Manufacturing & Service Operations Management

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Hedging Commodity Procurement in a Bilateral Supply Chain

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This paper explores the merits of hedging stochastic input costs (i.e., reducing the risk of adverse changes in costs) in a decentralized, risk-neutral supply chain. Specifically, we consider a generalized version of the well-known "selling-to-the-newsvendor" model in which both the upstream and the downstream firms face stochastic input costs. The firms' operations are intertwined—i.e., the downstream buyer depends on the upstream supplier for delivery and the supplier depends on the buyer for purchase. We show that if left unmanaged, the stochastic costs that reverberate through the supply chain can lead to significant financial losses. The situation could deteriorate to the point of a supply disruption if at least one of the supply chain members cannot profitably make its product. To the extent that hedging can ensure continuation in supply, hedging can have value to at least some of the members of the supply chain. We identify conditions under which the risk of the supply chain breakdown will cause the supply chain members to hedge their input costs: (i) the downstream buyer's market power exceeds a critical threshold; or (ii) the upstream firm operates on a large margin, there is a high baseline demand for downstream firm's final product, and the downstream firm's market power is below a critical threshold. In absence of these conditions there are equilibria in which neither firm hedges. To sustain hedging in equilibrium, both firms must hedge and supply chain breakdown must be costly. The equilibrium hedging policy will (in general) be a partial hedging policy. There are also situations when firms hedge in equilibrium although hedging reduces their expected payoff.

Keywords: risk management; commodity procurement; supply chain contracting; hedging
History: Received: July 25, 2011; accepted: September 27, 2014. Published online in *Articles in Advance*
February 19, 2015.

1. Introduction

Much of the extant supply chain and inventory literature takes demand as stochastic and production costs as fixed. The latter assumption, however, is in contrast to many industrial settings in which manufacturing firms are exposed to price changes of commodities that affect the direct costs of raw materials, components, subassemblies, and packaging materials as well as indirect costs from the energy consumed in operations and transportation. According to a survey of large U.S. nonfinancial firms (e.g., Bodnar et al. 1995), approximately 40% of the responding firms routinely purchase options or futures contracts in order to hedge[1] price risks such as those mentioned above. Why do these firms hedge?

To provide rationale for hedging, extant theories have relied on the existence of taxes (e.g., Smith and Stulz 1985), asymmetric information (e.g., DeMarzo and Duffie 1991), hard budget constraints and costly external capital (e.g., Froot et al. 1993, Caldentey and Haugh 2009), and risk aversion (e.g., Gaur and Seshadri 2005, Chod et al. 2010). All these papers take a firm as the basic "unit of analysis" and, broadly speaking, analyze situations in which the firm plans to either purchase or sell output at a later date while facing one of the above frictions. When viewed from today, the output's price and consequently the firm's transaction proceeds are random. However, by somehow eliminating the randomness in price—or by hedging—the firm is able to increase its (expected) payoff, which makes hedging beneficial to the firm.

The thesis behind this paper is that when several firms are embedded together in a supply chain while simultaneously facing the situation we described above, there will be an added level of complexity, which the existing theories cannot necessarily address. First, in a supply chain environment, firms will likely find themselves exposed not only to their own price risks but also to their supply chain partner's price risk. This is illustrated with an example from the auto industry (Matthews 2011). In 2011, steelmakers increased prices six times, for a total

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[1] Throughout the paper, to "hedge" means to buy or sell commodity futures as a protection against loss or failure because of price fluctuation. For details, see Van Mieghem (2003, p. 271).

increase of 30%. However, consistent with the extant theory, many car and appliance manufactures did not necessarily pay attention to steel prices because they did not directly purchase steel from steelmakers, only to find themselves in difficulty with some of their financially weaker suppliers, for whom steel represents their biggest cost. In a supply chain, price increases can therefore result in the supplier experiencing significant financial losses if it does not have the resources to manage price risk. The situation could deteriorate to the point of a supply disruption if the supplier cannot profitably make products for its buyer. Although firms can try to mitigate the risk of disruption by contractually imposing penalties, as will be seen, default penalties are often not a credible guarantee of supply unless they are also supported with hedging.

Second, unlike in the stand-alone firm's setting, supply chain firms cannot necessarily control any price risk they want by hedging alone. Consequently, firms will care whether and how their supply chain partners hedge, and any hedging activity will be impounded in transfer prices. Finally, the hedging behavior of their supply chain partners will affect their own hedging behavior, as will be seen.

Following the seminal contribution of Lariviere and Porteus (2001)—who point out that there is an intimate relationship between wholesale price, demand uncertainty, and the distribution of market power in the supply chain—this paper explores the merits of hedging stochastic input costs to ensure continuity of supply in a decentralized supply chain. Specifically, we consider a generalized version of the well-known "selling-to-the-newsvendor" model (Lariviere and Porteus 2001) in which both the upstream and the downstream firms face stochastic production costs. (In contrast to our paper, in Lariviere and Porteus 2001, production costs are linear with constant coefficients.) The intuition that underlies our model of hedging in a supply chain is as follows. By agreeing to sell their output at (contractually) predetermined prices before all factors affecting their productivity are known, both the upstream supplier and the downstream buyer subject themselves to default risk. The fact that a supply chain member may default in some states of the world will be, ex ante, impounded into the wholesale price as well as into the order quantity. However, if the firms could somehow "guarantee" that they will fulfill the supply contract (no matter what happens to their inputs costs), their supply chain counterpart may respond with a more favorable price or a more favorable order quantity. Either firm would like to guarantee supply contract performance if the expected profit associated with the guarantee is greater than the expected profit associated with default in some states. In such circumstances, the firm can commit to the guarantee by purchasing futures contracts on the underlying input commodity. The futures contracts will pay off precisely at the time when the firm's resources are strained. Hence, futures contracts have value because they prevent the firm from defaulting on a contractual obligation when not defaulting is (ex ante) important.

We identify the following conditions under which the supply chain firms should be expected to hedge in equilibrium: (i) the downstream buyer's market power exceeds a critical threshold; or (ii) the upstream firm operates on a large margin, there is a high baseline demand for downstream firm's final product, and the downstream firm's market power is below a critical threshold. In absence of these conditions there are equilibria in which neither firm hedges. To sustain hedging in equilibrium, both firms must hedge and supply chain breakdown must be costly. The equilibrium hedging policy will (in general) be a partial hedging policy. There are also situations when firms hedge in equilibrium although hedging reduces their expected payoff. (A partial hedging policy leaves the hedger exposed to some residual price risk. A full hedging policy eliminates all price risk.)

To summarize, we provide a rationale for hedging, which is based on a noncooperative behavior in a supply chain. We illustrate this point with an anecdotal example from a diversified global manufacturing company. The company's Residential Solutions Division produces a collection of home appliances and power tools, which are sold worldwide. Its Commercial Division manufactures industrial automation equipment and climate control technology. Many of these products are raw-material-intense. When it comes to supply chain management, the firm plays a variety of roles: Sometimes the firm is a supplier; other times, the firm plays a role of an end-product assembler. Recently, the company saw an increased volatility in the prices of metals, which affected its margins. To combat this, the firm hedged its raw material inputs, but only some. When asked when the firm hedges, a manager who presented the case replied that the firm's hedging decisions partly depend both on the product and on the firm's supply chain partners. For example, as a supplier, the firm fully hedges the raw material it uses to produce garbage disposals, which are sold through a major U.S. home-improvement store. This is because the firm's bargaining power against this retailer is small, and it is the retailer who dictates both production quantities and pricing. As a buyer, however, the firm tends not to hedge when it knows that it may face glitches in supply, because then it is exposed to risk of financial losses from its hedging activities. The existing risk-management literature does not always provide clear guidance on hedging in cases like these.

Either because it requires frictions (e.g., risk aversion), which do not apply to large multinationals (Carter et al. 2006, Jin and Jorion 2006), or because it does not allow hedgers to vary their hedging decisions across products and channel relationships. Whereas our model is a simple selling-to-the-newsvendor setting, this example illustrates that the equilibrium that we advance seems to mirror the facts of these cases (see part (i) of Proposition 2).

In what follows, we review related literature, present the model (§§3 and 4), and derive the firms' equilibrium behavior under the unhedged and hedged contracts (§§5.1 and 5.2). Then, by comparing the expected payoffs both firms can achieve under the unhedged and hedged contracts, we make predictions as to when offers of hedged wholesale contract should be expected in equilibrium (§5.3). Section 6 concludes.

2. Related Literature

Recent research in the finance literature has greatly improved our understanding of why and how individual firms should hedge. For instance, it has established that for hedging to be beneficial, the firm's payoff must be a concave function of the stochastic hedge-able exposure metric. In spirit, this result follows from basic convex-analysis theory: It means that the firm's *expected* payoff, a weighted average of a concave function, is lower when the level of variability of the exposure metric is higher. Thus, since hedging reduces variability, then it increases the firm's expected payoff.

Focusing on a *single-firm setting*, Smith and Stulz (1985) demonstrate that managerial risk aversion or market frictions such as corporate taxes cause a firm's payoff to become a concave function of its hedge-able exposure. One of the implications of this paper's result is that firms should hedge fully or not at all. Froot et al. (1993) expand the discussion to settings in which firms are exposed to multiple sources of uncertainty that are statistically correlated. They show that in such type of situations, it may become optimal for a firm to utilize partial hedging. Building upon this observation, a large body of the finance literature has studied hedging by taking a firm as the basic "unit of analysis" and exogenously *imposing* the aforementioned concavity property. This property has been commonly justified by assuming that the hedger's preferences on the set of its payoffs are consistent with some form of concave utility function. This literature provides insights into the mechanics of hedging using financial instruments such as futures and options. For instance, Neuberger (1999) shows how a risk-averse supplier may use short-maturity commodity futures contracts to hedge a long-term commodity supply commitment. A comprehensive summary of this research is given in Triantis (2005).

