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On the Impact of Uncertain Cost Reduction When Selling to Strategic Customers

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Abstract. Many products undergo cost reductions over their product life cycles. However, strategic customers may have more incentive to wait if they expect a cost reduction to lead to a price drop. A firm that does not face any uncertainty can use pricing strategies such as price commitment and price matching to alleviate the strategic waiting of customers. However, these pricing strategies provide less flexibility than dynamic pricing for a firm facing uncertainty. In this paper, we examine the impact of cost reduction under dynamic pricing, price commitment, and price matching when cost reduction can come from production learning or from technology advancement. The firm makes pricing decisions when facing uncertainty in future cost, and strategic customers decide whether to wait when facing uncertainty in future price. We show that in general the firm's profit is higher when future cost is more uncertain, but not necessarily when cost reduction is more significant. In addition, production learning and technology advancement can have opposite effects on the optimal pricing decisions and the choice of pricing strategy.

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Keywords: cost reduction • cost uncertainty • strategic customers • pricing strategy

1. Introduction

In August 2012, several months after the portable video game system Sony PS Vita was released to the market, Sony Worldwide Studios president Shuhei Yoshida said in an interview that "cost reduction is one area our engineering team is working on" and they "would like to address the pricing issue for some of the people who are waiting," but there had to be more reduction in cost before price could be decreased (Yin-Poole 2012). In February 2013, about a year after the product was released, Sony reduced the price of PS Vita in Japan by 30%, and sales quadrupled immediately (Sin 2013). This sharp increase in sales can be attributed to customers who were waiting for the price cut, and cost reduction apparently played an important role in the price cut.

Cost reduction may come from a reduction in the component cost as technology becomes more mature. For example, it was believed that a reason for the price cut of PS Vita was that "component prices dropped enough for it to adjust the retail price" (Humphries 2013). This type of cost reduction is independent of previous production quantities, and we call it the *technology advancement effect*. Another source of cost reduction comes from learning by doing. Workers become more familiar with their jobs and they may also have better knowledge on how to improve the process when experience accumulates. It is believed that "the cost to produce the consoles decreases according to the normal learning curve dynamics" (Wood 2013). This type of cost reduction increases with the quantity produced previously, and we call it the *production learning effect*. Although the technology advancement effect and the production learning effect help to reduce the future cost of a product, customers who expect a cost reduction and hence a future price reduction may strategically wait for a lower price. As lean manufacturing is widely adopted in practice, production is usually carried out just to meet current demand but not to build up inventories. In this case, the strategic waiting of customers directly reduces the firm's cost saving from production learning. Therefore, the firm's pricing and production planning decisions have to take into consideration not just the two types of cost reductions, but also the presence of strategic customers. In the presence of strategic customers, what effects do production learning and technology advancement have on prices, quantities, and the firm's profit? In particular, a firm is usually uncertain about the magnitude of cost reduction at the beginning, and will only know it as technology evolves. In this case, what is the impact of the uncertainty in cost reduction? How should a firm manage its pricing policy to increase profit? This paper aims to shed light on these questions.

In this paper, we consider a novel model in which a firm's later production cost may be reduced due to both production learning and technology advancement, but the magnitude of cost reduction is uncertain before production starts. This uncertainty in the magnitude of cost reduction leads to uncertainty in the firm's future production cost. Depending on the pricing strategy, the firm may decide to set the later price in response to the realized cost reduction (for example, under dynamic pricing). Thus, at the beginning of the selling season, the firm faces uncertainty in future cost, and customers may also face uncertainty in future price. The firm decides the pricing strategy, and strategic customers decide when to purchase under these uncertainties.

We first consider the case in which the firm uses a dynamic pricing strategy. Even though dynamic pricing is common in practice and provides the firm with a high degree of flexibility to change prices, it also provides incentives for customers to wait. Thus, we also study two other pricing strategies, namely, price commitment and price matching. Under price commitment, the firm commits to a preannounced future price. Under price matching, customers who purchase before a price cut (if there is any) are fully reimbursed the difference.

Our analysis leads to several interesting main results. First, the firm may be better off when uncertainty increases. When the firm faces greater uncertainty in cost reduction, customers also face higher uncertainty in future prices. Since greater uncertainty increases the chances for both higher and lower prices, it does not lead to more strategic waiting. Moreover, when the firm has the flexibility to update prices (under dynamic pricing and price matching), the firm is able to maximize its gain in case of a large cost reduction and to minimize the damage in case of a small cost reduction. Thus, the firm's profit is higher when the level of uncertainty is higher under these two strategies.

Our second main result is that a more significant production learning or technology advancement effect may lead to higher prices, lower quantities sold, and a lower profit for the firm. Under dynamic pricing, strategic waiting is intensified when customers expect a deeper price cut. In this case, the firm may charge higher prices, which leads to a smaller earlier quantity sold and hence a larger later cost. Thus, the total quantity sold is lower and so is the firm's profit. However, if the firm optimizes its strategy across dynamic pricing, price commitment, and price matching, its profit is always increasing in the significance of the production learning and technology advance effects. This suggests that a careful pricing strategy is needed to enhance return on cost reduction.

Our results also suggest that production learning and technology advancement can affect the optimal prices under individual pricing strategies and also the choice of pricing strategy in opposite ways. Cost reduction from production learning is affected by the early production quantity and hence is strongly dependent on customers' purchasing decisions. Thus, when the production learning effect becomes more significant, it is important to discourage strategic waiting of customers to capture a larger cost reduction, and the optimal pricing strategy may change from dynamic pricing to price commitment, but not the other way around. However, because cost reduction from technology advancement is not affected by strategic waiting of customers, the flexibility provided by dynamic pricing allows the firm to react to the actual cost reduction. Thus, when the technology advancement effect becomes more significant, the optimal pricing strategy may change from price commitment to dynamic pricing, but not the other way around.

The remainder of this paper is organized as follows. Section 2 reviews the literature. Section 3 introduces the model. Sections 4 analyzes the firm's problem under dynamic pricing, price commitment, and price matching, and §5 compares the three pricing strategies. Section 6 studies two extensions of the basic model through numerical exploration. Finally, §7 provides concluding remarks.

2. Literature Review

A growing body of literature on strategic behavior and its impact on a firm's pricing and operations decisions has stemmed from the pioneering work by Besanko and Winston (1990) and Su (2007). The survey by Shen and Su (2007) provides an excellent review of this emerging stream of literature. As demonstrated by Besanko and Winston (1990), strategic customers are more price sensitive than myopic customers. In equilibrium, the firm charges lower prices at the beginning of the planning horizon, which reduces its ability to price differentiate customers. Some studies suggest price commitment as a way to deter the strategic waiting of customers. Aviv and Pazgal (2008) and Dasu and Tong (2010) show that even with demand uncertainty, price commitment may sometimes be more profitable than dynamic pricing despite the flexibility the latter provides to respond to the uncertain demand. The conditions under which price commitment is more profitable than dynamic pricing are also identified in these two papers, which focus on the case of an exogenous production quantity. Su and Zhang (2008) and Cachon and Swinney (2009) study the case in which the production quantity is determined by the firm at the beginning of the planning horizon. Su and Zhang (2008) show that commitment can benefit the firm and propose some supply chain contracts to attain the same outcome as commitment. Cachon and Swinney (2009) study the value of the firm's option to place a second order after the demand uncertainty is resolved but before the prices are determined. They show that dynamic pricing is likely to be more profitable than price commitment regardless of whether the firm possesses the option to place a second order.

Price matching is another pricing policy that can be used to discourage strategic customers from waiting. Png (1991) and Lai et al. (2010) compare price matching and dynamic pricing when the firm sells to two groups of customers with different valuations. Png (1991) studies the case in which the initial inventory is exogenous, whereas Lai et al. (2010) study the case in which the initial inventory is an endogenous decision of the firm. Both of these papers identify cases in which price matching is more profitable. Li and Zhang (2013) also study the impact of price matching. They study the value of obtaining advance demand information through advance selling and examine the impact of price matching under the advance selling strategy.

Our paper differs from the aforementioned papers in several ways. First, our paper provides a unified comparison of price commitment, price matching, and dynamic pricing, whereas the aforementioned papers only compare dynamic pricing with either price commitment or price matching. Second, in all of these papers, strategic consumer behavior only affects the revenue of the firm. In our model, however, strategic consumer behavior also affects the production cost of the firm because early demand enhances production learning. Third, we focus on supply uncertainty instead of demand uncertainty. In our model, the future production cost is uncertain. Shou et al. (2011) also study a firm that faces both supply uncertainty and strategic customers. However, they focus on customers' incentives to stockpile when there is a risk of shortage in the future, whereas we focus on customers' incentives to delay purchase when there is a potential cost reduction in the future. In our model, customers are uncertain about future prices, rather than the availability of products, as studied by Shou et al. (2011).

Our paper is also related to the stream of literature on production learning by doing. The effect of production learning by doing has received a lot of attention in both empirical and analytic research since the seminal paper by Arrow (1962). Empirical research mainly focuses on identifying models to describe and estimate this effect and related phenomena (e.g., Hatch and Mowery 1998, Benkard 2000, Tucker 2008). In a recent paper, Lobel and Perakis (2011) develop a model to study the effect of production learning on consumers' purchase decisions on solar photovoltaic technology. On the analytic side, some papers study the macroeconomic effects of learning by doing (e.g., Romer 1986, Young 1991). Kalish (1983) and Cabral and Riordan (1994) study the effect of production learning on prices. Other studies focus on a firm's operations decisions when there is production learning, such as cost reduction investments (Fine and Porteus 1989, Bernstein and Kok 2009), quality decisions (Li and Rajagopalan 1998, Serela et al. 2003), and production quantities (Mazzola and McCardle 1997). Different from these papers, we take into account the existence of strategic or forward-looking consumers in our model and show that learning by doing may not be beneficial to the firm in this case.

3. Model

We consider a monopolistic firm selling to a large market consisting of a mass of infinitesimal customers over two periods. The size of the market is deterministic, and, without loss of generality, we normalize it to 1. Every customer represents a unit demand for the good. Customers' valuations are heterogeneous and distributed uniformly on the interval $[0,1]$. The selling season is divided into two periods and, similar to the models in Liu and van Ryzin (2008, 2011) and Swinney (2011), all potential customers are present at the beginning of the selling season. Each customer decides when to purchase, and the firm decides the price in each period, denoted by p_i, $i=1,2$. Customers behave strategically and they take into account the prices in both periods when making their purchase decisions. We assume that customers discount future utility over time by a factor of δ (where $0<\delta<1$); that is, for a customer with valuation v, the utility for waiting to purchase in the second period is $\delta(v-p_2)$.

The firm is risk neutral and makes pricing decisions to maximize its total profit in the two periods, with the profit in the second period discounted by a factor of θ, where $0<\theta<1$. Whereas the firm's production cost is fixed at c per unit in the first period (where $c<1$), the production cost in the second period may be lower due to technology advancement or improvement in the production process. However, the actual cost reduction is realized only at the beginning of the second period. In particular, the production cost for each unit of product in the second period is $c-(\alpha+\beta q_1+\epsilon)$, where q_1 is the selling quantity in the first period, α and β measure the significance of the technology advancement and production learning effects, respectively, and ϵ measures uncertainty in cost reduction and has a mean of 0 and variance σ^2. To ensure a nonnegative production cost and a nonnegative cost reduction in the second period, we assume that ϵ is distributed over the interval $[-\alpha, c-\alpha-\beta]$. Similar production learning models have been used in the literature to approximate the learning process. For example, Jin et al. (2004) use the linear learning model to study the spillover effects in learning in an industry with multiple competitors, and Hiller and Shapiro (1986) use a piecewise linear learning model to study capacity expansion decisions when there is production learning. We numerically test the robustness of our results in the case of an exponential learning curve in §6. Finally, we focus on a lean production system in which the production in a period covers only the demand in that period.

In this paper, we consider three pricing strategies: dynamic pricing (Model D), price commitment (Model C), and price matching (Model M). Under dynamic pricing, the firm decides the price in the second period after the realization of ϵ. With price commitment, the firm announces the prices in both periods before customers decide when to purchase. Price matching is similar to dynamic pricing except that the firm will reimburse customers who purchase in the first period the price difference between the two periods. Although dynamic pricing and price matching provide the flexibility to determine later price after uncertainty is realized, dynamic pricing does not deter strategic behavior of customers at all, and price matching comes with a cost of reimbursement. Without any uncertainty, this flexibility does not provide any benefit. In this case, price commitment dominates dynamic pricing, and the analysis reduces to a comparison between price commitment and price matching only. When cost uncertainty exists, the flexibility to update price after uncertainty is resolved becomes more valuable. In this case, all three pricing strategies can be optimal depending on different parameters.

4. Equilibrium Analysis Under Different Pricing Strategies

In this section, we study the firm's problem under the three different pricing strategies: dynamic pricing, price commitment, and price matching. For each pricing strategy, we characterize the firm's optimal decisions and how the firm's profit changes with some important parameters.

4.1. Dynamic Pricing

We start with the dynamic pricing strategy and solve the problem backward by first analyzing the firm's optimal price in the second period. Since customers discount utility for later purchases, it can be expected that in equilibrium customers with high valuations purchase early, whereas those with low valuations wait.[1] Suppose customers with valuations higher than $\bar{v}$ have purchased in the first period. Then, the realized production cost in the second period is $c_2 = c - [\alpha + \beta(1-\bar{v}) + \epsilon]$, and the firm's problem in the second period is to choose p_2 to maximize

$$\pi_2 = (\bar{v} - p_2)(p_2 - c_2). \tag{1}$$

Because the firm's objective function is concave in p_2, the first-order condition gives rise to the optimal decision,

$$p_2^*(\bar{v}, \epsilon) = \tfrac{1}{2}\left(\bar{v} + c - [\alpha + \beta(1-\bar{v}) + \epsilon]\right).$$

Strategic customers decide whether to wait based on what they think the firm's future price will be. In this regard, we apply the concept of rational-expectation equilibrium, which is used in many recent works on strategic customer behavior, such as Su (2007, 2010) and Liu and van Ryzin (2008). Under the rational expectation framework, the firm optimizes its pricing policy subject to its correct belief about customers' purchasing behavior, whereas each customer decides when to purchase subject to the correct anticipation of the firm's future pricing decision and other customers' behavior. It can be shown that if $\bar{v} < c_2$ with nonzero probability, the firm's profit is less than the case when it only sells in the second period. Thus, in equilibrium, $\bar{v} \geq c_2$ almost surely, implying that $\bar{v} \geq p_2$ almost surely. For a customer with valuation $\bar{v}$, the utility of purchasing in the first period is equal to that of waiting. In other words, $\bar{v} - p_1 = \delta(\bar{v} - E[p_2^*])$, or equivalently,

$$\bar{v} = \frac{2p_1 - \delta(c - \alpha - \beta)}{2 - \delta(1-\beta)}.$$

In this case, the firm's total expected discounted profit in the two periods is given by

$$\pi_1 = (1-\bar{v})(p_1 - c) + \theta E[\pi_2^*(p_1, \epsilon)], \tag{2}$$

where $\pi_2^*(p_1, \epsilon) = (2(1-\beta)p_1 + (\beta\delta - \delta + 2)\epsilon - 2(c - \alpha - \beta))^2 / (4(\beta\delta - \delta + 2)^2)$ represents the firm's profit in the second period. The following proposition characterizes the equilibrium prices in the two periods.

