



## Manufacturing & Service Operations Management

Publication details, including instructions for authors and subscription information:  
<http://pubsonline.informs.org>

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To cite this article:

Soo-Haeng Cho, Christopher S. Tang, (2013) Advance Selling in a Supply Chain Under Uncertain Supply and Demand. Manufacturing & Service Operations Management 15(2):305-319. <http://dx.doi.org/10.1287/msom.1120.0423>

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# Advance Selling in a Supply Chain Under Uncertain Supply and Demand

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We examine three selling strategies of a manufacturer who produces and sells a seasonal product to a retailer under uncertain supply and demand: (1) advance selling—presells the product before observing uncertain supply and demand; (2) regular selling—sells the product after supply and demand are realized; and (3) dynamic selling—combines both advance and regular selling strategies. We model the first two strategies as single-period Stackelberg games, and we model the last strategy as a two-period dynamic Stackelberg game. By comparing the equilibria of these games, we formalize our understanding of several intuitive results. For example, from the manufacturer's perspective, dynamic selling dominates advance selling and regular selling: having more selling opportunities is beneficial to the manufacturer. However, from the retailer's perspective, we find two counterintuitive results: (a) postponing the ordering decision can be detrimental—the retailer can be worse off under regular selling than under advance selling; and (b) more ordering opportunities can be detrimental—the retailer can be worse off under dynamic selling than under advance selling. In addition, we analyze the impact of supply and demand uncertainties under these strategies and find that both types of uncertainties can be beneficial to the retailer.

*Key words:* game theory; healthcare management; supply chain management

*History:* Received: January 9, 2012; accepted: October 4, 2012. Published online in *Articles in Advance* March 1, 2013.

## 1. Introduction

In the flu vaccine industry, mismatches of demand and supply occur regularly because production yield is highly uncertain owing to the biological nature of vaccine production. At the same time, vaccine demand is also highly uncertain because it is primarily determined by the prevalence and severity of unpredictable flu activities (Williams 2005). In the United States, for example, there was an oversupply of 18.4 million doses in the 2006–2007 flu season, whereas there was an undersupply of nearly 50 million doses in the 2004–2005 flu season due to contamination at the Chiron's plant (Health Industry Distributors Association 2007). This paper evaluates different selling strategies that flu vaccine manufacturers can choose in order to manage uncertain supply and demand.

Consider a manufacturer ("he") who produces and sells flu vaccines through a healthcare provider as a retailer ("she"). Whereas the capacity of the manufacturer is fixed, production yield and market demand are uncertain. Then should the manufacturer sell his vaccines before the uncertainties are resolved, after the uncertainties are resolved, or before and after

the uncertainties are resolved? At the same time, will the retailers benefit from postponing their orders after the uncertainties are resolved? Will they benefit from an additional ordering opportunity?

To examine these questions, we examine three selling strategies: (i) advance selling strategy—the manufacturer sells the product to retailers only before uncertain yield and demand are realized; (ii) regular selling strategy—the manufacturer sells the product only after yield and demand are realized; and (iii) dynamic selling strategy—the manufacturer combines both advance and regular selling strategies. There are several trade-offs arising from these three selling strategies. First, depending on the realized supply and demand, the prebook wholesale price that the manufacturer charges in equilibrium under advance selling can be higher or lower than the regular wholesale price. Second, the retailer bears the risks of overstocking and understocking under advance selling, but not under regular selling because the ordering decision is postponed until all uncertainties are resolved. On the other hand, the retailer's overstocking under advance selling is beneficial to the manufacturer because the retailer is committed

to her prebook order. Third, under the dynamic selling strategy, the manufacturer and the retailer need to examine the dynamic interactions associated with both advance and regular selling strategies.

We model these three selling strategies as three different Stackelberg games in which the manufacturer acts as the leader and the retailer acts as the follower. Our model captures the following characteristics of a flu vaccine supply chain, while other industries exhibit similar characteristics, for example, Xilinx reserves the capacity of a semiconductor foundry such as United Microelectronics Corporation through prepurchase commitment (see more examples in Erhun et al. 2008).

- *Uncertain supply and demand.* As described above, both supply and demand are fundamentally uncertain in this industry. Because the causes of supply and demand uncertainties are unrelated, we shall assume that yield and demand are not correlated.

- *Fixed capacity and full utilization.* The production capacity is known and fixed in each flu season because building or expanding the capacity is costly and time consuming (three to five years) due to the stringent U.S. Food and Drug Administration approval process (Matthews 2006). Also, the manufacturer often runs his plant at near full capacity because its fixed cost is much higher than its variable cost. One plant manager of a major flu vaccine manufacturer said in our interview: “We dive headfirst making as much as we can.”

- *No demand forecast updating.* Because of the long production lead time of six to eight months, the prebook orders of flu vaccines are placed long before the actual flu season. Thus, these prebook quantities have no information value. However, for tractability, we assume that the second period takes place after all uncertainties have been resolved. This assumption is commonly used in the related literature (e.g., Van Mieghem and Dada 1999, Cachon 2004, Chod and Rudi 2005, Erhun et al. 2008).

- *Stackelberg competition.* The Stackelberg vertical competition is appropriate in the flu vaccine industry where a large pharmaceutical manufacturer sells flu vaccines to smaller healthcare providers at the same price. For example, for the 2010–2011 season, GlaxoSmithKline offered the same wholesale price to smaller healthcare providers through an intermediary firm, MedAssets, and to a large healthcare provider in Pennsylvania (with annual revenues over \$1 billion).

- *Uncertain retail price.* Based on anecdotal evidence (e.g., Fine 2004) and our interview with practitioners, the retail price of flu vaccines as well as the regular wholesale price tends to be higher (or lower) than the average when the actual demand is higher (or lower) than the actual supply. The ex ante uncertain regular wholesale price is a significant concern for retailers

because a regular wholesale price can be substantially high during the shortage period; e.g., \$7 per dose as compared with \$3 per dose for the prebook order during the 2000–2001 season (United States General Accounting Office 2001).

Because of the intricacies of noncooperative games under uncertain supply and demand, it is unclear which strategy would yield a higher profit for the manufacturer or the retailer in equilibrium. By comparing the equilibrium outcomes of three different Stackelberg games, we formalize our understanding of several intuitive results. For example, from the manufacturer’s perspective, the dynamic selling strategy dominates both advance selling and regular selling strategies: having two selling opportunities is beneficial to the manufacturer. However, from the retailer’s perspective, we find two counterintuitive results: (a) postponing the ordering decision can be detrimental—although regular selling allows the retailer to place her order after all uncertainties are resolved, the retailer can be worse off under regular selling than under advance selling; and (b) more ordering opportunities can be detrimental—although dynamic selling offers two ordering opportunities, the retailer can be worse off under dynamic selling than under advance selling. In addition, from the manufacturer’s perspective, we find that demand uncertainty is beneficial, whereas supply uncertainty is detrimental. However, from the retailer’s perspective, counterintuitively, supply uncertainty can be beneficial to the retailer under all three strategies. We examine the conditions under which these counterintuitive results occur.

## 2. Literature Review

Our paper is related to four research streams: (1) advance booking/selling that is akin to quick response (QR), (2) price postponement, (3) production planning under random yield and demand, and (4) operations research in the flu vaccine industry.

(1) *Advance booking/selling.* Fisher and Raman (1996) are the first to introduce the idea of advance booking in the retail industry. In this stream of work, Iyer and Bergen (1997) analyze the benefit of QR when the retailer can place an order after observing partial demand information; Gurnani and Tang (1999) analyze the retailer’s two ordering decisions when the wholesale price for the prebook order is known but the wholesale price for the regular order is uncertain; Donohue (2000) examines how the wholesale price and returns policy can achieve channel coordination under two production modes; Brown and Lee (2003) analyze a situation in which the retailer can reserve the manufacturer’s capacity in advance and purchase an additional amount later at an extra price after

observing demand information; Özer and Wei (2006) study advance purchase contracts when there is information asymmetry about demand, and show that it can be optimal for the manufacturer to charge a higher wholesale price for the advance order than the regular order; and Özer et al. (2007) show that the dual purchase contract, through which the manufacturer provides a discount for the retailer's orders placed before her forecast update, can improve the supply chain efficiency over a wholesale price contract. In the context of risk allocation between members of a supply chain, Ferguson (2003) and Ferguson et al. (2005) examine how a part price and the timing of production and ordering decisions affect the distribution of supply chain profits between a buyer and a supplier. Using a two-period Newsvendor model, Cachon (2004) shows that a manufacturer can shift the risk of having excessive inventory to a retailer by offering a price discount for prebook orders to the retailer. The reader is referred to Choi and Sethi (2010) for a comprehensive review of this literature.

Our work contributes to this research stream in two ways. First, we analyze the value of advance selling in the presence of *both supply and demand uncertainties* and highlight the different impact of demand and supply uncertainties. We adopt a price-sensitive demand model to capture the effect of the imbalance between supply and demand due to uncertainties. Second, we determine the manufacturer's wholesale price and the retailer's order quantity in each period endogenously by analyzing a *two-period dynamic game*. In most existing models, the manufacturer's wholesale prices to the retailer are given exogenously or set much in advance of the selling season. Also, except Özer and Wei (2006), the prebook price is assumed or determined lower than the regular price. However, because of long production lead time, it is common for a manufacturer to postpone his pricing decision for the regular order. In this setting, because the regular wholesale price is not known to the retailer in advance, the retailer has an incentive to place a prebook order even without a price discount.

(2) *Price postponement*. In this literature, a firm is allowed to delay its pricing decision after uncertainty is resolved. Van Mieghem and Dada (1999) present a two-period model in which a firm makes three decisions: capacity, price, and production quantity. To evaluate the benefits of price and production postponements, they analyze various scenarios in which the firm can postpone part or all of those decisions after demand uncertainty is resolved in the second period. Chod and Rudi (2005) study the value of price postponement and resource flexibility. Both papers show that a firm can benefit from postponing its decision after the demand uncertainty is resolved, and that the benefit of price postponement tends to

increase as demand uncertainty increases. Recently, Tang and Yin (2007) analyze the issue of price postponement when supply is uncertain.