Whereas the finance literature focuses on mitigating pricing risk, the operations management literature predominantly considers demand and supply risks. An excellent survey of the early operations literature can be found in Van Mieghem (2003). More recent and influential papers in this area include Gaur and Seshadri (2005) and Caldentey and Haugh (2009). Gaur and Seshadri (2005) address the problem of using market instruments to hedge stochastic demand. The authors consider a newsvendor who faces demand that is a function of the price of an underlying financial asset. They show how firms can use price information in financial assets to set optimal inventory levels, and they demonstrate that financial hedging of demand risk can lead to higher return on inventory investment. Caldentey and Haugh (2009) show that financial hedging of stochastic demand can increase supply chain output. Note, however, that there are several important differences between the supply chain considered in Caldentey and Haugh (2009) and the supply chain in this paper. Their supply chain is centralized, the hedge-able risk in their chain is stochastic demand, and the risk is faced by only one of the firms in the supply chain. To provide a rationale for the hedging behavior, Gaur and Seshadri (2005) rely upon risk aversion; Caldentey and Haugh (2009) require market frictions, which lead to credit rationing.

Other papers in the operations management literature discuss and analyze hedging strategies for managing risky supply and ask when it makes a difference if firms hedge financially or operationally. For example, while intuition may suggest that operational and financial hedging may be substitutes, surprisingly Chod et al. (2010) show that they can be complements. Emergency inventory (e.g., Schmitt et al. 2010); emergency sourcing (e.g., Yang et al. 2009); and dual sourcing (e.g., Gümüş et al. 2012) are examples of operational hedging strategies capable of mitigating adverse supply shocks. Supplier subsidy (e.g., Babich 2010) is an example of a financial strategy designed to reduce the likelihood of supply contract abandonment. Haksöz and Seshadri (2007), in turn, put financial value on the supplier's ability to abandon a supply contract. We differ from the aforementioned literature by focusing on the management of input cost risk (rather than demand risk) and by extending the discussion of why and how risk-neutral firms hedge from the single-firm setting into a decentralized supply chain.

3. Preamble: A Model of a Vertically Integrated Supply Chain

Consider the problem of a risk-neutral manufacturer (final good assembler), M, who faces the newsven-

Figure 1   Inputs Required to Produce One Unit of Assembler's Final Product
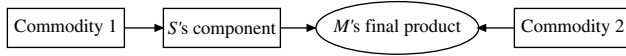
dor problem in that at time t_0 it must choose a production quantity, q, before it observes a single realization of stochastic demand, D, for its final product. Stochastic demand for the final product, D, has a distribution F and density f. The function F has support on $[\underline{d}, \bar{d}]$, i.e., there is always some demand (Cachon 2004, p. 225). The selling price of the final product is normalized to 1.

To assemble q units of the final product, M requires q units of commodity 2 as well as q components (subassemblies), which are produced by a component supplier, S. To produce q components, S requires q units of commodity 1. (Figure 1 summarizes the production inputs required to assemble 1 unit of the final product.) For simplicity, unmet demand is lost, unsold stock is worthless, and per unit production and holding costs are zero.

Because of operational frictions (e.g., capacity constraints and lead times), the subassembly production cannot be completed before time $t_1 > t_0$. Consequently, the final assembly cannot be completed before time $t_2 \geq t_1$. The manufacturer's, M's, sales revenue is realized at time $t_3 \geq t_2$ with the proviso that M cannot sell more at time t_3 than what it produced for at time t_2 and what the supplier, S, produced for at time t_1. Neither firm can produce more than what both firms committed to at time t_0.

The commodity $i = 1, 2$ per-unit price, S_i, is revealed at time t_i. Viewed from t_0, S_i is a random variable modeled on a complete probability space $(\Omega, \mathscr{F}, \mathbb{P})$ equipped with filtration $\{\mathscr{F}_t\}$, $t \in [t_0, t_3]$. S_1 and S_2 have a joint density function h. For simplicity, the discount rate between times t_0 and t_3 is taken to be zero. From CAPM, this is equivalent to assuming that the risk-free rate is zero and that both input commodities are zero beta assets. Relaxing these assumptions can be done without changing the main message of the results presented here.

We begin by observing that, in the absence of any working capital constraints, a *vertically integrated supply chain* in this context would actually take advantage of input price volatility and suspend production whenever input costs are unfavorable. To see this, for the case when $t_0 < t_1 = t_2 = t_3$, the time t_0 expected payoff of the integrated system is

$$\mathbb{E}_{t_0} \Pi(q) = \mathbb{E}_{t_0} \min\{q, D\}(1 - S_1 - S_2)^+. \tag{1}$$

The right-hand side of Equation (1) says that at time t_0, the integrated supply chain holds $\min\{q, D\}$ of put options, each with a time t_3 payoff of $(1 - S_1 - S_2)^+$.

Because of this optionality, the integrated system's expected payoff increases both in q and in input cost volatility. Consequently, the system's optimal production quantity will be $q = \bar{d}$. (When $t_0 < t_1 \leq t_2 \leq t_3$, the expression for $\mathbb{E}_{t_0} \Pi(q)$ becomes more complicated, but no new important insights are added.)

The vertically integrated supply chain design is contrasted with a *decentralized* supply chain design where M and S operate independently. Then,

• each firm will exercise its market power by pushing its price above its (expected) marginal cost, which will lead to double-marginalization;

• each firm will face a stochastic input cost, which will lead to an incentive to default when defaulting is not necessarily optimal for the integrated channel.

Under some conditions, the integrated channel payoff given in (1) can be replicated with the use of a *state-contingent contract*. A generalized version of the well-known revenue-sharing contract $\{w_r, \phi\}$ is an important example of such a contract, where $w_r = \phi S_1 - (1 - \phi)S_2$ is a wholesale price the assembler pays to the supplier at time t_2 and $0 \leq \phi \leq 1$ is the fraction of the supply chain revenue the assembler keeps at time t_3; so $(1 - \phi)$ is the fraction the supplier gets.

Notice that w_r is a function of both S_1 and S_2, which are revealed at times t_1 and t_2. Pricing this way is what allows the firms to share their production costs and thereby achieve the integrated channel outcome. As such, $w_r > 0$ if and only if $S_1 > S_2(1 - \phi)/\phi$; otherwise $w_r \leq 0$, which means that the supplier pays the assembler. If $t_1 = t_2 = t_3$, then the supplier's and the assembler's time t_3 payoffs, respectively, are as follows:

$$\Pi^S(q) = (1 - \phi)\min\{q, D\} + q(w_r - S_1) \quad \text{and}$$
$$\Pi^M(q) = \phi \min\{q, D\} - q(w_r + S_2). \tag{2}$$

A special case of this generalized revenue sharing contract is the well-known pass-through contract, which is obtained by setting ϕ to 1. Under the pass-through contract, raw material charges are simply passed through in full onto the downstream assembler.

EXAMPLE 1. Let $\phi = 1/2$ and suppose that viewed from time t_0, (S_1, S_2, D) is a Bernoulli random vector that takes on values $(4/5, 2/5, 5)$, $(2/5, 2/5, 5)$ with respective probabilities $\mathbb{P}\{(4/5, 2/5, 5)\} = 1/2$, $\mathbb{P}\{(2/5, 2/5, 5)\} = 1/2$. Then both the supplier's and the assembler's time t_3 payoffs are either 0 or 1/2, depending on the outcome of S_1. The total supply chain payoff is 0 or 1, which matches the integrated channel payoff given by the right-hand side of (1).

Example 1 illustrates a situation where the state-contingent contract addresses both double-marginalization and the adverse default incentive. That is, under the terms of the contract, the forward-looking

supplier defaults *exactly* when it is optimal to default for the integrated chain and the sum of both firm's expected payoffs is the same as the expected payoff of the integrated chain. However, as seen next, there are situations where the contract fails.

EXAMPLE 2 (CONTINUATION OF EXAMPLE 1). Everything is the same as in Example 1, except that viewed from time t_0, (S_1, S_2, D) is a Bernoulli random vector that takes on values $(2/5, 2/5, 5)$, $(2/5, 0, 5)$ with respective probabilities $\mathbb{P}\{(2/5, 2/5, 5)\} = 1/2$, $\mathbb{P}\{(2/5, 0, 5)\} = 1/2$, and $\phi = 2/5$.

Now, the state contingent contract of Example 1 no longer matches the integrated chain payoff for all outcomes of (S_1, S_2, D). To see this, suppose $(S_1, S_2, D) = (2/5, 2/5, 5)$. With $q = 5$, the integrated chain payoff will be $\min\{D, q\} - (4/5)q = 1 > 0$ while the decentralized chain's total payoff will be 0. The latter follows because $w_r = \phi S_1 - (1 - \phi)S_2 = -(2/25)$. Therefore by *not* producing, the assembler will collect $(2/25)q = 2/5 \ (= -w_r q)$ at time t_2. At the same time, the highest payoff the assembler can achieve by producing at time t_2 is $(2/25)q = 2/5$. The assembler, therefore, has no incentive to produce at time t_2. To avoid losing $(12/25)q = 12/5 \ (= qS_1 - qw_r)$, the forward-looking supplier breaks the contract at time t_1.

An obvious resolution to the situation described in Example 2 is to impose contractual *default penalties* that would induce the firms not to shirk when producing is profitable for the integrated supply chain. Had the assembler faced a default penalty greater than $(2/5)$ at time t_2, the state-contingent contract in Example 2 would again achieve the integrated channel payoff. A state-contingent contract with penalties, however, is still not a guarantee that all supply chain members will default *only* when it is optimal to default for the integrated supply chain.

EXAMPLE 3 (CONTINUATION OF EXAMPLE 2). Everything is the same as in Example 2, except that $t_1 \leq t_2$; S has zero assets; and when viewed from time t_0, (S_1, S_2, D) is a Bernoulli random vector that takes on values $(2/5, 2/5, 5)$, $(2/5, 4/5, 5)$ with respective probabilities $\mathbb{P}\{(2/5, 2/5, 5)\} = 1/2$, $\mathbb{P}\{(2/5, 4/5, 5)\} = 1/2$.

