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Supplier Competition in Decentralized Assembly Systems with Price-Sensitive and Uncertain Demand

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In a decentralized assembly supply chain, independent suppliers produce a set of *complementary* components from which an assembler assembles a final product and sells it to the market. In such a channel, several competitive forces interact with one another to affect the price and quantity decisions of the firms involved. These include: (1) the *direct* competition each supplier faces for producing the same component, (2) the *indirect* competition among the suppliers producing the set of complementary components needed for assembling the final product, and (3) the vertical interaction between the assembler and the component suppliers. This paper shows that the direct competition that one supplier faces helps improve the performance of the assembler and all the other suppliers in the channel; and surprisingly, it can help improve the performance of this particular supplier facing the competition as well. Second, the assembler benefits from a merger of suppliers producing different components in the complementary set. Furthermore, the assembler prefers a merger of suppliers with less direct competition over a merger of suppliers with more direct competition.

Key words: supply chain management; noncooperative games; assembly systems; price-production decisions
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1. Introduction

In a decentralized assembly supply chain, each firm decides on product price and production capacity/quantity by considering the strategic interactions among all firms involved at several levels. For example, in choosing the price of a component to sell to the assembler, a component supplier needs to take into account (1) the *direct competition* it faces for producing the same component, (2) the *indirect competition* from suppliers who produce other components in the complementary set that are needed by the assembler to assemble the final product, and (3) the reaction of the assembler to the wholesale prices offered by all suppliers to balance the production capacity/quantity with market demand that can be both price sensitive and uncertain.

The purpose of this paper is to present a stylized model to capture the strategic interactions among firms and to study their impact on firms' decision and performance. We consider an assembler who assembles a final product and sells it in a market with price-sensitive and uncertain demand. To assemble the final product, the assembler procures n components, each from an independent supplier. Without loss of generality, we assume that one unit of the final product consists of one unit of each of the n components. In addition, each of the n suppliers faces a direct competition from another supplier who produces the same component but at a higher production cost. (When there are more than two suppliers producing the same component, only the two lowest cost ones are relevant.) The cost difference between a supplier and its direct competitor measures the intensity of the direct competition. Suppliers simultaneously decide individual wholesale prices for their components, and the assembler then orders components from the suppliers and charges a retail price for the assembled products in the market. Firms make decisions before the resolution of demand uncertainty.

With some assumptions about the demand function, we characterize the equilibrium decision and performance of individual firms. While it is known that indirect competition among suppliers of different components in such systems harms the performance of all parties, we show in this paper that direct competition faced by each supplier improves performance. Specifically, as the direct competition for one supplier intensifies, profit increases for the assembler, the other suppliers, and the system. Surprisingly, as the direct competition intensifies gradually, the profit of this particular supplier facing the direct competition may increase, although it will decrease eventually. Second, we show that holding the total component production cost constant, the assembler prefers the suppliers of different components to merge and form a smaller supply base. Furthermore, the assembler prefers a merger of suppliers facing less intense direct competition over a merger of suppliers facing more intense direct competition.

This paper contributes to the research on contracting and coordination of decentralized assembly systems. Most papers on this topic assume that market demand for the final product is uncertain but price insensitive. See, for instance, Gerchak and Wang (2004), Granot and Yin (2008), Bernstein and DeCroix (2004, 2006), Tomlin (2003), Wang and Gerchak (2003), Gurnani and Gerchak (2007), Zhang (2006), Fang et al. (2008), Zhang et al. (2005), and Feng and Zhang (2005). With uncertain but price-insensitive demand, the relative profitability of component suppliers, in general, depends on their individual costs. Considering price sensitivity of demand, both Wang (2006), by using revenue-sharing contracts, and Yin (2006), by considering wholesale price contracts, reach the conclusion that all suppliers earn the same profit independent of their individual costs—a rather unexpected result. By introducing direct competition for producing each component, our model shows that suppliers make different profits, and their relative profitability depends on the intensity of the direct competition they each face. Finally, we point out that most of this literature, including ours, focuses on the noncooperative nature of decentralized assembly systems. There is a growing body of research work that analyzes the bargaining and cooperative aspects of such systems (see Nagarajan and Bassok 2008, Granot and Yin 2008, Yin 2006, Nagarajan and Sosic 2006, and the references therein).

In summary, this paper is the first to introduce suppliers' direct competition into a model of decentralized assembly systems, and to show that such competition helps improve system performance as well as individual firms' performance. Furthermore, for managing its supply base, the assembly firm has the incentive to encourage mergers of suppliers, especially a merger of those suppliers facing less direct competition.

Section 2 of this paper details the model setting, and §3 derives the model solution and discusses managerial insights. The appendix contains all mathematical proofs.

2. Model Assumptions

Consider n suppliers each producing one of a set of n components that are needed by an assembler to assemble a final product. Each supplier charges a wholesale price for its component. Let w_i be the wholesale price of supplier i, $i = 1, \ldots, n$, and define $W \equiv \sum_{i=1}^{n} w_i$ as the total wholesale price for one set of the n components.

