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Competitive Price-Matching Guarantees: Equilibrium Analysis of the Availability Verification Clause Under Demand Uncertainty

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Price-matching guarantees involve a retailer matching the lower price of a competitor for an identical product. In reality, retailers often make such guarantees contingent on the verification of product availability at the competitor's location, and decline a price-match request if the product is not available there. This creates certain consternation on the part of customers. In this paper, we investigate the availability contingency strategy from the perspectives of both the retailers and the customers. Our analysis shows that availability contingency clauses intensify inventory competition between retailers and reinstitutes price competition, which is otherwise eliminated by unconditional price-matching guarantees. Consequently, despite what customers may think about the availability verification, it actually *increases* their surplus. On the other hand, such a clause reduces profits and, hence, is *not* the equilibrium strategy for retailers. Subsequently, we discuss how a likely customer behaviour pattern may be a plausible explanation regarding the use of the clause by the retailers in practice.

Key words: price-matching guarantees; inventory; availability; stochastic demand; pricing; verification of availability
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1. Introduction

A price-matching guarantee (PMG) is a popular pricing strategy under which a retail store promises to match any lower price offered by a competitor for an identical merchandise. Under this strategy, customers have the opportunity to receive the lowest price in the marketplace at their preferred store. Consider a customer who prefers to shop at a particular retailer offering a PMG but realizes that a competing retailer is offering a lower price. Through price-match, the customer can get the lower price from her or his preferred retailer. A distinguishing feature of PMGs in practice is that retailers often make such guarantees contingent on product availability. Specifically, they reserve the right to verify the availability of the product at the competitor's location and decline the price-match request if the product is unavailable there. For instance, Best Buy (2012) explicitly states: "Tell us which competitor is offering the lower price; we will verify the price and that the item is in stock and available for immediate sale and delivery." Most of the retail giants, such as Home Depot, OfficeMax, Sears, Staples, Walmart, and Futureshop (in Canada), offer PMGs that are contingent on availability.

If retailers had inventory ready for sale at all times, then availability verification would be inconsequential; customers would always succeed in matching the price because the item would always be available at the competing retailer. In such a perfect-availability setting, PMGs have been well studied, and it has been shown that, despite seeming like a competition-enhancing device, they actually lead to *tacit collusion*, allowing retailers to collectively increase their prices (e.g., Hay 1981, Salop 1986). This is because if the competitor is offering a PMG, then a retailer cannot attract more customers via price reduction. Customers then ask for price matching at their preferred stores instead of visiting lower-priced ones. The collusion outcome prevails even when retailers make simultaneous (Sargent 1992) and sequential (Belton 1987) decisions, as well as in oligopolistic markets (Corts 1995, Doyle 1988). PMGs have also been demonstrated to enable retailers to price discriminate against customers who are uninformed about prices prior to their store visits (Baye and Kovenock 1994, Chen et al. 2001, Corts 1997, Edlin 1997, Png and Hirshleifer 1987) and to signal their low-cost or low-service-level position under horizontal and vertical differentiation

(Jain and Srivastava 2000, Moorthy and Winter 2006, Moorthy and Zhang 2006, Winter 2009).[1]

Unfortunately, perfect availability is not the reality in the retail industry; rather, stockouts are a serious problem. The worldwide stockout rate in the retail sector totals approximately 8% and reaches nearly 15% for promotional items (Gruen et al. 2002). It is an especially problematic issue for consumer electronics retailers (Holman and Buzek 2008). Stockouts, obviously, make the verification clause relevant and result in denial of some PMG requests. Consequently, customers often consider the clause to be an obstacle placed in their way by retailers to reject price-match requests. This is evidenced by a class action law suit filed against Best Buy in the U.S. District Court related to the company's price-matching practices (Silverberg 2009). Other consumer electronics retailers, such as Futureshop, have also been accused of dishonouring their promises (RedFlagDeals.com 2011).

In spite of its prevalence, the paper by Nalca et al. (2010) is the only paper in the extant literature studying the availability contingency clause. Assuming that such a clause is in effect, along with an exogenously specified frequency of inventory stockouts, they showed that the availability proviso enables retailers to price discriminate against customers who are informed about prices. In this paper, we endogenize the inventory decisions in an uncertain demand environment and address key operational and strategic issues related to the availability verification condition not studied in the extant PMG literature. Specifically, we shed light on the following two fundamental questions:

1. *What are the implications of availability verification proviso for retailers and customers*? (In particular, how are retail prices, ordering quantities, profits, and consumer surplus affected by the verification of availability as a PMG condition?)
2. *In equilibrium*, *should the retailers offer availability-contingent PMGs*?

We show that the verification of availability intensifies the inventory competition and leads to higher order quantities. Nonetheless, retailers still cannot provide perfect availability. The result is that customer price-matching requests could be refused. This *reinstitutes* the price competition between retailers that is otherwise *eliminated* by unconditional PMGs (i.e., the verification condition reduces prices). Consequently, customers enjoy higher levels of surplus. However, by the same token, the profits of the retailers are adversely affected such that, at the equilibrium, it is indeed *not* in their best interest to offer the verification clause.

[1] In addition to competitive PMGs, there is also an alternative price-guarantee mechanism, less related to our work, that is referred to as an internal price-matching guarantee (IPM). Under IPMs, retailers promise to refund the difference to a customer who has already purchased an item if a lower price is offered during some prespecified time period for that item. IPMs have been shown to facilitate collusion (Cooper 1986, Butz 1990) and to mitigate the strategic purchasing behaviour of customers (Aviv et al. 2009, Lai et al. 2010, Levin et al. 2007).

2. Model Framework

Consider two competing retailers, denoted as R_1 and R_2, selling identical short-life-cycle products with random demand. Each retailer's strategy consists of the type of PMG policy offered: the price (p_i) and the order quantity (q_i). A glossary of the notation is provided in the appendix. Retailers set their PMG policies by choosing from three strategies: no price-matching guarantee (C), unconditional price matching (PM), or price matching contingent on availability (PMA). Figure 1 facilitates our description of the policies by illustrating the final price charged to customers at each retailer when R_1 is the high-priced retailer. In the case of PM, proof of a lower price is sufficient for the price match. As a result, customers pay the lower of the two retail prices when they request a price match. In the case of PMA, a price match is contingent on the competitor's availability, and price is matched only if the item is available for immediate sale at the competing store. Note that PM and PMA policies are identical if the competing retailer has perfect availability.

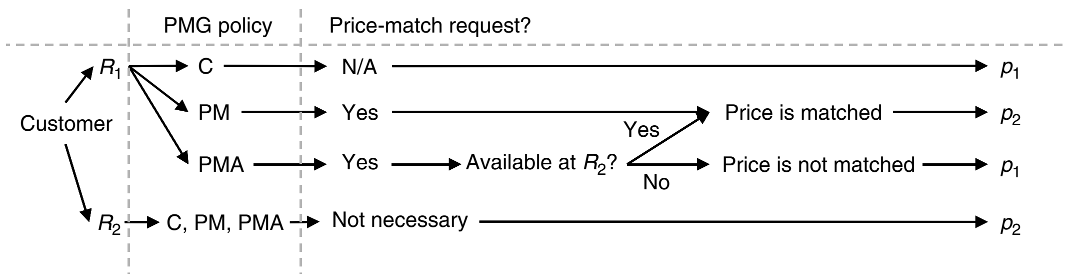
The game consists of three stages. In the first stage, the two retailers set their PMG policies. In the second stage, they choose prices and the amounts of inventory to stock. In the third stage, customers decide which retailer to visit. At each stage, involved parties make decisions simultaneously, and we seek subgame perfect equilibrium by solving the game backward.

2.1. Demand Model

Each retailer's demand is assumed to be equal to the aggregate demand multiplied by its market share, the latter of which depends on the PMG policy, price, and order quantity of both retailers. The total number of customers who want the product (i.e., the aggregate demand) is random, denoted by a positive random variable ε, which follows distribution F and density f. We assume that F has an increasing failure rate (see Lariviere 2006 for further reference).

The two retailers are located at the end points of a unit line, and potential customers are uniformly distributed between the retailers. Each customer has a constant valuation V for the product, purchases zero units or one unit of the product, and incurs a travel cost of k per unit distance traveled. In our setting, the travel cost is a proxy for the degree of differentiation between the retailers. Specifically, a high travel cost represents highly differentiated retailers, and vice

Figure 1 **Observed Price at Each Retailer When $p_1 > p_2$**



versa, which could be due to distinct store experiences, such as sales features and store design, or to a large geographical distance between them (Tirole 1988, p. 279).

All retail decisions (i.e., price, order quantity, and PMG policy) are observable by individual customers. Customers cannot verify product availability before visiting a retailer, but they are able to deduce an anticipated probability of finding the product in stock at each retailer, i.e., the anticipated fill rate of each retailer conditional on their visit to that retailer. Each customer is rational and makes a store choice to maximize her or his expected net utility. For this purpose, they account for the *price, fill rate, and the probability of being successful in their price-match requests given the availability proviso.* We assume that the cost of visiting a second retailer is arbitrarily high, and therefore customers visit just one retailer. Note that the demand is itself the result of a *rational expectations* game among customers because each customer's purchase decision is based on the probability of finding the product at the visited retailer and the probability of receiving a price match, both of which depend on the choices of all customers.

Later on, we shall show that there exists a unique rational expectations equilibrium to the game among customers. However, for the time being, to provide the details of the customer choice process and its relation to different PMG policies, we will work with the anticipated fill rate at R_i, $i \in \{1, 2\}$ (denoted by FR_i). The arguments of the anticipated fill rate are suppressed, but it should be kept in mind that it is a function of the number of customers that each customer anticipates will shop at each retailer as well as the PMG policies, prices, and order quantities of the retailers. We assume symmetry in the sense that all customers have the same probability of obtaining the product or receiving a price match independent of their proximity to retail locations.

2.1.1. Utility Functions. Without loss of generality, we place R_1 at origin and R_2 at the end of the unit line. In what follows, we derive the expected net utility of a representative customer visiting R_1 under each possible PMG policy. The expected utility of visiting R_2 can be obtained similarly. Let $u_i^{\mathcal{P}_i}(l)$ denote the expected utility obtained at R_i by a customer located at point l under policy $\mathcal{P}_i \in \{C, PM, PMA\}$, $i \in \{1, 2\}$. Based on the above description, the expected utility of visiting R_1 when R_1 is not offering a PMG, i.e., policy C, is

$$u_1^C(l) = (V - p_1) \cdot FR_1 - k \cdot l. \tag{1}$$

Consider now the case where R_1 offers PM. On one hand, if R_1 is the high-priced retailer, then customers will ask for a price match if they visit R_1. Each price match request is going to be granted because the PMG is not contingent on availability. On the other hand, if R_1 is the low-priced retailer, then there is no need to ask for a price match. This implies that the list price of R_1 can be replaced by the minimum of the two prices, $p_e \equiv \min\{p_1, p_2\}$, which we denote as the "effective price." The expected utility in this case is

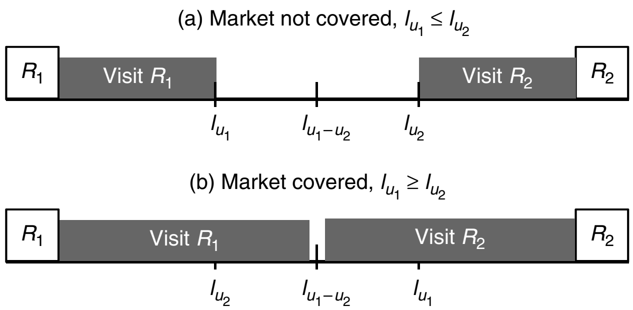
$$u_1^{PM}(l) = (V - p_e) \cdot FR_1 - k \cdot l. \tag{2}$$

Finally, suppose that R_1 offers PMA. Similar to the PM case, customers request a price match at R_1. However, the end result depends on the availability of the product at R_2. In particular, if the product is available at R_2, then customers receive the effective price and demand one unit of the product. If the product is not available at R_2, then they face the list price p_1 and demand one unit of the product as long as the price is less than their valuation of the product. In other words, the price charged by R_1 is p_e with probability FR_2 and p_1 with probability $1 - FR_2$, and the expected price is given by $\bar{p}_1 \equiv p_e \cdot FR_2 + p_1 \cdot (1 - FR_2)$. The expected utility in this case is

$$\begin{aligned} u_1^{PMA}(l) &= (V - \bar{p}_1) \cdot FR_1 - k \cdot l \\ &= \big(V - p_e \cdot FR_2 - p_1 \cdot (1 - FR_2)\big) \cdot FR_1 - k \cdot l. \end{aligned} \tag{3}$$

2.1.2. Retail Demand. Customers compare the above utilities and visit the retailer providing the highest expected net utility as long as it is also nonnegative. For instance, if none of the retailers offers a PMG, then each customer compares $u_1^C(l)$ and $u_2^C(l)$. In Figure 2, the customers located at $l_{u_1} = (V - p_1) \cdot FR_1/k$ and $l_{u_2} = 1 - (V - p_2) \cdot FR_2/k$ are indifferent

Figure 2 **Market Share of the Retailers**



between not buying and visiting R_1 and R_2, respectively. The customer located at $l_{u_1-u_2} = (k + (V - p_1) \cdot FR_1 - (V - p_2) \cdot FR_2)/(2k)$ is indifferent between visiting the two retailers. Depending on the problem parameters and the strategies of retailers, we may have an uncovered market, meaning that there is a nonempty set of customers who are unable to obtain a nonnegative expected utility from any of the retailers and therefore do not visit any of them (Figure 2(a)). In this case, the market share of R_i is going to be $((V - p_i) \cdot FR_i)/k$ for $i = 1, 2$. We may also have a covered market scenario where all customers obtain a nonnegative expected utility from at least one retailer (Figure 2(b)). In this case, the market shares of R_1 and R_2 are going to be $(k + (V - p_1) \cdot FR_1 - (V - p_2) \cdot FR_2)/(2k)$ and $1 - (k + (V - p_1) \cdot FR_1 - (V - p_2) \cdot FR_2)/(2k)$, respectively.

