Manufacturing & Service Operations Management

Publication details, including instructions for authors and subscription information:
http://pubsonline.informs.org

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Saibal Ray, Tamer Boyaci, Necati Aras,

MANUFACTURING & SERVICE OPERATIONS MANAGEMENT
Vol. 7, No. 3, Summer 2005, pp. 208–228
ISSN 1523-4614 | EISSN 1526-5498 | 05 | 0703 | 0208
DOI 10.1287/msom.1050.0080

Optimal Prices and Trade-in Rebates for Durable, Remanufacturable Products

Saibal Ray, Tamer Boyaci
Faculty of Management, McGill University, 1001 Sherbrooke Street West, Montreal, PQ, H3A 2M1, Canada
{saibal.ray@mcgill.ca, tamer.boyaci@mcgill.ca}

Necati Aras
Industrial Engineering Department, Boğaziçi University, Bebek, Istanbul, 34342, Turkey, arasn@boun.edu.tr

Most durable products have two distinct types of customers: first-time buyers and customers who already own the product, but are willing to replace it with a new one or purchase a second one. Firms usually adopt a price-discrimination policy by offering a trade-in rebate only to the replacement customers to hasten their purchase decisions. Any return flow of products induced by trade-in rebates has the potential to generate revenues through remanufacturing operations. In this paper, we study the optimal pricing/trade-in strategies for such durable, remanufacturable products. We focus on the scenario where the replacement customers are only interested in trade-ins. In this setting, we study three pricing schemes: (i) uniform price for all customers, (ii) age-independent price differentiation between new and replacement customers (i.e., constant rebate for replacement customers), and (iii) age-dependent price differentiation between new and replacement customers (i.e., age-dependent rebates for replacement customers). We characterize the roles that the durability of the product, the extent of return revenues, the age profile of existing products in the market, and the relative size of the two customer segments play in shaping the optimal prices and the amount of trade-in rebates offered. Throughout the paper we highlight the operational decisions that might influence the above factors, and we support our findings with real-life practices. In an extensive numerical study, we compare the profit potential of different pricing schemes and quantify the reward (penalty) associated with taking into account (ignoring) customer segmentation, the price-discrimination option, return revenues, and the age profile of existing products. On the basis of these results, we are able to identify the most favorable pricing strategy for the firm when faced with a particular market condition and discuss implications on the life-cycle pricing of durable, remanufacturable products.

Key words: trade-in rebates; pricing; remanufacturing; durable products; product-age profile
History: Received: January 10, 2003; accepted: May 7, 2005. This paper was with the authors 15 months for 4 revisions.

1. Motivation

Most of the pricing models in the literature are based on the assumption that the potential buyers are purchasing the product for the first time. This is indeed true for totally new products and consumables. However, for durable products a significant portion of the purchases could be replacement (or second purchase) in nature. In fact, for products with highly saturated markets, like refrigerators and electric heaters, the percentage of replacement purchases in the United States amounts to 70%–80% of annual sales (Fernandez 2001). The purchase decisions of both first-time and replacement buyers are obviously affected by the price. Replacement customers, however, are also influenced by what they perceive to be the "residual values"[1] of their existing products. This difference makes it important from a firm's perspective to adopt a price-discrimination policy between these two distinct sets of customers. One possible method for such price discrimination is to offer a special discount only to the repeat purchasers provided that they return their existing products. This special discount is referred to as a trade-in rebate.

There are a number of economic motivations for firms to offer trade-in rebates—to create switching

[1] In marketing literature this is often referred to as "mental book value" (Heath and Fennema 1996, Okada 2001).

costs for customers who are thinking of changing firms (Klemperer 1987), to disable the secondhand market of an old technology when introducing a new one (Levinthal and Purohit 1989), or just to increase the purchasing frequency (Ackere and Reyniers 1995). Examples of trade-in rebates can be found in products ranging from mature ones, such as cars, power tools, photocopiers, and refrigerators, to innovative ones like cell phones and computers. In this paper, we focus on durable products having a certain installed user base (i.e., a pool of replacement customers), and for which design changes in the products are not revolutionary in nature (i.e., low risk of obsolescence). Under these circumstances, the primary reason for offering trade-in rebates is to increase the frequency of purchase (Ackere and Reyniers 1995). By reducing the effective purchase price of a new product for replacement customers, firms reduce their unit profit from a sale, but compensate for this loss by inducing some customers to replace their products earlier. The replacement decisions are driven not only by the trade-in price, but also by the durability as well as the age of the existing product in use. Intuitively, the age of the product determines the residual value of the product, whereas durability has a bearing on how this value depreciates over time.

The return flow of products that are induced by trade-in rebates can serve as a significant source of revenue (or equivalently cost savings) for the firm. If customers can be persuaded to return their products before the end of their useful lives, firms might be able to generate some revenue either by totally remanufacturing these products and selling them as new, or by reusing some components, or even by recycling the material. Throughout this paper, we use the term remanufacturing to represent this broad family of revenue-generating product recovery options. It is perhaps because of these facts that manufacturers increasingly offer trade-in rebates and use the returned products for remanufacturing operations. One of the most well-known examples is Xerox, whose remanufacturing facility is based partly on returns from trade-ins and has resulted in cost savings of several hundred million dollars each year (WasteWi$e Update 1997, Fishbein et al. 2000). There are similar examples from industries like computers (e.g., IBM's Global Asset Recovery Services; see www.pprc.org), furniture (e.g., Herman Miller; see WasteWi$e Update 1997), carpets (e.g., Interface Inc.; see Fishbein et al. 2000), power tools (e.g., Bosch; see Klausner and Hendrickson 2000), and refrigerators (e.g., Electrolux; see Fishbein et al. 2000).

In this paper, we study the optimal pricing and trade-in rebate decisions for a profit-maximizing firm selling a durable, remanufacturable product. We focus on decisions of the firm at the particular point in time when it is announcing the trade-in offer. The potential market consists of a known population of two segments: first-time buyers and replacement buyers. We assume that the replacement customers are only considering trading in their existing products. At the end of the paper, we briefly discuss an extension where replacement customers are allowed the additional option of making new purchases without turning in their in-use products. The firm has some knowledge about the age profile of the products currently in use, and the revenue associated with a returned product, which is nonincreasing in the age of the return. We first develop an individual consumer choice framework that models each customer's purchasing and product return decisions based on the effective price (net of the trade-in rebate, if any) and the "remaining lifespan" (we will explain this in detail later on) of the existing product, if any. Aggregating these decisions over the two market segments, we obtain the overall profit function of the firm. Although static in nature, our integrated operations-marketing model enables us to address both analytically and numerically a number of managerial issues related to optimal pricing/trade-in strategy:

- What are the optimal prices/trade-in rebates under the following possible pricing schemes offered by the firm:

—A uniform price for both customer segments (i.e., no trade-in rebate);

—Age-independent price differentiation: One price for the new customers and a fixed, age-independent trade-in rebate (price) for the replacement segment;

—Age-dependent price differentiation: One price for the new customer segment and a continuous set of age-dependent trade-in rebates (prices) for the replacement customers.

• How do operational factors like durability, remanufacturability, quality of the product, and efficient handling of returns influence the optimal prices and the optimal trade-in rebates?

• What are the effects of the age profile of the currently owned products and the relative market size of first-time and replacement buyers on the pricing/trade-in rebate strategy?

• How do the above pricing schemes compare to each other in terms of profits? Under which operating and/or market conditions does it make the most sense for the firm to use a particular pricing scheme?

We are able to analytically determine the unique optimal prices and resulting trade-in rebates for all three pricing schemes. Subsequently, we focus on how the behavior of the optimal decisions are affected by the four salient features of our model: (i) the durability of the product, (ii) the extent of revenues generated from the returns, (iii) the age profile of the existing products, and (iv) the relative size of the two customer segments. We show that the optimal trade-in rebate is increasing in the return revenue, and decreases as the age profile gets older. Higher product durability increases the amount of age-dependent trade-in rebates. In the more common case of fixed rebates, however, the impact of product durability is more involved. For this scenario, we show that the rebate is nonmonotone with respect to durability—it increases for low-durability products, and decreases if the durability is high.

In a numerical study, we then provide a detailed comparison of the profits under each pricing scheme. This enables us to quantify the scale of profit improvement that can accrue to the firm from adopting these strategies, as well as how these gains change with respect to the four factors. From this analysis, we generate insights on the most suitable conditions for each scheme and how the firm should modify its pricing strategy as its product evolves through its life cycle. For example, comparing the uniform strategy with the age-independent policy, we demonstrate that pursuing customer segmentation and age-independent price discrimination between the two customer segments can benefit the firm significantly. Interestingly, this might be true even if the firm ignores return revenues or does not have complete distributional information about the age profile of its products in use. We show that such a pricing scheme is most beneficial for products with medium levels of durability, relatively new age profiles, high return revenues, and markets with comparable sizes of new and replacement customers. Age-dependent price discrimination, on the other hand, adds comparatively much less to the profit improvement even if the age of the returns can be determined readily. In reality, however, assessing the precise age of returns most often entails additional handling costs. If this handling cost is above a threshold value, then aggressive price differentiation might indeed be counterproductive for the firm. Analyzing this threshold, we show that an age-dependent pricing strategy is perhaps suitable only for highly durable products with rather old/uniform age profiles, substantial age-dependent return revenues, and markets with a high proportion of replacement customers. In general, return revenue and age-distribution information becomes more relevant as a firm becomes operationally efficient (higher product durability and/or return revenues), and more aggressive price differentiation becomes necessary as the product progresses through its life cycle.

Our aforementioned analysis and insights are based on the assumption that replacement customers are only considering trade-ins. Allowing the replacement customers the option to also buy a new product without turning in her/his old one induces a stronger coupling between the two customer segments. The resulting model is complicated and does not lend itself to analytical solutions or clear comparisons. In an attempt to understand how such a relaxation might affect our results, we conduct a numerical study of our central, age-independent pricing scheme in this extended setting. We show that although the price for new customers is naturally affected by the stronger dependence between the two customer segments, our major results regarding optimal trade-in rebates and their sensitivity to key parameters still remain valid.

