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We study the value of information, production flexibility, and supplier flexibility for a good for which an initial and a subsequent order may be placed. We consider a Bayesian model of demand in which the unknown mean demand rate is assumed to have a prior, which is a mixture of two normal distributions corresponding to the demand forecast for an innovative (fashion) good. We develop three models of production flexibility: a static model requiring initial placement of both orders, a partially dynamic model requiring a fixing of the time that the second order will be made, and a fully dynamic model with no restrictions on ordering. Supplier flexibility is modeled through supply lead times. We observe that the magnitude of the savings from the static to the fully flexible model, corresponding to the sum of the values of information and production flexibility, reflects all sources of variability: differences between demand means of the prior mixture, variability within each prior, and variability about the observed mean. We observe that as the difference between high and low demand cases increases, the value of information increases, though for long lead times, production flexibility is required to take advantage of the updated information. Further, we observe that the greater the uncertainty within each prior distribution, the greater the value of information relative to the value of production flexibility, particularly for long lead times. However, the greater the uncertainty around the mean demand, which is the uncertainty that cannot be resolved through observation, the lower the value of information. Finally, we observe that the value of supply flexibility grows initially in a concave then convex manner as a function of the supply lead times.
(*Supply Chain Management; Flexibility; Bayesian Forecasting; Lead Time; Flexibility*)

1. Introduction

Advances in information technology (client/server architectures, Internet, etc.) and related application tools (Enterprise Resource Planning (ERP) software, Web browsers, etc.) have had a profound impact on the effective management of supply-chain inventories. Companies in industries as diverse as computers, cosmetics, bicycles, automobiles, grocery, and food service have used the ready availability of online sales data and the power of ERP systems to provide supply-chain transactional transparency to better match supply and demand so as to reduce the costs of inventory and stock-outs. Detailed information in such initiatives can be found in recent supply-chain management textbooks; see Dornier et al. (1998), Handfield and Nichols (1999), and Simchi-Levi et al. (2000).

The biggest challenges in the matching of supply and demand are faced in markets characterized by tremendous product variety and ''innovative products'' (with this last term defined in Fisher (1997), a definition which accounts both for fashionable char-

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acteristics and fast technology progression as well as high service-level requirements in terms of delivery speed and customization). Examples of such market environments and their inventory management challenges are described in Fisher et al. (1994) and Feitzinger and Lee (1997). Quite often in such environments firms devise two-pronged strategies to deal with the matching of supply and demand challenges. On the one hand, they invest in information systems that accurately reflect the demand information over time. Such information systems could be point-of-sale (POS) systems that, via EDI or Web-based links, allow them to communicate with a representative sample of their stores or independent retailers. On the other hand, firms use flexibility in their production and sourcing practices to capitalize on available demand (or sales) information on a continuous basis. They do so by placing frequent orders, by adjusting order quantities, and by carefully managing the time in which orders are received. Examples of such practices are provided in Fisher and Raman (1996), Fisher et al. (1998), Eppen and Iyer (1997a and b), Iyer and Bergen (1997), Whang and Lee (1998), and Hayes (1995). For companies in such volatile market environments, investments in information technology and production and sourcing flexibility are intertwined in an effort to respond quickly and provide a high service level to their customers. The complementary nature of investments in the ability to update demand information and in abilities to exploit real time demand information via production and sourcing flexibility makes it hard to analyze their relative contributions. However, firms need to thoroughly understand these issues to prioritize investments that make them more responsive to customer needs while controlling relevant costs.

In this paper, we study the relative value of demand information updating and production and supplier flexibility for an innovative product in a continuous time, single season model with two ordering opportunities using a Bayesian model of demand. We consider three cases of ordering flexibility: (a) a *static* case, where two orders can be placed over a single season, but the quantity and timing (of receipt) of both orders are prespecified; (b) a *partially dynamic* case, where the timing of the second order is predetermined, but the quantity can be determined using updated demand information; and (c) a *fully dynamic* case, where both the timing and the quantity of the second order are determined with full usage of the updated demand information. Cost savings from the static case to the partially dynamic model measure the benefit contained in the updated information. The updated information reflects both the updated inventory level and the revised estimate of the demand process. Cost savings from the partially dynamic to the fully dynamic model measure the value of production flexibility. In a production environment, the partially dynamic case models the fixing of a production schedule while in the fully dynamic model; production can occur at any time. Alternatively, in an inventory purchasing environment, the difference between the models is analogous to the difference between a continuous and periodic review, albeit in a two-period model where the review period is predetermined in the partially dynamic model. Supplier flexibility is modeled through the lead time.

The main thesis of this paper is that the relative value of accurate demand information and production and supplier flexibility depend on the uncertainty of forecasts and the ability to act upon updated information. While not the focus of this paper, the continuous time models we develop are a contribution to the growing literature on nonstationary inventory models. Using these models, we derive several insights regarding the drivers of the relative values through numerical analysis. We show that the magnitude of the savings from the static to the fully flexible model, corresponding to the sum of the values of information and production flexibility, reflects all sources of variability: differences between distribution means of the prior mixture, variability within each prior, and variability about the observed mean. We observe that as the difference between high and low demand cases increases, the value of information increases, though for long lead times, production flexibility is required to take advantage of the updated information. Further, we observe that the greater the uncertainty within each prior distribution, the greater the value of information relative to the value of production flexibility, particularly for long lead times. However, the greater the uncertainty around the mean demand, which is the uncertainty that cannot be resolved through observation, the lower the value of information. Finally, we observe that the value of supply flexibility grows initially in a concave then convex manner as a function of the supply lead times.

The remainder of the paper is as follows. In §2, we review the related literature. In §3, we develop the model by presenting the ordering policies and their associated costs for the three ordering flexibility cases. We present numerical results in §4 and discuss the conceptual insights of our model in §5.

2. Literature Review

The early research in the area of incorporating information flows into inventory control models emphasized the use of historical data on the demand process to more accurately forecast the demand distribution using Bayesian updates. Examples of such research appears in the papers of Scarf (1959), Azoury and Miller (1984), Azoury (1985), and Iglehart (1964). More recent work emphasized the modeling aspects of inventory systems with nonstationary demand processes. Lovejoy (1990) studies a model in which a demand parameter is updated in a Bayesian fashion and provides conditions for the optimality of myopic policies. Song and Zipkin (1993 and 1996) study inventory control in an infinite horizon model where demand and/or supply varies according to a Markov process. Similarly, Sethi and Cheng (1997) model the demand process as dependent on exogenous variables, subject to stochastic fluctuations, and consider the optimality of (s, S) policies under these conditions. Heath and Jackson (1994) propose a generalized model with nonstationary demand. Graves (1999) considers an adaptive base-stock policy for a single item inventory system where the demand process is an integrated moving average for which an exponential weighted moving average provides the optimal forecast. Other research in inventory models in nonstationary demand settings appears in Aviv and Federgruen (1997) and Kapuscinski and Tayur (1998). In another approach Hariharan and Zipkin (1995) consider how demand lead times for firm orders can reduce the role of supply lead times. We compare (as described below) firm orders to orders with varying degrees of flexibility.

