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Project Characteristics, Incentives, and Team Production

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We develop a model to show how agency conflicts, free-rider effects, and monitoring costs interact to affect optimal team size and workers' incentive contracts. Team size increases with project risk, decreases with profitability, and decreases with monitoring costs as a proportion of output. Our predictions are consistent with empirical evidence that firm-specific risk has increased over time, average corporate earnings have declined, and firms' organizational structures have also flattened. The predicted effects of monitoring costs on team size are supported by evidence that improvements in information technology likely to lower monitoring costs lead to larger teams. Further, firms with relatively more intangible assets, where monitoring costs are likely to be higher, are smaller. Optimal incentive intensities decrease with risk and increase with profitability. The endogenous determination of team size accentuates the positive effects of a decline in risk and an increase in profitability on incentives.

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1. Introduction

A growing body of literature on the theory of the firm is devoted to the investigation of the determinants of the size and internal organization of firms. In two seminal studies, Marschak and Radner (1972) highlight the central role of teams in modern organizations and Alchian and Demsetz (1972) argue that a firm is an organization that is designed to deal with a "moral hazard in teams" problem. Holmstrom (1982) formalizes these arguments and derives an optimal multilateral incentive contract when only the aggregate output of the team is contractible. A significant strand of the subsequent literature focuses on the provision of team incentives to overcome concerns about "free riding" as well as to exploit the beneficial effects of competition among agents while mitigating the detrimental impact of collusion (see Chap. 8 of Bolton and Dewatripont 2005). Most studies in this literature, however, take the team size as given. A parallel literature on organizational design studies how factors such as uncertainty (e.g., Sah and Stiglitz 1986); information processing and communication costs (e.g., Radner 1992, Bolton and Dewatripont 1994, Garicano 2000); incentive provision and monitoring (e.g., Qian 1994, Melumad et al. 1997, Maskin et al. 2000, Mookherjee and Reichelstein 2001); and bargaining (Stole and Zwiebel 1996, Rajan and Zingales 2001) affect the internal organization of teams and firms.

We contribute to the literature by developing a model that integrates some of the features of frameworks analyzed in these relatively independent strands of the literature to show how characteristics of the external environment of a production team—the risk and profitability of its projects—interact with its internal characteristics—agency conflicts, free-rider effects, and monitoring costs—to affect the optimal size of the team and the incentives of its workers. Our results lead to the novel empirical implications that the optimal size of the team (i) increases with the risk of its projects, (ii) decreases with their productivity/profitability, and (iii) decreases with monitoring costs as a proportion of output. These implications are consistent with empirical evidence that firm-specific risk and earnings volatility have increased over time (Campbell et al. 2001, Wei and Zhang 2006) and average corporate earnings have declined (Wei and Zhang 2006) while firms' organizational structures have also "flattened" over time, suggesting an increase in the size of teams (Rajan and Wulf 2006). The predicted decline of team size with monitoring costs is supported by evidence that improvements in information technology that are likely to lower monitoring costs lead to flatter firms (Rajan and Wulf 2006) and with evidence that firms with more intangible assets, where monitoring costs are likely to be higher, are smaller (Chen 2014).

In our model, a manager controls a team of workers who execute a project. The project's total payoff equals the output generated by the workers' effort, which is unobservable by the manager, plus random, normally distributed noise whose variance represents the project's risk. There are positive synergies among workers arising from the benefits of complementarities, but there are decreasing returns to scale in such coordination so that the team's output is concave in the workers' effort choices. The project's total payoff is observable and verifiable by the manager if and only if she monitors the team. Monitoring costs increase with team size and output (e.g., see Garen 1985). Workers are risk averse with constant absolute risk aversion (CARA) preferences and receive explicit contracts contingent on the total payoff. The risk-neutral manager chooses the team size and the workers' incentive contracts to maximize her expected net payoff, which equals the project's expected total payoff net of the workers' expected compensation and expected monitoring costs. The workers' effort choices are determined in Nash equilibrium of the game in which each worker rationally anticipates the effort exerted by other workers. In the symmetric equilibrium, workers receive identical contracts and exert the same effort.

The optimal size of the team reflects the following trade-offs. First, increasing the team size increases the team's output. Increasing the team size, however, also increases the manager's monitoring costs. Because output is concave in workers' effort choices, increasing the number of workers also increases "free-rider" problems in that the *marginal* contribution of an additional worker *as a proportion* of the team's output declines. Since monitoring costs scale with output, the *marginal cost* of an additional worker as a proportion of output also declines with the number of workers. The decline is, however, less pronounced than that of the proportional marginal benefit because monitoring costs are a proportion (less than one) of output. The *net* proportional marginal benefit—the proportional marginal benefit minus cost—of an additional worker, therefore, declines with the number of workers and is zero at the optimal team size.[1]

The optimal size of the team increases with the project's risk, whereas the incentive intensity of each worker's contract, the output generated by each worker, and the output generated by the team decline. The intuition for the effects of project risk on team size hinges on the relative effects of changes in the project's risk on the net proportional marginal benefit of an additional worker. Keeping the team size fixed, an increase in the project's risk increases the costs of risk sharing, thereby decreasing the effort exerted by each worker and the team's output. Because output is concave in workers' effort, and output declines with risk, the net proportional marginal benefit of an additional worker increases with the project's risk. Put differently, because workers' effort and the team's output decline with project risk as a result of weaker incentives, free-rider problems and monitoring costs to deter expropriation become less significant at the margin relative to the beneficial impact of team size on output. Consequently, the optimal team size increases with project risk.

The positive effect of risk on team size is consistent with the combined implications of two sets of empirical findings. First, Campbell et al. (2001) document that firm-specific risk has increased over time. Wei and Zhang (2006) show that an increase in the volatility of firms' earnings explains the increase in idiosyncratic volatility. Second, Rajan and Wulf (2006) show that firms' organizational structures have flattened over time. Taken together, these findings suggest that team sizes increase with risk.

As indicated by the intuition above, because a decrease in the project's risk decreases the costs of risk sharing between the manager and the risk-averse workers, output increases. Because the team size decreases with a decrease in risk, the incentive intensity of each worker's compensation contract increases. The intuition also suggests that, if team size were not allowed to endogenously decrease with a decrease in risk, the increase in the effort and output per worker and, therefore, the incentive intensity of workers' contracts would be less pronounced. Consequently, our results suggest that the endogenous variation of team size with risk *accentuates* the positive impact of a decline in risk on incentives.[2]

Next, we demonstrate that an increase in the marginal product of the workers' effort—the *productivity* or *profitability*—decreases the optimal team size but increases the incentive intensity of each worker's contract, the output generated by each worker, and

[1] We ignore "integer" effects for simplicity.

[2] Because our model directly generalizes the classic Holmstrom and Milgrom (1987) model, it is an additive model in that the total output is additively affected by workers' effort and noise. Consequently, the relevant measure of the strength of incentives is the *dollar* sensitivity of a worker's compensation to output. In multiplicative models such as Edmans et al. (2009), Edmans and Gabaix (2011a), and Edmans and Gabaix (2011b), the relevant measure of the strength of incentives is the sensitivity of the *percentage* change in a worker's compensation to a *percentage* change in output. In the multiplicative model of the above studies, the percentage pay-performance sensitivity is not affected by risk or the agent's risk aversion, but only by the cost of effort.

the output of the team. Keeping the team size fixed, an increase in productivity increases the power of incentives that can be provided to each worker, thereby increasing the effort exerted by each worker and the output generated by the team. Because output is concave in workers' effort, the marginal benefit of an additional worker as a proportion of output—the proportional marginal benefit—decreases with productivity. As monitoring costs scale with output, the proportional marginal cost of an additional worker also declines with productivity, but the decline is less pronounced than that of the proportional marginal benefit. Put differently, as productivity increases, the increase in workers' effort and the team's output cause free-rider problems and monitoring costs to deter expropriation to become more pronounced at the margin relative to the beneficial impact of team size on total output. The optimal team size, therefore, decreases with productivity. Similar to the effects of risk on incentives, our results suggest that the endogenous variation of team size with productivity *enhances* the positive impact of an increase in productivity on incentives.

The predicted negative relation between team size and profitability is also consistent with empirical evidence. Wei and Zhang (2006) document that average corporate earnings have declined over time. This finding, along with the evidence in Rajan and Wulf (2006) of the flattening of firms' organizational structures, suggests that team size declines with profitability. The positive effect of productivity on incentives is in conformity with evidence that more productive agents at higher levels of a firm's hierarchy receive higher-powered incentives (Rajan and Wulf 2006, Wulf 2007, Kale et al. 2009).

Finally, we explore the effects of monitoring costs on the optimal team size, worker incentives, and output. The effects of changes in monitoring costs (as a proportion of output) are subtle because they could a priori have conflicting effects on team size and worker incentives. On the one hand, a decrease in expected monitoring costs as a proportion of output provides incentives to increase the size of the team. The intuition for our earlier results suggests that a decrease in expected monitoring costs as a proportion of output would lead to an increase in the team size but weaker incentives for each worker. On the other hand, a decrease in monitoring costs as a proportion of output could also be exploited to provide more powerful incentives to workers because the marginal product of effort is greater. The concavity of output in workers' effort would then imply that the net marginal benefit of an additional worker as a proportion of output declines, which provides incentives for the manager to decrease the size of the team and provide workers with stronger incentives. It turns out that the former "direct" effect dominates the latter "indirect" effect in determining the team size so that a decrease in monitoring costs as a proportion of output decreases the size of the team. The contrasting effects of the two "forces" described above on worker incentives, however, cause shifts in monitoring costs to have ambiguous effects on incentives.

Monitoring costs are (arguably) likely to be relatively lower in firms with more tangible assets and more easily verifiable output. In this context, the predicted decline of team size with monitoring costs is consistent with evidence that firms with a greater proportion of intangible assets are smaller (Chen 2014). Further, Rajan and Wulf (2006) argue that improvements in information technology over time have lowered monitoring costs and led to flatter firms. Lee and Mullineaux (2004) find that commercial lending syndicates are smaller and more concentrated when there is little information about the borrower so that monitoring costs are high.

We contribute to the literature by analyzing the effects of the interplay among moral hazard, free riding, and monitoring costs on the optimal size of a production team and the incentive contracts of its members. We derive the novel (to the best of our knowledge) predictions that team size increases with risk, decreases with profitability, and decreases with monitoring costs. Much of the literature following Holmstrom (1982) focuses on how free riding, the beneficial effects of competition among agents, and the negative effects of collusion interact to affect the incentives of agents but takes the number of agents as given (e.g., see Rasmusen 1987, Itoh 1991, McAfee and McMillan 1991, Legros and Matthews 1993, and the survey in Bolton and Dewatripont 2005, Chap. 8).

McLaughlin (1994) examines the determinants of the links between individual compensation and team performance using a framework in which agents have CARA preferences and output is Gaussian. Adams (2006) develops a model that incorporates productive complementarities among team members using a CES production function as in our framework. He shows that complementarities make the use of firm-wide incentives valuable, even in the presence of the induced free-rider problem.

Building on Calvo and Wellisz (1978), Qian (1994) develops a model of an organization in which managers are imperfectly monitored and the size of the organization is endogenously determined. Because agents' payoffs are deterministic in his framework, however, he abstracts away from risk and incentive considerations. Liang et al. (2008) study the determination of team size and incentives in a framework with risky payoffs and show that worker incentives are attenuated relative to the case when team size is exogenous. Another strand of the literature investigates the effects of bargaining and expropriation on organizational structure (e.g., Stole and Zwiebel 1996, Rajan and Zingales 2001, Harris and Raviv 2002). In these studies, however, payoffs are deterministic.

