



Manufacturing & Service Operations Management

Publication details, including instructions for authors and subscription information:
<http://pubsonline.informs.org>

Dynamic Assortment Customization with Limited Inventories

Fernando Bernstein, A. Gürhan Kök, Lei Xie

To cite this article:

Fernando Bernstein, A. Gürhan Kök, Lei Xie (2015) Dynamic Assortment Customization with Limited Inventories. *Manufacturing & Service Operations Management* 17(4):538-553. <http://dx.doi.org/10.1287/msom.2015.0544>

Full terms and conditions of use: <http://pubsonline.informs.org/page/terms-and-conditions>

This article may be used only for the purposes of research, teaching, and/or private study. Commercial use or systematic downloading (by robots or other automatic processes) is prohibited without explicit Publisher approval, unless otherwise noted. For more information, contact permissions@informs.org.

The Publisher does not warrant or guarantee the article's accuracy, completeness, merchantability, fitness for a particular purpose, or non-infringement. Descriptions of, or references to, products or publications, or inclusion of an advertisement in this article, neither constitutes nor implies a guarantee, endorsement, or support of claims made of that product, publication, or service.

Copyright © 2015, INFORMS

Please scroll down for article—it is on subsequent pages



INFORMS is the largest professional society in the world for professionals in the fields of operations research, management science, and analytics.

For more information on INFORMS, its publications, membership, or meetings visit <http://www.informs.org>

Dynamic Assortment Customization with Limited Inventories

Fernando Bernstein

Fuqua School of Business, Duke University, Durham, North Carolina 27708, fernando@duke.edu

A. Gürhan Kök

College of Administrative Sciences and Economics Koç University, 34450 Istanbul, Turkey, gkok@ku.edu.tr

Lei Xie

School of International Business Administration, Shanghai University of Finance and Economics,
200433 Shanghai, China, xie.lei@mai.shufe.edu.cn

We consider a retailer with limited inventory of identically priced, substitutable products. The retailer faces a market with multiple segments of customers that are heterogeneous with respect to their product preferences. Customers arrive sequentially, and the firm decides which subset of products to offer to each arriving customer depending on the customer's preferences, the inventory levels, and the remaining time in the season. We show that it is optimal to limit the choice set of some customers (even when the products are in stock), reserving products with low inventory levels for future customers who may have a stronger preference for those products. In certain settings, we prove that it is optimal to follow a threshold policy under which a product is offered to a customer segment if its inventory level is higher than a threshold value. The thresholds are decreasing in time and increasing in the inventory levels of other products. We introduce two heuristics derived by approximating the future marginal expected revenue by the marginal value of a newsvendor function that captures the substitution dynamics between products. We test the impact of assortment customization using data from a fashion retailer. We find that the potential revenue impact of assortment customization can be significant, especially when customer heterogeneity is high and when the products' inventory-to-demand ratios are asymmetric. Our findings suggest that assortment customization can be used as another lever for revenue maximization in addition to pricing.

Keywords: retailing; dynamic programming; pricing and revenue management; personalization; online retailing; customer-centric retailing

History: Received: February 12, 2014; accepted: March 21, 2015. Published online in *Articles in Advance* July 22, 2015.

1. Introduction

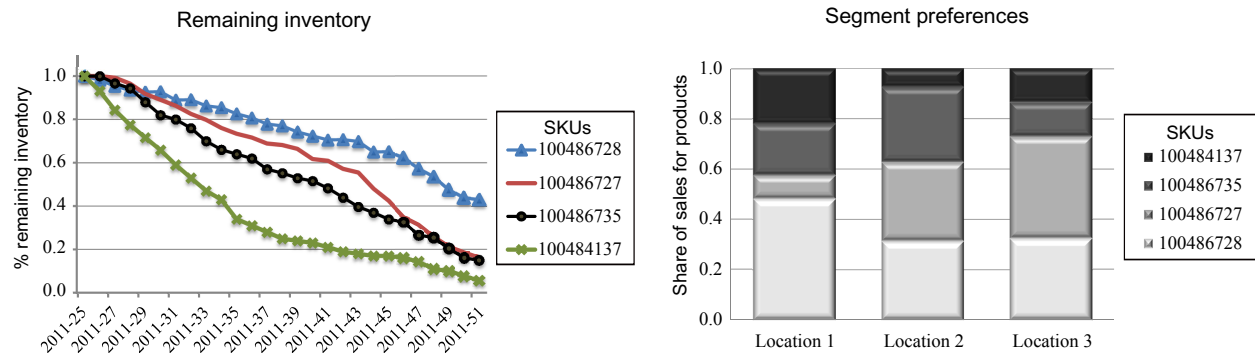
1.1. Motivation and Objective

Online retailers have been investing heavily in diverse technologies, such as sophisticated analytics and personalization tools. In contrast to traditional bricks-and-mortar stores, online retailers can not only easily collect personal information but also record their customers' purchase histories. As a result, online retailers are able to send personalized promotions or display customized recommendations to individual customers based on their personal information and browsing and purchasing history. This can effectively enhance the customers' shopping experience and influence their purchasing activities. Moreover, based on the customers' personal information and purchase history, a company can estimate the customers' preferences on several product characteristics. An online retailer can use this information to personalize the set of products made available to customers.

The goal of this paper is to explore the revenue impact of dynamic assortment customization. The

opportunity to engage in dynamic assortment customization arises as a mid-season tactic to increase revenue. At that stage in the selling season inventory imbalances tend to emerge, and the seller can employ a variety of strategies to stimulate demand and increase revenue (such as dynamic pricing, inventory reallocation, or cross-selling). We propose dynamic assortment customization as an alternative tactic to increase revenue (to enhance or in place of other strategies). To that end, we develop a stylized model of a multiproduct retailer and show that the optimal dynamic assortment policy involves offering *customized assortments* to individual customers (thus, rationing products to some customers) based on inventory conditions and the distribution of preferences of future customers. Furthermore, using a case study based on a large high-end fashion retailer's data set, we show that dynamic assortment customization has the potential to significantly increase revenue. The data suggests that demand, especially

Figure 1 (Color online) Inventory Depletion Curves and Preference Distribution by Location for Four Styles of Shoes



at the style level, is highly uncertain and inventory-demand imbalances start to emerge after the first few weeks of the season. The graph on the left in Figure 1 shows the remaining inventory over time for four styles of equally priced shoes sold by this retailer. (The curves are scaled by dividing the inventory of each product in each week by the initial inventory level for that product.) As can be noted, there are marked imbalances of inventory levels throughout the selling season. The graph on the right reports market shares for the four styles of shoes in three different geographical locations, based on data collected during the early part of the selling season. As the graph suggests, there are significant differences in the preferences of customers across regions.

1.2. Model and Results

We consider a firm that sells multiple products in a retail category. There are limited inventories of the products to sell over a finite selling season. The selling prices of the products are all equal (as in the case of different styles, colors, or sizes of the same garment). The customer base is heterogeneous and characterized by multiple segments with different product preference distributions. We consider settings in which the retailer can identify the segment of an arriving customer and customize the assortment to that particular customer without incurring additional cost. The customer then selects a product among those in the offered assortment or selects the no-purchase option. We formulate this as a dynamic assortment optimization problem in which the assortment decisions depend on the inventory levels, the current customer's segment, and the distribution of preferences of future customers.

Deriving the optimal dynamic assortment policy for settings with multiple products is complex because of the combinatorial nature of assortment problems. Indeed, research on choice-based network revenue management has generally focused on heuristic approaches to solve assortment problems. In this paper we analytically characterize the optimal policy

in specific settings. We find that if the retailer is not able to identify the types of arriving customers (and the preference profile of customers does not change over time), it is then optimal to offer every product to all customers. However, when such identification is possible, we show that the retailer may benefit from rationing products to some customer segments. In a setting with two products, we prove that a product is offered only if its inventory level is above a certain threshold. Hence, the firm has the potential to increase revenues by strategically restricting the set of product options it makes available to customers, even when all products are in stock and are identically priced. In other words, the company may conceal a product short on inventory in anticipation of future sales to other customers who may have a stronger preference for this product (and who are more likely to walk away if that product is not available).

1.3. Contributions of the Paper

Our paper is the first to propose the idea of assortment customization based on inventory conditions and in the presence of heterogeneous customer segments. We analytically characterize the optimal policy for the case of two products. The optimal policy involves rationing even when products' prices are equal. The analytical results in this paper suggest that rationing as a result of customer heterogeneity and supply conditions may be optimal in more general settings. Hence, we introduce two heuristics that are derived from the dynamic program used to characterize the optimal policy for the more stylized settings. The heuristics are derived by approximating the marginal expected revenue generated by each product in each period in the dynamic program by the marginal value of a newsvendor (single-period) function that captures the substitution dynamics between products. That is, the value of inventory of a product is assessed based on its total effective demand, which not only depends on the primary demand but also on the substitution demand that may spill over from other products that are low on inventory.

These heuristics capture the demand-supply conditions for each product (and in each period) based on newsvendor-type approximations. We utilize these heuristics to explore the revenue impact of assortment customization in cases with a large number of products.

We demonstrate the potential revenue impact of assortment customization with a case study based on data from Beymen—a privately held high-end fashion retailer in Turkey that operates online and offline channels. The company places orders from the wholesale arms of high-end fashion brands with firm purchasing commitments six to eight months in advance of the beginning of each season and then sells the products at full price until the clearance season. We use weekly item-level sales (units and prices) and inventory data for the category of women's shoes from the Fall–Winter 2011–2012 season. We demonstrate the potential impact of assortment customization by setting the parameters of our model based on the actual demand and inventory data and by comparing the expected revenue from assortment customization with the benchmark policy that involves offering all available products to any arriving customer in every period. We find that assortment customization can lead to significant benefits. Specifically, in this study, the revenue benefit from assortment customization can be as high as 5% relative to the benchmark policy.

