Management Science

Revenue Sharing in Airline Alliances

Xing Hu, René Caldentey, Gustavo Vulcano,

Please scroll down for article—it is on subsequent pages

Revenue Sharing in Airline Alliances

Xing Hu

Lundquist College of Business, University of Oregon, Eugene, Oregon 97403, xingh@uoregon.edu

René Caldentey, Gustavo Vulcano

Leonard N. Stern School of Business, New York University, New York 10012
{rcaldent@stern.nyu.edu, gvulcano@stern.nyu.edu}

We propose a two-stage game-theoretic approach to study the operations of an airline alliance in which independent carriers, managing different reservation and information systems, can collaboratively market and operate codeshare and interline itineraries. In the first-stage game, airlines negotiate fixed proration rates to share the revenues generated by such itineraries. In the second-stage game, airlines operate independent inventory control systems to maximize their own expected revenues. We derive a revenue-sharing rule that is (i) an admissible outcome of the first-stage negotiation, in the sense that no airline coalition has enough incentives to secede from the grand alliance, and (ii) efficient for the second-stage game, in the sense that the decentralized system can achieve the same revenues as a central planner managing the global alliance network. Our numerical study shows that the proposed proration rates can lead to a significant increase in revenues with respect to other rules commonly used in practice. Finally, because our proposal requires the disclosure of private demand information, we introduce a simple alternative rule that is based on public information. This heuristic performs remarkably well, becoming an interesting candidate to be pursued in practice.

Key words: revenue management; capacity control; contract design; cooperative game theory; Nash equilibrium
History: Received November 3, 2010; accepted April 25, 2012, by Martin Lariviere, operations management.
 Published online in *Articles in Advance* November 28, 2012.

1. Introduction

An *airline alliance* is an agreement between multiple independent partners to collaborate in various activities to streamline costs (e.g., by sharing sales offices, maintenance facilities, ground handling personnel, check-in and boarding staff, etc.) while expanding global reach and market penetration. The presence of alliances in the airline industry has followed an increasing trend since the first large airline alliance was formed in 1989 between Northwest and KLM. By March 2009, the three major alliances combined (Star, Sky Team, and Oneworld) flew nearly 73% of all passengers worldwide. The aggregate number of members evolved from 33 in 2003 to 52 in 2010. As the airline revenue management (RM) practice within the boundaries of a company is acknowledged to generate incremental revenues when switching inventory controls from leg-based to origin-destination (O-D)-based (2%–7% according to Vinod 2005), the formation of alliances scales up this benefit. At the same time, with the advent of airline alliances, the practice of O-D RM has become even more challenging.

One of the fundamental building blocks of an airline alliance is the ability to offer codeshare and interline flights. *Code sharing*, or *codeshare*, is the practice of one airline (the *marketing carrier*) marketing and selling its own itineraries and services on flights that are actually operated by a different airline (the *operating carrier*). *Interlining* permits a carrier to ticket another airline on its own ticket stock, leading to agreements that can be unilateral or bilateral. For example, within Oneworld, a one-stop itinerary from Buenos Aires (EZE) to Chicago (ORD) is operated by two airlines with the first leg Buenos Aires (EZE) to Miami (MIA) operated by LAN Airlines and the second leg MIA to ORD operated by American Airlines (AA). This itinerary is offered under both a codeshare agreement, where LAN is the marketing carrier of the AA flight but both legs are sold as LAN flights, and under an interline agreement, where LAN sells a sequence of two legs: a LAN leg and an AA leg. Thus, an itinerary as a combination of flight legs, may be operated by two or more companies, so that a request for a ticket will be accepted only if all airlines operating the legs agree to do so. The airlines involved generate revenues, and a transfer price is paid by the marketing airline to the operating airline(s) for the consumption of capacity. To illustrate the magnitude of the codeshare/interline practice, its revenues at Oneworld reached US$2.2 billion in 2010, an increase of 17% year-on-year. For some Oneworld members, these multicarrier revenues

account for more than 10% of the revenues they generate from overall passenger activities.

Within alliances, each airline member wants to maximize its own revenues by ensuring the optimal traffic flow in the global network, whose segments are now operated by several carriers. In the current RM practice, reservation systems of alliance members are still quite independent because of organizational and technical discrepancies, alliance exit-options, and revenue sharing and antitrust immunity considerations. Therefore, airline alliances trigger the need for the design of incentive and coordination mechanisms. In this paper we focus on contracts among airline partners that define the transfer prices between the marketing and operating carriers. These contracts should induce the right balance between the share per ticket and the volume of interline traffic to avoid situations such as a very asymmetric fare split, where the weak partner could often be the bottleneck for accepting interline or codeshare flow.

The rules used in practice to split revenues include static proration rates based on relative local fares published by the codesharing or interline partners, or on relative mileage with corrections for short feeder/defeeder flights. Airlines sometimes have joint ventures where not every single ticket is prorated but the collected money is distributed using some aggregated key figure such as available seat miles (ASM) per partner. There are also a few examples of bid price sharing implemented (or being considered) where the partners distribute the revenues not by fixed proration rates but by dynamic bid price values. Because this requires a major level of trust among partners, such methods are usually implemented between partners that have a deeper level of integration than the typical alliance relationship (e.g., between Lufthansa and Swiss, or between KLM and Air France). There are also IT incompatibility and legal barriers such as antitrust legislation in the United States to broadly expand the bid price exchange practice. In summary, several static and dynamic transfer price schemes have been proposed and it is unclear which will prevail in the future. Nevertheless, because of their widespread practice and simplicity, we focus on static proration rates in this paper.[1]

Although the revenue side of an airline alliance has been recently addressed in the literature, to our best knowledge this is the first paper that theoretically

Figure 1   **Tactical and Operational Levels of the Airline Alliance Game**
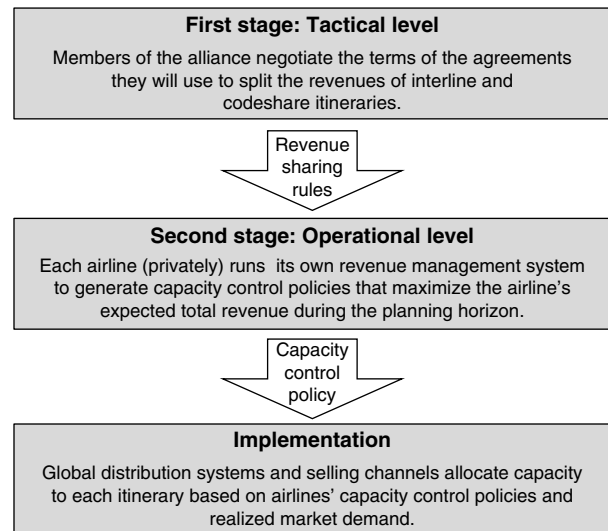
analyzes and characterizes equilibrium strategies for alliance members under incomplete information and various revenue-sharing parameters in a general network topology. Specifically, we attempt to characterize static proration rates that are simultaneously *efficient* from the perspective of the alliance (i.e., they maximize global revenues) and *admissible* in terms of providing enough incentives to individual airlines to sustain them as a contract. To this end, we model the airline alliance game using a two-stage hierarchical game theoretical approach, which is schematically depicted in Figure 1.

In the first stage (tactical level), we focus on admissibility. We use a cooperative game framework to model the output of the negotiation in which airlines decide the fixed proration rules that they will use to split the revenues of interline and codeshare itineraries. Our choice of a cooperative game to model this negotiation is consistent with what is arguably the main reason to form alliances, namely, the opportunity to collaborate with other carriers and share resources to increase revenues and reduce costs. The admissible sharing rule resulting from this negotiation must allow for an incentive compatible split of revenues so that there is no strict subset of airlines (i.e., a coalition) that can collectively obtain higher revenues by leaving the alliance to form an independent suballiance.

In the second stage (operational level), we consider efficiency by modeling the operation of the alliance as a noncooperative game in a decentralized network. In other words, we assume that once a fixed revenue-sharing rule has been selected in the first stage, each airline independently and privately manages its own reservation system so as to maximize its expected total revenue during the planning horizon. In our

---

[1] The creation of a Prorate Agency in the context of the International Air Transport Association (IATA) is an indicator of the interest in these types of agreements. The agency maintains the passenger and cargo prorate agreements on behalf of airlines signatory to the Prorate Agency Agreement. Part of the function of the Prorate Agency—since September 2008—is to publish a Prorate Manual Passenger (PMP) quarterly, which includes agreed weighted mileage formulas.

stylized model, each airline controls static partitioned booking limits (SBLs), which split the capacity among the different itinerary-fare-class combinations (products). Different from the traditional, monopolistic RM models, here the airline is allowed to share capacity with alliance partners to maximize expected revenues. To capture the interaction among players in the second stage in this decentralized network, we focus on strategies that are Nash equilibria. Technically, to solve the second-stage noncooperative game, we decompose the big fixed-point problem with multiple players into a collection of independent and simple subproblems. The Nash equilibrium that will be achieved depends on each airline's beliefs about other airlines' willingness to collaborate. Next, proceeding backward to stage one in our hierarchical approach, we derive closed-form fixed proration rates that can achieve the same performance of the centralized system, i.e., of a system run by a central planner that controls the entire global network. We provide intuition for why revenues should be shared in such a way among alliance members so that the system can perform optimally.

Despite the fact that the SBLs that we use in the analysis of the second stage are not used in practice, we consider them here to ensure tractability in the selection of proration rates that are efficient among the admissible rates explored in the first stage. We point out though that our numerical tests and final assessments are based on industry-based capacity control mechanisms such as bid prices and displacement adjusted virtual nesting (DAVN). Thus, our results can be viewed as providing a theoretical support for the use of simple fixed proration rates as an implementable revenue-sharing rule. One caveat is that our proposed proration rates require some degree of private information sharing among the partners, which could raise concerns about its practical feasibility. We circumvent this by providing an approximation that is based on publicly available fares and that exhibits small losses in our numerical examples.

Finally, note that our model and results can be extended beyond the scope of airline alliances to systems satisfying the following conditions: (a) multiple agents own sets of resources with finite capacity, (b) products are defined as subsets of resources, and agents must decide how much of each resource capacity under their control to allocate to these products, and (c) the production (supply) of a given product is determined by the minimum capacity that is allocated to the set of resources required in its production.

The remainder of this paper is organized as follows. We review the literature in §2. In §3, we describe our static network model, formulate the two stages of the game, and discuss the underlying assumptions.

Section 4 exposes the analysis of the two-stage hierarchical game, starting backward from the noncooperative inventory control game that takes place in the second stage, and following with the selection of proration rates in the first stage. We also present the heuristic proration rule that is based exclusively on publicly available fares. In §5, we numerically test our optimal and heuristic proposals under static and dynamic capacity control policies. We present our conclusions in §6. Proofs can be found in §A.1 in the online appendix (available at http://pages .uoregon.edu/xingh).

2. Literature Review

A distinguishing feature of our research is the two-stage approach combining cooperative and noncooperative game theory. Within the broad area of operations management, noncooperative game theory has been extensively used in supply chain management contexts. However, the use of cooperative game theory is much less prevalent (e.g., see Cachon and Netessine 2004, Nagarajan and Sošić 2008) and has been mainly related to inventory centralization. The use of two-stage games melding both approaches has been addressed in the biform games analyzed in Brandenburger and Stuart (2007). However, the sequence is reversed there: The first stage is noncooperative and is designed to describe strategic moves of the players. Each profile of strategic choices at the first stage leads to a second-stage cooperative game. The underlying justification for this order is the formalization of the idea that business strategies shape the competitive environment, and thereby the payoffs of the players. In our case, the players first agree on a competitive environment (through a revenue-sharing rule), and then operationalize the rules in a decentralized operation of their reservation systems. One of the novel aspects of our work is the application of both cooperative and noncooperative game theoretical approaches to the analysis of a RM problem.

Quantity-based, network RM involves controlling a fixed and perishable capacity of a set of resources over a finite horizon with the objective of maximizing revenues. Products are defined over the network, spanning one or more resources (i.e., legs). There is extensive literature on monopolistic RM, where a single airline owns all resources, and a variety of static and dynamic (i.e., depending on remaining time and capacity) control methods have been proposed. We suggest books by Talluri and van Ryzin (2004b) and Phillips (2005) for in-depth coverage of this work.

