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Guillermo Gallego, Ruxian Wang, Ming Hu, Julie Ward, Jose Luis Beltran

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No Claim? Your Gain: Design of Residual Value Extended Warranties Under Risk Aversion and Strategic Claim Behavior

Guillermo Gallego
Department of Industrial Engineering and Operations Research, Columbia University in the City of New York,
New York, New York 10027, gmg2@columbia.edu

Ruxian Wang
Johns Hopkins Carey Business School, Baltimore, Maryland 21202, ruxian.wang@jhu.edu

Ming Hu
Rotman School of Management, University of Toronto, Toronto, Ontario M5S 3E6, Canada, ming.hu@rotman.utoronto.ca

Julie Ward, Jose Luis Beltran
Hewlett-Packard Company, Palo Alto, California 94304 {jward@hp.com, jose-luis.beltran@hp.com}

Traditional one-price-for-all extended warranties do not differentiate customers according to their risk attitudes, usage rates, or operating environment. These warranties are priced to cover the cost of high-usage customers who have more failures and are willing to pay a risk premium for their risk aversion. That makes traditional warranties economically unattractive to low-usage customers and those who are less risk averse. These issues can be addressed by residual value warranties, which refund part of the up-front price to customers who have zero or few claims according to a predetermined refund schedule. Residual value warranties may induce strategic claim behavior, since customers may prefer to pay for small failures out of pocket rather than claim failures now and give up potential refunds later.

We design and price residual value warranties to maximize expected profits, taking into account strategic claim behavior and risk attitudes. For the constant absolute risk aversion model, we characterize customers' optimal claim strategy as well as the net value and support cost for residual value warranties. Surprisingly, the total support cost to the service provider, including repair costs and refunds, is lower for more risk-averse customers under the residual value warranties, whereas their willingness to pay is higher. As contingent contracts, residual value warranties can better price discriminate customers than traditional warranties. We identify conditions under which residual value warranties are strictly more profitable than traditional warranties in a homogeneous market, as well as in heterogeneous markets that differ in various dimensions, such as risk attitude, failure rate, and repair cost.

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1. Introduction

As product margins decline in increasingly competitive hardware markets, high-margin, high-revenue aftermarket services such as extended warranties are becoming critical to manufacturers' profitability (see Cohen et al. 2006, Gallego et al. 2014). Beyond direct profits, post-sales services have a decisive influence on customer loyalty, and in commodity product businesses, service quality and variety are important competitive differentiators. The number of complementary goods and services that are attached and sold together with each primary hardware product is known as the "service attach rate." Not surprisingly, raising service attach rates on hardware sales is a top strategic priority to original equipment manufacturers (OEMs).

The traditional warranty (TW) with a uniform price does not differentiate customers according to their risk attitudes, usage rates, or operating environment. These TWs are priced to cover the costs of highly risk-averse and high-usage customers who are more intolerant of risk and tend to have more failures. That makes the one-price-for-all TWs economically unattractive to less risk-averse or low-usage customers.

An OEM is faced with at least three important obstacles to selling more extended warranties. The first is competition from its own sales channel. Channel partners usually sell their own or third-party extended-service plans for the OEM's products because they earn much higher margins on services

than on hardware sales. A second challenge is a perception among some customers that warranties are not a good value (see Perton 2009). That perception may be because most warranties are offered at a uniform price regardless of risk attitude or usage. The price may be too high for less risk-averse or low-usage customers yet still less than the cost of supporting some high-usage customers. This leads us to the third challenge, which is how to price discriminate customers on the basis of risk aversion and expected support cost.

To overcome those difficulties, an OEM must design a compelling service offering that differentiates itself from its competitors' services, attracts customers from different market segments, and retains healthy profit margins. One way to achieve market segmentation with a single extended warranty contract without having to verify risk attitude or usage is through a residual value warranty (RVW). Since 2007, Hewlett-Packard has been offering an innovative "risk-free" Care Pack service with a *full* refund for its commercial customers in the PC business (see Hewlett-Packard 2010). In the automobile industry, some car dealers also offer extended warranties with a possible partial or full refund at the end of the warranty period if the warranty has not been used.

