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A General Multiple Distributed Lag Framework for Estimating the Dynamic Effects of Promotions

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Game attendance resulting from ticket sales is the single largest revenue stream for Major League Baseball (MLB) teams. We propose a general multiple distributed lag framework following the Koyck family of models for estimating MLB attendance drivers and focus specifically on the differential direct and carryover effects of in-game promotions. By setting various model constraints, the proposed framework incorporates different forms of serial correlation and promotion-specific dynamic effects. Using information model-selection heuristics, we select an optimal model of attendance drivers for the Pittsburgh Pirates' 2010–2012 MLB seasons. We demonstrate that our newly proposed model with an unrestricted serial correlation structure performs best. We find that although kids promotions have the highest direct effect on attendance, giveaway and entertainment promotions have substantial carryover effects and the largest total effects. We use our results to optimize the Pirates' promotional schedule and find that a reallocation of resources across promotional categories can increase profits between 39% and 88%.

Data, as supplemental material, are available at http://dx.doi.org/10.1287/mnsc.2013.1856.

1. Introduction

The estimated size of the U.S. sports industry in 2013 was $470 billion, and it is considered one of the 10 largest business sectors in the United States—twice the size of the auto industry and seven times as large as the movie industry (Plunkett Research 2013). However, research in the major academic marketing journals concerning sports is scarce and limited to areas such as player and team brands (Yang and Shi 2011, Yang et al. 2009), player salaries (Lewis 2008), sponsorship (Speed and Thompson 2000), and ticket pricing (Sainam et al. 2010, Soman and Gourville 2001). It is surprising that the role of in-game promotions (e.g., bobblehead dolls (or bobbleheads), fireworks, and T-shirts) has been overlooked in such prestigious academic journals considering the widespread use of promotions to enhance game-day attendance across a variety of sports at both the collegiate and professional levels (DeSarbo et al. 2012, DeSchriver and Jensen 2002, Hansen and Gauthier 1989, McDonald and Rascher 2000, Siegfried and Eisenberg 1980). In this paper, we focus on the role of in-game promotions and, more specifically, the dynamic effects of these promotions.

According to the U.S. Census Bureau (2013), an estimated $33.1 billion in revenue was generated from sport spectatorship in the United States in 2012.

Although this may seem impressive, revenue from sport spectatorship was down about 2% compared with 2011. The enormous revenues from sport spectatorship (and the implications thereof) are especially pertinent to Major League Baseball (MLB) because teams in the league have more ticket inventory to sell than any other professional sport; for example, whereas MLB teams sell 81 home games, the NBA and NHL only sell 41 home games, and the NFL only sells eight. In addition, ticket sales are the largest individual revenue source for most MLB teams (Fisher 2011), and they accounted for an average of 32% of total gross revenues for teams in 2012 (Badenhausen et al. 2013). MLB saw a 1% decrease in overall attendance in 2013, drawing 74,026,895 fans during the regular season (MLB 2013) (see Figure 1, where the two pronounced dips in attendance occurred in strike shortened seasons). Half the 30 MLB teams experienced a net decline in attendance, with the most severe loss surpassing 600,000 attendees (BaseballReference.com 2013a). Clearly, the success (or failure) of MLB attendance has major revenue implications regarding the overall profitability of these teams.

In an effort to bolster attendance to enhance revenue and profitability, it is important to understand the various factors that drive MLB attendance. After reviewing a fairly exhaustive collection of attendance

Figure 1   MLB Attendance over Time
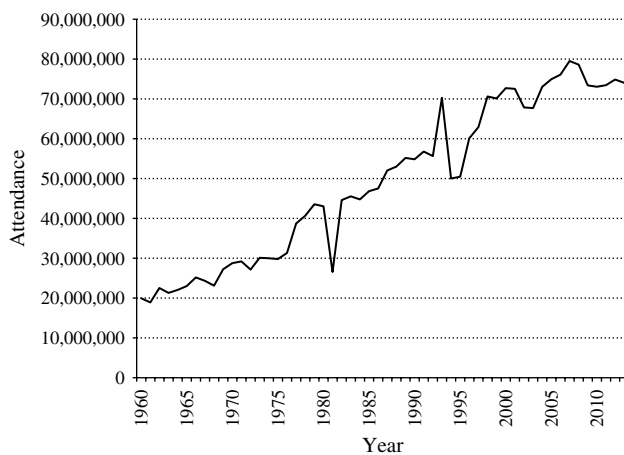
drivers identified in past research, we propose eight main drivers of attendance including in-game promotions, the opponent, team performance, weather, venue, media coverage, demographic and socioeconomic factors, and pricing. We develop a dynamic attendance response model with a specific interest in promotional activities and their role in increasing attendance because the promotional schedule is directly under the control of the chief marketing officer (CMO).

The promotional schedule—which is finalized and made public *before* the beginning of the season and publicized *throughout* the season in an effort to increase attendance—can make attending games more attractive and is likely to increase attendance, ceteris paribus. In addition, promotions may also have an impact on future attendance. On one hand, people visiting a game with a promotion may engage in postconsumption behavior because of a satisfactory experience, which may lead to increased loyalty, positive word of mouth, social bonding, and repurchase intentions (Anderson and Sullivan 1993, Garbarino and Johnson 1999, Lovelock 1983, Oliver 1999, Swan and Oliver 1989). On the other hand, visiting a game with a promotion may lead to satiation and people may engage in intertemporal substitution (Ahn and Lee 2007, Hartmann 2006). To examine the total effect of a promotion, it is necessary to understand both the *direct* and *carryover* effects of promotional expenditures as well as examine how these effects may vary by promotion. Thus, we develop an econometric framework for such promotional modeling based on the Koyck family of distributed lag models and test it on the promotional schedules for the Pittsburgh Pirates' 2010, 2011, and 2012 seasons.

The major methodological contribution of this research is the development of a general multiple distributed lag framework that permits the estimation of differential direct and carryover effects of various types of promotions, encompassing a number of nested special models together with model-selection heuristics and predictive validation techniques. Our proposed saturated (or most general) model is new. Substantively, this is the first research to our knowledge to estimate both the direct and carryover effects of in-game promotions for a major professional sports team. In addition, we also develop an optimization algorithm for the promotional schedule based on our findings.

Our results indicate that kids promotions have the largest direct effect on attendance, followed by giveaway and entertainment promotions. However, kids promotions have a negative carryover effect and the total effect is actually the smallest (a $1 increase in promotional spending increases attendance by 0.015). Taking average ticket prices into account, we find that it is not profitable to employ kids promotions. The total effect of giveaways is the largest (0.125), followed by entertainment promotions (0.069), and both giveaway and entertainment promotions are profitable based on the average ticket price. We use these findings to optimize the promotional schedule and find that a reallocation of resources across promotional categories can increase profits resulting from promotions between 39% and 88%.

The next section reviews the various drivers of game attendance. We then present a brief historical overview of the Pittsburgh Pirates MLB team and examine its attendance and promotional activities for the past three seasons (2010, 2011, and 2012). Next, we formally present our general multiple distributed lag framework, the nested relationships between the various models with associated citations in the marketing literature regarding special cases that have been employed, model-selection heuristics, and predictive validation techniques used to provide guidelines as to the most parsimonious model specification. We then apply our modeling framework to the Pittsburgh Pirates data and examine the direct and carryover effects of various promotions. Based on the findings, we calculate potential changes in profit as a result of adjusting the promotional schedule. We conclude by discussing managerial implications and provide directions for future research.

2. A Review of the Drivers of Professional Sports Attendance

Prior literature has examined a variety of drivers of game attendance. From this literature, we have identified eight major drivers of professional sports attendance, including in-game promotions, the opponent, team performance, weather, venue, media coverage, demographic and socioeconomic factors, and

Figure 2   MLB Attendance Drivers
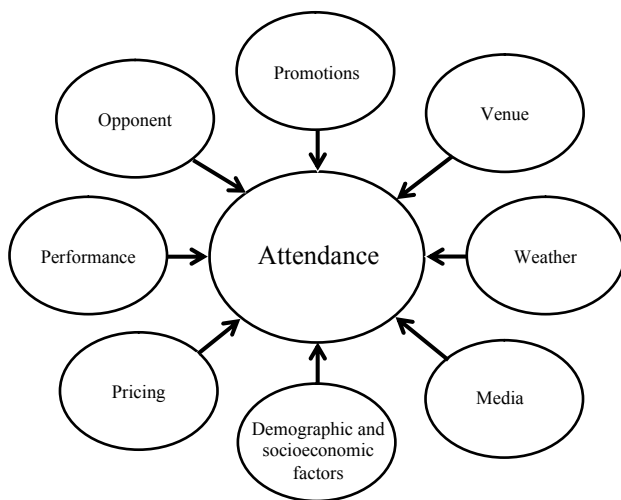
pricing (see Figure 2). Below, we discuss the first six drivers in greater detail. We omit further discussion of the last two drivers because we are unable to include them in our empirical analysis since we do not observe any meaningful variation in demographic/socioeconomic factors (i.e., our data represents only one team/geographic area) or pricing (i.e., dynamic pricing was not implemented uniformly within or across the three seasons and we observe limited game-to-game price variation in our data).

Perhaps the most widely studied attendance driver is that of team performance. This includes objective and relative (i.e., comparison with other teams) performance (Becker and Suls 1983) as measured by team success, competitiveness, and quality (Beckman et al. 2012, Greenstein and Marcum 1981, Hill et al. 1982, Lemke et al. 2010). Generally, attendance often reflects team performance. Some of the most common performance variables linked to changes in attendance include the team's winning percentage, the number of games back, and whether or not the team made the playoffs the previous season (e.g., Beckman et al. 2012, Lemke et al. 2010).

Closely related to team performance is the opponent. More attractive opponents (including higher-quality teams, teams with star players, or rivals) help strengthen game-day attendance (Baade and Tiehen 1990, Hill et al. 1982, Lemke et al. 2010, Marcum and Greenstein 1985). Also related to the opponent, a decrease in the distance between stadiums is associated with increases in attendance (Lemke et al. 2010).

Similarly, more attractive venue (or game schedule) characteristics (e.g., games played in the evening, on the weekend, in the summer, on holidays, or late in the season) serve as attendance boosters (Hansen and Gauthier 1989, Hill et al. 1982, Lemke et al. 2010, Marcum and Greenstein 1985). This driver also includes attractive facility characteristics that are

positively related to game-day attendance (Beckman et al. 2012). Some of the most common venue variables associated with increases in attendance include weekend games and games played in the summer (e.g., Beckman et al. 2012, Lemke et al. 2010).

Empirical evidence related to the impact of weather on attendance is mixed, with some studies demonstrating a negative effect of inclement weather on attendance (Siegfried and Hinshaw 1977, Trail et al. 2008, Welki and Zlatoper 1999) and others failing to establish any significant relationship (Baimbridge et al. 1996, Bird 1982). Typically, the two most common weather variables examined include game-day temperature and precipitation (e.g., Lemke et al. 2010).

As for media alternatives, evidence is also mixed on the impact of televised games on attendance, with some studies reporting a negative effect of televised games on attendance (Allan 2004, Baimbridge et al. 1996, Fizel and Bennett 1989), some reporting a positive effect of televised games on attendance (Kaempfer and Pacey 1986), and others failing to establish any relationship (Hill et al. 1982; Siegfried and Hinshaw 1977, 1979). Generally, the most commonly studied media variables include the availability of games on local and national television (e.g., Lemke et al. 2010). Also related to media alternatives, studies that examine substitute forms of entertainment (including other professional sports) consistently demonstrate a negative effect of substitutes on attendance (Baade and Tiehen 1990, Hill et al. 1982, Trail et al. 2008).

