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Buy Now and Match Later: Impact of Posterior Price Matching on Profit with Strategic Consumers

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With a posterior price matching (PM) policy, a seller guarantees to reimburse the price difference to a consumer who buys a product before the seller marks it down. Such a policy has been widely adopted by retailers. We examine the impact of a posterior PM policy on consumers' purchasing behavior, a seller's pricing and inventory decisions, and their expected payoffs, assuming that the seller cannot credibly commit to a price path, but can implement a posterior PM policy. We find that the PM policy eliminates strategic consumers' waiting incentive and thus allows the seller to increase price in the regular selling season. When the fraction of strategic consumers is not too small and their valuation decline over time is neither too low nor too high, the PM policy can substantially improve the seller's profit, as well as the inventory investment. In such situations, the strategic consumers' waiting incentive and the loss if they wait are both high. However, to adopt this policy, the seller also bears the refund cost. The seller must either pay the refund that consumers will claim or forgo the salvage value of any leftover inventory. The PM policy can be detrimental when there are only a few strategic consumers or the strategic consumers' valuation decline is very low or very high. We find that the performance of this policy is insensitive to the proportion of consumers who claim the refund. From the consumers' perspective, the PM policy generally reduces consumer surplus; however, there are cases where consumer surplus can be increased, typically when the variance of the potential high-end market volume is high. As a result, a Pareto improvement on both the seller's and the consumers' payoffs is possible. Finally, we find that the ability to credibly commit to a fixed price path is not very valuable when the seller can implement price matching.

Key words: strategic consumers; inventory management; pricing; discounts; rational expectations equilibrium
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1. Introduction

According to a *Wall Street Journal* article (Merrick 2001), three decades ago, marked-down goods only accounted for about 8% of sales in the U.S. retail industry; however, as of 2001, the ratio had risen to 20%. This widespread phenomenon has trained many consumers to take advantage of markdowns in sophisticated ways. For instance, a recent market study by Best Buy, Inc. shows that as many as 20% of its shoppers are "undesirable" (McWilliams 2004). They may reason strategically the best time to buy, search for deals, and rush in at the last minute. Today consumers are aided by advanced tools, such as online deal forums hosted by FatWallet.com, DealSea.com, and PriceGrabber.com where they can easily retrieve any information on price deals. This gaming behavior erodes the benefit from markdowns for sellers.

The markdown-and-waiting game has recently received attention from both practitioners and researchers. The common mechanisms suggested to mitigate this problem include committing to a fixed price (or price path) and avoiding substantial markdowns (e.g., Aviv and Pazgal 2008, Su and Zhang 2008), creating a sense of scarcity with low inventory levels (e.g., Liu and van Ryzin 2008, Su and Zhang 2008, O'Donnell 2006) or adjusting in-store display formats (Yin et al. 2008) and matching the demand more accurately to reduce the chance of markdowns (Cachon and Swinney 2008).

In this paper, we focus on an alternative marketing mechanism—posterior price matching (PM) policies—for dealing with this problem. A *posterior PM policy* is a marketing policy offered by a seller to match the lower prices if the seller marks down within a specified time. Posterior PM policies may effectively change consumers' purchasing behavior. For instance, Arbatskaya et al. (2004) provide a representative example where Circuit City, Inc. encouraged a father to "go ahead, take it" and purchase a 27-inch television for his son "now, without waiting" by offering a PM guarantee. With the refund promise, a consumer may be induced to buy early.

PM policies started to proliferate in the early 1980s in the business of consumer electronics, auto supplies, and general discount stores and then spread to the manufacturing field for matching wholesale prices and the financial market for matching interest rates (see Edlin 1997 for detailed examples). It was difficult to claim a refund in the early years because consumers needed to check the prices and claim the refund in person. However, technological advances have paved the way for the adoption of PM policies. Consumers can now easily retrieve price information through sellers' online channels and service-call centers. Some third-party websites such as RefundPlease.com even provide free services to track the prices of the sellers where a consumer makes the purchase. Furthermore, the reimbursement process has become much easier, given that most transactions are processed electronically. With these technological advances, PM policies have become more tractable and credible. We can easily find various PM policies in retailing and other industries, such as the policy offered by Staples, Inc. for office products, the policy offered by Best Buy, Inc. for electronic products, the policy offered by Gap, Inc. for fashion (apparel) products, and the policy offered by Priceline.com, Inc. for travel tickets.

Although PM policies have been addressed in previous research, the focus was mainly on competition, i.e., the impact of such policies on consumers' store choices. This is *concurrent* price matching, to match the verified competitors' lower prices for consumers at the time of the purchase. In the U.S. retail industry, there are retailers that offer concurrent and posterior price matching together, such as Circuit City, Inc.; there are also retailers that only offer the posterior price matching, such as Gap, Inc. In contrast, in this paper, we study one particular type of PM policy—posterior PM—within the markdown-and-waiting game. We also consider two typical concerns in operations management, i.e., uncertainty about the size of the market and long lead times for replenishment. These concerns necessitate inventory investment before the start of the season that will impact the markdown-and-waiting game. We set out the questions of how such a policy will influence consumers' purchasing behavior; how it will influence a seller's price and inventory decisions; and how it will impact the players' payoffs. We investigate these questions using a model where, in the presence of strategic consumers, a monopolist seller sells a product in two periods that correspond to the regular selling season and the season for potential markdown. In our model, the market can be divided into high- and low-end consumers in terms of their valuations for the product. The seller targets the high-end market, but its volume is uncertain when he invests one-time inventory in the first period. The seller may mark down in the second period if the realized sales in the first period do not deplete the inventory. A proportion of the high-end consumers are strategic and may wait to buy later; moreover, their valuation declines over time. Our base model assumes that the seller cannot preannounce any price path or inventory level but can credibly commit to a posterior PM policy in which he promises to refund the price difference if he marks down. We assume the strategic consumers and some of the myopic consumers will claim the refund.

This paper generates several managerial insights. First, we show that the posterior PM policy eliminates consumers' waiting incentive and thus allows the seller to increase the price in the regular selling season. Without this policy, the seller may need to set a low price to induce consumers to buy early. In contrast, the seller will always charge a premium price after adopting this policy. Second, we find that the posterior PM policy can substantially increase the seller's profit when the fraction of strategic consumers is not too small and their valuation decline over time is neither too low nor too high. In such situations, the strategic consumers' waiting incentive and the loss if they wait are both high. However, if there are only a few strategic consumers in the market or their valuation declines too much or too little over time, this posterior PM policy may hurt the seller. The performance of this policy is insensitive to the proportion of consumers who claim the refund.

Third, even though the consumer surplus in general decreases if the seller adopts the posterior PM policy, we show cases where this policy can also increase consumer surplus, typically when the variance of the volume of high-end consumers is high. Therefore, a Pareto improvement on both parties' payoffs is possible. Finally, we find that the ability to credibly commit to a fixed price path is not very valuable when the seller can implement price matching. When the fraction of strategic consumers is high, committing to a constant high price is the best commitment strategy, but it yields lower profits than no price commitment but with price matching. When the fraction of strategic consumers is low, committing to a markdown at the end of the season is the best commitment strategy, but no commitment and no price matching yields more profits.

The remainder of the paper is organized as follows. We review relevant literature in §2. Section 3 presents the model. We analyze the model without and with the posterior PM policy in §§4 and 5, respectively. We report in §6 our numerical studies and the key observations, and we conclude in §7.

2. Literature Review

Our paper relates to two bodies of research: the literature on PM policies and the literature on consumer behavior.

The research on PM policies has mainly been conducted in economics and marketing, addressing the impact of PM policies on competition. The literature was initiated by Salop (1986), Holt and Scheffman (1987), and Png and Hirshleifer (1987). They show that PM policies can mitigate the competition between sellers. Without a PM policy, competing sellers may lower their prices to compete for consumers. However, once PM policies are adopted, they are less afraid of losing their consumers, as they can match the competitors' prices. Although some literature (e.g., Corts 1996, Chen et al. 2001) that shows PM policies can also worsen sellers' profits if there are different segments of consumers with different search costs and PM hassle costs, a common understanding is that PM policies soften competition, as extensively discussed in Simons (1989), Sargent (1993), and Edlin (1997).

The above literature neither addresses posterior PM nor considers demand uncertainty. There are several papers that investigate posterior PM policies. Levin et al. (2007) study a revenue-management problem in which a monopolist seller can sell a price guarantee along with the product. A consumer who purchases the price guarantee with the product will receive a refund equal to the difference between the "strike" of the price guarantee and the lowest price at which the seller sells the product before the price guarantee expires if the latter is lower. Levin et al. explore the optimal decisions on the price path and the price guarantee policy (the fee and strike). However, their model does not consider consumers to be strategic but specifies an exogenously given consumer choice model. Moreover, their model assumes a given quantity of inventory, whereas the inventory decision is one of the interesting issues we measure. Butz (1990) studies the impact of a posterior PM policy on a durable-goods seller who produces and sells in an infinite horizon without capacity constraint. In his model, the seller offers a PM guarantee to buyers with a prespecified applicable duration. Butz investigates the seller's price decisions, but, by assuming exogenous demand functions, ignores strategic consumer behavior.

Png (1991) and Xu (2008) study posterior PM policies with forward-looking consumers. They both assume two segments of consumers with high and low valuations. Png (1991) considers a scenario in which a monopolist seller sells a quantity of capacity in two periods. In his model, the market has a fixed volume but the division of the two segments is ex ante random. Png examines the seller's price decision and profit with and without a posterior PM policy. However, Png assumes that the capacity is exogenous and common knowledge to consumers. Besides, Png does not consider the effect of consumers' valuation decline or the proportion of strategic consumers in the population, by assuming that the valuations are fixed and all consumers are strategic. Xu (2008) examines the selling-purchasing equilibrium in continuous time. In her model, Xu assumes that consumers' valuations decline once, simultaneously, but the time when they drop is stochastic. Xu explores the optimal design, the duration and refund ratio, of a posterior PM policy. However, by assuming a fixed number of consumers, Xu does not consider demand uncertainty or the inventory-investment decision.

Three main aspects characterize the difference between our paper and the existing PM literature: We study a posterior PM policy; we consider forward-looking consumers whose volume is random and whose valuation declines over time; and we address the inventory-investment decision in addition to the pricing issue. From the last two aspects, our paper is closely related to the recent literature in operations management studying consumer behavior. This stream of literature mainly explores two questions: How does consumer behavior affects a seller's operations (i.e., pricing and inventory decisions) and his performance? How can some mechanisms alleviate the negative impacts of consumer gaming?

Aviv and Pazgal (2008) address a revenue-management problem with strategic consumers. In their model, consumers' valuation continuously declines in time. In response to this, the seller may mark down at some given time during the selling horizon. However, strategic consumers aware of this potential may wait for that time. Aviv and Pazgal investigate the seller's pricing decision and performance in two main scenarios where the seller (a) does not commit to any price path and (b) does commit to a preannounced price path. They find situations in which the seller may earn higher profits with a preannounced path, in the presence of strategic consumers. However, their paper assumes both an exogenous inventory level and that a price path can be credibly committed. Similarly, Elmaghraby et al. (2008) and Su (2007, 2008) study dynamic pricing with strategic consumer behavior based on the revenue-management framework. Elmaghraby et al. (2008) address the optimal stream of markdown prices in the environment where the consumers may demand multiple units at each price step, and Su (2007, 2008) addresses the dynamic pricing problem by segmenting consumers into different classes according to their valuations, waiting costs, and holding costs. Yin et al. (2008) study the in-store display formats with preannounced price schedules. In their model, consumers arrive in a Poisson process and the seller can display the product one unit at a time during the regular season. This display format creates a sense of scarcity and reduces the consumers' expectations of inventory availability.

