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# Demand Uncertainty and the Bayesian Effect in Markdown Pricing with Strategic Customers

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This paper studies the role of demand uncertainty in temporal discrimination when the retailer applies markdown pricing facing strategic customers. We consider a model in which a retail firm announces a pair of declining prices for two selling periods, and customers with heterogeneous valuations each decide whether to buy a unit early, later or never. In this model, if the demand function is linear and its parameters are common knowledge, there *never* exist any markdown prices that achieve temporal discrimination for any feasible model parameters. Either all buying customers wait, or all buy early. By contrast, if the demand level is unknown, there always exists a temporally discriminating markdown pricing scheme for all feasible model parameters. We derive qualitative insights to the way demand uncertainty and Bayesian updating contribute to temporal discrimination, which broadly apply to nonlinear demand functions as well. We also show that in case of demand uncertainty, there always exists a temporally discriminating pricing scheme that yields a strictly higher profit to the retailer than the optimal static pricing scheme. Ironically, however, the retailer cannot implement the optimal scheme due to the same demand uncertainty.

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## 1. Introduction

We investigate the role of demand uncertainty in the design of markdown pricing by using a simple model in which a monopolistic retailer faces a large population of small customers. Consider a retail firm ("he") that is about to introduce a new product. The firm chooses and announces a pair of declining prices for the next two selling periods. Customers ("she" individually) have heterogeneous valuations of the product and decide whether to buy a unit, and if so, when to buy. The model highlights two aspects. First, customers are *strategic* in the sense that they can look ahead into the future, evaluate different options available now and later, and choose the best decision for themselves. In our context, while nonstrategic customers would decide whether to buy now or not in each period, strategic customers would consider another option—wait and buy later. In the process of making the decision, the customer evaluates the value of each option. The trade-off facing each customer is between availability and the price. By postponing the purchase, she will pay a lower price, but availability is not guaranteed. This requires her assessment of the probability of obtaining a unit in the second period.

Another highlight of the model is demand uncertainty and heterogeneous beliefs among customers. As a benchmark, we first study the complete information case. Demand is linear, and the retailer has complete information. Facing customers with different valuations, the retailer would possibly increase his profit by markdown pricing. High-valuation customers would buy first at the regular price, and lower-valuation customers would buy later at the discount price. However, in the complete information case, this policy would not segment the market, no matter what the price pair is. Either all buying customers buy early at the regular price, or all wait and buy later at the discount price. This holds under all feasible parameters of the model.

If, however, the demand intercept is unknown and customers develop their own beliefs, we have a different result. Suppose now that at the beginning both the retailer and customers have incomplete information of the market demand, each starting with a common prior. Based on the incomplete information, the retailer sets the prices for two periods. Next, each customer responds by developing her individual valuation for the product and incorporating it in the update of the demand before making a purchase decision. Through the process the "Bayesian bias" arises. If a particular customer likes the product a lot, she will believe that other customers may like the product a lot as well and raise her estimate of the market demand in the Bayesian fashion. The reverse also occurs: if her valuation turns out to be low, then she might infer that other customers would have similar feelings about the new product. The posterior thus developed is used for evaluating the chance of allocation in the second period, which drives her decision whether to buy early, later, or never. Then, in contrast with the complete information case, the market *always* has a pair of temporally discriminating prices. Thus, demand uncertainty plays a critical role in achieving market segmentation through its interactions with customers' strategic behavior.

The main objective of this paper is to develop an understanding of the contrast and derive several managerial insights to the design of markdown pricing. The intuitive explanation of the contrast is in the way Bayesian updating affects the purchase decision by customers. Consider the linear demand case with complete information. If anyone buys early, it would be the highest-valuation customers who are willing to pay a high price for guaranteed availability. But if she feels the chance of getting a unit later is high enough, she would rather wait and try to buy later. In assessing the probability of availability, she should be mindful of the next-highest-valuation customers who will wait and buy later. If there are many of them, her chance of getting a unit later is small, so she would buy early. In the linear case, however, the trade-off always directs them all in one way—either all wait or all buy early. Now turning to the incomplete information case, we find that the highest-valuation customers would overestimate the demand due to the above-mentioned Bayesian bias, which leads them to believe there would be more high-valuation customers than there actually are. Therefore, they choose to buy early, and the market is temporally segmented.

This paper offers several findings about markdown pricing. First, demand uncertainties in the presence of strategic customers can significantly affect the retailer's pricing decisions, as well as customers' purchase decisions. Many insights derived from the models of complete information and/or myopic customers may not apply. Second, demand uncertainties may contribute to the existence of temporally discriminating markdown price schemes. The Bayesian bias induces high-valuation customers to buy early even when they are better off waiting. Last, there always exists a markdown price scheme that strictly increases the retailer's profit over static pricing. But this does not imply that the retailer can implement it, due to demand uncertainties on his part.

The model is built on two critical assumptions. First, each individual customer develops a valuation of the product, which she uses as her reservation value and also applies in Bayesian updating. This is the source of the Bayesian bias that leads to the existence of temporal segmentation. Also, we assume that the retailer can commit to a pair of prices at the beginning. It thus does not capture the *dynamic* process of decision making by the retailer, where the retailer determines the second-period price after observing the sales in the first period. The commitment assumption has been widely used in the revenue management literature (e.g., Liu and van Ryzin 2008, Zhang and Cooper 2008), but it is restricted to certain circumstances like sales events regularly held by a large retailer (e.g., Neiman Marcus Last Call's 50%-off clearance sales that repeat at the end of each season). Noncommitment models also exist in the revenue management literature, and they are usually analyzed in the rational expectations framework (e.g., Cachon and Swinney 2009, Su and Zhang 2008, Swinney 2011). Rational expectation equilibrium requires analysis of signals implied in the dynamic actions of players, which is beyond the scope of our paper.

The rest of this paper is organized as follows. In §2 we provide a literature survey on strategic behavior and demand uncertainty in markdown pricing. In §3 we study the complete information case and show the nonexistence of temporally discriminating schemes, and in §4 we show the existence of temporally discriminating schemes under demand uncertainty. In §5 we discuss an extension to nonlinear demand and provide qualitative insights through the Bayesian bias. In §6 we summarize the results, discuss several limits and drawbacks of the model, and suggest future research direction.

## 2. A Literature Survey

Research in dynamic pricing has grown fast and wide in recent years, particularly after the seminal work by Gallego and van Ryzin (1994). Models of dynamic pricing vary in terms of *strategic* versus myopic customers, *fixed* versus dynamic discount, *risk-neutral* versus risk-averse customers, *uncertain* demand versus known demand, *all at once* arrivals of customers versus over multiple periods, continuous versus *discrete* time, *price commitment* versus contingent pricing, declining value versus *stationary* value, markup versus *markdown* policies, *large* market with nonatomic customers versus finite market, and homogeneous versus *heterogeneous* customers in terms of valuation. (Here, *italics* indicate the features of this paper.) General reviews of revenue management including markdown pricing are available in Bitran and Caldentey (2003) and Talluri and van Ryzin (2004).