Although all of the aforementioned attendance drivers are important for sports marketing managers to understand and acknowledge, the CMO has little to no control over the majority of these drivers. One potential attendance driver that the CMO does have direct control over, however, is in-game promotions, which is the focus of the current research. MLB teams have made extensive use of such promotions to incentivize fans to attend games as evidenced by the 767 giveaway (e.g., bobbleheads, caps, magnetic schedules, and T-shirts) and 1,505 nongiveaway (e.g., concerts, discounts, and fireworks) MLB promotional dates in 2012 (Broughton 2012).

The merit of engaging in promotional activity is also well supported by prior academic research that outlines the value of such promotions in terms of increasing attendance. For example, past work demonstrated that MLB attendance increases by an average of 14% when a promotion is offered (McDonald and Rascher 2000). Subsequent work is largely consistent with this finding, with Boyd and Krehbiel (2003) reporting an increase in attendance of 19.6% when a promotion is employed. In addition, promotions may be more valuable for poorer performing teams because research

has shown that nonperformance measures (e.g., promotions) have a much greater impact on attendance for losing teams than for winning teams (Marcum and Greenstein 1985). Thus, promotions can be an important marketing tool when on-field performance is poor. Importantly, some authors also recognized differences among attendance drivers (including promotions) for teams in different markets (Boyd and Krehbiel 1999, Lemke et al. 2010). Most recently, and consistent with this logic, DeSarbo et al. (2012) calibrated the differential impact of attendance based on different forms of in-game promotions for a single MLB team.

Whereas prior literature has examined the effectiveness of in-game promotions at professional sporting events, we distinguish ourselves from this literature in three ways. First, we investigate the dynamic effects of in-game promotions, which to our knowledge have not yet been examined in prior research. Second, in an effort to strengthen the methodological rigor of the study of promotions as well as the implications drawn from such work, we are one of the first to use promotional costs instead of dummy variables (see McDonald and Rascher 2000 for an exception in which giveaway costs were used as a proxy for promotional value). Third, few studies distinguish between different types of promotions and their influence on attendance above and beyond uncontrollable external drivers. Some previous work has examined differences between various promotions, but it is often unclear which promotions are included in such studies, and in some cases, a number of promotions are purposefully excluded from analysis (see Barilla et al. 2008, Boyd and Krehbiel 2003, Gifis and Sommers 2006, Lemke et al. 2010, Marcum and Greenstein 1985, McDonald and Rascher 2000, Siegfried and Eisenberg 1980). Therefore, every promotion employed by the Pirates from 2010 to 2012 was included in the data set and classified into one of the following four promotional categories (to be described in more detail later): entertainment, giveaway, kids, and price promotions.

3. The Pittsburgh Pirates

The Pittsburgh Pirates is a professional baseball team and a member of the Central Division in the National League of Major League Baseball. The Pirates played its first National League game in 1887 (Pittsburgh Pirates 2013b), and since the team's inception, it has made 14 playoff appearances and won nine pennants and five World Series championships: 1909, 1925, 1960, 1971, and 1979 (BaseballReference.com 2013b). Thirty-eight past Pirates Hall of Famers exist to date and include players such as Sparky Anderson, Roberto Clemente,

Willie Stargell, and Honus Wagner (Pittsburgh Pirates 2013b). While past team performance was strong, more recently, poor on-field performance led to the accumulation of 20 losing seasons, which is the record for consecutive losing seasons in North American professional sport (Fisher 2013).[1] The Pirates currently play in PNC Park, a riverfront stadium located on Pittsburgh's North Shore. PNC Park, which opened in 2001, is the team's fifth home (Pittsburgh Pirates 2013a) and was named best MLB ballpark in the nation by ESPN in 2006 (Caple 2006). PNC Park has a seating capacity of 38,362 and reached an attendance high of 39,585 (which includes standing room only tickets) in 2012 for the seasons under investigation.

The current study examines an extensive set of more than 100 variables related to attendance for the Pittsburgh Pirates' 2010, 2011, and 2012 seasons.[2] Based on the literature reviewed in §2, the drivers examined in this analysis include promotions (presence, type, category, and cost of); opponent characteristics (team, performance variables, rivalries, division, distance, population, and total home attendance of the opponent); performance variables (score, wins versus losses, winning percentage, winning streaks, and number of games back); weather (temperature and precipitation); venue (season, game date, month, day of the week, holidays, and the percentage of the stadium filled for each day of the week averaged across all other MLB teams); and media coverage (broadcast on local or national television and other concurrent professional sporting events played in Pittsburgh). Specifically, we want to assess the relationship between in-game promotions and their role in increasing attendance for the Pirates in conjunction with these other uncontrollable external drivers.

In the current study, the dependent variable of attendance is reported by MLB as the number of tickets sold for a particular game. The number of tickets sold includes both full-season and game-specific ticket holders (made up of single game and partial plan (i.e., a preselected package with a set number of games) buyers). Note that the number of season ticket holders is included in the attendance figure for every game and is constant throughout the season in our model. Although our measure of attendance does not accurately reflect the turnstile count (i.e., actual attendance at the gate), we focus on the number of

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[1] However, the Pirates recently put an end to this trend by securing a winning season in 2013 and qualifying for the playoffs.

[2] Secondary data included in the analyses were collected from the following websites (accessed September 30, 2013): http://www.mlb.com, http://www.pirates.com, http://www.baseball-reference.com, http://www.farmersalmanac.com/weather-history/, http://www.pro-football-reference.com/teams/pit/, http://www.hockey-reference.com/teams/PIT/.

Figure 3   The Relationship Between MLB Attendance Drivers and Attendance for the Pittsburgh Pirates
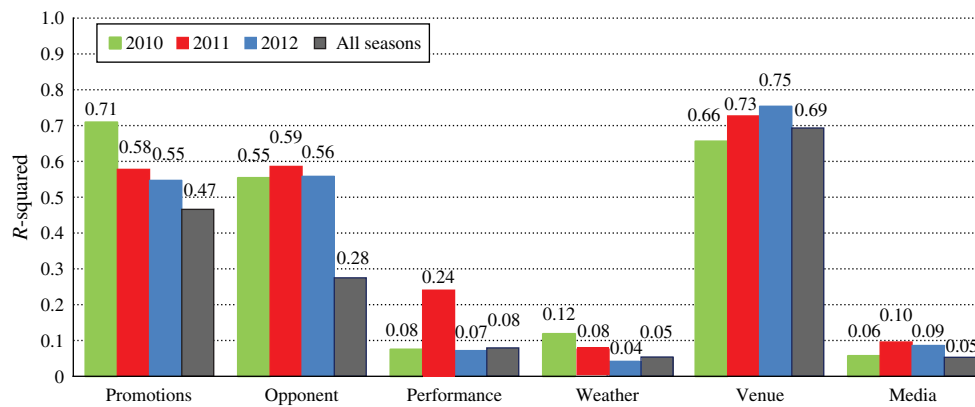
tickets sold because this is most directly related to team revenue.

In terms of promotions, Table 1 lists the 22 different types of in-game promotions that were utilized in the 2010, 2011, and 2012 seasons for the Pittsburgh Pirates. In 2010, 52 of the 81 home games involved one or more of the promotions listed in this table. In 2011, 42 of the 80 home games involved at least one of these promotions, and in 2012, this number decreased slightly to 39 of the 80 home games.[3] Thus, several of these individual promotions were repeated throughout the course of each season. In addition, there were some instances of multiple promotions employed in a single home game.

We start by presenting a preliminary analysis of the amount of variation in attendance explained by each of the six drivers (promotions, opponent, performance, weather, venue, and media coverage) that we conducted with the Pittsburgh Pirates. Figure 3 shows the R-squares obtained by separately regressing the indicators of each general driver on game attendance for each season and for the three seasons combined. Venue is the most important driver of attendance for 2011, 2012, and the three seasons combined, and promotions are the most important driver of attendance for 2010 as well as the second most important driver for the three seasons combined. The results in Figure 3 also illustrate that the various drivers of attendance are intercorrelated as the R-squares across the drivers for each season sum to more than one. Overall, the results in Figure 3 demonstrate that promotions (i.e., the only attendance driver directly under the CMO's control) can be considered a highly significant driver of attendance.

[3] For the 2011 and 2012 seasons, we only include 80 (versus 81) home games in our analysis because game 126 in 2011 and game 17 in 2012 were rescheduled as traditional double headers in which attendance data was not reported. There was no promotional event scheduled for either game date.

This finding is also supported by a careful examination of Figure 4, which displays game attendance for all the Pirates' home games for 2010, 2011, and 2012. To draw more than the Pirates' explicit goal of two million fans per season (designated by the horizontal line at 24,692 fans/game), some sort of promotion appeared necessary. Note that a promotion did not guarantee that an attendance above the target would be realized—we witness a substantial number of games with promotions that fell below the goal for all three seasons—but this trend seemed to decline as performance increased from 2010 to 2012 (with

Table 1   The 22 Different Types of Promotions Utilized by the Pittsburgh Pirates in 2010, 2011, and 2012

Promotion category	Promotion employed	2010	2011	2012
Entertainment	Concert	5	4	4
	Fireworks	8	9	8
	Irish/Italian/Polish celebration	3		
	Total entertainment promotions	**16**	**13**	**12**
Giveaway	Bag			1
	Beach towel	1		
	Blanket	1		1
	Bobblehead	1	1	1
	Bottle	1		
	Bowl	1		
	Bust/figurine	2	1	
	Canvas wrap	3	1	
	Cap	5	2	2
	Coffee clutch		1	
	Cooler	1		
	Gloves and scarf set	1		
	Magnetic schedule	4	2	2
	Mother's Day gift	1	1	1
	Poster	1	2	
	Stein	1	1	
	T-shirt	10	13	12
	Umbrella			1
	Total giveaway promotions	**34**	**25**	**21**
Kids	Kids day	12	13	13
	Total kids promotions	**12**	**13**	**13**
Total	*Total promotions*	**62**	**51**	**46**

Figure 4 Pittsburgh Pirates' Home Attendance for Games With and Without Promotions
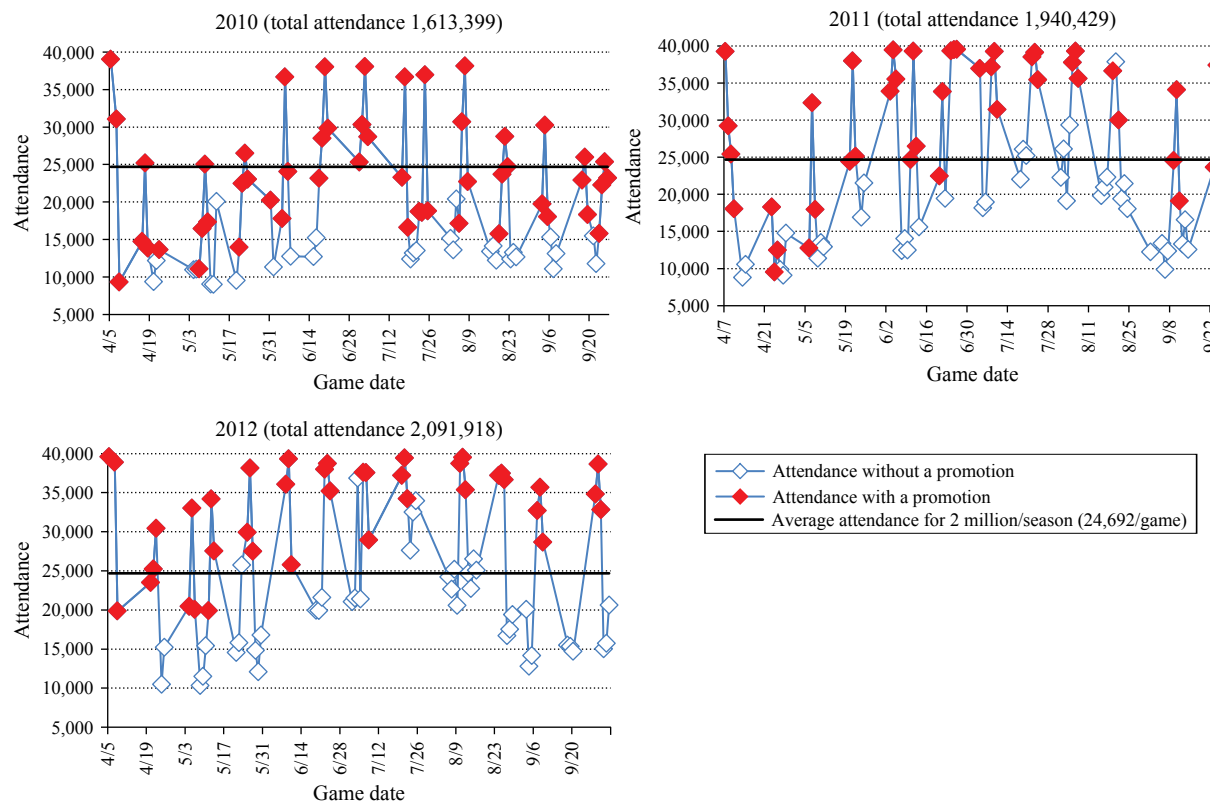
season-long winning percentages of 0.352, 0.444, and 0.488, respectively).

