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Consideration Set Formation with Multiproduct Firms: The Case of Within-Firm and Across-Firm Evaluation Costs

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We consider a theoretical setting in which firms carry multiple products and consumers incur evaluation costs not only across firms but also within firms. Consumers judiciously decide the number of firms to include in their consideration sets as well as how many products from those firms. This decision depends on the relative trade-offs of evaluating an additional product and whether it is from a firm already included in the consideration set or from an entirely new firm. The composition of consumers' consideration set affects how firms compete in prices and in the number of products to offer. Contrary to previous literature, we find that firm differentiation can reduce firms' product lines and within-firm evaluation costs have either a positive or a negative effect on firms' prices. Interestingly, we show that within-firm evaluation costs and across-firm evaluation costs are different constructs. The number of products firms offer in equilibrium can exceed the socially optimal level if within-firm evaluation costs are significant.

Key words: consideration set formation; multiproduct firms; within-firm search; product line; pricing
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1. Introduction

A consumer making a specialty purchase will often face a large number of alternatives, and determining which of these alternatives best satisfies her tastes may be nontrivial. Not only may there be many firms to consider, but each firm can carry a long line of products. If the consumer incurs a cost to inspect and evaluate each product, she will find it sensible to limit the number of options to consider (Hauser and Wernerfelt 1990). How many and which products to include in her consideration set depends on the relative trade-offs of evaluating an additional product, which may further depend on whether the additional product is from a firm that will already be included in the consideration set or from an entirely new firm. Two products from the same firm, for example, will contain similar features and be easier to evaluate than two products from differentiated firms. By the same token, the consumer improves her chances of finding a better fitting product if she expands her consideration set to include products from more firms. Therefore, when facing several competing firms, each offering multiple products, a consumer must decide how many firms to consider and the number of products from each firm to evaluate. In this way, the consumer's consideration set formation problem has two dimensions. Specifically, the consumer must examine the *within-firm* and *across-firm* trade-offs of adding another product to her consideration set.

To illustrate the consideration set formation in two dimensions, consider a simultaneous search situation in which a consumer contacts several firms to request product brochures. She first decides on the firms to contact and on the product brochures to request from each contacted firm. She then spends time contacting the intended firms to request brochures. After receiving all brochures from all contacted firms, she spends time evaluating the products. Finally, she makes her purchase decision. This example serves simply to illustrate the consumer's consideration set formation process as we model it and may not realistically capture the way a consumer typically collects information. In addition, firms may be able to strategically control the informational content of the brochures, in particular the products that firms choose to offer.

In general, the composition of the consumer's consideration set will depend on product evaluation costs and product differentiation (Roberts and Lattin 1991), both of which have a within-firm and an across-firm dimension. As noted above, evaluation costs will be lower for a second, third, or fourth product of a firm already included in the consideration set relative to the first product from another firm. Product differentiation governs the consumer's benefit from product evaluation. Because products from the same firm tend to be less differentiated than products from a different firm, the expected marginal benefit of within-firm evaluation is lower than that of across-firm evaluation. Consequently, the consumer has an incentive to sample more firms and fewer products from each firm when firms are more differentiated. And if firms are strategic, they will compete with knowledge of the makeup of the consumer's consideration set.

The composition of consumers' consideration sets also has implications for a firm's product line decision. Because maintaining a product line involves costs, a firm's product line decision will optimally consider the extent of evaluation within the firm. For instance, if consumers consider only a few products of a given firm, then the firm will find it optimal to maintain a short product line. Conversely, a firm's product line decision can have an impact on the consumer's consideration set. If, for example, product line costs are significant, then a firm may find it optimal to pare its product line to such an extent that consumers would evaluate additional products were they offered. Thus, in this case, a firm's product line decision can put an external constraint on consumers' consideration sets. The interaction of consumers' two-dimensional consideration set formation problem and firms' strategic price and product line decisions is at the heart of this research. Our objective is to study the implications of firm competition and product variety when consumers incur within-firm product evaluation costs in addition to across-firm evaluation costs. Extending the consumer's evaluation problem to include within-firm factors allows us to delineate two dimensions of the consideration set formation, offering new insights about how firms compete in product lines and how product evaluation costs affect firms' prices.

The first new insight regards the relationship between firm differentiation and product variety. Conventional intuition suggests that as firms become less differentiated, their incentives to invest in additional products are diminished by the corresponding decrease in margins (e.g., Cachon et al. 2008). But this intuition ignores a consumer's within-firm evaluation costs and her judicious selection of products to include in her consideration set. To illustrate this distinction, consider the following scenario. Suppose that several electronics manufacturers carry product lines of televisions, which vary by complex feature combinations (e.g., screen sizes, picture resolutions, and Internet connectivity). Yet the styles of each product line offered by manufacturers are relatively similar (e.g., all have the latest flat screen technology with a sleek appearance). With nonzero evaluation costs, the net expected benefit of evaluating a few products of many firms is outweighed by the net expected benefit of evaluating additional products from only a few firms. Strategic firms, aware of the consumer's desire for deeper within-firm evaluation, have a competitive incentive to expand their product lines to provide a better fit for the consumer. That is, less firm differentiation can induce more product variety.

The second new insight concerns the impact of within-firm evaluation costs on firms' prices and product lines. Intuitively, raising the consumer's cost of identifying the best product among a firm's product line implies lower potential surplus from shopping at that firm and therefore lower prices. But this intuition ignores the interplay between the two different dimensions of the product evaluation process when consumers face competitive multiproduct firms. We show that prices and profits can increase from larger within-firm evaluation costs. When small, within-firm evaluation costs play a role similar to conventional across-firm evaluation costs. In these cases, an increase in within-firm evaluation implies that it is more costly to evaluate an additional product at any inspected firm, which serves to raise the costs of evaluating an additional firm, implying softened price competition. In addition, similar to across-firm evaluation costs, an increase in within-firm evaluation costs shortens product line length. Thus, firms' price and profits increase. This is not the case when within-firm evaluation costs are large. In these situations, as we show, within-firm evaluation costs play a role opposite to conventional across-firm evaluation costs. For instance, an increase in within-firm evaluation costs induces the consumer to evaluate more firms. As a result, firms compete more aggressively in price. In addition, in contrast to across-firm evaluation costs, an increase in within-firm evaluation costs shortens product line length. These findings show that within-firm evaluation costs and across-firm evaluation costs can have either similar or opposite impacts on firms' price and product line competition.

These insights arise from our analysis of an equilibrium model in which consumers optimally form their consideration sets with two dimensions of evaluation costs, while firms choose their product line lengths and prices. The equilibrium exhibits the following fundamental property: Any firm's product line length always matches the number of products from that firm in consumers' consideration sets. Essentially, firms' have no incentive to provide more products than consumers are willing to consider. Similarly, consumers cannot evaluate more products than a firm provides in its product lines. This property turns out to be crucial to understanding our results because it demonstrates how the composition of consumers' consideration sets intrinsically depends on firms' incentives and vice versa. If within-firm evaluation costs are large, then the optimal number of firms and products at each evaluated firm is the interior optimum of the consumer's consideration set formation problem. In this case, firms' product line length decision in equilibrium is determined solely by the depth of consumers' within-firm evaluation. In contrast, when within-firm evaluation costs are small, consumers wish to evaluate more products at a considered firm than the firm provides in equilibrium. That is, because of product line costs, the firm finds it optimal to impose a binding constraint on consumers' consideration set formation problem. Consequently, an increase in within-firm evaluation costs does not have a direct effect on consumers' within-firm evaluation but rather indirectly induces firms to cut their product line because of competitive incentives.

As suggested above, consumers' within-firm evaluation costs can affect the amount of product variety in a competitive market. But does the competitive market provide too many or too few products relative to the social optimum? This is the final question we address in this paper. The literature on multiproduct firms has suggested that firms offer too few products because they do not fully account for the social benefit of providing a better fit for consumers (Anderson and de Palma 1992). However, when there are costs for inspecting additional products within a firm, it is natural to ask whether the consumer's judicious consideration set process mitigates this market inefficiency. By evaluating the equilibrium number of products offered by firms when consumers optimally collect product and firm information, we are able to assess the factors that affect the market level of product variety and how that compares to the socially optimal level. Our analysis shows, in fact, that small within-firm inspection costs have a correcting influence on the market failure identified in Anderson and de Palma (1992). However, when these costs are large, the previous result is overturned and firms provide too many products in equilibrium relative to the social optimum. This finding further highlights the distinctive implications of the within-firm dimension of product evaluation.

Notions of consideration set formation are typically based on either consumers' limited information processing ability or limited information gathering ability (Manrai and Andrews 1998, Mehta et al. 2003). Our research focuses on the latter notion, which recognizes that acquiring product information can be costly and, therefore, implies that consumers optimally stop short of inspecting all available options (Hauser and Wernerfelt 1990). In particular, we assume that consumers have a limited ability in gathering product-match information because of the costs associated with examining a product and evaluating its net benefit (Ratchford 1982). We extend this literature by distinguishing how these costs may differ depending on whether the consumer chooses to consider an additional firm or inspect an additional product with an evaluated firm. This is important because products from the same firm share similar traits, implying that the cost of inspecting another product within a firm already evaluated has lower costs than evaluating a whole new firm.

The marketing and economics literature has also recognized that the cost associated with acquiring product information has implications for price competition (e.g., Stigler 1961, Morgan and Manning 1985, Anderson and Renault 1999). This literature assumes that firms carry a single product and therefore does not account for within-firm product evaluation. This distinction is important with multiproduct firms because price competition is internalized for product comparisons at the same firm, implying that each dimension of evaluation costs affects competitive incentives differently. As we show, this has implications for the results on price competition.

Our paper also contributes to the growing literature on multiproduct firms and the incentives for providing product variety. Anderson and de Palma (1992) establish a useful consumer choice setting to model multiproduct firms that compete in prices and the number of products to offer. Their model generates a number of insights about competition among multiproduct firms and how well the market level of product variety compares to the social optimum. We utilize Anderson and de Palma's (1992) consumer choice framework to model firm competition and incorporate the notion of costly product evaluation. As we show here, the inclusion of costly product evaluation suggests new insights regarding product variety. In a competitive model with multiproduct firms, Cachon et al. (2008) demonstrates how lower evaluation (search) costs can lead to more product variety and higher prices. Evaluation costs in Cachon et al. (2008), however, apply only across firms. In contrast, we study the interplay of within-firm evaluation on the composition of consideration set and on firm competition. This allows us to identify that within-firm evaluation cost can have an anticompetitive effect in price and product line.