2. Literature Review

There are mainly two streams of research that are related to our work: (i) models from economics and marketing literature that deal with trade-in prices, but do not take into account the effects of the age profile or the revenues from returns, and (ii) models from the operations literature that take into account the revenues associated with returns, but have no pricing considerations. Our model bridges the gap between these two distinct streams. In what follows, we provide an overview of both research streams. We note, however, that our aim here is to position our work relative to the existing literature, rather than providing an exhaustive review.

There is an extensive literature in the marketing/economics area dealing with analytical modeling of different types of sales promotion efforts like coupons, trade promotions, etc. (Lilien et al. 1992, Blattberg and Neslin 1993). The volume of studies related to trade-ins is surprisingly low in spite of its obvious popularity. There are only a handful of papers in this area—Levinthal and Purohit (1989), Ackere and Reyniers (1995), Fudenberg and Tirole (1998), and Ferguson and Koenigsberg (2003)—addressing issues related to monopoly pricing and/or production policies of successive generations of a product. The common element of these papers is a two-period framework where the pricing decision in the first period divides the market into potential replacement customers and potential first-time buyers for the new generation. In the second period, the firm might decide to offer trade-in rebates for upgrades to repeat purchasers or to buyback some of the old models or to sell the old model at a discount. Obviously, all the products bought at the beginning of the first period are *equally* old when the second period starts. The primary objective of using trade-in rebates in these papers is to reduce the cannibalization of the new generation for short (technology) life-cycle products. Consequently, the authors take into account how the nature of information available to the firm and to the customers affects the pricing and product introduction decisions. In contrast, we assume a durable product for which the technology is relatively stable, and hence the firm's primary objective for the trade-in offer is to increase purchase frequency through early returns. Therefore, rather than explicitly modeling customer expectations in a dynamic setting, we capture the decisions of the firm only at the time of the trade-in offer. This allows us to address the impact of the age profile of existing products and the revenue potential of returns, which do not play any role in the earlier studies.

In the operations stream there are some models in which the age of the existing products and the revenues from returns are important elements. These are the replacement models in the maintenance literature that deal with determining the optimal age for replacement of a currently owned product (Wang 2002, Nahmias 2001). However, the focus in those models is on the perspective of an individual customer and not the firm. In a similar vein, our model is distinct from marketing models addressing how individual customers make their replacement decisions based on the "mental book value" (similar to perceived residual value) of their current product and their utility for the new product (Purohit 1995, Heath and Fennema 1996, Okada 2001). Our model of an individual customer's replacement decision is inspired by this literature, but this only serves as an input to our pricing model.

Our model is also related to the growing literature on remanufacturing and reverse logistics. The majority of this literature concentrates on inventory management and network design models (refer to Fleischmann 2001 and references therein). Nevertheless, some of these papers implicitly assume that the revenue from a return is a decreasing function of its age (Krikke et al. 1998, Ferrer and Whybark 2001, Heese et al. 2001). Although the usefulness of trade-in rebates in acquisition of returned products has been acknowledged by some works (Klausner and Hendrickson 2000, Heese et al. 2001, Tibben-Lembke 2002), an explicit modeling attempt like ours has not been made (Guide and van Wassenhove 2001). Perhaps the closest work in this stream is Guide et al. (2003), which considers an intermediary (remanufacturing) firm and studies optimal acquisition prices for used products and the selling price for remanufactured items. It is assumed that returned products can be categorized into quality grades (which determines remanufacturing costs), and the firm offers quality-dependent acquisition prices to maximize remanufacturing profits. Because there is no offering of new products, central issues of our paper, like customer trade-ins, rebates, age profiles, product durability, etc., are not addressed.

The organization of the remainder of this paper is as follows. In the next section (§3) we develop our consumer choice framework and the basic profit-maximization model. Section 4 presents the derivation and the sensitivity analysis of the optimal prices for the three pricing schemes considered. Section 5 compares the profit performance of the three schemes and illustrates our key results through a numerical study. Section 6 shows how the insights regarding the optimal decisions of the age-independent pricing scheme are affected when replacement customers are allowed the second purchase option. In §7 we present our concluding remarks and future research directions. In the interest of space, the detailed proofs of all propositions are provided in a separate online supplement (Ray et al. 2005).

3. Model Development

We model a profit-maximizing firm selling a durable product to a first-time-buyers (new customers) segment of size[2] M_N and a replacement segment of size M_R. We assume that the replacement customers have established their preferences regarding the option to either (i) keep their existing products and seek a second purchase at the price for a new product, or (ii) trade-in their existing products. In this respect, our basic framework models the M_R segment to contain only pure replacement customers who are not looking for a new purchase.[3] We also assume that customers have no knowledge about possible future trade-in offers; hence, anticipation does not play any role in their decisions. Consequently, the two customer segments act totally independently. We believe that this is a reasonable assumption for many durable products (e.g., refrigerators, power tools, photocopiers) for which customers are unlikely to discard their existing products and buy a brand-new one when there is a trade-in program in place.

Potential new customers usually make their purchase decisions based only on the price of the new product p_N. A customer from the replacement segment with an existing product of age t, on the other hand, must take into account both the price $p_R(t)$ ($\leq p_N$) being charged and the residual value of her/his product in use. This latter element is directly linked with the length of time during which a customer will mentally depreciate the full purchase price worth of the existing product (Heath and Fennema 1996, Okada 2001). We assume that there is a maximum length of use for the product (t_m) that will satisfy any customer. For a customer with a product of age t, the perceived residual value is assumed to be linearly proportional to the remaining useful lifespan ($t_m - t$). A number of consumer behavior studies support the hypothesis that mental depreciation for individuals shows strong linear dependence with respect to time of usage (see Heath and Fennema 1996, and references therein).

For a durable good with an installed user base, at any given point in time there will be a range of ages associated with the existing products. Regardless of whether the firm knows the identity of the product holders or not, the variability in the ages will result in an aggregate "product-age profile" that can best be represented as a random variable. Consequently, we model the age profile in the market as a random variable T having support on the finite interval $[0, t_m]$. The density function (pdf) and the distribution function (cdf) of T are denoted by $f(t)$ and $F(t)$, respectively. Let $\bar{F} = (1 - F)$. For analytical tractability, we assume that F is twice-differentiable in its domain, although our insights are also valid for discrete-age profiles (optimal prices then have to be found via exhaustive search).

We model the customer's purchase decisions using consumer surplus. We assume that each customer has a reservation price for the product. The reservation price P_0 differs among individuals (heterogeneous customers) and is represented by a uniform distribution in the interval $[0, a]$, where a represents the maximum price any customer is going to pay for the product (i.e., $P_0 \sim U[0, a]$). The uniform assumption provides an analytically tractable framework. At the same time, it represents a large degree of variability within the customer market, and has become almost a standard assumption in the related literature (see Ackere and Reyniers 1995, Desai and Purohit 1998, and the references therein). For a particular new customer paying p_N for the product, the consumer surplus can be expressed as

$$P_0 - p_N. \tag{1}$$

[2] We provide a comprehensive list of notations in the appendix.

[3] In this setting, any customer from the M_R segment who is looking for a second purchase without returning her/his existing product can be considered as a member of M_N.

The probability that a randomly chosen new customer from the M_N segment is willing to purchase the product, $\Pr_N$, would then be given by

$$\Pr_N = \Pr(P_0 - p_N > 0) = \frac{(a-p_N)^+}{a}. \tag{2}$$

On the other hand, the consumer surplus of a replacement customer having a product of age t depends on the price of the new product as well as the remaining lifespan of the product in use. Hence, if such a customer is charged a price $p_R(t)$, then her/his consumer surplus would be given by

$$P_0 - k(t_m - t) - p_R(t). \tag{3}$$

The term $k(t_m - t)$ is the "perceived residual value" and k is the rate at which this value changes with the remaining useful lifespan (i.e., $t_m - t$). For the replacement segment, a higher k means more residual value of the existing product (i.e., a lower consumer surplus from replacement), and hence a lower chance of replacement. In this sense, k is a measure of the *durability* of the product. Through this parameter we can analyze the impact of operational decisions, which might have a significant bearing on the durability of a product, on the optimal prices (refer to §5 for examples of such operational decisions).

Let $\Pr(t)_R$ represent the probability of a randomly chosen replacement customer from the M_R segment replacing her/his product of age t. Then,

$$\begin{aligned}\Pr(t)_R &= \Pr\big(P_0 - k(t_m - t) - p_R(t) > 0\big)\\ &= \frac{(a' + kt - p_R(t))^+}{a},\end{aligned} \tag{4}$$

where $a' = a - kt_m > 0$ is the difference in value to a customer of a new product versus a used one of age $t = 0$. It is obvious that a customer in the M_R segment will require a lower price compared to a new customer to induce replacement. Furthermore, the newer the product she/he owns, the less will be the price she/he is willing to pay. At the other extreme, a replacement customer who owns a product that has reached the end of its useful lifespan (i.e., $t = t_m$) will behave exactly like a new customer.

Observe that the aggregate customer demand that would result from the purchase probabilities (2) and (4) bears close resemblance to linear aggregate demand models (Lilien et al. 1992, Appendix C; Schmidt and Porteus 2000). As in all such models, there is a certain range of prices for which the demand will remain positive. As long as $0 \le p_N < a$, there will always be a positive fraction ($\Pr_N$) of the M_N segment who would be willing to buy. For the M_R segment, the proportion of customers willing to replace depends on the age profile. Specifically, none of the replacement customers whose current products are newer than a threshold age $t_{cr}(p_R(t)) = (p_R(t) - a')^+/k$ would be interested in replacing, while a positive fraction of the customers ($\Pr(t)_R$) with products older than $t_{cr}(p_R(t))$ would be ready to do so. This threshold age turns out to be important in our subsequent analysis.

To complete our model, we need to incorporate the operational benefits that accrue to the firm from product returns. It is generally accepted that remanufacturing is not only environmentally efficient, but also financially valuable. The revenue associated with a product return is precisely this value, which will decrease with the age of the return. This is because the deterioration of a product increases with its age, decreasing the chance of remanufacturing (Klausner and Hendrickson 2000). We model the net return revenue $R(t)$ accordingly, as a nonincreasing function of age t. For a brand-new product, the return revenue is the production cost c, net of the handling cost c_h associated with collection, inspection, and transportation of returns (i.e., $R(0) = c - c_h$). We also assume that $R(t) \ge 0\ \forall\, t \in [0, t_m]$, to provide enough incentive for collecting returns.[4] For a clear derivation and presentation of our analytical results, we suppose $R(t)$ to be *continuous*. However, it is indeed true that some remanufacturable products go through stages of incremental improvements (e.g., latest power tools use laser guides). This might give rise to *discrete* changes in the reusable values for returned items, depending on the generation from which they originate. All of our subsequent results with continuous $R(t)$ extend to such piecewise constant/decreasing revenue forms. We refer the interested reader to our supplement Ray et al. (2005) for more details (see §C).

