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Managing Storable Commodity Risks: The Role of Inventory and Financial Hedge

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We study how to manage commodity risks (price and consumption volume) via physical inventory and financial hedge in a multiperiod problem (with an interperiod utility function) for a risk-averse firm procuring a storable commodity from a spot market at a random price and a long-term supplier at a fixed price. The firm also has access to financial contracts written on the commodity price, such as futures contracts and call and put options. We examine different cases of financial hedging, for example, single-contract and multicontract hedges. For each case, we dynamically maximize the mean-variance utility of the firm's cash flow and characterize an optimal integrated policy of inventory and hedging, which is easy to compute and implement. We find that as long as futures are used in each period, alone or not, the optimal inventory policy is myopic. The optimal hedging policy, however, is never myopic, but depends on all the future optimal decisions. This is contrary to findings of the literature using intraperiod utility functions, which finds myopic hedging to be optimal. Moreover, we find that hedging may lead to inventory reduction in multiperiod problems. Thus the insights from the single-period studies in the literature—hedging leads to inventory increase—do not apply. Finally, insights are offered on the role and impact of inventory and financial hedge on profitability, variance control, and service level, using both analytical and numerical results.

Key words: stochastic inventory; commodity markets; futures; options; risk management; hedging; risk aversion
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1. Introduction

The sourcing, inventory storing, and processing of storable commodities, which are eventually sold in the form of differentiated goods to end-product markets, are cornerstone activities of many business strategies. Examples of commodities, storable and tradable on exchanges, are oil, steel, precious metals, corn, sugar, dynamic random access memories, etc. However, commodity risks can jeopardize even the best thought-out strategies (Tevelson et al. 2007). These days, commodity price risks are even more pronounced and unexpected than before because of shifts in supply-and-demand dynamics and global financial turmoil (Fisher and Kumar 2010). Prices of many commodities are now fluctuating as much in a single day as they did in a year in the early 1990s (Wiggins and Blas 2008).

For companies that rely on such commodities as production inputs and cannot pass cost increases to their customers, such volatility substantially increases their working capital needs and risks of financial distress. As a result, procurement organizations are playing pivotal roles in the financial success of such firms, and purchasing managers are expected to have skills never required before. For example, food companies are in search of procurement managers with commodity trading skills (Wiggins 2008). As argued and shown empirically by many finance researchers, hedging to reduce costly cash flow variability can increase the firm value (Froot et al. 1993).

Commodity inputs contribute to firms' cash flow volatility in terms of not only the *price risk* (the price volatility), but also the *consumption volume risk* (the volatility of consumed commodity volume to meet the uncertain end-product demand). Traditionally, these two types of risks are treated and hedged separately; the former is hedged by commodity derivatives, and the latter is hedged by physical inventories. This "soiled" approach, however, is a problem, especially in a large multibusiness organization (Fisher and Kumar 2010). As argued in

Kleindorfer (2009), integrating inventory and financial derivatives for an effective hedging of commodity risks creates research challenges. In particular, simultaneously optimizing inventory levels and derivative portfolios is a difficult problem, with only limited answers, yet mostly for nonstorable commodities like electricity. This paper attempts to offer some answers to this integrated risk management problem for storable commodities.

The fundamental setting of our problem is that a firm procures a storable commodity input from a long-term supplier and a commodity exchange (spot market), where selling the commodity is also possible. The firm also has access to financial contracts written on the commodity spot price. The firm processes the commodity input to make a differentiated end product, facing uncertain demands. The firm aims to hedge its cash flow volatility through both physical inventory and financial hedge for the commodity. For example, Emerson Motor Technologies, headquartered in St. Louis, Missouri, and selling a large product line of electromechanical motors for various applications, has a long-term contract (with a typical contract duration of three years) with U.S. Steel on purchasing various grades of steel with negotiated terms: a fixed quantity and a fixed price. To meet its regional needs (it has factories in the United States, Mexico, and China), the company works with various metal exchanges for spot purchase and sell. These exchanges include the London Metal Exchange, the CME Group and the China-based Shanghai Futures Exchange. Facing the rising commodity risks, Emerson Motor Technologies is interested in hedging using commodity derivatives or financial contracts, also available at these exchanges, with futures contracts being the most commonly offered. The above information was obtained from discussions with Ray Keefe, vice president of manufacturing, and Ken Poczekaj, vice president of global supply chain of Emerson, in June 2011.

In this paper, we dynamically maximize the firm's cash flow under a mean-variance (MV) criterion to determine jointly optimal policies for inventory and financial hedging. Note that futures and forward contracts, although possessing practical differences, are treated the same analytically (Geman 2005). Unlike forward contracts, which are typically custom bilateral contracts with banks, futures contracts are offered in standard forms and are tradable at exchanges and thus have better liquidity. Therefore, we consider futures rather than forward contracts in this paper. We investigate the interaction between inventory and financial hedging and their impacts on the firm's inventory level, mean profit, profit variance, and MV utility.

2. Literature Review

Our work falls under the general themes of "integrating physical and financial risk management in supply chains" and "hedging commodity risks in supply management," which are both expertly reviewed by Kleindorfer (2009, 2010). For an earlier literature review on supply contracts and spot markets, please see Kleindorfer and Wu (2003). The more general field of supply contracts is of passing relevance to our work, and we refer the readers to Cachon (2003). In this section, we first briefly review the research on integrated long-term and short-term (spot market) contracts. Note that the long-term contract terms (quantity and price) in this paper are exogenously given to reflect the practice at Emerson and similar businesses. We then review in detail the works most relevant to our paper and highlight our contributions to the literature.

A general single-period framework is presented in Wu and Kleindorfer (2005) for integrating long-term and short-term contract decisions for mostly nonstorable goods. Please see references therein for a more extensive review of the related work on nonstorable commodities. For storable commodities, much of the existing literature addresses various types of sourcing issues for risk-neutral decision makers, and thus hedging is unnecessary. For example, Lee and Whang (2002) were the first to integrate spot market considerations after sales within a newsvendor framework, and thus effectively endogenize the salvage value used in these models. Martínez-de-albéniz and Simchi-Levi (2005) use a multiperiod model to address the optimal creation of a portfolio of long-term contracts (including fixed commitment and flexibility contracts) integrated with potential spot market purchases. With rich institutional details of the fed-cattle supply chain, Boyabatli et al. (2011) offer a lucid picture of a beef processor's problem in these environments via a stylized single-period model, in the presence of spot market transaction costs, economies of scale in processing, quality differences, and correlated end-product demand. The work by Devalkar et al. (2011) is an example concerning risk aversion. Motivated by soybean processing, it analyzes the integrated procurement, processing, and selling decisions for a risk neutral/averse commodity processor in a multiperiod setting. Input commodities are procured from a spot market, processed, and then sold as commodities in a futures market. In contrast to ours, this work covers more operational details, but lacks concerns of financial hedging and demand uncertainty.

There is limited research on joint optimization of operational and hedging decisions. Most of it uses single-period settings to study the interaction between operational decisions (capacity and/or inventory) and hedging decisions under uncertainties in currency exchange rate, demand, weather, commodity prices, etc. For example, Ding et al. (2007) study the optimal policies for capacity investment and hedging on currency exchange rates for a risk-averse multinational newsvendor and find that the futures contract is the optimal hedge; Gaur and Seshadri (2005) study the optimal hedging for a risk-averse newsvendor with any given inventory level, based on the empirically proven assumption that the newsvendor's demand is correlated with the price of a financial asset. Both works show that financial hedging drives up inventory/capacity. Caldentey and Haugh (2006) extend the work of Gaur and Seshadri (2005) by allowing continuous trading in the financial market; Chod et al. (2010) extend the work of Gaur and Seshadri (2005) by implicitly characterizing the optimal multidimensional capacity investment and focusing on the complementarity/substitution effect between the operational (postponement and product) flexibility and financial hedging. Similar to our paper, Oum et al. (2006), Bodily and Palacios (2007), and Caldentey and Haugh (2009) study commodity procurement with financial hedging, but for nonstorable commodities like electricity and liquid natural gas. Note that since such commodities cannot be carried over to a next period, single-period models are sufficient for study. In contrast to the single-period works reviewed above, which all assume a fair-priced financial hedging, we study risk management for storable commodities using a multiperiod model without the fair-priced financial hedging assumption.

We next review the research on joint optimization of operational and hedging decisions in multiperiod settings. To the best of our knowledge, the works by Smith and Nau (1995), Chen et al. (2007), and Zhu and Kapuscinski (2011) are the only three in this stream. Among them, the one by Smith and Nau (1995) is most general in terms of joint optimization of operational and hedging decisions, but in a generic project valuation setting; the other two have specific operational settings. Chen et al. (2007) study an inventory problem with multiple supply contracts and simply apply the optimal hedging results of Smith and Nau (1995). Zhu and Kapuscinski (2011) study a complex capacity allocation problem, but the optimal hedging results of Smith and Nau (1995) are still applicable. To effectively contrast these works to ours in terms of model formulation, we first introduce two different types of utility function for multiperiod risk-averse problems: interperiod and intraperiod utility functions (for more detailed definitions and comparisons, see Alexander and Sobel 2006). We then compare our work to the three works mentioned above to demonstrate our contributions in terms of model formulation and managerial insights.

