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Rational Speculators, Contrarians, and Excess Volatility

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The vector autoregressive approach for testing present value models is applied to a heterogeneous-agent asset pricing model using historical observations of the S&P 500 index. Besides fundamentalists, who value assets according to expected dividends, the model features rational and contrarian speculators. Agents choose their strategy based on evolutionary considerations. Supplementing the standard present value model with speculative agents dramatically improves the model's ability to replicate observed market dynamics. In particular, the existence of contrarians can explain some of the most volatile episodes including the 1990s bubble, suggesting this was not a rational bubble.

Data, as supplemental material, are available at http://dx.doi.org/10.1287/mnsc.2014.1937.

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1. Introduction

Prices of financial assets are typically more volatile than dividends. Linear present value models, in which asset prices reflect discounted expectations of future dividends, therefore have a hard time explaining the observed volatility of asset prices. This result, known as excess volatility, has been documented in many studies, such as Shiller (1981), Campbell and Shiller (1987), and West (1988), and the survey by Gilles and LeRoy (1991). To accommodate excess volatility, the assumptions underlying linear present value models need to be relaxed. The dominant paradigm in recent decades, surveyed and advocated by Cochrane (2011), is to relax the assumption of a constant discount factor such that asset prices may change not only because of dividend expectations but also because of time-varying discount factors (i.e., expected returns), which reflect perceptions of risk. An alternative behavioral approach is to relax the assumption that expectations are rational, for example, by allowing market participants to form biased expectations based on limited information sets and underparameterized models (see, e.g., De Long et al. 1990a, b; Barberis et al. 1998; Hong and Stein 1999).

In this paper, I consider a simple behavioral asset pricing model with three types of agents who are allowed to have heterogeneous investment horizons and may form heterogeneous expectations regarding short-term price changes. The model fits in the class of evolutionary heterogeneous-agent models introduced by Brock and Hommes (1998), in which multiple agents can select from a set of various expectation rules. Whereas the framework by Brock and Hommes (1998) allows in principle for any form of expectation rule, in this paper the expectation formation mechanisms of all agents are based on the same vector autoregressive (VAR) representation of prices and dividends. This approach has multiple advantages. First, it formalizes the notion that all agents hold identical information sets. The model features long-term investors, who base their decisions on expected dividends, as well as short-term speculators, who are instead mainly interested in price changes. Both of these agents, however, form their predictions while conditioning on the history of both prices and dividends.

A second reason for using VAR-based expectations is that the model can be evaluated empirically using the well-known VAR approach for testing present value models, pioneered by Campbell and Shiller (1987, 1988), for which I use a data set containing annual observations on the S&P 500 index and underlying dividends for the period 1871–2011.[1] The results indicate that even if there is no disagreement at all among the agents regarding expected dividends, the model is able to generate prices far more volatile than the standard present value model. Statistical tests indicate that the model is preferred to alternative representative-agent models in which only one of the considered expectation formation mechanisms exists.

A widely heard critique on behavioral asset pricing is that after departing from full rationality, too many degrees of freedom become available (e.g., Barberis

[1] The source of this information is the home page of Robert J. Shiller (http://www.econ.yale.edu/~shiller, accessed September 11, 2012).

and Thaler 2003). However, by anchoring expectations to the VAR, I impose substantial discipline on the modeling of expectations. Moreover, the alternative approach of engineering a time-varying discount factor that fits the data has not necessarily fewer degrees of freedom. As Arrow (1986) argues, many predictions by rational-expectations models do not come from the assumption of rationality per se, but instead from various auxiliary assumptions. Cochrane (2011) makes a related point, by observing that many recent rational-expectations models have "exotic preferences" that make them difficult to distinguish from behavioral models.

Cochrane (2011) also notes that any behavioral model, with distorted expectations, can be expressed as an equivalent rational-expectations model with a time-varying discount factor. This observational equivalence does not, however, imply that modeling discount factors instead of expectations is always the more sensible strategy. Behavioral asset pricing models are particularly appealing when trying to capture the boom–bust cycles observed in financial markets, as well as various other market anomalies for which rational-expectations models struggle to find a convincing explanation. The results presented in this paper show that after a simple and straightforward behavioral extension (allowing for heterogeneous horizons and expectations), the present value model with a constant discount factor is well able to replicate the observed dynamics of stock prices. Specifying an economic model for the evolution of a stochastic discount factor that is able to accomplish similar results could instead be a rather complex task. As a robustness check, I extend the model in §5 with three simple time-varying discount factors based on consumption, risk-free returns, and volatility. Comparing the results with the benchmark model with a constant discount factor, I find that after allowing for heterogeneous expectations, introducing time variation in the discount factor does not significantly improve the empirical performance of the model.

The three types of agents in the model are fundamentalists, rational speculators, and contrarian speculators (contrarians). The first two agent types both act in accordance with the standard present value model. The only characteristic separating these agents is their investment horizon. The fundamentalist values assets according to the cash flows (dividends) that the asset is expected to generate. These agents can be thought of as following a buy-and-hold strategy, such that dividend yields are the primary investment objective. The second type is only interested in one-period returns, so that the main determinant of the asset's value is the expected selling price in the next period. This speculative behavior is similar to that of trend followers or momentum traders considered in the literature, for example, by Brock and Hommes (1998) or Hong and Stein (1999). However, whereas those agents are in general positive-feedback traders who form expectations based on a simple univariate model and a limited information set, typically by extrapolating recent returns, the speculators considered in this paper form more informed expectations by using exactly the same model and information set as the fundamentalists. Although these agents are not particularly interested in expected dividends, they do observe past dividends and use them for making predictions. I therefore refer to these agents as rational speculators.

Both the fundamentalists and the rational speculators are, strictly speaking, boundedly rational. Their expectation formation mechanism is represented by a VAR model. These expectations would be fully rational if the VAR were the true data-generating process. I show that although the VAR provides an appropriate characterization of the data, it remains only an approximation, which does not take all aspects of the data-generating process, such as the existence and strategies of other agents, explicitly into account.

The third type of agent also follows a short-term strategy. Regarding expected price changes, however, this type takes the exact opposite, or contrarian, stance from the rational speculators. These agents are therefore referred to as contrarian speculators, or contrarians. When the rational speculators expect an $x\%$ increase in the price, the contrarians expect an $x\%$ decrease, and vice versa. Hence, the contrarian strategy can be described as betting against the historic relationship between prices and dividends, as represented by the VAR.

Contrarians have been discussed in the finance literature before, typically as the counterpart of momentum traders. Using microlevel data, Kaniel et al. (2008) provide evidence that many investors indeed behave as contrarians. Laboratory experiments by Bloomfield et al. (2009) indicate that, in particular, uninformed investors may select contrarian strategies. Focusing on Finnish markets, Grinblatt and Keloharju (2000) find that domestic investors often follow contrarian strategies, whereas more sophisticated foreign investors typically rely on momentum trading. Lakonishok et al. (1994), as well as Jegadeesh and Titman (1995), argue that markets have a tendency to overreact to news announcements in the short run, which yields profitable opportunities for contrarian strategies in the long run. Dechow and Sloan (1997) find that a significant part of contrarian profits arise because these strategies exploit naive analysts' earnings forecasts, which are reflected in asset prices. Park and Sabourian (2011) provide a theoretical justification of contrarian behavior by arguing that such behavior is optimal when agents expect moderate outcomes to be more likely than extreme ones. In addition, Park and Sabourian (2011) show that contrarian behavior leads to higher volatility of prices.

