Management Science

Publication details, including instructions for authors and subscription information:
http://pubsonline.informs.org

Commissions and Sales Targets Under Competition

Guillermo Gallego, Masoud Talebian

MANAGEMENT SCIENCE
Vol. 60, No. 9, September 2014, pp. 2180–2197
ISSN 0025-1909 (print) | ISSN 1526-5501 (online)

http://dx.doi.org/10.1287/mnsc.2013.1885

Commissions and Sales Targets Under Competition

Guillermo Gallego
Department of Industrial Engineering and Operations Research, Columbia University, New York, New York 10027,
ggallego@ieor.columbia.edu

Masoud Talebian
School of Mathematics, University of Newcastle, Callaghan, New South Wales 2038, Australia,
masoud.talebian@newcastle.edu.au

We consider a game between two capacity providers that compete for customers through a broker who earns commissions on sales and sells to both loyal and nonloyal customers. The providers compete by selecting commission margins and sales targets above which the margins on total sales increase. We study the contract form in equilibrium and the effect that sales targets have on the profit split between the providers and the broker. We show that in equilibrium, contracts require positive sales targets that can be best described as a mechanism for the larger provider to profit at the expense of the smaller provider. The effect of sales targets is different when commission margins are exogenous and the providers compete by setting targets. In this case, it is the low-margin provider who benefits from sales targets at the expense of the broker, who in this context resists the imposition of targets.

Keywords: provider–broker competition; contract theory; quantity discount; game theory
History: Received March 8, 2011; accepted October 10, 2013, by Yossi Aviv, operations management. Published online in *Articles in Advance* April 2, 2014.

1. Introduction

Brokers play an important intermediary role between capacity providers and consumers, and in some service industries they are responsible for a large portion of sales. Providers rely on brokers because they are closer to customers, whereas customers prefer brokers because they view them as one-stop shops where they can purchase products from different providers.

Capacity providers and brokers have different incentives resulting in decentralized management. Revenue-splitting contracts between different business agents facilitate decentralized management. In this paper we study sales commission contracts, which are common between capacity providers and brokers in service industries. In this type of contract, sales occur through brokers, who receive a commission from providers for each unit sold.

Our work is motivated by our observation of business contracts, where commission margins depend on sales volumes. A common practice is to impose minimum sales volumes to increase commission margins. In other words, the broker is paid only a partial, often zero, commission margin unless sales exceed a set threshold. If sales exceed the target level set by a provider, the broker is paid the commission margin on all units sold, not just the units above the sales target. Sales targets (thresholds) distort the effort that the broker exerts on customers, because it may be optimal for the broker to steer demand to a provider to reach or exceed the target.

A practical motivation for our study is the relationship between competing service providers, e.g., airlines and hotels, which sell their capacity through travel agents, both traditional brick-and-mortar stores and online travel agents. Although in the U.S. market the commissions on simple domestic flights have vanished, they are still a major source of revenue for travel agents in other regions. Imposing positive sales targets is becoming a common practice in this industry. Alamdari (2002) reports a survey in which all eight participating Asian airlines indicated that they use override commission or back-end incentives. These remuneration programs offer agents bonus commissions if their sales exceed some thresholds set by airlines.

In practice, multiple providers of different services, e.g., flights and hotels, interact with multiple brokers to sell their products. We focus on a stylized model to study the interactions between two competing capacity providers and a single broker. We believe that the study of this stylized model provides some insights into what the different players can expect as the outcome of introducing sales targets into commission contracts.

We concentrate on how sales targets and margins are decided when the broker can influence the purchase decisions of some customers. On the basis of the providers' commission schedules, the broker decides how many units of product to sell from each provider. We show that in equilibrium providers impose positive targets, although this may harm them relative to payoffs from a game where sales targets are exogenously set at zero. We also show that competition is the main driver of positive sales targets. More precisely, if demand is large relative to aggregate capacity, providers do not need to compete for customers, and both commission margins and targets are set at zero in equilibrium.

We also study the effect of the size of the market and the power of the broker to influence demand. The broker's power is a measure of the ability to influence customers to buy one product over the other, by allocating his sales effort. We show that the broker always benefits from more power, but not necessarily from market growth; more explicitly, the broker may suffer from market growth. The reason for this is that when demand is abundant relative to capacity, the providers can pay low commission margins to generate sales. We also show that a powerful broker coupled with relative large capacities results in high commission margins in equilibrium. This justifies large margins in service industries with capacities that are large relative to demand.

1.1. Model Description and Assumptions

Our model consists of two providers with fixed capacities under competition and one broker with access to customers' demand. This results in both horizontal competition between providers and vertical competition between providers and the broker. This setting allows us to study the effect that sales targets have on the way revenue is split among the providers and the broker.

We assume that products are partially substitutable; there are loyal customers who are interested in only one of the providers and nonloyal customers that can be influenced by the broker to buy from either of the providers. The broker's power is measured by the ratio of nonloyal to total demand, and this ratio is assumed to be common knowledge to all players. Demand can be influenced by nonprice incentives such as advertising, reward points, more information about one product, attractive shelf space, and guiding consumer purchases with sales personnel.

As products become closer substitutes, customers become less loyal and the broker more powerful. In such settings, providers need to rely more on the broker to sell their products. In a commission contract, one common mechanism to persuade the broker to sell more of the provider's product is to impose sales targets to trigger commission margins.

Providers face a dilemma in setting sales targets. If a provider sets the target too low or too high, she loses sales as the broker's sales efforts may be directed toward the other provider. In equilibrium, the broker may find that it is profitable to buy more units than he can sell to reach the sales target, and then discard unsold units.

We use game theory to analyze the player's interactions and the resulting equilibria on the basis of the players' set of strategies. Although the game involves both of the providers and the broker, it can be reduced to a game between the providers by taking into account how the broker responds to sales targets and commission margins.

We compare how revenue splits before and after the introduction of sales targets to compare and contrast the effects of introducing sales targets. In particular, we study when sales targets are beneficial to at least one of the providers and who benefits and who loses by introducing sales targets into commission contracts.

We assume that products prices, providers' capacities, and the broker's power are exogenously determined and fixed over the contracting horizon. The fixed prices, capacities, and demand structure are a result from competition at a higher strategic level, which makes them exogenous during the contract designing. We also consider a deterministic demand model. These assumptions allow us to focus our attention on analyzing the players' interactions, the revenue split between them, and strategic effects of requiring the sales targets. In this setting, we present a new justification for commission contracts with sales targets by showing that such contracts arise as the only equilibrium in competition.

We assume that the broker's and the providers' costs are negligible and they prefer to fulfill demand even if they do not make a profit.[1] In the case of the broker, this can be justified when selling costs are negligible compared to the ill will of unsatisfied customers. For providers, it can be justified since they have a sunk investment in capacity, and we are assuming that commissions are the only costs associated with sales. In this setting, when the fixed costs are sunk and variable production and distribution costs are negligible, maximizing profits results in maximizing revenues. Notice that the commissions are neither negligible nor fixed, and providers maximize profits net of commissions paid to the broker, so we could use the term profit for the providers. However, we use the term revenue following the tradition of the revenue management literature to mean revenues net of commissions.

[1] In other words, we assume that the players do not have any reservation. If the players have a positive reservation, conditioned to be small enough, side payments can be set to satisfy it without changing the setting.

1.2. Analogy Between Commission Contracts and Sales Contracts

Notice that a commission contract is similar to a sales contract, where a retailer buys the products from a supplier to sell them to customers. Requiring minimum sales volumes to trigger a commission increase is similar to all-units quantity discounts, where the retailer receives a discount based on the size of his order. In this setting, the suppliers set a wholesale price and give discounts if the retailer reaches the set targets. Knowing the suppliers' quantity discount schedules, the retailer decides optimal purchase quantities and determines sales that maximizes his revenue. The suppliers maximize their revenue by optimizing their discount schedules. The discount schedule affects how the supply chain's revenue is distributed among the players.

Since our primary source of motivation is analyzing the service industry, where commission contracts are prevalent, for the rest of this paper we utilize the provider/broker terminology. However, because of this similarity, some of our results and insights may apply to quantity discount contracts and supplier/retailer relationship. The main difference is that in our setting we do not need to worry about inventory carrying costs.

From an economics perspective this research is related to the classical Bertrand model of oligopoly competition. The Bertrand model considers competing providers that sell their products directly to the market and compete on offered prices to customers. In our setting, providers sell their products through a common broker and compete on offered commission margins to the broker. In other words, the Bertrand model studies the business-to-customers relationship, whereas we study a parallel model of the business-to-business relationship.

The original Bertrand model does not consider the capacity constraints. In the Bertrand–Edgeworth model, the Bertrand model is generalized by considering cases where firms have capacity constraints, or more generally, the marginal production cost increases as capacity increases. Without capacity constraints, competition drives down the prices to marginal costs. Our model extends the Bertrand–Edgeworth model in several directions. First, we investigate a supply chain, where the providers are in indirect contact with the market through a broker. Second, we consider a more general form of wholesale price contract. And third, we consider products that are partially substitutable, whereas the original Bertrand model assumes that the products are identical and all customers buy the product with the lowest price.

The rest of this paper is organized as follows. After a review of the relevant supply chain coordination literature in §2, we provide variables definition and model formulation in §3. In §4, we analyze the effects of requiring sales targets to trigger a commission increase. In §5, we summarize our findings and provide conclusions and avenues for further research.

2. Literature Review

There is much evidence in theory and practice that shows supply chains are not necessarily coordinated, so the supply chain's profit in a decentralized system is less than the optimal profit in a centralized system. This situation puts all members in a moral hazard situation because seeking their own profit costs other members. There are two streams of literature that focus on this subject: contract theory in economics and supply chain coordination in operations management. Spengler (1950) provides an initial identification and review of supply chains coordination failure.

One of the factors that can make a chain uncoordinated with a linear wholesale price is double marginalization. One of the initial studies about double marginalization is by Jeuland and Shugan (1983). They consider a deterministic demand rate that is a decreasing function of the retail price. The supplier chooses the wholesale price, and the retailer chooses the retail price. They show that this simple setting results in a retail price that is higher than the chain optimal one. They extend this problem to vertical chains with at least two members. However, they find it complicated to extend the model to multiple suppliers or multiple retailers when they are competing with each other. Lariviere (1999) study low stocking factor by considering a newsvendor setting where the demand is stochastic, but the retail price is exogenous. In this setting, it can be shown that the retailer orders less than the chain's optimal amount, so the double marginalization problem manifests itself through low stocking.

One approach to increase coordination is the use of sophisticated contracts, which are the focus of our paper. The extensive literature on supply chain coordination is surveyed by Cachon (2003). In addition to all-units and incremental quantity discounts, some of the most widely used supply chain contracts to coordinate a newsvendor chain are revenue sharing, full return (buyback), and sales rebates (target rebates).

Hax and Candea (1984) investigate cases where the incremental quantity discount is equivalent to a two-part tariff. Weng (1995) considers a setting in which fixed ordering costs affect order quantity and, as a result, inventory and operating cost, and drives to an uncoordinated chain. He shows that two types of quantity discount contracts, all-units and incremental, perform identically in presence of fixed ordering costs. Moreover, he shows how quantity discounts and franchise fees can coordinate a chain with fixed ordering costs. In addition to achieving coordination, quantity discounts also can be used for price discrimination; see Kinter (1970) for more discussion.

Most of the current coordination literature focuses on the monopoly case, or noncompeting multiple suppliers and/or retailers. The studies that consider suppliers' competition usually ignore sophisticated contracts. An exception is the study by Cachon and Kok (2010), who analyze the effect of two-part tariff and a subset of incremental quantity discount and compare them to wholesale price with price sensitive customers. We focus on a different contract, the all-units quantity discount, and in addition, we consider suppliers with limited capacity. Nevertheless, they also reach the conclusion that sophisticated contracts are not necessarily beneficial for suppliers in equilibrium.

As a brief review of the literature has shown, sophisticated contracts are usually recommended to increase the total revenue of a supply chain. However, we focus our attention on how supply chain revenues are split under competition assuming fixed exogenous selling prices and sales effort, zero marginal fulfillment costs within capacity, and deterministic demands. These assumptions allow us to focus on competition instead of coordination as the source of sophisticated contracts.