In an influential paper Fisher and Raman (1996) provided a dynamic programming framework as a planning approach for ''accurate response'' practices of manufacturers of short lifecycle products. Accurate response is a risk-based production strategy that uses ''speculative'' production capacity (production scheduled before the observation of early indication of demand) for low-risk products and postpones production of higher-risk products after early indicators of demand are observed (''reactive'' production capacity). This paper documents the benefits of application of such procedures in a fashion skiwear firm. The follow-up paper of Fisher et al. (1998) elaborates on other managerial levers (e.g., excess capacity, supplier choice, etc.) that can be used to enhance the effectiveness of ''accurate response.'' In the above papers the formal dynamic programming model is set in a simplified two-period setting with two production quantities to be determined and uses demand distributions that are either multidimensionally normal, or estimated from historical data of the skiwear application.

In a spirit similar to Fisher and Raman (1996), Eppen and Iyer (1997a and b), and Iyer and Bergen (1997) present order quantity models for a two-period setting. The Iyer and Bergen paper uses the first period for collection of information on early demand indicators, but no production commitment is made in the first period. Using these early demand indicators, the demand distribution forecast is updated in a Bayesian fashion and a firm production order is placed. The emphasis of the paper is placed on understanding the benefits of such practices (an integral part of Quick Response strategies in the apparel industry) on the profitability of the supplier and the manufacturer. Similar issues, but for a specific contractual agreement (buyback contracts as commonly used in the catalog mail industry), are analyzed in Eppen and Iyer (1997b).

Our work differs from the above research in several dimensions. Our modeling framework allows for the observation of the evolution of the demand process in continuous time. Thus, we permit the dynamic timing of the second order in the fully dynamic model presented below. In doing so we explicitly account for production flexibility and supplier lead times. Further, we account for demand in two periods in contrast with several of the previous models that effectively had only a single period during which actual demand occurs. Using the control policies we derive, we numerically study the value of information and production flexibility.

We follow the spirit of the modeling of demand evolution in continuous time discussed in Whang and Lee (1998). The authors emphasize the limitations imposed by models assuming that customer orders over multiple time periods are represented as independent identically distributed (IID) random variables, as this implies that no forecasting is required over time. They use both IID and Martingale demand models to explain the value of postponement strategies in supply-chain practices, while our work uses similar models to better understand the value of information and production/sourcing flexibility in accurate response environments.

Other research incorporating information dynamics in supply chains places emphasis on information sharing practices between manufacturers and retailers. Papers in this research stream develop models to understand how the manufacturer can better reflect information about the retailer's inventory position and ordering policies in their production planning policies and then estimate the potential benefits for manufacturers and retailers. Such practices are important for "functional" products (again, we use the definition of the term as is used in Fisher (1997)) in efficiency driven chains. Representative papers in this research stream are Cachon and Fisher (1997), Gavirneni et al. (1999), Kapuscinski and Tayur (1998), Moinzadeh and Bassok (1998), Moinzadeh (1999), and Lee et al. (1999).

Finally, a research stream that emphasized the study of sourcing flexibility analyzed ordering decisions within sourcing contracts that allowed for ordering adjustments, but mostly in stationary demand environments. For representative papers of this research stream see Tsay et al. (1999) and references therein, Moinzadeh and Nahmias (1997), and Milner and Rosenblatt (1999).

3. Model

We will use the following notation:

T = total length of the horizon.
τ = time of placement of the second order.
L = lead time.
Q_i = amount ordered for time segment i of our planning horizon, $i = 1, 2$.
c = cost per unit.
h = holding cost per unit per unit time.
p = shortage cost per unit.
D_i = the random variable of demand in the ith time segment of our planning horizon, $i = 1, 2$.
z = the realized demand from time 0 to τ.
ξ = the realized lead time demand (demand from time τ to $\tau + L$).
y = the realized demand in the second time segment.
x = the inventory on hand at the end of the first time segment $(Q_1 - z)^+$.
$\bar{\rho}$ = the estimated demand rate at time τ.
$f_{D_1;t}$ = pdf of demand in first time segment of length t.
$f_{D_2|D_1;s,t}$ = conditional pdf of demand in a period of length s given demand D_1 during the initial t time units.
g_τ = pdf of τ, the time of placing the second order.
$\Phi(t)$ $(\phi(t))$ = the CDF (pdf) of the standard normal random variable.

Additional notation is introduced as needed.

We consider a single season model for a horizon of length T that is divided into two segments with an order placed for each segment. Note that we consider a continuous model of time so that the second segment can begin at any time. An initial order is placed prior to the season and received before any additional information is received. This order is used to fill demand during the first segment. A subsequent order is placed for the second segment. Depending on the model of flexibility, the second order will either be placed prior to the season for receipt at a specified time, at a prespecified time, or at any time. There is a fixed lead time for receipt (the lead time does not affect the static model). The second segment begins at the time when the order is received. The total cost reflects the unit cost, c, the holding cost rate, h, and the shortage cost, p. We define pdf's $f_{D_1;t}$ as the demand over a period of length t and $f_{D_2|D_1;s,t}$ as the demand over a period of length s, given observed demand D_1 over the initial t time units. We formulate the problem as a cost minimization. An equivalent profit maximization formulation may be made; however, doing so would obscure the relative differences between the models we study. We do include the unit costs in our study because better information may allow the firm to economize on purchases.

We make the following assumptions. We assume the holding cost over a segment of length t, given initial inventory Q, is

$$h\int_0^t (Q - \mu s)\, ds = h\left(Qt - \frac{1}{2}\mu t^2\right)$$

where μ is the expected demand rate in the period. This is the single period analog of the approximate holding cost used in the standard, infinite horizon, continuous review inventory model (see, e.g., Hadley and Whitin 1963, p. 165). Expected inventory at the end of the first segment is included in the initial inventory of the second segment. We assume observed lost sales in both segments to reflect the short selling season and to make the first and second segments of the period equivalent in how costs are accounted.

While the static and partially dynamic cases are solved in standard ways, we approximate the total cost of the fully dynamic case. To do so we assume the ordering policy to be an $(s(\tau), Q_2(\tau))$ policy where the second order for $Q_2(\tau)$ units is placed if the inventory at time τ is $s(\tau)$. We then approximate $s(\tau)$ and $Q_2(\tau)$ through a marginal analysis. Given $s(\tau)$, we approximate the distribution of the time τ when the inventory equals $s(\tau)$ (starting from an initial investment level $Q_1 > s(\tau)$). We present the details of these approximations below.

