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# Price and Time Competition for Service Delivery

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Many service firms use delivery time guarantees to compete for customers in the marketplace. In this research we develop a stylized model to analyze the impact of using time guarantees on competition. Demands are assumed to be sensitive to both the price and delivery time guarantees, and the objective of each firm is to select the best price and time guarantee to maximize its operating profit. We first analyze the optimization problem for the individual firms and then study the equilibrium solution in a multiple-firm competition. Using a numerical study, we further illustrate how the different firm and market characteristics would affect the price and delivery time competition in the market. Our results suggest that the equilibrium price and time guarantee decisions in an oligopolistic market with identical firms behave in a similar fashion as the optimal solution in a monopolistic situation from a previous study. However, when there are heterogeneous firms in the market, these firms will exploit their distinctive firm characteristics to differentiate their services. Assuming all other factors being equal, the high capacity firms provide better time guarantees, while firms with lower operating costs offer lower prices, and the differentiation becomes more acute as demands become more time-sensitive. Furthermore, as time-attractiveness of the market increases, firms compete less on price, and the equilibrium prices of the firms increase as a result. Our findings provide important implications about firm behaviour under price and time competition.

(*Time-Based Competition; Price Competition; Service Guarantees; Competitive Games*)

## 1. Introduction

Price and timely delivery are two important factors for success for service providers in today's competitive markets, and many service companies are offering time guarantees as a marketing weapon to attract customers in a time-sensitive market. For instance, the recent marketing effort of the United Parcel Service (UPS) promotes the notion "What's most important to you? Guaranteed on-time delivery? Or savings? Would you be willing to settle for both?" One major supermarket chain in California, Lucky, has been promoting itself as the low price leader as well as its "three's a crowd" service, guaranteeing that no more than three people should wait in any checkout line until a new counter will be open. The notion of time-based competition has been accepted in many industries; see Stalk and Hout (1990) and Blackburn (1991).

Price and time performance are closely related. Some customers are willing to pay a price premium for timely service delivery, e.g., Federal Express's next day delivery service. From the firm's perspective, it could be costly to deliver superior time performance. Since delivery time performance generally depends on the available capacity and operating efficiency of the system, a firm might need to increase the capacity or improve the efficiency of its service delivery system in order to achieve the desired time performance. Either approach can increase the operating cost of the firm,



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which in turn will affect the price and the overall profitability of the firm. Therefore, it is important to assess the impact of time performance and pricing strategy on demand as well as to understand their implications on capacity or related operation decisions.

There is substantial amount of research work that looks at the impact of time performance on pricing and capacity decisions. Recent works include Dewan and Mendelson (1990), Mendelson and Whang (1990), and Stidham (1992), where they studied the internal pricing and capacity selection for internal service operations, taking into account both users' delay cost and the capacity cost. Hill and Khosla (1992) used a deterministic model for expressing the demand as a function of the delivery lead time and price to study the impact of lead time reduction. Several researchers have studied the delivery time performance in a competitive environment. Kalai et al. (1992), studied the role of processing capacity in a competitive environment, where two firms compete for a stream of customers via their choices of processing speed. Li (1992) developed a model to examine the use of inventory for superior delivery time by firms to compete for orders in a market. Li and Lee (1994) studied the price competition between two service firms where customers consider both their prices and delivery speeds in selecting their services. Their model assumes that the customers have information about the service speed and current workload of the two firms. Lederer and Li (1997) developed an analytical model that captures the effect of time performance on prices, demands, and profit. They consider the situation where firms compete by choosing prices and service rates for multiple types of customers with different delay costs. Their results show how the different firms would choose their price and time performance for these different customer types.

This paper focuses specifically on the use of time guarantees for service delivery for firms to compete in a market. Classical examples include FedEx's next day guaranteed delivery and Domino Pizza's guaranteed 30-minute delivery. By marketing the importance of time, these companies promise some uniform delivery time performance for their services as a competitive weapon to attract customers in the market. These delivery time guarantees usually apply to any customer order at all times, and are attractive from the customer standpoint as they eliminate the time uncertainty in receiving the needed services.

While the use of these time guarantees could be an attractive marketing weapon, firms also realize the risk of failing to deliver the guarantee. The loss of goodwill and credibility will hurt future business. In some cases, there could be hefty lateness penalties associated with missing the guaranteed delivery time window. As discussed earlier, price and time performance are closely related, and a firm's capacity and its operating efficiency determine whether the system can deliver the time performance as guaranteed. Therefore, it is useful to develop an analytical model to understand how firms would jointly choose their pricing and time guarantees in a competitive market. The model can then be used to generate useful insights for several important questions on price and time competition for service delivery.

The basic research question we attempt to address in this paper is how firms will select their prices and time guarantees in a competitive market where demands are sensitive to both price and time guarantees. Would firms select these prices and time guarantees very differently under competition than in a monopolistic situation? When there are heterogeneous firms in the market, how would these firms use their distinctive firm characteristics to select price and time guarantees in order to differentiate their services? Furthermore, in a market where demands are increasingly time-sensitive, how would it affect the firm competition?

There are several recent papers that study the issues of quoting uniform delivery time. So and Song (1998) developed a model to study the optimal selection of price, uniform delivery time and capacity expansion to maximize the overall profit, where demands were assumed to be sensitive to both the price and delivery time. A single-firm framework was developed to study the interaction among the three decision variables and to evaluate how the firm and demand characteristics affect the optimal decisions. Palaka, et al. (1997) studied similar issues using a linear demand function, and their model also included system congestion costs and lateness penalty. Ho and Zheng (1995) developed a market-share model to study the impact of delivery time guarantee, but they did not consider the pricing issue.

Our modeling framework basically extends the previous single firm model developed by So and Song (1998) to a multiple firm setting. We consider size and operating efficiency as the two major firm characteristics for setting price and delivery time guarantee, and capture these two characteristics through their capacity and unit operating cost of each firm in our model. Demands are assumed to be sensitive to both the prices and time guarantees offered by the firms. We assume the overall market size is constant and use a multiplicative competitive interaction function to model the market share given the prices and time guarantees of the firms. Each firm is assumed to select the best decisions to maximize its resulting profit. We first analyze the optimization problem for the individual firms, and then study the equilibrium solution in a multiple-firm competition. We show that a unique Nash equilibrium solution exists and devise a simple iterative procedure to compute the equilibrium solution. When all competing firms are identical, we further characterize the equilibrium solution and analyze how the different parameters of the model affect the equilibrium. Through a numerical study, we provide several important insights regarding how the different firm and market characteristics would affect the price and delivery time competition in the market.

We summarize several key findings here. Our results suggest that the equilibrium price and time guarantee decisions in an oligopolistic market with identical firms behave in a similar fashion as the optimal solution in a monopolistic situation in a previous study. However, when there are heterogeneous firms in the market, these firms will exploit their distinctive firm characteristics to differentiate their services. Assuming all other factors being equal, the high capacity firms provide better time guarantees, while firms with lower operating costs offer lower prices, and the differentiation becomes more acute as demands become more time-sensitive. Furthermore, as time-attractiveness of the market increases, firms compete less on price, and the equilibrium prices of the firms increase as a result. These results provide important implications about firm behaviour in price and time competition.

The paper is organized as follows. In §2 we present our model and analysis. We first study the decision problem for an individual firm given the prices and time guarantees of other firms are fixed, and derive several analytic results. Then, using the results for the individual firm problem, we study the $N$-firm problem. We show that a unique Nash equilibrium exists and present an iterative procedure to compute the equilibrium. For identical firms, we further obtain structural results about the equilibrium solution. In the next two sections, we provide some numerical results that address the key research questions for our study. In §3 we present the results of a numerical study that examines the impact of several key model parameters on the equilibrium characteristics. In §4, we use a numerical experiment to examine the competitive advantage of a dominant firm. Finally in §5 we provide some concluding remarks.

## 2. Model Formulation and Analysis

Consider $N$ service providers (firms) competing for customers in a market. Customers choose the services provided by these firms based on their prices and delivery time guarantees, plus other factors such as the firm reputation or other service quality dimensions. Hence, a firm offering the lowest price and shortest time guarantee does not necessarily capture the entire market. To focus on price and time competition, we only model explicitly the impact of price and time guarantee on the demands, but assume that the other competitive factors are fixed and do not consider them as possible firm decisions for competition. All competing firms choose and announce the prices as well as some constant delivery time guarantee for their service to all potential customers. Denote the price and service delivery time guarantee for firm $i$ by $p_i$ and $t_i$, respectively.

Demands are sensitive to both the price and delivery time guarantee provided by each firm. We assume that the market size is fixed and is given by $\lambda$. In other words, the pricing and delivery time guarantee decisions of the firms affect only the resulting market share of each firm, but do not change the overall size of the market. We use the following multiplicative competitive interaction (MCI) model to represent the market demand of each firm:

$$\lambda_i = \lambda\left(\frac{L_i p_i^{-a} t_i^{-b}}{\sum_{j=1}^{N} L_j p_j^{-a} t_j^{-b}}\right). \tag{1}$$

Here the term $L_i p_i^{-a} t_i^{-b}$, with $a$, $b \geq 0$, represents the *attraction* of firm $i$, and its market share is given by its attraction relative to the total attraction of all firms combined. (We refer the reader to Cooper and Nakanishi (1988) or Lilien, et al. (1992) for additional discussions on this or other market share models.) The attraction of firm $i$ corresponds to how customers feel toward its service given its price and service delivery guarantee plus other competitive factors (e.g., the firm's reputation or the convenience of service locations), where the parameter $L_i > 0$ represents the combined effects of these other factors, and a larger $L_i$ corresponds to a higher attraction. We refer to the constants $a$ and $b$ as the price and time attraction factors of the market, respectively. A larger value of $a$ (or $b$) corresponds to a higher customer's attraction to the price (or time) attribute of the available services. For the assumption of constant market size to be appropriate, we impose some maximum price $\bar{p}$ that can be charged for the available services in the market. For instance, this could correspond to the situation where there are multiple market segments as defined by the different price ranges supported by each segment, and our model looks at the price and time competition among service providers only within the same market segment. Thus, the model is intended to capture each individual market segment in which the services provided by each firm are mainly differentiated by its price and time guarantee.

