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Inventory Management in Humanitarian Operations: Impact of Amount, Schedule, and Uncertainty in Funding

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Funding for humanitarian operations in the global health sector is highly variable and unpredictable. We study the problem of managing inventory in the presence of funding constraints over a finite planning period. Our goal is to determine the optimal procurement policy given the complexities associated with funding and also to analyze the impact of funding amount, funding schedule, and uncertainty around the funding timing on operations. We use a multiperiod stochastic inventory model with financial constraints and demonstrate that despite the funding complexities, the optimal replenishment policy is a state-independent policy that can be easily implemented. We also provide analytical results and several insights based on our computational study regarding the effect of funding timing uncertainty and variability on the operating costs and fill rates. Among other results, we find that receiving funding early is beneficial in underfinanced systems while avoiding funding delays is critical in fully financed systems. Our analysis also indicates that receiving less overall funding in a timely manner might actually be better than delayed full funding.

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1. Introduction

Financial flows play an important role in humanitarian operations and impact their scope, effectiveness, and efficiency. The total amount of donations received can impact the efficacy of such operations, but the timing, predictability, and flexibility of usage around those funds also have a strong influence (Wakolbinger and Toyasaki 2011). In a global health context, unpredictability and delays in donor funding are often cited as the reasons behind impaired supply chain management and reduced coverage. A recent study by the Brookings Institution (Lane and Glassman 2008) estimates that for every dollar received in funding, 7¢ to 28¢ is lost due to funding delays.

Motivated by the ready-to-use therapeutic food (RUTF) supply chain in Africa, we study the problem of managing inventory in the presence of funding constraints over a finite horizon (Swaminathan 2010). In the problem that motivated this study, the country office of UNICEF that procured RUTF was constrained by the timing of the receipt of previously promised funding from donor agencies. The unpredictable nature of donor funding is typical of the funding situation in many global health programs. Funding is received in installments throughout the planning period, and even in the best of situations, there might be uncertainty in terms of timing of receipt of those installments; in others, both the installment amounts and timing could be uncertain. An important question that arises is how to effectively manage inventory, taking into account both the current financial position as well as the funds that are due to arrive in the future periods.

In this paper, we model the above situation with a multiperiod stochastic inventory model with financial constraints. Our goal is to (i) determine the optimal ordering policy given the funding complexities and (ii) characterize the impact of funding timing, funding level, and funding uncertainty on the operating costs. Among other results, we show that despite the uncertainty in funding, the optimal replenishment policy is a state-independent modified base stock policy, which greatly enhances the appeal and implementation of the optimal policy. We also prove that uncertainty in funding timing (in comparison to a deterministic financial schedule) increases operating costs and so does the stochastically dominated late arrival of funds. Finally, we also show that increased variability in the funding timing (as measured by convex ordering) leads to higher costs.

Through an extensive numerical study, we offer insights into several issues including the impact of funding patterns, funding level (total funding received as a percentage of the amount required to meet total expected demand) and uncertainty in funding. Our analysis provides the following computational insights. (1) Front-loading (where a majority of the total funding is received in the initial periods) brings significant benefits in underfinanced systems ($< 100\%$ funding level), whereas avoiding back-loading (where a major chunk of the total funding is received in the later periods) is critical in fully financed systems (100% funding level). (2) Front-loaded funding at 75% funding level is better than back-loaded funding at 100% funding level. Depending on the level of front- and back-loading at 75% and 100% funding levels, the operating costs under back-loaded funding at 100% funding level vary between 1.5 and 5.5 times the operating costs under front-loaded funding at 75% funding level. (3) Front-loading offers significant benefits in programs offering products with high penalty costs, while the gains are relatively modest in low-penalty environments with a low demand uncertainty. (4) Additional funding is more valuable in operating environments with high-penalty costs, but demand uncertainty undermines the benefit of additional funding. (5) There is a nonlinear increase in costs with increased uncertainty in funding. Further, this effect decreases with demand uncertainty. However, the relationship between funding uncertainty and fill rates is not monotone, as one would expect.

1.1. Literature Review

Our work is related to the following three streams of literature.

The Interface Between Operations and Finance. In this stream of literature, a firm's available capital at the start of any given period is endogenously dependent on the revenues generated in the previous period; e.g., Archibald et al. (2002), Babich and Sobel (2004), Buzacott and Zhang (2004), Gaur and Seshadri (2005), Xu and Birge (2006), Chao et al. (2008), Hu and Sobel (2008), Caldentey and Haugh (2009), Protopappa-Sieke and Seifert (2010), and Li et al. (2013). Our work is more closely related to Chao et al. (2008), who also study inventory management under financial constraints. They study the inventory replenishment problem faced by a self-financing retailer whose objective is to maximize terminal wealth. For their model, they show that a capital-dependent base stock policy is optimal. One aspect that distinguishes our work from the existing literature is the presence of a donor funding stream that is exogenous to realized demand. In our work, demand fulfilled in the previous periods does not generate any revenue due to the nonprofit nature of the business.