The research closest to ours is in studies by Su and Zhang (2008), Liu and van Ryzin (2008), and Cachon and Swinney (2008), based on a newsvendor-like framework. In Su and Zhang (2008), the consumers have waiting incentive in the regular selling season, expecting that the newsvendor will salvage leftover inventory. Su and Zhang (2008) show that in equilibrium, the newsvendor will invest in less inventory and charge a lower regular sales price. The newsvendor's performance is substantially affected by the consumers' waiting behavior. To alleviate this impact, Su and Zhang study two mechanisms—quantity commitment and price commitment—embedded in supply chain management. They show that those commitments can be achieved by various supply contracts (e.g., wholesale price, markdown money, and buy-back) that can effectively improve the newsvendor's and the supply chain's performance. Liu and van Ryzin (2008) also study a situation in which the seller can commit to a preannounced price path and fill rate. They provide conditions under which it is optimal to create shortages by understocking products. Cachon and Swinney (2008) study another mechanism: the quick response to mitigate consumers' waiting behavior. With the ability of quick response, the newsvendor can procure less initially but replenish quickly if the demand is strong. This makes the supply match the demand more accurately, reduces the chance of markdown, and thus induces consumers to buy early. In contrast, our work considers the case where the newsvendor is not able to preannounce a price path or a fill rate or apply quick response. We study a marketing instrument—price matching—that provides an alternative approach to deal with consumers' waiting behavior.

3. Problem Description

We consider a monopolist seller who sells a product to potential consumers in two periods, labeled $t \in \{1, 2\}$, corresponding to a regular selling season and a season for potential markdown. We use the notations "PM" and "NP" to indicate association, where necessary, with the price-matching model and the no-price-matching model. Below, we explain the players' decisions in detail.

3.1. Posterior PM Policy

The PM policy, if offered, is a 100% difference refund policy triggered by consumers. The seller will refund a consumer the difference between what the consumer paid and the latest price if the latter is lower, but the consumer must submit the claim for the refund. Such a policy is widely applied in practice (see, for example, the policies offered by Circuit City, Inc., Staples, Inc., Best Buy, Inc., and Gap, Inc.). The decision on the PM policy $\vartheta = \{NP, PM\}$ needs to be made before the selling season. Once it is offered, the seller cannot revoke it. We assume that there is no hassle cost to process the refund claim.

3.2. Market

Potential consumers can be divided into two groups in terms of their valuations for the product: high- and low-end consumers. The high (low)-end consumers have a valuation V_H (V_L), where $V_L < V_H$. We consider a large volume of potential high-end consumers such that an individual consumer has a negligible mass compared with the total volume. We indicate the total volume of high-end consumers by means of λ, which is a continuous variable in $\mathbb{R}^+$. With a slight abuse of notation, λ denotes a stochastic variable with a continuous distribution $F(\lambda)$ (density $f(\lambda)$) having mean μ and standard deviation σ. λ is realized during the first period.

The valuation of the high-end consumers, V_H in the first period, declines to V_h in the second period, where $V_h \in [V_L, V_H]$. The difference $V_H - V_h$ is the value loss to the high-end consumers caused by delayed consumption of the product. This could correspond to the loss of utility in the first period, or, to the reduced appeal of "not being among the first consumers." This setting is appropriate for fashionable and seasonal items. To keep the analysis simple, we assume that the value depreciation is the same for all high-end consumers. See, for example, Cachon and Swinney (2008) or Aviv and Pazgal (2008) for the analysis with a heterogeneous value decrease. We assume that the low-end consumer's valuation V_L, which is already low, remains constant over the two periods. Because the number of the low-end consumers is generally much larger than that of high-end consumers, we assume that the number of low-end consumers is infinite. Furthermore, we assume that high-end consumers always have a higher demand to obtain the product than the low-end consumers, because the chance to obtain a product usually will be higher if the consumer is more eager and arrives much earlier. This assumption only plays a role when we introduce the second-period expected utility for the high-end consumer. Su and Zhang (2008) and Cachon and Swinney (2008) make similar assumptions.

3.3. Seller

The seller sets the first-period price p_1 and then invests in inventory Q before the market volume λ is realized. Unlike Cachon and Swinney (2008), we assume that there is no chance for the seller to replenish inventory during the selling season. The unit acquisition cost is c and satisfies $V_L < c < V_H$. Sales arise after setting the first-period price and the inventory level. At the beginning of the second period, the seller sets the price p_2. The seller incurs a holding cost, $h = \rho c$, per unit of inventory carried over from the first period to the second period, where $\rho \in [0, 1]$. To simplify the problem, the holding cost is incurred only once on all the leftover inventory after the first-period sales. If inventory remains after the second period, it can be disposed at zero cost.

3.4. Purchasing Behavior

We introduce the following heterogeneity with respect to the purchasing behavior of the high-end consumers: A fraction, ϕ, of the high-end consumers are strategic; they can decide to delay their purchasing decision to the second period. We denote the delaying decision by the (mixing) probability $q \in [0, 1]$. Then $1 - q$ is the probability that the strategic consumer decides to buy in the first period. Strategic consumers make their purchasing decisions, to buy early or wait, based on the associated payoffs which are determined by the utility for the product (V_H in the first period and V_h in the second period), the price in the first period, the expectation of the second-period price, and the availability of the product. When strategic consumers are indifferent between buying in the first or second period, they may randomize ($q \in (0, 1)$); otherwise, if delaying their purchasing decision yields strictly more (less) utility in expectation, they delay with probability $q = 1$ (0). As all strategic high-end consumers are identical, per force, the waiting decision, q, will be the same. When all strategic consumers decide q, as the total volume of high-end consumers is λ and each consumer is infinitesimally small, the volume of strategic high-end consumers that wait follows $\lambda q \phi$ (based on the law of large numbers).

The remaining high-end consumers $1 - \phi$ are myopic: They purchase the product immediately in the first period. The myopic consumers are the ones who do not consider any future utility when making a purchasing decision in the first period. Hence, their purchasing decision takes place in the first period. However, if the PM policy is offered, there is a fraction, $0 \le \gamma \le 1$, of the myopic consumers that may claim the refund in the second period. This consumer segment does not behave strategically when making a purchasing decision, but they will behave opportunistically when a refund is offered. Hence, this market segment needs to be taken into account for the analysis. Table 1 shows the segmentation of the high-end consumers.

As $c > V_L$, it is obvious that the seller will not set the first-period price below V_L. As a result, the low-end consumers only have a chance to purchase the product in the second period. Because the sale ends in the second period, the low-end consumers purchase the product immediately if it is available and the price p_2 is no higher than their valuation, V_L.

3.5. Information Structure

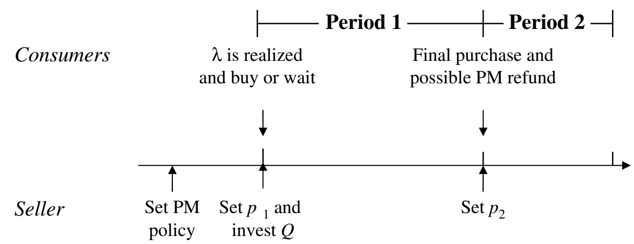
The seller's PM and pricing decisions, ϑ, p_1, and p_2, are observable to consumers, but the inventory level Q is not observable (assuming that the inventory is observable to the consumers would not substantially change the insights obtained in this paper; for a more detailed discussion, see Appendix E (all appendixes available online)). A strategic consumer's waiting strategy, q, is observable neither to the seller nor to the other consumers. The realization of λ is also unobservable to all players during both periods, but the seller can observe the realized sales, s, in the first period. All the other parameters and functions are common knowledge.

Table 1 Segments of High-End Consumers in Terms of Purchasing Behavior

Segment	Claim Refund	
Strategic (ϕ)	Yes	
Myopic ($1-\phi$)	Yes (γ)	No ($1-\gamma$)

Note. Fractions are shown in parentheses.

Figure 1 Timeline of Game



3.6. Formulation of Game

The timeline of the events is described in Figure 1. Following this timeline, we formulate the game in four stages. In the first stage, the seller makes the decision ϑ whether to introduce the PM policy. In the second stage, the seller sets the first-period price, p_1, and invests in inventory Q. Then the high-end market volume λ is realized. Strategic consumers who observe ϑ and p_1 (not Q or λ) simultaneously decide their waiting strategy q. In the third stage, the seller sets the second-period price, p_2, after observing the first-period sales, s. In the last stage, given p_2, all existing consumers who have not made the purchase in the first period decide to buy or leave. The strategic consumers and part of the myopic consumers who have made the purchase in the first period claim the refund if it is available.

3.7. Equilibrium Conditions

In Table 2, we show for all players (strategic consumers and the seller) decisions, information sets, corresponding beliefs, and payoffs. Given that neither the high-end market volume, λ, nor the consumers' waiting strategy, q, is observable, the seller makes his first-period decisions based on the distribution $f(\lambda)$ and his belief, q_s, of the strategic consumer's waiting strategy. The seller's second-period decision is made given his first-period decisions, his belief q_s, and the realized sales s. Similarly, as a consumer cannot know the market volume λ, the other consumers' strategy q,

Table 2 Seller's and Strategic Consumers' Decisions, Information Sets, and Beliefs in Periods 1 and 2 (for Both Subgames, With and Without Price Matching)

	Seller				Strategic consumer			
Period	Action	Information set	Belief	Utility	Action	Information set	Belief	Utility
1st	(ϑ, p_1, Q)	$\varnothing$	q_s	π, Π	q	(ϑ, p_1)	(Q_c, q_c)	u_1, u_2
2nd	p_2	(s, ϑ, p_1, Q)	q_s	R_2				

or the inventory level Q, she makes her decision q based on the retrievable information set (ϑ, p_1) combined with her belief (Q_c, q_c) of the inventory and the other strategic consumers' waiting strategy. The beliefs and information sets determine the seller's and the consumers' payoffs. We impose the following conditions for the equilibrium (we use "^{o}" to denote the equilibrium actions and beliefs).

Definition 1. We have an equilibrium under the following conditions.

1. Rationality of the seller's second-period price: $p_2^o \in \arg\max_{p_2} R_2(p_2; \vartheta, p_1, Q, q_s, s)$, where R_2 is the seller's second-period profit.

2. Rational-expectations equilibrium:

a. $q^o \in \arg\max_q \{(1-q)u_1(\vartheta, p_1, Q_c, q_c, p_2^o) + qu_2(\vartheta, p_1, Q_c, q_c, p_2^o)\}$, where u_t is the expected utility of purchasing in period $t \in \{1, 2\}$ with p_2^o satisfying 1.

b. $Q^o \in \arg\max_Q \pi(Q; \vartheta, p_1, q_s, p_2^o)$, where π is the seller's two-period profit with p_2^o satisfying 1.

c. $q_s = q_c = q^o$ and $Q_c = Q^o$.

3. Rationality of the seller's first-period price: $p_1^o(\vartheta) \in \arg\max_{p_1} \Pi(p_1; \vartheta)$, where $\Pi(p_1; \vartheta)$ is equal to $\pi(Q^o; \vartheta, p_1, q^o, p_2^o)$, with (Q^o, q^o, p_2^o) satisfying conditions 1 and 2. Rationality of the seller's PM policy: $\vartheta^o = PM \Leftrightarrow \Pi_{PM} \geq \Pi_{NP}$; otherwise, $\vartheta^o = NP$, where $\Pi_{PM} = \Pi(p_1^o(PM); PM)$ and $\Pi_{NP} = \Pi(p_1^o(NP); NP)$.