Supplier i, $i = 1, \ldots, n$, produces its component at the marginal cost of \$$c_i$, and faces a direct competition from supplier $\hat{i}$ who produces, at the marginal cost of \$$\hat{c}_i$, a component that is perfectly substitutable with component i. We assume without loss of generality that $c_i \le \hat{c}_i$, meaning that supplier i has a cost advantage over supplier $\hat{i}$. The cost difference, $\hat{c}_i - c_i$, measures the intensity of the direct competition supplier i faces: the direct competition becomes more intense as the cost difference becomes smaller. Without loss of generality, we label the n suppliers in decreasing order of the competition intensities they each face such that $\hat{c}_1 - c_1 \le \hat{c}_2 - c_2 \le \cdots \le \hat{c}_n - c_n$. We let $C_{\{i,j\}} = \sum_{l=i}^{j} c_l$ and $\widehat{C}_{\{i,j\}} = \sum_{l=i}^{j} \hat{c}_l$, for $1 \le i \le j \le n$.

Suppliers i and $\hat{i}$ compete through how they price their components. Since their components are perfectly substitutable, the assembler buys only from the one who charges a lower price. Such a setting falls into the Bertrand price competition model. As a standard result of the Bertrand model (cf. Tirole 1988, Mas-Collel et al. 1995), in equilibrium, with a cost advantage, supplier i prices its component at or below supplier $\hat{i}$'s cost $\hat{c}_i$ to become the sole supplier of component i. Thus the existence of direct competition imposes an upper limit on each supplier i's choice of wholesale price such that $w_i \le \hat{c}_i$, for $i = 1, \ldots, n$.

The assembler buys the components from individual suppliers, assembles them into final products, and sells the final products to a market. The retail price the assembler charges, denoted by p, influences market demand during a selling season. We use a multiplicative function, $D(p, \varepsilon)$, to capture the price sensitivity and uncertainty of demand:

$$D(p, \varepsilon) = y(p) \cdot \varepsilon, \tag{1}$$

where $y(p)$ is a deterministic and decreasing function of price p, and ε is a random factor with general CDF $F(\cdot)$, PDF $f(\cdot)$, and a mean value of μ. Assume that $f(\cdot)$ has a support on $[A, B]$ with $B > A > 0$, so $\mu > 0$. We assume that $F(\cdot)$ satisfies the increasing generalized failure rate (IGFR) condition (Lariviere and Porteus 2001), i.e., the function $xf(x)/[1 - F(x)]$ being increasing in x. We further let $y(p)$ take the form of

$$y(p) = ap^{-b}, \quad \text{where } a > 0,\ b > 1, \tag{2}$$

as in Wang (2006). The parameter b in (2) is the price elasticity index of the (expected) demand. We assume $b > 1$ to focus on price-elastic products. For simplicity, we assume any unsold product at the end of the season bears neither salvage value nor disposal cost. Similarly, in case of shortage, the unsatisfied demand carries no penalty beyond the loss of sales revenue. Assuming nonzero salvage value and shortage penalty may cause complexities and change the solution structure of the problem, see Petruzzi and Dada (1999) for such models of single decision makers.

3. Equilibrium of Firms' Decisions and Managerial Insights

We solve the decision problem that the n suppliers first simultaneously choose their individual wholesale prices followed by the assembler who decides on a quantity q to order from each supplier and a retail price p to sell the assembled product. (Since it is assumed that a final product consists of one unit of each of the n components, it is obviously suboptimal for the assembler to order different quantities from different suppliers.) All the decisions are made before demand uncertainty is resolved.

We derive the equilibrium decisions in a backward manner. Given suppliers' wholesale prices $\{w_i\}$, the assembler's profit function is

$$\Pi_0(p, q \mid W) = p \cdot E[\text{Min}\{q, D(p, \varepsilon)\}] - (W + c_0)q. \tag{3}$$

Its decision for choosing an order quantity q and a retail price p is affected only by the total price of all suppliers, $W \equiv \sum_{i=1}^{n} w_i$. We can show that under the condition of IGFR about the demand distribution function, the assembler's optimal decisions, denoted by $(p_d(W), q_d(W))$, are

$$p_d(W) = \frac{W + c_0}{C_{\{1,n\}} + c_0} p_C^*, \tag{4}$$

$$q_d(W) = \frac{(C_{\{1,n\}} + c_0)^b}{(W + c_0)^b} q_C^*, \tag{5}$$

where (p_C^*, q_C^*) are the optimal decisions for a centralized system and their expressions can be found in Wang et al. (2004). From (5) and (6), one can see that in a decentralized channel, as long as the suppliers offer a total wholesale price W higher than the total component production cost $C_{\{1,n\}}$, the assembler will set a price p_d higher than the channel optimal price p_C^* together with a quantity q_d lower than the channel optimal quantity q_C^*, because of the well-known effect of double marginalization.

Knowing that the assembler responds to their wholesale prices according to (5), the n suppliers engage in a game to choose simultaneously their individual wholesale prices. Given the other suppliers' wholesale prices, $w_{-i} \equiv \{w_1, \ldots, w_{i-1}, w_{i+1}, \ldots, w_n\}$, supplier i's profit is $\Pi_i(w_i \mid W_{-i}) = (w_i - c_i)q_d(W)$, where $W_{-i} \equiv W - w_i$. Substituting $q_d(W)$ of (5), we can show that

$$\begin{aligned}\Pi_i(w_i \mid W_{-i}) &= (w_i - c_i)q_d(W) \\ &= (b-1)(C_{\{1,n\}} + c_0)^{b-1}\Pi_C^* \cdot (w_i - c_i)(W + c_0)^{-b},\end{aligned} \tag{6}$$

where Π_C^* is the centralized channel profit, the expression of which can again be found in Wang et al. (2004). Supplier i maximizes $\Pi_i(w_i \mid W_{-i})$ subject to the constraint of $w_i \le \hat{c}_i$, because of the direct competition it faces. Lemma 1 characterizes its best response to the other suppliers' wholesale prices.

