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Experiments on Compound Risk in Relation to Simple Risk and to Ambiguity

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We conduct experiments measuring individual behavior under compound risk, simple risk, and ambiguity. We focus on (1) treatment of compound risks relative to simple risks and (2) the relationship between compound risk attitudes and ambiguity attitudes. We find that compound risks are valued differently than corresponding reduced simple risks. These differences measure compound risk attitudes. These attitudes display more aversion as the reduced probability of the winning event increases. Like Halevy [Halevy Y (2007) Ellsberg revisited: An experimental study. *Econometrica* 75:503–536], we find an association between compound risk reduction and ambiguity neutrality. However, in contrast to the almost perfect identification in Halevy's data, we find a substantially weaker relation in both directions. First, a majority of our ambiguity-neutral subjects fail to reduce compound risk. Second, almost a quarter of our subjects who reduce compound risk are nonneutral to ambiguity. All of the latter come from the more quantitatively sophisticated part of our subject pool.

Data, as supplemental material, are available at <http://dx.doi.org/10.1287/mnsc.2014.1953>.

Keywords: ambiguity; Ellsberg paradox; reduction of compound lotteries

History: Received December 15, 2011; accepted January 26, 2014, by Peter Wakker, decision analysis. Published online in *Articles in Advance* February 4, 2015.

1. Introduction

We experimentally investigate behavior toward compound risk and its relation with behavior toward simple risk and toward ambiguity (i.e., subjective uncertainty about probabilities). In classical theories of decision making, such as expected utility (Savage 1954), compound risk is reduced to simple risk by multiplication of probabilities, and ambiguity is reduced to simple risk through assignment of subjective probabilities. Thus, behavior toward all three types of uncertainty is tightly linked in classical theory. We examine binary prospects and focus on two issues: (1) the treatment of compound risks relative to the corresponding reduced simple risks, and how this varies with the probability of the winning event, and (2) the relationship between compound risk attitude and ambiguity attitude, especially between reduction of compound risk and ambiguity neutrality, and whether this relationship depends on the quantitative background of the subjects.

Regarding the first issue, there has been little direct examination of how aversion toward compound risk relative to simple risk changes with the reduced

probability of the winning event. In our view, the lack of focus on behavior toward compound risk is surprising in light of the low (if any) cost a policy maker or firm incurs when presenting risk as compound versus simple to an economic agent. To the extent that behavior differs substantially toward compound risks compared with simple risks, the marketing, policy, and economic implications of understanding this behavior may be large. For example, if individuals dislike compound risk, then sellers of risky assets or products would benefit from presenting the risk in its reduced form.

We compare the certainty equivalents recorded from subjects' choices under simple risk and two types of compound risk. We find that compound risks are valued differently than corresponding reduced simple risks. These differences reflect compound risk attitudes. A main novel finding is that individuals become more compound risk averse as the reduced probability of the winning event increases. Such increasing aversion is consistent with greater insensitivity to changes in probability under compound risk than under simple

risk. Likelihood insensitivity is a central concept in the literature on behavioral decision theory that builds on prospect theory (Fox and Tversky 1995, Tversky and Wakker 1995, Wakker 2010). Comparative insensitivity across sources has been investigated both theoretically and experimentally (Einhorn and Hogarth 1985, Abdellaoui et al. 2011b), but not for compound risk.

Turning to the second issue, we find that most theories of decision making under ambiguity do not specify any connection between behavior toward ambiguity and behavior toward compound risk. However, some prominent recent theories explicitly connect the two. For example, Segal (1987) and Seo (2009) present theories equating the reduction of objective compound lotteries with ambiguity neutrality.¹ The theory of Halevy and Ozdenoren (2008) does not equate the two but says that compound risk reduction implies ambiguity neutrality. This implication, if valid, may have a large impact on the way ambiguity attitudes are viewed. If one sees failure to reduce compound risk as a departure from rationality, then according to these theories, ambiguity nonneutrality must be such a departure as well. In contrast, if compound risk and ambiguity are less tightly linked, there is room to reach separate normative and prescriptive conclusions concerning each. Note that a number of related models, including Klibanoff et al. (2005), Nau (2006), Ergin and Gul (2009), and Neilson (2010), model ambiguity-sensitive behavior using subjective two-stage structures that are not reduced. However, these models do not include objective compound lotteries among the objects of choice and do not rely on nonreduction of objective lotteries to generate ambiguity sensitivity.

Few papers have empirically examined the link between attitude toward ambiguity and attitude toward compound risk. There appear to be two main (and contradictory) empirical contributions dealing with this relationship. Bernasconi and Loomes (1992) use a compound risk version of the three-color urn of Ellsberg (1961), and, based on their finding of less Ellsberg-type behavior than that typically found under ambiguity, conclude the following:

Our findings that “ambiguous lotteries” in the sense of Ellsberg cannot be fully characterized by “distributed lotteries” as suggested by Segal also undermine the possibility of viewing ambiguity aversion and risk aversion as “the two sides of the same coin.” (p. 91)

More recently, Halevy (2007) finds that the two attitudes are related. In fact, Halevy goes even further and suggests that nonreduction of compound

risk is *necessary* for a nonneutral attitude toward ambiguity:

Subjects who reduced compound lotteries were almost always ambiguity neutral, and most subjects who were ambiguity neutral reduced compound lotteries appropriately. (p. 531)

The results suggest that failure to reduce compound (objective) lotteries is the underlying factor of the Ellsberg paradox, (p. 532)

An advantage of the design of Halevy (2007) is its allowance for within-subject comparisons. We build on this important prior work and further explore the relationship between attitudes toward ambiguity and compound risks using a design that similarly allows for within-subject comparisons. Additionally, to test for the possible influence of subjects’ quantitative sophistication and background on the relationship, our subject pool was divided into advanced engineering students at an elite institution and students from a cross section of nonengineering fields and institutions.

Similar to Halevy (2007) and Dean and Ortoleva (2012), we find an association between compound risk reduction/nonreduction and ambiguity neutrality/nonneutrality. However, unlike the almost perfect identification of compound risk reduction with ambiguity neutrality in Halevy’s data, we find a substantially weaker relation in both directions. With regard to ambiguity neutrality implying reduction of compound risk (as in the theory of Seo (2009) and as supported in Halevy’s data), we find less support among both engineers and nonengineers for this hypothesis. In fact, the majority of ambiguity-neutral subjects do not reduce compound risk. With regard to reduction of compound risk implying ambiguity neutrality, we find less support for this among engineers while obtaining support similar to that in Halevy among nonengineers. For the engineers, compound risk reduction appears to be compatible with nonneutral attitudes toward ambiguity.

The rest of this paper is as follows. Section 2 details the experimental design of the two main studies (Studies 1 and 2). Section 3 describes and analyzes the observed behavior of subjects toward compound risk in relation to simple risk using Study 1. Section 4 discusses the findings from Study 2 on the relationship between ambiguity and compound risk attitudes. Section 5 concludes. Appendices contain additional details concerning the experimental procedures and some supplementary analysis.

2. Experiments

2.1. Procedure

Each of our studies involves binary bets with a winning payoff of €50 and a payoff of €0 otherwise. In Study 1, we consider bets under simple risk and two types of compound risk. For each type of bet we examine

¹ An early precursor to these theories is Kahneman and Tversky (1975, pp. 30–33). Furthermore, early empirical literature on ambiguity commonly assumed it to be operationally identical to compound risk (Becker and Brownson 1964, Yates and Zukowski 1976, Kahn and Sarin 1988).

three different probabilities of winning: $1/12$, $1/2$, and $11/12$. In Study 2, we consider bets under simple risk, three compound risks, and ambiguity. All of the bets in Study 2 are symmetric (probability $1/2$ of winning for simple and compound risks, symmetric information about two colors for ambiguity).

