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Provision of Incentives for Information Acquisition: Forecast-Based Contracts vs. Menus of Linear Contracts

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In the producer–seller relationship, the seller, besides his role of selling, is often in an ideal position to gather useful market information for the producer's operations planning. Incentive alignment is critical to motivate both information-acquisition and sales efforts. Two popular contract forms are investigated. One is the forecast-based contract (FC) that requires the seller to submit a demand forecast: the seller obtains commissions from the realized sales but is also obliged to pay a penalty for any deviation of the sales from the forecast. The other is the classical menu of linear contracts (MLC), from which the seller can choose a contract that specifies a unique commission rate and a fixed payment. The conventional understanding suggests that the MLC is superior, but it is often assumed that information is exogenously endowed. In contrast, we find that, with an endogenous information-acquisition effort, the MLC may suffer from a conflicted moral hazard effect that creates friction between motivations for the two efforts. The FC can, however, decouple these two tasks and thus dominate the MLC. We further find that when ensuring interim participation is necessary (e.g., renegotiation cannot be prevented after information acquisition), the performance of the FC might be affected by the adverse selection effect because it is unable to effectively separate different types, at which the MLC excels. We show that when the demand and supply mismatch cost is substantial, the conflicted moral hazard effect dominates the adverse selection effect, and the FC is more efficient, and it is the converse otherwise. These findings can enrich the understanding of these two contract forms and are useful for sales and operations planning.

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1. Introduction

A prevalent way to organize supply chain activities is to have production and selling done by different entities. Manufacturers may engage third parties to market and sell their products on their behalf. For example, a maker of fashion apparel may engage a department store to sell its seasonal product lines; a high-end bicycle manufacturer may contract with a dealer to sell its new designs; a home appliance producer may sell its products through telemarketers. A commonly recognized issue in such producer–seller relationships is the incentive conflict between the producer's desire for more sales and the seller's aversion to exerting selling effort. Sales commissions are generally used to incentivize the sellers. However, in an environment with separate production and selling entities, the problem of incentive alignment often goes beyond simply motivating sales. It is almost always true that the seller has close contacts with the end customers and thus is in an ideal position to gather market information, and fresh information from the field helps the producer to plan the right production quantity. Here lies another incentive problem that is often overlooked: the fresh information from the field, although highly valuable to the producer, is costly for the seller to gather. The importance of aligned sales and operations planning (S&OP) is widely recognized. As well, a more complete characterization of the alignment problem should include both information acquisition and selling effort: how can the seller be motivated to exert costly effort to gather market information about the producer's product, share that information with the producer, and work hard to promote sales of the product?

Ideally, the producer would compensate the seller for the amount of effort the seller has devoted to the tasks, both for information acquisition and for selling. However, in many practical settings, such efforts are often not observable, and the producer has to contract with the seller based on the sales. The question hence is how an incentive contract should be designed so that it maximizes the producer's profit. To study this problem, we employ a principal–agent model in which a producer (she) engages a seller (he) to sell a product over a single sales season. The demand for the product in the season is influenced by the market conditions as well as the seller's sales effort. A key feature of our model is that the seller has an opportunity to collect a signal about the market conditions before the sales season and can improve the quality of the signal by expending an information-acquisition effort. The producer is interested in improving her knowledge about the market conditions for better production planning.

In general, many contract forms can be designed to maximize the producer's profit, but not every contract can be easily implemented in practice. In this study, we focus on two contract forms for their popularity and simplicity: the forecast-based contract (FC) and the menu of linear contracts (MLC). Both of these contracts have been widely studied in the literature. In particular, under an FC, the seller can exert an effort to improve the market information he obtains and is asked to submit a demand forecast before the sales season, based on which the producer can fine-tune her production decision. The seller is penalized if the final sales volume deviates from his forecast, and thus he has an incentive to invest in collecting better market information, which improves the accuracy of his demand forecast. In addition to the penalty term, the contract also contains a linear term that is increasing in the total sales, rewarding the seller for increasing sales and thus inducing his selling effort. This FC is a formalization of the Gonik (1978) scheme, which was originally designed to extract information and motivate selling effort from a sales force and was implemented by IBM's Brazilian unit many years ago. Interestingly, the Gonik scheme is alive and well. For instance, Turner et al. (2007) describe that such forecasted-based incentive schemes are being widely used in the pharmaceutical industry in Europe.

Under an MLC, on the other hand, the producer provides the seller with a list of contracts, and each contract is a distinct linear, nondecreasing function that maps the total sales to the seller's compensation. The seller has an opportunity to collect market information before choosing a contract from the menu. Because better information enables the seller to make a better contract choice, the seller is motivated to exert an information-acquisition effort. The market information is conveyed to the producer through the seller's contract choice, and the producer can plan her production accordingly. After the sales season has begun, the seller also has an incentive to exert a selling effort because the more he sells the higher his compensation.

Despite the popularity of the FC in practice, the academic literature seems to suggest that the MLC is superior. In particular, the MLC has been shown to be optimal under various settings where a risk-neutral agent holds private information; see, e.g., Laffont and Tirole (1986) and Rao (1990). In a context related to ours with operations planning, Chen (2005) shows that the MLC is more efficient than the FC even when the agent is risk averse. However, a common assumption made in this literature is that the agent is exogenously endowed with private information. In contrast, our paper introduces an extra incentive problem: the seller can exert a costly effort to improve the quality of the information. Although this addition does not change the fundamental incentive conflicts between the producer and the seller, it alters the producer's objective. Interestingly, we find that the FC can now substantially outperform the MLC, and it can even achieve optimality under our setting. The intuition is intriguing and useful for understanding the performance comparison between these two contract forms.

In our context, the producer wants to motivate the seller to exert both the sales effort and the information-acquisition effort. Although the MLC can be effective in eliciting information and motivating the sales effort alone, it fails to achieve efficiency if the producer also wants to improve the information quality. These two objectives are in conflict under the MLC. In particular, motivating the information-acquisition effort calls for a *broader* menu of contracts with more distinct commission rates, so it is important for the seller to improve the information quality to make more accurate contract choices. On the other hand, inducing the optimal sales effort requires a narrower menu of contracts with similar commission rates. This conflict causes a loss of efficiency, which we call the "conflicted moral hazard effect'' of the MLC. The FC, however, can achieve a separation of these two objectives. A unique feature of this contract is that the seller is requested to submit a forecast and a penalty is imposed on any deviation of the realized sales from the forecast. This penalty term acts to motivate the seller to improve the accuracy of his information and convey it to the producer. On the other hand, the linear commission plan in the contract can separately motivate the seller to exert the sales effort. The advantage of the FC is the largest when the cost of demand and supply mismatch is substantial and the effort to improve the information quality is intermediate so that the conflicted moral hazard effect is the most significant.

To have a deeper understanding, we further examine an extended case where ensuring interim participation is necessary. That is, after the seller obtains the market signal, the producer's original contract needs to ensure that the seller's expected profit is no less than his reservation profit for any possible signal value. This extra participation constraint generates an adverse selection problem, which the MLC can effectively deal with. The FC that contains a single commission plan, however, suffers from the information rents it has to yield to the seller. We call this effect the "adverse selection effect." We find that, when the demand and supply mismatch cost is large, the producer wants to induce a large information-acquisition effort, which leads to the dominance of the conflicted moral hazard effect. The FC is thus preferred to the MLC. The comparison is reversed when the demand and supply mismatch cost is small and the adverse selection effect dominates the conflicted moral hazard effect. The information-acquisition cost also plays an important role. The FC has the best chance of outperforming the MLC when the information-acquisition cost is intermediate. When the information-acquisition cost is either costless or extremely costly, the conflicted moral hazard effect will vanish, which renders the MLC dominant. These observations significantly enrich the understanding of these two popular contract forms. They demonstrate that, in determining an effective incentive alignment for sales and operations planning (S&OP), one needs to carefully examine the value of market information for operations planning, the cost of information acquisition, as well as the seller's participation condition.

The rest of the paper is organized as follows. Section 2 reviews the relevant literature. In §3, we describe the model. Sections 4– 6 contain, respectively, the analysis of the first-best solution, the MLC, and the FC. Section 7 discusses the implications of the interim participation constraint and concludes.

2. Literature

In a supply chain context, we study how the producer can use the incentive contracts to elicit information and motivate the seller to improve information acquisition. Related to our work, Shin and Tunca (2010) investigate how a producer can use a market-based pricing scheme to effectively induce multiple competing retailers to acquire demand information. Focusing on channel coordination, Fu and Zhu (2010) examine the performance of several commonly used contracts (e.g., buy-back, revenue sharing) under costly retailer information acquisition. These studies assume that the information once acquired by the retailer is observable to the producer. Differently, Guo (2009) explores a setting where the information acquired by the retailer is unobservable to the producer unless it is strategically disclosed. He investigates the effects of information acquisition and disclosure on the supply chain parties' profits under the wholesale price contract. Based on a similar setting but allowing nonlinear pricing, Li et al. (2014) show that disclosing just the information status (whether or not information was gained) instead of the content can be beneficial for the retailer. Different from the above studies, our work considers both information-acquisition and sales efforts and focuses on the incentive contracts to motivate such efforts.

Note that the seller's private information in our study is conveyed to the producer by his contract choice or his response to the contract terms. More strategically, information dissemination in supply chains might also be achieved through voluntary sharing. For instance, Li et al. (2014) and Zhang (2002) show that, when there is downstream competition, sellers might be willing to share their private demand information with their common supplier. Information sharing can also arise in competing supply chains when the sellers' incentives to share information are influenced by the contract choice and the accuracy of information as well as the production efficiency in the supply chains (Ha and Tong 2008, Ha et al. 2011). The aforementioned studies all assume that information is shared through installed facilities (e.g., shared database, joint marketing campaign) and thus that information sharing is always truthful. Some recent studies find that sellers might be able to truthfully share their demand forecast with their supply chain partners simply by communication when there are conflicting trade-offs; e.g., announcing a large demand forecast may induce large capacity investment but may also trigger a price increase or downstream competition (Chu et al. 2013, Shamir and Shin 2013). This information-sharing literature generally does not investigate the sellers' information-acquisition effort or sales effort.

