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Consignment Contracts with Revenue Sharing for a Capacitated Retailer and Multiple Manufacturers

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We consider a retailer with limited storage capacity selling n independent products. Each product is produced by a distinct manufacturer, who is offered a consignment contract with revenue sharing by the retailer. The retailer first sets a common revenue share for all products, and each manufacturer then determines the retail price and production quantity for his product. Under certain conditions on price elasticities and cost fractions, we find a unique optimal revenue share for all products. Surprisingly, it is optimal for the retailer not to charge any storage fee in many situations even if she is allowed to do so. Both the retailer's and manufacturers' profits first increase and then remain constant as the capacity increases, which implies that an optimal capacity exists. We also find that the decentralized system requires no larger storage space than the centralized system at the expense of channel profit. If products are complementary, as the degree of complementarity increases, the retailer will decrease her revenue share to encourage the manufacturers to lower their prices.

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1. Introduction

We consider a retailer with limited storage space selling n independent products over a single period. The total demand for each product over the selling period is price sensitive and uncertain. Each product is produced by a distinct manufacturer before the start of the selling period. The retailer offers a consignment contract with revenue sharing to each manufacturer. Under each contract, the ownership of a product belongs to its manufacturer when it is stored in the retailer's warehouse. No money is transacted until a unit of the product is sold. For each unit of any product sold, the retailer keeps a fraction $r \in [0, 1)$ of the revenue for herself and remits the rest $1-r$ to the corresponding manufacturer. After the retailer specifies the common *revenue share* r for all products, each manufacturer then determines the retail price and the production quantity for his product.

An example of the above setting can be found in Amazon.com, which runs an online marketplace where sellers list their products (such as DVDs, video games, books, metal parts, soft drinks, honey, pasta sauces, etc.) for sale. To save logistics costs, sellers can enroll in the Fulfillment by Amazon (FBA) program (see http://services.amazon.com). Sellers in the FBA program store their products in a fulfillment center managed by Amazon. Upon receiving a customer order from her website, Amazon picks, packs, and ships the order to the customer.

The FBA program provides customer service including handling customer inquiries, refunds, and returns to shoppers for the listed products. Each seller determines the retail price of his product and the number of units to list for sale. Amazon charges only when a unit of a seller's product is sold. For each unit of the product sold, Amazon deducts a certain percentage of its retail price and deposits the remaining balance to the seller's account. Units that are not sold after a period of time are returned to the seller and the listing is closed.

Amazon prefers this type of contract for the following reasons: (i) Amazon bears no overstocking risk. (ii) Unlike in traditional wholesale-price contracts, Amazon does not need to negotiate with the individual sellers or to determine the retail price and production quantity for every product, which could be tedious when there are many sellers. (iii) Although a consignment contract with revenue sharing requires every seller to monitor his sales, the implementation is straightforward in an online setting because every transaction is tracked, so splitting the revenue can be done automatically.

As a retailer, Amazon stores the products from many manufacturers (sellers) in its fulfillment center. Because of limited storage space, the retailer should take her storage capacity into consideration when she signs the contracts with the manufacturers. As we will see in our analysis, even with the storage capacity constraint, the retailer can still choose a common revenue share such that the manufacturers will set the prices and deliver the quantities that favor her interest.

Wang et al. (2004) study a consignment contract with revenue sharing between a retailer and a single manufacturer. They do not consider the storage capacity constraint. In contrast, we consider a retailer selling products for multiple manufacturers over a single period. The retailer signs a separate contract with each manufacturer under a common revenue share r subject to the storage capacity constraint. We investigate the firms' decisions and their profits in the above business setting. Specifically, we would like to answer the following questions:

1. How should the retailer set a common revenue share for all products to maximize her profit subject to the capacity constraint?
2. If the manufacturers are charged for storage space, how should the retailer simultaneously set the revenue share and the storage fee subject to the capacity constraint?
3. Is it always beneficial to the retailer and the manufacturers if the capacity is expanded?
4. How does the decentralized supply chain compare to a centralized system in terms of space requirement and profit?
5. If products are complementary (e.g., different parts of a documentary video), how does the degree of complementarity affect the retailer's and the manufacturers' decisions?

We model decision making of the firms as a Stackelberg game in which the retailer, who acts as a leader, offers each manufacturer a take-it-or-leave-it contract. Each contract specifies a common revenue share r for the retailer. Each manufacturer, acting as a follower, determines the retail price and the production quantity for his product. We assume that each manufacturer accepts the contract if he can earn positive profit (his reservation profit is normalized to zero).

Section 2 reviews the related literature. Sections 3 and 4 analyze a centralized system and a decentralized system, respectively. Specifically, we find sufficient conditions for the existence of a unique optimal revenue share for the decentralized system in §4.2. For many products, these conditions are not difficult to satisfy in practice. We then investigate the problem of simultaneously optimizing the revenue share and the storage fee in §4.3. Surprisingly, it is optimal for the retailer *not* to charge any storage fee in many situations. Section 5 compares the decentralized and the centralized systems. Section 6 studies a system with two complementary products. Section 7 gives some concluding remarks.

2. Literature Review

Under a pure consignment contract such as the one described in §1, each supplier bears all the overstocking risk for his product because he retains full ownership of the inventory. In contrast, a pure wholesale-price contract serves as the other extreme: The downstream retailer bears the full risk of overstocking because she owns the inventory under such a contract. To share the overstocking risk, one can use an inventory buyback or return policy. The effect of shared inventory ownerships on the supply chain's performance has been studied in several papers. Pasternack (1985) shows that under a newsvendor setting, channel coordination is achievable by properly designing an inventory return policy. Kandel (1996) investigates the effects of different factors on the choice of inventory return policy. Emmons and Gilbert (1998) consider a downstream retailer that makes both price and production decisions. They study the effect of inventory return on channel performance. Rubinstein and Wolinsky (1987) compare consignment with nonconsignment contracts when there are multiple sellers, middlemen, and buyers. Hackett (1993) considers a retailer that exerts a sales effort under consignment contracts.