We capture two major firm characteristics in our model: their size and efficiency. Firm size is measured by its operating capacity, and firm efficiency is measured by its operating cost per unit service provided. Specifically, let $\mu_i$ denote the capacity and $\gamma_i$ the unit operating cost for firm $i$. Both $\mu_i$ and $\gamma_i$ are considered to be fixed parameters in our model, while $p_i$ and $t_i$ are decision variables. In effect, the posted price and delivery time guarantee are considered to be factors that can be changed to react to competition constantly, whereas firm capacity is considered to be a long-term strategic decision and cannot be changed in the short run and the unit operating cost of a firm cannot be improved in the short run. Instead, the model allows for sensitivity analysis to understand how these two model parameters, capacity and unit operating cost, can affect the decisions of pricing and delivery time guarantees of each firm.

The dynamics of the market as defined in (1) state that the prices and time guarantees affect the market shares of the firms. However, it does not explicitly consider the impact of the reliability of delivering these time guarantees. Firms that constantly miss their guaranteed delivery will eventually lose their credibility with customers for future business, and the poor delivery performance would defeat the purpose of exploiting time guarantees to attract customers. Indeed, many service firms carefully monitor their delivery performance of meeting the promised delivery times. To take this into consideration in our model, we need to impose a service requirement that each firm must select its time guarantee with a high probability of delivering this time guarantee.

The average delivery performance generally depends on the overall system capacity and the system workload, while the actual delivery time of a service order depends on the system status at the time when the order is placed. To construct a tractable service reliability constraint for complex service systems, one reasonable approach is to approximate the actual delivery time in a general service system by the sojourn time (total delay plus service time) of an M/M/1 queueing system, which is exponentially distributed.[1] Therefore, the requirement that the probability of meeting the time guarantee for each firm must be at least $\alpha$ (e.g. 95%) can be captured in the following service reliability constraint for firm $i$ as

$$1 - e^{-(\mu_i - \lambda_i)t_i} \geq \alpha,$$

or equivalently,

$$(\mu_i - \lambda_i)t_i \geq -\log(1 - \alpha).$$

In our model, we shall restrict our discussions only to cases where the service reliability for each firm $\alpha$ is the

[1]Empirical results from Shanthikumar and Sumita (1988) have shown that the time spent in a job shop is asymptotically exponentially distributed, which provides some justification of this approximation. Karmarkar (1993) provides a thorough survey of similar approaches.

same. This is applicable in situations where there exists some industry standard and published consumer report where the delivery performance of the service reliability of each firm is readily available to customers. In this way, firms are discouraged from performing below the standard such that the market share is then mainly affected by their promised time guarantee as depicted by the market share model given in (1). Of course, our model and analysis also allow for different service reliabilities for different firms, but this would not allow for any meaningful comparison purposes.

An alternative approach to capturing the service reliability is to impose a penalty cost, say $z_i$, when firm $i$ fails to meet its time guarantee. Our formulation above using the service reliability constraint can be easily modified to model this alternate approach. To see this, we shall show in the next section that the reliability constraint must be tight at optimality, such that we can simply add an expected penalty of $z_i(1 - \alpha)$ to the unit operation cost parameter $\gamma_i$ in the above model formulation. (However, our model formulation does not allow for the situation where demands are sensitive to the amount of penalty offered, as the demand function (1) is assumed to be dependent on price and delivery time guarantee only.) This penalty cost $z_i$ could include some actual cash payment to the customer for late delivery plus a loss of goodwill cost, which is difficult to estimate in practice. Therefore, we simply focus on the service constraint in this paper.

The objective for firm $i$ is then to select the best price and time guarantee for its service so as to maximize its operating profit, taking into consideration the prices and time guarantees provided by other firms. Mathematically, the problem faced by firm $i$ can be formulated as

$$\max_{p_i,t_i} (p_i - \gamma_i)\lambda_i \quad \text{s.t. } (\mu_i - \lambda_i)t_i \geq k \text{ and } p_i \leq \bar{p}, t_i \geq 0,$$

where $k = -\log(1 - \alpha)$. For each $i$, let $\beta_i = (1/L_i) \Sigma_{j \neq i} L_j p_j^{-a} t_j^{-b} > 0$. Using the market share function (1), the optimization problem for firm $i$ can be written as

$$\max_{p_i,t_i} \prod_i(p_i, t_i) = \frac{(p_i - \gamma_i)\lambda p_i^{-a} t_i^{-b}}{p_i^{-a} t_i^{-b} + \beta_i} \tag{2}$$

$$\text{s.t.} \quad \left(\mu_i - \frac{\lambda p_i^{-a} t_i^{-b}}{p_i^{-a} t_i^{-b} + \beta_i}\right) t_i \geq k, \tag{3}$$

$$p_i \leq \bar{p} \qquad t_i \geq 0,$$

where $\Pi_i(\cdot, \cdot)$ represents the operating profit function of firm $i$. For a feasible solution, we require that $\lambda < \Sigma_{i=1}^N \mu_i$, i.e., the total combined firm capacity must exceed total market demand.

### 2.1. The Firm *i* Problem

Let us first analyze the optimization problem for firm $i$ developed above, given that the prices and delivery time guarantees of all other firms are fixed. Express the price $p_i$ as a function of the service delivery time $t_i$ along the boundary of the service reliability constraint as follows:

$$P_i(t) = \left\{\frac{k + \lambda t - \mu_i t}{t^b \beta_i(\mu_i t - k)}\right\}^{1/a}. \tag{4}$$

Observe that $P_i(t)$ is defined only for $k/\mu_i < t < k/(\mu_i - \lambda)$ when $\mu_i > \lambda$, and for $k/\mu_i < t$ when $\mu_i \leq \lambda$. It can be shown (see Lemma A1 in the Appendix) that $P_i(t)$ is strictly decreasing in $t$, such that the inverse of $P_i$, denoted by $P_i^{-1}$, is well defined and is strictly decreasing.

The following result characterizes the optimal price and delivery time guarantee for the firm $i$ problem when the prices and service delivery times of all other firms are given.

Proposition 1. *(a) There exists a unique optimal price and delivery time guarantee* $(p_i^*, t_i^*)$ *that maximizes the profit function* $\Pi_i(p, t)$ *of firm i.*

*(b) For* $a > 1$, $p_i^* = \min(P(\hat{t}), \bar{p})$ *and* $t_i^* = P_i^{-1}(p_i^*)$, *where* $\hat{t}$ *be the unique solution to* $H_i(t) = 0$ *and*

$$H_i(t) = \frac{-k\lambda t}{k + \lambda t - \mu_i t} - b(\mu_i t - k) + ka - \frac{\gamma_i k a}{P_i(t)}. \tag{5}$$

*(c) For* $a \leq 1$, $p_i^* = \bar{p}$ *and* $t_i^* = P_i^{-1}(\bar{p})$.

While there exists no closed form solution for the optimal price and delivery time decisions for firm $i$ given the prices and time guarantees of other firms, Proposition 1 states that we can compute the optimal solution numerically by solving for $H_i(t) = 0$ and then setting $p_i^* = \min(P_i(\hat{t}), \bar{p})$. (The result from Lemma A1(ii) given in the appendix further shows that $\hat{t}$ can be easily found by using a simple bisection method.) Proposition 1 also shows that when the price attraction factor $a$ is less than or equal to one, firm $i$ should always charge the maximum price $\bar{p}$. This result implies that when $a \leq 1$, all firms will charge the maximum price and there is no price competition in the market. In other words, when the price attraction factor is low, the time guarantee becomes the critical strategy for firms to compete for the market share. In view of Proposition 1, we shall consider only the interesting case where $a > 1$ for the reminder of this paper.

The next proposition presents a key result that will be useful for analyzing the multiple firm problem in the next section. This result shows that when the combined attraction of its competitors for their services increases, a firm will need to compete with a lower price and a shorter time guarantee.

PROPOSITION 2. *The optimal price $p_i^*$ is decreasing in $\beta_i$ and the optimal time guarantee $t_i^*$ is strictly decreasing in $\beta_i$.*

Proposition 2 suggests that in addition to lower prices, increased competition in a time-sensitive market can also result in shorter delivery time guarantees for services. For example, the fast-food service business has long recognized the urgency of customers during their lunch hours. Fast-food chains like Del Taco had introduced a time guarantee program for their drive-through customers at lunch hours. With generally an hour or less as a break during lunch, several major dine-in restaurant chains, including Black Angus and Red Lobster, had previously targeted this specific market segment and to compete with fast-food establishments for lunch business by guaranteeing short delivery times for limited items on their lunch menus.

The following proposition provides further results regarding how the various parameters in the model affect the optimal price $p_i^*$ and service delivery time guarantee $t_i^*$ for firm $i$.

PROPOSITION 3. *The optimal price $p_i^*$ and time guarantee $t_i^*$ have the following properties:*

(a) *When $L_i$ increases, both $p_i^*$ and $t_i^*$ increase.*
(b) *When $\gamma_i$ increases, $p_i^*$ increases and $t_i^*$ decreases.*
(c) *When $\mu_i$ increases, $t_i^*$ decreases.*
(d) *When $\lambda$ decreases, $t_i^*$ decreases.*

Proposition 3 provides the following managerial implications. Part (a) asserts that when the other attraction factors of a firm as measured by $L_i$ increase, say, due to better brand image or more convenient service locations, the firm can charge a higher price and provide a longer time guarantee. This suggests that when all other factors are equal, a less reputable firm with a less extensive network of service locations will tend to compete with a lower price and shorter time guarantee. Part (b) states that as its unit operating cost increases, a firm needs to increase its price, but then needs to offer a shorter time guarantee to compensate for the higher price. Part (c) confirms the intuitive result that a firm should shorten its time guarantee when its system capacity increases, as such a move allows for an increase in its market share to fill the additional capacity. Similarly, part (d) predicts that as the size of the overall market shrinks, a firm will offer a shorter time guarantee to increase its market demand in order to fill its available capacity, assuming that the price and time guarantee of its competitor remains unchanged. Obviously, in the situation of a shrinking market, other firms will also adjust their prices and time guarantees to compete. This leads to our next discussion about the multiple firm competition.