The main focus of our paper is the interaction between strategic customer behavior and demand uncertainty, so we provide a short literature survey on these two topics in the context of markdown pricing.

The literature on durable goods monopoly in economics may be the first to investigate markdown pricing with *strategic customers*. They study a rational expectations equilibrium that results from the interplay between the monopolist's pricing decisions and consumers' expectations in a durable good market. Starting with the seminal work by Ronald Coase (1972), the research work on durable goods monopoly includes Stokey (1981), Bulow (1982) and Gul et al. (1986). A key component of the model is the assumption that the monopolist has no power to commit to future prices. By contrast, our model assumes that the retailer can commit to a pair of prices for the present and the future.

Operations researchers also incorporated strategic customer behavior in dynamic pricing to understand how the presence of strategic customers changes the retailer's pricing and quantity decisions, as well as his profit. Examples include Su (2007), Zhang and Cooper (2008), Aviv and Pazgal (2008), Cachon and Swinney (2009), and Liu and van Ryzin (2008). Su (2007) considers strategic customers who are heterogeneous in valuations and patience. Depending on the composition of the population, the structure of optimal markdown pricing changes. Zhang and Cooper (2008) analyze the retailer's problem of determining both the prices and the quantity for the second period. In a market consisting of myopic and strategic customers, "rationing" (i.e., limiting the supply) enhances the profit if the price is set contingent on the leftover inventory, while it does not help in the case of pre-committed prices. In Aviv and Pazgal (2008), customers—all strategic and differing in valuations—arrive over two periods and consider buying a product. They study the impact of strategic customers on the retailer's pricing policy and his profit. Cachon and Swinney (2009) consider a similar model, but demand is uncertain to the retailer and customers. They derive a rational expectations equilibrium where strategic customers and the retailer make their respective optimal decisions based on their expectations of future prices and availability of inventory under various policy options. Perhaps most closely related to the present paper is the work by Liu and van Ryzin (2008), who investigate the stocking decision problem when the monopolist tries to sell a product to strategic customers over two periods at pre-committed prices. They show that under certain conditions, the retailer would adopt a rationing strategy. This way the retailer would induce high-valuation customers to buy early and low-valuation customers to buy later. Our model has a similar setup, including pre-committed markdown prices, rationing possibility and strategic customers in a large market. But we focus on demand uncertainty assuming risk-neutral customers, while their focus is on the retailer's capacity decision facing risk-averse customers under known market demand.

Different aspects of *demand uncertainty* in markdown pricing were studied by Lazear (1986), Su and Zhang (2008), Swinney (2011), and Mersereau and Zhang (2012). Lazear (1986) models markdown pricing as a demand revealing process to extract more consumer surplus. A retailer would sell one unique item to a stream of arriving customers. He faces nonstrategic customers whose valuation of the product is identical but unknown. By starting at one price and, if not sold, lowering it later, the retailer can update his estimate of the valuation and charge the best price. Su and Zhang (2008) consider strategic customers in a two-period markdown pricing model with demand uncertainty. The retailer determines the stocking quantity and the selling price based on an estimate of market demand, while the customer estimates the chance of winning a unit in the second period and decides when to buy. They derive a rational expectation equilibrium that reveals the true values. Swinney (2011) studies the value of quick response (QR) to a manufacturing firm when selling to strategic customers under demand uncertainty. QR enables the firm to produce additional units after the demand is revealed. He compares the value of QR in the cases of strategic customers and nonstrategic customers. Mersereau and Zhang (2012) investigate markdown pricing in a market where myopic and strategic customers coexist. The source of uncertainty in the model is the fraction of strategic customers. The retailer develops an estimate and sets the price, and the customers reverse-engineer the retailer's price. They propose a robust pricing policy that requires no knowledge of the fraction.

None of the above models study how demand uncertainty facilitates temporal discrimination. We investigate the role of demand uncertainty in temporal discrimination by comparing two models—one with known demand (§3), and the other with unknown demand (§§4 and 5).

## 3. The Model of Complete Information

Consider a monopolistic retail firm that markets a product to a continuum of small risk-neutral customers. The retailer has a fixed quantity $K$ ($>0$) of the product to sell over two periods, 1 (also termed "early") and 2 ("later"). All costs of procuring the units are sunk, and the retailer incurs no further cost in keeping, selling, or discarding them. The salvage value of an unsold unit is zero. A *markdown pricing scheme* consists of two prices in the two periods $p_1$ and $p_2$, where $p_1 > p_2 > 0$. We let $\mathbf{p} = (p_1, p_2)$. A customer buys at most one unit of the product. Customers differ in their valuation or willingness-to-pay of the unit, and this is captured by a downward-sloping market demand function. Specifically, we assume a linear (inverse) demand function $p = b - aQ$ (with $Q \in$

$[0, b/a]$ for $a, b > 0$), where $Q$ is the mass of the customers whose valuation is $p$ or higher. $p_K$ denotes the maximum price that clears all $K$ units in the market (i.e., $p_K := \max[b - aK, 0]$). All the aggregate-level information, such as $a$, $b$, and $K$, is common knowledge, while the individual customer's valuation of the product is privately known to her alone. We call a customer with valuation $v$ "type $v$." The business environment with the complete information structure can be summarized by the "market" $\mathcal{M}_0(a, b, K)$ with $(a, b, K) \in \mathcal{R}^{+3}$.

We model the sales process as a *distribution game* $\mathcal{G}_0$ played in $\mathcal{M}_0$ by the retailer and customers. Similar to the Stackelberg game, it is a two-stage sequential game where the retailer moves first by setting the prices $(p_1, p_2)$, and then customers follow by making their purchase decisions. We assume that the retailer has the power to commit to the two prices at the beginning. In stage 2, each customer decides whether to buy early, later, or never. The retailer's prices and each customer's purchase decision form the equilibrium of the game. Specifically, the equilibrium concept of the game is subgame-perfect equilibrium in the sense that one player's strategy constitutes a best response for every subgame of the original game (Selten 1975). This requires solving the game by backward induction. In the second stage, given an arbitrary pair of prices, each customer makes the purchase decision to maximize her (expected) net value. In anticipation of this response, the retailer would choose the pair of prices that maximize his (expected) profit in the first stage.

