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Horizontal Mergers in Multitier Decentralized Supply Chains

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The well-known economic theory predicts that consumer price will fall after a horizontal merger when the amount of marginal cost reduction from operating synergies exceeds the premerger markup of a merging firm. However, when a horizontal merger occurs in a multitier decentralized supply chain where a finite number of firms compete at each tier, we show that this result holds only when a merger occurs at the tier that acts as the leader in the supply chain. In this supply chain, a horizontal merger at any other tier will decrease consumer price when the cost reduction exceeds a certain threshold that is larger than the premerger markup. Moreover, this threshold is increasing as the supply chain gets longer and can be substantially larger than the premerger markup. When accounting for subsequent entry after a merger in long-run equilibrium, contrary to a common belief, a larger synergy from a merger does not necessarily benefit consumers more.

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1. Introduction

Mergers and acquisitions (M&As) play an important role in today's economy, often making headline news because of their impact on the market. Despite the recent economic recession, M&A activities peaked in 2011 with more than thirty thousand transactions worldwide, worth about three trillion dollars (WilmerHale 2012). Among different types of M&As, the particular focus of this paper is on a horizontal merger that involves two firms that compete in the same kind of business activity. In the remainder of this paper, unless specified otherwise, a merger means a horizontal merger.

Today, in many industries such as automotive, consumer appliances, electronic equipment, and apparel, firms are vertically disintegrated and compete in supply chains that consist of multiple tiers with several firms competing at each tier (Corbett and Karmarkar 2001). In such industries, a horizontal merger can be viewed as a consolidation of two firms at one tier within a multitier decentralized supply chain. Examples of horizontal mergers at different tiers abound. In the personal computer industry, two hard disk drive suppliers, Seagate and Maxtor, merged in 2006; two contract manufacturers (who buy parts from hard disk drive suppliers), Flextronics and Solectron, merged in 2007; and two original equipment manufacturers (who outsource assembly to contract manufacturers), Hewlett-Packard and Compaq, merged in 2001. A number of mergers among retailers have also occurred: for example, two retail giants, Sears and Kmart, merged in 2005.

These examples pose the following research question: Under what condition does a merger between two firms at one tier in a multitier supply chain increase overall industry outputs, thus not raising consumer price? If upstream or downstream firms were not strategic decision makers and hence did not respond to changes induced by a merger, one could argue that the answer to this question would be the same as in the case of a merger in a vertically integrated market. However, because firms at one tier buy parts/products from their upstream market and sell their own products to their downstream market, a merger at one tier not only affects the decision making of the merging firms and the other firms at the same tier, but it also affects firms in the upstream and downstream tiers. Therefore, the answer to the question raised above is not clear. Thus, the objective of this paper is to study how a merger of two firms at one tier affects: (1) the outputs and profits of merging firms and nonparticipant firms, and the total output and market price at the tier where the merger occurs; (2) the outputs and profits of existing firms, and the total output and market price at the upstream and downstream tiers; and (3) the overall supply chain profit. Certainly, the impact of a merger on consumer price is the most important concern to antitrust authorities. In addition, answers to these questions will be useful for all constituent firms within a supply chain to assess potential consequences of a merger and to form their strategies accordingly.

In answering these questions, we focus on analyzing the two most common effects of a merger: the competition effect and the synergy effect. First, every merger creates the usual *competition effect* by reducing the level of competition at the tier where the merger occurs. Second, a merger often creates the *synergy effect* through marginal cost reduction. Bascle et al. (2008) report that cost efficiency is the main rationale behind more than 70% of M&As, and several empirical studies (e.g., Houston et al. 2001, DeLong 2003) suggest that operating synergies are the most important determinant of successful M&As. Gains in cost efficiency can come from multiple sources. For example, a merged firm may spread fixed costs over increasing production levels through economies of scale, improve efficiency from learning curve effects, use complementary technologies and patents to produce more efficiently, or reduce the cost of capital through lower securities and transaction costs. In the examples above, according to companies' forward-looking statements,[1] "Synergies resulting from the combination [of Seagate and Maxtor] are expected to generate annual pre-tax cost savings of $300 million," and "The merger [of HP and Compaq] is expected to generate cost synergies reaching approximately $2.5 billion annually."

To evaluate the effect of a merger in a multitier decentralized supply chain, we need a benchmark in which firms compete at different tiers of the supply chain *before* the merger takes place. For this benchmark, we use the model of Corbett and Karmarkar (2001), in which multiple firms compete at each tier of a multitier supply chain in the Cournot fashion. In our base model of a two-tier supply chain, we first consider the case in which a merger of two firms occurs at the upstream tier (*upstream merger*), and then consider the case in which a merger occurs at the downstream tier (*downstream merger*). For each merger, we examine the synergy effect and the competition effect separately, and then combine the two effects to examine the aggregate effect of the merger. The base model is then extended to a three-tier supply chain. From this analysis, it is straightforward to understand how the results are naturally extended to more than three tiers.

The analysis above focuses on the synergy and competition effects of a merger on incumbent firms in the market, but such an analysis may also include the *entry* of new firms. Antitrust agencies consider the prospect of entry because such entry can alleviate concerns about adverse competitive effects of the merger by increasing competition.[2] In a vertically integrated market, one would expect that when a merger creates significant synergies, it will deter the entry of a new firm, because a postmerger firm will have a significant cost advantage. However, in a decentralized supply chain, depending on the level of synergies, a merger at one tier can induce entry to the other tiers as well as to the tier where the merger occurs because firms at the other tiers are also affected by that merger. Thus, it is unclear how a merger will induce entry to multiple tiers and consequently affect consumer price.

Our analysis highlights the importance of the position (upstream or downstream) in a supply chain where a merger takes place. The well-known result of Farrell and Shapiro (1990) predicts that consumer price will fall after a merger when the amount of marginal cost reduction from operating synergies exceeds the premerger markup of a merging firm. However, when firms compete in a multitier decentralized supply chain, we show that this result holds only when a merger occurs at the most upstream tier of the supply chain; whereas a merger at any other tier will decrease consumer price when the amount of marginal cost reduction exceeds a threshold that is larger than the premerger markup of a merging firm. The difference between this threshold and the premerger markup increases as the supply chain gets longer, and can be significant. Furthermore, when accounting for entry in long-run equilibrium, a larger synergy from a merger does not necessarily reduce consumer price. Finally, we find that an upstream merger is less likely to raise consumer price than a downstream merger both in the short run (ignoring entry) and in the long run (accounting for entry).

We extend our analysis in multiple directions. First, while our base model follows most existing models of supply chain competition by assuming that upstream firms move before downstream firms (e.g., McGuire and Staelin 1983, Tyagi 1999, Corbett and Karmarkar 2001, Lin et al. 2013), in practice, we also observe a supply chain where downstream firms lead; for example, Walmart places orders to Li & Fung, who in turn find suppliers who can fulfill the orders (Adida et al. 2012). For such a supply chain,

[1] *Sources*. http://www.sec.gov/Archives/edgar/data/711039/000119312505246708/d425.htm and http://www.hp.com/hpinfo/newsroom/press/2001/010904a.html (accessed August 17, 2013).

[2] According to §9 in the "Horizontal Merger Guidelines" (U.S. Department of Justice and the Federal Trade Commission 2010), the agencies examine the timeliness, likelihood, and sufficiency of entry. For timeliness, entry must be rapid enough that customers are not significantly harmed by the merger before the entry. Entry is likely if it would be profitable. Profitability depends on the output level, price, and cost per unit, which may depend on the scale at which the entrant would operate. Finally, entry by a single firm that will replicate at least the scale of one of the merging firms is sufficient.

upstream (respectively, downstream) firms play the role of downstream (respectively, upstream) firms in our base model. Consequently, the results in our base model are reversed. Thus, in general, a merger at the tier that acts as the *follower* leads to a higher threshold for consumer price to fall than the premerger markup. Second, we demonstrate that our main insights continue to hold under a broad range of demand and cost functions.

2. Related Literature

In operations management (OM) literature, many recent advances have been made to model competition among firms in decentralized supply chains. As Netessine (2009, p. 236) points out, however, "most of the voluminous supply chain literature still focuses on monopolistic situations. ... There is very limited literature on how the *entry* and *exit* of firms into various echelons of the supply chain affect the entire network." Despite the economic significance of a merger, however, this literature has paid no attention to how a *horizontal merger* of firms affects various echelons (tiers) of a supply chain. We contribute to this literature by studying the effect of a horizontal merger on members of a decentralized supply chain as well as on the overall supply chain performance. Below we first review the economic theory of a merger, and then we review the related OM literature on supply chain competition, models of estimating merger synergies, and cooperative supply chains.

Mergers have long been a concern for antitrust authorities and economists. A central issue for merger approval is whether the merger will increase consumer price, hence reducing consumer welfare. Early research in this area is related to the formation of a cartel among firms that make a collusive decision in a competitive market. Stigler (1950) considers the formation of a cartel in a Cournot market with linear demand. He shows that cartel members reduce their production outputs to increase market price. Yet because such an increase benefits external firms more, the cartel is not stable. To explain the observed formation of cartels or mergers in practice, researchers have considered situations in which horizontal mergers reduce the marginal production costs of merging firms through operating synergies. With synergies, from the social viewpoint, there is a trade-off between reduced competition and marginal cost reduction, which may be passed on to consumers. This trade-off was first articulated by Williamson (1968) and has been explored by many researchers since then (for a comprehensive review, see Bloch 2005 and Whinston 2007). Most notably, Perry and Porter (1985) and Farrell and Shapiro (1990) have provided formal analyses of this trade-off for settings with Cournot competition. Farrell and Shapiro (1990) show that a price will fall after a merger if and only if the amount of marginal cost reduction is greater than the premerger markup of a merging firm. This result applies to a merger between two vertically integrated firms or two firms for which upstream and downstream markets are perfectly competitive, but it is unclear whether the result holds in a multitier decentralized supply chain where a finite number of firms compete at each tier. In this case, we find that the result of Farrell and Shapiro (1990) holds only when a merger occurs at the tier that acts as the leader in a supply chain. Economists have also attempted to extend a merger analysis to a simple supply chain. For example, when two manufacturers sell differentiated products using a two-part tariff through two competing retailers, Ziss (1985) analyzes the effect of a merger (which creates either a monopolist manufacturer or a monopolist retailer) on consumer price. We provide a detailed comparison of our paper with Ziss (1985) in Appendix B.

To extend the analysis of a merger to a multitier decentralized supply chain, our work builds on the OM literature that studies the competition, entry, or exit of a firm in such a supply chain. Whereas the previous marketing literature on sales channels treats the number of firms in a downstream or upstream market as fixed, Tyagi (1999) examines the effects of downstream entry in a supply chain with one manufacturer and multiple retailers. Corbett and Karmarkar (2001) consider a multitier supply chain in which oligopolistic firms complete in the Cournot fashion at each tier of the supply chain. Using their framework, Carr and Karmarkar (2005) study multitier supply chains with an assembly structure. Perakis and Roels (2007) study the efficiency of price-only contracts in three-tier supply chains under uncertain demand for fixed consumer price. In their supply chains, multiple firms compete in one tier and there is a monopolist in the other tiers. Majumder and Srinivasan (2008) study the effects of contract leadership on the efficiency of multitier supply chains and analyze competition between separate supply chains. In a two-tier supply chain with multiple manufacturers and retailers, Adida and DeMiguel (2011) study the effects of product and retailer differentiation, stochastic demand, and retailer risk aversion on the decentralized supply chain equilibrium and its efficiency. Recently, Adida et al. (2012) study the role of intermediation in a supply chain where retailers lead. Whereas the competition effect of a merger is opposite to the effect of an entry studied in this literature, the synergy effect exists only in the case of a merger. By combining these two effects, we examine the aggregate effect of a merger. Furthermore, we analyze the entry-inducing effect of a merger.

Several operations researchers have developed optimization models that quantify the synergy effect of mergers (e.g., Gupta and Gerchak 2002, Alptekinoğlu and Tang 2005, Nagurney 2009). Our work complements this stream of research by analyzing the synergy effect of a merger in a competitive setting in which multiple firms compete at each tier of a decentralized supply chain. Although our model follows the economics literature on mergers by capturing the synergy effect of a merger through an exogenous marginal cost reduction, it could potentially take an estimated value of a synergy from this OM literature or from an industry-specific detailed analysis (e.g., annual cost savings of \$300 million for the Seagate-Maxtor merger and \$2.5 billion for the HP-Compaq merger). In addition to the analytical papers reviewed above, Zhu et al. (2011) empirically analyze the effects of a horizontal merger on the financial and inventory-related performance of firms.

Finally, we discuss the difference between our paper and the literature that studies the formation of coalitions among supply chain members using cooperative game theory. In this literature, firms maintain their independence but consider forming coalitions to obtain synergies, for example, via quantity discounts (e.g., Nagarajan et al. 2010) or price collusion/coordination (e.g., Nagarajan and Sošić 2007, Yin 2010). Although these papers consider multilateral agreements among all members of coalitions, Fang and Cho (2012) consider bilateral agreements where synergies accrue to pairs of firms forming a collaborative link. Because firms remain *independent*, the benefit from collaboration needs to be allocated in such a manner that firms have incentives not to secede from coalitions. Thus, the main focus in this stream of research is on investigating the impact of different allocation mechanisms on the formation of stable coalitions. On the other hand, in this paper, firms become *a single entity* after their merger, and a merger almost always involves only two firms. Our main focus is on analyzing the effect of a merger between two firms at one tier on firms at the same tier and at their upstream/downstream tiers, provided that such a merger occurs.

3. Premerger Model and Analysis in a Two-Tier Supply Chain

This section describes the premerger model and analysis of Corbett and Karmarkar (2001) that serve as our benchmark for examining the effects of a merger. Consider a supply chain with two tiers. For convenience, we will refer to a downstream tier as tier 1 and an upstream tier as tier 2. Let n_i denote the number of firms at tier i $(=1$ or $2)$. For ease of exposition, we suppose that downstream firms at tier 1 buy "parts" from the upstream market at tier 2 to produce final products. Without loss of generality, we assume that one part is required to produce one unit of a final product. Firms within each tier are identical and engage in quantity competition by selling homogeneous goods. Firms at tier 1 face a linear demand curve $p_1 = a_1 - b_1 Q_1$, where p_1 represents the price of a final product sold to consumers by tier 1 firms, Q_1 represents the total quantity supplied by tier 1 firms, a_1 (>0) represents the highest price when $Q_1 = 0$, which we will call the "market potential" throughout the paper, and b_1 (>0) represents "price sensitivity" at tier 1. The price of parts produced by tier 2 firms is p_2. Let v_i denote the variable cost of firms at tier i. Each firm j $(=1, 2, \ldots, n_i)$ at tier i $(=1, 2)$ chooses its quantity q_{ij} that maximizes its profit π_{ij}. Let $\Pi_i = \sum_{j=1}^{n_i} \pi_{ij}$ denote the sum of the profits of all firms at tier i, and let $\Pi = \Pi_1 + \Pi_2$ denote the total profit of the supply chain.

Let us first examine the decisions of firms at tier 1. Let $Q_{1,-j} \equiv Q_1 - q_{1j}$ denote the total quantity produced by tier 1 firms except firm j. Given p_2, the profit of any firm j at tier 1 is

$$\pi_{1j} = (p_1 - v_1 - p_2) q_{1j} = \{a_1 - b_1(q_{1j} + Q_{1,-j}) - v_1 - p_2\} q_{1j} \quad \text{for } j = 1, 2, \ldots, n_1.$$

Given $Q_{1,-j}$, each firm j chooses q_{1j} that maximizes its profit π_{1j}. By following the standard Cournot analysis, one can obtain the following equilibrium at tier 1:

$$q_{1j} = \frac{a_1 - v_1 - p_2}{b_1(n_1+1)} \quad \text{and} \quad \pi_{1j} = b_1 q_{1j}^2 = \frac{1}{b_1}\left(\frac{a_1 - v_1 - p_2}{n_1+1}\right)^2 \quad \text{for } j = 1, 2, \ldots, n_1; \tag{1}$$

$$Q_1 = \frac{n_1(a_1 - v_1 - p_2)}{b_1(n_1+1)} \quad \text{and} \quad p_1 = \frac{a_1 + n_1(v_1 + p_2)}{n_1+1}. \tag{2}$$

Next, consider the decisions of firms at tier 2. Because one part is required to produce one unit of a final product, the total quantity produced in each tier is the same; i.e., $Q_1 = Q_2$. By setting $Q_1 = Q_2$ in (2), one can derive the inverse demand function for the upstream market as follows:

$$p_2 = (a_1 - v_1) - b_1\left(\frac{n_1+1}{n_1}\right) Q_2 = a_2 - b_2 Q_2, \tag{3}$$

where $a_2 \equiv a_1 - v_1$ and $b_2 \equiv b_1(n_1+1)/n_1$. By repeating the same analysis as at tier 1, one can obtain the following equilibrium at tier 2:

$$q_{2j} = \frac{a_2 - v_2}{b_2(n_2+1)} \quad \text{and} \quad \pi_{2j} = \frac{1}{b_2}\left(\frac{a_2 - v_2}{n_2+1}\right)^2 \quad \text{for } j = 1, 2, \ldots, n_2; \tag{4}$$

$$Q_2 = \frac{n_2(a_2 - v_2)}{b_2(n_2+1)} \quad \text{and} \quad p_2 = \frac{a_2 + n_2 v_2}{n_2+1}. \tag{5}$$

We assume that $a_2 - v_2 > 0$, so that $q_{2j} > 0$ for all j. Substituting the expressions for a_2, b_2, and p_2 into the above results gives the following equilibrium outcomes at both tiers:

$$Q_1 = Q_2 = \frac{1}{b_1}\frac{n_1 n_2(a_1 - v_1 - v_2)}{(n_1+1)(n_2+1)}; \tag{6}$$

$$p_1 = \frac{(n_1+n_2+1)a_1 + n_1 n_2(v_1+v_2)}{(n_1+1)(n_2+1)}, \quad p_2 = \frac{a_1 - v_1 + n_2 v_2}{n_2+1}; \tag{7}$$

$$\pi_{1j} = b_1 q_{1j}^2 = b_1\left(\frac{Q_1}{n_1}\right)^2 = \frac{1}{b_1}\left(\frac{n_2}{n_2+1}\right)^2\left(\frac{a_1-v_1-v_2}{n_1+1}\right)^2 \quad \text{for } j = 1, 2, \ldots, n_1; \tag{8}$$

$$\pi_{2j} = b_2 q_{2j}^2 = b_2\left(\frac{Q_2}{n_2}\right)^2 = \frac{1}{b_1}\left(\frac{n_1}{n_1+1}\right)\left(\frac{a_1-v_1-v_2}{n_2+1}\right)^2 \quad \text{for } j = 1, 2, \ldots, n_2. \tag{9}$$

From these results, one can also compute the total profit of each tier i, Π_i, and the total supply chain profit, Π. Observe from (6)–(9) that the number of firms at tier i (n_i), as well as the variable cost of firms at tier i (v_i), affects the equilibrium outcomes of both tiers. Therefore, a merger of two firms at tier i that reduces n_i and v_i will affect the equilibrium outcomes at both tiers.