### 2.2. The $N$-Firm Problem

We now study the pricing and service delivery time decisions for the $N$-firm problem. One basic question is to investigate whether an equilibrium exists in an oligopolistic competition for this problem, and if so, how the equilibrium solution will change under different market conditions. The $N$-tuple decision vector

$$((p_1^*, t_1^*), (p_2^*, t_2^*), \ldots, (t_N^*, p_N^*)),$$

where $(p_i^*, t_i^*)$ is the price and service delivery time guarantee for firm $i$, is called a Nash equilibrium if for each $i$, $(p_i^*, t_i^*)$ is the best response by firm $i$ to the price and delivery time guarantee $(p_j^*, t_j^*)$ chosen for all other firms $j$. In other words, $(p_i^*, t_i^*)$ is the optimal solution to the firm $i$ problem when the price and delivery time guarantee for firm $j$, $j \neq i$, are fixed at $(p_j^*, t_j^*)$. The Nash equilibrium implies that no single firm can benefit by deviating from this equilibrium point unilaterally.

We next present a simple iterative procedure to compute the Nash equilibrium solution. Our iterative procedure is based on the results given in Proposition 1 (and Lemma A1) and works as follows:

(1). (*Initialization*). For each firm $i$, choose $t_i = k/\mu_i$ and $p_i = \gamma_i$.

(2). (*Iterative step*). Start with $i = 1$. Apply the result in Proposition 1 to find the optimal $t_i$ and the corresponding optimal $p_i = P(t_i)$ using the current values of $t_j$ for all other firms $j$. Update the value of $t_i$ and $p_i$. Repeat this for $i = 2, 3, \ldots, N$.

(3). (*Convergence criteria*). Repeat Step (2) until each of the $t_i$'s differs from their previous value by less than some predetermined tolerance level $\epsilon$.

The next proposition establishes the results that a unique Nash equilibrium exists for our problem and that the above iterative procedure always converges to this equilibrium solution.

PROPOSITION 4. *The iterative procedure given above converges to the unique Nash equilibrium solution for the* N*-firm problem.*

We can further characterize the equilibrium solution when all $N$ firms are identical. Specifically, we show that the (unique) Nash equilibrium solution is symmetric and further analyze how the different parameters in the model affect the equilibrium solution.

PROPOSITION 5. *Suppose that all* N *firms are identical.*

(a) *The Nash equilibrium solution is symmetric.*

(b) *The equilibrium time guarantee $t^*$, price $p^*$ and profit $\Pi^*$ are given by*

$$t^* = \frac{Nk}{N\mu_i - \lambda}, \tag{6}$$

$$p^* = \begin{cases} \min\left\{\dfrac{\gamma_i a}{a - \dfrac{N}{N-1} - \dfrac{b\lambda}{N\mu_i - \lambda}}, \bar{p}\right\} \\ \quad \text{if } a - \dfrac{N}{N-1} - \dfrac{b\lambda}{N\mu_i - \lambda} > 0, \\ \bar{p} \qquad \text{otherwise,} \end{cases} \tag{7}$$

*and*

$$\Pi^* = \frac{(p^* - \gamma_i)\lambda}{N}. \tag{8}$$

(c) *Furthermore, Table 1 summarizes how the different parameters of the model affect $p^*$, $t^*$ and $\Pi^*$.*

Observe from (7) that when $(a - (N/(N-1)) - (b\lambda/(N\mu_i - \lambda)))$ is small (or less than zero), we have the boundary solution, i.e., $p^* = \bar{p}$. This result provides the condition under which the price attraction factor $a$ is small relative to the time attraction factor $b$ and the relative workload $(N\mu_i - \lambda)$ such that all firms will charge the maximum price $\bar{p}$ as allowed by the market. We summarize the results for the case with multiple identical firms as follows:

**Table 1** **Sensitivity Analysis with Identical Firms**

| Parameter ↑ | $t^*$ | $p^*$ | $\Pi^*$ |
|---|---|---|---|
| $N$ | ↓ | ↓ | ↓ |
| $\mu_i$ | ↓ | ↓ | ↓ |
| $\gamma_i$ | – | ↑ | ↑ |
| $\lambda$ | ↑ | ↑ | ↑ |
| $a$ | – | ↓ | ↓ |
| $b$ | – | ↑ | ↑ |
| $\alpha$ or $k$ | ↑ | – | – |

(i) The impacts of the different model parameters on the behaviour of the equilibrium price and time guarantee given in Table 1 are rather intuitive. For instance, a higher firm capacity $\mu_i$ or lower total market size $\lambda$ would result in lower prices and shorter time guarantees. Also, as the price attraction factor $a$ increases, firms compete more intensely on price, resulting in lower prices. On the other hand, as the time attraction factor $b$ increases, firms are more willing to compete on time and so are less pressed to compete on price, resulting in higher prices. In either case, since the resulting market shares are the same for all firms, the equilibrium time guarantee remains unchanged.

(ii) Observe that the general behaviour of the equilibrium price and time guarantee is parallel to that of the optimal price and time guarantee as predicted by the single firm model in So and Song (1998) (see Proposition 2 therein). The similarity is rather interesting as in their single-firm model, the firm selects the price and time guarantee to maximize its resulting profit under a constant elasticity demand function, whereas in our multiple-firm model here, the total demand is fixed as a firm competes only for market share under the multiplicative competitive interaction model given in (1) to maximize its profit.

(iii) The firm profit $\Pi^*$, on the other hand, behaves rather differently between the single-firm case and multiple-firm case. For example, a higher firm capacity increases the profit in the single-firm case, while an increase in firm capacity in the multiple-firm case lowers the profit. This is expected, as the market size is assumed constant in the multiple-firm case such that an increased capacity leads to increased price pressure and lower profit for the firm, whereas an increased capacity in the single-firm case provides additional flexibility in choosing its price and time guarantee to increase its overall profit.

(iv) Equations (6) and (7) provide an interesting result when we increase the number of competing firms $N$ while keeping the total capacity ($N\mu_i$) in the market constant. This corresponds to situations where there are few large (identical) firms versus many small (identical) firms in the market. Our model predicts that the equilibrium solution will be lower price but longer time guarantee when there are many small firms in the market. The longer time guarantee is the direct result of the service constraint requirement, while the corresponding lower price appears to be consistent with the expected result that increased competition in a market should benefit the end customers.

Many traditional service providers differentiate their services more than just price and delivery time performance, such as convenience of locations or "friendliness" of service representatives. However, it is interesting to note that these other competitive factors can become much less important with the increasing popularity of internet shopping, and different service providers would appear more "identical" among internet shoppers. Furthermore, the internet decreases search cost and allows for easy comparisons across multiple service providers, whereby price and service delivery times can become the two critical competitive factors among service providers on the internet.

For the case with nonidentical firms, we can use our iterative procedure to compute the equilibrium solution to our model numerically. In the next section, we report the results of a set of numerical experiments that are designed to address several key research questions regarding how the different firm and market characteristics would affect a firm's decision in setting price and time guarantee to compete in the market.

# 3. Numerical Experiments with Nonidentical Firms

In this section we present a set of numerical experiments to illustrate the behaviour of the equilibrium pricing and time guarantee decisions, as predicted by our model, when different firms compete under various market conditions. Results from additional numerical experiments suggest that the major observations drawn from this numerical analysis should also be applicable in more general cases, even though we are unable to establish such results analytically.

Our model captures two major firm characteristics: capacity (size) and unit operating cost (efficiency). We study the impact of each of these two characteristics as well as their joint impact on the equilibrium price and time guarantee decisions. To focus only on the interaction between price and time competition, we shall assume, except specified otherwise, that all $L_i$ are equal in our numerical experiments so as to eliminate the impact of all other competitive factors on the market share of the firms. We investigate the impact of these other factors on price and time competition in the last subsection. Also, we set the common service reliability $\alpha = 95\%$ in all our numerical experiments. Market share numbers are expressed in percentages in all our figures.