In contrast, the existing literature on airline alliance collaboration and operation is quite limited. A major challenge faced by academic researchers and industry practitioners is the conflict between alliance initiatives

to collaboratively manage codeshare and interline itineraries, and the independent control of individual airline reservation systems, as pointed out in overview papers by Boyd (1998) and Vinod (2005). Because a perfect integration of revenue management and reservation systems of alliance members is impractical and may be subject to regulatory constraints, airline alliances behave like decentralized systems at the operational level. However, little attention has been paid to their game theoretical nature.

Netessine and Shumsky (2005) is one of the first papers to investigate the effect of horizontal and vertical competition in an airline revenue management setting. In the horizontal game, airlines are in direct competition on a given O-D pair through parallel flights, and seats on different flights are substitute resources. In the vertical game, each airline operates a different leg in a network of interconnected flights. Thus, in this case, seats on different legs become complementary resources. For both types of competition, Netessine and Shumsky (2005) analyze a noncooperative game with two fare classes per route (high and low), where each airline selects the booking limit for low fare reservations. For the most part, the analysis is restricted to the case of two airlines, mainly because of the complexity of modeling demand substitution effects (passenger overflow) in the horizontal game. The results in Netessine and Shumsky (2005) show that under some mild assumptions on the demand distribution a pure-Nash equilibrium exists for both types of competition. Also, when compared to the central planner's solution, noncooperative booking limits are lower (higher) under horizontal (vertical) competition. Our paper extends the analysis in Netessine and Shumsky (2005) to an arbitrary number of airlines, fare classes, and network topologies at the expense of imposing more restrictive assumptions on the demand distributions that rule out passenger overflows. We also investigate in more detail the structure of revenue-sharing agreements that coordinate the operations of the alliance.

Wright et al. (2010) study a two-airline alliance network and formulate the operational problem as a dynamic game: a transfer price is computed at each customer arrival epoch, and each airline dynamically controls its inventory to maximize its own revenue during the booking process. They provide a counter example to show the nonoptimality of Markovian transfer price schemes, derive equilibrium acceptance policies using the value functions from each airline's dynamic program, and conclude that the performance of each transfer price scheme and partner's behavior is significantly affected by the proportion of local requests in the total demand. Wright et al. (2010) suggest a two-stage hierarchical approach similar to ours,

but bypass the cooperative game analysis of the first stage and assume complete information in the second stage (as in Netessine and Shumsky 2005). They highlight the complexity of computing dynamic revenue-sharing policies in a decentralized environment and the difficulty of scaling up their analysis to realistic networks (see §5.7 therein). In contrast, by using SBLs, we characterized optimal strategies for an arbitrary alliance in terms of its network topology and number of airlines. Recently, in a follow-up paper, Wright (2011) extends the dynamic bid price sharing scheme in Wright et al. (2010, §5.1) to the case where partners exchange real-time bid prices under incomplete information. To reduce the computational burden of the dynamic programming formulation, Wright (2011) analyzes a (static) *constant bid-price* heuristic. In his numerical study, the performance of the constant bid-price scheme parallels the performance of the full-information scheme on which it is based.

In a recent paper, Cai and Lim (2011) study the dynamic version of the operational problem we solve in the second stage, where multiple decentralized agents allocate resources among requests that arrive stochastically over time. They characterize transfer prices that coordinate the system in the sense that centralized optimality is achieved. An iterative algorithm for decentralized computation of these prices is also proposed and shown to converge to optimality. However, the algorithm is impractical for large-scale problems because each step calls for the solution of a stochastic control problem by multiple agents. Our approach, built on the static approximation to the dynamic program, provides a candidate that is scalable for practical implementation.

In the context of cargo RM, Agarwal et al. (2009) study fairness in the allocation of revenues among carriers. They propose a cooperative game approach to measure various benefit/cost allocations for alliances in sea and air cargo industries. Chun et al. (2011) analyze the exchange of resource capacities among partners and how it affects the competition among the alliance members. They present a stochastic optimization model with equilibrium constraints to determine the optimal amount of each resource to be exchanged in a noncooperative game setting.

Other than the RM side, various issues of airline alliances have been examined by researchers of different fields. Oum and Park (1997) describe the government policy toward airline alliances. Park (1997) examines the effects of airline alliances on profits, airfare, and economic welfare. Park and Zhang (2000) and Bamberger et al. (2004) empirically investigate the effects on airfares, passenger volume, and consumer surplus of international and domestic airline alliances, respectively. Gayle (2007) analyzes how policy makers can use a structural

econometric framework to quantify the competitive effects of proposed codeshare alliances, in which potential alliance partners compete on overlapping routes in the prealliance industry. Gayle (2007) applies the framework over the consumers' convenience of the Delta/Continental/Northwest alliance, and does not find any significant change between post and prealliance fares. Czerny (2009) questions the overall usefulness of codeshare agreements from a welfare perspective, in the sense that interline passengers always benefit from these agreements while noninterline passengers do not. Wan et al. (2009) suggest that the changes in fare values on parallel routes may be insignificantly different from the prealliance setting, or may even drop. Li and Netessine (2011) study the competitive effects of domestic alliances by analyzing how flight networks change dynamically after an alliance. They analyze six major U.S. airlines from 1998 to 2006 and estimate the changes in their entry/exit/stay behavior. They show that, in the postalliance era, airlines are more likely to enter/stay and build higher capacity in markets where their alliance partners hold strong market power, compared to markets where nonpartners dominate.

In terms of how to position our work with respect to the existing literature in airline alliance RM, we believe that the distinguishing features of our framework are: (a) general network topology and demand distributions, (b) analysis of a two-stage hierarchical approach that allows to decouple the contractual side (i.e., establishing the proration rates) from the operational side (i.e., inventory control), (c) characterization of optimal proration rates that can be implemented in a decentralized setting under SBLs and incomplete information, and (d) proposal of a simple heuristic revenue-sharing rule that relies exclusively on publicly available fare information.

3. Model

3.1. Description

We consider a general alliance network defined by a quintuplet $\{\mathcal{K}, (\mathcal{M}_k: k \in \mathcal{K}), \mathcal{N}, A, C\}$, where $\mathcal{K}$ is the set of airlines in the alliance, $\mathcal{M}_k$ is the set of legs (resources) controlled by airline $k \in \mathcal{K}$, $\mathcal{N}$ is the set of O-D itineraries (products) offered by the alliance, A is the incidence matrix of the network mapping itineraries to resources, and C is the vector of initial capacities (number of seats) available on each resource. We also let $\mathcal{M}$ denote the set of all resources, that is, $\mathcal{M} = \bigcup_{k \in \mathcal{K}} \{\mathcal{M}_k\}$. Because two different legs can serve the same O-D pair, we will assume, without loss of generality, that each leg is operated by a single airline, that is, $\{\mathcal{M}_k\}$ forms a partition of $\mathcal{M}$. We let K, m and n be the cardinality of the sets $\mathcal{K}$, $\mathcal{M}$ and $\mathcal{N}$, respectively, and we index the members of these sets by $k = 1, \ldots, K$, $i = 1, \ldots, m$, and $j = 1, \ldots, n$.

We associate to each itinerary $j \in \mathcal{N}$ a quadruplet $(\mathcal{R}_j, \mathcal{K}_j, p_j, D_j)$, where $\mathcal{R}_j \subseteq \mathcal{M}$ is the route (i.e., the subset of legs used by j); $\mathcal{K}_j \subseteq \mathcal{K}$ is the set of airlines that operates j; $p_j > 0$ is the fare; and $D_j = (D_j(t): t \in [0, T])$ is the demand process over the selling horizon of length T. We assume that each itinerary consumes one unit of capacity (one seat) on each leg of its route. As a result, the incidence matrix takes the form $A = [A_{ij}] \in \{0, 1\}^{m \times n}$, where $A_{ij} = 1$ if and only if $i \in \mathcal{R}_j$. Itineraries are partitioned into *locals*, which are operated by a single airline (i.e., $|\mathcal{K}_j| = 1$) and *interlines*, which are operated by at least two airlines (i.e., $|\mathcal{K}_j| \geq 2$). Even though there are differences between codeshare and interline itineraries, for the ease of exposition we will call the airline that collects the entire fare the *marketing carrier* (in airline jargon, this is the *plating* carrier for interline itineraries), and *interline* to any itinerary spanning two or more operating carriers. We denote by $\mathcal{I} \subseteq \mathcal{N}$ the set of interline itineraries. We denote by $\mathcal{L}_k \subseteq \mathcal{N}$ the set of local itineraries operated by airline k, and by $\mathcal{I}_k \subseteq \mathcal{I}$ the set of interline itineraries in which airline k participates.

In general, we will use the convention that adding a subscript k to an arbitrary set (matrix or vector) must be interpreted as the restriction of the set (matrix or vector) to the components associated to airline k. For example, $\mathcal{N}_k \subseteq \mathcal{N}$ is the subset of itineraries in which airline k operates at least one leg (that is, $\mathcal{N}_k = \mathcal{L}_k \cup \mathcal{I}_k$), $C_k \geq 0$ is the vector of initial capacity that airline k operates, and A_k is the submatrix with rows corresponding to the legs operated by airline k.

Our analysis of the alliance game will assume that airlines use SBLs to compute the expected revenues they can collect as members of the alliance. Under this type of control, each airline k decides the number of seats that it will allocate to each itinerary at the beginning of the horizon, effectively partitioning its capacity C_k among (possibly) all itineraries. We will denote by $x_k = (x_{kj}: j \in \mathcal{N}_k)$ airline's k SBL allocation, where x_{kj} is the number of seats that airline k allocates to product j. To distinguish between local and interline itineraries, we write $x_k = (x_k^{\mathcal{L}}, x_k^{\mathcal{I}})$, where $x_k^{\mathcal{L}} = (x_{kj}: j \in \mathcal{L}_k)$ and $x_k^{\mathcal{I}} = (x_{kj}: j \in \mathcal{I}_k)$, and where the superscripts "$\mathcal{L}$" and "$\mathcal{I}$" stand for *Local* and *Interline*, respectively. We denote by x_{-k} the collection of SBL allocations of all airlines other than k. We say that an allocation x_k is feasible for airline k if $A_k x_k \leq C_k$.

On the demand side, we consider the traditional independent demand model for RM under which $\{D_j(t), j \in \mathcal{N}\}$ are independent stochastic processes. In particular, under SBL controls, we only need to know the distribution of the random variable $D_j(T)$, the cumulative demand for product j during the selling horizon $[0, T]$. We denote by F_j this cumulative distribution function (c.d.f.), which we assume is continuous and strictly increasing in $\mathbb{R}_+$, for all $j \in \mathcal{N}$. We

also define its right tail distribution $\bar{F}_j := 1 - F_j$ and its (finite) mean $\mu_j := \mathbb{E}[D_j(T)]$.

