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Going Bunkers: The Joint Route Selection and Refueling Problem

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Managing shipping vessel profitability is a central problem in marine transportation. We consider two commonly used types of vessels—"liners" (ships whose routes are fixed in advance) and "trampers" (ships for which future route components are selected based on available shipping jobs)—and formulate a vessel profit maximization problem as a stochastic dynamic program. For liner vessels, the profit maximization reduces to the problem of minimizing refueling costs over a given route subject to random fuel prices and limited vessel fuel capacity. Under mild assumptions about the stochastic dynamics of fuel prices at different ports, we provide a characterization of the structural properties of the optimal liner refueling policies. For trampers, the vessel profit maximization combines refueling decisions and route selection, which adds a combinatorial aspect to the problem. We characterize the optimal policy in special cases where prices are constant through time and do not differ across ports and prices are constant through time and differ across ports. The structure of the optimal policy in such special cases yields insights on the complexity of the problem and also guides the construction of heuristics for the general problem setting.

Key words: routing; shipping; refueling; stochastic prices; maritime transportation
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1. Introduction

Water transportation is an important component of the U.S. transportation industry, accounting for more than $23 billion in revenues, according to the U.S. Economic Census (U.S. Census Bureau 2002). The Marine Transportation System National Advisory Council (2000) estimates that the marine import-export trade alone accounts for nearly 7% of the U.S. gross domestic product. In recent years, increased competition and global downturn in the shipping industry have been putting downward pressure on the revenues of shipping companies while increased safety regulations and fuel prices continued to increase companies' operating costs. The changing economic environment has prompted many companies to abandon a status quo complacency and search for new ways to maintain and optimize their profitability.

A typical marine transportation company operates a fleet of ships that belong to one of two broad groups. *Liners* are vessels that follow the same cyclical route comprised of a string of ports, and *trampers* are vessels for hire for which the next destination is selected according to the set of available transportation jobs. The fundamental "routing" distinction between these two groups of vessels translates into important differences in how liners and trampers are managed. A liner brings in a steady stream of revenue, and a major managerial challenge is to minimize its refueling costs given a limited vessel fuel capacity and unknown future fuel prices. A tramper, in contrast, has a choice of ports to visit in the future and thus can control both its revenues and its refueling costs. Both types of vessels use "bunkers," a byproduct of the oil refining process, as a fuel—and the bunkers' prices are highly unpredictable and can exhibit significant variation across ports on the same date. Note that vessels typically also use diesel oil for auxiliary power, and although such a component could be included in the model, we do not consider it to keep the exposition clear.

In the present paper, we propose a model for bunkers' price dynamics in conjunction with a novel model that describes a single-vessel profit optimization problem. In the case of a liner, the profit optimization reduces to minimizing the refueling costs. We formulate this problem as a long-term average stochastic dynamic program where the decisions are the refueling amounts at each port the vessel visits on its fixed route. In the presence of random fuel prices, this problem becomes a variant of the stochastic capacitated inventory management problem, whose solution is shown to be a capacity-adjusted state-dependent buy-up-to policy. For a tramper, the refueling dynamic programming problem is blended with combinatorial optimization of the vessel's route. For a static (deterministic or stochastic) routing policy, the tramper problem reduces to a liner instance, but the addition of dynamic routing significantly complicates the analysis of optimal profit management policies. The fact that refueling and routing decisions are interconnected makes the tramper problem unique, because it cannot be reduced to classes of problems analyzed in the stochastic routing literature (we return to this point in the next section). In particular, a tramper in general cannot ignore bunkers' prices when selecting a route to maximize profitability, as price differences among ports located in the same region might exist. For example, on November 30, 2007, the following bunker prices (per metric ton) were recorded for a few ports in North and Central America: $473 in Philadelphia, $445 in Houston, $521 in Los Angeles, and $460 in Panama.

The main contributions of the present work can be summarized as follows:

1. We develop a new modeling framework to analyze a single-vessel profit optimization problem as a blend of combinatorial route selection and dynamic programming refueling problems in the presence of stochastic revenues and fuel prices.

2. For vessels that follow a static routing policy (liners), we show that the optimal refueling policy is of the buy-up-to form (Proposition 1) and that the value of the buy-up-to level necessarily belongs to a finite, potentially small set. Under additional assumptions on the stochastic monotonicity of the underlying fuel price processes, we also establish monotonicity properties of the optimal buy-up-to levels (Proposition 2).

3. For vessels that combine refueling with route selection (trampers), we demonstrate that although the optimal refueling quantities in general need not be monotone in onboard inventory, the optimal postrefueling inventory levels are (Proposition 3). In addition, we investigate several special cases of the tramper problem. In particular, when the bunker prices are constant across time and equal across ports, we show that the problem can be reduced to one of finding the best profit-to-time ratio cycle on a network of ports. Also, when the bunker prices are constant across time but differ across ports, we show that the solution forms a cycle in a generalized location-inventory space (Proposition 4). In addition, we investigate the impact of vessel capacity (Proposition 5) and price stochasticity on optimal routing polices.

4. Based on the above results, we develop heuristics for the general case of the tramper problem and derive performance bounds for those (Propositions 6 and 7).

Our analysis of the vessel profit optimization problem includes further modeling of the stochastic fuel prices in which, at each port, the influence of global oil price dynamics is augmented by the contribution of local Markovian demand-supply dynamics.

Our paper is organized as follows. The next section provides a review of related work. Section 3 introduces our vessel profit maximization model. Section 4 is devoted to modeling of the stochastic dynamics of bunker prices. The special case of liners is analyzed in §5, and §6 focuses on trampers. We conclude by discussing possible future research directions for our work.

2. Review of Related Work

Traditional marine fleet management research literature outlines a hierarchy of vessel management problems (Christiansen et al. 2004, 2005). At the top of the hierarchy are the models that study strategic, long-term decisions, such as fleet sizing and route design (Dantzig and Fulkerson 1954, Richetta and Larson 1997, Imai and Rivera 2001, Cho and Perakis 1996). On a more tactical level, fleet management models focus on routing and scheduling of a fixed number of vessels. In particular, existing papers on liner operations exclusively focus on deterministic models of fleet deployment and cargo booking (Perakis and

Jaramillo 1991, Powell and Perakis 1997). In the context of airlines, linear programming formulations for fuel management have been proposed in the literature (where prices are taken to be constant over time); see, e.g., Darnell and Loflin (1977) and Stroup and Wollmer (1992) and references therein. These studies relate to the liner study presented here, as the route of the flights is fixed. Although it is natural to take prices to be constant over the horizon of a route, given the short time elapsed between each stop in the context of airlines, stochasticity of prices plays a more important role in the context of marine shipping and is explicitly taken into account in our liner formulation.

In its most general form, profit maximization at the vessel level represents a blend of combinatorial optimization (route selection) and stochastic dynamic programming (minimization of refueling costs).

The problem of controlling refueling costs is related to a number of papers on inventory management in the presence of random prices. In the first paper in this area, Kalymon (1971) generalizes the classical inventory model by Scarf (1960) (formulated for the setting with convex holding and shortage costs and a setup cost) by allowing product prices to follow a nonstationary Markov process. In the finite-horizon setting, the optimal replenishment policy in period n is shown to be of the $(s_n(p), S_n(p))$ form, where p is the realized product price in that period. Song and Zipkin (1993) consider a continuous time infinite-horizon inventory model where the product demand rate varies in a Markov-modulated fashion with an underlying "state-of-the-world" variable: if the "world" is in state i, the demand is assumed to follow a Poisson process with rate λ_i. Under the assumptions of full demand backlogging, stochastic lead times, and convex ordering and holding costs, the optimality of a state-dependent base-stock policy (without the order setup cost) and of the $(s_n(i), S_n(i))$ policy (with the setup cost) is established. This framework was extended by Ozekici and Parlar (1999) to include the influence of the Markovian random environmental process on the demand, supply, and cost parameters. An interesting variant of the random-price setting is analyzed in Moinzadeh (1997), where a continuous-time infinite-horizon setting is used to study an inventory problem with fixed-price discounts offered at random times. The price-dependent two-bin (R, s, Q) replenishment and stocking policy is employed: When the deal is offered, an $(s, s+Q)$ replenishment rule is used, but when the inventory is completely depleted, the order of $R \leq s+Q$ units is placed. Whereas for a fixed vessel route, the refueling cost-minimization problem resembles classical inventory replenishment problems with random prices and deterministic demand values, two features distinguish it from the variants studied in the literature: the finite fuel capacity of a vessel and the availability of the entire set of prices at all ports at any point in time. In other words, we deal with a capacitated inventory problem for which the state information is a vector of fuel prices that includes the price "now" as well as indicators for future prices (at all ports in the network). The presence of geographical dependence of prices is an aspect that is specific to the problem we analyze and has not, to the best of our knowledge, been studied in the inventory management literature.

The stochastic nature of future potential rewards places route selection into the same class of problems as, for example, the stochastic vehicle-routing problem. However, an important distinction of the routing aspect of the problem under consideration from the classical stochastic vehicle-routing literature resides in the the formulation itself. In our setting, the objective of the decision maker is to maximize the long-term average profits with constraints coming into play only through the refueling decisions. In contrast, the typical stochastic vehicle-routing problem consists of minimizing the total cost of a travel plan such that *each* node in the network is visited at least once, with potential side constraints. We refer the reader to Gendreau et al. (1995) for a review of stochastic vehicle routing.

An important observation is that the tramper problem, in the absence of refueling and in the special case of deterministic rewards, actually reduces to the problem of finding the cycle with highest revenue per unit of time over a network; this was studied in Dantzig et al. (1966). We build on this connection to understand the structure of optimal policies in the presence of refueling decisions.