1.4. Organization of the Paper

Section 2 provides a review of the literature. Section 3 describes the model. Section 4 examines the dynamic assortment customization problem. Section 5 presents the optimal policy for the case of two products and introduces heuristics for more general cases. Section 6 discusses the impact of assortment customization in a case study. Section 7 concludes the paper. All proofs are provided in the online appendix (available as supplemental material at <http://dx.doi.org/10.1287/msom.2015.0544>).

2. Literature Review

Our work is related to several streams of research. The first one is the literature on retail assortment planning, with papers focusing on assortment and inventory decisions for a single customer segment. Kök et al. (2009) provide a review of this literature. Van Ryzin and Mahajan (1999) derive the optimal assortment policy for a category with homogeneous products. Cachon et al. (2005) incorporate consumer search costs in a similar context. Kök and Xu (2010) study assortment and pricing decisions in retail categories with multiple subgroups of products. Smith and Agrawal (2000) discuss an optimization

approach for assortment selection and inventory management in a multi-item setting with demand substitution. Kök and Fisher (2007) describe a methodology for estimation of demand and substitution rates and for assortment optimization using data from a supermarket chain. Mahajan and van Ryzin (2001) and Honhon et al. (2010) optimize starting inventory levels for a model with dynamic customer substitution (i.e., customers choose from those products that are available at the time of their arrival). In our model, customers also engage in dynamic substitution, but the set of products displayed to each customer is a decision variable in our case. Caro and Gallien (2007), Rusmevichientong et al. (2010), Saure and Zeevi (2013), and Ulu et al. (2010) study dynamic assortment selection with demand learning during a single selling season. Caro and Martinez-de-Albeniz (2009) find that renewing the assortment frequently can allow a firm to charge higher prices. The models in this stream of research do not consider multiple customer segments with different product preferences; therefore, the assortment policy does not involve any form of customization. Kim et al. (2002) develop a methodology for estimating the product preferences of households and propose that Web retailers such as Net Grocer and Peapod could offer customized assortments to each household—rather than the full assortment—to reduce the search cost of customers, which has been shown to negatively influence sales.

A second related stream of research includes work on choice-based network revenue management. Zhang and Cooper (2005) study revenue maximization in a setting where customers choose from a set of parallel flights. Talluri and van Ryzin (2004b) describe a framework for choice-based network revenue management models with multiple products (itineraries) and components (flight-legs), where product prices can be different. The authors characterize the optimal assortment policy for settings in which all products share the same resource (aircraft capacity for one flight-leg). As in other revenue management papers with a single flight-leg, the optimal policy is a booking-limit based policy, under which some products with lower fare prices are not offered if the remaining capacity is low. In these models, the products command different prices but share the same resources. In contrast, each product variant has its own dedicated inventory in our setting.

For general choice-based network revenue management models, Gallego et al. (2004) and Liu and van Ryzin (2008) use a choice-based linear programming (CDLP) model to approximate the dynamic control problem. In addition, Liu and van Ryzin (2008) propose a dynamic programming decomposition heuristic and characterize the efficient sets that are used

in the optimal policy. Zhang and Adelman (2009) use an affine function to approximate the value function of the dynamic program, and Chen and Homem-de-Mello (2010) develop an approximation that consists of a sequence of two-stage stochastic programs with simple recourse. Van Ryzin Vulcano (2008a, b) study virtual nesting policies in a similar context where the demand process consists of a stochastic sequence of heterogeneous customers. Miranda Bront et al. (2009) show that the CDLP model of the assortment problem with multiple segments is NP-hard and propose a column generation algorithm. Rusmevichientong and Topaloglu (2012) propose a robust formulation of the assortment optimization problem. The above papers consider multiple segments characterized by different choice probabilities for the different products, and these products are sold at different prices (i.e., fare-route options). However, in these papers, the decision maker cannot observe the type of an arriving customer and therefore, at any given time, all customers are offered the same assortment (which is a list of fare class and route combinations). Hence, customization is not possible in those settings, and the optimal assortment decisions are based on aggregate choice probabilities across segments, inventory levels, and price differences between products.

Assortment customization has begun to attract attention in academic research and in industry. Golrezaei et al. (2014) study an assortment customization problem similar to ours and develop an inventory-balancing algorithm that minimizes the asymptotic worst-case gap with an upper bound. Lederman and Saure (2013) consider a hierarchical customization problem in which a customer is offered product assortments in an incremental fashion.

A third stream of papers examines companies' strategies to increase revenue, including dynamic pricing, cross-selling, and product upgrading. Netessine et al. (2006) study a retailer with limited inventories that engages in cross-selling by dynamically selecting product packages to offer to each arriving customer (the packages consist of the specific product requested by the customer plus a so-called packaging complement). For each arriving customer, the firm makes a decision regarding the packaging complement and the price. Shumsky and Zhang (2009) analyze a dynamic capacity allocation problem in a setting with multiple demand classes and equal number of products that are differentiated by their quality. The goal of the paper is to determine the optimal allocation of capacity, which may involve upgrading (so-called one-way substitution). The paper evaluates the value of an optimal upgrading policy. The authors develop several heuristics and identify those that are effective in their setting. In a single product setting, Aydin and Ziya (2009) consider personalized dynamic

pricing after receiving a signal about each customer's willingness-to-pay. Fudenberg and Villas-Boas (2006) provide a review of the literature on personalized pricing. Personalized pricing is also closely related to price discrimination, which has been studied extensively in the marketing literature. Price discrimination is achieved by offering a vertically differentiated product line (Mussa and Rosen 1978) and by offering product bundles (e.g., Fay and Xie 2008). Usually, in these settings, a static assortment is offered to all customers.

The ability of a company to limit the assortment to its customers is a form of inventory rationing. Therefore, our paper is also related to the stream of research on this topic. Ha (1997a) considers a single-item, make-to-stock production system with several demand classes (characterized by the different prices they are willing to pay) and lost sales, and demonstrates that the optimal policy is characterized by rationing levels for each demand class. Ha (1997b) studies a similar system with two demand classes and backordering. de Véricourt et al. (2002) extend this model to a setting with multiple demand classes characterized by different backorder penalty costs.

3. Model Formulation

We consider a retailer that sells a set of identically priced product variants within a retail category over a finite selling season. The retailer decides an assortment to offer to each arriving customer from a set of horizontally differentiated products, denoted by $\mathcal{N} = \{1, \dots, N\}$. The selling season has T periods. There is no replenishment during the season. The initial amount of stock for all products is denoted by an N -dimensional vector $\mathbf{y}_0 = (y_{01}, \dots, y_{0N})$, and \mathbf{y} denotes a generic vector of inventory levels. This model is applicable, for example, to short-life-cycle products with long procurement lead times. In these settings, dynamic assortment customization arises as a mid-season tactic to stimulate demand and increase revenues.

There are M customer segments characterized by different product preferences. These segments are essentially customer clusters with similar purchase histories as described in Linden et al. (2003). We model customer preferences using a mixed logit model. That is, each arriving customer belongs to a segment with a certain probability, and the choice process of all customers in a segment follows a specific multinomial logit (MNL) model. (Mixed logit models are quite flexible in modeling demand elasticities and do not suffer from some of the main criticisms of the MNL model, such as the independence of irrelevant alternatives assumption.) Given an assortment $S \subset \mathcal{N}$, the utility derived from choosing product $i \in S$

for a customer in segment m is $u_{mi} + \xi_{mi}$, where u_{mi} is the expected utility derived from product i and ξ_{mi} is a random variable representing the heterogeneity of utilities across customers in the same segment. In addition, customers can always choose not to purchase any product, receiving a utility $u_{m0} + \xi_{m0}$. Each customer chooses the product that offers the maximum utility. We assume that ξ_{mi} are i.i.d. random variables following a Gumbel distribution with mean zero and variance $\pi^2/6$. The probability of a customer choosing product i that arises from this utility maximization problem is given by

$$q_{mi}(S) = \frac{\theta_{mi}}{\sum_{j \in S} \theta_{mj} + \theta_{m0}}, \quad i \in S \cup \{0\},$$

where $\theta_{mi} = e^{u_{mi}}$. See Anderson et al. (1992) for more details on the MNL model and K  k et al. (2009) for a comparison of the MNL model with other demand models. We refer to θ_{mi} as a segment m customer's preference for product i , and we let $\Theta_m = (\theta_{m1}, \theta_{m2}, \dots, \theta_{mN})$ be the preference vector of segment m . Without loss of generality, we set $\theta_{m0} = \theta_0 > 0$ for all m ; i.e., all segments have the same preference for the no-purchase option, unless otherwise stated. We assume that the retailer knows the preference vector for each customer segment. The retailer is able to estimate these preferences based on the customers' purchase history. Mixed logit models are commonly used by retailers and marketing firms to identify multiple latent customer segments in the customer base and to estimate purchasing behavior for each segment (see, e.g., Gupta and Chintagunta 1994, Wedel and Kamakura 1998). A common approach in online retailing is collaborative filtering (Linden et al. 2003), which measures similarity of customers to each other based on past history and infers preferences of an arriving customer for the category of interest.

We consider a Poisson arrival process and assume that at most one customer arrives in each period. The sequence of events in each period is as follows: At the beginning of the period, a customer arrives with probability λ and the arriving customer belongs to segment m with probability ρ_m , with $\sum_{i=1}^M \rho_i = 1$. Because of the identification process that takes place upon arrival, the retailer has perfect information on the customer's segment. The retailer offers an assortment (subset of the available products) to the customer. Next, the customer makes a purchasing decision according to the choice process, and the revenue is received if a product is sold. One can allow for an additional generic segment with product preferences matching those of the general population. If there is no sales history or information on the identity of a particular customer, this customer would be assumed to belong to this generic segment.

