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How a Base Stock Policy Using "Stale" Forecasts Provides Supply Chain Benefits

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Managers often engage in forecast updating with the expectation that forecast updating reduces expected shortage and inventory costs. One undesirable effect of forecast updating is that it may lead to the bullwhip effect, a phenomenon where the variability of demand increases as one moves up the supply chain. The bullwhip effect can be undesirable for the supplier because more volatile orders from the downstream stage can be very costly to the supplier. It can make it more difficult for the supplier to forecast demand, and it can lead to large fluctuations in supplier production levels from period to period. Using "stale" or old forecasts may sound foolish, but their judicious use in a two-stage supply chain can improve fulfillment from the upstream stage to the downstream stage and reduce the fluctuations in production levels. We study a two-stage supply chain where the demand process is nonstationary and both stages use an adaptive base stock policy. We propose a policy that uses old forecasts to set base stock levels at the downstream stage while using current forecasts to communicate upcoming orders from the downstream stage to the upstream stage. We study a decentralized supply chain setting, and we find that our policy can reduce the expected supply chain inventory and shortage costs and significantly reduce the fluctuations in production levels compared to that of using current information. We also study a cooperative supply chain setting and, surprisingly, we find in numerical examples that our proposed policy results in very small increases in the expected systemwide inventory and shortage costs compared to a systemwide optimal policy, while reducing the fluctuations in production levels.

Key words: inventory control; forecasting; forecast revision; bullwhip effect; production smoothing
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1. Introduction

Managers often engage in forecast updating with the expectation that forecast updating reduces expected shortage and inventory costs. One undesirable effect of forecast updating is that it may lead to the bullwhip effect, a phenomenon where the variability of demand increases as one moves up the supply chain (Lee et al. 1997, Chen et al. 2000, Graves 1999). The bullwhip effect can be undesirable for the supplier because more volatile orders from the downstream stage can be very costly to the supplier. It can make it more difficult for the supplier to forecast demand, leading to higher inventory and shortage costs for the supplier. In addition, it can lead to large fluctuations in supplier production levels from period to period. More variable (less smooth) production is likely to require more production resources such as equipment and labor.

What if managers did not use the most current forecast of demand to reduce the bullwhip effect? How much would the upstream stage benefit? Would the downstream stages' inventory and shortage costs increase significantly? Would the supply chain as a whole be better off? In this paper, we address these questions. We consider a two-stage supply chain consisting of a supplier (upstream stage) and a manufacturer (downstream stage), where both stages use an adaptive base stock inventory policy. We propose a policy that uses "stale" or old forecasts (rather than a forecast based on the most recent demand information) to set base stock levels at the manufacturer. We study this policy in two supply chain settings: (1) a decentralized setting, where the supplier provides a high service level to the manufacturer and (2) a cooperative setting, where the supply chain members set safety stock levels to minimize the expected inventory and shortage costs for the supply chain. Our performance measures are (1) the expected supply chain inventory and shortage costs and (2) the standard deviation of the supplier's period-to-period production changes. The latter metric is a production smoothing measure and serves as a surrogate for all the costs associated with fluctuations in production levels at the supplier. (We note that our performance measures are similar to that of Graves et al. 1998 who study the trade-off between production smoothness and inventory.) Our benchmark policy is a policy that minimizes the expected systemwide inventory and shortage costs under some reasonable assumptions.

The rationale for studying a policy that uses "stale" or old forecasts to set base stock levels at the manufacturer is that such a policy reduces the amount of forecast updating that the supplier must contend with, potentially reducing the bullwhip effect, which, in turn, can reduce the fluctuations in production levels and improve fulfillment from the supplier to the manufacturer. However, it is expected that using old forecasts to set base stock levels at the manufacturer would reduce the ability of the manufacturer to fulfill customer demand, thus, increasing costs at the manufacturer.

In the decentralized supply chain setting, we find that our policy of using old forecasts can reduce both the expected supply chain inventory and shortage costs and the variance of the supplier's period-to-period production changes compared to that of using current information. When compared to the benchmark policy, our policy in the decentralized setting results in higher expected inventory and shortage costs as expected, but the variance of the supplier's period-to-period production changes can be considerably less. In the cooperative supply chain, surprisingly, we find in numerical examples that our proposed policy results in very small increases in the expected systemwide inventory and shortage costs compared to that of the benchmark policy, while reducing the variance of period-to-period production. Our findings enable supply chain managers to consider a trade-off between inventory and shortage costs and fluctuations in production levels, without directly constraining production levels.

In §2, we review the literature related to forecast updating. In §3, we present the model, the benchmark policy, and the proposed policy. In §4, we analyze the decentralized setting, while in §5, we analyze the cooperative setting. Concluding remarks are made in §6.

2. Literature Review

We mentioned previously that forecast updating can contribute to the bullwhip effect. Lee et al. (1997) study a time-correlated demand process and show that demand forecasting can contribute to the bullwhip effect. Chen et al. (2000) extend this work by assuming that the exact form of the demand process is not known. They quantify the impact of demand forecasting on the bullwhip effect. Graves (1999) considers an adaptive base stock policy where the demand process is nonstationary. The forecast is updated every period using the minimum mean squared error forecast (MMSE). He finds that the demand process for the upstream stage is more variable than that for the downstream stage.

Forecasting updating and its impact on production costs has been studied by a number of researchers. Graves et al. (1986) propose a production smoothing model for production settings where it is expensive and/or difficult to change the production level. Using their production smoothing model, they find that, for a given level of safety stock, the more that production is smoothed, the greater the variability in service level from month to month. In a related paper, Graves et al. (1998) propose an approach for revising the production plan based on the cumulative forecast revisions over the forecast horizon. They require that the current cumulative forecast revisions over the forecast horizon be allocated to the production plan over the forecast horizon through a weighting scheme. Using this approach, they optimize the trade-off between production capacity and inventory for a single stage. Our work is similar to their work in that our proposed policy of using old forecasts offers a trade-off between period-to-period production variability and supply chain inventory and shortage costs. Our work is different in that our proposed policy does not directly constrain production levels. We control the inventory policy and measure the resulting period-to-period production variability. Other papers that study forecast updating and production/inventory planning include Hausman and Peterson (1972), Heath and Jackson (1994), Fisher and Raman (1996), Gurnani and Tang (1999), Donahue (2000), Kaminsky and Swaminathan (2001), Toktay and Wein (2001), Aviv (2001, 2002), Ferguson et al. (2002), and Milner and Kouvelis (2002, 2003).

We also note that several researchers have found that forecast updating is not always as beneficial as one might think. Cattani and Hausman (2000) found that item level demand forecasts do not necessarily become more accurate as they are updated from period to period. Ren et al. (2002) studied forecasts from a semiconductor manufacturer and found that the sharing of inflated and/or volatile forecasts can lead the manufacturer to prolong the delivery lead time. Iyer and Bergen (1997) studied a manufacturer-retailer supply chain with a single-period selling season, where the manufacturer builds to order and the retailer builds to stock. They find that the retailer's expected profits increase with more accurate forecasts, while the manufacturer's expected profits typically decrease. Miyaoka and Hausman (2003) study a two-stage single-period model where both stages build to stock and they find that more accurate demand information may lead to reduced expected profits for the supply chain.

To our knowledge, this work is the first to study a policy that uses "stale" or old forecasts to set base stock levels at the downstream stage in a periodic review setting.