Under these assumptions, the supplier will require $(2/5)q$ in cash-on-hand to procure production inputs at time t_1 and either $(2/25)q$ or $(8/25)q$ (depending on the state of the world at time t_2) to pay the assembler at time t_2.

Facing a hard budget constraint, the upstream supplier, S, could do one of the following: (a) borrow from a bank and incur an interest payment, which is a cost that the integrated supply chain could *avoid*; (b) borrow from the downstream assembler. To overcome the supplier's budget constraint, the assembler could simultaneously procure commodity 1 at time t_1,

worth $(2/5)q$, and put over the payment of $w_r q$ to time t_2. The problem is that option (b) is not incentive compatible because if $(S_1, S_2, D) = (2/5, 4/5, 5)$, then the assembler will earn a payoff of $-(2/5)q$. If, on the other hand, $(S_1, S_2, D) = (2/5, 2/5, 5)$, then the supplier will earn $2/5$. Since $(1/2)(2/5) - (1/2)(2/5)q \leq 0$ for all $q = 1, 2, \ldots$, then the assembler has no incentive to lend to the supplier. At the same time, the integrated channel can achieve a payoff of 1 with probability $1/2$.

In summary, characteristics of the industry or channel relationship could make implementing of the state-contingent contract either expensive or ineffective. Since a price-only contract (Lariviere and Porteus 2001) is a natural alternative to the state-contingent contract, evaluating performance under such a contract is a first step in determining whether the additional costs of a more sophisticated policy are worth bearing.

Similarly to the state-contingent, the traditional price-only contract too does not address the adverse default issue. The idea that we develop in the remainder of this paper is that hedging *together* with contractually imposed default penalties are a way to protect the entire supply chain from failing when one of its members faces high realized input costs while its supply chain partner would like to produce.

4. Model of a Decentralized Supply Chain

Our assumptions are exactly the same as in §3 with the provision that the supply chain is decentralized, implying that the supplier, S, is now selling to the newsvendor, M. Both firms are risk neutral and are protected by limited liability. Each firm has a reservation payoff $R_j \geq 0$ and $c_j \geq 0$, $j \in \{M, S\}$ dollars in cash-on-hand. Firms have no outside wealth. (As explained in Cachon 2003, p. 234, exogenous reservation payoffs, R_j, are a standard way to model a firm's market power.)

One can interpret the firms as playing a game over times $t_0 < t_1 \leq t_2 \leq t_3$. The supplier offers the assembler a wholesale contract at time t_0 when only the *distribution function* of future commodity input costs is known. Because of production lead times, the supplier produces the subassembly at time t_1 and the assembler performs the final assembly at time t_2. As before, an outcome of the supplier's commodity input cost, $S_1 \geq 0$, is revealed at time t_1; an outcome of an assembler's commodity input cost, $S_2 \geq 0$, is revealed at time t_2; the assembler's sales revenue is realized at time t_3. The discount rate between times t_0 and t_3 is taken to be zero.

As explained in §3, both firms may have separate incentives to default on the supply contract when their time t_i commodity input costs turn out be too

high in relation to their predetermined selling prices. We assume that if a firm defaults on the supply contract, then it incurs a contractually predetermined default penalty L_j, $j \in \{M, S\}$. Throughout the paper, we assume that the penalty, L_j, is payable to firm j's supply chain partner. (It is, however, rather straightforward to change this to a model, in which L_j represents an indirect, reputation cost.) If both firms choose to produce, then q is the *highest payoff* either firm can achieve, as will be seen. *To prevent a situation in which one firm experiences a financial windfall when its supply chain partner defaults, throughout the analysis we assume that $L_S \leq q$ and $L_M \leq q$.* (As an example of a supply contract penalty, see the Plymouth Rubber Company contract in §A of the online supplement (provided at http://dx.doi.org/10.1287/msom.2014.0514); there, L_j is set as a percentage of the wholesale price. In the Davita, Inc. contract, the penalty can be more than 100% of the wholesale price.)

One of the goals of this paper is to investigate whether firms should be expected to *hedge* their commodity input costs to avoid default on the underlying supply contract. For this reason, we suppose that for each commodity $i = 1, 2$ there exists a futures contract to purchase a specific amount of commodity i at time t_i for a prespecified (futures) price, F_i. As is convention, the futures price is set so that the value of the futures contract at inception is zero. The payoff to a futures contract on commodity $i = 1, 2$ is realized at time t_i, where the payoff is the difference between the futures price, F_i, and the time t_i spot price, S_i. We use the variable n_j, $j \in \{M, S\}$ to represent firm j's position in the futures market: $n_j > 0$ indicates that firm j has a "long position" of n_j futures contracts, and $n_j < 0$ indicates that it has a "short position." If the firm $j \in \{M, S\}$ buys—or is long in—n_j futures contracts, its time t_i payoff is $n_j(S_i - F_i)$, which will be *positive when input prices are high and negative when input prices are low*. In our setting, both firms will be hedging by taking long positions. Therefore, hereafter, $n_j \geq 0$, $j \in \{M, S\}$.

The futures contracts are negotiated at a futures exchange, which acts as an intermediary. One of the key roles of the exchange is to organize trading so that futures contract defaults are completely *avoided* (for additional discussion, see Hull 2014, §§2 and 3). For this reason, the exchange will limit the number of contracts a firm $j \in \{M, S\}$ can buy if it knows that firm j will not have sufficient ex post resources to pay off its long futures contract position when the realized input prices at time t_i, $i = 1, 2$ are low.

In summary, there are three key assumptions about penalties and futures contracts that underlie our model of supply chain hedging:

ASSUMPTION 1. *Any default on the underlying supply contract is costly; neither firm, however, stands to unusually profit from a default of its supply chain partner.*

ASSUMPTION 2. *To avoid pathological cases, the futures prices F_i, $i = 1, 2$ are low enough so that by hedging, neither firm effectively locks in a negative (expected) profit.*

ASSUMPTION 3. *The futures market is credible, meaning that neither firm $j \in \{M, S\}$ defaults on its futures contract position in all possible futures states of the world* (Hull 2014, §§2 and 3).

Later in the paper, we will translate Assumptions 1–3 into mathematical conditions. It is worth mentioning that all three assumptions are quite standard in the hedging literature, which includes Smith and Stulz (1985) and Froot et al. (1993).

4.1. Game Sequence
What follows is a sequence of stages in the game between the supplier and the assembler. At time t_0:
- Firms choose to take positions $N_j \geq 0$, $j \in \{M, S\}$ in the futures contracts.
- The futures exchange accepts $0 \leq n_j \leq N_j$, $j \in \{M, S\}$.
- Supplier, S, sets a wholesale price, w.
- Assembler, M, sets an order quantity, q.

This ends time t_0. Before time t_1 begins, the future spot price of commodity 1 price is revealed. At time t_1:
- The supplier produces q units for the assembler and incurs a production cost qS_1. (If the supplier chooses not to produce, then it incurs a penalty L_S.)

This ends time t_1. Before time t_2 begins, the future spot price of commodity 2 and the supplier's production decision at time t_1 are revealed. If the supplier produced at time t_1, then at time t_2:
- The assembler produces q units of the final product and incur a production cost $q(w + S_2)$. (If the assembler chooses not to produce, then it incurs a penalty L_M.)

This ends time t_2. Before time t_3 begins, the value of demand D is revealed. If the assembler produced at time t_2, then at time t_3 the sales revenue $\min\{q, D\}$ is realized.

In terms of the information structure, we assume that all market participants can observe the actions taken by all players and can observe all market outcomes, i.e., information is complete. This assumption can best be justified with emerging empirical research in finance, which reveals that companies commonly incorporate covenant restrictions into supply contracts (e.g., see Costello 2011) that help them reduce or eliminate asymmetric information and moral hazard.

5. Analysis of the Decentralized Supply Chain
As previously mentioned, one of the objectives of this paper is to identify conditions under which the supply chain firms should be expected to hedge their

commodity input costs. We will proceed with the analysis by first assuming that neither firm hedges its input costs and show that in this case both firms subject themselves to default risk, since they agree to sell their output at prices that are fixed before all factors affecting their productivity are known. We'll refer to this situation as the "unhedged wholesale contract." Where useful, the subscript "*U*" will be used to indicate association with this subgame.

We then analyze the case in which the supplier avoids defaulting on the supply contract by entering into $n_S > 0$ futures contracts to buy commodity 1 at time t_1 and the assembler avoids defaulting on the supply contract by entering into $n_M > 0$ futures contracts to buy commodity 2 at time t_2. We will refer to this subgame as the "hedged wholesale contract" and use the subscript "*H*" to represent it. In analyzing this subgame, we determine the number of futures contracts, n_j, $j \in \{M, S\}$, both firms will be purchasing in equilibrium and consider what happens if $N_j > 0$ and $N_k = 0$, $j, k \in \{M, S\}$, $j \neq k$, which is a case in which one of the firms attempts to hedge while its supply chain partner does not hedge.

As is standard, the model is solved in two stages using backward induction: First stage characterizes the equilibrium behavior of the assembler; second stage characterizes the equilibrium behavior of the supplier and identifies the equilibrium in each subgame. The final equilibrium in which both firms decide whether to hedge can be determined by simply comparing the expected profits the supply chain firms generate with and without hedging.