It should be noted that quantity discount contracts and their terms, like all contracts, have litigation aspects too. Tirole (1988) discusses that only verifiable parameters should be written into a contract so that in the case of a disagreement between the contracting parties, a court can intervene. This implies that just observation by both parties is not enough, and the parties should be able to prove their observations.

3. Model

In this section, we describe our model and present the formulations for each of the players. We have two capacity providers and a broker who plays the intermediary role between providers and the market. The market consists of loyal and nonloyal customers; nonloyal customers can be assigned by the broker to any of the providers. Each player's formulation is discussed after defining the following parameters and decision variables.

The parameters are as follows:

- c_i: capacity of provider i;
- p_i: price of provider i;
- d_i: loyal demand for provider i;
- d_0: nonloyal demand, which the broker can tilt.

The providers' decision variables are as follows:

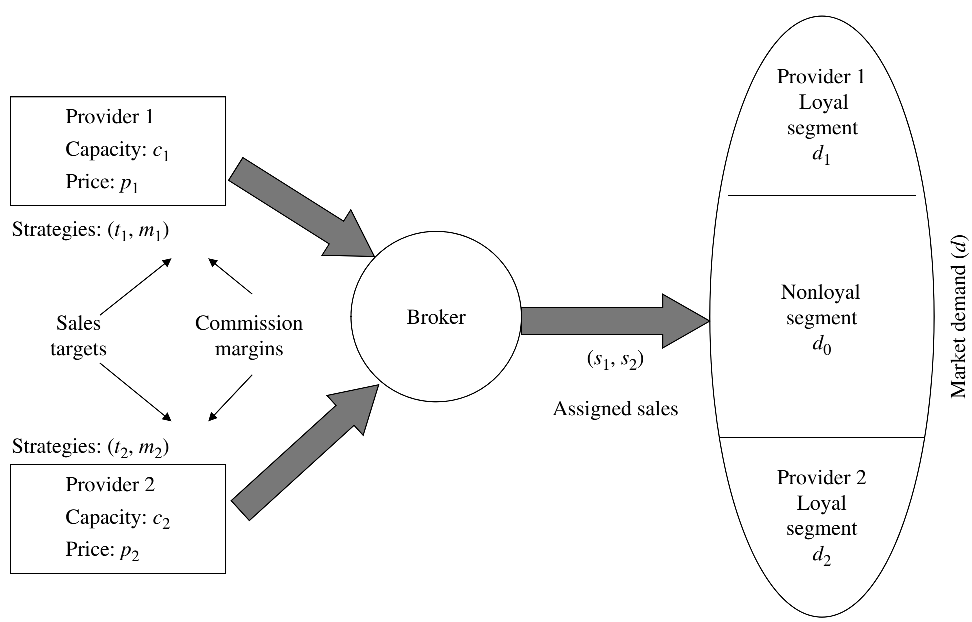
- $0 \le m_i \le p_i$: the full commission margin that provider i pays the broker on all units if sales exceed the sales target t_i;
- $0 \le t_i \le c_i$: the sales target required by provider i to obtain the full commission margin m_i on all units sold.

The broker's decision variable is as follows:

- s_i: units from provider i sold by the broker, $i = 1, 2$.

Figure 1 shows the model scheme, which is common knowledge to all players. The broker's power can be measured by the relative size of the nonloyal to the total demand and represents the proportion of the customers that can be influenced by thebroker. In our

Figure 1 **Game Scheme**



model, the providers differ in their capacity (c_i), their loyal market (d_i), and the price they charge for their products (p_i).

3.1. Player' Strategies

The strategy space for the providers consists of the pairs (t_i, m_i), $i = 1, 2$, where t_i denotes the sales target, and $m_i \in [0, p_i]$ is the per-unit commission margin that the broker obtains from provider i for all units sold when sales reach or exceed t_i. The broker obtains no commission from provider i if he sells fewer than t_i units, and obtains commission $m_i s_i$ if he sells $s_i \geq t_i$ units from provider i. Notice that the commission scheduled is described by only two parameters: the target t_i and the commission margin $m_i \in [0, p_i]$ that operates on sales $s_i \geq t_i$. Although this commission schedule seems very limited, it is possible to show (see Theorem 1) that a provider cannot do better by considering more general commission schedules with multiple sales targets.

The strategy of the broker is to assign sales of the nonloyal segment d_0 among the providers to maximize his profits from commissions. In principle, the broker's strategy may involve purchasing units beyond those he can sell (discarding) with the purpose of achieving sales targets. Theorem 1 shows that the broker can limit his strategy, without loss of optimality, to avoid discarding. Therefore, Theorem 1 justifies the reduction from a model with general commission schedules and discarding to a two-parameter commission structure without discarding.

Theorem 1.

- *For every nonnegative and nondecreasing commission schedule, there exists a two-parameter schedule (t_i, m_i) that results in the same revenue split between providers and the broker.*
- *For every equilibrium that allows discarding, there exists an equilibrium without discarding that results in the same revenue split between providers and the broker.*

Theorem 1 simplifies the analysis because it states that, without loss of generality (w.l.o.g.), we only need to consider two parameter policies for each provider instead of having to work with general commission schedules and the potential discarding of units purchased just for the purpose of reaching a target. The proof of Theorem 1 can be found in the appendix. In the formulation and analysis, we set $j = 3 - i$ without further notice to refer to the provider competing with provider $i = 1, 2$.

Definition 1. Define the following:

- $b_i = \min[c_i, d_0 + d_i]$;
- $r_i = \min[b_i, d - b_j]$.

Notice that b_i, as the minimum of the capacity and the maximum demand that can be channeled to provider i, equals the maximum sales of provider i, and r_i, as the minimum of the capacity and the residual demand, equals the minimum sales of provider i. One can also think of b_i and r_i as sales associated with taking first or second dibs on the market. More precisely, b_i is the amount that provider i would sell if the broker gave him priority over provider j. Conversely, r_i is the amount that provider i would sell if the broker gave priority to provider j. We will refer to b_i as the sellable capacity of provider i.

Throughout this paper, we label the primary provider, the one who has been given priority by the broker, as provider 1. The conditions for becoming provider 1 depend on the setting of the analysis. We will explain the priority scheme under each setting as we go through the different cases of the analysis.

Since each provider sells at least r_i, the only nontrivial sales targets are $t_i \geq r_i$, $i = 1, 2$. The case of zero targets, $t_i = 0$, will refer to the common situation where sales targets are not imposed. The case of zero targets is in fact equivalent to setting targets $t_i \in [0, r_i]$ for $i = 1, 2$. We refer to this case of inactive sales targets as "without sales targets," compared to the case of "with sales targets," which means that sales targets are decision variables selected by the providers.

Since the broker makes a decision on the basis of the providers' decisions, we can and will reduce the game to a duopoly after accounting for the broker's behavior. In this duopoly, the providers are the two players, and the broker's optimal policy determines their payoff functions.

3.2. Broker's Formulation

The broker, as an intermediary between providers and the market, can influence customers' choice by deciding how to distribute sales efforts and marketing capabilities between providers. The broker's power is measured by the size of the nonloyal demand d_0 relative to the total demand $d = d_0 + d_1 + d_2$. Extremes of $d_0 = 0$ and $d_0 = d$ represent a powerless and fully powerful broker, respectively. As $d_0/d \in [0, 1]$ increases from zero to one, competition between providers becomes more intense.

Let π_B denote the broker's revenue; the broker's formulation with nontrivial sales targets is given by

$$\begin{aligned}
\pi_B(t_i, m_i) = \max_{(s_i, k_i)} & \{k_1 m_1 s_1 + k_2 m_2 s_2\} \\
& r_i \leq s_i \leq b_i \quad \text{for } i = 1, 2, \\
& k_i t_i \leq s_i \qquad \text{for } i = 1, 2, \\
& s_1 + s_2 \leq d, \\
& k_i \in \{0, 1\} \quad \text{for } i = 1, 2.
\end{aligned}$$

In this formulation, k_i is a binary variable that determines whether or not the provider i's target is reached by the broker. If the broker does not reach the target, corresponding to $k_i = 0$, provider i does not pay a commission margin, getting a free ride. Note that in the broker's formulation, the broker decides what part of nonloyal demand should be assigned to each of the providers. For each solution s_i, the broker assigns $(s_i - d_i)^+$ of d_0 to provider i.

3.3. Providers' Formulations

Let $\pi_i(t_j, m_j)$ denote the optimal revenue of provider i as a function of provider j's strategies (t_j, m_j). Then,

$$\begin{aligned}\pi_i(t_j, m_j) = \max_{(t_i, m_i)} &(p_i - k_i m_i) s_i,\\ r_i \le\ &t_i \le c_i,\\ 0 \le\ &m_i \le p_i.\end{aligned}$$

As it has been modeled, the k's and the s's are external variables for the providers and functions of t's and m's, so providers can influence k's and s's by their decisions rather than directly setting them. The fact that the objective functions are discontinuous and nonmonotone in the strategies as a result of the binary sales target conditions makes classical results from standard game theory inapplicable.

3.4. Demand Formulation

In our analysis in the next section, we also discuss the effects of changes in the market size, d. For this purpose, we do not need to limit ourselves to any specific demand model. We only require that the demand model satisfies the assumption that as the market size, d, increases, d_0, d_1, and d_2 increase proportionally. As a result, the broker's power remains fixed as the demand scales up. This assumption is not restrictive and makes it possible to study the effect of the market size independent of the broker's power. We discuss the following two models as two examples that satisfy our assumption.

One way to model customers' behavior in choosing between different products is through the *multinomial logit* (*MNL*) *choice* framework. As Talluri and van Ryzin (2004) state, the MNL formulation models many real world situations closely, is analytically tractable, and is easy to estimate. Scaling all utilities based on customer elasticity, we define the following:

- u_i: the utility of each provider's product;
- v_i: the positive nonmonetary utility added in broker to the product of provider i;
- v: the broker's ability to affect the utility of customers, which is the bound on the utility that can be added by the broker, $(v_1 + v_2 \le v)$;
- $d_i(v_1, v_2)$: demand for each provider's product, conditioned that the broker assigns added utility v_i to each provider.

Then, the demand for provider i as a function of the broker's added utilities is

$$d_i(v_1, v_2) = \frac{\exp(u_i - p_i + v_i)}{\exp(u_1 - p_1 + v_1) + \exp(u_2 - p_2 + v_2)} d \quad \text{for } i = 1, 2.$$

We can represent the loyal demand for provider i as her minimal demand when all the effort is assigned to the other provider.[2] Note that in this model, the broker's power is equal to $(d - d_1(0, v) - d_2(v, 0))/d$, which increases by the v, and $v = \infty$ represents an extremely powerful broker.

Another way to model customers' behavior is *market segmentation*, where we define the following:

- α: nonloyal segment of the market;
- β_i: each provider's share of loyal segment of the market $(\beta_1 + \beta_2 = 1)$.

Let

$$d_i = \beta_i (1 - \alpha) d \quad \text{for } i = 1, 2.$$

Note that in this model, the broker's power is equal to α, where $\alpha = 1$ represents an extremely powerful broker.

4. Equilibrium

We first dispose of the case $b_1 + b_2 \le d$ of excess market demand. In this case, there exists a unique pure-strategy equilibrium such that $m_i = 0$ for $i = 1, 2$, which results in sales $s_i = r_i = b_i$ and revenue split $\pi_i = p_i b_i$ and $\pi_B = 0$. In this setting, the market is so large that there is no competition between providers, so commissions are zero and targets are irrelevant. As the market demand increases, the total revenue increases, but it does not change the broker's revenue, which stays at zero. The broker's revenue is also insensitive to the broker's power, but the revenues for both providers are increasing in the broker's power.

The above analysis shows that the only nontrivial case is $b_1 + b_2 > d$, where the collective sellable capacity is larger than demand, so the providers need to compete for the nonloyal customers. We assume from now on that this condition holds and give this a formal definition:

Definition 2. We call a market *competitive* if $b_1 + b_2 > d$.