The second-period price is rational when it maximizes the second-period profit, given the information set and the seller's belief about the strategic consumers' waiting strategy (Definition 1.1). The strategic consumers' waiting strategy and the seller's inventory investment form a rational-expectations equilibrium if any strategic consumer's waiting decision maximizes her expected utility or randomizes when indifferent, given her information set and belief about the seller's inventory level and all the other consumers' purchasing strategy, assuming rationality of the seller's second-period price (Definition 1.2a). The seller's inventory investment belongs to the set of inventory levels that maximize the seller's total profit, given his decisions on the PM policy and first-period price and his belief of the consumers' decisions, assuming rationality of his own second-period price (Definition 1.2b). The seller's belief, q_s, and any strategic consumer's belief q_c about the waiting decision are all the same and equal to the actual waiting decision, $q_s = q_c = q^o$, and the consumers' belief of the inventory is equal to the actually invested inventory: $Q_c = Q^o$ (Definition 1.2c). The outcomes must be consistent with the beliefs. The rational-expectations equilibrium concept (Muth 1961) has been applied in operational settings by, for example, Besanko and Winston (1990), Su and Zhang (2008), and Cachon and Swinney (2008). In a rational-expectations equilibrium, the equilibrium outcome is equal to the players' beliefs about the probability of delay. Finally, the first-period price and the decision on the PM policy maximize the expected profit based on the subgame equilibrium outcomes (Definition 1.3).

3.8. Technical Assumption

We make the following technical assumption, which provides a sufficient (though not necessary) condition to ensure the existence of a unique equilibrium. We assume that $F(\lambda)$ satisfies the following property.

Definition 2. A continuous nonnegative random variable X with density $f(x)$ satisfies the *restricted monotone scaled likelihood ratio* (restricted MSLR) if, for all $0 \leq \kappa \leq 1$ and x in the support of X, (a) $f(\kappa x)/f(x)$ is monotonic in x, and (b) $xf'(\kappa x)/f(x)$ is monotonic in x when $xf'(\kappa x)/f(x)$ is negative.

The assumption that $F(\lambda)$ is restricted MSLR will allow us later to obtain the monotonicity properties of the equilibrium. Otherwise, there is no straightforward interpretation of the restriction on the MSLR. We show, in Appendix A, that most of the common distribution functions, including uniform, exponential, Weibull, power, gamma, chi-squared, chi, and beta, satisfy this property. When the density function is increasing, the restricted MSLR coincides with the MSLR property defined by Cachon and Swinney (2008).

4. Model Without Posterior PM

This section characterizes the players' optimal decisions in the presence of strategic consumers but without the posterior PM policy, $\vartheta = NP$. We proceed by backward induction, analyzing the seller's second-period price, the strategic consumers' purchasing strategy, the seller's inventory decision, and the seller's first-period price in equilibrium.

4.1. Second-Period Price

Because the consumers will purchase the product in the second period if and only if p_2 is no higher than their valuations, the only rational prices for the seller are V_h and V_L: Sell the product only to the strategic consumers or mark down further and clear the inventory. (In fact, the seller shall set $p_2 = V_h - \varepsilon$ (or $V_L - \varepsilon$), where ε is strictly positive, but arbitrarily small. For sake of notational convenience, we will drop the ε, unless strictly required.) Given the first-period sales, s, if there is no inventory left (i.e., $s = Q$), the second-period price becomes irrelevant. If the sales are less than the inventory (i.e., $s < Q$), the seller can infer the volume of high-end consumers as follows. Recall that q_s denotes the seller's belief about the strategic consumers' waiting strategy and q denotes the consumers' actual action. Let $\beta(q) \doteq 1 - q\phi$ denote the fraction of high-end consumers who buy in the first period when their strategy is q. Then the first-period sales, s, are equal to $\beta(q)\lambda$. When observing $s < Q$, the seller's belief about the realized high-end market volume is $s/\beta(q_s)$ and, in his belief, the volume of strategic consumers who wait for the second period is $q_s\phi(s/\beta(q_s))$. We can now write the seller's second-period revenue according to these two cases as follows.

$$R_2 = \begin{cases} V_L(Q-s) & \text{if } p_2 = V_L, \\ V_h \min\left\{q_s\phi\dfrac{s}{\beta(q_s)}, (Q-s)\right\} & \text{if } p_2 = V_h. \end{cases} \tag{1}$$

For notational convenience here, and for the remainder of the paper, we do not show the dependency of R_2 on the intermediate or independent variables, except where it is strictly required.

The seller can compare the two revenues to set the second-period price. It is now easy to determine the rational price given the initial inventory, the sales, and the belief (see Definition 1.1):

Proposition 1. *Let* $\alpha(q) \doteq V_L/(V_L + q\phi(V_h - V_L))$. *Given* Q, s, *and* q_s, *the seller's second-period price follows*:

$$p_2^o = \begin{cases} V_L & \text{if } \dfrac{s}{\beta(q_s)} < \alpha(q_s)Q, \\ V_h & \text{if } \alpha(q_s)Q \le \dfrac{s}{\beta(q_s)} < \dfrac{Q}{\beta(q_s)}, \\ n/a & \text{if } s = Q. \end{cases} \tag{2}$$

Proof. All the proofs are provided in Appendix B. □

Proposition 1 reflects that the seller's incentive to mark down decreases in the volume of the high-end consumers who are present in the same period based on his belief. This is intuitive: When there are many high-end consumers, setting a high price is rational. Note that $\alpha(q) \le 1$ and $1/\beta(q) \ge 1$ for any $q \in [0, 1]$.

4.2. Purchasing and Inventory Equilibria

4.2.1. Purchasing Equilibrium. Suppose the strategic consumers hold a belief that the inventory is Q_c. To derive the purchasing equilibrium, we consider the strategy of one individual strategic consumer (the *focal* consumer) holding the belief that the other strategic consumers' decision is q_c. Note that when the focal consumer makes her decision, she needs to consider the seller's belief q_s of the strategic consumers' waiting decision. However, in equilibrium, we will impose (see Definition 1.2c) that $q_s = q_c = q^o$, and hence, any focal consumer's belief q_c of the other strategic consumers' decision will be the same as the seller's belief q_s.

The focal consumer's strategy q can be derived by comparing the two expected utilities of buying immediately and waiting for the second period. The expected utility of making an immediate purchasing decision is

$$u_1 = \left[F\left(\frac{Q_c}{\beta(q_c)}\right) + \int_{Q_c/\beta(q_c)}^{\infty} \frac{Q_c}{\beta(q_c)\lambda} f(\lambda)\,\mathrm{d}\lambda\right](V_H - p_1), \tag{3}$$

and the expected utility of delaying the purchasing decision is

$$u_2 = F(\alpha(q_c)Q_c)(V_h - V_L). \tag{4}$$

We explain these utilities as follows. First, consider the expected utility of purchasing in the first period. Based on the belief that the other strategic consumers' purchasing strategy follows q_c, the sum of the two terms in the bracket in Equation (3) is the probability that the focal consumer can obtain the product if she makes the purchase immediately: If the realization of the high-end market volume is low with $\beta(q_c)\lambda \leq Q_c$, she can obtain a unit for sure; if the realization is high with $\beta(q_c)\lambda > Q_c$, she will be rationed with a probability $Q_c/\beta(q_c)\lambda$ to obtain a unit. In either case, the surplus of buying the product is $V_H - p_1$.

Second, consider the expected utility of purchasing in the second period with the same belief. With q_c, the focal consumer makes a conjecture about the second-period price characterized by $\alpha(q_c)$. The consumer can obtain a positive utility only if $\lambda < \alpha(q_c)Q_c$. This is because if the realized volume of the high-end market is large such that the first-period sales $\beta(q_c)\lambda$ exceed the initial inventory (i.e., $\beta(q_c)\lambda \geq Q_c$), the product is sold out in the first period; if there is leftover inventory, but not a lot, $\alpha(q_c)Q_c \leq \lambda < Q_c/\beta(q_c)$, the seller will not mark down. In that case, the second-period price is equal to V_h, leaving no surplus to the strategic consumer. Only if there is a lot of leftover inventory, $\lambda < \alpha(q_c)Q_c$, will it be marked down. As the markdown price is V_L, and, by assumption (see §3) the high-end consumers always have priority over the low-end consumers, high-end consumers will have surplus $V_h - V_L$ when $\lambda \leq \alpha(q_c)Q_c$.

The rational decision of the focal consumer is $q = 0$ (1)—that is, buy in the first (second) period—if $u_1 > (<) u_2$ for any value q_c in $[0, 1]$. Then, with Definition 1.2a, the equilibrium strategy is $q^o = 0$ (1). If there exists some $q_E \in [0, 1]$ such that $u_1 = u_2$ when $q_c = q_E$, then any randomization strategy $q \in [0, 1]$ is rational for the focal consumer and hence, the purchasing equilibrium is reached at $q^o = q_E$.

From Equation (4), we can easily see that u_2 strictly decreases in q_c. When q_c increases, $\alpha(q_c)$ decreases; the probability that the seller will mark down decreases. This indicates that the other strategic consumers' waiting strategies exert a negative externality on the focal consumer because of the seller's markdown incentive in the second period: The more strategic consumers wait to purchase until the second period, the lower the incentive is for the seller to mark down and hence, the lower the focal consumer's expected utility. In contrast, we can verify from Equation (3) that u_1 increases in q_c: If more consumers choose to wait, then the chance of obtaining the product in the first period increases. However, u_1 may not be strictly increasing, because if $p_1 = V_H$, then $u_1 = 0$ always. Therefore, the equilibrium purchasing decision of the strategic consumers (see Definition 1.2a) follows.

Lemma 1. *Given p_1 and Q_c, there exists a unique purchasing equilibrium. The equilibrium waiting strategy q^o is*

$$q^o = \begin{cases} 0 & \text{if } u_1 > u_2 \text{ for any } q_c \in [0, 1], \\ q_E & \text{if there exists a } q_E \in [0, 1] \\ & \text{such that } u_1 = u_2 \text{ when } q_c = q_E, \\ 1 & \text{if } u_1 < u_2 \text{ for any } q_c \in [0, 1]. \end{cases} \tag{5}$$

Given that u_1 increases in q_c, but u_2 decreases strictly in q_c, if there exists a $q_E \in [0, 1]$ such that $u_1 = u_2$ holds when $q_c = q_E$, it is unique; otherwise, the equilibrium is reached on the boundaries, either 0 or 1.

4.2.2. Inventory Decision. Lemma 1 shows that for any given Q_c, there is a unique purchasing equilibrium q^o. To determine the seller's rational inventory investment, we take the consumers' belief Q_c and consequently q^o as given (Definition 1.2b) and optimize the seller's inventory decision Q:

$$\begin{aligned} \max_Q & \int_{Q/\beta(q^o)}^{\infty} p_1 Q f(\lambda)\,\mathrm{d}\lambda \\ & + \int_Q^{Q/\beta(q^o)} [p_1\beta(q^o)\lambda + V_h(Q - \beta(q^o)\lambda)] f(\lambda)\,\mathrm{d}\lambda \\ & + \int_{\alpha(q^o)Q}^{Q} [p_1\beta(q^o)\lambda + V_h(1 - \beta(q^o))\lambda] f(\lambda)\,\mathrm{d}\lambda \\ & + \int_0^{\alpha(q^o)Q} [p_1\beta(q^o)\lambda + V_L(Q - \beta(q^o)\lambda)] f(\lambda)\,\mathrm{d}\lambda \\ & - h\int_0^{Q/\beta(q^o)} (Q - \beta(q^o)\lambda) f(\lambda)\,\mathrm{d}\lambda - cQ. \end{aligned} \tag{6}$$

In Equation (6), the first term represents the revenue when $\lambda \geq Q/\beta(q^o)$ and all the inventory is cleared in the first period. The second term captures the revenue when $Q \le \lambda < Q/\beta(q^o)$ and some of the strategic consumers wait for the second period and the leftover inventory is cleared at price V_h in the second period. The third term is the revenue when $\alpha(q^o)Q \le \lambda < Q$ and some of the strategic consumers wait for the second period, but only part of the leftover inventory is sold at price V_h in the second period. The fourth term represents the revenue if $0 \le \lambda < \alpha(q^o)Q$: some of the strategic consumers wait for the second period and the leftover inventory is cleared at price V_L in the second period. The fifth term is the holding cost, and the last term is the inventory-investment cost.