Several authors have studied revenue-sharing schemes. Cachon and Lariviere (2001) consider various contracts offered by a downstream manufacturer to motivate an upstream supplier to build up production capacity. Under one of the contracts, the manufacturer offers a price to purchase components from the supplier while the retail price of the final product is fixed exogenously. The above contract represents a revenue-sharing scheme because the purchasing price offered to the supplier represents a "share" of the sales revenue. Gerchak and Wang (2004) consider a manufacturer that receives components from multiple suppliers to assemble a final product. Each supplier produces a different component for the final product. The manufacturer allocates the sales revenue between herself and the suppliers, who then determine their production quantities. The authors derive the equilibrium revenue-sharing allocation and production quantities. Wang and Gerchak (2003) extend the above model to determine production capacities. However, they assume the retail price of the final product is a constant.

Revenue sharing can be found in other business settings besides consignment. For example, in the video rental industry, a supplier offers a contract to a downstream retailer. Under such a contract, the supplier charges the retailer an upfront wholesale price plus a share of the sales revenue. The retailer then determines the order quantity or the retail price, or both. Cachon and Lariviere (2005) show that a supplier can coordinate a single retailer channel using such a contract. Dana and Spier (2001) analyze this contract when multiple downstream retailers face a perfectly competitive market. See Pasternack (2000), Mortimer (2002), and Gerchak et al. (2006) for other related work.

In our model, the upstream suppliers make production or inventory decisions. This is similar to a vendor-managed inventory program (see, e.g., Aviv and Federgruen 1998, Fry et al. 2001, and references therein). To implement such a program in practice, the downstream retailer may impose various constraints on the suppliers' production decisions such as minimum demand fill rates or bounds on production quantities. See Fry et al. (2001) for detailed discussions and examples in practice.

Wang et al. (2004) consider a retailer that offers an upstream supplier a consignment contract with revenue sharing. The retailer first specifies her revenue share for each unit of a product sold. Given the revenue share, the supplier then chooses the retail price and the production quantity for the product. The authors do not consider the storage capacity constraint. In contrast, we consider a retailer with limited storage capacity and multiple suppliers.

It is noteworthy that in our model the revenue share set by the retailer interacts with the retail prices (hence the total channel profit) set by the manufacturers. This is different from most channel models found in the marketing literature, where firms usually interact with each other through their individual profit margins. See, for example, Jeuland and Shugan (1983), Lal and Staelin (1984), Moorthy (1988), Choi (1991, 1996), and references therein.

3. The Centralized System

Consider a retailer that sells n different products over a single period. For each product i produced by manufacturer i, let m_i, d_i, and v_i denote the manufacturing cost (including the transportation cost to the warehouse) per unit for its manufacturer, the distribution cost (associated with handling and storage in the warehouse) per unit for the retailer, and the volume per unit, respectively, for $i = 1, \dots, n$. We assume the retailer charges a storage fee per unit volume s for the entire selling period. Thus, each unit of product i incurs a cost $c_i^M = m_i + sv_i$ for its manufacturer, a cost $c_i^R = d_i - sv_i$ for the retailer, and a total cost $c_i = c_i^M + c_i^R = m_i + d_i$. For each unit of product i, define $\alpha_i = c_i^R/c_i$ as its *cost fraction* for the retailer. The remaining fraction $1-\alpha_i$ is incurred at manufacturer i. We assume that m_i, d_i, and v_i are all positive and that $s \in [0, \min_i d_i/v_i]$.

Each manufacturer i delivers a quantity q_i of product i to the retailer. Because of space limitation, the retailer can only store a limited quantity of each product. Let V denote the total space capacity of the retailer, who is subject to the capacity constraint $\sum_{i=1}^n v_i q_i \le V$.

During the selling period, each product i has random and price-sensitive demand D_i, which has a multiplicative functional form: $D_i(p_i) = y_i(p_i)\varepsilon_i$, where $y_i(p_i)$ is a deterministic function of the retail price p_i, and ε_i is a random variable with probability density function $f_i(\cdot)$, cumulative distribution function $F_i(\cdot)$, failure rate $h_i(\cdot) = f_i(\cdot)/(1-F_i(\cdot))$, and mean μ_i, for $i = 1, \dots, n$. Assume that the probability distribution of ε_i has support on $[A_i, B_i]$ with $0 \le A_i < B_i$, and so $\mu_i > 0$. Note that B_i may be infinity. For each product i, we assume the increasing generalized failure rate (IGFR) condition holds: $d(xh_i(x))/dx = h_i(x) + xdh_i(x)/dx > 0$. This condition is satisfied by many distributions such as exponential, Weibull, and gamma distributions (see Cachon 2003), and is more general than the increasing failure rate (IFR) condition: $dh_i(x)/dx > 0$ (see Paul 2005).

We assume the function $y_i(p_i) = a_i p_i^{-b_i}$, where $a_i > 0$ and $b_i > 1$, for $i = 1, \dots, n$. (If $b_i < 1$, we can show that the optimal retail price p_i approaches infinity.) We call b_i the *price elasticity* of product i. We acknowledge that our results depend on this demand model and they may not hold generally.

We will compare the performance of a centralized system with that of a decentralized system under consignment contracts with revenue sharing. Specifically, we study the expected profit of the supply chain under each setting. We first analyze the centralized system in which a central decision maker coordinates the decision making process. He chooses the retail price p_i and the production quantity q_i for each product i to maximize the total profit of the entire supply chain. Following Petruzzi and Dada (1999), we define $z_i = q_i/y_i(p_i)$ as the *stocking factor* for product i. This definition of stocking factor is suitable for multiplicative demand models. Instead of determining p_i and q_i, the decision maker determines p_i and z_i. Let $\Pi_i(p_i, z_i)$ denote the profit generated from product i with retail price p_i and stocking factor z_i. Define $\mathbf{p} = (p_1, \dots, p_n)$ and $\mathbf{z} = (z_1, \dots, z_n)$. The total channel profit is $\Pi(\mathbf{p}, \mathbf{z}) = \sum_{i=1}^n \Pi_i(p_i, z_i)$. The objective is to

$$\max_{\mathbf{p},\,\mathbf{z}} \quad \Pi(\mathbf{p}, \mathbf{z}) = \sum_{i=1}^n \Pi_i(p_i, z_i),$$

$$\text{subject to} \quad \sum_{i=1}^n v_i z_i y_i(p_i) \le V,$$

where $\Pi_i(p_i, z_i) = -c_i q_i + p_i E[\min\{q_i, D_i(p_i)\}] = y_i(p_i)[p_i(z_i - \Lambda_i(z_i)) - c_i z_i]$, and $\Lambda_i(z_i) = \int_{A_i}^{z_i}(z_i - x) f_i(x)\,dx$, for $i = 1, \ldots, n$.

