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Managing Catastrophic Climate Risks Under Model Uncertainty Aversion

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Abstract. We propose a robust risk management approach to deal with the problem of catastrophic climate change that incorporates both risk and model uncertainty. Using an analytical model of abatement, we show how aversion to model uncertainty influences the optimal level of mitigation. We disentangle the role of preferences from the structure of model uncertainty, which we define by means of a simple measure of disagreement across models. With data from an expert elicitation about climate change catastrophes, we quantify the relative importance of these two effects and calibrate a numerical integrated assessment model of climate change. The results indicate that the structure of model uncertainty, and specifically how model disagreement varies in abatement, is the key driver of optimal abatement and that model uncertainty warrants a higher level of climate change mitigation.

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1. Introduction

This paper applies recently developed tools from decision theory and expert assessment data to discuss abatement strategies in the case of climate change under deep uncertainty. We distinguish the notion of risk, which characterizes situations in which probabilities of a random event are perfectly known, from the broader notion of (Knightian) uncertainty (also called ambiguity), which characterizes situations in which some events do not have an obvious, unanimously agreed upon probability assignment (Ghirardato et al. 2004).[1] More specifically, we focus on the notion of model uncertainty that corresponds to situations in which different data-generating mechanisms or models are considered as possible or plausible by the decision maker (DM). Throughout the paper, we consider the notion of model taken in its statistical sense, meaning that it is defined as a probability distribution over a sequence (or, here, over states of the world). Different models may exist, for example, because too little information is available, because different predictions exist (depending on different data sets, different techniques, etc.), or because the decision is based on the advice of experts who provide different assessments of probabilities for a given event (as will be the case in our application). We present the risk management problem as an intertemporal problem of optimal abatement under the possibility of a catastrophic climate event. As is the case with a vast range of economic problems, the climate change case illustrates particularly well a situation in which the probabilistic model is neither explicitly given nor can it be perfectly approximated or inferred with the available data and current scientific methods. Choosing among different climate policies in this situation is therefore essentially an exercise in risk management that has to be performed in a situation of deep uncertainty (Kunreuther et al. 2013). Therefore, it requires a robust decision-making approach that is less sensitive to initial assumptions, is valid for a wide range of futures, and keeps options open (Lempert and Collins 2007), rather than a formal approach that maximizes the expected utility mechanically.

More than ever before, it is now widely believed that our entire planet is undergoing climate change, and that this change is largely due to human activities (IPCC 2013). What is less clear is how this process, which is taking place over a very long time horizon, will unfold. Based on the available observations and on the current state of knowledge, scientific experts have constructed models in order to simulate and quantify the impact of human activity on the climate system and vice versa. However, a large degree of uncertainty surrounds these models. These uncertainties arise from both the underlying climate science (and the extreme complexity of the climatic system) and our inability to perfectly capture the way our socioeconomic system would respond and adapt to climate change (Heal and Millner 2014). This is particularly the case when we consider situations with potential catastrophic consequences, such as the collapse of the Atlantic thermohaline circulation or the melting of the Antarctic ice sheet. Such catastrophic events have not been encountered in recent history,[2] and their likelihood of occurrence is therefore extremely difficult to assess. From a decision maker's perspective, becoming aware of such occurrences leads to the expansion of the set of admissible states of the world. Therefore, the state space and the associated probabilities need to be adjusted for making decisions when considering such "unforeseen events" (Karni and Viero 2013). In recent years, a few studies have been undertaken to estimate these probabilities, consisting generally of experts' assessments.[3] Since climate science is currently unable to determine which of these estimates is the best or what the best combination of them is, and since these uncertainties are expected to persist even when better scientific models become available,[4] a decision maker confronted with this situation would find himself in a situation of model uncertainty rather than risk.

In view of this disagreement among experts or models, how should a rational policy decision maker proceed? If one follows the traditional Bayesian/subjective expected utility approach, one will simply aggregate the models by averaging them into a single representative model and then use the (subjective) expected utility framework (Newbold and Daigneault 2009). The problem with this approach is that the decision maker considers the resulting aggregated model in exactly the same way as one would consider an equivalent objective model representing a specific risk, and model uncertainty has therefore no impact on the decision-making process. This approach has, however, been seriously challenged in situations of deep uncertainty. The most famous example is that of Ellsberg (1961), who showed through different experiments that the choices of individuals cannot be rationalized under the traditional Bayesian expected utility paradigm, and that individuals usually manifest aversion toward situations in which probabilities are not perfectly known. In applied economic models, some recent contributions have started to apply nonexpected utility frameworks (i.e., alternative models of risk preferences and beliefs, most of which replace the expected utility formulation with alternative criteria) to the problem of climate change. In particular, these include applications of the smooth ambiguity model by Lange and Treich (2008), who provide comparative statics results of the role played by ambiguity in a simple two-period parametric model, and Millner et al. (2013) and Lemoine and Traeger (2014), both of whom propose numerical models under ambiguity aversion. Other contributions include applications of the macroeconomic technique of robust control (Hansen and Sargent 2008) by Athanassoglou and Xepapadeas (2012), who consider an analytical pollution control problem, and by Rudik (2016), who applies the concept in an integrated assessment model (IAM) including learning.[5] Finally, a different approach is taken by Drouet et al. (2015), who use the results of the most recent assessment of the Intergovernmental Panel on Climate Change (IPCC) to numerically disentangle model uncertainty and risks about mitigation costs, climate dynamics, and (continuous) climate damages. For what concerns the inclusion of catastrophic damages into integrated assessment models, our paper also extends the work of, among others, Gjerde et al. (1999), who show that taking into account a risk of catastrophe provides a rationale for current emission control, and Keller et al. (2004) and Lontzek et al. (2012), who model a collapse in ocean circulation as a permanent shock to the production function, and show that the optimal policy should be associated with immediate limitations on emissions.

In this paper, we take a step further in the direction of understanding the theoretical mechanisms underlying the results obtained in this literature, by applying the most recent robust tools developed in decision theory (Cerreia-Vioglio et al. 2013b, Marinacci 2015). We consider an alternative to expected utility models that allows us to incorporate both risk and model uncertainty. More specifically, we study the impact of model uncertainty aversion on optimal abatement policy. Our contribution is severalfold. We develop a two-period model of emission abatement with an endogenous probability of catastrophic climate change, which allows us to disentangle the contribution of preferences and the structure of model uncertainty on the level of first-period abatement. We show that a simple measure of the disagreement across models or experts is a sufficient statistic for determining the structure of model uncertainty that matters for abatement. We apply our theoretical results using an actual assessment of a major catastrophic climatic event with data from a recent expert's elicitation. Finally, we extend a widely used integrated assessment model (IAM) of climate change (dynamic integrated climate–economy (DICE) model; Nordhaus 1993) to include a tipping element in the climate response, in a framework where well-defined probabilities are unknown. This allows us to quantify the impact of deep uncertainty on the optimal level of emission abatement, addressing one of the criticisms of IAMs, which have been recently highlighted by Stern (2013), Pindyck (2013a, b, 2015), and Kunreuther et al. (2013). Our broader finding is that in most situations, a robust climate strategy implies stronger mitigation policies. In that sense, we show that deep uncertainty cannot be taken as an excuse for inaction, making a clear link to the precautionary principle.[6] We show that both preferences over model uncertainty, measured by ambiguity prudence, and the structure of the model uncertainty, measured by the decrease of the degree of model disagreement in abatement, determine the optimal abatement level. The data we use from expert elicitations indicate that it is the latter effect that is by far the most important, given that the disagreement across models or experts increases in global mean temperature.[7] Finally, the reformulated integrated assessment model allows the generation of quantitative estimates of the impact of risk and model uncertainty aversion on optimal emission reductions. Compared to the commonly used expected utility framework, model uncertainty raises abatement significantly. Our broader policy result corroborates the findings of the recent strand of research that has emphasized the importance of deep uncertainty and tipping points in quantitative climate policy making (Lemoine and Traeger 2014; Lontzek et al. 2012; Gjerde et al. 1999; Weitzman 2012, 2009; van der Ploeg and de Zeeuw 2014; Lempert and Collins 2007; Drouet et al. 2015).

Our results can be read in both positive and normative terms. Although we recognize the existence of a debate about the normative status of nonexpected utility models, and the predominance of the expected utility theory paradigm for normative purposes in decision making, we here follow the claim that there is nothing irrational about violating Savage's (1954) axioms in situations of deep uncertainty (Gilboa et al. 2008, 2009, 2012; Gilboa and Marinacci 2013).[8] The non-Bayesian framework we adopt is thus compatible with a normative assessment of optimal policies.

2. A Simple Model of Optimal Abatement Under Model Uncertainty

To investigate the effects of different types of uncertainty on emission abatement decisions, we construct a simple economic model of optimal abatement with two periods: today and the future. During the first period, the decision maker chooses a level of abatement a that is undertaken at cost $c(a)$. This abatement reduces available disposable income in such a way that consumption in period 1 is given by $C_1 = w_1 - c(a)$, where w_1 is the level of income of the first period. In the future, there are two possible categories of states of the world. One is catastrophic: the environment is severely affected so that the consumption in the second period C_2 is given by $w_2 - L_s$, where w_2 is the deterministic exogenous income, and L_s is the damage (loss) that occurs with probability π_s, conditionally on the fact that a catastrophe occurred (i.e., $\forall\, s \in S$, where S represents those catastrophic or unfavorable states). The other is one in which no catastrophe occurs, so that consumption is w_2 (i.e., favorable state). The probability $p(a)$ that such a catastrophic event will occur is endogenous and depends on the level of abatement chosen in the first period.[9] Consumption in the second period can therefore take $|S| + 1$ different values, and the abatement effort in the first period is the only choice variable in this model. It is conceptualized as an investment to reduce the risk of a catastrophic event that is difficult to compensate by ordinary savings (rather than an instrument used to optimally smooth consumption over time). As in most environmental economic models under uncertainty, intertemporal utility is assumed to be time separable, and future utility is discounted by a factor $\beta \in (0, 1]$.