Notice that when nonloyal demand is greater than zero, the competitive condition is equivalent to $c_i > d_i$ for $i = 1, 2$ and $c_1 + c_2 > d$. It is also easy to see that in a competitive market $r_i = d - b_j < b_i$. As a

[2] We assume that the broker exerts all his power, which is fixed exogenously. Notice that the broker's power does not affect the overall demand. Thus, for any (v_1, v_2) such that $v_1 + v_2 < v$, there exists a (v_1', v_2') such that $v_1' + v_2' = v$ and $d_i(v_1, v_2) = d_i(v_1', v_2')$.

result, if provider i is given priority, then sales for the providers are b_i and $d-b_i$, respectively, for provider i and j. In this case, the three players compete against each other to collect a larger share of the total revenue. The next two subsections deal with the competitive market problem with and without sales targets.

4.1. Analysis With Sales Targets

Based on Theorem 1, we can and do limit our study to zero partial commission margins so each provider has a two-dimensional strategy to set (t_i, m_i). Theorem 1 also justifies limiting our study to $r_i \le t_i \le b_i$. Indeed, if $b_i < t_i$, the broker can decide to reach the sales target, t_i, which results in discarding from provider i, and on the basis of Theorem 1, we can avoid studying this case. Alternatively, the broker can decide not to reach the target, which results in free riding. This case can be captured by setting $m_i = 0$.

Theorem 2. *There exists a pure-strategy equilibrium such that $m_i^* = ((b_i - r_i)/b_i)\min[p_1, p_2]$ and $t_i^* = b_i$ for $i = 1, 2$. The equilibrium results in revenue split*

$$\pi_i^* = p_i r_i + (p_i - p_j)^+(b_i - r_i) \quad \text{for } i = 1, 2;$$

$$\pi_B^* = \min[p_1, p_2](b_1 + b_2 - d).$$

Labeling the provider with the higher price *provider* 1, $s_1^* = b_1$ and $s_2^* = r_2$. Notice that provider 1 takes the first claim of the market, whereas provider 2 takes the residual demand and gets a free ride, with the broker making all his profit from provider 1. The broker's profit is equal to the excess of sellable capacity at the minimum of the two prices.

Theorem 2 also provides insights into push versus pull systems. In a pull system, customers already have chosen their brand and come to the broker to buy it, i.e., customers pull the products out of the system. In a push system, it is the broker who persuades customers and influences their choice, i.e., the products need to be pushed out of the system. Our results confirm the general intuition that whereas in a pull system commissions can be low, a push system requires high commissions to incentivize the broker. In practice, systems are usually a mix of push and pull systems, and the commission percentage depends on the mixture. The mixture between push and pull is related to the broker's power, d_0/d. As the broker's power increases, conditioned on existing enough capacity, the system moves toward a push system. The type of the system also depends on capacity versus demand; if capacity of provider i is binding such that $d_0 + d_i > c_i$, then as c_i increases, both commission margins increase, and the system moves toward a push system. If capacity is already abundant, no change is expected. The managerial implication is that a provider would choose her commission margin and sales target and as a result set up a push or pull system depending on the broker's power, size of her capacity, and also the competing provider's capacity.

Comparative statics based on Theorem 2 show that changes in providers' and the broker's profits as a result of demand changes are linear and capped by prices. It shows that the equilibrium solution is a smooth function of the problem parameters, and there are no jumps. In other words, if the coefficient of variation of demand is small, then changes in profits as a result of changes in demand are also small.

Theorem 2 also shows that market demand growth results in more revenue for the providers, but the revenues for the broker first increase and then decrease with the market size. The intuition is that an increase in aggregate demand can reduce the broker's revenues as the providers have less of an incentive to compete. Within the airline industry, it implies that as load factors increase, the commission margins decrease. Figure 2 shows each player's revenue as a function of market demand, when demand is not loyal.

Figure 3 shows how revenue splits between the providers and the broker as broker power increases. It confirms that the broker's power positively influences the broker and negatively affects the providers when the market is competitive. This is in sharp contrast to the noncompetitive case where, surprisingly, market power actually benefits the providers.

We consider the special case where the providers' capacities are large such that they are not binding factors, $c_i > d_0 + d_i$. This case closely represents service providers like insurance companies, where capacity is virtually infinite. Our study shows that as the broker's power increases and demand becomes nonloyal the broker's revenues increases. This can help explain the very large commission margins of brokers in the insurance industry when they sign in new customers.

Figure 2 **Effect of Market Size on Revenues with Sales Targets Under a Nonloyal Market;** $d_1 = d_2 = 0$, $c_1 = 3$, $p_1 = 5$, $c_2 = 8$, $p_2 = 3$

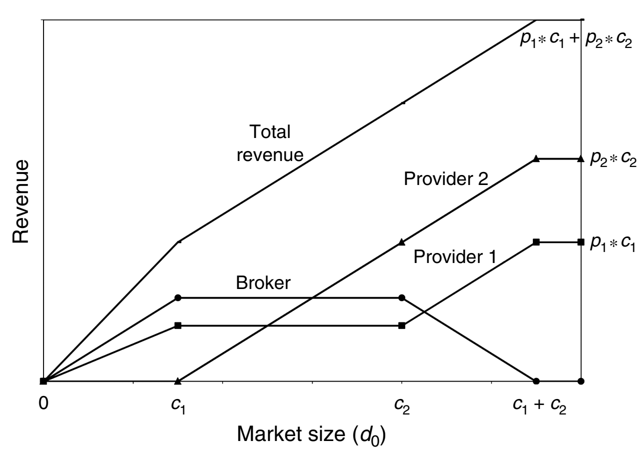
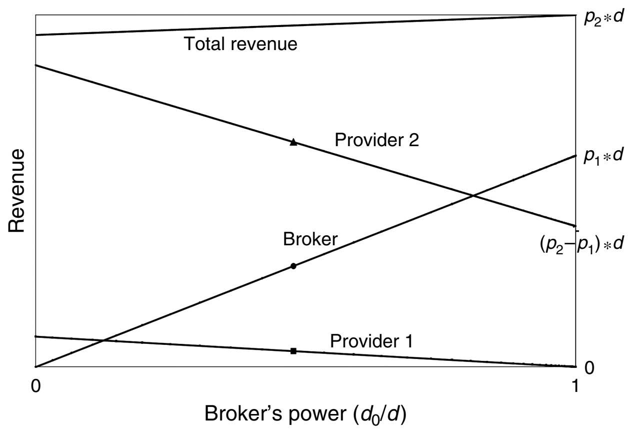


Figure 3 **Effect of Broker's Power on Revenues With Sales Targets Under Large Capacities;** $p_1 = 5$, $p_2 = 3$, $d = 11$, $c_1, c_2 \geq 11$



Providers may consider establishing direct sales channels to avoid paying commissions to the broker. Although direct sales can change the market structure, we focus here on the case where providers cannot reach the nonloyal segment through direct sales, and the size of the loyal market segment is fixed. Under this assumption, the incentive to open a direct sales channel is to avoid paying commission on the loyal part of the demand. Although this seems a compelling motive, we show that the providers cannot benefit from offering direct sales in this case. Let π_i^d be the expected profit to provider i assuming that loyal demand is sold through a direct sales channel.

Theorem 3. *If the broker is needed to attract nonloyal demand, then* $\pi_i^d = \pi_i^*$ *for* $i = 1, 2$.

Theorem 3 states that providers cannot benefit from direct sales if they still need a broker to attract the nonloyal part of the demand. The intuition here is that when a provider starts to sell directly to loyal customers, based on Theorem 2, she needs to offer higher commission margins and lower sales targets to incentivize the broker. In other words, selling directly to the loyal market puts the provider in a weaker position to compete for the nonloyal market, and the commission savings on loyal customers are offset by higher commissions paid for nonloyal customers. The result is independent of the market structure, i.e., the relative sizes of loyal and nonloyal markets.

The implication of Theorem 3 is that a hybrid business model that sells directly to loyal customers and via the broker to nonloyal customers can only be profitable for the provider if the direct sales channel can attract nonloyal customers. Theorem 3 suggests that the dual channel strategy can be justified only if the direct channel increases the size of the loyal market.

Theorem 3 is based on some assumptions that may not be applicable in practice. First, the demand is assumed to be deterministic, so the providers know exactly the size of loyal demand and can set their sales targets to avoid paying commission on loyal demand. Providers can not lower their cost of commissions by establishing direct channels for loyal customers because they are not paying commission for those sales anyway. Stochastic demands can change these results. Another restrictive assumption is that distribution costs are negligible for both providers and the broker. If the costs are not negligible and the direct channel can reduce costs, then a direct channel may be beneficial for the providers. On the other hand, small departures from our assumptions are unlikely to make a direct channel truly profitable unless it can help attract nonloyal customers.

4.2. Analysis Without Sales Targets

In this subsection, we assume sales targets are set to $t_i = 0$, and consequently the providers compete only with full margins. The providers choose commission margins that maximize their net revenues, and the broker always gives priority to the high margin provider. Through this section, we label the provider with a higher p_i/b_i as *provider* 1. A higher price and a lower capacity and/or loyal market share are criteria for determining provider 1. We use the superscript "o" to refer to the case without sales targets.

Theorem 4. *There exists a mixed-strategy equilibrium such that for* $m \in [0, ((b_i - r_i)/b_1)p_1]$, $P(m_i^o \leq m) = (1/(p_j - m))[(p_j b_i - p_i b_j)^+/b_i + m(d - b_i)/(b_i - r_i)]$. *Moreover, it results in revenue split*:

$$\pi_i^o = p_i r_i + \left(p_i - \frac{b_i}{b_j} p_j\right)^+ (b_i - r_i) \quad \textit{for } i = 1, 2;$$

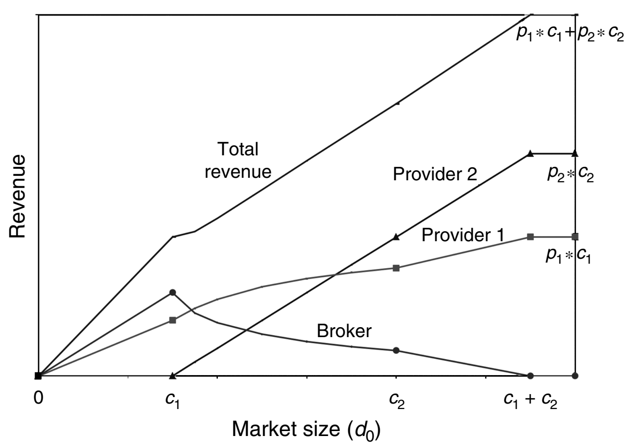
$$\pi_B^o = \begin{cases} p\dfrac{b_2}{b_1}(b_1 + b_2 - d) & \textit{if } p_1 = p_2 = p, \\[2ex] p_1 b_2 + \dfrac{p_1}{p_2 - p_1}(d - b_2)\left(p_2 \dfrac{b_2}{b_1 + b_2 - d} - p_1 \dfrac{b_2}{b_1}\right) & \\[2ex] \quad \cdot \ln\left(\dfrac{p_2(d - b_2)}{b_1 p_2 - (b_1 + b_2 - d)p_1}\right) & \textit{if } p_1 \neq p_2. \end{cases}$$

Theorem 4 states that if the market is competitive, then providers will randomize their strategy, and in every case, the broker prioritizes the provider who draws the largest margin. Notice that there exists a mass $(1/p_1)(p_1 b_2 - p_2 b_1)/b_2$ at $m_2^o = 0$ for provider 2.

Theorems 2 and 4 allow us to compare the revenue of the three players with and without targets. We postpone this discussion to §4.3, where we also discuss the strategic effects of sales targets.

