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Tiers in One-Sided Matching Markets: Theory and Experimental Investigation

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The design of a matching market may affect behavior in prematch stages. In some settings, forward-looking agents might purchase low-priced properties with the intention of trading up. From a design standpoint, such behavior is undesirable. We investigate a tiered structure as a potential solution. Using a model that endogenizes prematch acquisition decisions, we show that tiers promote exchange while protecting the primary market. In the laboratory, we find that both firm revenue and total social surplus are improved by tiered matching, and the amount of improvement depends on the exchange mechanism the firm uses. We focus on two popular mechanisms—deposit first and request first. We find that subjects are less likely to take advantage of the match under tier-free deposit first mechanism, possibly because of risk aversion. Thus, a tiered approach is more critical under the request first mechanism. We confirm that risk aversion partly explains deviations from theory.

Key words: one-sided matching; mechanism design; fractional ownership; state-dependant utility; economics experiments; risk aversion
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1. Introduction

The goal of a one-sided matching protocol is to match or reallocate properties to agents with complete private preferences and unit demand for the items. One-sided matching is useful when a money-based market mechanism is either unpalatable (e.g., "repugnant" transactions in Roth 2007), inefficient, or otherwise infeasible. Matches involving students, such as assigning housing or allocating courses to students (Abdulkadiroglu and Sönmez 1999, Krishna and Ünver 2008) and matches involving medical transplants (Roth et al. 2004, 2005) are prominent examples of successful matching protocols. Unlike other allocation tasks, both participation in a matching market and the revelation of private preferences are largely voluntary or at the very least difficult to enforce. Thus, to improve market efficiency, a matching program should invite as much participation and truthful revelation of preferences as possible. Typically, research into one-sided matching mechanisms involves a characterization of market outcomes resulting from the match, taking participants' prematch ownership as exogenous (e.g., Chen and Sönmez 2002, Wang and Krishna 2006). However, because market participants are forward looking, the design of the matching market can have a profound impact on their behavior in stages that precede the match. Extant research is silent on how to design the matching market while taking into consideration the impact of matching on decisions in a previous stage. The current work attempts to fill this gap in the literature.

Specifically, we examine the impact that the presence and design of a matching market can have on the decision of market participants to acquire properties in the first place. In an earlier stage, which we call the primary sales market, forward-looking agents make decisions on what property to purchase. The match, taking place in a secondary exchange market, provides participants the opportunity to exchange the usage right of their own property for a different one. This poses a potential problem because sales in the primary market may suffer when the secondary market is not properly designed. We propose that a tiered matching protocol allows the match-making firm, or the market maker, to maintain buyers' incentive to invest in quality and participate in the matching market. We characterize the properties of various two-tier systems with a focus on this dual purpose.

Tiered matching protocols are often found in fractional ownership markets, which is the working example used in this paper. In these markets, a household purchases a share of a property, which allows the owner to use the property for a specified period of time every year for the duration of the ownership. For example, a household can purchase week 47 (Thanksgiving week) of a three-bedroom house in Orlando. The household is then entitled to a week-long stay at the property every year in that specific week. This is different from the concept of rental (e.g., Purohit 1997, Lehman and Weinberg 2000) because the legal ownership of a property entitles the owner to participate in secondary matching markets or to resell. Fractional ownership is best known in the tourism and resort real estate industries under the term *time-share* (Pryce 2002), and has become increasingly popular in other markets for luxury goods such as private jets, yachts, and exotic cars.

When flexibility in consumption is desired, fractional owners often make use of secondary matching markets to exchange the usage rights of their properties. Although ownership remains unchanged, the owner would allow others to enjoy her own property once, in exchange for a one-time usage of another property. Thus, the secondary matching market creates additional surplus for property owners because it protects them against a future preference shock. A market maker could therefore benefit by providing that service and extracting some of that surplus. To facilitate such exchanges, market makers typically set up a pool of inventory so that owners can *withdraw* properties from this pool and, in return, *deposit* their own properties back into the pool. Exchange systems are very popular in fractional ownership markets. Ragatz Associates, an independent market research firm in the resort real estate industry estimates that about 66% of fractional homes are in an exchange program. It is not uncommon for participants in fractional ownership markets to attempt to game the system, as exemplified by the following advice: "It's not what you own that counts…stay in the best resorts…no matter what you own!" (Pierce 2012).

We study two key aspects of market design in these secondary matching markets: the exchange mechanism and the tier structure. As we show in this paper, both aspects of the design have significant implications for strategy choice and market outcomes. An *exchange mechanism* defines the sequence of deposits and withdrawals, which often takes one of two forms—request first and deposit first. Under the request first mechanism, owners turn in the usage right of their own property only after they have obtained a preferred property from the market maker. Hence this mechanism is incentive compatible. In contrast, deposit first mechanism requires owners to give up their own properties by making binding deposits prior to a search for potential withdrawals. The market maker also needs to make a critical decision on the *tier structure*. In reality, some firms (e.g., Timbers Resorts) permit all owners to make exchanges within the firm's entire network, whereas others (e.g., Netjets) classify properties into tiers, allowing owners to exchange only within their own tier or restricting trading up. We refer to the former approach as a tier-free structure, and the latter as a tiered structure.

To investigate the implications of various market designs, we present a theoretical framework that models forward-looking economic agents, referred to hereafter as *households*, who fall into two segments with different price sensitivity. Knowing the design of the secondary exchange market, households first make their acquisition decisions in the primary market and then make their entry decisions in the secondary market. Our theoretical analysis reveals that, regardless of the exchange mechanism, in a tier-free market households are likely to purchase lower quality properties in the primary market with the intention to upgrade in the secondary market. Such "gaming" behavior generates highly undesirable market outcomes. By restricting upgrades using a tiered structure, the market maker can help to restore primary sales because different segments are effectively separated, each segment purchasing the properties meant for it. Moreover, both segments would participate in the exchange market because it gives them additional utility.

To empirically test our theory about tier structure and to compare the benefit of adding tiers across the two most widely adopted exchange mechanisms—request first versus deposit first, we set up experimental markets with incentivized human subjects in a laboratory setting. We find strong evidence that market outcomes are more desirable in tiered markets than in tier-free markets, which supports our theory. Interestingly, we also find that the benefit of the tiered structure is much greater under the request first mechanism than under the deposit first mechanism. This is because, compared to a tier-free deposit first market, a tier-free request first market entices more participants to game the system by purchasing lower quality properties with the intention to trade up. Hence, empirical evidence shows that the choice of exchange mechanism is especially important when the market has a tier-free structure. Taken together, our results suggest that although the request first mechanism and tier-free structure may appear to be desirable because they both encourage participation in the match, these features can have detrimental effects on the markets that precede the match. Therefore, it is crucial to consider strategic behavior in both prematch and matching stages when studying the design of matching markets.

2. Background

We focus on a secondary one-sided matching market that follows a primary sales market. The matching market we study is not particularly different from other one-sided matching markets, where limited resources are allocated to individuals. For example, researchers have studied the allocation of houses to tenants (Abdulkadiroglu and Sönmez 1999; Carrillo and Singhal 2011; Chen and Sönmez 2002, 2004), the assignment of courses to students (Krishna and Ünver 2008, Sönmez and Ünver 2010), and the kidney exchange programs where kidneys are matched to patients (Roth et al. 2004, 2005; Zhang 2010). Most of the matching literature concerns the efficiency of the matching market, taking initial ownership as exogenously given. In some cases, exogenous ownership might be a reasonable assumption. For example, it might be a stretch to suggest that people would be less likely to take good care of their kidneys or livers because a match is made available to them (notwithstanding this logical stretch, there is an active debate about this exact issue, e.g., Cohen and Benjamin 1991). However, with college allocation decisions, from housing to course allocation, it is likely that the institution of a match will distort prematch decisions. Our research contributes to this literature by introducing a tiered structure into matching while endogenizing the prematch acquisition decision.

Considerations of decision stages prior to the match have been investigated in the literature on two-sided matching. In experimental investigations into medical labor clearinghouses (e.g., McKinney et al. 2005) and the law clerk clearinghouse (Haruvy et al. 2006), researchers typically allow for stages prior to the match in which employers can make exploding offers to potential employees. If characteristics of matching protocol are such that waiting for the match is not sufficiently appealing to both sides, early exploding offers will be made and some will be accepted. The process is self-reinforcing and could result in little or no entries into the match. This is known as unraveling—a dynamic process in which offers are made earlier from year to year. To our knowledge, attention to prematch periods in two-sided matching markets has not been extended to the one-side matching literature. Although we find unraveling to be interesting, we also believe the choice of which school to attend is not independent of the matching protocol used in the subsequent job market just as students' course choices are not independent of the course matching protocol. An example in a two-sided context is offered by Niederle and Roth (2003), who found that in the absence of a centralized match for gastroenterology fellowship positions, gastroenterologists are more likely to be employed at the same hospital in which they were internal medicine residents. This would suggest that both the preferences over and the decision on where to intern would fundamentally change with the introduction of a successful centralized match.

A second issue of importance in the present setting pertains to the fact that households purchase properties first and then may wish to temporarily exchange the usage rights of these properties due to idiosyncratic preference shocks. Hence, our work is closely related to the literature on state-dependent utility theory, which focuses on situations where the utility from consumption depends on future circumstances (Hauser and Wernerfelt 1990; Shugan and Xie 2000, 2005; Xie and Shugan 2001). Xie and Shugan (2001) pointed out that the utility from consumption can vary with mood, health, opportunities, conflicts, family situations, and so forth. Thus, in a particular year the household that bought week 47 in Orlando in the earlier example may prefer week 51 (the Christmas week) instead. They can try to exchange for week 51 for that year's vacation through a matching market. Following the literature on state-dependent utility, we model the impact of preference shocks on consumption utility via the density function of possible owner valuations.[1] This literature has mainly focused on primary sales, whereas our main concern is the design of the secondary exchange market from a market maker's perspective.

The present work is also a piece in an emerging literature on fractional ownership markets. From the operations perspective, researchers have explored solutions for effective scheduling in a fractional jet program (Yang et al. 2008) and a vacation timesharing program (Sampson 2008). Wang and Krishna (2006) identified the deficiencies of the request first and the deposit first mechanisms and proposed a "top trading cycles chains and spacebank" mechanism (TTCCS) to improve the efficiency of the secondary market. In the present work, the secondary matching market is structured much like that in Wang and Krishna (2006), but in addition we investigate the use of tier structure and its impact on the primary sales market. We focus on the deposit first and request first mechanisms because these two are widely used in fractional ownership markets, whereas TTCCS has not been widely adopted.[2]

Our setting involves decisions on whether or not to enter the secondary market. In that way our work ties directly into the market entry literature. In the experimental investigation on market entry, Rapoport et al. (2002) and Zwick and Rapoport (2002) studied situations where each agent must choose which

[1] Following this literature, we focus on shocks caused by personal factors such as health and family situations. We do not model uncertainty caused by external factors such as a hurricane that hits Florida. Such common shocks change all consumers' valuations in the same way, such that owners of Florida properties are effectively locked out of the exchange market.

