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This paper considers a production-scheduling problem arising when there are random yields and demands as well as two sequential production periods before demand occurs. A typical instance is the production of seed corn. The paper makes three contributions. First, we verify that the objective function for the problem is smooth and concave so that optimal solutions are easily computed. Second, by examining data that represents actual costs, prices, and yields encountered in the seed corn industry, we gain some insight into the value that the second production period provides. Third, for a representative sample of hybrids from a major seed corn producer, we show that margins could be enhanced considerably by using the model. The results of this paper will assist seed corn producers in making production-scheduling decisions.

(*Supply Chain; Optimization; Newsvendor Model; Production-Inventory Model; Agricultural Modeling*)

1. Introduction

In this paper we consider a two-period production problem in which production can occur in either or both periods. Production yield is random in each period and a random demand occurs at the end of the second production period. This supply-demand environment is encountered in the production of hybrid seed corn (as well as in the production of other seed crops).

A producer of hybrid seed corn is continually in the process of developing new hybrids. In fact, at any one time a producer can have more than one hundred hybrids available for sale. As corn hybrids are developed, the producer must harvest a sufficient amount of seed to meet demand. Product availability is essential to maintaining market share. Seed purchasers (primarily midwest farmers) have a relatively short time window for spring planting (in April or May) and will immediately go to a competitor if seed is not available. Seed production can only occur during previous growing seasons (quick-response strategies are not possible). Also, the producer faces risks associated with weather, disease, insects, and so forth. These risks result in an uncertain yield (bushels per acre planted) being faced by the producer. To mitigate these risks, large seed corn companies take advantage of the fact that there are different seed production seasons in North and South America. These two growing seasons define the two-period nature of our model. Finally, at the end of the second production period, the producer faces an uncertain demand for the seed.

It should be noted that there may be substantial fixed costs associated with the decision to move any

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production at all from North to South America. These include, but are not limited to, the costs of establishing a management structure for overseeing the planting, harvesting, and preliminary processing of corn and the costs associated with purchasing options to grow corn, etc. Thus, the decision to adopt the strategy of using a second (South American) production period is a strategic decision that is much different from the decision of whether or not to use the second period to produce a particular hybrid. The seed company with which we worked has already made this strategic decision (as have most other large seed companies) and incurred the associated costs which are, therefore, sunk and hence irrelevant to future decision making. This paper analyzes the narrower tactical decision of how to use the second-period planting opportunity for various hybrids once the strategic decision to make it available has already been made.

We were fortunate in that a major producer of hybrid seed was willing to share insights regarding their experiences with the two-period production problem described above. In early discussions with this company, we learned that a seed company can produce hybrid seed in the North American growing season of April through September, the South American growing season of October through March, or possibly both. At the time the second-period decision (number of South American acres) is made, the first period yields are known (or almost known). Whether produced in North or South America, realized product would be available for sale in the North American market the following spring. Thus, the producer uses the alternative growing seasons as two periods in which to produce seed.

Hybrid seed is produced by planting two ''parent varieties'' of corn in the same field. The two varieties are ''crossed'' by enforcing the pollination of the immature ears of one of the varieties with pollen from the other variety. The resulting pollinated ears provide the hybrid seed for sale. For several reasons (yield potential, etc.), corn grown from hybrid seed is not used as seed in future growing seasons. Thus, seed purchasers buy hybrid seed for planting each year.

The potential value of the second production period is quite large. *The USDA Feedgrain Yearbook,* for example, estimates that 1997 seed corn consumption in the United States was approximately 21 million bushels. Mr. Larry Svajgr, Executive Director of the Indiana Crop Improvement Association (1999) estimates that an average wholesale price for seed corn is about $34 per bushel. (It should be noted that the price of seed can vary considerably. Strains incorporating patented technology—genes for resistance to corn borers or genes for herbicide resistance, for example—sell for a price that includes a technology licensing fee, making them substantially more expensive than other varieties.) Combining these two facts, we see that the wholesale seed production industry (for hybrid corn alone) produces revenues in excess of $600 million per year. Because hybrid corn is but one of many different seed crops produced in large amounts, it is obvious that even a small percentage increase in margin (from having the option for a second planting period) may have very large impacts in total dollar terms.

The goal of this research is to analyze strategies for determining the number of production acres for each period. The overall objective is to maximize the expected margin. While our focus is on the production of hybrid seed corn, we observe similar issues in batch-processing environments where backup production sites are present and the product must be produced prior to the buying season. As noted above, neither quick-response strategies nor delayed customization strategies are possible for seed corn, although either may be for a manufactured product. As a result, the value of a second production period may well be higher with seed corn than it would be for manufactured products.

In the next section, we describe the problem in detail and explain how this research differs from the existing literature. We then formally present our model. Section 3 describes in detail the solution of the first- and second-period problems. Using the results of several solved problems, we provide managerial insights in §4. In §5, we characterize the value of a second chance for a specific example. Section 6 describes the results of an experiment conducted in conjunction with a major producer of seed corn, which tested what would have happened if the company had been using the model (and following its production recommendations) over a two-year period. Our conclusions follow in §7.

2. Problem Description

A producer of hybrid seed corn faces an unknown demand for a particular hybrid in the spring of each growing season. To meet this demand, the producer may contract for production acreage in both North and South America to take advantage of the different growing seasons. In North America, planting typically occurs in April and harvest is in September. Similarly, in South America planting occurs in October and harvest is in March. In either case, harvested seed will be dried, processed and made available for sale in North America in April.

Thus, the producer encounters a two-period sequential decision problem in which Period 1 is the North American production season and Period 2 is the South American production season. Processed seed corn is available for sale at the end of Period 2.

At the beginning of Period 1, the producer expects that demand at the end of Period 2 will follow a particular distribution, which we denote as $f(D)$. The demand distribution may be updated before deciding whether to use the second production opportunity. Consider, for example, a producer facing two possible scenarios, a high-demand scenario and a low-demand scenario, each with its associated probability distribution for demand. At the beginning of Period 1, the producer might not know which of these two scenarios would occur but might know the probability that each possible scenario will occur. Using these scenario probabilities and the scenario demand distributions, the company can easily compute an ''overall'' demand distribution based on the best information available at the beginning of Period 1. If, at the time of the second production decision, the actual scenario is known with certainty, the company would update its demand distribution by simply using the demand distribution for the actual scenario that occurred.