Proposition 1. *Under the dynamic pricing strategy, the firm sells in both periods when $\alpha/(1-c) < (\theta\beta - \delta - \theta + 2)/(\theta + \delta - \theta\beta)$ and sells only in the second period otherwise. Let p_1^D and p_2^D be the equilibrium prices. Then, p_1^D and p_2^D have the following properties.*

(1) *When the firm sells in both periods, both p_1^D and $E[p_2^D]$ can be increasing or decreasing in α and β.*

(2) *When the firm only sells in the second period, $E[p_2^D]$ is decreasing in α and independent of β.*

When the technology advancement effect is significant (i.e., α is large), the firm only sells in the later period, because strategic customers expect a significant price drop in the later period and hence will have a strong incentive to wait. Instead of significantly lowering the price in the early period to encourage early purchases, it is better for the firm to sell only in the later period because a lower cost will also lead to a higher profit margin.

The effect of production learning is completely different. The firm sells only in the later period when the production learning effect is insignificant. Whereas cost reduction from technology advancement is independent of quantity in the early period, cost reduction from production learning is greater when the quantity in the early period is higher. When the production learning effect is significant, the firm should sell in the early period to capture this type of cost reduction. Thus, the firm does not sell in the early period only when the production learning effect is insignificant.

Another observation is that prices may be higher when either the technology advancement or production learning effect becomes more significant (as shown in part (1) of Proposition 1). With a more significant technology advancement effect, customers already have a strong incentive to wait. In addition, the firm may prefer more customers to wait because the profit margin in the later period is higher. This is especially true when the production learning effect is insignificant. Thus, in this case, the price in the early period increases, which leads to more high valuation customers and hence a higher price in the later period. When the production learning effect becomes more significant, there are two opposing forces driving the prices of the firm. On the one hand, a more significant production learning effect provides a stronger motivation for the firm to stimulate early demand because each unit of early production now leads to a larger cost reduction in later production. On the other hand, stimulating early demand now requires a more substantial reduction in price in the early period because customers expect lower prices in the later period. The savings due to a larger cost reduction may not justify the loss due to a smaller profit margin in the early period. Thus, prices may increase or decrease.

Finally, the total output is given by $E[q_1^D + q_2^D] = 1 - E[p_2^D]$; therefore, the total output decreases when $E[p_2^D]$ increases. According to Proposition 1, the total output may decrease when the technology advancement or production learning effect becomes more significant. In particular, when the technology advancement (production learning) effect becomes more significant, the total output will decrease only when the production learning (technology advancement) effect is insignificant. The intuition behind this is that customers' strategic waiting behavior is intensified, which leads to more high valuation customers in the later period and also a higher cost due to a smaller cost reduction from production learning. The next proposition characterizes the firm's profit.

Proposition 2. *Let π_D be the firm's expected profit under the dynamic pricing strategy. Then, there exist $\tilde{\theta}^D$ and $\tilde{\alpha}^D$ such that π_D has the following properties.*

(1) *When the firm sells in both periods, π_D is increasing in both α and β if and only if $\theta \ge \tilde{\theta}^D$ or $\alpha/(1-c) \ge \tilde{\alpha}^D$. In addition, π_D is decreasing in δ and increasing in σ^2.*

(2) *When the firm only sells in the second period, π_D is increasing in α and σ^2, and it is independent of β and δ.*

Greater uncertainty in cost reduction always benefits the firm. On the one hand, the greater uncertainty does not induce more strategic waiting by customers. On the other hand, even though the chances of either a large or small reduction in cost are higher, the gain in the former case is larger than the loss in the latter case. The reason for this is that the firm can always use the price in the second period as a lever to retain the profit margin in the case of a smaller cost reduction.

A more significant technology advancement or production learning effect can lead to a lower profit for the firm because it induces strategic waiting. This happens, in particular, when the technology advancement effect is insignificant, because the later production cost is not much lower than the early cost. With more customers waiting until the later period, the revenue from these customers is lower, but the cost is not significantly lower. Thus, the firm's profit decreases.

Finally, the firm's profit decreases when customers are more patient (δ is larger). This is intuitive because the firm's ability to price differentiate customers decreases.

4.2. Price Commitment

Now we turn to the price commitment strategy, under which the firm announces both p_1 and p_2 in advance.[2] A customer with valuation v purchases the product in the first period if $v - p_1 \ge \delta(v - p_2)$; otherwise, he purchases in the second period if $v \ge p_2$. Thus, the demands in the first period and the second period are $1 - \bar{v}$ and $\bar{v} - p_2$, respectively, where $\bar{v} = (p_1 - \delta p_2)/(1-\delta)$. The firm's problem is to choose p_1 and p_2 to maximize its profit, which is given by

$$\pi = (1-\bar{v})(p_1 - c) + \theta(\bar{v} - p_2)(p_2 - E[c_2]),$$

where $c_2 = c - [\alpha + \beta(1-\bar{v}) + \epsilon]$. The following proposition identifies the equilibrium prices under a price commitment strategy.

Proposition 3. *Under the price commitment strategy, there exists $\tilde{\alpha}_1^C \le \tilde{\alpha}_2^C$ such that the firm sells in both periods when $\tilde{\alpha}_1^C < \alpha/(1-c) < \tilde{\alpha}_2^C$, it sells only in the first period when $\alpha/(1-c) \le \tilde{\alpha}_1^C$, and it sells only in the second period when $\alpha/(1-c) \ge \tilde{\alpha}_2^C$. Let p_1^C and p_2^C be the equilibrium prices. Then, p_1^C and p_2^C have the following properties.*

(1) *When the firm sells in both periods, p_1^C may be increasing or decreasing in α and β, p_2^C may be increasing or decreasing in β, but it is always decreasing in α.*

(2) *When the firm sells only in the first period, p_1^C is independent of α and β.*

(3) *When the firm sells only in the second period, p_2^C is decreasing in α, but independent of β.*

Different from the dynamic pricing strategy, the firm can commit to not offering a deep price cut in the later period under a price commitment strategy even if the realized cost reduction turns out to be large. This helps to discourage strategic customers from waiting. In fact, when the technology advancement effect is insignificant (α is small), the firm commits to not lowering the price at all. Under this uniform pricing policy, even though the firm is not able to benefit from cost reduction, the increase in profit in the first period is more than enough to compensate. When the technology advancement effect becomes more significant, it becomes more profitable to sell in both periods by capturing the cost savings in the later period. When the technology advancement effect is very significant, the firm only sells in the later period when the production cost is very low.

Similar to the case of dynamic pricing, prices may be higher when the technology advancement or production learning effect is more significant (as shown in part (1) of Proposition 3). The major difference is that the total output does not decrease when the technology advancement effect becomes more significant, because the preannounced second-period price can now be used as a tool to deter strategic waiting. As the firm can price differentiate customers more effectively by committing to lowering the price by only a small amount, the more significant technology advancement effect does not reduce the total output. When the production learning effect becomes more significant, the firm may even commit to a higher later price to induce customers to purchase early to reduce the later cost. In this case, the total output may decrease.

Proposition 4. *Let π_C be the firm's expected profit under the price commitment strategy. Then, π_C has the following properties.*

(1) *When the firm sells in both periods, π_C is increasing in α and β, decreasing in δ, and independent of σ^2.*

(2) *When the firm sells only in the first period, π_C is independent of α, β, σ^2, and δ.*

(3) *When the firm sells only in the second period, π_C is increasing in α, but independent of β, σ^2 and δ.*

Under the price commitment strategy, a more significant technology advancement or production learning effect will not hurt the firm. This is different from the case of the dynamic pricing strategy when the firm has less control over customers' strategic waiting behavior, because the firm can commit to a future price that is higher than what customers would expect under dynamic pricing. In fact, when the technology advancement effect is insignificant, the firm commits to not lowering the price at all. When the technology advancement effect is moderately significant, however, the firm commits to a markdown policy to take advantage of the cost reduction. In this case, committing to a certain magnitude of price reduction allows the firm to reduce the cost and effectively price differentiate customers at the same time. When the technology advancement effect is very significant, the firm sells only in the second period.

However, the price commitment strategy makes it impossible for the firm to gain from greater uncertainty in cost reduction, because the later price is independent of the realized cost reduction. Although the firm still benefits from a larger profit when the realized cost reduction is large, it cannot use the second-period price to stimulate more sales and further increase profit. Similarly, when the realized cost reduction is small, the firm cannot use the second-period price to retain the profit margin. As a result, the firm's expected profit remains the same when uncertainty in cost reduction increases.

Finally, the firm's profit also decreases in customer patience (δ) under price commitment because strategic waiting still exists. This is similar to the case of dynamic pricing, but different from the case of price matching, which is studied next.

4.3. Price Matching

We now analyze the strategy of price matching, in which the firm reimburses customers who purchase early when there is a markdown after their purchases. Under price matching, it is a common practice that the firm reimburses customers the full price difference.[3] Thus, similar to Png (1991) and Lai et al. (2010), we focus on the case in which the firm reimburses $p_1 - p_2$ to customers who purchase in the first period.

Given the price in the first period, p_1, and the expected price in the second period, $E[p_2]$, a customer with valuation v has a utility of $v - p_1 + \delta(p_1 - E[p_2])$ if he purchases in the first period and a utility of $\delta(v - E[p_2])$ if he purchases in the second period. Thus, this customer will purchase in the first period as long as $v \geq p_1$. In other words, price matching completely eliminates customers' strategic waiting, and hence customers' purchase decisions in the first period do not depend on the expected price in the second period.

However, this comes at a cost to the firm because the firm has to reimburse these customers the full price difference. Thus, given p_1, the firm's problem in the second period is to choose p_2 to maximize the profit in that period, given by

$$\pi_2 = (p_1 - p_2)(p_2 - c_2) - (p_1 - p_2)(1 - p_1),$$

where $c_2 = c - [\alpha + \beta(1 - p_1) + \epsilon]$ is the realized production cost in the second period. Since π_2 is concave in p_2, the firm's optimal price in the second period is given by

$$p_2^*(p_1, \epsilon) = \tfrac{1}{2}(\beta p_1 + c - \alpha - \epsilon - \beta + 1),$$

and the optimal profit in the second period is given by

$$\pi_2^*(p_1, \epsilon) = \tfrac{1}{4}(\alpha - c + \beta + \epsilon + 2p_1 - \beta p_1 - 1)^2.$$

The firm's problem in the first period is to choose p_1 to maximize its total expected discounted profit over the two periods, given by

$$\pi_1 = (1 - p_1)(p_1 - c) + \theta E[\pi_2^*(p_1, \epsilon)].$$

The following proposition characterizes the equilibrium prices in the two periods.

Proposition 5. *Under the price matching strategy, the firm sells in both periods when* $\alpha/(1-c) < (\theta\beta - 2\theta + 2)/(2\theta - \theta\beta)$*, and it sells only in the second period otherwise. Let* p_1^M *and* p_2^M *be the equilibrium prices. Then,* p_1^M *and* p_2^M *have the following properties.*

(1) *When the firm sells in both periods,* p_1^M *is increasing in* α*, but it can be increasing or decreasing in* β*. In addition,* $E[p_2^M]$ *is decreasing in* α *and* β*.*

(2) *When the firm sells only in the second period,* $E[p_2^M]$ *is decreasing in* α *and independent of* β*.*

Unlike price commitment, price matching allows a firm to completely eliminate strategic waiting. Hence, the expected total output $(1 - E[p_2^M])$ increases with the significance of the technology advancement and production learning effects. This, however, comes at a cost because the firm has to reimburse the intertemporal price difference to customers who purchase in the first period. Hence, when either effect becomes more significant, the firm may increase the early price to reduce the number of customers that the firm needs to reimburse. The following proposition characterizes the firm's expected profit under the price matching strategy.

Proposition 6. *Let* π_M *be the firm's expected profit under the price matching strategy. Then,* π_M *has the following properties.*

(1) *When the firm sells in both periods,* π_M *is increasing in* α*,* β *and* σ^2*, and it is independent of* δ*.*

(2) *When the firm sells only in the second period,* π_M *is increasing in* α *and* σ^2*, and it is independent of* β *and* δ*.*

Different from the case under dynamic pricing, a more significant technology advancement or production learning effect always benefits the firm under price matching; different from the case under price commitment, greater uncertainty can lead to higher profit for the firm. In other words, price matching allows the firm to eliminate strategic waiting without losing the flexibility to respond to the realized production cost in the later period. This, however, comes at the cost of losing the ability to price differentiate (because customers in the first period are reimbursed the price difference). As we show in the next section, this cost of reimbursement can be larger than the combined benefit of eliminating strategic waiting and retaining flexibility in some cases.

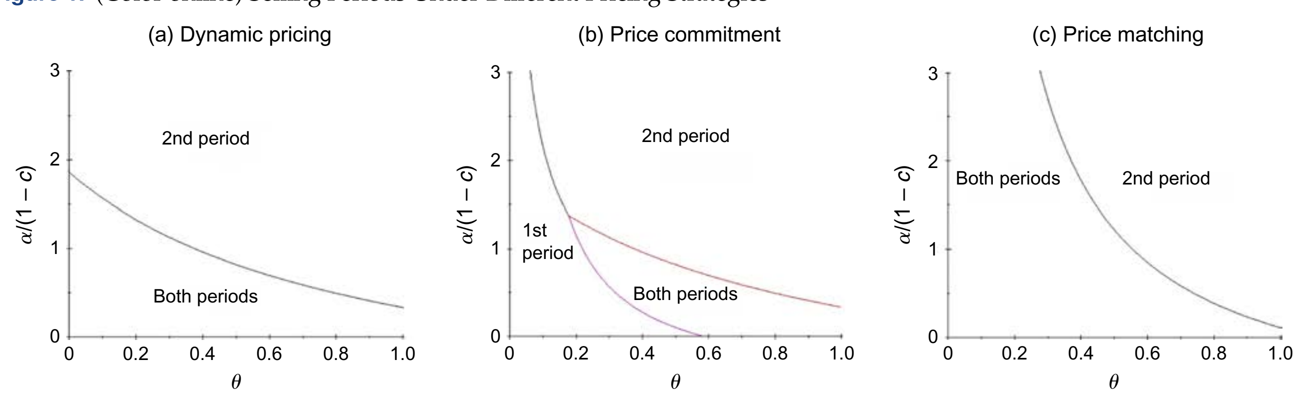
5. Comparisons of the Pricing Strategies

In this section, we compare the three pricing strategies. Because the firm may sell in only one period or in both periods, we first compare how the selling periods differ across the different pricing strategies. Figure 1 illustrates how the selling periods under dynamic pricing, price commitment, and price matching change in some parameters when $\beta = 0.2$ and $\delta = 0.7$, and the pattern is similar for other parameter values of β and δ.