Our paper extends this literature into three different directions as follows. First, we examine the value of postponement in the *two-level supply chain* in which the manufacturer and the retailer engage in vertical competition. Similar to Van Mieghem and Dada (1999) in the single-firm setting, we find that the manufacturer is better off under regular selling than advance selling. However, postponing the ordering decision under regular selling can be detrimental to the retailer. Second, we examine the value of postponement under *both supply and demand uncertainties*. The presence of both types of uncertainties leads to several unexpected results. For example, the benefit of postponement to the manufacturer increases with demand volatility, whereas it decreases with supply volatility. However, both demand and supply uncertainties can be beneficial to the retailer. Third, we examine dynamic interactions between two firms by analyzing a two-period *dynamic Stackelberg game*. We obtain the counterintuitive result that more ordering opportunities under dynamic selling can be detrimental to the retailer when there is a potential supply shortage. Although this result is similar to the result obtained by Erhun et al. (2008) (who also study the dynamic pricing and ordering decisions in a supply chain) under a binary demand distribution, our result is obtained under a more general setting with both supply and demand uncertainties that follow general probability distributions.

(3) *Production planning under random yield and demand*. The primary focus of this stream of research is to determine the optimal production quantity of a *single* manufacturer who faces both uncertainties when the cost and price of a final product are given exogenously (Yano and Lee 1995). Kazaz (2004) departs from this literature by assuming that the production cost and the sales price, while being still exogenous, are inversely impacted with the realized yield. Tomlin and Wang (2008) employ a utility-maximizing customer model and analyze the benefit of postponing the pricing and quantity decisions of two vertically differentiated products. As a related work, Federgruen and Yang (2008, 2009) study the procurement decision of a buyer from multiple suppliers under demand and supply risks when costs and prices are fixed. Although our model also involves both supply and demand risks, our research questions (firms' performance under advance, regular, or dynamic selling strategies) are quite different from those in this literature. Our model is also different in that a manufacturer *endogenously* determines his wholesale prices to retailers dynamically over two



periods, and a retail price is dependent on both random yield and demand.

(4) *Operations research in the flu vaccine industry.* Several researchers have examined various issues in this industry. For example, Chick et al. (2008) consider a cost-sharing contract between a manufacturer and a government when the government could determine how many individuals to vaccinate at the fixed administration cost. Deo and Corbett (2009) analyze the effect of yield uncertainty on competition among manufacturers. Cho (2010) characterizes the impact of the vaccine composition decision on subsequent production decisions, and vice versa. Our paper contributes to this literature by (a) examining tactical decisions in the downstream of this supply chain, namely, a manufacturer's pricing decisions and a retailer's ordering decisions; (b) examining dynamic pricing and ordering decisions over two periods; and (c) taking into account both demand and supply uncertainties and highlighting their different roles in this supply chain.

### 3. Model

Consider a two-level supply chain that is comprised of one manufacturer and one retailer. The manufacturer ( $M$ ) uses his production capacity  $k$  to produce a seasonal product and sells this product through the retailer ( $R$ ) over a short selling season. Because of the long production lead time, the manufacturer needs to start his production long before the start of the selling season. For simplicity, we assume that a variable production cost is zero, and that unsold products are discarded after the season is over. Each firm maximizes its own expected profit. Let  $\Pi_M$  and  $\Pi_R$  denote the manufacturer's profit and the retailer's profit, respectively.

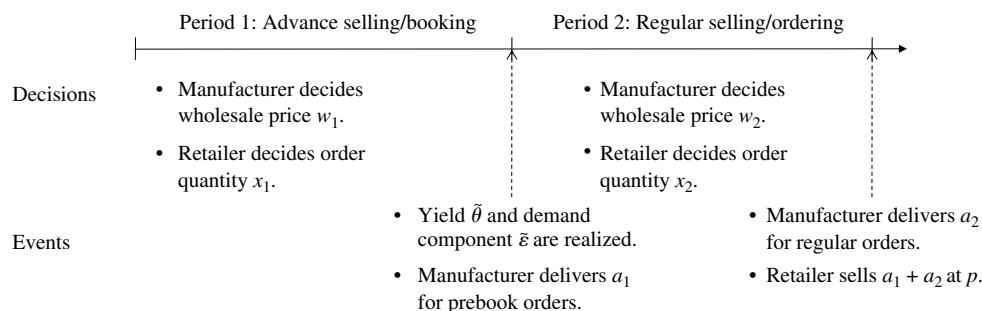
Although the capacity  $k$  is set a priori, the actual output is subject to a random production yield  $\theta$ . Using a proportional random yield model (e.g., Yano and Lee 1995, Cho 2010), we represent the manufacturer's actual supply as  $k\theta$ , where  $\theta$  follows a general cumulative distribution  $F$  with support over  $[\underline{\theta}, \bar{\theta}]$ , where  $\underline{\theta} \geq 0$ . We normalize  $E[\theta] = 1$  and allow

$\bar{\theta}$  to be greater than one. The retailer faces a linear demand curve so that the retail price  $p = 1 + \tilde{\varepsilon} - q$ , where  $\tilde{\varepsilon}$  represents a random demand component and  $q$  represents the quantity available for sales by the retailer. We assume the inventory "clearance" rule under which the retailer sells  $q$  to the market, while referring the reader to the online supplement (available at <http://dx.doi.org/10.1287/msom.1120.0423>) for our analysis of the "holdback" rule (also used in Erhun et al. 2008) under which the retailer can withhold some units procured from prebooking to sell them at a higher retail price. Similar to Chod and Rudi (2005), we find that the qualitative insights derived for the clearance rule are representative for the holdback rule. Note that the retail price  $p$  is *effectively random* because  $p$  depends on  $\tilde{\varepsilon}$  and  $q$ , and that  $q$  depends on the uncertain supply  $k\theta$ . We assume that  $\tilde{\varepsilon}$  follows a general cumulative distribution  $G$  with support over  $[\underline{\varepsilon}, \bar{\varepsilon}]$ , where  $E[\tilde{\varepsilon}] = 0$ . Thus, the expected market potential,  $E[1 + \tilde{\varepsilon}]$ , is normalized to one. To ensure  $p$  is positive ex post, the lower bound of  $\tilde{\varepsilon}$  should not be too small. It turns out that  $\underline{\varepsilon} \geq -0.75$  is sufficient to guarantee positive price in equilibrium (see the online supplement). We assume that  $\tilde{\varepsilon}$  and  $\theta$  are independent. The realized values of  $\tilde{\varepsilon}$  and  $\theta$  are denoted by  $\varepsilon$  and  $\theta$ , respectively. The distributions of  $\tilde{\varepsilon}$  and  $\theta$  as well as the capacity  $k$  are common knowledge to both firms.

The manufacturer sells the product to the retailer by using one of the following three strategies:

(1) *Dynamic selling strategy.* Both firms play a two-period *dynamic* Stackelberg game as depicted in Figure 1. In the first period, *before*  $\theta$  and  $\tilde{\varepsilon}$  are realized, the manufacturer and the retailer play a Stackelberg game: the manufacturer first sets his prebook wholesale price  $w_1$ , and the retailer then determines her prebook order quantity  $x_1$ . At the end of the first period, the manufacturer observes the realized  $\theta$  and delivers  $a_1 = \min\{k\theta, x_1\}$  to the retailer for her prebook order. If the actual supply  $k\theta$  is lower than the prebook order  $x_1$ , the manufacturer provides full refund for unfilled prebook quantity, and the game ends. Otherwise, the game proceeds to the second

Figure 1 Timeline of Decisions and Events Under the Dynamic Selling Strategy



period. In the second period, *after* observing the realized  $\theta$  and  $\varepsilon$ , the manufacturer and the retailer play a Stackelberg game: the manufacturer first sets his regular wholesale price  $w_2$ , and the retailer then determines her regular order quantity  $x_2$ . At the end of the second period, the manufacturer delivers  $a_2 = \min\{k\theta - a_1, x_2\}$  to the regular order by using his remaining supply ( $k\theta - a_1$ ). The total quantity available for sales by the retailer is  $q = a_1 + a_2$ , and the corresponding retail price is  $p = 1 + \varepsilon - (a_1 + a_2) = 1 + \varepsilon - \min\{k\theta, x_1\} - \max[0, \min\{k\theta - \min\{k\theta, x_1\}, x_2\}]$ .

(2) *Advance selling strategy*. Before  $\tilde{\theta}$  and  $\tilde{\varepsilon}$  are realized, both firms engage in the Stackelberg game that is similar to the first-period game under the dynamic selling strategy: the manufacturer acts as the leader who sets his prebook wholesale price  $w_1$ , and the retailer acts as the follower who determines her prebook order quantity  $x_1$ . After observing the realized  $\theta$  and  $\varepsilon$ , the manufacturer delivers  $a_1 = \min\{k\theta, x_1\}$  to the retailer. The retail price is then  $p = 1 + \varepsilon - a_1 = 1 + \varepsilon - \min\{k\theta, x_1\}$ .

(3) *Regular selling strategy*. After observing the realized  $\theta$  and  $\varepsilon$ , both firms engage in the Stackelberg game that is similar to the second-period game under the dynamic selling strategy: the manufacturer acts as the leader by setting his regular wholesale price  $w_2$ , and the retailer acts as the follower by determining her regular order quantity  $x_2$ . Then the manufacturer delivers  $a_2 = \min\{k\theta, x_2\}$  to the retailer, and the retail price is determined as  $p = 1 + \varepsilon - a_2 = 1 + \varepsilon - \min\{k\theta, x_2\}$ .

Note that the advance selling strategy (respectively, regular selling strategy) is *conceptually* a special case of the dynamic selling strategy: If we set the retailer's regular order quantity  $x_2$  (respectively, prebook order quantity  $x_1$ ) *arbitrarily* equal to zero, then the dynamic selling strategy becomes the advance (respectively, regular) selling strategy. However, in our later numerical study in §6, firms' *equilibrium* decisions under the advance selling strategy (respectively, the regular selling strategy) are never (respectively, rarely) the same as those under the dynamic selling strategy. Because firms behave differently in equilibrium under the dynamic selling strategy than under the other two strategies, one needs to determine the corresponding equilibrium outcomes by analyzing these strategies separately. Alternatively, the manufacturer may receive prebook orders after the uncertain yield (or demand) is realized. This is a special case of our model in which no supply (or demand) uncertainty exists.

