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Consumption Externality and Yield Uncertainty in the Influenza Vaccine Supply Chain: Interventions in Demand and Supply Sides

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We study the impact of yield uncertainty (supply side) and self-interested consumers (demand side) on the inefficiency in the influenza vaccine supply chain. Previous economic studies, focusing on demand side, find that the equilibrium demand is always less than the socially optimal demand because self-interested individuals do not internalize the social benefit of protecting others via reduced infectiousness (positive externality). In contrast, we show that the equilibrium demand can be greater than the socially optimal demand after accounting for the limited supply due to yield uncertainty and manufacturer's incentives. The main driver for this result is a second (negative) externality: Self-interested individuals ignore that vaccinating people with high infection costs is more beneficial for the society when supply is limited. We show that the extent of the negative externality can be reduced through more efficient and less uncertain allocation mechanisms. To investigate the relative effectiveness of government interventions on supply and demand sides under various demand and supply characteristics, we construct two partially centralized scenarios where the social planner (i.e., government) intervenes either on the demand side or the supply side, but not both. We conduct an extensive numerical analysis.

Key words: influenza vaccine; supply chain inefficiency; strategic consumer behavior; externality; yield uncertainty
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1. Introduction

Unlike pediatric vaccines, supply chain for the influenza (flu) vaccine in the United States is highly decentralized. Profit-maximizing firms decide the production quantity and bring the produced vaccine to the market while individuals decide whether to get vaccinated in a rational and self-interested manner based on the availability of the vaccine, severity of the infection, and vaccination costs (Vietri et al. 2008). The role of the government and its agencies, such as the Centers for Disease Control and Prevention (CDC), is limited to deciding the vaccine composition and recommending priority groups for vaccination (Institute of Medicine 2004, U.S. Government Accountability Office (GAO) 2004). On the supply side, potentially insufficient incentives for manufacturers together with a long production process (six to eight months) and the significant uncertainty in the production yield have contributed to shortages in the recent past (GAO 2004). On the demand side, *positive externality effect* associated with vaccination—each vaccinated individual decreases the infection risk of her close contacts, but fails to internalize this value while making her own vaccination decision (Brito et al. 1991)—has arguably contributed to vaccination coverage rates that are lower than the socially desired target rates (Harper et al. 2005).

In this paper, we develop an integrated model of production (yield) uncertainty and rational[1] consumer behavior, and investigate the interaction between these two potential sources of inefficiency. Motivated by the current practice, we study a decentralized system where the manufacturer determines

[1] We use the term "rational" or "self-interested" interchangeably to refer to an individual who maximizes her own expected net utility.

the production quantity and individuals make their vaccination decisions. We consider a single flu season and a population comprising two priority groups based on the infection disutility of its individuals. First, given the design of priority groups, the manufacturer decides the production quantity and incurs the related production cost. A random fraction of this production quantity is realized and brought to the market. The supply received at the beginning of the flu season is first allocated to the group with higher infection disutility. The remaining supply, if any, is then allocated to the group with lower infection disutility. Then, knowing the design of priority groups and observing the realized production, self-interested individuals decide whether to spend time and money to search (e.g., physical travel or information collection) for the vaccine. Because of limited supply, individuals that decide to search are not guaranteed to get the vaccine. Unvaccinated individuals, irrespective of whether they search for the vaccine, may be infected with some probability that depends on the vaccinated fraction of the population. Every infected individual incurs some infection disutility (e.g., payment for drugs, healthcare costs, lost wages, death). Vaccinated individuals become immune against influenza, but may incur some vaccination disutility (e.g., side effects, vaccine price, administration costs).

We also analyze a benchmark centralized system where a social planner decides the demand and the production quantity to maximize the total social welfare. We find that the production quantity in the centralized setting is always greater than that in the decentralized setting in accordance with previous supply chain models of yield uncertainty. However, contrary to existing economic models of vaccination, we find that the expected demand in the decentralized setting can be higher than that in the centralized setting.

This result is driven by the *negative externality effect*; i.e., when the vaccine supply is limited, each individual searching for the vaccine reduces the vaccine availability for some individuals with higher infection disutility, but does not internalize this cost. The negative externality effect causes inefficiency (i.e., lower social welfare) on the demand side under limited supply for two main reasons: (i) not all individuals searching for the vaccine can get it and hence wasteful search costs are incurred, and (ii) those who need vaccine the most (individuals with high infection disutility) may not get it. The first component is due to the uncertainty/randomness in the vaccine allocation; i.e., if individuals who failed to get the vaccine knew this a priori, they would not seek vaccination so that the equilibrium demand will be equal to the available supply. The second component is due to the inefficiency in the allocation of vaccines; i.e., if the vaccine was allocated to individuals in decreasing order of their infection disutility, individuals with higher infection disutility would be vaccinated first.

To understand the impact of prioritization, we analyze a model with a completely random allocation mechanism and a single group of individuals. We show that prioritization makes vaccine allocation more efficient and less uncertain by increasing the chance of obtaining the vaccine for individuals with high infection disutility. As a result, we find that the negative externality effect is smaller under the priority scheme with two groups than under a complete random allocation with no priority groups. Increasing the number of priority groups reduces the negative externality effect further by decreasing the inefficiency and uncertainty in the allocation mechanism.

In addition, we study how individuals accounting for the limited supply affect the total social welfare. For this purpose, we define *availability effect* as the difference in the equilibrium outcomes between the decentralized system and a reference case, where the fill rate is 1 and where the infection probability is based on unlimited supply. We find that this availability effect is beneficial to society when search costs are high and infection disutilities among individuals are low. In such cases, accounting for limited availability of vaccines decreases the incentive for individuals with low infection disutility to search for vaccines compared to the reference case. This results in lower wasteful search costs and better vaccine allocation for individuals with high infection disutility.

We also construct partially centralized models to inform the public policy debate over the relative effectiveness of government intervention on the demand side (Institute of Medicine 2004, Hinman 2005b) versus supply side (Grady 2004, Hinman et al. 2005). Under the demand-side intervention, the manufacturer chooses profit-maximizing production quantity, but the social planner chooses the demand to maximize the total social welfare. Under the supply-side intervention, the social planner decides the welfare-maximizing production quantity, but the demand is determined by self-interested individuals. We find that the demand-side intervention in some cases can provide too much incentive (i.e., expected marginal benefit) to the manufacturer and results in production quantity that is greater than that in the centralized solution. Similarly, the production quantity under supply-side intervention can be higher than that in the social optimum, especially at lower yield realizations, to compensate for the self-interested behavior of individuals. When a large fraction of the population has high infection disutility, the demand-side inefficiency due to negative externality effect is high and the manufacturer underproduces due to high supply risk and lower marginal benefits. In such cases,

we find that the social planner should intervene in the supply side to increase the production levels and consequently decrease the negative externality effect. On the contrary, when a large fraction of population has low infection disutility and the epidemic is less infectious, individuals are less willing to search for the vaccine. In such cases, the social planner can improve total social welfare more by intervening on the demand side.

2. Literature Review

In this section, we outline our contribution to three distinct streams of literature.

2.1. Operations Management Literature

Our consumer model contributes to the growing literature on supply chain management in the presence of strategic consumer behavior (Dana and Petruzzi 2001, Su and Zhang 2008, Cachon and Swinney 2009). Most papers in this stream assume that the valuation of the product for rational consumers is exogenously specified. To our knowledge, ours is among the first papers to consider a supply chain with consumption externality; i.e., consumers' valuation for the product is determined as a function of the fraction of market consuming the product in equilibrium. In a recent paper, Tereyağoğlu and Veeraraghavan (2012) analyze a model with consumption externality similar to ours. However, our paper differs significantly in various modeling elements such as yield uncertainty, heterogeneity of consumer valuations, allocation mechanism, and partially centralized scenarios.

Flu vaccine supply chain has drawn considerable attention from operations management researchers recently (Wu et al. 2005, Kornish and Keeney 2008, Chick et al. 2008, Deo and Corbett 2009, Cho 2010). Deo and Corbett (2009), using a model of Cournot competition with endogenous entry, show that yield uncertainty can explain the high concentration of the U.S. flu vaccine market. Chick et al. (2008) propose a cost-sharing contract to eliminate the supply-side inefficiency, i.e., a profit-maximizing manufacturer produces less vaccine compared to the socially optimal solution. We complement them by incorporating rational consumer behavior, examining the interaction between the demand side and supply side, and deriving relevant policy implications.

2.2. Health Economics Literature

Several papers in health economics analyze the externality arising from vaccination and find that the demand for vaccines is lower in equilibrium than in the socially optimal solution (Brito et al. 1991, Geoffard and Philipson 1996, Philipson 2000). In contrast, we show that when supply is endogenized, it can result in equilibrium demand that is higher than the socially optimal demand because of the negative externality arising from limited and stochastic supply. This effect acts counter to the positive externality effect from herd immunity, which results in lower than socially optimal demand. For some instances of yield uncertainty, the latter effect dominates the former, thereby resulting in higher demand.

2.3. Epidemiology Literature

Empirical evidence suggests that the decision to vaccinate is also strongly correlated with the vaccine effectiveness, flu severity, and vaccine side effects (Chapman and Coups 1999). Several studies (Bauch and Earn 2004, Reluga et al. 2006, Galvani et al. 2007) combine epidemiologic models and *deductive* game-theoretic models to analyze rational vaccination decisions by individuals in the presence of these factors. These models are also consistent with the health benefit model (Janz and Becker 1984), which has been shown to explain vaccination decisions of individuals (Larson et al. 1979). Given the strong empirical support, we also adopt a deductive[2] game approach, but incorporate the impact of supply-side factors such as yield uncertainty, which is missing from the earlier models.

3. Model and Assumptions

In this section, we describe various components of our model.

3.1. Supply Model

Injectable vaccines produced via embryonated chicken eggs still constitute the majority (over 97%) of flu vaccine produced and administered (Danzon et al. 2005, Palese 2006, GAO 2008). An important characteristic of this process is yield uncertainty resulting from uncertain growth characteristics of virus strains inside the chicken eggs and the possibility of bacterial contamination. In accordance with the models proposed in the literature (Palese 2006, Chick et al. 2008, Deo and Corbett 2009), we assume that the obtained number of vaccine doses, Q_r, is a stochastic proportion of the planned egg production Q (for models with proportional yield, see Yano and Lee 1995). That is, $Q_r = UQ$, where yield $U \in [0, \infty)$ is a random variable with mean μ, standard deviation σ, and coefficient of variation $CV = \sigma/\mu$. We assume that the yield, U, has a continuous, differentiable, and strictly increasing probability distribution, $M(\cdot)$. In addition, we let w denote the manufacturer's unit price for a dose of vaccine and c denote the cost of the manufacturer per planned egg. We assume that the salvage value of any vaccine remaining at

[2] For an alternative inductive game formulation, where individuals base their decisions on past experience only, see Breban et al. (2007).

the end of the season is zero because it cannot be used in subsequent seasons. Motivated by the FDA's regulatory requirements for all biological products (Food and Drug Administration 2008), we assume that accurate information on the quantity of vaccine produced is available before vaccination.

3.2. Demand Model

We consider a population of N individuals,[3] where each healthy individual enjoys utility $\bar{V}$. The health outcome of an infected individual may fall in one of the following categories: (i) no medical care sought, (ii) outpatient visit, (iii) hospitalization, and (iv) death, each with some probability. We use infection disutility δ to represent the expectation of all direct and indirect costs that an infected individual incurs in these four scenarios (Meltzer et al. 1999, Galvani et al. 2007). It has probability density function $g(\cdot)$ and cumulative probability distribution $G(\cdot)$, which is strictly increasing over the finite interval $[0, \bar{\delta}]$ and is common knowledge. Each individual searching for the vaccine also incurs a disutility, θ, which captures the time and money spent by the individual while searching.[4] Individuals receiving the vaccine incur some additional disutility, $r \geq w$, which includes vaccine price, administration costs, morbidity risk, and side effects from the vaccine (Meltzer et al. 1999, Galvani et al. 2007). We assume that θ and r are constant across the population for analytical tractability and because they are small compared to δ.

3.3. Allocation Mechanism

The CDC Advisory Committee on Immunization Practices (ACIP) divides the population into groups depending on the mortality and morbidity risk of individuals from infection (Meltzer et al. 1999, Galvani et al. 2007). Previous research has shown that the average δ of a high-risk group is higher than that of a low-risk group.[5] To ensure the analytical tractability, we consider the following stylized version of allocation mechanism. We prioritize individuals based on their infection disutility, i.e., individuals with $\delta \geq \beta$ have higher priority over individuals with $\delta < \beta$, where $\beta \in [0, \bar{\delta}]$. The entire vaccine supply arrives at the beginning of the season and is sequentially allocated, first to individuals with $\delta \geq \beta$, and then to those with $\delta < \beta$. Hence, an individual's ex post probability of obtaining the vaccine (ϕ) depends not only on the obtained number of doses, but also on his or her infection disutility and β. We assume that β is exogenously given, common knowledge, and does not depend on the number of vaccine doses produced, Q_r. This corresponds to the ACIP practice of announcing the priority groups before the realization of supply (Institute of Medicine 2004).

For analytical tractability, we assume that all individuals in the same priority group have an equal chance of getting the vaccine, i.e., complete random allocation within each priority group. It is conceivable that allocation within a priority group may be more efficient. For instance, those with higher disutility might be more perseverant in their search. However, CDC data show that this is not the case because of the problems in vaccine production and distribution (GAO 2008). For example, only about 33%–65% of high-risk individuals received vaccination in 2008–2009 flu season while about 11%–35% of low-risk individuals were vaccinated in the same year (Fiore et al. 2010).

3.4. Epidemiology Model

We consider an SIR (susceptible-infected-recovered) epidemic model in a closed homogeneous population with fixed size (N) in which vaccination is followed by the onset of (instantaneous) infections from exogenous sources (Chick et al. 2008). We assume that the vaccine is perfectly effective, i.e., all vaccinated individuals are immunized against the infection (Brito et al. 1991). Unvaccinated individuals may be infected with probability $p(h)$, that is continuous and nonincreasing in $h \in [0, 1]$, the vaccinated fraction of the population. Similar to Brito et al. (1991), we assume that $p(\cdot)$ is common knowledge. In addition, we make following technical assumptions for our subsequent analysis:

Assumption 1. (i) *The individual with largest infection disutility searches for the vaccine when nobody is vaccinated; i.e.,* $\bar{\delta}p(0) > r + \theta$.

(ii) *The infection probability is continuously differentiable and strictly decreasing for all* $h < h_{zr}$; *i.e.,* $p'(h)$ *is*

[3] We assume that each individual has negligible impact on the aggregate decision of population, which is referred to as the "fluid" or "continuous" approximation whose solution converges to that of the real system as the population size (N) becomes larger (Aumann 1964, Brito et al. 1991, Gallego and Şahin 2010, and references therein).

