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Evaluating corporate bonds and analyzing claim holders' decisions with complex debt structure☆

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ABSTRACT

Although many different aspects of debt structures such as bond covenants and repayment schedules are empirically found to significantly influence values of bonds and equity, many theoretical structural models still oversimplify debt structures and fail to capture phenomena found in financial markets. This paper proposes a carefully designed structural model that faithfully models typical complex debt structures containing multiple bonds with various covenants. For example, the ability for an issuing firm to meet an obligation is modeled to rely on its ability to meet previous repayments, and the default trigger is shaped according to the characteristics of its debt structure such as the amount and schedule of bond repayments. Thus our framework reliably provides theoretical insight and concrete quantitative measurements consistent with extant empirical research such as the shapes of yield spread curves under various firm's financial statuses, and the impact of payment blockage covenants on newly-issued and other outstanding bonds. We also develop the forest, a novel quantitative method to handle contingent changes in the debt structure due to premature bond redemptions. A forest consists of several trees that capture different debt structures, for instance those before or after a bond redemption. This method can be used to analyze how poison put covenants in the target firm's bonds influence the bidder's costs of debt financing for a leveraged buyout, or investigate how the presence of wealth transfer among the remaining claim holders due to a bond redemption influences the firm's call policy, or further reconcile conflicts among previous empirical studies on call delay phenomena.

1. Introduction

Corporate bonds are fundamental financing instruments that are widely held by institutional investors or fund managers. According to reports from the Securities Industry and Financial Markets Association (SIFMA), the amount of issuances (outstandings) in the US market grew from 343.7 billion (2247.9 billion) in 1996 to 1440.9 billion (7822.3 billion) in 2014.[1] Clearly corporate bonds play an important role in capital markets; their prevalence has further led academics and practitioner communities to devote their energies to the analysis of bond evaluations and relevant claim holders' decisions (e.g., premature redemptions of bonds).

While a *default-free* bond (e.g., a Treasury bond) can be *separately* evaluated without considering the presence of other simultaneously outstanding default-free bonds, the value of a corporate bond may be greatly influenced by the existence of other outstanding bonds of the same issuing firm due to the claim dilution effect. For example, Fama and Miller (1972) indicate that new bond issuances may dilute the values of previously-issued bonds. Ingersoll (1987) further points out that the issuances of short-term junior bonds may deteriorate the credit quality of previously-issued long-term senior bonds. Indeed, empirical investigations in Rauh and Sufi (2010) and Colla et al. (2013) show that most firms have very complicated debt structures, such as multiple outstanding bonds with different maturities, seniorities and embedded covenants. To analyze the relationships among a firm's debt structure, the values of its outstanding bonds and

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[1] See http://www.sifma.org/research/statistics.aspx.

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equity, and the relevant claim holders' decisions, we develop a quantitative framework that endogenously associates the firm's insolvency risk with its prevailing debt structure by taking advantage of the structural model pioneered by Merton (1974). This framework provides theoretical insight and concrete quantitative measurements for the empirical literature on debt structure.

To reduce mathematical or computational difficulty in modeling complex features of a firm's debt structure, many structural models oversimplify the debt structure; hence as examined in Jones et al. (1984), Eom et al. (2004) and Huang and Huang (2012), they perform poorly when evaluating corporate securities. For example, some models use a "representative bond" to stand for the overall complex debt structure (e.g., Merton, 1974; Kim et al., 1993; Leland, 1994); however, this simplification prevents us from analyzing the impacts of coexistent bonds with different covenants on the values of the firm's securities. Another popular approach, the "portfolio of zeroes approach", decomposes all outstanding bonds of the same firm into a portfolio of equal-priority zero-coupon bonds and evaluates them separately (e.g., Longstaff and Schwartz, 1995; Collin-Dufresne and Goldstein, 2001). Eom et al. (2004) indicate that this approach inaccurately estimates the default probability of each zero-coupon bond since each bond is evaluated without considering whether all previously-matured bonds are honored. In addition, many models put naive settings on default triggers to preserve mathematical tractability. For example, Black and Cox (1976) and Zhou (2001) assume that a firm defaults when its asset value falls below a unified default boundary without considering the debt repayment schedule implied by the firm's debt structure. Other work considers repayment schedules with naive financing settings. For example, Geske (1977) assumes that all debt repayments are financed by issuing new equities. Leland and Toft (1996) assume that a firm should keep the amounts of its outstanding bonds unchanged regardless of its financial status on repayment dates. However, Davydenko (2012) empirically shows that it is hard to identify a unified boundary level to exactly separate insolvent firms from solvent ones; in addition, much empirical evidence confirms that a firm's financing policy may depend on its current financial status, its investment opportunities, or the macroeconomic condition.[2]

Much effort has been devoted to enhancing the empirical validity of structural models. For example, Eom et al. (2004) empirically show that the "compound option approach" is much better than the aforementioned portfolio of zeros approach. Specifically, the former approach views the question of whether a due bond principal or coupon repayment is honored as an option on other options—whether previously-matured repayments are fulfilled. Thus the default probability for each repayment is evaluated conditionally on the default probabilities for previous repayments in the former approach; this is more reasonable than the latter approach's independent modeling of default events. In addition, some work elaborates structural models by considering the interdependence of a firm's investment policies and different facets of its debt structure such as bond maturities (e.g., Barclay et al., 2003), priorities (e.g., Hackbarth and Mauer (2012)), and leverage ratios (e.g., Kuehn and Schmid, 2014).

To appropriately associate a firm's insolvency risk with the different observable facets of its debt structure based on the compound option approach, we develop a novel evaluation framework by exploiting the tree method, a popular numerical technique proposed by Cox et al. (1979). With the flexibility of the tree method, our framework easily models the debt-structure-dependent default trigger shaped according to the repayment schedule and covenants embedded in the firm's outstanding bonds, thus providing theoretical insight into Davydenko (2012)'s observation that default triggers are widely dispersed among firms. Specifically, to measure a firm's ability to repay a certain obligation with internal or external funds under the burdens of previously-matured payments, we introduce "remaining assets", a novel proxy defined as the remainder of the firm's asset value after repaying all required payments that matured prior to that obligation. This proxy allows our framework to implicitly incorporate the spirit of the compound option approach without adopting naive financing settings, for instance, financing all repayments by raising new equities as in Geske (1977) or keeping the amounts of outstanding bonds stationary as in Leland and Toft (1996). To model the influence of repayment schedules and covenants, a default event is triggered once the firm's remaining asset value minus the values of the assets pledged for other outstanding bonds is less than its matured payment implied by its debt structure. As mentioned in Section 2, although introducing the concepts of remaining assets and the debt-structure-dependent default trigger complicates the resulting mathematical model, the flexibility of our framework overcomes these difficulties to provide reliable evaluations and theoretical insight into many empirical studies.

To demonstrate how simplifying debt structures and adopting naive financing settings generates inaccurate bond evaluation results, Fig. 1 illustrates the yield spread curves for simultaneously issued bonds (i.e., serial bonds) of the same firm extracted from empirical data and those generated from different structural models. The empirical studies in Helwege and Turner (1999) and Huang and Zhang (2008) suggest that most yield spread curves implied by serial bonds are upward-sloping despite the issuing firm's financial status.[3] Two typical examples of the yield spread curves for investment-grade issuer DIRECTV Holdings (in the black curve) and speculative-grade issuer Rockies Express Pipeline (in the gray curve) are illustrated in Fig. 1(a) to demonstrate this upward-sloping nature. We examine the reliability of different structural models by evaluating the issuing prices of three otherwise identical serial bonds issued by a hypothetical firm with maturities of 5, 10, and 20 years. In Fig. 1(b), the three serial bonds are either evaluated separately without considering the presence of other bonds (denoted by dashed curves) or by using our quantitative framework (solid curves). As noted by Jones et al. (1984, 1993), the former setting ignores the impacts of coexistent bonds and thus severely underestimates the bond yield spreads when the issuing firm is healthy as plotted in the black dashed curve. Furthermore, as the firm's creditworthiness deteriorates, the former setting generates a hump-shaped (plotted in dashed light gray) yield spread curve or a downward-sloping (dashed dark gray) one; these shapes are inconsistent with the upward-sloping nature found in the empirical studies. In contrast, our framework captures how the repayments of short-term bonds deteriorate the firm's solvency to further jeopardize the credit quality of the long-term bonds. Thus, it

[2] For example, Barclay et al. (2003) indicate that the issuing firm's choices of bond maturity are closely related to its investment opportunity. In addition, Rauh and Sufi (2010) and Hackbarth and Mauer (2012) show that unhealthy issuing firms may spread priority across debt classes. In addition, Chen et al. (2013) show that firms with high systematic risk favor long-term bond issuances and possess more stable debt maturity structure over the business cycle. Xu (2014) shows that speculative-grade firms actively extend their debt maturity structure in good times. Similarly, Kahl et al. (2015) show that, instead of continuing to use short-term bonds like commercial papers, firms with high rollover risk often issue long-term bonds to replace maturing short-term bonds.

[3] The empirical results in Helwege and Turner (1999) show that, in the primary market, over 80% of the yield spread curves implied by equal-priority bonds issued on the same day by the same speculative-grade issuer are upward-sloping; over 60% of the yield spread curves for those in the secondary market are also upward-sloping. The empirical investigation into the broader sample sets by Huang and Zhang (2008) shows that more than 80% of the yield spread curves for the cases of investment- and speculative-grade issuers are upward-sloping.

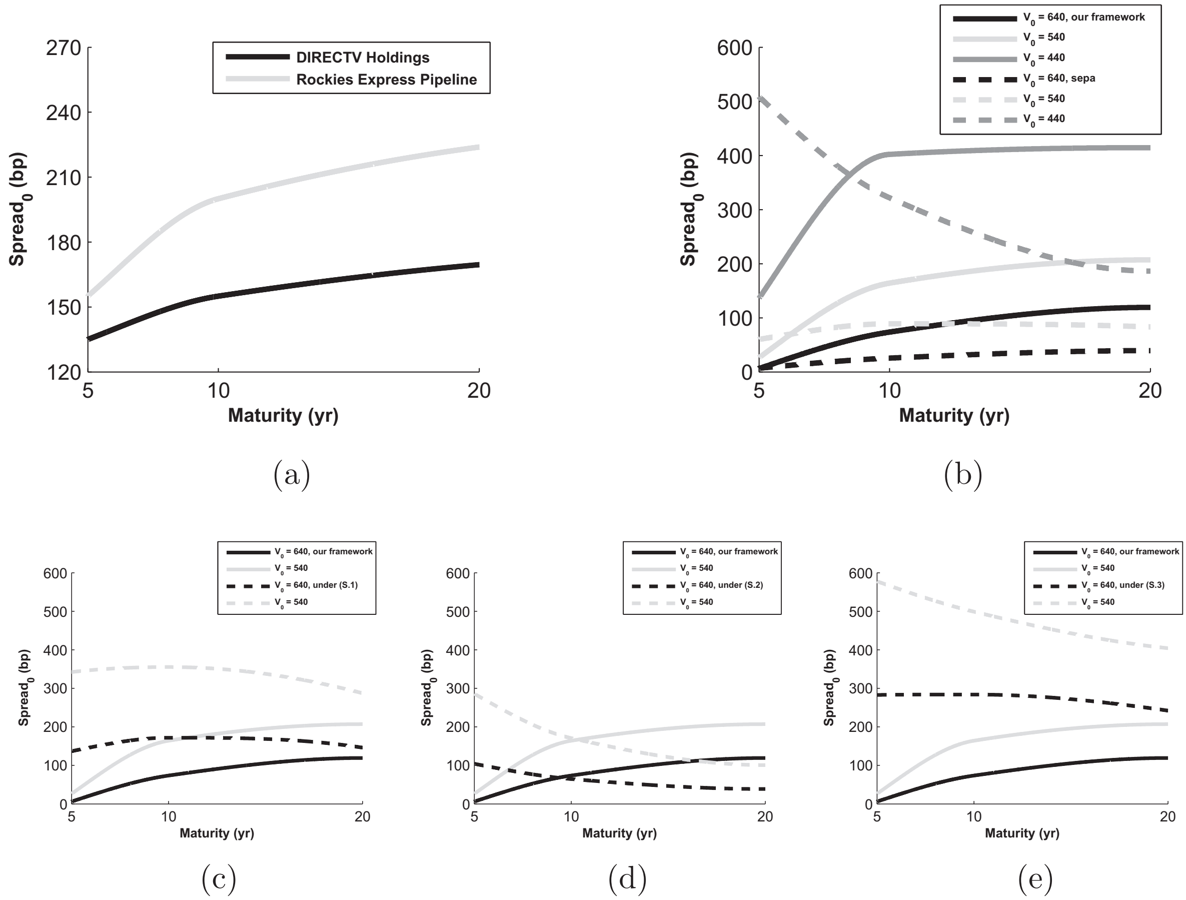


Fig. 1. *Shapes of yield spread curves.* Panel (a) illustrates the yield spread curves implied by equal-priority bonds issued on the same day by investment-grade issuer DIRECTV Holdings (TRACE CUSIP: 25459HAY1, 25459HBA2, 25459HAZ8) and speculative-grade issuer Rockies Express Pipeline (TRACE CUSIP: U75111AE1, U75111AF8, U75111AG6). The data is from Mergent Fixed Income Securities Database. The other panels illustrate how the structural models with different settings generate varied yield spread curves implied by three otherwise identical par bonds issued by a hypothetical firm with maturities of 5, 10 and 20 years. The risk-free interest rate is 2%, the firm value volatility is 20%, the lump sum of the face values of these three bonds is $300, the tax rate is 35%, and the loss ratio of the firm's asset value due to liquidation is 50%. The level of the firm's asset value at its issuance date V_0 is used as a proxy for the firm's prevailing financial status. The black and gray colors denote good and poor financial statuses of the prevailing firm, respectively. The solid curves denote the yield spread curves generated by our framework. The dashed curves in panel (b) are generated by evaluating these three bonds separately. The dashed curves in (c), (d), and (e) are generated by extant structural models with settings (S.1), (S.2), and (S.3).

does not significantly underestimate the bond yield spreads for a healthy firm (plotted as the black solid curve) and generates reasonable upward-sloping yield spread curves regardless of the issuing firm's financial status.

To ensure mathematical tractability when modeling complex debt structures, much literature adopts naive settings on default triggers and financing policies. Three typical simplified settings adopted widely in extant literature are examined under the aforementioned hypothetical scenario as illustrated in Fig. 1(c)–(e). The (S.1) setting adopts a unified default boundary without considering the repayment schedule implied by the issuing firm's debt structure (e.g., Black and Cox, 1976). (S.2) assumes that all future debt repayments are only financed by raising new equities (e.g., Geske, 1977), and (S.3) assumes that the issuing firm keeps its debt structure unchanged without considering its prevailing financial status (e.g., Leland and Toft, 1996).[4] Unlike the upward-sloping solid yield spread curves generated by our framework, adopting (S.1), (S.2) and (S.3) could lead to infeasible hump-shaped or downward-sloping curves as plotted by dashed curves denoted in Fig. 1(c)–(e), respectively.

To overcome computational difficulty in modeling complex debt structures, Broadie and Kaya (2007) and Wang et al. (2014) exploit the flexibility of the tree method. The former work considers the impacts of the reorganization processes under Chapter 11 of US bankruptcy code on bond prices, but the complex tree implementation leads to unstable pricing results due to nonlinearity errors (see Figlewski and Gao, 1999).[5] The latter work studies how to adjust the tree structure to stably evaluate multiple outstanding bonds of the same firm. Our framework incorporates the aforementioned consideration, such as the default trigger implied by a firm's debt structure, by exploiting the technique in the latter work. Thus it not only provides concrete quantitative measurements for the phenomena identified in extant empirical literature but also yields theoretical insight into these observations, such as reliable shapes for yield spread curves in Fig. 1, and the effect of the interest rate level (see Duffee, 1998), the firm value volatility (see Avramov et al., 2007), the firm's leverage ratio (see Collin-Dufresne et al., 2001; Flannery et al., 2012), and rollover risk (see Gopalan et al., 2014; Nagler, 2014) on bond yield spreads.[6] We also develop an indirect method to check the correctness of our framework by exploiting the capital structure irrelevance theory proposed by Modigliani and Miller (1958) as in Appendix B.

[4] The details of these three settings are discussed in Section 2.2.2.

[5] Broadie and Kaya (2007) demonstrate their oscillating numerical results in Figs. 5 and 6 of their paper. Indeed, analyzing the impacts of various reorganization procedures on the benefits and decisions of different claim holders without interference from numerical errors is an important topic for future study.

[6] See Appendix C and Section 4.1 for details.

The repayment schedule may change contingently due to triggers of bond covenants, and these changes increase the difficulty in analyzing bond values and claim holder' decisions. The following discussions show how our framework quantitatively models these covenants to provide theoretical insights and to explain the conflicts in past empirical studies. Linn and Stock (2005) empirically study the impact of a junior bond issuance on the previously-issued senior bond. They find that whether the junior bond matures earlier than the senior bond does not have a salient difference in the claim dilution effect as predicted in Ingersoll (1987), who argues the repayment schedule also implicitly influences the seniority of bonds. This might be because the holders of a previously-issued senior bond are usually granted limited rights to block certain payments to later-issued junior bonds to ensure their payments are fully repaid. Incorporating payment blockage covenants into our framework allows us to duplicate and to explain the phenomena observed in Linn and Stock (2005).

To capture contingent changes of a debt structure due to premature redemptions of callable or puttable bonds, a novel quantitative method, the "forest", is developed by systematically combining several trees arranged in layers; each tree captures one possible scenario of the debt structure, for instance before or after a bond premature redemption. This design is because each premature redemption can change the order of bond repayments and accordingly the default trigger, thus leading to wealth transfer among the firm's remaining claim holders. The forest method is, to our knowledge, the first quantitative method that can analyze complicated call policies of multiple outstanding callable bonds issued by the same firm; this is an open problem addressed in Jones et al. (1983).[7] The ability to evaluate the wealth transfer effect can be used to analyze the call delay phenomenon, in which a callable bond is not redeemed until its price far exceeds the call price. Our framework shows that the significance of the wealth transfer effect is influenced by the interest rate levels, the call price levels, the lengths of call protection periods, and the remaining time to maturities of callable bonds. These studies help us to reconcile the conflict between Longstaff and Tuckman (1994) and King and Mauer (2000) on the relationship between wealth transfer effects and call delay phenomena. In addition, our framework also analyzes how the wealth transfer effect due to triggers of poison put covenants embedded in the target firm's bonds decreases yield spreads of these bonds at the expense of the bidder's costs of debt financing for a leveraged buyout as studied in Cremers et al. (2007).

The rest of the paper is organized as follows. In Section 2, we describe our valuation framework, including the concepts of the remaining asset and the debt-structure-dependent default trigger. In Section 3, we first elaborate on the method in Wang et al. (2014) to implement the payment blockage covenant. Then we describe how a forest is developed to capture contingent changes of a firm's debt structure due to premature redemptions. Section 4 illustrates how our framework provides concrete quantitative measures as well as theoretical insight into the aforementioned empirical studies and further reconciles the conflicts in these studies. Section 5 concludes the paper.

2. Model settings

2.1. Fundamental settings of structural models

A structural model specifies the evolution of the market value of an issuing firm's assets and the conditions leading to default. We follow Merton (1974) by assuming that the firm's asset value at an arbitrary time t, V_t, obeys the following process under the risk-neutral probability measure:

$$dV_t = rV_t dt - P_t + \sigma V_t dz. \tag{1}$$

Here we follow Attaoui and Poncet (2013) by setting r to the long-term average interest rate since the impact of a stochastic interest rate can be negligible, as suggested in Ju and Ou-Yang (2006). P_t denotes the payout for dividends or contractually-obligated payments at time t; this depends on the different settings of future financing policies adopted in existent structural models, and is discussed in Section 2.2; σ denotes the volatility and can be viewed as a proxy for the firm's business risk as in Merton (1974). We follow Fan and Sundaresan (2000) by setting σ to a constant since the firm manager cannot alter the business risk arbitrarily due to restrictive covenants embedded in the outstanding bonds of the firm. dz denotes a standard Brownian motion.