Lemma 1. *Given the other suppliers' wholesale prices, $w_{-i} \equiv \{w_1, \ldots, w_{i-1}, w_{i+1}, \ldots, w_n\}$, supplier i's profit function $\Pi_i(w_i \mid W_{-i})$ is quasiconcave in its own price w_i and has the unique maximizer of*

$$w_i(W_{-i}) = \mathrm{Min}\left\{\frac{W_{-i} + bc_i + c_0}{b-1}, \hat{c}_i\right\}, \quad \text{for } i = 1, 2, \ldots, n. \tag{7}$$

According to (7), an individual supplier's optimal price increases (does not decrease) in its own cost, the other suppliers' wholesale prices, and the assembler's cost, which is rather intuitive. The equilibrium of the n suppliers' pricing game can be found by solving the n simultaneous equations of (7). The result is described in Theorem 1, where for convenience of notation, we define $c_{n+1} \equiv 0$ and $\hat{c}_{n+1} \equiv \infty$.

Theorem 1. *For $b > 1$, let m be the smallest integer in $\{0, 1, \ldots, n-1\}$ such that $b > n - m$; that is,*

$$m \equiv \mathrm{Min}\{j\colon\, j \in [0, n-1],\, b > n - j\}. \tag{8}$$

Let

$$k \equiv \mathrm{Min}\left\{j\colon\, j \in [m+1, n+1],\; \hat{c}_j \geq c_j + \frac{C_{\{j,n\}} + c_0 + \widehat{C}_{\{1,j-1\}}}{b - n + j - 1}\right\}, \tag{9}$$

where m is defined in (8). *The n suppliers' pricing game has the unique equilibrium of*

$$w_i^* = \begin{cases} \hat{c}_i, & i = 1, 2, \ldots, k-1 \\ c_i + \dfrac{c_0 + \widehat{C}_{\{1,k-1\}} + C_{\{k,n\}}}{b-n+k-1}, & i = k, k+1, \ldots, n. \end{cases} \tag{10}$$

To understand the structure of the equilibrium solution described in Theorem 1, we look at the externality that the n complementary suppliers exert *indirectly* on each other through their individual choices of wholesale prices: A price increase by any one supplier leads the assembler to reduce its order quantity from *all* the suppliers. When choosing a price, however, each supplier takes into account the net effect of a price increase *only* on its own profit but not on the others' profits. As a consequence, the n suppliers raise their wholesale prices, which in equilibrium leads the assembler to choose an order quantity that is suboptimal (too low) for the system.

More specifically, there are three forces that drive the equilibrium formation of wholesale prices in the system: (1) the number of complementary component suppliers, n, in the system, (2) the price elasticity b of market demand for the product, and (3) the intensity of the direct competition each supplier faces. As the system involves more component suppliers (i.e., as n increases), the externality effect of indirect competition among suppliers becomes more pronounced, which encourages each supplier to raise its price, as seen partially from the expressions for the equilibrium prices in (10). On the other hand, the price elasticity (sensitivity) b of market demand moderates the tendency of price increase by suppliers: as the value of b increases, suppliers lower their prices, as also seen from the equilibrium solution of (10). Third, the direct competition each supplier faces creates a cap that potentially limits the price each supplier can charge. The price cap is binding in equilibrium for the suppliers facing the most intensive direct competition. Depending on the relative magnitudes of the two opposing forces; namely, the number of suppliers n and the price elasticity b, *at least* m suppliers who face the most intense direct competition reach their respective price caps, with m determined by (8), and the actual number of $(k-1)$ suppliers reaching their price caps determined through (9).

Direct competition plays a major role in affecting the relative pricing power among suppliers. Not surprisingly, direct competition determines their *relative* profitability in equilibrium: suppliers facing more intense direct competition earn less profits than those facing less intense direct competition, and all those suppliers, if any, whose prices have not been capped by direct competition earn equal profits in equilibrium.

Proposition 1. *In equilibrium, the profits of the n suppliers are such that*

$$\Pi_1 \leq \Pi_2 \leq \cdots \leq \Pi_{k-1} \leq \Pi_k = \Pi_{k+1} = \cdots = \Pi_n,$$

where k is determined through (9).

The next proposition illustrates how the suppliers' profits will change as the intensity of the direct competition faced by any supplier i increases.

Proposition 2. *As the direct competition faced by supplier i, for some $i \in \{1, 2, \ldots, n\}$, intensifies, i.e., as $\hat{c}_i$ decreases,*

(1) *the profit of any other supplier* j, Π_j, *for* $j \in \{1, 2, \ldots, n\}$ *and* $j \neq i$, *increases* (*does not decrease*);

(2) *the profit of supplier* i, Π_i, *may first increase, but will eventually decrease and approach zero as* $\hat{c}_i$ *approaches* c_i.

Intuitively, as supplier i faces a more intense direct competition, it may need to lower its wholesale price. The lowered wholesale price induces the assembler to increase order quantity from all the suppliers, which benefits all the other suppliers. This is Part (1) of Proposition 2. Part (2) shows that supplier i itself may also benefit from a more intense direct competition. A plausible explanation is as follows: On the one hand, an intensified direct competition reduces supplier i's pricing power, and so hurts its profit. On the other hand, the reduction of its pricing power mitigates the negative externality effects exerted by all suppliers through indirect competition, resulting in an improvement to system efficiency. Such an improvement to system efficiency benefits every party, including supplier i. The benefit supplier i thus gains can outweigh its loss because of reduced pricing power, resulting in a net improvement in profit.