Before we proceed to our merger analysis, let us discuss a few issues of our model. First, while we take n_i as given exogenously in the above analysis, we derive n_i in equilibrium, in §5, by considering firms' entry decisions. Second, as is common in the OM literature, we consider a supply chain where upstream firms move before downstream firms, and we use a linear demand function with constant marginal costs in our base model. We extend the base model to a supply chain where downstream firms lead in §6.1, to nonlinear demands in §6.2, and to nonlinear costs in §6.3. Third, our analysis can be easily extended to a merger of more than two firms or simultaneous mergers at multiple tiers, albeit these are rare in practice. Fourth, we use the deterministic Cournot model of Corbett and Karmarkar (2001) as our benchmark because it enables us to analyze competition at multiple tiers, and to compare our results with the result of Farrell and Shapiro (1990), who also use the deterministic Cournot model in their single-tier analysis. Instead, suppose tier 1 firms determine their capacities under uncertain demand, and after observing realized demands, engage in Cournot competition by determining quantities to bring to the market. Goyal and Netessine (2007) analyze such a model under duopoly, and show that the capacities and profits of firms in equilibrium are the same as our deterministic model. Fifth, following Tyagi (1999) and Corbett and Karmarkar (2001), we assume that all firms within each tier are symmetric before a merger. A merger of two symmetric firms can still create synergies due to the reasons explained earlier (e.g., economies of scale). We can also consider a merger of two asymmetric firms. For example, suppose two firms with v_2 and v_2' ($< v_2$) are merged into one firm having v_2^M ($\le v_2'$). We can show numerically that the aggregate effect of this merger is similar to that of a merger between two symmetric firms (e.g., Figure 1 in §4.1). Our subsequent analysis focuses on the symmetric case.

Using the above premerger equilibrium, we next investigate the effect of a merger in this supply chain. We consider a merger without subsequent entry in §4, and then we examine the entry-inducing effects of a merger in §5. Proofs are presented in Appendix A.

4. A Merger in a Supply Chain Without Subsequent Entry

We investigate the effects of an upstream merger and a downstream merger in a two-tier supply chain in §§4.1 and 4.2, respectively. Our analysis is then extended to a supply chain with three or more tiers in §4.3.

4.1. An Upstream Merger in a Two-Tier Supply Chain

This subsection examines the effect of a merger between two firms in the upstream tier 2 when $n_2 \ge 2$. We index the merged firms by $j = 1, 2$ and refer to them as the *merging* firms. When the two merging firms become a single firm, we index this firm by $j = 1$ and refer to it as the *postmerger* firm. The other firms indexed by $j = 3, 4, \ldots, n_2$ are referred to as the *nonparticipant* firms. (Note that $j = 2$ is not used in the postmerger analysis.) The two effects of the merger are captured in our model as follows. With the synergy effect, the marginal production cost of the merging firms is reduced from v_2 to v_2^M ($< v_2$). Let $\Delta v_2 \equiv v_2 - v_2^M$ (> 0) denote the amount of marginal cost reduction. The competition effect is modeled by reducing the number of firms at tier 2 from n_2 to $n_2 - 1$. Next we examine each of these two effects in isolation, and then combine them to examine the aggregate merger effect. Table 1 summarizes the synergy and competition effects of an upstream merger.

4.1.1. The Synergy Effect. To isolate the synergy effect, consider a case in which the merging firms, firms 1 and 2 at tier 2, remain independent as in the premerger model, and hence make their decisions independently. We suppose that these firms have collaborated on various activities, and jointly reduced their marginal production cost from v_2 to v_2^M. In the literature (e.g., Bloch 2005), this form of interfirm collaboration is often referred to as "alliance."

Table 1 **Effects of an Upstream Merger in the Two-Tier Supply Chain When $n_2 \geq 2$**

		Synergy effect	Competition effect
Tier 2	$q_{21}+q_{22}$	$\Uparrow$	$\Downarrow$
	$\pi_{21}+\pi_{22}$	$\Uparrow$	$\Downarrow$ if $n_2>2$; $\Uparrow$ if $n_2=2$
	q_{2j}, π_{2j} $(j\geq 3)$	$\Downarrow$	$\Uparrow$
	p_2	$\Downarrow$	$\Uparrow$
	Π_2	$\Uparrow$	$\Uparrow$
Tier 1	q_{1j}, π_{1j}, Π_1	$\Uparrow$	$\Downarrow$
	p_1	$\Downarrow$	$\Uparrow$
Supply chain	$\Pi_1+\Pi_2$	$\Uparrow$	$\Uparrow$ if and only if $n_1 > 1+\dfrac{2n_2+1}{n_2^2-n_2-1}$

Note. $\Uparrow$ ($\Downarrow$) denotes increasing (decreasing) as a result of the upstream merger.

Given the part price p_2, firms at tier 1 compete in the same manner as in the premerger model. By following the same procedure as in §3, we can show that firms at tier 2 face the demand function given in (3): $p_2 = a_2 - b_2 Q_2$. Then the profit of firm j at tier 2 satisfies:

$$\pi_{2j} = \begin{cases} (p_2 - v_2^M)q_{2j} = (a_2 - b_2Q_2 - v_2^M)q_{2j} \\ \qquad \text{for } j = 1, 2, \\ (p_2 - v_2)q_{2j} = (a_2 - b_2Q_2 - v_2)q_{2j} \\ \qquad \text{for } j = 3, 4, \ldots, n_2. \end{cases} \tag{10}$$

By conducting an analysis similar to that in §3, we derive the closed-form expressions of all equilibrium outcomes. By comparing premerger and postmerger equilibria, we establish the synergy effect of the upstream merger presented in Table 1.

We first discuss the synergy effect of the upstream merger on its own tier. Because of the synergy effect, the merging firms increase their outputs and earn higher profits. The synergy of the merging firms brings negative externalities to the nonparticipant firms, who thus reduce their outputs and earn lower profits due to their competitive disadvantage in their marginal production cost. Because the average marginal cost of tier 2 firms, $((n_2-2)v_2 + 2v_2^M)/n_2$, is reduced, the total quantity produced by tier 2 firms increases, the market price decreases, and the total profit of tier 2 firms increases.

Next, we discuss the synergy effect of the upstream merger on the downstream tier. The synergy of the merger at tier 2 affects firms at tier 1 through the part price p_2. By reducing p_2, the synergy of the upstream merger improves the profit margin of downstream firms, which thereby increase their outputs. With more aggregate outputs, the market price of a final product falls. Overall, tier 1 firms benefit from the synergy effect of the upstream merger by earning higher profits.

Because the synergy effect of the upstream merger improves the profits of both tiers, it also improves the total supply chain profit.

4.1.2. The Competition Effect. Suppose that firms 1 and 2 at tier 2 merge and become a single entity. Then, the number of firms at tier 2 is reduced from n_2 to $n_2 - 1$. To isolate the competition effect, we consider a case in which there is no synergy from the merger, so that the marginal production cost of the postmerger firm remains v_2. In the literature (e.g., Bloch 2005), this form of a merger or interfirm collusion is often referred to as "cartel." Because there is no synergy from the merger, all firms are still symmetric after the merger. Thus, postmerger equilibrium outcomes without the synergy effect can be obtained simply by replacing n_2 with $n_2 - 1$ in the premerger equilibrium outcomes presented in §3. By comparing premerger and postmerger equilibria, we establish the competition effect of the upstream merger presented in Table 1.

We first discuss the competition effect of the upstream merger on its own tier. As a result of the upstream merger, the competitive intensity in this tier is reduced, so all firms at tier 2 increase their outputs and obtain higher profits. Note, however, that the output of the postmerger firm never exceeds the sum of the premerger outputs of the two merging firms. Also, unless the postmerger firm becomes a monopolist, the profit of the postmerger firm does not exceed the sum of the premerger profits of the two merging firms, implying that the competition effect alone does not justify a profitable upstream merger. This is analogous to the so-called "cartel instability" mentioned earlier (Stigler 1950). Because of the reduced competitive intensity, the total output of tier 2 firms is reduced, and thereby the part price at tier 2 is increased. Overall, the reduced level of competition will improve the total profits of tier 2 firms.

The competition effect of the upstream merger on the downstream tier is straightforward. Because the part price p_2 is increased as a result of the upstream merger, the profit margin of firms at tier 1 is reduced. Thus, firms at tier 1 reduce their outputs, and the market price of a final product rises. The upstream merger, therefore, reduces both individual and aggregate profits of tier 1 firms.

Table 1 shows that the supply chain profit can be either increased or decreased due to the competition effect. This is because the competition effect decreases the total profit of tier 1 firms but increases the total profit of tier 2 firms. Under the condition given in Table 1, the effect on tier 2 dominates the effect on tier 1, hence increasing the supply chain profit. One can show that this condition is satisfied for any

(n_1, n_2) except: $n_1 = 1$ or $(n_1, n_2) \in \{(2,3), (2,2), (3,2), (4,2), (5,2), (6,2)\}$. Because the merger that creates monopoly (i.e., $n_2 = 2$) would not be approved under the antitrust law, this condition holds unless $n_1 = 1$ or $(n_1, n_2) = (2, 3)$. Therefore, unless the competitive intensity is very low, the competition effect of the upstream merger will increase the supply chain profit.[3]

4.1.3. The Aggregate Effect. We now examine the aggregate effect of the upstream merger by combining the synergy and competition effects. Table 1 reveals that, although both effects increase the total upstream profit Π_2 and the total supply chain profit $\Pi_1 + \Pi_2$ under the mild condition, they affect all other equilibrium outcomes in the opposite directions. Thus, one cannot easily determine the aggregate effect by simply combining the two opposite effects. For those equilibrium outcomes, it is natural to ask: Under what condition does one effect outweigh the other effect? To answer this question, we derive the postmerger equilibrium outcomes denoted by a superscript M, and compare them with the premerger equilibrium outcomes presented in §3. For convenience, we define $u^{(1)} \equiv (((\sqrt{2}-1)n_2 - 1)/(n_2 - 1))((a_1 - v_1 - v_2)/(n_2 + 1))$, $u^{(2)} \equiv a_1 - v_1 - v_2$, and $u^{(3)} \equiv (a_1 - v_1 - v_2)/(n_2 + 1)$, where $u^{(1)}$, $u^{(2)}$ and $u^{(3)}$ are functions of (a_1, v_1, v_2, n_2). We suppress (a_1, v_1, v_2, n_2) in $u^{(1)}$, $u^{(2)}$, and $u^{(3)}$, and we discuss their properties at the end of this section. It is easy to see that $u^{(1)} < u^{(3)} < u^{(2)}$ for $n_2 \geq 3$.

To begin, we consider the following necessary conditions for the upstream merger.

Lemma 1. (C1) *For the postmerger firm,* $\pi_{21}^M \geq 2\pi_{21}$ *if and only if* $n_2 = 2$ *or* $\Delta v_2 \geq u^{(1)}$.

(C2) *For the nonparticipant firms* $(j = 3, 4, \ldots, n_2)$, $q_{2j}^M \geq 0$ *if and only if* $\Delta v_2 \leq u^{(2)}$.

The first condition (C1) concerns the incentive of the merging firms. To satisfy this condition, the postmerger firm should become a monopolist in the upstream market or the amount of marginal cost reduction, Δv_2, should be at least $u^{(1)}$. In the former case, it is unlikely that the merger would be approved under the antitrust law that prohibits monopolies (see §2 of the Sherman Act); thus, the condition on Δv_2 is likely to be binding. Under this condition, the synergy effect that increases the profit of the postmerger firm outweighs the competition effect that decreases the profit of the postmerger firm (see Table 1).[4] The second condition (C2) is the usual regularity condition that requires $q_{2j}^M \geq 0$ for all j. This is also a necessary and sufficient condition for the profit margin of the nonparticipant firms to be nonnegative. To satisfy this condition, the synergy effect should be bounded from above by $u^{(2)}$.

Theorem 1. *Under* (C1) *and* (C2), *the following results hold for an upstream merger with* Δv_2:

(a) (*Tier* 2) *For the postmerger firm,* $q_{21}^M > q_{21}$ *for any* $\Delta v_2 > 0$, *and* $q_{21}^M < 2q_{21}$ *if and only if* $\Delta v_2 < u^{(3)}$. *For the nonparticipant firms* $(j = 3, 4, \ldots, n_2)$, $\{q_{2j}^M > q_{2j}$ *and* $\pi_{2j}^M > \pi_{2j}\}$ *if and only if* $\Delta v_2 < u^{(3)}$. *Overall,* $\{Q_2^M < Q_2$ *and* $p_2^M > p_2\}$ *if and only if* $\Delta v_2 < u^{(3)}$; *whereas* $\Pi_2^M > \Pi_2$ *for any* $\Delta v_2 > 0$.

(b) (*Tier* 1) *For all* j, $\{q_{1j}^M < q_{1j}$, $\pi_{1j}^M < \pi_{1j}$, $Q_1^M < Q_1$, $p_1^M > p_1$, *and* $\Pi_1^M < \Pi_1\}$ *if and only if* $\Delta v_2 < u^{(3)}$.

(c) (*Supply Chain*) $\Pi^M > \Pi$ *if* $n_1 > 1 + (2n_2 + 1)/(n_2^2 - n_2 - 1)$.

We first discuss the effects of the upstream merger presented in Theorem 1 by using numerical examples given in Figure 1. We then discuss the condition $\Delta v_2 < u^{(3)}$ that appears in Theorem 1. Figures 1(a)–1(d) illustrate the aggregate effects of the upstream merger on production quantities, firms' profits, total profits, and prices, respectively, at both tier 1 and tier 2. Each figure is plotted against Δv_2 on the horizontal axis, which represents the degree of the synergy effect. When $\Delta v_2 = 0$, only the competition effect exists. So we can verify from Figure 1 the result presented earlier in the competition effect column in Table 1. As Δv_2 increases, the synergy effect changes the postmerger equilibrium outcomes in the direction prescribed in the synergy effect column in Table 1. When both effects are present, Theorem 1 shows that if $\Delta v_2 < u^{(3)}$, the competition effect outweighs the synergy effect, and vice versa.

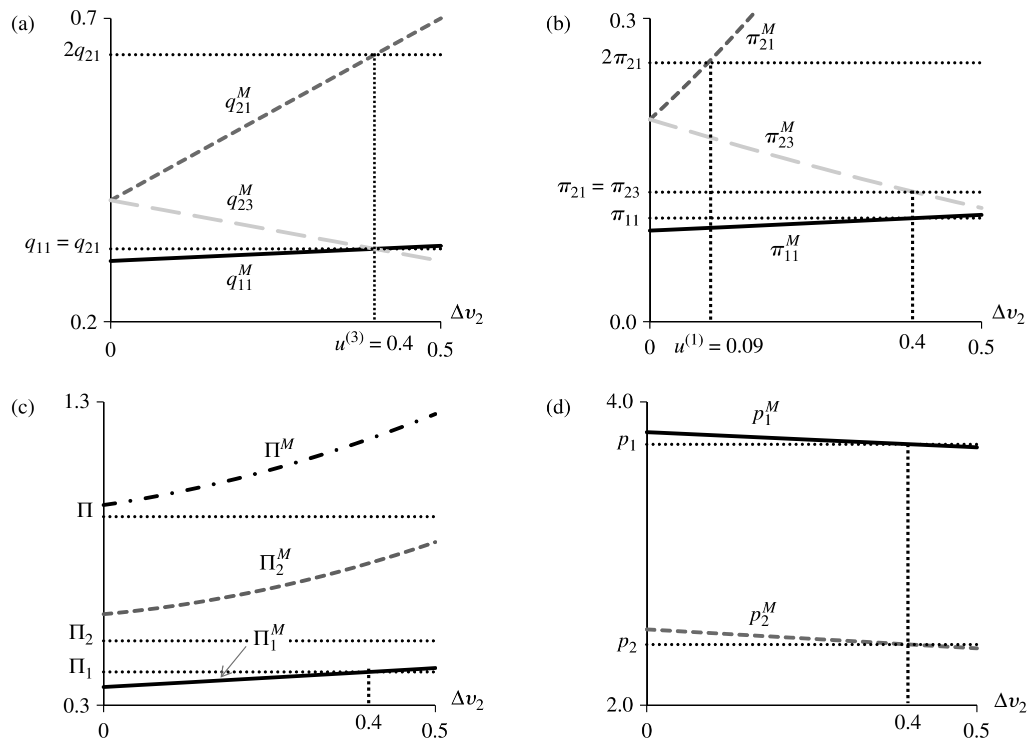
Following this logic, we can interpret Figure 1(a) as follows: When $\Delta v_2 = 0$, the output of the merging firms at tier 2 (q_{21}^M) is the same as that of the nonparticipant firms at tier 2 (q_{23}^M), which is larger than their premerger output (q_{21}) but smaller than $2q_{21}$. The output of a tier 1 firm (q_{11}^M) is smaller than its premerger output (q_{11}) (confirming the competition effect in Table 1). As Δv_2 increases, q_{21}^M and q_{11}^M increase, whereas q_{23}^M decreases (as prescribed by the synergy effect in Table 1). When Δv_2 exceeds $u^{(3)} = 0.4$ (i.e., the synergy effect is sufficiently high), the merging firms produce more than their premerger output $2q_{21}$; a tier 1 firm also produces more than its premerger output q_{11}, whereas the nonparticipant firms reduce their outputs below their premerger outputs q_{21} (confirming Theorem 1).

For the profits of individual firms, note that the general pattern shown in Figure 1(b) is similar to that in Figure 1(a). In this example, $n_2 = 4$, $u^{(1)} = 0.09$, and $u^{(2)} = 2$. Thus, conditions (C1) and (C2) given in Lemma 1 are satisfied when $\Delta v_2 \in [0.09, 2]$.

[3] Corbett and Karmarkar (2001) obtained a similar result, which states that the supply chain profit is maximized when $(n_1, n_2) = (2, 3)$ or $(3, 2)$, and increases with n_1 (respectively, n_2) when $n_2 = 1$ (respectively, $n_1 = 1$).

