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Resource Flexibility and Capital Structure

Jiri Chod, Jianer Zhou

Carroll School of Management, Boston College, Chestnut Hill, Massachusetts 02467
{chodj@bc.edu, jianer.zhou@bc.edu}

This paper examines how the optimal investment in the capacity of flexible and nonflexible resources is affected by financial leverage and, conversely, how a firm's resource flexibility affects its optimal capital structure. We consider a two-product firm that invests in the optimal capacity of product-flexible and product-dedicated resources in the presence of demand uncertainty. Before investing in capacity, the firm issues the optimal amount of debt, trading off the tax benefit and lower transaction cost of debt financing against the cost of financial distress and the agency cost associated with leverage. We show that in the presence of debt, resource flexibility has benefits in addition to reducing the mismatch between supply and demand. Namely, resource flexibility mitigates the shareholder–debtholder agency conflict as well as the risk of costly default. Most interestingly, we show that resource flexibility mitigates the underinvestment problem because it reduces the probability that a firm will go bankrupt with some of its capacity being fully utilized. When lenders anticipate that a firm will choose a relatively flexible capacity mix, they should provide more favorable credit terms, to which the firm should respond by issuing more debt. The main empirical predictions are that resource flexibility is negatively related to the cost of borrowing and positively related to debt.

Keywords: flexibility; capacity; leverage; capital structure; agency; asset substitution; underinvestment

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1. Introduction

This paper bridges the operations management and corporate finance literatures by examining the relation between a firm's resource flexibility, also known as product or mix flexibility, and its use of debt financing. One industry that shows the importance of both resource flexibility and financial leverage is car manufacturing. Between 1974 and 2004, the number of automobile models produced in North America grew from 90 to 165, whereas the number of assembly plants declined from 68 to 64 (Van Biesebroeck 2007). This led to a dramatic increase in the number of models produced side by side in a single plant, and often on a single assembly line.

The recent history of the U.S. automotive industry also illustrates the risks associated with excessive leverage. In 2009, following a dramatic decline in sales, General Motors (GM) declared bankruptcy under Chapter 11, claiming around \$82 billion in assets and around \$173 billion in debts (General Motors 2009). While wiping out existing shareholders, the bankruptcy allowed GM to reorganize and continue operations as a new company with a total debt reduction of \$92.7 billion. The following year, the new GM further reduced its financial leverage through a \$23 billion initial public offering (IPO). In its 2010 annual report, the company emphasized its intention to minimize financial leverage by using excess cash to

repay debt while maximizing capacity utilization and manufacturing flexibility (see General Motors 2010).

GM's efforts to enhance manufacturing flexibility following its reorganization and deleveraging are best illustrated by the company's recent investment initiatives. In 2011, the company announced plans to invest \$61 million in its idling plant in Spring Hill, Tennessee, for use as an "ultra-flexible" plant capable of producing multiple vehicles of similar size (Bunkley 2011). "The ultra-flexible plant... would serve as an overflow source of production for vehicles that GM is already building in other factories. When a vehicle like the Equinox, for example, gets hot in showrooms, GM could quickly crank up production to meet that extra demand, and then just as quickly ratchet it back down when sales start to cool off" (Muller 2011). In addition, GM announced an investment of \$68 million to prepare its Oshawa, Canada assembly plant to build the Chevrolet Impala on a "flex assembly line," which can be quickly retooled to make other models. "At the Oshawa plant, GM's move to shut down the traditional line in the first quarter of 2013 is part of a broader manufacturing plan to run more flex assembly lines..." (Nurwisah 2011).

The GM example raises several interesting questions regarding the relation between a firm's investment in flexible technology and its use of debt financing. How does debt affect a firm's optimal

mix of flexible and nonflexible capacity? And how do the availability and cost of flexible technology affect the firm's cost of capital and optimal capital structure? To address these questions, we consider a two-product firm that invests in product-flexible and product-dedicated capacity in the presence of product demand uncertainty. When choosing the optimal level of each type of capacity, the firm has to trade off the cost of excess capacity against the opportunity cost of lost sales. While flexible capacity is more expensive than nonflexible capacity, it is also more valuable because it can be used to produce either product depending on the realized demand. Prior to investing in capacity, the firm issues the optimal amount of fairly priced debt. The remaining capital required to finance the capacity investment is provided by shareholders. In choosing the optimal capital structure, the firm trades off the higher transaction cost of issuing equity and the tax benefit of issuing debt against the agency cost and the cost of financial distress associated with leverage.

Cost of issuing equity. The transaction cost of issuing equity is considerably higher than that of issuing debt. A substantial part of this cost is a fee paid to the underwriting bank known as the "spread," which tends to be a relatively stable percentage of the capital raised. The underwriter gross spread on equity tends to be 7% for initial public offerings (e.g., Chen and Ritter 2000, Hansen 2001) and 5.3% for seasoned equity offerings (SEOs) (Corwin 2003). The average gross spread on debt is considerably smaller, with estimates ranging between 0.92% (Roten and Mullineaux 2002) and 1.32% (Gande et al. 1999).

In addition to the spread, the direct cost of raising capital includes legal, auditing, and other expenses. According to Lee et al. (1996), the total direct cost of raising equity averages 11.0% and 7.1% of IPO and SEO proceeds, respectively, whereas the direct cost of raising debt averages 3.8% and 2.2% for convertible bonds and straight debt issues, respectively. Finally, new equity issues tend to involve a significant indirect cost due to short-run underpricing. Lee et al. (1996) report the average direct and indirect costs for IPOs to be 18.7% of the equity value. To reflect the above empirical regularities, we assume a transaction cost is involved in raising equity, which is a fraction of the equity issue, and we normalize the transaction cost of raising debt to zero.

Tax benefit of debt. To capture the tax benefit of debt, we assume that the firm's profit is taxed at a flat rate and that the interest is tax deductible. Even though some of our analytical results are derived for the special case of a zero tax rate, we verify that they continue to hold in the presence of tax in an extensive numerical study.

Agency cost of debt. Once debt is in place, a levered firm chooses capacity investment that maximizes the

value of equity rather than the total firm value, i.e., the value of equity plus the value of debt. Because debtholders anticipate the firm's investment strategy and price the debt accordingly, the loss of firm value, known as the agency cost of debt, is ultimately borne by shareholders through a higher cost of borrowing.

Cost of financial distress. When a firm cannot repay its debt obligation in full, creditors can accept debt renegotiation,¹ or they can force the firm into costly bankruptcy.² The larger the gap between the firm's debt obligation and its liquid assets, the more likely it is that debtholders will refuse renegotiation and force the firm into bankruptcy.³ We reflect the relation between the severity of the firm's insolvency and the likelihood of incurring the bankruptcy cost by assuming that the expected cost of financial distress increases with the shortfall between the value of the firm's liquid assets and the debt obligation. Because the cost of financial distress reduces the value of debt in default, it increases the interest required by debtholders and thereby reduces shareholder value.

We first examine the effect of leverage on capacity investment. When default is costly, to maximize the total firm value, a levered firm should invest in a more flexible capacity mix than an unlevered firm. This is because flexibility mitigates risk and thus the expected cost of financial distress. However, once debt is in place, a levered firm has exactly the opposite incentive. Because the loss realized in default states is borne by debtholders, a levered firm chooses a more risky and thus less flexible capacity mix than an unlevered firm, consistent with the asset substitution theory of Jensen and Meckling (1976).

Once the debt has been issued, the marginal increase in capacity investment is financed entirely by equity. At the same time, having additional capacity benefits the value of equity as well as the value of debt. Because a levered firm is concerned only with the benefit to equity, it underinvests in capacity, consistent with Myers (1977). Finally, because the distortion of the firm's capacity choice in terms of asset substitution and underinvestment can be anticipated by debtholders, it increases the cost of borrowing, which in turn decreases the amount of debt that the firm issues.

¹ For example, Gilson et al. (1990) report that about half of financially distressed publicly traded companies between 1978 and 1987 successfully restructured their debt outside bankruptcy protection.

² Whether the bankruptcy involves reorganization (Chapter 11) or liquidation (Chapter 7), it involves considerable legal and accounting fees as well as fees to government agencies (e.g., Ang et al. 1982).

³ For example, Bourgeon and Dionne (2013) model the dynamics of the strategic interaction between a firm and a bank, showing a bank is more likely to refuse debt renegotiation upon default when the firm has a low liquidation value.

To assess the impact of flexibility on the optimal amount of debt, we first examine the case in which it is optimal to choose only flexible capacity. In our model, the firm chooses full flexibility only when flexible and nonflexible capacity have the same cost. In reality, however, a firm can optimally choose 100% flexible capacity even if it is more expensive, provided there is a fixed cost associated with investing in each type of capacity, or capacity can be increased only by discrete quantities. We show that when a firm invests only in flexible capacity and product profit margins are not too asymmetric, it has an incentive to choose capacity and debt that jointly maximize total firm value.

Interestingly, we find that flexibility can eliminate not only asset substitution but also underinvestment. In fact, we show that the underinvestment problem as described by Myers (1977) does not apply to firms such as the standard single-product single-resource newsvendor whose capacity has no salvage or liquidation value.⁴ The reason is the following: If a firm does not default, the debt is worth exactly its face value. If a firm defaults, its capacity is underutilized, and the value of debt would not benefit from a marginally higher capacity either. Because a marginal change in capacity does not impact the value of debt, there is no conflict between maximizing the value of equity and maximizing total firm value at the capacity investment stage.

With multiple capacities dedicated to different products, a newsvendor-like firm can go bankrupt with one capacity being fully utilized, and hence, it faces underinvestment. In contrast, a fully flexible newsvendor cannot go bankrupt when its capacity is fully utilized, and thus, it does not face underinvestment. (An exception is the case in which the firm is highly levered and there is a large difference between product profit margins, which is not likely for products that rely on the same flexible capacity and, thus, are inherently similar.)

In addition to mitigating the agency cost, flexibility reduces the risk of costly default. Therefore, when lenders anticipate that a firm will invest in flexible capacity, they should require lower interest, and the firm should issue more debt. The operations literature has long recognized the value of flexibility in reducing the mismatch between supply and demand, but our paper is the first to show that resource flexibility also reduces the cost of borrowing by mitigating the agency cost and the risk of costly default associated with debt.

Because our analytical result regarding the effect of flexibility on debt is limited to the case of full flexibility, we verify its robustness in an extensive numerical study. Namely, we show, for a wide range of parameter values, that as the cost of flexible capacity decreases and the optimal capacity mix becomes more flexible, the optimal amount of debt increases monotonically.

The key managerial implications are the following. Because flexibility reduces the cost of borrowing, a firm relying on external financing should invest more in flexibility than suggested by models that implicitly assume pure equity financing. However, to enjoy this benefit of flexibility, the firm has to convince lenders of its commitment to flexible technology, e.g., by using bond covenants or sharing control over investment decisions with the lenders. When lenders anticipate that a firm will choose a relatively flexible capacity mix, they should provide more favorable credit terms, to which the firm should respond by issuing more debt. The main empirical predictions from our model are that resource flexibility is negatively related to the cost of borrowing and positively related to debt.

1.1. Relation to the Literature

The operations management literature on optimal investment in flexible capacity is extensive (e.g., Fine and Freund 1990, Van Mieghem 1998, Chod and Rudi 2005, Goyal and Netessine 2007), but it implicitly assumes that firms are financed entirely by equity. Our model of the optimal investment in a portfolio of flexible and nonflexible capacity builds on the seminal work of Van Mieghem (1998), but it allows capacity to be financed by equity as well as debt. Several studies model operational decisions in a framework similar to ours while considering external financing (e.g., Xu and Birge 2004, Kouvelis and Zhao 2011, Birge and Xu 2011), but they do not consider resource flexibility. An exception is Boyabatli and Toktay (2006 and 2011), who study a two-product firm that chooses whether to invest in a flexible or nonflexible technology under demand uncertainty while relying on external financing. Boyabatli and Toktay (2006) assume the cost of borrowing to be exogenous, and focus on the interaction between flexibility and hedging.

Boyabatli and Toktay (2011) endogenize the cost of borrowing and examine the effect of capital market imperfections on the firm's technology choice. Our model differs from theirs in several important ways. First, they consider a choice between a fully flexible and fully dedicated capacity, whereas we consider the optimal mix of flexible and dedicated capacity. Second, they assume market-clearing pricing that leads to full capacity utilization, whereas we consider a newsvendor setting in which capacity may be underutilized.

⁴ We use the term "newsvendor" in its narrow sense as a model with stochastic demand and predetermined price.

Furthermore, they consider a monopolist lender in addition to fairly priced debt. However, the key distinction is that in Boyabatli and Toktay (2011), leverage does not lead to agency conflict. They assume a given equity, in which case the marginal investment is financed by debt and underinvestment does not arise. Moreover, they assume a technology-specific loan contract that commits the firm to using a specific technology and therefore prevents asset substitution. The model of Boyabatli and Toktay (2011) thus captures very different aspects of the link between flexibility and debt. They show, among other things, that the prevalence of flexible technology is expected to be higher among firms using a secured loan, and that the higher the demand variability, the higher the prevalence of nonflexible technology.

Our contributions to the operations management literature are the following. First, we show how the optimal investment in flexible and nonflexible capacity is affected by financial leverage. Second, whereas the operations management literature has long recognized the value of flexibility in mitigating the mismatch between supply and demand, our paper is the first to show that resource flexibility also mitigates the shareholder–debtholder agency conflict and the risk of costly default. This means that firms investing in product-flexible capacity can expect more favorable credit terms and, therefore, should demand more debt.

The most novel finding of our paper in the light of the existing agency theory has to do with our insights into the underinvestment problem (Myers 1977). Our paper is, to the best of our knowledge, the first to show that the underinvestment problem, one of the most studied topics in the finance literature, does not apply to the classical single-product single-resource newsvendor when capacity has no liquidation or salvage value. The argument of Myers (1977) does not apply here because the newsvendor's capacity is always underutilized in default states. However, we show that a newsvendor does face the underinvestment problem when it relies on multiple resources dedicated to different products. Finally, we show that the underinvestment problem is mitigated when a firm invests in a single flexible resource instead of multiple product-dedicated resources.

Even though some finance literature links real flexibility and agency conflict (e.g., Myers 1977, Leland 1998, MacKay 2003), it focuses on a very different type of flexibility. In all of these studies, real flexibility is interpreted as the option to shift investment or production toward risky ventures, which exacerbates the shareholder–debtholder agency conflict. “Anticipating risk shifting and asset substitution, lenders respond to real flexibility with tight credit terms, causing flexible firms to demand less debt” (MacKay 2003, p. 1135).

In our context as well as in most of the operations management literature (e.g., Fine and Freund 1990, Van Mieghem 1998, Chod and Rudi 2005), flexibility is the ability to adapt after uncertainty resolution. The option to allocate flexible capacity to either product is exercised after uncertainty is resolved, and thus, cannot be used to increase risk. On the contrary, resource flexibility mitigates the risk of costly default as well as the agency cost associated with debt and therefore increases the optimal amount of debt financing.

Our model of capital structure and capacity choice is similar to Dotan and Ravid (1985) with several distinctions. First, they consider a firm facing stochastic price as opposed to stochastic demand. Second, they consider a single-product single-resource firm and thus the issue of resource flexibility does not arise. Finally, they assume a convex production cost and a fixed bankruptcy cost. They show that when debt is taken as given, the optimal capacity is decreasing in debt. This is consistent with our finding that a levered firm chooses less total capacity than an unlevered firm. They also show that the coordination of operational and financial decisions increases the optimal debt, which is consistent with our findings.

2. Unlevered Firm

In the following, \mathbb{E} denotes expectation with respect to the risk-neutral probability measure,⁵ the risk-free interest rate is normalized to zero, a prime denotes transpose, bold font denotes vectors, and the terms “increasing” and “decreasing” are used in a weak sense. We start by characterizing the optimal capacity investment in the absence of tax and the transaction cost of issuing equity, in which case a firm does not issue any debt.

