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Mirko Kremer, Laurens Debo

To cite this article:

Mirko Kremer, Laurens Debo (2016) Inferring Quality from Wait Time. *Management Science* 62(10):3023-3038. <http://dx.doi.org/10.1287/mnsc.2015.2264>

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# Inferring Quality from Wait Time

Mirko Kremer

Frankfurt School of Finance and Management, 60314 Frankfurt am Main, Germany, [m.kremer@fs.de](mailto:m.kremer@fs.de)

Laurens Debo

Tuck School of Business, Dartmouth College, Hanover, New Hampshire 03755, [laurens.g.debo@tuck.dartmouth.edu](mailto:laurens.g.debo@tuck.dartmouth.edu)

We study the impact of wait time on consumers' purchasing behavior when product quality is unknown to some consumers (the "uninformed consumers") but known to others (the "informed consumers"). In a capacitated environment, wait times act as a signal of quality for uninformed consumers because, due to informed consumers in the population, low (high) quality products tend to generate shorter (longer) wait times. Hence, longer wait times may increase uninformed consumers' perceived quality, and they may still purchase the product, even when the wait time is long. Similarly, short wait times decrease the consumers' perceived quality, and they may walk away despite the short wait—the "empty restaurant syndrome." This paper develops and tests a theory of observational learning that predicts these effects. We find that uninformed consumers' purchasing probability at short waits decreases in the presence of informed consumers. Furthermore, we find that relatively few informed consumers suffice to create this effect. Finally, we show that the purchasing frequency might even increase in the wait time.

Data, as supplemental material, are available at <http://dx.doi.org/10.1287/mnsc.2015.2264>.

**Keywords:** queues; quality signals; observational learning; experiments

**History:** Received August 3, 2012; accepted September 24, 2014, by Martin Lariviere, operations management.

Published online in *Articles in Advance* December 10, 2015.

## 1. Introduction

Wait times are generally badly perceived by firms and their customers alike, prompting firms to invest significant effort in reducing wait times for their customers (Larson 1987, Dakss 2007, Barnes 2010). The bulk of the operations management literature is concerned with mitigating costs caused by wait times for a product or for services. Folk wisdom, however, reveals that wait times may also influence the perception of the value of the good for which consumers have to wait. For example, observers associate long shipping delays of Apple's iPad with high demand and superior quality.<sup>1</sup> In service contexts such as restaurants, Raz and Ert (2008) empirically find a strong correlation between queue length at a restaurant and the choice of a restaurant, suggesting that uninformed customers (tourists) are more confident about the quality of restaurants with longer wait times. The flip-side, and a notion we will return to frequently, is the "empty restaurant syndrome"—a restaurant parlance that describes patrons not joining a restaurant when it is not busy, because they infer that the food quality is poor. These examples

have in common that a quality-related component of the product or service is not perfectly known to all consumers. The presence of informed consumers plays a key role in such environments because they will generate longer wait times on average for high-quality goods, turning wait times into informative signals for a priori uninformed consumers. Therefore, for uninformed consumers, finding an "empty" system may decrease the perceived value, and consumers may walk away despite the short wait time. On the other hand, long wait times may increase the perceived value and hence become more acceptable. In other words, wait times also influence the value perception of consumers, in addition to the (obvious) cost of waiting.

Our paper studies empirically the endogenous demand formation for a product whose quality is unknown to a part of the population. We first develop a theoretical equilibrium model that captures the key decision making and judgment dynamics in such an environment. We then extend the basic model to allow for decision error, using the framework of quantal response equilibrium (McKelvey and Palfrey 1995, Chen et al. 1997). The key prediction of our model analysis is that because of the presence of informed consumers, short/long wait times may decrease/increase the perceived value or quality

<sup>1</sup> See, for example, <http://www.ipadjailbreak.com/2011/03/lengthy-ipad-2-shipping-delays-already.html> (last accessed November 13, 2015).

of *uninformed* consumers. As a consequence, uninformed consumers are less likely to purchase at very short wait times (the “empty restaurant”) in the presence of informed consumers. On the other hand, when the perceived value increases faster than the waiting cost, it becomes possible that the purchasing probability increases in wait time. The model analysis supports a related comparative static result—wait times are more informative about quality when there are more informed consumers. Intuitively, for a tourist that is uninformed about quality, an empty restaurant is a stronger signal of low-quality service and food in a residential area with most patrons being familiar with the local restaurant scene, but it is less informative about quality in a tourist area where most patrons have little to no experience with the local restaurants.

We present the results from a set of laboratory experiments designed to test the key predictions from our model analysis. We consider a market with one firm and a given number of participants that are “potential consumers.” In each round of the experiment, Nature determines the quality of a product, and the type of each potential consumer (informed versus uninformed), both with some commonly known probability. Consumers are ranked in a random order and sequentially offered the possibility to purchase the product. The firm’s production capacity is normalized to one product per unit of time, which introduces a wait time (queue of orders) that is observed by the consumer before making a decision to purchase the product or not. The wait time depends on previous arrivals *and* decisions. After all orders have been collected, the firm starts producing and delivering the products to the consumers. We then vary the probability with which consumers are informed. The data provides strong support for the hypothesis that, in the presence of informed consumers, uninformed consumers learn value from wait times. Interestingly, a few informed consumers (5% on average) already influence significantly the uninformed consumers’ purchasing behavior, especially at short wait times, at which they over-infer low quality. Furthermore, we observe conditions where the purchasing probability locally *increases* in wait time.

Managerially, we illustrate the impact of individual level purchasing behavior on the firm’s sales. We find that the low-quality firm’s sales decrease as the presence of informed consumers increases, in accordance with equilibrium predictions. On the contrary, informed consumers may not always increase the high-quality firm’s sales. With only a few informed consumers in the population, the high-quality firm suffers because uninformed consumers over-infer low quality at a short wait time (as in the empty restaurant phenomenon), such that the system rarely generates

long wait times (at which uninformed consumers would infer high quality).

## 2. Literature Review

Our paper is related to work in consumer and social psychology that provide evidence for the effect of wait times on value perception, in contexts where the quality of the product or service is unknown or ambiguous. Giebelhausen et al. (2011) experimentally show that wait times can indeed be a signal about quality increasing both purchase intentions and experienced satisfaction. Similarly, Koo and Fischbach (2010) experimentally study how consumers’ value perception increases in the number of consumers waiting behind them. These papers investigate the value proposition of wait times, but they are silent on the issue of how wait times impact the actual decision of whether or not to buy the product or service. The purchasing decision is nontrivial, because longer wait times may not only indicate higher value but also higher waiting costs. Our study attempts to link the quality inferences from wait times to actual waiting decisions and, ultimately, to system behavior.

Our paper is also related to the theoretical and empirical literature on observational learning (see Chamley 2004 for an excellent review on social learning theories). Bikhchandani et al. (1991) and Banerjee (1992) describe situations in which investors observe privately some imperfect information about the return of an asset (that can either be a high-value or low-value asset), as well as the investment decisions taken by all earlier arriving investors. They show that, in the long run, rational investors will ignore their private information and follow their predecessors’ decision. That is, they “herd,” and may even all invest in the low-value asset instead of in the high-value asset. Anderson and Holt (1997) and Goeree et al. (2007) experimentally test the predictions of Bikhchandani et al. (1991). They find that subjects in the laboratory indeed herd on their predecessors’ decisions, but less than predicted by theory. Our study differs from these in two important regards. Importantly, the literature above assumes that decision makers observe the entire history of decisions that have been made before them. Our analysis of the unobservable rank case relaxes this assumption, similar to previous modeling work where the decision maker can observe only her immediate predecessor’s decision (Çelen and Kariv 2004), or some random samples from past decision (Banerjee and Fudenberg 2004, Smith and Sørensen 2013). In our setting, all an arriving consumer observes is the wait time, which is a natural indicator of how many previously arriving consumers have purchased the product (queue or orders). Another key distinction of our study concerns the typical assumption in the above models that

decisions do not cause any negative externalities on subsequent decision makers. For the congested environments that we study, this is generally not true; if a series of consumers joined a queue, their actions impose greater potential waiting cost for subsequent consumers.

Finally, our study is linked to recent modeling work in operations management. Building on observational learning theory, Debo et al. (2012a) consider a Markovian queueing system with some uninformed consumers that do not know the service quality but observe the length of the queue before deciding whether to buy the product. In the presence of other consumers that are perfectly informed about the quality, the authors show that the equilibrium queue joining frequency of uninformed consumers is non-monotone in the queue length. As a consequence of this behavior, Debo et al. (2012a) illustrate that the firm may even select a slow service rate to signal high quality via long lines. In a similar vein, Debo et al. (2012b) show that a firm may select an uninformative price and signal high quality via long lines instead.

In sum, these theoretical models predict that the presence of informed consumers may influence uninformed consumers' joining strategy in such a way that their queue joining probability does *not* monotonically decrease in queue length. The theories above leave open one important question: How do *human* decision makers that are uncertain about the quality react to wait times? Furthermore, how does such consumer behavior impact the firm's sales? The queueing games developed in Debo et al. (2012a, b) are infinite horizon games with an infinite number of players (consumers arrive according to a Poisson process) and exponentially distributed service times. To test predictions sharply in a laboratory environment, we develop in this paper a model with a finite number of players and deterministic service times. The qualitative features of the equilibrium analysis are similar to the literature.

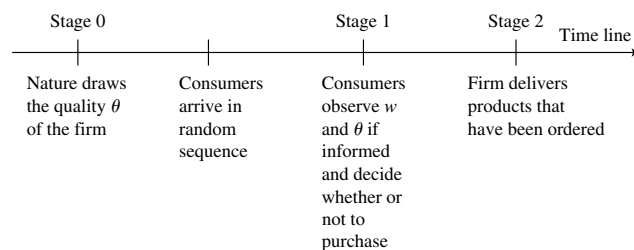
### 3. Theory

We next develop a simple game in which wait times can be a signal of quality information. The game helps us develop an intuition about the impact of informed consumers on quality beliefs and purchasing strategies of uninformed consumers. The sequence of events in the game is indicated in Figure 1.

