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Newsvendor Mergers

Soo-Haeng Cho,[a] **Xin Wang**[a]

[a] Tepper School of Business, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213
Contact: soohaeng@andrew.cmu.edu (S-HC); xinwang1@andrew.cmu.edu (XW)

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Abstract. This paper studies a merger between price-setting newsvendors in an oligopolistic market. It is well known that inventory pooling can greatly reduce inventory costs in a centralized distribution system because it helps reduce aggregate demand uncertainty. Although such statistical economies of scale are important benefits of a retail merger, the extant literature models cost savings from a merger only through reduction in a postmerger firm's marginal cost. In this paper, we develop a model of a retail merger under uncertain demand that distinguishes between cost savings from conventional economies of scale and those from statistical economies of scale, and we show that these two sources of cost savings have substantially different impacts on firms' decisions in a postmerger market. Contrary to the existing theory of mergers developed under deterministic demand, we find that although inventory pooling enables the postmerger firm to achieve cost savings, it always induces firms to raise their prices, and we find that marginal cost reduction induces firms to lower their prices only when it is substantial—consequently, larger cost synergies can benefit even nonparticipant firms. Finally, even if a merger induces all firms to raise their prices, it can still improve expected consumer welfare by increasing firms' service levels under uncertain demand.

History: Accepted by Serguei Netessine, operations management.

Keywords: price competition • demand uncertainty • horizontal merger • inventory pooling

1. Introduction

Potential benefits of mergers and acquisitions (M&As) are manifold. Through M&As, firms could increase their revenues by utilizing stronger market power and achieve cost savings by utilizing economies of scale and improving operational efficiency. M&As have been an important aspect of management strategy in modern economies—26,409 M&As that were worth 1.9 trillion dollars occurred in 2013 (WilmerHale 2014). Our focus in this paper is on studying the effects of a horizontal merger between retailers in an oligopolistic market.

In a retail business, price is a natural strategic variable in competing against other retailers. In their seminal study, Deneckere and Davidson (1985) show that in the absence of cost synergies, a merger of price-setting retailers will induce both merging and nonparticipant firms to raise their prices: by determining prices jointly that were set independently prior to the merger, merging parties will raise their prices, and this initial price increase will be followed by price increases from their competitors; the merging parties then further raise their prices, and so on, until all prices in the market will have risen, and all firms will be better off.

Since mergers that lead to such price increase will be unlikely to be approved by antitrust authorities, merging firms often argue that they can achieve cost savings through the merger, which in turn will be passed on to consumers. Several empirical studies (e.g., Houston et al. 2001, DeLong 2003) suggest that operating synergies are the most important determinant of successful M&As. According to Bascle et al. (2008), cost efficiency is the main rationale behind more than 70% of M&As. Because a postmerger firm can pool its resources at the two retail outlets that were managed independently prior to the merger, cost savings from a merger can come from pooling inventory as well as from conventional economies of scale (e.g., lower procurement cost from higher purchasing power, lower R&D cost, lower cost of capital), as illustrated in the following examples:

—"Zipcar agreed to sell itself to Avis Budget Group Inc., for about $500 million . . . . Avis expects the deal to lower the companies' combined costs by $50 million to $70 million a year. [Avis Chief Executive Officer Ronald L.] Nelson said the synergies were tied to three components: lower fleet costs, better fleet utilization and increased revenue . . . . The deal would allow Avis to reduce the number of cars at Zipcar locations during the week, but also to use Avis's excess weekend inventory to meet Zipcar's strong weekend demand" (Kell 2013).

—"Hertz would acquire Dollar Thrifty for about $2.3 billion in cash. . . . They expect the merger's synergies to include annual savings of $65.6 million in fleet costs, in part through sharing of vehicles across rental brands" (Sawyers 2012).

Savings in inventory costs from a merger are also substantial in various other industries such as auto dealerships (Gattorna 1998), office-supply stores such as Office Depot and OfficeMax (Office Depot 2013), and the airline industry (Seidenman and Spanovich 2011).

In the operations literature (e.g., Eppen 1979, Corbett and Rajaram 2006), it is well known that inventory pooling can greatly reduce inventory costs in a centralized distribution system because it helps reduce aggregate demand uncertainty. Although such *statistical* economies of scale are important benefits of a retail merger, the extant literature on mergers (e.g., Williamson 1968, Farrell and Shapiro 1990, Cho 2014) models cost synergies only through reduction in a postmerger firm's marginal cost; in other words, prior models of mergers treat both statistical economies of scale and conventional economies of scale in the same manner. A primary reason for this modeling choice is that researchers have analyzed the effects of a merger under *deterministic* demand, whereas a careful examination of statistical economies of scale requires a merger analysis under *uncertain* demand. Under uncertain demand, a merger analysis becomes more complex because firms use inventory as well as price as their strategic variables. Nevertheless, a retail business always entails uncertainty in consumer demand, and therefore demand uncertainty has been one of the most fundamental features in the literature of operations management (OM).

The objective of this paper is to study the effects of a merger on firms' prices and expected profits as well as consumer welfare under uncertain demand. Specifically, we consider a merger of two firms in an oligopolistic market in which firms determine their prices and inventory levels under uncertain demand. In the OM literature, such firms are often called price-setting competitive "newsvendors." Our focus is on examining the following three effects of a merger. First, the "collusion effect" arises as a result of the ability of a postmerger firm to set its prices jointly at the two retail outlets that were independent prior to the merger. Second, a merger creates the "pooling effect" when a postmerger firm can manage its inventory in a centralized manner. To save its inventory cost by utilizing statistical economies of scale, a postmerger firm may manage a single safety stock in a central warehouse that serves two retail outlets or in two warehouses by allowing transshipment between them. Third, the "synergy effect" exists when a postmerger firm can reduce its marginal cost; for example, a postmerger firm may spread fixed costs over a larger number of sales units through economies of scale or reduce the cost of capital from lower securities and transaction costs.

Our analysis highlights the important role of demand uncertainty and inventory pooling in evaluating a retail merger. As discussed above, the existing literature models cost savings from a merger through marginal cost reduction under deterministic demand, and it does not distinguish inventory cost savings as a result of statistical economies of scale from marginal cost reduction. The conventional wisdom that cost savings from a merger will drive firms' prices down has been proven by many economists—notably, Williamson (1968), Perry and Porter (1985), and Farrell and Shapiro (1990), and it has been regarded as the de facto standard result in the theory of mergers (see Whinston 2007 for a comprehensive review). As a result, firms justify their proposed mergers by emphasizing that their cost savings will be passed on to consumers, and it appears that antitrust agencies view such cost savings positively (e.g., see U.S. Department of Justice/Federal Trade Commission 2010). However, our results indicate that neither marginal cost reduction (from conventional economies of scale) nor inventory cost savings (from statistical economies of scale) will always induce firms to lower their prices. Furthermore, although both conventional and statistical economies of scale enable merging firms to reduce their expected costs, their impacts on firms' prices and expected profits are substantially different. Counterintuitively, consumer price is more likely to rise after a merger when the benefit of pooling is more significant, and larger cost synergies from a merger can benefit nonparticipant firms. Contrary to the previous literature studied under deterministic demand, we find that even if a merger induces all firms to raise their prices, it can still improve expected consumer welfare by increasing firms' service levels.

The rest of this paper is organized as follows. In Section 2 we review the related literature. In Section 3 we describe our premerger model. In Section 4 we present our postmerger model and analysis. In Section 5 we study several extensions of our base model. We conclude our paper in Section 6. Proofs are presented in Appendix A.

2. Related Literature

In this section, we first review the economic theory of a merger, and then we review the related OM literature on competitive models of newsvendors, inventory pooling, operational models of mergers, and cooperative networks.

Economists and antitrust agencies have long studied mergers, in particular focusing on how a merger affects price. Stigler (1950) considers the formation of a cartel among firms that make a collusive decision in a competitive market, and he shows that a cartel is not stable because an increase of a market price will benefit external firms more than cartel members. To explain the observed formation of cartels or mergers in practice, starting from Williamson (1968), economists have taken into account cost synergies of mergers that may induce merging firms to lower their prices. Most notably, Farrell and Shapiro (1990) show that if the amount of marginal cost reduction from a merger exceeds a certain threshold, then prices will fall after a merger. Whereas these papers and their subsequent extensions adopt the Cournot model of quantity competition among homogeneous goods (e.g., see a comprehensive review by Whinston 2007), Deneckere and Davidson (1985) analyze a merger in a differentiated market where firms engage in price competition. The analysis of a merger in such a market is particularly important because firms, especially retailers, are often price setters, and the nature of price competition is different from quantity competition (e.g., Vives 1999). For this reason, numerous papers have constructed their models by building on Deneckere and Davidson (1985), including Werden and Froeb (1994) and Davidson and Ferrett (2007). Since our work deals with retail mergers, we use Deneckere and Davidson (1985) as our benchmark model of deterministic demand. To the best of our knowledge, our paper is the first that evaluates a merger under uncertain demand and characterizes statistical economies of scale from a merger. Contrary to the existing results in this literature, our results show that marginal cost reduction from conventional economies of scale induces merging firms to lower their prices only when they are sufficiently large and that larger statistical economies of scale always induce both merging and nonparticipant firms to raise their prices.

To evaluate the effect of a merger under uncertain demand, we need a benchmark in which firms compete *before* the merger takes place. For this benchmark, our paper builds on the OM literature that studies competition among newsvendors. Traditional research on newsvendor models considers a monopolistic firm's decision on inventory under uncertain demand while taking demand and price as given exogenously. A major extension to this traditional approach is to consider a monopolistic newsvendor who sets its price and inventory simultaneously (e.g., Petruzzi and Dada 1999, Kocabıyıkoğlu and Popescu 2011). Another important extension is to introduce competition among newsvendors (e.g., Lippman and McCardle 1997, Netessine and Rudi 2003). Whereas these papers focus on the inventory decisions of competitive newsvendors, Zhao and Atkins (2008) consider a more general case in which each competitive newsvendor determines both price and inventory simultaneously. Our paper builds on Zhao and Atkins (2008) for the premerger model and examines the effect of a merger between two competitive newsvendors on merging firms, nonparticipant firms, and consumers.

Research on inventory pooling has a long tradition in operations management. The seminal paper by Eppen (1979) considers a multilocation newsvendor problem with normal demand at each location. He shows that inventory costs in a centralized system increase with the correlation between uncertain demands in different locations. Numerous extensions have followed Eppen (1979), including, among others, a decentralized system with one manufacturer and multiple retailers owned and operated by a single entity who can transship inventory between them (Dong and Rudi 2004), arbitrary dependence structure with nonnormal distributions (Corbett and Rajaram 2006), capacity-sharing joint ventures (Roels et al. 2012), and procurement contracts between two buyers and one common supplier (Hu et al. 2013). Whereas these papers consider the centralization of inventory among warehouses of a single firm or among monopolistic firms, Anupindi and Bassok (1999) and Wang and Gerchak (2001) analyze the centralization of inventory in a supply chain with one supplier and two competitive retailers and compare its performance with the decentralized system. There are two important differences between these papers and our work. First, although the previous papers consider the centralization of inventory (or stocking decisions) of all retailers in a market, in the context of a merger, such *complete* centralization will create a monopolist and hence will not be approved a priori by antitrust authorities. Instead, a merger usually involves only two firms, and it affects other nonparticipant firms in an oligopoly market—in this sense, a merger may be referred to as *partial* centralization. The essence of a merger analysis is to examine the competitive reactions of nonparticipant firms to the proposed merger, which in turn affect the decision of the post-merger firm, and so on; hence, the merger analysis is substantially different from the previous analyses that compare centralization with decentralization. Second, these papers assume *fixed* consumer prices of all retailers but consider stock-out substitution among retailers (i.e., a fraction of consumers who do not find the good at their local retailers look for the good at other retailers). By contrast, a central question in the analysis of a merger is how a merger affects consumer price. Therefore, in our work, we consider a *price-setting* competitive newsvendor model as our premerger model, which itself is hard to analyze. To maintain tractability, we assume initially that firms compete only through prices and examine stock-out substitution in a later extension numerically.

Despite the importance of mergers in practice, there is scant literature on mergers in the OM literature. Prior research in this literature mainly focuses on quantifying operating synergies from a merger in monopolistic markets (e.g., Gupta and Gerchak 2002, Nagurney 2009) or on vertical integration under deterministic demand (e.g., Corbett and Karmarkar 2001, Lin et al. 2014). Recently, Cho (2014) has studied a horizontal merger in a multitier decentralized supply chain in which firms engage in quantity competition at each tier. He characterizes the impact of a merger at one tier on strategic decisions of firms at different tiers of the supply chain. Unlike Cho (2014), who considers a deterministic setting, the main research question of the current paper is to characterize the impact of demand uncertainty and inventory pooling in a merger of price-setting firms. Different from the analytical papers reviewed above, Zhu et al. (2011) empirically study the effects of a horizontal merger on the financial and inventory-related performance of firms. Although they discuss the potential impact of demand uncertainty on firms' performance, they develop their hypotheses mainly from Deneckere and Davidson (1985), who characterize only the collusion effect under deterministic demand. Our paper complements their empirical work by providing theoretical results about the collusion, pooling, and synergy effects of a merger under uncertain demand.

Finally, in another stream of research, researchers use cooperative game theory to study the formation of resource-pooling or inventory-transshipment coalitions among firms. In this literature, firms maintain their independence but consider forming coalitions to obtain synergies or to reduce financial risk (e.g., Kemahlıoğlu-Ziya and Bartholdi 2011, Fang and Cho 2014, Huang et al. 2016). This literature focuses primarily on examining how stable coalitions can be formed by allocating the benefit from collaboration to independent firms appropriately. By contrast, the benefit from a merger need not be allocated between merging firms, since merging firms become a single entity after the merger. Our focus is on analyzing the effect of a merger on prices, expected profits, and consumer welfare, provided that such a merger occurs.

3. Premerger Model and Analysis

Consider n symmetric firms that sell products through different retail locations. Let p_i denote the price of firm i $(=1,2,\ldots,n)$, let $\mathbf{p}=(p_1,\ldots,p_n)$ denote the price vector, and let $\mathbf{p}_{-i}=(p_1,\ldots,p_{i-1},p_{i+1},\ldots,p_n)$ denote the price vector of all firms but firm i. The demand of firm i is $D_i(\mathbf{p})=L_i(\mathbf{p})+\tilde{\varepsilon}_i$, where $L_i(\mathbf{p})$ is the deterministic part of the demand and $\tilde{\varepsilon}_i$ is the random part of the demand. Following our benchmark model of deterministic demand in Deneckere and Davidson (1985), we assume

$$L_i(\mathbf{p})=a-bp_i+\gamma\left(\frac{1}{n}\sum_{j=1}^{n}p_j-p_i\right), \tag{1}$$

where a (>0) is the deterministic demand when all firms' prices are zero, and b (>0) captures the sensitivity of demand to firm i's own price p_i. The parameter γ (≥0) captures competition among firms in the following sense. When γ is close to zero, competition among firms is low, so that the difference between a firm's own price and other firms' prices has little impact on the demand; however, when γ is large, competition is intense. We assume that $\tilde{\varepsilon}=(\tilde{\varepsilon}_1,\tilde{\varepsilon}_2,\ldots,\tilde{\varepsilon}_n)$ follows a multivariate normal distribution $\mathrm{N}(0,\Sigma)$, where Σ represents the covariance structure of demand with $\mathrm{Var}(\tilde{\varepsilon}_i)=\sigma_i^2=\sigma^2$. We denote by $\phi(\cdot)$ and $\Phi(\cdot)$ the density and the cumulative distribution function of the standard normal random variable, respectively. To ensure positive demand, we require $a\gg\sigma$ so that $\Pr(a+\tilde{\varepsilon}_i<0)\approx 0$. Each firm i incurs a marginal cost $w_i=w$. We assume there is no salvage value for unsold goods.

