Management Science

Publication details, including instructions for authors and subscription information:
http://pubsonline.informs.org

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To cite this article:
Man Yu, Laurens Debo, Roman Kapuscinski (2016) Strategic Waiting for Consumer-Generated Quality Information: Dynamic Pricing of New Experience Goods. Management Science 62(2):410-435. http://dx.doi.org/10.1287/mnsc.2014.2134

MANAGEMENT SCIENCE
Vol. 62, No. 2, February 2016, pp. 410–435
ISSN 0025-1909 (print) | ISSN 1526-5501 (online)

http://dx.doi.org/10.1287/mnsc.2014.2134

Strategic Waiting for Consumer-Generated Quality Information: Dynamic Pricing of New Experience Goods

Man Yu
School of Business and Management, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong, manyu@ust.hk

Laurens Debo
Chicago Booth School of Business, University of Chicago, Chicago, Illinois 60637, laurens.debo@chicagobooth.edu

Roman Kapuscinski
Ross School of Business, University of Michigan, Ann Arbor, Michigan 48109, kapuscin@umich.edu

In this paper, we study the impact of consumer-generated quality information (e.g., consumer reviews) on a firm's dynamic pricing strategy in the presence of strategic consumers. Such information is useful, not only to the consumers that have not yet purchased the product but also to the firm. The informativeness of the consumer-generated quality information depends, however, on the volume of consumers who share their opinions and, thus, depends on the initial sales volume. Hence, via its initial price, the firm not only influences its revenue but also controls the quality information flow over time. The firm may either enhance or dampen the quality information flow via increasing or decreasing initial sales. The corresponding pricing strategy to steer the quality information flow is not always intuitive. Compared to the case without consumer-generated quality information, the firm may *reduce* the initial sales and *lower* the initial price. Interestingly, the firm may get strictly *worse* off due to the consumer-generated quality information. Even when the firm benefits from consumer-generated quality information, it may prefer *less* accurate information. Consumer surplus can also decrease due to the consumer-generated quality information, contrary to the conventional wisdom that word of mouth should help consumers. We examine extensions of our model that incorporate capacity investment, firm's private information about quality, alternative updating mechanisms, as well as multiple sales periods, and show that our insights are robust.

Keywords: strategic consumer behavior; price skimming; two-sided learning
History: Received September 26, 2013; accepted November 24, 2014, by Yossi Aviv, operations management. Published online in *Articles in Advance* July 15, 2015.

1. Introduction

In this paper, we focus on dynamic pricing of new experience goods. A new experience good is any good whose quality cannot be easily assessed ex ante by the firm and by consumers, for example, movies, theatre plays, books, or electronic gadgets that are just launched to market (Bergemann and Valimaki 1997, Liu and Schiraldi 2012, Bhalla 2012). Consumer-generated information can be disseminated via word of mouth or other platforms such as websites, smartphone applications, or social media. For example, Retrevo.com, Amazon.com, or Epinions.com gather reviews for new consumer electronic products. Similarly, movies, theatre plays, and books have their own review and discussion fora.

For conciseness, we use the terms "consumer-generated quality information" and "reviews" interchangeably. It is intuitive that the information generated through reviews should help consumers that have not yet experienced the product to better assess the product (Hu et al. 2015) and, thus, better time their purchasing decision (if any). However, such information is also available and relevant to the firm, who can learn from consumer experiences about its own quality and hence discover the price-demand relationship of the experience good. Such information can allow the firm to better price the good and increase its profits. Hence, through consumer-generated reviews, both the firm *and* the consumers learn about the product's quality, but, for different purposes. As a consequence, the reviews influence the dynamics of the firm-consumer interactions as quality information generated by some consumers impacts the behavior of other consumers and the firm over time. The detrimental impact of "strategic" consumers on the profits of a firm is notorious, even without reviews. When consumers are strategic and heterogeneous in their valuations, some consumers may delay their purchase in anticipation of the firm's price decrease, delaying the firm's revenues. With reviews, some consumers may also delay their purchase to free ride on better information about the product's quality generated by other consumers that bought earlier. The firm, on the other hand, may want to adjust its pricing strategy to prevent such consumers from waiting until later when the price drops or quality information becomes available.

Firms recognizing high (low) perceived quality from reviews might want to increase (decrease) the price. This is not inconsistent with observed practice. As an example, consider ticket prices of Broadway shows.[1] *The Book of Mormon* first started running in February 2011 with a highest official price of $252. It immediately garnered broad praise from the audience[2] and the critics.[3] In July 2011, the official highest box-office price almost doubled to $477 and the average price paid per admission increased by 30%.[4] This is different from prices of other shows. *Holler If Ya Hear Me* received fairly negative reviews[5] and shortly after its opening, its prices were heavily discounted.[6] Although the highest ticket price was $173, during the first three weeks of performance the audience only paid an average of $27.10 per ticket.[7] In fact, it became "the cheapest ticket on Broadway"[8] and closed just one month after opening.[9] Another example of an industry where reviews may influence the price is consumer electronic goods such as cameras. The ease of use of complex features of such goods can often best be learned via first-hand experience or shared experiences via reviews on platforms such as http://www.amazon.com or http://www.epinions.com with "Unbiased Reviews by Real People." Price changes for consumer-electronics products are extremely frequent.[10] Although this can be driven by a multitude of reasons, it is reasonable to expect that perceived quality would be one of the important factors. Finally, in the movie industry, some distributors would like to make admission ticket prices depend on perceived movie quality via, e.g., consumer reviews. However, inflexible upstream contracts with producers imposed by industry regulations prevent them from doing so. Industry observers estimate that the industry leaves millions of dollars on the table (Fritz 2012).

With the advent of more and cheap platforms to exchange consumer experiences, the understanding of consumers' and firms' incentives to disseminate information about quality is increasingly important but, to the best of our knowledge, no paper studied the impact of both waiting incentives (for a price decrease and for consumer-generated quality information) on the firm and consumer behavior jointly. In particular, it is not clear whether and how a firm could leverage the endogenously generated quality information and how it should adjust its pricing strategy, especially in the presence of strategic consumers. The central question we address in this paper is whether endogenously generated quality information hurts or enhances the firm's profit and consumer surplus. Also, whether the reviews provide an incentive for the firm to increase or decrease its initial price and, anticipating the firm's strategy, whether more or less consumers strategically delay their purchase compared to the no-review case.

To address these questions, we consider a dynamic pricing model where the consumers' valuations for a product depend on both the product's quality and consumers' private valuation. The product's quality represents a common value of the product and is initially unknown to the consumers *and* to the firm. The private valuation captures the heterogeneity in individual consumer's willingness to pay and is known only by the consumer. Based on the initial price, the consumers decide first when to buy the product (early or late). The consumers who buy early "experience" the product and report their experience via

[1] We are grateful to an anonymous referee for suggesting this example.

[2] http://www.entertainment-link.com/broadway/musicals/the-book-of-mormon.asp, http://www.broadwaybox.com/shows/the-book-of-mormon/reviews (accessed August 16, 2014).

[3] http://www.broadwayworld.com/article/Broadway-Review-Roundup-THE-BOOK-OF-MORMON-20110324 (accessed August 16, 2014).

[4] http://www.broadwayleague.com/index.php?url_identifier=nyc-grosses-11 (accessed August 16, 2014).

[5] http://www.nytix.com/news/tag/holler-if-ya-hear-me (accessed August 16, 2014).

[6] http://massappeal.com/holler-if-ya-hear-me-shutting-down/, http://www.businessweek.com/articles/2014-08-04/the-death-of-the-tupac-shakur-broadway-musical-solved, http://www.nytix.com/news/tag/holler-if-ya-hear-me (accessed August 16, 2014).

[7] http://www.broadwayleague.com/index.php?url_identifier=nyc-grosses-11, http://www.dailyactor.com/2014/06/the-business-of-broadway-holler-if-ya-hear-me-needs-to-holler-for-box-office-dollars/ (accessed August 16, 2014).

[8] http://massappeal.com/holler-if-ya-hear-me-shutting-down/ (accessed August 16, 2014).

[9] http://www.nytix.com/news/tag/holler-if-ya-hear-me (accessed August 16, 2014).

[10] For example, the price of a mini camcorder Canon VIXIA M500 increased significantly (from about $500 to $800, http://www.camelcamelcamel.com) between December 2012 and May 2014, which coincided with an increase of average ratings (http://www.amazon.com). Another example is the Canon EOS Rebel T3, a camera for beginners and professionals. It had decreasing average ratings between May 2011 and November 2012 and the average price of the Rebel dropped from about $630 to $380.

a "consumer report" that is made available to the remaining consumers in the market and to the firm. These reports reduce, but do not eliminate, the uncertainty about the quality.[11] The volume of the reviews influences their informativeness.[12] Upon receiving these reports, the firm and the remaining consumers update their belief about the product quality, the firm revises its price, and the remaining consumers decide whether to buy the product or not at the revised price.

A large volume of work in economics, operations management, and marketing focuses on *one-sided* learning, where only one side of the market, either sellers or buyers but not both, actively acquire information about the other side. The motivation for learning is obvious: to make better decisions (purchasing for consumers and operational decisions such as pricing for sellers). For example, from sales history a firm may infer consumers' arrival intensity (e.g., Aviv and Pazgal 2002); aggregate demand (e.g., Lariviere and Porteus 1999, Petruzzi and Dada 2002); or preferences over different products (e.g., Caro and Gallien 2007). Alternatively, consumers can obtain information on product quality by observing a firm's selling strategies (e.g., Bagwell and Riordan 1991); other consumers' purchasing decisions (e.g., Banerjee 1992, Debo et al. 2012, Debo and Veeraraghavan 2014); or their post-purchase reviews (e.g., Li and Hitt 2010, Ifrach et al. 2011, Papanastasiou et al. 2013). Recently, one-sided learning by the firm about market demand and by the consumers about their own valuations, *with* strategic consumers, has been studied by Levina et al. (2009), Aviv et al. (2012), Jing (2011a, b), and Bergemann and Valimaki (2006). Similarly, Ovchinnikov and Milner (2012) consider strategic consumers' one-sided learning about a firm's markdown policy. In general, strategic consumers reduce the effectiveness of learning. None of the studies above considers simultaneous learning by both seller and strategic buyers.

The seminal paper by Bergemann and Valimaki (1997) considers a *two-sided* learning model where both sides of the market, buyers and sellers, learn the true value of a new product through consumer experiences. By analyzing a dynamic duopoly competition, they show that information generated through consumer reviews is valuable to the seller and that, compared to the static equilibrium where all players ignore the informational effects of consumer experiments, the seller with unknown product quality induces higher initial sales to produce more information about quality. Bergemann and Valimaki (1999) extend Bergemann and Valimaki (1997) by considering market entry. They find that a monopolistic firm always increases initial sales to accelerate diffusion of quality information, however, a market entrant may reduce it if the new product has better quality than the existing product. Bergemann and Valimaki (2000) generalize Bergemann and Valimaki (1999) to the case where new product is launched in many distinct markets. Ifrach et al. (2013) focus on dynamic pricing with two-sided social learning through reviews with exogenously arriving consumers. When considering two-sided learning, the authors do not take strategic consumer behavior into account. They find that the firm always benefits from the (two-sided) learning.

There exist a few recent papers about two-sided learning with exogenously arriving consumers observing private, quality-related signals and historical sales (instead of reviews). Liu and Schiraldi (2012) examine a firm's decision on product launch sequence: sequential launch versus simultaneous launch. Bhalla (2013) extends Liu and Schiraldi (2012) by incorporating the firm's dynamic pricing decision. In these learning models, pricing plays a different role than it does in review-based learning models: Pricing is not used to influence precision of the reviews (as in our paper) but instead to screen consumers' private information. In general, the firm quotes a high initial price to promote learning, because high (low) historical sales at a high price convey high (low) quality, whereas historical sales at low price cannot convey any information (Bhalla 2013). The impact of historical sales on review information is quite different; more sales lead to more reviews and hence, more precise review information.

The fact that consumers are forward looking and strategic is an important component of our paper. Anticipating better information about the product quality based on other consumers' reviews, consumers may purposefully postpone their purchasing decisions. The existing literature on strategic consumers (e.g., Besanko and Winston 1990, Su 2007, Aviv and Pazgal 2008, Bhalla 2012, Altug and Aydinliyim 2013) mainly focuses on strategic consumer waiting for price markdowns. In particular, Bhalla (2012) examines a two-sided sales-based learning model with two consumers where the earlier-arriving consumer may delay purchase in the hope of buying at a lower price later. The paper finds that average

[11] As an illustration that reviews are not perfectly informative, consider the book that contains only numbers, *A Million Random Digits with 100,000 Normal Deviates*, by the RAND Corporation. The book has 559 customer reviews and has an average rating of 3.9 out of 5. Obviously, the reviews are all bogus.

[12] In practice, "[t]he sheer volume of reviews makes far more difference, according to Google's analysis of clicks and sales referrals. 'Single digits didn't seem to move the needle at all,' says Mr. AcAteer. [...] His company's research shows that visitors are more reluctant to buy until a product attracts a reasonable number of reviews and picks up momentum" (*The Economist* 2009). Hence, the relationship between informativeness and review volume (sales volume) is an important one.

prices may increase over time due to such consumer behavior. A few papers address consumers' strategic waiting for more information. Gallego and Sahin (2010), Swinney (2011), and Yu et al. (2015b) model consumers' option of delaying purchases till they become better informed about their own valuations of the product. Yu et al. (2015a) examine consumers' waiting for information on both individual valuations and product quality, and characterize its implication to the firm's signaling strategy. In all these papers, perfect information is *exogenously* revealed when consumption time approaches. In contrast, the review information in our paper is *endogenously* generated by consumer reviews and its precision is determined by the volume of initial sales, which is a function of the firm's pricing strategy. Indeed, one of the key elements of our paper is the interplay of consumers' strategic waiting for information and the firm's control over the information flow via pricing.

Our paper addresses two-sided learning from reviews with strategic consumers. To disentangle the incentives of the buyers and seller, we introduce two constructs: the option value of the firm and the option value of the consumers. We explain these option values in detail in the paper. We show how these constructs help us understand when reviews are profitable for the firm, when they increase consumer surplus, and how reviews impact the price and sales path. The constructs also allow us to understand the finding pertaining the impact of learning on profits, prices, and sales in a number of papers in the two-sided learning literature. The analysis of these option values leads to several interesting findings. First, although a larger volume of consumer reports generates more informative reviews, the firm may find it optimal to decelerate social learning by decreasing the initial sales, compared to the no-review case. This result is in contrast with the existing literature, e.g., Bergemann and Valimaki (1997, 1999), Ifrach et al. (2011), which shows that the firm always sells more initially. Furthermore, sometimes the firm's *reduction* in the initial sales is achieved via a *lower* initial price compared to the no-review case. The difference is primarily driven by consumers' strategic waiting behavior. We provide a detailed explanation based on an analysis of the option values of the firm and of the consumers due to consumer reviews.

Second, although the firm learns through consumer reviews, it may get strictly worse off due to the review information. This result is solely driven by strategic consumer behavior. This finding is in contrast with the recent literature on two-sided learning such as Ifrach et al. (2011). Also, the firm's loss due to reviews does not necessarily increase when consumers become more patient. Instead, it is most pronounced when consumers' patience level is intermediate. Interestingly, the firm sometimes gets *hurt* by more precise reviews. Even when the firm benefits from consumer reviews, it may prefer *less* precise reviews.

Third, although it is generally believed that word of mouth would not hurt consumers, we show that the consumers may end up with a *lower* aggregate surplus due to the reviews generated by themselves (as in Papanastasiou et al. 2013). In addition to allowing consumers to better assess the product, reviews also allow the firm to make more informed pricing decisions and to extract more consumer surplus. Similar to the firm, the consumers may also prefer reviews with *lower* precision.

We extend our base model by considering investment in capacity, multiple sales periods, a firm's private information, and alternative quality updating processes. We find that the main conclusions of our base model continue to hold. Interestingly, more flexibility (quick-response capacity) might decrease the firm's profit due to strategic consumers, and the effect is more pronounced with more informative reviews. Additional sales periods or increased pricing flexibility may also hurt the firm's profitability. The firm's private information about quality should provide the firm with a strategic advantage. However, signaling credibly private information about the quality via pricing might be expensive. Consequently, dissemination of quality information via endogenous discovery might be preferred compared to costly signaling. Finally, the firm's option value and its optimal pricing change significantly depending on proportion of population that writes or reads the reviews. Thus, the firm may benefit from tracking what fraction of the population is influenced by consumer reviews.

2. Model

We start with a brief depiction of the firm and the consumers, followed by a detailed description of the consumer-review model. We then introduce the dynamic game and describe the equilibrium.

2.1. The Firm and the Consumers

We consider a firm selling an experience good to a market with a continuum of consumers. The total volume of consumers is normalized to one. The common quality component of the experience good is the realization of a binary random variable, $\tilde{\Theta} \in \{0, 1\}$ with $p_0 = \Pr(\tilde{\Theta} = 1) \in (0, 1)$. Consumers' idiosyncratic preferences are represented by an individual quality component, v, which is uniformly distributed over $[0, 1]$. The gross utility of the experience good is $\tilde{\Theta} + v$ and the net utility is simply the gross utility minus the price the consumer pays. The consumers can purchase the experience good in one of two periods; $t \in \{1, 2\}$, where $t = 1$ represents "early" and $t = 2$ represents "late."[13] If a consumer does not purchase the good by the end of period 2, he obtains zero utility. Each consumer buys at most one unit of the product. All consumers and the firm know p_0, but not the realization of $\tilde{\Theta}$.