[4] Earlier work (Meltzer et al. 1999, Galvani et al. 2007) does not consider stochastic availability and hence implicitly assumes that all costs incurred by an individual are due to receiving the vaccine. However, a large fraction (over 35%) of these costs are incurred before an individual knows if the vaccine is available (Meltzer et al. 1999). We label these as search costs in this paper.

[5] The mortality costs dominate total costs in all other scenarios in which an infected individual may end up (Meltzer et al. 1999, Galvani et al. 2007). Thus, people having high mortality risk also have high infection disutility. As a result, our model also prioritizes the risk groups being prioritized in practice. For example, our model prioritizes elderly individuals over young individuals because they have higher infection disutility ($4,163 versus $567). The practice also prioritizes elderly individuals because they have higher mortality and morbidity risks (Galvani et al. 2007).

continuous and negative for all $h < h_{zr}$, which we term as the zero-risk vaccination fraction and is given by

$$h_{zr} \triangleq \begin{cases} \inf\{h \in [0,1] \mid p(h) = 0\} & p(1) = 0, \\ 1 & p(1) > 0. \end{cases} \tag{1}$$

(iii) *The expected number of infections $(N(1-h)p(h))$ is convex in h for all $h \in [0,1]$.*

Assumption 1(i) is sufficient to guarantee an interior solution in the decentralized equilibrium. Assumption 1(ii) ensures the uniqueness of the solution. By Assumptions 1(ii) and 1(iii), the number of infections is strictly convex for $h_{zr} = 1$, piecewise-linear convex for $h_{zr} \in [0,1]$, and $p(h)$ is constant for $h < h_{zr}$. These two special cases are considered in Chick et al. (2008).

4. Equilibrium in the Decentralized System

We model a decentralized supply chain consisting of a profit-maximizing manufacturer that produces the vaccine and sells directly to rational utility-maximizing individuals who make their own vaccination decisions. First, the manufacturer decides the production quantity. Second, each individual observes the obtained number of doses and decides whether to search for the vaccine. We use backward induction to characterize the subgame perfect equilibrium of this two-stage game.

4.1. Individual's Problem

In the second stage of the game, given the priority groups and obtained number of doses, each individual determines whether to search for the vaccine. An individual with infection disutility δ who decides not to search for the vaccine will remain healthy with probability $1 - p(h)$ and will enjoy utility $\bar{V}$. She will be infected with probability $p(h)$ and will enjoy utility $\bar{V} - \delta$, yielding an expected utility of $v_{ns}(h) = \bar{V} - \delta p(h)$. On the other hand, if she decides to search for the vaccine, she will incur a searching disutility θ. Moreover, she will be vaccinated with probability ϕ and will obtain a utility equal to $\bar{V} - r - \theta$; she will not be vaccinated with probability $1 - \phi$ and will obtain a net utility equal to $\bar{V} - \delta p(h) - \theta$, yielding an expected net utility of $v_s(h, \phi) = \bar{V} - \delta p(h) + \phi[\delta p(h) - r] - \theta$. Thus, an individual with infection disutility δ searches for the vaccine if

$$v_s > v_{ns} \quad \Leftrightarrow \quad \theta < \phi[\delta p(h) - r]. \tag{2}$$

The consumer model in Dana and Petruzzi (2001) is a special case of our model because individuals' valuation of the vaccine, $\delta p(h)$, depends on the consumption level and decreases in it. While our immediate motivation is flu vaccines, this model is applicable to a wider class of luxury products called "conspicuous goods" or "snob goods," where consumers value the good not only for its intrinsic functionality, but also for its prestige value that depends on the level of consumption (Amaldoss and Jain 2005).

Lemma 1. *In equilibrium, for a given Q_r, if an individual with infection disutility $\hat{\delta}$ does not search for the vaccine, then none of the individuals with $\delta < \hat{\delta}$ search for the vaccine.*

All proofs are presented in Appendix B. Lemma 1 implies the existence of a marginal individual who is indifferent to searching and not searching such that all individuals with infection disutility higher than that of marginal individual search for the vaccine. We define $\delta^E(\beta, Q_r)$ as the infection disutility of the marginal individual in equilibrium. In what follows, we suppress the arguments of $\delta^E(\beta, Q_r)$ to improve readability. The vaccine demand is given by $N[\bar{G}(\delta^E)]$ and the vaccinated fraction is given by

$$h(\delta^E, Q_r) = \min\left\{\bar{G}(\delta^E), \frac{Q_r}{N}\right\}, \tag{3}$$

where $\bar{G}(\cdot) = 1 - G(\cdot)$. In addition, the probability of being vaccinated for an individual with infection disutility $\delta \geq \delta^E$ is given by

$$\phi(\delta, \delta^E) = \begin{cases} \min\left\{1, \max\left\{\left(\dfrac{Q_r - N[\bar{G}(\beta)]}{N[G(\beta) - G(\delta^E)]}\right), 0\right\}\right\} & \delta < \beta, \\ \min\left\{1, \dfrac{Q_r}{N[\bar{G}(\beta)]}\right\} & \delta^E < \beta \leq \delta, \\ \min\left\{1, \dfrac{Q_r}{N[\bar{G}(\delta^E)]}\right\} & \beta \leq \delta^E. \end{cases} \tag{4}$$

From (2), none of the individuals searches for the vaccine ($\delta^E = \bar{\delta}$) when $Q_r = 0$ and $\theta > 0$. When $Q_r = 0$ and $\theta = 0$, any $\delta \in [0, \bar{\delta}]$ satisfies the inequality in (2). However, when $Q_r = 0$ and $\theta = 0$, we assume that $\delta^E = r/p(0)$ to ensure the continuity of δ^E in Q_r. Proposition 1 characterizes the equilibrium fraction of population searching for the vaccine for $Q_r > 0$.

Proposition 1. *Define δ_{zr} as follows*:

$$\bar{G}(\delta_{zr}) = h_{zr}, \tag{5}$$

where $h_{zr} \in [0,1]$ is the zero-risk vaccination fraction given by (1).

(i) *For all $Q_r > 0$, there exists a unique threshold value $\delta^E \in [0, \bar{\delta}]$ and the equilibrium fraction of population searching for the vaccine is given by*

$$\bar{G}(\delta^E) = \begin{cases} \bar{G}(\beta) & \dfrac{Q_r}{N}\left[\beta p\left(\dfrac{Q_r}{N}\right) - r\right] \geq \theta \bar{G}(\beta) \text{ and } \dfrac{Q_r}{N} \leq \min\{\bar{G}(\beta), \bar{G}(\delta_1^E)\}, \\ \bar{G}(\delta_2^E(\bar{\delta}, Q_r)) & \dfrac{Q_r}{N}\left[\beta p\left(\dfrac{Q_r}{N}\right) - r\right] < \theta \bar{G}(\beta) \text{ and } \dfrac{Q_r}{N} \leq \min\{\bar{G}(\beta), \bar{G}(\delta_1^E)\}, \\ \bar{G}(\delta_2^E(\beta, Q_r)) & \min\{\bar{G}(\beta), \bar{G}(\delta_1^E)\} < \dfrac{Q_r}{N} \leq \bar{G}(\delta_1^E) \\ \bar{G}(\delta_1^E) & \dfrac{Q_r}{N} > \bar{G}(\delta_1^E), \end{cases} \quad (6)$$

where $\delta_1^E \in (\delta_{zr}, \bar{\delta})$ and $\delta_2^E(\beta, Q_r)$ satisfy

$$\delta_1^E p(\bar{G}(\delta_1^E)) - r = \theta, \quad (7)$$

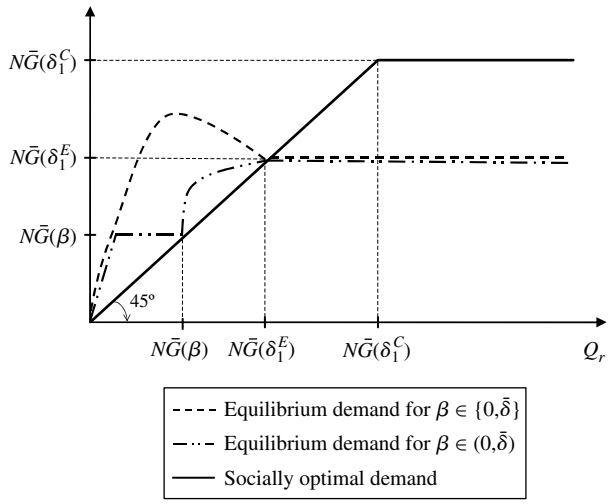
$$\frac{Q_r - N[\bar{G}(\beta)]}{N[G(\beta) - G(\delta_2^E(\beta, Q_r))]}\left[\delta_2^E(\beta, Q_r) p\left(\frac{Q_r}{N}\right) - r\right] = \theta. \quad (8)$$

(ii) *For $0 < Q_r < N[\bar{G}(\delta_1^E)]$, the ex post demand is always greater than or equal to the available supply, and they are equal only for $Q_r = N[\bar{G}(\beta)]$ when $\beta \geq \delta_1^E$.*

Figure 1 illustrates the equilibrium demand in the decentralized setting for $\beta \in \{0, \bar{\delta}\}$ and a $\beta \in (0, \bar{\delta})$ as a function of the obtained number of doses, Q_r. For $Q_r \geq N[\bar{G}(\delta_1^E)]$, the demand is less than or equal to the available supply. Hence, everyone searching for a vaccine gets it. Thus, individuals' decisions are independent of the available supply in this region. For $Q_r < N[\bar{G}(\delta_1^E)]$, vaccination is more valuable for each individual than that for $Q_r \geq N[\bar{G}(\delta_1^E)]$ since the infection risk is higher because of the limited supply. Hence, the equilibrium demand is more than the available supply as shown by Proposition 1(ii). Depending on the relative impact of the increase in infection risk and decrease in vaccine availability, the demand for high-obtained number of vaccine doses may be more or less than that for low-obtained number of vaccine doses.

Proposition 1(i) implies that the ex post vaccinated fraction in decentralized equilibrium is always less than the zero-risk vaccination fraction (h_{zr}) since $\delta_1^E \in (\delta_{zr}, \bar{\delta})$. For some functional forms of infection probability used in the literature (Bauch and Earn 2004, Breban et al. 2007, Chick et al. 2008), the zero-risk vaccination fraction in (1) is the same as the critical vaccination fraction in epidemiology that reduces the reproductive number, i.e., average number of individuals infected by an infected person, to below 1 (Hill and Longini 2003). This implies that the critical level of vaccination is not attained with self-interested individuals (Breban et al. 2007).

Figure 1 Demand in the Decentralized and Centralized Systems



Proposition 1 shows that the equilibrium demand is independent of the choice of β for $Q_r > N\bar{G}(\delta_1^E)$. Thus, prioritizing individuals with high infection disutility (δ) cannot affect individual decision making when all individuals searching for the vaccine can get it. However, when supply is limited, prioritization increases the availability of vaccine for individuals in a high priority group. This in turn may discourage some individuals in a low priority group from searching, thereby decreasing the equilibrium demand as shown by Figure 1. Proposition 2 formalizes this intuition.

Proposition 2. *For all $\beta \in [0, \bar{\delta}]$ and $Q_r < N[\bar{G}(\delta_1^E)]$, the ex post equilibrium demand with two patient groups is less than or equal to that in equilibrium with one group. However, the ex post vaccinated fraction of population is the same in both equilibria.*

Proposition 2 implies that, when the supply is limited, prioritization does not affect manufacturer's total sale (profit) but improves the social welfare by reducing wasteful search costs and increasing the probability that individuals with high infection disutility get the vaccine. Although some individuals in the low priority group may be worse off, total expected utility loss from infection decreases under prioritization. We analyze the magnitude of this benefit as a function of various problem parameters in §7.2.

Next, we consider an efficient allocation mechanism, where individuals are ordered in descending order of their infection disutility and an individual is vaccinated if, and only if, demand coming from individuals with greater infection disutility is satisfied.

Proposition 3. *Under efficient rationing, the ex post equilibrium demand is equal to the available supply for* $Q_r \le N[\bar{G}(\delta_1^E)]$ *and is equal to* $N[\bar{G}(\delta_1^E)]$ *for* $Q_r > N[\bar{G}(\delta_1^E)]$.

Proposition 3 implies that the efficient allocation mechanism completely eliminates wasteful searches and ensures that the right people get the vaccine when the supply is limited. Unfortunately, this allocation mechanism is not practical as it comprises a large number of priority groups, each with one individual.

4.2. Manufacturer's Problem

In the first stage of the game, the manufacturer determines the production quantity so that his expected profit is maximized. Thus, the manufacturer's problem is given by

$$\max_{Q\ge 0} \pi_E(Q) = wN\mathbb{E}_U[h(\delta^E, UQ)] - cQ, \tag{9}$$

where $h(\delta^E, Q_r))= \min\{\bar{G}(\delta_1^E), Q_r/N\}$ and U is the random yield. To eliminate the uninteresting case where the manufacturer does not produce at all, we assume that $w\mu > c$. The next proposition characterizes manufacturer's optimal production quantity.

Proposition 4. *Recall the definition of* δ_1^E *from* (7). *The unique optimal production quantity* Q_E *is given by*

$$w\int_0^{N[\bar{G}(\delta_1^E)]/Q_E} u dM(u) = c. \tag{10}$$

The right-hand side (RHS) and left-hand side (LHS) of (10) denote the marginal cost and expected marginal benefit to the manufacturer, respectively. Note that the marginal increase in revenue is wu if $uQ \le N[\bar{G}(\delta_1^E)]$ and zero otherwise. This is similar to Proposition 1 in Chick et al. (2008) where the government determines the equivalent of $\bar{G}(\delta_1^E)$. In our model, it is an outcome of the rational decision making by individuals.