Model uncertainty is introduced by relaxing the assumption that all the elements of the maximization program are objectively known or commonly agreed upon, so that the probability model over future consumption is no longer unique. We assume that a true climatic process is in place and generates observations, but that this true process and the probability model representing it are unknown to the decision maker. The observations generated by this model are, however, available and used by experts (scientists, climatologists, physicists, etc.) to construct predictive models that belong to a class M. The true process is assumed to belong to this class of models, and elements of M are interpreted by the DM as possible alternative models that could be selected by nature to generate observations. These possible models have a "Waldean" interpretation in the sense that the class M is regarded as a datum of the decision problem (Wald 1950). This implies that the models have to be consistent with objectively available information (note, however, that the information must be incomplete; otherwise M would be a singleton). This set therefore contains all the information the DM considers as "credible" in the sense that "states that are not given any weights by any of the relevant probability distributions are simply irrelevant" (Gajdos et al. 2008, p. 34). We assume that there are n different models (or experts). These different models P_θ are indexed by a parameter $\theta \in \{1, 2, \ldots, n\}$, so that $M = \{P_\theta\}_{\theta \in \{1, \ldots, n\}}$. Each P_θ describes a possible distribution (i.e., a possible risk) of second-period consumption: $\tilde{C}_2(a, \theta)$ (in what follows, only the probability of catastrophe will depend on θ). We also assume that the decision maker has a prior probability measure over the set of possible models; that is, $\tilde{\theta}$ has a probability distribution $q = (q_1, q_2, \ldots, q_n)$, so that $\tilde{\theta}$ takes value θ with probability q_θ. This second-order distribution $\tilde{\theta}$ reflects model uncertainty in the sense that the DM does not know which of the models P_θ is the true or the most accurate one and associates a subjective weight q_θ with each of them.[10]

In what follows, we consider different criteria for decision making under climate model uncertainty, and compare them in terms of optimal abatement. In Appendix B, we discuss the case of uncertainty about the economic impacts of a climate catastrophe (i.e., the size of L_s), showing that our results carry over (and are even strengthened) in this case. Although different existing models of ambiguity aversion could have been adapted to the presence of objective information (Marinacci 2015), we focus on the smooth model of model uncertainty aversion for its ability to characterize the notion of ambiguity neutrality, its mathematical convenience, and because it encompasses many of the alternative criteria as special or limit cases. Nonetheless, we explore alternative criteria, such as maxmin, in the online supplemental material, showing that they entail qualitatively similar results.

The traditional approach for addressing a problem in which the true distribution is unknown is to consider that agents use their probabilistic beliefs over the source of uncertainty in an expected utility maximization framework. Cerreia-Vioglio et al. (2013b) are the first to provide a decision theoretic derivation of this type of deep uncertainty presented in two layers. In particular, they enrich the standard Savage framework (Savage 1954) in the presence of objective information, and show that preferences satisfying Savage's axioms may be represented, in the context of our abatement model, by

$$W_{\mathrm{SEU}} = v(w_1 - c(a)) + \beta \mathrm{E}_\theta \mathrm{E} v(\tilde{C}_2(a, \tilde{\theta})). \tag{1}$$

In this expression, v is the per-period von Neumann–Morgenstern utility function reflecting both the decision maker's attitude toward risk and desire to smooth consumption over time;[11] E_θ is the expectation operator taken over prior distribution $\tilde{\theta}$, that is, $\mathrm{E}_\theta X(\tilde{\theta}) = \sum_{i=1}^n q_i X(i)$; and E is the expectation operator over second-period consumption in the different states of the world, conditional on model θ. This representation is called classical subjective expected utility (SEU) because it incorporates key objective pieces of information in Savage's subjective framework. In the context of this paper, the second-period expected utility for a given model θ may be written as

$$\mathrm{E}v(\tilde{C}_2(a,\theta)) \equiv p(a,\theta) \sum_{s\in S} \pi_s v(w_2 - L_s) + (1 - p(a,\theta)) v(w_2), \tag{2}$$

where we denote by $p(a,\theta)$ the probability of catastrophe as a function of abatement for model θ. For each prior distribution q, there exists an equivalent predictive distribution $\tilde{C}_2(a,\bar{\theta})$ such that $\mathrm{E}_\theta \mathrm{E}v(\tilde{C}_2(a,\tilde{\theta})) = \mathrm{E}v(\tilde{C}_2(a,\bar{\theta}))$, and it is therefore clear that the reduced form of representation (1) is nothing but the original "Savagian" subjective expected utility. The decision problem under uncertainty is then reduced to a simple decision problem under risk where the beliefs are subjective. On the other hand, when M is a singleton (i.e., when there is only one model everyone agrees on), there is no longer model uncertainty, so that the risky second-period consumption is $\tilde{C}_2(a)$ and we are back to the classical von Neumann–Morgenstern expected utility model. These different representations of the problem are observationally equivalent to someone who is not aware of the presence of objective information deriving from different experts' models.

In what follows, we consider the subjective expected utility representation as a benchmark. The economic problem of finding the level of abatement a^*_{SEU} that maximizes program (1) is easy to solve.[12] This level is implicitly given by equalizing the marginal cost and benefit of abatement:

$$v'(w_1 - c(a^*_{\mathrm{SEU}}))c'(a^*_{\mathrm{SEU}}) = -\beta \frac{\partial \mathrm{E}_\theta \mathrm{E}v(\tilde{C}_2(a^*_{\mathrm{SEU}}, \tilde{\theta}))}{\partial a}. \tag{3}$$

Although the classical subjective expected utility framework has the advantage of being easily tractable, it is unable to take into account different attitudes toward different types of uncertainty that surround the economics of climate change. We now introduce different attitudes toward different types of uncertainty. To investigate the relationship between risk aversion and model uncertainty aversion, we consider a criterion in which the function representing the agent's preferences toward model uncertainty is smooth and hence everywhere differentiable. By letting v represent attitude toward risk and h represent attitude toward model uncertainty, we can write the smooth criterion (Marinacci 2015) to be maximized as

$$W_{\mathrm{Smooth}} = v(w_1 - c(a)) + \beta (v \circ h^{-1})(\mathrm{E}_\theta (h \circ v^{-1})(\mathrm{E}v(\tilde{C}_2(a,\tilde{\theta})))). \tag{4}$$

This expression can be written equivalently as

$$W_{\mathrm{Smooth}} = v(w_1 - c(a)) + \beta v(CE(ce(a,\tilde{\theta}))), \tag{5}$$

where ce and CE both represent a certainty equivalent:

$$ce(a,\theta) \equiv v^{-1}(\mathrm{E}v(\tilde{C}_2(a,\theta))) \quad \text{and} \quad CE(ce(a,\tilde{\theta})) \equiv h^{-1}(\mathrm{E}_\theta h(ce(a,\tilde{\theta}))). \tag{6}$$

The first, $ce(a,\theta)$, corresponds to the certainty equivalent of wealth in the second period, if the abatement level is a and the expert's model considered is P_θ. Under model uncertainty, θ itself takes on different values, and so does the certainty equivalent $ce(a,\theta)$, which is computed conditionally on θ. A second-order certainty equivalent of these first-order certainty equivalents is then defined as CE by combining all models $\theta \in \{1, 2, \ldots, n\}$. The SEU representation (1) is then obtained in the special case in which the two certainty equivalents are evaluated using the same function v, so that the attitudes toward risk and model uncertainty are exactly the same.

The smooth model uncertainty criterion is mathematically equivalent to the two-period version of the ambiguity model developed by Klibanoff et al. (2009). The significant difference is that their model, as the vast decision theoretic literature dealing with ambiguity (see Gilboa and Marinacci 2013 for an excellent survey), has been developed in a purely subjective setup and therefore does not explicitly incorporate objective information à la Wald (1950). In particular, their representation is recovered by letting $\phi \equiv h \circ v^{-1}$ represent the ambiguity attitude. Klibanoff et al. (2005) associate ϕ being a concave function to ambiguity aversion and call the ratio $-(\phi''(x)/\phi'(x))$ the coefficient of absolute ambiguity aversion at x, a given level of expected utility. From representation (4), ambiguity aversion would correspond to h being more concave than v or, equivalently, model uncertainty aversion being stronger than risk aversion. Unsurprisingly, in the special case in which the DM manifests the same attitude toward risk and model uncertainty, the problem is reduced to the one considered by a classical subjective expected utility maximizer defined by representation (1). In this case, the decision problem may be reduced to a problem under risk.[13] The great flexibility of this decision rule, which is based on the smoothness of function h, implies different conditions in the comparative statics analysis of optimal abatement. In particular, one condition needed to sign the direction of the change resulting from higher aversion toward model uncertainty than toward risk is the notion of ambiguity prudence (Gierlinger and Gollier 2008, Berger 2014). This concept, which is closely related to the notion of risk prudence introduced by Kimball (1990), corresponds to a condition under which the individual is willing to save more because of the presence of ambiguity.[14] Equivalently, it expresses the sensitivity of the optimal choice to the combination of model uncertainty and risk. The notion of ambiguity prudence in this context corresponds to decreasing absolute ambiguity aversion, which is the analogue of the widely accepted notion of decreasing absolute risk aversion (DARA). Formally, the notions of constant, decreasing, and increasing absolute ambiguity aversion are defined depending on the monotonicity properties of the ratio $-(\phi''(x)/\phi(x))$. In what follows, we, respectively, use the abbreviations CAAA, DAAA, and IAAA to denote these cases.[15] Equipped with the ambiguity prudence property, which we investigate further in the next section, we now compare the optimal level of abatement chosen by a decision maker under the smooth criterion with the one chosen under classical subjective expected utility. First, let us recall the definition of comonotonic variables before summarizing the result in Lemma 1, which is reminiscent of Alary et al. (2013) and Berger (2016).[16]

Definition 1. Consider two random variables X and Y that are strictly monotonic transformations of a single random variable θ; that is, $(X, Y) = (f(\theta), g(\theta))$. The random variables X and Y are anticomonotonic if f is increasing and g is decreasing in θ and comonotonic if both f and g are increasing or decreasing in θ.

Lemma 1. *Assume that model uncertainty aversion is higher than risk aversion. In the optimal abatement model characterized by the maximization of program* (4)*, DAAA* (*IAAA*) *is sufficient to raise* (*decrease*) *the optimal abatement if* $\mathrm{E}v(\tilde{C}_2(a^*_{\mathrm{SEU}}, \theta))$ *and* $\partial \mathrm{E}v(\tilde{C}_2(a^*_{\mathrm{SEU}}, \theta))/\partial a$ *are anticomonotonic* (*comonotonic*).

Proof. See Appendix A.1. □

Lemma 1 tells us that the total effect on abatement not only depends on the ambiguity prudence condition but also on a second factor that concerns the way the second-period expected utility and the marginal benefit of abatement interact when different experts or models are considered. The question of whether the comonotonicity condition of Lemma 1 holds in practice is not trivial. However, the intuition is relatively simple. Consider the case of two models with different probability curves $p(a, \theta)$ that do not cross. When the more pessimistic one (e.g., the one with lower $\mathrm{E}v(\tilde{C}_2(a^*_{\mathrm{SEU}}, \theta))$) believes that the probability of catastrophe decreases faster in abatement (e.g., a higher $\partial \mathrm{E}v(\tilde{C}_2(a^*_{\mathrm{SEU}}, \theta))/\partial a$), the anticomonotonicity condition holds, and the condition of ambiguity prudence—stating that the DM is more willing to invest for the future when this future becomes more uncertain—is sufficient. In this case, it is equivalent to saying that the disagreement across models falls in abatement. For example, this would be the case if experts agreed that a high level of climate protection would give us good chances of avoiding a climate catastrophe but disagreed on the probabilities in the case of limited mitigation and thus higher global warming. To gain more intuition, we now disentangle the role of preferences from the structure of model uncertainty and study the two effects separately.

2.1. The Ambiguity Prudence Effect

In terms of attitudes toward risk and model uncertainty, the ambiguity prudence condition turns out to be nontrivial, as is summarized in the following proposition.