Inventory Management Under Capacity Constraints. Financial constraints on procurement are somewhat similar to supply-capacity constraints, and a lot of work has been done on inventory management under capacity constraints (see Federgruen and Zipkin 1986, Ciarallo et al. 1994, Wang and Gerchak 1996, Aviv and Federgruen 1997, Kapuscinski and Tayur 1998). The main difference between these models and ours is that whereas physical capacity constraints are rigid, financial constraints can be made flexible. Unlike production capacity, unused capital does not go to waste and can be utilized in the future periods.

Humanitarian Operations. Within humanitarian operations, a majority of the papers focus on disaster relief; e.g., Beamon and Kotleba (2006) and Duran et al. (2011) focus on inventory management during emergencies. In recent years, long-term public health issues have also received attention from the operations management community; e.g., Rashkova et al. (2011), Deo and Sohoni (2014), and Taylor and Xiao (2014). Our work is more closely related to Rashkova et al. (2011). Taking into account the funding and procurement delays in global health programs, they develop a simulation model to predict stockouts at the country level for a given funding schedule and inventory replenishment policy. The model is then used to compare different funding schedules and replenishment policies. In our work, we identify the optimal replenishment policy for any given funding schedule, and using the optimal policy, we analyze how funding delays and uncertainty in funding impact operating costs and fill rates.

The rest of this paper is organized as follows: In §2, we introduce the model and provide analytical results regarding the optimal policy and the impact of funding. In §3, we present computational results regarding the impact of funding timing and funding uncertainty. In §4, we offer some managerial insights and conclude the paper.

2. Model

We consider a finite-horizon, periodic-review inventory model for a single product. The planning horizon is divided into N periods with reverse time indexing, i.e., the first period is period N, followed by $N-1$, $N-2$ and so on. Demands in successive periods, ζ_t, $t = N, N-1, \ldots, 1$ are independent but not necessarily identically distributed with probability density function f_t and cumulative distribution function F_t. External funding is received in $m \le N$ installments. We denote the funding vector by $Z = (z_1, z_2, z_3, \ldots, z_m)$, where z_m and z_1 are the first and last installments, respectively. Therefore, the total promised funding over the horizon is $TPF = \sum_{j=1}^{m} z_j$. We do not impose any restrictions on the installment sizes (amount received in each installment), and they could be different from one another. The installment sizes are known beforehand, but the time of receipt of each installment could be uncertain. We refer to a scenario with uncertain funding timing as a *stochastic funding schedule*. A special case of the stochastic funding schedule is the *deterministic* *funding schedule* where both the installment amounts and timing are known beforehand. Let c be the unit purchase cost, h denote the unit holding cost per period, and b be the penalty cost per unit per period for any unsatisfied demand. We make the following assumptions.

1. Unmet demand is completely backlogged. The backlogging assumption is valid for a variety of health commodities, e.g., malaria bed nets and reproductive health supplies like contraceptives.

2. All the installments are received before the end of the planning horizon. Typically, donors make a commitment based on the funding proposals, and while the installment sizes may vary based on the donors' budget cycles, in most cases, the committed amount is received in full before the end of the planning period for which the donation was sought.

In addition, we also assume that one dose/unit of the product is sufficient to meet the needs of a customer/patient. The sequence of events in any period t is as follows: (1) Funding (if any) is received at the start of period t. (2) The procurement lead time is $\lambda \geq 0$ time periods, and procurement decisions are made subject to the capital available on-hand and the current inventory position. The procurement decision is made before receiving any shipments. We follow this convention to make the description consistent with the zero-lead time case. (3) Shipments due to arrive in period t are received. (4) Finally, demand is realized, and holding and backorder costs are calculated based on the ending on-hand inventory.

Let x_t denote the on-hand inventory before receiving shipments in period t, and r_t be the capital available at the start of period t, after receiving installments (if any) in period t. We denote the outstanding funding as of period t (after receiving funding at the beginning of the period) by OF_t and let $OF_t \mid OF_{t+1}$ represent the random variable corresponding to OF_t given OF_{t+1}. Note that $OF_{t+1} - OF_t$ is the funding received at the beginning of period t. Modeling funding inflows in this fashion makes our model general enough to capture a variety of funding scenarios, with stochastic and deterministic funding schedules being special cases.

The objective is to come up with an optimal ordering policy that minimizes the total cost incurred over the planning horizon subject to the funding constraints. Since we assume self-financing, the order quantity in period t cannot exceed r_t/c. Let $\tilde{G}_t^\lambda(x_t, w_t^0, w_t^1, w_t^2, \ldots, w_t^{\lambda-1}, r_t, OF_t)$ be the minimum expected cost with t periods to go, given x_t, r_t, OF_t, and w_t^i. Here, w_t^i is the order that will be received i periods into the future and w_t^0 will be received in period t. Then,

$$
\begin{aligned}
&\tilde{G}_t^\lambda(x_t, w_t^0, w_t^1, \ldots, w_t^{\lambda-1}, r_t, OF_t) \\
&= \min_{0\leq z\leq \frac{r_t}{c}} \{cz + b\mathsf{E}_{\zeta t}[\zeta_t - x_t - w_t^0]^+ + h\mathsf{E}_{\zeta_t}[x_t + w_t^0 - \zeta_t]^+ \\
&\quad + \mathsf{E}_{OF_{t-1}|OF_t}\mathsf{E}_{\zeta_t}\tilde{G}_{t-1}^\lambda(x_t + w_t^0 - \zeta_t, w_t^1, \ldots, w_t^{\lambda-1}, z, r_t \\
&\quad - cz + (OF_t - OF_{t-1}), OF_{t-1})\}.
\end{aligned}
$$