For tractability and to isolate the effects of customer heterogeneity and limited inventories, we focus on a model with identical prices for all products, denoted by p . This is usually the case in apparel, e.g., for items of different colors and sizes or for similar product styles. In a setting with nonidentical prices, there is a clear incentive for rationing (e.g., not offering a lower-priced product at certain levels of inventory) even with a homogeneous customer base. This additional incentive further complicates the derivation of analytical results in settings with nonidentical prices.

We assume that a product's price is the same for all customers and remains constant over time; that is, we do not consider customized pricing of individual products or dynamic pricing policies. It is a matter of debate whether customized pricing is legal with respect to antitrust laws (Ramasastry 2005). Phillips (2005) reports ethical, fairness, and legal concerns that are associated with dynamic pricing. Finally, without loss of generality, we assume that the salvage value for unsold units at the end of the season is zero. We use the following notation throughout the paper. We let e_i denote the i th unit vector. In addition, we let $\bar{S}(\mathbf{y})$ be the set of products with positive inventory and denote the cardinality of a set S as $|S|$.

4. The Dynamic Assortment Optimization Problem

In this section, we examine conditions under which it may be optimal to offer customized assortment. Our findings suggest that it is optimal to ration inventory when the customer base is heterogeneous and there is limited inventory of the available products.

Define the value function in period t as $V_t(\mathbf{y} | m)$, given the vector of inventory levels \mathbf{y} and that the customer arriving in this period is of segment m . Taking expectation across all customer segments, the value function at the beginning of period t is given by

$$V_t(\mathbf{y}) = \sum_{m \in \mathcal{M}} \rho_m V_t(\mathbf{y} | m).$$

Provided that the current arriving customer is in segment m , the goal is to select a customized assortment for each arriving customer to maximize revenue throughout the selling season. Thus, the optimality equation is given by

$$V_t(\mathbf{y} | m) = \max_{S \subset \bar{S}(\mathbf{y})} \left[\sum_{i \in S} \lambda q_{mi}(S) (p + V_{t+1}(\mathbf{y} - e_i)) + \lambda q_{m0}(S) V_{t+1}(\mathbf{y}) \right] + (1 - \lambda) V_{t+1}(\mathbf{y}). \quad (1)$$

The term inside the brackets is the value function for an arriving customer in segment m (an arrival occurs with probability λ). For a given assortment S , this

term accounts for the probability of selling one unit of product $i \in S$, earning a revenue of p from the sale, plus the revenue-to-go function in period $t + 1$ evaluated at the current inventory level minus the unit sold in period t . The term also accounts for the possibility that the customer does not make a purchase, in which case the revenue is the profit-to-go function in period $t + 1$ evaluated at the current vector of inventory levels. We maximize this term over all possible subsets of variants with positive inventory. The last term is the future value function if no customer arrives (with probability $1 - \lambda$). Because the product prices are equal, it is optimal to sell as many units of any product as possible throughout the selling season. Let $S_{mt}^*(\mathbf{y})$ denote the optimal assortment for a segment m customer at time t given inventory levels \mathbf{y} . The total optimal revenue over the selling season is given by $V_1(\mathbf{y}_0)$. The terminal condition of this dynamic program is the value function in period T . Because there are no more customers beyond the last period, the optimal policy is to offer all products with positive inventory to any arriving customer. Therefore, $V_T(\mathbf{y} | m) = \sum_{i \in \tilde{S}(\mathbf{y})} \lambda q_{mi}(\tilde{S}(\mathbf{y}))p$.

This formulation leads to a dynamic program with an N -dimensional state space and a large action space. (For each segment, there are $2^{|\tilde{S}(\mathbf{y})|}$ possible assortments that can be offered.) Thus, the above dynamic assortment optimization problem is intractable for large N . Note that the myopic solution to this dynamic program (which maximizes the current period reward ignoring the impact on future revenues) is to offer all products in every period, thus maximizing the probability of sales to any arriving customer. This policy serves as a benchmark for the value of dynamic assortment customization. We next discuss general properties of the optimal policy.

For $i \in \tilde{S}(\mathbf{y})$, define $\Delta_{t+1}^i(\mathbf{y}) = V_{t+1}(\mathbf{y}) - V_{t+1}(\mathbf{y} - \mathbf{e}_i)$ as the marginal expected revenue generated by the y_i -th unit of inventory of product i in period $t + 1$. Clearly, $0 \leq \Delta_{t+1}^i(\mathbf{y}) \leq p$. We rewrite the optimality equation in (1) as

$$V_t(\mathbf{y} | m) = \max_{S \subset \tilde{S}(\mathbf{y})} \left\{ \sum_{i \in S} \lambda q_{mi}(S)(p - \Delta_{t+1}^i(\mathbf{y})) \right\} + V_{t+1}(\mathbf{y}). \quad (2)$$

If product i is offered and sold in period t , then the revenue consists of the price p minus the lost opportunity revenue of selling this unit after period t , given by $\Delta_{t+1}^i(\mathbf{y})$. We denote the effective marginal price of product i in period t as $p_t^i(\mathbf{y}) = p - \Delta_{t+1}^i(\mathbf{y})$ if $i \in \tilde{S}(\mathbf{y})$ and note that $p_t^i(\mathbf{y}) = 0$ if $i \in \mathcal{N} \setminus \tilde{S}(\mathbf{y})$. Consider an ordering of the products in period t given inventory levels \mathbf{y} so that $p_t^{i_1}(\mathbf{y}) \geq \dots \geq p_t^{i_N}(\mathbf{y})$. Based on this ordering, we define a set consisting of the products with the largest effective marginal prices, given by $A_k(\mathbf{y}) = \{i_1, \dots, i_k\}$. The next result shows

that the optimal assortment for each customer segment is restricted to one of N possible sets $A_i(\mathbf{y})$.

LEMMA 1. *Given inventory levels \mathbf{y} in period t , the optimal assortment for a segment m customer is given by $S_{mt}^*(\mathbf{y}) \in \{A_1(\mathbf{y}), \dots, A_N(\mathbf{y})\}$.*

Lemma 1 is similar to Theorem 1 in Talluri and van Ryzin (2004a), although our setting involves an N -dimensional state-space (representing the separate inventories for each product) as opposed to a single resource in their setting. Moreover, our formulation also involves customized sets offered to each customer type (thus, the dependence on m in the optimal assortment set). Lemma 1 is helpful in the numerical study because it allows us to reduce the size of the action space from 2^N to N . However, Lemma 1 is ambiguous as to whether an arriving customer of segment m is offered all available products or only a subset of the available products, i.e., $S_{mt}^*(\mathbf{y}) = A_k(\mathbf{y})$ for $k < N$. In the remainder of this section, we examine conditions under which each of these assortment decisions may be optimal. To that end, we first discuss settings in which it is optimal to offer all products to an arriving customer.

PROPOSITION 1. (i) *Consider a setting in which it is not possible to segment customers or to identify the segment of arriving customers. Then, in every period, it is optimal to offer any arriving customer all products with positive inventory.* (ii) *If such identification is possible, but a single product is available, then it is optimal to offer this product to all arriving customers.* (iii) *In general, if the inventory level of product i in period t is large relative to the remaining time horizon, i.e., $y_i \geq T - t + 1$, then it is optimal to offer this product to all customers.*

When the firm is unable to identify the preference profile of arriving customers, all future customers have the same preference in expectation, so the firm cannot benefit from reserving a product in anticipation of future sales. Thus, it is optimal to offer all products. Thus, part (i) shows that heterogeneity in the customer base is a key contributor to customized assortments. Moreover, this result emphasizes the requirement that the retailer has the ability to identify different customer types upon arrival. Parts (ii) and (iii) indicate that assortment customization is relevant when there are multiple products and there is a limited amount of inventory of some of the available products. We next present an example to show that assortment customization may indeed be optimal when the customer base is heterogeneous and there are limited inventories.

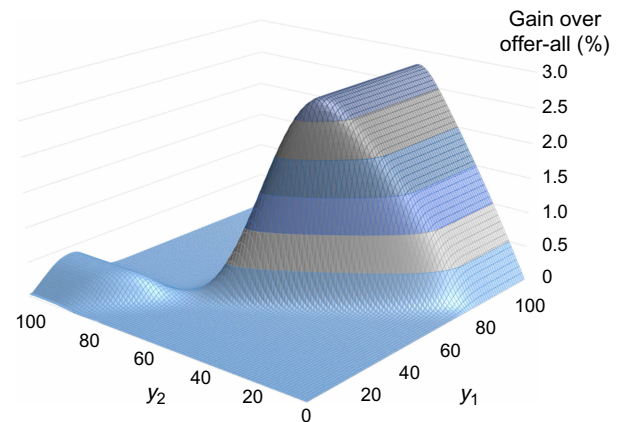
EXAMPLE 1. Consider the second to last period before the end of the horizon, i.e., $t = T - 1$, and let $\mathbf{y}_{T-1} = (y_1 = 1, y_2 = 2)$. The effective marginal prices in period $T - 1$ are $p_{T-1}^1(1, 2) = p - (V_T(1, 2) -$