Finally, we mention that our price model describes the evolution of bunker prices in terms of the dynamics of two independent Markov chains. Our approach to modeling bunker prices is related in spirit to the work of Hamilton (1989) that incorporates Markovian "regime changes" into the dynamics of energy prices. We refer the reader to Hamilton and Susmel (1994), Either and Mount (1998), and Noel (2007a, b) for more recent applications of this approach.

3. Optimizing Vessel Profits: The Model

We consider a vessel with fuel capacity C (measured, e.g., in metric tons (mts)) traveling through a network of N nodes, each representing a port. Because a day is a natural time unit in our setting, we use a discrete-time formulation for the vessel profit optimization problem. Each node in the network of ports is assumed to be connected to any other node by an arc. We use $\mathcal{N} = \{1, 2, \ldots, N\}$ to denote the set of all nodes and $\mathcal{A} = \{(i, j): i \in \mathcal{N}, j \in \mathcal{N}\}$ to denote the set of arcs connecting the nodes in $\mathcal{N}$. On the complete graph $(\mathcal{N}, \mathcal{A})$ we define the following set of measures:

1. $\mathbf{d} = \{d_{ij} > 0 \mid (i, j) \in \mathcal{A}\}$ and $\boldsymbol{\tau} = \{\tau_{ij} \in \mathbb{Z}_+ \mid (i, j) \in \mathcal{A}\}$ are the amount of fuel (e.g., in mts) and the time (in days), respectively, required to travel through the arcs of $\mathcal{A}$.

2. For each node $i \in \mathcal{N}$, the set $O(i) = \{j: (i, j) \in \mathcal{A}, d_{ij} \leq C\}$ denotes nodes that are "reachable" from i without stopping at any other port. Without loss of generality, we assume that the capacity of the vessel is such that any two nodes $i \in \mathcal{N}$ and $j \in \mathcal{N} \setminus O(i)$ are connected by a finite set of arcs, i.e., that there exists a finite set of nodes $i_1, \ldots, i_M$ such that $i_1 \in O(i)$, $i_2 \in O(i_1), \ldots, j \in O(i_M)$ for some $M \geq 1$.

3. For each time period $t = 1, 2, \ldots,$ the set of rewards associated with shipping contracts on every arc $(i, j) \in \mathcal{A}$ is denoted as $\mathbf{r}(t) = \{r_{ij}(t) \mid (i, j) \in \mathcal{A}\}$. The reward $r_{ij}(t)$ associated with traveling on a given arc $(i, j) \in \mathcal{A}$ at time period t is assumed to be a realization of a nonnegative bounded discrete i.i.d. random variable with a static probability distribution $p_{ij}(r) = \text{Prob}(r_{ij}(t) = r)$. In our analysis, we assume that each shipping contract is associated with a single arc (i, j)—an assumption valid for a large majority of contracts we observed in practice.

4. For each time period $t = 1, 2, \ldots,$ the set of fuel prices for all ports is denoted as $\mathbf{P}(t) = \{P_i(t) \mid i \in \mathcal{N}\}$. We assume that for any time period t and for any port $i \in \mathcal{N}$ the price $P_i(t)$ can only take one of the L values $\{P^{(1)}, \ldots, P^{(L)}\}$. In addition, we assume that the vector of prices $\mathbf{P}(t)$ follows a Markov process such that any price state can be reached from any other price state with a positive probability in a finite number of transitions.

Consider a vessel arriving at time t at port i with fuel inventory I, observing the values of fuel prices $\mathbf{P}$ and rewards $\mathbf{r}$. Let a stationary unichain policy π: $(i, I, \mathbf{P}, \mathbf{r}) \longmapsto (j, q)$ denote the choice of the next port to visit $j \in \mathcal{N} \setminus \{i\}$[1] and the refueling amount $q \in Q(i, j, I) := \{q \mid q \geq 0,\ d_{ij} \leq I + q \leq C\}$. Let $V_k^\pi(i, I, \mathbf{P}, \mathbf{r})$ be the total expected profit earned in the next k port-to-port transitions under a stationary policy π, and let $t_k^\pi(i, I, \mathbf{P}, \mathbf{r})$ be the expected time associated with these k transitions. In our analysis we use the long-run expected profit per unit of time

$$\lambda = \lim_{k \to \infty} \left(\frac{V_k^\pi(i, I, \mathbf{P}, \mathbf{r})}{t_k^\pi(i, I, \mathbf{P}, \mathbf{r})} \right) \tag{1}$$

as an optimization criterion for the vessel. Note that because we assume the travel times between ports to be deterministic, the analysis we conduct uses the standard approaches for semi-Markov decision processes as outlined in Bertsekas (2000). Clearly, under the assumptions on the reachability of nodes and on the reward and price dynamics presented above, the limit in (1) is well-defined, and its value is independent of the initial state. Following Bertsekas (2000), we can express the Bellman equation for λ as

$$h(i, I, \mathbf{P}, \mathbf{r}) = \max_{j \in O(i)} \Bigg(r_{ij} - \lambda \tau_{ij} + \max_{q \in Q(i, j, I)} \left(-P_i q + \mathbb{E}_{\mathbf{P}' \mid \mathbf{P}, \mathbf{r}'} [h(j, I + q - d_{ij}, \mathbf{P}', \mathbf{r}')] \right) \Bigg), \tag{2}$$

where $h(i, I, \mathbf{P}, \mathbf{r})$ is a function defined on the state space of the problem with $h(i_0, I_0, \mathbf{P}_0, \mathbf{r}_0) = 0$ for some (arbitrarily chosen) state $(i_0, I_0, \mathbf{P}_0, \mathbf{r}_0)$, and $\mathbf{P}' \mid \mathbf{P}$ denotes that the expectation with respect to $\mathbf{P}'$ (the vector of prices observed τ_{ij} units of time after $\mathbf{P}$ is observed) taken conditional on the value of $\mathbf{P}$.

The outer maximization operator in (2) represents the routing decision, and the inner maximization

[1] Note that here we assume that the ship never idles at a port. While this is typical of most practical settings, allowing for idling is possible at the expense of additional notation.

operator corresponds to the refueling decision. For simplicity, we assume that refueling is instantaneous.

The formulation as well as the analytical results to be presented are valid for *any* Markovian model for the fuel price dynamics, except when explicitly stated otherwise. In the next section, to finalize the specification of the model, we develop a particular fuel price model and estimate its parameters using actual pricing data for 18 ports for the period of January–June 2005. This model will then be used in our numerical experiments. We return to the analysis of the liner's and tramper's problems in §§5 and 6, respectively.

4. Modeling Stochastic Dynamics of Bunker Fuel Prices

In practice, selecting an appropriate price dynamics model is a challenging task because of strong time and geographical correlations in bunker prices. In our analysis, we have selected the general form of the Markov process $\{\mathbf{P}(t)\colon t \geq 0\}$ based on the following two fundamental features of the bunker price dynamics identified by practitioners. On the one hand, the main, global driver of the bunker fuel prices on a given day is crude oil price.[2] This dependence stems from the fact that the bunkers' fuel is a byproduct of the oil-refining process. In addition to this global effect, on the level of a particular port, the bunker price is also influenced by the interactions between supply of and demand for the fuel. This local supply effect is believed to be weaker than the global crude oil price effect.

We propose a model of bunker price dynamics that incorporates these two effects. Let $P_0(t)$ denote the spot price of crude oil (in \$ per barrel) in period t. Then the bunker fuel price $P_i(t)$ (in \$ per metric ton) at location i in period t is described as follows:

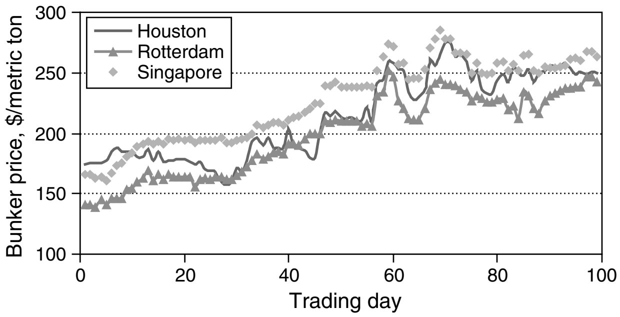
$$P_i(t) = \gamma P_0(t) + \alpha_i(t) + \epsilon_i(t), \tag{3}$$

where γ is a constant; $\alpha_i(t)$ are Markov processes independent of P_0 and independent across ports, each with m possible states; and $\epsilon_i(t)$ is a stationary random variable with $\mathbb{E}[\epsilon_i(t)] = 0$, $\mathbb{E}[\epsilon_i^2(t)] = \sigma_i^2 < \infty$ for all i and t. We also assume that all $\epsilon_i(t)$ are independent across time and across ports (and of P_0 and α_i's). Introducing a dependence on oil price in model (3) follows the established pricing models for other products of the oil refining process, such as gasoline or home heating oil (Borenstein et al. 1997, Asche et al. 2003, Kaufmann and Laskowski 2005); the introduction of location-dependent terms α_i and ϵ_i is an important addition. The interpretation of the model (3) is as follows: γ represents a global price-adjustment factor, α_i are local (geographical) supply correction factors for the bunkers prices, and $\epsilon_i(t)$ are geographical adjustments because of other factors. The role of the oil price and thus the presence of γ in (3) is straightforward in models of any oil-based products. The necessity to use $\epsilon_i(t)$ term stems from the presence of unexpected and potentially *short-lived* local factors such as sudden surge of the demand for bunkers caused by the unanticipated arrival of several tramper vessels at a particular port.