Let $\mathbf{p}^*(\mathbf{z}) = (p_1^*(\mathbf{z}), \ldots, p_n^*(\mathbf{z}))$ denote the optimal retail prices given stocking factors $\mathbf{z}$, and let $\mathbf{z}^* = (z_1^*, \ldots, z_n^*)$ denote the optimal stocking factors. For each product i, define $\tilde{p}_i(z_i) = (b_i c_i)/(b_i - 1) \cdot z_i/(z_i - \Lambda_i(z_i))$. The following theorem determines the optimal decisions for the centralized system. All proofs can be found in §A of the online supplement (available at http://dx.doi.org/10.1287/msom.2015.0543).

Theorem 1. *For any* $\mathbf{z}$ *such that* $z_j \in [A_j, B_j]$, $j = 1, \ldots, n$, *the optimal retail price of product* i *in the centralized system is*

$$p_i^*(\mathbf{z}) = \begin{cases} \tilde{p}_i(z_i), & \text{if } \sum_{j=1}^{n} v_j z_j a_j (\tilde{p}_j(z_j))^{-b_j} \le V; \\ \left(\dfrac{v_i}{c_i} \cdot \rho(\mathbf{z}) + 1\right) \tilde{p}_i(z_i), & \text{otherwise}; \end{cases}$$

where $\rho(\mathbf{z})$ *satisfies*

$$\sum_{j=1}^{n} v_j z_j a_j \left[\left(\frac{v_j}{c_j} \cdot \rho(\mathbf{z}) + 1\right) \tilde{p}_j(z_j)\right]^{-b_j} = V. \tag{1}$$

The optimal stocking factor z_i^* *is uniquely determined by* $F_i(z_i^*) = [z_i^* + (b_i - 1)\Lambda_i(z_i^*)]/(b_i z_i^*)$.

Given $\mathbf{z}^*$, the optimal retail price of product i in the centralized system is determined by

$$p_i^*(\mathbf{z}^*) = \begin{cases} \tilde{p}_i(z_i^*), & \text{if } \sum_{j=1}^{n} v_j z_j^* a_j (\tilde{p}_j(z_j^*))^{-b_j} \le V; \\ \left(\dfrac{v_i}{c_i} \cdot \rho(\mathbf{z}^*) + 1\right) \tilde{p}_i(z_i^*), & \text{otherwise}. \end{cases}$$

The optimal production quantity for product i is $q_i^* = a_i z_i^* (p_i^*(\mathbf{z}^*))^{-b_i}$, $i = 1, \ldots, n$. Note that the optimal stocking factors do not depend on the capacity V. Any changes in V are totally absorbed by adjusting the retail prices rather than changing the stocking factors. This is due to the multiplicative demand model and may not hold for other demand models.

4. The Decentralized System

In the decentralized system the retailer signs a consignment contract with each manufacturer. For each unit of any product sold, the retailer keeps a fraction r of the revenue and remits the rest, $1 - r$, to the corresponding manufacturer. After the retailer specifies the common revenue share r, each manufacturer chooses the retail price and production quantity for his product to maximize his own profit.

We model the decision process as a Stackelberg game where the retailer is the leader and the manufacturers are followers. The retailer first decides and announces a revenue share. Based on the announced revenue share, each manufacturer then chooses the retail price and production quantity (or, equivalently, the stocking factor) for his product to maximize his own profit. We will solve the overall problem backward: We first solve each manufacturer's problem to find his optimal response (price and quantity) to any revenue share offered by the retailer. Plugging each manufacturer's optimal response into the retailer's profit function, we then find the revenue share that maximizes the retailer's profit subject to her storage capacity constraint.

It is noteworthy that for our model setting, each manufacturer only needs to know his own demand function and cost parameters to make his price and quantity decisions. He does not need to know the retailer's cost parameters or other manufacturers' demand functions and cost parameters. The manufacturers hold the expectation, or are informed by the retailer explicitly, that all quantities that they deliver will be accepted by the retailer. This is consistent with Amazon's practice (see http://services.amazon.com).

The retailer, on the other hand, needs to know all information about the manufacturers. As such, the retailer can anticipate perfectly each manufacturer's optimal response to her revenue share offer. By considering her capacity constraint properly, the retailer can actually direct the manufacturers (through her choice of the revenue share) to choose production quantities such that their sum will be within her storage capacity.

Although the manufacturers do not consider directly the retailer's capacity constraint in their individual decisions, the outcome of the overall game will be able to sustain a fulfilled expectations equilibrium (Katz and Shapiro 1985); that is, in equilibrium, all quantities chosen by the manufacturers will be accepted by the retailer, and their sum will always satisfy the capacity constraint. Note that this is different from the subgame perfect equilibrium, which would also require the retailer's capacity constraint to be satisfied off the equilibrium path. As to be shown under our model assumptions, each manufacturer's optimal response is unique and the equilibrium of the game will be unique.

4.1. Manufacturers' Decisions

Expecting that the retailer will accept all units of his product, each manufacturer ignores the retailer's capacity constraint when he determines the retail price and the stocking factor for his product. Given any revenue share r, manufacturer i determines the retail price p_i and stocking factor z_i to maximize his expected profit:

$$\begin{aligned}\max_{p_i,z_i} M_{d,i}(r,p_i,z_i) &= -(1-\alpha_i)c_iq_i + (1-r)p_iE[\min\{q_i, D_i(p_i)\}]\\ &= y_i(p_i)[(1-r)p_i(z_i - \Lambda_i(z_i)) - (1-\alpha_i)c_iz_i].\end{aligned}$$

Lemma 1. *In the decentralized system, given any r and $z_i \in [A_i, B_i]$, the optimal response of manufacturer i is to set the retail price as $p^*_{d,i}(r,z_i) = ((1-\alpha_i)/(1-r))\cdot\tilde{p}_i(z_i)$ and the stocking factor as the unique optimal stocking factor of product i in the centralized system.*

Note that the equilibrium stocking factor z^*_i in the decentralized system is identical to the optimal stocking factor in the centralized system. Thus, for any revenue share r, the optimal production quantity for manufacturer i in the decentralized system is $q^*_{d,i}(r) = a_iz^*_i(p^*_{d,i}(r,z^*_i))^{-b_i}$, $i = 1,\ldots,n$.

