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Revenue Management of a Make-to-Stock Queue

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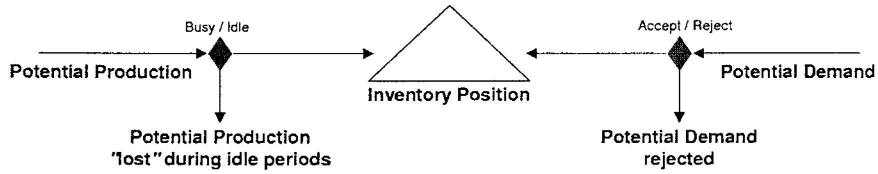
In this paper, we address the joint problem of order admission and inventory control for a single-product manufacturing system. We model the demand for the good as the combination of two exogenous and possibly correlated stochastic processes; one describes the arrival pattern of orders over time and the other describes the evolution of the market price. Electronic marketplaces (e.g., Keskinocak et al. 1999) are motivating examples. Here potential customers take the form of dynamically arriving e-orders, who post on the Internet a price they are willing to pay for a unit of product. On the manufacturing side, we use a single-server, make-to-stock queueing model. That is, production capacity is limited and stochastic and the manufacturer carries finished goods inventory to service demand. The manufacturer acts as a price taker and decides whether to accept or reject each arriving job at the time of its arrival. In addition, he has to decide how to control production and inventory levels. In this setting, the problem becomes how to adequately balance the benefits and costs associated with providing the good to the end customers by controlling both the admission of orders (accept/reject policy) and the levels of stock (traditional busy/idle production policy). A detailed formulation and analysis of this stochastic control problem can be found in Caldentey and Wein (2001). In what follows, we briefly summarize the main results.

Figure 1 Inventory Dynamics



In terms of the modelling, the novelty in our formulation is the use of a stochastic process to characterize the market price. In particular, we use a variation of the standard geometric Brownian motion (GBM). Our particular choice of a geometric Brownian motion is mainly influenced by the finance literature where GBM is the standard model used to represent price processes (e.g., in the derivation of the Black-Scholes option-pricing formula). On the other hand, the dynamics of the inventory position are determined by the potential demand and production processes together with the manager's decision. These dynamics are illustrates in Figure 1. The objective function that we consider has two main components: (1) a revenue for each order that is accepted and (2) a holding/backordering cost dependent on the inventory position. The trade-offs in our formulations are between the cost associated with rejecting orders and the cost of holding inventory or backlogging customers.