Solving this optimization problem, we have the following lemma:

Lemma 2. *Given $V_h - h \le p_1$ and q^o, the seller's profit function is unimodal in Q, and there is a unique optimal inventory decision, Q^o, that can be found from*

$$p_1 - c - (p_1 + h - V_h)F\left(\frac{Q}{\beta(q^o)}\right) - V_hF(Q) + V_LF(\alpha(q^o)Q) = 0. \tag{7}$$

The condition $p_1 \ge V_h - h$ in Lemma 2 is sufficient but not necessary to ensure the unimodality of the seller's profit function. In our numerical analysis with a gamma distribution (discussed in §6), we found that a unique solution of Equation (7) exists for cases where $p_1 < V_h - h$. Furthermore, this condition is not restrictive. In our numerical examples, the optimal first-period prices found are all higher than $V_h - h$. Intuitively, $V_h - h$ is the net revenue that the seller can always obtain from strategic consumers even if they wait (to charge V_h in the second period). Therefore, to charge a p_1 less than $V_h - h$ is rarely optimal. A rational-expectations equilibrium is reached if Q^o solved from the seller's inventory-optimization program matches the consumers' belief Q_c: $Q_c = Q^o$ (Definition 1.2c). (Note that in Lemma 2, q^o is a function of Q_c. Q^o solved from Equation (7) is a function of q^o. As a result, we impose $Q_c = Q^o(q^o(Q_c))$ for the equilibrium.) The following proposition shows the existence of a unique rational-expectations equilibrium. This result is obtained based on the sufficient condition that the probability distribution function $F(\lambda)$ is restricted MSLR, which ensures the monotonicity of the first-order condition (i.e., Equation (7)).

Proposition 2. *For any $V_h - h \le p_1$, there exists a unique equilibrium (Q^o, q^o) that satisfies Equations* (5) *and* (7) *simultaneously.*

It is worth noting that both $\alpha(q^o)$ and $\beta(q^o)$ in Equations (5) and (7) are functions of the product $q^o\phi$, which can be interpreted as the fraction of the high-end consumers who wait among the whole high-end population. Therefore, given p_1, we can construct a one-to-one mapping between Q^o and $q^o\phi$. No matter how q^o and ϕ change, if their product, $q^o\phi$, remains the same, then Q^o remains the same. Consequently, if $q^o\phi$ is fixed, the seller's expected profit is fixed, which can be easily found from Equation (6). This is also clear if we think about it conversely. Once Q^o is given, the product, $q^o\phi$, is determined. Then, given a ϕ, we can derive q^o, and vice versa. This property will be useful when we characterize the seller's optimal pricing decision and his expected profit.

4.3. First-Period Price and Expected Profit

Now we need to determine the first-period price in equilibrium (see Definition 1.3). We discuss several properties of the optimal p_1^o. The following lemma holds.

Lemma 3. *Given a set of parameters, there exists a pair, $\underline{z}$ and $\bar{z}$, where $V_L \le \underline{z} \le \bar{z} \le V_H$, such that*

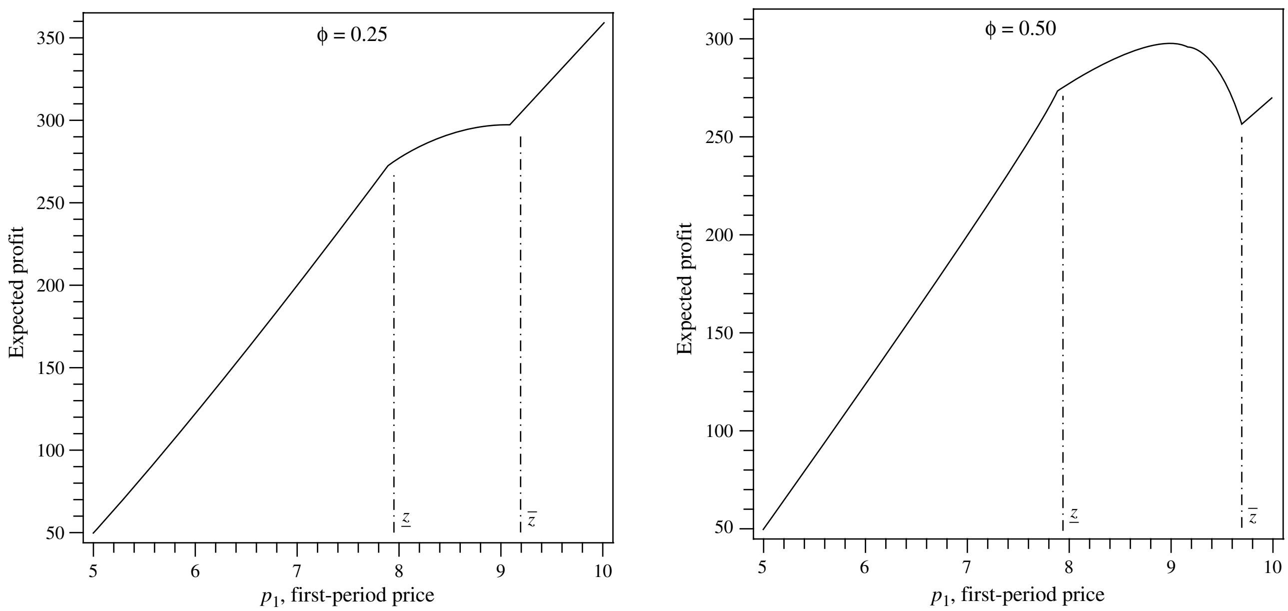
(i) $q^o = 0$ when $p_1 \in [V_L, \underline{z}]$ and $q^o = 1$ when $p_1 \in [\bar{z}, V_H]$

(ii) $\mathrm{d}\Pi(p_1; NP)/\mathrm{d}p_1 > 0$ when $p_1 \in [V_L, \underline{z}] \cup [\bar{z}, V_H]$

(iii) The optimal first-period price, p_1^o, equals either V_H or some point in the region $[\underline{z}, \bar{z})$.

Lemma 3 indicates that there is a low-price region where the strategic consumers will buy early, whereas if the first-period price falls into a high region, then all of them will wait. Given that the strategic consumers all buy early or wait, the seller's profit will increase with the first-period price (as long as it is within those two regions) because he can obtain more revenue from the sales with a higher price. However, when p_1 is intermediate (above the low-price region but below the high-price region), some of the strategic consumers wait while some of them buy immediately, complicating the situation. When p_1 increases, on the one hand, the margin from the first-period buyers increases, which results with a positive factor. On the other hand, q^o may increase (i.e., more

Figure 2 **Expected Profit, $\Pi(p_1; NP)$, Without PM Policy as a Function of p_1**



Note. λ follows a gamma distribution with $\mu = 100$ and $\sigma = 50$, $V_H = 10$, $V_L = 2$, $V_h = 5$, $c = 4$, $\rho = 0.04$, and $\gamma = 0.25$.

consumers may wait), giving a negative factor. The first-order derivative of the seller's profit function can be positive as well as negative when p_1 falls into the intermediate region. Figure 2 illustrates possible patterns of $\Pi(p_1; NP)$. We observe that $\Pi(p_1; NP)$ can increase or decrease in p_1 when p_1 is intermediate. As a result, to find the optimal first-period price, we need to search the region $[\underline{z}, \bar{z})$ and compare the maximum profit that the seller can achieve for a price within this region with the one under V_H.

Based on Lemma 3, we obtain Proposition 3.

Proposition 3. *There exists a threshold $\hat{\phi} \in [0, 1]$ such that if the fraction of strategic consumers satisfies $\phi \in [0, \hat{\phi})$, then $p_1^o = V_H$ is optimal for the seller; if $\phi \in [\hat{\phi}, 1]$, then there is a constant $p_I < V_H$ and $p_1^o = p_I$ is optimal for the seller.*

We interpret Proposition 3 as follows. When the volume of strategic consumers in the population is small, their waiting behavior will only have a minor impact on the seller's profit. The benefit from inducing them to buy early by lowering the price may not be sufficient to cover the loss of the margin from sales to myopic consumers. Charging the premium price V_H will be generally optimal when ϕ is small. However, under this premium price, (V_H), all strategic consumers will wait. This is because in buying early they obtain zero utility (see Equation (3)). However, if they wait, they can expect a positive utility because there always exists some probability that the seller will mark down in the second period (see Equation (4)). As ϕ increases, more and more consumers wait under the premium price, while their valuation declines over time. Therefore, there will be some threshold level, $\hat{\phi}$, at which letting the strategic consumers wait stops being optimal. It is now more beneficial for the seller to charge an interior price, $p_1^o < V_H$, to induce some consumers to buy early. The seller can find the corresponding subgame equilibrium (Q^o, q^o) at $\hat{\phi}$ with $q^o < 1$. Now, if the seller fixes this p_1^o for any $\phi \in [\hat{\phi}, 1]$, then the inventory level, Q^o, and the product, $q^o\phi$, in the subgame equilibrium will be also fixed no matter how ϕ changes within this region (as discussed after Proposition 2 in §4.2). This implies that the seller can always ensure the same profit for any $\phi \in [\hat{\phi}, 1]$ as the profit obtained under $\phi = \hat{\phi}$. However, it is intuitive that the seller cannot do better when ϕ increases. Therefore, keeping the price at p_1^o, which is optimal for $\phi = \hat{\phi}$, is optimal for any $\phi \in [\hat{\phi}, 1]$. This is interesting; once the fraction of strategic consumers grows to some level, an extra fraction of strategic consumers will not influence the seller's decisions or his profit.

We have thus obtained the following results in terms of the equilibrium inventory level, the fraction of consumers who wait, and the seller's expected profit.

Proposition 4. *If the seller prices optimally,* (i) *when* $\phi \in [0, \hat{\phi})$, *the equilibrium inventory level* Q^o *and the seller's expected profit* Π_{NP} *both decrease in* ϕ *and the strategic consumers' equilibrium waiting strategy is fixed at* $q^o = 1$; (ii) *when* $\phi \in [\hat{\phi}, 1]$, *the equilibrium inventory level* Q^o, *the seller's expected profit* Π_{NP}, *and the proportion* $q^o\phi$ *of the high-end consumers who wait are all fixed at some levels that do not vary as* ϕ *increases.*

5. Model with Posterior PM

This section analyzes the model with the PM policy, i.e., $\vartheta = PM$. In the following subsections we analyze the second-period price, the purchasing and inventory equilibrium, the first-period price, and the seller's expected profit.