Two observations are noteworthy here. First, the firm may sell only in the first period under price commitment when the technology advancement effect is insignificant or when the initial cost is low. This will never happen under dynamic pricing or price matching. Without a commitment to not lowering the price, the firm will always reduce the price in the second period due to a lower cost, and hence sales will always occur.

Second, a less farsighted firm (e.g., $\theta \le 0.7$ in our numerical study) is more likely to sell in both periods under price matching, whereas a more farsighted firm (e.g., $\theta > 0.7$ in our numerical study) is more likely to sell in both periods under dynamic pricing and price commitment (when the technology advancement effect α is intermediate). When the firm is more farsighted, the reimbursement under price matching becomes more expensive. In this case, the firm may prefer not to sell in the first period to avoid any reimbursement under price matching. When the firm is less farsighted, however, price matching can completely eliminate strategic waiting at a deeply discounted cost of reimbursement, but strategic waiting always exists under price commitment and dynamic pricing.

Figure 1. (Color online) Selling Periods Under Different Pricing Strategies



Next, we compare the firm's expected profit under the three pricing strategies in the following proposition.

Proposition 7. (1) *Suppose $\theta > \delta$. There exist $\tilde{\alpha}$, $\tilde{\sigma}_1$ and $\tilde{\sigma}_2$ such that the following statements hold.*

(a) *When $\alpha/(1-c) \geq (\theta\beta-\delta-\theta+2)/(\theta+\delta-\theta\beta)$, both dynamic pricing and price matching lead to the highest expected profit.*

(b) *When $\tilde{\alpha} < \alpha/(1-c) < (\theta\beta-\delta-\theta+2)/(\theta+\delta-\theta\beta)$, dynamic pricing leads to the highest expected profit if $\sigma^2 > \tilde{\sigma}_1$, and price commitment leads to the highest expected profit otherwise.*

(c) *When $\alpha/(1-c) \leq \tilde{\alpha}$, price matching leads to the highest expected profit if $\sigma^2 > \tilde{\sigma}_2$, and price commitment leads to the highest expected profit otherwise.*

(2) *Suppose $\theta \leq \delta$. Then, dynamic pricing and price matching both lead to the highest expected profit if $\alpha/(1-c) \geq (\theta\beta-2\theta+2)/(2\theta-\theta\beta)$, and price matching leads to the highest expected profit otherwise.*

When the firm is less farsighted ($\theta \leq \delta$), the discounted cost of reimbursement is low, and hence price matching dominates. When the technology advancement effect is very significant ($\alpha/(1-c)$ is large), dynamic pricing and price matching both dominate because the firm sells only in the second period when the cost is significantly reduced (as shown by parts (1a) and (2) in Proposition 7).

The more interesting cases take place when the firm is more farsighted ($\theta > \delta$) and the technology advancement effect is not too significant ($\alpha/(1-c) < (\theta\beta-\delta-\theta+2)/(\theta+\delta-\theta\beta)$). When the technology advancement effect is moderately significant (see part (1b) in Proposition 7), the discounted cost of reimbursement under price matching can be quite expensive. In this case, price commitment dominates when the level of uncertainty is low, and dynamic pricing dominates when the uncertainty is high because the latter allows more flexibility to respond to uncertainty. When the technology advancement effect is insignificant (part (1c) in Proposition 7), the reimbursement under price matching is less costly. In this case, price matching is the best strategy when the uncertainty is high, whereas price commitment still dominates when there is little uncertainty.

Although price matching can be optimal when the firm is more farsighted ($\theta > \delta$) and the technology advancement effect is not too significant ($\alpha/(1-c) < (\theta\beta-\delta-\theta+2)/(\theta+\delta-\theta\beta)$ such that it is optimal to sell in both periods under dynamic pricing), our numerical studies suggest that this is not common. In particular, we numerically compute the optimal pricing strategy for $c \in \{0.15, 0.35, 0.55\}$, $\alpha \in \{0.03, 0.09, 0.15\}$, $\beta \in \{0.03, 0.09, 0.15\}$, $\sigma^2 \in \{0.001, 0.005, 0.01\}$, $\theta \in \{0.2, 0.4, 0.6, 0.8\}$, and $\delta \in \{0.2\theta, 0.4\theta, 0.6\theta, 0.8\theta\}$. Out of the 816 instances in which the parameters are valid (to avoid negative production cost or negative cost reduction in the second period) and fall within the region of interest ($\theta > \delta$ and $\alpha/(1-c) < (\theta\beta-\delta-\theta+2)/(\theta+\delta-\theta\beta)$), price matching emerges as the optimal strategy only in six instances (0.74%) when both α and β are very small. This is because the discounted cost of reimbursement under price matching is high except when both the technology advancement and production learning effects are insignificant. Thus, we focus on the comparison between dynamic pricing and price commitment in this region.

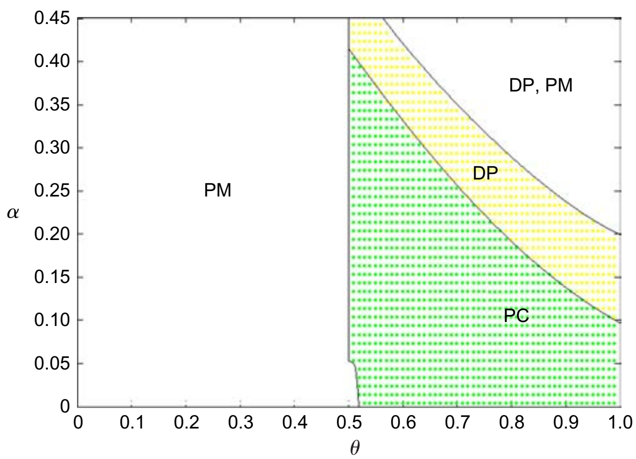
Our numerical studies suggest that the technology advancement effect and the production learning effect can have opposite effects on the optimal pricing strategy. In particular, we conduct numerical studies on the same set of parameter values on c, σ^2, θ, and δ, and α and β starting at 0.01 and incrementing by 0.01 (when $c = 0.15$), 0.02 (when $c = 0.35$), or 0.04 (when $c = 0.55$) until the parameters become invalid or until it is optimal to sell only in the second period under dynamic pricing. When the technology advancement effect becomes more significant, it is possible that the optimal pricing strategy changes from price commitment to dynamic pricing, but not the other way around. On the contrary, when the production learning effect becomes more significant, it is possible that the optimal pricing strategy changes from dynamic pricing to price commitment, but not the other way around. The reason behind this is that when the production learning effect becomes more significant, it is important to alleviate strategic waiting of customers to capture a larger cost reduction. However, cost reduction from technology advancement is not affected by strategic waiting. Hence, retaining the flexibility to respond to the realized cost reduction is more important when the technology advancement effect is significant. Table 1 illustrates the percentage improvement of profit under price commitment over that under dynamic pricing when $c = 0.55$, $\sigma^2 = 0.001$, $\theta = 0.6$, and $\delta = 0.48$, where "NA" denotes instances where the parameters are invalid. When $\beta \leq 0.17$, the optimal strategy changes from price commitment to dynamic pricing as α increases. When $\alpha = 0.33$, the optimal strategy changes from dynamic pricing to price commitment as β increases.

Now we summarize our findings regarding the choice of pricing strategy from both analytic and numerical explorations. Price matching is optimal when the firm is less farsighted. When the firm is more farsighted, price commitment is optimal when the technology advancement effect is insignificant, and dynamic pricing is optimal when the technology advancement effect is moderately significant and very significant. In addition, in the latter case, price matching is optimal only for a small set of possible parameter combinations when both the technology advancement

Table 1. Percentage Improvement of Profit Under Price Commitment Over Dynamic Pricing (%)

	α (%)									
β	0.01	0.05	0.09	0.13	0.17	0.21	0.25	0.29	0.33	0.37
0.01	6.05	4.55	3.27	2.21	1.39	0.77	0.35	0.07	−0.08	−0.14
0.05	5.89	4.47	3.26	2.25	1.46	0.86	0.43	0.14	−0.04	−0.12
0.09	5.74	4.40	3.25	2.29	1.52	0.94	0.50	0.20	0.01	−0.09
0.13	5.61	4.34	3.25	2.33	1.59	1.01	0.58	0.27	0.07	−0.06
0.17	5.49	4.29	3.25	2.37	1.65	1.09	0.66	0.35	0.13	−0.01
0.21	5.39	4.25	3.25	2.41	1.71	1.16	0.74	0.42	0.19	NA
0.25	5.30	4.21	3.26	2.45	1.77	1.23	0.81	0.49	NA	NA
0.29	5.22	4.18	3.27	2.48	1.83	1.30	0.89	NA	NA	NA

Figure 2. (Color online) Optimal Pricing Strategy



Note. PM, price matching; DP, dynamic pricing; PC, price commitment.

and production learning effects are very insignificant. Figure 2 illustrates the optimal pricing strategy when $c = 0.55$, $\beta = 0.1$, $\delta = 0.5$, and $\sigma^2 = 0.001$.

Other than the firm's profit, it is also interesting to compare the total production quantities induced by the three pricing strategies. A higher total quantity implies that more customers are able to enjoy the product, and thus the firm is delivering more value to society. In the following proposition, we consider the expected total quantity under different pricing strategies ($q^i = E[q_1^i + q_2^i]$, where $i = D, C, M$).

Proposition 8. (1) *Suppose $\theta \geq \delta$. Then, all three pricing strategies lead to the same expected total quantity if $\alpha/(1-c) \geq (\theta\beta - \delta - \theta + 2)/(\theta + \delta - \theta\beta)$, and dynamic pricing leads to the highest expected total quantity otherwise.*

(2) *Suppose $\theta < \delta$. Then, there exists $\tilde{\alpha}^q$ such that the following statement holds: all three pricing strategies lead to the same expected total quantity if $\alpha/(1-c) \geq (\theta\beta - 2\theta + 2)/(2\theta - \theta\beta)$, price matching leads to the highest expected total quantity if $\tilde{\alpha}^q \leq \alpha/(1-c) < (\theta\beta - 2\theta + 2)/(2\theta - \theta\beta)$, and dynamic pricing leads to the highest expected total quantity otherwise.*

As shown in part (1) in Proposition 8, dynamic pricing always leads to the highest output when the firm is more farsighted, because it offers the firm the highest flexibility to increase quantity sold in the second period when the cost reduction turns out to be high. Even though strategic waiting prevails, the firm's decision in the second period is not constrained by a committed price or affected by the concern of price matching reimbursement, which is quite costly in this case.

When the firm is less farsighted (part (2) in Proposition 8), however, price matching may lead to the highest output because the discounted cost of reimbursement is low. This happens when the technology advancement effect is significant. When the technology advancement effect is insignificant, cost reduction from production learning becomes more important. Because production learning requires a higher quantity in the early period, which is costly under price matching due to the larger number of customers receiving reimbursement, dynamic pricing leads to the highest output in this case.

Finally, it is noteworthy to discuss how the firm's profit changes with different parameters when the firm can choose among dynamic pricing, price commitment, and price matching. First, the firm's profit may still increase in the uncertainty in cost reduction when the firm can choose among dynamic pricing, price commitment, and price matching. In particular, this happens when dynamic pricing or price matching is the optimal strategy. In addition, under the optimal strategy, although the firm's profit always increases in the significance of the production learning effect or the technology advancement effect, the prices may still increase and the total output may still decrease in the significance of these two effects.

6. Extensions

In this section, we extend our results by altering some of the assumptions in the model. In the previous analysis, we assumed that the later production cost is linear in the early production quantity and all potential customers are present at the beginning of the selling season. Our first extension tests the robustness of our analytical results under the exponential learning curve. Our second extension considers the case in which some new customers arrive in the second period.

6.1. Exponential Learning Curve

The linear learning curve has the advantage of being easily understood by managers, as β can be interpreted as the reduction in unit cost for each unit of product produced in the first period. However, it is important to consider other forms of learning curve. Perhaps the most natural learning curve that will arise is the exponential learning curve, which is also widely used in the literature (e.g., Fine and Porteus 1989, Bernstein and Kok 2009). In this section, we study the case in which the unit cost in the second period is given by

$$\ln(c_2) = \ln(c) + \ln(1 - \alpha + \epsilon) - \beta \ln(1 + q_1),$$

where ϵ can be τ and $-\tau$, and each has a probability of 0.5. Thus, $0 \leq \tau < \min(\alpha, 1-\alpha)$ measures the uncertainty in cost reduction. For the basic settings, we consider cases with $c \in \{0.1, 0.3, 0.5\}$, $\tau \in \{0.05, 0.15, 0.25\}$, $\theta \in \{0.2, 0.4, 0.6, 0.8\}$, $\delta \in \{0.2, 0.4, 0.6, 0.8\}$, $\alpha \in \{0.1, 0.2, 0.3, 0.4\}$, and $\beta \in \{1, 2, 3\}$, and there are 1,296 cases in which the parameter values are valid. To study the effects of α and β on the pricing decisions, we study even more cases with larger ranges and smaller intervals for α and β.

The observations from the numerical results are mostly consistent with the insights from the analytic results concerning the optimal decisions under each pricing strategy. First, the firm sells in both periods or only in the later period under dynamic pricing or price matching, but may sell in the early period only, the later period only, or both periods under price commitment. Second, a more significant technology advancement effect (α) or production learning effect (β) may hurt the firm under dynamic pricing, but always benefits the firm under price commitment or price matching.

The results of comparing the different pricing strategies are also similar to our previous analytic results. If the firm is less farsighted (θ is small), price matching is the optimal strategy. If the firm is more farsighted, dynamic pricing is optimal when the technology advancement effect is relatively significant, and price commitment is optimal when the technology advancement effect is insignificant. In addition, a larger α may induce the optimal strategy to switch from price commitment to dynamic pricing, but not the other way around; a larger β may induce the optimal strategy to switch from dynamic pricing to price commitment, but not the opposite.

The only difference is that price matching does not arise as the optimal strategy in the cases in which the firm is more farsighted and the technology advancement effect is not very significant. In fact, price matching rarely arises as the optimal strategy (less than 1%) in the case of a linear learning curve according to our numerical study in the previous section.

