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How does the market variance risk premium vary over time? Evidence from S&P 500 variance swap investment returns ☆

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ABSTRACT

We explore whether the market variance risk premium (VRP) can be predicted. We measure VRP by distinguishing the investment horizon from the variance swap's maturity. We extract VRP from actual S&P 500 variance swap quotes and we test four classes of predictive models. We find that the best performing model is the one that conditions on trading activity. This relation is also economically significant. Volatility trading strategies which condition on trading activity outperform popular benchmark strategies, even once we consider transaction costs. Our finding implies that broker dealers command a greater VRP to continue holding short positions in index options in the case where trading conditions deteriorate.
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1. Introduction

The variance risk premium (VRP) is the reward required by a risk averse investor for being exposed to the risk stemming from random changes in the instantaneous variance of the risky asset and from jumps in its price (Todorov, 2010; Bollerslev and Todorov, 2011). Surprisingly, there is a paucity of research on whether the market VRP is predictable.

We investigate whether the Standard and Poor's (S&P) 500 VRP can be predicted. This question is of importance for two reasons. First, it enhances our understanding of the variables that predict the total equity risk premium. This includes the equity risk premium (arising from continuous fluctuations in the price of the risky asset) and VRP (Bollerslev et al., 2009; Chabi-Yo, 2012). Second, it helps variance traders to construct profitable volatility trading strategies and to avoid taking excessive risks.[2] Typically, short volatility positions are profitable (e.g., Coval and Shumway, 2001; Bakshi and Kapadia, 2003; Driessen and Maenhout, 2007;

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[2] Anecdotal evidence suggests that trading volatility has become particularly popular over the last decade. A number of products dependent on implied volatility indices such as volatility futures, volatility options and volatility exchange traded funds have been introduced. The new products improve upon the traditional class of volatility strategies conducted via index options (e.g., Alexander and Korovilas, 2013). The development of variance and volatility swap markets has also expanded the menu of volatility strategies even further. According to the Chicago Board of Options Exchange (CBOE) statistics, the trading volume in VIX options and VIX futures increased from 23,491 and 1731 contracts, respectively, in 2006 to 632,419 and 200,251 contracts, respectively, in 2014. In addition, the notional amount outstanding for index variance swaps was over USD 2 billion with USD 1.5 billion in S&P 500 VSs (Mixon and Onur, 2014).

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Ait-Sahalia et al., 2013) implying a negative market VRP. However, these positions are vulnerable to sharp increases in market volatility; this was highlighted over the recent 2008 crisis where the single names variance swap (VS) market dried up (Carr and Lee, 2009; Martin, 2013).[3]

To the best of our knowledge, only a few papers have examined whether the market VRP can be predicted. Adrian and Shin (2010) and Bekaert et al. (2013) document that broker dealers' funding liquidity and monetary policy predict VRP, respectively. However, both papers use synthesized rather than market VS rates to measure VRP. On the other hand, Bollerslev et al. (2011), Corradi et al. (2013) and Feunou et al. (2014) find that certain macro-variables, the business conditions and the term structure of the risk-neutral variance affect VRP, respectively. Nevertheless, their VRP measurement depends on the assumed parametric model and they focus on an in-sample setting. Finally, all the above studies but Feunou et al. (2014), focus on a one-month investment horizon whereas investors who trade volatility use longer investment horizons, as well.

We examine the predictability of the market VRP comprehensively. We take a unified approach by investigating whether VRP can be predicted by four model specifications: (1) the variation in the volatility of the S&P 500 returns model, (2) stock market conditions model, (3) economic conditions model, and (4) trading activity model. Theory and empirical evidence motivates this classification (see Section 2).

To address our research question, we define VRP as the conditional expectation of the return from a long position in a T-maturity S&P 500 VS contract held over an investment horizon $h \leqslant T$. The previous literature defines and measures VRP assuming that the position in a VS is held up to its maturity, i.e. $h = T$. However, in practice the position in a VS may be closed before its maturity. Our method takes this stylized fact into account and thus, it generalizes the conventional approach to measuring VRP. We calculate VRP by using a unique dataset of actual VS quotes written on the S&P 500. Previous studies measure VRP by employing synthetic VS rates synthesized using a particular portfolio of European options (e.g., Bollerslev et al., 2009, Bollerslev et al., 2014, Carr and Wu, 2009, Bekaert and Hoerova, 2014, Fan et al., 2013; Neumann and Skiadopoulos, 2013).[4]

We compute the market VRP from different T-maturity VS contracts and for different investment horizons h (term structure of VRP). Then, we conduct an in-sample as well as an out-of-sample analysis of the various model specifications. The out-of-sample setting is a useful diagnostic for the in-sample specification and it is of importance to an investor who is interested in using the models for market-timing. Hence, we perform the out-of-sample analysis using both a statistical as well as a VS trading strategy setting. Thus, we complement Egloff et al. (2010) and Ait-Sahalia et al. (2013) by providing evidence on the properties of investment strategies in the index VS markets. The previous literature has studied the performance of volatility strategies by focusing mainly on option and volatility futures markets (e.g., Coval and Shumway, 2001; Bakshi and Kapadia, 2003; Driessen and Maenhout, 2007; Konstantinidi et al., 2008).

We find that the model which conditions on trading activity variables (futures volume and TED spread) performs best. VRP increases in absolute terms (i.e. it becomes more negative) when trading activity deteriorates. We explain this as follows. Broker dealers are short in index options (Gârleanu et al., 2009) and they receive VRP as a compensation to hold these in their inventories. In the case where broker dealers face funding liquidity constraints, it is harder to take a short option position and hence, the long option investors need to offer them a greater VRP to entice them to do so. This relation holds across investment horizons and VS contracts' maturities, and it is both statistically and economically significant. VS strategies that take trading activity conditions into account outperform the buy-and-hold S&P 500 strategy and the short volatility strategy commonly used by practitioners even after we consider transaction costs. Our results extend the evidence from Adrian and Shin (2010) who find that broker dealer's funding liquidity predicts VRP.

The remainder of the paper is structured as follows. Section 2 describes the theoretical foundations and empirical evidence, which motivate the choice of models to explore the VRP predictability. Section 3 describes the data and Section 4 explains the proposed method to calculate the market VRP. Sections 5 and 6 present the in- and out-of-sample results on the statistical and economic significance of the predictors of VRP's evolution, respectively. The last section concludes.

2. Predictability of VRP: building the models

We classify the variables related to VRP in four categories: the variation of the S&P 500 volatility, the stock market conditions, the state of the economy and the trading activity. In this section, we outline briefly the relation of these variables to VRP from a theoretical as well as from an empirical perspective. For some variables, finance theory does not distinguish whether the relation between these variables and VRP is a contemporaneous or a predictive one. Nevertheless, we consider these variables as predictors to empirically test whether the relation holds in a predictive setting. For the purposes of our discussion, we fix the terminology hereafter as follows. Given that typically the market VRP is negative, we follow the VRP literature and define an increase in VRP to signify that the negative VRP becomes more negative.

2.1. Variation in the volatility of the S&P 500 returns

VRP is generated by random changes of the underlying asset's instantaneous variance stemming from two sources: the correlation (*Corr*) of variance changes with the S&P 500 returns (e.g., Cox, 1996) and the variance of volatility (*VoV*) of the S&P 500 returns (e.g., Heston, 1993; Eraker, 2008; Bollerslev et al., 2009; Drechsler and Yaron, 2011).

Corr has been documented to be negative (leverage effect). We define an increase in *Corr* to signify that the negative *Corr* becomes more negative. We expect an increase in *Corr* to increase VRP. An investor who holds a stock position pays a negative VRP as an insurance premium because the decline in the stock return can be hedged by a long position in a VS which benefits from the rise in volatility. The negative VRP becomes more negative the greater the negative *Corr* becomes because this increases the hedging effectiveness of the VS. Regarding *VoV*, we expect VRP to increase as *VoV* increases. because the greater the variation of the variance, the greater the insurance risk premium the investor is prepared to pay.

[3] A variance swap (VS) is a pure bet on variance and hence it is the natural instrument to trade variance. It is a contract that has zero value at inception. At maturity, the long side of the VS receives the difference between the realized variance over the life of the contract and a fixed rate, called the VS rate, determined at the inception of the contract (for a review of VSs, see Demeterfi et al., 1999).

[4] Alternatively, previous studies compute VRP by means of option trading strategies (e.g., Bakshi and Kapadia, 2003; Arisoy, 2010) or by taking a parametric approach where an assumed model is fitted either to market option prices (e.g., Bates, 2000, Chernov and Ghysels, 2000, Todorov, 2010; Bollerslev et al., 2011) or it is fitted to VS prices (Amengual, 2009; Egloff et al., 2010; Ait-Sahalia et al., 2013). There is also a number of studies which compute VRP by testing whether variance is priced in the cross-section of the asset returns (see e.g., Ang et al., 2006; Cremers et al., 2015). However, the computed VRP again depends on the assumed asset pricing model.

2.2. Stock market conditions

We consider VIX, the S&P 500 return, and the S&P 500 risk-neutral skewness as stock market variables expected to predict the VRP time variation.

We expect a negative relation between VRP and stock market volatility; an increase in VIX will increase VRP, i.e. it will make it more negative. Eraker (2008)'s and Chabi-Yo (2012)'s theoretical models derive such a relation, and Heston (1993) and Egloff et al. (2010) assume that the VRP magnitude increases as volatility increases. Therefore, considering the relation between VIX and VRP, tests this assumption.

We expect a positive relation between the S&P 500 return and the magnitude of VRP. This is because a decrease in the stock return will increase volatility due to the leverage effect. This will in turn increase the VRP magnitude (i.e. it will make it more negative), given the expected positive relation between the VRP magnitude and volatility.

Finally, we expect VRP to become more negative when the risk-neutral skewness becomes more negative because the latter captures the market participants fears for downward jumps in asset prices (Bakshi and Kapadia, 2003). In the occurrence of such a rare event, volatility will increase and the buyer of a VS will benefit. Hence, the buyer of the VS is willing to pay a greater VRP to take advantage of these downward jumps in S&P 500; Todorov (2010), Bollerslev and Todorov (2011) and Ait-Sahalia et al. (2013) also find that VRP reflects jump fears. Notice that we define the sign of the risk-neutral skewness variable within a conditional setting. This is because market risk-neutral skewness is negative. Therefore, we expect a positive relation between VRP and market risk-neutral skewness: VRP increases (i.e. it becomes more negative) when the market skewness increases (i.e. it becomes more negative).