Proposition 1. *Decision makers exhibit DAAA if and only if their preferences toward risk, captured by function u, and model uncertainty, captured by function h, are such that*

$$\frac{h'''}{h'} - \frac{v'''}{v'} \geq \left(-\frac{h''}{h'} + \frac{v''}{v'}\right)\left(-\frac{h''}{h'} - 2\frac{v''}{v'}\right). \tag{7}$$

Similarly, a decision maker exhibits CAAA if inequality (7) holds with an equality, and IAAA if inequality (7) is reversed.

Proof. See Appendix A.2. □

Intuitively, Proposition 1 tells us that a necessary and sufficient condition for DAAA is that the difference in downside model uncertainty and risk aversion (the left-hand side of (7)) is sufficiently high. To gain further insight about this condition, consider the following examples.

Example 1. When the isoelastic CRRA–CRMUA[17] specifications with relative risk aversion parameter ρ and relative model uncertainty aversion parameter $\mu \geq \rho$ (so that the DM is ambiguity averse) are considered, the ambiguity aversion function is given by $\phi(U) = (1/(1-\mu))[(1-\rho)U]^{(1-\mu)/(1-\rho)}$, the coefficient of absolute ambiguity aversion is $(\mu-\rho)/((1-\rho)U)$, and the DM exhibits DAAA when $\rho < 1$, CAAA when $\rho = 1$, and IAAA when $\rho > 1$.

Example 2. When the CARA-CAMUA[18] specifications are used with absolute coefficients of risk aversion and model uncertainty aversion, respectively, ρ and μ, the ambiguity function is $\phi(U) = -(-U)^{\mu/\rho}$, so that the DM always exhibits IAAA.

2.2. The Convergence of Agreement Effect

To study the structure of model uncertainty, let us simplify the notation in expression (2) above, and let $w_2 - L$ with $L > 0$ be the certainty equivalent consumption in the second period when the economy is hit by a catastrophe.[19] Remember that in this case, each model P_θ describes a possible risk of second-period consumption, which is fully characterized by $\tilde{C}_2(a,\theta) \sim [w_2 - L, p(a,\theta); w_2, 1-p(a,\theta)]$. An illustration of possible different models is depicted in the first row of Figure 1.

To further characterize the change in the optimal abatement decision, we now define the notion of degree of model disagreement. It is a measure of the variability across models (or, equivalently, of the disagreement among experts).

Definition 2. For any set of probability functions $\{p(a,\theta)\}_{\theta\in\{1,\dots,n\}}$ characterizing models $\{P_\theta\}_{\theta\in\{1,\dots,n\}}$, the degree of model disagreement is given by

$$\sigma^2(a) := \mathrm{Var}_\theta[p(a,\theta)],$$

for any given level of abatement a.

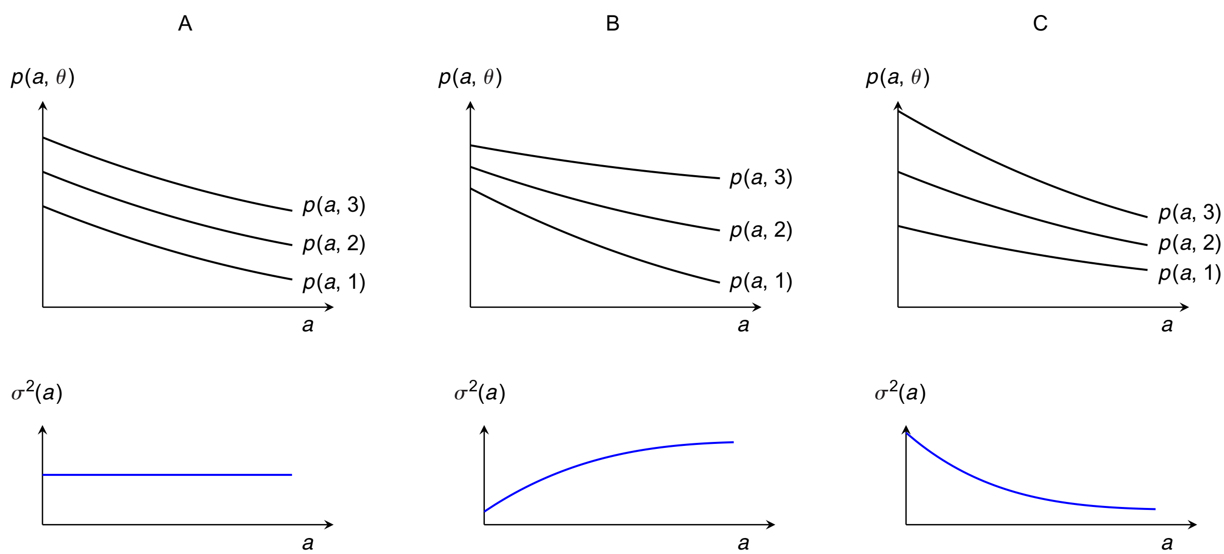
The degree of model disagreement is illustrated in the lower row of Figure 1. As can be seen, it can be (but is not limited to) constant (column A), increasing (column B), or decreasing (column C) in the level of abatement. In what follows, we will focus on the case where the degree of model disagreement is a monotonic function of abatement.[20] We will refer to "convergence of agreement," a situation in which the degree of model disagreement is decreasing. Using this simple metric, we can now relate the results of Lemma 1 to the structure of model uncertainty. First, note that, conditional on the true model being P_θ, we can write

$$\mathrm{E}v(\tilde{C}_2(a^*_{\mathrm{SEU}},\theta)) = v(w_2) - p(a,\theta)[v(w_2) - v(w_2 - L)] \tag{8}$$

$$\frac{\partial \mathrm{E}v(\tilde{C}_2(a^*_{\mathrm{SEU}},\theta))}{\partial a} = -p_a(a,\theta)[v(w_2) - v(w_2 - L)], \tag{9}$$

where $p_a \equiv \partial p/\partial a$. Lemma 1 therefore tells us that a DM exhibiting DAAA will always choose to abate more if $p(a,\theta)$ and $p_a(a,\theta)$ are anticomonotonic in θ, as is the case in column C of Figure 1, for example. In this case,

Figure 1. (Color online) Different Models or Experts $p(a,\theta)$ as Functions of the Abatement Level a, Under Constant (Column A), Increasing (Column B), or Decreasing (Column C) Degree of Model Uncertainty in Abatement



the degree of model uncertainty will intuitively be decreasing in abatement since abatement decreases the probability of catastrophe more strongly in more pessimistic models. The sufficient condition of Lemma 1 is, however, very restrictive, and Proposition 2 below tells us that the comonotonicity property does not necessarily have to hold for all the models considered. Instead, a simple and weaker condition on the degree of model disagreement can be used to determine the direction of the change induced by ambiguity aversion.

Proposition 2. *The degree of model disagreement* $\sigma^2(a)$ *is decreasing (increasing) in abatement if and only if* $\mathrm{Cov}_\theta(p(a,\theta);\, p_a(a,\theta)) \leq (\geq)\, 0$.

Proof. See Appendix A.3. □

With this intuition in mind, we can now introduce our main result, which does not require the relatively strong condition of comonotonicity.

Proposition 3. *In the optimal abatement problem under model uncertainty,*

(i) *a decision maker exhibiting CAAA always chooses to abate more (less) than an SEU maximizer if the degree of model disagreement decreases (increases) with abatement;*

(ii) *a decision maker exhibiting strict DAAA (IAAA) always chooses to abate strictly more (less) than an SEU maximizer if the degree of model disagreement decreases (increases) or is constant in abatement.*

This proposition tells us that if higher abatement leads to a reduction in the degree of model disagreement, a positive incentive is generated to abate more in the first period. Intuitively, the degree of model disagreement will be decreasing in abatement if abatement on average decreases the probability of a catastrophe more strongly in pessimistic models. This structural effect has, however, to be added to the ambiguity prudence effect to determine the direction of the total change in the abatement level. Ultimately, whether experts' disagreement decreases in abatement, and the extent to which the model structure effect interplays with ambiguity prudence in determining the optimal level of mitigation, can be determined only numerically. In the next section we bring the model to the data and analyze the direction and magnitude of both effects.

3. Empirical Evidence and Expert Judgments

The question of whether the degree of model disagreement is increasing or decreasing in the level of abatement is essentially an empirical one. In this section, we use the results of a recently published study to assess whether the conditions obtained from our theoretical model are met in practice. In particular, we study separately the two effects of ambiguity prudence and convergence of agreement we described in the previous section.

We use the data of Zickfeld et al. (2007). Their study presents the results from interviews with 12 leading climate scientists about the risk of a collapse of the Atlantic meridional overturning circulation (AMOC, also called thermohaline circulation) due to global climate change.[21] Specifically, the authors elicited the experts' probabilities[22] that a collapse of the AMOC will occur or will be irreversibly triggered as a function of global mean temperature increase realized by the year 2100. These probabilities are reproduced and approximated in a least-squares sense using a power function of the type $P(T) = k_1 T^{k_2}$, where T represents the change in global mean temperature, and k_1 and k_2 are the best-fit coefficients, in the upper panel of Figure 2. Note that, for $T = 0$, the probability of catastrophe $P(T)$ is set to zero for all experts. As can be seen, eight experts[23] assessed a nonzero probability of this catastrophic event. For an increase of 2°C in 2100 relative to 2000, four experts assessed the probability of a collapse as ≥5%, whereas for a warming of 4°C, three experts assigned a probability of ≥40%. Finally, if the increase in global warming reaches 6°C, the probability is 90% for two experts, ≥50% for four experts, and ≥10% for six experts. These curves represent the different probability functions $p(a,\theta)$ we introduced in §2, given that the abatement of greenhouse gas (GHG) emissions lowers expected global mean temperature. Although

Figure 2. (Color online) (Upper Panel) Experts' Probabilities as a Function of Global Mean Temperature in 2100; (Lower Panel) The Degree of Scientific Model Disagreement (σ^2)

