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Quantifying the Value of Leadtime Information in a Single-Location Inventory System

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This article studies a single-location inventory model with random leadtimes. Inventory is replenished by a single supplier who, at the time a replenishment order is received, knows exactly when the order will be delivered. In other words, the supplier knows the leadtime for every replenishment order. Suppose the single-location inventory system is managed by a retailer. The objective of this article is to quantify the value of the information about leadtimes to the retailer. This is achieved by considering and comparing the performances of two scenarios whether or not the supplier shares his leadtime information with the retailer. Numerical evidence suggests that the value of leadtime information can be significant.

Key words: stochastic inventory system; random leadtime; information sharing; value of information
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1. Introduction

If you ask an inventory manager to describe the things that make the job challenging, you will probably hear two perspectives. One has to do with unpredictable demand, and the other—well, what else?—uncertain supply. Probing further on the latter, you may learn how an order with the supplier may arrive, say, in anywhere from two weeks to four weeks, and it does not always come in one piece. One of the reasons for such uncertainties is lack of communication. For example, when the inventory manager places an order, the production manager at the supplier site may have some idea as to when that order will be filled based on his knowledge about the situation on the shop floor. When this knowledge is not shared with our inventory manager, supply uncertainty ensues.

The purpose of this article is to demonstrate the potential value of the information about delivery leadtimes. This is done in the context of a single-location inventory model with random leadtimes. (Each order arrives in one piece, albeit after a random leadtime.) In one scenario, the inventory manager knows the leadtime for each order placed. This represents complete information sharing between the inventory manager and the supplier, who is assumed to know exactly when an incoming order will be shipped. The other scenario is one where the inventory manager only has access to the history of order arrivals—when an order was placed, whether or not it has arrived, and if so, when—and can use this historical information to predict the current leadtime and make a replenishment decision accordingly. This is a case of no information sharing between the two parties in the supply chain. The difference in system performance between the above two scenarios represents the value of leadtime information (to the single-location inventory system). Numerical evidence shows that this value can be significant, with one example reaching a relative difference of at least 41%.

This article contributes to a growing body of literature on supply chain information sharing. Most of the existing work concentrates on the value of information that comes from the supply chain's downstream, e.g., sales information, inventory status at points of sales, etc. Very limited attention has been directed at the value of upstream information, such as the leadtime information considered here. It is interesting that the value of downstream information is typically small (see, e.g., Chen 2003), which stands in sharp contrast with the value of upstream information demonstrated in this paper. A closely related paper is Song and Zipkin (1996); these authors have considered the above full-information scenario and characterize the optimal policy for it. Our paper complements theirs by furnishing their analytical results with numerical evidence on the value of leadtime information.

The rest of the paper is organized as follows. Section 2 describes the model in detail. Section 3 deals with the complete information case, and §4 the incomplete-information case. Section 5 contains numerical examples. Section 6 concludes.

2. The Model

Consider the following single-location, periodic-review inventory model with an infinite planning horizon. A retailer buys a product from an outside supplier at the beginning of each period. Customer demand arises periodically, with demands in different periods being independent, identically distributed random variables. If demand exceeds the on-hand inventory in a period, the excess demand is backlogged. On-hand inventories incur holding costs, and customer backorders incur penalty costs. The retailer's objective is to choose a replenishment policy to minimize her long-run average holding and backorder costs.

We assume the following sequence of events in each period. At the beginning of the period an order, if any, is placed with the supplier, and any delivery (of a previous order) from the supplier due this period is received. During the period, demand occurs. At the end of the period, holding and backorder costs are assessed. Let h be the holding cost incurred per unit of inventory, and b the penalty cost incurred per unit of backorder. (The supplier charges a constant price for every unit ordered by the retailer, thus the long-run average purchasing cost is constant. We omit this cost in the analysis below.)