4.2. Retailer's Decision

Knowing the manufacturers' optimal responses $\mathbf{p}^*_d(r,\mathbf{z}^*) = (p^*_{d,1}(r,z^*_1),\ldots,p^*_{d,n}(r,z^*_n))$, the retailer needs to properly determine the revenue share r to maximize her expected profit $R_d(r)$ subject to her capacity constraint. The profit generated from product i is

$$\begin{aligned}R_{d,i}(r) &= -\alpha_ic_iq^*_{d,i}(r) + rp^*_{d,i}(r,z^*_i)\\ &\quad\cdot E[\min\{q^*_{d,i}(r), D_i(p^*_{d,i}(r,z^*_i))\}]\\ &= y_i(p^*_{d,i}(r,z^*_i))[rp^*_{d,i}(r,z^*_i)(z^*_i - \Lambda_i(z^*_i)) - \alpha_ic_iz^*_i],\end{aligned}$$

for $i = 1,\ldots,n$. The retailer's objective is to

$$\max_r \quad R_d(r) = \sum_{i=1}^n R_{d,i}(r),$$

$$\text{subject to} \quad \sum_{i=1}^n v_iz^*_iy_i(p^*_{d,i}(r,z^*_i)) \le V.$$

Let $\tilde{r}$ be a revenue share that satisfies the first-order condition:

$$\left.\frac{dR_d(r)}{dr}\right|_{r=\tilde{r}} = \sum_{i=1}^n \frac{a_ib_ic_iz^*_i}{(1-\alpha_i)^{b_i}(\tilde{p}_i(z^*_i))^{b_i}}\cdot(1-\tilde{r})^{b_i-2} \cdot\left[\frac{b_i-\alpha_i}{b_i-1}\cdot(1-\tilde{r}) - (1-\alpha_i)\right] = 0. \tag{2}$$

Let $\hat{r}$ be the revenue share such that the total volume required is V, that is,

$$\sum_{i=1}^n v_iz^*_ia_i\left(\frac{1-\alpha_i}{1-\hat{r}}\tilde{p}_i(z^*_i)\right)^{-b_i} = V. \tag{3}$$

The following theorem shows the retailer's optimal decision.

Theorem 2. *There exists a unique optimal revenue share $r^* = \max\{\tilde{r},\hat{r}\}$, if*

(i) $b_i = b$, $i = 1,\ldots,n$, or

(ii) $\max_i(((1-\alpha_i)(b_i-2))/(b_i-\alpha_i)) < \min_i(((1-\alpha_i)(b_i-1))/(b_i-\alpha_i))$.

Theorem 2 provides two sufficient conditions for the existence of a unique optimal revenue share. The first condition holds approximately for many products belonging to the same product category. For example, pasta sauces by Classico, Prego, and Ragu have price elasticities of 1.88, 1.85, and 1.83, respectively (Seo and Capps 1997), and most metal products have a price elasticity of 1.1 (Baumol and Blinder 2012).

It is also not difficult to satisfy the second condition. From the proof of Theorem 2, we have

$$\tilde{r}_i = 1 - \frac{(1-\alpha_i)(b_i-1)}{b_i-\alpha_i}, \tag{4}$$

which represents the optimal revenue share for a special case of the problem with a single product i and without the capacity constraint. Thus, the second condition of Theorem 2 can be rewritten as $\max_i(1-\tilde{r}_i)(b_i-2)/(b_i-1) < \min_i 1-\tilde{r}_i$. If $b_i \le 2$ for all i, this condition is satisfied. The price elasticity of many consumer goods falls between 1 and 2. For example, soft drinks and tea have price elasticities of 1.06 and 1.07, respectively (Bergtold et al. 2004), whereas jam and honey have price elasticities of 1.61 and 1.64, respectively (Helen and Willett 1986).

4.3. Changing the Storage Fee per Unit Volume s

In practice, the retailer can adjust the storage fee per unit volume s according to demand. For example, Amazon charges a higher storage fee near the end of a year. The following lemma guarantees the existence of optimal retailer's decisions if she sets r and s simultaneously.

Lemma 2. *There exist optimal decisions (r^*, s^*) for the retailer.*

To see how (r^*, s^*) respond to demands, we scale up demands for all products simultaneously such that $a_i = \lambda_ia_n$, for $i = 1,\ldots,n$. The following lemma shows that if demands are small relative to the capacity V, the optimal revenue share and storage fee remain constant. However, if demands are large, the retailer needs to increase the revenue share or storage fee according to the demands to satisfy the capacity constraint.

Lemma 3. *For demands satisfying $a_i = \lambda_ia_n$, $i = 1,\ldots,n$, there exists an $\bar{a}_n$ such that*

(i) *if $a_n \le \bar{a}_n$, then $(r^*, s^*) = (r^0, s^0)$, where r^0 and s^0 are independent of a_n;*

(ii) *otherwise, (r^*, s^*) depend on a_n, and $r^* > r^0$ or $s^* > s^0$.*

Figure 1 **Optimal Revenue Share r^* and Storage Fee s^***

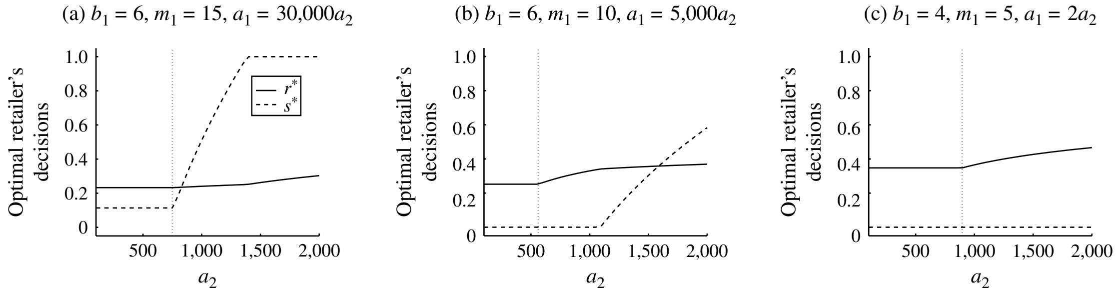


Figure 1 shows the optimal decisions (r^*, s^*) for a system with $n = 2$, $V = 10$, $v_1 = v_2 = 1$, $d_1 = d_2 = 1$, $b_2 = 4$, $m_2 = 5$, and $\varepsilon_i \sim \mathcal{N}(51, 8.33^2)$. There are three different scenarios: (a) The optimal s^* is always positive. (b) The optimal s^* first equals 0 and then increases with demand. (c) The optimal s^* always equals 0.