2. The Model

We model a team of workers who work on a project controlled by a manager. There are N workers indexed $1, 2, \ldots, N$. The number of workers, N, is later determined endogenously. Suppose worker j exerts effort e_j. The project's payoff or the total output is

$$P = \Lambda(e_1, \ldots, e_N) + \epsilon. \tag{1}$$

In the above, $\Lambda(e_1, \ldots, e_N)$ is the output the workers generate through their effort that is unobservable to the manager, where $\Lambda(e_1, \ldots, e_N)$ is strictly increasing, concave, and twice continuously differentiable in $(e_1, \ldots, e_N)$. This captures that there are positive synergies among workers arising from the benefits of the coordination of activities, but there are decreasing returns to scale in such coordination. For tractability, we assume a constant elasticity of substitution (CES) production function; that is,

$$\Lambda(e_1, \ldots, e_N) = \Theta\left[\sum_{j=1}^{N}(e_j)^{\alpha}\right]^{1/\alpha}, \quad 0 < \alpha \le 1, \tag{2}$$

where α determines the degree of complementarity between the workers' effort choices and Θ is the productivity of workers' effort that we hereafter refer to as the project's *productivity*. In (1), ϵ is a normal random variable with mean 0 and variance S^2 that is the project's *risk*.

The manager is risk neutral, whereas workers have identical CARA preferences with multiplicative disutilities of effort described by the utility function

$$U(x, e) = -\exp[-\gamma(x - \psi e^{\delta})], \quad \delta > 2, \tag{3}$$

where x is the worker's state-dependent payoff and e is his effort. The risk-aversion coefficient γ and the effort disutility function ψe^{δ} are identical across workers. The discount rates of the manager and workers are equal and normalized to zero.

The total output P is observable and verifiable by the manager if and only if she monitors the team. The monitoring costs incurred by the manager are

$$\text{Monitoring costs} = \mathcal{M}(N)P, \tag{4}$$

where $\mathcal{M}(\cdot)$ is strictly increasing. Monitoring costs, therefore, increase with team size and output reflecting the intuitive notions that, ceteris paribus, it is more difficult to monitor larger teams and, for a given team size, monitoring costs increase with the scale of the firm (e.g., see Garen 1985). In Appendix B, we extend the model to consider a setting in which a portion of the total output is verifiable even without manager monitoring. Specifically, each worker can take an additional action that we refer to as "expropriation" for concreteness (although it can be any action such as a "project choice") that lowers the verifiable output of the team but provides the worker with a private benefit. Under reasonable conditions, it is optimal for the manager to monitor workers and prevent expropriation in equilibrium so that the total output P given by (1) is verifiable with the manager incurring monitoring costs.

Workers receive explicit contracts from the manager that are contingent on P. The payoff P is normally distributed, the workers have CARA preferences, and the manager is risk neutral. By the results of Holmstrom and Milgrom (1987), we can, without loss of generality, restrict consideration to contracts that are affine functions of P. More precisely, a worker j is offered a contract that has a payoff

$$Q_j = a_j + b_j P. \tag{5}$$

For future reference, note that as in Holmstrom and Milgrom (1987), $b_j P$ is the *performance-sensitive* component of the worker's compensation and b_j is his *pay-performance sensitivity*, whereas a_j is the *performance-invariant* or *cash* component of his compensation.

Given the observable contracts offered to all workers, their effort choices are determined in Nash equilibrium of the game in which each worker rationally anticipates the effort exerted by all other workers in response to their incentives. Each worker's effort choice depends on his contract and his inferences about the effort choices of other workers. More precisely, suppose that a particular worker j anticipates that the effort choices of other workers are $e_{-j} = \{e_s; s \neq j\}$. By (3), given his contract parameters a_j, b_j, the worker's optimal effort $e(a_j, b_j, e_{-j})$ is given by

$$\begin{aligned} &e(a_j, b_j, e_{-j}) \\ &\quad = \arg\max_{e \ge 0} E\big[-\exp[-\gamma(a_j + b_j P - \psi e^{\delta})]\big] \\ &\quad = \arg\max_{e \ge 0} \big\{-\psi e^{\delta} + b_j \Lambda(e, e_{-j}) - \tfrac{1}{2}\gamma(b_j)^2 S^2\big\}, \end{aligned} \tag{6}$$

where the second equality follows from (1). By (6),

$$e(a_j, b_j, e_{-j}) = \arg\max_{e \ge 0} \big\{-\psi e^{\delta} + b_j \Lambda(e, e_{-j})\big\}. \tag{7}$$

By (7), the optimal effort exerted by the worker depends only on his pay-performance sensitivity b_j and the anticipated effort of other workers. It follows that in Nash equilibrium of the game for a given set of observable contracts offered to all workers, each worker's effort only depends on the pay-performance sensitivities of all workers. For future reference, let

$$e_j \equiv e_j(b_1, \ldots, b_N), \tag{8}$$

denote the *equilibrium* effort exerted by worker j given the pay-performance sensitivities, $(b_1, \ldots, b_N)$, of all workers. The corresponding output generated by the workers is $\Lambda(e_1, \ldots, e_N)$. In equilibrium, all workers correctly anticipate the effort choices, $(e_1, \ldots, e_N)$, of the team members and, therefore, the equilibrium output, $\Lambda(e_1, \ldots, e_N)$, they generate.

The manager's payoff is the project's payoff net of the workers' compensation and monitoring costs. Consequently, the expected payoff of the manager is

$$\tilde{V} = E\Bigg(\overbrace{P}^{\substack{\text{project}\\ \text{payoff}}} - \overbrace{\sum_{j=1}^{N}[a_j + b_j P]}^{\substack{\text{worker}\\ \text{compensation}}} - \overbrace{\mathcal{M}(N)P}^{\substack{\text{monitoring}\\ \text{costs}}}\Bigg). \tag{9}$$

The manager chooses the team size and a feasible set of contracts for the workers to maximize her expected payoff. Given the number of workers, N, the optimal contractual parameters for the workers must maximize the manager's expected payoff; that is,

$$\{a_j^*(N), b_j^*(N)\} \equiv \operatorname*{arg\,max}_{\{a_j, b_j\}} E\Bigg(\Bigg[\Lambda(e_1, \ldots, e_N) + \epsilon - \sum_{j=1}^{N}[a_j + b_j P] - \mathcal{M}(N)P\Bigg]\Bigg). \tag{10}$$

We normalize each worker's reservation payoff to zero so that the above optimization program is subject to the following participation constraints for the workers:

$$\overbrace{\left(a_j + b_j P - \psi(e_j)^{\delta} - \tfrac{1}{2}\gamma(b_j)^2 S^2\right)}^{\substack{\text{certainty equivalent contractual}\\ \text{payoff of worker}}} \geq 0, \quad j = 1, \ldots, N. \tag{11}$$

As noted above, the left-hand side of the constraint is a worker's certainty equivalent payoff from his contract. The constraint, which must be binding in equilibrium, determines the cash components $(a_1, \ldots, a_N)$ of the workers' compensation.

Let $V(N)$ denote the manager's maximum expected payoff for a given team size, N; that is, $V(N)$ is the manager's expected payoff when when the workers' contractual parameters are optimally chosen as in (10). The optimal size of the production team is then given by

$$N^* = \operatorname*{arg\,max}_{N} V(N). \tag{12}$$

3. The Equilibrium

Since workers are identical, we focus on the symmetric equilibrium in which all workers receive identical contracts and exert the same effort. The following proposition describes the determination of the optimal team size and the pay-performance sensitivity/incentive intensity of each worker's contract. We provide the proofs of selected propositions and lemmas in Appendix A.

Proposition 1 (Optimal Team Size and Incentives). *The optimal size of the production team, N^*, and the pay-performance sensitivity, b^*, of each worker are given by*

$$(N^*, b^*) \equiv \operatorname*{arg\,max}_{N, b} \Phi(N, b), \tag{13}$$

where

$$\Phi(N, b) = \overbrace{AN^{\chi} b^{\tau-1}}^{\text{team output}} - \overbrace{A\mathcal{M}(N)N^{\chi} b^{\tau-1}}^{\text{monitoring costs}} - \overbrace{BN^{\chi} b^{\tau}}^{\substack{\text{costs of effort}\\ \text{provision}}} - \overbrace{\tfrac{1}{2}\gamma N S^2 b^2}^{\substack{\text{costs of}\\ \text{risk sharing}}}, \tag{14}$$

where

$$\chi = \frac{\delta - \alpha}{\alpha(\delta - 1)}, \quad \tau = \frac{\delta}{\delta - 1}, \tag{15}$$

$$A = \Theta^{\delta/(\delta-1)}\left(\frac{1}{\delta\psi}\right)^{1/(\delta-1)}, \quad B = \Theta^{\delta/(\delta-1)}\psi\left(\frac{1}{\delta\psi}\right)^{\delta/(\delta-1)}. \tag{16}$$

By (13) and (14), we see that the optimal team size and pay-performance sensitivity of each worker, (N^*, b^*), jointly maximize the manager's expected payoff, $\Phi(N, b)$. As indicated in (14), the manager's expected payoff—the objective function—has four components. The first component is the total output generated by the team. The second component represents the manager's monitoring costs. The third component is the cost of effort provision by workers. The final component represents the costs of risk sharing between the risk-neutral manager and the risk-averse workers.

Let

$$\rho(N) = 1 - \mathcal{M}(N). \tag{17}$$

We see that $\rho(N)$ is strictly decreasing in N. We make the following additional assumption on the function $\rho(\cdot)$ to ensure the existence of an interior solution to the manager's optimization program.

Assumption 1. *The function $\rho(N)N^{\chi}$ is strictly quasi-concave, twice differentiable, and uniformly bounded for $N \in (0, \infty)$.*

An example of a function $\rho(\cdot)$ that satisfies Assumption 1 is $\rho(N) = \Gamma\xi^N$, where Γ, ξ are positive constants with $\xi < 1$.[3]

By (14), $\Phi(N, b) \to 0$ as $N \to 0$ or $b \to 0$. By (15), $\tau = \delta/(\delta - 1) > 1$ and $\tau - 1 = 1/(\delta - 1) < 1$ because $\delta > 2$. It immediately follows that for N fixed, $\lim_{b\to\infty} \Phi(N, b) = -\infty$. For b fixed, Assumption 1 ensures that $\lim_{N\to\infty} \Phi(N, b) = -\infty$. By the above discussion, the function $\Phi(N, b)$ attains an interior global maximum at (N^*, b^*). To avoid "integer" issues that complicate the notation without altering our main insights, we hereafter assume that the team size N takes continuous values so that the first-order conditions, $\partial\Phi(N^*, b^*)/\partial b = 0$; $\partial\Phi(N^*, b^*)/\partial N = 0$, must be satisfied. The second-order conditions for a global maximum hold for generic parameter values.[4] For our subsequent analysis, we assume that they hold strictly at (N^*, b^*) and that (N^*, b^*) varies smoothly with the model parameters.

