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# Economic Uncertainty, Disagreement, and Credit Markets

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**W**e study how the equilibrium risk sharing of agents with heterogeneous perceptions of aggregate consumption growth affects bond and stock returns. Although credit spreads and their volatilities increase with the degree of heterogeneity, the decreasing risk premium on moderately levered equity can produce a violation of basic capital structure no-arbitrage relations. Using bottom-up proxies of aggregate belief dispersion, we give empirical support to the model predictions and show that risk premia on corporate bond and stock returns are systematically explained by their exposures to aggregate disagreement shocks.

**Keywords:** credit risk; credit spreads; heterogeneous beliefs; uncertainty

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## 1. Introduction

Representative-agent structural models with time-additive preferences find it difficult to explain the behavior of corporate bond spreads. The so-called credit spread puzzle is due to the difficulty of generating realistic credit spreads for high-quality bonds, while keeping model-implied default and recovery rates at the empirically observed levels. This paper studies the equilibrium risk sharing of agents with heterogeneous perceptions of aggregate economic growth in an otherwise standard structural model.

We posit a frictionless structural Merton (1974)-type credit risk model, in which agents form individual forecasts about aggregate consumption growth, using an identical information set that leads them to agree to disagree on these beliefs. In contrast to most of the literature, we specify individual firm cash flows that are the sum of a firm-specific component and a systematic component proportional to aggregate consumption. Agents' disagreement about this common component produces systematic risk-sharing effects that explain an economically significant part of the cross section of asset risk premia.

Even though we assume homoskedastic firm dividend and consumption growth, agents' optimal learning and the equilibrium risk sharing generated by the common disagreement component imply heteroskedastic state prices. In equilibrium, we obtain an

endogenous firm-value process featuring a stochastic volatility and a stochastic risk-neutral skewness, which are both a direct function of the degree of heterogeneity in beliefs about consumption growth. For firms with positive cash-flow betas, we find that a higher degree of disagreement produces, at the same time, a lower firm value, a higher firm-value volatility, and a more negative firm-value risk-neutral skewness. Overall, these features increase the market price of default risk, bond risk premia, and credit spreads. However, since these effects are not primarily driven by the fundamental correlation between cash flows and consumption growth, the model does not imply an excessive physical probability of default.

Calibrating our model, we derive the following testable implications. First, we find that a higher heterogeneity of beliefs about consumption growth unambiguously increases credit spreads. Since this effect is primarily driven by the higher market price of default risk, and less through the probability of default, we can find a set of model parameters that imply realistic credit spreads and default probabilities.

Second, we show that the sign of the relation between belief heterogeneity and stock returns is ambiguous and leverage dependent. It is positive for high leverage firms, but it can turn negative for moderately levered firms. This feature follows from the different sensitivities of the default options embedded in the stock to changes in firm-value skewness: For

low (high) leverage companies, this option is far out of the money (closer to be in-the-money) and its value is more (less) sensitive to changes in skewness.

Third, in contrast to single-factor structural models of credit risk, the joint distribution of credit spreads and stock returns implies an ambiguous leverage-dependent sign for their correlation. This property implies that simple violations of basic capital structure no-arbitrage relations are possible and more likely for moderately levered firms when aggregate disagreement is large.

We empirically test the model predictions in a variety of specifications. Using earning forecast data from the Institutional Brokers Estimate System (I/B/E/S), we compute a bottom-up proxy for the aggregate cash-flow disagreement in our model, defined as the market-capitalization weighted average of individual firm disagreement proxies. Using this proxy of aggregate disagreement, we test our model predictions.

We first inspect whether there is any significant impact of aggregate disagreement onto bond returns and credit spreads. In line with our model, we find that firms with higher exposure to common disagreement shocks unambiguously have larger bond returns and credit spreads. For instance, the quintile of bonds with the largest exposure implies an average credit spread that is 32 basis points larger than the average credit spread of the quintile of bonds with the lowest exposure. This difference corresponds to about half the standard deviation of credit spreads in our sample. Similarly, the long-short spread portfolio between the quintile of bonds with the highest disagreement exposure and the quintile with the lowest exposure implies an economically and statistically significant annualized average excess return of about 0.38%. These findings are in line with the unambiguous positive relation between credit spreads and aggregate heterogeneity predicted by our model.

Second, we study the sign of the relation between stock returns and aggregate disagreement. When double-sorting stock returns with respect to firm leverage and the exposure to aggregate disagreement shocks, we find that for firms with medium and high leverage the relation is positive and significant, whereas for firms of moderate leverage it is significant and negative. This second set of findings is in line with the potentially ambiguous sign of the relation between stock returns and heterogeneity of beliefs in our model.

Third, using the two-step Fama and MacBeth (1973) methodology, we test whether aggregate disagreement risk is priced in the cross section of bond and stock returns. We find that common disagreement carries an economically and statistically significant risk premium of almost 1% per year for both bond

and stock returns. This empirical evidence is consistent with the main equilibrium risk-sharing mechanism driving our model predictions, which generates cross-sectional differences in asset risk premia via the stochastic discount factor effects of a frictionless economy with aggregate disagreement risk.

Finally, we test whether disagreement significantly increases the conditional likelihood of an empirical violation of capital structure no-arbitrage restrictions of single-factor models. A violation is defined by the joint realization of an increase (decrease) of the credit spread of some firm, together with a positive (negative) stock return. Our logit regression results show that a positive shock to the belief disagreement proxy significantly increases the probability of a violation, in a way that is robust to the inclusion of other control variables, such as leverage and the individual stock option-implied volatility. As predicted by our model, arbitrage violations are more frequent for firms with moderate leverage.

Our paper contributes to the literature that studies corporate credit spreads and equity returns in a structural framework. David (2008a, b) considers a structural Merton (1974)-type model with switching regimes, in which higher uncertainty increases credit spreads because of a convexity effect due to the time variation in the solvency ratio over the business cycle. Bhamra et al. (2010) and Chen (2010) study within a long-run risk framework how business cycle fluctuations affect firms' financing decisions and asset prices. Chen et al. (2009) investigate credit spreads and stock returns in an economy where the representative agents displays habit formation. In contrast to these papers, we specify a structural Merton-type economy with time-additive preferences and a very standard and simple default structure, in order to isolate the implications of disagreement in beliefs about aggregate economic conditions for the equilibrium price of default risk.

Our paper is also related to the empirical literature studying the link between proxies of heterogeneity in beliefs and asset returns. Most studies have focused on the relation between stock returns and firm-level proxies of belief disagreement.<sup>1</sup> The only paper that studies credit markets is Güntay and Hackbarth (2010), who find a positive relation between credit spreads and firm-specific measures of dispersion in analyst forecasts. However, little is known about whether this link is due to a systematic or idiosyncratic channel. Our paper is different from this literature along several dimensions. First, it introduces a novel set of structural predictions for the relation between aggregate measures of heterogeneity of

<sup>1</sup> Important examples in the context of the equity market include Diether et al. (2002) and Anderson et al. (2005).

beliefs, corporate credit spreads, and stock returns, which are based exclusively on the equilibrium risk sharing of heterogeneous investors in a frictionless economy. Second, it shows that aggregate disagreement is a priced risk factor for the cross sections of corporate bond and stock excess returns. Third, it helps to explain the joint comovement of credit spreads and stock returns.

Our paper also borrows from the theoretical asset pricing literature with heterogeneous beliefs. Detemple and Murthy (1994) and Basak (2000) specify heterogeneous prior beliefs about unobservable economic variables, and Dumas et al. (2009) model overconfidence in order to explain the excess volatility of stock returns. Buraschi and Jiltsov (2006) derive the option pricing implications of disagreement risk for the index option implied volatility smiles and trading volume. Finally, Chen et al. (2012) study the link between risk premia and belief heterogeneity in economies with disaster risk, in which the risk sharing produced by a small fraction of optimistic investors can generate a surprisingly large decline of the equity premium. Our model extends this literature by introducing an explicit default dimension, with corporate bonds and equity that are modeled as capital structure derivatives written on an endogenous firm value with skewed and heteroskedastic dynamics.

The remainder of this paper is organized as follows. Section 2 outlines the model and §3 presents a calibrated version of the model. Section 4 presents the data that are used to construct a bottom-up proxy of aggregate disagreement. Section 5 tests the theoretical hypotheses. Section 6 concludes the paper.

## 2. The Economy with Uncertainty and Disagreement

### 2.1. The Structural Model

We specify an extended version of Merton's (1974) structural model, by taking the dynamics of the asset cash flows of the firm as a primitive and assuming that investors disagree on it. The firm has a simple capital structure, consisting of equity and a defaultable bond. Asset cash flows,  $A(t)$ , exhibit the following dynamics:

$$dA(t)/A(t) = \beta dC(t)/C(t) + dI(t), \quad (1)$$

where aggregate consumption  $C(t)$  and process  $I(t)$  are such that  $E_t[dC(t)dI(t)] = 0$  and  $E_t(dI(t)) = 0$ . We interpret  $C(t)$  as a common component in the asset cash-flow growth and  $I(t)$  as a zero-mean idiosyncratic component. Parameter  $\beta$  captures cross-sectional differences in the sensitivity of cash flows to

aggregate consumption shocks. Therefore, it has the interpretation of a cash-flow beta.<sup>2</sup>

**REMARK 1.** We intentionally do not model the aggregation over a whole cross section of firms with cash-flow processes  $A_i(t)$ ,  $i = 1, \dots, N$ . The aggregate cash-flow dynamics that are consistent with the firm-level specification (1) are given by

$$dA(t)/A(t) = \left( \sum_{i=1}^N s_i(t)\beta_i \right) dC(t)/C(t) + \sum_{i=1}^N s_i(t)dI_i(t),$$

where  $s_i(t) = A_i(t)/A(t)$  is the total cash-flow share of firm  $i$ . In this setting, aggregate cash flows react to consumption shocks proportionally to the leverage parameter  $\phi(t) = \sum_{i=1}^N s_i(t)\beta_i$ , and the variance of the nonsystematic aggregate cash-flow component is  $\sum_{i=1}^N s_i^2(t)\sigma_{I_i}^2$ . For instance, a constant sensitivity of aggregate cash flows to consumption shocks (e.g.,  $\phi(t) = 1$ ) implicitly constrains the underlying share process  $s_i(t)$ ,  $i = 1, \dots, N$ . Furthermore, if  $\sup_{i=1, \dots, N} s_i(t) \rightarrow 0$  as  $N \rightarrow \infty$ , the nonsystematic aggregate cash-flow risk is diversifiable, provided that firm-specific risk  $\sigma_{I_i}^2$ ,  $i = 1, \dots, N$ , is uniformly bounded across firms.