2.3. Economic conditions

Regarding the economic conditions, we consider the credit spread and the forward variance as variables which affect VRP's dynamics. This is because VRP is counter-cyclical (Bollerslev et al., 2011; Bekaert et al., 2013; Corradi et al., 2013). The credit spread and the forward variance have been found to predict the state of the economy (see Gomes and Schmid, 2010; Bakshi et al., 2011, respectively). Bollerslev et al. (2011) also document that the credit spread drives the VRP dynamics. As the credit spread and the forward variance increase, VRP is expected to increase in magnitude, i.e. to become more negative; a higher credit spread and a higher forward variance predict a recession.

2.4. Trading activity

We investigate the predictive ability of two trading activity variables: (a) the trading volume of all S&P 500 futures contracts, and (b) the TED spread. We expect the VRP magnitude to decrease, i.e. VRP to become less negative, as the aggregate S&P 500 futures trading volume increases. This is because the latter implies lower volatility for the S&P 500 (Bessembinder and Seguin, 1992). Hence, the smaller volatility is, the smaller VRP will be (see Section 2.2).

The TED spread measures traders' funding liquidity. Hence, it is related to market liquidity (Brunnermeier and Pedersen, 2009) and therefore it is related to the trading activity. We expect VRP's magnitude to increase as the TED spread increases. This is because broker dealers are short in index options (Gârleanu et al., 2009) and they receive VRP as a compensation to hold these in their inventories. In the case where broker dealers face funding liquidity constraints, it is harder to take a short option position and hence, long option investors need to offer them a greater VRP to entice them to do so. Adrian and Shin (2010) confirm this prediction by finding that broker dealers' funding liquidity predicts VRP.

3. Data

3.1. Variance swap rates

We obtain daily closing quotes on over-the-counter VS rates (prices) quoted in volatility terms from a major broker dealer. The obtained VS quotes are written on the S&P 500 index and they correspond to different constant times-to-maturities (2 months, 3 months, 6 months, 1 year, and 2 years).[5] The VS data span January 4, 1996 to February 13, 2009.

Fig. 1 shows the evolution of the VS rates in volatility percentage points with time-to-maturity equal to 2, 3, 6 months, 1 and 2 years. We can see that the VS rates spike upward over periods of financial turmoil. For instance, VS rates peak in late 1998 (Russian debt and Long Term Capital Management crises), in September 2001 (World Trade Center attack), and in late 2008 (sub-prime debt crisis). Moreover, most of the time, the longer maturity VS rates are greater than the shorter maturity ones. However, over periods of financial turmoil, the long maturity VS rates are generally smaller than the shorter maturity VS rates. This implies that the term structure of VS rates is in contango (backwardation) in normal (crisis) periods. Table 1 reports summary statistics for the VS rates across the different maturities. We can see that the average VS rate increases as the contract's maturity increases. On the other hand, the variability of VS rates decreases as the contract's maturity increases.

3.2. Other variables

We employ data to measure variables expected to drive VRP. First, we obtain the daily closing prices of the S&P 500 index and the trading volume of S&P 500 futures from Bloomberg. Second, we obtain daily data on the VIX and the SKEW index from the Chicago Board of Options Exchange (CBOE) webpage. VIX and SKEW capture the risk-neutral expectation of the realized variance and the (negative) risk-neutral skewness of the S&P 500 returns over the next one-month, respectively. Increases in SKEW signify that the risk-neutral skewness becomes more negative. Third, we obtain daily data from the St. Louis Federal Reserve Bank website to measure the credit spread (difference between the yields of the Moody's AAA and BAA corporate bonds) and the TED spread (difference between the three-months Eurodollar rate and the three-months Treasury bill rate). Finally, we obtain daily data on all traded options written on the S&P 500 from the Ivy DB database of OptionMetrics to construct a number of option-based variables to be used in Section 5.

4. Measuring VRP from VS investment returns

4.1. The method

For an investment horizon h, we measure the market $VRP^{T}_{t\to t+h}$ extracted from a T-maturity VS held from t to $t+h$ ($h \leqslant T$) as:

[5] We acknowledge that our results are not directly comparable to the related literature on the VRP predictability (cited in the introduction) because our dataset does not include the one-month maturity VS contracts that the previous literature has focused on. However, we do not regard this as a limitation of our study for two reasons. First, even if we had actual data on one-month VS contracts, still our results would not be directly comparable to the previous literature. This is because any reported differences could be attributed to the different modeling assumptions, predictors and time periods employed by the previous studies. In addition, we use a much richer menu of maturities (T = 2 months, 3 months, 6 months, 1 year, and 2 years) and investment horizons (h = 1 month, 2 months, T months) which the previous literature has not considered. Therefore, we investigate the VRP predictability for fifteen cases rather than for one case as the previous literature has done.

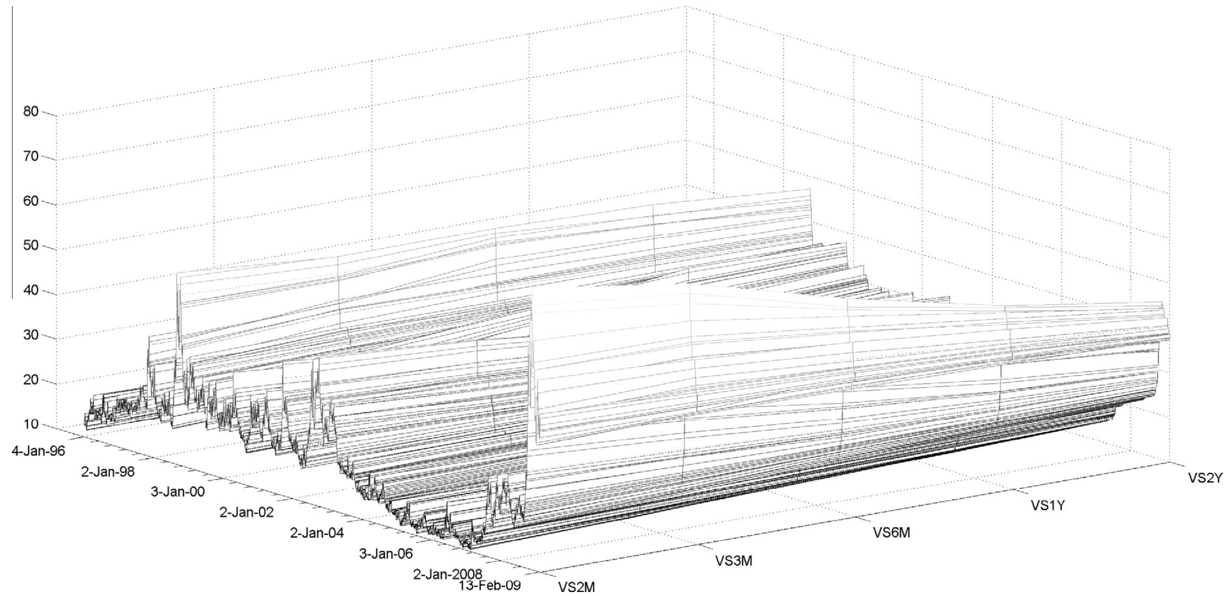

Fig. 1. Evolution of variance swap rates. Time series of the S&P 500 variance swap (VS) rates in volatility percentage points with times-to-maturity equal to 2 months (VS2M), 3 months (VS3M), 6 months (VS6M), 1 year (VS1Y) and 2 years (VS2Y). The sample spans January 4, 1996 to February 13, 2009.

Table 1
Summary statistics for the variance swap rates.

	VS2M	VS3M	VS6M	VS1Y	VS2Y
# Obs.	3302	3302	3302	3302	3302
Mean	21.7	21.79	22.19	22.77	23.26
Maximum	72.96	67.87	58.39	50.6	47.24
Minimum	10.34	10.92	11.94	13.11	14.01
Std. Dev.	8.32	7.92	7.27	6.67	6.23
Skewness	1.74	1.62	1.31	0.97	0.73
Kurtosis	8.04	7.44	5.8	4.32	3.28

Entries report summary statistics for the variance swap (VS) rates across different contract maturities. VS rates span January 4, 1996 to February 13, 2009.

$$VRP^T_{t\to t+h} = E^P_t[RetVS^T_{t\to t+h}] \quad (1)$$

where $RetVS^T_{t\to t+h}$ denotes the h-period return obtained from a long position on a T-maturity VS contract held from t to $t+h$. In line with Bondarenko (2014), we calculate $RetVS^T_{t\to t+h}$ as:

$$RetVS^T_{t\to t+h} = \frac{P\&L^T_{t\to t+h}}{VS_{t\to t+T}} \quad (2)$$

where $VS_{t\to t+T}$ is the VS rate and

$$P\&L^T_{t\to t+h} = e^{-r(T-h)}N[\lambda RV_{t\to t+h} + (1-\lambda)VS_{t+h\to t+T} - VS_{t\to t+T}] \quad (3)$$

is the profit & loss (P&L) obtained from holding a T-maturity VS contract over a horizon h, N is the notional value of the VS, r the risk-free rate, $\lambda = \frac{h}{T}$ is the proportion of the investment horizon over the time-to-maturity of the traded contract, and $RV_{t\to t+h}$ is the realized variance of the underlying asset's return distribution from t to $t+h$. Note that the S&P 500 VS contract specifications define $RV_{t\to t+h}$ to be:

$$RV_{t\to t+h} = \frac{252}{h}\sum_{i=1}^{h} ln\left(\frac{S_{t+i}}{S_{t+i-1}}\right)^2 \quad (4)$$

where S_t is the closing price of S&P 500 at time t.

Eq. (3) shows that the $P\&L^T_{t\to t+h}$ is a weighted sum of the "accrued" realized variance from t to $t+h$ and the "capital gain" which is difference in the two T-maturity VS rates prevailing at times t and $t+h$, respectively. Interestingly, the VS $P\&L^T_{t\to t+h}$ is analogous to the $P\&L^T_{t\to t+h}$ from a long position in a T-maturity bond held over an h period ($h < T$) which equals the accrued interest and the P&L from the marked-to-market bond position over the h-period.

Notice that our proposed approach to measure VRP generalizes the conventional VRP measure, which assumes that the investor holds the T-maturity contract up to its expiry (i.e. $h = T$).[6] This is important because in practice, the long position in a variance trade may be closed prior to its maturity, i.e. it can be held over an investment horizon $h < T$. Our proposed VRP measure takes this stylized fact into account and it distinguishes the investment horizon from the VS contract's maturity.

