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Understanding How Generation Flexibility and Renewable Energy Affect Power Market Competition

Majid Al-Gwaiz,[a] Xiuli Chao,[b] Owen Q. Wu[c]

[a] Saudi Aramco, Dhahran 31311, Saudi Arabia; [b] Department of Industrial and Operations Engineering, University of Michigan, Ann Arbor, Michigan 48109; [c] Kelley School of Business, Indiana University, Bloomington, Indiana 47405

Contact: majid.gwaiz@gmail.com (MA-G); xchao@umich.edu (XC); owenwu@indiana.edu (OQW)

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Abstract. We study supply function competition among conventional power generators with different levels of flexibility and the impact of intermittent renewable power generation on the competition. Inflexible generators commit production before uncertainties are realized, whereas flexible generators can adjust their production after uncertainties are realized. Both types of generators compete in an electricity market by submitting supply functions to a system operator, who solves a two-stage stochastic program to determine the production level for each generator and the corresponding market prices. We aim to gain an understanding of how conventional generators' (in)flexibility and renewable energy's intermittency affect the supply function competition and the market price. We find that the classic supply function equilibrium model overestimates the intensity of the market competition, and even more so when more intermittent generation is introduced into the system. The policy of economically curtailing intermittent generation intensifies the market competition, reduces price volatility, and improves the system's overall efficiency. Furthermore, these benefits of economic curtailment are most significant when the production-based subsidies for renewable energy are absent.

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Keywords: electricity market; supply function equilibrium • flexible/inflexible/variable generators • renewable energy • intermittent generation • economic curtailment • production-based subsidies

1. Introduction

The special nature of the electricity industry (quick and random fluctuations of demand, limited storage capability) requires production decisions to be automated and coordinated instantaneously. Thus, in an electricity market, the instruments of competition are supply functions, which specify the amount of electricity each firm is willing to generate at every market price. Based on the submitted supply functions, a system operator finds the most economical production schedule to meet the electricity demand and determines the payment to each firm. A set of supply functions constitutes a supply function equilibrium (SFE) if no firm can benefit by unilaterally changing its supply function. Klemperer and Meyer (1989) pioneered the effort in analyzing the SFE in general industrial contexts. Green and Newbery (1992) and Bolle (1992) were the first to employ the SFE framework to analyze electricity markets. These seminal studies and the subsequent stream of research provide important economic insights and policy recommendations, which we will review in Section 2.

Most SFE models for electricity markets assume that every firm has the flexibility to adjust its production to the point on its supply function corresponding to the market clearing price. This is a justifiable assumption in two situations. First, each firm owns a portfolio of power generators and offers an aggregate supply function. The portfolio consists of inflexible generators (e.g., nuclear and some coal-fired generators) and flexible generators (fueled by oil or gas); the aggregate output can be quickly adjusted in response to the price. This situation is studied by Green and Newbery (1992), Green (1996), Rudkevich (1999), and Baldick et al. (2004), among others. Second, in the real-time market, firms with flexible generators submit real-time supply offers to meet the energy imbalance (the energy that deviates from the day-ahead schedule). This situation is considered by, for example, Holmberg (2007, 2008). The theoretical framework of SFE is applicable to both situations.

The assumption of production flexibility, however, may not always be appropriate. As industry deregulation continues, firms downsize their portfolios by selling off parts of their generation assets, and independent power producers emerge to participate in the power markets. Furthermore, in many current electricity markets, every conventional generator that participates in the market is required to submit supply functions. As a result, generators with different levels of flexibility engage in a supply function competition.

The classic SFE model does not address the competitions involving inflexible generators, but in reality, (in)flexibility directly affects a generator's production and revenue. Thus, it is intriguing to ask how generators with different levels of flexibility behave in a supply function competition, and how the presence of inflexibility affects the equilibrium market price. Answers to these questions will help policy makers understand whether the classic SFE model may over- or underestimate the intensity of the market competition. The understanding of the effect of generation (in)flexibility on competition is especially important given the recent evolution of generation mix, with coal-fired generation retiring and more flexible generation fueled by natural gas coming in place.

An important part of this evolution is the increasing use of renewable energy sources, notably solar and wind power. Renewable energy generation displaces conventional flexible and inflexible generation, and potentially changes the competition between them. Thus, another question that this paper will address is how renewable energy penetration impacts the SFE between flexible and inflexible generators. The answer to this question depends on renewable energy subsidies and dispatch policies.

Renewable energy subsidies are prevalent around the world. The U.S. federal government provides a production tax credit of $23 for each megawatt hour generated by wind power plants during their first 10 years of operation. In addition, 29 states, the District of Columbia, and three territories have adopted renewable portfolio standards mandating the percentage of customer demand met with renewables, which translates into compliance markets where utilities purchase renewable energy certificates from renewable energy producers. Furthermore, the feed-in tariffs adopted by several U.S. states and other countries mandate the prices at which the utilities must offer to the renewable energy producers.

Because these subsidies are based on the output of renewable energy producers, *priority dispatch* policies have been in place to ensure that the renewable energy is given priority in dispatch. An international survey of wind energy curtailment practices by Rogers et al. (2010) shows that in most countries and regions, the only situation when renewable energy may be curtailed is when the excessive energy threatens power grid reliability, e.g., exceeding transmission capacity.

As the growth of renewable energy continues, however, curtailing renewable energy is found to have economic value, as shown by Ela (2009), Ela and Edelson (2012), Wu and Kapuscinski (2013), and Oggioni et al. (2014). Consequently, many system operators started to develop market mechanisms for *economic curtailment*. An important question is how renewable energy's dispatch policy (priority dispatch or economic curtailment) affects power market competition, which in turn affects the power generation cost and emissions. A caveat is that even if the economic curtailment policy is in effect, the production-based subsidies for renewable energy may reduce the amount of economic curtailment. Therefore, in addressing this question, we also analyze how the dispatch policy interacts with the production-based subsidies.

The objective of this paper is to address the questions raised above through theoretical and computational analysis of a stylized model. We consider a system consisting of inflexible generators, flexible generators, and variable (or intermittent) generators. The demand and the variable generators' output are uncertain. Before the uncertainties are realized, all generators submit supply offers to a system operator, who then decides a predispatch price and the inflexible generators' output. After the uncertainties are realized, the system operator decides the real-time price, the flexible generators' output, and the variable generators' output (if economic curtailment is allowed). For the tractability of analyzing SFE, our model does not include a comprehensive depiction of generators' physical constraints such as capacity, ramp rates, and minimum up/down times. However, our model captures the important realities of generation (in)flexibility and intermittent generation.

The main insights from this paper are summarized below. With the presence of generation inflexibility, the classic SFE model overestimates the intensity of the market competition. Inflexible generators do not directly compete with flexible generators in matching production with uncertain demand, leading to increased market power of flexible generators, which in turn results in a higher average price and a higher price volatility than predicted by the classic SFE model. Variable generation, when given priority in dispatch, further increases the flexible generators' market power.

Economic curtailment of variable generation provides the system operator with another lever to manage variability, in addition to using flexible generation for variability buffering. Thus, economic curtailment reduces the flexible generators' market power and intensifies the market competition. Economic curtailment also helps reduce the system's total operating cost, but may increase or decrease the total emission, depending on the generators' fuel types. The production-based subsidies for variable generation reduce the amount of curtailment, preventing the economic curtailment policy from achieving its full benefit of encouraging competition and reducing operating costs.

2. Literature Review

In their original work, Klemperer and Meyer (1989) show the existence of a family of SFEs for competing firms with identical cost functions and no capacity constraints. They characterize the SFEs by differential equations and show that, given the support of the uncertainty, the equilibria are independent of the distribution of the uncertainty. Since this seminal work, the SFE framework has been applied extensively to the research in electricity markets. Comprehensive reviews of this area are provided by Ventosa et al. (2005), Holmberg and Newbery (2010), and Li et al. (2011). Thus, we review below only the works most relevant to our paper.

Many studies focus on the case of symmetric equilibria, in which firms offer identical supply functions. Green and Newbery (1992) calibrate the SFE model for the British electricity industry, and their results suggest that the market power had been seriously underestimated by the policy makers. Rudkevich et al. (1998) study symmetric SFEs with inelastic demand and find that even with a relatively high number of competing firms, the market clearing prices are still significantly higher than perfectly competitive prices. Anderson and Philpott (2002) derive the conditions under which a supply function can represent a firm's optimal response to the offers of other firms and show that their model admits symmetric SFE. Holmberg (2008) proves the SFE is unique when power shortage occurs with positive probability and a price cap exists.

When firms differ in costs, the general asymmetric equilibria are difficult to find, and thus linear supply functions are often used to simplify the analysis. Green (1996) solves the asymmetric equilibrium with linear supply functions and studies the effects of various policies that could increase the competition in the electricity market. Rudkevich (1999) provides a more explicit solution to the SFE with linear supply functions and further finds that this equilibrium could be reached by a learning process. In this paper, we also analyze SFE with linear supply functions and extend the above studies by considering asymmetries in both cost and flexibility.

Physical constraints such as capacities and network transmission constraints are important areas in the SFE literature. SFE models with capacity constraints are considered by Green and Newbery (1992), Baldick et al. (2004), Holmberg (2007), Anderson and Hu (2008), Genc and Reynolds (2011), and Anderson (2013). SFE models with network transmission constraints are studied by Berry et al. (1999), Wilson (2008), and Holmberg and Philpott (2015). Constraints may also rise from market rules. Supatgiat et al. (2001) study the Nash equilibria when the price bids are restricted to a discrete set and each firm offers a single price–quantity pair. This paper complements these studies by focusing on firms' production-adjustment constraints, with the goal of understanding how inflexibility may contribute to the market power.

Many electricity markets operate in a two-settlement structure with a day-ahead market and a real-time market. In the day-ahead market, generators submit supply offers and the system operator makes unit commitment decisions based on the forecast for the demand and intermittent generation. In the real-time market, most uncertainties are revealed, fast-responding units are committed, and all units are dispatched. This two-stage decision process is often modeled as a two-stage stochastic program. Ruiz et al. (2009) compare the stochastic programming approach with the traditional reserve requirement approach. Papavasiliou and Oren (2013) employ a detailed two-stage stochastic program to study the impact of high wind penetration. Pritchard et al. (2010) devise a market settlement scheme based on stochastic programming and show that the scheme is revenue adequate in expectation. Khazaei et al. (2014) analyze a symmetric SFE in a market where generators bid a linear supply function as well as a penalty cost for deviation from the predispatch quantity. They compare the two-stage stochastic programming approach with the current market mechanism in New Zealand. It is also possible but complex to analyze a two-stage supply function game and subgame-perfect Nash equilibria; see the thesis by Anderson (2004). In this paper, we adopt stochastic programming and supply function competition frameworks to study the impact of generation inflexibility and renewable energy on SFE. The supply function competition occurs before the system operator solves the two-stage stochastic program.