The remainder of this paper is organized as follows. In §4, we analyze the games associated with the three selling strategies. We use superscripts  $AB$ ,  $A$ , and  $B$  to denote equilibrium outcomes of the games associated with dynamic selling, advance selling, and regular selling strategies, respectively. In §5, we compare

**Table 1** Summary of Notation

Symbol	Definition
$k$	Production capacity
$\tilde{\theta}$	Random production yield; distribution $F$ over $[\theta, \bar{\theta}]$ , where $\theta \geq 0$ and $E[\tilde{\theta}] = 1$
$\tilde{\varepsilon}$	Random demand component; distribution $G$ over $[\varepsilon, \bar{\varepsilon}]$ , where $\varepsilon \geq -0.75$ and $E[\tilde{\varepsilon}] = 0$
$\theta$	Realized value of random production yield $\tilde{\theta}$
$\varepsilon$	Realized value of random demand component $\tilde{\varepsilon}$
$p$	Retail price: $p = 1 + \varepsilon - q$
$q$	Quantity available for sales by the retailer: $q = a_1$ under advance selling, $q = a_2$ under regular selling, and $q = a_1 + a_2$ under dynamic selling
$(w_1, x_1)$	(Prebook wholesale price, prebook order quantity)
$(w_2, x_2)$	(Regular wholesale price, regular order quantity)
$a_1$	Quantity delivered for the prebook order: $a_1 = \min\{k\theta, x_1\}$
$a_2$	Quantity delivered for the regular order: $a_2 = \min\{k\theta, x_2\}$ under regular selling, and $a_2 = \max[0, \min\{k\theta - \min\{k\theta, x_1\}, x_2\}]$ under dynamic selling
$\Pi_M, \Pi_R$	Manufacturer's profit, retailer's profit
$AB, A, B$	Superscripts for dynamic selling, advance selling, and regular selling, respectively

firms' profits under the three strategies. In §6, we conduct a numerical study to generate further insights; this section also analyzes the impact of demand and supply uncertainties and compares supply chain profits and consumer welfare under the three selling strategies. In §7, we summarize our findings and discuss future research. All proofs are provided in the online supplement. Table 1 summarizes our notation.

## 4. Equilibrium Analysis

In this section, we use backward induction to derive subgame-perfect Nash equilibrium under each selling strategy. In §4.1, we analyze the two-period Stackelberg game under dynamic selling. In §4.2, by setting the retailer's regular order quantity  $x_2$  (respectively, prebook order quantity  $x_1$ ) *arbitrarily* equal to zero under dynamic selling, we analyze the single-period Stackelberg game under advance selling (respectively, regular selling).

### 4.1. Dynamic Selling Strategy

We first solve the second-period Stackelberg game conditioning on the realized supply  $k\theta$ , the realized demand component  $\varepsilon$ , and the decisions chosen in the first period. Then we solve the first-period Stackelberg game by embedding the second-period equilibrium outcomes into the firms' first-period expected profits.

In the second period, given regular wholesale price  $w_2$ , the prebook decisions  $(w_1, x_1)$  made in the first period, and information about the realized  $\theta$  and  $\varepsilon$ , the retailer determines her regular order quantity  $x_2$ . Let  $l'$  be the remaining supply after the first period, where  $l' \equiv k\theta - a_1 = k\theta - \min\{k\theta, x_1\}$ . If  $l' = 0$ , the game ends after the first period. Thus, we need to consider only the case when  $l' > 0$  so

that  $a_1 = x_1$ . Because the retailer's decision  $x_2$  affects (i) the quantity received for the regular order,  $a_2 = \min\{l', x_2\}$ ; and (ii) the retail price  $p = 1 + \varepsilon - x_1 - \min\{l', x_2\}$ , the objective of the retailer is to maximize her profit from selling the total quantity received for both advance and regular orders, which can be expressed as

$$\begin{aligned}\Pi_R(x_2, w_2; x_1, w_1) \\ = [(1 + \varepsilon - x_1 - \min\{l', x_2\}) - w_2] \min\{l', x_2\} \\ + [(1 + \varepsilon - x_1 - \min\{l', x_2\}) - w_1] x_1.\end{aligned}\quad (1)$$

LEMMA 1. For any given regular wholesale price  $w_2$  and prebook order  $x_1$ , the retailer's optimal regular order quantity is  $x_2^{AB}(x_1, w_2) = \max\{0, (m' - w_2)/2\}$ , where  $m' \equiv 1 + \varepsilon - 2x_1$ .

As one would expect the retailer's regular order quantity  $x_2^{AB}(x_1, w_2)$  decreases in the prebook order quantity  $x_1$  and the regular wholesale price  $w_2$ .

In anticipation of the retailer's regular order quantity  $x_2^{AB}(x_1, w_2)$ , the manufacturer sets his regular wholesale price  $w_2$ . By noting that the actual quantity delivered,  $a_2^{AB} = \min\{l', x_2^{AB}\}$ , takes on different values depending on the values of  $m'$ ,  $l'$ , and  $w_2$ , we can express the manufacturer's profit from the retailer's advance and regular orders as a function of  $w_2$  as follows:

$$\begin{aligned}\Pi_M(w_2; x_1, w_1, m', l') \\ = w_2 a_2^{AB} + w_1 a_1 \\ = \begin{cases} w_1 x_1 & \text{if } m' \leq 0 \text{ or } w_2 > m', \\ w_2 l' + w_1 x_1 & \text{if } m' > 0 \text{ and } w_2 \leq m' - 2l', \\ w_2 \frac{m' - w_2}{2} + w_1 x_1 & \text{if } m' > 0 \text{ and } m' - 2l' < w_2 \leq m'. \end{cases}\end{aligned}\quad (2)$$

The manufacturer determines his regular wholesale price  $w_2^{AB}$  that maximizes his profit given in (2). Note that the profit from the first period,  $w_1 x_1$ , does not affect this decision. After obtaining  $w_2^{AB}$ , we then substitute  $w_2^{AB}$  into  $x_2^{AB}(x_1, w_2)$  to find the equilibrium outcome of the retailer,  $x_2^{AB}(x_1)$ . The following lemma summarizes the results:

LEMMA 2. For any given prebook wholesale price  $w_1$  and prebook order  $x_1$ , the ex post equilibrium outcomes in the second-period game satisfy the following:

- (i) If  $m' > 0$  and  $l' \geq m'/4$ , then  $w_2^{AB}(x_1) = m'/2$ ,  $\Pi_M(x_1, w_1) = m'^2/8 + w_1 x_1$ ,  $x_2^{AB}(x_1) = m'/4$ , and  $\Pi_R(x_1, w_1) = m'^2/16 + (1 + \varepsilon - x_1 - w_1)x_1$ .
- (ii) If  $m' > 0$  and  $l' < m'/4$ , then  $w_2^{AB}(x_1) = m' - 2l'$  ( $> m'/2$ ),  $\Pi_M(x_1, w_1) = (m' - 2l')l' + w_1 x_1$  ( $< m'^2/8 + w_1 x_1$ ),  $x_2^{AB}(x_1) = l'$  ( $< m'/4$ ), and  $\Pi_R(x_1, w_1) = l'^2 + (1 + \varepsilon - x_1 - w_1)x_1$  ( $< m'^2/16 + (1 + \varepsilon - x_1 - w_1)x_1$ ).

- (iii) If  $m' \leq 0$ , then  $w_2^{AB}$  can be any positive value;  $\Pi_M(x_1, w_1) = w_1 x_1$ ,  $x_2^{AB}(x_1) = 0$ , and  $\Pi_R(x_1, w_1) = (1 + \varepsilon - x_1 - w_1)x_1$ .

Lemma 2 can be interpreted as follows. In case (i), the realized market potential for regular order,  $m'$ , is positive and the available supply for regular order,  $l'$ , is sufficiently high. Thus, both firms behave as if there is no supply constraint so that the equilibrium outcomes are independent of the capacity  $k$ . In case (ii), the realized market potential for regular order,  $m'$ , is positive, but the available supply for regular order,  $l'$ , is limited. To compensate for lower sales volume, the manufacturer sets a higher regular wholesale price  $w_2^{AB}$  than that in case (i). This causes the retailer to reduce her regular order quantity so that  $x_2^{AB} = l'$ . Because of the limited supply, both firms obtain lower profits in case (ii) than in case (i). In case (iii), the realized market potential for regular order,  $m'$ , is not positive so that the retailer orders nothing for regular order. This happens when the market turns out to be bad (i.e., low  $\varepsilon$ ) and/or when the retailer has prebooked and received too many units in the first period (i.e., high  $x_1$ ).

By using the ex post equilibrium outcomes of the second-period game given in Lemma 2, we now analyze the retailer's problem in the first-period game that takes place before the supply and demand uncertainties are resolved. Recall that the second period is reached only when the manufacturer has some remaining supply  $l' = k\tilde{\theta} - x_1 > 0$  to fill the retailer's regular order. When  $l' \leq 0$ , the retailer receives the entire supply  $k\theta$ , and the game ends after the first period. Combining this observation with the results stated in Lemma 2 for the case when  $l' > 0$ , we can represent the retailer's expected profit as a function of  $(x_1, w_1)$  as follows:

$$\begin{aligned}E_{\tilde{\theta}, \tilde{\varepsilon}}[\Pi_R(x_1, w_1)] \\ = E_{\tilde{\theta}, \tilde{\varepsilon}}\{[(1 + \tilde{\varepsilon} - \min\{k\tilde{\theta}, x_1\}) - w_1] \min\{k\tilde{\theta}, x_1\}\} \\ + \Pr\left[\tilde{m}' > 0, \tilde{l}' \geq \frac{\tilde{m}'}{4}\right] E_{\tilde{\theta}, \tilde{\varepsilon}}\left[\frac{(\tilde{m}')^2}{16} \mid \tilde{m}' > 0, \tilde{l}' \geq \frac{\tilde{m}'}{4}\right] \\ + \Pr\left[\tilde{m}' > 0, 0 < \tilde{l}' < \frac{\tilde{m}'}{4}\right] \\ \cdot E_{\tilde{\theta}, \tilde{\varepsilon}}\left[\tilde{l}'^2 \mid \tilde{m}' > 0, 0 < \tilde{l}' < \frac{\tilde{m}'}{4}\right],\end{aligned}\quad (3)$$

where  $\tilde{m}' \equiv 1 + \tilde{\varepsilon} - 2x_1$  and  $\tilde{l}' \equiv k\tilde{\theta} - x_1$  represent ex ante random variables for  $m'$  and  $l'$ , respectively. Similarly, by utilizing the results stated in Lemma 2, we can express the manufacturer's expected profit as a function of  $(x_1, w_1)$  as follows:

$$\begin{aligned}E_{\tilde{\theta}, \tilde{\varepsilon}}[\Pi_M(x_1, w_1)] \\ = E_{\tilde{\theta}, \tilde{\varepsilon}}[w_1 \min\{k\tilde{\theta}, x_1\}]\end{aligned}$$



$$\begin{aligned}
 & + \Pr \left[ \tilde{m}' > 0, \tilde{l}' \geq \frac{\tilde{m}'}{4} \right] E_{\tilde{\theta}, \tilde{\varepsilon}} \left[ \frac{(\tilde{m}')^2}{8} \mid \tilde{m}' > 0, \tilde{l}' \geq \frac{\tilde{m}'}{4} \right] \\
 & + \Pr \left[ \tilde{m}' > 0, 0 < \tilde{l}' < \frac{\tilde{m}'}{4} \right] \\
 & \cdot E_{\tilde{\theta}, \tilde{\varepsilon}} \left[ (\tilde{m}' - 2\tilde{l}')\tilde{l}' \mid \tilde{m}' > 0, 0 < \tilde{l}' < \frac{\tilde{m}'}{4} \right]. \quad (4)
 \end{aligned}$$