We assume that consumers and the firm are rational Bayesian decision makers that maximize their expected utility, given the available information and optimal behavior of all other decision makers. Consumers (the firm) discount(s) future utility at a rate of δ_c (δ). The firm incurs a cost c for serving each consumer. We assume that

$$1 < c < 1 + p_0; \tag{1}$$

i.e., the per-unit cost is strictly less than the gross utility of the highest-paying consumers in the first period, but is greater than the highest possible valuation in the market when the true quality is low ($\tilde{\Theta} = 0$). In the latter case, if the firm and the consumers knew its true quality is low, the firm would withdraw the product from the market.[14]

2.2. Consumer Reviews

Building on the classic bilateral-learning papers by Bergemann and Valimaki (1997, 1999, 2000), we model consumer reviews as an aggregate signal, g, which is the realization of a continuous random variable $\tilde{G} \mid \{\theta, x\}$ over $\mathbb{R}$, with a density that depends on the realization of the common quality component, $\theta \in \{0, 1\}$, and on the volume x of consumers that bought the product early. For example, g can be the average of individual scores that consumers assign to the product's quality. We follow Bergemann and Valimaki (1997) to assume that the density of the signal $\tilde{G} \mid \{\theta, x\}$ is $f_\theta(g, x) = (1/(\sigma\sqrt{2\pi x})) \cdot \exp(-\frac{1}{2}(g - x\theta)^2/(x\sigma^2))$ for $\theta \in \{0, 1\}$. Let $F_\theta(g, x)$ denote the cumulative density. Hence, conditional on the true quality, the generated report is normally distributed with mean $x\theta$ and standard deviation $\sqrt{x}\sigma$.[15]

We assume that $\sigma \in (0, +\infty)$. σ is an important parameter. We assume that σ is common knowledge.[16] Its reciprocal, $1/\sigma$ will be indicated by $\tau \in (0, +\infty)$, which is a measure of informativeness of the consumer reviews.

Posterior Quality. For given signal g and volume of consumers x, the posterior that the quality is high, $\Pr(\tilde{\Theta} = 1 \mid g, x)$, is via Bayes' rule:

$$p(g, x) = \frac{p_0 f_1(g, x)}{p_0 f_1(g, x) + (1 - p_0) f_0(g, x)} = \frac{p_0}{p_0 + (1 - p_0)\exp(-(g - x/2)\tau^2)} \quad \text{for } x \in (0, 1]. \tag{2}$$

For any finite signal realization g and volume x, the posterior is strictly in $(0, 1)$. Thus, the signal does not perfectly reveal the quality.

2.3. The Game

The timeline of the game is as follows: At time $t = 0$, nature determines the realization of $\tilde{\Theta}$; $\tilde{\Theta}$ is unobserved throughout the game. At time $t = 1$ (first period), the firm posts price P_1. Then, consumers observe P_1 and their individual valuations v and decide whether to buy at $t = 1$ or wait until the next (second) period, $t = 2$. Let x be the volume of consumers buying at $t = 1$. At the beginning of $t = 2$, the realization g of a continuous signal $\tilde{G} \mid \{\tilde{\Theta}, x\}$ is observed by all remaining consumers and by the firm. The firm then posts a price $P_2(g, P_1)$ and the remaining consumers decide whether to buy at $t = 2$ or quit without buying.

The Consumer Purchasing Strategy. We focus on threshold types of purchasing strategies: If a consumer type v decides to purchase in period $t = 1$, then all consumer types $v' > v$, who have higher individual valuations and, thus, are more "eager" to buy than v, also purchase.[17] For given price P_1, assume that $\bar{v}(P_1)$ is a threshold consumer type, above which consumers purchase in the first period, and below which consumers wait until the second period. In the beginning of the second period, consumer types $[0, \bar{v}(P_1)]$ remain in the market. After observing signal realization g of $\tilde{G}$ and the second-period price, P_2, let $\underline{v}(g, P_1, P_2) \in [0, \bar{v}(P_1)]$ be the threshold consumer type, above which all remaining consumers purchase in the second period, and below which remaining consumers quit; $[\bar{v}(P_1), \underline{v}(g, P_1, P_2)]$ fully specifies the consumer purchasing strategy.

[13] To be consistent with the multiperiod model in §6.2, the index $_0$ refers to the prior *at the beginning* of period $t = 1$.

[14] We discuss the implication of this assumption in §4.1.

[15] Consider the following discrete system in which there are N consumers indexed by $i \in \{1, 2, \ldots, N\}$. After consumption, every consumer i observes the following binary signal of quality: $\tilde{G}_i \in \{G_u, G_d\}$: $\Pr(\tilde{G}_i = G_u \mid \theta = 1) = q_1$ and $\Pr(\tilde{G}_i = G_u \mid \theta = 0) = q_0$, where

$$q_0 = \frac{2N\sigma^2 + 1/2 - (1/2)\sqrt{4N\sigma^2 + 1}}{4N\sigma^2 + 1}, \quad q_1 = 1 - q_0,$$

$$G_d = \frac{1}{2}\frac{1}{N}(1 - \sqrt{4N\sigma^2 + 1}) \quad \text{and} \quad G_u = \frac{1}{2}\frac{1}{N}(1 + \sqrt{4N\sigma^2 + 1}).$$

Let $X = xN$ and $G = \sum_{i=1}^{X} \tilde{G}_i$, then, it is easy to show that for $N \to \infty$, $G \mid \{x, \theta\}$ has mean $x\theta$ and standard deviation $\sqrt{x}\sigma$.

[16] Relaxing this assumption is challenging analytically, but does not alter our main insights.

[17] The proof is omitted for brevity. In the appendix, we show that such purchasing strategy is indeed an equilibrium.

The Firm's Pricing Strategy. In period $t=1$, the firm sets the initial price, P_1. The price in period $t=2$, $P_2(g, P_1)$, depends on the initial price as well as the realization of the consumer signal g; $[P_1, P_2(g, P_1)]$ fully specifies the firm's pricing strategy.

Equilibrium. We define equilibrium of the dynamic game, starting from period 2. In the beginning of period 2, all remaining consumers update their belief about product quality, based on the signal g and the volume $1-\bar{v}(P_1)$ of consumers that purchased early. By Equation (2), $p(g, 1-\bar{v}(P_1))$ is the posterior probability that quality is high (or simply posterior quality). The firm sets the price P_2 and consumers with nonnegative utility buy the product. Thus,

$$\begin{aligned} \underline{v}(g, P_1, P_2) = \min\{v \in [0, \bar{v}(P_1)]\colon\ & U(v, g, P_1, P_2) \\ & = p(g, 1-\bar{v}(P_1)) + v - P_2 \geq 0\} \end{aligned} \quad (3)$$

defines the threshold consumer type who buys in period 2. Let $\Pi_2(g, P_1, P_2)$ be the firm's second-period profit after observing signal g. The firm, based on the same information as consumers, uses $p(g, 1-\bar{v}(P_1))$ as the posterior quality and solves

$$\max_{P_2} \ \Pi_2(g, P_1, P_2), \quad (4)$$

which gives $P_2(g, P_1)$.

Consider now period 1, when the firm sets price P_1 and consumers decide whether to buy now or wait. Let $EU(v, P_1)$ denote $\mathbb{E}_{\tilde{G}}[U(v, \tilde{G}, P_1, P_2(\tilde{G}, P_1))] = \mathbb{E}_{\tilde{G}}[p(\tilde{G}, 1-\bar{v}(P_1)) + v - P_2(\tilde{G}, P_1)]^+$, which is consumer type v's expected future utility at price P_1.[18] Consumer v purchases in period $t=1$ if $p_0 + v - P_1 > \delta_c EU(v, P_1)$, otherwise, the consumer waits. The highest type that waits is $\bar{v}(P_1)$:

$$\bar{v}(P_1) = \max\{v \in [0,1]\colon\ p_0 + v - P_1 \leqslant \delta_c EU(v, P_1)\}. \quad (5)$$

The first-period price P_1 is determined by

$$\max_{P_1} \ \{\Pi_1(P_1) + \delta \mathbb{E}_{\tilde{G}}[\Pi_2^*(\tilde{G}, P_1) \mid P_1]\}, \quad (6)$$

where $\Pi_1(P_1)$ is the first-period profit, which is a function of $\bar{v}(P_1)$, i.e., the marginal consumer in period 1, and $\Pi_2^*(g, P_1) = \Pi_2(g, P_1, P_2(g, P_1))$ is the second-period optimal profit. Detailed derivations of the profit functions $\Pi_2^*(g, P_1)$ and $\Pi_1(P_1)$ are provided in the appendix.

In the subsequent analysis, we will focus on the equilibrium price path: P_1^* and $\bar{v}^*$ and $P_2^*(g)$, where $\bar{v}^*$ and $P_2^*(g)$ are shorthand notations for $\bar{v}^*(P_1^*)$ and $P_2^*(g, P_1^*)$.

[18] Note that $\tilde{G} = \tilde{G} \mid \tilde{\Theta}$ depends on P_1 via the volume of consumers that buy early.

2.4. Discussion of the Game

We make a few critical assumptions that warrant further discussion. First, the reviews are generated by consumers. In practice, there are many sources of information that a firm and consumers may consult before making a purchasing decision. For example, "experts" or consumer organizations may test new products and provide an opinion about the quality. Such information does not depend on the initial sales volume and can be conceptualized in our model as a posterior $\Phi(p, \hat{v})$ for some exogenously given $\hat{v}$. We compare this kind of exogenous information with consumer-generated review information in §4.2. In addition, such opinions may be biased and the biases are difficult to correct for, as financial ties with a firm for which the opinion is written may be difficult to uncover. User-generated reviews have been used for many categories of products, since presumably it is more difficult to influence them on a large scale.

Second, consumers are rational. Although we assume that consumers write reports about the common quality component Θ, one might argue that reviews reflect consumers' net experienced utility, which depends on the common quality component, on price, as well as on their private valuation (see, e.g., Li and Hitt 2010, Ifrach et al. 2013). In such a model, the reviews can be biased because, for example, the most eager consumers are more likely to submit a positive opinion (see, e.g., Papanastasiou et al. 2013). Nevertheless, this model can be shown to be equivalent to ours if consumers are rational in the following sense: They are able to "debias" or "correct" the reviews for price and private valuations. The debias involves adjustment of common price P_1 and private valuations v of those consumers who bought early (both known). To avoid an additional layer of transformation leading effectively to the same outcome, we assume that the reviews are already debiased.

Third, there may be many reasons for not *increasing* the price, even after receiving good reviews. Consumers may perceive price increases as unfair. However, price increases do take place and for some products are explicit, whereas for others the price increases can be subtle. For analytical tractability, we assume symmetries in consumer reactions to the firm's price changes.

Fourth, τ can be related to the complexity of the product. For more complex products, it may be more difficult to assess the common value component, hence the lower τ.

Fifth, we assume that the firm cannot control the dissemination of consumer reports. This is a realistic feature as third-party websites or social media act as aggregators of the dispersed consumer review information. It may be difficult for a firm to directly control the information flow.

3. Preliminaries

The initial sales, with volume sold x, in period 1 will generate reviews, which will then determine the posterior probability. It will be convenient to consider the posterior probability of high-quality product $\tilde{P}\mid\{x\}=p(\tilde{G},x)$ as a random variable for a given x. We obtain the distribution of $\tilde{P}\mid\{x\}$ as follows: Since $p(g,x)$ is increasing in g and thus invertible, let $\eta(p,x)=x/2-\ln(((1-p)p_0)/(p(1-p_0)))/\tau^2$ be the inverse of $p(g,x)$ for given x. Let $\Pr(\tilde{P}\leqslant p\mid x)=\Phi(p,x)$ be the distribution function of $\tilde{P}\mid\{x\}$, or $\Phi(p,x)=p_0F_1(\eta(p,x),x)+(1-p_0)F_0(\eta(p,x),x)$, which can be rewritten as

$$\Phi(p,x)=p_0F\left(\frac{\eta(p,x)-x}{\sqrt{x}/\tau}\right)+(1-p_0)F\left(\frac{\eta(p,x)}{\sqrt{x}/\tau}\right), \tag{7}$$

where $F(g)=\int_{-\infty}^{g}(1/\sqrt{2\pi})\exp(-r^2/2)\,dr$ is the cumulative standard normal distribution. Furthermore, because $\bar{v}(P_1)$ is invertible, instead of the initial price, P_1, it will be convenient to consider the highest remaining type in period $t=2$, $v=\bar{v}(P_1)$, as the decision variable. As shown in the appendix, with v as the decision variable, and $1-v$ customers purchasing in period 1, the firm's pricing problem becomes $\Pi^*=\max_{v\in[0,1]}\Pi(v)$, where

$$\Pi(v)\triangleq\Pi_0(v)-\delta_c C_W(v)+\delta\Pi_S(v) \tag{8}$$

and

$$\begin{cases}\Pi_0(v)=(1-v)(p_0+v-c),\\ C_W(v)=\displaystyle\int_0^1\underbrace{\tfrac{1}{2}(1-v)\{p+v-c\}^+}_{=\hat{C}_W(v,p)}d\Phi(p,1-v),\\ \Pi_S(v)=\displaystyle\int_0^1\underbrace{\tfrac{1}{4}(\{p+v-c\}^+)^2}_{=\hat{\Pi}_S(v,p)}d\Phi(p,1-v).\end{cases} \tag{9}$$

Here the first term is the firm's first-period profit if the consumers were myopic, the second term represents the firm's first-period loss due to forward-looking consumers, and the last term captures the firm's profit from selling in the second period. The operand of the second term $C_W(v)$ (related to strategic consumer behavior) is piecewise linear, and for the third term $\Pi_S(v)$ (related to firm's pricing) is piecewise quadratic; see Figure 1 (left panel). Observe that, due to strategic consumers and quality updates, the firm's myopic profit is adjusted by the expectation of a nonconvex function of the posterior quality; see Figure 1 (right panel). Solving problem (8), with Equations (9) and (7), yields v^*. Given v^*, the optimal initial price will then be given by $P_1(v^*)$, where

$$P_1(v)=p_0+v-\tfrac{1}{2}\delta_c\int_0^1\{p+v-c\}^+\,d\Phi(p,1-v). \tag{10}$$

Hence, Equations (8), (9), and (7) are the main equations for the base model.

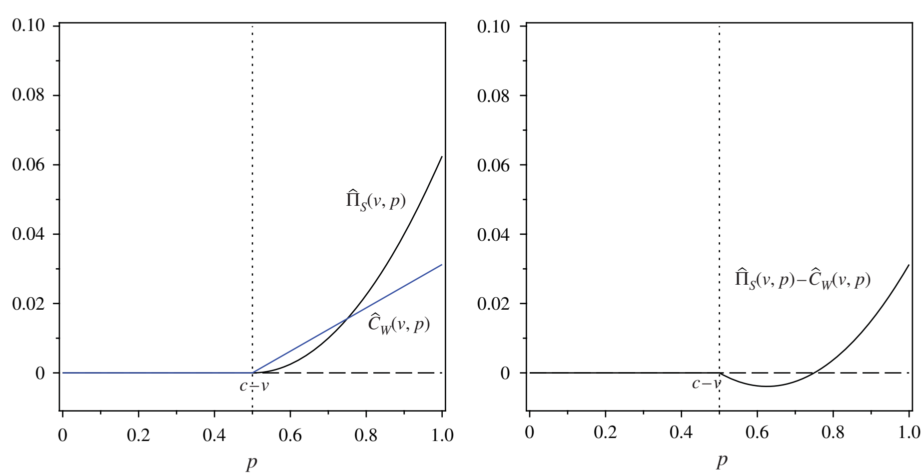
Now, we obtain two important properties of the distribution of the preposterior: let the density be $\phi(p,x)=(\partial/\partial p)\Phi(p,x)$ and the tail distribution be $\bar{\Phi}(p,x)=1-\Phi(p,x)$. Proposition 1 characterizes the impact of the volume of consumers, x, on the posterior quality.

Proposition 1. (i) *For any x, the expected future quality is equal to the expected current quality; $\int_0^1 r\,d\Phi(r,x)=p_0$ (martingale property).*

(ii) *$\tilde{P}\mid\{x\}$ second-order stochastically decreases in x, or $(\partial/\partial x)\int_0^p\bar{\Phi}(r,x)\,dr<0$ for any $p\in(0,1)$.*

Proposition 1(i) states that, irrespective of the early sales volume, the expected future quality is equal to the current quality. That is, the posterior satisfies the martingale property. Proposition 1(ii) states that the second-order stochastic dominance (SOSD) property captures how an increased volume of early consumers increases the spread in the posterior. This result is consistent with Bergemann and Valimaki (1997, 1999). Intuitively, smaller volumes of initial sales make the reports less informative. That is, when few reviews are posted, consumers will likely not change their prior after reading these reviews. In the extreme case,

Figure 1 (Color online) $\hat{\Pi}_S(v,p)$ and $\hat{C}_W(v,p)$ (Left) and $\hat{\Pi}_S(v,p)-\hat{C}_W(v,p)$ (Right) for $v=3/4$ and $c=5/4$



if the reports are not informative at all, the posterior equals to the prior with probability one.[19] To describe the effect of reviews, consider first a benchmark, the case without reviews. It actually corresponds to the limiting case of our model when the information precision approaches zero: $\tau \to 0^+$. When $\tau \to 0^+$, it is straightforward to show that $p(\tilde{G}, v)$ equals to p_0 almost surely. To facilitate future comparison, we denote the equilibrium marginal consumer as v^o and the corresponding profits and consumer surplus as $\Pi^o = \max_{v\in[0,1]} \Pi^o(v)$, with $\Pi^o(v) = \Pi_0(v) - \delta_c \hat{C}_W(v, p_0) + \delta\hat{\Pi}_S(v, p_0)$. The equilibrium price is determined by $P_1^o(v^o)$, where, as in Equation (10), $P_1^o(v) = p_0 + v - \frac{1}{2}\delta_c\{p_0 + v - c\}^+$.