To meet investment and finance requirements at different periods of time, a firm issues multiple bonds with different maturities, seniorities and covenants at different time points. Thus we model the firm's debt structure as comprised of N outstanding bonds: $B_1, \ldots, B_N$. The face value, the annual coupon payment, and the time to maturity for the ith bond B_i are denoted by F_i, C_i, and T_i, respectively. We set $0 < T_1 < \cdots < T_N = T$ for ease of later discussions. In a structural model, all bonds and the leveraged equity of the same issuing firm can be viewed as contingent claims on the firm's asset. For convenience, the values of these contingent claims at time t are denoted by $B_1(t, V_t), \ldots, B_N(t, V_t)$ and $E(t, V_t)$. To model the firm's value gained from tax shield benefits by using debt capital, we assume that the firm's coupon payments are tax-deductible at an exogenous tax rate τ, $\tau \in (0, 1)$. Raising debt capital, however, incurs a proportional cost of $k, k \in (0, 1)$, which is expressed as a fraction of the market value of the newly-issued bond (see Chen, 2010). Notice that k increases with the magnitude of the market recession and with the deterioration in the firm's creditworthiness.

Despite the tax shield benefits, using debt capital also incurs bankruptcy costs resulting from the costs of liquidation due to an inability to fulfill debt repayments. The key difference between the default triggers in our framework and in most existent structural models is the incorporation of the repayment schedule implied by the issuing firm's debt structure. This schedule is in practice an important factor when analyzing bond issuers' default risks.[8] Many existent structural models adopt unified default boundaries without considering the burdens due to scheduled repayments as discussed in the setting (S.1) in Section 2.2.2. In contrast, our framework follows Moody's definition of debt default as a firm misses a disbursement of a contractually-obligated interest or principal payment defined in bond indentures (see Ou et al., 2011). In addition, to focus on the disadvantages in oversimplifying debt structures and adopting naive financing policies, all structural models in this paper are analyzed based on the Chapter 7 proceedings as in Kuehn and Schmid (2014)[9]; that is, an insolvent firm is liquidated immediately after filing for bankruptcy. A constant fraction $\omega, \omega \in (0, 1)$, of the firm's asset value is lost as liquidation costs, such as the legal fees (see Leland, 1994). The leftover assets are then distributed according to the absolute priority rule as the evidence reported in Bris et al. (2006).

[7] Jones et al. (1983), without solving the problem, present a system of partial differential equations with complicated boundary conditions.

[8] For example, the Greek debt crisis has been analyzed with its payment schedule on the Economist website http://www.economist.com/blogs/graphicdetail/2015/04/daily-chart-7 and in many other related articles.

[9] Note that complex reorganization processes under Chapter 11 of US bankruptcy code are also widely studied (e.g., François and Morellec, 2004; Broadie et al., 2007; Galai et al., 2007). Implementing Chapter 11 using the tree method is preliminarily studied in Broadie and Kaya (2007); incorporating their implementation into our framework is a good topic for future study.

As fund suppliers, both bond and equity holders want to assure themselves of fair returns on their investments. Generally speaking, a firm manager acts for equity holders and tends to maximize their benefits at the expense of the bond holders (see Jensen and Meckling, 1976; Myers, 1977). This can be used to model the issuing firm's decisions, for instance, an optimal strategy for the early redemption of callable bonds analyzed in Section 4.3. To alleviate this agency problem (see Smith and Warner, 1979), the bond holders require collateral pledged for their bonds or restrictive covenants to ensure the security of their claims. Assets pledged as collateral influence the solvency of the firm; this factor is modeled by the debt-structure-dependent default trigger in our framework. Covenants protect the bond values either by restricting firm manager behaviors (e.g., in issuing new bonds) or by granting the bond holders the right to change the repayment order. Changing the repayment order causes wealth transfer among the holders of all outstanding claims and hence influences their decisions on enforcing covenants. To explain the relevant empirical phenomena analyzed in past studies and reconcile the conflicts among them, our framework quantitatively analyzes three such covenants: payment blockage covenants, poison put covenants, and call provisions. These three covenants are introduced as follows.

Payment blockage covenants grant senior bond holders the right to block scheduled payments to later-issued junior bonds in order to ensure that their payments are fully repaid before the payments to the junior bonds (see Linn and Stock, 2005; Davydenko, 2007). This covenant prevents the presence of payments to short-term junior bond holders from jeopardizing the effective seniority of long-term senior bonds. It can block all payments to junior bond holders during the blockage period $[t^* - \eta, t^*]$ to satisfy payments to senior bond holders if the firm defaults at time t^*.[10] In Section 4.2, we show that implementing this covenant explains the insignificant claim dilution effects on existing senior bonds due to short-term junior bond issuances found in Linn and Stock (2005).

Poison put covenants grant bond holders the right to sell the bonds back to the firm prematurely at a predetermined put price (abbreviated as PP) due to occurrences of predetermined unfavorable events such as a leveraged buyout (LBO).[11] Typically, the PP is equal to or exceeds the bond face value. In Section 4.4, we analyze how the wealth transfer effect resulting from triggering poison put covenants protects the holders of a target firm's bonds at the expense of the bidder's costs of debt financing for a LBO studied in Cremers et al. (2007).

Call provisions grant bond issuers the right to redeem the bonds prematurely at an effective call price (abbreviated as CP), which is defined as the call price predetermined in the bond's prospectus plus the accrued interest (see Thatcher, 1985). A callable bond usually contains the call protection period—the period that the bond is protected from being called prematurely—in order to protect the interests of bond holders. In Section 3.3, we develop the forest, a novel numerical method that analyzes complicated call policies of multiple outstanding callable bonds. The relationships between the wealth transfer effect due to premature redemptions and call delay phenomena that are widely examined by empirical literature are quantitatively analyzed in Section 4.3.

The value of bond B_i at time 0, denoted by $B_i(0, V_0)$, can be evaluated by different structural models introduced in the next subsection. Recall that the time to maturity for B_i is T_i and its yield to maturity, denoted by $Y_0^{T_i}$, can be expressed by the following equation, which makes the bond value equal to the lump sum of discounted future cash flows:

$$B_i(0, V_0) = F_i e^{-T_i Y_0^{T_i}} + \sum_{j=1}^{nT_i} \frac{C_i}{n} e^{-\frac{j}{n} Y_0^{T_i}}, \tag{2}$$

where n denotes the frequency of coupon payments per year. The corresponding yield spread $\text{Spread}_0^{T_i}$ can be derived by substituting $Y_0^{T_i}$ solved in Eq. (2) into the following equation:

$$\text{Spread}_0^{T_i} = Y_0^{T_i} - r.$$

2.2. Settings adopted in structural models

2.2.1. Our framework: the remaining assets and debt-structure-dependent default trigger

The key point that a compound option approach performs better (as mentioned in Eom et al., 2004) might be due to the fact that this approach takes into account the impacts of repaying coexistent outstanding bonds on other unmatured bonds. Note that bond repayments can be financed by either internal or external funds such as newly-raised equities or bonds. Thus financing decisions on repayments and hence default triggers are uncertain especially when the repayments are far into the future. Since adopting naive assumptions on financing policies and default triggers leads to inaccurate shapes of yield spread curves as in Fig. 1, our framework evaluates an issuing firm's bonds or equities with two novel concepts, remaining assets and debt-structure-dependent default triggers, to avoid making any assumptions on future financing policies.

Remaining assets can be viewed as a proxy for measuring a firm's ability to repay a certain obligation, for instance bond B_i's principal F_i at time T_i, under the burdens of previously-matured payments. It is defined as the *remainder* of the firm's asset value after repaying every required payment that occurs before time T_i. Specifically, let C_t^O denote the repayment amount that occurs at time t implied by the debt structure.[12] The process of remaining assets is then constructed by defining P_t in Eq. (1) as $C_t^O dS_t$, where S_t is a step function that increases by one at each repayment date. Therefore, the process of remaining assets follows a lognormal diffusion process between two adjacent payment dates and decreases by C_t^O at time t; in other words, the remaining asset value at a payment time t, V_t, can be expressed as $V_{t^-} - C_t^O$, where t^- denotes the time immediately preceding time t. The value decrements reflect the burdens of previous repayments and potentially lower the magnitude of V_{T_i}, which indicates a lower level of internal funds and a poorer ability to raise enough external funds (due to the low equity value or the debt overhang problem) to finance repayment F_i at time T_i.

We follow Moody's definition (see Ou et al., 2011) by defining a default event as the issuing firm's inability to fulfill repayments implied its debt structure. Specifically, a default occurs at payment time t once the level of its remaining assets V_{t^-} minus Λ_t, the frozen assets at time t due to restrictive covenants like collateral of secured bonds, cannot meet the repayment C_t^O; in other words, the firm defaults if V_{t^-} is lower than a debt-structure-dependent default boundary Θ_t, which is defined as $C_t^O + \Lambda_t$. Note that our framework is analogous to the compound option approach since whether each repayment is serviced depends on whether its previous payments are honored.

[10] The length of blockage period η usually ranges from 90 days to a year or more. See http://www.uccstuff.com/CLASSNOTES/SubordinatedDebt.shtml.

[11] Other unfavorable events include decapitalizations, recapitalizations, restructurings, mergers, acquisitions, share repurchases, leverage ratio increments, or downgraded credit ratings.

[12] To focus our analyses on the relationship between required payments and the firm's solvency, this paper does not consider dividend payments or other disbursements distributed to equity holders as in Ingersoll (1977a). Likewise, previous empirical literature (e.g., Collin-Dufresne et al., 2001; Avramov et al., 2007) does not consider dividend payments as explanatory variables of bond yield spreads.

In addition, issuances or rollover strategies in the near future are usually established; the impacts of these strategies on the yield spreads of a firm's existing bonds have also been widely studied empirically. Our framework incorporates these impacts by introducing the issuance cost k into the firm value process; in Section 4.1 we analyze empirical phenomena found in relevant literature. For example, issuing a new bond B with market value $B(t,V_t)$ at time t for investment purposes changes the leverage ratio and results in claim dilution as in Collin-Dufresne et al. (2001) and Flannery et al. (2012). Our framework evaluates all claims (including newly-issued bonds) as contingent claims on V_t, whose value is equal to the pre-issuance firm's asset value plus the net-of-cost proceeds from the issuance $C_t^I : V_{t-} + C_t^I$, where $C_t^I = (1-k)B(t,V_t)$. Gopalan et al. (2014) study the rollover risk for refinancing C_t^O by issuing new bonds; our framework adjusts the after-rollover firm value V_t as $V_{t-} - C_t^O + C_t^I$.

Prematurely redeeming a callable bond changes its prevailing debt structure and hence the repayment schedule, as well as the default trigger. Callable bonds tend to be redeemed at an optimal stopping time to maximize the benefits of equity holders; these redemptions redistribute wealth among remaining claim holders. To determine whether it is optimal to redeem a callable bond, two trees are required to evaluate the equity values before and after the redemption, respectively. The former tree evaluates all claims (including the callable bond) based on a debt structure that contains all unmatured payments of that callable bond. The latter tree evaluates all other claims (excluding the callable bond) based on a debt structure without that callable bond. The callable bond is redeemed prematurely if the equity value calculated in the latter tree is higher than its value in the former tree. Note that a premature redemption transfers the firm's status from the former to the latter tree, which as mentioned in Section 3.3 can be modeled by a novel numerical method, the forest, which combines these two trees.

2.2.2. Comparison with other frameworks

Instead of modeling a sophisticated default boundary based on the repayment schedule and restrictive covenants implied by an issuing firm's debt structure as mentioned above, most exogenous default boundary models exogenously specify "unified" default boundaries (see Davydenko, 2012) to preserve mathematical tractability as follows.[13]

(S.1) The default is triggered once the firm's asset value is lower than an exogenously given unified boundary constructed through simplified assumptions on covenants.

For example, many models adopt safety covenants (e.g., Black and Cox, 1976), maintenance covenants (e.g., Longstaff and Schwartz, 1995), or other similar covenants to unified default boundaries such as (the discounted value of) exogenously given constants (e.g., Kim et al., 1993), (a fraction of) market values of outstanding bonds (e.g., Briys and De Varenne, 1997; Ju and Ou-Yang, 2006), the face value of the firm's short-term bonds plus half of the face value of the long-term bonds (e.g., Crosbie and Bohn, 2002), 66% of the firm's total bond face value (e.g., Davydenko, 2012), and so on. Indeed, Davydenko (2012) notes that these types of covenants are rarely included in bond indentures in practice; therefore, they should be interpreted as simplified proxies for complex restrictive covenants implied by the firm's debt structure. However, he further notes that it is difficult to specify a unified boundary level to precisely separate insolvent firms from solvent firms, since empirically observed boundaries are widely dispersed among firms. Thus, even though these structural models can calibrate their unified default boundaries to perform reasonably well on average, relying on unified default boundaries may still contribute to poor cross-sectional prediction accuracy for bond yield spreads (see Eom et al., 2004). For example, following Davydenko (2012)'s suggestion to set the default boundary as 66% of the lump sum of all outstanding bonds' face values yields improper hump-shaped yield spread curves, as illustrated by the dashed curves in Fig. 1(c).

Other structural models determine their default boundaries endogenously to maximize equity values (e.g., Leland, 1994). However, the adopted financing policies for obligated payments significantly influence the reliability of these structural models. Two simplified policies that are widely adopted are listed as follows.

(S.2) Geske (1977) assumes that each bond repayment C_t^O that occurs at payment date t is simultaneously financed by issuing new equities without changing the firm's asset value; that is, $P_t = (C_t^O - C_t^I)dS_t = 0$ in Eq. (1). The firm files for bankruptcy at payment date t *once* it fails to raise new equities to fulfill the required payment C_t^O. This occurs when its equity value immediately prior to the payment date (i.e., $E(t^-, V_{t-})$) is less than the repayment C_t^O.

(S.3) Leland and Toft (1996) assume that the issuing firm rolls over matured bonds to keep the total amounts of outstanding bonds F and annual coupon payout C unchanged. The firm is also assumed to pay parts of its asset value V_t at a rate δ (i.e., $P_t = \delta V_t dt$ in Eq. (1)) to service bond repayments and dividends continuously. If the instantaneous payout $\delta V_t dt$ plus the gain of issuing a new bond $(1-k)B(t,V_t)dt$ exceeds the bond repayment $(F+C)dt$, the remaining part goes to equity holders as dividend payout.[14] Otherwise, the equity holders must absorb the deficit $(F+C)dt - \delta V_t dt - (1-k)B(t,V_t)dt$. Note that $Fdt - (1-k)B(t,V_t)dt$ denotes the rollover losses (or gains if it is negative) when replacing a matured bond with an otherwise identical new bond. The firm files for bankruptcy at time t *once* equity holders fail to absorb the deficiencies; in other words, when the equity value $E(t,V_t) = 0$.

(S.2) implicitly prevents an issuing firm from financing its required payments with internal funds or debt capital; this setting protects the payments to short-term bond holders from harming the values of long-term bonds. (S.2) also increases the incentive of equity holders to trigger default, because financing debt repayments with only equity capital significantly dilutes the value of the original equity holders. That may be why adopting (S.2) produces higher yield spreads for short-term bonds and generates downward-sloping term structures of yield spreads as illustrated by the dashed curves in Fig. 1(d).[15] In addition, unlike Merton (1974)'s model which significantly underestimates yield spreads for short-term bonds, adopting (S.2) could generate higher yield spreads for short-maturity bonds but be more likely to overestimate the spreads of short-maturity junk bonds (see Eom et al., 2004). However, Eom et al. (2004) indicate that such financing restrictions are rarely included in bond indentures. That could be why the downward-sloping yield spread curves generated by (S.2) do not fit the upward-sloping nature (see Fig. 1(a)

[13] Leland (2004) categorizes default boundaries in structural models into two types: exogenous default boundaries and endogenous ones.

[14] The new bond should have the same covenants—for instance the coupon rate and the face value—as other outstanding bonds. This stationary debt setting is widely adopted in academic literature such as Liu et al. (2006), Chen and Kou (2009) and He and Xiong (2012). The issuance price $B(t,V_t)$ depends on the firm's financial status V_t and other market conditions at time t.

[15] Similar pattern is predicted by Lando (2004).

studied in Helwege and Turner (1999) and Huang and Zhang (2008)).

Though (S.3) allows to finance debt repayments with internal funds and debt capital instead of solely using equity capital as in (S.2), its premise still causes structural models to generate improperly-shaped yield spread curves as illustrated by the dashed curves in Fig. 1(e). For relatively unhealthy issuing firms, equity holders must absorb the deficit $(F+C)dt - \delta V_t dt - (1-k)B(t, V_t)dt$ due to the very restrictive rollover and payout policies implied by (S.3). A poor financial status significantly lowers the price of newly-issued bonds $B(t, V_t)$ due to the fixed coupon requirement, and tremendous rollover losses cause the equity holders to precipitate bankruptcy which may push bond yield spreads to an incredibly high level. Eom et al. (2004) report that Leland and Toft (1996)'s model may overestimate bond yield spreads on average; this can be verified by observing that the dashed curves in Fig. 1(e) are higher than yield spread curves generated by other structural models. Eom et al. (2004) also claim that adopting (S.3) tends to overestimate the spreads of short-maturity bonds, which implies the downward-sloping term structures of yield spreads.

Contrary to the unified default boundary in (S.1), our framework portrays the default trigger according to the repayment schedule and covenants implied by the issuing firm's debt structure. Contrary to (S.2) and (S.3) which overly assume the firm's future financing policies, our framework preserves the nature of uncertainty about future debt financing policies unless the policies for the imminent due bonds are likely to be certain as in Gopalan et al. (2014). Adopting (S.1), (S.2), and (S.3) may produce improper hump-shaped or downward-sloping yield spread curves as illustrated by the dashed curves in Fig. 1(c)–(e), respectively. In contrast, the solid yield spread curves generated by our framework in Fig. 1(b) reflect the upward-sloping nature found in empirical studies as exhibited in Fig. 1(a).

3. Our quantitative framework

In a structural model, all outstanding bonds and equity of the same issuing firm can be viewed as contingent claims on the firm's asset value. Thus they can be evaluated by exploiting derivatives pricing methods. Our quantitative framework prices these bonds and equity by enhancing the tree method since it is a flexible and popular pricing method that easily handles complex debt structures as mentioned in Wang et al. (2014). In Section 3.1, we first show how the tree method models different financing assumptions and default triggers as discussed in Section 2.2. Next, we model the covenants that may change the repayment schedule to analyze their impact on the values of outstanding bonds and equity and hence the decisions of relevant claim holders. In Section 3.2, we implement the payment blockage covenant, which allows a previously-issued senior bond to block scheduled payments to newly-issued junior bonds. The resulting framework allows us to explain why the order of debt repayments is not necessarily a key determinant for yield spreads (see Linn and Stock, 2005) in Section 4.2. To model the premature redemption of bonds due to the exercising of embedded call options in bonds, we develop in Section 3.3 the forest, a novel method which consists of several trees to handle contingent changes of payment schedules due to premature redemptions. This framework is then used to analyze the wealth transfer effect among different claim holders and reconcile conflicts among past empirical studies on call delay phenomena in Section 4.3.