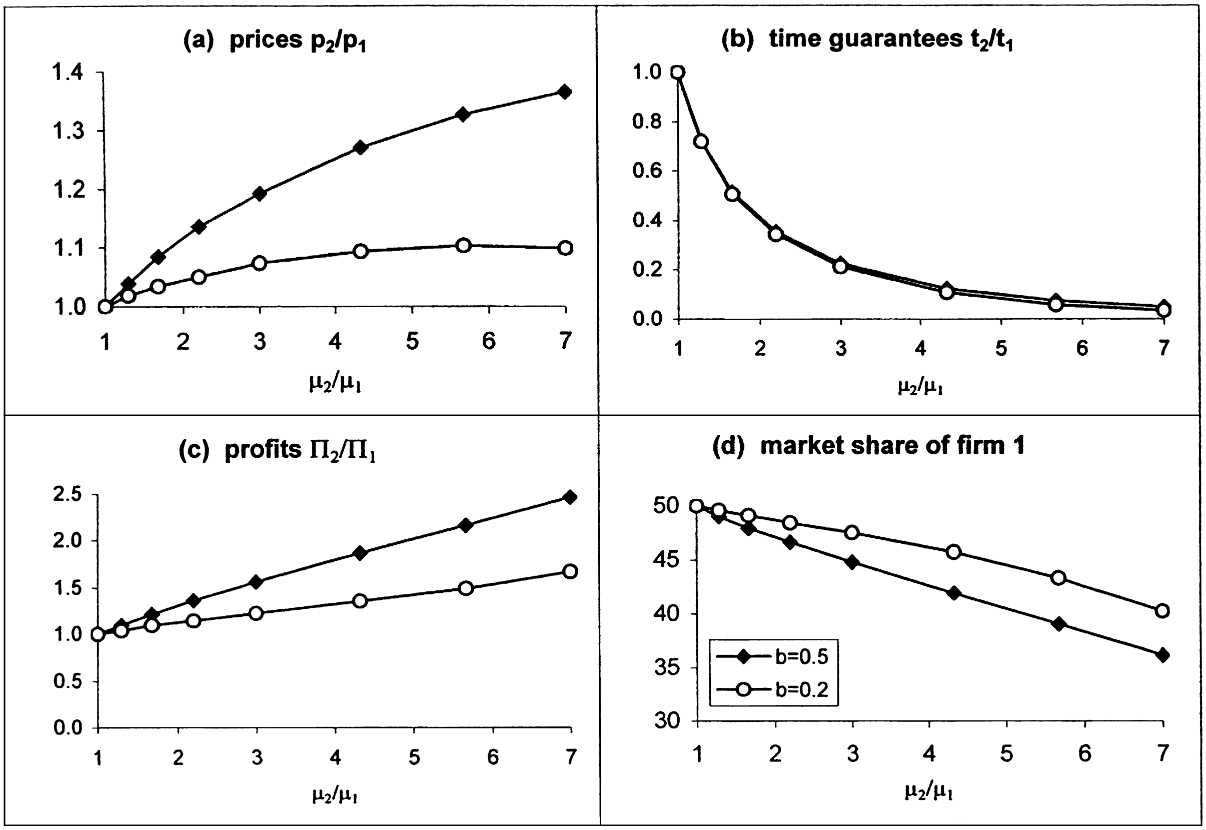
## 3.1. Impact of Firm Capacities

We first studied how the firm capacities would affect the price and time guarantee decisions. To do so, we considered two competing firms with a fixed total combined capacity of $\mu_1 + \mu_2 = 4$ and the unit operating cost of each firm $\gamma_1 = \gamma_2 = 1$. We set the market parameters at $\lambda = 1$ and $a = 3$ with two values of $b$ (0.2 and 0.5). We studied different cases with values of $\mu_1 \leq \mu_2$, and computed the ratios of their corresponding equilibrium prices, time guarantees and overall profits. We also compared these ratios for two different values of $b$ to see how the equilibrium solution changes when the market becomes more attracted to time guarantees. The results are given in Figure 1.

We summarize the major observations as follows:

(i) All other factors being equal, the firm with a higher capacity offers a shorter time guarantee and charges a higher price, resulting in a higher profit and

Figure 1 Impact of Capacity




market share than the firm with a lower capacity. Furthermore, the ratios of their prices and profits increase as the ratio of their capacities increases.

(ii) The comparative advantage of the higher capacity firm increases as the time attraction factor $b$ increases. The relative profit and market share of firm 2 increases as $b$ increases from 0.2 to 0.5. This shows that the ability to offer shorter time guarantees becomes increasingly beneficial to the larger capacity firm when the market is more sensitive to delivery time guarantee.

### 3.2. Impact of Unit Operating Costs

We next studied how the operating cost of the firms would affect the price and time guarantee decisions. We considered two competing firms with $\mu_1 = \mu_2 = 2$. We fixed $\gamma_1 = 1$ and consider different values of $\gamma_2 \geq 1$. Again, we set the market parameters at $\lambda = 1$ and $a = 3$ with two values of $b$ (0.2 and 0.5) and computed the ratios of their respective prices, time guarantees and profits for the two different values of $b$. The results are given in Figure 2.

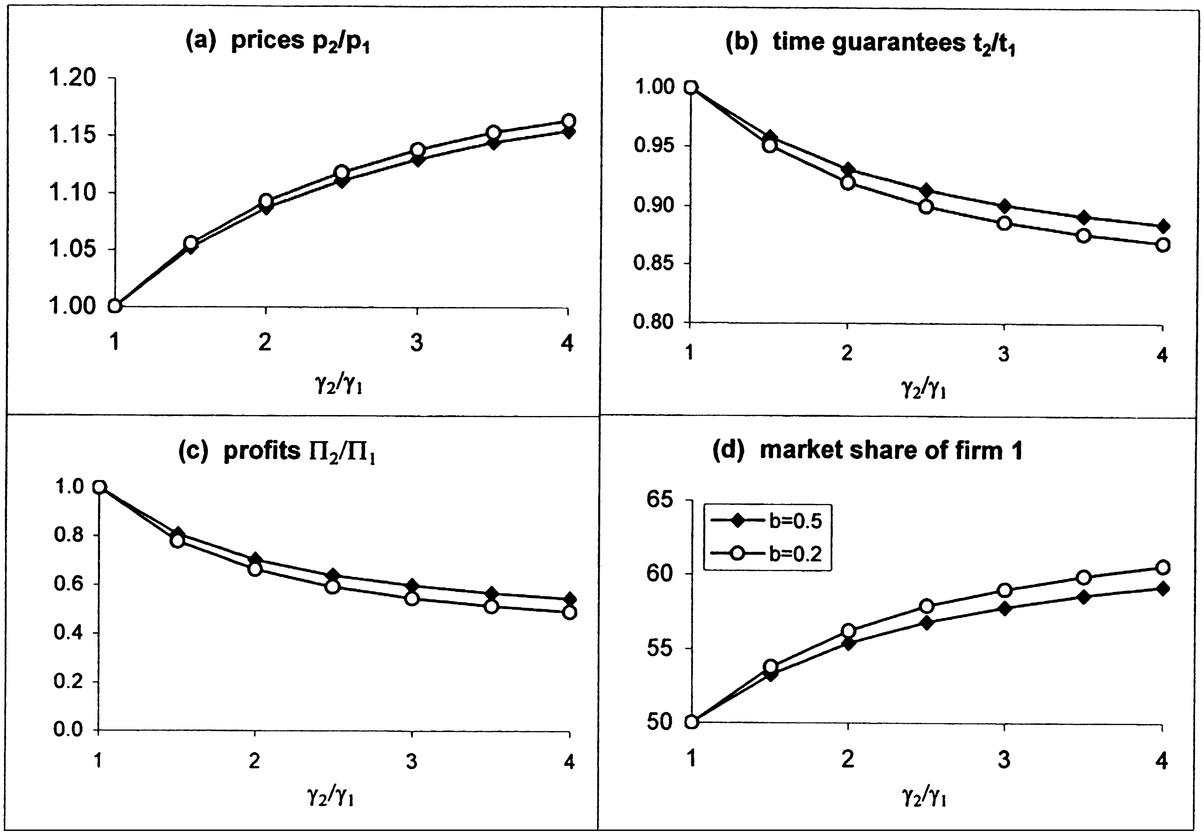
We summarize the major observations as follows:

(i) All other factors being equal, the firm with a lower operating cost offers a lower price but a longer time guarantee, resulting in a higher profit and market share than the firm with a higher operating cost. Furthermore, the relative profit and market share for the firm with the lower operating cost increases as the ratio $\gamma_2/\gamma_1$ increases.

(ii) However, the cost advantage of a firm diminishes in a more time-sensitive market. For example, the benefit of firm 1 due to its cost advantage over firm 2 decreases as the time attraction factor $b$ increases from 0.2 to 0.5.

Our model and results can be used to provide some important implications in operating decisions where the joint impact of capacity and operating cost factors