Because of the complexity of (3) and (4), there are no closed-form expressions for  $x_1^{AB}(w_1)$  that maximizes  $E_{\tilde{\theta}, \tilde{\varepsilon}}[\Pi_R(x_1, w_1)]$  for any given  $w_1$ , and for  $w_1^{AB}$  that maximizes  $E_{\tilde{\theta}, \tilde{\varepsilon}}[\Pi_M(x_1^{AB}(w_1), w_1)]$ . Even so, in §5.2, we are able to make analytical comparisons between the equilibrium outcomes under dynamic selling and those under advance or regular selling.

## 4.2. Advance or Regular Selling Strategy

We first analyze the advance selling strategy by considering the case when  $x_2 = 0$  under the dynamic selling strategy, and then we analyze the regular selling strategy by considering the case when  $x_1 = 0$  under the dynamic selling strategy.

**4.2.1. Advance Selling Strategy.** Under advance selling, firms engage in a Stackelberg game *before*  $\tilde{\theta}$  and  $\tilde{\varepsilon}$  are realized. Given prebook wholesale price  $w_1$ , the retailer decides her advance order quantity  $x_1^A(w_1)$  that maximizes her expected profit. By noting that the retail price  $p = 1 + \tilde{\varepsilon} - \min\{k\tilde{\theta}, x_1\}$ , we can express the retailer's expected profit as

$$\begin{aligned}
 & E_{\tilde{\varepsilon}, \tilde{\theta}}[\Pi_R(x_1, w_1)] \\
 & = E_{\tilde{\varepsilon}, \tilde{\theta}}[(1 + \tilde{\varepsilon} - \min\{k\tilde{\theta}, x_1\} - w_1) \min\{k\tilde{\theta}, x_1\}]. \quad (5)
 \end{aligned}$$

Observe that  $E_{\tilde{\varepsilon}, \tilde{\theta}}[\Pi_R(x_1, w_1)]$  in (5) is equal to  $E_{\tilde{\varepsilon}, \tilde{\theta}}[\Pi_R(x_1, w_1)]$  in (3) under dynamic selling when ignoring the second and third terms in (3) for the case when  $\tilde{m}' \equiv 1 + \tilde{\varepsilon} - 2x_1 > 0$ . These two terms represent the expected profit from regular order and they do not exist if one sets  $x_2 = 0$  arbitrarily. By anticipating the retailer's best response  $x_1^A(w_1)$ , the manufacturer determines his prebook wholesale price  $w_1^A$  that maximizes his expected profit given as

$$E_{\tilde{\varepsilon}, \tilde{\theta}}[\Pi_M(w_1)] = E_{\tilde{\varepsilon}, \tilde{\theta}}[w_1 \min\{k\tilde{\theta}, x_1^A(w_1)\}]. \quad (6)$$

Similar to  $E_{\tilde{\varepsilon}, \tilde{\theta}}[\Pi_R(x_1, w_1)]$ ,  $E_{\tilde{\varepsilon}, \tilde{\theta}}[\Pi_M(w_1)]$  in (6) is equal to  $E_{\tilde{\varepsilon}, \tilde{\theta}}[\Pi_M(w_1)]$  in (4) under dynamic selling when ignoring the expected profit from regular order. The following lemma characterizes equilibrium outcomes under advance selling:

**LEMMA 3.** (a) *For any given prebook wholesale price  $w_1$ , the retailer's optimal prebook order quantity is  $x_1^A(w_1) = \max\{0, (1 - w_1)/2\}$ .*

(b) *The equilibrium outcomes under advance selling satisfy the following:*

(i) *If  $k\theta \geq 0.25$ , then  $w_1^A = 0.5$ ,  $E\Pi_M^A = 1/8$ ,  $x_1^A(w_1^A) = 0.25$ , and  $E\Pi_R^A = 1/16$ .*

(ii) *If  $k\theta < 0.25$ , then  $w_1^A > 0.5$ ,  $E\Pi_M^A < 1/8$ ,  $x_1^A(w_1^A) < 0.25$ , and  $E\Pi_R^A < 1/16$ .*

Lemma 3 is analogous to Lemmas 1 and 2 for the second-period decisions under dynamic selling, but it is not exactly the same because the firms make their decisions under uncertainty in advance selling. Similar to Lemma 1, Lemma 3(a) shows that the retailer's optimal order quantity depends on neither capacity  $k$  nor uncertain yield  $\tilde{\theta}$ . This implies that the retailer can simply order the quantity she wishes to receive although she may not receive this quantity because of limited capacity and random yield. However, as shown in Lemma 3(b), the manufacturer's pricing decision depends on both capacity  $k$  and uncertain yield  $\tilde{\theta}$ —in particular, it hinges upon the minimum supply  $k\theta$ . In case (i), when there is ample supply ( $k\theta \geq 0.25$ ), both firms behave as if there is no supply constraint because the retailer can always receive her ideal quantity  $a_1^A(w_1^A) = \min\{k\theta, x_1^A(w_1^A)\} = x_1^A(w_1^A) = 0.25$  for any realized  $k\theta$ . The result in this case exhibits the classic double marginalization result. In case (ii), a supply shortage occurs when the realized supply  $k\theta$  is lower than the retailer's ideal quantity of 0.25. To compensate for lower expected sales volume, the manufacturer increases his prebook wholesale price by setting  $w_1^A > 0.5$ , causing the retailer to reduce her prebook order quantity so that  $x_1^A(w_1^A) < 0.25$ . Because of the potential supply shortage, both firms obtain lower expected profits in case (ii) than in case (i). Note that the closed-form expressions of equilibrium outcomes do not exist for general probability distributions for case (ii).

**4.2.2. Regular Selling Strategy.** Under regular selling, both firms enter a Stackelberg game *after* observing the realized supply  $k\theta$  and the realized demand component  $\varepsilon$ . Given regular wholesale price  $w_2$ , the retailer's objective is to find regular order quantity  $x_2^B(w_2)$  that maximizes her profit:

$$\Pi_R(x_2, w_2) = \{(1 + \varepsilon - \min\{k\theta, x_2\}) - w_2\} \min\{k\theta, x_2\}. \quad (7)$$

Observe that  $\Pi_R(x_2, w_2)$  in (7) is equal to  $\Pi_R(x_2, w_2; x_1, w_1)$  in (1) under dynamic selling when  $x_1 = 0$ . Anticipating the retailer's best response  $x_2^B(w_2)$ , the manufacturer sets his regular wholesale price  $w_2$  that maximizes his profit  $\Pi_M(w_2) = w_2 \min\{k\theta, x_2^B(w_2)\}$ . By setting  $x_1 = 0$  in the equilibrium outcomes under dynamic selling given in Lemma 2, we obtain the following (note that case (iii) in Lemma 2 will not happen under regular selling because  $m' = 1 + \varepsilon - 2x_1 = 1 + \varepsilon > 0$ ):

**COROLLARY 1.** *The ex post equilibrium outcomes under regular selling satisfy the following:*

(i) *If  $k\theta \geq (1 + \varepsilon)/4$ , then  $w_2^B = (1 + \varepsilon)/2$ ,  $\Pi_M^B = (1 + \varepsilon)^2/8$ ,  $x_2^B(w_2^B) = (1 + \varepsilon)/4$ , and  $\Pi_R^B = (1 + \varepsilon)^2/16$ .*



(ii) If  $k\theta < (1 + \varepsilon)/4$ , then  $w_2^B = 1 + \varepsilon - 2k\theta$  ( $> (1 + \varepsilon)/2$ ),  $\Pi_M^B = (1 + \varepsilon - 2k\theta)k\theta$  ( $< (1 + \varepsilon)^2/8$ ),  $x_2^B(w_2^B) = k\theta$  ( $< (1 + \varepsilon)/4$ ), and  $\Pi_R^B = (k\theta)^2$  ( $< (1 + \varepsilon)^2/16$ ).

Corollary 1 can be interpreted in the same manner as Lemma 2. By using the ex post equilibrium outcomes stated in Corollary 1, we can express the manufacturer's ex ante expected profit as  $E\Pi_M^B = \int_{\theta}^{\bar{\theta}} \int_{\varepsilon}^{\bar{\varepsilon}} \Pi_M^B(\tilde{\theta}, \tilde{\varepsilon}) dG(\tilde{\varepsilon}) dF(\tilde{\theta})$ , where  $\Pi_M^B(\tilde{\theta}, \tilde{\varepsilon}) = (1 + \tilde{\varepsilon})^2/8$  if  $k\tilde{\theta} \geq (1 + \tilde{\varepsilon})/4$ , and  $\Pi_M^B(\tilde{\theta}, \tilde{\varepsilon}) = (1 + \tilde{\varepsilon} - 2k\tilde{\theta})k\tilde{\theta}$  otherwise. Similarly, we can obtain  $E\Pi_R^B$ ,  $Ew_2^B$  and  $Ex_2^B$  in equilibrium.

## 5. Profit Comparison Under Advance, Regular, or Dynamic Selling

Using the equilibrium outcomes derived in §4, we now compare the firms' expected profits under different selling strategies: advance selling versus regular selling in §5.1, and dynamic selling versus advance or regular selling in §5.2.