When we have an uncovered market, each retailer acts as a local monopoly. This is an uninteresting case to analyze because retailers are then not competing for any of the customers in the market. So, in the sequel, we assume that the problem parameters are such that the market is covered in equilibrium. During our analysis (§3), we prove that a covered market equilibrium is guaranteed for a relatively low travel cost (k), i.e., retailers are close enough in terms of store characteristics and/or geographical proximity. Indeed, this assumption is consistent with the real-life implementation of PMGs because they are valid only for retail stores located in the same area. For instance, Sears Canada (2009) explicitly states: "We reserve the right to verify that the competitor is an authorized dealer located in Canada, that the advertisement is correct, and that the merchandise is identical, and is in-stock at the *competitor's local store*."

Let $\mathcal{S} = (\mathcal{S}_i, \mathcal{S}_j)$ represent the retail strategies, where $\mathcal{S}_i = (\mathcal{P}_i, p_i, q_i)$, and let $FR = (FR_i, FR_j)$ be the anticipated fill rates for retailers $i = 1, 2$ and $j = 3 - i$. We can then derive the market shares for any combination of PMG offers, and they are as follows:

$$d_1(\mathcal{S} \mid FR) = \frac{k + (V - p_1^{\mathcal{P}_1}) \cdot FR_1 - (V - p_2^{\mathcal{P}_2}) \cdot FR_2}{2k},$$

$$d_2(\mathcal{S} \mid FR) = \frac{k - (V - p_1^{\mathcal{P}_1}) \cdot FR_1 + (V - p_2^{\mathcal{P}_2}) \cdot FR_2}{2k},$$

where $p_i^C = p_i$, $p_i^{PM} = p_e$, and $p_i^{PMA} = \bar{p}_i = p_e \cdot FR_j + p_i \cdot (1 - FR_j)$, for $i = 1, 2$ and $j = 3 - i$, denote the anticipated prices given the PMG policies and the list prices.[2] Demand at each retailer, given the market size ε and conditional on the anticipated fill rate, is then $D_i(\mathcal{S} \mid FR) = \varepsilon \cdot d_i(\mathcal{S} \mid FR)$. Observed fill rate at R_i, conditional on the anticipated fill-rate, can be written as $\mathcal{FR}_i(\mathcal{S} \mid FR) = E[\min\{D_i(\mathcal{S} \mid FR), q_i\}/D_i(\mathcal{S} \mid FR)]$. In the following proposition, we show that customer anticipation is consistent with observations. All the proofs are provided in the appendix.

Proposition 1. *There is a unique rational expectations equilibrium to the game among customers.*

The unique equilibrium of the game among customers allows us to describe the market share, the random demand, and expected profit as a function of retail strategy, which we denote by $d_i(\mathcal{S})$, $D_i(\mathcal{S}) = \varepsilon \cdot d_i(\mathcal{S})$, and $\pi_i(\mathcal{S})$, respectively. The market share of R_1 in each subgame is given in Table 1.

2.2. Profit Functions

We assume that the retailers have identical purchasing costs (\$$c$/unit) and that leftover units have zero salvage value. We again focus on R_1; profit for R_2 can be deduced similarly. Suppose that R_1 is not offering any type of PMG (i.e., policy C). If customers decide to visit R_1, then they will face the list price, irrespective of the PMG policy offered by R_2, and receive one unit of the product as long as the item is available. Accordingly, the expected profit in this setting is

$$\pi_1(C, p_1, q_1, \mathcal{S}_2) = p_1 \cdot E[\min\{q_1, D_1(C, p_1, q_1, \mathcal{S}_2)\}] - c \cdot q_1.$$

Suppose now that R_1 is offering PM. While making their store decisions, customers will look at the effective price instead of the list price because the price-matching offer at R_1 is not conditional on availability. Accordingly, the expected profit function of R_1 is

$$\pi_1(PM, p_1, q_1, \mathcal{S}_2) = p_e \cdot E[\min\{q_1, D_1(PM, p_1, q_1, \mathcal{S}_2)\}] - c \cdot q_1.$$

Recall that, in case of PMA, some of the customers visiting R_1 will be able to get the effective price, whereas some others will receive the list price. We assume that the inventory is proportionally rationed in the event of a stockout (Tirole 1988, p. 213). Then, the expected profit at R_1 is

$$\pi_1(PMA, p_1, q_1, \mathcal{S}_2) = [p_e \cdot FR_2 + p_1 \cdot (1 - FR_2)] \cdot E[\min\{q_1, D_1(PMA, p_1, q_1, \mathcal{S}_2)\}] - c \cdot q_1.$$

[2] The market shares presented in this section are for given list prices, anticipated fill rates, and PMG policies of retailers. The equilibrium decisions are different under each scenario and are derived in §3.

Table 1 **Market Share d_1 of R_1 as a Function of Prices, Anticipated Fill Rates, and the PMG Policies in Effect**

	R_2		
R_1	C	PM	PMA
C	$\frac{(V-p_1)\cdot FR_1-(V-p_2)\cdot FR_2+k}{2k}$	$\frac{(V-p_1)\cdot FR_1-(V-p_e)\cdot FR_2+k}{2k}$	$\frac{(V-p_1)\cdot FR_1-(V-\bar{p}_2)\cdot FR_2+k}{2k}$
PM	$\frac{(V-p_e)\cdot FR_1-(V-p_2)\cdot FR_2+k}{2k}$	$\frac{(V-p_e)\cdot FR_1-(V-p_e)\cdot FR_2+k}{2k}$	$\frac{(V-p_e)\cdot FR_1-(V-\bar{p}_2)\cdot FR_2+k}{2k}$
PMA	$\frac{(V-\bar{p}_1)\cdot FR_1-(V-p_2)\cdot FR_2+k}{2k}$	$\frac{(V-\bar{p}_1)\cdot FR_1-(V-p_e)\cdot FR_2+k}{2k}$	$\frac{(V-\bar{p}_1)\cdot FR_1-(V-\bar{p}_2)\cdot FR_2+k}{2k}$

Because customers' purchase decisions are sensitive to product availability, the inventory decisions of the retailers have an effect on demand in all three scenarios. However, the inventory decision of the competing retailer has a direct effect on the price charged to customers *only under the PMA policy* when $\bar{p}_i = p_e \cdot FR_j + p_i \cdot (1-FR_j)$ for $i = 1, 2$ and $j = 3-i$.

3. Analysis

We seek subgame perfect equilibria. In the event of multiple equilibria, we use Pareto dominance as the refinement method; that is, if both retailers can increase their profits by choosing one equilibrium over another, they will do so. Our starting basis is the scenario with no PMGs (i.e., each retailer offers policy C). The model and the analysis for this scenario are similar to those of Deneckere and Peck (1995) and Krishnan and Winter (2010). As is well known from these works, the existence of pure strategy equilibrium is at risk when firms simultaneously select prices and order quantities under demand uncertainty. This is most often due to the failure of the quasi concavity of the profit functions. Specifically, if the travel cost is very low, then a firm can deviate from any potential equilibrium by undercutting the competitor's price and increasing the order quantity, which enables the firm to capture the entire market. In our model, a very high travel cost is not desirable either because, as discussed in §2.1, it leads to an uncovered market scenario. As a remedy to these risks, we prove in the appendix that there exists a range, $[\underline{k}, \bar{k}]$, for the travel cost where the above-mentioned problems can be avoided. In particular, $k \geq \underline{k}$ guarantees a pure strategy equilibrium, and $k \leq \bar{k}$ guarantees a covered market scenario. In the remainder, we restrict our attention to $k \in [\underline{k}, \bar{k}]$, implying that the retailers are not excessively but sufficiently differentiated (which is also the practically interesting scenario). We determine the impact of availability verification by comparing the equilibrium under C and the price-matching scenarios PM and PMA. For expositional brevity, we concentrate on subgames where both retailers offer the same policy. In the appendix, we prove that each subgame has a symmetric equilibrium, meaning that the decisions and profits of the two retailers are identical. So, we use $p^{\mathcal{P}}$, $q^{\mathcal{P}}$, $\pi^{\mathcal{P}}$, and $cs^{\mathcal{P}}$ to denote the equilibrium price, order quantity, retail profit, and consumer surplus under policy $\mathcal{P} \in \{C, PM, PMA\}$, respectively.

We start our discussion with the effects on retail decisions.

Proposition 2. *The following are true.*

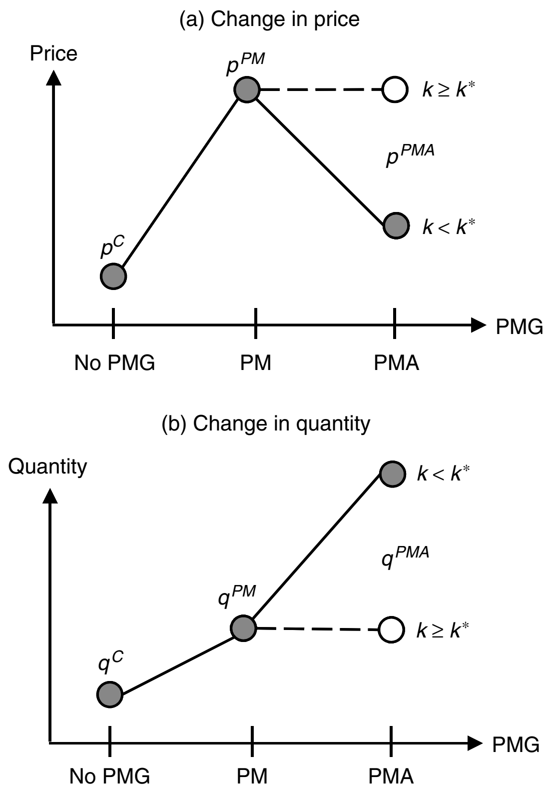
(i) *Offering an unconditional PMG increases the price and order quantity of the retailers compared to the no-PMG case.*

(ii) *An availability verification clause, compared to an unconditional PMG, results in lower prices and higher order quantity for the retailers if the travel cost is sufficiently low, and has no effect on decisions otherwise.*

Specifically, there exists a threshold $k^* \in [\underline{k}, \bar{k}]$ for the unit travel cost such that if $k < k^*$, then $p^C < p^{PMA} < p^{PM}$ and $q^C < q^{PM} < q^{PMA}$, and if $k \geq k^*$, then $p^C < p^{PMA} = p^{PM}$ and $q^C < q^{PM} = q^{PMA}$. In other words, the availability clause has a monotone impact when the travel cost is low and has no impact when the travel cost is high. Accordingly, in the remainder we use the terms *increasing* and *decreasing* in the weak sense. The price and order quantity for each scenario are depicted in Figure 3.