4.2. Implementation

Every day, we measure $RetVS^T_{t\to t+h}$ realized from investing in the T-maturity S&P 500 VS for different investment horizons h (h = 1, 2 and T months) by using Eq. (2). To this end, we assume that the notional value of the VS contract is one ($N = 1$) and that the risk-free rate of interest is zero. The latter assumption does not affect our results and it is in line with market practice. To calculate $P\&L^T_{t\to t+h}$ using Eq. (3), all terms but $VS_{t+h\to t+T}$ are observable because VS rates are quoted for constant times-to-maturity. In line with the industry practice for the daily mark-to-market of VSs and also following Carr and Wu (2009) and Egloff et al. (2010), we interpolate linearly in the total variance to obtain the value of $VS_{t+h\to t+T}$.[7]

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[6] In the special case where $h = T$, Eq. (1) becomes:

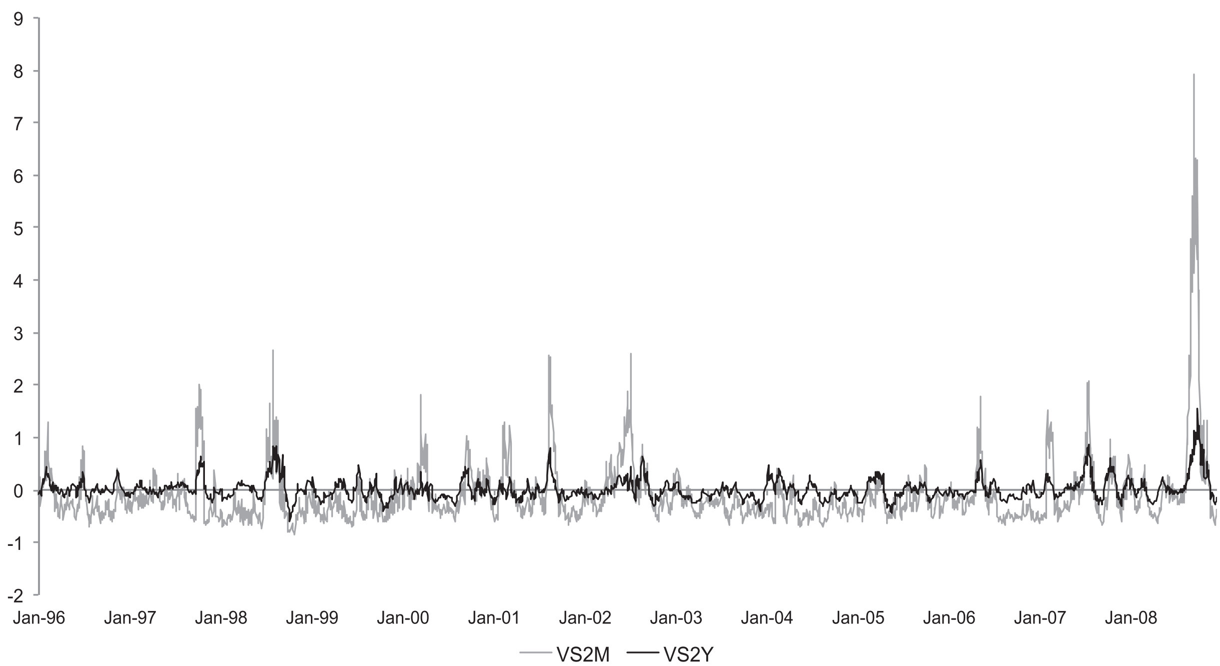
$$VRP^T_{t\to t+T} = \frac{1}{VS_{t\to t+T}}E^P_t(P\&L^T_{t\to t+h}) = \frac{1}{VS_{t\to t+T}}[E^P_t(RV_{t\to t+T}) - VS_{t\to t+T}]$$
$$= \frac{1}{VS_{t\to t+T}}[E^P_t(RV_{t\to t+T}) - E^Q_t(RV_{t\to t+T})]$$

because $VS_{t\to t+T} = E^Q_t(RV_{t\to t+T})$ and assuming $N = 1$. The last equation is the conventional VRP measure (defined in terms of VS returns) used in the literature.

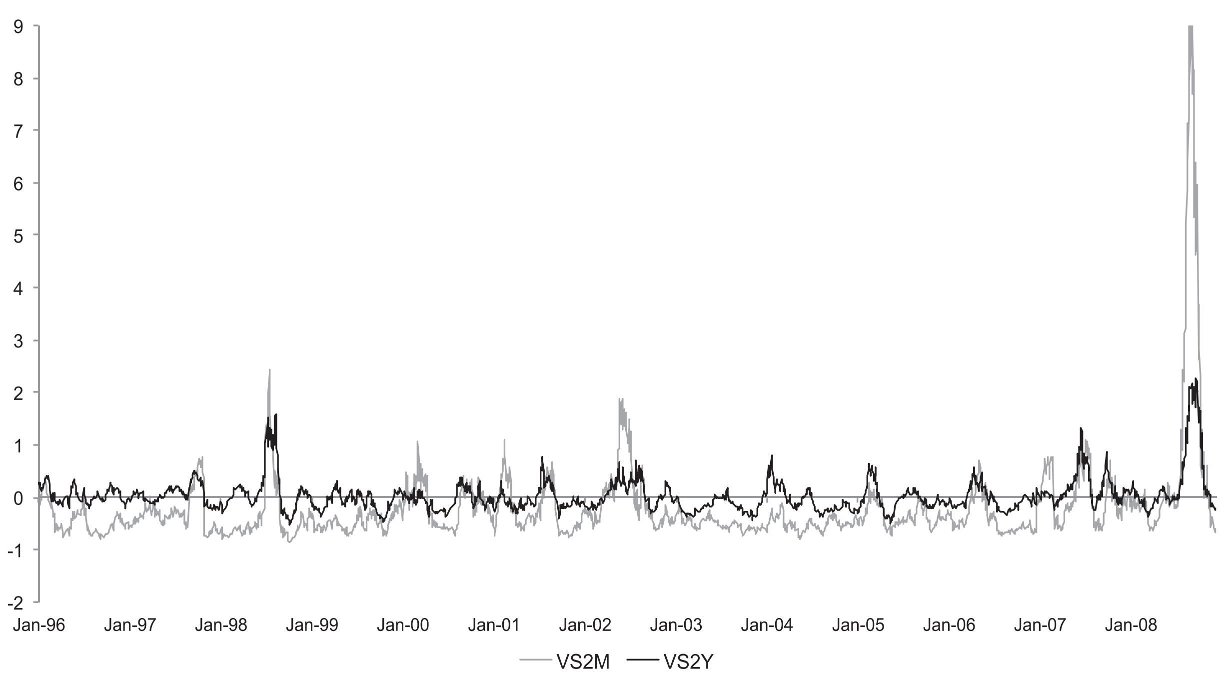
[7] At time t, the total variance interpolation method amounts to obtaining the T-maturity VS rate ($VS_{t\to t+T}$) from the traded T_i and T_{i+1}-maturity VS contracts as follows: $VS_{t\to t+T} = \frac{1}{T}[\frac{(T-T_i)}{(T_{i+1}-T_i)}(T_{i+1}VS_{t\to t+T_{i+1}} - T_iVS_{t\to t+T_i}) + T_iVS_{t\to t+T_i}]$

Fig. 2 shows indicatively the *RetVS*'s time variation from investing in two-months and two-years maturity VSs over $h = 1$ months and 2 months. We can see that the *RetVS* evolution is similar across different investment horizons and contract maturities. We can also see that the *RetVSs* become positive and jump upwards in late 2008 around the Lehman Brothers' default. This manifests the risks from taking short volatility trading positions; in the event of a stock market crisis when volatility increases, the short positions in volatility can be catastrophic.

Table 2 reports the summary statistics for the *RetVS* obtained from investing in VS contracts of different maturities and over different investment horizons ($h = 1, 2$ and T months, panels A, B and C, respectively); the previous literature has not examined the effect of the contract maturity and the effect of the investment horizon



Panel A: $RetVS$s from holding a VS over one month



Panel B: $RetVS$s from holding a VS over two months

Fig. 2. Evolution of S&P 500 variance swap returns. Evolution of the return time series (*RetVS*) obtained from holding over h months a T-maturity variance swap (VS) contract. Panels A and B show the *RetVSs* for $h = 1$ month and 2 months, respectively. The *RetVSs* are computed for two alternative maturity contracts: $T = 2$ months (gray line) and $T = 2$ years (black line). The *RetVSs* are recorded at time t, i.e. at the time where a position is opened, and span the period January 4, 1996 to December 12, 2008.

Table 2
Summary statistics for variance swaps *RetVS*s.

	VS2M	VS3M	VS6M	VS1Y	VS2Y
Investment horizon of h = 1 month					
Mean	**−0.105***	**−0.049***	**−0.015***	0.001	0.011
	(0.000)	(0.000)	(0.014)	(0.589)	(0.999)
Median	−0.257	−0.163	−0.089	−0.048	−0.025
Maximum	7.91	5.75	3.46	2.36	1.54
Minimum	−0.86	−0.75	−0.70	−0.66	−0.60
Std. Dev.	0.64	0.50	0.35	0.25	0.20
Skewness	5.04	4.67	3.73	2.72	1.76
Kurtosis	43.18	38.07	26.80	17.26	9.36
Investment horizon of h = 2 months					
Mean	**−0.202***	**−0.095***	**−0.025***	0.005	0.024
	(0.000)	(0.000)	(0.009)	(0.763)	(1.000)
Median	−0.418	−0.319	−0.172	−0.082	−0.035
Maximum	10.47	9.84	5.74	3.44	2.28
Minimum	−0.86	−0.84	−0.71	−0.64	−0.56
Std. Dev.	0.89	0.89	0.61	0.43	0.33
Skewness	6.63	5.78	4.91	3.93	2.84
Kurtosis	59.03	46.69	35.08	24.88	15.39
Investment horizon of h = T months					
Mean	**−0.202***	**−0.173***	**−0.143***	**−0.137***	−0.012
	(0.000)	(0.000)	(0.000)	(0.000)	(0.277)
Median	−0.418	−0.412	−0.375	−0.378	−0.283
Maximum	10.47	9.39	5.47	3.49	4.33
Minimum	−0.86	−0.86	−0.85	−0.86	−0.83
Std. Dev.	0.89	0.96	0.92	0.76	1.02
Skewness	6.63	6.02	3.89	2.44	2.54
Kurtosis	59.03	46.99	19.45	8.93	9.28

Entries report summary statistics for the returns (*RetVS*) from investing in VS contracts of different maturities and over different investment horizons h ($h = 1, 2$ and T months in panels A, B and C, respectively). One asterisk denotes rejection of the null hypothesis the average (unconditional) *RetVS* is zero against the alternative that it is negative at the 1% significance level; the bootstrapped p-values are shown within parentheses. The *RetVS*s are recorded at time t, i.e. at the time where a position is opened, and span the period from January 4, 1996 to December 12, 2008.

on VRP. A number of observations can be drawn. First, the average *RetVS* (i.e. the unconditional market VRP) is significantly negative. The evidence for a negative market VRP is in line with the negative S&P 500 VRP reported by the previous literature for $h = T = 1$ month (e.g., Carr and Wu, 2009; Neumann and Skiadopoulos, 2013) and it indicates that on average it is profitable to sell S&P 500 VSs. In particular, the market VRP reaches its maximum value in absolute terms by shorting the two-months maturity VS contract and holding this to its maturity (VRP of 20.2%); Bondarenko (2014) documents an unconditional S&P 500 VRP month with a similar order of magnitude for $h = T = 1$.