Consider the assembler interested in managing its supply base. Suppose suppliers i and j, for some $1 \le i < j \le n$, merge to become a single supplier who produces both components i and j at a total cost of $c_i + c_j$. (That is, as a base case, we assume there is no economy of scale in the system.) This supplier after the merger now offers a single price for selling both components to the assembler, but faces a direct competition that sets an upper limit on its price at $\hat{c}_i + \hat{c}_j$.

Proposition 3. *In a decentralized assembly system,*

(1) *the assembler benefits from a merger of suppliers;*

(2) *the assembler benefits more from a merger of two suppliers facing less intense direct competition than from two suppliers facing more intense direct competition.*

Intuitively, because a merger reduces the number of independent suppliers, it mitigates the negative externality that independent suppliers put on the system through indirect competition, and thus benefits the assembler. The direct competition each supplier faces, however, also plays a role in mitigating the negative externality of indirect competition, as discussed earlier. Consequently, the assembler is able to extract more (remaining) benefit from a merger of suppliers facing less intense direct competition than from a merger of suppliers facing more intense direct competition.

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Appendix. Mathematical Proofs

Proof of Lemma 1. From (6), it follows, after some algebra, that

$$\frac{d\Pi_i(w_i \mid W_{-i})}{dw_i} = (b-1)(C_{\{1,n\}}+c_0)^{b-1}\Pi_c^* \cdot (W+c_0)^{-b-1} L(w_i \mid W_{-i}),$$

where

$$L(w_i \mid W_{-i}) = W_{-i} - (b-1)w_i + bc_i + c_0.$$

Since $(W+c_0)^{-b-1} > 0$, the sign of $d\Pi_i(w_i \mid W_{-i})/dw_i$ is determined by that of $L(w_i \mid W_{-i})$. Now, $L(w_i \mid W_{-i})$ is decreasing in w_i. Furthermore, $L(w_i = c_i \mid W_{-i}) = W_{-i} + c_i + c_0 > 0$ and $L(w_i \to \infty \mid W_{-i}) < 0$. As a consequence, $\Pi_i(w_i \mid W_{-i})$ is quasiconcave in w_i and is maximized at the point uniquely determined by solving $L(w_i \mid W_{-i}) = 0$, which gives $w_i = (W_{-i} + bc_i + c_0)/(b-1)$. Quasiconcavity of the objective function guarantees the optimality of (7) under the constraint of $w_i \le \hat{c}_i$. □

Proof of Theorem 1. First, note that by definitions of (8) and (9), there always exists a unique set of m and k such that $0 \le m < k \le n+1$. For given wholesale prices of all the other suppliers, w_{-i}^*, supplier i's best response is, by Lemma 1, $w_i(w_{-i}^*) = \mathrm{Min}\{(bc_i + c_0 + \sum_{j\neq i} w_j^*)/(b-1), \hat{c}_i\}$.

We partition the suppliers into three subgroups: $\Omega_1 = \{1, 2, \ldots, m\}$, $\Omega_2 = \{m+1, \ldots, k-1\}$, and $\Omega_3 = \{k, \ldots, n\}$, and, for a supplier i in each subgroup, show that w_i^* is his best response wholesale price, given w_{-i}^* by all the other suppliers. First, we analyze the best response price of a supplier i in Ω_1, which is not empty if $m \ge 1$. Note that in this case, $n-m < b \le n-m+1$. Given $w_j^* = \hat{c}_j$ for $j = 1, \ldots, k-1$, $j \neq i$, and $w_j^* = c_j + (C_{\{k,n\}} + c_0 + \sum_{l=1}^{k-1} \hat{c}_l)/(b-n+k-1)$, for $j = k, \ldots, n$,

$$\begin{aligned}
&\frac{bc_i + c_0 + \sum_{j\neq i} w_j^*}{b-1} - \hat{c}_i \\
&\quad = \frac{b}{(b-1)(b-n+k-1)}\left(C_{\{i,n\}} + c_0 + \sum_{j=1}^{i-1}\hat{c}_j\right) \\
&\qquad + \left[\frac{n-k+1}{(b-1)(b-n+k-1)} - 1\right](\hat{c}_i - c_i) \\
&\qquad + \frac{b}{(b-1)(b-n+k-1)}\sum_{j=i+1}^{k-1}(\hat{c}_j - c_j)
\end{aligned}$$

$$
\begin{aligned}
&\geq \frac{b}{(b-1)(b-n+k-1)}\left(C_{\{i,n\}}+c_0+\sum_{j=1}^{i-1}\hat{c}_j\right)\\
&\quad+\left[\frac{n-k+1}{(b-1)(b-n+k-1)}-1\right](\hat{c}_i-c_i)\\
&\quad+\frac{b(k-1-i)}{(b-1)(b-n+k-1)}(\hat{c}_i-c_i)\\
&=\frac{b}{(b-1)(b-n+k-1)}\left(C_{\{i,n\}}+c_0+\sum_{j=1}^{i-1}\hat{c}_j\right)\\
&\quad+\frac{b(n+1-b-i)}{(b-1)(b-n+k-1)}(\hat{c}_i-c_i)\\
&\geq \frac{b}{(b-1)(b-n+k-1)}\left(C_{\{i,n\}}+c_0+\sum_{j=1}^{i-1}\hat{c}_j\right)\\
&\quad+\frac{b(n-m+1-b)}{(b-1)(b-n+k-1)}(\hat{c}_i-c_i)\geq 0,
\end{aligned}
$$

where the first inequality is because $\hat{c}_j-c_j\geq\hat{c}_i-c_i$, for $j=i+1,\ldots,k-1$; the second inequality follows since $i\leq m$, and the third inequality follows since $b\leq n-m+1$. So $w_i^*=\hat{c}_i$ is the best response price of supplier i.