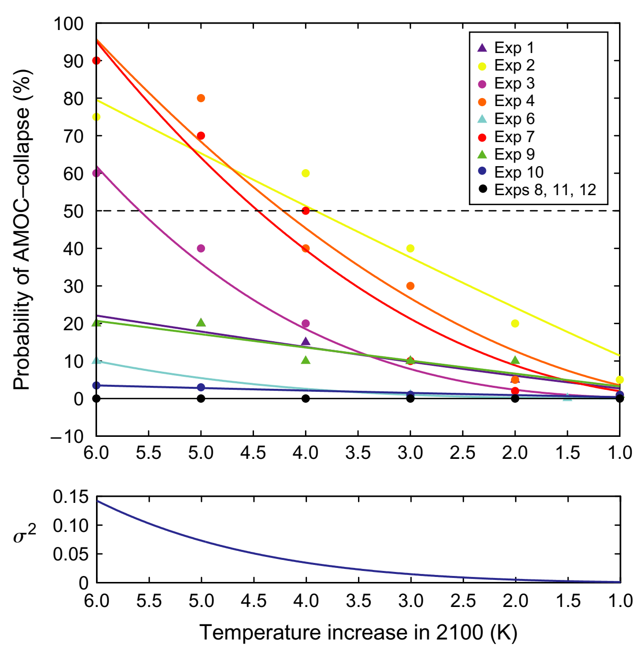
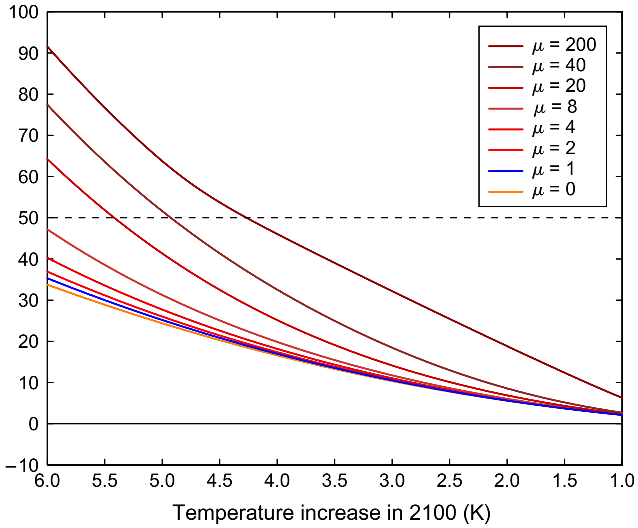


Figure 3. (Color online) Distorted Probabilities of the Catastrophe



Note. Specifications: v, CRRA ($\rho = 1$); and h, CRMUA (μ).

the link between cumulative emissions and temperature increase has been shown to be robustly described by a linear relationship (Matthews et al. 2009, IPCC 2013), the magnitude of the so-called carbon-climate response describing this relationship remains uncertain. In our framework, we have so far neglected this additional source of uncertainty. In the online supplemental material, we allow for different values of carbon-climate response and show that our results are robust to this alternative source of uncertainty.

In Figure 3, we provide the distorted probability functions for different values of model uncertainty aversion. Formally, this notion is defined as follows.

Definition 3. The distorted probability $\hat{p}(a)$ is the probability that would be equivalently considered under expected utility, and that is defined as

$$\begin{aligned}\hat{p}(a)v(w_2 - L) + (1 - \hat{p}(a))v(w_2) \\ = (v \circ h^{-1})\{\mathrm{E}_\theta (h \circ v^{-1})\{p(a, \tilde{\theta})v(w_2 - L) \\ + (1 - p(a, \tilde{\theta}))v(w_2)\}\}. \qquad (10)\end{aligned}$$

Note that for an individual exhibiting an equal attitude toward risk and model uncertainty, the distorted probability corresponds to the predictive probability of catastrophe: $\bar{p}(a) \equiv \mathrm{E}_\theta p(a, \tilde{\theta})$. On the contrary, under the smooth model uncertainty aversion criterion, the DM aggregates the different models depending on the degree of model uncertainty aversion relative to the risk aversion and acts like an expected utility maximizer considering only the distorted probability $\hat{p}(a)$. In particular, if aversion to model uncertainty is stronger than that to risk, it must be that $\hat{p}(a) \geq \bar{p}(a)\ \forall\, a$, leading any ambiguity-averse DM to overweight more pessimistic models. Since estimates of the potential loss L are hard to obtain, we follow van der Ploeg and de Zeeuw (2014) in assuming a 20% loss of GDP. This order of magnitude is rather speculative and is used essentially for illustrative purposes in the context of climate change, but it is based on the findings of Barro (2015), who shows that, historically, catastrophes (defined as losses of at least 10% of GDP) averaged approximately 20% of GDP. Finally, regarding the weights of different experts, we consider a uniform prior distribution, given that we do not have any information about the "qualifications" of the different experts.[24] We can now study separately the effect resulting from the degree of model disagreement (convergence of agreement effect) and the one resulting from the attitude toward model uncertainty (ambiguity prudence effect).

Let us begin with the former. The lower panel of Figure 2 tells us that the degree of model disagreement for the AMOC collapse is decreasing in the level of abatement. To compute the distorted probabilities in Figure 3, we use a utility function v of the type CRRA with a parameter of relative risk aversion $\rho = 1$ (i.e., log utility), and a function h of the CRMUA form, with a model uncertainty aversion parameter μ. From the properties of the CRRA–CRMUA functions discussed above, the DM exhibits CAAA, so that there is no ambiguity prudence effect. The total effect on abatement can therefore be entirely attributed to the decrease in the degree of model disagreement. For $\mu = 1$, the individual is ambiguity neutral and maximizes expected utility by considering only the probability depicted in blue. When $\mu = 0$, the DM is ambiguity loving and seems to be considering more optimistic experts, whereas when μ increases, more weight is attached to more pessimistic experts, and the probability of catastrophe increases for any fixed level of abatement. What Figure 3 indicates is that not only is the distorted probability of catastrophe higher when $\mu > 1$, but so is the slope of the distorted probability functions, therefore making abatement marginally more desirable. This change in the marginal benefit of abatement induces the DM to opt for a higher abatement level.

To isolate the ambiguity prudence effect, we artificially construct three different probability laws representing experts' assessments of the AMOC collapse (upper panel of Figure 4). The probability laws are constructed in such a way that, by considering a uniform prior over experts, an EU maximizer chooses exactly the same amount of abatement as in the case considering the data from the probabilities of Zickfeld et al. (2007), presented in Figure 2. Since these probability laws are perfectly parallel, the degree of model disagreement $\sigma^2(a)$ is constant in abatement (lower panel of Figure 4). The effect of higher aversion toward model uncertainty than toward risk on optimal abatement in this case therefore depends exclusively on the DM's

Figure 4. (Color online) (Upper Panel) Artificial Experts' Probabilities as a Function of Global Mean Temperature in 2100; (Lower Panel) The Degree of Model Disagreement ($\sigma^2(a)$)

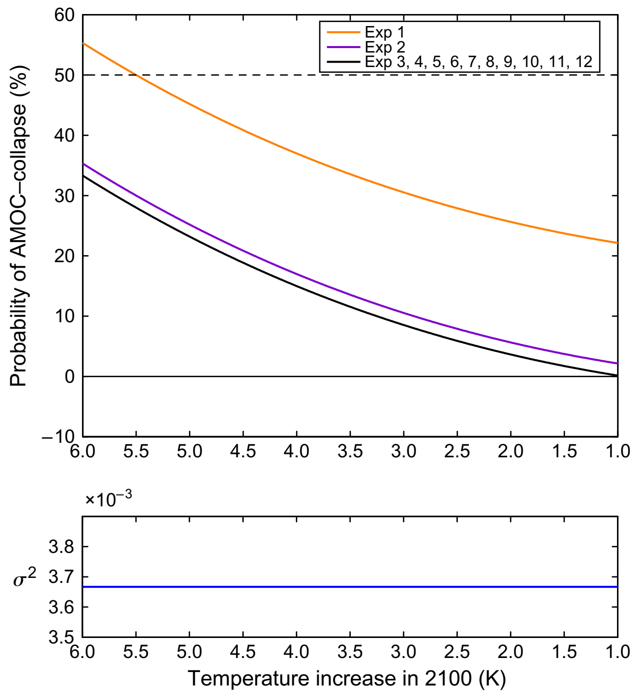
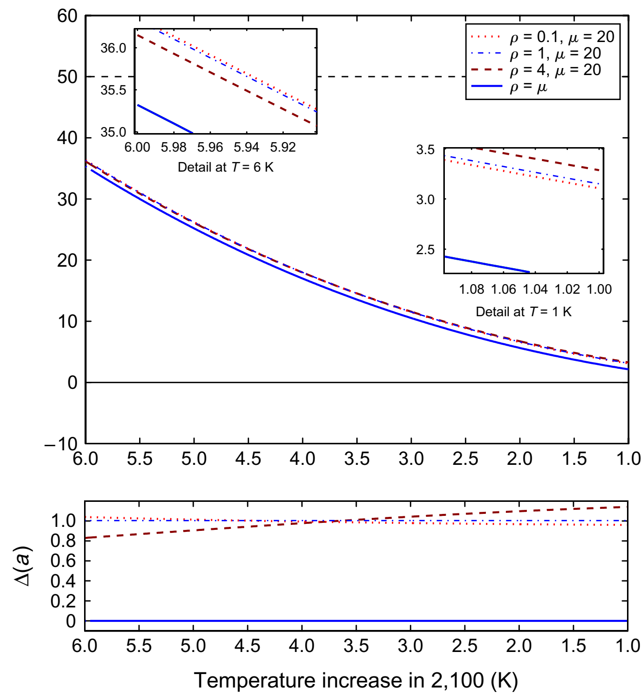


Figure 5. (Color online) (Upper Panel) Distorted Probabilities of Catastrophe for Different Specifications of v, CRRA (ρ), and h, CRMUA (μ); (Lower Panel) Difference ($\Delta(a)$) Between Distorted Probability and Probability Law Considered Under SEU



ambiguity-prudence attitude. In particular, from our model we know that DMs who exhibit IAAA abate less, DMs who exibit CAAA abate exactly the same amount, and DMs who exhibit DAAA abate more. To quantify the importance of this effect, we present the distorted probabilities with different specifications of the functions v and h in Figure 5. We consider the CRRA–CRMUA specification described above, spanning rather extreme values of relative risk aversion ρ and model uncertainty aversion μ. As before, when $\rho = \mu$, the DM acts as an expected utility maximizer and considers the average probability of catastrophe, depicted in solid blue. When model uncertainty aversion is higher than risk aversion ($\mu > \rho$), a DM exhibiting DAAA (respectively, CAAA and IAAA) considers the distorted probability represented by the red dotted line (respectively, the blue dash-dotted line and brown dashed line). The lower panel of Figure 5 shows that the difference between the distorted probabilities and the simple average, $\Delta(a) := \hat{p}(a) - \bar{p}(a)$, is, respectively, constant, decreasing, or increasing in abatement when CAAA, DAAA, or IAAA is considered. This gives an ambiguity-averse individual manifesting DAAA an incentive to abate more in order to prevent the realization of the bad state in the future, since the absolute slope of the probability curve (and hence the marginal benefit of abatement) is always higher in this situation than under expected utility or under CAAA. However, although the direction of the effect is the same as predicted by our model, its magnitude appears to be small, with discernible difference only for very high values of model uncertainty aversion.[25] This provides an empirically grounded assessment of the relative importance of the structure versus the attitude toward model uncertainty, showing that the former effect (namely the convergence of model agreement) has a bigger impact on the optimal climate policy decision. To provide a quantitative assessment of the combined effects of model uncertainty on optimal abatement, we now apply our framework to a general equilibrium model of climate change economics.