Like our paper, Baumol and Ide (1956), Villas-Boas (2009), and Kuksov and Villas-Boas (2010) consider within-firm evaluation costs and identify reasons that a firm may strategically limit product variety. Baumol and Ide (1956) study the trade-off faced by a retailer in stocking enough variety of products to make shopping worthwhile while being aware of consumers' burden from too many products to evaluate. Villas-Boas (2009) shows that a monopolist may curtail product variety to avoid holding up the consumer when she incurs an evaluation cost before the monopolist commits to a price. Kuksov and Villas-Boas

(2010) illustrate that too much product variety by a monopolist can force shoppers to engage in too much evaluation, causing them to avoid shopping altogether. Unlike these works, we study a competitive setting in which firms choose prices and product line length when consumers judiciously choose the number of firms to consider and how many of each firm's products to evaluate. In addition, we consider the trade-off of the interactive roles between consumers and firms by considering firms' product line cost. Doing so, we identify another incentive for firms to strategically limit product variety: if consumers limit the number of sampled products because of large within-firm evaluation costs, then firms cut costs by keeping product lines short.[1]

In the next section, we develop a model of the consumer's consideration set formation and firm competition and derive the equilibrium. In §3, we find the socially optimal outcome and compare that to the equilibrium outcome. Section 4 concludes.

2. Model

There are $n \geq 2$ firms, each selling a set of differentiated products. We denote x_i as the product variety, the number of products or product line length, offered by firm i. There is no systematic quality difference across firms or products within each firm. The mass of consumers is normalized to one, and each consumer has demand for one product. The timing of the model is as follows: First, firms decide the number of products to carry for their product lines. Second, after observing the number of products chosen by other firms, firms choose prices of their products. Firms observe each others' choices but consumers do not. Third, consumers form their consideration sets by choosing the number of firms to evaluate and the number of products to inspect from each firm. Finally, consumers purchase a product from their consideration set.

2.1. Consumer Choice and Consideration Set Formation

We assume the consumer initially has imperfect information about the attributes of the differentiated products. Specifically, product match is idiosyncratic to the consumer and she must evaluate a product to determine the utility from consuming it. Because evaluation is costly, a consumer judiciously forms a consideration set that does not include all products offered. We model the formation of her consideration set as a simultaneous search process (Morgan and Manning 1985) in which the consumer commits to evaluating a set of products before making a purchase. Such a process may not be very appealing in many important situations in which sequential search is possible, and this remains a potential issue for future research. Our setting implies that the consumer cannot explicitly observe firms' choices regarding prices and number of products offered. Although consumers do not observe firms' decisions, we assume they are able to deduce the equilibrium strategies of all firms. Nevertheless, the consumer must engage in costly product evaluation in order to determine her match utility (e.g., its color, styling, or fit). A product's match utility has a firm-level component and a product-level component, each of which requires a cost to learn. Specifically, a consumer incurs a cost for each firm from which she inspects at least one product and a cost for each additional product inspected at that firm.

A consumer's consideration set is determined before she evaluates products.[2] By assumption, products are a priori identical before evaluation. At this point, firms are also otherwise identical except perhaps in terms of the number of products they offer, x_i, and their corresponding prices. Recall that consumers cannot directly observe firms' decisions but can rationally deduce them. We can now describe the construction of the consumer's consideration set by the decision to evaluate $z_i \geq 0$ products from firm $i = 1, \ldots, n$. We assume that the first evaluated product of a considered firm does not cause any evaluation costs to the consumer. From this set, the consumer picks the best alternative from the $\sum_i z_i$ products.

The utility of product j from firm i is given by

$$u_{ij} = v - p_{ij} + \mu_1 \varepsilon_i + \mu_2 \varepsilon_{ij},$$

where v is a base level of utility,[3] p_{ij} is the product's price, and ε_i and ε_{ij} are random utility terms from the firm and product, respectively. We assume these terms are random variables with extreme value distributions and independent from each other and across products and firms. The coefficients μ_1 and μ_2 capture the across-firm and within-firm levels of heterogeneity, respectively. A large value of μ_1 reflects a high degree of differentiation between firms and a large value of μ_2 means that any given product is not very substitutable with the other products from the same firm. We assume that $\mu_1 \geq \mu_2$, which ensures that our multinomial logit (MNL) approach to

[1] Although firms in our model may cut their product lines to match consumers' limited within-firm inspection, our results indicate that product lines may be longer than the socially optimal length.

[2] We assume that firms have no direct means to influence the makeup of a consumer's consideration set. In many settings, however, a firm can attempt to be included in consumers' consideration sets. For example, Amaldoss and He (2010) study a firm's use of informative advertising to manipulate the content of consumers' consideration sets.

[3] We assume that v is sufficiently large to induce consumer search.

consumer choice (described below) is consistent with utility maximization (McFadden 1978, Ben-Akiva and François 1983).

Suppose the consumer contacts $m < n$ firms and inspects $1 \le z_i \le x_i$ products from each contacted firm $i = 1, \ldots, m$.[4] We only focus on the interior equilibria where not all firms are contacted by the consumer. This assumption, however, may not hold when there are only a few competing firms. She incurs a cost $\tau > 0$ for each firm contacted and the first product of each contacted firm and a cost $\sigma > 0$ for each additional product inspected. Because of this definition, it is natural to have $\tau > \sigma$. Thus, for a consideration set $\{z_i\}_{i=1}^m$, her total evaluation cost is $m(\tau - \sigma) + \sigma \sum_{i=1}^m z_i$. The number of firms and the number of products from each firm she has in her consideration set depends not only on these costs but also on the expected benefit of the choice resulting from the inspection of $\{z_i\}_{i=1}^m$ products. We model this choice as a multinomial nested logit model with nests $i = 1, \ldots, m$ and choices $j = 1, \ldots, z_i$ within nest i. The MNL approach allows us to derive a closed-form expression for consumer demand and provides us a mathematically tractable choice model from a set of multiproduct firms (Anderson and de Palma 1992). Table 1 summarizes all notations.

For every product in the consumer's consideration set $\{z_i\}_i^m$, the consumer knows its utility. From this set, she chooses the product with the highest utility. The demand for product j from firm i is its choice probability, which is derived as follows. Let $\mathbb{P}_i$ be the (unconditional) probability that the consumer buys from firm i and let $q_{j|i}$ be the (conditional) probability she chooses product j given that she's buying from i. Specifically, the probability of product j being chosen conditional that a consumer has chosen firm i and has inspected $z_i \le x_i$ of its products is

$$q_{j\,|\,i} = \frac{e^{(v-p_{ij})/\mu_2}}{\sum_{k=1}^{z_i} e^{(v-p_{ik})/\mu_2}}; \quad j = 1, \ldots, z_i, \tag{1}$$

where z_i is the number of products inspected at firm i. The probability for firm i to be chosen is

$$\mathbb{P}_i = \frac{e^{A_i/\mu_1}}{\sum_{l=1}^m e^{A_l/\mu_1}}; \quad i = 1, \ldots, m, \tag{2}$$

where $A_i = E(\max_{j \le z_i}(v - p_{ij} + \mu_2 \varepsilon_{ij}))$ as the attractiveness of firm i given that she will inspect z_i of its products. We can then write the demand of product j of firm i as $q_{ij} = \mathbb{P}_i\, q_{j|i}$.

We focus on symmetric equilibria of the form $(x, p) \in \mathbb{R}_+^2$, where x represents the product line length of all firms and p is the price of all products at

Table 1 Notation

Symbol	Definition
μ_1	Across-firm heterogeneity
μ_2	Within-firm heterogeneity
τ	Across-firm evaluation cost
σ	Within-firm evaluation cost
x	Product line length
c	Product line cost
z	Consumer within-firm evaluation depth
m	Consumer across-firm evaluation
p	Product price

each firm. Specifically, we assume that consumers believe that symmetric firms will play symmetric strategies. In light of this symmetry, firms are a priori undifferentiated and the consumer's optimal sampling decision can be expressed simply as selecting, at random, a number of firms, m, and the number of products $\{z_i\}_{i=1}^m$ to inspect from each considered firm. The consumer benefits from considering additional products because it improves the possibility of a better match to her idiosyncratic tastes. Given the utility formulation above, this benefit is expressed by $\mu_1 \ln\{\sum_i \exp[(\mu_1/\mu_2)\ln z_i]\}$. Because all products have the same price across all firms, we can characterize the consumer's optimal *sampling plan* $(\hat{m}, \{\hat{z}_i\})$ as the maximization of the following objective:

$$v - p + \mu_1 \ln\left\{\sum_i \exp\left[\left(\frac{\mu_2}{\mu_1}\right)\ln z_i\right]\right\} - m(\tau - \sigma) - \sigma \sum_i z_i, \tag{3}$$

subject to $\hat{z}_i \le x$ for all i. Two points about (3) are worth emphasizing. First, because (x, p) represents all firms' equilibrium strategies, consumers assume that no firm has an incentive to deviate in prices or product variety. Consequently, the consumer can take (x, p) as fixed when determining her optimal sampling plan. Second, from the perspective of the optimization of (3), firms are completely identical. Therefore, consumers are concerned only about the number of firms m to consider and the number of products $\{z_i\}$ from those firms to evaluate. Hence, the consumer's consideration set formation problem[5] is reduced to the optimization of (3) by the simultaneous choice of $(m, \{z_i\})$. The following lemma characterizes this optimization.

Lemma 1. *Given firms' symmetric product line lengths x and equal prices p, a consumer's optimal sampling plan is symmetric across firms ($\hat{z}_i = \hat{z}$ for all i) and therefore*

[4] To simplify index notation, we relabel the firm indices as $i = 1, \ldots, m$ for the firms in the consumer's consideration set.