[4] We make this assumption purely for expository reasons. Our analysis only requires that $E[R(t)] \ge 0$.

The objective of the firm is to maximize its total expected profit $\Pi(p_N, p_R(t))$, which is the sum of the expected profit from the new customers $\Pi_N(p_N)$ and the expected profit from the replacement customers $\Pi_R(p_R(t))$. Given the above definitions, the two profit components can be expressed as

$$\Pi_N(p_N) = M_N(p_N - c)\Pr_N = \frac{M_N}{a}(p_N - c)(a - p_N)^+ \quad (5)$$

and

$$\begin{aligned}
&\Pi_R(p_R(t)) \\
&= M_R E_T[(p_R(t) - c + R(t))\Pr(t)_R] \\
&= \frac{M_R}{a} E_T[(p_R(t) - c + R(t))(a' + kt - p_R(t))^+], \\
&= \frac{M_R}{a} \int_{(p_R(t)-a')^+/k}^{t_m} (p_R(t) - c + R(t))(a' + kt - p_R(t)) f(t)\, dt.
\end{aligned} \quad (6)$$

We remark that it is necessary to impose some mild parameter restrictions on the prices p_N and $p_R(t)$ to rule out cases in which the firm will never make a positive profit. These are identified and discussed in our supplement (see appendix).

4. Optimal Prices and Trade-in Rebates

When the firm ignores the replacement customer segment, it would charge the optimal price for the new customer segment to all customers. Optimizing $\Pi_N(p_N)$ given by (5), we can easily determine the optimal price p_N^* as

$$p_N^* = \frac{a + c}{2}. \quad (7)$$

In reality, however, firms dealing in durable, remanufacturable products are expected to take note of the customer segmentation as well as the possibility of price discrimination within and between the two segments. To characterize the effects of both factors on the optimal prices and assess their profit implications, we study three pricing schemes. In increasing order of degree of price differentiation, these are: (i) charging a uniform price p_S to all customers without any trade-in rebates, i.e., $p_N = p_R(t)\ \forall t \in [0, t_m] = p_S$; (ii) charging p_N to all new customers and a uniform age-independent trade-in price p_R for the replacement segment, i.e., $p_R(t) = p_R\ \ \forall t \in [0, t_m]$; and (iii) charging p_N to all new customers and age-dependent trade-in prices $p_R(t)$ (for age t) for the replacement group.

4.1. Uniform Pricing for Both Customer Segments with No Trade-in Rebates

In this section, we study the first pricing scheme where the firm recognizes the two customer segments and the age profile of the existing products, but does not pursue price differentiation and charges a single price p_S to all customers. Note that such a pricing scheme might be necessary for products for which there is a frictionless secondhand market, and consequently the firm is not able to distinguish between the two segments (buyer anonymity)—e.g., textbooks (Fudenberg and Tirole 1998). This scheme is not the central theme of our paper, but serves as a benchmark model for assessing the benefits of price differentiation later on. In determining the optimal uniform price p_S^*, we assume that the replacement customers still return their existing products and the firm collects the associated revenues. Naturally, there might be product categories (presumably those easy to discard) for which the replacement customers are more likely to dispose of their existing products in the absence of a trade-in rebate. The optimal uniform price for this scenario can be obtained from our analysis by simply setting $R(t) = 0\ \forall t \in [0, t_m]$. For a given p_S, the expected profit for the firm is $\Pi_S(p_S) = \Pi_N(p_S) + \Pi_R(p_S)$:

$$\begin{aligned}
\Pi_S(p_S) &= \frac{M_N}{a}(a - p_S)(p_S - c) + \frac{M_R}{a} \\
&\quad \cdot \int_{(p_S - a')^+/k}^{t_m} (p_S - c + R(t))(a' + kt - p_S) f(t)\, dt.
\end{aligned} \quad (8)$$

Proposition 1. *$\Pi_S(p_S)$ is unimodal in p_S if the age profile in the market falls within the class of nondecreasing hazard (failure) rate distributions. As a result, the optimal price p_S^* is the unique solution to the first-order condition:*

$$\begin{aligned}
&\frac{M_N}{a}(a - 2p_S^* + c) \\
&\quad + \frac{M_R}{a}\Bigg\{(a' - 2p_S^* + c)\bar{F}\left(\frac{(p_S^* - a')^+}{k}\right) \\
&\qquad + k\left(E[T] - \int_0^{(p_S^*-a')^+/k} tf(t)\, dt\right) \\
&\qquad - \left(E[R(t)] - \int_0^{(p_R^*-a')^+/k} R(t) f(t)\, dt\right)\Bigg\} = 0.
\end{aligned} \quad (9)$$

Furthermore, $p_S^* < p_N^*$.

The class of nondecreasing hazard rate distributions is very large, including most of the commonly used probability distributions like Uniform, Gamma, Beta (both parameters ≥ 1), Weibull, Normal, Lognormal, Exponential, Truncated Normal, Truncated Logistic (Rosling 2002). This means that practically any age profile could be analyzed, provided it is not bimodal. In fact, a limited number of statistics available on age profiles for household appliances shows that age profiles are indeed unimodal, and hence satisfy the desired property (www.eia.doe.gov/emeu/consumption).

Note that by definition $\Pi_S(p_S^*) \geq \Pi_S(p_N^*)$. The uniform pricing strategy results in some lost profit for the firm from the new customer segment due to a lower price compared to p_N^*. Conversely, this lower price induces more (and newer) returns from replacement customers. As a result, the sizes of the two customer segments play an integral role in shaping the optimal uniform price. We would expect that as the proportion of replacement customers compared to new customers in the market increases, the differences between the optimal prices $(p_N^* - p_S^*)$ and the optimal profits $(\Pi_S(p_S^*) - \Pi_S(p_N^*))$ would also increase. Managers might be especially concerned about the magnitude of these differences and how they are affected by the market conditions. These issues will be addressed in §5. Note that in case the firm does not collect any returns and the associated revenues, i.e., $R(t) = 0\ \forall\, t \in [0, t_m]$, it will charge a higher price and face significant loss of potential profit.

4.2. Age-Independent Price Differentiation

For this focal model of the paper we assume that the firm is charging p_N to the new customers and offering a fixed trade-in rebate of $(p_N - p_R)$ to all the replacement customers, irrespective of the ages of their current products (i.e., replacement price $= p_R\ \forall\, t$). Because the new and replacement customer segments act independently, the optimal new price will be $p_N^* = (a + c)/2$ as in (7). The firm then needs to determine the optimal price (hence the rebate) for the replacement customers (p_R^*) to maximize the profit from the M_R segment, i.e., $\Pi_R(p_R) = (M_R/a)\int_{(p_R-a')^+/k}^{t_m}(p_R - c + R(t))(a' + kt - p_R)f(t)\,dt$.

Proposition 2. *$\Pi_R(p_R)$ is unimodal in p_R if the age-profile distribution has a nondecreasing hazard (failure) rate. Consequently, the optimal price p_R^* is the unique solution to*

$$(a' - 2p_R^* + c)\bar{F}\left(\frac{(p_R^* - a')^+}{k}\right) + \left\{k\left(E[T] - \int_0^{(p_R^*-a')^+/k} tf(t)\,dt\right) - \left(E[R(t)] - \int_0^{(p_R^*-a')^+/k} R(t)f(t)\,dt\right)\right\} = 0. \quad (10)$$

Furthermore, p_R^* exhibits the following properties:

- $p_R^* < p_S^* < p_N^*$.
- *Consider two age profiles (T_1, T_2) with nondecreasing hazard rates and corresponding optimal replacement prices (p_R^{1*}, p_R^{2*}). Suppose that $T_1 >_{hr} T_2$; i.e., T_1 has a hazard rate that is larger than T_2. Then, $p_R^{1*} \geq p_R^{2*}$.*
- *Consider two return revenue structures $(R^1(t), R^2(t))$ with corresponding optimal replacement prices (p_R^{1*}, p_R^{2*}). Suppose that $R^2(t) > R^1(t)\ \forall\, t$. Then $p_R^{2*} < p_R^{1*}$.*

Closer examination of (10) reveals an immediate corollary.

Corollary 1. *If $-a' + (c - E[R(t)]) + kE[T] \leq 0$, then*

$$p_R^* = \frac{a' + (c - E[R(t)]) + kE[T]}{2} \leq a'. \quad (11)$$

Otherwise, $p_R^* > a'$.

The optimal age-independent price differentiation strategy is characterized as (p_N^*, p_R^*). Notice that the optimal uniform price p_S^* of the previous section expectedly lies between p_N^* and p_R^*. Price differentiation allows the firm to improve the profits from both segments. The lower price charged to the M_R segment induces more (and newer) replacements from them; on the other hand, the profit from the M_N segment is aided by the higher price. The extent of profit improvement from such differentiation (i.e., $\Pi_N(p_N^*) + \Pi_R(p_R^*) - \Pi_S(p_S^*)$) will be critically analyzed in our subsequent numerical study.