Bets were displayed on a computer monitor one at a time. Simple risk, compound risk, and ambiguity were each represented by Ellsberg-like urns containing colored balls. Subjects had to consider bets whose outcome depended on which color(s) were drawn from the urn(s). For the simple (or compound) risk, the subjects saw the color of the balls in one (correspondingly, in two successive stages of) urn(s) and thus could infer the probability of winning the bet. Section 2.3 describes and motivates the different types of compound risks. For ambiguity, the colors of the two balls in the urn were hidden by making the urn opaque. Section 2.2 describes how the composition of the ambiguous urn was determined. The bets used in Study 1 are displayed in Table B.1 in Appendix B. Similarly, Table B.2 in Appendix B displays the bets used in Study 2. We elicited certainty equivalents for each bet using an iterative choice list procedure adapted from Abdellaoui et al. (2011b). For each bet, subjects make choices between the bet and (an ascending range of) sure payments. See Appendix §C.1 for details and sample screen images of this choice list procedure.

The experiments were organized as follows: A group of subjects entered the room, and each subject was assigned a cubicle with a computer monitor and mouse. Once everyone in the group was seated, the instructions were displayed on each screen and the experimenter read them aloud. After the instructions, the subjects were allowed to ask the experimenter questions in case there was anything they did not understand. The computer then randomly assigned each subject to an order treatment. There were two order treatments for Study 1, determining whether a subject faced risk first or compound risk first, and six order treatments for Study 2, determining the sequence among risk, ambiguity, and compound risk.² Within each type of bet, the order of presentation was also randomized. Next came a training phase (see Appendix §C.2), in which each subject was presented with the first stimulus to be faced and was guided through the choice list comparisons to be made, as well as how payment was to be determined, but was not yet making choices that count. After this training, the experiment proper began, with bets presented sequentially and choices

recorded. Copies of the instructions for Studies 1 and 2 are provided in Appendix §§A.1 and A.2, respectively. At the end of the experiment, payments were made by using the random incentive system described in §2.4. Subjects were additionally paid a show-up fee of €5.

2.2. Ambiguity Implementation

The ambiguous urn for Study 2 was generated through a two-stage process involving a physical bag of balls. During the reading of the instructions, one of the participants was asked to generate the ambiguous urn by drawing (in front of everyone) two balls from a bag containing 12 balls. The subjects were told that the 12 balls could be of the two possible colors, red and black, but were not informed of the distribution of colors in the bag (the actual distribution was six red and six black). The balls were covered so that the colors of the two drawn balls could not be seen by either the experimenter or the subjects until the end of the experiment. These two balls are placed in a bag representing the ambiguous urn. Near the end of the instructions for Study 2, the computer prompted each subject to choose the color that will be associated with the €50 prize for the bets that will be presented.

This process has three main advantages. First, the final color composition of the ambiguous urns is unknown to both the subject and the experimenter, reducing possible comparative ignorance effects (Fox and Tversky 1995). Second, this process may induce a two-stage representation of ambiguity in subjects (however, it does not guarantee that subjects used such a representation); hence, one cannot argue that possible similarities in attitudes toward two-stage compound risk and ambiguity are undermined by the way ambiguity has been implemented. If anything, our setup may be biased in the direction of making such a connection stronger. Third, allowing subjects to choose which color is associated with the winning payoff reduces possible suspicion effects (Hey et al. 2010). A possible disadvantage of allowing the choice of color is that it may bias findings against ambiguity aversion for those subjects who view randomizing over choice of color as a hedge against ambiguity (Raiffa 1961, Dominiak and Schnedler 2011). Color choice may also bias upward valuation of all bets through an illusion of control (Langer 1975).

2.3. Choice of Compound Risks

The types of compound risk used in our studies and our motivation for including them are explained here. In compound risk with a degenerate second stage (CRD), all risk resolves in the first stage, since each possible second-stage urn contains only one color. CRD was included in Study 1 to test for violations of time neutrality (Segal 1987), i.e., for a pure effect of risk resolving in the first stage versus the second stage.

² Although the order was intended to be assigned in an exactly balanced way, we ended up with a slight imbalance of subjects across order treatments because a few subjects' computers froze at the instruction screen and needed to be rebooted, thus assigning them to the next order treatment.

In diverse uniform compound risk (CRU), the first stage generates a uniform distribution over all possible second-stage urns. Both CRU and CRD were included in Study 2 to mimic the two compound risks used in Halevy (2007). In hypergeometric compound risk (CRG), two balls are drawn (without replacement) from the first-stage urn to form the contents of the second-stage urn. We refer to such risk as hypergeometric because the number of red balls in the second-stage urn follows a hypergeometric distribution. CRG (included in all studies) was designed to be similar to the process used to generate the ambiguous urns in our study—in both cases, balls are drawn (without replacement) from a larger urn and placed in a new urn, a draw from which determines the outcome of the bet.

2.4. Incentives

Subjects performed the choice tasks knowing that, at the end of the session, one task for each subject would be randomly drawn and the choice the subject had indicated for this task would be implemented and paid. If the subject had chosen a (simple or compound) risk, the corresponding urn(s) were created and the subject physically selected the ball(s) that determined the payment. If the subject had chosen an ambiguous urn had been chosen, he or she physically drew the ball that determined the payment from the two-ball urn that was generated at the beginning of the experiment. If the subject had chosen a sure amount, that amount was paid. Thus, each subject's total payoffs ranged from €0 to €50 plus the €5 show-up fee.

Paying according to a random-lottery incentive system (RIS) is common in the literature and has many desirable properties. Yet it has also been criticized (Holt 1986). A key issue is whether subjects treat the entire experiment as one grand compound lottery/ambiguity; if they do so, because they may violate independence and/or compound reduction properties, they might choose differently in subproblems than they would if presented with that subproblem alone. If subjects, instead, treat each choice in isolation, then a RIS introduces no distortions. Experimental evidence on these points is mixed but is generally supportive of RIS in practice. See Starmer and Sugden (1991) and Cubitt et al. (1998) for supportive evidence and Harrison et al. (2013) for a finding of some distortion. Wakker (2007) provides an informative discussion of the issue.

2.5. Sample

As indicated in Table 1, 209 subjects are included in our sample—124 were students from Arts et Métiers ParisTech (an elite French graduate engineering school) and 85 were university students recruited from across nonengineering fields through the lab recruitment system (Laboratoire d'Economie Expérimentale de

Table 1 Pool of Subjects

Sample	Study 1	Study 2	Total
Engineers	49	75	124
Nonengineers	45	40	85
Total	94	115	209

Paris (LEEP)).³ What distinguishes engineers and nonengineering students is that the former are intensively quantitatively trained and highly selected on academic ability whereas the latter are less quantitatively trained and vary in the selectivity of their courses of study and institutions. Each subject participated in only one of the two studies.

3. Valuation of Compound Risk in Relation to Simple Risk

3.1. Attitudes Toward Compound Risk

One of the main assumptions of most economic models of decision making under risk is that the nature and the complexity of a lottery should not affect its evaluation. Most models implicitly or explicitly incorporate a reduction-of-compound-lotteries axiom requiring a decision maker to value a simple lottery exactly as any compound lottery with the same reduced probabilities. Table 2 reports statistics on the observed certainty equivalents for the simple and compound risks in Study 1.