#### 3.1. Basic Model Setup

We consider a single firm that brings a new product to the market. At stage 0, nature chooses the type of the firm ( $\theta$ ), which corresponds with the quality of the good produced by the firm. The type is high ( $\theta = h$ ) with probability  $p_0$  and low ( $\theta = \ell$ ) with probability

Figure 1 Time Line of the Game



$1 - p_0$ . The gross product value<sup>2</sup> depends on the quality,  $v_\theta$  for  $\theta \in \{\ell, h\}$ , where  $v_\ell < v_h$ . We refer to  $v_0 = p_0 v_h + (1 - p_0) v_\ell$  as the prior (gross) product quality. In the context of this paper, we will use the terms “quality,” “gross value,” and “value” interchangeably. The net utility is determined by the waiting cost. Every period that the consumer has to wait to obtain the product causes some inconvenience of  $c$  to the consumer. The firm operates in a make-to-order environment in which it can produce and deliver exactly one product per period. The market operates as follows. In stage 1, *all* orders are collected:  $\Lambda$  ( $\in \mathbb{N}$ ) potential consumers arrive in a random sequence and consider purchasing the firm's product (i.e., “placing an order”) or an outside product whose value we normalize to zero.

Before placing an order, a consumer observes the wait time,  $w \in \mathcal{W} \doteq \{1, 2, \dots, \Lambda\}$ , for the product. For example, when  $\Lambda = 4$ , the (randomly chosen) first consumer observes a wait time of  $w = 1$ . If that consumer decides to purchase the good, the second (randomly chosen) consumer observes a wait time of  $w = 2$ . Otherwise, the second consumer also observes a wait time of  $w = 1$ , and so on. Thus, the wait time,  $w$ , is a measure of the cumulative sales;  $w - 1$  is the number of other consumers that have decided to purchase the product already. In stage 2, the firm produces and delivers the product to the consumers that have purchased the product in the order that the purchases have been made. Thus, after all orders are collected, consumers that decided to purchase the product at a wait time  $w$  wait exactly  $w$  until their product is delivered.

In our model, with probability  $q$ , a consumer knows the realization of the quality of the firm,  $\theta$ . These are the *informed* consumers. With probability  $1 - q$ , the consumer is *uninformed* and does not know product quality realization, only the prior distribution of quality (i.e.,  $p_0$ ). Hence,  $q$  measures the presence of informed consumers in the market and is common knowledge. For uninformed consumers, wait time has

<sup>2</sup> To focus on wait times as a signal of quality, we keep the price exogenously fixed; hence,  $v_\theta$  is the gross value, excluding the waiting costs.



informational content because it reveals the cumulative sales, which depends on the quality via the informed consumers.

### 3.2. Equilibrium Analysis

Focusing on pure strategies, we characterize formally the purchasing strategy of each consumer type, either informed or uninformed, by means of the triplet  $\mathbf{x} = (x_{i,h}, x_{i,\ell}, x_u)$ , where  $x_{i,h}, x_{i,\ell}, x_u: \mathcal{W} \rightarrow \{0, 1\}$ . Thus, for any  $w \in \mathcal{W}$ :  $x_{i,\theta}(w) = 1$  (0) means that an informed consumer that observes  $w$ , and knows the quality is  $\theta$ , purchases (does not purchase). Similarly,  $x_u(w) = 1$  (0) means that an uninformed consumer that observes  $w$  purchases (does not purchase). We characterize formally the posterior belief of the uninformed consumers that the quality is high, because  $p(w, \mathbf{x})$  after observing  $w$  when the purchasing strategy is  $\mathbf{x}$ . We impose the following equilibrium conditions for all  $w \in \mathcal{W}$ : (i) The equilibrium purchasing strategy  $\mathbf{x}^*$  maximizes expected utility, given the beliefs; and (ii) posterior beliefs  $p^*(w, \mathbf{x}^*)$  about the quality follow Bayes' rule, where possible, given the strategy of all other consumers.

Because informed consumers know for sure that the quality is  $\theta$  (either  $\ell$  or  $h$ ), their strategy after observing  $w$  is trivial: purchase when  $x_{i,\theta}^* = 1 \Leftrightarrow v_\theta - cw \geq 0 \Leftrightarrow w \leq \lfloor v_\theta/c \rfloor$  for  $\theta \in \{\ell, h\}$ . As more consumers are expected to purchase when the quality is high than when the quality is low (because  $\lfloor v_\ell/c \rfloor \leq \lfloor v_h/c \rfloor$ ), uninformed consumers can infer quality from wait time and thus make purchasing decisions based on an updated quality belief. Their expected utility  $u(w, \mathbf{x})$  after observing  $w$  is

$$u(w, \mathbf{x}) = \underbrace{v_\ell + p^*(w, \mathbf{x})(v_h - v_\ell)}_{\text{posterior value}} - \underbrace{cw}_{\text{waiting cost}} \quad \text{for } w \in \mathcal{W}. \quad (1)$$

In Online Appendix E (online appendices available as supplemental material at <http://dx.doi.org/10.1287/mnsc.2015.2264>), we specify  $p^*(w, \mathbf{x})$  via Bayes' rule and provide expressions for the transient probabilities of the wait times (conditional on the product quality). Because the expected utility depends on the vector  $\mathbf{x}$ , the rational strategy for uninformed consumer when observing  $w \in \mathcal{W}$  is  $x_u^*(w) = 1$  (0) when  $u(w, \mathbf{x}^*) \geq 0$  ( $\leq 0$ ). Thus, we have a  $|\mathcal{W}|$ -dimensional fixed point condition for  $\mathbf{x}_u^*$  that completes the equilibrium characterization, similar to Debo et al. (2012a). Without loss of generality, for the remainder of this paper, we normalize the waiting costs:  $c = 1$ . Further, we set  $v_\ell = 0$  and  $v_h = \Lambda - 1/2$ . With these restrictions on the parameters, when the quality is low, no informed consumer would ever purchase at any wait time ( $w \geq 1$ ). When the quality is high, the market is “just large enough” such that up to  $\Lambda - 1$  informed consumers

would purchase. For these parameter assumptions,  $\mathbf{x}_{i,\ell}^*$  and  $\mathbf{x}_{i,h}^*$  are uniquely determined. We focus on the structure of the strategy of the uninformed consumers;  $\mathbf{x}_u^*$ . As a shortcut notation, let  $v^*(w) \doteq v_\ell + p^*(w, \mathbf{x}^*)(v_h - v_\ell)$  indicate the posterior (gross) value in equilibrium. Recall that without informed consumers ( $q = 0$ ), the posterior value of the uninformed cannot depend on the wait time<sup>3</sup>. Now, we describe the differential impact of informed consumers on the posterior value: At  $w = 1$ , the posterior value is non-increasing in  $q$ . However, there exists a  $\hat{w}$  such that the posterior value is non-decreasing in  $q$  at wait times that are strictly longer;  $w > \hat{w}$ . Consequently, the presence of informed consumers influences the uninformed consumers' purchasing decision.

**PROPOSITION 1.** When  $q > 0$ , there exists a wait time  $\hat{w} \in \{1, \dots, \Lambda\}$  such that

- (i) when  $w = \hat{w}$ , the posterior value  $v^*(\hat{w})$  is less than the waiting cost  $\hat{w}$ ;
- (ii) when  $\hat{w} < w \leq \Lambda$ , the posterior value  $v^*(w)$  increases from  $v_0$  to  $v_h$  when  $q$  increases from 0<sup>4</sup> to  $\varepsilon > 0$ ;
- (iii) when  $1 < \hat{w}$ , the posterior value  $v^*(1)$  is nonincreasing in  $q$  with  $w = 1$ .<sup>5</sup>

The key insight of Proposition 1 is that, in the presence of informed consumers, short waits bear “bad” news about the quality and long waits bear “good” news about the quality. We next develop the intuition behind the proposition, using the example in Figure 2 for illustration. The solid lines in the top panel of Figure 2 indicate the posterior gross value;  $v^*(w)$ . The dashed lines indicate the waiting costs;  $cw$  (with  $c = 1$ ). Hence, the difference between both lines is  $u(w, \mathbf{x}^*)$ , which determines the rational purchasing strategy, the solid lines in the bottom panel of Figure 2:  $x_u^*(w) = 1$  if  $v^*(w) > w$ . The intuition behind the proposition is based on the interaction between *negative* and *positive externalities* of wait times.