[2] Although TTCCS generates a more efficient outcome than deposit first and request first mechanisms in the secondary market, the key argument about how a tier-free structure diminishes the incentive of households to buy high quality properties in the primary market would still apply under TTCCS.

market to participate in, and the probability of winning a prize in each market decreases as the number of agents choosing that market increases. Our setting is more complex in that the probability of a household winning a better property is affected by the number of other households who choose to enter the market, the quality of the properties they own, and the design of the market. Incentivized laboratory experiments allow us to empirically test the theory and, furthermore, to measure the benefit of adding tiers under different exchange mechanisms. Thus, our paper represents another promising endeavor to experimentally explore strategic decision making in markets (Amaldoss and He 2009, Amaldoss et al. 2008, Amaldoss and Rapoport 2005, Chen and Sönmez 2006, Ho et al. 2006, Krishna and Ünver 2008, Lim 2010, Srivastava et al. 2000, Wang and Krishna 2006, Zwick et al. 2003).

3. Model

We use the time-share industry as a running example to examine the effects of market design on agents' behavior and market outcomes. In this industry, *households* purchase time-share *properties* from a property developer, which we call the *primary seller*. An independent firm in the secondary market acts as the *market maker* that designs and operates the matching system. The market maker's depository of properties is referred to as the *space bank*. Property owners can *deposit* their property into the space bank and *withdraw* a different property from it to temporarily exchange the usage rights of their properties. The market maker is an independent downstream channel member of the primary seller. Because the approval by the primary seller is imperative to the existence and success of the secondary matching market, the market maker operates under the constraints imposed by the primary seller.[3]

We investigate two exchange mechanisms that are widely used in the industry: the deposit first (DF) mechanism and the request first (RF) mechanism. The DF mechanism requires a binding deposit before a search for potential withdrawal. This suggests an owner may end up with a less desirable property than the one he deposited. In contrast, the RF mechanism requires the deposit only after a successful withdrawal. Hence, there is no risk of trading downward so that theoretically participation in the match weakly dominates any other strategy for all owners. The incentive compatibility (IC) of the RF mechanism is usually considered a desirable feature in matching markets.

In our model, properties fall into two discrete types with different quality: low quality properties and high quality properties. The qualities are denoted Q_L and Q_H, for low and high quality, respectively, such that $Q_H > Q_L > 0$. There is no uncertainty regarding property quality, which is objectively determined and observed by all agents in the market. Prices are exogenously given based on property quality, at P_L for low quality properties and P_H for high quality properties, such that $P_H > P_L > 0$. Price information is common knowledge in the market. The market maker takes prices as exogenous because it is assumed to be a downstream channel member of the primary seller.

There are N risk-neutral households, each with one unit of demand. Households seek to buy properties and then potentially exchange them (i.e., the property usage rights rather than the physical properties). A household belongs to one of two types: the *premium household* and the *value household*. There are N_P premium households and N_V value households, where $N_P + N_V = N$. The number of households of each type is exogenous and is common knowledge to both households and the market maker. A premium household h values a property j at $Q_j + s^P_{hj}$ $(j \in \{H, L\})$; Q_j is the property's objective quality, and s^P_{hj} represents the random shock a premium household h may experience in its preference over property j. The net utility a premium household h receives from purchasing and consuming property j is $U^P_{hj} = Q_j + s^P_{hj} - P_j/\bar{\theta}$ $(j \in \{H, L\})$. The parameter $\bar{\theta}$ is strictly positive and captures premium households' price sensitivity. Similarly, a value household h' values property j at $Q_j + s^V_{h'j}$. When it purchases the property at price P_j and consumes it, the net utility is $U^V_{h'j} = Q_j + s^V_{h'j} - P_j/\underline{\theta}$ $(j \in \{H, L\})$. We impose the condition $\bar{\theta} > \underline{\theta} > 0$ because the premium households are less price sensitive than the value households.

When households make purchase decisions, they only observe the deterministic elements of their utility—the qualities and prices of the properties. The random variables s^P_{hj} and $s^V_{h'j}$ represent the individual household-specific random shock in preference, or the variation in consumption utility caused by personal factors such as health, moods, opportunities, conflicts, and family situations. Random shocks are private and revealed only after purchase. For tractability, we assume the same distribution across households and across properties and denote all preference shocks as s_{hj}. We assume this shock to be zero mean, with the maximum value $s_{\max}$ and the

[3] We do not model the primary seller's role or its relationship with the market maker in great detail. In reality, the interaction between the primary seller and the market maker could be quite complex. In some cases, the match is a service sanctioned directly by the primary seller. In other cases, the market maker is an independent downstream channel member of the primary seller. These different possible relationships have different implications overall. But in general, an explicit or implicit approval by the primary seller is a key ingredient to the existence and success of the match-making firm.

Table 1 Three Stages of the Market

Stage	Decision maker	Choice
Stage 1: Market design	Market maker	Tier structure: tier-free or tiered Exchange mechanism: RF or DF
Stage 2: Primary market	Households	Purchase decision: buy either low quality or high quality property from the primary seller
Stage 3: Secondary market	Households	Entry decision: enter the exchange market or stay out

minimum value $s_{\min}$. We also assume $s_{\max} - s_{\min} < Q_H - Q_L$, that is, every household prefers any high quality property to any low quality property.

We view the problem from the perspective of the market maker, given the constraints set by the primary seller. Thus, the market maker's objective is to maximize exchanges, subject to maintaining incentive compatibility for premium households to buy high quality properties in the primary market. The primary seller prefers to separate the two segments of households so that each segment purchases the property meant for it, because this gives it higher profits from the sales of properties. The market maker operates a space bank that consists of properties with quality Q_L and Q_H. We assume that the number of low quality properties in the space bank, n_L, is exogenous and greater than zero.[4] The market maker has control over n_H, the number of high quality properties it places in the space bank.

The market proceeds in three stages (see Table 1). In stage 1, the market maker determines the design of the secondary exchange market. In stage 2, forward-looking households buy properties based on expected utility and future exchange prospects. In stage 3, uncertainty about individual preferences (s_{hj}) is resolved, and households can choose to stay out or to enter the secondary market to exchange the usage right of their properties.

3.1. Incentive Compatibility and Individual Rationality Conditions Without Prospect of Exchange

The model is set up such that in the absence of a secondary market, the premium households prefer to purchase high quality properties (Q_H), and the value households prefer to purchase low quality properties (Q_L). This is necessary for the two property levels to coexist in the primary market (and more generally in the resort real estate market). Therefore, the prices should support the incentive compatibility and individual rationality (IR) conditions for both household segments. For the premium households, IC in the primary market requires that the net utility from purchasing a high quality property is greater than the net utility from purchasing a low quality property. Formally, we express the IC condition for the premium households as follows:

$$\sigma = (\bar{\theta} \cdot Q_H - P_H) - (\bar{\theta} \cdot Q_L - P_L) > 0. \tag{1}$$

IR requires that for a premium household the expected net utility from purchasing high quality property is nonnegative:

$$\bar{\theta} \cdot Q_H - P_H \geq 0. \tag{2}$$

Similarly, for the value households, the IC and IR constraints are, respectively,

$$\underline{\theta} \cdot Q_L - P_L \geq \underline{\theta} \cdot Q_H - P_H, \tag{3}$$

$$\underline{\theta} \cdot Q_L - P_L \geq 0. \tag{4}$$

We impose conditions (1)–(4) throughout this paper. Note that when the two types of households separate in the primary market, the primary seller's profit is $N_V P_L + N_P P_H$.

3.2. Incentive Compatibility and Individual Rationality Conditions With Exchange

The introduction of a secondary exchange market can have an adverse impact on primary sales. We define "gaming the system" as households purchasing low quality properties with an intention to upgrade through the secondary market. In §3.4 we show that the existence of a tier-free exchange market might trigger the premium households to game the system. If this happens, profits of the primary seller decline from $N_V P_L + N_P P_H$ to $(N_V + N_P)P_L$. Therefore, as a downstream channel member, the market maker needs to fulfill exchanges while maintaining premium households' incentive to purchase high quality properties.

Households consider the four combinations of strategies as presented in Table 2. Denote by $G_h(j, \xi \mid n_H, n_L, \chi_H, \chi_L)$ the expected value for household h,

$$h = \begin{cases} P & \text{for premium households,} \\ V & \text{for value households,} \end{cases}$$

Table 2 Households Strategy Menu

Strategy	Description
A	Buy high quality property (Q_H) and stay out of the exchange market
B	Buy high quality property (Q_H) and enter the exchange market
C	Buy low quality property (Q_L) and stay out of the exchange market
D	Buy low quality property (Q_L) and enter the exchange market

[4] Relaxing this assumption does not affect our results.

from strategy (j, ξ), where the household purchases a property of quality Q_j, $j \in \{H, L\}$ in stage 2, and takes an action ξ,

$$\xi = \begin{cases} 0 & \text{if staying out,} \\ 1 & \text{if entering exchange,} \end{cases}$$

in stage 3. The expected value G is a function of the number of available properties in the space bank (n_H high quality properties; n_L low quality ones), the number of households that purchase high quality properties (χ_H), and the number of households that purchase low quality properties (χ_L). It also depends on the household's strategy (j, ξ) as well as the actions taken by other households.[5]

Conditions (5)–(8) characterize market equilibrium conditions. For the premium households, given the prospect of potential exchanges,

$$\text{IC:}\quad \bar{\theta} \cdot G_P(H, \xi_P \mid n_H, n_L, \chi_H, \chi_L) - P_H \geq \bar{\theta} \cdot G_P(L, \xi_P' \mid n_H, n_L, \chi_H, \chi_L) - P_L, \tag{5}$$

$$\text{IR:}\quad \bar{\theta} \cdot G_P(H, \xi_P \mid n_H, n_L, \chi_H, \chi_L) - P_H \geq 0. \tag{6}$$

The notations ξ_P and ξ_P' denote the optimal entry strategies for a premium household given its purchase decision. Equation (6) guarantees that premium households prefer purchasing high quality properties to not making any purchase. When Equation (5) holds, premium households prefer to purchase high quality properties over low quality properties. Recall that the market maker would want premium households to purchase high quality properties, and it would also want to encourage exchanges among these households. In other words, the market maker would prefer Equations (5) and (6) to hold simultaneously, and it wants the premium households to choose $\xi_P = 1$ after purchasing high quality properties.