$V_T(0, 2) = p - (V_T(1, 1) - V_T(0, 1))$ and $p_{T-1}^2(1, 2) = p - (V_T(1, 1) - V_T(1, 1)) = p$. (Note that $V_T(1, 2) = V_T(1, 1)$ and $V_T(0, 2) = V_T(0, 1)$, because the firm can sell at most one unit in the last period.) It follows from Lemma 1 that the optimal assortment in period $T - 1$ is either $\{2\}$ or $\{1, 2\}$ because $p_{T-1}^1(1, 2) < p_{T-1}^2(1, 2) = p$. Suppose that a customer from segment 2 arrives in period $T - 1$, and consider possible sample paths based on the realization of her preferences for the three options—purchase product 1, purchase product 2, or no purchase. If the customer's highest realized preference is either to purchase product 2 or the no-purchase option, then rationing product 1 has no effect on current or future revenues. Consider a sample path in which the customer's highest realized utility is that of product 1. If the offered assortment is $S_{2,T-1} = \{1, 2\}$, then the customer buys product 1, yielding a revenue of $p + V_T(0, 2)$. If the offered assortment is $S_{2,T-1} = \{2\}$, then she either buys product 2 with probability $\theta_{22}/(\theta_0 + \theta_{22})$, yielding a revenue of $p + V_T(1, 1)$, or chooses the no-purchase option, yielding a revenue of $V_T(1, 2)$. Hence, the assortment $S_{2,T-1} = \{1, 2\}$ yields a revenue of $p + V_T(0, 1)$ and $S_{2,T-1} = \{2\}$ yields a revenue of $p(\theta_{22}/(\theta_0 + \theta_{22})) + V_T(1, 1)$. Under this sample path, and with $S_{2,T-1} = \{2\}$, the firm loses some revenue in the current period, but it gains $V_T(1, 1) - V_T(0, 1)$ in the last period. Thus, in expectation, depending on the choice probabilities and the size of the segments, $S_{2,T-1} = \{2\}$ may be the optimal solution—implying rationing product 1. This is the case, for example, when θ_{22} is high relative to θ_0 or when segment 1 is relatively large (ρ_1 is close to one). In such cases, it is optimal to direct the segment 2 customer to buy product 2, reserving product 1 for the last period.

This example reveals how the presence of heterogeneity and limited inventories may result in product rationing, even in the absence of differences in prices. Based on the data from Beymen, we next show an example with two products and three customer segments that illustrates the magnitude of the benefits associated with assortment customization.

EXAMPLE 2. We consider an example with two products: SKUs 100486727 and 100486728 (see Figure 1). We calculate reliable estimates of popularity of both items (preference vectors) based on sales data; we defer these calculations to   6. Segmentation is based on the customers' locations. Figure 2 shows the percentage revenue increase caused by assortment customization relative to the offer-all policy as a function of the inventory levels. As can be seen from the graph, when the inventory levels of both products are large, the percentage revenue impact is small. In that case, the retailer is less likely to run out of stock, so there is no need for rationing inventory. Similarly,

Figure 2 (Color online) Percentage Revenue Increase Caused by Assortment Customization Relative to the Benchmark Policy That Involves Offering All Available Products to All Customers



Note. Parameters: $T = 100$, $p = 2,385$ TL, $\lambda = 1$, $\rho = (0.57, 0.15, 0.28)$, $\theta_1 = (1.68, 0.33)$, $\theta_2 = (1, 1)$, $\theta_3 = (0.8, 1.05)$, $\theta_0 = 0.2$.

when inventory levels of both products are low relative to the remaining demand in the season (low values of y_1 and y_2 in the graph), there is enough time to sell all units, so both products are likely to be offered in the optimal solution, and again the revenue impact is small. The revenue impact is more significant in the areas where the inventory level of one product is relatively high, whereas the inventory level of the other product is relatively low (a maximum of 2.8% in this example).

Example 2 highlights some key aspects of dynamic assortment customization. First, it can lead to substantial benefits. Because retailers usually operate with low profit margins, even a small percentage increase in revenue can have a significant impact on net profit. Also, whether or not a product is included in the offered assortment depends on the inventory level of that product relative to the inventory level of other products and relative to the demand conditions in the remaining selling season. We next present a structural result that formalizes these observations.

5. Structural Result and Heuristics

In this section, we first characterize the optimal dynamic assortment policy in a setting with two products and two customer segments. The analysis of this setting allows us to generate insights regarding the role of the products' inventory levels and the interplay of demand-supply conditions on the assortment decision. We next build on the dynamic program formulation of the general problem to construct two heuristics—so-called Sub_i and Sub_0 —that apply in general settings. These heuristics capture the demand-supply dynamics present in the optimal solution for the simpler setting. Moreover, the heuristic

Sub_0 generates a dynamic assortment customization policy that is similar to the optimal policy in a system with two products.

5.1. Optimal Policy: The Case of Two Products and Two Customer Segments

Consider a stylized setting with two products, two customer segments, and identical prices. The preference vectors for customer segments 1 and 2 are given by $(\theta_{11}, \theta_{12})$ and $(\theta_{21}, \theta_{22})$, respectively. The scenarios of interest are those in which the range of preferences for the two segments have some overlap, i.e., $\theta_{11} \geq \theta_{21}$ and $\theta_{22} \geq \theta_{12}$. Theorem 1 shows that the optimal policy involves inventory rationing and characterizes properties of the optimal solution.

THEOREM 1. *Consider a setting with $N = M = 2$ and preference vectors with $\theta_{11} \geq \theta_{21}$, $\theta_{22} \geq \theta_{12}$, and $\theta_{11} - \theta_{12} \geq |\theta_{21} - \theta_{22}|$. If both products are available in period t , then for a segment i customer, there exists a threshold level $y_{ji}^*(y_i)$ such that if $y_j \geq y_{ji}^*(y_i)$, then $S_{it}^*(y) = \{1, 2\}$; if $y_j < y_{ji}^*(y_i)$, then $S_{it}^*(y) = \{i\}$. Moreover, the threshold $y_{ji}^*(y_i)$ is increasing in y_i for $i \neq j$ and decreasing in t .*

The optimal policy in this setting is characterized by a set of thresholds y_{ji}^* and y_{ji}^* in each period t . A customer from segment i (who has a stronger preference for product i) is offered product j if the inventory level of that product is large enough. Otherwise, the customer is only offered product i . If the inventory level of product j is low, then it may be optimal to reserve that inventory for future arriving customers of segment j , because those customers are more likely to leave without purchasing any product if product j is not available. Theorem 1 suggests that the assortment decision is driven by the products' relative inventory levels and by the interplay of demand-supply conditions. Indeed, the result shows that the thresholds are increasing in the inventory level of the other product; i.e., given inventory levels y_i and y_j , if product i is offered to segment j , then it is also optimal to offer product i to segment j for lower inventory levels of product j . Thus, for a given inventory level of product i , it is more likely to offer that product to an arriving customer of segment j when the inventory level of product j is low. In this case, it is optimal to induce some segment j customers to substitute product j by product i to reduce the likelihood of running out of stock of product j in the future. On the other hand, when the inventory level of product j is high, it may be optimal to offer only product j to an arriving customer of segment j , thereby increasing the demand for that product. Theorem 1 also indicates that the interplay of demand-supply conditions—and not just the inventory levels—is relevant to the assortment customization decision. Indeed, the thresholds decrease over time—as time increases, the remaining

forecasted demand of all products decreases, changing the relative balance of demand and supply. In particular, less rationing occurs as time approaches the end of the selling season, because there is less concern about future sales; therefore, it is optimal to sell as many units as possible.

5.2. Heuristics

We next explore a general setting with multiple products and multiple customer segments. Because this problem is intractable because of the combinatorial aspect of assortment customization, we propose two heuristics. These heuristics are used in the case study discussed in §6.

The heuristics build on the dynamic program presented in §4. Recall that the optimality equation in period t is given by

$$V_t(y | m) = \max_{S \subset \tilde{S}(y)} \left\{ \sum_{i \in S} \lambda q_{mi}(S) (p - \Delta_{t+1}^i(y)) \right\} + V_{t+1}(y).$$

The goal of these heuristics is to approximate $\Delta_{t+1}^i(y)$ to solve the maximization problem efficiently. To that end, we approximate the marginal expected revenue generated by the y_i -th unit of inventory of product i in period $t + 1$ by the marginal value of a single-period newsvendor function that takes into account the potential future demand for this product and captures the substitution dynamics between products (i.e., considers spillover demand when products are out of stock). In particular, when a product is sold out, a portion of the excess demand substitutes the unavailable product with one that is in stock. Let y^t be the vector of inventory levels available at time t . We define

$$d_j^t = \lambda \sum_{m \in \mathcal{M}} \rho_m \frac{\theta_{mj}}{\sum_{k \in \tilde{S}(y^t)} \theta_{mk} + \theta_0},$$

and let D_j^t be a Poisson distributed random variable with rate $d_j^t * (T - t)$. Following the problem studied in Netessine and Rudi (2003), we model the first-order substitution demand (or effective demand) in period t as $D_i^{tS} = D_i^t + \sum_{j \in \tilde{S}(y^t) \setminus \{i\}} \alpha_{ji}^t (D_j^t - y_j^t)^+$. The coefficient α_{ji}^t approximates the fraction of customers that may substitute product i by product j if product i is not available. We define these fractions as

$$\alpha_{ji}^t = \sum_{m \in \mathcal{M}} \rho_m \left(\frac{\theta_{mj}}{\sum_{k \in \tilde{S}(y^t) \setminus \{i\}} \theta_{mk} + \theta_0} \right) \quad \text{for all } j \neq i.$$

Consistent with the MNL choice probabilities, the quantity in parentheses is the proportion of customers in segment m that chooses product j from the set $\tilde{S}(y^t) \setminus \{i\}$ of products available at time t when product i is excluded from the assortment. The approximate marginal expected revenue is given by the

derivative of $p * \sum_{i \in \bar{S}(\mathbf{y}^t)} E[\min\{D_i^{tS}, y_i^t\}]$ with respect to y_i^t , i.e., the marginal value of the y_i^t -th unit of inventory in a multiproduct newsvendor model with substitution. That is, we define

$$\begin{aligned} \tilde{\Delta}_{t+1}^i(\mathbf{y}^t) \\ = p * \left[\Pr(D_i^{tS} > y_i^t) - \sum_{j \neq i} \alpha_{ij}^t \Pr(D_j^{tS} \leq y_j^t, D_i^t > y_i^t) \right]. \end{aligned} \quad (3)$$

The first term in the squared brackets accounts for the probability that the effective demand of product i (primary demand plus substitution demand from other out-of-stock products) is greater than the current inventory level. The second term accounts for the probability that demand for product i in excess of the available inventory of that product will be served by inventory of other products in stock.