5.1. Second-Period Price

In contrast to the case without the PM policy, now some of the first-period buyers will claim the refund if the seller charges a second-period price lower than the first-period price. There are three potential choices on the second-period price that are rational for the seller: to charge $p_2 = V_L$, $p_2 = V_h$, or not to sell the product by charging any $p_2 > \max\{p_1, V_h\}$. We explain this as follows. Compared to any price lower than V_L, V_L leads to the highest revenue that the seller can extract without influencing the consumers' purchasing decisions (all the consumers present will buy the product for $p_2 \le V_L$). Moreover, V_L will lead to a lower refund cost. Compared to any price between V_L and V_h, V_h leads to a higher revenue and a lower refund cost without changing the volume of sales (only the high-end consumers will buy the product for $V_L < p_2 \le V_h$). Any price higher than $\max\{p_1, V_h\}$ will lead to the result with no sales as well as no refund cost. Because it is the same for any $p_2 > \max\{p_1, V_h\}$, we use a price $\max\{p_1, V_h\} + \varepsilon$ to indicate this choice without loss of generality. Therefore, the seller only needs to compare these three choices to set the optimal second-period price.

Recall that q_s denotes the seller's belief about the strategic consumers' waiting policy. When observing the first-period sales, $s(<Q)$, the seller infers the realization of the high-end market volume as $s/\beta(q_s)$. Hence, the volume of consumers that will claim the refund follows $(((1-q_s)\phi + (1-\phi)\gamma)s)/\beta(q_s)$, that is, those high-end strategic consumers who purchased in the first period, (fraction $(1 - q_s)\phi$), and those myopic consumers who care about the refund, (fraction $(1-\phi)\gamma$). Now we can write the seller's second-period revenue according to the above three choices

$$R_2 = \begin{cases} V_L(Q-s) - (p_1 - V_L)\dfrac{((1-q_s)\phi + (1-\phi)\gamma)s}{\beta(q_s)} \\ \qquad \text{if } p_2 = V_L, \\ V_h \min\left\{q_s\phi\dfrac{s}{\beta(q_s)}, (Q-s)\right\} \\ \qquad -(p_1 - V_h)^+ \dfrac{((1-q_s)\phi + (1-\phi)\gamma)s}{\beta(q_s)} \\ \qquad \text{if } p_2 = V_h, \\ 0 \quad \text{if } p_2 = \max\{p_1, V_h\} + \varepsilon, \end{cases} \tag{8}$$

where $x^+ = \max\{x, 0\}$. Before proceeding with the optimal second-period price, define $\eta(q)$, $\theta_I(q)$, and $\theta_{II}(q)$ as

$$\eta(q) = \frac{V_L}{((1-q)\phi + (1-\phi)\gamma)((p_1 - V_L) - (p_1 - V_h)^+) + q\phi V_h + (1-q\phi)V_L}$$

$$\theta_I(q) = \frac{V_h}{((1-q)\phi + (1-\phi)\gamma)(p_1 - V_h)^+ + (1-q\phi)V_h}$$

$$\theta_{II}(q) = \frac{V_L}{((1-q)\phi + (1-\phi)\gamma)(p_1 - V_L) + (1-q\phi)V_L}.$$

The following proposition characterizes the seller's optimal second-period price with the PM policy (see Definition 1.1).

Proposition 5. *With the PM policy, there is a unique* $q' \in [0, 1]$ *corresponding to the given parameters that determines the seller's optimal second-period price.*

(i) *If* $q_s > q'$,

$$p_2^o = \begin{cases} V_L & \text{if } \dfrac{s}{\beta(q_s)} < \eta(q_s)Q, \\ V_h & \text{if } \eta(q_s)Q \le \dfrac{s}{\beta(q_s)} < \theta_I(q_s)Q, \\ \max\{p_1, V_h\} + \varepsilon & \\ & \text{if } \theta_I(q_s)Q \le \dfrac{s}{\beta(q_s)} < \dfrac{Q}{\beta(q_s)}, \\ n/a & \text{if } s = Q. \end{cases} \tag{9}$$

(ii) *If* $q_s \le q'$,

$$p_2^o = \begin{cases} V_L & \text{if } \dfrac{s}{\beta(q_s)} < \theta_{II}(q_s)Q, \\ \max\{p_1, V_h\} + \varepsilon & \\ & \text{if } \theta_{II}(q_s)Q \le \dfrac{s}{\beta(q_s)} < \dfrac{Q}{\beta(q_s)}, \\ n/a & \text{if } s = Q. \end{cases} \tag{10}$$

Proposition 5 illustrates how the optimal second-period price depends on first-period sales and the strategic consumers' waiting strategy belief. If the seller believes that there are enough strategic consumers who will wait until the second period, ($q_s > q'$), the intuition of Proposition 5 is the following: When the realized sales are large ($\theta_I(q_s)Q \le s/\beta(q_s) < Q/\beta(q_s)$), the level of the leftover inventory will be low. At the same time, the refund that the first-period buyers may claim will be high if the seller marks down. Therefore, not selling the product will be the best strategy, i.e., setting the price just above $\max\{p_1, V_h\}$, is optimal. In contrast, when the realized sales are low, ($s/\beta(q_s) < \eta(q_s)Q$), there will be a large leftover inventory and a small volume of first-period buyers. In this situation, the salvage value of the inventory is high and the possible refund is low. Thus, charging V_L and clearing the inventory will be the best choice for the seller. For intermediate sales, the seller targets the high-end strategic consumers who waited in the first period with price V_h and pays the potential refund cost.

When the seller believes that there are not many strategic consumers in the second period ($q_s \le q'$), it is not rational for him to only target the high-end consumers in the second period; rather, the seller either salvages the inventory or does not sell the product.

5.2. Purchasing and Inventory Equilibria

5.2.1. Purchasing Equilibrium. Suppose the strategic consumers believe that the equilibrium inventory level is Q_c. To determine the purchasing equilibrium, we again consider a focal consumer who believes that the other strategic consumers' decision is q_c. The focal consumer's belief will be the same as the seller's belief: $q_s = q_c$ (Definition 1.2c). Then the focal consumer's strategy can be derived by comparing the two expected utilities of buying immediately in the first period and waiting for the second period. With the PM policy, given the seller's second-period price, the expected utilities for this consumer to buy immediately and wait are, respectively,

$$u_1 = \left[F\left(\frac{Q_c}{\beta(q_c)}\right) - F(\theta_i(q_c)Q_c) + \int_{Q_c/\beta(q_c)}^{\infty} \frac{Q_c}{\beta(q_c)\lambda} f(\lambda)\,d\lambda\right](V_H - p_1) + \int_0^{\theta_i(q_c)Q_c} (V_H - p_1 + (p_1 - p_2^o)^+) f(\lambda)\,d\lambda \tag{11}$$

and

$$u_2 = \int_0^{\theta_i(q_c)Q_c} (V_h - p_2^o)^+ f(\lambda)\,d\lambda, \tag{12}$$

where $i \in \{I, II\}$ is determined by Proposition 5 for $q_s = q_c$. The first term of Equation (11) is the expected utility for the cases where the consumer purchases the product in the first period but there is no refund. The second term of Equation (11) is the expected utility for the cases where the consumer purchases the product in the first period and could obtain a refund in the second period, i.e., when $\lambda < \theta_i(q_c)Q_c$ (see Proposition 5). In contrast, if the consumer chooses to buy in the second period, she will obtain a positive utility only when $\lambda < \theta_i(q_c)Q_c$ (see Proposition 5). This leads to Equation (12).

A comparison between these two utility functions shows that with the PM policy, the best response strategy for each strategic consumer is always to buy immediately in the first period. Now we can determine the strategic consumers' equilibrium purchasing decision (see Definition 1.2a):

Lemma 4. *Given p_1 and Q_c, the unique purchasing equilibrium is to buy immediately*: $q^o = 0$.

The intuition of Lemma 4 follows. The expected utility of a consumer is determined by the probability of obtaining the product and the surplus from consuming it. It is easy to see that the probability of obtaining the product in the first period is always higher than that of obtaining it in the second period, given that the seller cannot replenish inventory. Furthermore, if the belief about the other consumers' strategies is fixed, changing an individual consumer's decision cannot influence the seller's second-period price. Then, if the seller's second-period price is no lower than the first-period price, it is always beneficial to buy early as the consumer's valuation is declining. It is the same if the second-period price is lower. With the PM policy, the consumer can obtain the refund if the seller marks down. The consumer will effectively obtain the same low price. As a result, the best response of an individual consumer is always to choose to buy early.

5.2.2. Inventory Decision. Lemma 4 shows that for any given Q_c, there is a unique purchasing equilibrium $q^o = 0$. To solve the seller's inventory problem, we take the consumers' belief Q_c and consequently $q^o = 0$ as given and optimize the seller's inventory decision. In particular, when $q^o = 0$, the second-period price always follows Proposition 5(ii) and θ_{II} reduces to $V_L/(V_L + (\phi + (1-\phi)\gamma)(p_1 - V_L))$.

The seller's rational initial inventory investment is determined by (see Definition 1.2b)

$$\begin{aligned}\max_Q \int_Q^\infty p_1 Q f(\lambda)\,\mathrm{d}\lambda &+ \int_{\theta_{II}Q}^Q p_1\lambda f(\lambda)\,\mathrm{d}\lambda \\ &+ \int_0^{\theta_{II}Q} [p_1(1-\phi)(1-\gamma)\lambda \\ &\qquad + V_L(Q-(1-\phi)(1-\gamma)\lambda)] f(\lambda)\,\mathrm{d}\lambda \\ &- h\int_0^Q (Q-\lambda) f(\lambda)\,\mathrm{d}\lambda - cQ. \end{aligned} \tag{13}$$

In Equation (13), the first term is the revenue when $\lambda \geq Q$ and all the inventory is cleared in the first period. The second term captures the revenue when $\theta_{II}Q \leq \lambda < Q$ and the seller keeps the second-period price at least at the level p_1 and foregoes selling the leftover inventory. The third term represents the revenue when $0 \leq \lambda < \theta_{II}Q$: The seller clears the inventory at price V_L and offers the refund to the high-end consumers who claim it. The fourth term is the holding cost, and the last term is the inventory-investment cost. Lemma 5 demonstrates the seller's optimal inventory decision.

Lemma 5. *With the PM policy, given p_1 and q^o, the seller's profit function is unimodal in Q, and there is a unique optimal inventory decision Q^o that can be solved from*

$$p_1 - c - (p_1 + h)F(Q) + V_L F(\theta_{II}Q) = 0. \tag{14}$$

Given that the strategic consumers' purchasing decision is independent of the seller's inventory decision, the purchasing and inventory equilibria follow directly, based on Lemmas 4 and 5.

Proposition 6. *With the PM policy, given p_1, there exists unique purchasing and inventory equilibria (Q^o, q^o) that satisfy Equation* (14) *and $q^o = 0$.*

5.3. First-Period Price and Expected Profit

Because the high-end consumers will buy the product in the first period if the price is no higher than their valuation, the seller now does not need to lower his price to induce early purchasing. Proposition 7 demonstrates that setting p_1 equal to V_H is optimal for the seller (see Definition 1.3).

Proposition 7. *With the PM policy, the optimal first-period price $p_1^o = V_H$.*

Note that with the PM policy, the seller's first-period price and the strategic consumers' waiting strategy become independent of the fraction of strategic consumers' ϕ. The fraction, ϕ, only matters in the second period by influencing the salvage value of the leftover inventory, and then impacts the seller's initial inventory-investment decision. Furthermore, the impact of ϕ now becomes monotonic. Proposition 8 shows that the seller's expected profit always decreases in ϕ. This is because the strategic consumers will claim the refund if the seller marks down in the second period. Thus, if ϕ increases, the expected refund cost increases, or identically the expected salvage value of the leftover inventory decreases, which reduces the seller's expected profit.

