)$ is continuous, twice differentiable, and strictly concave in $x_2 \in X_2$.*

As a consequence of the structure of the best-response functions and Brouwer's fixed-point theorem, we have the following:

BGK Fact 1. *There exists a pure-strategy Nash equilibrium (c_1^*, x_1^*, x_2^*) in the simultaneous game.*

BGK Fact 2. *There are no mixed-strategy Nash equilibria in the simultaneous game.*

Thus, equilibrium in the simultaneous game is determined by solving the system of equations

$$\frac{\partial}{\partial x_1}\pi_1(x_1, x_2, c_1) = 0 \quad \text{and} \quad \frac{\partial}{\partial x_2}\pi_2(x_1, x_2) = 0, \tag{1}$$

and

$$\frac{\partial}{\partial c_1}\pi_1(x_1, x_2, c_1) = 0 \tag{2}$$

Next, we turn to the sequential game and study subgame-perfect equilibria. First, we need to ensure that following any history, c_1, the game is well behaved. Hence, we make the following assumption.

BGK Assumption 2. *For given c_1, there exists a unique Nash equilibrium $(x_1^*(c_1), x_2^*(c_1))$.*

Assumption 2 merely guarantees that equilibrium multiplicity and equilibrium selection do not play a strategic role in player 1's choice of c_1 in the first period.

BGK Assumption 3. *Profit function $\pi_1(c_1, x_1^*(c_1), x_2^*(c_1))$ is strictly concave in $c_1 \in [\underline{c}_1, \bar{c}_1]$.*

BGK Assumptions 1–3 imply that the sequential game has a unique subgame-perfect equilibrium, comprising a set of pure strategies $((c_1^{**}, x_1^*(c_1)), x_2^*(c_1))$. The program to find the subgame-perfect equilibrium entails: (1) solving

$$\frac{\partial}{\partial x_1}\pi_1(x_1, x_2, c_1) = 0 \quad \text{and} \quad \frac{\partial}{\partial x_2}\pi_2(x_1, x_2) = 0 \tag{3}$$

for x_1 and x_2 given c_1, (2) substituting the (unique) solution to the system given in (3) into player 1's objective function, and (3) choosing c_1 such that

$$\frac{\partial}{\partial c_1}\pi_1(x_1^*(c_1), x_2^*(c_1), c_1) = 0. \tag{4}$$

Costly Observation in the BGK Model. Now, let us amend the model and endogenize the observation decision. Suppose that player 2 gets to observe player 1's first-period action, c_1, if and only if he pays $\varepsilon > 0$. The observation decision is common knowledge. In this case, the payoff function for player 2 is $\Pi_2 = \pi_2(x_2, x_1) - I\varepsilon$.

Let x_i^{L}, $i = 1, 2$, denote player i's action conditional on player 2 observing ("looking"), whereas x_i^{NL} denotes player i's action conditional on player 2 not observing ("not looking"). Suppose that with probability p, player 2 observes player 1's choice of c_1. Then, player 1's problem corresponds to choosing c_1, x_1^{L}, and x_1^{NL} to maximize

$$E[\Pi_1] = p\pi_1(c_1, x_1^{\mathrm{L}}, x_2^{\mathrm{L}}) + (1-p)\pi_1(c_1, x_1^{\mathrm{NL}}, x_2^{\mathrm{NL}}).$$

By subgame perfection, we know that $x_1^{\mathrm{L}} = x_1^*(c_1)$ and $x_2^{\mathrm{L}} = x_2^*(c_1)$. Note that by BGK Assumption 1, we also know that x_2^{NL} is a pure strategy. Hence, player 1's problem reduces to choosing c_1 and x_1^{NL} to maximize

$$E[\Pi_1] = p\pi_1(c_1, x_1^*(c_1), x_2^*(c_1)) + (1-p)\pi_1(c_1, x_1^{\mathrm{NL}}, x_2^{\mathrm{NL}}). \tag{5}$$

The following technical lemma implies concavity of this problem.

Lemma 1. *The Hessian of $E[\Pi_1]$ in Equation* (5) *is negative definite on* $[\underline{c}_1, \bar{c}_1] \times [\underline{x}_1, \bar{x}_1]$.

Negative definiteness implies concavity. Hence, in any subgame-perfect equilibrium, c_1, x_1^{NL}, and x_2^{NL} are always in pure strategies.

From the fact that the systems (1) and (3) are identical, the following result is immediate.