3.1. Bayesian Demand Model

For the Bayesian model of information, we assume that demand follows a Brownian motion with an unknown drift parameter θ and known variance σ^2. We assume a prior for θ. Given θ, the Brownian motion assumption implies that the total demand from time 0 to time t is

$$f_{D_1;t}(x \mid \theta) \sim N(t\theta, t\sigma^2).$$

In this study we assume that the demand rate is either high or low. Such is often the case with style goods where the demand mean is uncertain though one of two scenarios is likely and once this source of uncertainty is resolved, good forecasts may be obtained. We consider various cases in our numerical studies below.

By observing demand over the first segment, the firm may update its forecast for the second segment and order accordingly. To model this, we assume the prior distribution $\pi(\theta) = \Sigma_{i=1,2}\, \alpha_i \pi_i(\theta)$ where

$$\pi_i(\theta) \sim N(\mu_i, \nu_i^2) \quad \text{for } i = 1, 2$$

and $\alpha_1 + \alpha_2 = 1$, $\alpha_i \geq 0$, $i = 1, 2$, i.e., $\pi(\theta)$ is a mixture of the two prior distributions $\pi_i(\theta)$. Therefore, the pdf for the demand until time τ is

$$\begin{aligned} f_{D_1;\tau}(x) &= \int_\theta f_{D_1;\tau}(x \mid \theta)\pi(\theta)\, d\theta \\ &= \sum_{i=1,2} \alpha_i \int_\theta f_{D_1;\tau}(x \mid \theta)\pi_i(\theta)\, d\theta \\ &= \sum_{i=1,2} \frac{\alpha_i}{\sigma_i'} \phi\left(\frac{x - \tau\mu_i}{\sigma_i'}\right) \end{aligned} \tag{1}$$

where $\sigma_i'^2 = \tau\sigma^2 + \tau^2\nu_i^2$ for $i = 1, 2$. So the mean demand until τ is $\tau\, \Sigma_{i=1,2}\, \alpha_i\mu_i$, and the variance is

$$\tau\sigma^2 + \tau^2\left(\sum_{i=1,2} \alpha_i(\mu_i^2 + \nu_i^2) - \left(\sum_{i=1,2} \alpha_i\mu_i\right)^2\right). \tag{2}$$

The last result is easy to establish.

Based on an observation of demand over a period of time, t, we determine a posterior distribution for the demand rate over the remainder of the period and calculate the marginal distribution of total demand. To do so, define $m(\rho)$ as the marginal distribution of the demand rate until t. Let $\hat{\sigma}_i^2 = \sigma^2/t + \nu_i^2$. From (1), we can find

$$m(\rho) = \sum_{i=1,2} \frac{\alpha_i}{\hat{\sigma}_i}\phi\left(\frac{\rho - \mu_i}{\hat{\sigma}_i}\right).$$

If the total demand by time t is z, the average demand rate is

$$\bar{\rho} = z/t,$$

which is a sufficient statistic for estimating θ.

Let $m_i(\bar{\rho}) = (1/\hat{\sigma}_i)\phi((\bar{\rho} - \mu_i)/\hat{\sigma}_i)$ for $i = 1, 2$. Based on the observed demand rate, $\bar{\rho}$, the posterior distribution of θ is

$$p(\theta \mid \bar{\rho}) = \sum_{i=1,2} w_i p_i(\theta \mid \bar{\rho})$$

where $p_i(\theta \mid \bar{\rho}) \sim N(\bar{\mu}_i, \bar{\nu}_i^2)$ with

$$\bar{\mu}_i = \frac{\sigma^2\mu_i + t\nu_i^2\bar{\rho}}{\sigma^2 + t\nu_i^2}, \qquad \bar{\nu}_i^2 = \frac{\sigma^2\nu_i^2}{\sigma^2 + t\nu_i^2}, \quad \text{and}$$

$$w_i = \frac{\alpha_i m_i(\bar{\rho})}{\sum_{i=1,2} \alpha_i m_i(\bar{\rho})} \tag{3}$$

for $i = 1, 2$. ($\bar{\mu}_i$ and $\bar{\nu}_i^2$ are the mean and variance of the posterior distributions for each prior in the mixture, and w_i is the updated weight given each distribution in the mixture.) See Carlin and Louis 1996.

Let $\tilde{\mu}_i(s) = s\bar{\mu}_i$ and $\tilde{\sigma}_i^2(s) = s\sigma^2 + s^2\bar{\nu}_i^2$. Then, similar to the marginal demand until time τ, (1), the conditional demand over a duration s, given observed demand $D_1 = z$ over a duration t, has pdf

$$f_{D_2|D_1;s;t}(y \mid z) = \sum_{i=1,2} \frac{w_i}{\tilde{\sigma}_i}\phi\left(\frac{y - \tilde{\mu}_i}{\tilde{\sigma}_i}\right). \tag{4}$$

Let the prior mean demand rate be $\mu = \alpha_1\mu_1 + \alpha_2\mu_2$ and the posterior mean rate be $\bar{\mu} = w_1\bar{\mu}_1 + w_2\bar{\mu}_2$, which reflects the demand by time t through $\bar{\rho}$ (cf. (3)).

3.2. Analysis of Ordering Policies

3.2.1. Static Case. In the static case, Q_1 and Q_2, as well as τ, are determined prior to the period. Without loss of generality we assume that $L = 0$ for the static case. For the static case, in the first segment the costs are:

$$cQ_1 + h\int_{t=0}^{\tau} (Q_1 - \mu t)\, dt + p\int_{Q_1}^{\infty} (z - Q_1) f_{D_1;\tau}(z)\, dz.$$

In the second segment, the initial inventory is $(Q_1 - z)^+ + Q_2$ where z is the first segment demand. The holding cost in the second segment reflects the initial inventory and the posterior demand rate, while the shortage cost reflects the initial inventory and conditional demand, $f_{D_2|D_1;T-\tau,\tau}$. Therefore the total cost for the two segments is

$$\begin{aligned}
TC^S&(Q_1, Q_2, \tau) \\
&= c(Q_1 + Q_2) + h\int_{t=0}^{\tau} (Q_1 - \mu t)\, dt \\
&\quad + p\int_{Q_1}^{\infty} (z - Q_1) f_{D_1;\tau}(z)\, dz \\
&\quad + h\int_{\tau}^{T}\int_{0}^{\infty} ((Q_1 - z)^+ + Q_2 - t\bar{\mu}) f_{D_1;\tau}(z)\, dz\, dt \\
&\quad + p\int_{z=0}^{\infty}\int_{y=(Q_1-z)^++Q_2}^{\infty} (y - (Q_1 - z)^+ - Q_2) \\
&\qquad\qquad \times f_{D_2|D_1;T-\tau,\tau}(y \mid z) \\
&\qquad\qquad \times f_{D_1;\tau}(z)\, dy\, dz.
\end{aligned}$$

Using standard convexity arguments, the value of Q_2 can be found for a fixed Q_1 and τ by solving a newsvendor-type equation:

$$\int_0^{Q_1} F_{D_2|D_1}(Q_1 + Q_2 - z \mid z) f_{D_1}(z)\, dz + \int_{Q_1}^{\infty} F_{D_2|D_1}(Q_2 \mid z) f_{D_1}(z)\, dz = \frac{p - c - h(T - \tau)}{p} \tag{5}$$

where we suppress the subscripts τ and $T - \tau$. In the first period, we can search for the optimal value of Q_1 for a given τ as convexity holds. We can then search over τ for the minimum total cost.