Because all the installments are received before the end of the horizon, $OF_1 = 0$ always. The terminal cost is $\tilde{G}_0^\lambda(\cdot) = 0\ \forall (x_0, w_0^0, w_0^1, \ldots, w_0^{\lambda-1}, r_0)$. In our analysis, we find it convenient to use a modified value function expressed in terms of the variable $R_t = r_t + cIP_t$ in place of r_t. Here, $IP_t = x_t + \sum_{j=0}^{\lambda-1} w_t^j$ is the inventory position at the beginning of period t. Define

$$
\begin{aligned}
\text{(P1)}\quad & G_t^\lambda(x_t, w_t^0, w_t^1, \ldots, w_t^{\lambda-1}, R_t, OF_t) \\
&= \min_{0\leq z\leq \frac{r_t}{c}} \{cz + b\mathsf{E}_{\zeta_t}[\zeta_t - w_t^0 - x_t]^+ + h\mathsf{E}_{\zeta_t}[x_t + w_t^0 - \zeta_t]^+ \\
&\quad + \mathsf{E}_{OF_{t-1}|OF_t}\mathsf{E}_{\zeta_t} G_{t-1}^\lambda(x_t + w_t^0 - \zeta_t, w_t^1, \ldots, w_t^{\lambda-1}, \\
&\quad z, R_t - c\zeta_t + (OF_t - OF_{t-1}), OF_{t-1})\}.
\end{aligned}
$$

The function G_t^λ is jointly convex in variables x_t and w_t^i, $i = 0, 1, \ldots, \lambda - 1$ for fixed R_t and OF_t (see Lemma 1 in the online appendix, available as supplemental material at http://dx.doi.org/10.1287/msom.2014.0497). Using this joint convexity, it is easy to show that a (R_t, OF_t)-dependent modified base stock policy is optimal in period t. However, we go one step further and demonstrate that the optimal replenishment policy is actually simpler—the optimal policy is a *state-independent modified base stock policy* where the order up-to levels depend only on t and not on R_t or OF_t.

Consider problem P2, with the same cost and demand parameters and replenishment lead time as problem P1, but without financial constraints. Let $NV_t^\lambda(x_t, w_t^0, w_t^1, \ldots, w_t^{\lambda-1})$ be the minimum expected cost with t periods to go for problem P2. Then,

$$
\begin{aligned}
\text{(P2)}\quad & NV_t^\lambda(x_t, w_t^0, w_t^1, \ldots, w_t^{\lambda-1}) \\
&= \min_{z\geq 0}\{cz + b\mathsf{E}_{\zeta_t}[\zeta_t - x_t - w_t^0]^+ + h\mathsf{E}_{\zeta_t}[x_t + w_t^0 - \zeta_t]^+ \\
&\quad + \mathsf{E}_{\zeta_t} NV_{t-1}^\lambda(x_t + w_t^0 - \zeta_t, w_t^1, \ldots, w_t^{\lambda-1}, z)\}.
\end{aligned}
$$

Let $NV_0^\lambda(\cdot) = 0\ \forall (x_0, w_0^0, w_0^1, \ldots, w_0^{\lambda-1})$. For P2, it is well known that there exists an optimal base stock level $y_t^{\lambda*}$ in each period such that if the inventory position in period t is below $y_t^{\lambda*}$, it is optimal to order up-to $y_t^{\lambda*}$, and not order otherwise. In Theorem 1, we prove that the unconstrained base stock levels $y_t^{\lambda*}, y_{t-1}^{\lambda*}, \ldots, y_1^{\lambda*}$, optimal for P2, are optimal for P1 with funding constraints as well.

Theorem 1. *Let $y_t^{\lambda*}, y_{t-1}^{\lambda*}, \ldots, y_1^{\lambda*}$ be the optimal base stock levels corresponding to problem P2. Let $(x_t, w_t^0, w_t^1, \ldots, w_t^{\lambda-1}, R_t, OF_t)$ be the system state in problem P1 at the beginning of period t. Then, the optimal ordering policy for problem P1 has the following simple structure:*

$$
\begin{array}{ll}
\textit{Order up-to } R_t/c & \textit{if } R_t/c \leq y_t^{\lambda*}, \\
\textit{Order upto } y_t^{\lambda*} & \textit{if } R_t/c > y_t^{\lambda*}, IP_t < y_t^{\lambda*}, \\
\textit{Do not order} & \textit{if } IP_t \geq y_t^{\lambda*}.
\end{array}
\qquad (1)
$$

Theorem 1 raises an important question: Why are the base-stock levels independent of OF_t and R_t? Recall that OF_t keeps track of the outstanding amount, and determines the level of uncertainty in future funding. However, because of backlogging, funding uncertainty does not impact the total demand met between t and the end of the horizon. The total demand satisfied depends only on the sum: c × (inventory position) + capital available on-hand + outstanding funding. Since an increase in OF_t implies a corresponding decrease in on-hand funding, the sum is independent of OF_t. Hence, the outstanding funding and future funding uncertainty impact only the actual costs incurred but not the incremental difference in costs obtained by changing the order quantity.