Given the relatively small number of observations (241 games across three seasons) relative to some 22 different types of promotions and the fact that many promotions are only employed a few times, we aggregate the promotions into three categories: *entertainment*, *giveaway*, and *kids* promotions (see Table 1).[4] The goal here is to examine whether there is differential effectiveness by category of promotion. For this endeavor, we introduce a new general multiple distributed lag framework capable of estimating a number of nested attendance response models, some of whose special cases have been published in the literature, as well as model-selection criteria to delineate which model is most parsimonious.

4. The Proposed Attendance Response Model Framework

The main goal of promotions in this MLB context is to increase game-day attendance, which we refer to as the *direct effect*. However, promotions may also have

an impact on future attendance, which we refer to as the *carryover effect*. The combination of the direct and carryover effects determines the *total effect* of promotions. Promotions may impact future attendance through several different mechanisms: purchase reinforcement, word of mouth, intertemporal substitution, satiation, etc. First, promotions can lead to purchase reinforcement (Givon and Horsky 1990). Garbarino and Johnson (1999) found that for occasional ticket buyers, satisfaction is the primary driver of future purchase intentions. Satisfaction may depend on the type of promotion. Satisfaction may also increase as a result of the quality of the promotion, the number of other fans attending a game (Lovelock 1983), the overall experience of the game itself, or team performance. Second, promotions may lead to positive or negative word of mouth from fans who attended the game (Westbrook 1987). Promotions are designed to make games more attractive, so we expect that positive word of mouth prevails. Third, customers may engage in intertemporal substitution (Ahn and Lee 2007). For example, they may attend a game on Sunday with an attractive promotion instead of a game on the following Monday without a promotion. Fourth, attending a game with a promotion may lead to satiation, whereby the marginal utility of visiting another game decreases immediately after attending the present game (Hartmann 2006).

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[4] We excluded the price promotions category from the analysis because it was only employed once per season (in the form of Buc Day/Night) and because we do not have cost information for this promotion.

These effects are greatest directly after the game experience and decline over time as spectators forget their experience and their enthusiasm/satiation decreases (Chessa and Murre 2007, Homburg et al. 2006, Westbrook 1987). Hence, these effects will typically have a greater impact on games in the near versus distant future, which needs to be reflected in the construction of the attendance response model. It is also important to allow for differential direct and carryover effects by type of promotion because promotions may vary in terms of quality and attractiveness.

Next, we discuss the proposed multiple distributed lag framework, estimation, model diagnostics, incorporation of control variables, potential endogeneity, and the pooling of data across seasons.

4.1. Multiple Distributed Lag Framework

To model the relationship between the Pittsburgh Pirates' game attendance and promotional activities, we want to measure the impact of promotions on both current and future games. As such, we devise a distributed lag model. Such a model captures *both* the direct effect of promotions to attract fans to the game as well as the potential carryover effects of purchase reinforcement, word of mouth, intertemporal substitution, and satiation that may impact future attendance. The model is given by

$$Y_t = \sum_{i=0}^{\infty} \beta_i X_{t-i} + \varepsilon_t, \tag{1}$$

where Y_t represents the attendance at game t and X_t represents the total promotional costs at game t. For now, we assume that the disturbances are independent and identically normally distributed. For ease of exposition, we leave out the constant as well as external variables in Equation (1). We will return to the assumptions on the error term and the inclusion of external variables later.

When the summation over i in Equation (1) runs to infinity, the model is called an *infinite* distributed lag model. When the summation runs over a finite number of periods P, the model is called a *finite* distributed lag model. The finite distributed lag model suffers from several drawbacks: (1) before estimation, one must specify the lag length p, which is typically unknown; (2) the higher the lag length, the fewer degrees of freedom remain; and (3) these models often suffer from multicollinearity between the lagged explanatory variables. One of the most popular finite distributed lag models is the Almon (1965) lag model, which approximates the β_i's by a low-degree polynomial. Given the limitations listed above, a finite distributed lag model seems ill suited for this study.

The infinite distributed lag model in Equation (1) cannot be estimated directly because it has an infinite

number of parameters. Therefore, one has to impose some structure on the β_i's. We assume that $-\infty < \sum_{i=0}^{\infty} \beta_i < \infty$. A popular structure imposed on the β_i's reflects an exponential decay, whereby after a certain time point $i = m_0$, the lag parameters follow the sequence

$$\beta_{m_0+m} = \lambda \beta_{m_0+m-1}, \tag{2}$$

where m_0, $m \geq 0$, and $0 < \lambda < 1$. The latter assumption is made to ensure that the lag structure has a smooth decreasing pattern. A negative λ would imply an alternating effect of promotions over time, which is not realistic in the vast majority of real-world applications. The value of m_0 can be determined empirically and is equal to zero in most cases in the literature. However, if one expects sign changes in the lagged effects or the exponential decay to set in only after a few periods, one can set $m_0 > 0$. The exponential decay in Equation (2) in combination with Equation (1) is commonly referred to as the geometric distributed lag (or Koyck) model and was originally proposed by Koyck (1954, pp. 20–25). The exponential decay structure reflects the fact that the strength of an attendee's experience diminishes exponentially over time (Mehta et al. 2004). The exponential decay of memory over time is supported by laboratory experiments and studies in neuroscience (see Chessa and Murre 2007 and the references therein).

Combining Equations (1) and (2), we obtain the following:

$$Y_t = \sum_{i=0}^{m_0-1} \beta_i X_{t-i} + \beta_m \sum_{i=m_0}^{\infty} \lambda^{i-m_0} X_{t-i} + \varepsilon_t. \tag{3}$$

At this point, it is convenient to introduce the lag operator L, whereby $L^i \cdot X_t = X_{t-i}$. To make Equation (3) estimable, we subtract $\lambda \cdot Y_{t-1}$ from Equation (3), commonly known as the Koyck transformation, and after rearranging terms, we get

$$Y_t = \frac{\beta_0}{1-\lambda L} X_t + \frac{\sum_{i=1}^{m_0} (\beta_i - \lambda \beta_{i-1}) X_{t-i}}{1-\lambda L} + \varepsilon_t, \tag{4}$$

which is equivalent to

$$Y_t = \beta_0 X_t + \sum_{i=1}^{m_0} (\beta_i - \lambda \beta_{i-1}) X_{t-i} + \lambda L Y_t + (1-\lambda L) \varepsilon_t. \tag{5}$$

This model is equivalent to an autoregressive moving average model with explanatory variables (ARMAX), noting that λ appears multiple times in Equation (5). When $m_0 = 0$, which means that the exponential decay starts immediately, Equation (5) reduces to

$$Y_t = \beta_0 X_t + \lambda L Y_t + (1-\lambda L) \varepsilon_t. \tag{6}$$

This model is frequently used in the literature. For example, Equation (6) is similar to the adaptive expectations model (Cagan 1956). The partial adjustment

model proposed by Nerlove (1958) is closely related to Equation (6) as well; the only difference is that the error term in the partial adjustment model is not a moving average. Our proposed framework includes both of these classic models as special cases.[5]

Depending on the intensity of the experience and where in the brain the memories are stored, promotions may have a different decay pattern (Chessa and Murre 2007). We can extend the geometric distributed lag model to multiple promotional variables, all of which have their own distributed lag function. Such a model was first suggested by Griliches (1967) and first applied in marketing by Peles (1971), who demonstrated that not accounting for differential distributed lag functions can lead to biased estimates. Other studies have followed this tradition (e.g., Berkowitz et al. 2001, Breuer et al. 2011, Herrington and Dempsey 2005, Leeflang et al. 1992, Montgomery and Silk 1972, Naik and Raman 2003), and several of these studies found differential lag functions across promotions.

Rewriting Equation (4) to allow for $j = 1, \ldots, J$ different promotional variables, X_{jt}, gives

$$Y_t = \sum_{j=1}^{J} \frac{\beta_{0j}}{1 - \lambda_j L} X_{jt}$$
$$+ \sum_{j=1}^{J} \frac{\sum_{i=1}^{m_{0j}} (\beta_{ij} - \lambda_j \beta_{i-1,j}) X_{j,t-i}}{1 - \lambda_j L} + \varepsilon_t, \quad (7)$$

which is equivalent to

$$\prod_{j=1}^{J} (1 - \lambda_j L) Y_t$$
$$= \sum_{j=1}^{J} \left(\prod_{k \neq j} (1 - \lambda_k L) \beta_{0j} X_{jt} \right)$$
$$+ \sum_{j=1}^{J} \sum_{i=1}^{m_{0j}} \prod_{k \neq j} (1 - \lambda_k L)(\beta_{ij} - \lambda_j \beta_{i-1,j}) X_{j,t-i}$$
$$+ \prod_{j=1}^{J} (1 - \lambda_j L) \varepsilon_t. \quad (8)$$

[5] Adding lagged X variables to Equation (6) and ignoring the moving average term leads to the autoregressive distributed lag model (ADL). The simplest version of this model is the ADL(1, 1) model that has an autoregressive order of one and contains contemporaneous and one-period lagged X variables. The ADL(1, 1) model can be rewritten as a stationary error correction model. Although this model is more flexible than the geometric distributed lag model, it is more difficult to empirically disentangle the carryover effects of promotions and the serial correlation because a moving average process of higher order can be closely approximated by an autoregressive term of order one. Disentangling the carryover effects of promotions and serial correlation becomes especially challenging when the model is extended to allow for differential effects of multiple promotional variables (which is our ultimate goal) as such a model contains a higher-order moving average process because of the Koyck transformation. Also note that Equation (5) with $m_0 = 1$ leads to a restricted form of the ADL(1, 1) model.