In general, an RVW is a finite-duration extended warranty for which customers may receive at expiration a partial or full refund in the form of cash or credit toward future purchases and where the size of the refund depends on the number of claims made against the warranty. Customers who are not entitled to a refund are still covered by the warranty. Similar to a TW, an RVW offers customers "peace-of-mind" protection without requiring them to make extra payments whenever they make a claim. The concept of an RVW is related to the practice in the insurance industry of offering a discounted premium for policyholders with good claim records. These programs are used in the automobile insurance market under various names such as "no-claims bonus," "bonus-malus," and "merit-rating" systems.

A warranty with residual values may induce strategic claim behavior, since customers may prefer to pay out of pocket for small failures rather than to claim failures now and give up potential refunds later. We use a stochastic control model to formulate customers' dynamical strategic claim behavior, taking into account their risk attitudes. The customers' optimal strategy has a threshold structure: when facing a failure, the customer will claim it against the warranty if and only if the out-of-pocket repair cost is greater than a time-dependent threshold. The customer intends to pay out of pocket for the failures as it approaches the end of the warranty horizon. We also conduct comparative statics for the thresholds with

respect to refund size, risk attitude, failure rate, and distribution of repair cost. To our knowledge this is the first research to design and price post-sale services by applying ideas from stochastic optimal control theory.

As customers may be heterogeneous in risk attitude, failure rate, or repair cost, their willingness to pay and the cost of supporting them can be different because of different failure realizations and different strategic claim behaviors, even though they face the RVW with the same up-front price and refund schedule. In particular, less risk-averse customers value the refund more and are less likely to claim failures against the warranty; low-usage customers make fewer claims on average and are therefore entitled to larger refunds. As a result, an RVW enables the provider to price discriminate on the basis of risk attitudes, usage rates, or operating conditions without the need to monitor individual customers' magnitude of risk attitude or usage. (The RVW can be attractive to risk-averse, even risk-seeking, customers because of the potential refunds. In practice, the general view of the warranties is that they are of little or no value to customers; hence the purchasers of TWs tend to be very risk averse. RVWs can expand the market coverage by selling to those less risk-averse or even risk-seeking customers.)

The price and refund schedule of an RVW must be carefully designed if these benefits are to be realized. One must consider the expected support costs to the manufacturer for customers who buy the RVW; they are affected by the distribution of risk attitudes and product usage rates in the customer population, as well as by the strategic claim behavior that may be induced by the prospect of refunds. We have created an analytical framework for designing and pricing RVWs that considers all of those factors. We show that the support cost to the warranty provider is decreasing in the risk attitude for risk-averse customers under strategic claim behavior while the willingness to pay is increasing, which implies potential profitability of RVWs. More importantly, we pinpoint specific conditions under which the RVWs are strictly more profitable than the TWs in a homogeneous market as well as in the heterogeneous markets that differ in various dimensions, such as risk attitudes, failure rates, and repair costs.

Because an RVW offers many advantages, it may attract a broad range of customers. But for a warranty provider who also sells a TW with one or more full years of coverage, the introduction of an RVW will likely cannibalize some or all of the demand for the TW. Therefore, the RVW should be carefully designed and priced to avoid eroding profits and, indeed, to improve profitability. We continue our discussion of the heterogeneous market and consider

a warranty menu that includes a TW and an RVW. Under the assumption of individual rationality, each customer will choose the option with the lowest total cost, taking into account the up-front price, the possible refund, and the out-of-pocket cost, given her risk attitude. Our analytical results and numerical studies clearly show that the warranty menu can outperform a single TW or RVW in a wide spectrum of primitives.

2. Literature Review

There are several veins of research related to this work. One of them is the design and pricing of warranties. In the warranty literature, a few papers illustrate how heterogeneity among customers can enable segmentation of the extended warranty market. For example, Padmanabhan and Rao (1993) consider pricing strategies in the presence of heterogeneous risk preferences and consumer moral hazard. Lutz and Padmanabhan (1994) consider income variation among customers, whereas Lutz and Padmanabhan (1998) examine how customers' differing utility of a functioning product makes market segmentation possible.