[4] As in the literature reviewed earlier, we do not consider the fixed cost of a merger. However, we can easily incorporate the fixed cost into condition (C1). This fixed cost does not affect subsequent results.

Figure 1 **Effects of the Upstream Merger on (a) Quantities, (b) Firms' Profits, (c) Total Profits, and (d) Prices**



Note. Parameters: $n_1 = n_2 = 4$, $a_1 = 5$, $b_1 = 1$, $v_1 = 1$, and $v_2 = 2$.

In this interval, the profit of the postmerger firm (π_{21}^M) is higher than its premerger profit ($2\pi_{21}$), and the profit of the nonparticipant firm (π_{23}^M) is nonnegative. Also, observe that the upstream merger reduces the profit of a tier 1 firm (i.e., $\pi_{11}^M < \pi_{11}$) when $\Delta v_2 < 0.4$, implying that the negative externalities brought by the competition effect outweigh the positive externalities brought by the synergy effect.

As Table 1 shows, both synergy and competition effects increase the total profit Π_2 of tier 2, at which the merger takes place. This is illustrated in Figure 1(c), which shows that $\Pi_2^M > \Pi_2$ for all Δv_2. In contrast, as Table 1 shows, the two effects have an opposite impact on the total profit Π_1 of tier 1; thus, the postmerger profit Π_1^M is greater than the premerger profit Π_1 only when the synergy effect is sufficiently high with $\Delta v_2 > 0.4$. Because $(n_1, n_2) = (4, 4)$ in this example satisfies the condition given in Theorem 1(c), the upstream merger improves the overall supply chain.[5]

Figure 1(d) can be interpreted similarly as follows: When $\Delta v_2 = 0$, postmerger prices p_1^M and p_2^M are higher than their respective premerger prices due to the competition effect. As Δv_2 increases, p_1^M and p_2^M start to decline, and when $\Delta v_2 > 0.4$, they fall below their premerger levels. This implies that when the synergy effect of an upstream merger is sufficiently high, consumers benefit from the merger. Note also that an upstream merger changes both p_1^M and p_2^M in the same direction (i.e., they are decreasing in Δv_2).

Finally, we discuss the condition that $\Delta v_2 < u^{(3)}$ given in Theorem 1. First, as noted earlier, $u^{(1)} < u^{(3)} < u^{(2)}$ for $n_1 \geq 3$, so there always exists $\Delta v_2 < u^{(3)}$ or $\Delta v_2 > u^{(3)}$ that satisfies the necessary conditions (C1) and (C2) (i.e., $\Delta v_2 \in [u^{(1)}, u^{(2)}]$). This suggests that the upstream merger can either increase or decrease the equilibrium outcomes that involve this condition in Theorem 1. This is because the synergy effect and the competition effect have opposite impacts on those outcomes, and neither effect always dominates the other. Second, $u^{(3)} = (a_1 - v_1 - v_2)/(n_2 + 1)$ increases as the market potential of a final product (a_1) increases, the marginal production cost of either tier (v_1 or v_2) decreases, and the number of firms at tier 2 (n_2) decreases. When $u^{(3)}$ increases, the interval of $\Delta v_2 < u^{(3)}$ expands. Because $p_1^M > p_1$ and $p_2^M > p_2$ when $\Delta v_2 < u^{(3)}$, this implies that as a_1 increases and v_1, v_2 or n_2 decreases, the upstream merger is more likely to raise the market price at either tier (p_1 and p_2), resulting in a welfare loss for consumers. Third, this condition can be re-written as $\Delta v_2 < u^{(3)} = p_2 - v_2$,

[5] The condition given in Theorem 1(c) is the same as the condition under which the supply chain profit is improved solely due to the competition effect. Because the synergy effect always improves the supply chain profit, this condition is sufficient but not necessary. As discussed earlier, this condition is satisfied in most realistic situations. We also provide the sufficient and necessary condition for $\Pi^M > \Pi$ in the proof.

where p_2 is the premerger part price given in (7). Thus, $u^{(3)}$ represents the premerger markup of an upstream firm. This condition is essentially the same as that derived by Farrell and Shapiro (1990).

4.2. A Downstream Merger in a Two-Tier Supply Chain

This subsection examines the effect of a merger between two firms in the downstream tier 1 when $n_1 \geq 2$. Similar to §4.1, we index two merging firms by $j = 1, 2$, a postmerger firm by $j = 1$, and nonparticipant firms by $j = 3, 4, \ldots, n_1$. The synergy effect is modeled by reducing the marginal production cost of the merging firms from v_1 to v_1^M $(< v_1)$ with $\Delta v_1 \equiv v_1 - v_1^M$ (> 0). The competition effect is modeled by reducing the number of firms at tier 1 from n_1 to $n_1 - 1$. We first examine each of these two effects in isolation (using superscripts A and C to denote equilibria under the synergy effect only and the competition effect only, respectively). We then combine the two effects to examine the aggregate effect (using superscript M to denote postmerger equilibria). Because the analysis proceeds similarly to that in §4.1, we highlight the difference between the upstream merger and the downstream merger so as to minimize repetition. Table 2 summarizes the synergy and competition effects of a downstream merger.

4.2.1. The Synergy Effect. Suppose that firms 1 and 2 at tier 1 have formed an alliance and have jointly reduced their marginal cost from v_1 to v_1^M. Given p_2, the profit of firm j at tier 1 is

$$\pi_{1j} = \begin{cases} (p_1 - v_1^M - p_2)q_{1j} = (a_1 - b_1 Q_1 - v_1^M - p_2)q_{1j} \\ \qquad \text{for } j = 1, 2, \\ (p_1 - v_1 - p_2)q_{1j} = (a_1 - b_1 Q_1 - v_1 - p_2)q_{1j} \\ \qquad \text{for } j = 3, 4, \ldots, n_1. \end{cases} \tag{11}$$

Table 2 **Effects of a Downstream Merger in the Two-Tier Supply Chain When $n_1 \geq 2$**

		Synergy effect	Competition effect
Tier 2	a_2	⇑	·
	b_2	·	⇑
	q_{2j}, π_{2j}, Π_2	⇑	⇓
	p_2	⇑	·
Tier 1	$q_{11} + q_{12}$	⇑	⇓
	$\pi_{11} + \pi_{12}$	⇑	⇓ if $n_1 > 2$; ⇑ if $n_1 = 2$
	q_{1j}, π_{1j} $(j \geq 3)$	⇓	⇑
	p_1	⇓	⇑
	Π_1	⇑	⇑
Supply chain	$\Pi_1 + \Pi_2$	⇑	⇑ if and only if $n_2 > 1 + \dfrac{2n_1 + 1}{n_1^2 - n_1 - 1}$

Similar to (1) in the premerger model, the total output of tier 1 firms in equilibrium is

$$Q_1 = \frac{n_1}{b_1(n_1 + 1)} \left\{ a_1 - p_2 - \frac{(n_1 - 2)v_1 + 2v_1^M}{n_1} \right\}. \tag{12}$$

By setting $Q_2 = Q_1$ in (12), we can derive the following inverse demand function for tier 2:

$$\begin{aligned} p_2 &= \left(a_1 - v_1 + \frac{2\Delta v_1}{n_1} \right) - b_1 \left(\frac{n_1 + 1}{n_1} \right) Q_2 \\ &= a_2^A - b_2^A Q_2, \end{aligned} \tag{13}$$

where $a_2^A = a_1 - v_1 + 2\Delta v_1 / n_1$ and $b_2^A = b_1(n_1 + 1)/n_1$. Compared with $a_2 = a_1 - v_1$ and $b_2 = b_1(n_1 + 1)/n_1$ in the premerger model, $a_2^A > a_2$ and $b_2^A = b_2$; that is, the synergy effect of a downstream merger increases the market potential for upstream firms, but it does not affect the price sensitivity in the upstream market. In response to such changes in the demand curve, each firm at tier 2 alters its premerger decision after the merger, which in turn affects the strategic decisions of tier 1 firms.

We next discuss the synergy effect of the downstream merger on the upstream tier presented in Table 2. With the expanded market potential a_2^A, firms at tier 2 increase their outputs and thereby earn higher profits. This is analogous to the synergy effect of the upstream merger on the downstream tier (shown in Table 1). Thus, in general, *the synergy effect of a merger at one tier brings positive externalities to firms at the other tier*. Because the downstream merger increases both a_2^A and Q_2, we observe from (13) that it may either increase or decrease p_2. However, we can show that $p_2^A - p_2 = 2\Delta v_1 / (n_1(n_2 + 1)) > 0$; that is, p_2 is always increased. This synergy effect is contrary to our earlier finding that the synergy effect of the upstream merger induces the downstream market price to fall (see Table 1).

Table 2 shows that the synergy effect of the downstream merger on its own tier is essentially the same as that of the upstream merger on its own tier as presented in Table 1. However, the underlying forces that drive such results differ in the following sense: In the case of the upstream merger, the total output of the upstream tier is increased because the synergy effect reduces the average marginal production cost of upstream firms. In the case of the downstream merger, in addition to this *direct* effect of marginal cost reduction, there is an *indirect* effect of marginal cost reduction that increases the part price p_2; this indirect effect pushes the total output of the downstream tier downward by reducing the profit margin of downstream firms. In spite of the presence of these two opposite effects, we can show $Q_1^A - Q_1 = 2n_2\Delta v_1 / (b_1(n_1 + 1)(n_2 + 1)) > 0$, implying that the direct effect dominates the indirect effect and thus increases the total output of the industry.

The profits of both tiers are improved due to the synergy effect of the downstream merger, as is the supply chain profit. This is the same as the synergy effect of the upstream merger.

4.2.2. The Competition Effect. Similar to the upstream merger, in the presence of only the competition effect, postmerger equilibrium outcomes can be obtained by replacing n_1 with n_1-1 in the premerger equilibrium outcomes presented in §3. Then we have the following inverse demand function for tier 2:

$$p_2=(a_1-v_1)-b_1\left(\frac{n_1}{n_1-1}\right)Q_2=a_2^C-b_2^CQ_2, \tag{14}$$

where $a_2^C=a_1-v_1$ and $b_2^C=b_1n_1/(n_1-1)$. Compared with $a_2=a_1-v_1$ and $b_2=b_1(n_1+1)/n_1$ in the premerger model, $a_2^C=a_2$ and $b_2^C>b_2$; that is, the reduced level of competition in the downstream tier does not affect the market potential for the upstream firms, but it does increase the price sensitivity in the upstream market. This change in the demand curve induces the competition effect of the downstream merger as summarized in Table 2.

Table 2 shows that the reduced level of downstream competition hurts the upstream firms by reducing their outputs and lowering profits. This is analogous to the competition effect of the upstream merger on the downstream firms (shown in Table 1). One may interpret this result in terms of buyer or supplier power: The downstream (respectively, upstream) merger increases buyer (respectively, supplier) power with one fewer firm in the market, which hurts the profits of the upstream (respectively, downstream) firms. However, in contrast to the upstream merger, which causes the product price p_1 to rise, the downstream merger does not affect the part price p_2.[6]

Table 2 also shows that the competition effect of the downstream merger on its own tier is essentially the same as the competition effect of the upstream merger on its own tier (see Table 1). Likewise, the competition effect of the downstream merger can increase or decrease the supply chain profit. The condition under which the supply chain profit is increased is symmetric: The downstream merger requires $n_2>1+(2n_1+1)/(n_1^2-n_1-1)$, whereas the upstream merger requires $n_1>1+(2n_2+1)/(n_2^2-n_2-1)$.

[6] Corbett and Karmarkar (2001) show that the concentration at the downstream tier does not affect the competitiveness of the upstream tier as measured by demand elasticity and the Lerner index at the upstream tier. In §6.2, following the lead of Tyagi (1999), we verify that this property holds under a more general class of demand functions. In §6.3, we show that this property no longer holds under nonlinear cost functions, but the aggregate effect of a downstream merger on consumer price is still the same under a certain condition.

4.2.3. The Aggregate Effect. We now examine the aggregate effect of the downstream merger by combining the synergy effect and the competition effect. Similar to the upstream merger, we observe from Table 2 that the synergy effect and the competition effect affect many equilibrium outcomes in the opposite direction. To determine the aggregate effect, we first derive postmerger equilibrium outcomes, and then compare them with the premerger equilibrium outcomes presented in §3.

We can derive the following inverse demand function for tier 2:

$$\begin{aligned}p_2&=\left(a_1-v_1+\frac{\Delta v_1}{n_1-1}\right)-b_1\left(\frac{n_1}{n_1-1}\right)Q_2\\&=a_2^M-b_2^MQ_2,\end{aligned} \tag{15}$$

where $a_2^M=a_1-v_1+\Delta v_1/(n_1-1)$ and $b_2^M=b_1n_1/(n_1-1)$. Compared with $a_2=a_1-v_1$ and $b_2=b_1(n_1+1)/n_1$ in the premerger model, $a_2^M>a_2$ and $b_2^M>b_2$; that is, the downstream merger increases both the market potential and the price sensitivity in the upstream market. This is due to the combination of the synergy effect (that increases the market potential; i.e., $a_2^A>a_2$) and the competition effect (that increases the price sensitivity; i.e., $b_2^C>b_2$). We next investigate how these changes in the demand function for tier 2 affect firms' decisions.

In preparation, we define the following:

$$d^{(1)}\equiv\frac{n_2(n_1-1)\{(\sqrt{2}-1)n_1-1\}}{n_1(n_1-2)(n_2+1)+n_2}\frac{a_1-v_1-v_2}{n_1+1},$$

$$d^{(2)}\equiv\frac{n_2n_1^2-n_2}{n_2n_1-n_2+n_1}\frac{a_1-v_1-v_2}{n_1+1},\quad d^{(3)}\equiv\frac{a_1-v_1-v_2}{n_1+1},$$

$$d^{(4)}\equiv\frac{a_1-v_1-v_2}{n_1+1+n_1\sqrt{(n_1+1)/(n_1-1)}},$$

$$d^{(5)}\equiv\frac{n_2(n_1-1)^2}{n_2(n_1-1)^2+n_1(n_1-2)}\frac{a_1-v_1-v_2}{n_1+1},$$

$$d^{(6)}\equiv\frac{n_2(n_1-1)}{n_2(n_1-1)+n_1}\frac{a_1-v_1-v_2}{n_1+1}.$$

We suppress the arguments (a_1,v_1,v_2,n_1,n_2) in $d^{(k)}$ for $k=1,2,\ldots,6$. We can show the following relation among these numbers, which will be useful in our subsequent discussions.

Lemma 2. *For any* (a_1,v_1,v_2,n_2) *and* $n_1\geq 3$, $0<d^{(1)}<d^{(6)}<d^{(5)}<d^{(3)}<d^{(2)}$ *and* $d^{(1)}<d^{(4)}<d^{(3)}$.

Using $d^{(k)}$, we first derive in Lemma 3 the necessary conditions for the downstream merger to occur, and then we present in Theorem 2 the aggregate effects of the downstream merger.

Lemma 3. (C3) *For the postmerger firm,* $\pi_{11}^M\geq 2\pi_{11}$ *if and only if* $n_1=2$ *or* $\Delta v_1\geq d^{(1)}$.

(C4) *For the nonparticipant firms* $(j=3,4,\ldots,n_1)$, $q_{1j}^M\geq 0$ *if and only if* $\Delta v_1\leq d^{(2)}$.

Lemma 3 states that, for a downstream merger to take place, the amount of marginal cost reduction, Δv_1, must be between $d^{(1)}$ and $d^{(2)}$ in realistic situations where $n_1 \geq 3$. This is consistent with the necessary conditions (C1) and (C2) derived in Lemma 1 for the upstream merger, although the lower and upper bounds are different; i.e., $d^{(1)} \neq u^{(1)}$ and $d^{(2)} \neq u^{(2)}$.

Theorem 2. *Under* (C3) *and* (C4), *the following results hold for a downstream merger with* Δv_1:

(a) (*Tier* 2) *For any* $\Delta v_1 > 0$, $p_2^M > p_2$. *For all* j, $\{q_{2j}^M < q_{2j}$ *and* $Q_2^M < Q_2\}$ *if and only if* $\Delta v_1 < d^{(3)}$. *For all* j, $\{\pi_{2j}^M < \pi_{2j}$ *and* $\Pi_2^M < \Pi_2\}$ *if and only if* $\Delta v_1 < d^{(4)}$.

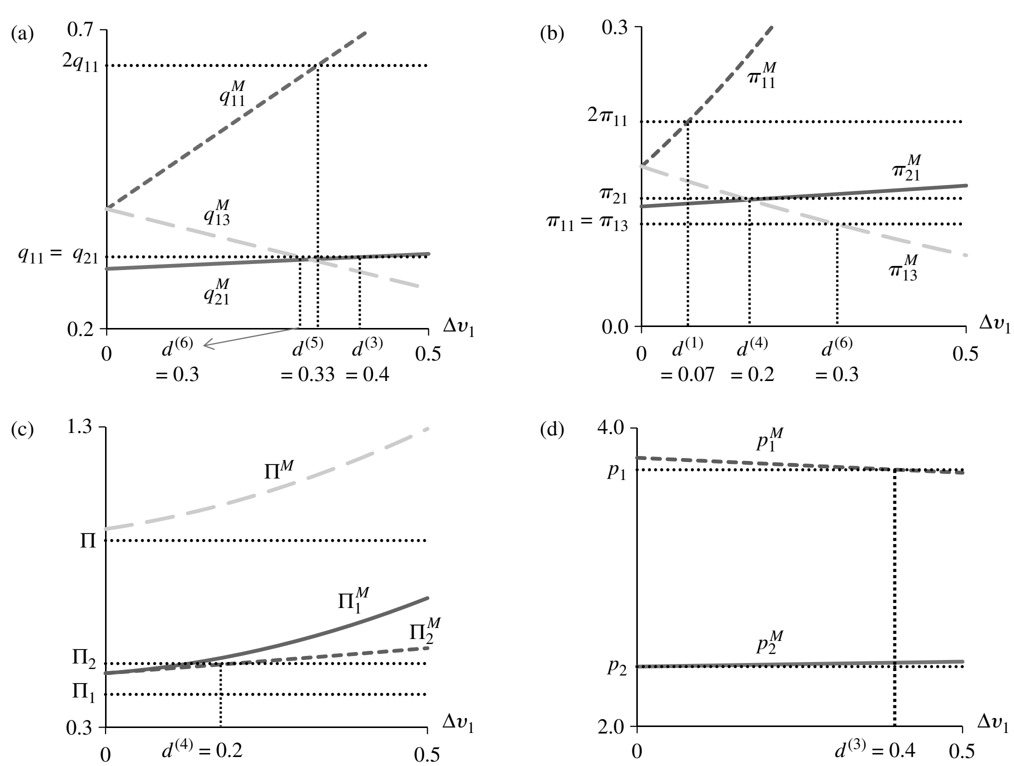
(b) (*Tier* 1) *For the postmerger firm,* $q_{11}^M - q_{11} > 0$ *for any* $\Delta v_1 > 0$; *and* $q_{11}^M - 2q_{11} < 0$ *if and only if* $\Delta v_1 < d^{(5)}$. *For the nonparticipant firms* ($j = 3, 4, \ldots, n_1$), $\{q_{1j}^M > q_{1j}$ *and* $\pi_{1j}^M > \pi_{1j}\}$ *if and only if* $\Delta v_1 < d^{(6)}$. *Overall,* $\{Q_1^M < Q_1$ *and* $p_1^M > p_1\}$ *if and only if* $\Delta v_1 < d^{(3)}$. *Finally,* $\Pi_1^M > \Pi_1$ *for any* $\Delta v_1 > 0$.