We first study the equilibrium at stage 2 of the game. At the beginning of this stage, facing arbitrary $(p_1, p_2)$ with $p_1 > p_2$, a customer of type $v$ considers buying one unit of the product. All customers are *strategic* and evaluate three options: (1) buy early in the first period, (2) buy later in the second period, or (3) never buy. Each customer makes the decision to maximize the expected net value, so the decision depends on the individual customer's valuation $v$, as well as the prices and demand parameters. Denoting by $\Delta(v)$ the type-$v$ customer's decision, we write $\Delta(v) = i$, if she chooses option $i$ $(\in \{1, 2, 3\})$. Each customer faces a trade-off between securing a unit at a higher price early and obtaining it later at lower price with availability risk. Thus, a type-$v$ customer with $v \geq p_1$ will choose to buy early, if and only if

$$v - p_1 \geq R^* \cdot (v - p_2), \tag{1}$$

where $R^*$ is the probability of getting a unit in the second period. We assume that any customer who wants to buy in the first period is guaranteed to get one—by expediting the order at higher cost and zero margin to the retailer, while in the second period there is no such supply available, so each customer faces a random chance of getting a unit if demand exceeds supply. In case demand exceeds supply, all customers who order have equal probabilities of allocation.

Rewriting the buy-early condition (1), we have

$$v - R^* \cdot (v - p_2) \geq p_1. \tag{2}$$

Following Su and Zhang (2008), we call the left-hand side of the inequality the type-$v$ customer's *reservation price* in $\mathcal{M}_0(a, b, K)$ and define it as the maximum first-period price at which the customer is willing to buy early, instead of waiting, for a given $p_2$. The reservation price strictly increases in $v$ (when $0 < R^* < 1$), so if anyone buys early, it would be the highest-valuation customers. Thus, the type-$v$ customer's optimal decision rule $\Delta^*(v)$ is given by the following: For some threshold valuation $v^* \in [p_1, \infty)$,

$$\Delta^*(v) = \begin{cases} 1 & \text{if } v \geq v^*, \\ 2 & \text{if } p_2 \leq v < v^*, \\ 3 & \text{if } v < p_2. \end{cases} \tag{3}$$

As a result, the equilibrium of the second stage of the game can be succinctly denoted by the threshold $v^*$. Once $v^*$ is determined, the strategy of each type-$v$ customer is determined by $\Delta^*(\cdot)$.

Note that if $v^* \in (p_1, b)$, the threshold $v^*$ represents the marginal customer, who is indifferent about buying early or later. Denote the reservation price by $s(v, v^*)$ $(:= v - R^* \cdot (v - p_2))$; then the marginal type $v^*$ can be characterized by the solution of Equation (1) in equality, or

$$t(v^*) := s(v^*, v^*) = p_1. \tag{4}$$

The allocation probability $R^*$ is the ratio of supply (quantity left) to demand (quantity ordered) in the second period; hence, we have

$$R^* = \begin{cases} 0 & \text{if } p_K > v^*, \\ \dfrac{v^* - p_K}{v^* - p_2} & \text{if } v^* \geq p_K > p_2, \\ 1 & \text{if } p_K \leq p_2. \end{cases} \tag{5}$$

The second-case equation of the right-hand side is derived since $(v^* - p_K)/a$ is the quantity left for the second period, and $(v^* - p_2)/a$ is the order quantity in the second period.

The quantities sold in the two periods are, respectively, $(b - v^*)^+/a$ and $\min[(v^* - p_2)^+/a, (v^* - p_K)^+/a]$. If a markdown pricing scheme segments the market with a strictly positive mass of purchasers in each period, we call it *temporally discriminating*. For example, if $v^* \geq b$ under $(p_1, p_2)$, no customer buys early. If $v^* \leq p_K$, all customers of higher-than-$p_1$ types buy early and consume all the capacity $K$, so the pricing scheme $(p_1, p_2)$ in both cases is not a temporally discriminating scheme.

The following Proposition—due to Liu and van Ryzin (2008) and Zhang and Cooper (2008)—characterizes the equilibrium of the distribution game $\mathcal{G}_0$.

Proposition 1. (a) *In stage* 2 *there does not exist any temporally discriminating markdown pricing scheme in* $\mathcal{M}_0(a, b, K)$ *for any feasible parameters* $a$, $b$, *and* $K$. *Given an arbitrary pair of prices* $(p_1, p_2)$, *either all buying customers buy in the first period, or all in the second period. The former happens if* $p_K \geq p_1$, *while the latter happens if* $p_K < p_1$.

(b) *In stage* 1 *the retailer's equilibrium pricing scheme is* $(p^* + \epsilon, p^*)$ *for an arbitrarily small positive* $\epsilon$, *where*

$$p^* = \begin{cases} b - aK & \text{if } K \leq b/(2a), \\ b/2 & \text{if } K > b/(2a). \end{cases}$$

*Any leftover stock* $(K - b/(2a))^+$ *will be discarded, so no units will be left for sales in the second period.*

(All proofs except Propositions 4 and 5 are available in the online supplement, available at http://dx.doi.org/10.1287/msom.2014.0499.)

In general, a temporally discriminating scheme may potentially help the retailer extract more consumer surplus than static pricing, but Proposition 1 reports that there exists no markdown pricing that can temporally segment the market in the present setting of linear demand and complete information. Given the nonexistence of temporally discriminating schemes, the retailer will maximize the profit for one period and forgo any sales in the other period. Thus, the retailer's optimal markdown pricing will degenerate to static pricing scheme $(p^*, p^*)$ as $\epsilon$ is set close to zero.

## 4. The Model of Incomplete Information

In the previous section we assumed that all consumers have complete information about all model parameters (e.g., $a$, $b$, and $K$). We now relax this assumption and study the impact of demand uncertainties on the existence of temporally discriminating schemes. The general setup remains the same as in §3, but now neither the retailer nor customers exactly know the demand of the new product. They all start with the same prior belief about market demand for the product. Based on the incomplete information, the retailer sets the prices for two periods, and each customer responds by making her purchase decision using her individual valuation. Below are the details of the mathematical model.

(A1) (*Demand Uncertainty*) We assume that both the retailer and customers have incomplete information of $b$—the intercept of the linear (inverse) demand function. Alternatively, all know that the valuations of potential customers in the market are *uniformly* distributed in $[0, \bar{u}]$, but they do not know the market size $Q^0$ or the range of their valuations (i.e., $\bar{u}$). These two models become identical by letting $b = \bar{u} = aQ^0$. Thus, the uncertainty facing the retailer is about the market size of the product (as well as the range of customers' valuations).