4. Quantification Using an Integrated Assessment Model

To quantify the theoretical predictions, we implement the model developed in this paper using the data of §3 in the most widely used integrated assessment model for the analysis of climate change, the DICE model (Nordhaus 1993). The DICE (dynamic integrated climate and economy) model is a numerical optimal growth model à la Ramsey, which integrates emissions and their mitigation in the production function, and which provides climate change feedback on the economy through climate and impact modules.[26] We extend the DICE model by reformulating it as a stochastic control problem and by implementing the endogenous possibility of a climate catastrophe based on the estimated experts' probability functions. Section 2 in the online supplemental material provides a more detailed description of the model. Following the expert elicitation of Zickfeld et al. (2007), we consider the case where the uncertainty is resolved at one single point in time, in the year 2100. That is, after the year 2100, either the catastrophe has hit the economy, leading to the crossing of a tipping point, or not. In the catastrophic state, an irreversible damage occurs in that an additional 20% of baseline GDP is lost for the remaining time horizon. This means that we extend the DICE damage function that expresses the economic impacts of climate change D in percent of GDP as the following random variable:

$$\tilde{D}_\theta(T) \sim \big[\kappa_1 T + \kappa_2 T^{\kappa_3} + L, p_\theta(T); \\ \kappa_1 T + \kappa_2 T^{\kappa_3}, (1 - p_\theta(T))\big], \qquad (11)$$

where T is the change in global mean temperature relative to the preindustrial level, and p_θ is the probability of suffering an additional catastrophic loss L, as given by expert θ.[27] The term $\kappa_1 T + \kappa_2 T^{\kappa_3}$ on the right-hand side of expression (11) represents the standard DICE damage function. For example, the default calibration of $\kappa_1 = 0, \kappa_2 = 0.00267$, and $\kappa_3 = 2$ yields a standard damage estimate of 5.4% of GDP for a 4.5°C temperature increase. The loss due to a catastrophic event ($L = 20\%$) adds to the standard damage function and occurs with a probability that depends on the temperature increase attained in the year 2100. Finally, although so far we have assumed that the elasticity of intertemporal substitution was equal to the inverse of the degree of relative risk aversion (an assumption that is maintained throughout the literature; see Klibanoff et al. 2009), we disentangle these two very different normative characteristics of the decision maker to obtain a more realistic representation of preferences. To do so, we modify DICE's utility function and adapt the generalized model of Hayashi and Miao (2011) to disentangle the three concepts of risk aversion, intertemporal elasticity of substitution, and model uncertainty aversion. As before, we represent risk aversion by the function v and model uncertainty aversion by h. The agent's intertemporal welfare at time t is represented by the following recursive form:

$$W_t = u^{-1}[(1-\beta)u(c_t) + \beta u(R_t(\tilde{W}_{t+1}(\tilde{\theta})))], \qquad (12)$$

where u characterizes the attitude toward consumption smoothing over time,[28] β is the discount factor, C_t is the consumption at time t, and $R_t(\tilde{W}_{t+1}(\tilde{\theta}))$ represents the double certainty equivalent. We define the double certainty equivalent as follows:

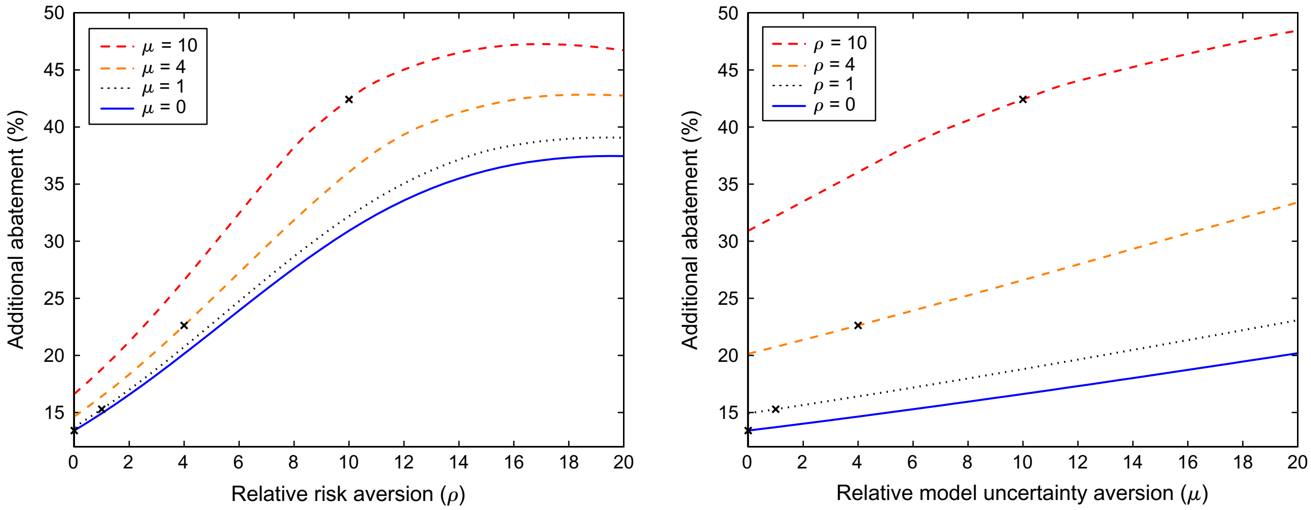
$$R_t(\tilde{W}_{t+1}(\tilde{\theta})) := h^{-1}(\mathrm{E}_{t,\theta}(h \circ v^{-1})(\mathrm{E}_t v(\tilde{W}_{t+1}(\tilde{\theta})))). \qquad (13)$$

In this expression, $\mathrm{E}_{t,\theta}$ is the expectation operator taken at time t over models, and E_t is the expectation operator taken at time t over future consumption, conditional on θ. For the implementation, we use a threefold isoelastic specification of the different functions: η is the inverse of the elasticity of intertemporal substitution, ρ is the constant relative risk aversion (CRRA) parameter, and μ is the degree of constant relative model uncertainty aversion (CRMUA). The main purpose of the threefold disentanglement is to allow varying risk preferences while keeping constant the certainty equivalent discount rate, as defined by the Ramsey rule.

The enhanced DICE model allows us to quantify the impact of model uncertainty aversion on abatement in light of the insights of the theoretical model proposed in §2. In particular, it shows how the combined effects of ambiguity prudence and the convergence of agreement effect impact the optimal abatement decisions. To do so, we compute the level of additional abatement, i.e., the extra reduction of cumulative emissions, under the possibility of a climate catastrophe, relative to that in the standard version of DICE without catastrophic climate change.[29] Figure 6 illustrates the results in terms of additional abatement realized during the period 2010–2100 for different parametrizations. For $\rho = \mu = 0$ (that is, for a risk- and model-uncertainty-neutral policy maker), the only difference with respect to the standard DICE is the presence of catastrophe, which is evaluated as its expected future consumption loss. In this case, optimal abatement increases by 13.5% in the sense that the cumulative emissions are further reduced from 3,813 to 3,301 $GtCO_2$, as reported in Table 1.

The additional abatement rises to 15%, 23%, and 42% when both risk and model uncertainty aversions are increased simultaneously ($\rho = \mu$) to 1, 4, and 10, respectively (black crosses in Figure 6). This situation of ambiguity neutrality corresponds to the Epstein–Zin–Weil version of the model. Differentiating the coefficients of risk aversion and model uncertainty aversion enables us to see that the additional abatement level is increasing in the risk aversion parameter ρ, though at a decreasing rate (left panel of Figure 6). For what concerns the model uncertainty aversion parameter μ, the additional abatement level monotonically increases. In terms of magnitude, the results suggest that the effect of model uncertainty aversion is approximately one-fourth to approximately one-half of the effect of risk aversion (as can also be inferred from Figure 6): starting from the case of $\rho = \mu = 1$ with additional abatement of 15% of emissions, increasing ρ to 10 roughly doubles the effort to 32%, whereas increasing μ to 10 increases abatement to 19%. Moreover, the results of the enhanced DICE model confirm what we observed

Figure 6. (Color online) Additional Abatement Based on the Modified Version of DICE with the Possibility of AMOC Collapse for Different Values of Relative Risk Aversion (ρ) (Left) and Relative Model Uncertainty Aversion (μ) (Right)



in the previous section concerning the relative importance of preferences and structure of model disagreement: since the experts' disagreement decreases in abatement, abatement always increases in the degree of model uncertainty aversion, even when $\rho \geq 1$.

Table 1 provides additional details of the scenario runs. In the third column, we report the social cost of carbon in 2015.[30] It is estimated to be \$17.7 per ton of CO_2 in the standard version of DICE without catastrophe and increases to \$20.4 when the possibility of a catastrophe is taken into account in a risk- and model-uncertainty-neutral environment. It further increases to \$27 when the relative risk and model uncertainty aversion parameters $\mu = \rho = 10$ are considered. Additional results concerning the temperature increase reached in 2100 are presented in the fourth column of Table 1. As can be seen, the introduction of a potential catastrophic climate change event reduces the admitted global temperature increase in 2100 (compared to the preindustrial level) from 3.1°C in the standard version of DICE, to 2.5°C (when $\mu = \rho = 10$). Both the risk and model uncertainty aversion parameters lead to a reduction of the optimal temperature increase.[31] Finally, the two last columns of

Table 1. Global Cumulative Emissions for the Period 2010–2100, Social Cost of Carbon in 2015, Temperature Increases (with Respect to Preindustrial Levels), and Average and Distorted Probabilities of Catastrophe Obtained with the Modified Version of DICE Under the Possibility of AMOC Collapse

	Cum. emissions in 2010–2100 ($GtCO_2$)	Additional optimal abatement (%)	Social cost of carbon in 2015 ($\$/tCO_2$) (\$)	Temperature increase in 2100 (°C)	Average prob. of catastrophe $\bar{p}(T^*)$ (%)	Distorted prob. of catastrophe $\hat{p}(T^*)$ (%)
$\mu = 0$						
$\rho = 0$	3,301	13.4	20.4	2.91	6.6	6.6
$\rho = 1$	3,244	14.9	20.7	2.89	6.5	6.4
$\rho = 4$	3,045	20.1	21.8	2.82	6.2	5.7
$\rho = 10$	2,634	30.9	24.1	2.67	5.5	3.9
$\mu = 1$						
$\rho = 0$	3,290	13.7	20.5	2.91	6.5	6.6
$\rho = 1$	3,230	15.3	20.8	2.89	6.4	6.4
$\rho = 4$	3,022	20.7	21.9	2.81	6.1	5.7
$\rho = 10$	2,585	32.2	24.4	2.65	5.4	4.0
$\mu = 10$						
$\rho = 0$	3,179	16.6	21.1	2.87	6.4	7.3
$\rho = 1$	3,096	18.8	21.5	2.84	6.2	7.2
$\rho = 4$	2,799	26.6	23.2	2.73	5.8	6.5
$\rho = 10$	2,195	42.4	27.0	2.50	4.8	4.8
Standard optimal version of DICE	3,813	0	17.7	3.1	0	0

Table 1 present the average $\bar{p}(T^*)$ and distorted $\hat{p}(T^*)$ probabilities of the AMOC collapse that is ultimately admitted by the DM. These values are computed at the optimal temperature endogenously calculated by the model. As expected, these probabilities are decreasing in both μ and ρ since the temperature is decreasing in both parameters. We also remark that $\bar{p}(T^*) < \hat{p}(T^*)$ as long as $\mu > \rho$, so that the DM always overestimates the probability of catastrophe when model uncertainty aversion is stronger than risk aversion (and vice versa). Overall, these results from the stochastic IAM confirm that model uncertainty plays an important role in quantitative terms when the convergence of agreement effect is important. Depending on the parametrization of preferences, the possibility of catastrophes leads to an additional mitigation effort of the cumulative emissions in the baseline scenario in the range of 13%–49%.