We note that the cost may not be convex in τ. For example, in a single period model assuming known normal demand (a special case of the Bayesian model), one can show that the optimal total cost is

$$TC_1 = \left(c + \frac{h}{2}\tau\right)\mu\tau + p\sigma\sqrt{\tau}\phi(k^*)$$

where $k^* = \Phi^{-1}(p - c - h\tau)/p$. In this case $d^2TC_1/d\tau^2 = h\mu - (1/4)p\sigma\phi(k^*)/\tau^{3/2} + h\sigma\tau^{1/2}(k^*/\tau + dk^*/d\tau)$, which is negative as $\tau \to 0$ (assuming $k^{*\prime} > 0$), so that the total cost in the single period is not convex in τ. In a two-segment model, the total cost would reflect the sum of two such terms so that it is possible that the costs will not be convex if one segment has a duration close to zero. However, as a small τ would imply that the second segment would have a long duration, in most practical settings the second segment costs would be highly convex, and so the overall costs would be convex in τ. In conducting our numerical study, see §4, we observed that the total cost is convex in τ.

3.2.2. Partially Dynamic Case. In the partially dynamic case, an initial order, Q_1, is placed at time 0 and a second order is placed at time τ with τ predetermined at time 0. The order arrives at time $\tau + L$. The second order, Q_2, reflects an order-up-to level. Because the second order is placed with knowledge of the updated demand information, the partially dynamic case reflects the value of the updated information over the static case. The total cost for the partially dynamic case can be found by dynamic programming. The solution follows a newsvendor setting for two order segments.

For presentation purposes we divide the first segment into Period 1*a*, consisting of time 0 to τ and Period 1*b* consisting of time τ to $\tau + L$, which is the lead time. The second segment begins at $\tau + L$. As in (4), we can write the pdf of the conditional demand over the lead time, $D_{1b} \mid D_{1a}$, as $f_{D_{1b}|D_{1a};L,\tau}$, where the parameters defining the right-hand side of (4) are defined using $\tilde{\mu}_i(L)$ and $\tilde{\sigma}_i^2(L)$. Similarly, we can define the conditional second segment demand given an observed total first segment demand of $z + \xi$ at time $\tau + L$ as $f_{D_2|D_1;T-\tau-L,\tau+L}(y \mid z + \xi)$ where $\tilde{\mu}_i(T - \tau - L)$ and $\tilde{\sigma}_i^2(T - \tau - L)$ are used, and $\bar{\mu}_i$, $\bar{\nu}_i^2$ and $\bar{\rho}$ are defined for $t = \tau + L$ with the demand observed by time $\tau + L$ equal to $z + \xi$. We abbreviate the conditional pdf of demand over the lead time and that over the second segment as $f_L(\xi \mid z)$ and $f_2(y \mid \xi + z)$, respectively.

We first define the costs associated with the second order, Q_2, dependent on the observed Period 1*a* demand z. The expected holding cost for the second segment reflects the order quantity, Q_2, plus the expected inventory remaining from the first segment:

$$\mathrm{Hold}_2(Q_2) = h\left(\left(Q_2 + \int_0^x (x - \xi) f_L(\xi|z)\, d\xi\right) \times (T - \tau - L) - \bar{\mu}(T - \tau - L)^2/2\right)$$

where $x = (Q_1 - z)^+$. The expected shortage cost in the second segment depends on whether a stock-out occurs before time $\tau + L$ or not. If $\xi < x$, then the inventory on hand at time $\tau + L$ is $Q_2 + x - \xi$, and if $\xi > x$, the inventory on hand is Q_2. The conditional demand in the second period reflects the demand by time $\tau + L$, $z + \xi$. Given the demand z at time τ, we take the expectation over the demand during the lead time to find the expected shortage cost:

$$\begin{aligned}\mathrm{Short}_2(Q_2) &= p\left(\int_{\xi=0}^{x} \int_{y=Q_2+x-\xi}^{\infty} (y - (Q_2 + x - \xi)) \times f_2(y|z + \xi) f_L(\xi|z)\, dy\, d\xi \right.\\ &\quad \left. + \int_{\xi=x}^{\infty} \int_{y=Q_2}^{\infty} (y - Q_2) f_2(y|z + \xi) f_L(\xi|z)\, dy\, d\xi\right).\end{aligned}$$

The total cost for the second segment given x, z, and τ is

$$TC_2^p(x; z, \tau) = \min_{Q_2 \geq 0} cQ_2 + \mathrm{Hold}_2(Q_2) + \mathrm{Short}_2(Q_2).$$

Assuming $Q_2 > 0$, we can substitute in $S_2 = Q_2 + x$ the order-up-to value at time τ and, differentiating with respect to S_2, we find the optimal order-up-to S_2^* solves the first order sufficient condition

$$\int_{\xi=0}^{x} F_2(S_2^* - \xi|z + \xi) f_L(\xi|z)\, d\xi + \int_{\xi=x}^{\infty} F_2(S_2^* - x|z + \xi) f_L(\xi|z)\, d\xi = \frac{p - c - h(T - \tau - L)}{p} \tag{6}$$

where $F_2(\cdot \mid z + \xi)$ is the CDF of $f_2(\cdot \mid z + \xi)$. If $x <$

S_2^*, $Q_2^* = S_2^* - x$; otherwise $Q_2^* = 0$. Comparing the LHS of (6) with the LHS of (5), we observe that x replaces Q_1 and $F_L(\xi \mid z)$ replaces $F_{D1}(z)$ in the partially dynamic model, reflecting the ability to update demand information.

The total expected holding and shortage cost for the first segment reflects the demand over $\tau + L$. Thus, the total expected cost given τ is found as

$$TC^P(\tau) = \min_{Q_1 \geq 0} cQ_1 + h(Q_1(\tau + L) - \mu(\tau + L^2/2) + p\left(\int_{u=Q_1}^{\infty} (u - Q_1) f_{D_1;\tau+L}(u)\, du\right) + E_{D_{1a}}[TC_2^P((Q_1 - D_{1a})^+;\, D_{1a}, \tau)]. \tag{7}$$

The minimum of (7) can be found by searching over τ.