3.1. Tree structures

We use the generic example illustrated in Fig. 2 to demonstrate how the tree method adjusts its structure to simulate the asset value process of an issuing firm with multiple outstanding bonds under different financing assumptions and default triggers. To keep the illustrated tree structure and the following discussions simple, the firm is assumed to issue bonds B_1 and B_2 with face values of F_1 and F_2 and times to maturity of T_1 and T_2, respectively.[16] For simplicity, we also ignore tree structure adjustments for modeling coupon payments. During the period without debt repayments, the evolution of the firm's asset value reflects its business risk and is mainly modeled by the well-known CRR binomial structure (denoted by solid lines) proposed by Cox et al. (1979). At a debt repayment date, for instance T_1, the "remaining assets" concept is implemented by subtracting the repayment $C^O_{T_1}$ (denoted by downward arrows) from the firm value $V_{T_1^-}$ (dashed circles) to obtain the remaining value $V_{T_1} \equiv V_{T_1^-} - C^O_{T_1}$ (boldfaced circles). The debt-structure-dependent default trigger is implemented by having the firm default if its value is less than the default boundary Θ_{T_1}, which is defined as the repayment $C^O_{T_1}$ plus the value of the frozen asset Λ_{T_1}. Specifically, the firm defaults and is liquidated if its asset value reaches the nodes I, K, or L at time T_1 and no outgoing branches emit from these nodes. Note that this design captures the merit of the compound option approach; that is, whether B_2's holder can receive principal payments at time T_2 depends on whether the repayment at time T_1 is fulfilled. The trinomial structure (dashed lines) proposed by Dai and Lyuu (2010) is adopted here to adjust the tree structure. For example, the outgoing trinomial structure from the root node at time 0 can make one tree node, for instance I, coincide with the default boundary Θ_{T_1} at time T_1 to avoid the nonlinearity error problem identified by Figlewski and Gao (1999) from causing the tree to produce unstable pricing results. In addition, the outgoing trinomial structure from bold-faced nodes at time T_1 prevents an uncombined tree structure at the succeeding time step and decreases the computational cost for evaluating bonds and equity (see Dai and Lyuu, 2010). Note that this tree construction technique can also be used to model different financing assumptions and default triggers as discussed in Section 2.2. For example, the unified default boundary (i.e., (S.1)) assumption can be modeled by setting the default boundary Θ to the unified function defined in previous literature and by adjusting the tree to have a node that coincides with the boundary at every time step to avoid unstable pricing results. Rolling over bond B_1 at time T_1 by issuing another new bond B_3 with time to maturity T_3 and face value F_3 can be done by extending the tree structure from time T_2 to time T_3. The repayment at time $T_3, C^O_{T_3}$, is set as the repayments of principal F_3 and the coupon. The decrement of the firm value at time T_1 is set as $C^O_{T_1}$ minus $C^I_{T_1}$, where the latter part denotes the funds raised by issuing B_3.

3.2. Modeling payment blockage covenants

Note that the ability to fulfill a debt repayment can be greatly influenced by its repayment order arranged according to the presence of other outstanding bonds of the same firm as discussed in Fig. 1. However, the repayment schedule may change due to enforcement of the payment blockage covenants—covenants that grant the holders of a previously-issued senior bond the right to block the payments to newly-issued junior bonds to ensure payments to senior bond holders. This covenant has a salient effect on determining the credit spreads as argued in Linn and Stock (2005) but to our knowledge has never been quantitatively analyzed in any structural models. Here we use the generic two-bond example in Fig. 2 to demonstrate our method. Let B_2 be a senior

[16] Cases with more than two bonds can be modeled by repeating the tree structure for simulating the payments and default triggers in Fig. 2.

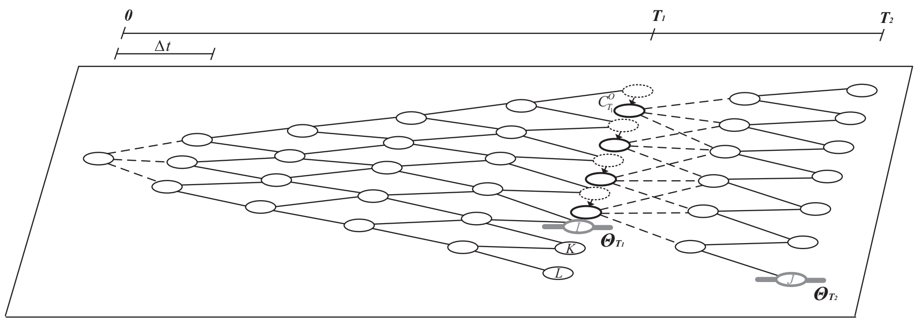


Fig. 2. *Tree that simulates the dynamics of the firm's asset value.* The firm is assumed to issue bonds B_1 and B_2 that mature at T_1 and T_2, respectively. Δt is the length of a time step. The CRR binomial structure proposed by Cox et al. (1979) is plotted by solid lines and the trinomial structure proposed by Dai and Lyuu (2010) is plotted by dashed lines. The boldfaced circles denote the remaining asset values after paying $C^O_{T_1}$ (marked by downward arrows) to redeem B_1. For simplicity, tree structure adjustments for modeling coupon payments are ignored. Nodes I and J are set to match the default boundaries (plotted by gray thick lines) to stabilize pricing results.

bond with a covenant to block payments to junior bond B_1. If the firm defaults at time t^*, then the payments to B_1's holder during the blockage period $[t^*-\eta,t^*]$ are blocked (if necessary) to satisfy the repayment of B_2, where η denotes the length of the blockage period contractually determined in the covenant.

To model how a payment blockage covenant transfers the value from B_1's holder to B_2's, we define $\texttt{BP}(t^*)$ as the value of the "blocked payments": the value of the total payments to B_1's holder that occur during the blockage period. We also define $\texttt{UP}(t^*)$ as the value of the "unblocked payments": the value of total payments to B_1's holder during the time interval $[t^*-\eta-\Delta t,t^*-\eta)$, where Δt denotes the length of a time step in the tree model. Payments that occur during this interval are no longer blocked and are guaranteed to be received by B_1's holder when we move from a node (in the tree) at time $t-\Delta t$ to the succeeding node at time t. The definitions of $\texttt{BP}(t^*)$ and $\texttt{UP}(t^*)$ can be used to clearly explain the real payments received by B_1's and B_2's holders under the potential enforcement of this covenant.

Since B_1, B_2, and equity E can be viewed as contingent claims on the firm's asset value, we apply the risk-neutral valuation method to evaluate them by summing the expected present values of the cash flows received by the claim holders. Here we denote the value of B_1, B_2, and E at time t with the remaining firm's asset value V_t by $B_1(t,V_t), B_2(t,V_t)$, and $E(t,V_t)$, respectively. The coupons C_1 and C_2 for bonds B_1 and B_2 are assumed to be paid semiannually. These three claims can be evaluated by applying the backward induction procedure in the tree illustrated in Fig. 2 sketched as follows.

Case 1. At the long-term bond B_2 maturity date T_2.

$$E(T_2,V_{T_2}) = \begin{cases} 0 & \text{if } V_{T_2} < \Theta_{T_2}, \\ V_{T_2} - F_2 - C_2/2 & \text{if } V_{T_2} \geq \Theta_{T_2}, \end{cases}$$

$$B_2(T_2,V_{T_2}) = \begin{cases} \min(\texttt{BP}(T_2) + (1-\omega)V_{T_2}, F_2 + C_2/2) & \text{if } V_{T_2} < \Theta_{T_2}, \\ F_2 + C_2/2 & \text{if } V_{T_2} \geq \Theta_{T_2}, \end{cases}$$

and

$$B_1(T_2,V_{T_2}) = \begin{cases} \texttt{UP}(T_2) + \max(\texttt{BP}(T_2) + (1-\omega)V_{T_2} & \text{if } V_{T_2} < \Theta_{T_2}, \\ \quad -F_2 - C_2/2, 0) & \\ \texttt{BP}(T_2) + \texttt{UP}(T_2) & \text{if } V_{T_2} \geq \Theta_{T_2}. \end{cases}$$

At time T_2, the firm survives if the firm value V_{T_2} is larger than the default boundary Θ_{T_2}; the residual of the firm value after repaying bonds goes to equity holders. Simultaneously, B_2's holder receives the payments in full and B_1's holder receives all blockage payments $\texttt{BP}(T_2)$ (payments to B_1's holder that occurred during period $[T_2-\eta,T_2]$) plus the unblocked payment $\texttt{UP}(T_2)$ (payments to B_1's holder that occurred during period $[T_2-\eta-\Delta t, T_2-\eta)$). Note that these two values can be zero if B_1's maturity date T_1 is earlier than $T_2-\eta-\Delta t$. Otherwise, the firm defaults and the residual firm value (minus the liquidation cost) $(1-\omega)V_{T_2}$ plus the blocked payments $\texttt{BP}(T_2)$ is first used to satisfy the payments to B_2's holder $F_2 + C_2/2$. The remaining value (if any) plus the unblocked payments $\texttt{UP}(T_2)$ goes to B_1's holder.

Case 2. $0 \leq t < T_2$

At any time t prior to T_2, the firm defaults if its asset value V_{t^-} cannot meet the default boundary Θ_t, defined as the repayment C^O_t plus the value of the assets frozen by restrictive covenants Λ_t. If the firm defaults, the firm is liquidated and equity holders receive nothing. The residual firm value (minus the liquidation cost) $(1-\omega)V_{t^-}$ plus the blocked payments $\texttt{BP}(t)$ is first used to satisfy B_2's required payments $\texttt{PV}_2(t)$, which is defined as the present value of all future unpaid payments to B_2's holder at time t. The remaining value (if any) plus the unblocked payments $\texttt{UP}(t)$ goes to B_1's holder.

The firm may be able to simultaneously fulfill the debt repayment C^O_t and satisfy the restrictive bond covenant. The remaining assets concept can be implemented by setting V_t to V_{t^-} minus C^O_t. The value of bond B_2 (or B_1) immediately prior to time t, denoted by $B_2(t^-,V_{t^-})$ (or $B_1(t^-,V_{t^-})$), is equal to the payments received by B_2's (or B_1's) holder at time t plus the continuation value—the expected present value of all B_2's (or B_1's) payments that occurred after time t. The value of the former part is equal to zero at a non-repayment date. The value of the latter part, denoted by $B_2(t,V_t)$ (or $B_1(t,V_t)$) for convenience, can be evaluated by the backward induction procedure which calculates the discounted expectation of the bond value at the next time step $B_2(t+\Delta t^-, V_{t+\Delta t^-})$ (or $B_1(t+\Delta t^-, V_{t+\Delta t^-})$). Similarly, equity value $E(t^-,V_{t^-})$ can be evaluated as the dividends paid at time t plus the continuation value $E(t,V_t)$ calculated by backward induction. Thus we have

$$E(t^-,V_{t^-}) = \begin{cases} 0 & \text{if } V_{t^-} \leq \Theta_t \text{ and } C^O_t > 0, \\ E(t,V_t) + \texttt{Dividend paid at time } t & \text{otherwise,} \end{cases}$$

$$B_2(t^-,V_{t^-}) = \begin{cases} \min(\texttt{BP}(t) + (1-\omega)V_{t^-}, \texttt{PV}_2(t)) & \text{if } V_{t^-} \leq \Theta_t \text{ and } C^O_t > 0, \\ B_2(t,V_t) + \texttt{Coupon paid at time } t & \text{otherwise,} \end{cases}$$

and

$$B_1(t^-,V_{t^-}) = \begin{cases} \texttt{UP}(t) + \max(\texttt{BP}(t) + (1-\omega)V_{t^-} & \text{if } V_{t^-} \leq \Theta_t \text{ and } C^O_t > 0, \\ \quad -\texttt{PV}_2(t^-), 0) & \\ \texttt{UP}(t) + B_1(t,V_t) & \text{otherwise.} \end{cases}$$

The coupon and the principal payments to B_1's holder can be blocked due to the payment blockage covenant embedded in B_2. For example, if the firm defaults at time t, the present value of all payments to B_1's holder that occur within the period $[t-\eta, t]$ (denoted by $\texttt{BP}(t)$) can be blocked to satisfy the payments to B_2's holder $\texttt{PV}_2(t)$. Payments that occur within the period $[t-\eta-\Delta t, t-\eta)$ (denoted by $\texttt{UP}(t)$) have just exited the blockage period and are received by B_1's holder.

Linn and Stock (2005) empirically study whether the repayment order implied by the maturities of both senior and junior bonds influences the credit quality of the senior bond. To analyze why their results contradict the arguments of Ingersoll (1987) in Section 4.2, we modify our framework slightly to analyze how the holder of the short-term senior bond can block the payments to that of the long-term junior bond. Let B_1 and B_2 be the short-term senior and long-term junior bonds, respectively. The payment blockage covenant ceases to exist when the payments to B_1's holder are fully satisfied at time T_1. So the usual backward induction without sophisticated designs for "BP" and "UP" is applied to time interval $(T_1, T_2]$. During interval $[0, T_1]$, the payment blockage covenant embedded in B_1 blocks the payments to B_2's holder; thus the procedure in the aforementioned Case 2 is applied by swapping the roles of B_1 and B_2.

3.3. Dealing with premature redemptions using a forest

Modeling premature bond redemptions by exercising call or put options embedded in bonds is still an open problem (see Jones et al., 1983) due to complicated analyses of the many possible debt structures and hence repayment schedules caused by contingent bond redemptions. For example, calling back a long-term callable bond before the maturity of a short-term bond changes the order of principal repayment and then transfers wealth among the equity, short-term bond, and long-term bond holders. To maximize the benefits of the equity holders, the issuing firm optimizes its redemption strategies by comparing the equity values under different debt structures, for instance before or after the premature bond redemption. To evaluate contingent claims and analyze optimal redemption strategies, we develop in this section the forest, a novel quantitative framework composed of several trees, each of which models one possible debt structure.

3.3.1. Forest construction

We describe the structure of a forest through a generic example that replaces the straight bond B_1 in Fig. 2 with an otherwise identical callable bond. The resulting forest illustrated in Fig. 3 is composed of two trees. As with the tree in Fig. 2, the upper-layer tree models the dynamics of the firm's asset value under the condition that B_1 is not yet called. Thus the firm either repays $C^O_{T_1}$ at time T_1 (marked by downward arrows) to redeem B_1 or default once its asset value is less than the default boundary Θ_{T_1}. The lower-layer tree models the dynamics of the firm's asset value under the premise that B_1 is called. Note that a firm under this condition is not required to redeem B_1 at time T_1 but must still redeem B_2 at time T_2, and default once its asset value is less than Θ_{T_2}.

For convenience, we define $\upsilon(\phi)$ as the firm's asset value at tree node ϕ. Calling B_1 back simultaneously changes the firm's asset value and the prevailing debt structure. For example, redeeming B_1 early at node U (located at time t in the upper-layer tree in Fig. 3) reduces the firm's asset value $\upsilon(U)$ by the effective call price $\texttt{CP}$; this change is denoted by a downward jump to node W. Note that B_1 is now removed from the debt structure; to reflect this removal, the outgoing trinomial branches from node W connect to its successor nodes, for instance X, Y, and Z, at time $t+\Delta t$ in the lower-layer tree. This trinomial structure is constructed by adopting Dai and Lyuu (2010)'s method which asymptotically simulates the firm value dynamics (see Eq. (1)) during the time interval $[t, t+\Delta t]$.

The above forest construction mechanism can be extended to more complicated scenarios. For example, if a callable bond is also refundable, then the firm can call back the bond using the proceeds from the issue of a new bond. Thus transiting from a upper-layer tree (before the redemption) to a lower-layer tree (after the

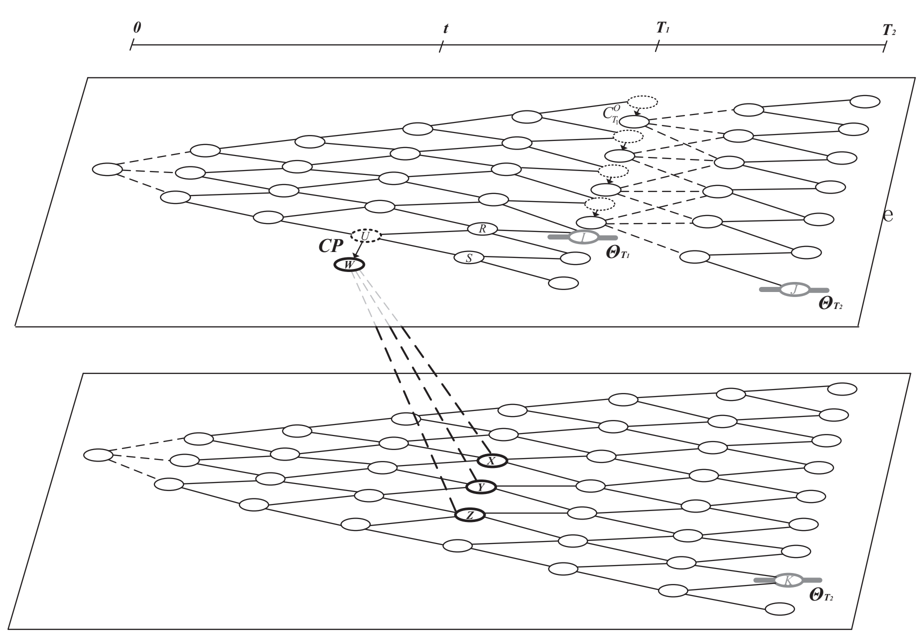


Fig. 3. *Two-layer forest for modeling a callable bond early redemption.* All debt structure settings follow those in Fig. 2 except for B_1, which is set as an otherwise identical callable bond. The evolution of the firm's asset value is modeled by the two-tree forest; the upper- and lower-layer trees model the evolution of the firm's asset value given B_1 is not called yet and B_1 is already called, respectively. The CRR binomial structure is plotted using solid lines and the Dai and Lyuu (2010)'s trinomial structure is plotted using dashed lines. To ensure pricing stability, nodes I, J, and K are set to match the default boundaries. Calling B_1 back reduces the firm value by the effective call price CP (denoted by a downward jump to node W) and the prevailing debt structure (denoted by the trinomial structure connected to the lower-layer tree).

redemption) changes the prevailing debt structure by replacing the callable bond with the new bond. The structure of the lower-layer tree is adjusted to reflect the change of the repayment schedule as discussed in Appendix A.1. In addition, a forest composed of multiple trees can be constructed based on the aforementioned two-layer forest to model debt structures with multiple outstanding callable bonds as discussed in Appendix A.2. These trees can model scenarios in which one or more callable bonds are redeemed prematurely, and thus resolve the intractable bond valuation problem raised in Jones et al. (1983). The resulting forest framework can be used to analyze bond redemption strategies, the wealth transfer among claim holders, and phenomena such as the conflicts found in empirical studies in Section 4.3.

3.3.2. Decision on premature redemptions

To maximize the benefits of equity holders, a callable bond issuer determines whether it is optimal to redeem an outstanding callable bond at each call date. Each tree in the forest simulates one possible debt structure caused by calling certain outstanding bonds back; thus it can be used to evaluate the values of equity and bonds under this debt structure. Hence an issuing firm can compare the equity values under different redemption strategies to identify the best one.

Here we use node U in the aforementioned generic example in Fig. 3 to demonstrate the process of decision making on calling B_1 back. Recall that the upper- and lower-layer trees model scenarios in which the B_1 is not yet or is already called, respectively. If the firm does not call the B_1 back at node U, the "**N**on-called" equity value, denoted by $E^N(t^-, \upsilon(U))$, is evaluated as the dividend paid at node U plus the continuation value. The continuation value represents time t's expected present value of future equity values at node U's successor nodes R and S, and this value can be calculated using backward induction. If, however, B_1 is redeemed at node U, then the firm's asset value jumps downward from node U to node W to reflect the burden of repaying the effective call price CP. The outgoing branches from W connect to nodes X, Y, and Z located in the lower-layer tree to reflect the removal of B_1 from the debt structure. Thus the "**C**alled" equity value, denoted as $E^C(t^-, \upsilon(U))$, is evaluated as the dividend paid at node U plus the continuation value calculated using backward induction from node W's successor nodes X, Y, and Z.