6.2. New Customer Arrivals

We now investigate the case in which new customers arrive in the later period. Let m denote the size of the new customers in the second period. Same as the customers arriving in the first period, these new customers have valuations that are heterogeneous and distributed uniformly on the interval $[0, 1]$. In addition, ϵ follows a two-point distribution, and it equals τ with a probability of $\frac{1}{2}$ and $-\tau$ with a probability of $\frac{1}{2}$. For the basic setting, consider cases with $m \in \{0.2, 1, 5\}$, $c \in \{0.15, 0.35, 0.55\}$, $\tau \in \{0.02, 0.06, 0.1\}$, $\theta \in \{0.25, 0.5, 0.75\}$, $\delta \in \{0.25, 0.5, 0.75\}$, $\alpha \in \{0.05, 0.15, 0.25\}$, and $\beta \in \{0.05, 0.15, 0.25\}$, and there are 756 cases in which the parameter values are valid. To study the effects of α and β on the pricing decisions, we study even more cases with larger ranges and smaller intervals for α and β. Note that the size of customers arriving in the first period is equal to 1; hence, $m = 0.2$, 1, and 5 represent cases where the size of new customers arriving in the second period is small, moderate, and large, respectively.

When $m = 0.2$ or 1, the only difference from the original model is that the firm will always sell in the second period even under price commitment (in contrast to the original model where the firm may sell only in the first period). The reason for this is that the new customers with high valuation in the second period will purchase even if the firm commits to not lowering the price.

When $m = 5$, we obtain another observation that is different from the analytical results without new customer arrivals: A more significant technology advancement or production learning effect always benefits the firm under all pricing strategies. The intuition behind this is that when the market size of early customers is much smaller than the size of customers arriving late, the loss of profit due to strategic waiting of the first group of customers is less important than the lower production cost in serving new customers in the second period.

7. Discussions

We study the impact of three important drivers, namely, cost reduction, uncertainty, and strategic customer behavior, on a firm's pricing strategy and profit. Our results demonstrate that the benefits of cost reduction are greatly influenced by strategic customer behavior. The more the cost is expected to be reduced, the greater the incentive for strategic customers to wait. Thus, a more significant technology advancement or production learning effect may not be beneficial to the firm. The comparison of dynamic pricing, price commitment, and price matching suggests that none of these strategies dominates the other two. In general, price matching is optimal only when the firm is less farsighted. Otherwise, price commitment or dynamic pricing will be a better strategy.

In practice, it has not been observed that Sony or other companies in the electronic gaming industry have used price matching to deter customers from waiting for markdown due to cost reduction. Our result that price matching is rarely optimal when the firm is farsighted can provide one explanation for this phenomenon. However, there are several occasions where Sony made announcements that price would not fall in the near future,[4] which can be viewed as a form of price commitment. Of course, one disadvantage of price commitment not studied in this paper is that it does not provide the firm with any flexibility to respond to competitors' pricing. Thus, it would be interesting to extend the comparison to situations where other drivers such as competition and market uncertainty exist.

In this paper, we assume that customers have rational expectations about the magnitude of price reduction due to production learning and technology advancement. For products, such as electronic goods, where the costs are more openly accessible, customers may be able to predict future price trends almost as accurately as the firm. However, for products of which the costs are less accessible, the information asymmetry between the firm and customers can be more significant. For price matching and price commitment, this is not an issue, because customers either do not need to know the future price or know exactly what it will be. In the case of dynamic pricing, customers may behave differently depending on their beliefs. First, they may try to obtain signals about future price trends based on the firm's early price. Empirical research on how customers estimate future prices from current prices would be interesting. In addition, customers may form beliefs from observing the firm's price reductions after multiple repeated interactions. It would also be interesting to study how a firm should respond to the realized cost reduction when this decision can affect customers' beliefs about future cost reductions.

In addition to waiting for markdown due to cost reduction, consumers may also wait for new product generations (which is considered in Lobel et al. 2016). The latter effect may not interfere with the former one for the case of the Sony PS Vita and other electronic gaming products since the life cycles of these products are typically years and much longer than the time it takes for cost to reduce.[5] For products with shorter product life cycles, however, it would be useful to consider the combined impact of both cost reduction and new product launches.

Finally, we study a two-period setting, which is common in the strategic customer behavior literature. We expect most of the insights from our model can be extended to a multiperiod setting. The difference is that in a multiperiod model, even when the firm produces nothing for a number of periods at the beginning to take advantage of the technology advancement effect, production learning still occurs in later periods. Although it is interesting to capture this effect, the multiperiod model presents challenges in analytical tractability. For price matching and price commitment, customers either do not need to know the future price or know exactly what it will be. For these two pricing strategies, the multiperiod model can be studied by analyzing a multiperiod dynamic programming problem (for price matching) or a single-period multivariable problem (for price commitment). For dynamic pricing, however, the multiperiod model will be equivalent to a multistage game where some uncertainty is resolved in each stage. The existence of a subgame perfect equilibrium is by itself a challenging problem. In the case with no cost reduction and no uncertainty, Besanko and Winston (1990) show the existence of a subgame perfect equilibrium and characterize the equilibrium. Although extending their problem to the case with cost reduction and cost uncertainty would clearly be interesting, this is a substantial problem that deserves a separate study.

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Appendix A. Proofs

Proof of Proposition 1. The firm's problem in the first period is

$$\begin{aligned}\underset{p_1}{\text{maximize}}\quad \pi_1 &= \left(1-\frac{2p_1-\delta(c-\alpha-\beta)}{2-\delta(1-\beta)}\right)(p_1-c)\\ &\quad+\frac{\theta(2(1-\beta)p_1-2(c-\alpha-\beta))^2}{4(\beta\delta-\delta+2)^2}+\frac{\theta\sigma^2}{4},\\ \text{s.t.}\quad &\frac{2p_1-\delta(c-\alpha-\beta)}{2-\delta(1-\beta)}\le 1.\end{aligned}$$

Note that π_1 is concave in p_1. By using the first-order condition, we get

$$\begin{aligned}p_1^* = \big(&(2\theta-2\delta+\delta^2-2\theta\beta-\beta\delta^2)\alpha\\ &+(-2\theta\beta^2-\beta\delta^2+2\beta\delta+2\theta\beta+\delta^2-4\delta+4)(1-c)\big)\\ &\cdot\big(-2(\theta+2\delta-2\theta\beta-2\beta\delta+\theta\beta^2-4)\big)^{-1}+c.\end{aligned}$$

Note that $(2p_1^*-\delta(c-\alpha-\beta))/(2-\delta(1-\beta))<1$ if and only if $\alpha/(1-c)<(\theta\beta-\delta-\theta+2)/(\theta+\delta-\theta\beta)$. Therefore, when $\alpha/(1-c)\ge(\theta\beta-\delta-\theta+2)/(\theta+\delta-\theta\beta)$, the firm only sells in the second period, and $E[p_2^D]=(1+c-\alpha)/2$, which is decreasing in α and independent of β. When $\alpha/(1-c)<(\theta\beta-\delta-\theta+2)/(\theta+\delta-\theta\beta)$, the firm sells in both periods, where $p_1^D=p_1^*$ and

$$\begin{aligned}E[p_2^D] = \big(&(2\theta\beta-3\delta-2\theta+\beta\delta+4)\alpha\\ &+(2\beta+\delta-2\theta\beta-3\beta\delta+2\theta\beta^2-2)(1-c)\big)\\ &\cdot\big(2(\theta+2\delta-2\theta\beta-2\beta\delta+\theta\beta^2-4)\big)^{-1}+c.\end{aligned}$$

In this case, we derive sensitivity analysis results as follows:

(a) $(\partial/\partial\alpha)p_1^D = ((-\delta^2 - 2\theta)\beta + (\delta^2 - 2\delta + 2\theta))/(-2(\theta + 2\delta - 2\theta\beta - 2\beta\delta + \theta\beta^2 - 4))$, which is positive if and only if $\beta < (\delta^2 + 2\theta - 2\delta)/(\delta^2 + 2\theta)$.

(b) $(\partial/\partial\alpha)E[p_2^D] = ((-2\theta - \delta)\beta + (2\theta + 3\delta - 4))/(-2(\theta + 2\delta - 2\theta\beta - 2\beta\delta + \theta\beta^2 - 4))$, which is positive if and only if $\beta < (2\theta + 3\delta - 4)/(2\theta + \delta)$.

(c) $(\partial/\partial\beta)p_1^D = -\theta(N_1^D\alpha + N_2^D(1-c))/(2(\theta + 2\delta - 2\theta\beta - 2\beta\delta + \theta\beta^2 - 4)^2)$, where $N_1^D = (\beta^2 - 2\beta + 1)\delta^2 + (4\beta - 4)\delta + 2\theta\beta^2 - 4\theta\beta + 2\theta + 8$ and $N_2^D = (2\beta^2 - 4\beta + 2)\theta + (\beta^2\delta^2 + 2\beta^2\delta - 2\beta\delta^2 + 8\beta + \delta^2 - 2\delta)$. Since N_1^D is convex and decreasing in $\delta \in [0,1]$ and $N_1^D|_{\delta=1} > 0$, we have $N_1^D > 0$. N_2^D is increasing in θ with $N_2^D|_{\theta=1} > 0$ and $N_2^D|_{\theta=0} = N_3^D = (\beta^2 - 2\beta + 1)\delta^2 + (2\beta^2 - 2)\delta + 8\beta$. Note that N_3^D is convex and decreasing in $\delta \in [0,1]$, and $N_3^D|_{\delta=1} \ge 0$ if and only if $\beta \ge \frac{2}{3}\sqrt{3} - 1$. So when $\beta \ge \frac{2}{3}\sqrt{3} - 1$, $N_3^D \ge 0$; when $\beta < \frac{2}{3}\sqrt{3} - 1$, $N_3^D \ge 0$ if and only if $\delta \le (1 + \beta - \sqrt{-6\beta + \beta^2 + 1})/(1-\beta)$. Therefore, when $\beta < \frac{2}{3}\sqrt{3} - 1$, $\delta > (1 + \beta - \sqrt{-6\beta + \beta^2 + 1})/(1-\beta)$, $\theta < -((\beta^2 - 2\beta + 1)\delta^2 + (2\beta^2 - 2)\delta + 8\beta)/(2\beta^2 - 4\beta + 2)$ and $\alpha/(1-c) < -N_2^D/N_1^D$, $(\partial/\partial\beta)p_1^D > 0$; otherwise, $(\partial/\partial\beta)p_1^D \le 0$. Define $\tilde{\alpha}_1^D$ as $-N_2^D/N_1^D$. We have $(\theta\beta - \delta - \theta + 2)/(\theta + \delta - \theta\beta) - \tilde{\alpha}_1^D = -2(\beta\delta - \delta + 2)(\theta + 2\delta - 2\theta\beta - 2\beta\delta + \theta\beta^2 - 4)/((\theta + \delta - \theta\beta)N_1^D) > 0$.

(d) $(\partial/\partial\beta)E[p_2^D] = -(N_4^D\alpha + N_5^D(1-c))/(2(\theta + 2\delta - 2\theta\beta - 2\beta\delta + \theta\beta^2 - 4)^2)$, where $N_4^D = (2\beta^2 - 4\beta + 2)\theta^2 + (8\beta + 5\delta - 6\beta\delta + \beta^2\delta)\theta + 4\delta^2 - 4\delta$, and $N_5^D = (2\beta^2 - 4\beta + 2)\theta^2 + (12\beta + 5\delta + 2\beta^2 - 6\beta\delta + \beta^2\delta - 6)\theta + 4\delta^2 - 12\delta + 8$. Note that N_4^D is convex and increasing in $\theta \in [0,1]$. Since $N_4^D|_{\theta=0} < 0$ and $N_4^D|_{\theta=1} > 0$, there exists $\theta_t^D \in (0,1)$ such that $N_4^D|_{\theta=\theta_t^D} = 0$ and $N_4^D > 0$ if and only if $\theta > \theta_t^D$. Note that N_5^D is convex in θ, $N_5^D|_{\theta=0} > 0$ and $N_5^D|_{\theta=1} > 0$. If $0 < (12\beta + 5\delta + 2\beta^2 - 6\beta\delta + \beta^2\delta - 6)/(-2(2\beta^2 - 4\beta + 2)) < 1$, the discriminant of N_5^D with respect to θ is positive, and $\tilde{\theta}_1^D < \theta < \tilde{\theta}_2^D$, that is, $\beta < \frac{2}{3}\sqrt{3} - 1$, $\delta > (2(\beta + 1)(9 - \beta^2 - 4\sqrt{2}\sqrt{3\beta^2 + 1}))/((\beta - 1)(-10\beta + \beta^2 - 7))$ and $\tilde{\theta}_1^D < \theta < \tilde{\theta}_2^D$, $N_5^D < 0$; otherwise, $N_5^D \ge 0$. Here, $\tilde{\theta}_1^D$ and $\tilde{\theta}_2^D$ are the two solutions to $N_5^D = 0$. When $N_4^D < 0$, $N_4^D\alpha + N_5^D(1-c) > (N_4^D((\theta\beta - \delta - \theta + 2)/(\theta + \delta - \theta\beta)) + N_5^D) \cdot (1-c) = (-2\theta(\beta + 1)(\theta + 2\delta - 2\theta\beta - 2\beta\delta + \theta\beta^2 - 4)(1-c))/(\theta + \delta - \theta\beta) > 0$, so $(\partial/\partial\beta)E[p_2^D] < 0$. Note that $N_5^D > N_4^D$ if and only if $\theta < 4(1-\delta)/(3 - 2\beta - \beta^2)$. Since $\theta_t^D < 4(1-\delta)/(3 - 2\beta - \beta^2)$, $N_5^D > 0$ when $\theta = \theta_t^D$ (i.e., $N_4^D = 0$). So $(\partial/\partial\beta)E[p_2^D] < 0$ when $N_4^D = 0$. When $N_4^D > 0$ and $N_5^D \ge 0$, $(\partial/\partial\beta)E[p_2^D] < 0$. Finally, when $N_4^D > 0$ and $N_5^D < 0$, we can show that $\theta_t^D < \tilde{\theta}_1^D$, so $(\partial/\partial\beta)E[p_2^D] > 0$ if and only if $\beta < \frac{2}{3}\sqrt{3} - 1$, $\delta > (2(\beta + 1) \cdot (9 - \beta^2 - 4\sqrt{2}\sqrt{3\beta^2 + 1}))/((\beta - 1)(-10\beta + \beta^2 - 7))$, $\tilde{\theta}_1^D < \theta < \tilde{\theta}_2^D$, and $\alpha/(1-c) < -N_5^D/N_4^D$. Define $\tilde{\alpha}_2^D$ as $-N_5^D/N_4^D$. We have $\tilde{\alpha}_2^D < (\theta\beta - \delta - \theta + 2)/(\theta + \delta - \theta\beta)$. □