In terms of revenue allocation, we consider fixed *proration rates* $\beta = [\beta_{kj}]$, where β_{kj} is the fraction of the revenues generated by itinerary j that is collected by airline k. We assume that an airline collects no revenue from those itineraries in which it does not operate at least one leg. We explain in §3.3 how we could also accommodate codeshare agreements. Within this framework, the set of feasible proration rates $\mathcal{B}$ is given by

$$\mathcal{B} := \left\{ \beta \geq 0 \text{ such that } \sum_{k \in \mathcal{K}} \beta_{kj} = 1 \text{ for all } j \in \mathcal{N} \text{ and} \right.$$
$$\left. \beta_{kj} = 0 \text{ if } k \notin \mathcal{K}_j \right\}. \quad (1)$$

Given a revenue-sharing rule $\beta \in \mathcal{B}$, and the distinction between local and interline itineraries, airline k's expected payoff for a feasible SBL allocation $x = (x_k, x_{-k})$ is equal to

$$\Pi_k(x_k, x_{-k}; \beta)$$
$$= \Pi_k^{\mathcal{L}}(x_k^{\mathcal{L}}) + \Pi_k^{\mathcal{I}}(x_k^{\mathcal{I}}, x_{-k}^{\mathcal{I}}, \beta)$$
$$= \sum_{j \in \mathcal{L}_k} p_j \mathbb{E}[\min\{D_j(T), x_{kj}^{\mathcal{L}}\}]$$
$$+ \sum_{j \in \mathcal{I}_k} \beta_{kj} p_j \mathbb{E}\left[\min\left\{ D_j(T), \min_{\hat{k} \in \mathcal{K}_j}\{x_{\hat{k}j}^{\mathcal{I}}\} \right\} \right]. \quad (2)$$

Airline k's total payoff is the sum of the expected revenues collected on its local itineraries, $\Pi_k^{\mathcal{L}}(x_k^{\mathcal{L}})$, and its interline itineraries, $\Pi_k^{\mathcal{I}}(x_k^{\mathcal{I}}, x_{-k}^{\mathcal{I}}, \beta)$. Note that local revenues are independent of the proration rates and other airlines' strategies. On the other hand, interline revenues depend on both. The term $\beta_{kj} p_j$ is the per seat revenue that airline k gets from itinerary j. The term $\min\{D_j, \min_{\hat{k} \in \mathcal{K}_j}\{x_{\hat{k}j}\}\}$ is the number of seats sold in itinerary j, i.e., the minimum between the demand D_j and the minimum number of seats allocated to itinerary j by those airlines operating this itinerary. It is worth noting that $x_{-k}^{\mathcal{I}}$ impacts the revenues that airline k can get on itinerary $j \in \mathcal{I}_k$ only through the value of the *meet* $\min\{x_{\hat{k}j}^{\mathcal{I}} : \hat{k} \in \mathcal{K}_j - \{k\}\}$. In §4.1 we will take advantage of this property to characterize the set of Nash equilibria of the alliance game with payoffs given by (2).

3.2. Formulation of the Game

Our analysis follows the two-stage hierarchical approach illustrated in Figure 1. In the first stage, we focus on admissibility using a cooperative game framework and studying the output of the negotiation in which airlines decide the terms of the revenue split for codeshare and interline itineraries. We assume that for a $\beta \in \mathcal{B}$ to be an *admissible* sharing rule resulting from this negotiation it must lead to payoffs (2)

that are in the *core* of the game so that there is no coalition that can collectively get higher revenues by leaving the grand alliance to form an independent suballiance. In the second stage, we consider *efficiency* by modeling the operations of the alliance as a noncooperative game in a decentralized network in which each airline must decide how to allocate its capacity among the different itineraries. We present below the models that we use to characterize the tactical and operational levels.

3.2.1. Tactical Level Cooperative Game. To formalize the first-stage equilibrium concept and the corresponding admissible proration rates, we compute the payoffs that coalitions can realize if they secede from the alliance. To tackle this problem, we follow the cooperative game literature on market and production games (see Shapley and Shubik 1969, Owen 1975 and references therein) and assume that the revenues that a coalition can generate equal those of a single airline that pools the resources of all its members. Some additional notation customary in cooperative game theory follows: For a given coalition $S \subseteq \mathcal{K}$, we let C_S be the vector of aggregated capacity of the legs operated by the airlines in S, and $\mathcal{M}_S := \bigcup_{k \in S} \mathcal{M}_k$ be the set of legs operated by the coalition. We say that coalition S operates product j, which we denote by $j \in \mathcal{M}_S$, if $\mathcal{R}_j \subseteq \mathcal{M}_S$. For any coalition S, we let $\beta_{Sj} := \sum_{k \in S} \beta_{kj}$ be the fraction of the revenue of itinerary j collected by the members of S. We define $v(S)$ to be the cumulative expected revenue that the coalition can get if all its members decide to form a suballiance independent of the airlines in $\mathcal{K} - S$. The function v is known as the *characteristic function* of the game, and is well defined under the independent-demand model and the feasible proration rules defined in (1). In other words, the payoffs of a coalition S depend exclusively on the set of resources and itineraries operated by the coalition.

We assume that airlines in S pool their resources and use SBLs on each itinerary to estimate the cumulative expected revenue that the coalition can generate. As a result, we define the characteristic function as follows:

$$v(S) := \max_{x \geq 0} \left\{ \sum_{j \in \mathcal{N}} p_j \mathbb{E}[\min\{D_j(T), x_j\}] \right\} \quad (3)$$

$$\text{subject to} \quad Ax \leq C_S. \quad (4)$$

The decision variable x_j is the number of seats that coalition S would allocate to itinerary j. Note that we are not imposing integrality constraints on these seat allocations. The optimization in (3)–(4) is known as the *probabilistic nonlinear programming problem* (PNLP), in this case associated to coalition S. Clearly, PNLP is a concave optimization problem with a separable objective function. Furthermore, because we have

assumed that the c.d.f. of $D_j(T)$ is strictly increasing in $\mathbb{R}_+$, it follows that the objective function is strictly concave and there is a unique solution, denoted by $x^C(S) = (x_j^C(S), j \in \mathcal{N})$, to (3)–(4). (The superscript "C" stands for *cooperative*.) We can find this solution solving the Karush–Kuhn–Tucker (KKT) conditions (see Rockafellar 1970, §28).[2] Note that for the grand coalition, $x^C(\mathcal{K})$ is the central planner's optimal static booking limit solution. We will thereafter refer to this solution as the *grand coalition* or the *central planner solution*.

The following definition formalizes our previous discussion on the type of admissible proration rates that can be considered a stable output of the negotiation in the first stage of the alliance game.

DEFINITION 1 (ADMISSIBLE REVENUE-SHARING RATES). A proration rate $\beta \in \mathcal{B}$ is admissible if for all coalition $S \subseteq \mathcal{K}$,

$$v(S) \leq \sum_{j \in \mathcal{N}} \beta_{Sj} p_j \mathbb{E}[\min\{D_j(T), x_j^C(\mathcal{K})\}], \qquad (5)$$

where $x^C(\mathcal{K})$ is the vector of booking limits that solves (3)–(4) for the grand coalition (i.e., the entire alliance). We define $\mathcal{B}^A \subseteq \mathcal{B}$ to be the set of admissible proration rates.

In other words, condition (5) requires that the revenue that any coalition S can realize by seceding and forming a suballiance (left-hand side) cannot exceed the revenues that S can collect if all its members stay in the grand alliance and receive a fraction of the revenues generated by each itinerary according to the proration rates β (right-hand side). Hence, because $v(\mathcal{K})$ is the maximum payoff that the alliance can generate, an admissible β gives each possible coalition enough economic incentive to stay within the grand coalition. Note that (5) holds with equality for the grand coalition, when $S = \mathcal{K}$. More generally, condition (5) clearly implies that any admissible proration rate $\beta \in \mathcal{B}^A$ produces an allocation of revenues that is in the core of the alliance game $(\mathcal{K}, v)$.[3] Indeed, for a given $\beta \in \mathcal{B}^A$, consider the allocation $\alpha = (\alpha_k: k \in \mathcal{K})$ such that

$$\alpha_k = \sum_{j \in \mathcal{N}} \beta_{kj} p_j \mathbb{E}[\min\{D_j(T), x_j^C(\mathcal{K})\}].$$

It follows from (5) that for any coalition S, $v(S) \leq \sum_{k \in S} \alpha_k$, with equality for $S = \mathcal{K}$.

3.2.2. Operational Level Noncooperative Game. In the second stage of our hierarchical approach, we wish to determine whether a particular feasible proration rate β can indeed sustain an efficient operation of the entire alliance as specified by the optimal vector of booking limits $x^C(\mathcal{K})$. Given that each airline privately operates its own inventory system, we model this second stage as a noncooperative game in which each airline chooses its own booking limits as a best response to those selected by the other airlines.

Our analysis of this second-stage game consists of two parts. We start by characterizing the Nash equilibria of the airline alliance game given a set of proration rates β, that is, we seek booking limit vectors $x^* = (x_1^*, x_2^*, \ldots, x_K^*) \in \mathcal{E}(\beta)$, where

$$x^* \in \mathcal{E}(\beta) \iff \text{ for all } k \in \mathcal{K},$$
$$x_k^* \in \arg\max_{x_k \geq 0}\{\Pi_k(x_k, x_{-k}^*; \beta): A_k x_k \leq C_k\}. \qquad (6)$$

Here, we do not impose integrality constraints on the seat allocations. Also, note that a Nash equilibrium x^*, as defined in (6), is parameterized by the vector of proration rates β and we use the notation $x^*(\beta)$ to emphasize this dependence whenever necessary. As we will see in §4.1, the set $\mathcal{E}(\beta)$ is nonempty. In particular, for any $\beta \in \mathcal{B}$, one can show that there exists a Nash equilibrium for which $x_{kj}^* = 0$ for all interline itineraries, and for which the payoffs $\Pi_k(x_k^*, x_{-k}^*; \beta)$ are independent of β. Hence, for this family of equilibria the choice of β is immaterial. However, the set $\mathcal{E}(\beta)$ is rich enough to allow for a nontrivial analysis of the effects of β on the performance of the alliance.

In the second part of our analysis of this noncooperative game, we investigate conditions under which a proration rate $\beta \in \mathcal{B}$ can implement an efficient operation of the alliance in the following sense.[4]

DEFINITION 2 (EFFICIENT PRORATION RATES). A revenue-sharing rule $\beta \in \mathcal{B}$ is efficient for the alliance if

$$x^C(\mathcal{K}) \in \mathcal{E}(\beta). \qquad (7)$$

We let $\mathcal{B}^E \subseteq \mathcal{B}$ be the set of efficient proration rates.

The last part of our analysis of the alliance game combines the results for the cooperative and noncooperative games and attempts to characterize proration rates in the set $\mathcal{B}^* := \mathcal{B}^A \cap \mathcal{B}^E$, i.e., proration rates that are admissible from a cooperative standpoint and that at the same time can implement an efficient allocation of resources in a noncooperative environment.

---

[2] No additional regularity conditions (e.g., Slater conditions) are needed because the constraints in (4) are linear.

[3] Recall that an allocation $\alpha = (\alpha_k: k \in \mathcal{K})$ is in the core of the game $(\mathcal{K}, v)$ if for all $S \subseteq \mathcal{K}$, $v(S) \leq \sum_{k \in S} \alpha_k$, with equality for $S = \mathcal{K}$.

[4] Note that the notion of efficiency used in Definition 2 is with respect to the allocation of capacity (seats) to the different itineraries as opposed to with respect to the design of an optimal set of itineraries to offer.

3.3. Discussion of the Model Assumptions

A potentially valid criticism of this model is that, in practice, airlines do not use static booking limits to control capacity. Rather, they allocate their capacity dynamically as time goes by and demand information gradually unfolds. There are, however, a few reasons that justify our choice of SBLs to model airlines' admission control strategies. First, there is mathematical tractability of modeling and computing dynamic capacity control policies. Even in the monopolistic case, network RM is a challenging problem for which only heuristic solutions (such as bid price or virtual nesting controls) have been successfully proposed. In our setting with multiple players, there is an extra layer of complexity associated with modeling the alliance game because a dynamic game-theoretical model would at least require including the information flow and the beliefs of each airline about other airlines' remaining capacities, demand forecasts, and future strategies. Yet we can extend the monopolistic analysis in Cooper (2002) to show that SBLs are asymptotically optimal as the size of the alliance (demand levels and leg capacities) grows proportionally large (see Hu 2011 for details). In addition, as we mentioned earlier, our main goal is to characterize "optimal" (i.e., admissible and efficient) proration rates β_{kj} that airlines can effectively negotiate in the first stage of the game. Given the nature and scope of this negotiation, we believe it is reasonable to assume that airlines only need at this point an estimate of the revenues they can get from a given revenue-sharing rule β. Hence, we view SBLs only as a convenient capacity control mechanism to reach this goal and not as an actual mechanism to be implemented in practice. As will be shown in our numerical experiments, our proposed proration rates work noticeably well even under more realistic control policies like bid prices and virtual nesting.

Another aspect of the model that deserves further discussion is our choice of a cooperative game to characterize the outcome (prorations rates) of the first-stage negotiation process. As mentioned earlier, we believe that cooperative game theory offers a parsimonious framework to capture the fundamental raison d'être of these airline alliances, namely, a platform for collaboration on resource use and (operating) cost reduction. There are, of course, other alternatives that we could have used to model this negotiation process such as a bargaining model. We will come back to this point in §4.3 where we show that our proposed *admissible* revenue-sharing rates obtained imposing the cooperative game stability concept in Definition 1 can also be derived using a Nash bargaining model. In this regard, our results for this first-stage negotiation process appear to be robust with respect to the equilibrium concept used to characterize the outcome of this game.