On the other hand, the α_i terms stand to represent the potentially *longer-term* effects on the bunker price stemming from the juxtaposition of global changes in the state of the oil-refining industry and local demand-and-supply conditions (the evolution of the α_is is related to the "regime changes" in Hamilton 1989). At any port, we describe the Markovian dynamics of the local supply correction factors using a simple two-state form: α_i is assumed to take "high" values when the fuel supply at the port is constrained and "low" otherwise; i.e., α_i, $i = 1, \ldots, N$ can have two possible states: $\alpha_i^{\pm} = A_i \pm \delta_i$, where A_i plays the role of the average price premium/discount over the value of $\gamma P_0(t)$ associated with port i, and δ_i, which characterizes the amplitude of the price response at port i to the changes in the local demand-supply balance. The transitions between these two supply states are described at port i by a local "inertia" parameter η_i, which stands for the probability of the local supply state remaining unchanged on the next day. Thus, if the port i is in state $A_i + \delta_i$ one day, it will remain in the state $A_i + \delta_i$ the next day with probability η_i and transfer to the state $A_i - \delta_i$ with probability $1 - \eta_i$. We note that according to (3), the state of the price system in period t is described by $2N + 1$ values: the oil price $P_0(t)$, the set of local supply corrections $\boldsymbol{\alpha}$, and the perturbation vector $\boldsymbol{\epsilon}$. The motivation for including

[2] The two main types of crude oil typically used by traders as indicators for the bunker prices are Brent (traded on London's Intercontinental Exchange) and Western Texas Intermediate, traded on the New York Mercantile Exchange.

Figure 1 Bunker Prices in World's Three Largest Ports, January–June 2005 (98 Bunker Fuel Trading Days)



the "inertia" terms into the pricing model (3) is illustrated in Figure 1, which plots actual bunker prices observed in three large ports—Rotterdam, Houston, and Singapore—during the period January–June 2005 (the total of 98 bunker trading days).

We observe that, in addition to the general common price trend and to the random daily shocks, the bunker prices for these three ports exhibit an absolute ranking with some longer-term stability. For example, for the first 10 or so trading days, the price ranking of three ports, from highest to lowest, is Houston-Singapore-Rotterdam. After that, the ranking becomes, almost uninterruptedly for the next 30 days, Singapore-Houston-Rotterdam. For the next five or so days, the ranking is Singapore-Rotterdam-Houston. After that, almost uninterruptedly, the Singapore-Houston-Rotterdam ranking is restored. The "inertial" Markovian structure of the local supply correction terms α_i helps model such behavior.

Bunker pricing data is not publicly available, and shipping companies resort to specialized firms for historical worldwide pricing information. We obtained actual process from such a firm for $N = 18$ ports for the period of January–June 2005 ($k = 98$ trading days). A spreadsheet with price information is enclosed as a part of an online appendix.

The details of the parameter estimation procedure for model (3) are provided in the online Appendix B. The value of the parameter γ representing a conversion factor between the time average of the oil price expressed in \$/barrel and the time-and-location average of bunker prices expressed in \$/mts is estimated as 4.43, and the values of other parameter estimates for three of the world largest ports—Rotterdam, Houston,

Table 1 Parameters of the Bunker Fuel Price Dynamics

Parameter	Rotterdam	Houston	Singapore	Max. (all ports)	Min. (all ports)	Mean (all ports)
A, \$/metric ton	−28.4	−13.4	−0.42	34.0	−31.6	0
δ, \$/metric ton	29.7	31.7	30.8	31.7	24.6	28.6
η	0.99	0.93	0.99	0.99	0.93	0.98
σ, \$/metric ton	14.8	13.9	14.2	18.1	10.9	14.2
σ/δ	0.50	0.44	0.46	0.61	0.38	0.50

and Singapore, as well as some descriptive statistics for all ports—are presented in Table 1. Note that, as Table 1 indicates, the expected price premia/discounts A_i vary substantially over the geographical locations: from a deep expected discount of \$31.60 (observed at the port of Antwerp, Belgium), to a heavy premium of \$34.00 (observed at the port of Yokohama, Japan). Yet these expected values do not tell the entire story: the amplitudes of the Markovian local supply corrections are around \$30, decisively influencing the resulting price ranking of ports. Finally, very high estimates for η_i, indicating extreme "stickiness" of the local supply corrections, are largely caused by a two-state nature of the Markovian supply correction model and high values of the amplitudes of supply corrections: Note that the transition between $A_i + \delta_i$ and $A_i - \delta_i$ prices involves a price change of $2\delta_i$, or, on average, of \$57. Clearly, a reasonable model should predict that such dramatic price changes happen very infrequently to produce reasonable values for the average price change caused by "jumps" between local supply states. For example, the average "inertia" coefficient equal to 0.98 and the average δ equal to \$28.60 correspond to equivalent daily price change of $(1 - 0.98) \times 2 \times \$28.60 = \$1.04$. An increase in a number of modeled possible states for the local bunkers supply chain will result in a reduction in the estimates for the "inertia" coefficients associated with each state.

To fully characterize the proposed model (3), we turn to the last remaining element of the stochastic dynamics of bunker prices, namely, the stochastic dynamics of crude oil prices. The earlier literature on the subject (Brennan and Schwartz 1985, Paddock et al. 1988, Smith and McCardle 1998) has established the geometric Brownian motion (GBM), in both continuous and discretized versions, as a standard for the description of the crude oil dynamics. In the discretized version of the GBM model, the relative changes in the oil prices for any two consecutive days are assumed to be i.i.d. normal random variables:

$$\frac{P_0(t+1)-P_0(t)}{P_0(t)} \sim N(\mu_0, \sigma_0). \tag{4}$$

More recently, however, several studies (Laughton and Jacoby 1995, Cortazar and Schwartz 1994, Dixit and Pindyck 1994, Smith and McCardle 1999) have argued that the unlimited asymptotic variance associated with the GBM model may not be a good descriptor of actual price dynamics. As an alternative, these studies suggested using mean-reverting processes, such as an $AR(1)$ process

$$P_0(t+1) = \lambda P_0(t) + (1-\lambda)\hat{P}_0 + \varepsilon_0(t), \tag{5}$$

where $\hat{P}_0$ is the long-term price level to which oil prices "revert," and $\varepsilon_0(t)$, $t = 0, 1, \ldots$ are i.i.d. normal random variables. By analyzing the actual crude oil data for the period January–June 2005 (see the estimation details in Appendix B), we have found that the mean-reverting model provides a better description of the crude oil price dynamics. It is this model that we use in our numerical studies.

5. The Case of a Liner

The fixed-route feature of a liner trajectory simplifies the analysis of the vessel profit management problem (2) and allows one to characterize optimal refueling policies.

For a liner, at each port i the destination $j \in O(i)$ is known and fixed. We let $j^{(0)}(i) = i$ and $j^{(k)}(i)$ denote the kth port visited when starting at port i.[3] With a slight abuse of notation, we let $j(i)$ denote $j^{(1)}(i)$. Similarly, we let $d^{(k)}(i)$ denote the consumption needed for the next k ports; i.e.,

$$d^{(k)}(i) = \sum_{m=0}^{k-1} d_{j^{(m)}(i) j^{(m+1)}(i)}.$$

When the route is fixed, the revenue stream is known and is unaffected by the refueling decisions of the firm. In this context, the objective of the firm is to minimize the long-run average fuel costs along the sequence of ports visited. If we let μ denote the optimal cost per unit of time, the Bellman equation can be written as follows:

$$w(i, I, \mathbf{P}) = -\mu\tau_{ij(i)} + \min_{q \in Q(i, j(i), I)} \left(P_i q + \mathbb{E}_{\mathbf{P}'|\mathbf{P}}[w(j(i), I+q-d_{ij(i)}, \mathbf{P}')]\right), \tag{6}$$

where $w(i, I, \mathbf{P})$ is a function defined on the state space of the problem with $w(i_0, I_0, \mathbf{P}_0) = 0$ for some (arbitrarily chosen) state $(i_0, I_0, \mathbf{P}_0)$.

The following result summarizes structural properties of the optimal cost function and the optimal refueling policy:

Proposition 1. (a) *The function* $w(i, I, \mathbf{P})$ *associated with the optimal cost is a convex function of fuel inventory* I.

(b) *There exists a price-dependent function* $S(i, \mathbf{P})$ *taking values in* $[d_{i, j(i)}, C]$ *such that an optimal refueling policy is given by*

$$Q^*(i, I, \mathbf{P}) = (S(i, \mathbf{P}) - I)^+. \tag{7}$$

(c) *The function* $S(i, \mathbf{P})$ *takes values in* $B(i)$, *where* $B(i) = \{d^{(l)}(i)\colon\ l = 1, 2, \ldots, l^*(i)\} \cup \{C\}$, *with* $l^*(i) = \max\{k\colon d^{(k)}(i) \le C\}$.

Proposition 1(b) describes a capacitated version of the base-stock inventory policy and admits a straightforward interpretation. On the one hand, when refueling is necessary at port i (corresponding to $I \le d_{ij^*(i)}$), it should be performed up to the level equal to $S(i, \mathbf{P}(t))$. On the other hand, if there is enough fuel to get to the next port $j^*(i)$—i.e., $I > d_{ij^*(i)}$—the refueling at i should be done if and only if $S(i, \mathbf{P}(t)) > I$. By Proposition 1(c), the optimal buy-up-to level belongs to the set $B(i)$; in other words, whenever bunkers are purchased, they are purchased either to arrive at one of the next ports on the route with exactly zero inventory or to fill up the tank up to capacity. This property allows one to restrict the set of possible values for $S(i, \mathbf{P})$ when searching for an optimal policy.