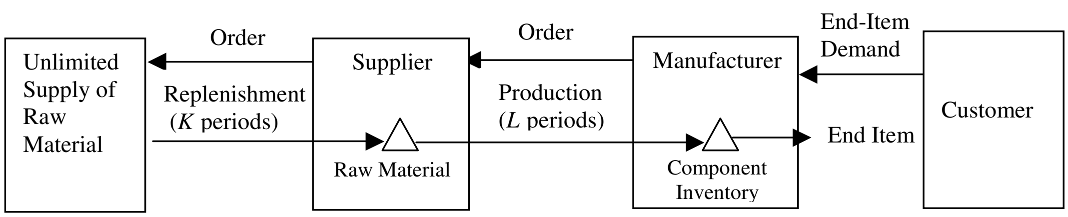
3. The Model

We consider a single-item, multiperiod, two-stage supply chain similar to that of Graves (1999). The supplier produces a component and the manufacturer converts the component into a finished good. We assume that the supplier orders raw material, holds it in stock, and then converts the raw material into the component based on actual orders from the manufacturer. The replenishment lead time for raw material is K periods, while component production has a lead time of L periods. We assume that the manufacturer receives components from the supplier immediately after completion at the supplier. (That is, the transportation time between the supplier and manufacturer is negligible.) Furthermore, we assume that the manufacturer is able to immediately convert the component into a finished good, and does so after receiving actual orders from the end customer (see Figure 1).

We assume that the supplier orders from an unlimited source of supply. Unsatisfied end-product demand is backordered at the manufacturer and unsatisfied orders are backordered at the supplier. At the end of each period, the manufacturer is charged a holding cost of h_1 per unit of inventory on hand plus a shortage penalty of b_1 per unit of backordered demand. At the end of each period, the supplier is charged a holding cost of h_2 per unit of inventory on hand.

The timing of events in each period is as follows. In period t, the manufacturer observes demand (d_t), determines his component order quantity (q_t), receives as many components as possible from his order placed L periods ago plus any filled backorders, and fills customer demand (including any backorders) as much as possible from inventory. In period t, the supplier observes the manufacturer's order (q_t), determines his raw material order quantity (p_t), receives raw material from his order

Figure 1 **Two-Stage Supply Chain Structure**



placed K periods ago (p_{t-K}), responds to the manufacturer's order and any backorders by releasing raw material into production (r_t), completes component production started L periods ago (r_{t-L}), and immediately delivers these completed components to the manufacturer.

Note that r_t represents the production start quantity in period t at the supplier, which is not necessarily equal to q_t because of raw material shortages at the supplier. (When the supplier's safety stock of raw material provides a high service level to the manufacturer so that the lead time seen by the manufacturer is L, then r_t is essentially equivalent to q_t.)

3.1. The Demand Process and Forecasting Model

We use the demand process and forecasting model in Graves (1999). The end-item demand process is an integrated moving average (IMA) process of order (0,1,1) as follows:

$$d_1 = \mu + \varepsilon_1 \quad \text{and}$$
$$d_t = d_{t-1} - (1-\alpha)\varepsilon_{t-1} + \varepsilon_t \quad \text{for } t = 2, 3, \ldots, \tag{1}$$

where d_t is the end-item demand in period t, μ and α are known constants, and ε_t is a normally distributed, identical and independent random error term with mean zero and variance σ^2. Using Equation (1) recursively, demand can also be expressed as

$$d_t = \varepsilon_t + \alpha\varepsilon_{t-1} + \alpha\varepsilon_{t-2} + \cdots + \alpha\varepsilon_1 + \mu. \tag{2}$$

As Graves (1999) points out, we can model a range of demand processes by varying α. When $\alpha = 0$, the demand process is a stationary i.i.d. process with mean μ and variance σ^2, and when $\alpha = 1$, the demand process is a random walk. When $0 < \alpha \leq 1$, the demand process is nonstationary and as α increases, the process depends more on the most recent demand realization. This demand process can result in negative demand. See the Appendix for a discussion of the likelihood of negative demand.

We assume that the forecasting technique is an exponential weighted moving average with smoothing constant α, which is the MMSE forecast for the stated demand process (Muth 1960, Box et al. 1994). The forecast, F_t, is the forecast generated in period $t-1$ for period t, after observing the end-item demand in period $t-1$.

$$F_1 = \mu \quad \text{and}$$
$$F_{t+1} = \alpha d_t + (1-\alpha)F_t \quad \text{for } t = 1, 2, \ldots. \tag{3}$$

Graves (1999) shows by induction that the forecast error is

$$d_t - F_t = \varepsilon_t \quad \text{for } t = 1, 2, \ldots. \tag{4}$$

Using (2) and (4), the forecast can be expressed in terms of the random noise terms

$$F_t = \alpha\varepsilon_{t-1} + \alpha\varepsilon_{t-2} + \cdots + \alpha\varepsilon_1 + \mu. \tag{5}$$

Note that at time $t-1$, the forecast for demand in any future period is the same as that for the next period.

3.2. The Benchmark Policy

In this section, we consider the optimization problem that minimizes the systemwide shortage and inventory costs. If we assume a "costless" return policy (Lee et al. 1997, Dong and Lee 2003), then a myopic solution is optimal and we can utilize the approach of Dong and Lee (2003) to show that an echelon base stock policy is optimal. In our two-stage supply chain, the "costless" return policy as described in Dong and Lee (2003) has the following meaning: at the manufacturer, if the target inventory level is lower than the physical inventory plus the quantity on order, then the manufacturer gets a refund at the original purchase price for each unit of excess inventory. This refund is paid by an outside party (e.g., a bank); however, the excessive inventory does not leave the system physically. It is not counted as inventory for the manufacturer, but is counted as inventory for the supplier for future replenishment to the manufacturer. At the supplier, excess inventory is returned to its outside source with a refund at the original purchase cost.

Unfortunately, this "costless" return policy is not realistic in many settings. For example, the outside party who pays the refund to the manufacturer is unlikely to provide this service without some cost. Also, returns from the supplier to its outside source are unlikely to be without cost. However, the optimal echelon base stock policy that results from this "costless" return assumption can provide an appropriate benchmark if excess inventory at both the manufacturer and supplier rarely occurs. In the following, we present this benchmark policy. In the Appendix, we discuss why this policy is unlikely to result in excess inventory occurrences under some reasonable assumptions.

The benchmark policy is as follows. The manufacturer uses an adaptive base stock inventory policy, where the order q_t^* is placed according to

$$q_t^* = d_t + L(F_{t+1} - F_t). \tag{6}$$

Let $e_t^1 = \sum_{i=1}^{L} d_{t+i} - LF_{t+1}$. Using the Graves (1999) approach, e_t^1 is normally distributed with $E[e_t^1] = 0$ and

$$\mathrm{Var}[e_t^1] = L\sigma^2\left(1 + \alpha(L-1) + \frac{\alpha^2(L-1)(2L-1)}{6}\right)$$

for $t \ge L$, where Var[ ] denotes the variance. The manufacturer's optimal safety stock level is $x_0^* = \mathrm{Std}[e_t^1]\Phi_N^{-1}((b_1+h_2)/(b_1+h_1))$, where Std[ ] denotes the standard deviation and $\Phi_N(\cdot)$ is the cumulative distribution function (CDF) for the standard normal random variable.