The cash flows reveal that, in general, each firm $j \in \{M, S\}$ will choose not to produce if the firm j's future spot price, S_i, $i = 1, 2$, is either too high in relation to firm j's selling price (retail or wholesale price) or too low in relation to firm j's futures price, F_i, $i = 1, 2$. We evaluate the issue of default in detail in Lemma 1 and present the results graphically in

Figure 2 (Color online) Supplier's and Assembler's Equilibrium Strategies

Figure 2. Given arbitrary futures contract positions, $N_M \geq 0$ and $N_S \geq 0$, cash flows to both firms are

$$
\Pi^M = \begin{cases}
N_M(S_2 - F_2) + \min\{q, D\} - qS_2 - wq \\
\quad \text{if } u_{L1}^S < S_1 \leq u_{H1}^S, u_L^M < S_2 \leq u_H^M, t_2 < t_3, \\
N_M(S_2 - F_2) + \min\{q, D\}(1 - S_2) - wq \\
\quad \text{if } u_{L1}^S < S_1 \leq u_{H1}^S, u_L^M < S_2 \leq u_H^M, t_2 = t_3, \\
\max\{N_M(S_2 - F_2) - L_M, -c_M\} \\
\quad \text{if } u_{L2}^S < S_1 \leq u_{H2}^S, S_2 > u_H^M, t_2 \leq t_3, \\
\max\{N_M(S_2 - F_2) - L_M, -c_M\} \\
\quad \text{if } u_{L1}^S < S_1 \leq u_{H1}^S, S_2 \leq u_L^M, t_2 \leq t_3 \\
\max\{N_M(S_2 - F_2) + \ell_S, -c_M\} \\
\quad \text{otherwise;}
\end{cases} \quad (3a)
$$

$$
\Pi^S = \begin{cases}
N_S(S_1 - F_1) + q(w - S_1) \\
\quad \text{if } u_{L1}^S < S_1 \leq u_{H1}^S, u_L^M < S_2 \leq u_H^M, t_1 \leq t_2, \\
N_S(S_1 - F_1) + \ell_M - qS_1 \\
\quad \text{if } u_{L2}^S < S_1 \leq u_{H2}^S, S_2 > u_H^M, t_1 \leq t_2, \\
N_S(S_1 - F_1) + \ell_M - qS_1 \\
\quad \text{if } u_{L1}^S < S_1 \leq u_{H1}^S, S_2 \leq u_L^M, t_1 \leq t_2, \\
\max\{N_S(S_1 - F_1) - L_S, -c_S\} \\
\quad \text{otherwise;}
\end{cases} \quad (3b)
$$

where

$$
\begin{aligned}
\ell_S &\equiv \min\{L_S, (c_S + N_S(S_1 - F_1))^+\} \quad \text{and} \\
\ell_M &\equiv \min\{L_M, (c_M + N_M(S_2 - F_2))^+\}.
\end{aligned} \quad (3c)
$$

Above, ℓ_j, $j \in \{M, S\}$ are expressions for penalties that each firm j winds up paying if it fails to produce. In (3c), $N_j(S_i - F_i)$, $j \in \{M, S\}$, $i = 1, 2$ is firm j's payoff from the futures contract position and $c_j + N_j(S_i - F_i)$ is firm j's total capital at time t_i. Taken together, the expressions for ℓ_j reflect the fact that both firms j have *limited liability*. The u's in Equations (3a) and (3b) are spot prices of commodities 1 and 2 at which the supplier and the assembler will break the underlying supply contract at times t_1 and t_2, respectively. Their values are as follows. (Formal proofs of all of the paper's results are included in §B of the online supplement.)

Lemma 1. *If $t_0 < t_1 \leq t_2 \leq t_3$ and $0 \leq N_j \leq q$, $j \in \{M, S\}$, then in (3), $u_{L1}^S = u_{L2}^S = 0$, $u_L^M = 0$, and*

$$
u_{H1}^S = \begin{cases}
\min\left\{\left(\dfrac{wq + c_S - N_S F_1}{q - N_S}\right)^+, w + \dfrac{L_S}{q}, 1\right\} \\
\quad \text{if } t_1 = t_2, \\
\min\left\{\left(\dfrac{c_S - N_S F_1}{q - N_S}\right)^+, (q w \, \mathbb{P}\{S_2 \leq u_H^M\} \right. \\
\quad \left. + \ell_M \, \mathbb{P}\{S_2 > u_H^M\} + L_S) \cdot (q)^{-1}, 1\right\} \\
\quad \text{if } t_1 < t_2;
\end{cases} \quad (4a)
$$

$$
u_{H2}^S = \begin{cases} \min\left\{ \dfrac{\ell_M + L_S}{q}, \left(\dfrac{\ell_M + c_S - N_S F_1}{q - N_S}\right)^+, 1 \right\} \\ \qquad \text{if } t_1 = t_2, \\ u_{H1}^S \qquad \text{if } t_1 < t_2; \end{cases} \tag{4b}
$$

$$
u_H^M = \begin{cases} \min\left\{ \left(\dfrac{\min\{q,D\} + c_M - N_M F_2 - w q}{\min\{q,D\} - N_M}\right)^+, \right. \\ \qquad \left. \left(\dfrac{\min\{q,D\} + L_M - w q}{\min\{q,D\}}\right)^+, 1 \right\} \\ \qquad \text{if } t_2 = t_3, \\ \min\left\{ \left(\dfrac{c_M - N_M F_2 - w q}{q - N_M}\right)^+, \right. \\ \qquad \left(\dfrac{\mathbb{E}_{t_0} \min\{q,D\} + L_M}{q} - w\right)^+, \\ \qquad \left. \dfrac{\mathbb{E}_{t_0} \min\{q,D\}}{q} \right\} \\ \qquad \text{if } t_2 < t_3. \end{cases} \tag{4c}
$$

If $t_0 < t_1 \le t_2 \le t_3$ and $0 \le q < N_j$, $j \in \{M, S\}$, then in (3):

$$
u_{H1}^S = \begin{cases} \max\left\{ u_{L1}^S, \min\left\{ w + \dfrac{L_S}{q}, 1 \right\} \right\} \\ \qquad \text{if } t_1 = t_2, \\ \max\left\{ u_{L1}^S, \min\left\{ (q w \, \mathbb{P}\{S_2 \le u_H^M\} \right. \right. \\ \qquad \left. \left. + \ell_M \mathbb{P}\{S_2 > u_H^M\} + L_S\right) \cdot (q)^{-1}, 1 \right\} \right\} \\ \qquad \text{if } t_1 < t_2; \end{cases} \tag{5a}
$$

$$
u_{L1}^S = \begin{cases} \left(\dfrac{N_S F_1 - w q - c_S}{N_S - q}\right)^+ & \text{if } t_1 = t_2, \\ \left(\dfrac{N_S F_1 - c_S}{N_S - q}\right)^+ & \text{if } t_1 < t_2; \end{cases} \tag{5b}
$$

$$
u_{H2}^S = \begin{cases} \max\left\{ u_{L2}^S, \dfrac{\ell_M + L_S}{q}, 1 \right\} & \text{if } t_1 = t_2, \\ u_{H1}^S & \text{if } t_1 < t_2; \end{cases} \tag{5c}
$$

$$
u_{L2}^S = \begin{cases} \left(\dfrac{N_S F_1 - c_S - \ell_M}{N_S - q}\right)^+ & \text{if } t_1 = t_2, \\ u_{L1}^S & \text{if } t_1 < t_2; \end{cases} \tag{5d}
$$

$$
u_H^M = \begin{cases} \max\left\{ u_L^M, \min\left\{ \dfrac{\min\{q,D\} + L_M - w q}{\min\{q,D\}}, 1 \right\} \right\} \\ \qquad \text{if } t_2 = t_3, \\ \left\{ u_L^M, \min\left\{ \left(\dfrac{\mathbb{E}_{t_0} \min\{q,D\} + L_M}{q} - w\right)^+, \right. \right. \\ \qquad \left. \left. \dfrac{\mathbb{E}_{t_0} \min\{q,D\}}{q} \right\} \right\} \\ \qquad \text{if } t_2 < t_3; \end{cases} \tag{5e}
$$

$$
u_L^M = \begin{cases} \left(\dfrac{w q + N_M F_2 - \min\{q,D\} - c_M}{N_M - \min\{q,D\}}\right)^+ \\ \qquad \text{if } t_2 = t_3, \\ \left(\dfrac{w q + n_M F_2 - c_M}{N_M - q}\right)^+ \\ \qquad \text{if } t_2 < t_3. \end{cases} \tag{5f}
$$

A careful inspection of the functions u specified in the above lemma reveals that each firm $j \in \{M, S\}$ will default on the supply chain contract if at least one of the following holds:

1. Firm j's expected profit from production is *strictly less* than the penalty L_j, $j \in \{M, S\}$ firm j agreed to pay for breaking the underlying supply contract. That is, if input prices at time t_i turn out to be high, and the cost of default, L_j, is low, then firm j will choose not to produce owing to insufficient expected payoff.

2. Firm j lacks sufficient resources to produce. To illustrate, suppose $t_1 < t_2$. Then, the supplier, S, will inevitably *default* at time t_1 if $c_S + N_S(S_1 - F_1) < q S_1$ because, in such a case, the time t_1 cost of production inputs, $q S_1$, exceeds the time t_1 available capital, $c_S + N_S(S_1 - F_1)$.

Figure 2, which illustrates the functions u graphically confirms that the supplier, S, will rationally choose to produce if it knows that the assembler will want to produce *and* the time t_1 spot price of commodity 1 is between u_{L1}^S and u_{H1}^S. In this case the supplier will earn a payoff of $q(w - S_1) + N_S(S_1 - F_1)$. However, the figure also reveals that the supplier will choose to produce if it knows that the assembler will *not* produce *and* if its time t_1 spot price of commodity 1 is between u_{L2}^S and u_{H2}^S. In this case, the supplier will receive a payoff of $N_S(S_1 - F_1) + \ell_M - q S_1$, which is in excess of the penalty payment ℓ_S. The assembler, on the other hand, will only produce if its commodity input cost is between u_L^M and u_H^M and if the supplier produces. (This is because the assembler's production process requires an input from the supplier.)

A closer examination at the functions u given in the lemma also reveals that the exogenously given capital endowment, $c_j \ge 0$, $j \in \{M, S\}$, causes "parallel shifts" to the functions u, but it does not qualitatively change how each firm j behaves as its endogenously chosen position in the futures market, N_j, and the spot price of commodity $i = 1, 2$ change (except when c_j is very large; then the role of the firm j's futures contract position, N_j, diminishes). *It is for this reason that hereafter we analyze the model in the analytically simplest way by taking $c_j = 0$.* Moreover, to allow firms to produce without having to borrow, we take $0 < t_1 = t_2 = t_3$. Then both firms incur revenues and costs simultaneously, and production is feasible without cash-on-hand and without borrowing. (Strictly speaking, to

avoid borrowing, all we require is that $(t_3 - t_1) \geq 0$ be no more than the standard contractual grace period, which allows payments to be received for a certain period of time after a payment due date. This is usually up to 21 days.)