Each firm i decides its price p_i and inventory q_i simultaneously with other firms before the random demand is realized. Let $q_i=L_i(\mathbf{p})+y_i$, where y_i is firm i's safety stock to hedge against demand uncertainty. As in Petruzzi and Dada (1999) and Zhao and Atkins (2008), it can be shown that a game with decision variables (p_i,q_i) is equivalent to a game with decision variables (p_i,y_i). With (p_i,y_i) as decision variables, a firm's strategy set can be unbounded. To ensure the compactness of a firm's strategy set for the existence of Nash equilibrium, we assume $p_i\in[\underline{p},\bar{p}]$ and $y_i\in[-\bar{y},\bar{y}]$, where $\underline{p}>w$, and $\bar{p}$ and $\bar{y}$ are sufficiently large numbers that do not constrain firms' decisions. Given $\mathbf{p}_{-i}$, firm i chooses p_i and y_i to maximize its expected profit given as

$$\begin{aligned}\pi_i(\mathbf{p},y_i)&=(p_i-w)L_i(\mathbf{p})-wy_i-p_iE(\tilde{\varepsilon}_i-y_i)^+\\&=(p_i-w)L_i(\mathbf{p})-wy_i-p_i\sigma R\left(\frac{y_i}{\sigma}\right),\end{aligned} \tag{2}$$

where $R(x)=\int_x^\infty(u-x)\phi(u)\,du$ is the standard normal loss function and the expected lost sale of firm i having safety stock y_i is given as $E(\tilde{\varepsilon}_i-y_i)^+=\int_{y_i}^\infty(u-y_i)\cdot(\phi(u/\sigma)/\sigma)\,du=\sigma R(y_i/\sigma)$. Since there are no lost sales under deterministic demand, for convenience, we define $\sigma R(y/\sigma)=0$ when $\sigma=0$. Let $\pi_i^d(\mathbf{p})=(p_i-w)L_i(\mathbf{p})$ and $c_i(p_i,y_i)=wy_i+p_i\sigma R(y_i/\sigma)$, representing the profit from the deterministic demand and the expected cost caused by demand uncertainty, respectively. Then $\pi_i(\mathbf{p},y_i)=\pi_i^d(\mathbf{p})-c_i(p_i,y_i)$. Note that $-c_i(p_i,y_i)$ can also be interpreted as the expected profit of a newsvendor who faces the demand of $\tilde{\varepsilon}_i$. In our subsequent analysis, we will focus on the case in which all firms earn positive expected profits.

Following Netessine and Rudi (2003) and Zhao and Atkins (2008), we can show that a unique pure-strategy Nash equilibrium exists under a certain condition (see Lemma A1 in Appendix A). The symmetric equilibrium price $p_1^{\mathrm{pre}}=p_2^{\mathrm{pre}}=\cdots=p_n^{\mathrm{pre}}$ is the unique solution of the following equation:

$$\begin{aligned}-\left(2b+\frac{n-1}{n}\gamma\right)p_1^{\mathrm{pre}}-\sigma R\left(\Phi^{-1}\left(1-\frac{w}{p_1^{\mathrm{pre}}}\right)\right)\\+a+\left(b+\frac{n-1}{n}\gamma\right)w=0.\end{aligned} \tag{3}$$

The equilibrium safety stock $y_1^{\mathrm{pre}} = y_2^{\mathrm{pre}} = \cdots = y_n^{\mathrm{pre}}$ is equal to $\sigma\Phi^{-1}(1 - w/p_1^{\mathrm{pre}})$, and it is also unique. The corresponding expected profit of firm i in equilibrium is denoted by π_i^{pre}.

Before proceeding to our postmerger analysis, we remark on our assumptions and later extensions. First, we consider symmetric firms in the premerger market in Sections 3 and 4, and we extend the analysis to asymmetric firms in Section 5.1. Our analysis in Sections 3 and 4 enables us to isolate the effect of demand uncertainty on the celebrated result of Deneckere and Davidson (1985), who also consider symmetric firms in the premerger market, and to compare the effect of a merger on merging firms with that on nonparticipant firms. Second, we assume that random demands follow a multivariate normal distribution, which is widely used in the literature that studies the effect of inventory pooling (e.g., Eppen 1979, Anupindi and Bassok 1999, Dong and Rudi 2004, Hu et al. 2013). Even in this case, no closed-form expressions for p_i^{pre}, y_i^{pre}, and π_i^{pre} exist, and our subsequent analysis deals with the implicit functions such as (3) that define these equilibrium outcomes. Nevertheless, we show in Section 5.2 that our results hold under a more general class of distributions. In Section 5.3, we consider a demand model with a general uncertainty structure. Finally, in Section 5.4, we examine the impact of stock-out substitution on our results.

4. Postmerger Model and Analysis

In Section 4.1, we present our postmerger model and describe the collusion, pooling, and synergy effects of a merger. We then characterize these effects of a merger on firms' prices and expected profits in Section 4.2 and on firms' service levels and expected consumer welfare in Section 4.3.

4.1. Postmerger Model

Suppose firms 1 and 2 in the premerger market described in Section 3 have merged. We refer to these two firms that are merged as the *merging* firms. When the two merging firms become a single firm in the postmerger market, we refer to this firm as the *postmerger* firm and refer to the other firms as the *nonparticipant* firms. We index the postmerger firm by $i = m$ and the nonparticipant firms by $i = 3, 4, \ldots, n$. We consider an oligopolistic market with $n \geq 3$ because a merger that creates a monopolist (i.e., $n = 2$) is unlikely to be approved by antitrust authorities.

The postmerger firm faces the demand of $L_m(p) + \tilde{\varepsilon}_m$, where $L_m(p) = L_1(\mathbf{p}) + L_2(\mathbf{p})$ represents the deterministic part of the demand and $\tilde{\varepsilon}_m$ represents the random part of the demand. The random part $\tilde{\varepsilon}_m$ is given as $\tilde{\varepsilon}_m = \tilde{\varepsilon}_1 + \tilde{\varepsilon}_2$. Since the linear combination of the components of the multivariate normal random vector $\tilde{\varepsilon}$ is normally distributed, $\tilde{\varepsilon}_m$ follows $\mathrm{N}(0, \sigma_m)$, where σ_m represents the postmerger firm's aggregate volatility of the uncertain demand. Letting ρ ($\in [-1, 1]$) denote the correlation coefficient between $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$, we obtain $\sigma_m = \sigma\sqrt{2 + 2\rho}$. Prior to the merger, firms 1 and 2 set prices p_1 and p_2 at their respective retail outlet independently, whereas the postmerger firm can set its prices p_1 and p_2 *collusively* at these two retail outlets. We call the effect of such price collusion on equilibrium the *collusion effect*. To examine the collusion effect, we assume that the postmerger firm maintains their two retail outlets after the merger—the same assumption is made by Deneckere and Davidson (1985) and subsequent extensions (e.g., Werden and Froeb 1994, Davidson and Ferrett 2007).

In addition to setting its prices at two retail outlets collusively, the postmerger firm may also manage its inventories at these two locations in a *centralized* manner. We model this *pooling effect* by allowing the postmerger firm to sell the same product at both retail outlets and to use a single safety stock y_m to hedge against the aggregate demand volatility σ_m. This is essentially the same as the centralization of inventory in the literature (e.g., Eppen 1979, Anupindi and Bassok 1999, Wang and Gerchak 2001, Corbett and Rajaram 2006). By contrast, when each retail outlet sells a different product and/or manages its stock *separately* even after the merger, there is no pooling effect. This corresponds to the centralization of stocking decisions in the literature (compared with the centralization of physical inventory) (e.g., Netessine and Rudi 2003, Netessine and Zhang 2005).[1] We can prove that this case is identical to the special case when the postmerger firm pools its inventories under perfectly correlated demand (i.e., $\rho = 1$) so that $\sigma_m = 2\sigma$. In the presence of the pooling effect (i.e., $\rho < 1$), when the demands of the two merging firms are highly correlated (i.e., high ρ), the postmerger firm faces high demand volatility σ_m because of the *low* pooling effect.

The postmerger firm often achieves cost synergies by utilizing economies of scale. Following the common approach in the merger literature (e.g., see Cho 2014 and references therein), we model the *synergy effect* of a merger by reducing the marginal cost of the postmerger firm from w to w_m ($\in (0, w]$). Let $s \equiv ((w - w_m)/w)$ ($\in [0, 1)$) denote a percentage of marginal cost reduction after a merger. When $s = 0$, there is no synergy effect. When $s > 0$, the synergy effect exists, and as s increases, the merger entails larger cost synergies. The synergy level s is an aggregate measure for cost synergies from various areas of operations, marketing, and administration, and its estimation often requires an industry-specific detailed analysis.

As in the premerger market, each firm i ($= m, 3, 4, \ldots, n$) in the postmerger market decides its price p_i and inventory q_i simultaneously before the random demand is realized. The postmerger firm decides its prices p_1 and p_2 as well as its safety stock y_m to maximize its expected profit π_m, which can be expressed similarly to (2) as follows:[2]

$$\pi_m(\mathbf{p}, y_m) = (p_1 - w_m)L_1(\mathbf{p}) + (p_2 - w_m)L_2(\mathbf{p}) - w_m y_m - (p_1 + p_2)\frac{\sigma_m}{2} R\left(\frac{y_m}{\sigma_m}\right). \quad (4)$$

Let $\pi_m^d(\mathbf{p}) = (p_1 - w_m)L_1(\mathbf{p}) + (p_2 - w_m)L_2(\mathbf{p})$ and $c_m(p_1, p_2, y_m) = w_m y_m + (p_1 + p_2)(\sigma_m/2)R(y_m/\sigma_m)$, so that $\pi_m(\mathbf{p}, y_m) = \pi_m^d(\mathbf{p}) - c_m(p_1, p_2, y_m)$. The expected profit of nonparticipant firm i $(=3, 4, \ldots, n)$ remains the same as $\pi_i(\mathbf{p}, y_i)$ given in (2). Similar to the premerger market, we can show that equilibrium prices p_m^{post} (where $p_1^{\text{post}} = p_2^{\text{post}} = p_m^{\text{post}}$) and p_3^{post} (where $p_3^{\text{post}} = p_4^{\text{post}} = \cdots = p_n^{\text{post}}$) are the unique solutions that satisfy two first-order conditions, and safety stocks in equilibrium are $y_i^{\text{post}} = \sigma\Phi^{-1}(1 - w/p_i^{\text{post}})$ for nonparticipant firm i and $y_m^{\text{post}} = \sigma_m\Phi^{-1}(1 - w_m/p_m^{\text{post}})$ for the postmerger firm. The corresponding expected profit of firm i is π_i^{post} for $i = m, 3, 4, \ldots, n$.

4.2. Postmerger Analysis: Price and Expected Profit

The prior literature on a merger of price-setting firms focuses on the collusion effect of the merger under *deterministic* demand. In this section, we will first investigate the collusion effect under *uncertain* demand. We then examine the impact of two sources of cost savings for a postmerger firm—namely, inventory pooling and cost synergies—on firms' prices and expected profits. Last, by combining the collusion, pooling, and synergy effects of a merger, we examine the aggregate effect of a merger on firms' prices and expected profits.

To isolate the impact of demand uncertainty on the *collusion* effect, we examine the same setting as Deneckere and Davidson (1985) except that firms face uncertain demand in our model. In this special case, no pooling and synergy effects exist; i.e., a postmerger firm decides on its prices at its two retail outlets, but the merger entails no cost savings through inventory pooling or marginal cost reduction (i.e., $\rho = 1$ and $w_m = w$).

Lemma 1. *When $\rho = 1$ and $w_m = w$, the collusion effect of a merger leads to the following results*:

(a) *The postmerger price of any firm is higher than its premerger price (i.e., $p_m^{\text{post}} > p_1^{\text{pre}}$ and $p_i^{\text{post}} > p_i^{\text{pre}}$ for $i = 3, 4, \ldots, n$). In addition, the price of the postmerger firm is higher than that of a nonparticipant firm (i.e., $p_m^{\text{post}} > p_i^{\text{post}}$ for $i = 3, 4, \ldots, n$).*

(b) *The postmerger expected profit of any firm is higher than its premerger expected profit (i.e., $\frac{1}{2}\pi_m^{\text{post}} > \pi_1^{\text{pre}}$ and $\pi_i^{\text{post}} > \pi_i^{\text{pre}}$ for $i = 3, 4, \ldots, n$). Furthermore, the postmerger expected profit of a merging firm is lower than that of a nonparticipant firm (i.e., $\frac{1}{2}\pi_m^{\text{post}} < \pi_i^{\text{post}}$ for any $i = 3, 4, \ldots, n$).*

Lemma 1 shows that the price collusion of the merging parties induces all firms to raise their prices and thereby earn higher expected profits. These results verify that the collusion effect of a merger on prices and profits under deterministic demand remains valid under uncertain demand. This suggests that the nature of competition that drives these results is unaffected by demand uncertainty. Specifically, a merging firm has an incentive to raise its price after the merger because its higher price has a positive externality on the other merging party. The increased prices of the merging firms in turn benefit nonparticipant firms by raising their demands (see (1)). Consequently, nonparticipant firms also raise their prices after the merger. However, they raise prices less so than the postmerger firm for the (technical) reason that a nonparticipant firm's best response function to the merged firm's price is upward sloping with a slope less than 1 (see Appendix A).

Having characterized the collusion effect of a merger under uncertain demand, we next examine how inventory pooling and cost synergies affect postmerger equilibrium. Since a merger always enables merging parties to collude on their prices, we examine these effects in the presence of the collusion effect. We first examine the pooling effect and then the synergy effect. As discussed in Section 4.1, the postmerger firm faces the aggregate volatility in its total demand, $\sigma_m = \sigma\sqrt{2+2\rho}$, which is increasing in the correlation coefficient ρ between the demands of two merging firms. Thus, when ρ is low (respectively, high), σ_m is low (respectively, high) because of the high (respectively, low) pooling effect.