Our study is naturally related to the literature that explores incentive contracts to motivate an agent to exert effort with private information. In a seminal work, Laffont and Tirole (1986) show that the MLC is the optimal contract form to motivate the risk-neutral agent to truthfully report his private cost information and exert cost reduction effort in the procurement context. This is affirmed in a separate study by Picard (1987). In an alternative sales force context, Rao (1990) also derives that the MLC achieves optimum results for motivating a risk-neutral sales agent to exert sales effort with private demand information. Under a setting similar to ours with operations planning, Chen (2005) compares the performance of the MLC with that of the FC to elicit information from the agent and motivate him to exert sales effort. He shows that the MLC is still superior to the FC even if the agent is risk averse. The MLC is also applied in Khanjari et al. (2014) as an efficient incentive scheme to elicit private information from a sales agent. They discuss whether the manufacturer or the retailer in the supply chain should hire the sales agent, considering the effect of information asymmetry among the supply chain parties. A common assumption made in these studies is that the agent is exogenously endowed with private information. One exception is Lewis and Sappington (1997) who relax the setting in Laffont and Tirole (1986) by assuming that the agent can decide whether or not to learn the production conditions before choosing a contract from the menu offered by the principal. They show that this agent's choice of whether to learn the market conditions can lead to significant modifications of the classical MLC.

Different from the above studies, in our sales and operations planning (S&OP) context the seller can exert an information-acquisition effort to improve the quality of the demand information he will receive, and our focus is on the comparison between the performances of the MLC and the FC. This reasonable addition to the incentive problem results in the interesting finding that the MLC can be inferior to the FC, especially in an environment where the information on the market conditions is important for the producer's operations planning. It is worth noting that the FC is also related to the so-called bottom-up approach used by many firms in practice to gain information from their sales force. Under this approach, the firm requires its sales force to submit forecast information and ties this information to their sales quotas and performance goals (see, e.g., Mantrala and Raman 1990, Mishra and Prasad 2004). Therefore, the FC can be viewed as a specific incarnation of the bottom-up approach.

3. The Model

Consider a risk-neutral producer who produces a single product and employs a risk-neutral seller to market and sell the product. The demand x is jointly determined by a market condition θ, the seller's sales effort a, and a random noise ε via the following additive form: $x=\theta+a+\varepsilon$. Here, the seller's effort can represent the activities of reaching out to potential customers and persuading them to purchase. Assume that θ and ε are two independent, normally distributed random variables with $\theta\sim N(\mu_\theta,\sigma_\theta^2)$ and $\varepsilon\sim N(0,\sigma_\varepsilon^2)$. The seller incurs a cost of $V(a)$ for exerting sales effort a. For simplicity, let $V(a)=a^2/2$.[1]

Before determining the level of sales effort, the seller has an opportunity to acquire a private signal s about the market condition. Assume that $s=\theta+\eta$, where η is independent of θ and ε with $\eta\sim N(0,\sigma_\eta^2)$. Let $\sigma^2\equiv\sigma_\eta^2/(\sigma_\eta^2+\sigma_\theta^2)$. From the conjugate property of the normal distribution, the posterior distribution of θ for a given signal s is

$$\theta\,|\,s\sim N(\mu_{ps},\sigma_{ps}^2), \tag{1}$$

where

$$(\mu_{ps},\sigma_{ps}^2)\equiv\big(\sigma^2\mu_\theta+(1-\sigma^2)s,\ \sigma_\theta^2\sigma^2\big). \tag{2}$$

Note that σ_η is a measure of the signal's precision in predicting the market condition. The larger the value of σ_η, the less information the signal has about the market condition. The seller can select a value of σ_η, or equivalently σ, by exerting information-acquisition effort. This effort represents the investment that the seller can make to improve the quality of the information (e.g., develop forecasting tools, hire marketing professionals, conduct customer surveys). If $\sigma=0$, the signal reveals the exact value of the market condition. On the other hand, as $\sigma\to 1$, the posterior distribution of the market condition is identical to its prior distribution; i.e., the signal contains no useful information. Let $\Gamma(\sigma)$ be the cost the seller incurs for collecting a signal of precision σ, $\sigma\in(0,1]$, with $\Gamma(1)=0$. We assume that $\Gamma(\sigma)$ is strictly decreasing and convex, i.e., $\Gamma'(\sigma)<0$ and $\Gamma''(\sigma)>0$ for $\sigma\in(0,1]$.[2]

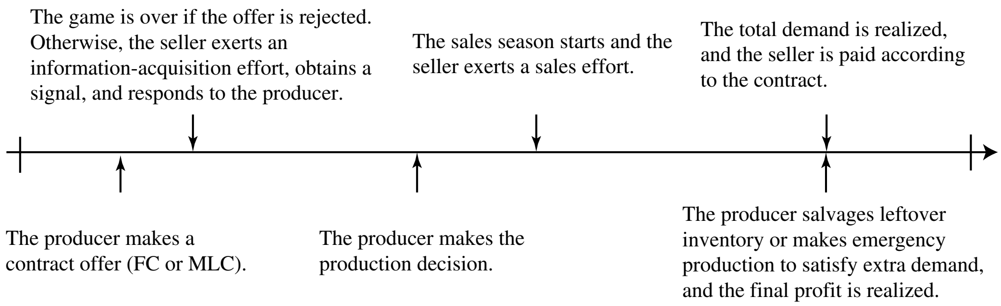
The producer must decide how much to produce before the sales season. This is often true where the production lead time is long relative to the length of the sales season, rendering a make-to-order system impractical. Denote the unit production cost by c. The sales price is $1+c$ (hence the profit margin is normalized to 1). If the total demand exceeds the initial production quantity, the excess demand is satisfied by emergency production with a unit cost c'; otherwise, the leftover inventory is salvaged for v per unit. Reasonably, $v<c<c'<1+c$. Thus, if the demand is x and the initial production quantity is q, then the producer's profit (excluding the seller's compensation) is $x-L(x-q)$, where $L(\cdot)$ represents the total cost of demand and supply mismatch, with $L(z)=(c'-c)\max\{0,z\}+(c-v)\max\{0,-z\}$. Clearly, the more accurate the market information the producer has, the less the mismatch cost she will incur.

However, as in standard agency theory, the seller's information-acquisition effort and sales effort are both unobservable to the producer. Moreover, the market signal the seller collects is also inaccessible to

[1] These assumptions are often made in agency models to evaluate the performance of specific contracts; see, e.g., Lal and Srinivasan (1993).

[2] A similar setting for information improvement is used in the literature; see, e.g., Winkler (1981).

Figure 1 **The Timeline of the Model**



the producer. As such, the only (indirect) measure of the seller's efforts in our model is the realized sales, upon which the producer can contract for the seller's service. We consider two types of contracts for their popularity and simplicity: the FC and the MLC. These contracts have been widely studied in the literature. Let $w(x, y)$ denote the compensation the seller receives under the producer's contract, which is a function of the realized sales x and a parameter y chosen by the seller according to the contract form (it is either a forecast submitted by the seller or the index of a contract selected from the menu). By observing the seller's response y, the producer may gain some information about the market conditions. Let I be the producer's information at the time of making the production decision. The producer's objective is to maximize her expected profit, $E[x-w(x,y)]-E[E[L(x-q)\,|\,I]]$. The seller's objective is to maximize his expected net income, which is equal to the compensation received (w) less the effort costs ($V(a)$ and $\Gamma(\sigma)$). Without loss of generality, the seller's minimum requirement for his expected net income is normalized to 0. Thus, the producer's contract offer must generate for the seller a nonnegative expected net income in order for him to accept it.

Figure 1 details the timeline of the model. First, the producer makes a contract offer to the seller (either an FC or MLC). If the offer is rejected, the game is over. Otherwise, the seller decides how much effort to expend to improve the quality of the information he will receive and then he obtains a signal. The seller responds to the producer according to the specification in the contract offer (either to submit a demand forecast under the FC or to select one compensation plan under the MLC). Based on this response, the producer makes her initial production decision. Then, the sales season starts and the seller decides his sales effort. After the total demand is realized, the producer salvages the leftover inventory if there is any or makes emergency production to meet the extra demand. The seller is paid according to the contract, and the producer's final profit is realized.

4. The First-Best Benchmark

We present the first-best benchmark in this section by assuming that the seller's efforts are contractible and the signal the seller obtains is observable to the producer. As such, the producer can implement any effort level and simply compensate the seller for his cost.

Suppose that the producer implements a contingent optimal sales effort $a(s)$ after the signal s of the market condition is obtained. The distribution of the market condition θ given s is normal with mean μ_{ps} and variance $\sigma_\theta^2\sigma^2$ (see (1) and (2)). Since $x=\theta+a(s)+\varepsilon$ and ε is a normal random variable independent of s, the distribution of the demand x given s is also normal with mean $\mu_{ps}+a(s)$ and variance $\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2$. Note that, in our model, the production decision facing the producer is a standard newsvendor problem. Hence, the producer's optimal production quantity can be expressed as $q(s)=\mu_{ps}+a(s)+\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}\,\Phi^{-1}((c'-c)/(c'-v))$, under which the demand and supply mismatch cost is $E[L(x-q(s))|s]=\rho\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}$, where $\rho=(c'-v)\phi(\Phi^{-1}((c'-c)/(c'-v)))$ with $\phi(\cdot)$ being the standard normal density function and $\Phi^{-1}(\cdot)$ the inverse of the standard normal distribution function. Notice that ρ can sufficiently capture the producer's operations cost structure (i.e., c, c', and v), which will thus be used throughout the analysis.

We can write the producer's optimization problem with respect to the effort levels as

$$\text{(P1)}\qquad \max_{\sigma\in(0,1],\,a(\cdot)\geq 0}\Big\{\mu_\theta+E_s[a(s)]-E_s[V(a(s))]-\Gamma(\sigma)-\rho\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}\Big\}.$$

Proposition 1. *The optimal solution* $\{\sigma_o, a_o(\cdot)\}$ *to* (P1) *is*

$$\sigma_o=\arg\min_{\sigma\in(0,1]}\Big\{\Gamma(\sigma)+\rho\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}\Big\} \tag{3}$$

$$a_o(\cdot)=a_o=\arg\max_{a\geq 0}\{a-V(a)\}=1, \tag{4}$$

based on which the producer's maximum expected profit is

$$\Pi_o=\mu_\theta+1/2-\Gamma(\sigma_o)-\rho\sqrt{\sigma_\theta^2\sigma_o^2+\sigma_\varepsilon^2}. \tag{5}$$

The benefit of having a more accurate signal is a smaller posterior variance of the market condition and thus a lower expected demand and supply mismatch cost. Hence, the optimal level of the signal precision is obtained by minimizing the total cost (see (3)). Due to the additive sales response, the optimal sales effort remains constant for any value of the realized signal. The optimal level of sales effort is derived by balancing the gross profit due to sales effort (profit margin is 1) and the cost of the effort. This explains (4). The producer's profit Π_o is simply her gross profit $(\mu_\theta + a_o)$ less the effort costs compensated to the seller and the expected demand and supply mismatch cost, as given in (5).