Without reviews, the firm's optimization problem (Π^o) yields insights similar to the classical durable-goods problem with forward-looking consumers (Besanko and Winston 1990). In particular, it is easy to show that strategic consumers (i) provide the firm an incentive to sell less initially, (ii) make the price path flatter, and (iii) induce the firm to delay all sales until the second period when both the firm and consumers are equally and very patient ($\delta = \delta_c = 1$).

4. Analysis: General Results

In this section, we introduce the firm's and consumers' option value (in §4.1), and examine from the firm's perspective, the impact of consumer reviews on the firm's profit and marginal profit (in §4.2).

4.1. Skimming the Market with Consumer Reviews

Our objective is to analyze the impact of informative reviews. Assume that the information precision is strictly positive, $\tau > 0$. The change in firm's profits, given the same first-period market segment v, can be expressed as

$$\Pi(v) - \Pi^o(v) = \delta \underbrace{\left(\int_0^1 \hat{\Pi}_S(v, p)\, d\Phi(p, 1-v) - \hat{\Pi}_S(v, p_0)\right)}_{\doteq O(v)\geq 0} - \delta_c \underbrace{\left(\int_0^1 \hat{C}_W(v, p)\, d\Phi(p, 1-v) - \hat{C}_W(v, p_0)\right)}_{\doteq O_c(v)\geq 0}.$$

We refer to $\delta O(v) - \delta_c O_c(v)$ as the firm's *net* discounted option value. This is a combination of two effects: $O(v)$ captures the firm's option value due to informative reviews, and $O_c(v)$ represents the consumers' option value due to reviews. Both option values are nonnegative: $O(v) \geq 0$ and $O_c(v) \geq 0$.[20] The firm's option value $O(v)$ is due to its price adjustment in reaction to consumer reviews, which is always beneficial for the firm. Specifically, the firm will not sell in the second period a product whose quality is bad for the highest remaining type, corresponding to the kink in the integrand of $O(v)$ ($\hat{\Pi}_S(v, p)$ in Figure 1, left panel). Otherwise, the firm offers the product in the second period, with price adjusted for posterior quality. The adjusted margin *and* the corresponding sales yield the quadratic part in the integrand of $O(v)$. The consumers' option value $O_c(v)$ captures consumers' gain from reviews if they delay purchase, i.e., they can choose not to buy if the reviews are bad, corresponding to the kink in the integrand of $O_c(v)$ ($\hat{C}_W(v, p)$ in Figure 1, left panel). Consumers' option value is always disadvantageous to the firm and manifests itself in additional price discount in the first period, paid to all $1 - v$ consumers who bought *early*. This difference between the firm's and consumers' option values determines how each will be able to gain advantage from reviews, as we discuss in §§5.1 and 5.2: Intuitively, the consumer option value (reducing the firm's profit) is purely driven by the possibility that the firm withdraws its product after bad reviews. The firm's option value is due to adjustment in the option value via its margin and volume adjustment and also by the possibility of withdrawal.

Consider now how the firm's net discounted option value changes in the initial segment v:

$$\begin{aligned}\frac{d}{dv}\Pi(v) - \frac{d}{dv}\Pi^o(v) &= \delta \underbrace{\left(\int_0^1 \frac{\partial\hat{\Pi}_S(v, p)}{\partial v}\, d\Phi(p, 1-v) - \frac{\partial\hat{\Pi}_S(v, p_0)}{\partial v}\right)}_{\doteq MO(v)\geqslant 0} + \delta \underbrace{\int_0^1 \hat{\Pi}_S(v, p)\frac{\partial}{\partial v}\phi(p, 1-v)\, dp}_{\doteq I(v)\leqslant 0} \\ &\quad - \delta_c \underbrace{\left(\int_0^1 \frac{\partial\hat{C}_W(v, p)}{\partial v}\, d\Phi(p, 1-v) - \frac{\partial\hat{C}_W(v, p_0)}{\partial v}\right)}_{\doteq MO_c(v)\geqslant 0} - \delta_c \underbrace{\int_0^1 \hat{C}_W(v, p)\frac{\partial}{\partial v}\phi(p, 1-v)\, dp}_{\doteq I_c(v)\leqslant 0}.\end{aligned}$$

[19] Consumer reports thus impact the dispersion on the posterior, similar to Johnson and Myatt (2006), who consider other marketing incentives that cause a larger dispersion in demand.

[20] This is due to Jensen's inequality; the integrands of $\hat{\Pi}_S(v, p)$ and $\hat{C}_W(v, p)$ are convex (see Figure 1) and, via the martingale property of the preposterior (Proposition 1), the prior, p_0, is the expected value of the preposterior. Hence, the option value as defined is nonnegative. Also, note that by Equation (1), for any given v, $p + v - c < 0$ for some $p \in [0, 1]$. Thus, this excludes the situation that $C_W(p, v) = (1 - v)(p + v - c)$ for all p and v, in which case $O_c(v) = 0$ trivially for all v.

The derivative of the firm's option value $O(v)$ can be decomposed into two factors, $MO(v)$ and $I(v)$, and the derivative of the consumers' option value is decomposed into another two factors, $MO_c(v)$ and $I_c(v)$. The first two factors, $MO(v)$ and $MO_c(v)$, are the direct effects of changing v on the option values, ignoring changes in the informativeness of the reviews. If the signal were exogenously generated, e.g., if the posterior were $\Phi(p, \hat{v})$ for some exogenously given $\hat{v}$, $MO(v)$ and $MO_c(v)$ would be the only effects. However, with consumer reviews, changing v also changes the informativeness of the signal via $\Phi(p, v)$. This effect is captured by $I(v)$ and $I_c(v)$. By the martingale and SOSD property of the posterior (Proposition 1), the signs of the four components are as follows:

- $MO(v) \geqslant 0$. Intuitively, the higher v is, the larger the *remaining* market with higher-valuation consumers waiting to purchase is. The firm can leverage the consumer review information on a larger market, which improves its profits.
- $MO_c(v) \leqslant 0$. It is interesting that the direct effect of a higher v on the consumers' option value is opposite to that on the firm's option value. Ignoring the dependence of Φ on v (which is captured in $I_c(v)$), $O_c(v)$ decreases in v because a higher v means (a) a lower volume of initial sales and thus a lower volume of consumers that enjoy the option value, and (b) a lower discount offered by the firm, as the marginal consumer v has a higher valuation and thus is more likely to buy in the second period, implying a lower option value for the individual consumer.
- $I(v) \leqslant 0$. A higher v means fewer early consumers post reviews, and thus the reviews become less informative and the firm benefits less from them.
- $I_c(v) \leqslant 0$. Similarly to $I(v)$, $I_c(v)$ is also negative: as v increases, reviews become less informative, and thus the consumers' option value due to reviews also decreases.

4.2. Summary and Discussion: The Impact of Consumer Reviews

We summarize the impact of consumer review on the firm's profit: $\Pi(v) - \Pi^o(v) = \delta O(v) - \delta_c O_c(v)$. Because both $O(v) \geq 0$ and $O_c(v) \geq 0$, it follows that when $\delta_c = 0$ and $\delta > 0$, for any v the profits with consumer reviews are higher than without. As a consequence, without strategic consumer behavior, informative consumer reviews are beneficial for the firm. The other way around also follows immediately: When $\delta = 0$ and $\delta_c > 0$, informative consumer reviews always hurt the firm. This is solely driven by strategic consumers, who anticipate the review information and have the option of waiting, which reduces their willingness to pay in the first period and dampens the firm's profitability. In the general case, with any

Table 1 **Summary of the Signs of the Marginal Effects in the Marginal Option Values**

	Marginal option value	Information incentive
Firm effect (δ)	$MO(v) \geqslant 0$	$I(v) \leqslant 0$
Consumer effect (δ_c)	$MO_c(v) \leqslant 0$	$I_c(v) \leqslant 0$

discount factors δ and δ_c, more information does not always increase the firm's profit; the relative patience, δ versus δ_c, is a key driver for the firm's benefit due to consumer reviews. In §5 we describe in more detail this interaction.

It is more subtle to determine the impact of consumer reviews on the firm's marginal profit:

$$\frac{d}{dv}\Pi(v) - \frac{d}{dv}\Pi^o(v) = \delta(MO(v) + I(v)) - \delta_c(MO_c(v) + I_c(v)).$$

Table 1 displays the signs and allows us to make several observations: When $\delta_c = 0$ and $\delta > 0$, because $MO(v)$ and $I(v)$ have different signs, the impact of consumer reviews on the marginal profit is ambiguous and the optimal initial sales may increase or decrease due to consumer reviews. However, when $\delta = 0$ and $\delta_c > 0$, because $MO_c(v)$ and $I_c(v)$ are both negative, consumer reviews reduce the marginal waiting cost, which will thus increase the firm's marginal profit. So, with patient consumers and an impatient firm, due to reviews, the firm will decrease the initial sales and more consumers delay purchase till the second period. As we noted, this result contrasts with the literature, e.g., Bergemann and Valimaki (1997, 1999), which shows that a monopolistic firm always sells more initially to promote fast learning. In our model, strategic consumers perceive a larger option value of waiting due to more informative reviews. Thus, the firm would have to substantially lower the initial price if it wanted to increase initial sales. This may not be worthy for the firm, especially when it is impatient and cares little about learning through reviews. Note that Bergemann and Valimaki (1997, 1999) consider independent cohorts of nonstrategic consumers that arrive in different periods. Hence, they do not consider the firm's marginal option value (i.e., $MO(v) = 0$) or consumer effect ($\delta_c = 0$).

Notice that the information terms $I(v)$ and $I_c(v)$ are intrinsically linked with the endogeneity of information generation that is typical for consumer reviews. If the posterior distribution was exogenously determined, the information incentives would be zero; $I(v) = 0$ and $I_c(v) = 0$, and the impact of exogenous information on the marginal profit would be determined by $MO(v) - MO_c(v)$, which is always positive. Thus, with exogenous quality information such as independent quality reviews from experts or consumer organizations, the firm's marginal profit increases, providing an incentive to sell less in period 1. This is intuitive: if quality information diffuses over time irrespective of the sales volume, then the firm is better off delaying the sales until more information becomes available and then setting a price in response to the information. With endogenously generated information, this strategy is not always optimal, because the initial sales determine the precision of review information.

5. Analysis: Further Results

In the previous section, we discussed general conditions under which the profit difference and the marginal profit difference is either positive or negative for all values of v. The results we obtained are general because they only depend on the SOSD and the martingale property of the preposterior.

In this section, we analyze how the relative patience (of the firm and consumers) and informativeness of the reviews, τ, determine the value of information. Furthermore, we identify how a firm should narrow or broaden its initial market segment $(1 - v^*)$ due to the presence of consumer reviews and how such strategy can be implemented by changing the initial price (P^*).

5.1. The Impact of Reviews on the Firm's Profit

Recall that the impact of consumer reviews on the firm's profit is influenced by the relative patience level (δ versus δ_c). As we noted, the clear outcome is that, when the firm is very patient (impatient) and the consumers are very impatient (patient), the firm gains (loses) from consumer reviews. Now, we consider environments in which the patience level is less polarized.

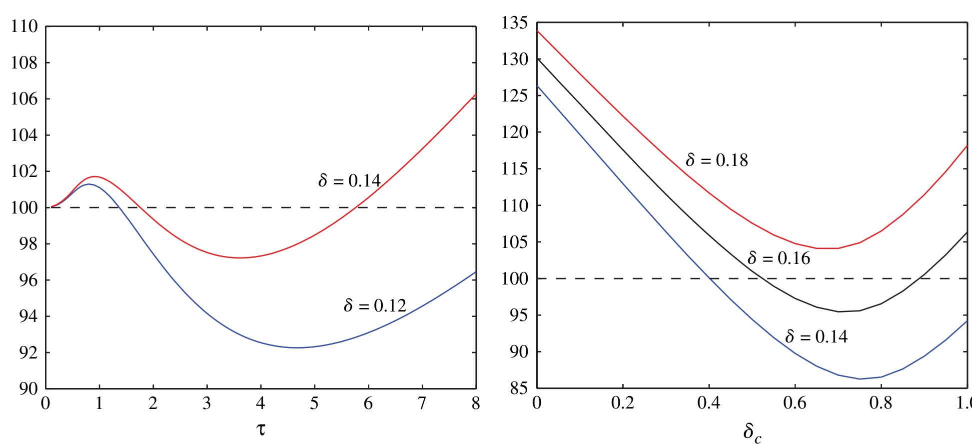
The left panel of Figure 2 shows that, for low levels of informativeness, the firm's gain from reviews is positive, even when the consumers are much more patient than the firm. In particular, the firm's gain first increases in τ. This may be intriguing. We delay explanation of this dynamic until Proposition 2. After reaching a maximum, the firm's gain starts to decrease. Thus, for some levels of informativeness, the firm may prefer *less* informative reviews. Intuitively, this is because more informative reviews reinforce consumers' incentive to wait. As τ becomes very large, reviews may benefit the firm again. This is when the firm shifts to selling a very small volume in the initial period. Because of high informativeness of the reviews, small early sales allow the firm to fine-tune the pricing policy in the following period and the firm is increasingly better off.

The right panel of Figure 2 shows the effect of the consumers' patience on the firm's profits, for $\tau \to +\infty$ and for various (fixed) levels of firm's patience δ. Intuitively, one would expect that as consumers become more strategic (increasing δ_c), the gains from reviews decrease because consumers have stronger incentives to wait for review information. It is mostly the case, but interestingly, for high enough δ_c, the opposite might be true: The firm's gain from reviews are greater for *more* strategic consumers. Note that since the reviews are very informative, the product quality can be learned with great certainty even with a few shared consumer experiences and, thus, the firm prefers to sell a very small quantity up front and the firm's profit barely depend on the patience of consumers. However, without reviews, the firm continuously gets worse with more patient consumers. Thus the benefit of the reviews is influenced by the benchmark case getting worse.

We formalize these observations and provide intuition for the left panel of Figure 2.

The Effect of Informativeness. We refer to reviews with strictly positive, but, low precision, $\tau \to 0^+$ as "noisy reviews." First we formalize the observation,

Figure 2 **(Color online)** Π^*/Π^o **(in %) as Functions of** τ **(Left) and Functions of** δ **and** δ_c **(Right):** $p_0 = 0.5$, $c = 1.3$, $\delta_c = 1$ **(Left),** $p_0 = 0.95$, $c = 1.9$ **and** $\tau = 100$ **(Right)**



from Figure 2 (left panel) that noisy reviews increase the firm's profit, *irrespective* of patience level of the firm and consumers:

Proposition 2. *Noisy reviews always increase the firm's profits, irrespective of the relative patience level of the firm and consumers. (Formally, for very small but, strictly positive τ and for any $\delta, \delta_c \in (0, 1)$: $\Pi^* > \Pi^o$.)*

Proposition 2 shows that the firm always benefits from consumer reviews that are noisy (i.e., not very accurate). This is an interesting observation and the intuition is an extension of the discussion in §3. Noisy reviews do not significantly change the prior perception, hence the chances that, based on very bad initial reviews, the product is unattractive to all consumers, are extremely slim. However, the consumers' option value is purely driven by the possibility of not purchasing the product after bad reviews. Consequently, with noisy reviews, consumer option value is almost zero, while the firm still benefits by adjusting price according to reviews and enjoys a strictly positive option value.[21]

The Effect of Impatience. Although the firm always gains from noisy reviews, it may get worse off by accurate ones when the consumers are relatively more patient than the firm. We formalize this observation in Proposition 3(i).

Proposition 3. *Assume that the consumer reports are perfectly informative (i.e., $\tau \to +\infty$).*

(i) *Reviews decrease the firm's profits under the following circumstances:*

- *When the firm's patience level is low and the consumers' patience level is high (formally, there exist a threshold $\underline{\delta} > 0$ and a function $\underline{\delta}_c(\delta)$, such that $\Pi^* < \Pi^o$ for $\delta < \underline{\delta}$ and $\delta_c \in (\underline{\delta}_c(\delta), 1]$).*

(ii) *Reviews increase the firm's profits under the following circumstances:*

- *When the firm's patience level is high, irrespective of the patience level of the consumers (formally, there exists a threshold $\bar{\delta} < 1$ such that for $\delta > \bar{\delta}$, $\Pi^* > \Pi^o$ for $\delta_c \in [0, 1]$).*
- *When the firm's patience level is low and the consumer is equally or less patient (formally, for $\delta < \underline{\delta}$, $\Pi^* > \Pi^o$ for $\delta_c \in [0, \delta]$).*

In §3 we showed that, when the firm is impatient while the consumers are patient, the firm's *net* option value will be negative. Proposition 3(i) extends this result to a broader set of situations (when the firm is impatient enough and consumers are very patient). Proposition 3(ii) analyzes the opposite case and states that when the firm is patient enough, reviews are always profitable for the firm. When the firm's patience level δ is high, the impact of forward-looking consumers is less important, because the firm is patient enough to delay most of the sales to the second period, with or without reviews. In such a case, the firm always benefits from reviews.

Interestingly, independently of the firm's patience level, if consumers are equally or less patient, reviews remain profitable for the firm. That is, even though the firm's profit is reduced by the consumers' option value, the firm's option value is always greater than the consumers' option value when they are equally patient. We expect that in realistic settings, the firm's patience level will be higher than that of consumers. This implies that the consumer-generated reviews, despite strategic behavior of consumers, continue to benefit the firm. Interestingly, numerical comparisons strengthen this claim. They show that for any patience level of the firm, when the consumers are equally patient, both the firm and the consumers benefit from reviews, independent of the informativeness of the reviews.