Proposition 1. *For any $w_m \in (0, w]$, the inventory pooling of a postmerger firm affects postmerger equilibrium as follows*:

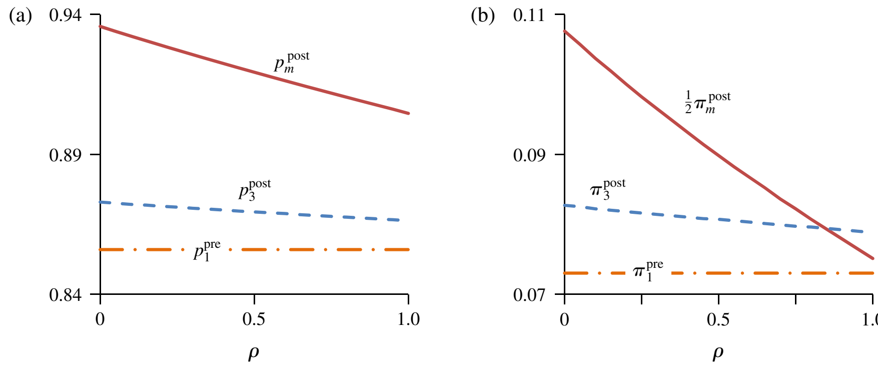
(a) *The postmerger price of any firm i, p_i^{post} $(i = m, 3, 4, \ldots, n)$, is decreasing in ρ.*

(b) *The postmerger expected profit of any firm i, π_i^{post} $(i = m, 3, 4, \ldots, n)$, is decreasing in ρ.*

Proposition 1(a) states that as the pooling effect becomes more substantial with lower ρ (i.e., the postmerger firm faces a lower aggregate volatility), the postmerger firm charges a higher price; see Figure 1(a). Because the postmerger firm saves its inventory cost from pooling inventories, one might anticipate that such cost savings will be passed on to consumers through reduced prices. In fact, the existing theory of mergers developed under deterministic demand posits that marginal cost reduction through cost synergies will induce firms to reduce their prices (see Section 2). However, our result indicates that although inventory pooling enables the postmerger firm to achieve cost savings, it always induces firms to raise their prices.

We can explain this result as follows. To prove that p_m^{post} decreases with σ_m as well as ρ (since $\sigma_m =$

Figure 1. (Color online) The Pooling Effect of a Merger on (a) Prices and (b) Expected Profits



Notes. The following parameter values are used: $n = 3$, $a = 1$, $b = 0.6$, $\gamma = 0.5$, $w = w_m = 0.5$, and $\sigma = 0.3$. These values are motivated by the U.S. rental car industry; see Appendix B.

$\sigma\sqrt{2+2\rho}$), we apply the implicit function theorem to two first-order conditions for p_m^{post} and p_3^{post}, and we obtain the following after simplifications (see the proof):

$$\frac{dp_m^{\text{post}}}{d\sigma_m} = \left(-\frac{\partial^2 \pi_m(\mathbf{p},\sigma_m)}{\partial p_1 \partial \sigma_m}\bigg|_{\mathbf{p}=\mathbf{p}^{\text{post}}}\right) \cdot \left[\frac{\partial}{\partial p_m^{\text{post}}}\left(\frac{\partial \pi_m(\mathbf{p},\sigma_m)}{\partial p_1}\bigg|_{\mathbf{p}=\mathbf{p}^{\text{post}}}\right) + \frac{\partial}{\partial p_3^{\text{post}}}\left(\frac{\partial \pi_m(\mathbf{p},\sigma_m)}{\partial p_1}\bigg|_{\mathbf{p}=\mathbf{p}^{\text{post}}}\right)\frac{dp_3^{\text{post}}}{dp_m^{\text{post}}}\right]^{-1}. \quad (5)$$

In (5), the first term in the denominator captures the shape of the postmerger firm's profit function π_m with respect to its own price, and the second term captures the competitive dynamics between postmerger and nonparticipant firms. Using Lemma 1 (as well as Lemma A1 in Appendix A, that shows $0 < dp_3^{\text{post}}/dp_m^{\text{post}} < 1$), we show in the proof that the denominator of (5) is negative. Thus, it suffices to show that the numerator of (5) is positive; i.e., $\partial^2\pi_m/\partial p_1\partial\sigma_m = \partial^2\pi_m^d/\partial p_1\partial\sigma_m - \partial^2 c_m/\partial p_1\partial\sigma_m < 0$ at $\mathbf{p}=\mathbf{p}^{\text{post}}$, which follows from $\partial^2\pi_m^d/\partial p_1\partial\sigma_m = 0$ and $\partial^2 c_m/\partial p_1\partial\sigma_m = R(\Phi^{-1}(1 - w_m/p_m^{\text{post}})) > 0$ at $\mathbf{p}=\mathbf{p}^{\text{post}}$. Note that the numerator is computed for fixed prices of all nonparticipant firms, and hence it is consistent with the result of the price-setting monopolistic newsvendor models (Mills 1959, Petruzzi and Dada 1999). Its intuition is as follows. Although demand volatility σ_m does not affect the profit from the deterministic demand, π_m^d, it does affect the expected cost as a result of demand uncertainty, c_m. It can be shown that c_m increases with price p_m^{post} as well as volatility σ_m. The result that $\partial^2 c_m/\partial p_1\partial\sigma_m|_{\mathbf{p}=\mathbf{p}^{\text{post}}} > 0$ suggests that the marginal cost of a higher demand volatility σ_m increases with price p_m^{post} because the lost revenue as a result of demand uncertainty increases with price p_m^{post}. Likewise, the marginal cost of a higher price p_m^{post} increases with volatility σ_m because more demand will be lost with a higher volatility σ_m. Taken as a whole, considering the impact of σ_m on its own expected profit π_m as well as the competitive response of nonparticipant firms, the postmerger firm raises its price in equilibrium when facing a lower σ_m. In response to the increased price of the postmerger firm, as discussed earlier in Lemma 1(a), nonparticipant firms raise their prices as well.

Proposition 1(b) shows, as expected, that a postmerger firm will obtain a higher expected profit by pooling its inventories. One might expect that the cost advantage of the postmerger firm may hurt its competitors. In contrast to this first intuition, Proposition 1(b) shows that the high pooling effect also benefits nonparticipant firms. This happens because the increased price of the postmerger firm will allow nonparticipant firms to raise their prices as well and to earn higher expected profits. Although the pooling effect benefits both postmerger and nonparticipant firms, Figure 1(b) illustrates that the pooling effect is more beneficial to a merging firm than a nonparticipant firm, and hence when ρ is sufficiently low, a merging firm earns a higher expected profit than that of a nonparticipant firm. This is contrary to the result of Deneckere and Davidson (1985), who show that a merger is always more beneficial to a nonparticipant firm under deterministic demand.

We next examine the impact of marginal cost reduction from merger synergies on the postmerger equilibrium. Although both inventory pooling and cost synergies enable a postmerger firm to reduce its expected cost, the following proposition shows that the synergy effect on the postmerger equilibrium differs substantially from the pooling effect presented earlier in Proposition 1.

Proposition 2. *For any $\rho \in [-1, 1]$, there exists a threshold $s^{(1)} \in [0, 1)$, which is nondecreasing in σ_m with $s^{(1)} = 0$ at $\sigma_m = 0$, such that*

(a) *the postmerger price of any firm i, p_i^{post} ($i = m, 3, 4, \ldots, n$), is decreasing in s if and only if $s > s^{(1)}$; and*

(b) *the expected profit of the postmerger firm, π_m^{post}, is always increasing in s, whereas the expected profit of a nonparticipant firm, π_i^{post} ($i = 3, 4, \ldots, n$), is decreasing in s if and only if $s > s^{(1)}$.*

Proposition 2(a) states that larger cost synergies of a merger do not necessarily induce firms to lower their prices under uncertain demand. This bears important implications for antitrust policies, since firms often use cost synergies to justify their proposed merger to antitrust authorities. Note that this result is not obtained under deterministic demand (since $s^{(1)} = 0$) as is the case in the existing literature. This result can be explained similarly to Proposition 1(a). In particular, p_m^{post} decreases with s if and only if $\partial^2\pi_m/\partial p_1 \partial w_m > 0$ at $\mathbf{p} = \mathbf{p}^{\text{post}}$, which is proven by showing that

$$\left.\frac{\partial^2 \pi_m^d}{\partial p_1 \partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{\text{post}}} = b + \gamma\frac{n-2}{n} > 0;$$
$$\left.\frac{\partial^2 c_m}{\partial p_1 \partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{\text{post}}} = \frac{\sigma_m w_m}{2\phi(\Phi^{-1}(1 - w_m/p_m^{\text{post}}))(p_m^{\text{post}})^2} \geq 0. \tag{6}$$

Under deterministic demand with $\sigma_m = 0$, the expected cost c_m is zero, so $\partial^2 c_m/\partial p_1 \partial w_m = 0$. In this case, since $\partial^2\pi_m/\partial p_1 \partial w_m > 0$, p_m^{post} decreases with s. This means that without demand uncertainty, cost synergies enable the postmerger firm to lower its price. However, under uncertain demand with $\sigma_m > 0$, observe from (6) that $\partial^2 c_m/\partial p_1 \partial w_m > 0$, and consequently, $\partial^2\pi_m/\partial p_1 \partial w_m$ can be either positive or negative. Proposition 2(a) provides the necessary and sufficient condition for $\partial^2\pi_m/\partial p_1 \partial w_m < 0$ at the equilibrium point, so that p_m^{post} increases with s. This condition requires that the synergy level s is lower than the threshold $s^{(1)}$.[3] The threshold $s^{(1)}$ is nondecreasing with the aggregate demand volatility σ_m. This implies that when the postmerger firm faces a higher demand volatility, it is more likely to observe the counterintuitive result that p_m^{post} increases with s; see Figure 2. The same condition applies to nonparticipant firms as well, since nonparticipant firms change their prices in the same direction as the postmerger firm (see Lemma 1(a)).

Interestingly, Proposition 2(b) shows that when the postmerger firm achieves larger cost synergies, nonparticipant firms can (but not always) also earn higher expected profits. We can explain this result in the same manner as the nonmonotonic change of the postmerger prices in Proposition 2(a). Although the price of a postmerger firm changes nonmonotonically with the synergy level s, Proposition 2(b) shows that as the synergy level s increases, the postmerger firm earns larger expected profit. This result is intuitive and also holds for the case under deterministic demand.

Finally, by combining Lemma 1 with Propositions 1 and 2, we examine the aggregate (collusion, pooling, and synergy) effect of a merger and compare premerger equilibrium with postmerger equilibrium. See Figure 3 for illustration.

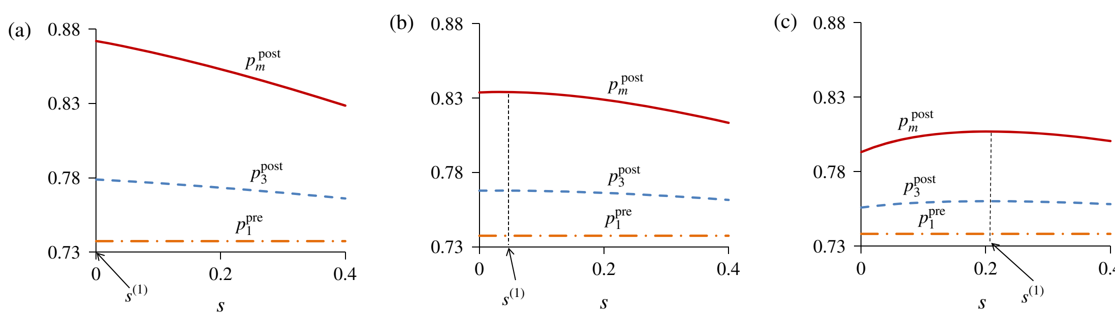
Proposition 3. *For any $\rho \in [-1, 1]$ and $w_m \in (0, w]$, there exist thresholds $s^{(2)} \in (s^{(1)}, 1]$ and $s^{(3)} \in [0, s^{(2)}]$ such that the following holds:*

(a) *The postmerger price of any firm is higher than its premerger price (i.e., $p_m^{\text{post}} > p_1^{\text{pre}}$ and $p_i^{\text{post}} > p_i^{\text{pre}}$ for $i = 3, 4, \ldots, n$) if and only if $s < s^{(2)}$. The price of the postmerger firm is higher than that of a nonparticipant firm (i.e., $p_m^{\text{post}} > p_i^{\text{post}}$ for $i = 3, 4, \ldots, n$) if and only if $s < s^{(2)}$; furthermore, $s^{(2)}$ is nonincreasing in ρ.*

(b) *The postmerger expected profit of a merging firm is higher than its premerger expected profit (i.e., $\frac{1}{2}\pi_m^{\text{post}} > \pi_1^{\text{pre}}$) for any s, whereas the postmerger expected profit of a nonparticipant firm is higher than its premerger expected profit (i.e., $\pi_i^{\text{post}} > \pi_i^{\text{pre}}$ for $i = 3, 4, \ldots, n$) if and only if $s < s^{(2)}$. Moreover, the postmerger expected profit of a merging firm is higher than that of a nonparticipant firm (i.e., $\frac{1}{2}\pi_m^{\text{post}} > \pi_i^{\text{post}}$ for $i = 3, 4, \ldots, n$) if $s > s^{(3)}$.*

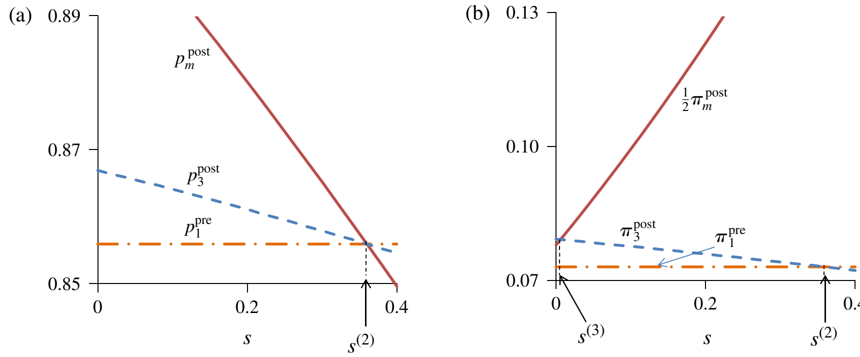
Proposition 3(a) states that a merger will cause firms' prices to drop only when the synergy level s is higher than $s^{(2)}$. This result combines the collusion effect (which causes prices to rise, as shown in Lemma 1(a)), the pooling effect (which causes prices to rise, as shown in Proposition 1(a)), and the synergy effect (which causes prices to drop only when $s > s^{(1)}$, as shown in

Figure 2. (Color online) The Synergy Effect of a Merger on Prices Under Uncertain Demand at (a) $\rho = 0$, (b) $\rho = 0.5$, and (c) $\rho = 1$



Note. The same parameter values are used as in Figure 1, except $\sigma = 0.5$.

Figure 3. (Color online) The Aggregate Effect of a Merger on (a) Prices and (b) Expected Profits



Note. The same parameter values are used as in Figure 1, except $\rho = 0.9$ to better illustrate $s^{(2)}$ and $s^{(3)}$.