The first part of Proposition 2 states that offering an unconditional PMG (i.e., PM policy) leads to higher prices and inventory levels. The intuition behind this result is as follows. At any solution with equal prices, R_1 cannot attract more customers through price reduction because R_2 automatically matches the lower price via its PMG. Also, R_1 has no incentive to increase the price because it is also offering PM and has to match R_2's price for all of its customers, i.e., demand and the profit margin remain the same even if R_1 increases price. In short, *offering a PM policy eliminates the price competition between the retailers, allowing them to collectively increase their prices*; that is, the tacit-collusion results related to PMGs in a deterministic setting (Salop 1986) continue to hold in the stochastic case as well. After the price increase, inventory is the only tool available to the retailers for attracting

Figure 3 **Comparison of the Price and Inventory Decisions**



more customers. So, they order more compared to the no-PMG case.

More interesting is that verification of availability as a PMG condition leads to lower prices and higher inventory levels compared to an unconditional PMG. Let us look at the implications of availability verification for R_1. First of all, such a condition allows R_1 to charge a higher price to its customers if there is a stockout at R_2. Suppose that the state of the demand is high and that the inventory of R_2 is exhausted. As soon as the last unit in R_2 is sold, R_1 can start declining the price-match requests via its verification clause. By declining the requests, R_1 can charge a higher price when customers are at the store asking for a price match, i.e., once their travel cost is sunk and there are no units available at R_2. This is beneficial for R_1 because it extracts a higher margin from these customers. On the other hand, there is also a cost attached to verifying the availability. When verifying the availability, R_1 is matching the price *only* if the product is available at R_2; that is, R_1 is now matching the price for *some* of its customers, as opposed to the PM policy, where the price is matched for *all* customers. Customers can infer this and switch to R_2 because they think that they would not be able to get a price match and have to pay the high list price at R_1 (this is related to the customer consternation about PMA discussed in §1). As such, R_1 loses some customers to R_2. We observe both effects while deriving the best-response mappings of the players. However, at equilibrium, the cost associated with the verification condition turns out to be stronger than its benefits. The PMA policy thus creates an incentive for the retailers to reduce their prices compared to the PM case to counteract the potential for customer loss associated with it; i.e., the retailers compete on prices under the PMA policy. Recall that offering PMA with perfect availability would be identical to PM and would allow high prices. In an attempt to imitate this behaviour under the PMA policy, the retailers then improve their availability by increasing order quantities.

Based on the above findings, we can establish the effects of verification availability on the consumers' surplus and the retailer profits. We measure the former as the expected net total utility obtained by all customers.

Proposition 3. *The availability verification clause increases consumers' surplus and decreases retail profits.*

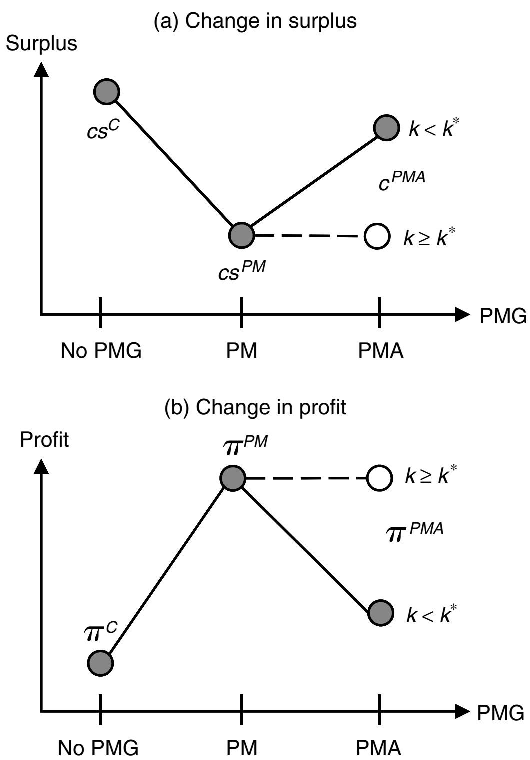
Because we have a symmetric equilibrium and a covered market scenario in each case, the comparison of the surplus boils down to the comparison of retail decisions. Accordingly, an availability verification clause *increases consumers' surplus* by accommodating lower prices and higher order quantities (Figure 4(a)). Even though customers may think the PMA policy is a hassle for them, the retail competition spawned by availability verification can indeed be beneficial if they are aware of the possibility of product unavailability and its effects while making store choices. Be that as it may, verification of availability leads to lower profits for retailers (Figure 4(b)). The main reason is that the availability clause increases the order quantity but at the same time reduces the profit margin and increases the cost of overstocking, which requires the retailer to reduce its order quantity.

Our analysis shows that verification of availability in PMGs decreases retail profits. However, a complete characterization also requires the equilibrium solution to the retailer's strategic decision as to which form of the PMG to adopt. The following proposition states that *retailers should not impose availability verification as a condition for granting price-match requests.* Rather, they should unconditionally match the price of the competing retailer.

Proposition 4. *Offering an unconditional PMG is the dominating equilibrium strategy for both retailers.*

Indeed, the results of this section are quite robust with respect to our modelling choices. In particular, verification of availability reduces prices, increases order quantities, reduces retail profits, and increases consumers' surplus compared to the PM policy in a variety of related model settings (the details of the

Figure 4 Comparison of Consumers' Surplus and Retail Profit



analysis are provided in the expanded working paper version of this paper, Nalca et al. 2012):

- Retailers make their price and inventory decisions sequentially rather than simultaneously.
- Retailers are asymmetric in terms of their product costs (although not extremely).
- Retailers offer an alternative PMA policy description, where the retailer announces that it will verify the availability for a certain fraction of the requests. In other words, for each price-match request, there is a predetermined probability that the retailer will exercise the availability contingency clause. This model is motivated by the fact that, in reality, retailers "reserve" the right to verify the availability and do not necessarily verify the availability for each price-match request.
- Under information heterogeneity, which is introduced through a segment called *uninformed customers*. Uninformed customers have no information regarding the PMGs, price, or product availability prior to their store visits. The related literature shows that unconditional PMGs allow price discriminating against uninformed customers (Png and Hirshleifer 1987). Our numerical analysis shows that the availability clause, in addition to increasing customer surplus and decreasing retail profits and in contrast to previous literature, may actually prevent retailers from price discriminating against customers by removing the price difference between the retailers.

4. Concluding Discussion

Retailers offering PMGs promise to match any lower price at the competitor for identical products. For many retailers, the price-matching offer is conditional on the availability of the product at the competing retailer. So, the retailer does not match the price unless the product is available for sale at the competing location. Although unconditional PMGs have been well analyzed in the extant literature, the effects of availability-contingent PMGs on retailers and customers have been a relatively neglected topic. In this paper, we enrich the literature by building and analyzing a theoretical framework that establishes such effects in an uncertain demand environment with rational customers.

Our paper provides strong theoretical support for retailers *not* to offer a price-match guarantee that is contingent on availability by showing that it leads to lower retail profits. The reason is that under an unconditional PMG, the retailer promises to match the price for all customers, and thus completely eliminates the risk of losing customers to competing stores due to lower prices. But, if the guarantee is contingent on availability, then customers foresee the risk of being declined because of product unavailability. So, offering lower prices again becomes a viable strategy to attract customers. This results in reinstitution of price competition between the retailers, leading to lower prices. Note that the availability clause would not have any effect on the equilibrium prices had the retailers been able to provide perfect availability. Therefore, investigating the availability clause in an uncertain demand environment improves our knowledge of inventory decisions as well. Indeed, when verifying the availability, retailers increase their order quantities, which leads to higher product availability and reduces the number of customers who are visiting the rival because of the concern of having their price-match requests declined. In fact, even though customers may see the availability verification as a hassle created by retailers to discourage them from requesting price matches, this strategy is beneficial for them, resulting in *higher surpluses*. We also analyze extensions to our basic framework to show that the above effects of the verification strategy are quite robust with respect to the modelling choices.

However, we observe widespread use of the availability verification clause by retailers in the real world. One plausible explanation for the above discrepancy may be that customers ignore or are not well informed about the clause; that is, while making store choice decisions, customers may assume that their price-match requests are always going to be granted. We consider a couple of alternatives to make this idea operational. In the first one, customers focus solely on the prices while making store choices and do not consider the possibility or impact of product unavailability, i.e., they ignore that retailers may be sold out at the time of their visits. In the second approach, when making store choice decisions, customers are aware that retailers may stock out; however, they are not cognizant about the verification of availability as a PMG proviso and therefore assume that their price-match requests will always be granted. Our analysis of these two settings clearly proves that the equilibrium strategy of the retailers is indeed to verify the availability at the competing store before granting a price match.[3] The verification clause then enables the retailers to *price discriminate* against customers. Retailers take advantage of scarcity in the market and obtain higher profits by verifying the availability and declining to match the price if the competitor is out of stock. In fact, an availability proviso leads to higher equilibrium prices, increases the retail profits, and decreases consumers' surplus.

In summary, in this paper we analyze the popular retail strategy of verifying the availability as a PMG condition. Customers consider this as a contentious retail tactic to avoid price-match promises. We show that such a condition negates any incentive for price collusion as long as customers incorporate the implications of the availability condition during their decision-making process. Consequently, contrary to public perception, an availability verification proviso actually hurts the retailers and benefits the customers.

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Appendix

Notation

i, j: Index for retailers $i \in \{1, 2\}$ and $j = 3 - i$
R_i: Retailer i
$\mathcal{P}_i$: Price-matching policy $\mathcal{P}_i \in \{C, PM, PMA\}$
c: Unit cost
p_i: List price at R_i
q_i: Order quantity at R_i
$S_i = (\mathcal{P}_i, p_i, q_i)$: Strategy of R_i
FR_i: Fill rate: probability of finding the product available at R_i
p_e: Effective price equal to $\min\{p_1, p_2\}$
$\bar{p}_i$: Expected price at R_i under the PMA policy equal to $p_e \cdot FR_j + p_i \cdot (1 - FR_j)$
$p_i^{\mathcal{P}}$: The price customers expect to pay at R_i when R_i offers policy $\mathcal{P}$
V: Valuation of the product sold at retailers
k: Travel cost per distance
$u_i^{\mathcal{P}_i}(l)$: Expected utility of visiting R_i with policy $\mathcal{P}_i$ from location l
ε: Aggregate number of customers (random) in the market
$f(\cdot)$, $F(\cdot)$: Density and the distribution function of the aggregate demand
CV: Coefficient of variation
$d_i(\mathcal{S}_i, \mathcal{S}_j)$: Market share of R_i
$D_i(\mathcal{S}_i, \mathcal{S}_j) = \varepsilon \cdot d_i(\mathcal{S}_i, \mathcal{S}_j)$: Random demand at R_i
$\pi_i(\mathcal{S}_i, \mathcal{S}_j)$: Expected profit of R_i
$p^{\mathcal{P}}, q^{\mathcal{P}}, \pi^{\mathcal{P}}$: Equilibrium price, quantity, and profit when both retailers offer $\mathcal{P}$ policy