Figure 1(a) suggests that r^* and s^* first remain constant and start increasing simultaneously with a_n when the capacity constraint is binding (at the vertical dotted line). This is consistent with Lemma 3. Note that $s^* > 0$ for all a_n. Figure 1(b) shows that under a different parameter setting, s^* first equals 0. The revenue share r^* starts increasing when the capacity constraint is binding, whereas s^* remains equal to 0. This is because the retailer gains a larger marginal profit when she raises the revenue share compared to increasing the storage fee. It is optimal not to charge any storage fee for small a_n. However, s^* starts increasing when a_n is sufficiently large. Figures 1(a) and 1(b) suggest that the retailer should charge a higher storage fee when the demand is large. This is consistent with the practice of Amazon, who charges a higher storage fee during a peak season (see http://services.amazon.com).

Figures 1(a) and 1(b) seem to suggest that it is optimal to charge a positive storage fee when demand is sufficiently large. However, Figure 1(c) shows an example where the retailer always sets $s^* = 0$. In fact, if all products have identical values for their parameters, except a_i, then it is always optimal for the retailer not to charge any storage fee. This result is summarized in the following theorem. We say a system is *symmetric* if the following conditions hold: $b_i = b$, $F_i = F$, $m_i = m$, $d_i = d$, and $v_i = v$, for $i = 1, \ldots, n$. These conditions can potentially hold for products belonging to the same family with common characteristics (e.g., DVDs).

Theorem 3. *For a symmetric system,*

(i) *the optimal storage fee per unit volume $s^* = 0$ for any $a_i > 0$, $i = 1, \ldots, n$, and*

(ii) *the optimal revenue share r^* first remains constant and then strictly increases with $\sum_{i=1}^{n} a_i$.*

Theorem 3 is surprising because it shows that under certain symmetry conditions, it is optimal not to charge any storage fee even if the system fully utilizes its capacity (see Figure 1(c)). This shows that it is more effective for the retailer to influence the manufacturers' production quantities through raising r than raising s. Since the system is always symmetric for $n = 1$, it is always optimal for the retailer to set $s^* = 0$ if she deals with only one manufacturer.

Corollary 1. *If $n = 1$, then*

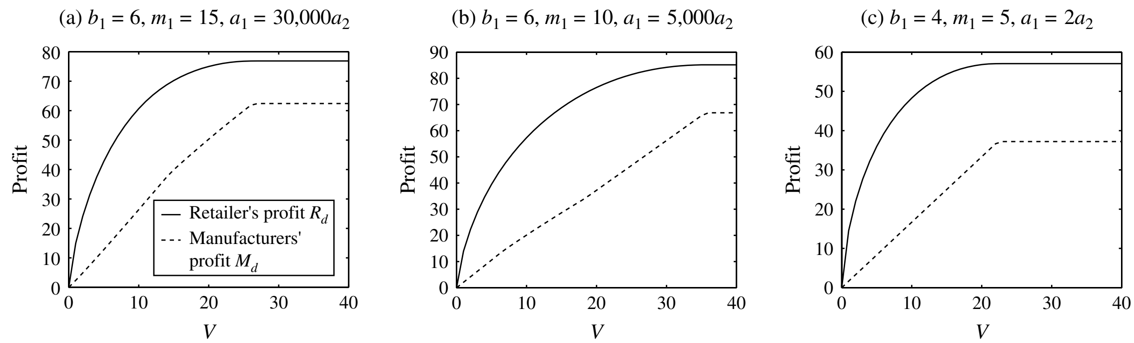
(i) *the optimal storage fee per unit volume $s^* = 0$ for any $a_1 > 0$, and*

(ii) *the optimal revenue share r^* first remains constant and then strictly increases with a_1.*

To check whether the retailer's optimal decisions (r^*, s^*) always follow the three patterns shown in Figure 1, we investigate the behavior of (r^*, s^*) numerically by enumerating various parameters. We set $n = 2$, $d_1 = d_2 = 1$, $v_1 = v_2 = 1$, and $\varepsilon_1, \varepsilon_2 \sim \mathcal{N}(51, 8.33^2)$. We consider $b_i = 1.5 + 0.3k$, $k = 0, 1, \ldots, 15$, and $m_i = 1 + 2k$, $k = 0, 1, \ldots, 7$, for $i = 1, 2$. Without loss of generality, we only consider cases where $b_1 \geq b_2$. This results in 8,704 combinations of (b_1, b_2, m_1, m_2). For each combination of parameters (b_1, b_2, m_1, m_2), we fix the ratio a_1/a_2 such that the optimal volumes of both products are comparable (to prevent the system from degenerating to the one-product case). We then find (r^*, s^*) for $a_2 \in [1, 50]$. To make capacity relevant, we set V such that the system fully utilizes its capacity if and only if $a_2 \geq 10$.

Out of the 8,704 parameter settings, 99.5% exhibit one of the three typical patterns shown in Figure 1: 21.6% follow Figure 1(a), 31.0% follow Figure 1(b), and 46.9% follow Figure 1(c). Note that among all the parameter settings, only 1.5% are symmetric ($b_1 = b_2$ and $m_1 = m_2$). This implies that 45.4% (46.9% − 1.5%) of the settings are asymmetric with $s^* = 0$, and this generally happens when m_1 and m_2 are close to each other. Thus, although Theorem 3 only applies to symmetric systems, our numerical results suggest that in many asymmetric systems (45.4%) it is also optimal for the retailer not to charge any storage fee.

Figure 2 Retailer's Profit and Manufacturers' Total Profit



The above observations can be summarized as follows. (i) If the manufacturing costs m_1 and m_2 are similar, it is usually optimal for the retailer not to charge any storage fee (see Figure 1(c)). In this case, it is more effective for the retailer to influence the manufacturers' production quantities through raising r than raising s. (ii) If the manufacturing costs are very different, it is usually optimal to charge a positive s^* when the system fully utilizes its capacity (see Figures 1(a) and 1(b)). (iii) If the manufacturing costs are very different, sometimes it is optimal to charge a positive s^* even if the system has not fully utilized its capacity (see Figure 1(a)).