5. Optimal Solution of the Centralized System

In the centralized system, a social planner such as the government or one of its agencies coordinates the production and vaccination to maximize the total expected social welfare. This centralized solution will serve as a benchmark to understand the sources and extent of inefficiencies in the decentralized setting. First, given the priority groups β, the planner determines the optimal egg production, Q, to maximize the total expected social welfare. Second, observing the obtained number of doses, Q_r, the planner determines the demand to maximize the ex post total social welfare. Here, the ex post utility per individual is given by

$$\begin{aligned} W(\delta, \beta, Q_r) &= \bar{V} - \theta[\bar{G}(\delta)] - rh(\delta, Q_r) \\ &\quad - p(h(\delta, Q_r))\int_0^{\delta} z dG(z) - p(h(\delta, Q_r)) \\ &\quad \times \left[(1-\phi(\max\{\delta,\beta\},\delta))\int_{\max\{\delta,\beta\}}^{\bar{\delta}} z dG(z)\right. \\ &\quad \left. + (1-\phi(\min\{\delta,\beta\}),\delta)\int_{\min\{\delta,\beta\}}^{\beta} z dG(z)\right], \end{aligned} \tag{11}$$

and the total sales revenue is given by

$$R(\delta, Q_r) = wNh(\delta, Q_r). \tag{12}$$

The first term on the RHS of (11) is the average utility of being healthy; the second term is the average utility loss of the individual from searching; the third term represents the average cost from receiving the vaccine; the fourth term is the average utility loss of individual from infection if she does not search for the vaccine; and the last term is the average utility loss of individual from infection if she cannot receive the vaccine when the supply is limited. Then the social planner's problem in the second stage is given by

$$\max_{\delta\in[0,\bar{\delta}]}\{NW(\delta,\beta,Q_r) + R(\delta, Q_r)\}. \tag{13}$$

Assumption 1 is sufficient to ensure that the objective function in (13) is strictly quasi concave and has a unique maximizer, δ^C, which is used to characterize the demand in the centralized system below.

Proposition 5. *Let the marginal benefit to the society from the vaccination of the individual with infection disutility* δ *be*

$$B(\delta) \triangleq= -p'(\bar{G}(\delta))\int_0^{\delta} z dG(z) + \delta p(\bar{G}(\delta)) - (r+\theta-w) \quad \forall\, \delta\in[0,\bar{\delta}]. \tag{14}$$

(i) *There exists a unique* $\delta^C \in [\delta_{zr}, \bar{\delta})$ *for all* $Q_r \ge 0$. *The fraction of population searching for the vaccine in the centralized system is*

$$\bar{G}(\delta^C) = \min\{\bar{G}(\delta_1^C), Q_r/N\}, \tag{15}$$

where

$$\delta_1^C = \begin{cases} \delta_{zr} & \lim_{\delta\to\delta_{zr}^+} B(\delta) \ge 0, \\ \tilde{\delta} & \lim_{\delta\to\delta_{zr}^+} B(\delta) < 0, \end{cases} \tag{16}$$

and $\tilde{\delta} \in (\delta_{zr}, \bar{\delta})$ *is the unique value satisfying*

$$B(\hat{\delta}) = 0. \tag{17}$$

(ii) *The ex post demand in the centralized system is greater than that in the decentralized system only for sufficiently large quantities, i.e.,* $\bar{G}(\delta^E) < \bar{G}(\delta^C)$ *if, and only if,* $Q_r > N[\bar{G}(\delta_1^E)]$.

Figure 1 also depicts the demand in the social optimum as a function of the obtained number of vaccine doses. Note that the first term of $B(\delta)$ in (14) represents the positive externality effect, i.e., the decrease in the utility loss of all unvaccinated individuals, while the rest of the expression is the direct benefits to the society when the individual with infection disutility δ is vaccinated. Observe that the net gain of society from vaccination of an individual with searching disutility $\delta > \delta_1^C$ is positive and increasing by Assumption 1(iii) and (16). Hence, in the case of unlimited supply, it is optimal for the social planner to vaccinate individuals with infection disutility greater than δ_1^C. However, if the supply is limited, i.e., $Q_r < N[\bar{G}(\delta_1^C)]$, it is optimal for society that all individuals with infection disutility $\delta \geq \bar{G}^{-1}(Q_r/N)$ search for the vaccine so that all the vaccine is used and everyone who searches for the vaccine gets it.

Proposition 5 implies that the socially optimal vaccination fraction may be less than the zero-risk vaccination fraction, which coincides with the critical vaccination for some basic epidemiological models when supply is unlimited and when $\lim_{\delta \to \delta_{zr}^+} B(\delta) < 0$ (Bauch and Earn 2004, Breban et al. 2007, Chick et al. 2008). This result is different from those previously available in the literature because of important differences in the underlying modeling assumptions. For instance, Hill and Longini (2003) show that critical vaccination fraction is socially optimal when the objective is minimization of number of infections instead of welfare maximization or cost minimization. Chick et al. (2008), using an objective similar to ours (minimization of total social costs), show that the critical vaccination fraction is socially optimal. However, they assume that all individuals have identical infection disutility and obtain positive expected net benefit from vaccination when coverage is less than the critical fraction. Also, because demand is determined before yield realization in their model, it may be higher than the critical vaccination fraction to compensate for low-yield realizations.

Remark 1 (Positive Externality Effect). Observe from Figure 1 that the demand as well as number of vaccinated individuals in the centralized system is higher than those in the decentralized system for $Q_r \geq N[\bar{G}(\delta_1^C)]$. For such obtained number of vaccine doses, the demand-side inefficiency is due to the positive externality of vaccination; i.e., the government considers the impact of an individual's vaccination on the likelihood of infection for others, but the individuals do not. For $N[\bar{G}(\delta_1^E)] < Q_r < N[\bar{G}(\delta_1^C)]$, the demand and number of vaccinated individuals in the centralized system are still higher than those in equilibrium, but the gap between the two is lower and depends on Q_r.

Remark 2 (Negative Externality Effect). Figure 1 shows that for a sufficiently low obtained number of vaccine doses, i.e., $Q_r < N[\bar{G}(\delta_1^E)]$, the equilibrium demand is actually higher than the demand in the centralized system, whereas the number of vaccinated individuals is equal in both systems. The demand-side inefficiency in this region stems from the negative externality effect, i.e., when the vaccine supply is limited, each individual searching for the vaccine reduces the vaccine availability for individuals with higher infection disutility and hence their expected net utility, but does not internalize it. The negative externality effect reduces the social welfare because of two characteristics of the allocation mechanism: (i) uncertainty/randomness, i.e., not everyone searching can get the vaccine and hence wasteful search costs are incurred, and (ii) inefficiency, i.e., those who need vaccine the most (consumers with high δ) may not get it. Note that (i) is due to the uncertainty/randomness in the vaccine allocation (i.e., only individuals who will get the vaccine would search if they knew that they will get it), whereas (ii) is due to the inefficiency in the allocation of vaccines (i.e., individuals with higher infection disutility are vaccinated first if the vaccine is allocated to individuals in decreasing order of their infection disutility). The second observation indicates that the demand-side inefficiency for $Q_r < N[\bar{G}(\delta_1^E)]$ would still exist if search costs were zero. Also, Propositions 2 and 3 show that the demand-side inefficiency due to negative externality effect can be substantially reduced by either appropriate choice of β for two priority groups or by increasing the number of priority groups. We explore this issue further in §7.2.

Remark 3 (Availability Effect). If consumers ignore the fact that supply is limited, similar to Brito et al. (1991), the equilibrium demand is equal to $N[\bar{G}(\delta_1^E)]$ for all obtained number of vaccine doses. However, when individuals consider limited supply, the equilibrium demand is not necessarily equal to $N[\bar{G}(\delta_1^E)]$ and depends on the available supply. We label the gap between the two demands as the availability effect. Figure 1 shows that the equilibrium demand for $Q_r < N[\bar{G}(\delta_1^E)]$, which is always more than the available supply, is less than $N[\bar{G}(\delta_1^E)]$ for some values of Q_r but higher for others. Thus, the availability effect is ex post beneficial for the society for some cases and harmful for other cases. As a result, ex ante, the availability effect could be beneficial or harmful depending on the yield distribution. Proposition 3 implies that under efficient rationing

the availability effect is always beneficial for society because it completely eliminates wasteful searches and does not change the number of vaccinations. We further explore the impact of allocation mechanism (through the choice of β) on the magnitude of the availability effect in §7.2.

Returning to the first stage, the social planner chooses the egg production, Q, to maximize the total expected social welfare. Hence, the social planner's first stage problem is given by

$$\max_{Q\geq 0} \mathcal{W}_C(Q) := N\mathbb{E}_U[W(\delta^C(UQ), \beta, UQ)] + \mathbb{E}_U[R(\delta^C(UQ), UQ)] - cQ. \quad (18)$$

Proposition 6 characterizes the optimal production quantity in the centralized system.

Proposition 6. (i) *The optimal production quantity for the centralized system* (Q_C) *is unique and given by*

$$\int_0^{N[\bar{G}(\delta_1^C)]/Q_C} B(\delta_2^C(uQ_C))dM(u) = c, \quad (19)$$

where $\delta_2^C(Q_r) = \bar{G}^{-1}(\min\{1, Q_r/N\})$ *and* $B(\delta)$ *is given by* (14).

(ii) *The optimal production quantity in the centralized system* (Q_C) *is greater than that in the decentralized system* (Q_E).

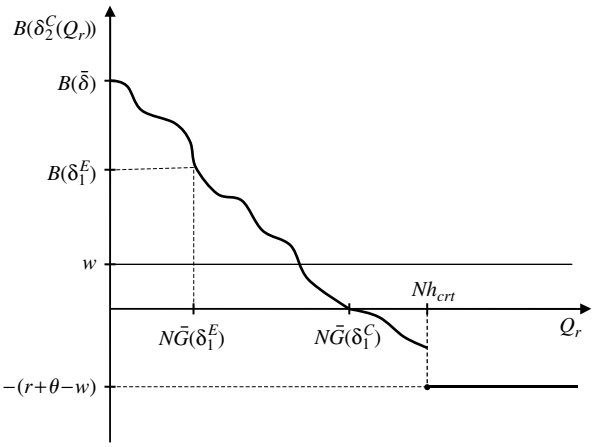
For an intuitive understanding of result (ii), note from Figure 2 that the marginal benefit to society is always greater than w, which is the marginal benefit to the manufacturer for $Q_r \leq N[\bar{G}(\delta_1^E)]$. This together with $\delta_1^C < \delta_1^E$ implies that at any $Q_r \leq Q_E$, the expected marginal benefit to the society (LHS of (19)) is greater than the expected marginal benefit to the manufacturer (LHS of (10)). However, marginal costs to society and to the manufacturer are the same. Hence, the government in the centralized system targets a higher production quantity than the manufacturer in the decentralized system.

Proposition 5 shows that ex post demand can be higher in equilibrium for some values of the realized quantity. Hence, it is conceivable that for some values of the parameters, Q_E and Q_C are such that the expected demand is greater in the decentralized setting. Because of analytical intractability, we verified this numerically. This observation is contrary to the existing results on products with positive externality with unlimited supply, which state that the consumers demand less than the socially optimal level (Brito et al. 1991).

6. Government Interventions

Despite general agreement on the need for a more active government role in coordinating the U.S. flu vaccine supply chain (Kilbourne 1991, Harris 2004), experts disagree on whether the government should modify consumer behavior on the demand side (Institute of Medicine 2004, Sloan et al. 2004, Hinman 2005a) or provide adequate incentives for manufacturers on the supply side (Pauly 2005, Hinman 2005a). We model these two broad policy suggestions as alternative intermediate scenarios of partial centralization.

Figure 2 **Marginal Social Benefit When Social Planner Decides the Demand Where $B(\delta)$ Is Given by (14)**



In the *demand-side intervention*, the government decides the socially optimal demand, given that the production quantity is determined by a profit-maximizing manufacturer. In the *supply-side intervention*, the government decides the welfare-maximizing production quantity while individuals decide whether to be vaccinated. Our objective is to characterize the relative effectiveness of these interventions as a function of different parameters and to isolate the inefficiency on each side. We do not aim to formulate concrete mechanisms for implementing them in the decentralized supply chain.

6.1. Demand-Side Intervention

A demand-side intervention could take the form of direct purchasing (*Wall Street Journal* 2003), compulsory vaccination (Philipson 2000, Sloan et al. 2004), or implementation of a tax/subsidy mechanism (Brito et al. 1991, Geoffard and Philipson 1996, Sloan et al. 2004). In the first stage, the manufacturer plans the production quantity to maximize expected profit, and in the second stage, after observing the obtained number of doses Q_r, the planner determines the demand to maximize ex post total social welfare, which is exactly the same as (13).[6] The total demand under

[6] Clearly, this definition of demand-side intervention is idealized because it requires that the social planner have sufficient information to determine the socially optimal demand, and can compel individuals to search. Nonetheless, it is a fair comparison for the supply-side intervention and thus can be used to compare the relative effectiveness of the two interventions.

demand-side intervention is $N[\bar{G}(\delta^C)]$, where $\bar{G}(\delta^C)$ satisfies (15). The manufacturer's problem in the first stage is given by

$$\max_{Q\geq 0} \pi_D(Q) = wN\mathbb{E}_U[h(\delta^C, UQ)] - cQ, \tag{20}$$

where $h(\delta^C, Q_r) = \bar{G}(\delta^C)$. Proposition 7 characterizes the equilibrium production quantity.

Proposition 7. (i) *The unique equilibrium production quantity Q_D is given by*

$$w\int_0^{N[\bar{G}(\delta_1^C)]/Q_D} udM(u) = c, \tag{21}$$

where δ_1^C satisfies (16).

(ii) *In equilibrium, the optimal production under demand-side intervention (Q_D) is greater than that in the decentralized system (Q_E).*

The government intervention in the demand side expands the market for higher realizations of Q_r and keeps it unchanged for lower realizations of Q_r. As a result, the marginal expected benefit to the manufacturer as seen in the LHS of (21) is greater than that in the decentralized system (10). Hence, the manufacturer plans to produce more under demand-side intervention because this increases his expected revenue.

Since Q_D is manufacturer's best response to socially optimal demand, its comparison with Q_C provides a measure of inefficiency on the supply side alone. Given that the demand-side intervention represents an intermediate level of centralization, one might expect that $Q_E < Q_D < Q_C$. However, Proposition 8 shows that this is not necessarily the case.

Proposition 8. *The optimal production quantity under the demand-side intervention, Q_D, is less (greater) than that in the centralized system, Q_C, if and only if*

$$\int_0^k B\left(\delta_2^C\left(\frac{uN[\bar{G}(\delta_1^C)]}{k}\right)\right) udM(u) > (<)c, \tag{22}$$

where $B(\delta)$ and δ_1^C are given by (14) *and* (16), *respectively, $\delta_2^C(Q_r) = \bar{G}^{-1}(\min\{1, Q_r/N\})$, and k satisfies*

$$w\int_0^k udM(u) = c. \tag{23}$$

These results suggest that the expected marginal benefit of the manufacturer can be greater than that of the social planner in some cases. Observe from Figure 2 that because of market expansion from demand-side intervention, there exists a $Q_w \in (N[\bar{G}(\delta_1^E)], N[\bar{G}(\delta_1^C)])$ such that the marginal benefit to society is less than that to the manufacturer (w) for $Q_r > Q_w$. Hence, depending on the value of Q_w, and the yield distribution, the expected marginal benefit to the society can be less than that to the manufacturer at $Q = Q_C$. These conditions on Q_w and the yield distribution are captured in (22).