5.1. Analysis of the Unhedged Contract

Suppose now that neither firm hedges its commodity input cost. The assembler's and supplier's time t_0 payoffs, Π_U^M and Π_U^S, can be recovered from Equations (3) and (4) and by setting $N_M = N_S = 0$. Since $c_M = c_S = 0$, then Equations (3c) imply $\ell_M = \ell_S = 0$ for all $L_S \geq 0$, $L_M \geq 0$, implying that *default penalties in the unhedged case are irrelevant*.

$$
\max_{\underline{d} \leq q \leq \bar{d}} \left\{ \int_0^{u_{H1}^S} \int_q^{\bar{d}} \int_0^{u_H^M(q,q)} q(1-y+w_U)f(x)h(y,z)\,dy\,dx\,dz \right.
$$
$$
\left. + \int_0^{u_{H1}^S} \int_{\underline{d}}^q \int_0^{u_H^M(x,q)} (x-xy-w_U q)f(x)h(y,z)\,dy\,dx\,dz \right\}. \quad (6)
$$

Using the necessary optimality conditions with respect to q, (e.g., Bertsekas 2003, Proposition 1.1.1) and the Leibniz rule, we can obtain the implicit inverse demand curve, $w_U(q)$, that the supplier faces:

$$
w_U(q) = \frac{\int_0^{u_{H1}^S} \int_q^{\bar{d}} \int_0^{u_H^M(q,q)} (1-y)f(x)h(y,z)\,dy\,dx\,dz}{\mathbb{P}\{S_1 \leq u_{H1}^S, S_2 \leq u_H^M\}}. \quad (7)
$$

By substituting the right-hand side of (7) for the wholesale price, w_U, into the maximand in (6), we can obtain an expression for the assembler's expected payoff (in terms of q):

$$
\mathbb{E}_{t_0} \Pi_U^M(q)
$$
$$
= \int_0^{u_{H1}^S} \int_{\underline{d}}^q \int_0^{u_H^M(x,q)} x(1-y)f(x)h(y,z)\,dy\,dx\,dz. \quad (8)
$$

Since the distribution of demand, F, has support on $[\underline{d}, \bar{d}]$, then $\mathbb{E}_{t_0} \Pi_U^M(q): [\underline{d}, \bar{d}] \to \mathbb{R}$, given by (8), is a continuous real-valued mapping, where $[\underline{d}, \bar{d}]$ is a nonempty subset of $\mathbb{R}$. There exists a quantity q_M^* such that $\mathbb{E}_{t_0} \Pi_U^M(q) \leq \mathbb{E}_{t_0} \Pi_U^M[q_M^*]$ for all $q \in [\underline{d}, \bar{d}]$ (Weierstrass proposition). It follows that if $\mathbb{E}_{t_0} \Pi_U^M[q_M^*] \leq R_M$, then the assembler will place no order.

$$
\mathbb{E}_{t_0} \Pi_U^S(q) = q\big(w_U(q) - \mathbb{E}_{t_0}(S_1 \mid S_1 \leq u_{H1}^S, S_2 \leq u_H^M)\big)
$$
$$
\cdot \mathbb{P}\{S_1 \leq u_{H1}^S, S_2 \leq u_H^M\}, \quad \text{or, equivalently,}
$$
$$
\mathbb{E}_{t_0} \Pi_U^S(q) = q \int_0^{u_H^M(q,q)} \int_q^{\bar{d}} \int_0^{u_{H1}^S} (1-y)f(x)h(y,z)\,dz\,dx\,dy
$$
$$
- q \int_0^{u_H^M(q,q)} \int_0^{u_{H1}^S} zh(y,z)\,dz\,dy. \quad (9)
$$

Since F has support on $[\underline{d}, \bar{d}]$, then $\mathbb{E}_{t_0} \Pi_U^S(q): [\underline{d}, \bar{d}] \to \mathbb{R}$ is a continuous real-valued mapping where $[\underline{d}, \bar{d}]$ is

a nonempty subset of $\mathbb{R}$. There exists a quantity q_U^* such that $\mathbb{E}_{t_0} \Pi_U^S(q) \leq \mathbb{E}_{t_0} \Pi_U^S[q_U^*]$ for all $q \in [\underline{d}, \bar{d}]$ (Weierstrass proposition). The supplier's optimal order quantity q_U^{**} is given by $\arg\max_{q \geq \underline{d}} \mathbb{E}_{t_0} \Pi_U^S(q)$ s.t. $\mathbb{E}_{t_0} \Pi_U^M(q) \geq R_M$ and $\mathbb{E}_{t_0} \Pi_U^S(q) \geq R_S$, where R_S, R_M are the firms' reservation payoffs (note that if R_M and R_S are excessively large, then q_U^{**} may not exist). It follows that if the supplier were able to choose any wholesale price, it would choose $w_U(q_U^*)$, where w_U solves Equation (7).

5.2. Analysis of the Hedged Contract

Suppose now that both firms hedge by entering into $n_j \geq 0$, $j \in \{M, S\}$ futures contracts to purchase commodity $i = 1, 2$ at time t_i. Each firm j's time t_i payoff from the futures contract position will be $n_j(S_i - F_i)$, which is positive when the realized future spot price, S_i, is high (i.e., when $0 \leq F_i < S_i \leq \infty$) and negative when it is low (i.e., when $0 \leq S_i < F_i < \infty$). Before we begin the Stage 1 analysis of the supply chain contract with hedging, we present a preliminary result that deals with firms' equilibrium long futures contract positions.

PROPOSITION 1. *Suppose Assumption 3 holds and* $c_j = 0$, $j \in \{M, S\}$. *The number of futures contracts,* $n_j \geq 0$, *that can be supported as a subgame equilibrium (SE) must satisfy the following constraints:*

$$
\frac{q L_S}{L_S + q(w_H - F_1)} \leq n_S \leq \min\left\{\frac{q w_H}{F_1}, \frac{L_M}{F_1}, q\right\}, \quad (10a)
$$

$$
\frac{\underline{d} L_M}{\underline{d}(1 - F_2) + L_M - w_H q}
$$
$$
\leq n_M \leq \min\left\{\frac{\underline{d} - q w_H}{F_2}, \frac{L_S(S_2 = 0)}{F_2}, q\right\}, \quad (10b)
$$

where $L_S(S_2 = 0)$ *denotes the value penalty* L_S *when* $S_2 = 0$. (*Of course, this is only relevant in cases when* L_S *is a function of* S_2.)

Reading from the left, the first upper bound on n_j given in (10) is in force when both supply chain members choose to produce at time t_i, $i = 1, 2$ and firm j's realized input price, S_i, at time t_i is less than the futures price, F_i. The bound ensures that firm j's operating profit is sufficient to offset any loss from firm j's futures contract position, $n_j(S_i - F_i) < 0$, $j \in \{M, S\}$, $i = 1, 2$. Otherwise firm j would be at risk of defaulting on its futures contract position because of insufficient resources.

The second upper bound on n_j given in (10) is in force when firm j's realized time t_i input price is again low (i.e., $S_i < F_i$) and its supply chain counterpart, firm $k \in \{M, S\}$, $j \neq k$, chooses *not* to produce at time t_l, $l = 1, 2$, $l \neq i$. This will occur exactly when firm k's realized input cost, S_l, at time t_l is high (infinite). In this

situation firm k will be contractually obligated to use the payoff from its futures contract position, $n_k(S_l - F_l) > 0$, to compensate firm j with a default penalty, $L_k \geq 0$. (Note that if firm k breaks the supply contract, the penalty payment, L_k, is firm j's *only* revenue.) As before, the upper bound on n_j ensures that L_k is sufficient to offset any loss from firm j's futures contract position, $n_j(S_i - F_i) < 0$. At the same time, the *lower* bound on n_k given in (10) ensures that firm k's payoff from its futures contract position, $n_k(S_l - F_l) > 0$, is enough to pay at least L_k to firm j.

In summary, Proposition 1 can be viewed as a set of necessary and sufficient conditions under which neither supply chain member defaults on its futures contract position in all possible futures states of the world—as specified in our Assumption 3.

Since the lower bounds on n_M and n_S are strictly positive, then Proposition 1 also reveals that, in equilibrium, either both firms $j \in \{M, S\}$ *simultaneously* enter into futures contract positions $n_j > 0$, or they do not hedge at all. This reflects the fact that a situation in which one firm hedges while its supply chain partner does not hedge, makes the hedger susceptible to default on the futures contract. This default occurs exactly when the realized future spot price of the firm that did not hedge is high (causing it to default on the underlying supply contract), and the realized future spot price of the firm that did hedge is low (causing it to default on the futures contract). Corollary 1 summarizes the result.

COROLLARY 1. *If $c_j = 0$, $j \in \{M, S\}$, then in equilibrium, either both firms j simultaneously enter into futures contract positions $n_j > 0$, or they do not hedge at all.*

Finally, to ensure that the ranges for n_j, $j \in \{M, S\}$ given in Proposition 1 are *nonempty*, we require the following two conditions:

CONDITION 1. $w_H q \leq L_M \leq q$ *and* $\min\{q, D\}(1 - S_2)^+ - w_H q \leq L_S \leq q$.

CONDITION 2. $F_1 \leq w_H$ *and* $F_2 \leq 1 - (q w_H)/\underline{d}$.