The concept of a contrarian strategy in this paper is different from that in the previous literature, in the sense that the contrarians in this paper do not simply act against recent price movements but instead against the VAR-based predictions. In §3, I show that this contrarian strategy can actually result in trend-following behavior, when the VAR predicts a reversal of recent trends. The basic intuition of going "against the crowd" or challenging "conventional wisdom" is, however, very similar to that in the earlier literature. I motivate the existence of these contrarians empirically by showing that observed market dynamics can be replicated rather well when a certain fraction of market participants is forming contrarian expectations. Whereas the existence of rational speculators can explain much of the volatility observed on financial markets, the contrarians turn out to be an essential element of the model to capture the direction in which observed prices move. Contrarian beliefs are particularly helpful in explaining the high valuations that the stock market reached at the end of the 1990s, mainly driven by technology stocks. Whether this episode constituted a bubble has been the subject of debate among many authors, including Ofek and Richardson (2003), Pástor and Veronesi (2006), Bradley et al. (2008), O'Hara (2008), and Phillips et al. (2011). The results in this paper indicate that dividend expectations are not the dominant factor in the observed price increases during the 1990s. In this sense, it could be justified to classify this event as a bubble. Nevertheless, it was not a rational bubble as defined by Blanchard and Watson (1982), since the results show that rational speculators would have driven the market in the opposite direction. Instead, the observed dynamics of the 1990s can be closely approximated by the contrarian valuation model, suggesting that nonrational beliefs inflated this bubble.

It is well known that financial variables exhibit regime-switching behavior (see, e.g., the survey by Ang and Timmermann 2012). By allowing agents to switch between strategies, heterogeneous-agent models provide an economic justification for these nonlinear dynamics. I assume that agents observe the recent performance of each strategy and choose their own strategy accordingly, following the evolutionary selection mechanism introduced by Brock and Hommes (1997, 1998). As Hommes (2013) discusses, evolutionary selection based on relative performance places further discipline on the expectation rules by maintaining a plausible and empirically relevant form of consistency between predictions and realizations. This endogenous switching mechanism has been applied previously in many theoretical and empirical studies of heterogeneous-agent models in finance, including those by Boswijk et al. (2007), Branch and Evans (2010), and Lof (2013).[2] Similar concepts, in which agents apply learning principles to update expectations, are considered by Timmerman (1994), Hong et al. (2007), and Branch and Evans (2011), among others. Hommes et al. (2005) and Bloomfield and Hales (2002) provide experimental evidence in favor of such principles being applied in the formation of expectations. Alternatively, the fractions of different types of agents may be held constant (Szafarz 2012) or vary according to an exogenous process, such as the business cycle (Lof 2012).

In this paper the expectations of different agents are empirically generated by a VAR process, which can be seen as a special case of the more general set of expectation rules considered by Brock and Hommes (1998). This VAR approach was also recently applied by Cornea et al. (2012) to a heterogeneous-agent model of the New Keynesian Phillips curve, in which price setters are allowed to switch between forward-looking and naive backward-looking inflation expectations. Cornea et al. generate only the expectations of the forward-looking price setters by a VAR. In this paper, on the other hand, I let all three types of agents form expectations based on the same VAR framework, such that all agents have the same information set. Nevertheless, despite having identical information sets, the agents do not form identical valuations of the asset. Since the expectations are derived from an unrestricted VAR, the valuation based on expected long-term dividends and the valuation based on expected short-term price changes do not necessarily align.

The rest of this paper proceeds as follows. The next section outlines the present value model, the concept of rational bubbles, and the log-linear approximation by Campbell and Shiller (1988). In §3, the VAR approach is reviewed and applied to three representative-agent models, in which the representative agent is either a rational long-term investor, a rational speculator, or a contrarian. In §4, these models are merged into one regime-switching model. The section further includes estimation results and specification tests. In §5, the model is generalized to allow for time-varying discount factors. Section 6 concludes.

2. The Present Value Model and Rational Bubbles

According to the standard present value model, the price of an asset should equal the discounted present value of the cash flows (dividends) that the asset is expected to generate:

$$P_t = \sum_{i=1}^{\infty} \delta^i E_t[D_{t+i}], \tag{1}$$

[2] See, e.g., Alfarano et al. (2005), Baur and Glover (2014), Eichholtz et al. (2012), ter Ellen and Zwinkels (2010), and Kouwenberg and Zwinkels (2011) for applications of endogenous switching models to other asset classes.

in which P_t refers to the asset price and D_t to its underlying dividend. The discount factor δ is, for simplicity, assumed to be constant, implying risk neutrality. In §5, I examine the validity of this assumption by considering several time-varying discount factors. Assuming rationality and market efficiency requires that the conditional expectation operator $E_t[\cdot]$ is the optimal prediction conditional on all available information. Because in Equation (1) the value is entirely determined by expected dividends, or fundamentals, this expression is sometimes referred to as the fundamental value, which will be equal to the observed market price if all agents are rational fundamentalists (e.g., Szafarz 2012).

Agents are not necessarily planning to hold the asset for a long period and may be more interested in the short-term trading profits rather than long-term dividend yields. If agents are planning to hold the asset for a short time only, say, one period, the value of the asset should equal the discounted sum of the expected dividend paid out in the next period and the expected price at which the asset can be sold subsequently:

$$P_t = \delta E_t[P_{t+1} + D_{t+1}]. \tag{2}$$

The long-term model (1) is the solution to the short-term model (2) under the following transversality condition:

$$\lim_{i\to\infty} \delta^i E_t[P_{t+i}] = 0. \tag{3}$$

Hence, under this transversality condition, the investment horizon of the agents should not have an impact on the price. However, Equation (2) has a more general solution that does allow for a discrepancy between Equations (1) and (2):

$$P_t = \sum_{i=1}^{\infty} \delta^i E_t[D_{t+i}] + C_t, \tag{4}$$

in which $E_t[C_{t+1}] = \delta^{-1} C_t$, or, equivalently, $C_t = \delta^{-t} M_t$, in which M_t may be any martingale process (i.e., $E_t[M_{t+1}] = M_t$). Because C_t constitutes a discrepancy between the fundamental value and the observed price, it may be referred to as a bubble. However, since the bubble exists due to a violation of the transversality condition rather than the a violation of rationality, Blanchard and Watson (1982) name it a *rational bubble*. The finding that rational dividend expectations are not sufficiently volatile to explain observed price volatility can be regarded as a rejection of the present value model (1) and is in the literature often interpreted as evidence in favor of rational bubbles (Gürkaynak 2008).

Two recent studies present theoretical analyses of asset pricing models in which long-term fundamentalists and short-term speculators coexist. Szafarz (2012) finds that the existence of multiple investment horizons is a potential source of price volatility. Anufriev and Bottazzi (2012), however, argue that variation in the investment horizon has a significant effect on market dynamics only when agents hold heterogeneous expectations about future prices. In this paper, I follow an empirical approach by applying the VAR-based tests of present value models by Campbell and Shiller (1987, 1988) to an asset pricing model with heterogeneity in both investment horizons and expectations. As will become evident in the next section, heterogeneity in investment horizons can explain the high level of volatility observed in stock prices. Nevertheless, heterogeneity in expectations appears to be a crucial element required for generating prices that do not only capture the volatility but also obtain a relatively high correlation with observed stock prices.