Second, the unconditional VRP increases in absolute terms as we move from the longer to the shorter maturity VS contracts for any given investment horizon h. Hence, on average, it is more profitable to short shorter than longer maturity VS. Moreover, the market VRP increases as the investment horizon increases for any given maturity. This implies that investors should not close their VSs positions prior to their maturity. Finally, *RetVS*s are not normally distributed; they exhibit a positive skewness and an excess kurtosis which are greater for the shorter maturities VSs. Unreported results show that the *RetVS*s are positively correlated across investment horizons and maturities; the smallest correlation is 0.16. The correlation is greater between the *RetVS*s from investing in VS contracts with maturities that are close to each other and they share the same investment horizon.

4.3. Actual versus synthetic VRPs: a comment on biases

The VRP computation requires the VS rate as an input which equals the $E^{Q}(RV)$. Given that data on actual VS rates are not available from data vendors, typically the previous literature computes VRP by synthesizing the VS rates via a trading strategy in European options and futures (for the theoretical underpinnings, see Britten-Jones and Neuberger (2000), Jiang and Tian (2005), Carr and Wu (2009) and references therein); the strategy mimics the VIX construction algorithm (Jiang and Tian, 2007). However, this may yield a bias in the calculation of VRP for at least three reasons.

First, the synthetic VS rate is a biased estimator of $E^{Q}(RV)$ in the presence of jumps in the underlying S&P 500 index (Demeterfi et al., 1999; Ait-Sahalia et al., 2013, Du and Kapadia, 2013; Bondarenko, 2014). More specifically, synthetic VS rates underestimate actual VS rates when downward jumps dominate with the bias being proportional to the jump intensity (Du and Kapadia, 2013).[8] Second, there are numerical errors in synthesizing the VS rates (Jiang and Tian, 2007). Finally, Andersen et al. (2011) document that the VIX algorithm creates artificially jumps and it is particularly unreliable during periods of market stress when it's informational content as a gauge of the investor's fear is needed most. The authors conclude that "*the quality of the risk premium measures [based on VIX] are similarly degraded*". Our computed VRP bypasses the above constraints because we implement Eqs. (2) and (3) by using actual VS quotes and hence we do not need to synthesize the VS rates.

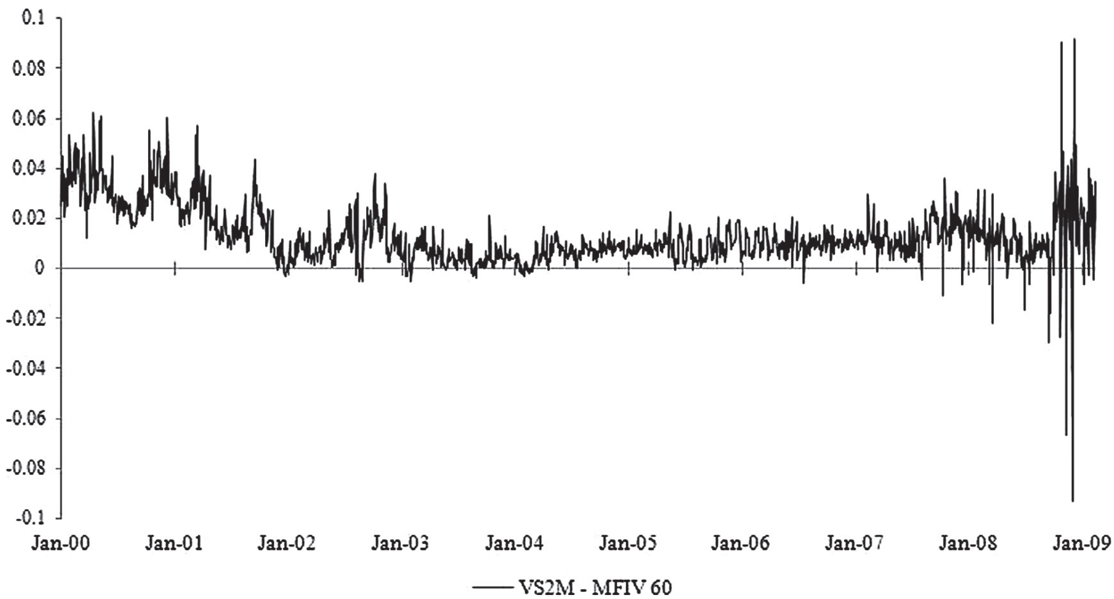
However, the computed VRP based on actual VS quotes is not bias-free either, because it may be subject to a liquidity bias. The actual VS quotes may be affected by low liquidity in the index VS markets. Unfortunately, we have no information about the trading volume and bid-ask spreads of the *individual* actual VS quotes, which would enable us to shed light on the effect of liquidity on the quality of our data. The lack of this information is also encountered in studies that employ actual S&P 500 VS quotes (e.g., Egloff et al., 2010; Ait-Sahalia et al., 2013; Nieto et al., 2014). The reason is that S&P 500 VS trade over-the-counter and hence, data on the liquidity of the *individual* quotes are not available. On the other hand, related evidence reassures us that illiquidity is not a major concern for the S&P 500 VS market. Mixon and Onur (2014) report that the interest of investors in S&P 500 VS is significant. In June 2014, the notional amount outstanding for index VSs was over USD 2 billion with USD 1.5 billion invested in S&P 500 VSs. In addition, anecdotal evidence based on conversations with practitioners who trade VS reveals that S&P 500 VSs like the ones we use in the paper continued exhibiting trading activity even over the 2008 crisis. Therefore, we feel reasonably comfortable about the quality of the employed data.

To examine whether the $RetVS^{T}_{t\to t+h}$ constructed from the actual VS rates may differ from the $RetVS^{T}_{t\to t+h}$ constructed from the synthetic ones, we synthesize the two- and three-months to maturity VS rates by following Carr and Wu (2009) approach. In sum, the approach replicates the VS rate by following four steps. First, we collect the prices of OTM S&P 500 European calls and puts with maturities surrounding any targeted constant maturity. Then, for each maturity, we perform a cubic spline interpolation across the obtained option prices as a function of the strike price to obtain a continuum of option prices. We force the spline to extrapolate smoothly in a horizontal manner (for a similar choice, see Jiang and Tian, 2005; Carr and Wu, 2009). Next, we calculate the integral of a certain portfolio of the collected options which yields the price of this portfolio; the portfolio price is the VS rate of the respective maturity. Finally, we derive the targeted constant maturity VS rate

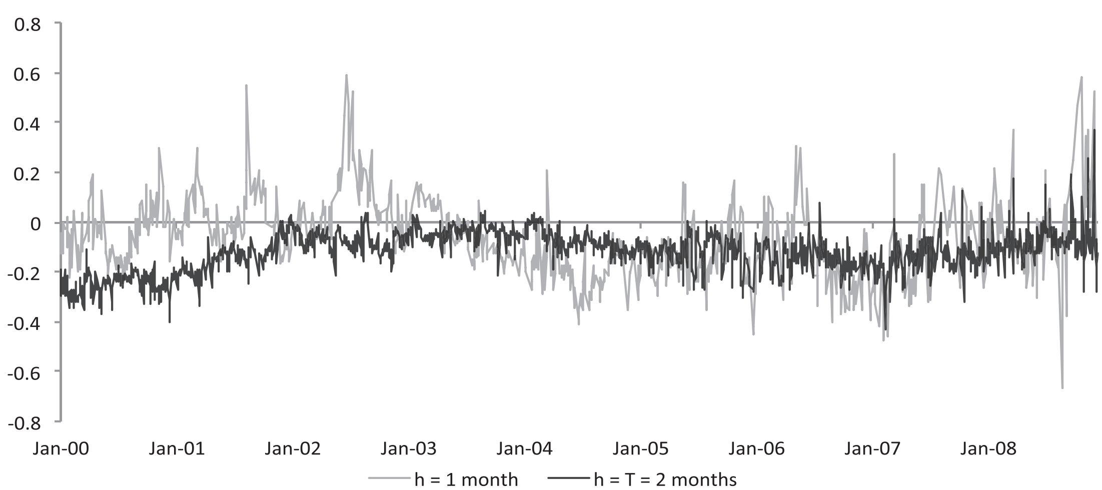
[8] Interestingly, three recent papers show that VS rates can be synthesized by market option prices even in the presence of jumps. This is feasible once either the payoff of the traded VS is proxied by a correlated payoff of a specific functional form and a certain trading strategy in options and in the underlying asset is followed (Martin, 2013; Mueller et al., 2013; Bondarenko, 2014) or the trading strategy in European option prices is modified (Du and Kapadia, 2013). However, both approaches require a continuum of traded options; this condition is not met in practice.

by interpolating linearly across the VS rates of the surrounding maturities.

Fig. 3 shows the difference between the two-month maturity actual and the synthetic VS rates in volatility percentage points (panel A) and the difference between the $RetVS^{T}_{t\to t+h}$ constructed from the actual and synthetic VS rates, respectively, (panel B) for $h = 1, 2$. We can see that the difference between the quoted and synthetic VS rates tends to be positive over time and it is 1.3 volatility points on average. A t-test suggests that the null hypothesis of a zero mean difference is rejected at a 1% level of significance (t-statistic = 51.7). Similarly, we can see that the *RetVS*s based on quoted and synthetic VS rates differ across both investment horizons. The unreported mean *RetVS* difference is negative and increases with the investment horizon (mean difference is −6.83% and −8.43% for the one- and two-months investment horizon, respectively). This suggests that on average, the *RetVS* based on the synthetic VS rates is greater than the *RetVS* based on actual quotes. A t-test also suggests that the average *RetVS* difference is significant for both investment horizons (t-statistic = −15.12 and −11.54 for $h = 1$ and 2, respectively). Again, these results do not imply that actual or synthetic VS rates should be preferred for the purposes of calculating VRP. They simply show that the two inputs differ. Analogous findings are documented for the cases where the compute *RetVS*s from other maturity contracts.



Panel A: Difference between the quoted and synthetic VS rate with two months time-to-maturity



Panel B: Difference between the $RetVS$s based on the quoted and the synthetic VS rates with two months time-to-maturity

Fig. 3. Bias in the synthetic variance swap rates. Panel A shows the difference between quoted and synthetic VS rates with two months time-to-maturity. Panel B shows the difference between *RetVS*s constructed from quoted and synthetic VS rates with two months time-to-maturity. The *RetVS*s are computed for two alternative investment horizons: $h = 1$ month (gray line) and $h = T = 2$ months (black line). The *RetVS*s are recorded at time t, i.e. at the time where a position is opened. Both figures refer to the period January 3, 2000 to February 13, 2009.