Next, we analyze the best response of a supplier i in Ω_2, which is nonempty when $k\geq m+2$. Given $w_j^*=\hat{c}_j$, for $j\in\{1,2,\ldots,k-1\}$, $j\neq i$, and $w_j^*=c_j+(C_{\{k,n\}}+c_0+\sum_{j=1}^{k-1}\hat{c}_j)/(b-n+k-1)$, for $j\in\{k,\ldots,n\}$,

$$
\begin{aligned}
&\frac{bc_i+c_0+\sum_{j\neq i}w_j^*}{b-1}-\hat{c}_i\\
&\quad=\frac{b(C_{\{i,n\}}+c_0+\sum_{j=1}^{i-1}\hat{c}_j)}{(b-1)(b-n+k-1)}+\frac{b(\sum_{j=i+1}^{k-1}\hat{c}_j-C_{\{i+1,k-1\}})}{(b-1)(b-n+k-1)}\\
&\qquad-\left[1-\frac{n-k+1}{(b-1)(b-n+k-1)}\right](\hat{c}_i-c_i)\\
&\quad>\left[\frac{b(b-n+i-1)+n-k+1}{(b-1)(b-n+k-1)}-1\right](\hat{c}_i-c_i)\\
&\qquad+\frac{b\sum_{j=i+1}^{k-1}(\hat{c}_j-c_j)}{(b-1)(b-n+k-1)}\\
&\quad>\frac{b(1+i-k)}{(b-1)(b-n+k-1)}(\hat{c}_i-c_i)\\
&\qquad+\frac{b(k-i-1)}{(b-1)(b-n+k-1)}(\hat{c}_i-c_i)=0,
\end{aligned}
$$

where the first inequality is because $\hat{c}_i<c_i+(C_{\{i,n\}}+c_0+\sum_{j=1}^{i-1}\hat{c}_j)/(b-n+i-1)$; and the second inequality follows since $\hat{c}_i-c_i\leq\hat{c}_j-c_j$ for $j=i+1,\ldots,k-1$. So $w_i^*=\hat{c}_i$ is the best response price of supplier i.

Last, we analyze the best response price of a supplier i in Ω_3, which is nonempty when $k\leq n$. Given $w_j^*=\hat{c}_j$ for $j\in\{1,2,\ldots,k-1\}$, and $w_j^*=c_j+(C_{\{k,n\}}+c_0+\sum_{j=1}^{k-1}\hat{c}_j)/(b-n+k-1)$ for $j\in\{k,\ldots,n\}\backslash\{i\}$,

$$
\begin{aligned}
&\frac{bc_i+c_0+\sum_{j\neq i}w_j^*}{b-1}\\
&\quad=c_i+\frac{1}{b-1}\left(C_{\{k,n\}}+c_0+\sum_{j=1}^{k-1}\hat{c}_j\right)\\
&\qquad+\frac{n-k}{(b-1)(b-n+k-1)}\left(C_{\{k,n\}}+c_0+\sum_{j=1}^{k-1}\hat{c}_j\right)\\
&\quad=c_i+\frac{C_{\{k,n\}}+c_0+\sum_{j=1}^{k-1}\hat{c}_j}{b-n+k-1}\leq\hat{c}_i,
\end{aligned}
$$

where the last inequality follows since $\hat{c}_i-c_i\geq\hat{c}_k-c_k\geq(C_{\{k,n\}}+c_0+\sum_{j=1}^{k-1}\hat{c}_j)/(b-n+k-1)$. So $w_i^*=(bc_i+c_0+\sum_{j\neq i}w_j^*)/(b-1)=c_i+(C_{\{k,n\}}+c_0+\sum_{j=1}^{k-1}\hat{c}_j)/(b-n+k-1)$ is the best response of supplier i. □

Proof of Proposition 1. Because the assembler orders the same quantity from each supplier, it suffices to show the results for the profit margins. Let $k\in[1,n+1]$ be defined in (9), in equilibrium, the profit margins of the suppliers are as follows.

$$
\pi_i=\begin{cases}\hat{c}_i-c_i & i=1,2,\ldots,k-1\\[2mm] \dfrac{C_{\{k,n\}}+c_0+\widehat{C}_{\{1,k-1\}}}{b-n+k-1} & i=k,\ldots,n.\end{cases}
$$

As we label them, $\{\hat{c}_i-c_i\}_{i=1}^n$ is an increasing sequence, so $\pi_i\geq\pi_{i-1}$, for $i=2,\ldots,k-1$. π_i is a constant for $i\in[k,n]$. By definition of k, $\hat{c}_k\geq c_k+(C_{\{k,n\}}+c_0+\widehat{C}_{\{1,k-1\}})/(b-n+k-1)$, so that $\pi_k\geq\pi_{k-1}$.
So $\pi_1\leq\pi_2\leq\cdots\leq\pi_{k-1}\leq\pi_k=\pi_{k+1}=\cdots=\pi_n$, and hence the claim. □