The consumption dynamics are

$$dC(t)/C(t) = \mu_C(t)dt + \sigma_C dW_C(t),$$

$$d\mu_C(t) = (a_{0C} + a_{1C}\mu_C(t))dt + \sigma_{\mu_C} dW_{\mu_C}(t).$$

In this setting,  $\mu_C(t)$  is a time-varying expected growth rate and  $\sigma_C > 0$  is a constant consumption volatility. In the  $\mu_C(t)$  dynamics, parameter  $-a_{1C}$  captures the speed of mean reversion in consumption expectations, and parameter  $-a_{0C}/a_{1C}$  determines the long-term mean of expected consumption growth.

Overall, shocks to the consumption dynamics are generated by the bivariate standard Brownian motion  $(W_C(t), W_{\mu_C}(t))$ . Investors form their beliefs about the growth rate of firm cash flows in Equation (1) consistently with their beliefs about overall expected consumption growth. Consumption shocks are observable and generate information about the systematic component of cash-flow growth. Thus, realized consumption growth is rationally used by agents as a signal to improve their inference about the expected growth of individual firm asset cash flows. This also implies that uncertainty about aggregate consumption growth  $\sigma_{\mu_C}$  affects uncertainty about individual firm asset cash flows.

<sup>2</sup> The motivation for this specification is similar to, e.g., Abel (1999) and Bansal and Yaron (2004), who model differences in risk compensation through different exposures of cash flows to consumption shocks. For example, Bansal et al. (2005) show that the cash-flow beta explains more than 60% of the cross-sectional variation in risk premia across various assets.



We posit a standard two-agent economy with heterogeneous beliefs, in which investors agree to disagree. Let  $\mathcal{F}_t^C$  be the information filtration generated by the process  $C(t)$ , which is a public signal. Investors have heterogeneous beliefs about  $\mu_C(0)$  and  $\sigma_{\mu_C}$ . They rationally update their beliefs about  $\mu_C(t)$ , starting from a Gaussian prior and using  $\mathcal{F}_t^i$ . Agents do not update  $\sigma_{\mu_C}$ .

Agents maximize their lifetime expected power utility

$$U^i = \sup_{c_i} \mathbb{E} \left[ \int_0^\infty e^{-\rho t} \frac{c_i(t)^{1-\gamma}}{1-\gamma} dt \middle| \mathcal{F}_0^i \right], \quad (2)$$

subject to the budget constraint, where  $c_i(t)$  is the consumption of agent  $i = 1, 2$  at time  $t$  and  $\gamma > 0$  ( $\rho > 0$ ) is the common relative risk aversion (time preference rate) of the two investors;  $\mathbb{E}[\cdot | \mathcal{F}_t^i]$  denotes expectations conditional to the filtration  $\mathcal{F}_t^i$ .

Bayesian posterior growth rates for each agent are obtained from a Kalman and Bucy filter. Heterogeneity in beliefs is summarized by the differences in posterior means and variances of future consumption growth. Let  $m_C^i(t) := \mathbb{E}[\mu_C(t) | \mathcal{F}_t^i]$ , under technical regularities detailed in Lipster and Shiryaev (2000), the dynamics of posterior expected consumption growth is

$$dm_C(t) = (a_{0C} + a_{1C}m_C(t))dt + \frac{\gamma(t)}{\sigma_C^2}(dC(t)/C(t) - m_C(t)dt),$$

where the variance  $\gamma^i(t) = \mathbb{E}[(dC(t)/C(t) - m_C(t))^2 | \mathcal{F}_t^i]$  solves the following Riccati differential equation:

$$\dot{\gamma}(t) = -\frac{\gamma(t)^2}{\sigma_C^2} + 2a_{1C}\gamma(t) + \sigma_{\mu_C}^2, \quad (3)$$

$$\gamma^i(0) = \mathbb{E}[(dC(0)/C(0) - \mu_C(0))^2 | \mathcal{F}_0^i].$$

We denote by  $dW_C^i(t)$  the innovation process implied by agent  $i$ 's belief,  $dW_C^i(t) = (dC(t)/C(t) - m_C^i(t)dt)/\sigma_C$ , and we specify agents' disagreement about consumption growth as

$$\Psi_C(t) \equiv \frac{m_C^1(t) - m_C^2(t)}{\sigma_C}.$$

Belief heterogeneity influences the prices of all financial assets, which—from the perspective of agent 1 or agent 2—can be written as functions of the filtered Brownian motion shocks  $dW_C^1(t)$  or  $dW_C^2(t)$ .<sup>3</sup>

<sup>3</sup> To focus on the equilibrium implications of aggregate disagreement, and for ease of notation, we assume that investors do not disagree on the firm-specific cash-flow component:  $\mathbb{E}_t^1[dI_i(t) | \mathcal{F}_t^{C, A_i}] = \mathbb{E}_t^2[dI_i(t) | \mathcal{F}_t^{C, A_i}] = 0$ . Equivalently, one could impose that investors disagree also on the firm-specific cash-flow component, but believe that nonsystematic cash-flow shocks are diversifiable and thus have a zero market price of risk. We thank an anonymous referee for suggesting this interpretation of our model.

In our economy, at least one risky financial asset is needed to complete the market. We consider an economy including corporate bonds and equity. We denote by  $r(t)$  the interest rate on the risk-less bond, in zero net supply; by  $S(t)$  the price of the stock of the firm, in positive net supply; and by  $B^d(t)$  the price of a corporate bond, in positive supply;  $V(t)$  denotes the market value of the assets  $A(t)$  of the given firm in our economy.

Optimal consumption takes the form  $c_i(t) = (y_i \xi^i(t))^{-1/\gamma}$ , where  $y_i$  is the Lagrange multiplier in the static budget constraint of agent  $i$  and  $\xi^i(t)$  is the state-price density of this agent. Closed-form expressions for  $\xi^i(t)$  follow with standard computations<sup>4</sup>

$$\begin{aligned} \xi^1(t) &= \frac{e^{-\rho t}}{y_1} C(t)^{-\gamma} (1 + \lambda(t)^{1/\gamma})^\gamma, \\ \xi^2(t) &= \frac{e^{-\rho t}}{y_2} C(t)^{-\gamma} (1 + \lambda(t)^{1/\gamma})^\gamma \lambda(t)^{-1}, \end{aligned} \quad (4)$$

where the stochastic weight  $\lambda(t) = y_1 \xi^1(t)/(y_2 \xi^2(t))$  has dynamics

$$\frac{d\lambda(t)}{\lambda(t)} = -\Psi_C(t) dW_C^1(t).$$

In contrast to Merton's (1974) structural model, the stochastic discount factor is a function of the additional state variable  $\Psi_C(t)$  that directly affects agents' optimal risk sharing. In the special case where  $\Psi_C(t) = 0$ , we obtain a structural economy with identical agents. In this setting, both  $\lambda(t)$  and the relative consumption of each agent are constant.

## 2.2. Pricing of Financial Assets

As in Merton's (1974) structural credit risk model, default occurs only at maturity of the corporate bond, if the asset value is below the face value of the bond. The price of the corporate bond can be written as the sum of the prices of the zero-coupon bond and the price of a short put option on the firm value. Similarly, the price of equity is the firm-value residual in excess of the price of corporate debt.

Given the individual state-price density  $\xi^i(t)$ , we can price any security by computing the expectation of its contingent payoffs weighted by  $\xi^i(t)$ . Denote by  $K$  the face value of the corporate debt and by  $B(t, T)$  the equilibrium price of the risk-free zero bond. Hence, the corporate bond price in our economy is given by

$$B^d(t, T) = KB(t, T) - \mathbb{E}_t^i \left[ \frac{\xi^i(T)}{\xi^i(t)} (K - V(T))^+ \right].$$

<sup>4</sup> The equilibrium is solved using the martingale approach, originally developed by Cox and Huang (1989). The extension to the case of heterogeneous beliefs has been studied, among others, by Cuoco and He (1994), Karatzas and Shreve (1998), and Basak and Cuoco (1998).

Note that the right-hand side of this equation depends on both the aggregate consumption process  $C(t)$  and the aggregate disagreement process  $\Psi_C(t)$ , via the state-price density  $\xi^i(t)$ . To compute  $B^d(t, T)$  and the price of all other derivatives on the capital structure of the firm, we need the joint density of  $C(t)$ ,  $\lambda(t)$  and the relevant contingent claim payoff. The joint density of  $(C(t), \lambda(t))$  is typically unavailable in closed form. However, we can calculate its Laplace transform analytically, which can be used, in a second step, to price more efficiently all securities based on Fourier transform methods.

The closed-form expression for the Laplace transform of  $(C(t), \lambda(t))$  in our model follows the approach developed in Dumas et al. (2009). This approach is based on the solutions  $\gamma^1(t) = \gamma^1$ ,  $\gamma^2(t) = \gamma^2$  of the steady-state version of the Riccati equation (3) for investor  $i = 1, 2$ :

$$0 = -\frac{(\gamma^i)^2}{\sigma_C^2} + 2a_{1C}\gamma^i + (\sigma_{\mu_C}^i)^2. \quad (5)$$

In our setup, we adopt the parametrization  $\sigma_{\mu_C}^1 < \sigma_{\mu_C}^2$ , meaning that the optimistic investor perceives a lower consumption growth uncertainty. With this parametrization, we obtain that  $\gamma_1 < \gamma_2$ , which implies a countercyclical dynamic for  $d\Psi_C(t)$ . This feature is consistent with the countercyclical properties of the bottom-up proxy of disagreement used in our empirical study.