5. Predicting VRP: in-sample evidence

To investigate whether VRP is predictable, we formulate the following regression model for each T-maturity VS contract and investment horizon h:

$$RetVS^T_{t\to t+h} = c^T_0 + c^T_1 X_t + \epsilon^T_{t+h} \tag{5}$$

where X is a $(n \times 1)$ vector of the VRP drivers, c^T_0 is a scalar constant, c^T_1 is a $(1 \times n)$ vector of constant coefficients and the superscript T reminds that $RetVS^T_{t\to t+h}$ is extracted from the T-maturity VS contract. The conditional expectation of the left-hand-side of Eq. (5) delivers the VRP obtained from t to $t+h$ from trading a T-maturity VS contract defined by Eq. (1) as a function of X_{it}, i.e.

$$VRP^T_{t\to t+h} = E^P_t[RetVS^T_{t\to t+h}] = c^T_0 + c^T_1 X_t \tag{6}$$

Eq. (6) shows that we examine the VRP time variation in a predictive setting because we use the information known up to time t to explain the VRP movements over the time interval $[t, t+h]$. In the case where elements in the c^T_1 vector are found to be significant, this would indicate that VRP is predictable and time varying in the sense that as the information set changes (i.e. the values of the X variables in the right hand side of Eq. (5)), VRP also changes; this notion of time variation is also used by the equity premium literature.

5.1. Models and results

We run four main regression models corresponding to the S&P 500 volatility, the stock market conditions, the state of the economy and the trading activity categories discussed in Section 2. Therefore, we choose the variables contained in vector X in Eq. (5) correspondingly.

We estimate Eq. (5) by using daily observations of the realized $RetVS^T_{t\to t+h}$ and X_{it}. We measure X_{it} from January 4, 1996 to December 29, 2000 in all cases, i.e. over a common period across all maturity VS contracts and all investment horizons. This corresponds to a sample period that differs for each investment horizon because $RetVS$s are observed on $t+h$ (and not t). Therefore, results are not comparable across investment horizons. The rest of the data will be used for the models out-of-sample evaluation to be conducted subsequently in Section 6. Our in-sample period includes the burst of the dot-com bubble (e.g., Ofek and Richardson, 2003). To fix ideas, we consider the following four models:

Volatility variation model:

$$RetVS^T_{t\to t+h} = c^T_0 + c^T_1 Corr_t + c^T_2 VoV_t + \epsilon^T_{t+h} \tag{7}$$

where $Corr_t$ is the pairwise correlation between S&P 500 daily returns and daily changes in VIX over the past year and VoV_t is the variance of variance at time t. We orthogonalize VoV_t by regressing VoV_t on $Corr_t$, and use the residuals as the orthogonalized VoV_t in Eq. (7).

We construct VoV_t as the difference between the VS rate measured in variance terms and the squared volatility swap rate ($VolS$) for a time-to-maturity equal to two months ($T = 2$). This is because under the Q-probability measure:

$$VoV_t = var^Q_t(\sigma) = E^Q_t(\sigma^2) - E^Q_t(\sigma)^2 = VS_{t\to t+T} - VolS^2_{t\to t+T} \tag{8}$$

We use the S&P 500 at-the-money (ATM) implied volatility as a measure of $VolS$ (see Appendix A for the construction methodology). This is because Carr and Lee (2009) show that $VolS$ is well approximated by the ATM implied volatility.

Stock market conditions model:

$$RetVS^T_{t\to t+h} = c^T_0 + c^T_1 VIX_t + c^T_2 R_{t-h\to t} + c^T_3 SKEW_t + \epsilon^T_{t+h} \tag{9}$$

where VIX_t is the CBOE VIX index measured at time t, $R_{t-h\to t}$ is the S&P 500 return between $t-h$ and t, and $SKEW_t$ is the CBOE skewness index at time t. We use the CBOE skew index to measure the risk-neutral skewness of the S&P 500 return distribution. According to the construction methodology of the SKEW, increases in its value signify that the risk-neutral skewness becomes more negative. Consequently, the relation between VRP and the CBOE SKEW index is expected to be negative.

Economic conditions model:

$$RetVS^T_{t\to t+h} = c^T_0 + c^T_1 CS_t + c^T_2 FV_t + \epsilon^T_{t+h} \tag{10}$$

where CS_t and FV_t denote the credit spread and the forward variance at time t, respectively. We orthogonalize FV_t by regressing FV_t on CS_t and use the residuals as the orthogonalized FV_t in Eq. (10).

At time t, we define the forward variance between $t+\tau_i$ and $t+\tau_{i+1}$ with $\tau_i \leqslant \tau_{i+1}$ as:

$$FV_{t+\tau_i\to t+\tau_{i+1}} = lnH_{t\to t+\tau_i} - lnH_{t\to t+\tau_{i+1}}$$

where $H_{t\to t+\tau}$ is the price of an exponential claim whose payoff is contingent on the exponential of integrated variance between t and $t+\tau$ and its price is given by:

$$\begin{aligned} H_{t\to t+\tau} &= e^{-r\tau} E^Q\left\{exp(-\int_t^{t+\tau} \sigma^2_u du)|F_t\right\} \\ &= e^{-r\tau} + \int_{K>S_t} w(K) C_{t\to t+\tau}(K) dK + \int_{K<S_t} w(K) P_{t\to t+\tau}(K) dK \end{aligned} \tag{11}$$

where $C_{t\to t+\tau}(K)$ $[P_{t\to t+\tau}(K)]$ is the price of a call (put) written on the S&P 500 at time t with a strike price K and time-to-maturity τ, r is the risk free rate, and $w(K) = -\frac{\frac{8}{\sqrt{14}}cos\left[arctan\left(\frac{1}{\sqrt{7}}\right)+\frac{\sqrt{7}}{2}ln\left(\frac{K}{S_t}\right)\right]}{\sqrt{S_t}K^{3/2}}$ is the weight of the options position. Note that when $\tau = 0$ the price of the exponential claim is one (i.e. $H_{t\to t} = 1$).

To implement Eq. (11), we follow the same procedure as in the case of the risk-neutral skewness (for more details, see Appendix B). We consider alternative times to maturity, namely $\tau_0 = 0$ months, $\tau_1 = 1$ month, $\tau_2 = 2$ months, $\tau_3 = 3$ months and $\tau_4 = 4$ months and we construct four forward variance measures (i.e. $FV_{t\to t+1M}, FV_{t+1M\to t+2M}, FV_{t+2M\to t+3M}$ and $FV_{t+3M\to t+4M}$). We use the first principal component obtained from the set of forward variances as a VRP predictor. Forward variances are highly correlated and the first principal component explains 94% of the total variation.

Trading activity model:

$$RetVS^T_{t\to t+h} = c^T_0 + c^T_1 TVol_t + c^T_2 TED_t + \epsilon^T_{t+h} \tag{12}$$

where $TVol_t = Volume_t/Volume_{t-1}$ is the growth of the aggregate S&P 500 futures trading volume and TED_t is the TED spread at time t.

Table 3 reports the estimated coefficients, the Newey–West t-statistics and the adjusted R^2 of Eqs. (7), (9), (10) and (12) for the various investment horizons across the various VS maturities. (panels A, B, C and D, respectively).[9] We can see that for the one-month investment horizon, the adjusted R^2 is below 10% which is comparable to R^2s documented in the equity predictability literature (e.g., Goyal and Welch, 2008). In addition, for a given VS contract

[9] An alternative way of conducting statistical inference in the presence of overlapping observations would be to employ Hodrick's (1992) standard errors. However, this is not possible in our case because of the nature of the dependent variable. More specifically, Hodrick's (1992) standard errors are based on the assumption that the regressand variable is measured over h periods and can be decomposed into the sum of single period variables (see Hodrick, 1992, pp. 361–362). This does not hold in our case though because $RetVS_{t\to t+h}$ is not equal to $RetVS_{t\to t+1} + RetVS_{t+1\to t+2} + \ldots + RetVS_{t+h-1\to t+h} = \frac{P\&L_{t\to t+1}}{VS_{t\to t+T}} + \frac{P\&L_{t+1\to t+2}}{VS_{t+1\to t+T}} + \ldots + \frac{P\&L_{t+h-1\to t+h}}{VS_{t+h\to t+T}}$.

Table 3
In-sample results.