### 5.1. Advance Selling vs. Regular Selling

Because no closed-form expressions exist for the firms' expected profits under advance selling, we examine the functional characteristics of the firm's expected profits under advance selling with those under regular selling. The following theorem summarizes the results.

**THEOREM 1.** (a) From the manufacturer's perspective, regular selling dominates advance selling; that is,  $E\Pi_M^B \geq E\Pi_M^A$ , where the equality holds when  $\Pr[\tilde{\theta} = 1] = \Pr[(1 + \tilde{\varepsilon})/4 \geq k\tilde{\theta}] = 1$  or  $\Pr[\tilde{\varepsilon} = 0] = \Pr[(1 + \tilde{\varepsilon})/4 \leq k\tilde{\theta}] = 1$ .

(b) From the retailer's perspective, if there is ample supply so that  $\Pr[(1 + \tilde{\varepsilon})/4 \leq k\tilde{\theta}] = 1$ , then regular selling dominates advance selling; that is,  $E\Pi_R^B \geq E\Pi_R^A$ , where the equality holds when  $\Pr[\tilde{\varepsilon} = 0] = 1$ . Otherwise, advance selling can dominate regular selling with  $E\Pi_R^A > E\Pi_R^B$ .

Theorem 1(a) asserts that the manufacturer is better off by postponing his pricing decision until all uncertainties are resolved. This result is due to the following two reasons. First, under regular selling, the manufacturer can utilize information about the realized supply to set his regular wholesale price. Better information enables the manufacturer to extract more surplus from the retailer. Second, we find that the value of the manufacturer's ability to set his regular wholesale price in response to the realized market potential increases with demand volatility. This result extends the result of Van Mieghem and Dada (1999) obtained in a single-firm setting with only demand uncertainty to the supply chain setting in which a manufacturer and a retailer engages in vertical competition under both demand and supply uncertainties.

Theorem 1(a) also shows the two conditions under which the manufacturer is indifferent between regular selling and advance selling. The first condition deals with the case when there is no supply uncertainty (i.e.,  $\Pr[\tilde{\theta} = 1] = 1$ ), and the "demand of the retailer" (i.e., the ideal quantity the retailer wishes to receive with abundant supply) always exceeds the supply of the manufacturer (i.e.,  $\Pr[(1 + \tilde{\varepsilon})/4 \geq k\tilde{\theta}] = 1$ ). The second condition deals with the case when there is no demand uncertainty (i.e.,  $\Pr[\tilde{\varepsilon} = 0] = 1$ ), and the supply is always sufficient to meet the demand of the retailer (i.e.,  $\Pr[(1 + \tilde{\varepsilon})/4 \leq k\tilde{\theta}] = 1$ ). In both cases, it can be shown that the minimum of the demand and the supply is always constant for any  $\tilde{\theta}$  and  $\tilde{\varepsilon}$ . Only when one of these conditions holds, the regular selling strategy does not provide the manufacturer with any information value regarding uncertain demand or supply as compared with the advance selling strategy. Therefore, in the presence of both uncertainties, the manufacturer is strictly better off under regular selling.

Theorem 1(b) shows that when there is ample supply, the retailer is also better off under regular selling by postponing her ordering decision until uncertainties are resolved. In this case, the manufacturer's ability to set his wholesale price based on information about the realized yield and demand is futile under regular selling. On the other hand, when there is a potential supply shortage so that  $\Pr[(1 + \tilde{\varepsilon})/4 > k\tilde{\theta}] > 0$ , Theorem 1(b) presents a counterintuitive result that the retailer can be better off by placing her order before the supply and demand uncertainties are resolved. The intuition from this result is as follows. Under regular selling, when the supply turns out to be scarce, the manufacturer sets a high wholesale price, which hurts the retailer. Under advance selling, this does not happen because the wholesale price is set before the supply shortage is observed, and therefore the retailer enjoys a moderate wholesale price even under supply shortage. Note, however, that the retailer is not always better off under advance selling with a potential supply shortage. To examine when this counterintuitive result occurs, let us consider a special case in which there is no supply uncertainty with  $\Pr[\tilde{\theta} = 1] = 1$ . In this case, we can obtain the closed-form expressions for  $E\Pi_R^A$  (from the proof of Lemma 3) and  $E\Pi_R^B$  (from Corollary 1) as follows:  $E\Pi_R^A = \min\{1/16, k^2\}$ , and  $E\Pi_R^B = E \min\{(1 + \tilde{\varepsilon})^2/16, k^2\}$ . It is easy to check that  $E\Pi_R^A = 1/16 < E[(1 + \tilde{\varepsilon})^2/16] = E\Pi_R^B$  when  $k$  is sufficiently large. This suggests that demand uncertainty helps the retailer earn a higher expected profit under regular selling with a high level of capacity. Thus, only when capacity  $k$  is tight and lies within a certain range so that  $\varepsilon < 4k - 1 < 0 < \bar{\varepsilon}$  (see Figure 2), we have  $E\Pi_R^B = \int_{\varepsilon}^{4k-1} ((1 + \tilde{\varepsilon})^2/16) dG(\tilde{\varepsilon}) + \int_{4k-1}^{\bar{\varepsilon}} k^2 dG(\tilde{\varepsilon}) < k^2 = E\Pi_R^A$ .

The graph shows the function  $\Pi_R^B(\varepsilon)$  plotted against  $\varepsilon$ . The x-axis has tick marks at  $-1$ ,  $\underline{\varepsilon}$ ,  $0$ ,  $\varepsilon$ , and  $1$ . The y-axis has tick marks at  $1/16$  and  $k^2$ . The function is defined as  $\Pi_R^B = \min\left\{k^2, \frac{(1+\varepsilon)^2}{16}\right\}$ . The curve is dashed for  $\varepsilon < \underline{\varepsilon}$  and  $\varepsilon > \varepsilon$ . It is solid and linear from  $(\underline{\varepsilon}, \frac{(1+\underline{\varepsilon})^2}{16})$  to  $(0, 1/16)$ . It is dashed and horizontal at  $y = k^2$  for  $\varepsilon > 0$ . A vertical line at  $\varepsilon = 0$  is labeled  $4k-1$ .

## 5.2. Dynamic Selling vs. Advance or Regular Selling

LEMMA 4. *Both firms obtain higher expected profits under dynamic selling than under regular selling; that is,  $E\Pi_M^{AB} \geq E\Pi_M^B$  and  $E\Pi_R^{AB} \geq E\Pi_R^B$ , where the equalities hold when  $x_1^{AB} = 0$ .*

In contrast, when the manufacturer determines his prebook price  $w_1$  under the dynamic selling strategy, he cannot credibly commit to setting his regular price  $w_2$  so high that the retailer chooses her regular order quantity  $x_2 = 0$  for every possible realization of  $\tilde{\theta}$  and  $\tilde{\varepsilon}$ . Thus, in terms of equilibrium outcomes, the advance selling strategy is not a special case of the dynamic selling strategy. (In our numerical experiments in §6, we have verified that two strategies never yield the same equilibrium outcomes.) Nevertheless, the following theorem shows that the manufacturer is always better off under dynamic selling than under advance selling, while the retailer is not.

(b) *From the retailer's perspective, if there is ample supply so that  $\Pr[(1 + \tilde{\epsilon})/4 \leq k\theta] = 1$ , then dynamic selling dominates advance selling; that is,  $E\Pi_R^{AB} \geq E\Pi_R^A$ , where the equality holds when  $x_1^{AB} = 0$  and  $\Pr[\tilde{\epsilon} = 0] = 1$ . Otherwise, advance selling can dominate dynamic selling with  $E\Pi_R^A > E\Pi_R^{AB}$ .*

From the retailer's perspective, we obtain the counterintuitive result in Theorem 2(b), which states that the option value of a regular order under dynamic selling can be negative unless there is ample supply. One might expect that the retailer is always better off under dynamic selling than under advance selling because the retailer has an additional opportunity to place a regular order under dynamic selling. However, when determining her prebook order quantity *ex ante* under dynamic selling, the retailer cannot credibly commit to not placing a regular order *ex post*. Therefore, more ordering opportunities can be detrimental to the retailer! In the next section, we examine numerically when  $E\Pi_{RB}^{AB} < E\Pi_R^A$  occurs.

## 6. Numerical Study

In the previous section, we have presented analytical results based on the *general probability distributions* of  $\tilde{\theta}$  and  $\tilde{\varepsilon}$ . To gain further insights, we now present a numerical study that is based on the following setting. We assume that  $\tilde{\theta}$  is uniformly distributed between  $1 - r$  and  $1 + r$ , where  $r \in (0, 1]$  represents the *degree of supply uncertainty*, and that  $\tilde{\varepsilon} = e$  (representing high demand) with probability 0.5 and  $\tilde{\varepsilon} = -e$  (representing low demand) with probability 0.5, where  $e \in (0, 0.75]$  represents the *degree of demand uncertainty*. We have examined 630 scenarios that are constructed by varying  $k$  from 0.1 to 1,  $e$  from 0.1 to 0.7, and  $r$  from 0.1 to 0.9 with an increment of 0.1. For each scenario that is based on a specific set of  $(k, e, r)$ , we have computed equilibrium outcomes under each selling strategy and compared them among the different selling strategies. To illustrate, we report the results of 27 scenarios in Table 2 (advance versus regular selling) and Table 3 (advance versus dynamic selling), which confirm the results in Theorems 1 and 2, respectively. Furthermore, Table 3 shows that  $E\Pi_R^{AB} < E\Pi_R^A$  is observed when the capacity  $k$  is tight as in the cases when  $E\Pi_R^B < E\Pi_R^A$  in Table 2. To explain this counterintuitive result, let us first notice that the remaining supply  $l' = k\theta - x_1$  after fulfilling the prebook order  $x_1$  tends to be very low when the capacity is tight under dynamic selling (see the last column in Table 3).