Three papers discuss usage heterogeneity in the context of warranty pricing. Padmanabhan (1995) shows how manufacturers can design and price a menu of warranty options in the presence of consumer moral hazard and usage heterogeneity and can satisfy the warranty demands of various customer segments. Hollis (1999) examines consumer welfare in the extended warranty market when customers vary in usage but the manufacturer cannot verify usage (and so cannot use usage as part of its warranty terms). Moskowitz and Chun (1994) study the design and pricing of a menu of usage and time-based warranties in the presence of usage variation among customers.

A stream of work studies the customer's optimal strategy at the expiration of a warranty and designs warranties that take the customer's optimal strategy into consideration. Jack and Murthy (2007) consider a setting where the manufacturer sets the price per unit of time of the extended warranty and the fixed markup on each out-of-warranty repair, and customers choose the starting time for the extended warranty and replacement time for the product. Hartman and Laksana (2009) study the optimal strategy for customers to make warranty-renewal and product-replacement decisions as the best response to the provider's service terms and pricing. Chun and Tang (1999) investigate the so-called two-attribute warranty policy, such as the age and mileage of an automobile. They propose several decision models to estimate the expected total cost incurred under various types of two-attribute warranty policies, including a

free-replacement warranty policy and a pro rata, two-attribute warranty plan.

A second related vein of research is on customers' strategic claim behavior. Several papers have studied customer's optimal claim strategies and expected costs under such systems. Von Lanzenauer (1974) and De Pril (1979) consider a merit-rating system in which policyholders are classified into a number of risk categories in each period of the finite insurance horizon. In a given period, a policyholder may have one or more accidents where the damage caused by each accident is a random variable and he must decide whether to file a claim for each such accident. At the end of each period, customers move from one risk category into another according to the number of claims they filed in the preceding period. These papers show that the policyholder's optimal claim strategy is a threshold-type policy, and they characterize the policyholder's critical claim thresholds and expected costs. Moreover, De Pril shows how to find the optimal critical claim sizes in closed form for the case of exponentially distributed repair costs.

The findings in our paper for a customer's optimal claim behavior under an RVW are related to those in De Pril (1979) and Kliger and Levikson (2002), although the customer's optimal claim decision is somewhat different. Similar to auto insurance policies, this type of extended warranty rewards customers for having few or no claims. Unlike our model, neither of these papers discusses the optimal design of insurance contracts. We do not attempt to provide a comprehensive review of this research area here, but instead we discuss a few selected papers and refer readers to Kliger and Levikson for a survey of this stream of research.

Warranty design in the supply chain context is the third related area of research. Other than the insurance rationale of extended warranties as segmentation instruments, Desai and Padmanabhan (2004) highlight the role of extended warranty in channel coordination. Gallego et al. (2010) study a supply chain where a supplier sells a basic product and an ancillary service such as extended warranties through a retailer, and they investigate channel coordination mechanisms in the coexistence of basic products and ancillary services. Given that both the manufacturer and retailer can offer the extended warranties, Hsiao et al. (2010) investigate the profitability of three strategies from the manufacturer's perspective in a distribution channel: deter, acquiesce, or foster. Hsiao and Chen (2012) consider the interplay between returns policy, pricing strategy, and quality risk. One of the interesting findings is that further improvement in mitigating the quality risk may not necessarily benefit

the seller. Dai et al. (2012) study how decisions about product quality and warranty coverage interact with one another and influence supply chain performance.

3. Residual Value Warranty and Customers' Risk Attitudes

We refer to a warranty with possible refunds as an RVW. The simplest RVW may take the following form: a customer who bought an RVW will receive a refund r at the end of the warranty term if she has not claimed any failure against the RVW; if the customer claims a failure, all other failures thereafter will still be covered by the RVW but she will not receive any refund. The RVW may induce strategic claim behavior since customers may prefer to have small failures repaired at their own expense rather than give up potential refunds.