5.2. The Impact of Reviews on Consumer Surplus

Although we used the term consumers' option, it was primarily used to explain the effect of strategic consumers on firm's profits. A natural question that arises in our framework, in which quality information is generated by consumers, is whether (and if so, to what degree) consumers actually benefit from such information, given that the firm may benefit from such information via price changes. To answer this question, we analyze the consumer surplus. For the marginal consumer, v, we can express the consumer surplus as $C(v) = \int_0^1 \hat{C}(p, v)\, d\Phi(p, v)$, where

$$\hat{C}(p, v) = \frac{(1-v)^2}{2} + \frac{\delta_c}{2}(1-v)\{p+v-c\}^+ + \frac{\delta_c}{8}(\{p+v-c\}^+)^2. \quad (11)$$

The surplus of the consumers that purchase early is captured by the first two terms. The surplus of the consumers that purchase late is captured by the last term. Let $C^* = C(v^*)$ indicate the equilibrium consumer surplus.

For noisy reviews, a similar logic as in Proposition 2 applies to the consumer surplus: noisy reviews are beneficial for consumers too, irrespective of the relative patience of the firm and consumers. Also, the consumers benefit when the firm and consumers are very patient:

Proposition 4. *When the reviews are noisy, or when the firm and consumers are all very patient, consumers are better off with reviews (formally, for very small τ and any $\delta, \delta_c \in (0, 1)$; or when $\delta = \delta_c = 1$: $C^* \geq C^o$).*

[21] The posterior is distributed closely around the prior p_0 and for p around p_0, the integrand of $C_W(v^o)$ is linear and the integrand of $\Pi_S(v^o)$ is convex, since $p_0 + v^o - c > 0$, which we show in the proof of Proposition 2.

In general, though, for imbalanced patience levels, or when reviews are significantly informative, reviews may either decrease or increase the consumer surplus. Furthermore, similar to the firm, consumers may also prefer less informative reviews. The underlying reason is that, although consumer-generated quality information helps other consumers better assess the product, it also allows the firm to make more informed pricing decisions and to extract more consumer surplus.

As we noted in the previous section, when both firm and consumers are equally patient, the firm always benefits from consumer reviews and the firm's gain increases in the common patience level, irrespective of the informativeness of the reviews.[22] Numerical experiments indicate that the consumers' gain from reviews is comparable to the firm's. This is expected since the second-period consumer surplus is a similar quadratic function of the posterior as the firm's second-period profit.

5.3. The Impact of Reviews on Initial Sales

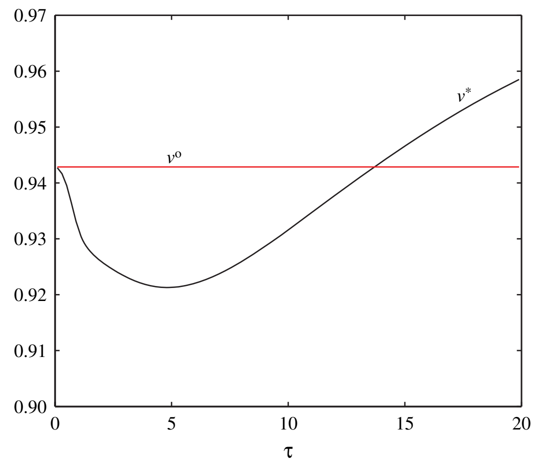
Recall from §4.2 that reviews increase the marginal profit when the consumers are patient and the firm is impatient ($\delta = 0$), or when the posterior distribution is exogenously determined such as via experts or consumer organizations. In these cases, reviews provide incentives to sell less initially; $1 - v^* < 1 - v^o$. In general, the impact of consumer reviews on the early sales volume is ambiguous. Figure 3 illustrates the impact of consumer reviews on the early volumes, for different levels of informativeness. Observe that the firm's initial sales increase first and then decrease due to reviews. In the following proposition, we describe environments in which consumer reviews make the initial sales either increase or decrease.

Proposition 5. (i) *The initial sales decrease for perfectly informative reviews (formally, when either $\delta < 1$ or $\delta_c < 1$, then $1 - v^* < 1 - v^o$ for $\tau \to +\infty$).*

(ii) *The initial sales increase for noisy reviews (formally, when $\delta > 0$ and either $\delta < 1$ or $\delta_c < 1$, then $1 - v^* > 1 - v^o$ for very small τ).*

Proposition 5(i) reinforces our previous observation that, with very informative consumer reviews, a very small volume of initial sales provides accurate information. Hence, the firm prefers to shift consumers to period 2, when pricing is based on better information about quality. Proposition 5(ii) indicates that, for noisy reviews, the firm sells more initially to obtain more information. This is consistent with the result in Bergemann and Valimaki (1997, 1999). Although their result

Figure 3 **(Color online) The Firm's Optimal Initial Segment (v^*) Is Not Monotonic in τ: $p_0 = 0.5$, $c = 1.3$, and $\delta = \delta_c = 0.75$**



of the firm increasing initial sales does not hold in general when the strategic consumers are present, we show that it is valid when the review precision is low.

5.4. The Impact of Reviews on Prices

The optimal changes in the volume of the initial sales are actually implemented via changes in initial price. Figure 4 illustrates how the firm should adjust its price in response to informative reviews. Note the existence of $\hat{\tau}$ at which reviews do not change the initial sales. For less informative reviews, initial sales are higher ($1 - v^* > 1 - v^o$) and the corresponding initial price is lower ($P^* < P^o$). For significantly more informative reviews, the opposite is true: The initial sales are lower ($1 - v^* < 1 - v^o$) and the initial price is higher ($P^* > P^o$). However, for intermediate levels of precision, it is optimal to *decrease* the initial sales ($1 - v^* < 1 - v^o$) and yet, *decrease* the initial price ($P^* < P^o$). Observe from the comparison of the left with the right panel of Figure 4 that the range when this occurs is larger for higher δ_c: strategic consumer behavior amplifies the region in which the firm needs to reduce initial price to "bribe" consumers to buy early.

The initial price can be formally captured. It equals the difference between the marginal consumer's expected gross utility and his expected utility from delaying the purchase till the second period. Specifically, from Equation (10), define the initial price without consumer reviews, as a function of v, by $P_1^o(v)$ (and $P_1^o = P_1^o(v^o)$) and that with consumer reviews by $P_1^* = P_1^*(v^*)$. We can now compare P_1^o and P_1^*:

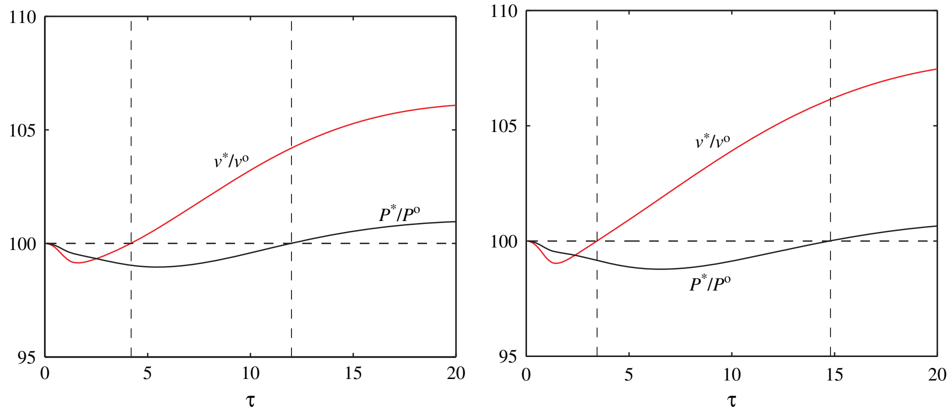
Proposition 6. (i) *If the initial sales volume increase with reviews,*[23] *then the initial price decreases (formally, when $1 - v^* > 1 - v^o$, then $P_1^* < P_1^o$).*

(ii) *There exists a level of informativeness such that the initial sales volume does not change, but the initial price*

[22] Recall that we prove the result for $\tau \to \infty$ in Proposition 3. For general τ, we numerically verify the result, as shown in the supplementary document (available at http://dx.doi.org/10.1287/mnsc.2014.2134).

[23] In Proposition 5, the initial sales increase for noisy reviews.

Figure 4 (Color online) Impact of Reviews on Initial Segment and Price: v^*/v^o and P^*/P^o (in %) as Functions of τ: $\delta_c = 0.5$ (Left), $\delta_c = 0.7$ (Right), $\delta = 0.5$, $p_0 = 0.5$, and $c = 1.1$



decreases (*formally, for* δ, $\delta_c \in (0, 1)$, *there exists a* $\hat{\tau} \in (0, +\infty)$ *for which* $1 - v^* = 1 - v^o$ *and* $P_1^* < P_1^o$).

Thus, Proposition 6 formally captures the behavior we illustrated and explained above.

6. Extensions

Below, we describe how some of the assumptions in the basic model can be relaxed. To reduce the notational burden, we do not differentiate the symbols across the extensions. All the proofs of the results in the extensions are provided in the supplementary document (available at http://dx.doi.org/10.1287/mnsc.2014.2134).

6.1. Capacity Constraints

So far, we have assumed that the firm has ample capacity to satisfy all demand. Capacity constraints, however, are very common, especially for new products, where quality and, consequently, the effective demand are unknown. Capacity constraint results in smaller flexibility of the firm. This in itself can influence the value of information, carried through reviews, to both the firm and to the consumers. Very often a firm's capacity constraint can be overcome by operational improvements, which allows a quick response to realized demand, at an additional cost. These could be, for example, overtime or use of expediting and are often labeled as "quick response." In this extension, we analyze the effect of the firm's capacity decision and the effect of quick response, on the firm's profit. Assume that the firm invests in capacity K_i in period i, for $i = 1, 2$, at a cost of k per unit of capacity. The capacity is not observable to consumers. Demand in each period can be fulfilled up to capacity limit, at unit cost c. In the second period, demand above capacity K_2 can be satisfied at a cost of c_2 ($> c$) per unit.[24] Increasing c_2 reduces the firm's ability of quick response and, when $c_2 = +\infty$, no excess demand can be satisfied.

We formally prove and, in some cases, illustrate numerically, that most of the properties continue to hold with capacity constraints, as well as with quick-response options. In the supplementary document, the model is formally presented. In particular, we explain in detail the consumer rational expectation model (following Su and Zhang 2008) and include additional properties and omitted proofs.

Note that, since the second-period price is determined *after* the reviews are posted (i.e., after the uncertainty about posterior is resolved), stock-outs never occur in the second period. Nevertheless, the quick-response capability can enhance the profit if the posterior, p, is high.

Similarly as in the base model we define the firm's and the consumers' option values as $O(v)$ and $O_c(v)$, respectively. Despite the fact that the option values become fairly complicated, we can still show the following:

Lemma 1. *The firm's and consumers' option values are nonnegative* (*formally,* $O(v) \geq 0$ *and* $O_c(v) \geq 0$).

Quick response is usually described as a tool to mitigate strategic consumer behavior (Cachon and Swinney 2009, Swinney 2011) and in our model we can show that a patient firm ($\delta > 0$) facing impatient consumers ($\delta_c = 0$) is better off by an improvement in its quick-response capability. Proposition 7 shows the opposite outcome when an impatient firm faces patient consumers:

Proposition 7. *An impatient firm facing patient consumers*

(i) *is worse off by an improvement in its quick-response capability* (*formally, if* $\delta = 0$ *and* $\delta_c > 0$, *for any* $k > 0$, Π^* *increases in* c_2);

(ii) *loses more due to quick response as the reviews become more accurate* (*formally, if* $\delta = 0$ *and* $\delta_c > 0$, $\Pi^*|_{c_2=\infty} - \Pi^*|_{c_2>0}$ *increases in* τ);

[24] In the first period, the demand is deterministic because no reviews have yet been observed; hence, the firm can plan its capacity equal to its demand.

(iii) *loses more due to strategic consumer behavior as the firm's quick-response capability is improved or as the reviews become more accurate (formally, if $\delta = 0$ and $\delta_c > 0$, $\Pi^*|_{\delta_c=0} - \Pi^*|_{\delta_c>0}$ decreases in c_2 and increases in τ).*

The intuition behind (i) is as follows: A cheaper quick-response supply (i.e., lower c_2) implies that the second-period price is lower and thus the consumers' expected surplus from postponing purchase is higher. As a result, a deeper price discount is needed to induce customers' purchases in the first period, and the firm's profit decreases because of the quick-response capability. As shown in (ii), the negative effect of quick response is amplified by informative reviews: Consumers become more willing to wait for information available in the second period. Consequently, both quick response and informative reviews reinforce consumers' waiting incentive and aggravate the firm's profit loss due to strategic consumers, as shown in (iii).

Our findings complement the literature of quick response. In a model with strategic consumers in the absence of learning, Cachon and Swinney (2009) show that quick response always benefits the firm and its benefit can be higher due to strategic consumers. In contrast to Cachon and Swinney (2009), Swinney (2011) considers a model augmented by strategic consumers learning their own valuations and finds that quick response may reduce the firm's profit and its value is smaller with strategic consumers. This is because quick response increases the expected fill rate in the second period and, thus, strengthens consumers' incentive to wait. Our result that quick response may hurt the firm is similar to the one in Swinney (2011), but the underlying reason is different: In our model with dynamic pricing, quick response does not influence fill rate in the second period, which always equals one. But quick response does increase second-period supply and, thus, leads to lower second-period prices, which reinforces consumers' willingness to procrastinate purchases and increases the waiting cost for the firm.

6.2. Additional Sales Periods

In this extension, we first consider an extended selling horizon of $T \geq 2$ time periods. Selling over multiple time periods allows the firm to earn extra (discounted) profits in future periods. To avoid any end-of-horizon effects, we allow T to be infinity and, therefore, we exclude discounts $\delta = 1$ and $\delta_c = 1$. Without reviews ($\tau = 0$), the model becomes that by Besanko and Winston (1990) and the state of the system is described by the highest consumer type in the market that did not yet purchase in period t, v_{t-1}. With reviews, the state in period t additionally includes the posterior on quality of the product, p_{t-1}, where p_0 is the given prior in period $t = 1$.

In a generic period, $\hat{p}$ is the prior and p is the preposterior (of the next period) when a volume x is sold. Equation (7) becomes

$$\Phi(p, \hat{p}, x) = \hat{p}F\left(\frac{\eta(p, \hat{p}, x) - x}{\sqrt{x}/\tau}\right) + (1-\hat{p})F\left(\frac{\eta(p, \hat{p}, x)}{\sqrt{x}/\tau}\right). \tag{7*}$$

For any $H(\cdot)$ defined over $[0, 1]$, for a given prior, $\hat{p}$ and volume sold x, define

$$\mathbb{E}_p[H(p) \mid \hat{p}, x] = \int_0^1 H(p)\, d\Phi(p, \hat{p}, x),$$

which is the expected value of $H(\cdot)$, over the preposterior, given prior $\hat{p}$ and volume sold x. In the supplementary document, we prove that Equations (9) and (8) become, respectively,

$$\begin{cases} \Pi_{t,0}(p_{t-1}, v_{t-1}, v_t) \\ \quad = (v_{t-1} - v_t)(p_{t-1} + v_t - c), \\ C_{t,W}(p_{t-1}, v_{t-1}, v_t) \\ \quad = (v_{t-1} - v_t)c_{t,W}(p_{t-1}, v_{t-1}, v_t), \\ \Pi_{t,S}(p_{t-1}, v_{t-1}, v_t) \\ \quad = \mathbb{E}_p[\Pi^*_{t+1}(p, v_t) \mid p_{t-1}, v_{t-1} - v_t] \end{cases} \tag{9*-t}$$

and

$$\Pi^*_t(p_{t-1}, v_{t-1}) = \max_{v_t \leq v_{t-1}} \Big\{\Pi_{t,0}(p_{t-1}, v_{t-1}, v_t) - \delta_c C_{t,W}(p_{t-1}, v_{t-1}, v_t) + \delta \Pi_{t,S}(p_{t-1}, v_{t-1}, v_t)\Big\} \tag{8*-t}$$

with solution $v^*_t(p_{t-1}, v_{t-1})$ and

$$c_{t,W}(p_{t-1}, v_{t-1}, v_t) = \mathbb{E}_p[\{\delta_c c_{t+1,W}(p, v_t, v^*_{t+1}(p, v_t)) + v_t - v^*_{t+1}(p, v_t)\}^+ \mid p_{t-1}, v_{t-1} - v_t],$$

with $c_T(p_{T-1}, v_{T-1}, v_T) = \Pi_{T,S}(p_{T-1}, v_{T-1}, v_T) = 0$. We report analytical results for an infinite horizon, $T \to \infty$ and perfect reviews, $\tau \to +\infty$. We complement our insights for general τ through numerical experiments.

As in the base model, we define $O_c(v) = C_{1,W}(p_0, 1, v) - C^o_{1,W}(1, v)$ and $O(v) = \Pi_{1,S}(p_0, 1, v) - \Pi^o_{1,S}(1, v)$ as the option value in the first period for the firm and consumer, respectively.

We confirm that key properties hold for multiple periods.[25] To estimate the impact of increasing the

[25] This includes properties such as for perfectly informative reviews, the firm's and consumers' option values are nonnegative. A sufficiently patient firm always gains from reviews, whereas a sufficiently impatient firm gains from reviews if consumers are equally or more impatient, otherwise, the impatient firm may lose from reviews. Also, for noisy reviews, the firm always gains from reviews. We numerically confirm that these key properties continue to hold in an infinite horizon.

length of the horizon, we compare the profits Π_∞^* (and Π_∞^o) for the infinite horizon and Π_2^* (and Π_2^o) for the two-period horizon (as in the base model).