We model the supply process with a finite-state Markov chain. Let L_t, a positive integer, be the leadtime for an order placed in period t. (Thus the order arrives in period $t + L_t$.) Assume that $\mathbf{L} \stackrel{\text{def}}{=} \{L_t\}$ is a Markov chain with state space $S = \{1, 2, \ldots, M\}$, where M is a fixed, positive integer. Moreover, the Markov chain is time homogeneous and ergodic. Denote the one-step transition probability from state i to state j by p_{ij}, $i, j \in S$. Let P be the transition matrix. The supply process is exogenous, i.e., the evolution of the Markov chain is independent of the operations of the retailer's inventory system. To ensure no order crossovers, we restrict the transition probabilities so that $p_{ij} = 0$ for any $j < i - 1$. (The transition matrix thus has a semiupper triangular form.) Finally, partial shipments of orders are not allowed. Song and Zipkin (1996) have given several examples of the above leadtime process. For other inventory models with random leadtimes, see, e.g., Kaplan (1970), Nahmias (1979), Ehrhardt (1984), Zipkin (1986), Svoronos and Zipkin (1991), Song (1994), and Robinson et al. (2000).

It is assumed that the supplier always knows the value of L_t at the beginning of period t, for all t. Below, we focus on the retailer's replenishment decisions, which, as we will see, depend on whether or not the supplier shares his leadtime information with the retailer.

3. Complete Information Sharing

Consider the scenario where the supplier shares his leadtime information with the retailer. The sequence of events in each period is as follows: At the beginning of each period t, the retailer learns the value of L_t and decides how much, if any, to order. Then, any order(s) due this period is(are) received, demand is realized, and holding and backorder costs are assessed. Because the value of L_t changes from period to period, it is conceivable that the optimal ordering decision depends on the leadtime value. Song and Zipkin (1996) have shown that a state-dependent, base-stock policy is optimal, i.e., that it is optimal to place an order in period t to increase the retailer's inventory position to a base-stock level that is a function of L_t, for all t.

We provide an iterative algorithm to compute the optimal state-dependent, base-stock levels. It is similar to the algorithm developed by Chen and Song (2001) for supply chain models with a Markov-modulated demand process. Below, we briefly describe our algorithm, omitting any proof (which can be easily fashioned by following Chen and Song 2001).

Define inventory level to be on-hand inventory minus backorders. Let $IL(t)$ be the inventory level at the end of period t. Thus, the holding and backorder costs incurred in period t are $hIL(t)^+ + bIL(t)^-$. Define inventory position to be inventory level plus all outstanding orders (i.e., orders placed but not yet received). Let $IP(t)$ be the inventory position at the beginning of period t after order placement, if any. Recall that an order placed in period t arrives in period $t+L_t$, and that the next order placed in period $t+1$ arrives in period $t+1+L_{t+1}$. The following are the well-inventory balance equations

$$IL(\tau) = IP(t) - D[t, \tau], \quad \tau = t + L_t, \ldots, t + L_{t+1},$$

where $D[t, \tau]$ is the total demand in periods $t, \ldots, \tau$. Therefore, the inventory position $IP(t)$ determines the distributions of the inventory levels $IL(\tau)$ for $\tau = t + L_t, \ldots, t + L_{t+1}$, and thus the expected costs in those periods. We consequently charge the expected costs incurred in $[t + L_t, t + L_{t+1}]$ to period t. (Note that it is possible that $L_{t+1} = L_t - 1$, in which case the set $[t + L_t, t + L_{t+1}]$ is empty and thus zero costs are charged to period t.) Let $G(m, y)$ be the expected costs charged to period t, given $L_t = m$ and $IP(t) = y$. (Due to the Markovian nature of the leadtime process and the i.i.d. demand process, it is clear that the expected costs charged to period t depend only on L_t and $IP(t)$. In other words, $G(m, y)$ does not depend on t.) Define for any $l \geq 0$,

$$g(l, y) = E\big[h(y - D[0, l])^+ + b(y - D[0, l])^-\big].$$

Therefore,

$$\begin{aligned} G(m, y) &= \sum_{m'=m}^{M} p_{mm'} \sum_{l=m}^{m'} g(l, y) \\ &= \sum_{l=m}^{M} \Pr(L_{t+1} \geq l \mid L_t = m) g(l, y). \end{aligned}$$

Note that $g(l, y)$ is convex in y for any l. Consequently, $G(m, y)$ is convex in y for any m.