	h = 1 month					h = 2 months					h = T months				
	VS2M	VS3M	VS6M	VS1Y	VS2Y	VS2M	VS3M	VS6M	VS1Y	VS2Y	VS2M	VS3M	VS6M	VS1Y	VS2Y
Panel A: volatility variation model															
c	0.362	**0.373****	**0.409***	**0.303***	**0.243***	−0.157	0.165	**0.523***	**0.468***	**0.398***	−0.157	−0.148	0.074	**0.257***	**0.966***
	(1.623)	(2.002)	(2.869)	(2.943)	(2.872)	(−0.875)	(0.932)	(3.643)	(4.222)	(4.154)	(−0.875)	(−0.974)	(0.583)	(2.633)	(13.192)
Corr	**0.709****	**0.611****	**0.582***	**0.403***	**0.299****	0.235	**0.511****	**0.807***	**0.647***	**0.504***	0.235	0.254	**0.567***	**0.833***	**1.728***
	(2.433)	(2.505)	(3.044)	(2.838)	(2.516)	(0.964)	(2.045)	(3.950)	(4.000)	(3.611)	(0.964)	(1.202)	(3.350)	(6.467)	(18.522)
VoV	**−4.383****	**−4.072****	−2.841	−1.173	−0.755	**−6.594***	**−8.292***	**−7.206***	**−5.533***	**−4.710***	**−6.594***	**−7.513***	**−8.159***	**−7.013***	**−5.807***
	(−2.101)	(−2.161)	(−1.466)	(−0.577)	(−0.380)	(−4.018)	(−4.306)	(−4.339)	(−3.774)	(−3.688)	(−4.018)	(−4.974)	(−6.000)	(−6.931)	(−6.689)
Adj. R^2	**0.038**	**0.043**	**0.050**	**0.034**	**0.026**	**0.039**	**0.053**	**0.081**	**0.078**	**0.078**	**0.039**	**0.063**	**0.180**	**0.429**	**0.681**
Panel B: stock market conditions															
c	0.706	0.312	−0.101	−0.260	−0.294	0.175	0.439	−0.194	−0.398	−0.485	0.175	0.509	**0.985****	**1.017***	**1.895***
	(1.145)	(0.581)	(−0.213)	(−0.664)	(−0.903)	(0.252)	(0.508)	(−0.253)	(−0.657)	(−1.002)	(0.252)	(0.788)	(2.158)	(5.140)	(9.284)
VIX	**−0.015****	**−0.015***	**−0.014***	**−0.009***	**−0.008****	**−0.017****	**−0.028***	**−0.027***	**−0.020***	**−0.016***	**−0.017****	**−0.018***	**−0.018***	**−0.018***	**−0.026***
	(−2.553)	(−3.270)	(−3.778)	(−2.870)	(−2.452)	(−2.381)	(−3.262)	(−3.891)	(−3.896)	(−4.036)	(−2.381)	(−2.670)	(−4.870)	(−7.828)	(−10.906)
Ret	−0.153	0.144	0.189	0.151	0.171	−0.652	**−0.903****	0.064	**0.486****	**0.643***	−0.652	−0.470	0.142	0.016	**0.309****
	(−0.186)	(0.224)	(0.357)	(0.357)	(0.499)	(−1.699)	(−2.194)	(0.217)	(2.086)	(3.323)	(−1.699)	(−1.374)	(0.502)	(0.123)	(2.218)
SKEW	−0.005	−0.001	0.003	0.004	0.004	−0.001	0.000	0.006	0.007	0.007	−0.001	−0.004	**−0.008****	**−0.008***	**−0.015***
	(−0.822)	(−0.119)	(0.751)	(1.146)	(1.430)	(−0.145)	(−0.013)	(0.787)	(1.173)	(1.572)	(−0.145)	(−0.563)	(−2.008)	(−4.581)	(−8.176)
Adj. R^2	**0.040**	**0.053**	**0.062**	**0.047**	**0.047**	**0.039**	**0.078**	**0.108**	**0.112**	**0.132**	**0.039**	**0.063**	**0.184**	**0.402**	**0.626**
Panel C: economic conditions															
c	**0.433****	**0.395****	**0.405***	**0.322***	**0.272***	0.294	**0.583***	**0.773***	**0.705***	**0.604***	0.294	**0.392****	**0.359***	**0.178***	−0.114
	(2.167)	(2.424)	(2.957)	(3.250)	(3.473)	(1.672)	(2.995)	(4.239)	(4.630)	(4.745)	(1.672)	(2.425)	(3.149)	(3.635)	(−1.664)
CS	**−86.348***	**−68.844***	**−61.319***	**−46.101***	**−36.739***	**−87.566***	**−112.811***	**−118.821***	**−100.365***	**−81.407***	**−87.566***	**−102.969***	**−98.920***	**−74.896***	**−23.827****
	(−3.516)	(−3.439)	(−3.624)	(−3.704)	(−3.667)	(−4.047)	(−4.692)	(−5.277)	(−5.294)	(−5.055)	(−4.047)	(−5.108)	(−6.614)	(−10.491)	(−2.470)
FV	−12.142	−12.750	−12.937	−8.187	−7.014	1.250	−14.968	**−21.849****	**−16.171****	**−13.733****	1.250	−0.704	−13.326	**−21.468***	**−49.760***
	(−0.979)	(−1.258)	(−1.581)	(−1.338)	(−1.428)	(0.108)	(−1.354)	(−2.319)	(−2.035)	(−2.020)	(0.108)	(−0.072)	(−1.948)	(−5.170)	(−11.328)
Adj. R^2	**0.057**	**0.062**	**0.081**	**0.081**	**0.081**	**0.073**	**0.100**	**0.165**	**0.175**	**0.168**	**0.073**	**0.116**	**0.199**	**0.496**	**0.684**
Panel D: trading activity															
c	−0.054	0.023	**0.099****	**0.112***	**0.116***	**−0.247***	−0.063	0.098	**0.155***	**0.174***	**−0.247***	**−0.241***	**−0.320***	**−0.199***	−0.006
	(−0.803)	(0.413)	(2.220)	(3.085)	(3.619)	(−3.689)	(−0.903)	(1.848)	(3.587)	(4.569)	(−3.689)	(−3.808)	(−5.988)	(−5.299)	(−0.149)
Tvol	0.002	0.002	0.001	0.001	0.001	0.006	0.007	**0.007****	0.005	0.004	0.006	0.005	0.002	**0.001****	0.001
	(1.130)	(1.143)	(1.003)	(0.848)	(0.820)	(1.754)	(1.809)	(1.970)	(1.950)	(1.837)	(1.754)	(1.390)	(1.031)	(2.106)	(1.475)
TED	**−0.216****	**−0.206****	**−0.233***	**−0.205***	**−0.181***	−0.169	**−0.287***	**−0.331***	**−0.313***	**−0.280***	−0.169	**−0.182****	−0.054	**−0.293***	**−0.545***
	(−2.077)	(−2.263)	(−3.046)	(−3.159)	(−3.152)	(−1.761)	(−2.929)	(−3.941)	(−4.233)	(−4.183)	(−1.761)	(−1.978)	(−0.611)	(−4.875)	(−9.840)
Adj. R^2	**0.011**	**0.015**	**0.030**	**0.040**	**0.046**	**0.013**	**0.022**	**0.040**	**0.050**	**0.058**	**0.013**	**0.015**	**0.001**	**0.106**	**0.252**

Entries report results from the in-sample estimated volatility variation model (Panel A), stock market conditions model (Panel B), economic conditions model (Panel C) and trading activity model (Panel D) for any given investment horizon across the various VS maturities. In the case of the volatility variation model and the economic conditions model, we orthogonalize VoV_t by regressing VoV_t on $Corr_t$, and FV_t by regressing FV_t on CS_t, respectively. Coefficient estimates and the Newey-West t-statistics (within parentheses) of each one of the predictor variables are reported, as well as the adjusted R^2 for any given model. One and two asterisks denote rejection of the null hypothesis of a zero coefficient at the 1% and 5% significance level, respectively. We measure all VRP predictors over the common period January 4, 1996 to December 29, 2000.

maturity, the in-sample R^2 increases with the investment horizon. This comes as no surprise given that volatility is a fast mean reverting process (e.g., Stein and Stein, 1991). Therefore, it is easier to predict the VRP in the long run given that the realized variance forms part of the VRP.

Regarding the volatility variation model, we can see that both *Corr* and *VoV* account for the VRP time-variation in almost all cases. In particular, *Corr* has a positive effect on *RetVS*, whereas has *VoV* has a negative effect on it as expected. These findings suggest that VRP stems from both the correlation between the stock market returns and volatility, and an independent factor that drives the stochastic evolution of the variance of the S&P 500 returns.

In the case of the stock market conditions model, we can see that VIX affects VRP whereas the S&P 500 return and the SKEW do not. Regarding the economic conditions model, we can see that *CS* is significant across all contract maturities and investment horizons whereas *FV* affects the *RetVS* only for longer maturity contracts and for investment horizons greater than two months. These results show that VRP is countercyclical. This extends the evidence in Corradi et al. (2013) and Neumann et al. (2014) who employ a parametric VRP measure though. Interestingly, the negative and statistically significant sign of *CS* can also be interpreted within a financial intermediaries setting (for a similar explanation, see also Fan et al., 2013). An increase in *CS* signifies that the financial intermediaries are not willing to take on excessive risk and as a result VRP has to increase in magnitude (i.e. to become more negative) to entice them to take on this risk. This can be the case over crisis periods where the broker dealers deleverage their balance sheets by selling risky corporate debt; this presses the bond prices down and as a result the corporate yield and hence *CS* increases.

Finally, regarding the trading activity model, we can see that only the TED spread affects VRP's time variation in most cases. VRP increases when the *TED* spread increases. This is consistent with a funding liquidity explanation where increases in the TED spread signify increases in funding liquidity risk. As a result, VRP also has to increase to compensate the suppliers of the VS for bearing the additional risk. This finding extends Adrian and Shin (2010) results who report that broker dealers' repos positions predict VRP measures constructed by assuming that the position in a variance trade is held up to the maturity of the contract.

5.2. In-sample evidence: robustness tests

In this section, we assess the robustness of the results documented in Section 5.1. First, we consider a shorter in-sample period spanning January 4, 1999 to December 31, 1999 that does not include the burst of the dot-com bubble. Second, we repeat our analysis by considering P&Ls (and not *RetVS*s) as the dependent variable in Eq. (5). Both robustness checks yield results that are qualitatively similar to those reported in Table 3 and hence, they are not reported due to space limitations.

Third, we consider alternative measures of volatility variation and stock market variables; we re-estimate Eqs. (7) and (9) by considering only one of the alternative measures each time. In the case of the volatility variation model, we measure *Corr* as the rolling correlation of the daily S&P returns and VIX changes over one month as opposed to one year used previously. We also examine various *VoV* measures. First, we construct *VoV* with a three months horizon (as opposed to two months used previously) by using Eq. (8). Second, we follow Baltussen et al. (2013) and define *VoV* alternatively as follows:

$$VoV_t^{\text{Baltussen etal. (2013)}} = \frac{\sqrt{\frac{1}{n}\sum_{i=1}^{n}(\sigma_i - \overline{\sigma}_t)^2}}{\overline{\sigma}_t} \tag{13}$$

where σ is a measure of stock return volatility, $\overline{\sigma}_t$ is the average volatility over the past month and $n = 21$ is the number of volatility observations over the past month. To construct this measure, we consider alternative volatility measures: ATM implied volatility and volatility forecasts derived from a GARCH (1,1) with constant mean return model. In the case of the ATM implied volatility and the GARCH (1,1) forecasts, we examine various horizons for the alternative volatility measures; one month for the former, and 1 month, 2 months, 3 months, 6 months and 1 year for the latter

Regarding the stock market conditions model, we examine alternative stock market volatility and risk-neutral skewness measures. First, we proxy stock market volatility with the ATM implied volatility (horizons of one, two and three months) and GARCH (1,1) volatility (horizons of 1 month, 2 months, 3 months, 6 months and 1 year). Second, we measure the risk-neutral skewness extracted from S&P 500 option prices using Bakshi et al. (2003) model-free methodology (one and two months horizon, see Appendix B for the construction methodology) as an alternative to the CBOE SKEW variable.

Unreported results show that the findings reported in Section 5.1 are robust to various alternative volatility variation and stock market condition variables. In the case of the volatility variation model, both *Corr* and *VoV* are significant in most of the cases; VRP increases (i.e. becomes more negative) when the volatility variation increases. In the case of the stock market conditions model, VRP increases (i.e. becomes more negative) when market volatility increases (i.e. the stock market conditions deteriorate) with the other predictors being insignificant just as was the case in Section 5.1.

6. Predicting VRP: out-of-sample analysis

In Section 5 we found that a number of variables for each one of the four model specifications (volatility variation, stock market, economic and trading activity conditions) predict VRP within an in-sample setting. In this section, we investigate whether these relations also hold in an out-of-sample setting to conclude which model specification explains best VRP's time-variation.