Lemma 2. *In any subgame-perfect equilibrium of the BGK model with observation costs, following the equilibrium action* c_1, $x_2^{\mathrm{NL}} = x_2^{\mathrm{L}}$, *and* $x_1^{\mathrm{NL}} = x_1^{\mathrm{L}}$.

Because, in equilibrium, player 2 takes the same action regardless of whether he observes, there is no value to observing. As observation is costly, we may conclude the following:

Lemma 3. *In any subgame-perfect equilibrium of the BGK model with observation costs, player* 2 *never observes player* 1*'s action*.

Together, these lemmas imply the following result.

Proposition 5. *In the BGK model with observation costs, indirect commitment has no value.*

Formally, any subgame-perfect equilibrium of the costly observation game is payoff equivalent to a Nash equilibrium of the simultaneous game.

The key insight from the proposition is that the fragility of commitment readily extends to the case where commitment is indirect. Indeed, many real-world examples of commitment, such as cost and branding spillovers, are special cases of this model.

5. Is Commitment Really Worthless?

We have shown that the value of commitment in many standard models is extremely fragile. That is, under a small and arguably realistic perturbation, the value of commitment completely vanishes in all subgame-perfect equilibria of these games. This seems to suggest that managers should forget about trying to gain a strategic advantage by precommitting to a course of action. However, perhaps this is the wrong conclusion. Instead, one can reinterpret our result as a warning against an excessive reliance on full rationality in business strategy. That is, the fragility result itself may in turn be fragile to a departure from full rationality in the direction of (more realistic) bounded rationality.

For instance, if player 1 is prone to making mistakes, then his actions will be noisy, and player 2 will have an incentive to look. One way to formalize this is by studying quantal-response equilibria of the sequential game.[1] Here, the idea is that players "tremble" (or, equivalently, have some random component to their payoffs) but are more likely to choose actions that are more profitable than actions that are less profitable. As an illustration, we revisit the linear demand Stackelberg–Cournot model with observation costs. Player 1 is assumed to be boundedly rational in the sense of quantal-response equilibrium, whereas for simplicity, player 2 remains fully rational. (The basic idea readily extends to the case where both players are boundedly rational, as well as to the general model.)

Specifically, suppose that the probability that player 1 chooses action x_1 is given by the density

$$T(E[\Pi_1(x_1, E[x_2(x_1)])]),$$

where T is a transformation function that is increasing in its argument and, when integrated over x_1, is equal to 1. The dependence of x_2 on x_1 reflects the fact that when player 2 chooses to observe, 2's choice depends on player 1's action.

We will identify conditions such that player 2 observing player 1's action is part of all quantal-response equilibria. First, notice that $x_2(x_1)$ is a function—namely, $x_2 = (1/2)(1 - x_1)$—and, hence, the distribution of player 1's actions is fully specified by T. This gives rise to a cumulative distribution, H, over actions for player 1.

When player 2 chooses to observe player 1's action and best responds, he expects to earn

$$\begin{aligned} E^{\mathrm{L}}[\Pi_2] &= E\left[\tfrac{1}{2}(1-x_1)\left(1-x_1-\tfrac{1}{2}(1-x_1)\right)\right] \\ &= \tfrac{1}{4}(1-2E[x_1]+E[x_1^2]), \end{aligned}$$

whereas his expected payoff of not observing is

$$E^{\mathrm{NL}}[\Pi_2] = \tfrac{1}{4}(1-2E[x_1]+E^2[x_1]).$$

The difference in payoffs between observing and not observing is

$$E^{\mathrm{L}}[\Pi_2] - E^{\mathrm{NL}}[\Pi_2] = \tfrac{1}{4}(E[x_1^2]-E^2[x_1]) = \tfrac{1}{4}\mathrm{Var}[x_1].$$

Hence, player 2 strictly prefers to observe if and only if

$$\varepsilon < \tfrac{1}{4}\mathrm{Var}[x_1]. \tag{6}$$

Of course, the variance will depend on the particulars of the T function. When empirically estimating quantal-response equilibria, it is common to use a multinomial logit specification, where a single parameter λ characterizes the degree of rationality. As $\lambda \to \infty$, the model approaches full rationality and the variance of actions goes to zero. At the other

[1] See McKelvey and Palfrey (1995, 1998) for the original formulation. Subsequent formulations of this solution concept amenable to sequential games with continuous actions may be found in Goeree et al. (2005). See Baye and Morgan (2004) for an application of quantal-response equilibrium in Bertrand games with continuous actions.