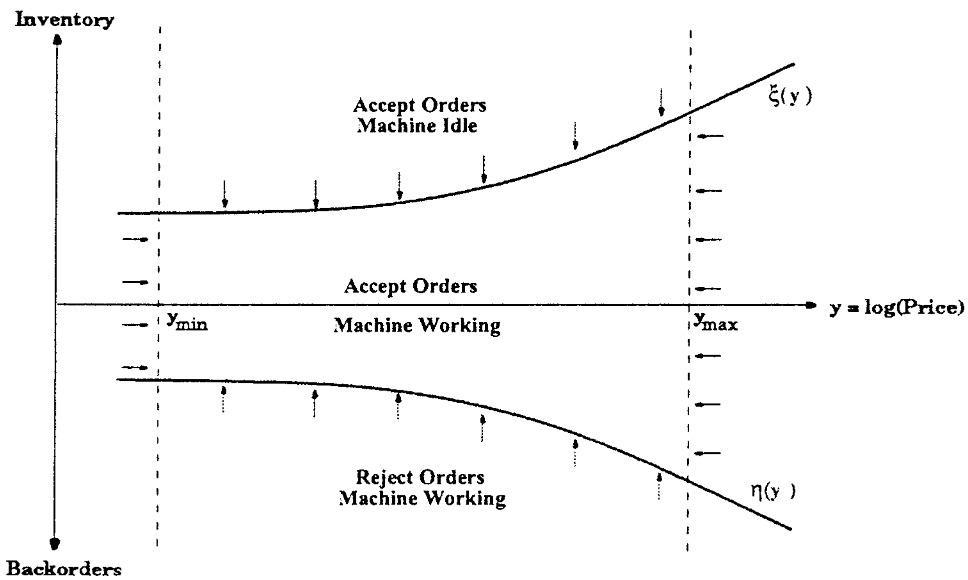
The exact formulation of the stochastic control problem appears to be analytically intractable for the general case, so we simplify it by considering its diffusion version. That is, using a *heavy traffic scaling* transformation of time and space, we are able to replace our original control problem with a much tractable diffusion control problem. In this case, the optimality conditions (Hamilton-Jacobi-Bellman equations) take the form of a two-dimensional[1] elliptic PDE system with free boundary conditions. Two price-dependent switching curves, η and ξ, characterize the optimal solution defining the accept/reject and busy/idle decisions, respectively. Figure 2 schematically shows the shape of this solution. Intuitively, this solution tells us the following. If the inventory is "under control" (i.e., it lies between the two switching curves), then the manager should be accepting orders and producing. If the inventory increases and reaches ξ, then production should stop. On the other hand, if backorders increase, reaching η, then demand should be rejected. The reflection field on Figure 2 describes the instantaneous directions along with the two-dimensional process (price and inventory) moves when hitting the boundary. Closed-form solutions for free boundary PDE problems are very rare in general. Unfortunately, our system is not an exception, and we have not been able to solve it analytically. For this reason, we approach the solution through numerical and approximated methods.

Our numerical method is an iterative procedure that combines the *finite difference approximation* technique, developed by H. Kushner (1977), and a new method, proposed by Kumar and Muthuraman (2000), for solving singular control problems. Starting with an initial solution (η^0, ξ^0) we generate a sequence of solutions (η^k, ξ^k) that converges in a finite amount of time. For a given solution (η^k, ξ^k), Kushner's Markov chain approximation of the diffusion process is used to compute the value function and the expected cost. With this information, Kumar and Muthuraman's method is used to check optimality and to generate the next solution in the sequence in case of suboptimality.

On the other hand, to get approximate solutions we note that the main obstacle for solving the problem is connected to the particular reflection field on the boundaries that we have (see Figure 2). However, a major simplification arises if we replace this reflection by the standard Neumann condition; i.e., the inward unit normal vector field. This transformation allows us to approximate the steady-state distribution of our two-dimensional process by an exponential and more importantly to replace the PDE system by a much more tractable ODE system. Using standard calculus of variation techniques, we solve the ODE system and

[1] The two dimensions are given by the market price and inventory processes.

Figure 2 Shape of the Optimal Solution



Arrows indicate the reflection field on the boundary.

obtain our *proposed* solution. For this *proposed* policy, the switching curve ξ ends up being a base-stock policy independent of the price. This, perhaps, surprising result turns out to be consistent with the numerical solution. Moreover, the assumption that the reflection field is normal to the boundary holds automatically in this situation. On the other hand, the *proposed* rejection switching curve, η, depends on the price in a way that admission increases linearly with the price and its first derivative. Intuitively, this *proposed* policy suggests that price variability should not affect the level of safety stock but only the admission process. In other words, it is not optimal to hold more inventory when the price is high; it is simply better to accept more orders.

To assess the performance of this *proposed* policy with respect to common practices in industry, we also consider the simplest *static* solution. This policy is obtained by assuming that admission and production decisions are independent of the price, that is, η and ξ are constant. From a managerial perspective this solution is easy to implement because it does not require any monitoring of the dynamics of the price. Computational experiments show that on average the suboptimality of the *proposed* policy with respect to the numerical solution is below 5%. In contrast, the *static* solution, which does not incorporate price variability, has an average error of above 40%. These preliminary results reveal that price fluctuations do have a significant impact on the performance of the system. This is particularly important if we consider that much of the inventory control literature assumes static price models.

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