3.1. Risk Attitude

Customer risk aversion is a common justification for charging risk premiums in the insurance industry, where losses are large enough to impose some curvature on the utility function. Similar to the prior works on warranty, we consider customer risk preferences. Let $U(v)$ denote the utility function of a customer at her current wealth level v. We assume that her utility function exhibits the property of the constant absolute risk aversion. Let γ denote the degree of the customer's risk attitude. A customer is risk averse if $\gamma > 0$, she is risk neutral if $\gamma = 0$, and she is risk seeking if $\gamma < 0$. The Arrow–Pratt measure of risk aversion (see, e.g., Desai and Padmanabhan 2004) implies that $\gamma = U''(v)/U'(v) = \partial \ln(U'(v))/\partial v$, where $U'(v)$ and $U''(v)$ are the respective first-order and second-order derivatives of the function $U(v)$ with respect to the wealth level v. Integrating both sides and simplifying the equation yields the familiar exponential utility function $U(v) = -e^{-\gamma v}$. The utility function may include a constant term, which can be ignored because it does not change the decision whether to buy a warranty or claim a failure.

We consider the RVW with a possible (partial or full) refund; i.e., if the customer has not claimed any failures against the warranty, she will receive a refund r at the end of the warranty term. The RVW reduces to the one-price-for-all TW if the refund is zero.

It is apparent that a customer who buys a TW will claim all the product failures regardless of the repair cost. However, the customer who has bought an RVW may have different claim behavior. With an RVW, the customer has the option of paying out of pocket a possibly random cost C_t for a repair at time t if she decides not to claim the failure. A rational customer

will dynamically and optimally decide whether to pay out of pocket for a repair or to claim a failure against the RVW. We will use the stochastic control model to formulate a customer's post-purchase strategic claim behavior, taking into account her risk attitude.

3.2. Strategic Claim Behavior

An RVW customer who has not claimed any failures has two options when facing a failure: make a claim against the RVW or pay out of pocket for a repair. A customer who has already claimed a failure against the warranty is not entitled to the refund, so there is no trade-off to be balanced for her and she will claim all the failures thereafter. We will consider a strategic customer who applies the optimal strategy to maximize her expected net utility of possessing a functional base product over a finite period of time given her risk attitude.

The warranty's coverage period is of duration T. Time is measured *backwards*, with t being the time to go until the end of the warranty. Similar to Von Lanzenauer (1974), Beichelt (1993), and Dimitrov et al. (2004), we assume that failures follow a nonhomogeneous Poisson process with instantaneous failure rate λ_t at time t. A failure at time t has random repair cost C_t with publicly known cumulative distribution function $F_t(\cdot)$. We denote the mean of random cost C_t by $c_t := E[C_t]$.

If a customer has already claimed a failure against the RVW, there is no further trade-off to be made: it is optimal for her to claim all future failures because she is no longer entitled to receive a refund. If she has not yet claimed any failure at time t, i.e., she has paid out of pocket for all failures up to time t, let $g(t; \gamma, r)$ be the net value of the RVW that is *certainty-equivalent* to the random benefit of holding an RVW with a possible refund r for her as a strategic customer with risk attitude γ.

The Hamilton-Jacobi-Bellman (HJB) equation arises as the limit of a discrete-time dynamic program as the time increment approaches zero. Since the failure process is a nonhomogeneous Poisson process, for a small δ, the probability that only one failure occurs in the interval $(t, t - \delta]$ is approximately $e^{-\lambda_t \delta} \lambda_t \delta \simeq \lambda_t \delta$, the probability of no failures is approximately $1 - \lambda_t \delta$, and the probability of two or more failures is $o(\delta)$. Assume that the customer's wealth level is v_t at time t, which is also assumed to include the utility of possessing a functional product. Later on, we will show that the optimal strategy is independent of the wealth level v_t. When facing a failure with cost C_t at time t, the customer will choose the option that maximizes her total expected utility from time t to