Proposition 2(a)). Since the synergy effect causes prices to drop only when $s > s^{(1)}$, even in the absence of the pooling effect, the threshold $s^{(2)}$ in Proposition 3(a) is higher than $s^{(1)}$. It is also possible that $s^{(2)} = 1$, implying that a merger will increase firms' prices for any synergy level s. This extreme case may happen when the premerger marginal cost w is so low that further reduction of the marginal cost from synergies does not outweigh the collusion and pooling effects of the merger on prices. Proposition 3(a) also reveals that when the synergy level is so high that the merger decreases prices (i.e., $s > s^{(2)}$), the price of the postmerger firm becomes lower than the price of nonparticipant firms; see Figure 3(a).

In addition, Proposition 3(a) shows that the threshold $s^{(2)}$ is nonincreasing in ρ. This happens because the pooling effect drives prices upward, as shown in Proposition 1(a). This result suggests that as the pooling effect becomes more significant, larger cost synergies are required for prices to drop after a merger. Therefore, ceteris paribus, consumer price is less likely to rise after a merger in an industry where firms' uncertain demands are highly correlated (e.g., household furniture, home appliances, and motor vehicle dealerships, where demand is closely related to business cycles; Berman and Pfleeger 1997)).

Since the postmerger firm benefits from each of the collusion, pooling, and synergy effects, a merger will increase the expected profit of a merging firm (Proposition 3(b)). On the other hand, a merger will increase the expected profit of a nonparticipant firm only when the merger induces all firms to raise their prices. This is consistent with our previous lemma and propositions. As illustrated in Figure 3(b), when the synergy level s is larger than $s^{(3)}$, the expected profit of a merging firm exceeds that of a nonparticipant firm.

4.3. Postmerger Analysis: Service Level and Expected Consumer Welfare

So far, we have focused on the effect of a merger on firms' prices and their expected profits, following the tradition of prior work on mergers studied under deterministic demand. However, when demand is uncertain, firms determine their stocking levels, which can affect the availability of products to consumers. In this section, we examine how the pooling and synergy effects of a merger affect firms' service levels and ultimately expected consumer welfare.

Following the convention of the operations management literature, we define firm i's service level l_i as its in-stock probability: $l_i = \Pr(D_i \le q_i) = \Pr(\tilde{\varepsilon}_i \le y_i) = \Phi(y_i/\sigma_i)$. The following proposition shows the pooling and synergy effects of a merger on firms' service levels (in the presence of the collusion effect) and compares service levels between premerger and postmerger markets.

Proposition 4. (a) *For any fixed* $w_m \in (0, w]$, *the postmerger service levels of all firms,* l_m^{post} *and* l_i^{post} $(i = 3, 4, \ldots, n)$, *are decreasing in* ρ.

(b) *For any fixed* $\rho \in [-1, 1]$, *the service level of the postmerger firm,* l_m^{post}, *is always increasing in* s, *whereas the service level of a nonparticipant firm,* l_i^{post} $(i = 3, 4, \ldots, n)$, *is increasing in* s *if and only if* $s < s^{(1)}$ (*where* $s^{(1)}$ *is defined in Proposition* 2).

(c) *The service level of the postmerger firm is always higher than its premerger service level (i.e.,* $l_m^{post} > l_1^{pre}$). *The service level of a nonparticipant firm is higher than its premerger service level (i.e.,* $l_i^{post} > l_i^{pre}$ *for* $i = 3, 4, \ldots, n$) *if and only if* $s < s^{(2)}$ (*where* $s^{(2)}$ *is defined in Proposition* 3).

Proposition 4(a) shows that when the pooling effect of a merger is significant with low ρ, firms raise their service levels. To understand this result, recall from Section 4.1 that the optimal safety stock is $y_i^{post} = \sigma_i \Phi^{-1}(1 - w_i/p_i^{post})$, at which the service level is $l_i^{post} = 1 - w_i/p_i^{post}$. Because a higher pooling effect (i.e., lower ρ) raises all firms' prices p_i^{post} $(i = m, 3, 4, \ldots, n)$ for any fixed w_i (Proposition 1(a)), it also raises their service levels l_i^{post} $(i = m, 3, 4, \ldots, n)$. With a higher service level, a firm's lost sales are decreased; i.e., $E(D_i - q_i)^+ = \sigma_i R(\Phi^{-1}(l_i))$ is decreasing with l_i.

Unlike the pooling effect, the synergy effect affects the service level of the postmerger firm $l_m^{\text{post}} = 1 - w_m/p_m^{\text{post}}$ via changes in both w_m and p_m^{post}. When the synergy level s is significant with $s > s^{(1)}$, the result is not straightforward because a higher s means a lower w_m, but it induces the postmerger firm to lower its price p_m^{post} (Proposition 2). It turns out that the price drop is always less than the cost reduction (i.e., $dp_m^{\text{post}}/dw_m < 1$; see the proof of Proposition 4(b)), so the service level l_m^{post} increases for any synergy level s. For nonparticipant firm i ($i = 3, 4, \ldots, n$), the synergy effect affects its service level $l_i^{\text{post}} = 1 - w_i/p_i^{\text{post}}$ only through a change in its price p_i^{post}. Since p_i^{post} increases with the synergy level s if and only if $s < s^{(1)}$ (Proposition 2), so does l_i^{post}.

The aggregate effect (collusion, pooling, and synergy effects) of a merger on a firm's service level is shown in Proposition 4(c). It always increases the postmerger firm's service level, but it increases the nonparticipant firm's service level only when the synergy level is low (i.e., $s < s^{(2)}$).

This result raises an interesting point. When the synergy level $s < s^{(2)}$, a merger not only induces all firms to raise their prices (Proposition 3(a)) but also induces all firms to raise their service levels. The former affects consumer negatively, whereas the latter affects consumers positively. Note that the latter effect exists only when demand is uncertain. A similar trade-off also exists when $s > s^{(2)}$ because consumers will benefit from lower prices as well as a higher service level of a postmerger firm but hurt from a lower service level of nonparticipant firms. Then how can we measure the overall impact of a merger on consumers? Now we propose expected consumer welfare as the aggregate measure that antitrust agencies and other interested parties may use in evaluating a merger. Compared with the standard approach of computing consumer welfare as an area under the demand curve of a single firm, special care must be taken to account for price-competing oligopoly as well as for potential stock-outs. We derive expected consumer welfare in the following two steps. First, we present the consumer utility function of a representative consumer that leads to our demand function in the oligopolistic market. Second, we use this utility function to define expected consumer welfare that takes into account potential stock-outs.

Following Shubik (1980), we can show that the following utility function of a representative consumer generates the demand function $D_i = L_i(\mathbf{p}) + \tilde{\varepsilon}_i$ (where $L_i(\mathbf{p})$ is given in (1)):

$$u(\mathbf{D}) = \sum_{i=1}^{n}\left\{\frac{1+\gamma/(nb)}{b+\gamma}\left(a+\tilde{\varepsilon}_i-\frac{1}{2}D_i\right) + \frac{\gamma}{nb(b+\gamma)}\sum_{j\neq i}^{n}\left(a+\tilde{\varepsilon}_j-\frac{1}{2}D_j\right)\right\}D_i, \quad (7)$$

where $\mathbf{D} = (D_1, D_2, \ldots, D_n)^T$ denotes a consumption bundle. In (7), the first term in the bracket represents the (direct) marginal utility from the product sold by firm i (hereinafter, product i in short), and the second term in the bracket is the marginal utility from substitution.

Total expected consumer welfare (or surplus) from the consumption bundle is denoted by $E[cs(\mathbf{D})]$, which is the sum of expected consumer surplus from product i, $E[cs_i(\mathbf{D})]$; i.e., $E[cs(\mathbf{D})] = \sum_{i=1}^{n} E[cs_i(\mathbf{D})]$. To derive $cs_i(\mathbf{D})$, we consider two different cases. In the first case when a realization of random demand component $\tilde{\varepsilon}_i$ is smaller than or equal to safety stock $y_i = \sigma_i\Phi^{-1}(l_i)$, all demand will be satisfied. In this case, by substituting the demand $D_i = L_i(\mathbf{p}) + \tilde{\varepsilon}_i$ to (7) and then subtracting the price paid $\sum_{i=1}^{n} p_i D_i$, we obtain the following ex post consumer surplus:

$$cs_i(\mathbf{D}) = cs_i(\mathbf{p}, \tilde{\boldsymbol{\varepsilon}}) = \frac{1}{2}\left\{\frac{1+\gamma/(nb)}{b+\gamma}(a+\tilde{\varepsilon}_i) + \frac{\gamma}{nb(b+\gamma)}\sum_{j\neq i}^{n}(a+\tilde{\varepsilon}_j) - p_i\right\}\{L_i(\mathbf{p})+\tilde{\varepsilon}_i\}. \quad (8)$$

In the second case when a realization of $\tilde{\varepsilon}_i$ is greater than y_i, some demand will be lost. Similar to the monopoly cases of Ovchinnikov and Raz (2015) and Cohen et al. (2016), we assume that customers are first-come, first-served, so that every customer faces the same probability of not getting a product, $\{L_i(\mathbf{p}) + y_i\}/\{L_i(\mathbf{p}) + \tilde{\varepsilon}_i\}$. In this case, the ex post consumer surplus is given as $cs_i(\mathbf{p}, \tilde{\varepsilon}_i)\{L_i(\mathbf{p}) + y_i\}/\{L_i(\mathbf{p}) + \tilde{\varepsilon}_i\}$. Putting the two cases together, we can express the expected consumer surplus from product i, $E[cs_i(\mathbf{p}, \tilde{\boldsymbol{\varepsilon}})]$, as follows:

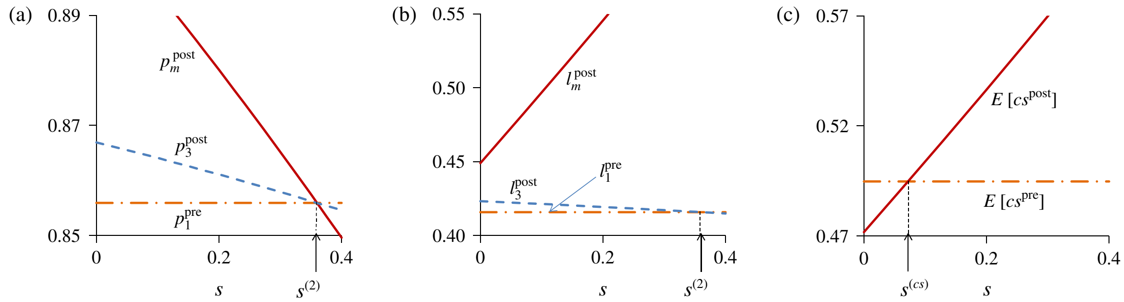
$$\begin{aligned} E[cs_i(\mathbf{p}, \tilde{\boldsymbol{\varepsilon}})] = &\int_{-\infty}^{\sigma_i\Phi^{-1}(l_i)}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} cs_i(\mathbf{p}, \boldsymbol{\varepsilon}) \\ &\cdot f(\boldsymbol{\varepsilon})\,d\varepsilon_1\cdots d\varepsilon_{i-1}d\varepsilon_{i+1}\cdots d\varepsilon_n d\varepsilon_i \\ &+\int_{\sigma_i\Phi^{-1}(l_i)}^{\infty}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} cs_i(\mathbf{p}, \boldsymbol{\varepsilon})\frac{L_i+\sigma_i\Phi^{-1}(l_i)}{L_i+\varepsilon_i} \\ &\cdot f(\boldsymbol{\varepsilon})\,d\varepsilon_1\cdots d\varepsilon_{i-1}d\varepsilon_{i+1}\cdots d\varepsilon_n d\varepsilon_i, \end{aligned} \quad (9)$$

where $f(\boldsymbol{\varepsilon}) = (2\pi)^{-n/2}|\Sigma|^{-1/2}e^{-\boldsymbol{\varepsilon}'\Sigma^{-1}\boldsymbol{\varepsilon}/2}$ is the n-dimensional joint density of the multivariate normal random variable $\tilde{\boldsymbol{\varepsilon}}$ and Σ is its covariance matrix. After a merger between firms 1 and 2, $cs_m(\mathbf{p}, \boldsymbol{\varepsilon})$ in (9) needs to be integrated over the aggregate uncertain component, $(\varepsilon_1 + \varepsilon_2)$.

Using the total expected consumer welfare $E[cs(\mathbf{D})]$ defined above, we now examine the aggregate impact of a merger on consumers (which essentially combines *all* the effects we have examined separately, including the collusion, pooling, and synergy effects on prices and service levels of both postmerger and nonparticipant firms).

Proposition 5. *Suppose* $b \geq (n-2)\sigma w(n\phi(\Phi^{-1}(l_3^{\text{post}}))\cdot(p_3^{\text{post}})^2)$. *Then there exists a threshold* $s^{(cs)} \in [0, s^{(2)}]$ *such*

Figure 4. (Color online) The Aggregate Effect of a Merger on (a) Prices, (b) Service Levels, and (c) Expected Consumer Welfare



Note. The same parameter values are used as in Figure 3.

that for any $s > s^{(cs)}$, the expected consumer welfare after a merger, $E[cs^{post}]$, is greater than that before the merger, $E[cs^{pre}]$.

The existence of the threshold $s^{(cs)}$ is intuitive. As discussed above, when $s > s^{(2)}$, consumers benefit from lower prices of all firms and a higher service level of a postmerger firm, although they hurt from a lower service level of nonparticipant firms. When the synergy level s is sufficiently high, the former positive effect outweighs the latter negative effect. However, compared with the earlier result in Proposition 3(a) that $s^{(2)}$ is nonincreasing in ρ, we observe $s^{(cs)}$ as well as $E[cs^{post}]$ change nonmonotonically in ρ. This is because the high pooling effect hurts consumers through increased prices, but at the same time, it benefits consumers through increased service levels. More importantly, even if a merger induces all firms to raise their prices, it can still improve expected consumer welfare by increasing firms' service levels. This is illustrated in Figure 4 when the synergy level s falls between $s^{(cs)}$ and $s^{(2)}$.[4]

The main takeaway from the above analysis is as follows. The extant literature has measured the impact of a merger on consumers via price changes, assuming consumer demand is deterministic. In reality, consumer demand is fundamentally uncertain. Our result indicates that under uncertain demand, it is crucial to take into account how a merger affects consumers via firms' service levels as well as their prices.

5. Extensions

This section examines four extensions of our base model. In Section 5.1, we consider asymmetric firms in a premerger market. In Section 5.2, we extend our results to nonnormal distributions. In Section 5.3, we analyze a demand model with a general uncertainty structure. Finally, in Section 5.4, we analyze the impact of stock-out substitution on mergers. For brevity, we focus on the pooling effect of a merger on prices and the synergy effect on nonparticipants' profits. The effects on service levels and expected consumer welfare can also be shown similarly.

5.1. Asymmetric Firms

So far, we have analyzed a merger in the premerger market in which firms are symmetric. Such a merger results in asymmetric competition between a postmerger firm and nonparticipant firms in the postmerger market. In this section, we examine the impact of a merger in the premerger market in which firms are asymmetric, and we demonstrate that our main results continue to hold.