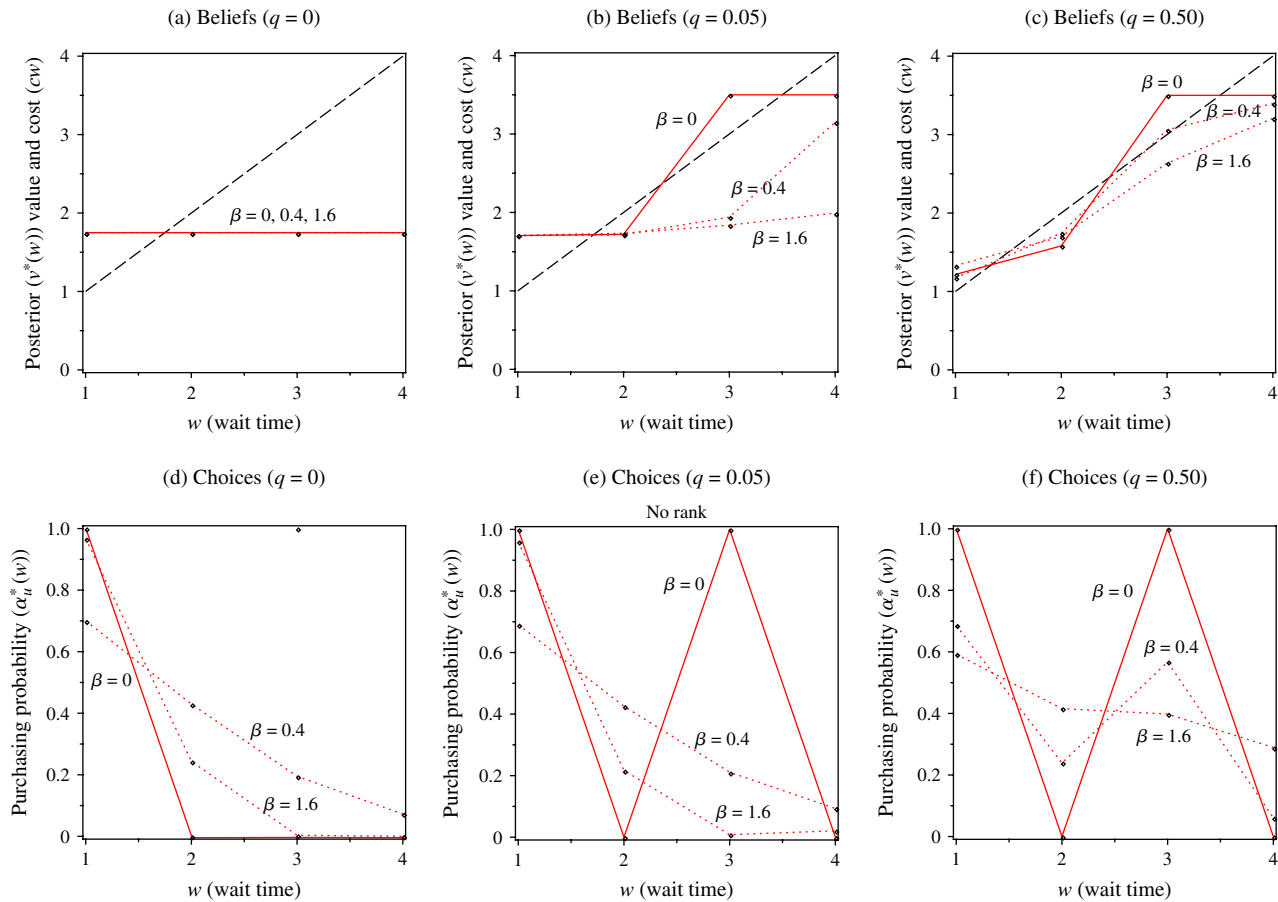
**No Informed Consumers—No News.** In the absence of informed consumers ( $q = 0$ , left panel in Figure 2), because wait times cannot contain any quality information, the posterior value is flat and equals the prior. Because there are only *negative externalities* (i.e., waiting cost), uninformed consumers purchase only when the posterior value ( $1/2 \times 0 + 1/2 \times 7/2 = 1.75$ ) exceeds the waiting cost,  $w$ . Equilibrium behavior of uninformed consumers changes structurally in the

<sup>3</sup> Note that it is possible with  $q = 0$  that some wait times are off the equilibrium path, where Bayes' rule has no bite. In such case, we set the posterior value equal to the prior value.

<sup>4</sup> A pure strategy equilibrium exists under mild restrictions on the parameters; see Equation (8) in Online Appendix B of the online document. These restrictions hold for low enough  $q$  and for all parameters of our experimental design.

<sup>5</sup> The intuition carries over to wait times  $1 < w < \hat{w}$ , however, restricted to low values of  $q$ ; see proof of Proposition 1.

**Figure 2** (Color online) Posterior Value and Cost, and Purchasing Probability for  $p_0 = 1/2$ ,  $v_l = 0$ ,  $v_h = 7/2$ ,  $\Lambda = 4$ ,  $\beta \in \{0, 0.4, 1.6\}$



Notes. When  $\beta = 0$ , consumers are rational. When  $q = 0.05$  or  $q = 0.50$ , the rational purchasing strategy is characterized by  $\hat{w} = 2$  (see Proposition 1).

presence of informed consumers ( $q > 0$ ), because the latter consumer type exert *positive externalities*, rendering wait times into informative signals of quality.

**Long Wait—Good News.** The wait time  $\hat{w}$ , which we label a “hole” (Debo et al. 2012a), plays a central role in Proposition 1. At  $\hat{w}$ , uninformed consumers do not purchase, and neither do informed consumers who know that the quality is low. Hence, as long as the probability that there is an informed consumer in the market is strictly positive ( $q = \epsilon > 0$ ), a wait time that is strictly higher than  $\hat{w}$  and is observed by an uninformed consumer can only be explained by the purchase of a consumer who is informed that the quality is high. This allows uninformed consumers to perfectly infer high quality after observing a long enough wait time (above  $\hat{w}$ ). In other words, they revise their prior from  $v_0$  to  $v_h (= \Lambda - 1/2)$ . Also, for wait times  $w \in \{\hat{w} + 1, \dots, \Lambda - 1\}$ , uninformed consumer purchase, because the posterior value exceeds the waiting cost. The center and right columns of Figure 2 illustrate this case. Uninformed consumers do not purchase at  $w = \hat{w} = 2$ . Hence, at  $w = 3$ , an uninformed consumer knows that an informed consumer

must have purchased at  $w = 2$ . Because this unambiguously indicates high quality of  $v_h = 3.5$  (panels (b) and (c)), the consumer purchases at  $w = 3$  (panels (e) and (f)). Of course, at  $w = \Lambda (= 4$  in our example), the consumer does not purchase even if the consumer can perfectly infer that the quality is high.

**Short Wait—Bad News.** At the lowest possible wait time,  $w = 1$ , the impact of informed consumers on the uninformed consumers’ posterior value is the opposite from the effect above the hole  $\hat{w}$ : The posterior is nonincreasing in  $q$ . Consider an uninformed consumer who observes a wait time of  $w = 1 < \hat{w}$  but still purchases in equilibrium (i.e.,  $x_u^*(1) = 1$ ).<sup>6</sup> When  $q > 0$ , there is a possibility that the consumer is the second (or later) one to arrive and observes  $w = 1$ . In presence of informed consumers ( $q > 0$ ), it *may* be the case that the earlier consumer was informed about the quality. The *only* reason why an informed consumer would not purchase at  $w = 1$  would be that the quality is low because, if the quality were high, the

<sup>6</sup> If the uninformed consumer would not purchase at the lowest possible wait time, no consumer would ever purchase the  $\ell$ -product.

informed consumer would have purchased for sure. As a consequence, the possible existence of informed consumers ( $q > 0$ ) makes the uninformed consumers observing  $w = 1$  revise downward their prior about the quality (even though they still purchase). This can be observed by comparing in Figure 2 the posterior value at  $w = 1$  in the left, middle, and right top panels.

*The Impact of  $q$ .* Figure 2, bottom panel, reveals an additional observation, which is important in the context of our study. Whereas the posterior quality belief at the empty system,  $v^*(1)$ , is decreasing monotonically in  $q$ , this does not necessarily translate into a difference in the choice pattern between *low*  $q$  (e.g., 0.05) and *high*  $q$  (e.g., 0.50) environments. The significant difference in purchasing strategy in the bottom panel of Figure 2 is between  $q = 0$  (left panel) and  $q = 0.05$  (middle panel). Without informed consumers ( $q = 0$ ), the wait time never exceeds  $w = 2$ . With informed consumers ( $q > 0$ ), with a strictly positive probability, the wait time of the high-quality product exceeds  $w = 2$ . In other words, any strictly positive measure of informed consumers suffices to create the effects in Proposition 1.

### 3.3. Quantal Response Equilibrium

The equilibrium analysis above provides the basic intuition behind the mechanics through which rational uninformed consumers learn value from wait times. However, we are interested in actual behavior, and there is little reason to believe that human decision makers would follow the predictions from Proposition 1 perfectly. To relax the rationality assumptions, while maintaining its basic decision making features, we next analyze our model through the lens of quantal response equilibrium (QRE). In the QRE (see, e.g., McKelvey and Palfrey 1995, as well as Su 2008 and Huang et al. 2013 for recent applications in related operations management contexts), each subject faces an individual payoff-relevant disturbance,  $\beta\epsilon$ , where  $\epsilon$  follows some distribution with mean zero and a standard deviation one, and  $\beta$  is the “bounded rationality” parameter. Whereas our theoretical results make no further distributional assumptions, for our numerical examples and structural estimations below we use the logistic distribution  $\Phi(z) = \{1 + e^{-z\sqrt{3}/\pi}\}^{-1}$ . The disturbance, or “noise,” could be interpreted in different ways. For example, it could represent an error because of bounded rationality or an individual-specific shock that occurs for other reasons. To obtain the QRE, the purchasing probabilities of informed consumers at a given wait time  $w$  are again straightforward:  $x_{i,\theta}^*(w) = \Phi((v_\theta - w)/\beta)$ . Given  $x_{i,\theta}^*$  for  $\theta \in \{\ell, h\}$ , the probability that the uninformed consumer purchases at a wait time of  $w$  is given by

$$x_u^*(w) = \Phi(u(w, \mathbf{x}^*)/\beta) \quad \text{for } w \in \mathcal{W}. \quad (2)$$

Similarly as in §3.2, Equation (2) is a  $|\mathcal{W}|$ -dimensional fixed point problem whose solution yields  $\mathbf{x}_u^*$ .<sup>7</sup> The parameter  $\beta$  is inversely proportional to the decision maker’s “degree of rationality.” On the one hand,  $\beta = 0$  models a rational decision maker who makes the decision of purchasing the queue strictly based on the sign of  $u(w, \mathbf{x}^*)$ , as in §3.2. On the other hand, as  $\beta$  grows to infinity, the purchasing probability approaches 1/2, which implies that the decision maker randomizes between purchasing and not purchasing with equal probability, irrespective of the wait time. It is in this sense that decision noise “smoothes” the sharp all-or-nothing predictions of the normative model where a consumer purchases either with probability 1 or 0.