5. The Menu of Linear Contracts

In this section, we characterize the optimal MLC for the environment where neither of the seller's two efforts is contractible nor is his received information observable. Clearly, offered with a menu of contracts, the seller will select one that yields him the highest expected compensation upon his receiving the market signal. Hence, to induce the seller to convey his information, the contracts in the menu should have sufficient differences so that the seller makes distinct choices for different signal values. Further, to motivate the seller to exert sales effort, each contract should reward him for more realized sales; on the other hand, to induce the information-acquisition effort, the overall menu should be designed in a way such that it is beneficial for the seller to make more accurate contract choices. In the following, we formulate the seller's and the producer's problems under the MLC.

Let $w(x, \mu_{ps}) = \alpha(\mu_{ps})x + \beta(\mu_{ps})$ denote the seller's compensation, which is a linear function of the realized sales x, with the commission rate $\alpha(\mu_{ps}) \geq 0$ and the fixed transfer $\beta(\mu_{ps})$ each being a function of μ_{ps}, the posterior mean of the market condition reported by the seller. We denote the menu by $\{\alpha(\cdot), \beta(\cdot)\}$. Recall from §2 that after accepting the menu $w(\cdot,\cdot)$, the seller first decides the signal precision σ, then observes the signal s and thus μ_{ps}, then chooses a contract (by reporting $\hat{\mu}_{ps}$), and finally makes the sales effort decision. Next we derive the seller's optimal decisions using backward induction.

We begin with the seller's sales effort decision. The seller who observed μ_{ps} is called the type μ_{ps} seller. Recall that $x \mid \mu_{ps} \sim N(\mu_{ps} + a, \sigma_\theta^2\sigma^2 + \sigma_\varepsilon^2)$. Thus, the type μ_{ps} seller's expected compensation is equal to $\alpha(\hat{\mu}_{ps})(\mu_{ps} + a) + \beta(\hat{\mu}_{ps})$, if he reports $\hat{\mu}_{ps}$ and exerts sales effort a. Subtracting the cost of sales effort $V(a) = a^2/2$ from the expected compensation, we have the type μ_{ps} seller's expected profit (excluding the sunk information-acquisition cost). Maximizing this profit over a, we obtain

$$\pi(\mu_{ps}, \hat{\mu}_{ps}) = \max_{a\geq 0}\big[\alpha(\hat{\mu}_{ps})(\mu_{ps} + a) + \beta(\hat{\mu}_{ps}) - a^2/2\big]$$

which is the type μ_{ps} seller's maximum expected profit if he reports $\hat{\mu}_{ps}$. From the revelation principle, we can without loss of generality restrict ourselves to the menus that induce the seller to truthfully report his type; i.e., $\pi(\mu_{ps}, \mu_{ps}) \geq \pi(\mu_{ps}, \hat{\mu}_{ps})$ for any μ_{ps} and $\hat{\mu}_{ps}$. Let $\pi(\mu_{ps}) \equiv \pi(\mu_{ps}, \mu_{ps})$. Clearly, with truth telling, the type μ_{ps} seller's optimal sales effort, denoted by $a(\mu_{ps})$, is equal to the commission rate of the chosen contract; i.e., $a(\mu_{ps}) = \alpha(\mu_{ps})$.

Now consider the seller's decision on σ, the signal precision. Let $\hat{\pi}(\sigma)$ be the seller's expected profit as a function of σ. Thus

$$\hat{\pi}(\sigma) = E_{\mu_{ps}}[\pi(\mu_{ps})] - \Gamma(\sigma),$$

where $\mu_{ps} \sim N(\mu_\theta, \sigma_\theta^2(1-\sigma^2))$ (see (2)). Let

$$\sigma = \arg\max_{\sigma' \in (0,1]} \hat{\pi}(\sigma'),$$

the optimal signal precision.

We are now ready to consider the producer's optimization problem. Let I represent the producer's information related to the market demand at the time of the production decision. Because of truth telling, $I = \{\mu_{ps}, \sigma\}$, where σ is inferred from the seller's optimization problem. We can then derive the minimum expected demand and supply mismatch cost $\rho\sqrt{\sigma_\theta^2\sigma^2 + \sigma_\varepsilon^2}$ (similar to the first-best solution). Consequently, the producer's expected profit under the menu $\{\alpha(\cdot), \beta(\cdot)\}$ can be written as

$$\Pi(\alpha(\cdot), \beta(\cdot)) = E_{\mu_{ps}}[\mu_{ps} + \alpha(\mu_{ps})] - E_{\mu_{ps}}[\alpha^2(\mu_{ps})/2] - \Gamma(\sigma) - \rho\sqrt{\sigma_\theta^2\sigma^2 + \sigma_\varepsilon^2} - \hat{\pi}(\sigma),$$

which can be interpreted as the supply chain's expected profit (the first four terms: gross profits, cost of sales effort, cost of information acquisition, and cost of demand and supply mismatch) minus the seller's expected profit (the last term). The optimal MLC is the solution to

$$\text{(P2)} \quad \max_{\alpha(\cdot)\geq 0,\, \beta(\cdot)} \big\{\Pi(\alpha(\cdot), \beta(\cdot))\big\}$$

$$\text{s.t.} \quad \pi(\mu_{ps}) \geq \pi(\mu_{ps}, \hat{\mu}_{ps}), \quad \forall\, \mu_{ps}, \hat{\mu}_{ps}, \qquad \text{(IC1)}$$

$$\sigma = \arg\max_{\sigma' \in (0,1]} \hat{\pi}(\sigma'), \qquad \text{(IC2)}$$

$$\hat{\pi}(\sigma) \geq 0. \qquad \text{(IR)}$$

Note that (IC1) and (IC2) are incentive-compatibility constraints, whereas (IR) is the participation constraint. Denote the optimal menu by $\{\alpha^*(\cdot), \beta^*(\cdot)\}$.

Before we present the optimal menu, define

$$\phi(z,\sigma) \equiv \frac{1}{\sigma_\theta\sqrt{1-\sigma^2}}\phi\left(\frac{z-\mu_\theta}{\sigma_\theta\sqrt{1-\sigma^2}}\right) \quad \text{and}$$

$$\bar{\Phi}(z,\sigma) \equiv \bar{\Phi}\left(\frac{z-\mu_\theta}{\sigma_\theta\sqrt{1-\sigma^2}}\right),$$

which are the density and the residual distribution of the normal random variable μ_{ps} with mean μ_θ and standard deviation $\sigma_\theta\sqrt{1-\sigma^2}$. Define $\lambda(\sigma) \equiv \{\lambda \mid \lambda\bar{\Phi}(-1/\lambda) = -\Gamma'(\sigma)\sqrt{1-\sigma^2}/(\sigma_\theta\sigma)\}$ for $\sigma \in (0,1]$. Note that $\lambda\bar{\Phi}(-1/\lambda)$ strictly increases from 0 to infinity as λ increases from 0 to infinity. Because $\Gamma'(\sigma) < 0$ and $\Gamma''(\sigma) > 0$, $-\Gamma'(\sigma)\sqrt{1-\sigma^2}/(\sigma_\theta\sigma)$ is positive and strictly decreases in σ for $\sigma \in (0,1]$. Therefore, $\lambda(\sigma)$ is well defined and strictly decreasing in σ. Let $x^+ \equiv \max\{x, 0\}$.

Proposition 2. *The optimal MLC is*

$$\alpha^*(\mu_{ps}) = \left[1 + \lambda(\sigma^*)\frac{\mu_{ps}-\mu_\theta}{\sigma_\theta\sqrt{1-\sigma^{*2}}}\right]^+$$

and

$$\beta^*(\mu_{ps}) = -\tfrac{1}{2}[\alpha^*(\mu_{ps})]^2 - \mu_{ps}\alpha^*(\mu_{ps}) + \Gamma(\sigma^*) - \int_{-\infty}^{+\infty}\bar{\Phi}(z,\sigma^*)\alpha^*(z)\,dz + \int_{-\infty}^{\mu_{ps}}\alpha^*(z)\,dz,$$

where

$$\sigma^* = \arg\max_{\sigma\in(0,1]}\left\{\int_{-\infty}^{+\infty}\left[\alpha(z,\sigma) - \frac{\alpha^2(z,\sigma)}{2}\right]\phi(z,\sigma)\,dz - \Gamma(\sigma) - \rho\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}\right\}, \tag{6}$$

with

$$\alpha(\mu_{ps},\sigma) \equiv \left[1 + \lambda(\sigma)\frac{\mu_{ps}-\mu_\theta}{\sigma_\theta\sqrt{1-\sigma^2}}\right]^+.$$

The seller will always accept this contract offer, choose signal precision σ^*, collect a market signal (and thus observe μ_{ps}), reveal μ_{ps} to the producer (and thus choose the linear contract with commission rate $\alpha^*(\mu_{ps})$ and fixed transfer $\beta^*(\mu_{ps})$), and finally exert sales effort $a^*(\mu_{ps}) = \alpha^*(\mu_{ps})$. The producer's maximum expected profit is

$$\Pi^*_{MLC} = \mu_\theta + \int_{-\infty}^{+\infty}\left[\alpha^*(z) - \frac{[\alpha^*(z)]^2}{2}\right]\phi(z,\sigma^*)\,dz - \Gamma(\sigma^*) - \rho\sqrt{\sigma_\theta^2\sigma^{*2}+\sigma_\varepsilon^2}. \tag{7}$$

The following corollaries point out the key differences between the optimal MLC and the first-best solution. Recall that $a_o(=1)$ and σ_o are the first-best sales effort and information-acquisition effort, respectively.