We consider a two-product firm that invests in the capacity of two product-dedicated resources and one product-flexible resource, labeled 1, 2, and 3, respectively. In modeling the capacity investment problem, we generally follow the seminal work of Van Mieghem (1998), although we assume all cost, revenue, and demand distribution parameters to be equal for the two products. (We discuss the case of asymmetric profit margins in §6.) While product demand is uncertain, the firm chooses the vector of capacity levels $\mathbf{K} = (K_1, K_2, K_3)'$. We assume a constant marginal capacity investment cost and denote the vector of unit capacity costs as $c = (c_N, c_N, c_F)'$, where subscripts N and F refer to nonflexible (product-dedicated) and flexible resources, respectively, and $c_F \in [c_N, 2c_N]$.

Once product demand is realized, the firm chooses the vector of production quantities $\mathbf{x} = (x_{11}, x_{22}, x_{13}, x_{23})'$, where x_{ij} is the output of product i produced

⁵ We assume that all cash flows can be replicated by a tradable portfolio so that they can be valued using an equivalent risk-neutral probability measure (see Harrison and Kreps 1979).

using resource j . The marginal cost of production is also assumed to be constant. After production is complete, the output is sold at a predetermined price, the firm is liquidated, and capacity has no residual value. Let p be the unit contribution margin, i.e., the output price net of the unit production cost.

The output vector is constrained by existing capacity as well as by realized demand. We assume the demand vector $\mathbf{D} = (D_1, D_2)$ to be drawn from an arbitrary distribution with a continuous density f over \mathbb{R}_+^2 , with mean vector (μ, μ) and covariance matrix $\begin{pmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{pmatrix}$. Given the optimal output decision, the firm's operating profit, i.e., the sales revenue net of the production cost, equals

$$\begin{aligned} \pi(\mathbf{D}, \mathbf{K}) = \max_{\mathbf{x} \in \mathbb{R}_+^3} & p(x_{11} + x_{22} + x_{13} + x_{23}), \\ \text{subject to} & x_{ii} + x_{i3} \leq D_i, \quad i = 1, 2, \\ & x_{ii} \leq K_i, \quad i = 1, 2, \quad \text{and} \\ & x_{13} + x_{23} \leq K_3. \end{aligned} \quad (1)$$

It is useful to partition the state space of the demand vector, \mathbb{R}_+^2 , into four events corresponding to four possible solutions to (1). These events, which we formally define in the proof of Proposition 1, are illustrated in Figure 1(a). If $\mathbf{D} \in \Omega_0$, both demands are relatively low and can be fully satisfied with existing capacity. If $\mathbf{D} \in \Omega_i$, $i = 1, 2$, demand for product i is so high that it cannot be fully satisfied even when the entire flexible capacity is allocated to this product, whereas demand for product $3 - i$ is relatively low and can be fully satisfied with the corresponding product-dedicated capacity. Finally, if $\mathbf{D} \in \Omega_3$, both demands are so high that some demand is lost even if all capacity is fully utilized.

Before demands are known, the firm chooses a capacity vector that maximizes firm value, denoted as V , which is equal to the expected operating profit

less the capacity investment cost, i.e., $V = \mathbb{E}[\pi(\mathbf{D}, \mathbf{K})] - \mathbf{c}'\mathbf{K}$. In the next proposition, we characterize the optimal capacity vector. The symmetry of product parameters together with the uniqueness of the optimal capacity vector implies that it is always optimal to set both nonflexible capacity levels as equal. We can therefore simplify the notation by letting $K_N \equiv K_1 = K_2$ and $K_F \equiv K_3$. Furthermore, we let $\Omega_{12} \equiv \Omega_1 \cup \Omega_2$, $\Omega_{123} \equiv \Omega_1 \cup \Omega_2 \cup \Omega_3$, etc.

PROPOSITION 1. *The optimal capacity investment of an unlevered firm, \mathbf{K}^{UL} , is characterized by the following necessary and sufficient conditions:*

(i) If $c_F = c_N$, then $K_N^{\text{UL}} = 0$, $K_F^{\text{UL}} > 0$, and K_F^{UL} satisfies $p \Pr(\Omega_3) = c_F$.

(ii) If $c_F \geq 2c_N - p \Pr(D_1 > \bar{K}_N, D_2 > \bar{K}_N)$, where \bar{K}_N is the unique solution of $p \Pr(D_1 > K_N) = c_N$, then $K_F^{\text{UL}} = 0$ and $K_N^{\text{UL}} = \bar{K}_N > 0$.

(iii) Otherwise, $K_N^{\text{UL}} > 0$, $K_F^{\text{UL}} > 0$, and they jointly satisfy

$$p \Pr(\Omega_{13}) = c_N \quad \text{and} \quad p \Pr(\Omega_{123}) = c_F. \quad (2)$$

PROOF. All proofs are in the appendix.

The first two cases correspond to the boundary solutions in which the firm chooses only flexible or only nonflexible capacity, respectively. The third case corresponds to the interior solution in which the firm chooses both types of capacity. The optimality conditions (2) set the expected marginal operating profit of each type of capacity equal to its marginal cost. The marginal operating profit of either type of capacity is the unit contribution margin, p , if this type of capacity is fully utilized, and zero otherwise. Capacity dedicated to product 1 (2) is fully utilized if demand for product 1 (2) is relatively high, namely, if $\mathbf{D} \in \Omega_{13}$ (Ω_{23}). Flexible capacity is fully utilized if demand for either product is relatively high, namely, if $\mathbf{D} \in \Omega_{123}$. Next we consider the optimal capacity investment of a levered firm together with the firm's choice of optimal capital structure.

Figure 1 (a) Partitioning of the Demand State Space as Defined in (22); (b) Partitioning of the Demand State Space as Defined in (24)

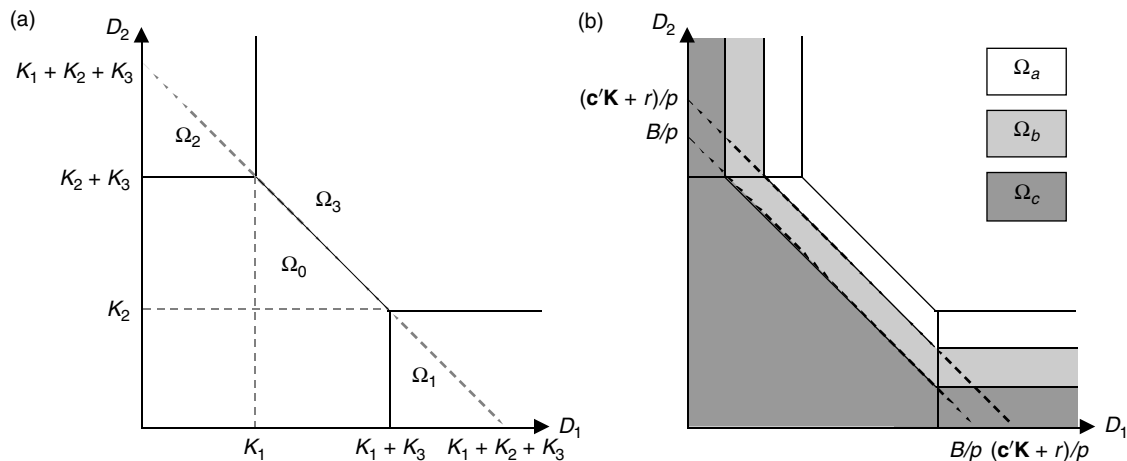
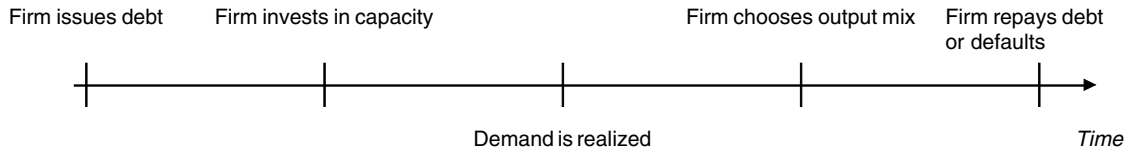


Figure 2 Sequence of Events



3. Levered Firm

In this section, we assume that prior to investing in capacity, the firm issues the optimal amount of debt. We refer to this sequence of events, which is illustrated in Figure 2, as the “base case.” We further assume that the firm is fully controlled by shareholders who maximize shareholder value, i.e., the expected terminal value of equity minus the shareholders’ initial equity investment.

Let B be the face value of the debt and r be the interest, so that the issue price of the debt is $B - r$. In other words, the firm borrows $B - r$ and promises to repay B . We consider two reasons why the firm issues debt: the tax shield provided by the interest payment and the transaction cost of issuing equity. To capture the tax benefit of debt, we assume that the firm’s profit is subject to tax with a flat rate t , and the interest payment is tax deductible. Because we consider a single-period model, we assume that no tax carrybacks or carryforwards are allowed, as is standard in the literature (e.g., DeAngelo and Masulis 1980, Dotan and Ravid 1985). Once production is complete and the firm generates operating profit π given by (1), one of the following three events occurs:

- Ω_a : If $\pi > c'K + r$, the firm earns a positive profit that is subject to tax, and is able to repay the full face value of the debt. Thus, the terminal value of equity is $\pi - B - t(\pi - c'K - r)$.
- Ω_b : If $B \leq \pi \leq c'K + r$, the firm incurs a loss and, thus, is not subject to tax, but it is able to repay the debt in full. Therefore, the terminal value of equity is $\pi - B$.
- Ω_c : If $\pi < B$, the firm’s realized operating profit is less than the face value of the debt, and the firm defaults. As discussed in the introduction, we assume that default is costly and that the higher the shortfall between the firm’s realized operating profit, π , and the debt obligation, B , the higher the cost of default. For analytical tractability, we further assume that the cost of default is proportional to the shortfall; i.e., it equals $b(B - \pi)$, where b parameterizes the cost of financial distress. In this case, equity becomes worthless, whereas debtholders receive the firm’s operating profit minus the cost of default, $\pi - b(B - \pi)$.

We denote the domains of the demand state space corresponding to these three events as Ω_a , Ω_b , and Ω_c ,

respectively. Superimposing these three events on the four events corresponding to different capacity allocations, $\Omega_0, \dots, \Omega_3$, so that, e.g., $\Omega_{0a} \equiv \Omega_0 \cap \Omega_a$, we can distinguish 10 events that are defined in the proof of Proposition 2 and illustrated in Figure 1(b). In this figure, the darkly shaded area indicates default outcomes, whereas the lightly shaded area corresponds to situations in which the firm loses money but does not default. Note that when the entire capacity is fully utilized, the firm has to make profit, i.e., $\Omega_3 \subset \Omega_a$, otherwise no capacity investment would have been made.

In addition to the tax benefit, another important advantage of debt financing is that raising equity involves a considerably higher transaction cost, including the usual 7% underwriter spread (e.g., Chen and Ritter 2000, Hansen 2001). We reflect this empirical regularity by assuming that there is a transaction cost of raising equity, which is a fraction ϕ of the equity issue, and we normalize the transaction cost of issuing debt to zero. Given the capacity investment cost $c'K$ and the borrowing $B - r$, the firm needs to raise $c'K - (B - r)$ from shareholders.⁶ Because fraction ϕ of shareholder investment is paid to the underwriting bank, the total amount that shareholders must invest in the firm to finance the capacity investment is

$$\frac{c'K - (B - r)}{1 - \phi}.$$

This means that the shareholder value, i.e., the expected terminal value of equity net of the money invested in the firm by shareholders, can be written as

$$V = \Pr(\Omega_a) \mathbb{E}(\pi - B - t(\pi - c'K - r) | \Omega_a) + \Pr(\Omega_b) \mathbb{E}(\pi - B | \Omega_b) - \frac{c'K - (B - r)}{1 - \phi}. \quad (3)$$

We assume that debt is fairly priced; i.e., its issue price, $B - r$, is equal to its expected value. Recall that debtholders are repaid the full face value, B , if $D \in \Omega_{ab}$, and they receive the operating profit minus

⁶ We assume that the firm is not allowed to pay dividends before repaying the debt. In practice, this is typically ensured through bond covenants requiring that dividends be paid only from earnings generated subsequent to borrowing or earnings above a given amount (see, e.g., Smith and Warner 1979). This means that the equity raised has to be nonnegative; i.e., we require $c'K \geq B - r$.

the cost of default, $\pi - b(B - \pi)$, if $\mathbf{D} \in \Omega_c$. Therefore, fair pricing requires that

$$B - r = \Pr(\Omega_{ab})B + \Pr(\Omega_c)\mathbb{E}(\pi - b(B - \pi) | \Omega_c) \\ \iff r = (1 + b)\Pr(\Omega_c)\mathbb{E}(B - \pi | \Omega_c). \quad (4)$$

The firm's choice of optimal debt and capacity is a two-stage decision problem. In the second stage, the firm chooses a capacity investment that maximizes shareholder value (3) for a given debt contract, i.e.,

$$\mathbf{K}^*(B, r) = \arg \max_{\substack{\mathbf{K} \geq 0 \\ c'\mathbf{K} \geq B - r}} V(\mathbf{K} | B, r). \quad (5)$$

In the first stage, the firm chooses a debt contract that maximizes shareholder value (3) subject to the fair-pricing constraint (4) and taking into account the optimal investment strategy (5). Formally,

$$(B^*, r^*) = \arg \max_{B, r \geq 0} V(\mathbf{K}^*(B, r), B, r) \quad \text{subject to (4)}. \quad (6)$$

When the firm chooses capacity, one of two possible scenarios takes place. If the total capacity cost exceeds the borrowing, the firm has to raise additional capital by issuing equity. Alternatively, the firm invests in capacity only the amount borrowed. We refer to these two scenarios as mixed debt-equity financing and full debt financing, respectively. In this and the next two sections, we focus on the mixed debt-equity financing, and we discuss the boundary case of full debt financing in §6. We also assume that $(1 - t)p > c_N(1/(1 - \phi) - t)$ to exclude the case when $(\mathbf{K}^*, B^*) = \mathbf{0}$.

The next proposition characterizes the optimal capacity investment (5). Unfortunately, it is difficult to prove that the objective is unimodal in \mathbf{K} even though our numerical experiments indicate that it is. Therefore, we state the optimality conditions only as necessary.

PROPOSITION 2. Suppose the optimal capacity investment $\mathbf{K}^*(B, r)$ relies on both debt and equity financing, i.e., $c'\mathbf{K}^*(B, r) > B - r > 0$.

(i) If $c_F = c_N$, then $K_N^*(B, r) = 0$, $K_F^*(B, r) > 0$, and $K_F^*(B, r)$ satisfies

$$(1 - t)p\Pr(\Omega_3) = c_F\left(\frac{1}{1 - \phi} - t\Pr(\Omega_a)\right). \quad (7)$$

If $c_F > c_N$, then $\mathbf{K}^*(B, r)$ has one of the following two distinct forms:

(ii) $K_F^*(B, r) = 0$, $K_N^*(B, r) > 0$, and $K_N^*(B, r)$ satisfies

$$(1 - t)p\Pr(\Omega_{13a}) + p\Pr(\Omega_{1b}) = c_N\left(\frac{1}{1 - \phi} - t\Pr(\Omega_a)\right), \quad (8)$$

(iii) $K_F^*(B, r) > 0$, $K_N^*(B, r) > 0$, and they jointly satisfy

$$(1 - t)p\Pr(\Omega_{123a}) + p\Pr(\Omega_{12b}) = c_F\left(\frac{1}{1 - \phi} - t\Pr(\Omega_a)\right), \quad (9)$$

and

$$(1 - t)p\Pr(\Omega_{13a}) + p\Pr(\Omega_{1b}) = c_N\left(\frac{1}{1 - \phi} - t\Pr(\Omega_a)\right). \quad (10)$$

Cases (i) and (ii) correspond to the boundary solutions in which the firm chooses only flexible capacity and only nonflexible capacity, respectively. Case (iii) describes the interior solution in which it is optimal to choose a positive amount of each capacity. Optimality conditions (9) and (10) ensure that the expected marginal operating profit of each type of capacity equals the marginal capacity cost, taking into account tax and the cost of issuing equity. Note that in assessing the marginal operating profit of capacity (the left-hand side of (9) and (10)), the firm takes into account only nondefault outcomes, Ω_a and Ω_b . This is because, due to limited liability, the loss realized in default states, Ω_c , does not affect the value of equity. It only affects the value of debt, which the firm does not take into account when choosing capacity.