3.2.3. Fully Dynamic Case. In the fully dynamic case, the firm places an initial order, Q_1, and places a second order, Q_2, at any time $\tau < T$ and for any quantity with a delivery lead time, L. We assume an ordering policy characterized by an initial order, Q_1, and a pair of functions $(s(\tau), Q_2(\tau))$, such that if the inventory level at time τ equals $s(\tau)$, an order of $Q_2(\tau)$ is placed. Let $g_\tau(t)$ be the pdf of the random variable τ measuring the time until the inventory first hits $s(\tau)$ and the second order is placed. We show how a pair $(s(\tau), Q_2(\tau))$ may be determined and then approximate the distribution $g_\tau(t)$.

We make no claim that an $(s(\tau), Q_2(\tau))$ policy is optimal nor that the functions we find are optimal. However, such a policy does seem a reasonable one, assuming $Q_2(\tau)$ is a critical fractile policy as given below, and may be seen as the continuous review counterpart to that considered in Graves (1999) for a periodic review system. Furthermore, Morton and Pentico (1995) have shown that critical fractile policies, though not always optimal, are near optimal for finite horizon inventory problems with nonstationary demand. We also note the similarity between our approximation and that given in Hadley and Whitin (1963, p. 169) for computing the reorder point/reorder quantity for an infinite horizon continuous review ordering policy for known, stationary demand. Our approximation determines an order quantity that reflects the demand determined implicitly by the reorder point.

Let $TC_2^F(\tau)$ be the cost in the second segment when an order is placed at time τ (with the second segment starting at time $\tau + L$). Then given a policy $s(\tau)$ and $Q_2(\tau)$,

$$\begin{aligned} TC_2^F(\tau) = {} & cQ_2(\tau) \\ & + h\Bigg(\bigg(Q_2(\tau) + \int_{\xi=0}^{s(\tau)} (s(\tau) - \xi) f_L(\xi \mid z(\tau))\, d\xi\bigg) \\ & \quad \times (T - \tau - L) - \frac{\bar{\mu}}{2}(T - \tau - L)^2\Bigg) \\ & + p \int_{\xi=0}^{s(\tau)} \int_{y=Q_2(\tau)+s(\tau)-\xi}^{\infty} (y - (Q_2(\tau) + s(\tau) - \xi)) \\ & \quad \times f_2(y \mid z(\tau) + \xi) \\ & \quad \times f_L(\xi \mid z(\tau))\, dy\, d\xi \\ & + p \int_{\xi=s(\tau)}^{\infty} \int_{y=Q_2(\tau)}^{\infty} (y - Q_2(\tau)) f_2(y \mid z(\tau)_\xi) \\ & \quad \times f_L(\xi \mid z(\tau))\, dy\, d\xi \end{aligned} \tag{8}$$

where $z(\tau) = Q_1 - s(\tau)$ and $f_L(\xi \mid z)$ and $f_2(y \mid z + \xi)$ are as defined for the partially dynamic model.

For a given τ and $s(\tau)$ we can solve for $Q_2(\tau)$ through the first-order sufficient condition (note we suppress the dependence on τ):

$$\int_{\xi=0}^{s} F_2(Q_2 + s - \xi \mid z + \xi) f_L(\xi \mid z)\, d\xi + \int_{\xi=s}^{\infty} F_2(Q_2 \mid z + \xi) f_L(\xi \mid z)\, d\xi = \frac{p - c - h(T - \tau - L)}{p}. \tag{9}$$

Comparing (9) and (6) we note that in the fully dynamic model, $s(\tau)$ is the on-hand inventory at the time of ordering and by definition $Q_2 > 0$.

An approximate value of $s(\tau)$ can be found by marginal analysis. Clearly if $s(\tau)$ is too small, there is a chance that there will be a shortage. The relevant cost is

$$p(\text{expected number short given order policy } s(\tau))$$
$$= p \int_{s(\tau)}^{\infty} (\xi - s(\tau)) f_L(\xi \mid z(\tau)) \, d\xi$$

so that the marginal cost is $p(1 - F_L(s(\tau) \mid z(\tau)))$. Next, if $s(\tau)$ is too large, there are extra holding charges incurred. As we are ordering $Q_2(\tau)$ units, the holding cost increases by $hQ_2(\tau)\Delta t$ if the units are ordered Δt time units earlier. But on the margin $\Delta t = \Delta s/\bar{\mu}$. So the marginal holding cost is $hQ_2/\bar{\mu}$. Therefore, equating the additional shortage and holding costs implies

$$F_L(s(\tau) \mid z(\tau)) = \frac{p - hQ_2(\tau)/\bar{\mu}}{p}. \tag{10}$$

By searching for values of $Q_2(\tau)$ and $s(\tau)$ which satisfy (9) and (10), we find a feasible ordering policy.

To complete the analysis of the fully dynamic case we need to approximate $g_\tau(t)$, the distribution of τ. Consider first the time until a stock-out given an initial inventory Q_1. Let Ω be this random variable. A simple approximation is to assume that the distribution of Ω given θ is normally distributed with mean Q_1/θ and variance $Q_1\sigma^2/\theta^3$ (see Feller 1968, p. 372). Because we are concerned not with the time until stocking out, but the time until we first reach $s(\tau)$, consider

$$h(t; \tau \mid \theta) = \frac{1}{\sigma\sqrt{(Q_1 - s(\tau))/\theta^3}} \phi\left(\frac{t - (Q_1 - s(\tau))/\theta}{\sigma\sqrt{(Q_1 - s(\tau))/\theta^3}}\right),$$

which is the conditional density function for the time, t, until demand in the first period equals $Q_1 - s(\tau)$ for a given value τ. Then the unconditional time is $h(t; \tau) = \int h(t; \tau \mid \theta)\pi(\theta) \, d\theta$. Now consider the function $h(t) = h(t; t)$ that traces the values of the pdf of the time until demand equals $Q_1 - s(t)$ for all t. Note that $h(t)$ is well defined but may not be a probability density function. We approximate $g_\tau(t)$ by normalizing $h(\tau)$ to find

$$g_\tau(t) = h(t) \bigg/ \int h(t) \, dt.$$

That is, we determine the time until the second order is placed as the normalized function, tracing the probability density that the order is placed at each point in time.