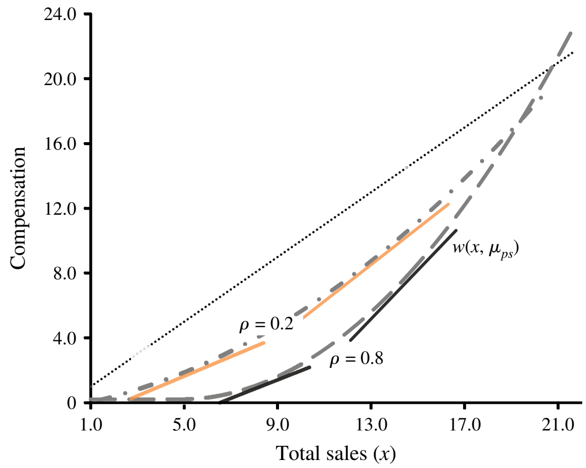
Corollary 1. *If $\rho > 0$, the optimal MLC leads to underinvestment in information acquisition; i.e., $\sigma^* > \sigma_o$. Moreover, $\lim_{\rho\to 0}\sigma^* = \lim_{\rho\to 0}\sigma_o = 1$.*

Corollary 2. *If $\rho > 0$, the optimal MLC leads to overinvestment in sales effort when the market signal is favorable, and underinvestment in sales effort otherwise; i.e., $a^*(\mu_{ps}) > a_o$ for $\mu_{ps} > \mu_\theta$ and $a^*(\mu_{ps}) < a_o$ for $\mu_{ps} < \mu_\theta$. Moreover, $\lim_{\rho\to 0} a^*(\mu_{ps}) = \lim_{\rho\to 0} a_o = 1$.*

Corollary 3. *Let $\Gamma_k(\sigma) \equiv k\Gamma(\sigma)$ for $k > 0$ and $\sigma \in (0,1]$. Let $\Pi_o(k)$ and $\Pi^*_{MLC}(k)$ be the producer's expected profit under the first-best solution and the optimal MLC, respectively, assuming that the seller's cost of information acquisition is $\Gamma_k(\cdot)$. Then $\lim_{k\to 0}\Pi^*_{MLC}(k) = \lim_{k\to 0}\Pi_o(k)$. In other words, as the cost of information acquisition decreases, the producer's optimal expected profit under the MLC approaches her first-best profit.*

Unlike the findings in the existing literature, the above corollaries clearly show that the optimal MLC does not achieve the first best in our context. To understand the intuition, recall that the producer needs to motivate the seller to exert two hidden efforts: gathering information and generating sales. Fundamentally, the MLC loses efficiency because these two goals are in conflict with each other. To motivate the seller to exert the first-best sales effort, all the linear contracts should have the same slope because that gives the marginal return for the seller's sales effort. On the other hand, to motivate the seller to gather information, the producer must offer a broader menu; i.e., the linear contracts in the menu should offer a wide range of slopes so that the optimal contract choice for the seller varies greatly, depending on the market signal. In other words, the contract the seller picks after receiving good news from the market should be very different from the contract chosen when the news is bad. It is thus beneficial to gather precise market information to make more accurate contract choices. As a result, under the MLC form, motivating the optimal sales effort calls for a narrower menu, whereas motivating the optimal information-acquisition effort requires a broader menu. It is this conflict that leads to the efficiency loss of the MLC, which we call the conflicted moral hazard effect. This conflicted moral hazard effect is more significant if the demand and supply mismatch cost is bigger, when the producer wants to motivate a larger information-acquisition effort. These observations are illustrated in Figure 2. We plot the locus of the optimal MLC under $\rho = 0.2$ and $\rho = 0.8$, respectively. Note that any tangent line of the locus represents a linear contract in the corresponding menu. The dotted diagonal line represents the commission rate that would induce the first-best sales effort. We can see that, because of the conflicted moral hazard

Figure 2 (Color online) Illustration of the Characteristics of the Optimal Menu of Linear Contracts



Note. The dash-dotted and the dashed curves represent the locus of the MLC where any tangent line is a linear contract in the menu. The parameters are $\mu_\theta = 10$, $\sigma_\theta = 3$, $\sigma_\varepsilon = 0$, and $\Gamma(\sigma) = 0.2(1/\sigma - 1)$.

effect, the slopes of the linear contracts in the optimal menu deviate from that of the dotted diagonal line, and both downward and upward deviations can arise. Moreover, we can observe that the curvature of the locus increases in ρ. Recall that the demand and supply mismatch cost positively depends on ρ. Therefore, when the mismatch cost increases, the difference among the contracts in the menu will increase, which implies that the conflicted moral hazard effect will be more significant.

The above discussions readily suggest that if motivating information acquisition becomes either less useful or easier, then the producer's profit will approach the first best. The intuition is clear because both of these scenarios make it unnecessary to have a broad menu. The theoretical arguments are provided in Corollaries 1 and 2 when the market information becomes less useful as the demand and supply mismatch cost decreases, and in Corollary 3 when the cost of information acquisition decreases.

6. The FC

In this section, we consider the FC that is obtained by adding a penalty term to a linear contract. Specifically, the seller is required to submit a sales forecast and his compensation is reduced if the actual sales differ from the forecast. Mathematically, the seller's compensation is determined by the following formula:

$$w(x, F) = \alpha x + \beta - \gamma h(x - F) \tag{8}$$

where x and F are the actual sales and the submitted forecast respectively, $\{\alpha, \beta, \gamma\}$ are contract parameters chosen by the producer, and $h(\cdot)$ is a penalty function with $h(0) = 0$, $h'(z) > 0$ for $z > 0$, and $h'(z) < 0$ for $z < 0$. Examples of the penalty function include $h(z) = |z|$, $h(z) = z^2$, etc. Although it appears as a single contract, this FC in fact is also a menu contract because the seller essentially chooses one contract from the menu by specifying his forecast F. Note that the Gonik (1978) scheme is a special case of the above contract form by specifying $h(z) = u_1 z$ for $z > 0$ and $h(z) = -u_2 z$ for $z < 0$ for some positive constants u_1 and u_2. We seek to characterize the optimal contract parameters $\{\alpha^*, \beta^*, \gamma^*\}$ that maximize the producer's expected profit for any general form of $h(\cdot)$.[3]

Suppose that the producer offers the contract (8) to the seller. Assume that the seller accepts the contract. The sellerthen faces a two-stage decision problem. The first stage is to decide the signal precision σ or equivalently the effort for improving the accuracy of the signal. The second stage is after collecting the market signal. Here the seller decides a sales forecast F and determines the sales effort a. Obviously, the forecast and the sales effort can both be functions of the market signal. As before, we substitute μ_{ps} for the market signal. At the second stage, given σ and μ_{ps}, the seller reports the forecast F and exerts sales effort a to maximize his expected profit (excluding the sunk information-acquisition cost); i.e.,

$$\pi(\mu_{ps}) = \max_{F, a \geq 0} \big[\alpha(\mu_{ps} + a) + \beta - \gamma E_x[h(x - F) \mid \mu_{ps}] - a^2/2\big] \tag{9}$$

where $\pi(\mu_{ps})$ is thus the seller's maximum expected profit going forward after observing μ_{ps}. Recall that $x \mid \mu_{ps} \sim N(\mu_{ps} + a, \sigma_\theta^2\sigma^2 + \sigma_\varepsilon^2)$. Thus we can rewrite $x = \mu_{ps} + a + \xi\sqrt{\sigma_\theta^2\sigma^2 + \sigma_\varepsilon^2}$, where $\xi \sim N(0, 1)$. Using this expression in (9) and defining $\Delta = F - \mu_{ps} - a$, we have

$$\pi(\mu_{ps}) = \max_{a \geq 0} \big[\alpha(\mu_{ps} + a) + \beta - a^2/2\big] - \min_{\Delta} \gamma E_\xi\big[h\big(\xi\sqrt{\sigma_\theta^2\sigma^2 + \sigma_\varepsilon^2} - \Delta\big)\big]. \tag{10}$$

Note that the optimization is greatly simplified because the objective function is separable in a and Δ. The optimal solution is $\tilde{a} = \alpha$ and

$$\tilde{\Delta} = \arg\min_{\Delta} E_\xi\big[h\big(\xi\sqrt{\sigma_\theta^2\sigma^2 + \sigma_\varepsilon^2} - \Delta\big)\big].$$

It is important to note that $\tilde{a}$ is independent of the first-stage decision (σ) and the market signal, and that $\tilde{\Delta}$ is independent of the market signal (but may depend on σ). Consequently, the seller's optimal forecast is $\tilde{F} = \mu_{ps} + \alpha + \tilde{\Delta}$. Having characterized the seller's decisions at stage 2, we step back to stage 1,

[3] It will be shown in Proposition 3 that the specific form of the penalty function does not affect the performance of the optimized FC.

where the decision is σ. Denote by $\hat{\pi}(\sigma)$ the seller's expected profit as a function of σ. Clearly,

$$\hat{\pi}(\sigma) = E_{\mu_{ps}}[\pi(\mu_{ps})] - \Gamma(\sigma), \tag{11}$$

where $\mu_{ps} \sim N(\mu_\theta, \sigma_\theta^2(1-\sigma^2))$. Maximizing the above expression over σ leads to the optimal precision level.

We proceed to consider the producer's optimization problem. Let I be the information the producer has that is related to the demand during the sales season at the time of the production decision. Because $\tilde{F} = \mu_{ps} + \alpha + \tilde{\Delta}$, the producer can first solve the seller's first- and second-stage problems to determine the value of $\tilde{\Delta}$, from which she infers the value of μ_{ps} from the seller's submitted forecast $\tilde{F}$. Therefore, $I = \{\mu_{ps}, \sigma\}$. Given this information, the minimum expected demand and supply mismatch cost is $\rho\sqrt{\sigma_\theta^2\sigma^2 + \sigma_\varepsilon^2}$ (see §3). This, together with the fact that the seller exerts sales effort $\tilde{a} = \alpha$, implies that the producer's expected profit, denoted by $\Pi(\alpha, \beta, \gamma)$, can be written as

$$\Pi(\alpha, \beta, \gamma) = \mu_\theta + \alpha - \frac{\alpha^2}{2} - \Gamma(\sigma) - \rho\sqrt{\sigma_\theta^2\sigma^2 + \sigma_\varepsilon^2} - \hat{\pi}(\sigma).$$

The producer's optimization problem can thus be formulated as

$$\text{(P3)} \qquad \max_{\alpha \geq 0,\, \beta,\, \gamma \geq 0} \{\Pi(\alpha, \beta, \gamma)\}$$
$$\text{s.t. } \sigma = \arg\max_{\sigma' \in (0,1]} \hat{\pi}(\sigma'),$$
$$\hat{\pi}(\sigma) \geq 0.$$