When there are no loyal customers and capacities are abundant, i.e., $c_1, c_2 > d = d_0$, we have a Bertrand model where the broker plays the intermediary role between providers and the market. In this case, provider 1 is the provider with the higher price

Figure 4 **Effect of Market Size on Revenues Without Sales Targets Under a Nonloyal Market;** $d_1 = d_2 = 0$, $c_1 = 3$, $p_1 = 5$, $c_2 = 8$, $p_2 = 3$



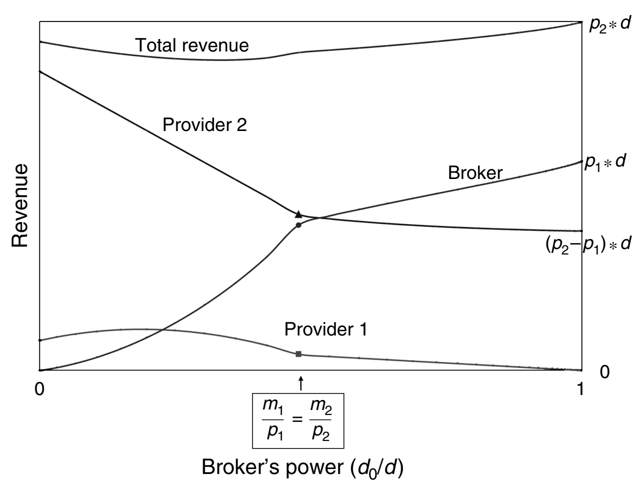
that pays a commission equal to p_2 and gains a profit equal to $(p_1 - p_2)d$. Provider 2 does not gain any profit, as in the classical Bertrand result.

Theorem 4 confirms that as the market size increases providers earn more profit, as illustrated in Figure 4, but the revenues for the broker first increase and then decrease with the market size. The broker gains from more power at the expense of the providers as illustrated in Figure 5.

4.3. Strategic Effects

In this section, we study the strategic effects of requiring sales targets. First, we notice that in presence of sales targets, a pure-strategy equilibrium exists, whereas without targets, there is no pure-strategy equilibrium, and providers randomize their commission margins. If the market is not competitive, the revenue split does not change by introducing sales targets. However, in a competitive market, the revenue split changes in the presence of sales targets, as stated in the following theorem.

Figure 5 **Effect of Broker's Power on Revenues Without Sales Targets Under Large Capacities;** $p_1 = 5$, $p_2 = 3$, $d = 11$, $c_1, c_2 \geq 11$



Theorem 5. *The introduction of sales targets has the following effects in a competitive market*:

- *The profit of the larger provider increases or stays the same.*
- *The profit of the smaller provider decreases or stays the same.*
- *The profit of the broker increases if the larger provider charges a lower price; otherwise, the broker's profit increases, decreases, or stays the same.*

Recall that our definition of the larger provider is the one with larger sellable capacity. It is interesting to see how the introduction of sales targets changes profits in a way that is not consistent with intuition. Intuition suggests that the broker should fear targets because with them he may have to work for free. Our analysis suggests that it is the smaller provider who stands to lose the most by the introduction of sales targets. Comparing Figures 4 and 2 over the interval $(c_1, c_1 + c_2)$ shows that the losses for the smaller provider can be very substantial in a nonloyal market and that the benefits go essentially to the broker. Notice also that the larger provider can introduce sales targets with the purpose of hurting the smaller provider even when he does not directly benefit from doing this.

4.4. Exogenous Commission Margins

We have seen that the broker may benefit from the introduction of sales targets, but in practice brokers adamantly resist the introduction of sales targets. Our best explanation for this paradox is to consider the framework where sales targets are introduced in an environment where margins are fixed.

Although the game with endogenously determined margins and sales targets seems like the natural and more general setting for our study, our initial motivation came from service industries where historical commission margins had been fixed for a long time and were paid on total sales. Indeed, travel agents operated in a world with fixed commissions without the need to meet sales volumes to earn them. As some service providers started experimenting with positive sales targets to trigger commissions, the commission margins were considered fixed.

The assumption of zero partial commission margins remains without loss of generality, as stated in the following theorem. It allows us to work with a more parsimonious model where providers only decide about sales targets.

Theorem 6. *There does not exist an equilibrium with positive partial commission margins that Pareto dominates the provider's revenues from an equilibrium with zero partial commission margins.*

Based on Theorem 6, we can assume, without loss of generality, that partial commission margins are zero. An important insight that can be gleaned from the proof is that having a positive partial commission margin helps the competitor, so both providers prefer to have zero commission margins below their sales targets.

However, buying and discarding unsold items may become necessary in equilibrium when margins are exogenous. Let δ_i denote the number of units of provider i discarded by the broker. The broker's formulation is then given by

$$\pi_B(t_i) = \max_{(s_i,\, k_i,\, \delta_i)} \sum_{i=1,2} [k_i m_i (s_i + \delta_i) - p_i \delta_i]$$

$$\begin{aligned} & r_i \le s_i \le b_i && \text{for } i = 1, 2, \\ & k_i t_i \le s_i + \delta_i \le c_i && \text{for } i = 1, 2, \\ & s_1 + s_2 \le d, && \\ & k_i \in \{0, 1\} && \text{for } i = 1, 2, \\ & 0 \le \delta_i && \text{for } i = 1, 2. \end{aligned}$$

For the providers, the only decision variable is the sales target t_i, so the providers' formulation is given by[3]

$$\pi_i(t_j) = \max_{t_i} (p_i - k_i m_i) s_i$$

$$\begin{aligned} & r_i \le t_i \le c_i \\ & 0 \le m_i \le p_i. \end{aligned}$$

In the case of a single provider, the monopolistic provider can sell as much as $\min[c, pd/(p-m)]$ and earn revenue of $\min[(p-m)c, pd]$. This implies that the provider, conditioned on having enough capacity, can inflate the demand by a factor of $p/(p-m)$ by requiring a sales target higher than the market size. On the basis of this intuition, we use the following definitions to analyze the problem under competition.

[3] In the model, targets have been capped by the capacities. It can be argued that sales targets should be capped by market demand, because it is not reasonable to use targets higher than market size. However, our model allows for scenarios where the providers use targets higher than the market size and induce the broker to buy and then discard unsold items. We have anecdotal evidence of this behavior happening in practice by online travel agents. Most of the time, agents will use these extra bought capacities as employee benefits. However, in some cases, it is not possible for the broker to discard extra units of the product to reach the sales targets if providers monitor sales and do not allow discarding. In this case, the broker is in a stronger position to argue that the targets should not exceed the market size. Without discarding, the broker's response set is more limited, and one can expect he should be worse off and the providers better off. However, as a counterintuitive result, it is possible to show that the providers' revenues stay fixed or decrease. For an analysis of this scenario, refer to Talebian (2010).

Definition 3.

- We say that provider i has *ample* capacity if $c_i > (p_i/(p_i - m_i))r_i$, and *scarce* capacity otherwise.
- Define $\hat{b}_i = \max[b_i, (p_i/(p_i - m_i))r_i]$ and $\hat{\pi}_i = m_i \hat{b}_i - p_i(\hat{b}_i - b_i)$.

As products become closer substitutes and loyal demands decrease, it is more likely that capacity exceeds loyal demand, intensifying competition for the nonloyal portion of the demand. Also, notice that $p_i/(p_i - m_i)$ decreases when p_i increases or m_i decreases. A lower $p_i/(p_i - m_i)$ implies more intense competition since providers can inflate their demand less. Therefore, as the market price increases or full margins decrease, competition intensifies. We focus on the case that both providers have ample capacity: $c_i > (p_i/(p_i - m_i))r_i$.

The quantities $\hat{b}_i$ and $\hat{\pi}_i$ are equal to b_i and $m_i b_i$, except when $d_0 + d_i < p_i/(p_i - m_i)r_i < c_i$. This corresponds to a situation where a provider can inflate her residual demand to a level higher than her potential demand, $d_0 + d_1$. In this case, $\hat{b}_i = p_i/(p_i - m_i)r_i$ and $\hat{\pi}_i = p_i(d_0 + d_i - r_i)$. The quantity $\hat{\pi}_i$ represents the maximum amount that provider i can and will pay the broker to get prioritized. We label the provider with a higher $\hat{\pi}_i$ as *provider* 1.

Theorem 7. *When commission margins are exogenous, there exists a pure-strategy equilibrium such that*

$$t_1^* = \min\left[c_1, \hat{b}_1 + \frac{\hat{\pi}_1 - \hat{\pi}_2}{p_1 - m_1}\right] \quad \text{and} \quad t_2^* = \frac{p_2}{p_2 - m_2} r_2,$$

which results in revenue split:

$$\pi_1^* = \min[(p_1 - m_1)c_1, p_1 b_1 - \hat{\pi}_2], \qquad \pi_2^* = p_2 r_2,$$
$$\pi_B^* = \max[p_1 b_1 - (p_1 - m_1)c_1, \hat{\pi}_2].$$

Note that each provider sets a sales target at least as high as $p_i r_i/(p_i - m_i)$, the broker reaches only provider 1's target, and provider 2 gets a free ride. Theorem 7 states that the provider with higher $\hat{\pi}$ becomes provider 1, so a larger full commission margin, higher available capacity, and a larger size of loyal market share are all criteria for becoming provider 1 and getting prioritized. We can interpret $\hat{\pi}_i$ as the maximum total commission fee that provider i is ready to pay the broker. As Theorem 7 confirms, the broker assigns the inflated residual demand to provider 2. It also confirms that when providers have ample capacity, discarding is inevitable when $c_1 > d_0 + d_1$, unless $p_1 r_1/(p_1 - m_1) < d_0 + d_1$ and $\hat{\pi}_1 = \hat{\pi}_2$.

It follows from the proof of Theorem 7 that although an equilibrium always exists, it is not necessarily unique. We observe that in certain cases, there are equilibria with and without free riding that generate the same revenue split for the providers and the broker. In other words, providers can attain as much revenue from more sales as they can attain from free riding, assuming selling is advantageous for the broker. We mentioned one equilibrium in the theorem; the set of all equilibria can be found in the proofs.

To be able to investigate the effect of sales targets, we study the case without sales targets and fixed commission margins. If both providers do not require targets, the provider with the higher full commission margin becomes provider 1, and revenue splits as $\pi_1^o = (p_1 - m_1)b_1$, $\pi_2^o = (p_2 - m_2)r_2$, and $\pi_B^o = m_1 b_1 + m_2 r_2$. This is an intuitive result since the broker allocates as much demand as possible to the high margin provider and gives the residual demand to the other provider. In other words, the broker's only criterion to prioritize a provider is a higher margin; loyal market share and capacity are not criteria. As expected, without sales targets, discarding does not happen.

Theorem 8. *In a market with exogenous commission margins,*

- *sales targets affect revenue split unless $c_i \le d_0 + d_i$ for $i = 1, 2$ and $c_1 + c_2 \le d$;*
- *when the sales targets affect revenue split, the profit of the broker decreases;*
- *when the sales targets affect revenue split, the profit of the low margin provider increases unless her capacity is less than her loyal demand;*
- *when the sales targets affect revenue split, the profit of the high margin provider can increase, decrease, or stay the same.*

Theorem 8 summarizes the strategic effects of sales targets and how the total revenue is split between the broker and the providers. The low margin provider wins and the broker loses when targets are used. The fate of the provider with higher commission margin depends on capacities and market structure, and is detailed in the appendix. This result shows that at least one of the providers has an incentive to introduce sales targets.

As the broker becomes more powerful the sales targets start to become effective. This insight can explain the introduction of sales targets in the airline industry: as some travel agents became powerful, capacity providers started experimenting with positive targets to trigger fixed commission margins.

The introduction of sales targets reduces the flexibility of the broker to allocate nonloyal demand among the providers. Without sales targets, if commission margins are equal, then any allocation of nonloyal demand among the providers results in the same profit for the broker. However with targets, even if the total commission fees are equal, the broker is likely to use an extreme point solution to allocate nonloyal demand.

This result is in sharp contrast to the case of endogenous margins, where the larger provider wins, the smaller loses, and the broker's fate depends on the market structure. However, with exogenous margins, the smaller provider wins at the expense of the broker by introducing sales targets. This result provides an explanation to the sales target paradox and justifies the broker's fears of sales target introduction. This justification relies on assuming shallow analytic capabilities of some brokers and not being able to study more complicated effects.

In addition to our method, there may exist other avenues to justify the paradox. One possible explanation may be based on a renegotiation element in contracts, the fact that if the broker and one of the providers find that they are both mutually better off by renegotiating the current contract to a different contract, they will credibly do so. The fact that, in addition to the Nash framework, our results are also valid in the Stackelberg framework may give some confidence that the current equilibria are negotiation proof. For a more detailed analysis about the Stackelberg framework, where the follower chooses her actions after observing the leader's actions, refer to Talebian (2010).