Before proceeding to estimation of the VAR, it is preferable to apply the log-linear approximation of the present value model derived by Campbell and Shiller (1988). The return on holding an asset for one period ($R_{t+1} = (P_{t+1} + D_{t+1})/P_t$) can be approximated by a linear equation:

$$r_{t+1} = \rho p_{t+1} - p_t + (1-\rho) d_{t+1} + k, \tag{5}$$

in which $p_t \equiv \log(P_t)$, $d_t \equiv \log(D_t)$, and $r_t \equiv \log(R_t)$. The parameter ρ is below, but close to, 1: it denotes the mean of the ratio $P_t/(P_t + D_t)$, which Campbell and Shiller (1988) assume to be approximately constant over time. Following Campbell and Shiller (1988), the constant term k is ignored in much of the analysis below, because explaining price movements rather than levels is the main objective of this study. Engsted et al. (2012) show by simulation that these log-linear returns are a close approximation to true returns even in the presence of rational bubbles.

The assumption of a constant discount factor as in Equations (1) and (2) implies that expected returns are constant:

$$E_t[R_{t+1}] = \frac{E_t[P_{t+1} + D_{t+1}]}{P_t} = \delta^{-1}. \tag{6}$$

Taking conditional expectations on both sides of Equation (5), substituting constant expected returns ($E_t[r_{t+1}] \equiv \bar{r}$), and rearranging gives

$$y_t = \rho E_t[y_{t+1}] + E_t[\Delta d_{t+1}] + k - \bar{r}, \tag{7}$$

in which $y_t \equiv p_t - d_t$ denotes the log price–dividend (PD) ratio. Equation (7) can be iterated forward to obtain the long-term interpretation of the present value model, in which the valuation of the asset is determined by expected future dividend growth rates:

$$y_t = \sum_{i=0}^{\infty} \rho^i E_t[\Delta d_{t+1+i}] + \frac{k - \bar{r}}{1-\rho}. \tag{8}$$

This solution requires the assumption of a transversality condition:

$$\lim_{i\to\infty} \rho^i E_t[y_{t+i}] = 0, \tag{9}$$

which, like condition (3), excludes the possibility of a rational bubble. Equation (8) can be interpreted as the log-linear equivalent of (1).

It is also possible to derive a short-term interpretation of the log-linear present value model, in which the value of an asset is determined by the expected return of holding the asset for one period. Subtracting ρy_t from Equation (7) and dividing both sides by $1-\rho$ gives

$$y_t = \frac{\rho}{1-\rho} E_t[\Delta y_{t+1}] + \frac{1}{1-\rho} E_t[\Delta d_{t+1}] + \frac{k-\bar{r}}{1-\rho}, \tag{10}$$

or, since $\Delta y_t = \Delta p_t - \Delta d_t$,

$$y_t = \frac{\rho}{1-\rho} E_t[\Delta p_{t+1}] + E_t[\Delta d_{t+1}] + \frac{k-\bar{r}}{1-\rho}. \tag{11}$$

In this model the PD ratio is entirely determined by one-period expectations of the change in the price and dividend. Since the parameter ρ is below but close to 1, the ratio $\rho/(1-\rho)$ is a rather large number, implying that the expected price change is the dominant factor in the valuation of the asset. Expectations on future dividends therefore only play a minor role in this short-term valuation model, akin to the models by Hong et al. (2007) and Branch and Evans (2010) in which agents have the option to omit dividends partly or entirely from their expectation formation mechanism. Nevertheless, in this model dividends are not irrelevant, since observed dividends play a role in the VAR-based expectations of price changes.

Unlike the long-term model (8), the short-term model (11) does not require the transversality condition (9), and therefore it allows for the existence of a rational bubble. In the next section, I evaluate both models (8) and (11) using the VAR approach proposed by Campbell and Shiller (1987, 1988).

3. The VAR Approach

Campbell and Shiller (1988) propose to test the log-linear present value model (8) based on an estimated VAR(q) for the log-PD ratio and the dividend growth rate (both measured in logs):

$$v_t \equiv \begin{bmatrix} y_t \\ \Delta d_t \end{bmatrix} = \sum_{i=1}^{q} A_i v_{t-i} + u_t. \tag{12}$$

Both the PD ratio and the dividend growth rate are demeaned so that intercept terms are not required and the parameters k and $\bar{r}$ in (8) can be disregarded. I estimate a VAR(2) for annual observations of the PD ratio and the dividend growth rate over the period 1872–2011. The lag length of $q=2$ was selected using the Akaike information criterion (AIC). This lag order is consistent with the results of Campbell and Shiller (1988). Table 1 depicts the AIC for different lag lengths, as well as diagnostic tests for the selected VAR(2). The second-order VAR seems to describe the data well, as there is no sign of autocorrelation or heteroscedasticity in the residuals. Moreover, the results of a Chow forecast test at several potential breakpoints indicate that parameter constancy cannot be rejected.

Table 1 **VAR Specification and Diagnostics**

Lags	1	2	3	4	5	6
AIC	−7.980	−7.986	−7.967	−7.953	−7.889	−7.889
Autocorrelation	17.63	(0.612)				
Heteroscedasticity	51.62	(0.231)				
Breakpoint	1890	1910	1930	1950	1970	1990
Chow FC	0.578	0.403	0.345	0.998	0.976	0.624

Notes. Values for VAR(q) model (12), with annual data for 1872–2011, are shown. The top two rows show lag selection based on the Akaike information criterion. The middle two rows show Langrange multiplier-type test statistics (p-values in parentheses) for autocorrelation (five lags) and multivariate autoregressive conditional heteroscedasticity (five lags) in residuals of VAR(2). The bottom two rows show p-values for the Chow forecast test (Chow FC) for parameter constancy. All three diagnostic tests are executed using JMulTi (Lütkepohl and Krätzig 2004).

To proceed with testing the present value model, it is convenient to consider the VAR(2) model in its companion form:

$$\begin{bmatrix} v_t \\ v_{t-1} \end{bmatrix} = \begin{bmatrix} A_1 & A_2 \\ I_2 & O_{2,2} \end{bmatrix} \begin{bmatrix} v_{t-1} \\ v_{t-2} \end{bmatrix} + \begin{bmatrix} u_t \\ O_{2,1} \end{bmatrix}, \tag{13}$$

or

$$z_t = B z_{t-1} + \epsilon_t, \tag{14}$$

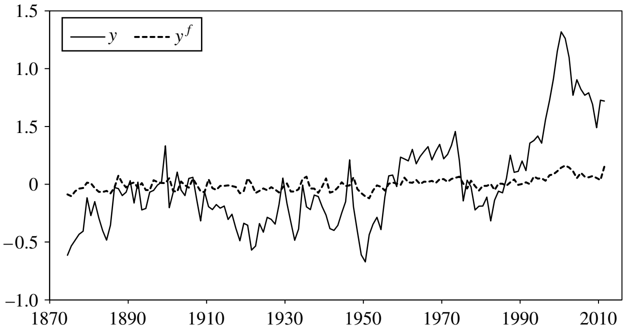
in which $z_t \equiv (v_t, v_{t-1})'$. If this VAR provides an accurate description of the data, which the diagnostics in Table 1 indeed suggest, the matrix of estimated parameters B can be used to replicate the conditional expectations in Equation (8) and to compute a time series of theoretical PD ratios,

$$y_t^f = \sum_{i=0}^{\infty} \rho^i E_t[\Delta d_{t+1+i}] = \sum_{i=0}^{\infty} \rho^i (e_2' B^i z_t) = e_2' B (I - \rho B)^{-1} z_t, \tag{15}$$

in which e_i is a vector of zeros in which the ith element is replaced by 1. A full derivation is provided by Campbell and Shiller (1988). The superscript f to the theoretical PD ratio indicates "fundamentalist." The generated theoretical PD ratio can be interpreted as an estimate of how the PD ratio would behave if all agents are fundamentalists, who value assets according to long-term dividend expectations.