In addition, the use of a cooperative game framework can be problematic from an antitrust perspective. In practice, however, there seem to be two contrasting views on this issue. On one hand, the Department of Transportation (DOT)—which grants international antitrust immunity (ATI) from U.S. laws to international air transportation—has argued that immunized alliances have led to pro-competitive changes in the industry increasing consumer benefits in the form of improved service and price reductions. On the other hand, the Department of Justice (DOJ) has expressed concerns about the impact of these alliances on fares and competition. For instance, a recent DOJ study (Gillespie and Richard 2011) reports that fares in nonstop trans-Atlantic flights are typically higher when competing carriers integrate their operations, for example in the form of a global alliance (an average increase of 7% for each reduction by one in the number of competitors). Similar findings are reported in Li and Netessine (2011). As a result, it seems that from an empirical (and regulatory) standpoint, there is no clear consensus as to whether collaboration (to set prices and capacity) among airlines in an alliance is increasing or decreasing consumer welfare. In the context of our research, we believe this issue of collusion and its (possible) negative effects on fares and competition is not particularly problematic. Our model assumes that both fares and capacities are fixed and airlines' collaboration is restricted exclusively to (1) defining the split (revenue-sharing rules) of the revenues of interline itineraries and (2) selecting capacity control policies (i.e., seat allocation among different itineraries).

In terms of the generality of our formulation, note that our proposed model of the alliance is flexible enough to allow for: (a) multiple fare classes on the same O-D route, which are identified as different products; (b) multiple airlines offering products on the same O-D pair; because different legs can connect a given O-D; and (c) pure marketing airlines collecting revenues on itineraries operated exclusively by other airlines. The latter point seems to contradict our definition of feasible proration rates in (1). However, we can accommodate a pure marketer in our model by introducing a fictitious leg over which it becomes the operating carrier.[5] Going back to our example in §1,

---

[5] Suppose airline $\hat{k}$ is a pure marketing airline for some itinerary $\hat{j}$, that is, $\hat{k} \notin \mathcal{K}_{\hat{j}}$. Since $\hat{j}$ is an itinerary in the network, there is already an incidence column in the matrix A describing it. Then, we can add a fictitious leg $\hat{i}$ with capacity $C_{\hat{k}\hat{i}} > \max\{C_{ki}: k \in \mathcal{K}_{\hat{j}}, i \in R_{\hat{j}}\}$, and modify the network structure adding a row to the incidence matrix for the new resource $\hat{i}$ with $A_{\hat{i}\hat{j}} = 1$ and $A_{\hat{i}h} = 0$ for $h \neq \hat{j}$. We also add the new leg to those operated by $\hat{k}$, i.e., $\mathcal{M}_{\hat{k}} \leftarrow \mathcal{M}_{\hat{k}} \cup \{\hat{i}\}$, and to those in the route of itinerary $\hat{j}$, i.e., $R_{\hat{j}} \leftarrow R_{\hat{j}} \cup \{\hat{i}\}$. In this new modified network, airline $\hat{k}$ is now an operating airline for itinerary $\hat{j}$. Note that the choice of capacity $C_{\hat{k}\hat{i}}$ is such that the fictitious resource $\hat{i}$ is never a bottleneck that limits the number of seats that can be sold in itinerary $\hat{j}$.

LAN could offer a single-leg ticket in the route MIA–ORD operated by AA (i.e., a pure codeshare itinerary).

Our definition of the set of feasible proration rates $\mathcal{B}$ in (1) requires that each airline collect all the revenues generated by its local network. In the absence of this condition there is a simple family of efficient revenue-sharing rules. Consider the family of *perfect sharing rules* defined by $\beta_{kj} = \tilde{\beta}_k$ for all itineraries $j \in \mathcal{N}$, where $(\tilde{\beta}_1, \tilde{\beta}_2, \ldots, \tilde{\beta}_K)$ is a K-dimensional vector such that $\sum_k \tilde{\beta}_k = 1$ and $\tilde{\beta}_k \geq 0$ for all $k \in \mathcal{K}$. Under this perfect sharing rule, airline k gets a fixed proportion $\tilde{\beta}_k$ of the revenues generated by any itinerary in the entire network. As a result, the expected payoff of each airline is a fixed proportion of the payoff of the aggregate system. For a perfect sharing rule $\tilde{\beta}$ to be in $\mathcal{B}^A$, the vector $(\tilde{\beta}_1, \tilde{\beta}_2, \ldots, \tilde{\beta}_K)$ must satisfy an additional participation constraint (i.e., each airline has to collect at least the same revenues that it could collect if it stays out of the alliance). Suppose $\alpha = (\alpha_k : k \in \mathcal{K})$ is an allocation in the core of the game $(\mathcal{K}, v)$ defined by (3)–(4), and consider the perfect sharing rule $\tilde{\beta}$ such that $\tilde{\beta}_k = \alpha_k/\alpha_{\mathcal{K}}$, where $\alpha_{\mathcal{K}} = \sum_{k \in \mathcal{K}} \alpha_k$. It follows that for any coalition S,

$$v(S) \leq \alpha_S = \alpha_S \frac{v(\mathcal{K})}{\alpha_{\mathcal{K}}} = \sum_{j \in \mathcal{N}} \frac{\alpha_S}{\alpha_{\mathcal{K}}} p_j \mathbb{E}[\min\{D_j(T), x_j^c(\mathcal{K})\}]$$

$$= \sum_{j \in \mathcal{N}} \tilde{\beta}_{Sj} p_j \mathbb{E}[\min\{D_j(T), x_j^c(\mathcal{K})\}],$$

and so $\tilde{\beta}$ is admissible. However, despite their theoretical value, these perfect sharing rules appear to be of limited practical interest because for their implementation each airline would have to monitor the sales process of every itinerary on the entire network, including those where it does not operate. On the other hand, for the set of feasible proration rates $\mathcal{B}$ in (1), each airline only needs to monitor the sales process of those itineraries that it operates at least a leg, a feature that certainly favors its implementation.

We also note that one feasible strategy for each airline is to set the booking limits for interline itineraries at $x_{kj} = 0$. This is equivalent to the airline seceding from the alliance to operate its own local network exclusively. Hence, it follows that each airline is willing (in a weak sense) to participate in the alliance independently of the proration rates β that are used to split interline and codeshare revenues.

Finally, our model is built on an assumption that has a long tradition in the RM literature, namely, demands for different itineraries are considered independent stochastic processes (e.g., see §7.1 in Talluri and van Ryzin 2004b for a discussion about this *independent demand model*). There is an ongoing trend in RM research toward enriching this model to account for demand substitution and choice behavior effects (e.g., Talluri and van Ryzin 2004a, Kunnumkal and

Topaloglu 2010). Although we do not attempt to capture demand substitution in our model because of the additional degree of complexity it entails, we do believe that extending our model and results in this direction can open promising venues for future research.

4. Analysis

We proceed backward in the analysis of the two-stage hierarchical approach, starting from the operational level (decentralized control) and following with the tactical level (cooperative stage).

4.1. Nash Equilibria for Fixed Proration Rates

In this section we characterize the set of Nash equilibria $x^* \in \mathcal{E}(\beta)$ solution to (6) for a fixed value $\beta \in \mathcal{B}$. To this end, we take advantage of the special structure of the airline alliance game to derive an alternative formulation that will prove useful in this characterization. For this, recall that x_{-k} impacts the payoff of airline k only through the value of the *meet* $\min\{x_{\hat{k}j} : \hat{k} \in \mathcal{K}_j - \{k\}\}$. To capture this special feature of the game, we introduce an auxiliary vector $w = (w_j, j \in \mathcal{I})$ and interpret w_j as a public signal announced by a coordinator[6] that specifies a quota on the maximum number of seats each airline should allocate to itinerary $j \in \mathcal{I}$. For a given a vector $w = (w_j, j \in \mathcal{I})$, we define the projection $w_k = (w_j, j \in \mathcal{I}_k)$ as the subset of signals associated to those interline itineraries operated by airline k. A vector of public signals w is feasible if $A_k w_k \leq C_k$ for all $k \in \mathcal{K}$.

The advantage of introducing w is that it allows us to decompose (6) into a series of simple subproblems. In particular, we can rewrite airline k's payoff as follows:

$$\Pi_k(w, \beta) := \max_{x_k^{\mathcal{L}}, x_k^{\mathcal{I}} \geq 0} \left\{ \sum_{j \in \mathcal{L}_k} p_j \mathbb{E}[\min\{D_j(T), x_{kj}\}] \right.$$

$$\left. + \sum_{j \in \mathcal{I}_k} \beta_{kj} p_j \mathbb{E}[\min\{D_j(T), x_{kj}^{\mathcal{I}}\}] \right\}, \quad (8)$$

$$\text{subject to} \quad A_k(x_k^{\mathcal{L}} + x_k^{\mathcal{I}}) \leq C_k, \quad x_k^{\mathcal{I}} \leq w_k. \quad (9)$$

Formulation (8)–(9) is similar to the right-hand side of (6) for a given k, except for the fact that the impact of other airlines' decisions on Π_k is now implicitly captured by the set of constraints involving the public signal w. These constraints make explicit the fact that in equilibrium an airline will never allocate more seats to an interline itinerary than the number of seats

---

[6] Although this coordinator is a fictitious entity that we introduce to explain our mathematical construction, major airline alliances have a central authority or *managing partner* responsible for coordinating products and services offered by its members that could play such a role.

allocated by the other airlines operating the itinerary. Note also that the optimization problem (8)–(9) is an extension of the PNLP in (3)–(4) to the airline alliance game and preserves many of its properties. In particular, it is a strictly concave optimization problem with a separable objective function.

The solution to (8)–(9), denoted by $x_k(w, \beta) = (x_k^{\mathscr{L}}(w, \beta), x_k^{\mathscr{I}}(w, \beta))$, is airline k's best response to the public signal w and leads to an alternative characterization of the set of Nash equilibria for the airline alliance game. To see this, we introduce the following set:

$$\mathscr{W}(\beta) := \big\{ w \geq 0 \colon x_{kj}^{\mathscr{I}}(w, \beta) = w_j \text{ for all } j \in \mathscr{I}_k$$

$$\text{and for all } k \in \mathscr{K} \big\}. \quad (10)$$

Intuitively, for $w \in \mathscr{W}(\beta)$, each airline follows the public signal allocating exactly w_j seats to those interline itineraries in which it participates. The following lemma shows that $\mathscr{W}(\beta)$ characterizes the set of all Nash equilibria, and relates it to the $\mathscr{E}(\beta)$ defined in (6).

LEMMA 1. *For any $w \in \mathscr{W}(\beta)$, there exists a unique Nash equilibrium $x(w) \in \mathscr{E}(\beta)$ such that $x_j(w) = w_j$ for all $j \in \mathscr{I}$. Conversely, for any $x \in \mathscr{E}(\beta)$ there exists a unique $w(x) \in \mathscr{W}(\beta)$ such that $w_j(x) = x_j$ for all $j \in \mathscr{I}$.*

Hereafter we use $\mathscr{W}(\beta)$ and $\mathscr{E}(\beta)$ interchangeably to denote the set of Nash equilibria. Lemma 1 implies that we can find the Nash equilibria of the alliance game by looking at those signals w for which the second inequality constraints in (9) are binding at optimality. Using this result, we provide an analytical representation of $\mathscr{W}(\beta)$ in the following proposition, whose statement requires some additional notation. For this, consider the following optimization problem:

$$\Pi_k^{\mathscr{L}}(w_k) := \max_{x_k^{\mathscr{L}} \geq 0} \left\{ \sum_{j \in \mathscr{L}_k} p_j \mathbb{E}[\min\{D_j, x_{kj}^{\mathscr{L}}\}] \right\} \quad (11)$$

$$\text{subject to} \quad A_k x_k^{\mathscr{L}} \leq C_k - A_k w_k,$$

which determines airline k's optimal allocation of resources within its local network after assigning a partition w_k to the interline itineraries (the right-hand side $C_k - A_k w_k$ is airline k's remaining capacity to operate its local network). Observe that (11) is a PNLP constrained to the local network, and hence it is also a strictly concave optimization problem. The revenue function $\Pi_k^{\mathscr{L}}(w_k)$ will prove useful in the analysis that follows as it provides airline k with an autonomous (i.e., independent of other airlines' actions) mechanism to price its resources, particularly those allocated to interline itineraries. These prices are the Lagrange multipliers (shadow prices) of the capacity constraints in (11), which we denote by $\lambda_k(w) = (\lambda_{ki}(w) \colon i \in \mathscr{M}_k)$.