We next illustrate the effect of decision noise, using the numerical example in Figure 2. Without informed consumers ( $q = 0$ , panel (d)), the purchasing probability equals 1 at  $w = 1$ , and 0 at  $w = 2, 3, 4$ . As  $\beta$  increases, essentially, purchasing probabilities converge toward 1/2. This smoothing effect carries over to environments with informed consumers ( $q > 0$ , panels (e) and (f)), but decision noise changes one of the key predictions of the normative model: Recall that rational uninformed consumers ( $\beta = 0$ ) can perfectly infer high quality beyond the “hole”  $\hat{w}$  ( $=2$  in the example), even if the measure of informed consumers,  $q$ , is very low (but strictly positive). However, for higher values of  $\beta$ , this nonmonotonicity in the purchasing strategy may be “washed away” with a small measure of informed consumers but not with a large measure of informed consumers. The intuition is the following: When observing a wait time of  $w = 3$ , uninformed consumers cannot be sure anymore that only an informed consumer knowing that the quality is high purchased at  $w = 2$ . It could be that an uninformed consumer *erroneously* purchased (because of the decision noise), or even that an informed consumer, knowing that the quality is low, *erroneously* purchased. Thus, whereas one would observe a local increase of purchasing probability at  $\hat{w}$  for the rational case with  $\beta = 0$ , in a QRE, the purchasing behavior with a small measure of informed consumers (e.g.,  $q = 0.05$ , middle panel in the bottom of Figure 2) behave almost monotonically decreasing (e.g., for  $\beta = 0.4$ ). With a large measure of informed consumers (e.g.,  $q = 0.50$ , right panel), nonmonotone purchasing strategies survive even relatively large degree of bounded rationality (e.g., for  $\beta = 0.4$ ), but will also vanish eventually (e.g., for  $\beta = 1.6$ ). As a consequence, decision noise may make the nonmonotonicity disappear when  $q$  is low and/or when  $\beta$  is high. In effect, the impact

<sup>7</sup> We indicate the QRE with an asterisk (\*).



of the informed consumers on the uninformed consumer's posterior value and purchasing is ambiguous in the QRE, because it depends on a complex interplay between  $q$  and  $\beta$ . However, the result of Proposition 1(iii) carries through unambiguously to the QRE; at the shortest possible wait time,  $w = 1$ , the QRE posterior value is nonincreasing in  $q$ :

**PROPOSITION 2.** *In any QRE with  $\beta > 0$ ,  $v^*(1)$  is non-increasing in  $q$ .*

Recall from the discussion of Figure 2 that the rational (i.e.,  $\beta = 0$ ) purchasing strategy at  $w = 1$  is the same for  $q = 0.05$  and  $q = 0.50$ , despite the decrease in posterior (Proposition 1(iii)). From Equation (2), an immediate consequence of Proposition 2 is that in the QRE (i.e.,  $\beta > 0$ ) the purchasing probability is non-increasing in the measure of informed consumers,  $q$ . Proposition 2 can be linked to the “empty restaurant” phenomenon; the presence of informed consumers ( $q > 0$ ) makes uninformed consumers less likely to purchase at short wait times.

## 4. Experiment

We designed an experimental study to test key predictions of how the presence of informed consumers affects the purchasing decisions of uninformed consumers.

### 4.1. Hypotheses

Our analyses predict that *informed consumers matter*, in the sense that their presence renders wait times into informative signals of value, which in turn affects the value inferences and choices of uninformed consumers. Specifically, our analyses predict two behavioral patterns that would not occur in the absence of informed consumers. First, *short wait times are bad news*—purchasing probabilities at the empty system decrease in the measure of informed consumers. Second, *long wait times are good news*—Proposition 1 shows the existence of a “hole,” implying the possibility that purchasing probabilities (locally) *increase* in wait times.

Finally, we are interested in shedding light on a more subtle question: Besides the mere existence of the predicted effects, how many informed consumers are enough to trigger them? On the one hand, an intriguing implication of Proposition 1(ii) is that any strictly positive measure of informed consumers suffices to trigger a local increase in purchasing probability. On the other hand, as illustrated in Figure 2, the notion of random noise may well make the local increase vanish, unless the measure of informed consumers,  $q$ , is large.

## 4.2. Experimental Design and Implementation

**4.2.1. Task.** Subjects in our experiment faced the task described in §3.1. We let a firm bring 26 completely independent products to the market, one after the other (round after round). The firm is facing a market of  $\Lambda = 4$  subjects as potential buyers of their products (A-pod, B-pod, ...). The quality of every product is determined in the beginning of every round. It is high with a publicly known prior probability  $p_0$ .

The firm can produce exactly one product per week and delivers the products to the subjects in the sequence that the purchases were made. In every round, first, all subjects arrive to the market in a random sequence. When a subject arrives, she observes the time the product will be delivered (i.e., the wait time), if she decided to purchase the product. In addition, subjects observe their type and, if informed, they also observe the product value (low or high). Based on this information, the subject must decide whether or not to purchase the product. After all subjects have made their decisions, the production and delivery process is simulated at a speed of five seconds per “week” to reinforce subjects’ understanding of the process.

Every subject receives a budget of \$4 in the beginning of each round. If a subject decided not to purchase the product, she would simply keep the \$4. If a subject decided to purchase at a wait time of  $w$  weeks, she keeps the \$4, *plus* the value of the product (either  $v_\ell = \$0$  or  $v_h = \$3.50$ ), minus the waiting costs  $cw$  (with  $c = \$1$ ). Hence, in the best case, when the quality is high and the wait time is low ( $w = 1$ ), the subject that purchases the product makes  $\$4 + 3.5 - 1 = \$6.5$ . In the worst case when the quality is low and the wait time is high ( $w = 4$ ), the subject that purchases makes  $\$4 + 0 - 4 = \$0$ , i.e., the budget of \$4 was specifically chosen to prevent losses. After completion of a round, subjects move on to the next round for another, independent, product.

**4.2.2. Design.** Does the presence of informed consumers matter to the uninformed ones? To test the predictions of our model under controlled conditions, our experimental design varies the measure of informed consumers; see Table 1 for the parameters and rational strategies. As an experimental baseline, we create an environment without informed consumers ( $q = 0$ ), henceforth labeled as condition  $\mathbf{Q}_{00}$ . In this case, because no subject knows the product value before the start of the production and delivery process, rational consumers simply make purchasing decisions based on the comparison of the wait time  $w$  and *expected* product value,  $v_0 = p_0 v_h + (1 - p_0) v_\ell$ . We then replicate condition  $\mathbf{Q}_{00}$ , the only change being that we inject informed consumers into



**Table 1** Treatments, Sample Sizes, and Equilibrium Strategies for  $\beta = 0$ 

	$p_0$	$q$	$N$	Cohorts	$(x_u^*(w), w \in \mathcal{W})$
$\mathbf{Q}_{00}$	0.50	0.00	32	8	(1,0,0,0)
$\mathbf{Q}_{05}$	0.50	0.05	56	14	(1,0,1,0)
$\mathbf{Q}_{50}$	0.50	0.50	68	17	(1,0,1,0)
$\mathbf{Q}'_{00}$	0.25	0.00	32	8	(0,0,0,0)
$\mathbf{Q}'_{05}$	0.25	0.05	48	12	(0,1,1,0)
$\mathbf{Q}'_{50}$	0.25	0.50	60	15	(0,1,1,0)

Note.  $\mathcal{W} = \{1, 2, 3, 4\}$ .

the market. Specifically, we consider two environments with 5% and 50% informed consumers on average (i.e.,  $q = 0.05$  and  $q = 0.5$ ), respectively (labeled  $\mathbf{Q}_{05}$  and  $\mathbf{Q}_{50}$ ). Both conditions allow for a direct test of the role of informed consumers on the purchasing behavior of uninformed consumers. Remember that, in the presence of learning about quality from wait times, the analyses from §3 predict that individual and aggregate purchasing behavior is different across the three conditions. In particular, we hypothesized that purchasing probability at the empty system,  $w = 1$ , decreases in the measure of informed consumers (i.e., as we move from  $\mathbf{Q}_{00}$  to  $\mathbf{Q}_{05}$  to  $\mathbf{Q}_{50}$ ). Further, we hypothesized that purchasing probability may not monotonically decrease in wait time  $w$ , in the presence of informed consumers (i.e.,  $\mathbf{Q}_{05}$  and  $\mathbf{Q}_{50}$ ). Condition  $\mathbf{Q}_{05}$  allows to test an intriguing possibility: Does a small proportion of informed consumers suffice to create the two key behavioral patterns that our theoretical analyses predicts—the “empty restaurant syndrome” and locally increasing (in wait time) purchasing frequency?

Our design includes both a high prior ( $p_0 = 0.50$ ) and a low prior ( $p_0 = 0.25$ ) condition, which entail expected product values  $v_0 = 1.75$  and  $v_0 = 0.875$ , respectively. We use “ $\prime$ ” notation to distinguish the low prior conditions ( $\mathbf{Q}'$ ) from the high prior conditions ( $\mathbf{Q}$ ). Including two levels for  $p_0$  in our design has two potential benefits. First, a broader set of parameters provides robustness to our results. Second, it can be illustrated numerically<sup>8</sup> that our QRE model predicts a relatively more pronounced local increase in purchasing probability (as wait time increases) in conditions with a low prior probability. Our design allows us to test this qualitative prediction directly.

**4.2.3. Product Value, Wait Time, Utility, and Money.** Our model is meant to represent consumers facing the trade-off between (anticipated) utility

derived from the product’s quality and disutility from waiting for delivery of the product. The hedonic nature of enjoying the use of a product or enjoying time not spent waiting is difficult to control. For example, the same physical measures of product quality—such as features, ease of handling, etc.—may be perceived or valued differently by different people. Similarly, waiting may be perceived differently depending on a consumer’s outside value of time. Therefore, we operationalize product quality and waiting cost with money, which is standard practice in experimental economics. The dollar award received after waiting for product delivery is a proxy for product quality.