For now, the parameter ρ is calibrated at a fixed value, as in Campbell and Shiller (1988). I set $\rho = 0.958$,

Figure 1 **Observed PD Ratio (y_t) and Theoretical PD Ratio (y_t^f) from the Long-Term/Fundamentalist Model (15), with $\rho = 0.958$**



Note. $\text{corr}(y_t, y_t^f) = 0.799$; $\sigma(y_t^f)/\sigma(y_t) = 0.135$.

which is the sample average of the ratio $P_t/(P_t + D_t)$. At the end of this section, I discuss the sensitivity of the results with respect to this calibration.

Figure 1 shows the theoretical PD ratio (y_t^f), as well as the realized PD ratio (y_t). The figure looks similar to the charts in Campbell and Shiller (1987). The theoretical PD ratio is quite strongly correlated with the realized PD ratio ($\text{corr}(y_t^f, y_t) = 0.799$), but the volatility of the theoretical PD ratio falls far behind of observed volatility. This is illustrated by the volatility ratio ($\sigma(y_t^f)/\sigma(y_t) = 0.135$), which expresses the standard deviation of the theoretical PD ratio as a fraction of the standard deviation of the realized PD ratio. The long-term present value model (15) therefore seems able to explain the direction of the stock market but lacks explanatory power regarding the observed volatility of the stock market. Already in the 1980s, Campbell and Shiller, among others, interpreted this excess volatility as a rejection of present value models. In fact, as Figure 1 shows, the discrepancy between the theoretical and observed PD ratio has only increased further since then, with an unprecedented rise in the PD ratio during the 1990s, which the present value model fails to capture.

The VAR approach can also be applied to the short-term model (11), which is the correct model if all agents are rational speculators. These agents are speculators, as they are mainly interested in short-term trading profits rather than in the dividends the asset generates in the long run. They can be considered (boundedly) rational, however, because they form expectations using the same information set and VAR model as the fundamentalists considered above. The conditional expectations of these rational speculators (*rs*) can therefore be replicated based on the estimated VAR, as above:

$$y_t^{rs} = \frac{\rho}{1-\rho} E_t[\Delta p_{t+1}] + E_t[\Delta d_{t+1}], \tag{16}$$

in which

$$E_t[\Delta d_{t+1}] = e_2' B z_t, \tag{17}$$

and

$$\begin{aligned} E_t[\Delta p_{t+1}] &= E_t[\Delta y_{t+1}] + E_t[\Delta d_{t+1}] \\ &= E_t[y_{t+1}] - y_t + E_t[\Delta d_{t+1}] \\ &= e_1'(B - I) z_t + e_2' B z_t. \end{aligned} \tag{18}$$

In addition, I consider the valuation model according to a second type of speculator: the contrarian speculator (*cs*), or simply, contrarian. Contrarian agents agree with the rational agents on expected dividends but form alternative expectations on price changes:

$$y_t^{cs} = \frac{\rho}{1-\rho} \tilde{E}_t^{cs}[\Delta p_{t+1}] + E_t[\Delta d_{t+1}]. \tag{19}$$

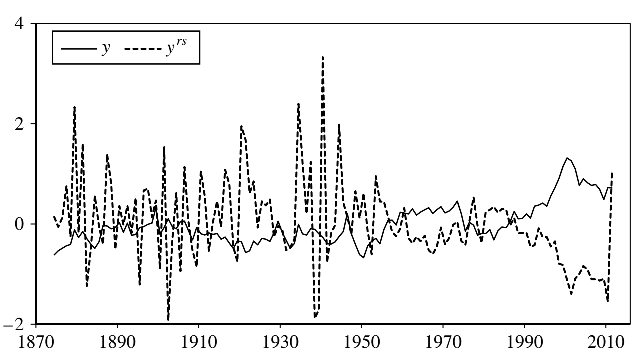
In fact, regarding expected price changes, contrarians take the exact opposite stance from the rational speculators:

$$\tilde{E}_t^{cs}[\Delta p_{t+1}] = -E_t[\Delta p_{t+1}]. \tag{20}$$

Figure 2 shows y_t^{rs} and y_t. The model with rational speculative expectations (16) appears able to generate large price fluctuations, with the volatility of the theoretical PD ratio even overshooting observed volatility ($\sigma(y_t^{rs})/\sigma(y_t) = 2.065$). Nevertheless, the correlation with the observed PD ratios is very weak, even negative ($\text{corr}(y_t^{rs}, y_t) = -0.403$). From Figure 2, it can be seen that during several episodes, most notably the 1990s, the theoretical PD ratio moves in the opposite direction from the observed PD ratio. The rational speculative model (16) therefore fails to explain the bull market in the 1990s any better than the long-term model (15) does.

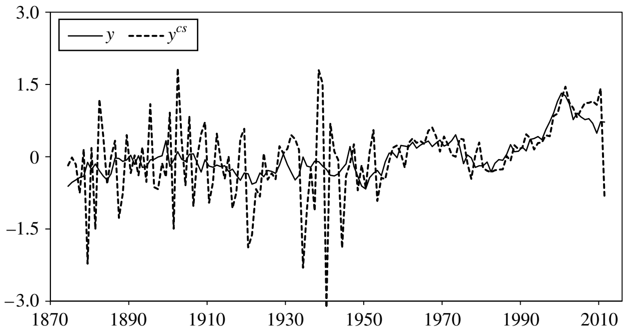
Figure 3 shows the empirical need for a model with contrarian expectations. The theoretical PD ratio y_t^{cs}, which is generated by model (19), nearly matches y_t^{rs} in terms of volatility: ($\sigma(y_t^{cs})/\sigma(y_t) = 1.977$). Unlike the rational speculative model, however, the contrarian model generates a PD ratio that is positively correlated with the observed PD ratio ($\text{corr}(y_t^{cs}, y_t) = 0.447$).

Figure 2 **Observed PD Ratio (y_t) and Theoretical PD Ratio (y_t^{rs}) from the Rational Speculative Model (16), with $\rho = 0.958$**



Note. $\text{corr}(y_t, y_t^{rs}) = -0.403$; $\sigma(y_t^{rs})/\sigma(y_t) = 2.065$.