The redemption decision is made by comparing $E^C(t^-, \upsilon(U))$ and $E^N(t^-, \upsilon(U))$. If the former is larger, the firm redeems B_1 at node U, and the continuation value for B_2 is evaluated as the discounted expectation of B_2's future values at nodes X, Y, and Z. If the latter is larger, B_1 remains outstanding, and the continuation value for B_1 (B_2) is evaluated as the discounted expectation of B_1's (B_2's) future values at nodes R and S. Note that the above process of decision making can be applied to all the nodes located at called dates in the upper-layer tree. The values of all claims at nodes that are not located at any call date can be evaluated using backward induction from the node's successor nodes in the upper-layer tree.

4. Numerical results with empirical implications

By exploiting the flexibility of the tree method, the proposed quantitative framework faithfully captures various aspects of complicated debt structures such as the repayment schedule and the seniority of an issuing firm's outstanding bonds, and hence matches and explains past empirical studies. We resolve the numeric unstability problem addressed in Figlewski and Gao (1999) to produce stable pricing results as illustrated in Appendix B. We also propose a method to indirectly verify the accuracy of our pricing results by taking advantage of the capital structure irrelevance theory proposed by Modigliani and Miller (1958). The experiments in Appendix C illustrate how our framework quantitatively analyzes factors that influence the yield spreads of straight (or callable) bonds to explain phenomena found in empirical studies such as Duffee (1998), Avramov et al. (2007) and Dass and Massa (2014).

The repayment schedule may change due to new bond issuances, triggers of bond covenants, or the exercise of embedded options in bonds. These changes make it difficult to evaluate claim holders' values and analyze their decisions. In this section we analyze four such scenarios using our quantitative framework discussed in Section 3. In Section 4.1, we analyze how the amounts, the seniorities, and the maturities of newly-issued bonds dilute the values of existing outstanding bonds for comparison with the empirical results in Collin-Dufresne et al. (2001) and Flannery et al. (2012). The impact of financing nearly-mature bonds with internal or external funds under different levels of market liquidity on the yield spreads of the existing bonds are also studied for comparison with Gopalan et al. (2014) and Nagler (2014). In Section 4.2, we examine the impact of bond replacements and payment blockage covenants on repayment orders and hence the values of existing outstanding bonds to explain the observations in Linn and Stock (2005). The forest method discussed in Section 3.3 models debt structures that contain callable or puttable bonds. Section 4.3 studies how the call delay phenomenon is caused by the interaction effect (see Acharya and Carpenter, 2002) and the wealth transfer effect (see Longstaff and Tuckman, 1994). We then attempt to reconcile the conflicts between Longstaff and Tuckman (1994) and King and Mauer (2000) by finding the reasons—for instance, the interest rate level during the sample period—that influence the significance of the empirical tests on an issuing firm's call policy considering the wealth transfer effect. Section 4.4 illustrates how a poison put covenant can protect the target firm's bonds against a leveraged buyout by increasing the bidder's cost to raise debt capital as argued in Cook and Easterwood (1994) and Cremers et al. (2007).

In the following experiments, all coupons are paid semiannually (i.e., $n = 2$ in Eq. (2)). We follow Welch (1997) and Gorton and Kahn (2000) and consider bank loans to be the most senior bond type in a corporate debt structure. All bonds are assumed to be debentures except for bank loans which are fully secured by the firm's assets. In addition, all newly-issued bonds are assumed to be issued at par for consistency. Numerical settings such as the interest rate r, the firm's asset volatility σ, the tax rate τ, the bankruptcy cost ω, and the bond issuance cost k basically follow Leland (1994) and He and Xiong (2012).

4.1. Analyzing planned issuances and rollovers

4.1.1. Issuance strategies and claim dilution effects

In addition to analyzing complicated debt structures, our framework can also be used to analyze the impact on the values of unmatured bonds and equities when new bonds are issued or when mature bonds are rolled over. This capability allows us to explore the factors that influence yield spreads and hence analyze bond issuance strategies. Note that a planned issuance now (or in the near future) increases the current (or future) leverage ratio of the issuing firm and hence increases future required payments, the default likelihood, as well as the yield spreads of bonds. This might partially explain why Collin-Dufresne and Goldstein (2001) argue that bond yield spreads contain information about the firm's current leverage and about investor expectations with respect to the firm's future leverage. In addition, Flannery et al. (2012) empirically identify that the impact of change in the expected future leverage on yield spreads is much more salient than that of change in the current leverage. To analyze the influence of current and expected future leverage on yield spreads, we introduce planned

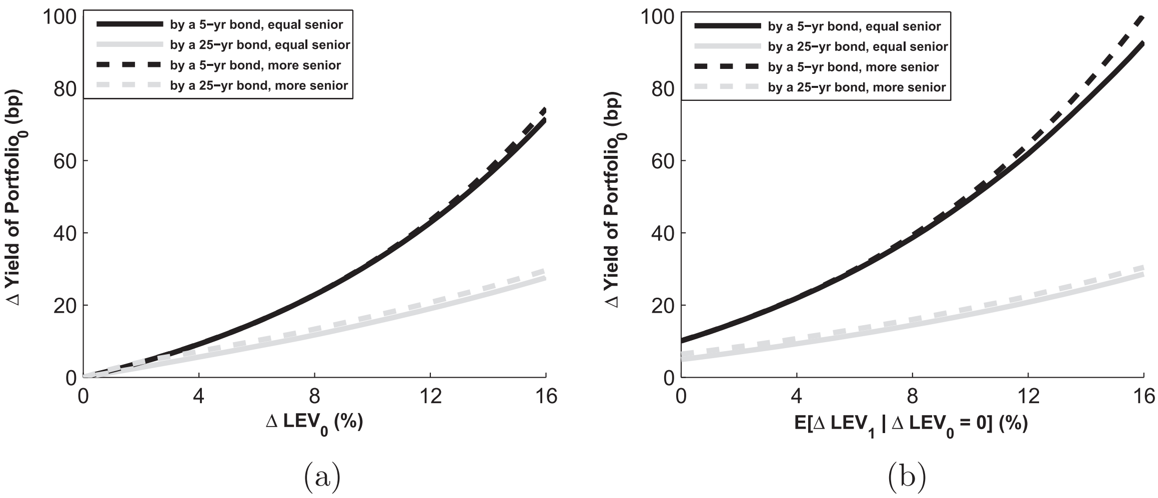


Fig. 4. *Impact of bond issuances on the portfolio yield of outstanding bonds.* Panels (a) and (b) denote scenarios in which the new bond issuance occurs immediately or one year later, respectively. The $\Delta \texttt{LEV}_t$ on the x-axis represents the change in the leverage ratio at time t due to a new bond issuance; the y-axis denotes the change in the portfolio yield of outstanding bonds due to the new bond issuance. Black and gray curves deNote 5- and 25-year maturities for the newly-issued bond, respectively. Solid and dashed curves denote new bond seniorities that are equal to or more senior than other outstanding bonds, respectively. The issuing firm is assumed to have five equal-priority \$100 outstanding bonds with remaining time to maturities of 1, 10, 16, 20 and 30 years and coupon rates of 7%, 8%, 10%, 11% and 14%, respectively. The initial firm value V_0 is 1000, its volatility $\sigma = 20\%$, the risk-free rate $r = 6\%$, the issuance cost $k = 1\%$, the liquidation cost $\omega = 50\%$, and the tax benefit $\tau = 35\%$.

bond issuances to change the leverage ratio and analyze the corresponding impact on yield spreads as illustrated in Fig. 4. We follow Collin-Dufresne et al., 2001 in calculating the leverage ratio at time $t, \texttt{LEV}_t$, as

$$\texttt{LEV}_t = \frac{(\texttt{Totalfacevalueofbonds})_t}{(\texttt{Totalfacevalueofbonds})_t + (\texttt{Marketvalueofequity})_t},$$

where $(\texttt{Marketvalueofequity})_t$ is evaluated by our framework. The expected future leverage in Fig. 4(b) is proxied by the expected future leverage one year ahead as in Flannery et al. (2012). We observe that the change of the issuing firm's leverage ratio is positively related to the change of yield for the portfolio of the firm's outstanding bonds regardless of the issuance date, seniority, and maturity date of the newly-issued bond. By comparing the curves in Fig. 4(b) with those in Fig. 4(a), our quantitative framework confirms Flannery et al. (2012)'s finding that increments in the expected future leverage ratio have a greater impact on the change of portfolio yield than increments in the current leverage ratio.

Our quantitative framework can also be used to analyze the impact of factors other than leverage ratios. By comparing the black curves with the gray curves in Fig. 4, we observe that new short-term (5-year) issuances result in a more significant claim dilution effect on other outstanding bonds than long-term (25-year) issuances. This observation is consistent with the argument in Ingersoll (1987) that issuances of short-term bonds may deteriorate the credit quality of unmatured bonds more. Similarly, by comparing the solid curves with the dashed curves, we observe that the credit quality of other outstanding bonds deteriorates with the seniority of the new issuance. Note that the impact of the maturity or seniority of newly-issued bonds becomes more significant with increments in the bond issuance size (or the leverage ratio). In addition, the rollover risk can also be estimated by our framework as in Fig. 4(b). Specifically, proceeds raised by issuing new bonds occurring at one year later are first used to refinance bonds that mature at year 1 (B_1) and thus the increment in the portfolio yield given the increment in the expected future leverage ratio is zero purely reflects the risk for rolling over B_1. A more detailed analysis for rollover risk is provided in the next subsection.

4.1.2. Rollover risk analyses

The impact of potential rollover risk as implied by the issuing firm's debt structure on its creditworthiness has been widely studied recently. Gopalan et al. (2014) empirically confirm that a firm with a greater proportion of bonds that will be rolled over within one year is more likely to experience severe credit deterioration; therefore, long-term bonds of the same firm are more likely to be traded at higher yield spreads. This phenomenon has been found to be more significant during recession years. Nagler (2014) studies how the illiquidity of bonds raises rollover risk and hence the entire term structure of bond yield spreads.

To examine the negative impact of rolling over short-term bonds on other outstanding bonds of the same firm, Fig. 5(a) shows that the yield spread of a 16-year bond (denoted by the y-axis) increases with increments in a 1-year bond's face value given the firm's the total amount of outstanding bonds is unchanged. Note that the dotted curves denote a scenario in which the 1-year bond is financed by internal funds; the differences between solid (or dashed) curves and dotted ones reflect the cost of issuing new bonds and the corresponding rollover risk. Obviously, yield spread differences increase with increments in the 1-year bond's face value, which is consistent with the aforementioned Gopalan et al. (2014)'s observation that credit deteriorates with increments in the proportion of short-term bonds being rolled over. Note that the firm's financial status (proxied by initial firm value V_0) is negatively related to the illiquidity of bonds and hence issuance cost k (defined in Section 2.2). Here we compare the rollover risk given a good financial status (with a high V_0 1000 and low k 1%) plotted in black curves and a poor status (low V_0 700 and high k 2%) plotted in gray curves. The yield spread difference for the unhealthy firm tends to be more significant than that for the healthy firm. This reveals an asymmetric effect of rollover risk on credit risk: rollover risk is much more devastating to unhealthy firms than to healthy firms. In addition, rollover risk is also influenced by the maturities of bonds issued to refinance debt repayments. Here we compare the rollover risk between a long-term 25-year bond issuance plotted in dashed curves with a short-term 5-year bond issuance plotted in solid curves. The yield spread difference tends to be more significant when issuing short-term bonds to refinance, which reveals another asymmetric effect: rollover risk is

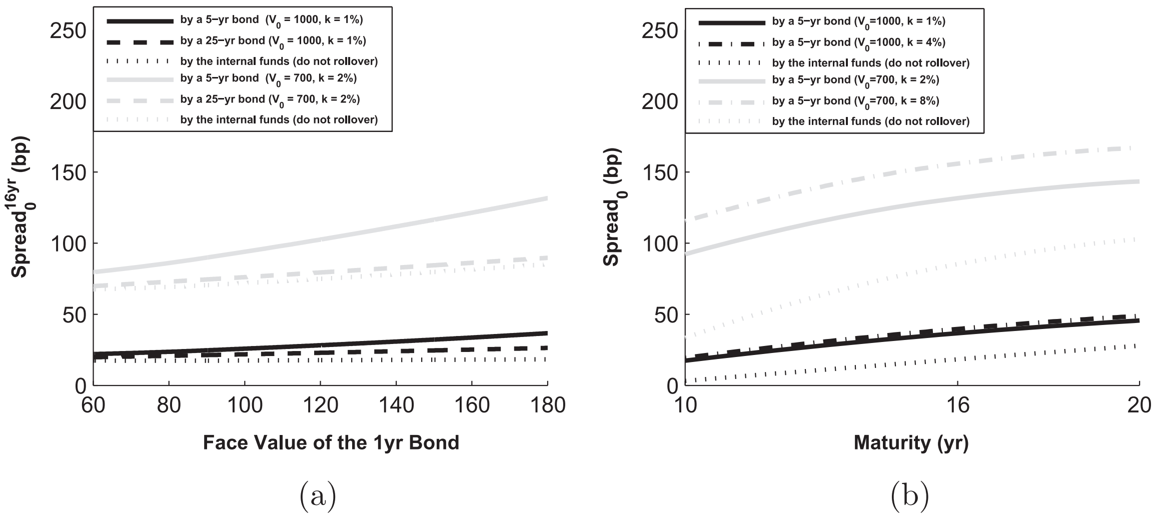


Fig. 5. *Rollover risk analyses.* The debt structure of the issuing firm is the same as the five-outstanding-bond structure studied in Fig. 4 except for the face value of the 1-year bond in panel (a) which is denoted by the *x*-axis, and in panel (b) is set to \$180. The face value of the 30-year bond is tuned such that the total amount of outstanding bonds is equal to \$500. In both panels, the black and gray colors represent a case with a good financial status ($V_0 = 1000$ with a low issuance cost of $k = 1\%$ or 4%) or with a poor status ($V_0 = 700$ with a high cost of 2% or 8%), respectively. Dot curves denote a scenario in which the 1-year bond is financed by the firm's internal funds. In panel (a), the *y*-axis denotes the yield spread for the 16-year bond. At the maturity of the 1-year bond, the repayment is refinanced by issuing a new 5-year equal-priority bond (denoted by solid curves), or by a new 25-year equal-priority bond (denoted by dashed curves). Panel (b) displays yield spread curves implied by coexistent 10-, 16- and 20-year bonds given that the 1-year bond is intended to be rolled over by another 5-year equal-priority bond. Solid curves and dashed-dotted curves denote low and high issuance costs due to different market liquidity conditions when the initial firm value is 1000 or 700. Other numerical settings are $r = 6\%, \sigma = 20\%, \tau = 35\%$ and $\omega = 50\%$.

much more salient, as the maturing bond is replaced by a short-term bond issuance. Combining these two asymmetric effects helps to explain Kahl et al. (2015)'s finding that short-term bonds are less likely to be used by relatively unhealthy firms to replace maturing bonds to alleviate the negative impact of rollover risk on creditworthiness.

Fig. 5(b) plots yield spread curves under different financing policies and issuing firm's financial statuses. It can be observed that rollover risks rise entire yield spread term structures. Moreover, issuance costs increase further in recession bond markets which increases bond yield spreads further (compare solid curves with dot-dashed curves).

4.2. Bond replacement and payment blockage covenants

The yield spread of a bond depends on its effective repayment priority determined by its seniority and the repayment schedule arranged according to the issuing firm's debt structure. For example, the repayment of a short-term junior bond can weaken the firm's financial status and thus deteriorate the effective repayment priority of another long-term senior bond. This effective priority may change due to replacement of other coexistent bonds or due to the existence of payment blockage covenants. Our framework measures the impact of changing effective priorities on bond yield spreads to explain the phenomena and conflicts found in previous studies. Linn and Stock (2005) show that replacing an existing bank loan (BL) with a new junior bond (NewJB) decreases the yield spread of another outstanding senior bond (SB), and that the magnitude of the decrement increases with the level of the replacement size. Apparently, this is because replacing the most senior BL with a NewJB improves the relative priority of the SB. Surprisingly, whether the NewJB matures earlier or later than the SB does not significantly affect the decrement magnitude of the SB's yield spread *on average* unless the firm's creditworthiness deteriorates further. This observation contradicts Ingersoll (1987)'s inference that the repayment of the NewJB prior to the maturity of the SB has a salient dilution effect on the SB. This may be because the previously-issued SB is protected by a payment blockage covenant which grants the SB's holders the right to block scheduled payments to the NewJB's holders that occur during the so-called payment blockage period to assure that payments to the SB are fulfilled. Our framework models this covenant as discussed in Section 3.2 to analyze the aforementioned empirical findings and conflicts as follows.

To examine the impact of bond replacement and the payment blockage covenant, we study a hypothetical five-bond debt structure slightly modified from the one considered in Section 4.1 to fit the scenario analyzed by Linn and Stock (2005) as illustrated in Fig. 6. Note that to focus our analysis solely on the impact of change in relative priority due to the bond replacement, the BL is replaced by a NewJB with the same face value and maturity. Both panels in Fig. 6 show that the bond replacement decreases the SB's yield spread (denoted by negative $\Delta\mathtt{Spread}_0^{10\mathrm{yr}}$); this phenomenon is more pronounced with the increment of the bond replacement size. These results are consistent with the observations in Linn and Stock (2005). To analyze the conflict between Ingersoll (1987) and Linn and Stock (2005) on the impact of the relative maturity defined as NewJB's maturity minus SB's, we compare the influence of the absence or existence of the payment blockage covenant in the SB with a 2-year blockage period in Fig. 6(a) and (b), respectively. In Fig. 6(a), whether the NewJB matures earlier or later than the SB (denoted by negative or positive relative maturity) significantly influences the magnitude of $\Delta\mathtt{Spread}_0^{10\mathrm{yr}}$, which is consistent with the aforementioned Ingersoll (1987)'s argument. However, introducing the payment blockage covenant as illustrated in Fig. 6(b) protects the benefits of SB's holders by blocking payments to NewJB that occur within the blockage period. Therefore, given that NewJB matures prior to the maturity of SB but within the blockage period (i.e., the relative maturity is within the range $[-2, 0)$), the decrement of $\Delta\mathtt{Spread}_0^{10\mathrm{yr}}$ in Fig. 6(b) is more significant than that illustrated in Fig. 6(a). This could explain why Linn and Stock (2005) find that the relative maturity is not a significant factor in explaining $\Delta\mathtt{Spread}_0^{10\mathrm{yr}}$.