Proposition 1 describes the structure of the optimal refueling policy for any general Markovian price process $\{\mathbf{P}(t)\colon t \ge 0\}$. Sharper characterization of the properties of the capacity-adjusted fuel-up-to levels $S(i, \mathbf{P}(t))$ is possible for specific models of the Markovian price dynamics. For example, based on the structure of the fuel pricing dynamics described in (3)

[3] Note that as stated, any liner has to travel through a cycle in the network of ports. The analysis easily extends to a case where the liner follows any repeated route (not necessarily a cycle in the network of ports) at the expense of additional notation.

in §4, one can analyze the properties of the fuel-up-to-levels $S(i, P_0, \boldsymbol{\epsilon}, \boldsymbol{\alpha})$ (note that because the values of the local supply correction terms α_i are assumed to be observable, expressing the state of the system in terms of the prices $\mathbf{P}$ is equivalent to specifying the values of corrections $\boldsymbol{\epsilon}$). In particular, let $F(P_0(t+1) \mid P_0(t))$ denote the cumulative distribution function of the oil price in period $t+1$ given that the oil price in period t is $P_0(t)$. Consider the following assumptions on the shape of F:

Assumption 1 (Crude Oil Price Dynamics). (i) *$F(P_0(t+1) \mid P_0(t))$ is a nonincreasing function of $P_0(t)$ for any $P_0(t+1)$.*

(ii) *$P_0(t) - \mathbb{E}(P_0(t+t') \mid P_0(t))$ is a nondecreasing function of $P_0(t)$ for all $t' \geq 0$.*

Assumption 1 can be rationalized as follows. Let $P_0(t+1) \mid P_0(t)$ be the (random) oil price in period $t+1$, given that in period t the oil price is $P_0(t)$. Assumption 1(i) states that $P_0(t+1) \mid P_0(t) = P_H$ stochastically dominates $P_0(t+1) \mid P_0(t) = P_L$ for all $P_L \leq P_H$. In particular, it implies that the conditional expectation $\mathbb{E}(P_0(t+1) \mid P_0(t))$ is a nondecreasing function of $P_0(t)$. Assumption 1(ii) limits the rate at which this conditional expectation grows as a function of $P_0(t)$. Note that both the mean-reverting model as well as GBM model with nonpositive drift satisfy this assumption. Assumption 1 serves as a sufficient condition for the monotonicity of the fuel-up-to levels $S(i, P_0, \boldsymbol{\epsilon}, \boldsymbol{\alpha})$:

Proposition 2. *Suppose that bunker fuel prices evolve according to* (3). *Then the fuel-up-to level $S(i, P_0, \boldsymbol{\epsilon}, \boldsymbol{\alpha})$ is a nonincreasing function of ε_i (or equivalently P_i for fixed P_0 and $\boldsymbol{\alpha}$). If, in addition, Assumption* 1 *holds, then the fuel-up-to level $S(i, P_0, \boldsymbol{\epsilon}, \boldsymbol{\alpha})$ is a nonincreasing function of P_0 for fixed $\boldsymbol{\epsilon}$ and $\boldsymbol{\alpha}$.*

In particular, we note that under Assumption 1, when the local supply corrections are constant across time, the values of the α_is do not influence the monotonicity of the optimal fuel-up-to levels. However, when Assumption 1 is not satisfied, it is easy to find examples in which the monotonicity of the fuel-up-to levels as functions of the oil price P_0 breaks down.

In online Appendix C, we illustrate numerically the properties of the optimal fuel-up-to levels described in Propositions 1 and 2 as well as the performance of simple heuristics, which require more moderate computational effort.

6. The Case of a Tramper

In this section we consider a profit maximization problem for a vessel whose route can be dynamically adjusted based on the set of available shipping jobs and the set of observed fuel prices. In particular, we focus on the case of a "spot tramper," for which the set of available jobs becomes known only after the completion of a previous shipping job, upon arrival at a discharge port. This reflects the practice of several tramper companies with which we collaborated.

The case of a tramper vessel is far more complex and less amenable to analysis because of the additional routing decision that needs to be made at every port. Section 6.1 illustrates some of the complications that arise in the context of the tramper problem when contrasting it to the liner. Section 6.2 analyzes the tramper problem in various special cases, where either the optimal solution can be characterized or simple policies can be shown to be near optimal. Based on the latter analysis, §6.3 develops potential heuristics for the general tramper problem and illustrates their performance.

6.1. Optimal Refueling Decisions: Monotonicity Properties

Below, we re-express the Bellman equation given in (2) as

$$h(i, I, \mathbf{P}, \mathbf{r}) = \max_{j \in O(i)} (r_{ij} - \lambda \tau_{ij} + D_{ij}(I, \mathbf{P})), \tag{8}$$

where

$$D_{ij}(I, \mathbf{P}) = \max_{q \in Q(i, j, I)} \left(-P_i q + \mathbb{E}_{\mathbf{P}' \mid \mathbf{P}, \mathbf{r}'}[h(j, I+q-d_{ij}, \mathbf{P}', \mathbf{r}')]\right), \tag{9}$$

with $Q(i, j, I) = \{q \mid q \geq 0, d_{ij} \leq I + q \leq C\}$. Note that although concavity of h with respect to inventory is preserved under the transformation on the right-hand side of (9), the combinatorial nature of the route selection problem in (8) breaks this concavity down. This, in turn, affects the monotonicity properties of the optimal refueling policies.

One can derive sensitivity properties for the optimal refueling values. In particular, assume that while in port i, node j is selected as the next destination, and let

$$q^*(i, j, I, \mathbf{P}) = \min\left(\arg\max_{q \in Q(i, j, I)} \left(-P_i q + \mathbb{E}_{\mathbf{P}' \mid \mathbf{P}, \mathbf{r}'}[h(j, I+q-d_{ij}, \mathbf{P}', \mathbf{r}')]\right)\right)$$

be the smallest optimal refueling quantity in this case. Then,

PROPOSITION 3. *$I + q^*(i, j, I, \mathbf{P})$ is a nondecreasing function of I for any $i = 1, \ldots, N$, $j \in O(i)$, and for any current price vector $\mathbf{P}$.*

Although the monotonicity of the fuel-up-to level with respect to the onboard inventory is to be expected, it may be possible for the refueling quantity itself to be nonmonotone because of the influence that the routing decisions exert on the refueling process.

6.2. Optimal and Near-Optimal Solution Structure: Special Cases

An important feature of the tramper problem is a strong interaction in general between the routing and the refueling decisions. Indeed, the latter decisions need not decouple in the general case, and the fact that they are selected jointly presents unique challenges. In this section, we develop an understanding of the solution structure in various special cases. This will highlight how various features of the tramper problem, independently of each other, create complex interactions between refueling and routing decisions. We focus on three key features associated with the tramper problem: (i) price heterogeneity (across ports); (ii) price stochasticity; and (iii) vessel capacity constraint. We will illustrate how each of these features introduces strong interactions between the refueling and routing decisions. In addition to these insights, the analysis of the special cases will also be the basis for the development of heuristics with reduced computational requirements in §6.3.

To analyze the various impacts of the model features above, we will start from a base case where all the features mentioned are absent and then study the impact of adding one of those features. In particular, our base case will be a variant of the vessel profit management problem (2) with deterministic reward values $\mathbf{r}$, deterministic, constant (across time), and uniform (across ports) fuel prices; i.e., $P_i = P_c$, $i \in \mathcal{N}$ for some common value P_c. In addition, we assume that the consumption between ports and corresponding travel times are proportional, i.e., that for all $(i, j) \in \mathcal{A}$, $d_{ij} = \nu\tau_{ij}$ for some $\nu > 0$.

6.2.1. Base Case: Deterministic Uniform Fuel Prices. In the base case, prices are equal across all ports and are constant; hence, without loss of generality, one can restrict attention to policies where the firm replenishes the exact amount required to reach the port it selects to travel to next. Under any such policy, the fuel on board on arrival at a port will always be exactly zero, and it is possible to associate a profit with each arc $(i, j) \in \mathcal{A}$ as follows:

$$\theta_{ij} = r_{ij} - P_c d_{ij}.$$

Based on the above, the tramper problem in the base case reduces to that of finding the cycle that maximizes the profit per unit of time in the network $(\mathcal{N}, \mathcal{A})$ where crossing arc (i, j) earns θ_{ij} units of profit and requires τ_{ij} days. This maximum profit-to-time ratio problem has been analyzed in the literature; see Dantzig et al. (1966) for an early reference. When θ_{ij} are integers, Ahuja et al. (1993, §5.7) describe a binary search algorithm to find the optimal revenue per unit of time, λ^D. The algorithm uses the fact that $\lambda^D \in [-\theta_{\max}, \theta_{\max}]$, where $\theta_{\max} = \max\{|\theta_{ij}|: (i, j) \in \mathcal{A}\}$. At a high level, the algorithm follows a sequence of iterations, and after each such iteration, one can halve the interval of possible values for the optimal profit per unit of time. Each iteration starts with an interval of possible values $[\underline{\mu}, \bar{\mu}]$ and concludes whether $\lambda^D > (\underline{\mu}+\bar{\mu})/2$, $\lambda^D < (\underline{\mu}+\bar{\mu})/2$, or $\lambda^D = (\underline{\mu}+\bar{\mu})/2$. In the first two cases, one can restrict attention in the next iteration to an interval with half the length of the one started with. In the last case, one has found the optimal profit per unit of time. At each iteration, one considers a network $(\mathcal{N}, \mathcal{A})$ with arc measures

$$\check{\theta}_{ij} = \theta_{ij} - \frac{\underline{\mu}+\bar{\mu}}{2}\tau_{ij}$$

and applies a shortest path label-correcting algorithm to detect the existence or nonexistence of a negative length cycle. If a negative length cycle exists, then $\lambda^D < (\underline{\mu}+\bar{\mu})/2$; if all cycles have positive length, then $\lambda^D > (\underline{\mu}+\bar{\mu})/2$; and if a zero length cycle exists, then $\lambda^D = (\underline{\mu}+\bar{\mu})/2$ and the cycle found is optimal in the original problem.