Proposition 1 collects the Laplace transform expression relevant for the computation of asset prices in our economy.

**PROPOSITION 1.** *The moment generating function for the joint distribution of  $C(t)$  and  $\lambda(t)$  under the measure of agent 1 is given by*

$$\begin{aligned} \mathbb{E}_{m_C^1, \Psi_C} \left[ \left( \frac{C(u)}{C(t)} \right)^\epsilon \left( \frac{\lambda(u)}{\lambda(t)} \right)^\chi \right] \\ = F_{m_C}(m_C^1, t, u, \epsilon) \times F_{\Psi_C}(\Psi_C, t, u, \epsilon, \chi), \end{aligned}$$

where

$$F_{m_C}(m_C^1, t, u, \epsilon) = \exp(A_m(\epsilon, u - t) + B_m(\epsilon, u - t)m_C^1)$$

and

$$\begin{aligned} F_{\Psi_C}(\Psi_C, t, u, \epsilon, \chi) \\ = \exp(A(\epsilon, u - t) + \epsilon\Psi_C B(\chi, u - t) + \Psi_C^2 C(\chi, u - t)), \end{aligned}$$

with functions  $A_m$ ,  $B_m$ ,  $A$ ,  $B$ , and  $C$  are given explicitly in the appendix.

**Table 1** Choice of Parameter Values and Benchmark Values of State Variables

Parameters for cash flow		
Long-term growth rate of cash-flow growth	$a_{0A}$	0.01
Mean-reversion parameter of cash-flow growth	$a_{1A}$	−0.01
Volatility of cash flow	$\sigma_A$	0.07
Initial level of cash flow	$A$	1.00
Initial level of cash-flow growth	$m_A^1$	0.01
Cash-flow beta	$\beta$	0.57
Agent specific parameters		
Relative risk aversion	$\gamma$	2.00
Time preference parameter	$\rho$	0.02
Difference in economic uncertainty	$\sigma_{\mu_C}^1 - \sigma_{\mu_C}^2$	−0.002

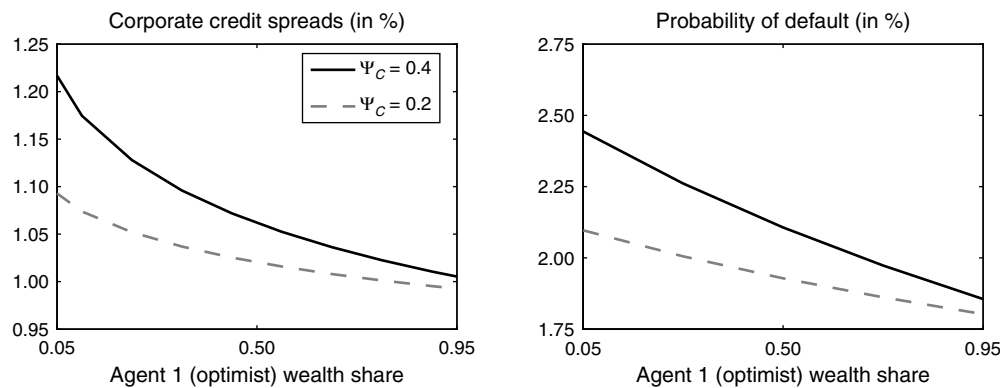
*Notes.* This table lists the parameter values used for all figures in this paper. We calibrate the model parameters to the mean and volatility of the time-series average of operating cash flow for all firms present in our database. Operating cash flow is earnings before extraordinary items (Compustat item 18) minus total accruals, scaled by average total assets (Compustat item 6), where total accruals are equal to changes in current assets (Compustat item 4) minus changes in cash (Compustat item 1), changes in current liabilities (Compustat item 5), and depreciation expense (Compustat item 14) plus changes in short-term debt (Compustat item 34). The initial values for the conditional variances are set to their steady-state variances.

### 3. Model Predictions

To derive a number of testable model predictions, we calibrate our economy to the cash-flow dynamics of a representative firm in our sample. Table 1 summarizes our main calibration parameters.

We assume a risk-aversion parameter of  $\gamma = 2$  and a cash-flow volatility of  $\sigma_A = 7\%$ . There are several ways to estimate a firm's consumption beta. We use data on operating cash flows to calculate a regression beta from firm specific operating cash flows onto consumption growth. Real consumption corresponds to real per-capita expenditures on nondurable goods and services. Data are taken from the Bureau of Economic Analysis. Running these regressions, we find that the average cash-flow beta is around 0.57.<sup>5</sup> Using these parameters, we then present comparative statics with respect to the amount of aggregate disagreement. We take values of  $\Psi_C(t)$  between 0 and 0.4, in line with the empirical properties of our bottom-up disagreement proxy presented later. The average leverage ratio in our database is 0.20. When we refer to low leverage, we study firms in the lower tercile of the distribution, i.e., with leverage below 11%. Medium leverage refers to firms with a leverage ratio of between 11% and 26%, and high leverage refers to firms with a leverage of 26% or larger.

<sup>5</sup> Another way to calculate cash-flow betas is to use a vector autoregression. Recent contributions include Campbell et al. (2012) and Bansal et al. (2013), who study an intertemporal capital asset pricing model with stochastic volatility. The average estimated cash-flow beta in Campbell et al. (2012) is 0.22 using data between 1963 and 2011. The estimates in Bansal et al. (2013) are smaller. However, these authors use a much longer time series that includes the Great Depression.

**Figure 1** Credit Spreads and Probability of Default: The Effect of Disagreement

Notes. The left (right) panel plots the credit spread (probability of default) of a 10-year bond as a function of the wealth share of the optimist for different levels of disagreement,  $\Psi_C = 0.2, 0.4$ . We calibrate the model to the average leverage ratio in our data set, which is 0.2. Other parameters used for the calibration are gathered in Table 1.

### 3.1. Credit Spreads and Equity Volatility

A well-known weakness of the Merton (1974) model is the difficulty to reconcile the empirically observed credit spreads of high-quality bonds with the empirical probabilities of default. To study whether priced disagreement risk can help explain the credit spread puzzle, we consider a firm with a 10-year maturity corporate bond and plot in Figure 1 the average credit spread (left panel) and probability of default (right panel) implied by our calibrated model, as a function of the degree of heterogeneity  $\Psi_C(t)$  and the consumption share  $s(t) = 1/(1 + \lambda(t)^{1/\gamma})$  of the optimistic investor at time  $t$ .

Figure 1, left panel, depicts the equilibrium credit spread, for different levels of  $\Psi_C(t) = 0.2, 0.4$ , as a function of the consumption share  $s(t)$ . We find that, as disagreement rises from 0.2 to 0.4, the average credit spread in an economy with symmetrically distributed consumption ( $s(t) = 1/2$ ) increases from about 103 to about 110 basis points. This variation corresponds to about one fifth of the average standard deviation of corporate credit spreads in our sample.

In the model, credit spreads also significantly increase in states where aggregate consumption is predominantly consumed by the pessimistic investor, i.e., in bad consumption states. For instance, when  $\Psi_C(t) = 0.2$ , the credit spread increases from 100 to 110 basis points as aggregate consumption is more and more consumed by the pessimistic investor. This is a direct consequence of the fact that in bad economic states, risk-sharing motives are particularly strong, while risk-sharing opportunities are more limited.

Overall, our calibrations indicate that a degree of heterogeneity in beliefs about aggregate consumption growth similar to the one in the data, together with plausible assumptions about the distribution of consumption across investors, are compatible with a

realistic level of credit spreads. In the right panel of Figure 1, we depict the associated probability of default again as a function of the share process. We note that for an equally distributed economy, the probability of default increases from 2% to almost 2.25% for an increase in disagreement from 0.2 to 0.4. These numbers are broadly in line with the data.<sup>6</sup>

A second prediction of our model is that a higher disagreement also increases the equity volatility, which creates an additional channel for a positive comovement of credit spreads and stock return volatility. In our calibration, we find that the equilibrium volatility of stock returns can rise from 7% to 15% when  $\Psi_C(t)$  increases from 0 to 0.4, in an economy with cross-sectional evenly distributed consumption. These properties are consistent with the well-established empirical evidence documenting a positive comovement between credit spreads and stock return volatilities; see Campbell and Taksler (2003), among others. In our economy, such positive comovement exceeds the comovement predicted by standard structural models and it derives mostly from the time variation of  $\Psi_C(t)$ .

### 3.2. The Equilibrium Price of Default Risk

To better understand the economics linking the heterogeneity in beliefs to the stock volatility and the price of default risk in our economy, it is convenient to investigate in more detail the equilibrium firm-value process. Whereas in Merton's (1974) model the firm-value process is homoskedastic, in our economy firm value depends directly on time-varying heterogeneity in beliefs and the cross-sectional wealth among agents. Using the calibrated model, we find that an increase of  $\Psi_C(t)$  from 0 to 0.2 can reduce the

<sup>6</sup> The empirical default frequency for the period between 1996 and 2007 is slightly higher than 2.73%, according to Moody's (2008).

equilibrium (asset) value of the firm, in an economy with symmetrically distributed consumption, from 160 to 155. When cross-sectional consumption is more concentrated on pessimistic investors, the effect is even larger.