A related question is as follows: Why is the base-stock level independent of R_t $(= r_t + cIP_t)$, which acts like a capacity constraint? Under capacity constraints, it is well known that the (capacity-dependent) order up-to levels are higher than the corresponding unconstrained base stock levels (see, e.g., Federgruen and Zipkin 1986). Although both funding and capacity constraints place an upper bound on the order quantity, there is a fundamental difference between the two. While unused capacity goes to waste, unused capital can be used in the later periods, i.e., it acts like transferable capacity. Intuitively, this is the reason why the unconstrained base stock levels are optimal even under funding constraints.

2.1. Impact of Funding Timing

Consider funding scenarios 1 and 2 such that

$$OF_t^2 \mid (OF_{t+1}^2 = i) \geq_{st} OF_t^1 \mid (OF_{t+1}^1 = i) \quad \forall\, t \in \{2, 3, \ldots, N-1\} \quad \text{and} \tag{2}$$

$$OF_t^n \mid (OF_{t+1}^n = i') \geq_{st} OF_t^n \mid (OF_{t+1}^n = i) \quad \forall\, t = 2, \ldots, N-1,\ n \in \{1, 2\},\ i' > i, \tag{3}$$

where $\geq_{st}$ means first-order stochastic dominance (see Shaked and Shanthikumar 2007). Condition (2) implies that the outstanding funding at the beginning of any period t is (stochastically) larger under funding scenario 2. Condition (3) says that, under both funding scenarios, the outstanding funding at the beginning of t stochastically increases in the outstanding funding at the beginning of $t+1$. We denote the value functions under scenarios 1 and 2 (satisfying conditions (2) and (3) stated above) by $G_t^{\lambda,1}$ and $G_t^{\lambda,2}$, respectively. In the following theorem, we demonstrate that (stochastically) early funding leads to lower costs due to increased procurement flexibility.

Theorem 2. *If conditions* (2) *and* (3) *hold, then* $G_t^{\lambda,2}(x_t, w_t^0, \ldots, w_t^{\lambda-1}, R_t, j) \geq G_t^{\lambda,1}(x_t, w_t^0, \ldots, w_t^{\lambda-1}, R_t, j)$ *for every* $j \in [0, TPF]$.

2.2. Impact of Variability in Funding Timing

Next, we focus on the variance aspect of the uncertainty in funding. For fixed total funding, we consider two funding scenarios such that

$$(OF_t - OF_{t-1}^2) \mid OF_t \geq_{cvx} (OF_t - OF_{t-1}^1) \mid OF_t \quad \forall\, t = 3, 4, \ldots, N \quad \text{and} \tag{4}$$

$$(OF_t - OF_{t-1}^n) \mid OF_t \text{ is SSCV in } OF_t \text{ for } n = 1, 2. \tag{5}$$

Condition (4) states that for fixed OF_t, the funding received in period $t-1$ (or equivalently the outstanding funding as of $t-1$) is more variable under funding scenario 2 than under scenario 1. The convex ordering implies that the mean funding received (or equivalently the mean outstanding funding) remains the same across the two scenarios. Condition (5) states that the funding received in period $t-1$ is strong stochastically concave (SSCV) in OF_t under both funding scenarios. Given that the expected funding received remains the same, the following theorem shows that a higher variability around the funding received in any given period drives up the operating costs.

Theorem 3. *Let conditions* (3), (4), *and* (5) *hold. Then,* $G_t^{\lambda,2}(x_t, w_t^0, \ldots, w_t^{\lambda-1}, R_t, j) \geq G_t^{\lambda,1}(x_t, w_t^0, \ldots, w_t^{\lambda-1}, R_t, j)$ *for every* $j \in [0, TPF]$.

3. Computational Study

For our computational study, we first consider deterministic funding schedules and analyze how funding patterns impact operating costs. Recall that under deterministic funding, both the installment amounts and the timing are fixed. Subsequently, we consider stochastic funding schedules to understand how uncertainty in funding timing affects system performance. Throughout the numerical study, we relate our results to common operating environments seen in humanitarian programs. For simplicity, we assume that the replenishment lead time is zero throughout the numerical study.

3.1. Deterministic Funding Schedules

3.1.1. Experimental Setup. We consider planning horizon, N, of different lengths, $N = 2$, 4, 6, 12, and 24. Purchase cost c was normalized to 1. For each N, we varied the following parameters.

Holding Cost: We chose four values for holding cost—$h = 0.01$, 0.05, 0.1, and 0.25.

Penalty Cost: For each h, the penalty cost b was varied so that the critical ratio (CR), $(b-c)/(b+h)$, took on values 0.2, 0.4, 0.6, 0.8, 0.9, and 0.95, respectively.

Demand: We consider uniform and truncated normal demand. To test the impact of demand variability, we considered $U \sim [70, 130]$ and $U \sim [25, 175]$ for uniform demand. For normal demand, the mean was fixed at 100 units and we used coefficient of variation values of

0.1 and 0.25. The normal distribution was truncated at three standard deviations. Thus, for each N, we have $4*6*4=96$ problem instances.