Proposition 2 and its corollary highlight additional structural properties of the optimal age-independent pricing strategy.[5] In particular, Corollary 1 indicates that when the optimal price for replacement customers is in the first range $[0, a']$, then the optimal

[5] We note that these qualitative insights also hold true for the uniform price p_S^*.

replacement price is dependent only on the *average* age of the existing products; otherwise, the firm needs to track the *entire* age-profile distribution. From the condition in Corollary 1, it is clear that the relative value of p_R^* to a' depends on three factors: age profile, durability, and return revenue. For specificity, consider a linear revenue structure $R(t) = c - c_h - \alpha t$. For a given product category (with identical unit costs c for the two firms), $R^2(t) > R^1(t)$ might then imply two fundamental structural differences. One possibility is that $c_h^2 < c_h^1$, which means that Firm 2 has better reverse logistics capabilities, i.e., it is more efficient in its collection and remanufacturing operations. It can also mean that Product 2 is better designed for remanufacturability. Another possible difference is $\alpha^2 < \alpha^1$, implying that Product 2 might be of higher "quality," and so its return value decreases less rapidly with time than that of Product 1. For this case, the condition in the corollary reduces to $-a' + (c - E[R(t)]) + kE[T] = -a + kt_m + c_h + (k+\alpha)E(T)$. Hence, the firm's optimal pricing policy is more likely to be dependent only on the mean of the age profile for products with: low durability (k) and/or high return revenues (i.e., low c_h or α) and/or newer age profiles (low $E(T)$). On the other hand, for sufficiently durable products with older age profiles and low return revenues, $p_R^* \in (a', a]$, and hence is affected by the entire age profile. This is because, depending on the reservation prices, in the former case there is a positive fraction of replacement customers returning for all ages, whereas in the latter scenario only a part of the customers who own products above a threshold age are willing to return. To determine this threshold level, it is important that the firm understands the complete age profile.

The effects of the age profile and the return revenue function on the optimal prices are quite intuitive. Customers who have older products are more likely to replace their products anyway, and hence there is less incentive to provide additional rebates. This effect is accentuated by the decrease in revenue obtained from older product returns. Therefore, we would expect customers from a market with an "older" age profile to receive lower trade-in rebates. This intuition is in general correct, but its proof requires hazard rate ordering (see Shaked and Shanthikumar 1994) of age distributions, which is slightly stronger than the weakest sense of stochastic dominance. In contrast, a firm that has higher potential for generating revenues from returned products (due to better design/system/quality) would provide a higher trade-in rebate and would induce more replacements.

The impact of product durability on the optimal pricing strategy is more involved. For a clear understanding of these effects, suppose that the age profile is uniformly distributed in $(0, t_m)$ and $R(t) = c - c_h - \alpha t$.[6] Then, the optimal trade-in price can be expressed in closed form as:

$$p_R^* = \begin{cases} \dfrac{a}{2} + \dfrac{(c-c')}{2} + \dfrac{t_m}{4}(\alpha - k) \le a' \\ \qquad \text{if } -a' + c_h + \dfrac{(k+\alpha)}{2} t_m \le 0, \\ \dfrac{a(k-\alpha) + 2k(\alpha t_m + (c-c'))}{(3k-\alpha)} > a' \\ \qquad \text{otherwise.} \end{cases}$$

Proposition 3. *The optimal trade-in price p_R^* (trade-in rebate $p_N^* - p_R^*$) is decreasing (increasing) in durability k for $p_R^* < a'$ and increasing (decreasing) in k for $p_R^* > a'$.*

The nonmonotone effect of the durability on the optimal price (and trade-in rebate) can be explained as follows. As the durability increases, the fraction of customers replacing their products decreases, and consequently the firm needs to offer a deeper rebate to induce replacements. In doing so, the firm also acquires newer used products from its customers, which provide higher return revenues. However, if the product durability further increases, the critical age above which customers start replacing their products also increases. This means that it becomes more difficult for the firm to acquire products for every marginal rebate offered. Hence, beyond a certain durability level, it does not pay off for the firm to offer higher rebates to acquire the few extra products; increasing the trade-in price is more profitable.

4.3. Age-Dependent Price Differentiation

Although, in practice, most trade-in rebate programs offer an age-independent rebate for all replacement

[6] The generality of these results will be illustrated in the following section.

customers, there might be situations (e.g., auto industry) in which the firm opts for a pricing scheme that offers rebates tied to the age of the returned products. In this section we derive the optimal age-dependent pricing strategy for replacement customers.

When the firm has the option of charging age-dependent prices, it would determine a unique optimal price $p_R^*(t)$ for any given product-age t, i.e., age-dependent trade-in rebate $= p_N^* - p_R^*(t)$. Clearly, $p_R^*(t)$ maximizes the firm's expected profit from a replacement customer with a product of age t, i.e.,

$$(p_R(t) - c + R(t))\frac{[a' + kt - p_R(t)]^+}{a}. \qquad (12)$$

Proposition 4. *For any product of age t, $0 \le t \le t_m$, the optimal trade-in price $p_R^*(t)$ is given by*

$$p_R^*(t) = \frac{a + c - k(t_m - t) - R(t)}{2}. \qquad (13)$$

Furthermore, $p_R^*(t)$ is increasing in product age, and decreasing in return revenue and durability.

Proposition 4 states that the optimal age-dependent pricing scheme is critically linked to the return revenue function. For example, if the return revenue function is linear, so is the optimal pricing strategy. On the other hand, a piecewise constant/linear revenue function would result in a piecewise linear pricing policy. The special case in which the returned products have no value to the firm (e.g., software upgrades) also results in a simple linear pricing structure of the form shown in (13) with $R(t) = 0\ \forall t$, and $p_R^*(0) \le p_R^*(t) \le p_R^*(t_m) = p_N^*$. Note that while the optimal price is independent of the age profile, the profit for the firm will surely be affected by the age profile. Furthermore, the older the product, the less the rebate will be. Higher return revenues and durability, however, increase the optimal rebate.

Inarguably, when everything else remains the same, an age-dependent pricing strategy has a clear profit advantage over an age-independent strategy. The magnitude of this profit improvement would naturally change depending on the operating conditions. However, in practice, determining the ages of individually returned products might require additional inspection. This might entail deployment of specialized equipment as well as labor at every location where product returns are accepted by the firm. Such an action could substantially increase the per-unit handling cost c_h (Klausner and Hendrickson 2000).[7] Consequently, there is a hidden operational disadvantage of age-dependent pricing strategy. This implies that when the cost of performing individual inspection/handling is sufficiently high, age-independent pricing would continue to be more profitable for the firm. We pinpoint such scenarios in the next section.

5. Numerical Study

The previous section provided analytical characterization of the optimal prices for the three pricing policies. Our numerical study in this section focuses on prescribing how managers should react when faced with specific market conditions. As noted earlier, we define the market condition in terms of four characteristics: age profile, durability, return revenue, and relative size of the two market segments. In order to put our insights in proper perspective, we first discuss how the four characteristics can be influenced by managerial actions for a given product. We also highlight variations among different products with respect to these factors, due to their inherent nature or market/technology characteristics. Additional examples are provided during our detailed discussions. Subsequently, we concentrate on how these factors affect managerial decisions and profit performance of the firm under various pricing schemes. We provide a summary of our results and a synthesis of the insights in the conclusion section.

Age profile. Firms can, to some extent, create "newer" age profiles through frequent take-back initiatives or by marketing strategies that reposition the product as a fashion item (e.g., cell phones). Naturally, the precise form and rate of change of the age profile over time would also depend on the relative length of the lifespan of the product to that of the whole product life cycle. Generally speaking, however, the age profile of a specific product is likely to become older as the product moves through its life cycle (e.g., plasma TV—new, Color TV—uniform, and B&W TV—mostly old).

[7] Although assessing age may be expensive, the precise knowledge might generate more revenue from a returned product.

Durability. As indicated earlier, firms can intentionally increase the durability of their products through their design, manufacturing, and operational strategies (e.g., filamentless design can increase the durability of light bulbs, www.energyandlight.com; also see Bras and Hammond 1996). Across different products, the relative degree of product durability is very much determined by the nature of the product (e.g., carpets versus printers). Even within a particular category, products might display different durability levels (e.g., dishwashers are considered less durable than freezers, www.whirlpoolappliances.ca).

Return revenue. Firms can also increase their return revenues by design and operational strategies, e.g., better design for remanufacturability (e.g., Interface, Xerox) or better reverse logistics network (e.g., Bosch). Across products, the return revenue depends on the overall value and durability of the product, as well as how amenable it is for collection and remanufacturing (e.g., cameras versus refrigerators).

Relative size of market segments. The relative size of new and replacement customer segments is closely linked to the age profile. A product recently introduced to a market would have mostly new customers. Mature products, however, have a varying but more visible distinction of new and replacement customers (e.g., cell phones, printer cartridges).

In order to demonstrate the wide applicability of our results, we conduct the numerical study with a beta distribution, $B(\beta_1, \beta_2)$, defined on $(0, t_m)$. This distribution, as desired, has the property of increasing hazard rate (for $\beta_1,\ \beta_2 \geq 1$). Also, by fixing β_1 (at $\beta_1 = 2$) and varying β_2, we can conveniently represent different product-age profiles. Note that the lower the value of β_2, the older the products in the market are. For example, $\beta_2 \approx \beta_1$ corresponds to products with ages distributed almost uniformly over the entire interval; lower values ($\approx$1) imply that most of the products in the market are quite old, while high values (say, 5, 8, or 13) represent that the market has "mostly new" products. We assume a continuous linear return revenue function of the form $R(t) = c - c_h - \alpha t$. Different combinations of α and c_h then enable us to represent varying levels of return revenues. Specifically, we suppose $c = 45$, $t_m = 10$, and the (c_h, α) combinations are taken as follows: $(5, 0)$, $(5, 4)$, $(15, 3)$, $(25, 2)$, and $(45, 0)$. Clearly, $R(t) \geq 0\ \forall\, t \in [0, t_m]$ in all cases, and $R(t)$ is decreasing over the five pairs for any $t \in [0, t_m]$. Moreover, note that the first and last pairs represent scenarios where the return revenues are independent of the age of the return t (because $\alpha = 0$). On the other hand, for the intermediate three pairs ($\alpha \neq 0$), the sensitivity of the return revenue to the age of the return decreases as c_h increases and α decreases. We reiterate that all the results of this section also hold true for revenue functions with discrete changes.