The total cost is then

$$\begin{aligned} TC^F = \min_{Q_1} \, & cQ_1 \\ & + h \int_{\tau=0}^{T-L} \left(Q_1(\tau + L) - \frac{\bar{\mu}(\tau + L)^2}{2} \right) g_\tau(\tau) \, d\tau \\ & + p \int_{\tau=0}^{T-L} \int_{y=s(\tau)}^{\infty} (y - s(\tau)) f_L(y \mid z(\tau)) g_\tau(\tau) \, dy \, d\tau \\ & + E_\tau[TC_2^F(\tau)]. \end{aligned} \tag{11}$$

4. Numerical Results

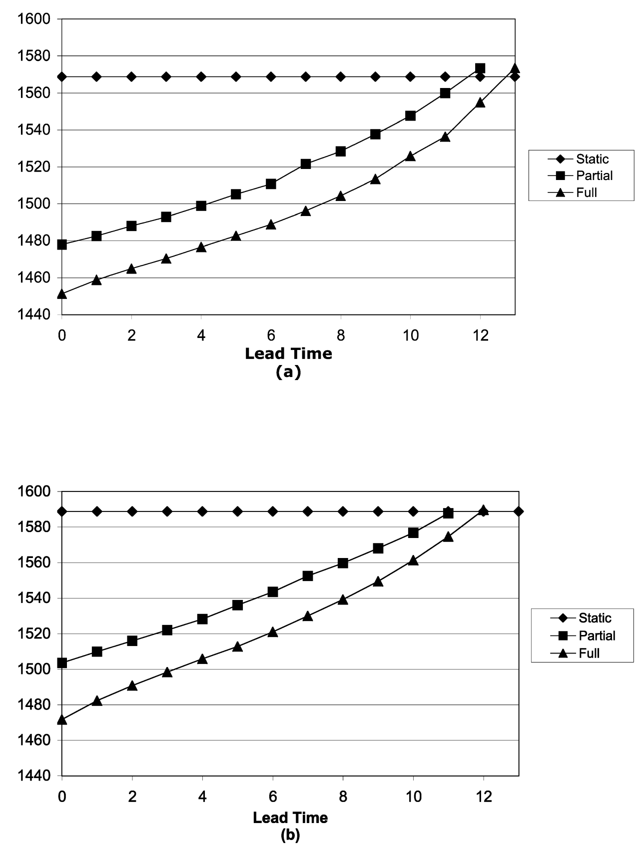
In this section we discuss the results of numerical tests that provide an understanding of the relationship between the value of updated demand information and production (ordering) and supplier flexibility. We present a representative set of tests and sensitivity analysis to derive insights related to several important parameters of the Bayesian model, namely, μ_1, μ_2, σ^2, ν_1^2, and υ_2^2 and $\alpha_1 = 1 - \alpha_2$.

To assure a high service level we use the following parameter values: $c = 3$ per unit, $p = 40$ per unit, and $h = 0.1$ per unit per unit time. In the models studied, we set $T = 20$. Note that in a deterministic model, a unit purchased would be held on average $T/4 = 5$ periods so that the average holding cost would be $h' = 0.5$ per unit. This implies a critical fractile value of $(p - c)/(p + h') = 0.91$. In Table 1 we present the parameter values and the resulting mean, variance (Var), and C.V. of demand for $t = 10$. We also present the percentage savings of the fully dynamic model from the static model, % savings = $(TC^S - TC^F)/TC^F \times 100\%$, for $L = 0$. The variance of the demand is comprised of two parts (cf. (2)). We present the percentage of the variance associated with uncertainty in the mean, $\%\text{Var}_\mu = (\text{Var} - t\sigma^2)/\text{Var} \times 100\%$ because $t\sigma^2$ is the variance associated with uncertainty for a known mean. Finally, we present the percentage value of information, i.e., the percentage of the savings from the static model to the fully dynamic model attributable to the partially dynamic model, $\%V_{INF} = (TC^S - TC^P)/(TC^S - TC^F) \times 100\%$, for $L = 0$ ($\%V_{INF}(L)$ for a lead time L). As discussed in §1, we attribute this share of the savings to the value of the updated demand information, while the share from the partially dynamic to the fully dynamic model is attributed to the value of production flexibility. Thus, we can also infer the value of production flexibility from the table, i.e., 100% − (%V_{INF}).

As is to be expected, we observe that the relative savings of the fully dynamic model increases with the C.V. of the demand. That is, the total value of updated demand information and production and supplier flexibility combine for an increasing share of the static model's total cost as the variability in the model increases. This reflects the general intuition that information and production flexibility have increasing value as the degree of uncertainty increases. We also observe that the share of the savings attributable to information increases as the percentage of the variability attributable to uncertainty in the mean increases. We note, though, that as %Var_μ decreases, the value of information remains considerable (cf. Case 8).

In Figures 1 (a) and (b), we plot the total cost versus lead time for the three ordering flexibility models (static and partially and fully dynamic) for the cases of $\sigma^2 = 4.0$ and $\sigma^2 = 8.0$, respectively (Cases 2 and 3 in Table 1). The total cost in the latter case is approximately 1.4% higher in the fully dynamic case for $L = 0$ than in the former as would be expected with greater variability and is similarly higher for the other models and other lead times. We observe that as the lead time increases, the partially dynamic cost curve increases in a convex manner. The fully dynamic cost curve initially increases in a concave manner until approximately the point where lead time equals five, after which it increases in a convex manner (the static case is clearly insensitive to lead time). When the lead time is very long (greater than 13), the partial and fully dynamic models are equivalent to the static model. In that case, there is insufficient time to make an observation regarding the mean of the demand and place an updated order. For shorter lead times we observe that there are cost savings between the static model and the dynamic models (such savings represent the summed up value of demand information and production flexibility), with the maximum observed value of such savings being approximately 7.5% of the static cost for the case of no lead time. We observe that increasing the variability around the mean, σ^2, increases both the partially and fully dynamic model costs and so shortens the lead time over which they provide benefit over the static model.

Figure 1 Total Cost Versus Lead Time for the Three Ordering Flexibility Cases (Static, Partially, and Fully Dynamic). (a) $\sigma^2 = 4.0$; (b) $\sigma^2 = 8.0$

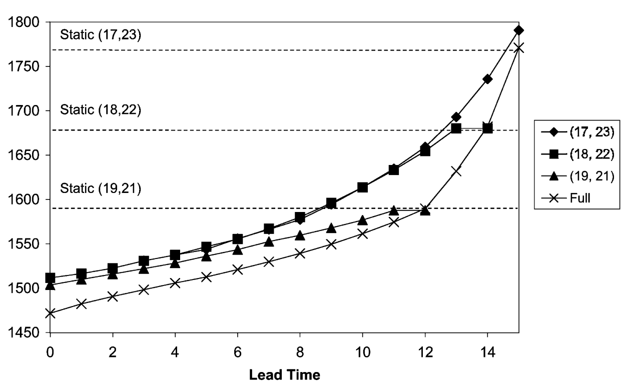


Next, we investigate how varying μ_1 and μ_2 affects the total cost. We present in Figure 2 the static and partially dynamic cost curves for three cases (Cases 4, 9, and 10 in Table 1), maintaining a mean demand of 20 units per unit time. We also present the fully dynamic total cost for Case 10 (the fully dynamic cost curve for Cases 4 and 9 differ from this cost curve by no more than 1% and have been left out of the figure for clarity). We observe that the static model's cost increases with the difference between μ_1 and μ_2, as would be expected given the increasing demand var-

Table 1 Parameter Values for Initial Test Cases with Mean, Variance, and C.V. of Demand for $t = 10$