The equilibrium price of the corporate bond is directly determined by the distribution of the firm value, because corporate bond payoffs contain a short put position on the firm value. To understand the equilibrium pricing of corporate bonds and other capital structure derivatives, it is useful to write each agent equilibrium stochastic discount factor (4) in terms of agents' stochastic share  $s_i(t)$  of total consumption:

$$\xi^i(t) = \frac{1}{y_i} e^{-\rho t} C(t)^{-\gamma} s_i(t)^{-\gamma},$$

where  $s_i(t) = c_i(t)/C(t)$  is investor's  $i$  consumption share. The consumption share is greater (lower) in good (bad) aggregate consumption states, when the marginal utility of the optimist has a larger (lower) impact on the stochastic discount factor. Since in good (bad) states the marginal utility of the optimist (pessimist) is lower, the present value of the cash flows of a typical firm with positive cash-flow beta is lower than in the economy with homogeneous beliefs.

A second key property of the stochastic discount factor (4) is an endogenous stochastic volatility that is increasing in the degree of belief heterogeneity  $\Psi_C(t)$ :

$$\begin{aligned} d\xi_i(t)/\xi_i(t) - E_t[d\xi_i(t)/\xi_i(t)] \\ = -(\gamma\sigma_C + (1 - s_i(t))\Psi_C(t))dW_C^i(t). \end{aligned}$$

As the volatility of future state prices increases with the degree of belief heterogeneity, we find that the firm-value volatility also increases with the degree of belief disagreement. Within our calibrated model, this implies that the equilibrium firm-value volatility increases from 7% to approximately 11% in an economy with a symmetric cross-sectional consumption. Since equity is a call option on the firm value, we also find that the volatility of stock returns is mainly driven by the firm-value volatility. This link generates the positive relation between belief heterogeneity  $\Psi_C(t)$  and stock return volatility, which was highlighted in §3.1.

A third characteristic of the stochastic discount factor (4) is a positive asymmetric relation between state prices and state price volatility. In our calibrated model, this feature arises because consumption shocks  $dC(t)$  are negatively correlated with disagreement shocks  $d\Psi_C(t)$ , i.e., aggregate disagreement is countercyclical.<sup>7</sup> A negative shock to aggregate consumption increases the state price  $\xi_1(t)$ , and it also

increases the product  $(1 - s_i(t))\Psi_C(t)$  in the calibrated economy:

$$\text{Cov}_t[d\xi_1(t)/\xi_1(t), d((1 - s_1(t))\Psi_C(t))] > 0. \quad (6)$$

Such a positive correlation generates positively skewed state prices, i.e., asymmetrically larger state prices for low aggregate consumption states.

The skewed distribution of state prices is linked to a negative firm-value risk-neutral skewness, which is a manifestation of the large price requested by investors for selling default insurance. The left panel of Figure 2 quantifies the link between belief heterogeneity and negative risk-neutral skewness in the calibrated model. We find that an increase of  $\Psi_C(t)$  from 0.2 to 0.4 can significantly lower the firm-value risk-neutral skewness, on average from  $-0.43$  to  $-0.6$ , in an economy with symmetrically distributed cross-sectional consumption.<sup>8</sup> Overall, such an asymmetric structure of state prices can generate a significant premium component for default risk, despite the modest physical probability of default in the calibrated economy.

### 3.3. Stock Returns and Capital Structure Arbitrage

A tight constraint of single-factor structural models inspired by Merton's (1974) approach is the unambiguous negative correlation between stock returns and credit spreads.<sup>9</sup> This negative correlation is at the hearth of various capital structure arbitrage strategies, which can be implemented with portfolios of different capital structure derivatives, such as credit default swaps. An interesting property of our economy, is that the sign of the comovement of stock returns and credit spreads is unrestricted and dependent on the degree of belief heterogeneity, with a price of equity that can both increase or decrease with disagreement, depending on the relevant firm characteristics. Figure 2 documents the ambiguous effect of an increase in disagreement  $\Psi_C(t)$  on the comovement of credit spreads and stock returns, in dependence of firm leverage.

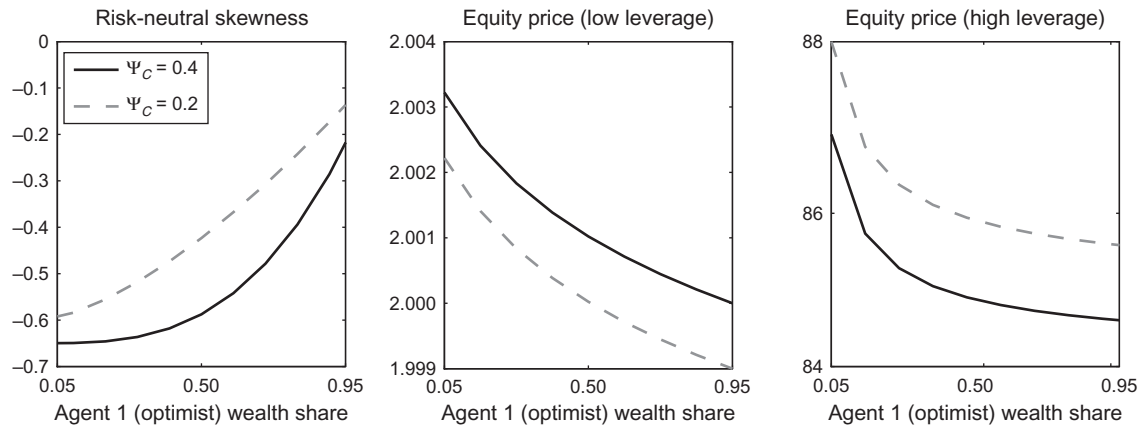
For high leverage firms, we find that an increase of  $\Psi_C(t)$  from 0.2 to 0.4 lowers the price of equity by 3 %, whereas for low leverage firms it increases the price of equity by 1 % in a symmetric economy. Since in the

<sup>7</sup> A sufficient condition for the countercyclicity of  $\Psi_C(t)$  is  $\gamma^2 > \gamma^1$ ; i.e., the optimistic investor perceives a lower consumption growth uncertainty than the pessimistic investor.

<sup>8</sup> We compute the risk-neutral skewness following the methodology developed in Bakshi and Madan (2000), who show that any payoff function can be spanned by a continuum of out-of-the-money calls and puts.

<sup>9</sup> In this approach, corporate debt and equity are (nonlinear but monotonic) contingent claims collateralized by the same balance sheet. Since the firm's cash flows act as the single pricing factor, negative cash-flow shocks reduce the value of the equity claim in the capital structure. The correlation between corporate bond spreads and equity prices is restricted, by no arbitrage, to be negative at all times and states.



**Figure 2** Risk-Neutral Skewness and Equity Price

*Notes.* The left panel plots the risk-neutral skewness of the firm value as a function of the wealth share of the optimist for different levels of disagreement,  $\Psi_C$ . The right two panels plot the equity value for two different values of leverage. Low (high) leverage refers to a firm with a leverage ratio of 5% (35%). Other parameters used for the calibration are gathered in Table 1.

first case stock prices and credit spreads comove negatively, whereas in the second case the comovement is positive, we obtain a standard hedge ratio that can even change sign.

The economics for the complex comovement features of stock returns and credit spreads in our model are better understood by recalling that the stock price is the price of a portfolio consisting of a long position in the firm value  $V(t)$ , a short position in  $K$  risk-free zero bonds, with price  $ZCB(t)$ , and a long out-of-the-money put on the firm value, with strike  $K$  and price  $P(t, K)$ , where  $K$  is the face value of the corporate debt:

$$S(t) = V(t) - K \cdot ZCB(t) + P(t, K).$$

Although the firm value  $V(t)$  is independent of leverage and decreasing in disagreement, the zero coupon price  $ZCB(t)$  decreases with disagreement at the calibrated parameters. Thus, the effects of the first two components on the price of equity tend to offset each other, with the effect of the second component that increases proportionally to firm leverage.

The price of the put option  $P(t, K)$  has a positive impact on the price of equity, but the size of the effect depends in a nonmonotonic way on the degree of firm leverage. Moreover, we find that there exist regions of leverage where the positive price impact of the put option component can be large enough to offset the negative impact of the change in the value of the firm:

$$\frac{dS}{d\Psi} = \frac{dV}{d\Psi} - K_1 \cdot \frac{dZCB}{d\Psi} + \underbrace{\left[ \frac{dP}{dV} \cdot \frac{dV}{d\Psi} \right]}_{\text{Delta: +}} + \underbrace{\left[ \frac{dP}{d\sigma_V} \cdot \frac{d\sigma_V}{d\Psi} \right]}_{\text{Vega: +}} + \underbrace{\left[ \frac{dP}{dSk_V} \cdot \frac{dSk_V}{d\Psi} \right]}_{\text{Skewness: +}}. \quad (7)$$

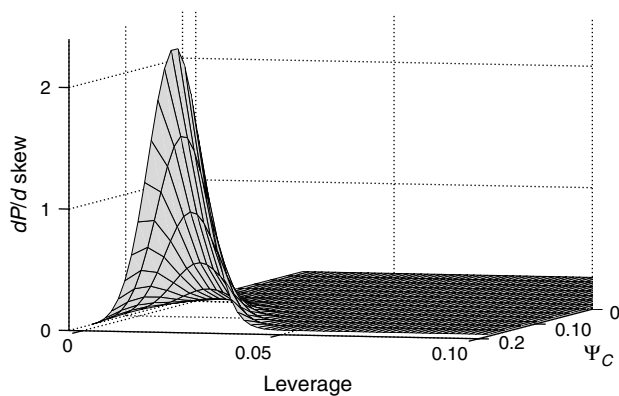
Typically, such an offsetting positive impact of the put option component arises for firms that are not highly levered. When leverage is high, the dominating effect on the price of equity comes from the first two terms in Equation (7), because the Delta, Vega, and Skewness sensitivities of the put price to changes in disagreement are small. For a leverage near to zero, the put option price and the price of the position in the zero bond are a negligible fraction of the firm value, so that the price of equity is dominated by the first term in Equation (7). However, for more intermediate leverage regions the skewness sensitivity of the out-of-the-money put to changes in disagreement can be large and the resulting impact on the price of equity substantial. Figure 3 illustrates the decomposition of the total put option effect on the price of equity, in terms of the resulting skewness put option sensitivity, as  $\Psi_C(t)$  varies from 0 to 0.2.<sup>10</sup>

We find that for a leverage between 0 and 0.03, the large skewness sensitivity of the put option component can make the price of equity increase in the degree of belief dispersion. This effect tends to vanish for low levels of disagreement and is zero in the homogeneous economy. The region for which disagreement and stock prices have a positive comovement depends on firm characteristics and, more generally, on the preference parameters in the model.