The supplier uses an adaptive base stock inventory policy, where the order p_t^* is placed according to

$$p_t^* = d_t + (L+K)(F_{t+1} - F_t). \tag{7}$$

Let $e_t^2 = \sum_{i=1}^{L+K} d_{t+i} - (L+K)F_{t+1}$. Similar to above, using the Graves (1999) approach, e_t^2 is normally distributed with $E[e_t^2] = 0$ and

$$\mathrm{Var}[e_t^2] = (L+K)\sigma^2\left(1 + \alpha(L+K-1) + \frac{\alpha^2(L+K-1)(2(L+K)-1)}{6}\right)$$

for $t \ge L+K$.

By using the approach in Dong and Lee (2003), the supplier's safety stock level, y_0^* is the y_0 that satisfies the following condition:

$$h_2 - (h_2+b_1)\Phi_N\left(\frac{-(y_0 - x_0^*)}{\hat{\sigma}}\right) + (h_1+b_1)\Phi_N\left(\frac{-(y_0 - x_0^*)}{\hat{\sigma}}, \frac{y_0}{\mathrm{Std}[e_t^2]}, -\frac{\hat{\sigma}}{\mathrm{Std}[e_t^2]}\right) = 0,$$

where $\hat{\sigma} = \sqrt{\mathrm{Var}[e_t^2] - \mathrm{Var}[e_t^1]}$ and $\Phi_N(x_1, x_2, \rho)$ denotes the CDF of the pair of standard normal random variables (x_1, x_2) with correlation ρ. The expected inventory and shortage costs per period, excluding the expected work-in-process (WIP) inventory costs per period, for this benchmark policy is in the Appendix. (The expected WIP inventory cost per period is equal to the expected demand over the production lead time multiplied by the WIP inventory holding cost per unit, thus, is a constant and not a factor in the optimization problem. We choose to exclude these inventory costs to facilitate a comparison among policies.)

We now consider our second measure, the standard deviation of the period-to-period production changes that result from this benchmark policy. In the benchmark policy, the supplier may face a raw material shortage, which means that the supplier's production start quantity in a given period may not be equal to the current order from the manufacturer. If the supplier is unable to produce the entire amount of the current order, then backorders will be produced in a future period. The supplier's net inventory at the end of period t is

$$\begin{aligned} y_t &= \left[(L+K)F_{t-K+1} + y_0^*\right] - \left[LF_{t-K+1} + x_0^*\right] - \sum_{i=1}^{K} q_{t-K+i}^* \\ &= (y_0^* - x_0^*) - e_{t-K}^2 + e_t^1. \end{aligned}$$

Recall that r_t denotes the production start quantity in period t at the supplier. In period t, the supplier desires to produce the quantity on backorder at the end of period $t-1$,

$$\max(-y_{t-1}, 0) = \max\left((e_{t-K-1}^2 - y_0^*) - (e_{t-1}^1 - x_0^*), 0\right)$$

plus the current order q_t^*; however, the supplier may not be able to produce this desired quantity because of raw material shortages in period t. The raw material shortage in period t is the quantity

$$\max(-y_t, 0) = \max\left((e_{t-K}^2 - y_0^*) - (e_t^1 - x_0^*), 0\right).$$

Thus,

$$\begin{aligned} r_t &= \max\left((e_{t-K-1}^2 - y_0^*) - (e_{t-1}^1 - x_0^*), 0\right) \\ &\quad + q_t^* - \max\left((e_{t-K}^2 - y_0^*) - (e_t^1 - x_0^*), 0\right). \end{aligned} \tag{8}$$

Let $\delta_t = r_t - r_{t-1}$.

$$\begin{aligned} \delta_t &= q_t^* - q_{t-1}^* + 2\max\left((e_{t-K-1}^2 - y_0^*) - (e_{t-1}^1 - x_0^*), 0\right) \\ &\quad - \max\left((e_{t-K}^2 - y_0^*) - (e_t^1 - x_0^*), 0\right) \\ &\quad - \max\left((e_{t-K-2}^2 - y_0^*) - (e_{t-2}^1 - x_0^*), 0\right). \end{aligned} \tag{9}$$

It can be shown that $E[\delta_t] = E[\Delta_t] = 0$ for $t \geq L + K + 2$. While the expression in (9) is complex, we can observe that $\mathrm{Var}[\delta_t]$ is independent of t for $t \geq L + K + 2$. One way to obtain an estimate of $\mathrm{Var}[\delta_t]$ is to numerically evaluate it; however, the calculations required are significant. Another way to obtain an estimate of $\mathrm{Var}[\delta_t]$ is through simulation, which is the approach that we will use later when comparing our proposed policy to the benchmark policy.

3.3. The Proposed Policy

In the following, we propose our inventory policy, which uses old forecasts to set base stock levels at the manufacturer.

3.3.1. The Manufacturer's Inventory Replenishment Policy. We assume that the manufacturer uses an adaptive base stock inventory policy, where the order-up-to level is determined from a forecast of demand. We propose the following ordering policy:

$$q_t(s) = d_t + L(F_{t+1-s} - F_{t-s}), \tag{10}$$

where $q_t(s)$ is the order placed by the manufacturer in period t when $s \geq 0$ is the age (in periods) of the forecast used by the manufacturer to set his base stock level. There are two components to the order quantity $q_t(s)$. The first component (d_t) replenishes the demand in the current period, while the second component ($L(F_{t+1-s} - F_{t-s})$) adjusts the base stock level. When $s = 0$, the policy in (10) is equivalent to the inventory control policy in Graves (1999).

The policy in (10) permits negative order quantities. However, similar to the discussion in the Appendix about the likelihood of negative demand, the probability of negative order quantities from the manufacturer is small under some reasonable assumptions.

We define $e_t^1(s) = \sum_{i=1}^{L} d_{t+i} - LF_{t+1-s}$, as the cumulative forecast error in period t of demand over the next L periods when $s \geq 0$ is the age (in periods) of the forecast used by the manufacturer to set his base stock level.

3.3.2. The Supplier's Inventory Replenishment Policy. We assume that the supplier also uses an adaptive base stock policy. Let $p_t(s)$ be the supplier's order quantity in period t when $s \geq 0$ is the age (in periods) of the forecast used by the manufacturer to set his base stock level. We propose the following policy for the supplier:

$$p_t(s) = q_t(s) + \sum_{i=1}^{K} G_{t,\,t+i}(s) - \sum_{i=1}^{K} G_{t-1,\,t-1+i}(s), \tag{11}$$

where $G_{t,\,t+i}(s)$ is the MMSE forecast of the manufacturer order for period $t+i$, $i \geq 1$ prepared in period t when $s \geq 0$ is the age (in periods) of the forecast used by the manufacturer to set his base stock level. Similar to $q_t(s)$, there are two components to the supplier's order quantity $p_t(s)$. The first component ($q_t(s)$) replenishes the "demand" seen by the supplier in the current period, while the second component $\left(\sum_{i=1}^{K} G_{t,\,t+i}(s) - \sum_{i=1}^{K} G_{t-1,\,t-1+i}(s)\right)$ adjusts the base stock level.

The policy in (11) permits negative quantities, which can represent returns from the supplier to its raw material vendor. However, similar to the discussion in the Appendix about the likelihood of negative demand, the probability of negative order quantities from the supplier is small under some reasonable assumptions.