[5] The consumer's consideration set formation process can be equally interpreted as a simultaneous search process (Morgan and Manning 1985).

can be characterized by the pair $(\hat{m}, \hat{z})$, *where*

$$\hat{z}(x) = \min\left\{x, \left(\frac{\tau-\sigma}{\mu_1-\mu_2}\right)\frac{\mu_2}{\sigma}\right\} \quad \textit{and}$$

$$\hat{m}(x) = \max\left\{\frac{\mu_1-\mu_2}{\tau-\sigma}, \frac{\mu_1}{\tau-\sigma+\sigma x}\right\}.$$

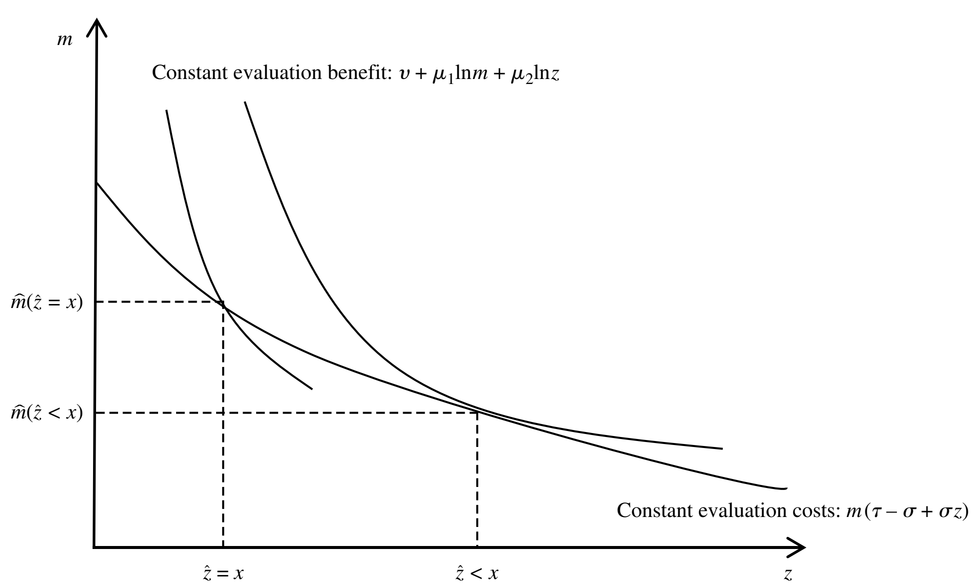
An important implication of Lemma 1 is that the consumer's optimal sampling plan involves inspecting the same number of products $\hat{z}$ at each firm. Consequently, the consumer's evaluation decision reflects the simple trade-off between considering fewer products at more firms or considering more products at fewer firms. In addition, even though a consumer never considers all firms, she may evaluate all products within a firm.

The lemma describes two situations: First, when the consumer is unconstrained by the number of products offered by the firm. In this case, her optimal sampling plan is the interior solution to (3), $\hat{z}(x) = ((\tau-\sigma)/(\mu_1-\mu_2))(\mu_2/\sigma) < x$ and $\hat{m}(x) = (\mu_1-\mu_2)/(\tau-\sigma)$. In this unconstrained case, the marginal cost and benefit of evaluating an additional product within a firm is $m\sigma$ and μ_2/z, respectively. Thus, the first-order condition of z at the symmetric interior optimum implies that the total number of inspected products depends only on the ratio of *within*-firm evaluation parameters: $\hat{m}\hat{z} = \mu_2/\sigma$. As this ratio increases, the consumer benefits more from evaluating more products at each firm and more products in total. An increase in $(\mu_1-\mu_2)/(\tau-\sigma)$, however, implies greater relative benefits from considering more firms and evaluating fewer products within each firm.

The second situation occurs when the consumer would optimally evaluate more products if firms offered more. In this case, the maximization of (3) is the boundary solution with $\hat{z}(x) = x < ((\tau-\sigma)/(\mu_1-\mu_2))(\mu_2/\sigma)$ and $\hat{m}(x) = \mu_1/(\tau-\sigma+\sigma x)$. Because the consumer cannot evaluate more products within the firms' product lines, she expands her consideration across firms beyond the interior optimal number of firms. Here the number of products evaluated is $\hat{m}\hat{z} = \mu_1 x/(\tau-\sigma+\sigma x)$. Similar to the unconstrained case, the consumer samples fewer products as within-firm evaluation costs increases. Unlike in the unconstrained case, however, the number of products evaluated decreases in across-firm evaluation costs, τ, and increases in product line length, x, and in firm-level differentiation, μ_1.

Figure 1 provides a graphical illustration of the consumer's optimal sample plan and helps highlight the important distinction between the interior and boundary solutions to the consumer's sampling problem. A typical iso-cost curve is drawn as the flatter curve. Its downward slope represents the positive costs of each dimension of evaluation. Its slight curvature indicates that evaluation costs are not a linear combination of m and z. Specifically, recall that evaluating m firms requires the cost $m(\tau-\sigma+\sigma z)$. A pair of iso-benefit curves are also drawn on the figure. At the interior optimum, the consumer considers $\hat{m}$ firms and evaluates $\hat{z} \leq x$ products at each firm such that the iso-cost and iso-benefit curves are tangent. At the corner solution, the iso-benefit curve cuts the iso-cost curve "from the top-left," which implies that the marginal benefit of within-firm evaluation exceeds the marginal cost. This is an important feature of the corner solution. As within-firm evaluation costs increase, the iso-cost curve shifts downward. Holding $\hat{z} = x$ fixed, the optimal number of firms

Figure 1 **Consumer's Optimal Sample Plan with Corner Solution ($\hat{z} = x$) and Interior Solution ($\hat{z} < x$)**



considered *decreases*. Even though σ increases the cost of evaluating an additional product within the firm, the consumer does not reduce z. However, because the cost of evaluating another firm, $\tau - \sigma + \sigma \hat{z}$, also increases, the consumer reduces the number of firms considered. This observation plays a crucial role in the results of §3.

2.2. Firms' Game in Prices and Product Line Length

We now consider the game played by the n firms. Each i of the n firms chooses its product line length, x_i, and a price p_{ij} for each of its $j = 1, \ldots, x_i$ products. We assume each firm has zero marginal cost but incurs an operational cost $c(x) = cx$, $c > 0$ to maintain a product line of length x products. Firms $i = 1, \ldots, n$ simultaneously choose x_i. Then, firms $i = 1, \ldots, n$ observe the product line lengths of their rivals and choose prices $\{p_{ij}\}$, $j = 1, \ldots, x_i$. We do not consider the realistic possibility that firms have the ability to control the positioning of their products.

We next determine the equilibrium of this game given that consumers acquire product information as described above. In deriving the equilibrium, several points should be noted. First, given that there are no systematic differences in quality among a firm's products, each firm sets the same price for all of its products in equilibrium, a fact established in Lemma 2:

$$p_{ij} = p_i \quad \text{for all } j = 1, \ldots, x_i.$$

This further implies that the demand for each product at firm i is the same for each j and h:

$$q_{ij} = q_{ih} \quad \text{for all } j, h = 1, \ldots, x_i \text{ and } j \neq h.$$

Second, consumers' sampling plans are based on rational expectations at the symmetric equilibrium and, therefore, do not react to deviations from any symmetric equilibrium. This permits us to consider deviations from a potential equilibrium without considering corresponding changes to consumer sampling behavior. Consumers' purchases, on the other hand, may depend on such deviations because purchases occur after they have inspected the mz products.

Given Lemma 1, firms expect consumers to employ a symmetric sampling plan. Based on this observation, we analyze firms' equilibrium behavior for all possible symmetric sampling plans (m, z). To do so, we utilize the result of Anderson and de Palma (1992), who analyze a game without consumer evaluation costs in which symmetric firms first choose the number of products to offer and then set prices. There are three key differences to keep in mind when applying the result. The first key difference is that a consumer cannot observe firms' actions when forming her consideration set. We must, therefore, specify the consumer's beliefs about how a firm chooses its product line length. (Beliefs about how firms choose prices need not be specified because a consumer's sampling plan does not depend on prices under our symmetry condition.) We assume that consumers believe that firms would not choose product line lengths such that a slightly longer product line would simultaneously benefit consumers and firms.[6] Second, in our model, consumers consider only a subset of existing firms and products. However, because firm deviations from equilibrium do not alter sampling decisions, we can, under the belief structure above, apply the Anderson and de Palma (1992) result to a game of m firms taking into account that there is a $m/n < 1$ probability that a firm will be considered by a consumer. The last key difference is that a firm may constrain its product line length to be shorter than that dictated by the interior optimum length whenever consumers constrain their evaluation within a firm. The following definitions help clarify the arguments.

Definition 1. An (m, z)-*equilibrium* is a symmetric equilibrium in firms' strategies $(\hat{x}(m, z), \hat{p}(m, z))$ when consumers employ the sampling plan (m, z).

As we argue below, there is a unique (m, z)-equilibrium for each sampling plan (m, z). Furthermore, firms' strategies in any equilibrium of the overall game must be an (m, z)-equilibrium. Note that only symmetric equilibria of the overall game are considered.

To determine an (m, z)-equilibrium, we focus on firm i, assuming all other firms employ the strategy (x, p) and consumers employ the sampling plan (m, z). Under this condition, we can directly compute the conditional demand of firm i's products as a function of strategic choices p_i and x_i using (1) and (2):

$$q_i(x_i, p_i \mid x, p, m, z) = \begin{cases} \dfrac{e^{(\mu_2/\mu_1)\ln(x_i)}}{(m-1)e^{[(\mu_2/\mu_1)\ln(x)-(p-p_i)/\mu_1]} + e^{(\mu_2/\mu_1)\ln(x_i)}} & \text{if } x_i, x \leq z, \\[2ex] \dfrac{1}{(m-1)e^{(p-p_i)/\mu_1} + 1} & \text{if } x_i, x > z, \end{cases} \tag{4}$$

where the probability of firm i being evaluated is m/n. Firm i's expected profit is then expressed:

$$\pi_i(x_i, p_i \mid x, p, m, z) = p_i(m/n)q_i(x_i, p_i \mid x, p, m, z) - cx_i. \tag{5}$$

[6] This assumption rules out equilibria in which firms' choices are constrained by arbitrary consumers' beliefs about the number of products carried by each firm. For instance, an equilibrium in which firms carry no products can be supported by a consumer belief that firms carry no products.

Firm i chooses p_i to maximize (5) given its prior choice of x_i, other firms' choice (x, p), and consumers' sampling plan (m, z). Accounting for reactions in p_i, firm i first chooses x_i. For any sampling plan (m, z), we define firms' symmetric equilibrium product line length as $\hat{x}(m, z)$ and price as $\hat{p}(m, z)$.

We report in Lemma 2 the necessary conditions for the n firms' pricing and product line strategies $(\hat{x}, \hat{p})$ in equilibrium.

Lemma 2. *For any sampling plan (m, z), there exists a unique (m, z)-equilibrium, which is characterized as follows*:

(i) $\hat{x}(m, z) = \min\left\{z, \dfrac{\mu_2}{cn}\dfrac{m^2 - m}{m^2 - m + 1}\right\}$; *and*

(ii) $\hat{p}(m) = \mu_1 \dfrac{m}{m-1}$.

Part (i) of this lemma ensures us that in equilibrium, firms never extend their product line beyond what consumers are willing to evaluate: $\hat{x}(m, z) \le z$. This implies that if consumers significantly limit their evaluation within firms, the optimal product-line length will be a corner solution to the optimization of (5). Part (ii) states that the equilibrium price is a function only of firm heterogeneity, μ_1, and the number of firms considered, m. In particular, the ability of firms to set prices depends on the extent to which consumers make price comparisons, as reflected by m, but not on the number of products evaluated within the firm, z. (Hence, we abbreviate notation to $\hat{p}(m)$.) Because firms' pricing strategies in equilibrium do not depend on z, $\hat{p}(m)$ does not depend directly on the constraint $\hat{x} \le z$.