Proof of Proposition 1

Consider a customer located at l. Given the price, anticipated fill rate, and the PMG policy of each retailer, the expected utility provided by visiting R_i is given by $u_i^{\mathcal{P}_i}(l) = (V - p_i^{\mathcal{P}_i}) \cdot FR_i - t_i(l)$, where $p_i^C = p_i$, $p_i^{PM} = p_e = \min\{p_1, p_2\}$, and $p_i^{PMA} = \bar{p}_i = p_e \cdot FR_j + p_i \cdot (1 - FR_j)$ for $i = 1, 2$ and $j = 3 - i$. Accordingly, given the fill rate that each customer expects to observe at the retailers, the market share of each retailer as a function of retailer strategies is given by $d_i(\mathcal{S} \mid FR) = \{l \mid u_i^{\mathcal{P}_i}(l) \geq 0 \text{ and } u_i^{\mathcal{P}_i}(l) \geq u_j^{\mathcal{P}_j}(l)\}$. Note that $u_1^{\mathcal{P}_1}(l) \geq 0$ if and only if $l \leq (V - p_1^{\mathcal{P}_1}) \cdot FR_1/k$, $u_2^{\mathcal{P}_2}(l) \geq 0$ if and only if $l \geq 1 - (V - p_2^{\mathcal{P}_2}) \cdot FR_2/k$, and $u_1^{\mathcal{P}_1}(l) \geq u_2^{\mathcal{P}_2}(l)$ if and only if $l \leq [(V - p_1^{\mathcal{P}_1}) \cdot FR_1 - (V - p_2^{\mathcal{P}_2}) \cdot FR_2 + k]/(2k)$. On one hand, if

$$\frac{(V - p_1^{\mathcal{P}_1}) \cdot FR_1}{k} \leq 1 - \frac{(V - p_2^{\mathcal{P}_2}) \cdot FR_2}{k},$$

then

$$\frac{(V - p_1^{\mathcal{P}_1}) \cdot FR_1}{k} \leq \frac{(V - p_1^{\mathcal{P}_1}) \cdot FR_1 - (V - p_2^{\mathcal{P}_2}) \cdot FR_2 + k}{2k} \leq 1 - \frac{(V - p_2^{\mathcal{P}_2}) \cdot FR_2}{k},$$

and we have an uncovered market scenario where $d_1(\mathcal{S} \mid FR) = (V - p_1^{\mathcal{P}_1}) \cdot FR_1/k$ and $d_2(\mathcal{S} \mid FR) = (V - p_2^{\mathcal{P}_1}) \cdot FR_2/k$ (also depicted in Figure 2(a)). On the other hand, if

$$\frac{(V - p_1^{\mathcal{P}_1}) \cdot FR_1}{k} \geq 1 - \frac{(V - p_2^{\mathcal{P}_2}) \cdot FR_2}{k},$$

then

$$\frac{(V - p_1^{\mathcal{P}_1}) \cdot FR_1}{k} \geq \frac{(V - p_1^{\mathcal{P}_1}) \cdot FR_1 - (V - p_2^{\mathcal{P}_2}) \cdot FR_2 + k}{2k} \geq 1 - \frac{(V - p_2^{\mathcal{P}_2}) \cdot FR_2}{k},$$

and we have a covered market scenario where

$$d_1(\mathcal{S} \mid FR) = \frac{(V - p_1^{\mathcal{P}_1}) \cdot FR_1 - (V - p_2^{\mathcal{P}_2}) \cdot FR_2 + k}{2k}$$

and

$$d_2(\mathcal{S} \mid FR) = 1 - \frac{(V - p_1^{\mathcal{P}_1}) \cdot FR_1 - (V - p_2^{\mathcal{P}_2}) \cdot FR_2 + k}{2k}$$

[3] The details of this extension are available in the expanded working paper version of this paper (Nalca et al. 2012).

(also depicted in Figure 2(b)). Combining the two cases,

$$d_1(\mathcal{S} \mid FR) = \min\left\{\frac{(V-p_1^{\mathcal{P}_1})\cdot FR_1}{k}, \frac{(V-p_1^{\mathcal{P}_1})\cdot FR_1-(V-p_2^{\mathcal{P}_2})\cdot FR_2+k}{2k}\right\},$$

$$d_2(\mathcal{S} \mid FR) = \min\left\{\frac{(V-p_2^{\mathcal{P}_2})\cdot FR_2}{k}, \frac{(V-p_2^{\mathcal{P}_2})\cdot FR_2-(V-p_1^{\mathcal{P}_1})\cdot FR_1+k}{2k}\right\}.$$

Also note that the market share of each retailer is nondecreasing in its own anticipated fill rate FR_i and nonincreasing in the opponents anticipated fill rate because we have

$$\frac{\partial}{\partial FR_i}\left(\frac{(V-p_i^{\mathcal{P}_i})\cdot FR_i}{k}\right) = \frac{(V-p_i^{\mathcal{P}_i})}{k} > 0,$$

$$\frac{\partial}{\partial FR_i}\left(\frac{(V-p_i^{\mathcal{P}_i})\cdot FR_i-(V-p_j^{\mathcal{P}_j})\cdot FR_j+k}{2k}\right) = \frac{(V-p_i^{\mathcal{P}_i})}{2k} > 0,$$

$$\frac{\partial}{\partial FR_j}\left(\frac{(V-p_i^{\mathcal{P}_i})\cdot FR_i}{k}\right) = 0, \quad \text{and}$$

$$\frac{\partial}{\partial FR_j}\left(\frac{(V-p_i^{\mathcal{P}_i})\cdot FR_i-(V-p_j^{\mathcal{P}_j})\cdot FR_j+k}{2k}\right) = \frac{-(V-p_j^{\mathcal{P}_j})}{2k} < 0.$$

If the total number of customers in the market is ε, then each customer expects the demand at retailers to be $D_i(\mathcal{S} \mid FR) = \varepsilon \cdot d_i(\mathcal{S} \mid FR)$. If the total number of customers in the market follows the probability distribution $g(\varepsilon)$, then the probability that a customer receives one unit of the product after visiting R_i, that is, the fill-rate at R_i, is $\int_0^\infty \min\{1, q_i/D_i\}\cdot g(\varepsilon)\cdot d\varepsilon = \int_0^\infty \min\{1, q_i/(\varepsilon\cdot d_i)\}\cdot g(\varepsilon)\,d\varepsilon$. Following Deneckere and Peck (1995) and Dana (2001), a random customer who wants the good conditions his belief about the distribution of ε on his own realized demand. The conditional distribution of ε given that a random customer wants the good is $g(\varepsilon) = \varepsilon\cdot f(\varepsilon)/\mu$. Therefore, the observed fill rate at retailer R_i, given its inventory level and market share, can be driven as follows:

$$\begin{aligned} &E[\min\{1, q_i/D_i(\mathcal{S} \mid FR)\}] \\ &= \int_0^{q_i/d_i(\mathcal{S}|FR)} 1\cdot\frac{\varepsilon\cdot f(\varepsilon)}{\mu}\,d\varepsilon + \int_{q_i/d_i(\mathcal{S}|FR)}^\infty \frac{q_i}{d_i(\mathcal{S} \mid FR)\cdot\varepsilon} \cdot\frac{\varepsilon\cdot f(\varepsilon)}{\mu}\,d\varepsilon \\ &= \frac{1}{\mu}\int_0^{q_i/d_i(\mathcal{S}|FR)} \varepsilon\cdot f(\varepsilon)\,d\varepsilon + \frac{1}{\mu}\int_{q_i/d_i(\mathcal{S}|FR)}^\infty \frac{q_i}{d_i(\mathcal{S}|FR)}\cdot f(\varepsilon)\,d\varepsilon \\ &= \frac{1}{\mu}\cdot E[\min\{\varepsilon, q_i/d_i(\mathcal{S} \mid FR)\}] = \frac{E[\min\{D_i(\mathcal{S} \mid FR), q_i\}]}{d_i(\mathcal{S} \mid FR)\cdot\mu}. \end{aligned}$$

Let

$$\Psi_1(FR) = \frac{E[\min\{\varepsilon\cdot d_1(\mathcal{S} \mid FR), q_1\}]}{d_1(\mathcal{S} \mid FR)},$$

and let

$$\Psi_2(FR) = \frac{E[\min\{\varepsilon\cdot d_2(\mathcal{S} \mid FR), q_2\}]}{d_2(\mathcal{S} \mid FR)}.$$

So, the mapping $\Psi(FR) = (\Psi_1(FR), \Psi_2(FR))$ takes the anticipated fill rate of the customers and gives us the observed fill rates. Brouwer's fixed-point theorem guarantees the existence of a rational expectations equilibrium because the domain (FR_1, FR_2) is nonempty, compact, and convex, and the mapping $\Psi(FR)$ is nonempty and continuous. Furthermore, we have

$$\frac{\partial\Psi_i(FR)}{\partial FR_i} = -\frac{\partial d_i(\mathcal{S} \mid FR)}{\partial FR_i}\cdot\frac{q_i\cdot\mathbb{P}\{q_i/d_i(\mathcal{S} \mid FR)\ge\varepsilon\}}{[d_i(\mathcal{S} \mid FR)]^2} < 0$$

and

$$\frac{\partial\Psi_i(FR)}{\partial FR_j} = -\frac{\partial d_i(\mathcal{S} \mid FR)}{\partial FR_j}\cdot\frac{q_i\cdot\mathbb{P}\{q_i/d_i(\mathcal{S} \mid FR)\ge\varepsilon\}}{[d_i(\mathcal{S} \mid FR)]^2} > 0$$

because the market share of R_i is increasing in the anticipated fill rate FR_i and decreasing in the anticipated fill rate FR_j for all possible PMG scenarios. Furthermore, we have a unique fixed point. The proof of uniqueness is by contradiction. Suppose that there are two fixed points, (a, b) and (c, d). Next, we show that having $a < b$ and $c < d$ or $a > b$ and $c < d$ is impossible.

Case 1. If $a < b$ and $c < d$, then (i) $\Psi_1(a, d) < \Psi_1(a, c)$ because $\Psi_1(x, y)$ is increasing in y, (ii) $\Psi_1(b, d) < \Psi_1(a, d)$ because $\Psi_1(x, y)$ is decreasing in x, (iii) $\Psi_2(b, c) < \Psi_2(a, c)$ because $\Psi_2(x, y)$ is increasing in x, and (iv) $\Psi_2(b, d) < \Psi_2(a, d)$ because $\Psi_2(x, y)$ is decreasing in y. Accordingly we must have $\Psi_1(a, d) < \Psi_1(a, c) = a < b = \Psi_1(b, d) < \Psi_1(a, d)$ and $\Psi_2(b, d) < \Psi_2(a, c) = c < d = \Psi_2(b, d) < \Psi_2(a, d)$, which is a contradiction.

Case 2. If $a > b$ and $c < d$, then (i) $\Psi_1(a, c) < \Psi_1(b, c) < \Psi_1(b, d)$ because $\Psi_1(x, y)$ is decreasing in x and increasing in y, and (ii) $\Psi_2(a, c) > \Psi_2(a, d) > \Psi_2(b, d)$ because $\Psi_2(x, y)$ is decreasing in y and increasing in x. Accordingly, we must then have $a = \Psi_1(a, c) < \Psi_1(b, c) < \Psi_1(b, d) = b$ and $c = \Psi_2(a, c) > \Psi_2(a, d) > \Psi_2(b, d) = d$, but this is also a contradiction.

The unique equilibrium to the game among customers allows us to describe the market share, the random demand, and expected profit of each retailer, which we denote by $d_i(\mathcal{S})$, $D_i(\mathcal{S}) = \varepsilon\cdot d_i(\mathcal{S})$, and $\pi_i(\mathcal{S})$, respectively, as a function of the retailers' strategy. Let the stocking factor z_i be the ratio of the order quantity to the market share of the retailer. Accordingly, we can substitute the stocking factor instead of the order quantity decision and obtain $FR_1 = \Theta(z_1)/\mu$ and $FR_2 = \Theta(z_2)/\mu$, where $\Theta(z) \equiv E[\min\{\varepsilon, z\}]$. Note that for the rest of the analysis, we will focus on the case where the market is covered. The resulting market share of R_1 under each PMG scenario, $d_1(\mathcal{P}_1, p_1, z_1, \mathcal{P}_2, p_2, z_2)$, is given in Table A.1. Note that because we assume a covered market scenario, $d_2(\mathcal{P}_2, p_2, z_2, \mathcal{P}_1, p_1, z_1) = 1 - d_1(\mathcal{P}_1, p_1, z_1, \mathcal{P}_2, p_2, z_2)$.

Let $\Pi(p, z) \equiv p\cdot\Theta(z) - c\cdot z$ denote the expected profit rate of the retailers per unit market share. Accordingly, the resulting expected profit of R_1, $\pi_1(\mathcal{P}_1, p_1, z_1, \mathcal{P}_2, p_2, z_2)$, under each PMG scenario is as follows: $\pi_1(C, p_1, z_1, \mathcal{S}_2) = d_1(C, p_1, z_1, \mathcal{S}_2)\cdot\Pi(p_1, z_1)$, $\pi_1(PM, p_1, z_1, \mathcal{S}_2) = d_1(PM, p_1, z_1, \mathcal{S}_2)\cdot\Pi(p_e, z_1)$, and $\pi_1(PMA, p_1, z_1, \mathcal{S}_2) = d_1(PMA, p_1, z_1, \mathcal{S}_2)\cdot\Pi(\bar{p}_1, z_1)$. □

Proof of Proposition 2

The proof involves three steps: derivation of best-response mappings for each subgame, derivation of equilibrium for each subgame, and the comparison of retail decisions.