Proposition 8. *With the PM policy, the seller's equilibrium inventory level Q^o and his optimal expected profit Π_{PM} both monotonically decrease in ϕ.*

6. Profitability of PM Policies

Now we analyze when the PM policy would result in more profits. This is the final step in the determination of the equilibrium (see Definition 1.3). The comparison of the equilibrium profits with and without the PM policy is not trivial, as we do not have closed-form expressions in both cases. Based on the properties discussed in the previous sections, we first obtain several analytical insights and then use a comprehensive numerical study to explore the impact of this policy. In the experiments, we chose the main parameters shown in Table 3 (in some of the experiments, ϕ, V_h, and γ may vary, which will be explained). As the optimal first-period price, p_1^o, in the model without

Table 3 Parameters in Experiments

Parameters	Values
Demand distribution	Gamma
μ	100
σ	$\{25, 50, 100\}$
V_H	10
V_L	2
V_h	$\{5, 8\}$
c	$\{4, 6, 8\}$
ρ	4%
ϕ	$\{0.25, 0.5, 0.75\}$
γ	0.25

the PM policy cannot be analytically determined, we discretized p_1^o over $[c, V_H]$ and applied a line search to locate the price that provides the close-to-maximum profit. In particular, we varied p_1 by 0.01 per step from c to V_H (i.e., with potentially $100(V_H - c)$ sample points).

We analyze a special case where there are no strategic consumers in Appendix C, which gives the best performance that the seller can achieve. We use this as the benchmark and report how much the PM policy can improve relative to this reference. In §§6.1, 6.2, and 6.3, we examine the impact of this policy on the seller's expected profit, the seller's pricing and inventory decisions, and the consumer surplus, respectively. In §6.4, we discuss the relationship with price commitment.

6.1. Value of PM

The fraction of strategic consumers, ϕ, is the key factor that influences the seller's profit. If $\phi = 0$, then the PM policy becomes irrelevant and the seller can achieve the newsvendor profit. When ϕ increases, the negative impact of the strategic consumers' waiting behavior appears. Proposition 4 has shown that the seller's profit without the PM policy decreases as ϕ increases as long as ϕ is low enough and, for higher values, remains constant. Proposition 8 has shown that with the PM policy the profit monotonically decreases in ϕ. Based on the insights of Propositions 4 and 8, Proposition 9 provides a comparison between the two models for $\gamma = 0$, i.e., no myopic consumers will claim the refund. Recall that $\hat{\phi}$ is the threshold fraction of strategic consumers from which the seller's price, inventory investment, and profit become fixed in the model without the PM policy.

Proposition 9. *When* $\gamma = 0$, (i) $\Pi_{PM} = \Pi_{NP}$ *when* $\phi = 0$;

(ii) *If there exists a* $\underline{\phi} \in (\hat{\phi}, 1]$ *such that* $\Pi_{PM} = \Pi_{NP}$ *when* $\phi = \underline{\phi}$, *then* $\Pi_{PM} > \Pi_{NP}$ *for* $\phi \in (0, \underline{\phi})$ *and* $\Pi_{PM} < \Pi_{NP}$ *for* $\phi \in (\underline{\phi}, 1]$

(iii) *If such a* $\underline{\phi}$ *does not exist*, $\Pi_{PM} > \Pi_{NP}$ *for all* $\phi \in (0, 1]$.

Point (i) of Proposition 9 is intuitive. If there are no strategic consumers and none of the myopic consumers claims a refund, the models with and without the PM policy will be identical. The seller can always charge a premium price in the first period and mark down in the second period without any cost.

Proposition 9 (points (ii) and (iii)) also asserts that when $\gamma = 0$ the PM policy can improve the seller's expected profit when there are strategic consumers and their fraction is lower than $\hat{\phi}$ (because even if a $\underline{\phi}$ exists, it is within the region $(\hat{\phi}, 1]$). To understand this result, first recall from Proposition 4 that when $\phi \in (0, \hat{\phi})$, the seller sets $p_1^o = V_H$ in the model without the PM policy. As a result, the first-period prices in the two models are equal. Now, suppose the seller uses the same inventory level in both models. We compare the corresponding revenues. In the model without this policy, strategic consumers wait for the second period, given $p_1^o = V_H$. The seller then collects a revenue, V_H, from the myopic consumers in the first period. In the second period, the seller's decision is to mark down and clear the inventory at price V_L or to not mark down and sell the product only to the strategic consumers at price V_h. In comparison, in the model with this policy, the seller collects a revenue, V_H, from the high-end consumers in the first period. In the second period, the seller decides to mark down or to not sell the product. If the seller marks down, he needs to reimburse $V_H - V_L$ to the strategic consumers and in effect sells the product to them at the price V_L. If he does not mark down, he keeps the revenue, V_H, from the strategic consumers. We can view this as if the strategic consumers waited for the second period and the seller decided to either mark down and clear the inventory at V_L or to sell the product only to the strategic consumers at V_H $(>V_h)$.

It is obvious that the revenue in the model with the PM policy will be higher. The difference here is that with the PM policy, the seller can capture V_H rather than V_h from the strategic consumers with the decision not to mark down. Furthermore, the seller saves part of the holding cost as the strategic consumers buy early. We call these two benefits the *early cash in* effect of the PM policy. This reasoning reveals a key insight: Without the PM policy, the seller has to gain part of the cash return in the second period, while the consumers' valuation is declining and the holding cost is incurred. With the PM policy, the seller shifts that part of sales into the first period and avoids those inventory cost losses. The above discussion assumes that the seller does not change the inventory level after adopting the PM policy. The seller cannot do worse when optimizing the inventory decision. The other result of Proposition 9 is straightforward. From Proposition 4 we know that for $\phi > \hat{\phi}$, the seller's profit is fixed at some level in the model without the PM policy, and his profit with this policy monotonically decreases in ϕ. As a result, these two profit curves can at most cross once, and if they cross, it happens within the region $(\hat{\phi}, 1]$. The left subplot in Figure 3 provides a demonstration of this result.

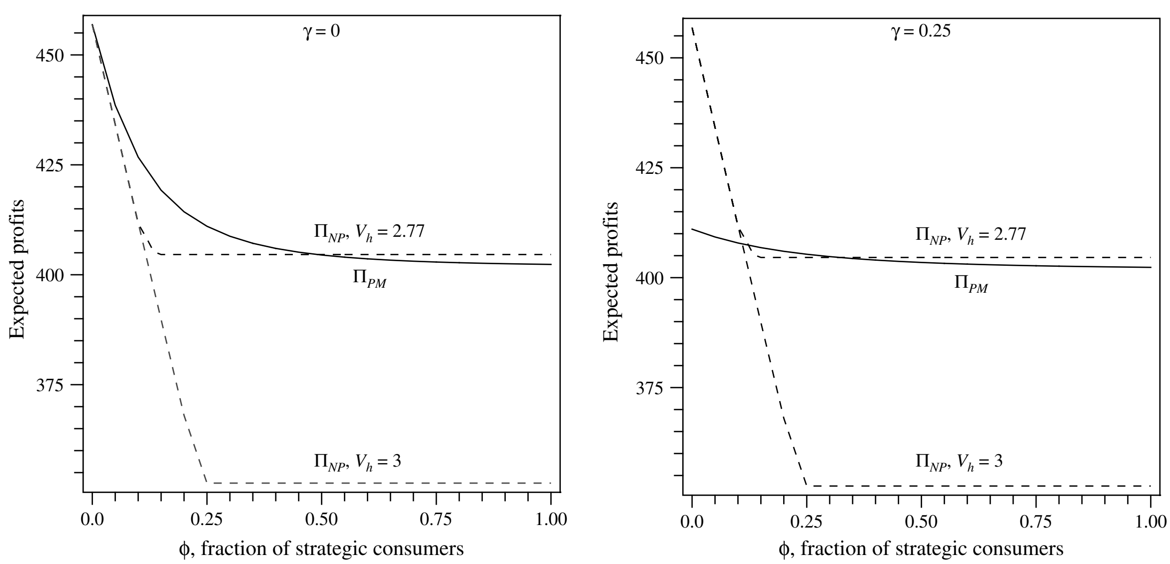
Now we turn to the case when the myopic consumers also claim the refund (i.e., $\gamma > 0$). It is straightforward that the myopic consumers' claims will reduce the benefit of the PM policy, given that the seller has to refund them the price difference in the case of markdown. The seller's profit curve Π_{PM} moves down when γ increases (see the right subplot in Figure 3), and the profit curve Π_{NP} is independent of γ. In the next proposition, we study a limit result when the fraction of strategic consumers vanishes:

Proposition 10. *When* $\gamma > 0$, $\Pi_{PM} < \Pi_{NP}$ *when* $\phi = 0$.

Because the benefit of the PM policy comes from inducing strategic consumers to buy early, if there are few strategic consumers, ($\phi \to 0$), the impact of their waiting behavior on the seller's profit is low and thus the value of this policy is low. In contrast, once the seller commits to this policy, he has to refund the price difference when he marks down. If many myopic consumers claim a refund, the cost of the refund can outweigh the benefit from early cash in. Propositions 9 and 10 reveal the following insight in the profitability of this policy with myopic consumers claiming a refund: The PM policy may no longer be optimal when myopic consumers claim a refund and the fraction of strategic consumers is low.

Propositions 9 and 10, however, do not reveal the profitability of this policy when the fraction of strategic consumers is very high ($\phi \to 1$). This policy is generally beneficial when there are many strategic consumers (as revealed by numerical experiments). However, it also depends on another factor—the consumers' valuation decline, $\Delta \doteq V_h - V_L (\in [0, V_H - V_L])$.

Figure 3 **Potential Scenarios Between Expected Profits of the Two Models**



Notes. No myopic consumers claim the refund in the left subplot (i.e., $\gamma = 0$), and 25% of the myopic consumers (i.e., $\gamma = 0.25$) claim the refund in the right subplot. λ follows a gamma distribution with $\mu = 100$ and $\sigma = 50$, $V_H = 10$, $V_L = 2$, $c = 4$, and $\rho = 0.04$.

The following proposition analyzes the limit when V_h declines very quickly (i.e., V_L) or very slowly (i.e., V_H).

Proposition 11. *When* $\gamma > 0$, $\Pi_{PM} < \Pi_{NP}$ *when* (i) $\Delta = 0$ *or* (ii) $\Delta = V_H - V_L$ *and* $h = 0$.

Imagine that V_h goes to V_L (e.g., for a very fashionable product). The strategic consumers will have very little incentive to wait because the surplus $(V_h - V_L)$ from obtaining the product in the second period, even if the seller marks down, will be very small. Then the seller can induce them to buy early by just lowering the first-period price slightly and, hence, the impact from their waiting behavior will be low. Similarly, if V_h goes to V_H (e.g., for a durable product), although the strategic consumers' incentive to wait may be strong, the impact on the seller's profit will be low because even if they wait, the seller can still capture a relatively high margin from them in the second period by charging a price equal to V_h. In the extreme case with no valuation loss $(V_h = V_H)$ and no holding cost loss $(h = 0)$, inducing early purchasing with a PM guarantee will never increase revenue from the strategic consumers, but it will incur the additional refund cost that myopic consumers may claim. Note that the above analysis does not concern the fraction of strategic consumers (as long as their share in the high-end market is sufficiently large to influence the seller's profits). It follows thus that the PM policy can only be profitable for intermediate value declines; either a steep or weak value decrease hurts the PM profitability.

To obtain further insights, we conducted two sets of experiments where we fix γ at 0.25 (i.e., 25% of the myopic consumers will claim the refund). In the first set of experiments, we used all the combinations of the parameters in Table 3. We report the results in Table 5 of Appendix D (all the tables of the data from the experiments are provided in Appendix D). In the second set of experiments, we examine the impact of ϕ and V_h where we varied ϕ from 0 to 1 and V_h from 2 to 10. Figure 4 presents the results.