Briefly speaking, *interperiod utility* functions count in the cash flow correlations across periods, whereas *intraperiod utility* functions do not. An example of intraperiod utility is the additive exponential utility used by all the multiperiod works mentioned above, where "additive" means that the utility of the total cash flow in multiple periods is a sum of the discounted utility of each period's cash flow. In contrast, we use an interperiod utility function. More importantly, as shown by Alexander and Sobel (2006), intraperiod utility functions under frequently encountered conditions may imply risk neutrality, and thus interperiod utility functions are the most appropriate ones to use for risk-averse decision makers in multiperiod settings. Thus, although more challenging to handle, our formulation is new and richer in capturing the cash flow correlations across periods and fully reflecting risk aversion in the multiperiod context.

Smith and Nau (1995) study profitable integration of option pricing and decision analysis methods in project valuation, where, however, the investment decisions for a project to be valued and the financial decisions are conceptually equivalent to the inventory decisions and the hedging decisions in our problem, respectively. They prove the separation theorem. This theorem is directly applied by Chen et al. (2007). Zhu and Kapuscinski (2011) study a multiperiod joint optimization problem of operational (initial capacity allocation and periodic production and transshipment) and hedging (derivatives written on exchange rates) for a multinational newsvendor. Unlike our paper, it is assumed that no inventory is carried from period to period, and thus the only link between different periods is the exchange rates; the interaction between the optimal operational and hedging decisions is not studied analytically. Despite the complex operational decisions, the separation theorem is also applicable to their model. In contrast, we show that the separation theorem is not applicable to our model as we use an interperiod utility function (a different, but more appropriate, type of utility function). Note that application of the separate theorem to our model would have implied that the optimal hedging is myopic, which differs from our findings.

In summary, our model, when restricted to its one-period version, is consistent with previous single-period newsvendor-like results, while offering sharper insights on the relationship between financial hedge and inventory. However, our model elucidates the importance of treating such problems by fully reflecting multiperiod considerations. The single-period insights are not transferrable to the multiperiod context in the presence of risk aversion and hedging concerns. In general, inventory and financial hedge might not be separable, and this is definitely the case if futures contracts are not used.

When separable, the optimal inventory policies are myopic; the optimal financial hedging policies are not myopic (which is different from the single-period insights as well as the past multiperiod insights). Previous multiperiod results are heavily driven by the use of intraperiod utility functions, which limits the derived insights when cash flow correlations across periods and genuine risk-aversion concerns of decision makers enter the picture. Our work contributes to the literature and practice for managing storable commodity risks with tractable optimal policies and new managerial insights.

Last, we recognize methodological similarities to a recent pure finance paper. Basak and Chabakauri (2010) employ an interperiod MV utility model, in continuous and discrete time, for an asset allocation problem, where the asset includes a risky stock and a bond. Because of our consideration of additional factors including physical inventory, demand uncertainty, and financial hedging, the exact methodology and results of this paper are not applicable to our problem. However, by following a similar approach of dynamically solving an interperiod MV utility function, our model also avoids the time-inconsistency issue; this is the extent of overlap of the two papers.

3. Problem Description

A firm procures and processes (or manufactures) a single commodity to make an end product, which is then sold at an exogenous market price. We first list the sequence of events for each period. At the start of each period, the firm procures the commodity from the long-term supply and trades (buys or sells) in the spot market. The commodity is then processed in make-to-order fashion to meet the uncertain demand. Unmet demand is assumed lost, and excess inventory of the commodity is carried over to the next period. To account for the significant amount of setup and processing time in each period, we do not allow spot trading in the middle of a period. Each period, the firm needs to make two operational decisions: (1) the final inventory level of the commodity and (2) how much commodity to process (or how many end products to produce). If profitable, that is, if the gross margin (the revenue less the procurement and processing cost) is positive, the firm should process to best meet the end-product demand and procure the commodity primarily for production. However, if the gross margin is zero or negative, the firm should not produce, but may still procure the commodity for a better utility. We will elaborate on this case in §4.1.

We next list our notation and assumptions for period n, $n = 1, \ldots, N$. The decision variables are described in the last two bulleted items. We follow the convention of denoting random variables by uppercase letters and their realizations by corresponding lowercase letters.

- $\alpha = 1/(1+r_f) \in (0, 1)$: the period discount factor for the firm's cash flow, where r_f is the risk-free interest rate for each period. We assume that the firm is using the risk-free interest rate to discount its cash flows and the periods are of equal length. The former assumption will be relaxed in §8; the latter assumption can be easily relaxed by adjusting r_f according to the actual duration of each period.
- $D_n, S_n \geq 0$: the end-product demand in period n and the commodity spot price at the start of period n, respectively. They are defined on probability space $(\Omega, \mathcal{F}, Q)$, where any state in Ω can be written as a vector of a state of demand and a state of spot price, and $\mathcal{F}$ is generated by $\{D_n\}_{1\le n\le N}$ and $\{S_n\}_{1\le n\le N+1}$. We define the probability measure, Q, by following the assumptions for the "partially complete" market Smith and Nau (1995) introduced, in which the market and private uncertainties are equivalent to the spot price and demand uncertainties in our paper, respectively. In other words, we assume that Q is indeed a combination of the real-world probability measure on demands and the risk-neutral probability measure (or equivalent martingale measure) on spot prices. Note that the risk-neutral probability measure is commonly used in the literature (see Chod et al. 2010). We will relax this Q-measure assumption in §8 and consider the real-world probability measure, P.

—S_n: We assume that $\{S_n\}_{1\le n\le N+1}$ is Markovian and the spot market operates with zero bid–ask spread and zero transaction cost. Although S_n and D_n are allowed to correlate, given $S_n = s_n$, D_n is assumed independent of $S_{n+1}, \ldots, S_{N+1}$. In other words, S_n can serve as a sufficient statistic for the entire past when predicting D_n and S_{n+1}. In addition, knowing D_n will not alter the firm's forecast on $S_{n+1}, \ldots, S_{N+1}$. This is consistent with the "partially complete" market assumption in Smith and Nau (1995). To understand the feasibility of such an assumption, let us look at an example. Suppose $\{S_n\}_{1\le n\le N+1}$ follows geometric Brownian motion (GBM), that is, $S_{n+1} = (S_n/\alpha)e^{\sigma_s B_n - \sigma_s^2/2}$ for all n, where σ_s is the volatility and $B_1, \ldots, B_N$ are independent and identically distributed (i.i.d.) standard Normal random variables. Note that B_n is independent of S_n for all n. Suppose D_n and S_n are correlated because they both depend on B_{n-1}, say $D_n = G(B_{n-1})$, for all n, where $G(\cdot)$ is an arbitrary positive valued function. Thus, although there is correlation between D_{n+1} and S_{n+1}, between S_{n+1} and S_n, and between S_n and D_n, we still have $D_{n+1} = G(B_n)$ being independent of $D_n = G(B_{n-1})$ (as B_n and B_{n-1} are independent).

—D_n: We assume that $D_1, \ldots, D_N$ are mutually independent for technical tractability. The cumulative distribution function (cdf) of D_n, denoted by $F_n(d \mid s_n)$, is an increasing function of d for any s_n, where

"$|s_n$" is used to reflect the correlation between S_n and D_n. Note that an increasing cdf is only assumed for the strictly increasing or decreasing properties of the optimal inventory levels. For any cdf, these properties should be modified to nondecreasing or nonincreasing, respectively.

- $\lambda \geq 0$: the absolute risk aversion of the firm used in MV utility functions ($U = E[\cdot] - (\lambda/2)V[\cdot]$, where $E[\cdot]$ and $V[\cdot]$ stand for the expectation and the variance of cash flows, respectively).
- $w, q \geq 0$: the exogenous wholesale price and fixed order quantity, respectively, specified in the long-term contract. Note that although the firm would prefer a low wholesale price, final contract terms should be negotiated with the supplier.
- $r_n \geq 0$: the exogenous unit revenue of the end-product sold in period n, excluding the processing cost. In other words, we assume the firm is a price taker in the end-product market.
- $h_n \geq 0$: the unit inventory holding cost for the commodity input in period n.
- $K_{i,n}, \beta_{i,n} > 0$: the strike price and the no-arbitrage price paid upon transaction, respectively, for hedging contract (or hedge) i available for trading at the start of period n, where $i = c$ (call option), p (put option). A call (put) option is the right, but not obligation, to buy (sell) the commodity at the strike price on the expiration date.

—We assume for model simplicity that the financial contracts considered expire in exactly one period. For interested readers, please find the discussion, results, and formal proof for the relaxed problem after the proof for Proposition 5.1 in the online supplement (available at http://dx.doi.org/10.1287/msom.2013.0433).

—$\chi_i(S_{n+1})$: the payoff function for hedge i, $i = f$ (futures contract), c, p, where,

- for the futures contract, $\chi_f(S_{n+1}) = S_{n+1} - E_Q[S_{n+1} \mid s_n] = S_{n+1} - (s_n/\alpha)$;
- for the call option, $\chi_c(S_{n+1}) = (S_{n+1} - K_{c,n})^+ - \beta_{c,n}/\alpha$, where $\beta_{c,n} = \alpha E_Q[(S_{n+1} - K_{c,n})^+ \mid s_n]$;
- for the put option, $\chi_p(S_{n+1}) = (K_{p,n} - S_{n+1})^+ - \beta_{p,n}/\alpha$, where $\beta_{p,n} = \alpha E_Q[(K_{p,n} - S_{n+1})^+ \mid s_n]$.