For the value households, we have

$$\text{IC:}\quad \underline{\theta} \cdot G_V(L, \xi_L \mid n_H, n_L, \chi_H, \chi_L) - P_L \geq \underline{\theta} \cdot G_V(H, \xi_L' \mid n_H, n_L, \chi_H, \chi_L) - P_H, \tag{7}$$

$$\text{IR:}\quad \underline{\theta} \cdot G_V(L, \xi_L \mid n_H, n_L, \chi_H, \chi_L) - P_L \geq 0. \tag{8}$$

Equation (8) suggests that the value households prefer purchasing low quality properties to not purchasing any property. They prefer purchasing low quality properties to high quality properties when condition (7) holds.

[5] It is important to note that in function $G_h(\cdot)$, n_H is a decision variable for the market maker. Moreover, for exposition purposes, we leave others' actions out of function $G_h(\cdot)$. We will detail the value of n_H and others' actions when computing the expected valuation G in our derivation.

3.3. Interpretation of the Household Strategies Under RF and DF

3.3.1. Request First. Under the RF mechanism, a household "enters the exchange" when it inspects the space bank for a potential withdrawal. For tractability, we assume that each household is allowed to inspect the space bank once, at which point a withdrawal request can be made. Entries are sequential. All households in a tier-free exchange market (or in the same tier in a tiered structure) are treated equally, such that they have equal likelihood of being the nth household to access the space bank. At its turn, a household makes a withdrawal request if it finds a property that is preferred to its own. Once the withdrawal request is fulfilled, the household's original property is immediately released into the space bank for future withdrawal by others. If a household does not find anything desirable in the space bank, it keeps its original property. In either case, the household exits the market after this step. Because RF is incentive compatible with no risk of downgrading, strategy B (buy a high quality property and enter the exchange) weakly dominates strategy A (buy a high quality property and stay out of the exchange), and strategy D (buy a low quality property and enter the exchange) weakly dominates strategy C (buy a low quality property and stay out of the exchange).

3.3.2. Deposit First. Under the DF mechanism, a household enters the exchange market by making a binding deposit. All households must make the entry decision simultaneously before any withdrawal occurs. Withdrawals are sequential. All households that enter a tier-free market (or in the same tier in a tiered market) are treated equally when making withdrawals, as in the RF system. Thus, a household in the DF market either makes no exchange (if it chooses not to deposit) or a single exchange (if it enters the exchange market by depositing its own property). Under DF, strategies A and C of Table 2 involve property owners electing to stay out of the exchange *no matter how undesirable the random shock s_{hj} turns out to be for their own property*. Strategies B and D of Table 2 involve *a conditional entry into the exchange market*, where property owners compare the expected value from the exchange market with the realized value of their own property and enter the exchange if the former value is higher. Thus, under DF, strategies B and D are interpreted as *households having a positive probability of entering the exchange*.

3.4. Equilibrium Derivation

Our main theoretical proposition is that in the absence of tiers, under both RF and DF mechanisms, *an initial depository of high quality properties in the space bank* gives an incentive to households to game the system, thus causing the primary market to collapse.

In this context, a tiered matching structure can serve the dual purpose of maintaining the primary sales and facilitating the secondary exchanges. In what follows, we present the most likely market outcomes—pooling equilibria in tier-free systems and separating equilibria in tiered systems. For every proposition we also provide a theoretical prediction on the expected number of exchanges.

3.4.1. Request First. 1. *Tier-Free RF*. In a tier-free structure, all households are treated equally in exchanges irrespective of what they own. We consider the situation where $n_L > 0$ and $0 < n_H < N$.[6] This means owners of high quality properties can potentially make exchanges. However, there is also a positive probability for owners of low quality properties to trade up to high quality properties. In this scenario, a pooling equilibrium is likely to emerge, where both types of households purchase the low quality property knowing that they are likely to obtain a high quality property in the subsequent exchange market. We formally present the sufficient condition and the pooling equilibrium in Proposition 1.

Proposition 1. *In a tier-free RF market, when $\sigma/\bar{\theta} + s_{\max} \le (n_H/N)(Q_H - Q_L)$, there exists a pooling equilibrium where both types of households purchase the low quality properties and then enter the exchange market. In equilibrium, the expected number of exchanges is $n_H + (N - n_H) \cdot (n_L + n_H)/(n_L + n_H + 1)$.*

All detailed proofs are available in Online Appendix A (A.1.1).[7] The sketch of the proof is as follows. Because there is no risk involved in RF, households only choose between strategy B (purchase a high quality property and enter exchange) and strategy D (purchase a low quality property and enter exchange). Strategy D is the dominant strategy for value households because for them the expected value from purchasing high quality properties is never high enough to justify its high price, P_H.

When condition $\sigma/\bar{\theta} + s_{\max} \le (n_H/N)(Q_H - Q_L)$ holds for the premium households, purchasing high quality properties does not increase their expected values enough to justify the price difference. This condition can be rewritten as $n_H/N + (P_H - P_L)/(\bar{\theta} \cdot (Q_H - Q_L)) \ge 1 + s_{\max}/(Q_H - Q_L)$, which suggests that if it is more likely to upgrade (i.e., n_H/N is larger), or if the price difference is too big as compared to the quality difference (i.e., ratio of $(P_H - P_L)/(Q_H - Q_L)$ is large), then the premium households would also purchase low quality properties, causing the primary seller's profitability to plummet.

2. *Tiered RF*. For a tiered market, because no upgrading from a low quality property to a high quality property is allowed and no owners of high quality properties would willingly downgrade, the high versus low quality exchange tiers can be seen as two simultaneous but strictly separate markets.

For tiered RF markets, we define $\gamma_{L(0,\, n_L)} \equiv G_h(L, 1 \mid n_H = 0, n_L > 0, \chi_H, \chi_L) - Q_L$,[8] which represents *the expected added value from exchange for owners of low quality properties*. This is essentially the difference between the expected payoff of strategy D (purchasing a low quality property and entering the exchange market) and that of strategy C (purchasing a low quality property and staying out). Similarly, we define $\gamma_{H(n_H,\, 0)} \equiv G_h(H, 1 \mid n_H > 0, n_L = 0, \chi_H, \chi_L) - Q_H$, which represents *the expected added value from exchange for owners of high quality properties* in tiered RF markets. For details about the computation of these values, refer to Online Appendix A (A.1.2).

In Proposition 2 below we formally present the separating equilibrium in tiered RF markets.

Proposition 2. *In a tiered RF market with $n_H > 0$, there exists a separating equilibrium in which value households purchase low quality properties and enter the exchange market in the low quality tier, and the premium households purchase high quality properties and enter the exchange market in the high quality tier. This equilibrium exists when $\gamma_{L(0,\, n_L)} \ge \gamma_{H(n_H,\, 0)}$ and $\sigma \ge \bar{\theta}(\gamma_{L(0,\, n_L)} - \gamma_{H(n_H,\, 0)})$. In equilibrium, the expected number of exchanges is $(N_V \cdot n_L)/(n_L + 1)$ for the N_V value households and $(N_P \cdot n_H)/(n_H + 1)$ for the N_P premium households.*

The condition $\gamma_{L(0,\, n_L)} \ge \gamma_{H(n_H,\, 0)}$ ensures that the potential value from exchanging high quality properties in the high quality tier is insufficient to entice the value households to purchase high quality properties. Note that this is not a necessary condition because for value households the IC constraint is not binding. Therefore, even when the condition $\gamma_{L(0,\, n_L)} \ge \gamma_{H(n_H,\, 0)}$ is not satisfied, value households may still prefer to purchase low quality properties.

Condition $\sigma \ge \bar{\theta}(\gamma_{L(0,\, n_L)} - \gamma_{H(n_H,\, 0)})$ is critical because it ensures that the premium households do not switch to the low quality segment. Intuitively, σ captures the extent to which premium households prefer purchasing high quality properties over low quality properties without considering the added value from the exchange market. When the added value from the low

[6] Setting $n_H = 0$ would completely eliminate the possibility of gaming. However, for any exchange to take place for owners of high quality properties, n_H has to be positive. Moreover, if $n_H \ge N$, then there are enough high quality properties in the initial space bank for all households to withdraw. This would entice all households to buy the low quality properties (at a lower price) and then trade up. We do not consider this trivial case.

[7] All appendices to this paper can be found at http://www.utdallas.edu/~eeh017200/papers/Appendices_TieredMatching.pdf and are also available from the authors upon request.

[8] An upgrade in the exchange market is not possible. For owners of low quality properties it is as if $n_H = 0$.

quality tier is *not* much higher than that of the high quality tier, this preference should not reverse and the two types of households would separate.

3.4.2. Deposit First. Because DF requires a binding deposit before withdrawal, a household enters the exchange market only when the expected value from exchange is higher than the realized value of its own property. This suggests that the optimal strategy in the secondary market is characterized by cutoff points. Let $s_H^*(s_L^*)$ be the cutoff point for owners of high (low) quality properties such that a household $h(h')$ with high (low) quality property j (j') enters the exchange market when $s_{hj} < s_H^*$ $(s_{h'j'} < s_L^*)$.

1. *Tier-Free DF*. There are two possibilities for the cutoff point s_H^*: (1) $s_H^* \leq s_{\min}$, so that owners of high quality properties would not enter the exchange no matter how they value their own properties, or (2) $s_H^* > s_{\min}$, so that owners of high quality properties have a positive probability $\Pr(s < s_H^*)$ of entering the exchange. A closer look at possibility (2) reveals that when the two quality tiers are far apart (as in our context), this is a situation where even the premium households do not want to pay the high price for high quality properties because they can purchase low quality properties at a much lower price and expect the same value (i.e., $Q_H + s_H^*$) from the tier-free exchange market. Thus, like the value households, the premium households would prefer to purchase low quality properties and enter the exchange, which suggests that there will be no owners of high quality properties in the exchange market to sustain such $s_H^* > s_{\min}$. Therefore, possibility (2) cannot be sustained in equilibrium.

The above argument implies that in the tier-free DF, market households only consider two feasible strategies: strategy A (buying a high quality property and staying out of the exchange) and strategy D (buying a low quality property and entering the exchange when $s_{hj} \leq s_L^*$). An important implication of this conclusion is that exchanges only happen among the owners of low quality properties (for complete analysis, see Online Appendix A.2.1).