Overall, if $\tau_{\min} = \min\{\tau_{ij}: (i, j) \in \mathcal{A}\}$, it is possible to show that $O(\log(\tau_{\min}\theta_{\max}))$ iterations will suffice to find the optimal profit per unit of time and the associated cycle.

In the base case, refueling decisions are trivial and the optimal solution dictates that one follow a cycle in the space of port locations. In particular, it is straightforward to establish that the cycles that maximize revenues and profits coincide. In that sense, the refueling and routing decisions are decoupled.

As highlighted in the introduction, an important feature associated with many networks of ports is that prices differ across ports. Next, we analyze how the structure of an optimal solution changes in the presence of price heterogeneity.

6.2.2. Deterministic Nonuniform Fuel Prices. In this section, we assume again that prices are deterministic, but we turn attention to the more general case in which prices differ across ports. Note that, although in the case of uniform prices the long-term optimal trajectory of a tramper is always a cycle on the graph $(\mathcal{N}, \mathcal{A})$, in this case, the optimal trajectory need not be, in general, a cycle in the location space. However, as we show below, one can build on the analysis of the previous section to search for an optimal solution in the case of nonuniform prices, which can still be represented as a cycle, but in a generalized, location-inventory space. In the analysis below, it is convenient to introduce the following notation. Let $\widetilde{\mathcal{N}} := \mathcal{N} \otimes \mathcal{I}$ denote the set of possible location-inventory combination pairs and let $\tilde{\mathcal{A}} := \{(z, z')\colon z \neq z', z, z' \in \widetilde{\mathcal{N}}\}$ denote the set of all possible arcs connecting different elements of $\widetilde{\mathcal{N}}$. A node z defines both a geographic location and an inventory position; hence moving from a node $z = (i, I)$ to a node $z' = (i', I')$ implies that the ship purchases $I' - I + d_{ii'}$ at price P_i at node i. Note that $(\widetilde{\mathcal{N}}, \tilde{\mathcal{A}})$ defines a complete graph built on $N \times (C+1)$ nodes. For any $z = (i, I) \in \widetilde{\mathcal{N}}$, let $\widetilde{O}(z) := \{z' = (i', I')\colon i' \in O(i), I - d_{ii'} \leq I' \leq C - d_{ii'}\}$ denote the set of nodes in $z' = (i', I') \in \widetilde{\mathcal{N}}$ reachable in one transition from z. For each arc (z, z') belonging to $\tilde{\mathcal{A}}$, we can introduce the following measure

$$\tilde{\theta}_{z, z'} = r_{ii'} - P_i(I' - I + d_{ii'}), \tag{10}$$

which fully characterizes the profit contribution earned when moving from $z = (i, I)$ to $z' = (i', I')$. In addition, we let $\tilde{\tau}_{z, z'} = \tau_{ii'}$ for any $(z, z') \in \tilde{\mathcal{A}}$. Consider a cycle $\mathscr{C}$ on $(\widetilde{\mathcal{N}}, \tilde{\mathcal{A}})$ consisting of the nodes $z_1 = (i_1, I_1)$, $z_2 = (i_2, I_2), \ldots, z_k = (i_k, I_k)$, z_1, where k is a positive integer, and let

$$\Theta_{\mathscr{C}} := \tilde{\theta}_{z_k, z_1} + \sum_{m=1}^{k-1} \tilde{\theta}_{z_m, z_{m+1}}, \tag{11}$$

$$T_{\mathscr{C}} := \tilde{\tau}_{z_k, z_1} + \sum_{m=1}^{k-1} \tilde{\tau}_{z_m, z_{m+1}} \tag{12}$$

be the accumulated profit and the travel time associated with the cycle $\mathscr{C}$, respectively. Then the profit per unit of time associated with $\mathscr{C}$ is given by

$$\lambda_{\mathscr{C}} = \frac{\Theta_{\mathscr{C}}}{T_{\mathscr{C}}}. \tag{13}$$

The next result provides a generalization of the location connectivity assumption for $(\mathcal{N}, \mathcal{A})$ introduced in §3 and recasts the deterministic tramper problem as a problem of finding the cycle with maximum profit per unit of time on the network $(\widetilde{\mathcal{N}}, \tilde{\mathcal{A}})$.

Proposition 4. (a) *Suppose that any two nodes $i \in \mathcal{N}$ and $j \in \mathcal{N} \backslash O(i)$ can be connected by a finite set of arcs in $\mathcal{A}$. Consider any cycle $\mathscr{C}$: $z_1 = (i_1, I_1)$, $z_2 = (i_2, I_2), \ldots, z_k = (i_k, I_k)$, z_1 in $(\widetilde{\mathcal{N}}, \tilde{\mathcal{A}})$ and any node $z \in \widetilde{\mathcal{N}}$, $z \notin \mathscr{C}$. Then any point in the cycle is reachable from z in a finite number of transitions.*

(b) *For any point $z \in \widetilde{\mathcal{N}}$ and any point z_i on a cycle $\mathscr{C}$, let $\mathscr{P}(z, z_i)$ denote a path from z to z_i with the smallest number of transitions. Further, let $F(z) = (i_F(z), I_F(z))$ denote the follower of point z on $\mathscr{P}(z, z_i)$, where $i_F(z) \in \mathcal{N}$ is the location of the next point and $I_F(z)$ is the inventory level corresponding to $F(z)$. Let $\mathscr{C}^*$: $z_1^* = (i_1^*, I_1^*)$, $z_2^* = (i_2^*, I_2^*)$, $z_k^* = (i_k^*, I_k^*)$, z_1^* be the cycle that achieves the highest profit per unit of time $\lambda_{\mathscr{C}^*}$ on the network $(\widetilde{\mathcal{N}}, \tilde{\mathcal{A}})$. Then, when at point $z = (i, I) \in \widetilde{\mathcal{N}}$, the optimal policy for the tramper problem is given by*

$$j^*(z) = \begin{cases} i_{m+1}^* & \text{if } z = z_m^*,\ m = 1, \ldots, k-1, \\ i_1^* & \text{if } z = z_k^*, \\ i_F(z) & \text{if } z \notin \{z_1^*, \ldots, z_k^*\}, \end{cases} \tag{14}$$

$$q^*(z) = \begin{cases} I_{m+1}^* + d_{i, i_{m+1}^*} - I & \text{if } z = z_m^*, m = 1, \ldots, k-1, \\ I_1^* + d_{i, i_1^*} - I & \text{if } z = z_k^*, \\ I_F(z) + d_{i, i_F(z)} - I & \text{if } z \notin \{z_1^*, \ldots, z_k^*\}, \end{cases} \tag{15}$$

and $\lambda = \lambda_{\mathscr{C}^*}$.

The proof of Proposition 4(a) is contained in the online appendix and provides an explicit method of reaching any point in the cycle z_i, $i = 1, \ldots, k$ from any other point z. The result of Proposition 4(b) indicates that, under deterministic assumptions, a tramper should get to the "optimal" cycle $\mathscr{C}^*$ as fast as possible and then remain on it. Under such a policy, the profit values earned before the tramper reaches the cycle $\mathscr{C}^*$ do not contribute to the long-run value of the profit per unit of time—thus, the key element of the optimal location-refueling policy is the cycle $\mathscr{C}^*$.

To find the optimal cycle $\mathscr{C}^*$, the binary search algorithm discussed in the context of the base case can be adapted to the case of the expanded network $(\widetilde{\mathscr{N}}, \tilde{\mathscr{A}})$. The optimal solution is now a cycle in the generalized location-inventory space. The need to search for cycles beyond the location space is a consequence of the presence of different prices across ports.

6.2.3. Stochastic Uniform Prices. In this section, we analyze the impact of price stochasticity on the optimal solution structure. We assume that prices are uniform across all ports but that the common price P_c is stochastic. We illustrate through an example that the optimal solution need not be a cycle in the space of locations, as in the base case.

Example 1. Consider a setting with three ports. Suppose that $r_{12} = r_{21} = r_{23} = r_{32} = \$20,000$ and $r_{13} = r_{31} = \$34,000$, $\tau_{12} = \tau_{21} = 1$ day, $\tau_{23} = \tau_{32} = 1$ day, $\tau_{13} = \tau_{31} = 2$ days, $d_{12} = d_{21} = 100$ mts, $d_{23} = d_{32} = 100$ mts, $d_{13} = d_{31} = 200$ mts, and $C = 1,000$ mts. Suppose also that the price P_c can only take two values—$P_L = \$50$ and $P_H = \$100$—and that the transition probability matrix is given by $[0.1, 0.9; 0.9, 0.1]$. Note that when P_c is equal to the time average of these two prices, \$75, the optimal routing reduces to the cycle $(1, 3)$. When P_c follows the two-value stochastic dynamics described above, the routing decisions, when in Ports 1 and 3, become conditional on the state of the system (Table 2). In particular, when in Port 3, the optimality of the "go to Port 1" decision now depends on either the observed price being low or, if it is high, on the value of the onboard fuel inventory also being high enough to eliminate the need for refueling in the near future; in the cases when the fuel price is high and the need for refueling is strong, the best routing decision is to go to Port 2. Because the problem data are symmetric, a similar routing policy applies when the vessel is in Port 1.