**Table 2** Equilibrium Outcomes: Advance Selling vs. Regular Selling

Parameters			Equilibrium outcome comparisons			
$k$	$e$	$r$	$E\Pi_M^B - E\Pi_M^A$	$E\Pi_R^B - E\Pi_R^A$	$Ex_2^B - w_1^A$	$Ex_2^B - x_1^A$
0.1	0.1	0.1	0.001	-0.001	0.010	-0.005
0.1	0.1	0.4	0.005	-0.003	0.039	-0.020
0.1	0.1	0.7	0.007	-0.004	0.067	-0.034
0.1	0.4	0.1	0.001	-0.001	0.010	-0.005
0.1	0.4	0.4	0.005	-0.003	0.039	-0.020
0.1	0.4	0.7	0.007	-0.004	0.069	-0.034
0.1	0.7	0.1	0.002	-0.003	0.035	-0.017
0.1	0.7	0.4	0.006	-0.005	0.065	-0.033
0.1	0.7	0.7	0.009	-0.007	0.099	-0.050
0.4	0.1	0.1	0.001	0.001	0.000	0.000
0.4	0.1	0.4	0.001	0.002	-0.004	0.002
0.4	0.1	0.7	0.005	0.005	-0.011	0.005
0.4	0.4	0.1	0.020	0.010	0.000	0.000
0.4	0.4	0.4	0.019	0.005	0.013	-0.007
0.4	0.4	0.7	0.019	0.008	0.006	-0.003
0.4	0.7	0.1	0.060	0.020	0.026	-0.013
0.4	0.7	0.4	0.055	0.013	0.048	-0.024
0.4	0.7	0.7	0.051	0.015	0.041	-0.020
0.7	0.1	0.1	0.001	0.001	0.000	0.000
0.7	0.1	0.4	0.001	0.001	0.000	0.000
0.7	0.1	0.7	0.002	0.002	-0.006	0.003
0.7	0.4	0.1	0.020	0.010	0.000	0.000
0.7	0.4	0.4	0.020	0.010	0.000	0.000
0.7	0.4	0.7	0.019	0.009	0.001	-0.001
0.7	0.7	0.1	0.061	0.031	0.000	0.000
0.7	0.7	0.4	0.061	0.031	0.000	0.000
0.7	0.7	0.7	0.058	0.024	0.015	-0.007

In this case, a regular wholesale price can be much higher than a prebook wholesale price. For example, when  $(k, e, r) = (0.1, 0.4, 0.4)$ , the highest possible value of  $w_2^{AB}$  occurs when the realized  $\varepsilon = e$  (highest demand) and the realized  $\theta = 1 - r$  (lowest yield). When this happens,  $w_2^{AB} = 1.28 > w_1^{AB} = 0.80 > w_1^A = 0.76$ , implying that the retailer pays higher wholesale prices for both prebook and regular orders under dynamic selling. Because of this risk, when the capacity is tight, the retailer is actually better off by placing a larger prebook order at a lower prebook wholesale price under advance selling than under dynamic selling; hence,  $x_1^{AB} + Ex_2^{AB} < x_1^A$  and  $E\Pi_R^{AB} < E\Pi_R^A$ .

In the remainder of this section, we present comparative statics in §6.1 to examine the effects of capacity, demand uncertainty, and supply uncertainty on equilibrium, and discuss how different selling strategies affect the supply chain profit and consumer welfare in §6.2.

### 6.1. Comparative Statics

To examine the impact of supply and demand uncertainties under general probability distributions of  $\tilde{\theta}$  and  $\tilde{\varepsilon}$ , we use stochastic ordering relations. Specifically, we write  $\tilde{\theta}_2 \geq_v \tilde{\theta}_1$  when  $\tilde{\theta}_2$  is *more variable* than  $\tilde{\theta}_1$  (which implies  $\text{Var}[\tilde{\theta}_2] \geq \text{Var}[\tilde{\theta}_1]$  when  $E[\tilde{\theta}_2] = E[\tilde{\theta}_1]$ ). As it turns out, several effects are not necessarily monotonic. For this reason, we also conduct a numerical study to explore dominant effects and to compare comparative statics across the three selling strategies.

Our numerical results are summarized in Table 4. To examine the effect of capacity  $k$ , we compute the difference in the equilibrium outcomes associated with the adjacent values of  $k$  for any given  $(r, e)$ . Because there are 63 possible pairs of  $(r, e)$  and nine incremental differences of  $k$ , there are 567 scenarios for which we can examine whether the equilibrium outcome increases or decreases with an increase of  $k$ . For example, the first three entries in the first row of Table 4 show that as  $k$  increases, the prebook quantity  $x_1^A$  under the advance selling strategy has increased in 336 scenarios, decreased in 0 scenarios, and unchanged in 231 scenarios. Similarly, we can examine the impact of supply uncertainty  $r$  in 560 scenarios and the impact of demand uncertainty  $e$  in 540 scenarios.

We observe an intuitive result from Table 4: the expected profits of both firms are nondecreasing in capacity  $k$ . Thus, we focus our discussion on the impact of supply uncertainty and demand uncertainty in each of the three selling strategies.

(1) *Advance selling strategy.* For general probability distributions of  $\tilde{\theta}$  and  $\tilde{\varepsilon}$ , we establish the following:

**PROPOSITION 1.** Suppose that  $\tilde{\theta}_2 \geq_v \tilde{\theta}_1$ . Then,  $E_{\tilde{\theta}_1}[\Pi_M^A] \geq E_{\tilde{\theta}_2}[\Pi_M^A]$ . However,  $E_{\tilde{\theta}_1}[\Pi_R^A] - E_{\tilde{\theta}_2}[\Pi_R^A]$  can be positive, zero, or negative.

**Table 3** Equilibrium Outcomes: Dynamic Selling vs. Advance Selling

Parameters			Equilibrium outcome comparisons					
$k$	$e$	$r$	$E\Pi_M^{AB} - E\Pi_M^A$	$E\Pi_R^{AB} - E\Pi_R^A$	$w_1^{AB} - w_1^A$	$x_1^{AB} - x_1^A$	$x_1^{AB} + Ex_2^{AB} - x_1^A$	Prob <sup>a</sup>
0.1	0.1	0.1	0.001	−0.001	0.010	−0.105	−0.005	1.000
0.1	0.1	0.4	0.005	−0.003	0.039	−0.120	−0.020	1.000
0.1	0.1	0.7	0.007	−0.004	0.067	−0.134	−0.034	1.000
0.1	0.4	0.1	0.001	−0.001	0.010	−0.105	−0.005	1.000
0.1	0.4	0.4	0.005	−0.003	0.039	−0.120	−0.020	1.000
0.1	0.4	0.7	0.007	−0.004	0.073	−0.116	−0.034	0.961
0.1	0.7	0.1	0.003	−0.003	0.050	−0.075	−0.010	0.500
0.1	0.7	0.4	0.006	−0.005	0.074	−0.090	−0.027	0.685
0.1	0.7	0.7	0.009	−0.007	0.107	−0.104	−0.045	0.714
0.4	0.1	0.1	0.017	0.012	0.062	−0.125	0.063	0.000
0.4	0.1	0.4	0.013	0.010	0.062	−0.143	0.048	0.193
0.4	0.1	0.7	0.015	0.012	0.040	−0.119	0.045	0.330
0.4	0.4	0.1	0.029	0.016	0.094	−0.177	0.034	0.164
0.4	0.4	0.4	0.027	0.012	0.067	−0.153	0.031	0.245
0.4	0.4	0.7	0.028	0.014	0.053	−0.130	0.033	0.320
0.4	0.7	0.1	0.061	0.020	0.058	−0.229	−0.007	0.473
0.4	0.7	0.4	0.059	0.015	0.087	−0.182	−0.002	0.340
0.4	0.7	0.7	0.056	0.018	0.080	−0.153	0.007	0.306
0.7	0.1	0.1	0.017	0.012	0.062	−0.125	0.063	0.000
0.7	0.1	0.4	0.017	0.012	0.062	−0.125	0.063	0.000
0.7	0.1	0.7	0.015	0.013	0.054	−0.127	0.058	0.101
0.7	0.4	0.1	0.036	0.022	0.062	−0.125	0.063	0.000
0.7	0.4	0.4	0.036	0.022	0.062	−0.125	0.063	0.000
0.7	0.4	0.7	0.032	0.019	0.059	−0.131	0.052	0.101
0.7	0.7	0.1	0.077	0.042	0.062	−0.125	0.063	0.000
0.7	0.7	0.4	0.075	0.040	0.068	−0.140	0.054	0.054
0.7	0.7	0.7	0.069	0.032	0.069	−0.141	0.039	0.137

<sup>a</sup>Prob represents  $\Pr\{\tilde{I}' \leq 0 \text{ or } 0 < \tilde{I}' \leq \tilde{m}'/4\}$  under the dynamic selling strategy.

Proposition 1 suggests that a higher level of supply uncertainty always hurts the manufacturer's expected profit  $E\Pi_M^A$  under advance selling. This result is intuitive because, as supply uncertainty increases, the

**Table 4** Numerical Results: Comparative Statics

	$k$			$r$			$e$		
	↑	↓	·	↑	↓	·	↑	↓	·
Advance selling									
$x_1^A$	336	0	231	98	238	224	0	0	540
$w_1^A$	0	336	231	238	98	224	0	0	540
$E\Pi_R^A$	336	0	231	56	280	224	0	0	540
$E\Pi_M^A$	336	0	231	0	336	224	0	0	540
Regular selling									
$Ex_2^B$	408	0	159	0	358	202	0	352	188
$EW_2^B$	0	408	159	358	0	202	352	0	188
$E\Pi_R^B$	408	0	159	99	305	156	396	121	23
$E\Pi_M^B$	408	0	159	0	404	156	517	0	23
Dynamic selling									
$x_1^{AB}$	433	7	127	180	258	122	75	340	125
$Ex_2^{AB}$	389	60	118	110	331	119	81	346	113
$x_1^{AB} + Ex_2^{AB}$	449	0	118	32	407	121	20	406	114
$w_1^{AB}$	43	395	129	338	93	129	327	73	140
$EW_2^{AB}$	0	449	118	370	69	121	403	23	114
$E\Pi_R^{AB}$	449	0	118	131	347	82	387	130	23
$E\Pi_M^{AB}$	449	0	118	0	445	115	517	0	23

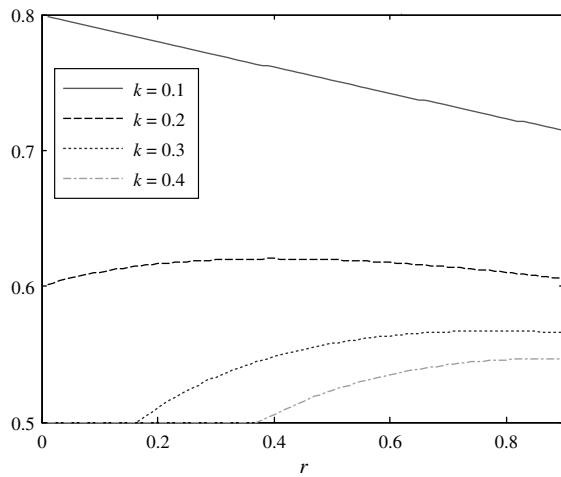
manufacturer bears more risk because he receives all orders before the uncertainties are resolved. This result is also verified in Table 4: when yield  $\hat{\theta}$  follows a uniform distribution in  $[1 - r, 1 + r]$ ,  $E\Pi_M^A$  is nonincreasing in  $r$ . From the retailer's perspective, Proposition 1 and Table 4 reveal a seemingly counterintuitive result: Supply uncertainty can be beneficial to the retailer under advance selling. To understand why this can occur, see Figure 3, which shows that when  $k$  is low and/or  $r$  is high,  $w_1^A$  is decreasing in  $r$ . In this case, as the supply risk becomes more eminent, the manufacturer reduces his prebook wholesale price to secure a larger prebook order from the retailer. Because the retailer can acquire a larger prebook order quantity at a lower price, a higher level of supply uncertainty could improve the retailer's expected profit.