Several empirical studies have been conducted to compare the SFE prediction with actual market data. Sioshansi and Oren (2007) find evidence that generators in the Texas electricity market bid less competitively than predicted by the SFE model. Willems et al. (2009) find similar evidence in the German electricity market and include constant correction terms in their model. Some insights from our model are consistent with these empirical findings.

Integration of variable generation into electricity systems has received substantial research attention over the past decade. The National Renewable Energy Laboratory completed two large variable generation integration studies: the Western Wind and Solar Integration Study (GE Energy 2010) and the Eastern Wind Integration and Transmission Study (EnerNex 2011). Reviews of these and other renewable energy integration studies are provided by Smith et al. (2007), Ela et al. (2009), and Hart et al. (2012). Most of the integration studies focus on quantifying system cost reduction due to variable generation, as well as the additional cost in balancing against variable generation.

The impact of variable generation on the SFE in electricity markets has been considered recently. Green and Vasilakos (2010) assume wind power has priority in dispatch and study the impact of wind power on the market price generated from an SFE among symmetric conventional generators. They find that wind power significantly increases the hourly price volatility and that market power raises both average price and price volatility. Sioshansi (2011) considers a Stackelberg game with wind power generators deciding output followed by a supply function competition among conventional generators. Assuming wind power generators are price takers and have priority, Buygi et al. (2012) analyze an SFE with linear supply functions and find that although the intermittency of wind power tends to increase the market price, the net impact of wind power is a lower market price. In this paper we also treat variable generators as price takers and study their impact on average price and price volatility. We further consider the impact of subsidies and dispatch policies on SFE and market prices. For an analysis of supply function competition among variable generators, the reader is referred to Sunar and Birge (2015).

The role of economic curtailment policy has been investigated in several studies. Ela (2009) explores the network effects of economic curtailment. Ela and Edelson (2012) analyze the benefit of curtailment on relieving physical constraints of generation resources, thereby bringing substantial cost savings. Wu and Kapuscinski (2013) analyze the impact of economic curtailment on cycling cost and peaking cost and find that curtailing wind power can be both economically and environmentally beneficial under certain situations. Oggioni et al. (2014) compare the priority dispatch policy with the economic curtailment for the European electricity market and find that priority dispatch policy may lead to market collapse under certain market organizations, and economic curtailment can remove such problems. Henriot (2015) uses a stylized model to analyze how the optimal level of curtailment varies with system parameters and how the efficiency gains from curtailment would be shared among all stakeholders. This paper complements these works by studying the impact of economic curtailment on market competition. We find that economic curtailment increases market competition, a benefit of economic curtailment that has not been previously identified.

3. The Model

In the classic SFE model, facing the uncertain demand, each firm commits a supply function; after the demand shock is realized, each firm produces at the point on its supply function corresponding to the market clearing price. Our model encompasses two important extensions that reflect the electricity market reality: First, inflexible generators are unable to adjust production quickly and their production levels have to be determined before uncertainty is realized. Second, renewable generation introduces additional uncertainty into the market, and it is possible to curtail (waste) some renewable generation under oversupply situations. These modeling elements are detailed below.

3.1. Uncertainties and Generator Types

We consider a two-stage model for electricity markets. In the first stage, generators engage in supply function competition under uncertainties in demand and renewable generation. In the second stage, the uncertainties are realized and the generators produce power to meet the demand. The uncertainties are modeled by a pair of random variables (L, W), where L represents the price-insensitive demand (or load), and W denotes the *potential* output of variable generators (VGs). VGs may be instructed to adjust their actual output below the potential output, known as curtailment (see details in Section 3.3). Let $\mathcal{S}$ denote the set of all possible realizations of the uncertainties. Under scenario $s \in \mathcal{S}$, the realization of (L, W) is denoted as (L_s, W_s). We assume that (L, W) has a joint continuous distribution.

The system consists of two types of conventional generators: inflexible and flexible generators. The sets of inflexible and flexible generators are denoted by G^I and G^F, respectively. Our model for generation flexibility follows Pritchard et al. (2010):

- Inflexible generators (IGs), indexed by $i \in G^I$, cannot adjust their output in response to the realization of the uncertainties. The output of generator $i \in G^I$, denoted as $q_i \geq 0$, is determined by the system operator based on the supply function competition in the first stage. Let $C_i^o(q_i)$ denote generator i's cost of producing q_i.
- Flexible generators (FGs), indexed by $j \in G^F$, can adjust their output in the second stage after the uncertainties are realized. Let $q_{j,s} \geq 0$ denote the output of generator $j \in G^F$ under scenario s, and let $C_j^o(q_{j,s})$ denote the associated production cost.

The costs of the generators satisfy the following assumption.

Assumption 1. (i) *For any generator $k \in G^I \cup G^F$, the cost function $C_k^o(q)$ is convex, strictly increasing, and continuously differentiable in q, and $C_k^o(0) = 0$.* (ii) *VGs produce energy at negligible operating cost and receive a subsidy of $r \geq 0$ per unit of output that is not curtailed.*

The convexity and monotonicity in Assumption 1(i) reflect the production cost function in reality. Assumption 1(ii) states that VGs receive a production-based subsidy and it implies that the marginal production cost of VGs is $-r$.

3.2. Supply Functions and Stated Costs

In the first stage, IGs and FGs simultaneously submit supply functions to the system operator. Generator $k \in G^I \cup G^F$ submits a supply function $S_k(p)$, which specifies the output it is willing to produce when the price is p ($p \in \Re$, the set of real numbers). Based on these supply functions, the system operator determines a first-stage price (or predispatch price), denoted as p_0, which in turn decides generator k's *planned* production $S_k(p_0)$ for the second stage and associated payment $S_k(p_0)p_0$, for all $k \in G^I \cup G^F$. In the second stage, IG $i \in G^I$ produces $S_i(p_0)$ and receives a payment of $S_i(p_0)p_0$, whereas the FGs' actual output and payment may be adjusted as uncertainties unfold. When scenario $s \in \mathcal{S}$ is realized, the system operator decides a second-stage price p_s and instructs FG $j \in G^F$ to produce $S_j(p_s)$. The adjusted quantity $S_j(p_s) - S_j(p_0)$ is settled at price p_s. Thus, FG j receives a total payment of $S_j(p_0)p_0 + (S_j(p_s) - S_j(p_0))p_s$. The first- and second-stage prices resemble the day-ahead and real-time prices in the electricity markets. In our model, FGs do not resubmit supply functions in the second stage; the supply function competition occurs only in the first stage and involves both IGs and FGs. The supply functions satisfy the following condition.

Assumption 2. *For any $k \in G^I \cup G^F$* (i) *there exists $p_k^{\min} \geq 0$ such that $S_k(p) = 0$ for $p \leq p_k^{\min}$;* (ii) *$S_k(p)$ strictly increases in p for $p \geq p_k^{\min}$; and* (iii) $\lim_{p\to 0} S_k(p) = 0$.

Assumption 2(i) implies that no IG or FG is willing to produce when the price is too low. Part (ii) is consistent with practice; e.g., the Midcontinent Independent System Operator's (2016, p. 86) business practice manual "Energy and Operating Reserve Markets" states that the price–quantity pairs that form a supply function must be (weakly) increasing for price and strictly increasing for quantity. Part (iii) is automatically satisfied if $p_k^{\min} > 0$ due to part (i); when $p_k^{\min} = 0$, part (iii) states that no generator, flexible or inflexible, is willing to produce when the price drops to nearly zero. The commonly used affine supply function $S_k(p) = \beta_k(p - p_k^{\min})^+$, where $\beta_k > 0$, satisfies Assumption 2. (Throughout this paper, $x^+ = \max\{x, 0\}$ for any $x \in \Re$.)

Based on the submitted supply function $S_k(p)$, the system operator computes the stated cost function of generator k as follows:

$$C_k(q) \stackrel{\text{def}}{=} \int_0^q S_k^{-1}(x)\,dx, \quad \forall\, k \in G^I \cup G^F, \tag{1}$$

where $S_k^{-1}(q) \stackrel{\text{def}}{=} \inf\{p\colon S_k(p) > q\}$ is the inverse supply function. Note that $C_k(q)$ is the stated cost function, which should be distinguished from the true cost function $C_k^o(q)$. These two cost functions are equal if generator k submits its inverse marginal cost function as its supply function.

Unlike IGs and FGs, VGs are unable to guarantee an output because of their inherent intermittency. Under the economic curtailment policy, each VG submits a price offer for its potential output, i.e., a price at or below which it is willing to be curtailed. To focus on analyzing the strategic interactions between IGs and FGs, we assume that there are many VGs, and each individual VG's output has negligible influence on the market price. Therefore, each VG offers a price equal to its marginal cost $-r$ (see Assumption 1(ii)). Consequently, the VGs' stated cost of output q is $-rq$. In the second stage, VGs will completely curtail output when $p_s < -r$, produce the potential output W_s when $p_s > -r$, and may produce any quantity in $[0, W_s]$ when $p_s = -r$.

3.3. System Operator's Problem

The objective of the system operator is to minimize the expected total stated cost of serving the load. This objective is consistent with the practice (see, e.g., Midcontinent Independent System Operator 2016, Attachment B, Section 4.1.5) and the literature (see, e.g., Anderson and Philpott 2002).