Given that each leg is operated by a single airline, we can unambiguously define $\lambda^{\mathscr{L}}(w) = (\lambda_i^{\mathscr{L}}(w) \colon i \in \mathscr{M})$, where $\lambda_i^{\mathscr{L}}(w) = \lambda_{ki}(w)$ for $i \in \mathscr{M}_k$.

PROPOSITION 1. *Consider a feasible w, that is $w \geq 0$ and $A_k w_k \leq C_k$ for all $k \in \mathscr{K}$. Then, $w \in \mathscr{W}(\beta)$ if and only if there exists a vector of Lagrange multipliers $\lambda^{\mathscr{L}}(w)$ for problem (11) such that*

$$\beta_{kj} p_j \bar{F}_j(w_j) \geq \mathbb{1}(w_j > 0) \sum_{i \in \mathscr{M}_k} A_{ij} \lambda_i^{\mathscr{L}}(w)$$

$$\text{for all } k \in \mathscr{K} \text{ and } j \in \mathscr{I}_k. \quad (12)$$

The left-hand side of the inequality is the additional revenue that airline k realizes by marginally increasing the allocation of capacity to the interline itinerary j. The right-hand side represents the opportunity cost of the capacity used by itinerary j if it were used locally by airline k. Hence, in a Nash equilibrium characterized by w, no airline has an incentive to deviate from the current allocation and transfer capacity from its interline network to its local itineraries. Note that there is no need to account for an allocation deviation from an interline to other interline itineraries because they are fixed by the decision of the other alliance members.

Clearly, $w = 0$ (trivially) satisfies the conditions in Proposition 1, and so the set of Nash equilibria is guaranteed to be nonempty. The equilibrium with $w = 0$ corresponds to the case in which there is no cooperation among the members of the alliance and each airline allocates its entire capacity to its local network. In general, it is hard to fully characterize the set $\mathscr{W}(\beta)$ (e.g., it is not even convex in general, see Hu 2011 for an example). The following proposition highlights an important feature of the set $\mathscr{W}(\beta)$.

PROPOSITION 2. *If $w, w' \in \mathscr{W}(\beta)$, and $w' \leq w$, then w Pareto dominates w' in the sense that the payoff of each airline under w is greater than or equal to the payoff under w'.*

The result establishes that if two ordered levels of collaboration are sustainable in equilibrium, then the greater level generates higher revenues to each alliance member. In this respect, the game exhibits *strategic complementarity*.

To illustrate the analysis of the second stage, the following running example is used throughout this section.

RUNNING EXAMPLE. Consider an alliance with two airlines, each controlling a single leg. The alliance offers four itineraries; itinerary i is a local itinerary for airline i, $i = 1, 2$, while itineraries 3 and 4 are interline and use one unit of capacity on each leg. The alliance network is schematically depicted in Figure 2. The incidence matrix $A = [A_{ij}] \in \{0, 1\}^{2 \times 4}$ satisfies $A_{ij} = 1$ if and only if $(i, j) \notin \{(1, 2), (2, 1)\}$. The details about

Figure 2 An Alliance with Two Airlines, Each Controlling One Leg (Airline 1 Controls Leg X–Y and Airline 2 Controls Leg Y–Z) and Four Itineraries
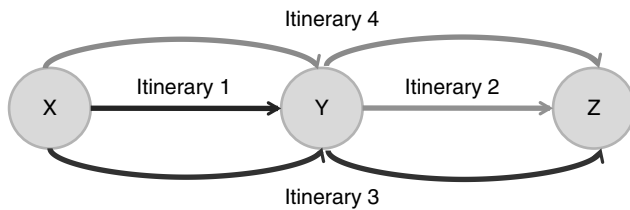
Note. Itineraries 1 and 2 are local, while itineraries 3 and 4 are interline, each using both legs.

Table 1 Network Data for the Example in Figure 2

	Revenue per seat ($\beta_{kj} p_j$) ($)				Capacity C_{kd}
	Itinerary 1	Itinerary 2	Itinerary 3	Itinerary 4	
Airline 1	200	0	250	100	300
Airline 2	0	300	100	250	250
Demand distribution	Unif[0, 350]	Unif[0, 250]	Unif[0, 200]	Unif[0, 200]	

Note. In this example, the price vector is $p = (200, 300, 350, 350)$, and the proration rates are $\beta_1 = (1, 0, 5/7, 2/7)$ and $\beta_2 = (0, 1, 2/7, 5/7)$ for Airlines 1 and 2, respectively.

demand distributions, capacities, and prices are summarized in Table 1.

Note that despite the fact that itineraries 3 and 4 consume the same set of resources, they generate different revenues for the two airlines; Airline 1 receives $250 per seat sold in itinerary 3 and only $100 per seat sold in itinerary 4. This could be an example in which itineraries 3 and 4 are marketed by Airline 1 and Airline 2, respectively.

In this example, the public signal is a two-dimensional vector $w = (w_3, w_4)$, each component representing the allocation of capacity to each of the interline itineraries. The left panel in Figure 3 depicts the set of Nash equilibria $\mathcal{W}(\beta)$. This set takes the form of a polyhedron in the positive orthant when demands are uniformly distributed (see Hu 2011 for details). In this case, the active constraints (edges) of $\mathcal{W}(\beta)$ are given by the nonnegativity constraints on w (segments A–B and A–D), Airline 1's participation constraint on itinerary 4 (segment B–C)[7] and Airline 2's participation constraint on itinerary 3 (segment C–D). The right panel in Figure 3 shows the set of achievable payoffs for the two airlines for the set of Nash equilibria $\mathcal{W}(\beta)$. There is a correspondence between the regions in both panels. Note that the region of payoffs is not convex for the set of pure-strategy Nash equilibria $\mathcal{W}(\beta)$, hence, it is possible to (strictly) enlarge this region using randomization.

In terms of efficiency, the noncollaboration equilibrium in point A serves as a benchmark to evaluate the value of the alliance. Although this value depends on the specific equilibrium under consideration, it is clear that both airlines are better off engaging in some degree of collaboration. Pareto efficiency is reached on the upper-right boundary B–C–D; it follows more generally from Proposition 2. Point C is the most efficient equilibrium in the set $\mathcal{W}(\beta)$ in the sense that the sum of the payoffs of the two airlines ($\Pi_1 + \Pi_2$) is

maximized at C. Note, however, that point C does not characterize the first-best allocation of capacity from the alliance perspective, which is achieved under a centralized control (we will return to this point in the next section). If we compare the payoffs of the two airlines in the extreme cases, point A (no collaboration) and point C (full collaboration), we conclude that collaboration increases the payoff of the alliance by approximately 18%. For the individual airlines, collaboration increases payoffs for Airlines 1 and 2 by 16% and 21%, respectively.

The discussion in the previous example unveils a number of challenges that need to be addressed in the coordination of the alliance, most notably the fact that the game has multiple equilibria leading to a wide range of outcomes. It seems certain that the members of the alliance should look for ways to reach an equilibrium that lies on the Pareto frontier. At the same time, the question of which specific equilibrium should be selected on this frontier has a less obvious answer and depends on the relative bargaining power of the different airlines. A simple mechanism that the alliance coordinator can use to address this issue is proposed in Hu (2011).

4.2. Efficient Revenue-Sharing Rules

In this section we explore the characteristics of a revenue-sharing rule $\beta \in \mathscr{B}$ under which the performance of the alliance is maximized. Specifically, we provide a sufficient condition under which $\beta \in \mathscr{B}^E$, where $\mathscr{B}^E$ is the set of efficient prorations described in Definition 2.

To this end, recall that $x^c(\mathcal{K}) = (x_j^c: j \in \mathcal{N})$ is the optimal vector of booking limits when all the members of the alliance form a single (grand) coalition that solves the central planner's problem in (3)–(4). For the remainder of this section, we will drop the dependence of $x^c(\mathcal{K})$ on $\mathcal{K}$. We can use the result in Proposition 1 to see whether $\beta \in \mathscr{B}^E$. For this, let us define $w^c = (w_j^c: w_j^c = x_j^c$ for $j \in \mathscr{I})$. Since x^c is optimal, and therefore feasible for problem (3)–(4), it follows that $Aw^c \leq C$. As a result, Proposition 1 implies the following result.

---

[7] In other words, in the boundary B–C, Airline 1 is indifferent between interchanging capacity between itinerary 4 and its local itinerary 1; that is, condition (12) holds with equality for w over that segment.

Figure 3   Nash Equilibria (Left Panel) and Airlines' Payoffs (Right Panel) for the Example in Figure 2 and the Data in Table 1, Given Fixed Proration Rates β
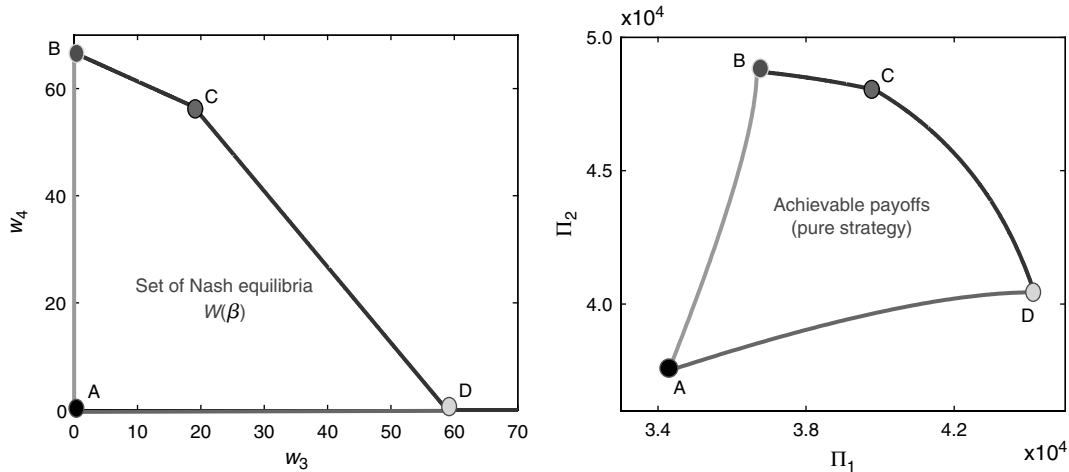
Note. Following the labeling of the vertices in the left panel, each edge of $\mathscr{W}(\boldsymbol{\beta})$ maps into a portion of the boundary of the region of achievable payoffs.

THEOREM 1. *Let* $\lambda^{\mathrm{c}} = \{\lambda_i^{\mathrm{c}}: i \in \mathcal{M}\}$ *be a vector of Lagrange multipliers for the grand coalition problem in* (3)–(4) *(with* $S = \mathcal{K}$*). Suppose* $\beta^* \in \mathcal{B}$ *satisfies*

$$\beta_{kj}^* = \frac{\sum_{i \in \mathcal{M}_k} A_{ij} \lambda_i^{\mathrm{c}}}{\sum_{i \in \mathcal{M}} A_{ij} \lambda_i^{\mathrm{c}}} \quad \text{for all } j \in \mathcal{N} \text{ such that } x_j^{\mathrm{c}} > 0. \quad (13)$$

Then, β^* *belongs to the set of efficient proration rates* $\mathcal{B}^E$.