- $E[\chi_i(S_{n+1}) \mid s_n]/\beta_{i,n}$: the risk premium for hedge i traded at the start of period n, $i = c, p$. The risk premium for hedge f traded at the start of period n is $E[\chi_f(S_{n+1}) \mid s_n]/s_n$ (Dothan 1990). Note that under the Q-measure (in §§3–7), the risk premium of any financial hedge is zero; under the P-measure (in §8), however, the risk premium for some financial hedges may be positive or negative.
- $z_n \geq 0$: the final inventory level of the commodity (i.e., the maximum end-product inventory level available to fill demand D_n) at the start of period n. Unlike equity markets, spot markets do not allow short selling, and thus z_n is assumed to be nonnegative.

—z_n^{i*}: the optimal commodity level when only hedge i, $i = f, c, p$, is used.

—z_n^*: the optimal commodity level when a multicontract hedge is used.

- $y_{i,n} \in \mathbb{R}$: the quantity of hedge i, $i = f, c, p$, traded at the start of period n, where $y_{i,n} < 0$ if contract i is sold and $y_{i,n} > 0$ if contract i is purchased. Without loss of generality, we assume a common unit, for example, tons of steel, is used for z_n and $y_{i,n}$.

—$y_{i,n}^{1*}$: the optimal quantity of hedging contract i when a single-contract hedge is used.

—$y_{i,n}^*$: the optimal quantity of hedging contract i when a multicontract hedge is used.

At the start of period n, $n = 1, \ldots, N$, the firm, upon observing s_n, optimizes the inventory level, z_n, and the quantity of hedge i, $y_{i,n}$, $i = f, c, p$, by maximizing the MV utility of the net present value of the profit-to-go or the total cash flows earned in periods $n, \ldots, N$. At the end of the horizon, we assume that the firm receives no long-term supply. Having no demand to fill, the firm should simply sell all the excess inventory to the spot market. Please note that analytical tractability of such a complex multiperiod problem is mainly enabled by (1) the firm's ability to sell back to the spot market with no transaction cost and zero bid–ask spread, and (2) the firm's choice of the commonly used MV utility function.

4. Optimal Policy for Single-Contract Financial Hedging

We first study the inventory and hedging policies for the firm when it chooses to employ a single-contract hedge every period, a hedge containing a single financial contract, such as the futures contract or a call or a put option. The same type of contract is used across all periods. The analysis for this case with single-contract hedges is important because it reflects the current practice in which the futures contract is the most popular single contract used for hedging.

4.1. The Firm's Utility Function with Single-Contract Financial Hedging

At the start of period n, observing the current spot price, s_n, the firm needs to make the inventory decision, $z_n \geq 0$, and the hedging decision, $y_{i,n} \in \mathbb{R}$, $i = f, c, p$. It then processes the commodity in make-to-order fashion to meet the uncertain demand D_n if profitable.

Recall that r_n represents the unit revenue excluding the processing cost. Thus the firm's gross margin is $r_n - s_n$, based on which the firm decides whether to produce to fill its customer demand. Note that the money paid to the long-term supplier each period is a sunk cost.

Let $\pi_n^i(z_n \mid s_n, y_{i,n-1})$ denote the firm's (random) profit function in period n (if the firm uses hedge

i, $i = f, c, p$ every period) with commodity inventory level $z_n \geq 0$, given any observed spot price s_n and quantity of hedge i traded in the previous period $y_{i,n-1}$. Although z_n and $y_{i,n}$ are determined simultaneously, they have one-period difference for their effective times, due to the one-period time lag between the transaction and the exercise of the hedging contracts. When the gross margin $r_n - s_n$ is positive, the firm should process to best meet the demand, that is, the production level should be $z_n \wedge D_n$. Because of the assumption of zero bid–ask spread and transaction cost for the spot market, we can rewrite the model as if all the excess commodity at the end of each period were sold to the spot market; the discounted payoff of this sale can be counted as part of the profit earned in the current period. As a result, the firm will always start the next period with zero on-hand inventory:

$$\begin{aligned}\pi_n^i&(z_n \mid s_n, y_{i,n-1})\\&= y_{i,n-1}\chi_i(s_n) + (s_n - w)q + (r_n - s_n)(z_n \wedge D_n)\\&\quad + (\alpha S_{n+1} - s_n - h_n)(z_n - D_n)^+. \qquad (1)\end{aligned}$$

When the gross margin $r_n - s_n$ is zero or negative, since production is not profitable, the firm should not produce, but may make speculative purchases of the commodity only to improve its utility. In this case, the firm's profit function becomes

$$\begin{aligned}\pi_n^i(z_n \mid s_n, y_{i,n-1}) = y_{i,n-1}\chi_i(s_n) &+ (s_n - w)q\\&+ (\alpha S_{n+1} - s_n - h_n)z_n. \qquad (2)\end{aligned}$$

At the end of the horizon, facing no customer demand, the firm simply sells all the excess commodity to the spot market (i.e., $z_{N+1} \equiv 0$). Thus, regardless of the sign of $r_n - s_n$, we have

$$\pi_{N+1}^i(z_{N+1} \equiv 0 \mid s_{N+1}, y_{i,N}) = y_{i,N}\chi_i(s_{N+1}). \qquad (3)$$

Using π_n^i as a short notation for the profit earned in period n, we formally define the firm's MV utility function for period n, $n = 1, \ldots, N+1$, by

$$\begin{aligned}U_n&(z_n, y_{i,n} \mid s_n, y_{i,n-1})\\&= E\left[\sum_{k=n}^{N+1} \alpha^{k-n}\pi_k^i \,\middle|\, s_n\right] - \frac{\lambda}{2}V\left[\sum_{k=n}^{N+1}\alpha^{k-n}\pi_k^i \,\middle|\, s_n\right], \qquad (4)\end{aligned}$$

where $\pi_{n+1}^i = \pi_{n+1}^i(Z_{n+1}^{i*} \mid S_{n+1}, y_{i,n})$ and $\pi_k^i = \pi_k^i(Z_k^{i*} \mid S_k, Y_{i,k-1}^{1*})$, $k \geq n+2$. Note that $Z_k^{i*} = z_k^{i*}(S_k)$ and $Y_{i,k}^{1*} = y_{i,k}^{1*}(S_k)$ are random variables representing the optimal decisions for period k, $k \geq n+1$ (randomness coming from S_k only, not from $S_{n+1}, \ldots, S_{k-1}, D_n, \ldots, D_{k-1}$, due to the assumptions of Markovian price process and zero bid–ask spread and transaction cost for the spot market).

At the end of the horizon, facing no risks, the firm should not consider hedging, that is, $y_{i,N+1}^{1*} = 0$, and thus, using (3), the firm's utility function can be simplified to

$$U_{N+1}(z_{N+1}^{i*} = 0, y_{i,N+1}^{1*} = 0 \mid s_{N+1}, y_{i,n}) = y_{i,n}\chi_i(s_{N+1}). \qquad (5)$$

4.2. Optimal Policy for Each Period

We characterize optimal inventory and hedging policies for period n in this section using the iterative method. Using the assumption of zero bid–ask spread for the spot market and the MV utility form, we are able to first prove that the utility function $U_n(z_n, y_{i,n} \mid s_n, y_{i,n-1})$ is concave in $y_{i,n}$ and thus characterize the optimal solution $y_{i,n}^{1*}$, for any given z_n. We then prove that $U_n(z_n, y_{i,n}^{1*} \mid s_n, y_{i,n-1})$, as a function of z_n, displays a property that is weaker than concavity but guarantees a single optimum, z_n^{i*} (the solution to $(d/dz_n)U_n(z_n, y_{i,n}^{1*} \mid s_n, y_{i,n-1}) = 0$). For readers' convenience, we present the optimal policies by first simplifying some notation. First, we denote the *future profit* for period n, $n < N$, for the case of using hedge i, $i = f, c, p$, as the single-contract hedge by

$$\begin{aligned}\Pi_n^i = \pi_{n+1}^i&(Z_{n+1}^{i*} \mid S_{n+1}, y_{i,n}) - y_{i,n}^{1*}\chi_i(S_{n+1})\\&+ \sum_{k=n+2}^{N+1}\alpha^{k-n-1}\pi_k^i(Z_k^{i*} \mid S_k, Y_{i,k-1}^{1*}). \qquad (6)\end{aligned}$$

Note that Π_n^i is random, but $\Pi_N^i \equiv 0$. Second, the first-order condition (FOC) $(d/dz_n)U_n(z_n, y_{i,n}^{1*} \mid s_n, y_{i,n-1}) = c_n^u(z_n, s_n)\bar{F}_n(z_n \mid s_n) - c_n^o(z_n, s_n)F_n(z_n \mid s_n) = 0$, where $\bar{F}(\cdot) = 1 - F(\cdot)$, displays similarities to that from which the standard newsvendor solution is derived. Thus, we define $c_n^o(z_n, s_n)$ and $c_n^u(z_n, s_n)$ as the *overage cost* and the *underage cost*, respectively:

$$\begin{aligned}c_n^o(z_n, s_n) &= c_n^{o1}(s_n) + c_n^{o2}(z_n, s_n) \quad \text{and}\\ c_n^u(z_n, s_n) &= c_n^{u1}(s_n) + c_n^{u2}(z_n, s_n),\end{aligned} \qquad (7)$$

where for $i = f, c, p$,

$$\begin{aligned}c_n^{o1}(s_n) &= h_n + \lambda\alpha^2\,\mathrm{Cov}(S_{n+1}, \Pi_n^i \mid s_n) - c_n^{h1}(s_n),\\ c_n^{o2}(z_n, s_n) &= \lambda\alpha^2 V[S_{n+1} \mid s_n]E[(z_n - D_n)^+ \mid s_n]\\&\quad - c_n^{h2}(z_n, s_n),\\ c_n^{u1}(s_n) &= r_n - s_n,\\ c_n^{u2}(z_n, s_n) &= -\lambda((r_n + h_n - s_n)^2 + \alpha^2 V[S_{n+1} \mid s_n])\\&\quad \cdot E[(z_n - D_n)^+ \mid s_n],\end{aligned} \qquad (8)$$

in which the hedging-related terms in the overage costs are

$$\begin{aligned}c_n^{h1}&(s_n)\\&= \lambda\alpha^2\frac{\mathrm{Cov}(S_{n+1}, \chi_i(S_{n+1}) \mid s_n)\mathrm{Cov}(\chi_i(S_{n+1}), \Pi_n^i \mid s_n)}{V[\chi_i(S_{n+1}) \mid s_n]},\\ c_n^{h2}&(z_n, s_n) = \lambda\alpha^2\frac{\mathrm{Cov}^2(S_{n+1}, \chi_i(S_{n+1}) \mid s_n)}{V[\chi_i(S_{n+1}) \mid s_n]}\\&\quad \cdot E[(z_n - D_n)^+ \mid s_n].\end{aligned} \qquad (9)$$