**REMARK 2.** When  $\gamma$  is integer, the stochastic discount factor of the representative investor in our economy can be written as  $\xi(t) = (\xi^1(t)^{1/\gamma} + \xi^2(t)^{1/\gamma})^\gamma$ .

<sup>10</sup> We do not plot the effects coming from the firm value ( $dV/d\Psi$ ) and the zero coupon bond ( $dZCB/d\Psi$ ) because the first effect is just a constant that is independent from leverage and the second effect is an increasing function of leverage that does not add to the changing sign. Notice that the Delta and Vega effects do not contribute to the changing sign either.

**Figure 3** Change in the Equity Price



*Notes.* This figure plots the change in the equity price due to the skewness effect as in Equation (7) as a function of the bond face value (leverage) and the level of disagreement,  $\Psi_C$ . We split the total variation of equity into three main effects: The Delta effect, which is due to a change in the firm value; the Vega effect, which is due to a change in the firm-value volatility; and a Skewness effect, which is due to a change in the risk-neutral skewness. The figure plots the skewness effect.

For  $\gamma = 2$ , as in our calibrations, one obtains  $\xi(t) = \xi^1(t) + 2\xi^1(t)^{1/2}\xi^2(t)^{1/2} + \xi^2(t)$ ; see Yan (2008) and Bhamra and Uppal (2013), among others. Substituting this decomposition of the stochastic discount factor in the valuation formula for the put default option, i.e.,  $E_t((\xi(T)/\xi(t))(K - V(T))^+)$ , we can quantify the contribution of each term to the put price. In contrast to the first and the last term, the middle term in the decomposition depends on a mixed interaction between the stochastic discount factors of the two agents in our model.<sup>11</sup> We decompose the total effect shown in Figure 3 in these three terms. Indeed, we find that the dominating term in generating the large skewness sensitivity of the put default option is due to the mixed interaction between stochastic discount factors. As the initial level of disagreement increases, this term becomes quite significant, especially for small values of leverage.

#### 4. Data Sets

To test the main implications of our model, we merge four data sets and match, for each firm, information on professional earnings forecasts, balance-sheet data, corporate bond spreads, and stock returns. The merged data set contains monthly information on 526 firms for the period from January 1996 to December 2007.

*Bond Data.* The bond data are obtained from the Fixed Income Securities Database (FISD) on corporate bond characteristics and the National Association of Insurance Commissioners (NAIC) database

on bond transactions. We eliminate all bonds with embedded optionalities, such as callable, puttable, exchangeable, convertible securities, bonds with sinking fund provisions, nonfixed coupon bonds, and asset-backed issues. To compute credit spreads, we use zero-coupon yields available from the Center for Research in Security Prices (CRSP).

*Disagreement Proxies.* To obtain our proxies of belief disagreement, we use forecasts of earnings per share, from the Institutional Brokers Estimate System (I/B/E/S) database. These data contain individual analyst's forecasts organized by forecast date and the last date the forecast was revised and confirmed as accurate. To circumvent the problem of using stock-split adjusted data, we use unadjusted data. We extend each forecast date to its revision date. If an analyst makes more than one forecast per month, we take the last forecast that was confirmed. In our model, agents disagree about the aggregate firm's earnings. We construct such a disagreement measure by using a market-capitalization weighted average of individual firm disagreement proxies. For each firm  $i$  and month  $t$ , we compute the median and the mean absolute deviation of analysts' earnings per share forecasts, obtained from the unadjusted I/B/E/S summary database. The market capitalization of stock  $i$  at the end of month  $t$  is defined as (the monthly stock closing price)  $\times$  (the number of shares outstanding), where these data are obtained from CRSP. The common disagreement proxy is then defined as

$$\overline{\text{DiB}}(t) = \sum_i \text{Mktcap}_i(t) \text{DiB}_i(t) / \sum_i \text{Mktcap}_i(t),$$

where  $\text{Mktcap}_i(t)$  is the market capitalization of firm  $i$  and  $\text{DiB}_i(t)$  is the individual cash-flow disagreement proxy for that firm.

*Other Control Variables.* Leverage is defined as total debt divided by the sum of total debt and the book value of shareholders' equity. Data are from Compustat. We proxy bond liquidity with the Fontaine and Garcia (2012) funding liquidity measure, which is the difference between prices of on the run and off the run Treasury bonds.

#### 5. Empirical Analysis

In this section, we test the main predictions of our model, using a consolidated panel data set of corporate credit spreads and stock returns. As a first test, we take our measure of aggregate disagreement and examine the model implied time series of corporate credit spreads. We then test the model predictions using linear regressions. First, we study the positive link between bottom-up measures of disagreement risk and credit spreads within a panel regression

<sup>11</sup> All terms in the decomposition of  $\xi(t)$  depend on  $\Psi_C(t)$ , since the optimal consumption of each agent depends on the beliefs of the other agent in equilibrium.

setting. Second, using a portfolio double-sorting approach, we analyze whether exposure to aggregate disagreement risk helps to systematically explain cross-sectional differences in bond and stock returns. Third, based on a two-step Fama and MacBeth (1973) regression approach, we formally test the hypothesis that disagreement is a priced risk factor for corporate bond and stock returns, while controlling for other well-known risk factors in the literature. Finally, we test whether the likelihood of a no-arbitrage violation in credit markets is increased in states of larger aggregate disagreement.

### 5.1. Model Implied Credit Spreads

To assess the ability of our calibrated model to generate empirically plausible time series of aggregate credit spreads, we take our proxy of aggregate disagreement,  $\overline{\text{DiB}}(t)$ , and back out the model-implied credit spreads in the time series. The result is presented in Figure 4, where we plot the model-implied credit spread together with the spread between the Moody's Baa and the Moody's Aaa bond yields. We depict the model-implied credit spread for different levels of wealth of the optimist. In the first case, we assume that the optimist holds 0.9 of the wealth. The second case looks at equal shares, and the last case shows what happens when the optimist holds 0.1 of the total wealth. In the last part of the sample, model-implied spreads underestimate their empirical counterpart. However, note that we calibrate the model to the average firm in our sample, using an average fixed cash-flow beta of 0.57. The unconditional correlation between the model-implied credit spread and the average credit spread in the data is 41%. The model-implied credit spreads pick up quite nicely the two spikes that occurred during the two National Bureau of Economic Research recessions in our sample, indicated by the gray bars. At the same time, we find that the large increase in credit spreads following the long-term capital management default cannot be explained by disagreement about aggregate consumption growth. Chen (2010) reports a long-term average

recovery rate of 41.4%. Using our calibrated parameters, we find that the average probability of default as implied by our model is 2%, which underestimates marginally the 2.7% in the data. Accordingly, our calibrated model implies an expected recovery rate of 52.3%. Again, these numbers are based on average values of leverage (0.2), disagreement (0.2), and an equally distributed wealth between optimists and pessimists.

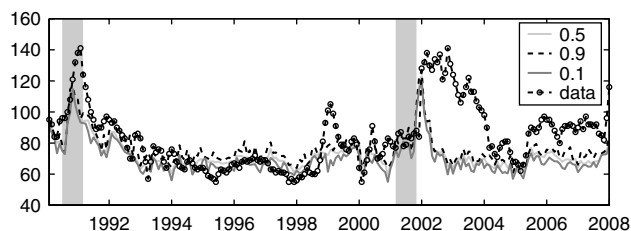
### 5.2. Credit Spread Panel Regression

Our model predicts an unambiguously positive link between the level of aggregate disagreement and the level of corporate credit spreads. It also implies a positive relation between credit spreads and equity volatility. To test these predictions, we estimate a simple panel regression for credit spreads. In addition to our bottom-up measure of aggregate disagreement ( $\overline{\text{DiB}}$ ) and to the VIX volatility index ( $\text{VIX}(t)$ ) as a proxy of aggregate stock market volatility, the regression includes other well-established control variables known to have explanatory power for corporate credit spreads ( $\text{CS}_i(t)$ )—the Fontaine and Garcia (2012) proxy of funding liquidity ( $\text{Liq}(t)$ ) and firm leverage ( $\text{Leverage}_i(t)$ ):

$$\text{CS}_i(t) = \beta_0 + \beta_1 \overline{\text{DiB}}(t) + \beta_2 \text{Leverage}_i(t) + \beta_3 \text{Liq}(t) + \beta_4 \text{VIX}(t) + \epsilon_i(t).$$

The results reported in Table 2 show that  $\overline{\text{DiB}}$  has a statically and economically significant positive impact on the level of corporate credit spreads. All other control variables are also significant and carry the correct sign. For instance, a higher liquidity lowers the average credit spread, and a higher level of VIX increases

**Figure 4** Model Implied and Empirical Credit Spreads



**Notes.** This figure plots the model implied credit spread for different levels of wealth of the optimist (0.1, 0.5, and 0.9) together with an aggregate measure of credit spreads that is the difference between the Moody's Baa and the Moody's Aaa bond yields. Data are monthly and run from 1990 to 2007.