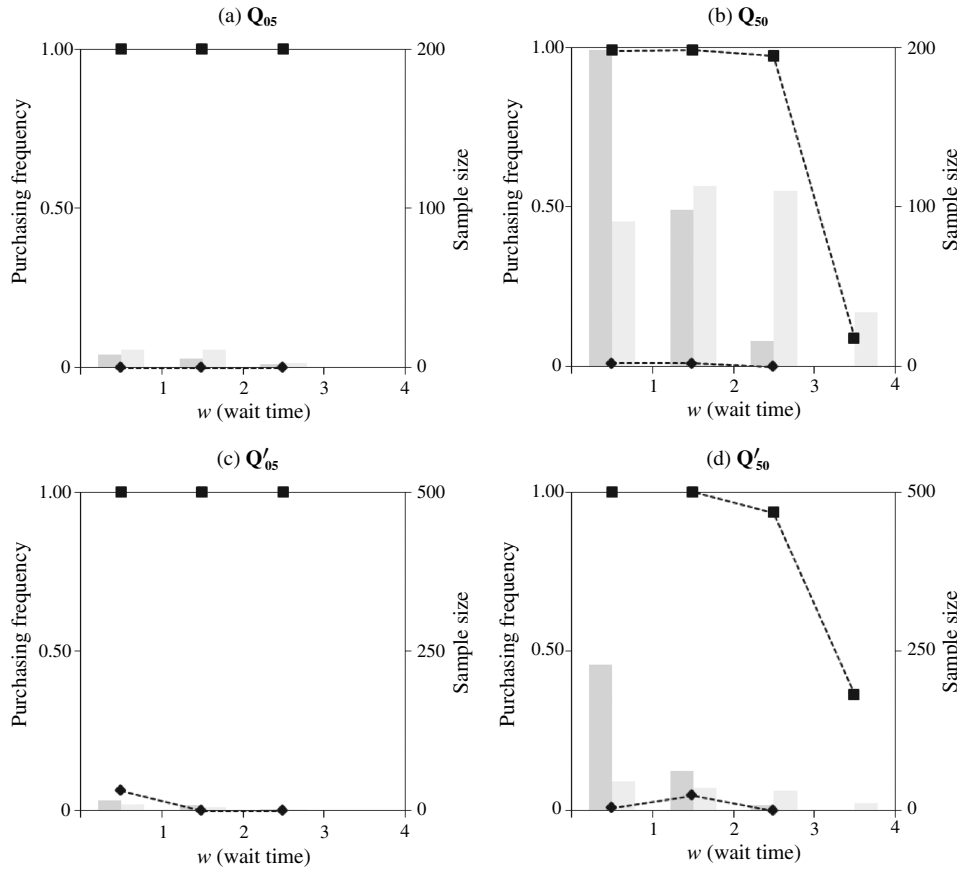
**4.2.4. Prior Information, Sample Information, Rank Information.** At the beginning of the experiment, we provide participants with full knowledge about the relevant structure and parameters of the environment ( $\Lambda, p_0, q, v_\ell, v_h, c$ ). During the experiment, participants were exposed to unbiased random samples from all relevant stochastic populations of the setting described in §3.1. In particular, these random elements are consumer type (informed with probability  $q$ , observed at time of arrival), product value (high with probability  $p_0$ , observed by *all* subjects after production and delivery has begun), and consumers’ order of arrival (never directly observed). Further, for the empirical test of our theory, it is critical that an arriving subject does not know her rank in the order of arrivals.<sup>9</sup> To implement the assumption of unobservable ranks, at the beginning of each new round a random amount of time (uniformly distributed between three and 10 seconds) is added to the arrival of the first subject. Whereas the addition of random noise renders rank inferences a rather difficult task, and hence is a reasonable implementation of our theoretical setup, we provide a more detailed discussion and various robustness checks on the issue of rank inferences in §4.4.2.

**4.2.5. Software, Recruitment, and Payment.** The experiment was implemented in the experimental software z-Tree (Fischbacher 2007). Throughout the experiment, the computer screen would display all relevant game information ( $\Lambda, p_0, q, v_\ell, v_h, c$ ). Further, subjects had access to a history box containing all relevant information from past rounds, such as the wait time offer, their consumer type, their purchasing decisions, as well as true product value. We recruited subjects from an experimental subject pool at Pennsylvania State University. Sessions were conducted at the Laboratory for Economic Management

<sup>8</sup> This is also consistent with the literature (see, e.g., Debo et al. 2012a, §4.2, Equation (7)) about the impact of a low prior on the location of the “hole” in their model.

<sup>9</sup> Of course (and this is part of the equilibrium predictions described in §3.1), subjects can make *some* inferences; for example, if the offered wait time is  $w = 3 = \Lambda - 1$ , then all other consumers have already decided to order the product.

**Figure 3** Purchasing Frequencies of Informed Consumers with Diamonds for the  $\ell$ -Products (Rational Strategy Is  $\alpha_\ell^* = (0, 0, 0, 0)$ ) and Squares for the  $h$ -Products (Rational Strategy Is  $\alpha_h^* = (1, 1, 1, 0)$ )



Note. The number of (informed) subjects observing a specific  $w$  indicated by bars (dark:  $v_l$ ; light:  $v_h$ ).

and Auctions (LEMA) at the Smeal College of Business. After arriving at the laboratory facilities, participants were given the instructions to read, and then they watched a brief presentation with the snapshots of the experimental interface. Subjects then played four training rounds to familiarize themselves with the task environment and the computer interface with the option to clarify any comprehension issues before proceeding to the actual experiment. We implemented cohorts of  $\Lambda = 4$  in a “partner design,” i.e., subjects played within the same cohort for every round of the experiment. The number of cohorts per condition is given in Table 1. We chose this mechanism to reduce strategic uncertainty (about other subjects’ behaviors) and thus facilitate learning.<sup>10</sup> Each session lasted about 45 minutes. Subjects were compensated based on the average revenue earned across the 26 rounds, at a conversion rate of \$4 US dollars for each \$1 laboratory currency. Compensation varied from \$12.61 to \$19.92 with an average of \$16.80, and participants were paid in private at the end of the session; cash was the only incentive offered.

<sup>10</sup> Note that our task environment does not provide any incentives to collude.

### 4.3. Results

To describe the structure of our data, let  $w_{ict}$  denote the wait time observed by subject  $i$  in round  $t$  of cohort  $c$ . The collected data is then  $x_{u,ict}(w_{ict}) \in \{0, 1\}$ , where 1 indicates purchasing when subject  $i \in \{1, 2, 3, 4\}$  is uninformed and presented wait time  $w_{ict} \in \{1, 2, 3, 4\}$  in round  $t$  of cohort  $c \in \{1, 2, \dots, C\}$ . Similarly, the collected data is  $x_{i,ict}(\theta_{ct}, w_{ict}) \in \{0, 1\}$  when subject  $i$  in round  $t$  of cohort  $c$  is informed that the quality is  $\theta_{ct} \in \{\ell, h\}$ .

**4.3.1. Informed Consumers.** Remember that, when offered a wait time of  $w$ , an informed consumer simply needs to compare  $cw$  with  $v_\theta$  and purchases when  $v_\theta \geq cw$  (recall that  $c = 1$ ). Whereas the choice for informed consumers is obviously trivial, it is a key driver of the type of behavior our model predicts; if informed consumers disregarded their information about product value, wait times would not contain any information for uninformed consumers. Figure 3 shows the averaged purchasing decisions as a function of the wait time  $w$ , separate by  $v_\theta$ . When consumers know that the quality is high ( $v_h = 3.5$ ), the purchasing frequency drops from slightly below 1 to slightly above 0 when the wait time increases

from 3 to 4. When consumers know that the quality is low ( $v_\ell = 0$ ), they almost never purchase. As a consequence, because the wait time does depend on quality via the informed consumers, wait times are theoretically informative for uninformed consumers.

**4.3.2. Uninformed Consumers.** We now turn to our main question: Does the presence of informed consumers have a substantive effect on the purchasing decisions of uninformed consumers? The design of our experiment allows us to answer this question simply by comparing observed purchasing behavior across conditions: if uninformed consumers do *not* infer expected product value from the wait time they are offered, we would not observe any differences between conditions with ( $q = 0.05, q = 0.50$ ) and without ( $q = 0$ ) informed consumers. A visual inspection of the purchasing patterns in Figure 4, which shows the averaged purchasing decisions for all six conditions of our experimental design, suggests quite clearly that this is not the case.

To formally test the observed pattern, we estimate a probit dummy variable regression model with the purchasing decision as a dependent variable,

$$\Pr(x_{u,ict}(w_{ict}) = 1) = \Phi(\alpha_1 + \alpha_w \mathbf{I}_w + \alpha_{w,q} \mathbf{I}_{w,q} + v_c), \quad (3)$$

which includes a cohort-level random effect  $v_c$  in the error structure. The constant  $\alpha_1$  provides the baseline at  $w = 1$ , and  $\mathbf{I}_w$  is a set of (three) dummy variables for larger wait time (e.g.,  $I_2 = 1$  if  $w_{it} = 2$ ). Using  $\mathbf{Q}_{05}$  as the baseline condition, the set of (eight) dummy variables  $\mathbf{I}_{w,q}$  for conditions  $\mathbf{Q}_{00}$  and  $\mathbf{Q}_{50}$  allows us to distinguish conditions at each wait time  $w$ . Because our main hypotheses do not require the statistical contrasting along the “ $p_0$  factor” of our experimental design, we estimate the model independently for our low and high prior conditions to avoid the addition of another layer of dummy variables (including interactions). We provide the estimation details in Table 5 in Online Appendix B, and report here instead the predicted purchasing probabilities (in Table 2), because these provide a more direct mapping to Figure 4. The probabilities are calculated as the combination of the

estimated coefficients. For example, given the specification of Equation (3), the predicted purchasing probability at wait time  $w = 2$  for  $\mathbf{Q}_{50}$  is given by  $\Phi(\alpha_1 + \alpha_2 + \alpha_{2,50}) = 0.46$ ;  $\alpha_1$  is the estimated base line coefficient at  $w = 1$ ,  $\alpha_2$  corresponds with the dummy for  $w = 2$ , and  $\alpha_{2,50}$  with the dummy for  $\mathbf{Q}_{50}$  at  $w = 2$ . To indicate the statistical difference between “adjacent” purchasing probabilities, Table 2 includes the results from a series of two-sided Wald tests, based on the relevant coefficient estimates. For example, for  $\mathbf{Q}_{50}$ , purchasing probability is different at wait time 1 (0.68) and 2 (0.46) ( $\alpha_{1,50}$  vs.  $\alpha_2 + \alpha_{2,50}$ ,  $\chi^2(1) = 32.84$ ,  $p < 0.01$ ).