The *game setting* in Chick et al. (2008) corresponds to our demand-side intervention, and their *system setting* corresponds to our centralized system. In contrast to Proposition 8, they show that the production in the system setting is always greater than the production in the game setting. In their system setting, similar to our centralized system, the demand and production quantity are determined so as to maximize the total social welfare, which includes the manufacturer's profit. In their *game setting*, the government is the purchaser of vaccines and hence maximizes total utility of all individuals. In contrast, in our demand-side intervention, the government is the social planner and hence maximizes the total social welfare, which is the sum of the utilities of all individuals and the profit of the manufacturer.[7]

6.2. Supply-Side Intervention

A supply-side intervention can be implemented via direct control of the flu vaccine production (Hinman et al. 2005), or by implementing appropriate contracts such as buying all remaining vaccine back at the end of vaccination season or by sharing the production costs (Chick et al. 2008).

Under the supply-side intervention, given priority classes (β), the government plans the production quantity in the first stage to maximize the ex ante total social welfare. In the second stage, each individual decides to search or not to maximize her expected net utility given β and Q_r. Clearly, identical to the second stage of the decentralized system analyzed in §4, the total demand under supply-side intervention is $N[\bar{G}(\delta^E)]$, where $\bar{G}(\delta^E)$ satisfies (6).

The government's problem in the first stage, similar to §5, is given by

$$\max_{Q\geq 0} \mathscr{W}_S(\beta, Q) := N\mathbb{E}_U[W(\delta^E, \beta, UQ)] + \mathbb{E}_U[R(\delta^E, UQ)] - cQ, \tag{24}$$

where $W(\delta, \beta, Q_r)$ and $R(\delta, Q_r)$ satisfy (11) and (12), respectively. Because the function $\mathscr{W}_S(\beta, Q)$ is not necessarily unimodal in Q, the optimal production in supply-side intervention (Q_S) cannot be characterized analytically. Therefore, we conduct the analysis of supply-side intervention via numerical examples where we find Q_S by exhaustive line search. Similar to Proposition 8, we find several examples where $Q_S > Q_C$, i.e., the optimal production quantity in the supply-side intervention is higher than that at the social optimum.

[7] Numerical examples show for low $\bar{\delta}$ that an alternative demand-side intervention that maximizes only the total utilities of all individuals results in reduction of the total social welfare by 1% to 6% for parameter values considered in our study.

7. Numerical Study

In this section, we use our analytical results to serve two main purposes. In §7.1, we study the relative effectiveness of demand-side and supply-side interventions as a function of key supply chain characteristics such as yield uncertainty and infectiousness of the disease. In §7.2, we investigate the robustness of availability effect and negative externality effect with respect to priority groups (β) and various other parameters.

Similar to Bauch and Earn (2004), we consider an infection probability of the following form that satisfies Assumption 1:

$$p(h) = \begin{cases} 1 - \dfrac{1}{R_0(1-h)} & h < h_{zr}, \\ 0 & h \geq h_{zr}, \end{cases} \tag{25}$$

where R_0 is the average number of individuals infected by an infected person, and $h_{zr} = 0$ if $R_0 \leq 1$ and $h_{zr} = 1 - 1/R_0$ if $R_0 > 1$. Using the first term in (14) and (25), it can be seen that the positive externality effect decreases as R_0 increases. To make our welfare-maximizing formulation consistent with the cost-minimizing formulations in the literature (Galvani et al. 2007, Chick et al. 2008), we normalize $\bar{V}$ to 0. In this case, $-\mathscr{W}_C(Q_C)$ is the ex ante total social cost in the centralized system. Also, we normalize the population size to $N = 100$.

We choose the parameters in our numerical study based on published estimates from previous studies wherever possible, and consider a range of values for sensitivity (Table 1). See Appendix A for details. Note that $CV = 0$ in Table 1 corresponds to the case with deterministic[8] yield where $Q_r = Q$.

7.1. Value of Supply Chain Interventions

We measure the value of the completely centralized solution as the percentage reduction in the total social costs compared to those in the decentralized equilibrium, i.e., $[1 - \mathscr{W}_C(Q_C)/\mathscr{W}_E(\beta, Q_E)] \times 100\%$. Similarly, the value of demand-side and supply-side interventions is defined as $[1 - \mathscr{W}_D(Q_D)/\mathscr{W}_E(\beta, Q_E)] \times 100\%$ and $[1 - \mathscr{W}_S(Q_S)/\mathscr{W}_E(\beta, Q_E)] \times 100\%$, respectively. Figures 3(a) and 3(b) illustrate these entities as a function of the coefficient of variation of the yield distribution (CV) and the infectiousness of the disease (R_0), respectively, for two $\bar{\delta}$ values.

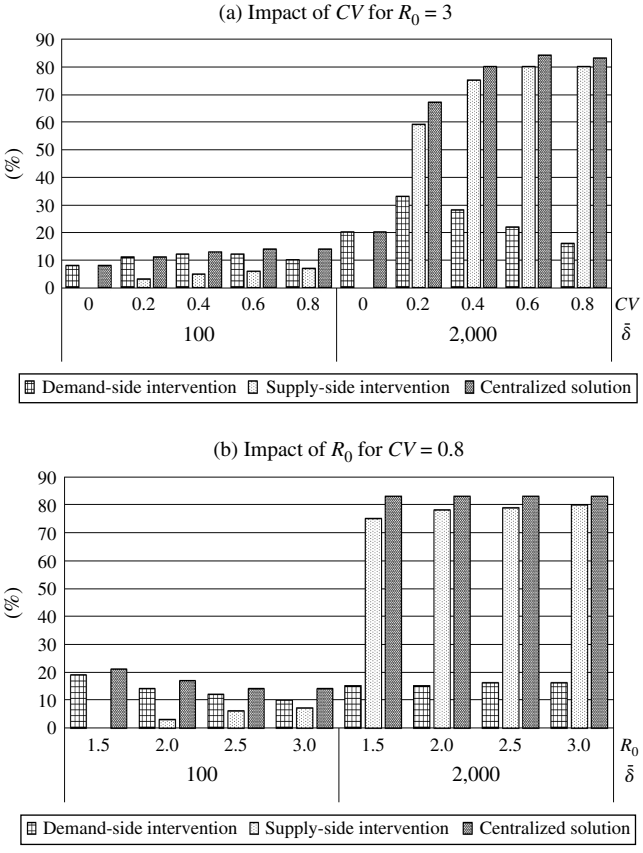
We observe from Figure 3 that the supply-side intervention is more valuable when the infection disutilities among the population are high (high $\bar{\delta}$). To understand the underlying intuition, note for high $\bar{\delta}$ that the positive externality effect is lower and the negative externality effect is higher. The former comes from more individuals behaving close to the social optimum for high number of obtained vaccine doses. The latter comes from more people searching but failing to obtain the vaccine for low number of obtained vaccine doses. The supply-side intervention

[8] We analyze all four scenarios with the deterministic yield and find that the demand-side inefficiency is independent of supply-side factors when the yield is deterministic indicating that yield uncertainty is the main driver of supply-side inefficiency. We do not present the analysis of deterministic yield in this paper for brevity, but it is available from the authors upon request.

Table 1 **Values of Input Parameters**

Parameter	Value	Source
μ	1	Chick et al. (2008)
CV	{0, 0.2, 0.4, 0.6, 0.8}	Chick et al. (2008)
$\bar{\delta}$ ($)	{100, 500, 2000}	Galvani et al. (2007)
β ($)	{0, 0.25$\bar{\delta}$, 0.4$\bar{\delta}$, 0.6$\bar{\delta}$}	GAO (2001, 2008)
R_0	{1.5, 2, 2.5, 3}	Breban et al. (2007), Chick et al. (2008)
θ ($)	{0, 10, 20}	Meltzer et al. (1999), Galvani et al. (2007)
c ($)	{1.5, 3}	Deo and Corbett (2009), Koh and Paxson (2009)
w/c	{2, 5}	CDC (2009)
r ($)	$w + 23$	Meltzer et al. (1999), Chick et al. (2008)

Figure 3 **Value of Centralized Solution, and Supply-Side and Demand-Side Interventions for $w/c = 2$, $c = 3$, $\theta = 10$, and $\beta = 0.25\bar{\delta}$**



results in larger target egg production and consequently a large number of obtained vaccine doses. This, in turn, lowers both positive and negative externality effects in expectation. Figure 3(b) shows that the effectiveness of supply-side intervention further increases in the infectiousness of the disease (R_0) for high $\bar{\delta}$ values. As the disease becomes more infectious, the expected marginal benefit of the social planner increases more than that of the manufacturer. Therefore, the extent of underproduction in equilibrium and hence the value of supply-side intervention is higher for higher R_0 values.

On the other hand, as observed in Figure 3, the demand-side intervention is more effective when infection disutilities among the population are low (low $\bar{\delta}$). The positive externality effect is high and the negative externality effect is low in such cases. Because of lower benefits, the manufacturer, in line with social optimum, targets a low target egg production when $\bar{\delta}$ is low. The demand-side intervention induces and reduces the demand for high and low number of obtained vaccine doses, respectively. This results in lower positive and negative externality effects. Further, Figure 3 shows that the effectiveness of demand-side intervention for low $\bar{\delta}$ first increases then decreases in the level of yield uncertainty. As the yield uncertainty increases, the total social costs both in equilibrium and in the demand-side intervention increase while the gap between them first increases then decreases. Consequently, the value of demand-side intervention first increases then decreases in the yield uncertainty. Thus, a model ignoring yield uncertainty, which is the source of supply-side inefficiency, may underestimate or overestimate the value of the demand-side intervention.

7.2. Robustness of Availability and Negative Externality Effects

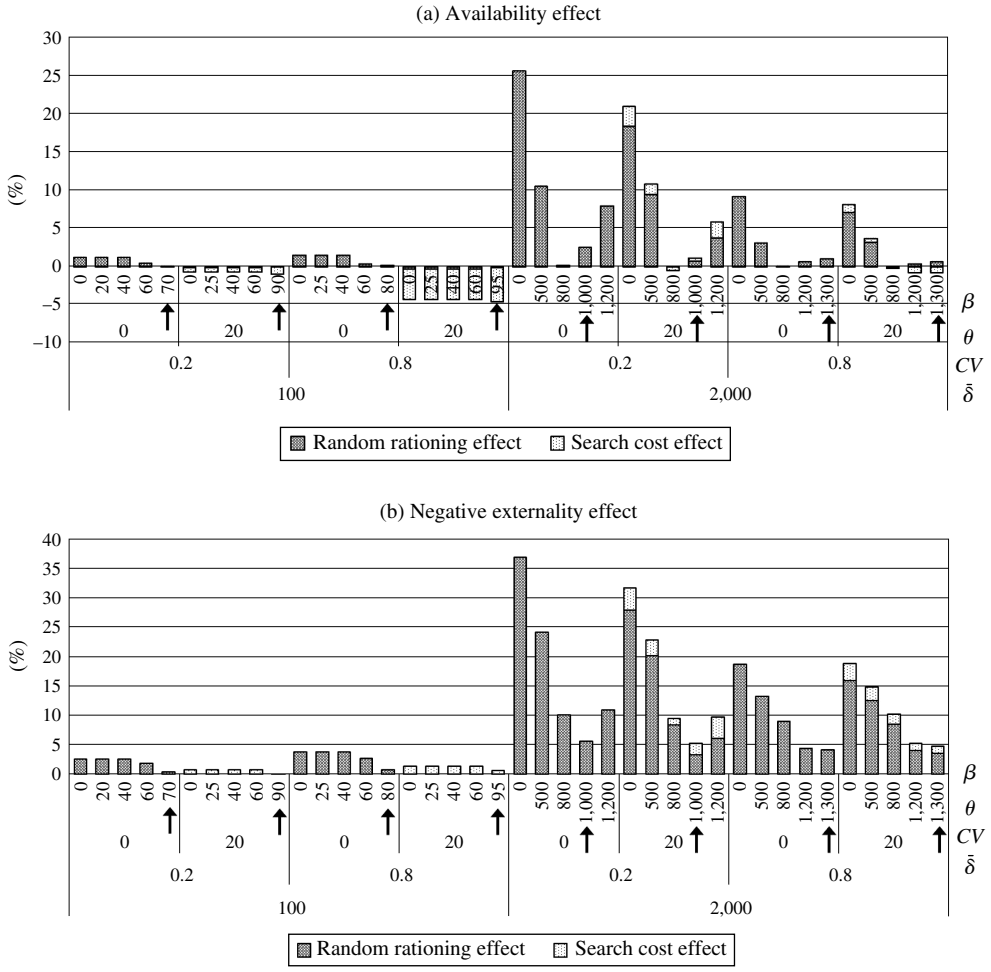
We quantify the magnitude of availability effect and analyze its dependence on various problem parameters. To do this, we construct a reference case where individuals ignore the vaccine supply being limited so that the ex post demand is equal to $N[\bar{G}(\delta_1^E)]$ for all realizations of the production quantity (Brito et al. 1991). However, the optimal production quantity for the manufacturer is still equal to Q_E because manufacturer's ex post sales do not change. We let $\mathcal{W}_{E,NA}(Q_E)$ denote the total social welfare in this reference case and measure the availability effect as the percentage gap between the social costs in the equilibria with and without the availability effect, i.e., $[1 - \mathcal{W}_{E,NA}(Q_E)/\mathcal{W}_E(\beta, Q_E)] \times 100\%$. Recall that both $\mathcal{W}_{E,NA}(Q_E)$ and $\mathcal{W}_E(\beta, Q_E)$ are negative in our numerical examples since we normalized $\bar{V}$ to zero. Hence, the availability effect is harmful for society when $[1 - \mathcal{W}_{E,NA}(Q_E)/\mathcal{W}_E(\beta, Q_E)] \times 100\%$ is positive. In addition to the four β values in Table 1, we include the results for optimal β found by line search on the interval $[0, \bar{\delta}]$.

Overall, the impact of the availability effect can be divided into two categories (i) impact on wasteful search costs, (ii) impact on vaccine availability for individuals with high infection disutility. Figure 4(a) illustrates how each of these components vary in β for different combinations of θ, CV, and $\bar{\delta}$ values. The availability effect is most harmful for low values of yield uncertainty and for high values of $\bar{\delta}$, i.e., when the infection is very expensive for more individuals. For these parameter combinations, the increase in the expected costs of infection dominates the decrease in vaccine availability when each individual accounts for limited supply. As a result, vaccine demand for limited supply values increases thereby increasing wasteful search costs and decreasing vaccine availability for individuals with high infection disutility. Thus, the availability effect becomes harmful to society when infection is very expensive for more individuals. On the other hand, the availability effect is beneficial to the society when the infection is not very expensive (low $\bar{\delta}$) and search is relatively costly (high θ). This is because limited availability decreases the incentive for individuals with low infection disutility to search for vaccines compared to the reference case. This results in lower wasteful search costs and better vaccine allocation for individuals with high infection disutility.