Proof of Proposition 2. When the firm sells in both periods, $\pi_D = -((\delta^2 + 4\theta)\alpha^2 + (2\delta^2 - 4\delta + 4\theta + 4\theta\beta)(1-c)\alpha + (\delta^2 - 4\delta + 4\theta\beta + 4)(1-c)^2)/(4(\theta + 2\delta - 2\theta\beta - 2\beta\delta + \theta\beta^2 - 4)) + \theta\sigma^2/4$. By taking the derivative of π_D with the corresponding parameters, we can have $(\partial/\partial\alpha)\pi_D > 0$ if and only if $\theta \ge (2\delta - \delta^2)/(2(1+\beta))$ or $\alpha/(1-c) > -(\delta^2 - 2\delta + 2\theta + 2\theta\beta)/(\delta^2 + 4\theta)$; $(\partial/\partial\beta)\pi_D > 0$ if and only if $\theta \ge (2\delta - \delta^2)/(2(1+\beta))$ or $\alpha/(1-c) > -(\delta^2 - 2\delta + 2\theta + 2\theta\beta)/(\delta^2 + 4\theta)$; $(\partial/\partial\delta)\pi_D < 0$; and $(\partial/\partial(\sigma^2))\pi_D > 0$. In part (1), the results follow by defining $\tilde{\theta}^D = (2\delta - \delta^2)/(2(1+\beta))$ and $\tilde{\alpha}^D = -(\delta^2 - 2\delta + 2\theta + 2\theta\beta)/(\delta^2 + 4\theta)$. When the firm only sells in the second period, $\pi_D = \theta(((1 - c + \alpha)^2 + \sigma^2)/4)$, which is increasing in α and σ^2 but independent of β. □

Proof of Proposition 3. The firm's problem is

$$\begin{aligned}\underset{p_1,p_2}{\text{maximize}}\quad \pi &= \left(1 - \frac{p_1 - \delta p_2}{1-\delta}\right)(p_1 - c) + \theta\left(\frac{p_1 - \delta p_2}{1-\delta} - p_2\right)\\ &\quad\cdot\left(p_2 - \left(c - \alpha - \beta\left(1 - \frac{p_1 - \delta p_2}{1-\delta}\right)\right)\right),\\ \text{s.t.}\quad & 1 - \frac{p_1 - \delta p_2}{1-\delta} \ge 0,\\ & \frac{p_1 - \delta p_2}{1-\delta} - p_2 \ge 0.\end{aligned}$$

We can prove that $(\partial^2/\partial p_1^2)\pi < 0$, $(\partial^2/\partial p_2^2)\pi < 0$, $(\partial^2/\partial p_1\partial p_2)\pi > 0$, and $((\partial^2/\partial p_1^2)\pi)((\partial^2/\partial p_2^2)\pi) - ((\partial^2/\partial p_1\partial p_2)\pi)^2 = -(1/(\delta-1)^2)((\beta^2 - 2\beta + 1)\theta^2 + (2\delta - 2\beta\delta - 4)\theta + \delta^2)$. Note that $(\beta^2 - 2\beta + 1)\theta^2 + (2\delta - 2\beta\delta - 4)\theta + \delta^2$ is convex and decreasing in $\theta \in [0,1]$, and it is positive when $\theta = 0$ and negative when $\theta = 1$. So there exists $\theta_t^C \in (0,1)$ such that π is jointly concave in (p_1, p_2) if and only if $\theta > \theta_t^C$.

Suppose $\theta > \theta_t^C$. By using the first-order condition, we have $p_1^* = (\theta(\delta - \theta + \theta\beta)\alpha + \theta(2\delta - \theta\beta - \beta\delta + \theta\beta^2 - 2) \cdot (1-c))/(\theta^2\beta^2 - 2\theta^2\beta + \theta^2 - 2\theta\beta\delta + 2\theta\delta - 4\theta + \delta^2) + c$ and $p_2^* = -(\theta(\theta + \delta - \theta\beta - 2)\alpha + (\theta\beta + 1 - \delta)(\theta + \delta - \theta\beta)(1-c))/(\theta^2\beta^2 - 2\theta^2\beta + \theta^2 - 2\theta\beta\delta + 2\theta\delta - 4\theta + \delta^2) + c$. For the first constraint, $1 - (p_1^* - \delta p_2^*)/(1-\delta) > 0$ if and only if $\alpha/(1-c) < (\theta\beta - \delta - \theta + 2)/(\theta + \delta - \theta\beta)$. For the second constraint, when $\theta \ge \delta/(1+\beta)$, $(p_1^* - \delta p_2^*)/(1-\delta) - p_2^* > 0$; otherwise, $(p_1^* - \delta p_2^*)/(1-\delta) - p_2^* > 0$ if and only if $\alpha/(1-c) > -(\theta + \theta\beta - \delta)/(2\theta)$. Note that $\theta_t^C < \delta/(1+\beta)$. Therefore, (p_1^*, p_2^*) is an optimal solution in one of the following two cases: (a) $\theta \ge \delta/(1+\beta)$ and $\alpha/(1-c) < (\theta\beta - \delta - \theta + 2)/(\theta + \delta - \theta\beta)$; (b) $\theta_t^C < \theta < \delta/(1+\beta)$ and $-(\theta + \theta\beta - \delta)/(2\theta) < \alpha/(1-c) < (\theta\beta - \delta - \theta + 2)/(\theta + \delta - \theta\beta)$. In the remaining parameter ranges, the optimal solution is on the boundaries. When $(p_1 - \delta p_2)/(1-\delta) - p_2 = 0$, the firm sells only in the first period, and the optimal price and profit are, respectively, $p_1^{B1} = (1+c)/2$ and $\pi^{B1} = (1-c)^2/4$. When $1 - (p_1 - \delta p_2)/(1-\delta) = 0$, the firm sells only in the second period, and the optimal price and profit are, respectively, $p_2^{B2} = (1 + c - \alpha)/2$ and $\pi^{B2} = \theta((1 - c + \alpha)^2)/4$. By comparing π^{B1} and π^{B2}, we have the following: (a) when $\theta > \theta_t^C$ and $\alpha/(1-c) \ge (\theta\beta - \delta - \theta + 2)/(\theta + \delta - \theta\beta)$, the firm sells only in the second period; (b) when $\theta_t^C < \theta < \delta/(1+\beta)$ and $\alpha/(1-c) \le -(\theta + \theta\beta - \delta)/(2\theta)$, the firm sells only in the first period.

Suppose $\theta \le \theta_t^C$. In this case, π is not jointly concave in (p_1, p_2). Thus, (p_1^*, p_2^*) is the only extreme point of π, and it is interior only if $-(\theta + \theta\beta - \delta)/(2\theta) < \alpha/(1-c) < (\theta\beta - \delta - \theta + 2)/(\theta + \delta - \theta\beta)$, which contradicts $-(\theta + \theta\beta - \delta)/(2\theta) \ge (\theta\beta - \delta - \theta + 2)/(\theta + \delta - \theta\beta)$. So (p_1^*, p_2^*) cannot be optimal, and the optimal solution comes from the boundaries. By comparing the profits on the two boundaries, we have the following: (a) when $\alpha/(1-c) \ge 1/\sqrt{\theta} - 1$, the firm sells only in the second period; (b) when $\alpha/(1-c) < 1/\sqrt{\theta} - 1$, the firm sells only in the first period.

Define

$$\tilde{\alpha}_1^C = \begin{cases} \dfrac{1}{\sqrt{\theta}} - 1, & \text{if } \theta \le \theta_t^C,\\[2ex] \dfrac{-(\theta + \theta\beta - \delta)}{2\theta}, & \text{otherwise,}\end{cases}$$

and

$$\tilde{\alpha}_2^C = \begin{cases} \dfrac{1}{\sqrt{\theta}} - 1, & \text{if } \theta \le \theta_t^C, \\ \dfrac{\theta\beta - \delta - \theta + 2}{\theta + \delta - \theta\beta}, & \text{otherwise.} \end{cases}$$

Thus, $\tilde{\alpha}_1^C \le \tilde{\alpha}_2^C$. Since $-(\theta + \theta\beta - \delta)/(2\theta) \le 0$ when $\theta \ge \delta/(1+\beta)$, we have three possible outcomes summarized as follows: (1) when $\tilde{\alpha}_1^C < \alpha/(1-c) < \alpha_2^C$, the firm sells in two periods, and $(p_1^C, p_2^C) = (p_1^*, p_2^*)$; (2) when $\alpha/(1-c) \le \tilde{\alpha}_1^C$, the firm sells only in the first period, and $p_1^C = (1+c)/2$; (3) when $\alpha/(1-c) \ge \alpha_2^C$, the firm sells only in the second period, and $p_2^C = (1+c-\alpha)/2$.

For outcome (1), $(\partial/\partial\alpha)p_1^C = \theta(\delta - \theta + \theta\beta)/(\theta^2\beta^2 - 2\theta^2\beta + \theta^2 - 2\theta\beta\delta + 2\theta\delta - 4\theta + \delta^2)$, which is positive if and only if $\beta < 1 - \delta/\theta$.

And $(\partial/\partial\beta)p_1^C = -\theta((\theta N_1^C\alpha + N_2^C(1-c))/((\theta^2\beta^2 - 2\theta^2\beta + \theta^2 - 2\theta\beta\delta + 2\theta\delta - 4\theta + \delta^2)^2))$, where $N_1^C = (\beta^2 - 2\beta + 1)\theta^2 + (2\beta\delta - 2\delta + 4)\theta - 3\delta^2$ and $N_2^C = (\theta^3 + \delta\theta^2)\beta^2 - 2\theta \cdot (-2\theta + \theta^2 + \delta^2)\beta + (\theta+\delta)(\theta-\delta)^2$. There exists $\theta_{t1}^C > \theta_t^C$ such that $N_1^C|_{\theta=\theta_{t1}^C} = 0$ and $N_1^C < 0$ if and only if $\theta < \theta_{t1}^C$. We can also show that $N_2^C > 0$. Thus, when $\theta \ge \theta_{t1}^C$, $(\partial/\partial\beta)p_1^C < 0$; when $\theta_t^C < \theta < \theta_{t1}^C$, $(\partial/\partial\beta)p_1^C < 0$ if and only if $\alpha/(1-c) < N_2^C/(-\theta N_1^C)$. Define $\hat{\alpha}_1^C$ as $N_4^C/(-\theta N_3^C)$. We further find that $\hat{\alpha}_1^C \ge (\theta\beta - \delta - \theta + 2)/(\theta + \delta - \theta\beta)$ if and only if $\theta \ge \tilde{\theta}^C$, where $\tilde{\theta}^C \in (\theta_t^C, \theta_{t1}^C)$. Besides, when $\theta < \delta/(1+\beta)$, we can show $\hat{\alpha}_1^C > -(\theta + \theta\beta - \delta)/(2\theta)$. Thus, p_1^C is decreasing in β if and only if $\theta \ge \tilde{\theta}^C$ or $\alpha/(1-c) < \hat{\alpha}_1^C$.

In addition, $(\partial/\partial\beta)p_2^C = -\theta((\theta N_3^C\alpha + N_4^C(1-c))/(\theta^2\beta^2 - 2\theta^2\beta + \theta^2 - 2\theta\beta\delta + 2\theta\delta - 4\theta + \delta^2)^2)$, where $N_3^C = (\beta^2 - 2\beta + 1)\theta^2 + (4\beta + 2\delta - 2\beta\delta)\theta + \delta^2 - 4\delta$ and $N_4^C = (\theta^3 + \theta^2)\beta^2 + (6\theta^2 - 2\theta^3 - 2\theta\delta - 2\theta^2\delta)\beta + (\theta+1)\delta^2 + (2\theta^2 - 6\theta)\delta + (\theta^3 - 3\theta^2 + 4\theta)$. There exists $\theta_{t2}^C > \theta_t^C$ such that $N_3^C|_{\theta=\theta_{t2}^C} = 0$ and $N_3^C < 0$ if and only if $\theta < \theta_{t2}^C$. We can show that $N_4^C > 0$. Thus, when $\theta \ge \theta_{t2}^C$, $(\partial/\partial\beta)p_2^C < 0$; when $\theta_t^C < \theta < \theta_{t2}^C$, $(\partial/\partial\beta)p_2^C < 0$ if and only if $\alpha/(1-c) < N_4^C/(-\theta N_3^C)$. Define $\hat{\alpha}_2^C$ as $N_4^C/(-\theta N_3^C)$. We further find that $\hat{\alpha}_2^C \ge (\theta\beta - \delta - \theta + 2)/(\theta + \delta - \theta\beta)$ if and only if $\theta \ge \delta/(1+\beta)$. Besides, when $\theta < \delta/(1+\beta)$, we can show $\hat{\alpha}_2^C > -(\theta + \theta\beta - \delta)/(2\theta)$. Thus, p_2^C is decreasing in β if and only if $\theta \ge \delta/(1+\beta)$ or $\alpha/(1-c) < \hat{\alpha}_2^C$. The other results are straightforward and we omit the details. □

Proof of Proposition 4. When the firm sells in both periods, $\pi_C = -(\theta(\theta\alpha^2 + (\theta - \delta + \theta\beta)\alpha(1-c) + (\theta\beta - \delta + 1)(1-c)^2))/(\theta^2\beta^2 - 2\theta^2\beta + \theta^2 - 2\theta\beta\delta + 2\theta\delta - 4\theta + \delta^2)$; when the firm sells only in the first period, $\pi_C = (1-c)^2/4$; when the firm sells only in the second period, $\pi_C = \theta((1-c+\alpha)^2/4)$. The other results can be obtained by taking the derivative of π_C with the corresponding parameters and we omit the details. □

Proof of Proposition 5. The firm's problem in the first period is

$$\underset{p_1}{\text{maximize}} \quad \pi_1 = (1-p_1)(p_1 - c) + \theta\frac{(\alpha - c + \beta + 2p_1 - \beta p_1 - 1)^2 + \sigma^2}{4},$$
$$\text{s.t.} \quad p_1 \le 1.$$

Note that π_1 is concave in p_1. By using the first-order condition, we have $p_1^* = -((2-\beta)\theta\alpha + (3\theta\beta - \theta\beta^2 - 2\theta + 2) \cdot (1-c))/(4\theta - 4\theta\beta + \theta\beta^2 - 4) + c$. We have $p_1^* < 1$ if and only if $\alpha/(1-c) < (\theta\beta - 2\theta + 2)/(2\theta - \theta\beta)$. Therefore, when $\alpha/(1-c) \ge (\theta\beta - 2\theta + 2)/(2\theta - \theta\beta)$, the firm sells only in the second period, and $E[p_2^M] = (1+c-\alpha)/2$, which is decreasing in α and independent of β. When $\alpha/(1-c) < (\theta\beta - 2\theta + 2)/(2\theta - \theta\beta)$, the firm sells in both periods, $p_1^M = p_1^*$, and $E[p_2^M] = -(-(\theta\beta - 2\theta + 2)\alpha + (3\theta\beta - \beta - 2\theta - \theta\beta^2 + 2)(1-c))/(\theta\beta^2 - 4\theta\beta + 4\theta - 4) + c$. In this case, by taking the derivatives of p_1^M and $E[p_2^M]$ with the corresponding parameters, we can have $(\partial/\partial\alpha)p_1^M > 0$; $(\partial/\partial\beta)p_1^M < 0$ if and only if $\theta \ge 4(1-\beta)/(\beta^2 - 4\beta + 4)$ or $\alpha/(1-c) > -((\beta^2 - 4\beta + 4)\theta + (4\beta - 4))/(\theta\beta^2 - 4\theta\beta + 4\theta + 4)$; $(\partial/\partial\alpha)E[p_2^M] < 0$; and $(\partial/\partial\beta)E[p_2^M] < 0$. □

Proof of Proposition 6. When the firm sells in both periods, $\pi_M = -(\theta\alpha^2 + \theta\beta\alpha(1-c) + (\theta\beta - \theta + 1)(1-c)^2)/(\theta\beta^2 - 4\theta\beta + 4\theta - 4) + \theta\sigma^2/4$; when the firm sells only in the second period, $\pi_M = \theta(((1-c+\alpha)^2 + \sigma^2)/4)$. The other results can be obtained by taking the derivative of π_M with the corresponding parameters and we omit the details. □

The following Lemma 1 is useful for the proof of Proposition 7.