Condition 1 corresponds to Assumption 1, which requires that a default on the underlying supply contract be costly; neither firm, however, should unusually profit from a default of its supply chain partner. Interestingly, the penalties that satisfy the bounds given in Condition 1 not only ensure that neither firm $j \in \{M, S\}$ defaults on its futures contract position, but also that each firm j is indifferent as to whether its supply chain partner chooses to produce or not. To see this, consider the case when $L_M = w_H q$, and $L_S = \min\{q, D\}(1 - S_2)^+ - w_H q$ and when both firms hedge. Then the supplier is guaranteed to earn a payoff of $q w_H(q) - \min\{q S_1, \min\{q, D\}(1 - S_2)^+\} + n_S(S_1 - F_1)$, and the assembler is guaranteed to receive a payoff of $\min\{q, D\}(1 - S_2)^+ - q w_H + n_M(S_2 - F_2)$,

whatever action its supply chain counterpart takes. It is worth mentioning that penalties that satisfy Condition 1 are not inconsistent with what one can observe in empirical practice. For example, in the Davita, Inc. contract included in §A of the online supplement, the penalty can be more than 100% of the wholesale price.

Condition 2 corresponds to Assumption 2, which requires upper bounds on the futures prices of commodities 1 and 2. Without these upper bounds, firms cannot make money in expectation, and hedging is essentially infeasible.

LEMMA 2. *If Conditions 1 and 2 hold, then the range for n_j, $j \in \{M, S\}$ given in (10) is nonempty for all $0 \leq S_1 < \infty$, $0 \leq S_2 < \infty$, $\underline{d} \leq D \leq \bar{d}$, and $c_j = 0$.*

We illustrate Proposition 1 with the following simple example.

EXAMPLE 4. Suppose that S_1 and S_2 are independent and log-normally distributed with volatilities $\sigma_1 = \sigma_2 = 60\%$. Lead time, i.e., $t_1 - t_0$, is six weeks. $F_1 = F_2 = \$0.25$. Demand, D, is uniformly distributed on the interval $[100, 150]$. Using Condition 1, we assume $L_M = w_H q$ and $L_S = \min\{q, D\}(1 - S_2)^+ - w_H q$, where the (equilibrium) wholesale price w_H is given by (12), which will be derived later in the paper. Using the result in Proposition 1, the following table summarizes the range for n_S and n_M that can be supported as an SE.

Order quantity, q	Range for n_S required by (10a)	Range for n_M required by (10b)
100	$\dfrac{100}{3} \leq n_S \leq 100$	$n_M = 100$
105	$\dfrac{2{,}667}{59} \leq n_S \leq 105$	$91 \leq n_M \leq 105$
110	$\dfrac{1{,}738}{29} \leq n_S \leq 110$	$\dfrac{242}{3} \leq n_M \leq 110$
115	$\dfrac{4{,}439}{57} \leq n_S \leq 115$	$69 \leq n_M \leq 115$
120	$\dfrac{696}{7} \leq n_S \leq 120$	$56 \leq n_M \leq 120$

Note that when $q = 100$, then the assembler faces no demand risk, but the range for n_M reduces to a single point, implying that the assembler can only hedge by purchasing q futures contracts. The range for n_M, however, opens up as the order quantity, q increases. Therefore for $q > 100$, the assembler can hedge by adopting different hedging policies that achieve the same expected payoff.

Figure 3 illustrates Proposition 1 graphically for the case when $q = 120$. Figure 3(a) illustrates a hypothetical nonequilibrium situation in which neither of the firms purchased a sufficient number of futures contracts and as a consequence there exist $S_1 \times S_2 \times D$ combinations for which at least one of the firms

Figure 3 (Color online) Range of S_1 and S_2 in Which Both Firms Can Sustain Not Defaulting on Their Futures Contract Positions

(a) $n_M = 55$, $n_S = 85$ (b) $n_M = n_S = 110$
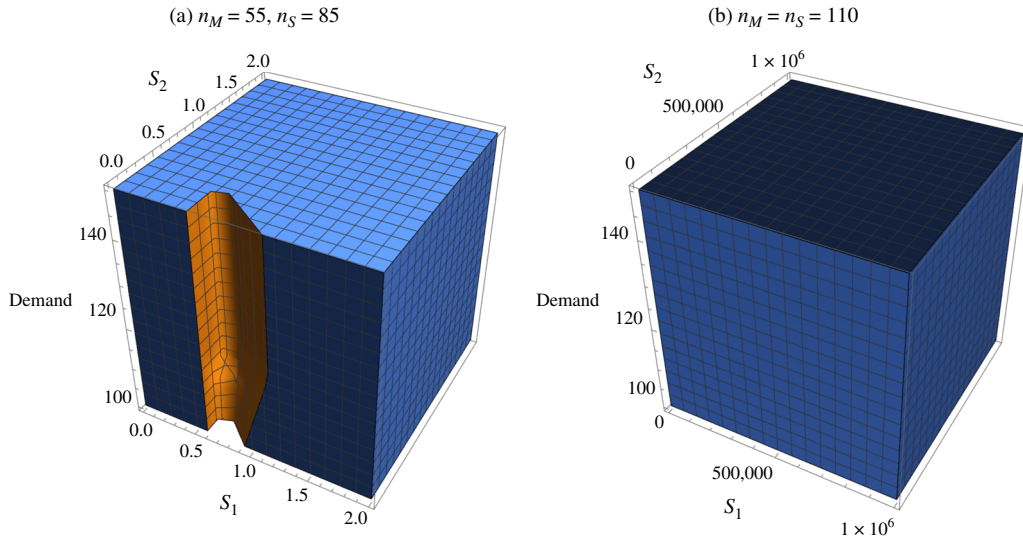
Note. Assumes $F_1 = F_2 = \$0.25$, $D \sim \mathcal{U}[100, 150]$, $q = 120$, $L_M = w_H q$, and $L_S = \min\{q, D\}(1 - S_2)^+ - w_H q$.

should be expected to default on its futures contract position (these combinations are represented by the hollow space in Figure 3(a)). In Figure 3(b), both firms purchased a correct number of futures contracts and can rationally sustain *not* to default for all $S_1 \times S_2 \times D$.

5.2.1. Partial vs. Complete Hedging. Following SFAS 52 and 80, which are generally accepted accounting principles (GAAP) for the accounting treatment of hedged transactions (see Mian 1996, p. 425), one could adopt the assumption that a futures trade is *not* considered to be a hedge unless the underlying transaction is a *firm commitment*. Under, this assumption, both firms would hedge by taking futures contract positions $n_j = q$, $j \in \{M, S\}$ and thus completely insulate their market values from hedge-able risks.

Here, however, we allow that firms adopt the "take-the-money-and-run" strategy under which firms default on the underlying supply contract and sell their commodity inputs on the open market—whenever it is profitable to do so. There is some empirical evidence for this conduct: Hillier et al. (2008, p. 768), for example, describe a case where in 2001 an aluminum producer Alcoa temporarily shut down its smelters and sold its electricity futures contracts on the open market (since it could make more money by selling electricity than by selling aluminum).

Allowing firms to adopt the take-the-money-and-run strategy implies that the hedged supply contract is no longer a firm commitment and the results in Proposition 1 reveal that in equilibrium firms may only hedge partially, meaning that $0 < n_j < q$, $j \in \{M, S\}$.

The hedged contract is operationalized as follows: In Stage 1, both firms infer the equilibrium quantity, say $q_H^{**} \in H$, and adopt hedging positions n_j, $j \in \{M, S\}$

that satisfy Proposition 1. The equilibrium quantity that firms infer can be derived by adapting results from the existing literature (for example, Lariviere and Porteus 2001). Using Equations (3), the assembler's *expected* payoff is given by

$$
\mathbb{E}_{t_0} \Pi_H^M(q, w_H) = \int_0^\infty \int_q^{\bar{d}} \int_0^1 q(1-y)f(x)h(y, z)\, dy\, dx\, dz
$$
$$
+ \int_0^\infty \int_{\underline{d}}^q \int_0^1 (1-y)xf(x)h(y, z)\, dy\, dx\, dz
$$
$$
- qw_H + n_M \underbrace{(\mathbb{E}_{t_0} S_2 - F_2)}_{=0}. \tag{11}
$$

Using the fact that $F_2 = \mathbb{E}_{t_0} S_2$, the assembler's optimal order quantity implicitly defined by

$$
\int_0^\infty \int_q^{\bar{d}} \int_0^1 (1-y)f(x)h(y, z)\, dy\, dx\, dz - w_H = 0,
$$

from which we can solve for the inverse demand, say $w_H(q)$, the supplier faces

$$
w_H(q) = \min\left\{ \int_0^\infty \int_q^{\bar{d}} \int_0^1 (1-y)f(x)\, h(y, z)\, dy\, dx\, dz, \right.
$$
$$
\left. \frac{\underline{d}(1-F_2)}{q} \right\}. \tag{12}
$$

We can then use Equations (11) and (12) to yield

$$
\mathbb{E}_{t_0} \Pi_H^M(q)
$$
$$
= \max\left\{ \int_0^\infty \int_{\underline{d}}^q \int_0^1 x(1-y)f(x)h(y, z)\, dy\, dx\, dz, \right.
$$
$$
\int_0^\infty \int_q^{\bar{d}} \int_0^1 q(1-y)f(x)h(y, z)\, dy\, dx\, dz
$$

$$+ \int_0^\infty \int_{\underline{d}}^q \int_0^1 (1-y) x f(x) h(y,z) \, dy \, dx \, dz$$
$$- \underline{d}(1 - F_2) \Bigg\}, \quad (13)$$

which is the assembler's expected payoff in terms of q. Differentiation reveals that the assembler's payoff is increasing in q. To see this, the derivatives of the first and the second expression on the right-hand side of (13) are

$$\int_0^\infty \int_0^1 q(1-y) f(q) h(y,z) \, dy \, dz \geq 0 \quad \text{and}$$
$$\int_0^\infty \int_q^{\bar{d}} \int_0^1 (1-y) f(x) h(y,z) \, dy \, dx \, dz \geq 0. \quad (14)$$

The supplier's *expected* profit is

$$\mathbb{E}_{t_0} \Pi_H^S(q) = q \, w_H(q) - \mathbb{E}_{t_0} \min\{q S_1, \min\{q, D\}(1-S_2)^+\}$$
$$+ n_S \underbrace{(\mathbb{E}_{t_0} S_1 - F_1)}_{=0}. \quad (15)$$

The supplier's most preferred order quantity, q_H^*, is given by $\arg\max_{q \geq \underline{d}} \mathbb{E}_{t_0} \Pi_H^S(q)$; the supplier's optimal order quantity q_H^{**} is given by $\arg\max_{q \geq \underline{d}} \mathbb{E}_{t_0} \Pi_H^S(q)$ s.t. $\mathbb{E}_{t_0} \Pi_H^M(q) \geq R_M$ and $\mathbb{E}_{t_0} \Pi_H^S(q) \geq R_S$, where R_S, R_M are the firms' reservation payoffs. In special case, the results in Lariviere and Porteus (2001) show that $\mathbb{E}_{t_0} \Pi_H^S(q)$, given by (15), is *unimodal* in q.