According to theories assuming reduction, within each winning probability treatment, and for each subject, the certainty equivalents for the three bets should be equal. A multivariate analysis of variance (MANOVA) with repeated measures rejects even the weaker hypothesis that within each probability treatment the average certainty equivalents for the three bets are equal ($p < 0.05$). Thus, our data reject reduction of compound lotteries. Miao and Zhong (2012) also find evidence of nonreduction of compound lotteries, whereas Harrison et al. (2013) find mixed evidence on reduction.

Given nonreduction, it is interesting to investigate attitudes toward compound risk by comparing certainty equivalents between compound risk treatments and a simple risk treatment giving the same reduced probability, because such a comparison controls for attitudes toward simple risk. For this purpose, define the *compound risk premium* (for a given compound lottery) as the certainty equivalent for the corresponding reduced lottery minus the certainty equivalent for

³ These numbers do not include those subjects who at any point violated dominance by preferring €0 to a bet or preferring a bet to €50 or who violated monotonicity by preferring a lower sure amount to the bet but the bet to a higher sure amount. In Study 1, there were eight such subjects; in Study 2, there were two. These 10 subjects were not included in any of our calculations or results.

Table 2 Certainty Equivalents of Simple Risk and Compound Risks

Winning probability →		1/12			6/12			11/12		
Expected value		4.17			25			45.83		
Certainty equivalents (€) ↓	Sample size (<i>n</i>)	Mean	Median	SD	Mean	Median	SD	Mean	Median	SD
Risk	94	9.91	5.50	9.97	22.65	24.50	9.23	37.74	40.50	11.29
CRD	94	9.85	5.50	9.35	22.17	24.50	9.85	36.30	40.50	12.39
CRG	94	9.25	5.50	8.30	20.72	22.50	9.34	34.72	39.00	12.36
Pooled across Risk, CRD, and CRG	282	9.67	5.50	9.20	21.85	24.50	9.47	36.26	40.00	12.04

the compound lottery. Dillenberger (2010) calls this a gradual resolution premium. We say a subject is *compound risk averse*, *compound risk neutral*, or *compound risk seeking* when the compound risk premium is positive, zero, or negative, respectively. On average (across probability treatments and the two compound risks), our subjects are compound risk averse (mean = 1.27, SD = 8.55, MANOVA; $p < 0.01$). Average time neutrality (indifference to risk resolving entirely in the first stage of two rather than in one stage) is not rejected (the average compound risk premium for CRD is 0.663, t -test; $p = 0.17$).

Importantly, there is systematic variation in compound risk premia across probability treatments. The first row of Table 3 reports the average compound risk premia by probability treatment (pooled across the two compound risks). Note that these average premia appear to increase with the probability of winning. Furthermore, the within-subject variation provides strong evidence that compound risk premia increase with the probability of winning (Page's L -test for increasing trend, $p < 0.01$).⁴

Recent literature has not investigated the relationship between the probability of winning and compound risk aversion. Older experimental literature examining ambiguity operationalized as compound risk could be interpreted as providing some evidence for the pattern of more compound risk aversion for high probabilities than for low probabilities (Kahn and Sarin 1988) and others not supporting that pattern (Larson 1980). Evidence that compound risk seeking is common for low probability levels is found by Friedman (2005), who examines only low probability compound lotteries (with winning probabilities ranging from 0.0625 to 0.5625) and finds that individuals tend to value the compound lotteries more than their reduced simple risks. This is also found by Kahn and Sarin (1988). See

Budescu and Fischer (2001) for additional studies and discussion of behavior toward compound lotteries.

This pattern of becoming more averse as winning probabilities increase has been previously identified for other types of uncertainty including simple risk (see the references in Wakker (2010, Chap. 7, Footnote 2), as well as Table 7 in Web Appendix SW.1 (available at <http://ssrn.com/abstract=2495076>), showing that Study 1 finds this pattern for simple risk) and ambiguity (Tversky and Fox 1995, Abdellaoui et al. 2011b). The most prominent explanation in the literature of such patterns is based on insensitivity to changes in probabilities (Wakker 2010, Chap. 7). In this light, our findings could be interpreted as evidence that when moving from simple to compound risk, subjects become more insensitive to changes in the winning probability, and this leads them to effectively discount the increases from 1/12 to 1/2 and from 1/2 to 11/12.

3.2. Sensitivity Analysis: Effects of Type of Compound Risk, Subject's Sophistication, and Order of Presentation

How sensitive are these patterns in compound risk aversion/insensitivity to different varieties of compound risk and to differences in the technical sophistication of subjects? Relatedly, do these patterns depend on whether or not one is first presented with simple risk?

Table 3 starts from the average compound risk premia pooled across the two types of compound risk, across order, and across engineering and nonengineering subjects, and then disaggregates. The table may seem to suggest that the pattern of increasing compound risk aversion with probability treatments is driven primarily by the less quantitatively sophisticated (nonengineering) subjects who were presented with compound risk before simple risk. However, these are only cross-subject averages. The more important pattern of increasing compound risk aversion is the within-subject pattern. Here, Page's L -test for increasing trend, which strongly indicated a within-subject increasing trend on the pooled data, continues to show a within-subject increasing trend for both engineers and nonengineers ($p < 0.05$ and $p < 0.01$, respectively). Within both groups of subjects and for both compound risks, however, the increasing trend appears significant only for subjects who were presented with compound

⁴ Page's L -test (Page 1963) has as its null hypothesis that, for each subject, all orderings of premia across treatments are equally likely (this description ignores treatment of ties in orderings, for which the test itself accounts). The alternative is that treatments with higher winning probabilities tend to have premia assigned higher ranks. An advantage of this test is that it does not make parametric distributional assumptions and, most importantly, accommodates heterogeneity in distributions of premia across subjects since only within-subject rankings are used.

Table 3 Average Compound Risk Premia by Probability Treatment, Sophistication, and Order Effect

Winning probability →	Sample size (<i>n</i>)	1/12	1/2	11/12	Page's <i>L</i> -test <i>p</i> -value for within-subject increasing trend
Average compound risk premia	94	0.36	1.20**	2.23**	0.000***
Average premia for CRG		0.66	1.93***	3.02**	0.006***
Engineers	49	1.18	0.29	1.92	0.133
Nonengineers	45	0.09	3.72***	4.23**	0.007***
Average premia for CRD		0.06	0.48	1.45	0.162
Engineers	49	−0.51	0.00	0.31	0.059*
Nonengineers	45	0.69	1.00	2.69	0.583
Average CR premium engineers		0.34	0.14	1.11	0.012**
Risk presented first	27	1.06	1.57*	0.31	0.170
CR presented first	22	−0.55	−1.61	2.09	0.001***
Average CR premium nonengineers		0.39	2.36***	3.46**	0.007***
Risk presented first	22	1.84	2.45**	1.36	0.440
CR presented first	23	−1.00	2.26*	5.46*	0.000***
Average CR premium if Risk presented first		1.41**	1.97***	0.79	0.210
Engineers, CRG	27	2.74**	2.04	2.37	0.500
Engineers, CRD	27	−0.63*	1.12*	−0.29	0.472
Nonengineers, CRG	22	1.59	4.05***	3.32	0.225
Nonengineers, CRD	22	2.09*	0.86	−1.74	0.959
Average CR premium if CR presented first		−0.78	0.37	3.81**	0.000***
Engineers, CRG	22	−0.73	−1.86	1.36	0.048**
Engineers, CRD	22	−0.36	−1.36	2.82	0.012**
Nonengineers, CRG	23	−1.35	3.39**	5.09	0.003***
Nonengineers, CRD	23	−0.65	1.13	5.83*	0.081*

***Significant at 1%; **significant at 5%; *significant at 10%.

risk first ($p < 0.05$ and $p < 0.05$ for engineers presented with CRG and CRD first, respectively; $p < 0.01$ and $p < 0.1$ for nonengineers presented with CRG and CRD first, respectively).