(c) (*Supply Chain*) $\Pi^M > \Pi$ *if* $n_2 > 1 + (2n_1 + 1)/(n_1^2 - n_1 - 1)$.

Figures 2(a)–2(d) illustrate the aggregate effects of the downstream merger on production quantities, firms' profits, total profits, and prices, respectively, at both tier 1 and tier 2. Each figure is plotted against Δv_1 on the horizontal axis, which represents the degree of the synergy effect. We can interpret Figure 2 similarly to Figure 1 as follows: When $\Delta v_1 = 0$, only the competition effect exists; as Δv_1 increases, the synergy effect changes the postmerger equilibrium outcomes in the direction prescribed in Table 2; when Δv_1 is sufficiently high, the synergy effect outweighs the competition effect. Whereas Theorem 1 shows that the condition for Δv_2 in an upstream merger involves only one threshold number, $u^{(3)}$, Theorem 2 shows that the corresponding conditions for Δv_1 involve multiple threshold numbers such as $d^{(3)}$, $d^{(4)}$, $d^{(5)}$, and $d^{(6)}$.

We can explain this difference between an upstream merger and a downstream merger by comparing their impact on the prices p_1 and p_2. First, we discuss their impact on p_2. For the upstream merger, Figure 1(d) shows that p_2^M is decreasing in Δv_2 and is lower than p_2 when $\Delta v_2 > u^{(3)}$. In contrast, for the downstream merger, Figure 2(d) shows that p_2^M is increasing in Δv_1 and is always higher than p_2. That is, a downstream merger always increases the part price p_2 by changing a demand function for the upstream market; it increases the market potential (a_2^M) and the price sensitivity (b_2^M) via the synergy effect and the competition effect, respectively (see Table 2). Next, we compare the impact of an upstream merger on the consumer price p_1 with that of a downstream merger. For an upstream merger, Theorem 1(b) states that $p_1^M - p_1 > 0$ if and only if $\Delta v_2 < u^{(3)} = p_2 - v_2$. This is consistent with the insightful result of Farrell and Shapiro (1990). For a downstream merger,

Figure 2 **Effects of the Downstream Merger on (a) Quantities, (b) Firms' Profits, (c) Total Profits, and (d) Prices**



Note. The same parameters are used as in Figure 1.

Theorem 2(b) shows that $p_1^M - p_1 > 0$ if and only if $\Delta v_1 < d^{(3)}$. Using the premerger prices p_1 and p_2 given in (7), we can rewrite $d^{(3)} = (a_1 - v_1 - v_2)/(n_1+1)$ as $d^{(3)} = ((n_2+1)/n_2)(p_1 - v_1 - p_2)$, where $p_1 - v_1 - p_2$ represents the premerger markup of a tier 1 firm. Thus, *the result of Farrell and Shapiro* (*1990*) *does not extend to a downstream merger in a two-tier supply chain.* Moreover, if a merger is approved when the amount of marginal cost reduction is greater than a firm's premerger markup according to Farrell and Shapiro (1990), then consumer price will be more likely to rise after a downstream merger than an upstream merger. The difference between the threshold $d^{(3)}$ and the premerger markup can be substantial: For example, $d^{(3)}$ is twice of the premerger markup when $n_2 = 1$.

This difference between the upstream merger and the downstream merger arises from the underlying mechanisms by which a merger at one tier affects strategic decision making of firms at the other tier. The upstream merger affects downstream firms by either increasing or decreasing the part price p_2^M: If the part price p_2^M is increased, the profit margin of downstream firms is reduced, resulting in a decrease in the total output and an increase in the consumer price p_1^M. On the other hand, the downstream merger increases the part price p_2^M by changing the demand curve for the upstream market, which in turn reduces the profit margin of downstream firms. If one does not consider the effect of the downstream merger on the part price p_2^M, this condition would be the same as that of Farrell and Shapiro (1990); that is, $p_1^M - p_1 > 0$ if and only if $\Delta v_1 < (a_1 - (v_1 + p_2))/(n_1+1) = p_1 - v_1 - p_2$. By noting that $d^{(3)} > p_1 - v_1 - p_2$, we assert that *consumer price is more likely to rise after a downstream merger in a vertically disintegrated market than in a vertically integrated market*.

Building on the above discussion about the different price effects of the upstream and downstream mergers, we can explain various conditions for Δv_1 for postmerger production quantities presented in Theorem 2. Note from Theorem 2(a) that the condition for $q_{21}^M < q_{21}$ at tier 2 is the same as that for $p_1^M > p_1$. This is because $p_1^M > p_1 \Leftrightarrow Q_1^M < Q_1 \Leftrightarrow Q_2^M < Q_2 \Leftrightarrow q_{21}^M < q_{21}$. However, Theorem 2(b) shows that the condition for $q_{11}^M < 2q_{11}$ or for $q_{13}^M > q_{13}$ at tier 1 differs from that for $p_1^M > p_1$. This is because the downstream merger raises the part price p_2^M, and hence reduces the margin of a postmerger firm ($p_1^M - v_1^M - p_2^M$) and that of a nonparticipant firm ($p_1^M - v_1 - p_2^M$) for any given p_1^M. Since $d^{(3)} > d^{(5)} > d^{(6)}$ from Lemma 2 (also observed in Figure 2(a)), $q_{13}^M > q_{13}$ implies $q_{11}^M < 2q_{1j}$, which in turn implies $q_{21}^M < q_{21}$. The profits of individual firms shown in Figure 2(b) can be explained similarly.

Finally, the effects of a downstream merger on the total profit at each tier and the overall supply chain profit (as illustrated in Figure 2(c)) are similar to those of an upstream merger (as illustrated in Figure 1(c)) in the following ways: (i) a merger improves the total profit of the tier at which the merger takes place; (ii) a merger improves the total profit of the other tier only when its synergy effect is sufficiently high; and (iii) a merger improves the overall supply chain profit as long as the competitive intensity of each tier is not very low.

4.3. Supply Chains with Three or More Tiers

Consider a three-tier supply chain with n_i firms at tier $i = 1, 2$, or 3. We number the tiers by 1 (downstream), 2 (intermediary), and 3 (upstream). For example, tier 1 firms may represent original equipment manufacturers, tier 2 firms may represent contract manufacturers, and tier 3 firms may represent part suppliers. For this supply chain, Corbett and Karmarkar (2001) provide the premerger model and equilibrium that serve as a benchmark for our study. We examine the effects of a merger of two firms at each tier i in the following order: an upstream merger at tier 3, a downstream merger at tier 1, and an intermediary merger at tier 2. We briefly discuss key results below.

4.3.1. Upstream Merger. The effects of the upstream merger in the three-tier supply chain are analogous to those of the upstream merger in the two-tier supply chain presented in Theorem 1. In the two-tier supply chain, Theorem 1 shows that the upstream merger at tier 2 will decrease the total output Q_2^M and increase the price p_2^M if and only if $\Delta v_2 < (a_1 - v_1 - v_2)/(n_2+1) = p_2 - v_2$. Similarly, in the three-tier supply chain, the upstream merger at tier 3 will decrease the total output Q_3^M and increase the price p_3^M if and only if $\Delta v_3 < (a_1 - v_1 - v_2 - v_3)/(n_3+1) = p_3 - v_3$. The effect of the upstream merger at tier 3 cascades to firms at tiers 1 and 2 through the part price p_3^M: If p_3^M goes up or down, so does p_1^M and p_2^M. The effects of the upstream merger on the total profits are also similar to those presented in Theorem 1: The upstream merger increases the total profit of tier 3, whereas it increases the total profit of tier 1 or tier 2 only when its synergy effect is sufficiently high.

4.3.2. Downstream Merger. Similar to our previous analysis of the two-tier supply chain, we obtain the following inverse demand functions for tier 2 and tier 3, respectively:

$$p_2^M = \left(a_1 - v_1 + \frac{\Delta v_1}{n_1 - 1}\right) - b_1\left(\frac{n_1}{n_1 - 1}\right) Q_2^M = a_2^M - b_2^M Q_2^M,$$

$$p_3^M = (a_2^M - v_2) - b_2^M\left(\frac{n_2+1}{n_2}\right) Q_3^M = a_3^M - b_3^M Q_3^M.$$

Compared with $a_2 = a_1 - v_1$ and $b_2 = b_1(n_1+1)/n_1$ in the premerger model, $a_2^M > a_2$ and $b_2^M > b_2$, respectively; similarly, $a_3^M > a_3 = a_2 - v_2$ and $b_3^M > b_3 =$

$b_2(n_2+1)/n_2$. Thus, the downstream merger at tier 1 increases the market potentials and price sensitivities of both tiers 2 and 3. Therefore, the effects of the downstream merger in the three-tier supply chain are analogous to those of the downstream merger in the two-tier supply chain. For example, the downstream merger will raise the market prices of tiers 2 and 3, p_2^M and p_3^M, while $p_1^M > p_1$ if and only if $\Delta v_1 < (a_1 - v_1 - v_2 - v_3)/(n_1+1) = ((n_2+1)(n_3+1)/(n_2 n_3))(p_1 - v_1 - p_2)$. Similar to the upstream merger, the downstream merger increases the total profit of tier 1, whereas it increases the total profit of tier 2 or tier 3 only when its synergy effect is sufficiently high.

4.3.3. Intermediary Merger. We examine the effects of the intermediary merger at tier 2 on the upstream tier 3, the downstream tier 1, and its own tier 2, respectively. First, the effects of the intermediary merger on tier 3 are essentially the same as the effects of the downstream merger on the upstream tier in the two-tier supply chain summarized in Theorem 2(a); the downstream merger affects the upstream market by expanding its market potential and increasing its price sensitivity. Second, the effects of the intermediary merger on tier 1 are essentially the same as the effects of the upstream merger on the downstream tier in the two-tier supply chain summarized in Theorem 1(b); the upstream merger affects the downstream market by changing the procurement cost of downstream firms. Third, the effects of the intermediary merger on its own tier 2 are similar to the effects of the downstream merger on its own tier summarized in Theorem 2(b).

From the above analysis of the three-tier supply chain, it is straightforward to understand how it can be naturally extended to more than three tiers. In particular, the following corollary presents the general condition under which a merger at one tier in a multitier supply chain leads to an increase of consumer price.

Corollary 1. *Suppose that two firms at tier i $(=1, 2, \ldots,$ or $N)$ in the N-tier supply chain have merged into one firm and the postmerger firm has reduced its marginal cost to v_i^M $(< v_i)$. Then $p_1^M > p_1$ if and only if $\Delta v_i < (a_1 - \sum_{j=1}^N v_j)/(n_i+1)$, where $(a_1 - \sum_{j=1}^N v_j)/(n_N+1) = p_N - v_N$ for $i = N$ and $(a_1 - \sum_{j=1}^N v_j)/(n_i+1) = ((n_{i+1}+1)(n_{i+2}+1)\ldots(n_N+1)/(n_{i+1}n_{i+2}\ldots n_N))(p_i - v_i - p_{i+1})$ for $i < N$.*[7]

[7] Although our results are extended to any N (≥ 2) tiers, in reality, firms and antitrust agencies may limit their attention to the first m $(< N)$ tiers when information about upstream firms above tier m is not readily available or when a large number of firms exist above tier m (e.g., perfectly competitive market). In the latter case, a merger at tier m will have a negligible impact on upstream prices. Therefore, a firm at the most upstream tier could be thought of as the firm that buys inputs from a perfectly competitive market.

5. Entry-Inducing Effects of Mergers in a Supply Chain

So far, we have analyzed the effects of a merger in a multitier supply chain, assuming that the merger does not induce the entry of new firms. Although this is a valid assumption in the short run, a merger may induce entry in the long run. In this section, we assume that the supply chain has been in free-entry, long-run equilibrium before a merger, and examine how the merger affects this equilibrium by inducing entry to each tier of a supply chain. We focus our analysis on a two-tier supply chain, and at the end of this section, we discuss how the analysis can be extended to a longer supply chain. Following the literature (e.g., Werden and Froeb 1998, Tyagi 1999, Corbett and Karmarkar 2001), we answer the question of whether entry is profitable at the same scale as that of a premerger incumbent firm. As discussed in §1, antitrust agencies also examine the "sufficiency" of an entry by considering a single firm that replicates at least the scale of one of the merging firms.

Again, we use the premerger long-run equilibrium of Corbett and Karmarkar (2001). Let F_i (>0) denote the fixed cost of entry to tier i, and let $\pi_i(n_1, n_2)$ denote the postentry profit of a firm at tier i when n_i firms compete at each tier $i \in \{1, 2\}$. Then any pair (n_1, n_2) in equilibrium will satisfy (i) $\pi_1(n_1, n_2) \geq F_1$ and $\pi_2(n_1, n_2) \geq F_2$, and (ii) $\pi_1(n_1+1, n_2) < F_1$ and $\pi_2(n_1, n_2+1) < F_2$. Condition (i) requires that the operating profits of entrants should cover their cost of entry; condition (ii) ensures that no more firms will enter each tier. Define $W = \{(n_1, n_2) \in \mathcal{N}^2 \mid \pi_i(n_1, n_2) \geq F_i \text{ for } i = 1, 2\}$. This set represents the set of *viable* supply chain structures in the sense that when $(n_1, n_2) \in W$, all firms in the supply chain can earn profits at least as high in postentry competition as their entry cost. This set can be rewritten as $W = \{(n_1, n_2) \in \mathcal{N}^2 \mid g_1(n_1) \leq n_2 \leq g_2(n_1)\}$, where $g_1(n_1)$ and $g_2(n_1)$ can be derived from the following inequalities:

$$\pi_1(n_1, n_2) = \frac{1}{b_1}\left(\frac{n_2}{n_2+1}\right)^2\left(\frac{a_1 - v_1 - v_2}{n_1+1}\right)^2 \geq F_1$$

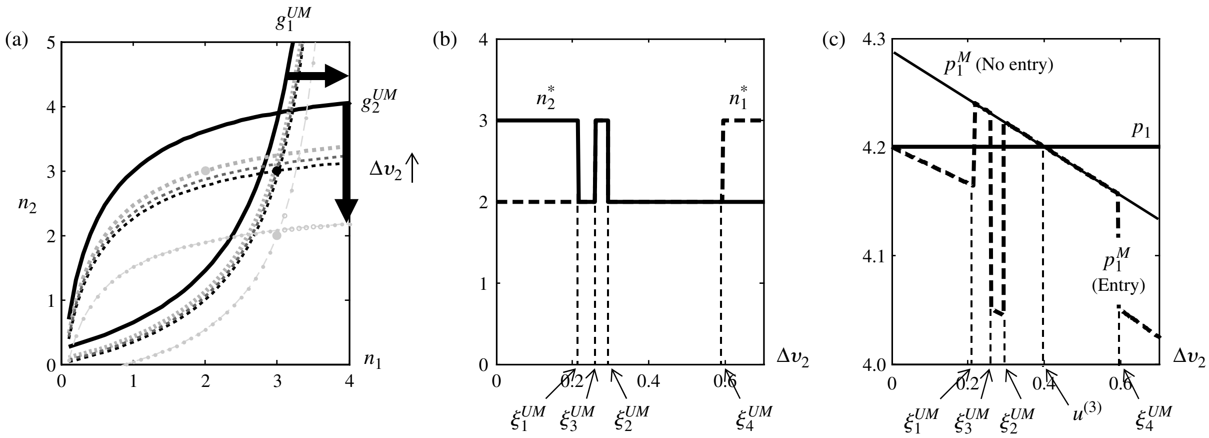
$$\Leftrightarrow \quad n_2 \geq \frac{(n_1+1)\sqrt{b_1 F_1}}{a_1 - v_1 - v_2 - (n_1+1)\sqrt{b_1 F_1}} \equiv g_1(n_1),$$

$$\pi_2(n_1, n_2) = \frac{1}{b_1}\left(\frac{n_1}{n_1+1}\right)\left(\frac{a_1 - v_1 - v_2}{n_2+1}\right)^2 \geq F_2$$

$$\Leftrightarrow \quad n_2 \leq \sqrt{\frac{n_1}{n_1+1}}\frac{a_1 - v_1 - v_2}{\sqrt{b_1 F_2}} - 1 \equiv g_2(n_1),$$

where $\pi_1(n_1, n_2)$ and $\pi_2(n_1, n_2)$ are from (8) and (9), respectively. Corbett and Karmarkar (2001) show that nonempty W is a bounded lattice; its maximal

Figure 3 **(a) Viable Structures $W^{UM} = \{(n_1, n_2) \in \mathcal{N}^2 \mid g_1^{UM}(n_1) \le n_2 \le g_2^{UM}(n_1)\}$, Where $g_1^{UM}(n_1)$ (Respectively, $g_2^{UM}(n_1)$) from Left to Right (Respectively, Top to Bottom) When $\Delta v_2 = 0$, $\xi_1^{UM} (= 0.21)$, $\xi_3^{UM} (= 0.26)$, ξ_2^{UM} $(= 0.29)$, and $\xi_4^{UM} (= 0.60)$; (b) Equilibrium Number of Firms (n_1^*, n_2^*); and (c) Consumer Price: p_1^M (Entry), p_1^M (No Entry), and p_1**



Note. Parameters: $a_1 = 1$, $b_1 = 0.05$, $v_1 = v_2 = 1.7$, $F_1 = 10$, and $F_2 = 8$.

element $(\bar{n}_1, \bar{n}_2)$ is the only stable farsighted equilibrium.[8]

We first consider an upstream merger with the cost synergy of Δv_2 as in §4.1. Immediately after this merger, the number of firms in the supply chain will become $(\bar{n}_1, \bar{n}_2 - 1)$. We assume that potential entrants will make their entry decisions simultaneously with foresight. Define the viable structure $W^{UM} = \{(n_1, n_2) \in \mathcal{N}^2 \mid \pi_i^M(n_1, n_2) \ge F_i \text{ for } i = 1, 2\}$, where $\pi_i^M(n_1, n_2)$ denotes the postentry profit of a nonparticipant firm at tier i when n_i firms compete at each tier $i \in \{1, 2\}$ after the upstream merger. Similar to W, W^{UM} can be rewritten as $W^{UM} = \{(n_1, n_2) \in \mathcal{N}^2 \mid g_1^{UM}(n_1) \le n_2 \le g_2^{UM}(n_1)\}$, where $g_1^{UM}(n_1)$ and $g_2^{UM}(n_1)$ can be computed similarly to $g_1(n_1)$ and $g_2(n_1)$, respectively, as shown in Appendix A. If nonempty W^{UM} is a bounded lattice, according to Corbett and Karmarkar (2001), it contains a maximal element (n_1^*, n_2^*). In that case, if $n_1^* > \bar{n}_1$ (respectively, $n_2^* > \bar{n}_2 - 1$), the upstream merger will be followed by the entry of a new firm to tier 1 (respectively, tier 2) in the long run.[9] The next theorem shows when there will be subsequent entry of a new firm to either or both tiers after the upstream merger.