(A2) (*The Game*) The sequence of the events in the new game $\mathcal{G}_1$ is as follows. In stage 0, nature draws a realization of the intercept $b$, but no one observes it until the end of the game. We assume that $b$ is from a Pareto distribution $\mathcal{P}(\alpha, \gamma)$ with scale parameter $\alpha(> 0)$ and shape parameter $\gamma(> 2)$. In our Bayesian model $\mathcal{P}(\alpha, \gamma)$ serves as the conjugate prior distribution to the uniform distribution (DeGroot 1986). Its probability density function is given by $f(b) = \gamma\alpha^\gamma / b^{\gamma+1}$ on the support $[\alpha, \infty)$ with its mean $E(b) = \gamma\alpha/(\gamma - 1)$ and variance $V(b) = \gamma\alpha^2/((\gamma-1)^2(\gamma-2))$. Also, we have $\alpha \leq b$, since otherwise the prior contradicts the truth. In stage 1, since neither the retailer nor customers know $b$, they all start with the common prior $\mathcal{P}(\alpha, \gamma)$ on $b$. Based on this prior, the retailer selects and announces the price pair $(p_1, p_2)$ with $p_1 > p_2$. Next, in stage 2, each customer develops her private valuation $v$ of the product and decides whether and when to purchase a unit. The valuation $v$ is a random draw from the uniform distribution $[0, b]$, and the customer uses it to update her estimate of $b$ in the Bayesian fashion. Note that her valuation $v$, known only to the customer herself throughout the game, does not only represent the intrinsic value of the unit to herself, but also serves as a private *signal* to help her estimate the market demand. We assume that her valuation is intrinsically determined without being influenced by other customers' decisions and remains the same over the two periods. From standard Bayesian statistics we have the posterior of $b$ as follows:

$$(b \mid v) \sim \mathcal{P}(\max[\alpha, v], \gamma + 1). \tag{6}$$

Note that $\max(\alpha, v)$ is a sufficient statistic for the scale parameter of the Pareto distribution. Thus, a private signal $v$ can be either *informative* or *noninformative*. If a customer receives a signal $v$ larger than $\alpha$, she will find it informative and update the scale parameter $\alpha$ to $v$ in her posterior, while if $v < \alpha$, she will discard the noninformative signal. Also note that as she observes a higher value of $v$, her estimate of $b$ (weakly) increases. Finally, the market outcome is realized; that is, units are allocated according to the rule, and the retailer realizes his profit.

Note that in the model all customers and the retailer alike start with the same prior, but later each customer observes a private signal and develops her individual posterior on the unknown $b$. This *Common Prior Assumption* or the *Harsanyi's Doctrine* is originally due to Harsanyi (1967) and widely used (despite some resistance) in game-theoretic models to capture heterogeneous beliefs among players (see, for example, Aumann 1976 or Morris 1995). It postulates that the difference in subjective probabilities among individuals is driven by information gained later, as opposed to the fundamental differences in their priors.

In this incomplete information setting the market can be denoted by $\mathcal{M}_1(a, K, \alpha, \gamma)$. The equilibrium concept of the Bayesian game in $\mathcal{M}_1$ is Bayesian Nash equilibrium where all players respectively update their beliefs using the Bayes' rule in the course of the game and choose a best response to maximize their expected payoff. See, for example, Rasmusen (1994) or Fudenberg and Tirole (1993) for details. More specifically, the seller's problem can be formulated as

$$\max_{(p_1, p_2):\ p_1 > p_2} \{p_1\mu_1(\Delta^\dagger, K) + p_2\mu_2(\Delta^\dagger, K)\} \tag{7}$$

$$\text{subject to}\quad \Delta^\dagger(v) \in \arg\max_{\Delta\in\{1,2,3\}} u_\Delta(v), \quad \text{for each } v \in (0, b), \tag{8}$$

where $\mu_i(\Delta^\dagger, K)$ is the expected number of customers choosing option $i$ $(=1, 2)$ under customers' decision rule $\Delta^\dagger$ and capacity $K$, and $u_\Delta(v)$ is the expected payoff to a type-$v$ customer choosing option $\Delta$. Thus, each type-$v$ customer chooses her best option $\Delta^\dagger(v)$ by solving (8) for a given pair of prices, and the seller chooses a best pair of prices by solving (7) in anticipation of $\Delta^\dagger(\cdot)$.

We start with the decision problems facing customers. Similarly to (1), a type-$v$ customer with $v \geq p_1$ will buy in the first period if and only if

$$v - p_1 \geq E(R^\dagger \mid v)\cdot(v - p_2), \tag{9}$$

where $R^\dagger$ is the equilibrium probability of getting an allocation in the second period. If we assume for now (and verify later in Proposition 4) that $E(R^\dagger \mid v)$ is weakly decreasing in $v$, this condition leads to a threshold policy like (3), according to the same logic used in deriving (5). That is,

$$\Delta^\dagger(v) = \begin{cases} 1 & \text{if } v \geq v^\dagger, \\ 2 & \text{if } p_2 \leq v < v^\dagger, \\ 3 & \text{if } v < p_2, \end{cases} \tag{10}$$

for some positive threshold $v^\dagger (\geq p_1)$. To completely determine the equilibrium strategy profile for each type-$v$ customer, we have yet to derive the equilibrium threshold $v^\dagger$.

In §4.1–4.4 we first analyze the stage 2 equilibrium of $\mathcal{G}_1$ in three steps—derivation of the equilibrium threshold $v^\dagger$ that determines customers' decision rules, the equilibrium outcome of markdown prices, and the existence of temporally discriminating prices in the market $\mathcal{M}_1$. Next, we turn to the retailer's problem and check if a temporally discriminating markdown scheme always exists that offers a higher profit to the retailer than static pricing.

### 4.1. Customers' Decision Rule

To obtain the threshold $v^\dagger$ and the equilibrium allocation probability $R^\dagger$, consider the marginal type-$v^\dagger$ customer who is indifferent between buying early and later. The threshold $v^\dagger$ $(\geq p_1)$ must satisfy

$$v^\dagger - p_1 = E(R^\dagger \mid v^\dagger)\cdot(v^\dagger - p_2). \tag{11}$$

Note that

$$R^\dagger = \begin{cases} 0 & \text{if } p_K > v^\dagger, \text{ or equivalently, } b > v^\dagger + aK; \\ \dfrac{v^\dagger - b + aK}{v^\dagger - p_2} & \text{if } v^\dagger \geq p_K > p_2, \text{ or } p_2 + aK < b \leq v^\dagger + aK; \\ 1 & \text{if } p_K \leq p_2, \text{ or } 0 < b \leq p_2 + aK. \end{cases} \tag{12}$$

This allocation probability $R^\dagger$ is a function of the random variable $b$, so each customer will take its expectation using her posterior distribution.

We separate the analysis into two cases depending on whether or not the marginal customer's signal is informative (i.e., $v^\dagger \geq \alpha$). Suppose first that $v^\dagger \geq \alpha$. From (12) we have

$$\begin{aligned} E(R^\dagger \mid v^\dagger) &= \int_{v^\dagger}^{p_2+aK} 1\cdot dF_1(b) + \int_{p_2+aK}^{v^\dagger+aK} \frac{v^\dagger - b + aK}{v^\dagger - p_2}\, dF_1(b) \\ &= F_1(p_2 + aK) \\ &\quad + \int_{p_2+aK}^{v^\dagger+aK} \frac{v^\dagger - b + aK}{v^\dagger - p_2}\, dF_1(b), \end{aligned} \tag{13}$$

where $F_1$ is the distribution function of $\mathcal{P}(v^\dagger, \gamma+1)$.