2.3. The Equilibrium and Its Key Properties

The previous sections illustrate (i) how consumers form their consideration set and their final product choice given a fixed set of prices and product lines and (ii) how firms compete in prices and product line lengths given a fixed sampling plan of consumers. In this section, we bring these two pieces together, deriving the equilibrium of the overall model and then discuss its key properties.

To define the equilibrium of the overall model, we use Lemmas 1 and 2.

Definition 2. A *symmetric equilibrium* is a quadruple (z^*, m^*, x^*, p^*) such that x^* and p^* constitute an (m^*, z^*)-equilibrium where (m^*, z^*) is the optimal sampling plan given firm strategies x^* and p^*. Equivalently: $m^* = \hat{m}(x^*), z^* = \hat{z}(x^*), p^* = \hat{p}(m^*)$, and $x^* = \hat{x}(m^*, z^*)$.

It is immediate from the previous results that a unique symmetric equilibrium exists. Also evident from these two lemmas is that in this equilibrium, either firms or consumers will be at the corner solution of their respective optimization problems. Furthermore, the properties of this equilibrium depend on whether $x^* \le z^*$ is binding for the firm or $z^* \le x^*$ is binding for the consumers. Understanding which agent has the binding condition is crucial for understanding the comparative statics properties of the equilibrium. Lemma 3 demonstrates how these two different regimes depend on the size of within-firm evaluation costs.

Lemma 3. *There exists a unique threshold $\tilde{\sigma} \in (0, \tau)$ with the following properties.*

(i) *For large within-firm evaluation costs, $\tau > \sigma \ge \tilde{\sigma}$, consumers' equilibrium sampling plan (m^*, z^*) is characterized by the interior solution of* (3), *with $x^* = z^*$ a corner solution to the maximization of* (5).

(ii) *Otherwise, for small within-firm evaluation costs, $\sigma \le \tilde{\sigma}$, the consumers' equilibrium sampling plan (m^*, z^*) is a corner solution of* (3), *with $x^* = z^*$ satisfying the first-order condition of an interior maximizer of* (5).

The threshold $\tilde{\sigma}$ is defined by the parameter constellation that equates the interior solutions of the firms and consumers. Specifically, $\tilde{\sigma}$ uniquely solves

$$\frac{\mu_2}{cn}\frac{m^{*2} - m^*}{m^{*2} - m^* + 1} = \frac{\tau - \tilde{\sigma}}{\mu_1 - \mu_2}\frac{\mu_2}{\tilde{\sigma}},$$

where $m^* = (\mu_1 - \mu_2)/(\tau - \tilde{\sigma})$. Part (i) of Lemma 3 implies that for large values of within-firm evaluation costs, $\tau > \sigma \ge \tilde{\sigma}$, the consumers limit their evaluation within firms to such an extent that it constrains firms' product line. Conversely, part (ii) means that for small within-firm evaluation costs, $\sigma \le \tilde{\sigma}$, firms' costs dictate the extent of their product line even though consumers have an incentive to evaluate more products within firms. Finally, note that Lemma 3 also implies that in the knife-edge case of $\sigma = \tilde{\sigma}$, the symmetric equilibrium is an interior solution to both the firm's and consumer's respective optimization. Therefore, the symmetric equilibrium outcome (m^*, z^*, x^*, p^*) is continuous in σ at $\tilde{\sigma}$. This continuity property will be useful when comparing the equilibrium with the social optimum.

The threshold $\tilde{\sigma}$ is a function of market parameters. This threshold is increasing in the firms' cost of extending the product line, c. As this cost increases, firms' interior optimal product line length shortens and tends to constrain consumers' within-firm evaluation. On the other hand, as the consumer's relative benefit of across firms, $(\mu_1 - \mu_2)/(\tau - \sigma)$, increases, consumers substitute more across-firm evaluation for within-firm inspection, which tends to constrain firms' product line length decision.

Lemma 3 can also be used to show the distinct impacts of each dimension of evaluation costs. To do so, first consider the extreme case when across-firm evaluation cost and within-firm evaluation cost are equal ($\tau = \sigma$). Because $\tilde{\sigma} < \sigma$, consumers' equilibrium sampling plan is characterized by the interior solution of sampling plan. That is, consumers' evaluation decision dictates firms' product line length. In the alternative extreme, when the within-firm evaluation cost is zero ($\sigma = 0 < \tilde{\sigma}$), firms' equilibrium product line length, x^*, is characterized by the interior solution and consumers' equilibrium sampling plan is constrained. In this case, firms' incentives alone determine product line length. Hence, as these two extreme cases demonstrate, product line length is dictated by different agents reacting to different incentives depending on which dimension of the two evaluation costs is present. Later we illustrate the differential impacts of these two types of evaluation costs when both are present.

To understand the equilibrium properties further, we respectively examine the equilibrium outcomes when consumers' within-firm evaluation costs are relatively large, $\tau > \sigma > \tilde{\sigma}$, and when they are relatively small, $\sigma < \tilde{\sigma}$. Our objective is to characterize the equilibrium outcomes in this setting and discuss their key comparative statics properties.

Proposition 1 characterizes the equilibrium choices of consumers and firms categorized based on the magnitude of consumers' within-firm evaluation costs.

Proposition 1. (i) *With large within-firm evaluation cost $\tau > \sigma > \tilde{\sigma}$, the symmetric equilibrium has firms choosing price p^* and product line length x^*, where*

$$p^* = \mu_1 \frac{m^*}{m^* - 1} \quad x^* = z^*,$$

and consumers considering m^* firms and evaluating z^* products at each firm, where

$$z^* = \frac{\tau - \sigma}{\mu_1 - \mu_2} \frac{\mu_2}{\sigma} \quad m^* = \frac{\mu_1 - \mu_2}{\tau - \sigma}.$$

(ii) *With smaller within-firm evaluation cost $\sigma < \tilde{\sigma}$, the symmetric equilibrium has firms choosing price p^* and product line length x^*, where*

$$p^* = \mu_1 \frac{m^*}{m^* - 1} \quad x^* = \frac{\mu_2}{nc} \frac{m^{*2} - m^*}{m^{*2} - m^* + 1},$$

and consumers considering m^* firms and evaluating z^* products at each firm, where

$$z^* = x^* \quad m^* = \frac{\mu_1}{\tau - \sigma + \sigma x^*}.$$

With large within-firm evaluation cost ($\tau > \sigma > \tilde{\sigma}$), the impacts of parameters μ_1, μ_2, σ and τ on consumer evaluation are seen directly from the expressions in part (i) of the proposition. Specifically, from Lemma 3 we know that consumers' sampling plan (m^*, z^*) is an interior solution to their optimal consideration set formation problem. Therefore, in equilibrium, changes in evaluation costs and product heterogeneity parameters follow the same logic as discussed after Lemma 1. With small yet positive within-firm evaluation costs ($\sigma < \tilde{\sigma}$), the equilibrium product line length and price (x^*, p^*) correspond to the interior maximum of firms' profits, whereas the consumers' optimal sampling plan (m^*, z^*) is a corner solution to (3). Consequently, as exhibited by part (ii) of the proposition, the product line length x^* depends directly only on the product heterogeneity parameter μ_2 and indirectly on the parameters σ, τ, μ_1, which occurs through the equilibrium number of considered firms, m^*. An increase in within-firm product differentiation induces the firm to expand its product line to better match consumer tastes. However, for all other parameters, the impact on product line length is a competitive reaction to the amount of firms the consumer has considered. If the consumer considers more firms, then a firm competes more aggressively on product line length to encourage purchase from that firm.[7]

Proposition 1 implies several interesting comparative statics results. The following two corollaries present the key comparative statics properties of the equilibrium with respect to evaluation cost parameters (Corollary 1) and firm differentiation (Corollary 2). We provide the complete set of comparative statics results in §2.4.

Corollary 1 (Evaluation Costs). (i) *If $\tau > \sigma > \tilde{\sigma}$, then an increase in within-firm evaluation cost lowers prices and shortens firms' product line length, whereas an increase in across-firm evaluation cost has the opposite effect.*

(ii) *If $\sigma < \tilde{\sigma}$, then an increase in within-firm evaluation cost raises prices and shortens firms' product line length, whereas an increase in across-firm evaluation cost has the same effect.*

The insights coming from Corollary 1 relate to how firms' prices and product line lengths respond to changes in evaluation costs. Part (i) of Corollary 1 demonstrates that when within-firm evaluation cost is large, an increase in σ induces consumers to evaluate few products at each firm, inducing firms to pare their product lines. In addition, they consider more firms, implying lowered prices.[8] Note that these effects are

[7] In fact, the term $\xi(m) \equiv (m^2 - m)/(m^2 - m + 1) < 1$ can be interpreted as an inverse measure of the degree of market power each firm has with respect to its product line as a function of m. $\lim_{m\to\infty} \xi(m) = 1$.

[8] It is possible to connect this result to that of Kuksov (2004), who shows that lower (across-firm) search costs induce competitive firms to invest in product differentiation. Our model suggests that lower (within-firm) evaluation costs lead to more investment in product line length.

opposite to the conventional effects of across-firm evaluation costs.

This is not the case when within-firm evaluation costs is small, $\sigma < \tilde{\sigma}$, as indicated in part (ii) of the corollary. When within-firm evaluation costs are small, consumers' sampling plan is at boundary solution. Thus, larger within-firm evaluation costs increase the cost of evaluating another firm and thus consumers react as if there are larger across-firm evaluation costs. This results in higher prices and shortened product line lengths.

We turn to the impact of firm differentiation on the equilibrium in the following corollary.