Next we characterize the optimal debt level in case the optimal \mathbf{K}^* is interior, leaving the discussion of full flexibility and no flexibility to §5.

PROPOSITION 3. When the optimal capacity investment \mathbf{K}^* relies on both debt and equity financing, i.e., $c'\mathbf{K}^* > B^* - r^* > 0$, and includes both types of capacity, i.e., $K_F^* > 0$ and $K_N^* > 0$, the optimal debt level B^* is characterized by the following first-order condition:

$$\underbrace{t(1 + b)\Pr(\Omega_a)\Pr(\Omega_c)}_{\text{tax benefit}} + \underbrace{\phi\Pr(\Omega_{ab})\frac{1}{1 - \phi}}_{\text{cost of issuing equity}} \\ = \underbrace{b\Pr(\Omega_c)\frac{1}{1 - \phi}}_{\text{cost of default}} \\ - \underbrace{\Pr(\Omega_{12c})(1 + b)p\frac{d(K_N^* + K_F^*)}{dB}\left(\frac{1}{1 - \phi} - t\Pr(\Omega_a)\right)}_{\text{agency cost}}. \quad (11)$$

Optimality condition (11) equates the marginal benefit and the marginal cost of debt. The marginal benefit of debt (the left-hand side of (11)) involves two terms, which correspond to the tax benefit of debt and to the saving of the transaction cost of issuing equity, respectively. The marginal cost of debt (the right-hand side of (11)) also has two components. First, issuing more debt increases the risk of costly default. Although the cost of default is ex post borne by debtholders, it is priced into the cost of borrowing, and therefore, it reduces the optimal amount of debt.

Second, issuing debt creates an agency conflict between shareholders and debtholders, which reduces firm value by distorting the firm's incentives in the capacity investment stage. A levered firm chooses

capacity that maximizes shareholder value rather than total firm value (shareholder value plus the value of debt). The fact that the firm's capacity choice ignores the value of debt is priced into the cost of borrowing and, as a result, reduces the optimal amount of debt. In the next section, we examine the agency conflict and its effect on the optimal capacity mix and capital structure in more detail.

4. Agency Cost of Debt

To evaluate the impact of agency conflict on the firm's capacity investment and capital structure, we consider the "first-best" scenario in which a firm chooses capacity and debt that *jointly* maximize shareholder value (3). With fairly priced debt, this is equivalent to maximizing total firm value. Formally,

$$(\mathbf{K}^{\text{FB}}, B^{\text{FB}}, r^{\text{FB}}) = \arg \max_{\substack{\mathbf{K}, B, r \geq 0 \\ c'\mathbf{K} \geq B - r}} V(\mathbf{K}, B, r) \quad \text{subject to (4).} \quad (12)$$

By definition, $V(\mathbf{K}^{\text{FB}}, B^{\text{FB}}, r^{\text{FB}}) \geq V(\mathbf{K}^*, B^*, r^*)$, where the difference is the *cost of agency*. What makes the first-best scenario difficult to implement is that once the first-best debt contract is written, the firm has an incentive to deviate from the first-best capacity choice because it no longer maximizes shareholder value:

$$\mathbf{K}^{\text{FB}} \neq \arg \max_{\substack{\mathbf{K} \geq 0 \\ c'\mathbf{K} \geq B^{\text{FB}} - r^{\text{FB}}}} V(\mathbf{K} | B^{\text{FB}}, r^{\text{FB}}).$$

Because debtholders can anticipate the firm's behavior, they will not agree on the first-best debt contract in the first place, unless the firm can credibly commit to the first-best capacity choice through bond covenants, shared control with debtholders, etc. The first-best scenario is nevertheless a useful benchmark that enables us to analyze the impact of agency on debt and capacity. In the next proposition, we characterize the first-best capacity investment and debt, focusing again on the case of mixed debt-equity financing and leaving the discussion of full debt financing to §6.

PROPOSITION 4. Suppose the first-best capacity investment relies on both debt and equity financing, i.e., $c'\mathbf{K}^{\text{FB}} > B^{\text{FB}} - r^{\text{FB}} > 0$.

(i) If $c_F = c_N$, then $K_N^{\text{FB}} = 0$, $K_F^{\text{FB}} > 0$, and K_F^{FB} satisfies

$$(1-t)p\Pr(\Omega_3) = c_F \left(\frac{1}{1-\phi} - t\Pr(\Omega_a) \right). \quad (13)$$

If $c_F > c_N$, then \mathbf{K}^{FB} has one of the two distinct forms:

(ii) $K_F^{\text{FB}} = 0$, $K_N^{\text{FB}} > 0$, and K_N^{FB} satisfies

$$(1-t)p\Pr(\Omega_{13a}) + p\Pr(\Omega_{1b}) = (c_N - (1+b)p\Pr(\Omega_{1c})) \left(\frac{1}{1-\phi} - t\Pr(\Omega_a) \right), \quad (14)$$

(iii) $K_F^{\text{FB}} > 0$, $K_N^{\text{FB}} > 0$, and they jointly satisfy

$$(1-t)p\Pr(\Omega_{13a}) + p\Pr(\Omega_{1b}) = (c_N - (1+b)p\Pr(\Omega_{1c})) \left(\frac{1}{1-\phi} - t\Pr(\Omega_a) \right), \quad (15)$$

and

$$(1-t)p\Pr(\Omega_{123a}) + p\Pr(\Omega_{12b}) = (c_F - (1+b)p\Pr(\Omega_{12c})) \left(\frac{1}{1-\phi} - t\Pr(\Omega_a) \right). \quad (16)$$

In each case, the first-best debt level B^{FB} is characterized by the following first-order condition:

$$\underbrace{t(1+b)\Pr(\Omega_a)\Pr(\Omega_c)}_{\text{tax benefit}} + \underbrace{\phi\Pr(\Omega_{ab})\frac{1}{1-\phi}}_{\text{cost of issuing equity}} = \underbrace{b\Pr(\Omega_c)\frac{1}{1-\phi}}_{\text{cost of default}}. \quad (17)$$

Compared to the base case, the optimality conditions for first-best capacity levels include additional terms that capture the impact of capacity on the loss realized in default states Ω_{1c} and Ω_{2c} . In the first-best scenario, the firm takes this loss into consideration because it affects the value of debt. The firm's "consideration for debtholders" pays off because it reduces the cost of borrowing. The optimality condition for the debt level is the same as that in the base case except that no agency cost is associated with debt.

The next proposition compares the optimal capacity mix and debt in the base-case, first-best, and all-equity scenarios⁷ assuming that the capacity investment problem has an interior solution and $t = 0$. (The cases of full flexibility and no flexibility are discussed in §5, the case of full debt financing is discussed in §6, and the case of $t > 0$ is examined numerically in §5.)

PROPOSITION 5. Suppose that $t = 0$, and the optimal solution satisfies $K_F > 0$, $K_N > 0$ and $c'\mathbf{K} > B - r$ in each scenario. The following is true:

(i) In the first-best scenario, a firm chooses less nonflexible, more flexible, and more total capacity than an unlevered firm.

(ii) In the base-case scenario, a firm chooses more nonflexible, less flexible, and less total capacity than an unlevered firm.

(iii) In the base-case scenario, a firm chooses less debt than in the first-best scenario.

Formally, we have

$$K_N^* \geq K_N^{\text{UL}} \geq K_N^{\text{FB}}, \quad K_F^* \leq K_F^{\text{UL}} \leq K_F^{\text{FB}}, \\ K_F^* + 2K_N^* \leq K_F^{\text{UL}} + 2K_N^{\text{UL}} \leq K_F^{\text{FB}} + 2K_N^{\text{FB}}, \quad \text{and} \quad B^* \leq B^{\text{FB}}.$$

⁷ To make this a fair comparison, we assume $\phi > 0$ even in the all-equity case.

According to part (i), a firm following the first-best strategy should choose a more flexible capacity mix and install more total capacity than an unlevered firm. This is because for any given debt, flexibility and extra capacity mitigate the risk of costly default that an unlevered firm does not face. According to part (ii), a levered firm that maximizes shareholder value has exactly the opposite incentives. It chooses less flexibility, less capacity, and, therefore, a lower investment than an unlevered firm.

The fact that leverage reduces flexibility is a manifestation of the asset substitution problem first noted by Jensen and Meckling (1976). They argue that leverage induces risk-seeking behavior on the part of shareholders who are protected from the downside by limited liability. In our context, nonflexible capacity is more risky than flexible capacity. Whereas a unit of nonflexible capacity is less expensive and thus provides a higher potential payoff than a unit of flexible capacity, this payoff is more uncertain because nonflexible capacity is less likely to be utilized. As a result, leverage induces the firm to replace the less risky flexible capacity with the more risky nonflexible capacity.

The fact that leverage reduces the total capacity investment is in the spirit of Myers (1977). Let V_E and V_D be the expected terminal value of equity and debt, respectively. Myers (1977) argues that when a levered firm makes an incremental discretionary investment I that requires a fresh commitment of equity capital, to maximize the total firm value, $V_E + V_D$, the firm should invest as long as $d(V_E + V_D)/dI > 1$. However, shareholders who ignore the effect of investment on the value of debt invest as long as $dV_E/dI > 1$. Thus, they invest less than what would maximize the total firm value. As Myers notes, underinvestment occurs only if $dV_D/dI > 0$. Let \tilde{V}_D be the realized value of debt so that

$$\begin{aligned} V_D &= \mathbb{E} \tilde{V}_D = \Pr(\text{default}) \mathbb{E}(\tilde{V}_D | \text{default}) \\ &\quad + \Pr(\text{nondefault}) \mathbb{E}(\tilde{V}_D | \text{nondefault}), \quad \text{and} \\ dV_D/dI &= \Pr(\text{default}) \mathbb{E}(d\tilde{V}_D/dI | \text{default}) \\ &\quad + \Pr(\text{nondefault}) \mathbb{E}(d\tilde{V}_D/dI | \text{nondefault}). \end{aligned}$$

When the firm does not default, debtholders receive exactly the face value of debt, i.e., $\tilde{V}_D = B$ and $d\tilde{V}_D/dI = 0$. Therefore,

$$dV_D/dI = \Pr(\text{default}) \mathbb{E}(d\tilde{V}_D/dI | \text{default}).$$

Thus, underinvestment arises only when investment affects the value of debt in default states.

Interestingly, this is not the case when a newsvendor-type firm invests in a single type of capacity with zero liquidation value. Whenever a newsvendor

with a single type of capacity defaults, its capacity is underutilized, and therefore, additional capacity has no value for debtholders; i.e., $d\tilde{V}_D/dI = 0$. Therefore, $dV_D/dI = 0$, and no agency conflict or underinvestment arises.

In the newsvendor setting, underinvestment occurs only when there are multiple types of capacity. In that case, there is a positive probability that some capacity will be fully utilized in default states, and thus, a larger capacity investment would benefit debtholders. This is exactly the case we study. A firm may default due to low demand for one product while capacity dedicated to the other product and flexible capacity are fully utilized. A larger capacity investment would increase the value of debt in these default states, and therefore, $dV_D/dI > 0$. Because shareholders ignore the effect of capacity investment on the value of debt, the firm underinvests.

Finally, part (iii) of the proposition stipulates that the agency conflict reduces the amount of debt that a firm issues. This is because the distortion of the firm's investment strategy toward less flexibility and less capacity increases the cost of borrowing. In the next section, we examine how optimal debt level depends on the relative costs of flexible and nonflexible capacity.

5. Flexibility and Optimal Debt Level

In evaluating the impact of flexibility on optimal capital structure, we cannot measure flexibility directly by the proportion of flexible capacity in the firm's optimal capacity portfolio because it is determined endogenously. However, we can take advantage of the fact that this proportion decreases in the cost of flexible capacity c_F relative to the cost of nonflexible capacity c_N . At a sufficiently high c_F , the firm invests exclusively in nonflexible capacity. As c_F decreases, the optimal proportion of flexible capacity increases, and it reaches 100% when $c_F = c_N$. Thus, to assess the effect of flexibility, we can take c_N as given and examine the effect of decreasing c_F . We first characterize the case when c_F is so high that the firm chooses only nonflexible capacity. After that, we examine what happens to agency cost, cost of borrowing, and optimal debt when c_F drops all the way to c_N and the firm switches to full flexibility. We then show, using numerical analysis, that these effects are monotone in c_F . For now, we continue to assume that mixed debt-equity financing is optimal, i.e., $c'K^* > B^* - r^*$.

PROPOSITION 6. *If $t = 0$ and $c_F = 2c_N$ so that it is optimal to invest only in nonflexible capacity, the following is true:*

(i) *In the first-best scenario, a firm chooses more capacity than an unlevered firm.*

- (ii) In the base-case scenario, a firm chooses less capacity than an unlevered firm.
(iii) In the base-case scenario, a firm chooses less debt than in the first-best scenario.
Formally, we have

$$K_N^{\text{FB}} \geq K_N^{\text{UL}} \geq K_N^* \quad \text{and} \quad B^{\text{FB}} \geq B^*.$$

When investing in two nonflexible and thus equally risky resources, the firm cannot engage in asset substitution. However, agency cost still exists due to underinvestment. The first-best strategy of a levered firm is to build more capacity relative to an unlevered firm to mitigate the risk of costly default. However, the incentive of a levered firm in the base case is exactly the opposite. Because the firm does not care about the impact of capacity on the value of debt, it underinvests. Interestingly, in the other boundary case when it is optimal to choose only flexible capacity, the agency cost disappears.

PROPOSITION 7. *If $c_F = c_N$ so that it is optimal to invest only in flexible capacity, the firm chooses the first-best capacity investment and debt, and therefore, there is no agency cost associated with leverage. Formally, we have*

$$K^* = K^{\text{FB}}, \quad B^* = B^{\text{FB}}, \quad \text{and} \\ V(K^*, B^*, r^*) = V(K^{\text{FB}}, B^{\text{FB}}, r^{\text{FB}}).$$

The agency cost of debt stems from asset substitution and underinvestment. When there is no cost advantage to using nonflexible capacity, the firm has no incentive to engage in asset substitution. Interestingly, investing in a single type of capacity eliminates the underinvestment problem as well. As discussed in §4, the underinvestment problem stems from the fact that shareholders ignore the impact of capacity investment on the value of debt. With a single

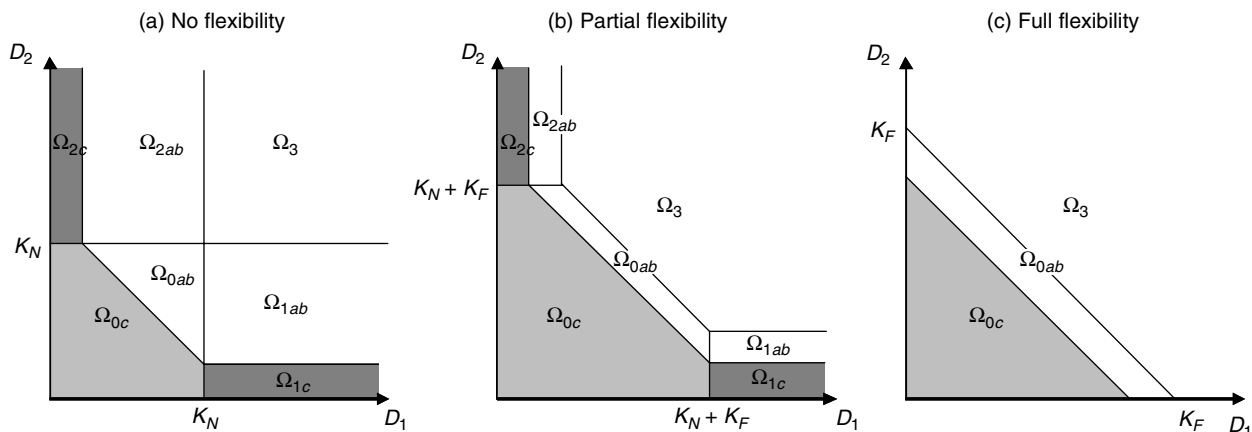
type of capacity, the investment level has no impact on the value of debt. In nondefault states, debtholders are repaid exactly the face value of the debt. In default states, the capacity is underutilized, and therefore, a higher capacity investment would not benefit debtholders either.