5.3. Equilibrium Among Unhedged and Hedged Contracts

We now allow the firms to enter into the supply chain contract by choosing whether they wish to hedge. Depending on the model parameters, it is possible to have equilibria where (a) neither firm hedges,

or (b) both firms hedge. Because of Proposition 1, case (a) will occur if one of the firms in the model decides *not* to hedge at time t_0. If, on the other hand, there exists a subgame perfect Nash equilibrium (SPNE) in which

$$\mathbb{E}_{t_0} \Pi_H^S(q_H^{**}) \geq \mathbb{E}_{t_0} \Pi_U^S(q_U^{**}), \quad (16a)$$

then the supplier, S, will hedge, and as will be seen in Lemma 3, part (i), the assembler will hedge as well. Similarly, the supplier will hedge in cases where the unhedged wholesale contract may be *infeasible*. This occurs when

$$\mathbb{E}_{t_0} \Pi_U^M(q_M^*) \leq R_M, \quad \text{and there exists}$$
$$q \in H \text{ for which}$$
$$R_S \leq \mathbb{E}_{t_0} \Pi_H^S(q) \quad \text{and} \quad R_M \leq \mathbb{E}_{t_0} \Pi_H^M(q). \quad (16b)$$

The following Lemma 3 is needed to establish that conditions (16a) and (16b) can hold in equilibrium. Parts (i) and (ii) describe how the risk of supply contract default affects both the assembler's expected payoff and the wholesale price. Parts (iii) and (iv) give sufficient conditions under which (for both the supplier and the assembler) there exist expected payoff levels that may only be achieved via the hedged wholesale contract.

LEMMA 3. *Let* $w_U(q)$, $\mathbb{E}_{t_0} \Pi_U^M(q)$, $\mathbb{E}_{t_0} \Pi_U^S(q)$, $w_H(q)$, $\mathbb{E}_{t_0} \Pi_H^M(q)$, *and* $\mathbb{E}_{t_0} \Pi_H^S(q)$, *respectively, be given by* (7), (8), (9), (12), (13), *and* (15). *If* $c_j = 0$ *and Conditions 1 and 2 hold. Then*:

(i) $0 \leq \mathbb{E}_{t_0} \Pi_U^M(q) \leq \mathbb{E}_{t_0} \Pi_H^M(q)$.

(ii) $w_H(q) \leq w_U(q)$ (*if* S_1 *and* S_2 *are independent*).

(iii) *The set* $H_M := \{q \in H \mid \mathbb{E}_{t_0} \Pi_U^M(q_M^*) \leq \mathbb{E}_{t_0} \Pi_H^M(q)\}$ *is nonempty.* (*An example of the set* H_M *can be seen graphically in Figure* 4(a).)

Figure 4   (Color online) Expected Payoffs Under Unhedged and Hedged Wholesale Contracts
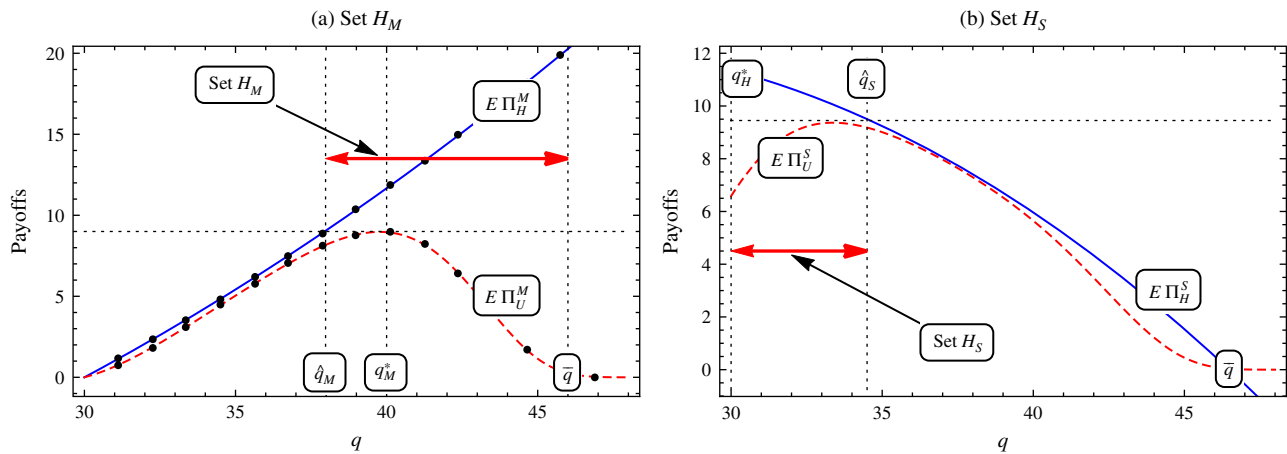
(a) Set H_M

(b) Set H_S

Notes. $D \sim \mathcal{U}(30, 60)$; S_1 and S_2 are independent and log-normally distributed; time t_0 prices of commodities 1 and 2 are \$0.3; lead-time $= t_1 - t_0 = 6$ weeks; $t_2 = t_1$; annual volatility of commodity 1 and 2 spot prices is 60%. *Definitions.* (i) For the order quantities $\hat{q}_S$ and $\hat{q}_M$, respectively, we have $\mathbb{E}_{t_0} \Pi_H^S[\hat{q}_S] = \mathbb{E}_{t_0} \Pi_U^S(q_U^*)$ and $\mathbb{E}_{t_0} \Pi_H^M[\hat{q}_M] = \mathbb{E}_{t_0} \Pi_U^M(q_M^*)$. (ii) For the order quantity $q = \hat{q}_S$, we have $\mathbb{E}_{t_0} \Pi_H^S[\hat{q}_S] = \mathbb{E}_{t_0} \Pi_U^S(q_U^*)$; for the quantity $q = \hat{q}_M$, we have $\mathbb{E}_{t_0} \Pi_H^M[\hat{q}_M] = \mathbb{E}_{t_0} \Pi_U^M(q_M^*)$. (iii) For the order quantity $q = \bar{q}$, we have $\mathbb{E}_{t_0} \Pi_H^S[\bar{q}] = 0$.

(iv) $H_S := \{q \in H \mid \mathbb{E}_{t_0} \Pi_U^S(q_U^*) \leq \mathbb{E}_{t_0} \Pi_H^S(q)\}$ *is non-empty if*

$$\mathbb{P}\{S_1 \leq_{H1}^S, S_2 \leq u_H^M(q_U^*, q_U^*)\}$$
$$+ \left(\int_0^1 \frac{z h_Z(z)}{\overline{F}(q_U^*)} \, dz + \int_0^1 y h_Y(y) \, dy \right)$$
$$\leq \mathbb{P}\{S_2 \leq 1\}$$
$$+ \left(\int_0^{u_{H1}^S} \int_0^{u_H^M(q_U^*, q_U^*)} \left(\frac{z}{\overline{F}(q_U^*)} + y \right) h(y, z) \, dy \, dz \right), \quad (17)$$

where $h_Y(y)$ and $h_Z(z)$ denote marginal densities. (An example of the set H_S can be seen graphically in Figure 4(b).)

For the case when S_1 and S_2 are independent, part (ii) of the lemma formally establishes that the equilibrium wholesale price is lower when the downstream assembler hedges. This result reflects the fact that by hedging, the assembler guarantees the supply contract performance and the rational supplier responds to this guarantee by reducing the wholesale price.

Parts (iii) and (iv) give sufficient conditions under which there exist expected payoff levels that can only be achieved via the hedged wholesale contract. As a consequence of part (i) of the lemma, the set H_M will always be nonempty.

The set H_S will be nonempty if $\underline{d}$ and the supplier's margin are sufficiently large. The former is consistent with a situation in which there is a high baseline demand for the assembler's final product. The latter is consistent with a situation in which F_1 is sufficiently low and w_H is sufficiently high. Under these conditions, reducing the probability that the assembler defaults on the supply contract is ex-ante important to the supplier. Mathematically, this situation reduces to condition (17). We illustrate this in the following example, which reveals that the supplier can be better off or worse under the hedged contract.

EXAMPLE 5 (CONTINUATION OF EXAMPLE 4). If $F_1 = F_2 = \$0.25$, $q_U^* = 105$ and condition (17) reduces to $1.446 \leq 1.477$. In fact, with $R_M = R_S = 0$, the highest profit the supplier can attain without hedging is $\mathbb{E}_{t_0} \Pi_U^S(q_U^*) = \$39.89$. However, if the supplier and the assembler, respectively, purchase $2{,}667/59 \leq n_S \leq 105$ and $91 \leq n_M \leq 105$ futures contracts, the supplier's expected payoff increases from \$39.89 to \$42.86. Clearly, hedging is beneficial to the supplier (Note that the assembler will join the supplier in hedging because of Lemma 3, part (i).) On the other hand, if $F_1 = F_2 = \$0.45$, then $q_U^* = 102$ and condition (17) reduces to $1.38001 \nleq 1.3488$ and the highest profit the supplier can attain without hedging is $\mathbb{E}_{t_0} \Pi_U^S(q_U^*) = \$8.22$. With hedging, however, the supplier's expected payoff would decrease from \$8.22 to \$4.83. The supplier is, therefore, better off without hedging.

The primary use of Lemma 3 is in establishing the next Proposition 2.