Another possible explanation of the paradox relies on relaxing some of the fixed parameters in our model. In a more general setting, the providers can decide not only about commission margin schedules, but also about their capacities, loyal shares, and prices. We can think of a two-stage game in which providers decide about these parameters in the first stage and then decide about their commission margins. Then, based on equilibrium decisions, it can be investigated whether the broker wins or loses. We expect this analysis can differentiate industries; whereas in some industries larger providers are able to charge higher prices, in some others, larger providers decide to charge lower prices to benefit from their excess capacity.

5. Conclusions and Further Research

We have analyzed a setting in which two providers sell partially substitutable products through a broker and decide on their commission schedules. This analysis enabled us to consider horizontal competition between providers as well as vertical competition between providers and the broker.

We have shown that without loss of generality we can limit the study to schedules where a sales target triggers full commission margins for all units sold. In this setting, we investigated the effect of sales targets on how the revenue splits between providers and the broker. In a competitive market, targets affect how revenue is split between the players. We have demonstrated that in equilibrium the larger provider wins at the expense of the smaller provider. Our results show that the broker either prospers or languishes from the introduction of sales targets, depending on the market structure. Whereas the loss cases are more intuitive, the gains can be explained when providers compete both on targets and commission margins, as the two competition levers increase competition. Comparative statics show that the changes in profit as a result of demand changes are capped by prices, and the equilibrium is smooth to changes in demand. This suggests that our results may be robust to demand misspecifications.

When margins are exogenous, at least one provider benefits from positive sales targets. The case of fixed margins arises in practice when full margins are fixed and the providers start manipulating sales targets. The apparent gains obtained under the assumption of exogenous margins are just a mirage, since this equilibrium is not sustainable because providers then have an incentive to change their commission margins. In other words, the benefits are not sustainable, and there are reversals in fortunes by the introduction of sales targets that would render the long-term tradition of fixed margins and zero sales targets a better solution for at least one of the providers. Comparing equilibria with and without targets under endogenous commission margins shows that imposing sales targets is not Pareto optimal and one of the providers loses.

In both settings, as was expected, the equilibrium depends on the comparative size of the market to capacities and the market price. As observed, sales targets are effective in a competitive market, where supply is large compared to demand. This implies that if the market demand is large relative to the available capacities, then there is no competition between providers, and targets are not effective.

Whereas the providers' revenue may decrease or increase as the broker becomes more powerful, in all cases the broker's revenue increases as his power increases. Although this confirms our intuition, the result is nontrivial, since the providers change their policies as the broker power increases. Another observation corresponds to the broker's revenue change as the demand increases. Growth in market demand is not necessarily beneficial for the broker, since more demand results in less competition and lower commission margins.

As discussed, contracts without sales targets are unstable, and in equilibrium the providers require minimum sales targets, which means that competition can justify the use of sophisticated contracts. In contrast, the main reason to use sophisticated contracts in the operations management literature is to increase coordination, e.g., avoid double marginalization in pricing and newsvendor settings. We pay special attention to the specific case of equal prices. In this case, the total revenue of a competitive market is fixed, and does not depend on targets. It depends only on the capacities and the size of the market, and there are no revenue losses because of decentralized decision making. Our study confirms that competition, without any notion of coordination, justifies requiring targets.

We believe that this work can shed light on the effects of business contracts in a competitive setting. We think our approach to competition may question some of the core insights of traditional models, in particular, the emphasis on negotiation on allocating coordination gains. Negotiation plays an important role in most contracts without competition because usually there is not a unique way to distribute the additional profits gained from the coordination between the chain members. However, in a competitive setting, the gains are divided specifically because of a more limited number of equilibria, and this lowers the effect of negotiation on the outcome of the game.

In our work, competition, not coordination, is the main driver for the use of sophisticated contracts. This result introduces a research setting where coordination and competition coexist. An avenue of research is to make the model more realistic by considering the providers' marginal costs of production, which can be heterogeneous and production volume dependent. Similarly, one can consider the broker's distribution costs. Another setting in which the chain coordination can be investigated is a game where the broker has pricing power and/or should decide the level of his costly sales effort. These extensions would make the model more realistic at the cost of adding significant complexity to a model that already requires intricate analysis.

Also, a stochastic model, where the demand has a probabilistic distribution, can be considered. We have shown that the results change smoothly in regard to demand volume; however, the equilibrium with random demand can be quite different from that of a deterministic model. Finally, the identification of the sales target paradox, the fact that in some settings the broker benefits from sales target introduction but still resists it, has the potential to attract more research attention. We identified a few possibilities in §4.4.

Acknowledgments
The authors benefited greatly from discussions with colleagues at Columbia University; they specifically thank Jay Sethuraman and Ozge Sahin for helpful comments. The authors are also grateful to the department editor, associate editor, and reviewers for helpful comments and suggestions for extending the model.

Appendix

For the proofs, we define $0 \le w_i \le m_i$ as the partial commission margin that provider i pays the broker in the case that he does not reach the target, and δ_i as the number of units of provider i discarded by the broker. Incorporating partial commission margins and discarding, the complete formulations for the broker and the providers with endogenous full commission margins are given by

$$\begin{aligned}
&\pi_B(w_1, w_2, t_1, t_2, m_1, m_2)\\
&\quad = \max_{(s_i, k_i, \delta_i)} \sum_{i=1,2} [(1-k_i)w_i(s_i+\delta_i) + k_i m_i(s_i+\delta_i) - p_i\delta_i],\\
&\qquad r_i \le s_i \le b_i \qquad \text{for } i=1,2,\\
&\qquad k_i t_i \le s_i + \delta_i \le c_i \qquad \text{for } i=1,2,\\
&\qquad s_1 + s_2 \le d,\\
&\qquad k_i \in \{0,1\} \qquad \text{for } i=1,2,\\
&\qquad 0 \le \delta_i \qquad \text{for } i=1,2;
\end{aligned}$$

$$\begin{aligned}
\pi_i(w_j, t_j, m_j) = \max_{(w_i, t_i, m_i)} &[p_i - (1-k_i)w_i - k_i m_i](s_i+\delta_i)\\
&r_i \le t_i \le c_i,\\
&0 \le w_i \le m_i \le p_i.
\end{aligned}$$

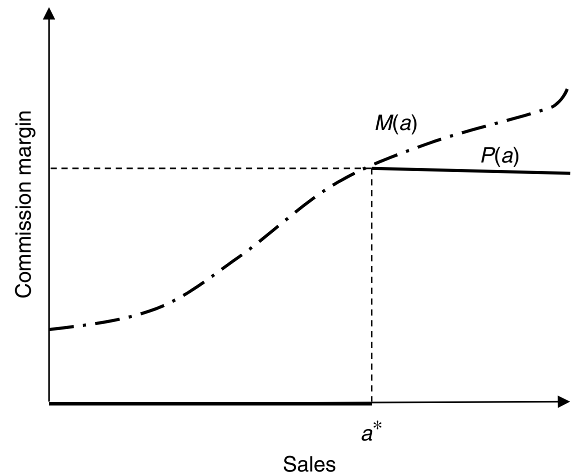
We use $a_i = s_i + \delta_i$ in the proofs to make the notation easier to follow. Notice that a_i equals all units that provider i delivers through the broker, whether sold or discarded.

Proof of Theorem 1. The theorem has two statements, and we prove each in the following bullet points.

• Consider a nonnegative and nondecreasing commission margin function $M(a)$ that results in a sales of a^* and commission $a^*M(a^*)$ for the broker. Function M can have jumps and need not be either left or right continuous. Consider a new schedule $P(a)$ defined by $P(a) = 0$ for $a < a^*$ and $P(a) = M(a)$ for all $a \ge a^*$. Since $P \le M$, the provider is at least as well off under P as under M for any given value of a. We claim that $a = a^*$ is still optimal for the broker under P. Indeed, if $a < a^*$, then the broker makes 0 under the new schedule. But $0 \le aM(a)$, which was rejected by the broker under the old schedule. On the other hand, if $a > a^*$, then the broker makes $aP(a)$ under P, which is less than $aM(a)$, which was rejected by the broker under M. Consequently, $a = a^*$ remains optimal, generating commission $a^*P(a^*) = a^*M(a^*)$ for the broker. Note that broker does not increase or decrease sales because he is facing the same or a smaller margin. Figure A.1 shows the conversion.

• Assume that in equilibrium the broker discards $\delta_i > 0$ units of provider i. Since the unit cost of discarding exceeds the commission margin, it must be that discarding is used to reach the sales target t_i imposed by provider i. Consequently, $a_i = t_i = s_i + \delta_i$, resulting in profit $\pi_i = (p_i - m_i)a_i$ to provider i, and revenue $m_i a_i - p_i\delta_i$ to the broker from provider i. Provider i can modify the contract by setting $t_i' = t_i - \delta_i$ and $m_i' = m_i - (p_i - m_i)\delta_i/s_i$. Under the new contract, the choice $a_i' = t_i'$ brings revenue $m_i't_i' = m_i t_i - p_i\delta_i \ge 0$ to the broker and revenue $(p_i - m_i')a_i' = (p_i - m_i)a_i$ to provider i, so the new contract brings the same revenue to both the broker and provider i if $a_i' = t_i'$. The choice $a_i' < t_i'$ brings zero

Figure A.1 **Converting a Continuous Margin Schedule to a Simple Schedule**



revenue to the broker, which is suboptimal. On the other hand, if the broker selects $a_i > t_i$, this increases the profit of provider i, showing that the new contract is at least as good as the old one for provider i. In other words, the new strategy, which includes a lower commission margin and a lower target, dominates the previous strategy for provider i. □

Proof of Theorem 2. Notice that if $(t_1^*, m_1^*, t_2^*, m_2^*)$ is a Nash equilibrium, then (t_1^*, t_2^*) is a Nash equilibrium for fixed (m_1^*, m_2^*). Consequently, we can use the results of Theorem 7 for the case of exogenous margins to aid us with this proof. To keep the appendix order as in the main text, we will discuss the proof of 7 later, but use its results here. Recall from Definition 3 that $\hat{b}_i = \max[b_i, (p_i/(p_i - m_i))r_i]$ and $\hat{\pi}_i = m_i\hat{b}_i - p_i(\hat{b}_i - b_i)$, which are, respectively, the upper bound on sales for provider i and the revenue for the broker if provider i is given priority. In applying Theorem 7, we assume without loss of generality that both providers have $c_i > p_i r_i/(p_i - m_i)$. This clearly holds for all sufficiently small $m_i \ge 0$. To see that it holds at optimal margins, notice that when the inequality fails, the provider has an incentive to reduce the margin.

We provide the proof in the following steps:

• The provider with higher $\hat{\pi}$ sells her sellable capacity because Theorem 1 states that there is no discarding. By Theorem 7, if this is the provider i, then $\hat{\pi}_i \ge \hat{\pi}_j$ and $s_i^* = \min[c_i, \hat{b}_i + (\hat{\pi}_i - \hat{\pi}_j)/(p_i - m_i)] = b_i$. This implies that in cases where $c_i > b_i = d_0 + d_i$, $s_i^* = \hat{b}_i + (\hat{\pi}_i - \hat{\pi}_j)/(p_i - m_i) = b_i$.

• If $\hat{\pi}_i \ge \hat{\pi}_j$, then provider i earns $(p_i - m_i)b_i$. Otherwise, she earns $p_i(d - b_j)$. Provider i is has incentive to increase $\hat{\pi}_i$ over the competitor as long as $p_i(d - b_j) < (p_i - m_i)b_i$ or $m_i b_i < p_i(b_1 + b_2 - d)$.

• We claim that $\hat{\pi}_1 = \hat{\pi}_2$. Otherwise, it is optimal for the provider with higher $\hat{\pi}$ to decrease her commission margin and increase her revenue. This follows because, based on the first step, the provider with highest $\hat{\pi}$, say, provider i, sells b_i, and her revenue is $(p_i - m_i)b_i$, which can be increased by decreasing m_i.