After some straightforward algebra, we have

$$\begin{aligned} E(R^\dagger \mid v^\dagger) = 1 &- \left(\frac{v^\dagger}{p_2 + aK}\right)^\gamma \frac{v^\dagger}{\gamma(v^\dagger - p_2)} \\ &+ \left(\frac{v^\dagger}{v^\dagger + aK}\right)^\gamma \frac{v^\dagger}{\gamma(v^\dagger - p_2)}. \end{aligned} \tag{14}$$

By incorporating (14) into (11), we obtain $v^\dagger = \alpha^\dagger(\mathbf{p})$ as a unique solution to

$$\begin{aligned} H(v) := -p_1 + p_2 &+ \left(\frac{v}{p_2 + aK}\right)^\gamma \frac{v}{\gamma} \\ &- \left(\frac{v}{v + aK}\right)^\gamma \frac{v}{\gamma} = 0. \end{aligned} \tag{15}$$

This holds only if $\alpha^\dagger(\mathbf{p}) \geq \alpha$. In addition, one can show (see the online supplement) that the marginal type is always higher than $p_1$.

Next, if $\alpha^\dagger(\mathbf{p}) < \alpha$, the marginal analysis of (13)–(15) fails to hold, since the marginal customer's posterior $\mathcal{P}(\max(v^\dagger, \alpha), \gamma+1)$ would be $\mathcal{P}(\alpha, \gamma+1)$, instead of $\mathcal{P}(v^\dagger, \gamma+1)$. By reflecting this change on (13)–(15), we have in this case

$$E(R^\dagger \mid v^\dagger) = 1 - \left(\frac{\alpha}{p_2 + aK}\right)^{\gamma} \frac{\alpha}{\gamma(v^\dagger - p_2)} + \left(\frac{\alpha}{v^\dagger + aK}\right)^{\gamma} \frac{\alpha}{\gamma(v^\dagger - p_2)}. \tag{16}$$

If $v^\dagger > p_1$, this together with (11) yields $v^\dagger = \beta^\dagger(\mathbf{p})$, where

$$\beta^\dagger(\mathbf{p}) := \frac{\alpha}{\left(-\dfrac{\gamma}{\alpha}(p_1 - p_2) + \left(\dfrac{\alpha}{p_2 + aK}\right)^{\gamma}\right)^{1/\gamma}} - aK. \tag{17}$$

Since the marginal customer must satisfy $v^\dagger > p_1$, we obtain in this case

$$v^\dagger = \max[\beta^\dagger(\mathbf{p}), p_1]. \tag{18}$$

Below is the summary of the above results on the threshold $v^\dagger$.

Proposition 2. *In $\mathcal{M}_1(a, K, \alpha, \gamma)$, the (unique) stage 2 equilibrium threshold $v^\dagger$, given the markdown pricing scheme $(p_1, p_2)$, is*

$$v^\dagger = v^\dagger(\mathbf{p}) := \begin{cases} \alpha^\dagger(\mathbf{p}) & \text{if } \alpha < \alpha^\dagger(\mathbf{p}), \\ \beta^\dagger(\mathbf{p}) & \text{if } \alpha^\dagger(\mathbf{p}) \leq \alpha < \delta(\mathbf{p}), \\ p_1 & \text{if } \alpha \geq \delta(\mathbf{p}), \end{cases} \tag{19}$$

*where $\alpha^\dagger(\mathbf{p})$ is defined in Equation (15), $\beta^\dagger(\mathbf{p})$ is defined in Equation (17), and*

$$\delta(\mathbf{p}) := \left[\frac{\gamma(p_1 - p_2)}{(p_2 + aK)^{-\gamma} - (p_1 + aK)^{-\gamma}}\right]^{1/(\gamma+1)}. \tag{20}$$

The optimal decision rule $\Delta^\dagger(\cdot)$ of Equation (10), with the equilibrium threshold $v^\dagger$, completely determines the equilibrium strategy of each type-$v$ customer, while her belief is given by $\mathcal{P}(\max(v, \alpha), \gamma+1)$. Since $v^\dagger$ does not depend on $b$, all customers (as well as the retailer) can derive $v^\dagger$ from public information and develop her decision $\Delta^\dagger(\cdot)$. Figure 1 shows the trajectory of $v^\dagger$ as $p_1$ changes in $\mathcal{M}_1(a = 10, K = 4, \alpha = 65, \gamma = 3)$ where $p_2$ is fixed at 20. Note that the threshold $v^\dagger$ increases in $p_1$, since a high $p_1$ will drive more customers to option 2 (i.e., wait).

### 4.2. Equilibrium Outcome of Markdown Pricing

Given customers' *strategies*, we can derive the equilibrium *outcome*, that is, how the market is segmented. An important difference between a strategy and an equilibrium outcome is in the realization of the random variable $b$. The strategy is chosen before the uncertainty is resolved, while the equilibrium outcome depends on its realization. We would like to investigate if a given markdown pricing scheme is temporally discriminating and under what conditions such a scheme exists. But whether or not a scheme is temporally discriminating depends on the intercept $b$. To this end, we need a specific value of $b$. For the sake of exposition, therefore, we consider the "conditional" market $\mathcal{M}'_1(a, b, K, \alpha, \gamma)$ with $(a, b, K, \alpha, \gamma) \in \mathcal{R}^{+5}$, $\alpha \leq b$, and $\gamma > 2$, where nature's move is conditioned on $b$ (unknown to all players) leaving everything else as is. Note that since $b$ is unknown throughout the game, the existence of a temporally discriminating scheme does not mean that the retailer can actually implement it, but instead says that there is a chance of implementing the scheme or a similar one in its neighborhood. By contrast, if there does not exist any temporally discriminating scheme, there is no way to implement it.

Proposition 2 readily leads to the following corollary.

Corollary 3. *A markdown pricing scheme $(p_1, p_2)$ is temporally discriminating in the market $\mathcal{M}'_1(a, b, K, \alpha, \gamma)$ if and only if $v^\dagger \in (p_K, b)$, where $v^\dagger$ is defined in Proposition 2.*