Next, we conduct a similar sensitivity analysis for the negative externality effect. We construct a reference case where individuals internalize the negative externality arising from limited availability of vaccine. Thus, the ex post equilibrium demand for $Q_r \geq N[\bar{G}(\delta_1^E)]$ is the same as that in decentralized system, but the demand for $Q_r < N[\bar{G}(\delta_1^E)]$ is equal to the socially optimal demand (Q_r). The optimal production in this case is also equal to Q_E since ex post sales of manufacturer stay the same. Defining $\mathcal{W}_{E,NE}(Q_E)$ as the total social welfare in this reference case, we measure the negative externality effect by $[1 - \mathcal{W}_{E,NE}(Q_E)/\mathcal{W}_E(\beta, Q_E)] \times 100\%$. Figure 4(b) displays the negative externality effect and its components for different parameter combinations.

We observe from Figures 4(a) and 4(b) that, for high $\bar{\delta}$, the availability and negative externality effects are not sensitive to search cost (θ) because a large component of both effects is due to random allocation within priority groups. Figures 4(a) and 4(b), in line with our discussion in §5, show that when appropriately determined, the prioritization may significantly reduce the availability and negative externality effects compared to the complete random allocation case ($\beta = 0$), especially for high $\bar{\delta}$ and low CV values. Finally, we observe from Figure 4 that, under

Figure 4 **Sensitivity Analysis of Availability and Negative Externality Effects When $R_0 = 3$**



Note. The socially optimal β in each case is indicated by an arrow.

socially optimal β, the harmful impact of the availability and negative externality effects is the highest when infection disutilities among the population are high (high $\bar{\delta}$). For any β, the demand for vaccines and consequently the number of people seeking vaccination but not getting it is higher when the infection is more costly. Therefore, the availability and negative externality effects are larger for high $\bar{\delta}$.

8. Conclusion and Future Research

In this paper, we present the first joint model of demand-side and supply-side inefficiencies resulting from rational consumer behavior and yield uncertainty at a profit-maximizing manufacturer, respectively. We find that combining the two effects generates several novel insights through negative externality and availability effects. First, we find that, contrary to the wisdom from health economics, demand for the vaccine in equilibrium can be higher than the socially optimal demand if the negative externality effect dominates the positive externality effect. This has important implications for public policy because it suggests that, in some cases, curbing rather than inducing demand for vaccines might be appropriate. We show that more efficient allocation mechanisms that prioritize individuals with high infection disutility can mitigate the negative externality effect and curb the demand for limited supply values. In addition, the appropriateness of curbing or inducing the demand depends critically on the supply-side parameters such as yield uncertainty.

Our model highlights the interaction between demand-side and supply-side inefficiencies. We find that the value of supply-side intervention depends on the infectiousness of the disease, whereas the value of demand-side intervention depends on the yield uncertainty of the production process. This suggests that mechanisms to coordinate the entire supply chain would need to combine demand-side interventions (tax-subsidy mechanisms as in Brito et al. 1991) and supply-side interventions (cost-sharing contracts as in Chick et al. 2008) in a nonobvious way. We derive one

such coordinating mechanism in a companion paper (Arifoğlu et al. 2011).

Our work provides several interesting avenues for future study. A natural extension is to consider a two-period model of the flu season, where the manufacturer decides the quantities to bring to market in each of the two periods and the consumers decide the timing of their vaccination. The unique characteristics of the flu vaccine context that make this model interesting are as follows: (i) consumer valuations of the vaccine in the second period are uncertain because they depend on the number of infections in the first period, and (ii) consumers exit the market at the end of the first period not only based on vaccination but also based on infection. This setup could be used to understand when one might expect to see a mad rush for vaccine at the beginning of a flu season accompanied with excess stock at the end of the season.

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Appendix A. Choice of Parameter Values
In this appendix, we explain how we chose the parameter values for our numerical study in §7. We assume that the stochastically proportional yield, U, has a gamma distribution (Chick et al. 2008), which is truncated at $\bar{U} = 4$ with $\mu = 1$. To model various levels of yield uncertainty, we consider five different values of the coefficient of variation $CV \in \{0, 0.2, 0.4, 0.6, 0.8\}$, where $CV = 0$ corresponds to the deterministic case.

In all numerical examples, we assume that δ is uniformly distributed over the finite interval $[0, \bar{\delta}]$. Weycker et al. (2005) estimate the average direct costs due to infection as \$96 over all population groups, whereas Galvani et al. (2007) estimate the total (direct and indirect) costs due to infection as \$567 for the young and \$4,163 for the elderly. We present our results only for $\bar{\delta} \in \{\$100, \$500, \$2{,}000\}$ because our qualitative insights remain the same for larger $\bar{\delta}$ values.

High priority population segments, based on CDC recommendations, have increased from about 40% in the 2001–2002 flu season (GAO 2001) to about 75% in the 2006–2007 flu season (GAO 2008). For sensitivity analysis, we also consider cases where 60% and 100% of the population is in the high priority group. These correspond to four different β values given in Table 1.

The searching disutility θ includes travel costs and time costs incurred by an individual during her search. Galvani et al. (2007) estimate the average searching disutility as $\theta = \$20$ (travel costs (\$4) and time costs (\$16)). To check the robustness of our findings, we also consider lower values of $\theta = 0$ and $\theta = 10$. For seasonal flu epidemic, R_0 is in $[1.5, 3]$ (Breban et al. 2007, Chick et al. 2008). Hence, we consider four R_0 values, i.e., $R_0 \in \{1.5, 2, 2.5, 3\}$.

The production cost per dose for epidemic flu vaccine was in the range of \$1.50 to \$2.50 during 1990s, but increased later (O'Mara et al. 2003). The production cost per dose for pandemic flu vaccine is in the range of \$3 to \$4.50 per dose (Koh and Paxson 2009). Moreover, each dose of seasonal flu vaccine requires 1–2 eggs (Hartgroves 2009). Hence, we assume that $c \in \{1.5, 3\}$, but present the results only for $c = \$3$ for brevity. The wholesale prices for different manufacturers were in the range of \$6 to \$14 in the 2009–2010 flu season (CDC 2009). Hence, we consider two values of $w/c = 2$ and $w/c = 5$ here; however, present the results for $w/c = 2$ for brevity.

Meltzer et al. (1999) estimate costs from vaccine side effects as approximately \$3 per dose, whereas Chick et al. (2008) assume a vaccine administration cost equal to \$20 per dose. Hence, we assume the cost of receiving vaccine as $r = w + 23$ in all numerical examples.

Appendix B. Proofs

Proof of Lemma 1
Assume that $\tilde{\phi}_h$, $\tilde{\phi}_l$ ($\tilde{\phi}_h \geq \tilde{\phi}_l$) and $\tilde{h}$ are, respectively, the availability probabilities for high and low priority classes, and vaccinated fraction in the equilibrium. Next consider two cases: $\hat{\delta} < \beta$ and $\hat{\delta} \geq \beta$.

Case 1 ($\hat{\delta} < \beta$): Because the individual with $\hat{\delta}$ does not search and she belongs to the lower priority class, we have $\tilde{\phi}_l(\hat{\delta}p(\tilde{h}) - r) < \theta$, i.e., $\hat{\delta}p(\tilde{h}) < \theta/\tilde{\phi}_l + r$. Notice that for any $\delta < \hat{\delta}$, this condition is satisfied and hence individuals with $\delta < \hat{\delta}$ do not search for the vaccine.

Case 2 ($\hat{\delta} \geq \beta$): In this case, the individual with $\hat{\delta}$ will be in the high priority class. Since this individual does not search for the vaccine, we have $\tilde{\phi}_h(\hat{\delta}p(\tilde{h}) - r) < \theta$, i.e., $\hat{\delta}p(\tilde{h}) < \theta/\tilde{\phi}_h + r$. First, consider individuals with $\delta \in [\beta, \hat{\delta})$. These individuals are also in the high priority class; therefore, the availability probability for them is $\tilde{\phi}_h$. Obviously, any $\delta \in [\beta, \hat{\delta})$ satisfies the last inequality; therefore, all individuals with $\delta \in [\beta, \hat{\delta})$ do not search for the vaccine. For individuals with $\delta \in [0, \beta)$, availability probability is $\tilde{\phi}_l$. Note by $\hat{\delta} > \beta$ and $\tilde{\phi}_h \geq \tilde{\phi}_l$ that $\delta p(\tilde{h}) < \theta/\tilde{\phi}_l + r$, as a result, $\tilde{\phi}_l(\delta p(\tilde{h}) - r) < \theta$ for $\delta \in [0, \beta)$. Hence, no individual with $\delta \in [0, \beta)$ searches for the vaccine.

Proof of Proposition 1
We prove both claims in Proposition 1 separately.

Part (i): Lemma 1 implies that there exists a marginal individual ($\delta^E(\beta, Q_r)$) who is indifferent between searching and not searching. In what follows, we first characterize the threshold valuation level $\delta^E(\beta, Q_r)$ for $Q_r > 0$, and then we show that the fraction searching for the vaccine satisfies (6).

a. Characterizing the threshold valuation level ($\delta^E(\beta, Q_r)$): We define $\psi(\delta, \beta, Q_r)$ as the excess of utility to the marginal individual with valuation δ who is indifferent between searching and not searching. It turns out that

$$\psi(\delta, \beta, Q_r) = \phi(\delta, \delta, \beta, Q_r)[\delta p(h(\delta, Q_r)) - r] - \theta \quad \text{(B1)}$$

for all $Q_r \geq 0$, and $\delta, \beta \in [0, \bar{\delta}]$, where h and ϕ satisfy (3) and (4), respectively. Then, by definition, $\delta^E(\beta, Q_r) = \sup\{\delta \in [0, \bar{\delta}]: \psi(\delta, \beta, Q_r) \leq 0 \text{ for } \beta \in [0, \bar{\delta}] \text{ and } Q_r \geq 0\}$. Below, we consider two cases: $Q_r \in (0, N[\bar{G}(\beta)]]$ and $Q_r > N[\bar{G}(\beta)]$.

Case a.1 $(Q_r > N[\bar{G}(\beta)])$: We define $\delta^*(Q_r) = \bar{G}^{-1}(\min\{1, Q_r/N\})$, where $\bar{G}^{-1}$ is the inverse of $\bar{G}$. Certainly, $\bar{G}^{-1}$ exists since G is strictly increasing and continuous. Next, we consider two subcases: $\delta_{zr} \le \delta^*(Q_r)$ and $\delta_{zr} > \delta^*(Q_r)$.

Subcase a.1.1 $(\delta_{zr} \le \delta^*(Q_r))$: By (B1), it turns out that

$$\psi(\delta, \beta, Q_r) = \begin{cases} \dfrac{1}{G(\beta) - G(\delta)} \varphi_2(\delta, \beta, Q_r) & \delta < \delta^*(Q_r), \\ \varphi_1(\delta) & \delta^*(Q_r) \le \delta \le \bar{\delta} \end{cases}$$

for all $Q_r > N[\bar{G}(\beta)]$ and $\beta \in [0, \bar{\delta}]$, where

$$\varphi_1(\delta) = \delta p(\bar{G}(\delta)) - (r + \theta), \tag{B2}$$

$$\varphi_2(\delta, \beta, Q_r) = \frac{Q_r - N[\bar{G}(\beta)]}{N}\left[\delta p\left(\frac{Q_r}{N}\right) - r\right] - \theta[G(\beta) - G(\delta)]. \tag{B3}$$

Observe that $\psi(\delta, \beta, Q_r)$ is continuous in δ for all $Q_r > N[\bar{G}(\beta)]$ and $\beta \in [0, \bar{\delta}]$, and that

$$\frac{d\varphi_1(\delta)}{d\delta} = p(\bar{G}(\delta)) - \delta g(\delta) p'(\bar{G}(\delta)) \ge 0, \tag{B4}$$

$$\frac{\partial \varphi_2(\delta, \beta, Q_r)}{\partial \delta} = \frac{Q_r - N[\bar{G}(\beta)]}{N} p\left(\frac{Q_r}{N}\right) + \theta g(\delta) > 0. \tag{B5}$$

Note that $\delta^*(Q_r) < \beta$ in this case since $Q_r > N[\bar{G}(\beta)]$. In addition, we let δ_1^E satisfy $\varphi_1(\delta_1^E) = 0$. Note by Assumption 1, (5) and (B2), that $\lim_{\delta \downarrow \delta_{zr}} \varphi_1(\delta) < 0$, $\lim_{\delta \uparrow \bar{\delta}} \varphi_1(\delta) > 0$ and $d\varphi_1(\delta)/d\delta > 0$ for $\delta \in (\delta_{zr}, \bar{\delta})$. As a result, there exists such a unique $\delta_1^E \in (\delta_{zr}, \bar{\delta})$. Now, we consider two different cases: $\delta_1^E \le \delta^*(Q_r)$ and $\delta_1^E > \delta^*(Q_r)$.

First, suppose that $\delta_1^E \le \delta^*(Q_r)$. Note that $\psi(\delta, \beta, Q_r) = \varphi_1(\delta) > 0$ for all $\delta > \delta^*(Q_r)$ since $\varphi_1(\delta) > 0$ for all $\delta > \delta_1^E$. On the other hand, $\psi(\delta, \beta, Q_r) = \varphi_2(\delta, \beta, Q_r)/(G(\beta) - G(\delta))$ for all $\delta < \delta^*(Q_r)$. Observe that $G(\beta) - G(\delta) > 0$ for all $\delta < \delta^*(Q_r)$ since $Q_r > N[\bar{G}(\beta)]$. This implies that whether $\psi(\delta, \beta, Q_r)$ for $\delta < \delta^*(Q_r)$ is positive or negative depends on the sign of $\varphi_2(\delta, \beta, Q_r)$. Note that $\lim_{\delta \downarrow 0} \varphi_2(\delta, \beta, Q_r) < 0$ and $\lim_{\delta \uparrow \delta^*(Q_r)} \varphi_2(\delta, \beta, Q_r) \ge 0$, where the first inequality follows from $\delta < \delta^*(Q_r) < \beta$ and the second inequality follows from the continuity of $\psi(\delta, \beta, Q_r)$ and $\varphi_1(\delta) \ge 0$ for all $\delta \ge \delta_1^E$. Note that the second inequality holds as an equality for $\delta^*(Q_r) = \delta_1^E$, and it is strict for $\delta^*(Q_r) > \delta_1^E$. This by (B5) implies that there exists $\delta_2^E(\beta, Q_r) \in (0, \delta^*(Q_r))$ $(\delta_2^E(\beta, Q_r) \in (0, \delta^*(Q_r))$ for $\delta^*(Q_r) > \delta_1^E)$ such that $\varphi_2(\delta_2^E(\beta, Q_r), Q_r) = 0$, where $\delta_2^E(\beta, Q_r)$ is given by (8). Then, $\psi(\delta, \beta, Q_r)$ is negative (positive) for $\delta < \delta_2^E(\beta, Q_r)$ $(\delta > \delta_2^E(\beta, Q_r))$; moreover, $\psi(\delta_2^E(\beta, Q_r), \beta, Q_r) = 0$. Therefore, $\delta^E(\beta, Q_r)$ is unique and equal to $\delta_2^E(\beta, Q_r)$ if $\delta_1^E \le \delta^*(Q_r)$.