Figure 3 **Observed PD Ratio (y_t) and Theoretical PD Ratio (y_t^{cs}) from Contrarian Model (19), with $\rho = 0.958$**

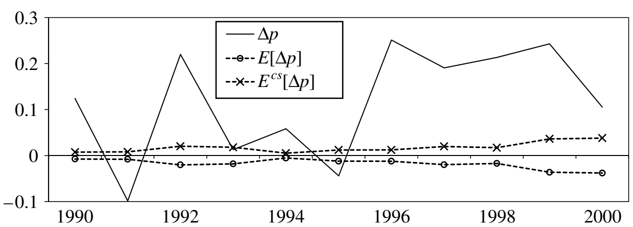


Note. $\mathrm{corr}(y_t, y_t^{cs}) = 0.447$. $\sigma(y_t^{cs})/\sigma(y_t) = 1.977$.

Although this correlation remains quite low compared with the long-term model (15), it is evident from Figure 3 that for recent decades the contrarian model traces the observed PD ratio remarkably well.

Based on Figure 1, it can be argued that the bull market in the 1990s was a bubble. It was, however, not a rational bubble, as in that case the rational speculative model (Figure 2) should be able to replicate the bubble. Instead, I find that the model requires nonrational, or contrarian, beliefs to explain the 1990s bubble. Figure 4 shows the expected price changes according to both rational speculators and contrarians, as well as the realized price changes over the period 1990–2000. With the exception of 1991 and 1995, the S&P 500 index increased every year, even exceeding 20% in several years. The rational speculators predicted that prices would decrease throughout this decade, as the market was overvalued according to the VAR representation. As a result, the contrarians, who take the opposite stance from the VAR-based predictions, made more accurate predictions during this period. The contrarian model (Figure 3) is therefore closer to the observed true prices than the rational speculative model (Figure 2).

Figure 4 **Realized Price Changes (Δp_t) and Expected Price Changes According to Rational Speculators ($E_{t-1}[\Delta p_t]$) and Contrarians ($\tilde{E}_{t-1}^{cs}[\Delta p_t]$) During 1990–2000**



Boswijk et al. (2007), also using data on the S&P 500 index, estimate a heterogeneous-agent model with fundamentalists and trend followers. These trend followers form expectations by simply extrapolating recent trends, instead of the VAR framework of this paper. Boswijk et al. find that the 1990s bubble was strongly amplified by trend-following behavior. This interpretation is different, but not necessarily inconsistent, with the results presented in this paper, because Figure 4 shows that during this episode, the contrarians actually behaved as positive-feedback traders by expecting continuing price increases, just like the trend followers in Boswijk et al. (2007).

It is evident from Figures 1–3 that the performance (or fit) of the three alternative models changes over time, which could indicate misspecification of the VAR as a result of the existence of structural breaks or time-varying parameters. The diagnostic tests presented in Table 1, however, indicate that the VAR is correctly specified. In addition, I estimate the VAR and generate y_t^f, y_t^{rs}, and y_t^{cs} again for the last 40 years in the sample only; these results are presented in Figure 5. These plots tell a roughly similar story as in Figures 1–3, suggesting that the time-varying performance of the three models is not the result of misspecification of the VAR.

Instead, the time-varying fit of the three models could indicate that the market is subject to regime-switching behavior, with agents switching between the fundamental long-term strategy based on expected

Figure 5 **Observed PD Ratio (y_t) and Theoretical PD Ratios (y_t^f, y_t^{rs}, and y_t^{cs}) from Models (15), (16), and (19) During 1972–2011**

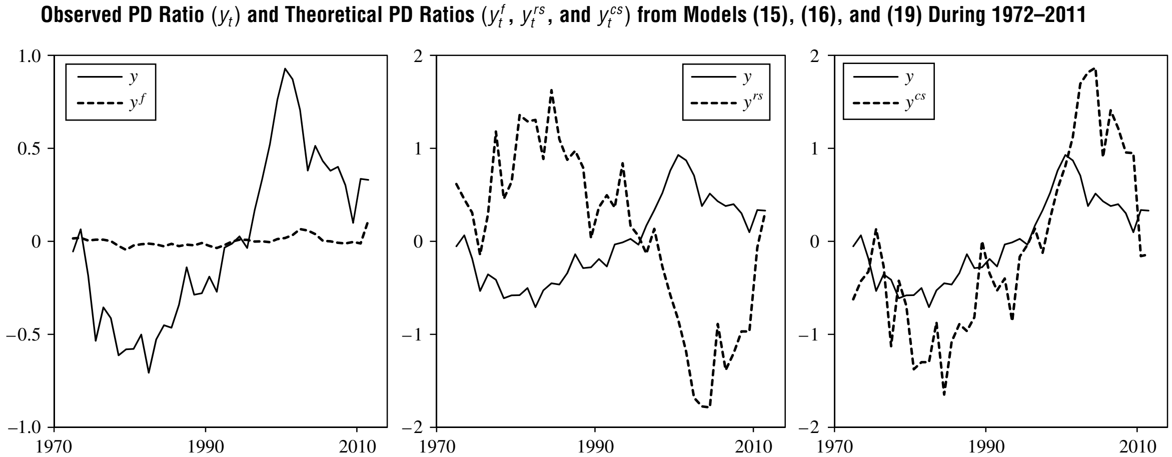
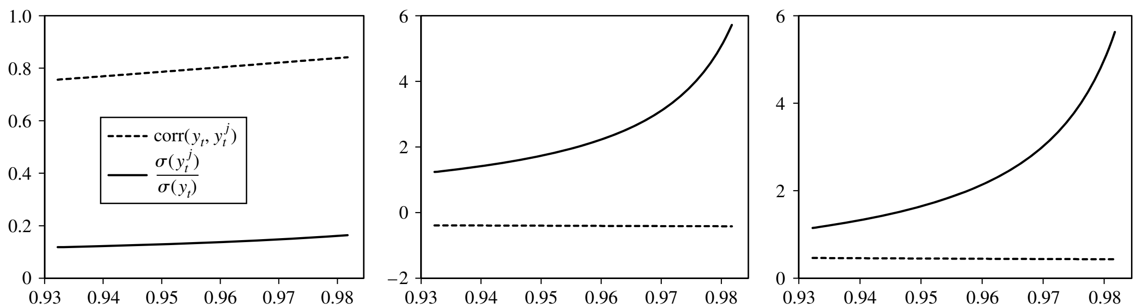


Figure 6 **Correlation and Volatility Ratio for Different Values of ρ for $j = f$ (Left), $j = rs$ (Middle), and $j = cs$ (Right)**



dividends and more speculative (rational or contrarian) strategies. In the next section, I therefore combine Equations (15), (16), and (19) into one regime-switching model, in which the asset price is determined by the interaction of fundamentalists, rational speculators, and contrarians.

So far, the parameter ρ is calibrated at the sample average of the ratio $P_t/(P_t + D_t)$. The obtained results are somewhat sensitive to this calibration. This is illustrated in Figure 6, which shows volatility ratios and the correlation between realized and theoretical PD ratios for different values of ρ for all three models. For the long-term model, the sensitivity with respect to ρ is rather modest. Campbell and Shiller (1988) make the same observation. For the speculative models, however, small changes in ρ do have a great impact. Calibrating ρ and disregarding its uncertainty therefore seems inappropriate. Instead, I estimate ρ in the remainder of this paper jointly with the other parameters in the model.