We assume that the producer has some notion of the selling price p per bushel sold and the salvage value v per bushel not sold. The salvage value includes any costs associated with holding the seed until the following year, when it can once again be used to satisfy potential demand. If demand during the selling period exceeds supply, a shortage cost of π per bushel demanded, but not available, will be incurred. This shortage cost includes the cost of future lost sales (dissatisfied customer). If a particular hybrid is not available to a customer, that customer will typically purchase another (competing product) and may simply choose to continue using that alternative product in succeeding years. We denote the distribution function of yield of seed (in bushels per acre) in period one as $g(y_1)$. There is a per-acre cost in Period 1 for land rental, cultivation, planting, application of herbicides, etc., which we denote as K_1. This per-acre cost is incurred regardless of the ultimate yield that will be realized. Similarly, we assume that the producer knows the distribution function of yield, $g(y_2)$, in Period 2 as well as the per-acre cost K_2. In addition to the per-acre cost, there is also a per-bushel processing cost in each period. We denote this cost as c_1 for Period 1 and c_2 for Period 2. With this knowledge, the producer will decide on the quantity Q_1 of acreage to be planted in Period 1.

At the end of Period 1 (September), the producer realizes a yield of y_1 and will process Q_1y_1 bushels at a cost of c_1 per bushel. With knowledge of the quantity of product on hand along with possibly updated knowledge regarding $f(D)$, the producer must then decide on the acreage, denoted by Q_2, to be planted in Period 2. At the end of Period 2, the producer will incur a processing cost of c_2 for each bushel harvested (a total of Q_2y_2 bushels). The seed harvested and processed from both periods can then be used to meet demand.

In general, first- and second-period parameters such as K, c and $g(\cdot)$ are not the same because they are incurred in different regions of the world. Also, whereas we will generally refer to c_1 and c_2 as processing costs, c_1 also includes the cost to store the seed from the end of Period 1 until the end of Period 2. Similarly, c_2 would include any applicable transportation cost, etc., in moving the seed from South to North America.

The environment described above can be viewed as a variant of the standard ''newsboy'' problem (Hadley and Whitin 1963) with three distinguishing features. First, although demand is experienced only once (at the end of the second production period), production can occur twice before incurring demand. Second, the two production yields, as well as consumer demand, are variable. Third, the cost structure incorporates a per-acre cost that is incurred regardless of eventual yield, as well as a variable cost of processing that depends linearly upon the yield.

Several production inventory planning models with variable yield have appeared in the literature previously. In fact, Yano and Lee (1995) provide a review (along with a list of over 100 references) of the literature on lot sizing when procurement quantities are uncertain. Perhaps the earliest model was developed by Karlin in Chapter 8 of the book by Arrow et al. (1958). Although Karlin specifically couches his model in terms of agricultural production, our model differs substantially from his because his model considers only one period, restricts choice of acreage planted to a finite set, and assumes a different cost structure from our model. Additional work on these problems can be found in Shih (1980), Erhardt and Taube (1987), Gerchak et al. (1986, 1988), Henig and Gerchak (1990), Grosfeld-Nir and Gerchak (1990, 1996), and Gerchak and Grosfeld-Nir (1998). Recent work since the publication of the Yano and Lee review includes Hsu and Bassok (1999) and Bollapragada and Morton (1999).

Supply chain models where there are multiple production/supply opportunities have captured the interest of several researchers. Some of the more recent results (and this list is by no means meant to be exhaustive) include those of Eppen and Iyer (1997), Ernst and Cohen (1992), Fisher and Raman (1996), Iyer and Bergen (1997), Anupindi and Bassok (1998), Barnes-Schuster et al. (1998), and Donohue (1998). The focus of much of this research has been on devising different contractual mechanisms for coordinating relations between upstream and downstream members in the supply chain. Our research focus is entirely different because of the structure of the agricultural seed industry, in which a small number of producers face a market composed of a large number of relatively small purchasers who have virtually no market or contractual power.

Perhaps most closely related to our research is the work of Yan et al. (1998), Gerchak et al. (1988), and Gerchak and Grosfeld-Nir (1998). Yan et al. consider a model with two possible production periods (with the distribution on demand being updated because of additional information after the first period). Their model, however, assumed yield to be deterministic, incorporated a different cost structure, and assumed that the lower bound on demand was zero. Gerchak and Grosfeld-Nir consider the problem of multiple production opportunities with random yield to meet uncertain demand. They find that an optimal policy is of a *control limit* type; they stop production if and only if the stock of nondefective items exceeds a control limit. Gerchak et al. provide extensive theoretical results that we will make use of in the analysis of our problem.

A recent note by Ridder et al. (1998) describes scenarios where higher variability may lead to lower costs in the newsvendor problem. We have also observed instances of nonintuitive behavior in our problem and will describe them along with examples.

In modeling the decision problem faced by the seed producer, we use the following notation to reflect the cost parameters, distribution functions, and decision variables.

Cost Parameters

p—the price per bushel for which the seed is sold

π—the shortage cost per bushel for unmet demand

v—the salvage value per bushel for any not sold seed at the end of Period 2

c_i—cost of processing seed at the end of period i (includes holding or shipping as applicable)

K_i—cost per acre in period i

Distribution Functions

$f(D)$—distribution of demand; we assume that $0 < D_L \leq D \leq D_U$. (We allow for the possibility of updating this distribution as the selling season nears.)

$g(y_i)$—distribution of yield in period i; we assume that $0 < y_{iL} \leq y_i \leq y_{iU}$, $i = 1, 2$.

F, G—cumulative distribution functions of f and g, respectively.

Decision Variables

Q_1—number of acres to plant during period one (first growing season)

Q_2—number of acres to plant during period two (second growing season)

We conduct our analysis assuming upper and lower limits on both demand and yield in each period. Each year, many seed customers will buy a variety that they have used in the past and had success with, if it is available. This tendency implies that some positive demand will occur. Also, a particular hybrid will be discontinued in the market place not when demand goes to zero, but rather when demand is so low that production can no longer be economically justified. Thus, we safely assume that the smallest possible demand is positive. At the same time, demand obviously must have an upper limit.