Let $s_m^0 = \arg\min_y G(m, y)$, $m \in S$. That is, s_m^0 is the myopic base-stock level when the leadtime is m. The myopic base-stock policy is one whereby we order to increase the inventory position to the myopic level in every period; if the inventory position before ordering is already above the myopic level, we do not place an order. This policy may be suboptimal because the myopic base-stock level is, well, myopic, and does not take into account its impact on the costs in future periods. It is also clear that it is never optimal to increase the inventory position to a level higher than the myopic level. Hence the myopic base-stock levels must be adjusted downward to arrive at the optimal base-stock levels. The algorithm is simply a systematic way to do this.

The optimal state-dependent, base-stock levels can be determined in M iterations. (Recall M is the number of leadtime states.) In iteration i, $i = 1, \ldots, M$, the state space S of the Markov chain $\{L_t\}$ is partitioned into two subsets, U^i and V^i, and an accounting scheme is given for assessing holding and backorder costs. The accounting scheme is specified by a set of functions $G^i(m, \cdot)$, $m \in V^i$, and works as follows: To period t we charge zero costs if $L_t \in U^i$, and charge $G^i(m, y)$ if $L_t = m \in V^i$ and $IP(t) = y$. The algorithm is initialized with $U^1 = \varnothing$, $V^1 = S$, and $G^1(m, y) = G(m, y)$, for all $m \in V^1$ and all y. Each iteration identifies a state and determines the optimal base-stock level for that state. More specifically, define

$$y_m^i = \arg\min_y G^i(m, y), \quad m \in V^i, \ i = 1, \ldots, M.$$

(Therefore, $s_m^0 = y_m^1$, $\forall m \in S$.) Let i^* be the leadtime state with the smallest value of y_m^i, $m \in V^i$, i.e.,

$$i^* = \arg\min_{m \in V^i} y_m^i.$$

Then, $s^*(i^*) \stackrel{\text{def}}{=} y_{i^*}^i$ is the optimal base-stock level for leadtime state i^*. The intuition for this is that ordering up to $s^*(i^*)$ is myopically optimal and does not create any problem for the other leadtime states in V^i, because their ideal base-stock levels are all greater than $s^*(i^*)$. The state i^* is then moved from V^i to U^i, i.e., $U^{i+1} = U^i \cup \{i^*\}$ and $V^{i+1} = V^i \backslash \{i^*\}$. Therefore, the V set always contains the leadtime states for which the optimal base-stock levels are yet to be determined, and the U set contains the states with known optimal base-stock levels. In the last iteration, there is only one leadtime state in V^M, which is M^*. Consequently, the minimum long-run average systemwide cost is

$$\pi(M^*) \min_y G^M(M^*, y) = \pi(M^*) G^M(M^*, s^*(M^*)),$$

where $\pi(\cdot)$ is the stationary distribution of the Markov chain $\mathbf{L}$. The part of the algorithm that computes $G^{i+1}(\cdot, \cdot)$ from $G^i(\cdot, \cdot)$ is exactly the same as in the Chen-Song algorithm, and is omitted here.

Proposition 1. *In the complete information case, the retailer's optimal replenishment strategy is to follow a state-dependent, base-stock policy with order-up-to level* $s_m^* \stackrel{\text{def}}{=} s^*(i^*)$ *with* $i^* = m$, $m \in S$. *That is, if the leadtime state is* m, *order to increase the inventory position to* s_m^*, *and if the inventory position before ordering is already above* s_m^*, *do not order. Denote by* C_C *the long-run average systemwide costs under the optimal policy.*

4. No Information Sharing

We now consider the case where the supplier does not share with the retailer the leadtime information. As a result, at the beginning of any period t, the retailer does not know the exact value of L_t. However, the retailer has access to the history of order arrivals, i.e., the orders placed with the supplier in the past, whether or not these orders have arrived and if so, when. Because the retailer knows that L_t is generated by a Markov chain, he can use the Markov structure and the order history to infer some information about the current leadtime and make replenishment decisions accordingly. As we will see, the retailer's replenishment decision is much more complex than in the complete-information case. Below, we consider two scenarios that bound the system performance under incomplete information.