This is illustrated in Figure 3. In the case of no flexibility and partial flexibility, there is a chance that some capacity will be fully utilized upon default (the darkly shaded areas), so more capacity would benefit debtholders. With full flexibility, the capacity constraint is never binding upon default, and thus, additional capacity would not increase the value of debt. As a result, capacity that maximizes shareholder value also maximizes total firm value.

Note that the result is not driven by the assumption of zero cost difference between flexible and nonflexible capacity. The agency cost disappears as soon as a firm chooses a single type of capacity. It is true that in our model (in which both flexible and nonflexible capacity are available, there is no fixed cost of investing in specific capacity, and capacity is infinitesimally divisible), the only situation in which a firm chooses a single type of capacity is when $c_F = c_N$. In practice, a firm can optimally invest in 100% flexible capacity in many circumstances even when flexibility comes at a cost, e.g., when there is a fixed cost associated with each type of capacity, when capacity can be added only in discrete increments, when the choice is between fully flexible technology or fully dedicated technology, or when there is no such thing as product-dedicated capacity. The result of Proposition 7 holds in all these cases, i.e., agency conflict disappears as long as it is optimal for the firm to include only one type of capacity in its portfolio.

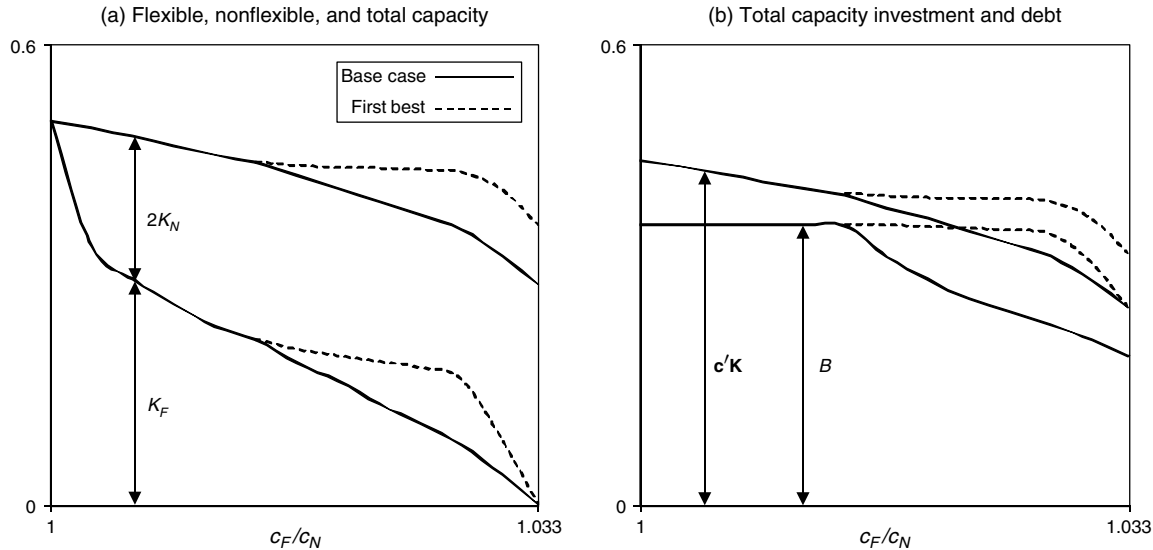
Next we characterize the effect of full flexibility on the cost of borrowing and optimal debt when $t = 0$.

Figure 3 Partitioning of the Demand State Space Defined in (24) in the Case of (a) No Flexibility, (b) Partial Flexibility, and (c) Full Flexibility



Note. The darkly shaded regions, Ω_{1c} and Ω_{2c} , correspond to default states in which some capacity is fully utilized.

Figure 4 Optimal Capacity Levels, Capacity Investment, and Debt as a Function of $c_F/c_N \in (1, 1.033)$ for $\mu = 1$, $\sigma = 1$, $\rho = 0$, $p = 1$, $c_N = 0.9$, $\phi = 0.05$, $t = 0.3$, and $b = 10$



PROPOSITION 8. If $t = 0$, full resource flexibility leads to the lowest interest (4) for any $(B, K^*(B))$, and to the highest optimal debt level B^* .

When lenders anticipate that the firm will invest in entirely flexible capacity, they require the lowest interest for any given loan amount. This is because full flexibility prevents the firm's "misbehavior" in terms of asset substitution and underinvestment, and mitigates the risk of costly default. Because full flexibility minimizes the cost of borrowing, it also maximizes the amount of debt that a firm issues.

Our numerical experiments indicate that the effect of flexibility on optimal debt is monotone and that it continues to hold in the presence of tax, as illustrated in Figure 4. The figure shows the optimal levels of flexible and total nonflexible capacity, K_F and $2K_N$, total capacity investment, $c'K$, and debt, B , as a function of the relative cost of flexible capacity, c_F/c_N , in the base-case and first-best scenarios.

As c_F/c_N increases, flexible capacity becomes relatively more expensive, and the firm gradually replaces flexible capacity with nonflexible capacity. At the same time, the optimal debt decreases, consistent with Proposition 8. Interestingly, a moderate level of flexibility is enough to maximize the debt level. At low levels of flexibility, the agency conflict reduces the optimal level of flexible capacity ($K_F^* \leq K_F^{FB}$) as well as the total capacity investment ($c'K^* \leq c'K^{FB}$) due to asset substitution and underinvestment, as predicted by Proposition 5. In addition, the agency conflict reduces the optimal debt ($B^* \leq B^{FB}$), also consistent with Proposition 5. At high levels of flexibility, the agency conflict disappears and the first-best and base-case scenarios are equivalent, consistent with

Proposition 7. Again, a relatively moderate level of flexibility is enough to eliminate agency cost.

5.1. Numerical Analysis

We examined the effect of the relative cost of flexible capacity, c_F/c_N , on optimal capacity mix, debt, and leverage in an extensive numerical study. Because we did not prove concavity of the objective functions, we used the Global Search algorithm in Matlab, which is designed to search for global solutions to problems that contain multiple local maxima or minima without relying on the first-order conditions. To find each point on the graph, we simulated 20,000 demand scenarios drawn from a bivariate lognormal distribution. We used the following baseline parameter values: $\mu = 1$, $\sigma = 1$, $\rho = 0$, $p = 1$, $c_N = 0.9$, $\phi = 0.05$, $t = 0.3$, and $b = 10$, with c_F varying from c_N up to the point where the optimal level of flexible capacity drops to zero. To ensure robustness with respect to profit margin, demand variability and correlation, the cost of issuing equity, tax rate, and the cost of financial distress, we varied the model parameters as follows: $c_N \in (0.7, 0.9)$, $\sigma \in (0.6, 1.8)$, $\rho \in (-0.4, 0.8)$, $\phi \in (0.01, 0.1)$, $t \in (0, 0.3)$, and $b \in (0, 20)$.

These model parameters are motivated by the U.S. automotive industry. For example, the average profit margin of GM (Ford) over the last five years has been 4.73% (3.05%).⁸ Assuming that the profit margin is 5% of revenue and capacity cost is half of total cost, capacity cost = $47.5/(100 - 47.5) = 90\%$ of revenue

⁸ http://ycharts.com/companies/GM/profit_margin and http://ycharts.com/companies/F/profit_margin (accessed March 2013).

minus variable production cost, consistent with our baseline parameter choice $c_N = 0.9p$.⁹

The relatively high coefficient of variation (CV) of product demand ($\sigma/\mu = 1$) is chosen to ensure a realistic value of sales variability for which empirical data are available. With our baseline parameters, the CV of sales varies between 0.029 and 0.039 (as a function of c_F), averaging 0.032.¹⁰ Therefore, it has the same order of magnitude as the CV of sales of major U.S. automakers. For example, using the eight years of monthly sales data at 75 GM, Ford, and Daimler Chrysler assembly plants reported by Goyal et al. (2012), we estimated the average CV of annual sales at the plant level to be 0.035. In addition, the CV of detrended annual sales of GM (Ford) over the 10 years preceding the automotive industry crisis (1998–2007), which we calculated using COMPUSTAT data, is 0.048 (0.046).

The cost of issuing equity $\phi = 0.05$ reflects the typical difference between the total direct cost of seasoned equity offerings and straight debt issues, which are 7.1% and 2.2%, respectively, as discussed in §1. The tax rate $t = 0.3$ reflects the U.S. Federal corporate tax rate, which varies between 15% and 35%.

Finally, the bankruptcy cost parameter $b = 10$ ensures that the expected bankruptcy cost relative to the firm value, i.e., $b\mathbb{E}\max(B - \pi, 0)/V$, is consistent with the empirical evidence. In our baseline scenario, this ratio ranges from 1% to 19% (as c_F varies), averaging around 10%. For example, Altman (1984) estimated the sum of direct bankruptcy costs (such as legal and accounting fees) and indirect bankruptcy costs (defined predominately as loss of profits) to range from 12% to 17% of firm value over the three years immediately preceding the bankruptcy announcement. Andrade and Kaplan (1998) estimate the cost of financial distress for a sample of 31 highly leveraged transactions that subsequently became financially distressed to be 10% of firm value. More recently, Bhabra and Yao (2011) estimate indirect bankruptcy costs for a sample of large corporate bankruptcies in the United States between 1997 and 2004 to range from 2% to 14.9% of firm value over the three years immediately preceding the bankruptcy. Together with the direct costs reported in Altman (1984), this brings the total bankruptcy related costs to between 6% and 17.4% of firm value over the corresponding three years. With these parameter values,

⁹ Recall that p is the unit revenue minus the unit variable cost of production.

¹⁰ The discrepancy between the high CV of demand and the low CV of sales is due to the low profit margin. With a 10% profit margin, the optimal newsvendor capacity is fully utilized, i.e., sales equal capacity, with 90% probability irrespective of demand variability. Thus, a relatively high CV of demand translates into a much lower CV of sales.

the leverage ratio, $(B - r)/c'K$, in our baseline scenario ranges from 0.75 to 0.9 depending on flexibility.

Whereas we did not observe any cases where it is optimal to use only equity financing, we found instances where it is optimal to use full debt financing (Figures 5(b) and 5(d)), which we discuss in more detail in §6. Most important, as shown in Figure 5, the optimal debt increases monotonically with flexibility in all cases, confirming the central insight of the paper. Whereas more flexibility leads to more debt, it does not necessarily lead to higher leverage, defined as the issue price of debt over the book value of total assets, i.e., $(B - r)/c'K$. This is because in addition to increasing debt, flexibility affects the total capacity investment. Namely, the risk-pooling effect of flexibility induces the firm to set total capacity closer to expected demand. With our parameter values, total capacity is below expected demand, and therefore, flexibility leads to larger capacity investment. Finally, because flexibility increases debt as well as total capacity investment, its effect on leverage is generally nonmonotone. As shown in Figure 5, the leverage ratio always initially increases and then decreases (or stays flat) as a function of flexibility.

However, we observe a monotone relationship between leverage and several model parameters. As expected, financial leverage increases in the tax rate and the cost of issuing equity, and it decreases in the bankruptcy cost parameter b . Furthermore, financial leverage is negatively related to demand variability and correlation, which increase the risk of costly default and therefore make external financing less attractive.

6. Extensions

In this section, we discuss the boundary solution of full debt financing and the case of asymmetric profit margins.

6.1. Full Debt Financing

In Figure 5, we have seen instances of a 100% leverage ratio. To get a more precise picture of when full debt financing is optimal, we plot the optimal leverage, $(c'K^*)/(B^* - r^*)$, while varying the tax rate $t \in (0, 0.3)$, the cost of issuing equity $\phi \in (0, 0.1)$, and the bankruptcy cost $b \in (0, 2)$ in Figure 6. For example, Figure 6(a) shows that at baseline parameter values $t = 30\%$ and $\phi = 5\%$, the optimal leverage reaches 100% as the bankruptcy cost b falls below 2. Because full debt financing can be optimal for plausible parameter values, it is worthwhile to examine it more closely. We start by characterizing the optimal capacity investment that is fully financed by debt.

PROPOSITION 9. Suppose the optimal capacity investment $K^*(B, r)$ relies exclusively on debt financing, i.e., $c'K^*(B, r) = B - r > 0$.

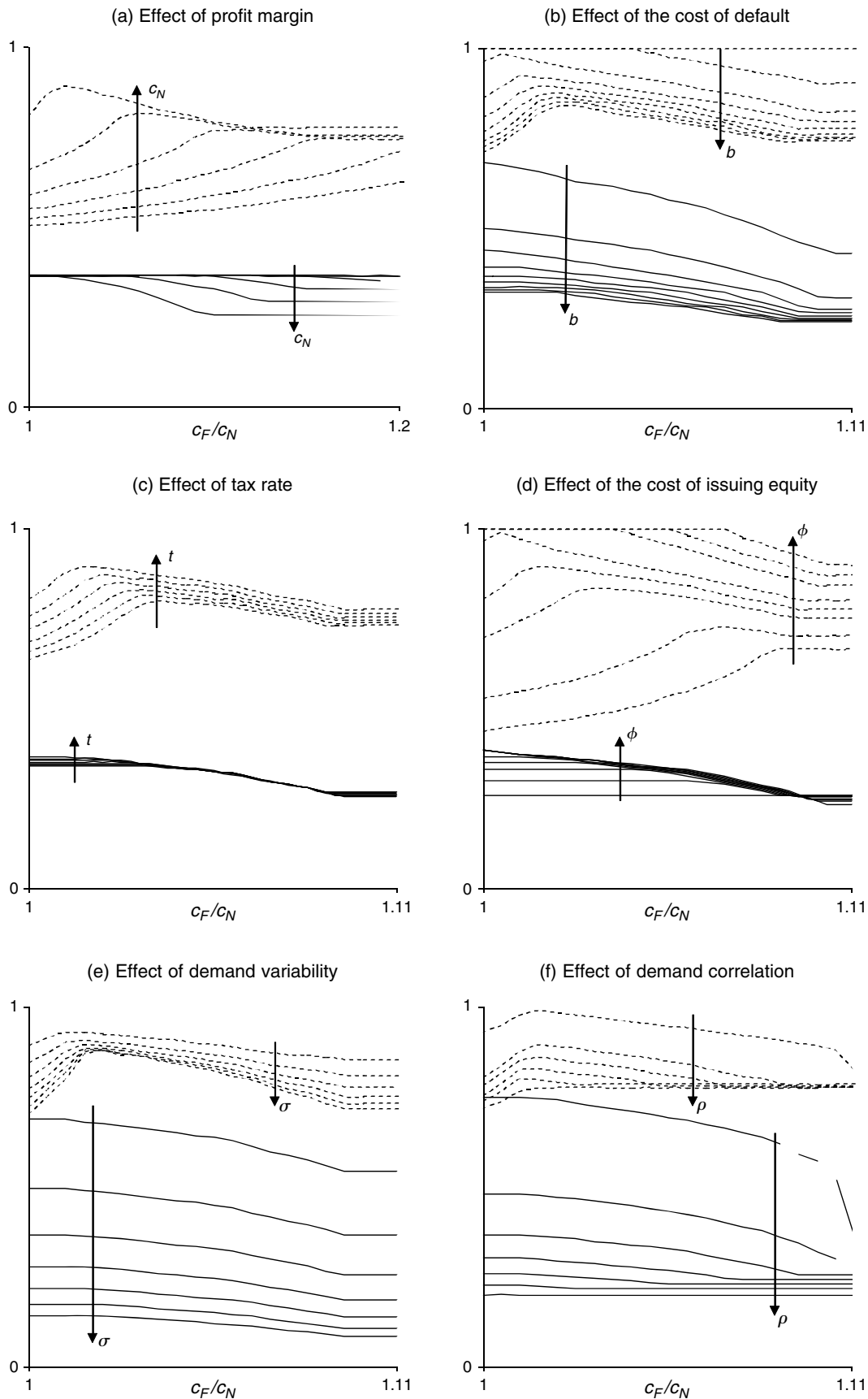
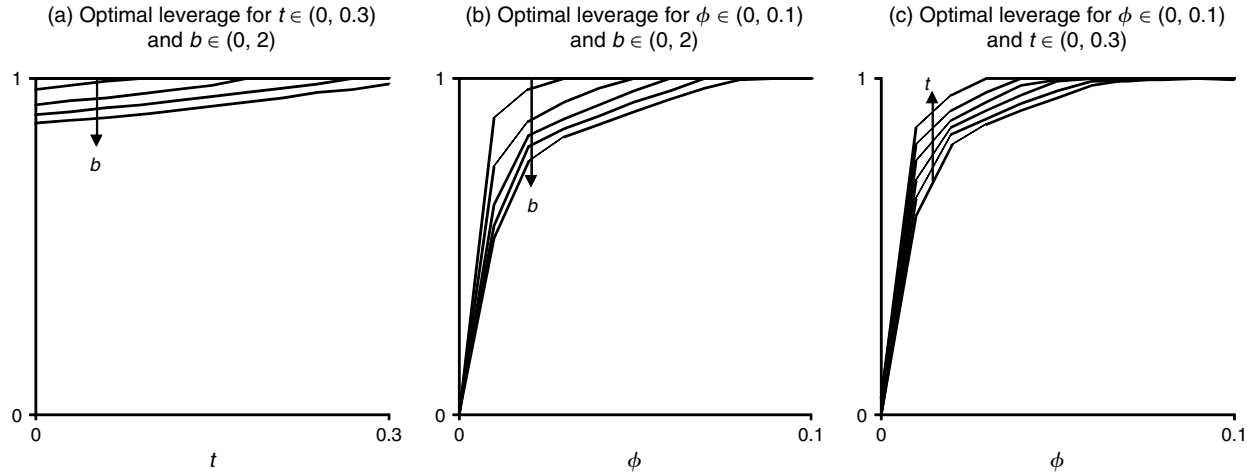
Figure 5 Optimal Debt, B (Solid Line) and Leverage, $(B - r)/c'K$ (Dashed Line) as Functions of c_F/c_N for Different Values of Model Parameters