the end of the warranty horizon. By the principle of optimality, we have

$$-e^{-\gamma(v_t + g(t; \gamma, r))}$$

$$= \lambda_t \delta E[\max(-e^{-\gamma(v_t + g(t-\delta; \gamma, r) - C_t)}, -e^{-\gamma v_t})]$$
$$+ (1 - \lambda_t \delta)(-e^{-\gamma(v_t + g(t-\delta; \gamma, r))}) + o(\delta),$$

where $E[\cdot]$ takes expectation with respect to the random repair cost C_t. We assume that after the instantaneous repair, the failure rate and the utility of possessing a functional product return to the levels they were at before the failure. Rearranging the above equation results in

$$\frac{e^{-\gamma g(t; \gamma, r)} - e^{-\gamma g(t-\delta; \gamma, r)}}{\delta}$$

$$= \lambda_t e^{-\gamma g(t-\delta; \gamma, r)} (E[e^{\gamma \cdot \min(C_t, g(t-\delta; \gamma, r))}] - 1) + \frac{o(\delta)}{\delta}.$$

Taking the limit at both sides as δ goes to zero and rearranging the terms, we obtain the following differential equation:

$$g'(t; \gamma, r) = -\frac{\lambda_t}{\gamma} (E[e^{\gamma \cdot \min(C_t, g(t; \gamma, r))}] - 1), \qquad (1)$$

with the boundary condition $g(0; \gamma, r) = r$ for any given γ and r.

Intuitively, this strategic claim policy captured by Equation (1) will depend on the time to go, risk attitude, refund size, the failure probabilities, and the random repair costs.

THEOREM 1 (OPTIMAL CLAIM POLICY). *The optimal claim policy for a customer with risk attitude γ facing an RVW with refund r has the following structures.*

(a) *Optimal Policy: When a failure with repair cost C_t occurs at time t, it is optimal for a customer with risk attitude γ to make a claim if and only if $C_t \geq g(t; \gamma, r)$ and to pay out of pocket if $C_t < g(t; \gamma, r)$. Moreover, $g(t; \gamma, r)$ is decreasing in t.*

(b) *Comparative Statics: The function $g(t; \gamma, r)$ is decreasing in γ while it is increasing in r. Moreover, $g(t; \gamma, r) \to r$ as $\gamma \to -\infty$; $g(t; \gamma, r) \to 0$ as $\gamma \to \infty$ for any positive time t.*

All the proofs in this paper are available as supplemental material in the online appendix (http://dx.doi.org/10.1287/msom.2014.0501). For any given refund, the longer the remaining time is, the less possible it is that the customer will receive a refund, because she may experience more failures later. Moreover, the more risk averse a customer is, the less valuable the RVW is to her. As a result, the customer is more likely to claim failures against the warranty. On the other hand, a customer who is less risk averse or more risk seeking prefers to receive the refund and therefore claim fewer failures. We consider the case

with γ approaching zero. The differential equation (1) becomes

$$g'(t; 0, r) = \lim_{\gamma \to 0} g'(t; \gamma, r)$$

$$= \lim_{\gamma \to 0} -\frac{\lambda_t}{\gamma} (E[e^{\gamma \cdot \min(C_t, g(t; \gamma, r))}] - 1)$$

$$= -\lambda_t E[\min(C_t, g(t; 0, r))].$$

This corresponds to the risk-neutral case.

As a direct application of the properties of $g(t; \gamma, r)$, we can obtain the closed-form solution for the constant repair cost; i.e., $\Pr(C_t = c) = 1$. To avoid the trivial case, we assume that the constant repair cost c is less than the refund r, i.e., $c < r$; otherwise, the problem is trivial since it is optimal to claim all the failures. Moreover, we can also derive the closed-form solution to the HJB equation (1) when the repair cost is exponentially distributed. We summarize these results in the following proposition.