Linn and Stock (2005) also show that the issuing firm's financial status could influence the explanatory power of the relative maturity on $\Delta\mathtt{Spread}_0^{10\mathrm{yr}}$; we analyze this by using different initial

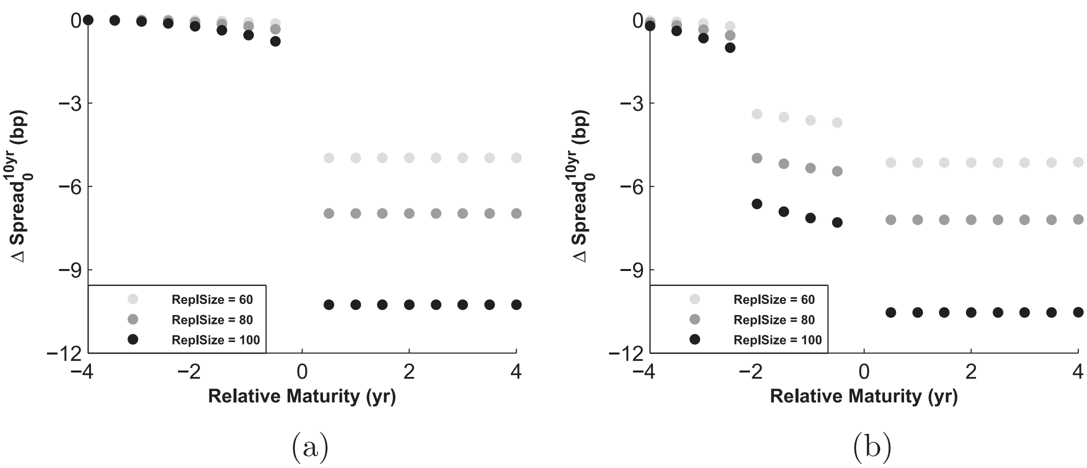


Fig. 6. *Impact of bond replacement and payment blockage covenant on* SB *yield spread change.* The debt structure is the same as the five-bond structure studied in Fig. 4 except that 1-year bond is changed to a bank loan (BL), the 10-year bond is a senior bond (SB), and other bonds are junior bonds. For comparisons, three different face values of BL (or the replacement size "ReplSize" by issuing NewJB) are denoted by different colors illustrated in the lower left corners in both panels. The face value of the 30-year bond is tuned such that the total amount of outstanding bonds is equal to $500. The "Relative Maturity" on the x-axis is defined as NewJB's maturity minus SB's. The $\Delta\mathtt{Spread}_0^{10yr}$ on the y-axis denotes the change of SB's yield spread by replacing BL with NewJB. Panels (a) and (b) consider the absence or existence of the payment blockage covenant in the SB with a 2-year blockage period, respectively. Other numerical settings are $V_0 = 1000, r = 6\%$, $\sigma = 20\%, k = 1\%, \tau = 35\%$, and $\omega = 50\%$.

firm values V_0 to proxy different statuses as in Fig. 7. In Fig. 7(a), the decrement of $\Delta\mathtt{Spread}_0^{10yr}$ becomes more significant with decrements of V_0, which implies that the improvement in the SB's credit quality provided by the bond replacement becomes more significant with the deterioration of the firm's financial status. To imitate regression analysis which estimates the relationship among different factors by aggregating the impacts of these factors' values from collected samples, we calculate the average of $\Delta\mathtt{Spread}_0^{10yr}$, denoted by Ave. $\Delta\mathtt{Spread}_0^{10yr}$, under different scenarios in Fig. 7(b). For example, node A at the solid black curve denotes a scenario in which NewJB matures later than SB, V_0 is 800, and SB contains a payment blockage covenant. The value of Ave. $\Delta\mathtt{Spread}_0^{10yr}$ is calculated by averaging the values of eight sample points in the dotted rectangle in Fig. 7(a). This allows us to measure the average impact of V_0 on the $\Delta\mathtt{Spread}_0^{10yr}$ under different scenarios listed in the bottom right corner in Fig. 7(b). Given that the payment blockage covenant is present (denoted by solid curves), the difference between the scenario in which the relative maturity is negative (denoted by the gray solid curve) and positive (black solid curve) becomes more significant with decrements of V_0; this confirms Linn and Stock (2005)'s finding that the explanatory power of the relative maturity becomes more significant when the firm's financial status deteriorates. In addition, the credit enhancement for the SB provided by its payment blockage covenant can be measured by the differences between dashed curves and solid curves. We see that this effect is insignificant when NewJB matures later than SB by comparing the black dashed and solid curves. By comparing gray dashed and solid curves, this effect becomes more significant with the deterioration of the firm's financial status when NewJB matures earlier than SB.

4.3. Optimal call policies for complex debt structures

The forest method introduced in Section 3.3 can model the contingent change of an issuing firm's debt structure due to the exercise of options embedded in its outstanding bonds. Thus we can theoretically analyze claim holders' exercise decisions to explain phenomena and conflicts found in past studies. This section analyzes call policies for American-style callable bonds that can be redeemed by the issuing firm at any time prior to maturity. Brennan and Schwartz (1977) and Ingersoll (1977a) suggest that a callable bond should be redeemed immediately once its market value exceeds the effective call price CP (i.e., the call price plus the accrued interest). This policy, termed the "textbook policy" by Longstaff and Tuckman (1994), can explain the empirical phenomenon that a low interest rate environment entails high bond values and thus a high likelihood for early redemptions. However, many empirical studies find that the market value of an about-to-call bond is usually higher than its CP, which entails that the issuing firm tends to defer the call decision. In addition to market frictions like the tax mentioned in Ingersoll (1977b), the call delay phenomenon might result from the interaction effect and the wealth transfer effect. The former effect proposed by Acharya and Carpenter (2002) states that the issuing firm tends to defer call decisions since an immediate redemption decision also ruins the firm's option to potentially default on the callable bond in the future. The latter effect, studied by Jones et al. (1984); (Longstaff and Tuckman (1994), hereafter LT) states that, under a complex debt structure with multiple outstanding bonds, an early redemption of one bond may redistribute wealth to holders of other outstanding bonds other than equity holders. This effect defers the firm's call decision to protect the interests of equity holders. However, the empirical studies in (King and Mauer (2000), hereafter KM) empirically reject LT's argument. The following experiments will first quantitatively examine these two effects and then explore the reason for the inconsistent empirical results made by LT and KM.

To capture the impacts of the interaction and wealth transfer effects on call decisions, Table 1 displays the evaluation results for different bond and equity holders under hypothetical five-bond debt structures. All outstanding bonds are straight bonds except for B_3 which is a callable bond under two call policies denoted as $\mathcal{D}_c^T$ and $\mathcal{D}_c^M$, in which the superscripts "T" and "M" denote that the call policy for B_3 is the "**T**extbook" policy and is the policy that "**M**aximizes" the equity value, respectively. By comparing the values of bonds and equity in case $\mathcal{D}_c^T$ with those in the otherwise identical five-straight-bond case $\mathcal{D}_s$, the potential early redemption of B_3 under the textbook policy significantly decreases B_3's value; corresponding benefits are redistributed to holders of B_4 and B_5 which mature later than B_3. This is because the burden of repaying the principal of B_3 mainly deteriorates the credit quality of B_4 and B_5 rather than B_1 and B_2; thus the benefit of redeeming B_3 early

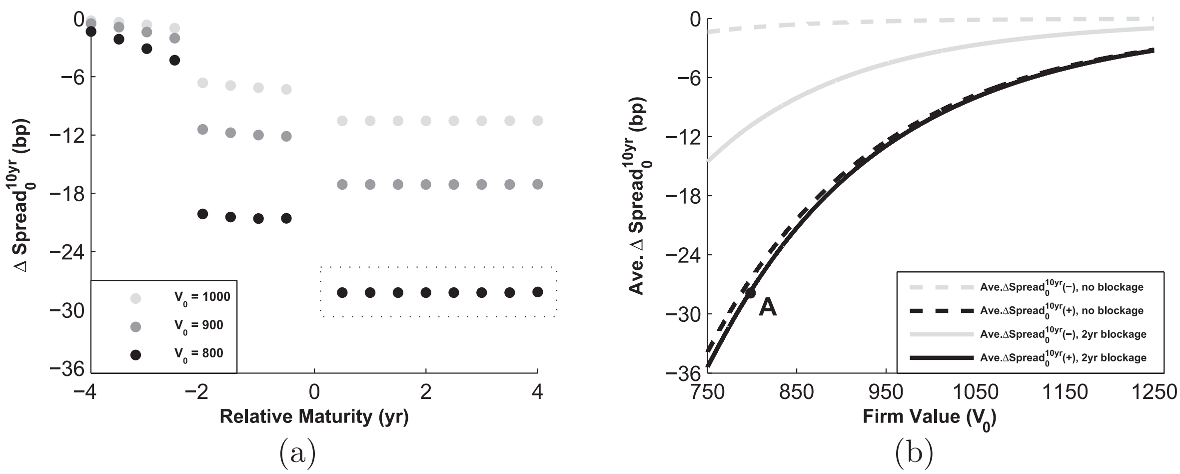


Fig. 7. *Issuing firm's financial status, relative maturity, and significance of* SB *yield spread change.* The debt structure is identical to the structure studied in Fig. 6 except for the face value of BL (or the bond replacement size) which is set to \$100. Panel (a) displays the impact of the relative maturity on the yield spread change of SB with a 2-year blockage covenant under different financial statuses (proxied by the various V_0 listed in the lower left corner). Panel (b) illustrates the impact of the issuing firm's financial status and the payment blockage covenant on the average yield spread change of SB (denoted by $\mathtt{Ave.\Delta Spread}_0^{\mathtt{10yr}}$ on the y-axis). In the bottom right corner, "−" and "+" denote that the ranges of relative maturity lie within $[-4,0)$ and $(0,4]$, respectively. Solid and dashed curves indicate that SB contains or does not contain a payment blockage covenant, respectively. Other numerical settings are $r = 6\%, \sigma = 20\%, \tau = 35\%$ and $\omega = 50\%$.

mainly compensates the holders of B_4 and B_5. The textbook policy might even harm the benefit of equity holders since the early redemption benefits are mainly absorbed by the holders of B_4 and B_5. Obviously, the textbook policy is suboptimal to equity holders when there are multiple outstanding bonds in the debt structure.[17]

The call policy to maximize the benefits of equity holders seems to be more reasonable and can simultaneously explain the call delay phenomenon. In a comparison of the values of bonds and equity in case $\mathcal{D}_c^{\mathrm{M}}$ with those in $\mathcal{D}_c^{\mathrm{T}}$, the values of callable bond B_3 in the former case are larger than those in the latter. This value increment seems due to the fact that the firm tends to delay the call decision until the market value of B_3 greatly exceeds its CP. This call policy also decreases the values of B_4 and B_5 and increases the value of E, which suggests that it alleviates the wealth transfer effect (compared to the textbook policy) to improve the benefit of equity holders. In addition, by comparing the values of B_3 in the case $\mathcal{D}_c^{\mathrm{M}}$, the interaction effect can be captured by observing that B_3 appreciates when V_0 slightly decreases. This is because the firm tends to delay the call decision in poor financial conditions to avoid ruining its option to default on callable bonds in the future.[18]

To isolate the impact of the interaction effect on call delay, we consider a simple debt structure containing the single callable bond B'. The impact of the wealth transfer effect can be measured by comparing this single-bond debt structure with the five-bond structure. The latter structure contains 3 straight bonds and 2 callable bonds with times to maturity of 10 years (short-term callable bond B_2) and 16 years (long-term callable bond B_3) and is used to analyze call policies for multiple callable bonds empirically studied in KM. Recall that under the textbook policy, a callable bond is redeemed once its market value exceeds the accrued interest plus the call price, which in our experiment is set to the face value of the callable bond. Thus a callable bond should be redeemed once the interest rate r is lower than its coupon rate. However, all call boundaries implied by the equity-value-maximization policy illustrated in Fig. 8(a) show that the firm tends to defer the call decision until r reaches a much lower level. For example, the call delay phenomenon of B' can be illustrated by the difference between the call boundary of B' plotted as the dashed curve and the call boundary of the textbook policy $r = 10\%$ (the coupon rate of B').[19] This difference also purely reflects the impact of the interaction effect on the call delay phenomenon. In addition, the call delay phenomenon is more pronounced with decrements in the prevailing firm's asset value V_5 due to the firm's concern about the value of the default option, which increases with decrements in V_5.

The impact of the wealth transfer effect on the call delay phenomenon can be illustrated by the difference between the single-bond debt structure's call boundary (denoted by the dashed curve) and the five-bond structure's boundaries (solid curves). It can be observed that the firm with multiple outstanding bonds tend to defer the call decision until r reaches a much lower level. Similarly, KM observe that long-term callable bonds are prone to be redeemed earlier than the short-term one; this can be confirmed by comparing the call boundaries for B_2 (denoted by the gray solid curve) and for B_3 (black solid curve). Note that both boundaries overlap when the interest rate level r is low, illustrating that both bonds are simultaneously redeemed.

KM empirically reject the relationship between the wealth transfer effect and the call delay phenomenon by introducing a new measure: premium over effective call price (abbreviated as PoCP). This is defined as the market value for an about-to-call bond (or the bond value at a selected time point if the bond is not redeemed prematurely during the sample period) minus the effective call price. We standardize PoCP (abbreviated as SPoCP) by dividing it by the CP and use it to analyze the call delay phenomenon as KM do in Fig. 8(b). KM argue that the relation between SPoCP and V_0 (abbreviated as SPoCP-V_0 relation) would form a hump-shaped curve if the wealth transfer effect were a key determinant of call policy; this is because a callable bond might appreciate with a slight deterioration in the firm's creditworthiness due to the firm's call delay decisions to alleviate the wealth transfer effect.[20] Note that this argument also explains that

[17] Actually, LT state that the textbook policy is optimal only if the issuer's capital structure does not change during the early redemption process. That is, "the bond has to be refunded with an issue that has exactly the same remaining interest payments, sinking fund provisions, and option features as the original issue." However, "many callable bonds are not refundable."

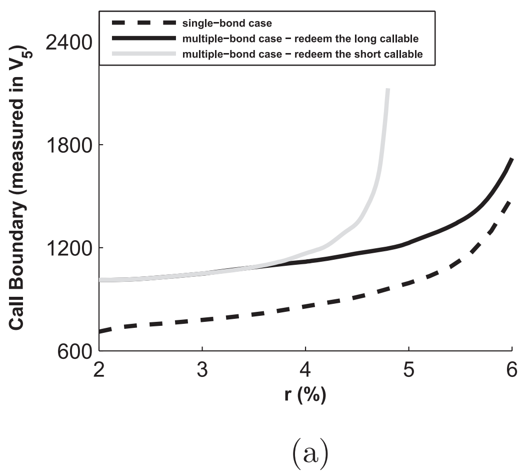
[18] Jacoby and Shiller (2010) also empirically confirm that the presence of the issuer's default risk indeed affects its call policy.

[19] This is not shown in Fig. 8(a) so we can zoom in and compare another three call boundaries.

[20] Note that the bond would eventually depreciate due to the default risk if the firm's creditworthiness deteriorated further.

Table 1
Wealth transfer and interaction effects. The debt structure is identical to the structure studied in Fig. 4 except for the maturity of B_1 which is changed to 5 years and B_3 which is set to a callable but non-refundable bond with a call price of $100 in cases $\mathcal{D}_c^T$ and $\mathcal{D}_c^M$. Specifically, $\mathcal{D}_s$ denotes the case in which all outstanding bonds are straight ones. $\mathcal{D}_c^T$ and $\mathcal{D}_c^M$ denote cases where the callable bond B_3 is redeemed under the textbook policy and under the policy that maximizes the benefits of equity holders, respectively. The numerical settings are the same as those in Fig. 4 except for V_0 which defined in the first row. All outstanding bond values and the equity value E for each scenario are listed under that scenario. The bold bond values represent the values of the otherwise identical callable bond B_3 with the textbook call policy (in $\mathcal{D}_c^T$ columns) or with the call policy maximizing the equity value (in $\mathcal{D}_c^M$ columns). In addition, the red-colored bond values in $\mathcal{D}_c^T$ columns denote the case that the equity values under the textbook call policy are lower than the equity values in $\mathcal{D}_s$ columns, whereas the blue-colored bond values in $\mathcal{D}_c^M$ columns mean the opposite.

V_0	1000			1250			1500		
	$\mathcal{D}_s$	$\mathcal{D}_c^T$	$\mathcal{D}_c^M$	$\mathcal{D}_s$	$\mathcal{D}_c^T$	$\mathcal{D}_c^M$	$\mathcal{D}_s$	$\mathcal{D}_c^T$	$\mathcal{D}_c^M$
B_1 (5 − yr)	103.87	103.87	103.87	103.87	103.87	103.87	103.87	103.87	103.87
B_2 (10 − yr)	113.23	113.23	113.23	113.90	113.90	113.90	114.07	114.07	114.07
B_3 (16 − yr)	134.67	**101.90**	**125.36**	137.59	**101.90**	**122.65**	138.72	**101.90**	**117.16**
B_4 (20 − yr)	148.04	150.95	148.09	152.40	154.25	152.49	154.34	155.44	154.47
B_5 (30 − yr)	194.18	198.36	194.32	201.13	204.00	201.36	204.46	206.58	204.76
E	477.20	469.96	477.48	715.82	712.83	716.34	960.81	960.13	961.61



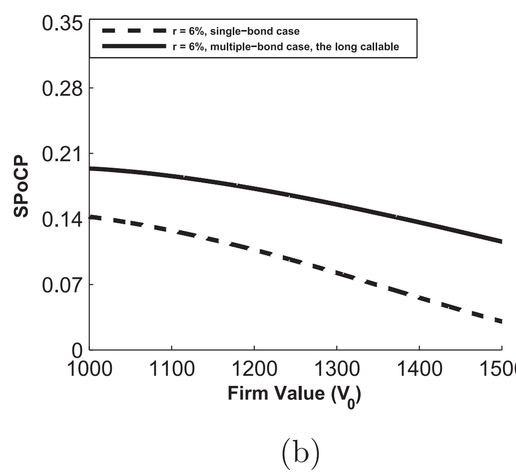


Fig. 8. *Measuring call delay phenomena with call boundaries and* SPoCP. The five-bond debt structure is identical to the structure studied in Table 1 except for B_2 and B_3, which are both set as callable bonds with a call price of $100. The single-bond debt structure contains only one 16-year callable bond B' with a face value of $500 and a coupon rate of 10% that is equal to the total amount of outstanding bonds and the coupon rate of B_3, respectively, in the five-bond debt structure. Panel (a) measures the call boundary in terms of the prevailing firm value V_5 and the interest rate level r at year 5. For example, it is optimal to redeem B' in the single-bond structure if the firm value V_5 exceeds the call boundary denoted by the dashed curve, or if the interest rate falls below the boundary. The call boundaries for B_3 and B_2 in the five-bond structure are denoted by the black and gray solid curves, respectively. Panel (b) displays the relation between the SPoCP and the initial firm value V_0 for B' and B_3 plotted in the dashed and solid curves, respectively, given the prevailing interest rate $r = 6\%$. The SPoCP is defined as B' and B_3's premium over effective call price divided by their effective call price. Other numerical settings are $\sigma = 20\%, \tau = 35\%$ and $\omega = 50\%$.

the interaction effect would cause the hump-shaped relation as illustrated by the dashed curve. In addition, the wealth transfer effect strengthens this relation as illustrated by the solid curve. However, KM empirically show that the SPoCP-V_0 relation is significantly monotonically increasing, and that the hump-shaped relation is insignificant *on average*.