Figure 6 Optimal Leverage as a Function of $t \in (0, 0.3)$, $\phi \in (0, 0.1)$, and $b \in (0, 2)$ for the Baseline Parameter Values $\mu = 1$, $\sigma = 1$, $\rho = 0$, $p = 1$, $c_N = 0.9$, $c_F = 0.95$, $\phi = 0.05$, $t = 0.1$, and $b = 1$



(i) If $c_F = c_N$, then $K_N^*(B, r) = 0$ and $K_F^*(B, r) = (B - r)/c_F$.

If $c_F > c_N$, then $\mathbf{K}^*(B, r)$ has one of the following two distinct forms:

(ii) $K_F^*(B, r) = 0$ and $K_N^*(B, r) = (B - r)/2c_N$,

(iii) $K_F^*(B, r) > 0$, $K_N^*(B, r) > 0$, and they jointly satisfy $c' \mathbf{K}^*(B, r) = B - r$ and

$$\frac{\Pr(\Omega_{13a})}{\Pr(\Omega_{123a})} = \frac{c_N}{c_F}. \quad (18)$$

Cases (i) and (ii) correspond to the situations in which the firm buys only flexible capacity and only nonflexible capacity, respectively. In case (iii), the firm splits the borrowed capital between the two types of capacity so that the marginal dollar spent on each capacity has the same payoff, which is guaranteed by (18). To identify the impact of agency conflict, we next characterize the first-best capacity investment.

PROPOSITION 10. Suppose the first-best capacity investment \mathbf{K}^{FB} relies exclusively on debt financing, i.e., $c' \mathbf{K}^{\text{FB}} = B^{\text{FB}} - r^{\text{FB}} > 0$.

(i) If $c_F = c_N$, then $K_N^{\text{FB}} = 0$ and $K_F^{\text{FB}} = (B^{\text{FB}} - r^{\text{FB}})/c_F$.

If $c_F > c_N$, then \mathbf{K}^{FB} has one of the following two distinct forms:

(ii) $K_F^{\text{FB}} = 0$ and $K_N^{\text{FB}} = (B^{\text{FB}} - r^{\text{FB}})/2c_N$,

(iii) $K_F^{\text{FB}} > 0$, $K_N^{\text{FB}} > 0$, and they jointly satisfy $c' \mathbf{K}^{\text{FB}} = B^{\text{FB}} - r^{\text{FB}}$ and

$$\frac{\Pr(\Omega_{13a})}{\Pr(\Omega_{123a})} = \frac{c_N - (1 + b) \Pr(\Omega_{1c})p}{c_F - (1 + b) \Pr(\Omega_{12c})p}. \quad (19)$$

Note that the ratio according to which the firm divides the borrowed capital between the two types of capacity in the first-best scenario (19) is slightly different from the corresponding ratio in the base case (18). The difference is that in the first-best scenario, the firm takes into account the impact of the capacity mix

on the value of debt in default states Ω_{1c} and Ω_{2c} . In the base case, the firm ignores this impact and thus deviates from the first best. As in the case of mixed debt-equity financing, the agency conflict disappears under full flexibility.

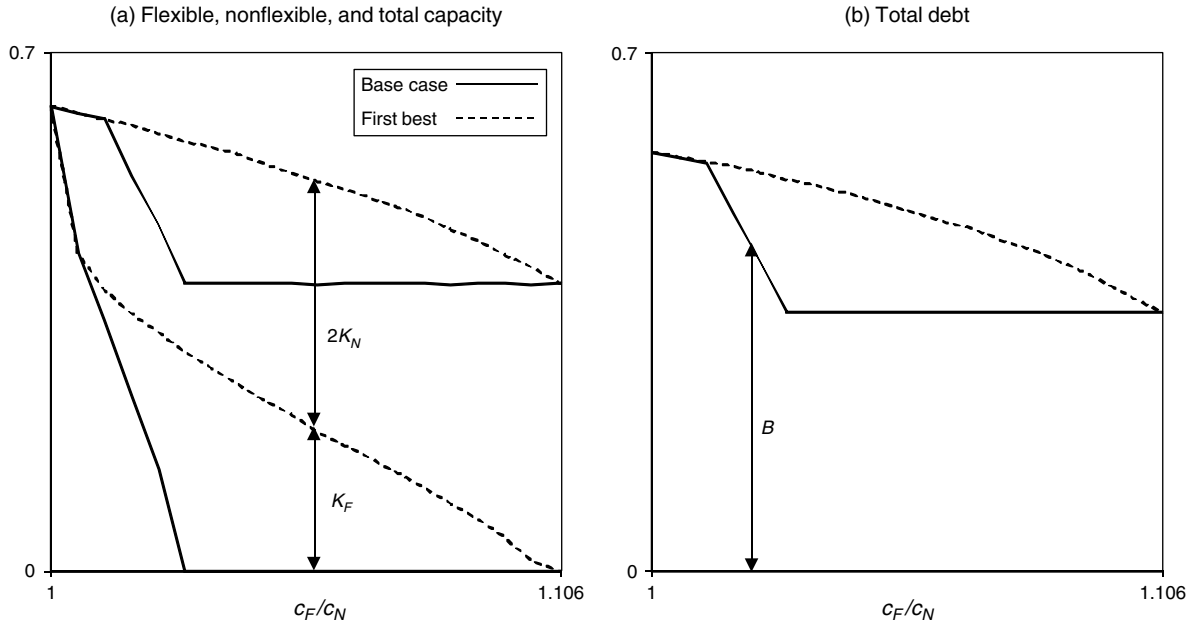
PROPOSITION 11. In the case of full flexibility, the firm chooses the first-best capacity investment and debt, and therefore, there is no agency cost associated with leverage even with full debt financing. Formally, we have

$$\mathbf{K}^* = \mathbf{K}^{\text{FB}}, \quad B^* = B^{\text{FB}}, \quad \text{and} \\ V(\mathbf{K}^*, B^*, r^*) = V(\mathbf{K}^{\text{FB}}, B^{\text{FB}}, r^{\text{FB}}).$$

The logic is the same as in the case of mixed debt-equity financing. With full flexibility, the firm can only default when capacity is underutilized. Hence a marginal increase in capacity has no impact on the value of debt and the capacity investment that maximizes the value of equity also maximizes total firm value. Figure 7 shows optimal capacity and debt as a function of flexibility in both the base case and the first-best scenario when full debt financing is optimal. Figure 7 is analogous to Figure 4 except that the bankruptcy cost parameter b is set at a sufficiently low value so that it is optimal to use full debt financing. The effects of agency conflict and flexibility are similar to those in the case of mixed debt-equity financing. As the relative cost of flexible capacity, c_F/c_N , increases, the firm gradually replaces flexible capacity with nonflexible capacity. At the same time, optimal debt decreases. The agency conflict reduces the optimal levels of flexible capacity ($K_F^* \leq K_F^{\text{FB}}$), total capacity ($2K_N^* + K_F^* \leq 2K_N^{\text{FB}} + K_F^{\text{FB}}$), and debt ($B^* \leq B^{\text{FB}}$). Finally, at high levels of flexibility, the agency cost disappears.

One difference from the case of mixed debt-equity financing is that under full debt financing the effect of

Figure 7 Optimal Capacity Levels and Debt as Functions of $c_F/c_N \in (1, 1.106)$ for $\mu = 1$, $\sigma = 1$, $\rho = 0$, $p = 1$, $c_N = 0.9$, $t = 0.3$, $b = 1$, and $\phi > 0.05$ When 100 Leverage Is Optimal



agency conflict is generally more pronounced. In particular, as flexible capacity becomes more expensive, the optimal level of flexible capacity K_F^* drops to zero much faster than K_F^{FB} . Another difference from mixed debt-equity financing is that when the first-best strategy is to choose zero flexibility ($K_F^{\text{FB}} = 0$), agency conflict disappears as well ($K_N^* = K_N^{\text{FB}}$ and $B^* = B^{\text{FB}}$). When the first-best strategy is to choose only nonflexible capacity, the firm cannot engage in asset substitution. Furthermore, when the first-best investment equals the borrowing, the firm cannot underinvest because it cannot invest less than the borrowing. In other words, once a firm issues the first-best amount of debt, B^{FB} , it cannot deviate from choosing the first-best capacity $K_N^{\text{FB}} = (B^{\text{FB}} - r^{\text{FB}})/2c_N$.

6.2. Asymmetric Profit Margins

In this section, we show that with asymmetric product profit margins, there is a possibility, albeit a small one, that agency cost exists even under full flexibility. Suppose that product 1 has a higher contribution margin than product 2, i.e., $p_1 > p_2$, and $c_F = c_N$ so that the firm chooses only flexible capacity. Because $p_1 > p_2$, the optimal capacity allocation prioritizes product 1, and the operating profit is

$$\pi(\mathbf{D}, \mathbf{K}) = p_1 \min(D_1, K_F) + p_2 \min(D_2, K_F - \min(D_1, K_F)). \quad (20)$$

To understand how flexible capacity is allocated between the two products, it is useful to partition the state space of the demand vector, \mathbb{R}_+^2 , into three events, Ω_0 , Ω_4 , and Ω_5 , which are formally defined

in the proof of Proposition 12 and illustrated in Figure 8(a). If $\mathbf{D} \in \Omega_0$, both demands can be fully satisfied with existing capacity. If $\mathbf{D} \in \Omega_4$, capacity is enough to fully satisfy the demand for product 1, but not enough to fully satisfy the demand for product 2. Finally, if $\mathbf{D} \in \Omega_5$, even the demand for product 1 cannot be fully satisfied. In Figure 8(b) these events are further combined with events Ω_a , Ω_b , and Ω_c , which correspond to the firm (a) earning profit, (b) losing money but not defaulting, and (c) defaulting, respectively.

The interesting event is Ω_{4c} , which is nonempty if $B > p_2 K_F$; i.e., the firm is highly levered and the lower contribution margin is very low. This event corresponds to the situation in which the firm defaults even though its capacity is fully utilized because most of it is used for the low-margin product. If there is a risk of default with full capacity utilization, the agency conflict exists.

PROPOSITION 12. When $\Pr(\Omega_{4c}) > 0$, the agency cost exists even under full flexibility; i.e.,

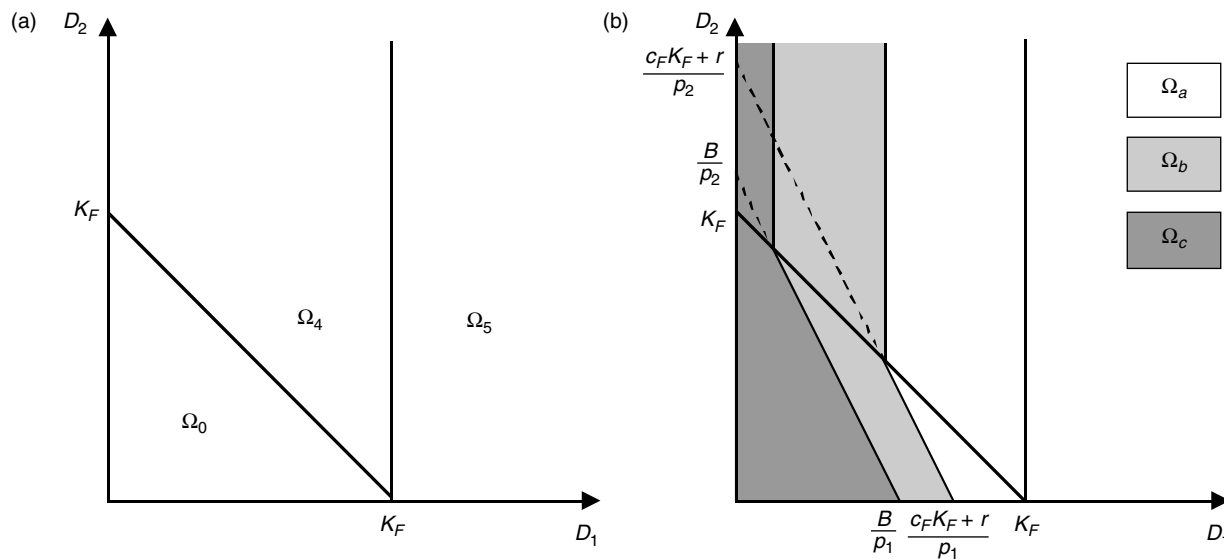
$$K_F^* \neq K_F^{\text{FB}}, \quad B^* \neq B^{\text{FB}}, \quad \text{and}$$

$$V(K_F^*, B^*, r^*) \neq V(K_F^{\text{FB}}, B^{\text{FB}}, r^{\text{FB}}) \quad \text{even if } c_F = c_N.$$

When the firm uses both debt and equity financing, the optimal capacity, K_F^* , is given by $\partial V / \partial K_F = 0$, whereas the first-best capacity, K_F^{FB} , is given by $dV / dK_F = 0$, where

$$\begin{aligned} \frac{dV}{dK_F} &= \frac{\partial V}{\partial K_F} + \frac{\partial r}{\partial K_F} \frac{\partial V}{\partial r} \\ &= \frac{\partial V}{\partial K_F} + (1+b) \Pr(\Omega_{4c}) p_2 \left(\frac{1}{1-\phi} - \Pr(\Omega_a) t \right). \end{aligned}$$

Figure 8 Partitioning of the Demand State Space as Defined in (46) When $K_N = 0$ and $p_1 > p_2$



The difference between the base case and the first best is that in the base case, the firm ignores the marginal capacity payoff generated in default state Ω_{4c} , and thus, underinvests in capacity. However, in most practical situations, the probability of Ω_{4c} is likely to be small or zero. Products relying on the same capacity (such as two car models that are made on the same assembly line) are not likely to have such different contribution margins that the firm would go bankrupt when its entire capacity is fully utilized only because a large proportion is allocated to the lower-margin product.