We note that the lower limit on yield is generally positive. Producers of hybrid seed corn frequently will plant in a number of locations in each period. This helps them to reduce the risks of severe weather, crop disease, and so forth, which might wipe out the entire crop if it had been planted in a concentrated geographical area. This practice effectively provides for a nonzero yield for the crop. On the other hand, yield obviously has an upper limit. Note that when we refer to yield, we mean the average yield that is observed across the different planting sites used.

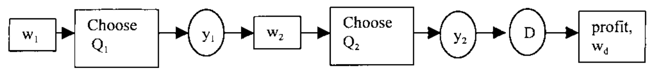
For the problem to be well posed, we make two assumptions regarding the problem parameters, where $E(\cdot)$ is expected value.

(i) $$v \leq \frac{K_i}{E(y_i)} + c_i \qquad i = 1, 2$$

(ii) $$\frac{K_i}{E(y_i)} + c_i \leq p + \pi, \quad \text{for at least one } i \in \{1, 2\}.$$

Assumption (i) states that the salvage value of seed must be less than or equal to its expected cost of production. In the absence of this assumption, the expected margin of the producer would be unbounded. This condition must hold for both Period 1 and Period 2.

Figure 1 **Timing of Events**



Assumption (ii) states that the expected per-bushel cost of production must be less than or equal to the total gain (avoidance of penalty costs plus revenue) that can be earned from selling seed. Violation of this assumption would imply that the producer's optimal choice would be to not produce. This condition must hold for at least one of the two periods.

Denote w_i as the number of bushels available at the beginning of period i. At the beginning of Period 1, the producer has w_1 bushels available (the quantity of product carried over from the previous year). At the beginning of Period 2, the producer has $w_2 = w_1 + Q_1y_1$ bushels available. Finally, after second-period production and demand D has occurred, the producer has $w_d = \max(0, (w_2 + Q_2y_2) - D)$ bushels which will be carried forward to the following year. The salvage value applies to these w_d bushels.

Thus, we have a two-period model in which the producer makes sequential decisions. Figure 1 summarizes the sequence of events for our problem.

As the growing season in the United States arrives, the producer has some notion as to what the demand for the seed will be one year later, and also obviously knows the current amount w_1 of that hybrid which is on hand. The producer must now select a quantity Q_1 of acres to plant during Period 1. At the end of Period 1, the producer observes that yield y_1 has occurred. At this point, he/she may also have some updated information regarding the Period 2 yield distribution, as well as demand for the following spring. Thus, expectations for both $f(y_2)$ and $g(D)$ may differ from the beginning of Period 1 to the beginning of Period 2.

Based on the yield y_1 observed, and the (possibly updated) distributions of Period 2 yield and demand, the producer selects an acreage Q_2 to be planted in

Period 2. At the end of Period 2, the producer observes yield y_2. The harvested $Q_2 y_2$ bushels will then be processed and, along with the quantity w_2, are available to meet demand. The result will involve some positive margin (or possibly loss) and possibly some seed to carry into the next year.

We now give an overview of the process to find a solution—an optimal value of Q_1 and optimal values of Q_2 (conditioned on $w_2 = w_1 + Q_1 y_1$)—that maximizes expected margin. A solution to the problem requires that the producer first decide on a value for Q_1 and then given some subsequent realization of y_1, make a decision regarding Q_2. Thus, the second-period problem is a problem with recourse. Denote $V_2(Q_2, w_2)$ as the expected value of the resulting second-period margin when there are w_2 units available at the beginning of the second period and the producer plants Q_2 acres. Also, $V_1(Q_1, w_1)$ is the first-period expected margin given that w_1 units are available prior to the first-period decision, Q_1 is the first period decision, and an *optimal* choice of Q_2 is made following realization of the random variable y_1. Because the optimal value of Q_2 depends on w_2, we let $Q_2^*(w_2)$ denote this optimal value. In addition, we note that the optimal Q_1 depends upon w_1 (supply left over from the previous selling period). However, to simplify our discussion, we assume henceforth that w_1 is zero. Modification to the case where $w_1 > 0$ is straightforward. (In particular, it can be shown that the desirable properties, e.g., concavity of the value functions, are preserved with nonzero w_1.) Also in §6 we report the results of solving several problems where $w_1 > 0$ in each case. With $w_1 = 0$, we write Q_1^* as the optimal value of the first-period decision. Finally, we write $V_2^*(w_2)$ for $V_2(Q_2^*(w_2), w_2)$ and V_1^* for $V_1(Q_1^*)$.

To solve the overall problem, we need to initially solve the second-period problem, where the optimal solution $Q_2^*(\cdot)$ is conditioned on w_2. Thus, we must find $Q_2^*(w_2)$ for all potential values of w_2. With this information available we can then evaluate $V_2^*(w_2)$ and use this information to find Q_1^*.

Several analytical results will be needed to show that the overall problem is well behaved and amenable to relatively efficient solution procedures. In the next section, starting with the second period, we establish that $V_2(Q_2, w_2)$ is concave in Q_2 for a fixed w_2. We then show that $V_2^*(\cdot)$ is concave in Q_1, and we will achieve this by showing that $V_2^*(\cdot)$ is concave in w_2. Note that this is sufficient because $w_2 = Q_1 y_1$ (recall we have assumed $w_1 = 0$) is linear in Q_1. The first result provides us with the ability to efficiently find Q_2^* given some value for w_2. The second result is useful for solving the first-period problem.

To conclude this section, we briefly describe an EXCEL-based optimization program that we have developed for the problem considered in this paper. The program uses the What's Best! (Lindo Systems, Inc.) solver, and can be used to solve either a two-period problem (e.g., at the beginning of Period 1), or a one-period problem (e.g., at the beginning of Period 2.) As mentioned previously, the objective is to maximize expected margin.