We refer to the models in Equations (7) and (8) as the *multiple geometric distributed lag model*. We can now allow for a more flexible form of serial correlation by relaxing the restriction that the moving average (MA) parameters are the same as the parameters found elsewhere in the model. For example, there may already be serial correlation present in Equation (1) before applying the Koyck transformation. This results in the new and most flexible multiple geometric distributed lag model, which we denote as Model A:

$$\prod_{j=1}^{J} (1 - \lambda_j L) Y_t$$
$$= \sum_{j=1}^{J} \left(\prod_{k \neq j} (1 - \lambda_k L) \beta_{0j} X_{jt} \right)$$
$$+ \sum_{j=1}^{J} \sum_{i=1}^{m_{0j}} \prod_{k \neq j} (1 - \lambda_k L)(\beta_{ij} - \lambda_j \beta_{i-1,j}) X_{j,t-i}$$
$$+ \prod_{j=1}^{J} (1 - \rho_j^* L) \varepsilon_t, \quad (9)$$

where $\rho_j^* = \rho_j + \lambda_j$ and ρ_j represents the additional serial correlation on top of the "artificial" serial correlation caused by the Koyck transformation.

In our MLB application, we consider the effect of promotional spending for three different categories ($J = 3$) on attendance: entertainment, giveaway, and kids promotions. Based on the most flexible model in Equation (9) (Model A), we can test the various restricted models (cf. Leeflang et al. 1992, Weiss and Windal 1980). Table 2 provides an overview of all the different models one can construct from this general framework, the parameter restrictions imposed in Equation (9), and a (nonexhaustive) overview of where these models are employed in the literature. Note that most of the literature assumes $m_{0j} = 0$ for all j (except for some early applications of the Koyck model, e.g., Bass and Clarke 1972, Massy and Frank 1965, Montgomery and Silk 1972).

Model B in Table 2 restricts the moving average parameters to be similar to the distributed lag parameters in line with the original Koyck model (see Equation (8)). Peles (1971) already proposed this model but estimated a simplified version. Model C assumes no serial correlation and has been used by Breuer et al. (2011) and Herrington and Dempsey (2005). However, these studies did not properly test for serial correlation. Models D–F assume a similar carryover effect for all promotional variables but differ in their serial correlation structure. To the best of our knowledge, Model D, with an unrestricted serial correlation structure, has not been used in the literature. The serial correlation structure in Model E is again in line with the original Koyck model, and Model F assumes no serial

Table 2   Model Specifications and Restrictions Compared with Model A

Model	Restrictions	Equation	Literature
A	—	$(1 - \lambda_1 L)(1 - \lambda_2 L)(1 - \lambda_3 L)Y_t$ $= (1 - \lambda_2 L)(1 - \lambda_3 L)\beta_{01}X_{1t} + (1 - \lambda_1 L)(1 - \lambda_3 L)\beta_{02}X_{2t} + (1 - \lambda_1 L)(1 - \lambda_2 L)\beta_{03}X_{3t} + \sum_{i=1}^{m_{01}}(1 - \lambda_2 L)(1 - \lambda_3 L)(\beta_{i1} - \lambda_1\beta_{i-1,1})X_{1,t-i}$ $+ \sum_{i=1}^{m_{02}}(1 - \lambda_1 L)(1 - \lambda_3 L)(\beta_{i2} - \lambda_2\beta_{i-1,2})X_{2,t-i} + \sum_{i=1}^{m_{03}}(1 - \lambda_1 L)(1 - \lambda_2 L)(\beta_{i3} - \lambda_3\beta_{i-1,3})X_{3,t-i}$ $+ (1 - (\rho_1 + \lambda_1)L)(1 - (\rho_2 + \lambda_2)L)(1 - (\rho_3 + \lambda_3)L)\varepsilon_t$	NEW
B	$\rho_1 = \rho_2 = \rho_3 = 0$	$(1 - \lambda_1 L)(1 - \lambda_2 L)(1 - \lambda_3 L)Y_t$ $= (1 - \lambda_2 L)(1 - \lambda_3 L)\beta_{01}X_{1t} + (1 - \lambda_1 L)(1 - \lambda_3 L)\beta_{02}X_{2t} + (1 - \lambda_1 L)(1 - \lambda_2 L)\beta_{03}X_{3t} + \sum_{i=1}^{m_{01}}(1 - \lambda_2 L)(1 - \lambda_3 L)(\beta_{i1} - \lambda_1\beta_{i-1,1})X_{1,t-i}$ $+ \sum_{i=1}^{m_{02}}(1 - \lambda_1 L)(1 - \lambda_3 L)(\beta_{i2} - \lambda_2\beta_{i-1,2})X_{2,t-i} + \sum_{i=1}^{m_{03}}(1 - \lambda_1 L)(1 - \lambda_2 L)(\beta_{i3} - \lambda_3\beta_{i-1,3})X_{3,t-i} + (1 - \lambda_1 L)(1 - \lambda_2 L)(1 - \lambda_3 L)\varepsilon_t$	Leeflang et al. (1992)
C	$\rho_1 + \lambda_1 = \rho_2 + \lambda_2 = \rho_3 + \lambda_3 = 0$	$(1 - \lambda_1 L)(1 - \lambda_2 L)(1 - \lambda_3 L)Y_t$ $= (1 - \lambda_2 L)(1 - \lambda_3 L)\beta_{01}X_{1t} + (1 - \lambda_1 L)(1 - \lambda_3 L)\beta_{02}X_{2t} + (1 - \lambda_1 L)(1 - \lambda_2 L)\beta_{03}X_{3t} + \sum_{i=1}^{m_{01}}(1 - \lambda_2 L)(1 - \lambda_3 L)(\beta_{i1} - \lambda_1\beta_{i-1,1})X_{1,t-i}$ $+ \sum_{i=1}^{m_{02}}(1 - \lambda_1 L)(1 - \lambda_3 L)(\beta_{i2} - \lambda_2\beta_{i-1,2})X_{2,t-i} + \sum_{i=1}^{m_{03}}(1 - \lambda_1 L)(1 - \lambda_2 L)(\beta_{i3} - \lambda_3\beta_{i-1,3})X_{3,t-i} + \varepsilon_t$	Berkowitz et al. (2001), Breuer et al. (2011), Herrington and Dempsey (2005)
D	$\lambda = \lambda_1 = \lambda_2 = \lambda_3$; $\rho = \rho_1$; $\rho_2 = \rho_3 = 0$	$(1 - \lambda L)Y_t = \beta_{01}X_{1t} + \beta_{02}X_{2t} + \beta_{03}X_{3t} + \sum_{i=1}^{m_{01}}(\beta_{i1} - \lambda\beta_{i-1,1})X_{1,t-i} + \sum_{i=1}^{m_{02}}(\beta_{i2} - \lambda\beta_{i-1,2})X_{2,t-i} + \sum_{i=1}^{m_{03}}(\beta_{i3} - \lambda\beta_{i-1,3})X_{3,t-i} + (1 - (\rho + \lambda)L)\varepsilon_t$	NEW
E	$\lambda = \lambda_1 = \lambda_2 = \lambda_3$; $\rho_1 = \rho_2 = \rho_3 = 0$	$(1 - \lambda L)Y_t = \beta_{01}X_{1t} + \beta_{02}X_{2t} + \beta_{03}X_{3t} + \sum_{i=1}^{m_{01}}(\beta_{i1} - \lambda\beta_{i-1,1})X_{1,t-i} + \sum_{i=1}^{m_{02}}(\beta_{i2} - \lambda\beta_{i-1,2})X_{2,t-i} + \sum_{i=1}^{m_{03}}(\beta_{i3} - \lambda\beta_{i-1,3})X_{3,t-i} + (1 - \lambda L)\varepsilon_t$	Leeflang et al. (1992)
F	$\lambda = \lambda_1 = \lambda_2 = \lambda_3$; $\rho_1 + \lambda_1 = \rho_2 + \lambda_2 = \rho_3 + \lambda_3 = 0$	$(1 - \lambda L)Y_t = \beta_{01}X_{1t} + \beta_{02}X_{2t} + \beta_{03}X_{3t} + \sum_{i=1}^{m_{01}}(\beta_{i1} - \lambda\beta_{i-1,1})X_{1,t-i} + \sum_{i=1}^{m_{02}}(\beta_{i2} - \lambda\beta_{i-1,2})X_{2,t-i} + \sum_{i=1}^{m_{03}}(\beta_{i3} - \lambda\beta_{i-1,3})X_{3,t-i} + \varepsilon_t$	Massy and Frank (1965), Montgomery and Silk (1972), Naik and Raman (2003)
G	$\beta_i = \beta_{ij} = \beta_{ik}$ if $i \le \min\{m_{0j}, m_{0k}\}$, for $j, k = 1, 2, 3$; $\lambda = \lambda_1 = \lambda_2 = \lambda_3$; $\rho = \rho_1$; $\rho_2 = \rho_3 = 0$	$(1 - \lambda L)Y_t = \beta_0 X_t + \sum_{i=1}^{m_{01}}(\beta_i - \lambda\beta_{i-1})X_{1,t-i} + \sum_{i=1}^{m_{02}}(\beta_i - \lambda\beta_{i-1})X_{2,t-i} + \sum_{i=1}^{m_{03}}(\beta_i - \lambda\beta_{i-1})X_{3,t-i} + (1 - (\rho + \lambda)L)\varepsilon_t$	Palda (1964), Winer (1979)
H	$\beta_i = \beta_{ij} = \beta_{ik}$ if $i \le \min\{m_{0j}, m_{0k}\}$, for $j, k = 1, 2, 3$; $\lambda = \lambda_1 = \lambda_2 = \lambda_3$; $\rho_1 = \rho_2 = \rho_3 = 0$	$(1 - \lambda L)Y_t = \beta_0 X_t + \sum_{i=1}^{m_{01}}(\beta_i - \lambda\beta_{i-1})X_{1,t-i} + \sum_{i=1}^{m_{02}}(\beta_i - \lambda\beta_{i-1})X_{2,t-i} + \sum_{i=1}^{m_{03}}(\beta_i - \lambda\beta_{i-1})X_{3,t-i} + (1 - \lambda L)\varepsilon_t$	Bass and Clarke (1972), Palda (1964), Zellner and Geisel (1970)
I	$\beta_i = \beta_{ij} = \beta_{ik}$ if $i \le \min\{m_{0j}, m_{0k}\}$, for $j, k = 1, 2, 3$; $\lambda = \lambda_1 = \lambda_2 = \lambda_3$; $\rho_1 + \lambda_1 = \rho_2 + \lambda_2 = \rho_3 + \lambda_3 = 0$	$(1 - \lambda L)Y_t = \beta_0 X_t + \sum_{i=1}^{m_{01}}(\beta_i - \lambda\beta_{i-1})X_{1,t-i} + \sum_{i=1}^{m_{02}}(\beta_i - \lambda\beta_{i-1})X_{2,t-i} + \sum_{i=1}^{m_{03}}(\beta_i - \lambda\beta_{i-1})X_{3,t-i} + \varepsilon_t$	Palda (1964), Winer (1979)

correlation. Models G–I are most similar to the original Koyck model and assume that the direct effect of a promotion, β, is similar across promotions. These particular models are widely used in the marketing literature, initiated by Palda (1964). Note that Model H is similar to the original adaptive expectations model (Cagan 1956) and Model I is similar to the original partial adjustment model (Nerlove 1958).

4.2. Estimation

In the models that allow for serial correlation (Models A–B, D–E, and G–H), ordinary least squares is biased and inconsistent because the lagged dependent variable is correlated with the disturbance term. The maximum-likelihood method proposed by Zellner and Geisel (1970) is frequently used in the literature to estimate such geometric lag models. It involves a grid search over a subset of the parameters, before estimating the other parameters conditional on the initial subset of parameters. However, this conditional estimation does not provide standard errors around the parameters on which a grid search is performed and underestimates the asymptotic standard errors of the other parameters. Alternative estimation methods that provide unbiased and consistent estimates are instrumental variables (IV) and maximum-likelihood estimation. The IV method requires suitable instruments and moreover renders inefficient estimates. Full maximum-likelihood estimation, with the appropriate parameter restrictions, is more challenging to perform because of the nonlinearity of the equation, but it provides asymptotically efficient estimates.