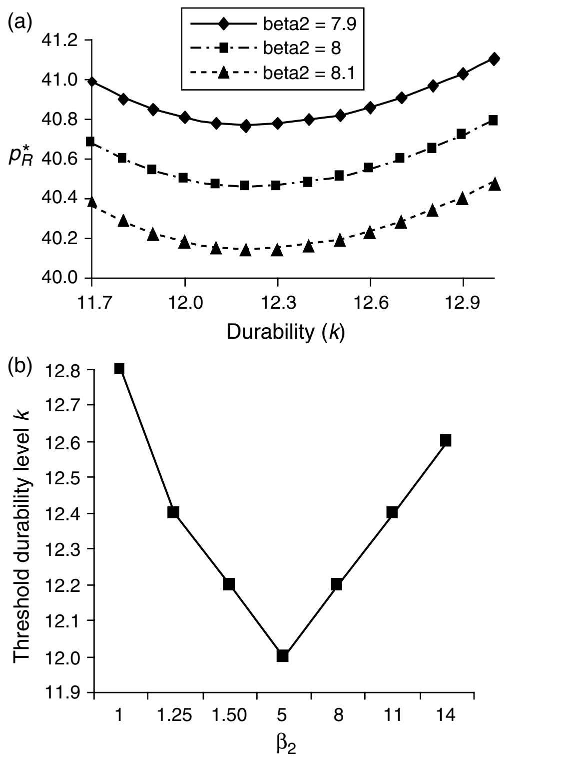
In §5.1 we illustrate the sensitivity of the optimal trade-in prices/rebates toward the four factors. Section 5.2 compares the three pricing schemes and identifies conditions that favor the use of each scheme. Note that throughout this section $a = 200$ and $\beta_1 = 2$.

5.1. Sensitivity Analysis of the Optimal Prices

Our purpose in this section is to illustrate the generality of our earlier analytical results regarding the sensitivity of optimal prices, and to generate further insights through numerical studies. We also provide supporting real-life evidence for our claims, wherever possible. We restrict attention primarily to our focal, age-independent pricing model. The sensitivity of the uniform price p_S^* is qualitatively similar to that of p_R^* (except for the effect of M_R/M_N, which we present below). The sensitivity of the age-dependent price $p_R^*(t)$, on the other hand, is evident from Proposition 4. For the analysis in this section, the following is the base parameter set: $M_R = 20{,}000$; $M_N = 10{,}000$; $t_m = 15$; $c = 50$; $c_h = 20$; and $\alpha = 2$.

Figure 1(a) shows the effects of durability and age profile on the optimal trade-in price p_R^*. Note that, regardless of the durability of the product, the optimal trade-in price decreases in β_2. That is, the older the products in the market, the less the trade-in rebate should be, confirming Proposition 2. The figure also validates Proposition 3 regarding the effect of product durability (k): For any product-age profile, the optimal trade-in price first decreases in k, and after a threshold k, it starts increasing. Hence, firms dealing in products of medium durability should announce the maximum trade-in rebates. We also noted (see Figure 1(b)) that the threshold durability level (or "turning point") itself is affected by the age profile. Specifically, the turning point first decreases as the products in the market get newer, and then

Figure 1 (a) Change in Optimal Age-Independent Replacement Price p_R^* w.r.t. Durability (k) and Age Profile (β_2), (b) Change in Threshold Durability (Turning Point) w.r.t. Age Profile (β_2)



starts increasing, with the lowest values observed for average-age/somewhat new products.

Real-life instances (Fishbein et al. 2000) support our claims about the nonintuitive effects of durability on the optimal pricing policy. For example, Xerox has recently started using gold rather than lead for electrical contacts in "medium" durability products like high-end laser printers, as a part of its remanufacturing-oriented design/manufacturing philosophy. This action has resulted in simultaneous increase of up-front cost and durability. However, Xerox could afford to increase its discounts for these "more" durable products and induce earlier returns, because these products (or their components) can now be reused through several lifespans. Conversely, competitors like Pitney-Bowes have not been successful in improving the durability of their products, and consequently do not aggressively promote trade-in discounts for their "lower" durability products. At the other extreme, experience in the carpet industry has demonstrated that it is difficult to induce early returns of such highly durable products unless providing substantial (even loss-making) discounts. Otherwise, trade-ins in this industry result in the return only of quite old products, which do not have much remaining value. Hence, firms in the carpet industry have mostly shied away from providing any significant trade-ins.

We proved in §4.2 that for products with low durability, the optimal trade-in price p_R^* depends only on the mean of the age profile, while highly durable products require information about the entire age distribution. Because this latter information is sometimes costly to obtain, managers might be tempted to use only the mean value while deciding on the optimal trade-in price (rebate) even for a highly durable product. It is then of interest to understand under what conditions the distributional information provides the maximum value for the firm, and the scale of profit gains from such information. For this analysis, we have focused on medium/high-durability products, where the optimal prices are actually dependent on the entire age profile (i.e., $p_R^* > a'$). We then compared the optimal profit with that of using a suboptimal price, derived only from the mean age. Figure 2 illustrates the extent of profit gain for different durability and age profiles.

As evident from Figure 2, the profit gain is always increasing in durability, but might be increasing (for high k) or nonmonotone (for low k) with respect to the age profile. We also noted that the profit improvement is almost insensitive to the relative market size (M_R/M_N) and increases with the potential for high

Figure 2 Value of Using Entire Age-Profile Distribution Rather Than Average Age

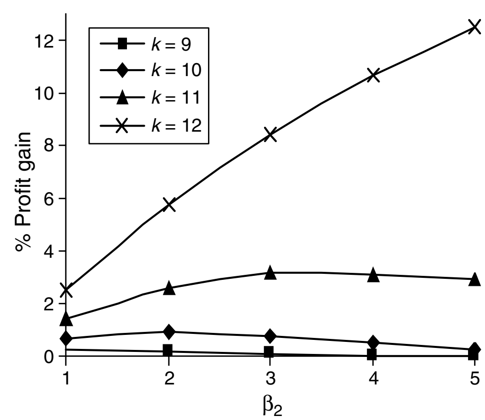
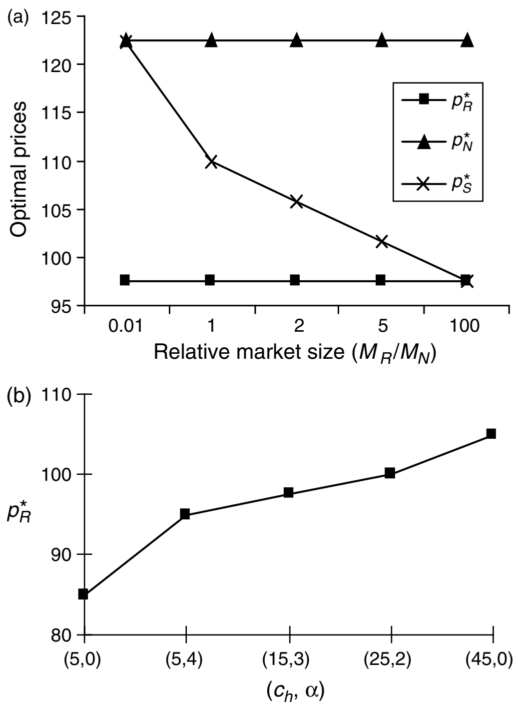


Figure 3 (a) Effects of Relative Market Size (M_R/M_N) on Optimal Prices, (b) Change in Optimal Trade-in Price w.r.t. $R(t)$



return revenues ($R(t)$). Hence, if most of the products in the market are new and highly durable and the firm has efficient reverse logistics operations, the profit increase from distributional information might indeed be considerably large. Our examples indicate that this benefit can reach 10%–12%. On the other hand, for markets with mostly old products and low return revenues, it is justified to utilize the mean value as an approximation even for highly durable products.

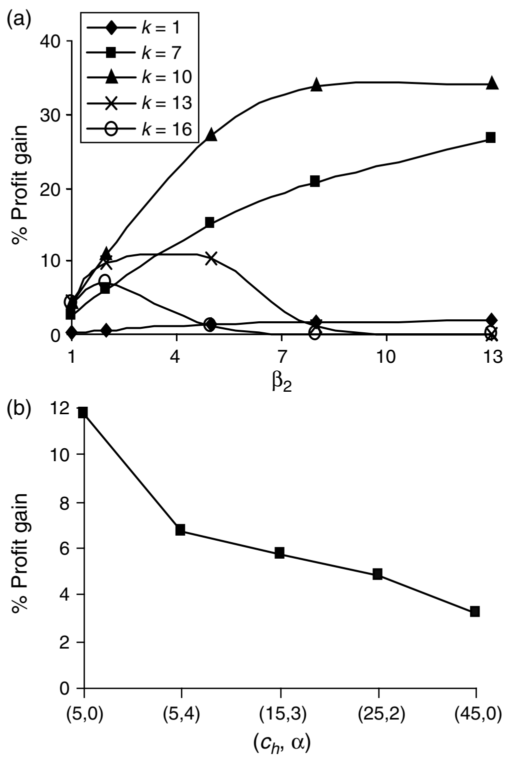
In our numerical study, we also investigated the effects of the relative market sizes (M_R/M_N) and the scale of return revenues $R(t)$ on the optimal decisions. Figure 3(a) shows that $p_R^* < p_S^* < p_N^*$ (Proposition 2), and p_S^* itself is decreasing in M_R/M_N. Hence, if the replacement market segment is very small compared to the new customer segment, then $p_S^* \approx p_N^*$, while for large replacement segment $p_S^* \approx p_R^*$. Figure 3(b), on the other hand, confirms Proposition 2 that the optimal replacement price p_R^* decreases (i.e., trade-in rebate increases) as the returned products are able to generate higher revenues (note that $R(t)$ decreases over the horizontal axis in Figures 3(b), 4(b), 5(b), and 8(b)). This effect is also corroborated by real-life managerial practices. Most profitable remanufacturing firms nowadays locate their "asset recovery engineers" with product designers during the development of new products so that products/components retain substantial value when returned. Such endeavors have resulted in accomplishments like Xerox developing a powder-based ink for toners that avoids ink contamination and can easily be removed from returned machines. In the carpet industry, although most firms cannot afford significant trade-ins, companies like Interface have made some headway by introducing a new material (Solenium) that considerably slows down the deterioration process in carpets. This allows substantial recycling of old carpet fiber in production of new carpets. In fact, Interface's Déjà Vu carpet is made from 72% recycled material, one of the highest recycled contents in the industry. These types of modifications are targeted toward increasing return revenues, which enables firms to provide higher discounts (Bras and Hammond 1996, Fishbein et al. 2000). Consequently, these firms attract more early returns of durable products from customers (which bring higher return revenues), resulting in more successful remanufacturing initiatives, and thereby creating an ever-improving loop.