When a pattern occurs with one ordering but not the other, a natural question is whether the pattern remains evident in a setting where just one type of bet is presented. Arguably, the only certainty equivalents in our study reflecting such an evaluation are those for the type of bets the subjects see first. Is the same pattern of increasing compound risk premia present if only these certainty equivalents are used? Since it is impossible for any subject to see both simple and compound risk first, some cross-subject comparisons are necessary to address this question. With this caveat in mind, Table 4 shows the compound risk premia based on differences in average certainty equivalents (the average certainty equivalent for simple risk when simple risk is seen first, minus the average certainty equivalent for compound risks when compound risk is seen first) by probability treatment. The increasing pattern is strongly evident (one-tailed *t*-test for 1/2 versus 1/12, $p < 0.01$; one-tailed *t*-test for 11/12 versus 1/2, $p < 0.01$).⁵ This supports the notion that insensitivity/increasing compound risk aversion across probability treatments is a genuine aspect of subjects' compound risk evaluations and not merely a product of order effects.

⁵ These tests use the within-subject increases in certainty equivalents across probability treatments for the type of bet seen first. The alternative hypothesis is that the population average within-subject increases are smaller for the population seeing compound risk first.

Table 4 Average Certainty Equivalent for Simple Risk When Simple Risk Is Seen First, Minus Average Certainty Equivalent for Compound Risks When Compound Risk Is Seen First

	1/12	1/2	11/12
Average cross-subject premia for compound risk	−0.92	3.34	8.33

3.3. Summary

A key finding is that increases in the probability of the good outcome increase aversion toward compound risk. In other words, raising the probability of winning increases the valuation of compound risk by less than it does for the corresponding simple risk. This, together with prior results relating to simple risk, highlights the importance of controlling for probability when measuring risk aversion. It also suggests that any descriptive model intended to apply to the full range of risky situations must allow aversion to increase with probabilities.

4. The Relationship Between Compound Risk Attitudes and Ambiguity Attitudes

Segal (1987) (as well as antecedents such as Becker and Brownson (1964); Kahneman and Tversky (1975, pp. 30–33); Yates and Zukowski (1976); and others) suggests that ambiguous bets can be represented as a two-stage risk, where the first-stage lottery describes the probabilities of getting various lotteries in the

This is equivalent to the population versions of the premia in Table 4 increasing with probability treatment.

second stage. This model relies on the hypothesis of nonreduction of two-stage lotteries to generate ambiguity-sensitive behavior. More recent theories explicitly using the violation of reduction of objective compound lotteries to model ambiguity attitudes include Halevy and Ozdenoren (2008) and Seo (2009). In these papers, the reduction of objective compound lotteries implies neutrality to ambiguity. The posited link between the nonreduction of compound lotteries and Ellsberg-type behavior motivates Halevy (2007).

4.1. A Comparison with Halevy (2007)

The strongest and most striking evidence in Halevy (2007) for the identification of ambiguity attitude with compound risk attitude is a contingency table relating neutrality/nonneutrality toward ambiguity and reduction/nonreduction of compound risk. All bets in Halevy either have objective probability of winning of $1/2$ or, in the case of ambiguity, win if one of the two possible colors is drawn. Accordingly, in Study 2, we present subjects with a two-ball risky urn, a two-ball ambiguous urn, and three compound urns each having reduced probability of winning of $1/2$. Four of the five bets are analogous to the four bets used by Halevy (a minor difference is that our final-stage urns contain 2 balls rather than 10). The fifth (hypergeometric compound risk) was included because it is a plausible model of the actual process used to generate and draw from the ambiguous urn.

In Table 5, we construct a contingency table for our data from Study 2 alongside the table reported by Halevy (2007). Subjects are classified as reducing compound risk if the reported certainty equivalents for the simple risk and each of the compound risks are equal. Subjects are classified as ambiguity neutral if the reported certainty equivalents for the simple risk and for the ambiguous bet are equal. In the table, “Count” indicates the number of subjects in each category, whereas “Expected” indicates the number of subjects

in each category expected if the joint distribution over cells is equal to the product of the observed marginal distributions (e.g., $4.5 = (28/142) \times (23/142) \times 142$).

Both sets of data show a relationship between reduction/nonreduction and ambiguity neutrality/nonneutrality. Halevy’s data suggest something close to identification in the direction of reduction implying neutrality: conditional on reducing compound risk, 1 of 23 subjects is nonneutral toward ambiguity, whereas conditional on ambiguity neutrality, 6 of 28 subjects fail to reduce the compound risks. However, our data are less extreme: conditional on reducing compound risk, 4 of 17 subjects are nonneutral toward ambiguity (all 4 are ambiguity averse), whereas conditional on ambiguity neutrality, 17 of 30 subjects fail to reduce the compound risk. The difference between 1 of 23 and 4 of 17 is significant (two-sample test of proportions, $p < 0.07$), as is the difference between 6 of 28 and 17 of 30 ($p < 0.01$). Similar conclusions hold when reduction in Study 2 is defined without the hypergeometric compound risk (which was not present in Halevy (2007)).

4.2. Variation with Subject’s Background

In Study 2, as in Study 1, some of our subjects were advanced engineering students from an elite program while others were from a broader cross section of nonengineering fields and institutions. In part, this was motivated by questions from reviewers about data we collected from an earlier sample (Study 3, described in Web Appendix W.3) composed almost entirely of advanced engineering students. That data resulted in contingency tables even less similar to Halevy’s (Abdellaoui et al. 2011a), and one hypothesis put forward was that the quantitative sophistication of the subjects was instrumental in generating that difference. To this end, Table 6 splits our data from Table 5 into separate contingency tables for the advanced engineers and nonengineers.

Interestingly, almost the same proportion of engineers and nonengineers reduce compound risks (14.7%

Table 5 Contingency Table Relating Ambiguity and Compound Risk Attitudes

Ambiguity attitudes	Compound risk attitudes					
	Halevy (2007)			Study 2		
	Reduce	Do not reduce	Total	Reduce	Do not reduce	Total
Neutral						
Count	22	6	28	13	17	30
Expected	4.5	23.5		4.4	25.6	
Nonneutral						
Count	1	113	114	4	81	85
Expected	18.5	95.5		12.6	72.4	
Total	23	119	142	17	98	115

Note. In both Study 2 and Halevy (2007), p -value = 0.000 by Fisher’s exact (two-tailed) test.

Table 6 Contingency Table Relating Ambiguity and Compound Risk Attitudes by Background

Ambiguity attitudes	Compound risk attitudes					
	Nonengineers			Engineers		
	Reduce	Do not reduce	Total	Reduce	Do not reduce	Total
Neutral						
Count	6	9	15	7	8	15
Expected	2.2	12.8		2.2	12.8	
Nonneutral						
Count	0	25	25	4	56	60
Expected	3.8	21.2		8.8	51.2	
Total	6	34	40	11	64	75

Note. For both engineers and nonengineers, p -value = 0.001 by Fisher’s exact (two-tailed) test.

versus 15%, $p = 0.96$). Fewer engineers are ambiguity neutral (20% versus 37.5%, $p < 0.05$). Thus, considering our advanced engineers as the more “sophisticated” subjects, we find that more sophistication does not result in more ambiguity neutrality nor in more reduction of compound risks.