4.1. A Constant Base-Stock Policy

The simplest policy the retailer can use is a base-stock policy with an order-up-to level that is independent of the history of order arrivals. Let y be the constant order-up-to level. Thus, in each period the retailer places an order to increase its inventory position to y. Note that under this policy the long-run average costs for the retailer are $\sum_{m\in S} \pi(m)G(m, y)$. (Recall that $\pi(\cdot)$ is the steady-state distribution of Markov chain $\mathbf{L}$.) This is a convex function of y. Minimizing it over y, we obtain the optimal constant base-stock level Y and the corresponding long-run average costs C_S. Clearly, C_S is an upper bound on the minimum long-run average costs the retailer can achieve under incomplete information.

4.2. A History-Dependent Base-Stock Policy

A more sophisticated replenishment strategy should make use of the historical information about order arrivals. The retailer's knowledge about the leadtime process at the beginning of period t can be characterized by the vector $\{(\tau, \mathrm{Ind}_\tau, l_\tau),\ \tau = t - M + 1, \ldots, t - 1\}$, where $\mathrm{Ind}_\tau = 0$ if no order was placed in period τ and $\mathrm{Ind}_\tau = 1$ otherwise, and l_τ contains information about L_τ, i.e., if $\mathrm{Ind}_\tau = 0$ then there is no information about L_τ, and if $\mathrm{Ind}_\tau = 1$ then if the order has arrived by period t $l_\tau = L_\tau$, otherwise the information is simply that $L_\tau > t - \tau$. It is conceivable that there is a different optimal target inventory position for every possible value of the vector. Clearly, however, the number of possible values for the above vector is very large, rendering the computation of the optimal policy intractable.

Below, we consider a fictitious scenario where an order is placed in every period for the purpose of characterizing the retailer's knowledge about the leadtime process (whether or not a real order has been placed). Under this fictitious scenario, the above indicator function Ind_τ is always equal to 1. Then, l_τ either equals the realized leadtime (i.e., the order has arrived by time t) or indicates that the order has not yet arrived. Due to the Markov nature of the leadtime process, it is only necessary to track the most recent order arrival, i.e., when it was placed and when it arrived. In other words, the retailer's knowledge in period t about the leadtime process, denoted by H_t, can be completely represented by two parameters (τ, l), which means that the most recent order arrival took place in period $t-\tau+l$, the order was placed in period $t-\tau$, and $L_{t-\tau+1} > \tau - 1$ (or the order placed in period $t-\tau+1$ has not yet arrived by period t). The fictitious scenario thus drastically simplifies the representation of the retailer's knowledge about the leadtime process.

It is easy to see that the fictitious scenario effectively increases the retailer's information about the leadtime process. Therefore, the minimum long-run average cost achievable under the fictitious scenario provides a lower bound on the minimum long-run average cost achievable for the original incomplete-information case. We next characterize such a lower bound.

Define $\mathbf{H} = \{H_t\}$. Because τ can only take values $1, \ldots, M$ and for each τ, $1 \le l \le \tau$, there are $M(M+1)/2$ possible states for $\mathbf{H}$. Moreover, $\mathbf{H}$ is Markovian, as we will see next. Suppose $H_t = (\tau, l)$.

The next state H_{t+1} depends on whether or not there is an order arrival at time $t+1$. If there is no order arrival, then $H_{t+1}=(\tau+1, l)$. Otherwise, if there is an order arrival at time $t+1$, then $H_{t+1}=(i, i)$ if the order was placed i periods ago (i.e., at time $t+1-i$), $i=1, \ldots, \tau$. From Bayes' rule,

$$\begin{aligned}\Pr\big(H_{t+1}&=(\tau+1, l)\mid H_t=(\tau, l)\big)\\&=\Pr\big(L_{t-\tau+1}>\tau \mid L_{t-\tau}=l,\, L_{t-\tau+1}\geq\tau\big)\\&=\frac{c_{l,\tau+1}}{c_{l\tau}},\end{aligned}$$

where $c_{mm'}=\Pr(L_1\geq m' \mid L_0=m)$ for all m, m'. Similarly, for $i=1, \ldots, \tau$,

$$\begin{aligned}\Pr\big(H_{t+1}&=(i, i)\mid H_t=(\tau, l)\big)\\&=\Pr\big(L_{t-\tau+1}=\tau,\, L_{t-\tau+2}=\tau-1, \ldots, L_{t-i+1}=i,\\&\qquad\qquad L_{t-i+2}\geq i \mid L_{t-\tau}=l,\, L_{t-\tau+1}\geq\tau\big)\\&=\frac{p_{l\tau}p_{\tau,\tau-1}\cdots p_{i+1,i}c_{ii}}{c_{l\tau}}.\end{aligned}$$

The probability of making a transition from (τ, l) to any other state is zero.