Hence, we employ Gaussian full maximum-likelihood estimation of all the parameters, taking into account that the autoregressive and moving average parameters may appear multiple times in the model. Our most extensive model (Model A) is of order $j + \max(m_{0j})$, so we drop the first $j + \max(m_{0j})$ observations from our estimation sample. We do this for all models to facilitate the comparison across models, even though some of the more restricted models have a smaller order. We set the initial values for the moving average terms equal to zero. Also, we transform the parameters λ and ρ to ensure that $0 < \lambda < 1$ and $-1 < \rho < 1$. An estimate of λ below zero would imply an alternating effect of promotions over time, which is unrealistic in this application, and a λ larger than one implies that the dependent variable is evolving. A unit root test showed that this is not the case in our data. Finally, the data do not suffer from the data interval bias (Clarke 1976, Tellis and Franses 2006) because it is in its most disaggregate form (i.e., the individual game level).

4.3. Diagnostics

Equation (1) may contain serial correlation because of omitted variables. Although we discuss the inclusion

of variables related to the other drivers of attendance in the next section, Equation (1) may still exhibit serial correlation in which case the model would render biased parameters if not properly accounted for. Although Equation (9) may already capture this serial correlation, it is important to test all models in our framework for remaining serial correlation. In our estimation, we test for remaining serial correlation using the Breusch–Godfrey test.[6] This test is also known as the Lagrange multiplier test and performs well in small samples. If this test provides evidence of serial correlation, we should increase the order of the moving average terms.[7]

To select the best model among those listed in Table 2, we compute an out-of-sample and several in-sample performance diagnostics. The in-sample fit is assessed by comparing three common measures across the models: (1) the log-likelihood, (2) the Akaike information criterion (AIC), and (3) the Schwarz information criterion (SIC). Both information criteria trade off the fit of the model with the number of parameters, with the SIC imposing a higher penalty for additional parameters which performs well for dynamic models (Rust et al. 1995). However, for each model we first select the optimal value for m_{0j}, for $j = 1, \ldots, J$, based on the AIC and SIC.

To test out-of-sample fit, we break up the sample into an estimation sample and a holdout sample. Because we have a limited number of observations available per season, we use *leave-one-out* validation.[8] For every observation, we estimate the model based on all other observations that do not use the left-out observation as an explanatory variable (recognizing the order of the model) and predict the attendance for the left-out observation. As our models are at a maximum of order $j + \max(m_{0j})$, we do not use the $j + \max(m_{0j})$ observations following the left-out observation in our estimation sample, and we set the resulting

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[6] A frequently used test for serial correlation is the Durbin–Watson test. However, in autoregressive models, this test is biased toward rejecting serial correlation (Gujarati 2003). Another alternative to test for serial correlation is Durbin's h test, which is unbiased in autoregressive models but has lower power than the Breusch–Godfrey test.

[7] Alternatively, one can correct for remaining serial correlation using an autoregressive or an autoregressive moving average lag structure (see Carter and Zellner 2004 for an extensive discussion).

[8] Although using a holdout sample at the end of the observation period to test the fit of the model is most common in time-series analysis, this method may not be informative given the nature of our data. In our application, the last games of the final season in our sample show an atypical pattern for two reasons: (1) the Pirates were in playoff contention until late in the season, and (2) the Pirates had an opportunity to put an end to 20 consecutive losing seasons up until the last home stand (however, several games before the end of the season they failed to achieve either a playoff berth or finish the season at or above 0.500).

missing values for the moving average terms equal to zero. We use the root mean squared prediction error (RMSPE) and the mean absolute percentage error (MAPE) as measures of out-of-sample fit. In addition to these fit statistics, we compute a likelihood ratio test to formally test the parameter restrictions in Table 2 compared with Model A.

Thus, the methodological contribution of the current work is to provide a general framework for the construction of an entire family of Koyck distributed lag models of which Model A is the most general form. We demonstrate how setting various model parameter restrictions allows for the estimation of more traditional Koyck model forms as seen in prior literature. In addition, we propose a set of model-selection heuristics in order to select the most parsimonious model given a specific empirical application.

4.4. Other Drivers of Attendance Variables

As discussed in §2 and shown in Figure 3, there are other significant drivers of attendance related to the opponent, performance, weather, venue, and media coverage. In our estimation, we add a subset of variables per driver directly to the initial distributed lag model before applying the Koyck transformation (cf. Peles 1971). This ensures that the estimates for the other explanatory variables can be interpreted as direct effects on attendance. We select the explanatory variables based on three criteria to ensure parsimony of the model because of the limited number of observations.

First, we look to the previous literature discussed in §2, which identifies the most important variables within each of the six drivers. Guided by the importance of the various drivers as shown in Figure 3, we select at least one variable (the most relevant) within each driver and select additional variables for the more important drivers. Second, we include several variables that are highly correlated to the promotional variables of interest as well as attendance to prevent confounding effects. Specifically, we include separate dummy variables for Friday, Saturday, and Sunday to avoid confounding between the effects of promotions and day of the week (average attendance across the remaining days of the week is very similar). Third, because we are interested in the dynamic effects of promotions, the carryover effect of promotions in one of the last games of the season on the first games of the next season likely differs from the carryover effect of promotions on subsequent games during the same season. In addition, attendance at the first game of the season is typically very high. Therefore, we include a dummy variable for the opening game of each season to control for these effects. We also include season-specific constants to control for both unobserved changes across seasons (e.g., team

roster) and the season-specific number of season ticket holders, which are included as constants in attendance for each game within a season.

As a result of these three criteria, we selected the following variables within each driver: cost of entertainment promotions, cost of giveaway promotions, cost of kids promotions (promotions); distance to the opponent's stadium (opponent); winning percentage of the home team (performance); temperature (weather); opening day 2011, opening day 2012, 2011 season, 2012 season, Friday, Saturday, Sunday, July, August, and September (venue) dummy variables; and a dummy indicating whether or not the game is broadcast on national TV (media).

4.5. Endogeneity

Based on discussions with the Pirates' management team, we understand the managerial decision-making process for the promotional schedule as follows: Although the MLB season starts around early April every year, the promotional schedule is discussed by team marketing executives and made public in November/December to aid in advanced ticket sales. Once published, there are rarely any changes made to the promotional schedule during the season. The marketing team considers past promotions, other teams' promotions, promotional costs and budget, gives a priority to fill up weekend games, and avoids repeating the same promotions sequentially. However, no quantitative analysis is involved in this process. Endogeneity may arise if we omit variables from the attendance response model that affect both the promotional scheduling decisions and attendance. We control for potential endogeneity based on observable variables by including the variables related to the various drivers of attendance directly in the attendance response model. However, there may also be unobserved variables influencing both the promotional schedule and attendance.

We address this potential endogeneity by using a parametric control function approach. The control function approach extends the classic treatment effect model to correct for the endogeneity of discrete treatments (Heckman et al. 1997, Shaver 1998) and allows for the correction of endogeneity for different types of endogenous variables (e.g., continuous, ordered, and count variables) (Blundell et al. 2005, Petrin and Train 2010). The basic idea behind the control function approach is that it corrects for endogeneity by including extra control variables for each category in the attendance model to condition out the variation in the disturbance term that is not independent from the endogenous variable(s) (Petrin and Train 2010). Conditional on appropriate control variables, endogeneity is no longer an issue and estimation of the attendance equation is straightforward.

In a first step, we estimate a control function model that explains the decision to spend a particular amount of money on a promotional category for a certain game using a linear model (a censored linear model to accommodate the fact that promotional expenditures cannot be negative yields similar results). In the second step, we include the error term resulting from each promotional category as additional explanatory variables in our attendance model. The control function corrects for potential endogeneity under the following three assumptions (Heckman et al. 1997). First, the *conditional independence assumption* states that potential attendance is independent of the decision to allocate a certain amount of promotional expenditures to a game, conditional on a set of covariates. Second, the *overlap condition* assumes appropriate overlap in the covariate distributions of the observations with a positive amount spent on a promotional category and a game without that category of promotion. This is effectively an assumption on the joint distribution of observable variables W_t (Imbens 2004) and implies that, conditional on W_t, each promotion has positive support everywhere in the interval $[0, \max(\mathrm{Prom}_{jt})]$. This is also referred to as the *common support condition*. Third, identification of the selection bias in a linear parametric control function comes from appropriate exclusion restrictions. These restrictions are variables that have an impact on promotional spending but not attendance. In line with the managerial decision-making process, we include an extensive set of variables related to the temporal spacing between different categories of promotions. Specifically, we include the promotional costs for a particular promotional category in the previous/future games or weeks. Note that we can use instruments from the past as well as the future because the promotional schedule is completed well in advance of the season.

4.6. Pooling Data Across Seasons

We have data on three consecutive seasons for the Pirates (2010, 2011, and 2012). To obtain more reliable results concerning the promotional effects, we test whether we can pool the data across seasons. We test for pooling using the Chow test. We estimate all nine models within our multiple distributed lag framework for each season separately and for the data pooled across seasons. In each model, we include the selected variables from all six drivers of attendance. If the Chow test does not allow pooling of the data across seasons, we can incorporate additional season-specific variables until pooling is allowed. We only exclude the first $j + \max(m_{0j})$ observations for the first season for the Chow test because of the $j + \max(m_{0j})$ lagged variables in our most extensive model. For subsequent seasons, we use all observations and the lagged observations from the previous season for the

first $j + \max(m_{0j})$ games of the new season. Hence, the total number of observations is equal across the pooled and unpooled models.

5. Empirical Results

5.1. Control Function Model

Table 3 shows the results for the control function model to correct for potential endogeneity resulting from unobserved factors that may impact both the promotional schedule and attendance for the models of the expenditures on entertainment, giveaway, and kids promotions for all three seasons combined. (We also estimated these models for each season separately but do not report results here.) The models contain several exclusion restrictions related to the temporal spacing of each category of promotion that the Pirates' management team considered important in crafting the promotional schedule. The R-squares indicate good fits for the models across each promotional category.

Entertainment promotions always take place on Saturdays in our data and hence the dummy for Saturday indicates the base promotional expenditure. Results show that entertainment promotions cost \$8,309.60 more in the summer months. The expenditures on the previous (−0.30) and following (−0.23) Saturday games have a significant negative effect on current promotional expenditures. This reflects the temporal variation in promotions and the fact that the most expensive entertainment promotion (i.e., Skyblast—a concert with fireworks) is not repeated week after week. Average giveaway costs are positive on Thursday, Friday, and Saturday, with the highest expenditures observed on Friday (\$70,600.11). Furthermore, we find that the giveaway expenditures two games before have a significant positive effect (0.11) and that the expenditures of giveaway promotions are lower when entertainment expenditures in the same week are higher. Note that we have not included the giveaway promotions for the game directly before (and after) the focal game to minimize the chance that the instruments impact attendance (via carryover effects). Finally, kids promotions are only employed on Sundays. The expenditures on kids promotions on the previous Sunday game (0.14) and the next Sunday game (0.16) have a significant positive impact on expenditures of kids promotions in the focal game.

Overall, we conclude that the variables related to the temporal spacing of promotions satisfy the exclusion restrictions because several of them (especially the promotional costs in future games) significantly impact promotions but not attendance. Hence, we use the error terms resulting from these models as explanatory variables in our attendance response models.