In contrast to the standard newsvendor solution, the overage cost (the underage cost) in our problem consists of two parts, a constant cost, $c_n^{o1}(s_n)$ ($c_n^{u1}(s_n)$), and a variable cost, $c_n^{o2}(z_n, s_n)$ ($c_n^{u2}(z_n, s_n)$). Indeed, $c_n^{o2}(z_n, s_n)$ increases in z_n (except when $i = f$, $c_n^{o2}(z_n, s_n) = 0$), whereas $c_n^{u2}(z_n, s_n)$ decreases in z_n. More interpretation of these costs is provided later in this section. Last, let $z_n^u(s_n)$ and $z_n^o(s_n)$ denote the unique nonnegative values of z_n satisfying $c_n^u(z_n, s_n) = 0$ when $r_n - s_n > 0$ and $c_n^o(z_n, s_n) = 0$ when $c_n^{o1}(s_n) \le 0$, respectively. We show that a base-stock policy is optimal for every period.

Proposition 4.1. *Given spot price s_n, $n \le N$, for $i = f, c, p$, the optimal quantities are as follows:*
Hedging:

$$y_{i,n}^{1*} = -\{E[(z_n^{i*} - D_n)^+ \mid s_n]1_{\{r_n > s_n\}} + z_n^{i*}1_{\{r_n \le s_n\}}\} \cdot \frac{\mathrm{Cov}(S_{n+1}, \chi_i(S_{n+1}) \mid s_n)}{V[\chi_i(S_{n+1}) \mid s_n]} - \frac{\mathrm{Cov}(\chi_i(S_{n+1}), \Pi_n^i \mid s_n)}{V[\chi_i(S_{n+1}) \mid s_n]},$$

$$y_{i,N}^{1*} = \{E[(z_N^{i*} - D_N)^+ \mid s_N]1_{\{r_N > s_N\}} + z_N^{i*}1_{\{r_N \le s_N\}}\} \cdot \frac{\mathrm{Cov}(S_{N+1}, \chi_i(S_{N+1}) \mid s_N)}{V[\chi_i(S_{N+1}) \mid s_N]}, \quad \textit{and}$$

$$|y_{f,N}^{1*}| \le |y_{j,N}^{1*}|, \quad j = c, p.$$

Inventory:

- *When $r_n - s_n > 0$, we have the following cases:*
  —*If $c_n^{o1}(s_n) > 0$, $z_n^{i*} (\in (0, z_n^u(s_n)))$ is the unique solution to the FOC, that is, $c_n^o(z_n^{i*}, s_n)F_n(z_n^{i*} \mid s_n) = c_n^u(z_n^{i*}, s_n)\bar{F}_n(z_n^{i*} \mid s_n)$.*
  —*If $c_n^{o1}(s_n) \le 0$, then*
    - *if $z_n^u(s_n) > z_n^o(s_n)$, $z_n^{i*} (\in (z_n^o, z_n^u(s_n)))$ is the unique solution to the FOC;*
    - *if $z_n^u(s_n) \le z_n^o(s_n)$, $z_n^{i*} (\in [z_n^u(s_n), z_n^o(s_n)])$ satisfies the FOC or $z_n^{i*} = +\infty$.*
- *When $r_n - s_n \le 0$, the firm is speculative, and*

$$z_n^{i*} = 0 \vee \frac{-c_n^{o1}(s_n)}{\lambda\alpha^2\left(V[S_{n+1} \mid s_n] - \frac{\mathrm{Cov}^2(S_{n+1}, \chi_i(S_{n+1}) \mid s_n)}{V[\chi_i(S_{n+1}) \mid s_n]}\right)}.$$

We start by interpreting the hedging result. Financial hedging contributes nothing to the mean profit because of the Q-measure assumption. For the last period, the hedgeable profit risk is reflected by the profit term $\alpha S_{N+1}(z_N - D_N)^+$ (part of the current period profit earned from selling the excess commodity of quantity $(z_N - D_N)^+$). Thus, the firm should hold a short position, that is, should sell the futures contract or a call or buy a put, of quantity

$$|y_{i,N}^{1*}| = E[(z_N^{i*} - D_N)^+ \mid s_N] \cdot \left|\frac{\mathrm{Cov}(S_{N+1}, \chi_i(S_{N+1}) \mid s_N)}{V[\chi_i(S_{N+1}) \mid s_N]}\right| \quad \text{if } r_N > s_N.$$

This means that for each unit of the excess commodity, the absolute hedging amount is

$$\left|\frac{\mathrm{Cov}(S_{N+1}, \chi_i(S_{N+1}) \mid s_N)}{V[\chi_i(S_{N+1}) \mid s_N]}\right| \quad (\ge 1, = 1 \text{ when } i = f).$$

Note that in §6 we show that the futures contract is indeed the best single-contract hedge for the last period. For any other period, however, the firm hedges also the risk in the future profit $\alpha\Pi_n^i$. Thus the optimal hedging quantity $y_{i,n}^{1*}$ consists of two parts, used to hedge $\alpha S_{n+1}(z_n - D_n)^+$ in the current period profit and the future profit $\alpha\Pi_n^i$, respectively. Since $\alpha\Pi_n^i$ is independent of z_n, the hedging decision only interacts with the inventory decision z_n for the same period through the term $\alpha S_{n+1}(z_n - D_n)^+$.

As discussed previously, our model can be treated as the firm starting each period with zero on-hand inventory. Thus the current period inventory decision has no impact on the future decisions. Conversely, if the firm's utility function includes the mean profit only, the future decisions should not influence the current inventory decision. This, however, no longer holds when the profit variance is also included in the utility function. Note that the current inventory decision and the future decisions are linked or interacted via the term $\alpha S_{n+1}(z_n - D_n)^+$. Indeed, their dependence is captured by the covariance between the current period profit and the future profit $\alpha^2 \mathrm{Cov}(S_{n+1}(z_n - D_n)^+, \Pi_n^i \mid s_n)$. Thus, in general, the optimal policy is not myopic and requires the knowledge of optimal decisions for all future periods. It is important to note that since the futures contract "perfectly" hedges the covariance, the corresponding optimal inventory policy is myopic. More discussion of this myopic inventory policy is provided in §5 for the multicontract hedge case.

Note that because of the use of the multiperiod MV utility and incorporation of financial hedging, our inventory solution is more complex than the standard newsvendor solution. It, however, reveals similar (but new) insights on the trade-off between the overage cost, $c_n^o(z_n, s_n)$, and the underage cost, $c_n^u(z_n, s_n)$, defined by (7)–(9). Note that the underage cost is the profit loss, $r_n - s_n$, less the utility gain or profit variance reduction due to underage inventory (less inventory leads to a smaller profit variance). The overage cost equals the holding cost, h_n, plus two types of utility loss, where one is inventory independent (showing the effect of overage inventory and hedging on the mean and covariance terms involving z_n) and the other is inventory dependent (showing the effect of overage inventory and hedging on the variance terms involving z_n). Note that financial hedging affects the overage cost only as it is effective only when excess commodity exists and is sold to the spot market. Indeed, for the last period, any financial hedge reduces the overage cost, resulting in an inventory increase (as shown in Proposition 7.2). For any other period, because the future profit also requires hedging, financial hedge may not reduce the overage cost itself, but it does reduce the increasing rate of the overage cost, $(\partial/\partial z_n)c_n^o(z_n, s_n)$. Finally, when the firm is risk neutral ($\lambda = 0$), the inventory solution reduces to a standard newsvendor solution ($c_n^o(z_n, s_n) = h_n$ and $c_n^u(z_n, s_n) = r_n - s_n$).

The understanding of the overage and underage costs helps us uncover and classify the firm's motives (production and/or speculation) behind its actions. Specifically, these motives are explained by its gross margin, $r_n - s_n$, and the constant part in the overage cost, $c_n^{o1}(s_n)$. The firm has pure production motives when $r_n - s_n > 0$ and $c_n^{o1}(s_n) > 0$, production and speculation motives when $r_n - s_n > 0$ and $c_n^{o1}(s_n) \le 0$, and pure speculation motives when $r_n - s_n \le 0$ and $c_n^{o1}(s_n) \le 0$. Only with a positive gross margin would the firm find it profitable to produce. If beneficial, the firm may also speculate and stock more than needed for production. If the gross margin is, however, zero or negative, the firm makes speculative purchases only when it improves its utility (i.e., when $c_n^{o1}(s_n)$ is negative). In this case, financial hedges also serve as speculative tools.