Now consider the retailer's replenishment decisions under the above fictitious information scenario. As in the complete information case, one can show that a state-dependent, base-stock policy is optimal. The algorithm for computing the optimal base-stock levels is the same as that described in §3, after making simple changes to the algorithm's input. The only changes are to replace the Markov chain $\mathbf{L}$ with the Markov chain $\mathbf{H}$, and charge $G((\tau, l), y)$ to period t if $H_t=(\tau, l)$ and $IP(t)=y$, where

$$G((\tau, l), y)\stackrel{\text{def}}{=}\sum_{m\in S}\Pr\big(L_t=m\mid H_t=(\tau, l)\big)G(m, y).$$

To determine $\Pr(L_t=m\mid H_t=(\tau, l))$ for all $m\in S$, first compute the conditional probability distribution of $L_{t-\tau+1}$ given H_t:

$$\begin{aligned}\varphi(l')&\stackrel{\text{def}}{=}\Pr\big(L_{t-\tau+1}=l'\mid H_t=(\tau, l)\big)\\&=\Pr\big(L_{t-\tau+1}=l'\mid L_{t-\tau}=l,\, L_{t-\tau+1}\geq\tau\big)\\&=\begin{cases}0 & l'<\tau\\ p_{ll'}/c_{l\tau} & l'\geq\tau\end{cases}\end{aligned}$$

for all $l'\in S$. Then, $(\varphi(1), \ldots, \varphi(M))\cdot P^{\tau-1}$ gives the conditional probability distribution of L_t given H_t. The algorithm has a total of $M(M+1)/2$ iterations, the number of states for $\mathbf{H}$.

Proposition 2. *When the supplier does not share his leadtime information with the retailer, the minimum long-run average cost for the retailer's inventory system, denoted by C_I (unknown), can be bounded by the minimum long-run average costs in the following two systems. In one, the retailer uses a constant base-stock level, effectively ignoring any information the retailer has about the leadtime process. The minimum long-run average cost in this system, denoted by C_S, is an upper bound on C_I. The other system assumes a fictitious scenario that gives the retailer more leadtime information than she actually has. The optimal replenishment strategy for this system is a state-dependent, base-stock policy with order-up-to levels s_h^*, $h\in\mathscr{H}$, where $\mathscr{H}$ is the state space of $\mathbf{H}$. The corresponding minimum long-run average cost, denoted by C_H, is a lower bound on C_I.*

5. Numerical Examples

The main purpose of this section is to compare the retailer's long-run average cost under complete information sharing with that under no information sharing in numerical examples. The results provide numerical evidence on the value of leadtime information.

Recall that the system parameters consist of the holding and penalty costs, the demand distribution, and the transition matrix of the leadtime Markov chain. We set $h=1$ and $b=1, 10, 20$. We used a negative binomial distribution for the demand in each period. We divided the examples into two classes depending on the mean demand per period. The high-volume examples have a mean demand around 25, and the low-volume examples have a mean demand less than 1. Table 1 summarizes the demand parameters used, where μ is the mean demand per period and σ is the standard deviation.[1]

The following queueing model describes the supplier's production process.[2] Imagine that the supplier

[1] Note that for the negative binomial distribution, $\mu\leq\sigma^2\leq\mu(\mu+1)$. Therefore, $1/\sqrt{\mu}\leq\sigma/\mu\leq\sqrt{(\mu+1)/\mu}$. As a result, a large coefficient of variation is possible only with low-volume items. The fact that the negative binomial distribution has an integer parameter also limits the values of μ and σ.

[2] This model is adapted from an example given in Song and Zipkin (1996).