7. Conclusion

This paper examines the relation between resource flexibility and debt financing. We consider a two-product firm that issues the optimal amount of debt and subsequently invests in the optimal mix of product-flexible and product-dedicated capacity in the presence of demand uncertainty. Once demand uncertainty is resolved, the firm uses its capacity to produce the optimal mix of the two products. Our model extends that of Van Mieghem (1998) by allowing the capacity investment to be financed by the optimal combination of equity and debt. The firm's optimal capital structure trades off the tax benefit of debt and the higher transaction cost of issuing equity against the cost of financial distress and the agency cost associated with debt.

The agency cost of debt stems from the fact that a levered firm chooses a capacity mix that maximizes shareholder value as opposed to total firm value and, as a result, underinvests in capacity and chooses a less flexible capacity mix. Because the firm's behavior can be anticipated by lenders, it ultimately reduces

shareholder value by increasing the cost of borrowing. We show that as flexible capacity becomes relatively cheaper and the firm's optimal capacity mix becomes more flexible, the agency cost of debt as well as the risk of costly default decrease. This reduces the cost of borrowing and therefore increases the optimal amount of debt. This finding is in contrast with the notion that real flexibility exacerbates the agency conflict by allowing shareholders to increase risk (e.g., Myers 1977, Leland 1998, MacKay 2003). The real option embedded in resource flexibility, i.e., the option to allocate capacity to either product, is exercised after demand uncertainty is resolved, and thus, cannot be used by shareholders to take more risk. On the contrary, resource flexibility mitigates the downside risk as well as the agency conflict between shareholders and debtholders. Our paper is, to the best of our knowledge, the first to show that the underinvestment problem does not apply to the classical single-product single-resource newsvendor when the resource has zero salvage value. However, a newsvendor does face the underinvestment problem when it relies on multiple resources dedicated to different products. Replacing multiple product-dedicated resources with a single flexible resource therefore mitigates underinvestment.

The managerial implications for a firm investing in capacity are the following. Flexibility has another benefit in addition to risk pooling in that it reduces the cost of borrowing. Thus, a firm relying on external financing should invest in flexibility more than suggested by models that implicitly assume pure equity financing. However, to enjoy this benefit of flexibility, the firm must convince lenders of its commitment to flexible technology. For example, a firm can

start building up flexible capacity using its own capital before requesting additional external financing, it can commit to flexible technology using a bond covenant, or it can share control over investment decisions with lenders. A firm willing to invest in flexibility will benefit from monitoring associated with private debt, i.e., debt from commercial banks that perform comprehensive credit evaluations before a debt issue and monitor firm performance after a debt issue (Krishnaswami et al. 1999). The managerial implication for lenders is also clear. When lenders anticipate that a firm will choose a relatively flexible capacity mix, they should provide more favorable credit terms, to which the firm will respond by issuing more debt.

The main empirical predictions from our work are that resource flexibility is negatively related to the cost of borrowing and positively related to debt. A possible measure of resource flexibility that could be used to test these predictions would be the number of products produced per plant as in Goyal et al. (2012). In addition, our numerical analysis provides empirical predictions linking financial leverage to several product market characteristics. For example, we expect financial leverage to be negatively related to demand variability and correlation.

The result that full flexibility eliminates agency conflict holds only if the firm cannot go bankrupt when its capacity is fully utilized. We have seen that with a large difference between product profit margins and a high leverage, this may not be the case and agency conflict may persist even under full flexibility. However, this is a highly improbable scenario because products relying on the same flexible capacity (e.g., two car models using the same platform) are unlikely to have such different profit margins that an unfavorable allocation of fully utilized capacity would lead to bankruptcy. A key assumption of our model is that product prices are given. An interesting extension of our work would be to examine a price-setting firm facing uncertain demand curves. Another interesting extension would be to consider the impact of competition, which is known to affect the value of flexibility as well as the value of debt.

Acknowledgments

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Appendix

PROOF OF PROPOSITION 1. The optimal profit (1) can be written as

$$\pi(\mathbf{D}, \mathbf{K}) = \begin{cases} p(D_1 + D_2) & \text{if } \mathbf{D} \in \Omega_0, \\ p(K_1 + K_3 + D_2) & \text{if } \mathbf{D} \in \Omega_1, \\ p(K_2 + K_3 + D_1) & \text{if } \mathbf{D} \in \Omega_2, \\ p(K_1 + K_2 + K_3) & \text{if } \mathbf{D} \in \Omega_3, \end{cases} \quad (21)$$

where

$$\begin{aligned} \Omega_0(\mathbf{K}) &= \{\mathbf{D} \geq \mathbf{0}: D_1 + D_2 < K_1 + K_2 + K_3, D_i \leq K_i + K_3, i=1,2\}, \\ \Omega_1(\mathbf{K}) &= \{\mathbf{D} \geq \mathbf{0}: D_1 > K_1 + K_3, D_2 \leq K_2\}, \\ \Omega_2(\mathbf{K}) &= \{\mathbf{D} \geq \mathbf{0}: D_2 > K_2 + K_3, D_1 \leq K_1\}, \\ \Omega_3(\mathbf{K}) &= \{\mathbf{D} \geq \mathbf{0}: D_1 + D_2 \geq K_1 + K_2 + K_3, D_i > K_i, i=1,2\}. \end{aligned} \quad (22)$$

Firm value V is jointly concave in \mathbf{K} , and the Kuhn-Tucker conditions (KTC), which are necessary and sufficient, can be written as in Proposition 1. For details, see Van Mieghem (1998). \square

PROOF OF PROPOSITION 2. The KTC for problem (5) are

$$\begin{aligned} \frac{\partial}{\partial K_N} (V + uK_F + vK_N + w(\mathbf{c}'\mathbf{K} - B + r)) \\ = \frac{\partial V}{\partial K_N} + v + 2wc_N = 0, \\ \frac{\partial}{\partial K_F} (V + uK_F + vK_N + w(\mathbf{c}'\mathbf{K} - B + r)) \\ = \frac{\partial V}{\partial K_F} + u + wc_F = 0, \\ uK_F = 0, \quad vK_N = 0, \quad w(\mathbf{c}'\mathbf{K} - B + r) = 0, \quad \text{and} \\ u, v, w \geq 0. \end{aligned} \quad (23)$$

To obtain $\partial V/\partial K_N$ and $\partial V/\partial K_F$, we need to distinguish 10 events resulting from the intersection between events Ω_a , Ω_b , and Ω_c and events $\Omega_0, \dots, \Omega_3$:

$$\begin{aligned} \Omega_{0a} &= \{\mathbf{D} \geq \mathbf{0}: (\mathbf{c}'\mathbf{K} + r)/p \leq D_1 + D_2 < 2K_N + K_F \text{ and} \\ &\quad D_i < K_N + K_F, i=1,2\}, \\ \Omega_{0b} &= \{\mathbf{D} \geq \mathbf{0}: B/p \leq D_1 + D_2 < (\mathbf{c}'\mathbf{K} + r)/p \text{ and} \\ &\quad D_i < K_N + K_F, i=1,2\}, \\ \Omega_{0c} &= \{\mathbf{D} \geq \mathbf{0}: D_1 + D_2 < B/p \text{ and} \\ &\quad D_i < K_N + K_F, i=1,2\}, \\ \Omega_{1a} &= \{\mathbf{D} \geq \mathbf{0}: K_N + K_F \leq D_1 \text{ and} \\ &\quad (\mathbf{c}'\mathbf{K} + r)/p - K_N - K_F \leq D_2 < K_N\}, \\ \Omega_{1b} &= \{\mathbf{D} \geq \mathbf{0}: K_N + K_F \leq D_1 \text{ and} \\ &\quad B/p - K_N - K_F \leq D_2 < (\mathbf{c}'\mathbf{K} + r)/p - K_N - K_F\}, \\ \Omega_{1c} &= \{\mathbf{D} \geq \mathbf{0}: K_N + K_F \leq D_1 \text{ and} \\ &\quad D_2 < B/p - K_N - K_F\}, \\ \Omega_{2a} &= \{\mathbf{D} \geq \mathbf{0}: K_N + K_F \leq D_2 \text{ and} \\ &\quad (\mathbf{c}'\mathbf{K} + r)/p - K_N - K_F \leq D_1 < K_N\}, \\ \Omega_{2b} &= \{\mathbf{D} \geq \mathbf{0}: K_N + K_F \leq D_2 \text{ and} \\ &\quad B/p - K_N - K_F \leq D_1 < (\mathbf{c}'\mathbf{K} + r)/p - K_N - K_F\}, \\ \Omega_{2c} &= \{\mathbf{D} \geq \mathbf{0}: K_N + K_F \leq D_2 \text{ and} \\ &\quad D_1 < B/p - K_N - K_F\}, \\ \Omega_3 &= \{\mathbf{D} \geq \mathbf{0}: D_1 + D_2 \geq 2K_N + K_F \text{ and} \\ &\quad D_i \geq K_N, i=1,2\}. \end{aligned} \quad (24)$$

Using (3) and (21), we obtain

$$\begin{aligned} \frac{\partial V}{\partial K_N} &= (1-t) \Pr(\Omega_{13a}) 2p \\ &\quad + \Pr(\Omega_{12b}) p - 2c_N \left(\frac{1}{1-\phi} - t \Pr(\Omega_a) \right), \end{aligned}$$

and

$$\frac{\partial V}{\partial K_F} = (1-t) \Pr(\Omega_{123a})p + \Pr(\Omega_{12b})p - c_F \left(\frac{1}{1-\phi} - t \Pr(\Omega_a) \right).$$

When the constraint $\mathbf{c}'\mathbf{K} - B + r \geq 0$ is not binding, there are four possible solutions to the KTC (23):

1. $\mathbf{c}'\mathbf{K} - B + r > 0$, $K_N = 0$, and $K_F = 0$. The KTC contradict our assumption that $(1-t)p > c_N(1/(1-\phi) - t)$.
2. $\mathbf{c}'\mathbf{K} - B + r > 0$, $K_N = 0$, and $K_F > 0$. The KTC imply that $c_N = c_F$ and K_F satisfies (7).
3. $\mathbf{c}'\mathbf{K} - B + r > 0$, $K_N > 0$, and $K_F > 0$. The KTC imply that $c_F > c_N$ and \mathbf{K} satisfies (9) and (10).
4. $\mathbf{c}'\mathbf{K} - B + r > 0$, $K_N > 0$, and $K_F = 0$. The KTC imply that $c_F > c_N$ and K_N satisfies (8). \square

PROOF OF PROPOSITION 3. The problem of choosing optimal debt (6) can be written as

$$B^* = \arg \max_{B \geq 0} V(B, \mathbf{K}^*(B), r^*(B)),$$

where V is given by (3), and $\mathbf{K}^*(B)$ and $r^*(B)$ are given by (9) and (10) together with (4). Because V is continuous in B , our assumption that $\mathbf{K}^* > 0$ and $\mathbf{c}'\mathbf{K}^* > B^* - r^*$ guarantees that $\mathbf{K}^*(B) > 0$ and $\mathbf{c}'\mathbf{K}^*(B) > B - r^*(B)$ in some neighborhood of B^* . Hence, $\mathbf{K}^*(B)$ is given by (9) and (10) for all B in the neighborhood of B^* . Therefore, $\mathbf{K}^*(B)$ as well as $V(B, \mathbf{K}^*(B), r^*(B))$ are differentiable at B^* , and the optimal $B^* > 0$ satisfies the first-order condition $dV/dB = 0$. We have

$$\frac{dV}{dB} = \frac{\partial V}{\partial B} + \frac{dr}{dB} \frac{\partial V}{\partial r} + \frac{dK_N}{dB} \frac{\partial V}{\partial K_N} + \frac{dK_F}{dB} \frac{\partial V}{\partial K_F}.$$

Because $\partial V/\partial K_N = \partial V/\partial K_F = 0$, we have

$$\frac{dV}{dB} = \frac{\partial V}{\partial B} + \frac{dr}{dB} \frac{\partial V}{\partial r} = \frac{\partial V}{\partial B} + \left(\frac{\partial r}{\partial B} + \frac{dK_N}{dB} \frac{\partial r}{\partial K_N} + \frac{dK_F}{dB} \frac{\partial r}{\partial K_F} \right) \frac{\partial V}{\partial r}.$$

Taking the partial derivatives and applying some algebra gives

$$\begin{aligned} \frac{dV}{dB} = & t(1+b) \Pr(\Omega_a) \Pr(\Omega_c) + \phi \Pr(\Omega_{ab}) \frac{1}{1-\phi} - b \Pr(\Omega_c) \frac{1}{1-\phi} \\ & + \Pr(\Omega_{12c})(1+b)p \frac{d(K_N + K_F)}{dB} \left(\frac{1}{1-\phi} - t \Pr(\Omega_a) \right), \end{aligned}$$

and the optimality condition, $dV/dB = 0$, can be written as (11). \square

PROOF OF PROPOSITIONS 4. The first-best problem (12) can be written as

$$\begin{aligned} (\mathbf{K}^{\text{FB}}, B^{\text{FB}}) = & \arg \max_{\mathbf{K}, B} V(\mathbf{K}, B, r(\mathbf{K}, B)) \\ \text{s.t. } & \mathbf{K} \geq 0, B \geq 0, \text{ and } \mathbf{c}'\mathbf{K} \geq B - r(\mathbf{K}, B), \end{aligned}$$

where V is given by (3) and $r(\mathbf{K}, B)$ is given by (4). The optimal $B^{\text{FB}} > 0$ must satisfy the KTC, which can be written as follows:

$$\begin{aligned} \frac{d}{dK_N} (V + uK_F + vK_N + w(\mathbf{c}'\mathbf{K} - B + r)) &= 0, \\ \frac{d}{dK_F} (V + uK_F + vK_N + w(\mathbf{c}'\mathbf{K} - B + r)) &= 0, \end{aligned}$$

$$\begin{aligned} \frac{d}{dB} (V + uK_F + vK_N + w(\mathbf{c}'\mathbf{K} - B + r)) &= 0, \quad uK_F = 0, \\ vK_N &= 0, \quad w(\mathbf{c}'\mathbf{K} - B + r) = 0, \quad \text{and } u, v, w \geq 0, \end{aligned}$$

where $dY/dX = \partial Y/\partial X + (\partial r/\partial X)(\partial Y/\partial r)$. Taking the derivatives and using some algebra, this becomes

$$\begin{aligned} (1-t) \Pr(\Omega_{13a})2p + \Pr(\Omega_{12b})p + \left(\Pr(\Omega_a)t - \frac{1}{1-\phi} + w \right) \\ \cdot (2c_N - (1+b) \Pr(\Omega_{12c})p) + v = 0, \\ (1-t) \Pr(\Omega_{123a})p + \Pr(\Omega_{12b})p + \left(\Pr(\Omega_a)t - \frac{1}{1-\phi} + w \right) \\ \cdot (c_F - (1+b) \Pr(\Omega_{12c})p) + u = 0, \\ (1-t) \Pr(\Omega_a) + \Pr(\Omega_b) + \left(\Pr(\Omega_a)t - \frac{1}{1-\phi} + w \right) \\ \cdot (1 - (1+b) \Pr(\Omega_c)) = 0, \quad uK_F = 0, \quad vK_N = 0, \\ w(\mathbf{c}'\mathbf{K} - B + r) = 0, \quad \text{and } u, v, w \geq 0. \end{aligned} \quad (25)$$