To analyze the robustness of our results, we perform an extensive sensitivity analysis with respect to the most relevant model parameters and specifications. In particular, we take into account a different timing of the catastrophic event, different values for the equilibrium climate sensitivity, different utility discount rates, and different values of the economic losses of the catastrophe. Although the full set of results is available in the online supplemental material, a summary of the results can be found in Table 2. We focus on the effect of model uncertainty aversion, while maintaining an intermediate value of $\rho = 4$ for the parameter of risk aversion.

Overall, the results show that the qualitative effect of model uncertainty aversion is robust throughout the specifications considered. For a lower value of the economic loss from the catastrophe (10% of GDP), a very low value of the climate sensitivity, or a comparable high utility discount rate of 3%, the effect of model uncertainty is attenuated but still leads to a notable increase in optimal abatement. If, on the other hand, the values are set to the other side of the spectrum, model uncertainty raises precautionary mitigation effort significantly, and more than proportionally. For example, an impact L of 30% as opposed to 20% raises the social cost of carbon by approximately \$7/tCO2. Finally, regarding the timing of learning and the potential occurrence of the catastrophic event, we find that the increase in abatement is higher for earlier occurrences and that it diminishes over time.

5. Conclusion

This paper aims at understanding and quantifying the impact of model uncertainty aversion on optimal abatement decisions, for the policy-relevant case of catastrophic climate change. This attempt stems from the recognition that, although it is now fully recognized that the presence of these uncertainties represents an essential datum of the climate change issue, the way they are treated and integrated in the models used to make predictions or to design public policies remains unsatisfactory. By evaluating the optimal strategy for responding to an uncertain threat, the model we present in this paper has the advantage of treating policy analysis like a robust risk management problem.

In particular, we consider situations in which the actions we take today (such as choosing the level of

Table 2. Sensitivity Analysis

	Increased abatement for $\mu = 0 \rightarrow 10$ (%)	Cum. emissions ($\mu = 10$) during 2010–2100 ($GtCO_2$)	Social cost of carbon ($\mu = 10$) in 2015 ($/$tCO_2$) ($)	Temperature increase ($\mu = 10$) in 2100 (°C)
Impact L				
$L = 10\,\%$	1.3	3,442	19.6	2.96
$L = 30\,\%$	19.8	1,953	30.0	2.37
Climate sensitivity				
$ECS = 1.5$	2.4	4,820	8.5	1.96
$ECS = 4.5$	9.6	1,540	42.5	3.16
Discount rate				
prstp= 0.001	10.3	922	83.0	1.86
prstp = 0.03	3.3	4,306	9.1	3.20
Time of resolution[a]				
2075	8.5	2,091	26.8	2.32
2100	5.4	2,410	23.2	2.41
2125	3.5	2616	21.1	2.46
Standard version ($\rho = 4, \mu = 10$)	8.1	2,799	23.2	2.73

Notes. Differences in relative abatement are given in percentage points comparing the model run with $\mu = 10$ to $\mu = 0$, keeping $\rho = 4$ and everything else constant. Cumulative emissions, social cost of carbon, and temperature increases are reported for the high model uncertainty aversion case ($\mu = 10$). ECS, equilibrium climate sensitivity; prstp, initial rate of social time preference per year.

[a]For these cases, cumulative emissions and temperature are reported for/until the year 2075.

abatement) affect the probability of incurring a high-damage event (of catastrophic nature) in the future. The selection of optimal policies in this sense is essentially an exercise in risk management. However, the particularity of this exercise is that it is carried out under partial ignorance: the decision makers we study admit that they do not know the exact relationship between their action and the probability of catastrophe. Rather, what the decision makers have available to help them make a choice is a collection of models or experts' estimates of this relationship. In contrast with purely risky situations in which the probabilities are known, the situations we study are therefore deeply uncertain or ambiguous. The ambiguity results precisely from the combination of risk and model uncertainty, and the decision maker's attitude toward ambiguity naturally results from the composition of attitudes toward these two distinct sources of uncertainty.

We compare this robust decision-making approach with the standard expected utility approach and show that the latter is not capable of differentiating distinct attitudes toward different types of uncertainty: it implicitly treats a situation in which experts have different dogmatic beliefs exactly the same way as a situation of pure risk. Rather, if the policy maker is ambiguity averse by being more sensitive to model uncertainty than to risk, we show that the policy maker will undertake more abatement effort if the combination of the ambiguity prudence effect and the convergence of the agreement effect is positive. The former condition is directly related to a condition about the functions representing preferences, whereas the latter is a characteristic of the available expert elicitation or model data. The intuition behind this result is that the desirability of preventive efforts is measured not only by the reduction in the expected damages, but also by the value of the associated reduced uncertainties. A degree of model disagreement that is decreasing in abatement effort is asking for a policy limiting global warming to relatively lower levels because it gives a precautionary policy maker an extra incentive for a more stringent mitigation policy, in the spirit of the precautionary principle. Finally, in contributing to answering the need to integrate the treatment of deep uncertainties and of possible catastrophic events in integrated assessment models, we apply our insights to the DICE model and show that robust precautionary climate policies require a significantly higher abatement level. Although the risk-neutral consideration of a catastrophic risk leads to a comparably low increase in abatement effort, this increase is magnified for reasonable degrees of both risk and model uncertainty aversion.

Although the proposed framework allows us to generate a set of original insights, many limitations remain. For example, we abstracted from the possibility of learning. Although it is unclear how much we can actually learn about these extreme climatic outcomes and what the implications are of learning on optimal abatement (IPCC 2014), several insightful applications emphasizing the role of learning in integrated assessment models with tipping elements have been recently proposed (Rudik 2016, Lemoine and Traeger 2014). Our framework also requires calibrating parameters for which few estimates exist, such as the model uncertainty aversion, potentially limiting its practical use. Nonetheless, we believe that the flexibility of the model uncertainty decision framework, the results about the importance of the structure of model uncertainty, and the simplicity of the proposed metric of model disagreement, are widely applicable and can be fruitfully extended to other policy objectives and data-generating processes such as additional tipping elements, climate engineering, technological change, or even non-climate-related policy issues.

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Appendix A. Proofs

A.1. Proof of Lemma 1

The proof directly follows from Proposition 4 in Berger (2016). The condition to observe a higher (lower) abatement due to ambiguity aversion may be written as

$$\mathrm{E}_\theta\left[\phi'(\mathrm{E}v(\tilde{C}_2(a,\tilde{\theta})))\frac{\partial \mathrm{E}v(\tilde{C}_2(a,\tilde{\theta}))}{\partial a}\right] \geq (\leq)\phi'\big(\phi^{-1}(\mathrm{E}_\theta\phi(\mathrm{E}v(\tilde{C}_2(a,\tilde{\theta}))))\big)\mathrm{E}_\theta\frac{\partial \mathrm{E}v(\tilde{C}_2(a,\tilde{\theta}))}{\partial a}. \quad \text{(A.1)}$$

Analogously to the risk theory literature, it can moreover be shown that CAAA is equivalent to $\phi'(\phi^{-1}(\mathrm{E}\phi(\tilde{U}))) = \mathrm{E}\phi'(\tilde{U})$, strict DAAA is equivalent to $\phi'(\phi^{-1}(\mathrm{E}\phi(\tilde{U}))) < \mathrm{E}\phi'(\tilde{U})$, and strict IAAA is equivalent to $\phi'(\phi^{-1}(\mathrm{E}\phi(\tilde{U}))) > \mathrm{E}\phi'(\tilde{U})$. By letting $A \equiv \mathrm{E}v(\tilde{C}_2(a,\tilde{\theta}))$ and $B \equiv \partial \mathrm{E}v(\tilde{C}_2(a,\tilde{\theta}))/\partial a$, we can rewrite condition (A.1) as $\mathrm{Cov}_\theta(\phi'(A);B) \geq (\leq) 0$, or $\mathrm{Cov}_\theta(A;B) \leq (\geq) 0$, since ϕ' is decreasing under ambiguity aversion. In the case of strict DAAA, we can use the chain of inequalities

$$\mathrm{E}_\theta[\phi'(A)B] \geq \mathrm{E}_\theta\phi'(A)\mathrm{E}_\theta B > \phi'(\phi^{-1}(\mathrm{E}_\theta\phi(A)))\mathrm{E}_\theta B, \quad \text{(A.2)}$$

so that the left-hand side (LHS) of (A.1) is greater than the right-hand side (RHS) if $\mathrm{Cov}_\theta(A;B) \leq 0$; in the case of strict IAAA, we can use the chain of inequalities

$$\mathrm{E}_\theta[\phi'(A)B] \leq \mathrm{E}_\theta\phi'(A)\mathrm{E}_\theta B < \phi'(\phi^{-1}(\mathrm{E}_\theta\phi(A)))\mathrm{E}_\theta B \quad \text{(A.3)}$$

to show that the RHS of (A.1) is greater than the LHS if $\mathrm{Cov}_\theta(A;B) \geq 0$. From Kimball (1951), it follows that this covariance is negative (positive) if A and B are anticomonotonic (comonotonic). □

A.2. Proof of Proposition 1

Considering the ambiguity aversion function $\phi(U) = (h \circ v^{-1})(U)$, where U represents the expected utility computed in the presence of risk (i.e., $U \equiv \mathrm{E}v(\tilde{x})$), it is easy to compute the index of absolute ambiguity aversion as

$$-\frac{\phi''(U)}{\phi'(U)} = -\frac{v'h'' - h'v''}{(v')^3}\frac{v'}{h'} = \frac{1}{v'}\left[-\frac{h''}{h'} + \frac{v''}{v'}\right], \tag{A.4}$$

where, in a slight abuse of notations, we let $h \equiv h(v^{-1}(U))$ and $v \equiv v(v^{-1}(U))$. As expected, this ratio is positive if model uncertainty aversion is higher than risk aversion. Analogously to risk theory literature, DAAA means that $-\phi'''(U)/\phi''(U) \geq -\phi''(U)/\phi'(U)$, which is the case if and only if

$$\frac{h'''}{h'} + 2\left(\frac{-v''}{v'}\right)^2 \geq \frac{v'''}{v'} + \left(\frac{-h''}{h'}\right)^2 + \left(\frac{-h''}{h'}\right)\left(\frac{-v''}{v'}\right). \quad \square \tag{A.5}$$

A.3. Proof of Proposition 2

The result is obtained by decomposing $\sigma^2(a) = \mathrm{E}_\theta[p(a,\theta)^2] - p(a,\bar{\theta})^2$, where $\bar{p}(a) \equiv \mathrm{E}_\theta p(a,\tilde{\theta})$, and deriving this expression with respect to a: $\partial\sigma^2(a)/\partial a = 2\mathrm{E}_\theta[p(a,\theta)p_a(a,\theta)] - 2p(a,\bar{\theta})p_a(a,\bar{\theta}) = 2\mathrm{Cov}_\theta(p(a,\theta); p_a(a,\theta))$. $\square$

Appendix B. Economic Impact Uncertainty

Impact (or socioeconomic) uncertainty results from our "imperfect understanding of the impacts of climate change on human societies and of how these societies will respond" (Heal and Millner 2014, p. 121). In the context of our abatement model, imagine that there is a scientific consensus on the link between the probability of a catastrophic event and the temperature increase (or abatement levels) given by a particular probability function $p(a)$. Even in this far-from-realistic situation of limited scientific uncertainty, there would still be room for model uncertainty to play a significant role because of the remaining uncertainty concerning the economic impacts of a climate catastrophe. What, for example, would be the economic loss associated with a sea level rise of one meter? Would it be possible to construct protective dikes to save the most vulnerable places, and if so, at what cost? Alternatively, what would be the cost associated with relocation and reconstruction? All of these costs correspond to what we have called the economic loss associated with the catastrophic event and are far from being perfectly known.[32] Different experts or studies may disagree on the total impact of a possible catastrophe, and this disagreement among economic models may potentially affect the decision made by a policy maker.