Table 1 Demand Parameters

μ	σ^2	σ/μ
0.6	0.72	1.41
0.2	0.24	2.45
0.1	0.105	3.24
24	600	1.02
26	164	0.49
25	69	0.33

accepts orders from our retailer as well as the outside world and puts them in a queue according to the sequence in which they were received. Assume that the orders generated by our retailer are only a very small fraction of the supplier's total business. In other words, an order from our retailer is so small in size that the leadtime for the order is essentially the time the order spends waiting in the production queue (assuming zero transportation time). Let X_t be the total orders the supplier receives from the outside world in period t. Assume that the X_ts are i.i.d. Let $f(k) = \Pr(X_t = k)$ and $\bar{F}(k) = \Pr(X_t > k)$, $k = 0, 1, 2, \ldots$. Let Q_t be the queue length, i.e., the total size of the orders in queue, at the beginning of period t. The production capacity per period is C, a positive integer. Moreover, assume that the supplier starts rejecting orders from the outside world the moment the queue length reaches N, a positive integer. Therefore

$$Q_{t+1} = \min\{(Q_t - C)^+ + X_t, N\}.$$

(An implicit assumption here is that production planning for a period is done at the beginning of the period and then frozen for the rest of the period.)

Now suppose the retailer places an order at the beginning of period t. The supplier will be able to deliver this order once the current production queue, of size Q_t, is cleared. Thus

$$L_t = 1 + \min\{l\colon (l+1)C \geq Q_t \text{ and } l \text{ nonnegative integer}\}.$$

For the numerical examples, we set $C = 1$ and $N = 4$. Thus, $L_t = \max\{1, Q_t\}$, with $S = \{1, 2, 3, 4\}$. To obtain the transition probabilities p_{ij}, first note that $p_{ij} = 0$ for all $i, j \in S$ with $j < i - 1$. This is simply because orders do not cross. Now consider p_{ij} for any $i \geq 2$ and $j \geq i - 1$. Given $L_t = i \geq 2$, we have $Q_t = i$. Therefore, $Q_{t+1} = \min\{i - 1 + X_t, N\}$. Consequently, $p_{ij} = f(j - i + 1)$ for $j = i - 1, \ldots, N - 1$ and $p_{iN} = \bar{F}(N - i)$. Finally, consider p_{1j} for $j = 1, \ldots, N$. Given $L_t = 1$, it must be that $Q_t \leq 1$. Therefore, $Q_{t+1} = \min\{X_t, N\}$. Note that $L_{t+1} = 1$ if and only if $Q_{t+1} \leq 1$ or $X_t \leq 1$. Thus $p_{11} = f(0) + f(1)$. Now take any $j = 2, \ldots, N - 1$. Note that $L_{t+1} = j$ if and only if $Q_{t+1} = j$ or $X_t = j$. Thus $p_{1j} = f(j)$, $j = 2, \ldots, N - 1$. Similarly, $p_{1N} = \bar{F}(N - 1)$. This completes the transition matrix.

For the numerical examples, we assumed that the orders from the outside world arrived according to a Poisson process with rate λ. Consequently, $f(k) = e^{-\lambda}\lambda^k/k!$ for $k = 0, 1, \ldots$. We used three different values of λ. For $\lambda = 1$, the steady state distribution of the leadtime process is $\pi = (0.31, 0.23, 0.23, 0.23)$. We call this case *flat*, referring to the shape of the distribution. For $\lambda = 2$, $\pi = (0.01, 0.03, 0.16, 0.80)$, and we refer to this case as skewed high or simply high. Finally, for $\lambda = 0.5$, $\pi = (0.83, 0.12, 0.04, 0.01)$, and is referred to as skewed low or simply low. (We believe that it is the shape of the steady state distribution of the leadtime process that plays a key role in the value of leadtime information.)

The results for the low-volume items are summarized in Table 2. Notice that the average of C_H/C_C is