Consider a premerger market in which firms differ in demand and cost parameters. Specifically, firm i $(= 1, 2, \ldots, n)$ faces its demand $D_i(\mathbf{p}) = L_i(\mathbf{p}) + \tilde{\varepsilon}_i$, where $L_i(\mathbf{p}) = a_i - b_i p_i + \gamma(\sum_{j=1}^n p_j/n - p_i)$ and $\tilde{\varepsilon} = (\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_n)$ follows a multivariate normal distribution $N(0, \Sigma)$ with $\text{Var}(\tilde{\varepsilon}_i) = \sigma_i^2$; firm i incurs a marginal cost w_i. We consider a situation where firms sell homogeneous products and differentiation among firms occurs at the retail level for the reasons such as locations, consumer characteristics, and store characteristics. After a merger between firms 1 and 2, the deterministic part of the postmerger firm's demand becomes $L_m(\mathbf{p}) = L_1(\mathbf{p}) + L_2(\mathbf{p})$, assuming that two retail outlets maintain their differentiation. As before, we measure the pooling effect of a merger in terms of the correlation coefficient ρ between $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$. Unlike the symmetric case, the postmerger firm may set two different prices at retail outlets 1 and 2 *in equilibrium*, so we denote firm i's postmerger price by p_i^{post} for $i = 1, 2, \ldots, n$ (instead of using subscript m). As for the synergy effect, when firms are symmetric in the premerger market, in Section 4 we have defined the synergy level as $s \equiv (w - w_m)/w$. However, when asymmetric firms compete in the premerger market, merging firms' marginal costs w_1 and w_2 may differ, so we refine our previous definition of s to $s \equiv (\min\{w_1, w_2\} - w_m)/\min\{w_1, w_2\}$. The synergy level s is nonnegative when the marginal cost of the postmerger firm w_m is at least as small as $\min\{w_1, w_2\}$. Note that when w_1 and w_2 are different, a merger leads to a change in the marginal cost of at least one merging firm. For this reason, we cannot isolate the collusion effect from the synergy effect as we did in Lemma 1 for the symmetric case. The following corollary shows that the pooling and synergy effects of a merger in the premerger market of asymmetric firms are consistent with those in the premerger market of symmetric firms.

Corollary 1. (a) *For any fixed $w_m \in (0, w]$, all postmerger prices p_i^{post} $(i = 1, 2, \ldots, n)$ are decreasing in ρ.*

(b) *For any fixed $\rho \in [-1, 1]$, there exists a threshold $s_{\text{asym}}^{(1)} \in [0, 1)$ such that a nonparticipant firm's profit π_i^{post} $(i = 3, 4, \ldots, n)$ as well as all postmerger prices p_i^{post} $(i = 1, 2, \ldots, n)$ is increasing in s if $s < s_{\text{asym}}^{(1)}$.*

5.2. Nonnormal Distributions

In this section, we consider a case in which the demand of a firm follows a general distribution. We denote by $f(\cdot)$ and $F(\cdot)$ the density of $\tilde{\varepsilon}_i$ with $E(\tilde{\varepsilon}_i) = 0$ and its cumulative distribution, respectively, and we denote by $f_m(\cdot)$ and $F_m(\cdot)$ the density of $\tilde{\varepsilon}_m = \tilde{\varepsilon}_1 + \tilde{\varepsilon}_2$ and its cumulative distribution. In the premerger market, the expected profit of firm i can be expressed as $\pi_i(\mathbf{p}, y_i) = (p_i - w)L_i(\mathbf{p}) - wy_i - p_i R_f(y_i)$, where $R_f(y_i) \equiv \int_{y_i}^{\infty} (t - y_i) f(t)\, dt$ represents the expected lost sales of firm i having safety stock y_i. Similarly, the expected profit of the postmerger firm is given as $\pi_m(\mathbf{p}, y_m) = (p_1 - w_m)L_1(\mathbf{p}) + (p_2 - w_m)L_2(\mathbf{p}) - w_m y_m - ((p_1 + p_2)/2) \cdot R_{f_m}(y_m)$.

We next present the definition of dispersive ordering and a failure rate (e.g., see Müller and Stoyan 2002). Then we use these properties to generalize our results in Section 4.

Definition. (a) A random variable X is smaller than Y in dispersive ordering, written as $X \leq_{\text{disp}} Y$, if $F^{-1}(\tau_2) - F^{-1}(\tau_1) \leq G^{-1}(\tau_2) - G^{-1}(\tau_1)$ for all $0 < \tau_1 < \tau_2 < 1$, where F and G are the distribution functions of X and Y, respectively.

(b) Let X be a random variable with density f and cumulative distribution F. The failure rate of X is defined as $h(x) = f(x)/\{1 - F(x)\}$. The random variable X has an increasing failure rate (IFR) if $h(x)$ is increasing for all x such that $F(x) < 1$.[5]

Corollary 2. (a) *Let $\tilde{\varepsilon}_m$ and $\tilde{\xi}_m$ be two random variables with $E[\tilde{\varepsilon}_m] = E[\tilde{\xi}_m] = 0$. Let p_i^{post} or $\hat{p}_i^{\text{post}}$ $(i = m, 3, 4, \ldots, n)$ be the equilibrium price of firm i when the postmerger firm faces random demand component $\tilde{\varepsilon}_m$ or $\tilde{\xi}_m$, respectively. Then, $\tilde{\varepsilon}_m \leq_{\text{disp}} \tilde{\xi}_m$ implies $p_i^{\text{post}} \geq \hat{p}_i^{\text{post}}$.*

(b) *If $\tilde{\varepsilon}_m$ follows an IFR distribution, then there exists a threshold $s_{\text{non}}^{(1)} \in [0, 1]$ such that a nonparticipant firm's profit π_i^{post} $(i = 3, 4, \ldots, n)$, as well as all postmerger prices p_i^{post} $(i = m, 3, 4, \ldots, n)$, is increasing in s if and only if $s < s_{\text{non}}^{(1)}$.*

Corollary 2(a) generalizes the pooling effect of a merger presented earlier in Proposition 1(a). The condition $\tilde{\varepsilon}_m \leq_{\text{disp}} \tilde{\xi}_m$ implies that the expected lost sales $R_{f_m}[F_m^{-1}(1 - w_m/p_m)]$ $(= 2\,\partial c_m/\partial p_1)$ are smaller for a less dispersive demand. Because a marginal loss from raising a price is lower for a less dispersive demand, the postmerger firm charges a higher price in equilibrium, which is followed by an increase in prices of nonparticipant firms. This is consistent with Proposition 1(a), since a larger ρ—or equivalently, a larger σ_m under a normally distributed demand—results in a more dispersive $\tilde{\varepsilon}_m$. Corollary 2(b) shows that the synergy effect of a merger presented earlier in Proposition 2 holds for an IFR distribution that includes many commonly used distributions such as normal, uniform, gamma, and Weibull (with a shape parameter greater than 1 for gamma and Weibull); see, e.g., Kocabıyıkoğlu and Popescu (2011).

5.3. Demand Function with a General Uncertainty Structure

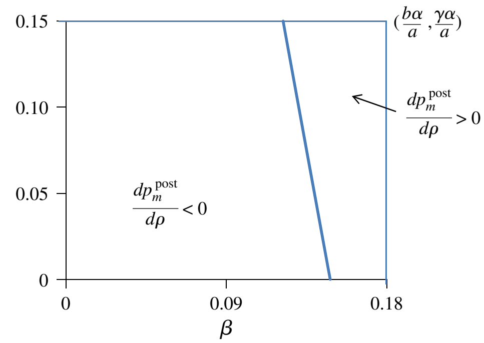
In this section, we consider a more general model in which firm i's demand is $D_i(\mathbf{p}) = L_i(\mathbf{p}) + \delta_i(\mathbf{p})\tilde{\varepsilon}_i$, where $L_i(\mathbf{p})$ is given in (1), $\delta_i(\mathbf{p}) = \alpha - \beta p_i + \theta((1/n)\sum_{j=1}^{n} p_j - p_i)$ ($\alpha \geq 0$, $\beta \geq 0$ and $\theta \geq 0$), and $\tilde{\varepsilon} = (\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_n)$ follows a multivariate normal distribution with $E(\tilde{\varepsilon}_i) = 0$ and $\text{Var}(\tilde{\varepsilon}_i) = 1$. This model takes the following two commonly used models in the literature as special cases: (1) when $(\beta, \theta) = (0, 0)$, $D_i(\mathbf{p}) = L_i(\mathbf{p}) + \alpha\tilde{\varepsilon}_i$, which is the additive demand function in the main body with $\sigma = \alpha$; and (2) when $(\beta, \theta) = (b\alpha/a, \gamma\alpha/a)$, $D_i(\mathbf{p}) = L_i(\mathbf{p})(1 + \alpha\tilde{\varepsilon}_i/a)$, which is a multiplicative demand function.[6] After a merger between firms 1 and 2 takes place, the postmerger firm faces the demand of $D_m(\mathbf{p}) = L_1(\mathbf{p}) + L_2(\mathbf{p}) + \delta_1(\mathbf{p})\tilde{\varepsilon}_1 + \delta_2(\mathbf{p})\tilde{\varepsilon}_2$.

Corollary 3. (a) *For any fixed $w_m \in (0, w]$ and $\theta \geq 0$, there exists a threshold $\hat{\beta}$ (≥ 0) such that if $\beta < \hat{\beta}$, all postmerger prices p_i^{post} $(i = m, 3, 4, \ldots, n)$ are decreasing in ρ.*

(b) *Suppose $\gamma > \theta\phi(\Phi^{-1}(1 - w/p_3^{\text{post}}))(1 - w/p_3^{\text{post}})^{-1}$. Then, for any fixed $\rho \in [-1, 1]$, there exists a threshold $s_{\text{gen}}^{(1)} \in [0, 1)$ such that a nonparticipant firm's profit π_i^{post} $(i = 3, 4, \ldots, n)$, as well as all postmerger prices p_i^{post} $(i = m, 3, 4, \ldots, n)$, is increasing in s if and only if $s < s_{\text{gen}}^{(1)}$.*

Corollary 3(a) shows that the pooling effect of a merger on firms' prices is the same as that in our base model as long as the impact of demand uncertainty on price sensitivity is sufficiently small (i.e., β and θ are small). See Figure 5 for illustration, in which the lower left corner at $(\beta, \theta) = (0, 0)$ corresponds to the additive demand case and the upper right corner at $(\beta, \theta) = (b\alpha/a, \gamma\alpha/a)$ corresponds to the multiplicative demand case. The intuition from this result is as follows. To hedge against the risk of uncertainty as a result of high ρ, in the additive case, a postmerger firm wants to reduce its price to decrease the coefficient of variance without affecting the variance; however, in the multiplicative case, a postmerger firm wants to increase its price to reduce the variance without affecting the coefficient of variance (see Petruzzi and Dada 1999). When β and θ are sufficiently small, the effect of controlling

Figure 5. (Color online) The Pooling Effect of a Merger Under a General Uncertainty Model



Note. The same parameter values are used as in Figure 1.

the coefficient of variance dominates the effect of controlling the variance, hence inducing a postmerger firm to decrease its price with ρ as in the additive case. Next, Corollary 3(b) shows that the nonmonotonic relationship between the postmerger firm's prices p_i^{post} and the synergy level s is preserved in the general demand model. Consequently, when synergy level s is small, larger cost synergies from a merger benefit nonparticipant firms. The condition on γ guarantees that a higher price of a merging firm has a positive externality on a nonparticipant firm as in the base model.[7]

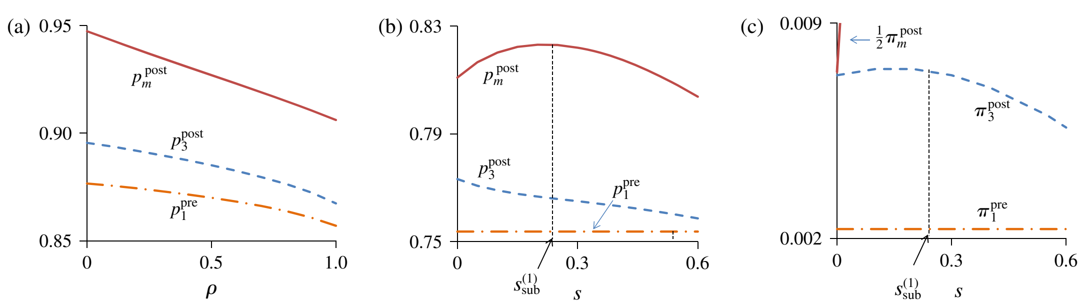
5.4. Stock-out Substitution

This section examines the case in which a fraction of consumers who do not find a product at their local retailers look for the product at other retailers. We follow the standard approach of modeling this stock-out substitution (Netessine and Rudi 2003, Zhao and Atkins 2008, and references therein) by assuming that a part of excess demand is reallocated to other retailers in deterministic proportions and that the sales are lost if reallocated demand cannot be satisfied. Then, the total demand of firm i is the sum of its original demand and a fraction of lost demands from other firms; i.e., $\hat{D}_i(\mathbf{p}, \mathbf{y}_{-i}) = L_i(\mathbf{p}) + \tilde{\varepsilon}_i + (\kappa/(n-1)) \sum_{j\neq i}(D_j - q_j)^+$, where κ (≤ 1) is a fraction of the excess demand from firm j spilled to other firms, and $(\kappa/(n-1)) \sum_{j\neq i}(D_j - q_j)^+$ is referred to as "spill demand." Random variable $\hat{D}_i$ is the sum of n random variables—a normal $\tilde{\varepsilon}_i$ and $(n-1)$ truncated normal $(\kappa/(n-1))(D_j - q_j)^+$ for $j \neq i$—which are correlated with each other in a complex manner. Thus the analytical characterization of even premerger equilibrium is intractable (Netessine and Rudi 2003, Zhao and Atkins 2008). For this reason, we study stock-out substitution numerically.

We first examine how stock-out substitution affects the pooling effect of a merger. Figure 6(a) uses the same parameter values as in Figure 1 except $\kappa = 0.2$. By comparing these two figures, we observe that under stock-out substitution inventory pooling continues to induce all firms to raise their prices (i.e., p_m^{post} and p_3^{post} decrease with ρ). This is because the main driver for the pooling effect (i.e., the demand volatility of the postmerger firm is increasing with ρ) exists with or without stock-out substitution. Although the correlation between firms' demands does not affect *premerger* equilibrium in Figure 1, we observe in Figure 6(a) that p_1^{pre} is decreasing with ρ under stock-out substitution. To understand this result, suppose that ρ is high. Then when the demand of one firm, say, firm 1, is high, there is a high chance that the demands of the other firms are also high. In this case, when the other firms experience stock-outs, it is likely that firm 1 also experiences a stock-out and cannot satisfy the spill demand from the other firms. As a result, higher ρ reduces the expected spill demand a firm can satisfy.[8] Therefore, with higher ρ, firms compete more intensely by reducing their prices.