extreme, when $\lambda = 0$, player 1's choice consists of a uniform distribution over the support of possible actions. Let us restrict attention to the case where ε is small enough so that the cost of observation does not overwhelm all possible benefit. In the logit case, this amounts to $\varepsilon < (1/4)(\bar{x}_1 - \underline{x}_1)^2/12$; i.e., if the first mover chooses his action with uniform probability over its support, the second mover prefers to observe. Our next proposition shows that, in this setup, the value of commitment is restored in all quantal-response equilibria if either ε or λ is sufficiently small.

Proposition 6. 1. *Fix some* $\lambda > 0$. *In the Stackelberg model with costly observation, for* ε *sufficiently small, the value of commitment is restored in all quantal-response equilibria.*

2. *Fix some* $\varepsilon \in (0, (1/4)(\bar{x}_1 - \underline{x}_1)^2/12)$. *In the Stackelberg model with costly observation, for* λ *sufficiently small, the value of commitment is restored in all quantal-response equilibria.*

The point of the proposition is that when the cost of observation is small, minor departures from full rationality are sufficient to overturn our negative conclusions regarding the fragility of commitment. Hence, although one typically thinks of the fully rational model as a useful approximation of economic decision making by firms, when studying the value of commitment, correctly modeling the appropriate degree of rationality is essential. Indeed, the boundedly rational model suggests that, in fact, there is a threshold cost above which the value of commitment is lost and below which it is preserved.

Although the quantal-response model shows how bounded rationality on the part of the leader can restore the value of commitment, a different form of bounded rationality on the part of the follower can serve the same role. Suppose that the follower incurs computation costs to determine his optimal action. Obviously, the cost of calculating his best response to a given set of beliefs is unaffected by whether he observes; however, when the follower chooses not to observe, he incurs additional processing costs to determine his beliefs about the leader's action. Paying to observe economizes on these costs. Again, when observation costs are sufficiently low, this will be worthwhile and the value of commitment is restored.

A conventional view holds that commitment strategies are most effective when both parties are strategically sophisticated and fully understand the implications of strategic moves. Our analysis suggests the opposite. When players are fully rational, the value of commitment disappears. Bounded rationality, on the other hand, can serve as a form of metacommitment that allows the leader to (correctly) believe that the follower will pay to observe his action. This restores the value of commitment.

6. Conclusions

An implicit assumption of most models of commitment is that a follower not just can, but *must* observe the leader's actions. However, when observation is costly, the follower will only observe when doing so is informationally valuable, which requires uncertainty about the leader's choice of action. Provided the game is "well behaved," the leader cannot credibly introduce this uncertainty by randomizing his actions. Hence, the follower fails to observe, and the value of commitment unravels. Indeed, for a broad class of games, small costs of observation are outcome and payoff equivalent to circumstances where observation costs are infinite.

To restore the value of commitment, the leader must ensure that the costs of observation are zero or even negative. Many business strategies that, at first blush, appear to be pure extravagance may in fact be attempts to achieve this. Lavish, celebrity-strewn launch parties are often seen as a perk for a firm's most valued stakeholders. Yet, their true target may be rivals who find it impossible to ignore the publicity surrounding these events.[2] In that sense, we offer a novel channel for how advertising can achieve commitment.

Bounded rationality offers another path to restore the value of commitment: on the part of the leader, when bounded rationality leads to decision errors; and on the part of the follower, when observation economizes on mental computation costs. In either case, when costs are sufficiently small, the follower will observe and thereby restore the value of commitment. In our view, bounded rationality models provide a more realistic description of firm behavior than the fully rational model. Hence, our results suggest that the value of commitment is a rare instance of a game-theoretic idea that works better in practice than in theory.

Acknowledgments
The authors thank Mitch Hoffman, Martin Kolmar, Dana Sisak, two anonymous referees, and the department editor for helpful comments and suggestions.

Appendix A. n-Player Stackelberg–Cournot Games

In this appendix, we show that Proposition 1 extends to n players. The economic environment is as specified in §2.1. However, instead of two, there are now n identical firms $i = 1, 2, \ldots, n$ competing in the market. First, firm 1 chooses its quantity x_1. Next, firms $2, 3, \ldots, n$ simultaneously decide whether to pay ε to observe firm 1's output.