Table A.1 Market Share of R_1 Under All Possible PMG Scenarios

	R_2		
R_1	C	PM	PMA
C	$\frac{(V-p_1)\cdot\Theta(z_1)-(V-p_2)\cdot\Theta(z_2)+k\mu}{2k\mu}$	$\frac{(V-p_1)\cdot\Theta(z_1)-(V-p_e)\cdot\Theta(z_2)+k\mu}{2k\mu}$	$\frac{(V-p_1)\cdot\Theta(z_1)-(V-\bar{p}_2)\cdot\Theta(z_2)+k\mu}{2k\mu}$
PM	$\frac{(V-p_e)\cdot\Theta(z_1)-(V-p_2)\cdot\Theta(z_2)+k\mu}{2k\mu}$	$\frac{(V-p_e)\cdot\Theta(z_1)-(V-p_e)\cdot\Theta(z_2)+k\mu}{2k\mu}$	$\frac{(V-p_e)\cdot\Theta(z_1)-(V-\bar{p}_2)\cdot\Theta(z_2)+k\mu}{2k\mu}$
PMA	$\frac{(V-\bar{p}_1)\cdot\Theta(z_1)-(V-p_2)\cdot\Theta(z_2)+k\mu}{2k\mu}$	$\frac{(V-\bar{p}_1)\cdot\Theta(z_1)-(V-p_e)\cdot\Theta(z_2)+k\mu}{2k\mu}$	$\frac{(V-\bar{p}_1)\cdot\Theta(z_1)-(V-\bar{p}_2)\cdot\Theta(z_2)+k\mu}{2k\mu}$

Derivation of the Stage-Two Subgame Best-Response Mappings. Recall that we have nine stage-two subgames. Let $\{\mathcal{P}_1 - \mathcal{P}_2\}$ denote the subgame where R_1 and R_2 offer policies $\mathcal{P}_1$ and $\mathcal{P}_2$, respectively. In this part, we derive the best response of R_1 under all possible *PMG* scenarios. To avoid repetitions, we investigate the best response, if necessary, by dividing the strategy space into two mutually exclusive sets, $A_1 = \{(p_1, p_2) \mid p_1 \geq p_2\}$ and $A_2 = \{(p_1, p_2) \mid p_1 \leq p_2\}$. In the following, let $p_i^{\mathcal{P}_i|\mathcal{P}_j}(p_j, A_k)$ denote the best-response price of R_i in region A_k $(k = \{1, 2\})$ when R_i offers policy $\mathcal{P}_i$ and R_j offers policy $\mathcal{P}_j$ and price p_j, where $\mathcal{P}_i \in \{C, PM, PMA\}$, $i \in \{1, 2\}$, $j = 3 - i$. Also observe that $\Pi(p, z) = [p \cdot \Theta(z) - c \cdot z]$ is concave in z because $\partial^2\Pi(p, z)/\partial z^2 = -p \cdot f(z) < 0$ and linear increasing in p because $\partial\Pi(p, z)/\partial p = \Theta(z)$. Lemma 1 at the end of this section shows that these two observations are sufficient to prove that the simultaneous solution of the first-order conditions $\partial\Pi(p, z)/\partial z = 0$ and $\partial\Pi(p, z)/\partial p = 0$ provide us the optimal solution.

1. *Best response of R_1 in the $\{C - C\}$ game.* If none of the retailers offers a PMG, then the expected profit for R_1 is $\pi_1(C, p_1, z_1, C, p_2, z_2) = d_1(C, p_1, z_1, C, p_2, z_2) \cdot \Pi(p_1, z_1)$, where

$$d_1(C,p_1,z_1,C,p_2,z_2)=\frac{(V-p_1)\cdot\Theta(z_1)-(V-p_2)\cdot\Theta(z_2)+k\mu}{2k\mu}$$

is the market share of R_1, and $\Pi(p, z) \equiv [p \cdot \Theta(z) - c \cdot z]$. Observe that $d_1(C, p_1, z_1, C, p_2, z_2)$ is linear decreasing in p_1 and concave increasing in z_1 because

$$\frac{\partial d_1(C,p_1,z_1,C,p_2,z_2)}{\partial p_1}=\frac{-\Theta(z_1)}{2k\mu}<0,$$

$$\frac{\partial d_1(C,p_1,z_1,C,p_2,z_2)}{\partial z_1}=\frac{(V-p_1)\cdot\bar{F}(z_1)}{2k\mu}>0,$$

and

$$\frac{\partial^2 d_1(C,p_1,z_1,C,p_2,z_2)}{\partial z_1^2}=\frac{-(V-p_1)\cdot f(z_1)}{2k\mu}<0.$$

Consequently, by Lemma 1, there exists a unique best response satisfying the first-order conditions

$$\frac{\partial\pi_1(C,p_1,z_1,C,p_2,z_2)}{\partial z_1} = \frac{(V-p_1)\cdot\bar{F}(z_1)}{2\mu k}[p_1\cdot\Theta(z_1)-c\cdot z_1] + \frac{(V-p_1)\cdot\Theta(z_1)-(V-p_2)\cdot\Theta(z_2)+k\mu}{2\mu k}[p_1\cdot\bar{F}(z_1)-c]=0$$

and

$$\frac{\partial\pi_1(C,p_1,z_1,C,p_2,z_2)}{\partial p_1} = \frac{(V-p_1)\cdot\Theta(z_1)-(V-p_2)\cdot\Theta(z_2)+k\mu}{2\mu k}\Theta(z_1) - \frac{\Theta(z_1)\cdot[p_1\cdot\Theta(z_1)-c\cdot z_1]}{2\mu k}=0.$$

Simultaneous solution provides us the best response of R_1 as

$$BR_1(p_2,z_2) = \left\{(p_1,z_1)\,\middle|\, \frac{\partial\pi_1(C,p_1,z_1,C,p_2,z_2)}{\partial z_1} \text{ and } p_1=\min\left\{\frac{k\mu+c\cdot z_1+V\cdot\Theta(z_1)-(V-p_2)\cdot\Theta(z_2)}{2\cdot\Theta(z_1)},\; V-\frac{\mu k-(V-p_2)\cdot\Theta(z_2)}{\Theta(z_1)}\right\}\right\}.$$

Recall that we are imposing a market covered scenario, therefore, we also need

$$1-\frac{(V-p_2)\cdot\Theta(z_2)}{\mu k}\leq\frac{(V-p_1)\cdot\Theta(z_1)}{\mu k},$$

which holds if and only if

$$p_1\leq V-\frac{\mu k-(V-p_2)\cdot\Theta(z_2)}{\Theta(z_1)}.$$

During the derivation of equilibrium, we will make sure that problem parameters are such that the condition is guaranteed to hold. Denote the best-response price by

$$p_1^{C|C}(p_2)\equiv\frac{k\mu+c\cdot z_1+V\cdot\Theta(z_1)-(V-p_2)\cdot\Theta(z_2)}{2\cdot\Theta(z_1)}.$$

It will be useful to fully describe the best-response price function in regions A_1 and A_2. Let p^C denote the fixed point of the price response, i.e.,

$$p^C=\left\{p\,\middle|\,p=\frac{k\mu+c\cdot z_1+V\cdot\Theta(z_1)-(V-p)\cdot\Theta(z_2)}{2\cdot\Theta(z_1)}\right\}.$$

Note that $p_1^{C|C}(p_2)$ is increasing in p_2; therefore, $p_1^{C|C}(p_2)\geq p_2$ for $p_2\leq p^C$, and $p_1^{C|C}(p_2)\leq p_2$ for $p_2\geq p^C$. Thus, $p_1^{C|C}(p_2,A_1) = \max\{p_1^{C|C}(p_2),p_2\}$, and $p_1^{C|C}(p_2,A_2) = \min\{p_1^{C|C}(p_2),p_2\}$.

2. *Best response of* R_1 *in the* $\{PM-C\}$ *game.* We have $\pi_1(PM, p_1, z_1, C, p_2, z_2) = d_1(PM, p_1, z_1, C, p_2, z_2)[p_e \cdot \Theta(z_1) - c \cdot z_1]$. A detailed look verifies that if $p_1 \geq p_2$, then $\pi_1(PM, p_1, z_1, C, p_2, z_2) = \pi_1(C, p_2, z_1, C, p_2, z_2)$. In other words, in region A_1, R_1 practically accepts the lower price of R_2 and decides on the stocking factor z_1 only. If $p_1 \leq p_2$, then $\pi_1(PM, p_1, z_1, C, p_2, z_2) = \pi_1(C, p_1, z_1, C, p_2, z_2)$; that is, the profit of R_1 is identical to that of the $\{C-C\}$ game and so is the best response in A_2. Combining these two observations, if the best response of R_1 has a lower price than R_2 in the $\{C-C\}$ game, then it is also the best response in the $\{PM-C\}$ game. If the best response of R_1 has higher price than R_2 in the $\{C-C\}$ game, then the best response in the $\{PM-C\}$ is to accept the price of R_2. We have $p_1^{PM|C}(p_2) = \min\{p_1^{C|C}(p_2), p_2\}$. We have established in the derivation of best response in $\{C-C\}$ game that $\pi_1(C, p_1, z_1, C, p_2, z_2)$ is unimodal in z_1. Therefore,

$$BR_1(p_2, z_2) = \left\{(p_1, z_1) \,\middle|\, p_1 = \min\{p_1^{C|C}(p_2), p_2\} \text{ and } \frac{\partial \pi_1(PM, p_1, z_1, C, p_2, z_2)}{\partial z_1} = 0\right\}.$$

3. *Best response of* R_1 *in the* $\{PMA-C\}$ *game.* We have $\pi_1(PMA, p_1, z_1, C, p_2, z_2) = d_1(PMA, p_1, z_1, C, p_2, z_2) \cdot \Pi(\bar{p}_1, z_1)$, where

$$d_1(PMA, p_1, z_1, C, p_2, z_2) = \frac{(V-\bar{p}_1)\cdot\Theta(z_1) - (V-p_2)\cdot\Theta(z_2) + k\mu}{2k\mu}$$

and $\bar{p}_1 = p_e \cdot FR_2 + p_1 \cdot (1-FR_2)$. Similar to the previous game, if $p_1 \leq p_2$, then $\pi_1(PMA, p_1, z_1, C, p_2, z_2) = \pi_1(C, p_1, z_1, C, p_2, z_2)$; that is, in A_2, the profit of R_1 is identical to that of the $\{C-C\}$ game and so is the best response. Let us have a closer look at $p_1 \geq p_2$. In region A_1, $d_1(PMA, p_1, z_1, C, p_2, z_2)$ is linear decreasing in p_1 and concave increasing in z_1 because

$$\frac{\partial d_1(PMA, p_1, z_1, C, p_2, z_2)}{\partial p_1} = \frac{-\Theta(z_1)\cdot(1-FR_2)}{2k\mu} < 0,$$

$$\frac{\partial d_1(PMA, p_1, z_1, C, p_2, z_2)}{\partial z_1} = \frac{(V-\bar{p}_1)\cdot\bar{F}(z_1)}{2k\mu} > 0,$$

and

$$\frac{\partial^2 d_1(PMA, p_1, z_1, C, p_2, z_2)}{\partial z_1} = \frac{-(V-\bar{p}_1)\cdot f(z_1)}{2k\mu} < 0.$$

Also, $\Pi(p, z)$ is concave in z and linear increasing in p. Consequently, by Lemma 1, the unique best response of R_1 satisfies the first-order conditions

$$\frac{\partial \pi_1(PMA, p_1, z_1, C, p_2, z_2)}{\partial z_1} = \frac{(V-\bar{p}_1)\cdot\bar{F}(z_1)}{2\mu k}[\bar{p}_1\cdot\Theta(z_1) - c\cdot z_1] + \frac{(V-\bar{p}_1)\cdot\Theta(z_1) - (V-p_2)\cdot\Theta(z_2) + k\mu}{2\mu k}[\bar{p}_1\cdot\bar{F}(z_1) - c] = 0$$

and

$$\frac{\partial \pi_1(PMA, p_1, z_1, C, p_2, z_2)}{\partial p_1} = \frac{(V-\bar{p}_1)\cdot\Theta(z_1) - (V-p_2)\cdot\Theta(z_2) + k\mu}{2\mu k}\cdot\Theta(z_1) - [\bar{p}_1.\Theta(z_1) - c\cdot z_1]\frac{\Theta(z_1)}{2\mu k} = 0.$$