To formulate the system operator's problem, we define the aggregate output and aggregate stated cost functions. Let q^I, q_s^F, and q_s^V denote the aggregate output in scenario s for IGs, FGs, and VGs, respectively. The VGs' aggregate stated cost is $-rq_s^V$. The aggregate stated cost functions for IGs and FGs are computed by optimizing the allocations of q^I and q_s^F among individual generators, as follows:

$$C^I(q^I) \stackrel{\text{def}}{=} \min\left\{\sum_{i\in G^I} C_i(q_i)\colon q_i \geq 0,\ \sum_{i\in G^I} q_i = q^I\right\}, \tag{2}$$

$$C^F(q_s^F) \stackrel{\text{def}}{=} \min\left\{\sum_{j\in G^F} C_j(q_{j,s})\colon q_{j,s} \geq 0,\ \sum_{j\in G^F} q_{j,s} = q_s^F\right\}. \tag{3}$$

The following lemma summarizes the properties of $C^I(q)$ and $C^F(q)$ and their relationship with the aggregate supply functions, defined as

$$S^I(p) \stackrel{\text{def}}{=} \sum_{i\in G^I} S_i(p), \quad \text{and} \quad S^F(p) \stackrel{\text{def}}{=} \sum_{j\in G^F} S_j(p). \tag{4}$$

Lemma 1. *The aggregate stated cost functions $C^I(q)$ and $C^F(q)$ are convex, strictly increasing, and continuously differentiable in q. Furthermore, $(C^I)'(q) = (S^I)^{-1}(q)$ and $(C^F)'(q) = (S^F)^{-1}(q)$.*

Lemma 1 confirms that the aggregate supply functions in (4) are consistent with the inverse marginal stated cost functions. The proof for the lemma and all other technical results are included in the online appendix.

The total generation and the load should be balanced under any scenario $s \in \mathcal{S}$. Imbalance leads to extra operating cost. In the case of oversupply, the system operator must take mitigating actions, such as reducing generation to an emergency minimum level, or even shutting down some IGs at significant wear-and-tear costs. Following the approach employed in practice, we model the extra costs for handling oversupply situations using a penalty function $h(e)$, which represents the extra cost when the total output exceeds the load by $e \geq 0$. For example, in the Texas electricity system, a penalty for violating the power balance constraint is included in the objective function of the security-constrained economic dispatch problem (Electric Reliability Council of Texas 2012, p. 24).

Assumption 3. *The oversupply penalty rate function $h(e)$ is strictly convex, strictly increasing, and continuously differentiable in $e \geq 0$, and $h(0) = 0$.*

The system operator's problem can be formulated as a two-stage stochastic program, similar to Ruiz et al. (2009), Pritchard et al. (2010), and Papavasiliou and Oren (2013), among others. In the first stage, the IGs' output q^I is decided before the uncertainties are realized; in the second stage, when $s \in \mathcal{S}$ is realized, the FGs' and VGs' outputs q_s^F and q_s^V are determined. The system operator's problem in the first stage is to minimize the expected total stated cost. Under the *economic curtailment* policy, this problem can be written as

$$\min \quad C^I(q^I) + \mathsf{E}[C^F(q_s^F) - r q_s^V + h(e_s)] \tag{5}$$

$$\text{s.t.} \quad e_s \equiv q^I + q_s^F + q_s^V - L_s \geq 0, \quad \forall s \in \mathcal{S}, \tag{6}$$

$$q_s^V \leq W_s, \quad \forall s \in \mathcal{S}, \tag{7}$$

$$q^I, q_s^F, q_s^V \geq 0, \quad \forall s \in \mathcal{S}, \tag{8}$$

where the expectation in (5) is taken over all scenarios in $\mathcal{S}$. The inequality in (6) ensures sufficient supply to meet the load, whereas excess supply (if $e_s > 0$) is penalized in the objective (5). The system operator will derive the first- and second-stage prices p_0 and p_s from (5)–(8); see the analysis in Section 4.

Under the *priority dispatch* policy, the system operator sets $q_s^V = W_s$ in (7); i.e., all potential VG output is used. The analysis of the two dispatch policies, economic curtailment and priority dispatch, can be unified by recognizing that when r approaches infinity (here r is treated as a parameter and should not be interpreted as subsidy), the optimal solution to (5)–(8) has $q_s^V = W_s$, and therefore coincides with the solution under the priority dispatch policy.

Similar to Pritchard et al. (2010) and Khazaei et al. (2014), we assume away intertemporal dynamics to analyze supply function equilibria. The equilibrium analysis will become intractable if we include intertemporal dynamics such as start-up, shutdown, ramping, and minimum on/off times (see also the discussion in Li et al. 2007).

Note that the FGs' stated cost function $C^F(q)$ is used in the objective function (5) to compute FGs' production cost. Because FGs are completely flexible, no production adjustment cost is involved in this paper. The reader is referred to Khazaei et al. (2014) for an analysis involving production adjustment cost.

4. Optimal Generation and Market Prices

In this section, we characterize the optimal solution for the system operator's problem in (5)–(8). We first solve for the optimal output of FGs and VGs under any given IG output $q^I \geq 0$ and scenario $s \in \mathcal{S}$, and then decide the optimal IG output q^{I*} that minimizes the expected total stated cost. The first- and second-stage prices that induce the optimal generation levels are also derived.

4.1. Optimal Flexible and Variable Generation Under Given IG Production

In the second stage, under the IG output q^I decided in the first stage and the realized scenario $s \in \mathcal{S}$, the system operator decides the optimal flexible and variable generation by solving the following problem:

$$\hat{C}(q^I, L_s, W_s) \stackrel{\text{def}}{=} \min_{\{q_s^F, q_s^V\}} \quad C^F(q_s^F) - r q_s^V + h(e_s) \tag{9}$$

$$\text{s.t.} \quad e_s \equiv q^I + q_s^F + q_s^V - L_s \geq 0, \tag{10}$$

$$q_s^V \leq W_s, \quad q_s^F, q_s^V \geq 0, \tag{11}$$

where $\hat{C}(q^I, L_s, W_s)$ is the optimal total cost excluding the IGs' cost under given q^I and scenario s.

The following theorem provides an explicit solution to the problem in (9)–(11).

Theorem 1. *For a given IG output $q^I \geq 0$, under a realized load L_s and VG potential output W_s, the optimal FG and VG production levels are*

$$\begin{aligned} q_s^{F*} &= (L_s - q^I - W_s)^+ \quad \textit{and} \\ q_s^{V*} &= \min\{W_s, (L_s - q^I + \mu(r))^+\}, \end{aligned} \tag{12}$$

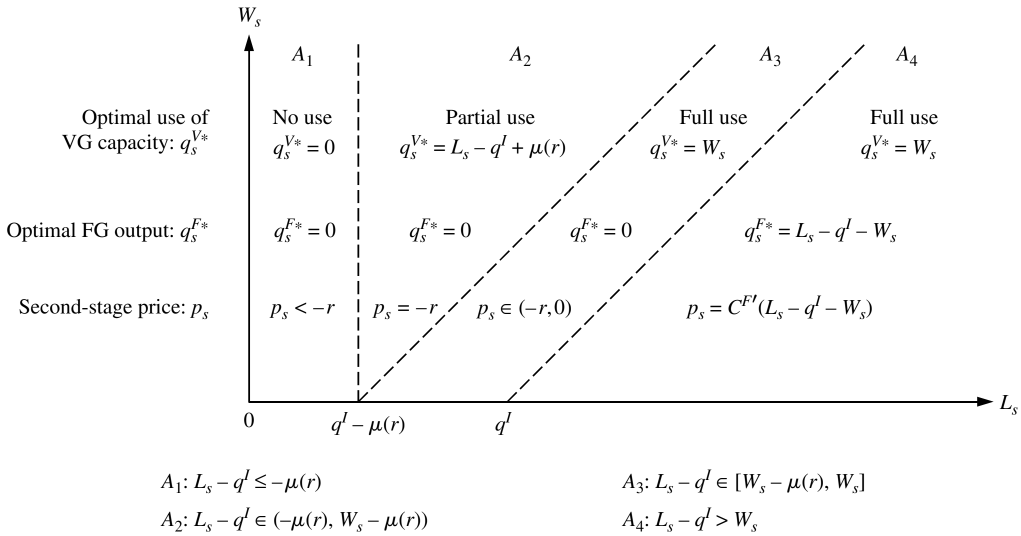
where $\mu(r) \stackrel{\text{def}}{=} (h')^{-1}(r) = \inf\{q \geq 0: h'(q) > r\}$. Furthermore, $\hat{C}(q^I, L_s, W_s)$ defined in (9) is jointly convex in (q^I, L_s, W_s).

The optimal solutions in (12) under various realized values of L_s and W_s are illustrated in Figure 1. If the load L_s is so low that the marginal oversupply penalty exceeds the per-unit VG subsidy, $h'(q^I - L_s) \geq r$ (i.e., $q^I - L_s \geq \mu(r)$ or event A_1), then any VG output will only increase the system's cost in (9), and thus VG output is completely curtailed. Otherwise, some or all of the VG potential output is used, corresponding to the next three cases.

In event A_2, VG output is partially curtailed such that the per-unit subsidy equals the marginal oversupply penalty, $r = h'(e_s)$ or $\mu(r) = e_s = q^I + q_s^{V*} - L_s$. In event A_3, the per-unit subsidy outweighs the marginal oversupply penalty when all VG potential output is used, $r \geq h'(q^I + W_s - L_s)$, and thus, no curtailment occurs. In event A_4, IGs and VGs cannot meet the entire load, and FGs serve the remaining load.

Figure 1 also shows the second-stage price, p_s, which is equal to the system's marginal cost, i.e.,

Figure 1. Second-Stage Operating Decisions and Prices



the cost of serving an additional unit of load. When the load exceeds the combined output of IGs and VGs (event A_4), the additional load is served by FGs, and thus the price is the FGs' marginal cost: $p_s = C^{F\prime}(L_s - W_s - q^I) > 0$. Using Lemma 1, we can also write $p_s = (S^F)^{-1}(L_s - W_s - q^I)$.

When the load can be met by IGs and VGs, the price becomes zero or negative:

(a) The price is zero when VG output is partially curtailed (event A_2 occurs) and no subsidy is provided ($r = 0$), because an additional unit of load can be served by VGs at zero cost.

(b) The price is negative when an additional load lowers the total stated cost by either reducing the oversupply penalty (in A_1 and A_3) or increasing VG output (in A_2 with $r > 0$). In A_1, no VG output is used, oversupply is $q^I - L_s$, and price is $p_s = -h'(q^I - L_s) < -r$. In A_2, VG is partially curtailed and $p_s = -r$. In A_3, no curtailment occurs and $p_s = -h'(q^I + W_s - L_s) \in (-r, 0)$.