[8] Corbett and Karmarkar (2001) assume that firms make their entry decisions simultaneously because there is an equilibrium that cannot result from the sequential entry process. They further show that only the maximal element is farsighted stable; that is, firms keep entering as long as there is some further entry pattern that produces profits, even though they might lose money in the current structure.

[9] In this section, we assume that the conditions given in Lemmas 1 and 3 are satisfied so that a merger is profitable and does not induce the exit of nonparticipant firms. If (C2) or (C4) is violated, all nonparticipant firms will exit from the industry and no new firms will enter.

Theorem 3. *Nonempty W^{UM} is a bounded lattice. There exist four threshold numbers ξ_1^{UM}, ξ_2^{UM}, ξ_3^{UM}, and ξ_4^{UM} with $0 < \xi_1^{UM} < \xi_2^{UM}$ and $0 < \xi_3^{UM} < \xi_4^{UM}$ such that the upstream merger with cost synergy Δv_2 will be followed by*

(i) *the entry of a new firm to each of tiers 1 and 2 if $\Delta v_2 \in U_{12} = \{\Delta v_2 \mid \xi_3^{UM} \le \Delta v_2 \le \xi_2^{UM}\}$, where $U_{12} = \varnothing$ when $\xi_2^{UM} < \xi_3^{UM}$;*

(ii) *the entry of a new firm to only tier 2 if $\Delta v_2 \in U_2 = \{\Delta v_2 \mid \Delta v_2 \le \xi_1^{UM}$ and $\Delta v_2 \notin U_{12}\}$;*

(iii) *the entry of a new firm to only tier 1 if $\Delta v_2 \in U_1 = \{\Delta v_2 \mid \Delta v_2 \ge \xi_4^{UM}$ and $\Delta v_2 \notin U_{12}\}$;*

(iv) *no entry if $\Delta v_2 \notin U_1 \cup U_2 \cup U_{12}$.*

Figure 3(a) illustrates how the viable structure W^{UM} changes with Δv_2. As shown, $g_1^{UM}(n_1)$ and $g_2^{UM}(n_1)$ are increasing in n_1 for any given Δv_2, and decreasing in Δv_2 for any given n_1 (hence, as Δv_2 increases, $g_1^{UM}(n_1)$ is shifting right and $g_2^{UM}(n_1)$ is shifting down). The thresholds ξ_1^{UM}, ξ_2^{UM}, ξ_3^{UM}, and ξ_4^{UM} are the values of Δv_2 at which $g_2^{UM}(\bar{n}_1) = \bar{n}_2$, $g_2^{UM}(\bar{n}_1 + 1) = \bar{n}_2$, $g_1^{UM}(\bar{n}_1 + 1) = \bar{n}_2$, and $g_1^{UM}(\bar{n}_1 + 1) = \bar{n}_2 - 1$, respectively. In Figure 3(a), for example, $g_2^{UM}(2) = 3$ at $\Delta v_2 = \xi_1^{UM} = 0.21$, $g_2^{UM}(3) = 3$ at $\Delta v_2 = \xi_2^{UM} = 0.29$, and so on.

We now discuss how the cost synergy of the upstream merger, Δv_2, affects subsequent entry. First, with no synergy (i.e., $\Delta v_2 = 0$), it is obvious that a new entrant at the upstream tier will replace the one that has vanished after the merger, hence restoring the premerger equilibrium (e.g., in Figure 3(b), $(n_1^*, n_2^*) = (\bar{n}_1, \bar{n}_2) = (2, 3)$ when $\Delta v_2 \le \xi_1^{UM}$, as in Theorem 3(ii)). As the cost synergy becomes larger, the postmerger firm will become more competitive. When Δv_2 exceeds ξ_1^{UM}, a new firm will not enter the upstream tier (provided it does not induce the entry of a downstream firm as we shall discuss below) because it can no longer recoup its entry cost in postentry competition (e.g., in Figure 3(b), $(n_1^*, n_2^*) = (2, 2)$ when $\Delta v_2 \in (\xi_1^{UM}, \xi_3^{UM})$, as in Theorem 3(iv)). These observations are consistent with the entry-inducing effect of a merger in a vertically integrated market; specifically, we show that if $\Delta v > a - v - \sqrt{bF}\lfloor (a-v)/\sqrt{bF} \rfloor\ (\geq 0)$, no entry will follow a merger in a vertically integrated market. Second, unlike a vertically integrated market, Theorem 3 shows that when cost synergy is substantial, the upstream merger can induce the entry of a downstream firm (e.g., in Figure 3(b), $(n_1^*, n_2^*) = (3, 2)$ when $\Delta v_2 \geq \xi_4^{UM}$, as in Theorem 3(iii)). To understand why this occurs, recall from Table 1 that downstream firms benefit from the synergy effect of the upstream merger so that their profits increase with Δv_2. When Δv_2 is sufficiently high, the downstream tier can support one more firm in free-entry equilibrium. Therefore, *unlike in a vertically integrated market, a large cost synergy does not necessarily preclude entry in a decentralized supply chain*. Third, if $\xi_3^{UM} \leq \xi_2^{UM}$, the moderate level of cost synergy induces the entry of both upstream and downstream firms (e.g., in Figure 3(b), $(n_1^*, n_2^*) = (3, 3)$ when $\Delta v_2 \in [\xi_3^{UM}, \xi_2^{UM}]$, as in Theorem 3(i)). In this case, the equilibrium number of firms at each tier is nonmonotonic with the level of cost synergy, Δv_2 (see Figure 3(b)). We explain this result using the competition effect of an upstream or downstream merger. As Tables 1 and 2 show, the competition effect of a merger at one tier hurts the profits of firms at the other tier. Because the entry effect of a new firm is opposite to the competition effect of a merger, the entry of a new firm to one tier increases the profits of firms at the other tier. Thus, when the merged firm has the cost synergy with $\Delta v_2 > \xi_1^{UM}$, a new firm can still enter the upstream tier provided that it accompanies the entry of a new downstream firm (which happens when $\Delta v_2 \in [\xi_3^{UM}, \xi_2^{UM}]$). Similarly, the entry of a new upstream firm can enable the downstream tier to support an additional firm at a lower level of cost synergy than ξ_4^{UM}.

Finally, we examine the aggregate effect of the upstream merger on consumer price that takes into account subsequent entry. The effects on production quantities, profits, and part price can be examined similarly. When $\Delta v_2 \in U_{12}$, as in Theorem 3(i), with the entry of a new firm to each tier, the number of firms at tier 2 will become the same as before the merger, while the number of firms at tier 1 will be greater than before the merger. Thus, the combination of the synergy effect of the upstream merger shown in Table 1 and the opposite of the competition effect of the downstream merger shown in Table 2 will lead consumer price to fall after this merger (e.g., in Figure 3(c), $p_1^M(Entry) < p_1$ when $\Delta v_2 \in [\xi_3^{UM}, \xi_2^{UM}]$). When $\Delta v_2 \in U_2$, as in Theorem 3(ii), the entry of a new firm to tier 2 will deter the competition effect of the upstream merger. Consequently, only its synergy effect presented in Table 1 will remain, resulting in a price fall (e.g., in Figure 3(c), $p_1^M(Entry) < p_1$ when $\Delta v_2 \leq \xi_1^{UM}$). When $\Delta v_2 \in U_1$, as in Theorem 3(iii), with the entry of a new firm to tier 1, the opposite of the competition effect of the downstream merger shown in Table 2 (which pushes consumer price downward) will counteract the aggregate effect of the upstream merger presented in Theorem 1. Thus, consumer price will be more likely to fall than in the case of no entry; specifically, we show that $p_1^M > p_1 \Leftrightarrow \Delta v_2 \leq \{1 - (n_2/(n_1+1))^2\}((a_1 - v_1 - v_2)/(n_2+1))$ (cf. $\Delta v_2 \leq (a_1 - v_1 - v_2)/(n_2+1) = u^{(3)}$ in Theorem 1). In the example shown in Figure 3(c), $\{1 - (n_2/(n_1+1))^2\}(a_1 - v_1 - v_2)/(n_2+1) < u^{(3)} < \xi_4^{UM}$, so $p_1^M(Entry) < p_1$ when $\Delta v_2 \geq \xi_4^{UM}$. Finally, when $\Delta v_2 \notin U_1 \cup U_2 \cup U_{12}$, as in Theorem 3(iv), with no entry, the previous result in Theorem 1 continues to hold (e.g., in Figure 3(c), $p_1^M(Entry) = p_1^M$ *(No Entry)* when $\Delta v_2 \in (\xi_1^{UM}, \xi_3^{UM}) \cup (\xi_2^{UM}, \xi_4^{UM})$). From this analysis, we find that *although a larger cost synergy from a merger always reduces consumer price without entry, this is not necessarily true when accounting for subsequent entry after a merger.*

Next, consider a downstream merger with the cost synergy of Δv_1 as in §4.2. Define the viable structure W^{DM} similarly to W^{UM}. The next theorem shows the condition under which nonempty W^{DM} is a bounded lattice, and it shows when there will be subsequent entry to either or both tiers after this merger.

Theorem 4. *Suppose* $\Delta v_1 < \min\{\Delta v_1 < n_1(2n_1+1) \cdot (a_1 - v_1 - v_2), n_1^2(n_2+1)\sqrt{b_1 F_1}\}$ *for* $(n_1, n_2) \in W^{DM}$. *Then nonempty* W^{DM} *is a bounded lattice and there exist four threshold numbers* ξ_1^{DM}, ξ_2^{DM}, ξ_3^{DM}, *and* ξ_4^{DM} *with* $0 < \xi_1^{DM} < \xi_2^{DM}$ *and* $0 < \xi_3^{DM} < \xi_4^{DM}$ *such that the downstream merger with cost synergy* Δv_1 *will be followed by*

(i) *the entry of a new firm to tiers* 1 *and* 2 *if* $\Delta v_1 \in D_{12} = \{\Delta v_1 \mid \xi_3^{DM} \leq \Delta v_1 \leq \xi_2^{DM}\}$, *where* $D_{12} = \varnothing$ *when* $\xi_2^{DM} < \xi_3^{DM}$;

(ii) *the entry of a new firm to only tier* 1 *if* $\Delta v_1 \in D_1 = \{\Delta v_1 \mid \Delta v_1 \leq \xi_1^{DM}$ *and* $\Delta v_1 \notin D_{12}\}$;

(iii) *the entry of a new firm to only tier* 2 *if* $\Delta v_1 \in D_2 = \{\Delta v_1 \mid \Delta v_1 \geq \xi_4^{DM}$ *and* $\Delta v_1 \notin D_{12}\}$;

(iv) *no entry if* $\Delta v_1 \notin D_1 \cup D_2 \cup D_{12}$.

The condition for bounded lattice W^{DM} is innocuous; For example, if $(2, 2) \in W^{DM}$, this condition becomes $\Delta v_1 < \min\{10(a_1 - v_1 - v_2), 12\sqrt{b_1 F_1}\}$, where $(a_1 - v_1 - v_2)$ is larger than the premerger markup $p_1 - v_1 - p_2$ (see §4.2) and the entry cost F_1 tends to be much larger than the unit production cost v_1. Note also that the entry conditions given in Theorem 4 have the same structure as those given in Theorem 3. Therefore, the insights discussed for the upstream merger apply to the downstream merger as well.

Because the threshold numbers in Theorems 3 and 4 are different, it is unclear which merger (upstream or

Table 3 **Numerical Results: Upstream vs. Downstream Merger**

	Upstream merger (%)					Downstream merger (%)				
The amount of marginal cost reduction as % of premerger markup	20	60	100	140	180	20	60	100	140	180
% of scenarios under which a merger induces entry to:										
Tier 2	75	28	0	0	0	0	0	10	21	32
Tier 1	0	0	0	8	19	73	24	0	0	0
Both tiers	3	4	0	0	0	3	5	0	0	0
% of scenarios under which a merger raises consumer price:										
Without entry (short run)	100	100	0	0	0	100	100	100	4	0
With entry (long run)	22	68	0	0	0	24	71	90	2	0

Notes. Parameters: $a_1 = 5$, $b_1 = 0.01$, F_1 and $F_2 \in \{5, 5.5, \ldots, 10\}$, v_1 and $v_2 \in \{1, 1.1, \ldots, 2\}$, where a_1 and b_1 are fixed because they appear as $(a_1 - v_1 - v_2)$ and $\sqrt{b_1 F_i}$ in equilibrium outcomes. Out of 14,641 scenarios, this table used 14,040 scenarios that resulted in $\bar{n}_1 \geq 2$ and $\bar{n}_2 \geq 2$.

downstream) induces more entries and how it affects consumer price. Because of the complexity of these thresholds (which are derived in Appendix A), we examine these questions numerically. The results are summarized in Table 3, from which we draw the following observations:

(1) An upstream merger tends to induce more entry to the tier at which a merger takes place, but less entry to the other tier than a downstream merger. There is no significant difference in entries to both tiers between an upstream merger and a downstream merger.

(2) An upstream merger tends to raise consumer prices less often than a downstream merger in the short run (ignoring entry) and in the long run (accounting for entry).

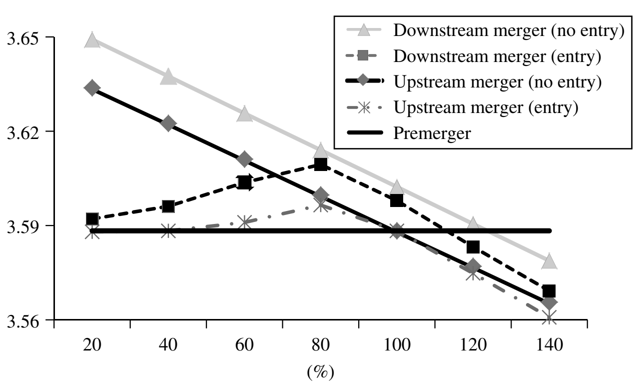
When using the same set of parameter values as in Table 3, Figure 4 illustrates how average consumer price changes as cost synergy increases. From Figure 4, we can verify the following:

(1) As cost synergy increases, the average postmerger consumer price declines in the short run (ignoring entry), but increases and then decreases in the long run (accounting for entry). As expected, the average postmerger consumer price is higher with no entry than with entry.

(2) For the same level of cost synergy (as a percentage of premerger markup), the average consumer price after an upstream merger is lower than that after a downstream merger both in the short run and in the long run. Thus, a larger cost synergy is required for a downstream merger to induce prices to fall than for an upstream merger.

It is not difficult to see how the analysis above can be extended to a supply chain with more than two tiers. In such a supply chain, a merger at any tier can not only induce entry to the same tier (when its cost synergy is sufficiently low) but also to all other tiers (when its cost synergy is sufficiently high). Entry to any tier will counteract the adverse competitive effect of a merger, so the merger will be more beneficial (or less harmful) to consumers.

Figure 4 **Average Postmerger Consumer Price with Respect to the Amount of Marginal Cost Reduction as Percentage of Premerger Markup**



6. Extensions

This section examines three extensions of our base model presented in §§3 and 4. In §6.1, we consider supply chains where downstream firms lead. In §6.2, we show that our previous results obtained under linear demand continue to hold for a broad class of demand functions. In §6.3, we examine the impact of increasing or decreasing marginal costs.

6.1. Supply Chains Where Retailers Lead

Our base model follows most existing models of supply chain competition by assuming that upstream firms move before downstream firms. While this assumption is reasonable for many real-world supply chains, Adida et al. (2012) recently considered a supply chain where downstream firms lead. Such a supply chain is relevant to the setting where large retailers (e.g., Walmart) place orders to intermediary firms (e.g., Li & Fung) who in turn find suppliers who can fill the orders. In this section, instead of using the model of Corbett and Karmarkar (2001), we use the model of Adida et al. (2012) as our premerger benchmark.