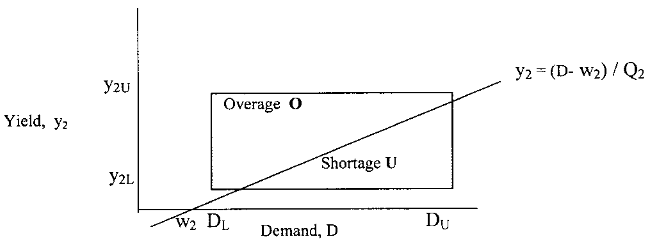
At the beginning of Period 1, the user enters w_1 (inventory carried over from the previous year), and ''best estimates'' on p, π, v, $f(D)$, as well as K_i, c_i, and $g(y_i)$, for $i = 1, 2$. The program then finds Q_1^*, the first-period (North American) acreage that maximizes expected margin. The expectation is taken over $g(y_1)$, $g(y_2)$, and $f(D)$. At end of Period 1, knowing actual North American harvest, the user solves a one-period problem with the program. At this time, ''incoming supply'' includes the recently realized North American harvest, and the user enters (possibly updated) values for p, π, v; South American costs and yield data; as well as (possibly updated) $f(D)$. The program then finds South American acreage that maximizes the expected margin. The expectation is taken over the South American yield distribution and $f(D)$.

3. Analysis

3.1. Second-Period Problem

We first describe how the optimal acreage for the second period is found for a given value of w_2. Clearly, given some choice of Q_2 and an observed second-period yield y_2, as well as customer demand D, if $D > (w_2 + Q_2 y_2)$, a shortage occurs; and if $D < (w_2 + Q_2 y_2)$ an overage occurs, and some seed will be salvaged. For insight, consider Figure 2, which shows possible demand values on the x-axis and possible yield values

Figure 2 Supply Versus Demand



on the y-axis resulting in the rectangle shown. Because we assume that both yield and demand are bounded from above and are strictly positive, the left and right, and top and bottom edges of the rectangle are at D_L and D_U, and y_{2L} and y_{2U}, respectively. For a given value of Q_2, the line represents those points where the total units available (i.e., $w_2 + Q_2 y_2$) are exactly equal to demand. The line shown in Figure 2 intersects the demand axis at the point (w_2, 0) and has slope $1/Q_2$. The area in the rectangle above the line represents those instances (combination of y_2 and D) where an overage occurs (i.e., supply exceeds demand, and w_d is positive). The area below the line indicates that a shortage has occurred. Thus, the rectangle represents the set of all possible demand-yield pairs, and for each such pair, there is some resulting positive margin (or loss).

Throughout the remainder of this subsection, we will focus on the second-period problem. For simplicity, denote the area of overage (region of the rectangle above the line) as O. Similarly, denote the area of shortage (underage) as U. With this notation, the second period margin function for fixed Q and w is:

$$V_2(Q_2, w_2) = -K_2 Q_2 - c_2 Q_2 E(y_2) + \int_O \{pD + v(Q_2 y_2 + w_2 - D)\} f(D)\, g(y_2)$$
$$+ \int_U \{p(Q_2 y_2 + w_2) - \pi(D - (Q_2 y_2 + w_2))\} f(D)\, g(y_2), \quad (1)$$

where the integral is actually a double integral over D and y_2, and $E(\cdot)$ is the expected value operator. In what follows, we assume that all functions are differentiable, although the results can be proven using only directional derivatives. If a derivative with respect to some nonnegative variable or parameter at zero does not exist, we define the derivative to be the right derivative.

We now have:

THEOREM 1. *With $w_2 \geq 0$ fixed,*

(a) $V_2(Q_2, w_2)$ is concave in Q_2.

(b) If (i) and (ii) hold, $Q_2^*(w_2)$ exists and is strictly positive.

PROOF.

(a) This result is proven in Gerchak et al. (1988).

(b) (i), and (ii) imply that $\partial V_2/\partial Q_2 > 0$ at $Q_2 = 0$, and that $\partial V_2/\partial Q_2 < 0$ at $Q_2 \geq (D - w_2)/y_L$.

Based on Theorem 1 we find that $Q_2^*(w_2)$ is straightforward. The following example illustrates how Q_2^* can change as the variation of yield changes.

EXAMPLE 1. Let $p = 3$, $c = 1$, $\pi = 9$, $v = 1$, $K = 150$, and $w = 0$. Suppose that the yield distribution is discrete and assumes either the value of $150 - t$ or $150 + t$, $0 \leq t \leq 150$, each with probability ½. Demand is a constant, 150. Table 1 shows that Q_2^* changes as the variance of yield increases. (Each yield distribution has an expected value of 150.)

We see that as the variation of yield increases from zero, i.e., t increases from zero, Q_2^* initially increases. However, when the variance gets sufficiently large, i.e., t is large, we see that additional increases in variation cause Q_2^* to decline.

To understand this behavior, first note that it is not hard to show that, for this example, the optimal solution must be either (because our earlier assumptions rule out a zero or infinite solution)

(a) plant enough acreage so that if yield is low,

Table 1 Changes in Q_2^* as a Function of Yield Variance

t	Q_2^*
0	1.000
20	1.154
40	1.364
80	2.143
120	5.000
122.7	5.495
122.7273	0.659
140	0.517

demand will be exactly satisfied ($Q^* = 150/(150 - t)$), or

(b) plant enough acreage so that if yield is high, demand will be exactly satisfied, ($Q^* = 150/(150 + t)$).

Consider a marginal analysis conducted from the acreage specified under Solution Type B. In this case, the expected marginal benefit to be obtained from planting an additional δ acres would be:

$$\begin{aligned} \mathrm{MB} &= \{(½)(p + \pi)(D - t) + (½)(v)(D + t)\}\delta \\ &= \{975 - 5.5t\}\delta. \end{aligned}$$

The marginal cost incurred by planting the additional δ acres would be:

$$\mathrm{MC} = \{K + (c)(½)[(150 - t) + (150 + t)]\}\delta = 300\delta.$$

Notice that, when t is small, the marginal benefit of planting an additional δ acres exceeds the marginal cost, so planting additional acreage is advantageous so long as we have not reached the acreage specified in Solution Type (a). Because in this case marginal benefit is decreasing in t, we can solve for the value of t at which the two are equated. This value is given by: $t = 675/5.5 \approx 122.7273$. Thus, for values of t less than this critical value, the optimal solution is of Type (a). For larger values of t, the optimal solution is of Type (b). This explains why the optimal acreage is an increasing function of yield variance until the critical value is reached, and why it then decreases in a discontinuous fashion. It should be noted that using continuous instead of discrete distributions will smooth out the discontinuity, but the nonmonotonicity will remain. Furthermore, even more complex behavior is possible when demand is allowed to be random instead of constant.

The above example is interesting, if not surprising, in light of an approximation to Q_2^* provided by Henig and Gerchak (1990) for a model similar to ours. Their approximate Q_2^* was shown to be monotonically decreasing in yield variance.