We next examine how stock-out substitution affects the synergy effect of a merger. Figures 6(b) and 6(c) use the same parameter values as in Figure 2, except $\kappa = 0.2$. From these two figures, we observe that the synergy effect on the postmerger firm is consistent in both cases with or without stock-out substitution: larger cost synergies increase the postmerger firm's price only when s is lower than a certain threshold (denoted by $s_{sub}^{(1)}$ in Figure 6(b)), although it always increases the postmerger firm's expected profit. For the nonparticipant firm, when s is small, π_3^{post} increases with s, confirming that larger cost synergies can benefit nonparticipant firms. However, unlike Proposition 2

Figure 6. (Color online) Under Stock-out Substitution, (a) the Pooling Effect on Prices, (b) the Synergy Effect on Prices, and (c) the Synergy Effect on Expected Profits



(showing that without stock-out substitution, p_3^{post} and π_3^{post} decrease in s if and only if $s > s^{(1)}$), Figures 6(b) and 6(c) show that p_3^{post} and π_3^{post} can decrease in s even when $s < s_{\text{sub}}^{(1)}$. We can explain this result intuitively as follows. With larger cost synergies, the postmerger firm increases its safety stock, which not only reduces its own expected lost sales as a result of stock-outs but also reduces its spill demand to the nonparticipant firm. Therefore, with stock-out substitution, larger cost synergies create an additional force that induces the nonparticipant firm to charge a lower price and to stock less, causing π_3^{post} to be decreasing in s further.

6. Conclusion

M&As have been employed by many firms as major strategies to create competitive advantages. A merger enables merging parties not only to cooperate with each other in their decision making but also to achieve cost savings by improving operational efficiencies. Whether such competitive advantages created by a merger will benefit consumers is a central concern of antitrust agencies. Popular defensive arguments used by firms have been that merger synergies will lower the cost of a postmerger firm and thus be ultimately passed on to consumers. Whereas the existing theory of mergers has been supportive of those arguments, this paper shows they are not necessarily true.

While building on the competitive models established in the rich literature on mergers, our model features two novel operational elements: uncertain demand and statistical economies of scale. Clearly, a retail business entails uncertainty in consumer demand, and therefore demand uncertainty has been one of the most fundamental features in the literature of operations management. Under uncertain demand, a merger can create statistical economies of scale by reducing the aggregate volatility of combined demands in addition to conventional economies of scale that lead to marginal cost reduction. Such statistical economies of scale allow a postmerger firm to reduce inventory costs by managing their stocks in a centralized manner.

Our analysis shows that cost savings from statistical economies of scale have substantially different impacts on firms' prices and expected profits compared with cost savings from conventional economies of scale. First, we find that statistical economies of scale (i.e., pooling effect) indeed reduce the expected cost of a postmerger firm (hence increasing its expected profit), but contrary to a common belief, they always induce both postmerger and nonparticipant firms to raise their prices. Second, although the existing theory has shown that cost synergies resulting from conventional economies of scale (i.e., synergy effect) lead to price reduction under deterministic demand, our analysis shows that this is no longer true under uncertain demand. When a postmerger firm faces highly uncertain demand or its cost synergies are not significant, it is better off raising its price. Interestingly, larger cost synergies of a postmerger firm can benefit nonparticipant firms when accompanying a price increase. Finally, when a postmerger firm can utilize both conventional and statistical economies of scale, consumer price is less likely to rise after a merger in the industries that exhibit higher correlation among firms' uncertain demands. Furthermore, a merger may induce firms to raise their service levels and ultimately benefit consumers even if prices are increased.

We have considered various extensions of our base model and analysis. However, because of the inherent complexity of analyzing competitive price-setting newsvendors and their mergers, we have made some simplifying assumptions such as a linear demand function, no supply chain consideration, and a single product with no economies of scope. Relaxing these assumptions will enrich our findings, but the incorporation of these features may require simplification of other parts. Our results also provide several important theoretical findings that may be tested empirically.

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Appendix A. Main Proofs

We use a superscript * to denote a firm's best response to the other firms' prices. For example, the joint best response price of all nonparticipant firms to the postmerger firm's price p_m is denoted by $p_3^*(p_m)$. To conserve space, we omit the proofs of Lemmas A1 and 1, Proposition 3, and Corollary 3 as well as other detailed proofs. These omitted proofs are available from the authors upon request.

Lemma A1. *Suppose the following condition is satisfied*:

$$2b + \frac{n-2}{n}\gamma - \frac{\sigma w^2}{\phi(\Phi^{-1}(1-w/\underline{p}))\,\underline{p}^3} > 0. \tag{A1}$$

(a) $0 < dp_3^*/dp_m < 1$, $0 < dp_m^*/dp_3 < 1$ *and* $d\pi_3^*/dp_m > 0$.

(b) *There exists a unique pure-strategy Nash equilibrium in the premerger market in which the symmetric equilibrium prices* $p_1^{\text{pre}} = p_2^{\text{pre}} = \cdots = p_n^{\text{pre}}$ *are the unique solution of* (3).

(c) *There exists a unique pure-strategy Nash equilibrium in the postmerger market in which* $p_1^{\text{post}} = p_2^{\text{post}} = p_m^{\text{post}}$ *and* $p_3^{\text{post}} = p_4^{\text{post}} = \cdots = p_n^{\text{post}}$ *are the unique solutions of the following equations:*

$$a - \left(b + \gamma\frac{n-2}{n}\right)(2p_m^{\text{post}} - w_m) + \gamma\frac{n-2}{n}p_3^{\text{post}} - \frac{\sigma_m}{2}R\left(\Phi^{-1}\left(1 - \frac{w_m}{p_m^{\text{post}}}\right)\right) = 0; \tag{A2}$$

$$a-\left(2b+\frac{n+1}{n}\gamma\right)p_3^{\text{post}}+\left(b+\frac{n-1}{n}\gamma\right)w +\gamma\frac{2}{n}p_m^{\text{post}}-\sigma R\left(\Phi^{-1}\left(1-\frac{w}{p_3^{\text{post}}}\right)\right)=0. \tag{A3}$$

Proof of Proposition 1. (a) We compute $dp_m^{\text{post}}/d\sigma_m$ and $dp_3^{\text{post}}/d\sigma_m$ by applying the implicit function theorem to (A2) and (A3) as follows:

$$\begin{bmatrix}\dfrac{dp_m^{\text{post}}}{d\sigma_m}\\ \dfrac{dp_3^{\text{post}}}{d\sigma_m}\end{bmatrix} = -J^{-1}\times\begin{bmatrix}\dfrac{\partial}{\partial\sigma_m}\left(\dfrac{\partial\pi_m(\mathbf{p},\sigma_m)}{\partial p_1}\bigg|_{\mathbf{p}=\mathbf{p}^{\text{post}}}\right)\\ \dfrac{\partial}{\partial\sigma_m}\left(\dfrac{\partial\pi_3(\mathbf{p})}{\partial p_3}\bigg|_{\mathbf{p}=\mathbf{p}^{\text{post}}}\right)\end{bmatrix} = -J^{-1}\times\begin{bmatrix}\dfrac{\partial^2\pi_m(\mathbf{p},\sigma_m)}{\partial p_1\partial\sigma_m}\bigg|_{\mathbf{p}=\mathbf{p}^{\text{post}}}\\ 0\end{bmatrix},$$

where

$$J^{-1} = \left(\begin{bmatrix}\dfrac{\partial}{\partial p_3^{\text{post}}}\left(\dfrac{\partial\pi_3(\mathbf{p})}{\partial p_3}\bigg|_{\mathbf{p}=\mathbf{p}^{\text{post}}}\right) & -\dfrac{\partial}{\partial p_3^{\text{post}}}\left(\dfrac{\partial\pi_m(\mathbf{p},\sigma_m)}{\partial p_1}\bigg|_{\mathbf{p}=\mathbf{p}^{\text{post}}}\right)\\ -\dfrac{\partial}{\partial p_m^{\text{post}}}\left(\dfrac{\partial\pi_3(\mathbf{p})}{\partial p_3}\bigg|_{\mathbf{p}=\mathbf{p}^{\text{post}}}\right) & \dfrac{\partial}{\partial p_m^{\text{post}}}\left(\dfrac{\partial\pi_m(\mathbf{p},\sigma_m)}{\partial p_1}\bigg|_{\mathbf{p}=\mathbf{p}^{\text{post}}}\right)\end{bmatrix}\right) \cdot\left(\frac{\partial}{\partial p_m^{\text{post}}}\left(\frac{\partial\pi_m(\mathbf{p},\sigma_m)}{\partial p_1}\bigg|_{\mathbf{p}=\mathbf{p}^{\text{post}}}\right)\frac{\partial}{\partial p_3^{\text{post}}}\left(\frac{\partial\pi_3(\mathbf{p})}{\partial p_3}\bigg|_{\mathbf{p}=\mathbf{p}^{\text{post}}}\right) -\frac{\partial}{\partial p_3^{\text{post}}}\left(\frac{\partial\pi_m(\mathbf{p},\sigma_m)}{\partial p_1}\bigg|_{\mathbf{p}=\mathbf{p}^{\text{post}}}\right)\frac{\partial}{\partial p_m^{\text{post}}}\left(\frac{\partial\pi_3(\mathbf{p})}{\partial p_3}\bigg|_{\mathbf{p}=\mathbf{p}^{\text{post}}}\right)\right)^{-1}.$$

Substituting J^{-1} into the above and using

$$\frac{dp_3^{\text{post}}}{dp_m^{\text{post}}} = -\frac{\partial}{\partial p_m^{\text{post}}}\left(\frac{\partial\pi_3(\mathbf{p})}{\partial p_3}\bigg|_{\mathbf{p}=\mathbf{p}^{\text{post}}}\right)\bigg/\frac{\partial}{\partial p_3^{\text{post}}}\left(\frac{\partial\pi_3(\mathbf{p})}{\partial p_3}\bigg|_{\mathbf{p}=\mathbf{p}^{\text{post}}}\right),$$

we can simplify $dp_m^{\text{post}}/d\sigma_m$ and $dp_3^{\text{post}}/d\sigma_m$ into

$$\frac{dp_m^{\text{post}}}{d\sigma_m} = \left(-\frac{\partial^2\pi_m(\mathbf{p},\sigma_m)}{\partial p_1\partial\sigma_m}\bigg|_{\mathbf{p}=\mathbf{p}^{\text{post}}}\right) \cdot\left(\frac{\partial}{\partial p_m^{\text{post}}}\left(\frac{\partial\pi_m(\mathbf{p},\sigma_m)}{\partial p_1}\bigg|_{\mathbf{p}=\mathbf{p}^{\text{post}}}\right) +\frac{\partial}{\partial p_3^{\text{post}}}\left(\frac{\partial\pi_m(\mathbf{p},\sigma_m)}{\partial p_1}\bigg|_{\mathbf{p}=\mathbf{p}^{\text{post}}}\right)\frac{dp_3^{\text{post}}}{dp_m^{\text{post}}}\right)^{-1} \tag{A4}$$

and $\dfrac{dp_3^{\text{post}}}{d\sigma_m} = \dfrac{dp_3^{\text{post}}}{dp_m^{\text{post}}}\dfrac{dp_m^{\text{post}}}{d\sigma_m}$.

We now show $dp_m^{\text{post}}/d\sigma_m<0$ by examining its denominator and numerator, respectively. To compute the denominator of $dp_m^{\text{post}}/d\sigma_m$, we use (A2) and $dR(t)/dt=\Phi(t)-1$, getting

$$\frac{\partial}{\partial p_m^{\text{post}}}\left(\frac{\partial\pi_m(\mathbf{p},\sigma_m)}{\partial p_1}\bigg|_{\mathbf{p}=\mathbf{p}^{\text{post}}}\right) = -2\left(b+\gamma\frac{n-2}{n}\right)+\frac{\sigma_m w_m^2}{2\phi(\Phi^{-1}(1-w_m/p_m^{\text{post}}))(p_m^{\text{post}})^3}<0,$$

and $\partial/\partial p_3^{\text{post}}(\partial\pi_m(\mathbf{p},\sigma_m)/\partial p_1|_{\mathbf{p}=\mathbf{p}^{\text{post}}})=((n-2)/n)\gamma>0$. Using (A1), we can show that

$$\frac{\partial}{\partial p_m^{\text{post}}}\left(\frac{\partial\pi_m(\mathbf{p},\sigma_m)}{\partial p_1}\bigg|_{\mathbf{p}=\mathbf{p}^{\text{post}}}\right)+\frac{\partial}{\partial p_3^{\text{post}}}\left(\frac{\partial\pi_m(\mathbf{p},\sigma_m)}{\partial p_1}\bigg|_{\mathbf{p}=\mathbf{p}^{\text{post}}}\right)<0.$$

Because $dp_3^{\text{post}}/dp_m<1$ from Lemma A1(a),

$$\frac{\partial}{\partial p_m^{\text{post}}}\left(\frac{\partial\pi_m(\mathbf{p},\sigma_m)}{\partial p_1}\bigg|_{\mathbf{p}=\mathbf{p}^{\text{post}}}\right) +\frac{\partial}{\partial p_3^{\text{post}}}\left(\frac{\partial\pi_m(\mathbf{p},\sigma_m)}{\partial p_1}\bigg|_{\mathbf{p}=\mathbf{p}^{\text{post}}}\right)\frac{dp_3^{\text{post}}}{dp_m^{\text{post}}}<0.$$

To compute the numerator of $dp_m^{\text{post}}/d\sigma_m$, we first find the expression of $\pi_m(\mathbf{p})=\pi_m^d(\mathbf{p})-c_m(p_1,p_2)$. By substituting $y_m=\sigma_m\Phi^{-1}(1-2w_m/(p_1+p_2))$, $L_1(\mathbf{p})$ and $L_2(\mathbf{p})$ into $\pi_m^d(\mathbf{p})$ and $c_m(p_1,p_2,y_m)$ in (4), respectively, we have

$$\pi_m^d(\mathbf{p}) = \sum_{i=1}^{2}(p_i-w_m) \cdot\left\{a-\left(b+\frac{n-1}{n}\gamma\right)p_i+\frac{\gamma}{n}\sum_{j\neq i}^{n}p_j\right\},\quad\text{and} \tag{A5}$$

$$c_m(p_1,p_2) = w_m\sigma_m\Phi^{-1}\left(1-\frac{2w_m}{p_1+p_2}\right) +\frac{p_1+p_2}{2}\sigma_m R\left(\Phi^{-1}\left(1-\frac{2w_m}{p_1+p_2}\right)\right). \tag{A6}$$

From (A5), $\partial^2\pi_m^d(\mathbf{p},\sigma_m)/(\partial\sigma_m\partial p_1)=0$. From (A6), by using $dR(t)/dt=\Phi(t)-1$ we obtain $\partial^2 c_m(p_1,p_2,\sigma_m)/\partial\sigma_m\partial p_1$ as follows:

$$\frac{\partial^2 c_m(p_1,p_2,\sigma_m)}{\partial\sigma_m\partial p_1} = \frac{\partial}{\partial\sigma_m}\left\{w_m\sigma_m\frac{\partial\Phi^{-1}(1-2w_m/(p_1+p_2))}{\partial p_1} -\frac{p_1+p_2}{2}\sigma_m\frac{2w_m}{p_1+p_2}\frac{\partial\Phi^{-1}(1-2w_m/p_1+p_2)}{\partial p_1} +\frac{\sigma_m R(\Phi^{-1}(1-2w_m/(p_1+p_2)))}{2}\right\} = \frac{1}{2}R\left(\Phi^{-1}\left(1-\frac{2w_m}{p_1+p_2}\right)\right)>0.$$

Thus,

$$-\frac{\partial^2\pi_m(\mathbf{p},\sigma_m)}{\partial p_1\partial\sigma_m}\bigg|_{\mathbf{p}=\mathbf{p}^{\text{post}}} = -\frac{\partial^2\pi_m^d(\mathbf{p},\sigma_m)}{\partial\sigma_m\partial p_1}+\frac{\partial^2 c_m(p_1,p_2,\sigma_m)}{\partial\sigma_m\partial p_1}>0.$$

Finally, because the denominator of $dp_m^{\text{post}}/d\sigma_m$ is negative and its numerator is positive, we get $dp_m^{\text{post}}/d\sigma_m<0$, hence $dp_m^{\text{post}}/d\rho=(\sigma/\sqrt{2+2\rho})(dp_m^{\text{post}}/d\sigma_m)<0$.