Before proceeding to derive our results, we describe the economic forces that determine the optimal size of the production team. First, increasing the number of workers increases the team's output (the first term in (14)). Increasing the number of workers, however, also increases monitoring costs as a proportion of output; the costs due to workers' effort provision; and the costs of risk sharing (the second, third, and fourth terms in (14)). Our results are primarily driven by the effects of the first two terms in (14), so we focus on them for clarity.

Because output is concave in workers' effort choices, increasing the number of workers increases free-rider problems in that the marginal contribution of an additional worker as a proportion of the team's output declines. As expected monitoring costs scale with output, the marginal cost of an additional worker as a proportion of output also declines with the team size, but the decline is less than that of the proportional marginal benefit because expected monitoring costs are a proportion (less than one) of output. The net proportional marginal benefit of an additional worker, therefore, declines with the number of workers. At the optimal team size, the net proportional marginal benefit of an additional worker is zero.

4. Main Results

In this section, we derive the main implications of the model. The optimal team size and the performance sensitivity of each worker together solve (13).

4.1. Effects of Risk

We first explore the effects of the project's risk, S, on the optimal team size, the incentives of the workers, and the output of the team.

Proposition 2 (Risk and Team Size). *The optimal team size increases with the project risk, S.*

Recall from our earlier discussion that the net proportional marginal benefit of an additional worker is zero at the optimal team size. The variation in the team size with the project's risk depends on the relative effects of changes in the project's risk on the net proportional marginal benefit of an additional worker. Consider the effects of an increase in the project's risk. Keeping the team size fixed, an increase in the project's risk increases the costs of risk sharing, thereby decreasing the effort exerted by each worker and, therefore, the team's output. Because output is concave in workers' effort, and output declines with risk, the marginal benefit of an additional worker as a proportion of output increases with risk. The marginal cost of an additional worker as a proportion of output due to monitoring costs also increases with risk, but the increase is less than the increase in the proportional marginal benefit because expected monitoring costs are a proportion (less than one) of output. Consequently, the net marginal benefit of an additional worker as a proportion of output increases with the project's risk. In other words, as risk increases, the decline in the team's output makes costs arising from free-rider effects and monitoring costs to deter expropriation less significant at the margin so that it is optimal to increase the team size.

To further clarify the intuition above, we turn to an illustrative numerical example. We consider two different project risk levels, S_1 and S_2. For each risk level, we compute the proportional marginal benefit and cost of an additional worker. The "benefit" of workers when the project's risk is S_i, $i = 1, 2$, is given by the first term in (14) with the incentive intensity set to the optimal incentive intensity, that is, $AN(S_i)^\chi b_i^*(N(S_i))^{\tau-1}$. Consequently, the marginal benefit of an additional worker as a proportion of output is

$$\text{Proportional marginal benefit} = \frac{AN(S_i)^\chi b_i^*(N(S_i)+1)^{\tau-1} - AN(S_i)^\chi b_i^*(N(S_i))^{\tau-1}}{AN(S_i)^\chi b_i^*(N(S_i))^{\tau-1}}. \quad (18)$$

Note that in the numerator of the expression on the right-hand side above, the optimal incentive intensity, $b_i^*(N(S_i)+1)$, when there are $N(S_i)+1$ workers differs from the optimal incentive intensity, $b_i^*(N(S_i))$, when there are $N(S_i)$ workers. The "cost" of workers is given

[3] In this case, each worker's bargaining power, $v(N) = (1-\Gamma\xi^N)/N$, is, in general, nonmonotonic in N. In our microfoundation for workers' bargaining power in Appendix B, $v(N)$ could, in general, be increasing, decreasing, or nonmonotonic over the feasible range of values of N.

[4] More precisely, the subset of parameter values for which the second-order conditions do not hold strictly has measure zero.

Table 1 Parameter Values for Numerical Example

α	δ	ψ	γ	S_1	S_2	ξ	Γ
0.5	2.1	10	2	0.5	5.0	0.9	1

by the sum of the second, third, and fourth terms in (14) with the incentive intensity set to $b_i^*(N(S_i))$; that is,

$$\begin{aligned}\mathbb{C}(N(S_i)) &= A\Psi(N(S_i))N(S_i)^\chi b_i^*(N(S_i))^{\tau-1}\\&\quad+BN(S_i)^\chi b_i^*(N(S_i))^\tau+\tfrac{1}{2}\gamma N(S_i)S_i^2 b_i^*(N(S_i))^2\\&= A(1-\rho(N(S_i)))N(S_i)^\chi b_i^*(N(S_i))^{\tau-1}\\&\quad+BN(S_i)^\chi b_i^*(N(S_i))^\tau+\tfrac{1}{2}\gamma N(S_i)S_i^2 b_i^*(N(S_i))^2.\end{aligned}$$

Consequently, the marginal cost of an additional worker as a proportion of output is

$$\text{Proportional marginal cost} = \frac{\mathbb{C}(N(S_i)+1)-\mathbb{C}(N(S_i))}{AN(S_i)^\chi b_i^*(N(S_i))^{\tau-1}}. \tag{19}$$

We set $\rho(N)=\Gamma\xi^N$, where $\xi<1$. We plot the proportional marginal benefit and cost as a function of the number of workers at the two different risk levels. Table 1 lists the parameter values we assume in the example.

Figures 1 and 2 show the proportional marginal benefit and cost curves, respectively, at the two risk levels. As discussed earlier, we see that the proportional marginal benefit and cost at each level decline with the number of workers. Further, the proportional marginal benefit and cost at the lower risk level are, respectively, lower than the proportional marginal benefit and cost at the higher risk level.

In Figure 3, we plot the difference between the proportional marginal benefits at the high and low project risks as well as the difference between the proportional marginal costs. As the figure shows, for a fixed team size, the proportional marginal benefit and cost of an additional worker increases with risk. However, the proportional marginal benefit increases to a greater extent than the proportional marginal cost because monitoring costs are a proportion, $\mathscr{M}(N)$, of their output (compare the first and second terms in (14)). Hence, the *net* proportional marginal benefit of an additional worker increases with risk.

Figure 4 plots the *net* proportional marginal benefit (proportional marginal benefit minus cost) of an additional worker versus the team size for each project risk level. As discussed above, keeping the team size fixed, the net proportional marginal benefit increases with risk. Consequently, the optimal team size, which is the point at which the net proportional marginal benefit curve intersects the x axis, increases with risk.

By the above discussion, three key features of the model—the concavity of output in workers' effort, the presence of agency conflicts, and monitoring costs—play central roles in driving the positive effect of risk on team size. We discuss the impact of each in turn. The concavity of output ensures that the marginal benefit of an additional worker as a proportion of output increases as each worker's effort declines. The presence of costs of risk sharing arising from agency conflicts causes each worker's effort to decline with risk so

Figure 1 (Color online) Variation of Proportional Marginal Benefits with Team Size

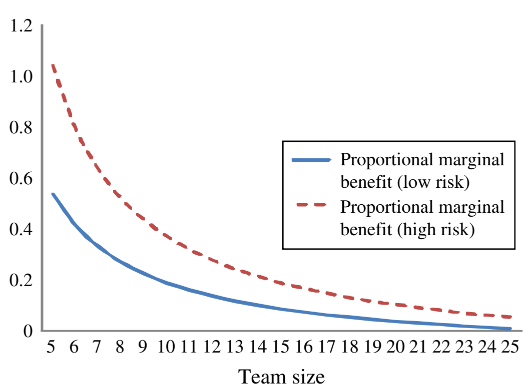


Figure 2 (Color online) Variation of Proportional Marginal Costs with Team Size

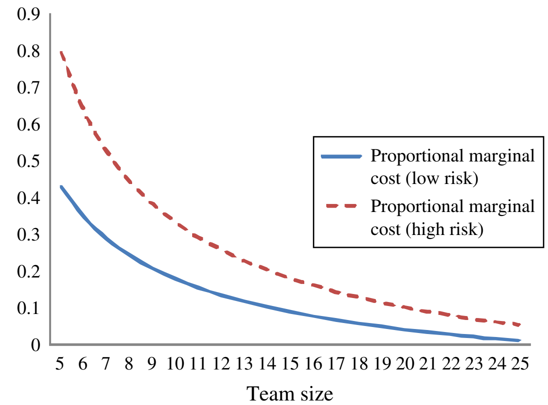


Figure 3 (Color online) Difference Between Proportional Marginal Effects: High Risk Minus Low Risk

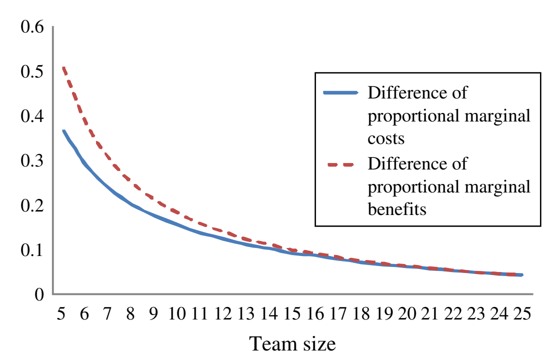
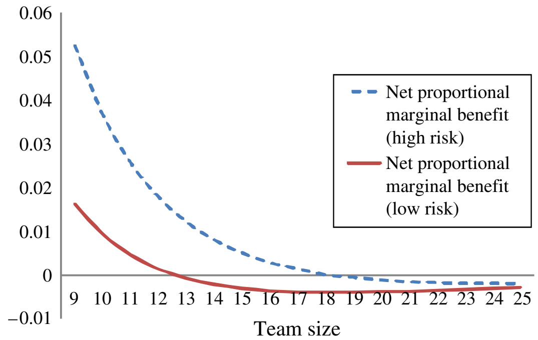


Figure 4 **(Color online) Variation of Net Proportional Marginal Benefits with Team Size**



that the proportional marginal benefit of an additional worker increases. Finally, monitoring costs are a proportion of output. Consequently, the optimal team size is determined by the intersection of the proportional marginal benefit and cost curves. Because monitoring costs are a proportion less than one of output, the *net* proportional marginal benefit of an additional worker increases with risk.

The prediction of Proposition 2, that team size increases with risk, is consistent with the combined implications of two sets of empirical findings in the literature. First, Campbell et al. (2001) document that firm-specific risk has increased over time. Wei and Zhang (2006) show that an increase in the volatility of firms' earnings explains the increase in idiosyncratic volatility. Second, Rajan and Wulf (2006) show that firms' organizational structures have flattened over time. Taken together, these findings suggest that team sizes increase with risk. The predicted positive effect of risk on team size can be more directly tested with more detailed data on the sizes of production teams and using earnings volatility as a proxy for risk.

The following lemma describes the effects of risk on the incentives of each worker and the team's output.

Lemma 1 (Risk, Incentives, and Output). *The optimal pay-performance sensitivity of each worker and the output generated by the team decline with the project risk, S.*

As discussed in the intuition for Proposition 2, an increase in the project's risk increases the costs of risk sharing and, therefore, decreases the output generated by the team in equilibrium (this follows immediately from the envelope theorem). Because the output generated by the team declines with risk, but the size of the team increases with risk, the output generated by each worker declines. This, in turn, implies that the power of incentives provided to each worker declines with risk. Note that in contrast with traditional principal–*single agent* models, the optimal team size and the incentive structures of workers are simultaneously and endogenously determined. The predicted effects of risk on incentives, therefore, do not directly follow from the results of these models.