We assume that $G_{t,\,t+i}(s)$ is prepared by the manufacturer and provided to the supplier for $i = 1, 2, \ldots, K$. To calculate $G_{t,\,t+i}(s)$, consider the expression q_{t+i} in (10). If $i > s \geq 0$, then the best forecast of $F_{t+i+1-s} - F_{t+i-s}$ is $F_{t+1} - F_{t+1} = 0$. In other words, if $i > s \geq 0$, then the adjustment in the base stock level that will occur when the manufacturer places his order in period $t+i$ is a random variable with mean zero. If $1 \leq i \leq s$, then the forecasts $F_{t+i+1-s}$ and F_{t+i-s} are known at time t, which means that the adjustment in the base stock level that will occur when the manufacturer places his order in period $t+i$ is known with certainty at period t. Thus, we see that the use of old forecasts by the manufacturer can reduce the uncertainty of the second component of the manufacturer's order (that is, the part that adjusts the base stock level).

In period t, the best forecast of d_{t+i} is F_{t+1}. Hence, for $s \geq 0$,

$$G_{t,\,t+i}(s) = \begin{cases} F_{t+1} + L(F_{t+i+1-s} - F_{t+i-s}) & \text{if } 1 \leq i \leq s \\ F_{t+1} & \text{otherwise.} \end{cases} \tag{12}$$

Thus, the manufacturer uses the most recent forecast of demand when communicating the forecast of the first component of its order quantity (that is, the part that replenishes demand). When $s = 0$, the policy in (11) is equivalent to the inventory control policy in Graves (1999).

We define $e_t^2(s) = \sum_{i=1}^{K}(q_{t+i}(s) - G_{t,\,t+i}(s))$ as the cumulative forecast error in period t of manufacturer orders across the next K periods, when $s \geq 0$ is the age (in periods) of the forecast used by the manufacturer to set his base stock level.

4. Setting 1—Decentralized Supply Chain with High Supplier Service Level

In this setting, we assume that the supplier's raw material safety stock provides a high service level to the manufacturer. This high service level assumption was made by Graves (1999) and it allows for a closed-form characterization of the inventory random variables at each stage in the supply chain. These characterizations of the inventory random variables allow for a good approximation of expected underage and overage costs at both the manufacturer and the supplier in this setting.

Let $x_t(s)$ represent the manufacturer's *net* inventory at the end of period t when the manufacturer uses forecasts that are old by $s \geq 0$ periods to set his base stock level. The inventory balance equation for this setting is $x_t(s) = x_{t-1}(s) - d_t + q_{t-L}(s)$ for $t = 1, 2, \ldots,$ where $x_0(s)$ is the initial inventory level (safety stock). Using the Graves (1999) approach, it can be shown that $x_t(s)$ is a normally distributed random variable with mean $x_0(s)$ and

$$\mathrm{Var}[x_t(s)] = L\sigma^2\left(1 + \alpha(L-1) + \frac{\alpha^2(L-1)(2L-1)}{6} + \alpha^2 Ls\right) \tag{13}$$

for $t \geq L + s$. It is easy to show that $x_t(s) = x_0(s) - e_{t-L}^1(s)$. Thus, $\mathrm{Var}[e_t^1(s)] = \mathrm{Var}[x_t(s)]$, $t \geq L + s$. That is, the variance of the manufacturer's inventory random variable is equivalent to the variance of the cumulative forecast error of demand over the lead time L.

Let $J_t(x_0(s))$ represent the manufacturer's expected underage and overage costs in period t when $x_0(s)$ is the safety stock level and the manufacturer uses forecasts old by $s \geq 0$ periods to set his base stock level. Then,

$$J_t(x_0(s)) = E\big[h_1(x_t(s))^+ + b_1(0 - x_t(s))^+\big]. \tag{14}$$

Minimizing (14) with respect to $x_0(s)$ results in the classic newsvendor problem with the solution $x_0^*(s) = z_1\,\mathrm{Std}[x_t(s)]$, where $z_1 = \Phi_N^{-1}(b_1/(h_1 + b_1))$. The manufacturer's optimal expected costs in period t are $J_t(x_0^*(s)) = \mathrm{Std}[x_t(s)][h_1 z_1 + (b_1 + h_1)I_N(z_1)]$, where $I_N(\cdot)$ is the loss function for the standard normal random variable. Because $\mathrm{Std}[x_t(s)]$ is independent of t (for $t \geq L + s$), we can drop the t index and denote the manufacturer's optimal expected costs per period as $J(x_0^*(s))$.

We now consider the supplier's inventory random variable. Let $y_t(s)$ represent the supplier's *net* inventory of raw material at the end of period t when the manufacturer uses forecasts that are old by $s \geq 0$ periods to set his base stock level. The inventory balance equation for this setting is $y_t(s) = y_{t-1}(s) - q_t(s) + p_{t-K}(s)$ for $t = 1, 2, \ldots,$ where $y_0(s)$ is the initial inventory level (safety stock). Again, using Graves' (1999) approach, it can be shown that $y_t(s)$ is a normally distributed random variable with mean $y_0(s)$ and

$$\mathrm{Var}[y_t(s)] = \begin{cases} \sigma^2 K\left(1 + \alpha(K-1) + \dfrac{\alpha^2(K-1)(2K-1)}{6}\right) \\ \quad + \sigma^2[(K-s)L\alpha(2 + L\alpha) \\ \qquad + L\alpha^2(K(K-1) - s(s-1))] & \text{for } s < K \\ \sigma^2 K\left(1 + \alpha(K-1) + \dfrac{\alpha^2(K-1)(2K-1)}{6}\right) & \text{for } s \geq K \end{cases} \tag{15}$$

for $t \geq K + L + s$. It is easy to show that $y_t(s) = y_0(s) - e_{t-K}^2(s)$. Thus, $\mathrm{Var}[e_t^2(s)] = \mathrm{Var}[y_t(s)]$, $t \geq K + L + s$. That is, the variance of the supplier's inventory random variable is equivalent to the variance of the cumulative forecast error of manufacturer orders over the lead time K.

For $s \geq 0$, let $H_t(y_0(s))$ represent the supplier's expected overage costs in period t when $y_0(s)$ is the safety stock level. Then

$$H_t(y_0(s)) = E[h_2(y_t(s))^+]. \tag{16}$$

Minimizing (16) with respect to $y_0(s)$ subject to a minimum service level β (which represents the probability of fulfilling the manufacturer's order), results in the solution $y_0^*(s) = z_2 \operatorname{Std}[y_t(s)]$, where $z_2 = \Phi_N^{-1}(\beta)$. The supplier's optimal expected costs in period t are $H_t(y_0^*(s)) = h_2 \operatorname{Std}[y_t(s)][z_2 + I_N(z_2)]$. Again, we can drop the t index and denote the supplier's optimal expected costs per period as $H(y_0^*(s))$.

To study how $J(x_0^*(s))$ and $H(y_0^*(s))$ behave as a function of $s \geq 0$, we first study $\operatorname{Std}[x_t(s)]$ and $\operatorname{Std}[y_t(s)]$.

Proposition 1. *For $s \geq 0$,*

(a) $\operatorname{Std}[x_t(s)]$ *is concave increasing in s.*

(b) $\operatorname{Std}[y_t(s)]$ *is concave decreasing in s for $s < K$ and unchanging in s for $s \geq K$.*

(c) $\operatorname{Std}[y_t(K)] = \operatorname{Std}[\sum_{i=1}^{K} d_{t+i} - KF_{t+1}]$.