Finally, $dp_3^{\text{post}}/d\sigma_m=(dp_3^{\text{post}}/dp_m^{\text{post}})(dp_m^{\text{post}}/d\sigma_m)<0$ because $dp_m^{\text{post}}/d\sigma_m<0$ from above and $dp_3^{\text{post}}/dp_m>0$ from Lemma A1(a). So $dp_3^{\text{post}}/d\rho=(\sigma/\sqrt{2+2\rho})(dp_3^{\text{post}}/d\sigma_m)<0$.

(b) We first prove $d\pi_m^{\text{post}}/d\rho<0$. From (4), we can write

$$\frac{d\pi_m^{\text{post}}}{d\rho} = \left(\frac{\partial\pi_m^{\text{post}}}{\partial p_1}+\frac{\partial\pi_m^{\text{post}}}{\partial p_2}\right)\frac{dp_m^{\text{post}}}{d\rho}+\frac{\partial\pi_m^{\text{post}}}{\partial y_m}\frac{dy_m^{\text{post}}}{d\rho} +\frac{\partial\pi_m^{\text{post}}}{\partial\rho}+\frac{dp_3^{\text{post}}}{d\rho}\sum_{j=3}^{n}\frac{\partial\pi_m^{\text{post}}}{\partial p_j}=\frac{\partial\pi_m^{\text{post}}}{\partial\rho} +\frac{dp_3^{\text{post}}}{d\rho}\sum_{j=3}^{n}\frac{\partial\pi_m^{\text{post}}}{\partial p_j},$$

since $\partial\pi_m^{\text{post}}/\partial p_1=\partial\pi_m^{\text{post}}/\partial p_2=0$ and $\partial\pi_m^{\text{post}}/\partial y_m=0$ at $(p_1,p_2,y_m)=(p_m^{\text{post}},p_m^{\text{post}},y_m^{\text{post}})$ (by the envelope theorem).

From (4), using $R(t) = \phi(t) - t(1 - \Phi(t))$ and $dR(t)/dt = \Phi(t) - 1$, we have $\partial\pi_m^{\text{post}}/\partial p_j = (2\gamma/n)(p_m^{\text{post}} - w_m)$ and

$$\begin{aligned}\frac{\partial\pi_m^{\text{post}}}{\partial\rho} &= -\frac{p_m^{\text{post}}\sigma}{\sqrt{2+2\rho}} R\left(\frac{y_m^{\text{post}}}{\sigma_m}\right) - p_m^{\text{post}}\sigma_m \frac{d}{d\rho} R\left(\frac{y_m^{\text{post}}}{\sigma_m}\right)\\ &= -\frac{p_m^{\text{post}}\sigma}{\sqrt{2+2\rho}}\phi\left(\frac{y_m^{\text{post}}}{\sigma_m}\right).\end{aligned}$$

Finally, substituting $\partial\pi_m^{\text{post}}/\partial p_j$ and $\partial\pi_m^{\text{post}}/\partial\rho$ into $d\pi_m^{\text{post}}/d\rho$, we obtain $d\pi_m^{\text{post}}/d\rho = -(p_m^{\text{post}}\sigma/\sqrt{2+2\rho})\phi(y_m^{\text{post}}/\sigma_m) + (2\gamma/n)\cdot(p_m^{\text{post}} - w_m)(n-2)(dp_3^{\text{post}}/d\rho) < 0$, where the inequality is due to $dp_3^{\text{post}}/d\rho < 0$ from part (a). Next, $d\pi_3^{\text{post}}/d\rho = (d\pi_3^{\text{post}}/dp_m)\cdot(dp_m^{\text{post}}/d\rho) < 0$ because $d\pi_3^{\text{post}}/dp_m > 0$ by Lemma A1(a) and $dp_m^{\text{post}}/d\rho < 0$ by part (a). □

Proof of Proposition 2. (a) We focus on proving that $dp_m^{\text{post}}/ds < 0$ if and only if $s > s^{(1)}$, and that $s^{(1)}$ is nondecreasing in σ_m with $s^{(1)} = 0$ at $\sigma_m = 0$. Then the result about p_3^{post} follows easily: $dp_3^{\text{post}}/ds = (dp_3^{\text{post}}/dp_m)(dp_m^{\text{post}}/ds) < 0$ if and only if $s > s^{(1)}$ because $dp_3^{\text{post}}/dp_m > 0$ from Lemma A1(a) and $dp_m^{\text{post}}/ds < 0$ if and only if $s > s^{(1)}$. The proof proceeds in three steps. In Step 1, we get the expression of dp_m^{post}/dw_m. In Step 2, we prove there exists a unique $w_m^{(1)}$ such that $dp_m^{\text{post}}/dw_m > 0$ if and only if $w_m < w_m^{(1)}$. In Step 3, we use $w_m^{(1)}$ to compute $s^{(1)}$.

Step 1. Similar to Proposition 1(a), we apply the implicit function theorem to obtain

$$\begin{aligned}\frac{dp_m^{\text{post}}}{dw_m} = &\left(-\frac{\partial^2\pi_m(\mathbf{p}, w_m)}{\partial p_1\partial w_m}\bigg|_{\mathbf{p}=\mathbf{p}^{\text{post}}}\right)\\ &\cdot\left(\frac{\partial}{\partial p_m^{\text{post}}}\left(\frac{\partial\pi_m(\mathbf{p}, w_m)}{\partial p_1}\bigg|_{\mathbf{p}=\mathbf{p}^{\text{post}}}\right)\right.\\ &\left.+\frac{\partial}{\partial p_3^{\text{post}}}\left(\frac{\partial\pi_m(\mathbf{p}, w_m)}{\partial p_1}\bigg|_{\mathbf{p}=\mathbf{p}^{\text{post}}}\right)\frac{dp_3^{\text{post}}}{dp_m^{\text{post}}}\right)^{-1}. \quad \text{(A7)}\end{aligned}$$

Since the denominator of (A7) is negative (see the proof of Proposition 1(a)), dp_m^{post}/dw_m has the same sign as

$$\begin{aligned}\frac{\partial^2\pi_m(\mathbf{p}, w_m)}{\partial p_1\partial w_m}\bigg|_{\mathbf{p}=\mathbf{p}^{\text{post}}} = &\frac{\partial^2\pi_m^d(\mathbf{p}, w_m)}{\partial w_m\partial p_1}\bigg|_{\mathbf{p}=\mathbf{p}^{\text{post}}}\\ &-\frac{\partial^2 c_m(p_1, p_2, \sigma_m)}{\partial\sigma_m\partial p_1}\bigg|_{\mathbf{p}=\mathbf{p}^{\text{post}}},\end{aligned}$$

where $\partial^2\pi_m^d(\mathbf{p}, w_m)/\partial w_m\partial p_1 = b + ((n-2)/n)\gamma$ from (A5) and $\partial^2 c_m(p_1, p_2, \sigma_m)/\partial\sigma_m\partial p_1 = \sigma_m w_m/(2\phi(\Phi^{-1}(1 - 2w_m/(p_1+p_2)))\{(p_1+p_2)/2\}^2)$ from (A6) and $dR(t)/dt = \Phi(t) - 1$. Thus,

$$\begin{aligned}&\frac{\partial^2\pi_m(\mathbf{p}, w_m)}{\partial p_1\partial w_m}\bigg|_{\mathbf{p}=\mathbf{p}^{\text{post}}}\\ &= b + \frac{n-2}{n}\gamma - \sigma_m\left\{2h\left(\Phi^{-1}\left(1 - \frac{w_m}{p_m^{\text{post}}}\right)\right)p_m^{\text{post}}\right\}^{-1}, \quad \text{(A8)}\end{aligned}$$

where $h(t) = \phi(t)/\{1 - \Phi(t)\}$ is the failure rate of a standard normal variable.

Step 2. We can show the existence and uniqueness of $w_m^{(1)}$ by proving the following three statements: (i) when $w_m \to 0$, $\partial^2\pi_m(\mathbf{p}, w_m)/\partial p_1\partial w_m|_{\mathbf{p}=\mathbf{p}^{\text{post}}} > 0$, (ii) when w_m is sufficiently large, $\partial^2\pi_m(\mathbf{p}, w_m)/\partial p_1\partial w_m|_{\mathbf{p}=\mathbf{p}^{\text{post}}} < 0$, and (iii) $d\{\partial^2\pi_m(\mathbf{p}, w_m)/\partial p_1\partial w_m|_{\mathbf{p}=\mathbf{p}^{\text{post}}}\}/dw_m < 0$ when $\partial^2\pi_m(\mathbf{p}, w_m)/\partial p_1\partial w_m|_{\mathbf{p}=\mathbf{p}^{\text{post}}} = 0$. Then from (i) and (ii), there exists at least one $w_m^{(1)}$, and the value of $w_m^{(1)}$ is determined by $\partial^2\pi_m(\mathbf{p}, w_m)/\partial p_1\cdot\partial w_m|_{\mathbf{p}=\mathbf{p}^{\text{post}}} = 0$. In addition, from (iii), $w_m^{(1)}$ is unique because $\partial^2\pi_m(\mathbf{p}, w_m)/\partial p_1\partial w_m|_{\mathbf{p}=\mathbf{p}^{\text{post}}}$ can cross zero only once as w_m increases. To show how $w_m^{(1)}$ changes with σ_m, we compute $d\sigma_m/dw_m^{(1)}$. By solving $\partial^2\pi_m(\mathbf{p}, w_m)/\partial p_1\partial w_m|_{\mathbf{p}=\mathbf{p}^{\text{post}}} = 0$ from (A8), we have $\sigma_m = 2(b + (n-2)/n\gamma)h(\Phi^{-1}(1 - w_m^{(1)}/p_m^{\text{post}}))p_m^{\text{post}}$. Then,

$$\begin{aligned}\frac{d\sigma_m}{dw_m^{(1)}} = &-\frac{2(b + (n-2)/n\gamma)h'(\Phi^{-1}(1 - w_m^{(1)}/p_m^{\text{post}}))}{\phi(\Phi^{-1}(1 - w_m^{(1)}/p_m^{\text{post}}))}\\ &+\frac{\partial\{2(b + (n-2)/n\gamma)h(\Phi^{-1}(1 - w_m^{(1)}/p_m^{\text{post}}))p_m^{\text{post}}\}}{\partial p_m^{\text{post}}}\\ &\cdot\frac{dp_m^{\text{post}}}{dw_m}\bigg|_{w_m = w_m^{(1)}},\end{aligned}$$

where the first term is negative because of the inequality $h'(\Phi^{-1}(1 - w_m^{(1)}/p_m^{\text{post}})) > 0$ and the second term is zero due to $dp_m^{\text{post}}/dw_m|_{w_m = w_m^{(1)}} = 0$. Therefore, $dw_m^{(1)}/d\sigma_m < 0$.

Step 3. If $w < w_m^{(1)}$, then $dp_m^{\text{post}}/dw_m > 0$ and $dp_m^{\text{post}}/ds < 0$ for any $w_m \le w$. We define $s^{(1)} = 0$ in this case. If $w \ge w_m^{(1)}$, then we define $s^{(1)} = (w - w_m^{(1)})/w$. In this case, $dp_m^{\text{post}}/ds < 0$ if and only if $s > s^{(1)}$. Since $dw_m^{(1)}/d\sigma_m < 0$, $ds^{(1)}/d\sigma_m > 0$ in this case. Since $w_m^{(1)}$ solves $\partial^2\pi_m(\mathbf{p}, w_m)/(\partial p_1\partial w_m)|_{\mathbf{p}=\mathbf{p}^{\text{post}}} = 0$, from (A8), $s^{(1)}$ is the maximum of 0 and the unique s that solves $b + (n-2)/n\gamma - \sigma_m\{2h(\Phi^{-1}(1 - w(1-s)/p_m^{\text{post}}))p_m^{\text{post}}\}^{-1} = 0$.

(b) Note that $d\pi_3^{\text{post}}/ds = (dp_m^{\text{post}}/ds)(d\pi_3^{\text{post}}/dp_m) < 0$ if and only if $s > s^{(1)}$ because $d\pi_3^{\text{post}}/dp_m > 0$ by Lemma A1(a) and $dp_m^{\text{post}}/ds < 0$ if and only if $s > s^{(1)}$ by part (a). In the rest of the proof, we prove $d\pi_m^{\text{post}}/ds > 0$ for all s by showing $d\pi_m^{\text{post}}/dw_m < 0$ for all w_m in each of the following two cases: $dp_m^{\text{post}}/dw_m \le 0$ (Case I) and $dp_m^{\text{post}}/dw_m > 0$ (Case II).

Case I. From (4),

$$\begin{aligned}\frac{d\pi_m^{\text{post}}}{dw_m} &= \left(\frac{\partial\pi_m^{\text{post}}}{\partial p_1} + \frac{\partial\pi_m^{\text{post}}}{\partial p_2}\right)\frac{dp_m^{\text{post}}}{dw_m} + \frac{\partial\pi_m^{\text{post}}}{\partial y_m}\frac{dy_m^{\text{post}}}{dw_m} + \frac{\partial\pi_m^{\text{post}}}{\partial w_m}\\ &\quad+\frac{dp_3^{\text{post}}}{dw_m}\sum_{j=3}^{n}\frac{\partial\pi_m^{\text{post}}}{\partial p_j}\\ &= \frac{\partial\pi_m^{\text{post}}}{\partial w_m} + \frac{dp_3^{\text{post}}}{dp_m}\frac{dp_m^{\text{post}}}{dw_m}\sum_{j=3}^{n}\frac{\partial\pi_m^{\text{post}}}{\partial p_j},\end{aligned}$$

where $\partial\pi_m^{\text{post}}/\partial p_1 = \partial\pi_m^{\text{post}}/\partial p_2 = 0$ and $\partial\pi_m^{\text{post}}/\partial y_m = 0$ at $(p_1, p_2, y_m) = (p_m^{\text{post}}, p_m^{\text{post}}, y_m^{\text{post}})$ by the envelope theorem. By computing $\partial\pi_m^{\text{post}}/\partial p_j$ and $\partial\pi_m^{\text{post}}/\partial w_m$ from (4) and substituting them into $d\pi_m^{\text{post}}/dw_m$, we get

$$\frac{d\pi_m^{\text{post}}}{dw_m} = -q_m^{\text{post}} + 2\gamma\frac{n-2}{n}(p_m^{\text{post}} - w_m)\frac{dp_3^{\text{post}}}{dp_m}\frac{dp_m^{\text{post}}}{dw_m} < 0,$$

where the inequality is due to $q_m^{\text{post}} > 0$, $dp_m^{\text{post}}/dw_m \le 0$ by the premise and $dp_3^{\text{post}}/dp_m > 0$ by Lemma A1(a).