5. Optimal Policy for Multicontract Financial Hedging

We now consider the case in which the firm adopts all financial contracts available in the market, referred to as the multicontract hedge case. It is important to note that since the multicontract hedge always includes the futures contract (always available in the market), the single-contract case is not a special case of the multicontract case. Note that any financial contract with twice differentiable payoff functions can be replicated by bonds, futures contracts, and call and put options (see details in Oum et al. 2006, §3.2; Carr and Madan 2001). Thus a theoretical optimal multicontract hedge should include futures contracts and call and put options with a continuum of strike prices; a practical optimal multicontract hedge should involve these contracts available in the market. In this section, we focus on the practical optimal multicontract hedge.

We first define some additional parameters and decision variables needed for this section. Let $K_{c,n,i}$ and $\chi_{c,i}(S_{n+1})$, $i = 1, \ldots, n_c$, and $K_{p,n,j}$ and $\chi_{p,j}(S_{n+1})$, $j = 1, \ldots, n_p$, denote the strike prices and payoff functions for the call and put options available at the start of period n, respectively. Without loss of generality, we assume these financial contracts are not replicating each other. Let $\mathbf{y}_n = [y_{f,n},\ y_{c,n,1}, \ldots, y_{c,n,n_c}, y_{p,n,1}, \ldots, y_{p,n,n_p}]$ denote the array of the corresponding hedging quantities. Let $\pi_n(z_n \mid s_n,\ \mathbf{y}_{n-1})$, Π_n, and $U_n(z_n, \mathbf{y}_n \mid s_n, \mathbf{y}_{n-1})$ denote the firm's current period profit, future profit, and MV utility function for period n, respectively. They are similar to the corresponding functions in the single-contract hedge case, except that the hedging payoff is now the sum of the payoffs from each hedge; that is, $y_{i,n-1}\chi_i(S_n)$ is replaced by $y_{f,n-1}\chi_f(S_n) + \sum_{j=1}^{n_c} y_{c,n-1,j}\chi_{c,j}(S_n) + \sum_{j=1}^{n_p} y_{p,n-1,j}\chi_{p,j}(S_n)$.

Our analysis shows that the derivatives of the utility function with respect to $\mathbf{y}_n$ are

$$\frac{\partial U_n(z_n, \mathbf{y}_n \mid s_n, \mathbf{y}_{n-1})}{\partial \mathbf{y}_n} = -\lambda\alpha^2(\boldsymbol{\Sigma}_n(s_n)\cdot \mathbf{y}_n - \boldsymbol{\Psi}_n(z_n, s_n))$$

and

$$\frac{\partial^2 U_n(z_n, \mathbf{y}_n \mid s_n, \mathbf{y}_{n-1})}{\partial (\mathbf{y}_n)^2} = -\lambda\alpha^2\boldsymbol{\Sigma}_n(s_n).$$

Note that $\boldsymbol{\Sigma}_n(s_n)$ is the conditional (on s_n) covariance matrix for $S_{n+1}, (S_{n+1} - K_{c,n,1})^+, \ldots, (S_{n+1} - K_{c,n,n_c})^+, (S_{n+1} - K_{p,n,1})^-, \ldots, (S_{n+1} - K_{p,n,n_p})^-$; it is invertible as these corresponding financial contracts are not replicating each other. Also, $\boldsymbol{\Psi}_n(z_n, s_n) = [\psi_{f,n}, \psi_{c,n,1}, \ldots, \psi_{c,n,n_c}, \psi_{p,n,1}, \ldots, \psi_{p,n,n_p}]$, where, for example, $\psi_{c,n,i} = -E[(z_n - D_n)^+ \mid s_n]\mathrm{Cov}(\chi_{c,i}(S_{n+1}), S_{n+1} \mid s_n) - \mathrm{Cov}(\chi_{c,i}(S_{n+1}), \Pi_n \mid s_n)$. It is not difficult to see that the Hessian matrix of the utility function, $-\lambda\alpha^2\boldsymbol{\Sigma}_n(s_n)$, is negative semidefinite, and thus the utility function is concave in $\mathbf{y}_n$. This eventually leads to the optimality of the base-stock policy, as explained in the single-contract hedge case.

Proposition 5.1. *Given spot price s_n, $n \le N$, the optimal quantities are as follows:*
Hedging: $\mathbf{y}_n^* = (\boldsymbol{\Sigma}_n(s_n))^{-1} \cdot \boldsymbol{\Psi}_n(z_n^*, s_n)$ and $\mathbf{y}_N^* = [-E[(z_N^* - D_N)^+ \mid s_N], 0, \ldots, 0]$.
Inventory: $z_n^* \equiv z_n^{f*}$ and the overage cost $c_n^o(z_n, s_n) = h_n > 0$. When $r_n - s_n \le 0$, $z_n^* = 0$; when $r_n - s_n > 0$, z_n^* is the unique solution to

$$h_n F_n(z_n^* \mid s_n) = (r_n - s_n - \lambda((r_n + h_n - s_n)^2 + \alpha^2 V[S_{n+1} \mid s_n]) \cdot E[(z_n^* - D_n)^+ \mid s_n])\bar{F}_n(z_n^* \mid s_n). \tag{10}$$

This proposition implies that for the last period, the optimal multicontract hedge contains futures only. For any other period, however, it also contains call options with strike prices lower than the futures' price and put options with strike prices higher than the futures' price. Like the single-contract hedge case, the optimal quantity of each financial contract in the hedge also consists of two parts, which are used to hedge the current period profit and the future profit, respectively.

It is important to note that the optimal inventory level, z_n^*, is myopic and identical to z_n^{f*} (the optimal inventory level when using futures alone). As discussed in §4.2, as long as the futures contract is utilized, alone or not, the covariance between the current period profit and the future profit is "perfectly" hedged, and thus the inventory policy is myopic. As a result, financial hedging helps completely eliminate both types of utility loss in the overage cost; thus we have $c_n^o(z_n, s_n) = h_n > 0$. Since the overage cost is always positive, the firm has no speculative motives—it procures for production only when the gross margin is positive. Moreover, the property $z_n^* = z_n^{f*}$ is because the covariances between the hedging payoff and the hedgeable current period profit, $S_{n+1}(z_n - D_n)^+$, for the corresponding two cases are equal, that is, $y_{f,n}^* V[S_{n+1} \mid s_n] + \sum_{i=1}^{n_c} y_{c,n,i}^* \operatorname{Cov}(\chi_{c,i}(S_{n+1}), S_{n+1} \mid s_n) + \sum_{j=1}^{n_p} y_{p,n,j}^* \operatorname{Cov}(\chi_{p,j}(S_{n+1}), S_{n+1} \mid s_n) = y_{f,n}^{1*} V[S_{n+1} \mid s_n]$. However, since the hedging payoff also affects other parts of the utility function, using the futures contract alone is not optimal for any period besides the last.

Last, we consider a special case: the infinite-horizon case with the futures contract included in the multicontract hedge. Our analysis for the finite-horizon case indicates that the above results of the optimal inventory (myopic) and hedging policy are indeed derivable without requiring any specific property of the utility functions for the future periods. This observation implies that the optimal policy for the finite-horizon case is also optimal for the infinite-horizon case.

6. How to Select a Single-Contract Hedge

We now compare and rank hedges, single-contract or multicontract, based on their contribution to the utility, with emphasis on single-contract hedges. Let χ and $\chi(S_{n+1})$ denote any financial hedge, single-contract or multicontract, traded in period n and its payoff function, respectively. Let χ_f, $\chi_{c(K_c)}$, and $\chi_{p(K_p)}$ denote the futures contract, the call option with strike price K_c, and the put option with strike price K_p, respectively. Let $U_n(z_n, y_{\chi_1,n} \mid s_n, y_{\chi,n-1})$, similar to $U_n(z_n, y_{f,n} \mid s_n, y_{f,n-1})$, denote the firm's utility function, given that hedge χ_1 is traded in period n and optimal hedges (which may or may not include the same type(s) of contract as χ_1, unlike the single-contract hedge case) are applied for all future periods. Formally we define the ordering of hedges based on their contribution to the firm's utility. For any two hedges χ_1 and χ_2, we say $\chi_1 \succeq \chi_2$ (i.e., χ_1 is not worse than χ_2) if $U_n(z_n^{\chi_1*}, y_{\chi_1,n}^* \mid s_n, y_{\chi,n-1}) \geq U_n(z_n^{\chi_2*}, y_{\chi_2,n}^* \mid s_n, y_{\chi,n-1})$. It is important to note that such a definition on ordering, assuming the use of same optimal hedges for all future periods, promotes a fair comparison for the hedges. We say $\chi_1 \succ \chi_2$ (i.e., χ_1 is better than χ_2) if $U_n(z_n^{\chi_1*}, y_{\chi_1,n}^* \mid s_n, y_{\chi,n-1}) > U_n(z_n^{\chi_2*}, y_{\chi_2,n}^* \mid s_n, y_{\chi,n-1})$ and $\chi_1 \simeq \chi_2$ (i.e., χ_1 and χ_2 are equivalent) if $U_n(z_n^{\chi_1*}, y_{\chi_1,n}^* \mid s_n, y_{\chi,n-1}) = U_n(z_n^{\chi_2*}, y_{\chi_2,n}^* \mid s_n, y_{\chi,n-1})$.