**Figure 2** Impact of Operating Cost



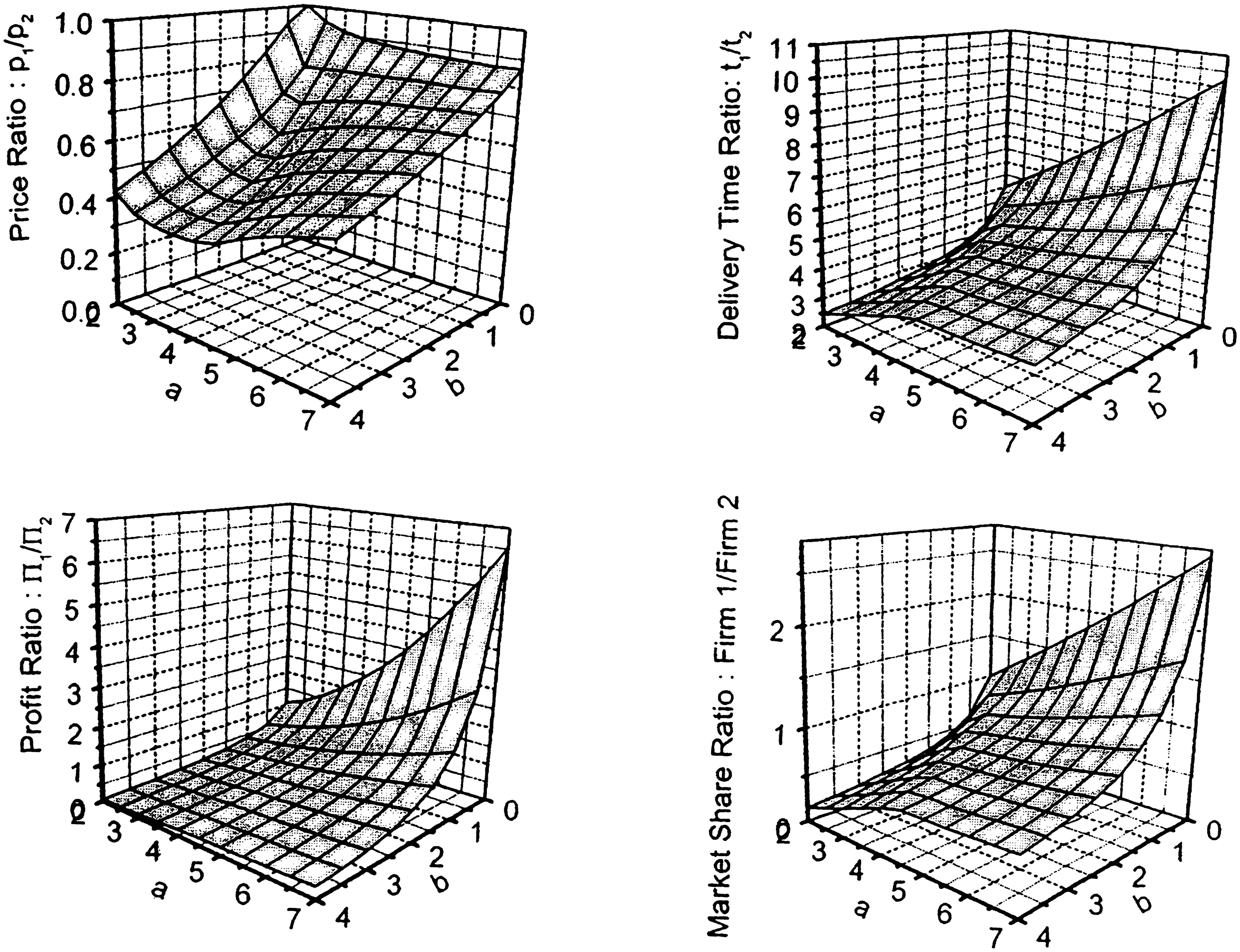

needs to be assessed. Consider the capacity expansion decision, where capacity and operating costs are correlated due to the possible economies of scale in production and/or facility expansion in an already tight labor market. We can analyze the joint impact of increased capacity and the possible changes in operating cost for such decisions to understand how the various possible options would change the underlying competitive position of the firm in a price and time sensitive market.

To illustrate, consider a simple example with two identical competing firms with $\mu_1 = \mu_2 = 2$ and $\gamma_1 = \gamma_2 = 1$. Firm 2 now considers two possible options to double its current capacity to $\mu_2 = 4$, where Option 1 is to expand its existing facility and Option 2 is to replace its facility with a new one in a different location. Assume that the labor market is tight at its current location so that Option 1 would increase its existing unit operation cost by 25%, while the new facility can reduce its existing unit operation cost by 25% under Option 2. Assume $\lambda = 1$, $a = 3$ and $b = 0.5$. The equilibrium solutions for both options are given below:

| Option | $p_2^*$ | $t_2^*$ | Profit | Market Share |
|---|---|---|---|---|
| 1 | 4.53 | 0.927 | 2.52 | 51.3% |
| 2 | 3.49 | 0.944 | 2.26 | 55.1% |

One interesting observation here is that under Option 2, firm 2 would obtain in a lower profit, despite a lower unit operating cost associated with this option. Apparently, this can be explained by the fact that in a duopoly, the lower operating cost entices firm 2 to compete by offering lower prices, driving the profit of both firms down. Observe that under Option 2, firm 2 is able to grab a higher market share than that under Option 1.

**Figure 3** Joint Impact of Price and Time Attraction Factors




### 3.3. Impact of Price and Time Attraction Factors

Our above results show that higher capacity firms offer shorter time guarantees, whereas firms with lower unit operating costs compete by charging lower prices. This suggests that firms will differentiate their services based on their respective capacity and cost advantage under time and price competition. Furthermore, the results show that the time attraction factor $b$ is one key factor in determining the comparative advantages between these two firm characteristics. Therefore, an important issue is to understand how firms with distinctive characteristics would differentiate their services under different market conditions.

To address this issue, we considered two competing firms, one with a higher capacity and the other with a lower unit operating cost, by setting $(\mu_1, \gamma_1) = (1, 1)$ and $(\mu_2, \gamma_2) = (3, 2)$. We set $\lambda = 1$ and considered different values of $a$ and $b$, where $a$ ranges from 2 to 7 and $b$ ranges from 0 to 4. Figure 3 shows the ratio curves of the equilibrium characteristics for the two competing firms for different values of $a$ and $b$.

We summarize the major observations as follows:

(i) With all other factors being equal, the two firms will differentiate their services according to their relative capacity and cost advantages. Specifically, Figure 3 shows that the low-cost firm (firm 1) always charges a lower price while the higher-capacity firm (firm 2) always offers a shorter time guarantee.

(ii) As the price attraction factor $a$ increases, the prices of both firms decrease, whereas the time guarantee of firm 1 increases but the time guarantee of firm 2 decreases. In other words, as the market becomes more price-sensitive, both firms lower their prices to compete. Furthermore, the price gap between the two firms generally narrows as $a$ increases (except for the case when firm 2 charges the maximum price.) On the other hand, the difference between the time guarantees of the two firms widens as firm 2 provides an increasingly shorter time guarantee to compete with the price (cost) advantage enjoyed by firm 1.

(iii) As the time attraction factor $b$ increases, the time guarantee of firm 1 shortens while the time guarantee of firm 2 slightly increases, resulting in a decrease of the difference between the time guarantees offered by the two firms. At the same time, the prices offered by the two firms generally increase and the difference in their prices also increases (except in the case when firm 2 charges the maximum price). In other words, as the market becomes more time-sensitive, the two firms will tend to differentiate more on their prices. Essentially, the effects of the price and time attraction factors complement each other.

(iv) The results in Figure 3 also show that the profit and market share become increasingly beneficial to the low-cost firm (firm 1) as the price attraction factor $a$ increases, while they become increasingly beneficial to the high-capacity firm as the time attraction factor $b$ increases.

### 3.4. Impact of Other Competitive Factors

In our numerical experiments so far, we assume that all other competitive factors of the firms are the same to isolate the impact of the firm characteristics ($\gamma_i$ and $\mu_i$) and market characteristics ($a$ and $b$) on the equilibrium price and time guarantee decisions. It is an interesting question to ask whether the difference in these other competitive factors, as captured by $L_i$ in our model, would change the basic observations in previous subsections in any qualitative manner.

To address this question, we again considered two competing firms with $(\mu_1, \gamma_1) = (1, 1)$ and $(\mu_2, \gamma_2) = (3, 2)$. We set $\lambda = 1$, $a = 3$ and $b = 0.5$, and computed the equilibrium solution for different values of the ratio $L_1/(L_1 + L_2)$, ranging from 0.1 to 0.9. Observe that $L_1/(L_1 + L_2) = 0.5$ corresponds to the case where these competitive factors of the two firms are equal. When $L_1/(L_1 + L_2) > 0.5$, firm 1 has an advantage over firm 2, e.g., more convenient service locations, and vice versa. The results are given in Figure 4.

We summarize the major observations as follows:

(i) As $L_1/(L_1 + L_2)$ increases (the competitive position of firm 1 increases), firm 1 can increase its price and lengthen its time guarantee while firm 2 needs to lower its price (with time guarantee remaining fairly constant) to compete. In particular, when $L_1/(L_1 + L_2) > 0.7$ in this example, firm 1 offers both lower price and shorter time guarantee than those of firm 2. Thus, the results suggest that the other competitive factors have significant impact, especially on the equilibrium price and profit of the firms.

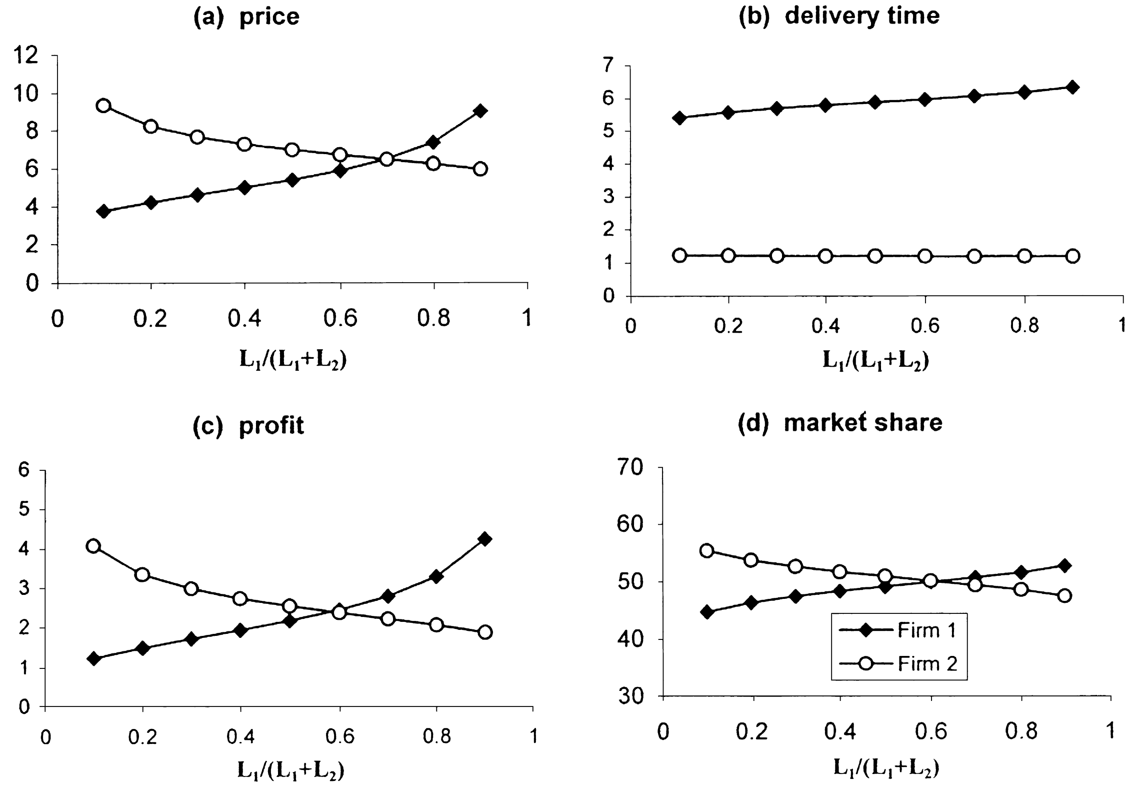
(ii) As $L_1/(L_1 + L_2)$ increases, both the profit and market share of firm 1 increase while those of firm 2 decrease. Furthermore, firm 2 offers both lower price and shorter time guarantee to maintain its market share. For instance, when $L_1/(L_1 + L_2) = 0.9$, firm 2 still maintains a market share of 47%, compared to only a 10% market share if its price and time guarantee were the same as those offered by firm 1.