Proof. See Appendix. □

Since $\operatorname{Std}[x_t(s)] = \operatorname{Std}[e_t^1(s)]$, we can conclude that the manufacturer's ability to predict demand over the lead time L worsens at a decreasing rate as $s \geq 0$ increases. And because $\operatorname{Std}[y_t(s)] = \operatorname{Std}[e_t^2(s)]$, we can conclude that the supplier's ability to predict manufacturer orders over the lead time K improves at an increasing rate as $s \geq 0$ increases for $s < K$. When $s = K$, the supplier's ability to predict manufacturer orders over the lead time K is equivalent to the manufacturer's ability to predict demand over the lead time K with current forecasts. *In other words, the amplification of the variance of the forecast error as one moves up the supply chain is eliminated when $s = K$.* Proposition 2 follows from Proposition 1.

Proposition 2. *For $s \geq 0$,*

(a) $J(x_0^*(s))$ *is concave increasing in s.*

(b) $H(y_0^*(s))$ *is concave decreasing in s for $s < K$ and unchanging in s for $s \geq K$.*

(c) *Let $TC(s) = J(x_0^*(s)) + H(y_0^*(s))$. The solution to the problem* $\min_{s\geq 0} TC(s)$ *is*

$$s^* = \begin{cases} 0 & \text{if } TC(0) \leq TC(K) \\ K & \text{otherwise.} \end{cases}$$

The manufacturer's expected inventory and shortage costs are increasing at a decreasing rate as $s \geq 0$ increases, while the supplier's expected inventory and shortage costs are decreasing at an increasing rate at $s \geq 0$ increases. The concavity of $J(x_0^*(s))$ and $H(y_0^*(s))$ for $s < K$, together with the result that $TC(s)$ is increasing in s for $s \geq K$, ensures that $TC(s)$ is minimized at either $s = 0$ or $s = K$.

Because the supplier provides a high service level, the supplier's production start stream, $\{r_t(s), t \geq 1\}$ can be approximated by the manufacturer's order stream, $\{q_t(s), t \geq 1\}$. Let $\Delta_t(s) = q_t(s) - q_{t-1}(s)$.

Proposition 3. (a) *For $s \geq 0$, $\Delta_t(s)$ is normally distributed with mean 0 and*

$$\operatorname{Var}[\Delta_t(s)] = \begin{cases} \sigma^2\big[(L\alpha+1)^2 + ((L-1)\alpha+1)^2\big] & \text{for } s=0 \\ \sigma^2\big[1+((L+1)\alpha-1)^2 + L^2\alpha^2\big] & \text{for } s=1 \\ \sigma^2\big[1+(\alpha-1)^2+2L^2\alpha^2\big] & \text{for } s\geq 2 \end{cases} \tag{17}$$

for $t \geq 2+s$.

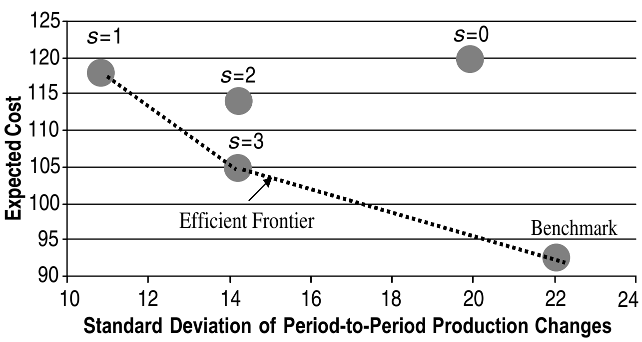
(b) *For $s \geq 2$, $\operatorname{Var}[\Delta_t(1)] \leq \operatorname{Var}[\Delta_t(s)] \leq \operatorname{Var}[\Delta_t(0)]$ for $t \geq 2+s$.*

Proposition 3 shows that the variance of the difference in orders is smallest when $s = 1$ and largest when $s = 0$.

The above analysis suggests that the manufacturer would prefer $s = 0$, however, the supplier would prefer $s = K$ from an expected inventory and shortage cost standpoint and $s = 1$ from a production variability standpoint. To explore how the supply chain members may evaluate the trade-off between costs and fluctuating production levels, we consider an example.

Example 1—Decentralized Setting. $\mu = 100$, $\sigma = 8$, $\alpha = 0.3$, $b_1 = 10$, $h_1 = 2$, $\beta = 0.98$, $h_2 = 1$, $K = 3$, and $L = 3$. The expected inventory and shortage costs are approximated by using $J(x_0^*(s))$ and $H(y_0^*(s))$ for $s = 0, 1, 2$, and 3. The standard deviation of the difference in production from period to period is approximated by using (17) for $s = 0, 1, 2$, and 3. In Figure 2, we plot the standard deviation of the difference in production from period to period versus the expected costs for $s = 0, 1, 2$, and 3 and the benchmark policy. For the benchmark case, the estimate of the standard deviation comes from a simulation of 1,000 periods repeated enough times to generate a 95% confidence interval with a range not exceeding plus or minus 1.2% of the estimate. (We also simulated the proposed policy for $s = 0, 1, 2$, and 3 to show that the approximations are appropriate. See the Appendix for the simulation results.)

Figure 2 **Example 1—Decentralized Setting**



From Figure 2, we see how our proposed policy can provide the supply chain members with some options regarding inventory and shortage costs and period-to-period production variability. The benchmark policy provides the lowest expected costs per period, however, it results in the highest period-to-period production variability. The proposed policy with $s=1$ provides the lowest period-to-period production variability, however, its expected inventory and shortage costs are 28% higher than that of the benchmark policy. While offering neither the lowest expected costs or period-to-period production variability, the proposed policy with $s=3$ provides a 36% reduction in the standard deviation of the period-to-period production change, while incurring only a 14% increase in the expected costs (in comparison to that of the benchmark policy). The supplier may be able to offer the manufacturer a side payment to participate in this policy. Note that when we compare the proposed policy using current forecasts ($s=0$) with those using old forecasts ($s>0$), both the expected supply chain inventory and shortage costs and the variability of period-to-period production changes can be reduced by selecting stale forecasts with $s=1, 2$, or 3.

5. Setting 2—Cooperative Supply Chain

In this setting, we relax the assumption of high supplier service levels. We further assume that the manufacturer and supplier restrict themselves to the adaptive base stock (order-up-to) policies in (10) and (11), respectively, and they set their safety stock levels to minimize the expected costs per period of the supply chain. We follow the approach used by Aviv (2002). The optimization problem can be represented as

$$\min_{y_0(s)}\Big\{E\big[h_2(y_t(s))^+\big] + \min_{x_0(s)} E\big[h_1(x_{t+L}(s))^+ + b_1(0-x_{t+L}(s))^+\big]\Big\}. \quad (18)$$

The supplier's net inventory at the end of period t is

$$\begin{aligned} y_t(s) &= \sum_{i=1}^{K} G_{t-K,\,t-K+i}(s) + y_0(s) - \sum_{i=1}^{K} q_{t-K+i} \\ &= y_0(s) - e^2_{t-K}(s). \end{aligned} \quad (19)$$

Because the supplier does not necessarily provide a high service level to the manufacturer, shortages at the supplier will impact the manufacturer's on-hand inventory. The manufacturer's on-hand inventory at the end of period $t+L$ is equal to the manufacturer's order-up-to point at the beginning of period $t+1$, less the amount of the supplier's backlog and less the total end-product demand over periods $t+1, t+2, \ldots, t+L$. That is,

$$\begin{aligned} x_{t+L}(s) &= LF_{t+1-s} + x_0(s) \\ &\quad - \max\big(e^2_{t-K}(s) - y_0(s), 0\big) - \sum_{i=1}^{L} d_{t+i} \\ &= x_0(s) - \max\big(e^2_{t-K}(s) - y_0(s), 0\big) - e^1_t(s). \end{aligned} \quad (20)$$

In Proposition 4, we show that $(e^2_{t-K}(s), e^1_t(s))$ has a bivariate normal distribution.