Consider a two-tier supply chain in which tier 1 firms as leaders first determine their quantities, and tier 2 firms are margin-takers with respect to tier 1 firms. This is the variant of the model in Choi (1991) that Adida et al. (2012) have proposed. In the original model of Choi (1991), there is only one firm at tier 1 (retailer) and two firms at tier 2 (manufacturers). This variant model considers multiple competing firms at each tier. Let $m_1 = p_1 - p_2 - v_1$ denote the margin of a tier 1 firm. Then, given m_1, the profit of firm j $(=1, 2, \ldots, n_2)$ at tier 2 is $\pi_{2j} = (p_2 - v_2)q_{2j} = (a_1 - b_1 Q_1 - m_1 - v_1 - v_2)q_{2j}$, where $Q_1 = Q_2 = q_{2j} + Q_{2,-j}$. Given $Q_{2,-j}$, each firm j chooses q_{2j} that maximizes its profit π_{2j}. By following the analysis similar to our base analysis, we obtain the total quantity in equilibrium given by $Q_2 = n_2(a_1 - m_1 - v_1 - v_2)/(b_1(n_2+1))$, from which we derive $m_1 = a_1 - v_1 - v_2 - ((n_2+1)/n_2)b_1 Q_2$. Then each firm j $(=1, 2, \ldots, n_1)$ at tier 1 chooses q_{1j} that maximizes its profit $\pi_{1j} = m_1 q_{1j} = \{a_1 - v_1 - v_2 - ((n_2+1)/n_2)b_1(q_{1j} + Q_{1,-j})\}q_{1j}$. For convenience, we use superscript (S) to denote the equilibrium outcomes in our base model in §3. Then we can easily show that the equilibrium in this model satisfies the following:

$$Q_1 = Q_1^{(S)}, \quad p_1 = p_1^{(S)}, \quad \text{and} \quad p_2 < p_2^{(S)};$$
$$\pi_{1j} = \frac{n_2+1}{n_2}\pi_{1j}^{(S)} > \pi_{1j}^{(S)}, \quad \pi_{2j} = \frac{n_1}{n_1+1}\pi_{2j}^{(S)} < \pi_{2j}^{(S)},$$
$$\text{and} \quad \Pi = \Pi^{(S)}.$$

Because this model essentially reverses the role of leaders and followers in a supply chain as compared with Corbett and Karmarkar (2001), it results in the same total quantity Q_1 (hence p_1) and the same supply chain profit Π in equilibrium. Because tier 2 firms are now followers, they earn lower profit π_{2j} than $\pi_{2j}^{(S)}$, and command lower price p_2 than $p_2^{(S)}$. Similarly, tier 1 firms as leaders earn higher profit π_{1j} than $\pi_{1j}^{(S)}$. Using this premerger equilibrium, we can derive the following conditions under which consumer price will rise after an upstream or downstream merger:

Corollary 2. *Consider the supply chain where downstream firms lead.*

(a) *An upstream merger with cost synergy Δv_2 will result in $p_1^M > p_1$ if and only if $\Delta v_2 < u^{(3)} = (a_1 - v_1 - v_2)/(n_2+1) = ((n_1+1)/n_1)(p_2 - v_2)$.*

(b) *A downstream merger with cost synergy Δv_1 will result in $p_1^M > p_1$ if and only if $\Delta v_1 < d^{(3)} = (a_1 - v_1 - v_2)/(n_1+1) = p_1 - p_2 - v_1$.*

Corollary 2 shows that the condition for $p_1^M > p_1$ is the same as Farrell and Shapiro (1990) for the downstream merger, whereas the threshold $u^{(3)}$ for the upstream merger is greater than the premerger markup of a tier 2 firm (i.e., $p_2 - v_2$). This is the opposite of our results in the base model. Therefore, we can refine our intuition obtained under the base model as follows: A merger at the tier that acts as the follower leads to a higher threshold than the premerger markup.[10]

[10] The premerger model described above assumes that the demand function ($p_1 = a_1 - b_1 Q_1$) is known to upstream firms. Adida et al. (2012) have proposed an alternative supply chain model where retailers lead. Our analysis shows that the main results of Corollary 2 continue to hold in this model.

6.2. Nonlinear Demand Functions

Our base model assumes that firms at tier 1 face the linear demand curve, $p_1 = a_1 - b_1 Q_1$. Below we examine whether the results obtained under this demand function continue to hold under more general nonlinear demand functions.

We first consider the upstream merger in the two-tier supply chain, as analyzed in §4.1. As Theorem 1(a) shows, the upstream market price will rise if and only if the marginal cost reduction is lower than the premerger markup of the merging firms. Farrell and Shapiro (1990) show that the same condition applies to other demand functions in their single-tier model as long as they satisfy the stability conditions for Cournot equilibrium (Dixit 1986). Furthermore, Theorem 1(b) shows that the upstream merger affects the downstream market by changing the procurement cost of downstream firms, so that the downstream market price will fall if and only if the upstream market price falls. This result holds irrespective of the type of demand curve. Thus, Theorems 1(a) and 1(b) hold as long as the demand functions in the upstream market satisfy the stability conditions for Cournot equilibrium. However, the condition provided in Theorem 1(c) for the supply chain profit depends on a specific functional form of the demand.

We next consider the downstream merger in the two-tier supply chain, as analyzed in §4.2. Theorem 2(a) shows that the upstream market price always increases as a result of the downstream merger. This is due to the combination of the synergy effect that increases the upstream market price and the competition effect that does not affect the upstream market price (see Table 2). Tyagi (1999) shows that this competition effect holds for the following class of commonly used demand functions: $p_1 = a_1 - b_1 Q_1^k$ where $a_1 \geq 0$, $b_1 > 0$, and $0 < k < 3$. These demand functions satisfy the conditions that guarantee the existence and stability of Cournot equilibrium (Dixit 1986). Note that this class of demand functions includes linear $(k = 1)$, convex $(0 < k < 1)$, and concave $(1 < k < 3)$ functions. The following corollary shows that our previous results continue to hold under this class of demand functions.

Corollary 3. *Suppose that the inverse demand function satisfies $p_1 = a_1 - b_1 Q_1^k$, where $a_1 \geq 0$, $b_1 > 0$, and $0 < k < 3$. Then, a downstream merger with cost synergy Δv_1 will result in $p_2^M > p_2$ for any $\Delta v_1 > 0$, whereas $p_1^M > p_1$ if and only if $\Delta v_1 < ((k/(n_1+k))(a_1 - v_1 - v_2) = ((n_2+k)/n_2)(p_1 - v_1 - p_2)$.*

The above result is consistent with our earlier result in that a downstream merger will raise consumer price if and only if Δv_1 is lower than $((n_2+k)/n_2)(p_1 - v_1 - p_2)$, which is larger than the premerger markup of a downstream firm. Moreover, as a demand function becomes more concave (i.e., k increases),

$((n_2+k)/n_2)(p_1-v_1-p_2)$ increases, so a downstream merger is more likely to raise consumer price.

6.3. Nonlinear Cost Functions

While a stylized fact of U.S. industry is that marginal costs are typically constant (e.g., Werden and Froeb 1998), it is worth investigating how increasing or decreasing marginal costs affect our results. To this end, we consider a more general cost function, and we assume that the marginal production cost of firm j $(=1,2,\ldots,n_i)$ at tier i is $v_i+w_iq_{ij}$ before a merger takes place. This quadratic cost function includes constant ($w_i=0$), increasing ($w_i>0$), and decreasing ($w_i<0$) marginal cost functions that are frequently used in the literature. For premerger equilibrium to exist, we assume that the marginal cost is not decreasing too fast; specifically, $w_1>-b_1$ and $w_2>-b_2$ (also, $w_2>-b_2^M$ for postmerger equilibrium). By following the same procedure as in the base model, we obtain the following equilibrium prices in the two-tier supply chain:

$$p_1=a_1-b_1Q_1=a_1-(a_1-v_1-v_2)\cdot\left(\frac{(n_1+1)(n_2+1)}{n_1n_2}+\frac{2(n_2+1)w_1}{n_1n_2b_1}+\frac{2w_2}{n_2b_1}\right)^{-1}; \quad (16)$$

$$p_2=a_2-b_2Q_1=(a_1-v_1)-\left\{\frac{(n_1+1)b_1}{n_1}+\frac{2w_1}{n_1}\right\}Q_1$$
$$=a_1-v_1-\frac{a_1-v_1-v_2}{(n_2+1)/n_2+(2n_1w_2)/(n_2\{(n_1+1)b_1+2w_1\})}. \quad (17)$$

As expected, increasing marginal costs ($w_1>0$ or $w_2>0$) reduce firms' incentives to produce large quantity, leading to higher p_1 in (16) than p_1 in (7) in the base model where $w_1=w_2=0$. Also, while p_2 in (7) in our base model is independent of n_1, p_2 in (17) is increasing in n_1 for positive w_1 and w_2. Thus, under increasing marginal costs, the competition effect of a downstream merger will cause the part price p_2 to fall. Note, however, that increasing marginal costs not only affect the competition effect of a downstream merger on p_2 but also its synergy effect on p_2 (as well as its synergy and competition effects on p_1). Thus, it is not immediately clear how increasing marginal costs will affect the conditions presented earlier in Theorems 1 and 2 under which an upstream or downstream merger will raise consumer price p_1.

Corollary 4. *Suppose that the marginal production cost of firm j $(=1,2,\ldots,n_i)$ at tier i is $v_i+w_iq_{ij}$ (where $w_i>-b_i$) before a merger takes place.*

(a) *An upstream merger with cost synergy Δv_2 (≥ 0) and Δw_2 $(\in[0,b_2+w_2))$ will result in $p_1^M>p_1$ if and only if*

$$\Delta v_2<\frac{[b_1(n_1+1)+2w_1+2n_1w_2-4n_1\Delta w_2](a_1-v_1-v_2)}{(n_2+1)\{b_1(n_1+1)+2w_1\}+2n_1w_2}=p_2-v_2-4q_{2j}\Delta w_2.$$

(b) *A downstream merger with cost synergy Δv_1 (≥ 0) and Δw_1 $(\in[0,b_1+w_1))$ will result in $p_1^M>p_1$ if and only if*

$$\Delta v_1<\frac{(n_2+1)(b_1+2w_1-4\Delta w_1)(a_1-v_1-v_2)}{(n_2+1)\{b_1(n_1+1)+2w_1\}+2n_1w_2}=\frac{n_2+1}{n_2}(p_1-v_1-p_2-4q_{1j}\Delta w_1).$$

From Corollary 4, we can verify the following insights obtained under the base model. First, it shows that consumer price will rise after the merger when cost synergy is sufficiently small. Second, ceteris paribus, consumer price is more likely to rise after a downstream merger than an upstream merger. To see this, we rewrite the conditions given in Corollary 4, (a) and (b), as $\Delta v_2+4q_{2j}\Delta w_2<p_2-v_2$ and $\Delta v_1+4q_{1j}\Delta w_1<p_1-v_1-p_2+\Delta v_1/(n_2+1)$, respectively, where $4q_{ij}\Delta w_i$ is the amount of marginal cost reduction evaluated at the premerger output level of two merging firms, $2q_{ij}$, and $p_i-v_i-p_{i+1}$ (where $p_3=0$) is the premerger markup of a tier i firm in the base model without the additional cost term associated with w_i. Consistent with the results in the base model, the right-hand side of the condition for a downstream merger is larger than $p_1-v_1-p_2$, while the corresponding right-hand side for an upstream merger is equal to p_2-v_2.

In summary, this section refines the results from the base model for the supply chain where downstream firms lead, and it also shows that our main insights continue to hold under nonlinear demand and cost functions.

7. Concluding Remarks

We now summarize the policy and managerial implications of our results. First, in a decentralized supply chain in which firms interact strategically with other firms across different tiers, antitrust agencies should take into account how a merger at one tier affects strategic decisions of firms at different tiers, which in turn affect consumer price. Our results suggest that: (1) consumer price is less likely to fall when a merger occurs in a vertically disintegrated industry rather than in a vertically integrated industry, and (2) when upstream firms lead the supply chain, an upstream merger tends to raise consumer price less often than a downstream merger both in the short run (without entry) and in the long run (accounting for entry). Therefore, antitrust agencies should not only pay attention to whether a merger occurs in a vertically integrated or disintegrated market but also to whether a supply chain is led by upstream or downstream firms, and to where a merger takes place in a supply chain.

Second, Table 1 (respectively, Table 2) helps managers understand the synergy and competition effects of an upstream merger (respectively, a downstream merger) in a two-tier supply chain. For both upstream and downstream mergers, the synergy effect creates negative externalities to nonparticipant firms (who thus reduce their outputs and earn lower profits), but creates positive externalities to firms at the other tier (who thus increase their outputs and earn higher profits); whereas the competition effect creates positive externalities to nonparticipant firms, but creates negative externalities to firms at the other tier. However, the effects of an upstream merger on prices differ from those of a downstream tier: (i) The synergy effect of an upstream merger induces the downstream market price to fall, whereas that of a downstream merger raises the upstream market price. (ii) The competition effect of an upstream merger causes a downstream price to rise, whereas that of a downstream merger does not affect an upstream price. When the two effects of a merger change equilibrium outcomes in opposite directions, the synergy effect outweighs the competition effect when the amount of marginal cost reduction is sufficiently high.

Third, when accounting for subsequent entry after a merger in the long run, a larger cost synergy from a merger does not necessarily benefit consumers more. This happens because a more competitive postmerger firm deters the entry of new firms to the tier at which the merger takes place. However, the synergy effect of a merger at one tier brings positive externalities to firms at the other tiers, and thus induces the entry of new firms at the other tiers when cost synergy is sufficiently large. In this case, the entry moderates the adverse competitive effect of a merger, and helps reduce price.

We have demonstrated the robustness of our main results by considering the supply chain where downstream firms lead as well as nonlinear demand and cost functions. Clearly, there are other opportunities to extend this research. For example, while it is common in the literature that a proposed merger is exogenous, there are some papers (e.g., Gowrisankaran 1999) that try to endogenize the set of mergers that will occur in a market without any antitrust constraint. In this dynamic context, the currently proposed merger may be followed by other mergers; thus, its impact may depend on the possibility of future mergers. However, Nocke and Whinston (2010) show that a myopic merger review policy that ignores the possibility of future mergers is optimal in most cases. Whether this result will hold in a decentralized supply chain deserves further investigation. Other extensions include product substitution, demand uncertainty, assembly structure, capacity constraints, bargaining, multiple products, and economies of scope.

Overall, this paper contributes to the supply chain literature by analyzing the effects of a merger in a decentralized supply chain, and to the theory of mergers by analyzing the effects of a merger at one tier on its upstream and downstream tiers, and by re-evaluating the effects of a merger on its own tier, taking into account the cross-tier impacts of a merger.

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Appendix A. Proofs

Proof of Theorem 1. We derive the closed-form expressions of postmerger equilibrium outcomes and compare them with the corresponding premerger equilibrium outcomes (see Table A.1). □

Proof of Theorem 2. Similar to the proof of Theorem 1, we have Table A.2. □

Proof of Lemma 2. $d^{(1)} < d^{(4)}$:

$$\frac{n_2(n_1-1)\{(\sqrt{2}-1)n_1-1\}}{n_1(n_1-2)(n_2+1)+n_2} < \frac{(n_1-1)\{(\sqrt{2}-1)n_1-1\}}{n_1(n_1-2)+1}.$$

Thus, it suffices to show that

$$\frac{(n_1-1)\{(\sqrt{2}-1)n_1-1\}}{n_1(n_1-2)+1} < \frac{1}{1+n_1/\sqrt{n_1-1}}.$$

Then, $(n_1-1)\{(\sqrt{2}-1)n_1-1\}((1+n_1/\sqrt{n_1-1})) - \{n_1(n_1-2)+1\} = ((n_1\sqrt{n_1+1})/\sqrt{n_1-1})\{(\sqrt{2}-2)\sqrt{n_1^2-1}+(\sqrt{2}-1)n_1-1\} < ((n_1\sqrt{n_1+1})/\sqrt{n_1-1})\{(\sqrt{2}-2)\sqrt{n_1^2-1}+(\sqrt{2}-1)n_1\}$, where $(\sqrt{2}-2)\sqrt{n_1^2-1}+(\sqrt{2}-1)n_1 < 0$ when $n_1^2 > 2$.

$d^{(4)} < d^{(3)}$: $d^{(6)}/d^{(3)} = (n_1+1)/(n_1\sqrt{(n_1+1)/(n_1-1)}+n_1+1) < 1$.

$d^{(1)} < d^{(6)}$:

$$\begin{aligned}\frac{d^{(6)}}{d^{(1)}} &= \frac{(n_1-2)n_1(n_2+1)+n_2}{\{(\sqrt{2}-1)n_1-1\}n_1(n_2+1)-(\sqrt{2}-1)n_1n_2+n_2}\\ &> \frac{(n_1-2)n_1(n_2+1)+n_2}{\{(\sqrt{2}-1)n_1-1\}n_1(n_2+1)+n_2} > 1,\end{aligned}$$

where the first inequality is because $-(\sqrt{2}-1)n_1n_2 < 0$ and the second inequality is because $(n_1-2) > \{(\sqrt{2}-1)n_1-1\}$.

$d^{(6)} < d^{(5)}$: $d^{(6)}/d^{(5)} = ((n_1-1)^2n_2+n_1(n_1-2))/((n_1-1)^2n_2+n_1(n_1-1)) < 1$.

$d^{(5)} < d^{(3)}$: $d^{(5)}/d^{(3)} = (n_2(n_1-1)^2)/(n_2(n_1-1)^2+n_1(n_1-2)) < 1$.