In summary, we can express the second-stage price p_s as a function of q^I, L, and W, defined in (13) below. The dependence on supply function $S^F(\cdot)$ is also emphasized in (13):

$$\begin{aligned} &P(q^I, L, W, S^F) \\ &\quad \overset{\text{def}}{=} (S^F)^{-1}(L - W - q^I)\mathbf{1}_{A_4} - h'(q^I + W - L)\mathbf{1}_{A_3} \\ &\qquad - r\mathbf{1}_{A_2} - h'(q^I - L)\mathbf{1}_{A_1}, \end{aligned} \tag{13}$$

where $\mathbf{1}_{A_i}$ is the indicator function for event A_i. A more compact expression for the second-stage price is given in the following lemma.

Lemma 2. *The second-stage price in* (13) *can be expressed as*

$$\begin{aligned} &P(q^I, L, W, S^F) \\ &\quad = \inf\{p\colon q^I + S^F(p) + W\mathbf{1}_{\{p \ge -r\}} - \mu(-p) \ge L\}. \end{aligned} \tag{14}$$

Lemma 2 provides a supply function-based method for finding the second-stage price. On the left side of the inequality in (14), $S^F(p)$ is the FGs' supply function, $W\mathbf{1}_{\{p \ge -r\}}$ is the VGs' supply function (VGs offer the entire potential output whenever the price is at least $-r$), and the oversupply function $\mu(-p)$ gives the oversupply level when the price is $p < 0$. According to (14), the second-stage price is the minimum price such that the supply minus the oversupply meets the demand.

4.2. Optimal Inflexible Generation

After finding the minimum cost $\hat{C}(q^I, L_s, W_s)$ in (9) for each scenario $s \in \mathcal{S}$ and given IG output q^I, we can rewrite the system operator's first-stage problem in (5)–(8) as

$$\min_{q^I \ge 0} C^I(q^I) + \mathsf{E}[\hat{C}(q^I, L, W)]. \tag{15}$$

From Lemma 1 and Theorem 1, $C^I(q^I)$ and $\hat{C}(q^I, L_s, W_s)$ are convex in q^I, which implies that the objective function in (15) is convex in q^I.

The system operator minimizes the total cost in (15) by deciding an optimal IG production q^{I*}. If $q^{I*} > 0$, then the system operator announces a first-stage price $p_0 = (S^I)^{-1}(q^{I*})$, where $S^I(p)$ is IGs' aggregate supply function defined in (4). In response to the first-stage price p_0, IG $i \in G^I$ commits to producing $S_i(p_0)$ for the second stage, while FG $j \in G^F$ plans to produce $S_j(p_0)$, knowing that its actual output may deviate from the plan after uncertainties are realized in the second stage.

A natural question is the following: What is the relationship between the first-stage price p_0 and the second-stage price $P(q^I, L, W, S^F)$ defined in (13)? Theorem 2 provides a way to link these two prices.

Theorem 2. *An IG production, q^{I*} is optimal if and only if*

$$q^{I*} = S^I(\bar{P}(q^{I*}, S^F)), \tag{16}$$

where the average price function is defined as

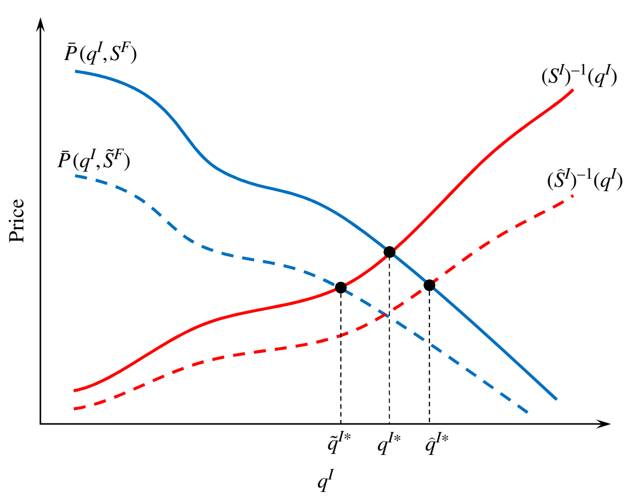
$$\bar{P}(q^I, S^F) \stackrel{\text{def}}{=} \mathsf{E}[P(q^I, L, W, S^F)]. \quad (17)$$

If $q^{I*} > 0$, then (16) can be equivalently written as $(S^I)^{-1}(q^{I*}) = \bar{P}(q^{I*}, S^F)$ or $p_0 = \mathsf{E}[p_s]$, which means that the first-stage price that incentivizes IGs to commit producing q^{I*} is equal to the expected second-stage price under q^{I*}. The practical meaning of this result is that the predispatch price (e.g., day-ahead price) is equal to the expected real-time price. This relation is not an assumption, but a result of the system operator's total cost minimization. Note that (16) holds even if $q^{I*} = 0$ (see the proof of Theorem 2), but the following discussions focus on the more realistic case of $q^{I*} > 0$.

The average price function $\bar{P}(q^I, S^F)$ defined in (17) is a function of the IG output q^I for given FG supply function. Thus, $\bar{P}(q^I, S^F)$ can be interpreted as IGs' inverse residual demand function. (Note that $\bar{P}(q^I, S^F)$ decreases in q^I because $P(q^I, L, W, S^F)$ in (13) decreases in q^I due to the monotonicity of $S^F(\cdot)$ and $h'(\cdot)$.) With this interpretation, Equation (16) means that q^{I*} is the intersection of the IGs' inverse supply function $(S^I)^{-1}(q^I)$ and the IGs' inverse residual demand function $\bar{P}(q^I, S^F)$. These two functions are depicted as the solid curves in Figure 2.

Figure 2 allows us to see the effect of supply functions on IG production q^{I*}. When IGs bid more competitively by increasing their supply function to $\hat{S}^I(p)$ or decreasing their inverse supply function to $(\hat{S}^I)^{-1}(q^I)$, the IG production q^{I*} rises to $\hat{q}^{I*}$; i.e., IGs' market share increases. When FGs bid more competitively by increasing their supply function to $\tilde{S}^F(p)$, the price decreases according to (14), and the average price decreases to $\bar{P}(q^I, \tilde{S}^F)$. Consequently, q^{I*} decreases to $\tilde{q}^{I*}$. In both cases, more competitive supply offers lead to a lower market price.

Figure 2. (Color online) Optimal IG Production q^{I*} and Supply Functions



Finally, we reiterate the payment scheme that resembles the two-settlement process in reality. Based on the first-stage price p_0, each generator $k \in G^I \cup G^F$ receives a payment of $S_k(p_0)p_0$. For IG $i \in G^I$, $S_i(p_0)p_0$ is the final payment because $S_i(p_0)$ is IG i's actual output. After the uncertainties are realized and the second-stage price p_s is announced, FG $j \in G^F$ produces $S_j(p_s)$, and the deviation $S_j(p_s) - S_j(p_0)$ is settled at price p_s. The total payment to FG j is $S_j(p_0)p_0 + (S_j(p_s) - S_j(p_0))p_s$.

With the knowledge of the system operator's dispatch decisions and the payment scheme, the IGs and FGs compete in a Nash game through supply functions, analyzed in the next section.

5. Supply Function Competition

This section analyzes the generator-level problem and the supply function equilibrium. A key feature of the generator-level problem is that the optimality condition (16) for the system operator's problem serves as a constraint in each generator's profit-maximization problem. Thus, our model shares similar features with mathematical programs with equilibrium constraints (MPECs; see, e.g., Hobbs et al. 2000), which typically involve nonconvex optimization problems.

In the classic SFE model, each generator optimizes its profit with respect to its residual demand function, and the SFEs are characterized by a system of differential equations (Klemperer and Meyer 1989). The differential equation approach is analytically challenging, especially when generators are asymmetric. Thus, SFEs with linear supply functions are considered in the classic works by Klemperer and Meyer (1989) and Rudkevich (1999), among others; see Baldick et al. (2004) for a summary of the advantages of linear SFEs. Green (1996, 1999) and Baldick et al. (2004) use linear SFEs to derive economic insights and policy implications. Khazaei et al. (2014) analyze a linear SFE under a stochastic settlement scheme where each generator incurs a cost for deviating from its predispatch quantity.

In the following analysis, we also focus on solving for linear SFEs. Solving for the general SFEs with asymmetric generators is difficult in the classic setting, and more difficult in our setting where generators differ not only in their costs but also in their flexibility.

5.1. Linear Supply Functions and the System's Optimal Decisions

From this point onward, we assume each generator's (true) production cost is quadratic in its output:

$$C_k^o(q) = \tfrac{1}{2}c_k q^2, \quad k \in G^I \cup C^F,\ c_k > 0,\ q \geq 0, \quad (18)$$

which implies a (true) linear marginal cost $(C_k^o)'(q) = c_k q$. Hence, in a perfectly competitive market, generator k would submit a supply function $S_k(p) = c_k^{-1}p^+$.

In an imperfect competition, we assume the generators submit

$$S_k(p) = \beta_k p^+, \quad k \in G^I \cup C^F, \beta_k > 0, p \in \mathfrak{R}; \tag{19}$$

that is, when the price is positive, the output that a generator is willing to produce is linear in price.

We assume a quadratic oversupply penalty function:

$$h(e) = a_h e + \tfrac{1}{2} c_h e^2, \quad a_h \geq 0, c_h > 0, e \geq 0, \tag{20}$$

which gives $\mu(r) = (h')^{-1}(r) = (r - a_h)^+ / c_h$.

The aggregate IG and FG supply functions are

$$S^I(p) = \beta^I p^+ \quad \text{and} \quad S^F(p) = \beta^F p^+,$$

where $\beta^I \overset{\text{def}}{=} \sum_{i \in G^I} \beta_i$ and $\beta^F \overset{\text{def}}{=} \sum_{j \in G^F} \beta_j$. Then, we can express the second-stage price functions in (13) and (17) as functions of the aggregate supply function slope β^F, written as follows:

$$\begin{aligned} P(q^I, L, W, \beta^F) = {} & \frac{1}{\beta^F}(L - W - q^I)\mathbf{1}_{A_4} \\ & - [a_h + c_h(q^I - L + W)]\mathbf{1}_{A_3} \\ & - r\mathbf{1}_{A_2} - [a_h + c_h(q^I - L)]\mathbf{1}_{A_1}, \end{aligned} \tag{21}$$

$$\bar{P}(q^I, \beta^F) = \mathsf{E}[P(q^I, L, W, \beta^F)]. \tag{22}$$

In most of the practical situations, the system operator instructs IGs to produce a positive output. Thus, we focus on the parameter settings where $q^{I*} > 0$. Then, Equation (16) that determines q^{I*} can be written as

$$q^{I*} = \beta^I \bar{P}(q^{I*}, \beta^F). \tag{23}$$

Thus, for given aggregate supply function slopes β^I and β^F, (23) uniquely determines q^{I*}. In other words, (23) defines an implicit function $q^{I*}(\beta^I, \beta^F)$.