**Table 2** Panel Regressions for Credit Spreads

Constant	0.657 (8.44)	0.652 (8.78)	0.574 (8.78)	0.675 (8.56)
DiB	0.701 (5.03)	0.634 (4.39)	0.657 (4.73)	0.674 (4.65)
Leverage		0.289 (5.01)	0.301 (5.23)	0.356 (4.89)
Liq			−0.108 (−1.98)	−0.124 (−2.12)
VIX				0.887 (5.65)
Adj. $R^2$	0.12	0.35	0.36	0.45

**Notes.** Using monthly data running from January 1996 to December 2007, we regress credit spreads on corporate bonds on a set of variables:

$$\text{CS}_i(t) = \beta_0 + \beta_1 \overline{\text{DiB}}(t) + \beta_2 \text{Leverage}_i(t) + \beta_3 \text{Liq}(t) + \beta_4 \text{VIX}(t) + \epsilon_i(t),$$

where  $\text{CS}_i(t)$  is the credit spread of firm  $i$ ,  $\overline{\text{DiB}}(t)$  is our proxy of common disagreement,  $\text{Leverage}_i(t)$  is leverage of firm  $i$ ,  $\text{Liq}(t)$  is the Fontaine and Garcia (2012) funding liquidity proxy, and  $\text{VIX}(t)$  is the implied volatility of options on the S&P 500. All estimations use autocorrelation and heteroskedasticity-consistent  $t$ -statistics, which are reported in parentheses.

bond spreads. As expected, a higher leverage also increases firm credit spreads.

The results are also economically meaningful in supporting the implications of the theoretical model. For example, the estimates in the last column of Table 2 suggest that credit spreads increase by five basis points ( $0.072 \times 0.674$ ) for any one standard deviation change in disagreement. This is comparable to the impact of a one standard deviation change in the VIX, which increases credit spreads on average by six basis points ( $0.064 \times 0.887$ ). Overall, these results support two of the theoretical predictions of our model. First, the positive relation between aggregate disagreement and credit spreads. Second, the positive link between credit spreads and equity volatility.

### 5.3. Disagreement Exposure, Credit Spreads, and Stock Returns

To investigate whether expected bond and stock returns are related to an exposure to aggregate disagreement shocks, we adopt a straightforward portfolio approach to create a universe of portfolio returns with a sufficient degree of dispersion in their beta with respect to aggregate disagreement shocks.

At the end of each month, we regress individual bond and stock excess returns ( $rx_i(t)$ ) onto the contemporaneous variation in the bottom-up disagreement proxy ( $\overline{\text{DiB}}(t)$ ), the VIX volatility ( $\Delta\text{VIX}(t)$ ), and the funding liquidity ( $\Delta\text{Liq}(t)$ ), using a window of 36 monthly returns:<sup>12</sup>

$$rx_i(t) = \beta_0^i + \beta_1^i \Delta\overline{\text{DiB}}(t) + \beta_2^i \Delta\text{VIX}(t) + \beta_3^i \Delta\text{Liq}(t) + \epsilon_i(t). \quad (8)$$

In this way, at the end of each month, we obtain an estimate of the conditional bond and stock exposure  $\beta_1^i$  to aggregate disagreement shocks, which is based on time-series information only up to time  $t$ .

At the end of each month, we sort bond and stock returns by their estimated disagreement exposure  $\hat{\beta}_1^i$  and assign them to five bond and stock portfolios. For each of these portfolios, we compute the portfolio postranking return over the following month, after which we repeat the estimation and portfolio formation procedure. This procedure gives rise to five monthly time series of sorted bond and stock portfolio returns. The postranking disagreement betas for each of these portfolios are collected in the last row of panel A in Table 3.

Portfolio disagreement betas are obtained by estimating regression (8) for each quintile portfolio return

over the full sample period. The postranking betas increase across bond and stock quintiles and generate a good degree of variation in exposure to aggregate disagreement shocks. For instance, the difference between the largest and smallest portfolio betas is strongly significant and amounts to 1.26 and 1.80, respectively, for the bond and stock portfolios.

The first row of panel A in Table 3 reports for each bond and stock quintile portfolio the estimated average excess return, together with its  $t$ -statistic. We find an average return that increases almost monotonically with the postranking disagreement beta of the bond and stock portfolios. For instance, the long-short spread portfolio between the bonds with the highest and lowest disagreement beta has a significant average annualized excess return of about 0.38%. The long-short portfolio between the stocks with highest and lowest disagreement beta implies a larger average excess return of about 0.71%. Moreover, as expected, portfolios of bonds with a larger exposure to disagreement shocks systematically imply larger credit spreads. For instance, while the average credit spread of bonds in the top quintile of disagreement exposure is 62 basis point, the one in the bottom quintile is only 30 basis points. This difference corresponds to about half the sample standard deviation of credit spreads in our sample.

In summary, this evidence supports the main model prediction that different exposures to aggregate disagreement risk are systematically linked to cross-sectional differences in excess bond and stock returns. In our model, disagreement increases bond returns and credit spreads independently of the leverage level. For stock returns, however, a potentially reversed relationship can arise for low leverage vis-à-vis average or high leverage firms. To test this model prediction in more detail, we follow a portfolio approach based on a double-sorting procedure. After having computed in each month preranking disagreement betas for individual bonds and stocks based on regression model (8), we double sort bond and stock returns into terciles of disagreement beta and terciles of firm leverage and assign them to nine bond and stock portfolios. For each portfolio, we then compute the postranking return over the following month, after which we repeat the estimation and portfolio formation procedure. The results of this double-sorting approach are reported in panel B of Table 3.

For bond returns, we find in line with the model that average portfolio returns increase with disagreement betas, irrespective of the average degree of leverage in the portfolio. For stock returns, we find a different result for the lowest leverage tercile. Precisely, whereas the portfolios in the top and medium leverage terciles imply average returns that monotonically increase with disagreement betas, those in

<sup>12</sup> As a robustness check, we also used additional risk factors in our regressions, such as the standard Fama and French factors, but the results on the link between disagreement exposure, bond and stock returns are unaffected by this choice.



**Table 3** Priced Disagreement Risk

Panel A: Disagreement-sorted bond and stock return portfolios														
	Bond returns					Stock returns								
	D1 (low)	D2	D3	D4	D5 (high)	D1 (low)	D2	D3	D4	D5 (high)				
Mean	0.240 (2.88)	0.291 (3.05)	0.375 (2.58)	0.597 (2.35)	0.621 (3.61)	0.41 (2.33)	0.53 (3.12)	0.89 (2.87)	0.94 (3.01)	1.12 (2.46)				
Mean CS	0.298	0.406	0.547	0.598	0.622									
Post $\beta$	0.125	0.253	0.456	0.903	1.384	0.187	0.839	0.714	1.373	1.984				
Panel B: Double-sorted bond and stock return portfolios														
	Bond returns				Stock returns									
	D1 (low)	D2	D3 (high)	D1-D3	D1 (low)	D2	D3 (high)	D1-D3						
L1 (low)	0.155 (2.98)	0.209 (3.05)	0.228 (4.11)	−0.073 (−1.68)	L1 (low)	0.75 (2.42)	0.64 (1.81)	0.31 (2.01)	0.44 (1.87)					
L2	0.485 (2.22)	0.607 (3.57)	0.711 (3.14)	−0.226 (−2.22)	L2	0.85 (1.87)	0.98 (4.01)	1.05 (2.14)	−0.20 (−2.05)					
L3 (high)	0.651 (4.12)	0.716 (4.26)	1.002 (4.87)	−0.351 (−4.36)	L3 (high)	0.91 (3.88)	1.03 (2.85)	1.14 (3.16)	−0.23 (−3.66)					
L1-L3	−0.496 (−4.35)	−0.507 (−3.85)	−0.774 (−3.05)		L1-L3	−0.16 (−2.37)	−0.39 (−2.45)	−0.83 (−3.84)						
Panel C: Cross-sectional regression estimates														
	Bond returns							Stock returns						
	$\lambda^{\text{Liq}}$	$\lambda^{\text{VIX}}$	$\lambda^{\text{DIB}}$	$\lambda^{\text{MRKT}}$	$\lambda^{\text{SMB}}$	$\lambda^{\text{HML}}$	$R^2$	$\lambda^{\text{Liq}}$	$\lambda^{\text{VIX}}$	$\lambda^{\text{DIB}}$	$\lambda^{\text{MRKT}}$	$\lambda^{\text{SMB}}$	$\lambda^{\text{HML}}$	$R^2$
Estimate	−0.035 (−3.12)	0.025 (5.65)	0.062 (4.22)	−0.057 (−1.99)	−0.024 (−1.32)	−0.063 (−1.93)	0.84	−0.032 (−2.17)	−0.055 (−10.78)	0.074 (6.95)	0.097 (2.01)	−0.051 (−1.38)	−0.003 (−0.89)	0.78

*Notes.* Using monthly data running from January 1996 to December 2007, we sort corporate bond and stock returns into portfolios according to their exposure to common disagreement as in Equation (8) (panel A). D1 (D5) represents those portfolios that have a low (high) estimated beta. The numbers are average monthly spreads and returns, respectively. We also report the average credit spread of firms in any particular bin and postsorting disagreement betas in the last row. In panel B, we double sort according to the assets exposure to common disagreement and leverage. D1 (D3) represents those assets with a low (high) disagreement exposure, L1 (L3) represent those assets that have low (high) leverage. D1-D3 (L1-L3) represent the low minus high disagreement (leverage) portfolio. Panel C reports the prices of risk in the second step of the Fama and MacBeth (1973) regression. The  $t$ -statistics are in parentheses.

the bottom tercile imply a monotonically decreasing relation. For instance, although the long–short spread portfolio of stocks with high and low disagreement betas (portfolio D3-D1) has a positive average return of 0.20% and 0.23%, respectively, in the medium and top leverage terciles, the same portfolio has a negative return of −0.44% in the lowest leverage tercile.

In summary, this last set of results is consistent with the potentially ambiguous relation between stock returns and disagreement risk predicted by our model. At the same time, it confirms the unambiguously positive relation between disagreement risk, credit spreads, and bond returns predicted by our model.