Corollary 2 (Firm Differentiation). (i) *If $\tau > \sigma > \tilde{\sigma}$, then greater firm differentiation (μ_1) decreases product line length. If $\sigma < \tilde{\sigma}$, then greater firm differentiation increases product line length.*

(ii) *If $\tau > \sigma > \tilde{\sigma}$, then prices and profits are nonmonotonic in firm differentiation. Furthermore, profits may decrease in μ_1 even if prices are increasing. If $\sigma < \tilde{\sigma}$, then prices are increasing in firm differentiation, whereas the impact on profits is ambiguous.*

Part (i) of the corollary focuses on the impact of firm differentiation on product line length. With large within-firm evaluation costs, we know from Lemma 2 that the firm provides exactly the number of products in its product line that the consumer optimally evaluates. Thus, increases in the relative benefit of considering an additional firm by expanding firm differentiation μ_1 induces the consumer to consider more firms and evaluate fewer products within each firm. As a result, firms reduce the length of the product line. With small within-firm evaluation costs, the equilibrium product line length corresponds to the interior maximum of firms' profits. As a result, the product line length, x^*, depends indirectly on firm differentiation, μ_1, which occurs through the competitive reaction to the amount of firms the consumer has considered. With small within-firm evaluation costs, the consumer considers more firms as firms become more differentiated. Thus, a firm competes more aggressively on product line length to encourage purchase from that firm.[9]

As part (ii) of Corollary 2 indicates, firm differentiation has nonmonotonic effect on prices and profits for large within-firm evaluation costs ($\tau > \sigma > \tilde{\sigma}$). Obviously, greater firm differentiation grants firms more pricing power. But additional firm differentiation induces consumers to expand their search to include additional firms, which puts competitive pressure on prices. This latter effect can dominate and implies that prices decrease in firm differentiation when $\mu_1/(\tau-\sigma)$ is small. For larger values of $\mu_1/(\tau-\sigma)$, the former effect dominates and prices react to firm differentiation in the expected way. This relationship between prices and firm differentiation has been established in earlier work on single product firms in Anderson and Renault (1999). There is a noteworthy distinction, however, with multiproduct firms. Namely, equilibrium profits may decrease in firm differentiation despite increasing prices. The distinction has to do with how firms modify their product line length to changes in firm differentiation. Specifically, the fact that consumers consider more firms as a result of larger firm differentiation implies that they compete more aggressively on product lines, whose extra costs can outweigh the extra revenue from higher prices. This result holds for intermediate values of $\mu_1/(\tau-\sigma)$.[10] For small within-firm evaluation costs ($\sigma < \tilde{\sigma}$), then prices react to firm differentiation in the expected way. The complexity of the equilibrium expression for profits prevents us from deriving the comparative statics result with respect to μ_1.

Numerical examples further demonstrate the two main results from our equilibrium model. The table below illustrates the first main result: As the bolded numbers indicate, for $\tau > \sigma > \tilde{\sigma}$, within-firm evaluation costs induce consumers to evaluate more firms, which implies lowered prices. Across-firm evaluation costs have the opposite effects. For $\sigma < \tilde{\sigma}$, within-firm evaluation costs induce consumers to evaluate fewer firms, which implies higher prices. In addition, within-firm evaluation costs always reduce product line length.

	$z^* = x^*$	m^*	p^*	σ	τ	μ_1	μ_2
$\tau > \sigma > \tilde{\sigma}$	**5**	**20**	**13.68**	0.11	**0.21**	13	11
$nc = 1$	**10**	**10**	**14.44**	0.11	**0.31**	13	11
$\tau > \sigma > \tilde{\sigma}$	**20**	**5**	**13.75**	**0.1**	0.3	11	10
$nc = 0.4$	**5**	**10**	**12.22**	**0.2**	0.3	11	10
$\sigma < \tilde{\sigma}$	**8.1**	**5**	**625**	**0**	100	500	400
$nc = 47$	**5.97**	**2.11**	**1,000**	**27.5**	100	500	400

The table below illustrates the second main result: As the bolded numbers indicate, product line length decreases in μ_1 when $\tau > \sigma > \tilde{\sigma}$ and increases in μ_1 when $\sigma < \tilde{\sigma}$.

	$z^* = x^*$	m^*	p^*	σ	τ	μ_1	μ_2
$\tau > \sigma > \tilde{\sigma}$	**10**	10	13.33	0.11	0.21	**12**	11
$nc = 1$	**5**	20	13.68	0.11	0.21	**13**	11
$\sigma < \tilde{\sigma}$	**2.2**	1.95	1,560	0	400	**780**	700
$nc = 200$	**3**	3	1,800	0	400	**1,200**	700

[9] Another interpretation of this result is possible. Similar to Anderson and Renault (1999), under certain conditions, more firm differentiation implies more firm evaluation, which intensifies competition. In this case, more competition can lead to less product variety.

[10] The exact conditions on $\mu_1/(\tau-\sigma)$ are given in Corollary 3.

2.4. Additional Comparative Statics

In this section we provide the complete set of comparative statics results.

Corollary 3. *The equilibrium has the following comparative statics properties*:

	σ	τ	μ_1	μ_2
$\tau > \sigma > \tilde{\sigma}$				
$z^* = x^*$	$-$	$+$	$-$	$+$
m^*	$+$	$-$	$+$	$-$
p^*	$-$	$+$	$\pm^{a}$	$+$
π^*	?	$\pm^{c}$	$\pm^{b}$	$\pm^{d}$
$\sigma < \tilde{\sigma}$				
$x^* = z^*$	$-$	$-$	$+$	$+$
m^*	$-$	$-$	$+$	$-$
p^*	$+$	$+$	$+$	$+$
π^*	$+$	$+$	?	$\pm^{d}$

Positive iff

(a) $\mu_1 > \tau - \sigma + \mu_2 + \sqrt{(\tau-\sigma)^2 + (\tau-\sigma)\mu_2}$;

(b) $\dfrac{\mu_1}{\tau-\sigma} < m^*(m^*-1) + \dfrac{cn\mu_2}{\tau\sigma}\left(\dfrac{m^*-1}{m^*}\right)^2$;[11]

(c) $\tau > \sigma + (\mu_1 - \mu_2)\left(1 - \dfrac{1}{\sqrt{(nc/\sigma)(\mu_2/\mu_1)}}\right)$; *and*

(d) $\dfrac{1}{n}\left(\dfrac{m^*}{m^*-1}\right)^2 > \dfrac{c}{\sigma}$.

Evident from Corollary 3 is that when $\tau > \sigma > \tilde{\sigma}$, an increase in across-firm evaluation costs raises firms' profits for the usual reasons so long as product line cost c is not too large. (See condition (c).) Recall that an increase in τ induces firms to offer longer product lines. If c is large, firms compete too aggressively on product lines and actually see profits decrease despite reduced number of firms evaluated.

Interestingly, with large within-firm evaluation costs, within-firm product differentiation μ_2 has similar comparative statics properties as τ. In our model of two-dimensional consumer evaluation, changes in μ_2 and τ have similar consequences for consumers' sampling incentives. But, unlike firm evaluation costs, μ_2 has a slightly larger effect on number of products evaluated than τ. Consequently, the condition (d) for firm profits to increase in μ_2 is stronger than condition (c). Finally, Corollary 3 also provides the exact conditions required for the nonmonotonic relationship between profits and firm differentiation discussed after Corollary 2. Note that condition (a) is stronger than condition (b), so prices increasing in firm differentiation are sufficient, but not necessary, for profits to increase in firm differentiation. However, firm differentiation μ_1 is quite different in comparative statics properties from σ.

In addition, an increase in within-firm product differentiation (μ_2) always raises prices but may decrease profits when c/σ is large. Greater benefit from within-firm evaluation, as reflected by an increase in μ_2, encourages the consumer to consider fewer firms in favor of evaluating more products within each considered firm. Firms react by competing less aggressively on price (raise p^*) and more aggressively on product line (expand x^*). Similar to the discussion after Proposition 1, more aggressive competition in product line raises firms' costs and can actually lower profits if the cost of product line expansion is severe (large c).

[11] Note that m^* is a function of μ_1. Thus, the necessary condition for (b) to hold requires μ_1 to be large. We do not give the exact range to avoid messy algebra.

3. Equilibrium vs. the Social Optimum

We now turn to the question of whether firms' market incentives induce them to offer product lines that are excessively long or short relative to the social welfare maximizing length. A firm's choice of extending the product line by one product does not fully account for the social benefit of a consumer's improved fit from the additional product nor the consumers' costs to evaluate products. On the consumer side, the decision to evaluate an additional product is based on the relative costs and benefits of product evaluation and not on the firm's cost to extend the product line. So although it is perhaps intuitive that the product line length chosen by the market does not coincide with the socially optimal length, it is not clear on which side it lies.

A second and related question of market performance is whether the consumer evaluates more or fewer products than socially optimal. This question is not obvious because the consumer makes a tradeoff between within-firm and across-firm evaluations. Thus, the consumer may substitute too few or too many firms in her overall sampling plan relative to the social optimum. The fact that either the firm or consumer is constrained in its choice of products, produced or evaluated, respectively, further complicates the question. We explore these two questions in this section.

We start by defining social welfare. Because firms are symmetric, we assume that all firms carry the same length of product line, x. And by virtue of the argument in Lemma 1, we can restrict ourselves to the symmetric case in which consumers evaluate the same number of products, z, at each of the m firms considered. Under this symmetric setting, social welfare (SW) is defined as

$$\mathrm{SW} = v + \mu_1 \ln m + \mu_2 \ln z - \tau m - \sigma(z-1)m - ncx, \quad (6)$$

and the social optimum as the maximization of SW over the choices m, z, and $x \geq z$, which we denote by (m^{so}, z^{so}, x^{so}). Note that we ignore price because it is simply a transfer of surplus between firms and consumers and, therefore, does not affect social welfare. Observe that a benevolent social planner has no reason to set $z^{so} \neq x^{so}$ because otherwise there would be excessive production or excessive product evaluation. Therefore, $x^{so} = z^{so}$. The first order conditions for the maximization of SW in (6) with respect to m and z subject to $x = z$ gives the socially optimal number of evaluated products:

$$m^{so} z^{so} = \frac{\mu_2}{\sigma} - \frac{cn}{\sigma} \cdot z^{so}. \tag{7}$$

This expression reflects the social trade-off with respect to both dimensions of evaluation as well as the cost of maintaining a product line. We now compare the optimal number of product evaluations given in (7) with the equilibrium number of products inspected, $m^* x^*$. For large within-firm evaluation costs, $\tau > \sigma > \tilde{\sigma}$, the consumer's optimal sample plan corresponds to the interior optimum of (3). Note as well that the only difference between (3) and (6) is the term ncx, which is the total social cost of a symmetric product line of length x. In the case of $\sigma > \tilde{\sigma}$, we have $m^* x^* = \mu_2/\sigma$, which is the first term on the right-hand side of (7). That is, $m^* x^* - m^{so} z^{so} = (cn/\sigma) z^{so}$.

The difference arises because the consumer ignores the firm's cost of expanding the product line and evaluates too many of each firm's products relative to the social optimum. In particular, even though this cost is internal to the firm, the firm has no incentive to cut the product line because its marginal profit exceeds c when $\tau > \sigma > \tilde{\sigma}$. As for the number of firms considered, recall from Lemma 1 that this number is independent of within-firm evaluation costs. The social optimal m^{so}, however, depends on σ and exceeds m^* when within-firm evaluation σ is costly.