[2] We thank an anonymous referee for this observation. Some evidence for this effect is found in MacMillan et al. (1985), who show that a more visible product launch leads to a greater delay in rivals entering the market.

Finally, these follower firms simultaneously choose quantities $x_2, x_3, \ldots, x_n$, and the market price and profits are realized.

Because firms $2, 3, \ldots, n$ are symmetric, we focus on equilibria where these firms play symmetric (but possibly mixed) strategies. Let x^{L} denote the equilibrium output of a follower firm that chose to observe x_1, and let x^{NL} denote the equilibrium output of a follower firm that chose not to observe x_1. Suppose that, in equilibrium, follower firms observe with probability p.

Because demand is linear, a follower's payoff only depends on the expected total output of all other firms. (Of course, conditional on observing, x_1 takes the place of $E[x_1]$.) Also notice that, because followers' payoffs are strictly concave in their own outputs x^{L} and x^{NL}, these equilibrium outputs must be in pure strategies.

From a follower firm's perspective, the expected number of other firms choosing to observe firm 1's output is $\alpha = (n-2)p$. Similarly, the expected number of other firms choosing not to observe x_1 is $\beta = n - 2 - \alpha$. Thus, when a follower firm chooses to observe, its best response, x^{L}, satisfies

$$x^{\mathrm{L}} = \tfrac{1}{2}(1 - x_1 - \alpha x^{\mathrm{L}} - \beta x^{\mathrm{NL}}). \tag{A1}$$

When a follower firm chooses not to observe, its best response, x^{NL}, satisfies

$$x^{\mathrm{NL}} = \tfrac{1}{2}(1 - E[x_1] - \alpha E[x^{\mathrm{L}}] - \beta x^{\mathrm{NL}}). \tag{A2}$$

Notice that $E[x^{\mathrm{L}}]$ represents the expectation of x^{L} taken over the realizations of x_1. This expectation must be consistent with the expression for x^{L} in Equation (A1). Taking expectations in Equation (A1) over realizations of x_1 yields

$$E[x^{\mathrm{L}}] = \tfrac{1}{2}(1 - E[x_1] - \alpha E[x^{\mathrm{L}}] - \beta x^{\mathrm{NL}}). \tag{A3}$$

Conditional on a quantity choice x_1 by firm 1, an equilibrium in the subgame between follower firms $2, 3, \ldots, n$ consists of a simultaneous solution to Equations (A1)–(A3). In matrix form, this system of equations may be written as

$$\begin{bmatrix} 1+\frac{1}{2}\alpha & \frac{1}{2}\beta & 0 \\ 0 & 1+\frac{1}{2}\beta & \frac{1}{2}\alpha \\ 0 & \frac{1}{2}\beta & 1+\frac{1}{2}\alpha \end{bmatrix} \begin{bmatrix} x^{\mathrm{L}} \\ x^{\mathrm{NL}} \\ E[x^{\mathrm{L}}] \end{bmatrix} = \begin{bmatrix} \frac{1}{2}(1-x_1) \\ \frac{1}{2}(1-E[x_1]) \\ \frac{1}{2}(1-E[x_1]) \end{bmatrix}.$$

Inverting the matrix, we find

$$\begin{bmatrix} x^{\mathrm{L}} \\ x^{\mathrm{NL}} \\ E[x^{\mathrm{L}}] \end{bmatrix} = \begin{bmatrix} \dfrac{2}{\alpha+2} & -\dfrac{\beta}{\alpha+\beta+2} & \dfrac{\alpha\beta}{(\alpha+2)(\alpha+\beta+2)} \\ 0 & \dfrac{\alpha+2}{\alpha+\beta+2} & -\dfrac{\alpha}{\alpha+\beta+2} \\ 0 & -\dfrac{\beta}{\alpha+\beta+2} & \dfrac{\beta+2}{\alpha+\beta+2} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2}(1-x_1) \\ \frac{1}{2}(1-E[x_1]) \\ \frac{1}{2}(1-E[x_1]) \end{bmatrix}.$$

Hence,

$$x^{\mathrm{L}} = \frac{1}{n} - \frac{x_1 - (\beta/n)E[x_1]}{\alpha+2} \quad \text{and} \quad x^{\mathrm{NL}} = \frac{1}{n} - \frac{E[x_1]}{n}.$$

Taking expectations over x_1 using the solution for x^{L}, it may be readily verified that the expression of $E[x^{\mathrm{L}}]$ given in the matrix is indeed consistent. Therefore, we have found the unique symmetric equilibrium as a function of x_1 and $E[x_1]$.