Having established that the second-period problem is one of maximizing a concave function, we now consider the problem faced by the producer in the first planting period.

3.2. First-Period Problem

We now show that the first-period objective function is concave in the first-period decision variable Q_1 (the number of acres to plant in the first period). The producer's objective is to choose Q_1 to maximize the expected margin function:

$$\begin{aligned} &V_1(Q_1, w_1) \\ &\quad = -K_1Q_1 + \int_{y=y_{1L}}^{y_{1U}} \{-c_1Q_1y_1 + V_2^*(w_1 + Q_1y_1)\}\, g(y_1). \end{aligned} \tag{2}$$

The first term in (2) accounts for the planting cost in Period 1, while the integral evaluates the expected first-period production (harvesting) cost as well as the expected second-period optimal-value function *assuming an optimal acreage choice in the second period.* We now have:

Theorem 2. *With y_1 and w_1 fixed, $V_2^*(w_1 + Q_1y_1)$, the second-period optimal value function, is concave in Q_1.*

Proof. See Gerchak et al. (1988).

Using Theorem 2, it is now straightforward to establish that:

Theorem 3. *$V_1(Q_1, w_1)$ is concave in the variable Q_1.*

Proof. The first term in (2) is linear and therefore concave. From Theorem 2, the term in braces in the integrand is concave in Q_1, for a fixed value of y_1. The concavity of $V_1(Q_1, w_1)$ now follows from Theorem 5.7 of Rockafellar (1970).

From Theorem 3 we see that the producer's decision problem is well behaved in the sense that a locally optimal (maximizing) solution is also a global solution. In our analysis, we considered continuous probability distributions on yields as well as demand, but of course similar results are obtainable with discrete distributions.

4. Managerial Insights

In this section we present numerical examples based on yield, cost, and price data that are representative of industry experiences. In constructing these examples, we relied heavily on advice and input from Mr. Larry Svajgr, and from Mr. Sonny Beck, who is CEO of Beck's Superior Hybrids (a privately held producer of hybrid seed corn). In all, we present results from nine different problem scenarios representing different combinations of demand and supply variance. The cost and price data for all nine examples are identical and are summarized in Table 2. These values reflect an average of nonproprietary and company-sensitive data.

In reality, costs, prices, yields, and demand estimates can be expected to change from Period 1 to Period 2. Costs of second-period production (in South America), for example, are typically higher than first-period production costs (in North America). The wholesale price of the resulting seed corn, while set by the seed company, must take account of such market factors as availability and prices of close substitutes marketed by competitors. Thus, the expected wholesale price may change from the start of Period 1 to the start of Period 2 if market conditions change. Yield distributions are typically different from one production site to the next. One site, for example, may have irrigation, thereby reducing the potential for problems from drought, while another site has no irrigation. Because the production sites for the two different production periods may be on different continents, their expected yields may very well differ. Finally, the demand by farmers for seed corn next spring will be influenced by their experiences with it in the current summer (prior to the beginning of Period 2), as well as by the introduction of new hybrids by competitors. Thus, demand estimates may change substantially from Period 1 to Period 2.

Table 2 Cost and Price Data

K—Cost per acre*	\$900
c—Variable processing cost per bushel	\$10
π—Shortage cost per bushel	\$27.50
v—Salvage value per bushel	\$23.50
p—Price per bushel	\$60

*Includes land use, planting, fertilizing, detasseling, etc.

While our model allows for variability in each of these parameters, in the examples presented in this section, we assume that all remain constant over the two production periods. Our reason for keeping costs, demand and yield distributions, etc. identical in both periods is that we did not want changes in these values to cloud our examination of the effect on margin, total acres planted, etc., of having the second production opportunity. It should also be noted that, although the examples presented here use symmetric unimodal distributions on demand and yields, such distributions are not required by the model. Section 6 of the paper, for example, presents a study of data from one seed company in which the distributions are not symmetric and in which most parameters vary from one period to the next.

We considered three different yield scenarios based on high, medium, and zero variance of yield. Table 3 summarizes the yield scenarios. In every example, the expected yield of seed corn was the same—40 bushels per acre. This value for average yield is typical for many modern hybrids. The coefficients of variation for the three yield distributions are 0.19, 0.11, and 0, respectively, for High, Medium, and Low.

In a similar fashion, we constructed three scenarios for demand, each of which had an expected demand of 210,000 bushels. The figure of 210,000 was determined by using (arbitrarily) 1% of the *USDA Feedgrain Yearbook* total seed corn consumption estimate of 21 million bushels. Table 4 summarizes the demand scenarios. The coefficients of variation for the three demand distributions are 0.05, 0.03, and 0, respectively, for High, Medium, and Low.

With three demand scenarios and three yield scenarios, nine different combinations are possible. Table 5 presents the results of our optimization program on each of these nine examples. In the table, the three

Table 3 Probabilities of Different Yields Under Three Scenarios

Scenario	Yield (bushels/acre) 20	25	30	35	40	45	50	55	60
High Variance	0.01	0.04	0.1	0.2	0.3	0.2	0.1	0.04	0.01
Medium Variance	0	0	0.05	0.2	0.5	0.2	0.05	0	0
Low (zero) Variance	0	0	0	0	1	0	0	0	0

yield scenarios are given in the columns and the three demand scenarios are given in the rows. Thus, the table contains nine "major cells" corresponding to each of these nine examples.

Each "major cell" is divided into two columns. The first column, titled "1 period" presents results from an optimization in which there is only one production period allowed. The second column, titled "2 period" presents results from an optimization in which two production periods are allowed. Each major cell also has five rows titled *E*(*Margin*), *COV Margin*, *Acres*, *E*(*Supply*), and *COV Supply*.

The *E*(*Margin*) row reports the optimal expected margin. The *Acres* row reports optimal acreage planted in the first production period and (following the "/" for the two-period problems only) the *expected* optimal acreage to be planted in the second production period. Even when following an optimal production plan, there is some variance in yield. Because supply equals the product of (yield per acre) $\times$ (acres planted), it follows that supply will be a random variable with some standard deviation. The *E*(*Supply*) row reports expected seed corn supply, and the *COV Supply* row reports the coefficient of variation of supply (the standard deviation of supply divided by expected supply). Margin depends on the outcomes of yield and demand, and thus has variation. *E*(*Margin*) is the expected margin, while *COV Margin* is the ratio of the standard deviation of margin divided by the expected margin.