Proposition 2. *When $\tau > \sigma \geq \tilde{\sigma}$, the symmetric equilibrium always leads to socially excessive product lines and insufficient number firms considered: $z^{so} < x^*$ and $m^{so} > m^*$. The total number of products evaluated is excessive in equilibrium $z^{so} m^{so} < x^* m^*$.*

The comparison of the equilibrium and social optimum is not as direct for the case of small within-firm evaluation costs ($\sigma < \tilde{\sigma}$). Because the consumer's sample plan constitutes a corner solution to (3), we are not able to make a comparison of (z^{so}, m^{so}) and (x^*, m^*) for all values of $\sigma < \tilde{\sigma}$. We can, however, utilize the analysis above and the continuity of the equilibrium at $\sigma = \tilde{\sigma}$ to make the comparison in the neighborhoods of $\sigma = 0^+$ and $\tilde{\sigma}^-$.

From Lemma 1, we know that when $\sigma = \tilde{\sigma}$, the equilibrium expressions in the two regimes (as described in Proposition 1) coincide. Therefore, the equilibrium is continuous at $\sigma = \tilde{\sigma}$. Furthermore, Proposition 3 indicates excessive product evaluation in equilibrium for $\tau > \sigma \geq \tilde{\sigma}$. Thus, we have excessive product evaluation for values of σ close to but smaller than $\tilde{\sigma}$. In contrast to the intuition from Proposition 2, however, the economic inefficiency arises because firms do not internalize the social cost of additional consumer evaluation when expanding their product lines. And although the consumer does internalize this cost, the benefit of additional within-firm evaluation exceeds the nontrivial social cost $\sigma > 0$.

But what about very small within-firm evaluation costs? We examine the case $\sigma = 0$ as a benchmark. It is straightforward to show that the socially optimal amount of products evaluated at each firm reflects the within-firm evaluation benefit and product line costs: $z^{so} = \mu_2/(nc)$. The firm's choice of product line length, however, depends on its internal benefit, which is its margin, rather than the entire surplus. Therefore, in equilibrium $x^* = (\mu_2/nc)((m^{*2} - m^*)/(m^{*2} - m^* + 1)) < z^{so}$. Hence, there is a socially insufficient amount of products evaluated in equilibrium when within-firm evaluation costs are absent. By continuity, this argument applied to positive within-firm evaluation costs in the neighborhood of $\sigma = 0^+$. To compare the number of firms evaluated, note from the solution to the maximization of (6) and Proposition 1, $m^{so} - m^* = \mu_1/(\tau - \sigma + \sigma z^{so}) - \mu_1/(\tau - \sigma + \sigma x^*) < 0$ because $z^{so} > x^*$. Thus, the socially insufficient amount of product variety at a given firm drives the consumer to evaluate a socially excessive number of firms. Proposition 3 formalizes the above results.

Proposition 3. *When $\sigma < \tilde{\sigma}$, there exists values $\bar{\sigma}_1, \bar{\sigma}_2 \in (0, \tilde{\sigma})$ with the following properties.*

(i) *If $0 \leq \sigma < \bar{\sigma}_1$, then competitive firms provide socially insufficient product lines $z^{so} > x^*$ and the number of firms considered is excessive $m^{so} < m^*$ in equilibrium with insufficient product evaluation $z^{so} m^{so} > x^* m^*$.*

(ii) *If $\bar{\sigma}_2 < \sigma \leq \tilde{\sigma}$, then the symmetric equilibrium always has socially excessive product lines and an insufficient number of firms considered: $z^{so} < x^*$ and $m^{so} > m^*$. The total number of products evaluated is excessive in equilibrium $z^{so} m^{so} < x^* m^*$.*

As Propositions 2 and 3(ii) demonstrate, significant within-firm evaluation costs can lead to excessively long product lines. Either the firm does not internalize the consumers' within-firm evaluation costs when dictating product line length (Proposition 3(ii)) or the consumer does not internalize the firm's cost of expanding the product line (Proposition 2). This is in contrast to the result with little or no within-firm evaluation costs (part (i) of Proposition 3 and Anderson and de Palma 1992), which states that firms keep product lines too short relative to the social optimum because firms do not fully account for the consumers' benefit from a better fit.

4. Conclusion

It is natural to assume that in many specialty purchase situations, consumers incur a positive cost to compare products. Generally, the existing literature supposes that consumers incur an evaluation cost when comparing firms when each firm carries exactly one product. However, because firms often offer multiple products, it is reasonable to suppose additional evaluation costs within the firm as well. This paper has proposed a modeling framework for analyzing a setting of consumer evaluation in two dimensions: within and across firms. And, as our model shows, the addition of a second dimension of product evaluation leads to new results not obtained in the classic literature on one-dimensional sampling. For example, higher evaluation costs within a firm can actually either increase or decrease firms' prices. Also, firm differentiation decreases product line length. Within-firm evaluation costs and across-firm evaluation costs can have either similar effects or opposite effects in firms' product lines or prices.

Finally, we compared the equilibrium outcome with the social optimum. We found that firms can provide too many products relative to the level that maximizes social welfare when consumers face significant within-firm evaluation costs. This result counters the extant literature on multiproduct firms. When within-firm evaluation costs are small, however, firms' product lines are too short relative to the social optimum.

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Appendix

The appendix contains all proofs for all propositions, lemmas, and corollaries.

Proof of Lemma 1. A consumer's maximization of (3) subject to $\hat{z}_i \le x, i = 1, \ldots, m$ implies the following Lagrangian

$$\begin{aligned}\mathcal{L}&(m, z_1, \ldots, z_m, \lambda_1, \ldots, \lambda_m)\\ &= \mu_1 \ln\left\{\sum_i \exp[(\mu_2/\mu_1)\ln z_i]\right\} - (\tau-\sigma)m\\ &\quad - \sigma\sum_i z_i - \sum_i \lambda_i(z_i - x).\end{aligned}$$

We ignore integer constraints and assume that for an optimal sample plan $(\hat{m}, \{\hat{z}_i\})$ there exist $\hat{\lambda}_i \ge 0$, such that $\hat{\lambda}_i(\hat{z}_i - x) = 0$ for $i = 1, \ldots, \hat{m}$, and

$$\frac{\partial \mathcal{L}}{\partial m} = 0,$$

$$\frac{\partial \mathcal{L}}{\partial z_i} = 0 \quad \text{for all } i = 1, \ldots, \hat{m}.$$

We first show that the optimal number of products $\hat{z}_i$ evaluated at each considered firm is identical across firms. Define the benefit of sample plan $(m, \{z_i\}_i^m)$, gross of evaluation costs, as $G(z_1, z_2, \ldots, z_m) = v - p + \mu_1 \ln\{\sum_k \exp[(\mu_2/\mu_1)\ln z_k]\}$. Suppose $\hat{z}_i < x$ and $\lambda_i = 0$ for all i. Then $\partial\mathcal{L}/\partial z_i = 0$ implies $\partial G/\partial z_i = \sigma$ for all i. The only solution has $\hat{z}_1 = \hat{z}_2 = \cdots = \hat{z}_m$. Denote the common value as $\hat{z}$. Now consider the boundary case in which the consumer evaluates all products at the m firms: $\hat{z}_i = x$ for all i. Then for all i, $\partial G/\partial z_i = \sigma + \hat{\lambda}$, for some $\hat{\lambda} > 0$. Now suppose the asymmetric case by assuming that $\hat{z}_j < \hat{z}_k = x$ for some pair j, k. Then

$$\frac{\partial \mathcal{L}}{\partial z_j} = 0 \implies (\hat{z}_j)^{\mu_2/\mu_1 - 1} = \frac{\sigma}{\mu_1\mu_2} G(\hat{z}_1, \ldots, \hat{z}_{\hat{m}}) \quad \text{and}$$

$$\frac{\partial \mathcal{L}}{\partial z_k} = 0 \implies (\hat{z}_k)^{\mu_2/\mu_1 - 1} = \frac{\sigma + \hat{\lambda}}{\mu_1\mu_2} G(\hat{z}_1, \ldots, \hat{z}_{\hat{m}}).$$

Because $\mu_2/\mu_1 < 1$, we have $\hat{z}_j > \hat{z}_k$, which is a contradiction. Therefore, for any m, the consumer's optimal sample plan has the same number of products $\hat{z}$ evaluated at each of the m firms. Using the simplification that

$$G(z, z, \ldots, z) = v - p + \mu_1 \ln m + \mu_2 \ln z,$$

we find the explicit solution to (3) by maximizing

$$v - p + \mu_1 \ln m + \mu_2 \ln z - (\tau - \sigma)m - \sigma m z,$$

with respect to (m, z) subject to $z \le x$. This yields

$$z < x \implies \begin{cases} \hat{Z} = \dfrac{\mu_2}{\sigma}\left(\dfrac{\tau-\sigma}{\mu_1-\mu_2}\right), \\ \hat{m} = \dfrac{\mu_1-\mu_2}{\tau-\sigma}, \end{cases} \quad \text{and}$$

$$z = x \implies \begin{cases} \hat{z} = x, \\ \hat{m} = \dfrac{\mu_1}{\tau - \sigma + \sigma x}. \end{cases}$$

Note that $\hat{z}$ and $\hat{m}$ are weakly decreasing and increasing in x. Thus, the optimal sampling plan can be expressed as in the statement of the lemma. □

Proof of Lemma 2. Using the argument of Anderson and de Palma (1992), we establish that any firm finds it optimal to charge a single price for all its products. Suppose consumers adopt the sampling plan (m, z). Because consumers do not observe firms' strategies and their sampling plans cannot respond to deviations from any equilibrium, firms' pricing game in our setting is nearly identical to the one analyzed in Anderson and de Palma (1992). The only difference is that consumers may not evaluate all firms $(m < n)$ and products offered by each firm $(z \le x)$. Nevertheless, each product from any firm is inspected with equal probability. Therefore, firms' pricing incentives in our setting are identical to those in Anderson and de Palma (1992). Hence, as in Anderson and de Palma (1992), $p_{ij} = p_i$ for all firms $i = 1, \ldots, n$ and all products $j = 1, \ldots, x$.

Now suppose consumers expect all firms to play (x, p) and implement the sampling plan (m, z). We start by supposing firm i deviates in the product line choice with x_i and all other firms x. Maximizing (5) with respect to p_i subject to (4) implies

$$p_i = \mu_1/(1-q_i).$$

Similarly,

$$p = \mu_1/(1-q)$$

where $q_i + (m-1)q = 1$. An argument similar to the proof of the lemma (p. 266) of Anderson and de Palma (1992) establishes that (p_i, p) is a unique equilibrium to the game in which firm i sells x_i products and all other firms sell x.