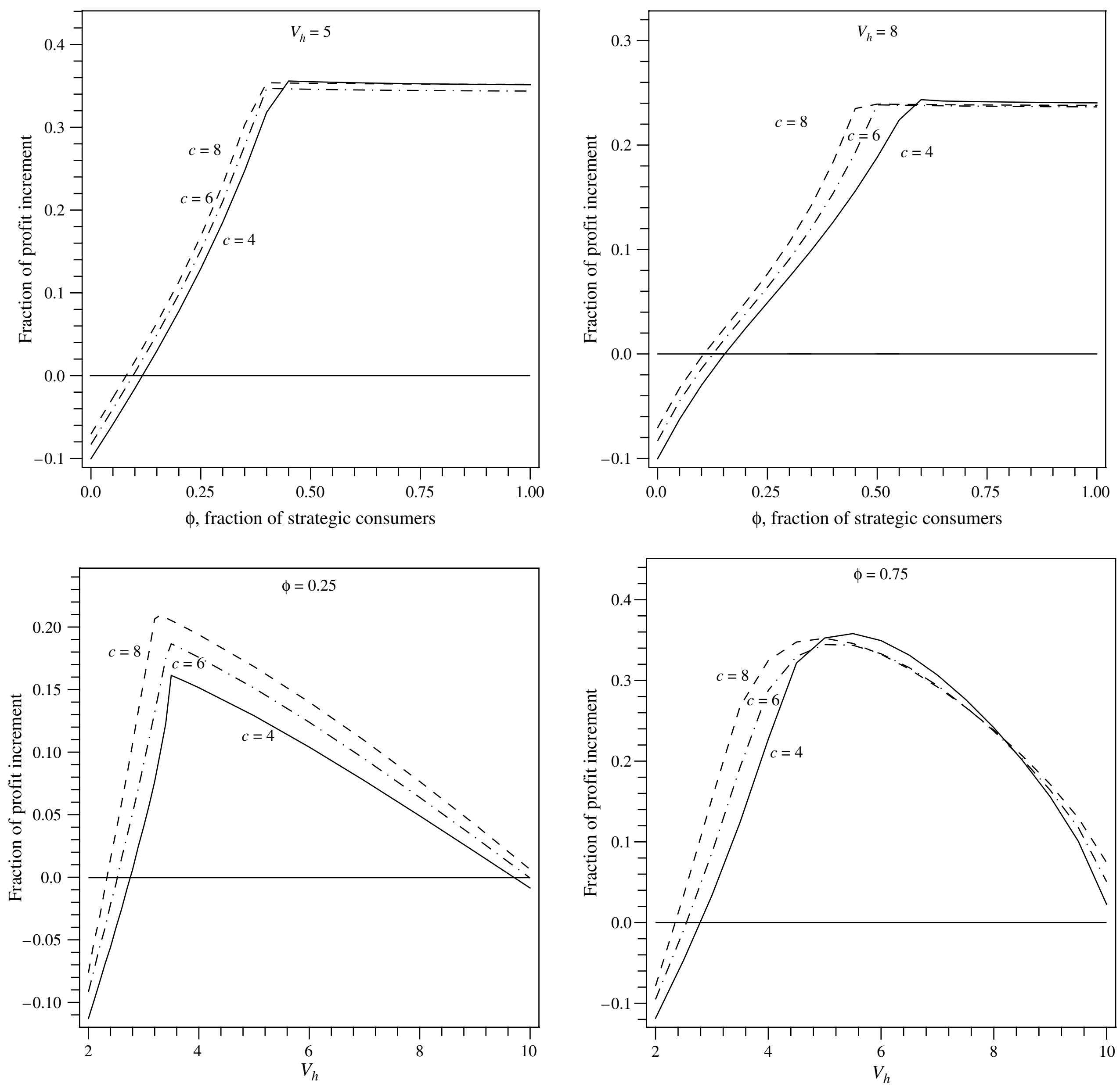
Observation 1. The PM policy can increase the seller's expected profit in many situations, especially when there is a considerably large fraction of strategic consumers and their valuation decline by waiting is neither too low nor too high. However, when the fraction is small or the valuation decline is very slight or very steep, this policy may not always be beneficial.

From the experiments, we observe that when V_h is close to the middle of V_L and V_H and ϕ is not too low, the impact of the strategic consumers' waiting behavior on the seller's profit is substantial (see Table 5 in Appendix D, where we compare the seller's profit in the model without the PM policy with the benchmark case). In such situations, the PM policy can help the seller achieve a large profit increment. In some instances, the profit by committing to this policy can even be more than doubled (see the case where $\phi = 0.75$, $c = 8$, $\sigma/\mu = 1$, and $V_h = 5$ in Table 5 in Appendix D). The seller's performance with this policy in many cases of our experiments is much closer to the benchmark than the one without the policy. Note that when the valuation decline is moderate, this policy is generally beneficial for large ϕ. However, as discussed above, when ϕ is small or V_h is close to either V_L or V_H, we clearly observe from Figure 4 that this policy may turn to be detrimental. Therefore, the PM policy should be used when there is a considerably large fraction of strategic consumers and their valuation decline is neither too weak nor too deep.

Note that besides ϕ, γ is another parameter that segments consumer purchasing behavior (see Table 1). In the above examination, we chose a particular γ (=0.25). In the numerical analysis, we also investigated the impact of different choices on γ. We found that the seller's profit with the PM policy declines as γ increases; however, the decline ratio is slight. (Recall that the model without the PM policy is independent of γ.) In particular, we report in Table 6 in Appendix D the best and worst cases for the seller with the PM policy where $\gamma = 0$ and 1—that is, myopic consumers do not and do claim the refund, respectively. We see that the seller's profit changes slightly in most cases (except when the fraction of strategic consumers ϕ is small and the coefficient of variation σ/μ is high. With a small ϕ, the fraction of myopic consumers is large. With a high variance of the market volume, the probability of a mismatch between the inventory and demand is high and thus the chance of markdown is high. Then, the myopic consumers' behavior on refund claiming will influence the seller's profit relatively strongly). Therefore, we make a second observation.

Observation 2. The seller's profit with the PM policy is relatively insensitive to the fraction of myopic consumers that claim the refund, γ.

Figure 4 **Fraction of Profit Increment** $((\Pi_{PM} - \Pi_{NP})/\Pi_{NP})$ **by PM Policy**



Note. λ follows a gamma distribution with $\mu = 100$ and $\sigma = 50$, $V_H = 10$, $V_L = 2$, $\rho = 0.04$, and $\gamma = 0.25$.

6.2. Impact on Price and Inventory

We compare the first-period prices and the inventory levels in the two models using all the combinations of the parameters in Table 3. We report the results in Tables 7 and 8 in Appendix D.

Observation 3. The PM policy always increases the first-period price and increases the inventory level in most cases.

Without the PM policy, the seller may have to lower the first-period price to induce early purchasing (see Table 7 in Appendix D), especially when the fraction of strategic consumers is high. The seller would lose much profit if he let the strategic consumers wait. With a low price, the margin from the sales will be low, which reduces the inventory investment. Moreover, the seller has the incentive to set a relatively low inventory level to push the strategic consumers to buy early. From Table 8 in Appendix D, we observe that the inventory level is substantially low in the model without the PM policy compared to the benchmark.

Nevertheless, once the PM policy is established, the strategic consumers are induced to buy early. The seller can lift the price to V_H, the level in the benchmark. The inventory investment increases as the margin increases. Moreover, it becomes unnecessary to intentionally lower the inventory investment to create the sense of scarcity as suggested by Liu and van Ryzin (2008). As a result, the inventory level increases substantially in most of the cases after the seller adopts the PM policy.

6.3. Impact on Consumer Surplus

Another interesting measure is to examine the consumers' expected surplus. Because the low-end consumers are only able to buy the product at the price equal to their valuation (the seller will never charge a price lower than V_L under our model), their surplus is always zero. Therefore, we look at the high-end consumers and measure their total expected surplus. In the model without the PM policy, the total expected surplus can be calculated as

$$\begin{aligned} S_{NP} = &\int_{Q^o_{NP}/\beta(q^o)}^{\infty} (V_H - p_1) Q^o_{NP} f(\lambda)\, \mathrm{d}\lambda \\ &+ \int_0^{Q^o_{NP}/\beta(q^o)} (V_H - p_1)\beta(q^o)\lambda f(\lambda)\, \mathrm{d}\lambda \\ &+ \int_0^{\alpha(q^o)Q^o_{NP}} (V_h - V_L) q^o \phi\lambda f(\lambda)\, \mathrm{d}\lambda. \end{aligned} \tag{15}$$

The first term in Equation (15) is the expected consumer surplus if the inventory is not enough to satisfy all the demand in the first period (and thus there are no second-period sales). The sum of the second and third terms is the total expected consumer surplus if there is excess inventory, where the second (third) term is the expected surplus of the first- (second-) period buyers.

In the model with the PM policy, the high-end consumers buy in the first period at price V_H, and thus there is a positive surplus only if the seller marks down in the second period. Therefore, the expected surplus is

$$S_{PM} = \int_0^{\theta_{II} Q^o_{PM}} (V_H - V_L)(\phi + (1-\phi)\gamma)\lambda f(\lambda)\, \mathrm{d}\lambda. \tag{16}$$

Table 9 in Appendix D presents the data of the consumers' expected surplus where we use all the combinations of the parameters in Table 3. Note that in the benchmark case the consumer surplus is always zero, as they are myopic consumers and always buy at V_H in the first period.

Observation 4. Although the PM policy generally reduces the consumer surplus, it can also increase consumer surplus in some cases, typically when the variance of the volume of high-end consumers is high.

Without the PM policy, if the seller charges a first-period price lower than V_H, then the myopic consumers as well as the strategic consumers who buy early will obtain a positive surplus immediately. The strategic consumers who wait can expect to obtain the same surplus; otherwise, they would choose to buy early. In contrast, with the PM policy, the seller always charges a price V_H. As a result, most of the myopic consumers (except for those who will claim the refund) will obtain zero surplus. Moreover, the seller will become more reluctant to mark down in the second period considering the refund cost. Therefore, the PM policy generally will reduce consumer surplus.

However, there exist cases where the PM policy can improve the expected consumer surplus. Imagine that the seller also charges $p_1^o = V_H$ in the model without the PM policy. The situation will be different. First, in such a situation, the myopic consumers will obtain zero surplus. However, in the model with the PM policy, part of them may obtain a positive surplus because they can claim the refund when the seller marks down. Note that, from Table 7 in Appendix D, there is a high chance that the seller will charge $p_1^o = V_H$ when the variance of the high-end market volume is high in the model without the PM policy. Second, as we discussed in §6.2 the seller will increase inventory investment in many cases after he adopts the PM policy. Then if the market variance is high, this inventory increment may substantially increase the probability that the seller will mark down. As a result, in the situations with high volatility of the high-end market volume, the consumer surplus may increase after the seller adopts the PM policy. This implies that a Pareto improvement for both the seller's and the consumers' payoffs is possible under the PM policy.

6.4. Relationship to Price Commitment

So far, we have assumed that the seller cannot commit to a price path; that is, the second-period price is subgame perfect (see Definition 1.1). Our second-period rational price corresponds to Aviv and Pazgal's (2008) (inventory) contingent discount. This assumption is usually made in the durable goods literature (Coase 1972, Stokey 1981, Bulow 1982) as price commitment is typically not credible. Not being able to commit to a price path leads to a loss in profits for the seller and in the extreme case can lead to a monopolist selling a product at the marginal cost (see Stokey 1981). Commitment to a PM policy, however, is weaker than committing to a fixed price path and hence is easier to implement, as can be inferred from the many practical examples. In this subsection, we assume that commitment is achievable and study commitment to a fixed price path, which corresponds to Aviv and Pazgal's (2008) fixed price discount. It is interesting to investigate how the models without and with the PM policy compare with price commitment.