In our simple optimal abatement problem under impact uncertainty about the economic impacts L_s, the second-period expected utility for a given model P_θ is now written as

$$\mathrm{E}v(\tilde{C}_2(a,\theta)) = p(a)\sum_{s\in S}\pi_s(\theta)v(w_2 - L_s) + (1-p(a))v(w_2), \tag{B.1}$$

where $\pi_s(\theta)$ denotes the probability according to expert θ of the loss L_s. The expected marginal benefit of abatement can be obtained as

$$\frac{\partial \mathrm{E}v(\tilde{C}_2(a,\theta))}{\partial a} = -p_a(a)\left[v(w_2) - \sum_{s\in S}\pi_s(\theta)v(w_2 - L_s)\right]. \tag{B.2}$$

Given that the probability of catastrophe is assumed to be decreasing in abatement, it is clear that expressions (B.1) and (B.2) will always be anticomonotonic, which leads us to the following result.

Proposition 4. *In the optimal abatement problem under model uncertainty about impacts, an agent considering the smooth criterion and exhibiting CAAA or DAAA always chooses to abate more than an expected utility maximizer .*

Proof. The result directly follows from Lemma 1. $\square$

On the contrary, if the DM exhibits IAAA, it is impossible to unambiguously sign the final effect of model uncertainty aversion since it will depend on which of the two effects (degree of model disagreement or ambiguity prudence) dominates.

Endnotes

[1] This definition comprises the definition of deep uncertainty given by Lempert et al. (2006). In this paper, we use the terms "Knightian uncertainty," "ambiguity," and "deep uncertainty" interchangeably.

[2] These phenomena have been referred to as "tipping elements" because they imply abrupt climate change occurring "when the climate system is forced to cross some threshold, triggering a transition to a new state" (Lenton et al. 2008, p. 1786). The corresponding critical point at which the future state of the system is qualitatively altered is called a "tipping point."

[3] Remark that these experts' assessments may be the result of using different climatic models, different physical parameters, different methodologies, or different databases.

[4] Even if climate scientists have recently made a great deal of progress in understanding and describing the physical mechanisms involved in the climate change phenomenon, many uncertainties still remain. Some of them will eventually be resolved with future scientific progress, whereas others may be in the realm of "unknowable" (Pindyck 2013a) or "unquantifiable" (Heal and Millner 2014).

[5] Note that the preferences used by Hansen and Sargent (2008) can be seen as a special case of the smooth ambiguity preferences. In their case, the ambiguity function is of the exponential or constant absolute ambiguity aversion type (Cerreia-Vioglio et al. 2011, Marinacci 2015).

[6] The precautionary principle states that "When an activity raises threats of harm to human health or the environment, precautionary measures should be taken even if some cause and effect relationships are not fully established scientifically" (Science and Environmental Health Network 1998).

[7] Climate data allow the prediction of a low level of future warming with more confidence than a high level of warming, as mentioned by Allen and Frame (2007, p. 582) who write, "once the world has warmed by 4°C, conditions will be so different from anything we can observe today that it is inherently hard to say when the warming will stop."

[8] In situations where information is scarce, alternative decision-theory models may, on the contrary, perform better in the sense that they have better explanatory power or are able to provide better predictions and guidelines.

[9] This type of model, with endogenous probability to model mitigation, is referred to as a "self-protection model" in the risk literature. An alternative is to consider the case of adaptation or self-insurance in which the loss in the second period depends on the abatement level. Given the limited scope for adaptation in reducing catastrophic impacts, we decided not to consider the latter case in this paper. It can, however, be shown that our main results hold and would even

be reinforced by the presence of adaptation to the catastrophe or standard continuous damages in our framework (Berger 2016).

[10] As mentioned earlier, a parallelism can be made between the uncertainty that follows this decomposition into model (or epistemic) uncertainty and risk (also called aleatory or physical uncertainty) and what is generally referred to in the decision theory literature as *ambiguity* (i.e., situations in which "a decision maker does not have sufficient information to quantify through a single probability distribution the stochastic nature of the problem he is facing" (Cerreia-Vioglio et al. 2013a, p. 975).

[11] The two features could easily be disentangled using Kreps and Porteus (1978) and Selden (1978) preferences. For the sake of expositional clarity and simplicity, we only consider this specification in the quantification part of the paper (see §4).

[12] The maximization programs we consider in this paper are assumed to be convex. Sufficient conditions for concavity of (1) are that the cost function is increasing ($c'(a) > 0$) and convex ($c''(a) > 0$) in the level of abatement, and that the probability function is decreasing and convex ($p'(a) < 0$, $p''(a) > 0$). More generally, sufficient conditions for concavity of (4) may be found in Proposition 3 in Berger (2016).

[13] Note also that the maxmin criterion presented in the online supplemental material is recovered from this formulation in the special case of infinite model uncertainty aversion.

[14] Formally, an agent is said to be ambiguity prudent if the introduction of ambiguity through a mean-preserving spread in the space of conditional second-period expected utility raises the agent's optimal level of saving (Berger 2014).

[15] Note that DAAA encompasses the most widely used functional forms of power and exponential ϕ (the former is usually referred to as "constant relative ambiguity aversion" (CRAA) and the latter corresponds to CAAA). It is stronger than requiring $\phi''' > 0$, but this should not be surprising, given that future utility in program (4) is represented by the ϕ certainty equivalent of the expected utilities rather than by its expected ϕ valuation.

[16] In a recent contribution, Millner et al. (2013) also proposed a model of abatement with an endogenous probability in a one-period framework similar to Alary et al. (2013). When two periods are considered, however, the comonotonicity condition only concerns the second period, and general conclusions may be drawn for the more realistic cases in which the effort exerted in the first period also reduces the ambiguity.

[17] A utility function has the constant relative risk aversion (CRRA) property if it takes the form $v(x) = x^{1-\rho}/(1-\rho)$, where ρ is the coefficient of relative risk aversion (note that when $\rho = 1$, it collapses to $v(x) = \ln x$). Constant relative model uncertainty aversion (CRMUA) is defined similarly for function h, in the sense that $h(x) = x^{1-\mu}/(1-\mu)$, where μ represents the coefficient of relative model uncertainty aversion.

[18] A utility function exhibits constant absolute risk aversion (CARA) if it has the form $v(x) = -e^{-\rho x}$, where ρ is the coefficient of absolute risk aversion. Cconstant absolute model uncertainty aversion (CAMUA) is defined analogously, so that $h(x) = -e^{-\mu x}$, where μ is the coefficient of absolute model uncertainty aversion.

[19] More precisely, this certainty equivalent is implicitly defined by $\sum_{s\in S} \pi_s v(w_2 - L_s) = v(w_2 - L)$.

[20] Remark that this assumption is less restrictive than the one requiring the set of models $\{P_\theta\}$ to be monotonic in θ for all levels of abatement equivalent to the one used in Alary et al. (2013) and Berger (2016). In particular, our assumption does not require probability curves to not cross each other.

[21] The AMOC is a major current in the Atlantic Ocean that transports heat energy from the tropics and Southern Hemisphere toward the North Atlantic. Changes in this ocean circulation could have an important impact on many aspects of the global climate system, including changes in the carbon cycle. A collapse of the AMOC is defined in Zickfeld et al. (2007, p. 249) as a "reduction in AMOC strength by more than 90% relative to present-day." Such an event may potentially have catastrophic consequences such as changes in sea level in the North Atlantic up to 1 meter (Zickfeld et al. 2007, Figure 7) and reductions in crop production or water availability with consequent impacts (IPCC 2007, Table 12.4). The list of scientists selected for this study can be found in Zickfeld et al. (2007). It includes experts with different scientific backgrounds (observationalists, palaeoclimatologists, modelers), geographic origins, and schools of thought. These experts were selected based on different criteria (authors' knowledge of the field, review of recent publications, advice from scientists in the field).

[22] Expert elicitation is a tool for systematically gathering and projecting scientific information on complex policy problems that is increasingly recognized to play a valuable role for informing climate policy decisions (Kriegler et al. 2009).

[23] Note that expert 5 did not answer the question.

[24] This view is supported by Zickfeld et al. (2007, p. 239) who wrote, "the process of choosing experts for inclusion in this study is fundamentally different from the process of sampling to estimate some uncertain value such as a physical quantity, or polling the public to predict the results of an election. The route to scientific truth is not a matter of voting. One of the outliers among the respondents may be correct, and those who appear to be in close agreement may all be wrong."

[25] It may be shown that this result of almost no ambiguity-prudence effect is robust to a reasonable range of GDP losses.

[26] See Nordhaus and Sztorc (2013) and Nordhaus (2014) for a description of all assumptions, equations, and data used for the latest version of the model, DICE2013R.

[27] An adjustment factor of 0.7°C, representing the global mean increase in temperature between the years 1900 and 2000 (Hansen et al. 2006), is used since the DICE standard damage function considers 1900 as a reference for temperatures.

[28] Similar to what is proposed by Epstein and Zin (1989) and Weil (1990), we consider the particular case in which u is isoelastic, with a parameter η representing the inverse of the elasticity of intertemporal substitution.

[29] Unless stated explicitly otherwise, we keep the standard specifications of the latest version of the DICE model (see Nordhaus and Sztorc 2013, Nordhaus 2014) unaltered. This, for example, means that the inverse of the elasticity of intertemporal substitution is fixed to $\eta = 1.45$ and that the pure rate of time preference equals 1.5% per year. In the standard scenario without the possibility of a catastrophe, the global temperature increase by 2100 is approximately 3.1°C, and the global cumulative CO_2 emissions, also called the cumulative carbon budget, amount to 3,813 gigatonnes of carbon dioxide ($GtCO_2$) for the period 2010–2100 (see the last row of Table **??**). We will refer to additional abatement for any further relative reduction of this carbon budget.