We now compare single-contract hedges that include the futures, call, and put options. Unlike the futures contract, there are many tradable call and put options with a common strike time but different strike prices. Applying the definition of ordering, we can characterize an optimal call (with the best strike price $K_{c,n}^*$) or an optimal put (with the best strike price $K_{p,n}^*$). The following comparison results are mainly based on the observation that the use of any hedge χ in period n reduces the profit variance by $\operatorname{Cov}^2(\chi(S_{n+1}), S_{n+1}(z_n - D_n)^+ + \Pi_n \mid s_n)/V[\chi(S_{n+1}) \mid s_n]$, where Π_n represents the corresponding future profit and is defined similarly to Π_n^i in (6).

Proposition 6.1. *For period n, we have the following:*

(1) $\chi_f \simeq \chi_{c(0)} \simeq \chi_{p(\infty)}$; $\chi_1 \succeq \chi_2$ *if and only if (iff)*

$$\frac{\operatorname{Cov}^2(S_{N+1}, \chi_1(S_{N+1}) \mid s_N)}{V[\chi_1(S_{N+1}) \mid s_N]} \geq \frac{\operatorname{Cov}^2(S_{N+1}, \chi_2(S_{N+1}) \mid s_N)}{V[\chi_2(S_{N+1}) \mid s_N]}$$

for $n = N$ and iff

$$\frac{\operatorname{Cov}^2(\chi_1(S_{n+1}), S_{n+1}(z_n^{\chi_1*} - D_n)^+ + \Pi_n \mid s_n)}{V[\chi_1(S_{n+1}) \mid s_n]} \geq \frac{\operatorname{Cov}^2(\chi_2(S_{n+1}), S_{n+1}(z_n^{\chi_2*} - D_n)^+ + \Pi_n \mid s_n)}{V[\chi_2(S_{n+1}) \mid s_n]}$$

for $n < N$.

(2) *For $n = N$, $\chi_f \succeq \chi$, $\forall \chi$; $\chi_{c(K_{c1})} \succ \chi_{c(K_{c2})}$ iff $K_{c1} < K_{c2}$ and thus $K_{c,N}^* = 0$; $\chi_{p(K_{p1})} \succ \chi_{p(K_{p2})}$ iff $K_{p1} > K_{p2}$ and thus $K_{p,N}^* = +\infty$; and $\chi_1 \succeq \chi_2$ iff $z_N^*(\chi_1) \geq z_N^*(\chi_2)$ when $r_N > s_N$ and $c_N^{o1}(S_N) > 0$.*

For the last period, this proposition implies that the optimal hedge is the futures contract or its equivalent contracts, such as the call with the lowest strike price and the put with the highest strike price (referred to as the deep-in-the-money options). Intuitively, the cash flow risk exists in the current period profit only and is reflected by the term $\alpha S_{N+1}(z_N - D_N)^+$. Since D_N is independent of S_{N+1} and thus cannot be hedged at all, perfect hedges for the cash flow risk do not exist. But an optimal hedge exists and should result in the maximum reduction of the profit variance, where the reduction is expressed by

$$\frac{\operatorname{Cov}^2(S_{N+1}, \chi(S_{N+1}) \mid s_n) E^2[(z_N - D_N)^+ \mid s_n]}{V[\chi(S_{N+1}) \mid s_n]}.$$

Since the maximum reduction occurs when the hedge's payoff function is perfectly correlated with S_{N+1}, the futures contract and its equivalent options are optimal. The equivalence between the futures and the deep-in-the-money options remains for any other period.

Moreover, we prove for the last period a monotonic relationship: the lower (higher) the strike price, the better the call (put) option. Furthermore, we show that the use of a better hedge leads to a higher inventory level and a bigger profit variance reduction. These results, however, do not apply to any other period. This is because the profit variance reduction may not be monotonic to the inventory level or the strike price if a call or put is used as a single-contract hedge. Thus, a better hedge, though improving the utility, may or may not lead to a higher inventory level. Our numerical study shows that in some cases the futures contract is not the best single-contract hedge, but results in the highest inventory level.

7. Role of the Operational and Financial Hedges

Since the physical inventory, well known for effectively dealing with the supply-demand mismatch risk, also contributes to hedging the firm's cash flow in our model, we refer to it as the operational hedge. In this section, we study the role and interaction of the operational and financial hedges in dealing with various risks. In our model, inventory comes from two sources: the long-term supply (with fixed quantities and prices) and the spot market procurement (buying or selling). By nature, the long-term supply with locked prices protects the firm against the commodity price risk, whereas the spot market procurement protects the firm against the commodity consumption risk (or the end-product demand risk). Since the money paid for the long-term supply is a sunk cost, it can be removed from the MV utility without affecting the optimization. Thus the real effective operational hedge is the inventory procured from the spot market. Different financial hedges we study include single-contract and multicontract hedges, where the contracts are futures, call, and put options. It is well know that they hedge the commodity price risk directly. In our paper, however, they also hedge the demand risk indirectly via their influence on inventory levels (which is made through hedging quantities and correlations between demands and spot prices).

We next examine, analytically and numerically, the impact of different financial hedges on the effectiveness of the operational hedge. The impact is measured by the inventory level change: more effective operational hedges lead to higher inventory levels. We then study how the operational hedge and different financial hedges affect the firm's financial performance, characterized by the mean, variance, and utility of the cash flows. Specifically, we compare some different practical scenarios, which include SS (single sourcing from long-term supply), DS (dual sourcing from long-term supply and spot market), DS + i (dual sourcing with hedge i), $i = f$ (futures), c (call), p (put), and DS + HP (dual sourcing with a hedging portfolio).

7.1. Numerical Study Setup

To supplement the analytical results provided in this paper, we performed a two-period numerical study with identical periods (and thus, in this section, we remove the subscript n from the relevant cost parameters) and correlated spot prices and demands. Both spot prices and demands follow Lognormal distribution or GBM when treated as continuous time stochastic processes. Specifically, let $S_{n+1} = (S_n/\gamma)e^{\sigma_s B_{n+1} - \sigma_s^2/2}$ and $D_n = \mu e^{\sigma_d W_n - \sigma_d^2/2}$, where B_n's and W_n's are i.i.d. standard Normal random variables, and (B_n, W_n) is bivariate Normal with correlation $\rho \in [-1, 1]$ (used to represent the correlation between S_n and D_n), $n = 1, 2$. To choose reasonable values for parameters γ and σ_s, we fit the spot price distribution to the U.S. hot-rolled coil steel spot prices from January 2009 to June 2010. The fitting indicates that the steel spot price is approximately GBM with monthly discount factor $\gamma = 0.9758$ and monthly drift parameter $\sigma_s = 0.114$. We set $\alpha = \gamma$ to reflect the Q-measure, that is, to guarantee $E[\alpha S_{n+1} \mid S_n] = S_n$. In particular, our numerical study parameters are as follows: $\alpha = 0.9758$; $w = 3$; $q = 60, 80, \ldots, 200$; $s_1 = 3$; $r = 4, 5, 6, 7, 8$; $h = 0.6$; $\sigma_s = \sigma_d = 0.114$; $\rho = -0.5, 0, 0.5$; $\mu = 100$; and $\lambda = 0.002$, 0.006, 0.01, 0.02, 0.06, 0.1.

Note that we set the risk aversion, λ, appropriately small enough to avoid a pathological behavior that may occur when maximizing the MV utility: reducing inventory when seeing a higher unit revenue (which magnifies the profit variance's negative effect on utility). Analytically, we can show that z_n^* increases in r if

$$\lambda \leq \frac{1}{2(r+h-s_n)E\big[\big(F_n^{-1}\big(\frac{r-s_n}{r+h-s_n}\big) - D_n\big)^+\big]}.$$

Applying this to our numerical study setup, we should set λ below or at 0.1 to prevent the pathological behavior.

7.2. Impact on the Service Level

Let z_n^{0*} denote the firm's optimal inventory level for period n for case DS, where the superscript 0 stands for no hedging; it satisfies the FOC with the overage and underage costs modified by omitting the hedging-related terms in (8). Below we show the impact of financial hedging and risk aversion (λ) on the optimal inventory (service) level. For its significance, we present a more detailed sensitivity analysis result for $z_n^*(= z_n^{f*})$.

Proposition 7.1. *For period n, $n \leq N$, we obtain the following*:

(1) *For risk-neutral firms, that is, when $\lambda = 0$,*

$$z_n^{i*} = F_n^{-1}\left(\frac{r_n - s_n}{r_n + h_n - s_n}\right) \triangleq \bar{z}_n^*, \quad i = f, c, p.$$

(2) *Sensitivity analysis results are as follows*:
- $z_n^*(=z_n^{f*})$ *is decreasing in* λ, h_n, *and* $V[S_{n+1} \mid s_n]$;
- z_n^{i*}, $i=c,p$, *is decreasing in* λ *if*

$$\operatorname{Cov}(S_{n+1}, \Pi_n^i \mid s_n) \geq \frac{\operatorname{Cov}(S_{n+1}, \chi_i(S_{n+1}) \mid s_n)\operatorname{Cov}(\chi_i(S_{n+1}), \Pi_n^i \mid s_n)}{V[\chi_i(S_{n+1}) \mid s_n]};$$

- z_n^{0*} *is decreasing in* λ *if* $\operatorname{Cov}(S_{n+1}, \Pi_n \mid s_n) \geq 0$ (*automatically held for* $n=N$).

(3) *Service level comparison is* $\bar{z}_N^* \geq z_N^* \geq z_N^{c*}$, $z_N^{p*} > z_N^{0*}$.