Proposition 8. *For a patient firm* ($\delta > 0$), *we have the following*:

(i) *With perfectly informative, or noisy reviews, when the consumers are equally or relatively more patient than the firm, extending the sales horizon from two periods to infinite periods hurts the firm (formally, a slightly stronger claim holds: for* $\tau \to \infty$, $\Pi_\infty^* < \Pi_2^*$, *and* $\Pi_\infty^o < \Pi_2^o$ *if and only if* $1-\delta_c < \sqrt{1-\delta}$).

(ii) *Over an infinite horizon, the value of perfectly informative reviews is positive when both the firm and consumers are equally patient (formally, for* $\tau \to \infty$, $\Pi_\infty^* > \Pi_\infty^o$ *when* $\delta_c = \delta > 0$).

For the case when the firm and the consumers are equally patient $\delta_c = \delta$, we numerically confirm the results of Proposition 8 for general τ, as shown in Table 2.

As shown in Proposition 8 and Table 2, when the firm and consumers are equally patient ($\delta = \delta_c$), extending the horizon is *not* profitable, irrespective of the informativeness of the reviews. Despite the increased opportunities to skim the market, a longer selling horizon strengthens strategic consumers' incentive to wait, due to more buying opportunities and better information about quality of the product. As a result, the firm needs to offer a deeper price discount to induce early purchases of consumers, which reinforces consumers' expectations and hurts the firm's total profit. This behavior is consistent with the classical durable-goods results. Without reviews ($\tau = 0$), when the firm and consumers are very and equally patient (i.e., $\delta = \delta_c \to 1$), the firm's (monopoly) profits in an infinite selling horizon tend to be zero due to the strategic consumer behavior (Besanko and Winston 1990). We confirm that this result continues to hold for perfect reviews ($\tau \to +\infty$).

We report an additional result pertaining to the pricing strategy over an infinite horizon with perfectly informative reviews. In the price-skimming literature, without reviews, the price decreases over time. With reviews, the benefit of learning may lead to nonmonotone prices. This is best illustrated when reviews are very informative. Conditional on the quality being high, we obtain the following:

Proposition 9. *For* $\delta_c = \delta$, *conditional on the quality being high, the expected price may increase and then decrease when the reviews are perfectly informative (formally, for* $\tau \to \infty$ *and* $\delta = \delta_c$, *conditional on* $\Theta = 1$: $P_1^* < P_2^*$ *and* $P_2^* > P_3^* > P_4^* > \cdots$).

In the supplementary document, we provide the conditions under which the price path is nonmonotone. Prices and sales are directly linked, although they are not perfectly mimicking each other. As the states are determined by a Markov chain governed by the equilibrium segmentation strategy and distribution of the signals, we consider the probability density of being in state (p, v) at time t, $\pi_t^*(p, v)$, for an initial condition at $t = 0$, $(p_0, 1)$. The expected sales in period t are, therefore, $S_t^* = \int_0^1 \int_0^1 \{v_{t-1} - v_t^*(p_{t-1}, v_{t-1})\} \pi_t^*(p, v)\, dp\, dv$. Whereas sales with market skimming (and uniform distribution of types) typically are monotone decreasing over time (see, e.g., Besanko and Winston 1990), with consumer reviews, the sales pattern might change over time:

Proposition 10. *Conditional on the quality being high, with sufficiently high cost c, the expected sales reaches a peak in the second period when the reviews are perfectly informative (formally, for* $\tau \to \infty$, *conditional on* $\Theta = 1$, *when c is sufficiently high,* $S_1^* < S_2^*$ *and* $S_2^* > S_3^* > S_4^* > \cdots$).

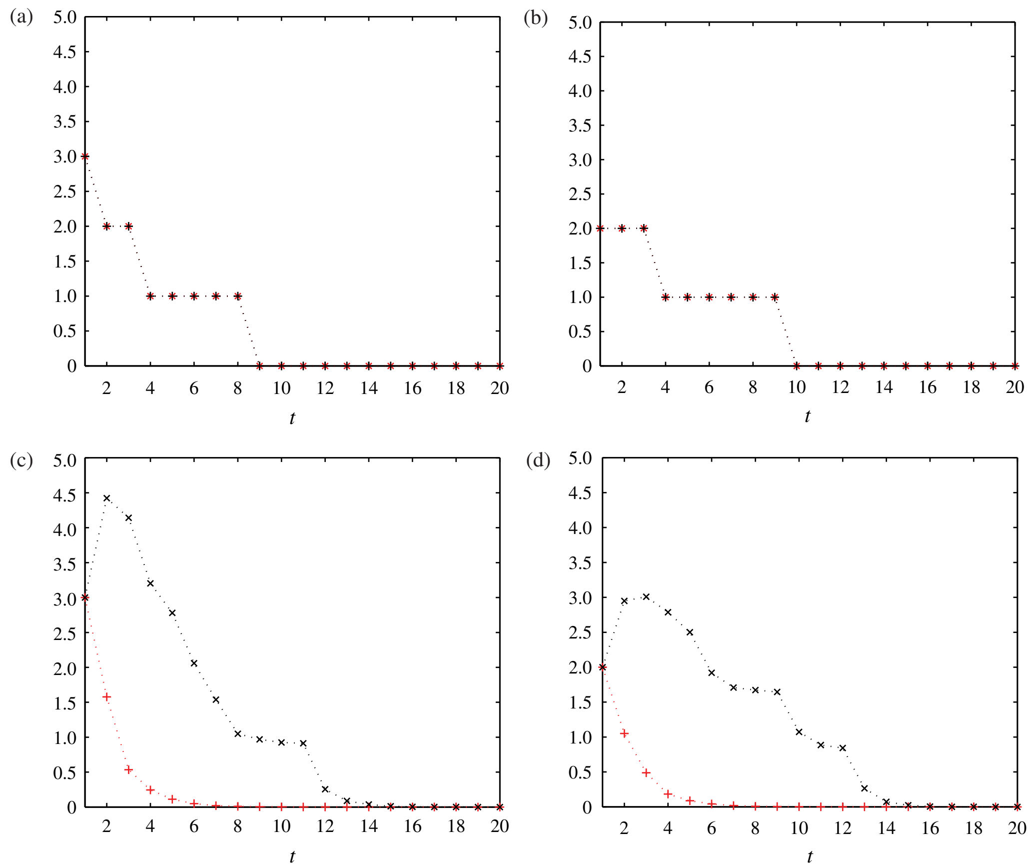
Figure 5 illustrates the expected sales (with discrete consumer types) over an infinite horizon, as a function of the informativeness of the reviews τ and the patience level of the consumers, δ_c. For noisy reviews $\tau \to 0^+$, the sales decrease over time. For informative reviews $\tau > 0$, sales build up gradually to a peak and then taper off. Notice that such a sales pattern is not unrealistic, but could never be obtained in the durable-goods literature. Also, note that when consumers become less strategic (lower δ_c), most sales occur earlier. Thus, strategic consumer behavior and reviews push the sales to a later time in the season.

So far, we considered extending the length of the horizon by *additional time periods*. To differentiate between the extended length of the horizon versus increased number of opportunities to change prices, we consider now multiple prices in a fixed selling horizon, say $t \in [0, 1]$. Formally, assuming N discrete consumer types in the market, we allow the firm to set n prices at $t \in \{0, 1/(n-1), 2/(n-1), \ldots, (n-1)/(n-1)\}$, where $2 \le n \le N$. The discount factors at $t = k/(n-1)$ equal to $\delta^{k/(n-1)}$ and $\delta_c^{k/(n-1)}$ for the firm and consumers, respectively. By definition, $n = 1$ corresponds to a firm that *commits* to a single price at

Table 2 **Profit Comparisons and Value of Reviews in the Infinite-Horizon Model (All in %):** $c = 1.1$, $p_0 = 0.5$, $N = 21$

	$\delta=\delta_c=0.1$		$\delta=\delta_c=0.5$		$\delta=\delta_c=0.9$	
	Π_∞^*/Π_2^*	$\Pi_\infty^*/\Pi_\infty^o$	Π_∞^*/Π_2^*	$\Pi_\infty^*/\Pi_\infty^o$	Π_∞^*/Π_2^*	$\Pi_\infty^*/\Pi_\infty^o$
$\tau=0$	99.69	100.00	93.52	100.00	52.03	100.00
$\tau=1$	99.73	100.76	90.06	100.05	53.12	105.60
$\tau=2$	99.65	102.05	91.34	108.73	58.77	127.58
$\tau=4$	99.45	104.11	93.75	124.35	55.25	146.44
$\tau=6$	99.45	106.49	93.96	138.55	58.19	180.45
$\tau=8$	99.27	107.36	91.93	145.78	55.00	189.69

Figure 5 **(Color online) Expected Sales Over Time, Where the Points Marked with "+" Represent Expected Sales Conditional on Bad Quality, and Those with "×" Are Expected Sales Conditional on Good Quality:** $N=31$, $p_0=0.5$, $c=1.1$, $\delta=0.9$, $\tau=0$, $\delta_c=0$ **(a)**, $\tau=0$, $\delta_c=0.9$ **(b)**, $\tau=6$, $\delta_c=0$ **(c)**, **and** $\tau=6$, $\delta_c=0.9$ **(d)**



$t=0$ and does not provide any other selling opportunity in $(0,1]$.[26] Also, $n=2$ is our base model in §4. As n increases, the firm has more pricing "flexibility."[27] We summarize observations from a numerical experiment (details are in the supplementary document): In absence of strategic consumers ($\delta_c=0$), more pricing flexibility allows the firm to extract more profits. In addition, the benefit of pricing flexibility increases with reviews informativeness (increasing τ). These observations are intuitive as the firm has more skimming opportunities and can adjust prices in response to review outcomes. However, with equally patient strategic consumers ($\delta_c=\delta$), limited pricing flexibility ($2\le n<N$) is optimal for the firm. This is interesting because, with reviews and strategic consumers, a firm should neither commit to a single price (which is optimal without reviews), nor revise prices too often (which is optimal without strategic consumers).

[26] When $n=1$, the firm solves $\max_{v\in[0,1]} \Pi_0(v)$.

[27] This case is similar to a "responsive" pricing policy as in Aviv and Pazgal (2008).

6.3. Asymmetric Quality Information

In this extension, we introduce asymmetric information between the firm and consumers. Assume that the firm acquires, e.g., via consumer focus groups, information about the quality of its product in the form of a signal, s, that is imperfectly correlated with the quality and observed privately. Formally, we assume that before the initial period, the firm privately observes the realization of a binary signal, $s\in\{G,B\}$ that is imperfectly correlated with the true quality as follows: $\Pr(s=G\mid\Theta=1)=\Pr(s=B\mid\Theta=0)=q\in[1/2,1)$. After observing signal s, but before observing reviews, the firm's posterior becomes

$$p_s=\begin{cases}\dfrac{p_0q}{p_0q+(1-p_0)(1-q)}, & s=G,\\[2ex] \dfrac{p_0(1-q)}{p_0(1-q)+(1-p_0)q}, & s=B.\end{cases}$$

For $q>1/2$, we have $p_G>p_B$. The firm will update its prior p_0 to p_s after observing signal s and, thus, the firm's initial price will depend on p_s, which may signal to the consumers the information that the firm obtained. The firm with a bad signal (B-firm) has an incentive to mimic the firm with a good signal (G-firm) and, thus, in order to credibly communicate its private signal, the G-firm needs to distort its price, which is costly.

Assume that the firm observes signal s and the consumers believe that the firm observed signal $\hat{s}$ (which may or may not be equal to s). With asymmetric information between the consumers and the firm, the preposterior becomes a function of the consumer's belief, $p_{\hat{s}}$, about the quality, which may be different from the firm's belief about the quality, p_s. The firm's belief about the consumer's preposterior $p(\tilde{G}, p_{\hat{s}}, x)$ is based on its own assessment of the product quality p_s, and the consumer's belief is based on the prior $p_{\hat{s}}$. These beliefs directly impact the firm's skimming profit and waiting cost. The details of formulation are included in the supplementary document.

Since the firm's equilibrium profit increases in consumers' prior, the B-firm will have an incentive to make the consumers believe it observed signal $s=G$. Thus, in order to be credible, the G-firm will distort the price (initial segment) such that the B-firm has no incentive to imitate the price (initial segment) of the G-firm. We qualitatively describe the signaling equilibrium based on numerical experiments.

Numerical Illustration. In Figure 6 we illustrate the equilibrium initial segments, initial prices, and total profits, as functions of q, the informativeness of the firm's private signal. The terms with asterisks are for the asymmetric-information model and those without are for the symmetric-information model. To convince consumers about a good signal of quality, the G-firm reduces its initial price $P_{1G}^* < P_{1G}$ (increasing initial sales $1-v_G^* > 1-v_G$), compared to its symmetric-information strategy, as shown in Figures 6(a) and 6(b). Even though lowering the initial price hurts G-firm, by selling more in the first period, the firm expects more positive consumer reviews, which will boost the sales and profits in the second period. B-firm is hurt more if it follows the same strategy, as it anticipates many "thumb downs" from the first-period reviews.

Signaling via the initial price, albeit effective, turns out to be very costly. Figure 6(c) illustrates that the cost of signaling $(\Pi_G - \Pi_G^*)$ may be very high. In classical signaling models (e.g., Milgrom and Roberts 1986), the cost of signaling is determined by the difference in cost structure. For example, when the unit cost difference between a high-quality and a low-quality firm is large enough, the cost of signaling can even be zero. In our model, there is no difference in cost between the high- and low-quality product (because the firm does not know the quality very well). The only difference between the high-quality and imitating low-quality firm is due to differences in belief about the *future* skimming profits. When the difference is relatively small, signaling becomes very expensive for the G-firm. As depicted in Figure 6(d), the two expected profit functions, $\Pi_G^G(v)$ and $\Pi_B^G(v)$, are very close to each other, where $\Pi_s^{\hat{s}}(v)$ is the s-firm's profit from convincing consumers that it observed signal $\hat{s}$. Thus, to prevent imitating, the G-firm needs to select a price (market segment, v_G^*) such that the B-firm is indifferent between mimicking G-firm and revealing itself truthfully: $\Pi_B = \Pi_B^G(v_G^*)$.

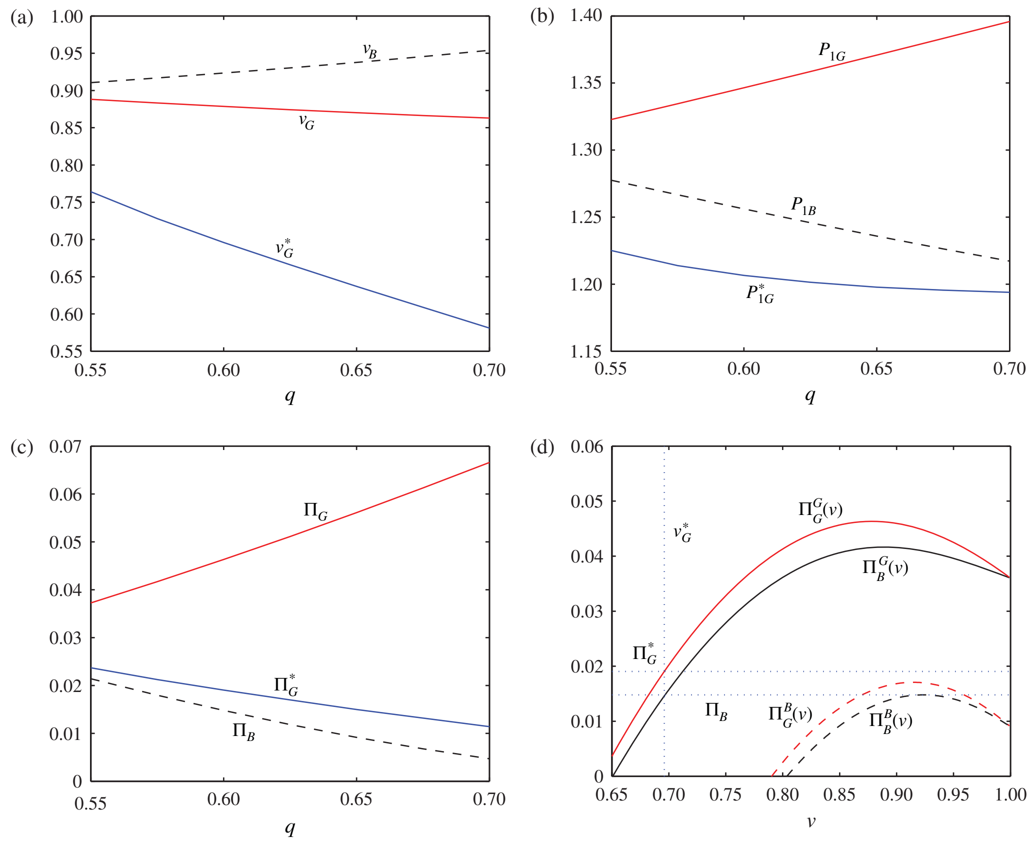
Notice from Figure 6(c) that the G-firm's equilibrium profits decrease in the precision of its private signal.[28] Our analysis highlights a trade-off that firms may need to consider when the quality of the products they sell is unknown: They may either rely on the consumers who discover and communicate quality information via reviews or, alternatively, they can acquire (purchase) quality information themselves and communicate it to their consumers via its price. The information cost to the firm in the former case is fully captured by the consumer's option value (O_c). For the latter case, the firm's private information may reduce the consumers' option value but, in order to communicate private information credibly via its price, the firm incurs signaling costs. We find that the latter might be more expensive.