Using this approximation, and given a vector of inventory levels \mathbf{y} , we solve

$$\max_{S \subset \bar{S}(\mathbf{y})} \left\{ \sum_{i \in S} \lambda q_{mi}(S) (p - \tilde{\Delta}_{t+1}^i(\mathbf{y})) \right\},$$

using the result in Lemma 1. We refer to this heuristic as Sub_t .

The second heuristic is a variant of Sub_t , in which we approximate the expected marginal revenue as follows:

$$\begin{aligned} \hat{\Delta}_{t+1}^i(\mathbf{y}^t) = p * \left[\Pr(D_i^{tOS} > y_i^t) \right. \\ \left. - \sum_{j \neq i} \alpha_{ij}^t \Pr(D_j^{tOS} \leq y_j^0, D_i^0 > y_i^0) \right], \end{aligned} \quad (4)$$

where y_j^0 is product j 's inventory level at the beginning of the selling season, D_j^0 is a Poisson distributed random variable with rate $d_j^0 * T$, and

$$\begin{aligned} D_j^{tOS} &= D_i^0 + \sum_{j \neq i} \alpha_{ji}^0 (D_j^0 - y_j^0)^+, \\ D_j^{tOS} &= D_i^t + \left(\frac{T-t}{T} \right) \sum_{j \in \bar{S}(\mathbf{y}^t) \setminus \{i\}} \alpha_{ji}^t (D_j^0 - y_j^0)^+. \end{aligned}$$

We refer to this heuristic as Sub_0 . Unlike Sub_t , under which $\tilde{\Delta}_{t+1}^i(\mathbf{y}^t)$ depends on the entire vector of inventory levels in period t , the approximate marginal expected revenue under Sub_0 depends on \mathbf{y}^t only through the set $\bar{S}(\mathbf{y}^t)$ of products with positive inventory level in period t . This enables significant preprocessing (prior to initiating the heuristic), resulting in shorter running times.

Both heuristics are extremely effective, as demonstrated in   6. Moreover, Sub_0 results in a threshold-type policy, as the optimal policy for two products derived in Theorem 1.

PROPOSITION 2. *The dynamic assortment policy that emerges under the heuristic Sub_0 is a threshold-type policy; i.e., in every period t and for each product i and customer segment m , there exists a threshold $y_{it}^m(\mathbf{y})$ such that product i is in the assortment offered to an arriving customer of segment m if $y_i \geq y_{it}^m(\mathbf{y})$. Moreover, $y_{it}^m(\mathbf{y})$ is increasing in y_j for $j \neq i$.*

The proof of this result follows from a similar logic to that used in the proof of Theorem 1. In the heuristic Sub_t , the approximate marginal value depends on the entire vector of inventory levels. Under this heuristic, one can show that the effective marginal price of each product j increases with the inventory level of product i , i.e., $p_j^i(\mathbf{y}) \leq p_j^i(\mathbf{y} + e_i)$ for all j . However, a threshold-type result requires a stronger condition that involves the joint effect of a change in the inventory level of product i on the effective marginal prices of all products. This threshold-type result holds for the optimal policy in the case of two products, because the dynamic program takes the full substitution effect into account.

6. Value of Dynamic Assortment Customization

In this section, we demonstrate the potential impact of assortment customization on revenue with a case study. We measure the impact on revenue of employing the optimal policy by calculating the percentage revenue increase caused by assortment customization relative to a benchmark (offer-all) policy under which all available products are offered to any arriving customer. We also assess the efficiency of the heuristics developed in   5 and discuss limitations of the model.

6.1. Data Set and Estimation

Beymen is a privately held high-end fashion retailer in Turkey. The retailer operates 21 department stores and mono-brand stores in Turkey, and an online sales channel. The company places orders from the wholesale arms of high-end fashion brands with firm purchasing commitments six to eight months in advance of the beginning of each season and then sells the products at full price until the clearance season. There are two main buying cycles every year, resulting in two distinct selling seasons: Fall–Winter and Spring–Summer. There is little to no replenishment opportunity during the season for most of the brands, and leftover inventory at the end of the season is liquidated at outlet stores. The company's buyers make product assortment and chain-level inventory decisions (at brand level and at style level) to minimize the occurrence of stockouts and the cost of leftover inventory. However, demand is highly uncertain, and inventory-demand imbalances start to emerge after the first few weeks of the season. Because the

markdown period starts only at the end of the sixth month of the season, the merchandising team reviews the remaining inventory-to-demand ratios every week and manages demand–inventory imbalances using various methods. One method involves increasing visibility of some product groups in the online channel, either by showing them more prominently on opening pages or by assigning higher rankings in customer search results. This is in line with the practice of many online retailers that personalize marketing messages by emphasizing certain products in email messages sent to their customers or on their webpages. We next provide details on the data used for this case study.

1. *Data*: We have countrywide weekly item-level sales (units and prices) and inventory data for the top seven women's shoe brands (Christian Louboutin, Miu Miu, Prada, Valentino, YSL, Tory Burch, Tod's) from the Fall–Winter 2011–2012 season. The season starts in July 2011 and ends in February 2012. All available products were offered during the selling season. The markdown period starts around January 1, 2012. Because there are significant price differences across products, we rank all styles with respect to their prices and create three price clusters that contain styles with similar prices. Average prices within clusters are 1,000 Turkish Lira (TL), 500 TL, and 200 TL, respectively. It is reasonable to assume that there is no substitution across products from different price classes.

2. *Segmentation*: A segmentation approach used by the company is based on the customer's location. When a customer logs in to the firm's website, the location of the customer is identified either through the customer login information or the IP address. We consider three location-based segments: the more affluent neighborhoods of Istanbul (segment 1), other neighborhoods in Istanbul (segment 3), and other cities in Turkey (segment 2). The graph on the right of Figure 1 illustrates the preferences of the three segments for four selling shoe styles (referred to by their SKU numbers) for price group 3 (the highest-priced styles). We estimate the segment probabilities from the relative volume of sales for each location based on historical data.

3. *Demand estimation*: To simulate a setting in which our model could be applied, we suppose that we are in a given week w into the selling season and estimate demand for the remainder of the season. Sales from early season can be used to compute high-quality estimates for full-season demand, as demonstrated by Fisher and Raman (1996). We estimate seasonality from the total shoe sales across the chain. Let $x(w)$ denote the share of week w 's demand to full-season demand, which can be estimated from the previous year's same season data. We denote demand

of item k from segment j up to period w by $d_{jk}(w)$ and the estimate of demand from week w until the end-of-season W as $\bar{d}_{jk}(w, W)$, or simply as \bar{d}_{jk} when the relevant time periods are clear from the context. At time point w , $d_{jk}(w)$ can be observed from data and $\bar{d}_{jk}(w, W) = \alpha(w)d_{jk}(w)$, where $\alpha(w)$ is the ratio of demand after week w to demand up to week w and is given by $(\sum_{i=w}^W x(i))/(\sum_{i=1}^{w-1} x(i))$. The season starts on week 25 of 2011, and all products are offered at regular prices until week 53. We set w equal to the 44th week of the year. Given the demand estimates for each product-segment combination, we estimate the segment's choice probabilities (preferences) by equating the ratio of preferences to the ratio of estimated demands; see (5) in §6.2. For further discussion on the identification of latent customer segments and estimation of their preferences, we refer to Gupta and Chintagunta (1994) and Wedel and Kamakura (1998).

4. *Inventory*: Inventory of product k at time w is denoted by $y_k(w)$ and can be observed from the data provided by Beymen.

Two key ideas emerge from this data set, which are more broadly applicable to fashion retailing in general: (1) Some form of segmentation is possible and segments' preferences can be easily estimated from the sales data. (2) Significant demand–inventory imbalances across products tend to develop within the season because of the uncertainty of product preferences (as illustrated in Figure 1). In what follows, we demonstrate the potential impact of assortment customization by estimating the parameters of our model based on the company's data and by comparing the expected profit from assortment customization with the expected profit under the offer-all policy.

6.2. Revenue Impact of Assortment Customization

We discuss two case studies in this section. Case Study 1 considers all possible combinations of four products chosen among the 20 top-selling shoe styles across the chain (the top 20 products are determined based on sales until week 44 of the year 2011).