We construct out-of-sample h-period *RetVS* forecasts based on each one of the multiple predictor models (5) described in Section 5.1. We estimate each model at each point in time for any given T-maturity by using a rolling window of four years (i.e. 1009 daily observations) and we generate the out-of-sample *RetVS* forecast. At each time step, we measure all predictor variables over a common in-sample period across models, maturities and investment horizons. The first in-sample dataset corresponds to predictors observed from January 4, 1996 to December 29, 2000. The last in-sample dataset corresponds to predictors observed from November 24, 2004 December 12, 2008.

6.1. Out-of-sample evaluation: statistical evaluation

We evaluate the out-of-sample forecasting performance of Eq. (5) for each model by using the out-of-sample R^2 (Campbell and Thompson, 2008). The sign of the out-of-sample R^2 shows whether the variance explained by the ith model is greater or smaller than the variance explained by a benchmark model. Given that we explore whether there is evidence of predictability from a statistical perspective, we choose the random walk (RW) as a benchmark model. RW is consistent with the notion of no predictability in asset returns and it is defined as

$$RetVS_{t\to t+h}^{T} = c_0^T + c_1^T RetVS_{t-h\to t}^{T} + \epsilon_{t+h}^{T} \tag{14}$$

Table 4
Out-of-sample R^2.

	VS2M	VS3M	VS6M	VS1Y	VS2Y
Panel A: Investment horizon h = 1 month					
Volatility variation model	0.279	0.277	0.261	0.265	0.306
Stock market conditions model	0.234	0.202	0.169	0.176	0.249
Economic conditions model	−0.185	−0.151	−0.075	0.043	0.205
Trading activity model	0.291	0.278	0.282	0.300	0.340
Panel B: Investment horizon h = 2 months					
Volatility variation model	0.347	0.356	0.366	0.385	0.405
Stock market conditions model	0.368	0.363	0.344	0.343	0.370
Economic conditions model	−0.094	−0.013	−0.049	−0.013	0.096
Trading activity model	0.409	0.403	0.407	0.423	0.447
Panel C: Investment horizon h = T months					
Volatility variation model	0.347	0.382	0.162	−0.372	0.171
Stock market conditions model	0.368	0.372	0.125	−0.601	0.080
Economic conditions model	−0.094	0.051	−0.176	−1.002	−0.060
Trading activity model	0.409	0.421	0.246	−0.445	−0.128

Entries report the out-of-sample R^2 for any given model specification across the various VS contracts and investment horizons (panels A, B and C, respectively). Each model is estimated using a rolling window of 1,261 daily observations, At each time step, VRP's predictors are measured over a common in-sample period. The first in-sample period for all VRP predictors spans January 4, 1996 to December 29, 2000, whereas the last one spans November 24, 2004 to December 12, 2008.

Then, the out-of-sample R^2 is defined as:

$$R_i^2 = 1 - \frac{var\left(E_t^i\left[RetVS_{t\to t+h}^T\right] - RetVS_{t\to t+h}^T\right)}{var\left(E_t^{RW}\left[RetVS_{t\to t+h}^T\right] - RetVS_{t\to t+h}^T\right)} \tag{15}$$

where $E_t^i\left[RetVS_{t\to t+h}^T\right]$ is the forecasted $RetVS_{t\to t+h}^T$ obtained from the ith model ($i = 1$ for the volatility variation model, 2 for the stock market conditions model, 3 for the economic conditions model, and 4 for the trading activity variables model) and $E_t^{RW}\left[RetVS_{t\to t+h}^T\right] = RetVS_{t-h\to t}^T$ is the forecasted $RetVS_{t\to t+h}^T$ obtained from the RW model. Positive (negative) values of the out-of-sample R_i^2 indicate that the ith model outperforms (underperforms) the RW model and they would imply that there is (no) evidence of statistical predictability; the RW model implies that VS returns are martingales.

Table 4 reports the out-of-sample R^2 for any given model specification across the various VS maturities and investment horizons. All but the economic conditions model outperform the RW in the vast majority of investment horizons and contract maturities. In particular, the trading activity model yields the greatest out-of-sample R^2 in most cases. The only exception occurs for the one year- or a two years-maturity contract where the trading activity model yields a negative out-of-sample R^2 and hence, it is outperformed by the RW model.

6.2. Out-of-sample evaluation: trading strategy

We investigate whether the evidence of in-sample statistical predictability is economically significant. We consider trading strategies based on the out-of-sample *RetVS* forecasts constructed from each one of the multiple predictor models examined in Section 5.1 and a particular filter value F_t^T to avoid trading on noisy signals (for a similar approach, see also Gonçalves and Guidolin, 2006; Ait-Sahalia et al., 2013). To fix ideas, at time t we construct an out-of-sample forecast for the $P\&L_{t\to t+h}^T$ based on any given model specification. If the forecasted $P\&L_{t\to t+h}^T$ is greater (less) than a filter F_t^T $(-F_t^T)$, then we go long (short) the VS contract and we keep this position up to $t+h$. On the other hand, if the forecasted $P\&L_{t\to t+h}^T$ lies between F_t^T and $-F_t^T$, we stay out of the market. We implement this strategy over the out-of-sample period. We use a time varying filter which equals the standard deviation of the P&Ls used for the in-sample estimation for any given model specification at each time step.

To evaluate the economic significance of a given trading strategy, we calculate the Sharpe ratio (SR) by taking transaction costs into account. To this end, we use each strategy's excess returns $RetVS_{t\to t+h}^T$ after transaction costs defined as follows:

$$RetVS_{t\to t+h}^T = \frac{P\&L_{t\to t+h}^T \text{ after } TC}{VS_{t\to t+T} + TC} \tag{16}$$

where TC is the transaction cost in variance points. P&L correspond to excess returns assuming that the notional value of the VS contract is fully collateralized; this is a typical assumption in the literature on the computation of futures returns. Note that in the case where we keep our position over a horizon $h < T$, we incur the transaction cost twice, whereas when we hold our position to maturity $(h = T)$ we incur the transaction cost only once, i.e.:

$$P\&L_{t\to t+h}^T \text{ after } TC = \begin{cases} Position_t \times P\&L_{t\to t+h}^T - 2TC \text{ when } h < T \\ Position_t \times P\&L_{t\to t+h}^T - TC \text{ when } h = T \end{cases} \tag{17}$$

where $Position_t$ equals 1 (-1) when we enter a long (short) position in a T-maturity VS contract at time t and $P\&L_{t\to t+h}^T$ is the realized P&L after transaction costs of a position opened at t and held up to $t+h$. We set the VS transaction costs to 0.5 volatility points (i.e. 0.25 variance points) which is the typical VS bid-ask spread (e.g., Egloff et al., 2010; we have also confirmed it following discussions with practitioners).

We compare the SR of any given strategy to the SRs of two respective benchmark strategies. First, following Ait-Sahalia et al. (2013) we consider a buy-and-hold strategy in the S&P 500 over various horizons h. Second, we consider a naive short volatility strategy where at time t, the investor opens a short position on a T-maturity VS contract ($T =$ 2 months, 3 months, 6 months, 1 year, 2 years) and she keeps this position up to $t+h$ ($h = 1$ month, 2 months, T months). This is a popular strategy because it is well documented that the average market VRP is negative and hence, shorting VSs is profitable on average.

Table 5 reports the SR once we take transaction costs into account obtained by any given model specification and for the benchmarks across the various maturities and investment horizons $h = 1, 2$ and T (Panels A and B, respectively). We can see that the

Table 5
Sharpe ratios after transaction costs.

	VS2M	VS3M	VS6M	VS1Y	VS2Y
Investment horizon of h = 1 month					
Volatility variation model	−0.310	−0.258	0.053	0.161	−0.446
Stock market conditions model	0.006	−0.013	0.134	0.043	0.026
Economic conditions model	−0.248	−0.224	−0.130	−0.055	0.049
Trading activity model	0.370	0.208	0.370	0.387	0.239
	(0.000)	(0.000)	(0.000)		
Buy-and-hold (Long S&P 500)	−0.077	−0.077	−0.077	−0.077	−0.077
Short VS	0.063	0.031	0.001	−0.029	−0.061
Investment horizon of h = 2 months					
Volatility variation model	−0.047	−1.768	−1.277	−1.048	−1.232
Stock market conditions model	−0.169	−0.064	−0.006	0.036	−0.057
Economic conditions model	−0.158	−0.377	−0.379	−0.064	0.077
Trading activity model	0.243	0.251	0.305	0.488	0.565
	(0.001)	(0.000)			
Buy-and-hold (Long S&P 500)	−0.092	−0.092	−0.092	−0.092	−0.092
Short VS	0.110	0.021	−0.014	−0.047	−0.084
Investment horizon of h = T months					
Volatility variation model	−0.047	−0.564	−0.347	0.342	−0.434
Stock market conditions model	−0.169	−0.304	−0.407	−0.637	−0.708
Economic conditions model	−0.158	−0.343	−0.460	−0.307	−0.837
Trading activity model	0.243	0.222	0.284	−0.062	−0.233
	(0.000)	(0.000)			
Buy-and-hold (Long S&P 500)	−0.092	−0.142	−0.135	−0.039	−0.022
Short VS	0.110	0.059	−0.002	−0.036	−0.237

Entries report the adjusted for transaction costs Sharpe ratio for any given model specification and benchmark strategies across the various variance swap contract maturities and investment horizons, as well as the *p*-values (within parentheses) obtained from testing $H_0 : SR_{\text{Best Model}} = SR_{\text{Benchmark}}$ against $H_a : SR_{\text{Best Model}} > SR_{\text{Benchmark}}$ by using the Jobson and Korkie (1981) test as corrected by Memmel (2003). We report the *p*-values only in the case where our best performing model and the benchmark model have both a positive Sharpe ratio. This occurs only when the naive short volatility strategy serves as the benchmark. The best performing model is the trading activity model. The employed trading strategy is the following: Go long (short) a variance swap contract when the forecasted return (*RetVS*) is greater (less) than the filter value (minus the filter value) and keep this contract for an *h* investment horizon (h = 1, 2, and T months in panels A, B and C respectively). If this trading condition is not met, the investor stays out of the market. The filter equals one standard deviation of the *RetVS*s used for the in-sample estimation of each model specification at each time step. Transaction costs equals 0.5 volatility points per transaction.

trading conditions model yields the greatest SRs among the four categories across VS maturities and investment horizons. It yields SRs between 0.21 and 0.39 for $h = 1$ month, between 0.24 and 0.57 for $h = 2$ months and up to 0.28 $h = T$. In addition, it outperforms the buy-and-hold S&P 500 strategy and the VS short strategy across all investment horizons and for most maturity contracts.