Using the expressions (1) and (2), profits to firm i when it sells x_i products and all other firms sell x, simplifies to

$$\pi_i(x_i, x) = \frac{m}{n}\frac{\mu_1}{m-1}e^{(p-p_i)/\mu_1}\Gamma(x_i, x) - cx_i,$$

where

$$\Gamma(x_i, x) = \begin{cases} (x_i/x)^{\mu_1/\mu_2} & \text{if } x_i, x < z, \\ (x_i/z)^{\mu_1/\mu_2} & \text{if } x_i < z \le x, \\ (z/x)^{\mu_1/\mu_2} & \text{if } x < z \le x_i, \\ 1 & \text{if } z < x, x_i. \end{cases}$$

Denote $\xi(m) = (m^2-m)/(m^2-m+1)$ and $\eta(m) = m/(m-1)$. If $x_i, x < z$, then $d\pi_i/dx_i = 0$ with $x_i = x$ implies $x = (\mu_2/(cn))\xi(m) < z$ and $p_i = p = \mu_1\eta(m)$. An argument similar to Proposition 1 (p. 268) of Anderson and de Palma (1992) proves (x, p) is a unique equilibrium to this game.

If $x_i, x \ge z$, then $d\pi_i/dx_i = -c < 0$. Because $d\pi_i/dx_i < 0$ for $x_i > z$, it is optimal for firm i to set $x_i = z$. Therefore, $x = z$ and $p = \mu_1\eta(m)$ is the unique equilibrium.

The case $x_i < z \le x$ leads to the outcome for $x_i, x > z$.

The case $x < z \le x_i$ leads to the outcome for $x_i, x < z$.

Hence, for a given sampling plan (m, z), the unique equilibrium as expressed in the lemma. □

Proof of Lemma 3. The boundary $\tilde{\sigma}$ is determined by the equation

$$\frac{\mu_2}{cn}\frac{m^{*2}-m^*}{m^{*2}-m^*+1} = \frac{\tau-\sigma}{\mu_1-\mu_2}\frac{\mu_2}{\sigma},$$

where $m^* = (\mu_1-\mu_2)/(\tau-\sigma)$.

It implies

$$\sigma = cn\frac{\tau-\sigma}{\mu_1-\mu_2}\left(1+\frac{1}{m^{*2}-m^*}\right).$$

Solving σ explicitly, there is one solution $\tilde{\sigma}$ in real value.

It is straightforward that $\tilde{\sigma} > 0$ because otherwise the right-hand side of the expression above would be negative. Also $\tilde{\sigma} < \tau$ because otherwise $\tilde{\sigma} < 0$, which is debunked above. Thus, $0 < \tilde{\sigma} < \tau$.

Parameterize the equilibrium by σ: $(x^*(\sigma), z^*(\sigma), m^*(\sigma), p^*(\sigma))$. By definition of $\tilde{\sigma}$

$$\hat{x}(m, z) = \frac{\mu_2}{nc}\xi(m) = \frac{\mu_2}{\sigma}\frac{\tau-\sigma}{\mu_1-\mu_2} = \hat{z}(x),$$

is satisfied for $\hat{m}(x) = (\mu_1-\mu_2)/(\tau-\sigma) = \mu_1/(\tau-\sigma+\sigma x)$. Thus,

$$m^*(\tilde{\sigma}) = \frac{\mu_1-\mu_2}{\tau-\tilde{\sigma}} = \frac{\mu_1}{\tau-\tilde{\sigma}+\tilde{\sigma}x^*(\tilde{\sigma})},$$

$$x^*(\tilde{\sigma}) = z^*(\tilde{\sigma}) = \frac{\mu_2}{nc}\xi(m^*(\tilde{\sigma})) = \frac{\mu_2}{nc}\frac{\tau-\tilde{\sigma}}{\mu_1-\mu_2},$$

$$p^*(\tilde{\sigma}) = \mu_1\eta(m^*(\tilde{\sigma})).$$

At $\sigma = \tilde{\sigma}$, both the consumers and firms simultaneously operate at the corner solution and satisfy the conditions for an interior optimum. For all $\sigma \ne \tilde{\sigma}$, exactly one of either the consumers or the firms operate at their respective interior optimum.

Let $\sigma > \tilde{\sigma}$. By Lemma 1,

$$m^*(\sigma) \ge \frac{\mu_1-\mu_2}{\tau-\tilde{\sigma}} = m^*(\tilde{\sigma}).$$

This implies

$$\begin{aligned}\frac{\mu_2}{nc}\xi(m^*(\sigma)) &\ge \frac{\mu_2}{nc}\xi(m^*(\tilde{\sigma})) = \frac{\mu_2}{\tilde{\sigma}}\frac{\tau-\tilde{\sigma}}{\mu_1-\mu_2} \\ &> \frac{\mu_2}{\sigma}\frac{\tau-\sigma}{\mu_1-\mu_2} \ge z^*(\sigma) \ge x^*(\sigma),\end{aligned}$$

where the first inequality is because $\xi(m)$ is nondecreasing in m, the equality by definition of $\tilde{\sigma}$, the strict inequality by assumption $(\sigma > \tilde{\sigma})$, and the last two inequalities by Lemmas 1 and 2, respectively. Hence, by virtue of the strict inequality, firms optimally operate on the boundary of their profit maximization when $\sigma > \tilde{\sigma}$.

Finally, let $\sigma < \tilde{\sigma}$.

We show that the equilibrium sampling plan $(m^*(\sigma), z^*(\sigma))$ cannot be the interior solution to consumers' optimal consideration set formation problem. Suppose it were. Then

$$m^*(\sigma) = \frac{\mu_1-\mu_2}{\tau-\sigma} \quad \text{and} \quad z^*(\sigma) = \frac{\mu_2}{\sigma}\frac{\tau-\sigma}{\mu_1-\mu_2}.$$

We now can write

$$m^*(\tilde{\sigma}) = \frac{\mu_1-\mu_2}{\tau-\tilde{\sigma}} > \frac{\mu_1-\mu_2}{\tau-\sigma} = m^*(\sigma),$$

which follows from Lemma 1 and $z^* = x^*$ in equilibrium. Hence, $m^*(\tilde{\sigma}) > m^*(\sigma)$. This last fact implies $\xi(m^*(\sigma)) < \xi(m^*(\tilde{\sigma}))$. Thus,

$$\begin{aligned}\frac{\mu_2}{\sigma}\frac{\tau-\sigma}{\mu_1-\mu_2} &= z^*(\sigma) = x^*(\sigma) \le \frac{\mu_2}{nc}\xi(m^*(\sigma)) \\ &< \frac{\mu_2}{nc}\xi(m^*(\tilde{\sigma})) = x^*(\tilde{\sigma}) = \frac{\mu_2}{\tilde{\sigma}}\frac{\tau-\tilde{\sigma}}{\mu_1-\mu_2},\end{aligned}$$

where the inequality holds from Lemma 2. The order of the two extreme expressions contradicts $\sigma < \tilde{\sigma}$. Therefore, the equilibrium sampling plan cannot be an interior solution to the consumers' consideration set formation problem. □

Proof of Proposition 1. By Lemma 3, $\tau > \sigma > \tilde{\sigma}$ implies the equilibrium is determined by the interior solution to the consumers' consideration set formation problem. Thus, m^* and z^* are expressed as indicated. We must have $x^* = z^*$ in any equilibrium and Lemma 2 implies the expression for p^*.

Also by Lemma 3, $\sigma < \tilde{\sigma}$ implies the equilibrium is determined by the interior solution to the firm's profit maximization. Thus, $x^* = (\mu_2/(nc))\xi(m^*)$, where (m^*, z^*) must be the boundary solution to the consumers' optimal consideration set problem when $\hat{z}(x^*) = x^*$. Lemma 2 implies the expression for p^*. □

Proof of Corollary 1. The impact of changes σ on p^* and π^* are determined simply through the changes in the equilibrium sampling plan. That is,

$$\frac{\partial p^*}{\partial \sigma} = \mu_1\eta'(m^*)\frac{\partial m^*}{\partial \sigma} \quad \text{and}$$
$$\frac{\partial \pi^*}{\partial \sigma} = \frac{\mu_1}{n}\eta'(m^*)\frac{\partial m^*}{\partial \sigma} - c\frac{\partial x^*}{\partial \sigma}. \tag{8}$$

For $\sigma > \tilde{\sigma}$, the comparative statics of m^*, x^*, and p^* with respect to σ follow directly from inspecting the expression given in Proposition 1. However, the impact of σ on π^* is ambiguous.

For $\sigma < \tilde{\sigma}$, because the consumer's optimal sample plan is characterized by an implicit function, we must first solve the comparative statics on m^* and x^* simultaneously.

For σ, the pair equations

$$\frac{\partial m^*}{\partial \sigma} = \frac{-\mu_1}{(\tau-\sigma+\sigma x^*)^2}\left(x^*-1+\sigma\frac{\partial x^*}{\partial \sigma}\right) \quad \text{and}$$
$$\frac{\partial x^*}{\partial \sigma} = \frac{\mu_2}{nc}\xi'(m^*)\frac{\partial m^*}{\partial \sigma}$$

imply $\partial m^*/\partial\sigma, \partial x^*/\partial\sigma < 0$.

Equation (8) implies $\operatorname{sgn}(\partial p^*/\partial\sigma) = -\operatorname{sgn}(\partial m^*/\partial\sigma)$.

Finally, evaluating the comparative statics of π^* is direct for σ:

$$\frac{\partial p^*}{\partial \sigma} > 0 > \frac{\partial x^*}{\partial \sigma} \implies \frac{\partial \pi^*}{\partial \sigma} > 0.$$

For $\sigma > \tilde{\sigma}$, the comparative statics of m^*, z^*, and p^* with respect to τ follow from directly inspecting the expression implied in Proposition 1.

The comparative statics of π^* with respect to τ can be deduced from evaluating (8) with the equilibrium expressions from Proposition 1. That is,

$$\frac{\partial \pi^*}{\partial \tau} = \frac{\mu_1}{n}\eta'(m^*)\left(-\frac{m}{\tau-\sigma}\right) - \frac{c}{\sigma}\frac{\mu_2}{\mu_1-\mu_2} > 0$$
$$\iff \frac{1}{n}\left(\frac{m^*}{m^*-1}\right)^2 > \frac{c}{\sigma}\frac{\mu_2}{\mu_1}.$$

For $\sigma < \tilde{\sigma}$,

$$\frac{\partial m^*}{\partial \tau} = \frac{-\mu_1}{(\tau-\sigma+\sigma x^*)^2}\left(1+\sigma\frac{\partial x^*}{\partial \tau}\right) \quad \text{and}$$
$$\frac{\partial x^*}{\partial \tau} = \frac{\mu_2}{nc}\xi'(m^*)\frac{\partial m^*}{\partial \tau},$$

imply $\partial m^*/\partial\tau$, $\partial x^*/\partial\tau < 0$.

Equation (8) implies $\operatorname{sgn}(\partial p^*/\partial\tau) = -\operatorname{sgn}(\partial m^*/\partial\tau)$.

Finally, evaluating the comparative statics of π^* is direct for τ using (10) and the above results.