5.2. Generators' Strategic Interactions

The generators' strategic variables are the supply function slopes β_k, $k \in G^I \cup G^F$. A larger β_k implies a more competitive supply offer. An upper bound for β_k is c_k^{-1}, which is the supply function slope under the perfect competition. In Section 5.3, we will show that β_k has a strictly positive lower bound, denoted as $\beta_k^{\min} > 0$. Thus, generator k's (pure) strategy space is $[\beta_k^{\min}, c_k^{-1}]$.

To understand the generators' strategic interactions, it is useful to first consider a special case where IGs and VGs do not exist and FGs compete by choosing supply function slopes β_j, $j \in G^F$. Because there is no inflexible output or variable generation, the second-stage price is simply $p_s = L_s/\beta^F$. Assuming the first-stage price is $p_0 = \mathsf{E}[p_s]$ (see the discussion after Theorem 2), we can write generator j's expected profit function as

$$\begin{aligned} \pi_j &= \beta_j p_0^2 + \mathsf{E}\big[\beta_j(p_s - p_0)p_s - \tfrac{1}{2}c_j(\beta_j p_s)^2\big] \\ &= \mathsf{E}\big[\beta_j p_s^2 - \tfrac{1}{2}c_j(\beta_j p_s)^2\big] \\ &= \underbrace{\frac{\beta_j(1 - c_j\beta_j/2)}{(\beta^F)^2}}_{\substack{\text{Competition}\\ \text{factor}}} \underbrace{\mathsf{E}[L^2]}_{\substack{\text{Market size}\\ \text{factor}}}. \end{aligned} \tag{24}$$

In (24), generator j's expected profit consists of two factors: a competition factor capturing the interactions between generator j and the other FGs and a market size factor that is the second moment of the load served by FGs. It can be shown that the best response of generator j satisfies $\beta_j = (1 - c_j\beta_j)(\beta^F - \beta_j)$, which is consistent with Green (1996) and Rudkevich (1999).

Next, we consider the competition between FGs and IGs. For notational convenience, we let $\lambda(j, G^F)$ represent the competition factor in (24) and define $\lambda(i, G^I)$ in a similar form:

$$\lambda(j, G^F) \overset{\text{def}}{=} \frac{\beta_j(1 - c_j\beta_j/2)}{(\beta^F)^2}, \quad \lambda(i, G^I) \overset{\text{def}}{=} \frac{\beta_i(1 - c_i\beta_i/2)}{(\beta^I)^2}.$$

Theorem 3. *In the linear supply function competition between FGs and IGs, the expected profits of generators are*

$$\pi_i = \lambda(i, G^I)(q^{I*}(\beta^I, \beta^F))^2, \quad i \in G^I, \tag{25}$$

$$\pi_j = \lambda(j, G^F)\mathsf{E}[((L - W - q^{I*}(\beta^I, \beta^F))^+)^2], \quad j \in G^F. \tag{26}$$

The IGs and FGs compete only through the market size factor. The IGs' market size $q^{I*}(\beta^I, \beta^F)$ is strictly increasing in β^I and strictly decreasing in β^F.

The above result characterizes the nature of the competition when the generators are asymmetric in their flexibility. Two forms of competition are compounded:

- *Between-group competition*. The two groups of generators, IGs and FGs, compete only through the market size factor. The IGs' market size $q^{I*}(\beta^I, \beta^F)$ is determined by (23), and the FGs' market size $(L - W - q^{I*}(\beta^I, \beta^F))^+$ is determined by Theorem 1. The monotonicity of $q^{I*}(\beta^I, \beta^F)$ stated in Theorem 3 qualitatively characterizes the market size competition.
- *Within-group competition*. Within each group, the generators compete in the standard manner captured by the competition factors $\lambda(i, G^I)$ and $\lambda(j, G^F)$.

Theorem 3 also suggests that the variabilities in load and VG potential output play no direct role in IGs' profit function (25). Thus, IGs do not directly compete with FGs in meeting the variable demand. On the other hand, the variabilities in L and W directly affect FGs' profit function in (26). Hence, FGs compete among themselves to meet the variable demand.

The between-group competition also renders the generators' best responses dependent on the distribution of the uncertainties. The distributions of L and W affect the optimal choice of β_j, $j \in G^F$, in (26), which in turn affects the decisions of all other generators. (Without the between-group competition, $q^{I*}(\beta^I, \beta^F)$ would be constant, and the distributions of L and W would not affect FG j's best response.) This feature contrasts with the classic SFE model, in which supply function equilibria are found to be independent of the demand distribution; see, e.g., Klemperer and Meyer (1989), Green (1996), Holmberg (2007), and Anderson and Hu (2008).

It is worth analyzing a special case where the second-stage price is always positive. In this special case, we can express the market size $q^{I*}(\beta^I, \beta^F)$ in explicit form and substitute it into (25)–(26), allowing us to further investigate the composite effect of the two forms of competition.

Corollary 1. *Let $D \stackrel{\text{def}}{=} L - W$ and suppose $D > q^{I*}$ with probability one. Then, the generators' expected profits can be expressed as*

$$\pi_i = \lambda(i, G^I \cup G^F)(\mathsf{E}[D])^2, \quad i \in G^I, \tag{27}$$

$$\pi_j = \lambda(j, G^I \cup G^F)(\mathsf{E}[D])^2 + \lambda(j, G^F)\,\mathsf{Var}[D], \quad j \in G^F, \tag{28}$$

where

$$\lambda(k, G^I \cup G^F) = \frac{\beta_k(1 - c_k\beta_k/2)}{(\beta^I + \beta^F)^2}, \quad k \in G^I \cup G^F.$$

Corollary 1 reveals that if the net demand $D = L - W$ exceeds q^{I*} almost surely, the composite effect of the between- and within-group competitions is that all generators compete to serve the expected net demand, while FGs compete among themselves to meet the net demand variability.

If we ignored inflexibility and treated all IGs as if they were FGs, then (24) implies that the expected profit of generator k would be

$$\pi_k = \lambda(k, G^I \cup G^F)\,\mathsf{E}[D^2],$$

which can be written as

$$\pi_k = \lambda(k, G^I \cup G^F)(\mathsf{E}[D])^2 + \lambda(k, G^I \cup G^F)\,\mathsf{Var}[D], \quad k \in G^I \cup G^F. \tag{29}$$

Comparing (27) with (29) suggests that IGs are unable to profit from the variabilities. Comparing (28) with (29) confirms that FGs compete among themselves to meet the net demand variability: the competition factor associated with $\mathsf{Var}[D]$ is $\lambda(j, G^F)$ in (28), while it is $\lambda(j, G^I \cup G^F)$ in (29).

5.3. Existence of Equilibrium Under Normally Distributed Uncertainties

Proving the existence and uniqueness of the equilibrium for the game specified in Section 5.2 presents analytical challenges. In general, the existence of a pure-strategy Nash equilibrium can be established by proving one of the following: (a) the best response functions constitute a contraction mapping and the decision space is compact; (b) each player's payoff function is quasiconcave in its own decision, and the decision space is compact, convex, and independent of other players' decisions; and (c) each player's payoff function is supermodular with respect to its own decision and other players' decisions, and the decision space is a lattice.

As shown in Theorem 3, the IGs' and FGs' payoff functions (25) and (26) depend on the system operator's decision on q^{I*} in (23). Thus, the generator-level optimization problem belongs to the class of MPEC problems that are highly complex. However, we are able to use approach (b) to prove the existence under the assumption that the load and VG potential output are jointly normally distributed. Due to the aforementioned complexity, we are unable to prove the uniqueness of the equilibrium, but our extensive numerical results show that the equilibrium is unique, which we will discuss in Section 6.

The joint normal distribution is denoted as $L \sim \mathcal{N}(\mu_L, \sigma_L^2)$ and $W \sim \mathcal{N}(\mu_W, \sigma_W^2)$, with a correlation coefficient ρ. Then, the net demand $D = L - W \sim \mathcal{N}(\mu_D, \sigma_D^2)$, where $\mu_D = \mu_L - \mu_W$ and $\sigma_D^2 = \sigma_L^2 + \sigma_W^2 + 2\rho\sigma_L\sigma_W$. Although wind power generation at an individual location may not be normally distributed, the aggregate generation from a large number of geographically dispersed wind power turbines is approximately normal due to the central limit theorem (see similar assumptions in Ortega-Vazquez and Kirschen 2009). As is common with models using normal distributions to approximate nonnegative random variables, the results in this section are proven when the variance of the normal distribution is not too large.

Proving quasiconcavity of the profit functions (25) and (26) under general conditions is difficult due to the complicated structure of the price function in (13), which renders the average price in (17) neither convex nor concave in q^I. However, if the probability of $q^I < L - W$ (event A_4 in Figure 1) is sufficiently high, then the average price function is approximately linear, which bounds its second-order derivative with respect to q^I and leads to the quasiconcavity of the profit functions. The formal proof requires a lemma, stated below.

A lower bound for β_k can be obtained by solving a less competitive game with only IGs or only FGs, which is essentially the classic supply function game in Klemperer and Meyer (1989). In that setting, Rudkevich (1999) studies the linear SFEs for these games and shows that the slopes of the equilibrium supply functions are strictly positive and independent of the demand distribution. Hence, β_k has a strictly positive lower bound, denoted as $\beta_k^{\min} > 0$, which is independent of the distribution of the uncertainties. Consequently, the slopes of the aggregate supply functions are bounded by

$$\beta^{I\min} = \sum_{i\in G^I} \beta_i^{\min}, \quad \beta^{I\max} = \sum_{i\in G^I} c_i^{-1},$$

$$\beta^{F\min} = \sum_{j\in G^F} \beta_j^{\min}, \quad \beta^{F\max} = \sum_{j\in G^F} c_j^{-1}.$$

Using Theorem 3, q^I is bounded above by $q^{I\max} = q^{I*}(\beta^{I\max}, \beta^{F\min})$.