PROPOSITION 2. *If $c_j = 0$ and Conditions 1 and 2 hold, then there exist SPNEs in which both firms will hedge. Both firms will hedge if* (i) $R_M \geq \mathbb{E}_{t_0} \Pi_U^M(q_M^*)$; *or* (ii) *if $R_M = 0$ and the set H_S is nonempty (see part 3 of Lemma 3).*

PREDICTION 1. Offers of the hedged wholesale contract should be expected if the downstream firm's market power exceeds a critical threshold.

Prediction 1 essentially identifies situations when hedging is mainly important to the downstream assembler. A second empirical prediction regarding the use of a hedged contract can be made using part (ii) of Proposition 2. Prediction 2 identifies situations when hedging is mainly important to the upstream supplier.

PREDICTION 2. Offers of the hedged wholesale contract should be expected if the upstream firm operates on a large margin, there is a high baseline demand for downstream firm's final product, and the downstream firm's market power is below a critical threshold.

Table 1 presents some data on how much better the assembler can do with the hedged contract: If the *unhedged* equilibrium order quantity is q, then column (L) of Table 1 shows the percentage profit increase the assembler experiences if it is offered an alternative *hedged* contract that leaves the supplier no worse off than the original unhedged contract. The data shows that the assembler's expected payoff can increase from 0.34% to 75%. Column (G) of Table 1 presents comparable data for the supplier. As might be expected, the value derived from using the hedged contract will depend on lead times, spot price volatilities, and spot price correlation. In computing the results in Table 1, we assumed that the spot prices of commodities 1 and 2 were independent and chose annual volatilities ranging from 30% to 60% (a range comparable to equities; e.g., see Hull 2014, §11). Interestingly, in practice, annual volatility of commodity prices can easily exceed 60%. To illustrate, silver, an industrial metal used in electrical contacts and in catalysis of chemical reactions, was trading at around \$18 per ounce in April and May of 2010. Less than a year later, silver almost tripled in value to \$49 per ounce during the final week of April 2011 (see Zsidisin and Hartley 2012). Our lead times, namely the difference between t_1 and t_0, were 4 to 12 weeks.

We can also circle back some of our predictions both to results found in the research existing literature and to the existing industry practices. In Example 5, we demonstrate that hedging can be unprofitable

Table 1 The Impact of Switching to the Supplier's (Assembler's) Best Hedged Contract That Leaves the Assembler (Supplier) No Worse Off

Lead time (weeks) (A)	Annual volatility (%) (B)	q (C)	$w_U(q)$ (D)	Supplier's best hedged contract $(q', w_H(q'))$ subject to $\mathbb{E}_{t_0}\Pi_H^M(q') \geq \mathbb{E}_{t_0}\Pi_U^M(q)$					Assembler's best hedged contract $(q'', w_H(q''))$ subject to $\mathbb{E}_{t_0}\Pi_H^S(q'') \geq \mathbb{E}_{t_0}\Pi_U^S(q)$				
				q' (E)	$w_H(q')$ (F)	$\frac{(\mathbb{E}_0\Pi_H^S(q'))}{(\mathbb{E}_0\Pi_U^S(q))}-1$ (%) (G)	$\frac{(\mathbb{E}_0\Pi_H^S(q)/\mathbb{E}_0\Pi_U^S(q))}{+\mathbb{E}_0\Pi_H^M(q'))}$ (%) (H)	$\frac{(\mathbb{E}_0\Pi_H^S(q')/(\mathbb{E}_0\Pi_H^S(q')}{+\mathbb{E}_0\Pi_H^M(q'))}$ (%) (I)	q'' (J)	$w_H[q']$ (K)	$\frac{(\mathbb{E}_0\Pi_H^M(q')}{(\mathbb{E}_0\Pi_U^M(q))-1}$ (%) (L)	$\frac{(\mathbb{E}_0\Pi_U^M(q)/\mathbb{E}_0\Pi_U^S(q)}{+\mathbb{E}_0\Pi_H^M(q))}$ (%) (M)	$\frac{(\mathbb{E}_0\Pi_H^M(q')/(\mathbb{E}_0\Pi_H^S(q')}{+\mathbb{E}_0\Pi_H^M(q'))}$ (%) (N)
4	30	32.00	0.63	31.82	0.64	8.57	84.37	85.37	33.78	0.57	≥75	15.38	28.07
		34.00	0.57	33.97	0.57	0.88	70.38	70.50	34.09	0.56	3.50	29.41	30.07
		36.00	0.50	36.00	0.50	0.34	57.89	57.90	36.00	0.50	0.43	41.93	41.96
		38.00	0.43	37.94	0.44	0.98	46.49	46.66	38.02	0.43	1.42	53.37	53.64
8	45	32.00	0.63	31.56	0.65	19.57	84.74	86.84	35.30	0.52	≥75	15.01	38.84
		34.00	0.57	33.67	0.58	5.83	70.52	71.61	34.70	0.54	30.60	29.27	35.04
		36.00	0.50	35.72	0.51	2.73	57.79	58.37	36.08	0.50	7.18	42.04	43.67
		38.00	0.43	37.36	0.45	6.58	47.00	48.51	38.06	0.43	10.89	52.86	55.36
12	60	32.00	0.63	31.39	0.65	22.39	85.22	87.52	35.31	0.52	≥75	14.52	40.82
		34.00	0.57	33.25	0.59	9.34	71.49	73.20	34.80	0.54	51.96	28.30	37.44
		36.00	0.50	35.09	0.53	5.71	59.19	60.45	35.86	0.50	16.73	40.64	44.35
		38.00	0.43	36.55	0.48	10.14	48.80	51.14	37.62	0.45	18.49	51.05	55.21

Notes. $D \sim \mathcal{U}(30, 60)$; S_1 and S_2 are independent and log-normally distributed; time t_0 prices of commodities 1 and 2 are $0.3. Column (A): Lead-time $= t_1 = t_0$. The example assumes $t_0 < t_1 = t_2 = t_3$ (see ""Game Sequence" in §4.1). Column (B): Measures standard deviations of price returns. Columns (C) and (D): Equilibrium unhedged wholesale contract, $(q, w_U(q))$. Columns (E) and (F): Supplier's best hedged contract $(q', w_U(q'))$ subject to $\mathbb{E}_{t_0}\Pi_H^M(q') \geq \mathbb{E}_{t_0}\Pi_U^M(q)$. Columns (J) and (K): Assembler's best hedged contract $(q'', w_H(q''))$ subject to $\mathbb{E}_{t_0}\Pi_H^S(q'') \geq \mathbb{E}_{t_0}\Pi_U^S(q)$.

for the upstream supplier. However, such a supplier may still hedge in equilibrium if $\mathbb{E}_{t_0}\Pi_U^M(q_M^*) \leq R_M$ (i.e., if the supplier's counterpart has a high reservation payoff). This observation is not necessarily consistent with the existing theories of hedging, which imply that hedging should increase firms' market values. It does, however, appear to be consistent with the empirical findings in Jin and Jorion (2006), who report that although many firms hedge, hedging does not necessarily increase their market values. Likewise, it appears to be consistent with anecdotal examples from auto parts supply chains (e.g., Hakim 2003, Matthews 2011), food processing supply chains (e.g., Eckblad 2012), and energy supply chains (e.g., Thakkar 2013), which appeared in popular business press.

6. Conclusion

The production processes of many firms depend on raw materials whose prices can be highly volatile (e.g., Carter et al. 2006). Moreover, many firms who purchase raw materials also depend on suppliers who purchase raw materials for their own production. Of course, suppliers experience price volatility, too, and they may request price increases and surcharges. In some cases, if commodity prices significantly increase, the suppliers may not be able to fulfill contractual requirements, causing a breakdown in the supply chain. It is possible to find examples of this phenomenon from various industries, including auto parts, e.g., Hakim (2003), Matthews (2011); food processing, e.g., Eckblad (2012); energy and utilities, e.g., Thakkar (2013); and heavy manufacturing, e.g., Matthews (2011). Given the prevalence of this issue, one might guess that the topic of risk management would command a great deal of attention from researchers in finance, and that practitioners would therefore have a well-developed body of wisdom from which to draw in formulating hedging strategies.

Such a guess would, however, be at best only partially correct. Finance theory does a good job of instructing *stand-alone* firms on the implementation of hedges. Finance theories, however, are generally silent on the implementation of hedges in situations where a firm operates in a supply chain and its operations are exposed to price fluctuations not only through direct purchases of commodities but also through its suppliers and the suppliers of its suppliers. This is precisely the question that we study in this paper.

Specifically, we ask the following: (1) When should supply chain firms hedge their stochastic input costs? (2) Should they hedge fully or partially? (3) Do the answers depend on whether the firms' supply chain partners hedge? To answer these questions, we consider a simple supply chain model—the "selling-to-the-newsvendor" model (Lariviere and Porteus

2001)—and generalize it by assuming that both the upstream and the downstream firms face stochastic production costs. The stochastic costs could represent the raw material costs. We show that the stochastic costs reverberate through the supply chain and will be, ex ante, impounded into the wholesale price. In some cases, if input costs significantly increase, one of the supply chain members may not be able to fulfill its contractual requirements, causing the entire supply chain to break down. We identify conditions under which the risk of the supply chain breakdown and its impacts on the firms' operations will cause the supply chain members to hedge their input costs: (i) the downstream buyer's market power exceeds a critical threshold; or (ii) the upstream firm operates on a large margin, there is a high baseline demand for downstream firm's final product, and the downstream firm's market power is below a critical threshold. In absence of these conditions there are equilibria in which neither firm hedges. To sustain hedging in equilibrium, both firms must hedge and supply chain breakdown must be costly. The equilibrium hedging policy will (in general) be a partial hedging policy. There are also situations when firms hedge in equilibrium although hedging reduces their expected payoff.

As an extension, one could consider the case when firms' operations are financed with borrowing and show that hedging can be profitable even in the absence of breakdown risk. There, the equilibrium hedging policy would be a full hedging policy. Future research may consider the role coordinating contracts, financing, and hedging play in the effective management of decentralized supply chains in the presence of stochastic demands and stochastic input cost.

Supplemental Material

Acknowledgments

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