Case Study 1. The case study consists of a total of $\binom{20}{4} = 4,845$ combinations. Each combination represents an experiment with four products (out of the 20 top-selling styles) and three segments. Total estimated future demand is given by $D = \sum_{j=1}^3 \rho_j \sum_{k=1}^4 \bar{d}_{jk}$, where the \bar{d}_{jk} estimates are obtained based on sales data up to week 44, and inventory levels at week 44 are given by y_k for $k = 1, \dots, 4$. We define the load factor as the ratio of total demand to total inventory, given by $D/\sum_{k=1}^4 y_k$. Based on our discussion about the demand-supply conditions that tend to lead to higher gains from assortment customization (see Example 2), we focus on the 1,974 cases (out of the 4,845 combinations) in which the load factor is between 0.8 and 1.3. We set $q_0 = 0.1$, $\theta_0 = 10$, and solve

for the set of preferences θ_{jk} for all j, k , to satisfy the demand ratio equations and the given no purchase probability,

$$\begin{aligned} \frac{\theta_{jk}}{\sum_{i=1}^4 \theta_{ji}} &= \frac{\bar{d}_{jk}}{\sum_{i=1}^4 \bar{d}_{ji}}, \quad j=1,2,3, k=1,2,3,4 \quad \text{and} \\ \frac{\theta_0}{\theta_0 + \sum_{i=1}^4 \theta_{ji}} &= q_0 = 0.1, \quad j=1,2,3. \end{aligned} \quad (5)$$

Note that q_0 is difficult to estimate because the no-purchase alternative is not observed (see, e.g., K  k and Fisher 2007, Vulcano et al. 2012). In general, $1 - q_0$ determines the probability to accept a substitute, which has been shown to be around 40% for supermarket categories (Gruen et al. 2002) and is suspected to be much lower for online retail settings. We set $T = 50$ for the dynamic program, implying an average of 45 potential buyers in the remaining weeks of the season. Since D may well exceed 45, we rescale inventories to match the scaling of demand, i.e., $\hat{y}_k = (T(1 - q_0)/D)y_k$ for $k = 1, \dots, 4$. (If D is equal to 45, then inventory levels remain the same; otherwise, inventory and demand are both scaled down in the same proportion.) Note that the load factor remains the same before and after the rescaling described above.

For the experiments in this case study, we compute the optimal assortment policy by solving the dynamic program and making use of Lemma 1. Figure 3 shows the percentage increase in revenue of assortment customization relative to offer-all as a function of the load factor. The impact of assortment customization is nonnegligible when the load factor is close to 1. The average improvement over the offer-all policy across the 1,974 experiments is 0.21%, with a maximum of 4.92%. In particular, 231 cases yield an impact higher than 0.5% and 102 cases yield an impact higher than 1%.

The actual scale of demand and inventory levels is generally higher than that which can be handled in a

dynamic program. In the next case study, we demonstrate the impact of assortment customization in settings with a larger number of products and longer time horizons (larger T) by evaluating the offer-all policy and the heuristics introduced in   5 using simulation. We also compare the performance of the heuristic to an upper bound introduced by Golrezaei et al. (2014). The revenue achieved under the optimal assortment customization policy is bounded by the optimal value of a linear programming formulation of the problem under which demand is deterministic. A constraint guarantees that the total fulfilled demand of a product does not exceed its initial inventory. We refer to Lemma 1 in Golrezaei et al. (2014) for more details. For the experiments in Case Study 1, the average revenue of the optimal solution relative to the upper bound is 97.9%.

Case Study 2. This case study consists of 200 experiments with $N = 4$, 200 experiments with $N = 10$, and 100 experiments with $N = 20$. The experiments with $N = 4$ are based on a subsample of 100 instances from Case Study 1 with load factors between 1 and 1.3, and with (i) $T = 90$ and $q_0 = 0.5$ and (ii) $T = 180$ and $q_0 = 0.75$. We also consider all possible combinations of 10 products chosen among the 20 top-selling shoe styles and select a subsample of 100 experiments with load factors between 1 and 1.1. We have also run two sets of experiments with this subsample, with (i) $T = 90$ and $q_0 = 0.5$, (ii) $T = 180$, and $q_0 = 0.75$. Finally, we selected a sample of 100 experiments with $N = 20$, load factors between 0.9 and 1.3, and with $T = 180$ and $q_0 = 0.75$.

For these 500 experiments, we run the offer-all policy, the two heuristics introduced in   5, i.e., Sub_t and Sub_0 , and a heuristic proposed in Golrezaei et al. (2014), which we denote by GNR. The GNR heuristic is based on a penalty function that uses supply information (i.e., information about the prevailing inventory levels) to determine the set of products to offer to each arriving customer. The heuristic achieves the best worst-case asymptotic performance (as $T \rightarrow \infty$) within the class of single-variable decreasing penalty functions. The advantage of the GNR heuristic is that it does not require demand forecasting. On the other hand, unlike the optimal policy and the heuristics Sub_t and Sub_0 , the GNR heuristic ignores demand-supply imbalances across products and the substitution dynamics between products. The numbers reported in Table 1 represent the percentage revenue improvement over the offer-all policy denoted by OA.

The results in Table 1 suggest that the heuristics introduced in   5 lead to significant revenue improvement (including settings with a large number of products and long time horizon), provided that the load factors are around 1. Moreover, the heuristic

Figure 3 (Color online) Percentage Revenue Increase Relative to the Offer-All Policy as a Function of the Load Factor

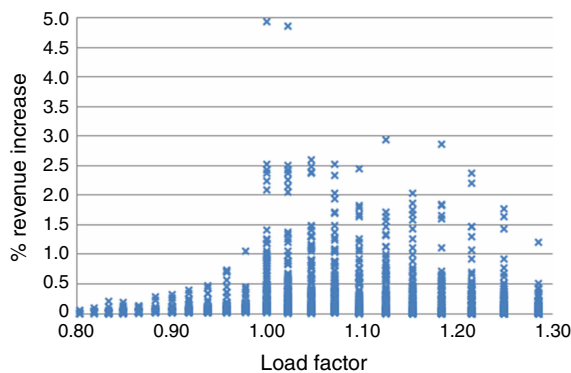


Table 1 Impact of Assortment Customization for Case Study 2

N	T	q ₀	Sub ₀ vs. OA (%)		Sub _t vs. OA (%)		GNR vs. OA (%)	
			Avg.	Max	Avg.	Max	Avg.	Max
4	90	0.5	0.47	1.71	0.34	1.27	0.24	1.12
	180	0.75	0.23	1.34	0.21	1.01	−0.02	1.26
10	90	0.5	0.23	1.05	−0.04	0.66	−0.55	0.98
	180	0.75	0.20	1.15	0.15	0.98	0.13	0.98
20	180	0.75	0.11	0.95	0.07	0.71	0.03	0.79

Sub₀ dominates the other heuristics both in terms of the average and maximum revenue improvement. Unlike GNR, the Sub₀ and Sub_t heuristics explicitly account for demand-supply imbalances and the interplay between the products' inventory levels by accounting for the substitution dynamics between products. As demonstrated in Theorem 1, the optimal policy in the case of two products also exhibits these properties. In the Sub_t heuristic, the approximate marginal value of inventory is re-solved in each period to account for the prevailing inventory levels and updated forecasted future demand. In contrast, in the Sub₀ heuristic, the approximate marginal value of inventory for a given product does not depend on the updated inventory levels or updated demand forecasts of other products (it only updates the set of products with positive inventory and recomputes the substitution rates and the effective demand). In this respect, the superior performance of Sub₀ over that of Sub_t is consistent with the finding of Cooper (2002) that frequent updating or re-solving is not necessarily better. (In the context of a resource allocation problem in revenue management, Cooper (2002) finds that re-solving may lead to strictly worse average revenue performance.)

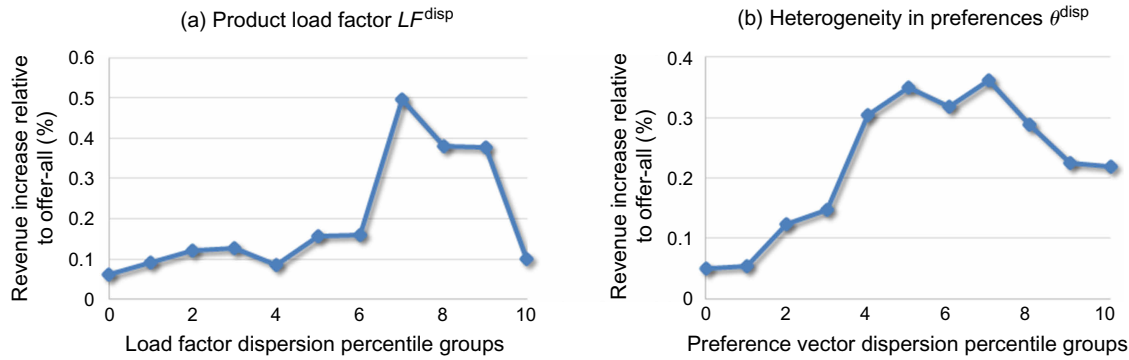
In addition to comparing the performance of the heuristics to the offer-all policy, we also report the gap between the revenue generated in each instance under the Sub₀ heuristic and the revenue corresponding to the upper bound from Golrezaei et al. (2014) discussed earlier. The average revenue under the Sub₀ heuristic relative to the upper bound is 97.3% for $N = 4$, 99.0% for $N = 10$, and 98.5% for $N = 20$.

6.3. Inventory Imbalance and Market Heterogeneity

In this section, we explore the impact of imbalances in the inventory levels and of market heterogeneity on the value of assortment customization. To examine the impact of inventory imbalances, we define a metric to measure the dispersion in the products' load factors and use the data from Case Study 1 to evaluate the impact of this metric on the value of assortment customization. Let $LF_k = (\sum_{j=1}^M \rho_j \bar{d}_{jk})/y_k$ be the load factor for product k , defined as the forecasted

demand for that product across all segments until the end-of-season divided by the prevailing inventory level. The products' load factor dispersion metric is $LF^{\text{disp}} = \sum_{k=1}^N \sum_{j \neq k} |LF_k - LF_j|$. The load factor dispersion metric ranges from 0.34 to 31.22 in the data corresponding to Case Study 1. A larger value of LF^{disp} suggests that there are significant demand-supply imbalances across products—i.e., relative to future forecasted demand, some products have sufficient or excess inventory, whereas others are low in stock—and vice versa. We divide the experiments in 10 equal-sized groups in that range and calculate the average gain over the offer-all policy for all instances in each group. Figure 4(a) exhibits the value of assortment customization as the product load factor dispersion metric changes across groups within that range. As can be noted in the graph, the value of assortment customization first increases with LF^{disp} , consistent with the idea that inventory imbalances lead to higher gains, and then experiences a sharp decrease when LF^{disp} is high—those are scenarios in which inventory is so low for some products and/or so high for others that customization does not bring much additional value. This observation is consistent with Figure 2: Setting a fixed total amount of inventory in that example, say $y_1 + y_2 = 90$, and moving along that diagonal line from the point it intersects the 45° angle up to the y_1 -axis, one obtains a cross section of the graph that resembles the one in Figure 4(a).