Here we again focus our analysis on the scenario where the market maker initially provides some high quality properties in the space bank $(0 < n_H < N)$. In this situation a pooling equilibrium where all households purchase low quality properties is likely to emerge. Proposition 3 below formally presents this equilibrium.

Proposition 3. *In a tier-free DF exchange market, when $(n_H/N) \cdot (Q_H - Q_L) > \max\{s_{\max}, \sigma/\bar{\theta}\}$, there exists a pooling equilibrium where both types of households purchase low quality properties. In equilibrium, all N households enter the exchange market by depositing their low quality property.*

The sketch of the proof is as follows. By design, purchasing high quality properties is dominated for the value households so they will purchase low quality properties. Having bought low quality properties, entering the exchange may or may not be optimal depending on one's realized value. However, when condition $(n_H/N) \cdot (Q_H - Q_L) > s_{\max}$ holds, even providing one unit of high quality property in the space bank $(n_H = 1)$ tilts the balance in favor of entering the exchange market.

For the premium households, this decision is less straightforward and crucially depends on the price difference and quality difference for the two types of properties. As we discussed earlier, the premium households have to decide between buying a high quality property at a higher price and staying out of the exchange versus buying a low quality property at a lower price in the hope of exchanging it for a high quality property from the exchange market. When the probability of upgrading is sufficiently high and the price difference is sufficiently large (i.e., $(n_H/N) \cdot (Q_H - Q_L) > \sigma/\bar{\theta}$, which can be rewritten as $n_H/N + (P_H - P_L)/(\bar{\theta} \cdot (Q_H - Q_L)) > 1$), the premium households opt for the latter, causing the primary market to collapse.

2. *Tiered DF*. Similar to the tiered RF market, in the tiered DF market the two quality tiers can be seen as simultaneous but strictly separate exchange markets. In the low quality tier there exists a cutoff point $s_L^* > s_{\min}$, and in the high quality tier there exists another cutoff point $s_H^* > s_{\min}$.

Define in the tiered DF markets $\alpha_{L(0,\, n_L,\, \chi_L)} \equiv G_h(L, 1 \mid n_H = 0, n_L > 0, \chi_H, \chi_L) - Q_L$; $\alpha_{L(0,\, n_L,\, \chi_L)}$ is the counterpart of $\gamma_{L(0,\, n_L)}$ in tiered RF, representing *the expected added value from exchange for owners of* Q_L, when there are n_L low quality properties in the initial space bank and χ_L owners of Q_L. Similarly for its high quality tier, define $\alpha_{H(n_H,\, 0,\, \chi_H)} \equiv G_h(H, 1 \mid n_H > 0, n_L = 0, \chi_H, \chi_L) - Q_H$; $\alpha_{H(n_H,\, 0,\, \chi_H)}$ is the counterpart of $\gamma_{H(n_H,\, 0)}$ in tiered RF and captures *the expected added value from exchange for owners of high quality properties* when there are n_H high quality properties in the space bank and χ_H owners of Q_H.

To quantify the additional value households obtain from exchange, α, one needs to compute the cutoff points s_L^*, s_H^*. This is not trivial, but we are able to characterize them (details about the computation are available in Online Appendix A.2.2). In this paper we take these cutoff points as given when showing our major results.

For owners of low (high) quality properties, define the probability of entering the low (high) quality tier of the DF exchange market as $p_L^* \equiv \Pr(s < s_L^*)$ $(p_H^* \equiv \Pr(s < s_H^*))$. In the low quality tier, households that enter the exchange have an expected value of $Q_L + s_L^*$. Those who lie above the cutoff point keep their original (higher valued) property and stay out of the exchange market. So the equations below characterize $\alpha_{L(0,n_L,\chi_L)}$:

$$G_h(L,1\mid n_H=0,n_L>0,\chi_H,\chi_L) = p_L^*\cdot(Q_L+s_L^*)+(1-p_L^*)\cdot E(Q_L+s\mid s\ge s_L^*),$$
$$\alpha_{L(0,n_L,\chi_L)} = p_L^*\cdot s_L^*+(1-p_L^*)\cdot E(s\mid s\ge s_L^*)>0,$$
$$p_L^*=\Pr(s<s_L^*).$$

Similarly, in the high quality tier of DF markets we have

$$G_h(H,1\mid n_H>0,n_L=0,\chi_H,\chi_L) = p_H^*\cdot(Q_H+s_H^*)+(1-p_H^*)\cdot E(Q_H+s\mid s\ge s_H^*),$$
$$\alpha_{H(n_H,0,\chi_H)} = p_H^*\cdot s_H^*+(1-p_H^*)\cdot E(s\mid s\ge s_H^*)>0,$$
$$p_H^*=\Pr(s<s_H^*).$$

Proposition 4 describes a separating equilibrium in the tiered DF market.

Proposition 4. *In a tiered DF exchange market, there exists a separating equilibrium where the value households purchase low quality properties and enter the exchange market in the low quality tier, and the premium households purchase high quality properties and enter the exchange market in the high quality tier. This equilibrium exists when* $\alpha_{L(0,n_L,\chi_L=N_V)} \ge \alpha_{H(n_H,0,\chi_H=N)}$ *and* $\sigma \ge \bar{\theta}(\alpha_{L(0,n_L,\chi_L=N)} - \alpha_{H(n_H,0,\chi_H=N_P)})$. *In equilibrium, the expected number of exchanges is* $N_V\cdot\Pr(s<s_L^*)$ *for the* N_V *value households and* $N_P\cdot\Pr(s<s_H^*)$ *for the* N_P *premium households.*

The proof for Proposition 4 is in Online Appendix A.2.2 and follows the same logic as in Proposition 2. The condition $\alpha_{L(0,n_L,\chi_L=N_V)} \ge \alpha_{H(n_H,0,\chi_H=N)}$ implies that the value from purchasing high quality properties and entering the high quality exchange tier is not sufficiently large to justify the higher price of high quality properties for value households, and so value households prefer to purchase low quality properties and participate in the low quality exchange tier. Again, this is not a necessary condition for this equilibrium because the IC condition is not binding for the value households. Condition $\sigma \ge \bar{\theta}(\alpha_{L(0,n_L,\chi_L=N)} - \alpha_{H(n_H,0,\chi_H=N_P)})$ suggests that the added value from the low quality exchange tier is insufficient to entice premium households to purchase low quality properties. This is a critical condition for the separating equilibrium to exist. In this equilibrium, $\chi_H=N_P$ and $\chi_L=N_V$. In the secondary market, there are two separate exchange tiers each having its own cutoff point, $s_L^*>s_{\min}$ and $s_H^*>s_{\min}$, respectively.

3.4.3. Alternative Equilibria. In tier-free markets, the market maker can compel the premium households to invest in quality by limiting the number of high quality properties in the initial space bank; that is, if n_H is very small so that upgrading is highly unlikely in a tier-free market, then the premium households may prefer purchasing high quality properties over purchasing low quality properties and gaming the system, a desirable outcome for the primary seller. However, notice that n_H also has to be sufficiently large for any exchange to take place between owners of high quality properties. Therefore, when $n_H<\hat{n}_H=1$ and $\sigma>\bar{\theta}\cdot\gamma_{L(0,n_L)}$ in tier-free RF markets, or when $\hat{n}_H>n_H\ge 0$ where $\hat{n}_H=(N_V+1)(\sigma/\bar{\theta}-s_{\max})/(Q_H-Q_L)$ in tier-free DF markets, there is an undesirable degenerate separating equilibrium where the value households purchase low quality properties and enter the exchange market, and the premium households purchase high quality properties but do not enter the exchange. This means that the exchange market does not generate any additional value for owners of high quality properties. For completion of the theory, we present these less likely and undesirable alternative equilibria in Online Appendix A (A.1.3 and A.2.3).

This analysis highlights the fact that in a tier-free system the objectives of the primary versus secondary markets are fundamentally conflicting, which makes it very difficult for the market maker to ensure both the separation of purchase and the success in exchanges.

On the other hand, in tiered markets the premium households could pool with the value households in purchasing low quality properties and making exchanges in the low quality tier if the high quality exchange tier provides too little additional value compared to the low quality tier. This underscores the importance of not only adopting a tiered structure in exchange, but also initializing the space bank with a reasonably large pool of high quality properties. However, because gaming is no longer an issue in tiered markets, the market maker can avoid this undesirable pooling outcome simply by increasing n_H. This alternative equilibrium is also presented in Online Appendix A (A.1.3 and A.2.3).

4. Experiments

Our theory generates important predictions for various designs of the secondary matching market. Next, we test Propositions 1–4 and compare the benefits of the tiered structure under RF versus DF using controlled laboratory experiments. Specifically, we explore (1) how our theory performs in predicting agents' behavior and market outcomes, (2) whether subjects learn from their past experience (Camerer and Ho 1999, Camerer et al. 2002), and (3) whether subjects' behavior is impacted by gender and risk attitude (Charness and Gneezy 2010;

Table 3 **Experimental Design**

Market design	RF mechanism		DF mechanism	
2 × 2	4 sessions	4 sessions	4 sessions	4 sessions
Tier-free structure	Markets 1–15 Proposition 1	Markets 16–30 Proposition 2	Markets 1–15 Proposition 3	Markets 16–30 Proposition 4
Tiered structure	Markets 16–30 Proposition 2	Markets 1–15 Proposition 1	Markets 16–30 Proposition 4	Markets 1–15 Proposition 3

Charness et al. 2007; Croson and Gneezy 2009; Eckel and Grossman 2008; Holt and Laury 2002, 2005). Although the theory assumes risk neutrality, we consider risk attitudes and gender effects in the empirical investigation as many strategic decisions in our context are made under uncertainty. We also allow participants to make decisions in repeated markets, and we expect their behavior to converge toward the theoretically predicted equilibrium. Repeated markets also allow us to gather sufficient data from a manageable number of experimental sessions.

Ninety-six participants were recruited from a subject pool in a Southwestern university. There were 16 experimental sessions, each consisting of six participants. The experiment lasted for about 75 minutes, and the individual payment was between \$11 and \$28, with an average of \$18.50. This includes a \$7 participation fee.

4.1. Experimental Manipulations

The experiment had a 2 × 2 design: RF versus DF (between subject) × tier-free versus tiered structure (within subject). The two mechanisms (RF or DF) had a between-subject design, such that for any session we implemented only one of the two exchange mechanisms. There were eight sessions for RF and eight sessions for DF. We broke each session into two disjoint sets of 15 consecutive markets, with one set testing the tier-free structure and the other testing the tiered structure. Thus, each session involved 30 markets in total, starting with one of the tier structures for markets 1–15 and then shifting (with the same group of participants) to the other structure for markets 16–30.[9] We varied the order of the tier structures across sessions to check for potential order effect. The experimental design is summarized in Table 3.