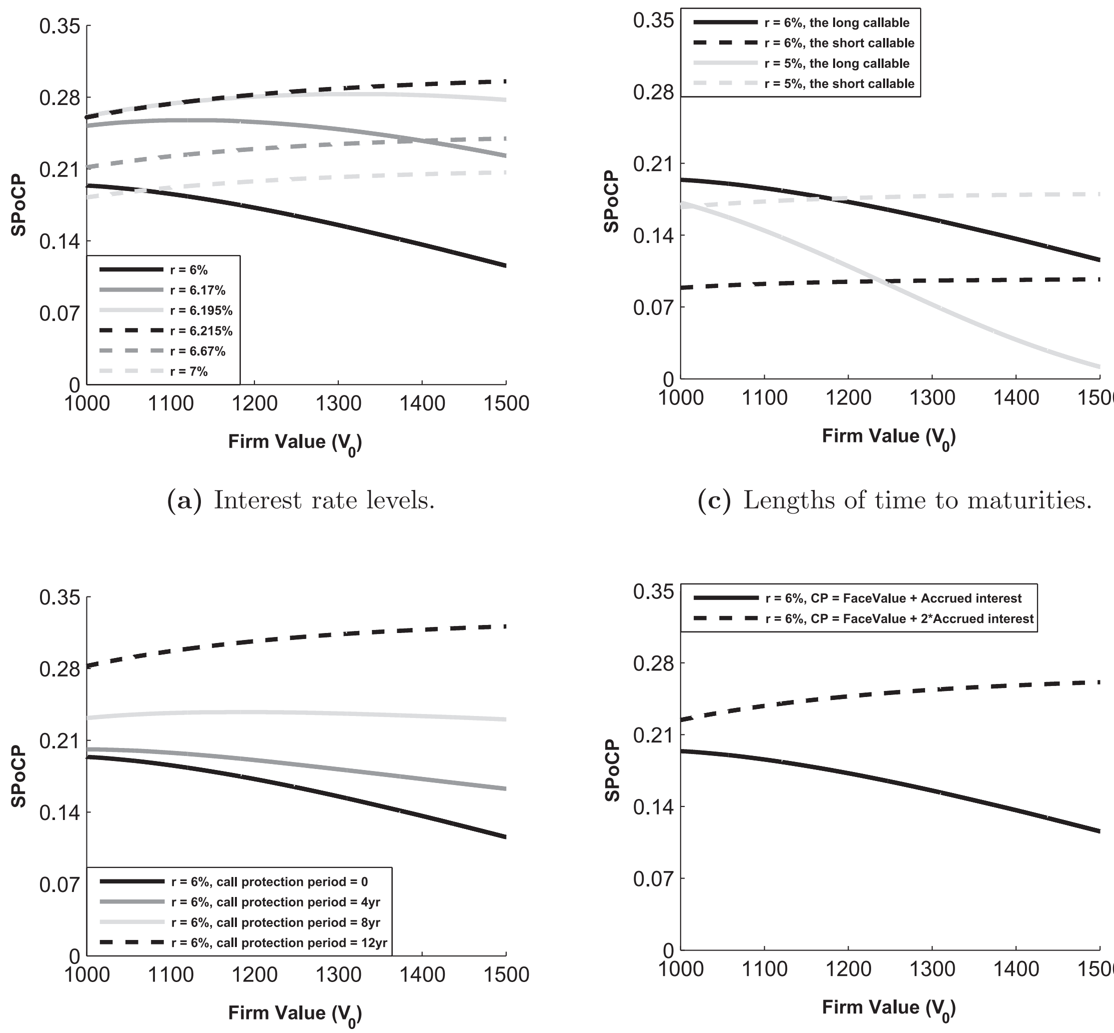
The conflict between LT and KM could be because the proportion of callable bonds that are not redeemed early in KM's sample set is much larger than this proportion in LT's set. Since the characteristics of callable bonds that are not redeemed early are similar to those of straight bonds, KM's empirical studies tend to capture the upward-sloping SPoCP-V_0 relation implied by straight bonds.[21] The SPoCP curves observed in our quantitative experiments in Fig. 9 change from hump-shaped to upward-sloping with the decrements in the early redemption likelihood; this could be influenced by four factors: the interest rate levels, the lengths of call protection periods, the remaining time to maturities of callable bonds, and the call price levels. By combining our experiments with the descriptions of LT's and KM's data sets, we discover some explanations for their conflicts.

Increasing the interest rate level r could reduce the likelihood of early redemption and hence change the SPoCP-V_0 relation from the hump-shaped pattern (denoted by solid curves) to the positively-related pattern (dashed curves) as illustrated in Fig. 9(a). It is difficult to accurately measure the average interest rate levels of LT's and KM's data sets; instead, we measure the average 10-year Treasury yields of their sample periods as shown in Fig. 10. The callable bonds sampled by the former study are from August 1991 to August 1992, whereas the bonds sampled by the latter study are from January 1975 to March 1994. The average 10-year Treasury yield for the former sample period 7.31% is significantly lower than that for the latter sample period 9.22%. Since a higher interest rate level implies that callable bonds are less likely to be redeemed early and that tends to form positive SPoCP-V_0 relation as illustrated in Fig. 9(a), this might explain why LT and KM observe different SPoCP-V_0 relations.

There are two other possible reasons that cause KM to render hump-shaped relations insignificant. First, they claim that the average call protection period of their callable bonds samples accounts for more than 65% of the average maturity of these bonds (6.41 years out of 9.83 years).[22] Our experiment shows that increasing the call protection period could decrease the early redemption likelihood and change the SPoCP-V_0 relation from the

[21] Note that the value of a straight bond increases monotonically with the firm's creditworthiness.

[22] See King (2002) for information about the call protection period.



(a) Interest rate levels.

(c) Lengths of time to maturities.

(b) Lengths of call protection periods.

(d) Levels of effective call prices.

Fig. 9. *Sensitivity analyses of the relation between* SPoCP *and* V_0. The debt structure is identical to the five-bond structure studied in Fig. 8. The relation of the 16-year callable bond's SPoCP and V_0 is drawn in panels (a), (b) and (d). Panel (c) plots SPoCPs for coexistent 10-year (short-term) and 16-year (long-term) callable bonds. The solid and dashed patterns denote hump-shaped and upward-sloping curves, respectively. All other numerical settings are the same as those in Fig. 8(b) unless stated otherwise.

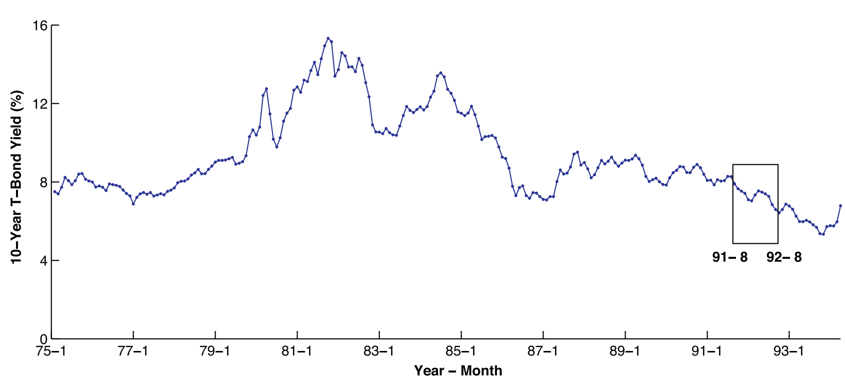


Fig. 10. *10 year treasury yields as proxies for interest rate levels.* The full time span and that marked by the rectangle denote the sample periods of KM's and LT's data sets, respectively.

hump-shaped pattern to the positively-related one as illustrated in Fig. 9(b). Second, as illustrated in Fig. 8(a), the long-term callable bond is prone to be redeemed earlier than the short-term one in a complicated debt structure with multiple outstanding bonds. The differences in early redemption likelihoods lead to hump-shaped and upward-sloping SPoCP-V_0 relations for long-term and short-term callable bonds, respectively, as illustrated in Fig. 9(c). Note that KM seems to sample many callable bonds by issuers with

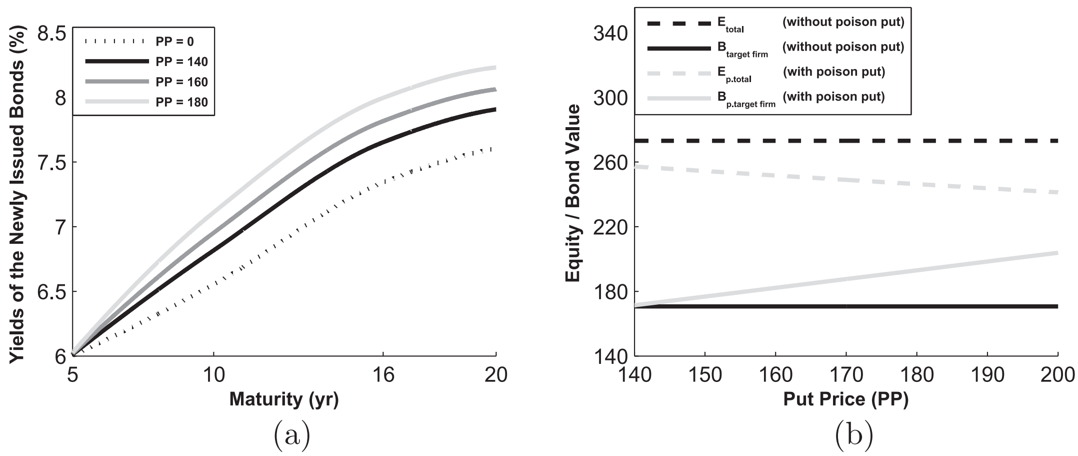


Fig. 11. *Impact of the poison put.* The target firm is assumed to have one 30-year outstanding bond with a face value of $100 and a coupon rate of 14%. The bidder finances the LBO by issuing another 4 otherwise identical serial bonds with face values of $100 and maturities of 5, 10, 16, and 20 years. After the LBO, all serial bonds are senior to the 30-year bond. Panel (a) displays the change of yield spread curves for serial bonds with the change of the PP. In panel (b), $E_{p.total}$ and E_{total} denote the total equity value after the LBO with or without the poison put embedded in the 30-year bond, respectively. $B_{p.targetfirm}$ and $B_{targetfirm}$ denote the value of the 30-year bond with or without the poison put, respectively. Other numerical settings are $V_0 = 1000, r = 6\%, \sigma = 20\%, k = 1\%, \tau = 35\%$ and $\omega = 50\%$.

multiple outstanding callable bonds.[23] That may be another reason why in their study they conclude that hump-shaped relations are insignificant. In additional to the aforementioned factors that change the SPoCP-V_0 relation, our experiment in Fig. 9(d) also shows that the increments in CP also decrease the early redemption likelihood and tend to produce positively-related patterns.[24]

4.4. Poison puts and the bidder's costs of debt financing

A poison put is a bond covenant that gives bond holders the right to demand redemption at a pre-specified put price PP prior to maturity given the occurrence of a certain event such as a leveraged buyout (LBO). As argued in Cook and Easterwood (1994), this covenant can protect bond holders of the target firm, increase the bidder's cost for raising new bonds, and have a negative effect on equity holders. This is because triggering the poison put once or after the occurrence of a LBO can alter the bidder's debt repayment schedule to improve the effective repayment priority of the bonds originally issued by the target firm at the expense of other outstanding bonds of the bidder. Cremers et al. (2007) claim that bond holders' governance through poison put covenants efficiently mitigates the asset substitution problem.

Our forest model quantitatively analyzes how a poison put can avoid asset substitution actions like a LBO as illustrated by the hypothetical scenario in Fig. 11. Consider a poison put that requires the bidder to redeem the 30-year bond originally issued by the target firm once the bidder's asset value after the LBO falls below the sum of the face values of its outstanding bonds.[25] The bidder issues serial bonds to finance the LBO; the impact of different levels of PP on the yield spreads are illustrated in Fig. 11(a). For comparison, we also illustrate the case in which the 30-year bond is not protected by the poison put (denoted by PP = 0). Potential premature redemptions significantly raise the yields of newly-issued serial bonds (compare the dot curve with the solid curves), because they change the bidder's debt repayment order and deteriorate the effective priorities of serial bonds. The magnitude of the PP can be viewed as the level of protection for the 30-year bond at the expense of holders of serial bonds and equity; observe that the yield curve of serial bonds increases with the PP. Fig. 11(b) illustrates the impact of the PP's magnitude on the values of equity and the 30-year bond denoted by the gray dashed and gray solid curves, respectively. For comparison, the values of the equity and the bond without the poison put protection are denoted by the black dashed and the black solid horizontal lines. Note that the increment of the PP increases the protection for the 30-year bond (compare the black solid lines with the gray solid lines) and deprives the wealth of equity holders (compare the black dashed lines with the gray dashed lines) due to the high cost for raising debt capital reflected by the yields of serial bonds.

5. Conclusion

Many existent structural models oversimplify the issuing firm's debt structure and fail to capture phenomena found in empirical studies. To provide theoretical insight and reliable evaluations consistent with existent empirical studies, we develop a quantitative framework by exploiting the flexibility of the tree method to faithfully capture various aspects of a complicated debt structure. The risk for a firm to default on a repayment is modeled to rely on its ability to repay previous repayments (i.e., the compound option approach); the default boundary is shaped according to the repayment amounts, the repayment schedule, and bond covenants (i.e., debt-structure-dependent default boundaries). Our quantitative framework also does not make excessive assumptions about the firm's future financing policies and implements cases in which the due bond is financed using internal or external funds. The repayment order determined by seniority of bonds or payment blockage covenants is implemented by a sophisticated backward induction method. Thus our framework faithfully captures the impact of factors such as the interest rate level, the firm's financial status, the repayment schedule, and refinancing strategies on yield spreads of the firm's outstanding bonds studied in past empirical literature. The accuracy of our evaluation results can also be indirectly verified by taking advantage of the capital structure irrelevance theory.

The payment schedule changes contingently due to the issuing firm's call back decisions or triggers of bond covenants such as poison puts. To evaluate values of claim holders under different

[23] According to the report in KM, among the 1642 calls made by 530 firms, "294 firms called (possibly multiple bonds) one time, 108 firms called bonds two different times, 30 firms called three different times, 33 firms called four times and 21 firms called more than four times during the sample period."

[24] CP varies from bond to bond. It depends on the provisions contracted in callable bonds (see Tewari et al., 2015).

[25] This is similar to the positive net worth covenant adopted in academic literature like Leland (1994) and Longstaff and Schwartz (1995).

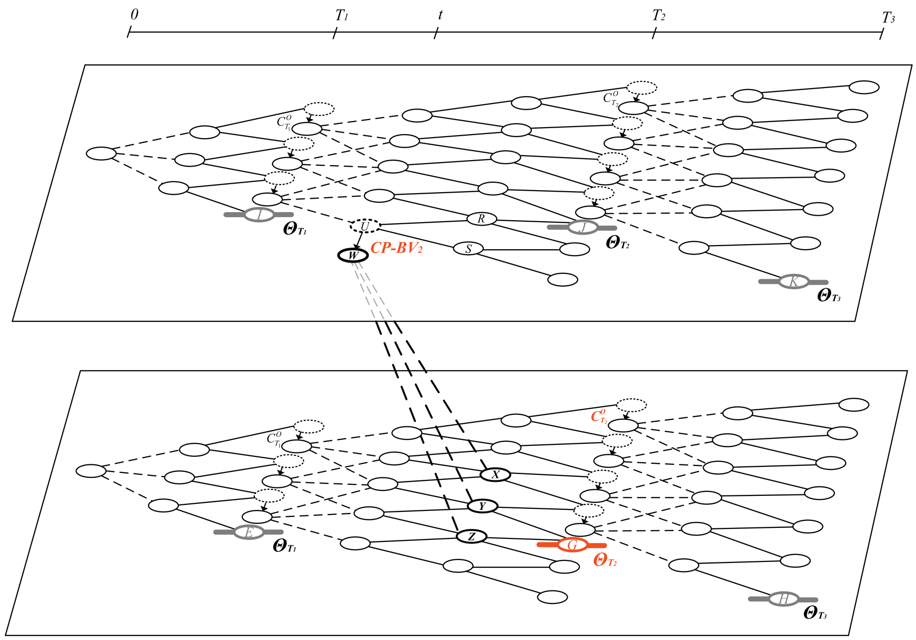


Fig. 12. *Two-layer forest for modeling callable bond refunding.* In this generic three-bond scenario, B_1,B_2, and B_3 mature at time T_1,T_2, and T_3, respectively. The forest is composed of two trees; the upper (lower) layer tree models the scenario in which B_2 is not yet refunded (already refunded) by issuing B_2'. Refunding B_2 changes the issuing firm's asset value (denoted by a downward jump from node U to W) and the prevailing debt structure (denoted by transferring from the upper-layer tree to the lower-layer one.) Terms colored in red (including BV_2, $C^O_{T_2}$, and Θ_{T_2}) depend on the payment and value of the new issuance B_2'. Cox et al. (1979)'s binomial structures are plotted using solid lines and Dai and Lyuu (2010)'s trinomial structures are plotted using dashed lines. Boldfaced circles denote assets remaining after debt repayments. Nodes I,J,K,E,G and H are decided to exactly match the default boundaries marked by thick lines. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

repayment schedules and hence analyze the decisions to exercise the options embedded in bonds, we construct a novel quantitative method, the forest, which is composed of several trees. Each tree simulates the values of claim holders under a possible repayment schedule; transitions between two trees denote contingent changes in the schedule. The forest method can analyze complicated call policies of multiple callable bonds, which is a longstanding open problem. We analyze how the interaction effect and the wealth transfer effect result in call delay phenomena. We also resolve conflicts among past empirical literature by finding the reasons, for example the interest rate level over the sample period, that might influence regression results. The forest method can also illustrates how a poison put can protect a target firm's bonds against a leveraged buyout by increasing the bidder's cost to raise debt capital.

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Appendix A. Forests in more complex forms

To explore how a forest framework can grow in a more complex form to deal with complicated debt structures and covenants, the following scenarios are studied based on a generic three-bond example: bonds B_1, B_2, and B_3 mature at time T_1, T_2, and T_3, respectively. Note that for simplicity, coupon payments are neglected in the following forest structure constructions.

A.1. Refunding a callable bond

The forest construction method studied in Section 3.3 to model non-refundable callable bonds can be slightly modified to deal with refundable callable bonds as illustrated in Fig. 12. Consider the case in which B_2 is a refundable callable bond which can be called back using the proceeds from issuing a new bond B_2' with the same maturity date T_2. On each call date, issuers can decide whether they should refund B_2 under the burdens for repaying the straight bonds B_1 and B_3. Thus the forest is composed of two trees: the upper (lower) layer tree models a scenario in which B_2 is not yet refunded (already refunded) by issuing B_2'. These two trees can be constructed using the methods from Section 3.1 described as follows.

In the upper-layer tree, the issuer repays the coupons plus the principal payments of B_1, B_2, and B_3 at these bonds' maturities. The concept of remaining assets is implemented by subtracting the debt repayments $C^O_{T_1}$ and $C^O_{T_2}$ (denoted by downward arrows) from $V_{T_1^-}$ and $V_{T_2^-}$ (denoted by dashed circles) to get the remaining value V_{T_1} and V_{T_2} (denoted by boldfaced circles), respectively. The firm defaults if its asset value cannot meet the debt-structure-dependent default boundaries $\Theta_{T_1}, \Theta_{T_2}$, and Θ_{T_3} (plotted by gray thick lines), which are equal to the values of frozen assets plus the repayment at these three repayment dates. To prevent price instability due to nonlinearity errors as studied in Figlewski and Gao (1999), tree nodes I, J, and K are decided to match the boundaries. The lower-layer tree follows a similar structure except for the payment to the callable bond B_2 which is replaced by a payment to the new bond B_2' (denoted by $C^O_{T_2}$). The default boundary Θ_{T_2} is redefined as the values of the frozen assets plus the repayments of

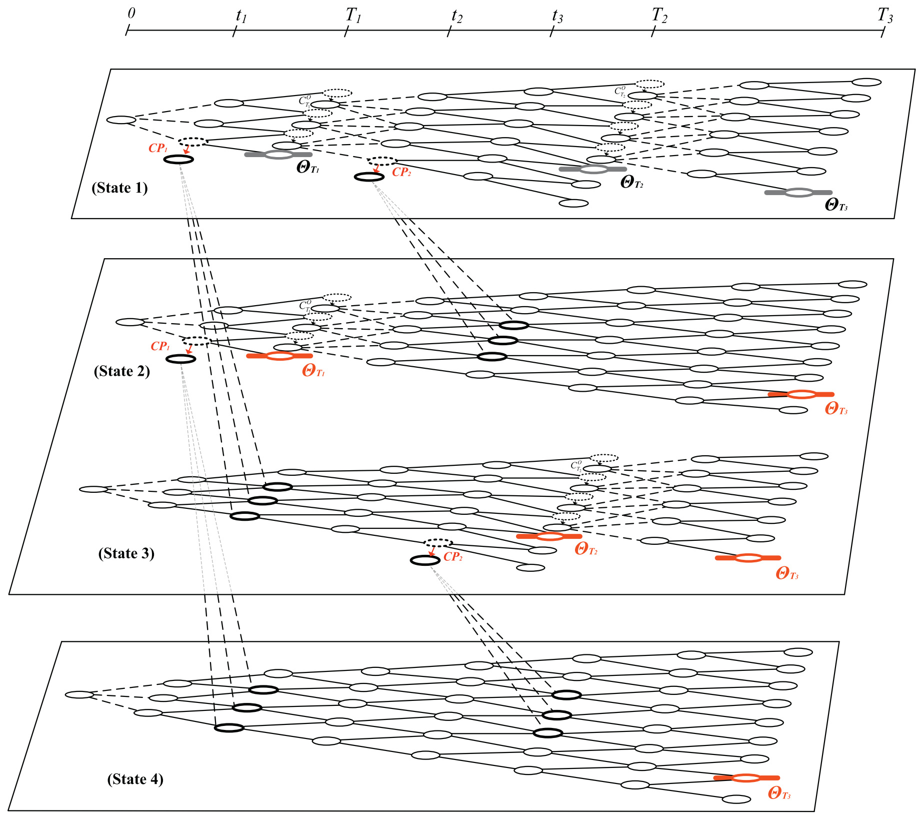

Fig. 13. *Three-layer forest for modeling possible redemption scenarios for two callable bonds.* The debt structure settings are the same as those in Fig. 12 except for B_1 and B_2 which are non-refundable callable bonds. The forest is composed of three layers with four trees marked by State 1 $\cdots$ State 4. CRR binomial branches are plotted by solid lines and Dai and Lyuu (2010)'s trinomial branches are plotted by dashed lines. Gray/red nodes are decided to exactly match the default boundaries marked by thick gray/red lines. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

the bond B'_2. Note that tree nodes E, G, and H are likewise decided to match default boundaries to ensure stable pricing results.