We now study the impact of heterogeneity by focusing on two market characteristics: the relative size of segments and the dispersion of preferences across segments. To measure the effect of segment sizes on the benefit of assortment customization, we define the following segment distance metric: $\rho^{\text{dist}} = \sum_{j=1}^M |\rho_j - 1/M|$. A lower ρ^{dist} value indicates a more heterogeneous population with M relatively equal-sized segments. A higher value of ρ^{dist} implies a less heterogeneous population concentrated primarily in one large segment. We apply this metric to the experiments in Case Study 1 and divide the experiments into three equal-sized groups with low, medium, and high values of the metric ρ^{dist} . The minimum, average, and maximum ρ^{dist} values are 0.41, 0.86, and 1.25, respectively. Figure 5 reports the average percentage gain from assortment customization relative to offer-all for each group, controlling for the load factor. Although the net impact depends on the relationship between the sizes of the segments and the preference vectors of each segment, Figure 5 clearly suggests that the impact of assortment customization is highest in settings with more heterogeneous populations. This finding makes sense, because there would be no benefit to assortment customization if all customers belonged to the same segment.

Figure 4 (Color online) Average Percentage Revenue Increase Relative to Offer-All as a Function of LF^{disp} and θ^{disp} 

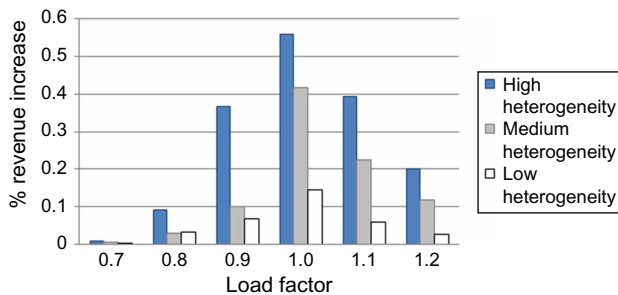
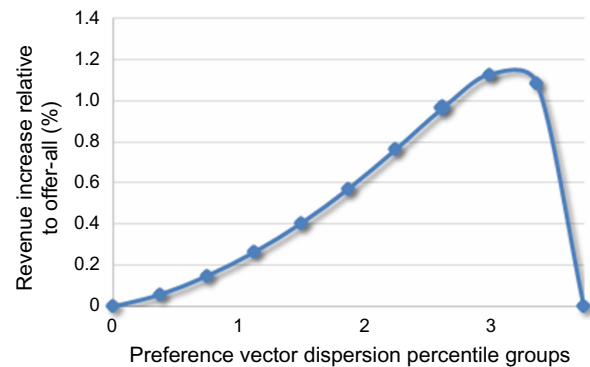
To examine the effect of heterogeneity in the customers' preferences on the value of assortment customization, we define $\theta^{\text{disp}} = \sum_{m=1}^M \sum_{n=1, n \neq m}^M \rho_m \rho_n \cdot \sum_{i=1}^N |\theta_{mi} - \theta_{ni}|$ as a measure of the dispersion in the customers' preferences (weighted by the relative sizes of the segments). We first apply this metric in a stylized setting in which customer demands are symmetric. This condition is formalized in Proposition 3, which presents the optimal policy in that setting.

PROPOSITION 3. Consider a setting with N products and M customer segments but with symmetric product demands. Specifically, assume that $\sum_{m \in S} \rho_m q_{ml}(S) = \sum_{m \in S'} \rho_m q_{ml}(S')$ for any two subsets S, S' and any two products l, l' , with $l \in S, l' \in S'$ (or $l = l' = 0$), and $|S| = |S'|$. Then, for an arriving customer of segment m in period t , there exists a threshold $y_{mt}^*(\mathbf{y})$ such that product i is in the assortment offered to this customer if $y_i \geq y_{mt}^*(\mathbf{y})$.

The following setting constitutes an example of a symmetric demand scenario: N products and $M = N$ customer segments with $\rho_m = 1/N$ for all m ; customer segment $m = 1, \dots, N$, has a preference vector with $\theta_{mm} = \theta$ and $\theta_{mj} = \theta'$ for $j \neq m$. In such settings, Proposition 3 indicates that the optimal dynamic assortment policy is a threshold policy. Products low on inventory tend to be rationed to more customer segments. (The same policy is also optimal if segments only

differ in terms of the value they assign to the outside option.) For the above demand scenario, we have that $\theta^{\text{disp}} = (2(N-1)/N^2)|\theta - \theta'|$. That is, for a given number of products the dispersion is highest when $\theta' = 0$, and it decreases in θ' . However, as exhibited in Figure 6, the benefit associated with assortment customization relative to offer-all is nonmonotone in θ^{disp} . Indeed, the thresholds derived in Proposition 3 are $y_{mt}^*(\mathbf{y}) = 0$ when either $\theta' = 0$ or $\theta' = \theta$, i.e., when there is either no overlap in preferences (each segment likes a single product) or full overlap (all customers like all products equally). In those cases, assortment customization has no impact on revenue because customer segments do not interact with each other. The value of customization is highest when there is some overlap but the segment preferences are distinctively different.

The stylized example with symmetric demands is helpful because it evaluates the impact of heterogeneity in preferences in a controlled environment with equal segment sizes and equal overall demand for each product, so that only the relative dispersion of preferences matters. When the θ^{disp} metric is applied to Case Study 1, we do observe a similar effect. In Figure 4(b), we divide the experiments from Case Study 1

Figure 5 (Color online) Average Percentage Revenue Increase Relative to the Offer-All Policy as a Function of the Size of Segments and the Load Factor**Figure 6** (Color online) Revenue Impact of Assortment Customization Relative to the Offer-All Policy as a Function of θ^{disp} 

Note. Parameters: $T = 50$, $N = 4$, $y_1 = 17$, $y_2 = 11$, $y_3 = 3$, $y_4 = 2$, and symmetric demands with $\rho_1 = \rho_2 = \rho_3 = \rho_4 = 0.25$, $\theta = 10$, $0 \leq \theta' \leq 10$.

in 10 equal-sized groups and plot the average gain of assortment customization for instances in each group against the θ^{disp} metric. The result is an increasing-decreasing shape that resembles the one in Figure 6.

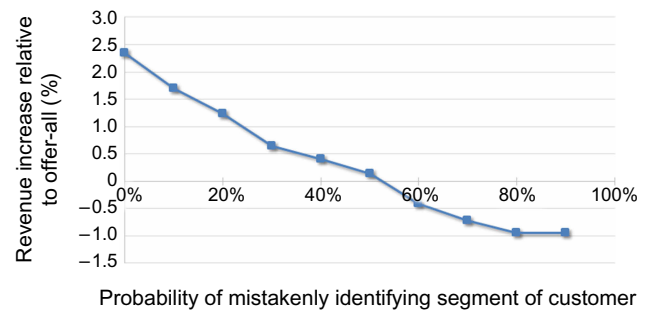
6.4. Discussion of Model and Case Study

We now comment on the main limitations of the model and case study. The idea of assortment customization relies on the ability of the firm to accurately forecast customer preferences and identify the segment of arriving customers. Inaccurate predictions can reduce or possibly eliminate the benefits of customization. A recent article on data-driven online shopping (Wood 2014) provides examples of companies that offer a personalized shopping experience to their customers based on estimates derived from customer data collected on the Web. This suggests that online retailers are working towards the development of reliable estimates of customer preferences. In this respect, the goal of our paper is to show the benefits of assortment customization for those firms that are able to segment customers (and estimate their preferences) with some accuracy. In the Beymen case study, the proposed segmentation is based on the location of the customer. Hence, the misidentification error (i.e., a customer of segment m_1 is identified as belonging to segment m_2 and is offered the set of products that would be optimal for segment m_2) is unlikely to occur because the customer login information carries the location ID. Nevertheless, to explore the potential loss associated with misidentification, we consider an instance of Example 2 with inventory levels $y_1 = 11$ and $y_2 = 25$ and a range of scenarios reflecting an increasing proportion of arriving customers that are misidentified. We then simulate such systems following the optimal policy that would arise if the identification of segments were completely accurate. The gain from assortment customization starts at 2.3% when identification is accurate, and it is eroded as the proportion of misidentified customers increases. In this example, a high inaccuracy error (of over 50%) is needed to completely eliminate the benefits of customization; see Figure 7.

We have assumed throughout the paper that product prices are equal. In addition to gaining analytical tractability, this assumption allows us to isolate the impact of customer heterogeneity and limited inventories on the value of assortment customization. It is of interest to extend our results to settings with non-identical and dynamically optimized prices.

The case study presents the following limitations: (i) As discussed in §6.2, the proportion of customers that do not buy any product (the no-purchase option) is not observed from the data. One can use general industry estimates instead. (ii) Along the same lines, our data reflects sales (not demand) and therefore our preference estimates are based on sales

Figure 7 (Color online) Impact of Inaccurate Identification of Customers



data. This does not create a significant bias in our case study because we estimate demand at mid-season, before significant stockouts occur. (iii) A model of active learning could help refine the preference estimates when assortment customization is implemented. (iv) Customer choices may have been influenced by other considerations not captured in our model, such as advertisement and competitors' actions.