5.2. Profit Comparison of Pricing Strategies

In this section, we illustrate the profit impact of employing the different pricing schemes. This enables us to isolate and quantify the value of three integral dimensions of differentiation represented in these pricing schemes: customer segment, price, and age profile. We also investigate how the magnitude of the profit impact changes with the four key factors studied and identify conditions under which the benefits to the firm are most significant. The base set of parameters used in this section is: $M_R = 20{,}000$, $M_N = 10{,}000$, $t_m = 10$, $c = 45$, $c_h = 15$, $\alpha = 3$, $k = 7$, and $\beta_2 = 2$.

5.2.1. Value of Age-Profile-Based Uniform Pricing. The simplest pricing scheme that the firm can follow is to charge the new price $p_N^* = (a + c)/2$ uniformly to all customers without any regard for the age of the existing products. This pricing scheme might be necessary in scenarios where the firm has no knowledge about the size or profile of the customer segments. In most situations, however, the firm should

Figure 4 Value of Customer Segmentation: (a) w.r.t. Durability (k) and Age Profile (β_2), (b) w.r.t. $R(t)$



be able to obtain information about the sizes of the two distinct sets of customers as well as the age profile of existing products, enabling it to charge the uniform optimal price p_S^* to both customer segments. The percent-profit gain $(=(\Pi_S(p_S^*)-\Pi_S(p_N^*))/\Pi_S(p_N^*)\times 100)$ from adopting the optimal uniform price p_S^* can then be interpreted as the value of customer segmentation. Figure 4 illustrates this profit gain for different durability, age profile, and return revenue structures.

From Figure 4(a) we can see that the profit gain is nonmonotone in both durability (k) and age profile (β_2), while Figure 4(b) depicts it to be increasing in $R(t)$. The firm can be significantly better off (10%–35%) in markets with mostly new products, high return revenues, and medium levels of durability by recognizing the two customer segments. Conversely, if the durability of the product is extreme (very high or very low) and/or the return revenue is low, the profit penalty for using the new customer price p_N^* as the uniform price might not be substantial regardless of the age profile of existing products. Although not shown here, we note that the profit gain would also increase as the proportion of the replacement customer segment in the market increases, in which case p_S^* drifts further away from p_N^* (see Figure 3(a)). Evidently, using p_N^* might be justifiable when a firm introduces a product (especially a low-durability one) and does not have much know-how about remanufacturing operations or increasing durability. However, as the firm improves its reverse logistics and makes the product more durable (or when the product is by nature quite durable), it should charge p_S^* as the uniform price, rather than p_N^*.

5.2.2. Value of Age-Independent Price Differentiation. Price discrimination is the primary reason underlying trade-in rebates. In this section, we investigate the profit impact of pursuing (age-independent) price discrimination between new and replacement customers, in addition to customer segmentation of the previous section (by tracking the age profile of existing products). Because we assume that the product is remanufacturable, another salient characteristic of this strategy that deserves attention is the incorporation of return revenues.

The percent-profit gain of price differentiation can be expressed as

$$\frac{(\Pi_R(p_R^*)+\Pi_N(p_N^*))-(\Pi_R(p_S^*)+\Pi_N(p_S^*))}{(\Pi_R(p_S^*)+\Pi_N(p_S^*))}\times 100\%,$$

where p_S^* is the uniform price offered to all customers without any discrimination. Clearly, the above ratio assumes that the firm is taking into account the return revenues in setting prices. It is possible that the firm can pursue differentiation without incorporating product returns and associated revenues while deciding on the optimal prices. We can express the profit gain from incorporating *only* return revenues as

$$\frac{(\Pi_R(p_R^*)+\Pi_N(p_N^*))-(\Pi_R(p_{R1}^*)+\Pi_N(p_N^*))}{(\Pi_R(p_{R1}^*)+\Pi_N(p_N^*))}\times 100\%,$$

where p_{R1}^* is the optimal price for the replacement segment if $R(t)$ is ignored. In a similar fashion, the joint effect of the two features can be represented as

$$\frac{(\Pi_R(p_R^*)+\Pi_N(p_N^*))-(\Pi_R(p_{S1}^*)+\Pi_N(p_{S1}^*))}{(\Pi_R(p_{S1}^*)+\Pi_N(p_{S1}^*))}\times 100\%,$$

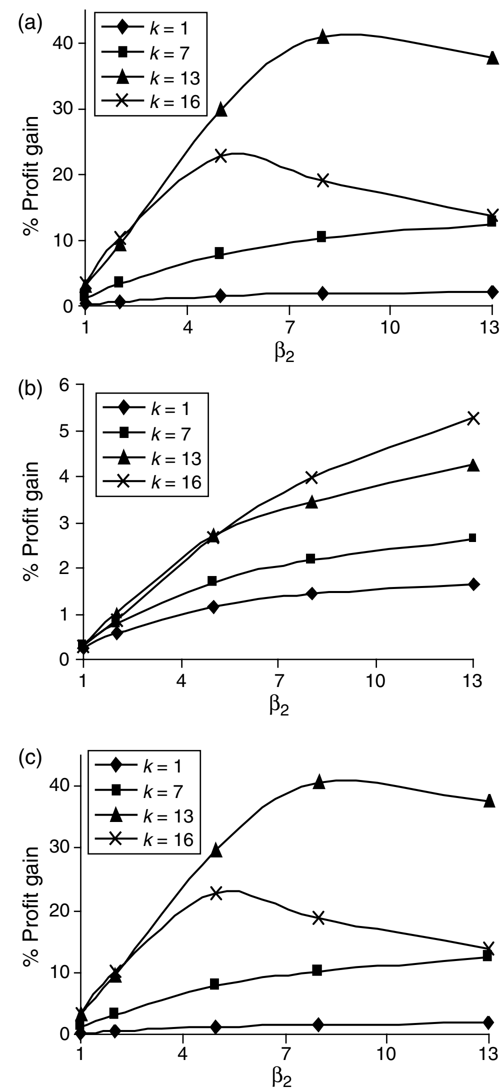
where p^*_{S1} is the uniform price for all customers if $R(t)$ is ignored. We present the three ratios for different durability (k) and age-profile (β_2) values in Figure 5, while the ratios for varying M_R/M_N and $R(t)$ are shown in Figure 6. We note that the relative magnitude of $R(t)$ is crucial in these comparisons. For our base-case scenario $p^*_R = 97.5$, while the revenue generated from the return of a product of average lifespan (because $t_m = 10$, $\beta_1 = 2$, and $\beta_2 = 2$, average lifespan $= 5$) is 15, which is 15.4% of p^*_R.[8] Obviously, because values of both p^*_R and $R(t)$ change during our experiments, the percentage might be greater or smaller than 15.4%, depending on the circumstances.

We can make a number of observations from Figures 5 and 6. First of all, note that the profit impact of price differentiation is higher than incorporating return revenues for almost all age-profile, durability, and return revenue structures, except for a very high proportion of replacement customers. Our numerical evidence suggests that by price-discriminating new and replacement customers, the firm can generate incremental profits on the scale of 10%–40%, highlighting the potential profitability of trade-in offers. Although not comparable in scale, ignoring the value of returned products can also be quite costly for the firm under certain circumstances. Our studies show that up to 5%–6% profit gain is due to setting prices taking into account the return revenues. Interestingly, as illustrated in Figure 6(b), the two factors are positively correlated—the higher the return revenue, the higher the profit gain from price differentiation.

Figures 5 and 6 also demonstrate how the value of age-independent price differentiation changes with the four critical factors we study. From Figures 5(a) and 5(c), it is evident that the profit gain from price differentiation is nonmonotone with respect to durability and age profile, and is obviously higher with the inclusion of return revenues. This suggests that price-discriminating new and replacement customers is most profitable for products with medium durability and comparatively new age profile, while Figure 5(b) shows the importance of taking into account the return revenue aspect while deciding on prices

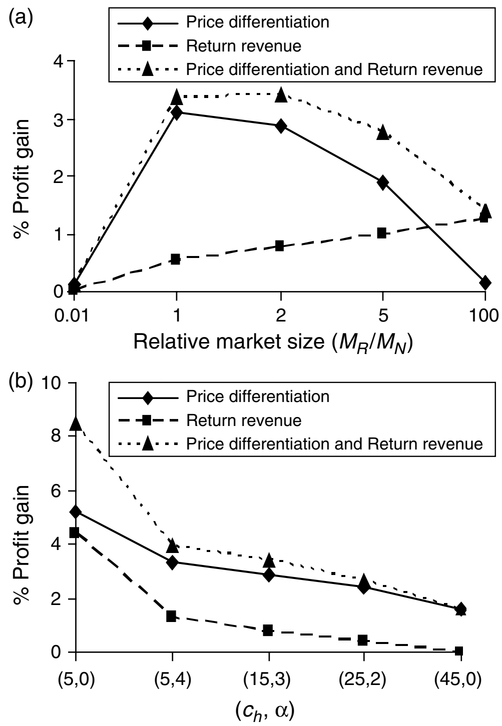
[8] This percentage is empirically supported by data from Klausner and Hendrickson (2000) for the power tool industry in Germany.

Figure 5 (a) Value of Price Differentiation, (b) Value of Return Revenue, (c) Joint Value of Price Differentiation and Return Revenue (w.r.t. Durability (k) and Age Profile (β_2))



for products with high durability and new age profile. Note that the latter are the same conditions under which the entire distributional information for the age profile is most valuable for the firm. In contrast, for industries with low-durability products and rather old/uniform age profile, the loss from ignoring even both the price differentiation option and return revenues might not be very significant (Figure 5(c)). As far as the effects of the relative market size of the two customer segments (M_R/M_N) are concerned (Fig-

Figure 6 Value of Price Differentiation, Return Revenue, and Their Joint Value: (a) w.r.t. Relative Market Size (M_R/M_N), (b) w.r.t. $R(t)$



ure 6(a)),[9] it is rather intuitive that a large replacement segment will make the issue of return revenues more important. On the other hand, price differentiation is most important when the sizes of the two customer segments are relatively similar. Because extreme values of (M_R/M_N) bring p_S^* closer to either p_N^* or p_R^* (Figure 3(a)), such conditions make price differentiation less crucial. Finally, Figure 6(b) confirms that the profit penalty of ignoring return revenue increases with $R(t)$, which also increases the value of price discrimination. In the extreme case when $R(t)$ is substantial ($c_h = 5$, $\alpha = 0$; i.e., $R(t)/p_R^* \approx 0.47\ \forall\, t$), the profit gain of incorporating $R(t)$ into the pricing decisions is almost as significant as that of price discrimination.