• In the previous step, we demonstrated that provider i has an incentive to increase π_i as long as $m_i b_i \le p_i(b_1 + b_2 - d)$. The inequality $m_i b_i \le p_i(b_1 + b_2 - d)$ and the fact that

Table A.1 Sensitivity Analysis

	$\partial\pi_i/\partial d$	$\partial\pi_B/\partial d$	$\partial\pi_i/\partial d_0$	$\partial\pi_B/\partial d_0$
$c_1 \le d_0+d_1$, $c_2 \le d_0+d_2$	$\min[p_1,p_2]$	$-\min[p_1,p_2]$	0	0
$c_1 > d_0+d_1$, $c_2 > d_0+d_2$	$(d_i/d)p_i+(d_0/d)(p_i-p_j)^+$	$(d_0/d)\min[p_1,p_2]$	$-p_i[0,1]+(p_i-p_j)^+$	$\min[p_1,p_2]$
$c_i \le d_0+d_i$, $c_j > d_0+d_j$	$(d_i/d)\min[p_1,p_2]$	$(d_i/d)\min[p_1,p_2]$	$[-\min[p_i,p_j],0]$	$[0,\min[p_1,p_2]]$
$c_i > d_0+d_i$, $c_j \le d_0+d_j$	$p_i-(d_i/d)(p_i-p_j)^+$	$(d_i/d)\min[p_1,p_2]$	$[-(p_i-p_j)^+,0]$	$[0,\min[p_1,p_2]]$

$\hat{\pi}_1=\hat{\pi}_2$ imply that $m_ib_i=\min[p_1,p_2](b_1+b_2-d)$ or $m_i=\min[p_1,p_2]((b_1+b_2-d)/b_i)$.

- Setting $m_i=\min[p_1,p_2]((b_1+b_2-d)/b_i)$ in Theorem 7 results in $t_i=b_i$ as claimed.

Note that in this case, the provider with the highest price becomes primary because he can offer more to the broker for nonloyal demand. Therefore, if $p_i>p_j$, then $\pi_i=(p_i-p_j((b_1+b_2-d)/b_i))b_i$, $\pi_j=p_j(d-b_i)$, and $\pi_B=p_j(b_1+b_2-d)$.

At end of the proof, we also provide the sensitivity analysis based on the results of Theorem 2. As we claimed in this paper, Table A.1 shows that changes in providers' and the broker's profits as a result of demand changes are linear and capped by prices.

As the demand increases,
- $\partial\pi_i/\partial d\in[0,p_i]$;
- $\partial\pi_B/\partial d\in[-\min[p_1,p_2],\min[p_1,p_2]]$.

As the broker's power increases,
- $\partial\pi_i/\partial d_0\in[-\max[\min[p_1,p_2],(p_i-p_j)^+],(p_i-p_j)^+]$;
- $\partial\pi_B/\partial d_0\in[0,\min[p_1,p_2]]$. □

Proof of Theorem 3. If provider i sells directly to loyal customers, she collects all the revenue from them, which equals p_id_i. To analyze the revenue from selling through the broker, it should be noticed that the total demand that is sold through the broker has been reduced to $d-d_i$. By Theorem 2, the revenue of provider i from the broker's sales of nonloyal demand is $p_i((d-d_i)-b_j)+(p_i-p_j)^+((b_i-d_i)+b_j-(d-d_i))$. This means that

$$\begin{aligned}\pi_i^d&=p_id_i+p_i(d-d_i-b_j)+(p_i-p_j)^+(b_i+b_j-d)\\&=p_i(d-b_j)+(p_i-p_j)^+(b_i+b_j-d)=\pi_i^*.\quad\square\end{aligned}$$

Proof of Theorem 4. Notice that providers can guarantee an income equal to p_ir_i by taking the residual demand. Consequently, margins must be selected so $(p_i-m_i)b_i\ge p_ir_i$, which implies

$$m_i\le\frac{b_1+b_2-d}{b_i}p_i.$$

Since each provider's margin needs to increase just to be higher than the other provider's margin, the cap of margins is the minimum of the two caps. Since we labeled the provider with higher p_i/b_i provider 1,

$$\frac{p_2}{b_2}\le\frac{p_1}{b_1}\quad\Rightarrow\quad m_i\le\frac{b_1+b_2-d}{b_2}p_2.$$

This means that provider 1 never chooses a margin in the interval $(((b_1+b_2-d)/b_2)p_2,((b_1+b_2-d)/b_1)p_1]$ because they are dominated by $((b_1+b_2-d)/b_2)p_2$, which is high enough to beat provider 2.

In this case, there does not exist any pure-strategy Nash equilibrium, because both players' commissions adjust up to $((b_1+b_2-d)/b_2)p_2$, and then the first provider jumps down to 0, resulting in the second provider also decreasing her commission margin. Therefore, only a mixed-strategy equilibrium exists. Since the providers should be indifferent among all strategies in the support set of a mixed strategy, the revenues in the support set should be equal; provider j should be indifferent to offering margin $m\in[0,((b_1+b_2-d)/b_2)p_2]$:

$$\begin{aligned}&P(m_i^o\le m)(p_j-m)b_j+P(m_i^o>m)(p_j-m)(d-b_i)\\&\quad=\left(p_j-\frac{b_1+b_2-d}{b_1}p_1\right)b_j.\end{aligned}$$

Solving for $F_i(m)=P(m_i^0\le m_i)$, we obtain

$$\begin{aligned}&F_i(m)(p_j-m)b_j+(1-F_i(m))(p_j-m)(d-b_i)\\&\quad=[b_ip_j-b_jp_i]^+\frac{b_1+b_2-d}{b_i}+p_j(d-b_i),\\&F_i(m)=\frac{1}{p_j-m}\left[\frac{(p_jb_i-p_ib_j)^+}{b_i}+\frac{m(d-b_i)}{b_1+b_2-d}\right].\end{aligned}$$

Also,

$$\begin{aligned}&P(m_1^o<m_2^o)\\&=\int_0^{((b_1+b_2-d)/b_1)p_1}F_1(m)f_2(m)\,dm\\&=\int_0^{((b_1+b_2-d)/b_1)p_1}\frac{1}{p_2-m}\left(\frac{p_2b_1-p_1b_2}{b_1}+\frac{m(d-b_1)}{b_1+b_2-d}\right)\\&\quad\cdot\frac{d-b_2}{b_1+b_2-d}\frac{p_1}{(p_1-m)^2}\,dm\\&=\int_0^{((b_1+b_2-d)/b_1)p_1}\frac{1}{p_2-m}\left(\frac{p_2b_1-p_1b_2}{b_1}+\frac{m(d-b_1)}{b_1+b_2-d}\right)\\&\quad\cdot\frac{d-b_2}{b_1+b_2-d}\frac{p_1}{(p_1-m)^2}\,dm\\&=\frac{p_2b_1-p_1b_2}{b_1}\frac{(d-b_2)p_1}{b_1+b_2-d}\int_0^{((b_1+b_2-d)/b_1)p_1}\frac{1}{(p_2-m)(p_1-m)^2}\,dm\\&\quad+\frac{(d-b_1)p_1}{b_1+b_2-d}\frac{d-b_2}{b_1+b_2-d}\int_0^{((b_1+b_2-d)/b_1)p_1}\frac{m}{(p_2-m)(p_1-m)^2}\,dm\\&=\frac{(p_2b_1-p_1b_2)(d-b_2)p_1}{b_1(b_1+b_2-d)}\\&\quad\cdot\frac{p_2-p_1+(p_1-m)\ln((p_1-m)/(p_2-m))}{(p_1-p_2)^2(p_1-m)}\Bigg|_0^{((b_1+b_2-d)/b_1)p_1}\\&\quad+\frac{(d-b_1)(d-b_2)p_1}{(b_1+b_2-d)^2}\end{aligned}$$

Table A.2 Strategic Effects

		Before targets (π^o)	After targets (π^*)	Comparison
$p_1 \le p_2$	1	$p_1(d-b_2)$	$p_1(d-b_2)$	Unchanged
	2	$p_2(d-b_1)+\big(p_2-(b_2/b_1)p_1\big)(b_1+b_2-d)$	$p_2(d-b_1)+(p_2-p_1)(b_1+b_2-d)$	Lost
$p_1 > p_2$	1	$p_1(d-b_2)$	$p_1(d-b_2)+(p_1-p_2)(b_1+b_2-d)$	Win
$(b_1/p_1) > (b_2/p_2)$	2	$p_2(d-b_1)+\big(p_2-(b_2/b_1)p_1\big)(b_1+b_2-d)$	$p_2(d-b_1)$	Lost
$p_1 > p_2$	1	$p_1(d-b_2)+\big(p_1-(b_1/b_2)p_2\big)(b_1+b_2-d)$	$p_1(d-b_2)+(p_1-p_2)(b_1+b_2-d)$	Win
$(b_1/p_1) \le (b_2/p_2)$	2	$p_2(d-b_1)$	$p_2(d-b_1)$	Unchanged

$$\cdot \left.\frac{p_1(p_2-p_1)+p_2(p_1-m)\ln((p_1-m)/(p_2-m))}{(p_1-p_2)^2(p_1-m)}\right|_0^{((b_1+b_2-d)/b_1)p_1}$$

$$= \frac{(p_2b_1-p_1b_2)(d-b_2)p_1}{b_1(b_1+b_2-d)}\left[\frac{b_1+b_2-d}{(p_2-p_1)(d-b_2)p_1} - \frac{\ln((d-b_2)p_2/(b_1p_2-(b_1+b_2-d)p_1))}{(p_1-p_2)^2}\right]$$

$$+\frac{(d-b_1)(d-b_2)p_1}{(b_1+b_2-d)^2}\left[\frac{b_1+b_2-d}{(p_2-p_1)(d-b_2)} - \frac{p_2\ln((d-b_2)p_2/(b_1p_2-(b_1+b_2-d)p_1))}{(p_1-p_2)^2}\right]$$

$$= \frac{p_2b_1-p_1b_2}{b_1(p_2-p_1)} + \frac{(d-b_1)p_1}{(b_1+b_2-d)(p_2-p_1)} - \left[\frac{(d-b_1)(d-b_2)p_1p_2}{(b_1+b_2-d)^2} + \frac{(p_2b_1-p_1b_2)(d-b_2)p_1}{b_1(b_1+b_2-d)}\right] \cdot \frac{\ln((d-b_2)p_2/(b_1p_2-(b_1+b_2-d)p_1))}{(p_1-p_2)^2}.$$

Total revenue of the chain is equal to

$$\begin{aligned}
\pi_{\text{chain}} &= P(m_1^o < m_2^o)(p_1r_1+p_2b_2) + P(m_2^o < m_1^o)(p_1b_1+p_2r_2)\\
&= p_1b_1+p_2(d-b_1)+(b_1+b_2-d)(p_2-p_1)P(m_1<m_2)\\
&= p_1b_1+p_2(d-b_1)+\frac{(p_2b_1-p_1b_2)(b_1+b_2-d)}{b_1}\\
&\quad +(d-b_1)p_1\\
&\quad -\left[\frac{(d-b_1)(d-b_2)p_1p_2}{(b_1+b_2-d)} + \frac{(p_2b_1-p_1b_2)(d-b_2)p_1}{b_1}\right]\\
&\quad \cdot\frac{\ln((d-b_2)p_2/(b_1p_2-(b_1+b_2-d)p_1))}{p_1-p_2}\\
&= p_1d+p_2b_2-\frac{p_1b_2(b_1+b_2-d)}{b_1}\\
&\quad -\frac{(d-b_2)p_1}{p_1-p_2}\left[\frac{(d-b_1)p_2}{(b_1+b_2-d)} + \frac{(p_2b_1-p_1b_2)}{b_1}\right]\\
&\quad \cdot\ln\frac{(d-b_2)p_2}{b_1p_2-(b_1+b_2-d)p_1},
\end{aligned}$$

$$\begin{aligned}
\pi_B &= \pi_{\text{chain}} - \pi_1 - \pi_2\\
&= p_1b_2 - \frac{(d-b_2)p_1}{p_1-p_2}\left[\frac{(d-b_1)p_2}{(b_1+b_2-d)} + \frac{(p_2b_1-p_1b_2)}{b_1}\right]\\
&\quad \cdot\ln\frac{(d-b_2)p_2}{b_1p_2-(b_1+b_2-d)p_1}.
\end{aligned}$$

If $p_1 = p_2 = p$, then the $(1/(p_1-p_2))\ln((d-b_2)p_2/(b_1p_2-(b_1+b_2-d)p_1))$ is undefined. We find the limit, using L'Hospital's rule, which results in

$$\pi_B = p\frac{b_2}{b_1}(b_1+b_2-d). \quad \square$$

Proof of Theorem 5. The providers' revenue comparison before and after targets are presented in Table A.2, assuming $b_1 > b_2$. If $b_1 = b_2$, the providers' revenues do not change.