Now, recall that firm 1's expected profits are

$$E[\Pi_1] = x_1(1 - x_1 - (n-1)px^{\mathrm{L}} - (n-1)(1-p)x^{\mathrm{NL}}).$$

Substituting for x^{L} and x^{NL} and simplifying, we obtain

$$E[\Pi_1] = x_1\left(1 - x_1 - (n-1)p\left(\frac{1}{n} - \frac{x_1 - (\beta/n)E[x_1]}{\alpha+2}\right) - (n-1)(1-p)\frac{1-E[x_1]}{n}\right).$$

This expression is strictly concave in x_1. Hence, in any equilibrium, firm 1 must be choosing a pure strategy. This implies that none of the follower firms will pay to observe firm 1's output and hence, in essence, the game is reduced to one with simultaneous moves. As a result, there is no value of commitment.

To summarize, we have shown the following:

Proposition 7. *In the n-player Stackelberg model with observation costs, there is no first-mover advantage. That is, in the unique symmetric subgame-perfect equilibrium, the Cournot outcome obtains.*

Appendix B. Proofs

Proof of Proposition 1. Following firm 1's quantity choice, the continuation play in any subgame-perfect equilibrium is as follows: Firm 2 chooses to observe 1's quantity with probability p. If firm 2 observes, then it best responds to firm 1's action by selecting a quantity

$$x_2(x_1) = \tfrac{1}{2}(1 - x_1).$$

If firm 2 does not observe, then 2 selects a quantity x_2 according to some cdf F.

Now consider the optimization problem of firm 1 given its beliefs about the continuation. Firm 1's expected profits from choosing x_1 are

$$E[\Pi_1] = x_1\left(p(1 - x_1 - x_2(x_1)) + (1-p)\int_{X_2}(1 - x_1 - t)\,dF(t)\right).$$

Substituting for $x_2(x_1)$ gives

$$E[\Pi_1] = x_1\left(p\tfrac{1}{2}(1-x_1) + (1-p)(1 - x_1 - E[x_2])\right). \tag{B1}$$

Notice that this is a quadratic expression in x_1. Because it is strictly concave, this function attains a unique global maximum. Therefore, optimizing play by firm 1 always entails choosing a pure strategy.

Now consider firm 2's situation. It knows that firm 1 plays a pure strategy. Moreover, in equilibrium, firm 2 correctly anticipates what this pure strategy is. Hence, it is not a best response for firm 2 to pay the observation cost $\varepsilon > 0$. Of course, firm 1 is aware of this and realizes that changes in its quantity provoke no reaction from firm 2. Hence, in equilibrium, both firms choose their quantities as in the simultaneous game, and there is no value of commitment. □

Proof of Proposition 2. Firm 1's expected profit is

$$E[\Pi_1] = x_1\left(p(1-x_1+x_2(x_1))+(1-p)\int_{X_2}(1-x_1+t)\,dF(t)\right)$$
$$= x_1\left(p\left(1+\tfrac{1}{2}(1-x_1)\right)+(1-p)(1-x_1+E[x_2])\right). \quad \text{(B2)}$$

Equation (B2) is analogous to Equation (B1). Thus, using identical arguments as in the Stackelberg–Cournot case, it follows that, in the unique subgame-perfect equilibrium of the sequential Bertrand game with observation costs, the price levels are identical to those chosen in the simultaneous move game. Hence, commitment has no value, and there is no second-mover advantage. □

Proof of Proposition 3. First, we show that player 1 always plays a pure strategy in all subgame-perfect equilibria. Then we argue that player 2 never pays to observe player 1's move. Finally, we conclude that the value of commitment is lost completely.

With probability p, player 2 observes player 1's action, x_1. Conditional on observing, subgame perfection implies that player 2 plays his unique best response $x_2(x_1)$. With probability $(1-p)$, player 2 does not observe player 1's action. In that case, we represent player 1's beliefs about player 2's action by the cdf $F(x_2)$. Player 1's expected profits, $E[\Pi_1(x_1)]$, are

$$E[\Pi_1(x_1)] = p\pi_1(x_1, x_2(x_1)) + (1-p)\int_{X_2}\pi_1(x_1, x_2)\,dF(x_2).$$

In this expression, the integral is strictly concave in x_1 because $\pi_1(x_1, x_2)$ is strictly concave in x_1 for each x_2. Moreover, the function $\pi_1(x_1, x_2(x_1))$ is strictly concave by assumption. Finally, as a convex combination of two expressions that are strictly concave in x_1, $E[\Pi_1(x_1)]$ is also strictly concave.