Lemma 3. *If*

$$\sigma_D \le \sigma_D^* \equiv \sqrt{2\pi}\beta^{F\min}\left[\frac{\mu_D}{\beta^{I\max}} + \frac{\min\{r, a_h\}}{2}\right],$$

then $q^{I\max} < \mu_D$.

Lemma 3 shows that for a sufficiently small σ_D, the IG production is bounded above by μ_D. Indeed, the IGs' aggregate output does not exceed the average net demand μ_D for most situations in practice. The condition given in Lemma 3 is not stringent. For example, if $a_h = 0$ and $\beta^{F\min}$ is one-tenth of $\beta^{I\max}$ (which, according to our numerical tests, is on the conservative end), then $\sigma_D^* = \sqrt{2\pi}\mu_D\beta^{F\min}/\beta^{I\max} \approx 0.25\mu_D$. Thus, the condition holds if the standard deviation of the net demand is within 25% of its mean, which is a mild assumption in most practical situations. Lemma 3 leads to the following equilibrium existence theorem.

Theorem 4. *When generators compete using linear supply functions and the standard deviation of the net demand σ_D is sufficiently small, there exists a (pure strategy) supply function equilibrium.*

In Theorem 4, the upper bound on σ_D that ensures the existence of a linear SFE is provided in the proof in the online appendix. Our numerical experiments further demonstrate that the equilibrium exists for a wider range of σ_D and for other forms of load and VG output distributions.

6. Numerical Analysis

In this section, we first compute SFE based on the model analyzed in Sections 4–5 and compare our results with the classic SFE model by Klemperer and Meyer (1989) and Green (1996). We then analyze the effects of increasing VG penetration, VG dispatch policy, and the production-based subsidies on SFE. Our analysis aims to derive qualitative insights and provide policy recommendations.

6.1. Setups and Computational Procedure

We consider a market consisting of four IGs indexed by $i \in G^I = \{1,2,3,4\}$ and four FGs indexed by $j \in G^F = \{5, 6, 7, 8\}$. Their production cost functions are given in (18): $C_k^o(q) = \frac{1}{2}c_k q^2$, where q is measured in megawatt hours (MWh) and c_k is measured in \$/(MWh)^2. To facilitate the comparison between IGs' and FGs' equilibrium behavior, we keep generators identical within each generator type. We assume the cost coefficients are $c_i = 1/3$ for IG $i \in G^I$ and $c_j = 2/3$ for FG $j \in G^F$. The system's oversupply penalty is assumed to be $h(e) = \frac{1}{2}c_h e^2$, with $c_h = 1$. We have examined other cost parameters with $c_i < c_j$, where $c_i, c_j \in \{1/6, 1/4, 1/3, 1/2, 2/3, 1, 2\}$, and $c_h \in \{1/3, 1/2, 1, 2, 5, 8\}$. We find that the qualitative results described in this section are robust across all problem instances we examined.

We assume the load and VG potential output follow independent normal distributions, with $\mu_L = 1{,}200$, $\sigma_L = 180$, $\mu_W = 60$, and $\sigma_W = 20$, measured in MWh. The VG penetration level is $\mu_W/\mu_L = 5\%$, close to the current VG penetration in the United States. In addition to this base case, we also consider various VG penetration levels. Following Wu and Kapuscinski (2013), when VG penetration increases by m times (μ_W increases to $m\mu_W$), the standard deviation σ_W increases to $m\sigma_W$ if the existing and added VG outputs are perfectly correlated, or to $\sqrt{m}\,\sigma_W$ if they are independent. The realistic case is likely in between, and we assume that σ_W increases to $m^{0.75}\sigma_W$. Specifically, we consider five VG penetration levels: 0%, 5%, 15%, 30%, and 50%; that is, $m = 0, 1, 3, 6, 10$.

We consider the following policies for VGs: priority dispatch for VG (no curtailment), full economic curtailment for VG (when subsidy $r = 0$), and partial economic curtailment for VG (when subsidy $r = 20$ or 40 \$/MWh).

The generators submit supply functions $S_k(p) = \beta_k p^+$, as in (19). The following iterative procedure is used to compute the equilibrium supply function slopes:

Step 1. Set $n = 0$ and set an initial slope $\beta_k^0 \in [\beta_k^{\min}, c_k^{-1}]$ for every generator $k \in G^I \cup G^F$.

Step 2. Increase n by 1. For every generator $k \in G^I \cup G^F$, find $\beta_k^n \in [\beta_k^{\min}, c_k^{-1}]$ that maximizes generator k's expected profit, assuming that none of the other generators modify their supply functions (i.e., use β_l^{n-1} for generator $l \ne k$).

Step 3. If $\max_{k\in G^I\cup G^F}\{|\beta_k^n - \beta_k^{n-1}|/\beta_k^{n-1}\} < \varepsilon$, then terminate the procedure, and the equilibrium supply function slopes are $\{\beta_k^n\}$; otherwise, go to Step 2.

In Step 2, we numerically find that each generator's objective function is quasiconcave. In Step 3, we use $\varepsilon = 10^{-7}$, and the above procedure typically takes six to nine iterations to converge.

To numerically examine the uniqueness of the equilibrium, we initiate the procedure with many different starting points in Step 1 and find that the procedure always converges to the same equilibrium. Furthermore, when all generators are assumed to be flexible, the procedure produces exactly the same results as the classic SFE model with linear supply functions.

6.2. FG–IG Equilibrium vs. Klemperer–Meyer Equilibrium

The supply function slope β_k is useful for the theoretical analysis in Section 5, but for the purpose of describing the insights from our numerical results, it is more intuitive to use *price offer slope* $\gamma_k \overset{\text{def}}{=} 1/\beta_k \in [c_k, 1/\beta_k^{\min}]$. A lower value of γ_k means a more competitive price offer. Generator k's marginal cost is $c_k q$, and the price markup is $(\gamma_k - c_k)q$.

As our model extends the classic SFE model to include the flexibility asymmetry, we first compare the SFE in our model (referred to as FG–IG equilibrium) with the SFE by Klemperer and Meyer (1989) (referred to as KM equilibrium), focusing on the SFE with linear supply functions.

The KM model ignores the generators' inflexibility and treats generators 1–4 as if they were flexible to find the equilibrium supply functions. Following Green (1996) and Rudkevich (1999), we solve the KM equilibrium with linear supply functions under the cost functions specified in Section 6.1. We find that in the KM model, the price offer slopes of generators 1–4 are 0.412 $\$/(\text{MWh})^2$ and the price offer slopes of generators 5–8 are 0.739 $\$/(\text{MWh})^2$, shown in Figure 3(a).

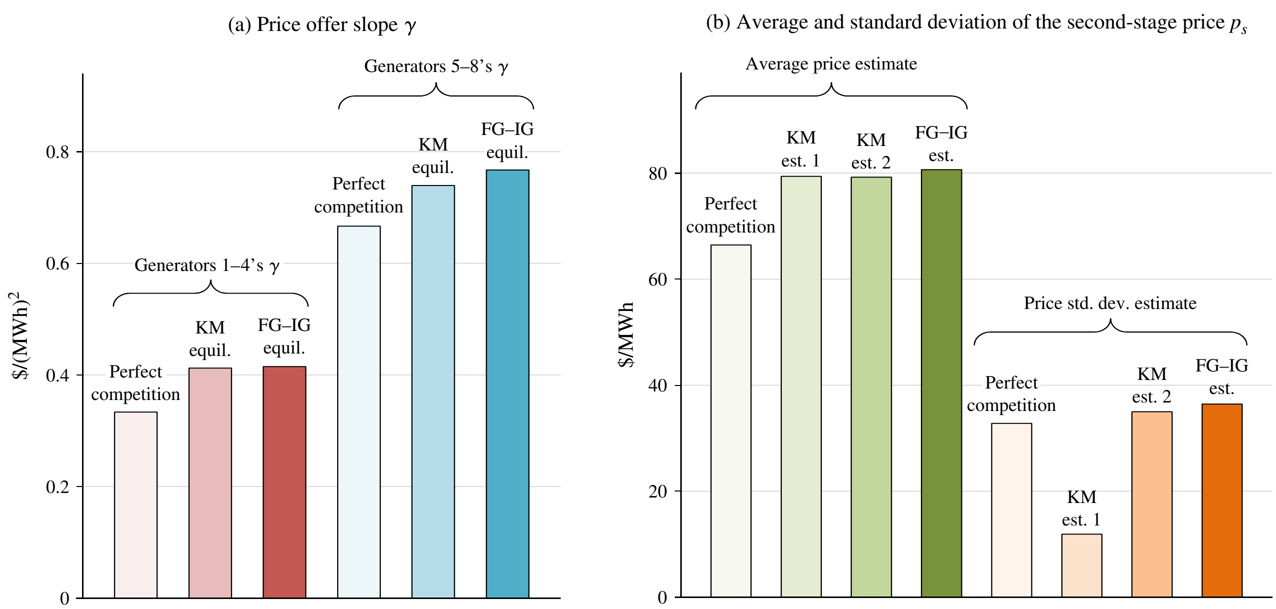
Using the iterative procedure in Section 6.1, we compute the FG–IG equilibrium without VGs in the system. Because our model recognizes the inflexibility of generators 1–4, we had expected IGs 1–4 to behave more differently than FGs 5–8 compared to the KM equilibrium. However, in the FG–IG equilibrium, each IG's price offer slope is 0.415 $\$/(\text{MWh})^2$, which is only 0.7% higher than in the KM model, whereas each FG's price offer slope is 0.767 $\$/(\text{MWh})^2$, 3.8% higher than in the KM model. In terms of the price markup $(\gamma_k - c_k)q$, FGs' markups are 38% higher than their markups in the KM model; IGs' markups are only 3.6% higher than in the KM model. These price offer slopes are shown in Figure 3(a).