#### 5.4. The Market Price of Disagreement Risk

In the previous section, we have relied on the spread in the average return of sorted bond and stock portfolios to infer the sign of the expected return premium for aggregate disagreement risk exposure. Based on a two-step cross-sectional Fama and MacBeth (1973)

approach, we now estimate these premia and the market price of aggregate disagreement risk using the nine double-sorted portfolio returns in §5.3.

In a first step, we follow the standard procedure in, e.g., Ang et al. (2006), among many others, to construct three factor mimicking portfolios for the untraded risk factors  $\Delta \text{DiB}(t)$ ,  $\Delta \text{Liq}(t)$  and  $\Delta \text{VIX}(t)$  in model (8). We denote the return of these factor mimicking portfolios by  $\text{FDiB}(t)$ ,  $\text{FLiq}(t)$  and  $\text{FVIX}(t)$ , respectively.

In a second step, we regress the time series of returns of the double-sorted portfolios on the factor mimicking portfolio returns for disagreement, liquidity, VIX, the aggregate stock market return ( $\text{RM}(t)$ ), and the two Fama and French risk factors ( $\text{SMB}(t)$  and  $\text{HML}(t)$ ):

$$rx_i(t) = \alpha^i + \beta_1^i \text{FDiB}(t) + \beta_2^i \text{FVIX}(t) + \beta_3^i \text{FLiq}(t) + \beta_4^i \text{RM}(t) + \beta_5^i \text{SMB}(t) + \beta_6^i \text{HML}(t) + e_i(t),$$

where  $rx_i(t)$  is the excess return of the double-sorted portfolios  $i = 1, \dots, 9$  in §5.3. Finally, we regress the average excess return of the double-sorted portfolios on the vector of estimated factor exposures, in order to test the pricing equation:

$$E(rx_i(t)) = \lambda_{\Psi_C} \beta_{\Psi_C}^i + \lambda_{VIX} \beta_{VIX}^i + \lambda_{Liq} \beta_{Liq}^i + \lambda_{RM} \beta_{RM}^i \\ + \lambda_{SMB} \beta_{SMB}^i + \lambda_{HML} \beta_{HML}^i,$$

where  $\lambda = (\lambda_{\Psi_C}, \lambda_{VIX}, \lambda_{Liq}, \lambda_{RM}, \lambda_{SMB}, \lambda_{HML})'$  denotes the vector of market prices of factor risk.

The estimated market prices of factor risk are presented in panel C of Table 3. Both for the cross section of corporate bond and stock returns, we find that the risk factors for disagreement, market volatility, liquidity, and market risk are linked to an economically and statistically significant nonzero factor price of risk. In contrast, the only Fama and French factor that is significantly priced is HML( $t$ ) for the cross section of bond returns.

Interestingly, the market price of disagreement and liquidity risk is positive and negative, respectively, for both the cross section of corporate bond and stock returns. In contrast, the market price of VIX volatility risk and market risk has a different sign for the cross sections of bond and stock returns.

We find that aggregate disagreement carries a statistically and economically significant risk premium of 0.062% and 0.074%, on a monthly basis, for bond and stock returns, respectively. Given the positive bond and stock disagreement betas in panel A of Table 3, this finding is in line with the positive risk premia for disagreement risk implied by the long–short spread portfolios D5–D1 in panel A of Table 3.

In summary, our findings support the main model implications of a priced aggregate disagreement risk component in the cross section of asset returns, generating a cross section of bond and stock risk premia that is broadly consistent with the empirical evidence.

### 5.5. Capital Structure No-Arbitrage Violations

Capital structure arbitrage has become increasingly popular among long–short, multistrategy, and event driven hedge funds. The success of these strategies depends on the empirical realism of key assumptions about the joint behavior of the value of debt and equity, such as the positive sensitivity of corporate bond prices to changes in the price of equity. Anecdotal evidence suggests that this relationship can fail in the data. The last prediction of our model, which we test in this section, is the existence of a systematic link between the likelihood of a no-arbitrage violation, the level of aggregate disagreement, and firm leverage.

To study how belief disagreement relates to the conditional probability of a violation in credit markets, defined by either a positive or a negative common

variation in credit spreads and stock returns, we estimate a set of logit panel regressions. In these regressions, the binary variable  $y_i(t)$  indicates the event of a violation at time  $t$  for firm  $i$ . We specify the probability of a no-arbitrage violation with the following logit model:

$$P(y_i(t) = 1) = F\left(\beta_0 + \beta_1 \log \overline{\text{DiB}}(t) + \beta_2 \log \text{Liq}(t) + \sum_{j=1}^2 \delta_j \log S^{ij}(t)\right), \quad (9)$$

where  $F$  is the cumulative distribution function of a logistic distribution,  $\beta_1$  the loading on the common disagreement proxy,  $\beta_2$  the loading on the liquidity proxy, and  $\delta_j$ ,  $j = 1, 2$ , the loadings on the firm option-implied volatility and leverage ( $S^{i1}(t)$  and  $S^{i2}(t)$ ). We estimate model (9) by maximum-likelihood and collect the estimation results in Table 4.

Consistent with the model predictions, we find that common disagreement is positively related to the conditional probability of a violation, with an estimated coefficient that is highly significant and stable across different leverage terciles. Firm leverage also positively and significantly impacts the conditional probability of a violation. In contrast, liquidity risk and the firm option-implied volatility fail to generate a significant relation across the different leverage terciles.

In summary, these findings are consistent with the theoretical predictions of our model on the positive link between the frequencies of credit market no-arbitrage violations and aggregate disagreement risk.

**Table 4** Logit Regression of Arbitrage Violations on Credit Markets

	Low	Medium	High
Constant	−0.12 (−2.12)	−0.205 (−3.15)	−0.265 (−2.87)
Common DiB	0.156 (3.25)	0.169 (3.06)	0.166 (2.98)
Liquidity	0.053 (1.00)	0.065 (1.22)	0.058 (0.87)
IV	0.124 (2.01)	0.115 (1.78)	0.117 (1.69)
Leverage	0.133 (2.38)	0.184 (2.41)	0.126 (2.55)
Pseudo $R^2$	0.15	0.16	0.15

*Notes.* This table summarizes the logit regression results for the violation frequency in credit markets for different levels of leverage. Low leverage refers to firms with a leverage ratio of 11% or lower, medium leverage are firms with a leverage ratio of between 11% and 26%, and high leverage refers to firms with a leverage of 26% and larger. The probability that a violation event occurs is specified as  $P(y^i(t) = 1) = F(\beta_0 + \beta_1 \log \overline{\text{DiB}}(t) + \beta_2 \log \text{Liq}(t) + \sum_{j=1}^2 \delta_j \log S^{ij}(t))$ , where  $\overline{\text{DiB}}(t)$  is the common disagreement,  $\text{Liq}(t)$  is the Fontaine and Garcia (2012) funding liquidity, and  $S^{i1}(t)$  and  $S^{i2}(t)$  are the implied volatility and leverage of firms 1 and 2, respectively. The  $t$ -statistics are in parentheses.

## 6. Conclusion

We specify a frictionless Merton (1974)–type credit risk model, in which we derive the implications of the equilibrium risk-sharing mechanism between agents with heterogeneous beliefs about aggregate consumption growth. In contrast to most of the literature, we specify individual firm cash flows that are the sum of a firm specific and a systematic component proportional to aggregate consumption. Agents' disagreement about this common component produces systematic risk-sharing effects that explain an economically significant part of the cross section of credit spreads and asset risk premia. Using the model solutions, we obtain a number of testable empirical predictions.

First, we find that a higher disagreement about aggregate consumption growth unambiguously increases credit spreads and stock return volatilities. Second, it can reduce the expected excess stock return of low leverage firms, whereas it increases the stock risk premium of average and highly levered firms. Third, the positive link between the stock returns of low leverage firms and disagreement risk can explain the likelihood of a credit market no-arbitrage violations of single factor models.

Using a consolidated panel of individual firm earnings forecasts, credit spreads, and stock returns, we test the model predictions using a set of cross-sectional, panel, and logit regressions. Our empirical study produces a number of results that are in line with the theoretical model predictions. First, sorting credit spreads according to their exposure to bottom-up measures of aggregate disagreement produces a significant positive difference between the average credit spreads of high and low exposure bond portfolios. The same exercise applied to stock returns implies a reversed result for firms in the lowest leverage tercile. This feature helps to explain the likelihood of an empirical violation of capital structure arbitrage strategies, which is found to be positively related to the level of aggregate disagreement based on a panel logit regression. Finally, using the two-step Fama and MacBeth (1973) methodology, we test whether aggregate disagreement risk is priced in the cross section of bond and stock returns. We find that exposure to aggregate disagreement shocks is linked to an economically and statistically significant annual risk premium of almost 1% for both bond and stock returns.