Points (i) and (iii) above can be explained as follows. Since the retailer can only set a common r for all products, she prefers a group of manufacturers with similar $\tilde{r}_i$ because this will reduce her profit loss caused by setting a common r. Since $\tilde{r}_i$ is very sensitive to α_i (see Equation (4)), the retailer prefers a group of manufacturers with similar α_i. Note that $\alpha_i = 1 - c_i^M/(c_i^M + c_i^R) = 1 - (m_i + sv_i)/(m_i + d_i)$. If the manufacturing costs m_i are close to each other, then α_i are close to each other even with $s = 0$ (this explains point (i)). However, if m_i are very different from each other, the retailer tends to make α_i more homogeneous by increasing s (this explains point (iii)).

4.4. Changing the Capacity V

The retailer can expand her storage capacity to maximize her profit (note that capacity expansion cannot be done in a short time and is generally planned in advance). Interestingly, if $a_i = \lambda_i a_n$ for all i, the impact of increasing V on the optimal revenue share and storage fee is effectively equivalent to that of reducing demand a_n. Lemma 3 implies the following corollary.

Corollary 2. *For demands satisfying* $a_i = \lambda_i a_n$, $i = 1, \dots, n$, *there exists a* $\bar{V}$ *such that*

(i) *if* $V \geq \bar{V}$, *then* $(r^*, s^*) = (r^0, s^0)$, *where* r^0 *and* s^0 *are independent of* V;

(ii) *otherwise*, (r^*, s^*) *depend on* V, *and* $r^* > r^0$ *or* $s^* > s^0$.

Similarly, Theorem 3 implies the following corollary.

Corollary 3. *For a symmetric system*,

(i) *the optimal storage fee per unit volume* $s^* = 0$ *for any* V, *and*

(ii) *the optimal revenue share* r^* *first strictly decreases with* V *and then remains constant*.

Since expanding capacity (e.g., building a warehouse) comes with a cost, how much capacity should the retailer invest? Suppose the retailer has initial capacity V_0, and assume it incurs a constant cost κ to expand a unit volume. It is important to first study how the retailer's profit changes with capacity V. Figures 2(a)–2(c) show the retailer's profit in the three parameter settings corresponding to Figures 1(a)–1(c), respectively. We set $a_2 = 3{,}000$. Figures 2(a) and 2(b) correspond to asymmetric systems, whereas Figure 2(c) corresponds to a symmetric system. In all cases, the retailer's profit first increases concave in V and then remains constant. Figure 2 also shows that the manufacturers' total profit $M_d = \sum_{i=1}^n M_{d,i}$ first strictly increases in V and then remains constant. This suggests that expanding capacity may benefit not only the retailer, but also the manufacturers.

Lemma 4. *For a symmetric system, we have the following results*:

(i) *The retailer's profit first strictly increases concave in* V *and then remains constant*.

(ii) *Each manufacturer's profit first linearly increases in* V *and then remains constant*.

(iii) *Given an initial capacity* V_0 *and a constant expansion cost per unit volume* κ, *the retailer's optimal additional capacity is* $\max\{V^* - V_0, 0\}$, *where*

$$V^* = \left(\sum_{i=1}^n a_i\right) v z^* \left[\left(\frac{\kappa v}{c} + \frac{b-\alpha}{b-1}\right)\tilde{p}(z^*)\right]^{-b}.$$

If $V_0 \geq V^*$, the retailer does not need to expand her capacity. Otherwise, part (i) of Lemma 4 implies that the retailer should expand her capacity to V^*, where her marginal profit equals κ. This is summarized in part (iii) of Lemma 4. Furthermore, parts (i) and (ii) show that in a symmetric system both the retailer and the manufacturers may benefit from the capacity

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expansion. Our numerical results suggest that this is also generally true for asymmetric systems. Thus, not only Amazon but also the sellers in the FBA program may benefit from the former's capacity expansion.

5. Comparing the Decentralized and the Centralized Systems

We use the centralized system as a benchmark to evaluate the decentralized system, where the retailer sets both the revenue share and storage fee given a fixed capacity V. It is generally hard to compare the stocking factors and retail prices of the decentralized and the centralized supply chains across multiple products, but we can compare them for a symmetric system.

Lemma 5. *For a symmetric system, the equilibrium stocking factor (retail price) of each product in the decentralized supply chain is the same as (no less than) the optimal stocking factor (retail price) of the product in the centralized supply chain.*

We further compare the decentralized and the centralized systems in other aspects as follows.

5.1. Space Requirement and Channel Efficiency

Define $\phi = (R_d^* + \sum_{i=1}^{n} M_{d,i}^*)/\Pi^*$ as the *channel efficiency* of the decentralized system, where R_d^* and $M_{d,i}^*$ represent the equilibrium profits of the retailer and manufacturer i, respectively, in the decentralized system, and Π^* is the centralized system's optimal profit.

Lemma 6. *For a symmetric system, we have the following results:*

(i) *Under equilibrium decisions, the decentralized system always requires less space than the centralized system unless both systems fully utilize their capacity.*

(ii) *The channel efficiency ϕ is larger than $2/e \approx 0.736$.*

Although part (i) of Lemma 6 holds only for symmetric systems, our numerical studies suggest that this result also holds for asymmetric systems generally. In each graph of Figure 3, the dashed line shows the ratio of the decentralized system's volume requirement to the centralized system's volume requirement. We use the same parameter settings in Figures 1(a)–1(c) for Figures 3(a)–3(c), respectively.

In all the three cases, the volume ratio never exceeds 1. The decentralized system requires no larger space than the centralized system. The volume ratio is first constant in demand (a_2) when the capacity is not fully used in both the centralized and decentralized systems. The ratio starts to increase with demand as the centralized system fully utilizes its capacity (at the left vertical dotted line). As demand continues to grow, both the centralized and decentralized systems fully utilize their capacity (at the right vertical dotted line), and the volume ratio becomes 1.