$d^{(3)} < d^{(2)}$: $d^{(2)}/d^{(3)} = (n_2n_1^2-n_2)/(n_2n_1-n_2+n_1) > 1$ because $(n_2n_1^2-n_2)-(n_2n_1-n_2+n_1) = n_1\{n_2(n_1-1)-1\} > 0$ $\forall\, n_1(\geq 3), n_2(\geq 1)$. □

Proof of Theorem 3. First, we characterize $(\bar{n}_1, \bar{n}_2)$ in premerger equilibrium. Before the merger, $\bar{n}_1$ and $\bar{n}_2$

Table A.1 The Aggregate Effect of an Upstream Merger on Equilibrium Outcomes

	Postmerger equilibrium	Postmerger vs. premerger
Tier 2	$q_{21}^M=\frac{a_2-v_2+(n_2-1)\Delta v_2}{b_2n_2}$	$q_{21}^M-q_{21}=\frac{a_2-v_2+(n_2^2-1)\Delta v_2}{b_2n_2(n_2+1)}>0$
		$q_{21}^M-2q_{21}=-\frac{(n_2-1)\{a_2-v_2-(n_2+1)\Delta v_2\}}{b_2n_2(n_2+1)}$
	$\pi_{21}^M=\frac{1}{b_2}\left\{\frac{a_2-v_2+(n_2-1)\Delta v_2}{n_2}\right\}^2$	$\pi_{21}^M-\pi_{21}=\frac{\{(2n_2+1)(a_2-v_2)+(n_2^2-1)\Delta v_2\}\{a_2-v_2+(n_2^2-1)\Delta v_2\}}{b_2n_2^2(n_2+1)^2}>0$
		$\pi_{21}^M-2\pi_{21}$ (*)
	$q_{2j}^M=\frac{a_2-v_2-\Delta v_2}{b_2n_2}\quad (j\geq 3)$	$q_{2j}^M-q_{2j}=\frac{a_2-v_2-(n_2+1)\Delta v_2}{b_2n_2(n_2+1)}$
	$\pi_{2j}^M=\frac{1}{b_2}\left\{\frac{a_2-v_2-\Delta v_2}{n_2}\right\}^2\quad (j\geq 3)$	$\pi_{2j}^M-\pi_{2j}=\frac{(2n_2+1)(a_2-v_2)^2+(n_1+1)^2\{-2(a_2-v_2)\Delta v_2+(\Delta v_2)^2\}}{b_2n_2^2(n_2+1)^2}$
	$Q_2^M=Q_1^M=\frac{(n_2-1)(a_2-v_2)+\Delta v_2}{b_2n_2}$	$Q_2^M-Q_2=-\frac{a_2-v_2-(n_2+1)\Delta v_2}{b_2n_2(n_2+1)}$
	$p_2^M=\frac{a_2+(n_2-2)v_2+v_2^M}{n_2}$	$p_2^M-p_2=\frac{a_2-v_2-(n_2+1)\Delta v_2}{n_2(n_2+1)}$
	$\Pi_2^M=\frac{(n_2-1)(a_2-v_2)^2+(n_2^2-n_2-1)\Delta v_2^2+2(a_2-v_2)\Delta v_2}{b_2n_2^2}$	$\Pi_2^M-\Pi_2=\frac{(n_2^2-n_2-1)\{(a_2-v_2)^2+\Delta v_2^2\}+2(a_2-v_2)\Delta v_2}{b_2n_2^2(n_2+1)^2}>0$
Tier 1	$q_{1j}^M=\frac{(n_2-1)(a_1-v_1-v_2)+\Delta v_2}{b_1(n_1+1)n_2}\quad \forall j$	$q_{1j}^M-q_{1j}=-\frac{a_1-v_1-v_2-(n_2+1)\Delta v_2}{b_1n_2(n_1+1)(n_2+1)}$
	$\pi_{1j}^M=\frac{1}{b_1}\left\{\frac{(n_2-1)(a_1-v_1-v_2)+\Delta v_2}{(n_1+1)n_2}\right\}^2\quad \forall j$	$\pi_{1j}^M-\pi_{1j}=\frac{\{(-2n_2^2+1)(a_1-v_1-v_2)-(n_2+1)\Delta v_2\}\{a_1-v_1-v_2-(n_2+1)\Delta v_2\}}{b_1n_2^2(n_1+1)^2(n_2+1)^2}$
	$p_1^M=a_1-\frac{n_1(n_2-1)(a_1-v_1-v_2)+n_1\Delta v_2}{(n_1+1)n_2}$	$p_1^M-p_1=\frac{n_1\{a_1-v_1-v_2-(n_2+1)\Delta v_2\}}{n_2(n_1+1)(n_2+1)}$
	$\Pi_1^M=\frac{n_1}{b_1}\left\{\frac{(n_2-1)(a_1-v_1-v_2)+\Delta v_2}{(n_1+1)n_2}\right\}^2$	$\Pi_1^M-\Pi_1=\frac{n_1\{(-2n_2^2+1)(a_1-v_1-v_2)-(n_2+1)\Delta v_2\}\{a_1-v_1-v_2-(n_2+1)\Delta v_2\}}{b_1n_2^2(n_1+1)^2(n_2+1)^2}$
Supply chain	$\Pi^M=\Pi_1^M+\Pi_2^M$	$\Pi^M-\Pi=(\Pi_1^M+\Pi_2^M)-(\Pi_1+\Pi_2)$ (**)

(*) Since $\pi_{21}^M=b_2(q_{21}^M)^2$ and $\pi_{2j}=b_2(q_{21})^2$, $\pi_{21}^M\geq 2\pi_{21}$ if and only if

$$q_{21}^M-\sqrt{2}q_{21}=\frac{-\{(\sqrt{2}-1)n_2-1\}(a_2-v_2)+(n_2^2-1)\Delta v_2}{b_2n_2(n_2+1)}\geq 0,$$

which is simplified to

$$\Delta v_2\geq\frac{(\sqrt{2}-1)n_2-1}{n_2-1}\,\frac{a_2-v_2}{n_2+1}=u^{(1)}.$$

(**) After simplification, we obtain the following:

$$\Pi^M-\Pi=\frac{(n_2^2-n_2-1)(n_1-1-(2n_2+1)/(n_2^2-n_2-1))(a_2-v_2)^2}{b_2(n_1+1)n_2^2(n_2+1)^2}+\frac{(n_2+1)^2\Delta v_2\{\Delta v_2(n_2^2-n_2-1)(n_1+1+1/(n_2^2-n_2-1))+2(a_2-v_2)(3n_1-1)\}}{b_2(n_1+1)n_2^2(n_2+1)^2},$$

from which we can express a necessary and sufficient condition for $\Pi^M-\Pi>0$. A sufficient condition for $\Pi^M-\Pi>0$ is the condition for $\Pi^M-\Pi>0$ given in Table 1.

are the largest integers that satisfy $n_2\geq g_1(n_1)$ and $n_2\leq g_2(n_1)$. It is straightforward to show $\partial g_1(n_1)/\partial n_1>0$ and $\partial g_2(n_1)/\partial n_1>0$. Thus, $n_2\geq g_1(n_1)$ can be rewritten as $n_1\leq g_1^{-1}(n_2)=(n_2/(n_2+1))((a_1-v_1-v_2)/\sqrt{b_1F_1})-1$. Then $(\bar{n}_1,\bar{n}_2)$ is the unique solution that solves: $\bar{n}_1=\lfloor g_1^{-1}(\bar{n}_2)\rfloor$ and $\bar{n}_2=\lfloor g_2(\bar{n}_1)\rfloor$.

Next, we prove the existence of (n_1^*,n_2^*) in post-merger equilibrium by showing that nonempty W^{UM} is a bounded lattice. After the upstream merger occurs, $\pi_{2j}^M=(1/b_2)\{(a_2-v_2-\Delta v_2)/n_2\}^2$ $(j\geq 3)$ in Table A.1 represents the profit of a nonparticipant firm j at tier 2 when there are n_1 firms at tier 1 and n_2-1 firms at tier 2. Thus,

$$\pi_2^M(n_1,n_2)=\frac{1}{b_2}\left\{\frac{a_2-v_2-\Delta v_2}{n_2+1}\right\}^2\geq F_2$$

$$\Leftrightarrow\quad n_2\leq\sqrt{\frac{n_1}{n_1+1}}\,\frac{a_1-v_1-v_2-\Delta v_2}{\sqrt{b_1F_2}}-1\equiv g_2^{UM}(n_1),$$

Table A.2 The Aggregate Effect of a Downstream Merger on Equilibrium Outcomes

	Postmerger equilibrium	Postmerger vs. premerger
Tier 2	$q_{2j}^M = \dfrac{(n_1-1)(a_1-v_1-v_2)+\Delta v_1}{b_1 n_1(n_2+1)} \quad \forall j$	$q_{2j}^M - q_{2j} = -\dfrac{a_1-v_1-v_2-(n_1+1)\Delta v_1}{b_1 n_1(n_1+1)(n_2+1)}$
	$\pi_{2j}^M = \dfrac{\{(n_1-1)(a_1-v_1-v_2)+\Delta v_1\}^2}{b_1 n_1(n_1-1)(n_2+1)^2} \quad \forall j$	$\pi_{2j}^M - \pi_{2j}$ (*)
	$p_2^M = \dfrac{a_1-v_1+(\Delta v_1)/(n_1-1)+n_2 v_2}{n_2+1}$	$p_2^M - p_2 = \dfrac{\Delta v_1}{(n_1-1)(n_2+1)}$
	$\Pi_2^M = \dfrac{n_2\{(n_1-1)(a_1-v_1-v_2)+\Delta v_1\}^2}{b_1 n_1(n_1-1)(n_2+1)^2}$	$\Pi_2^M - \Pi_2$ (*)
Tier 1	$q_{11}^M = \dfrac{n_2(a_1-v_1-v_2)+((\Delta v_1)/(n_1-1))\{n_1(n_1-2)(n_2+1)+n_2\}}{b_1 n_1(n_2+1)}$	$q_{11}^M - q_{11} = \dfrac{n_2(a_1-v_1-v_2)}{b_1 n_1(n_1+1)(n_2+1)} + \dfrac{\Delta v_1\{n_1(n_1-2)(n_2+1)+n_2\}}{b_1 n_1(n_1-1)(n_2+1)} > 0$
		$q_{11}^M - 2q_{11} = -\dfrac{n_2(n_1-1)(a_1-v_1-v_2)}{b_1 n_1(n_1+1)(n_2+1)} + \dfrac{\Delta v_1\{n_1(n_1-2)(n_2+1)+n_2\}}{b_1 n_1(n_1-1)(n_2+1)}$
	$\pi_{11}^M = \dfrac{[n_2(a_1-v_1-v_2)+((\Delta v_1)/(n_1-1))\{n_1(n_1-2)(n_2+1)+n_2\}]^2}{b_1 n_1^2(n_2+1)^2}$	$\pi_{11}^M - \pi_{11}$ (**)
		$\pi_{11}^M - 2\pi_{11}$ (**)
	$q_{1j}^M = \dfrac{n_2(a_1-v_1-v_2)+((\Delta v_1)/(n_1-1))\{-n_1(n_2+1)+n_2\}}{b_1 n_1(n_2+1)} \quad (j\ge 3)$	$q_{1j}^M - q_{1j} = \dfrac{n_2(n_1-1)(a_1-v_1-v_2)-(n_1+1)\{n_2(n_1-1)+n_1\}\Delta v_1}{b_1 n_1(n_1+1)(n_2+1)(n_1-1)}$
	$\pi_{1j}^M = \dfrac{[n_2(a_1-v_1-v_2)+((\Delta v_1)/(n_1-1))\{-n_1(n_2+1)+n_2\}]^2}{b_1 n_1^2(n_2+1)^2} \quad (j\ge 3)$	$\pi_{1j}^M - \pi_{1j}$ (**)
	$Q_1^M = Q_2^M = \dfrac{n_2\{(n_1-1)(a_1-v_1-v_2)+\Delta v_1\}}{b_1 n_1(n_2+1)}$	$Q_1^M - Q_1 = -\dfrac{n_2\{a_1-v_1-v_2-(n_1+1)\Delta v_1\}}{b_1 n_1(n_1+1)(n_2+1)}$
	$p_1^M = a_1 - \dfrac{n_2\{(n_1-1)(a_1-v_1-v_2)+\Delta v_1\}}{n_1(n_2+1)}$	$p_1^M - p_1 = \dfrac{n_2\{(a_1-v_1-v_2)-(n_1+1)\Delta v_1\}}{n_1(n_1+1)(n_2+1)}$
	$\Pi_1^M = \dfrac{(n_1-1)n_2^2(a_1-v_1-v_2)^2+2n_2^2(a_1-v_1-v_2)\Delta v_1}{b_1 n_1^2(n_2+1)^2} + \dfrac{\Delta v_1^2\{(n_1(n_1-2)(n_2+1)+n_2)^2+(n_1-2)(-n_1(n_2+1)+n_2)^2\}}{b_1 n_1^2(n_1-1)^2(n_2+1)^2}$	$\Pi_1^M - \Pi_1 > 0$ (***)
Supply chain	$\Pi^M = \Pi_1^M + \Pi_2^M$	$\Pi^M - \Pi = (\Pi_1^M + \Pi_2^M) - (\Pi_1 + \Pi_2)$ (****)

(*) After simplification, we obtain the following:

$$\pi_{2j}^M - \pi_{2j} = -\frac{\{a_1-v_1-v_2-(n_1+1+n_1\sqrt{(n_1+1)/(n_1-1)})\Delta v_1\}\{a_1-v_1-v_2-(n_1+1-n_1\sqrt{(n_1+1)/(n_1-1)})\Delta v_1\}}{b_1 n_1(n_2+1)^2(n_1+1)}.$$

Observe that $\{a_1-v_1-v_2-(n_1+1-n_1\sqrt{(n_1+1)/(n_1-1)})\Delta v_1\} > 0$ because $n_1+1-n_1\sqrt{(n_1+1)/(n_1-1)} = n_1+1-\sqrt{(n_1+1)^2+(n_1+1)/(n_1-1)} < 0$. Thus, $\pi_{2j}^M - \pi_{2j} < 0$ if and only if $a_1-v_1-v_2-(n_1+1+n_1\sqrt{(n_1+1)/(n_1-1)})\Delta v_1 > 0$, which is simplified to $\Delta v_1 < (a_1-v_1-v_2)/(n_1+1+n_1\sqrt{(n_1+1)/(n_1-1)}) = d^{(4)}$. Since $\Pi_2^M - \Pi_2 = n_2(\pi_{2j}^M - \pi_{2j})$, the sign of $\Pi_2^M - \Pi_2$ is the same as that of $\pi_{2j}^M - \pi_{2j}$.

(**) Since $\pi_{1j}^M = b_1(q_{1j}^M)^2$ and $\pi_{1j} = b_1(q_{1j})^2$ for all j, $q_{11}^M > q_{11}$ implies $\pi_{11}^M > \pi_{11}$; for $j \ge 3$, $\pi_{1j}^M > \pi_{1j}$ if and only if $q_{1j}^M > q_{1j}$; and $\pi_{11}^M \ge 2\pi_{11}$ if and only if

$$q_{11}^M - \sqrt{2}q_{11} = \frac{n_2\{-(\sqrt{2}-1)n_1+1\}(a_1-v_1-v_2)}{b_1 n_1(n_1+1)(n_2+1)} + \frac{\Delta v_1\{n_1(n_1-2)(n_2+1)+n_2\}}{b_1 n_1(n_1-1)(n_2+1)} \ge 0,$$

which is simplified to

$$\Delta v_1 \ge \frac{n_2(n_1-1)\{(\sqrt{2}-1)n_1-1\}}{n_1(n_1-2)(n_2+1)+n_2}\,\frac{a_1-v_1-v_2}{n_1+1} = d^{(1)}.$$

Similarly, we can derive the conditions that involve $d^{(2)}$, $d^{(3)}$, $d^{(5)}$, and $d^{(6)}$ from this table.

(***) When $\Delta v_1 = 0$, $\Pi_1^C - \Pi_1 > 0$ from Table 2. Observe that Π_1^M increases with Δv_1. Therefore, $\Pi_1^M - \Pi_1 > 0$ for any $\Delta v_1 > 0$.

(****) After simplification, we obtain that $\Pi_1^M - \Pi_1 = (1/(b_1 n_1^2(n_2+1)^2))[((\{n_1(n_1-1)-1\}n_2^2)/(n_1+1)^2)(a_1-v_1-v_2)^2 + 2n_2^2(a_1-v_1-v_2)\Delta v_1 + ((\Delta v_1^2)/(n_1-1)^2)\{(n_1(n_1-2)(n_2+1)+n_2)^2+(n_1-2)(-n_1(n_2+1)+n_2)^2\}]$. Combining $\Pi_1^M - \Pi_1$ and $\Pi_2^M - \Pi_2 = n_2(\pi_{2j}^M - \pi_{2j})$, we can express a necessary and sufficient condition for $\Pi^M - \Pi = (\Pi_1^M - \Pi_1) + (\Pi_2^M - \Pi_2) > 0$. A sufficient condition for $\Pi^M - \Pi > 0$ is the condition for $\Pi^M - \Pi > 0$ given in Table 2.

where $\partial g_2^{UM}(n_1)/\partial n_1 > 0$. Similarly, using π_{1j}^M in Table A.1, we obtain the following:

$$\pi_1^M(n_1, n_2) = \frac{1}{b_1}\left\{\frac{n_2(a_1 - v_1 - v_2) + \Delta v_2}{(n_1+1)(n_2+1)}\right\}^2 \geq F_1$$

$$\Leftrightarrow \quad n_2 \geq \frac{a_1 - v_1 - v_2 - \Delta v_2}{a_1 - v_1 - v_2 - (n_1+1)\sqrt{b_1 F_1}} - 1 \equiv g_1^{UM}(n_1).$$

Observe that $\partial g_1^{UM}(n_1)/\partial n_1 > 0$, so that $n_2 \geq g_1^{UM}(n_1)$ can be rewritten as $n_1 \leq (g_1^{UM})^{-1}(n_2) = (1/\sqrt{b_1 F_1})\{(n_2/(n_2+1)) \cdot (a_1 - v_1 - v_2) + \Delta v_2/(n_2+1)\} - 1$. The viable structure W^{UM} can be defined using g_1^{UM} and g_2^{UM} as follows: $W^{UM} = \{(n_1, n_2) \in \mathcal{N}^2 \mid n_1 \leq (g_1^{UM})^{-1}(n_2)$ and $n_2 \leq g_2^{UM}(n_1)\}$. A sufficient condition for nonempty W^{UM} to be a lattice is that $\partial g_2^{UM}(n_1)/\partial n_1 > 0$ and $\partial g_1^{UM}(n_1)/\partial n_1 > 0$ (Corbett and Karmarkar 2001), which is shown above. Moreover, W^{UM} is bounded because $n_2 \leq (a_1 - v_1 - v_2 - \Delta v_2)/\sqrt{b_1 F_2} - 1$ and $n_1 \leq (1/\sqrt{b_1 F_1})\{(a_1 - v_1 - v_2) + \Delta v_2\} - 1$. This guarantees the existence of the maximal element (n_1^*, n_2^*) in W^{UM}.

Finally, we define four threshold numbers and derive the conditions for entry. Define ξ_1^{UM}, ξ_2^{UM}, ξ_3^{UM}, and ξ_4^{UM} as follows:

$$\xi_1^{UM} = a_1 - v_1 - v_2 - (\bar{n}_2+1)\sqrt{\frac{(\bar{n}_1+1)b_1 F_2}{\bar{n}_1}},$$

$$\xi_2^{UM} = a_1 - v_1 - v_2 - (\bar{n}_2+1)\sqrt{\frac{(\bar{n}_1+2)b_1 F_2}{\bar{n}_1+1}},$$

$$\xi_3^{UM} = -\bar{n}_2(a_1 - v_1 - v_2) + (\bar{n}_1+2)(\bar{n}_2+1)\sqrt{b_1 F_1},$$

$$\xi_4^{UM} = -(\bar{n}_2-1)(a_1 - v_1 - v_2) + (\bar{n}_1+2)\bar{n}_2\sqrt{b_1 F_1}.$$

Then, $\Delta v_2 \leq \xi_1^{UM} \Leftrightarrow g_2^{UM}(\bar{n}_1) \geq \bar{n}_2$, $\Delta v_2 \leq \xi_2^{UM} \Leftrightarrow g_2^{UM}(\bar{n}_1+1) \geq \bar{n}_2$, $\Delta v_2 \geq \xi_3^{UM} \Leftrightarrow (g_1^{UM})^{-1}(\bar{n}_2) \geq \bar{n}_1 + 1$, and $\Delta v_2 \geq \xi_4^{UM} \Leftrightarrow (g_1^{UM})^{-1}(\bar{n}_2 - 1) \geq \bar{n}_1 + 1$. We can easily verify $0 < \xi_1^{UM} < \xi_2^{UM}$ and $0 < \xi_3^{UM} < \xi_4^{UM}$. Using these threshold numbers, we next prove the conditions under which a new firm will enter tier 2 in each of the following three intervals: (1) $\Delta v_2 \leq \xi_1^{UM}$, (2) $\xi_1^{UM} < \Delta v_2 \leq \xi_2^{UM}$, and (3) $\Delta v_2 > \xi_2^{UM}$.