A few comments about this result and implications are in order. First, a key observation that follows from the proof of Theorem 1 is that we only need to guarantee that the allocation of capacity to the interline itineraries in the grand coalition, $(x_j^{\mathrm{c}}: j \in \mathcal{I})$, can be sustained in the decentralized system. Once the allocation of capacity to these interline itineraries has been decided, the optimization problem of how to distribute the remaining capacity among the local itineraries is the same for the centralized and decentralized systems. Note also that according to Theorem 1 if an itinerary $j \in \mathcal{I}$ is not profitable for the alliance as a whole (i.e., $x_j^{\mathrm{c}} = 0$), there is no need to impose any constraint on how to allocate the revenues of this interline itinerary among the operating airlines (β_{kj}^* can be arbitrarily chosen). This follows from the fact that the allocation $x_j^{\mathrm{c}} = 0$ can always be implemented in a Nash equilibrium. On the other hand, if itinerary j is profitable for the grand coalition (i.e., $x_j^{\mathrm{c}} > 0$) β_{kj}^* is set in such a way that each airline in the decentralized system has enough incentives to allocate the optimal number of seats to itinerary j in equilibrium. These incentives are determined using the grand coalition bid prices in a way that each airline gets exactly the proportion of the opportunity cost that it contributes to the itinerary. We note that our proration rates β^* are an approximation to

the (dynamic) *bid price proration scheme* analyzed in (Wright et al. 2010, §5.4.2).

The efficient proration rates identified in Theorem 1 have some direct practical implications on the type of agreements that the members of the alliances should be trying to negotiate.

COROLLARY 1. 1. *Proration rates should be defined at the route level and not at the product level. That is,* $\beta_{kj}^* = \beta_{kj'}^*$ *for any pair of itineraries* j *and* j' *that cover the same route* $(\mathcal{R}_j = \mathcal{R}_{j'})$ *and that are both open in the centralized solution* $(x_j^{\mathrm{c}} > 0$ *and* $x_{j'}^{\mathrm{c}} > 0)$.

2. *According to* (13), *the airline operating an arbitrary leg* $i \in \mathcal{M}$ *should collect a fraction* $(A_{ij}\lambda_i^{\mathrm{c}})/\sum_{h \in \mathcal{M}} A_{hj}\lambda_h^{\mathrm{c}}$ *of the revenues generated by an arbitrary itinerary* $j \in \mathcal{N}$ *in compensation for using leg* i.

3. *If a resource* i *is not binding in the solution to the grand coalition problem, then* $\lambda_i^{\mathrm{c}} = 0$, *and that leg would not be compensated. In particular, if that leg stands for a fictitious leg defining a pure codeshare product* $\hat{\jmath}$, *then* $\beta_{k\hat{\jmath}}^* = 0$.

Recall that a *route* is a sequence of flight legs, potentially involving several fare-class combinations. Typically the number of routes in the network is significantly smaller than the number of itineraries offered by the alliance, so the first point can prove very useful in practice. The second point claims that since λ^{c} depends exclusively on the grand coalition solution and not on how resources are distributed among airlines, it follows that the revenue-sharing rule in Theorem 1 is independent of the ownership structure within the alliance. This feature argues that our efficient proration rates are robust to potential dominant positions among alliance members. The last point asserts that pure marketing airlines that sell

products that they do not operate should get no compensation for their services.[8] From an economic perspective, this point is intuitive because we are not modeling the direct costs associated with marketing efforts to promote services strictly operated by others. At the same time, codesharing is a growing trend in the airline industry, and so an interesting extension to our model would be to formalize these marketing costs and measure their impact on these efficient proration rates.

Theorem 1 unveils another complicating factor in the coordination of the alliance game, namely, the potential multiplicity of feasible and efficient proration rates β^*, despite the fact that the efficient vector of SBLs x^c is unique. This multiplicity is rooted in the potential multiplicity of Lagrange multipliers λ_i^c. However, under relatively mild conditions on the grand coalition solution x^c (i.e., every airline allocates some local traffic to the legs it operates), the following result guarantees the existence of unique efficient proration rates β_{kj}^* for the relevant itineraries.

PROPOSITION 3. *Suppose that for every leg $i \in \mathcal{M}$ there is an itinerary $j_i \in \mathcal{N}$ such that $\mathcal{R}_{j_i} = \{i\}$ (that is, j_i is a single-leg itinerary) such that $x_{j_i}^c > 0$. Then, there is a unique efficient way to choose the proration rates β_{kj}^* for those interline itineraries j for which $x_j^c > 0$, which is given by*

$$\beta_{kj}^* = \frac{\sum_{i \in \mathcal{M}_k} A_{ij} p_{j_i} \bar{F}_{j_i}(x_{j_i}^c)}{\sum_{i \in \mathcal{M}} A_{ij} p_{j_i} \bar{F}_{j_i}(x_{j_i}^c)}$$

for all $j \in \mathcal{N}$ such that $x_j^c > 0$. (14)

Note that as a feasible proration rate in the sense of (1), under β^* each airline can compute its payoff by monitoring only the sales process of those itineraries in which it operates at least one leg. However, computing β^* as an efficient proration rate in the sense of Theorem 1 and Proposition 3 depends on the primal and dual solutions of the grand coalition problem, which involves demand information of all itineraries in the network. So, unless a central authority (such as the alliance managing partner) can compute this solution and airlines are willing to share private information about demand forecasts for their local itineraries, it can be difficult for the members of the alliance to agree on an efficient vector of proration rates. To address this issue, in §4.2.1 we propose a simple mechanism that produces a vector of proration rates that is close (and in some cases belongs) to the

set of efficient proration rates, and that at the same time does not require sharing private information or computing the grand solution.

4.2.1. A Nearly Efficient Proration Rule. Recall that, independent of the revenue-sharing rule used, airlines are always better off (in a weak sense) joining the alliance than staying out of it. At the same time, each individual airline would like to implement a proration rule that maximizes its own revenue. This tension may lead to a complicated negotiation process, one that a formal analysis is beyond the scope of this paper. Instead, we will propose a simple proration rate rule that relies on observable data, and that airlines can use to reach a solution close to an efficient solution.

Let us first discuss the similarities between the proration rates in Proposition 3 and one of the alternatives that is commonly used in practice, namely, proration based on local fares. Under the latter, the fraction of the fare that an operating airline receives from an interline itinerary is proportional to the local fares of the corresponding single-leg itineraries. To be more precise, consider an interline itinerary $j \in \mathcal{I}$ and let p_i be the fare of a single-leg itinerary $i \in \mathcal{R}_j$, then the proration rates based on local fares are computed as follows:

$$\beta_{kj}^F := \frac{\sum_{i \in \mathcal{M}_k} A_{ij} p_i}{\sum_{i \in \mathcal{M}} A_{ij} p_i}.$$ (15)

Note that β_{kj}^F may not be well defined because it is possible to have multiple single-leg itineraries using leg i and so the fare p_i is not uniquely specified (e.g., different fare classes on the same flight). In practice, this ambiguity is resolved by establishing a mapping from the set of interline itineraries to the set of single-leg itineraries based on booking classes with similar services and restrictions. For example, fare class Y in an interline itinerary may be mapped to fare class Y in the corresponding single-leg itineraries.[9]

From the result in Proposition 3, we can see the resemblance between the proration rule β^* and the one defined by local fares β^F. The following result formalizes this connection.

PROPOSITION 4. *Consider an interline itinerary $j \in \mathcal{I}$ such that $x_j^c > 0$, and suppose that for each of the legs $i \in \mathcal{R}_j$ there exists a single-leg local itinerary $\tilde{J}_i$ such that the fill rates of these local itineraries are all equal in the centralized solution, that is, there exists a constant α_j such*

---

[8] This applies to the fictitious legs we add to the network to model codeshare itineraries as explained in Footnote 5. These resources are never bottlenecks and therefore are never binding. Following up with our LAN–AA case in §1, even though our model allows LAN to sell a single-leg, codeshare ticket MIA–ORD, it should not get any compensation for it under optimal proration rates.

[9] It is common practice in the airline industry to use capital letters to denote booking classes that impose similar restrictions on a ticket; such as advance purchase requirement, minimum length of stay, change of date fees, etc. For example, class Y usually refers to "full fare unrestricted economy class."

that $\bar{F}_{\tilde{J}_i}(x^c_{\tilde{J}_i}) = \alpha_j$ *for all* $i \in \mathcal{R}_j$. *Suppose* β^F_{kj} *is computed using the prices of these single-leg itineraries*

$$\beta^F_{kj} := \frac{\sum_{i \in \mathcal{M}_k} A_{ij} p_{\tilde{J}_i}}{\sum_{i \in \mathcal{M}} A_{ij} p_{\tilde{J}_i}}.$$

Then, $\beta^*_{kj} = \beta^F_{kj}$.

The condition on the fill rates required in the proposition is rather restrictive and unlikely to be met in practice. However, the previous result suggests a simple heuristic that the members of the alliance can use to approximate β^*. According to this proposition, it would be enough to identify a set of single-leg local itineraries with similar fill rates, for instance products with a small fill rate. Usually, within the set of booking fares associated with a particular route, those with lower fares (and more restrictive conditions) are generally closed first, a fact suggesting that their fill rates are consistently low. This strategy of closing low-fare classes first is also consistent with the low-before-high pattern of demand typically observed in the airline industry (as in other *Class A* services in terms of Desiraju and Shugan 1999; see also Talluri and van Ryzin 2004b, Chap. 2). The following heuristic proposes a simple mechanism to compute the proration rates and is motivated by this feature of low fare products.

DEFINITION 3 (HEURISTIC REVENUE-SHARING RULE). For each leg $i \in \mathcal{M}$, let the operating airline identify the corresponding single-leg itinerary using leg i with the lowest fill rate. Using a slight abuse of notation, let us define this itinerary by $\tilde{J}_i$. Then, for any itinerary $j \in \mathcal{N}$, define the proration rates

$$\beta^H_{kj} := \frac{\sum_{i \in \mathcal{M}_k} A_{ij} p_{\tilde{J}_i}}{\sum_{i \in \mathcal{M}} A_{ij} p_{\tilde{J}_i}}. \tag{16}$$

Our proposal β^H has some noticeably desirable features: its computation is distribution free, and only requires as input the mapping $\{\tilde{J}_i: i \in \mathcal{M}\}$. We acknowledge that its implementation requires some data processing efforts: The lowest-fare open classes $p_{\tilde{J}_i}$ are public information but change with different departure dates and times. Therefore, the alliance members should agree on a common reading data source, which is usually provided by third parties (e.g., QL2 Software, LLC).

The following result provides a uniform bound for the estimation error of this revenue-sharing rule.

PROPOSITION 5. *For each leg* i, *let* $\delta_i := \bar{F}_{\tilde{J}_i}(x^c_{\tilde{J}_i})$ *so that* $1 - \delta_i$ *is the fill rate for local itinerary* $\tilde{J}_i$ *under a centralized solution. Then,*

$$\frac{|\beta^*_{kj} - \beta^H_{kj}|}{\beta^H_{kj}} \le \frac{1}{\underline{\delta}} - 1 \quad \text{for all } j \text{ such that } x^c_j > 0,$$

where $\underline{\delta} := \min\{\delta_i: i \in \mathcal{M}\}$.

RUNNING EXAMPLE (CONTINUED). Under the proration original rates $\beta_1 = (1, 0, 5/7, 2/7)$ and $\beta_2 = (0, 1, 2/7, 5/7)$, the most efficient Nash equilibrium (point C in Figure 3) produces payoffs $\Pi_1 = \$40,640$ and $\Pi_2 = \$47,318$ for Airlines 1 and 2, respectively. Using the result in Theorem 1, we can compute the vectors of efficient proration rates $\beta^*_1 = (1, 0, 0.402, 0.402)$ and $\beta^*_2 = (0, 1, 0.598, 0.598)$, which generate the payoffs $\Pi^*_1 = \$41,095$ and $\Pi^*_2 = \$50,211$. Finally, using the proposed revenue-sharing rule, we obtain the proration rates $\beta^H_1 = (1, 0, 0.4, 0.4)$ and $\beta^H_2 = (0, 1, 0.6, 0.6)$, with associated payoffs $\Pi^H_1 = \$41,023$ and $\Pi^H_2 = \$50,282$. The inefficiency of the sharing rule described in Table 1, measured as $1 - (\Pi_1 + \Pi_2)/(\Pi^*_1 + \Pi^*_2)$, is equal to 3.70%. On the other hand, the inefficiency of the proposed rule β^H is only 0.001%.

4.3. Admissible Revenue-Sharing Rules

We conclude our analysis of the airline alliance game investigating conditions under which a revenue-sharing rule is admissible, in the sense of Definition 1. Our main result in this section shows that the efficient proration rates identified in Theorem 1 are also admissible. We also show that these admissible proration rates can be derived using an alternative equilibrium concept for the first-stage game based on the concept of Nash bargaining solution. We start with the following intermediate result.