The solid line in each graph of Figure 3 shows the channel efficiency, which is always above 0.7 for all the three cases. Part (ii) of Lemma 6 provides a lower bound on the channel efficiency for a symmetric system (Figure 3(c)). The result of this special case is similar to Proposition 5 in Wang et al. (2004). On the other hand, we find that some asymmetric systems (e.g., when m_1 is extremely different from m_2) give arbitrarily low channel efficiency.

For all the three cases, the channel efficiency first remains constant in a_2 when the capacity is not fully used in both the centralized and decentralized systems. The channel efficiency starts to increase with a_2 as the centralized system fully utilizes its capacity, and it continues to grow until both systems fully utilize their capacity.

When both the centralized and decentralized systems fully utilize their capacity, the channel efficiency behaves differently for different cases. For the symmetric case (Figure 3(c)), the decentralized system achieves perfect channel efficiency and it is called *coordinated*. This is because all manufacturers set the same retail price, which is equal to the common retail price in the centralized system. This leads to the same total profit for both the centralized and decentralized systems because they have identical stocking factors $\mathbf{z}^*$. The channel efficiency is below 1 for asymmetric cases (Figures 3(a) and 3(b)) because the retailer

Figure 3 **Volume Ratio and Channel Efficiency**

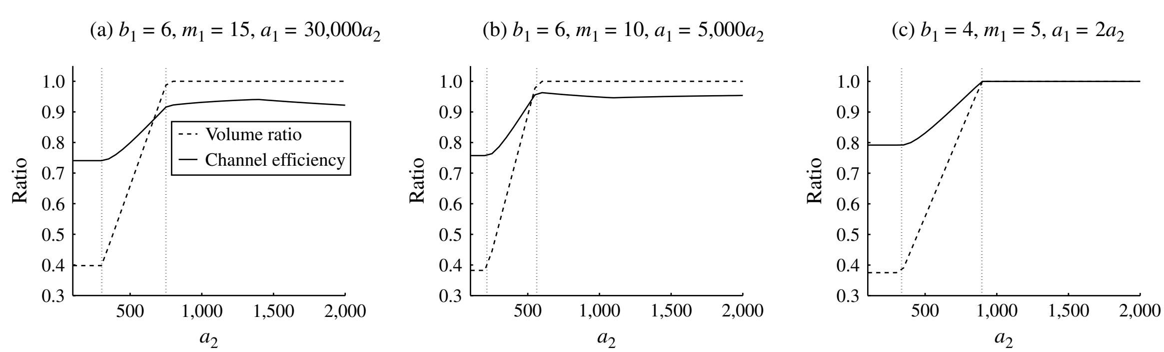
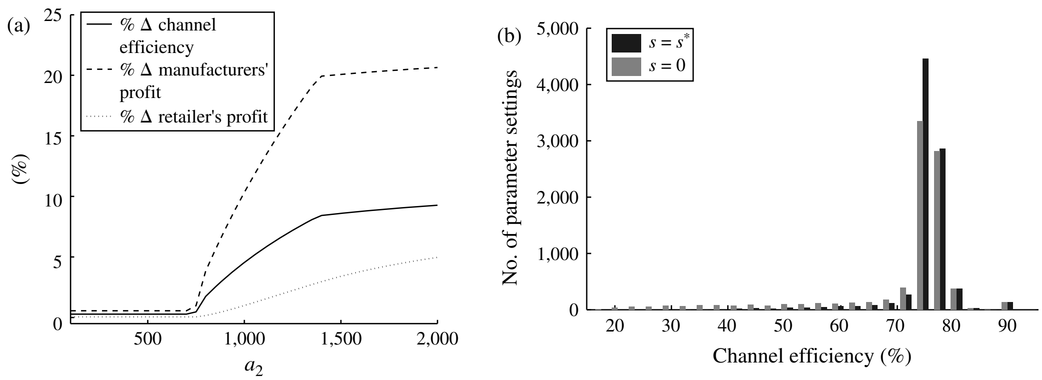


Figure 4 Advantages of Charging a Storage Fee



controls only two variables r and s to influence the manufacturers' retail prices in the decentralized system. In contrast, the centralized system enjoys the flexibility of directly setting every product's price.

In summary, the decentralized system requires a smaller storage space but provides less channel profit than the centralized system.

5.2. The Advantages of Charging a Storage Fee

What are the advantages of charging a storage fee? The solid line in Figure 4(a) shows the increase in channel efficiency when the retailer sets $s = s^* \geq 0$ instead of $s = 0$. We use the same parameter setting as in Figure 1(a). The dashed and the dotted lines show that the increases in the manufacturers' and retailer's profits can be as large as 20% and 5%, respectively. Surprisingly, charging storage fee benefits not only the retailer, but also the manufacturers. Although they pay the storage fee per unit volume s^*, the manufacturers enjoy a higher percentage increase in their total profit than the retailer. This is because if $s = s^* > 0$, not only does the channel gain more revenue, but also the retailer tends to set a lower r, which yields a larger revenue share $1 - r$ for each manufacturer.

We use the same 8,704 parameter settings in §4.3 to further investigate the impact of storage fee. In each parameter setting, we set V such that the capacity is fully used if and only if $a_2 \geq 10$. Figure 4(b) shows the histograms of the channel efficiency for both $s = s^*$ and $s = 0$. For most of the parameter settings, the channel efficiency is larger than 0.736—the lower bound for a symmetric system established in Lemma 6. The figure also suggests that charging a storage fee improves channel efficiency. For all 8,704 parameter settings, charging $s = s^*$ always results in higher (or equal) channel efficiency, retailer's profit, and manufacturers' total profit compared to charging $s = 0$. Thus, the current Amazon's practice of charging a storage fee benefits not only itself, but also the sellers in the FBA program. For 98% of the settings, the manufacturers enjoy a higher (or equal) percentage increase in their total profit compared to the retailer. This suggests that charging the optimal storage fee is more advantageous to the manufacturers than to the retailer.