Simultaneous solution provides us $\bar{F}(z_1) = c/V$ and

$$p_1 = \max\left\{p_2, \frac{k\mu + c\cdot z_1 + V\cdot\Theta(z_1) - (V-p_2)\cdot\Theta(z_2)}{2\cdot\Theta(z_1)\cdot(1-FR_2)} - \frac{p_2\cdot FR_2}{1-FR_2}\right\} = \max\left\{p_2, \frac{p_1^{C|C}(p_2) - p_2\cdot FR_2}{1-FR_2}\right\}.$$

Note that

$$\frac{p_1^{C|C}(p_2) - p_2\cdot FR_2}{1-FR_2} \geq p_2 \;\Leftrightarrow\; p_1^{C|C}(p_2) \geq p_2 \;\Leftrightarrow\; p^C \geq p_2,$$

meaning that if the best-response price of R_1 in the $\{C-C\}$ is higher than R_2, then it is also higher in the $\{PMA-C\}$ game, and vice versa. So,

$$p_1^{PMA|C}(p_2) = \max\left\{p_1^{C|C}(p_2), \frac{p_1^{C|C}(p_2) - p_2\cdot FR_2}{1-FR_2}\right\} = \begin{cases} [p_1^{C|C}(p_2) - p_2\cdot FR_2]\dfrac{1}{1-FR_2} & \text{if } p_2 \leq p^C, \\ p_1^{C|C}(p_1) & \text{if } p_2 \geq p^C. \end{cases}$$

Combining these two observations,

$$BR_1(p_2, z_2) = \left\{(p_1, z_1) \mid \bar{F}(z_1) = \frac{c}{V} \text{ and } p_1 = \max\left\{p_1^{C|C}(p_2), \frac{p_1^{C|C}(p_2) - p_2\cdot FR_2}{1-FR_2}\right\}\right\}.$$

4. *Best response of* R_1 *in the* $\{C-PM\}$ *game.* We have $\pi_1(C, p_1, z_1, PM, p_2, z_2) = d_1(C, p_1, z_1, PM, p_2, z_2)\Pi(p_1, z_1)$. If $p_1 \geq p_2$, then $\pi_1(C, p_1, z_1, PM, p_2, z_2) = \pi_1(C, p_1, z_1, C, p_2, z_2)$; that is, in A_1, the profit of R_1 is identical to that of the $\{C-C\}$ game and so is the best response. If $p_1 \leq p_2$, then $\pi_1(C, p_1, z_1, PM, p_2, z_2) = \pi_1(C, p_1, z_1, C, p_1, z_2)$. In this scenario R_2 matches the price of R_1, and hence R_1 is not capable of cutting the price. Combining these two observations, if the best-response price of R_1 is higher price R_2 in the $\{C-C\}$ game, then it is also higher in the $\{C-PM\}$ game. If the best-response price of R_1 is lower than R_2 in the $\{C-C\}$ game, then they are equal in the $\{C-PM\}$ game; $p_1^{C|PM}(p_2) = \max\{p_1^{C|C}(p_2), p_2\}$. Lemma 1 proves that $\pi_1(C, p_1, z_1, C, p_2, z_2)$ is unimodal in z_1. Therefore,

$$BR_1(p_2, z_2) = \left\{(p_1, z_1) \,\middle|\, p_1 = \max\{p_1^{C|C}(p_2), p_2\} \text{ and } \frac{\partial \pi_1(PM, p_1, z_1, C, p_2, z_2)}{\partial z_1} = 0\right\}.$$

5. *Best response of* R_1 *in the* $\{PM - PM\}$ *game.* We have $\pi_1(PM, p_1, z_1, PM, p_2, z_2) = \pi_1(C, p_e, z_1, C, p_e, z_2)$. In other words, each retailers' price is equal to the effective price in the market. Therefore, $p_1^{PM|PM}(p_2) = p_2$, and the stocking factor can be found from the first-order condition.

6. *Best response of* R_1 *in the* $\{PMA - PM\}$ *game.* We have $\pi_1(PMA, p_1, z_1, PM, p_2, z_2) = d_1(PMA, p_1, z_1, PM, p_2, z_2)\Pi(\bar{p}_1, z_1)$, where

$$d_1(PMA, p_1, z_1, PM, p_2, z_2) = \frac{(V - \bar{p}_1)\cdot\Theta(z_1) - (V - p_e)\cdot\Theta(z_2) + k\mu}{2k\mu}$$

is the market share of R_1, and $\bar{p}_1 = p_e \cdot FR_2 + p_1 \cdot (1 - FR_2)$. If $p_1 \geq p_2$, then $\pi_1(PMA, p_1, z_1, PM, p_2, z_2) = \pi_1(PMA, p_1, z_1, C, p_2, z_2)$. In A_1, the profit of R_1 is identical to that of the $PMA - C$ game and so is the best response. If $p_1 \leq p_2$, then $\pi_1(PMA, p_1, z_1, PM, p_2, z_2) = \pi_1(C, p_1, z_1, PM, p_2, z_2)$. In A_2, the profit of R_1 is identical to that of the $C - PM$ game and so is the best response. Combining the two regions,

$$p_1^{PMA|PM}(p_2) = \max\left\{[p_1^{C|C}(p_2) - p_2\cdot FR_2]\frac{1}{1 - FR_2},\, p_2\right\},$$

and the stocking factor comes from the first-order condition.

7. *Best response of* R_1 *in the* $\{C - PMA\}$ *game.* If $p_1 \geq p_2$, then $\pi_1(C, p_1, z_1, PMA, p_2, z_2) = \pi_1(C, p_1, z_1, C, p_2, z_2)$; that is, the profit of R_1 is identical to that of the $\{C - C\}$ game and so is the best response. However, if $p_1 \leq p_2$, then the analysis is a bit more involved. Observe that

$$\frac{\partial d_1(C, p_1, z_1, PMA, p_2, z_2)}{\partial p_1} = \frac{\Theta(z_1)}{2k\mu}\,\frac{\Theta(z_2) - \mu}{\mu} < 0 \quad \text{and}$$

$$\frac{\partial d_1(C, p_1, z_1, PMA, p_2, z_2)}{\partial z_1} = \frac{\bar{F}(z_1)}{2k\mu}[V - p_1\cdot(1 - FR_2) - p_2\cdot FR_2] > 0$$

because $p_1 < p_2 < V \Rightarrow p_1\cdot(1 - FR_2) + p_2\cdot FR_2 < V$, and

$$\frac{\partial^2 d_1(C, p_1, z_1, PMA, p_2, z_2)}{\partial z_1^2} = \frac{-f(z_1)}{2k\mu}[V - p_1\cdot(1 - FR_2) - p_2\cdot FR_2] < 0;$$

that is, the market share of R_1 is linear decreasing in p_1 and concave increasing in z_1. Also observe that $\Pi(p, z)$ is concave in z and linear increasing in p. Consequently, by Lemma 1, given the strategy of R_2, there exists a unique best response of R_1 satisfying $\partial\pi_1(C, p_1, z_1, PMA, p_2, z_2)/\partial z_1 = 0$ and $\partial\pi_1(C, p_1, z_1, PMA, p_2, z_2)/\partial p_1 = 0$. Simultaneous solution in A_2 provides us $\bar{F}(z_1) = c(1 - FR_2)/(V - FR_2\cdot p_2)$ and

$$p_1 = \min\{p_2, (k\mu + c\cdot z_1 + V\cdot\Theta(z_1) - (V - p_2)\cdot\Theta(z_2) - FR_2\cdot[p_2\cdot\Theta(z_1) + c\cdot z_1])\cdot(2\cdot(1 - FR_2)\cdot\Theta(z_2))^{-1}\}.$$

Let

$$p^{PMA} = \left\{p \,\middle|\, p = (k\mu + c\cdot z_1 + V\cdot\Theta(z_1) - (V - p)\cdot\Theta(z_2) - FR_2\cdot[p\cdot\Theta(z_1) + c\cdot z_1])\cdot(2\cdot(1 - FR_2)\cdot\Theta(z_2))^{-1} \text{ and } \bar{F}(z_1) = \frac{c(1 - FR_2)}{V - FR_2.p}\right\}$$

denote the fixed price point of the response function. We can easily show that

$$\frac{k\mu + c\cdot z_1 + V\cdot\Theta(z_1) - (V - p_2)\cdot\Theta(z_2) - FR_2\cdot[p_2\cdot\Theta(z_1) + c\cdot z_1]}{2\cdot(1 - FR_2)\cdot\Theta(z_2)} < p_2$$

if and only if $p_2 > p^{PMA}$. When we combine the observations from areas A_1 and A_2,

$$p_1^{C|PMA}(p_2) = \begin{cases} p_1^{C-C}(p_2) & \text{if } p_2 \leq p^C, \\ p_2 & \text{if } p^C \leq p_2 \leq p^{PMA}, \\ (k\mu + c\cdot z_1 + V\cdot\Theta(z_1) - (V - p_2)\cdot\Theta(z_2) \\ \quad - FR_2\cdot[p_2\cdot\Theta(z_1) + c\cdot z_1])\cdot(2\cdot(1 - FR_2)\cdot\Theta(z_2))^{-1} \\ & \text{if } p^{PMA} \leq p_2, \end{cases}$$

and the stocking factor can be found from the first-order condition.

8. *Best response of* R_1 *in the* $\{PM - PMA\}$ *game.* Note that $\pi_1(PM, p_1, z_1, PMA, p_2, z_2) = \pi_1(PM, p_1, z_1, C, p_2, z_2)$ if $p_1 \geq p_2$, and $\pi_1(PM, p_1, z_1, PMA, p_2, z_2) = \pi_1(C, p_1, z_1, PMA, p_2, z_2)$ if $p_1 \leq p_2$. Consequently,

$$p_1^{PM|PMA}(p_2) = \min\{p_2, (k\mu + c\cdot z_1 + V\cdot\Theta(z_1) - (V - p_2)\cdot\Theta(z_2) - FR_2\cdot[p_2\cdot\Theta(z_1) + c\cdot z_1])\cdot(2\cdot(1 - FR_2)\cdot\Theta(z_2))^{-1}\}.$$

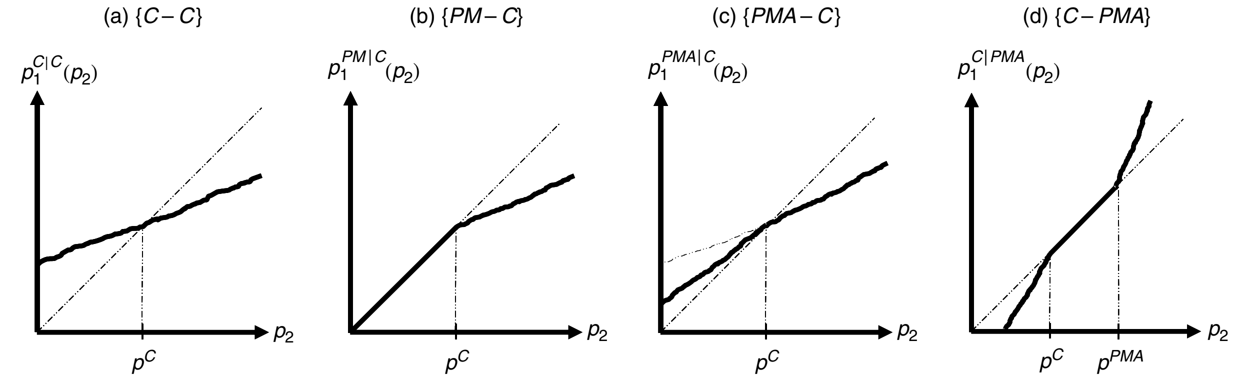
9. *Best response of* R_1 *in the* $\{PMA - PMA\}$ *game.* We have $\pi_1(PMA, p_1, z_1, PMA, p_2, z_2) = \pi_1(PMA, p_1, z_1, C, p_2, z_2)$ when $p_1 \geq p_2$ and $\pi_1(PMA, p_1, z_1, PMA, p_2, z_2) = \pi_1(C, p_1, z_1, PMA, p_2, z_2)$ when $p_1 \leq p_2$. Consequently, the best response is identical to that in $\{PMA - C\}$ game.

Figure A.1 provides a plot of the price values of the best-response mappings of R_1 under the following subgames: $\{C - C\}$, $\{PM - C\}$, $\{PMA - C\}$, and $\{C - PMA\}$. These figures are helpful and sufficient to visualize the best-response price behavior and the equilibrium structure.