Table 3 **Control Function Model for Entertainment, Giveaway, and Kids Promotional Costs**

	Entertainment		Giveaway		Kids	
	Mean	SD	Mean	SD	Mean	SD
Constant			5,231.73	3,411.51		
Thursday			19,194.51**	4,741.56		
Friday			70,600.11**	4,126.18		
Saturday	143,980.70**	15,445.56	18,002.65**	4,000.40		
Sunday					12,588.45**	2,646.12
Dummy June, July, and August	8,309.60*	4,143.11				
Entertainment previous Saturday game	−0.30**	0.06				
Entertainment two Saturday games before	0.01	0.06				
Entertainment next Saturday game	−0.23**	0.06				
Entertainment two Saturday games after	0.12	0.06				
Giveaway $(t-2)$			0.11*	0.05		
Giveaway $(t-3)$			0.00	0.05		
Giveaway $(t+2)$			0.03	0.04		
Giveaway $(t+3)$			−0.03	0.05		
Entertainment current week			−8,139.43**	2,798.71		
Kids previous Sunday game					0.14*	0.07
Kids two Sunday games before					0.06	0.07
Kids next Sunday game					0.16*	0.07
Kids two Sunday games after					0.09	0.07
R-squared	0.52		0.60		0.80	
Adjusted *R*-squared	0.51		0.58		0.80	
AIC	24.24		22.66		19.58	
SIC	24.32		22.80		19.65	

Notes. All models are based on 238 observations. SD indicates the standard deviation.
*Significant at the 0.05 level, **significant at the 0.01 level.

5.2. Model Selection

We estimated the attendance response model including the variables selected from the six drivers of attendance and the control function terms for all three seasons together and for each season separately.[9] First, we select optimal values for m_{0j}, for $j = 1, \ldots, J$. We select these values based on the pooled model, and using the AIC and SIC, we select $m_{01} = m_{02} = 0$ and $m_{03} = 1$ for Models A–F. For Models G–I, we select $m_0 = 0$. We use the Chow test with equal variances to test whether we can pool the data across seasons for each model within our multiple distributed lag framework. We confirm that the variances are equal across seasons, and results indicate that we cannot reject the null hypothesis of equal parameters across seasons for all the different models within our framework (*p*-values ranged from 0.08 to 0.56). Therefore, we can pool the data; results for the series of pooled models are reported in Table 4.

The newly proposed and most general multiple geometric distributed lag model (Model A) is presented on the left side of Table 4, followed by the various restricted models. Before assessing the fit of the various models, we examine whether there

[9] Note that we estimated separate control functions for each season for the unpooled models first. We also estimated a control function for Models G, H, and I, with the sum of the promotional costs as the dependent variable.

is remaining serial correlation after estimation using the Breusch–Godfrey test. At the bottom of Table 4, we report the test for first-order serial correlation. A *p*-value of less than 0.05 indicates significant first-order serial correlation. The results in Table 4 show that Models F and H suffer from remaining serial correlation. Higher-order serial correlation tests (not reported here) led to similar conclusions.

Next, we inspect the in-sample fit of the models. Model A fits best; it has the highest log-likelihood (−2,295.93) and the lowest AIC (4,649.85) across all models. The SIC, which imposes a greater penalty based on the number of parameters in the model, favors Model G (4,731.52). In assessing out-of-sample fit across models by performing leave-one-out validation, Model A outperforms the other models based on the RMSPE (4,320.12) and the MAPE (16.10%). Finally, we formally test the parameter restrictions in nested Models B–H compared with Model A (see the second column of Table 2) using the likelihood ratio test. The last row of Table 4 provides evidence that the parameter restrictions are rejected at the 0.05 level for all models, favoring Model A. In sum, using both in-sample and out-of-sample fit performance diagnostics, our newly proposed Model A appears to perform best overall. We therefore discuss the results obtained from Model A in more detail.

The direct effect of entertainment is 0.009. This can be interpreted as the effect of spending $1 on entertainment promotions on attendance on the day of

Table 4 **Parameter Estimates (and Standard Errors) of the Distributed Lag Models**

Model	A	B	C	D	E	F	G	H	I
Constant	157.47	−887.59	−3,571.17	178.16	−2,629.44*	−3,693.31	421.61	−2,292.30*	−3,587.20
	300.68	807.57	3,557.56	918.93	1,026.56	3,487.61	691.46	946.19	3,185.11
Any promotion							0.006	0.013*	0.015
							0.01	0.01	0.01
Entertainment	0.009[a]	0.014*	0.012	0.007	0.010	0.015			
	0.01	0.01	0.01	0.01	0.01	0.01			
Giveaway	0.048*	0.058*	0.076	0.025	0.030**	0.071*			
	0.02	0.02	0.05	0.01	0.01	0.04			
Kids	0.133	0.242	0.151	0.043	0.186	0.122			
	0.22	0.20	0.22	0.21	0.17	0.21			
Kids ($t-1$)	−0.100**	−0.07	−0.04	−0.05	0.04	−0.06			
	0.04	0.05	0.04	0.05	0.04	0.05			
Carryover				0.86**	0.87**	0.28**	0.88**	0.85**	0.28*
				0.07	0.05	0.07	0.07	0.05	0.06
Carryover entertainment	0.87**	0.84**	0.28**						
	0.11	0.10	0.07						
Carryover giveaway	0.62**	0.60**	0.00						
	0.23	0.18	—[b]						
Carryover kids	0.16	0.04	0.00						
	0.20	0.26	—[b]						
MA(1)	−0.12			−0.24**			−0.24**		
	0.07			0.06			0.06		
MA(2)	0.07								
	0.13								
MA(3)	0.11								
	0.09								
Season 2011	2,405.63	−369.61	2,396.52	1,742.36	−1,684.81	2,360.05	1,792.56	−971.01	2,437.99
	2,676.14	1,460.62	1,402.19	2,260.52	1,434.29	1,357.64	2,334.66	1,284.73	1,340.31
Season 2012	3,848.06	466.69	3,701.00*	3,331.03	−415.07	3,680.40*	3,116.05	−208.24	3,741.30*
	3,000.63	1,657.65	1,569.98	2,504.88	1,422.35	1,473.42	2,759.87	1,423.47	1,463.45
Opening day 2011	25,242.66**	21,418.03	22,653.33	25,007.61**	23,005.45	22,658.00	25,107.24**	21,935.42	22,699.95
	6,011.39	19,132.03	29,724.53	8,414.63	22,030.02	34,534.48	9,390.08	16,222.09	23,835.17
Opening day 2012	31,489.68**	43,703.92**	33,016.11*	31,464.21	49,091.22**	32,980.62**	30,840.86*	45,096.69**	32,655.56
	9,675.32	15,403.35	15,519.20	16,973.92	18,303.89	11,768.91	12,426.81	16,992.94	17,514.82
Friday	8,364.54**	8,132.58**	6,435.72*	9,840.79**	10,436.39**	6,589.69**	11,168.69**	11,185.18**	10,525.40**
	1,632.79	1,795.30	3,017.34	1,064.67	1,039.41	2,336.17	741.95	897.64	1,179.57
Saturday	13,945.71**	13,490.08**	14,795.46**	15,172.13**	15,661.68**	13,354.18**	16,765.99**	16,044.59**	15,573.44**
	1,281.45	1,417.64	1,790.85	1,206.36	1,248.97	2,024.45	1,072.76	1,228.31	2,230.24
Sunday	4,200.51	1,618.56	5,307.87	6,943.70	4,467.63	5,237.30	8,944.69**	8,165.88**	8,830.33**
	5,198.62	4,834.82	5,262.89	4,913.26	4,177.41	5,095.62	881.85	931.56	1,133.38
July	4,236.83**	4,119.56**	5,257.40**	3,843.45*	3,350.77**	5,330.11**	3,715.92*	3,618.14**	4,988.62**
	1,487.75	1,126.34	1,262.43	1,723.93	1,065.14	1,244.46	1,668.41	1,043.27	1,243.19
August	4,523.34*	4,127.89**	4,644.54**	4,279.22*	4,903.81**	4,735.59**	4,175.54	3,917.69**	4,583.49**
	2,031.28	1,247.29	1,439.50	2,100.92	1,194.43	1,405.48	2,200.66	1,111.36	1,410.67
September	1,847.01	1,200.50	438.09	1,482.14	2,525.72	610.21	1,212.14	807.52	379.04
	1,784.87	1,350.21	1,471.55	1,959.56	1,492.42	1,470.00	1,969.28	1,193.37	1,435.96
Winning percentage	15,770.84	41,635.94**	20,956.69*	17,780.51	49,640.70**	21,373.57**	15,160.93	44,551.41**	20,262.23**
	10,967.36	8,117.65	8,265.80	10,970.85	7,525.58	7,935.52	9,696.28	6,388.94	7,283.20
Distance	−1.50**	−1.90**	−1.78*	−1.51*	−1.87**	−1.77*	−1.62**	−2.01**	−1.86**
	0.54	0.56	0.70	0.64	0.56	0.69	0.58	0.57	0.67
Temperature	22.01	148.33**	139.48**	53.85	123.80**	138.07**	44.98	140.96**	144.27**
	44.65	44.25	45.27	47.15	42.55	44.58	45.57	37.16	43.10
National TV	4,427.62*	4,745.05*	4,954.51*	4,528.81	4,887.78*	4,886.17*	4,547.24*	4,448.64*	4,614.92*
	2,205.70	1,958.25	2,398.98	2,564.17	2,044.89	2,227.06	2,069.73	1,839.00	2,003.60
CF entertainment	0.02	0.01	0.01	0.01	0.01	0.01			
	0.01	0.01	0.01	0.01	0.01	0.01			

Table 4 **(Continued)**

Model	A	B	C	D	E	F	G	H	I
CF giveaway	0.02	0.01	−0.02	0.04*	0.04*	0.00			
	0.03	0.03	0.05	0.02	0.02	0.04			
CF kids	−0.25	−0.37*	−0.25	−0.16	−0.30*	−0.24			
	0.20	0.17	0.19	0.19	0.15	0.19			
CF all promotions							0.02*	0.01	0.01
							0.01	0.01	0.02
Selected m_{0j}	$m_{03} = 1^c$	$m_{03} = 1^c$	$m_{03} = 1^c$	$m_{03} = 1^c$	$m_{03} = 1^c$	$m_{03} = 1^c$	$m_0 = 0$	$m_0 = 0$	$m_0 = 0$
BG test (p-value)	0.23	0.25	0.14	0.21	0.40	0.01	0.72	0.02	0.16
Log-likelihood	−2,295.93	−2,311.13	−2,312.10	−2,302.99	−2,313.43	−2,311.51	−2,311.08	−2,321.86	−2,320.68
AIC	4,649.85	4,674.27	4,676.19	4,655.99	4,674.87	4,671.01	4,662.16	4,681.73	4,679.36
SIC	4,750.42	4,764.44	4,766.36	4,742.69	4,758.10	4,754.24	4,731.52	4,747.62	4,745.25
RMSPE	4,320.12	5,147.06	4,822.69	4,727.89	5,387.42	4,849.88	5,897.45	5,971.60	5,656.06
MAPE (%)	16.10	17.84	17.47	17.39	18.25	17.53	21.40	21.38	20.57
Likelihood ratio test (p-value)	—	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00

Notes. Both models are based on 237 observations. The BG (Breusch–Godfrey) test reports the *p*-value for first-order serial correlation. CF refers to the control function terms.

[a]We report the promotional effects with an increased level of precision compared with the other variables.

[b]This parameter is restricted between zero and one. As it converges to the lower boundary, we treat this variable as redundant and do not obtain a standard error.

[c]$m_{01} = m_{02} = 0$, whereas $m_{03} = 1$.