Funding Patterns: For each combination of N, h, b and demand, we consider five funding patterns and four funding levels. The funding patterns are extremely front-loaded (EFL) funding, moderately front-loaded (MFL) funding, evenly spread (ES) funding, moderately back-loaded funding (MBL) funding, and extremely back-loaded (EBL) funding. The holding cost and funding patterns were chosen to be consistent with an earlier study that analyzes the RUTF supply chain in the Horn of Africa (Swaminathan et al. 2009). For backorder costs, because of lack of precise estimates, we carry out a sensitivity analysis over a wide range of critical ratios. For all the funding vectors, the total funding received is equal to $N*\text{funding level}*\text{mean demand}$. In our experiments, we consider 25%, 50%, 75%, and 100% funding levels and we label them as severely underfinanced, moderately underfinanced, mildly underfinanced, and fully financed systems, respectively. For example, a severely underfinanced system receives $N*0.25*\text{mean demand}$ over the planning period. We use $N=4$, $U\sim[70,130]$ demand and 100% funding to explain the difference between the funding patterns.

- EFL: The entire funding ($=N*\text{mean demand}$) is received upfront; i.e., the funding vector is $(0,0,0,400)$. Recall that we count time in the reverse order.
- MFL: In the first $N/2$ periods, the installment size is $1.5*\text{mean demand}$ followed by $0.5*\text{mean demand}$ in the last $N/2$ periods. For the specific case considered, the funding vector is $(50,50,150,150)$.
- ES: Every installment is equal to mean demand; i.e., the funding vector is $(100,100,100,100)$.
- MBL: In the first $N/2$ periods, the installment size is $0.5*\text{mean demand}$ followed by $1.5*\text{mean demand}$ in the last $N/2$ periods. The funding vector would be $(150,150,50,50)$.
- EBL: The entire funding is back-loaded to the last period; i.e., the funding vector is $(400,0,0,0)$.

Note that as we move from EBL funding to EFL funding, additional funds are received in the initial periods. Compared to ES funding, front-loading can be viewed as a funding advance and back-loading can be considered a funding delay. Using ES funding as a benchmark, we investigate how back-loading and front-loading the funding impacts operating costs. We compute the relative percentage cost difference for a particular funding pattern, say EBL funding, as follows: $100*(cost_{\text{EBL}}-cost_{\text{ES}})/cost_{\text{ES}}$.

Table 1 Average Percentage Cost Difference for Different Funding Patterns Relative to ES Funding

EBL	MBL	MFL	EFL
948.56	248.10	−29.76	−31.5

3.1.2. Impact of Funding Pattern. From Table 1, we see that operating costs increase almost exponentially with funding delays (back-loading). However, contrary to common perception, ES funding is not the optimal funding pattern since, under ES funding, there is little flexibility to deal with large demand surges upfront. Having flexibility upfront is valuable, and we see that even moderate front-loading results in savings of 29.8%. However, pushing additional funds to the initial periods yields little to no return and, even in case of EFL funding, the savings is only 31.5%.

Next, we analyze if the insights obtained from Table 1 change with the operating environment. We focus on two parameters, critical ratio and demand uncertainty. We visualize different operating environments as being elements of the 2×2 matrix in Table 2, corresponding to high and low levels of demand uncertainty and critical ratio, respectively. Some illustrative examples of products that fit into each category are provided in the table (see Swaminathan et al. 2009; USAID 2009, 2010). For illustration, we label the demand distributions with range $[70,130]$ and $[25,175]$ as low variability and high variability demand, respectively. We pick $CR=0.2$ and $CR=0.95$ to represent a low and high critical ratio, respectively.

Effect of the Operating Environment on the Relative Costs of Different Funding Patterns. In Table 3, we pick $N=24$ and normal demand to present the insights, but the results hold for other N and uniform demand as well. From the table, we see that the benefits of front-loading increase with the critical ratio, and this effect is amplified under demand uncertainty. Under back-loading, the results are different. We see that critical ratio amplifies the negative impact of back-loading, whereas demand uncertainty has a mitigating effect. Overall, Table 3 illustrates that avoiding back-loading is paramount to performance improvement in all operating environments. On the other hand, front-loading offers significant gains for products with

Table 2 Matrix of Different Operating Environments

	Critical ratio	
	L	H
Demand uncertainty		
L	Essential medicines, e.g., aspirin tablets	Depo-Provera contraceptive injections
H	Disposable gloves, cotton wool packs	Ready-to-use therapeutic foods

Table 3 Effect of Demand Variability and Critical Ratio on the Percentage Cost Difference (Relative to ES Funding) for Different Funding Patterns at 100% Funding Level, $N = 24$

		EBL	MBL	MFL	EFL
$CR = 0.2$	$N \sim [70, 130]$	1,239.10	312.82	−12.69	−12.84
	$N \sim [25, 175]$	999.43	240.82	−24.75	−25.84
$CR = 0.6$	$N \sim [70, 130]$	2,222.64	561.51	−23.39	−23.67
	$N \sim [25, 175]$	1,566.74	378.13	−39.90	−41.61
$CR = 0.95$	$N \sim [70, 130]$	6,325.19	1,598.94	−68.61	−69.44
	$N \sim [25, 175]$	2,936.11	709.73	−76.94	−80.33

high critical ratios but the gains are relatively modest at low critical ratios, especially in environments with low demand uncertainty. Hence, the decision to invest in front-loading initiatives needs to be reconciled with the nature of the product being offered by the program.