Table 2 Optimal Routing Decision at Port 3 as Function of the Fuel Price and Onboard Fuel Inventory

	Next port when in Port 3										
Onboard fuel inventory (in 100's of mts)	0	1	2	3	4	5	6	7	8	9	10
Fuel price $= P_H$	2	2	2	1	1	1	1	1	1	1	1
Fuel price $= P_L$	1	1	1	1	1	1	1	1	1	1	1

As our example indicates, even in the absence of price dispersion across ports, a modest degree of stochasticity in fuel prices creates, in general, a substantial degree of complexity in optimal routing decisions, strongly coupling routing and refueling decisions.

6.2.4. Coupling Between Routing and Refueling Decisions: The Role of Capacity. In this section we investigate the role of another factor, vessel capacity, on the coupling between routing and refueling decisions. We focus on the case of deterministic but nonuniform fuel prices.

Let $\mathscr{C}^{(r)}$ be the cycle that maximizes revenues per unit of time (in the location space), where the revenues accumulated over a cycle are given by the sum of the rewards over that cycle. Let $\mathscr{C}^{(r)}$: $i_1, i_2, \ldots, i_m, i_1$,[4] $\bar{d} = \max_{(i,j)\in\mathscr{A}} d_{ij}$, and $i_{\min} = \arg\min_{i\in\mathscr{N}}\{P_i\}$. We suppose that $C > \bar{d}$. Consider the policy $\pi_{\mathscr{C}^{(r)}}$ that separates refueling and routing decisions as follows: it follows the cycle $\mathscr{C}^{(r)}$ until the reserve of fuel on board minus the fuel needed to reach the next port $I - d_{ii'}$ drops below $\bar{d}$. When this occurs, the vessel travels to port $i_{\min}$, replenishes up to C, and travels back to i and resumes following the route of the cycle. Let $m^* = \arg\min\{d_{i_k, i_{\min}}, k = 1, \ldots, m\}$ and $I_{\mathscr{C}^{(r)}} = \sum_{i=1}^{m} d_{i,i+1} + d_{i_{m^*}, i_{\min}} + d_{i_{\min}, i_{m^*}}$. More formally, the policy can be defined as follows:

$$j^*(z) = \begin{cases} i_{l+1} & \text{if } i = i_l \in \{i_1, i_2, \ldots, i_m\}\backslash\{i_{m^*}\}, \\ i_{m^*+1} & \text{if } i = i_{m^*} \text{ and } i_{\min} = i_{m^*}, \\ i_{\min} & \text{if } i = i_{m^*},\ i_{\min} \neq i_{m^*}, \text{ and} \\ & I < I_{\mathscr{C}^{(r)}} - d_{i_{\min}, i_{m^*}}, \\ i_{m^*+1} & \text{if } i = i_{m^*},\ i_{\min} \neq i_{m^*}, \text{ and} \\ & I \geq I_{\mathscr{C}^{(r)}} - d_{i_{\min}, i_{m^*}}, \\ i_{m^*} & \text{if } i \notin \{i_1, \ldots, i_m\}; \end{cases} \tag{16}$$

[4] We adopt the convention that i_{m+1} refers to i_1 to simplify notation.

$$q^*(z)=\begin{cases}(d_{i_l,i_{l+1}}-I)^+ & \text{if } i=i_l\in\{i_1,i_2,\dots,i_m\}\backslash\{i_{m^*}\},\\ (d_{i_{m^*},i_{\min}}-I)^+ & \text{if } i=i_{m^*},\, i_{\min}\neq i_{m^*}, \text{ and } I<I_{\mathscr{C}(r)}-d_{i_{\min},i_{m^*}},\\ 0 & \text{if } i=i_{m^*},\, i_{\min}\neq i_{m^*}, \text{ and } I\geq I_{\mathscr{C}(r)}-d_{i_{\min},i_{m^*}},\\ (d_{i,i_{m^*}}-I)^+ & \text{if } i\notin\{i_1,\dots,i_m\}\cup\{i_{\min}\},\\ C-I & \text{if } i=i_{\min}.\end{cases} \tag{17}$$

The next result establishes the near optimality of this policy in cases where the capacity is large relative to the maximum fuel consumption required for a single interport transition. In particular, define

$$\zeta=\frac{\max_{(i,j)\in\mathscr{A}} d_{ij}}{C}$$

as the proportion of the total capacity needed for the most fuel-consuming interport transition. We have the following result.

Proposition 5. *Let $\lambda^D>0$ be the optimal profit per unit of time. Suppose that $C\geq I_{\mathscr{C}(r)}$. Then the long run profit per unit of time associated with the policy $\pi_{\mathscr{C}(r)}$, $\lambda_{\pi_{\mathscr{C}(r)}}$, satisfies*

$$\frac{\lambda_{\pi_{\mathscr{C}(r)}}}{\lambda^D}\geq\frac{1-4\zeta}{1-2\zeta}. \tag{18}$$

Hence, as ζ converges to zero, one can essentially decouple refueling and routing decisions. Having ζ "small" corresponds to a tramper operating in a region with a high number of ports that are close to each other. This would be the case for a tramper restricting its operation to a small region of the world, such as the Mediterranean. For $\zeta=1/8$, one can lower bound the ratio of the profit of the proposed policy to the optimal profit by two-thirds and for $\zeta=1/12$ by four-fifths.

6.3. Heuristic Profit Management Policies and Their Performance

In practice, even small-size instances of the dynamic program (8)–(9) are often computationally intractable. In particular, it is nearly impossible to solve such a problem to optimality for a network with more than three ports because the size of the state space grows exponentially with the number of ports. In such a setting, developing heuristics with lower computational requirements becomes critical.

In addition, as illustrated in the previous section, the resulting optimal profit management policies may take a complex and nonintuitive form in the general case. This section focuses on the development of insights on the optimal policy and of two potential heuristic profit management policies.

6.3.1. Optimal Location Cycle and Optimal Location-Inventory Cycle Heuristics. The first profit management heuristic policy uses the approach described in §6.2.1 to decouple the routing and refueling decisions. For any port $i\in\mathscr{N}$, let $\underline{P}_i$ denote the smallest value that the price at port i can take and let $\underline{P}=\min_{i\in\mathscr{N}}\underline{P}_i$. The heuristic proceeds in two steps. In the first step, it is assumed that all fuel prices are constant over time and equal to a common value $P_c=\underline{P}$, and all arc rewards are constant equal to their expectation: Using this simplification, the profit-maximizing location cycle is determined. In the second step, the optimal refueling policy is determined for a liner following that location cycle. We refer to this heuristic as the optimal location cycle (OLC). In summary, the OLC heuristic tackles the combinatorial nature of the dynamic routing decisions by separating them from the refueling decisions. Note that the OLC heuristic still requires solving a liner problem over a given cycle, which might imply significant computational requirements if the cycle has many ports.

we provide a bound for the performance of the OLC heuristic. For any arc $(i,j)\in\mathscr{A}$, let $\bar{r}_{ij}$ denote the largest possible value of the reward on arc (i,j). In addition, let $\mathscr{S}_C$ denote the set of all cycles in the location space $(\mathscr{N},\mathscr{A})$ and define

$$\Delta_r=\max_{(i,j)\in\mathscr{A}}\{\bar{r}_{ij}-\mathbb{E}[r_{ij}]\},$$

$$\Delta_p=\max_{i\in\mathscr{N}}\{\mathbb{E}_\infty[P_i]-\underline{P}\},$$

$$\omega=\max_{\mathscr{C}\in\mathscr{S}_C}\frac{\sum_{(i,j)\in\mathscr{C}}1}{\sum_{(i,j)\in\mathscr{C}}\tau_{ij}},$$

$$\rho=\max_{\mathscr{C}\in\mathscr{S}_C}\frac{\sum_{(i,j)\in\mathscr{C}}d_{ij}}{\sum_{(i,j)\in\mathscr{C}}\tau_{ij}},$$

where $\mathbb{E}_\infty$ corresponds to the expectation with respect to the steady-state distribution of prices.

Proposition 6. *Let λ^* and λ_{OLC} denote the profit per unit of time values achieved by an optimal policy and by the OLC heuristic, respectively. Then*

$$\lambda^* - \lambda_{\mathrm{OLC}} \le \omega \Delta_r + \rho \Delta_p. \tag{19}$$

The bound presented in Proposition 6 is the sum of two components. Each of the components is the product of two terms, where the first one, ω (respectively ρ), depends exclusively on the characteristics of the network of ports. Δ_r represents the influence of the stochastic nature of rewards, and Δ_p represents the influence of the price heterogeneity across ports. Note that the bound (19) emphasizes the near optimality of the OLC heuristic in settings where rewards are mildly stochastic and where prices do not significantly differ across ports, i.e., where Δ_r and Δ_p are small compared to the average values of rewards and prices, respectively. Note also that the decoupling between routing and refueling followed by this heuristic implies that when there is significant price heterogeneity across ports (even if prices are constant across time), one runs the risk of "missing" the ports at which fuel is sold at particularly low prices, which might have severe impacts on performance.