Interval 1. By the definition of ξ_1^{UM}, $\bar{n}_2 \leq g_2^{UM}(\bar{n}_1)$. Because $(\bar{n}_1, \bar{n}_2) \in W$, $\bar{n}_1 \leq g_1^{-1}(\bar{n}_2)$. Since $(g_1^{UM})^{-1}(n_2) \geq g_1^{-1}(n_2)\ \forall n_2$ for $\Delta v_2 \geq 0$, $\bar{n}_1 \leq (g_1^{UM})^{-1}(\bar{n}_2)$. Thus, $(\bar{n}_1, \bar{n}_2) \in W^{UM}$. Since (n_1^*, n_2^*) is the maximal element in W^{UM}, $n_2^* \geq \bar{n}_2$ (i.e., a new firm will enter tier 2).

Interval 2. By the definition of ξ_1^{UM} and ξ_2^{UM}, $g_2^{UM}(\bar{n}_1) < \bar{n}_2 \leq g_2^{UM}(\bar{n}_1+1)$. If $\Delta v_2 \geq \xi_3^{UM}$ so that $\bar{n}_1 + 1 \leq (g_1^{UM})^{-1}(\bar{n}_2)$, then $(\bar{n}_1+1, \bar{n}_2) \in W^{UM}$, and therefore $n_1^* \geq \bar{n}_1 + 1$ and $n_2^* \geq \bar{n}_2$ (i.e., new firms will enter both tiers).

Interval 3. By the definition of ξ_2^{UM}, $\bar{n}_2 > g_2^{UM}(\bar{n}_1+1)$, so $(\bar{n}_1+1, \bar{n}_2) \notin W^{UM}$. Also, since $\partial g_2^{UM}(n_1)/\partial n_1 > 0$, $\bar{n}_2 > g_2^{UM}(\bar{n}_1)$, so $(\bar{n}_1, \bar{n}_2) \notin W^{UM}$.

By following the same procedure as above, we can also prove the conditions under which a new firm will enter tier 1. □

Remark. We can easily modify this proof for the case when multiple new firms can enter each tier (although such a case has not been observed in our numerical study). For illustration, consider the case when two new firms might enter tier 1. Define ξ_5^{UM} ($> \xi_2^{UM}$) and ξ_6^{UM} ($> \xi_4^{UM}$) such that $\Delta v_2 \leq \xi_5^{UM} \Leftrightarrow g_2^{UM}(\bar{n}_1+2) \geq \bar{n}_2$ and $\Delta v_2 \geq \xi_6^{UM} \Leftrightarrow (g_1^{UM})^{-1}(\bar{n}_2) \geq \bar{n}_1 + 2$, respectively. Then Interval 3 needs to be divided into two intervals: (3-1) $\xi_2^{UM} < \Delta v_2 \leq \xi_5^{UM}$, and (3-2) $\Delta v_2 > \xi_5^{UM}$. In Interval 3-1, if $\Delta v_2 \geq \xi_6^{UM}$, then $(\bar{n}_1+2, \bar{n}_2) \in W^{UM}$ and $n_2^* \geq \bar{n}_2$. In Interval 3-2, $(\bar{n}_1+2, \bar{n}_2) \notin W^{UM}$.

Proof of Theorem 4. The proof proceeds similarly to that of Theorem 3. We first prove the existence of (n_1^*, n_2^*) in postmerger equilibrium by showing that nonempty W^{DM} is a bounded lattice under the specified condition. From Table A.2, $\pi_{2j}^M = \{(n_1-1)(a_1 - v_1 - v_2) + \Delta v_1\}^2/(b_1 n_1(n_1-1) \cdot (n_2+1)^2)$ when there are $(n_1 - 1, n_2)$ firms. Thus,

$$\pi_2^M(n_1, n_2) = \frac{\{n_1(a_1 - v_1 - v_2) + \Delta v_1\}^2}{b_1 n_1(n_1+1)(n_2+1)^2} \geq F_2$$

$$\Leftrightarrow \quad n_2 \leq \frac{n_1(a_1 - v_1 - v_2) + \Delta v_1}{\sqrt{n_1(n_1+1)b_1 F_2}} - 1 \equiv g_2^{DM}(n_1).$$

Then, $\partial g_2^{DM}/\partial n_1 = (n_1(2n_1+1)(a_1 - v_1 - v_2) - \Delta v_1)/(2\{n_1(n_1+1)\}^{1.5}(b_1 F_2)^{0.5}) > 0$ if and only if $\Delta v_1 < n_1(2n_1+1) \cdot (a_1 - v_1 - v_2)$. Similarly, we obtain the postentry profit of a nonparticipant firm at tier 1 from Table A.2 as follows:

$$\pi_1^M(n_1, n_2) = \frac{[n_2(a_1-v_1-v_2)+((\Delta v_1)/n_1)\{-(n_1+1)(n_2+1)+n_2\}]^2}{b_1(n_1+1)^2(n_2+1)^2} \geq F_1$$

$$\Leftrightarrow \quad n_1 \leq \frac{n_2}{n_2+1}\frac{a_1-v_1-v_2}{\sqrt{b_1 F_1}} - 1 - \frac{\Delta v_1}{\sqrt{b_1 F_1}}\left\{1 + \frac{1}{n_1(n_2+1)}\right\}$$

$$\equiv f_1^{DM}(n_1, n_2).$$

Define $h(n_1, n_2) \equiv n_1 - f_1^{DM}(n_1, n_2) = n_1 + ((\Delta v_1)/\sqrt{b_1 F_1})\{1 + 1/(n_1(n_2+1))\} - g_1^{-1}(n_2) = 0$. By the implicit function theorem, there exists $g_1^{DM}(n_2)$ such that $h(g_1^{DM}(n_2), n_2) = 0$ and $\partial g_1^{DM}(n_2)/\partial n_2 = -(\partial h/\partial n_2)/(\partial h/\partial n_1)$. We can compute $\partial h/\partial n_2 = -\Delta v_1/(n_1(n_2+1)^2\sqrt{b_1 F_1}) - \partial g_1^{-1}(n_2)/\partial n_2 < 0$, and $\partial h/\partial n_1 = 1 - \Delta v_1/(n_1^2(n_2+1)\sqrt{b_1 F_1})$. Thus, if $\Delta v_1 < n_1^2(n_2+1)\sqrt{b_1 F_1}$, then $\partial g_1^{DM}(n_2)/\partial n_2 > 0$. In this case, $\partial g_1^{DM}(n_2)/\partial \Delta v_1 = -(\partial h/\partial \Delta v_1)/(\partial h/\partial n_1) < 0$ for any fixed n_2, hence $g_1^{DM}(n_2) \leq g_1^{-1}(n_2)$ for any $\Delta v_1 \geq 0$ (because $g_1^{DM}(n_2) = g_1^{-1}(n_2)$ when $\Delta v_1 = 0$). Define the viable structure W^{DM} using g_1^{DM} and g_2^{DM} as follows: $W^{DM} = \{(n_1, n_2) \in \mathcal{N}^2 \mid n_1 \leq g_1^{DM}(n_2)$ and $n_2 \leq g_2^{DM}(n_1)\}$. If $\Delta v_1 < \min\{\Delta v_1 < n_1(2n_1+1)(a_1 - v_1 - v_2), n_1^2(n_2+1)\sqrt{b_1 F_1}\}$, then $\partial g_2^{DM}/\partial n_1 > 0$ and $\partial g_1^{DM}/\partial n_2 > 0$, so W^{DM} is a lattice. In this case, W^{DM} is bounded because $n_2 \leq ((a_1 - v_1 - v_2) + \Delta v_1)/\sqrt{b_1 F_2} - 1$ and $n_1 \leq (a_1 - v_1 - v_2)/\sqrt{b_1 F_1} - 1$. This guarantees the existence of the maximal element (n_1^*, n_2^*) in W^{DM}.

Next, we define four threshold numbers ξ_1^{DM}, ξ_2^{DM}, ξ_3^{DM}, and ξ_4^{DM} as follows:

$$\xi_1^{DM} = \frac{\bar{n}_1(\bar{n}_2+1)\sqrt{b_1 F_1}}{\bar{n}_1(\bar{n}_2+1)+1}\left\{\frac{\bar{n}_2(a_1 - v_1 - v_2)}{(\bar{n}_2+1)\sqrt{b_1 F_1}} - (\bar{n}_1+1)\right\},$$

$$\xi_2^{DM} = \frac{\bar{n}_1(\bar{n}_2+2)\sqrt{b_1 F_1}}{\bar{n}_1(\bar{n}_2+2)+1}\left\{\frac{(\bar{n}_2+1)(a_1 - v_1 - v_2)}{(\bar{n}_2+2)\sqrt{b_1 F_1}} - (\bar{n}_1+1)\right\},$$

$$\xi_3^{DM} = -\bar{n}_1(a_1 - v_1 - v_2) + (\bar{n}_2+2)\sqrt{\bar{n}_1(\bar{n}_1+1)b_1 F_2},$$

$$\xi_4^{DM} = -(\bar{n}_1-1)(a_1 - v_1 - v_2) + (\bar{n}_2+2)\sqrt{\bar{n}_1(\bar{n}_1-1)b_1 F_2}.$$

Then, $\Delta v_1 \leq \xi_1^{DM} \Leftrightarrow g_1^{DM}(\bar{n}_2) \geq \bar{n}_1$, $\Delta v_1 \leq \xi_2^{DM} \Leftrightarrow g_1^{DM}(\bar{n}_2+1) \geq \bar{n}_1$, $\Delta v_1 \geq \xi_3^{DM} \Leftrightarrow g_2^{DM}(\bar{n}_1) \geq \bar{n}_2 + 1$, and

$\Delta v_1 \geq \xi_4^{DM} \Leftrightarrow g_2^{DM}(\bar{n}_1 - 1) \geq \bar{n}_2 + 1$. We can easily verify that $0 < \xi_1^{DM} < \xi_2^{DM}$ and $0 < \xi_3^{DM} < \xi_4^{DM}$. The conditions for subsequent entry to either tier can be derived in the same manner as in the proof of Theorem 3. □

The proofs of Tables 1 and 2 and Corollaries 1–4 follow the procedure similar to the above proofs and hence are omitted. These proofs are available upon request.

Appendix B. Extended Literature Review

Similar to our paper, Ziss (1995) compares the effects of an upstream merger with those of a downstream merger. However, our model is substantially different from that of Ziss (1995) in the following dimensions. First, our model deals with a more general supply chain in which n_i firms compete at each tier i $(=1, 2, \ldots, N)$. In contrast, Ziss (1995) is limited to two competing firms at two tiers; thus, a merger at one tier creates a monopolist in that tier. Existence of only two firms at each tier is a serious drawback in a merger analysis because the antitrust agency is unlikely to approve a merger that creates a monopolist. Also, it is important to understand how a merger affects nonparticipant firms because this will impact consumers as well. Second, the nature of competition is different. In our model, firms at each tier of a supply chain compete in the Cournot fashion. In contrast, in Ziss (1985), two manufacturers sell differentiated products and offer two-part tariffs to downstream retailers. Third, we explicitly model the cost synergy similarly to Farrell and Shapiro (1990), whereas Ziss (1995) does not. Finally, we analyze entry-inducing effects of a merger, whereas Ziss (1995) does not.

Next, let us compare our results in a two-tier supply chain without entry with the results of Ziss (1985). We examine the synergy effect and the competition effect separately, and combine them to measure the aggregate effect. Because of the opposite impact of the synergy and competition effects on consumer price, both the upstream merger and the downstream merger raise consumer price only when the cost synergy is sufficiently low. In contrast, Ziss (1985) shows that an upstream merger always raises consumer price (anticompetitive), whereas a downstream merger always lowers consumer price (procompetitive). The result of an upstream merger in Ziss (1985) is analogous to the competition effect of an upstream merger in our paper, but it is different from the aggregate effect in our paper because of the synergy effect that Ziss (1985) does not take into account. Ziss (1985, p. 69) explains the effect of a downstream merger as follows: "For example if products are perfect substitutes and marginal costs are constant then a multi-outlet retailer will assign all the output to the low cost product and the other product will remain unsold. Increased accommodation by retailers expands the scope for manufacturers to shift rents via their wholesale pricing policy and thus induces the latter to be more aggressive. In the linear case the reductions in the wholesale prices more than offset the collusive output setting effects resulting from downstream merger thereby resulting in more output." The economies of scope considered in Ziss (1985) (which allows a merged retailer to choose only a low-cost product between two products) are analogous to the synergy effect of a downstream merger in our paper. However, unlike Ziss (1985), the synergy effect does not always dominate the competition effect in our model. Our result is more consistent with our observation of the practice because it is unlikely that a downstream merger is always procompetitive.

References

Adida E, DeMiguel V (2011) Supply chain competition with multiple manufacturers and retailers. *Oper. Res.* 59(1):156–172.

Adida E, Bakshi N, DeMiguel V (2012) Supply chain intermediation when retailers lead. Working paper, University of Illinois at Chicago, Chicago.

Alptekinoğlu A, Tang CS (2005) A model for analyzing multichannel distribution systems. *Eur. J. Oper. Res.* 163(3):802–824.

Bascle I, Duthoit C, Goodall S, Matthiessen J, Strüven P, Tevelson R (2008) Thinking laterally in PMI: Optimizing functional synergies. *BCG Focus*. Accessed August 17, 2013, http://www.bcg.co.jp/documents/file15156.pdf.

Bloch F (2005) Group and network formation in industrial organization: A survey. Demange G, Wooders M, eds. *Group Formation in Economics: Networks, Clubs and Coalitions* (Cambridge University Press, Cambridge, UK), 335–353.

Carr SM, Karmarkar US (2005) Competition in multiechelon assembly supply chains. *Management Sci.* 51(1):45–59.

Choi SC (1991) Price competition in a channel structure with a common retailer. *Marketing Sci.* 10(4):271–296.

Corbett CJ, Karmarkar US (2001) Competition and structure in serial supply chains with deterministic demand. *Management Sci.* 47(7):966–978.

DeLong G (2003) Does long-term performance of mergers match market expectations? Evidence from the US banking industry. *Financial Management* 32(2):5–25.

Dixit A (1986) Comparative statics for oligopoly. *Internat. Econom. Rev.* 27(1):107–122.

Fang X, Cho S (2012) Stability and endogenous formation of inventory transshipment networks. Working paper, Carnegie Mellon University, Pittsburgh.

Farrell J, Shapiro C (1990) Horizontal mergers: An equilibrium analysis. *Amer. Econom. Rev.* 80(1):107–126.

Gowrisankaran G (1999) A dynamic model of endogenous horizontal mergers. *RAND J. Econom.* 30(1):56–83.

Goyal M, Netessine S (2007) Strategic technology choice and capacity investment under demand uncertainty. *Management Sci.* 53(2):192–207.

Gupta D, Gerchak Y (2002) Quantifying operational synergies in a merger/acquisition. *Management Sci.* 48(4):517–533.

Houston JF, James CM, Ryngaert MD (2001) Where do merger gains come from? *J. Financial Econom.* 60:285–331.

Lin Y-T, Parlaktürk A, Swaminathan JM (2013) Vertical integration under competition: Forward, backward, or no integration? *Production Oper. Management*, ePub ahead of print May 16, http://onlinelibrary.wiley.com/doi/10.1111/poms.12030/abstract.

Majumder P, Srinivasan A (2008) Leadership and competition in network supply chains. *Management Sci.* 54(6):1189–1204.

McGuire TW, Staelin R (1983) An industry equilibrium analysis of downstream vertical integration. *Marketing Sci.* 2(2):161–191.

Nagarajan M, Sošić G (2007) Stable farsighted coalitions in competitive markets. *Management Sci.* 53(1):29–45.

Nagarajan M, Sošić G, Zhang H (2010) Stable group purchasing organization. Working paper, University of British Columbia, Vancouver.

Nagurney A (2009) A system-optimization perspective for supply chain network integration: The horizontal merger case. *Transportation Res. E* 45(1):1–15.

Netessine S (2009) Supply webs: Managing, organizing, and capitalizing on global networks of suppliers. Kleindorfer P, Wind YJ, eds. *The Network Challenge: Strategy, Profit, and Risk in* *an Interlinked World*, Chap. 13 (Wharton School Publishing, Philadelphia).

Nocke V, Whinston MD (2010) Dynamic merger review. *J. Political Econom.* 118(6):1201–1251.

Perakis G, Roels G (2007) The price of anarchy in supply chains: Quantifying the efficiency of price-only contracts. *Management Sci.* 53(8):1249–1268.

Perry MK, Porter RH (1985) Oligopoly and the incentive for horizontal merger. *Amer. Econom. Rev.* 75(1):219–227.

Stigler JG (1950) Monopoly and oligopoly by merger. *Amer. Econom. Rev.* 40(2):23–34.

Tyagi RK (1999) On the effects of downstream entry. *Management Sci.* 45(1):59–73.

U.S. Department of Justice and the Federal Trade Commission (2010) Horizontal merger guidelines. Accessed August 17, 2013, http://www.justice.gov/atr/public/guidelines/hmg-2010.html.

Werden G, Froeb LM (1998) The entry-inducing effects of horizontal mergers: An exploratory analysis. *J. Indust. Econom.* 46(4):525–543.

Whinston M (2007) Antitrust policy toward horizontal mergers. Armstrong M, Porter RH, eds. *Handbook of Industrial Organization*, Vol. 3, Chap. 36 (North-Holland, Amsterdam).

Williamson OE (1968) Economies as an antitrust defense: The welfare tradeoffs. *Amer. Econom. Rev.* 58(1):18–36.

WilmerHale (2012) 2012 M&A report. Accessed August 17, 2013, http://www.wilmerhale.com/uploadedFiles/WilmerHale_Shared_Content/Files/PDFs/2012_MA_Report.pdf.

Yin S (2010) Alliance formation among perfectly complementary suppliers in a price-sensitive assembly system. *Manufacturing Service Oper. Management* 12(3):527–544.

Zhu J, Boyaci T, Ray S (2011) Horizontal mergers and supply chain performance. Working paper, McGill University, Montreal.

Ziss S (1995) Vertical separation and horizontal mergers. *J. Indust. Econom.* 43(1):63–75.