Demand uncertainty  $e$  does not affect the equilibrium outcomes under advance selling. This is because the retailer maximizes her expected profit that does not depend on  $e$ ; see (5).

(2) *Regular selling strategy.* For general probability distributions of  $\tilde{\theta}$  and  $\tilde{\varepsilon}$ , we establish the following:

**PROPOSITION 2.** (a) Suppose that  $\tilde{\theta}_2 \geq_v \tilde{\theta}_1$ . Then,  $E_{\tilde{\theta}_1}[\Pi_M^B] \geq E_{\tilde{\theta}_2}[\Pi_M^B]$ ,  $E_{\tilde{\theta}_1}[w_2^B] \leq E_{\tilde{\theta}_2}[w_2^B]$ , and  $E_{\tilde{\theta}_1}[x_2^B] \geq$



**Figure 3** Prebook Wholesale Price  $w_1^A$  Under Advance Selling

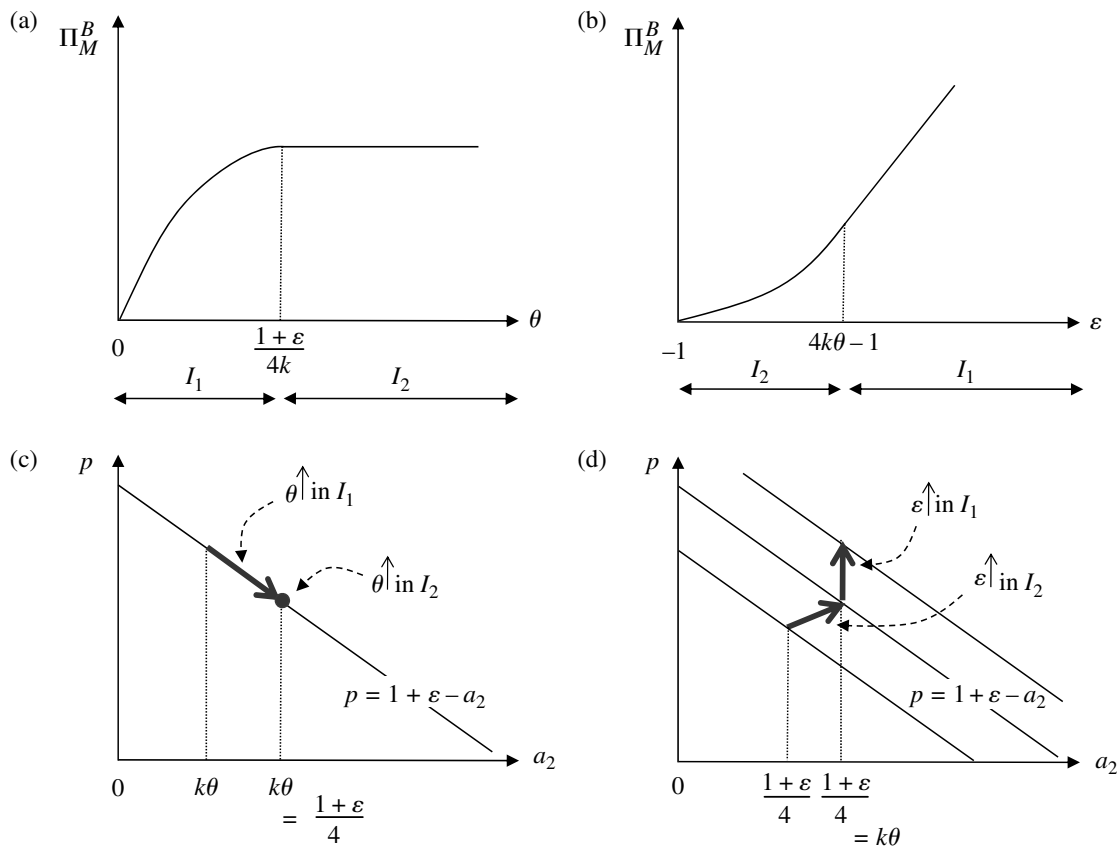
$E_{\tilde{\theta}_2}[x_2^B]$ . However,  $E_{\tilde{\theta}_1}[\Pi_R^B] - E_{\tilde{\theta}_2}[\Pi_R^B]$  can be positive, zero, or negative.

(b) Suppose that  $\tilde{\varepsilon}_2 \geq_v \tilde{\varepsilon}_1$ . Then,  $E_{\tilde{\varepsilon}_1}[\Pi_M^B] \leq E_{\tilde{\varepsilon}_2}[\Pi_M^B]$ ,  $E_{\tilde{\varepsilon}_1}[w_2^B] \leq E_{\tilde{\varepsilon}_2}[w_2^B]$ , and  $E_{\tilde{\varepsilon}_1}[x_2^B] \geq E_{\tilde{\varepsilon}_2}[x_2^B]$ . However,  $E_{\tilde{\varepsilon}_1}[\Pi_R^B] - E_{\tilde{\varepsilon}_2}[\Pi_R^B]$  can be positive, zero, or negative.

Proposition 2(a) states that supply uncertainty is always detrimental to the manufacturer under regular

selling, whereas supply uncertainty can be beneficial to the retailer. This result is consistent with Proposition 1. However, unlike advance selling, the ex ante expected regular wholesale price  $Ew_2^B$  as well as the ex ante expected regular order quantity  $Ex_2^B$  change monotonically. To understand this result, note that under regular selling, each firm chooses an action that maximizes his or her ex post profit after the uncertainties are resolved. Thus, the supply risk under advance selling that causes the manufacturer to reduce his prebook wholesale price with an increase of  $r$  does not exist under regular selling.

Interestingly, Proposition 2(b) reveals that the effect of demand uncertainty on the manufacturer's expected profit  $E\Pi_M^B$  is opposite to that of supply uncertainty. To investigate this counterintuitive result, let us examine the manufacturer's ex post profit  $\Pi_M^B$  presented in Corollary 1. Figures 4(a) and 4(b) illustrate  $\Pi_M^B$  against realized  $\theta$  and  $\varepsilon$ , respectively. In Figure 4, the supply is short relative to the demand of the retailer in interval  $I_1$  (where  $k\theta < (1+\varepsilon)/4$  as in Corollary 1(ii)), whereas the supply is sufficient relative to the demand in interval  $I_2$  (where  $k\theta \geq (1+\varepsilon)/4$  as in Corollary 1(i)). As illustrated in Figure 4(a),  $\Pi_M^B$  is nondecreasing and concave in  $\theta$  because (i) in  $I_1$ , the marginal benefit of increasing  $\theta$  is positive but

**Figure 4** (a)  $\Pi_M^B$  Against Yield  $\theta$ ; (b)  $\Pi_M^B$  Against Demand Component  $\varepsilon$ ; (c) Demand Curve When Yield  $\theta$  Increases; (d) Demand Curves When Demand Component  $\varepsilon$  Increases

decreasing because additional units produced with higher  $\theta$  will be sold to customers with lower reservation prices (see Figure 4(c)); and (ii) in  $I_2$ , it is zero because additional units will not be sold. On the other hand, Figure 4(b) shows that  $\Pi_M^B$  is strictly increasing and convex in  $\varepsilon$  because (i) in  $I_2$ , the marginal benefit of increasing  $\varepsilon$  is increasing because additional units will be sold to customers at a higher price  $p = 1 + \varepsilon - a_2$ ; and (ii) in  $I_1$ , it is a positive constant because no additional units will be sold with an increase of  $\varepsilon$ , but units will still be sold at a higher price  $p$  (see Figure 4(d)). Therefore, the manufacturer's ex ante expected profit  $E\Pi_M^B$  decreases with supply volatility due to the concavity of  $\Pi_M^B$  in  $\theta$ , and  $E\Pi_M^B$  increases with demand volatility due to the convexity of  $\Pi_M^B$  in  $\varepsilon$ .

In contrast to  $E\Pi_M^B$ , Proposition 2 and Table 4 reveal that the retailer's ex ante expected profit  $E\Pi_R^B$  does not change monotonically against supply and demand uncertainties. One can see this from Corollary 1, which shows that the retailer's ex post profit  $\Pi_R^B$  is neither convex nor concave in  $\theta$  or  $\varepsilon$ . This result is consistent with the previous result that  $E\Pi_R^A$  is not monotonic in  $r$  under advance selling.

Although the effect of demand uncertainty on  $E\Pi_M^B$  is opposite to that of supply uncertainty, Proposition 2(b) shows that the effects of demand uncertainty on  $Ew_2^B$  and  $Ex_2^B$  are the same as those of supply uncertainty. Similar to the effects on  $\Pi_M^B$ , we can explain this result by showing that  $w_2^B$  (respectively,  $x_2^B$ ) is convex (respectively, concave) in both  $\theta$  and  $\varepsilon$ .