4.2. Experimental Procedure

In each session, six participants were seated in front of computer terminals networked through a *z*-Tree client-server program (Fischbacher 2007). Participants were informed that there was a \$7 participation fee and they would also earn tokens from 30 experimental markets. Earnings would depend on the decisions they and others in the market made. The tokens were converted to monetary payment at the end of the experiment.

In any market, there were three premium households and three value households. A participant's role stayed fixed within a market but was rotated at the beginning of a new market, thus balancing out participants' compensation. Participants learned about the exchange mechanism (RF or DF) for that particular session as well as the tier structure (tiered or tier free) for markets 1–15. They were allowed to go through two trial markets to gain familiarity with the interface. Performance in the trial markets did not affect participants' payment. Any question a participant raised was publicly repeated and answered. Then the first set of markets (1–15) was run. Upon finishing market 15, the participants were notified of the change in tier structure and then went through markets 16–30.

In each market, in line with the setup of our model, every participant made one purchase decision in the primary market and one entry decision in the secondary market. (See Online Appendix B for details.) At the end of each market, participants learned from their computer screen how much profit they made and how many tokens they had accumulated so far.

After the 30 experimental markets, participants made a sequence of paired lottery-choice decisions between what we call "lottery A" and "lottery B." This is a procedure designed to elicit an individual's risk attitude (Holt and Laury 2002). Table 4 illustrates the payoff table presented to participants. In this table, a participant chose lottery A or lottery B in each row, and one row was later selected at random for payout for that participant. Because lottery B involves more extreme payoffs, it is more risky than lottery A. Thus, in each choice (except for choice 1, which involves no uncertainty and serves as a test for participants' understanding of the instructions), there is a trade-off between expected (maximum) payoff and the risk involved. The expected payoff from lottery A becomes greater than the expected payoff from lottery B as one proceeds down the table. Most participants would be expected to switch from B to A on some row in the table, and this switching point can then be used to infer their risk attitude. The logic behind this test is that only risk-loving participants would choose lottery B in the bottom rows, and only risk-averse participants would choose lottery A in

[9] Similar experimental designs were adopted by Kagel and Roth (2000) and McKinney et al. (2005).

Table 4 **Ten Binary Lottery Choices**

Choice	Lottery A	Lottery B
1	$2.00 with probability 1.0; $1.60 with probability 0.0	$3.85 with probability 1.0; $0.10 with probability 0.0
2	$2.00 with probability 0.9; $1.60 with probability 0.1	$3.85 with probability 0.9; $0.10 with probability 0.1
3	$2.00 with probability 0.8; $1.60 with probability 0.2	$3.85 with probability 0.8; $0.10 with probability 0.2
4	$2.00 with probability 0.7; $1.60 with probability 0.3	$3.85 with probability 0.7; $0.10 with probability 0.3
5	$2.00 with probability 0.6; $1.60 with probability 0.4	$3.85 with probability 0.6; $0.10 with probability 0.4
6	$2.00 with probability 0.5; $1.60 with probability 0.5	$3.85 with probability 0.5; $0.10 with probability 0.5
7	$2.00 with probability 0.4; $1.60 with probability 0.6	$3.85 with probability 0.4; $0.10 with probability 0.6
8	$2.00 with probability 0.3; $1.60 with probability 0.7	$3.85 with probability 0.3; $0.10 with probability 0.7
9	$2.00 with probability 0.2; $1.60 with probability 0.8	$3.85 with probability 0.2; $0.10 with probability 0.8
10	$2.00 with probability 0.1; $1.60 with probability 0.9	$3.85 with probability 0.1; $0.10 with probability 0.9

the top rows. A risk-neutral participant would switch from choosing B to choosing A when the expected payoffs are about the same, i.e., she would choose B for the first six rows and A thereafter.

Finally, participants filled out a short checkout form, received their cash payment, were thanked, and left the computer lab. Online Appendix C provides detailed experimental instructions for RF sessions starting with the tier-free structure. The other three sets of instructions were similar.

4.3. Parameter Values and Theoretical Predictions

We selected a single set of parameter values that supports all market equilibria as predicted by Propositions 1–4.[10] To do this, we used the equations that characterize the equilibrium outcomes to compute cutoff points, probability of entry, and added values from exchange to ensure that all conditions in the four propositions are met simultaneously. We summarize these parameter values in Table 5.

Note that we have a strong test of the theory as n_H is set to 1 (i.e., initially there is only one high quality property in the space bank). With a larger number of high quality properties in the space bank, it would become obvious that the premium households should pool with value households on purchasing low quality properties in a tier-free structure. Comparatively, with $n_H = 1$, the benefit of pooling on purchasing low quality properties for the premium households is subtle.

Given the parameters in Table 5, without exchange the expected payoff for value households is 2 ($20 \times 1 - 18$) from purchasing low quality properties and -20 ($70 \times 1 - 90$) from purchasing high quality properties. Even considering the potential exchanges in the secondary market, value households should not purchase high quality properties.

For premium households, without exchange, the expected payoff is 12 ($20 \times 1.5 - 18$) from purchasing low quality properties and 15 ($70 \times 1.5 - 90$) from purchasing high quality properties. However, premium households' optimal purchasing strategy would be determined by the design of the secondary market. For example, tier-free RF markets support the pooling equilibrium proposed in Proposition 1, and we expect unraveling over time to no purchase of high quality properties by premium households; that is, all six households should purchase low quality properties. Moreover, all households should enter the exchange market to try to upgrade. We expect a first exchange from a low quality property to a high quality property, followed by an average of 3.75 exchanges from households' original low quality properties to more desirable low quality properties; that is, 79.17% (4.75 out of 6) of the participants should make successful exchanges.

The tiered RF markets support the separating equilibrium proposed in Proposition 2, so we expect the three premium households to purchase high quality properties and the three value households to purchase low quality properties. Assuming that three households end up in the low (high) quality tier, 88.89% (92.31%) are expected to enter the exchange, and 66.67% (50.00%) should make successful withdrawals.[11]

We also make specific predictions for the DF markets that are in line with Proposition 3 and Proposition 4. Table 5 provides a summary of all theoretical predictions given our parameter values.

4.4. Experimental Results

Across all four market conditions, we have 2,880 observations. For each observation, we know the

[10] The conditions that support Propositions 1–4 are met by a broad range of reasonable parameter values. We use a single set of parameter values in all four experimental conditions so that any difference we observe across conditions in agents' behavior and market outcomes can then be attributed to the design of the market.

[11] Under tiered RF, a household in a given tier should stay out of the exchange when endowed with the maximum possible value in that tier. Under tiered DF, a household should stay out when her state-dependent component s is above the cutoff value in her tier. Table 5 provides percentage predictions on these entry decisions.

Table 5 Parameter Values and Theoretical Predictions for Value and Premium Households

	RF mechanism	DF mechanism
Number of households	$N_V = 3$; $N_P = 3$	$N_V = 3$; $N_P = 3$
Quality of properties	$Q_L = 20$; $Q_H = 70$	$Q_L = 20$; $Q_H = 70$
Prices	$P_L = 18$; $P_H = 90$	$P_L = 18$; $P_H = 90$
Price sensitivity	$\underline{\theta} = 1$; $\bar{\theta} = 1.5$	$\underline{\theta} = 1$; $\bar{\theta} = 1.5$
Initial space bank	$n_L = 2$; $n_H = 1$	$n_L = 2$; $n_H = 1$
s	U[−4, 4] integers	U[−4, 4] integers

Stage 1: Market design	Tier-free RF		Tiered RF		Tier-free DF		Tiered DF	
	Value households	Premium households	Value households	Premium households	Value households	Premium households	Value households	Premium households
Stage 2: Predicted outcome in primary market	Purchase low quality properties	Purchase low quality properties	Purchase low quality properties	Purchase high quality properties	Purchase low quality properties	Purchase low quality properties	Purchase low quality properties	Purchase high quality properties
Stage 3: Predicted entries in secondary market	100% enter the exchange market (6 out of 6)		88.89%[a] enter low quality tier (8 out of 9)	92.31%[a] enter high quality tier (12 out of 13)	100% enter the exchange market (6 out of 6)		78.59%[b] enter low quality tier	72.10%[b] enter high quality tier
Predicted number of exchanges	79.17% (4.75 out of 6)		66.67% in low quality tier (2 out of 3)	50% in high quality tier (1.5 out of 3)	100% (same as the number of entries)		78.59% in low quality tier	72.10% in high quality tier
Predicted number of successful exchanges	79.17% (4.75 out of 6)		66.67% in low quality tier (2 out of 3)	50% in high quality tier (1.5 out of 3)	N/A		N/A	N/A

[a]In the tiered RF markets, 1/9 (1/13) of the value (premium) households are expected to be endowed with the maximum possible value for the property they purchase, and hence should have no incentive to enter the exchange market. The percentages of entries are calculated accordingly.

[b]In the tiered DF markets, households in each quality tier should enter the exchange market when their endowed value is below a cutoff point and should stay out of the exchange market otherwise. The percentages of entry are calculated accordingly.

participant's household type (value or premium), purchase decision, entry decision, and the payoff earned in the market. We also measured risk attitude and identified the gender of each participant.

Observed actions in the primary and secondary markets can be described as one of the following: "buy a high quality property and stay out," "buy a high quality property and enter," "buy a low quality property and stay out" and "buy a low quality property and enter." In Figure 1 (RF markets) and Figure 2 (DF markets) we present the percentages of households who exhibited each type of these actions. To illustrate how participants were learning to adapt to the market, we divide data into three blocks. Block 1 consists of observations from the first five markets (markets 1–5 and 16–20). Block 2 consists of the four markets in the middle (markets 6–9 and 21–24). Block 3 consists of observations from the last six markets (markets 10–15 and 25–30).[12]

Theoretically optimal actions are highlighted with circles along the horizontal axis in Figures 1 and 2.[13] In theory, all households are expected to game the system in a tier-free structure by purchasing low quality properties and entering the exchange market for an upgrade; in a tiered structure, premium (value) households should purchase high quality properties (low quality properties) and the majority of households should enter the exchange market. We can see in the figures that value households' actual behavior was quite consistent with these predictions. However, premium households attempted to game the system much less frequently than predicted in the tier-free structure. Premium households also entered the exchange less frequently than predicted in the tiered structure, especially in markets operating under the DF mechanism.