Second, assume that $\delta_1^E > \delta^*(Q_r)$. In this case, we have $\lim_{\delta \downarrow 0} \varphi_2(\delta, \beta, Q_r) < 0$ and $\lim_{\delta \uparrow \delta^*(Q_r)} \varphi_2(\delta, \beta, Q_r) < 0$, where the first inequality follows from $Q_r > N[\bar{G}(\beta)]$, and the second inequality follows from $\delta_1^E > \delta^*(Q_r)$ and $\varphi_1(\delta) < 0$ for all $\delta < \delta_1^E$. By (B5), it follows that $\psi(\delta, \beta, Q_r)$ is negative for $\delta \in [0, \delta^*(Q_r))$. On the other hand, $\psi(\delta, \beta, Q_r) = \varphi_1(\delta)$ for $\delta \in [\delta^*(Q_r), \bar{\delta}]$. Recall that $\delta_1^E \in (\delta_{zr}, \bar{\delta})$ is unique and satisfies $\varphi_1(\delta_1^E) = 0$. This implies that $\delta^E(\beta, Q_r)$ is unique and equal to δ_1^E when $\delta_1^E > \delta^*(Q_r)$.

Subcase a.1.2 $(\delta_{zr} > \delta^*(Q_r))$: By (B1), it turns out that

$$\psi(\delta, \beta, Q_r) = \begin{cases} -r\dfrac{Q_r/N - \bar{G}(\beta)}{G(\beta) - G(\delta)} - \theta & \delta < \delta^*(Q_r), \\ -(r + \theta) & \delta^*(Q_r) \le \delta \le \delta_{zr}, \\ \varphi_1(\delta) & \delta_{zr} < \delta \le \bar{\delta} \end{cases}$$

for all $Q_r > N[\bar{G}(\beta)]$ and $\beta \in [0, \bar{\delta}]$, where $\varphi_1(\delta)$ is given by (B2). Note that $\psi(\delta, \beta, Q_r) < 0$ for all $\delta \le \delta_{zr}$. Then, $\delta_1^E \in (\delta_{zr}, \bar{\delta})$ being unique δ which satisfies $\varphi_1(\delta) = 0$ implies that $\delta^E(\beta, Q_r)$ is unique and equal to δ_1^E.

Case a.2 $(Q_r \in (0, N[\bar{G}(\beta)]])$: For brevity, we sketch the proof of this case; however, it is very similar to that of Case a.1. By Lemma 1, vaccine availability for the marginal individual with valuation δ is zero if she is in the low priority group in this case, i.e., $\phi_l(\delta, \delta, \beta, Q_r) = 0$ for $\delta < \beta$. Hence, $\psi(\delta, \beta, Q_r) = -\theta$ for all $\delta < \beta$ and $Q_r \in (0, N[\bar{G}(\beta)]]$. This implies $\delta^E(\beta, Q_r) \ge \beta$ for $Q_r \in (0, N[\bar{G}(\beta)]]$. We define $\delta^*(Q_r)$ and δ_1^E as in Case a.1. Note that $\bar{G}(\delta^*(Q_r)) = Q_r/N$ and $\delta^*(Q_r) \ge \beta$ since $Q_r \le N[\bar{G}(\beta)]$ in this case. Then, it turns out that for all $Q_r \in (0, N[\bar{G}(\beta)]]$ and $\beta \in [0, \bar{\delta}]$,

$$\psi(\delta, \beta, Q_r) = \begin{cases} -r\dfrac{Q_r}{N[\bar{G}(\delta)]} - \theta & \beta \le \delta < \delta^*(Q_r), \\ -(r + \theta) & \delta^*(Q_r) \le \delta \le \delta_{zr}, \\ \varphi_1(\delta) & \delta_{zr} < \delta \le \bar{\delta} \end{cases}$$

if $\delta_{zr} > \delta^*(Q_r)$,

$$\psi(\delta, \beta, Q_r) = \begin{cases} \dfrac{1}{\bar{G}(\delta)} \varphi_2(\delta, \bar{\delta}, Q_r) & \beta \le \delta < \delta^*(Q_r), \\ \varphi_1(\delta) & \delta^*(Q_r) \le \delta \le \bar{\delta} \end{cases}$$

if $\delta_{zr} \le \delta^*(Q_r)$, where $\varphi_1(\delta)$ and $\varphi_2(\delta, \bar{\delta}, Q_r)$ are, respectively, given by (B2) and (B3). Similar to Case a.1.2, one can show that $\delta^E(\beta, Q_r)$ is unique and equal to δ_1^E when $Q_r \in (0, N[\bar{G}(\beta)]]$ and $\delta^*(Q_r) < \delta_{zr}$. When $\delta^*(Q_r) \ge \delta_{zr}$, we need to consider two subcases: $(Q_r/N)[\beta p(Q_r/N) - r] < \theta \bar{G}(\beta)$ and $(Q_r/N)[\beta p(Q_r/N) - r] \ge \theta \bar{G}(\beta)$. Similar to Case a.1.1, one can show that when $(Q_r/N)[\beta p(Q_r/N) - r] < \theta \bar{G}(\beta)$, the threshold value $\delta^E(\beta, Q_r)$ is unique and it is equal to $\delta_2^E(\bar{\delta}, Q_r) \in (\beta, \delta^*(Q_r))$ if $\delta_1^E \le \delta^*(Q_r)$ and is equal to δ_1^E if $\delta_1^E > \delta^*(Q_r)$). On the other hand, when $(Q_r/N)[\beta p(Q_r/N) - r] \ge \theta \bar{G}(\beta)$, net utility from searching is positive for all individuals with $\delta \ge \beta$, i.e., all individuals in the high priority group search for the vaccine. Therefore, the threshold value $\delta^E(\beta, Q_r)$ is unique and equal to β when $(Q_r/N)[\beta p(Q_r/N) - r] \ge \theta \bar{G}(\beta)$. In addition, note by $\lim_{\delta \downarrow 0} \varphi_2(\delta, \bar{\delta}, Q_r) < 0$, $\lim_{\delta \downarrow \beta} \varphi_2(\delta, \bar{\delta}, Q_r) \ge 0$ and $\partial \varphi_2(\delta, \bar{\delta}, Q_r)/\partial \delta > 0$ that $\delta_2^E(\bar{\delta}, Q_r)$ satisfying (8) is in $(0, \beta]$ when $(Q_r/N)[\beta p(Q_r/N) - r] \ge \theta \bar{G}(\beta)$.

In summary, using $\delta_1^E > \delta_{zr}$, the threshold valuation level $\delta^E(\beta, Q_r)$ for all $\beta \in [0, \bar{\delta}]$ and $Q_r > 0$ satisfies

$$\delta^E(\beta, Q_r) = \begin{cases} \beta & \dfrac{Q_r}{N}\left[\beta p\left(\dfrac{Q_r}{N}\right) - r\right] \geq \theta\bar{G}(\beta) \text{ and} \\ & \dfrac{Q_r}{N} \leq \min\{\bar{G}(\beta), \bar{G}(\delta_{zr})\}, \\ \delta_2^E(\bar{\delta}, Q_r) & \dfrac{Q_r}{N}\left[\beta p\left(\dfrac{Q_r}{N}\right) - r\right] < \theta\bar{G}(\beta) \text{ and} \\ & \dfrac{Q_r}{N} \leq \min\{\bar{G}(\beta), \bar{G}(\delta_1^E)\}, \\ \delta_2^E(\beta, Q_r) & \bar{G}(\beta) < \dfrac{Q_r}{N} \leq \bar{G}(\delta_1^E), \\ \delta_1^E & \text{otherwise.} \end{cases} \tag{B6}$$

b. Finding the equilibrium demand: Here, using (B6), we show that the equilibrium fraction searching for the vaccine is given by (6). First of all, consider $Q_r > N[\bar{G}(\delta_1^E)]$. When $Q_r/N \leq \min\{\bar{G}(\beta), \bar{G}(\delta_{zr})\}$, we have

$$\begin{aligned} \beta p\left(\frac{Q_r}{N}\right) - (r+\theta) &\leq \delta^*(Q_r) p\left(\frac{Q_r}{N}\right) - (r+\theta) \\ &= \varphi_1(\delta^*(Q_r)) < 0 \end{aligned}$$

for $Q_r > N[\bar{G}(\delta_1^E)]$, where the last inequality comes from $\varphi_1(\delta)$ being strictly increasing in δ for $\delta > \delta_{zr}$. This implies by (B6) that $\delta^E(\beta, Q_r) = \beta$ is never possible if $Q_r > N[G(\delta_1^E)]$; moreover, $\delta^E(\beta, Q_r) = \delta_1^E$ for all $Q_r > N[\bar{G}(\delta_1^E)]$. As a result, $\bar{G}(\delta^E(\beta, Q_r)) = \bar{G}(\delta_1^E)$ for all $Q_r > N[\bar{G}(\delta_1^E)]$.

Next, we assume $Q_r \in (0, N[\bar{G}(\delta_1^E)]]$ and consider two cases: $\beta \leq \delta_1^E$ and $\beta > \delta_1^E$.

Case b.1 ($\beta < \delta_1^E$): In this case, $Q_r < N[\bar{G}(\beta)]$ for $Q_r \leq N[\bar{G}(\delta_1^E)]$. Moreover,

$$\min\{\bar{G}(\beta), \bar{G}(\delta_1^E)\} = \min\{\bar{G}(\beta), \bar{G}(\delta_{zr})\} \geq \bar{G}(\delta_1^E)$$

since $\delta_1^E \in (\delta_{zr}, \bar{\delta})$. Then, it follows by (B6) that, for $Q_r \leq N[\bar{G}(\delta_1^E)]$, $\bar{G}(\delta^E(\beta, Q_r)) = \bar{G}(\beta)$ if $(Q_r/N)[\beta p(Q_r/N) - r] \geq \theta\bar{G}(\beta)$, and

$$\bar{G}(\delta^E(\beta, Q_r)) = \bar{G}(\delta_2^E(\bar{\delta}, Q_r))$$

if $(Q_r/N)[\beta p(Q_r/N) - r] < \theta\bar{G}(\beta)$.

Case b.2 ($\beta \geq \delta_1^E$): Since $\delta_1^E > \delta_{zr}$, we have

$$\min\{\bar{G}(\beta), \bar{G}(\delta_1^E)\} = \min\{\bar{G}(\beta), \bar{G}(\delta_{zr})\} = \bar{G}(\beta)$$

in this case. By (B6), $\bar{G}(\delta^E(\beta, Q_r)) = \bar{G}(\delta_2^E(\beta, Q_r))$ for all $Q_r \in (N[\bar{G}(\beta)], N[\bar{G}(\delta_1^E)]]$. For $Q_r \leq N[\bar{G}(\beta)]$, $\bar{G}(\delta^E(\beta, Q_r)) = \bar{G}(\beta)$ if $(Q_r/N)[\beta p(Q_r/N) - r] \geq \theta\bar{G}(\beta)$, and $\bar{G}(\delta^E(\beta, Q_r)) = \bar{G}(\delta_2^E(\bar{\delta}, Q_r))$ if $(Q_r/N)[\beta p(Q_r/N) - r] < \theta\bar{G}(\beta)$.

Part (ii): By (6), $\bar{G}(\delta^E(\beta, Q_r)) = \bar{G}(\delta_1^E)$ if $Q_r > N[\bar{G}(\delta_1^E)]$. Also note by (7), (B6), and $\delta_1^E \in (\delta_{zr}, \bar{\delta})$ that for $Q_r = N[\bar{G}(\delta_1^E)]$

$$\begin{aligned} &\beta p\left(\frac{Q_r}{N}\right) - (r+\theta) \\ &\quad = \beta p(\bar{G}(\delta_1^E)) - (r+\theta) < \delta_1^E p(\bar{G}(\delta_1^E)) - (r+\theta) = 0 \end{aligned}$$

if $\delta_1^E > \beta$. This implies that $\delta^E(\beta, Q_r) = \beta$ for $Q_r = N[\bar{G}(\delta_1^E)]$ only if $\beta = \delta_1^E$. This together with (B6) and $\delta_2^E(\bar{\delta}, Q_r) = \delta_2^E(\beta, Q_r) = \delta_1^E$ for $Q_r = N[\bar{G}(\delta_1^E)]$ imply that $\delta^E(\beta, Q_r) = \delta_1^E$ for $Q_r = N[\bar{G}(\delta_1^E)]$ for all $\beta \in [0, \bar{\delta}]$. As a result, the supply is always more than the equilibrium demand for $Q_r > N[\bar{G}(\delta_1^E)]$ and they are equal for $Q_r = N[\bar{G}(\delta_1^E)]$. In the remaining of this part, we assume that $0 < Q_r < N[\bar{G}(\delta_1^E)]$ and consider two cases: $\beta < \delta_1^E$ and $\beta \geq \delta_1^E$.

Case 1 ($\beta < \delta_1^E$): In this case, by (6), $\bar{G}(\delta^E(\beta, Q_r)) = \bar{G}(\beta)$ for $Q_r < N[\bar{G}(\delta_1^E)]$ if $(Q_r/N)[\beta p(Q_r/N) - r] \geq \theta\bar{G}(\beta)$. Since $\beta < \delta_1^E$, the equilibrium demand is more than the available supply when $(Q_r/N)[\beta p(Q_r/N) - r] \geq \theta\bar{G}(\beta)$ and $0 < Q_r < N[\bar{G}(\delta_1^E)]$. On the other hand, if $(Q_r/N)[\beta p(Q_r/N) - r] < \theta\bar{G}(\beta)$, then $\bar{G}(\delta^E(\beta, Q_r)) = \bar{G}(\delta_2^E(\bar{\delta}, Q_r))$ for $0 < Q_r < N[\bar{G}(\delta_1^E)]$. Recall from Case a.2 in Part (i) that $\delta_2^E(\bar{\delta}, Q_r) \in (\beta, \delta^*(Q_r))$ for $0 < Q_r < N[\bar{G}(\delta_1^E)]$, where $\bar{G}(\delta^*(Q_r)) = \min\{1, Q_r/N\}$, when $(Q_r/N)[\beta p(Q_r/N) - r] < \theta\bar{G}(\beta)$. Then, it turns out that $\bar{G}(\delta^E(\beta, Q_r)) > Q_r/N$ for $0 < Q_r < N[\bar{G}(\delta_1^E)]$ if $\beta < \delta_1^E$ and $(Q_r/N)[\beta p(Q_r/N) - r] < \theta\bar{G}(\beta)$.