6. Two Complementary Products

We also consider two complementary products (e.g., two parts of a documentary video) with demand functions $D_i(p_i, p_j) = y_i(p_i, p_j)\varepsilon_i$, where $y_i(p_i, p_j) = a_i(p_i + \beta p_j)^{-b_i}$, $\beta \in [0, 1]$, $i, j \in \{1, 2\}$, and $i \neq j$. Note that $\partial D_i(p_i, p_j)/\partial p_j \leq 0$, which is consistent with the definition of complementary products in the economics literature (see, e.g., Stiglitz 1993). These demand functions are inspired by the log-linear demand model (Bell 1968). They also generalize the model used by Wang (2006) (which is a special case with $\beta = 1$) and the model in the previous sections (which is a special case with $\beta = 0$). In this section, we consider a symmetric system for tractability. We assume $b > 1 + \beta$ and the IFR condition $dh(x)/dx > 0$ holds.

We first determine the manufacturers' decisions. Given any revenue share r and the retail price p_j, the objective of manufacturer i is to

$$\begin{aligned}\max_{p_i, z_i} M_{d,i}(r, p_i, z_i, p_j) &= -(1-\alpha)cq_i + (1-r)p_i E[\min\{q_i, D_i(p_i, p_j)\}] \\ &= a_i(p_i + \beta p_j)^{-b}[(1-r)p_i(z_i - \Lambda(z_i)) - (1-\alpha)cz_i].\end{aligned}$$

Lemma 7. *The equilibrium stocking factor of each product equals z^*, which is uniquely determined by $F(z^*) = [(1+\beta)z^* + (b-1-\beta)\Lambda(z^*)]/(bz^*)$. Given any revenue share r, the optimal retail price of each product is $p_d^*(r) = ((1-\alpha)bc/((1-r)(b-1-\beta))) \cdot z^*/(z^* - \Lambda(z^*))$.*

If $\beta = 0$, z^* and $p_d^*(r)$ reduce to their counterparts in Lemma 1 for a symmetric system. As β increases, z^* and $p_d^*(r)$ deviate from that of Lemma 1. In particular, z^* increases with β.

Corollary 4. *The equilibrium stocking factor z^* is strictly increasing in β.*

Knowing the manufacturers' optimal responses, the retailer chooses the revenue share r to maximize her expected profit $R_d(r) = R_{d,1}(r) + R_{d,2}(r)$, where $R_{d,i}(r) = y(p_d^*(r), p_d^*(r))[rp_d^*(r)(z^* - \Lambda(z^*)) - \alpha cz^*] = a_i[(1+\beta)p_d^*(r)]^{-b}[((1-\alpha)b/(b-1-\beta)) \cdot (r/(1-r)) - \alpha]cz^*$, representing the profit generated from product i. The retailer's decision is determined as follows.

Theorem 4. *The optimal revenue share is* $r^* = \max\{\tilde{r}, \hat{r}\}$, *where* $\tilde{r} = [\alpha(b-2-\beta)+1]/[b-(1+\beta)\alpha]$ *and* $\hat{r} = 1 - [V/(vz^*(a_1+a_2))]^{1/b} \cdot ((1+\beta)(1-\alpha)bc/(b-1-\beta)) \cdot (z^*/z^* - \Lambda(z^*))$.

Lemma 7 shows that each retail price decreases as the revenue share decreases. Furthermore, in this complementary demand model, reducing the price p_j increases not only the demand for product j, but also the demand for product i. As β gets larger, any price reduction will increasingly benefit the retailer. In this situation the retailer should decrease her revenue share to encourage the manufacturers to lower their prices. This is confirmed by the following corollary.

Corollary 5. *The optimal revenue share r^* is strictly decreasing in β.*

Corollaries 4 and 5 show the monotonic behavior of the stocking factor and revenue share in β. However, the retail price does not have any monotonic behavior in β.

If the retailer can change the storage fee per unit volume s, it can be shown that Theorem 3 continues to hold for two complementary products. The proof is similar to that of Theorem 3 and is therefore omitted. Thus, in a symmetric system it is always optimal for the retailer to set $s^* = 0$ for n independent products or for two complementary products.

7. Conclusion

We study a retailer that has limited storage space selling products for n manufacturers under consignment contracts with revenue sharing. Knowing the manufacturers' optimal responses, the retailer sets a common revenue share to maximize her profit subject to the storage capacity constraint. We show that there exists a unique optimal revenue share if one of the following conditions is satisfied: (i) All products have identical price elasticity. (ii) All products have price elasticity no larger than 2.

We have three major findings for independent products. First, if the products have similar manufacturing costs, we obtain a counterintuitive result that the retailer generally should not charge any storage fee even if demand is high. We prove that for a symmetric system the optimal storage fee per unit volume is $s^* = 0$. Surprisingly, we also find that $s^* = 0$ for many asymmetric systems in our numerical studies if the products have similar manufacturing costs. In this situation, it is sufficient for the retailer to adjust only the revenue share when demand increases. If the products have very different manufacturing costs, we find that charging storage fee benefits not only the retailer, but also the manufacturers.

Second, both the retailer and manufacturers may benefit from the retailer's capacity expansion. For a symmetric system, we prove that both the retailer's and the manufacturers' profits first increase and then remain constant as the capacity increases. We also observe these behaviors in numerical studies for asymmetric systems. Furthermore, by taking the capacity cost into account, we determine the retailer's optimal capacity for a symmetric system.

Third, the decentralized system requires no larger space than the centralized system. We prove this result for a symmetric system, and it also holds for asymmetric systems in numerical studies. Although the decentralized system generates less profit than the centralized system, it attains at least 0.736 channel efficiency if the system is symmetric. Thus, the decentralized system uses less storage space at the expense of channel profit.

We have one major finding for two complementary products. As the degree of complementarity β increases, the retailer will decrease her revenue share. This is because if β is large, any reduction in price will greatly benefit the retailer. In this situation, the retailer should reduce the revenue share to encourage the manufacturers to lower their prices.

We would like to highlight that even though the manufacturers may approach the retailer at different times in practice, the retailer still has to determine the common revenue share in advance (e.g., Amazon publishes the revenue share on its website). This paper provides a model for the retailer to set the revenue share and storage fee, given multiple manufacturers sharing her limited storage space during an extended period of time (say, one year). Our model serves as an approximation of the actual problem when the system reaches a steady state, where the number of manufacturers at any point in time is roughly constant. The retailer can forecast this constant and allocate a fixed volume of storage space for this number of manufacturers. Our model provides guidance and insights to the retailer to determine the revenue share, the storage fee, and the capacity in this setting. The model also helps the manufacturers to determine the retail prices and production quantities for their products.

Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/msom.2015.0543.

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