## 4. Competitive Advantage of a Dominant Firm

Our numerical results in the previous section suggest that when one firm dominates the other firm with both a higher capacity and lower unit operating cost in a two-firm competition, the dominant firm would offer both a better time guarantee and a lower price, assuming the other competitive factors are equal. This raises the interesting issue whether this result would extend to a situation when there are more than two firms competing in the market. In particular, would the firm with highest capacity and lowest operating cost always offer the shortest time guarantee and the lowest price? In this section, we use our model in a competitive scenario analysis to generate some insight for this question.

We constructed a scenario consisting of three competing firms in the market, with $(\mu_1, \gamma_1) = (1, 1)$, $(\mu_2, \gamma_2) = (3, 2)$, and $(\mu_3, \gamma_3) = (3, 1)$. Thus, firm 3 is dominant and has the highest capacity and lowest unit operating cost. Firm 1 has the same unit operating cost

**Figure 4** Impact of Other Attraction Factors: $(\mu_1, \gamma_1) = (1, 1)$ and $(\mu_2, \gamma_2) = (3, 2)$




and firm 2 has same capacity as firm 3, but both firm 1 and 2 cannot match firm 3 on the other characteristics. We set $\lambda = 1$ and $a = 3$ with the values of $b$ ranging from 0 to 1. The results are given in Figure 5.

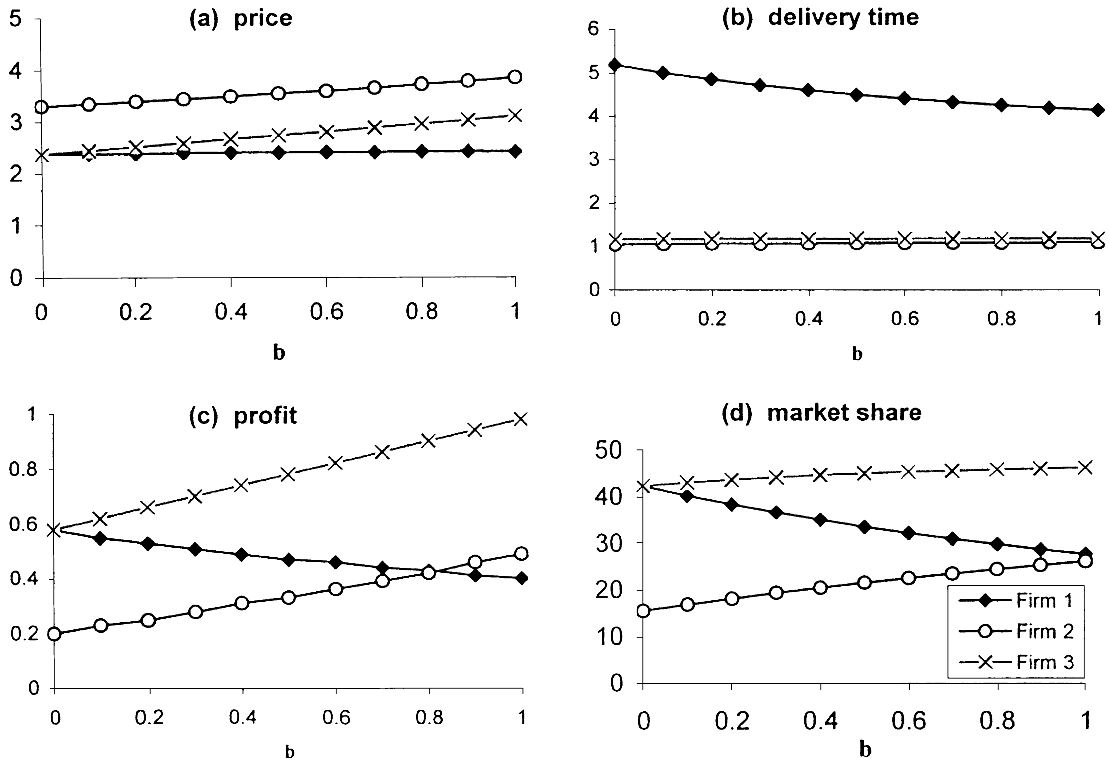
We summarize the major observations as follows:

(i) The dominant firm 3, with the highest capacity and lowest operating cost, has the largest profit and market share. The profits and market shares of both firms 1 and 2 are adversely affected due to the addition of firm 3 in the market, as compared to the results in Figure 3. Furthermore, the adverse impact on firm 2, which enjoys a higher capacity over firm 1, is especially severe when $b$ is large. In other words, firm 2 can no longer enjoy its previous dominance in capacity to allow for a high price premium, especially when the time attraction factor $b$ is large.

(ii) The dominant firm (firm 3) does not necessarily offer the lowest price nor the best time guarantee. This is somewhat surprising. One plausible explanation is that since firms 1 and 2 have the advantage in only one dimension (cost or capacity), they are more pressed to differentiate their services along that dimension. Consequently, firm 1 always offers the lowest price and firm 2 always provides the shortest time guarantee. In contrast, firm 3 has the advantage in both dimensions so that it need not rely on solely one aspect to compete, and as a result, its price and time guarantee are not (but are very close to) the lowest. This suggests that the weaker firms will more likely choose to compete in one or a few competitive dimensions (niche) for their services in the market.

(iii) The benefit of offering short time guarantees in a time-sensitive market is clearly illustrated. Observe that when $b = 0$, the differences in prices, profits and market shares among the three firms are purely due to their differences in operating costs of the firms, such that the $b = 0$ case can serve as a reference point here. In particular, firms 1 and 3 offer the same lowest price when $b = 0$. However, as $b$ increases, all three firms increase their prices, but the price increase of firm 3 is

**Figure 5** **Competition Among 3 Firms:** $(\mu_1, \gamma_1) = (1, 1)$, $(\mu_2, \gamma_2) = (3, 2)$ **and** $(\mu_3, \gamma_3) = (3, 1)$




much higher than that of firm 1 due to its ability to provide a shorter time guarantee for its service.

We extended the scenario where firm 3 is dominating both firm 1 and firm 2 in cost and capacity. Specifically, consider the case where $(\mu_3, \gamma_3) = (6, 0.5)$ such that firm 3 has twice the capacity of its next competitor in this dimension (firm 2) while its unit operating cost is only half of its next competitor in this dimension (firm 1). In this case, firm 3 still does not necessarily offer the lowest price and the shortest time guarantee. In particular, firm 3 only offers the lowest price and the shortest time guarantee when the time attraction factor $b$ is small. For $b \geq 0.3$, firm 1 offers the lowest price. This indicates that as timely guarantee becomes an increasingly competitive factor, firm 3 can raise its price much more than that of firm 1, together with a much shorter time guarantee that firm 1 cannot afford to compete.

We briefly discuss the opposite case where firm 3 is now being dominated in cost and capacity by firm 1 and firm 2, respectively. In particular, we set $(\mu_3, \gamma_3) = (1, 2)$. Parallel to the case when firm 3 is dominant, the equilibrium results show that firm 3 does not necessarily offer the highest price nor the worst time guarantee. In this case, firm 1 always offers the lowest price and firm 2 always provides the shortest time guarantee, while the price and time guarantee offered by firm 3 are always between those by firm 1 and firm 2. We omit the details here.

## 5. Conclusions

Price and timely delivery are two important factors for success for service providers in today's competitive markets, and many service companies are offering time guarantees to compete in a time-sensitive market. Using a stylized model, we analyze the interaction between price and delivery time guarantee and their impact on competition. In our model, we capture the essence of several major characteristics of the market and firms. Through a numerical study, we illustrate how firms would choose their pricing and time guarantee strategy to compete in a market. Our results help to identify the key market factors and important firm characteristics for success in price and time competition, and have important implications on how firms should choose to compete under different market environments.

Our results suggest that different firm characteristics play a key role in differentiating their services in a time-sensitive market. With all other factors being equal, a high capacity firm will compete with shorter time guarantee, while a firm with lower unit operating cost will compete with lower price. The differentiation becomes increasingly acute as demands become more sensitive to time guarantees. This is consistent with the observation in practice that some restaurants (such as Pizza Hut, Black Angus) are willing to offer time guarantees for their lunch services but not their dinner services as lunch meals are more time-sensitive. The express mail delivery service market is so time-sensitive that only a few major firms can participate in the most time-critical next-day/overnight delivery services. The time-attractiveness of the market is one major element in determining the comparative advantages of firms with different characteristics.

Several extensions to our modeling framework are worth further investigation. First, our model and results are based on a specific market share function and linear cost structure. So, it would be interesting to see whether the results can be extended to other market share functions or more general cost structure. We remark that Lederer and Li (1997) studied the effect of price and time performance in a competitive environment, under which firms compete by choosing prices and service rates for multiple types of customers with different delay costs, and our qualitative results parallels to theirs. Second, the service reliability level $\alpha$, i.e., the probability of meeting the service delivery time guarantee is assumed to be given and fixed in our model. Instead, one can consider $\alpha$ to be a decision variable that can also affect the market share function. While our model can be easily modified (to the unit operating cost parameter) to include some fixed penalty cost for late delivery, we do not assume that the penalty amount will affect the demand or the market share function. One can relax this assumption to also consider the penalty cost as a decision variable that can be set by the firms. It will be interesting to see whether these extensions would change the basic observations in any qualitative fashion. Finally, an empirical study on how these delivery time guarantees affect demands and firm profitability would also be an interesting topic.