Proposition 4. *The joint distribution of* $e^2_{t-K}(s), e^1_t(s)$ *is bivariate normal.* $E[e^1_t(s)]=0$, *and* $E[e^2_{t-K}(s)]=0$. *For* $t\ge K+L+s$, $\mathrm{Var}[e^1_t(s)]$ *is per (13),* $\mathrm{Var}[e^2_{t-K}(s)]$ *is per (15), and* $\mathrm{Cov}[e^2_{t-K}(s), e^1_t(s)] = L\alpha\sigma^2 s(1+\alpha(s-1)/2)$.

Proof. See Appendix. □

Because the distribution of $e^2_{t-K}(s), e^1_t(s)$ is independent of $t\ge K+L+s$, we let $(\xi_2(s), \xi_1(s))$ be a pair of random variables that represent the process $\{e^2_{t-K}(s), e^1_t(s){:}\ t\ge K+L+s\}$.

The objective in the inner minimization problem in (18) is to find the safety stock level $x_0(s)$ that minimizes the expected costs at the manufacturer for a given value of $y_0(s)$. By substituting (20) into (18), the inner minimization problem results in a newsvendor problem with $x_0(s)$ representing the "order quantity" and $\xi_1(s)+\max(\xi_2(s)-y_0(s),0)$ representing the random "demand." Thus, we want to find the $x_0(s)$ that satisfies

$$\Pr\left(\left\{\xi_1(s)+\max(\xi_2(s)-y_0(s),0)\le x_0(s)\right\}\right)=\frac{b_1}{b_1+h_1}. \tag{21}$$

We can solve (21) for a given $y_0(s)$ by numerical evaluation. A line search on $y_0(s)$ can be done to complete the minimization problem in (18). (We note that the problem can also be solved using a simulation-based algorithm (see Aviv 2002). The authors used the simulation-based algorithm to greatly reduce the search region. The authors recommend this simulation-based algorithm rather than numerical evaluation because accurate results can be obtained and it can be performed on a spreadsheet, greatly simplifying the computations required.) For $s\ge 0$, the expected supply chain inventory and shortage costs per period (excluding the expected WIP inventory costs) resulting from the optimal safety stocks, $x_0^*(s)$ and $y_0^*(s)$ are

$$\begin{aligned}A(s)&=h_2E\left[(y_0^*(s)-\xi_2(s))^+\right]\\&\quad+h_1E\left[(x_0^*(s)-\max(\xi_2(s)-y_0^*(s),0)-\xi_1(s))^+\right]\\&\quad+b_1E\left[(-x_0^*(s)+\max(\xi_2(s)-y_0^*(s),0)+\xi_1(s))^+\right].\end{aligned} \tag{22}$$

We now consider our second measure, the standard deviation of the period-to-period production changes. Similar to the benchmark policy, in this setting, the supplier can face raw material shortages, which means that the supplier's production start quantity in a given period may not be equal to the current order from the manufacturer. Let $r_t(s)$ denote the supplier's production start quantity in period t. In period t, the supplier desires to produce the quantity on backorder, $\max(-y_{t-1}(s),0)=\max(e^2_{t-K-1}(s)-y_0^*(s),0)$ plus the current order $q_t(s)$, however, the supplier may not be able to produce this desired quantity due to raw material shortages in period t. The shortages in period t are the quantity $\max(-y_t(s),0)=\max(e^2_{t-K}(s)-y_0^*(s),0)$. Thus,

$$\begin{aligned}r_t(s)&=\max\left(e^2_{t-K-1}(s)-y_0^*(s),0\right)\\&\quad+q_t(s)-\max\left(e^2_{t-K}(s)-y_0^*(s),0\right).\end{aligned} \tag{23}$$

Let $\delta_t(s)=r_t(s)-r_{t-1}(s)$.

$$\begin{aligned}\delta_t(s)&=q_t(s)-q_{t-1}(s)+2\max\left(e^2_{t-K-1}(s)-y_0^*(s),0\right)\\&\quad-\max\left(e^2_{t-K}(s)-y_0^*(s),0\right)\\&\quad-\max\left(e^2_{t-K-2}(s)-y_0^*(s),0\right).\end{aligned} \tag{24}$$

It is easy to show that $E[\delta_t(s)]=E[\Delta_t(s)]=0$ for $t\ge K+L+s+2$. And it can be shown that $\mathrm{Var}[\delta_t(s)]$ is independent of t for $t\ge K+L+s+2$. One way to obtain an estimate of $\mathrm{Var}[\delta_t(s)]$ is to numerically evaluate it; however, the calculations required are significant. Another way to obtain an estimate of $\mathrm{Var}[\delta_t(s)]$ is through simulation. We will use the latter in subsequent examples.

We consider four examples. The first example considered, "Example 1—Cooperative Setting," is the same as "Example 1—Decentralized Setting" considered in §5.

Example 1—Cooperative Setting. $\mu=100$, $\sigma=8$, $\alpha=0.3$, $b_1=10$, $h_1=2$, $h_2=1$, $K=3$, and $L=3$.

Example 2. $\mu=100$, $\sigma=8$, $\alpha=0.1$, $b_1=10$, $h_1=2$, $h_2=1$, $K=3$, and $L=1$.

Example 3. Same as Example 2, except that $\alpha=0.5$.

Example 4. Same as Example 2, except that $b_1=100$.

In Table 1, we summarize the results of the expected supply chain inventory and shortage costs for these four examples. The number in parenthesis is the percentage increase in expected costs compared to that of the benchmark policy.

The benchmark policy and the proposed policy with $s=0$ both use the most current demand

Table 1 **Expected Supply Chain Inventory and Shortage Costs per Period**

Example	Benchmark	$s=0$	$s=1$	$s=2$	$s=3$
1	92.1	92.1 (0.0%)	93.8 (1.9%)	95.0 (3.2%)	95.1 (3.3%)
2	45.6	45.7 (0.2%)	45.7 (0.2%)	45.7 (0.2%)	45.7 (0.2%)
3	65.7	65.7 (0.0%)	67.1 (2.1%)	68.0 (3.5%)	68.2 (3.8%)
4	69.9	70.0 (0.1%)	70.1 (0.3%)	70.1 (0.3%)	70.2 (0.4%)

information when setting base stock levels. The difference between these two policies is that, in the benchmark policy, the supplier sets his base stock levels using the echelon inventory position, while in the proposed policy, the supplier sets his base stock levels using his local inventory position. From Table 1, we see negligible expected inventory and shortage cost increases from using the proposed policy with $s = 0$ compared to that of the benchmark policy. This result is not that surprising based on work by Gallego and Zipkin (1999). They consider several local policy heuristics for a multistage serial supply chain and find that local base stock policies can perform well.