4. Heterogeneous Agents

The results in the previous section indicate that the long-term present value model (15) can explain the direction of stock market movements but not its excess volatility. The speculative models (16) and (19) are able to generate sufficient volatility, but their correlation with the observed market falls short of the long-term model. In an attempt to specify a model that is able to capture both correlation and volatility, I consider an economy in which all three agents (fundamentalists, rational speculators, and contrarians) are present:

$$y_t^{ha} = G_t^f y_t^f + G_t^{rs} y_t^{rs} + G_t^{cs} y_t^{cs}, \tag{21}$$

where the subscript *ha* denotes heterogeneous agents. The fractions of each type of agent are denoted by G_t^f, G_t^{rs}, and G_t^{cs} and are allowed to vary over time. This process of switching between agent types or regimes is modeled based on evolutionary selection following Brock and Hommes (1998), such that the fraction of each type of agents increases when its predictions outperform the other types. The predictions of each type are evaluated by a measure of fitness representing the distance between the theoretical PD ratio and the realized PD ratio in the previous period:

$$U_t^j = -(y_{t-1}^j - y_{t-1})^2 \quad j \in \{rl, rs, cs\}. \tag{22}$$

The fractions of each type are then determined by multinomial logit probabilities:

$$G_t^j = \frac{\exp(\beta^j U_t^j)}{\sum_k \exp(\beta^k U_t^k)} \quad j, k \in \{f, rs, cs\}, \tag{23}$$

such that the fractions of the three types sum to 1. The parameters β denote the intensity of choice, which indicate the willingness of agents to switch between strategies. A higher value of β^j implies that agents are more willing to switch to another type. Whereas Brock and Hommes (1998) hold β constant across types, I allow for type-specific intensities of choice. This setting accommodates the idea by Hong et al. (2007) that agents may hold heterogeneous thresholds for switching between strategies. For comparison, I also estimate the model under the restriction that β is constant across types.

To obtain estimates of β and ρ, I estimate the following model by nonlinear least squares (NLS):

$$y_t = y_t^{ha} + \varepsilon_t. \tag{24}$$

The top row of Table 2 shows the parameter estimates, whereas Figure 7 shows a plot of the theoretical PD ratio y_t^{ha}. The generated PD ratio is highly correlated with the realized PD ratio, $\text{corr}(y_t^{ha}, y_t) = 0.759$, which is of the same magnitude as the correlation coefficient for the fundamental long-term model considered in §3. The volatility ratio for the heterogeneous-agent model is, however, much larger ($\sigma(y_t^{ha})/\sigma(y_t) = 0.752$). Unlike the representative-agent models considered in §3, the heterogeneous-agent model is able to explain both the direction and the volatility of the observed PD ratio to a large extent.

The fitted values of model (24), $\hat{y}_t^{ha}$, are used to estimate the following regression by ordinary least squares:

$$y_t = \phi \hat{y}_t^{ha} + \epsilon_t. \tag{25}$$

Table 2 Estimation Results

	ρ	β^f	β^{rs}	β^{cs}	ϕ	$\sigma(y_t^j)/\sigma(y_t)$	$\text{corr}(y_t, y_t^j)$	R^2	AIC
ha	0.966	0.799	5.175	1.125	0.962	0.752	0.759	0.548	−2.557
	(0.004)	(0.599)	(6.156)	(0.401)	(0.029)				
ha (∗)	0.949	1.552	—	2.332	0.956	0.704	0.678	0.454	−2.384
	(0.239)	(1.630)		(2.480)	(0.310)				
ha (∗∗)	0.954	1.849	—	—	1.045	0.630	0.711	0.467	−2.422
	(0.072)	(1.418)			(0.138)				
f	1.000	—	—	—	4.474	0.193	0.865	0.297	−2.138
	(0.073)				(0.548)				
rs	0.000	—	—	—	3.933	0.080	0.317	0.044	−1.830
	(0.000)				(0.497)				
cs	0.000	—	—	—	3.933	0.080	0.317	0.044	−1.830
	(0.202)				(0.568)				

Notes. NLS estimates and measures of fit for model (21)–(24) are shown. *ha*, heterogeneous agents (three types) and evolutionary dynamics (22)–(23); *ha* (∗), heterogeneous-agent model with only fundamentalists and contrarians; *ha* (∗∗), heterogeneous-agent model under restriction $\beta^f = \beta^{rs} = \beta^{cs}$; *f*, $G_t^f = 1$, $G_t^{rs} = G_t^{cs} = 0$; *rs*, $G_t^{rs} = 1$, $G_t^f = G_t^{cs} = 0$; *cs*, $G_t^{cs} = 1$, $G_t^f = G_t^{rs} = 0$. The parameter ϕ is estimated by model (25). Annual data are for 1872–2011. Standard errors (in parentheses) are computed using 10,000 bootstrap replications.

Table 2 reports the estimate and standard error of ϕ, showing that the null hypothesis that $\phi = 1$ cannot be rejected.

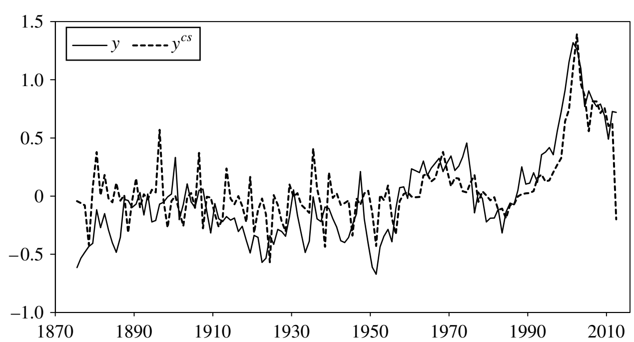
To take into account the uncertainty underlying the estimated parameters in the VAR model (12), all standard errors in Table 2 are based on the following bootstrap procedure:

1. Generate simultaneously an artificial series ($T + 100$ observations) of dividend growth rates from the VAR model (12) using the parameter estimates $\hat{B}$ and an artificial series ($T + 100$ observations) of PD ratios from the model (21)–(24) using the parameter estimates $\hat{\beta}$ and $\hat{\rho}$. The innovations to both series are drawn (with resampling) from the fitted residuals $e_2'\hat{u}_t$ and $\hat{\varepsilon}_t$.
2. Use the last T observations from both artificial series to estimate models (24) and (25). Store the estimates $\tilde{\beta}$, $\tilde{\rho}$, and $\tilde{\phi}$.
3. Repeat Steps 1 and 2 R times. For each parameter, the standard deviation of the R artificial estimates is reported in Table 2 as the parameter's standard error.

For this procedure, I set $T = 138$ equal to the sample size in the estimations and the number of replications as $R = 10{,}000$.