Intuitively, a more risk-averse firm usually keeps a lower inventory level despite the option of financial hedging. A risk-neutral firm has the highest inventory level, $\bar{z}_n^*$. Our numerical study indicates that, with or without financial hedging, risk-averse firms with risk aversion $\lambda \leq 0.002$ carry as much inventory as risk-neutral firms (see Figures 1–3). When futures contracts are used, alone or not, as holding inventory becomes more expensive, a risk-averse firm lowers its inventory level to increase its mean profit and at the same time decrease its profit variance. Facing a higher spot price volatility, a risk-averse firm keeps a lower inventory level to reduce its profit variance.

For the last period, our results are consistent with the results of the current literature for single-period problems (Ding et al. 2007, Gaur and Seshadri 2005). In particular, risk-averse firms stock less than risk-neutral firms; the use of financial hedging helps raise risk-averse firms' inventory levels. In addition, we show that a better hedge (e.g., a call with a lower strike price or a put with a higher strike price) leads to a higher inventory level because of its better control of the profit variance.

Figure 1 **Optimal Inventory Level** ($\rho=-0.5$) **Against** $\lambda/2=0.001,\ldots,0.01$

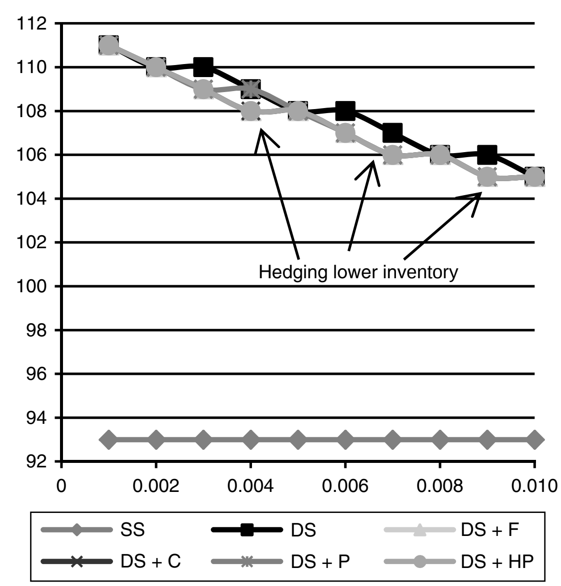
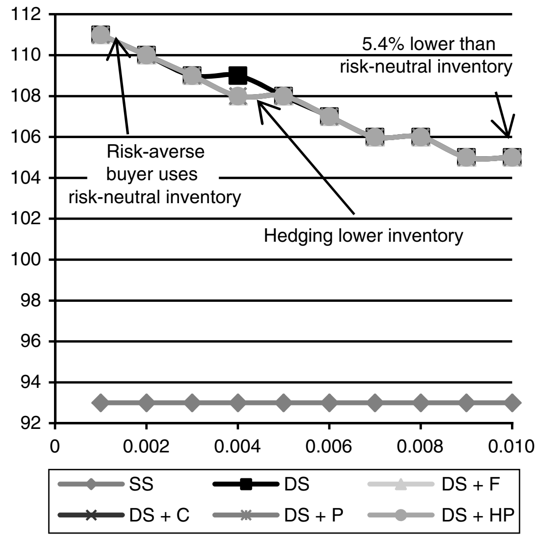
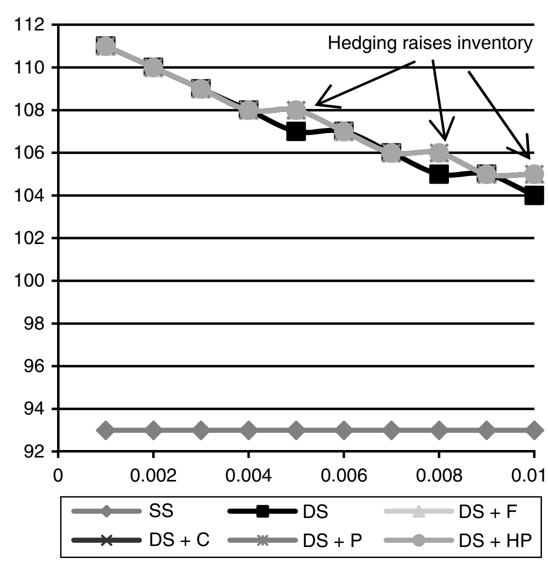


Figure 2 **Optimal Inventory Level** ($\rho=0$) **Against** $\lambda/2=0.001,\ldots,0.01$



For any other period, however, our results differ from the results of the current literature for multiperiod problems (Smith and Nau 1995, Chen et al. 2007, Zhu and Kapuscinski 2011). As mentioned in §2, all of these works use an intraperiod utility function, which ignores the cash flow correlations across periods. They show that operation and hedging decisions are separable and the optimal financial hedging is myopic (meaning that the optimal hedge is the futures when applied to the context of our problem). In contrast, using an interperiod MV utility function, we prove that the optimal financial hedging is not myopic and also includes various call and put options. In addition, we observe that the

Figure 3 **Optimal Inventory Level** ($\rho=0.5$) **Against** $\lambda/2=0.001,\ldots,0.01$



use of financial hedging may lower the risk-averse firm's inventory level (shown in Figure 1). Unlike the last period, there are future period profits to hedge, and thus the monotonic relationship no longer holds between the ranking of financial hedge and the optimal inventory level. We observe in the numerical study that a better hedge may lead to a lower inventory level. In addition, comparing Figures 1–3, we find that how financial hedging affects the inventory level may also depend on ρ, the correlation between demands and spot prices.

7.3. Impact on the Mean, Variance, and Utility

We now discuss how the operational hedge (the spot market procurement) and different financial hedges affect the firm's financial performance characteristics including the mean, variance, and utility of the cash flows.

Proposition 7.2. *For period n, $n \leq N$, we have the following results.*

(1) *Ranking by the utility value (from best to worst) is DS + HP, DS + i, DS, and SS (where the comparison of the last two is in expectation).*

(2) *When $z_n \leq \bar{z}_n^*$ (the feasible region for the risk-averse firm), as z_n increases,*

- *$E[\sum_{k=n} \alpha^{k-n} \pi_k]$ and $V[\sum_{k=n} \alpha^{k-n} \pi_k]$ both increase if the futures contract is used;*
- *$E[\pi_N]$ and $V[\pi_N]$ both increase regardless of financial hedging.*

We find that it is better to use multicontract hedges. The operational hedge is more effective (in terms of improving the mean profit) than any financial hedge because the benefit of making real-time decisions is salient. In addition, we use the numerical results to illustrate the effectiveness of operational and financial hedging in reducing the profit variance. As shown in Figures 4 and 5, for the case with positively correlated demand and spot price ($\rho = 0.5$), the profit variance reduction achieved by switching from SS to DS is between 25% and 50% (showing the impact of operational hedging); further profit variance reduction achieved by adding financial hedging is between 10% and 16% (showing the impact of financial hedging). For most cases, a higher inventory level corresponds to both a higher mean profit and a higher profit variance. Thus, an optimal inventory level is chosen to balance the mean and the variance. As the firm becomes more risk averse, the profit variance becomes more dominant, and thus the optimal inventory level is reduced accordingly.

We next use the efficient frontier (see Figures 6–8) to compare the impacts of different financial hedges

Figure 4 **Variance Reduction from SS to DS Against** $\lambda/2 = 0.001, \ldots, 0.01$

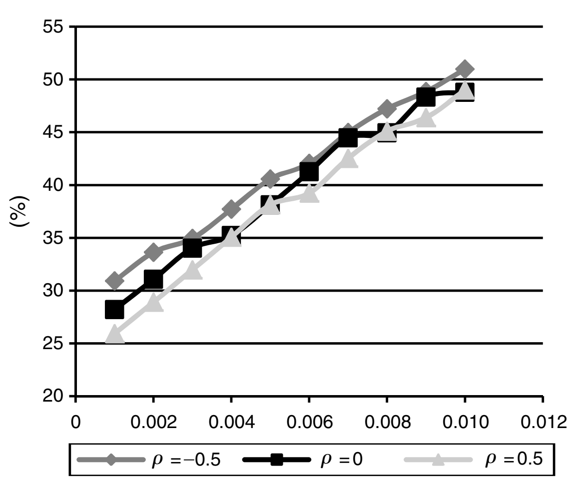


Figure 5 **Variance Reduction from DS to DS + HP Against** $\lambda/2 = 0.001, \ldots, 0.01$

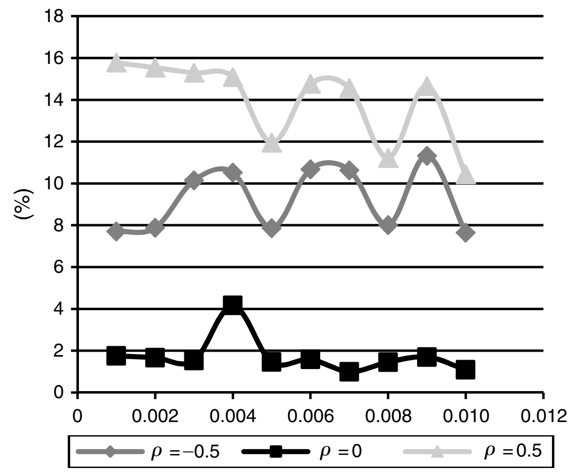


Figure 6 **Efficient Frontier (Variance vs. Mean) with the Demand and Spot Price Correlation** $\rho = -0.5$