Based on the above discussion, we can make further conclusions. We observe that the favorable conditions for customer segmentation (charging p_S^* rather than p_N^*) are very similar to those for price discrimination. Consequently, when the firm recognizes the two distinct customer segments, it should also discriminate these segments in terms of price. The incremental value of price discrimination is, in general, higher than customer segmentation, even if the firm does not take into account return revenues in setting prices. Nevertheless, as the firm engages in design modifications to increase product durability and/or achieves higher efficiency in reverse logistics operations, it can substantially improve its gains by taking into account both return revenues and the complete information on the age-profile distribution.

5.2.3. Value of Age-Dependent Price Differentiation. When the firm has the capability of easily determining the age of a returned product, it may decide to discriminate replacement customers by offering age-dependent rebates $p_N^* - p_R^*(t)$. As noted earlier, such a pricing scheme is always more profitable than an age-independent one. Using the same base parameter set as the last one, in this section we investigate the magnitude of profit improvement $((\Pi_R(p_R^*(t)) + \Pi_N(p_N^*)) - (\Pi_R(p_R^*) + \Pi_N(p_N^*)))/(\Pi_R(p_R^*) + \Pi_N(p_N^*)) \times 100$/from pursuing this age differentiation. We present the percent-profit gain for different age-profile and durability values in Figure 7(a) and for different $R(t)$ values in Figure 7(b).

Interestingly, Figure 7 suggests that age-dependent price discrimination does not add much to the profit gain. The incremental value of age differentiation is, in general, considerably smaller (<10%) compared to price differentiation and even customer segmentation. Specifically, the gain is increasing in durability, but nonmonotone with respect to age profile and return revenue. As far as the effect of return revenue potential is concerned, from Figure 7(b) we can conclude that the value of age-dependent pricing increases as $R(t)$ increases and becomes more dependent on the age of the returns (i.e., $\alpha \neq 0$, and increasing; refer to the discussion at the beginning of §5). Although not shown here, we note that the relative market size (M_R/M_N) has no considerable effect on the value of age differentiation. In summary, based on our numerical study it seems that operationally efficient firms dealing with highly durable products and having a substantial base of installed customers

[9] Note that the comparatively low values in Figures 6(a) and 6(b) are due to the fact that in those two figures $\beta_2 = 2$ and $k = 7$ (base parameter set). If $\beta_2 = 7 - 10$ and $k \approx 13$, then the percent-profit gains would be higher, as evident from Figure 5(a).

(old/uniform age profile) should be the only candidates for adopting this aggressive price-differentiation strategy.

The desirability of adopting age-dependent pricing diminishes even further if we take into account the informational requirements for implementing this strategy. As pointed out in §4.3, individual inspection required to assess the exact age of returns is likely to increase the return-handling cost c_h. If the handling cost (with additional inspection) is higher than a threshold value, say c_h', it might actually be preferable for the firm to offer age-independent trade-ins. The effects of product durability and age profile on c_h' are shown in Figure 8(a) (recall that the base $c_h = 15$), whereas Figure 8(b) demonstrates the effects of return revenue. Note that for the latter analysis, because we vary c_h, we report the threshold cost c_h' in relation to the original cost c_h. Clearly, the results reinforce our previous discussion. The threshold handling cost is high (and hence age-dependent rebates can be implemented) only when the products in the market are almost uniformly distributed in age and/or durability is high. Because uniform age distribution translates into high variability in the return revenues, and age-profile information is more valuable for more durable products, in these situations being precise about how much to pay for trade-ins provides the maximum benefit to the firm. This is especially true when the return revenues are strongly dependent on the age of the returns (i.e., $\alpha \neq 0$).

Figure 7 **Value of Age Differentiation: (a) w.r.t. Durability (k) and Age Profile (β_2), (b) w.r.t. $R(t)$**

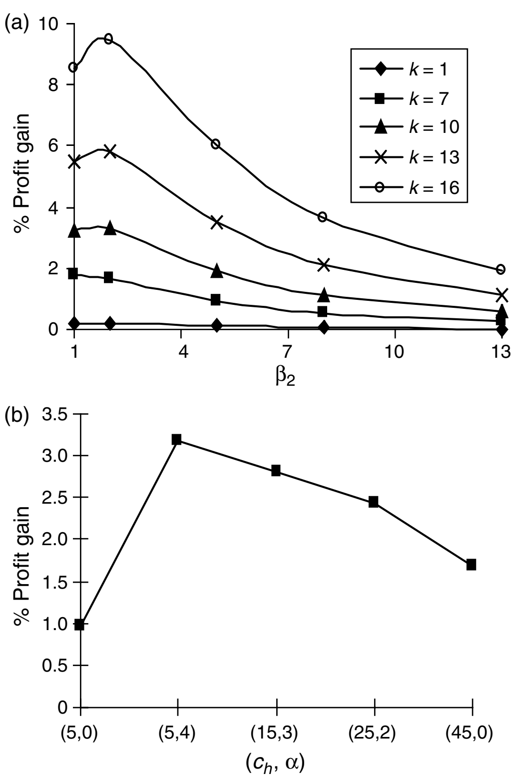
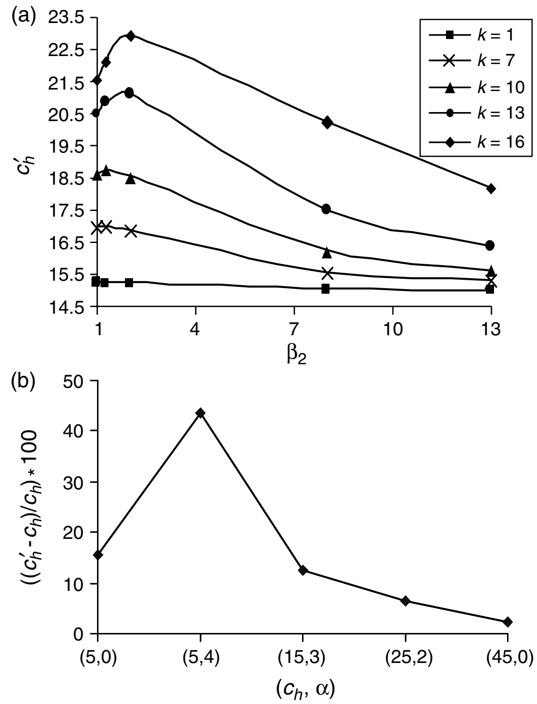


Figure 8 **Threshold Handling Cost (c_h'): (a) w.r.t. Durability (k) and Age Profile (β_2), (b) w.r.t. $R(t)$**



6. Model Extension with Second Purchase Option

Our basic pricing framework in §4 and subsequent numerical study assumed that the replacement customers do not have the option to purchase a new product without turning in their old one. In this section we relax this assumption and allow for stronger dependency between the pricing of the two customer segments. It turns out that the analytical study of such a framework is rather involved. Hence, we focus on developing the model, and numerically investigate whether our insights regarding the optimal pricing (trade-in) decisions from previous sections remain valid. Because an age-independent pricing scheme is the one most prevalent in real life, we concentrate on it in this section.

Note that a second purchase option has no impact on the new customer segment. However, any customer in the M_R segment owning a product of age t now has the option to buy a second one without trading in the existing one. Consequently, she/he will have a consumer surplus of $(P_0 - p_N)$ from the new purchase plus an additional $k(t_m - t)$ from the existing one. A trade-in customer, on the other hand, will have a surplus of $P_0 - k(t_m - t) - p_R$. It can be easily verified that the decision to purchase a second new product will be governed by a threshold age $t^P_{cr}(p_R, p_N)$:

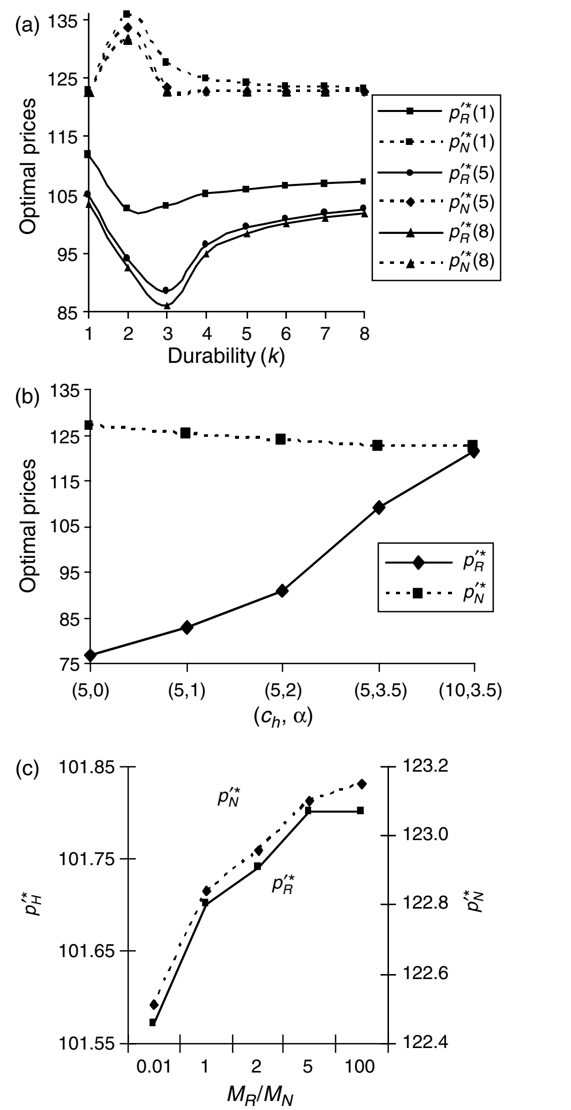
$$t^P_{cr}(p_R, p_N) = \left(t_m - \frac{(p_N - p_R)}{2k}\right)^+, \tag{14}$$

such that a randomly chosen replacement customer will prefer the second purchase option if the age of her/his product $t \le t^P_{cr}(p_R, p_N)$, and the trade-in option if the age $t > t^P_{cr}(p_R, p_N)$. This means essentially that a part of the original replacement customer segment will now become a part of the "new" customer segment. The firm can influence the volume of this shift using both prices as levers, and hence the two market segments and the prices become closely coupled. The overall profit of the firm becomes

$$\begin{aligned}\Pi^P(p_N, p_R) &= \frac{M_N}{a}(p_N - c)(a - p_N)^+ \\ &+ \frac{M_R}{a}\int_0^{t^P_{cr}(p_R, p_N)} (a - p_N)^+(p_N - c)f(t)\,dt \\ &+ \frac{M_R}{a}\int_{t^P_{cr}(p_R, p_N)}^{t_m} (p_R - c + R(t))(a' + kt - p_R)f(t)\,dt.\end{aligned} \tag{15}$$

The first term of the above equation is for the real "new" customers who decide to buy, the second term represents those replacement customers who opt for a second purchase without returning, while the last term denotes the trade-in customers. Optimizing (15) is evidently more involved, but optimal prices (say p'^*_N, p'^*_R) can easily be found through numerical search. We have done many experiments to compare the results of this extended model with our basic model.