Thus, for the problem with high variance in both demand and yield, if limited to one production period, it is optimal to plant 6.1 acres for an expected margin of $5,075. With two production periods, we would plant 4.5 acres in the first production period. Of course, the actual planting acreage in Period 2 depends upon the yield in Period 1 but has an expected value of 1.1 acres. The expected margin from the two-period problem would be $5,587 (an increase over the one-period option of more than 10%).

Most managers would agree that *stability* of both supply and margin is also an important consideration. Note that in the case of high variance in both demand and yield, the coefficient of variation for both of these measures is considerably reduced by having two production periods as opposed to only one period.

There are a number of conclusions one can draw from examining these data. First, the effect of a second production period in mitigating the effects of variability in yield is substantial. With a second production period available, the producer can hold first-period acreage substantially lower than would otherwise be the case, while reserving the second period of production as an "adjustment" mechanism. Notice that for a fixed-demand distribution, more expected

Table 4 Probabilities of Different Demands Under Three Scenarios

Scenario	Demand (bushels $\times$ 1000) 190	195	200	205	210	215	220	225	230
High Variance	0.01	0.04	0.1	0.2	0.3	0.2	0.1	0.04	0.01
Medium Variance	0	0	0.05	0.2	0.5	0.2	0.05	0	0
Low (zero) Variance	0	0	0	0	1	0	0	0	0

Table 5

	Yield Variance					
	High		Medium		Zero	
Demand Variance	1 period	2 period	1 period	2 period	1 period	2 period
High						
E(Margin)	5075	5587	5373	5647	5666	5666
COV Margin	0.29	0.16	0.14	0.10	0.04	0.04
Acres	6.1	4.5/1.1	−6	4.6/0.9	5.5	5.5/0
E(Supply)	246	224	240	2.20	220	220
COV Supply	0.19	0.06	0.11	0.02	0	0
Med						
E(Margin)	5086	5612	5390	5686	5714	5714
COV Margin	0.29	0.15	0.13	0.09	0.02	0.02
Acres	6.1	4.8/0.8	6	4.8/0.6	5.4	5.4/0
E(Supply)	246	223	240	217	215	215
COV Supply	0.19	0.07	0.11	0.03	0	0
Zero						
E(Margin)	5102	5640	5409	5722	5775	5775
COV Margin	0.3	0.15	0.13	0.09	0	0
Acres	6.0	4.7/0.8	6	4.7/0.7	5.3	5.3
E(Supply)	136	51	49	12	0	0
COV Supply	0.19	0.07	0.11	0.03	0	0

acreage is used for the adjustment as yield variance increases. Second, note that the COV of supply is significantly reduced in all cases in which yield variance is nonzero.

The magnitude of the increase in expected margin by having a second period of production depends largely upon the variance in yield and not on the variation in demand. Note that margin improvement is roughly constant at about $520 in the high-yield variance cells and at $290 in the medium-yield variance cells. The zero-yield variance cells, of course, show no potential for margin improvement: This is exactly what we should expect in this set of examples because cost and yield data remain the same from period to period, and there is no updating of the demand distribution. As with supply, the COV of margin is reduced in those cases with nonzero-yield variance.

In the next section we pursue further, via a specific example, the value of the second production opportunity. We also characterize this value as a function of yield variation.

5. The Value of a Second Chance

In this section we present a numerical example that illustrates the impact a second production opportunity can have on expected margin. We consider a problem with *fixed* demand, but variable yield at each production opportunity. Our goal is to focus on the impact of having two production opportunities instead of one. By fixing demand, we avoid those instances where the decision for the second production period is influenced more by the change in demand expectations during the first production period than by the random yield that occurred in that period. Although nothing in our modeling approach precludes having different economic parameters as well as different yield distributions from one production period to the next, the numerical example presented here has stationary economic parameters and probability distributions. Related work can be found in Grosfeld-Nir and Gerchak (1990, 1996) where multiple production opportunities are possible, but fixed demand must be satisfied (shortages are not allowed).

The numerical example is specified by the following economic parameters: $K = v = \pi = 0$, $p = 2$, $c = 1$. The yield is uniformly distributed over an interval $[\bar{y} - t, \bar{y} + t]$. Note that this is equivalent to minimizing the "expected loss due to uncertainty," $E\{F(Q)\}$, where $F(Q) = |D - S|$. The demand $D > 0$ and $S = Qy$ is the supply resulting from choosing Q and realizing a random yield of y. For simplicity, we assume that the beginning inventory is zero.

We consider two cases. In Case 1, there is only a single production period, while Case 2 has two production periods. As before, the second selection of Q is made once the yield of the first period is known. In both cases (and in both periods of Case 2), we assume that yield is uniformly distributed on $[\bar{y} - t, \bar{y} + t]$, where $0 < t \leq \bar{y}$. We use the parameter t to perform sensitivity analysis. (We use the uniform distribution for yield to simplify our analysis. Our numerical experiments with other yield distributions have resulted in outcomes similar to those reported in this section.) In what follows, we assume that $t > 0$ because otherwise there is no advantage for having two production periods. Also, F_1 and F_2 refer to the objective functions in Case 1 and Case 2, respectively.

Case 1. With only one production period, it can be shown that

$$Q^* = \alpha D, \quad \text{and} \quad E\{F_1(Q^*)\} = \gamma D, \tag{3}$$

where

$$\alpha = (\bar{y}^2 + t^2)^{-0.5}, \quad \text{and} \quad \gamma = (\alpha^{-1} - \bar{y})/t. \tag{4}$$