Case	μ_1	μ_2	σ^2	$v_1^2 = v_2^2$	α_1	Mean	Var.	C.V.	% Savings	% Var_μ	% $V_{INF}(0)$
1	19	21	1	0.2	0.5	200	130	0.057	8.0	92.3	83.2
2	19	21	2	0.2	0.5	200	140	0.059	7.7	85.7	81.7
3	19	21	4	0.2	0.5	200	160	0.063	7.5	75.0	77.3
4	19	21	8	0.2	0.5	200	200	0.070	7.8	60.0	72.8
5	19	21	16	0.2	0.5	200	280	0.084	8.1	42.8	65.2
6	19	21	32	0.2	0.5	200	440	0.105	9.2	27.3	61.4
7	19	21	64	0.2	0.5	200	760	0.138	10.9	15.9	60.0
8	19	21	128	0.2	0.5	200	1400	0.187	13.7	8.6	59.8
9	18	22	8	0.2	0.5	200	500	0.112	12.4	84.0	82.2
10	17	23	8	0.2	0.5	200	1000	0.158	16.9	92.0	86.1
11	19	21	4	0.1	0.5	200	150	0.061	7.2	73.3	76.9
12	19	21	4	0.5	0.5	200	160	0.069	9.0	75.0	78.1
13	19	21	4	1	0.5	200	240	0.077	10.8	83.3	80.5
14	19	21	4	2	0.5	200	340	0.092	13.3	88.2	81.1
15	19	21	4	4	0.5	200	540	0.116	17.1	92.6	83.0
16	10	20	6.25	4	0.1	190	1363	0.194	16.8	95.4	93.0
17	10	20	6.25	4	0.3	170	2563	0.298	24.9	97.6	94.7
18	10	20	6.25	4	0.5	150	2963	0.363	33.6	97.9	93.3
19	10	20	6.25	4	0.7	130	2563	0.389	41.6	97.6	91.3
20	10	20	6.25	4	0.9	110	1363	0.335	44.5	95.4	86.8

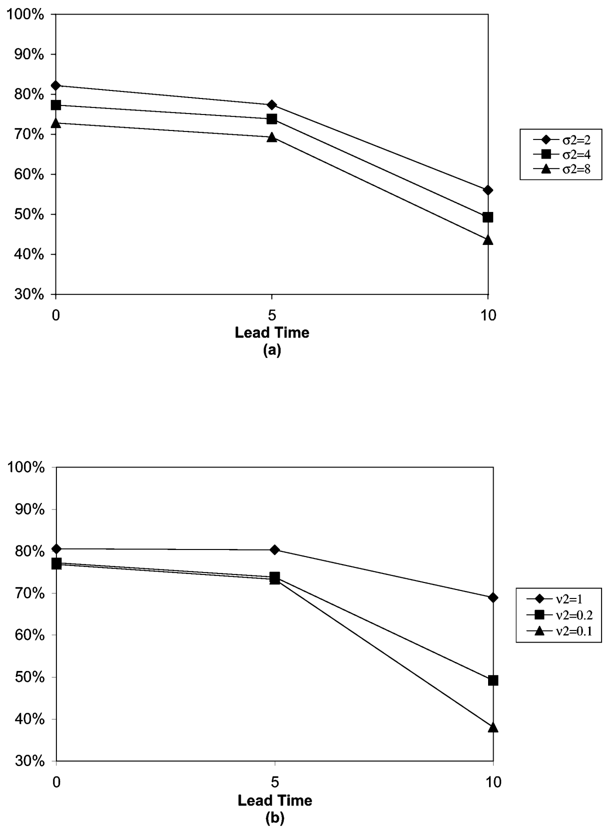
Figure 2 Total Cost Versus Lead Time Varying μ_1 and μ_2 Symmetrically



iance. For small lead times, there is little difference among the partially dynamic costs in the three cases (and, as previously noted, the fully dynamic costs). Only with long lead times does a difference between the three cases emerge. Thus, for the case of short lead times most of the uncertainty regarding the mean of the demand (whether it is closer to μ_1 or μ_2) may be resolved prior to placing the second order. Short lead times allow adequate demand observation, even for a partially dynamic model, as long as there is any reasonable choice of the timing of the second order. When lead times are long, however, the available demand observation period is seriously constrained, making it more difficult to resolve the mean demand uncertainty. This proves increasingly costly the greater the spread between μ_1 and μ_2, as a wrong guess of a low (high) mean demand leads to stockouts (high inventory) costs. Thus the value of production flexibility increases for long lead times as the spread increases.

We next consider how changing σ^2, the uncertainty around the mean, and ν^2, the uncertainty in the mean, differ in their effect on the relative value of information. In Figure 3(a) we graph the percentage value of information as a function of the lead time ($\%V_{INF}(L)$). We observe that the relative value of information is decreasing with lead time. Further, we observe that increasing uncertainty around the mean further reduces the value of information. Because σ^2 measures the uncertainty in the demand that cannot be resolved through observation, as it increases, the value of information obtained decreases. However, the val-

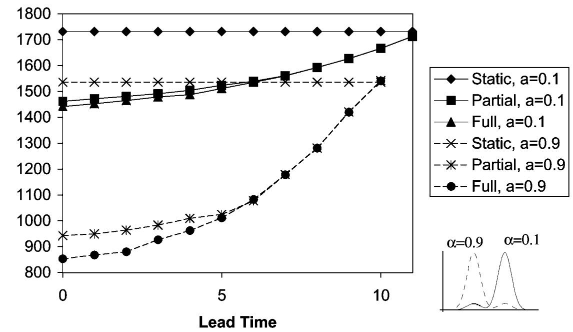
Figure 3 Percentage of Total Savings Attributable to Information Varying as a Function of Lead Time ($\%V_{INF}(L)$) (a) σ^2; (b) ν^2



ue of reducing σ^2 has the same impact on both the partially and fully dynamic model. Therefore, there is little difference in the relative value of information for different σ^2's at the different lead times.

Letting $\nu^2 = \nu_1^2 = \nu_2^2$, we present in Figure 3(b) the percentage value of information as a function of lead time ($\%V_{INF}(L)$) for several cases of ν^2. Recall that ν^2 measures the uncertainty in the prior distribution. For small lead times the value of ν^2 has little effect on the value of information, while for long lead times, as ν^2 decreases, the value of information decreases (or, in contrast, the value of production flexibility increases). In the case of short lead times, there is adequate time to reveal the true nature of the demand rate distribution leading to a high value of information. In the case of long lead times, because of the forced early placement of the second order, production flexibility

Figure 4 Total Cost Versus Lead Time for Asymmetric Cases $\alpha_1 = 0.1$ and $\alpha_1 = 0.9$



is increasingly valuable as the true nature of the demand mean is known. By reducing the uncertainty in the mean, the firm can use the production flexibility available in the fully dynamic model to order at the appropriate time and thus increase the relative value of production flexibility. To sum up: The greater the uncertainty *around* the mean, the lower the value of information relative to the value of production flexibility; the greater the uncertainty *in* the value of the true mean demand, the greater the value of information, particularly for long lead times.