As is apparent from the counts, more engineers who reduce compound lotteries are ambiguity nonneutral compared with the nonengineers, to Halevy’s data, and to the “coarsened” Halevy data (two-sample test of proportions; $p < 0.1$, $p < 0.05$, and $p < 0.05$, respectively). Such a contrast between engineers and nonengineers is not evident with regard to the proportion of those who are ambiguity neutral failing to reduce compound risk (two-sample test of proportions, $p = 0.71$). Thus, it appears that the tendency for those who reduce compound risks to be ambiguity neutral is much weaker among the quantitatively sophisticated population of engineers than in a broader cross section of students, such as our nonengineers or Halevy’s subjects.⁶ On the other hand, the tendency in the other direction (ambiguity neutrality implying reduction of compound risk) is much weaker in our data than in Halevy (2007) and almost identical across engineers/nonengineers.

For further analysis and robustness checks related to Study 2, see Web Appendix §W.2.

5. Conclusion

We provide new evidence on behavior toward compound risk in relation to simple risk and ambiguity. We confirm that reduction of compound risk generally fails; i.e., nonneutrality toward compound risk is typical. Specifically, behavior toward compound risk relative to simple risk displays systematic variation as the (reduced) probability of winning a binary bet increases. We find that the predominant pattern is increasing aversion to compound risk. This behavior is consistent with more likelihood insensitivity for compound risk than for simple risk. This pattern is strongest when the compound risk is evaluated before seeing the comparable simple risk. These results suggest that a descriptively valid theory of decision making under uncertainty should account for compound risk attitude and simple risk attitude as distinct aspects of preference and allow each to display aversion increasing in the probabilities of winning as, for example, through source-dependent likelihood sensitivity.

In regard to ambiguity, we investigated the relationship between reduction of compound risk and

ambiguity neutrality, finding, for a quantitatively sophisticated population (advanced engineers), a weaker relation between the two than prior literature did. Even for nonengineer subjects, conditional on ambiguity neutrality, the tendency to reduce compound risk was weaker than previously found. These findings caution against modeling ambiguity nonneutrality through nonreduction of objective compound lotteries (Segal 1987, Halevy and Ozdenoren 2008, Seo 2009) when applying such models to sophisticated subjects. We hope that our findings also contribute productively to further investigations of individual sensitivity to forms of uncertainty (e.g., simple risk, compound risk, and ambiguity) and the role that educational background, especially quantitative sophistication, may play.

Supplemental Material

Supplemental material to this paper is available at <http://dx.doi.org/10.1287/mnsc.2014.1953>; the Web appendix is available at <http://ssrn.com/abstract=2495076>.

Acknowledgments

The authors thank Aurélien Baillon, Michèle Cohen, Emre Ozdenoren, Marciano Siniscalchi, Jean-Marc Tallon, and the audiences at numerous seminars and conferences for helpful discussions. They especially thank Olivier Armantier, Yoram Halevy, and Peter Wakker for detailed written comments and advice on previous drafts. The authors also thank Yoram Halevy for emphasizing to them the parallel between a hypergeometric compound risk and the process of creating and drawing from an ambiguous urn. L. Placido acknowledges the Fulbright Commission for supporting a visit to Northwestern University in 2011 and thanks the Managerial Economics and Decision Sciences Department for their hospitality. P. Klibanoff acknowledges the hospitality and partial support of the Paris School of Economics, University Paris 1, and the University of Cergy-Pontoise during a visit to Paris in 2013. He acknowledges the Zell Center for Risk Research at the Kellogg School of Management, Northwestern University, for providing research funds used to pay for Studies 1 and 2.

Appendix A

A.1. Instructions for Study 1

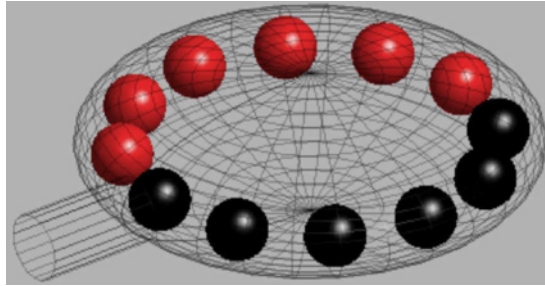
Aim of the Study. You are participating in a study on decision making under uncertainty.

Procedure. There are nine different uncertain scenarios; for each scenario you are asked to make choices between two options:

- Option 1 is “Having the amount of money you receive determined by the outcome of the uncertain scenario.”
- Option 2 is “Receiving a given amount of money for sure.”

Each uncertain scenario consists of an urn (or a combination of urns) from which one or two balls is (are) drawn, resulting in either €0 or €50 depending on whether the color of the ball is black or red (or resulting in an additional draw from another urn and €0 or €50 depending on whether the color of the ball on this second draw is red or black).

⁶ Halevy (2012) finds that 20%–30% of subjects maintain ambiguity nonneutrality after being taught to reduce compound risk. Our findings support the hypothesis that subjects with greater quantitative sophistication will be overrepresented in this 20%–30% segment, because the advanced engineers are the subjects for whom the link between compound risk reduction and ambiguity neutrality is the weakest in our data.



(Color online) Example of an urn

The sure amounts of money in Option 2 also range between €0 and €50.

For each of the nine uncertain scenarios and each whole euro amount between 0 and 50, you must choose between Option 1 and Option 2. To help you decide between Options 1 and 2, we proceed iteratively, in three steps, with the uncertain scenario remaining the same during these steps.

Step 1. For each amount €0, €10, €20, €30, €40, and €50 (Option 2), you have to ask yourself whether you prefer to be paid according to the uncertain scenario or whether you prefer to be given this amount of money for sure. For instance, you can begin to ask yourself the following questions:

- For €0, do I prefer to be paid according to the uncertain scenario resulting in €0 or €50, or to be given €0?
- For €10, do I prefer to be paid according to the uncertain scenario resulting in €0 or €50, or to be given €10?
- ...
- For €50, do I prefer to be paid according to the uncertain scenario resulting in €0 or €50, or to be given €50?

Step 2. You are asked to refine your choice to the nearest euro. Imagine you chose Option 1 for €10 and Option 2 for €20. You will be asked to assess your preference between Option 1 and Option 2 for the amounts ranging between €10 and €20 (11, 12, ..., 19). If you choose Option 1 for a euro amount from 0 to x , and Option 2 for an amount from $x + 1$ to 50, we will say that x is your switching point.

Step 3. This step summarizes all of your choices between one uncertain option and each of 51 sure amounts, illustrating your switching point. If you wish to go back and revise some of these choices at this stage, you may. Otherwise, simply confirm that these are the choices you would like to submit.

Payment. After the experiment, you will be paid a show-up fee of €5 plus an amount of money that depends on the choices you have made in the experiment. For each participant, first, one of the nine uncertain scenarios from the experiment is randomly selected with an equal chance of each one, and the choices you made for this scenario are displayed. Next, a number between 0 and 50 is randomly selected with an equal chance for each one. This number is compared with your switching point x .

- If the number selected is between 0 and x , you will draw a ball from the corresponding physical urn(s) and learn whether the result is €0 or €50, depending on the color of the ball drawn.
- If the number selected is between $x + 1$ and 50, as you chose the sure monetary amount, the result is the number selected in euros. If the result is z euros, you will be paid z (in addition to the show-up fee). Only you and the experimenter will know your final payment.

A.2. Instructions for Study 2

Aim of the Study. You are participating in a study on decision making under uncertainty.