Lemma 1. *Let* $f(x, y) = g(x, y)\cdot h(x, y)$ *where* (i) $g(x, y)$ *is nonnegative, linear decreasing in* x *and increasing concave in* y, *and* (ii) $h(x, y)$ *is nonnegative, linear increasing in* x *and concave in* y. *Then* $f(x, y)$ *has a unique maximizing point that satisfies the first-order conditions* $\partial f(x, y)/\partial x = 0$ *and* $\partial f(x, y)/\partial y = 0$.

Proof. Let $f_x(x, y) \equiv \partial f(x, y)/\partial x$, $f_y(x, y) \equiv \partial f(x, y)/\partial y$, $f_{xx}(x, y) \equiv \partial^2 f(x, y)/\partial x^2$, $f_{xy}(x, y) \equiv \partial^2 f(x, y)/\partial x\partial y$, and $f_{yy}(x, y) = \partial^2 f(x, y)/\partial y^2$. The first- and second-order partial derivatives with respect to the stocking factor are $f_y(x, y) = g_y(x, y)\cdot h(x, y) + g(x, y)\cdot f_y(x, y)$ and $f_{yy}(x, y) = g_{yy}(x, y)\cdot h(x, y) + 2\cdot g_y(x, y)\cdot h_y(x, y) + g(x, y)\cdot h_{yy}(x, y)$, respectively. First-order condition $f_y(x, y) = 0$ is satisfied only if $h_y(x, y) < 0$ because $g(x, y)$ is increasing in y and $h(x, y) > 0$. The second-order derivative is negative when the first-order condition is satisfied because (i) $g(x, y)$ is concave in y, (ii) $h_y(x, y) < 0$ when the first-order condition is satisfied, and (iii) $h(x, y)$ is concave in y. Therefore, $f_y(x, y)$ changes sign once from positive to negative. Thus there exists a unique $y(x)$ satisfying $f_y(x, y) = 0$. Differentiating both sides with respect to x, we have $y'(x) = -(f_{xy}(x, y)/f_{yy}(x, y))|_{y=y(x)}$. Next, we

Figure A.1 Best-Response Price Behavior of R_1



plug $y(x)$ back into $f(x,y)$ rewrite it as a function of price only; $f(x,y(x)) = g(x,y(x)) \cdot h(x,y(x))$. Then, the first-order derivative is given by $f_x(x,y(x)) = [f_x(x,y) + y'(x) \cdot f_y(x,y)]|_{y=y(x)}$. But recall that $y(x)$ satisfies $f_y(x,y)=0$; thus the first-order derivative with respect to price is $f_x(x,y(x)) = f_x(x,y)|_{y=y(x)} = [g_x(x,y) \cdot h(x,y) + g(x,y) \cdot h_x(x,y)]|_{y=y(x)}$, and the second-order derivative with respect to price is $f_{xx}(x,y(x)) = [g_{xx}(x,y) \cdot h(x,y) + 2g_x(x,y) \cdot h_x(x,y) + g(x,y) \cdot h_{xx}(x,y)]|_{y=y(x)} + y'(x)[g_{xy}(x,y) \cdot h(x,y) + g_x(x,y) \cdot f_y(x,y) + g_y(x,y) \cdot h_x(x,y) + g(x,y) \cdot h_{xy}(x,y)]|_{y=y(x)}$. If we substitute $y'(x)$ and recall that $g(x,y)$ is linear decreasing and $h(x,y)$ is linear increasing in price x, then we have

$$f_{xx}(x,y(x)) = \left[2g_x(x,y) \cdot h_x(x,y) + \frac{[f_{xy}(x,y)]^2}{f_{yy}(x,y)}\right]\Bigg|_{y=y(x)},$$

which is negative because $f(x,y)$ is unimodal in y and $g_x(x,y) \cdot h_x(x,y) < 0$ when the first-order condition is satisfied. Consequently, $\arg\max_{(x,y)} f(x,y) = \{(x,y) \mid f_y(x,y)=0$ and $f_x(x,y)=0\}$ $\square$

Equilibrium Solutions for Stage-Two Subgames. Here, we derive the equilibrium for stage-two subgames. Because some of them have identical solutions, we will present solutions to the following: $\{C-C\}$, $\{PM-C\}$, $\{PM-PM\}$, $\{PMA-C\}$, and $\{PMA-PMA\}$.

1. *The* $\{C-C\}$ *game.* The best responses of R_1 and R_2 satisfy

$$BR_1(p_2,z_2) = \left\{(p_1,z_1) \,\middle|\, \bar{F}(z_1) = \frac{c}{V} \text{ and } p_1 = \frac{k\mu + c \cdot z_1 + V \cdot \Theta(z_1) - (V-p_2) \cdot \Theta(z_2)}{2 \cdot \Theta(z_1)}\right\}$$

and

$$BR_2(p_1,z_1) = \left\{(p_2,z_2) \,\middle|\, \bar{F}(z_2) = \frac{c}{V} \text{ and } p_2 = \frac{k\mu + c \cdot z_2 + V \cdot \Theta(z_2) - (V-p_1) \cdot \Theta(z_1)}{2 \cdot \Theta(z_2)}\right\},$$

respectively. Solving the best-response mappings simultaneously, the unique symmetric equilibrium solution is

$$(p^C, z^C) = \left\{(p,z) \,\middle|\, \bar{F}(z) = \frac{c}{V} \text{ and } p = \frac{k\mu + c \cdot z}{\Theta(z)}\right\}.$$

The expected profit for each retailer is $\pi^C = (\mu k)/2$. To guarantee an interior solution with a covered market, we need $0 < k \le \bar{k} = (2c/(3\mu))[\Theta(z)/\bar{F}(z) - z]$, where $\bar{F}(z) = c/V$.

2. *The* $\{PM-C\}$ *game.* We show that the equilibrium of the $\{PM-C\}$ game is identical to that of the $\{C-C\}$ game. In this subgame, the best responses of R_1 and R_2 are, respectively,

$$BR_1(p_2,z_2) = \left\{(p_1,z_1) \,\middle|\, p_1 = \min\left\{\frac{k\mu + c \cdot z_1 + V \cdot \Theta(z_1) - (V-p_2) \cdot \Theta(z_2)}{2 \cdot \Theta(z_1)}, p_2\right\} \text{ and } \frac{\partial \pi_1(PM,p_1,z_1,C,p_2,z_2)}{\partial z_1} = 0\right\}$$

and

$$BR_2(p_1,z_1) = \left\{(p_2,z_2) \,\middle|\, p_2 = \max\left\{\frac{k\mu + c \cdot z_2 + V \cdot \Theta(z_2) - (V-p_1) \cdot \Theta(z_1)}{2 \cdot \Theta(z_2)}, p_1\right\} \text{ and } \frac{\partial \pi_2(C,p_2,z_2,PM,p_1,z_1)}{\partial z_2} = 0\right\}.$$

Their simultaneous solution provides $(p^C, z^C) = \{(p,z) \mid \bar{F}(z) = c/V$ and $p = (k\mu + c \cdot z)/\Theta(z)\}$ as the unique equilibrium solution.

3. *The* $\{PM-PM\}$ *game.* In this game, each retailers' price is equal to the effective price because $\pi_1(PM,p_1,z_1,PM,p_2,z_2) = \pi_1(C,p_e,z_1,C,p_e,z_2)$ and $\pi_2(PM,p_2,z_2,PM,p_1,z_1) = \pi_2(C,p_e,z_2,C,p_e,z_1)$. Therefore, we have price symmetry at equilibrium. If prices are equal, then so are the stocking factors, because of retail symmetry. Consider a symmetric equilibrium point where retailers are making nonnegative profits. None of the retailers have an incentive to unilaterally deviate because (i) increasing the list price does not change the effective price due to price matching, and (ii) decreasing the price does not increase demand because the price will be matched by the competitor. Retailers select (p,z) that maximizes $\pi_1(PM,p,z,PM,p,z) = \pi_2(PM,p,z,PM,p,z) = (1/2)[p\Theta(z) - cz]$ subject to the constraints $(V-p_e) \cdot \bar{F}(z)[p_e \cdot \Theta(z) - c \cdot z] + k\mu[p_e \cdot \bar{F}(z) - c] = 0$

and $2(V-p)\cdot\Theta(z)/(k\mu)\geq 1$, which gives us $(p^{PM}, z^{PM}) = \{(p,z) \mid (V-p)\cdot\bar{F}(z)[p\cdot\Theta(z) - c\cdot z] + k\mu[p\cdot\bar{F}(z) - c] = 0$ and $p = V-(k\cdot\mu)/(2\cdot\Theta(z))\}$. The expected profit is $\pi^{PM} = (1/2)[p^{PM}\cdot\Theta(z^{PM}) - c\cdot z^{PM}] = (1/2)[V\cdot\Theta(z^{PM}) - c\cdot z^{PM} - \mu k/2]$.

4. *The* $\{PMA-C\}$ *game.* We show that the equilibrium of the $\{PMA-C\}$ game is identical to that of the $\{C-C\}$ game. In this subgame, the best response of R_1 is

$$\begin{aligned}
&BR_1(p_2, z_2)\\
&= \Bigg\{(p_1, z_1) \,\Bigg|\, \bar{F}(z_1) = \frac{c}{V} \text{ and}\\
&\qquad p_1 = \max\Bigg\{\frac{k\mu + cz_1 + V\Theta(z_1) - (V-p_2)\Theta(z_2)}{2\Theta(z_1)},\\
&\qquad\qquad \frac{k\mu + cz_1 + V\Theta(z_1) - (V-p_2)\Theta(z_2)}{2\Theta(z_1)} - \frac{p_2 FR_2}{1-FR_2}\Bigg\}\Bigg\}.
\end{aligned}$$

The best response of R_2 satisfies

$$p_2^{C|PMA}(p_1) = \begin{cases}
(k\mu + c\cdot z_2 + V\cdot\Theta(z_2) - (V-p_1)\cdot\Theta(z_1))\cdot(2\cdot\Theta(z_2))^{-1} & \text{if } p_1 \leq p^C,\\
p_1 & \text{if } p^C \leq p_1 \leq p^{PMA},\\
(k\mu + c\cdot z_2 + V\cdot\Theta(z_2) - (V-p_1)\Theta(z_1) - FR_1 \cdot [p_1\cdot\Theta(z_2) + c\cdot z_2])(2\cdot(1-FR_1)\cdot\Theta(z_1))^{-1} & \text{if } p^{PMA} \leq p_2,
\end{cases}$$

and the stocking factor can be found from the first-order condition. Consequently, the simultaneous solution of the best responses provides us $(p^C, z^C) = \{(p,z) \mid \bar{F}(z) = c/V$ and $p = (k\mu + c\cdot z)/\Theta(z)\}$ as the unique equilibrium solution.

5. *The* $\{PMA-PMA\}$ *game.* Based on the best-response functions of the players, we have again a continuum of equilibrium solutions for $p\in[p^C, p^{PMA}]$. The Pareto-dominant solution among them is $(p^{PMA}, z^{PMA}) = \{(p,z) \mid \bar{F}(z) = (c(1-FR(z)))/(V - pFR(z))$ and $p = (k\mu + c\cdot z\cdot(1-FR(z)))/((1-FR(z))\cdot\Theta(z))$ and $FR(z) = \Theta(z)/\mu\}$. Integrating the market coverage assumption, we have (p^{PMA}, z^{PMA}) if $[1-FR(z^{PM})][p^{PM}\cdot\Theta(z^{PM}) - cz^{PM}] \geq \mu\cdot k$ and (p^{PM}, z^{PM}) otherwise.

Now let us have a closer look at the condition

$$\begin{aligned}
&[1-FR(z^{PM})][p^{PM}\cdot\Theta(z^{PM}) - cz^{PM}] \geq \mu\cdot k \;\Leftrightarrow\; k\\
&\quad \leq \frac{1-FR(z)}{3-FR(z)}\frac{2\cdot c}{\mu}\left[\frac{\Theta(z)}{\bar{F}(z)} - z\right] \leq \frac{2c}{3\mu}\left[\frac{\Theta(z)}{\bar{F}(z)} - z\right] = \bar{k},
\end{aligned}$$

where $\bar{F}(z) = c/V$. If we let

$$k^* = \frac{1-FR(z)}{3-FR(z)}\frac{2\cdot c}{\mu}\left[\frac{\Theta(z)}{\bar{F}(z)} - z\right],$$

then the equilibrium solution for the nine stage-two subgames can be summarized as in Table A.2.