From Table 1, we also see that the cost penalties for using old forecasts ($s = 1, 2, 3$) are small. (We note that if we had included the expected WIP inventory costs, the cost percentage differences would be substantially less.) To offer some insight into this counterintuitive result, we take a closer look at the inner minimization problem in (18). Recall that this problem is essentially a newsvendor problem with $x_0(s)$ representing the "order quantity" and $\xi_1(s) + \max(\xi_2(s) - y_0(s), 0)$ representing the random "demand." Notice that this "demand" is a function of both the supplier and manufacturer's cumulative forecast errors for a given $s \geq 0$. As s increases, the standard deviation of the supplier's cumulative forecast error decreases, while the standard deviation of the manufacturer's cumulative forecast error increases. Therefore, in this problem, the decision maker faces random "demand" that is composed of two components, one where the standard deviation decreases as s increases and another where the standard deviation increases as s increases.

Since the use of old forecasts to set base stock levels at the manufacturer impacts the ability of the manufacturer to fulfill customer demand, one might expect that the proposed policy with $s > 0$ would not perform well under high shortage costs. Surprisingly, we did not find this to be the case. In Example 4, we set the shortage cost $b_1 = 100$ (10 times that of Example 2), and similar to Example 2, we see almost no cost penalty of using old forecasts. Again, we point out the inner minimization problem in (18), where we see that the uncertain "demand" is not the end-product demand, but rather a random variable that is a function of both the supplier and manufacturer's cumulative forecast errors.

We expect that current demand information would be more beneficial as the demand process becomes more nonstationary, because as the process becomes more nonstationary, demand depends more on the most recent demand information. We do see that this is the case from our examples. The cost penalties of using old forecasts for Example 1 ($\alpha = 0.3$) and Example 3 ($\alpha = 0.5$) are higher than that of Examples 2 and 4 ($\alpha = 0.1$).

We now consider the potential benefit of the proposed policy, which is the reduction in the variance of the difference in production from period to period. We simulated Example 1 for 1,000 time periods to obtain an estimate of the standard deviation of the difference in production from period to period, and then repeated the simulation enough times so that we could generate a 95% confidence interval with a range not exceeding plus or minus 2% of the estimate. We simulated Example 2 for 2,000 time periods to obtain an estimate of the standard deviation of the difference in production from period to period, and then repeated the simulation enough times so that we could generate a 95% confidence interval with a range not exceeding plus or minus 1% of the estimate. (The reason for running each trial of Example 1's simulation for only 1,000 time periods is that in Example 1, $\alpha = 0.3$, and we wanted to avoid generating negative demand values. See discussion of this issue in the Appendix.) In Figure 3 (Example 1) and Figure 4 (Example 2), we plot the standard deviation of the difference in production

Figure 3 **Example 1—Cooperative Setting**

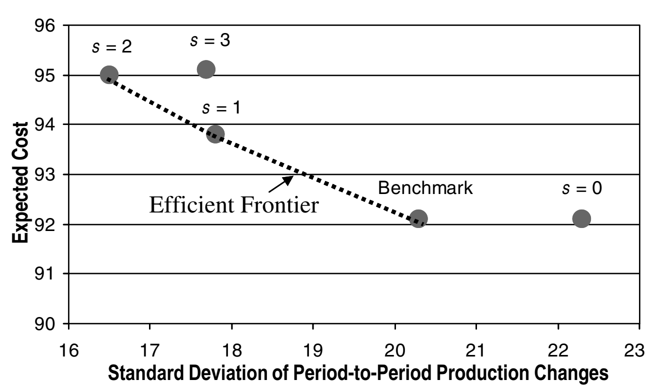
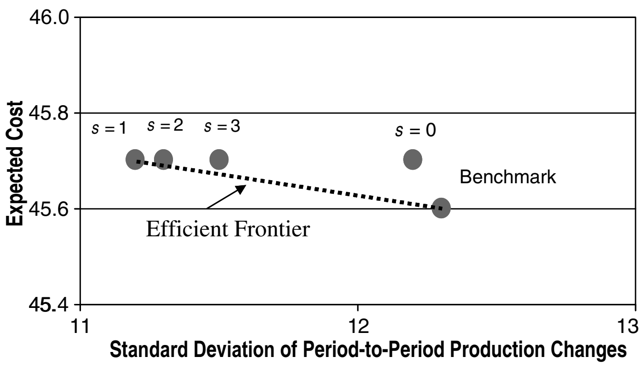


Figure 4 Example 2



from period to period versus the expected costs for $s = 0, 1, 2$, and 3 and the benchmark policy to illustrate the trade-off between expected inventory and shortage costs and the period-to-period production variability. In Example 1, the proposed policy with $s = 2$ can achieve an 18.8% decrease in the standard deviation of the period-to-period production changes in exchange for a 3.2% increase in expected inventory and shortage costs over that of the benchmark policy. In Example 2, the proposed policy with $s = 1$ can achieve a 9.1% decrease in the standard deviation of the period-to-period production changes with essentially no increase in expected inventory and shortage costs over that of the benchmark policy.

6. Conclusion

In this paper we consider a two-stage, multiperiod, supply chain where the demand process is nonstationary. Both stages use an adaptive base stock policy, and we study a proposed policy that uses old forecasts to set base stock levels at the manufacturer. We study this proposed policy in two different supply chain settings and compare the results to that of a benchmark policy, which minimizes the expected systemwide inventory and shortage costs under some reasonable assumptions.

In the decentralized setting, it is assumed that the supplier provides a high level of service to the manufacturer. In this setting, we find that our proposed policy can reduce both the expected supply chain inventory and shortage costs and the variability of period-to-period production changes compared to that of using current information in a decentralized setting. When compared to the benchmark policy, our policy results in higher expected inventory and shortage costs as expected, but it can considerably reduce the variability of period-to-period production changes. In the cooperative setting, where we relax the assumption of a high supplier service level, we find that our proposed policy can result in very small increases in the expected systemwide inventory and shortage costs compared to the benchmark policy, while reducing the variability of period-to-period production changes. Our findings enable supply chain managers to consider a trade-off between inventory and shortage costs and period-to-period production variability, without directly constraining production levels.

We point out that our proposed policy is just one way of using old forecasts to set base stock levels at the manufacturer. There certainly can be other approaches of using forecast information to offer benefits to the production facility without significantly increasing inventory and shortage costs. These are subjects of future research.

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Appendix

Discussion of the Likelihood of Negative Demand Occurrences

This demand process in (2) can result in negative demand. Graves (1999) shows that $\mathrm{Var}[d_{t+j} \mid F_t] = (1 + j\alpha^2)\sigma^2$ (where $\mathrm{Var}[\,]$ denotes the variance and F_t is the forecast generated in period $t-1$ for period t, after observing the end-item demand in period $t-1$) and suggests that a comparison of the mean with the standard deviation gives a good proxy for the likelihood that d_{t+j} will be negative. Note that if the factor $j\alpha^2$ is large, then $\mathrm{Std}[d_{t+j} \mid F_t]$ (where $\mathrm{Std}[\,]$ denotes the standard deviation) might be large in comparison to F_t. Our experience with simulating this demand process over 2,000 periods is that when α is small (say, $\alpha = 0.1$), negative demand rarely occurs. However, when α is larger (say, $\alpha = 0.5$), in some trial runs, negative demand never occurs; in other trial runs, negative demand occurs a few times; and in other trial runs, the demand process ends up staying in negative territory for a significant period of time! Reducing the number of periods of the simulation run can significantly reduce the occurrences of negative demand. For example, we found that negative demand rarely occurred with parameters $\mu = 100$, $\sigma = 8$, and $\alpha = 0.5$ over a simulation of 250 periods. Given this drawback in the demand process, some judgment is required as to applicability of this demand process to a given product.