LEMMA 2. *Consider a general alliance game* $(\mathcal{K}, \mathcal{N}, \{\mathcal{M}_k: k \in \mathcal{K}\}, A, C)$ *(as presented in §3) and let* β^* *be an efficient revenue-sharing rule satisfying the conditions in Theorem* 1. *Suppose that airlines* k_1 *and* k_2 *form a coalition and merge their resources to become a new airline* $\hat{k}$. *In the resulting alliance game* $(\hat{\mathcal{K}}, \mathcal{N}, \{\mathcal{M}_k: k \in \hat{\mathcal{K}}\}, A, C)$, *define the proration rate* $\hat{\beta}^*$ *by*

$$\hat{\beta}^*_{\hat{k}j} = \beta^*_{k_1 j} + \beta^*_{k_2 j} \quad \text{and} \quad \hat{\beta}^*_{kj} = \beta^*_{kj} \quad \text{for all } k \ne \hat{k}.$$

Then, $\hat{\beta}^*$ *is also an efficient revenue-sharing rule in this new alliance game. As a result, the grand coalition solution* x^c *is a strong Nash equilibrium*[10] *under* β^*.

The proof of this lemma is straightforward once one realizes that the values of the Lagrange multipliers λ^c used in Theorem 1 to define β^* depend exclusively on the grand coalition optimization problem and remain unchanged after the merge between airlines k_1 and k_2. It follows then that β^* is efficient for any arbitrary reorganization of the ownership structure within the alliance and not just for the merger of two individual

---

[10] A strategy x^* is said to be a *strong Nash equilibrium* if for any coalition $S \subseteq \mathcal{K}$ there does not exist a strategy x_S such that each member in S is strictly better off if all members in S play x_S instead of x^*_S while assuming that players in $\mathcal{K} - S$ play x^*_{-S}.

members (see also point 2 in Corollary 1). From this, we can immediately conclude that x^c is a strong Nash equilibrium. As a direct consequence of Lemma 2, we have the following result.

THEOREM 2. *Let β^* be an efficient revenue-sharing rule satisfying the conditions in Theorem 1. Then, β^* is admissible in the sense of Definition 1. As a result, airlines' payoffs under β^* correspond to an allocation of the central planner's total payoffs in the core.*

Theorem 2 reveals a remarkable feature of our alliance game, namely, that a simple mechanism such as fixed revenue-sharing proration rates can achieve full coordination of the alliance in a noncooperative mode of operation (efficiency) and at the same time produce a payoff allocation that is immune to deviations by coalitions (admissibility). Both features support β^* as an output of an eventual negotiation and implementation. Furthermore, under the plausible conditions in Proposition 3, such β^* is unique.

As a final remark, note that β^* can also be obtained as an optimal solution of a Nash bargaining game among the airlines in the alliance. Let us fix a particular itinerary j and define the Nash bargaining solution for the allocation of the revenues generated by itinerary j as follows:

$$\max_{\beta_{kj}} \left\{ \prod_{k \in \mathcal{K}} (\beta_{kj} p_j)^{\theta_{kj}} \right\},$$

where θ_{kj} is the relative bargaining power of airline k with respect to itinerary j. The key step in connecting this bargaining model with our admissible proration rates is given by the choice of these relative bargaining powers. In particular, consider the following choice of bargaining power for airline k:

$$\theta_{kj} = \sum_{i \in \mathcal{M}_k} A_{ij} \lambda_i^c,$$

which is a measure of the opportunity cost that airline k incurs when selling an itinerary j. It is a matter of simple calculation to show that under this choice of $\{\theta_{kj}\}$ the optimal solution to the Nash bargaining model above coincides with β^* in Theorem 1.

5. Numerical Experiments

As pointed out earlier, in our theoretical analysis we assume SBL as the (unrealistic) capacity control policy implemented by the airlines. The purpose of this section is to evaluate the performance of the alliance members under our proposed proration rates when making decisions based on more realistic, widely used dynamic control policies. To this end, we compare the revenues obtained when applying four different static proration rates, including our optimal and heuristic schemes.

Figure 4   Four-Airline Network in Williamson (1992)
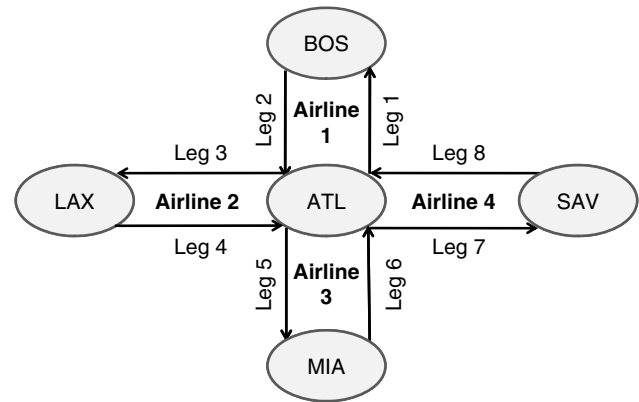
Note. Each airline controls a pair of legs from/to the hub.

5.1. Experimental Setup

We consider the hub-and-spoke network in Williamson (1992) (see Figure 4), which is a representative example of an airline network that has been used extensively in the literature. The alliance is built on such a network with multiple airline companies each controlling a set of flight legs. For example, Figure 4 shows one possible configuration with four airlines each controlling a pair of round-way flight legs between the hub and a spoke. There are local and interline itineraries, and on each possible route there are four fare classes denoted by Y, M, B, and Q. This gives a total of 80 possible itineraries: 32 local and 48 interline. In §5.2.2, we consider a set of alternative configurations in terms of ownership of the flight legs.

Demand for each itinerary is a Poisson process with mean as in Table 2. The table also includes the fare (or revenue) of each itinerary and the corresponding mileage and capacity.

In our computational experiments, we test the performance of a set of policies that differ in two dimensions: (1) the vector of proration rates used to split interline revenues, and (2) the airlines' admission control mechanism used to manage capacity. In terms of revenue-sharing prorations, our tests include the following.

• β^*: Our revenue-sharing proration proposed in Theorem 1. For the case of a class M ticket in a LAX–MIA itinerary, our proposed proration rates based on the dual solution of the grand coalition problem allocates \$142 to Airline 2 and the remaining \$97 to Airline 3.

• β^H: Heuristic prorations based on local fares computed in Equation (15). For example, the revenue from fare class M on the itinerary from LAX to MIA is split between Airlines 2 and 3 proportionally to the lowest local fare (i.e., class Q fare): \$83 and \$60, respectively. In this case, Airline 2 gets \$139 and Airline 3 gets \$100.

Table 2   **Description of Williamson's (1992) Network**

O-D market	Revenue				Mean demand				Mileage	Capacity	Operating carrier(s)
	Y	M	B	Q	Y	M	B	Q			
ATL–BOS/BOS–ATL	435	373	257	50	15	5	7	75	945	320	{2}
ATL–LAX/LAX–ATL	565	495	398	83	16	10	17	75	1,940	370	{1}
ATL–MIA/MIA–ATL	355	269	174	60	12	9	15	58	596	345	{4}
ATL–SAV/SAV–ATL	245	195	110	99	20	10	7	55	215	315	{3}
BOS–LAX/LAX–BOS	575	380	159	139	9	4	11	53	2,885	320	{1, 2}
BOS–MIA/MIA–BOS	403	314	124	89	11	5	14	51	1,541	320	{2, 4}
BOS–SAV/SAV–BOS	569	450	309	180	5	7	10	42	1,160	315	{2, 3}
LAX–MIA/MIA–LAX	477	239	139	119	15	7	10	70	2,536	345	{1, 4}
LAX–SAV/SAV–LAX	700	550	354	234	7	5	6	53	2,155	315	{1, 3}
MIA–SAV/SAV–MIA	526	468	294	149	13	4	7	53	811	315	{3, 4}

Notes. There are four operators and 20 one-way O-D pairs. The mileage of an interline itinerary is the sum of the two legs involved, and its capacity is the minimum of both leg capacities.

• β^M: Mileage-based proration under which an airline gets a split of revenues that is proportional to the total mileage of the itinerary that it operates. For example, according to Table 2, the revenues of an itinerary between LAX and MIA are split between Airlines 2 and 3 proportionally to the mileage they cover: 1,940 and 596 miles, respectively. Hence, out of the $239 from class M ticket, Airline 2 gets $183 and Airline 3 gets $56.

• β^F: Fare-based proration under which each fare class on an interline itinerary is mapped to the same fare class on each local leg in the itinerary so that the revenue is split proportionally to these local fare classes. For example, the $239 revenue of a class M ticket from LAX to MIA is split between Airlines 2 and 3 proportionally to the corresponding class M local fares: $495 and $269, respectively. As a result, Airline 2 gets $155 and Airline 3 gets $84.

In terms of capacity control, we test each revenue-sharing proration scheme under three policies: SBLs and two commonly used dynamic policies: dynamic bid price (DBP) and displacement adjusted virtual nesting (DAVN) controls. DBP and DAVN are dynamic in the sense that the booking horizon is divided into a number of subperiods (10 in our case) and the control parameters are revised at the beginning of each time window after updating the information on the demand forecast and the remaining capacity. The three policies are briefly described as follows (more details can be found in §A.2 in the online appendix).

• *SBL*: Each airline decides, at time 0, the number of seats that it will allocate to each itinerary, effectively partitioning its capacity among (possibly) all itineraries (see §3 for more details). This is the policy we used to compute β^*.

• *DBP*: Each airline estimates an opportunity cost (bid price) for each of the resources it operates, and accepts a request for an itinerary if and only if the revenue from this itinerary exceeds its opportunity cost (sum of bid prices of the resources it uses) and there

is enough capacity available. At each resolving point, we calculate these bid prices as the dual variables of a deterministic linear program (DLP) associated with the alliance network (e.g., see Chap. 3 of Talluri and van Ryzin 2004a for bid price controls in a monopolistic setting). For the alliance network, we compute this policy assuming that there is no inventory information exchange between the airlines. Under DBP, airlines compute a bid price for each leg without taking into account partners' inventories, as if any request for a seat on partners' legs will always be accepted, or equivalently, assuming infinite capacity and zero bid price for the partners.[11]

• *DAVN*: This is a network decomposition scheme where each airline first clusters or indexes different products on each resource into a fixed number of *virtual classes* according to the net benefit of accepting a request for the product on the resource. Then, the airline sets protection levels for these virtual classes on each resource. During the booking horizon, the protection levels will be updated at each resolving point. A request for a product is accepted only if it is accepted on each resource based on the corresponding protection levels and the remaining capacities. As with the bid price policy, in our alliance network, we consider a heuristic DAVN where we assume that each airline does not take into account the partners' inventories when setting the protection levels.

5.2. Experimental Results

5.2.1. Performance of Proration Rates. In our first set of experiments, we compare the performance of the different revenue-sharing prorations and inventory controls described earlier. In total, there are 12 (proration, inventory control) combinations. Table 3

---

[11] We are aware of commercial airlines that have been using this inventory control heuristic mainly due to the lack of flexibility in their RM systems to incorporate partners' capacity and admission control strategies.

Table 3   Relative Performance of Different Control Policies and Proration Schemes with Respect to the Upper Bound Provided by DLP

Proration	DF = 1			DF = 2			DF = 3		
	SBL	DBP	DAVN	SBL	DBP	DAVN	SBL	DBP	DAVN
β^*	0.9425	0.9860	0.9850	0.9467	0.9833	0.9768	0.9500	0.9800	0.9698
β^H	0.8912	0.9863	0.9849	0.8956	0.9842	0.9766	0.8935	0.9767	0.9669
β^M	0.8942	0.9835	0.9849	0.7784	0.9028	0.9396	0.6995	0.8560	0.8729
β^F	0.8947	0.9849	0.9849	0.8743	0.9602	0.9546	0.8430	0.9429	0.9380

shows the ratio of the alliance total revenues under a particular (proration, inventory control) policy to the upper bound for the revenues obtained solving the DLP for the grand coalition (see Cooper 2002 for details of an asymptotic analysis in a monopolistic network RM setting). This relative performance is computed for three different network workloads, defined by the demand factors

$$\mathrm{DF} := \frac{\sum_{j \in \mathcal{N}} |\mathcal{R}_j| \mu_j}{\sum_{i \in \mathcal{M}} c_i},$$

where μ_j is the mean demand for itinerary j, $\mathcal{R}_j$ is the set of legs used by j, and c_i is the capacity of leg i. The base case DF = 1 corresponds to the original values in Table 2, and DF = 2, 3 is obtained by scaling the mean demands by corresponding factors. Each value in Table 3 is an average computed over 500 simulations of demand instances for seven different asymptotic regimes, that is,

$$\frac{1}{7} \sum_{r \in \Gamma} \frac{1}{500} \sum_{\omega=1}^{500} \frac{\Pi(r, \omega, \beta^i, control)}{\Pi(r, \omega, \beta^*, \mathrm{SBL})},$$

where r is the scale of an asymptotic regime, and w is the index of a sample path, respectively. For each asymptotic regime, the capacity on each leg and the demand on each itinerary are multiplied by r, for $r \in \Gamma = \{0.8, 1, 1.2, 1.4, 1.6, 1.8, 2\}$.