To test for between-condition differences, we perform a series of joint tests on the coefficients of Equation (3). For the sharpest comparison, consider conditions without ( $q = 0$ ) and with many ( $q = 0.50$ ) informed consumers, for which the data shows significant difference in both the high prior conditions  $\mathbf{Q}_{00}$  and  $\mathbf{Q}_{50}$  ( $\chi^2(4) = 68.99$ ,  $p < 0.01$ ),<sup>11</sup> and the low prior conditions  $\mathbf{Q}'_{00}$  and  $\mathbf{Q}'_{50}$  ( $\chi^2(4) = 107.56$ ,  $p < 0.01$ ). Similarly, behavior of uninformed consumers is different between the conditions with a few ( $q = 0.05$ ) and many ( $q = 0.50$ ) informed consumers, for both the high prior conditions  $\mathbf{Q}_{05}$  and  $\mathbf{Q}_{50}$  ( $\chi^2(4) = 27.07$ ,  $p < 0.01$ ), and the low prior conditions  $\mathbf{Q}'_{05}$  and  $\mathbf{Q}'_{50}$  ( $\chi^2(4) = 17.35$ ,  $p < 0.01$ ). Importantly, we find that behavior is significantly different from the  $q = 0$  baseline already with a few informed consumers ( $q = 0.05$ ), for both  $\mathbf{Q}_{00}$  versus  $\mathbf{Q}_{05}$  ( $\chi^2(4) = 28.67$ ,  $p < 0.01$ ), and  $\mathbf{Q}'_{00}$  versus  $\mathbf{Q}'_{05}$  ( $\chi^2(4) = 56.08$ ,  $p < 0.01$ ). In other words, a few informed consumers in the population is sufficient to change the behavior of the uninformed consumers.

**Short Wait—Bad News.** We next turn to the purchasing probability of uninformed consumers at the shortest wait time;  $w = 1$ . Based on Proposition 2, we predict that this probability decreases in the fraction of informed consumers. Table 2 shows that in both the high and low prior conditions, purchasing probabilities at the empty system,  $w = 1$ , are significantly lower in conditions with few informed consumers ( $\mathbf{Q}_{05}, \mathbf{Q}'_{05}$ ) than without informed consumers ( $\mathbf{Q}_{00}, \mathbf{Q}'_{00}$ ), but statistically indistinguishable from the environments with significantly more informed consumers ( $\mathbf{Q}_{50}, \mathbf{Q}'_{50}$ ). These observations are striking: increasing the presence of informed consumers from  $q = 0$  to  $q = 0.05$  decreases significantly the observed purchasing frequency at  $w = 1$ , but a further increase from  $q = 0.05$  to  $q = 0.5$  does not have a significant additional effect. Thus, our results indicate that subjects may overweigh small probabilities ( $q = 0.05$ ), which is in line with the literature on probability

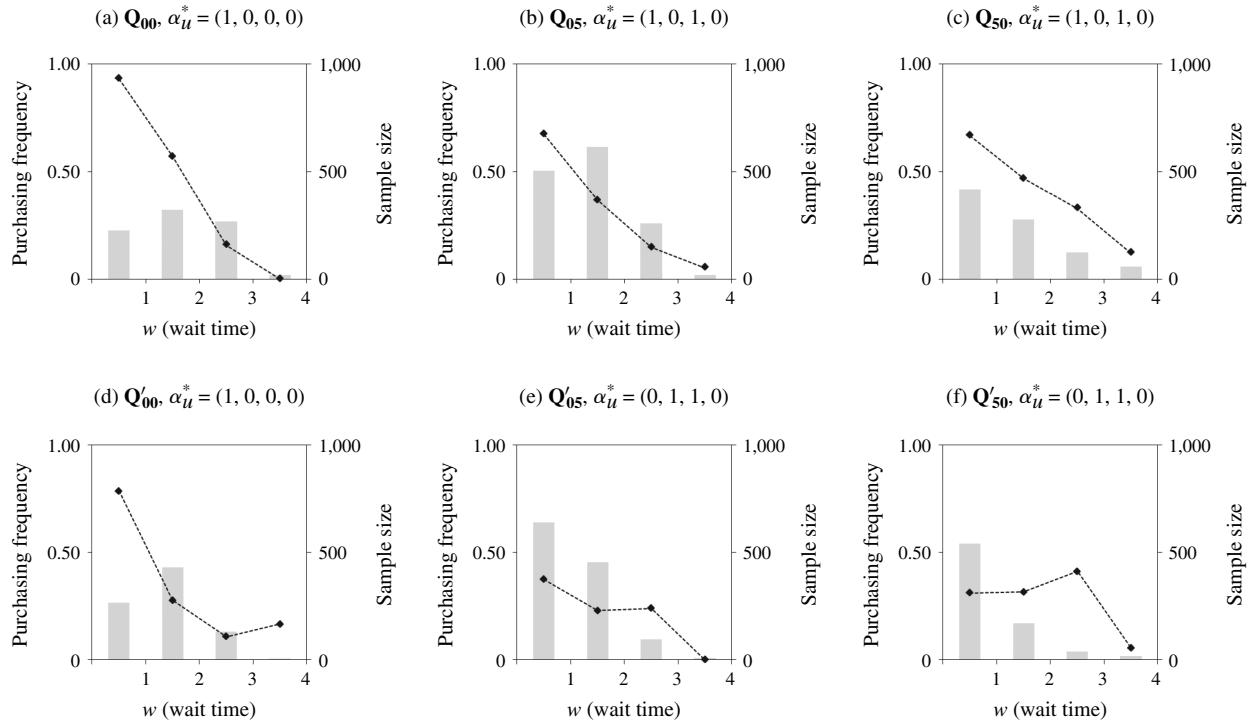
**Table 2** Predicted Probabilities

$w$	$p_0 = 0.50$				$w$	$p_0 = 0.25$			
	1	2	3	4		1	2	3	4
$\mathbf{Q}_{00}$	0.95	> 0.60	> 0.13	— 0	$\mathbf{Q}'_{00}$	0.80	> 0.28	> 0.09	— 0.13
	✓	✓				✓		^	
$\mathbf{Q}_{05}$	0.72	> 0.36	> 0.10	— 0.02	$\mathbf{Q}'_{05}$	0.41	> 0.18	— 0.18	— 0
			^					^	
$\mathbf{Q}_{50}$	0.68	> 0.46	> 0.31	> 0.10	$\mathbf{Q}'_{50}$	0.32	— 0.24	— 0.36	> 0.03

*Note.* “>”s indicate significance at the 5% level, and “—” indicates nonsignificant.

<sup>11</sup> Based on joint test on  $(\alpha_{1,0} = \alpha_{1,50}, \alpha_{2,0} = \alpha_{2,50}, \alpha_{3,0} = \alpha_{3,50}, \alpha_{4,0} = \alpha_{4,50})$ .

Figure 4 Purchasing Frequencies of Uninformed Consumers



Note. The number of (uninformed) subjects observing a specific  $w$  indicated by bars.

weighting in the context of decision making under risk (see, e.g., Barron and Erev 2003, Hertwig and Erev 2009, Ungemach et al. 2009). We explore this possibility further in §4.3.3. In addition, our results show that the difference between the conditions with a few informed ( $q = 0.05$ ) and many ( $q = 0.5$ ) informed consumers is driven by queue purchasing at wait times  $w > 1$ .

**Long Wait—Good News.** Besides the “empty restaurant syndrome,” another key element of our observational learning model is the notion that purchasing probability may *not* monotonically decrease in wait time. To assess how the wait time impacts the purchasing frequency, we conduct a series of one-sided Wald tests. Excluding  $w = 3$  (as for  $w = 4$ , we have practically no data), we test the null hypothesis that purchasing probability does *not* decrease in the wait time:  $\alpha_w \leq \alpha_{w+1}$  (in the baseline conditions for  $q = 0.05$ ) and  $\alpha_w + \alpha_{w,q} \leq \alpha_{w+1} + \alpha_{w+1,q}$  (for  $q = 0$  and  $q = 0.5$ ). At  $w = 1$ , we reject this hypothesis for all conditions except  $Q'_{50}$ . In other words, purchasing probability decreases as the wait time increases to  $w = 2$ , although it is noteworthy that Figure 4 suggests this decrease is generally less pronounced in the presence of informed consumers ( $q > 0$ ). In contrast, at  $w = 2$ , we reject the hypothesis for all conditions except  $Q'_{05}$  ( $\chi^2(1) = 0$ ,  $p = 0.98$ ) and  $Q'_{50}$  ( $\chi^2(1) = 2.02$ ,  $p = 0.92$ ). In other words, in our low prior conditions with informed consumers (i.e.,  $p_0 = 0.25$ ,  $q > 0$ ), purchasing probability does *not* monotonically decrease in

wait time, which is suggestive of the types of value inferences our theory predicts. This can be observed directly in the purchasing probabilities of Table 2: The predicted purchasing probability in condition  $Q'_{50}$  increases from 0.24 at  $w = 2$  to 0.36 at  $w = 3$ .

Furthermore, our experimental design allows us to test the idea that long wait times signal quality by comparing purchasing frequencies at long wait times *between* conditions. The following analysis tests whether the purchase probability at longer wait times increases in the proportion of informed consumers, because the latter render long wait times into signals of high quality. For a sharp comparison, consider the purchasing probability at the longest reasonable wait time,  $w = 3$ ,<sup>12</sup> where uninformed consumers are significantly more likely to purchase in the presence of many ( $q = 0.50$ ) than without ( $q = 0$ ) informed consumers, in the high prior condition ( $\chi^2(1) = 8.49$ ,  $p < 0.01$ , compare the purchasing probability of 0.31 with 0.13 in Table 2) as well as the low prior condition ( $\chi^2(1) = 9.89$ ,  $p < 0.01$ , compare the purchasing probability of 0.36 with 0.09 in Table 2). These results support the idea that because of the presence of informed consumers, uninformed consumers increase their value expectation when two other consumers have purchased the product already.