Proof of Proposition 2. By Theorem 1, the suppliers' profits in equilibrium can be calculated as follows

$$
\Pi_l=\begin{cases}(\hat{c}_l-c_l)q_C^*\dfrac{(C_{\{1,n\}}+c_0)^b(b-n+k-1)^b}{(C_{\{k,n\}}+c_0+\widehat{C}_{\{1,k-1\}})^b b^b} & l=1,\ldots,k-1\\[4mm] q_C^*\dfrac{(C_{\{1,n\}}+c_0)^b(b-n+k-1)^{b-1}}{(C_{\{k,n\}}+c_0+\widehat{C}_{\{1,k-1\}})^{b-1}b^b} & l=k,\ldots,n.\end{cases}
$$

Consider supplier i, for some $i\in[1,n]$, and let $\hat{c}_i$ decrease, so that it faces intensified direct competition. (If more than one supplier faces the same intensity as supplier i, we let i be the smallest label.) Let $\hat{c}_i$ drop to $\hat{c}_i-\Delta$ and $\Delta>0$ be small enough to maintain the labeling of the suppliers by the intensities of direct competition. There are two possible consequences on the value of k, defined in (9), of such a change: it either remains unchanged or increases by one. In the following, we evaluate suppliers' profits in the two situations separately.

Case 1. The drop in $\hat{c}_i$ does not change k. If $i \in [k, n]$, the profits of all suppliers are unchanged. If $i \in [1, k-1]$, then the profits of the suppliers other than supplier i are

$$\Pi_l^N(\Delta) = \begin{cases} (\hat{c}_l - c_l) q_C^* \dfrac{(C_{\{1,n\}} + c_0)^b (b-n+k-1)^b}{(C_{\{k,n\}} + c_0 + \widehat{C}_{\{1,k-1\}} - \Delta)^b b^b} \\ \quad l = 1, \ldots, k-1, l \neq i \\ q_C^* \dfrac{(C_{\{1,n\}} + c_0)^b (b-n+k-1)^{b-1}}{(C_{\{k,n\}} + c_0 + \widehat{C}_{\{1,k-1\}} - \Delta)^{b-1} b^b} \\ \quad l = k, \ldots, n \end{cases}$$

where the superscript "N" emphasizes that these are the new equilibrium profits. It is straightforward to see that $\Pi_l^N(\Delta) > \Pi_l^N(0) = \Pi_l$, for $l \neq i$. Supplier i's profit after $\hat{c}_i$ decreases by Δ is

$$\Pi_i^N(\Delta) = (\hat{c}_i - c_i - \Delta) q_C^* \frac{(C_{\{1,n\}} + c_0)^b (b-n+k-1)^b}{(C_{\{k,n\}} + c_0 + \widehat{C}_{\{1,k-1\}} - \Delta)^b b^b}.$$

$$\frac{d\Pi_i^N(\Delta)}{d\Delta} = q_C^* \frac{(C_{\{1,n\}} + c_0)^b (b-n+k-1)^b}{b^b (C_{\{k,n\}} + c_0 + \widehat{C}_{\{1,k-1\}} - \Delta)^{b+1}} M(\Delta),$$

where $M(\Delta) = b(\hat{c}_i - c_i) - (b-1)\Delta - (C_{\{k,n\}} + \widehat{C}_{\{1,k-1\}} + c_0)$. $M(\Delta)$ decreases in Δ, and its sign determines that of the first-order derivative of $\Pi_i^N(\Delta)$. Suppose $\widehat{C}_{\{i+1,k-1\}} - C_{\{i+1,k-1\}} \leq ((n-i)/(b-n+i-1))(C_{\{i,n\}} + c_0 + \widehat{C}_{\{1,i-1\}})$. Then, $M(0) \geq 0$ and $\Pi_i^N(\Delta)$ first increases in Δ if $\hat{c}_i - c_i \in [(C_{\{k,n\}} + c_0 + \widehat{C}_{\{1,k-1\}})/b, (C_{\{i,n\}} + c_0 + \widehat{C}_{\{1,i-1\}})/(b-n+i-1)]$; while $M(0) < 0$ and $\Pi_i^N(\Delta)$ profit monotone decreases in Δ otherwise.

Case 2. The decrease in $\hat{c}_i$ causes k to increase by one. It must be that $i = k$ and $\hat{c}_k - c_k = (C_{\{k,n\}} + c_0 + \widehat{C}_{\{1,k-1\}})/(b-n+k-1)$, so that for any $\Delta > 0$, we have

$$\hat{c}_k - \Delta - c_k < \frac{C_{\{k,n\}} + c_0 + \widehat{C}_{\{1,k-1\}}}{b-n+k-1}, \quad \text{and}$$

$$\hat{c}_{k+1} - c_{k+1} \geq \frac{C_{\{k+1,n\}} + c_0 + \widehat{C}_{\{1,k\}} - \Delta}{b-n+k}.$$

After $\hat{c}_i$ drops to $\hat{c}_i - \Delta$, the profits of the suppliers other than supplier i are as follows.

$$\Pi_l^N(\Delta) = \begin{cases} (\hat{c}_l - c_l) q_C^* \dfrac{(C_{\{1,n\}} + c_0)^b (b-n+k)^b}{(C_{\{k+1,n\}} + c_0 + \widehat{C}_{\{1,k\}} - \Delta)^b b^b} & l = 1, \ldots, k-1 \\ q_C^* \dfrac{(C_{\{1,n\}} + c_0)^b (b-n+k)^{b-1}}{(C_{\{k+1,n\}} + c_0 + \widehat{C}_{\{1,k\}} - \Delta)^{b-1} b^b} & l = k+1, \ldots, n. \end{cases}$$

Since $\hat{c}_k - \Delta - c_k < (C_{\{k,n\}} + c_0 + \widehat{C}_{\{1,k-1\}})/(b-n+k-1)$, $(b-n+k)/(C_{\{k+1,n\}} + c_0 + \widehat{C}_{\{1,k\}} - \Delta) > (b-n+k-1)/(C_{\{k,n\}} + c_0 + \widehat{C}_{\{1,k-1\}})$.
So $\Pi_l^N(\Delta) > \Pi_l$, for $l \neq i$.