The reasons underlying the difference between the FG–IG and KM equilibria stem from the reduced competition due to inflexibility. All eight generators are treated as flexible in the KM model, but only four of them are actually FGs. IGs do not compete with FGs in matching production with the variable demand; FGs compete among themselves to serve the variable demand (see the theory and discussions in Section 5.2). Thus, the competition facing an FG in our model is less intense than that in the KM model, allowing FGs to mark up their prices significantly above what the KM model predicts. On the other hand, IGs' price offers in our equilibrium are only slightly higher than in the KM model, because every IG faces direct competition from all other generators. In particular, IGs still compete with FGs for the market share (see Theorem 3). These findings suggest that the KM model underestimates generators' price offers, more significantly so for FGs.

Next, we compare how the two models differ in estimating the second-stage (real-time) electricity price. The estimation involves two steps: first, estimate the equilibrium supply functions using an SFE model; second, compute the price statistics under the estimated supply functions. A forecaster who uses the classic SFE model in the first step may or may not consider the inflexibility of IGs in the second step. We refer to the estimates without recognizing inflexibility in either step as "KM est. 1" and the estimates with inflexibility consideration in only the second step as "KM est. 2." Our FG–IG equilibrium model recognizes inflexibility in both steps. As a benchmark, we also estimate the price under the perfect competition with inflexibility consideration in the second step.

Figure 3(b) shows that the mean and standard deviation of the price estimated by the KM model (KM est. 1 and est. 2) are lower than those estimated by the FG–IG model. KM est. 1 for the price standard deviation is considerably lower, because ignoring inflexibility in the second step leads to an incorrect assumption that all generators can mitigate load variability.

Figure 3. (Color online) Klemperer–Meyer Equilibrium vs. FG–IG Equilibrium Without Variable Generation



KM est. 2 for the average price is lower than that in the FG–IG model because the KM model underestimates the equilibrium price offers for both IGs and FGs. KM est. 2 also underestimates the price variability for two reasons. First, the KM model underestimates FGs' price offer slopes, and it thus underestimates the magnitude of price fluctuations when the price is positive. Second, because the KM model underestimates the price offer slopes more for FGs than for IGs, it underestimates IGs' market share and, it thus underestimates the magnitude of oversupply and the resulting negative prices.

In view of both the generators' price offers and the equilibrium price, the KM model overestimates the intensity of the competition in a market with inflexible generators. This result is robust across all cost parameters we have examined.

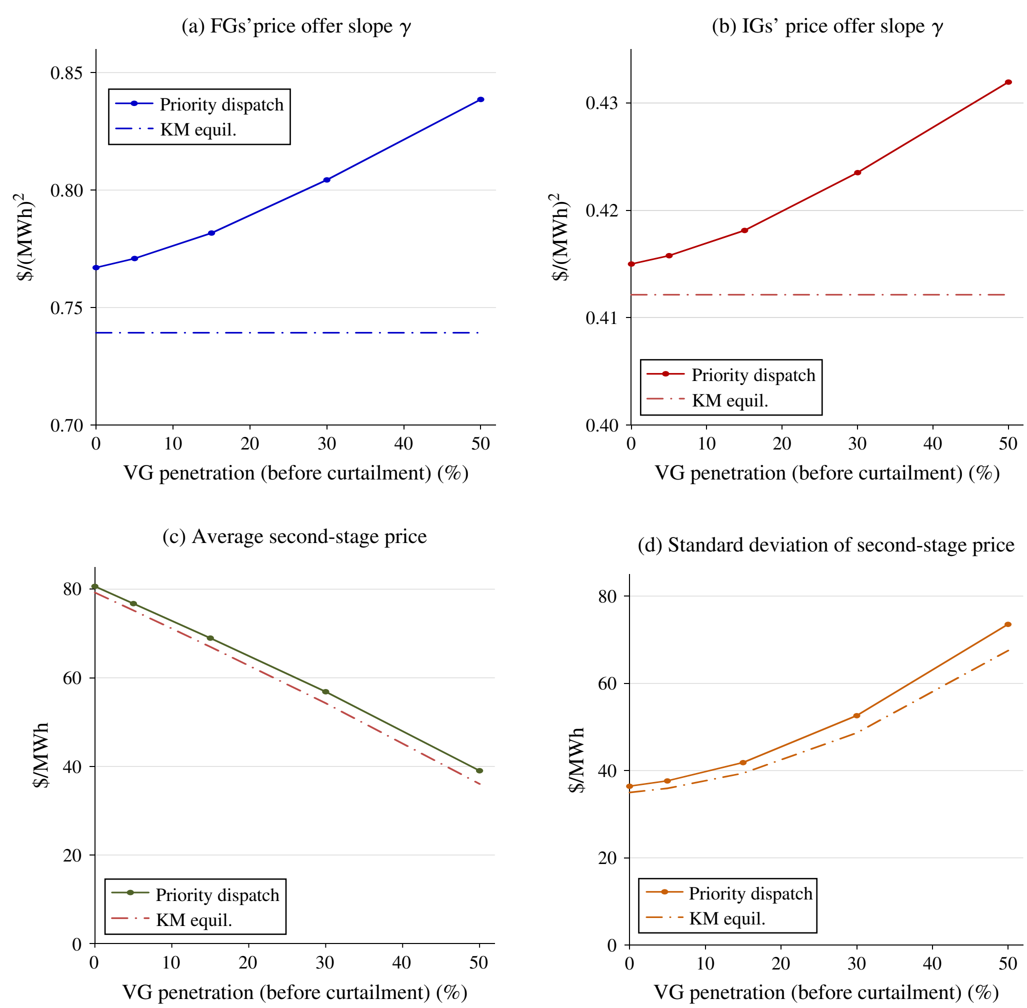
6.3. Impact of Variable Generation on SFE Under Priority Dispatch Policy

In the KM model, the equilibrium linear supply functions are independent of the distribution of the uncertainties and, thus, variable generation has no impact on the price offers in the KM equilibrium. In Figure 4(a) and 4(b), the price offer slopes in the KM equilibrium are invariant to the VG penetration levels.

In the FG–IG equilibrium, when the VG penetration increases, the overall variability facing the system increases. Under the priority dispatch policy for VGs, the only lever for balancing against the increased variability is adjusting FGs' output. Increased supply variability also increases the chance of oversupply, which makes a lower IG output more desirable for the system. Both of these reasons give FGs an advantage in the market-share competition with IGs. To profit from this advantage, FGs raise their price offers as the VG penetration increases, which is confirmed in Figure 4(a).

On the other hand, as the VG output increases, IGs face a price–quantity trade-off: They can either increase price offers to raise the equilibrium price but get a smaller market share, or lower their price offers to gain more market share. Because a low IG output is desirable for the system to mitigate the oversupply penalty when VGs have priority, IGs' strategy of lowering price offers may not lead to a market share

Figure 4. (Color online) Effects of Variable Generation on the Equilibrium Under Priority Dispatch



increase that is sufficient to raise their profits. Thus, IGs prefer raising price offers, as confirmed by Figure 4(b). Hence, under the priority dispatch policy for VGs, both IGs and FGs raise their price offers as the VG penetration increases, and the KM model increasingly underestimates generators' price offers and overestimates the intensity of the market competition.

Although inflated price offers tend to raise the market price, the increased VG penetration reduces the average net demand and tends to reduce the price. The equilibrium price is a result of the combination of these two effects. The second effect dominates in determining the average price, as shown in Figure 4(c): the average price declines as the VG penetration increases.

Under the priority dispatch policy, increasing VG penetration makes the real-time price more volatile, as revealed in Figure 4(d). The reasons are twofold. At a high VG penetration level, the VG output can still occasionally drop to a low level, requiring FGs to ramp up production, which escalates the real-time price due to FGs' increased price offers. When the VG output surges, however, the system has to use all the VG output under the priority dispatch policy, resulting in possibly very negative real-time prices at high VG penetration levels.

We have examined the above results under various cost parameters. The qualitative trends discussed in this section are robust.

6.4. Impact of Economic Curtailment Policy and Production-Based Subsidies

The analysis in Section 6.3 assumes VGs have priority. In this section, we consider the economic curtailment policy. We focus on the full economic curtailment case under zero subsidy ($r = 0$) and compare it with the partial economic curtailment cases under subsidies $r = 20$ and 40 $/MWh.

The FG–IG equilibrium price offer slopes under economic curtailment are shown in Figure 5(a) and 5(b). The economic curtailment policy encourages both IGs and FGs to offer more competitive prices compared to the priority dispatch policy. As the VG penetration increases, Figure 5(a) shows that the FGs' price offer slopes increase slower than under the priority dispatch policy, while Figure 5(b) reveals that the IGs' price offer slopes decline (when $r = 0$) and may even drop below the price offer slope predicted by the KM model.

The economic curtailment policy increases the market competition in two ways. First, economic curtailment provides the system operator with an additional lever to manage uncertainty, and thus the system operator allocates less production to FGs than under the priority dispatch policy. As a result, FGs offer more competitive prices to compete for the market share. Second, economic curtailment significantly reduces the oversupply penalty, thereby altering the price–quantity trade-off facing IGs (this trade-off is described in Section 6.3). Consequently, IGs' strategy of lowering price offers can yield a market share increase that is sufficient to increase IGs' profits. These two effects of the economic curtailment policy reinforce each other in equilibrium, because IGs reduce their price offers in response to FGs' reduced price offers and vice versa.

Figure 5 also shows the effect of production-based subsidies. Subsidies effectively grant priority to VGs to some extent and thus lead to less competitive price offers. The price offer slopes in the cases of $r = 20$ and 40 $/MWh lie in between the price offer slopes in the priority dispatch and full economic curtailment cases.

For all the problem instances we examined, under the economic curtailment policy without the subsidy ($r = 0$), we find that as VG penetration increases, FGs either raise their price offers or first reduce their price offers and then raise them, whereas IGs always reduce their price offers. Under $r = 20$ or 40 $/MWh, IGs and FGs may increase or decrease their price offers, but their offers always lie between the priority dispatch and full economic curtailment cases.