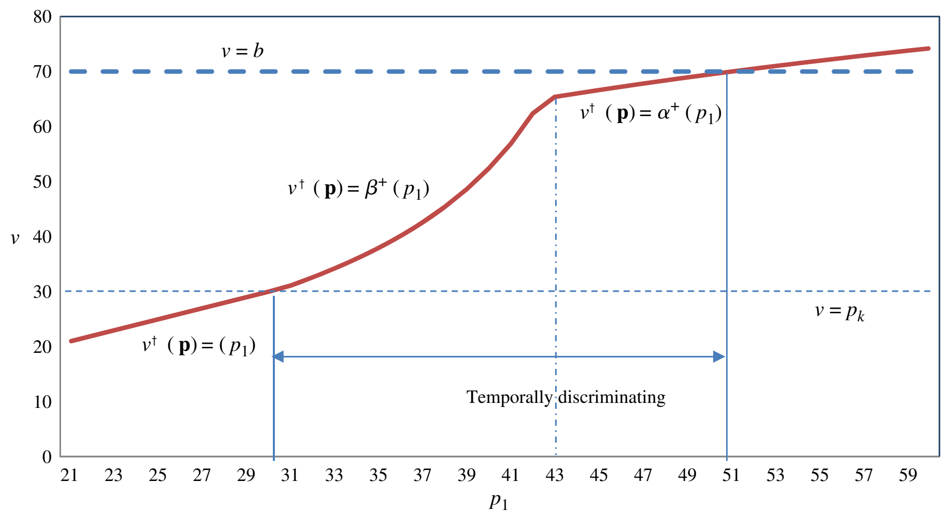
Figure 1 demonstrates the corollary in the market $\mathcal{M}_1(10, 4, 65, 3)$ for a specific realization of $b = 70$, i.e., in $\mathcal{M}'_1(10, 70, 4, 65, 3)$; $\mathcal{M}'_1$ has a temporally discriminating pricing scheme $(p_1, p_2)$ when $30 < p_1 < 51$ and $p_2 = 20$. We have obtained $p_1 = 30$ by solving $v^\dagger(\mathbf{p}) = p_K$, and $p_1 = 51$ by solving $v^\dagger(\mathbf{p}) = b$. Generally speaking, a temporally discriminating scheme requires that the two prices should not be too close or too far apart. If they are too close (i.e., $p_1 \leq 30$ and $p_2 = 20$), all high-valuation customers buy in the first period. If they are too far apart (i.e., $p_1 \geq 51$ and $p_2 = 20$), no customer buys in the first period.

### 4.3. Existence of Temporally Discriminating Markdown Prices

In the previous subsections we characterized the equilibrium strategy of customers and studied temporally discriminating prices *if they exist*. We now analyze the *existence* of temporally discriminating pricing schemes in a given market $\mathcal{M}_1$. We report the following:

Proposition 4. (a) *In the market $\mathcal{M}_1(a, K, \alpha, \gamma)$, there always exists a temporally discriminating pricing scheme*

**Figure 1** (Color online) Threshold at Different First-Period Prices




*Notes.* In $\mathcal{M}_1'(a = 10, b = 70, \alpha = 65, y = 3, K = 4)$ with $p_2 = 20$, the equilibrium threshold $v^{\dagger}$ ($\mathbf{p}$) increases in $p_1$ (Proposition 2). It also shows the range of $p_1$ that makes $(p_1, p_2)$ temporally discriminating (Corollary 3).

$(p_1, p_2)$ *(with* $p_1 > p_2 > 0$*) for any feasible parameters* $a, K, \alpha,$ *and* $\gamma$*. (b) Furthermore, in* $\mathcal{M}_1'(a, b, K, \alpha, \gamma)$*, there always exists a temporally discriminating markdown pricing scheme* $(p_1, p_2)$ *for any* $p_2 \in [p_K, b)$ *with* $p_1 > p_2 > 0$.

Proof. We first prove part (b). Consider $\mathcal{M}_1'(a, b, K, \alpha, \gamma)$. An arbitrary type-$v$ customer assesses the probability of allocation as

$$E(R^{\dagger} \mid v) = \begin{cases} 1 - \left(\dfrac{\alpha}{p_2 + aK}\right)^{\gamma} \dfrac{\alpha}{\gamma(v^{\dagger} - p_2)} \\ \quad + \left(\dfrac{\alpha}{v^{\dagger} + aK}\right)^{\gamma} \dfrac{\alpha}{\gamma(v^{\dagger} - p_2)} & \text{if } v \le \alpha, \\ 1 - \left(\dfrac{v}{p_2 + aK}\right)^{\gamma} \dfrac{v}{\gamma(v^{\dagger} - p_2)} \\ \quad + \left(\dfrac{v}{v^{\dagger} + aK}\right)^{\gamma} \dfrac{v}{\gamma(v^{\dagger} - p_2)} & \text{if } v > \alpha, \end{cases} \tag{21}$$

where $v^{\dagger}$ is defined in Proposition 2. Note that different types of customers may make different assessments of the probability of getting an allocation, based on the private signal. This "perceived" probability $E(R^{\dagger} \mid v)$ weakly decreases in $v$; that is, the customer who observes a high signal assesses a lower probability of allocation. Next, consider the stochastic version of the reservation price $S(v, v^{\dagger})$ in $\mathcal{M}_1(a, K, \alpha, \gamma)$, which we define for a given $p_2$ as

$$S(v, v^{\dagger}) := E[s(v, v^{\dagger})] = v - E(R^{\dagger} \mid v)(v - p_2). \tag{22}$$

Then, Equation (21) yields (for $v$ and $v^{\dagger}$ equal to, or greater than, $p_2$)

$$S(v, v^{\dagger}) = \begin{cases} p_2 + \dfrac{\alpha}{\gamma}\left(\dfrac{v - p_2}{v^{\dagger} - p_2}\right)\left(\dfrac{\alpha}{p_2 + aK}\right)^{\gamma} \\ \quad - \dfrac{\alpha}{\gamma}\left(\dfrac{v - p_2}{v^{\dagger} - p_2}\right)\left(\dfrac{\alpha}{v^{\dagger} + aK}\right)^{\gamma} & \text{if } v \le \alpha, \\ p_2 + \dfrac{v}{\gamma}\left(\dfrac{v - p_2}{v^{\dagger} - p_2}\right)\left(\dfrac{v}{p_2 + aK}\right)^{\gamma} \\ \quad - \dfrac{v}{\gamma}\left(\dfrac{v - p_2}{v^{\dagger} - p_2}\right)\left(\dfrac{v}{v^{\dagger} + aK}\right)^{\gamma} & \text{if } v > \alpha. \end{cases} \tag{23}$$

A sufficient condition for the existence of temporally discriminating prices is as follows: (i) a solution $v^{\dagger}$ to $T(v^{\dagger}) := S(v^{\dagger}, v^{\dagger}) = p_1$ exists for some $p_1 \in (p_K, b)$ and (ii) $S(\cdot, v^{\dagger})$ is continuous and increasing for a given $v^{\dagger}$. To find a temporally discriminating pricing scheme $(p_1, p_2)$, we note that for a given $p_2 (\in [p_K, b))$, $T(p_2) = p_2 \ge p_K$ from (23), and that $T(v^{\dagger})$ is continuous and strictly increasing in $v^{\dagger} \in (p_2, b)$. Hence, there exists an interior point $p_1$ $(\in (p_K, b))$ satisfying $T(v^{\dagger}) = p_1$ for some $v^{\dagger} \in (p_K, b)$. Also, one can see from (23) that $S(\cdot, v^{\dagger})$ is continuous and increasing for any given $v^{\dagger}$, so the sufficient condition is met, and the proof is complete for $\mathcal{M}_1'(a, b, K, \alpha, \gamma)$. Part (a) comes as a corollary. The set $[p_K, b)$ is nonempty (since $b > p_K$), so there always exists a $p_2$ in $[p_K, b)$. By applying part (b) here, we prove part (a). Proposition 4(a) holds in $\mathcal{M}_1'(a, b, K, \alpha, \gamma)$ for any $b$, so it also holds in $\mathcal{M}_1(a, K, \alpha, \gamma)$ for any feasible parameters. □

The incomplete information case offers a contrast to the complete information case. The complete information market $\mathcal{M}_0$ *never* has any temporally discriminating price pair for any feasible model parameters, while the incomplete information market $\mathcal{M}_1$ *always* has a temporally discriminating price pair for any feasible model parameters. Due to demand uncertainty, therefore, temporal discrimination becomes feasible (but not always implementable).