Let $\alpha^* = 1$, γ^* be the solution to

$$\arg\min_{\sigma \in (0,1]} \left\{ \gamma^* \min_{\Delta} E_\xi\left[h\left(\xi\sqrt{\sigma_\theta^2\sigma^2 + \sigma_\varepsilon^2} - \Delta\right)\right] + \Gamma(\sigma) \right\} = \sigma_o$$

where $\xi \sim N(0,1)$ and σ_o is the first-best signal precision, and

$$\beta^* = -\mu_\theta - \tfrac{1}{2} + \gamma^* \min_{\Delta} E_\xi\left[h\left(\xi\sqrt{\sigma_\theta^2\sigma_o^2 + \sigma_\varepsilon^2} - \Delta\right)\right] + \Gamma(\sigma_o).$$

Proposition 3. *The FC with parameters $\{\alpha^*, \beta^*, \gamma^*\}$ characterized above is acceptable to the seller. Under this contract, the seller's optimal decisions on the signal precision and the sales effort are $\sigma^* = \sigma_o$ and $a^* = 1$. Moreover, for any given μ_{ps}, the seller's optimal forecast is $F^* = \mu_{ps} + a^* + \Delta^*$, where*

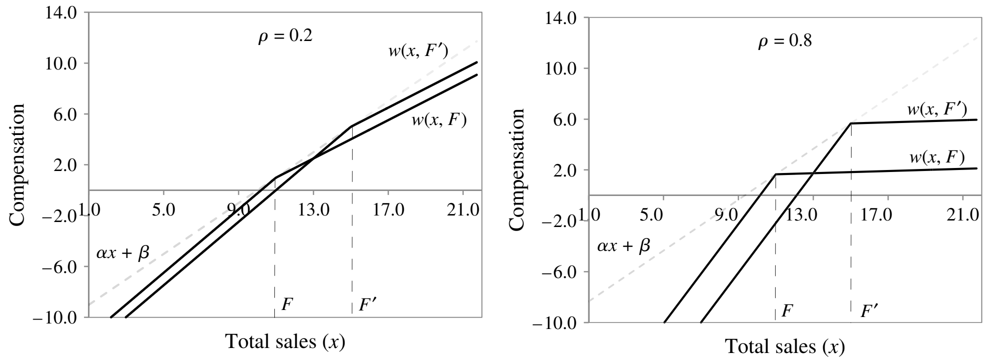
$$\Delta^* = \arg\min_{\Delta} E_\xi\left[h\left(\xi\sqrt{\sigma_\theta^2\sigma_o^2 + \sigma_\varepsilon^2} - \Delta\right)\right]$$

with $\xi \sim N(0,1)$. Therefore, the producer can fully infer the value of μ_{ps} from the seller's forecast. Finally, under this contract, the seller's expected profit is equal to his reservation profit, and the producer's expected profit, denoted by Π_{FC}^*, is equal to the first-best profit, i.e., $\Pi_{FC}^* = \Pi_o$.

We saw in the previous section that the MLC suffers from the conflicted moral hazard effect and loses efficiency at optimum compared to the first-best solution. Therefore, it is all the more interesting to see that the FC can achieve the first best and thus outperforms the MLC. To understand the intuition, it is important to note that the FC consists of two parts: a linear commission plan based on the realized sales and a penalty term based on the difference between the forecast and the realized sales. These two parts can successfully decouple the producer's two objectives in our context. In particular, the first part can motivate the seller to exert sales effort, and the second part can motivate the seller to improve the quality of information and convey it to the producer. When the forecast provided by the seller includes the sales effort that he plans to exert, the potential deviation of the realized sales from the forecast will not be affected by the sales effort under the additive demand structure, and, thus, the penalty term in the contract will have no influence on the seller's decision on his sales effort. This makes it possible for the single linear commission plan to motivate the seller to exert the first-best effort irrespective of the signal he receives. On the other hand, it is clearly in the seller's interest to always include his sales effort in his demand forecast because that will minimize the potential deviation of the realized sales from the forecast and hence minimize the penalty he will incur. It is also in his interest to include the posterior mean of the market condition (μ_{ps}) in the forecast to minimize the potential penalty, which conveys the signal he receives to the producer. Furthermore, through the penalty term, the producer can control the seller's information-acquisition decision (σ) by adjusting the coefficient γ, to achieve an ideal balance between the demand and supply mismatch cost and the information-acquisition cost. Such a decoupling of two objectives overcomes the conflicted moral hazard effect suffered by the MLC and makes it possible to achieve the first best for the FC.

We illustrate the characteristics of the optimal FC in Figure 3. The dashed linear line in each plot is the sum of the fixed payment and commission included in the FC. Therefore, the gap between the solid curve (the actual compensation plan) and the dashed line represents the penalty for the deviation of the actual sales from the reported forecast (the gap is 0 when the actual sales coincide with the forecast). As noted above, the seller's optimal forecast always includes his planned sales effort and the posterior mean of the market conditions. Moreover, the optimal forecast also includes a constant term Δ^* that depends only on the shape of the penalty function and is independent of the sales effort (Proposition 3). Let x (x') be the demand associated with forecast F (F'). Then, the deviation $x - F$ will have exactly the same distribution as $x' - F'$, regardless of the seller's selling effort. Therefore, the expected penalty the seller incurs as a

Figure 3 **Illustration of the Characteristics of the Optimal Forecast-Based Contract**

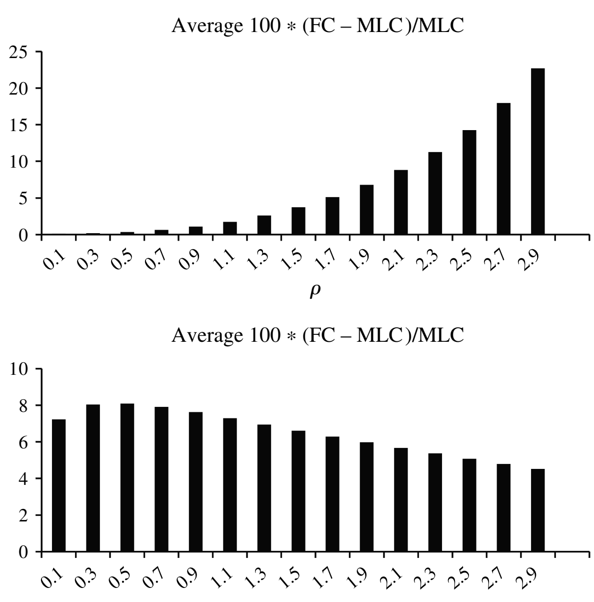


Note. The parameters are $\mu_\theta = 10$, $\sigma_\theta = 3$, $\sigma_\varepsilon = 0$, $\Gamma(\sigma) = 0.2(1/\sigma - 1)$, $h(x-F) = |x-F|$, $\alpha = 1$, $\gamma = 0.246$, and $\beta = -10.01$ (left plot), $\gamma = 0.957$, and $\beta = -9.34$ (right plot).

result of any discrepancy between his forecast and the actual sales, i.e., $E[h(x-F)]$ or $E[h(x'-F')]$, is independent of his sales effort decision. This means that when the seller is making the sales effort decision, he only needs to focus on the dashed linear line, i.e., the linear part of the compensation plan, and can safely ignore his forecast decision. In contrast, under the MLC, the seller's response (i.e., the selection of a contract) will affect the seller's incentive to exert sales effort (we can see from Figure 2 that the slopes of the contracts are all different). Furthermore, we can also observe from the two plots in Figure 3 that a change of the demand and supply mismatch cost does not affect the slope of the dashed linear line (α) but changes the penalty coefficient term (γ). This assures us that the first-best sales effort can always be induced, whereas the change of the penalty term induces the seller to choose a different forecasting precision. Clearly, a larger mismatch cost (captured by a larger ρ) calls for a higher precision, which needs to be motivated by a larger penalty term (the solid curves in Figure 3 have a larger curvature in the right plot compared to those in the left plot).

To better understand the performance comparison between these two contract forms, we conducted a numerical study. We consider 675 instances with $\mu_\theta = 10$, $\sigma_\theta = \{1,2,3\}$, $\sigma_\varepsilon = 0$, $\rho = \{0.1, 0.3, 0.5, \ldots, 2.9\}$, $\Gamma(\sigma) = k(1/\sigma - 1)$ with $k = \{0.1, 0.3, 0.5, \ldots, 2.9\}$, where the penalty function in the FC takes the form of $h(x-F) = |x-F|$. For each instance, we derived the optimal MLC and the associated producer profits. We also computed the optimal FC and the corresponding producer profits (essentially the first-best profits). The numerical study reveals several useful insights. First, we observe that the FC can substantially outperform the MLC and the improvement increases in ρ (or equivalently, the demand and supply mismatch cost). Figure 4 shows that the average percentage improvement across the instances can reach 22.7% when $\rho = 2.9$. Indeed, when the mismatch cost increases, the conflicted moral hazard effect will be more significant for the MLC, and the producer will sacrifice the sales effort to focus more on motivating information acquisition (in the worst case, the producer just gives up inducing any sales effort). Second, we observe that the FC outperforms the MLC the most when the information-acquisition cost is intermediate (see the second panel in Figure 4). This is because if the information-acquisition cost is very small, it will be easy for the producer to induce the seller to exert the information-acquisition effort, and if the information-acquisition cost is very large, the producer will not have a strong incentive to induce the information-acquisition effort. Either way, the conflicted moral hazard effect in these scenarios will not be as significant as when the information-acquisition is intermediate.

Figure 4 **The Comparison Between the Performances of the Optimal Menu of Linear Contracts and the Forecast-Based Contract**



6.1. Incentive-Proof Sales Effort Inducement

Thus far, we have implicitly assumed that the sales effort is a one-time decision of the seller and the decision is made before the demand uncertainty is resolved. It is therefore possible that the actual sales exceed the forecast. Without any restriction on the penalty function, $h(\cdot)$, the penalty for exceeding the forecast may be so large that the seller is actually worse off for higher sales. In practice, if the seller can dynamically adjust his effort as the demand uncertainty unfolds, he will reduce his effort once the sales have reached the forecast level. This may lead to an effort level that is lower than the first best. Fortunately, there exist penalty functions so that the above scenario does not happen. To ensure that the FC continues to incentivize the seller to exert sales effort even after the demand exceeds the forecast, all we need is to have $\alpha > \gamma h'(x-F)$ for all $x \geq F$ (see (8)). The following is such an example.