The second heuristic policy we consider is based on the analysis conducted for the deterministic case in §6.2.2. In particular, consider the maximum profit-to-time ratio cycle in the network $(\widetilde{\mathcal{N}}, \tilde{\mathcal{A}})$ with profits

$$\tilde{\theta}_{z,z'} = \mathbb{E}[r_{ii'}] - \mathbb{E}_\infty[P_i](I' - I + d_{ii'}). \tag{20}$$

Let π_{OLIC} denote the policy that leads the vessel through this location-inventory cycle, and let $\mathcal{P}_{\mathrm{OLIC}}$ denote the subset of stationary policies that form cycles in the location-inventory network. The analysis performed in §6.2.2 implies that for the general stochastic tramper problem, π_{OLIC} achieves the highest possible long-term average profit among all policies in $\mathcal{P}_{\mathrm{OLIC}}$. In summary, the optimal location-inventory cycle (OLIC) policy captures both the heterogeneity in prices as well as the combination of route selection with refueling decisions but ignores stochastic variations of prices and rewards. Note that this feature of the OLIC policy is in contrast with the OLC heuristic that captures price stochasticity across the ports it visits.

Below, we provide a bound for the performance of the OLIC heuristic. In addition to the notation introduced above, let $\widetilde{\mathcal{S}}_C$ denote the set of all cycles in the location-inventory space $(\widetilde{\mathcal{N}}, \tilde{\mathcal{A}})$ and define

$$\tilde{\Delta}_p = \max_{i\in\mathcal{N}}\{\mathbb{E}_\infty[P_i] - \underline{P}_i\},$$
$$\tilde{\omega} = \max_{\mathscr{C}\in\widetilde{\mathcal{S}}_C} \frac{\sum_{(z,z')\in\mathscr{C}} 1}{\sum_{(z,z')\in\mathscr{C}} \tau_{ii'}},$$
$$\tilde{\rho} = \max_{\mathscr{C}\in\widetilde{\mathcal{S}}_C} \frac{\sum_{(z,z')\in\mathscr{C}} d_{ii'}}{\sum_{(z,z')\in\mathscr{C}} \tau_{ii'}}.$$

Proposition 7. *Let λ^* and λ_{OLIC} denote the profit per unit of time values achieved by an optimal policy and by the OLIC heuristic, respectively. Then*

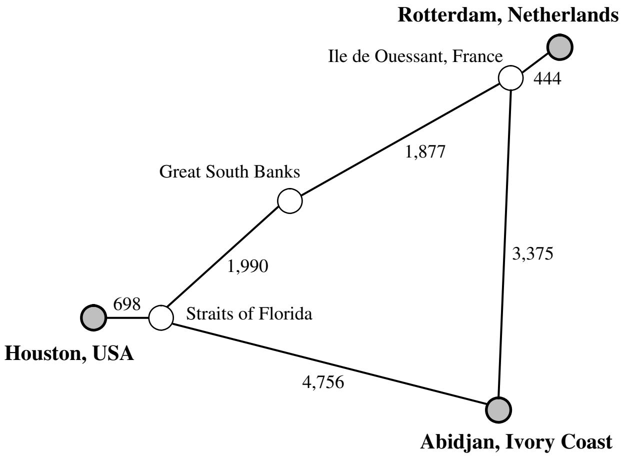
$$\lambda^* - \lambda_{\mathrm{OLIC}} \le \tilde{\omega}\Delta_r + \tilde{\rho}\tilde{\Delta}_p. \tag{21}$$

Note that the bound (21) emphasizes the near optimality of the OLIC heuristic in mildly stochastic settings, i.e., where Δ_r and $\tilde{\Delta}_p$ are small compared with the average values of rewards and prices, respectively. Contrasting the OLIC and OLC heuristics, one observes that the former captures price heterogeneity across ports and is not exposed to the risk of potentially ignoring ports where fuel is sold at a discount; this is reflected in the performance bounds through the dependence on $\tilde{\Delta}_p$ for the OLIC heuristic (which is driven exclusively by price stochasticity) as opposed to Δ_p for the OLC heuristic (which is driven by price heterogeneity).

Although we will restrict attention to these two heuristics, it is worth noting that the analysis of §6.2.4 can be a basis for the development of additional heuristics, where one could determine a cycle that maximizes average accumulated rewards per unit of time and then only deviate from such a cycle to replenish at ports where fuel prices are heavily discounted.

6.3.2. Comparing the Performance of Heuristics. In this section we report the results of a numerical study designed to gain insights on the optimal policy and the performance of the two heuristics. The problem setting for our numerical study is designed to test heuristics in a real-life setting representing a tramper traversing a small network of three ports. The choice of a small network for testing purposes is dictated by the computational challenges associated with obtaining the optimal policies for problems with more than three ports (the heuristics can be applied to real-life

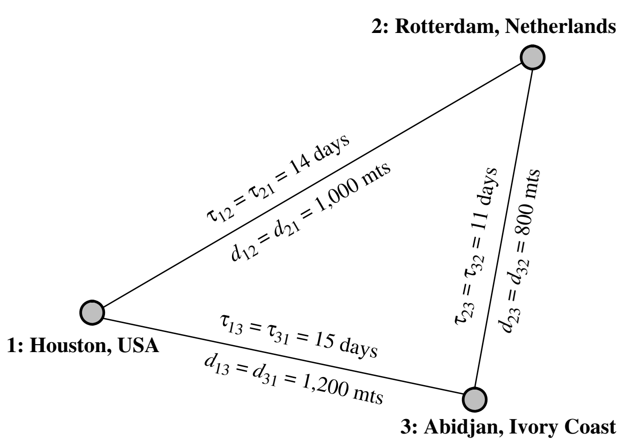
Figure 2 Schematic Geographic Representation of Distances Between Ports (in Nautical Miles)



problems with a larger number of ports). For example, a simple problem instance with four ports, 15 crude oil price levels, 10 possible fuel inventory levels, 2 levels of local Markovian price corrections at each port, 2 levels of local i.i.d. price corrections, and 2 revenue levels on each arc has 38,400 states. Such a problem instance takes several days to solve using our tool of choice, Mathematica (it takes around 10 hours to solve a typical tramper problem with three ports).

We consider a tramper with capacity of 6,000 mts operating between the ports of Houston (Port 1), Rotterdam (Port 2), and Abidjan (Port 3). Figures 2 and 3 show a schematic representation of the distances between these ports expressed in nautical miles and of the corresponding network parameters, respectively.

Figure 3 Travel Times and Fuel Consumption Values for Ports



We note that the distances between the ports are calculated assuming that the tramper follows the standard interport trajectories passing through the so-called junction points as specified by the National Imagery and Mapping Agency (2001). For example, in the case of travel between Rotterdam and Houston, a tramper vessel would pass from Rotterdam to the junction point at Ile de Ouessant, then to the junction point at Great South Banks, followed by the junction point at the Straits of Florida, and, finally, by the port of Houston, resulting in a travel distance of 5,009 nautical miles. The travel times and fuel consumption values for the trips between ports were calculated using the average travel speed of 15 nautical miles per hour and the average fuel consumption rate of 75 mts per day. The resulting travel time values were rounded to the nearest day and the resulting fuel consumption values to the nearest multiple of 200 mts. Note that we assume, for the sake of simplicity, that the travel times and the fuel consumption values are symmetric; i.e., $\tau_{ij} = \tau_{ji}$ and $d_{ij} = d_{ji}$ for all $i, j = 1, 2, 3$, $i \neq j$. In summary, the fuel consumptions and travel times between the three ports are given by

$$\mathbf{d} = \begin{bmatrix} 0 & 1{,}000 & 1{,}200 \\ 1{,}000 & 0 & 800 \\ 1{,}200 & 800 & 0 \end{bmatrix}, \quad \boldsymbol{\tau} = \begin{bmatrix} 0 & 14 & 15 \\ 14 & 0 & 11 \\ 15 & 11 & 0 \end{bmatrix}. \tag{22}$$

To reduce the computational effort, we assume that the ship can only replenish fuel in multiples of 200 mts. We use the price dynamics model outlined in (3) and the parameter estimation procedure described in online Appendix B. As before, the value of the parameter $\hat{\gamma}$ representing a conversion factor between the time average of the oil price expressed in \$/bbl and the time-and-location average of bunker prices expressed in \$/mts is estimated as 4.43, and the values of other parameter estimates are presented in Table 3.

Table 3 Parameters of Fuel Price Dynamics for Ports of Houston, Rotterdam, and Abidjan

Parameter	Houston (Port 1)	Rotterdam (Port 2)	Abidjan (Port 3)
A, \$/mts	−13.42	−28.44	9.62
δ, \$/mts	31.71	29.72	19.21
η	0.93	0.99	0.90
σ, \$/mts	13.94	14.83	11.91

As Table 3 indicates, the port of Rotterdam has, on average, a pronounced price advantage over the other two ports, whereas refueling at Abidjan is, in general, undesirable. As in the liner case, the dynamics of oil price P_0 are described by (B9) in online Appendix B. This parameter combination results in Markovian price dynamics under which the high degree of inertia in price values and the high variance in supply corrections allow for various orderings of prices among the three ports. We use a simple form for random price correction terms by assuming that ε_i at port i takes two values, $\pm\sigma_i$, w.p. 0.5 for each value. The oil price dynamics are taken to follow the mean-reverting model (B9) with $\sigma_0 = 1.25$, $\lambda = 0.96$, $P_0^{\min} = 46$, and $P_0^{\max} = 62$.

In practice, the revenues associated with transporting cargo between ports are established in the process of negotiations, their exact values being closely guarded commercial information. In our numerical study, we assume a simple setting with constant revenue matrix

$$\mathbf{r} = \begin{bmatrix} 0 & r_{12} & r_{13} \\ r_{21} & 0 & r_{23} \\ r_{31} & r_{32} & 0 \end{bmatrix}, \tag{23}$$

where r_{ij}, $i \neq j$ are taken to be proportional to the distances between ports, $r_{ij} = \kappa_{ij} d_{ij}$, with κ_{ij} being the parameters that we varied. We take $\kappa_{12} = \kappa_{21} = \kappa_{13} = \kappa_{31} = \kappa_{23} = \kappa_{32} = 250$ as the base scenario for our study. Note that for such values of revenue parameters, the profit rate generated by the optimal tramper policy turns out to be \$794/day, a very modest amount roughly corresponding to the price of 4 metric tons of fuel, or less than 5% of typical daily fuel cost. Thus, the base scenario value of revenue parameters corresponds to a reasonable lower bound on revenues that guarantee profitability.