It is useful to examine how endogenizing the team size influences the effect of risk on incentives. Suppose that the risk level is S and the corresponding optimal pay-performance sensitivity (PPS) and team size are $b^*(S)$ and $N^*(S)$, respectively. Suppose now that the risk decreases to S', but the team size is fixed at $N^*(S)$. How does the optimal PPS with the team size fixed at $N^*(S)$ compare with the *globally* optimal PPS, $b^*(S')$, when the team size also endogenously changes to reflect the lower risk level? Given that the team size is suboptimally fixed at $N^*(S)$ when the risk level declines to S', the resulting output is lower than the optimal output at the risk level S'. Because the optimal team size declines with a decline in risk by Proposition 2, the effort and output per worker are lower than the optimal effort and output per worker at the risk level S'. It follows that the optimal PPS when the team size is exogenously fixed at $N^*(S)$ is lower than the globally optimal PPS when the team size endogously decreases to the optimal team size, $N^*(S')$. In other words, endogenizing the team size *accentuates* the positive effect of a decline in risk on incentives.

Corollary 1 (Endogenous Team Size and Risk-Incentives Relation). *Suppose that $S' < S$. Then*

$$b^*(S') > b^*(N^*(S), S'),$$

where $b^*(N^*(S), S')$ is the optimal PPS of workers when the team size is fixed at $N^*(S)$, but the risk level is S'.

It is important to note here that because our model directly generalizes the classic Holmstrom and Milgrom (1987) model, it is an additive model in that the total output is additively affected by workers' effort and noise. Consequently, the relevant measure of the strength of incentives is the *dollar* sensitivity of a worker's compensation to output. In multiplicative models such as Edmans et al. (2009), Edmans and Gabaix (2011a), and Edmans and Gabaix (2011b), the relevant measure of the strength of incentives is the sensitivity of the *percentage* change in a worker's compensation to a *percentage* change in output. In the above studies, the percentage pay-performance sensitivity is not affected by risk or the agent's risk aversion but only by the cost of effort because the agent learns the shock realization before exerting effort. These studies demonstrate that the multiplicative model can more easily reconcile empirical evidence of the scaling of incentives with firm size.

Figure 5 (Color online) Variation of Proportional Marginal Benefits with Team Size

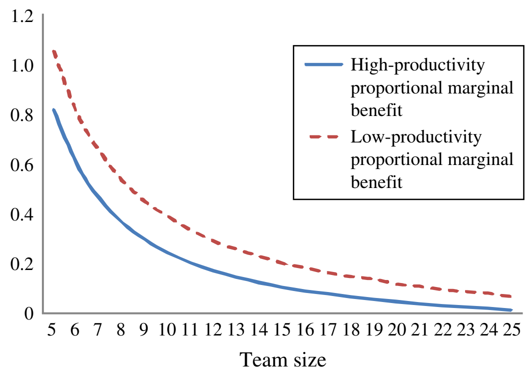
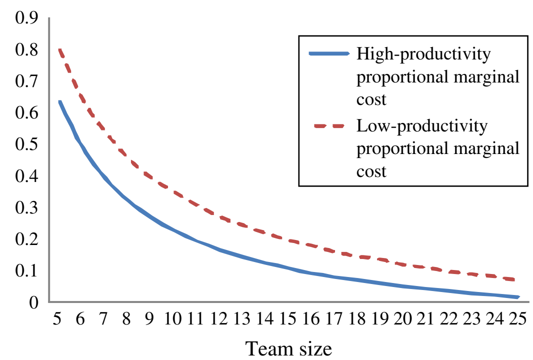


Figure 6 (Color online) Variation of Proportional Marginal Costs with Team Size



4.2. Effects of Productivity

We now investigate the effects of the project's productivity, Θ, (see (2)) on the optimal team size, Incentives, and output.

Proposition 3 (Productivity and Team Size). *The optimal team size decreases with the project's productivity,* Θ.

The intuition for Proposition 3 can be understood using arguments analogous to those underlying the intuition for Proposition 2. Keeping the team size fixed, an increase in the project's productivity increases the power of incentives that can be provided to each worker, thereby increasing the effort exerted by each worker and the output generated by the team. Because output is concave in workers' effort, and output increases with productivity, the marginal benefit of an additional worker as a proportion of output—the proportional marginal benefit—decreases with productivity. The proportional marginal cost of an additional worker due to monitoring costs also decreases with productivity, but the decrease is less than that of the proportional marginal benefit because monitoring costs are a proportion less than one of output. Consequently, the net proportional marginal benefit of an additional worker decreases with productivity. In other words, as productivity increases, the increase in the team's output makes free-rider effects and monitoring costs to deter expropriation *more significant* at the margin so that it is optimal to decrease the team size.

As in §4.1, we illustrate illustrate the intuition above by showing the effects of productivity on the proportional marginal benefit and cost of an additional worker. The parameter values are shown in Table 2.

Table 2 Parameter Values for Numerical Example

α	δ	ψ	γ	S	ξ	Γ	Θ_1	Θ_2
0.5	2.1	10	2	2	0.9	1	0.2	2

Figures 5 and 6 show the proportional marginal benefit and cost curves at two different productivity levels. Consistent with our previous discussion, the graphs show that the proportional marginal benefit and cost decline with team size. Moreover, the proportional marginal benefit and cost at the high productivity level are lower than their counterparts at the low productivity level.

Figure 7 plots the difference between the proportional marginal benefits at the high and low productivity levels as well as the difference between the proportional marginal costs. It shows that the proportional marginal benefit and cost of an additional worker decrease with the productivity level and the proportional marginal benefit decreases to a larger extent than the proportional marginal cost. The combined effects cause the net proportional marginal benefit of an additional worker to decline with productivity.

Finally, we plot the net proportional marginal benefit (the proportional marginal benefit minus cost) at the two productivity levels in Figure 8. The optimal team size, which is the intersection point of the net proportional marginal benefit curve and the x axis, is *higher* at the *lower* productivity level.

Figure 7 (Color online) Difference Between Proportional Marginal Effects: Low Productivity Minus High Productivity

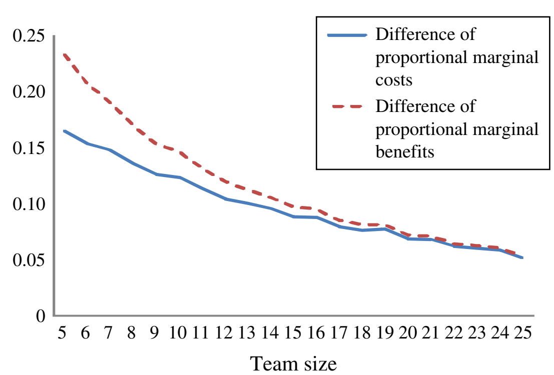
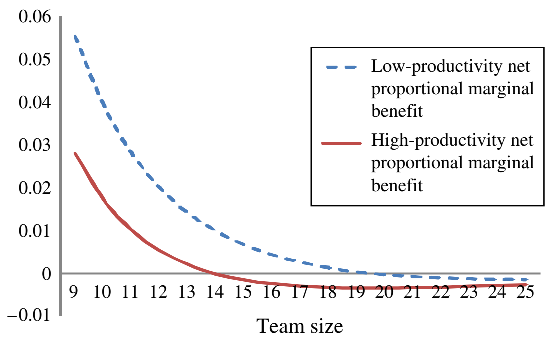


Figure 8 **(Color online) Variation of Net Proportional Marginal Benefits with Team Size**



The following lemma shows the effects of productivity on incentives and output.

Lemma 1 (Productivity, Incentives and Output). *The optimal pay-performance sensitivity of each worker and the output generated by the team increase with the project's productivity,* Θ.

As with the effects of risk on incentives and output, the impact of productivity is consistent with the predictions of principal–single agent models but is not a priori obvious because team size and incentives are simultaneously determined. Similar to the effects of risk, it is worth exploring how endogenizing the team size influences the positive effect of an increase in productivity on incentives.

Suppose that the productivity is Θ and the corresponding optimal PPS and team size are $b^*(\Theta)$ and $N^*(\Theta)$, respectively. Suppose now that the productivity increases to Θ', but the team size is *fixed* at $N^*(\Theta)$. How does the optimal PPS with the team size fixed at $N^*(\Theta)$ compare with the globally optimal PPS, $b^*(\Theta')$, when the team size also endogenously changes to reflect the higher productivity? Given that the team size is suboptimally fixed at $N^*(\Theta)$ when the productivity increases to Θ', the resulting output is lower than the globally optimal output at the productivity level Θ'. Because the optimal team size decreases with productivity by Proposition 3, the effort and output per worker are lower than the optimal effort and output per worker at the productivity level Θ'. It follows that the PPS when the team size is exogenously fixed at $N^*(\Theta)$ is lower than the PPS when the team size endogously decreases to the optimal team size, $N^*(\Theta')$. In other words, endogenizing the team size *accentuates* the positive effect of an increase in productivity on incentives.

Corollary 2 (Endogenous Team Size and Productivity-Incentives Relation). *Suppose that* $\Theta' > \Theta$. *Then*

$$b^*(\Theta') > b^*(N^*(\Theta), \Theta'),$$

where $b^*(N^*(\Theta), \Theta')$ *is the optimal PPS of workers when the team size is fixed at* $N^*(\Theta)$*, but the productivity level is* Θ'.

The implications of Propositions 3 and 2 are also consistent with empirical evidence. Wei and Zhang (2006) document that average corporate earnings have declined over time, whereas Rajan and Wulf (2006) show that firms' organizational structures have flattened. Taken together, the above findings suggest that team size declines with profitability, which is consistent with the implications of Propositions 3 and 2.

The predicted increase in the power of incentives with productivity is also supported by a number of empirical findings. For example, Wulf (2007) shows that corporate officers with broader decision-making authority have greater pay-performance sensitivities than do nonofficers, and Rajan and Wulf (2006) find that division managers who move closer to CEOs receive higher pay and greater long-term incentives. Kale et al. (2009) document evidence that CEOs receive relatively greater equity-based compensation than lower level managers. The predicted negative impact of productivity on team size can be more directly tested using detailed data on team size and using average earnings as a proxy for profitability/productivity.

4.3. Effects of Monitoring Costs

As discussed in the intuition for Proposition 2, the function $\mathcal{M}(\cdot)$, which determines the monitoring costs as a proportion of output, plays an important role in determining the optimal team size. The following proposition shows the effects of a *decrease* in $\mathcal{M}(\cdot)$ of the form

$$\tilde{\mathcal{M}}(N) = \mathcal{M}(N) - \varsigma \tag{20}$$

for all N, where $\varsigma > 0$ is fixed.

Proposition 4 (Monitoring Costs and Team Size). *A decrease in monitoring costs as a proportion of output increases the optimal team size and output.*

The effects of shifts in the monitoring cost function $\mathcal{M}(\cdot)$ are subtle because they could potentially be exploited in ways that have opposing effects on the optimal team size and worker incentives. On the one hand, a decrease in $\mathcal{M}(\cdot)$ lowers the proportion of the total output expended in monitoring costs and thereby provides incentives to increase the size of the team. The intuition for Propositions 2 and 1 then suggests that a decrease in monitoring costs as a proportion of output leads to an increase in the team size but weaker incentives for each worker. On the other hand, a decrease in monitoring costs as a proportion of output could also be exploited to provide more powerful incentives to workers because the marginal product of worker effort is greater (see (14)). The concavity of output in workers' effort would then imply that the proportional marginal benefit of an additional worker declines, which provides incentives for the manager to decrease the team size and provide workers with stronger incentives. It turns out that the former "direct" effect dominates the latter "indirect" effect in determining the number of workers so that a decrease in $\mathcal{M}(\cdot)$ increases the size of the team.