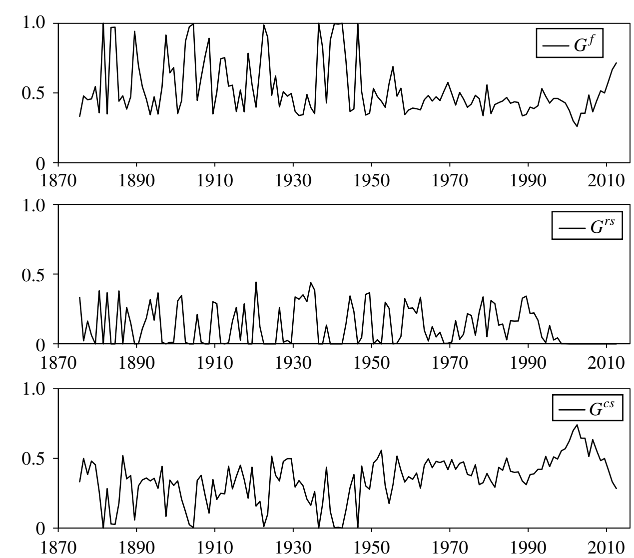
Figure 8 shows the estimated fractions of each type of agent over time. Fundamentalists are always present in the economy, with their fraction of the total population fluctuating for most of the time between roughly 40% and 100%. After 1950, their fraction stays close to the lower bound of this interval, suggesting that expected dividends have lost relevance as a determinant of asset prices. This is consistent with the finding of decreasing dividend yields reported by Fama and

Figure 7 Observed PD Ratio (y_t) and Theoretical PD Ratio (y_t^{ha}) from Heterogeneous-Agent Model (21), with ρ and β Estimated by NLS (See Table 2)



Note. $\text{corr}(y_t, y_t^{ha}) = 0.759$; $\sigma(y_t^{ha})/\sigma(y_t) = 0.752$.

Figure 8 Time-Varying Fractions of Fundamentalists (Top), Rational Speculators (Middle), and Contrarians (Bottom)



French (2001). The fraction of contrarians is relatively high during this period and increases further during the buildup of the 1990s bubble. The fraction of rational speculators stays rather low during the entire sample period.

As a robustness check, the second row in Table 2 shows estimates of the model with only two types of agents by excluding the rational speculators entirely from the model. The two-type model fits the data rather well, providing further evidence that only a relatively small fraction of the population follows a rational speculative strategy. Nevertheless, the benchmark model with all three types of agents is the preferred specification in terms of correlation, volatility, and overall goodness of fit. The third row in Table 2 presents the heterogeneous-agent model estimated under the restriction that the intensities of choice are equal for all investors: $\beta^f = \beta^{rs} = \beta^{cs}$. Although this restricted version of the heterogeneous-agent model outperforms the representative-agent models discussed below, it underperforms with respect to the benchmark model according to all measures. Judging from the AIC, the costs of estimating two extra parameters is outweighed by the increased fit of the benchmark model. This result provides empirical support for the assumption of type-specific intensities of choice, in line with the heterogeneous switching thresholds by Hong et al. (2007). The intensity of choice parameters β are estimated rather imprecisely, with relatively large standard errors for all three heterogeneous-agent models. This is, however, a well-known problem for these nonlinear smooth-transition models (see, e.g., Boswijk et al. 2007) and for small samples in particular.

Table 2 further shows estimates of the representative-agent models considered in §3, with the difference that the parameter ρ is now estimated using NLS. These models can be seen as a restricted version of the model (21)–(24). Instead of the evolutionary dynamics (22) and (23), the fractions G_t^{rl}, G_t^{rs}, and G_t^{cs} are restricted to either 0 or 1. The parameters β therefore drop from the model. The correlation coefficients, volatility ratios, and goodness-of-fit measures reported in Table 2 suggest that the heterogeneous-agent model is the preferred specification. The fundamentalist valuation model generates a higher correlation coefficient ($\mathrm{corr}(y_t^f, y_t) > \mathrm{corr}(y_t^{ha}, y_t)$), but in all other cases, the heterogeneous-agent model generates higher correlation and volatility as well as a better overall fit. The null hypothesis that $\phi = 1$ is rejected for all three alternatives.

The parameter ρ is estimated under the restriction $0 \le \rho \le 1$. For the heterogeneous-agent model, the estimate of ρ is rather close to the calibration in §3. For the representative-agent models, however, a corner solution is reached with ρ estimated at either 0 or 1. In the log-linear approximation by Campbell and Shiller (1988), the parameter ρ represents the mean of the ratio $P_t/(P_t + D_t)$. Of course, this mean can never be 0 or 1 because this implies that either prices or dividends are always equal to 0. It is furthermore easy to see that the two speculative models (16) and (19) reduce to identical models in which one-period dividend expectations are the sole determinant of prices in the case that $\rho = 0$. The finding that highly unrealistic values of ρ are required to obtain the best fit can be interpreted as an economic rejection of the three representative-agent models.

For a formal statistical comparison of the heterogeneous-agent model and the three representative-agent models, I rely on the test for nonnested nonlinear regression models developed by Davidson and MacKinnon (1981). The test is based on the following regression:

$$y_t = (1-\alpha) y_t^{H1} + \alpha \hat{y}_t^{H2} + \eta_t, \qquad (26)$$

in which y_t^{H1} and y_t^{H2} are two nonnested nonlinear regression models, such as the different models considered above. The parameters of y_t^{H1} are estimated jointly with α by NLS, and the test regression further includes the fitted values from NLS estimation of the model y_t^{H2}. The hypothesis H_0: $\alpha = 0$ is equivalent to the hypothesis that y_t^{H1} is the correct data-generating process. Table 3 shows the estimates and standard errors of α from testing y_t^{ha} against y_t^f, y_t^{rs}, and y_t^{cs}, and vice versa. The top row shows the result when $y_t^{H1} = y_t^{ha}$. The hypothesis that y_t^{ha} is correct cannot be rejected against any of the three alternatives. Moreover, the bottom row of Table 3 shows that the hypotheses that y_t^f, y_t^{rs}, and y_t^{cs} are correct are all rejected against the alternative $y_t^{H2} = y_t^{ha}$.

5. Time-Varying Discount Factors

I have so far assumed a constant discount factor and, as a result, constant expected returns. The log-linear approximation by Campbell and Shiller (1988) does, however, allow for time-varying discount factors. If discount factors are allowed to vary over time, Equation (7) becomes (disregarding the constant term k)

$$y_t = \rho E_t[y_{t+1}] + E_t[\Delta d_{t+1}] - E_t[r_{t+1}]. \qquad (27)$$

Table 3 **Nonnested Hypothesis Tests**

	f	*rs*	*cs*
H1: *ha*	0.792	0.611	0.611
	(0.662)	(4.468)	(4.342)
H2: *ha*	0.787	0.927	0.927
	(0.028)	(0.024)	(0.027)

Notes. NLS estimates of α in model (26) are shown. For H1, $y_t^{H1} = y_t^{ha}$ and $\hat{y}_t^{H2} = \hat{y}_t^j$, $j \in \{f, rs, cs\}$. For H2, $y_t^{H1} = y_t^j$, $j \in \{f, rs, cs\}$ and $\hat{y}_t^{H2} = \hat{y}_t^{ha}$. Rejection of H_0: $\alpha = 0$ implies rejection of y_t^{H1} (Davidson and MacKinnon 1981). Annual data are for 1872–2011. Standard errors (in parentheses) are computed using 10,000 bootstrap replications.