Case 2. In this case, we denote the choices of Q at Periods 1 and 2 by Q_1 and Q_2, respectively. Also, let y_1 and y_2 be the random yield values at these two periods. In addition, let $S_i = Q_i y_i$, $i = 1, 2$, so that $S = S_1 + S_2$ is the final supply. It can be shown that

$$Q_1^* = \beta D, \quad \text{and that} \tag{5}$$

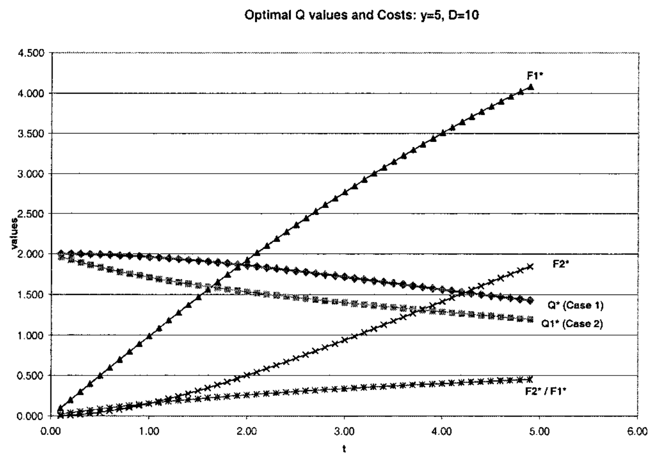
$$Q_2^* = 0, \quad \text{if } S_1 > D; \qquad \alpha(D - S_1), \tag{6}$$

otherwise, where

$$\beta = (\gamma + 1)/(\gamma(\bar{y} - t)^2 + (\bar{y} + t)^2)^{0.5}. \tag{7}$$

Finally, we have

Figure 3 Optimal *Q* Values and Expected Outcomes for Cases 1 and 2



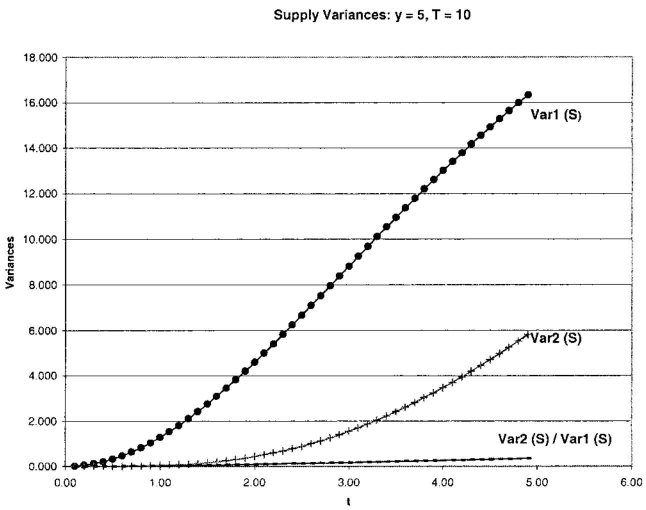
$$E\{F_2(Q_1^*, Q_2^*)\}$$
$$= (D/2t)[(\gamma + 1)/\beta - (\gamma + 1)\bar{y} + (\gamma - 1)t]. \tag{8}$$

We now analyze these two cases. For notational convenience, we denote $E\{F_1(Q^*)\}$ as F_1^*, and $E\{F_2(Q_1^*, Q_2^*)\}$ as F_2^*. First, it can be shown that $Q_1^* < Q^*$, and that $F_1^* > F_2^*$. These results are intuitive because having two production periods allows the decision maker to approach D in two steps instead of one. Also, domination in objective function values is expected because one strategy for Case 2 is to set $Q_1 = 0$. However, this reduces Case 2 to Case 1.

The discrepancy between Q_1^* and Q^* (and between F_1^* and F_2^*) depends on t. Figure 3 shows these four values as a function of t when $D = 10$, $\bar{y} = 5$, and t ranges from 0.1 to 4.9. Also, we have plotted the ratio F_2^*/F_1^*. Note that both Q_1^* and Q_2^*, as well as the difference between Q_1^* and Q^* (and between F_1^* and F_2^*), *increases* with t. With t larger (larger variance on yield), there is a higher probability of overshooting D for any value of Q, and so the decision maker becomes more conservative in the first period. Also, similar to the results reported in Table 5, the advantage of having two production periods increases as the variance on yield increases.

Perhaps the main reason that having a second production opportunity greatly improves overall performance is demonstrated in Figure 4. In the figure, the data series Var1(S) and Var2(S), are the variances of

Figure 4 Supply Variances for Cases 1 and 2



final supply for Cases 1 and 2, respectively, when $D = 10$, $\bar{y} = 5$, and t ranges from 0.1 to 4.9. These variance values are computed as follows (explicit formulas are available from the authors): Var1(S) is the variance of the random variable αDy (see (3)). Var2(S) is the variance of the random variable $S_1 + Q_2^* y_2$, where $S_1 = \beta D y_1$ (see (5)), and Q_2^* is given by (6). Obviously, when $t = 0$, both Var1(S) and Var2(S) are identically zero because there is no uncertainty. Also, the figure contains the plot of the ratio Var2(S)/Var1(S). It is interesting to note that, as expected, both Var1(S) and Var2(S) increase as t increases, but that Var1(S) increases at a much faster rate as t approaches 5. Clearly, the figure illustrates once again the fact that the second production period can be viewed as an opportunity to fine-tune overall supply.

6. An Experiment

This section presents the results of a study performed for a major producer of seed corn in which four different hybrids claimed by the producer to be representative of their overall production were analyzed over a two-year period. The purpose of the study was to answer two questions. First, would using the model have produced substantially different production plans than those that were actually used; and if so, would the model's production plans have differed in any systematic way from those actually used by the company? Second, would using the model have had any substantial impact on the company's gross margins? Although the data set is limited to eight observations, surely not a large enough sample from which to draw strong, statistically sound conclusions, we believe the results do allow for some tentative conclusions to be drawn. In particular, we note that the managers of the seed company with which we partnered in this study found the results to be sufficiently compelling that they had decided to use the model as an input to their planning process.

The seed company was able to provide complete data on its costs, distributions of production yields, etc., but views these data as proprietary, so we are unable to give precise figures. Estimating distributions on demands was somewhat more involved. To do this, we obtained company data records that provided over 200 observations of *forecasted* demand as well as *actual* demand for different hybrids. First, we normalized the data by dividing, in each case, the actual demand by the forecasted demand. We then estimated a normalized demand distribution by constructing a histogram from the normalized data. This histogram shows, for example, what percentage of the time actual demand was between, say, 90% and 100% of forecasted demand. The actual demand distributions used in the model were derived by multiplying forecasted demand, a datum, by the normalized demand distribution described immediately above.