For the broker, consider the case where $b_1 \ge b_2$ and $p_1 \le p_2$; then

$$\begin{aligned}
\pi_B^o &= p_1b_2 + \frac{p_1}{p_2-p_1}(d-b_2)\left(p_2\frac{b_2}{b_1+b_2-d} - p_1\frac{b_2}{b_1}\right)\\
&\quad \cdot\ln\frac{p_2(d-b_2)}{b_1p_2-(b_1+b_2-d)p_1}\\
&\le p_1b_2 + \frac{p_1}{p_2-p_1}(d-b_2)\left(p_2\frac{b_2}{b_1+b_2-d} - p_1\frac{b_2}{b_1}\right)\\
&\quad \cdot\frac{-(p_2-p_1)(b_1+b_2-d)}{b_1p_2-(b_1+b_2-d)p_1} \quad \text{(because of } \ln(x) \le x-1)\\
&= p_1b_2 + p_1(d-b_2)\left(-p_2\frac{b_2}{b_1p_2-(b_1+b_2-d)p_1}\right.\\
&\quad \left.+p_1\frac{b_2}{b_1}\frac{b_1+b_2-d}{b_1p_2-(b_1+b_2-d)p_1}\right)\\
&\le p_1b_2 + p_1(d-b_2)\left(-p_2\frac{b_2}{b_1p_2-(b_1+b_2-d)p_2}\right.\\
&\quad \left.+p_2\frac{b_2}{b_1}\frac{b_1+b_2-d}{b_1p_2-(b_1+b_2-d)p_2}\right) \quad (p_1 \le p_2)\\
&= p_1\frac{b_2}{b_1}(b_1+b_2-d)\\
&\le p_1(b_1+b_2-d) \quad (b_2 \le b_1)\\
&= \min[p_1,p_2](b_1+b_2-d) = \pi_B^*. \quad \square
\end{aligned}$$

We need several lemmas to establish the remaining theorem. We first study the case where both partial and full commission margins $0 \le w \le m$ are exogenous, and providers decide only sales targets. The following lemma looks at the single provider case.

Lemma 1. *Under the assumption that commission margins are exogenous, an optimal sales target for the single provider* *is given by* $t^* = \min[c, ((p-w)/(p-m))d]$, *which results in sales*

$$s^* = b, \quad \delta^* = t^* - b,$$

and revenue split

$$\pi^* = \min[(p-m)c, (p-w)d], \quad \pi_B^* = \max[mc - p(c-d), wd].$$

Proof. Recall that $b = \min(c, d)$, where d is the total demand. Since $w \le m$, the form of t^* implies that $\delta^* = t^* - b \ge 0$. The broker prefers to reach the target whenever $mt - p(t-d)^+ \ge w\min[t, d]$ by setting $a = t$. Since the provider earns $(p-m)a$, it is optimal for the provider to choose the largest $t \le c$, which satisfies the above inequality. Therefore, the provider chooses $t^* = \min[c, ((p-w)/(p-c))d]$, and earns $(p-m)t^* = \min[(p-m)c, (p-w)d]$, which leaves the broker with the remaining revenue of $pd - \min[(p-m)c, (p-w)d] = \max[pd - (p-m)c, wd]$.

If $c > ((p-w)/(p-m))d$, then the provider also can set higher targets $t^* \in (((p-w)/(p-m))d, c]$. In this case, the broker does not reach the target and sets $a^* = d$, so the provider gets a free ride. However, this does not change the revenue split.

If $c < d$, then setting a target is not important for the provider, and $t^* \in [0, c]$. □

Definition 4. With exogenous margins, we say that provider i has *ample* capacity if $c_i > ((p_i - w_i)/(p_i - m_i))r_i$, and has *scarce* capacity otherwise.

Lemma 2. *Suppose that provider* 1 *has scarce capacity. Then there exists a pure-strategy equilibrium such that* $t_1^* = c_1$ *and* $t_2^* = \min[c_2, ((p_2 - w_2)/(p_2 - m_2))r_2]$, *which results in sales*

$$s_1^* = b_1, \quad \delta_1^* = c_1 - b_1, \quad s_2^* = r_2,$$

$$\delta_1^* = \min\left[c_2, \frac{p_2 - w_2}{p_2 - m_2} r_2\right] - r_2$$

and revenue split

$$\pi_1^* = (p_1 - m_1)c_1 \quad \pi_2^* = \min[(p_2 - m_2)c_2, (p_2 - w_2)r_2],$$

$$\pi_B^* = m_1c_1 - p_1(c_1 - b_1) + \max[m_2c_2 - p_2(c_2 - r_2), w_2r_2].$$

Proof. Notice that $c_1 \le ((p_1 - w_1)/(p_1 - m_1))r_1 \Rightarrow w_1r_1 \le m_1c_1 - p_1(c_1 - r_1)$, and hence the broker is always better off reaching the first provider's target. Since $\pi_1 = (p_1 - m_1)s_1 = (p_1 - m_1)t_1$, provider 1 sets t_1 as high as possible, i.e., to $t_1^* = c_1$. This means the broker can maximize his revenue by selling $a_1^* = t_1^* = c_1$. After selling the first provider's capacity, the second provider faces a monopolistic market of size r_2, so based on Lemma 1, $t_2^* = \min[c_2, ((p_2 - w_2)/(p_2 - m_2))r_2]$. □

Lemma 3. *Consider a case where both providers have ample capacity with exogenous commission margins. Then, w.l.o.g., we can limit the strategy of provider* i *to* $t_i > ((p_i - w_i)/(p_i - m_i))r_i$. *In addition, the broker reaches at most one of the providers' targets in equilibrium, and if he reaches the target of provider* i, *then* $s_i + \delta_i \ge b_i$.

Proof. W.l.o.g., $t_i > ((p_i - w_i)/(p_i - m_i))r_i$:

Since the providers have ample capacity, provider i can guarantee a minimum revenue of $(p_i - w_i)r_i$ by setting $t_i > ((p_i - w_i)/(p_i - m_i))r_i$. If provider i chooses $t_i \le ((p_i - w_i)/(p_i - m_i))r_i$ and the broker reaches her target by setting $a_i = t_i$, her revenue is given by $(p_i - m_i)t_i$, which is less than or equal to the minimum guaranteed, $(p_i - w_i)r_i$, and this implies that $t_i > ((p_i - w_i)/(p_i - m_i))r_i$. Notice that the setting sales target to $((p_i - w_i)/(p_i - m_i))r_i$ may be optimal in some cases, but always there exists a $t_i > ((p_i - w_i)/(p_i - m_i))r_i$ that brings the same revenue.

The broker never reaches both providers' targets in equilibrium. Label the provider with higher price p_1. Notice the following:

- Reaching both targets, ($t_1 \le a_1$, $t_2 \le a_2$):

$$\begin{aligned}\pi_B &= p_1s_1 - (p_1 - m_1)a_1 + p_2s_2 - (p_2 - m_2)a_2 \\ &< p_1s_1 - (p_1 - m_1)a_1 + p_2s_2 - (p_2 - w_2)r_2 \\ &= p_1s_1 - (p_1 - m_1)a_1 + p_2(s_2 - r_2) + w_2r_2 \\ &= p_1b_1 - (p_1 - m_1)a_1 + w_2r_2.\end{aligned}$$

- Reaching the target of provider 1, ($t_1 \le a_1, a_2 < t_2$): In this case, one option of the broker is setting $a_1 = \max[t_1, b_1]$ and $a_2 = t_2$. This option's profit is the lower bound for the broker's optimal profit:

$$\pi_B \ge p_1b_1 - (p_1 - m_1)(\max[t_1, b_1]) + w_2r_2.$$

To prove the claim that the broker reaches at most one of the providers' targets in equilibrium by contradiction, assume that reaching both targets is better than reaching only the target of provider 1 in equilibrium. Then,

$$\begin{aligned}&p_1b_1 - (p_1 - m_1)a_1 + w_2r_2 \\ &\quad > p_1b_1 - (p_1 - m_1)(\max[t_1, b_1]) + w_2r_2 \\ &\Rightarrow \; a_1 < \max[t_1, b_1] \\ &\Rightarrow \; a_1 < b_1 \quad (\text{because } t_1 \le a_1).\end{aligned}$$

In this case, provider 1 has an incentive to increase her target to more than her current sales and still be sure that the broker reaches her target. It shows that the current point is not an equilibrium and results in a contradiction.

W.l.o.g., we can assume that in equilibrium $a_i \ge t_i \Rightarrow a_i \ge b_i$.

Assume for a contradiction that $t_i \le a_i < b_i$. Since provider i has responded optimally, if she increases t_i up to $a_i + \epsilon$, the broker does not reach her target. One of the following cases must happen:

- The broker reaches none of the targets, $m_ia_i + w_j(d - a_i) = \max[w_ir_i + w_jb_j, w_ib_i + w_jr_j] = w_ir_i + w_jb_j$.

Provider j can put $t_j' = ((p_j - w_j)/(p_j - m_j))b_j$ and be sure that the broker reaches her target, because $m_jl_j' - p_j(t_j' - b_j) + w_ir_i = w_jb_j + w_ir_i = m_ia_i + w_j(d - a_i)$. This means the current point is not an equilibrium, noting $(p_j - w_j)b_j > (p_j - w_j) \cdot (d - a_i)$.

- The broker reaches the target of provider j, $m_ia_i + w_j(d - a_i) = w_ir_i + m_j\max[t_j, b_j] - p_j(t_j - b_j)^+$.

This means the broker is indifferent between satisfying t_i by selling a_i and satisfying t_j by selling $\max[t_j, b_j]$, and both are equilibria. □

Definition 5. Let $\hat{b}_i = \max[b_i, ((p_i - w_i)/(p_i - m_i))r_i]$ and $\hat{\pi}_i = m_i\hat{b}_i - p_i(\hat{b}_i - b_i) + w_jr_j$.

One can think of $\hat{\pi}_i$ as the revenue cap for the broker, conditioned on reaching the target of provider i. Lemma 4 studies the duopoly case where both providers have ample capacity, based on the results on Lemma 3 and Definition 5. We label the provider with the higher $\hat{\pi}_i$ *provider* 1.

Lemma 4. *Consider the case where both providers have ample capacity with exogenous commission margins. Then there exists a pure-strategy equilibrium such that* $t_1^* = \min[c_1, \hat{b}_1 + (\hat{\pi}_1 - \max[\hat{\pi}_2 - w_2 r_2, w_1 b_1])/(p_1 - m_1)]$ *and* $t_2^* = ((p_2 - w_2)/(p_2 - m_2))r_2$, *which results in sales*

$$s_1^* = b_1, \quad \delta_1^* = t_1^* - b_1, \quad s_2^* = r_2, \quad \delta_2^* = 0$$

and revenue split

$$\pi_1^* = \min[(p_1 - m_1)c_1, p_1 b_1 - \max[\hat{\pi}_2 - w_2(d - b_1), w_1 b_1]],$$
$$\pi_2^* = (p_2 - w_2) r_2,$$
$$\pi_B^* = \max[m_1 c_1 - p_1(c_1 - b_1), \hat{\pi}_2 - w_2 r_2, w_1 b_1] + w_2 r_2.$$

Proof. We define π_{Ri} as the broker's revenue from provider i if he reaches her target. Based on Lemma 3, the broker does not reach both targets in equilibrium; the broker chooses one of the four actions after observing the providers' targets:

$$\pi_B = \max[\pi_{R1}, \pi_{R2}, w_1 b_1 + w_2 r_2, w_1 r_1 + w_2 b_2].$$

The first two alternatives correspond to reaching the target of the first or the second provider. The last two alternatives correspond to not reaching any of the targets and giving priority to the first or the second provider. On the basis of Lemma 3, we know that in equilibrium, $a_i < t_i$ or $a_i \geq \hat{b}_i$. We now show that $\pi_{Ri} \leq \hat{\pi}_i$. Since if $a_i \geq t_i$, then $a_i \geq \hat{b}_i$, the broker's revenue from provider i is capped by $m_i \hat{b}_i - p_i(\hat{b}_i - b_i) = m_i b_i - (p_i - m_i)(\hat{b}_i - b_i)$, and his revenue from provider j is capped by $w_j r_j$. Therefore, his total revenue after reaching t_i is capped by $\hat{\pi}_i$.