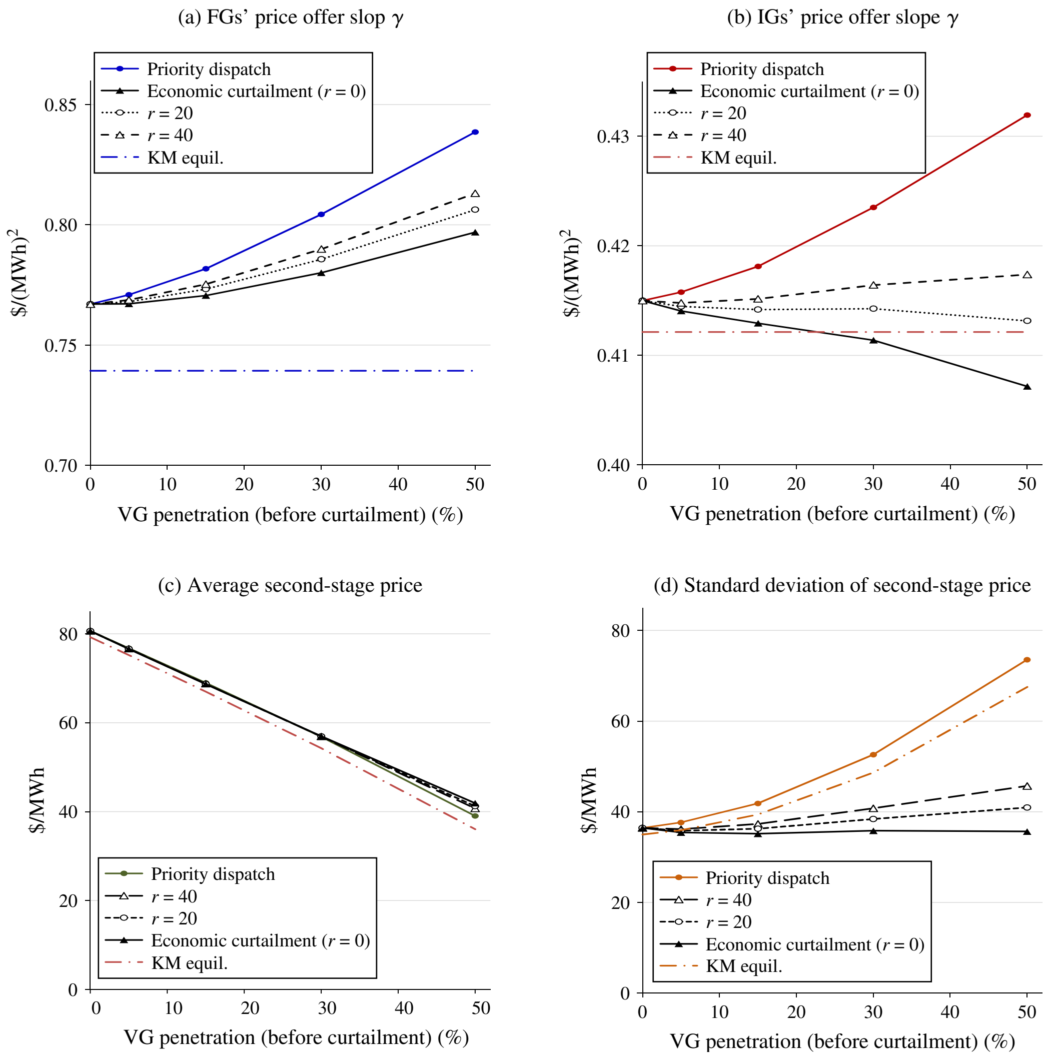
Economic curtailment has little effect on the average price, but its impact on the price variability is significant. Figure 5(c) shows that the average prices under various VG policies are indistinguishable for low and medium VG penetration levels. At high VG penetration levels, the average price under economic curtailment is slightly higher because the curtailment reduces the severity of the negative prices. In contrast, Figure 5(d) reveals that the real-time price variability under the economic curtailment policy is considerably lower than under the priority dispatch policy, because economic curtailment reduces extreme prices by making the market more competitive when the price is high and reducing the oversupply penalty when the price is negative.

6.5. Effects of Economic Curtailment and Subsidies on Efficiency and Emission

Finally, we study how the system efficiency and emissions change with VG policies. The system efficiency is measured by the average electricity generation cost per MWh, i.e., the total operating costs (including the true production costs and the oversupply penalty) divided by the total load. Table 1 lists the average cost per MWh and the cost saving per MWh of economic curtailment.

Not surprisingly, the average cost per MWh declines significantly as more free energy from VGs is used. Importantly, the benefit of the economic curtailment policy over the priority dispatch policy grows with the VG penetration. At 5% VG penetration, economic curtailment reduces the system operating cost by only 0.1% (from $32.44 to $32.40 per MWh), whereas at 30% VG penetration, the cost reduction is 2% (from $19.31 to $18.93 per MWh), and at 50% VG, the cost reduction grows to 10.7%. Interestingly, at all VG penetration

Figure 5. (Color online) Effects of Economic Curtailment and Subsidies on the Equilibrium



levels, 1 MWh of economic curtailment consistently reduces the total operating costs by about \$74.

Table 1 also reveals that the subsidies for VG reduce the amount of curtailment, but increase the cost of electricity. Interestingly, a higher subsidy also means more cost saving per MWh of economic curtailment. For example, with 5% VG and $r = \$20/\text{MWh}$, 1 MWh of curtailment reduces the system operating cost by \$92; with $r = \$40/\text{MWh}$, the cost saving per MWh of curtailment increases to \$110. This result is again consistent across all VG penetration levels. The implication is that the benefit of economic curtailment may be very high in countries and regions where intermittent renewable generation is heavily subsidized based on the amount of renewable energy generated.

Using the output of VGs can reduce CO_2 emission due to the displacement of the conventional energy by the renewable energy. Table 1 confirms that the average amount of CO_2 emission per MWh significantly decreases as the VG penetration increases.

The impact of economic curtailment on CO_2 emission, however, is not as obvious and depends on the generators' fuel types. Because economic curtailment increases IG output and reduces FG output, if IGs have a higher (lower) CO_2 emission rate than FGs, economic curtailment may increase (decrease) total CO_2 emission. Table 1 demonstrates that when IGs are coal-fired generators and FGs are natural gas combustion turbines, economic curtailment increases CO_2 emission, but when IGs are nuclear power generators, economic curtailment reduces the emission.

7. Conclusion

Electricity markets have been gradually evolving toward deregulated structures that intend to encourage competition and improve efficiency. The research in deregulated electricity markets, especially the supply function competition, has provided considerable insights into generators' bidding behavior and

Table 1. Effects of Economic Curtailment and Subsidies on Cost and CO_2 Emission

Metrics	VG dispatch policy and subsidies	VG penetration 5%	15%	30%	50%
VG penetration	$r = 40$	4.98%	14.91%	29.71%	48.96%
(after economic curtailment)	$r = 20$	4.97%	14.88%	29.61%	48.64%
	$r = 0$	4.96%	14.84%	29.48%	48.18%
Average electricity generation cost	Priority dispatch	32.44	26.60	19.31	12.51
($/MWh)	$r = 40$	32.41	26.51	18.98	11.32
	$r = 20$	32.41	26.49	18.95	11.22
	$r = 0$	32.40	26.49	18.93	11.17
System cost saving per MWh	$r = 40$	110.3	110.3	112.2	114.3
of economic curtailment ($/MWh)	$r = 20$	92.1	91.9	93.3	94.3
	$r = 0$	73.9	73.4	74.1	73.4
CO_2 emission with	Priority dispatch	917.0	740.5	509.7	268.5
coal-fired IGs (kg/MWh)	$r = 40$	918.0	743.9	518.4	285.1
	$r = 20$	918.5	745.1	521.4	290.9
	$r = 0$	919.0	746.8	525.5	299.6
CO_2 emission with	Priority dispatch	178.8	151.2	119.2	91.6
nuclear IGs (kg/MWh)	$r = 40$	178.1	149.2	113.8	78.8
	$r = 20$	177.9	148.5	112.1	75.2
	$r = 0$	177.6	147.5	109.7	70.2

Notes. CO_2 emission factor: 97.5 kg/mmBtu of coal; 53.1 kg/mmBtu of natural gas; no emission for nuclear power generators. Fuel price: $2.5/mmBtu of coal; $4/mmBtu of natural gas.

market power. This paper provides new results that address how the competition is affected by generators' (in)flexibility and intermittent renewable generation. We find that inflexibility contributes to the market power and that the economic curtailment of renewable generation increases the market competition and the system efficiency, but the production-based subsidies reduce the benefits of economic curtailment.

Inflexibility contributes to the market power in the following way. Inflexible generators do not directly compete with flexible generators in matching production with uncertain demand, leading to increased market power of flexible generators, which in turn results in higher average price and price volatility than predicted by the classic SFE model.

Variable generation, when given priority in dispatch, exacerbates the effect of inflexibility on market competition. As the rising VG penetration increases the overall variability facing the system, if VGs have priority, the system operator has to balance the increased variability by using FGs rather than IGs, which allows FGs to have an advantage in the market-share competition with IGs. To profit from this advantage, FGs reduce their supply functions, i.e., lower output at every price.

Economic curtailment of VG provides the system operator with an additional lever to balance against variability and serves as a partial substitute for FGs. Thus, economic curtailment intensifies the market competition: IGs and FGs offer more competitive supply functions than if VGs have priority. Economic curtailment has only small impact on the average price, but substantially reduces the price volatility. Furthermore, economic curtailment improves the system efficiency by reducing the oversupply penalty and increasing the use of low-cost inflexible generation. However, emissions may increase or decrease depending on the generators' fuel types.

The production-based subsidies increase the priority of variable generation and reduce the amount of curtailment. Thus, in the presence of the subsidies, the economic curtailment policy cannot achieve its full benefit of encouraging market competition and improving system efficiency.

The insights from this paper also provide several recommendations for regulators and policy makers. First, in assessing the competitiveness of the electricity market, it is important to incorporate generators' flexibility/inflexibility. Flexible generators compete in balancing against variability and often set the market price. Encouraging the development of more flexible generators (e.g., fueled by natural gas) enhances the overall competitiveness of the electricity market. Second, in assessing the benefit of the economic curtailment policy, it is important to recognize that economic curtailment helps increase market competition and reduce price volatility. Policy makers need to revisit the policy of giving priority to intermittent renewable generation and consider a full range of benefits of economic curtailment. Other benefits of economic curtailment include reduced cycling cost and peaking cost (Wu and Kapuscinski 2013) and improved economic dispatch in a network (Ela 2009). Third, policy makers need to reconsider the design of incentives aimed to promote renewable energy. The design of subsidies should facilitate economic curtailment and avoid unintended consequences. Investment in research and development can push technology advancement that makes renewable energy generation more competitive in the future even without subsidies.

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