In terms of the choice of proration rates, both our proposed solution β^* and its heuristic approximation β^H perform substantially better than the other two, particularly under highly congested networks. Between fare-based and mileage-based prorations, β^F has a better performance and is more robust in most scenarios under both static and dynamic controls. Even though it was derived under SBL, our heuristic proration β^H performs quite well under dynamic controls, with a relative gap with respect to β^* of less than 0.35%. Also, across all inventory control policies, β^H generally outperforms β^M and β^F, and does so by 9% and 2.2%, respectively, on average. These results suggest that our simple heuristic is a good candidate for practical implementation.

In terms of capacity control, Table 3 confirms the intuitive result that the use of dynamic policies has a positive impact on revenues compared to SBL. On average, DBP increases the total revenue by 9.03%

and DAVN increases the total revenue by 9.10%. Note that, consistent with the literature (e.g., see Talluri and van Ryzin 2004b), there is no clear dominance between bid price controls and displacement adjusted virtual nesting. Note also that as network congestion increases, our benchmark policy (β^*, SBL) can outperform dynamic policies that use suboptimal revenue-sharing rules, such as (β^M, DBP) and (β^M, DAVN). This suggests that in highly congested networks the choice of the revenue-sharing scheme can be more critical than the use of a dynamic capacity control policy.

5.2.2. Network Ownership. In this section, we test the impact of different network ownership structures based on variations of Figure 4. We consider five network ownership structures denoted by vectors (4 4), (6 2), (4 2 2), (2 4 2), (2 2 2 2), respectively. The number of elements in a vector represents the number of airlines in the alliance, and each element represents the number of flight legs the corresponding airline operates. For example, (2 4 2) represents a network with three airlines, Airline 1 operates legs {1, 2}, Airline 2 operates legs {3, 4, 5, 6}, and Airline 3 operates legs {7, 8}. The base case depicted in Figure 4 is (2 2 2 2).

As before, for each network configuration we consider the same 12 (proration, inventory control) combinations and seven asymptotic regimes. In this case, we compute the average revenue over 1,000 simulations for each asymptotic regime and for each (proration, inventory control) pair. Table 4 shows the relative performance of these revenue-sharing prorations with respect to the upper bound provided by the DLP solution of the grand coalition, for each of the five structures under consideration. The demand factor is set at DF = 2.

When tested under SBL, β^* dominates the other proration rules because of its self-consistency with the executed control policy (recall that β^* is in fact derived from SBL). Because the demand volume is high, the load factor is not affected much under SBL. Yet when tested under DBP and DAVN, both β^* and β^H perform quite closely and generally beat the performance under SBL. Under DBP, both β^* and β^H consistently outperform the mileage-based and fare-based prorations, and are off by less than 2% from the upper bound. Under DAVN, they also tend to dominate the other two (except for the network (6 2)). Also,

Table 4 **Performance of Different Revenue-Sharing Prorations in Five Network Ownership Structures for the Topology in Figure 4**

Control policy	Proration rule	Ownership structure				
		(4 4)	(6 2)	(4 2 2)	(2 4 2)	(2 2 2 2)
SBL	β^*	0.9460	0.9460	0.9460	0.9460	0.9460
	β^H	0.8159	0.7949	0.8562	0.8443	0.8953
	β^M	0.6617	0.7647	0.7775	0.7738	0.7778
	β^F	0.8026	0.7644	0.8333	0.8345	0.8736
DBP	β^*	0.9832	0.9803	0.9817	0.9831	0.9826
	β^H	0.9819	0.9894	0.9809	0.9836	0.9832
	β^M	0.8449	0.9369	0.9123	0.9221	0.9025
	β^F	0.9678	0.9600	0.9578	0.9582	0.9593
DAVN	β^*	0.9647	0.9128	0.9609	0.9510	0.9764
	β^H	0.9634	0.9172	0.9628	0.9563	0.9763
	β^M	0.8791	0.9416	0.9423	0.9375	0.9389
	β^F	0.9618	0.9142	0.9521	0.9360	0.9539

the alliance performance under β^* and β^H appears to be robust with respect to different market concentrations, which suggests that from the alliance perspective, changing the ownership structure through merges or splits may not affect overall system performance. This is consistent with our theoretical results (see point 2 in Corollary 1 and Lemma 2).

6. Conclusions

In this paper we propose a framework to study revenue-sharing schemes in airline alliances, posed as a two-stage hierarchical game. In the first stage, alliance members agree on proration rates that they use to split the revenues generated by their interline and codeshare itineraries. In the second stage, given these proration rates, each airline manages its own capacity in a noncooperative fashion so as to maximize its expected cumulative revenue during the booking horizon.

Our study of the alliance contractual and operational problem proceeds backward. First, we characterize equilibrium revenue-sharing prorations that are implemented in the second stage. The challenge here is to find *efficient* rates in the sense that the decentralized operation of the alliance achieves the first best solution, i.e., the revenues that would be obtained in case a central planner controls the whole network. This central planner can be seen as the managing partner of a grand coalition among the alliance members. Next, for the first stage, we seek *admissible* proration rates; i.e., rates under which no airline coalition has incentive to break off from the grand alliance. Assuming that airlines use SBLs to control capacity, we characterize optimal proration rates in the sense that they are simultaneously admissible and efficient. Although these rates turn out to be unique under relatively mild conditions, the downside is that their implementation requires the disclosure of private demand information. To circumvent this, we propose a simple alternative rule that is based on public fares.

Partitioned booking limits are chosen for the tractability of the analysis, but are seldom used in the real world. To test the practical potential of our optimal rule, and more importantly, of our heuristic rule, we use two dynamic control policies currently implemented by airlines: bid prices and DAVN. The optimal and heuristic proration rules perform notably well under both the original SBLs and dynamic control policies, and are almost consistently better than two other proration rules discussed in the industry: mileage-based and fare-based prorations. These observations position our heuristic proration rule as a very interesting candidate to be pursued in real world implementations.

Our model and results can be extended in a number of directions. First, in this paper we have not discussed the negotiation process of the contractual stage, but rather characterize the outcome of such a process. Negotiation is complex and could be approached from bargaining theory. Another aspect of the model that deserves more attention is our assumption of independent demands. There is a growing interest in the RM literature to model demand using consumer choice models. Bringing these ideas into an airline alliance setting can prove very challenging (for instance, we can no longer decouple local and interline itineraries as we did in this paper), but we think it is an important extension to pursue. Finally, static proration rules are simple to implement and acknowledged by practitioners as an interesting first step to pursue (e.g., see Vinod 2005), but the use of dynamic proration schemes could lead to additional revenue increases and is worthy of further exploration.

thank Richard Ratliff (Sabre Holdings) and Stefan Poelt (Lufthansa) for providing detailed comments on an early version of the manuscript and up-to-date information about the industry practice of airline alliances. Finally, the authors thank Vivek Farias (Massachusetts Institute of Technology) for pointing them to the connection between their admissible proration rates and the solution to the Nash bargaining model discussed in §4.3.

References

Agarwal R, Ergun Ö, Houghtalen L (2009) Collaboration in cargo transportation. Chaovalitwongse W, Furman KC, Pardalos PM, eds. *Optimization Logistics Challenges in the Enterprise*, Vol. 30 (Springer, New York), 373–409.

Bamberger G, Carlten D, Neumann L (2004) An empirical investigation of the competitive effects of domestic airline alliances. *J. Law Econom.* 47:195–222.

Boyd A (1998) Airline alliance revenue management. *OR/MS Today* 25:28–31.

Brandenburger A, Stuart H (2007) Biform games. *Management Sci.* 53:537–549.

Cachon G, Netessine S (2004) Game theory in supply chain analysis. Simchi-Levi D, Wu SD, Shen Z-J, eds. *Handbook of Quantitative Supply Chain Analysis: Modeling in the E-Business Era*, Chap. 2 (Springer, New York), 13–66.

Cai H, Lim A (2011) Decentralized control of stochastic dynamic resource allocation problems. Working paper, Department of Industrial Engineering and Operations Research, University of California, Berkeley, Berkeley.

Chun SY, Kleywegt AJ, Shapiro A (2011) Resource exchange seller alliances. Working paper, H Milton Stewart School of Industrial and Systems Engineering, Georgia Institute of Technology, Atlanta.

Cooper W (2002) Asymptotic behavior of an allocation policy for revenue management. *Oper. Res.* 50:720–727.

Czerny A (2009) Codesharing, price discrimination and welfare losses. *J. Transport Econom. Policy* 43:193–210.

Desiraju R, Shugan S (1999) Strategic service pricing and yield management. *J. Marketing* 63:44–56.

Gayle P (2007) Airline code-share alliances and their competitive effects. *J. Law Econom.* 50:781–819.

Gillespie W, Richard OM (2011) *Antitrust Immunity and International Airline Alliances* (Antitrust Division, U.S. Department of Justice, Washington, DC).

Hu X (2011) Collaborative revenue management of airline alliance. Unpublished doctoral dissertation, Leonard N. Stern School of Business, New York University, New York, http://pages.uoregon.edu/xingh.

Kunnumkal S, Topaloglu H (2010) A new dynamic programming decomposition method for the network revenue management problem with customer choice behavior. *Production Oper. Management* 19:575–590.

Li J, Netessine S (2011) Partnering with competitors: An empirical analysis of airline alliances and multimarket competition. Working paper, INSEAD, France.

Nagarajan M, Sošić G (2008) Game-theoretic analysis of cooperation among supply chain agents: Review and extensions. *Eur. J. Oper. Res.* 187:719–745.

Netessine S, Shumsky R (2005) Revenue management games: Horizontal and vertical competition. *Management Sci.* 51: 813–831.

Oum T, Park J (1997) Airline alliances: Current status, policy issues, and future directions. *J. Air Transport Management* 3:133–144.

Owen G (1975) On the core of linear production games. *Math. Programming* 9:358–370.

Park J (1997) The effect of airline alliances on markets and economic welfare. *Transportation Res. Part E* 33:181–195.

Park J, Zhang Y (2000) An empirical analysis of global airline alliances: Cases in North Atlantic markets. *Rev. Indust. Organ.* 16:367–384.

Phillips R (2005) *Pricing and Revenue Optimization* (Stanford University Press, Palo Alto, CA).

Rockafellar RT (1970) *Convex Analysis* (Princeton University Press, Princeton, NJ).

Shapley L, Shubik M (1969) On market games. *J. Econom. Theory* 1:9–25.

Talluri K, van Ryzin G (2004a) Revenue management under a general discrete choice model of consumer behavior. *Management Sci.* 50:15–33.

Talluri K, van Ryzin G (2004b) *The Theory and Practice of Revenue Management* (Springer, New York).

Vinod B (2005) Alliance revenue management. *J. Revenue Pricing Management* 4:66–82.

Wan X, Zou L, Dresner M (2009) Assessing the price effects of airline alliances on parallel routes. *Transportation Res. Part E* 45:627–641.

Williamson E (1992) Airline network seat inventory control: Methodologies and revenue impacts. Unpublished doctoral dissertation, MIT, Cambridge, MA.

Wright CP (2011) A bid-price decomposition of airline alliances under incomplete information. Working paper, College of Business Administration, Niagara University, Lewiston, NY.

Wright CP, Groenevelt H, Shumsky R (2010) Dynamic revenue management in airline alliances. *Transportation Sci.* 44:15–37.