Table 2 Comparison of Long-Run Average Costs for Low-Volume Items

$\pi(\cdot)$	σ/μ	μ	b/h	s_1^*	s_2^*	s_3^*	s_4^*	C_C	C_H/C_C (%)	Y	C_S/C_C (%)
flat	3.2	0.1	1	1	1	1	1	0.79	100.0	1	100.0
flat	2.4	0.2	1	1	1	1	1	0.78	100.0	1	100.0
flat	1.4	0.6	1	1	2	2	2	1.24	102.5	2	105.5
flat	3.2	0.1	10	1	1	1	1	1.34	100.0	1	100.0
flat	2.4	0.2	10	2	2	2	2	2.08	100.0	2	100.0
flat	1.4	0.6	10	3	4	5	5	3.43	104.3	4	108.5
flat	3.2	0.1	20	1	1	2	2	1.77	102.5	2	104.7
flat	2.4	0.2	20	2	2	3	3	2.58	102.5	3	105.7
flat	1.4	0.6	20	4	5	6	6	4.15	104.7	5	107.3
low	3.2	0.1	1	1	1	1	1	0.74	100.0	1	100.0
low	2.4	0.2	1	1	1	1	1	0.80	100.0	1	100.0
low	1.4	0.6	1	1	2	2	3	1.45	101.5	3	101.7
low	3.2	0.1	10	1	1	1	1	1.68	100.0	1	100.0
low	2.4	0.2	10	2	2	2	2	2.44	100.0	2	100.0
low	1.4	0.6	10	4	5	5	6	3.87	101.4	5	101.4
low	3.2	0.1	20	1	2	2	2	1.91	100.0	2	100.0
low	2.4	0.2	20	2	3	3	3	2.86	100.0	3	100.0
low	1.4	0.6	20	5	5	6	6	4.65	100.2	6	100.2
high	3.2	0.1	1	1	1	1	1	0.83	100.0	1	100.0
high	2.4	0.2	1	1	1	1	1	0.78	100.0	1	100.0
high	1.4	0.6	1	1	1	2	2	0.95	100.6	1	100.6
high	3.2	0.1	10	1	1	1	1	1.09	100.0	1	100.0
high	2.4	0.2	10	1	2	2	2	1.78	101.4	1	102.1
high	1.4	0.6	10	3	4	4	5	2.79	101.5	3	101.9
high	3.2	0.1	20	1	1	1	1	1.38	100.0	1	100.0
high	2.4	0.2	20	2	2	2	2	2.13	100.0	2	100.0
high	1.4	0.6	20	4	4	5	6	3.45	101.2	4	101.2

101% and the average of C_S/C_C is 102%. Therefore, the leadtime information has limited value when demand is low.

The results for the high-volume items are in Table 3. The average of C_H/C_C is 111%, and the maximum is 141%. Note that C_H/C_C represents a lower bound on the (relative) value of leadtime information. Therefore, the value of leadtime information can be significant when demand is high.

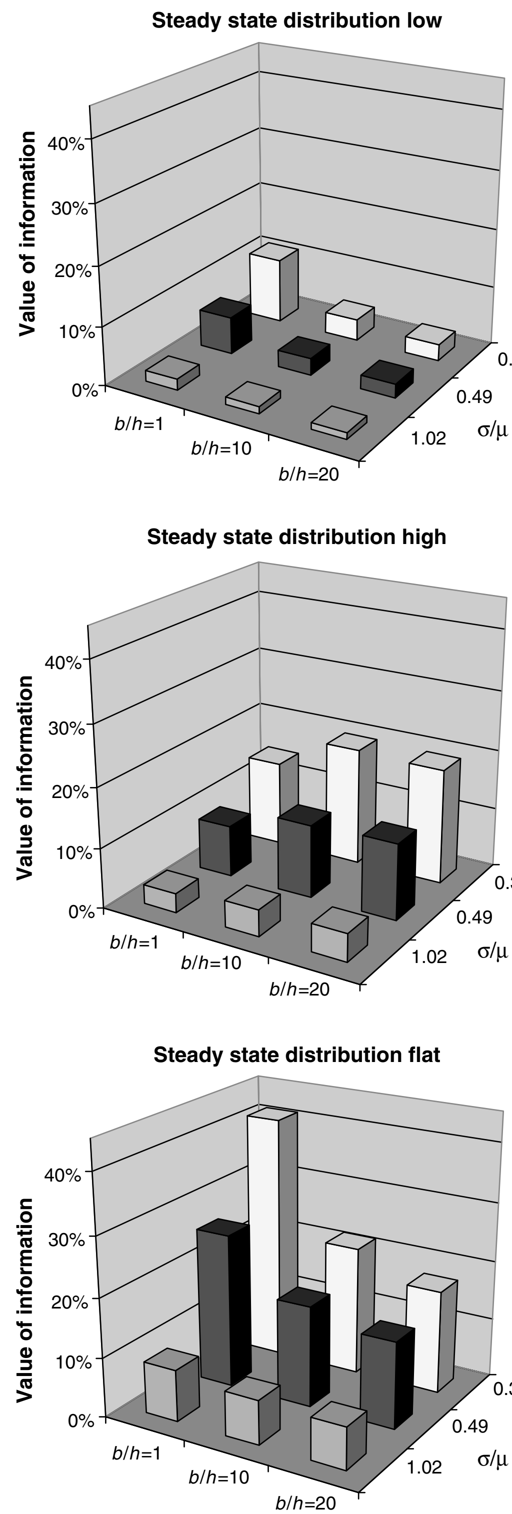
Figure 1 plots the ratio C_H/C_C against several system parameters for the high-volume items. The results suggest that the value of leadtime information increases as the demand coefficient of variation decreases. The relationship with the backorder cost depends on the shape of the steady state leadtime distribution. Finally, the value of leadtime information is highest when the steady state leadtime distribution is flat.