When the constraint $\mathbf{c}'\mathbf{K} - B + r \geq 0$ is not binding, there are four possible solutions to the KTC (25):

1. $\mathbf{c}'\mathbf{K} - B + r > 0$, $K_N = 0$, and $K_F = 0$. The KTC contradict our assumption that $(1-t)p > c_N(1/(1-\phi) - t)$.
2. $\mathbf{c}'\mathbf{K} - B + r > 0$, $K_N = 0$, and $K_F > 0$. The KTC imply that $c_N = c_F$, K_F satisfies (13), and B satisfies (17).
3. $\mathbf{c}'\mathbf{K} - B + r > 0$, $K_N > 0$, and $K_F > 0$. The KTC imply that $c_F > c_N$, \mathbf{K} satisfies (15) and (16), and B satisfies (17).
4. $\mathbf{c}'\mathbf{K} - B + r > 0$, $K_N > 0$, and $K_F = 0$. The KTC imply that $c_F > c_N$, K_N satisfies (14), and B satisfies (17). \square

PROOF OF PROPOSITION 5. We first prove that $K_N^{\text{FB}} \leq K_N^{\text{UL}}$ and $K_F^{\text{FB}} + 2K_N^{\text{FB}} \geq K_F^{\text{UL}} + 2K_N^{\text{UL}}$, which immediately leads to $K_F^{\text{FB}} \geq K_F^{\text{UL}}$. With $t = 0$, the optimality conditions for \mathbf{K}^{FB} (15) and (16) simplify into

$$\begin{aligned} \Pr(\Omega_{13ab}) + \Pr(\Omega_{1c}) \frac{1+b}{1-\phi} &= \frac{c_N}{p(1-\phi)} \quad \text{and} \\ \Pr(\Omega_{123ab}) + \Pr(\Omega_{12c}) \frac{1+b}{1-\phi} &= \frac{c_F}{p(1-\phi)}. \end{aligned}$$

Using the fact that $\Pr(\Omega_{1i}) = \Pr(\Omega_{2i})$ (from the symmetry of the two products) and $\Pr(\Omega_{3ab}) = \Pr(\Omega_3)$ (see Figure 1(b)), these conditions imply that

$$\begin{aligned} \Pr(\Omega_1) &= \frac{c_F - c_N}{p(1-\phi)} - \Pr(\Omega_{1c}) \frac{b+\phi}{1-\phi} \quad \text{and} \\ \Pr(\Omega_3) &= \frac{2c_N - c_F}{p(1-\phi)}. \end{aligned} \quad (26)$$

After incorporating ϕ , the optimality conditions for \mathbf{K}^{UL} (2) become

$$\Pr(\Omega_{13}) = \frac{c_N}{p(1-\phi)} \quad \text{and} \quad \Pr(\Omega_{123}) = \frac{c_F}{p(1-\phi)}.$$

Using the symmetry $\Pr(\Omega_1) = \Pr(\Omega_2)$, these conditions become

$$\Pr(\Omega_1) = \frac{c_F - c_N}{p(1-\phi)} \quad \text{and} \quad \Pr(\Omega_3) = \frac{2c_N - c_F}{p(1-\phi)}. \quad (27)$$

Comparing (26) and (27), we have

$$\Pr(\Omega_3(\mathbf{K}^{\text{FB}})) = \Pr(\Omega_3(\mathbf{K}^{\text{UL}})) \quad \text{and} \quad (28)$$

$$\Pr(\Omega_1(\mathbf{K}^{\text{FB}})) \leq \Pr(\Omega_1(\mathbf{K}^{\text{UL}})). \quad (29)$$

Suppose that $K_N^{\text{FB}} > K_N^{\text{UL}}$. Inspecting Figure 1(a), this together with (29) implies that $K_F^{\text{FB}} + 2K_N^{\text{FB}} \geq K_F^{\text{UL}} + 2K_N^{\text{UL}}$. Again inspecting Figure 1(a), this together with $K_N^{\text{FB}} > K_N^{\text{UL}}$ contradicts (28). Hence, $K_N^{\text{FB}} \leq K_N^{\text{UL}}$. This together with (28) implies that $K_F^{\text{FB}} + 2K_N^{\text{FB}} \geq K_F^{\text{UL}} + 2K_N^{\text{UL}}$.

Next, we show that $K_N^{\text{UL}} \leq K_N^*$ and $K_F^{\text{UL}} + 2K_N^{\text{UL}} \geq K_F^* + 2K_N^*$, which immediately leads to $K_F^{\text{UL}} \geq K_F^*$. With $t = 0$, the optimality conditions for \mathbf{K}^* (9) and (10) simplify into

$$\Pr(\Omega_{13ab}) = \frac{c_N}{p(1-\phi)} \quad \text{and} \quad \Pr(\Omega_{123ab}) = \frac{c_F}{p(1-\phi)}.$$

Using the symmetry $\Pr(\Omega_{1ab}) = \Pr(\Omega_{2ab})$, and the fact that $\Pr(\Omega_{3ab}) = \Pr(\Omega_3)$ (from Figure 1(b)), these conditions imply that

$$\Pr(\Omega_1) = \frac{c_F - c_N}{p(1-\phi)} + \Pr(\Omega_{1c}) \quad \text{and} \quad \Pr(\Omega_3) = \frac{2c_N - c_F}{p(1-\phi)}. \quad (30)$$

Comparing (27) and (30), we have

$$\Pr(\Omega_3(\mathbf{K}^{\text{UL}})) = \Pr(\Omega_3(\mathbf{K}^*)) \quad \text{and} \quad (31)$$

$$\Pr(\Omega_1(\mathbf{K}^{\text{UL}})) \leq \Pr(\Omega_1(\mathbf{K}^*)). \quad (32)$$

Suppose that $K_N^{\text{UL}} > K_N^*$. Inspecting Figure 1(a), this together with (32) implies that $K_F^{\text{UL}} + 2K_N^{\text{UL}} \geq K_F^* + 2K_N^*$. Again inspecting Figure 1(a), this together with $K_N^{\text{UL}} > K_N^*$ contradicts (31). Hence, $K_N^{\text{UL}} \leq K_N^*$. This together with (31) implies that $K_F^{\text{UL}} + 2K_N^{\text{UL}} \geq K_F^* + 2K_N^*$.

It remains to prove that $B^* \leq B^{\text{FB}}$. To do so, we need to establish that $d(K_N^*(B) + K_F^*(B))/dB < 0$. The optimal capacity, $\mathbf{K}^*(B)$, is given by $(\partial V/\partial K_N, \partial V/\partial K_F) = (0, 0)$. Therefore,

$$\frac{dK_F^*}{dB} = \frac{\frac{\partial^2 V}{\partial B \partial K_N} \frac{\partial^2 V}{\partial K_N \partial K_F} - \frac{\partial^2 V}{\partial K_N^2} \frac{\partial^2 V}{\partial B \partial K_F}}{\det(\nabla_{\mathbf{K}}^2 V)},$$

and

$$\frac{dK_N^*}{dB} = \frac{\frac{\partial^2 V}{\partial K_F \partial K_N} \frac{\partial^2 V}{\partial B \partial K_F} - \frac{\partial^2 V}{\partial B \partial K_N} \frac{\partial^2 V}{\partial K_F^2}}{\det(\nabla_{\mathbf{K}}^2 V)}, \quad (33)$$

where

$$\det(\nabla_{\mathbf{K}}^2 V) = \frac{\partial^2 V}{\partial K_N^2} \frac{\partial^2 V}{\partial K_F^2} - \frac{\partial^2 V}{\partial K_N \partial K_F} \frac{\partial^2 V}{\partial K_N \partial K_F}$$

is the determinant of the Hessian of $V(\mathbf{K})$. Because the objective function, $V(\mathbf{K})$, must be locally concave at \mathbf{K}^* (otherwise $V(\mathbf{K}^*)$ would not be a maximum), we have $\det(\nabla_{\mathbf{K}}^2 V) \geq 0$ at \mathbf{K}^* . When $t = 0$, differentiating (3) with respect to K_N and K_F gives

$$\begin{aligned} \frac{\partial V}{\partial K_N} &= 2p\Pr(\Omega_{13ab}) - 2c_N/(1-\phi) \\ &= 2p \int_{K_N}^{K_N+K_F} \int_{2K_N+K_F-x}^{\infty} f(x, y) dy dx \\ &\quad + 2p \int_{K_N+K_F}^{\infty} \int_{B/p-K_N-K_F}^{\infty} f(x, y) dy dx - 2c_N/(1-\phi) \end{aligned} \quad (34)$$

and

$$\begin{aligned} \frac{\partial V}{\partial K_F} &= p\Pr(\Omega_{123ab}) - c_F/(1-\phi) \\ &= p \int_{K_N}^{K_N+K_F} \int_{2K_N+K_F-x}^{\infty} f(x, y) dy dx \end{aligned}$$

$$\begin{aligned} &+ p \int_{K_N+K_F}^{\infty} \int_{B/p-K_N-K_F}^{\infty} f(x, y) dy dx + p \\ &\cdot \int_{K_N+K_F}^{\infty} \int_{B/p-K_N-K_F}^{K_N} f(x, y) dy dx - c_F/(1-\phi). \end{aligned} \quad (35)$$

Further differentiating (34) and (35) with respect to K_N , K_F , and B gives

$$\begin{aligned} \frac{\partial^2 V}{\partial K_N^2} &= 2p \int_{K_N+K_F}^{\infty} f(x, B/p-K_N-K_F) dx \\ &\quad - 2p \int_{B/p-K_N-K_F}^{K_N} f(K_N+K_F, y) dy - 2p \int_{K_N+K_F}^{\infty} f(K_N, y) dy \\ &\quad - 4p \int_{K_N}^{K_N+K_F} f(x, 2K_N+K_F-x) dx, \\ \frac{\partial^2 V}{\partial K_N \partial K_F} &= 2p \int_{K_N+K_F}^{\infty} f(x, B/p-K_N-K_F) dx \\ &\quad - 2p \int_{B/p-K_N-K_F}^{K_N} f(K_N+K_F, y) dy \\ &\quad - 2p \int_{K_N}^{K_N+K_F} f(x, 2K_N+K_F-x) dx, \\ \frac{\partial^2 V}{\partial K_F^2} &= 2p \int_{K_N+K_F}^{\infty} f(x, B/p-K_N-K_F) dx \\ &\quad - 2p \int_{B/p-K_N-K_F}^{K_N} f(K_N+K_F, y) dy \\ &\quad - p \int_{K_N}^{K_N+K_F} f(x, 2K_N+K_F-x) dx, \quad \text{and} \\ \frac{\partial^2 V}{\partial B \partial K_N} &= \frac{\partial^2 V}{\partial B \partial K_F} = -2 \int_{K_N+K_F}^{\infty} f(x, B/p-K_N-K_F) dx. \end{aligned}$$

Substituting these expressions into (33), we obtain

$$\begin{aligned} \frac{dK_F^*}{dB} &= -\det(\nabla_{\mathbf{K}}^2 V)^{-1} 4p \int_{K_N+K_F}^{\infty} f(x, B/p-K_N-K_F) dx \\ &\quad \cdot \left(\int_{K_N}^{K_N+K_F} f(x, 2K_N+K_F-x) dx + \int_{K_N+K_F}^{\infty} f(x, K_N) dx \right), \\ \frac{dK_N^*}{dB} &= \det(\nabla_{\mathbf{K}}^2 V)^{-1} 2p \int_{K_N+K_F}^{\infty} f(x, B/p-K_N-K_F) dx \\ &\quad \cdot \int_{K_N}^{K_N+K_F} f(x, 2K_N+K_F-x) dx, \quad \text{and} \\ \frac{d(K_F^* + K_N^*)}{dB} &= -\det(\nabla_{\mathbf{K}}^2 V)^{-1} 2p \int_{K_N+K_F}^{\infty} f(x, B/p-K_N-K_F) dx \\ &\quad \cdot \left(\int_{K_N}^{K_N+K_F} f(x, 2K_N+K_F-x) dx \right. \\ &\quad \left. + 2 \int_{K_N+K_F}^{\infty} f(x, K_N) dx \right) \leq 0. \end{aligned}$$

When $t = 0$, the optimality conditions for B^* and B^{FB} , (11) and (17), can be written as

$$(b + \phi) \Pr(\Omega_c) = \phi + (1 + b)p \Pr(\Omega_{12c}) \frac{d(K_N^*(B) + K_F^*(B))}{dB}$$

and $(b + \phi) \Pr(\Omega_c) = \phi$, respectively.

Because $d(K_N^*(B) + K_F^*(B))/dB < 0$, we have

$$\Pr(\Omega_c(B^*, \mathbf{K}^*)) \leq \Pr(\Omega_c(B^{\text{FB}}, \mathbf{K}^{\text{FB}})). \quad (36)$$

Furthermore, $K_N^{FB} \leq K_N^*$ together with $K_F^{FB} + 2K_N^{FB} \geq K_F^* + 2K_N^*$ implies that $K_F^{FB} + K_N^{FB} \geq K_F^* + K_N^*$. This, together with (36) implies that $B^{FB} \geq B^*$. \square

PROOF OF PROPOSITION 6. When $c_F = 2c_N$, then $K_F^{UL} = K_F^* = K_F^{FB} = 0$. Using Propositions 1, 2, and 4, and the fact that $t = 0$, we can write the optimality conditions for K_N^{UL} , K_N^* , K_N^{FB} , and B^{FB} as

$$\Pr(\Omega_{13})p = \frac{c_N}{1-\phi}, \quad (37)$$

$$\Pr(\Omega_{13ab})p = \frac{c_N}{1-\phi}, \quad (38)$$

$$\Pr(\Omega_{13ab})p + \frac{1+b}{1-\phi} \Pr(\Omega_{1c})p = \frac{c_N}{1-\phi}, \quad (39)$$

$$\text{and } (b+\phi)\Pr(\Omega_c) = \phi, \quad (40)$$

respectively. Finally, the optimality condition for B^* , which can be derived similarly to (11), is

$$(b+\phi)\Pr(\Omega_c) = \phi + \Pr(\Omega_{12c})(1+b)p \frac{dK_N^*}{dB}. \quad (41)$$

We first prove that $K_N^{UL} \geq K_N^*$. It follows from (37) and (38) that $\Pr(\Omega_{13}(K_N^{UL})) = \Pr(\Omega_{13ab}(K_N^*, B^*))$. Because $\Pr(\Omega_{13}(K_N^*)) \geq \Pr(\Omega_{13ab}(K_N^*, B^*))$ and $\Pr(\Omega_{13}(K_N))$ is decreasing in K_N , we have $K_N^{UL} \geq K_N^*$. Next, we prove that $K_N^{FB} \geq K_N^{UL}$. It follows from (37) and (39) that

$$\Pr(\Omega_{13ab}(K_N^{FB}, B^{FB})) + \frac{1+b}{1-\phi} \Pr(\Omega_{1c}(K_N^{FB}, B^{FB})) = \Pr(\Omega_{13}(K_N^{UL})).$$

Because $(1+b)/(1-\phi) \geq 1$, this implies $\Pr(\Omega_{13ab}(K_N^{FB}, B^{FB})) + \Pr(\Omega_{1c}(K_N^{FB}, B^{FB})) \leq \Pr(\Omega_{13}(K_N^{UL}))$, which can be rewritten as $\Pr(\Omega_{13}(K_N^{FB})) \leq \Pr(\Omega_{13}(K_N^{UL}))$. Because $\Pr(\Omega_{13}(K_N))$ is decreasing in K_N , we have $K_N^{FB} \geq K_N^{UL}$.