*Significant at the 0.05 level, **significant at the 0.01 level.

the promotion. In other words, spending an additional \$1,000 on an entertainment promotion increases attendance by nine. However, this effect is statistically insignificant. The direct effect of giveaways is significant and positive (0.048), and the direct effect of kids promotions is positive but statistically insignificant (0.133). The carryover effect for entertainment promotions is most substantial (0.87), followed by the carryover effect of giveaways (0.62), both of which are statistically significant. For kids promotions, the geometric lag begins one game after the promotion was employed. Kids promotions have a negative effect on the next game (−0.100) and do not have a significant decay parameter (0.16).

Based on these findings, we can compute the total effects of each promotion given by $\sum_{i=0}^{m_{0j}-1} \beta_{ij} + \beta_{m_{0j}, j}/(1 - \lambda_j)$. The total effect is highest for giveaways (0.125), followed by entertainment (0.069) and kids (0.015) promotions. It is important to note that because entertainment promotions are always on Saturdays and kids promotions are always on Sundays, their effects are conditional on the fact that they take place on their respective day of the week; however, as giveaway promotions show substantial variation with respect to day of the week, their effect is not conditional on a specific day of the week.

We can compare the effects of promotional spending with the only other empirical study we are aware of that incorporated costs into the analysis. McDonald and Rascher (2000) found that increasing the value of an individual promotional item by \$1 led to an increase in attendance of 2,688 spectators. Based on their average of 15,000 promotional items per game,

we can compute the effect of a \$1 increase in total promotional spending per game, leading to a promotional effect of 0.18, which is somewhat higher than what we find based on our proposed model. However, the cost of attendance during McDonald and Rascher's (2000) period of analysis was lower than in our data set, and moreover, they do not include dynamic effects and do not correct for endogeneity, which may explain why their effect is higher.

Table 4 also demonstrates that opening day for the 2011 and 2012 seasons drew a substantially higher number of spectators to the stadium. (Recall that opening day for 2010 was excluded in the analysis.) In addition, the 2011 and 2012 seasons had a higher average attendance compared with 2010. Day of the week also mattered, with Fridays drawing an additional 8,364.54 fans, Saturdays drawing an additional 13,945.71 fans, and Sundays drawing an additional 4,200.51 fans, although the effect of Sunday is insignificant. (It is important to control for these day of the week effects; otherwise, they are confounded with entertainment and kids promotions.) The summer months also saw an increase in attendance with an additional 4,236.83 spectators per game in July, an increase of 4,523.34 in August, and an increase of 1,847.01 in September, although the effect of September is insignificant. The Pirates' winning percentage captures the positive effect of team performance on attendance (15,770.84), although this effect is not statistically significant. The distance (in miles) between the Pirates' and the opponent's stadiums had a negative impact on attendance (−1.50). This effect captures both the attendance driven by fans of the

opposing team as well as increased attendance as a result of games against rivals. The effect of temperature is insignificant (22.01), but this effect may already be partially captured by the positive effects for the summer months. There is a significant positive impact of games broadcast on national TV on attendance (4,427.62), which can be explained by the fact that nationally televised games typically represent highly competitive games played by teams in contention for playoff spots or between rivals. Finally, the control function terms for the three categories of promotions are not statistically significant, indicating that endogeneity may not play a major role here. However, statistical insignificance may be driven by the relatively small sample size, which is why we still include the control function terms in each of our models.

5.3. Profit Implications

We can use our findings to calculate the profitability of the promotional schedule using the proprietary promotional cost information, the average ticket price, and the average amount of money spent by spectators at a game. The average ticket price for the Pirates was $15.39 in 2010, $15.30 in 2011, and $16.11 in 2012. The average amount of spectator spending based on attendance was calculated by dividing the fan cost index (FCI) by four. The FCI estimates the average cost for a family of four to attend a game and includes the cost of four tickets, two beers, four soft drinks, four hot dogs, parking, two programs, and two adult-size caps (see http://www.teammarketing.com). The FCI per spectator for the Pirates was $31.64 in 2010, $31.93 in 2011, and $38.10 in 2012.

Table 5 provides the total profits resulting from each of the three promotional categories (taking direct as well as total effects into account). Columns one

Table 5   Profits per Season (in Millions of Dollars) by Promotional Category

	Based on ticket price			Based on fan cost index		
	(1) 2010	(2) 2011	(3) 2012	(4) 2010	(5) 2011	(6) 2012
Direct profit entertainment	−1.3	−1.2	−1.1	−1.0	−1.0	−0.9
Direct profit giveaway	−0.4	−0.4	−0.3	0.8	0.8	0.8
Direct profit kids	0.3	0.3	0.3	0.9	1.1	1.1
Direct profit all promotions	**−1.4**	**−1.3**	**−1.1**	**0.7**	**0.9**	**0.9**
Total profit entertainment	0.1	0.1	0.1	1.7	1.7	2.1
Total profit giveaway	1.4	1.4	1.4	4.5	4.5	5.1
Total profit kids	−0.2	−0.3	−0.2	−0.2	−0.2	−0.1
Total profit all promotions	**1.3**	**1.2**	**1.3**	**6.1**	**5.9**	**7.1**

through three show profits based on ticket prices after accounting for promotional costs, providing a lower bound on profits. Columns four through six show profits based on the FCI per spectator after accounting for promotional costs. It is not clear exactly how the revenues based on the FCI translate into profits (because we do not have information on the team's total costs), so the absolute profits based on the FCI per spectator provide an upper bound on profits.

Table 5 provides several insights. First, not all promotions are profitable when only taking the direct effects into account. Based on ticket prices, expenditures for entertainment and giveaway promotions lead to losses, but kids promotions increase profitability. Based on the FCI per spectator, giveaway and kids promotions are profitable. Second, taking the total effects of promotions into account leads to different conclusions compared with focusing on just the direct effects—as is typically done in the academic literature. Based on ticket prices and FCI per spectator, giveaway promotions are the most profitable followed by entertainment promotions, and kids promotions are no longer profitable. Interestingly, kids promotions are employed on all but one Sunday during our observation period; however, it is worth noting that the Pirates also spent five times less on kids promotions than on entertainment and giveaway promotions.
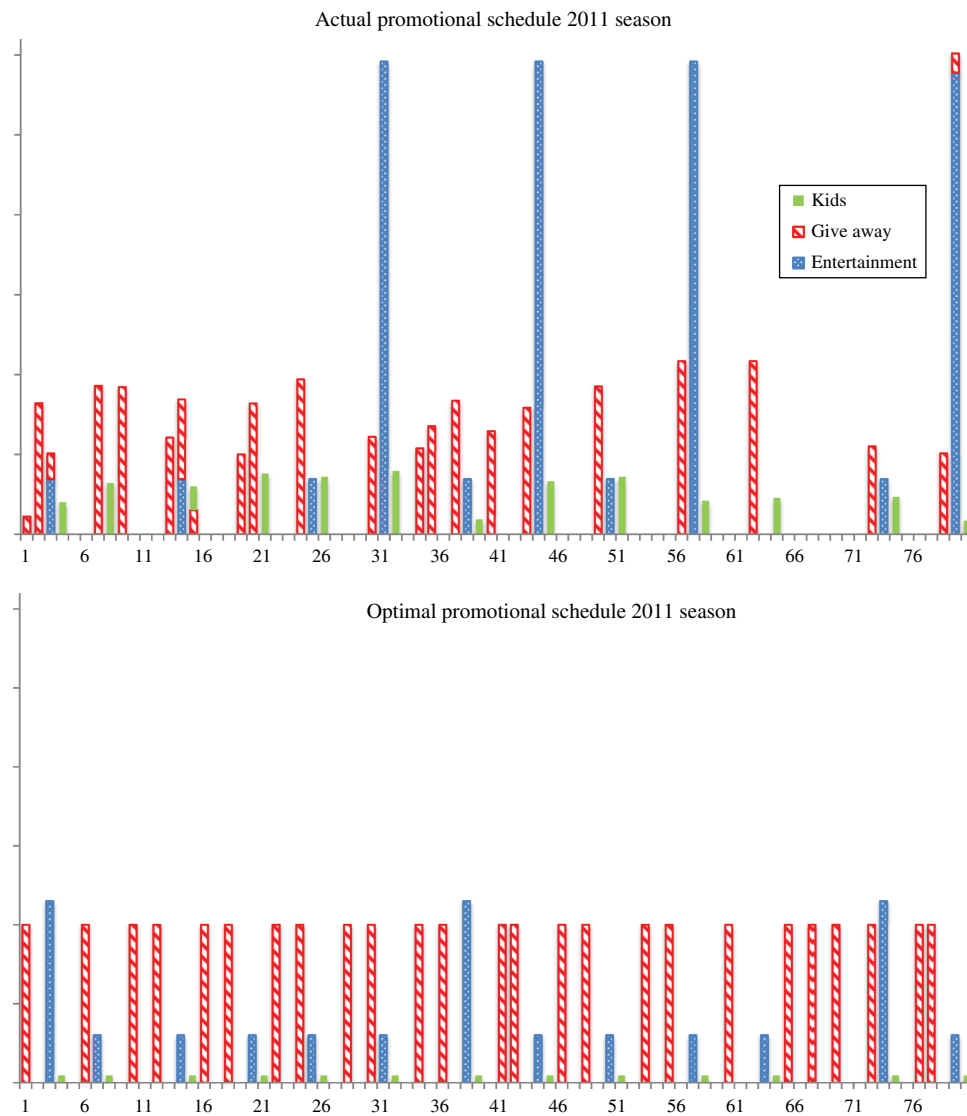
We can use the profits in Table 5 as a baseline to calculate profit changes under various alternative scenarios, subject to designated constraints. As a simple demonstration of the potential for increased profits based on small changes in the promotional schedule, we examine giveaway promotions. In 2010, every Thursday game featured a giveaway promotion. In 2011 and 2012 this trend was not observed, reducing the total number of giveaways by six in 2011 and seven in 2012. We can calculate the impact of this decision assuming that the expenditures on these additional giveaways are equal to the average expenditures on giveaways employed on Thursdays in 2010. If the Pirates continued these giveaway promotions on Thursdays in 2011 and 2012, total profits would have increased by 19% in 2011 and 22% in 2012 based on ticket prices. The next few scenarios use a formal optimization routine (see the appendix for more details) that allows us to incorporate a number of constraints and set the optimal schedule in terms of profit maximization.

For the first scenario, we compute the optimal allocation of resources across promotions. We restrict the number of promotions within each category to be similar to what we observe in our data set for each season. Also, we set the minimum and maximum amount spent on promotions within each category equal to the minimum and maximum we observe in our data set within each season. Finally, we restrict

Figure 5   Actual and Optimal Promotional Schedules for the 2011 Pittsburgh Pirates' Season

Note. The scale of the *y*-axis is the same across both panels; however, we cannot display the size of the expenditures due to the proprietary nature of the promotional cost information.

entertainment promotions to Saturdays and kids promotions to Sundays and maintain the same budget. This reallocation of expenditures across different types of promotions leads to an increase in profits of 88% based on ticket prices. Using Monte Carlo simulations, we calculate the uncertainty around profits resulting from reallocation. The lower quartile of the increase in profits is 48% and the upper quartile is 176%. Based on the FCI per spectator, the increase in profits is 39% (with a lower quartile of 8% and an upper quartile of 86%). Note that the optimal schedules in our application based on ticket prices and the FCI per spectator are similar. To highlight the key differences between schedules, Figure 5 illustrates the actual and optimal promotional schedules for the 2011 season. Although we cannot reveal the actual

promotional costs, both panels of Figure 5 have the same scale and can be directly compared. In the new schedule, resources are shifted away from the unprofitable kids promotions and the less profitable entertainment promotions to more expensive giveaways.