Case 2 ($\beta \geq \delta_1^E$): By (6) and Case a.1.1 in Part (i), $\delta^E(\beta, Q_r)$ is equal to $\delta_2^E(\beta, Q_r) \in (0, \delta^*(Q_r))$ for all $Q_r \in (N[\bar{G}(\beta)], N[\bar{G}(\delta_1^E)])$. As a result, $\bar{G}(\delta^E(\beta, Q_r)) > Q_r/N$ for $Q_r \in (N[\bar{G}(\beta)], N[\bar{G}(\delta_1^E)])$ if $\beta \geq \delta_1^E$. For $Q_r \in (0, N[\bar{G}(\beta)])$, we have $\bar{G}(\delta^E(\beta, Q_r)) = \bar{G}(\delta_2^E(\bar{\delta}, Q_r))$ if $(Q_r/N)[\beta p(Q_r/N) - r] < \theta\bar{G}(\beta)$, and $\bar{G}(\delta^E(\beta, Q_r)) = \bar{G}(\beta)$ if $(Q_r/N)[\beta p(Q_r/N) - r] \geq \theta\bar{G}(\beta)$. However, both β and $\delta_2^E(\bar{\delta}, Q_r)$ are smaller than $\delta^*(Q_r)$ when $Q_r < N[\bar{G}(\beta)]$ and $\beta \geq \delta_1^E$ ($\delta_2^E(\bar{\delta}, Q_r) \in (\beta, \delta^*(Q_r))$ from Case a.2 in Part (i)). Therefore, $\bar{G}(\delta^E(\beta, Q_r)) > Q_r/N$ for $Q_r \in (0, N[\bar{G}(\beta)])$. Finally, for $Q_r = N[\bar{G}(\beta)]$, $\bar{G}(\delta^E(\beta, Q_r)) = \bar{G}(\beta)$ since $(Q_r/N)[\beta p((Q_r/N)) - r] \geq \theta\bar{G}(\beta)$ if $\beta \geq \delta_1^E$. As a result, $\bar{G}(\delta^E(\beta, Q_r)) = Q_r/N$ when $Q_r = N[\bar{G}(\beta)]$ and $\beta \geq \delta_1^E$. This completes the proof.

Proof of Proposition 2

Note that there are no priority groups if $\beta = 0$ or $\beta = \bar{\delta}$. By Proposition 1 (i), the equilibrium demand for both $\beta = 0$ or $\beta = \bar{\delta}$ is the same. The equilibrium fraction searching for the vaccine when there are no priority group is given by

$$\bar{G}(\delta^E(Q_r)) = \begin{cases} \bar{G}(\delta_2^E(\bar{\delta}, Q_r)) & 0 < Q_r \leq N[\bar{G}(\delta_1^E)], \\ \bar{G}(\delta_1^E) & Q_r > N[\bar{G}(\delta_1^E)], \end{cases} \tag{B7}$$

where $\delta^E(Q_r)$ denotes the threshold value with no priority groups, and δ_1^E and $\delta_2^E(\bar{\delta}, Q_r)$ are, respectively, given by (7) and (8). Recall from Part (ii) of proof of Proposition 1 that $\bar{G}(\delta^E(\beta, Q_r)) = \bar{G}(\delta_1^E)$ for $Q_r = N[\bar{G}(\delta_1^E)]$ and $\beta \in [0, \bar{\delta}]$. Then it follows for $Q_r \geq N[\bar{G}(\delta_1^E)]$ that the demand in equilibrium with and without priority groups are equal to $N[\bar{G}(\delta_1^E)]$. Hence, for $Q_r \geq N[\bar{G}(\delta_1^E)]$, the vaccinated fraction of population in both equilibria are equal to $N[\bar{G}(\delta_1^E)]$. If $Q_r = 0$, nobody searches for the vaccine and hence vaccinated fraction in both equilibria are equal to zero.

Next, we prove Proposition 2 for $0 < Q_r < N[\bar{G}(\delta_1^E)]$. First, recall by Proposition 1(ii) that the equilibrium demand is at least as much as the available supply for all $\beta \in [0, \bar{\delta}]$ when $0 < Q_r < N[\bar{G}(\delta_1^E)]$. This implies that, for $0 < Q_r < N[\bar{G}(\delta_1^E)]$, the vaccinated fraction in equilibrium with two groups is equal to that with no groups, and they are both equal to Q_r/N. By (B7), for $0 < Q_r < N[\bar{G}(\delta_1^E)]$, the demand in the equilibrium with no priority group is equal to $N[\bar{G}(\delta_2^E(\bar{\delta}, Q_r))]$. Next, we consider two cases: $\beta < \delta_1^E$ and $\beta \geq \delta_1^E$.

Case 1 ($\beta < \delta_1^E$): By (6), the equilibrium demand with two groups is equal to $N[\bar{G}(\delta_2^E(\bar{\delta}, Q_r))]$ if $(Q_r/N)[\beta p(Q_r/N) - r] < \theta\bar{G}(\beta)$. Thus, the equilibrium demand with and without priority groups are equal when $(Q_r/N)\ [\beta p(Q_r/N) - r] < \theta\bar{G}(\beta)$. By (6), the equilibrium demand with two priority groups is equal to $N[\bar{G}(\beta)]$ if $(Q_r/N)[\beta p(Q_r/N) - r] \geq \theta\bar{G}(\beta)$. Recall from proof of Proposition 1 (Case a.2 in Part (i)) that $\delta_2^E(\bar{\delta}, Q_r) \in (0, \beta]$ if $(Q_r/N)[\beta p(Q_r/N) - r] \geq \theta\bar{G}(\beta)$. Thus, the equilibrium demand with no priority group is greater than or equal to that with two priority groups when $(Q_r/N)[\beta p(Q_r/N) - r] \geq \theta\bar{G}(\beta)$.

Case 2 ($\beta \geq \delta_1^E$): Similar to Case 1, one can show by (6) that, for $Q_r \leq N[\bar{G}(\beta)]$, the equilibrium demand with two priority groups is less than or equal to that with no priority group. For $Q_r \in (N[\bar{G}(\beta)], N[\bar{G}(\delta_1^E)])$, the equilibrium demand with two priotiy groups is equal to $N[\bar{G}(\delta_2^E(\beta, Q_r))]$. Note by (8) that

$$\frac{\partial \delta_2^E(\beta, Q_r)}{\partial \beta} = -\frac{g(\beta)[\delta_2^E(\beta, Q_r)p(Q_r/N) - (r+\theta)]}{((Q_r - N[\bar{G}(\beta)])/N)p(Q_r/N) + \theta g(\delta_2^E(\beta, Q_r))}$$

for all $Q_r \in (N[\bar{G}(\beta)], N[\bar{G}(\delta_1^E)])$. Recall from Case a.1.1 in the proof of Proposition 1 that $\delta_2^E(\beta, Q_r) \in (0, \delta^*(Q_r))$ for $Q_r \in (N[\bar{G}(\beta)], N[\bar{G}(\delta_1^E)])$. This implies that $Q_r/N - \bar{G}(\beta) < G(\beta) - G(\delta_2^E(\beta, Q_r))$ for $Q_r \in (N[\bar{G}(\beta)], N[\bar{G}(\delta_1^E)])$. It follows by (8) that $\delta_2^E(\beta, Q_r)p(Q_r/N) - (r + \theta) > 0$ for $Q_r \in (N[\bar{G}(\beta)], N[\bar{G}(\delta_1^E)])$, which further implies that $\partial\delta_2^E(\beta, Q_r)/\partial\beta < 0$ since G is strictly increasing. Then, it turns out for all $Q_r \in (N[\bar{G}(\beta)], N[\bar{G}(\delta_1^E)])$ that $\delta_2^E(\bar{\delta}, Q_r) < \delta_2^E(\beta, Q_r)$ and hence the demand in the equilibrium with two priority groups is less than that in equilibrium with no priority group. This completes the proof.

Proof of Proposition 3

Under efficient allocation, Lemma 1 is still valid because individuals with high infection disutilities have at least the same probability of getting the vaccine compared to those with lower infection disutilities. This indicates that under efficient allocation there is also a marginal individual who is indifferent between searching and not searching. Let $\delta^{ER}(Q_r)$ for $Q_r \geq 0$ be the disutility of this marginal individual. We need to characterize $\delta^{ER}(Q_r)$ for $Q_r \geq 0$ to find the equilibrium demand. For this purpose, we define $\psi(\delta, Q_r)$ as the excess of utility to the marginal individual with valuation δ when searching over not searching. Note that, for $Q_r \geq N$, $\psi(\delta, Q_r) = \varphi_1(\delta)$ for all $\delta \in [0, \bar{\delta}]$, where $\varphi_1(\delta)$ is given by (B2). Then, using definition of δ_1^E from Case a.1.1 in the proof of Proposition 1, we obtain $\delta^{ER}(Q_r) = \delta_1^E$ for $Q_r \geq N$.

Next, we assume that $0 \leq Q_r < N$. Define $\delta^*(Q_r) = \bar{G}^{-1}(\min\{1, Q_r/N\})$ for $Q_r \geq 0$. Note that there exists a unique $\delta^*(Q_r) \in (0, \bar{\delta}]$ in this case since $0 \leq Q_r < N$. Also note that under efficient allocation each individual knows for sure whether the vaccine is available for her since they observe Q_r and the vaccine is sequentially allocated based on δ. Then, it turns out that

$$\psi(\delta, Q_r) = \begin{cases} -\theta & \delta \leq \delta^*(Q_r), \\ \varphi_1(\delta) & \delta^*(Q_r) < \delta \leq \bar{\delta} \end{cases}$$

for all $Q_r \in [0, N)$, and $\delta \in [0, \bar{\delta}]$, where h and $\varphi_1(\delta)$ are, respectively, given by (3) and (B2). Recall from Case a.1.1 in the proof of Proposition 1 that there exists a unique $\delta_1^E \in (\delta_{zr}, \bar{\delta})$ such that $\varphi_1(\delta_1^E) = 0$, and $\varphi_1(\delta)$ is positive (negative) for all $\delta > \delta_1^E$ ($\delta < \delta_1^E$). Thus, if $\delta_1^E > \delta^*(Q_r)$ (i.e., $Q_r > N[\bar{G}(\delta_1^E)]$), $\psi(\delta_1^E, Q_r) = 0$, and $\psi(\delta, Q_r)$ is positive (negative) for all $\delta > \delta_1^E$ ($\delta < \delta_1^E$) and $Q_r \in [0, N)$. Hence, the equilibrium demand under efficient allocation is equal to $N[\bar{G}(\delta_1^E)]$ when $Q_r \in (N[\bar{G}(\delta_1^E)], N)$. On the other hand, if $\delta_1^E \leq \delta^*(Q_r)$ (i.e., $Q_r \leq N[\bar{G}(\delta_1^E)]$), $\psi(\delta, Q_r)$ is positive (negative) for all $\delta > \delta^*(Q_r)$ ($\delta \leq \delta_1^E$) and $Q_r \in [0, N)$. Thus, the equilibrium demand under efficient allocation is equal to available supply (Q_r) when $Q_r \leq N[\bar{G}(\delta_1^E)]$.

Proof of Proposition 4

First and second derivatives of $\pi_E(Q)$ in (9) are

$$\frac{d\pi_E(Q)}{dQ} = w\int_0^{N[G(\delta_1^E)]/Q} u dM(u) - c,$$

$$\frac{d^2\pi_E(Q)}{dQ^2} = -w\frac{N^2G^2(\delta_1^E)}{Q^3}m\left(\frac{N[G(\delta_1^E)]}{Q}\right) < 0$$

for $Q \geq 0$ since yield distribution is strictly increasing. The inequality implies that $\pi_E(Q)$ is strictly concave. Moreover, by $w\mu > c$, we have $\lim_{Q\downarrow 0} d\pi_E(Q)/dQ > 0$ and $\lim_{Q\uparrow\infty} d\pi_E(Q)/dQ = -c < 0$. Then, it follows that there exists a finite, positive, and unique Q_E which maximizes $\pi_E(Q)$. By first order condition, Q_E satisfies (10).

Proof of Proposition 5

We let $\delta^*(Q_r)$ satisfy $\bar{G}(\delta^*(Q_r)) = \min\{1, Q_r/N\}$ for all $Q_r \geq 0$. Note that $\delta^*(Q_r)$ is unique and in $(0, \bar{\delta}]$ for $Q_r \in [0, N)$; moreover, $\delta^*(Q_r) = 0$ for $Q_r \geq N$. $\bar{G}(\delta^*(Q_r))$ denotes the maximum fraction of population that the available supply can satisfy.

Part (i): We first characterize the threshold level in the centralized system($\delta^C(Q_r)$) and then show that the fraction searching for the vaccine satisfies (15). For this purpose, we let $H(\delta, \beta, Q_r)$ denote the objective function in (13). Next, we consider two cases: $\delta_{zr} \leq \delta^*(Q_r)$ and $\delta_{zr} > \delta^*(Q_r)$, where δ_{zr} is given by (5).