Procedure. There are five different uncertain scenarios; for each scenario you are asked to make choices between two options:

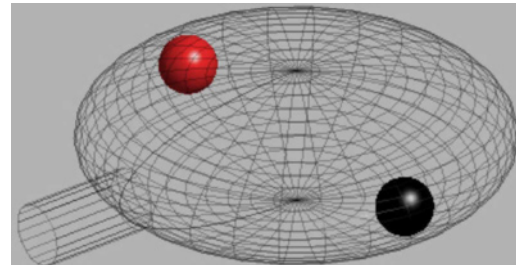
- Option 1 is "Having the amount of money you receive determined by the outcome of the uncertain scenario."
- Option 2 is "Receiving a given amount of money for sure."

Each uncertain scenario consists of an urn (or a combination of urns) from which one or two balls is (are) drawn, resulting in either €0 or €50, depending on whether the color of the ball is black or red (or resulting in an additional draw from another urn and €0 or €50, depending on whether the color of the ball on this second draw is red or black).

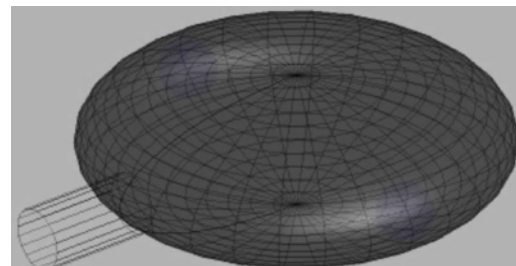
For four scenarios, the urn(s) is (are) transparent and you can observe how many balls there are of each color in the urn(s). For one scenario, the urn is opaque and you cannot observe how many balls there are of each color in the urn. The balls in the opaque urn are determined by a draw of 2 balls from a larger urn containing 12 red and black balls in unknown proportion. The experimenter will blindly perform this draw before the experiment begins so that neither you nor the experimenter knows how many of the 2 balls in the opaque urn are red and how many are black.

[At this point, the experimenter asks one subject to come up and, in view of everyone, carry out the blind draws to determine the two balls in the opaque urn.]

After the composition of the opaque urn is determined, you will be allowed to choose whether you want black or red to be the color associated with €50 when drawing from that urn.



(Color online) Transparent urn: it contains 2 balls and you observe its composition



(Color online) Opaque urn: it contains 2 balls but you cannot observe its composition

The sure amounts of money in Option 2 also range between €0 and €50.

For each of the five uncertain scenarios and each whole euro amount between 0 and 50, you must choose between Option 1 or Option 2. To help you decide between Options 1

and 2, we proceed iteratively, in three steps, the uncertain scenario remaining the same during these steps.

Step 1. For each amount €0, €10, €20, €30, €40, and €50 (Option 2), you have to ask yourself whether you prefer to be paid according to the uncertain scenario or whether you prefer to be given this amount of money for sure. For instance, you can begin to ask yourself the following questions:

- For €0, do I prefer to be paid according to the uncertain scenario resulting in €0 or €50, or to be given €0?
- For €10, do I prefer to be paid according to the uncertain scenario resulting in €0 or €50, or to be given €10?
- ...
- For €50, do I prefer to be paid according to the uncertain scenario resulting in €0 or €50, or to be given €50?

Step 2. You are asked to refine your choice to the nearest euro. Imagine you chose Option 1 for €10 and Option 2 for €20. You will be asked to assess your preference between Option 1 and Option 2 for the amounts ranging between €10 and €20 (11, 12, ..., 19). If you choose Option 1 for a euro amount from 0 to x euros, and Option 2 for an amount from $x + 1$ to 50, we will say that x is your switching point.

Step 3. This step summarizes all of your choices between one uncertain option and each of 51 sure amounts, illustrating your switching point. If you wish to go back and revise some of these choices at this stage, you may. Otherwise, simply confirm that these are the choices you would like to submit.

Appendix B

Payment. After the experiment, you will be paid a show-up fee of €5 plus an amount of money that depends on the choices you have made in the experiment. For each participant, first, one of the five uncertain scenarios from the experiment is randomly selected with an equal chance of each one, and the choices you made for this scenario are displayed. Next, a number between 0 and 50 is randomly selected with an equal chance for each one. This number is compared with your switching point x .

- If the number selected is between 0 and x , you will draw a ball from the corresponding physical urn(s) and learn whether the result is €0 or €50, depending on the color of the ball drawn.
- If the number selected is between $x + 1$ and 50, as you chose the sure monetary amount, the result is the number selected in euros. If the result is z euros, you will be paid z divided by 5 (in addition to the show-up fee). Only you and the experimenter will know your final payment.



(Note: the words in square brackets above were not included in the instructions but are intended to document when an action is performed during the experiment.)

Table B.1. (Color online) Bets Faced by a Subject Participating in Study 1 (Shown Here with Red as the Winning Color) Organized by Probability of Winning

Panel A: Simple risk	
Probability of winning 1/12	1/2
11/12	

Table B.1. (Continued)

Panel B: CRD	
Probability of winning 1/12	
1/2	
11/12	

Table B.1. (Continued)