## Acknowledgments

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## A. Appendix

Proof of Proposition 1. For simpler notation, we shall drop the subscript $i$ on $P(t)$, $H(t)$ and $\Pi(t)$ for the remainder of this section without causing any confusion. We first provide some useful properties about $P(t)$ and $H(t)$. For our results, we consider only $k/\mu_i < t < k/(\mu_i - \lambda)$ when $\mu_i > \lambda$, and $k/\mu_i < t$ when $\mu_i < \lambda$, under which $P(t)$ and $H(t)$ are defined. We first establish the following result

Lemma A1. (i) $P'(t) < 0$.
(ii) *For $a > 1$, there exists a unique $\hat{t}$ such that $H(\hat{t}) = 0$. Furthermore, $H(t) > 0$ when $t < \hat{t}$ and $H(t) < 0$ when $t > \hat{t}$.*
(iii) *For $a \le 1$, $H(t) < 0$.*

Proof of Lemma A1. (i) Observe that $P(t) > 0$ and $(\mu_i t - k)(k + \lambda t - \mu_i t) > 0$ for $k/\mu_i < t < k/(\mu_i - \lambda)$ when $\mu_i > \lambda$, and for $k/\mu_i < t$ when $\mu_i < \lambda$. Thus,

$$P'(t) = \frac{1}{a}\left\{\frac{k + \lambda t - \mu_i t}{t^b \beta_i(\mu_i t - k)}\right\}^{(1/a)-1} \cdot \frac{t^b \beta_i(\mu_i t - k)(\lambda - \mu_i) - (k + \lambda t - \mu_i t)[b t^{b-1}\beta_i(\mu_i t - k) + t^b \beta_i \mu_i]}{[t^b \beta_i(\mu_i t - k)]^2}$$

$$= \frac{1}{a}\left\{\frac{k + \lambda t - \mu_i t}{t^b \beta_i(\mu_i t - k)}\right\}^{(1/a)-1} \frac{-k\lambda t - b(\mu_i t - k)(k + \lambda t - \mu_i t)}{t^{b+1}\beta_i(\mu_i t - k)^2}$$

$$- P(t)\left\{\frac{k\lambda t + b(\mu_i t - k)(k + \lambda t - \mu_i t)}{at(\mu_i t - k)(k + \lambda t - \mu_i t)}\right\} < 0.$$

(ii) Since $P'(t) < 0$, we obtain

$$H'(t) = \frac{-k^2\lambda}{(k + \lambda t - \mu_i t)^2} - b\mu_i + \frac{\gamma_i k a P'(t)}{P(t)^2} < 0.$$

Furthermore, $H(k/\mu_i)^- = k(a - 1) > 0$, and $H(k/(\mu_i - \lambda))^+ \to -\infty$ when $\mu_i > \lambda$ and $H(\infty) \to -\infty$ when $\mu_i \le \lambda$. This implies that there exists a unique solution $\hat{t}$ to $H(t) = 0$ with $H(t) > 0$ when $t < \hat{t}$ and $H(t) < 0$ when $t > \hat{t}$.

(iii) For $a \le 1$, $H(k/\mu_i)^- = k(a - 1) \le 0$. Therefore, $H(t) < 0$ for all.

To prove Proposition 1, it is straightforward to show that for any fixed $p > \gamma_i$, the profit function $\Pi(p, t)$ given in (2) is strictly decreasing in $t$. Also, one can easily show that the left side of the service reliability constraint (4) is increasing in $t$ for any fixed $p$. These two facts imply that the optimal solution must lie on the boundary of constraint (4). Therefore, the optimization problem can be reduced to a one-dimensional search along the boundary.

Using (4), we can rewrite the boundary of (3) as

$$p = \left\{\frac{\beta_i(\mu_i t - k)}{t^{-b}(k + \lambda t - \mu_i t)}\right\}^{-1/a} = P(t).$$

By substituting $p = P(t)$ into (2), the profit function can be expressed as a function of $t$ only, and is given by

$$\Pi(t) = \Pi(p, t) = (P(t) - \gamma_i)\left(\mu_i - \frac{k}{t}\right). \tag{9}$$

Then,

$$\begin{aligned}\Pi'(t) &= P'(t)\left(\mu_i - \frac{k}{t}\right) + (P(t) - \gamma_i)\frac{k}{t^2}\\ &= -P(t)\left\{\frac{k\lambda t + b(\mu_i t - k)(k + \lambda t - \mu_i t)}{at^2(k + \lambda t - \mu_i t)}\right\} + \frac{P(t)k}{t^2} - \frac{\gamma_i k}{t^2}\\ &= \frac{P(t)H(t)}{at^2},\end{aligned}$$

from the definition of $H(t)$ given in (5).

Consider $a > 1$. It follows from Lemma A1(ii) and the fact $P(t) > 0$ that $\Pi'(\hat{t}) = 0$, and that $\Pi'(t) > 0$ when $t < \hat{t}$ and $\Pi'(t) < 0$ when $t > \hat{t}$, where $\hat{t}$ is the unique solution to $H(t) = 0$. Therefore, if $\hat{p} = P^{-1}(\hat{t}) \le \bar{p}$, then $(\hat{p}, \hat{t})$ is the unique optimal solution. Otherwise, $p^* = \bar{p}$ with $t^* = P^{-1}(\bar{p})$.

For $a \le 1$, Lemma A1(iii) implies that $\Pi'(t) < 0$ for all $t$, so $\Pi(t)$ is decreasing in $t$. Since $P^{-1}(t)$ is decreasing in $t$, it follows that the profit is increasing in $p$, and so $p^* = \bar{p}$ with $t^* = P^{-1}(\bar{p})$. □

Proof of Proposition 2. Observe from (4) that $P(t)$ is strictly decreasing in $\beta_i$, which implies that $H(t)$ is strictly decreasing in $\beta_i$. It then follows from Lemma 1(ii) that $\hat{t}$ is strictly decreasing in $\beta_i$. Furthermore, for fixed $p$, the corresponding $t$ on the boundary of the service constraint (4) is strictly decreasing in $\beta_i$. From Proposition 1, $t^* = \max(\hat{t}, P^{-1}(\bar{p}))$ is strictly decreasing in $\beta_i$.

From (5) and the definition of $\hat{t}$ that $H(\hat{t}) = 0$, we obtain

$$P(\hat{t}) = \gamma_i ka\left\{\frac{-k\lambda\hat{t}}{k + \lambda\hat{t} - \mu_i\hat{t}} - b(\mu_i\hat{t} - k) + ka\right\}^{-1}.$$

Observe that

$$\frac{-k\lambda\hat{t}}{k + \lambda\hat{t} - \mu_i\hat{t}} = \frac{-k\lambda}{(k/\hat{t}) + \lambda - \mu_i}$$

is strictly decreasing in $\hat{t}$ as $(k + \lambda\hat{t} - \mu_i\hat{t}) > 0$. Therefore, $P(\hat{t})$ is strictly increasing in $\hat{t}$. Since $\hat{t}$ is strictly decreasing in $\beta_i$, it follows that $P(\hat{t})$ is strictly decreasing in $\beta_i$. Thus, $p^* = \min(P(\hat{t}), \bar{p})$, $p^*$ is decreasing in $\beta_i$. □

Proof of Proposition 3. (a) Since $\beta_i = (1/L_i)\sum_{j\neq i} L_j p_j^{-a} t_j^{-b}$ and $L_j > 0$ for all $j$, this result follows immediately from Proposition 2.

(b) Observe from (4) and (5) that $P(t)$ is independent of $\gamma_i$ while $H(t)$ is decreasing in $\gamma_i$. Following the similar argument as (a), we deduce that $\hat{t}$ is decreasing in $\gamma_i$, which implies that $t^*$ is decreasing in $\gamma_i$. Since $P(t)$ is decreasing in $t$, $p^* = \min(P(\hat{t}), \bar{p})$ is increasing in $\gamma_i$.

(c) Both $P(t)$ and $H(t)$ decrease as $\mu_i$ increases. This implies that $\hat{t}$, and thus $t^*$, increases as $\mu_i$ increases.

(d) Similarly, both $P(t)$ and $H(t)$ decrease as $\lambda$ decreases. Therefore, $t^*$ decreases as $\lambda$ decreases. □

Proof of Proposition 4. We first show that the iterative procedure always converges to a Nash equilibrium. Let $p_i^{(k)}$ and $t_i^{(k)}$ be the solution given by the procedure at the $k$th iteration. We shall prove by induction that both $p_i^{(k)}$ and $t_i^{(k)}$ are increasing in $k$. Since $p_i^{(k)}$ is bounded above by $\bar{p}$, this establishes the result that $p_i^{(k)}$ (and thus $t_i^{(k)}$) converges.

By definition, $p_i^{(0)} = \gamma_i$ and $t_i^{(0)} = k/\mu_i$. It is obvious that $p_i^{(k)} > \gamma_i$ and $t_i^{(k)} > k/\mu_i$ for all $k$. In particular, $p_i^{(1)} > p_i^{(0)}$ and $t_i^{(1)} > t_i^{(0)}$ for all $i$. Assume now that $p_i^{(k)} \ge p_i^{(k-1)}$ and $t_i^{(k)} \ge t_i^{(k-1)}$ for all $i$ and $k < n$. Consider $k = n$.