(3) *Dynamic selling strategy.* Under dynamic selling, Table 4 shows that most equilibrium outcomes do not change monotonically with a change of any parameter value. This is primarily because of the nonmonotonicity of the prebook wholesale price  $w_1^{AB}$ . Despite this nonmonotonicity, Table 4 reveals that the general patterns observed under dynamic selling are similar to those obtained under advance or regular selling.

## 6.2. Supply Chain Performance and Consumer Welfare

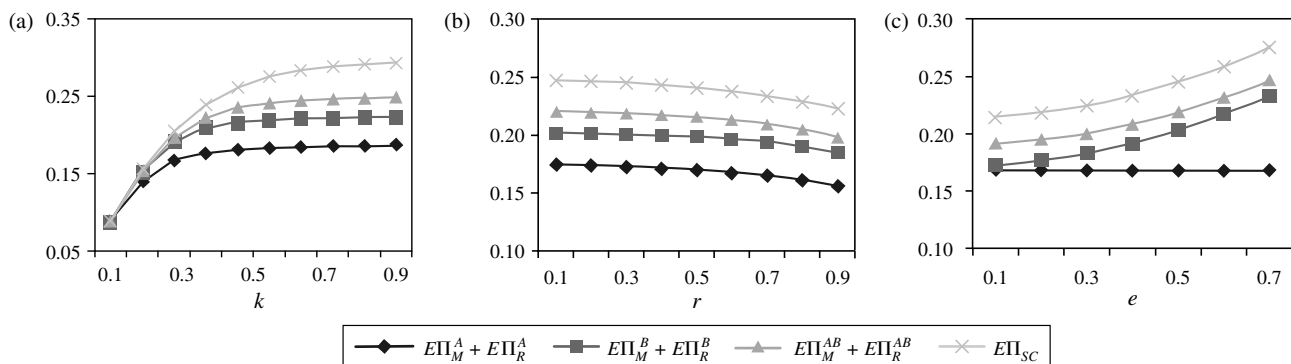
So far we have focused our analysis on the manufacturer's pricing decisions, the retailer's ordering decisions, and the resulting profits. We now examine how different selling strategies affect the overall supply chain profit and consumer welfare.

First, we examine how vertical competition in a decentralized supply chain affects supply chain performance as compared with a centralized supply chain. The centralized firm chooses the quantity  $q$  to bring to the market after observing realized yield  $\theta$  and realized demand component  $\varepsilon$ . Its optimal ex post profit  $\Pi_{SC}$  is given as  $\Pi_{SC} = \max_{0 \leq q \leq k\theta} (1 + \varepsilon - q)q$ . It is easy to find that the optimal quantity is  $\min\{(1 + \varepsilon)/2, k\theta\}$ . Observe from Corollary 1 that the ex post supply chain profit under regular selling is  $\Pi_M^B + \Pi_R^B = (1 + \varepsilon - x_2^B)x_2^B$ , where  $x_2^B = \min\{(1 + \varepsilon)/4, k\theta\}$ . Therefore, as expected, the ex post supply chain profit cannot be lower in the centralized supply chain under regular selling. Under advance or dynamic selling, the ex post supply chain profit is also higher in the centralized supply chain because the centralized firm can always replicate the equilibrium choices of independent firms. To examine the magnitude of difference, we conduct numerical experiments by considering the same 630 scenarios that are used throughout this section. The results are graphically shown in Figure 5. The graphs in this figure show average profits; for example, when  $k = 0.1$ , the expected profit of the supply chain under advance selling is computed by averaging  $(E\Pi_M^A + E\Pi_R^A)$  over 63 scenarios of  $(e, r)$  for varying  $e$  from 0.1 to 0.7 and  $r$  from 0.1 to 0.9 with an increment of 0.1. We draw the following observations from Figure 5:

(1) The decentralized supply chain performs the best on average under dynamic selling, and it performs the worst under advance selling (even with the holdback option discussed earlier in §3).

(2) There is a loss of efficiency in the decentralized supply chain under all selling strategies because of

Figure 5 Expected Profits of the Supply Chain Against (a) Capacity  $k$ , (b) Supply Uncertainty  $r$ , and (c) Demand Uncertainty  $e$



vertical competition between the manufacturer and the retailer. However, dynamic selling reduces this loss substantially from the average of 30% (or 18%) under advance selling (or regular selling) to 11% under dynamic selling, where the loss is computed as  $1 - (E\Pi_M + E\Pi_R)/E\Pi_{SC}$ .

(3) When capacity is tight, the efficiency loss under dynamic selling is not significant; for example, 0.5%, 2.9%, and 6.3% when  $k = 1, 2$  and  $3$ , respectively. The efficiency loss is increasing with capacity  $k$ , and it is not sensitive to demand uncertainty  $e$  or supply uncertainty  $r$ .

We have also examined how different selling strategies perform in terms of consumer welfare. This is a particularly relevant question when dealing with products like flu vaccine that have an impact on public health. We use the standard definition of consumers' surplus (which is the area under the demand curve):  $CS = \int_0^q (1 + \varepsilon - \hat{q})d\hat{q} = (1 + \varepsilon)q - q^2/2$ , where  $q$  represents the quantity brought to the market:  $q = \min\{k\theta, x_1^A\}$  under advance selling,  $q = \min\{k\theta, x_2^B\}$  under regular selling, and  $q = \min\{k\theta, x_1^{AB}\} + \max[0, \min\{k\theta - \min\{k\theta, x_1^{AB}\}, x_2^{AB}\}]$  under dynamic selling. Similar to supply chain profits, we have found that ex ante expected consumer welfare is the highest under dynamic selling and the lowest under advance selling, and that it changes with  $k$ ,  $r$ , and  $e$  similarly to Figure 5.

## 7. Concluding Remarks

In this paper we have examined three selling strategies (advance, regular, and dynamic) for a manufacturer who sells a seasonal product through a retailer under uncertain supply and demand. We have modeled these strategies as three different Stackelberg games in which the random yield and the random demand follow general probability distributions.

From the manufacturer's perspective, we have shown that the dynamic selling strategy dominates the other two strategies:  $E\Pi_M^{AB} \geq E\Pi_M^B \geq E\Pi_M^A$ . The first result ( $E\Pi_M^{AB} \geq E\Pi_M^B$ ) is due to the nonnegative "option value" of a prebook order under dynamic selling as compared with regular selling. This result is intuitive because under dynamic selling, the manufacturer can always set his prebook wholesale price sufficiently high to induce the retailer not to place any prebook orders. More interestingly, the manufacturer is strictly better off under dynamic selling (i.e.,  $E\Pi_M^{AB} > E\Pi_M^B$ ) when the retailer prebooks some units in equilibrium. This is because the retailer's prebook order enables the manufacturer to reduce his regular wholesale price and to receive a larger total order from the retailer. The second result ( $E\Pi_M^B \geq E\Pi_M^A$ ) is due to two factors: (i) value of price postponement—the manufacturer's ability to set his

regular wholesale price after observing the realized supply provides nonnegative information value; and (ii) convex profit function and Jensen's inequality—the value of the manufacturer's ability to set his regular wholesale price in response to the realized market potential increases with demand volatility. Furthermore, in the presence of both uncertainties, the manufacturer is *strictly* better off under dynamic or regular selling than under advance selling (i.e.,  $E\Pi_M^{AB} > E\Pi_M^A$  and  $E\Pi_M^B > E\Pi_M^A$ ).

From the retailer's perspective, as one may expect, the dynamic selling strategy dominates the regular selling strategy ( $E\Pi_R^{AB} \geq E\Pi_R^B$ ) because the dynamic selling strategy offers an option for the retailer to place a prebook order in addition to a regular order. If there is ample supply, the dynamic selling strategy also dominates the advance selling strategy; otherwise, the retailer can be better off under the advance selling strategy (i.e.,  $E\Pi_R^{AB} < E\Pi_R^A$ ). This implies seemingly counterintuitive results: (i) postponing the ordering decision can be detrimental to the retailer (i.e.,  $E\Pi_R^B < E\Pi_R^A$ ), and (ii) more ordering opportunities can be detrimental to the retailer (i.e.,  $E\Pi_R^{AB} < E\Pi_R^A$ ). Our extensive numerical experiments show that these counterintuitive results occur when the manufacturer's capacity is tight. Thus, when there is vertical competition between the manufacturer and the retailer, the value of the option to place a regular order (in addition to a prebook order) under the dynamic selling strategy can be negative to the retailer. In this situation, the retailer wants to commit to placing no regular order, but this is not a credible threat to the manufacturer under dynamic selling. However, the retailer may be able to do so by playing this dynamic game repeatedly. Thus, when the manufacturer chooses his preferred strategy of dynamic selling, the retailer should consider assuring the manufacturer that she will continue to place orders in the future. The formal analysis of such a repeated game would be interesting future research.

When comparing the impact of supply and demand uncertainties on the firms' expected profits, we have shown that the expected profit of the supply chain as well as that of the manufacturer tends to increase with demand volatility, whereas it tends to decrease with supply volatility. However, counterintuitively, supply uncertainty can be beneficial to the retailer under all three strategies by reducing the manufacturer's pricing power.

There are future research avenues to enrich our findings. First, instead of the wholesale price contract, one may consider examining other contracts such as buyback and quantity flexible contracts. For example, one flu vaccine manufacturer allows retailers to return up to a certain limit depending on the delivery date (which is uncertain a priori). For

tractable analysis, one may focus on a prebook contract between one manufacturer and one retailer. Second, one may consider analyzing a multiperiod dynamic game with multiple manufacturers and multiple retailers. In this case, our two-period dynamic Stackelberg game model may serve as a building block. When dealing with multiple manufacturers and multiple retailers, one needs to develop a new allocation mechanism that guarantees the existence of equilibrium in such settings.

### Electronic Companion

An electronic companion to this paper is available as part of the online version at <http://dx.doi.org/10.1287/msom.1120.0423>.

### Acknowledgments

The authors appreciate valuable comments from editor Steve Graves, the associate editor, and reviewers. The authors are grateful to Alex Brown and several other practitioners (who must remain anonymous) for sharing their in-depth knowledge about various advance selling strategies in practice. The authors also benefited from helpful discussions with Sushil Bikhchandani, Steven Lippman, Kevin McCardle, and seminar participants at the University of Michigan, Maryland University, and the Chinese University of Hong Kong. This work is partially supported by the Berkman Faculty Development Fund at Carnegie Mellon University and the Edward Carter Research Fund at the University of California, Los Angeles.

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