### 4.4. The Retailer's Pricing Strategy

We move backward to stage 1 of $\mathcal{G}_1$ where the retailer sets the prices. Proposition 4 reports that there always exists a temporally discriminating markdown pricing scheme in $\mathcal{M}_1(a, K, \alpha, \gamma)$, but now we ask how good it is. Our objective here is to investigate if a temporally discriminating markdown pricing scheme always exists that yields a higher profit to the retailer than static pricing for any $b$ and other market parameters. To this end, we first find the "first-best" temporally discriminating markdown pricing scheme in $\mathcal{M}'_1$ and compare it with the "first-best" static pricing scheme. We define "the first-best" as the solution attainable by the retailer in the most favorable condition where he knows $b$, customers do not know that he knows $b$, and he knows that customers do not know that he knows $b$, and so on. While the first-best solution is not always attainable by the retailer who does not know the true value of $b$, it will serve our needs to show the *existence* of such markdown pricing schemes. The problem of finding the first-best temporally discriminating scheme in $\mathcal{M}'_1(a, b, K, \alpha, \gamma)$ can be formulated as

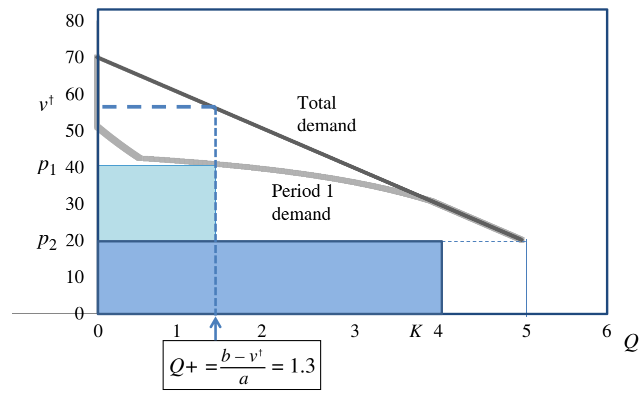
$$\max_{\mathbf{p}: p_1 > p_2 > 0,\, p_K < v^\dagger(\mathbf{p}) < b} \pi(\mathbf{p})$$

$$:= (p_1 - p_2)\min\left[\left(\frac{b - v^\dagger(\mathbf{p})}{a}\right)^+, K\right] + p_2 \min\left[\left(\frac{b - p_2}{a}\right)^+, K\right],$$

subject to $v^\dagger$ as defined in Proposition 2. The first term of the objective is the additional profit made at the high price in the first period, so it is the price premium $(p_1 - p_2)$ multiplied by first-period sales quantity. The second term is the profit over the two periods from the discount price—the low price times the total quantity sold.

To gain qualitative insights, see Figure 2, which is based on the same numerical example as Figure 1 (i.e., in $\mathcal{M}'_1(10, 70, 4, 65, 3)$). It shows the first-period demand curve—the plot of the points $(Q^\dagger, p_1)$ for a fixed $p_2$ and different values of $p_1 \in (p_2, b]$, where $Q^\dagger$ denotes the mass of higher-than-$v^\dagger$ customers, i.e., $Q^\dagger = (b - v^\dagger(\mathbf{p}))^+/a$. From the figure we make several observations. First, it shows the existence of first-period demand, unlike the complete information case. For example, $\mathbf{p}^\dagger := (41, 20)$ is a temporally discriminating scheme. Second, we note that the first-period demand is a fraction of the total demand (e.g., 1.3 out of 2.9 at $\mathbf{p}^\dagger$). The figure illustrates a comparison between the two cases of strategic and myopic customers. In the latter case, the same $\mathbf{p}^\dagger$ would lead to a profit 27% higher. Lastly, we confirm that the existence of temporally discriminating schemes *may* not necessarily imply their optimality. In the present numerical example, the optimal static pricing scheme (35, 35) is 14% better than this specific temporally discriminating scheme. We then ask, "Does a temporally discriminating markdown pricing scheme always exist that yields a higher profit to the retailer than static pricing?" The following proposition provides the answer.[1]

**Figure 2** (Color online) First-Period Demand Curve




*Notes.* To derive the period 1 demand curve for a given $p_2$ (e.g., 20) in $M'_1(a = 10, b = 70, \alpha = 65, y = 3, K = 4)$, first choose a value of $p_1$ (e.g., 41) and derive $v^+ = 57$, which gives $Q^+ = (b - v^+)/a = 1.3$. Thus, we obtain one point $(Q^+, p_1) = (1.3, 41)$ on the period-1 demand. Next, change $p_1$ and repeat the process.

Proposition 5. *In any feasible market $\mathcal{M}_1(a, K, \alpha, \gamma)$ there always exists a temporally discriminating markdown pricing scheme that yields a strictly higher profit to the retailer than any static pricing scheme.*

Proof. Let the optimal static price be denoted by $(p^*, p^*)$ where $p^* = \max(b/2, p_K)$. Then, consider a markdown pricing scheme $(p_1, p_2)$ where $p_2 = p^*$. Since $p_2 = p^* \geq p_K$, Proposition 4(b) applies, and there exists a $p_1 \in (p^*, b)$, for which the markdown pricing scheme $(p_1, p^*)$ is temporally discriminating. Then, by setting $p_1$ slightly above $p^*$, we obtain a markdown pricing scheme $(p_1, p^*)$ that is both temporally discriminating (from Proposition 4(b)) and profit-enhancing (due to the strictly higher margin in the first period price without reducing the total quantity sold). □

In any market $\mathcal{M}_1(a, K, \alpha, \gamma)$, therefore, the first-best pricing scheme $(p_1, p_2)$ is always a markdown pricing scheme with $p_1 > p_2$. To have a sense of the

[1] We thank the reviewer who suggested this approach.

magnitude of improvement by the first-best markdown pricing scheme, we conducted a set of numerical tests on $\mathscr{M}_1'(10, b, K, \alpha, \gamma)$ with $b = \{70, 80\}$, $K = \{3, 4\}$, $\alpha \in \{20, 40, 65\}$ and $\gamma \in \{3, 10\}$. The profit increase by the first-best markdown pricing over the first-best static pricing ranges from 0.4% to 12.8%, with the average of 3.6%. In this numerical analysis, we observe that (i) tight capacity (i.e., small $K$) leads to a higher profit improvement through temporal segmentation and (ii) higher degree of demand uncertainty (i.e., lower $\gamma$) leads to a higher profit improvement through temporal segmentation. When capacity is tight and/or demand is highly uncertain, high-valuation customers would buy early to secure a unit ahead of competition, so that temporal segmentation may be more effective in generating a higher profit. But since the retailer cannot implement the first-best markdown prices, the average improvement would be lower than 3.6%, or even negative.