Lemma 1. (1) *Suppose $\theta > \delta$. There exist $\tilde{\alpha}_1$, $\tilde{\sigma}_1$, and $\tilde{\sigma}_2$ such that the following statements hold.*

(a) *If $\alpha/(1-c) \ge (\theta\beta - \delta - \theta + 2)/(\theta + \delta - \theta\beta)$, then $\pi_D = \pi_M > \pi_C$.*

(b) *If $\tilde{\alpha}_1 < \alpha/(1-c) < (\theta\beta - \delta - \theta + 2)/(\theta + \delta - \theta\beta)$, then*
 (i) *$\pi_D > \pi_M > \pi_C$ when $\sigma^2 > \tilde{\sigma}_2$,*
 (ii) *$\pi_D > \pi_C \ge \pi_M$ when $\tilde{\sigma}_1 < \sigma^2 \le \tilde{\sigma}_2$,*
 (iii) *$\pi_C \ge \pi_D > \pi_M$ when $\sigma^2 \le \tilde{\sigma}_1$.*

(c) *If $\alpha/(1-c) \le \tilde{\alpha}_1$, then*
 (i) *$\pi_M \ge \pi_D > \pi_C$ when $\sigma^2 > \tilde{\sigma}_1$,*
 (ii) *$\pi_M > \pi_C \ge \pi_D$ when $\tilde{\sigma}_2 < \sigma^2 \le \tilde{\sigma}_1$,*
 (iii) *$\pi_C \ge \pi_M \ge \pi_D$ when $\sigma^2 \le \tilde{\sigma}_2$.*

(2) *Suppose $\theta \le \delta$. There exist $\tilde{\alpha}_2$ and $\tilde{\sigma}_3$ such that the following statements hold.*

(a) *If $\alpha/(1-c) \ge (\theta\beta - 2\theta + 2)/(2\theta - \theta\beta)$, then $\pi_M = \pi_D > \pi_C$.*

(b) *If $\tilde{\alpha}_2 \le \alpha/(1-c) < (\theta\beta - 2\theta + 2)/(2\theta - \theta\beta)$ or $\sigma^2 > \tilde{\sigma}_3$, then $\pi_M > \pi_D > \pi_C$.*

(c) *If $\alpha/(1-c) < \tilde{\alpha}_2$ and $\sigma^2 \le \tilde{\sigma}_3$, then $\pi_M > \pi_C \ge \pi_D$.*

Proof of Lemma 1. For the threshold values related to $\alpha/(1-c)$ in the three pricing strategies, we have (a) $(\theta\beta - \delta - \theta + 2)/(\theta + \delta - \theta\beta) > (\theta\beta - 2\theta + 2)/(2\theta - \theta\beta)$ if and only if $\theta > \delta$, (b) $(\theta\beta - \delta - \theta + 2)/(\theta + \delta - \theta\beta) > 1/\sqrt{\theta} - 1 > -(\theta + \theta\beta - \delta)/(2\theta)$ if and only if $\theta > \theta_t^C$, where θ_t^C is defined in the proof of Proposition 3, and (c) $(\theta\beta - 2\theta + 2)/(2\theta - \theta\beta) > 1/\sqrt{\theta} - 1$. Next we conduct a pairwise comparison of the three pricing strategies. Denote dynamic pricing, price commitment, and price matching by DP, PC, and PM, respectively.

(1) Compare π_M and π_C. Suppose $\theta > \delta$. When $\alpha/(1-c) \ge (\theta\beta - \delta - \theta + 2)/(\theta + \delta - \theta\beta)$, the firm sells only in the second period under both PM and PC, and $\pi_M > \pi_C$. When $(\theta\beta - 2\theta + 2)/(2\theta - \theta\beta) \le \alpha/(1-c) < (\theta\beta - \delta - \theta + 2)/(\theta + \delta - \theta\beta)$, the firm sells in two periods under PC, but sells only in the second period under PM. From direct comparison, we have $\pi_M > \pi_C$ if and only if $\sigma^2 > -(((\theta + \delta - \theta\beta)\alpha - (\theta\beta - \delta - \theta + 2)(1-c))^2)/(\theta^2\beta^2 - 2\theta^2\beta + \theta^2 - 2\theta\beta\delta + 2\theta\delta - 4\theta + \delta^2)$. When $\alpha/(1-c) < (\theta\beta - 2\theta + 2)/(2\theta - \theta\beta)$, the firm sells in two periods under both PM and PC, and

$\pi_M-\pi_C=\theta\sigma^2/4+((1-c)^2(\theta-\delta)N^{CM})/((\theta\beta^2-4\theta\beta+4\theta-4)\cdot(\theta^2\beta^2-2\theta^2\beta+\theta^2-2\theta\beta\delta+2\theta\delta-4\theta+\delta^2))$, where $N^{CM}=(3\theta^2+\theta\delta-2\theta^2\beta)(\alpha/(1-c))^2+(4\theta^2-\theta^2\beta-\theta^2\beta^2+\delta\theta\beta-4\theta)(\alpha/(1-c))+\delta-\theta+\theta^2-\theta^2\beta^2-2\theta\beta-\theta\delta+\theta^2\beta+\theta\beta\delta$. We can show that $N^{CM}<0$ for $\theta>\delta$ and $\alpha/(1-c)<(\theta\beta-2\theta+2)/(2\theta-\theta\beta)$. Thus, $\pi_M>\pi_C$ if and only if $\sigma^2>-4(\theta-\delta)\cdot(1-c)^2N^{CM}/(\theta(\theta\beta^2-4\theta\beta+4\theta-4)(\theta^2\beta^2-2\theta^2\beta+\theta^2-2\theta\beta\delta+2\theta\delta-4\theta+\delta^2))$.

Suppose $\theta_t^C<\theta\le\delta$. When $\alpha/(1-c)\ge(\theta\beta-2\theta+2)/(2\theta-\theta\beta)$, the firm sells only in the second period under both PM and PC, and $\pi_M>\pi_C$. When $(\theta\beta-\delta-\theta+2)/(\theta+\delta-\theta\beta)\le\alpha/(1-c)<(\theta\beta-2\theta+2)/(2\theta-\theta\beta)$, the firm sells in two periods under PM, but sells only in the second period under PC, and it is straightforward to show that $\pi_M>\pi_C$. When $-(\theta+\theta\beta-\delta)/(2\theta)<\alpha/(1-c)<(\theta\beta-\delta-\theta+2)/(\theta+\delta-\theta\beta)$, $\pi_M-\pi_C=\theta\sigma^2//4+(1-c)^2(\theta-\delta)N^{CM}/((\theta\beta^2-4\theta\beta+4\theta-4)(\theta^2\beta^2-2\theta^2\beta+\theta^2-2\theta\beta\delta+2\theta\delta-4\theta+\delta^2))$, where N^{CM} is given above. We can show that $N^{CM}<0$ for $\theta_t^C<\theta\le\delta$ and $-(\theta+\theta\beta-\delta)/(2\theta)<\alpha/(1-c)<(\theta\beta-\delta-\theta+2)/(\theta+\delta-\theta\beta)$. Thus, $\pi_M>\pi_C$. When $\alpha/(1-c)\le-(\theta+\theta\beta-\delta)/(2\theta)$, the firm sells in both periods under PM, but sells only in the first period under PC, and it is straightforward to show that $\pi_M>\pi_C$.

Suppose $\theta\le\theta_t^C$. Here the firm sells only in the first or second period under PC. We can easily get $\pi_M>\pi_C$.

By defining

$$\tilde{\sigma}_2=\begin{cases}(-4(\theta-\delta)((3\theta^2+\theta\delta-2\theta^2\beta)\alpha^2\\ \quad+(4\theta^2-\theta^2\beta-\theta^2\beta^2+\delta\theta\beta-4\theta)\alpha(1-c)\\ \quad+(\delta-\theta+\theta^2-\theta^2\beta^2-2\theta\beta-\theta\delta+\theta^2\beta+\theta\beta\delta)(1-c)^2))\\ \quad\cdot(\theta(\theta\beta^2-4\theta\beta+4\theta-4)\\ \qquad\cdot(\theta^2\beta^2-2\theta^2\beta+\theta^2-2\theta\beta\delta+2\theta\delta-4\theta+\delta^2))^{-1},\\ \hfill \dfrac{\alpha}{1-c}<\dfrac{\theta\beta-2\theta+2}{2\theta-\theta\beta},\\ -\dfrac{((\theta+\delta-\theta\beta)\alpha-(\theta\beta-\delta-\theta+2)(1-c))^2}{\theta^2\beta^2-2\theta^2\beta+\theta^2-2\theta\beta\delta+2\theta\delta-4\theta+\delta^2},\\ \hfill \dfrac{\alpha}{1-c}\ge\dfrac{\theta\beta-2\theta+2}{2\theta-\theta\beta},\end{cases}$$

we have the following: (a) In the case of $\theta>\delta$, when $\alpha/(1-c)\ge(\theta\beta-\delta-\theta+2)/(\theta+\delta-\theta\beta)$, $\pi_M>\pi_C$; otherwise, $\pi_M>\pi_C$ if and only if $\sigma^2>\tilde{\sigma}_2$. (b) In the case of $\theta\le\delta$, $\pi_M>\pi_C$.

(2) Compare π_D and π_C. Suppose $\theta>\theta_t^C$. When $\alpha/(1-c)\ge(\theta\beta-\delta-\theta+2)/(\theta+\delta-\theta\beta)$, the firm sells only in the second period under both DP and PC, and $\pi_D>\pi_C$. When $-(\theta+\theta\beta-\delta)/(2\theta)<\alpha/(1-c)<(\theta\beta-\delta-\theta+2)/(\theta+\delta-\theta\beta)$, the firm sells in two periods under both DP and PC. From direct comparison, we get $\pi_D>\pi_C$ if and only if $\sigma^2>\delta^2((\theta\beta-\delta-\theta+2)(1-c)-(\theta+\delta-\theta\beta)\alpha)^2/(\theta(\theta+2\delta-2\theta\beta-2\beta\delta+\theta\beta^2-4)(-4\theta+\theta^2+\delta^2+\theta^2\beta^2+2\theta\delta-2\theta^2\beta-2\theta\beta\delta))$. When $\alpha/(1-c)\le-(\theta+\theta\beta-\delta)/(2\theta)$, the firm sells in two periods under DP, but sells only in the first period under PC. From direct comparison, we get $\pi_D>\pi_C$ if and only if $\sigma^2>((\delta^2+4\theta)\alpha^2+(2\delta^2-4\delta+4\theta+4\theta\beta)(1-c)\alpha+(\theta\beta^2-2\beta\delta+2\theta\beta+\delta^2-2\delta+\theta)(1-c)^2)/(\theta(\theta+2\delta-2\theta\beta-2\beta\delta+\theta\beta^2-4))$.

Suppose $\theta\le\theta_t^C$. When $\alpha/(1-c)\ge1/\sqrt{\theta}-1$, the firm sells only in the second period under both DP and PC, and $\pi_D>\pi_C$. When $(\theta\beta-\delta-\theta+2)/(\theta+\delta-\theta\beta)\le\alpha/(1-c)<1/\sqrt{\theta}-1$, the firm sells only in the second period under DP, but sells only in the first period under PC. From direct comparison, we get $\pi_D>\pi_C$ if and only if $\sigma^2>(-\theta\alpha^2-2\theta(1-c)\alpha+(1-\theta)(c-1)^2)/\theta$. When $\alpha/(1-c)<(\theta\beta-\delta-\theta+2)/(\theta+\delta-\theta\beta)$, the firm sells in two periods under DP, but sells only in the first period under PC. From direct comparison, we get $\pi_D>\pi_C$ if and only if $\sigma^2>((\delta^2+4\theta)\alpha^2+(2\delta^2-4\delta+4\theta+4\theta\beta)(1-c)\alpha+(\theta\beta^2-2\beta\delta+2\theta\beta+\delta^2-2\delta+\theta)\cdot(1-c)^2)/(\theta(\theta+2\delta-2\theta\beta-2\beta\delta+\theta\beta^2-4))$.

By defining

$$\tilde{\sigma}_1=\begin{cases}((\delta^2+4\theta)\alpha^2+(2\delta^2-4\delta+4\theta+4\theta\beta)(1-c)\alpha\\ \quad+(\theta\beta^2-2\beta\delta+2\theta\beta+\delta^2-2\delta+\theta)(1-c)^2)\\ \quad\cdot(\theta(\theta+2\delta-2\theta\beta-2\beta\delta+\theta\beta^2-4))^{-1},\\ \hfill \dfrac{\alpha}{1-c}\le\dfrac{-(\theta+\theta\beta-\delta)}{2\theta},\\ (\delta^2((\theta\beta-\delta-\theta+2)(1-c)-(\theta+\delta-\theta\beta)\alpha)^2)\\ \quad\cdot(\theta(\theta+2\delta-2\theta\beta-2\beta\delta+\theta\beta^2-4)\\ \qquad\cdot(-4\theta+\theta^2+\delta^2+\theta^2\beta^2+2\theta\delta-2\theta^2\beta-2\theta\beta\delta))^{-1},\\ \hfill \dfrac{-(\theta+\theta\beta-\delta)}{2\theta}<\dfrac{\alpha}{1-c}<\dfrac{\theta\beta-\delta-\theta+2}{\theta+\delta-\theta\beta},\end{cases}$$

$$\hat{\sigma}_1=\begin{cases}((\delta^2+4\theta)\alpha^2+(2\delta^2-4\delta+4\theta+4\theta\beta)(1-c)\alpha\\ \quad+(\theta\beta^2-2\beta\delta+2\theta\beta+\delta^2-2\delta+\theta)(1-c)^2)\\ \quad\cdot(\theta(\theta+2\delta-2\theta\beta-2\beta\delta+\theta\beta^2-4))^{-1},\\ \hfill \dfrac{\alpha}{1-c}<\dfrac{\theta\beta-\delta-\theta+2}{\theta+\delta-\theta\beta},\\ \dfrac{-\theta\alpha^2-2\theta(1-c)\alpha+(1-\theta)(c-1)^2}{\theta},\\ \hfill \dfrac{\theta\beta-\delta-\theta+2}{\theta+\delta-\theta\beta}\le\dfrac{\alpha}{1-c}<\dfrac{1}{\sqrt{\theta}}-1,\end{cases}$$

we have the following: (a) In the case of $\theta>\theta_t^C$, when $\alpha/(1-c)\ge(\theta\beta-\delta-\theta+2)/(\theta+\delta-\theta\beta)$, $\pi_D>\pi_C$; otherwise, $\pi_D>\pi_C$ if and only if $\sigma^2>\tilde{\sigma}_1$. (b) In the case of $\theta\le\theta_t^C$, when $\alpha/(1-c)\ge1/\sqrt{\theta}-1$, $\pi_D>\pi_C$; otherwise, $\pi_D>\pi_C$ if and only if $\sigma^2>\hat{\sigma}_1$.