The contrasting effects of the two "forces" described above on incentives, however, cause shifts in $\mathcal{M}(\cdot)$ to have ambiguous effects on worker incentives in general. If team size were kept fixed, a decrease in monitoring costs as a proportion of output strengthens incentives because the marginal product of worker effort is greater. However, a decline in proportional monitoring costs increases the size of the team, which has a negative effect on incentives. The net effect of a shift in proportional monitoring costs on incentives, therefore, depends on which effect dominates and this, in turn, depends on the nature of the function $\mathcal{M}(\cdot)$.

Let $b^*(\varsigma)$ and $N^*(\varsigma)$ denote the optimal PPS and team size as functions of the parameter, ς, and $b^*(N, \varsigma)$ denote the optimal PPS when the team size is fixed at N. Then

$$\frac{db^*(\varsigma)}{d\varsigma} = \overbrace{\frac{\partial b^*(N^*(\varsigma), \varsigma)}{\partial \varsigma}}^{\text{direct effect}} + \overbrace{\frac{\partial b^*(N^*(\varsigma), \varsigma)}{\partial N}\frac{dN^*(\varsigma)}{d\varsigma}}^{\text{indirect effect}}.$$

The first term on the right-hand side above is the direct effect of ς on the PPS, whereas the second term is the indirect effect arising through the effect on team size. Recall from (20) that an increase in ς lowers proportional monitoring costs. Since $\partial b^*(N^*(\varsigma), \varsigma)/\partial\varsigma > 0$, $\partial b^*(N^*(\varsigma), \varsigma)/\partial N < 0$, and $dN^*(\varsigma)/d\varsigma > 0$, the direct effect is positive, whereas the indirect effect is negative. If $\partial b^*(N^*(\varsigma), \varsigma)/\partial\varsigma > |(\partial b^*(N^*(\varsigma), \varsigma)/\partial N) \cdot (dN^*(\varsigma)/d\varsigma)|$, then an increase in ς increases incentives, whereas the reverse is true if $\partial b^*(N^*(\varsigma), \varsigma)/\partial\varsigma < |(\partial b^*(N^*(\varsigma), \varsigma)/\partial N)(dN^*(\varsigma)/d\varsigma)|$. Because $N^*(\varsigma)$ and $b^*(\varsigma)$ must simultaneously satisfy (13), these conditions can be expressed in terms of the objective function Φ.

Corollary 3 (Monitoring Costs and Incentives). *The sensitivity $db^*(\varsigma)/d\varsigma > 0$ if and only if*

$$\frac{\chi}{\tau - 1}\frac{b^*(\varsigma)}{N^*(\varsigma)} < \frac{\partial^2\Phi(N^*(\varsigma), b^*(\varsigma))/\partial N^2}{\partial^2\Phi(N^*(\varsigma), b^*(\varsigma))/\partial b\partial N}.$$

Monitoring costs are likely to be lower in firms with a greater proportion of tangible assets and more easily verifiable output. In this context, the predicted decline of team size with monitoring costs is supported by recent evidence that firms with a greater proportion of intangible assets are smaller (Chen 2014). The prediction is also consistent with the arguments in Rajan and Wulf (2006) that improvements in information technology have lowered monitoring costs and led to flatter firms. Relatedly, Lee and Mullineaux (2004) find that commercial lending syndicates are smaller and more concentrated when there is little information about the borrower so that monitoring costs are high.

5. Conclusions and Empirical Implications

We analyze the impact of project characteristics on the optimal size of a production team and the incentives of its workers. The manager of the team optimally chooses its size and the workers' incentives by trading off the benefits of synergies among workers' effort choices against the detrimental impact of monitoring costs on the manager's expected payoff. Our results lead to the following novel empirical implications that relate the risk and productivity of a project as well as monitoring costs to the size of the team. (i) The optimal team size increases with the project's risk. (ii) The optimal team size decreases with the project's productivity/profitability. (iii) The optimal team size decreases with monitoring costs as a proportion of output.

The first two predictions are consistent with evidence that firm-specific volatility has increased over time and average corporate earnings have declined, whereas firms' organizational structures have flattened, suggesting an increase in team sizes. The implications are also consistent with evidence that firms typically have pyramidal organizational structures in which more productive agents are grouped in smaller teams at higher layers of the hierarchy. The predictions can be more directly tested in future empirical research using more detailed data on the sizes of production teams and using the mean and volatility of earnings as proxies for profitability and risk, respectively. The third prediction is consistent with evidence that firms with more intangible assets that are likely to have greater monitoring costs are smaller as well as with evidence that improvements in information technology that have lowered monitoring costs have also led to flatter firms.

The optimal incentive intensity of a worker's compensation contract decreases with risk and increases with productivity. The endogenous variation of team size with risk and productivity accentuates the positive impact of a decline in risk or an increase in productivity on incentives.

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Appendix A. Proofs

Proof of Proposition 1. Fix the team size N. Suppose that workers receive the contract (a, b). By (7), the effort e exerted by each worker must satisfy the following first-order conditions:

$$-\delta\psi(e)^{\delta-1} + b\frac{\partial\Lambda}{\partial e_j}\bigg|_{e_1=e_2=\cdots=e_N=e} = 0; \quad j = 1, \dots, N. \tag{A1}$$

Using (2), we obtain

$$e(b) = \left(\frac{\Theta b}{\delta\psi}\right)^{1/(\delta-1)} N^{(1-\alpha)/(\alpha(\delta-1))}, \tag{A2}$$

where we explicitly denote the dependence of the worker's effort on his pay-performance sensitivity, b. From (10) and (11), the manager optimally chooses the pay-performance sensitivities of the workers and adjusts the fixed components of their compensation so that their participation constraints are binding. To derive the optimal pay-performance sensitivity, we proceed in a manner similar to Holmstrom and Milgrom (1987) by first deriving the fixed component of a worker's compensation for a given pay-performance sensitivity. From the workers' binding participation constraints (11), we obtain

$$E[a + bP] = \tfrac{1}{2}\gamma(b)^2 S^2 + \psi e(b)^\delta. \tag{A3}$$

From the above, we obtain the fixed component, a, of a worker's compensation as a function of his pay-performance sensitivity b. Substituting (A3) in (10) and from (17) the optimal pay-performance sensitivity of the workers is

$$b^*(N) = \arg\max_b \Big\{\rho(N)\Lambda(e(b), \dots, e(b)) - N\big[\psi(e(b))^\delta + \tfrac{1}{2}\gamma S^2(b)^2\big]\Big\}.$$

From (2) and (A2), we obtain

$$\begin{aligned} b^*(N) &= \arg\max_b \left\{\rho(N)N^{1/\alpha}\Theta e(b) - N\left(\psi(e(b))^\delta + \frac{1}{2}\gamma S^2(b)^2\right)\right\} \\ &= \arg\max_b \left\{\rho(N)N^{(\delta-\alpha)/(\alpha(\delta-1))}\Theta^{\delta/(\delta-1)}\left(\frac{b}{\delta\psi}\right)^{1/(\delta-1)} - \psi N^{(\delta-\alpha)/(\alpha(\delta-1))}\left(\frac{\Theta b}{\delta\psi}\right)^{\delta/(\delta-1)} - \frac{1}{2}\gamma S^2 N b^2\right\} \\ &= \arg\max_b \left\{A\rho(N)N^\chi b^{\tau-1} - BN^\chi b^\tau - \frac{1}{2}\gamma N S^2 b^2\right\}, \end{aligned} \tag{A4}$$

where the constants, A, B, χ, and τ are given by (15) and (16). It immediately follows that the optimal size of the team and the pay-performance sensitivity of each worker solve (13). Q.E.D.

Proof of Proposition 2. We first note that the necessary first-order conditions for the optimal team size, N^*, and the pay-performance sensitivity of each worker, b^*, are

$$0 = \frac{\partial\Phi(N^*, b^*)}{\partial b} = A(\tau-1)\rho(N)(N^*)^\chi(b^*)^{\tau-2} - B\tau(N^*)^\chi(b^*)^{\tau-1} - \gamma N^* S^2(b^*), \tag{A5}$$

$$0 = \frac{\partial\Phi(N^*, b^*)}{\partial N} = [\chi\rho(N)(N^*)^{\chi-1} + \rho'(N^*)(N^*)^\chi]A(b^*)^{\tau-1} - B\chi(N^*)^{\chi-1}(b^*)^\tau - \frac{1}{2}\gamma S^2(b^*)^2. \tag{A6}$$

The optimal team size when the project risk is S^2, $N^*(S)$ and their pay-performance sensitivity, $b^*(S)$ solve (A5) and (A6). We hereafter drop the argument denoting their dependence on the project risk. We need to show that $\partial N^*/\partial(S^2) > 0$. The team size and the workers' incentive intensity are *jointly determined* by the solutions of (A5) and (A6). By (A5), (A6), and the implicit function theorem,

$$\frac{\partial N^*}{\partial(S^2)} = -\left(\frac{\partial^2\Phi}{\partial b\,\partial(S^2)}\frac{\partial^2\Phi}{\partial b\,\partial N}\bigg|_{N=N^*,\,b=b^*} - \frac{\partial^2\Phi}{\partial N\,\partial(S^2)}\frac{\partial^2\Phi}{\partial b^2}\bigg|_{N=N^*,\,b=b^*}\right) \cdot \left(\left(\frac{\partial^2\Phi}{\partial b\,\partial N}\bigg|_{N=N^*,\,b=b^*}\right)^2 - \frac{\partial^2\Phi}{\partial b^2}\frac{\partial^2\Phi}{\partial N^2}\bigg|_{N=N^*,\,b=b^*}\right)^{-1}. \tag{A7}$$

The denominator in the fraction on the right-hand side above is negative by the second-order conditions for the maximum of the function Φ. Recall that, by Assumption 1 and the subsequent discussion, Φ is guaranteed to have a maximum and the second-order conditions hold strictly for generic parameter values.