Strict concavity of $E[\Pi_1(x_1)]$ implies that player 1 has a unique best response to player 2's anticipated behavior. In turn, this implies that player 1 cannot be mixing and must be playing a pure strategy. Because player 1 is playing a pure strategy, in equilibrium, player 2 can perfectly predict player 1's action. Therefore, it is not rational for player 2 to pay any positive amount, no matter how small, to observe player 1's action and merely confirm his (correct) beliefs.

The fact that player 2 never observes player 1's action reduces the game to one of, essentially, simultaneous moves. Hence, any subgame-perfect equilibrium of the sequential game with observation costs must be payoff equivalent to a Nash equilibrium of the simultaneous move game. □

Proof of Proposition 4. We prove the result by showing that when m is sufficiently large, the second player chooses to never observe the first player's action.

Clearly, in any equilibrium where player 1 plays a pure strategy, player 2 never observes. Next, we note that as m goes to infinity, player 1's payoffs in the discrete version of the sequential game converge pointwise to player 1's payoffs when actions are continuous. We will show that this implies that for any sequence of mixed-strategy equilibria, the maximum distance between actions in the support of player 1's equilibrium mixture goes to zero as m goes to infinity. That is, the equilibrium mixed strategies converge to a pure strategy as the grid becomes arbitrarily fine.

Let $x_{1,0}(m)$ and $x_{1,1}(m)$ denote the upper and lower end points of the support of player 1's equilibrium mixed strategy when the grid is m. By contradiction, suppose that the distance between $x_{1,0}(m)$ and $x_{1,1}(m)$ converges to some $\delta > 0$ when $m \to \infty$. Then, for m sufficiently large, either player 1 earns strictly more from playing $x_{1,0}(m)$ than $x_{1,1}(m)$ (or the reverse), or there exists an action $x'(m) \in (x_{1,0}(m), x_{1,1}(m))$ which, by pointwise convergence and strict concavity of player 1's payoffs, must yield player 1 a strictly greater payoff than $x_{1,0}(m)$ and $x_{1,1}(m)$. Both cases are incompatible with player 1 mixing over $x_{1,0}(m)$ and $x_{1,1}(m)$. Hence, it must be that $|x_{1,1}(m) - x_{1,0}(m)| \to 0$ as $m \to \infty$.

As the distance between player 1's equilibrium actions becomes arbitrarily small, by continuity of player 2's payoffs, player 2 is strictly better off not expending ε to observe player 1's action for m sufficiently large. This proves the result. □

Proof of Lemma 1. First, notice that by BGK Assumptions 1 and 3,

$$\frac{\partial^2 E[\Pi_1]}{\partial c_1^2} = p\frac{\partial^2\pi_1(c_1, x_1^*(c_1), x_2^*(c_1))}{\partial c_1^2} + (1-p)\frac{\partial^2\pi_1(c_1, x_1^{\mathrm{NL}}, x_2^{\mathrm{NL}})}{\partial c_1^2} < 0.$$

Next, notice that by BGK Assumption 1,

$$\frac{\partial^2 E[\Pi_1]}{(\partial x_1^{\mathrm{NL}})^2} = (1-p)\frac{\partial^2\pi_1(c_1, x_1^{\mathrm{NL}}, x_2^{\mathrm{NL}})}{(\partial x_1^{\mathrm{NL}})^2} < 0.$$

Finally, notice that

$$\frac{\partial^2 E[\Pi_1]}{\partial x_1^{\mathrm{NL}}\partial c_1} = (1-p)\frac{\partial^2\pi_1(c_1, x_1^{\mathrm{NL}}, x_2^{\mathrm{NL}})}{\partial x_1^{\mathrm{NL}}\partial c_1}.$$

Hence, it remains to show that

$$\frac{\partial^2 E[\Pi_1]}{\partial c_1^2}\frac{\partial^2 E[\Pi_1]}{(\partial x_1^{\mathrm{NL}})^2} > \frac{\partial^2 E[\Pi_1]}{(\partial x_1^{\mathrm{NL}}\partial c_1)^2}.$$