In what follows, we first describe how agents learn over time. Then, for each household segment, we compare observed behavior with theoretical predictions and explore how risk attitude and gender affect

[12] The three blocks have slightly different numbers of observations. This is because the main focus of our analysis is on the last block where individual decision making and market outcomes have stabilized. We code the last six (instead of five) markets as block 3 so that each of the 96 participants contributed the same number of observations as a premium or a value household. However, dividing the 15 markets equally into three blocks gives us the same results qualitatively.

[13] A circle with double solid lines indicates that in theory 100% of households should choose the action. A circle with a single solid line indicates this is the best action for the majority of households, whereas a circle with dashed line is used for the best action for a small percentage of households. Refer to theoretical predictions in Table 5 for details.

Figure 1 Participants' Decisions in Markets Under RF ($n = 1{,}440$)

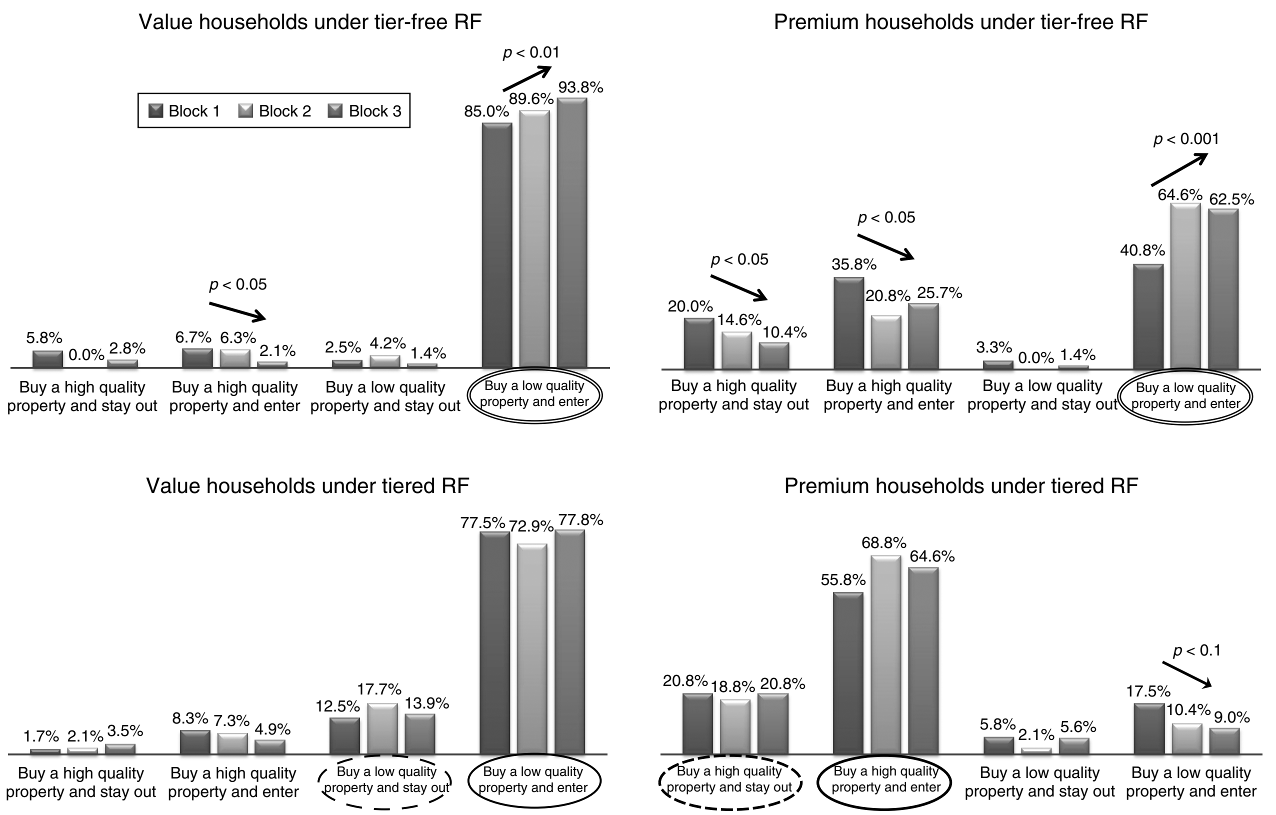

Figure 2 Participants' Decisions in Markets Under DF ($n = 1{,}440$)

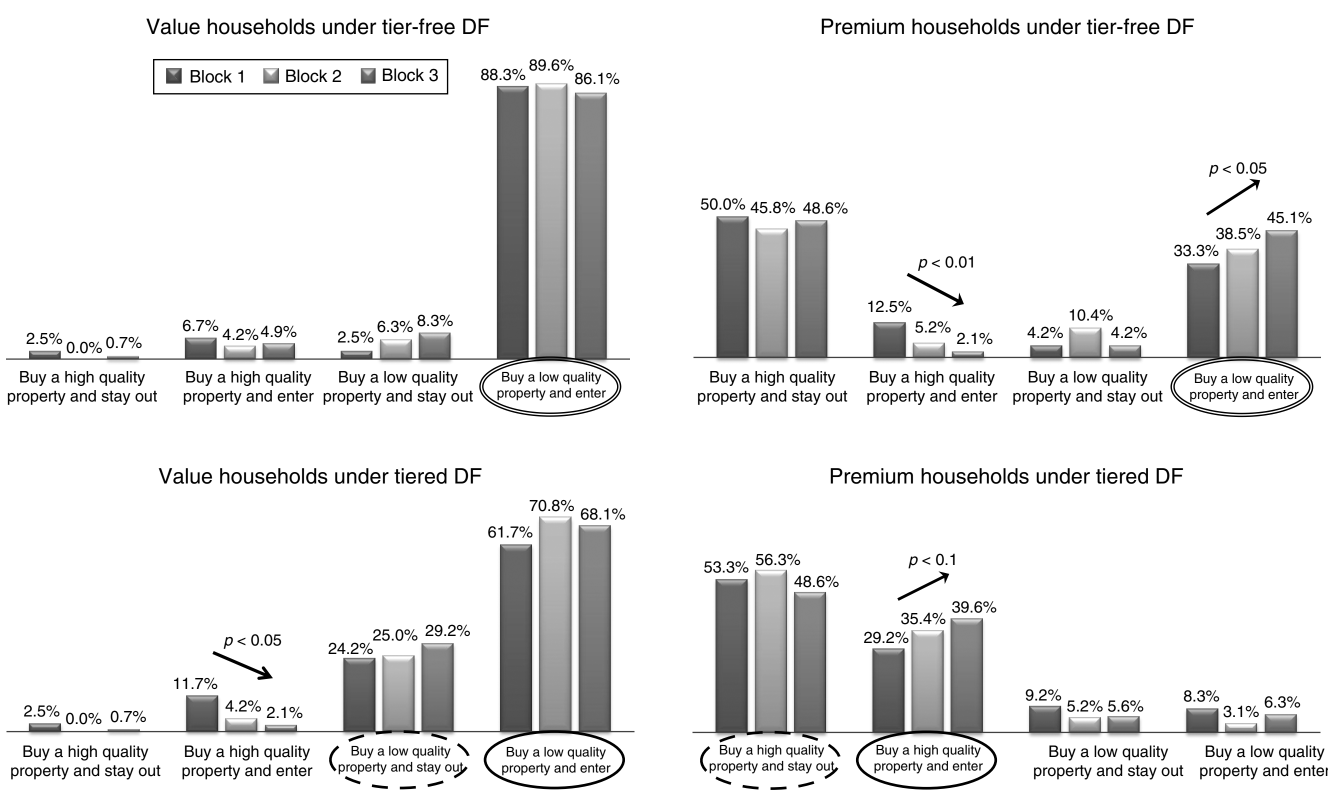

Table 6 Theory vs. Observed Behavior in Block 3 for Value and Premium Households

	RF mechanism				DF mechanism			
	Tier-free RF		Tiered RF		Tier-free DF		Tiered DF	
	Theory	Observed behavior	Theory	Observed behavior	Theory	Observed behavior	Theory	Observed behavior
Value households' behavior ($n = 144$/cell)								
Buy high quality properties and stay out (*mistake*)	0	**2.78**	0	3.47	0	0.69	0	0.69
Buy high quality properties and enter (*mistake*)	0	2.08	0	**4.86**	0	**4.86**	0	2.08
Buy low quality properties and stay out	0	1.39	11.11	13.89 (15.15)	0	**8.33**	21.41	29.17 (30.00)
Buy low quality properties and enter	100	**93.75** (98.54)	88.89	**77.78** (84.85)	100	**86.11** (**91.18**)	78.59	**68.06** (70.00)
Premium households' behavior ($n = 144$/cell)								
Buy high quality properties and stay out	0	**10.42**	7.69	**20.83** [**24.39**]	0	**48.61**	27.90	**48.61** [**55.12**]
Buy high quality properties and enter (*mistake in tier-free DF*)	0	**25.69**	92.31	**64.58** [**75.61**]	0	2.08	72.10	**39.58** [**44.88**]
Buy low quality properties and stay out	0	1.39	0	**5.56**	0	4.17	0	**5.56**
Buy low quality properties and enter	100	**62.50** (97.82)	0	**9.03**	100	**45.14** (**91.54**)	0	**6.25**

Notes. All numbers are percentages. Percentages in bold are significantly different from theoretical predictions ($p < 0.05$). Numbers in parentheses indicate entry decisions *conditional on purchasing a low quality property* (Q_L), i.e., the theoretically optimal purchase decision given the market condition and the role of participants. Numbers in square brackets indicate entry decisions *conditional on purchasing a high quality property* (Q_H), i.e., the theoretically optimal purchase decision given the market condition and the role of participants.

choices.[14] Finally, we compare market-level performance across different market conditions.

4.4.1. Learning. Across all four market conditions, participants made relatively few choices that led to losses. Moreover, such costly mistakes declined over time. For example, for premium households in tier-free DF markets, the action "buy a high quality property and enter" resulted in an average loss of 33 tokens. For these participants, a logistic regression with *buy a high quality property and enter* as the dependent variable and *block* as the independent variable shows a lower likelihood of choosing this action in later blocks ($z = -3.11$, $p < 0.01$). We run similar regressions for all four actions in each condition. Statistically significant findings are indicated in Figures 1 and 2. Overall, the patterns confirm that participants understood the market and learned to avoid costly mistakes.