To establish that $\partial N^*/\partial(S^2) > 0$, we need to show that

$$\frac{\partial^2\Phi}{\partial b\,\partial(S^2)}\frac{\partial^2\Phi}{\partial b\,\partial N}\bigg|_{N=N^*,\,b=b^*} - \frac{\partial^2\Phi}{\partial N\,\partial(S^2)}\frac{\partial^2\Phi}{\partial b^2}\bigg|_{N=N^*,\,b=b^*} > 0. \tag{A8}$$

By (14),

$$\begin{aligned} \frac{\partial^2\Phi}{\partial b\,\partial(S^2)}(N^*, b^*) &= -\gamma N^* b^*, \\ \frac{\partial^2\Phi}{\partial N\,\partial(S^2)}(N^*, b^*) &= -\frac{1}{2}\gamma(b^*)^2. \end{aligned} \tag{A9}$$

Plugging the above into (A8), it suffices to establish that

$$-N^*b^*\frac{\partial^2\Phi}{\partial b\,\partial N}(N^*, b^*) > -\frac{1}{2}(b^*)^2\frac{\partial^2\Phi}{\partial b^2}(N^*, b^*). \tag{A10}$$

By (14) and (A5), we have

$$b^*\frac{\partial\Phi}{\partial b}(N^*, b^*) = A(\tau-1)\rho(N^*)(N^*)^\chi(b^*)^{\tau-1} - B\tau(N^*)^\chi(b^*)^\tau - \gamma N^*(S^2)(b^*)^2, \tag{A11}$$

$$N^*\frac{\partial\Phi}{\partial N}(N^*, b^*) = A\chi\rho(N^*)(N^*)^\chi(b^*)^{\tau-1} - B\chi(N^*)^\chi(b^*)^\tau - \frac{1}{2}\gamma N^* S^2(b^*)^2 + A\rho'(N^*)(N^*)^{\chi+1}(b^*)^{\tau-1}. \tag{A12}$$

Next, we note that

$$\begin{aligned} -N^*b^*\frac{\partial^2\Phi}{\partial N\,\partial b}(N^*, b^*) &= -A\chi(\tau-1)\rho(N^*)(N^*)^\chi(b^*)^{\tau-1} \\ &\quad + B\chi\tau(N^*)^\chi(b^*)^\tau + \gamma N^*(S^2)(b^*)^2 \\ &\quad - A(\tau-1)\rho'(N^*)(N^*)^{\chi+1}(b^*)^{\tau-1}. \end{aligned} \tag{A13}$$

By (A5), $(\partial\Phi/\partial b)(N^*, b^*) = 0$ so that the right-hand side of (A11) is zero. Using this fact to simplify the right-hand side of (A13), we obtain

$$-N^* b^* \frac{\partial^2\Phi}{\partial N\partial b}(N^*, b^*) = -A(\tau-1)\rho'(N^*)(N^*)^{\chi+1}(b^*)^{\tau-1} - (\chi-1)\gamma N^* S^2(b^*)^2. \tag{A14}$$

From (A11), we have

$$\begin{aligned}-(b^*)^2\frac{\partial^2\Phi}{\partial b^2}(N^*, b^*) &= -A(\tau-1)(\tau-2)\rho(N^*)(N^*)^{\chi}(b^*)^{\tau-1} \\ &\quad + B\tau(\tau-1)(N^*)^{\chi}(b^*)^{\tau} + \gamma N^* S^2(b^*)^2 \\ &= -A(\tau-1)^2\rho(N^*)(N^*)^{\chi}(b^*)^{\tau-1} \\ &\quad + A(\tau-1)\rho(N^*)(N^*)^{\chi}(b^*)^{\tau-1} \\ &\quad + B\tau(\tau-1)(N^*)^{\chi}(b^*)^{\tau} \\ &\quad + \gamma N^* S^2(b^*)^2. \end{aligned} \tag{A15}$$

Since $(\partial\Phi/\partial b)(N^*, b^*) = 0$ by (A5), it follows from (A11) that

$$-A(\tau-1)^2\rho(N^*)(N^*)^{\chi}(b^*)^{\tau-1} + B\tau(\tau-1)(N^*)^{\chi}(b^*)^{\tau} = -\gamma(\tau-1)N^* S^2(b^*)^2.$$

Substituting the above in (A15), we have

$$-(b^*)^2\frac{\partial^2\Phi}{\partial b^2}(N^*, b^*) = A(\tau-1)\rho(N^*)(N^*)^{\chi}(b^*)^{\tau-1} - \gamma(\tau-2)N^* S^2(b^*)^2. \tag{A16}$$

To establish (A10), it follows from (A14) and (A16) that it suffices to establish that

$$\begin{aligned}&-A(\tau-1)\rho'(N^*)(N^*)^{\chi+1}(b^*)^{\tau-1} - (\chi-1)\gamma N^* S^2(b^*)^2 \\ &> \tfrac{1}{2}A(\tau-1)\rho(N^*)(N^*)^{\chi}(b^*)^{\tau-1} - \tfrac{1}{2}\gamma(\tau-2)N^* S^2(b^*)^2. \end{aligned} \tag{A17}$$

By (A6), $\partial\Phi/\partial N = 0$ so that

$$\begin{aligned}&-A(\tau-1)\rho'(N^*)(N^*)^{\chi+1}(b^*)^{\tau-1} \\ &= A\chi(\tau-1)\rho(N^*)(N^*)^{\chi}(b^*)^{\tau-1} - B\chi(\tau-1)(N^*)^{\chi}(b^*)^{\tau} \\ &\quad - \frac{(\tau-1)}{2}\gamma N^* S^2(b^*)^2. \end{aligned} \tag{A18}$$

Substituting (A18) in (A17) and simplifying, we need to show that

$$\begin{aligned}&A\chi(\tau-1)\rho(N^*)(N^*)^{\chi}(b^*)^{\tau-1} - B\chi(\tau-1)(N^*)^{\chi}(b^*)^{\tau} \\ &\quad - \left(\chi - 1 + \frac{\tau-1}{2}\right)\gamma N^* S^2(b^*)^2 \\ &> \frac{1}{2}A(\tau-1)\rho(N^*)(N^*)^{\chi}(b^*)^{\tau-1} - \frac{1}{2}\gamma(\tau-2)N^* S^2(b^*)^2 \end{aligned} \tag{A19}$$

or

$$\begin{aligned}&A\left(\chi-\tfrac{1}{2}\right)(\tau-1)\rho(N^*)(N^*)^{\chi}(b^*)^{\tau-1} \\ &\quad - B\chi(\tau-1)(N^*)^{\chi}(b^*)^{\tau} - \left(\chi-\tfrac{1}{2}\right)\gamma N^* S^2(b^*)^2 > 0. \end{aligned} \tag{A20}$$

Because the right-hand side of (A11) is zero by (A5), we have

$$\begin{aligned}&A\left(\chi-\tfrac{1}{2}\right)(\tau-1)\rho(N^*)^{\tau}(N^*)^{\chi}(b^*)^{\tau-1} - \left(\chi-\tfrac{1}{2}\right)\gamma N^* S^2(b^*)^2 \\ &= B\left(\chi-\tfrac{1}{2}\right)\tau(N^*)^{\chi}(b^*)^{\tau}. \end{aligned} \tag{A21}$$

Substituting (A21) in (A20), it remains to show that

$$\left(\chi-\tfrac{\tau}{2}\right)B(N^*)^{\chi}(b^*)^{\tau} > 0, \tag{A22}$$

which will follow if $\chi > \tau/2$. By (15), however, $\chi > \tau/2$ iff $2\delta - 2\alpha > \alpha\delta$ or $\delta > 2\alpha/(2-\alpha)$, which is true because $\delta > 2$ and $\alpha \le 1$. This establishes the assertion.

Proof of Lemma 1. We need to show that $\partial b^*/\partial(S^2) < 0$. By (A6), (A5), and the implicit function theorem,

$$\begin{aligned}\frac{\partial b^*}{\partial(S^2)} &= -\left(\frac{\partial^2\Phi}{\partial b\partial(S)^2}\frac{\partial^2\Phi}{\partial N^2}\bigg|_{(N^*,\,b^*)} - \frac{\partial^2\Phi}{\partial N\partial(S)^2}\frac{\partial^2\Phi}{\partial b\partial N}\bigg|_{(N^*,\,b^*)}\right) \\ &\quad \cdot\left(\frac{\partial^2\Phi}{\partial b^2}\frac{\partial^2\Phi}{\partial N^2}\bigg|_{(N^*,\,b^*)} - \left(\frac{\partial^2\Phi}{\partial b\partial N}\bigg|_{(N^*,\,b^*)}\right)^2\right)^{-1}. \end{aligned} \tag{A23}$$

The denominator in the fraction on the right-hand side above is positive by the second-order conditions for the maximum of the function Φ. We need to show that

$$\frac{\partial^2\Phi}{\partial b\partial(S)^2}\frac{\partial^2\Phi}{\partial N^2}(N^*, b^*) - \frac{\partial^2\Phi}{\partial N\partial(S)^2}\frac{\partial^2\Phi}{\partial b\partial N}(N^*, b^*) > 0.$$

By (A9), it suffices to establish that

$$-(N^*)\frac{\partial^2\Phi}{\partial N^2}(N^*, b^*) > -\frac{1}{2}(b^*)\frac{\partial^2\Phi}{\partial b\partial N}(N^*, b^*). \tag{A24}$$

In the proof of part (i), we have established (A10). By the second-order condition for the maximum of the function Φ,

$$\frac{\partial^2\Phi}{\partial b^2}\frac{\partial^2\Phi}{\partial N^2}(N^*, b^*) - \left(\frac{\partial^2\Phi}{\partial b\partial N}(N^*, b^*)\right)^2 > 0. \tag{A25}$$

From (A10), we know that

$$-N^*\frac{\partial^2\Phi}{\partial b\partial N}(N^*, b^*) > -\frac{1}{2}(b^*)\frac{\partial^2\Phi}{\partial b^2}(N^*, b^*),$$

so that we must have

$$-b^*\frac{\partial^2\Phi}{\partial b\partial N}(N^*, b^*) < -2(N^*)\frac{\partial^2\Phi}{\partial N^2}(N^*, b^*) \tag{A26}$$

for (A25) to hold. But (A26) is equivalent to (A24). It follows that $\partial b^*/\partial(S^2) < 0$. It immediately follows from (14) and the envelope theorem that $\partial\Phi/\partial(S^2) < 0$. Consequently, the surplus generated by workers decreases with project risk. Q.E.D.

Proof of Proposition 3. The optimal team size when the productivity is Θ, $N^*(\Theta)$ and their pay-performance sensitivity, $b^*(\Theta)$ solve (A5) and (A6). In (16), define

$$\Delta = \Theta^{\delta/(\delta-1)}. \tag{A27}$$

We need to show that $\partial N^*/\partial\Delta < 0$. We have

$$\begin{aligned}\frac{\partial N^*}{\partial\Delta} &= -\left(\frac{\partial^2\Phi}{\partial b\partial\Delta}\frac{\partial^2\Phi}{\partial b\partial N}\bigg|_{N=N^*,\,b=b^*} - \frac{\partial^2\Phi}{\partial N\partial\Delta}\frac{\partial^2\Phi}{\partial b^2}\bigg|_{N=N^*,\,b=b^*}\right) \\ &\quad \cdot\left(\left(\frac{\partial^2\Phi}{\partial b\partial N}\bigg|_{N=N^*,\,b=b^*}\right)^2 - \frac{\partial^2\Phi}{\partial b^2}\frac{\partial^2\Phi}{\partial N^2}\bigg|_{N=N^*,\,b=b^*}\right)^{-1} \end{aligned} \tag{A28}$$

The denominator in the fraction on the right-hand side above is negative by the second-order conditions for the maximum of the function Φ. To establish that $\partial N^*/\partial\Delta < 0$, we need to show that

$$\frac{\partial^2\Phi}{\partial b\partial\Delta}\frac{\partial^2\Phi}{\partial b\partial N}\bigg|_{N=N^*,\,b=b^*} - \frac{\partial^2\Phi}{\partial N\partial\Delta}\frac{\partial^2\Phi}{\partial b^2}\bigg|_{N=N^*,\,b=b^*} < 0. \tag{A29}$$

By (14), (16), and (A27)

$$\begin{aligned}\frac{\partial^2\Phi}{\partial b\partial\Delta}(N^*,b^*) &= \frac{1}{\Delta}A(\tau-1)\rho(N^*)(N^*)^\chi(b^*)^{\tau-2}\\ &\quad -\frac{1}{\Delta}B\tau(N^*)^\chi(b^*)^{\tau-1}, \qquad \text{(A30)}\\ \frac{\partial^2\Phi}{\partial N\partial\Delta}(N^*,b^*) &= \frac{1}{\Delta}A\chi\rho(N^*)(N^*)^{\chi-1}(b^*)^{\tau-1}\\ &\quad +\frac{1}{\Delta}A\rho'(N^*)(N^*)^\chi(b^*)^{\tau-1}\\ &\quad -\frac{1}{\Delta}B\chi(N^*)^{\chi-1}(b^*)^\tau. \qquad \text{(A31)}\end{aligned}$$

By (A5) and (A6), we have

$$\begin{aligned}\frac{\partial^2\Phi}{\partial b\partial\Delta}(N^*,b^*) &= \gamma\frac{S^2}{\Delta}N^*b^*,\\ \frac{\partial^2\Phi}{\partial N\partial\Delta}(N^*,b^*) &= \frac{1}{2}\gamma\frac{S^2}{\Delta}(b^*)^2.\end{aligned} \tag{A32}$$

Plugging the above into (A29), it suffices to establish that

$$-N^*b^*\frac{\partial^2\Phi}{\partial b\partial N}(N^*,b^*) > -\frac{1}{2}(b^*)^2\frac{\partial^2\Phi}{\partial b^2}(N^*,b^*). \tag{A33}$$

The above is, however, identical to (A10) that we have already established in the proof of Proposition 2. This completes the proof. Q.E.D.