In the second scenario, we calculate the impact of optimizing the promotional schedule for the previous scenario based on only the direct effects and compare it with the optimal schedule based on the total effects. For the naïve model with only the direct effects, we assume that the carryover effects are equal to zero and optimize the promotional schedule. As one may expect, based on the direct and total effects of the different promotions, the optimization resulting from this naïve model leads to dramatic changes. When the promotional schedule is optimized based on the

direct effects only and we compute profitability using the total effects, the promotional schedule leads to negative profitability and an estimated loss of half a million dollars over the three seasons based on ticket prices. This is a result of shifting money away from entertainment and giveaway promotions toward kids promotions, and as we now know, kids promotions are unprofitable in the long run.

5.4. Additional Insights and Robustness

Our results offer some additional insights into the usefulness of the proposed nested model framework (see Table 4). First, the model fit improves substantially when promotions are allowed to have differential direct effects, especially with respect to the out-of-sample fit. If a data set has a limited number of observations, Models D and F are reasonable alternatives to Model A. Second, it is important to test various serial correlation structures because two out of nine models do not pass the Breusch–Godfrey test of no remaining serial correlation. Although not explicitly shown, using the biased Durbin–Watson test for serial correlation (as is commonly done in the literature) often leads to the misguided conclusion that serial correlation is not a problem. Third, Koyck models with no or restricted serial correlation terms (Models B, C, E, F, H, and I) may lead to seriously biased estimates. Fourth, allowing and testing for different values of m_0 (the period after which the exponential decay starts) in a Koyck model widens the appeal of the Koyck model and may prevent one from obtaining biased insights.

We also performed a number of robustness checks (results available upon request). One assumption underlying our infinite distributed lag model is that once the geometric decay begins, the sign of the promotional effect is similar over time and follows a geometric decay. Another assumption is that our current models do not allow for lead effects. To investigate the validity of these assumptions, we estimated an unconstrained Almon lag model with the same explanatory variables as our main model (in Table 4) and a sufficiently high order to allow for negative and positive lag/lead effects for each category of promotion. Specifically, we allowed for a one-period lead effect and a five-period lagged effect using a fourth-order polynomial. Our best model (Model A) compares favorably with this model based on the AIC, SIC, and out-of-sample fit. Results also indicate that entertainment and giveaway promotions show a small, but insignificant, negative lead effect. However, the total effects of the different promotions resulting from such a flexible model are largely similar to those resulting from Model A in Table 4, which strengthens our confidence in our findings. In addition, we tested and rejected the autoregressive current effects model

(Weiss and Windal 1980) used to assess whether promotions only have an effect on the current game. We also tested various alternative day-of-the-week variables in our model and the results are robust. Finally, we tested whether we should model attendance in levels or in logs using a P_E test (MacKinnon et al. 1983). Estimating the model in logs would estimate the effects of promotions as relative effects. Based on the P_E test, however, we fail to reject the null hypothesis that the linear model is preferential to a logarithmic model ($p = 0.36$). The null hypothesis that the logarithmic model is preferred over the linear model has a p-value of 0.06. Although we do not reject the latter null hypothesis based on conventional levels of significance, we interpret this as limited evidence that the linear model is preferred.

6. Discussion

This research proposes a new general multiple distributed geometric lag framework to select the optimal model based on in-sample and out-of-sample fit to measure the differential direct and carryover effects of in-game promotions on attendance for the Pittsburgh Pirates MLB team. Methodologically, our findings demonstrate that it is important to test the appropriateness of various models, which can have a substantial impact on results. Our modeling framework helps researchers select the best model to estimate the differential dynamic effectiveness of in-game promotional activities that can be used for other teams in MLB as well as other leagues and sports in general.

Several managerial insights for MLB teams can also be gleaned from the results of this study. First, our modeling framework can be used to design more profitable promotional schedules by identifying the most effective promotions. Coupled with an easy-to-use optimization framework, our results show that a reallocation of resources across the different promotional categories can increase team profits between 39% and 88% in our application.

Second, in addition to the differential direct effects of promotions on potential team profit, the current work also emphasizes the importance of differential carryover effects of promotions. Ignoring differential direct and carryover effects of different types of promotions leads to seriously biased insights. For example, based on direct effects alone, our results demonstrate that kids promotions have the largest effect on attendance. However, kids promotions have a negative carryover effect leading to a smaller (and unprofitable) total effect compared with entertainment and giveaway promotions.

Third, although MLB promotional schedules are typically created using various manager heuristics (or rules of thumb), the current work shows that

establishing promotional schedules based on scientific modeling can improve team attendance and subsequent profits. Our proposed econometric framework allows teams to garner objective quantitative insights into the effectiveness of different types of promotions. However, the standard errors are relatively large for some of the promotional effects in our results. Hence, an important implication from the current analysis is that MLB teams can learn more about the effectiveness of various promotions if they experiment with their promotional schedules and provide more variation in the data that will increase the precision of the estimates. For example, because entertainment promotions always occur on Saturdays and kids promotions always occur on Sundays, we cannot evaluate whether they are equally effective during the rest of the week. Thus, CMOs could benefit from scientific experimentation in their promotional schedule to learn about the differential effectiveness of in-game promotions (exploration) and use the resulting knowledge to optimize their promotional schedule (exploitation).

Given the scarcity of published research in the sports context in the major academic marketing journals in general, and on the effects of in-game promotions specifically, the current research has several limitations and provides a number of opportunities for future research. First, there are many different types of promotions utilized within a season—22 of them for the seasons under investigation just for the Pittsburgh Pirates. In this study, we classified promotions into three categories (entertainment, giveaway, and kids promotions) because of the lack of degrees of freedom. To gain more specific insights about the effectiveness of different types of promotions, it would be worthwhile to investigate the impact of each of the different types of promotions employed. This will be no easy task, however, as evidenced by the fact that the Pirates had 52 games that contained at least one of the 22 different promotions in 2010 alone. DeSarbo et al. (2014) recently developed constrained stochastic extended redundancy analysis (CSERA) to address this problem. CSERA groups many predictor variables into a limited number of conceptual factors and estimates the effect of the conceptual factors as well as their specific variables. However, CSERA does not incorporate dynamic effects. Second, it would also be interesting to see if there are interactions between the different categories of promotions, as previous research revealed positive effects of stacking promotions (i.e., offering more than one category of promotion per game) (Boyd and Krehbiel 2006). In the current data set, only 12 out of 241 games offered promotions from two different categories on the same

game date, leaving us with too few degrees of freedom to estimate any potential interaction effects. Third, it would be interesting to investigate the potential for nonlinear effects of promotions because we did not have the degrees of freedom in our model or variation in our data necessary to explore more advanced transformations (e.g., s-shaped effect). Such modeling efforts could shed light on a number of questions related to the scheduling of various in-game promotions; for example, is it better to have one expensive entertainment promotion or two less expensive entertainment promotions? Fourth, we did not allow the promotional effects to change over time. However, some literature argues that the effects of promotions may change based on day of the week, team performance, etc. Investigating these moderators of promotional effectiveness may prove to be a valuable area for further research. Last, given the resources and time required for data collection and team collaboration, we only tested our model using data from one MLB team. Extensions to other MLB teams (30 in total) as well as other sports (e.g., basketball, football, ice hockey, and soccer) would be beneficial to the development of this particular literature stream.

Supplemental Material

Acknowledgments

Appendix
The team wants to maximize its profits Π each season given by

$$\max \Pi(X_{jt}) = \sum_{t=1}^{T}\left[\left(\sum_{i=0}^{m_{0j}-1} \beta_{ij} + \beta_{m_{0j},j}\Big/(1-\lambda_j)\right)X_{jt}Price_t - X_{jt}\right],$$

where we omit the season-specific subscript, T indicates the total number of games in a season, $\sum_{i=0}^{m_{0j}-1}\beta_{ij} + \beta_{m_{0j},j}/(1-\lambda_j)$ represents the total effect of \$1 spent on promotion j, $Price_t$ is the ticket price (or fan cost index) for one attendee, and X_{jt} is the cost of promotion j at time t.

The maximization problem is subject to a number of constraints:
- $X_{1t} = 0$ if the game does not take place on a Saturday,
- $X_{3t} = 0$ if the game does not take place on a Sunday,
- $A_{\min,j} \leq \sum_{t=1}^{T} I_{X_{jt}>0} \leq A_{\max,j}$, for $j=1,\ldots,J$,
- $C_{\min,j} \leq X_{jt} \leq C_{\max,j}$ if $I_{X_{jt}>0}=1$, for $j=1,\ldots,J$,
- $\sum_{t=1}^{T}\sum_{j=1}^{J} X_{jt} \leq B$,

where the first two constraints ensure that entertainment and kids promotions are only employed on Saturday and Sunday, respectively; $A_{\min, j}$ and $A_{\max, j}$ represent a minimum and maximum number of promotions for a promotional category; $C_{\min, j}$ and $C_{\max, j}$ represent a minimum and maximum expenditure for a promotion in promotional category j, conditional on the fact that a promotion takes place; I represents an indicator function taking the value 1 if the condition is true and 0 otherwise; and B indicates the season-specific budget constraint.

The mathematical programming problem provided above can be solved using the following three steps:

(1) Satisfy the minimum constraints: To satisfy the minimum constraints on the number of promotions ($A_{\min, j}$) and the minimum expenditures ($C_{\min, j}$) for an individual promotion in promotional category j, assign $C_{\min, j}$ to $A_{\min, j}$ games for $j = 1, \ldots, J$. This initial assignment of promotions should also satisfy the day of the week constraints and the budget constraint.

(2) Calculate which promotion is most effective in the long term, i.e.,

$$\max \sum_{i=0}^{m_{0j-1}} \beta_{ij} + \beta_{m_{oj}, j} \Big/ (1 - \lambda_j)$$

for $j = 1, \ldots, J$. If

$$\max \left(\sum_{i=0}^{m_{0j-1}} \beta_{ij} + \beta_{m_{oj}, j} \Big/ (1 - \lambda_j) \right) Price_t > 1$$

(which indicates that the promotion is cost effective), and allocate additional promotional expenditures for the most effective type of promotion to the promotional schedule using the following steps (note that steps 2(a) and 2(b) can be reversed), otherwise terminate:

(a) Increase the number of promotional games by assigning $C_{\min, j}$ to these games until $A_{\max, j}$ is reached or B is reached. If B is reached, terminate.

(b) Increase the promotional expenditures per promotional game until $C_{\max, j}$ is reached for each promotional day or until B is reached. If B is reached, terminate.
Otherwise, terminate.

(3) Repeat step 2 for the remaining promotional categories. If there are no more promotional categories left, terminate.

The strength of this simple solution is that it provides a great deal of managerial control over the optimization problem. Note that the optimization is linear in its decision variables and has linear constraints, so it will always render a corner solution. In case more complex constraints are introduced or the optimization function is nonlinear, it may be necessary to resort to more complex mathematical programming methods.

There are a number of extensions that can be incorporated into this basic optimization problem. Because the effect of the promotional expenditures on attendance is linear, the effect of employing one promotion of size S is equal to the sum of the effects of two promotions of size $S/2$. In practice, however, teams have specific promotions in mind, and this allows them some flexibility in the allocation of specific promotions. Alternatively, the team can set stricter constraints on the total number of or expenditures on promotions. In addition, we can add more constraints on the temporal spacing of promotions. This will not affect the profits for the optimal solution but may be valuable in practice to prevent repetition of the same type of promotion over time. Finally, the current optimization does not account for the maximum seating capacity of the stadium. This can be incorporated by including the expected demand based on all other attendance drivers in the optimization and adding a capacity constraint for each game.

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