Discussion of the Likelihood of Excess Inventory Occurrences in the Benchmark Policy

We discuss why the benchmark policy (§3.2) is unlikely to result in excess inventory occurrences under some reasonable assumptions. First, consider the manufacturer's problem in isolation, assuming that the manufacturer does not face any supply delays (that is, in each period, the manufacturer always receives his order from L periods ago). In this case, excess inventory occurs at the manufacturer in period t when q_t^* is negative. From (6), we see that the probability that q_t^* is negative is equivalent to the probability that $d_t + L\alpha\varepsilon_t$ is negative. Similar to our argument regarding negative demand (see above), the probability of negative orders from the manufacturer is small as long as L is not too large, σ is sufficiently smaller than μ and the time horizon is not too long.

Now consider the two-stage supply chain and the possibility of shortages at the supplier. We use the same argument as in Aviv (2001) to show that excess inventory occurrences are less likely to occur under the possibility of supplier shortages than when the manufacturer does not face any supply delays. Under the possibility of supplier shortages, excess inventory occurs at the manufacturer in period t when the following quantity is negative: the manufacturer's base stock level in period t less the inventory position in period t just prior to ordering, excluding those items that will not reach the manufacturer by period $t + L$ (because of supplier shortages). That is, excess inventory occurs at the manufacturer in period t when q_t^*, plus those items that will not reach the manufacturer by period $t+L$ is negative. Clearly, the excess inventory under supply shortages is no greater than that resulting from no supply delays.

Because the supplier orders from an unlimited supply of raw material, the supplier does not face any supply delays. Thus, excess inventory occurs at the supplier in period t when p_t^* is negative. From (7), we see that the probability that p_t^* is negative is equivalent to the probability that $d_t + (L+K)\alpha\varepsilon_t$ is negative. Again, as long as L and K are not too large, σ is sufficiently smaller than μ, and the time horizon is not too long, the probability of negative demand is small.

We also note that Johnson and Thompson (1975) study stationary and nonstationary demand processes as described by Box et al. (1994) (including the IMA process studied here) and show that in a single echelon system, excess inventory occurrences will not occur if we assume that demand is always positive, and that we can place limits on the demand in any period. They use these two assumptions to show that for a single echelon, a myopic policy is optimal in each period.

Expected Inventory and Shortage Costs for the Benchmark Policy

The expected supply chain inventory and shortage costs per period (excluding the expected WIP inventory costs) resulting from the optimal safety stocks, x_0^* and y_0^* are

$$\begin{aligned} A = {} & h_2 E\big[(y_0^* - x_0^* - \xi_2)^+\big] \\ & + h_1 E\big[(x_0^* - \max(\xi_2 - (y_0^* - x_0^*), 0) - \xi_1)^+\big] \\ & + b_1 E\big[(-x_0^* + \max(\xi_2 - (y_0^* - x_0^*), 0) + \xi_1)^+\big], \end{aligned}$$

where (ξ_2, ξ_1) is a pair of random variables that represent the process $\{e_{t-K}^2 - e_t^1, e_t^1\colon t \ge L + K\}$. It can be shown that the random variables ξ_2 and ξ_1 are independent and normally distributed. $E[\xi_1] = 0$ and $\mathrm{Var}[\xi_1] = \mathrm{Var}[e_t^1]$. $E[\xi_2] = 0$ and $\mathrm{Var}[\xi_2] = \mathrm{Var}[e_t^2] - \mathrm{Var}[e_t^1]$.

Proof of Proposition 1.

(a) For $s \ge 0$, $t \ge L + s + 1$, $\mathrm{Var}[x_t(s+1)] - \mathrm{Var}[x_t(s)] = L^2\alpha^2\sigma^2$, which shows that $\mathrm{Var}[x_t(s)]$ is linearly increasing in s and, thus, concave increasing in s. Since the square root function is a concave nondecreasing function, the result follows.

For $s \ge 0$, $t \ge K + L + s + 1$,

$$\mathrm{Var}[y_t(s+1)] - \mathrm{Var}[y_t(s)] = \begin{cases} -L\alpha\sigma^2(L\alpha + 2 + 2\alpha s) & \text{for } s < K \\ 0 & \text{for } s \ge K, \end{cases}$$

which shows that $\mathrm{Var}[y_t(s)]$ is decreasing in s for $s < K$ and unchanging in s for $s \ge K$.

(b) For $0 \le s < K$, $t \ge K + L + s + 1$,

$$\begin{aligned} & \big(\mathrm{Var}[y_t(s+1)] - \mathrm{Var}[y_t(s)]\big) \\ & \quad - \big(\mathrm{Var}[y_t(s)] - \mathrm{Var}[y_t(s-1)]\big) = -2L\alpha^2\sigma^2, \end{aligned}$$

which shows that $\mathrm{Var}[y_t(s)]$ is concave in s for $s < K$. Since the square root function is a concave nondecreasing function, the result follows.

(c) $y_t(K) = y_0(K) - \sum_{i=0}^{K-1}(1 + i\alpha)\varepsilon_{t-i} = y_0(K) - (\sum_{i=1}^{K} d_{t-K+i} - KF_{t-K+1})$ and the result follows. □

Simulation Results from Example 1—Decentralized Setting

In Tables 2 and 3, we compare the simulation results of Example 1—Decentralized Setting to that of the approximations derived in §4.

Table 2 Expected Supply Chain Costs for Example 1—Decentralized Setting

s	Estimate from simulation	95% Confidence interval	$TC(s)$
0	118.8	+/−1.4%	119.8
1	117.2	+/−1.8%	117.8
2	113.3	+/−2.3%	113.4
3	105.6	+/−3.0%	105.2

Table 3 Standard Deviation of Period-to-Period Production Changes for Example 1—Decentralized Setting

s	Estimate from simulation	95% Confidence interval	Std[$\Delta_t(s)$]
0	20.0	+/−0.9%	19.9
1	11.2	+/−1.7%	10.9
2	14.2	+/−1.2%	14.1
3	14.5	+/−1.6%	14.1

Proof of Proposition 4.

$$e_t^1(s) = \sum_{i=0}^{L-1}(1+\alpha i)\varepsilon_{t+L-i} + L\alpha\sum_{i=0}^{s-1}\varepsilon_{t-i} \quad \text{and}$$

$$e_{t-K}^2(s) = \sum_{i=0}^{s-1}(1+\alpha i)\varepsilon_{t-i} + \sum_{i=s}^{K-1}(1+i\alpha+\alpha L)\varepsilon_{t-i}.$$

Since $\varepsilon_t \sim N(0,\sigma^2)$ for $t=1,2,\ldots$, the random variables $e_t^1(s)$, $e_{t-K}^2(s)$ have a bivariate normal distribution. $E[e_t^1(s)]=0$. $E[e_{t-K}^2(s)]=0$. For $t\geq K+L+s$, $\text{Var}[e_t^1(s)]$ is per (13) and $\text{Var}[e_{t-K}^2(s)]$ is per (15).

$$\begin{aligned}\text{Cov}\big[e_t^1(s), e_{t-K}^2(s)\big] &= \text{Cov}\left[L\alpha\sum_{i=0}^{s-1}\varepsilon_{t-i}, \sum_{i=0}^{s-1}(1+\alpha i)\varepsilon_{t-i}\right]\\ &= L\alpha\sigma^2 s\left(1+\frac{\alpha(s-1)}{2}\right)\\ &\qquad \text{for } t\geq K+L+s. \quad \square\end{aligned}$$

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