Proof of Lemma 2. We need to show that $\partial b^*/\partial\Delta > 0$. By (A6), (A5), and the implicit function theorem,

$$\begin{aligned}\frac{\partial b^*}{\partial\Delta} &= -\bigg(\frac{\partial^2\Phi}{\partial b\partial\Delta}\frac{\partial^2\Phi}{\partial N^2}\bigg|_{(N^*,b^*)} - \frac{\partial^2\Phi}{\partial N\partial\Delta}\frac{\partial^2\Phi}{\partial b\partial N}\bigg|_{(N^*,b^*)}\bigg)\\ &\quad\cdot\bigg(\frac{\partial^2\Phi}{\partial b^2}\frac{\partial^2\Phi}{\partial N^2}\bigg|_{(N^*,b^*)} - \bigg(\frac{\partial^2\Phi}{\partial b\partial N}\bigg|_{(N^*,b^*)}\bigg)^2\bigg)^{-1}.\end{aligned} \tag{A34}$$

The denominator in the fraction on the right-hand side above is positive by the second-order conditions for the maximum of the function Φ. We need to show that

$$\frac{\partial^2\Phi}{\partial b\partial\Delta}\frac{\partial^2\Phi}{\partial N^2}(N^*,b^*) - \frac{\partial^2\Phi}{\partial N\partial\Delta}\frac{\partial^2\Phi}{\partial b\partial N}(N^*,b^*) < 0.$$

By (A32), it suffices to establish that

$$-(N^*)\frac{\partial^2\Phi}{\partial N^2}(N^*,b^*) > -\frac{1}{2}(b^*)\frac{\partial^2\Phi}{\partial b\partial N}(N^*,b^*). \tag{A35}$$

The above is, however, identical to (A24) that we have already established in the proof of Proposition 1. Q.E.D.

Proof of Proposition 4. Consider a decrease in the monitoring cost function, $\tilde{\mathcal{M}}(N) = \mathcal{M}(N) - \varsigma$, where $\varsigma > 0$, which corresponds to an increase in the function $\rho(N) = 1 - \mathcal{M}(N)$ defined in (17); that is,

$$\tilde\rho(N) = \rho(N) + \varsigma.$$

It suffices to show that

$$\frac{\partial N^*}{\partial\varsigma} > 0, \quad \frac{\partial\Phi}{\partial\varsigma} > 0. \tag{A36}$$

By (A6), (A5), and the implicit function theorem,

$$\begin{aligned}\frac{\partial N^*}{\partial\varsigma} &= -\bigg(\frac{\partial^2\Phi}{\partial b\partial\varsigma}\frac{\partial^2\Phi}{\partial b\partial N}\bigg|_{(N^*,b^*)} - \frac{\partial^2\Phi}{\partial N\partial\varsigma}\frac{\partial^2\Phi}{\partial b^2}\bigg|_{(N^*,b^*)}\bigg)\\ &\quad\cdot\bigg(\bigg(\frac{\partial^2\Phi}{\partial b\partial N}\bigg|_{(N^*,b^*)}\bigg)^2 - \frac{\partial^2\Phi}{\partial b^2}\frac{\partial^2\Phi}{\partial N^2}\bigg|_{(N^*,b^*)}\bigg)^{-1}.\end{aligned} \tag{A37}$$

The denominator of the expression on the right-hand side above is negative by the second-order condition for the maximum of the function Φ. Consequently, to show that $\partial N^*/\partial\varsigma > 0$, we need to show that

$$\frac{\partial^2\Phi}{\partial b\partial\varsigma}\frac{\partial^2\Phi}{\partial b\partial N}\bigg|_{(N^*,b^*)} - \frac{\partial^2\Phi}{\partial N\partial\varsigma}\frac{\partial^2\Phi}{\partial b^2}\bigg|_{(N^*,b^*)} > 0. \tag{A38}$$

By (14),

$$\begin{aligned}\frac{\partial\Phi}{\partial b}(N^*,b^*) &= A(\tau-1)\tilde\rho(N^*)(N^*)^\chi(b^*)^{\tau-2} - B\tau(N^*)^\chi(b^*)^{\tau-1}\\ &\quad -\gamma N^*S^2(b^*)\\ &= A(\tau-1)[\rho(N^*)+\varsigma](N^*)^\chi(b^*)^{\tau-2}\\ &\quad -B\tau(N^*)^\chi(b^*)^{\tau-1} - \gamma N^*S^2(b^*)\end{aligned} \tag{A39}$$

so that

$$\frac{\partial^2\Phi}{\partial b\partial\varsigma}(N^*,b^*) = A(\tau-1)(N^*)^\chi(b^*)^{\tau-2}. \tag{A40}$$

By (14),

$$\begin{aligned}\frac{\partial\Phi}{\partial N}(N^*,b^*) &= A\chi\tilde\rho(N^*)(N^*)^{\chi-1}(b^*)^{\tau-1} - B\chi(N^*)^{\chi-1}(b^*)^\tau\\ &\quad -\frac{1}{2}\gamma S^2(b^*)^2 + A\tilde\rho'(N^*)(N^*)^\chi(b^*)^{\tau-1}\\ &= A\chi[\rho(N^*)+\varsigma](N^*)^{\chi-1}(b^*)^{\tau-1} - B\chi(N^*)^{\chi-1}(b^*)^\tau\\ &\quad -\frac{1}{2}\gamma S^2(b^*)^2 + A\rho'(N^*)(N^*)^\chi(b^*)^{\tau-1}\end{aligned}$$

so that

$$\frac{\partial^2\Phi}{\partial N\partial\varsigma}(N^*,b^*) = A\chi(N^*)^{\chi-1}(b^*)^{\tau-1}. \tag{A41}$$

Plugging (A40) and (A41) into (A38), we need to show that

$$\begin{aligned}&A(\tau-1)(N^*)^\chi(b^*)^{\tau-2}\frac{\partial^2\Phi}{\partial b\partial N}\bigg|_{(N^*,b^*)}\\ &\quad -A\chi(N^*)^{\chi-1}(b^*)^{\tau-1}\frac{\partial^2\Phi}{\partial b^2}\bigg|_{(N^*,b^*)} > 0.\end{aligned}$$

Dividing both sides of the above expression by $A(N^*)^{\chi-1}(b^*)^{\tau-3}$, we need to show that

$$(\tau-1)(N^*)(b^*)\frac{\partial^2\Phi}{\partial b\partial N}(N^*,b^*) - \chi(b^*)^2\frac{\partial^2\Phi}{\partial b^2}(N^*,b^*) > 0. \tag{A42}$$

Plugging (A14) and (A16) into the above and simplifying, we need to show that

$$\begin{aligned}&A(\tau-1)\chi\rho(N^*)(N^*)^\chi(b^*)^{\tau-1} + A(\tau-1)^2\rho'(N^*)(N^*)^{\chi+1}(b^*)^{\tau-1}\\ &\quad +(1+\chi-\tau)\gamma S^2(N^*)(b^*)^2 > 0.\end{aligned} \tag{A43}$$

We first note that by (15), $1+\chi>\tau$ because $\delta>2\alpha$ (as $\delta>2$ and $\alpha<1$). Hence, the third term on the left-hand side of (A43) is positive. It, therefore, suffices to show that

$$\begin{aligned}&A(\tau-1)\chi\rho(N^*)(N^*)^{\chi}(b^*)^{\tau-1}\\&\quad+A(\tau-1)^2\rho'(N^*)(N^*)^{\chi+1}(b^*)^{\tau-1}>0\quad\text{or}\\&A\chi\rho(N^*)(N^*)^{\chi}(b^*)^{\tau-1}\\&\quad+A(\tau-1)\rho'(N^*)(N^*)^{\chi+1}(b^*)^{\tau-1}>0.\end{aligned}\tag{A44}$$

By (A6) and (A12),

$$\begin{aligned}&A\chi\rho(N^*)(N^*)^{\chi}(b^*)^{\tau-1}+A\rho'(N^*)(N^*)^{\chi+1}(b^*)^{\tau-1}\\&\quad=B\chi(N^*)^{\chi}(b^*)^{\tau}+\tfrac{1}{2}\gamma N^*S^2(b^*)^2>0\end{aligned}$$

because both terms in the last expression above, $B\chi(N^*)^{\chi}(b^*)^{\tau}$ and $\frac{1}{2}\gamma N^*S^2(b^*)^2$, are positive. By (15), $\tau-1<1$ because $\delta>2$. Since $\rho'(N^*)<0$, it then follows that

$$\begin{aligned}&A\chi\rho(N^*)(N^*)^{\chi}(b^*)^{\tau-1}+A\rho'(N^*)(N^*)^{\chi+1}(b^*)^{\tau-1}>0\\&\implies A\chi\rho(N^*)(N^*)^{\chi}(b^*)^{\tau-1}\\&\qquad+A(\tau-1)\rho'(N^*)(N^*)^{\chi+1}(b^*)^{\tau-1}>0,\end{aligned}$$

which is exactly (A44). This establishes (A38) and, therefore, the assertion that $\partial N^*/\partial \varsigma>0$. Q.E.D.