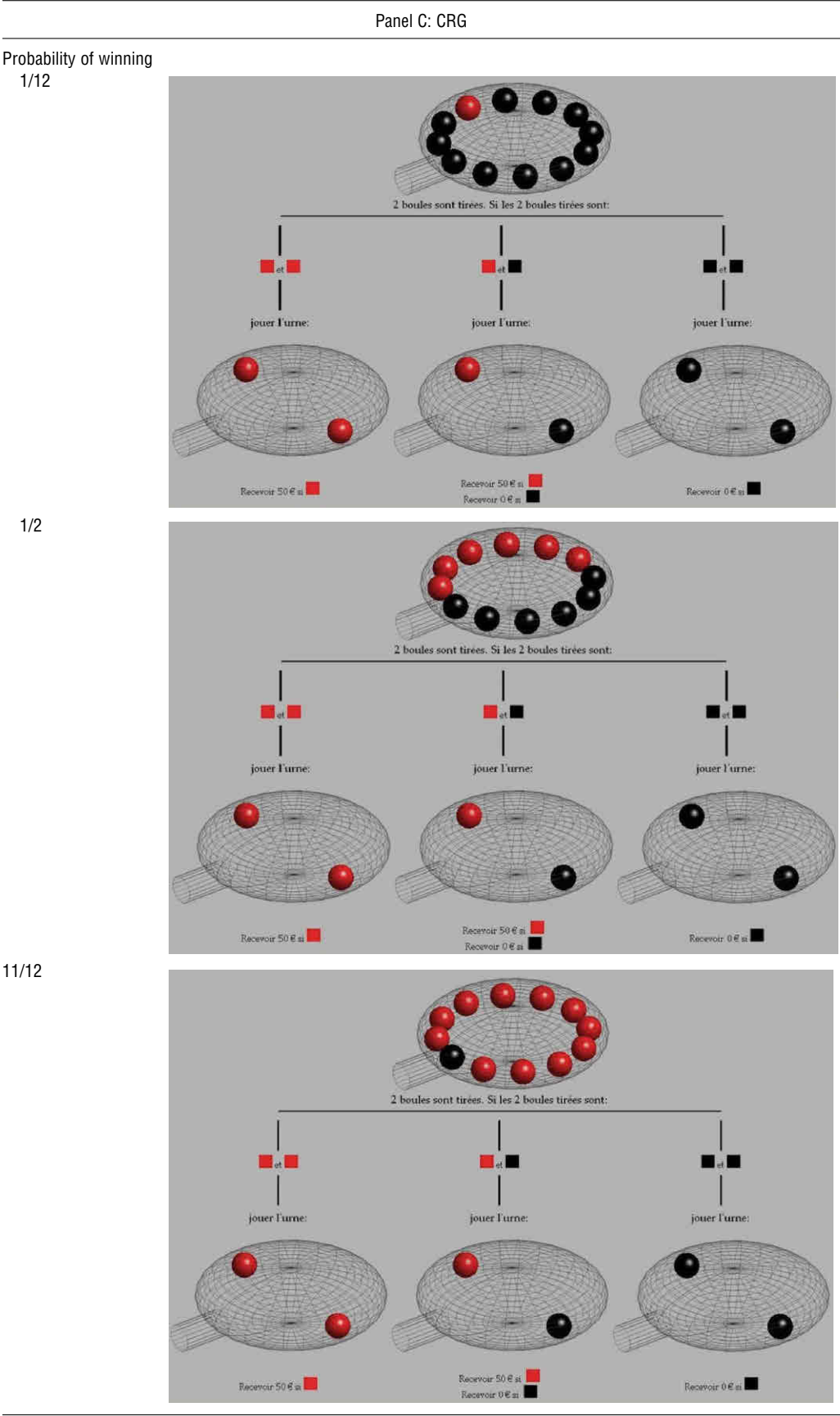


Table B.2. (Color online) Bets Faced by a Subject Participating in Study 2 (Shown Here with Black as the Winning Color) Organized by Type of Risk

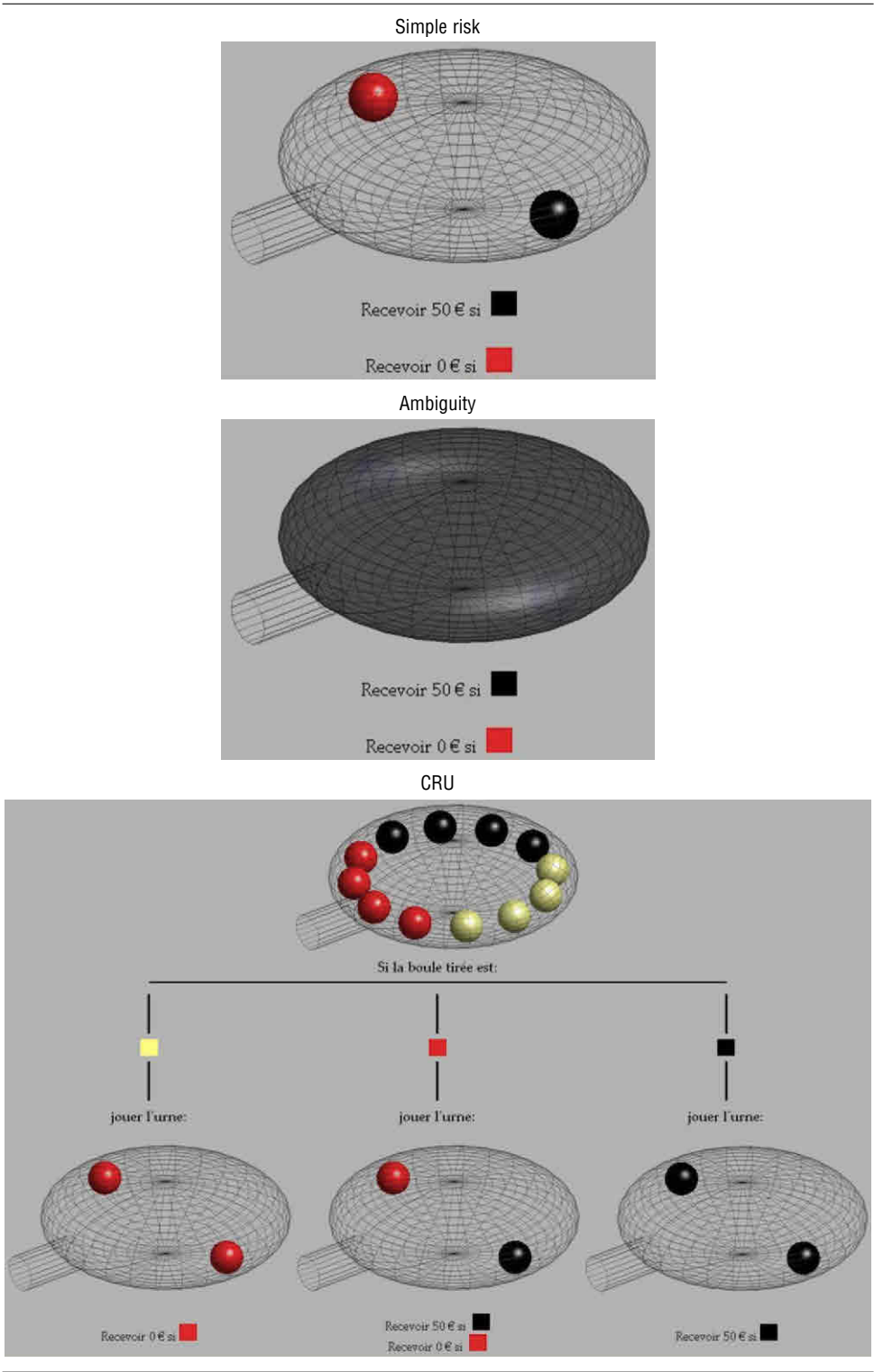
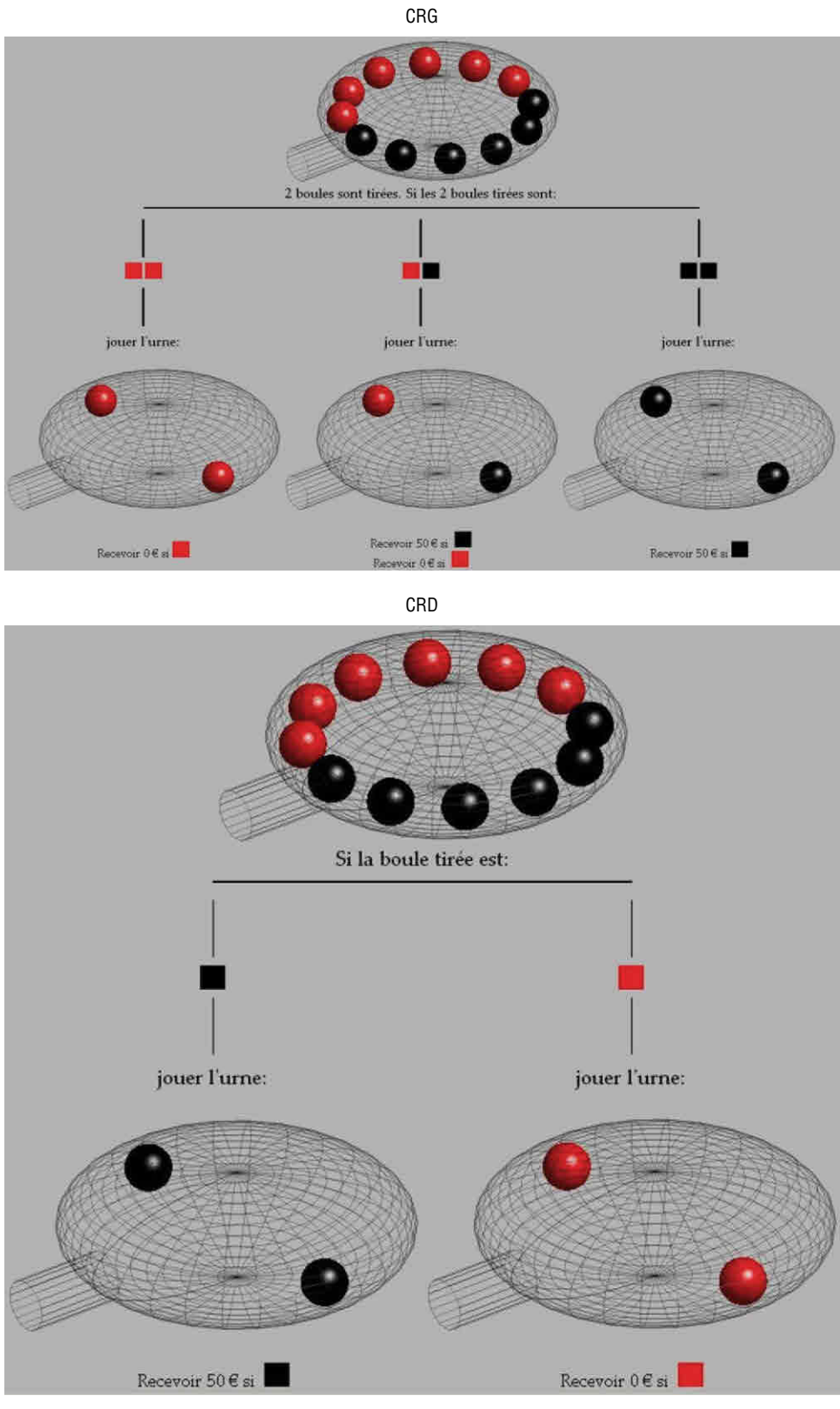