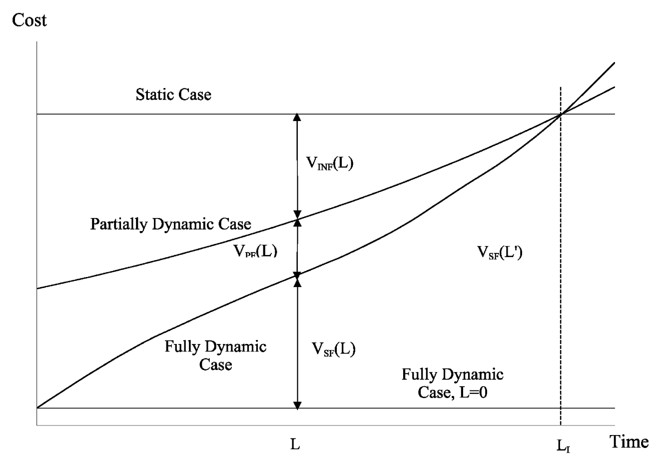
We have also studied the effects of asymmetry in the demand rate distribution. (Symmetry implies two equally uncertain normal prior distributions around μ_1 and μ_2.) We present two complementary asymmetric cases in Figure 4 (Cases 16 and 20 in Table 1). The priors are depicted in the figure. In the case of $\alpha_1 = 0.1$ (see the previous definitions of α_1 and α_2 given in §3.1), demand is expected to be high, but there is a small chance that it will be low, and the case of $\alpha_1 = 0.9$ is the converse. We observe that with $\alpha_1 = 0.1$ the maximum savings from the static to the fully dynamic model with no lead time is similar to the symmetric cases discussed, approximately 16.8%. This follows intuitively from the expectation that the firm would purchase inventory to cover the expected high demand, and if in fact the demand is low, the firm will not reorder. In the case of small α_1, the extra inventory costs might penalize an inappropriate assumption of high demand, but they are not so significant for reasonable length seasons because a canceled second order can temper them. However, in the case of $\alpha_1 = 0.9$, the difference from the static to the fully dynamic case is approximately 45% of the static case's total cost. In this case, the firm should plan for low demand. If, however, high demand is observed, the partially dynamic model gains its advantage over the static case by correctly sizing the second order. The fully dynamic model further improves upon it through avoiding some first segment stock-outs. With long lead times, for both $\alpha = 0.1$ and $\alpha = 0.9$, we observe little difference between the partially and fully dynamic cases. The savings in the first period provided by the fully dynamic model over the partially dynamic model disappear; the particular demand case is fully resolved so that the relative value of information in this case is high.

Finally, we would like to remark that in all our experiments for a wide range of parameter values, we observed a strongly robust result: The fully dynamic cost curve increases initially with supply lead times in a concave manner (typically for lead time values less than ¼ of the season length), and for larger lead time values, it assumes a convex and increasing shape. The reader can clearly see this result in Figures 1 and 2. We speculate that this behavior may be explained by considering the variance over the lead time, $\sigma_B^2 L + L^2(\sum_{i=1,2} w_{i}\mu_i^2 - (\sum_{i=1,2} w_i\mu_i)^2) + L^2 \sum_{i=1,2} w_i\bar{\nu}_i^2$, where the weights w_i and $\bar{\nu}_i^2$ reflect the uncertainty in the demand mean as given in (3). If the cost reflects this uncertainty, it increases with L according to $\sigma_B\sqrt{L} + \psi(L)L$, where $\psi(L)$ depends on the uncertainty of the demand at the time the order is placed. Observe that as L increases, the time at which the second order is placed, τ, decreases. We argue that $\psi(L)$ grows superlinearly as there is a factor τ in the denominator of each term (note the dependence of w_i on $\hat{\sigma}_i$). For small values of L (large values of τ), the uncertainty does not change greatly, so that the fully dynamic cost curve rises as $\sigma_B\sqrt{L}$. For intermediate values of L, the curve is approximately linear, and for large values of L, the uncertainty grows quickly as τ decreases, resulting in the convex curve.

5. Discussion

The framework developed in this study allows us to better understand some of the factors that influence the relative value of investment in information updates, production flexibility and supplier flexibility. Some fundamental insights of our framework are graphically represented in Figure 5. We interpret the cost difference between the static case and the fully dynamic case with no lead time constraint as being comprised of three values. We define the value of the updated demand information, $V_{INF}(L)$, to be the difference between the static case and the partially dynamic case; the value of production flexibility, $V_{PF}(L)$, as the difference between the partially and fully dynamic models; and the value of supplier's flexibility, $V_{SF}(L)$, to represent the difference between the fully dynamic model with lead time L and that achievable in a fully dynamic model with no lead time. We explain these values and their relationship to the demand process using Figure 5.

Figure 5 **Supplier Lead Time Effects on the Value of Updated Information, Production Flexibility, and Cost (or Value) of Supplier Inflexibility (or Flexibility)**



The total magnitude of the value of information and production and supply flexibility reflects in an intuitive way the sources of variability in the model. We studied the main sources of variability: differences between high and low demand cases, the uncertainty regarding the mean demand value, ν^2, and the uncertainty about the mean demand, σ^2. We found that the total of V_{INF}, V_{PF}, and V_{SF} increases with the C.V. of the demand over each segment (assuming an equal division of the period). Increasing the difference between high and low demand cases greatly increases V_{INF}, though only slightly alters the V_{PF} and V_{SF}—the ability to adjust orders to reflect the true demand provides most of the benefit. In general, we found that the division between V_{INF} and V_{PF} is in rough accordance with the division between variability associated with the prior and that inherent in the demand. For the cases we studied, this indicates a larger value of information than production flexibility. The division between the two, however, clearly depends on the lead time. We observe that production flexibility is increasingly valuable when the true demand mean can be resolved and lead times are long, constraining the value of updating demand information. The relative value of information is increasingly valuable as the uncertainty around the demand mean decreases. This is the uncertainty remaining if the true demand were known.

We observe that the value of supply flexibility grows with lead time, L, first in a concave manner and then in a convex manner. For small lead time values, the total cost of the fully dynamic model behaves in qualitatively similar ways to traditional inventory models (growing like $\sqrt{L}$). However, for large lead time values, the total cost exhibits the convex increasing behavior with respect to lead time effects as in recent research studies of inventory models with nonstationary demand processes. The resulting initially concave—subsequently convex—shape of the total cost curve as a function of lead time supports, at least on the potential benefits side, an emphasis on investment in supplier flexibility and information updating for supply chains with longer lead times. Investment in production flexibility need only be made after supply lead times have been substantially reduced. However, this qualitative observation is not meant to serve as a substitute to a rigorous analysis including both investment costs and derived benefits for all alternative flexibility investment options.

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