There are several ways to model time-varying discount factors. Campbell and Shiller (1988) evaluate three simple specifications of the discount factors based on short-term interest rates, consumption, and volatility of the S&P 500 index, in addition to a constant discount factor. With a time-varying discount factor, expected returns are computed as follows:

$$E_t[r_{t+1}] = \gamma E_t[x_{t+1}], \tag{28}$$

in which γ is the risk aversion coefficient and x_t denotes interest rates, consumption, or stock-market volatility. In the first case, x_t is the log-yield on Treasury bills (T-bills) as a proxy for the risk-free rate of return. In the second case, x_t is the log-growth rate of consumption such that the model (27) becomes a consumption-based asset pricing model with constant relative-risk aversion utility function. In the third case, x_t is the squared (lagged) log-return of the S&P 500 index, as a simple measure of market volatility or risk.

The constant discount factor is nested in the time-varying specifications. When $\gamma = 0$, it is easily seen that the expected return drops out of Equation (27), reducing it to the constant discount factor models considered in the previous sections.

I evaluate the three specifications of the time-varying discount factor in the heterogeneous-agent model (21). Following Campbell and Shiller (1988), I add x_t as a third variable to the VAR model (12), after which the long-term model (15) with the time-varying discount factor becomes

$$\begin{aligned} y_t^f &= \sum_{i=0}^{\infty} \rho^i (E_t[\Delta d_{t+1+i}] - E_t[r_{t+i+1}]) \\ &= (e_2' - \gamma e_3') B (I - \rho B)^{-1} z_t, \end{aligned} \tag{29}$$

in which $z_t \equiv (v_t, v_{t-1})'$ and $v_t \equiv (y_t, \Delta d_t, x_t)'$. The speculative models (16) and (19) become

$$y_t^{rs} = \frac{\rho}{1-\rho} E_t[\Delta p_{t+1}] + E_t[\Delta d_{t+1}] - \frac{1}{1-\rho} E_t[r_{t+1}] \tag{30}$$

and

$$y_t^{cs} = \frac{\rho}{1-\rho} \tilde{E}_t^{cs}[\Delta p_{t+1}] + E_t[\Delta d_{t+1}] - \frac{1}{1-\rho} E_t[r_{t+1}], \tag{31}$$

in which

$$E_t[r_{t+1}] = \gamma e_3' B z_t. \tag{32}$$

Because of limited data availability, the models with time-varying discount factors can be estimated only for the period 1891–2009. Campbell and Shiller (1988) find that these three time-varying discount factors are not helpful in explaining stock price movements in the long-term model. The results presented in Table 4 confirm that this finding also holds for the heterogeneous-agent model considered here. Of the four specifications, the constant discount factor is the preferred option. Table 4 shows the correlation, volatility ratio, and goodness-of-fit measures for the estimated heterogeneous-agent models (21) with different time-varying discount factors as well as a constant discount factor over this period. The table further shows the NLS estimate of the risk aversion coefficient γ. Using the discount factor based on either interest rates or consumption, the restriction $\gamma = 0$ (i.e., a constant discount factor) cannot be rejected. These specifications are therefore not preferred to the model with constant discount factor. Although the volatility ratio for the consumption-based model is slightly higher than for the model with constant discount factor, the latter yields a higher correlation and a better fit overall.

Table 4 **Time-Varying Discount Factors**

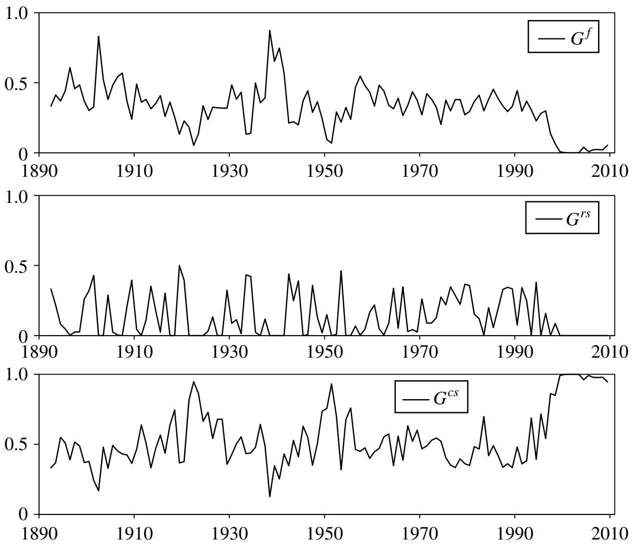
	γ	$\sigma(y_t^j)/\sigma(y_t)$	$\mathrm{corr}(y_t, y_t^j)$	R^2	AIC
Constant	—	0.777	0.797	0.621	−2.719
T-bill	−0.013	0.690	0.687	0.467	−2.361
	(0.304)				
Consumption	0.138	0.858	0.767	0.564	−2.561
	(0.210)				
Volatility	0.824	0.714	0.794	0.618	−2.693
	(0.157)				

Notes. NLS estimates and measures of fit for model (21)–(24), with the constant discount factor or time-varying discount factor (28) based on interest rates, consumption, or volatility, are shown. Annual data are for 1891–2009. Standard errors (in parentheses) are computed using 10,000 bootstrap replications.

In the case of a volatility-based discount factor, γ is significant, but Table 4 shows that also this model is not an improvement in terms of correlation, volatility ratio, or R^2 with respect to the constant discount factor model. Besides not improving the fit of the model nor increasing the volatility of replicated prices, including a time-varying discount factor based on volatility does not diminish the empirical need for heterogeneous horizons and expectations. As Figure 9 shows, with a volatility-based discount factor the estimated fractions of the different types evolve following a similar path as with a constant discount factor (Figure 8). In fact, the estimated fraction of contrarians is often even higher than with a constant discount factor.

Of course, various, more complex discount factor specifications besides these three examples could be considered. Nevertheless, the results presented in this paper show that allowing for heterogeneous agents is at least a more fruitful approach than the three simple discount rate specifications considered in this section. Finding another parametric process for the evolution of a discount factor in a rational-expectations model that is able to match the empirical results of the heterogeneous-agent model in this paper is presumably not an easy task.

Figure 9 Time-Varying Fractions of Fundamentalists (Top), Rational Speculators (Middle), and Contrarians (Bottom), with Volatility-Based Time-Varying Discount Factor



6. Conclusion

I develop an empirical asset pricing model in which the expectations of all agents are derived from a VAR representation for price–dividend ratios and dividend growth rates. Taking into account the performance of each strategy in the previous period, agents choose between a fundamental long-term strategy, valuing assets based on expected dividends, and two types of short-term strategies, valuing assets mainly based on expected price changes. This heterogeneous-agent model is able to generate far more volatile PD ratios than a standard present value model, thereby tackling a considerable part of the excess volatility puzzle.

The existence of speculators can explain the volatility of stock prices. Nevertheless, heterogeneity in expectations among the speculators is required to approximate observed prices in terms of volatility as well as correlation. In particular, to replicate the stock market during the 1990s accurately, a large fraction of market participants needs to adopt contrarian beliefs. Contrarian expectations were actually consistent with trend-following behavior during the 1990s, because the market was overvalued according to the VAR such that the "rational" VAR-based predictions suggested decreasing prices. I argue therefore that the 1990s bubble was not a rational bubble.

The introduction of time-varying discount factors into the model does not significantly alter the results. Overall, the results suggest that observed excess volatility with respect to the standard present value model is better explained by nonstandard expectations rather than by time-varying discount factors.

Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/mnsc.2014.1937.

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