6.4. Extensions to the Quality Updating Process

In this extension, we consider two separate scenarios: (1) only a fraction α of initial consumers who purchase the product write a review; (2) a fraction β of consumers that remain in the second period take reviews into account for their purchasing decision. For the case when all consumers read reviews, but only a portion α of consumers who have bought the product in the first period write reviews, we show that, by redefining the review informativeness as $\tau' = \sqrt{\alpha}\tau$, we recover the base model. That is, when some consumers do not write reviews, the information precision reduces proportionally. Hence, all of the results in the base model apply to this extended case. In addition, the effects of increasing α on the firm and on the consumers are exactly the same as those of increasing τ. When all consumers write reviews, but only a portion β of consumers who have waited to the second period read reviews, assume that β is also

[28] In an extreme case, when $q=1$, the firm knows the true quality of its product after observing the private signal. In such a case, only the consumers need to learn about quality. In a separating equilibrium, consumers can perfectly deduce the true quality from the first-period price, and thus consumer reviews become irrelevant. In such a case, we find that signaling by pricing is so costly that in equilibrium the high-quality firm obtains the same profit as the low-quality firm. That is, one-sided learning by the consumers via pricing eliminates any profit advantage of the superior quality.

Figure 6 (Color online) (a) Initial Segments, (b) Initial Prices, and (c) Profits, for the G-Firm and B-Firm in Equilibrium, as Functions of the Informativeness of the Firm's Private Signal q: $\delta=\delta_c=0.9$, $p_0=0.5$, $c=1.2$, $\tau=3$; (d) Profits as Functions of v for $q=0.6$



the probability that a consumer will read the reviews if he or she chooses to postpone purchase. In such a case, the second-period market consists of two groups of heterogeneous consumers, those who read reviews and those who do not. For intermediate values of β, the sizes of the two groups of consumers (readers and nonreaders) are significant and the level of consumer heterogeneity is high. Because of the considerable portion of nonreaders, the firm's second-period price becomes less responsive to reviews. Hence, the firm's option value due to reviews increases in the fraction of readers (β). On the other hand, consumers' option value may increase or decrease in β: As β increases, consumers care more about reviews because they are more likely to read; meanwhile, a higher β implies that, with more review-sensitive price, the firm can absorb a larger portion of the benefit from quality learning and leave less surplus for the waiting consumers. Hence, the effect of β on the firm's net discounted option value is ambiguous. As a consequence, a firm might want to carefully track what fraction of the population is actually influenced by consumer reviews and tailor its pricing strategy accordingly.

7. Conclusion

In this paper, we consider the effect of consumer-generated quality information, e.g., consumer reviews, on consumers' buying behavior and on firm's pricing. A classical model of price skimming (e.g., Besanko and Winston 1990) shows that forward-looking (strategic) consumers have an incentive to delay their purchase. With quality information, forward-looking consumers have an additional incentive to delay their purchase, as later they will learn more about the product quality via early reviews. However, the information in early consumer reviews is also available to the firm and the firm may adjust price of the product. Furthermore, the accuracy of the information generated via consumer reviews depends on the volume of consumers that purchase early (as in Bergemann and Valimaki 1997, 1999, 2000), which is influenced by firm's pricing policy. Thus, it becomes nonobvious which party (consumers, the firm, both, or none) benefits from the reviews. To the best of our knowledge, no previous paper studies the price skimming strategy with endogenously generated quality information and strategic consumers. Yet, with the advent of cheap and efficient platforms on which consumers can exchange information, the observability of quality information as well as ability to adjust pricing strategies accordingly becomes widespread.

The presence of smart consumers alters the insights obtained in the literature about double and single-sided learning. Bergemann and Valimaki (1997, 1999) find that consumer reviews always increase initial sales. In their model, however, consumers are myopic. With strategic consumers, we find that the initial sales may decrease. Ifrach et al. (2011) and Papanastasiou et al. (2013) find that, when the firm knows the quality, consumer reviews always improve the firm's profit. When the firm learns about reviews, we find that consumer reviews can actually be detrimental for the firm's profits. We also find that the consumer surplus may decrease.

The main drivers of these results are the firm's and consumers' option values. The firm's option value is fundamentally linked to its ability to change the price in the future period, after the quality information becomes available, which impacts both the firm's margin and the demand volume. The situation is quite different for consumers. Although the consumers may have surplus via better information available in the second period, the firm wants to prevent strategic waiting and offers a price reduction in the first period, before the quality information becomes available. Thus, early consumers "cash in" the option value. The (aggregate) consumer's option value is only linked to future surplus and not to the future demand volume. The fundamental difference between the firm's and consumer's option value implies that characteristics of consumers and of the firm determine the impact of consumer reviews on initial sales, the firm's profit, and consumer surplus.

The impact of the firm's and consumers' option values critically depend on the firm's and consumers' patience levels (discount factors) and on the informativeness of the reviews. Our paper indicates that managers should consider factors such as the relative patience level as well as the confidence levels that consumers attach to such reviews (i.e., the information precision). The consumer patience level has to do with how "hip" or trendy the product is. One would expect that for very fashionable items (e.g., consumer gadgets, movies), strategic consumers are impatient, allowing the firm to enjoy a higher option value. For less fashionable products such as some consumer electronics or books, consumers may be more patient, and, hence, reviews could decrease the firm's profit for these items. The information precision can be associated with how complex the product is and according to how many dimensions consumers can share their experiences with other consumers. A new consumer-electronics product could probably more easily be described using features and functionalities, making consumer reviews quite informative, even if they are small in number. A movie, on the other hand, or a book may be more difficult to review.

Based on these two key drivers (patience level and informativeness of reviews), we obtain a number of managerial and policy recommendations that may not be obvious without formally studying the firm's pricing strategy with endogenously generated information.

When one party is significantly more patient than the other party, the former (latter) is better (worse) off. Therefore, our model predicts that an impatient firm (e.g., under short-term profit generation pressure from its investors) selling less trendy products to a patient consumer base will have natural incentives to prevent diffusion of quality information via consumer reviews. The firm may try to limit the information available or consider alternative sources of information about quality.[29]

Interestingly, we find that when the firm is equally patient as the consumer, endogenously generated information leads to a win-win situation; both consumer surplus and the firm's profit increase.

As the size of the initial segment influences the informativeness of the reviews, we identify whether the firm should expand or contract the initial market segment. The choice of strategy depends on the firm's environment. With noisy reviews, the firm needs to increase the initial market segment. Typically, this can be achieved via an initial price decrease. For more informative reviews, the firm may want to contract the initial market segment. The contraction of the initial market segment is supported by an increase of the initial price—when sufficient information is obtained from the initial reviews, the firm increases profit by selling in a bigger market later. Interestingly, for intermediate levels of informativeness of the reviews, our model predicts that the contraction is coupled with a decrease of the initial price. We also find that noisy reviews may actually be preferred by both the firm and consumers.

We verify that our results about the profitability of reviews are robust with respect to (i) quick-response replenishment, (ii) multiple periods, (iii) asymmetric information, and (iv) boundedly rational consumers.

Although the critical lessons are not changed, we find that an impatient firm facing patient consumers may be hurt by a better operational environment, such as quick-response replenishment or additional sales periods. Quick replenishment increases availability of product, thus decreasing prices in both

[29] An extreme example would be if the firm could buy review fora and discontinue these (to decrease the diffusion of information). Clearly this is multifaceted issue. When reviews increase the consumer surplus, a government agency could ensure that consumers can express their experiences and share them with other consumers.

periods. Additional sales periods provide the consumers more opportunities to delay purchases until better information is available. Although firms may be tempted to acquire private information about quality, we show that this choice may be disadvantageous to the firm. Having consumers discover the quality of the product allows the firm to save on price signaling costs. Although in our base model, consumers always take review information into account and all write reviews after purchasing the product, we consider the case in which not all consumers write or read reviews. We show that the main insights continue to hold, although the speed and the strength of the interactions may be decreased.

Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/mnsc.2014.2134.

Acknowledgments

The authors thank department editor Yossi Aviv, an anonymous associate editor, and three anonymous referees for constructive comments that led to significant improvements of the paper. The work of Man Yu was supported by the Hong Kong Research Grant Council [Grants RGC647312 and RGC16504014].

Appendix A. Derivation of the Profit Functions in §§2 and 3

For convenience of notation, we introduce $C_W^o(v)=\hat{C}_W(v,p_0)$ and $\Pi_S^o(v)=\hat{\Pi}_S(v,p_0)$.

First, we determine the firm's second-period pricing strategy and profits. Given posterior belief $p(g,1-\bar{v}(P_1))$ and threshold $\bar{v}(P_1)$, at the beginning of the second period, the firm determines the second-period price P_2. Equivalently, the firm chooses a threshold $\underline{v}$ to maximize its second-period profit:

$$\Pi_2^*(g,P_1)=\max_{\underline{v}\in[0,\bar{v}(P_1)]}(\underline{v}+p(g,1-\bar{v}(P_1))-c)(\bar{v}(P_1)-\underline{v}).$$

It is easy to obtain $\underline{v}^*(g,P_1)=\min\{\bar{v}(P_1),(\bar{v}(P_1)-p(g,1-\bar{v}(P_1))+c)/2\}$. The optimal second-period price $P_2^*(g,P_1)$ immediately follows (proof omitted):

Lemma 2. *For a given signal realization g, when the posterior quality is $p(g,1-\bar{v}(P_1))$ and the remaining market valuations are in $[0,\bar{v}(P_1)]$, the firm's optimal second-period price is*

$$P_2^*(g,P_1)=\min\left\{p(g,1-\bar{v}(P_1))+\bar{v}(P_1),\ \frac{p(g,1-\bar{v}(P_1))+\bar{v}(P_1)+c}{2}\right\}. \tag{A1}$$

Since $\bar{v}(P_1)$ is the highest remaining consumer type in the beginning of the second period, the sales are zero when $(p(g,1-\bar{v}(P_1))+\bar{v}(P_1)+c)/2-p(g,1-\bar{v}(P_1))>\bar{v}(P_1)$, or $c>p(g,1-\bar{v}(P_1))+\bar{v}(P_1)$. That is, for low realizations of g, the firm withdraws the product from the market, since it would not be able to sell because of a negative margin; $(p(g,1-\bar{v}(P_1))+\bar{v}(P_1)+c)/2<c$. The firm's profit in $t=2$ is $\Pi_2^*(g,P_1)=\frac{1}{4}[\{p(g,1-\bar{v}(P_1))+\bar{v}(P_1)-c\}^+]^2$, where $x^+=\max(x,0)$. In period $t=1$, the firm's profit is $\Pi_1(P_1)=(1-\bar{v}(P_1))(P_1-c)$, where by definition of $\bar{v}(P_1)$ and Equation (A1), $\bar{v}(P_1)$ is determined by

$$\bar{v}(P_1)=\max\{v\in[0,1]:\ p_0+v-P_1 \leqslant \tfrac{1}{2}\delta_c\mathbb{E}_{\tilde{G}|\{\tilde{\Theta},1-v\}}[(p(\tilde{G},1-v)+v-c)^+]\}, \tag{A2}$$

where $\frac{1}{2}\delta_c\mathbb{E}_{\tilde{G}|\{\tilde{\Theta},1-v\}}[(p(\tilde{G},1-v)+v-c)^+]$ is the expected discounted utility of waiting for consumer v when all the consumers in the segment $(v,1]$ choose to buy early.

In the first period, the firm chooses P_1 to maximize the total profit over two periods: $\Pi_1(P_1)+\delta\mathbb{E}_{\tilde{G}|\{\tilde{\Theta},1-v\}}[\Pi_2^*(\tilde{G},P_1)]$. Now, instead of maximizing the total profit over P_1, we use the following result (proof in the supplementary document):

Proposition 11. *There exists a unique equilibrium for consumers' buy-or-wait decisions in the first period. Specifically,*

(a) *for a given P_1, there exists a unique threshold $\bar{v}(P_1)$ such that all consumer types $v>\bar{v}(P_1)$ purchase in the first period and all consumer types $v\le\bar{v}(P_1)$ wait till the second period; and*

(b) *there exist two thresholds, $\underline{P}$ and $\bar{P}$, such that $\bar{v}(P_1)=0$ if $P_1<\underline{P}$, $\bar{v}(P_1)=1$ if $P_1>\bar{P}$, and for $P_1\in[\underline{P},\bar{P}]$, $\bar{v}(P_1)$ satisfies*

$$p_0+v-P_1=\tfrac{1}{2}\delta_c\mathbb{E}_{\tilde{G}|\{\tilde{\Theta},1-v\}}[(p(\tilde{G},1-v)+v-c)^+], \tag{A3}$$

and $\bar{v}(P_1)$ is monotone increasing in P_1.

With the results of Proposition 11, it suffices to focus on $P_1\in[\underline{P},\bar{P}]$, for which $\bar{v}(P_1)$ is invertible. Instead of solving the firm's problem for P_1, we consider $v=\bar{v}(P_1)$ as the firm's decision variable, where v is the valuation of the marginal consumer that is indifferent between purchasing in $t=1$ and $t=2$. Aviv et al. (2012) used a similar redefinition. The volume of consumers that purchase in the first period, x, is equal to $1-v$, the volume of consumers with a type above the threshold v. With a minor abuse of notation,[30] we redefine $f_\theta(g,x)$ $(F_\theta(g,x))$ as $f_\theta(g,v)$ $(F_\theta(g,v))$. Similarly, we define $p(g,v)$ and $\tilde{P}\,|\,\{v\}$, which is distributed according to $\Phi(p,v)$ (with density $\phi(p,v)$). With v as the decision variable, the firm's pricing problem is as given in Equations (7)–(9).

Appendix B. Proof of Proposition 1

(i) Let $g=\eta(r,x)$, or $r=p(g,x)$. Apply this change of variable to the integration

$$\begin{aligned}\int_0^1 r\,d\Phi(r,x)&=\int_{-\infty}^{\infty}p(g,x)\,d\Phi(p(g,x),x)\\&=\int_{-\infty}^{\infty}\frac{p_0f_1(g,x)}{p_0f_1(g,x)+(1-p_0)f_0(g,x)}\\&\quad\cdot[p_0f_1(g,x)+(1-p_0)f_0(g,x)]\,dg\\&=\int_{-\infty}^{\infty}p_0f_1(g,x)\,dg\\&=p_0.\end{aligned}$$

[30] Consistent notation would have been $f_\theta(g,1-v)$ and $F_\theta(g,1-v)$, which we replace by $f_\theta(g,v)$ and $F_\theta(g,v)$.

(ii) To show $(\partial/\partial x)\int_0^p \bar{\Phi}(r,x)\,dr<0$, first note that

$$p(\tilde{G},x)\ge r \;\Leftrightarrow\; \frac{p_0}{p_0+(1-p_0)[\exp(-(\tilde{G}-x/2)\tau^2)]}\ge r$$
$$\Leftrightarrow\; \tilde{G}\ge \frac{x}{2}-\frac{1}{\tau^2}\ln\left(\frac{p_0/r-p_0}{1-p_0}\right).$$

Thus,

$$\begin{aligned}
&\int_0^p \bar{\Phi}(r,x)\,dr\\
&=\int_0^p \Pr[p(\tilde{G},x)\ge r]\,dr\\
&=\int_0^p \Pr\left[\tilde{G}\ge \frac{x}{2}-\frac{1}{\tau^2}\ln\left(\frac{p_0/r-p_0}{1-p_0}\right)\right]dr\\
&=\int_0^p\left(\int_{\frac{x}{2}-\frac{1}{\tau^2}\ln\frac{p_0/r-p_0}{1-p_0}}^{+\infty}[p_0f_1(y)+(1-p_0)f_0(y)]\,dy\right)dr\\
&\quad\text{(change order of integration)}\\
&=\int_{\frac{x}{2}-\frac{1}{\tau^2}\ln\frac{p_0/p-p_0}{1-p_0}}^{+\infty}\int_0^p[p_0f_1(y)+(1-p_0)f_0(y)]\,dr\,dy\\
&\quad+\int_{-\infty}^{\frac{x}{2}-\frac{1}{\tau^2}\ln\frac{p_0/p-p_0}{1-p_0}}\int_0^{\frac{p_0f_1(y)}{p_0f_1(y)+(1-p_0)f_0(y)}}[p_0f_1(y)+(1-p_0)f_0(y)]\,dr\,dy\\
&=\int_{\frac{x}{2}-\frac{1}{\tau^2}\ln\frac{p_0/p-p_0}{1-p_0}}^{+\infty}p[p_0f_1(y)+(1-p_0)f_0(y)]\,dy\\
&\quad+\int_{-\infty}^{\frac{x}{2}-\frac{1}{\tau^2}\ln\frac{p_0/p-p_0}{1-p_0}}p_0f_1(y)\,dy\\
&=(p-1)p_0\bar{F}_1\left(\frac{x}{2}-\frac{1}{\tau^2}\ln\left(\frac{p_0/p-p_0}{1-p_0}\right)\right)\\
&\quad+p(1-p_0)\bar{F}_0\left(\frac{x}{2}-\frac{1}{\tau^2}\ln\left(\frac{p_0/p-p_0}{1-p_0}\right)\right)+p_0,
\end{aligned}$$

where $\bar{F}_\theta=1-F_\theta$ for $\theta\in\{0,1\}$. Hence,

$$\begin{aligned}
&\frac{d}{dx}\int_0^p\bar{\Phi}(r,x)\,dr\\
&\quad=(p-1)p_0\frac{d}{dx}\bar{F}_1\left(\frac{x}{2}-\frac{1}{\tau^2}\ln\left(\frac{p_0/p-p_0}{1-p_0}\right)\right)\\
&\quad\quad+p(1-p_0)\frac{d}{dx}\bar{F}_0\left(\frac{x}{2}-\frac{1}{\tau^2}\ln\left(\frac{p_0/p-p_0}{1-p_0}\right)\right).
\end{aligned}\tag{B1}$$