Case II. Suppose that w_m is reduced by $dw_m (>0)$. Let $p_3^{\text{post}} - dp_3$, $p_m^{\text{post}} - dp_m$, $y_m^{\text{post}} - dy_m$, and $\pi_m^{\text{post}} - d\pi_m$ denote new equilibrium outcomes associated with the change of dw_m. Let π'_m denote the postmerger firm's expected profit at the new marginal cost $w_m - dw_m$ when $p_m = p_m^{\text{post}} - dp_m$, $p_3 = p_4 = \cdots =$

$p_n = p_3^{\text{post}} - dp_3$, and $y_m = y_m^{\text{post}}$. Note that $\pi_m' \le \pi_m^{\text{post}} - d\pi_m$ because $y_m = y_m^{\text{post}}$ is chosen instead of $y_m = y_m^{\text{post}} - dy_m$. To prove $d\pi_m^{\text{post}}/dw_m < 0$, we will show $\pi_m' > \pi_m^{\text{post}}$, so that $\pi_m^{\text{post}} - d\pi_m \ge \pi_m' > \pi_m^{\text{post}}$. From (4), we can compute π_m^{post}, π_m' and then $\pi_m' - \pi_m^{\text{post}}$ as follows:

$$\begin{aligned}
\pi_m^{\text{post}} &= (p_m^{\text{post}} - w_m)L_m^{\text{post}} - w_m y_m^{\text{post}} - p_m\sigma_m R\left(\frac{y_m^{\text{post}}}{\sigma_m}\right);\\
\pi_m' &= \{p_m^{\text{post}} - w_m + (dw_m - dp_m)\}\\
&\quad \cdot\left\{L_m^{\text{post}} + 2b\,dp_m + \frac{n-2}{n}2\gamma(dp_m - dp_3)\right\}\\
&\quad - (w_m - dw_m)y_m^{\text{post}} - (p_m^{\text{post}} - dp_m)\sigma_m R\left(\frac{y_m^{\text{post}}}{\sigma_m}\right);\\
\pi_m' - \pi_m^{\text{post}} &= \{p_m^{\text{post}} - w_m + (dw_m - dp_m)\}\\
&\quad \cdot\left\{2b\,dp_m + \frac{n-2}{n}2\gamma(dp_m - dp_3)\right\}\\
&\quad + (dw_m - dp_m)L_m^{\text{post}} + y_m^{\text{post}}dw_m + \sigma_m R\left(\frac{y_m^{\text{post}}}{\sigma_m}\right)dp_m\\
&> (dw_m - dp_m)L_m^{\text{post}} + y_m^{\text{post}}dw_m + \sigma_m R\left(\frac{y_m^{\text{post}}}{\sigma_m}\right)dp_m\\
&= (L_m^{\text{post}} + y_m^{\text{post}})dw_m + \left\{\sigma_m R\left(\frac{y_m^{\text{post}}}{\sigma_m}\right) - L_m^{\text{post}}\right\}dp_m,
\end{aligned}$$

where the inequality follows from $dp_m^{\text{post}}/dw_m > 0$ by the premise, $dp_m^{\text{post}}/dw_m < 1$ from Step 2 of part (a), and $dp_3^{\text{post}}/dp_m < 1$ by Lemma A1(a). Using $dw_m > dp_m > 0$, we can simplify this inequality into $\pi_m' - \pi_m^{\text{post}} > \{y_m^{\text{post}} + \sigma_m R(y_m^{\text{post}}/\sigma_m)\}dp_m$. Finally, we complete the proof by showing that $y_m^{\text{post}} \ge -\sigma_m R(y_m^{\text{post}}/\sigma_m)$. Since $y_m - (y_m - \tilde{\varepsilon}_m)^+ = \tilde{\varepsilon}_m - (\tilde{\varepsilon}_m - y_m)^+$, $y_m = E(y_m - \tilde{\varepsilon}_m)^+ - E(\tilde{\varepsilon}_m - y_m)^+$. Using $E(y_m - \tilde{\varepsilon}_m)^+ \ge 0$ and $E(\tilde{\varepsilon}_m - y_m)^+ = \sigma_m R(y_m/\sigma_m)$, $y_m^{\text{post}} \ge -\sigma_m R(y_m^{\text{post}}/\sigma_m)$. □

Proof of Proposition 4. (a) We have here $dl_m^{\text{post}}/d\rho = (\partial l_m^{\text{post}}/\partial p_m^{\text{post}})(dp_m^{\text{post}}/d\rho) = (w_m/(p_m^{\text{post}})^2)(dp_m^{\text{post}}/d\rho)$. Because of $dp_m^{\text{post}}/d\rho < 0$ from Proposition 1(a), we get $dl_m^{\text{post}}/d\rho < 0$. Similarly, we can prove $dl_3^{\text{post}}/d\rho < 0$.

(b) We have

$$\frac{dl_m^{\text{post}}}{dw_m} = -\frac{1}{p_m^{\text{post}}} + \frac{w_m}{(p_m^{\text{post}})^2}\frac{dp_m^{\text{post}}}{dw_m} = \left(\frac{w_m}{p_m^{\text{post}}}\frac{dp_m^{\text{post}}}{dw_m} - 1\right)\frac{1}{p_m^{\text{post}}}.$$

Since $w_m < p_m^{\text{post}}$ and $dp_m^{\text{post}}/dw_m < 1$ from the proof of Proposition 2(a), $dl_m^{\text{post}}/dw_m < 0$. Therefore,

$$\frac{dl_m^{\text{post}}}{ds} = \frac{dl_m^{\text{post}}}{dw_m}\frac{dw_m}{ds} = \frac{dl_m^{\text{post}}}{dw_m}(-w) > 0.$$

Next, $dl_3^{\text{post}}/ds = (w/(p_3^{\text{post}})^2)(dp_3^{\text{post}}/ds) > 0$ if and only if $s < s^{(1)}$ from Proposition 2(a).

(c) Since l_m^{post} decreases with ρ and increases with s, it suffices to show that $l_m^{\text{post}} > l_1^{\text{pre}}$ at $\rho = 1$ and $s = 0$, which follows directly from $l_m^{\text{post}} = 1 - w/p_m^{\text{post}}$, $l_1^{\text{pre}} = 1 - w/p_1^{\text{pre}}$ and $p_m^{\text{post}} > p_1^{\text{pre}}$ at $\rho = 1$ and $s = 0$. Since $l_3^{\text{post}} = 1 - w/p_3^{\text{post}}$ increases with p_3^{post}, the result follows from Proposition 3(a). □

Proof of Proposition 5. A sketch of the proof is as follows. We first find a lower bound of $E[cs^{\text{post}}]$, denoted by $E[cs_{\text{lbd}}]$. We then show that $E[cs_{\text{lbd}}] > E[cs^{\text{pre}}]$ at $s = s^{(2)}$ and that $E[cs_{\text{lbd}}]$ increases with s for $s \in [s^{(2)}, 1)$. Therefore, there exists a threshold $s^{(cs)} \in [0, s^{(2)}]$ such that $E[cs^{\text{post}}] > E[cs^{\text{pre}}]$ for any $s > s^{(cs)}$. □

Proof of Corollary 1. We provide a sketch of the proof for part (a). The proof of part (b) follows a similar procedure. We first get the n first-order conditions for the postmerger firm: $\partial\pi_m/\partial p_1 = 0$, $\partial\pi_m/\partial p_2 = 0$, and $\partial\pi_i/\partial p_i = 0$, $i = 3, 4, \ldots, n$. Using the implicit function theorem and Cramer's rule, we obtain $dp_i^{\text{post}}/d\rho = -|J_i^\rho|/|J|$, where J is the Jacobian matrix of the n first-order conditions, and J_i^ρ is the matrix formed by replacing the ith column of J with the vector $(\partial^2\pi_m/\partial p_1\partial\rho|_{\mathbf{p}=\mathbf{p}^{\text{post}}}, \partial^2\pi_m/\partial p_2\partial\rho|_{\mathbf{p}=\mathbf{p}^{\text{post}}}, 0, \ldots, 0)^T$. We can show that the sign of $|J|$ is $(-1)^n$. In addition, we can show that $\partial^2\pi_m/\partial p_1\partial\rho|_{\mathbf{p}=\mathbf{p}^{\text{post}}} < 0$ and $\partial^2\pi_m/\partial p_1\partial\rho|_{\mathbf{p}=\mathbf{p}^{\text{post}}} < 0$, and thus the sign of $|J_i^\rho|$ is also $(-1)^n$. So $dp_i^{\text{post}}/d\rho = -|J_i^\rho|/|J| < 0$. □

Proof of Corollary 2. We provide a sketch of the proof for part (a). We compare $\partial\pi_m/\partial p_1$ for the same price vector under two different demands $\tilde{\varepsilon}_m$ and $\tilde{\xi}_m$. We show that if the postmerger firm keeps its prices constant but chooses its safety stock optimally, then the expected lost sales are smaller for the less dispersive demand $\tilde{\varepsilon}_m$. This results in a larger $\partial\pi_m/\partial p_1$ for the less dispersive demand. Consequently, if the prices are set at the equilibrium point for the more dispersive demand (resulting in $\partial\pi_m/\partial p_1 = 0$ for this demand), then $\partial\pi_m/\partial p_1 > 0$ for the less dispersive demand. The rest of the proof follows the procedure similar to the proof of Lemma 1(a). The proof of part (b) follows the procedure similar to the proof of Proposition 2(a). □

Appendix B. Parameter Values Used in Figures

The parameter values used in our numerical examples are motivated by the U.S. rental car market. In Figure 1, we used the following parameter values: $n = 3$, $a = 1$, $b = 0.6$, $\gamma = 0.5$, $w = 0.5$, and $\sigma = 0.3$. We use $n = 3$ to reflect the number of major rental car companies in the United States (i.e., Hertz, Avis, and Enterprise). We used the normalized value of $a = 1$ for the demand intercept. The parameters b and γ are related to the price sensitivity of a firm's demand to its own price and other competitors' prices. According to McCarthy (1996), the own-price elasticity for the U.S. auto market is between −1.06 and −1.85, and the cross-price elasticity is between 0.28 and 0.86. We chose the values of b and γ to yield the own-price elasticity of −1.64 and the cross-price elasticity of 0.59, which are consistent with McCarthy. We chose $w = 0.5$ to yield a profit margin of 42% in the premerger market. From the quarterly financial reports of Hertz, we found that its gross profit margin was between 37% and 50% from 2006 to 2013. The profit margin in our numerical example is in the middle of this range. We used $\sigma = 0.3$ to add moderate uncertainty to demand.

Figure 2 is plotted over different levels of synergy s for a fixed value of ρ. For illustration, we provide three different figures for $\rho = 0$, 0.5, and 1, respectively. The case when $\rho = 1$ can be viewed as a benchmark case in which firm 1 and firm 2 maintain separate inventories after their merger. Other parameter values are the same as in Figure 1 except $\sigma = 0.5$. We used a larger σ to better illustrate the property that $s^{(1)}$ is nondecreasing with σ_m.

Endnotes

[1] In other words, the pooling effect does not exist when two merging firms sell two distinct products after the merger. Although this is quite plausible for a merger of *manufacturers*, it may not be common for a merger of *retailers*, which is the main focus of this paper.

[2] As is common in the literature, we do not consider a fixed cost of a merger. However, we can easily incorporate this cost into (4). Since the fixed cost does not affect the functional characteristic of π_m, it has no impact on subsequent analyses.

[3] Intuitively, when a postmerger firm decides on its price p_1, it considers a trade-off between a marginal gain in a deterministic profit from increasing p_1 and a marginal gain from hedging against uncertainty by reducing p_1. It turns out that the former is increasing linearly with w_m, and the latter is increasing convexly with w_m. Thus, when w_m is high, a small reduction of w_m has a larger impact on the marginal gain from hedging against uncertainty.

[4] The technical condition given in Proposition 5 is a sufficient condition for $s^{(cs)} \le s^{(2)}$, which is observed in all of our extensive numerical experiments with the following parameter values: $n = 3$, $a = 1$, $b \in \{0.1, 0.6, 1, 2\}$, $\gamma \in \{0.1, 0.5, 1, 2\}$, $w \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$, $\sigma \in \{0.05, 0.1, 0.3, 0.5\}$, and $\rho \in \{0, 0.2, 0.4, 0.6, 0.8, 1\}$. These scenarios include a set of the parameter values used in Figure 1 and also cover various possible scenarios.

[5] Dispersive ordering and an IFR have the following relation. Let X be a random variable with support on (a, ∞) where $a \ge -\infty$. For any $t \in (a, \infty)$, let $X_t = [X - t \mid X \ge t]$ denote the residual lifetime. The following statements are equivalent (Pellerey and Shaked 1997): (i) X has an IFR, (ii) $X_t \le_{\text{disp}} X$ for all t, and (iii) $X_t \le_{\text{disp}} X_s$ for all $s < t$.

[6] Young (1978) and Petruzzi and Dada (1999) use a similar demand function for a monopoly case: $D(p) = L(p) + \delta(p)\tilde{\varepsilon}$. Since we analyze an oligopoly model, we replace the price p with a vector of prices of all firms, $\mathbf{p}$. To be consistent with the main body, we assume that $L_i(\mathbf{p})$ is given in (1) and that $\delta_i(\mathbf{p})$ takes a similar form to $L_i(\mathbf{p})$.

[7] A demand function with additive uncertainty is more amenable to modeling consumer behavior from stock-out substitution as we discuss in Section 5.4. It is also worth noting that a demand function with multiplicative uncertainty does not satisfy the conditions necessary to guarantee utility maximization by a representative consumer (Krishnan 2010), and hence it cannot be used for welfare analysis in Section 4.3.

[8] Higher ρ also increases $\text{Var}(\hat{D}_1)$, since $\text{Var}(\hat{D}_1) = \text{Var}(\tilde{\varepsilon}_1) + (\kappa^2/4) \cdot \sum_{j=2}^{3} \text{Var}((\tilde{\varepsilon}_j - y_j)^+) + \kappa \sum_{j=2}^{3} \text{Cov}(\tilde{\varepsilon}_1, (\tilde{\varepsilon}_j - y_j)^+) + (\kappa^2/2)\text{Cov}((\tilde{\varepsilon}_2 - y_2)^+, (\tilde{\varepsilon}_3 - y_3)^+)$, where both $\text{Cov}(\tilde{\varepsilon}_1, (\tilde{\varepsilon}_j - y_j)^+)$ and $\text{Cov}((\tilde{\varepsilon}_2 - y_2)^+, (\tilde{\varepsilon}_3 - y_3)^+)$ are increasing in ρ.

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