We have also examined the performance of the myopic base-stock policy under complete information. We used simulation to evaluate the long-run average cost of the myopic policy. A robust conclusion

Table 3 **Comparison of Long-Run Average Costs for High-Volume Items**

$\pi(\cdot)$	σ/μ	μ	b/h	s_1^*	s_2^*	s_3^*	s_4^*	C_C	C_H/C_C (%)	Y	C_S/C_C (%)
flat	1.02	24	1	47	72	94	112	35.6	109	71	115
flat	0.49	26	1	57	86	110	129	21.5	126	85	146
flat	0.33	25	1	54	82	105	124	15.9	141	81	173
flat	1.02	24	10	114	147	176	197	102.8	107	157	112
flat	0.49	26	10	99	131	153	171	58.1	117	144	128
flat	0.33	25	10	89	118	136	150	44.0	122	131	135
flat	1.02	24	20	137	172	202	224	125.9	107	184	111
flat	0.49	26	20	113	144	165	182	69.9	115	158	124
flat	0.33	25	20	101	128	144	157	52.8	117	140	127
low	1.02	24	1	57	79	97	112	41.8	102	106	102
low	0.49	26	1	68	93	113	129	23.1	106	124	107
low	0.33	25	1	64	89	108	124	15.8	111	119	112
low	1.02	24	10	135	160	180	197	114.9	101	192	101
low	0.49	26	10	122	142	157	171	57.3	103	167	103
low	0.33	25	10	111	128	140	150	37.8	104	147	104
low	1.02	24	20	160	186	206	224	139.2	101	218	101
low	0.49	26	20	137	156	169	182	67.8	102	179	103
low	0.33	25	20	124	138	148	157	44.2	103	154	103
high	1.02	24	1	42	65	91	112	28.0	103	45	103
high	0.49	26	1	53	81	107	129	16.5	109	54	110
high	0.33	25	1	50	78	103	124	11.6	114	52	116
high	1.02	24	10	103	135	170	197	85.2	104	109	104
high	0.49	26	10	84	120	149	171	45.6	112	92	118
high	0.33	25	10	72	107	132	150	33.7	120	81	130
high	1.02	24	20	123	157	195	223	105.5	105	131	105
high	0.49	26	20	95	132	161	182	55.9	113	105	120
high	0.33	25	20	81	117	140	157	42.2	119	93	131

Figure 1 **Value of Leadtime Information**



is that myopic policy is close to being optimal. This is useful because the myopic policy is much easier to compute.

6. Concluding Remarks

This article has shown that the value of leadtime information can be significant. This is especially true when the leadtime distribution exhibits high variability or when the demand is high volume. It is important to point out that the value of leadtime information here is purely the result of information sharing between the supplier and the retailer, without changing the underlying leadtime (or production and transportation) process.

The algorithm developed here can be easily adapted for models with more general leadtime processes. For example, it is plausible that sometimes even the supplier himself does not have complete information about the leadtime for a particular order. In this case, sharing information about the supplier's production process only gives the retailer distributional knowledge about leadtime, as opposed to the actual leadtime value. The optimal policy can be computed by simply modifying the G functions but following the same iterative procedure (very much in the fashion of §4.2).

Finally, it would be interesting to examine the supplier's incentive to share with the retailer his information about the production process. What is the impact of such information sharing on the supplier's operations?

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