We also find that participants chose optimal strategies more frequently as they gained more experience. For example, for premium households in the tier-free RF market, a logistic regression with *buy a low quality property and enter* as the dependent variable and *block* as independent variable shows that participants were more likely to choose this optimal strategy in later blocks ($z = 3.80$, $p < 0.01$).

Learning is also present when we check for order effects. Participants who began with a tier-free structure were more likely to make mistakes in tier-free structure (i.e., the first set of markets for them) than those who began with a tiered structure, and vice versa.[15] These analyses consistently show that participants exhibited reasonable understanding and learning in all markets. Over time they made fewer mistakes and were more likely to behave optimally.

4.4.2. Comparison Between Observed Behavior and Theory. Next, we focus on data from block 3 (last six markets) where participants' behavior had stabilized. In Table 6 we compare observed behavior with theoretical predictions.

Consistent with theory, on average only 5.38% of value households purchased high quality properties

[14] Because each individual participant made multiple decisions, in all individual-level data analyses we cluster standard errors at the participant level.

[15] For instance, for value households an analysis of variance with *purchase decision* as the dependent variable and *tier structure*, *order*, and their interaction as independent variables reveals that the interaction term has a significant effect in block 1 ($F = 7.48$, $p < 0.01$). This is because 15.83% (5.83%) of value households who began with a tier-free (tiered) structure purchased Q_H in the tier-free structure, whereas 9.17% (15.00%) of them made the same costly mistake in the tiered structure. However, this interaction is not found in block 2 or block 3 ($p > 0.1$), suggesting that the order effect diminished over time. As residual order effect could be present in certain markets, we control for it in all analysis that follows.

(although $5.38\% > 0$, $p < 0.05$), and the probabilities of making this mistake were not statistically different across the four market conditions. We have the following findings about value households who purchased low quality properties.

First, entry by value households was consistent with theory (i.e., 100% entry) with the only exception of tier-free DF markets, where value households were less likely to enter the exchange than predicted ($91.18\% < 100\%$, $p < 0.05$). This could be due to the substantial risk involved in this particular market, which deterred entry even for owners of low quality properties.

Second, logistic regressions reveal that value households were much more likely to enter the exchange in a tier-free market than in a tiered market ($z = -3.39$, $p < 0.001$ in RF; $z = -4.80$, $p < 0.001$ in DF).[16] This is evidence showing that value households indeed attempted to game the tier-free markets under both exchange mechanisms.

Third, value households entered exchange more frequently under tier-free RF than tier-free DF (98.54% versus 91.18%, $z = 2.37$, $p < 0.05$), controlling for all other relevant factors. This is an interesting finding not predicted by theory, implying that with a tier-free market structure the RF mechanism invites more gaming than the DF mechanism among value households.

Overall, value households' behavior was quite consistent with theoretical predictions: although the tier structure did not affect their purchase decision, it had a significant effect on entry decisions. Evidently, the value households entered the tier-free markets much more frequently because there was the potential for upgrading.

Premium households' behavior was in line with theory, with some deviations. As theory predicts, premium households' purchase decisions were significantly different in tiered versus tier-free structures ($z = 5.61$, $p < 0.01$ in RF; $z = 4.32$, $p < 0.01$ in DF). Whereas 63.89% (49.31%) purchased low quality properties in the tier-free RF (DF) markets, very few purchased low quality properties when a tiered structure was in place. Therefore, the tiered structure curbed gaming and helped to restore primary sales in the premium segment of the market. This finding renders strong support for our theory. Moreover, consistent with what we find among value households, premium households gamed the system more in tier-free RF markets than in tier-free DF markets (62.50% versus 45.14%, $z = 1.69$, $p < 0.10$), suggesting that the tier-free RF mechanism was more likely to invite attempts to game the system.

On the other hand, we find that the premium households gamed the system in tier-free markets less than predicted (62.50% in tier-free RF; 45.14% in tier-free DF; theoretical prediction is 100% for both). This could be due to the fact that, unlike value households whose dominating strategy was straightforward, premium households expected a higher payoff from purchasing a high quality property (15) than purchasing a low quality property (12) if they did not consider any potential exchange. Furthermore, in the tiered structure most premium households purchased high quality properties as predicted by theory, but they entered the exchange market less frequently than predicted ($75.61\% < 92.31\%$, $p < 0.05$ in tiered RF; $44.88\% < 72.10\%$, $p < 0.05$ in tiered DF). This is partly due to participants' risk attitudes, as we show in §4.4.3.[17]

To summarize, analysis on individual behavior reveals that the tiered structure restored purchase of high quality properties in the premium segment by preventing gaming in both household segments. Interestingly, we also find that in tier-free markets, RF invited more gaming than DF from both types of households.

4.4.3. Effects of Risk Attitude and Gender. On average, our participants made 5.6 safe choices (i.e., lottery A in Table 4) reflecting slight risk aversion as classified by Holt and Laury (2002), which is consistent with findings in previous studies (e.g., Goeree and Holt 2004, Goeree et al. 2003). This measure of risk attitude was not significantly different for our male versus female participants ($t = 1.34$, $p = 0.18$), although we expected males to be significantly more risk seeking. This could be due to self-selection or the imbalanced number of females versus males in our sample (35 versus 61). We also did not have other controls (such as age, race, and professional experience) that were shown to impact risk attitude in predictable ways (Croson and Gneezy 2009, Dwyer et al. 2002, Finucane et al. 2000).

Using data from block 3, we closely examine the effects of risk attitude and gender on individual participants' choice of strategies across four different market conditions. For value households, we run logistic regressions to analyze how risk attitude and gender impact households' entry decisions conditional on their purchase of low quality properties and controlling for the initial value of their own property. For premium households, analysis is different depending on the tier structure. In tier-free markets, our focus is on the gaming behavior (i.e., purchasing low quality properties and then

[16] In these regressions, we controlled for the potential effect of order, original value of property, risk attitude, and gender.

[17] Loss aversion may also explain why premium households who purchased high quality properties would stayed out of the match more often than theoretical predictions. Given our data, we are not able to tease apart the roles of risk aversion and loss aversion in explaining such behavior.

Table 7 Effects of Risk Attitude and Gender in Block 3 for Value and Premium Households

	RF mechanism		DF mechanism	
	Tier-free RF	Tiered RF	Tier-free DF	Tiered DF
Value households Gender effect	Insignificant ($p > 0.70$)	Insignificant ($p > 0.80$)	Insignificant ($p > 0.90$)	After purchasing a low quality property: *Male* more likely to enter the DF exchange market ($p < 0.05$)
Risk effect	*Risk-averse* participants more likely to “game” the RF market (buy a low quality property and try to upgrade) ($p < 0.001$)	After purchasing a low quality property: *Risk-averse* participants more likely to enter the RF exchange market ($p < 0.01$)	Insignificant ($p > 0.30$)	After purchasing a low quality property: *Risk-seeking* participants more likely to enter the DF exchange market ($p < 0.01$)
Premium households Gender effect	Insignificant ($p > 0.80$)	Insignificant ($p > 0.70$)	*Male* more likely to “game” the DF market (buy a low quality property and try to upgrade) ($p < 0.01$)	After purchasing a high quality property: *Male* more likely to enter the DF exchange market ($p < 0.10$)
Risk effect	Insignificant ($p > 0.30$)	After purchasing a high quality property: *Risk-averse* participants more likely to enter the DF exchange market ($p < 0.05$)	*Risk-seeking* participants more likely to “game” the DF market (buy a low quality property and try to upgrade) ($p = 0.053$)	After purchasing a high quality property: *Risk-seeking* participants more likely to enter the DF exchange market ($p < 0.01$)

Notes. For value households we only include observations where participants purchased low quality properties because purchasing high quality properties is considered a mistake for this role. Note that by doing so we are considering a large majority of the observations. For premium households under tiered structure, we only include observations where participants purchased high quality properties. Note that by doing so we are considering a large majority of the observations.

entering the exchange). Therefore, we run logistic regressions to estimate whether risk attitude and gender are significant predictors of such gaming. In tiered markets, purchasing low quality properties is not beneficial for these households. Thus, we analyze the effects of risk attitude and gender on entry decisions conditional on purchasing high quality properties and controlling for the initial value of their own property. Table 7 presents our main findings.

As Table 7 indicates, risk attitude affects individual decisions differently under different exchange mechanisms. Recall that RF involves no risk for entrants, whereas DF does. Therefore, as expected, the general observed pattern is that risk aversion leads to *more* entry into the RF markets (more gaming for tier-free RF) but *less* entry into the DF markets. Normally, as RF invites more participation, it is considered more preferable over DF. But when the possibility of gaming the system exists, it may be beneficial to have a market that is somewhat risky to enter, because this discourages gaming. This is exactly what we observe in our experimental markets.

Interestingly, gender seems to impact decisions only in the DF markets (where there is substantial risk of trading for a worse property), and its effect is in line with the prevailing knowledge about the relationship between risk preference and gender. Specifically, male participants (who are documented as being more risk seeking in the literature; e.g., Croson and Gneezy 2009, Powell and Ansic 1997) were more likely to game the system by purchasing low quality properties and attempting to upgrade when they took the role of premium household in the risky tier-free DF markets. Furthermore, no matter which role they took, male participants were more likely to enter the risky tiered DF exchange markets. This seems to suggest that male are more willing to take risks in the context we study.[18]

4.4.4. Test of Theory at the Market Level. Now we focus on the most direct test of our main proposal that the tier-free structure would cause primary sales to collapse such that sales revenue would be much lower, compared to the tired structure, which encourages the two types of households (1) to separate in their purchase so that total sales revenue improves and (2) to participate in exchanges in the secondary market. Moreover, experimental findings from both household segments indicate that gaming

[18] This suggests that gender has an added explanatory power, over the Holt and Laury (2002) measure, that is consistent with males being more risk seeking than females.