Now, in response to provider i, provider j should decide about t_j, which determines π_{Rj}. Provider j has two options: fight back to have a higher $\hat{\pi}$ or allow provider i to have a higher $\hat{\pi}$, which correspond, respectively, to

1. $\pi_{Rj} \geq \max[\pi_{Ri}, w_1 b_1 + w_2 r_2, w_1 r_1 + w_2 b_2]$, $\pi_i = (p_i - w_i) r_i$, $\pi_j = p_j b_j - \pi_{Rj}$;
2. $\pi_{Rj} < \max[\pi_{Ri}, w_1 b_1 + w_2 r_2, w_1 r_1 + w_2 b_2]$, $\pi_i = p_i b_i - \pi_{Ri}$, $\pi_j = (p_j - w_j) r_j$.

It is easy to show that the strategy 1 is always more beneficial for provider j, and hence she always chooses strategy 1 if she can. This implies that providers i and j set π_{Ri} and π_{Rj} as high as is necessary to beat the other provider. Therefore, the result of the game will depend on the relation between $\hat{\pi}_i$ and $\hat{\pi}_j$. Since we have labeled provider 1 to have the highest revenue cap, $\hat{\pi}_1 \geq \hat{\pi}_2$, we have

$$\pi_{R1} \geq \max[\pi_{R2}, w_1 b_1 + w_2 r_2, w_1 r_1 + w_2 b_2]$$
$$\Rightarrow \quad \pi_{R1} \geq \max[\hat{\pi}_2, w_1 b_1 + w_2 r_2] \quad (w_1 r_1 + w_2 b_2 \leq \hat{\pi}_2)$$
$$\Rightarrow \quad p_1 b_1 - (p_1 - m_1) t_1 + w_2 r_2 \geq \max[\hat{\pi}_2, w_1 b_1 + w_2 r_2]$$
$$\Rightarrow \quad t_1 \leq \frac{p_1 b_1 - \max[\hat{\pi}_2 - w_2 r_2, w_1 b_1]}{p_1 - m_1}$$
$$\Rightarrow \quad t_1 = \min\left[c_1, \frac{p_1 b_1 - \max[\hat{\pi}_2 - w_2 r_2, w_1 b_1]}{p_1 - m_1}\right].$$

After selling the capacity of provider 1, the second provider faces a monopolistic market of size r_2, so by Lemma 1, $t_2^* = \min[c_2, ((p_2 - w_2)/(p_2 - m_2)) r_2]$. □

Theorem 6 studies the case where only full commission margins, m_1 and m_2, are fixed, and providers can decide t and w. In this case, we show that without loss of generality the providers can limit the analysis to $w = 0$. More precisely, we show that for any equilibrium with positive w, there is one with $w = 0$. We need the following lemma before proving Theorem 6. We use $t_i(w_i, w_j)$ to denote the optimal target for given partial commission margins, which can be found based on Lemmas 2 and 4.

Lemma 5. *Under the assumption that full commission margins are exogenous, the following inequalities hold*:

$$\pi_1(0, w_2, t_1(0, w_2), t_2(0, w_2))$$
$$\geq \pi_1(w_1, w_2, t_1(w_1, w_2), t_2(w_1, w_2)),$$
$$\pi_1(w_1, w_2, t_1, t_2(w_1, w_2)) \geq \pi_1(w_1, w_2, t_1, t_2(0, w_2)).$$

Proof. The first inequality can be proved by noticing that if provider 1 does not have ample capacity with $w_1 > 0$, she would not have ample capacity with $w_1 = 0$. Based on Definition 5, $\hat{\pi}_i$ is a function of (w_i, w_j). Since $\hat{\pi}_1(w_1, w_2) \leq \hat{\pi}_1(0, w_2)$ and $\hat{\pi}_2(w_1, w_2) \geq \hat{\pi}_2(0, w_2)$, if the first provider gets priority with (w_1, w_2), she also gets priority with $(0, w_2)$. On the basis of these observations, by considering all different cases and some algebra, it can be shown that the lemma is true in each case. For a detailed table, refer to Talebian (2010).

The proof of the second inequality is similar to the first one, noting that $a_2(w_1, w_2) \leq a_2(0, w_2)$ and $t_2(w_1, w_2) \leq t_2(0, w_2)$. □

Proof of Theorem 6. Based on the Lemma 4, we know $t_1(w_1, w_2)$ and $t_2(w_1, w_2)$ are Nash equilibrium strategies, when (w_1, w_2) are fixed. Now, we find Nash equilibria when (w_i, t_i) are chosen simultaneously. First note that any Nash equilibrium for the two-dimensional game is in the format $(w_1, w_2, t_1(w_1, w_2), t_2(w_1, w_2))$. If $(w_1^*, w_2^*, t_1^*, t_2^*)$ is a Nash equilibrium for the two-dimensional simultaneous game, then, (t_1^*, t_2^*) are equilibrium strategies for a one-dimensional game in which (w_1, w_2) are fixed at w_i^*, and players only choose (t_1, t_2). On the basis of this necessary condition, we can take advantage of Lemma 4, as if $(w_1^*, w_2^*, t_1^*, t_2^*)$ is a Nash equilibrium, then (t_1^*, t_2^*) is a Nash equilibrium if $w_1 = w_1^*$ and $w_2 = w_2^*$.

Now, to justify that we can limit our study to find a Nash equilibrium of the two-dimensional game to the case where $w_1 = w_2 = 0$, we show the following:

- $(0, 0, t_1(0, 0), t_2(0, 0))$ is a Nash equilibrium:

$$\pi_1(0, 0, t_1(0, 0), t_2(0, 0))$$
$$\geq \pi_1(w_1, 0, t_1(w_1, 0), t_2(w_1, 0)) \quad \text{(part 1 of Lemma 5)}$$
$$\geq \pi_1(w_1, 0, t_1, t_2(w_1, 0)) \quad (t_1(w_1, 0) \text{ is the best response})$$
$$\geq \pi_1(w_1, 0, t_1, t_2(0, 0)) \quad \text{(part 2 of Lemma 5).}$$

- In any other Nash equilibrium, the players do not earn more. Using the lemma above, it is enough to show that $\pi_i(w_1, t_1(w_1, w_2), w_2, t_2(w_1, w_2)) \leq \pi_i(0, t_1(0, 0), 0, t_2(0, 0))$ for $i = 1, 2$ and $\forall\, 0 \leq w_i \leq m_i$.

The proof is similar to that for Lemma 5.

• In any other Nash equilibrium, one player can deviate to zero without losing:

$$\pi_1(w_1, w_2, t_1(w_1, w_2), t_2(w_1, w_2))$$
$$\le \pi_1(0, w_2, t_1(0, w_2), t_2(0, w_2)) \text{ (part 1 of Lemma 5)}$$
$$\le \pi_1(0, w_2, t_1(0, w_2), t_2(w_1, w_2)) \text{ (part 2 of Lemma 5).} \quad \square$$

Notice that Theorem 6 investigates the two-dimensional game, where the providers can change both the lower commission margins and the targets. Theorem 6 states that, without loss of optimality, we can limit our study to cases with zero lower commission margins, and this allows us to reduce the two-dimensional strategy space (w_i, t_i) to a one-dimensional space t_i. On the basis of this fact, we can use Lemmas 1, 2, and 4 for the special case that $w_i = 0$.

Proof of Theorem 7. The proof follows from Lemma 4 when $w_i = 0$ for $i = 1, 2$.

The case without targets follows from Lemma 4 applied to $w_i = m_i$. $\square$

Proof of Theorem 8. We prove each of the theorem's statements in the following bullet points.

• If $c_i \le d_0 + d_i$ and $c_1 + c_2 \le d$, there will not be any competition between providers, and $a_i^* = c_i$.

• The broker loss can be confirmed through the formulation. Note that any feasible solution for the with-targets scenario is a solution or can be capped by a solution for the without-targets scenario.

$$\bullet\ c_2 < d_2 \quad \Rightarrow \quad a_2^* = c_2 \quad \text{(with and without targets)},$$
$$d_2 < c_2 \quad \Rightarrow \quad a_2^* = r_2 \quad \text{(without targets)},$$
$$a_2^* \ge \min\left[\frac{p_2}{p_2 - m_2} r_2, c_2\right] \quad \text{(with targets)},$$

• We have $a_1^* = b_1$ without sales targets. Now, we investigate what happens with sales targets. The following cases can happen:

—Both providers have ample capacity

∘ $\hat{\pi}_1 \ge \hat{\pi}_2$: $a_1^* = \min[c_1, \hat{b}_1 + (\hat{\pi}_1 - \hat{\pi}_2)/(p_1 - m_1)]$

• $d_0 + d_1 < c_1$: $a_1^* > b_1$ unless $\hat{\pi}_1 = \hat{\pi}_2$ and $(p_1/(p_1 - m_1))r_1 < d_0 + d_1$

• $d_0 + d_1 > c_1$: $a_1^* = b_1$

∘ $\hat{\pi}_1 \le \hat{\pi}_2$: in this case, it needs to be $a_1^* = (p_1/(p_1 - m_1))r_1 < d_0 + d_1$, so the provider loses

—At least one of the providers does not have ample capacity

∘ $d_0 + d_1 < c_1$: $a_1^* = \min[c_1, (p_1/(p_1 - m_1))r_1] > b_1$ if $d_0 + d_1 < (p_1/(p_1 - m_1))r_1$; otherwise, it is lower

∘ $c_1 \le d_0 + d_1$: if $c_1 \le (p_1/(p_1 - m_1))r_1$, then $a_1^* = c_1 = b_1$; if $c_1 > (p_1/(p_1 - m_1))r_1$, then $a_1^* = (p_1/(p_1 - m_1))r_1 < b_1$. $\square$

References

Alamdari F (2002) Regional development in airlines and travel agents relationship. *J. Air Transport Management* 8(5): 339–348.

Cachon GP (2003) Supply chain coordination with contracts. de Kok AG, Graves SC, eds. *Handbooks in Operations Research and Management Science: Supply Chain Management: Design, Coordination and Operation* (North-Holland, Amsterdam), 229–340.

Cachon GP, Kok AG (2010) Competing manufacturers in a retail supply chain: On contractual form and coordination. *Management Sci.* 56(3):571–589.

Hax AC, Candea D (1984) *Productiona and Inventory Management* (Prentice-Hall, Inc., Englewood Cliffs, NJ).

Jeuland AP, Shugan SM (1983) Managing channel profits. *Marketing Sci.* 2(3):239–272.

Kinter EW (1970) *A Robinson-Patman Primer* (Macmillan, New York).

Lariviere MA (1999) Supply chain contracting and coordination with stochastic demand. Ganeshan R, Tayur S, Magazine M, eds. *Quantitative Models of Supply Chain Management* (Kluwer Academic Publishers, Boston), 233–268.

Spengler JJ (1950) Vertical integration and anti-trust policy. *J. Political Econom.* 58(4):347–352.

Talebian M (2010) Essays on pricing and contract theory. Ph.D. dissertation, Columbia University, New York.

Talluri KT, van Ryzin GJ (2004) *The Theory and Practice of Revenue Management* (Kluwer Academic Publishers, Dordrecht, The Netherlands).

Tirole J (1988) *The Theory of Industrial Organization* (MIT Press, Boston).

Weng ZK (1995) Channel coordination and quantity discount. *Management Sci.* 41(9):1509–1522.