Figure 9 **Optimal Prices: (a) w.r.t. Durability (k) and Age Profile ($\beta_2 = 1, 5, 8$), (b) w.r.t. Return Revenue $R(t)$, (c) w.r.t. Relative Market Size (M_R/M_N)**



From Figure 9 we notice that the optimal price for a new product p'^*_N for this model depends on a number of factors. The nonmonotone nature of p'^*_N with respect to durability, as shown in Figure 9(a), is supportive of our earlier results. Higher p'^*_N enables the firm to offer higher rebates initially. Beyond a threshold durability level, however, it becomes harder for the firm to induce replacements. Coupled with the detrimental effects of high prices on new purchases (first-time or second), it becomes more profitable for the firm to reduce p'^*_N. Figure 9 also confirms the intuition that $p_N'^*$ should be increasing when there are potentially more customers making second purchases (i.e., higher M_R/M_N, older age profile), and vice versa. With respect to $R(t)$, $p_N'^*$ is relatively insensitive (decreases slightly), as the effects of $R(t)$ are reflected mostly on the replacement price.

The behavior of the trade-in price $p_R'^*$, on the other hand, is insensitive to the model framework and remains essentially unchanged. The only new effect comes from the relative market size—$p_R'^*$ increases in M_R/M_N. We remark that the behavior of the *optimal rebate* in terms of durability, age profile, and return revenues also remains exactly the same as in the basic model. It is evident from the figures that the optimal rebate is the highest for medium durability and minimum for high or low values of k (Proposition 3), and decreases as the age profile gets older (Proposition 2). Furthermore, in line with Proposition 2, higher rebates are possible for firms that can garner higher return revenues. The only new insight is again related to the relative market size, which indicates that firms should offer higher rebates if their replacement markets get larger.

The above discussion suggests that although the value of the optimal trade-in price might change if replacement customers are allowed second purchases, the key properties of the optimal rebate remain essentially the same. Also note that under certain circumstances, especially for low-durability products, even the optimal values are insensitive to the model framework,[10] which implies that managers can in such cases assume the two market segments to be independent.

7. Conclusions and Future Research Opportunities

In this paper, we developed and analyzed a pricing framework for a durable, remanufacturable product. Derived from the individual customer's choice behavior, this framework integrates pricing decisions with four defining characteristics of such a product—age profile of the ones currently in use, the durability of the product, the extent of age-dependent revenues associated with the returns, and the relative size of the new and replacement customer segments. Our model helps managers to determine the optimal price for new customers and the optimal trade-in rebate for replacement customers (at any given point in time assuming that the two segments are independent), for quite a general class of age-profile distributions.

We first establish the role each characteristic plays in shaping the structure and behavior of the optimal pricing/trade-in policy. We show that if the firm is dealing with a low-durability product, then it is sufficient to know only the average age of the products in use to determine the optimal age-independent rebate. Moreover, this rebate increases with durability. Conversely, if the firm is able to increase the durability of its product (e.g., via design modifications) or is dealing with an inherently highly durable product (e.g., carpet), it needs to utilize the entire distributional information of the age profile to determine the optimal rebate, and the rebate in this case decreases with durability. As far as the impact of the product-age profile is concerned, as the proportion of new products in the market increases (either through deliberate firm actions or due to changes in technology/life-cycle phase), the firm should provide higher rebates. Similarly, if firms are able to design products and establish recovery networks that can capture higher benefit out of the returns, they should offer better trade-in deals, and thereby improve product acquisition for remanufacturing operations.

In a subsequent numerical study, we compare four different pricing schemes to assess the incremental values of customer segmentation, price differentiation, and age differentiation represented in our pricing strategies. This study reveals that, in general, the firm stands to gain more from price discrimination than customer segmentation or age differentiation. We find that the extent of profit gain from differentiating replacement and new customers through fixed (age-independent) rebates can reach 40%. Furthermore, for operationally efficient firms offering highly durable products with considerable potential for return revenues and having relatively new age profiles, the exact distribution of the age profile and the extent of return revenues are both important concerns in setting these rebates. Otherwise, the firm can concentrate only on the average age of products and ignore the potential revenues generated from returns. We find that age differentiation can also benefit the firm, but the scale of these gains is normally low (up to 10% in

[10] This happens when $t_{cr}^P(p_R, p_N) = 0$. Then the extended model is equivalent to a particular instance of the basic one.

our experiments) even if the additional handling cost of determining the precise age of returns is ignored. This is a useful practical insight, keeping in mind the fact that an age-dependent pricing strategy might be expensive and difficult to implement.

Going a step further, through extensive comparisons we are able to demonstrate how the four characteristics affect the incremental values of the pricing schemes. This allows us to identify the most favorable conditions (i.e., combinations of parameters) for the use of each pricing strategy.

- The "naive" strategy of charging a uniform price p_N^* based only on new customers: under the extreme conditions of very new age profile, low durability and return revenue, and high proportion of new customers.
- A uniform price p_S^* based on customer segmentation taking into account the age profile of in-use products: when there is buyer anonymity, so that the firm is not able to price discriminate (e.g., for textbooks), and/or when the proportion of replacement customers is quite high.
- Age-independent price discrimination (p_N^* for the new customer segment and p_R^* for the replacement segment): New age profiles, medium durability, high return revenues, and comparable sizes of new and replacement customers. It is worthwhile to point out that for such cases, it is also advisable that the firm take into account the entire age-profile distribution and return revenue in setting the prices.
- Age-dependent price discrimination (p_N^* for the new customer segment and $p_R^*(t)$ for a replacement customer returning a product of age t): Only for highly durable products with a large proportion of replacement customers, old/uniform age profiles, and high (and strongly age-dependent) return revenues.

The above results have interesting implications on the life-cycle pricing of durable, remanufacturable products. Note that during the incubation phase, a product is likely to have a large proportion of new customers and a very new age profile for existing products. The design/technology of the product is likely to be less stable as well. Our study indicates that customer segmentation and price differentiation are not critical under such conditions. Hence, charging the new price p_N^* uniformly to all customers is reasonable. During the growth phase, the installed base of users increases. The firm also gains more expertise in the product/technology, enabling it to improve durability as well as the efficiency of remanufacturing operations. Our model advocates the adoption of a price-discrimination mechanism, and offering differentiated prices (p_N^*, p_R^*) and, thereby, (age-independent) trade-in rebates for replacement customers. When the product completely diffuses into the marketplace and reaches maturity, the firm is likely to have improved its design and operational efficiency further, and it is appropriate to further differentiate replacement customers by offering age-dependent rebates $(p_N^* - p_R^*(t))$. Obviously, how frequently the firm should change its pricing strategy depends on the nature of the product/market as well as the emphasis placed by the firm in managing the four salient factors that drive these decisions.

There are a number of possible extensions beyond the second purchase option for replacement customers that can deepen our understanding of the pricing of durable, remanufacturable products. One major extension is to consider a dynamic, temporal setting. Such a multiperiod model can help us to investigate the impact of past trade-in promotions on the future decisions of the firm. In addition, although our research shows the indirect effect of various operational decisions (e.g., product design) on the optimal pricing of durable goods, it might be interesting to include a detailed operations framework to address the direct impact of such decisions. We believe that our static model can act as a building block for this latter extension. Another worthwhile research direction is to extend our ideas to the case of short-life-cycle products with high rates of innovations and new product introductions. This would naturally entail a multiperiod, dynamic model. Similarly, mature durable products like automobiles, which have strong secondhand markets, need to be analyzed in a competitive framework.

Acknowledgments

The authors would like to thank two anonymous referees, the Senior Editor, and the Editor-in-Chief for their exceptional efforts, which significantly improved the relevance, positioning, and presentation of the paper. This research is supported by grants from the Natural Sciences and Engineering Research Council (NSERC) of Canada and Fonds de Recherche sur la Société et al Culture (FQRSC) of Québec.

Appendix. Glossary of Notations

M_N, M_R = size of new and replacement customer segments, respectively;

p_N, p_S = unit price for new customers and uniform price/unit for both segments, respectively;

$p_R(t)$, p_R = age-dependent and age-independent price/unit for the replacement segment, respectively;

t_m = useful lifespan for the product;

a = maximum price any customer would be willing to pay for the product;

k = durability of the product;

f, F = density and distribution function of the age profile, respectively;

P_0 = reservation price of the customers for the product $\sim U(0, a)$;

$\Pr(t)_R$ = probability that a randomly chosen customer having a product of age t from the M_R segment would be willing to replace (or fraction of the M_R segment who would replace);

$\Pr_N$ = probability that a randomly chosen customer from the M_N segment would be willing to purchase (fraction of the M_N segment who would purchase);

$t_{cr}(p_R(t))$ = threshold age that makes $\Pr(t)_R = 0$;

$R(t)$ = revenue (cost savings) from remanufacturing a returned product of age t;

α = rate of change of $R(t)$ with respect to age t;

c, c_h = unit production and handling costs, respectively;

c'_h = handling cost that makes age-dependent and age-independent pricing profits equal;

Π_N, Π_R = expected profit from the new and replacement segment, respectively;

Π_S = expected total profit when all customers are charged a flat price p_S.

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