Finally, we prove that $B^{FB} \geq B^*$. Analogously to showing that $d(K_N^* + K_F^*)/dB \leq 0$ in the proof of Proposition 5, we can show that $dK_N^*/dB \leq 0$. Using this fact together with (41) and (40), we have $\Pr(\Omega_c(K_N^*, B^*)) \leq \Pr(\Omega_c(K_N^{FB}, B^{FB}))$. Because $\Pr(\Omega_c(K_N, B))$ is decreasing in K_N and $K_N^{FB} \geq K_N^*$, we have $\Pr(\Omega_c(K_N^*, B^*)) \leq \Pr(\Omega_c(K_N^*, B^{FB}))$. Because $\Pr(\Omega_c(K_N, B))$ is increasing in B , we have $B^{FB} \geq B^*$. \square

PROOF OF PROPOSITION 7. The optimality condition for B^* in case $c_F = c_N$, which can be derived analogously to (11), is

$$t(1+b)\Pr(\Omega_a)\Pr(\Omega_c) + \phi\Pr(\Omega_{ab})\frac{1}{1-\phi} = b\Pr(\Omega_c)\frac{1}{1-\phi}. \quad (42)$$

We observe that when $c_F = c_N$, the optimality conditions for K_F^* and B^* given in (7) and (42), respectively, and those for K_F^{FB} and B^{FB} given in (13) and (17) are identical. \square

PROOF OF PROPOSITION 8. We first prove that full flexibility minimizes interest for any B . Consider any given B , and let $\tilde{\Omega}_i$, \tilde{K} , $\tilde{\pi}$, and \tilde{r} be $\Omega_i(\mathbf{K}^*(B), B)$, $\mathbf{K}^*(B)$, $\pi(\mathbf{K}^*)$, and $r(\mathbf{K}^*(B), B)$ for any $c_F > c_N$; and let $\tilde{\Omega}_i$, \tilde{K} , $\tilde{\pi}$, and \tilde{r} be $\Omega_i(\mathbf{K}^*(B), B)$, $\mathbf{K}^*(B)$, $\pi(\mathbf{K}^*)$, and $r(\mathbf{K}^*(B), B)$ for $c_F = c_N$. From (4), we have

$$r = (1+b)\Pr(\Omega_{0c})\mathbb{E}(B-\pi | \Omega_{0c}) + (1+b)\Pr(\Omega_{12c})\mathbb{E}(B-\pi | \Omega_{12c}).$$

Using the topology of Figures 3(b) and 3(c), and the fact that $\tilde{\pi} = \pi$ for any $\mathbf{D} \in \tilde{\Omega}_{0c} \subset \tilde{\Omega}_{0c}$ (no capacity constraint is binding), we have

$$\begin{aligned} \tilde{r} - \bar{r} &= (1+b)\Pr(\tilde{\Omega}_{12c})\mathbb{E}(B-\pi | \tilde{\Omega}_{12c}) \\ &\quad - (1+b)\Pr(\tilde{\Omega}_{0c} \cap \tilde{\Omega}_{12c})\mathbb{E}(B-\pi | \tilde{\Omega}_{0c} \cap \tilde{\Omega}_{12c}). \end{aligned}$$

Given that $B - \tilde{\pi} \geq 0$ if $\mathbf{D} \in \tilde{\Omega}_{12c}$, we have

$$\begin{aligned} \tilde{r} - \bar{r} &\geq (1+b)\Pr(\tilde{\Omega}_{0c} \cap \tilde{\Omega}_{12c})\mathbb{E}(B-\pi | \tilde{\Omega}_{0c} \cap \tilde{\Omega}_{12c}) \\ &\quad - (1+b)\Pr(\tilde{\Omega}_{0c} \cap \tilde{\Omega}_{12c})\mathbb{E}(B-\pi | \tilde{\Omega}_{0c} \cap \tilde{\Omega}_{12c}) \\ &= (1+b)\Pr(\tilde{\Omega}_{0c} \cap \tilde{\Omega}_{12c})\mathbb{E}(\tilde{\pi} - \pi | \tilde{\Omega}_{0c} \cap \tilde{\Omega}_{12c}). \end{aligned}$$

If $\mathbf{D} \in \tilde{\Omega}_{0c}$, $\tilde{\pi}$ is unconstrained by capacity and thus $\tilde{\pi} \geq \pi$. Therefore,

$$(1+b)\Pr(\tilde{\Omega}_{0c} \cap \tilde{\Omega}_{12c})\mathbb{E}(\tilde{\pi} - \pi | \tilde{\Omega}_{0c} \cap \tilde{\Omega}_{12c}) \geq 0$$

and $\tilde{r} \geq \bar{r}$.

Next, we prove that full flexibility maximizes B^* . Let \tilde{B} , $\tilde{K}(B)$, \tilde{K} , and $\tilde{\Omega}_i$ be B^* , $\mathbf{K}^*(B)$, \mathbf{K}^* , and $\Omega_i(\mathbf{K}^*, B^*)$ for any $c_F > c_N$; and let \tilde{B} , $\tilde{K}(B)$, \tilde{K} , and $\tilde{\Omega}_i$ be B^* , $\mathbf{K}^*(B)$, \mathbf{K}^* , and $\Omega_i(\mathbf{K}^*, B^*)$ for $c_F = c_N$. We only consider the case $\tilde{K} > 0$; i.e., we prove that debt under full flexibility, \tilde{B} , is higher than debt in the mixed capacity case, \tilde{B} . It can be shown similarly that debt under full flexibility is higher than debt in the fully nonflexible case. With $t = 0$, and using $d(K_N^*(B) + K_F^*(B))/dB \leq 0$, the optimality condition for \tilde{B} given in (11) can be written as

$$\phi = (\phi+b)\Pr(\tilde{\Omega}_c) + (1+b)p\Pr(\tilde{\Omega}_{12c}) \left| \frac{d(\tilde{K}_N(\tilde{B}) + \tilde{K}_F(\tilde{B}))}{d\tilde{B}} \right|. \quad (43)$$

With $t = 0$, the optimality condition for \tilde{B} given in (42) becomes $\phi = (b+\phi)\Pr(\tilde{\Omega}_c)$. Combining this with (43), we have

$$\begin{aligned} (b+\phi)\Pr(\tilde{\Omega}_c) &= (\phi+b)\Pr(\tilde{\Omega}_c) \\ &\quad + (1+b)p\Pr(\tilde{\Omega}_{12c}) \left| \frac{d(\tilde{K}_N(\tilde{B}) + \tilde{K}_F(\tilde{B}))}{d\tilde{B}} \right|. \quad (44) \end{aligned}$$

Suppose that $\tilde{B} < \tilde{B}$. From the topology of Figures 3(b) and 3(c), we have $\tilde{\Omega}_c \subset \tilde{\Omega}_c$. Therefore, $\Pr(\tilde{\Omega}_c) < \Pr(\tilde{\Omega}_c)$ and (44) cannot hold. This is a contradiction, and thus, $\tilde{B} \geq \tilde{B}$. \square

PROOF OF PROPOSITION 9. When the constraint $\mathbf{c}'\mathbf{K} - B + r \geq 0$ is binding, there are four possible solutions to the KTC (23):

1. $\mathbf{c}'\mathbf{K} - B + r = 0$, $K_N = 0$, and $K_F = 0$. The KTC contradict our assumption that $(1-t)p > c_N(1/(1-\phi) - t)$.
2. $\mathbf{c}'\mathbf{K} - B + r = 0$, $K_N = 0$, and $K_F > 0$. This implies that $K_F = (B-r)/c_F$, and the KTC imply that $c_N = c_F$.
3. $\mathbf{c}'\mathbf{K} - B + r = 0$, $K_N > 0$, and $K_F > 0$. The KTC imply that $c_F > c_N$ and \mathbf{K} satisfies (18).
4. $\mathbf{c}'\mathbf{K} - B + r = 0$, $K_N > 0$, and $K_F = 0$. This implies $K_N = (B-r)/2c_N$, and the KTC imply that $c_F > c_N$. \square

PROOF OF PROPOSITION 10. When the constraint $\mathbf{c}'\mathbf{K} - B + r \geq 0$ is binding, there are four possible solutions to the KTC (25):

1. $\mathbf{c}'\mathbf{K} - B + r = 0$, $K_N = 0$, and $K_F = 0$. The KTC contradict our assumption that $(1-t)p > c_N(1/(1-\phi) - t)$.
2. $\mathbf{c}'\mathbf{K} - B + r = 0$, $K_N = 0$, and $K_F > 0$. This implies that $K_F = (B-r)/c_F$, and the KTC imply that $c_N = c_F$.

3. $c_F K - B + r = 0$, $K_N > 0$, and $K_F > 0$. The KTC imply that $c_F > c_N$, and K satisfies (19).

4. $c_F K - B + r = 0$, $K_N > 0$, and $K_F = 0$. This implies $K_N = (B - r)/2c_N$, and the KTC imply that $c_F > c_N$. \square

PROOF OF PROPOSITION 11. When $c_F = c_N$, we have $K_N^* = K_N^{FB} = 0$. To prove that $(K_F^*, B^*, r^*) = (K_F^{FB}, B^{FB}, r^{FB})$, it is enough to show that $K_F^*(B^{FB}, r^{FB}) = K_F^{FB}$. Using (5) and (12), this means showing that

$$\begin{aligned} \arg \max_{K_F} V(K_F, B^{FB}, r^{FB}) &= \arg \max_{K_F} V(K_F, B^{FB}, r^{FB}) \\ \text{s.t. } c_F K_F &\geq B^{FB} - r^{FB} & \text{s.t. } c_F K_F &\geq B^{FB} - r^{FB} \text{ and} \\ & & r^{FB} &= (1+b)\Pr(\Omega_c) \\ & & & \cdot \mathbb{E}(B^{FB} - \pi | \Omega_c). \end{aligned} \quad (45)$$

The only difference between the two optimization problems is the additional fair-pricing constraint $r^{FB} = (1+b)\Pr(\Omega_c)\mathbb{E}(B^{FB} - \pi | \Omega_c)$. For any K_F such that $c_F K_F \geq B^{FB} - r^{FB}$, we have $d\pi/dK_F = 0$ for any $\mathbf{D} \in \Omega_c$, and therefore, $(d/dK_F)[(1+b)\Pr(\Omega_c)\mathbb{E}(B^{FB} - \pi | \Omega_c)] = 0$. Because the fair-pricing constraint is satisfied at $(K_F^{FB}, B^{FB}, r^{FB})$, it is satisfied for any K_F such that $c_F K_F > B^{FB} - r^{FB}$. Therefore, the fair-pricing constraint is redundant in (45) and the two formulations are equivalent. \square

PROOF OF PROPOSITION 12. It is useful to write the operating profit (20) as $\pi = p_1 D_1 + p_2 D_2$ if $\mathbf{D} \in \Omega_0$, $\pi = p_1 D_1 + p_2 (K_F - D_1)$ if $\mathbf{D} \in \Omega_4$, and $\pi = p_1 K_F$ if $\mathbf{D} \in \Omega_5$, where

$$\begin{aligned} \Omega_0(\mathbf{K}) &= \{\mathbf{D} \geq 0: D_1 + D_2 \leq K_F\}, \\ \Omega_4(\mathbf{K}) &= \{\mathbf{D} \geq 0: D_1 + D_2 > K_F, D_1 \leq K_F\}, \text{ and} \\ \Omega_5(\mathbf{K}) &= \{\mathbf{D} \geq 0: D_1 > K_F\}. \end{aligned} \quad (46)$$

We consider the base case first. When $K_N = 0$, the second-stage problem (5) becomes

$$\begin{aligned} K_F^*(B, r) &= \arg \max_{K_F} V(K_F | B, r) \\ \text{s.t. } c_F K_F &\geq B - r, \end{aligned}$$

where V is given by (3). We focus on the interior solution, where $c_F K_F > B - r > 0$, and thus, $K_F^*(B, r)$ satisfies $\partial V/\partial K_F = 0$. Taking the derivative, we can write the optimality condition as

$$\begin{aligned} \frac{\partial V}{\partial K_F} &= (1-t)\Pr(\Omega_{5a})p_1 + (1-t)\Pr(\Omega_{4a})p_2 + \Pr(\Omega_{4b})p_2 \\ &\quad - c_F \left(\frac{1}{1-\phi} - \Pr(\Omega_a)t \right) = 0. \end{aligned} \quad (47)$$

The first-stage problem (6) can be written as

$$B^* = \arg \max_{B \geq 0} V(B, K_F^*(B), r^*(B)),$$

where V is given by (3) and $r^*(B)$ and $K_F^*(B)$ are given jointly by (4) and (47). Because the constraint $c_F K_F \geq B - r$ is not binding at B^* , and $V(B)$ is continuous in B , the constraint is not binding in a neighborhood of B^* . Thus, $K_F^*(B)$ is given by (47), and $V(B, K_F^*(B))$ is differentiable in the

neighborhood of B^* . Therefore, the optimal $B^* > 0$ has to satisfy $dV/dB = 0$. We have

$$\frac{dV}{dB} = \frac{\partial V}{\partial B} + \frac{dr}{dB} \frac{\partial V}{\partial r} + \frac{dK_F}{dB} \frac{\partial V}{\partial K_F}.$$

Because $\partial V/\partial K_F = 0$, this becomes

$$\frac{dV}{dB} = \frac{\partial V}{\partial B} + \frac{dr}{dB} \frac{\partial V}{\partial r} = \frac{\partial V}{\partial B} + \left(\frac{\partial r}{\partial B} + \frac{dK_F}{dB} \frac{\partial r}{\partial K_F} \right) \frac{\partial V}{\partial r},$$

where V is given by (3) and r is given by (4). Taking the partials and applying some algebra gives

$$\begin{aligned} \frac{dV}{dB} &= t(1+b)\Pr(\Omega_a)\Pr(\Omega_c) + \frac{\phi}{1-\phi}\Pr(\Omega_{ab}) - \frac{b}{1-\phi}\Pr(\Omega_c) \\ &\quad - (1+b)\Pr(\Omega_{4c})p_2 \frac{dK_F}{dB} \left(\Pr(\Omega_a)t - \frac{1}{1-\phi} \right) = 0. \end{aligned} \quad (48)$$

Next, we consider the first-best scenario (12). When $K_N = 0$, the problem becomes

$$\begin{aligned} (K_F^{FB}, B^{FB}) &= \arg \max_{K_F, B} V(K_F, B, r(K_F, B)) \\ \text{s.t. } B &\geq 0 \text{ and } c_F K_F \geq B - r(K_F, B), \end{aligned}$$

where V is given by (3) and $r(K_F, B)$ is given by (4). We focus again on the interior solution, where $c_F K_F^{FB} > B^{FB} - r^{FB} > 0$, and thus, (K_F^{FB}, B^{FB}) has to satisfy $dV/dK_F = 0$ and $dV/dB = 0$. Using the facts that

$$\frac{dV}{dK_F} = \frac{\partial V}{\partial K_F} + \frac{\partial r}{\partial K_F} \frac{\partial V}{\partial r} \text{ and } \frac{dV}{dB} = \frac{\partial V}{\partial B} + \frac{\partial r}{\partial B} \frac{\partial V}{\partial r},$$

taking the partials, and applying some algebra, we obtain the following optimality conditions:

$$\begin{aligned} \frac{dV}{dK_F} &= (1-t)\Pr(\Omega_{5a})p_1 + (1-t)\Pr(\Omega_{4a})p_2 \\ &\quad + \Pr(\Omega_{4b})p_2 - c_F \left(\frac{1}{1-\phi} - \Pr(\Omega_a)t \right) \\ &\quad - (1+b)\Pr(\Omega_{4c})p_2 \left(\Pr(\Omega_a)t - \frac{1}{1-\phi} \right) = 0 \end{aligned} \quad (49)$$

and

$$\begin{aligned} \frac{dV}{dB} &= t(1+b)\Pr(\Omega_a)\Pr(\Omega_c) + \frac{\phi}{1-\phi}\Pr(\Omega_{ab}) \\ &\quad - \frac{b}{1-\phi}\Pr(\Omega_c) = 0. \end{aligned} \quad (50)$$

The comparison of the optimality conditions for K_F^* and B^* , (47) and (48), with those for K_F^{FB} and B^{FB} , (49) and (50), gives the desired result. \square

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