In our numerical study we used a standard value iteration approach to solve the tramper dynamic program. Table 4 illustrates the optimal route-selection decisions for the base scenario in the states with $P_0 =$ \$54 and zero onboard fuel inventory. In this table we have used a shorthand notation "$\pm\delta_i \pm \sigma_i$" to denote the price state $P_i = \gamma P_0 + A_i \pm \delta_i \pm \sigma_i$ at the "current" port $i = 1, 2, 3$, (i.e., at the port where the routing decision is made) and "$\pm\delta_i$" to denote the price state at other ports.

Table 4 Optimal Route Selection Decisions for the Base Scenario in States with $P_0 = \$54$ and Zero Onboard Fuel Inventory

Next port at Houston				
Prices at Rotterdam and Abidjan	Price at Houston			
	$-\delta_1-\sigma_1$	$+\delta_1-\sigma_1$	$-\delta_1+\sigma_1$	$+\delta_1+\sigma_1$
$(-\delta_2, -\delta_3)$	Abidjan	Rotterdam	Rotterdam	Rotterdam
$(-\delta_2, +\delta_3)$	Abidjan	Rotterdam	Rotterdam	Rotterdam
$(+\delta_2, -\delta_3)$	Abidjan	Rotterdam	Abidjan	Rotterdam
$(+\delta_2, +\delta_3)$	Abidjan	Rotterdam	Abidjan	Rotterdam
Next port at Rotterdam				
Prices at Houston and Abidjan	Price at Rotterdam			
	$-\delta_2-\sigma_2$	$+\delta_2-\sigma_2$	$-\delta_2+\sigma_2$	$+\delta_2+\sigma_2$
$(-\delta_1, -\delta_3)$	Abidjan	Houston	Abidjan	Houston
$(-\delta_1, +\delta_3)$	Abidjan	Houston	Abidjan	Houston
$(+\delta_1, -\delta_3)$	Abidjan	Houston	Abidjan	Houston
$(+\delta_1, +\delta_3)$	Abidjan	Houston	Abidjan	Houston
Next port at Abidjan				
Prices at Houston and Rotterdam	Price at Abidjan			
	$-\delta_3-\sigma_3$	$+\delta_3-\sigma_3$	$-\delta_3+\sigma_3$	$+\delta_3+\sigma_3$
$(-\delta_1, -\delta_2)$	Rotterdam	Rotterdam	Rotterdam	Rotterdam
$(-\delta_1, +\delta_2)$	Houston	Houston	Houston	Houston
$(+\delta_1, -\delta_2)$	Rotterdam	Rotterdam	Rotterdam	Rotterdam
$(+\delta_1, +\delta_2)$	Houston	Houston	Houston	Rotterdam

As Table 4 shows, the resolution of a trade-off between low fuel prices and high revenue values often takes an intuitive form. For example, when in Houston, the choice between higher revenue rate associated with the Houston-Abidjan link and cheaper fuel in Rotterdam is resolved as follows: when the fuel price in Houston is at its lowest possible level, immediate refueling allows the tramper to benefit from the revenue advantage of going to Abidjan: However, when the fuel price in Houston is at its highest possible level, the prospective of cheaper refueling in Rotterdam outweighs the revenue considerations. For the intermediate values of fuel prices at Houston, the trade-off becomes more subtle: Rotterdam is preferred unless the price in Houston is cheap enough to make immediate refueling in Houston more advantageous than the later refueling in Rotterdam and, thus, to allow the routing decision to be based solely on revenue rate consideration. The subtlety of such a trade-off is clearly illustrated in the optimal routing decisions when in Abidjan: whereas in the majority of cases the nextport to visit is decided based on the expected fuel price, in the setting where the prices at both Houston and Rotterdam are expected to remain high, the situation is "too close to call" without accounting for the fuel price in Abidjan itself. In particular, as the last line of the Table 4 indicates, the slight expected price advantage of Rotterdam is canceled out by a slight revenue rate advantage associated with the Abidjan-Houston link, unless the price in Abidjan is at its highest level as well.

In our numerical study we consider four different variations of the base scenario, each variation designed to emphasize the influence of a particular location cycle within the three-port network we consider. In particular, in the first variation of the base scenario we increase the revenue values associated with Houston-Rotterdam-Houston routing cycle by considering $\kappa_{12} = \kappa_{21} = 250, 275, \ldots, 350$ (the rest of revenue values remain at their base scenario values). Similarly, the second variation raises the importance of the Houston-Abidjan-Houston routing cycle by setting $\kappa_{13} = \kappa_{31} = 250, 275, \ldots, 350$. The third and forth settings emphasize the Rotterdam-Abidjan-Rotterdam and Houston-Rotterdam-Abidjan-Houston routing cycles by setting $\kappa_{23} = \kappa_{32} = 250, 275, \ldots, 350$ and $\kappa_{12} = \kappa_{23} = \kappa_{31} = 250, 275, \ldots, 350$, respectively. Note that in the latter case, the rewards on arcs are no longer symmetric. The relative performance of the two heuristic profit management approaches defined above (optimal location cycle and optimal location-inventory cycle) under these four problem variations is illustrated in Table 5. As we can see, both heuristics "latch on" to the appropriate route in the majority of cases studied. The optimal location cycle heuristic exhibits a very robust performance across all the problem instances—with the worst case relative performance gap of 13.1%. However, the best location-inventory cycle heuristic falls far behind in its performance even in cases when the location component of the location-inventory cycle is properly identified. Such inadequate performance is to be expected in settings where the stochasticity of the fuel prices is pronounced and the failure to capitalize on the optimal refueling policies significantly hinders the profit generation process. Indeed, the OLC heuristic adopts an optimal (state-dependent) refueling policy over the cycle that it selected, and the OLIC heuristic refuels according to what is prescribed by the location-inventory cycle, independently of current price realizations. At the same time, note that the cases with particularly high values of the relative performance gap stem from the low-profitability settings (i.e., where λ_{opt} is small) in which we test the heuristics.

Table 5 **Relative Performance Gaps Between Optimal Policy and "Optimal Location Cycle"** ($\varepsilon_{OLC} = (1 - (\lambda_{OLC}/\lambda_{opt}) \times 100\%)$ **and "Optimal Location-Inventory Cycle"** ($\varepsilon_{OLIC} = (1 - (\lambda_{OLIC}/\lambda_{opt}) \times 100\%)$ **Heuristics**

	ε_{OLC}	Locations visited: OLC	ε_{OLIC}	Locations visited: OLIC
$\kappa_{12} = \kappa_{21}$				
250	3.1	(1, 2)	51.5	(1, 3, 2)
275	0.0	(1, 2)	37.8	(1, 2)
300	0.0	(1, 2)	28.7	(1, 2)
325	0.0	(1, 2)	23.1	(1, 2)
350	0.0	(1, 2)	19.4	(1, 2)
$\kappa_{13} = \kappa_{31}$				
250	3.1	(1, 2)	51.5	(1, 3, 2)
275	13.1	(1, 3)	24.1	(1, 3)
300	3.6	(1, 3)	28.7	(1, 3)
325	0.0	(1, 3)	12.1	(1, 3)
350	0.0	(1, 3)	6.8	(1, 3)
$\kappa_{23} = \kappa_{32}$				
250	3.1	(1, 2)	51.5	(1, 3, 2)
275	0.0	(2, 3)	29.7	(2, 3)
300	0.0	(2, 3)	22.1	(2, 3)
325	0.0	(2, 3)	17.7	(2, 3)
350	0.0	(2, 3)	14.7	(2, 3)
$\kappa_{12} = \kappa_{23} = \kappa_{31}$				
250	3.1	(1, 2)	51.5	(1, 3, 2)
275	0.9	(1, 2, 3)	37.2	(1, 2, 3)
300	0.4	(1, 2, 3)	28.9	(1, 2, 3)
325	0.2	(1, 2, 3)	24.0	(1, 2, 3)
350	0.1	(1, 2, 3)	20.7	(1, 2, 3)

7. Summary and Future Work

In this study we have developed a profit optimization model for a marine shipping company that owns a fleet of liners (vessels with fixed route) and trampers (vessels for which the route can be selected in a dynamic fashion). A model was also proposed for the dynamics of bunker prices and was calibrated using the actual pricing data for a network of 18 ports for a period of 6 months. For the liner case, we formulate the vessel refueling problem as a long-term average stochastic dynamic program and prove that the optimal refueling policy has a capacitated price-dependent buy-up-to form. The monotonicity of the optimal fuel-up-to levels is established under additional assumptions on the stochastic properties of the fuel price dynamics. In the tramper case, the task of refueling is blended with a combinatorial route selection. We develop insights based on several special cases that allow one to isolate the impact of the features associated with this problem: heterogeneity of prices across ports, stochasticity of prices, and capacity constraints.

The single-vessel approach developed in this paper can serve as a good initial step to optimizing fleet profits. At the same time, real business settings are often characterized by the presence of volume refueling discounts, which can only be fully exploited using more than one vessel. Thus, the development of multiple-vessel profit management models represents a challenging research direction that will be of immediate interest to practitioners.

Electronic Companion

An electronic companion to this paper is available on the *Manufacturing & Service Operations Management* website (http://msom.pubs.informs.org/ecompanion.html).

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