Furthermore, for $\theta\in\{0,1\}$, by definition of the signal distribution,

$$\begin{aligned}
&\bar{F}_\theta\left(\frac{x}{2}-\frac{1}{\tau^2}\ln\frac{p_0/p-p_0}{1-p_0}\right)\\
&\quad=\int_{\frac{1}{\sqrt{x}}\left(\frac{1}{2}-\theta x\tau-\frac{1}{\tau}\ln\frac{p_0/p-p_0}{1-p_0}\right)}^{+\infty}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}z^2\right)dz,
\end{aligned}$$

where $z=\tau(y-x\theta)/\sqrt{x}$. Hence,

$$\begin{aligned}
&\frac{d}{dx}\bar{F}_\theta\left(\frac{x}{2}-\frac{1}{\tau^2}\ln\left(\frac{p_0/p-p_0}{1-p_0}\right)\right)\\
&\quad=-\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{1}{\sqrt{x}}\left(\frac{1}{2}-\theta\right)x\tau-\frac{1}{\tau}\ln\frac{p_0/p-p_0}{(1-p_0)}\right)^2\right)\\
&\quad\quad\cdot\left(\frac{1}{\sqrt{x}}\left(\frac{1}{2}-\theta x\tau-\frac{1}{\tau}\ln\frac{p_0/p-p_0}{1-p_0}\right)\right)'_x\\
&\quad=-\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\left[\left(\frac{1}{2}-\theta\right)\sqrt{x}\tau\right]^2\right.\right.\\
&\quad\quad\left.\left.+\left[\frac{1}{\sqrt{x}\tau}\ln\left(\frac{p_0/p-p_0}{1-p_0}\right)\right]^2\right)\right)\\
&\quad\quad\cdot\left(\frac{p_0/p-p_0}{1-p_0}\right)^{1/2-\theta}\\
&\quad\quad\cdot\left(\frac{1}{\sqrt{x}}\left(\frac{1}{2}-\theta\right)x\tau-\frac{1}{\tau}\ln\frac{p_0/p-p_0}{1-p_0}\right)'_x.
\end{aligned}$$

Substituting this result into Equation (B1) and noting $[(\frac{1}{2}-\theta)\sqrt{x}\tau]^2=[\sqrt{x}\tau/2]^2$ when θ equals to either 0 or 1, we have

$$\begin{aligned}
&\frac{d}{dx}\int_0^p\bar{\Phi}(r,x)\,dr\\
&=-\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\left[\frac{\sqrt{x}\tau}{2}\right]^2+\left[\frac{1}{\sqrt{x}\tau}\ln\left(\frac{p_0/p-p_0}{1-p_0}\right)\right]^2\right)\right)\\
&\quad\cdot\left\{(p-1)p_0\left(\frac{p_0/p-p_0}{1-p_0}\right)^{-1/2}\left(\frac{1}{\sqrt{x}}\left(-\frac{1}{2}x\tau-\frac{1}{\tau}\ln\frac{p_0/p-p_0}{1-p_0}\right)\right)'_x\right.\\
&\quad+p(1-p_0)\left(\frac{p_0/p-p_0}{1-p_0}\right)^{1/2}\\
&\quad\left.\cdot\left(\frac{1}{\sqrt{x}}\left(\frac{1}{2}x\tau-\frac{1}{\tau}\ln\frac{p_0/p-p_0}{1-p_0}\right)\right)'_x\right\}\\
&=-\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\left[\frac{\sqrt{x}\tau}{2}\right]^2+\left[\frac{1}{\sqrt{x}\tau}\ln\left(\frac{p_0/p-p_0}{1-p_0}\right)\right]^2\right)\right)\\
&\quad\cdot\sqrt{(1-p)p(1-p_0)p_0}\\
&\quad\cdot\left\{-\left(\frac{1}{\sqrt{x}}\left(-\frac{1}{2}x\tau-\frac{1}{\tau}\ln\frac{p_0/p-p_0}{1-p_0}\right)\right)'_x\right.\\
&\quad\left.+\left(\frac{1}{\sqrt{x}}\left(\frac{1}{2}x\tau-\frac{1}{\tau}\ln\frac{p_0/p-p_0}{1-p_0}\right)\right)'_x\right\}\\
&=-\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\left[\frac{\sqrt{x}\tau}{2}\right]^2+\left[\frac{1}{\sqrt{x}\tau}\ln\left(\frac{p_0/p-p_0}{1-p_0}\right)\right]^2\right)\right)\\
&\quad\cdot\sqrt{(1-p)p(1-p_0)p_0}\,(\sqrt{x}\tau)'_x<0.\quad\square
\end{aligned}$$

Appendix C. Proof of Proposition 3

We first solve the firm's decision problem for two limiting cases: no reviews ($\tau\to 0^+$) and perfectly informative reviews ($\tau\to+\infty$). To explicitly recognize the dependence of $\Pi(v)$ and v^* on τ, within this proof we write $\Pi_\tau(v)$ and v^*_τ.

• No reviews ($\tau\to 0^+$): by Equation (2), $p(\tilde{G},v)=p_0$ almost surely when $\tau\to 0^+$. Thus, by Equations (8) and (9),

$$\begin{aligned}
\Pi^o(v)\;=\;\lim_{\tau\to 0^+}\Pi_\tau(v)&=(1-v)(p_0+v-c)+\frac{\delta}{4}[(p_0+v-c)^+]^2\\
&\quad-\frac{\delta_c}{2}(1-v)(p_0+v-c)^+,
\end{aligned}\tag{C1}$$

which is strictly concave with a unique maximizer in $[0,1]$:

$$v^o=\frac{2-\delta_c-(2-\delta-\delta_c)(p_0-c)}{4-(\delta+2\delta_c)}.\tag{C2}$$

Thus, $\Pi^o = \lim_{\tau\to 0^+}\Pi_\tau(v_\tau^*) = \Pi^o(v^o)$. Furthermore, it is easy to show that $v^o > c - p_0$ and that $v^o = 1$ if and only if $\delta = \delta_c = 1$.

- Perfectly informative reviews ($\tau\to+\infty$): by the definition of signal distributions, $\tilde G\mid\{\theta, v\} = (1-v)\theta$ almost surely when $\tau\to+\infty$. Thus, from Equation (2), given any $v<1$, $p(\tilde G, v) = 1$ almost surely if $\theta = 1$ and $p(\tilde G, v) = 0$ almost surely if $\theta = 0$. Substituting this result into Equations (8) and (9), we have

$$\lim_{\tau\to+\infty}\Pi_\tau(v) = \begin{cases}\Pi^1(v), & 0\le v<1;\\ \Pi^o(v), & v=1,\end{cases} \tag{C3}$$

where $\Pi^1(v) = (1-v)(p_0+v-c) + (\delta/4)p_0[(1+v-c)^+]^2 - (\delta_c/2)\cdot(1-v)p_0(1+v-c)^+$. Note that when $\tau\to+\infty$, $\Pi_\tau(v)$ becomes discontinuous at $v=1$. Therefore, we need to compare in the limit $\Pi^o(1)$ with the maximum of $\Pi^1(v)$ over $[0,1)$. However, it is easy to see that $\Pi^1(1) > \Pi^o(1)$. Therefore, in the limiting case, we can simply optimize $\Pi^1(v)$ over $[0,1]$. It is easy to see that $\Pi^1(v)$ is strictly concave with a unique maximizer $v\in[0,1]$:

$$v^* = \min\left[\frac{2+(2-(\delta+\delta_c)p_0)c-(2-\delta)p_0}{4-(\delta+2\delta_c)p_0}, 1\right]. \tag{C4}$$

Whenever $\max_{v\in[0,1]}\Pi^1(v)$ is maximized at $v^*=1$, this means that for large, but, finite τ, there exist an $\varepsilon>0$, such that $|v_\tau^*-1|<\varepsilon$. The optimal profit, $\Pi_\tau(v_\tau^*)$, however, is arbitrarily close to $\Pi^1(1)$. Hence, $\Pi^* = \lim_{\tau\to+\infty}\Pi_\tau(v_\tau^*) = \Pi^1(v^*)$. Furthermore, it is easy to show $v^* > c-1$.

The following technical lemma describes a few properties of the profit difference $\Pi^*-\Pi^o$. Its proof is provided in the supplementary document.

Lemma 3. (i) $\Pi^*-\Pi^o$ *decreases for low* δ_c *and increases for high* δ_c;
(ii) $\Pi^*-\Pi^o$ *strictly increases in* $\delta<1$ *for given* δ_c;
(iii) $\Pi^*-\Pi^o>0$ *for* $\delta=\delta_c>0$.

Utilizing the results above, we now prove the proposition.

(i) First recall that when $\delta=0$, $\Pi^*-\Pi^o<0$ for any $\delta_c>0$. Also, when $\delta_c=0$, $\Pi^*-\Pi^o>0$ for any $\delta>0$. Hence, by Lemma 3(i) and (ii), there exist a threshold $\underline{\delta}>0$ and a switching function $\underline{\delta}_c(\delta)$ such that for $\delta<\underline{\delta}$, $\Pi^*-\Pi^o>0$ if $\delta_c<\underline{\delta}_c(\delta)$ and $\Pi^*-\Pi^o<0$ if $\delta_c>\underline{\delta}_c(\delta)$.

(ii) To show the threshold $\bar\delta$, note that by Lemma 3(ii), for given δ_c, there exists a switching function $\tilde\delta(\delta_c)$ such that $\Pi^*-\Pi^o>0$ if $\delta>\tilde\delta(\delta_c)$ and $\Pi^*-\Pi^o\le 0$ if $\delta\le\tilde\delta(\delta_c)$. Further, by Lemma 3(iii), $\tilde\delta(\delta_c)<\delta_c$. Hence, when $\delta=1$, $\Pi^*-\Pi^o>0$ for all $\delta_c\in[0,1]$. The existence of $\bar\delta$ then follows by continuity of the problem. Furthermore, by part (i) and Lemma 3(iii), $\underline{\delta}_c(\delta)>\delta$. Hence, for $\delta<\underline{\delta}$, $\Pi^*-\Pi^o>0$ if $\delta_c\le\delta$. $\square$

Appendix D. Proof of Proposition 2

We first examine the firm's profit difference $\Pi^*-\Pi^o$. By definition of Π^o, $\Pi^*=\Pi^o$ when $\tau\to 0^+$. Thus, to show $\Pi^*>\Pi^o$ for very small τ, it suffices to prove $\partial\Pi^*/\partial\tau>0$ for τ sufficiently small. Meanwhile, it is easy to show that when $\delta>0$, $v^o\in(c-p_0,1)$. Hence, by the continuity of the problem in τ and the envelop theorem, it suffices to show that for any $v\in(c-p_0,1)$,

$$\lim_{\tau\to 0}\left(\frac{\partial}{\partial\tau}\Pi(v)\right)\cdot\left(\exp\left(-\frac{1}{2}\left(\left[\frac{\sqrt{1-v}\,\tau}{2}\right]^2 + \left[\frac{1}{\sqrt{1-v}\,\tau}\ln\left(\frac{p_0/(c-v)-p_0}{1-p_0}\right)\right]^2\right)\right)\right)^{-1} > 0.$$

To this end, we first derive $(\partial/\partial\tau)\Pi(v)$. By Equations (8) and (9),

$$\begin{aligned}
\Pi(v) &= (1-v)(p_0+v-c) - \int_{c-v}^1(\delta\Pi_S(p,v)-\delta_c C_W(p,v))\,d\bar\Phi(p,v)\\
&\quad\text{(integrate by parts)}\\
&= (1-v)(p_0+v-c) - \Big[(\delta\Pi_S(p,v)-\delta_c C_W(p,v))\bar\Phi(p,v)\big|_{p=c-v}^1\\
&\quad - \int_{c-v}^1(\delta\Pi_S'(p,v)-\delta_c C_W'(p,v))\bar\Phi(p,v)\,dp\Big]\\
&= (1-v)(p_0+v-c) + \int_{c-v}^1(\delta\Pi_S'(p,v)-\delta_c C_W'(p,v))\,d\left[\int_0^p\bar\Phi(r,v)\,dr\right]\\
&\quad\text{(integrate by parts again)}\\
&= (1-v)(p_0+v-c) + (\delta\Pi_S'(p,v) - \delta_c C_W'(p,v))\int_0^p\bar\Phi(r,v)\,dr\big|_{p=c-v}^1\\
&\quad - \int_{c-v}^1[(\delta\Pi_S''(p,v)-\delta_c C_W''(p,v))\int_0^p\bar\Phi(r,v)\,dr]\,dp\\
&= (1-v)(p_0+v-c) + \Big[(\delta\Pi_S'(1,v)-\delta_c C_W'(1,v))p_0 + \frac{\delta_c}{2}(1-v)\int_0^{c-v}\bar\Phi(r,v)\,dp'\Big]\\
&\quad - \frac{\delta}{2}\int_{c-v}^1\left[\int_0^p\bar\Phi(r,v)\,dr\right]dp.
\end{aligned} \tag{D1}$$

Define an auxiliary variable $\xi=\sqrt{1-v}\,\tau$. Take derivative of $\Pi(v)$ with respect to τ:

$$\begin{aligned}
\frac{\partial}{\partial\tau}\Pi(v) &= \frac{\delta_c}{2}(1-v)\frac{\partial\int_0^{c-v}\bar\Phi(r,v)\,dr}{\partial\xi}\frac{\partial\xi}{\partial\tau}\\
&\quad - \frac{\delta}{2}\int_{c-v}^1\frac{\partial[\int_0^p\bar\Phi(r,v)\,dr]}{\partial\xi}\frac{\partial\xi}{\partial\tau}\,dp,
\end{aligned}$$

where $d\xi/d\tau=\sqrt{1-v}$ and by proof of Proposition 1,

$$\frac{d}{d\xi}\int_0^p\bar\Phi(r,v)\,dr = -\frac{\sqrt{p(1-p)p_0(1-p_0)}}{\sqrt{2\pi}}\cdot\exp\left(-\frac{1}{2}\left(\left[\frac{\xi}{2}\right]^2+\left[\frac{1}{\xi}\ln\left(\frac{p_0/p-p_0}{1-p_0}\right)\right]^2\right)\right).$$

Hence,

$$\frac{\partial}{\partial\tau}\Pi(v) = -\frac{\delta_c}{2}(1-v)^{3/2}\frac{\sqrt{(c-v)(1-c+v)p_0(1-p_0)}}{\sqrt{2\pi}}$$

$$\cdot\exp\left(-\frac{1}{2}\left(\left[\frac{\xi}{2}\right]^2+\left[\frac{1}{\xi}\ln\left(\frac{p_0/(c-v)-p_0}{1-p_0}\right)\right]^2\right)\right)$$
$$+\frac{\delta}{2}\sqrt{1-v}\int_{c-v}^{1}\frac{\sqrt{p(1-p)p_0(1-p_0)}}{\sqrt{2\pi}}$$
$$\cdot\exp\left(-\frac{1}{2}\left(\left[\frac{\xi}{2}\right]^2+\left[\frac{1}{\xi}\ln\left(\frac{p_0/p-p_0}{1-p_0}\right)\right]^2\right)\right)dp,\ \text{and}$$

$$\frac{(\partial/\partial\tau)\Pi(v)}{\exp(-(1/2)([\xi/2]^2+[(1/\xi)\ln((p_0/(c-v)-p_0)/(1-p_0))]^2))}$$
$$=-\frac{\delta_c}{2}(1-v)^{3/2}\frac{\sqrt{(c-v)(1-c+v)p_0(1-p_0)}}{\sqrt{2\pi}}$$
$$\cdot\exp\left(-\frac{1}{2}\left(\left[\frac{\xi}{2}\right]^2+\left[\frac{1}{\xi}\ln\left(\frac{p_0/(c-v)-p_0}{1-p_0}\right)\right]^2\right)\right)$$
$$+\frac{\delta}{2}\sqrt{1-v}\int_{c-v}^{1}\frac{\sqrt{p(1-p)p_0(1-p_0)}}{\sqrt{2\pi}}$$
$$\cdot\exp\left(-\frac{1}{2\xi^2}\left\{\left[\ln\left(\frac{p_0/p-p_0}{1-p_0}\right)\right]^2-\left[\ln\left(\frac{p_0/(c-v)-p_0}{1-p_0}\right)\right]^2\right\}\right)dp,$$

where, as $\tau\to 0$, $\xi\to 0$ and the first term goes to zero. Below, we show that the second term goes to $+\infty$ as $\xi\to 0$:

$$\int_{c-v}^{1}\frac{\sqrt{p(1-p)p_0(1-p_0)}}{\sqrt{2\pi}}\exp\left(-\frac{1}{2\xi^2}\left\{\left[\ln\left(\frac{p_0/p-p_0}{1-p_0}\right)\right]^2-\left[\ln\left(\frac{p_0/(c-v)-p_0}{1-p_0}\right)\right]^2\right\}\right)dp$$
$$=\int_{c-v}^{p_0}\frac{\sqrt{p(1-p)p_0(1-p_0)}}{\sqrt{2\pi}}\exp\left(-\frac{1}{2\xi^2}\left\{\left[\ln\left(\frac{p_0/p-p_0}{1-p_0}\right)\right]^2-\left[\ln\left(\frac{p_0/(c-v)-p_0}{1-p_0}\right)\right]^2\right\}\right)dp$$
$$+\int_{p_0}^{1}\frac{\sqrt{p(1-p)p_0(1-p_0)}}{\sqrt{2\pi}}\exp\left(-\frac{1}{2\xi^2}\left\{\left[\ln\left(\frac{p_0/p-p_0}{1-p_0}\right)\right]^2-\left[\ln\left(\frac{p_0/(c-v)-p_0}{1-p_0}\right)\right]^2\right\}\right)dp. \quad \text{(D2)}$$

Note that $[\ln((p_0/p-p_0)/(1-p_0))]^2$ decreases in p for $p\le p_0$ and increases in p for $p\ge p_0$. Thus, when $\xi\to 0$, the first term in Equation (D2) goes to $+\infty$, and the second term goes to zero. Hence,

$$\lim_{\tau\to 0}\left(\frac{\partial}{\partial\tau}\Pi(v)\right)\cdot\left(\exp\left(-\frac{1}{2}\left(\left[\frac{\sqrt{1-v}\tau}{2}\right]^2+\left[\frac{1}{\sqrt{1-v}\tau}\ln\left(\frac{p_0/(c-v)-p_0}{1-p_0}\right)\right]^2\right)\right)\right)^{-1}$$
$$=\lim_{\xi\to 0}\left(\frac{\partial}{\partial\tau}\Pi(v)\right)\cdot\left(\exp\left(-\frac{1}{2}\left(\left[\frac{\xi}{2}\right]^2+\left[\frac{1}{\xi}\ln\left(\frac{p_0/(c-v)-p_0}{1-p_0}\right)\right]^2\right)\right)\right)^{-1}=+\infty. \qquad \square$$

Appendix E. Proof of Proposition 4

When $\delta=\delta_c=1$, by Equation (C2), $v^o=1$. Since $v=1$ is also feasible for the problem with consumer reviews, $\Pi^*\ge\Pi(1)=\Pi^o$. Further, by Equations (9) and (11),

$$C(v)=\frac{\Pi(v)}{2}+\underbrace{\left[\begin{array}{l}\dfrac{(1-v)(c-p_0-2v+1)}{2}\\ +\dfrac{3}{4}(1-v)\displaystyle\int_0^1\{p+v-c\}^+\,d\Phi(p,v)\end{array}\right]}_{\doteq f(v)}.$$

It is easy to show that $f'(v)<(c-p_0+2v-3)/4<0$ for $v\in[0,1]$. Since $v^*\le v^o=1$ and $\Pi^*\ge\Pi^o$, $C^*=C(v^*)\ge\Pi^*/2+f(1)\ge\Pi^o/2=C^o$.