Case 1 ($\delta_{zr} \leq \delta^*(Q_r)$): In this case, for all $\beta \in [0, \bar{\delta}]$, we have

$$H(\delta, \beta, Q_r) = \begin{cases} N\left(\bar{V} - \theta\bar{G}(\delta) - (r-w)\dfrac{Q_r}{N} - p\left(\dfrac{Q_r}{N}\right)\displaystyle\int_0^\delta z dG(z)\right) \\ \quad - Np\left(\dfrac{Q_r}{N}\right)\left[\left(1 - \dfrac{Q_r}{N[\bar{G}(\beta)]}\right)\displaystyle\int_\beta^{\bar{\delta}} z dG(z) + \int_\delta^\beta z dG(z)\right] \\ \qquad\qquad \delta < \beta, \\ N\left(\bar{V} - \theta\bar{G}(\delta) - (r-w)\dfrac{Q_r}{N} - p\left(\dfrac{Q_r}{N}\right)\displaystyle\int_0^\delta z dG(z)\right) \\ \quad - N\left(1 - \dfrac{Q_r}{N[\bar{G}(\delta)]}\right)p\left(\dfrac{Q_r}{N}\right)\displaystyle\int_\delta^{\bar{\delta}} z dG(z) \\ \qquad\qquad \beta \leq \delta < \delta^*(Q_r), \\ N(\bar{V} - (r+\theta-w)\bar{G}(\delta) - p(\bar{G}(\delta))\int_0^\delta z dG(z)) \\ \qquad\qquad \delta^*(Q_r) \leq \delta \leq \bar{\delta} \end{cases}$$

when $Q_r \in [0, N[\bar{G}(\beta)]]$, and

$$H(\delta, \beta, Q_r) = \begin{cases} N\left(\bar{V} - \theta\bar{G}(\delta) - (r-w)\dfrac{Q_r}{N} - p\left(\dfrac{Q_r}{N}\right)\displaystyle\int_0^\beta z dG(z)\right) \\ \quad + \dfrac{Q_r - N[\bar{G}(\beta)]}{G(\beta) - G(\delta)} p\left(\dfrac{Q_r}{N}\right)\displaystyle\int_\delta^\beta z dG(z) & \delta < \delta^*(Q_r), \\ N(\bar{V} - (r+\theta-w)\bar{G}(\delta) - p(\bar{G}(\delta))\displaystyle\int_0^\delta z dG(z)) & \\ & \delta \geq \delta^*(Q_r) \end{cases}$$

when $Q_r > N[\bar{G}(\beta)]$. Then, it turns out that for all $\beta \in [0, \bar{\delta}]$

$$\frac{\partial H(\delta, \beta, Q_r)}{\partial \delta} = \begin{cases} N\theta g(\delta) & \delta < \beta, \\ Ng(\delta)\left[\theta - \dfrac{Q_r}{N[\bar{G}(\delta)]} p\left(\dfrac{Q_r}{N}\right)\left(\delta - \displaystyle\int_0^\delta z \frac{dG(z)}{\bar{G}(\delta)}\right)\right] & \beta \leq \delta < \delta^*(Q_r), \\ Ng(\delta)\left[r + \theta - w + p'(\bar{G}(\delta))\displaystyle\int_0^\delta z dG(z) - \delta p(\bar{G}(\delta))\right] & \\ & \delta \geq \delta^*(Q_r) \end{cases}$$

for all $Q_r \in [0, N[\bar{G}(\beta)]]$, and

$$\frac{\partial H(\delta, \beta, Q_r)}{\partial \delta} = \begin{cases} Ng(\delta)\left[\theta - \dfrac{Q_r/N - \bar{G}(\beta)}{G(\beta) - \bar{G}(\delta)} p\left(\dfrac{Q_r}{N}\right)\left(\delta - \displaystyle\int_\delta^\beta z \frac{dG(z)}{G(\beta) - \bar{G}(\delta)}\right)\right] & \delta < \delta^*(Q_r) \\ Ng(\delta)\left[p'(\bar{G}(\delta))\displaystyle\int_0^\delta z dG(z) - \delta p(\bar{G}(\delta)) + r + \theta - w\right] & \\ & \delta \geq \delta^*(Q_r) \end{cases}$$

for all $Q_r > N[\bar{G}(\beta)]$. Note that for $a < b$,

$$a < \int_a^b z \frac{dG(z)}{G(b) - G(a)} \tag{B8}$$

since G is strictly increasing. This implies that $\partial H(\delta, \beta, Q_r)/\partial\delta > 0$ for all $Q_r \geq 0$, $\beta \in [0, \bar{\delta}]$ and $\delta < \delta^*(Q_r)$. Observe from (14) that the sign of $\partial H(\delta, \beta, Q_r)/\partial\delta$ for $\delta \geq \delta^*(Q_r)$ depends on whether $B(\delta)$ is positive or negative since G is strictly increasing. By $r > w$, (5) and Assumption 1, we have $B(\delta) < 0$ for all $\delta \in [0, \delta_{zr}]$, $\lim_{\delta\downarrow\delta_{zr}^+} B(\delta) = -(\lim_{\delta\downarrow\delta_{zr}^+} p'(\bar{G}(\delta)))\int_0^{\delta_{zr}} z dG(z) - (r + \theta - w)$ and $\lim_{\delta\uparrow\bar{\delta}} B(\delta) > 0$. Then, $dB(\delta)/d\delta = 0$ for $\delta \in [0, \delta_{zr}]$ and

$$\begin{aligned} \frac{dB(\delta)}{d\delta} &= g(\delta)\left(-2\delta p'(\bar{G}(\delta)) + p''(\bar{G}(\delta))\int_0^\delta z dG(z)\right) \\ &= g(\delta)\left(2p'(\bar{G}(\delta))\left[\int_0^\delta z \frac{dG(z)}{G(\delta)} - \delta\right]\right. \\ &\quad \left. - [2p'(\bar{G}(\delta)) - G(\delta)p''(\bar{G}(\delta))]\int_0^\delta z \frac{dG(z)}{G(\delta)}\right) > 0 \end{aligned}$$

for all $\delta \in (\delta_{zr}, \bar{\delta}]$, where the inequality follows from Assumption 1(iii) and (B8). Note that $p(h)$ may not be differentiable at $h = h_{zr}$ so that $\lim_{\delta\downarrow\delta_{zr}^+} p'(\bar{G}(\delta_{zr}))$ is not necessarily zero. Therefore, both $\lim_{\delta\downarrow\delta_{zr}^+} B(\delta) < 0$ and $\lim_{\delta\downarrow\delta_{zr}^+} B(\delta) \geq 0$ are possible. This implies that $\tilde{\delta}$ such that $B(\tilde{\delta}) = 0$ may not always exists. Hence, we define $\delta_1^C = \inf\{\delta \in [0, \bar{\delta}]: B(\delta) \geq 0\}$. Since $dB(\delta)/d\delta > 0$ for all $\delta \in (\delta_{zr}, \bar{\delta}]$ and $\lim_{\delta\uparrow\bar{\delta}} B(\delta) > 0$, we have $\delta_1^C = \delta_{zr}$ if $\lim_{\delta\downarrow\delta_{zr}^+} B(\delta) \geq 0$, and $\delta_1^C = \tilde{\delta}$, where $\tilde{\delta} \in (\delta_{zr}, \bar{\delta})$ is unique and satisfies $B(\tilde{\delta}) = 0$, if $\lim_{\delta\downarrow\delta_{zr}^+} B(\delta) < 0$. Note that $B(\delta) > 0$ ($B(\delta) < 0$) for all $\delta > \delta_1^C$ ($\delta < \delta_1^C$).

If $B(\delta^*(Q_r)) \geq 0$, i.e., $\delta^*(Q_r) \geq \delta_1^C$, then $B(\delta) > 0$, so that $H(\delta, \beta, Q_r)$ is strictly decreasing in δ, for all $\delta > \delta^*(Q_r)$. Thus, $\delta^C(Q_r) = \delta^*(Q_r)$ for $Q_r \leq N[G(\delta_1^C)]$ when $\delta^*(Q_r) \geq \delta_{zr}$. On the other hand, if $B(\delta^*(Q_r)) < 0$, i.e., $\delta^*(Q_r) < \delta_1^C$, then $B(\delta)$ is positive (negative) for all $\delta > \delta_1^C$ ($\delta \in [\delta^*(Q_r), \delta_1^C]$). This implies that $H(\delta, \beta, Q_r)$ is strictly increasing (decreasing) in δ for all $\delta < \delta_1^C$ ($\delta > \delta_1^C$) if $\delta^*(Q_r) < \delta_1^C$. As a result, $\delta^C(Q_r) = \delta_1^C$ for all $Q_r > N[G(\delta_1^C)]$ when $\delta^*(Q_r) \geq \delta_{zr}$.

Case 2 ($\delta_{zr} > \delta^*(Q_r)$): For all $\delta \in [0, \bar{\delta}]$ and $Q_r \geq 0$, we have

$$H(\delta, \beta, Q_r) = \begin{cases} N\left(\bar{V} - \theta\bar{G}(\delta) - (r-w)\dfrac{Q_r}{N}\right) & \delta < \delta^*(Q_r), \\ N(\bar{V} - (r+\theta-w)\bar{G}(\delta)) & \delta^*(Q_r) \leq \delta \leq \delta_{zr}, \\ N\left(\bar{V} - (r+\theta-w)\bar{G}(\delta) - p(\bar{G}(\delta))\displaystyle\int_0^\delta z dG(z)\right) & \\ & \delta > \delta_{zr} \end{cases}$$

in this case. It follows that for all $\delta \in [0, \bar{\delta}]$ and $Q_r \geq 0$

$$\frac{\partial H(\delta, \beta, Q_r)}{\partial \delta} = \begin{cases} N\theta g(\delta) & \delta < \delta^*(Q_r), \\ Ng(\delta)(r + \theta - w) & \delta^*(Q_r) \leq \delta \leq \delta_{zr}, \\ -Ng(\delta)B(\delta) & \delta > \delta_{zr}, \end{cases}$$

where $B(\delta)$ is given by (14). Observe that $\partial H(\delta, \beta, Q_r)/\partial\delta > 0$ for all $\delta \leq \delta_{zr}$ and the sign of $\partial H(\delta, \beta, Q_r)/\partial\delta$ for $\delta > \delta_{zr}$ depends on whether $B(\delta)$ is positive or negative since G is strictly increasing. If we recall the definition of δ_1^C from Case 1, then $H(\delta, \beta, Q_r)$ is strictly increasing (decreasing) in δ for all $\delta < \delta_1^C$ ($\delta > \delta_1^C$) if $\delta_{zr} > \delta^*(Q_r)$. As a result, $\delta^C(Q_r) = \delta_1^C$ when $\delta_{zr} > \delta^*(Q_r)$.

In summary, the threshold value $\delta^C(Q_r)$ for $Q_r \geq 0$ satisfies $\delta^C(Q_r) = \delta^*(Q_r)$ for $Q_r < N[G(\delta_1^C)]$ and $\delta^C(Q_r) = \delta_1^C$ for $Q_r \geq N[G(\delta_1^C)]$ since $\delta_1^C \in (\delta_{zr}, \bar{\delta})$ and $\delta^*(Q_r) = \delta_1^C$ for $Q_r = N[G(\delta_1^C)]$. This implies that $\bar{G}(\delta^C(Q_r)) = \min\{Q_r/N, \bar{G}(\delta_1^C)\}$.

Part (ii): We first show that $\delta_1^C < \delta_1^E$. Recall from Proposition 1 that $\delta_1^E \in (\delta_{zr}, \bar{\delta})$. Thus, if $\delta_1^C = \delta_{zr}$, i.e., $\lim_{\delta\downarrow\delta_{zr}^+} B(\delta) \geq 0$, then $\delta_1^C < \delta_1^E$. Next, we show that $\delta_1^C < \delta_1^E$ if $\delta_1^C = \tilde{\delta}$, i.e., $\lim_{\delta\downarrow\delta_{zr}^+} B(\delta) < 0$. Note by (7) and (14) that $B(\delta_1^E) = -p'(\bar{G}(\delta_1^E))\int_0^{\delta_1^E} z dG(z) + w > 0$. Recall that $dB(\delta)/d\delta > 0$ for all $\delta \in (\delta_{zr}, \bar{\delta}]$ and $B(\delta_1^C) = 0$ if $\delta_1^C = \tilde{\delta}$. This implies that $\delta_1^C < \delta_1^E$.

Now, consider two cases: $Q_r \in [0, N[\bar{G}(\delta_1^E)]]$ and $Q_r > N[\bar{G}(\delta_1^E)]$. If $Q_r \in [0, N[\bar{G}(\delta_1^E)]]$, then $\bar{G}(\delta^E(\beta, Q_r)) \geq Q_r/N$ by Proposition 1(ii) and $\bar{G}(\delta^C) = Q_r/N$ by Part (i) since $\delta_1^C < \delta_1^E$. On the other hand, if $Q_r > N[\bar{G}(\delta_1^E)]$, then

$\bar{G}(\delta^E(\beta, Q_r)) = \bar{G}(\delta_1^E)$ by Proposition 1 and $\bar{G}(\delta^C(Q_r)) = \min\{\bar{G}(\delta_1^C), Q_r/N\} > \bar{G}(\delta_1^E)$ since $\delta_1^C < \delta_1^E$.

Proof of Proposition 6

For brevity, we sketch proof of the first part. Using (18), $\delta_1^C \geq \delta_{zr}$, and properties of functions B, G, and M, we first prove that $\mathscr{W}_C(Q)$ is strictly convex. Then, using Assumption 1(ii) and $w\mu > c$, we show that $\mathscr{W}_C(Q)$ has a finite and positive maximizer (Q_C). Finally, by first order optimality condition, Q_C satisfies (19).

Next, we prove the second part by contradiction. Suppose, contrary to our claim, that $Q_E \geq Q_C$. By (14), we have

$$c > \int_0^{N[\bar{G}(\delta_1^E)]/Q_C} B(\delta_2^C(uQ_C))udM(u) > w\int_0^{N[\bar{G}(\delta_1^E)]/Q_E} udM(u).$$

We get the first inequality above by (19), and by $\delta_1^C < \delta_1^E$ and $B(\delta) > 0$ for $\delta > \delta_1^C$ (see proof of Proposition 5). The last inequality follows from $B(\delta)$ being strictly increasing in δ for $\delta > \delta_1^C$, $B(\delta_2^C(N[\bar{G}(\delta_1^E)])) > w$ and our supposition that $Q_E \geq Q_C$. However, the final result is a contradiction to Proposition 4 and hence $Q_E < Q_C$.

Proof of Proposition 7

We skip the proof of the first part for brevity. However, it is very similar to that of Proposition 4. Next, we prove the second part. Note by (10) and (21) that $N[\bar{G}(\delta_1^E)]/Q_E = N[\bar{G}(\delta_1^C)]/Q_D$. Then, by $\delta_1^C < \delta_1^E$ (see proof of Proposition 5(ii)) and G being strictly increasing, it turns out that $Q_D > Q_E$.

Proof of Proposition 8

First, we show that if $Q_D < Q_C$, then the inequality in (22) holds with greater than sign. Hence, suppose that $Q_D < Q_C$. Recall that $\mathscr{W}_C(Q)$ is strictly concave in Q and hence the LHS of (19) is strictly decreasing in Q. This together with $Q_D < Q_C$ implies that

$$\int_0^{N[\bar{G}(\delta_1^C)]/Q_D} \left(\delta_2^C(uQ_D)p\left(\frac{uQ_D}{N}\right) - p'\left(\frac{uQ_D}{N}\right) \cdot \int_0^{\delta_2^C(uQ_D)} zdG(z) - (r+\theta-w)\right) udM(u) > c.$$

Moreover, by (21) and (23), we have $N[\bar{G}(\delta_1^C)]/Q_D = k$. If we substitute k in above inequality, we obtain the condition in (22) with greater than sign.

Next, we prove the other direction. Suppose that the inequality in (22) holds with greater than sign. In this case, by using (21) and (23), we obtain $\partial\mathscr{W}_C(Q_D)/\partial Q > 0$. Finally, $\mathscr{W}_C(Q)$ being strictly concave implies that $Q_D < Q_C$. Similarly, one can prove that $Q_D > Q_C$ if and only if the inequality in (22) holds with less than sign.

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