Supplier i's profit can be written as $\Pi_i^N(\Delta) = (\hat{c}_i - c_i - \Delta) q_C^* (((C_{\{1,n\}} + c_0)^b (b-n+i)^b)/((C_{\{i+1,n\}} + c_0 + \widehat{C}_{\{1,i\}} - \Delta)^b b^b))$.

$$\frac{d\Pi_i^N(\Delta)}{d\Delta} = q_C^* \frac{(C_{\{1,n\}} + c_0)^b (b-n+i)^b}{b^b (C_{\{i+1,n\}} + c_0 + \widehat{C}_{\{1,i\}} - \Delta)^{b+1}} M(\Delta), \quad \text{where}$$

$$M(\Delta) = (b-1)(\hat{c}_i - c_i - \Delta) - (C_{\{i,n\}} + c_0 + \widehat{C}_{\{1,i-1\}}).$$

$M(\Delta)$ decreases in Δ, and its sign determines that of the derivative of $\Pi_i^N(\Delta)$. Note that $M(0) = (b-1)(\hat{c}_i - c_i) - (C_{\{i,n\}} + c_0 + \widehat{C}_{\{1,i-1\}}) = ((n-i)/(b-n+i-1))(C_{\{i,n\}} + c_0 + \widehat{C}_{\{1,i-1\}}) > 0$. So $\Pi_i^N(\Delta)$ will first increase in Δ. However, as $\hat{c}_i \to c_i$, $\Pi_i^N(\Delta)$ will decrease since the profit margin approaches zero. □

Proof of Proposition 3. To facilitate exposition, we define $\Delta_l \equiv \hat{c}_l - c_l$ as the intensity of the direct competition at supplier l, for $1 \leq l \leq n$. Further, we define $B_l \equiv (C_{\{l,n\}} + c_0 + \widehat{C}_{\{1,l-1\}})/(b-n+l-1)$ and call it the reference price; we let $B_l = \infty$ if $b-n+l-1 \leq 0$. By Theorem 1, in an n-component system with suppliers labeled by increasing order of Δ_l's, k in (9) identifies the label of the first supplier whose competition intensity is not less than the reference price; we call such a supplier the threshold supplier, and k the threshold label. In such an n-component system, the assembler's profit is $\Pi_0 = A((b-n+k-1)^{b-1}/(C_{\{1,n\}} + c_0 + \sum_{l=1}^{k-1} \Delta_l)^{b-1})$, where $A \equiv \Pi_C^*((C_{\{1,n\}} + c_0)^{b-1}/b^{b-1})$.

Now, suppose suppliers i and j, for $1 \leq i < j \leq n$, merge. Let the new supplier be v, who faces a direct competition with intensity $\Delta_i + \Delta_j$. Define $u \equiv \text{Min}\{l \in [1, n+1]/\{i,j\}\colon \Delta_l \geq \Delta_i + \Delta_j\}$ as the lowest labeled supplier facing less direct competition than supplier v among the suppliers other than i and j. To study the effect of such a merger on the assembler, we analyze four scenarios when suppliers facing different levels of direct competition merge. We identify the threshold supplier in each scenario, and comparatively study the assembler's profit as the identities of merging suppliers change. Because of space limit, we provide analysis for selected scenarios, and sketch other main steps. We add subscripts on the equilibrium results after the merger of suppliers to differentiate the scenarios.

Scenario 1. $k < i < j \leq n$: Both merging suppliers face less direct competition. After the merger, the reference price of supplier l, for $l = 1, 2, \ldots, k$, is $B_{l,1} = (C_{\{l,n\}} + c_0 + \widehat{C}_{\{1,l-1\}})/(b-n+l) \leq (C_{\{l,n\}} + c_0 + \widehat{C}_{\{1,l-1\}})/(b-n+l-1) = B_l$; and his competition intensity is Δ_l. Since $\Delta_k \geq B_k > B_{k,1}$, by Theorem 1, the updated threshold label, k_1, satisfies $k_1 \leq k$, and $\Pi_{0,1} = A((b-n+k_1)^{b-1}/(C_{\{1,n\}} + c_0 + \sum_{l=1}^{k_1-1} \Delta_l)^{b-1})$. If $k_1 = k$, obviously $\Pi_{0,1} > \Pi_0$. If $k_1 \leq k-1$, still $\Pi_{0,1} \geq \Pi_0$, since

$$\frac{b-n+k_1}{C_{\{1,n\}} + c_0 + \sum_{l=1}^{k_1-1} \Delta_l} - \frac{b-n+k-1}{C_{\{1,n\}} + c_0 + \sum_{l=1}^{k-1} \Delta_l}$$
$$\geq \frac{(k-k_1)(-C_{\{1,n\}} - c_0 - \sum_{l=1}^{k_1-1} \Delta_l + (b-n+k_1)\Delta_{k_1})}{(C_{\{1,n\}} + c_0 + \sum_{l=1}^{k_1-1} \Delta_l)(C_{\{1,n\}} + c_0 + \sum_{l=1}^{k-1} \Delta_l)} \geq 0.$$

So the assembler earns higher profit after suppliers merge. Note that specific identities of the two merging suppliers in this scenario do not affect the equilibrium.