Table 8 Comparison of Market Outcomes Across Four Market Designs

Market outcome variables	(1) Tier-free DF	(2) Tiered DF	(3) Tier-free RF	(4) Tiered RF	(5) Paired t-test for (1) vs. (2) (df = 7) (p-value) Signed rank (p-value)	(6) Paired t-test for (3) vs. (4) (df = 7) (p-value) Signed rank (p-value)	(7) 2-sample t-test for (1) vs. (3) (df = 14) (p-value) Rank-sum (p-value)	(8) 2-sample t-test for (2) vs. (4) (df = 14) (p-value) Rank-sum (p-value)
Primary seller revenue	229.5	304.5	196.5	310.5	$t = $ **−3.64** **(0.008)** $S = $ **−16** **(0.023)**	$t = $ **−6.44** **(< 0.001)** $S = $ **−18** **(0.008)**	$t = $ **2.18** **(0.046)** $Z = $ **−1.754** **(0.080)**	$t = -0.38$ (0.706) $Z = 0.588$ (0.566)
Total consumer surplus	112.5	61.0	111.4	56.6	$t = $ **18.96** **(< 0.001)** $S = $ **18** **(0.008)**	$t = $ **21.32** **(< 0.001)** $S = $ **18** **(0.008)**	$t = 0.40$ (0.696) $Z = -0.788$ (0.431)	$t = 1.51$ (0.152) $Z = -1.313$ (0.209)
Total social surplus	417.0	503.0	382.9	504.6	$t = $ **−4.21** **(0.004)** $S = $ **−17** **(0.016)**	$t = $ **−7.24** **(< 0.001)** $S = $ **−18** **(0.008)**	$t = $ **2.51** **(0.025)** $Z = $ **−2.048** **(0.041)**	$t = -0.10$ (0.920) $Z = 0.368$ (0.178)
Number of entries	4.1	3.5	5.5	4.7	$t = $ **3.09** **(0.018)** $S = $ **10.5** **(0.031)**	$t = $ **5.09** **(0.001)** $S = $ **18** **(0.008)**	$t = $ **−7.39** **(< 0.001)** $Z = $ **3.275** **(0.001)**	$t = $ **−4.38** **(< 0.001)** $Z = $ **3.217** **(0.006)**
(*Predicted number of entries*)	(6)	(4.25)	(6)	(5.44)	—	—	—	—
Number of successful exchanges	3.4	2.8	3.7	2.9	$t = $ **2.76** **(0.028)** $S = $ **14.5** **(0.047)**	$t = $ **3.47** **(0.010)** $S = $ **16** **(0.031)**	$t = -1.07$ (0.305) $Z = 0.951$ (0.342)	$t = -0.46$ (0.656) $Z = 0.266$ (0.790)
(*Predicted number of successful exchanges*)	(N/A)	(N/A)	(4.75)	(3.5)	—	—	—	—

Notes. Tests are at the session aggregate level for the last six markets (block 3). The number of observations is 8 in every cell. Tests are paired (within-subject) for tier-free vs. tiered structure within an exchange mechanism (DF or RF). Tests are two-sample (between-subjects) for DF vs. RF under a given tier structure because each participant experienced either DF or RF, but not both. A t corresponds to the t-test statistic, S corresponds to the Wilcoxon signed rank test statistic, and Z corresponds to the Mann–Whitney–Wilcoxon two-sample rank-sum score (normal approximation). Bold numbers indicate significant differences.

in the tier-free structure is more severe under RF than under DF. Thus, it is critical to compare the market-level performance of RF and DF, and to explicitly measure the benefits of introducing a tiered structure under both mechanisms. We do so by using data from block 3 (last six markets) to compare primary seller revenue (sum of prices paid by all six participants in a market), total consumer surplus (sum of earnings for all six participants in a market), total social surplus (sum of primary seller revenue, total consumer surplus, and the remaining value of space bank properties[19]), number of entries (number of participants who entered the exchange market), and number of successful exchanges (number of participants who ended up with a better property than the original one) across different market designs. The summaries are in Table 8.[20]

[19] Properties that remain in the space bank should be included in calculating social surplus under the assumption that they have an expected rental value for the market maker.

[20] We replicated the results of Table 8 using only the data from the first 15 markets of each session. Refer to Online Appendix D for details.

From Table 8, we see that primary seller revenue is significantly higher in tiered structure than in tier-free structure under both DF ($p < 0.01$) and RF ($p < 0.01$). In particular, tier-free RF performs the worst among all four designs ($p < 0.05$). On the flip side, total consumer surplus is higher in tier-free structure (because they pay less there) under both DF ($p < 0.01$) and RF ($p < 0.01$). Combining these effects, total social surplus is significantly higher in tiered structure than in tier-free structure for both DF ($p < 0.01$) and RF ($p < 0.01$). Total social surplus is the lowest under tier-free RF ($p < 0.05$). These results, collectively, are consistent with the higher level of gaming we observe in tier-free markets (especially in tier-free RF markets) at the individual level.

Although across the board we observe slightly fewer entries and successful exchanges than theoretically predicted, the overall pattern supports the theory quite well: There are more entries into the exchange market under RF than under DF ($p < 0.01$ in both tiered and tier-free structure), and more entries in tier-free than in tiered structure ($p < 0.05$ under both DF and RF). There are also slightly more successful exchanges in tier-free than in tiered structure. However, there is still a significant amount of successful exchanges in markets with tiered structure, suggesting that the restoration of primary sales is achieved without adversely affecting exchanges to a great extent.

5. Conclusions

When secondary markets provide upgrade opportunities, incentives to purchase high quality properties diminish. Some households may purchase low quality properties with an intent to upgrade in the secondary market—a strategy we referred to as "gaming the system." By adopting a tiered approach that restricts gaming, the market maker can facilitate exchanges for property owners without reducing primary sales of higher quality properties.

In the present investigation, we focused on pooling equilibria resulting in the tier-free exchange markets and separating equilibria in the tiered exchange markets. These are not the only possible equilibria that could emerge in such markets. Alternative outcomes such as separating equilibria in the tier-free markets and pooling equilibria in the tiered markets may emerge. However, we showed that any design that involves the tier-free structure will have undesirable ramifications one way or the other, because the initial pool of properties in the space bank cannot simultaneously satisfy the requirements for both primary and secondary markets. On the one hand, high quality properties are needed in the initial space bank to allow owners of such properties to make exchanges, but this will invite gaming and hurt primary sales. On the other hand, if the initial space bank only includes very few high quality properties, then the premium (or less price-sensitive) households will choose to invest in quality. But after the purchase, they will have little or even no opportunity for exchanges. Both are undesirable outcomes from the marker maker's perspective. The tiered structure effectively limits upgrading, so that the market maker can easily separate different types of households in the primary market as well as give all property owners the opportunity for making exchanges. Combining these insights, we argue that, in general, a tiered structure will outperform a tier-free structure in the context we study.

From a theoretical perspective, both the tier-free structure and the RF exchange mechanism can potentially encourage property owners' participation in the secondary market, and higher participation in matching usually leads to higher market efficiency. In this sense, the tier-free structure and the RF mechanism should both be considered desirable features in market design. However, we show that when forward-looking agents strategically choose their initial entitlement *given the design of the matching market*, these features can become problematic because they invite agents to game the system. More generally, our work highlights the fact that a market design that seems desirable for the secondary matching market alone can in fact distort decisions in a previous stage and thus becomes undesirable.

We investigate how well the theory predicts individual behavior and market outcome using a series of controlled laboratory experiments. In these markets, participants gradually move toward equilibrium predictions as they gain more experience. We find some deviation from theory, especially in the premium segment, which is partially explained by risk aversion and gender. Most importantly, our main argument regarding the tier structure is strongly supported by experimental data. Under both the request first and the deposit first mechanisms, we find evidence that both household segments intentionally game the system when a tier-free structure is in place. In particular, participants in the premium segment tend to purchase high quality properties much less frequently in tier-free markets than in tiered markets. At the market level, primary seller revenue and total social welfare are significantly higher in tiered markets than in tier-free markets.

Interestingly, we also find evidence that the benefit of adopting a tiered structure is greater under the request first mechanism than under the deposit first mechanism. With a tier-free structure, the incentive compatible request first mechanism performs significantly worse than the non-incentive-compatible deposit first mechanism, because the request first mechanism does not involve any risk of downgrading, thus inducing more gaming behavior from all participants.

The working example in the analysis throughout was fractional ownership markets such as timesharing of vacation homes. In a sense, this is a fairly narrow example of a much broader set of insights regarding the design of matching markets in the presence of a previous property acquisition stage. Our insights could easily be applied to most if not all one-sided matching applications, including housing and course allocation. However, fractional ownership markets are where we see tiered systems being the most prevalent, and the RF and DF systems being commonly adopted. Thus, the application of the current insights to those markets would be most direct. Specifically, match-making firms should consider operating tiered programs that restrict upgrading if such programs are not already used. Although we compared a single-tier market to a two-tiered market, when the range of quality is wider, managers may prefer to introduce more tiers to discourage upgrading activities that might hurt primary sales and the industry as a whole. If they decide not to pursue a tiered approach, then they should consider a DF mechanism over an RF mechanism because DF discourages risk-averse participants from gaming the system, whereas RF does not. Overall, managers should be made aware of the importance of market design in the secondary market and how it affects outcomes in both primary and secondary markets. With a carefully designed market, firms can encourage purchase as well as exchange, boosting the profitability of their fractional ownership programs.

This research can be extended in many ways. First, a possible complicating factor is that with uncertainty about future preferences and the possibility of exchanging one's property at a later stage, households might prefer to obtain a higher quality property due to its exchange value rather than its expected intrinsic value to them. In other words, preference for flexibility (Kreps 1979, Dekel et al. 2001) might be driving choices in the primary market. In the present framework, we keep the preference shocks sufficiently small so that a household would never wish to downgrade a high quality property to a lower quality one in a future state, so the effect of flexibility on purchase decision does not take place in our setting. This feature keeps the analysis tractable and allows us to focus on the conflict between the primary market and the secondary market. Future research can explore how preference for flexibility affects purchases and the subsequent exchanges.

Moreover, larger preference shocks that translate to an overlap of tiers also suggest that households' preferences at the time of consumption are more heterogeneous. Forcing a tiered structure in this type of market can make mutually beneficial exchanges impossible. This can cause a serious conflict between market efficiency and primary seller profitability.

Second, under our framework, one could further explore the optimal level of resources a firm should make available in the space bank, incorporating an explicit transaction fee and holding cost (or opportunity cost) for each unit of property in the space bank.

Third, there are other mechanisms being utilized in exchange markets that could be investigated. For example, some markets are based on a "point" system, or a combination of one-to-one exchange with points as a side transfer.

Finally, in our model the market maker is a downstream channel member of the firms that operate in the primary sales market. If the market maker is also the seller of properties or if the market maker is sufficiently large in size to affect market clearing prices of the properties, then the prices of properties would likely be endogenous to the optimization problem. Endogenous prices would mean that not only quantity demanded of high quality properties would drop, but also prices of high quality properties would fall and prices of low quality properties would rise if gaming were possible (i.e., the demand curve would shift). From the primary seller's perspective, the demand curve shift is undesirable, so incentive compatibility conditions would be violated either way. Thus, the present abstraction of exogenous prices seems to capture the essence of the problem, although future research could model the pricing decisions to explore the effect on acquisition and subsequent matching outcomes.

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