Proof of Corollary 3. By (A6), (A5), and the implicit function theorem,

$$\begin{aligned}\frac{\partial b^*}{\partial\varsigma}&=-\left(\frac{\partial^2\Phi}{\partial b\partial\varsigma}\frac{\partial^2\Phi}{\partial N^2}\bigg|_{(N^*,b^*)}-\frac{\partial^2\Phi}{\partial N\partial\varsigma}\frac{\partial^2\Phi}{\partial b\partial N}\bigg|_{(N^*,b^*)}\right)\\&\quad\cdot\left(\frac{\partial^2\Phi}{\partial b^2}\frac{\partial^2\Phi}{\partial N^2}\bigg|_{(N^*,b^*)}-\left(\frac{\partial^2\Phi}{\partial b\partial N}\bigg|_{(N^*,b^*)}\right)^2\right)^{-1}.\end{aligned}\tag{A45}$$

The denominator in the fraction on the right-hand side above is positive by the second-order conditions for the maximum of the function Φ. Consequently, $\partial b^*/\partial\varsigma>0$ iff

$$\frac{\partial^2\Phi}{\partial b\partial\varsigma}\frac{\partial^2\Phi}{\partial N^2}\bigg|_{(N^*,\,b^*)}<\frac{\partial^2\Phi}{\partial N\partial\varsigma}\frac{\partial^2\Phi}{\partial b\partial N}\bigg|_{(N^*,\,b^*)}.$$

By (A40) and (A41), the above can be rewritten as

$$\begin{aligned}&A(\tau-1)(N^*)^{\chi}(b^*)^{\tau-2}\frac{\partial^2\Phi(N^*,b^*)}{\partial N^2}\\&\quad<A\chi(N^*)^{\chi-1}(b^*)^{\tau-1}\frac{\partial^2\Phi(N^*,b^*)}{\partial b\partial N}.\end{aligned}$$

Because $\partial^2\Phi/\partial N^2<0$, the above inequality is equivalent to

$$\frac{\chi}{\tau-1}\frac{b^*}{N^*}<\frac{\partial^2\Phi(N^*,b^*)/\partial N^2}{\partial^2\Phi(N^*,b^*)/\partial b\partial N}.$$

Appendix B. Partially Verifiable Output and Monitoring Costs

In the basic model, we assume that the total output P of the team is observable and verifiable by the manager if and only if she monitors the team and incurs monitoring costs. For the sake of completeness, we now extend the model to consider the scenario in which a portion of the team's output is verifiable even without manager monitoring. Specifically, each worker can take an additional action that affects the team's observed output. We refer to the action as "expropriation" for concreteness, although its interpretation can be quite general. (For example, it can be any other costly action such as a project choice that affects output.)

A worker chooses the action *after* he exerts effort. Each worker i chooses action $x_i\in\{0,1\}$. For given vectors of action choices $(x_1,\ldots,x_N)$ and effort choices $(e'_1,\ldots,e'_N)$, the total *verifiable* output if the manager does not monitor the team is

$$\text{Verifiable output}=\left(\frac{1}{N}\sum_{i=1}^{N}(1-\omega(N))x_i\right)P(e'_1,\ldots,e'_N).\tag{B1}$$

The verifiable output of the team is the only verifiable variable and, therefore, the only variable on which contracts can be contingent. To simplify the notation, we hereafter drop the argument denoting the dependence of the total output $P(e'_1,\ldots,e'_N)$ on the vector of effort choices wherever there is no danger of confusion. From the above, we note that if all the workers choose the action 1, the verifiable output is lower than the total output by the proportion $\omega(N)$, where the argument indicates that the reduction in verifiable output could depend on team size. Hence, $\omega(N)$ is the combined proportion of output that workers can expropriate in the absence of any monitoring by the manager. It is reasonable to assume that $\omega(N)$ increases with the team size N, reflecting the notion that larger teams can expropriate a greater proportion of output in the absence of any monitoring.[5] If a worker chooses $x_i=1$, he receives the output that he expropriates as a private benefit, that is,

$$\begin{aligned}&\text{Output expropriated by worker}\\&\quad=\text{Worker private benefit}=\frac{\omega(N)}{N}P.\end{aligned}\tag{B2}$$

The manager can choose to monitor the workers (after they have exerted effort) to prevent them from expropriating output. The manager cannot commit to a monitoring policy ex ante, that is, the manager chooses whether or not to monitor and prevent expropriation ex post (i.e., after workers have exerted effort) if it is in her interest to do so. The monitoring cost increases with the amount of output expropriated by workers. Specifically, given an action profile, $(x_1,\ldots,x_N)$, the amount of output expropriated by workers is $((1/N)\sum_{i=1}^{N}\omega(N)x_i)P$ as shown by (B1). The manager's monitoring cost is $\nu(N)((1/N)\sum_{i=1}^{N}\omega(N)x_i)P$, where $\nu(N)\in(0,1)$ is the marginal cost of monitoring, that is, the cost of preventing the expropriation of one unit of output. It is reasonable to assume that the marginal cost of monitoring, $\nu(N)$, increases with team size reflecting the intuition that it is more difficult to monitor as team size increases. Hence, the total monitoring costs incurred by the manager if all the workers choose to expropriate are given by $\omega(N)\nu(N)P=\mathcal{M}(N)P$, where $\mathcal{M}(N)=\omega(N)\nu(N)$ is strictly increasing in N.

We now show that if the marginal cost of monitoring, $\nu(N)$, is sufficiently less than one, it is optimal for the manager to monitor and prevent expropriation by workers in

[5] Our analysis holds even if $\omega(\cdot)$ is only weakly increasing. In particular, it could be constant.

equilibrium. Recall that the manager cannot commit to her monitoring policy ex ante. Given the linearity of the manager's payoff function, it is easy to see that the manager either chooses to monitor ex post and completely prevent expropriation by all workers or not monitor at all.

Suppose that the manager chooses not to monitor in equilibrium. Let us first show that all workers expropriate. Consider a worker i who has a contract (a, b). The worker's expected utility if he exerts effort e_i and chooses to expropriate given an action profile (x_{-i}) of the other workers is

$$E\bigg[-\exp\bigg\{-\gamma\bigg[a+b\bigg(\frac{1-\omega(N)}{N}+\frac{1}{N}\sum_{j=1,\,j\neq i}^{N}(1-\omega(N))x_j\bigg)P - \psi(e_i)^\delta+\frac{\omega(N)}{N}P\bigg]\bigg\}\bigg]$$

$$= E\bigg[-\exp\bigg\{-\gamma\bigg[a+b\overbrace{\bigg(\frac{1}{N}+\frac{1}{N}\sum_{j=1,\,j\neq i}^{N}(1-\omega(N))x_j\bigg)P}^{\text{total verifiable output if worker } i \text{ does not expropriate}} - \psi(e_i)^\delta + \overbrace{(1-b)\frac{\omega(N)}{N}P}^{\text{additional worker benefit from expropriation}}\bigg]\bigg\}\bigg].$$

Define

$$\Gamma(x_1,\ldots,x_N)=\frac{1}{N}\sum_{j=1}^{N}(1-\omega(N))x_j. \tag{B3}$$

The worker's certainty equivalent payoff if he chooses expropriation is

$$\begin{aligned} &a+b\Gamma(0,x_{-i})\Lambda-\psi(e_i)^\delta+(1-b)\frac{\omega(N)}{N}\Lambda \\ &\quad -\frac{1}{2}\gamma S^2\bigg[b\Gamma(0,x_{-i})+(1-b)\frac{\omega(N)}{N}\bigg]^2 \\ &= \overbrace{a+b\Gamma(0,x_{-i})\Lambda-\psi(e_i)^\delta-\frac{1}{2}\gamma S^2 b^2\Gamma(0,x_{-i})^2}^{\text{certainty equivalent payoff with no expropriation}} \\ &\quad +\overbrace{(1-b)\frac{\omega(N)}{N}\Lambda}^{\text{expected benefit from expropriation}} \\ &\quad \overbrace{-\frac{1}{2}\gamma S^2\bigg[2b(1-b)\Gamma(0,x_{-i})\bigg(\frac{\omega(N)}{N}\bigg)+(1-b)^2\bigg(\frac{\omega(N)}{N}\bigg)^2\bigg]}^{\text{additional cost of risk of expropriated output}}. \end{aligned}$$

From the above, we can show that if b is sufficiently less than 1 and $\gamma S^2<\Lambda$ (which are true for reasonable parameterizations of the model), it is a dominant strategy for each worker to expropriate.

Let us now investigate the workers' optimal contracts in the hypothesized "no monitoring" equilibrium. We focus on the symmetric equilibrium where all workers receive the same contract. Suppose all workers receive the contract (a, b). Define

$$b' = \overbrace{b(1-\omega(N))}^{\text{sensitivity of contractual payoff to } P} + \overbrace{\frac{\omega(N)}{N}}^{\text{sensitivity of private benefit to } P}. \tag{B4}$$

Since all workers expropriate in equilibrium, the verifiable output is $(1-\omega(N))P$ by (B1). It follows from (B2) that b' is each worker's *total sensitivity* to the total output P, that is, the sensitivity of his contractual payoff plus the sensitivity of his private benefit to the total output, P. Using arguments identical to those used to derive (A2), the optimal effort exerted by each worker is

$$e(b')=\bigg(\frac{\Theta b'}{\delta\psi}\bigg)^{1/(\delta-1)}N^{(1-\alpha)/(\alpha(\delta-1))}. \tag{B5}$$

Proceeding as in the proof of Proposition 1, and using the fact that the manager incurs no monitoring costs in the hypothesized equilibrium, we can then show that the optimal PPS of each worker's contract solves

$$\begin{aligned} \bar b(N) &= \arg\max_b\{N^{1/\alpha}\Theta e(b')-N(\psi(e(b'))^\delta+\tfrac{1}{2}\gamma S^2(b')^2)\} \\ &= \arg\max_b\{AN^\chi(b')^{\tau-1}-BN^\chi(b')^\tau-\tfrac{1}{2}\gamma NS^2(b')^2,\} \end{aligned} \tag{B6}$$

where the constants A, B, χ, and τ are as defined in (15) and (16). The manager's expected payoff in the hypothesized "no monitoring" equilibrium is, therefore,

$$\begin{aligned} Q^{\text{no monitoring}} &= AN^\chi(\bar b'(N))^{\tau-1}-BN^\chi(\bar b'(N))^\tau \\ &\quad -\tfrac{1}{2}\gamma NS^2(\bar b'(N))^2, \end{aligned} \tag{B7}$$

where $\bar b'(N)$ is related to $\bar b(N)$ by (B4).

The hypothesized "no monitoring" equilibrium is, in fact, an equilibrium if and only if it is *not* in the manager's interest to monitor the workers ex post and prevent expropriation. If the manager chooses to monitor ex post and prevent expropriation, his total expected payoff is

$$\tilde Q = Q^{\text{no monitoring}} + \overbrace{(1-N\bar b(N))\omega(N)P}^{\text{output saved from expropriation}} - \overbrace{\nu(N)\omega(N)P}^{\text{monitoring costs}}.$$

From the above, we see that, if

$$\nu(N) < 1-N\bar b(N), \tag{B8}$$

it is optimal for the manager to monitor ex post and prevent expropriation. Hence, the hypothesized "no monitoring" equilibrium is, in fact, not an equilibrium if the above condition holds.

We now examine the conditions under which it is, indeed, an equilibrium for the manager to monitor and prevent expropriation. If the manager monitors in equilibrium, each workers' optimal PPS must be $b^*(N)$ that solves (A4). It is then easy to see that it is, indeed, optimal for the manager to monitor ex post and prevent expropriation if

$$\nu(N) < 1-Nb^*(N). \tag{B9}$$

Combining (B8) and (B9), we see that the necessary and sufficient condition for the existence of a unique symmetric equilibrium that involves monitoring by the manager and no expropriation by workers is

$$\nu(N) < \min(1 - N\bar{b}(N), 1 - Nb^*(N)). \tag{B10}$$

For typical firms, the total equity stake held by owners far exceeds the stake held by employees and workers. For reasonable parametrizations, therefore, the total PPS of all workers in the absence and presence of monitoring—$N\bar{b}(N)$ and $Nb^*(N)$, respectively—would be significantly less than one. Consequently, as asserted earlier, if the marginal cost of monitoring, $\nu(N)$, is sufficiently less than one, the manager monitors and prevents expropriation in equilibrium.

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