3.1.3. Impact of Additional Funding. In Figure 1, we use the cost incurred under no funding constraints (NFC) as the benchmark. Note from Figure 1 that the benefits of additional funding remain fairly constant at all funding levels under evenly spread and back-loaded funding. However, the benefits of additional funding reduce significantly with the funding level for front-loaded funding, and the rate of reduction increases with the degree of front-loading.

Figure 1 also demonstrates that the losses from back-loading (gap between the lines corresponding to MBL/EBL and ES funding) are monotone increasing in the funding level, whereas the benefits of front-loading follow a U-shaped pattern, with maximum benefits seen either in moderately or mildly underfinanced

Figure 1 (Color online) Average Percentage Cost Difference for Different Funding Patterns at Different Funding Levels Relative to NFC

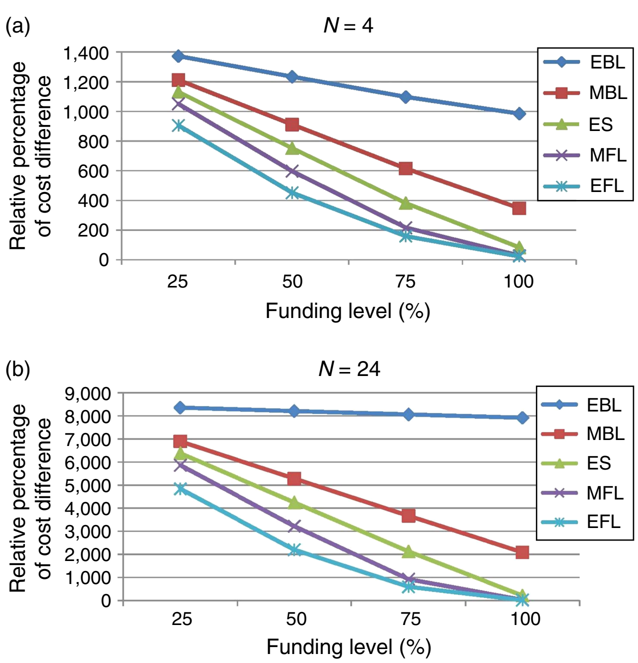

Table 4 Effect of the Operating Environment on the Incremental Benefits of Additional Funding (Between 75% and 100% Funding Level, Relative to NFC) for Different Funding Patterns, $N = 24$

		EBL	MBL	ES	MFL	EFL
$CR = 0.2$	$N \sim [70, 130]$	6.45	272.39	350.07	141.08	85.18
	$N \sim [25, 175]$	5.83	262.54	316.72	138.39	83.55
$CR = 0.95$	$N \sim [70, 130]$	488.28	4,984.15	6,307.62	2,774.44	1,825.21
	$N \sim [25, 175]$	440.32	4,680.53	5,597.83	2,656.82	1,737.29

systems. The intuition is that in severely underfinanced systems, the amount pushed to the initial periods is relatively less, and in fully financed systems there is sufficient cash already available for the additional funding to make a large impact.

Table 4 provides insights regarding how the benefits of additional funding change with respect to the operating environment. From Table 4, we see that the benefit of additional funding increases with the critical ratio but demand uncertainty undermines the benefits. Furthermore, the table also reveals that for products with a low critical ratio, additional funding has low impact when funding is extremely back-loaded. In this case, it might be more beneficial for an organization to first work on reducing funding delays before trying to secure more funding.

3.1.4. Funding Level vs. Funding Pattern. We refer back to Figure 1 to understand the relative importance of funding level vis-à-vis funding pattern. From the figure, we see that at low funding levels (25% and 50% funding levels), the funding pattern is almost inconsequential—except for extreme back-loading, all other funding patterns at 100% funding level perform better than extreme front-loading at 25% and 50% funding levels. For a mildly underfinanced system, the results are drastically different. From Figure 1, we see that back-loaded funding at 100% funding level performs significantly worse compared to front-loaded funding at 75% funding level. However, ES funding at 100% funding level outperforms even EFL funding at 75% funding level. This demonstrates that, at reasonably high funding levels, funding pattern is critical and an increase in overall funding should not be traded for a delay in funding.

3.2. Stochastic Funding Schedules

3.2.1. Experimental Setup. Except for funding patterns, the experimental setup remains unchanged from deterministic funding. As we mentioned in §2, funding is received in $m \leq N$ installments. For a fixed N, we consider several values of m for the stochastic funding case, details of which are given in Table 5.