Now, we prove that the difference in consumer surplus, C^*-C^o, is positive for small τ. Let $C^o(v)=\hat{C}(p_0,v)$. Note that, by Equations (9) and (11),

$$C(v)=\frac{\delta_c}{2\delta}\Pi(v)+\underbrace{\left[\begin{array}{l}\dfrac{1-v}{2}\left(1-v-\dfrac{\delta_c}{\delta}(p_0+v-c)\right)\\ +\dfrac{\delta_c^2+2\delta\delta_c}{4\delta}\cdot(1-v)\displaystyle\int_0^1\{p+v-c\}^+\,d\Phi(p,v)\end{array}\right]}_{\doteq h(v)}, \quad \text{(E1)}$$

where, by SOSD and martingale properties of the posterior,

$$h'(v)=\frac{1}{2}\left[-(1-v)\left(2+\frac{\delta_c}{\delta}\right)+\frac{\delta_c}{\delta}(p_0+v-c)\right]$$
$$+\frac{\delta_c^2+2\delta\delta_c}{4\delta}\left[(1-v)\left(\Pr[p+v-c\ge 0]+\int_0^1\{p+v-c\}^+\,d\frac{\partial\Phi(p,v)}{\partial v}\right)-\int_0^1\{p+v-c\}^+\,d\Phi(p,v)\right]$$
$$<\frac{1}{2}\left[-(1-v)\left(2+\frac{\delta_c}{\delta}\right)+\frac{\delta_c}{\delta}(p_0+v-c)\right]+\frac{\delta_c^2+2\delta\delta_c}{4\delta}[(1-v)-(p_0+v-c)]$$
$$=\frac{2\delta+2\delta_c-2\delta\delta_c-\delta_c^2}{2\delta}v+\frac{\delta_c^2+2\delta\delta_c}{4\delta}(c-p_0+1)-\frac{\delta_c}{2\delta}-\frac{1}{2\delta}\delta_c(c-p_0)-1.$$

Since $2\delta+2\delta_c-2\delta\delta_c-\delta_c^2>0$, for $v<v^o$,

$$h'(v)<\frac{2\delta+2\delta_c-2\delta\delta_c-\delta_c^2}{2\delta}v^o+\frac{\delta_c^2+2\delta\delta_c}{4\delta}(c-p_0+1)-\frac{\delta_c}{2\delta}-\frac{1}{2\delta}\delta_c(c-p_0)-1$$
$$=-\frac{(2-\delta_c)(1+p_0-c)(4-\delta_c-2\delta)}{4(4-\delta-2\delta_c)}<0.$$

Hence, $h(v)$ decreases in v for $v<v^o$. Furthermore, for small τ, $v^*<v^o$ by Proposition 5 (ii) and $\Pi^*>\Pi^o$ as proved above. Thus, for small τ,

$$C^*=C(v^*)=\frac{\delta_c}{2\delta}\Pi(v^*)+h(v^*)$$
$$>\frac{\delta_c}{2\delta}\Pi^o(v^o)+h(v^o)>\frac{\delta_c}{2\delta}\Pi^o(v^o)+h^o(v^o)=C^o,$$

where the last inequality is by Jensen's inequality and

$$h^o(v) \doteq \frac{1-v}{2}\left[1-v-\frac{\delta_c}{\delta}(p_0+v-c)\right] + \frac{\delta_c^2+2\delta\delta_c}{4\delta}(1-v)(p_0+v-c). \quad \square$$

Appendix F. Proof of Proposition 5

(i) Same as in the proof of Proposition 3, we explicitly write v_τ^* within this proof. First note that when either $\delta<1$ or $\delta_c<1$, by Equation (C2), $v^o<1$. Furthermore, by Equation (C4), if $v^*=1$, v_τ^* is arbitrarily close to one as $\tau\to+\infty$ and thus $\lim_{\tau\to+\infty} v_\tau^*>v^o$. On the other hand, if

$$v^* = \frac{2+(2-(\delta+\delta_c)p_0)c-(2-\delta)p_0}{4-(\delta+2\delta_c)p_0},$$

$$\lim_{\tau\to+\infty} v_\tau^*-v^o = v^*-v^o = \frac{(1-p_0)(-2\delta+2c\delta+(2-\delta-\delta_c)(\delta+2\delta_c)p_0)}{(4-\delta-2\delta_c)(4-\delta p_0-2\delta_c p_0)} \geq \frac{(1-p_0)p_0(2-\delta-\delta_c)(\delta+2\delta_c)}{(4-\delta-2\delta_c)(4-\delta p_0-2\delta_c p_0)}>0.$$

(ii) To show that $v^*<v^o$ for very small τ, it suffices to show that for very small τ, $(\partial/\partial v)\Pi(v)<0$ for all $v\geq v^o$. Since $v^o\in(c-p_0,1)$ when either $\delta<1$ or $\delta_c<1$ and $(\partial/\partial v)\Pi^o(v)<0$ for all $v\geq v^o$, it then suffices to show that for all $v\in(c-p_0,1)$, $\partial^2\Pi(v)/\partial v\partial\tau<0$ for small τ. Since $\partial^2\Pi(v)/\partial v\partial\tau$ is continuous in τ, it then suffices to show that for all $v\in(c-p_0,1)$, $\lim_{\tau\to 0}\partial^2\Pi(v)/\partial v\partial\tau<0$. Similarly as in the proof of Proposition 2, define $\xi=\sqrt{1-v}\tau$. Since $\xi\to 0$ as $\tau\to 0$, it then suffices to show that for any $v\in(c-p_0,1)$,

$$\lim_{\xi\to 0}\left(\xi^2\frac{\partial^2}{\partial v\partial\tau}\Pi(v)\right)\cdot\left(\exp\left(-\frac{1}{2}\left(\left[\frac{\xi}{2}\right]^2+\left[\frac{1}{\xi}\ln\left(\frac{p_0/(c-v)-p_0}{1-p_0}\right)\right]^2\right)\right)\right)^{-1}=-\infty.$$

To this end, we first derive $(\partial^2/\partial v\partial\tau)\Pi(v)$. Take the derivative of the expression of $\Pi(v)$ in Equation (D1) with respect to v and τ:

$$\begin{aligned}\frac{d^2}{dvd\tau}\Pi(v) =\ & \frac{\delta_c}{2}(1-v)\left[\frac{\partial^2[\int_0^{c-v}\bar\Phi(r,v)\,dr]}{\partial\xi^2}\frac{\partial\xi}{\partial v}\frac{\partial\xi}{\partial\tau} + \frac{\partial[\int_0^{c-v}\bar\Phi(r,v)\,dr]}{\partial\xi}\frac{\partial^2\xi}{\partial v\partial\tau}\right]\\ &-\frac{\delta}{2}\int_{c-v}^1\left[\frac{\partial^2[\int_0^p\bar\Phi(r,v)\,dr]}{\partial\xi^2}\frac{\partial\xi}{\partial v}\frac{\partial\xi}{\partial\tau} + \frac{\partial[\int_0^p\bar\Phi(r,v)\,dr]}{\partial\xi}\frac{\partial^2\xi}{\partial v\partial\tau}\right]dp\\ &-\frac{\delta+\delta_c}{2}\frac{\partial\int_0^{c-v}\bar\Phi(p,v)\,dp}{\partial\xi}\frac{\partial\xi}{\partial\tau}\\ &-\frac{\delta_c}{2}(1-v)\frac{\partial\bar\Phi(c-v,v)}{\partial\xi}\frac{\partial\xi}{\partial\tau},\end{aligned}$$

where

$$\begin{aligned}&\frac{\partial^2[\int_0^p\bar\Phi(r,v)\,dr]}{\partial\xi^2}\frac{\partial\xi}{\partial v}\frac{\partial\xi}{\partial\tau}+\frac{\partial[\int_0^p\bar\Phi(r,v)\,dr]}{\partial\xi}\frac{\partial^2\xi}{\partial v\partial\tau}\\ &=\frac{\sqrt{p(1-p)p_0(1-p_0)}}{2\sqrt{2\pi(1-v)}}\cdot\exp\left(-\frac{1}{2}\left(\left[\frac{\xi}{2}\right]^2+\left[\frac{1}{\xi}\ln\left(\frac{p_0/p-p_0}{1-p_0}\right)\right]^2\right)\right)\cdot\left[1-\frac{\xi^2}{4}+\left[\frac{1}{\xi^2}\ln\left(\frac{p_0/p-p_0}{1-p_0}\right)\right]^2\right]\end{aligned}$$

$$\begin{aligned}&\frac{\partial\int_0^{(c-v)}\bar\Phi(p,v)\,dp}{\partial\xi}\frac{\partial\xi}{\partial\tau}\\ &=-\frac{\sqrt{(c-v)(1-c+v)p_0(1-p_0)(1-v)}}{\sqrt{2\pi}}\cdot\exp\left(-\frac{1}{2}\left(\left[\frac{\xi}{2}\right]^2+\left[\frac{1}{\xi}\ln\left(\frac{p_0/(c-v)-p_0}{1-p_0}\right)\right]^2\right)\right)\end{aligned}$$

$$\begin{aligned}&\frac{d}{d\xi}\bar\Phi(p,v)\\ &=-\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\left[\frac{\xi}{2}\right]^2+\left[\frac{1}{\xi}\ln\left(\frac{p_0/p-p_0}{1-p_0}\right)\right]^2\right)\right)\\ &\quad\cdot\sqrt{p_0(1-p_0)p(1-p)}\left\{\frac{1}{2}\left(\frac{1}{p}-\frac{1}{1-p}\right)+\frac{1}{\xi^2}\ln\left(\frac{p_0/p-p_0}{1-p_0}\right)\left(\frac{1}{p}+\frac{1}{1-p}\right)\right\}.\end{aligned}$$

Therefore,

$$\begin{aligned}&\frac{\xi^2\partial^2\Pi(v)/\partial v\partial\tau}{\exp(-(1/2)([\xi/2]^2+[\ln((p_0/(c-v)-p_0)/(1-p_0))/\xi]^2))}\\ &=\frac{\delta_c}{2}\frac{\sqrt{(c-v)(1-c+v)p_0(1-p_0)(1-v)}}{2\sqrt{2}}\cdot\left[\xi^2-\frac{\xi^4}{4}+\left[\ln\left(\frac{p_0/(c-v)-p_0}{1-p_0}\right)\right]^2\right]\\ &\quad-\frac{\delta}{2}\int_{c-v}^1\frac{\sqrt{p(1-p)p_0(1-p_0)}}{2\sqrt{2\pi(1-v)}}\cdot\exp\left(-\frac{1}{2\xi^2}\left\{\left[\ln\left(\frac{p_0/p-p_0}{1-p_0}\right)\right]^2-\left[\ln\left(\frac{p_0/(c-v)-p_0}{1-p_0}\right)\right]^2\right\}\right)\\ &\quad\cdot\left[\xi^2-\frac{\xi^4}{4}+\left[\ln\left(\frac{p_0/p-p_0}{1-p_0}\right)\right]^2\right]dp\\ &\quad+\frac{\delta+\delta_c}{2}\frac{\sqrt{(c-v)(1-c+v)p_0(1-p_0)(1-v)}}{\sqrt{2\pi}}\\ &\quad+\frac{\delta_c}{2}(1-v)^{3/2}\frac{\sqrt{p_0(1-p_0)(c-v)(1-c+v)}}{\sqrt{2\pi}}\cdot\left\{\frac{1}{2}\left(\frac{1}{c-v}-\frac{1}{1-c+v}\right)\xi^2+\ln\left(\frac{p_0/(c-v)-p_0}{1-p_0}\right)\frac{1}{(c-v)(1-c+v)}\right\},\end{aligned}$$

where, as $\xi\to 0$, all the terms converge to finite numbers, except the term on the second line, which goes to $+\infty$

(without the minus sign upfront), as proved below:

$$\begin{aligned}
&\int_{c-v}^{1} \frac{\sqrt{p(1-p)p_0(1-p_0)}}{2\sqrt{2\pi(1-v)}} \\
&\quad\cdot \exp\left(-\frac{1}{2\xi^2}\left\{\left[\ln\left(\frac{p_0/p-p_0}{1-p_0}\right)\right]^2 - \left[\ln\left(\frac{p_0/(c-v)-p_0}{1-p_0}\right)\right]^2\right\}\right) \\
&\quad\cdot \left[\xi^2-\frac{\xi^4}{4}+\left[\ln\left(\frac{p_0/p-p_0}{1-p_0}\right)\right]^2\right]dp \\
&= \int_{c-v}^{p_0} \frac{\sqrt{p(1-p)p_0(1-p_0)}}{2\sqrt{2\pi(1-v)}} \\
&\quad\cdot \exp\left(-\frac{1}{2\xi^2}\left\{\left[\ln\left(\frac{p_0/p-p_0}{1-p_0}\right)\right]^2 - \left[\ln\left(\frac{p_0/(c-v)-p_0}{1-p_0}\right)\right]^2\right\}\right) \\
&\quad\cdot \left[\xi^2-\frac{\xi^4}{4}+\left[\ln\left(\frac{p_0/p-p_0}{1-p_0}\right)\right]^2\right]dp \\
&\quad+ \int_{p_0}^{1} \frac{\sqrt{p(1-p)p_0(1-p_0)}}{2\sqrt{2\pi(1-v)}} \\
&\quad\cdot \exp\left(-\frac{1}{2\xi^2}\left\{\left[\ln\left(\frac{p_0/p-p_0}{1-p_0}\right)\right]^2 - \left[\ln\left(\frac{p_0/(c-v)-p_0}{1-p_0}\right)\right]^2\right\}\right) \\
&\quad\cdot \left[\xi^2-\frac{\xi^4}{4}+\left[\ln\left(\frac{p_0/p-p_0}{1-p_0}\right)\right]^2\right]dp. \qquad \text{(F1)}
\end{aligned}$$

Note that $[\ln((p_0/p-p_0)/(1-p_0))]^2$ decreases in p for $p \le p_0$ and increases in p for $p \ge p_0$. Thus, when $\xi \to 0$, the first term in Equation (F1) goes to $+\infty$, and the second term goes to zero. Hence,

$$\lim_{\xi\to 0} \frac{\partial^2 \Pi(v)}{\partial v \partial \tau} \cdot \left(\exp\left(-\frac{1}{2}\left(\left[\frac{\xi}{2}\right]^2 + \left[\frac{1}{\xi}\ln\left(\frac{p_0/(c-v)-p_0}{1-p_0}\right)\right]^2\right)\right)\right)^{-1} = -\infty. \quad \square$$

Appendix G. Proof of Proposition 6

(i) Define two functions of $v \in [0,1]$:

$$P_1(v) = p_0 + v - \frac{\delta_c}{2}\int_0^1 (p+v-c)^+ \, d\Phi(p,v), \quad \text{and}$$

$$P_1^o(v) = p_0 + v - \frac{\delta_c}{2}(p_0+v-c)^+.$$

We note two properties of the two functions: first, $P_1(v)$ strictly increases in v since, by SOSD of $\tilde{P} \mid \{1-v\}$,

$$\frac{dP_1(v)}{dv} = 1 - \frac{\delta_c}{2}[\Pr[\tilde{P} \mid \{1-v\}+v-c \ge 0] + d\mathbb{E}[(\tilde{P} \mid \{1-v\}+v'-c)^+]/dv\,|_{v'=v}] > 0;$$

and second, for given v, $P_1(v) \le P_1^o(v)$ by Jensen's inequality. Based on these facts, when $v^* < v^o$, $P_1^* = P_1(v^*) < P_1(v^o) \le P_1^o(v^o) = P_1^o$.

(ii) The existence of $\hat{\tau}$ for which $v^* = v^o$ follows from Proposition 5(i) and (ii). Furthermore, by part (i), $P_1^* < P_1^o$ when $v^* = v^o$ and $\delta_c > 0$. $\square$

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