This follows from

$$\begin{aligned}
&\frac{\partial^2 E[\Pi_1]}{\partial c_1^2}\frac{\partial^2 E[\Pi_1]}{(\partial x_1^{\mathrm{NL}})^2}\\
&= (1-p)^2\frac{\partial^2\pi_1(c_1, x_1^{\mathrm{NL}}, x_2^{\mathrm{NL}})}{\partial c_1^2}\frac{\partial^2\pi_1(c_1, x_1^{\mathrm{NL}}, x_2^{\mathrm{NL}})}{(\partial x_1^{\mathrm{NL}})^2}\\
&\quad + p(1-p)\frac{\partial^2\pi_1(c_1, x_1^*(c_1), x_2^*(c_1))}{\partial c_1^2}\frac{\partial^2\pi_1(c_1, x_1^{\mathrm{NL}}, x_2^{\mathrm{NL}})}{(\partial x_1^{\mathrm{NL}})^2}\\
&> (1-p)^2\frac{\partial^2\pi_1(c_1, x_1^{\mathrm{NL}}, x_2^{\mathrm{NL}})}{\partial c_1^2}\frac{\partial^2\pi_1(c_1, x_1^{\mathrm{NL}}, x_2^{\mathrm{NL}})}{(\partial x_1^{\mathrm{NL}})^2}\\
&> (1-p)^2\left(\frac{\partial^2\pi_1(c_1, x_1^{\mathrm{NL}}, x_2^{\mathrm{NL}})}{\partial x_1^{\mathrm{NL}}\partial c_1}\right)^2 = \left(\frac{\partial^2 E[\Pi_1]}{\partial x_1^{\mathrm{NL}}\partial c_1}\right)^2,
\end{aligned}$$

where the last inequality is a consequence of BGK Assumption 1. This establishes negative definiteness of the Hessian on $[\underline{c}_1, \bar{c}_1] \times [\underline{x}_1, \bar{x}_1]$. □

Proof of Proposition 6. Commitment is fully restored if and only if player 2 observes with probability 1. Suppose, to the contrary, that there exists a quantal-response equilibrium where, for all ε (respectively, λ) player 2 sometimes does not observe. This behavior by player 2 induces a quantal response by player 1 with variance $V > 0$.

Part 1. Equation (6) then implies that, for all $\varepsilon < (1/4)V$, player 2 is better off always observing. This is a contradiction.

Part 2. As $\lambda \to 0$, player 1's action becomes uniformly distributed over its support. Hence, $\lim_{\lambda \to 0} V = (\bar{x}_1 - \underline{x}_1)^2/12$. By Equation (6), player 2 is then better off always observing. Again, this is a contradiction. □

References

Bagwell K (1995) Commitment and observability in games. *Games Econom. Behav.* 8:271–280.

Baye M, Morgan J (2004) Price dispersion in the lab and on the Internet: Theory and evidence. *RAND J. Econom.* 35:449–466.

Bulow J, Geanakoplos J, Klemperer P (1985) Multimarket oligopoly: Strategic substitutes and strategic complements. *J. Political Econom.* 93:488–511.

Dixit AK, Skeath S (2004) *Games of Strategy*, 2nd ed. (W. W. Norton & Company, New York).

Goeree J, Holt C, Palfrey T (2005) Regular quantal response equilibrium. *Experiment. Econom.* 8:347–367.

MacMillan I, McCaffery M, Van Wijk G (1985) Competitors' responses to easily imitated new products-exploring commercial banking product introductions. *Strategic Management J.* 6:75–86.

McKelvey R, Palfrey T (1995) Quantal response equilibria for normal form games. *Games Econom. Behav.* 10:6–38.

McKelvey R, Palfrey T (1998) Quantal response equilibria for extensive form games. *Experiment. Econom.* 1:9–41.

Morgan J, Várdy F (2004) An experimental study of commitment and observability in Stackelberg games with observation costs. *Games Econom. Behav.* 49:401–423.

Morgan J, Várdy F (2007) The value of commitment in contests and tournaments when observation is costly. *Games Econom. Behav.* 60:326–338.

Van Damme E, Hurkens S (1997) Games with imperfectly observable commitment. *Games Econom. Behav.* 21:282–308.

Várdy F (2004) The value of commitment in Stackelberg games with observation costs. *Games Econom. Behav.* 49:374–400.

von Stackelberg H (1934) *Marktform und Gleichgewicht* (Springer-Verlag, Vienna/Berlin).