For stochastic funding, we assume 100% funding level. To capture the uncertainty in the funding timing, we vary the number of installments. Depending on m, the amount received in each installment varies

Table 5 Number of Installments Considered for Each N

N	m
2	1, 2
4	1, 2, 3, 4
6	1, 2, 3, 4, 6
12	1, 2, 3, 4, 6, 12
24	1, 2, 3, 4, 6, 12, 24

($= N/m *$ mean demand in each period). We assume that the number of installments received in a period is uniformly distributed between 0 and the number of outstanding installments. To understand what happens when we increase the number of installments, consider the example of $N = 4$, $m = 1$ and $U \sim [70, 130]$ demand. When $m = 1$, the four extreme funding scenarios, namely $(400, 0, 0, 0)$, $(0, 400, 0, 0)$, $(0, 0, 400, 0)$ and $(0, 0, 0, 400)$, are equally likely. When we increase m to 2, the probability of extreme funding scenarios like $(400, 0, 0, 0)$ reduce from $1/4$ to $1/16$ while the probability of a more evenly spread funding vector like $(200, 0, 200, 0)$ increases to $1/8$. Hence, the probability of the funding being more evenly spread out increases with the number of installments, thereby reducing the volatility in funding received until any given period.

3.2.2. Impact of Funding Uncertainty. From Figure 2, we see that reducing the funding volatility, and making the funding more smooth and evenly spread out, lowers operating costs. However, such benefits of reducing funding uncertainty show diminishing rates of return, i.e., as m increases, the marginal value of receiving the funding in an additional installment decreases.

To understand why reducing funding uncertainty leads to lower costs, consider the case where $m = 1$ and $N = 6$. The single installment could be received in any of the six periods with equal probability. Of course, there is nothing better than receiving the installment in the first period (probability $1/6$), but we also need

Figure 2 (Color online) Average Percentage Cost Difference Due to Funding Timing Uncertainty Relative to a Funding Schedule with $m = N$

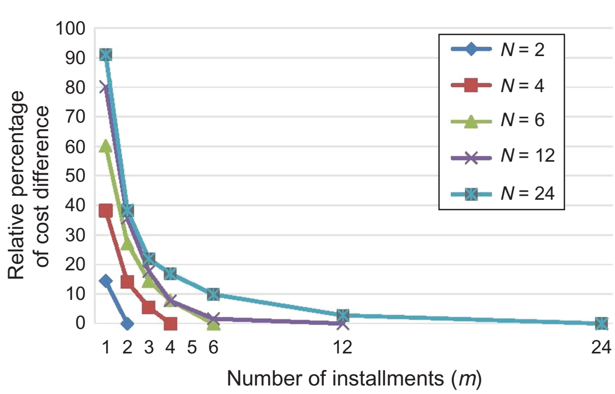

Table 6 Effect of Demand Variability and Critical Ratio on the Average Relative Percentage Cost Difference Due to Funding Uncertainty (Relative to a Funding Schedule with $m = N$), $N = 24$

		$m=1$	$m=2$	$m=3$	$m=4$	$m=6$	$m=12$	$m=24$
$CR = 0.2$	$N \sim [70, 130]$	78.82	33.50	19.27	14.91	8.86	2.69	0
	$N \sim [25, 175]$	76.54	32.20	18.33	14.12	8.25	2.38	0
$CR = 0.95$	$N \sim [70, 130]$	106.45	45.29	26.20	20.27	11.94	3.52	0
	$N \sim [25, 175]$	102.78	43.29	24.80	19.09	11.05	3.09	0

to take into account the other possibilities including an extreme back-loading with probability $1/6$. Considered together, receiving funding in one installment is no longer ideal—in fact, it is the worst. In general, when the number of installments decreases, it increases the possibility of both front- and back-loading. However, the nonlinear increase in costs as we move from front-loaded to back-loaded funding implies that in expectation, the operating costs increase.

From Table 6, we see that the benefit of reducing funding uncertainty increases with the critical ratio but demand uncertainty undermines the benefits. The undermining effect is more pronounced at higher critical ratios. Table 6 further illustrates that to achieve comparable performance (in terms of relative costs), humanitarian organizations offering products with high penalty costs need to reduce funding uncertainty even further when compared to low critical ratio products. This highlights the greater need to work with donors, financial institutions, and other stakeholders to reduce funding uncertainty in high-penalty environments like RUTF programs.

3.2.3. Funding Level vs. Funding Uncertainty. In this section, we aim to address the following question: Which of the two has a greater impact on system performance—funding level or funding uncertainty? Comparing rows 1 and 2 in Table 7 with Table 8, we see that at low (25% and 50%) funding levels, the funding pattern is inconsequential. Receiving less overall funding severely hurts performance, making even the most uncertain funding ($m = 1$) at 100% funding level attractive in comparison in almost all cases (an exception being EFL funding at 50% funding level).