[30] Put simply, this important concept measures the market price of emissions of GHGs. More formally, the social cost of carbon at time t is defined as the ratio of the marginal impact of emissions on welfare over the marginal welfare value of a unit of aggregate consumption (see Nordhaus 2014 for more details).

[31] Additional graphs concerning the social cost of carbon and the stochastic evolution of global temperature can be found in the online supplemental material.

[32] Stern (2007), for example, estimates the total loss for a high climate change scenario with nonmarket impacts and the risk of a catastrophe to be between 2.9% and 35.2% of GDP per capita in 2200 (see Figure 6.5 in Stern 2007 for more details).

References

Alary D, Gollier C, Treich N (2013) The effect of ambiguity aversion on insurance and self-protection. *Econom. J.* 123(573):1188–1202.

Allen MR, Frame DJ (2007) Call off the quest. *Science* 318(5850): 582–583.

Athanassoglou S, Xepapadeas A (2012) Pollution control with uncertain stock dynamics: When, and how, to be precautious. *J. Environ. Econom. Management* 63(3):304–320.

Barro RJ (2015) Environmental protection, rare disasters and discount rates. *Economica* 82(325):1–23.

Berger L (2014) Precautionary saving and the notion of ambiguity prudence. *Econom. Lett.* 123(2):248–251.

Berger L (2016) The impact of ambiguity and prudence on prevention decisions. *Theory Decision* 80(3):389–409.

Cerreia-Vioglio S, Maccheroni F, Marinacci M, Montrucchio L (2011) Uncertainty averse preferences. *J. Econom. Theory* 146(4): 1275–1330.

Cerreia-Vioglio S, Maccheroni F, Marinacci M, Montrucchio L (2013a) Ambiguity and robust statistics. *J. Econom. Theory* 148(3): 974–1049.

Cerreia-Vioglio S, Maccheroni F, Marinacci M, Montrucchio L (2013b) Classical subjective expected utility. *Proc. Natl. Acad. Sci. USA* 110(17):6754–6759.

Drouet L, Bosetti V, Tavoni M (2015) Selection of climate policies under the uncertainties in the Fifth Assessment Report of the IPCC. *Nature Climate Change.* 5:937–940.

Ellsberg D (1961) Risk, ambiguity, and the Savage axioms. *Quart. J. Econom.* 75(4):643–669.

Epstein LG, Zin SE (1989) Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework. *Econometrica* 57(4):937–969.

Gajdos T, Hayashi T, Tallon J-M, Vergnaud J-C (2008) Attitude toward imprecise information. *J. Econom. Theory* 140(1):27–65.

Ghirardato P, Maccheroni F, Marinacci M (2004) Differentiating ambiguity and ambiguity attitude. *J. Econom. Theory* 118(2): 133–173.

Gierlinger J, Gollier C (2008) Socially efficient discounting under ambiguity aversion. Working paper, Toulouse School of Economics, Toulouse, France.

Gilboa I, Marinacci M (2013) Ambiguity and the Bayesian paradigm. Acemoglu D, Arellano M, Dekel E, eds. *Advances in Economics and Econometrics: Theory and Applications, Tenth World Congress of the Econometric Society* (Cambridge University Press, Cambridge, UK), 179–242.

Gilboa I, Postlewaite A, Schmeidler D (2009) Is it always rational to satisfy Savage's axioms? *Econom. Philos.* 25(3):285–296.

Gilboa I, Postlewaite A, Schmeidler D (2012) Rationality of belief or why Savage's axioms are neither necessary nor sufficient for rationality. *Synthese* 187(1):11–31.

Gilboa I, Postlewaite AW, Schmeidler D (2008) Probability and uncertainty in economic modeling. *J. Econom. Perspect.* 22(3): 173–188.

Gjerde J, Grepperud S, Kverndokk S (1999) Optimal climate policy under the possibility of a catastrophe. *Resource Energy Econom.* 21(3–4):289–317.

Hansen J, Sato M, Ruedy R, Lo K, Lea DW, Medina-Elizade M (2006) Global temperature change. *Proc. Natl. Acad. Sci. USA* 103(39): 14288–14293.

Hansen L, Sargent T (2008) *Robustness* (Princeton University Press, Princeton, NJ).

Hayashi T, Miao J (2011) Intertemporal substitution and recursive smooth ambiguity preferences. *Theoret. Econom.* 6(3):423–472.

Heal G, Millner A (2014) Uncertainty and decision in climate change economics. *Rev. Environ. Econom. Policy* 8(1):120–137.

IPCC (Intergovernmental Panel in Climate Change) (2007) *Climate Change 2007: Impacts, Adaptation and Vulnerability. Contribution of Working Group II to the Fourth Assessment Report of the Intergovernmental Panel on Climate Change* Parry ML, Canziani OF, Palutikof J, van der Linden P, Hanson C, eds. (Cambridge University Press, Cambridge, UK).

IPCC (Intergovernmental Panel in Climate Change) (2013) *Climate Change 2013: The Physical Science Basis. Contribution of Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change* Stocker TF, Qin D, Plattner G-K, Tignor M, Allen SK, Boschung J, Nauels A, Xia Y, Bex V, Midgley PM, eds. (Cambridge University Press, Cambridge, UK).

IPCC (Intergovernmental Panel in Climate Change) (2014) *Climate Change 2014: Mitigation of Climate Change: Working Group III Contribution to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change* (Cambridge University Press, Cambridge, UK).

Karni E, Viero ML (2013) "Reverse Bayesianism": A choice-based theory of growing awareness. *Amer. Econom. Rev.* 103(7): 2790–2810.

Keller K, Bolker BM, Bradford DF (2004) Uncertain climate thresholds and optimal economic growth. *J. Environ. Econom. Management* 48(1):723–741.

Kimball AW (1951) On dependent tests of significance in the analysis of variance. *Ann. Math. Statist.* 22(4):600–602.

Kimball M (1990) Precautionary saving in the small and in the large. *Econometrica* 58(1):53–73.

Klibanoff P, Marinacci M, Mukerji S (2005) A smooth model of decision making under ambiguity. *Econometrica* 73(6):1849–1892.

Klibanoff P, Marinacci M, Mukerji S (2009) Recursive smooth ambiguity preferences. *J. Econom. Theory* 144(3):930–976.

Kreps DM, Porteus EL (1978) Temporal resolution of uncertainty and dynamic choice theory. *Econometrica* 46(1):185–200.

Kriegler E, Hall JW, Held H, Dawson R, Schellnhuber HJ (2009) Imprecise probability assessment of tipping points in the climate system. *Proc. Natl. Acad. Sci. USA* 106(13):5041–5046.

Kunreuther H, Heal G, Allen M, Edenhofer O, Field CB, Yohe G (2013) Risk management and climate change. *Nature Climate Change* 3(5):447–450.

Lange A, Treich N (2008) Uncertainty, learning and ambiguity in economic models on climate policy: Some classical results and new directions. *Climatic Change* 89(1):7–21.

Lemoine D, Traeger C (2014) Watch your step: Optimal policy in a tipping climate. *Amer. Econom. J. Econom. Policy* 6(1):137–166.

Lempert RJ, Collins MT (2007) Managing the risk of uncertain threshold responses: Comparison of robust, optimum, and precautionary approaches. *Risk Anal.* 27(4):1009–1026.

Lempert RJ, Groves DG, Popper SW, Bankes SC (2006) A general, analytic method for generating robust strategies and narrative scenarios. *Management Sci.* 52(4):514–528.

Lenton TM, Held H, Kriegler E, Hall JW, Lucht W, Rahmstorf S, Schellnhuber HJ (2008) Tipping elements in the Earth's climate system. *Proc. Natl. Acad. Sci. USA* 105(6):1786–1793.

Lontzek TS, Cai Y, Judd KL (2012) Tipping points in a dynamic stochastic IAM. RDCEP Working paper 12-03, Chicago.

Marinacci M (2015) Model uncertainty. *J. Eur. Econom. Assoc.* 13: 1022–-1100.

Matthews HD, Gillett NP, Stott PA, Zickfeld K (2009) The proportionality of global warming to cumulative carbon emissions. *Nature* 459(7248):829–832.

Millner A, Dietz S, Heal G (2013) Scientific ambiguity and climate policy. *Environ. Resource Econom.* 55(1):21–46.

Newbold SC, Daigneault A (2009) Climate response uncertainty and the benefits of greenhouse gas emissions reductions. *Environ. Resource Econom.* 44(3):351–377.

Nordhaus W (2014) Estimates of the social cost of carbon: Concepts and results from the DICE-2013R model and alternative approaches. *J. Assoc. Environ. Resource Economists* 1(1): 273–312.

Nordhaus W, Sztorc P (2013) DICE 2013R: Introduction and user's manual. http://www.econ.yale.edu/~nordhaus/homepage/documents/DICE_Manual_103113r2.pdf

Nordhaus WD (1993) Optimal greenhouse-gas reductions and tax policy in the "DICE" model. *Amer. Econom. Rev.* 83(2):313–317.

Pindyck RS (2013a) Climate change policy: What do the models tell us? *J. Econom. Literature* 51(3):860–872.

Pindyck RS (2013b) The climate policy dilemma. *Rev. Environ. Econom. Policy* 7(2):219–237.

Pindyck RS (2015) The use and misuse of models for climate policy. NBER Working paper 21097, National Bureau of Economic Research, Cambridge, MA.

Rudik I (2016) The social cost of carbon when damages are unknown. Working paper, Iowa State University, Ames.

Savage L (1954) *The Foundations of Statistics*, second revised edition (John Wiley & Sons, New York).

Science and Environmental Health Network (1998) The Wingspread Consensus on the Precautionary Principle. Accessed February 1, 2015, http://www.sehn.org/wing.html.

Selden L (1978) A new representation of preferences over "certain × uncertain" consumption pairs: The "ordinal certainty equivalent" hypothesis. *Econometrica* 46(5):1045–1060.

Stern N (2007) *The Economics of Climate Change: The Stern Review* (Cambridge University Press, Cambridge, UK).

Stern N (2013) The structure of economic modeling of the potential impacts of climate change: Grafting gross underestimation of risk onto already narrow science models. *J. Econom. Literature* 51(3):838–859.

van der Ploeg F, de Zeeuw A (2014) Climate tipping and economic growth: Precautionary saving and the social cost of carbon. OxCarre research paper 118, University of Oxford, Oxford, UK.

Wald A (1950) *Statistical Decision Functions* (Wiley, Oxford, UK).

Weil P (1990) Nonexpected utility in macroeconomics. *Quart. J. Econom.* 105(1):29–42.

Weitzman M (2009) On modeling and interpreting the economics of catastrophic climate change. *Rev. Econom. Statist.* 91(1):1–19.

Weitzman ML (2012) GHG targets as insurance against catastrophic climate damages. *J. Public Econom. Theory* 14(2): 221–244.

Zickfeld K, Levermann A, Morgan MG, Kuhlbrodt T, Rahmstorf S, Keith DW (2007) Expert judgments on the response of the Atlantic meridional overturning circulation to climate change. *Climatic Change* 82(3–4):235–265.