(3) Compare π_D and π_M. Suppose $\theta>\delta$. When $\alpha/(1-c)\ge(\theta\beta-\delta-\theta+2)/(\theta+\delta-\theta\beta)$, the firm sells only in the second period under both DP and PM, and $\pi_D=\pi_M$. When $(\theta\beta-2\theta+2)/(2\theta-\theta\beta)\le\alpha/(1-c)<(\theta\beta-\delta-\theta+2)/(\theta+\delta-\theta\beta)$, the firm sells in two periods under DP, but sells only in the second period under PM, so $\pi_D>\pi_M$. When $\alpha/(1-c)<(\theta\beta-2\theta+2)/(2\theta-\theta\beta)$, the firm sells in both periods under DP as well as PM, and $\pi_D-\pi_M=-(1-c)^2N^{DM}/(4(\theta\beta^2-4\theta\beta+4\theta-4)(\theta+2\delta-2\theta\beta-2\beta\delta+\theta\beta^2-4))$, where $N^{DM}=N_1^{DM}(\alpha/(1-c))^2-N_2^{DM}(\alpha/(1-c))+N_3^{DM}$, $N_1^{DM}=(12-8\beta)\theta^2+(\beta^2\delta^2-4\beta\delta^2+8\beta\delta+4\delta^2-8\delta)\theta-4\delta^2$, $N_2^{DM}=(4\beta^2+4\beta-16)\theta^2+(8\beta\delta^2-4\beta^2\delta-2\beta^2\delta^2-8\beta\delta-8\delta^2+16\delta+16)\theta+8\delta^2-16\delta$, and $N_3^{DM}=(4\beta-4\beta^2+4)\theta^2+(\beta^2\delta^2+4\beta^2\delta-4\beta\delta^2-8\beta+4\delta^2-8\delta-4)\theta+8\delta-4\delta^2+8\beta\delta$. We can show that N_1^{DM} and N_2^{DM} are both positive when $\delta<\theta<1$. Since the discriminant of N^{DM} is positive and $N^{DM}|_{\alpha/(1-c)=(\theta\beta-2\theta+2)/(2\theta-\theta\beta)}=4(\theta-\delta)^2\cdot(\theta\beta^2-4\theta\beta+4\theta-4)/(\theta^2(\beta-2)^2)<0$, $N^{DM}=0$ has two solutions, $\tilde{\alpha}_1$ and $\hat{\alpha}_1$, which satisfy $\tilde{\alpha}_1<(\theta\beta-2\theta+2)/(2\theta-\theta\beta)<\hat{\alpha}_1$. So $\pi_D>\pi_M$ if and only if $\alpha/(1-c)>\tilde{\alpha}_1$.

Suppose $\theta\le\delta$. Similar to the above analysis, when $\alpha/(1-c)\ge(\theta\beta-2\theta+2)/(2\theta-\theta\beta)$, $\pi_D=\pi_M$; when $(\theta\beta-\delta-\theta+2)/(\theta+\delta-\theta\beta)\le\alpha/(1-c)<(\theta\beta-2\theta+2)/(2\theta-\theta\beta)$, $\pi_M>\pi_D$. When $\alpha/(1-c)<(\theta\beta-\delta-\theta+2)/$

$(\theta+\delta-\theta\beta)$, $\pi_D-\pi_M=-(1-c)^2N^{DM}/(4(\theta\beta^2-4\theta\beta+4\theta-4)\cdot(\theta+2\delta-2\theta\beta-2\beta\delta+\theta\beta^2-4))$, where N^{DM} is given above. We can show that $N^{DM}|_{\alpha/(1-c)=0}=N_3^{DM}>0$ for $\theta\le\delta$ and $N^{DM}|_{\alpha/(1-c)=(\theta\beta-\delta-\theta+2)/(\theta+\delta-\theta\beta)}>0$. If $N_1^{DM}>0$ and $N_2^{DM}\le 0$, $N^{DM}>0$; if $N_1^{DM}>0$ and $N_2^{DM}>0$, $(\theta\beta-\delta-\theta+2)/(\theta+\delta-\theta\beta)$ is shown to be less than the extreme point of N^{DM}, so $N^{DM}>0$ for $\alpha/(1-c)<(\theta\beta-\delta-\theta+2)/(\theta+\delta-\theta\beta)$; if $N_1^{DM}\le 0$, N^{DM} is concave or linear, then $N^{DM}>0$ for $\alpha/(1-c)<(\theta\beta-\delta-\theta+2)/(\theta+\delta-\theta\beta)$. Therefore, given $\theta\le\delta$, $\pi_M>\pi_D$ for $\alpha/(1-c)<(\theta\beta-\delta-\theta+2)/(\theta+\delta-\theta\beta)$.

Combining the above two cases, we have the following: (a) Suppose $\theta>\delta$. When $\alpha/(1-c)\ge(\theta\beta-\delta-\theta+2)/(\theta+\delta-\theta\beta)$, $\pi_D=\pi_M$; when $\tilde{\alpha}_1<\alpha/(1-c)<(\theta\beta-\delta-\theta+2)/(\theta+\delta-\theta\beta)$, $\pi_D>\pi_M$; when $\alpha/(1-c)\le\tilde{\alpha}_1$, $\pi_D\le\pi_M$. (b) Suppose $\theta\le\delta$. When $\alpha/(1-c)\ge(\theta\beta-2\theta+2)/(2\theta-\theta\beta)$, $\pi_D=\pi_M$; otherwise, $\pi_D<\pi_M$.

Finally, note that $\tilde{\sigma}_2>\tilde{\sigma}_1$ if and only if $\alpha>\tilde{\alpha}_1$. The results follow from combining the above analysis and defining

$$\tilde{\alpha}_2=\begin{cases}\dfrac{1}{\sqrt{\theta}}-1, & \text{if } \theta\le\theta_t^C,\\[2ex] \dfrac{\theta\beta-\delta-\theta+2}{\theta+\delta-\theta\beta}, & \text{otherwise,}\end{cases}$$

and

$$\tilde{\sigma}_3=\begin{cases}\hat{\sigma}_1, & \text{if } \theta\le\theta_t^C,\\ \tilde{\sigma}_1, & \text{otherwise.}\end{cases}\qquad\square$$

Proof of Proposition 7. The results follow from Lemma 1 by defining $\tilde{\alpha}=\tilde{\alpha}_1$. $\square$

The following Lemma 2 is useful for the proof of Proposition 8.

Lemma 2. (1) *Suppose $\theta\ge\delta$.*

(a) *When $\alpha/(1-c)\ge(\theta\beta-\delta-\theta+2)/(\theta+\delta-\theta\beta)$, $q^D=q^C=q^M$.*

(b) *When $\alpha/(1-c)<(\theta\beta-\delta-\theta+2)/(\theta+\delta-\theta\beta)$, $q^D>q^C\ge q^M$.*

(2) *Suppose $\theta<\delta$. There exists $\tilde{\alpha}_1^q<\tilde{\alpha}_2^q<(\theta\beta-2\theta+2)/(2\theta-\theta\beta)$ such that the following statements hold.*

(a) *When $\alpha/(1-c)\ge(\theta\beta-2\theta+2)/(2\theta-\theta\beta)$, $q^M=q^D=q^C$.*

(b) *When $\tilde{\alpha}_2^q\le\alpha/(1-c)<(\theta\beta-2\theta+2)/(2\theta-\theta\beta)$, $q^M>q^D=q^C$.*

(c) *When $\tilde{\alpha}_1^q\le\alpha/(1-c)<\tilde{\alpha}_2^q$, $q^M\ge q^D>q^C$.*

(d) *When $\alpha/(1-c)<\tilde{\alpha}_1^q$, $q^D>q^M>q^C$.*

Proof of Lemma 2. By comparing q^M and q^C, we have the following: (a) If $\theta\ge\delta$, $q^M=q^C$ when $\alpha/(1-c)\ge(\theta\beta-\delta-\theta+2)/(\theta+\delta-\theta\beta)$, and $q^M\le q^C$ when $\alpha/(1-c)<(\theta\beta-\delta-\theta+2)/(\theta+\delta-\theta\beta)$. (b) If $\theta<\delta$, $q^M=q^C$ when $\alpha/(1-c)\ge(\theta\beta-2\theta+2)/(2\theta-\theta\beta)$, and $q^M>q^C$ when $\alpha/(1-c)<(\theta\beta-2\theta+2)/(2\theta-\theta\beta)$.

By comparing q^D and q^C, we have the following: (a) If $\theta>\theta_t^C$, $q^D=q^C$ when $\alpha/(1-c)\ge(\theta\beta-\delta-\theta+2)/(\theta+\delta-\theta\beta)$, and $q^D>q^C$ when $\alpha/(1-c)<(\theta\beta-\delta-\theta+2)/(\theta+\delta-\theta\beta)$. (b) If $\theta\le\theta_t^C$, $q^D=q^C$ when $\alpha/(1-c)\ge 1/\sqrt{\theta}-1$, and $q^D>q^C$ when $\alpha/(1-c)<1/\sqrt{\theta}-1$.

For the comparison between q^D and q^M, there exists $\tilde{\alpha}_1^q$ such that the following statement holds: (a) If $\theta\ge\delta$, $q^D=q^M$ when $\alpha/(1-c)\ge(\theta\beta-\delta-\theta+2)/(\theta+\delta-\theta\beta)$, and $q^D>q^M$ when $\alpha/(1-c)<(\theta\beta-\delta-\theta+2)/(\theta+\delta-\theta\beta)$. (b) If $\theta<\delta$, $q^D=q^M$ when $\alpha/(1-c)\ge(\theta\beta-2\theta+2)/(2\theta-\theta\beta)$, $q^D\le q^M$ when $\tilde{\alpha}_1^q\le\alpha/(1-c)<(\theta\beta-2\theta+2)/(2\theta-\theta\beta)$, and $q^D>q^M$ when $\alpha/(1-c)<\tilde{\alpha}_1^q$.

The results follow from combining the above analysis and defining $\tilde{\alpha}_2^q$ in the same way as $\tilde{\alpha}_2$ in Lemma 1. $\square$

Proof of Proposition 8. The results follow from Lemma 2 by defining $\tilde{\alpha}^q=\tilde{\alpha}_1^q$. $\square$

Appendix B. Tables and Graphs for Extensions

Here, we present some tables and graphs to illustrate the results for the two extensions discussed in §6.

Exponential Learning Curve

Figure B.1 illustrates how the firm's profit changes with the technology advancement effect and the production learning effect under different pricing strategies when $c=0.3$, $\theta=0.2$, $\delta=0.8$, and $\tau=0.05$. From the figure, the firm's profit can be always decreasing in the two effects under dynamic

Figure B.1. (Color online) Effects of Technology Advancement and Production Learning on Profits Under Different Pricing Strategies

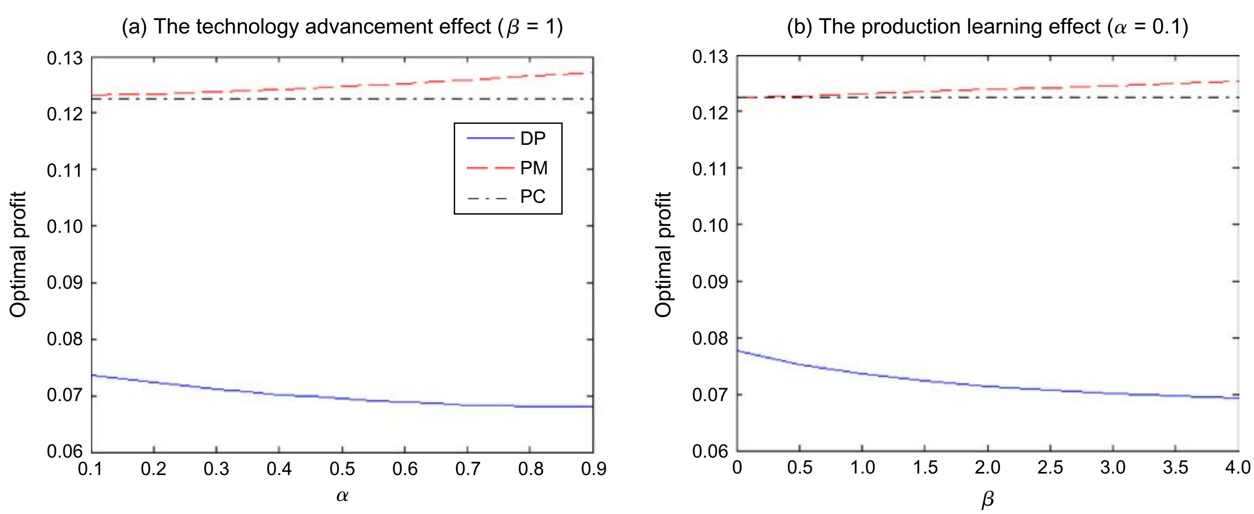


Figure B.2. (Color online) Optimal Strategy Switch Between Dynamic Pricing and Price Commitment

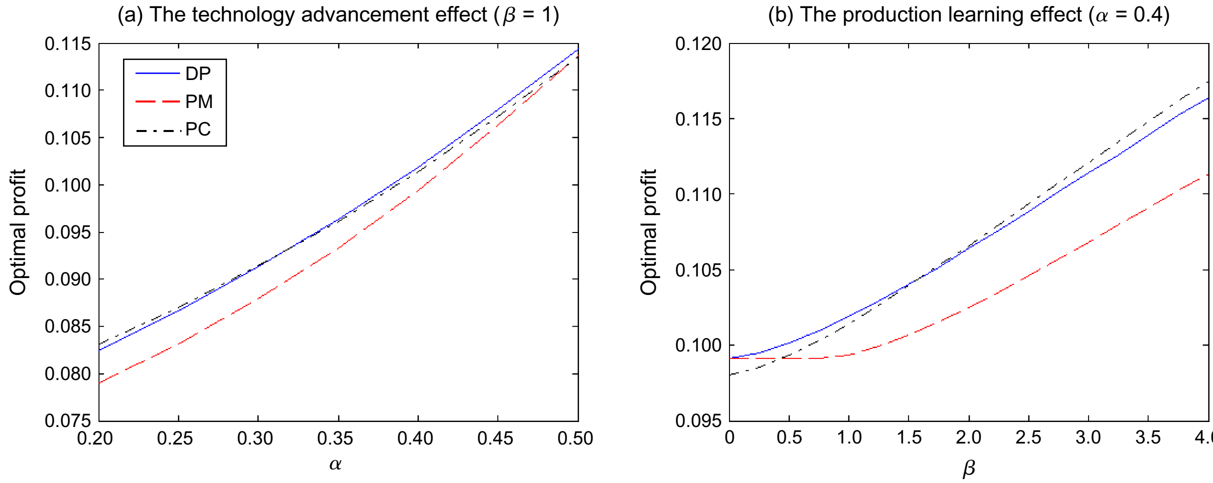


Table B.1. Percentage of Cases Where Each Pricing Strategy Is Optimal (Exponential Learning Curve)

	DP (%)	PC (%)	PM (%)	Total (%)
$\theta \le \delta$	0	0	100	100
$\theta > \delta$	8.85	91.15	0	100
Total	3.32	34.18	62.50	100

Table B.2. Percentage of Cases Where Each Pricing Strategy Is Optimal (New Customer Arrivals)

	DP (%)	PC (%)	PM (%)	Total (%)
$m = 0.2$	16.27	18.65	65.08	100
$m = 1$	19.44	15.08	65.48	100
$m = 5$	34.52	3.17	62.30	100
Total	23.41	12.30	64.29	100

pricing, but not under price commitment or price matching. from dynamic pricing to price commitment when β increases (from 1.5 to 2).