Comparison of Equilibrium Decisions. We first compare the stocking factors and prices in the $\{C-C\}$ and $\{PM-PM\}$ games. Recall that z^{PM} satisfies

$$\left.\frac{\partial\pi_1(PM, p_1, z_1, PM, p_2, z_2)}{\partial z_1}\right|_{p_1=p_2=p^{PM},\, z_1=z_2=z^{PM}} = 0.$$

Table A.2 **Equilibrium Solution Under PMG Scenarios**

	R_2		
R_1	C	PM	PMA
	(a) Case I: $k\leq k^*$		
C	(p^C, z^C)	(p^C, z^C)	(p^C, z^C)
PM	(p^C, z^C)	(p^{PM}, z^{PM})	(p^{PMA}, z^{PMA})
PMA	(p^C, z^C)	(p^{PMA}, z^{PMA})	(p^{PMA}, z^{PMA})
	(b) Case II: $k^*\leq k\leq\bar{k}$		
C	(p^C, z^C)	(p^C, z^C)	(p^C, z^C)
PM	(p^C, z^C)	(p^{PM}, z^{PM})	(p^{PM}, z^{PM})
PMA	(p^C, z^C)	(p^{PM}, z^{PM})	(p^{PM}, z^{PM})

Note. Note that each subgame has a symmetric solution.

Observe that

$$\begin{aligned}
&\left.\frac{\partial\pi_1(PM, p_1, z_1, PM, p_2, z_2)}{\partial z_1}\right|_{p_1=p_2=p^{PM},\, z_1=z_2=z^{PM}}\\
&\quad = \frac{\mu k}{2\cdot\Theta(z^{PM})}\left[V\cdot\Theta(z^{PM}) - \frac{\mu k}{2} - c\cdot z^{PM}\right]\\
&\qquad + k\mu\left[V\cdot\bar{F}(z^{PM}) - \frac{\mu k}{2\Theta(z^{PM})}\bar{F}(z^{PM}) - c\right].
\end{aligned}$$

And we also have

$$\begin{aligned}
&\left.\frac{\partial\pi_1(PM, p_1, z_1, PM, p_2, z_2)}{\partial z_1}\right|_{p_1=p_2=p^{PM},\, z_1=z_2=z^C}\\
&\quad = \frac{\mu k}{2\cdot\Theta(z^C)}\left[V\cdot\Theta(z^C) - \frac{\mu k}{2} - c\cdot z^C\right]\\
&\qquad + k\mu\left[V\cdot\bar{F}(z^C) - \frac{\mu k}{2\Theta(z^C)}\bar{F}(z^C) - c\right]\\
&\quad = c\left[\frac{\Theta(z^C)}{\bar{F}(z^C)} - z^C\right] = \frac{\mu k3}{2} > 0.
\end{aligned}$$

So we have $z^{PM} > z^C$. Second, let us compare the equilibrium prices. We have $p^{PM} = V - (k\cdot\mu)/(2\cdot\Theta(z^{PM}))$ and $p^C = (\mu k + cz^C)/\Theta(z^C)$, where $\bar{F}(z^C) = c/V$. So, $p^{PM} = c/\bar{F}(z^C) - k\cdot\mu/(2\cdot\Theta(z^{PM}))$. Now

$$p^{PM} = \frac{c}{\bar{F}(z^C)} - \frac{k\cdot\mu}{2\cdot\Theta(z^{PM})} > \frac{\mu k + cz^C}{\Theta(z^C)} = p^C$$

because

$$\begin{aligned}
&\frac{c}{\bar{F}(z^C)} - \frac{k\cdot\mu}{2\cdot\Theta(z^{PM})} > \frac{\mu k + cz^C}{\Theta(z^C)}\\
&\quad\Leftrightarrow\; c\left[\frac{\Theta(z^C)}{\bar{F}(z^C)} - z^C\right] \geq \mu\cdot k\left[1 + \frac{\Theta(z^C)}{2\Theta(z^{PM})}\right]
\end{aligned}$$

and

$$c\left[\frac{\Theta(z^C)}{\bar{F}(z^C)} - z^C\right] \geq \frac{3\mu k}{2} \geq \mu\cdot k\left[1 + \frac{\Theta(z^C)}{2\Theta(z^{PM})}\right].$$

So we have $p^{PM} > p^C$. Now, let us add subgame $\{PMA-PMA\}$ into the comparison. We first compare $z(p)$ at p^{PM}

and p^{PMA}. Note that $z'(p) = \Omega[\mu k + (V-p)\Theta(z(p)) - [p\cdot\Theta(z(p)) - c\cdot z(p)]]$, where

$$\Omega = (-\bar{F}(z(p)))\cdot\big[-\mu kpf(z(p)) + (V-p)\bar{F}(z(p)) \cdot[p\cdot\bar{F}(z(p)) - c] - f(z(p))(V-p)[p\cdot\Theta(z(p)) - c\cdot z(p)]\big]^{-1} > 0$$

and $z''(p)|_{z'(p)=0} = -2\Omega\Theta(z(p)) < 0$; thus, $z(p)$ is unimodal in p. Consequently, $z^{PM} = z(p)$ provides us two roots, $p = p^{PM}$ and $p = (2\cdot c\cdot z(p) + 3\mu k)/(2\Theta(z(p)))$. Now observe that

$$z'(p)\Big|_{z=z^{PM}} = \Omega\cdot\left[\frac{3\mu k}{2} - p^{PM}\cdot\Theta(z^{PM}) - c\cdot z^{PM}\right] < 0.$$

So, this tells us that we have $z(p) > z^{PM}$ for all $p \in [(2\cdot c\cdot z(p) + 3\mu k)\cdot(2\Theta(z(p))), p^{PM}]$. We know that $p^{PMA} < p^{PM}$, so if we show that $(2\cdot c\cdot z(p) + 3\mu k)/2\Theta(z(p)) \le p^{PMA}$, then we know that $z^{PMA} = z(p^{PMA}) > z(p^{PM}) = z^{PMA}$. Now, $(2\cdot c\cdot z(p) + 3\mu k)/(2\Theta(z(p))) \le p^{PMA}$ if and only if $FR(z^{PMA}) \ge 1/3$, and this holds because

$$\begin{aligned}1 - FR(z^{PMA}) &= \frac{\mu k}{[p^{PMA}\cdot\Theta(z^{PMA}) - c\cdot z^{PMA}]}\\ &< \frac{\mu k}{[p^{PM}\cdot\Theta(z^{PM}) - c\cdot z^{PM}]} < \frac{\mu k}{3\mu k/2}\\ &= \frac{2}{3}.\end{aligned}$$

Recall that z^C satisfies

$$\bar{F}(z^C) = \frac{c}{V} > \frac{c(1 - FR(z^{PMA}))}{V - p^{PMA}\cdot FR(z^{PMA})} = \bar{F}(z^{PMA})$$

for any $p^{PMA} < V$ and $FR(z^{PMA}) < 1$. Therefore, $z^{PMA} > z^C$. □

Proof of Proposition 3

For the comparison of profits, note that z^{PM} satisfies

$$\begin{aligned}&\frac{\mu k}{2\cdot\Theta(z^{PM})}\left[V\cdot\Theta(z^{PM}) - \frac{\mu k}{2} - c\cdot z^{PM}\right]\\ &\quad + k\mu\left[V\cdot\bar{F}(z^{PM}) - \frac{\mu k}{2\Theta(z^{PM})}\bar{F}(z^{PM}) - c\right]\\ &= \frac{\bar{F}(z^{PM})}{2\cdot\Theta(z^{PM})}\left[V\cdot\Theta(z^{PM}) - c\cdot z^{PM} - \frac{3\mu k}{2}\right]\\ &\quad + [V\cdot\bar{F}(z^{PM}) - c] = 0.\end{aligned}$$

Thus we must have $[V\cdot\Theta(z^{PM}) - c\cdot z^{PM} - (3\mu k)/2] > 0$ because $\bar{F}(z^{PM}) < \bar{F}(z^C) = c/V$. Consequently, $\pi^{PM} = (1/2)\big[V\cdot\Theta(z^{PM}) - c\cdot z^{PM} - (\mu k)/2\big] \ge \mu k/2 = \pi^C$. Recall that the symmetric solution under the $\{PMA - PMA\}$ game is (p^{PMA}, z^{PMA}) if $[1 - FR(z^{PM})][p^{PM}\cdot\Theta(z^{PM}) - cz^{PM}] \ge \mu\cdot k$ and (p^{PM}, z^{PM}) otherwise. Note that the solution to the $\{PM - PM\}$ game is the best that the retailers can do under a covered market scenario; that is, $(p^{PM}, z^{PM}) = \arg\max_{p,z}(1/2)[p\cdot\Theta(z) - cz] = \arg\max_p(1/2)[p\cdot\Theta(z(p)) - cz(p)]$, where $z(p) = \{z \mid (V-p)\cdot\bar{F}(z)[p\cdot\Theta(z) - c\cdot z] + k\mu[p\cdot\bar{F}(z) - c] = 0\}$ and subject to $2(((V-p)\cdot\Theta(z(p)))/(k\mu)) \ge 1$. Furthermore, observe that $z^{PMA}(p) = z^{PM}(p) = z^C(p) = z(p)$. So by definition, if the PMA solution is different from the PM solution, it is suboptimal; $\pi^{PMA} < \max_{p,z}(1/2)[p\cdot\Theta(z^{PMA}(p)) - cz^{PMA}] = \max_{p,z}(1/2)[p\cdot\Theta(z^{PM}(p)) - cz^{PM}] = \pi^{PM}$, concluding that the expected retail profits are smaller under PMA, i.e., $\pi^{PMA} < \pi^{PM}$. Also recall that the first-order condition with respect to price satisfies $-[1 - FR(z^{PMA})][p^{PMA}\cdot\Theta(z^{PMA}) - c\cdot z^{PMA}] + k\mu = 0$, which gives us $[p^{PMA}\cdot\Theta(z^{PMA}) - c\cdot z^{PMA}] = (k\mu)[1 - FR(z^{PMA}))> \mu\cdot k$. Therefore, $\pi^{PMA} = (1/2)[p^{PMA}\cdot\Theta(z^{PMA}) - c\cdot z^{PMA}] > k\cdot\mu/2 = \pi^C$.

We next show that consumers' surplus is higher if retailers verify the availability, i.e., under the PMA policy. In each subgame, we have a symmetric equilibrium, that is, the market is covered and equally shared by retailers. So, in each subgame, customers located on $[0, 1/2]$ visit R_1, and customers located on $[1/2, 1]$ visit R_2. Thus the travel cost is identical in each subgame. The net utility comparison obtained by each customer depends on the expected utility obtained by visiting the retailers, $(V-p)\cdot\Theta(z)$. Based on the equilibrium solutions, we know that $(V - p^C)\cdot\Theta(z^C) = V\cdot\Theta(z^C) - \mu k - cz^C$, where z^C satisfies $\bar{F}(z^C) = c/V$. Also, $(V - p^{PMA})\cdot\Theta(z^{PMA}) = V\cdot\Theta(z^{PMA}) - (\mu k/(1 - FR(z^{PMA}))) - cz^{PMA}$. Note that $V\cdot\Theta(z) - cz$ is concave in z and maximized at $\bar{F}(z) = c/V$. Therefore, $V\cdot\Theta(z^C) - cz^C \ge V\cdot\Theta(z^{PMA}) - cz^{PMA}$. Furthermore, $\mu k/(1 - FR(z^{PMA})) > \mu k$, thus $(V - p^C)\cdot\Theta(z^C) > (V - p^{PMA})\cdot\Theta(z^{PMA})$. Also, $(V - p^{PMA})\cdot\Theta(z^{PMA}) \ge (V - p^{PM})\cdot\Theta(z^{PM})$ because $p^{PM} \ge p^{PMA}$ and $z^{PM} \le z^{PMA}$. In summary, $(V - p^C)\cdot\Theta(z^C) \ge (V - p^{PMA})\cdot\Theta(z^{PMA}) \ge (V - p^{PM})\cdot\Theta(z^{PM})$. □

Proof of Proposition 4

The proof follows from Propositions 2 and 3. Note that Table A.2 of Table A.2 of Proposition 2 provides the strategic game to determine the equilibrium that $\pi^C \le \pi^{PMA} \le \pi^{PM}$. Consequently, offering the PM policy is the weakly dominating strategy for both retailers. □

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