<sup>12</sup> The wait time  $w = 3$  is above the hole for  $p_0 = 0.5$  and  $p_0 = 0.25$ ,  $\hat{w}$  as for  $p_0 = 0.5$ ,  $\hat{w} = 2$  and for  $p_0 = 0.25$ ,  $\hat{w} = 1$ ; see Table 1.



*Prior Belief  $p_0$ .* Our data is more suggestive of non-monotonicities in uninformed consumers' purchasing probability in the low prior conditions than in the high prior conditions, as illustrated in Figure 4. The theory developed in §3 might be helpful in understanding these observations: In the  $p_0 = 0.5$  conditions with informed consumers ( $\mathbf{Q}_{05}$  and  $\mathbf{Q}_{50}$ ), the hole,  $\hat{w}$ , identified in Proposition 1 is at a wait time of two weeks, whereas in the  $p_0 = 0.25$  conditions ( $\mathbf{Q}'_{05}$  and  $\mathbf{Q}'_{50}$ ), the hole is at one week (see Table 1). With a high prior, the predicted rational purchasing frequency increases from two to three weeks and drops from three to four weeks (because  $v_h = 3.5 < 4$ ). With a low prior, the predicted rational purchasing frequency increases from one week to two weeks, remains high at three weeks, and then drops from three to four weeks. Thus, the local peak above the hole for the high prior condition is much narrower than for the low prior condition. As a consequence, it is more subtle for the uninformed subjects in the high prior conditions to find out the wait time *above* which to infer high quality. Especially, in the presence of possible erroneous decisions by other subjects (as captured by the QRE), the inference about quality from a long wait time is more subtle when the prior is high than when the prior is low. Intuitively, with a low prior, even at "reasonable" wait times, the uninformed subject is "surprised" observing such wait time. The latter leads to an upward revision of the subject's prior belief about quality.

**4.3.3. Structural Estimation (QRE).** What (if anything) do uninformed consumers infer from wait time, relative to some normative benchmark, and how sensitive are these inferences across environments characterized by different measures of informed consumers? The previous analysis is informative to answer the question of "does  $q$  matter" (it does). To provide a sense of the "degree to which  $q$  matters," and to link the empirical data more tightly to our theoretically founded model of judgment and decision making, we next estimate the beliefs on uninformed consumers structurally via the QRE (see §6). Recall that, formally, uninformed consumers form posterior quality beliefs via the posterior utility  $u(w, \mathbf{x}, q)$ , which is specified in Online Appendix E. The notation makes explicit that quality inferences of uninformed consumers depend on the true measure of informed consumers,  $q$ . We model possible deviations from  $q$  succinctly as  $q^\gamma$ , such that  $\gamma = 1$  indicates that quality beliefs are properly calibrated, and  $\gamma > 1$  ( $0 < \gamma < 1$ ) indicates under-inference (over-inference). We structurally estimate the following equilibrium choice model:

$$\Pr(x_u(w) = 1) = \Phi(u(w, \hat{\mathbf{x}}(\beta, \gamma), q^\gamma)/\beta),$$

where we let the purchasing strategy  $\hat{\mathbf{x}}(\beta, \gamma)$  be an explicit function of the parameters  $(\beta, \gamma)$ :  $\hat{\mathbf{x}}(\beta, \gamma) = (\mathbf{x}_{i,\ell}^*(\beta), \mathbf{x}_{i,h}^*(\beta), \hat{\mathbf{x}}_u(\beta, \gamma))$ .  $\hat{\mathbf{x}}_u(w, \beta, \gamma)$  is the solution of the following  $|\mathcal{W}|$ -dimensional fixed point problem:

$$\hat{\mathbf{x}}_u(w, \beta, \gamma) = \Phi(u(w, \hat{\mathbf{x}}(\beta, \gamma), q^\gamma)/\beta) \quad \text{for } 1 \leq w \leq \Lambda.$$

The purchasing probability for informed consumers is  $x_{i,\ell}^*(w, \beta) = \Phi((v_\theta - cw)/\beta)$ , which is, of course, independent of  $\gamma$ .

Let  $\mathbf{X} = \{x_{u,ict}(w_{ict}), x_{i,ict}(\theta_{ct}, w_{ict}) \mid i = 1, 2, 3, 4; c = 1, 2, \dots, C; t = 1, 2, \dots, T\}$  denote the observed purchasing decisions in our experimental data, where  $I$  is the total number of subjects in an experimental condition, and  $T$  is the number of decision rounds in a session. The joint likelihood function is

$$\begin{aligned} L(\beta, \gamma \mid \mathbf{X}) = & \prod_{i,c,t} \{\Phi(u(w_{ict}, \mathbf{x}(\beta, \gamma), q^\gamma)/\beta)\}^{a_{u,ict}(w_{ict})} \\ & \cdot \{\bar{\Phi}(u(w_{ict}, \mathbf{x}(\beta, \gamma), q^\gamma)/\beta)\}^{1-x_{u,ict}(w_{ict})} \\ & \times \{\Phi((v_{\theta_t} - cw_{ict})/\beta)\}^{x_{i,ict}(w_{ict}, \theta_{ct})} \\ & \cdot \{\bar{\Phi}((v_{\theta_{ct}} - cw_{ict})/\beta)\}^{1-x_{i,ict}(w_{ict}, \theta_{ct})}. \end{aligned} \quad (4)$$

We maximize  $L(\cdot)$  over the two parameters  $\beta$  and  $\gamma$ . Table 3 summarizes the estimations.<sup>13</sup> For ease of interpretation, we present the results directly in terms of  $q^\gamma$ . For  $\mathbf{Q}_{50}(\mathbf{Q}'_{50})$ , we infer  $q^\gamma = 0.51(0.44)$ , which is statistically indistinguishable from the true measure of informed consumers. (Table 3 includes the relevant Wald tests for  $H_0: \hat{\gamma} = 1$ .) For the  $\mathbf{Q}_{05}$  condition, we estimate  $q^\gamma = 0.17$ , which is larger than the true value of  $q = 0.05$  ( $\chi^2(1) = 4.65, p < 0.05$ ). Similarly, for the  $\mathbf{Q}'_{05}$  condition, we estimate  $q^\gamma = 0.40$  ( $\chi^2(1) = 33.29, p < 0.01$ ), which supports the conclusion from the nonstructural analysis above; a few informed consumers may suffice to induce a purchasing pattern (of uninformed consumers) significantly different from the case without informed consumers.<sup>14</sup> In addition, in the low prior condition, the purchasing pattern with a few informed consumers ( $\mathbf{Q}'_{05}$ ) is similar to that in an environment with many informed consumers ( $\mathbf{Q}'_{50}$ ).

#### 4.3.4. Illustration and Estimation of Throughput.

So far, we discussed the purchasing behavior, *conditional* on observing a certain wait time. A key performance measure is the actual throughput (or sales) of the system as a function of the quality. The sales, conditional on the quality of the firm, depend thus on the

<sup>13</sup> The main insights remain unchanged if we use a probit link function (with normally distributed errors,  $\Phi$ ) instead.

<sup>14</sup> Interestingly, the estimate of probability overweighing in, for example, Wu and Gonzalez (1999),  $q^B/(q^B + (1-q)^B) = 0.0164$  for  $B = 0.56$  with  $q = 0.05$ , which is in the ballpark of our QRE estimations. We thank an anonymous referee for pointing this out.

Table 3 QRE Estimation Results

	$\rho_0 = 0.50$			$\rho_0 = 0.25$		
	$Q_{00}$	$Q_{05}$	$Q_{50}$	$Q'_{00}$	$Q'_{05}$	$Q'_{50}$
$\hat{\beta}$	0.61** (0.08)	0.60** (0.11)	0.31** (0.06)	1.19** (0.18)	0.76** (0.15)	0.35** (0.05)
$\hat{\gamma}$	—	0.59** (0.19)	0.95** (0.26)	—	0.31** (0.12)	1.18** (0.23)
$q^{\hat{\gamma}}$	—	0.17	0.51	—	0.40	0.44
$LL$	−435.00	−849.11	−681.15	−494.15	−730.81	−584.38
$\chi^2(1)^{\dagger}$	—	4.65*	0.03	—	33.29**	0.65
$N$	832	1,456	1,786	832	1,248	1,560

Note. Standard errors clustered at the cohort level.

$^{\dagger}H_0: \hat{\gamma} = 1$ . \* $p < 0.05$ ; \*\* $p < 0.01$ .

conditional wait time distributions. Now, we discuss how the presence of informed consumers, through their influence on purchasing behavior of uninformed consumers, affect the firms' sales. Table 4 reports the observed average sales of both firm types and includes two-sided  $t$ -tests with per-round cohort-level sales as the unit of analysis.

Without informed consumers, sales of the  $h$ -firm equals sales of the  $\ell$ -firm, as they should. As expected, for the  $\ell$ -firm, sales decrease monotonically in the fraction of informed consumers. A mere 5% of informed consumers on average causes a sales drop of about 16% (27%), and another drop of 34% (45%) as the measure of informed consumers increases to 50% in the high (low) prior condition. We can decompose the effect into a direct effect and an indirect effect. As more consumers learn that the quality is low, which makes the net utility of purchasing strictly negative at any wait time ( $\$0 - w < 0$ ), the purchasing frequency decreases via the direct effect. However, there is also an indirect effect; the remaining uninformed consumers are more reluctant to purchase as the presence of informed consumers increases. This effect is strong, even for  $q = 0.05$ .

Interestingly, more informed consumers *do not* monotonically increase the sales of a  $h$ -firm. Even though there is a direct effect (more consumers observe that the value is high and therefore would be willing to purchase even when the wait time is three weeks;  $\$3.5 - w > 0$  for  $w \leq 3$ ), the indirect effect of a

few informed consumers (on average) and the corresponding reduction in purchasing frequency at short waiting times significantly reduce the  $h$ -firm's sales when the presence of informed consumers is low. Hence, for an  $h$ -firm, our results suggest that some informed consumers may actually be more harmful than helpful.