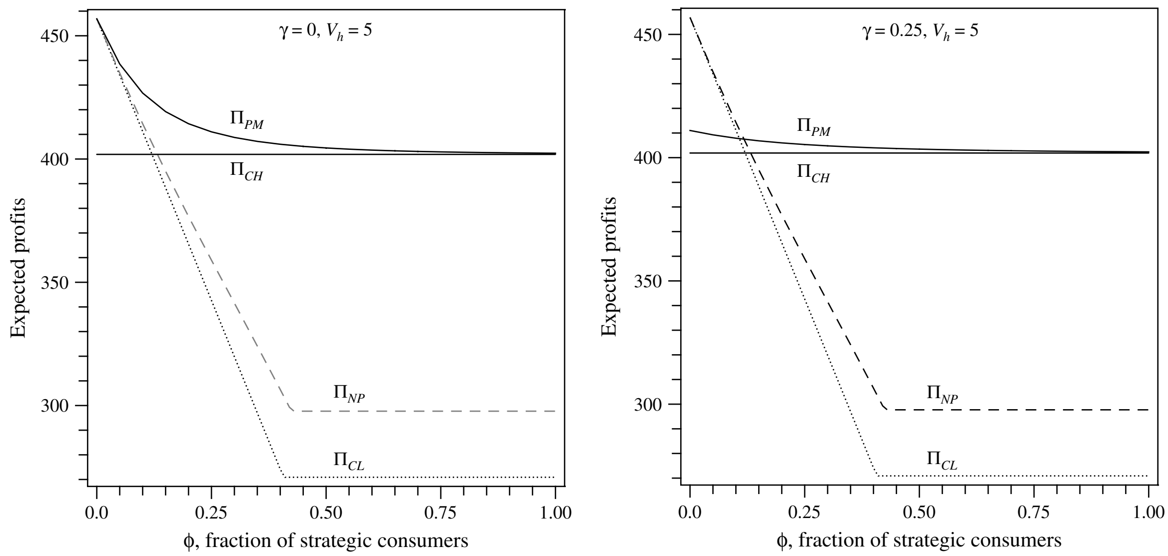
It can be easily seen that there are only two rational price commitment choices for the seller: either he commits to marking down (i.e., to set $p_2 = V_L$) and clears any leftover inventory, or he commits to keeping the price high (i.e., to set $p_2 > V_h$) such that the high-end consumers will never benefit from waiting. We call the former *committing to markdown* and the latter *committing to high price*. Any second-period price below V_L leaves extra revenue that the seller can extract, and any price from V_L to V_h is not able to attract the low-end consumers (i.e., not able to salvage the inventory) but creates the incentive for the high-end strategic consumers to wait. Proposition 12 follows from a comparison of these two price commitments with the models without and with the PM policy.

Proposition 12. *For the seller,* (i) *the expected profit by committing to markdown is dominated by the expected profit in the model without the PM policy, and* (ii) *the expected profit by committing to high price is dominated by the expected profit in the model with the PM policy.*

Proposition 12 can be explained as follows. First, it is easy to see that committing to mark down is always worse than no commitment. Without commitment, the seller may or may not mark down in the second period, depending on the inventory level at the end of the season. The consumers have to consider the price uncertainty when deciding to wait. This price concern, however, will be reduced if the seller commits to always marking down. It is natural that the consumers' waiting incentive will increase with a guaranteed low price, which is harmful for the seller. Second, committing to a high price reduces the consumers' waiting incentive and pushes them to buy early. This has been intuitively explained by Aviv and Pazgal (2008) as an effect of "burning the bridges." However, committing to not mark down could result in a loss of value from the leftover inventory. In contrast, the PM policy provides "a pie" to induce the consumers to buy early without burning the bridges. Similarly, waiting will not occur in equilibrium under this policy. But the seller still retains the option to mark down when it leads to higher profit. As a result, the PM policy is more beneficial than committing to a high price. A similar result is also discussed in Su and Zhang (2008).

However, it is worth noting that in our model committing to mark down may outperform the PM policy. For instance, if the fraction of strategic consumers is very small, their waiting behavior has a little impact on the seller's profit. In such a situation, the seller cannot benefit much from the PM policy but has to forgo the leftover inventory or incurs the refund cost that the myopic consumers may also claim. In contrast, committing to a high price may also lead to a higher profit for the seller than with no commitment when consumers are strategic. A high second-period price can deter the consumers from waiting and benefit the seller. Figure 5 demonstrates the relationship among committing to mark down, committing to a high price, committing to the PM policy, and no commitment. We find that price commitment even if it is achievable may not be the best choice for the seller in our model; choosing either no commitment or a PM policy can be more beneficial for the seller. This conclusion holds because of the stochastic nature of the high-end market size. If that were not the case, all policies would yield the same profits, where the seller can invest in the inventory level that exactly matches the demand. When the fraction of strategic consumers is high, PM yields more profits than committing to a high price. When the fraction of strategic consumers

Figure 5 Comparison Between Expected Profits of Committing to Mark Down (Denoted by "Π_{CL}"), Committing to High Price (Denoted by "Π_{CH}"), Committing to Price Matching (Denoted by "Π_{PM}"), and No Commitment (Denoted by "Π_{NP}")



Notes. No myopic consumers claim the refund in the left subplot (i.e., $\gamma = 0$), and 25% of the myopic consumers (i.e., $\gamma = 0.25$) claim the refund in the right subplot. λ follows a gamma distribution with $\mu = 100$ and $\sigma = 50$, $V_H = 10$, $V_L = 2$, $c = 4$, and $\rho = 0.04$.

is low, no PM yields more profits than committing to a markdown at the end of the season.

Note that when the fraction of strategic consumers is large, the profits of PM are close to the profits of committing to a fixed (and high) price path. Aviv and Pazgal (2008) found that committing to a fixed price path yields higher profits than not committing when the value decline is neither high nor low when all consumers are strategic. Furthermore, they found that the profits were close to the profits when the price remained constant over two periods. Their observations are thus consistent with our Observation 1 (stating that PM is significantly better than no PM when the value decline is neither high nor low and there are sufficient strategic consumers). In our model the inventories are optimized by the seller, whereas in Aviv and Pazgal (2008) inventories are exogenously given. On the other hand, their customer valuation is heterogenous and follows a continuous distribution; ours is bivalued.

7. Conclusion

Our paper shares the common interest of the ongoing research in operations management addressing strategic consumer behavior. We focus on a widely applied marketing tool: posterior PM policies. Our analysis shows that a posterior PM policy can eliminate strategic consumers' waiting incentive and thus allow the seller to increase the price. We find that the seller with a PM policy improves his profit substantially if the fraction of strategic consumers and their valuation decline by waiting are both moderate. Without the posterior PM policy, strategic consumers wait because, despite the decrease of their valuation, they will capture more surplus in the second period if the seller clears his inventory. With the posterior PM policy, the seller can make strategic consumers buy early when their reservation price is still high. This early cash-in benefit for the seller is enhanced by an inventory reoptimization: typically, more inventory is necessary. Naturally, the benefit from the posterior PM policy will be low if there are few strategic consumers and if they have little incentive to wait when their valuation declines too much by waiting. The situation is similar if strategic consumers' valuation does not decline. In those cases, the seller does not gain much by inducing strategic consumers to buy early by the posterior PM policy but must bear the refund burden later on. We find that the fraction of those myopic consumers who will also claim the refund does not have a strong impact on the profitability of the PM policy.

In terms of consumer surplus, we find that the posterior PM policy generally reduces consumer surplus because the seller charges a high price at the beginning and is reluctant to mark down later on. However, we also find cases where this policy increases consumer surplus, typically when the uncertainty of the high-end market volume is high. In such situations, the chance of markdown is high, allowing the strategic consumers and part of the myopic consumers to claim the refund.

Finally, we show that price commitment, even if it is credible, will not be the best choice for a seller. A commitment to mark down is always worse than no commitment because a guaranteed low price will drive strategic consumers to wait; a commitment to never mark down is dominated by the posterior PM policy, typically because of the option value of markdown in the PM policy.

Our paper contributes to the literature on strategic consumer behavior. We investigate the impact of a posterior PM policy on the markdown-and-waiting game and identify its benefit. We show that posterior PM policies can be one of the mechanisms that sellers use to mitigate the markdown-and-waiting game. This enriches the literature on strategic consumer behavior (e.g., Aviv and Pazgal 2008; Liu and van Ryzin 2008; Su 2007, 2008; Su and Zhang 2008; Cachon and Swinney 2008; Yin et al. 2008). Furthermore, contrary to the previous literature, such as Liu and van Ryzin (2008), Su and Zhang (2008), and Cachon and Swinney (2008), which suggest mechanisms with a fixed price or a low inventory level to alleviate consumers' waiting behavior, we show that with a posterior PM policy, for the seller, a strategy of first increasing the sales price and inventory level and marking down when beneficial can be a better way to extract profit. Furthermore, we extend the PM literature (e.g., Salop 1986, Holt and Scheffman 1987, Png and Hirshleifer 1987, Png 1991, Levin et al. 2007, Xu 2008) by broadening the focus to examine the role of PM policies on inducing consumers to buy early and the impact on operational decisions. We show that a monopolist seller can also benefit from a posterior PM policy. Moreover, complementary to the existing PM literature, we include market uncertainty and the inventory decision in our problem. This leads to one of our results—that a posterior PM policy may also increase consumer surplus in situations when the market uncertainty is high. This is in contrast to the common finding in the existing literature that shows that PM policies always reduce consumer surplus. Therefore, our paper contributes to the literatures on both strategic consumer behavior and PM policies.

We conclude by discussing the assumptions in our model and indicating directions for future research. PM policies usually have two roles—to induce consumers to buy *here* and *now*. At this point, we do not consider the role of *here* by assuming that the seller is a monopolist. This allows us to examine the seller's pricing and inventory decisions solely impacted by consumers' strategic waiting behavior. If competition exists, those decisions are also related to the impact of the competition and the other role of PM policies, which influences the consumers' store choices. To examine the combined effect of those two roles is an interesting direction for future research. We have assumed that there is no replenishment possibility during the season. This assumption simplifies the analysis, as only the initial inventory investment needs to be determined. In the case that a replenishment in the middle of the season were possible, potential cost reductions would create another barrier to implementation of posterior PM policies: If the seller wanted to decrease his price, following the cost decrease, a PM policy would imply that early consumers who bought a product that was expensive to produce now would effectively pay the price of the product with a lower production cost. Steep cost decreases would then limit severely the applicability of posterior PM policies. We have assumed that the high-end consumers are homogeneous in terms of their valuations and the number of the low-end consumers is infinite to simplify the analysis. To introduce heterogeneous valuations and a finite (or also uncertain) number of low-end consumers could enrich the managerial insights. We have normalized the hassle cost of PM to zero. Although a hassle cost will not change our results qualitatively as long as it is no larger than the valuation decline of the strategic consumers, it will make a PM policy comparably less attractive for the seller because he has to pay that cost when consumers claim the refund. Finally, we have assumed that in the second period when the inventory is limited, the high-end consumers always have priority over the low-end consumers. When that is not the case, more elaborate rationing mechanisms need to be considered. These will likely impact the equilibrium outcome. We think that rationing limited inventory between different classes of strategic consumers (e.g., high versus low end) is an interesting avenue for further research.

Electronic Companion

An electronic companion to this paper (containing appendixes) is available on the *Manufacturing & Service Operations Management* website (http://msom.pubs.informs.org/ecompanion.html).

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