Corollary 4. *If the penalty function has the form $h(z) = u_1 \max\{0, z\} + u_2 \max\{0, -z\}$, then there exists a set of $\{\alpha^*, \beta^*, \gamma^*, u_1^*, u_2^*\}$ for the FC that can implement the first best and is incentive proof.*

According to Proposition 3, we must have $\alpha = 1$ to motivate the first-best sales effort. Furthermore, when the penalty function is in the form of $h(z) = u_1 \max\{0, z\} + u_2 \max\{0, -z\}$, we must have $\gamma(u_1+u_2) \cdot \phi(\Phi^{-1}(u_1/(u_1+u_2))) = \rho$ to motivate the first-best information-acquisition effort. Finally, in order to be incentive proof as discussed above, the parameters need to satisfy $\alpha > \gamma u_1$. We show in the proof of Corollary 4 that there exist such parameters that satisfy the above conditions simultaneously (based on which, β^* is uniquely determined). The result of Corollary 4 is useful because such an incentive-proof contract with the piecewise linear penalty function can be easily implemented in practice. It is worth noting that, to ensure perfect information extraction from the seller, the condition $h'(z) > (<)\, 0$ for $z > (<)\, 0$ specified early in the model for the penalty function is important. A one-side flat penalty function (e.g., either $u_1 = 0$ or $u_2 = 0$ for the above penalty function) will prevent information extraction when the seller submits the forecast.

6.2. Unbiased Demand Forecast Inducement

Proposition 3 shows that, under the optimal FC, the seller's reported forecast is $F^* = \mu_{ps} + a^* + \Delta^*$. Also, the conditional mean of demand given the market signal is $\mu_{ps} + a^*$. Clearly, if $\Delta^* \neq 0$, the reported forecast is a biased estimate of demand. In practice, it might be useful to induce an unbiased estimate, because the producer can then directly use the seller's forecast to start the production planning, without having to first debias the forecast (by calculating the value of Δ^*). The following corollary provides a sufficient condition to guarantee that the seller's forecast is unbiased.

Corollary 5. *If $h(z) = h(-z)$ for all z, then $\Delta^* = 0$ and the seller's reported forecast is an unbiased estimate of demand.*

From Proposition 3, we have

$$\Delta^* = \arg\min_{\Delta} E_\xi\left[h\left(\xi\sqrt{\sigma_\theta^2\sigma_o^2 + \sigma_\varepsilon^2} - \Delta\right)\right]$$

with $\xi \sim N(0,1)$. Note that the distribution of $\xi\sqrt{\sigma_\theta^2\sigma_o^2 + \sigma_\varepsilon^2}$ is symmetric at 0. It therefore follows that if the penalty function is also symmetric at 0, then the optimal Δ^* is 0.

7. Discussion and Conclusion

7.1. Interim Participation Constraint

It is quite remarkable that, in our model, the FC outperforms the MLC, and the former can even achieve the first best. This is in sharp contrast with the previous literature that often shows the contrary: the MLC is either the optimal contract form or simply better than the FC (see, e.g., Laffont and Tirole 1986, Rao 1990, Chen 2005). To understand these different conclusions, it is helpful to note that in our model the seller needs to decide whether or not to accept the producer's contract offer before he collects any information of the market condition; once he agrees, the seller cannot renege regardless of the realized market signal he receives. These assumptions are reasonable in the supply chain context on which our study focuses. First, to conduct a demand forecast may require significant investment in market study and might also be time consuming, which makes it necessary for the two parties to first agree on the contract terms before the seller takes any action. Second, the penalties for breaching the contract in such business relationships can be sufficiently severe that the seller will always honor the contract. The penalties may take various forms, such as damage to the seller's reputation, a court-enforced monetary payment, the termination of a long-term relationship, etc. In such circumstances, the producer only needs to guarantee that the seller receive an expected profit no less than his reservation profit in order for him to accept the contract. This is often referred to as the "ex ante participation constraint" in the principal–agent literature. Consequently, the producer's problem is not subject to the adverse selection effect. As a result, once a contract overcomes the conflicted moral hazard effect, it becomes feasible to implement the first-best solution as the FC does.

Most of the existing studies assume that the agent's private information is exogenously endowed and, moreover, that his participation needs to be ensured for any received information (which is often referred to as the "interim participation constraint"). This would correspond to the following setting in our context: the seller does not have the capability to improve the quality of the information he obtains and can renege after observing a bad signal. Clearly, in such a situation, the conflicted moral hazard effect disappears, while the adverse selection effect will come into play since the seller may have an incentive to misreport and his interim participation condition needs to be satisfied. As demonstrated by the existing literature, the MLC is effective in dealing with the adverse selection effect, which can contain a complete set of commission and fixed payment pairs, corresponding to the set of signals the seller may receive. One can adjust the commission rates and the fixed payments (a high commission rate is paired with a low fixed payment, and vice versa) to prevent the seller from misreporting (see the tangent lines in Figure 2 that correspond to the linear contracts). The information rent paid to the seller is the least under the MLC. The FC, however, will suffer from the adverse selection effect. Given that it contains only a single commission and fixed payment pair, the seller will obtain a higher expected profit when he receives a good signal than a bad one. With the interim participation constraint, the producer needs to ensure the participation of the seller with the most negative signal, which implies that the seller will receive information rent whenever he observes a better signal. This adverse selection effect degrades the performance of the FC.

Hence, it becomes clear that, in the contexts of the existing literature, without the conflicted moral hazard effect, the adverse selection effect hurts the performance of the FC, which leads to the dominance of the MLC. In contrast, in our setting, the adverse selection effect is absent, and the MLC suffers from the conflicted moral hazard effect, which results in the dominance of the FC.

Although the setup of our model is reasonable in a supply chain context, it is still interesting to consider the performances of the above two contract forms when both the conflicted moral hazard effect and the adverse selection effect are present. For this purpose, we relax our original setting by allowing the seller to renege after receiving the signal, while keeping the assumption that it is always profitable for the producer to engage the seller regardless of the value of the signal. As a result, in addition to the ex ante participation constraint, the producer also needs to guarantee that the expected profit the seller will obtain under any signal he receives be no less than his reservation profit. In such a scenario, the producer needs to balance the costs due to moral hazard and adverse selection. We should point out that this is a significantly more challenging problem because it nests two moral hazard subproblems separated by adverse selection. Deriving the optimal contract is difficult even numerically. However, we are able to obtain some structural results.

Proposition 4. *With the addition of the interim participation constraint, there exist two thresholds $\underline{\rho} < \bar{\rho}$ such that the FC dominates the MLC when $\rho > \bar{\rho}$ and it is the converse when $\rho < \underline{\rho}$.*

Specifically, when the demand and supply mismatch cost is substantial (i.e., ρ is large), the producer wants to induce a large information-acquisition effort, which leads to the dominance of the conflicted moral hazard effect over the adverse selection effect. Hence, the FC is superior to the MLC. Conversely, if the demand and supply mismatch cost is small (i.e., ρ is small), the accuracy of the signal the seller receives becomes less important. In this situation, the producer is content with a small information-acquisition effort but keen on reducing the information rents. As a result, the adverse selection effect dominates, and the MLC is preferred to the FC.

Besides the demand and supply mismatch cost, the insight we discussed in the previous section with respect to the information-acquisition cost still holds. The FC will have the best chance of outperforming the MLC when the information-acquisition cost is intermediate and the conflicted moral hazard effect is the most significant. When the information-acquisition effort is either costless or extremely costly, the conflicted moral hazard effect will vanish and thus the MLC will dominate the FC in the presence of the interim participation constraint. Table 1 summarizes these results.

7.2. Concluding Remarks

Most studies of decentralized supply chains assume that the actors are simply endowed with information (some may be private) about the operating environment. However, the reality is often that valuable information needs to be gathered and doing so is costly. This paper fills that important gap by considering a producer–seller relationship where the seller, besides his selling role, is uniquely positioned to gather market information that can be used to improve production planning by the producer. We study two popular contract forms for the incentive alignment problem in such a context. One is the FC, under which the seller is required to report a demand forecast and gets commissions from the sales but also needs to pay a penalty if the sales differ from the forecast. The other is the MLC, each of which specifies a unique commission rate and a fixed payment. Our analysis reveals an important difference between these two contract forms in their abilities to manage the tension

Table 1 **MLC vs. FC**

Interim participation?	Intermediate cost of information acquisition	Costless or extreme cost of information acquisition
No	FC $\geq$ MLC	FC $=$ MLC
Yes	FC $\leq$ MLC if ρ is small FC $\geq$ MLC if ρ is large	FC $\leq$ MLC

between motivating for information acquisition and motivating for sales effort. Specifically, the MLC suffers from the conflicted moral hazard effect that creates friction between these two tasks: motivating information acquisition calls for a broader menu of contracts, but for motivating the sales effort it is better to have a narrower menu. In contrast, we find that the FC can effectively decouple these two motivation tasks. As a result, when it is not critical to ensure the interim participation condition for the seller (i.e., once the seller accepts the contract offer, he has to participate regardless of the information he receives), the FC is superior andcan substantially outperform the MLC in certain circumstances. However, when the producer has to ensure the seller's interim participation condition (e.g., renegotiation is difficult to prevent), the FC might suffer from the adverse selection effect because it is unable to effectively separate different types, at which the MLC excels. In such an environment, if the demand and supply match cost is substantial and the information-acquisition cost is neither costless nor extremely costly, the conflicted moral hazard effect dominates the adverse selection effect and hence the FC is more efficient in aligning the seller's incentives than is the MLC; it is the converse otherwise. These observations significantly enrich the current knowledge about these two contract forms and can provide useful insights for sales and operations planning (S&OP) in practice.

Future research can attempt to relax some of the key assumptions made in this paper. One such assumption is the additive form of demand, which has been widely used in the literature (see, e.g., Laffont and Tirole 1986, Picard 1987, Chen 2005, Khanjari et al. 2014). With this specific form of demand, the sales effort only changes the mean of the demand distribution; it does not affect the variance. This simple demand structure allows the FC to completely decouple the tasks of motivating the seller for sales effort and motivating him for information acquisition. If, on the other hand, the sales effort can also influence the demand variance (as in a multiplicative form), then the motivation for the sales effort will be intertwined with the motivation for the information-acquisition effort because both efforts will affect the demand and supply mismatch cost. In this case, the first-best solution may no longer be achievable with the simple FC. The analysis can be technically challenging. Nevertheless, the qualitative insight from the FC, i.e., decoupling the two motivational tasks, should point in a right direction. The second key assumption is the normal distribution, for both the market uncertainty and the signal. This gives us the conjugate property for simple Bayesian updating, which allows us to derive some elegant results. It is possible to use general demand distributions, however, so long as there is a simple mechanism for Bayesian updating. The same qualitative insights should still work; i.e., the two contract forms will exhibit different abilities in managing the conflicted moral hazard effect and the adverse selection effect, and which of these contract forms is preferred will depend on which of the two effects is more pronounced. Finally, the paper has confined itself to only two simple contract forms. Other more sophisticated contract forms are, of course, possible and can be considered in future research.