#### 4.4. Robustness Checks

**4.4.1. Robustness Check 1: Task Experience.** As an important robustness check, we next address the question of whether our main results continue to hold after subjects have accumulated experience with the task. To this end, we first reestimate our main regression model from Equation (3) independently for the data from the first half ( $t = 1 - 13$ ) and the second half ( $t = 14 - 26$ ) of the experiment. Table 5 in Online Appendix B provides the results from this exercise, displaying no apparent learning effects, because we observe no systematic changes in the estimated coefficients. Importantly, we can show that the main effects (empty restaurant syndrome; purchasing probability nonmonotonic in wait time) continue to hold when we constrain the analysis to the second half of the experiment, i.e., after subjects have gathered substantial experience with the decision task.

In a similar spirit, we repeatedly reestimate our QRE model of Equation (4) over the data from a "rolling window" with a fixed size of 13 rounds (i.e.,  $t \in \{1, \dots, 13\}, \{2, \dots, 14\}, \dots, \{14, \dots, 26\}$ ). With the exception of condition  $Q'_{50}$ , we observe that the "bounded rationality" parameter  $\beta$  tends to decrease as the data window is "rolled forward" (see Table 6 in Online Appendix B for a comparison for first half,  $t \in \{1, \dots, 13\}$ , versus second half,  $t \in \{14, \dots, 26\}$ ). This suggests that randomness in decision making abates as experience grows, which is consistent with the results reported in Chen et al. (2012). Whereas, not surprisingly, the estimated parameter  $\gamma$  (or, equivalently,  $q^{\gamma}$ ) is not perfectly constant over all windows, the important observation is that our main results continue to hold when we constrain the analysis to the data from the last window ( $t = 14 - 26$ ).

Table 4 Sales as a Function of Quality

	$\rho_0 = 0.50$		$\rho_0 = 0.25$	
	$\theta = \ell$	$\theta = h$	$\theta = \ell$	$\theta = h$
$Q_{00}$	2.07 (0.10)	2.09 (0.09)	1.64 (0.07)	1.66 (0.12)
$Q_{05}$	1.74 (0.09)	1.74 (0.10)	1.19 (0.15)	1.31 (0.10)
$Q_{50}$	1.11 (0.07)	2.70 (0.04)	0.66 (0.10)	2.33 (0.13)

Note. ">"s ("−"s) indicate significance at the 5% level (nonsignificant).

**4.4.2. Robustness Check 2: The Issue of Rank Inferences.** We next provide several robustness checks to address the possibility that subjects may have inferred something about their ranks despite our precautions described in §4.2.4. For sake of brevity, we present comprehensive details in Online Appendices A and C and highlight here only the key insights. We deal with the rank issue in three ways: theoretically, experimentally, and statistically. First, we develop our theory for the case with observable rank, i.e., a situation where arriving customers know their rank. The key predictions of our unobservable rank theory from §3 are robust to the knowledge of rank, and we make the same qualitative predictions regarding the main behavioral effects—purchasing behavior at the empty system (short wait—bad news) and local increases in purchasing probability at longer wait times (long wait—good news).

Second, we conducted additional laboratory experiments in which we reveal the rank information to arriving subjects. We find that the main behavioral patterns persist when subjects have perfect knowledge of their rank, which is not surprising, because our theoretical analyses predict exactly that. On the other hand, we observe behavioral differences on a more subtle level, and the observable rank data from the additional experiments (observable rank) allows us to refute the idea that subjects in our original experiments (unobservable rank) may have correctly inferred their ranks. In particular, we add rank dummy variables to our main regression model Equation (3). For the unobservable rank data, we find that the estimated rank coefficients vary unsystematically in sign, magnitude, and significance, and that overall model fit does not improve. In contrast, adding the same rank dummies to the regression analysis of the new (“observable rank”) data improves fit significantly.

In a similar spirit, we structurally estimated both QRE models (unobservable rank and observable rank) to both data sets (unobservable rank and observable rank). For the most part, we find that the (un)observable rank model fits best the (un)observable data sets. More importantly, with few exceptions, the estimated coefficients are of similar magnitude, which is not surprising, given how both models (unobservable and observable rank) make very similar predictions.

**4.4.3. Robustness Check 3: Out-of-Sample Predictions.** To investigate the predictive validity of the QRE model, we also use the data of conditions  $Q_{00}$ ,  $Q_{05}$ , and  $Q_{50}$  (with prior  $p_0 = 0.5$ ) to make *out-of-sample* predictions for conditions  $Q'_{00}$ ,  $Q'_{05}$ , and  $Q'_{50}$  (with prior  $p_0 = 0.25$ ). Online Appendix C provides procedures and detailed results. The QRE model makes some important qualitative predictions that are in line

with our data, although we note that it is neither our conclusion nor the objective of this exercise to tout the QRE model as the “best” in terms of describing behavior. First, the QRE predicts that purchasing frequencies at  $w = 1$  decrease in  $q$ , as observed in the experimental data ( $Q_{00}$ ,  $Q_{05}$ , and  $Q_{50}$ ). Further, even though there is no significant increase in purchasing probability with  $p_0 = 0.5$ , the *out-of-sample* predictions suggest such an increase for the conditions with  $p_0 = 0.25$ , which is in line with what we observe in the experimental data (Figure 4).

## 5. Conclusions

This paper provides a theoretical rationale, as well as experimental evidence, for the hypothesis that uninformed consumers infer product (or service) quality merely from wait times, *in the presence of* informed consumers. Consistent with the predictions of our theoretical analysis, the purchasing probability of uninformed consumer may even increase in wait time under certain conditions. On the other hand, purchasing probability at the shortest wait time is markedly lower in the presence of informed consumers than in their absence, an analogue to the “empty restaurant syndrome.” Surprisingly, a relatively low presence of informed consumers suffices to create these effects.

Managerially, our study links individual behavior to system metrics that are relevant for a firm (i.e., sales), in contrast to existing consumer research that focuses mostly on the cognitive processes through which wait times might impact the value perception (Koo and Fischbach 2010, Giebelhausen et al. 2011). Importantly, we investigate *individual and system behavior* as a function of a key parameter describing the business environment, i.e., the presence of consumers informed about product quality. We show that, as predicted, the low-quality firm never benefits from an increasing presence of informed consumers in the population. However, in the case that consumers do not observe their rank, our results suggest the high quality firm benefits from informed consumers only when they are sufficiently large in numbers. These results suggest opportunities for future research on the possible action paths a firm can take when consumers exhibit behavioral patterns as documented in our study. For example, start-up firms that developed truly high-quality new products (and are likely capacity constrained) may want to ensure that they invest sufficiently in informative marketing (such as free educational seminars) to avoid the “empty-restaurant syndrome” that scares away potential consumers (or patrons) when the business does not pick up (by chance). Our research reveals that catering to consumers’ negative reaction to empty systems may be as important as thriving on “full” systems. The reason is that, because of the behavioral pattern that we



describe in this paper, uninformed consumers' overreaction to empty systems will prevent the system from ever becoming full.

Behaviorally, the benchmark model of perfectly rational Bayesian agents has little descriptive validity in our setting. Our study identifies two refinements of the normative benchmark model. Analyzing our data through the lens of quantal response equilibrium (QRE) allows us to capture the well-established idea of random choice errors in a relatively parsimonious modeling framework, while maintaining internal consistency through its equilibrium conditions. Whereas the basic QRE model predicts well a number of comparative statics, we identify an important deviation from this noisy equilibrium model, namely, that a few informed consumers might suffice to create observational learning effects similar in strength to environments with many informed consumers.

Our study has limitations that point toward interesting opportunities for future research. On the behavioral side, our model predicts well the first-order effect of informed consumers, but the rich task environment that we study leaves room for a more nuanced investigation of decision making and judgment biases. A natural candidate is the consideration of risk preferences that relax the risk-neutrality assumption made throughout our analysis. Note that for uninformed consumers in our experiments, the choice to purchase essentially is a lottery ticket with subjective probabilities for the two quality outcomes (low or high). Another behavioral extension concerns possible judgment errors associated with the base rate of product quality. Whereas our experiments exposed subjects to sequences of serially uncorrelated realizations of random outcomes (i.e., product quality), many decision makers are prone to making flawed inferences from streaks of random outcomes (Asparahouva et al. 2009, Rabin and Vayanos 2010). We would welcome such refinements to the basic behavioral theory presented in this paper, in particular if they contributed to a better understanding and prediction of aggregate level behavior. On the modeling side, we make a number of assumptions for the sake of analytical tractability and empirical testability. For example, as is common in the observational learning literature, our model uses a bi-valued notion of quality. Further, our experiments provided subjects with all relevant parameters describing a simplified task environment. In many practical settings, however, the task environment is more complex, the quality is typically not bi-valued, and important parameters (such as the measure of informed consumers) can only be learned by experience. Consumers might have some sense about their rank via the timing of their purchasing decision (e.g., five weeks after the product launch), but they might not

know exactly how many other consumers considered purchasing before them. The end goal of this research would be to understand and possibly predict wait time-related phenomena outside the laboratory (as those suggested in Raz and Ert 2008 and Lu et al. 2013). By tightly controlling the key structural elements of our observational learning model, we hope that our laboratory study is a first step in this direction.

## Supplemental Material

Supplemental material to this paper is available at <http://dx.doi.org/10.1287/mnsc.2015.2264>.

## Acknowledgments

The authors are grateful to the three reviewers, an associate editor, and department editor Martin Lariviere for their constructive and insightful comments that helped to improve the paper significantly. The authors are also grateful to many seminar participants at various institutions and conferences where the authors presented this research. The authors also acknowledge the Civil, Mechanical, and Manufacturing Innovation Division of the National Science Foundation [Grant CMMI-1301234] for funding of the authors' research. Most of the research has been conducted while L. Debo was at the University of Chicago Booth School of Business of which he acknowledges summer support. M. Kremer acknowledges support by a grant from the Smeal College of Business. Finally, the authors thank the Laboratory for Economic Management and Auctions at Pennsylvania State University.

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