To assess the statistical significance of the difference in the SRs obtained from our best performing strategy (trading activity model) versus the corresponding one obtained from each benchmark model, we employ the Jobson and Korkie (1981) test as corrected by Memmel (2003). The null hypothesis is that the SRs of each pair of competing strategies is equal ($H_0 : SR_{\text{BestModel}} = SR_{\text{Benchmark}}$) and the alternative hypothesis is that the SR obtained from our best performing model is greater than the SR obtained from the benchmark model ($H_a : SR_{\text{BestModel}} > SR_{\text{Benchmark}}$). Table 5 also reports the *p*-values (within parentheses) obtained from testing our null hypothesis. We report the *p*-values only in the case where our best performing model and the benchmark model have both a positive Sharpe ratio, i.e. only when the naive short volatility strategy serves as the benchmark.

We can see that the reported *p*-values also confirm that the strategy based on the trading activity model outperforms the naive short volatility strategy. The outperformance of the trading activity model over the passive short volatility strategy can be explained by the fact that the former is a dynamic strategy based on a fast responsive to market conditions variable, the TED spread. This captures market makers' funding illiquidity. Market makers are net short in index options and they receive VRP as compensation for taking the other side of long investors demand (Gârleanu et al., 2009). Therefore, in the case where funding illiquidity increases, this will increase VRP because market makers will have a difficulty funding their short positions thus requiring a greater compensation to continue keeping them. Interestingly, our results extend the evidence from Adrian and Shin (2010) who find that broker dealer's funding liquidity measured by their repos and reverse repos positions predicts VRP.

Finally, we increase the VS transaction costs from 0.5 to 5 volatility points (i.e. 25 variance points) to check the robustness of our trading strategy results. The results are similar to these reported in Table 5 and are not reported for the sake of brevity; the trading activity model continues performing best across all investment horizons.

7. Conclusions

We examine whether the S&P 500 variance risk premium (VRP) can be predicted. To this end, first, we measure VRP by disentangling the investment horizon from the variance swap (VS) contract maturity. This approach is novel and it generalizes the conventional way to measure VRP. We compute the market VRP by employing actual over-the-counter S&P 500 VS quotes. Finally, we address our research question by employing a unified predictive setting.

We find that a trading activity model predicts the market VRP best among all alternative predictive models. VRP increases (i.e. becomes more negative) when funding illiquidity increases, i.e. trading conditions deteriorate. Our findings are statistically significant both in- and an out-of-sample. They are also economically significant; trading strategies with VSs which condition on trading activity, outperform the popular buy-and-hold S&P 500 and short volatility strategies even once we take transaction costs into account.

Our findings open at least two avenues for future research. First, future research should examine whether factors related to ambiguity aversion could also help predicting VRP; Miao et al. (2012) show that ambiguity aversion can explain a sizable portion of the observed VRP. From an empirical perspective, this is challenging because ambiguity aversion is not observable and hence, it needs to be measured. Second, the distinction between the investment horizon and the maturity of the VS contract prompts to determining the investor's horizon endogenously. However, the determination of the time at which the investor will exit her investment strategy is a complex problem from a modeling perspective because there is a number of factors which affect this decision (e.g., behavior of asset prices, investor's preferences, liquidity concerns, shocks to the initial endowment and shocks to the consumption process) and it has to be derived within a dynamic asset allocation setting (Karatzas and Wang, 2000; Blanchet-Scaillet et al., 2008). These questions are beyond the scope of the current paper but they deserve to become topics for future research.

Appendix A. Construction the at-the-money (ATM) implied volatility

To construct the ATM implied volatility, we consider only ATM options, i.e. options with moneyness between 0.97 and 1.03. We also filter the S&P 500 options data by applying filters commonly used in the literature (e.g., Neumann and Skiadopoulos, 2013). First, we incorporate only options with non-zero bid prices and premiums, measured as the midpoint of best bid and offer, greater than 3/8$. Second, we remove options with time-to-maturity less than five or more than 180 trading days. Third, we discard options with implied volatilities less than zero and greater than 100%. Fourth, we remove options with zero open interest and zero trading volume. Fifth, we discard options violating Merton's (1973) arbitrage bounds. Finally, we exclude options that form vertical and butterfly spreads with negative prices.

Subsequently, we follow the VXO methodology adopted by the Chicago Board Options Exchange (CBOE) to extract the ATM implied volatility (e.g., Carr and Wu, 2006). In particular, we use eight options with expirations that straddle and are closest to the desired constant time-to-maturity. For any given maturity, we consider two calls and two puts that are nearest-to-the-money and straddle the spot S&P 500 index level. For any given strike price, we calculate the average implied volatility from the call and the put. Then, we interpolate linearly between the implied volatility of the two strike prices to obtain the ATM implied volatility for the given maturity. Finally, we interpolate along the maturity dimension to obtain the constant maturity ATM implied volatility. We consider a constant time-to-maturity of two months.

Appendix B. Construction of model-free risk-neutral skewness

To extract the risk-neutral skewness, we consider only out-of-the-money (OTM) and at-the-money (ATM) options and we clean the S&P 500 options data on any given day by applying the same filters described in Appendix A. Subsequently, we construct the risk-neutral skewness by implementing Bakshi et al. (2003) model-free methodology. Let E_t^Q denote the conditional expected value operator under the risk-neutral measure formed at time t, r the risk-free rate, $C(t,\tau;K)$ $[P(t,\tau;K)]$ the price of a call [put] option with time to expiration τ and strike price K and $R(t,\tau)=\ln(S_{t+\tau}/S_t)$ be the continuously compounded rate of return at time t over a time period τ. Let also $V(\bullet), W(\bullet)$ and $X(\bullet)$

$$V(t,\tau)\equiv E_t^Q\left[e^{-r\tau}R(t,\tau)^2\right] \tag{B.1}$$

$$W(t,\tau)\equiv E_t^Q\left[e^{-r\tau}R(t,\tau)^3\right] \tag{B.2}$$

$$W(t,\tau)\equiv E_t^Q\left[e^{-r\tau}R(t,\tau)^4\right] \tag{B.3}$$

denote the fair values of three respective contracts with corresponding payoff functions $H[S]$

$$H[S]=\begin{cases} R(t,\tau)^2 \\ R(t,\tau)^3 \\ R(t,\tau)^4 \end{cases} \tag{B.4}$$

Let
$\mu(t,\tau)\equiv E_t^Q\{\ln(S_{t+\tau}/S_t)\}\approx e^{r\tau}-1-\frac{e^{r\tau}}{2}V(t,\tau)-\frac{e^{r\tau}}{6}W(t,\tau)-\frac{e^{r\tau}}{24}X(t,\tau)$
be the mean of the log-return over period τ. The risk-neutral skewness (SKEW) extracted at time t with horizon τ can be expressed in terms of the fair values of the three artificial contracts, i.e.

$$\begin{aligned} SKEW(t,\tau)&=\frac{E_t^Q\left\{\left[R(t,\tau)-E_t^QR(t,\tau)\right]^3\right\}}{E_t^Q\left\{\left[R(t,\tau)-E_t^QR(t,\tau)\right]^2\right\}^{3/2}} \\ &=\frac{e^{r\tau}W(t,\tau)-3\mu(t,\tau)e^{r\tau}V(t,\tau)+2\mu(t,\tau)^3}{\left[e^{r\tau}V(t,\tau)-\mu(t,\tau)^2\right]^{3/2}} \end{aligned} \tag{B.5}$$

Bakshi et al. (2003) show that the arbitrage-free prices of $V(t,\tau), W(t,\tau)$ and $X(t,\tau)$ are given by

$$\begin{aligned} V(t,\tau)=&\int_{S_t}^{\infty}\frac{2(1-\ln(K/S_t))}{K^2}C(t,\tau;K)dK \\ &+\int_0^{S_t}\frac{2(1+\ln(S_t/K))}{K^2}P(t,\tau;K)dK \end{aligned} \tag{B.6}$$

$$\begin{aligned} W(t,\tau)=&\int_{S_t}^{\infty}\frac{6\ln(K/S_t)-3\ln(K/S_t)}{K^2}C(t,\tau;K)dK \\ &+\int_0^{S_t}\frac{6\ln(S_t/K)+3\ln(S_t/K)}{K^2}P(t,\tau;K)dK \end{aligned} \tag{B.7}$$

$$\begin{aligned} X(t,\tau)=&\int_{S_t}^{\infty}\frac{12[\ln(K/S_t)]^2-4[\ln(K/S_t)]^3}{K^2}C(t,\tau;K)dK \\ &+\int_0^{S_t}\frac{12[\ln(S_t/K)]^2+4[\ln(S_t/K)]^3}{K^2} \end{aligned} \tag{B.8}$$

Thus, the price of each contract can be computed as a linear combination of out-of-the-money call and put options. Based on these prices, the risk-neutral skewness is computed in a model-free manner.

The implementation of Eqs. (B.6)–(B.8) requires a continuum of out-of-the-money calls and puts across strike prices. Since there is only a finite number of discrete strike prices traded in the market, we interpolate to create a continuum of OTM call and put strikes. We do this on each day and for any given contract maturity for which at least two OTM puts and two OTM calls are traded. We discard maturities that do not satisfy this requirement.

To fix ideas, we follow the subsequent steps to create a continuum of OTM call and put strikes. We convert the strike prices of the remaining options with a given maturity into call deltas using Merton's (1973) model. Then, for any given traded maturity we interpolate across the implied volatilities to obtain a continuum of implied volatilities as a function of delta. In particular, we interpolate on a delta grid with 1000 grid points ranging from 0.01 to 0.99 using a cubic smoothing spline. We calculate deltas by using the ATM implied volatility (i.e. the average of the closest-to-the-money call and put implied volatility). This ensures that the ordering of deltas is the same as the ordering of strike prices. We discard options with deltas above 0.99 and below 0.01 as these correspond to far OTM options that are not actively traded. For deltas beyond the largest and smallest available delta, we extrapolate horizontally using the respective boundary implied volatility.

Subsequently, we need to extract constant maturity risk-neutral skewness. To this end, we interpolate across the implied volatilities of the various expirations for any given delta grid value by using a cubic smoothing spline. Then, from the resulting interpolated volatility term structures, we select the respective implied volatilities for a targeted expiration. Next, we obtain the constant maturity implied volatility curve by fitting a cubic spline through these implied volatilities. If the target expiration is below the smallest available traded expiration, a constant maturity implied volatility curve is not constructed; extrapolating in the time dimension domain yields time series of implied moments that exhibit artificially created spikes.

Finally, we convert the delta grid and the corresponding constant maturity implied volatilities to the associated strike and option prices, respectively, using Merton's (1973) model. We compute the constant maturity risk-neutral skewness by evaluating the integrals in Eqs. (C.6)–(C.8) using the trapezoidal rule. We extract the one and two months constant maturities.

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