Let $\beta_i^{(k)}$ denote the combined attraction of other firms for the firm $i$ problem at the $k$-iteration. Then,

$$\begin{aligned}\beta_1^{(n)} &= \frac{1}{L_1}\sum_{j=2}^{N} L_j(p_j^{(n-1)})^{-a}\,(t_j^{(n-1)})^{-b}\\ &\le \frac{1}{L_1}\sum_{j=2}^{N} L_j(p_j^{(n-2)})^{-a}\,(t_j^{(n-2)})^{-b} = \beta_1^{(n-1)},\end{aligned}$$

where the inequality follows from the inductive assumption. It then follows from Proposition 2 that $p_1^{(n)} \ge p_1^{(n-1)}$ and $t_1^{(n)} \ge t_1^{(n-1)}$. Suppose that $p_j^{(n)} \ge p_j^{(n-1)}$ and $t_j^{(n)} \ge t_j^{(n-1)}$ for all $j = 1, 2, \ldots, l - 1$. Then, it follows again from the inductive assumption that

$$\begin{aligned}\beta_l^{(n)} &= \frac{1}{L_l}\sum_{j<l} L_j(p_j^{(n)})^{-a}\,(t_j^{(n)})^{-b} + \frac{1}{L_l}\sum_{j>l} L_j(p_j^{(n-1)})^{-a}\,(t_j^{(n-1)})^{-b}\\ &\le \frac{1}{L_l}\sum_{j<1} L_j(p_j^{(n-1)})^{-a}\,(t_j^{(n-1)})^{-b} + \frac{1}{L_l}\sum_{j>l} L_j(p_j^{(n-2)})^{-a}\,(t_j^{(n-2)})^{-b}\\ &= \beta_l^{(n-1)}\end{aligned}$$

Therefore, using Proposition 2 again, we deduce that $p_l^{(n)} \ge p_l^{(n-1)}$ and $t_l^{(n)} \ge t_l^{(n-1)}$. This implies that $p_i^{(n)} \ge p_i^{(n-1)}$ and $t_i^{(n)} \ge t_i^{(n-1)}$ for all $i$, which completes our induction.

We now show that the Nash equilibrium must be unique. Let us express the equilibrium solution as a function of the attraction of the individual firm $A_i = L_i p_i^{-a} t_i^{-b}$. Observe that for any given $(A_1, A_2, \ldots, A_N)$, all $t_i$ are uniquely defined by the tight service constraint (3) and all $p_i$ are then uniquely defined (4).

Suppose that there exists two different equilibrium solutions, denoted by $\Phi = (A_1, A_2, \ldots, A_N)$ and $\Phi' = (A_1', A_2', \ldots, A_N')$. Express $A_i' = (1 + r_i)A_i$. By numbering the firms and the two solutions appropriately, we can assume that $r_1 \ge r_2 \ge \cdots \ge r_N$ and $r_1 > 0$.

First, we must have $r_2 > 0$. Otherwise, this implies that $r_i \le 0$ for all $i \geq 2$, or equivalently, $A_i \geq A_i'$ for all $i \geq 2$. Since both $\Phi$ and $\Phi'$ are equilibrium solutions, the optimal price and time guarantee under $\Phi$ for the firm 1 problem must be lower than those under $\Phi'$ using Proposition 2. This would imply that

$$A_1 = L_1(p_1)^{-a}(t_1)^{-b} \geq L_1(p_1')^{-a}(t_1')^{-b} = A_1' = (1 + r_1)A_1,$$

contradicting $r_1 > 0$.

Consider the firm 1 problem with the attraction of individual firm $A_i'' = (1 + r_2)A_i$ for all $i \geq 2$. Let $(p_1'', t_1'')$ be the corresponding optimal solution. Then, from the tight service constraint (3), we have

$$\left(\mu_1 - \frac{\lambda A_1''}{A_1'' + (1 + r_2)(A_2 + \cdots + A_N)}\right) t_1'' = k. \tag{10}$$

On the other hand, since $\Phi = (A_1, A_2, \ldots, A_N)$ is an equilibrium solution, we also have

$$\left(\mu_1 - \frac{\lambda A_1}{A_1 + A_2 + \cdots + A_N}\right) t_1 = k, \tag{11}$$

where $(p_1, t_1)$ is the equilibrium solution associated with $A_1$ given in $\Phi$. Since $r_2 > 0$, we have $A'' > A_i$ for all $i \geq 2$. Using Proposition 2 again, $t_1 > t_1'' > 0$ since $A_i'' = (1 + r_2)A_i > A_i$ for all $i \geq 2$. It then follows from (10) and (11) that

$$\mu_1 - \frac{\lambda A_1''}{A_1'' + (1 + r_2)(A_2 + \cdots + A_N)} > \mu_1 - \frac{\lambda A_1}{A_1 + A_2 + \cdots + A_N},$$

which implies that

$$A_1'' < (1 + r_2)A_1. \tag{12}$$

Next, consider the firm 1 problem under the equilibrium solution $\Phi'$. Since $r_j \leq r_2$ for $j > 2$, we have $A_i' \leq A_i''$ for all $i \geq 2$. From Proposition 2, we obtain $p_1' \geq p_1''$ and $t_1' \geq t_1''$. Thus,

$$A_1'' = L_1(p_1'')^{-a}(t_1'')^{-b} \geq L_1(p_1')^{-a}(t_1')^{-b} = A_1' = (1 + r_1)A_1. \tag{13}$$

Combining (12) and (13), we then have

$$(1 + r_2)A_1 > A_1'' \geq A_1' = (1 + r_1)A_1,$$

contradicting the assumption that $r_1 \geq r_2$. Therefore, the equilibrium solution must be unique. □

Proof of Proposition 5. (a) Assume the contrary that the equilibrium solution is not symmetric. Denote the equilibrium solution by the $N$-tuple $((p_1, t_1), (p_2, t_2), (p_3, t_3), \ldots, (p_N, t_N))$ such that not all $p_i$ are the same or not all $t_i$ are the same. Without loss of generality, assume that $(p_1, t_1) \neq (p_2, t_2)$. But then, this implies that another $N$-tuple $((p_2, t_2), (p_1, t_1), (p_3, t_3), \ldots, (p_N, t_N))$ must also be a Nash equilibrium since all firms are identical, contradicting the uniqueness of the Nash equilibrium established in Proposition 4.

(b) Since the equilibrium is symmetric, i.e., $p_i = p^*$ and $t_i = t^*$ for all firm $i$, all $N$ firms have equal market share $1/N$ and $\lambda_i = \lambda/N$ for all $i$. Substituting this into the profit function (2) we obtain (8). Also, since the service constraint (3) must be tight at optimality for the firm $i$ problem, this implies that

$$t^* = \frac{k}{\mu_i - \lambda/N} = \frac{Nk}{N\mu_i - \lambda}.$$

From Proposition 1, when $p^*$ is an interior solution to firm $i$ problem, i.e., $p^* < \bar{p}$, $(p^*, t^*)$ must satisfy (5), i.e.,

$$\frac{-k\lambda t^*}{k + \lambda t^* - \mu_i t^*} - b(\mu_i t^* - k) + ka - \gamma_i ka \left\{\frac{k + \lambda t^* - \mu_i t^*}{(t^*)^b (N - 1)(p^*)^{-a}(t^*)^{-b}(\mu_i t^* - k)}\right\}^{-1/a} = 0.$$

Substituting (6) and (4) into the above equation and after simplification, we obtain the result given by (7).

(c) The results in Table 1 follow directly by inspecting the corresponding equations $t^*$, $p^*$ and $\Pi^*$ given in (6), (7) and (8), respectively. □

## References

Blackburn, J. D. 1991. *Time-Based Competition: The Next Battle Ground in American Manufacturing*. Richard D. Irwin, Inc.

Cooper, L. G., M. Nakanishi. 1988. *Market-Share Analysis*. Kluwer, Norwell, Massachusetts.

Dewan, S., H. Mendelson. 1990. User delay costs and internal pricing for a service facility. *Management Sci.* **36**(12) 1502–1517.

Hill, A., I. Khosla. 1992. Models for optimal lead time reduction. *Prod. Oper. Management* **1**(2) 185–197.

Ho, T., Y. S. Zheng. 1995. Setting customer expectation in service delivery: An integrated marketing-operations perspective. Working paper, Anderson Graduate School of Management, UCLA.

Kalai, E., M. I. Kamien, M. Rubinovitch. 1992. Optimal service speeds in a competitive environment. *Management Sci.* **38** 1154–1163.

Karmarkar, U. 1993. Manufacturing lead times, order release and capacity loading, Chapter 6. S. Graves, A. Rinnooy Kan and P. Zipkin (eds.) *Handbooks in Operations Research and Management Science, Vol. 4, Logistics of Production and Inventory*, North-Holland, Amsterdam.

Lederer, P. J., Li, L. 1997. Pricing production, scheduling, and delivery-time competition. *Oper. Res.* **45**(3) 407–420.

Li, L. 1992. The role of inventory in delivery-time competition. *Management Sci.* **38**(2) 182–197.

——, Y. S. Lee. 1994. Pricing and delivery-time performance in a competitive environment. *Management Sci.* **40**(5) 633–646.

Lilien, G. L., P. Kother, K. S. Moorthy. 1992. *Marketing Models*. Prentice-Hall, Englewood Cliffs, New Jersey.

Mendelson, H., S. Whang. 1990. Optimal incentive-compatible priority pricing for the $M/M/1$ queue. *Oper. Res.* **38** 870–883.

Palaka, K., S. Erlebacher, D. H. Kropp. 1997. Lead-time setting, capacity utilization, and pricing decisions under lead-time dependent demand. Working paper, Olin School of Business, Washington University.

Shanthikumar, J. G., U. Sumita. 1988. Approximations for the time spent in a dynamic job shop with applications to due date assignment. *Internat. J. Production Res.* **26** 1329–1352.

So, K. C., J. S. Song. 1998. Optimal pricing, leadtime and capacity expansion decisions. *European J. Oper. Res.* **111** 28–49.

Stalk, Jr., G., T. M. Hout. 1990. Competing Against Time: How Time-Based Competition is Reshaping Global Markets. Free Press.

Stidham, S. 1992. Pricing and capacity decisions for a service facility: Stability and multiple local optima. *Management Sci.* **38**(8) 1121–1139.

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