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## Appendix

### Proofs

PROOF OF PROPOSITION 1. For simplicity, we drop time indices. We want to compute

$$F(C, \eta, m_C^1, \Psi_C, \epsilon, \chi) = \mathbb{E}_{m_C^1, C}^1(C^\epsilon, \eta^\chi),$$

which satisfies the following partial differential equation:

$$0 \equiv \mathfrak{L}F(C, \eta, m_C^1, \Psi_C, \epsilon, \chi) + \frac{\partial F}{\partial t}(C, \eta, m_C^1, \Psi_C, \epsilon, \chi),$$

where  $\mathfrak{L}$  is the differential generator of  $(C, \eta, m_C^1, \Psi_C)$  under the probability measure of agent 1. We then get

$$\begin{aligned} 0 = & \frac{\partial F}{\partial C} C m_C^1 + \frac{\partial F}{\partial m_C^1} (a_{0C} + a_{1C} m_C^1) + \frac{\partial F}{\partial \Psi_C} \left( a_{1C} + \frac{\gamma^2}{\sigma_C^2} \right) \Psi_C \\ & + \frac{1}{2} \frac{\partial^2 F}{\partial C^2} (C \sigma_C)^2 + \frac{1}{2} \frac{\partial^2 F}{\partial (m_C^1)^2} \left( \frac{\gamma^1}{\sigma_C} \right)^2 + \frac{1}{2} \frac{\partial^2 F}{\partial \Psi_C^2} \left( \frac{\gamma^1 - \gamma^2}{\sigma_C^2} \right)^2 \\ & + \frac{1}{2} \frac{\partial^2 F}{\partial \eta^2} (\eta \Psi_C)^2 + \frac{\partial^2 F}{\partial C \partial m_C^1} \gamma^1 C + \frac{\partial^2 F}{\partial C \partial \Psi_C} \left( \frac{\gamma^1 - \gamma^2}{\sigma_C} \right) C \\ & - \frac{\partial^2 F}{\partial C \partial \eta} C \eta \Psi_C \sigma_C + \frac{\partial^2 F}{\partial m_C^1 \partial \Psi_C} \frac{\gamma^1 (\gamma^1 - \gamma^2)}{\sigma_C^3} \\ & - \frac{\partial^2 F}{\partial m_C^1 \partial \eta} \eta \Psi_C \frac{\gamma^1}{\sigma_C} - \frac{\partial^2 F}{\partial \Psi_C \partial \eta} \Psi_C \left( \frac{\gamma^1 - \gamma^2}{\sigma_C^2} \right) \eta + \frac{\partial F}{\partial t}. \end{aligned}$$

The solution to this partial differential equation takes the functional form

$$F = C^\epsilon \eta^\chi \tilde{F}.$$

Therefore, by factoring out  $\eta$  and  $C$ , we get

$$\begin{aligned} 0 = & \tilde{F} \epsilon m_C^1 + \frac{\partial \tilde{F}}{\partial m_C^1} (a_{0C} + a_{1C} m_C^1) + \frac{1}{2} \epsilon (\epsilon - 1) \tilde{F} \sigma_C^2 \\ & + \frac{1}{2} \frac{\partial^2 \tilde{F}}{\partial (m_C^1)^2} \left( \frac{\gamma^1}{\sigma_C} \right)^2 + \frac{\partial \tilde{F}}{\partial m_C^1} \epsilon \gamma^1 + \frac{\partial \tilde{F}}{\partial \Psi_C} \left( a_{1C} + \frac{\gamma^2}{\sigma_C^2} \right) \Psi_C \\ & + \frac{1}{2} \frac{\partial^2 \tilde{F}}{\partial \Psi_C^2} \left( \frac{\gamma^1 - \gamma^2}{\sigma_C^2} \right)^2 + \frac{1}{2} \chi (\chi - 1) \Psi_C^2 \tilde{F} \\ & + \frac{\partial \tilde{F}}{\partial \Psi_C} \epsilon \left( \frac{\gamma^1 - \gamma^2}{\sigma_C} \right) - \epsilon \chi \Psi_C \sigma_C \tilde{F} + \frac{\partial^2 \tilde{F}}{\partial m_C^1 \partial \Psi_C} \frac{\gamma^1 (\gamma^1 - \gamma^2)}{\sigma_C^3} \\ & - \frac{\partial \tilde{F}}{\partial m_C^1} \chi \Psi_C \frac{\gamma^1}{\sigma_C} - \frac{\partial \tilde{F}}{\partial \Psi_C} \Psi_C \chi \left( \frac{\gamma^1 - \gamma^2}{\sigma_C^2} \right) + \frac{\partial \tilde{F}}{\partial t}. \end{aligned}$$

Collecting terms, we find that

$$F_{m_C^1}(m_C^1, \epsilon) = \exp(A_m + B_m m_C^1),$$

where

$$A_m = \frac{\epsilon(\exp(a_{1C}) - 1)}{a_{1C}}$$

and

$$B_m = \frac{1}{2} \epsilon (\epsilon - 1) \sigma_C^2 + \frac{1}{a_{1C}} (a_{0C} + \epsilon \gamma^1) \exp(-a_{1C}) \\ + \frac{1}{a_{1C}} \left( \frac{\gamma^1}{\sigma_C} \right)^2 \left( \frac{3}{2} \exp(a_{1C}) - a_{1C} \right).$$

Solution of the rest is then

$$F_{\Psi_C}(\Psi_C, t, s, \epsilon, \chi) = \exp(A + B\Psi_C + C\Psi_C^2).$$

We know that

$$\frac{\partial \tilde{F}}{\partial \Psi_C} = \tilde{F}(B + 2C\Psi_C), \quad \frac{\partial^2 \tilde{F}}{\partial \Psi_C^2} = \tilde{F}((B + 2C\Psi_C)^2 + 2C), \\ \frac{\partial \tilde{F}}{\partial t} = -\tilde{F}(A' + B'\Psi_C + C'\Psi_C^2).$$

We, therefore, get

$$0 = (B + 2C\Psi_C) \left( a_{1C} + \frac{\gamma^2}{\sigma_C^2} \right) \Psi_C \\ + \frac{1}{2} ((B + 2C\Psi_C)^2 + 2C) \left( \frac{\gamma^1 - \gamma^2}{\sigma_C^2} \right)^2 \\ + \frac{1}{2} \chi (\chi - 1) \Psi_C^2 + (B + 2C\Psi_C) \left( \frac{\gamma^1 - \gamma^2}{\sigma_C} \right) - \chi \Psi_C \sigma_C \\ + A_m (B + 2C\Psi_C) \frac{\gamma^1 (\gamma^1 - \gamma^2)}{\sigma_C^3} - A_m \chi \Psi_C \frac{\gamma^1}{\sigma_C} \\ - (B + 2C\Psi_C) \Psi_C \chi \left( \frac{\gamma^1 - \gamma^2}{\sigma_C^2} \right) - (A' + B'\Psi_C + C'\Psi_C^2).$$

We know, therefore, that the parameters must solve the following ordinary differential equations:

$$C' = cC^2 - 2bC + c, \quad (10)$$

$$B' = B(aC - b) + 2(ne^{a_{1C}} + m)C - \chi \left( \frac{e^{a_{1C}}}{a_{1C}} \frac{\gamma^1}{\sigma_C} + \sigma_C \right), \quad (11)$$

$$A' = B \left( \frac{a}{4} B + m + n \frac{e^{a_{1C}}}{a_{1C}} \right), \quad (12)$$

where

$$a = \frac{1}{2} \chi (\chi - 1), \quad b = \chi \left( \frac{\gamma^1 - \gamma^2}{\sigma_C^2} - \left( a_{1C} + \frac{\gamma^2}{\sigma_C^2} \right) \right), \\ c = 2 \left( \frac{\gamma^1 - \gamma^2}{\sigma_C^2} \right), \quad m = \frac{\gamma^1 - \gamma^2}{\sigma_C^2}, \quad n = \frac{\gamma^1 (\gamma^1 - \gamma^2)}{\sigma_C^3}.$$

Except for the first one, all the ordinary differential equations are first-degree linear and can be easily solved. Equation (10) is quadratic, but can be linearized using Radon's lemma.

## Risk-Neutral Skewness

It follows from Bakshi and Madan (2000) that the entire collection of twice-continuously differentiable payoff functions with bounded expectation can be spanned algebraically. Applying this result to the firm value  $V(t)$  (or equivalently to the stock value  $S(t)$ ) yields

$$G(V) = G(\tilde{V}) + (V - \tilde{V})G_V(\tilde{V}) + \int_{\tilde{V}}^{\infty} G_{VV}(K)(V - K)^+ dK \\ + \int_0^{\tilde{V}} G_{VV}(K)(K - V)^+ dK,$$

where  $G_V$  is the partial derivative of the payoff function  $G(V)$  with respect to  $V$ , and  $G_{VV}$  is the corresponding second-order partial derivative. By setting  $\tilde{V} = V(t)$ , we obtain the final formula for the firm-value risk-neutral skewness, after applying the same steps as in Bakshi et al. (2003, Theorem 1).

**PROPOSITION 2.** Let  $v(t, T) = \ln(V(t+T)) - \ln(V(t))$  be the firm-value return between time  $t$  and  $T$ . The risk-neutral skewness of  $v(t, T)$  is given by

$$\text{skew}(t, T) = \frac{E_t((v(t, T) - E_t(v(t, T)))^3)}{(E_t(v(t, T) - E_t(v(t, T)))^2)^{3/2}} \\ = \frac{e^{rT} W(t, T) - 2\mu(t, T)e^{rT} R(t, T) + 2\mu(t, T)^3}{(e^{rT} R(t, T) - \mu(t, T)^2)^{3/2}},$$

where

$$R(t, T) = \int_{V(t)}^{\infty} \frac{2(1 - \ln(K/V(t)))}{K^2} (V(T) - K)^+ dK \\ + \int_0^{V(t)} \frac{2(1 + \ln(V(t)/K))}{K^2} (K - V(T))^+ dK,$$

$W(t, T)$

$$= \int_{V(t)}^{\infty} \frac{6\ln(K/V(t)) - 3(\ln(K/V(t)))^2}{K^2} (V(T) - K)^+ dK \\ - \int_0^{V(t)} \frac{6\ln(V(t)/K) - 3(\ln(V(t)/K))^2}{K^2} (K - V(T))^+ dK,$$

and

$$X(t, T) = \int_{V(t)}^{\infty} \frac{12(\ln(K/V(t)))^2 - 4(\ln(K/V(t)))^3}{K^2} (V(T) - K)^+ dK \\ - \int_0^{V(t)} \frac{12(\ln(V(t)/K))^2 - 4(\ln(V(t)/K))^3}{K^2} (K - V(T))^+ dK,$$

$$\mu(t, T) = E_t \left( \ln \left( \frac{V(t+T)}{V(t)} \right) \right) \\ \approx e^{rT} - 1 - \frac{e^{rT}}{2} R(t, T) - \frac{e^{rT}}{6} W(t, T) - \frac{e^{rT}}{24} X(t, T).$$

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