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Appendix

Proof of Proposition 1. Given that the problem is concave, the optimal solutions of the effort levels are unique and can be derived: $a_o = \arg\max_{a\geq 0}\{a - V(a)\} = 1$ and

$$\sigma_o = \arg\min_{\sigma\in(0,1]}\{\rho\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}+\Gamma(\sigma)\}.$$

The producer can also specify a compensation plan contingent on the efforts to motivate the seller. □

Proof of Proposition 2. The proof is carried out in two main steps. First, we derive a constraint that must be satisfied if (IC1) and (IC2) are both satisfied. The producer's objective function, together with this new constraint, forms a relaxed optimization problem, whose optimal objective is clearly an upper bound of that of (P2). Second, we solve the relaxed problem and prove that its optimal solution satisfies all the constraints of (P2) and thus is also an optimal solution to (P2).

It follows from the Envelope Theorem and (IC1) that

$$\pi'(\mu_{ps}) = \frac{\partial \pi(\mu_{ps}, \hat{\mu}_{ps})}{\partial \mu_{ps}}\bigg|_{\hat{\mu}_{ps}=\mu_{ps}} = \alpha(\mu_{ps}),$$

which by integration, leads to

$$\pi(\mu_{ps}) = \int_{-\infty}^{\mu_{ps}} \alpha(z)\,dz + \pi(-\infty). \tag{13}$$

Substituting the above expression of $\pi(\mu_{ps})$ into (IC2), we can rewrite (IC2) as follows:

$$\begin{aligned}\sigma &= \arg\max_{\sigma'\in(0,1]}\left\{E_{\mu_{ps}}\int_{-\infty}^{\mu_{ps}}\alpha(z)\,dz - \Gamma(\sigma')\right\}\\ &= \arg\max_{\sigma'\in(0,1]}\left\{\int_{-\infty}^{+\infty}\int_{-\infty}^{y}\alpha(z)\phi(x,\sigma')\,dz\,dy - \Gamma(\sigma')\right\}\\ &= \arg\max_{\sigma'\in(0,1]}\left\{\int_{-\infty}^{+\infty}\alpha(z)\bar{\Phi}(z,\sigma')\,dz - \Gamma(\sigma')\right\},\end{aligned}$$

where the second equality is due to $\mu_{ps} \sim N(\mu_\theta, \sigma_\theta^2(1-\sigma'^2))$ and the last equality is by integration by part. Therefore, the first-order necessary condition of (IC2) is

$$\int_{-\infty}^{+\infty} \alpha(z)\frac{(z-\mu_\theta)\sigma}{(1-\sigma^2)}\phi(z,\sigma)\,dz + \Gamma'(\sigma) = 0,$$

or equivalently,

$$\int_{-\infty}^{+\infty} \alpha(z)\frac{(z-\mu_\theta)}{\sigma_\theta\sqrt{1-\sigma^2}}\phi(z,\sigma)\,dz + \Gamma'(\sigma)\sqrt{1-\sigma^2}/(\sigma_\theta\sigma) = 0. \tag{14}$$

Note that, at the optimal solution, $\hat{\pi}(\sigma)=0$, because otherwise the solution can be improved by decreasing $\beta(\cdot)$ by a small value. The new derived constraint (14), together with the objective function of (P2), forms the following relaxed problem, denoted by (P2′):

$$\text{(P2′)} \quad \max_{\alpha(\cdot)\ge 0,\,\sigma\in(0,1]} E_{\mu_{ps}}[\alpha(\mu_{ps}) - \alpha^2(\mu_{ps})/2] - \Gamma(\sigma) - \rho\sqrt{\sigma_\theta^2\sigma^2 + \sigma_\varepsilon^2}, \quad \text{s.t. (14)},$$

where we remove from the objective function the constant terms that are independent of the decision variables.

Because the objective function of (P2′) is quadratic and the constraint is linear in $\alpha(\cdot)$, we can solve (P2′) by using the Karush–Kuhn–Tucker (KKT) condition. Let $\lambda(\sigma)$ be the Lagrange multiplier associated with the constraint. It follows from the KKT condition (taking the derivative of the Lagrange function with respect to $\alpha(z)$) that the optimal value of $\alpha(z)$ for any given σ, denoted by $\alpha(z,\sigma)$, is

$$\alpha(z,\sigma) = \left[1 + \lambda(\sigma)\frac{z-\mu_\theta}{\sigma_\theta\sqrt{1-\sigma^2}}\right]^+.$$

Substituting the above expression for $\alpha(z)$ in the constraint and after some algebra, we have $\lambda(\sigma) = \{\lambda \mid \lambda\bar{\Phi}(-1/\lambda) = -\Gamma'(\sigma)\sqrt{1-\sigma^2}/(\sigma_\theta\sigma)\}$, which is the same as the notation $\lambda(\sigma)$, introduced before Proposition 2. The results then follow because we can substitute the above expression for $\alpha(z)$ in the objective function of (P2′) so as to reduce (P2′) to the following unconstrained optimization problem:

$$\max_{\sigma\in(0,1]}\left\{\int_{-\infty}^{+\infty}\left[\alpha(z,\sigma) - \frac{\alpha^2(z,\sigma)}{2}\right]\phi(z,\sigma)\,dz - \Gamma(\sigma) - \rho\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}\right\}.$$

Consequently, the optimal solution to (P2′), denoted by $\{\sigma^*, \alpha^*(\cdot)\}$, takes the form given in Proposition 2.

Next we construct $\beta^*(\cdot)$ based on (13) and the binding (IR) constraint. It follows from the definition of $\pi(\mu_{ps})$ that

$$\pi(\mu_{ps}) = \alpha(\mu_{ps})\left(\mu_{ps} + \frac{\alpha(\mu_{ps})}{2}\right) + \beta(\mu_{ps}).$$

This, together with (13), leads to

$$\beta(\mu_{ps}) = -\alpha(\mu_{ps})\left(\mu_{ps} + \frac{\alpha(\mu_{ps})}{2}\right) + \int_{-\infty}^{\mu_{ps}} \alpha(z)\,dz + \pi(-\infty). \tag{16}$$

It follows from the binding (IR) constraint that

$$\begin{aligned} 0 &= \hat{\pi}(\sigma) \\ &= E_{\mu_{ps}}\int_{-\infty}^{\mu_{ps}} \alpha(z)\,dz + \pi(-\infty) - \Gamma(\sigma) \\ &= \int_{-\infty}^{+\infty} \alpha(z)\bar{\Phi}(z,\sigma)\,dz + \pi(-\infty) - \Gamma(\sigma), \end{aligned}$$

which implies that

$$\pi(-\infty) = -\int_{-\infty}^{+\infty} \alpha(z)\bar{\Phi}(z,\sigma)\,dz + \Gamma(\sigma). \tag{17}$$

By (16) and (17), we have

$$\beta(\mu_{ps}) = -\alpha(\mu_{ps})\left(\mu_{ps} + \frac{\alpha(\mu_{ps})}{2}\right) + \Gamma(\sigma) - \int_{-\infty}^{+\infty}\alpha(z)\bar{\Phi}(z,\sigma)\,dz + \int_{-\infty}^{\mu_{ps}}\alpha(z)\,dz.$$

Therefore, we can construct $\beta^*(\cdot)$ from the above equation by replacing $\{\sigma, \alpha(\cdot)\}$ with $\{\sigma^*, \alpha^*(\cdot)\}$.

Now it remains to verify that the constructed solution $\{\alpha^*(\cdot), \beta^*(\cdot)\}$ satisfies (IC1) and (IC2). First, it follows from the standard result in adverse selection that the sufficiency of (IC1) is ensured if $\alpha^*(\cdot)$ is monotone. The monotonicity property is easily verifiable from the definition of $\alpha^*(\cdot)$. Thus, (IC1) is satisfied. Second, to verify (IC2), we need to show that

$$\sigma^* \in \arg\max_{\sigma\in(0,1]}\left\{\int_{-\infty}^{+\infty}\alpha^*(z)\bar{\Phi}(z,\sigma)\,dz - \Gamma(\sigma)\right\}.$$

Because σ^* satisfies the first-order necessary condition of the above maximization problem, it suffices to show that the above objective function is concave in σ, which follows because its first part is concave (by Lemma 1) and its second part $\Gamma(\sigma)$ is convex. □

Lemma 1. *Let*

$$H(\sigma) \equiv \int_{-\infty}^{+\infty}(1+kz)^+\bar{\Phi}\left(\frac{z}{\sqrt{1-\sigma^2}}\right)dz$$

with $k \ge 0$. Then $H(\sigma)$ is nonincreasing and concave; i.e., $H'(\sigma) \le 0$ and $H''(\sigma) \le 0$ for every $\sigma \in (0,1]$.