It is worth noting that all the data used in constructing the normalized demand distribution were chosen for years and hybrids in which there was actually inventory left on hand after the sales period. This is important because otherwise we would not have been able to say with certainty what actual demand was—had ending inventory been zero, all we could have said is that demand exceeded supply. Furthermore, the four hybrids and two years chosen for the case study also satisfied this condition.

In the study, we ran the model for each hybrid in each of the two years of the study. For each model run, we used the yield distributions and demand distribution that could have been used at the time the acreage decisions were made. Yield distributions and cost data were different for North and South America. Once the model-computed values of Q were available, we made the assumption that realized yields for the model scenario would have been the same as the yields that were actually observed. To go into this in more detail, let us consider Hybrid A for Year 1 of the study period. The seed company forecasted a demand of 67,000 bushels in Year 1 for this particular hybrid. They expected a yield, based on prior harvest data, of 41.2 bushels per acre and actually planted 1,507 acres in North America and 0 acres in South America. Their actual yield was 46 bushels per acre for a total production of 69,322 bushels. Their total supply (including inventory carried over) was 98,000 bushels.

For the model run for this problem, we used a yield distribution that ranged from 31.2 to 51.2 bushels per acre, with an expected yield of 41.2. The demand distribution was generated by multiplying 67,000 (expected demand) times the normalized distribution described above. Using the model, the production plan for North America was 1,844 acres. If the company had planted this many acres, using the 46 bushels per acre figure, the total production would have been 84,802 bushels. Note that, because the actual North American yield of 46 bushels per acre was substantially larger than the expected yield of 41.2 bushels per acre, the model's second-period production plan, computed after Period 1 yield is known, called for zero acres in South America. Combined with carryover, using the model's production plan would have given a total supply of 113,400 bushels. Because actual demand was 72,000, the company experienced an actual carryover to the next year of 26,000 bushels. Using the model's production plan would have led to a carryover of 41,480 bushels.

In both cases supply was sufficient to meet demand, so revenue was the same. To determine margin, we subtracted planting and harvesting costs as well as carryover costs from revenues. In this case, the model's production plan incurred extra planting and harvesting costs as well as extra inventory-carrying costs, so the Year 1 actual margin realized by the company was greater than what they would have earned had they used the model.

Continuing on to Year 2, the company forecasted a demand of 164,000 bushels, planted 4,697 acres in North America, and eventually actually sold 146,000 bushels. The model's production plan called for 3,687 acres to be planted in North America. The model's lower second-period acreage is due in part to the fact that the carryover from Year 1 would have been larger using the first-period acreage specified by the model.

Thus, although using the model's suggested acreage decision in Year 1 would have led to a decrease in year 1 margin (relative to what was actually done), it turned out that the improvement that would have occurred in Year 2 for this hybrid would have given an overall (over the two-year interval) margin increase of 13%.

Table 6 summarizes the results from the study. In the table, the entries ''Model Inventory'' and ''Actual Inventory'' are the number of bushels, after sales, that are carried into the next year. Thus, for example, for Hybrid B in Year 1, the 65,000 actual inventory number represents incoming (into Year 1) inventory plus production (both periods) minus sales the spring following Year 1. Note that the model does not always outperform the actual results. Table entries in bold represent preferred values (low inventory, high margin). This, of course, is not surprising because there is a great deal of uncertainty in the decision process. However, using the model would have improved margin substantially, by 24% on average. Similarly, inventory carryover would have been, on average, 27% lower using the model.

These data, limited though they are, suggest that using the model would indeed produce production plans substantially different from those actually adopted by the company. On average, the model's production plans produce less inventory carryover and greater margin than those actually adopted. The model's production plans also differ systematically from those actually adopted in two other significant ways. First, almost implied by the smaller average inventory carryover, the model tended to plant a smaller acreage than was done in actuality. Second, we found that the year-to-year variation in acreage planted for any particular hybrid tends to be lower under the model's production plans. If very different

Table 6 **A Comparison of Four Hybrids over Two Years**

Hybrid	Year	Model Inv.	Actual Inv.	Model Margin	Actual Margin
A	yr 1	41,480	**26,000**	$3,239,502	**$3,836,480**
A	yr 2	**68,471**	103,000	**$6,650,799**	$4,930,685
B	yr 1	**29,201**	65,000	**$28,431,510**	$25,443,461
B	yr 2	**384,862**	521,000	**$3,737,587**	$134,456
C	yr 1	32,400	**3,408**	**$861,025**	−$292,320
C	yr 2	6,109	**4,000**	$567,040	**$1,548,000**
D	yr 1	**8,261**	67,000	**$6,256,907**	$3,505,308
D	yr 2	**70,226**	89,400	−$219,274	**$791,600**
Totals		**641,010**	878,808	**$49,525,096**	$39,897,670

% Inventory Reduction = 27%, % Margin Improvement = 24%
Bolded numbers represent preferred values.

amounts of production land are needed from one year to the next, the producer must make different arrangements each year to obtain the necessary land. If the land requirements are more predictable, obtaining this land is less of an issue for the producer.

As a result of our experiments, the seed corn company with which we partnered in this study decided to use the model as an input to the planting decisions for the production year 2000. After first determining production decisions on their 18 largest-selling hybrids using their normal procedures, the model was run on the same 18 hybrids. Although the model's production plans were not followed blindly, in some cases production plans were adjusted up or down based on the model's results. Final results from this test will not be available until late spring or early summer of 2001, when the seed corn produced in 2000 is actually sold.

7. Conclusions

This paper has presented a two-period model with random yield and random demand in which production can occur in either or both periods. We have shown that the model can be solved optimally as a sequential decision problem and have provided the results of representative sample problems (for seed corn) to demonstrate that the two-period production strategy has substantial economic payoff for the seed industry.

The model we presented addresses the value of having the flexibility to produce in either one or two periods, not the cost of obtaining this flexibility. We noted that several companies in the hybrid seed corn industry have already made the strategic decision to obtain this flexibility. If managers in other industries wish to explore the potential for production in multiple periods, our model could be used as a starting point for that analysis.

In additional work that is currently in progress, the authors consider a multiproduct model to address the issue that demands for various varieties of seed corn are actually correlated. That is, the combined demand for seed corn of all varieties is made up by summing the demands for all varieties of seed corn. If there were only two varieties, for example, a rising demand for one variety would imply either a rise in total demand, a fall in demand of the other variety, or a combination thereof. A subsequent paper will address these issues.

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