Scenario 2. $i < k \le j \le n$: One merging supplier faces relatively less direct competition, while the other faces relatively more direct competition. With the updated reference price for each supplier, we can show that the threshold label in this scenario, k_2, satisfies $k_2 \le k$, and $\Pi_{0,2} \le \Pi_{0,1}$. In particular, if $k_2 \le i-1$, $k_2 = k_1$, and $\Pi_{0,2} = \Pi_{0,1}$; if $k_2 \ge i+1$, $k_2 \ge k_1$, and $\Pi_{0,2} = A((b-n+k_2-1)^{b-1}/(C_{\{1,n\}} + c_0 + \sum_{l=1}^{k_2-1}\Delta_l - \Delta_i)^{b-1}) \le \Pi_{0,1}$, where the inequality follows since

$$\frac{b-n+k_2-1}{C_{\{1,n\}}+c_0+\sum_{l=1}^{k_2-1}\Delta_l-\Delta_i}-\frac{b-n+k_1}{C_{\{1,n\}}+c_0+\sum_{l=1}^{k_1-1}\Delta_l}$$
$$\le \frac{-(C_{\{1,n\}}+c_0+\sum_{l=1}^{k_1-1}\Delta_l-\Delta_i)+(b-n+k_1-1)\Delta_i}{(C_{\{1,n\}}+c_0+\sum_{l=1}^{k_2-1}\Delta_l-\Delta_i)(C_{\{1,n\}}+c_0+\sum_{l=1}^{k_2-1}\Delta_l)}(k_2-k_1)\le 0.$$

We next investigate the effects of the identities of merging suppliers. Since $k_2 \le k$, the identity of j in $[k, n]$ does not affect the equilibrium in this scenario. As i decreases, however, the threshold label will not decrease, but the assembler's profit will not increase. To see this, let i decrease to $i-1$; we add superscript N on equilibrium results after such a change. That i decreases to $i-1$ does not affect the reference price of supplier $l \in [1, i-2]$, but raises the reference price of supplier $l \in [i+1, k]$, so the new threshold label must satisfy $k_2^N \ge k_2$. We consider three cases as follows:

Suppose $k_2 \in [1, i-2]$ before i decreases to $i-1$, then $k_2^N = k_2$, and $\Pi_{0,2}^N = \Pi_{0,2}$.

Suppose $k_2 = i-1$ before i decreases to $i-1$, then $k_2^N \ge i$. If $k_2^N = i$, the assembler's profit will remain as $\Pi_{0,2}^N = \Pi_{0,2} = A((b-n+i-1)^{b-1}/(C_{\{1,n\}} + c_0 + \sum_{l=1}^{i-2}\Delta_l)^{b-1})$. If $k_2^N > i$, then, by definitions of k_2^N and k_2, $\Pi_{0,2}^N < \Pi_{0,2}$.

Suppose $k_2 \in [i+1, k]$ before i decreases to $i-1$. If $k_2^N = k_2$, then assembler's profit decreases from $\Pi_{0,2} = A((b-n+k_2-1)^{b-1}/(C_{\{1,n\}} + c_0 + \sum_{l=1}^{k_2-1}\Delta_l - \Delta_i)^{b-1})$ to $\Pi_{0,2}^N = A((b-n+k_2-1)^{b-1}/(C_{\{1,n\}} + c_0 + \sum_{l=1}^{k_2-1}\Delta_l - \Delta_{i-1})^{b-1})$, since $\Delta_{i-1} \le \Delta_i$; if $k_2^N > k_2$, then, by definitions of k_2^N and k_2, we can show that $\Pi_{0,2}^N \le A((b-n+k_2-1)^{b-1}/(C_{\{1,n\}} + c_0 + \sum_{l=1}^{k_2-1}\Delta_l - \Delta_{i-1})^{b-1})$. So the assembler's profit will not increase as i decreases.

Scenario 3. $i < j < k \le u$: Both merging suppliers face relatively more direct competition, but the intensity at the new supplier v is relatively low. We can show that the threshold label, k_3, satisfies that $k_3 \le k$. Moreover, if $k_3 \le j-1$, then $k_3 = k_2$ and $\Pi_0 \le \Pi_{0,3} = \Pi_{0,2}$; if $k_3 \ge j+1$, then $k \ge k_3 \ge k_2$ and $\Pi_0 \le \Pi_{0,3} \le \Pi_{0,2}$. Further, as either i or j decreases, we can use similar steps as those in Scenario 2 to show that k_3 will not decrease, but $\Pi_{0,3}$ will not increase.

So, the profit of assembler in this scenario is lower than that in Scenario 2, but higher than that in the original n-component system; given the identity of one merging supplier, the assembler earns higher profit if the other merging supplier faces less direct competition.

Scenario 4. $i < j < u \le k-1$: Both merging suppliers face relatively more direct competition, and the intensity faced by the new supplier v is strong as well. The threshold label in this scenario, k_4, satisfies $k_4 = k$, and the assembler's profit $\Pi_{0,4} = \Pi_0$. So, the assembler's profit is unaffected by the merger. The specific identities of suppliers i and j do not affect the equilibrium, as long as $u \le k-1$.

The analyses in Scenarios 1–4 combine to show that the assembler earns a higher profit as suppliers merge; and it earns a higher profit as the merging suppliers face less direct competition. □

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