Calling back B_2 using the proceeds from issuing B'_2 at a node, for example U, changes the prevailing debt structure and can be modeled by transferring from the upper-layer tree to the lower-layer one. Refunding changes the firm's asset value (i.e., $\upsilon(U) - \upsilon(W)$) by $\mathtt{CP} - \mathtt{BV}_2$ (marked by the downward arrow from node U to W), where $\mathtt{CP}$ denotes the effective call price and $\mathtt{BV}_2$ denotes the issuance price of B'_2. The trinomial structure emitted from node W to nodes X, Y, and Z is constructed by following the method of Dai and Lyuu (2010) to simulate the firm value dynamics during the time interval $[t, t + \Delta t]$. The issuance price $\mathtt{BV}_2$ should be equal to the value B'_2 calculated by backward induction at node W. Note that $\mathtt{BV}_2$ depends on the issuing firm's prevailing financial status (proxied by $\upsilon(W)$) and the time to maturity of B'_2. In addition, the aforementioned process can be slightly modified to consider the issuance cost of B'_2 as stated in Mauer (1993). Let each issuance incur a fixed cost ν_F plus k proportion of the issuance price. Then the change of the firm's asset value (i.e., $\upsilon(U) - \upsilon(W)$) due to refunding is instead expressed as $(\mathtt{CP} - \mathtt{BV}_2) + (\mu_F + k\mathtt{BV}_2)$.

To maximize equity values, the issuing firm finds the optimal strategy for refunding B_2 with the new issuance B'_2 at each B_2's call date as the method for making the optimal call decision discussed in Section 3.3.2. We use node U in the upper-layer tree to demonstrate this process. If the firm refunds B_2 at node U, the "**R**efunded" equity value $E^R(t^-, \upsilon(U))$ is evaluated as the dividend paid at node U plus the continuation value that represents time t's expected present value of future equity values at node W's successor nodes X, Y, and Z in the lower-layer tree to reflect the replacement of B_2 with B'_2. Otherwise, B_2 remains alive and the "**N**on-refunded" equity value $E^N(t^-, \upsilon(U))$ is evaluated as the dividend paid at node U plus the continuation value that represents time t's expected present value of future equity values at node U's successor nodes R and S in the upper-layer tree. If $E^R(t^-, \upsilon(U))$ is larger than $E^N(t^-, \upsilon(U))$, the firm refunds B_2 by issuing B'_2; the continuation values of B_1, B'_2, and B_3 are evaluated as the discounted expectations of their future values at nodes X, Y, and Z. Otherwise, the firm does not call B_2 back and the continuation values of all claims (except B'_2) are evaluated as the discounted expectations of their future values at nodes R and S. Note that all the aforementioned continuation values can be calculated using backward induction.

A.2. Multiple callable bonds

To analyze premature redemptions of one or more callable bonds from a complicated debt structure with multiple callable bonds, we use a forest with multiple layers to model the possible debt structure transitions due to different redemption strategies. Now we consider a slight modification of the aforementioned three-bond example by setting B_1 and B_2 as non-refundable callable bonds. The resulting forest illustrated in Fig. 13 can be decomposed into three layers with four trees. The upper layer contains only one tree (marked as State 1) which models the dynamics of the firm's asset value under the condition that both B_1 and B_2 are not yet called. The middle layer contains two trees marked by State 2 and State 3. The former (latter) tree models the dynamics of the asset value given that only B_2 (B_1) has been called. The lower layer contains one tree (marked as State 4) which models a scenario in which both B_1 and B_2 have already been called. The structure of each tree is constructed according to the method in Section 3.1 to calibrate the payment schedule and the default

Table 2

Robustness checks. Bond prices, corresponding bond yield spreads (listed in parentheses), and equity values E (in the last row) generated under our framework are examined under three different scenarios denoted by the first row. Five outstanding \$100 bonds, B_1,B_2,B_3,B_4, and B_5 are assumed to be issued by the same hypothetical issuer with coupon rates of 7%,8%,10%,11%, and 14%, and the remaining time to maturities of 5, 10, 16, 20, and 30 years, respectively. The yield spreads for the junior bond in the "Payment blockage" scenario and callable bonds in the "Multiple callable bonds" scenario are colored in red. The current issuing firm's asset value V_0 is 1000, its volatility σ is 20%, and the risk-free rate r is 6%. `Time steps` in the second row denotes the number of time steps used to partition the one-year time span in our framework. The blue-colored values in parentheses represent the yield spreads for straight bonds. The red-colored values in "Payment blockage" scenario represent the yield spreads for the otherwise identical bonds B_2 with payment blockage covenants, and those in "Multiple callable bonds" scenario represent the yield spreads for the otherwise identical bonds B_2 and B_3 with call provisions.

Scenario	Straight bonds			Payment blockage			Multiple callable bonds		
`Time steps`	32	128	512	32	128	512	32	128	512
B_1(5yr)	103.85 (0.30)	103.86 (0.29)	103.85 (0.30)	103.86 (0.29)	103.86 (0.27)	103.86 (0.29)	103.86 (0.30)	103.85 (0.34)	103.85 (0.33)
B_2(10yr)	110.62 (43.24)	110.66 (42.73)	110.68 (42.46)	107.99 (76.55)	107.96 (77.01)	107.95 (77.06)	104.20 (126.51)	104.24 (125.88)	104.29 (125.31)
B_3(16yr)	127.36 (98.59)	127.30 (99.10)	127.31 (99.05)	128.24 (91.12)	128.20 (91.41)	128.22 (91.29)	104.55 (322.66)	104.81 (319.72)	104.85 (319.36)
B_4(20yr)	138.83 (115.36)	138.80 (115.58)	138.77 (115.77)	139.71 (109.10)	139.70 (109.14)	139.68 (109.27)	143.23 (84.44)	143.08 (85.48)	143.04 (85.74)
B_5(30yr)	180.16 (121.52)	180.15 (121.59)	180.13 (121.65)	181.03 (117.31)	181.05 (117.25)	181.04 (117.28)	186.75 (90.57)	186.58 (91.35)	186.54 (91.53)
E	339.17	339.24	339.25	339.17	339.24	339.25	357.41	357.43	357.43

boundary implied by the tree's debt structure. For example, the debt repayment at time T_2 (T_1) is missing in tree State 2 (State 3) since in that tree, B_2 (B_1) has been called. Debt repayments at times T_1 and T_2 are missing in tree State 4 since both B_1 and B_2 have been called already.

Each transition between two trees from different layers denotes a change in the debt structure due to a premature redemption of callable bond(s). Transitions are modeled by a change in the firm's asset value (marked by a downward jump from a dotted circle to a boldfaced one) to reflect the redemption payment and by a trinomial structure connecting to successor nodes in another tree to reflect the change of the debt structure. The transition structure can be constructed for each tree node at a call date according to the method in Section 3.3.1.[26]

As described in Section 3.3.2, the issuing firm selects optimal bond redemption strategies to maximize the value of equity holders. The values of bonds and equity under different debt structures can be evaluated using backward induction in their corresponding trees. Therefore, at each node located at a call date, the firm picks the best redemption strategy by comparing their corresponding equity values. Take the redemption decision for a node at tree State 1 for example: the firm may choose not to redeem any callable bonds, to redeem B_1 and B_2 simultaneously (i.e., transit from State 1 to State 4), or to redeem either B_1 or B_2 (i.e., from State 1 to either State 3 or State 2). Note that the forest structure can also analyze the benefits of sequential redemptions. For example, the firm may first redeem B_1 and then redeem B_2 (i.e., transit from State 1 to State 3 and then to State 4), or vice versa (i.e., transit from State 1 to State 2 and then to State 4). The analyses of sequential redemptions can explain call strategies and in consequence the call delay phenomena (see Section 4.3) found in empirical studies such as King and Mauer (2000).

Appendix B. Robustness checks

Before applying the framework to compare or to analyze empirical observations, it is critical that we verify that the quantitative framework produces accurate and stable pricing results. Although pricing results generated by tree methods are expected to converge to theoretical values with the increment in the number of the time steps of the tree or forest (see Duffie, 1996), inappropriate tree structure adjustments or backward inductions can degrade accuracy. Previous literature such as Broadie and Kaya (2007) and Wang et al. (2014) examine the robustness of their methods by showing that their pricing results converge to those generated by analytical formulas under certain simple debt structures. However, analytical formulas are not available for complicated debt structure scenarios and hence the correctness for their complex tree implementations is difficult to verify.

Instead of directly verifying the correctness of each pricing result, we suggest indirectly checking the rationality of the pricing results as a whole by taking advantage of the capital structure irrelevance theory proposed by Modigliani and Miller (1958). Essentially, in a perfect and frictionless market, the market value of a firm is not influenced by the capital structure, and our framework should produce the same market value regardless of changes in debt structures under otherwise identical conditions. Indeed, given the capital market is frictionless (i.e, no bond issuance costs, taxes, and bankruptcy costs), our experiment in Table 2 shows that the leveraged firm value, which is equal to the lump sum of all contingent claims' values generated by our framework, is essentially equal to the initial firm value V_0 (1000 in this experiment) under the different debt structure scenarios listed in the first row. The "Straight bonds" scenario assumes that all outstanding bonds are equal-priority straight bonds. It can be observed that the lump sum of these five bond values listed in the second column (i.e., 103.85, 110.62, ..., 180.16) plus the equity value 339.17 is about 1000. By checking our numerical results in other columns, we verify that this irrelevant property also holds under other settings.

In the "Payment blockage" scenario, the payments to junior bond B_2 that occur within two years prior to default (i.e., the length of the blockage period η is two years) can be blocked to fulfill the payments to other unmatured senior bonds. This scenario can be evaluated by the method discussed in Section 3.2. By comparing this scenario with the otherwise identical scenario "Straight bonds", the payment blockage covenants clearly increase the values (decrease the yield spreads listed in parentheses) of bonds that mature later than junior bond B_2 (i.e., B_3, B_4 and B_5) at the expense of B_2's value.

In the "Multiple callable bonds" scenario, all settings are the same as those in the "Straight bond" scenario except for B_2 and B_3 which are callable bonds. The forest method discussed in Section 3.3 and Appendix A.2 can be applied to analyze the complex call strategies that are claimed to be intractable in Jones et al. (1983).

[26] This does not include the nodes in State 4 since no callable bonds exist in its corresponding debt structure.

Compared with the "Straight bonds" scenario, the call options embedded in B_2 and B_3 decrease their bond values (increasing the bond yield spreads) and benefit the holders of equity and bonds that mature later than B_3.

In addition, tree (or forest) methods may produce oscillating pricing results as in Figs. 5 and 6 in Broadie and Kaya (2007) due to the nonlinearity error problem (see Figlewski and Gao, 1999). Oscillating numerical errors may interfere with the analyses of bond covenants' impacts and claim holders' decisions. Fortunately, Wang et al. (2014) alleviate the oscillating problem by making some tree nodes coincide with the default boundaries and show that their pricing results converge smoothly with the increments in the number of time steps of their tree method. In our framework, we adopt their tree adjustment technique to generate stable numerical results. The fast and smooth convergence property can be verified by observing that all pricing results change little (less than 0.4%) when `Time steps` (listed in the second row) increases from 32 to 512. This good property allows us to analyze many empirical phenomena, for instance the existence of the payment blockage covenants with respect to reducing the yield spreads of B_3, B_4, and B_5.

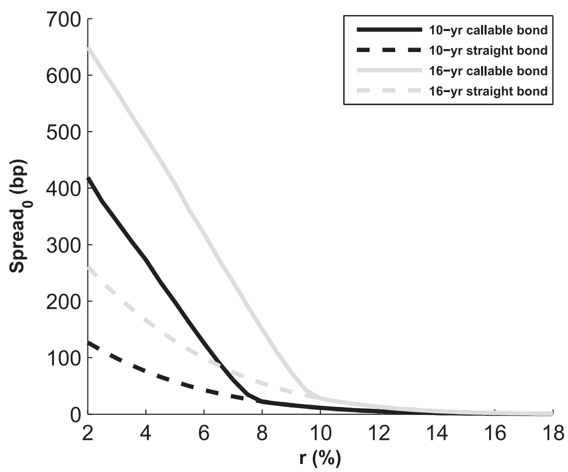

Fig. 14. *Impact of changing interest rate levels on yield spreads.* This figure illustrates the yield spreads of the 10-year bond (denoted in black color) and the 16-year one (gray color) under the "Straight bonds" (dashed curves) and "Multiple callable bonds" scenarios (solid curves) studied in Table 2. All numerical settings are identical to those in Table 2 except for the interest rate level which is denoted by the *x*-axis.

Appendix C. Extensive quantitative experiments on yield spreads

In addition to generating feasible upward-sloping yield spread curves as illustrated in Fig. 1, our framework quantitatively analyzes past empirical research on the determinants of bond yield spreads. For example, Duffee (1998) studies the impact of the interest rate level and the issuing firm's creditworthiness on the yield spreads of straight and callable bonds, yielding the following main empirical results:

(i) Yield spreads for straight bonds decrease with increments of the interest rate level.
(ii) Relation (i) is more pronounced for callable bonds.
(iii) Relation (i) is more pronounced for callable bonds when interest rate levels are low.
(iv) Relation (i) is more pronounced for straight bonds with lower credit ratings.

Avramov et al. (2007) studies the relationship among the firm's creditworthiness, its asset value volatility, and bond yield spreads. Their empirical results are listed as follows:

(v) Bond yield spreads increase with increments in the firm's asset value volatility.
(vi) Relation (v) is more pronounced for straight bonds with lower credit ratings.

Our framework provides theoretical insights and reliable quantitative measurements for the above empirical observations as illustrated in Figs. 14 and 15. The negative relation between yield spreads for straight bonds and the interest rate level can be observed from the dashed curves as illustrated in Fig. 14; this rela-

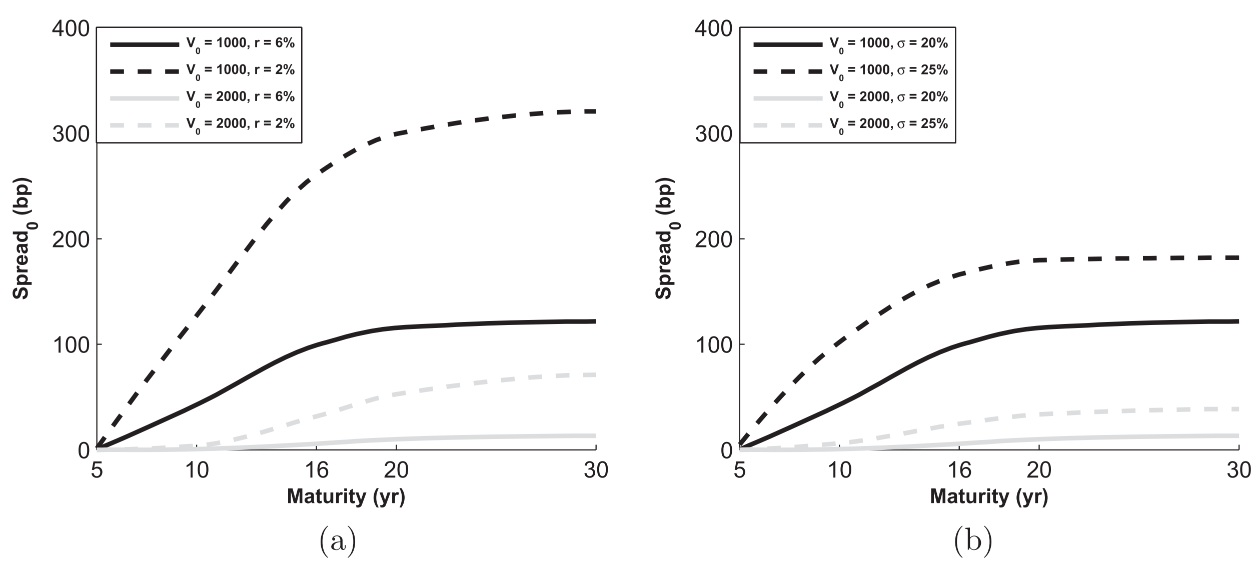

Fig. 15. *Impact of changing issuing firm's financial status, asset value volatility, and interest rate level on yield spread curve.* We illustrate yield spread curves for the five-bond debt structure described in the "Straight bonds" scenario in Table 2. All numerical settings are identical to those in Table 2 except the following settings. The black and gray colors denote a low initial firm value $V_0 = 1000$ and a high 2000, respectively. In panel (a), the solid and dashed curves denote a high interest rate $r = 6\%$ and a low 2%, respectively. The asset value volatility σ is set to 20%. In panel (b), the dashed and solid curves denote a high asset value volatility $\sigma = 25\%$ and a low 20%, respectively. The interest rate level r is set to 6%.

tion confirms property (i) mentioned above. In addition, the yield spread for the 16-year bond (denoted by the gray dashed curve) is greater than that of the 10-year bond (black dashed curve) since a greater insolvency burden is borne by the holder of the long-term 16-year bond than the holder of the 10-year bond. This phenomenon is also consistent with the finding of the upward-sloping yield spread curve illustrated in Fig. 1.

The yield spread for a callable bond reflects the combination of the issuing firm's insolvency risk and the bond holder's call risk due to the firm's premature redemption. The call risk can be evaluated by the option-adjusted spreads (OAS), which can be interpreted as the difference between the solid curves (which denote the yield spreads for the callable bonds) and the dashed curves (yield spreads for otherwise identical straight bonds) in Fig. 14. Given high enough interest rate levels, the black (gray) solid curve coincides with the black (gray) dashed curve, reflecting that OAS approaches zero. This is because a callable bond that is unlikely to be redeemed early is similar to an otherwise identical straight bond. On the other hand, decrements in the interest rate level increase the likelihood that the callable bond is redeemed early and hence increase the magnitude of the OAS. Since both OAS and the yield spread for the otherwise identical straight bond increase with decrements in the interest rate level, we find a more pronounced negative relation between the yield spread for a callable bond and the interest rate level (see property (ii)). We also find that this relation is more pronounced with decrements in the interest rate level (see property (iii)).