Table B.1 summarizes the percentage of cases that each pricing strategy is optimal, and the results suggest that price matching is always optimal when $\theta \le \delta$, but never when $\theta > \delta$.

Figure B.2 illustrates the case when $c = 0.5$, $\theta = 0.8$, $\delta = 0.6$, and $\tau = 0.15$. The figure demonstrates that the impact of the technology advancement effect and the production learning effect can be different. When α increases from 0.3 to 0.35, the optimal strategy switches from price commitment to dynamic pricing. However, the optimal strategy changes

New Customer Arrivals

Figure B.3 illustrates how the firm's profit changes with the technology advancement effect under dynamic pricing strategy when $c = 0.15$, $\theta = 0.25$, $\delta = 0.75$, $\tau = 0.05$, and $\beta = 0.05$. According to the figure, the firm's profit decreases with α when $m = 0.2$ or 1, but not when $m = 5$.

Figure B.4 illustrates the impact of the production learning effect when $c = 0.35$, $\theta = 0.25$, $\delta = 0.75$, $\tau = 0.05$, and $\alpha = 0.05$. Similarly to the impact of α, the firm's profit can decrease in β when $m = 0.2$ or 1, but not when $m = 5$.

Table B.2 shows the percentage of cases that each pricing strategy is optimal. According to the table, the percentage of cases that dynamic pricing is optimal increases when m increases. The intuition is that when number of customers arriving in the second period increases, the profit in the second period becomes more important and the impact of strategic waiting becomes smaller. Thus, the flexibility provided by dynamic pricing also becomes more beneficial.

Figure B.3. (Color online) The Effect of Technology Advancement on Optimal Profit Under Dynamic Pricing

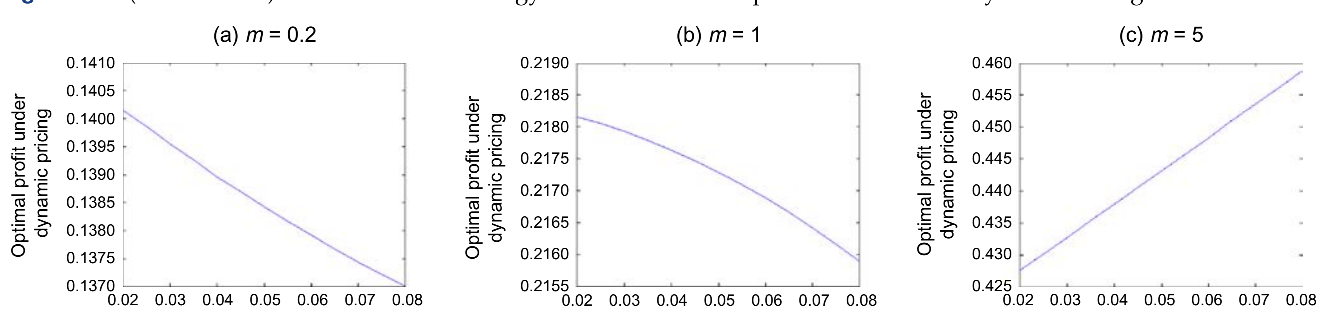
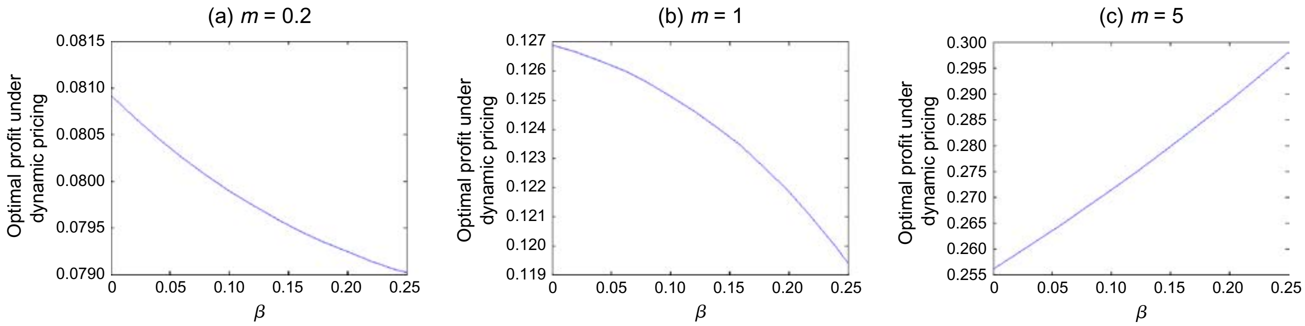


Figure B.4. (Color online) The Effect of Production Learning on Optimal Profit Under Dynamic Pricing



Endnotes

[1] Specifically, it can be shown that $(v - p_1) - \delta E[v - p_2]^+$ is increasing in v for any p_1 and random variable p_2.

[2] Price commitment has been adopted by, for example, a Nashville-based chain of retail stores called Bargain Hunt. Once a product has been in store for 30 days, the retailer starts marking down a product 10% every 10 days up to 90% off (Chattanoogan 2013). Other retailers that have used price commitment include Sam's Club (Elmaghraby et al. 2008) and Pricetack.com (Yousefi et al. 2014).

[3] See, for example, Dell's price guarantee policy (http://www.dell.com/learn/us/en/6099/campaigns/free-shipping-easy-returns-eep, accessed February 6, 2015), Fry's 30-Day Price Match Promise (http://www.frys.com/onlineads/0001507075, accessed February 6, 2015), Amazon's now-discontinued price matching policy (http://www.refundplease.com/, accessed February 6, 2015), and Google's Price Protection policy (Abent 2012).

[4] For example, Sony announced in August 2012 that the price of PS Vita would not drop by the end of the year (Yin-Poole 2012). Another example is that Sony announced in June 2014 that price cut for PS4 "could be some way off" (Johnson 2014).

[5] For example, Sony revealed in its 2014 first-quarter report that its PS4 was already getting a cost reduction, shortly after its release into the market in November 2013 (Usher 2014, Leack 2014).

References

Abent E (2012) Google announces price protection for devices on Play Store. *SlashGear* (November 9), http://www.slashgear.com/google-announces-price-protection-for-devices-on-play-store-09256405.

Arrow KJ (1962) The economic implications of learning by doing. *Rev. Econom. Stud.* 29(3):155–173.

Aviv Y, Pazgal A (2008) Optimal pricing of seasonal products in the presence of forward-looking consumers. *Manufacturing Service Oper. Management* 10(3):339–359.

Benkard CL (2000) Learning and forgetting: The dynamics of aircraft production. *Amer. Econom. Rev.* 90(4):1034–1054.

Bernstein F, Kok AG (2009) Dynamic cost reduction through process improvement in assembly networks. *Management Sci.* 55(4): 552–567.

Besanko D, Winston WL (1990) Optimal price skimming by a monopolist facing rational consumers. *Management Sci.* 36(5): 555–567.

Cabral L, Riordan M (1994) The learning curve, market dominance and predatory pricing. *Econometrica* 62(5):1115–1140.

Cachon G, Swinney R (2009) Purchasing, pricing, and quick response in the presence of strategic consumers. *Management Sci.* 55(3): 497–511.

Chattanoogan (2013) Essex Bargain Hunt opens its 1st Bargain Hunt Superstore location in Chattanooga. (October 16), http://www.chattanoogan.com/2013/10/16/261487/Essex-Bargain-Hunt-Opens-Its-1st.aspx.

Dasu S, Tong C (2010) Dynamic pricing when consumers are strategic: Analysis of posted and contingent pricing schemes. *Eur. J. Oper. Res.* 204(3):662–671.

Elmaghraby W, Gulcu A, Keskinocak P (2008) Designing optimal preannounced markdowns in the presence of rational customers with multiunit demands. *Manufacturing Service Oper. Management* 10(1):126–148.

Fine CH, Porteus EL (1989) Dynamic process improvement. *Oper. Res.* 37(4):580–591.

Hatch NW, Mowery DC (1998) Process innovation and learning by doing in semiconductor manufacturing. *Management Sci.* 44(11):1461–1477.

Hiller R, Shapiro J (1986) Optimal capacity expansion planning when there are learning effects. *Management Sci.* 32(9):1153–1163.

Humphries M (2013) Sony announces a PS Vita price cut for Japan. *GEEK* (February 18), http://www.geek.com/games/sony-announces-a-ps-vita-price-cut-for-japan-1540051/.

Jin J, Perote-Pena J, Troege M (2004) Learning by doing, spillovers and shakeouts. *J. Evolutionary Econom.* 14(1):85–98.

Johnson L (2014) Sony: PS4 price cut still "some way off." *Trusted Reviews* (June 13), http://www.trustedreviews.com/news/sony-ps4-price-cut-still-some-way-off.

Kalish S (1983) Monopolist pricing with dynamic demand and production cost. *Marketing Sci.* 2(2):135–159.

Lai G, Debo LG, Sycara K (2010) Buy now and match later: Impact of posterior price matching on profit with strategic consumers. *Manufacturing Service Oper. Management* 12(1):33–55.

Leack J (2014) PS4 manufacturing costs are down, leaving room for an aggressive price cut. *CRAVE* (July 31), http://www.craveonline.com/gaming/articles/735969-ps4-manufacturing-costs-are-down-leaving-room-for-an-aggressive-price-cut.

Li C, Zhang F (2013) Advance demand information, price discrimination, and pre-order strategies. *Manufacturing Service Oper. Management* 15(1):57–71.

Li G, Rajagopalan S (1998) Process improvement, quality, and learning effects. *Management Sci.* 44(11):1517–1532.

Liu Q, van Ryzin G (2008) Strategic capacity rationing to induce early purchases. *Management Sci.* 54(6):1115–1131.

Liu Q, van Ryzin G (2011) Strategic capacity rationing when customers learn. *Manufacturing Service Oper. Management* 13(1): 89–107.

Lobel R, Perakis G (2011) Consumer choice model for forecasting demand and designing incentives for solar technology. Working paper, MIT Sloan School of Management, Cambridge, MA.

Lobel I, Patel J, Vulcano G, Zhang J (2016) Optimizing product launches in the presence of strategic consumers. *Management Sci.* 62(6):1778–1799.

Mazzola JB, McCardle KF (1997) The stochastic learning curve: Optimal production in the presence of learning-curve uncertainty. *Oper. Res.* 45(3):440–450.

Png IPL (1991) Most-favored-customer protection versus price discrimination over time. *J. Political Econom.* 99(5):1010–1028.
Romer PM (1986) Increasing returns and long-run growth. *J. Political Econom.* 94(5):1002–1037.
Serela DA, Dadab M, Moskowitzb H, Plante RD (2003) Investing in quality under autonomous and induced learning. *IIE Trans.* 35(6):545–555.
Shen ZM, Su X (2007) Customer behavior modeling in revenue management and auctions: A review and new research opportunities. *Production Oper. Management* 16(6):713–728.
Shou B, Xiong H, Shen ZM (2011) Consumer panic buying and fixed quota policy. Working paper, City University of Hong Kong, Kowloon Tong.
Sin B (2013) PS Vita price cut results in sales quadrupling. *SlashGear* (March 5), http://www.slashgear.com/ps-vita-price-cut-results-in-sales-quadrupling-05272698/.
Su X (2007) Intertemporal pricing with strategic customer behavior. *Management Sci.* 53(5):726–741.
Su X (2010) Intertemporal pricing and consumer stockpiling. *Oper. Res.* 58(4):1133–1147.
Su X, Zhang F (2008) Strategic customer behavior, commitment, and supply chain performance. *Management Sci.* 54(10):1759–1773.
Swinney R (2011) Selling to strategic consumers when product value is uncertain: The value of matching supply and demand. *Management Sci.* 57(10):1737–1751.
Tucker C (2008) Identifying formal and informal influence in technology adoption with network externalities. *Management Sci.* 54(12): 2024–2038.
Usher W (2014) PS4 hardware costs have been reduced; price-cut coming soon? *Cinema Blend* (July), http://www.cinemablend.com/games/PS4-Hardware-Costs-Have-Been-Reduced-Price-Cut-Coming-Soon-66526.html.
Wood J (2013) Teardown of Xbox, PS4 reveal tight margins. *CNBC* (November 27), http://www.cnbc.com/id/101230904.
Yin-Poole W (2012) Sony engineers working on PS Vita "cost reduction" for post-2012 price cut. *Euro Gamer* (August 16), http://www.eurogamer.net/articles/2012-08-16-sony-engineers-working-on-ps-vita-cost-reduction-for-post-2012-price-cut.
Young A (1991) Learning by doing and the dynamic effects of international trade. *Quart. J. Econom.* 106(2):369–405.
Yousefi S, Rui H, Whinston A (2014) Optimal markdown and priority pricing with demand uncertainty. Working paper, Development Prospects Group (DECPG), World Bank, Washington, DC.