Specifically,

$$\frac{\partial p^*}{\partial \tau} > 0 > \frac{\partial x^*}{\partial \tau} \implies \frac{\partial \pi^*}{\partial \tau} > 0. \quad \square$$

Proof of Corollary 2. Form Lemma 2, $\hat{p}(m)$ decreases in $m^*(\eta' < 0)$. However, an increase in μ_1 increases the consumer's benefit from considering more firms $(\eta(m^*) > 0)$ from Lemma 1. Thus, the comparative statics of p^* and $\pi^* = p^*/n - cx^*$ with respect to μ_1 both have a direct effect and an indirect effect.

Specifically,

$$\frac{dp^*}{d\mu_1} = \underbrace{\eta(m^*)}_{+} + \underbrace{\mu_1\eta'(m^*)\frac{dm^*}{d\mu_1}}_{-} \tag{9}$$

or

$$\frac{dp^*}{d\mu_1} > 0 \iff \frac{\mu_1}{\tau-\sigma} < m^*(m^*-1) \quad \text{when } \sigma > \tilde{\sigma}.$$

For π^*, note that μ_1 affects not only p^* and m^*, but also x^*. The total impact can be seen by the following:

$$\frac{d\pi^*}{d\mu_1} > 0 \iff \frac{dp^*}{d\mu_1} > nc\left(\frac{dx^*}{d\mu_1}\right). \tag{10}$$

Specifically,

$$\frac{\partial \pi^*}{\partial \mu_1} = \frac{1}{n}\left[\frac{m^*}{m^*-1} - \frac{\mu_1}{\tau-\sigma}\left(\frac{1}{m^*-1}\right)^2\right] - c\frac{\mu_2}{\sigma}\left(-\frac{\tau-\sigma}{(\mu_1-\mu_2)^2}\right)$$

or

$$\frac{\partial \pi^*}{\partial \mu_1} > 0 \iff \frac{\mu_1}{\tau-\sigma} < m^*(m^*-1) + \frac{cn\mu_2}{(\tau-\sigma)\sigma}\frac{m^*-1}{m^*}$$
$$\text{when } \sigma > \tilde{\sigma}.$$

For $\sigma > \tilde{\sigma}$, the comparative statics of z^* with respect to μ_1 follow directly from inspecting the expression given in Proposition 1:

$$\frac{dx^*}{d\mu_1} < 0 \quad \text{if } \sigma > \tilde{\sigma} \quad \text{implies}$$
$$\frac{dp^*}{d\mu_1} > 0 \implies \frac{d\pi^*}{d\mu_1} > 0,$$

and it also implies μ_1 may decrease price but increase profit.

For $\sigma < \tilde{\sigma}$,

$$\frac{\partial m^*}{\partial \mu_1} = \frac{1}{\tau-\sigma+\sigma x^*} - \frac{\mu_1}{(\tau-\sigma+\sigma x^*)^2}\sigma\frac{\partial x^*}{\partial \mu_1} \quad \text{and}$$
$$\frac{\partial x^*}{\partial \mu_1} = \frac{\mu_2}{nc}\xi'(m^*)\frac{\partial m^*}{\partial \mu_1}$$

imply $\partial m^*/\partial\mu_1$, $\partial x^*/\partial\mu_1 > 0$.

Rearranging the terms of (9), it can be shown that

$$\frac{\partial p^*}{\partial \mu_1} > 0$$
$$\iff \frac{\mu_1}{(\tau-\sigma+\sigma x^*)^2 + \mu_1\mu_2\sigma\xi'(m^*)/(nc)} < \frac{m^*(m^*-1)}{\tau-\sigma+\sigma x^*}.$$

The fact that x and $\xi'(m)$ are positive implies that

$$\frac{\mu_1}{\tau-\sigma+\sigma x^*} \le m^*(m^*-1) \tag{11}$$

is sufficient for the positive derivative. Using the expression for m^* in Proposition 1 and noting that $m^* = \mu_1/(\tau-\sigma+\sigma x^*)$, it can be shown that (11) holds for all $m^* \ge 2$, which is assumed. $\square$

Proof of Corollary 3. The impact of changes in μ_2 on p^* and π^* are determined simply through the changes in the equilibrium sampling plan. That is,

$$\frac{\partial p^*}{\partial \mu_2} = \mu_1\eta'(m^*)\frac{\partial m^*}{\partial \mu_2} \quad \text{and}$$
$$\frac{\partial \pi^*}{\partial \mu_2} = \frac{\mu_1}{n}\eta'(m^*)\frac{\partial m^*}{\partial \mu_2} - c\frac{\partial x^*}{\partial \mu_2}.$$

For $\sigma > \tilde{\sigma}$, the comparative statics of m^*, x^*, and p^* with respect to μ_2 follow directly from inspecting the expression given in Proposition 1.

For π^*,

$$\frac{\partial \pi^*}{\partial \mu_2} = \frac{\mu_1}{n}\eta'(m^*)\left(-\frac{1}{\tau-\sigma}\right) - \frac{c}{\sigma}\frac{\tau-\sigma}{\mu_1-\mu_2}\left(1+\frac{\mu_2}{\mu_1-\mu_2}\right) > 0$$
$$\iff \frac{1}{n}\left(\frac{m^*}{m^*-1}\right)^2 > \frac{c}{\sigma}.$$

For $\sigma < \tilde{\sigma}$, because the consumer's optimal sample plan is characterized by an implicit function, we must first solve the comparative statics on m^* and x^* simultaneously.

For μ_2, the pair of equations

$$\frac{\partial m^*}{\partial \mu_2} = \frac{-\mu_1}{(\tau-\sigma+\sigma x^*)^2}\sigma\frac{\partial x^*}{\partial \mu_2} \quad \text{and}$$
$$\frac{\partial x^*}{\partial \mu_2} = \frac{1}{nc}\xi(m^*) + \frac{\mu_2}{nc}\xi'(m^*)\frac{\partial m^*}{\partial \mu_2}$$

imply $\partial m^*/\partial \mu_2 < 0 < \partial x^*/\partial \mu_2$.

Equation (8) implies $\operatorname{sgn}(\partial p^*/\partial \mu_2) = -\operatorname{sgn}(\partial m^*/\partial \mu_2)$.

For μ_2,

$$\frac{\partial \pi^*}{\partial \mu_2} = \frac{\mu_1}{n}\eta'(m^*)\frac{\partial m^*}{\partial \mu_2} - c\frac{\partial x^*}{\partial \mu_2}$$
$$= \frac{\partial x^*}{\partial \mu_2}\left[\frac{\sigma}{n}\left(\frac{m^*}{m^*-1}\right)^2 - c\right],$$

which is positive when the squared-bracketed term is positive because $\partial x^*/\partial \mu_2 > 0$. □

Proof of Proposition 2. First note that the social optimum is independent of whether σ is greater or less than $\tilde{\sigma}$. The maximizer of (6) subject to $z = x$ is explicitly expressed as follows:

$$m^{so} = \frac{1}{\sigma}\Big[(2cn\sigma u_2)\cdot(-cn(\tau-\sigma)-\sigma u_1+\sigma u_2 + \sqrt{4cn\sigma(\tau-\sigma)u_2+(cn(\tau-\sigma)+\sigma u_1-\sigma u_2)^2})^{-1} - cn\Big], \tag{12}$$

$$z^{so} = \frac{1}{2cn\sigma}\Big[\sqrt{4cn\sigma(\tau-\sigma)u_2+(cn(\tau-\sigma)+\sigma u_1-\sigma u_2)^2} - cn(\tau-\sigma)-\sigma u_1+\sigma u_2\Big]. \tag{13}$$

Using these expressions, we can immediately compute the expression for $m^{so}z^{so}$ in (7) and establish the comparison with m^*x^* when $\sigma > \tilde{\sigma}$. Showing that $m^{so} > m^*$ is equivalent to showing that

$$(2cnu_2)\cdot\Big(-cn(\tau-\sigma)-\sigma u_1+\sigma u_2 + \sqrt{4cn\sigma(\tau-\sigma)u_2+(cn(\tau-\sigma)+\sigma u_1-\sigma u_2)^2}\Big)^{-1} - \frac{cn}{\sigma} > \frac{u_1-u_2}{\tau-\sigma},$$

which reduces to the condition $(2cn\sigma(\tau-\sigma)\mu_2)^2 > 0$ and thus always holds. Finally, it must be the case that $z^{so} < z^*$ because $m^{so}z^{so} < m^*z^*$ and $m^{so} > m^*$. □

Proof of Proposition 3. The maximization of (6) implies the system of first-order conditions:

$$z^{so} = \frac{\mu_2}{m^{so}\sigma + nc} \quad \text{and} \quad m^{so} = \frac{\mu_1}{z^{so}\sigma+\tau-\sigma}.$$

Then compare with the corresponding equilibrium values in the limit $\sigma \to 0^+$:

$$z^{so} - x^* = \frac{\mu_2}{\tau-\sigma+\sigma m^{so}} - \frac{\mu_2}{nc}\xi(m^*)$$
$$\longrightarrow \frac{\mu_2}{nc}\left[1-\xi\left(\frac{\mu_1}{\tau}\right)\right] > 0, \tag{14}$$

because $\xi(m) < 1$ for all m. Thus, there exists $\bar{\sigma}_1 > 0$ such that $z^{so} > x^*$ for all $\sigma \in [0, \bar{\sigma}_1)$. In addition,

$$m^{so} - m^* = \frac{\mu_1}{\tau-\sigma+\sigma z^{so}} - \frac{\mu_1}{\tau-\sigma+\sigma x^*} \longrightarrow 0.$$

For any $\sigma \in [0, \bar{\sigma}_1)$, $z^{so} > x^*$ implies $m^{so} < m^*$. For the difference in total number of products evaluated

$$m^{so}z^{so} - m^*x^* \xrightarrow[\sigma\to 0^+]{} \frac{\mu_1\mu_2}{\tau nc} - \frac{\mu_1\mu_2}{\tau nc}\xi\left(\frac{\mu_1}{\tau}\right)$$
$$= \frac{\mu_1\mu_2}{\tau nc}\left(1-\xi\left(\frac{\mu_1}{\tau}\right)\right).$$

This limit expression is strictly positive for $\sigma \in [0, \bar{\sigma}_1)$. (See (14).) This establishes part (i). To establish part (ii), recall that the symmetric equilibrium outcome (m^*, z^*, x^*, p^*) is continuous in σ at $\tilde{\sigma}$. By Proposition 2, $m^{so}z^{so} < m^*z^*$ at $\sigma = \tilde{\sigma}$. Therefore, there exists a $\bar{\sigma}_2 < \tilde{\sigma}$ such that $m^{so}z^{so} < m^*z^*$ for all $\sigma \in (\bar{\sigma}_2, \tilde{\sigma}]$. □

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