Proof of Lemma 1. Note that

$$\begin{aligned} H'(\sigma) &= -\int_{-\infty}^{+\infty}(1+kz)^+\phi\left(\frac{z}{\sqrt{1-\sigma^2}}\right)\frac{z\sigma}{(1-\sigma^2)\sqrt{1-\sigma^2}}\,dz \\ &= -\int_{-\infty}^{+\infty}\left(1+k\sqrt{1-\sigma^2}y\right)^+\phi(y)\frac{y\sigma}{\sqrt{1-\sigma^2}}\,dy \\ &= -\int_{-1/k\sqrt{1-\sigma^2}}^{+\infty}\left(1+k\sqrt{1-\sigma^2}y\right)\phi(y)\frac{y\sigma}{\sqrt{1-\sigma^2}}\,dy \\ &= -k\sigma\bar{\Phi}\left(-\frac{1}{k\sqrt{1-\sigma^2}}\right), \end{aligned}$$

where the last equality follows from the fact that $\int_\Delta^{+\infty}\phi(y)y\,dy = \phi(\Delta)$ and $\int_\Delta^{+\infty}\phi(y)y^2\,dy = \Delta\phi(\Delta) + \bar{\Phi}(\Delta)$ for any Δ. Clearly, $H'(\sigma) \le 0$ and $H''(\sigma) \le 0$. □

Proof of Corollary 1. By definition of σ^* in Proposition 2,

$$\begin{aligned}\sigma^* &= \arg\max_{\sigma\in(0,1]}\left\{\frac{1}{2}\int_{-1/\lambda(\sigma)}^{+\infty}(1-\lambda^2(\sigma)y^2)\phi(y)\,dy - \Gamma(\sigma)-\rho\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}\right\}\\ &= \arg\min_{\sigma\in(0,1]}\left\{-\frac{1}{2}\int_{-1/\lambda(\sigma)}^{+\infty}(1-\lambda^2(\sigma)y^2)\phi(y)\,dy + \Gamma(\sigma)+\rho\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}\right\}\\ &\geq \arg\min_{\sigma\in(0,1]}\{\Gamma(\sigma)+\rho\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}\}=\sigma_o,\end{aligned}$$

where the inequality follows from the fact that

$$-\frac{1}{2}\int_{-1/\lambda(\sigma)}^{+\infty}(1-\lambda^2(\sigma)y^2)\phi(y)\,dy$$

strictly decreases in σ (because $\lambda(\sigma)$ strictly decreases in σ). The latter part of Corollary 1 follows directly from the definition of σ^* and σ_o. □

Proof of Corollary 2. The result follows directly from the fact that $a^*(\mu_{ps})=\alpha^*(\mu_{ps})$ and from the definition of $\alpha^*(\mu_{ps})$ in Proposition 2. □

Proof of Corollary 3. As $k\to 0$, $\lambda(\sigma)$ goes to 0 by its definition and thus σ^* goes to 0 by (6). It then follows from (5) and (7) that $\lim_{k\to 0}\Pi^*_{\mathrm{MLC}}(k)=\lim_{k\to 0}\Pi_o(k)=\mu_\theta+1/2-\rho\sigma_\varepsilon$. □

Proof of Proposition 3. Under the FC $\{\alpha^*,\beta^*,\gamma^*\}$, it follows from the analysis before Proposition 3 that the type μ_{ps} seller's optimal sales effort and forecast are

$$a(\mu_{ps})=\alpha^*=1$$

and

$$F(\mu_{ps})=\mu_{ps}+a(\mu_{ps})+\Delta(\sigma)=\mu_{ps}+1+\Delta(\sigma),$$

where $\Delta(\sigma)=\arg\min_\Delta E_\xi[h(\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}\xi-\Delta)]$.

Now consider the seller's decision on the signal precision. Given the seller's optimal decisions on sales effort and forecast, we can simplify (11) as follows:

$$\begin{aligned}\sigma &= \arg\min_{\sigma\in(0,1]}\{\gamma^*E_\xi[h(\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}\xi-\Delta(\sigma))]+\Gamma(\sigma)\}\\ &= \arg\min_{\sigma\in(0,1]}\{\gamma^*\min_\Delta E_\xi[h(\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}\xi-\Delta)]+\Gamma(\sigma)\}\\ &=\sigma_o,\end{aligned}$$

where the second equality is by definition of $\Delta(\sigma)$, and the last equality follows from the definition of γ^*.

Finally, it remains to show the existence of γ^*. Let $H(\gamma,\sigma)\equiv\gamma\min_\Delta E_\xi[h(\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}\xi-\Delta)]+\Gamma(\sigma)$. The existence of γ^* is ensured by the following two observations. First, as γ goes to 0, $\arg\min_{\sigma\in(0,1]}H(\gamma,\sigma)$ goes to 1. This observation follows from the fact that $\Gamma(\sigma)$ is decreasing in σ. Second, as γ goes to infinity, $\arg\min_{\sigma\in(0,1]}H(\gamma,\sigma)$ goes to 0. To prove the second observation, it suffices to show that $K(\sigma)\equiv\min_\Delta E_\xi[h(\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}\xi-\Delta)]$ strictly increases in σ. By the Envelope Theorem, we have that

$$\begin{aligned}K'(\sigma) &= E_\xi\left[\frac{\sigma_\theta^2\sigma}{\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}}\xi h'(\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}\xi-\Delta(\sigma))\right]\\ &= E_{\tilde\xi}\left[\frac{\sigma_\theta^2\sigma}{\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}}\tilde\xi h'(\tilde\xi)\right],\end{aligned}$$

where $\tilde\xi=\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}\xi-\Delta(\sigma)$ and the last equality follows from the first-order necessary condition that $E_\xi[h'(\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}\xi-\Delta(\sigma))]=0$. Because $h'(z)<0$ for $z<0$ and $h'(z)>0$ for $z>0$, the above equation implies that $K'(\sigma)>0$ for $\sigma\in(0,1]$. □

Proof of Corollary 4. Consider a special form of the optimal FC $\{\alpha^*,\beta^*,\gamma^*\}$, where the contract parameters are defined before Proposition 3 and the penalty function $h(x-F)=u_1^*\max\{0,x-F\}+u_2^*\max\{0,F-x\}$ with $u_1^*=c'-c$ and $u_2^*=c-v$. It follows from Proposition 3 that this FC achieves the first-best solution. Further, in determining the optimal forecast, the seller solves the classical newsvendor problem with the overforecast cost $c-v$ and the underforecast cost $c'-c$. Hence, the seller's optimal forecast under any given signal μ_{ps} is $F(\mu_{ps})=\mu_{ps}+a(\mu_{ps})+\Delta(\sigma)=\mu_{ps}+1+\Delta(\sigma)$ with

$$\Delta(\sigma)=\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}\,\Phi^{-1}\left(\frac{c'-c}{c'-v}\right),$$

under which the expected forecast penalty is $\gamma^*\rho\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}$. Therefore, to ensure that the seller exerts the first-best forecast effort, it suffices to set $\gamma^*=1$, under which the seller's optimal signal precision is

$$\sigma^*=\arg\min_{\sigma\in(0,1]}\{\Gamma(\sigma)+\rho\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}\},$$

which is indeed equal to the first-best signal precision σ_o. The corollary follows because $\gamma^*u_1^*=c'-c<1=\alpha^*$, implying that this specific form of FC not only achieves the first-best solution but also ensures that the seller's total compensation always increases in the realized sales volume x. □

Proof of Corollary 5. To determine his forecast, the seller solves $\Delta^*=\arg\min_\Delta E_\xi[h(\xi\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}-\Delta)]$ where $\xi\sim N(0,1)$. Given that the distribution of $\xi\sqrt{\sigma_\theta^2\sigma_o^2+\sigma_\varepsilon^2}$ is symmetric at 0, if $h(z)=h(-z)$, then $\Delta^*=0$ (recall we have assumed that the penalty function satisfies $h(0)=0$, $h'(z)>0$ for $z>0$, and $h'(z)<0$ for $z<0$). Hence, the seller's optimal forecast under any given signal μ_{ps} is $F(\mu_{ps})=\mu_{ps}+a(\mu_{ps})$ that is unbiased. □

Proof of Proposition 4. When $\rho=0$, the optimal FC takes the form of a single linear contract, which is weakly dominated by the MLC. This suggests that there exists a threshold $\underline{\rho}$ such that the MLC is better than the FC when $\rho\leq\underline{\rho}$.

Consider the other extreme case where ρ is sufficiently large. Because the inclusion of the interim participation constraint reduces the producer's expected profit, Π^*_{MLC} (defined in (7)) is an upper bound on the producer's expected profit under the optimal MLC. On the other hand, under the FC $\{0, \beta^* + 1/2, \gamma^*\}$ where β^* and γ^* are defined before Proposition 3, the seller chooses the first-best signal precision and exerts no sales effort regardless of the signal. Thus the seller's interim participation constraint is ensured by his ex ante participation constraint. The producer's expected profit under this contract, denoted by $\underline{\Pi}_{\rm FC}$, is $\underline{\Pi}_{\rm FC} = \mu_\theta - \min_\sigma\{\Gamma(\sigma) + \rho\sqrt{\sigma_\theta^2\sigma^2 + \sigma_\varepsilon^2}\}$. Clearly, $\underline{\Pi}_{\rm FC}$ is a lower bound on the producer's expected profit under the optimal FC. It remains to show that $\underline{\Pi}_{\rm FC} > \Pi^*_{\rm MLC}$ when ρ is sufficiently large.

It follows from the definition of $\alpha(z,\sigma)$ (in Proposition 2) that there exists $\bar\sigma$ such that

$$\int_{-\infty}^{+\infty}\left[\alpha(z,\sigma) - \frac{[\alpha(z,\sigma)]^2}{2}\right]\phi(z,\sigma)\,dz < 0, \tag{18}$$

for any $\sigma \in (0, \bar\sigma)$. Let ρ be sufficiently large so that

$$\Gamma(\sigma)+\rho\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}-\min_\sigma\{\Gamma(\sigma)+\rho\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}\} > \tfrac{1}{2} \tag{19}$$

for any $\sigma \in [\bar\sigma, 1]$. If $\sigma^* \in (0, \bar\sigma)$, then

$$\begin{aligned}
\Pi^*_{\rm MLC} &= \mu_\theta + \int_{-\infty}^{+\infty}\left[\alpha^*(z) - \frac{[\alpha^*(z)]^2}{2}\right]\phi(z,\sigma^*)\,dz \\
&\quad - \Gamma(\sigma^*) - \rho\sqrt{\sigma_\theta^2\sigma^{*2}+\sigma_\varepsilon^2} \\
&< \mu_\theta - \Gamma(\sigma^*) - \rho\sqrt{\sigma_\theta^2\sigma^{*2}+\sigma_\varepsilon^2} \qquad \text{(by (18))} \\
&\le \mu_\theta - \min_\sigma\{\Gamma(\sigma)+\rho\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}\} \\
&= \underline{\Pi}_{\rm FC}.
\end{aligned}$$

If $\sigma^* \in [\bar\sigma, 1]$, then

$$\begin{aligned}
\Pi^*_{\rm MLC} &= \mu_\theta + \int_{-\infty}^{+\infty}\left[\alpha^*(z) - \frac{[\alpha^*(z)]^2}{2}\right]\phi(z,\sigma^*)\,dz \\
&\quad - \Gamma(\sigma^*) - \rho\sqrt{\sigma_\theta^2\sigma^{*2}+\sigma_\varepsilon^2} \\
&\le \mu_\theta + \tfrac{1}{2} - \Gamma(\sigma^*) - \rho\sqrt{\sigma_\theta^2\sigma^{*2}+\sigma_\varepsilon^2} \\
&\le \mu_\theta - \min_\sigma\{\Gamma(\sigma)+\rho\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}\} \qquad \text{(by (19))} \\
&= \underline{\Pi}_{\rm FC}. \quad \square
\end{aligned}$$

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CORRECTION

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