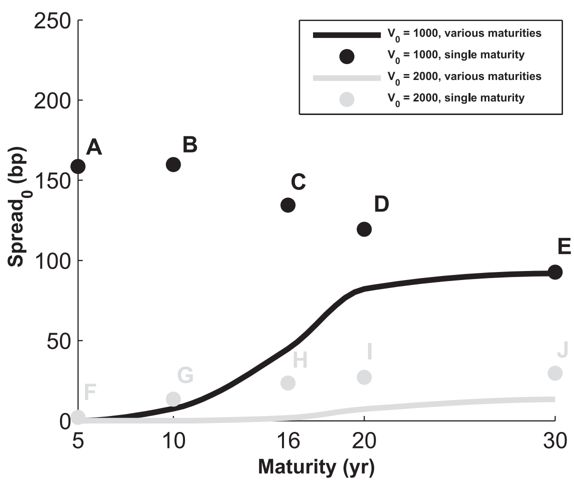

Fig. 16. *Yield spreads for issuing serial bonds or a single bond.* This figure illustrates yield spread curves for the issue of five otherwise identical par bonds with face values of $100 and maturities of 5, 10, 16, 20, and 30 years. The black and the gray colors denote a poor firm's financial statues (proxied by a low $V_0 = 1000$) and a good one (proxied by $V_0 = 2000$), respectively. Each circle at time t indicates the yield spread for the issue of a $500 par bond with maturity t. For example, the yield spread for the issue of a $500 10-year par bond is more than 150 basis points (denoted by node B) when the firm's financial status is poor. Other numerical settings are the same as those in Fig. 1 of our paper.

Fig. 15(a) exhibits the impact of changes in the interest rate level r and the firm's current asset value V_0 on the yield spread curve. Property (i) can be verified again by observing that the yield spread curve shifts downward from dashed curves to solid ones when the interest rate level increases from 2% to 6%. The deterioration of the firm's creditworthiness, which is proxied by decreasing the firm's asset value from 2000 (denoted by the gray color) to 1000 (black color), brings out more clearly the negative relation between the yield spread and the interest rate level (i.e., property (i)); this is consistent with property (iv). Fig. 15(b) exhibits different yield spread curves evaluated by changing the firm's asset value V_0 and its asset value volatility σ, which are usually used as the proxies for the firm's creditworthiness and its business risk, respectively (see Merton, 1974). Property (v) can be verified by observing that yield spread curves shift upward from solid curves to dashed ones when the firm's business risk σ increases from 20% to 25%. The shift magnitude becomes more significant with deteriorations in the firm's creditworthiness proxied by decreasing V_0 from 2000 (denoted by the gray color) to 1000 (black color), which is consistent with property (vi).

Dass and Massa (2014) identify that a bond-issuing firm can reduce bond yields by offering serial bonds to attract institutional investors. With our framework, we verify their empirical finding and offer a cause from the viewpoint of the credit risk as illustrated in

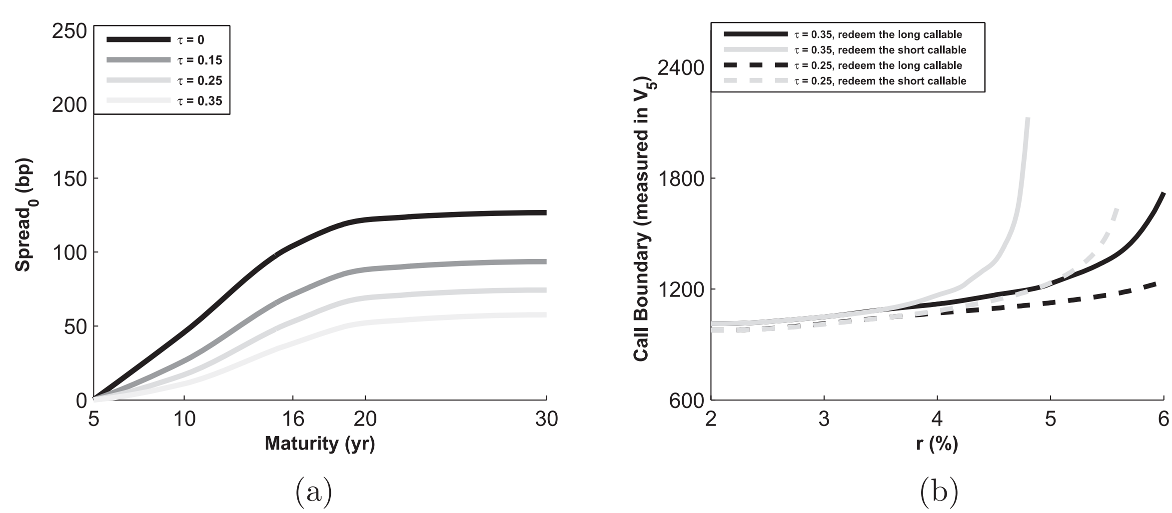

Fig. 17. *Impact of diminishing tax benefits on yield spread curves and call boundaries.* All numerical settings are the same as those in Fig. 8 except for the tax rate τ which is defined in the legends in the upper parts of both panels. Panel (a) illustrates the yield spread curves implied by five otherwise identical par bonds with maturities of 5, 10, 16, 20 and 30 years under different tax rates. The face values of these five straight bonds are $100. Panel (b) illustrates call boundaries for two callable bonds from the five-bond debt structure studied in Fig. 8. Black curves denote the boundaries for the long-term 16-year callable bonds and gray curves denote the boundaries for the short-term 10-year callable bonds. Call boundaries at year 5 are measured in terms of the prevailing firm value V_5 and the interest rate level r. A callable bond is redeemed immediately once the prevailing interest rate drops below the call boundary, or once the prevailing firm value exceeds the boundary. Call boundaries implied by high ($\tau = 0.35$) and low ($\tau = 0.25$) tax rates are denoted by solid and dashed curves, respectively.

Fig. 16. Here we compare the yield spread for the issue of a single bond (denoted by a circle) or the yield spread curve for the issue of serial bonds. For example, given that the issuing firm's financial status is poor (proxied by a low initial firm value $V_0 = 1000$), the spread for the issue of a 10-year bond with face value \$500 is denoted by node *B*, and the yield spread curve for the issue of five \$100 bonds with different maturities is denoted by the black curve. Note that the yield spread curve is lower than the yield spread of the single bond regardless of the change in the single bond's maturity and initial firm value V_0. This might be because dispersing principal repayments can alleviate the firm's insolvency risk on each bond maturity date and hence decrease the yield spreads of these bonds. Similar implications are also empirically consolidated by Norden et al. (2016).

This paper assumes that coupon payments are tax-deductible at an exogenous constant tax rate τ as long as the firm is solvent. This assumption is widely adopted by existent structural models, such as Leland (1994), Leland and Toft (1996), Hackbarth and Mauer (2012) and Kuehn and Schmid (2014). Even though a loss firm does not enjoy the interest tax deductibility immediately, its net operating losses can be carried forward for a long time, for instance 20 years in the US (see Graham, 2000). These losses can eventually be used as tax shields once the firm becomes profitable. The following experiments show that diminishing tax benefits slightly change the evaluation results without influencing our framework's ability to capture phenomena found in empirical studies.

The impact of diminishing tax benefits can be examined by decreasing the tax rate τ without changing other settings as illustrated in Fig. 17. Fig. 17(a) illustrates that diminishing tax benefits increase bond yield spreads without changing the shape of yield spread curves from upward-sloping to other shapes. This experiment is analogous to the experiment in Fig. 1. The upward-sloping pattern is consistent with the empirical studies such as Huang and Zhang (2008). The increment in yield spreads is due to the diminishment of tax benefits decreasing the benefits of using debt capital. Similar results can be found in Broadie and Kaya (2007). Fig. 17(b) illustrates the effect of diminishing tax benefits on call delay phenomena; this experiment is analogous to the experiment in Fig. 8(a). Note that the height of the call boundary denotes the significance of the call delay phenomenon as discussed in Section 4.3. King and Mauer (2000) observe that long-term callable bonds are prone to be redeemed earlier than the short-term one; this can be confirmed by observing that, ceteris paribus, the call boundary of short-term bonds (denoted by the gray color) is higher than the boundary of long-term ones (black color). In addition, diminishing tax benefits precipitate calls because the benefits of continuing to use debt capital decrease. This can be confirmed by observing that, ceteris paribus, the call boundary implied by a high tax rate (solid curve) is higher than the boundary implied by a low tax rate (dashed curve). Similar results can be found in Sarkar (2003) and Chen et al. (2013).

References

Acharya, v., Carpenter, J., 2002. Corporate bond valuation and hedging with stochastic interest rates and endogenous bankruptcy. Review of Financial Studies 15 (5), 1355–1383.

Attaoui, S., Poncet, P., 2013. Capital structure and debt priority. Financial Management 42 (4), 737–775.

Avramov, D., Jostova, G., Philipov, A., 2007. Understanding changes in corporate credit spreads. Financial Analysts Journal 90–105.

Barclay, M.J., Marx, L.M., Smith Jr, C.W., 2003. The joint determination of leverage and maturity. Journal of Corporate Finance 9 (2), 149–167.

Black, F., Cox, J., 1976. Valuing corporate securities: some effects of bond indenture provisions. Journal of Finance 31 (2), 351–367.

Brennan, M.J., Schwartz, E.S., 1977. Convertible bonds: valuation and optimal strategies for call and conversion. Journal of Finance 32 (5), 1699–1715.

Bris, A., Welch, I., Zhu, N., 2006. The costs of bankruptcy: chapter 7 liquidation versus chapter 11 reorganization. Journal of Finance 61, 1253–1303.

Briys, E., De Varenne, F., 1997. Valuing risky fixed rate debt: an extension. Journal of Financial and Quantitative Analysis 32 (2), 239–248.

Broadie, M., Kaya, O., 2007. A binomial lattice method for pricing corporate debt and modeling chapter 11 proceedings. Journal of Financial and Quantitative Analysis 42, 1253–1303.

Broadie, M., Chernov, M., Sundaresan, S., 2007. Optimal debt and equity values in the presence of chapter 7 and chapter 11. Journal of Finance 62 (3), 1341–1377.

Chen, H., 2010. Macroeconomic conditions and the puzzles of credit spreads and capital structure. Journal of Finance 65 (6), 2171–2212.

Chen, N., Kou, S., 2009. Credit spreads, optimal capital structure, and implied volatility with endogenous default and jump risk. Mathematical Finance 19 (3), 343–378.

Chen, H., Xu, Y., Yang, J., 2013. Systematic risk, debt maturity, and the term structure of credit spreads. Working Paper.

Chen, N., Dai, M., Wan, X., 2013. A nonzero-sum game approach to convertible bonds: tax benefit, bankruptcy cost, and early/late calls. Mathematical Finance 23 (1), 57–93.

Colla, P., Ippolito, F., Li, K., 2013. Debt specialization. Journal of Finance 68 (5), 2117–2141.

Collin-Dufresne, P., Goldstein, R.S., 2001. Do credit spreads reflect stationary leverage ratios? Journal of Finance 56 (5), 1929–1957.

Collin-Dufresne, P., Goldstein, R.S., Martin, J.S., 2001. The determinants of credit spread changes. Journal of Finance 56 (6), 2177–2207.

Cook, D.O., Easterwood, J.C., 1994. Poison put bonds: an analysis of their economic role. Journal of Finance 49 (5), 1905–1920.

Cox, J.C., Ross, S.A., Rubinstein, M., 1979. Option pricing: a simplified approach. Journal of Financial Economics 7 (3), 229–263.

Cremers, K.M., Nair, V.B., Wei, C., 2007. Governance mechanisms and bond prices. Review of Financial Studies 20 (5), 1359–1388.

Crosbie, P., Bohn, J., 2002. Modeling default risk. Working Paper KMV Corp.

Dai, T.-S., Lyuu, Y.-D., 2010. The bino-trinomial tree: a simple model for efficient and accurate option pricing. Journal of Derivatives 17 (4), 7–24.

Dass, N., Massa, M., 2014. The variety of maturities offered by firms and institutional investment in corporate bonds. Review of Financial Studies 2219–2266.

Davydenko, S.A., 2007. When do firms default? A study of the default boundary. Working Paper, August 2007.

Davydenko, S.A., 2012. When do firms default? a study of the default boundary. Working Paper, November 2012.

Duffee, G.R., 1998. The relation between treasury yields and corporate bond yield spreads. Journal of Finance 53 (6), 2225–2241.

Duffie, D., 1996. Dynamic Asset Pricing Theory. Princeton University Press.

Eom, Y.H., Helwege, J., Huang, J.-Z., 2004. Structural models of corporate bond pricing: an empirical analysis. Review of Financial Studies 17 (2), 499–544.

Fama, E.F., Miller, M.H., 1972. The Theory of Finance, vol. 3. Dryden Press Hinsdale, IL.

Fan, H., Sundaresan, S.M., 2000. Debt valuation, renegotiation, and optimal dividend policy. Review of Financial Studies 13 (4), 1057–1099.

Figlewski, S., Gao, B., 1999. The adaptive mesh model: a new approach to efficient option pricing. Journal of Financial Economics 53 (3), 313–351.

Flannery, M.J., Nikolova, S.S., Öztekin, Ö., 2012. Leverage expectations and bond credit spreads. Journal of Financial and Quantitative Analysis 47 (04), 689–714.

François, P., Morellec, E., 2004. Capital structure and asset prices: some effects of bankruptcy procedures. Journal of Business 77 (2), 387–411.

Galai, D., Raviv, A., Wiener, Z., 2007. Liquidation triggers and the valuation of equity and debt. Journal of Banking and Finance 31 (12), 3604–3620.

Geske, R., 1977. The valuation of corporate liabilities as compound options. Journal of Financial and Quantitative Analysis 12 (04), 541–552.

Gopalan, R., Song, F., Yerramilli, V., 2014. Debt maturity structure and credit quality. Journal of Financial and Quantitative Analysis 49 (04), 817–842.

Gorton, G., Kahn, J., 2000. The design of bank loan contracts. Review of Financial Studies 13 (2), 331–364.

Graham, J.R., 2000. How big are the tax benefits of debt? Journal of Finance 55 (5), 1901–1941.

Hackbarth, D., Mauer, D.C., 2012. Optimal priority structure, capital structure, and investment. Review of Financial Studies 25 (3), 747–796.

He, Z., Xiong, W., 2012. Rollover risk and credit risk. Journal of Finance 67 (2), 391–430.

Helwege, J., Turner, C.M., 1999. The slope of the credit yield curve for speculative–grade issuers. Journal of Finance 54 (5), 1869–1884.

Huang, J.-Z., Huang, M., 2012. How much of the corporate-treasury yield spread is due to credit risk? Review of Asset Pricing Studies 2 (2), 153–202.

Huang, J.-Z., Zhang, X., 2008. The slope of credit spread curves. Journal of Fixed Income 18 (1), 56–71.

Ingersoll, J.E., 1977. A contingent-claims valuation of convertible securities. Journal of Financial Economics 4 (3), 289–321.

Ingersoll, J.E., 1977. An examination of corporate call policies on convertible securities. Journal of Finance 32 (2), 463–478.

Ingersoll, J.E., 1987. Theory of Financial Decision Making, vol. 3. Rowman & Littlefield.

Jacoby, G., Shiller, I., 2010. Corporate bond pricing and the effects of endogenous default and call options. Journal of Fixed Income 20 (2), 80–100.

Jensen, M.C., Meckling, W.H., 1976. Theory of the firm: managerial behavior, agency costs and ownership structure. Journal of Financial Economics 3 (4), 305–360.

Jones, E.P., Mason, S.P., Rosenfeld, E., 1983. Contingent claims valuation of corporate liabilities: theory and empirical tests. National Bureau of Economic Research Working Paper.

Jones, E.P., Mason, S.P., Rosenfeld, E., 1984. Contingent claims analysis of corporate capital structures: an empirical investigation. Journal of Finance 39 (3), 611–625.
Ju, N., Ou-Yang, H., 2006. Capital structure, debt maturity, and stochastic interest rates*. Journal of Business 79 (5), 2469–2502.
Kahl, M., Shivdasani, A., Wang, Y., 2015. Short-term debt as bridge financing: evidence from the commercial paper market. Journal of Finance 70 (1), 211–255.
Kim, I., Ramaswamy, K., Sundaresan, S., 1993. Does default risk in coupons affect the valuation of corporate bonds?: a contingent claims model. Financial Management 22 (3), 117–131.
King, T.-H.D., 2002. An empirical examination of call option values implicit in us corporate bonds. Journal of Financial and Quantitative Analysis 37 (04), 693–721.
King, T.-H.D., Mauer, D.C., 2000. Corporate call policy for nonconvertible bonds. Journal of Business 73 (3), 403–444.
Kuehn, L.-A., Schmid, L., 2014. Investment-based corporate bond pricing. Journal of Finance 69 (6), 2741–2776.
Lando, D., 2004. Credit Risk Modeling: Theory and Applications. Princeton University Press.
Leland, H.E., 1994. Corporate debt value, bond covenants, and optimal capital structure. Journal of Finance 49 (4), 1213–1252.
Leland, H.E., 2004. Predictions of default probabilities in structural models of debt. Working Paper.
Leland, H.E., Toft, K.B., 1996. Optimal capital structure, endogenous bankruptcy, and the term structure of credit spreads. Journal of Finance 51 (3), 987–1019.
Linn, S.C., Stock, D.R., 2005. The impact of junior debt issuance on senior unsecured debt's risk premiums. Journal of Banking and Finance 29 (6), 1585–1609.
Liu, S.X., Qi, H., Wu, C., 2006. Personal taxes, endogenous default, and corporate bond yield spreads. Management Science 52 (6), 939–954.
Longstaff, F.A., Schwartz, E.S., 1995. A simple approach to valuing risky fixed and floating rate debt. Journal of Finance 50 (3), 789–819.
Longstaff, F.A., Tuckman, B.A., 1994. Calling nonconvertible debt and the problem of related wealth transfer effects. Financial Management 23 (4), 21–27.
Mauer, D.C., 1993. Optimal bond call policies under transactions costs. Journal of Financial Research 16 (1), 23–37.
Merton, R.C., 1974. On the pricing of corporate debt: the risk structure of interest rates. Journal of Finance 29 (2), 449–470.
Modigliani, F., Miller, M.H., 1958. The cost of capital, corporation finance and the theory of investment. American Economic Review 48 (3), 261–297.
Myers, S.C., 1977. Determinants of corporate borrowing. Journal of Financial Economics 5 (2), 147–175.
Nagler, F., 2014. Rolling over corporate bonds: how market liquidity affects credit risk. Working Paper.
Norden, L., Roosenboom, P., Wang, T., 2016. The effects of corporate bond granularity. Journal of Banking and Finance 63, 25–34.
Ou, S., Chlu, D., Metz, A., 2011. Corporate Default and Recovery Rates, 1920–2010. Moody's Investors Service.
Rauh, J.D., Sufi, A., 2010. Capital structure and debt structure. Review of Financial Studies 23 (12), 4242–4280.
Sarkar, S., 2003. Early and late calls of convertible bonds: theory and evidence. Journal of Banking and Finance 27 (7), 1349–1374.
Smith, C.W., Warner, J.B., 1979. On financial contracting: an analysis of bond covenants. Journal of Financial Economics 7 (2), 117–161.
Tewari, M., Byrd, A., Ramanlal, P., 2015. Callable bonds, reinvestment risk, and credit rating improvements: role of the call premium. Journal of Financial Economics 115 (2), 349–360.
Thatcher, J.S., 1985. The choice of call provision terms: evidence of the existence of agency costs of debt. Journal of Finance 40 (2), 549–561.
Wang, C.-J., Dai, T.-S., Lyuu, Y.-D., 2014. Evaluating corporate bonds with complicated liability structures and bond provisions. European Journal of Operational Research 237 (2), 749–757.
Welch, I., 1997. Why is bank debt senior? a theory of asymmetry and claim priority based on influence costs. Review of Financial Studies 10 (4), 1203–1236.
Xu, Q., 2014. Kicking the maturity down the road: Early refinancing and maturity management in the corporate bond market. Working Paper.
Zhou, C., 2001. The term structure of credit spreads with jump risk. Journal of Banking and Finance 25 (11), 2015–2040.