Table B.2. (Continued)



Appendix C

C.1. Description of the Iterative Choice List Method and Calculation of Certainty Equivalents

The iterative choice list procedure for eliciting certainty equivalents is borrowed from Abdellaoui et al. (2011b). For each bet, three screens were presented sequentially to the subject. On the first screen, they chose, for each of six amounts evenly spaced in increments of €10 between €0 and €50, between the bet and the sure amount (see Figure C.1). On the second screen, for the same bet, they chose, for each of 11 amounts in increments of €1 between the highest amount for which they chose the bet on the first screen and the lowest amount for which they chose the sure amount on the first screen, between the bet and the sure amount (see Figure C.2). On the third screen, the choices between the bet and sure amounts in increments of €1 between €0 and €50 implied by the choices from the first two screens and monotonicity were displayed for the subject (see Figure C.3). At that point, the subject was given the opportunity to modify any of these 51 choices on the third screen if desired, and then the final response for that bet was recorded. While the first two screens forced the responses to have at most one switching point from Option 1 to 2, the third screen allowed subjects to depart from this if desired. Note that the first two screens

are simply a method to allow the subject to initially populate the choices on the third screen through refinement—only the 51 choices on the third screen are recorded and eligible to be selected and paid through the random incentive system. For those subjects who had a single switching point, we calculate the certainty equivalent for the bet by taking the midpoint between the highest sure payment rejected and the lowest sure payment accepted in the third step. In the example below, this recorded certainty equivalent is €8.5.

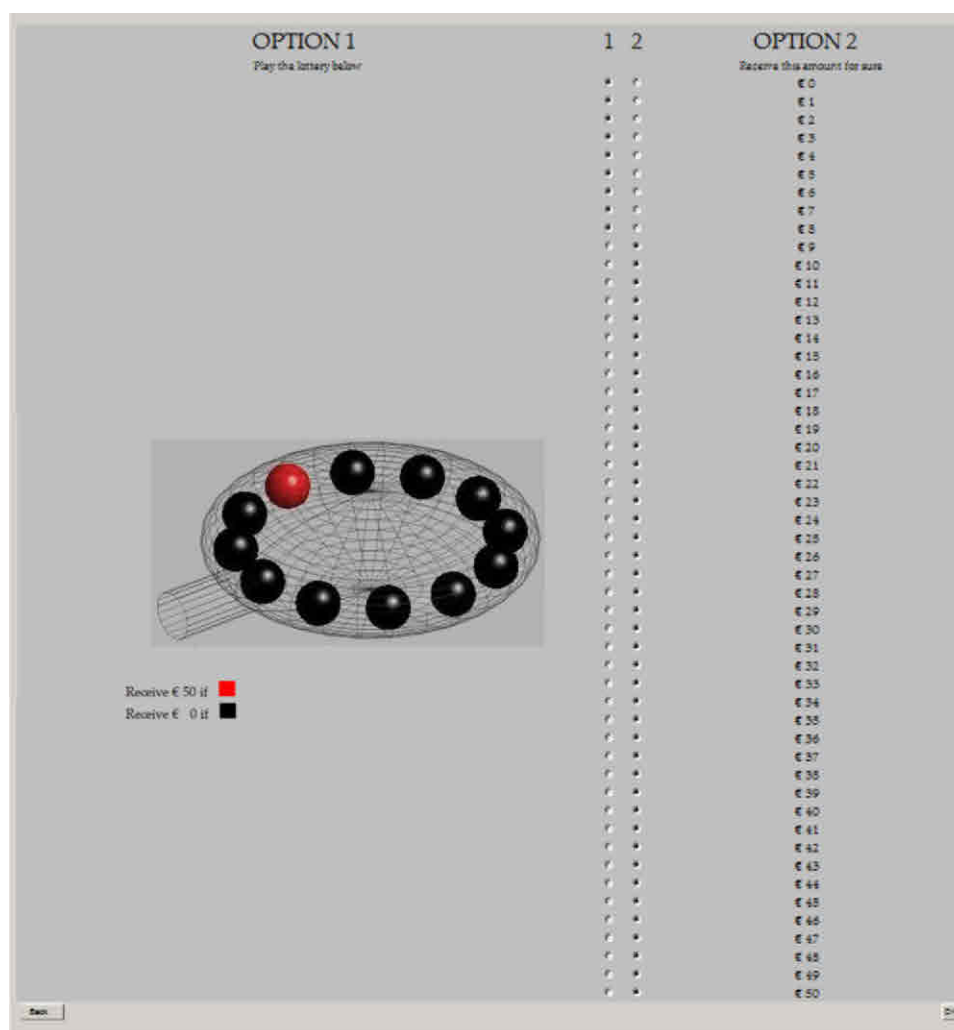
C.2. Description of the Training Phase

A training phase is launched by the experimenter after the reading of the instructions. At the beginning of the training phase, each subject is presented with a screen similar to Figure C.1, except that Option 1 is replaced by the first bet that the subject will see as determined by his or her study, order treatment, and any prior color choice, and the word “Training” appears prominently at the top of the screen. Since each subject may be presented with a different bet, the experimenter, so as to be able to guide all subjects, projects on a screen at the front of the room a version of Figure C.1 that replaces the bet and colors with rectangles containing the words “urn(s)” and “color 1” and “color 2” written on them. Then each screen of the iterative choice list (Figures C.1–C.3) is explained by the experimenter; subjects cannot move

Figure C.1 (Color online) Simple Risk for Probability 1/12 Treatment (First List)

Figure C.2 (Color online) Simple Risk for Probability 1/12 Treatment (Second List: Refinement of the First)

Figure C.3 (Color online) Simple Risk for Probability 1/12 Treatment (Third Step: Confirmation)



to the next training screen until the experimenter does so. Once subjects complete the three screens, they click on a button, and one choice from those on their third screen is randomly selected by the software and highlighted in red. The experimenter then explains, based on the choice selected, whether the bet or the sure amount would determine payment if this were the real experiment and not the training phase. Note that no bets have their outcomes revealed during the training phase. After again giving subjects the opportunity to ask questions, the experimenter ends the training phase and instructs the software to start the experiment proper on each subject's computer.

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