For 75% funding level, the results are very different (compare row 3 in Table 7 with Table 8). While the

Table 7 Average Relative Percentage Cost Difference ($100 * (Cost - Cost_{NFC})/Cost_{NFC}$) for Different Deterministic Funding Patterns at Different Funding Levels

Funding level (%)	EBL	MBL	ES	MFL	EFL
25	8,357.25	6,894.14	6,377.74	5,861.40	4,837.84
50	8,208.85	5,282.63	4,250.12	3,220.79	2,200.73
75	8,060.46	3,671.19	2,125.99	923.60	596.72
100	7,917.89	2,086.40	213.73	26.65	16.80

Table 8 Average Relative Percentage Cost Difference ($100 * (Cost - Cost_{NFC})/Cost_{NFC}$) Due to Funding Timing Uncertainty

$m=1$	$m=2$	$m=3$	$m=4$	$m=6$	$m=12$	$m=24$
2,800.83	1,968.97	1,710.29	1,630.68	1,518.65	1,407.39	1,363

back-loaded vectors at 75% funding level perform significantly worse than even the most uncertain funding at 100% funding level, front-loaded funding at 75% funding level outperforms uncertain funding at 100% funding level, even when the uncertainty is considerably reduced ($m = 24$). This demonstrates that at relatively high funding levels, the choice between deterministic funding and an even larger overall but uncertain funding is not straightforward.

3.3. Fill Rates

Operational efficiency is paramount to improving aid effectiveness, but meeting demand in a timely fashion is also an important consideration in humanitarian settings. We define fill rate as the percentage of demand that is met on time without being backlogged. From Table 9, we see that under deterministic funding, front-loading holds the key to increasing fill rates in underfinanced systems while avoiding back-loading is important in fully funded systems. This result is consistent with our earlier discussion with respect to operating costs. Interestingly, under stochastic funding, we see from Table 10 that the relationship between funding uncertainty and fill rate is not monotone as one would expect. The probability of both front- and back-loading increase with funding uncertainty and the results demonstrate that, in expectation, the positive impact of front-loading could dominate the negative effects of back-loading when it comes to fill rates. However, fill rates only tell one part of the story—what fraction of the demand is met on-time without backlogging. For backlogged demand, an analysis of our data shows that the average waiting time actually decreases with the number of installments and so does the cumulative waiting time for the population.

Table 9 Average Fill Rates for Different Funding Patterns at Several Funding Levels, $N = 24$

		EBL	MBL	ES	MFL	EFL
Funding level	0.25	0.00	0.57	1.18	1.87	25.13
	0.5	0.00	1.18	2.87	8.59	50.09
	0.75	0.00	1.89	8.77	59.96	75.09
	1.0	2.76	10.44	76.06	96.91	97.89

Table 10 Average Fill Rates for Different Number of Installments for $N = 24$

$m=1$	$m=2$	$m=3$	$m=4$	$m=6$	$m=12$	$m=24$
50.06	51.14	51.24	52.43	51.69	53.72	53.70

4. Conclusions and Managerial Insights

Incorporating funding flows into operational decisions is necessary for making optimal and operationally feasible decisions. In this paper, we study the problem of managing inventory of a health commodity subject to variable funding constraints. Our work brings out several important insights that would be valuable to humanitarian supply chain managers. To begin with, we find that preventing funding delays should be the top-most priority for humanitarian organizations. Our study suggests that front-loading initiatives need to be reconciled with the system funding level. Moderate front-loading is beneficial at all funding levels, but extreme front-loading brings little to no additional benefits in a fully financed system. Moreover, for both moderate and extreme front-loading, the benefits of front-loading are nonmonotone in the funding level. Our study also demonstrates that the benefits of front-loading are significantly lower for products with a low critical ratio, especially in low-demand uncertainty environments. Managers need to exercise caution and use careful judgment when deciding the level of front-loading in such situations.

Oftentimes, humanitarian organizations make an all-out effort to raise as much funding as possible to support the various programs, but our analysis shows that such an approach is not the most effective one. Our results indicate that even if the funding level is lower, performance may be better if the funding is received earlier or in a steady fashion.

Finally, managers also need to pay close attention to their operating environment when taking steps to improve the funding situation. One such aspect is the volatility of the underlying demand. Our analysis shows that demand uncertainty undermines the benefits of additional funding. Moreover, whereas the magnitude of the benefits of front-loading increase with demand volatility, the opposite is true regarding the savings resulting from reducing the funding uncertainty. Front-loading initiatives are likely to yield significant benefits in highly unpredictable environments like RUTF programs, while reducing the funding uncertainty can be expected to result in substantial savings in case of health commodities like reproductive supplies, which have a relatively more stable demand pattern.

Although we have several interesting computational results in this paper, some of those are worthy of deeper analytical analysis. In particular, our results show that the benefits of front-loading follow a U-shaped pattern with respect to the funding level, with maximum benefits seen in moderately or mildly underfinanced systems. A potential avenue for future work would be to analytically characterize the threshold funding level at which the benefits of front-loading start to decline. Such a characterization could be useful in identifying the tipping point at which humanitarian supply chain managers can expect to see a "diminishing rate of return" on initiatives to secure additional funding from donors. Furthermore, our results demonstrate that the trade-off between early funding and additional funding is not straightforward at relatively high funding levels. It would be potentially useful to develop an analytical model to more completely characterize the benefits of early vis-à-vis additional funding. Finally, since demand uncertainty undermines the benefits of reducing the funding uncertainty, it could be potentially valuable to model demand and funding uncertainty in greater detail and explore the right combination of efforts to direct towards reducing demand uncertainty vis-à-vis reducing funding uncertainty.

Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/msom.2014.0497.

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