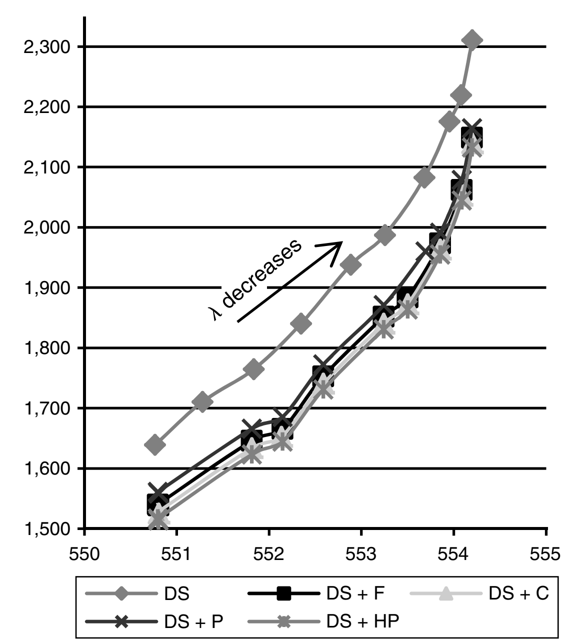
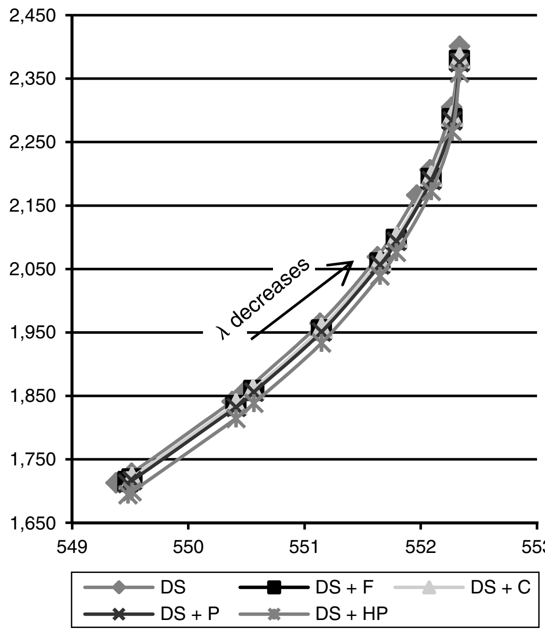


Figure 7 Efficient Frontier (Variance vs. Mean) with the Demand and Spot Price Correlation $\rho = 0$



on the firm's mean profit and profit variance. Since SS has a significantly lower mean but a higher variance than all other cases, it is removed from the figures for a clearer view of the other cases. For example, as shown by Figures 9 and 4, for the case with $\rho = 0.5$, by switching from SS to DS, the mean increase is between 7.1% and 7.7%, and the variance reduction is between 25% and 50%. As its risk aversion increases, the firm

Figure 8 Efficient Frontier (Variance vs. Mean) with the Demand and Spot Price Correlation $\rho = 0.5$

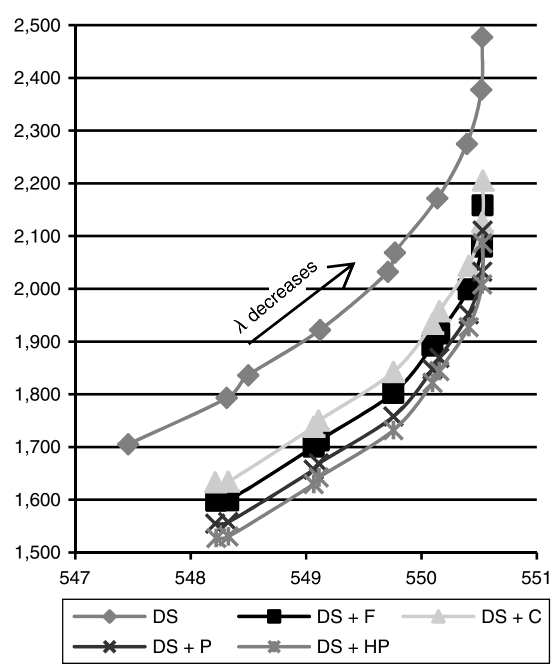
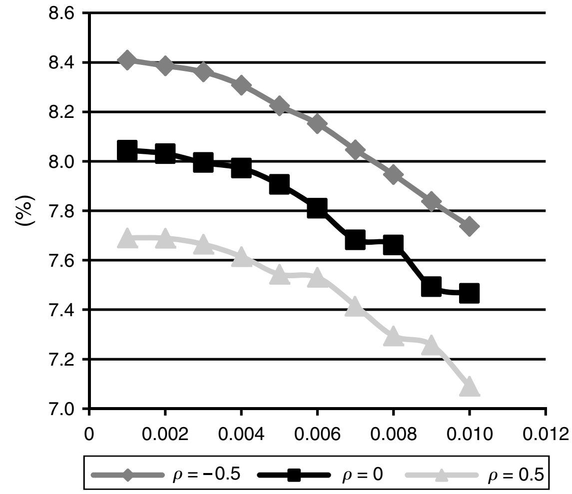


Figure 9 Mean Increase from SS to DS Against $\lambda/2 = 0.001, \ldots, 0.01$



moves to the optimal position with a lower mean and a lower variance. We observe that the curves for cases DS and DS + HP are at the top and the bottom of the graph area, respectively. The curves for cases DS + i, $i = f$, c, and p are sitting in the middle with alternating positions as the risk aversion increases. This clearly shows that it is always beneficial to adopt financial hedging, and the choice of a good hedge does have a significant impact on the firm's performance. Moreover, we note that multicontract hedges (which include the futures contract) are better than single-contract hedges in terms of not only effectiveness (of improving utility), but also computational convenience (for the optimal inventory decisions). As shown in Proposition 5.1, the optimal inventory decisions are myopic, and the optimal hedging quantities can be easily determined by solving a system of linear equations. The above observations back up our recommendation for the use of multicontract hedges in practice.

Last, we discuss our numerical observations on the impact of the correlation between demands and spot prices ($\rho = -0.5, 0, 0.5$). As Figure 5 demonstrates, the (multicontract) financial hedge is most effective when $\rho = 0.5$ and least effective when $\rho = 0$; it is not surprising. For the operational hedge, as shown in Figure 4, it is most effective when $\rho = -0.5$ and least effective when $\rho = 0.5$. Note that a negative correlation means that higher demands are more likely to occur together with lower spot prices, which translates to a higher profit margin. This may explain why the operational hedge is most effective in the case of negative correlation.

8. Extension to a General Probability Measure

We extend our model to accommodate the real-world probability measure, P. We have so far focused on the Q-measure, under which any financial hedge has zero risk premium (or zero expected payoff). Under the P-measure, the firm may have nonzero risk premium for some financial hedges. Accordingly, the firm may apply an exogenously given discount factor $\alpha = 1/(1+r)$, where r is often the weighted average cost of capital (Luenberger 1998). As discussed in §3, the Q-measure is defined as a combination of the real-world probability measure on demands and the risk-neutral probability measure on spot prices. Thus, when applying to uncertain demands, the P-measure and Q-measure yield the same results. When applying to uncertain spot prices, however, the P-measure and Q-measure yield different results.

We find that the results shown in previous sections remain unchanged, except for some minor changes on the expression of some parameters to reflect the nonzero expected payoff of financial hedges, for example, $E[\chi_i(S_{n+1}) \mid s_n] \neq 0$ and $E[\alpha S_{n+1} \mid s_n] - s_n \neq 0$. A detailed description of the changes is included in the online supplement. Comparing these to the previous results, we find that financial hedges now also contribute to the mean profit, and thus the gain/loss of mean profit due to hedging should also be considered when determining the optimal hedging quantities. Consequently, financial hedging adds an additional effect on the overage cost, which can be interpreted as the cost of hedging for a unit of overage inventory. More importantly, since the cost of hedging can be positive or negative, financial hedging may lead to less inventory. This new and important insight differs from what the past literature captures based on the Q-measure.

9. Managerial Insights and Conclusions

Our paper offers a multiperiod integrated risk management framework for storable commodities with a new problem formulation: one using an interperiod MV utility function, which considers the cash flow correlations across periods and appropriately characterizes risk aversion. Our results thus offer useful and new insights on the joint optimal inventory and financial hedging policies.

Effective implementation of such policies requires cross-functional decision coordination and information sharing among operations and financial managers. Our results indicate that the information burden is lower for operations managers in making sourcing and inventory decisions. The setting of optimal inventory (base-stock) levels requires awareness of the firm's commitment to financial hedging and the type of financial contract to be used, without, however, requiring the details of hedging quantities. We find that the optimal inventory level is myopic and robust to financial hedging, as long as the futures contract is included. Consistent with the exiting literature, we find that, for single-period (last-period) problems with zero risk premium, financial hedging raises the inventory levels. New to the literature, we find, for multiperiod problems or single-period problems with nonzero risk premium, that financial hedging may lower the inventory levels. Financial managers implementing an integrated commodity risk management are required to know the inventory levels for effectively choosing the type and quantity of the financial hedge. The good news is that the optimal hedging portfolio is obtained by simply solving a system of linear equations. Consistent with the existing literature, we find that futures contracts are all we need for single-period problems. Contrary to the existing literature for multiperiod problems, we find that the optimal hedge is not myopic and also includes various call and put options.

Our work clearly shows the role played by the operational and financial hedges. Spot market procurement is the effective operational hedge, used to deal with the demand risk; it is similar to, but more effective than, the financial hedge if only one type of hedge is allowed for use. When used jointly, they play distinct roles, where the operational hedge focuses on improving the mean profit, whereas the financial hedge controls the profit variance. More importantly, they help each other: better financial hedges make the operational hedge more effective (reflected by higher inventory levels). However, these observations may not hold for nonzero risk premium and multiperiod problems.

Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/msom.2013.0433.

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