7. Conclusion

We consider a retailer with limited inventory of substitutable products in a category with equal selling prices. The retailer faces a heterogeneous customer base, consisting of multiple segments characterized by different preferences for the products. The retailer can identify the segment of each arriving customer and therefore offer a customized assortment based on that segment's preferences. We formulate this problem as a dynamic assortment customization problem.

If the retailer is not able to identify the segment of an arriving customer, then it is optimal to offer all available products to any arriving customer. When the retailer has the ability to identify customer types, it may be optimal to ration products to some customers. This result applies when the following factors are present: heterogeneous segments, limited inventory, and multiperiod dynamics. For a setting with two products and two segments, we show that it is optimal to offer a product to a customer segment only if the inventory level of that product is higher than a threshold level. Otherwise, the product is not offered to that segment to reserve those units for future customers who may have a stronger preference for that product. The threshold levels are increasing in the other product's inventory level and decreasing in time, highlighting the impact of inventory levels and demand-supply conditions on the assortment policy. We conjecture that a similar threshold policy is optimal when there are more than two products.

We introduce two heuristics based on an approximation of the marginal expected revenue generated

by each product and in each period in the dynamic program by the marginal value of a newsvendor (single-period) function that captures the substitution dynamics between products. These heuristics internalize the interplay of demand-supply conditions for each product (and in each period) based on newsvendor-type approximations.

Assortment customization can be implemented as soon as reliable estimates of demand are available. We demonstrate the potential revenue impact of assortment customization with a case study based on a high-end fashion retailer. This study reveals the potential impact of assortment customization by comparing the revenue derived from the optimal policy and the heuristics with that of the myopic policy that involves offering all available products to any arriving customer. The revenue benefit from assortment customization can be significant. Moreover, we find that assortment customization is most beneficial when customers are heterogeneous and when the products' inventory-to-demand ratios are asymmetric.

Our analytical results and case study suggest that dynamic assortment customization is an effective lever for revenue maximization in online retailing and in other environments in which the identification and customization processes are feasible.

Supplemental Material

Supplemental material to this paper is available at <http://dx.doi.org/10.1287/msom.2015.0544>.

Acknowledgments

The authors thank Stephen C. Graves and the review team for constructive comments that helped improve the paper. They also thank Arian Aflaki for computational assistance. The second author's research is partially supported by an ICI-Unilever research grant by ECR Europe. The third author's research is partially supported by the National Natural Science Foundation of China [Grant 71202070].

References

Anderson S, de Palma A, Thisse J (1992) *Discrete Choice Theory of Product Differentiation* (MIT Press, Cambridge, MA).

Aydin G, Ziya S (2009) Personalized dynamic pricing of limited inventories. *Oper. Res.* 57(6):1523–1531.

Cachon G, Terwiesch C, Xu Y (2005) Retail assortment planning in the presence of consumer search. *Manufacturing Service Oper. Management* 7(4):330–346.

Caro F, Gallien J (2007) Dynamic assortment with demand learning for seasonal consumer goods. *Management Sci.* 53(2):276–292.

Caro F, Martinez-de-Albeniz V (2009) Strategic assortment rotation. Working paper, Anderson School of Management, University of California, Los Angeles.

Chen L, Homem-de-Mello T (2010) Re-solving stochastic programming models for airline revenue management. *Ann. Oper. Res.* 177(1):91–114.

Cooper W (2002) Asymptotic behavior of an allocation policy for revenue management. *Oper. Res.* 50(4):720–727.

de Véricourt F, Karaesmen F, Dallery Y (2002) Optimal stock allocation for a capacitated supply system. *Management Sci.* 48(11):1486–1501.

Fay S, Xie J (2008) Probabilistic goods: A creative way of selling products and services. *Marketing Sci.* 27(4):674–690.

Fisher M, Raman A (1996) Reducing the cost of demand uncertainty through accurate response to early sales. *Oper. Res.* 44(1):87–99.

Fudenberg D, Villas-Boas JM (2006) Behavior-based price discrimination and customer recognition. Hendershott TJ, ed. *Economics and Information Systems*, Handbook in Information Systems, Vol. 1 (Emerald Group Publishing, Bingley, UK), 377–436.

Gallego G, Iyengar G, Phillips R, Dubey A (2004) Managing flexible products on a network. Working paper, Columbia University, New York.

Golrezaei N, Nazerzadeh H, Rusmevichientong P (2014) Real-time optimization of personalized assortments. *Management Sci.* 60(2):1532–1551.

Gruen T, Corsten D, Bharadwaj S (2002) *Retail Out-of-Stocks: A Worldwide Examination of Extent, Causes and Consumer Responses* (Grocery Manufacturers of America, Washington, DC).

Gupta S, Chintagunta P (1994) On using demographic variables to determine segment membership in logit mixture models. *J. Marketing Res.* 31(1):128–136.

Ha A (1997a) Inventory rationing in a make-to-stock production system with several demand classes and lost sales. *Management Sci.* 43(8):1093–1103.

Ha A (1997b) Stock-rationing policy for a make-to-stock production system with two priority classes and backordering. *Naval Res. Logist.* 44(5):458–472.

Honhon D, Gaur V, Seshadri S (2010) Assortment planning and inventory decisions under stockout-based substitution. *Oper. Res.* 58(5):1364–1379.

Kim J, Allenby G, Rossi P (2002) Modeling consumer demand for variety. *Marketing Sci.* 21(3):229–250.

Kök AG, Fisher M (2007) Demand estimation and assortment optimization under substitution: Methodology and application. *Oper. Res.* 55(6):1001–1021.

Kök AG, Xu Y (2010) Optimal and competitive assortments with endogenous pricing under hierarchical choice models. *Management Sci.* 57(9):1546–1563.

Kök AG, Fisher ML, Vaidyanathan R (2009) Assortment planning: Review of literature and industry practice. Agrawal N, Smith SA, eds. *Retail Supply Chain Management* (Springer, New York), 99–153.

Lederman R, Saure D (2013) Hierarchical customization of product assortments under ranking-based consumer choice. Working paper, IBM T.J. Watson Research Center, Yorktown Heights, NY.

Linden G, Smith B, York J (2003) Amazon.com recommendations: Item-to-item collaborative filtering. *Internet Comput., IEEE* 7(1):76–80.

Liu Q, van Ryzin G (2008) On the choice-based linear programming model for network revenue management. *Manufacturing Service Oper. Management* 10(2):288–310.

Mahajan S, van Ryzin G (2001) Stocking retail assortments under dynamic consumer substitution. *Oper. Res.* 49(3):334–351.

Miranda Bront J, Mendez-Diaz I, Vulcano G (2009) A column generation algorithm for choice-based network revenue management. *Oper. Res.* 57(3):769–784.

Mussa M, Rosen S (1978) Monopoly and product quality. *J. Econom. Theory* 18(2):301–317.

Netessine S, Rudi N (2003) Centralized and competitive inventory models with demand substitution. *Oper. Res.* 51(2):329–335.

Netessine S, Savin S, Xiao W (2006) Dynamic revenue management through cross-selling in e-commerce retailing. *Oper. Res.* 54(5):893–913.

- Phillips R (2005) *Pricing and Revenue Optimization* (Stanford University Press, Stanford, CA).
- Ramasastri A (2005) Web sites change prices based on customers' habits. *CNN* (June 24), <http://edition.cnn.com/2005/LAW/06/24/ramasastri.website.prices/>.
- Rusmevichientong P, Topaloglu H (2012) Robust assortment optimization in revenue management under the multinomial logit choice model. *Oper. Res.* 60(4):865–882.
- Rusmevichientong P, Shen Z, Shmoys D (2010) Dynamic assortment optimization with a multinomial logit choice model and capacity constraint. *Oper. Res.* 58(6):1666–1680.
- Saure D, Zeevi A (2013) Optimal dynamic assortment planning with demand learning. *Manufacturing Service Oper. Management* 15(3):387–404.
- Shumsky R, Zhang F (2009) Dynamic capacity management with substitution. *Oper. Res.* 57(3):671–684.
- Smith S, Agrawal N (2000) Management of multi-item retail inventory systems with demand substitution. *Oper. Res.* 48(1):50–64.
- Talluri K, van Ryzin G (2004a) Revenue management under a general discrete choice model of consumer behavior. *Management Sci.* 50(1):15–33.
- Talluri K, van Ryzin G (2004b) *The Theory and Practice of Revenue Management* (Kluwer Academic Publishers, New York).
- Ulu C, Honhon D, Alptekinoglu A (2010) Learning consumer tastes through dynamic assortment. *Oper. Res.* 60(4):833–849.
- van Ryzin G, Mahajan S (1999) On the relationship between inventory costs and variety benefits in retail assortment. *Management Sci.* 45(11):1496–1509.
- van Ryzin G, Vulcano G (2008a) Computing virtual nesting controls for network revenue management under customer choice behavior. *Manufacturing Service Oper. Management* 10(3): 448–467.
- van Ryzin G, Vulcano G (2008b) Simulation-based optimization of virtual nesting controls for network revenue management. *Oper. Res.* 56(4):865–880.
- Vulcano G, van Ryzin GJ, Ratliff R (2012) Estimating primary demand for substitutable products from sales transaction data. *Oper. Res.* 60(2):313–334.
- Wedel M, Kamakura W (1998) *Market Segmentation: Conceptual and Methodological Foundations* (Kluwer Academic Publishers, Boston).
- Wood M (2014) A new kind of e-commerce adds a personal touch. *New York Times* (August 13), http://www.nytimes.com/2014/08/14/technology/personaltech/data-driven-shopping-with-the-personal-touch.html?_r=0.
- Zhang D, Adelman D (2009) An approximate dynamic programming approach to network revenue management with customer choice. *Transportation Sci.* 43(3):381–394.
- Zhang D, Cooper W (2005) Revenue management for parallel flights with customer-choice behavior. *Oper. Res.* 53(3):415–431.