## 5. Extension to Nonlinear Demands and the Bayesian Effect

In the previous sections we used restrictive assumptions on demand uncertainties. The private signal was assumed drawn from a uniform distribution with unknown support, and the Pareto distribution was used as the prior. But the qualitative result of the model—that demand uncertainties and Bayesian updating will distort the perceived demand and facilitate temporal discrimination—is broadly upheld in general settings. The key driver of the result is what might be termed "the Bayesian bias" or "the Bayesian effect," that is, the way Bayesian updating changes the perceived (expected) probability $E(R^\dagger \mid v)$ of allocation. As Equation (21) illustrates, $E(R^\dagger \mid v)$ weakly decreases in $v$. A customer of high type overestimates the demand and underestimates the probability of getting an allocation, while a low-type customer does the opposite.

To intuitively understand how the Bayesian effect contributes to the existence of temporally discriminating prices in a more general setting, suppose that the demand function is *nonlinear* and unknown, but that all have a common prior with $E(R^\dagger \mid v)$ decreasing in $v$. Recall that $T(v^\dagger)$ is defined as the marginal customer's reservation price $S(v^\dagger, v^\dagger)$. From the proof of Proposition 4, one of the two conditions for the existence of temporally discriminating prices is condition (i): an interior solution $v^\dagger$ to $T(v^\dagger) = p_1$ exists for some $p_1 \in (p_K, b)$. Condition (i) is met if $T(\cdot)$ is a continuous increasing function, so we check

$$\frac{dT(v^\dagger)}{dv^\dagger} = (1 - E(R^\dagger \mid v^\dagger)) - (v^\dagger - p_2)\frac{\partial E(R^\dagger \mid v)}{\partial v^\dagger}\bigg|_{v=v^\dagger} - (v^\dagger - p_2)\frac{\partial E(R^\dagger \mid v)}{\partial v}\bigg|_{v=v^\dagger}. \quad (24)$$

The first term on the right-hand side is positive. The second term is negative, since a higher threshold leads to higher allocation probability as a combined result of more units available later and more customers competing over them. The last term is positive as it captures the Bayesian effect. Thus, $T(v^\dagger)$ is not necessarily increasing in $v^\dagger$. Contrast this with the *complete* information case, where there is no Bayesian effect. The counterpart of (24) in complete information has only the first two terms; i.e.,

$$\frac{dt(v^*)}{dv^*} := \frac{ds(v^*, v^*)}{dv^*} = (1 - R^*) - (v^* - p_2)\frac{dR^*}{dv^*}, \quad (25)$$

which is less likely to be positive than in the incomplete information case, due to the missing Bayesian effect. In general, therefore, Bayesian updating—through $E(R^\dagger \mid v)$ decreasing in $v$—helps meet condition (i) in the incomplete information case (in fact, condition (ii) as well). As a result, the market under demand uncertainty is more likely to have temporally discriminating prices than the corresponding market without demand uncertainty. The *linear* demand function serves as the tipping point, where Equation (25) is uniformly zero (in the nontrivial case $p_K \geq p_2$), and Equation (24) is always positive. This explains how demand uncertainty leads to temporally discriminating prices under linear demand.

Temporal segmentation is related to the shape of the demand curve. If the (inverse) demand function is strictly *convex* decreasing, Equation (25) is always positive in the nontrivial case, and the market always has temporally discriminating markdown prices in both complete and incomplete information cases. On the other hand, if the demand function is strictly *concave* decreasing, Equation (25) is always negative in the nontrivial case, so that temporally discriminating schemes do not exist in the complete information case. Thus, *convexity* is a critical factor driving temporal discrimination. A convex demand function implies a small fraction of very-high-type customers who are willing to buy at a very high price and a larger fraction of next-high types of customers who stand by waiting for a price reduction. The abundance of the latter types makes the chance of later allocation slim, so the retailer can induce the former to buy early through a properly chosen pair of prices. Thus, a temporally discriminating scheme comes to exist under convex demand functions. In a sense, the Bayesian effect creates a *convexity-like* effect under demand uncertainty. It leads a few high-type customers in unknown linear demand to overestimate the number of next-high types of customers and act like very-high-type customers in known convex demand. In summary, demand uncertainty creates the Bayesian effect, which in turn generates the convexity-like effect, thereby facilitating temporal discrimination.

## 6. Concluding Remarks

We have studied the role of demand uncertainty in temporal discrimination when the retailer applies markdown pricing facing strategic customers. By developing a stylized model, we derived qualitative insights to the way demand uncertainty and Bayesian updating contribute to temporal discrimination, which broadly apply to nonlinear demand functions as well. We also showed that there always exists a temporally discriminating markdown pricing scheme that yields a strictly higher profit to the retailer than the optimal static pricing scheme in a market with uncertain demand. Ironically, however, the retailer cannot implement the optimal scheme due to the same demand uncertainty.

We have kept the model simple to make analysis tractable and derive sharper insights. As its byproduct, the model has important limits and drawbacks. First of all, the common prior assumption (CPA), a critical element of our model, has been both pervasive and controversial among game-theoretic economists. For example, Morris (1995) notes that CPA is "implicit or explicit in the vast majority of the differential information literature in economics and game theory." We chose the CPA for lack of alternative ways to model nonhomogeneous beliefs. And yet, there exist some compelling challenges to the assumption (see Morris 1995, Gul 1998, Feinberg 2000). For example, we may ask, "How do customers come to have the same prior with the retailer and among themselves?" In reply, it might be that the retail firm has established a certain reputation in the market, or the new product was featured and reviewed in public media. And yet, whether all customers would actually derive a common prior from various public sources before adding their private information is a different question to which we do not have a clear answer.

Another critical assumption of the model is that the retailer is never stocked out in the first period. This simplification is widely adopted in the literature involving strategic customers—such as Liu and van Ryzin (2008) and Su and Zhang (2008). Admittedly, a more natural way of modeling this process would be to allocate the limited stock to customers if demand exceeds supply in the first period. If we follow the latter path, Proposition 1 remains valid; that is, no temporally discriminating markdown pricing scheme exists in the complete information case. For the incomplete information case, however, analysis changes and gets complicated because the trade-off between buying early and later (lower price) changes. We were unable to obtain any closed-form solutions (especially, $v^{\dagger}$) to the counterparts of Equation (13) and onwards. But given the intuition behind the present analysis, we conjecture that the same qualitative results may apply in this case. Overall, we believe our setup is a fair choice in the trade-off between analytic tractability and realism.

### Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/msom.2014.0499.

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