PROPOSITION 1 (CLOSED-FORM OPTIMAL CLAIM POLICY). *Let $\Lambda(t)$ be the expected total number of failures, i.e., $\Lambda(t) = \int_0^t \lambda_s \, ds$ for any $0 \leq t \leq T$.*

(a) *Constant Repair Cost: If $C_t = c$ for all t, then there exists a time threshold t^* such that it is optimal to claim a failure at time t if and only if $t \geq t^*$, and*

$$g(t; \gamma, r) = \begin{cases} r - \dfrac{1}{\gamma}(e^{\gamma c} - 1)\Lambda(t) & t < t^*, \\[2mm] -\dfrac{1}{\gamma} \ln\big(1 - e^{-\Lambda(t) + \Lambda(t^*)} \\[1mm] \qquad\quad \cdot (1 - e^{-\gamma c})\big) & t \geq t^*, \end{cases} \quad (2)$$

where t^* is the unique point satisfying $\Lambda(t^*) = \gamma(r-c)/(e^{\gamma c} - 1)$.

(b) *Exponential Repair Cost: If C_t is an independently and identically distributed (i.i.d.) exponential random variable with mean $1/\mu$ and $\mu > \gamma$, and independent of the Poisson failure process, then*

$$g(t; \gamma, r) = \frac{1}{\mu - \gamma} \ln\big[(1 - e^{-\Lambda(t)}) + e^{(\mu - \gamma)r - \Lambda(t)}\big]. \quad (3)$$

In particular, $g(t; \gamma, r) \to e^{-\Lambda(t)} r$ as $\gamma \to \mu$.

Suppose that a customer adopts the *claim-all* policy; i.e., she claims all the failures no matter how much the out-of-pocket repair cost is. Since she claims any failure, she loses the opportunity to receive the refund. We note that the claim-all policy, a bounded rational behavior, becomes nearly optimal when the risk attitude is sufficiently high, because $g(t; \gamma, r) \to 0$ as $\gamma \to \infty$ by Theorem 1. Interestingly, $g(t; \infty, r)$ is different from the expected refund under the claim-all policy, which can be expressed by $e^{-\Lambda(t)} r$, independent of her risk attitude, whereas $g(t; \infty, r)$ takes the risk attitude into account. Surprisingly, by Proposition 1, the

expected refund of the RVW under the claim-all policy is equal to the value under the strategic claim policy for the exponentially distributed repair cost with parameter equal to the risk attitude. We will further investigate the claim-all policy in §5.2.

4. Provider's Problem

The literature on warranties often relies on risk aversion as a motivation for customers to buy warranties (see, e.g., Padmanabhan 1995, Hollis 1999, Desai and Padmanabhan 2004). That is also one of the reasons why warranty providers charge high premiums, thereby earning significant profits. For simplicity of exposition, we assume that a failure with repair cost C_t to the customer (she) also incurs the same cost to the provider (he) if the customer claims it against the warranty. In fact, the cost to the provider may be less because of economies of scale, i.e., he may incur a cost βC_t with $0 \leq \beta \leq 1$, which makes selling warranties even more profitable.

Under the strategic claim behavior described in Theorem 1, let $h(t; \gamma, r)$ represent the warranty provider's expected costs of repairs and refunds from time t to the end of the warranty duration for a customer with risk attitude γ who has not yet claimed any failure against the RVW. If the customer has already claimed a failure against the RVW, she will claim all the failures thereafter, so the expected cost to the provider is equal to $E[R(t)]$ for $0 \leq t \leq T$, where $R(t)$ is the total repair cost from time t to the end of the duration. Since the failures follow a nonhomogeneous Poisson process, if the repair cost C_t is i.i.d. and is independent of the failure process, then the total repair cost is compound Poisson, and $E[R(t)] = \Lambda(t)c$, where $E[C_t] = c$.

Similarly to the customer's problem (1), for a small δ, the probability that only one failure occurs in the interval $(t, t - \delta]$ is approximately $\lambda_t \delta$: the strategic customer will claim a failure of repair cost C_t with probability $\Pr(C_t \geq g(t; \gamma, r))$ according to Theorem 1. If so, the expected support cost to the provider is equal to $E[C_t \mid C_t \geq g(t; \gamma, r)] + E[R(t)]$, noting that the failures thereafter will also be covered by the RVW. Then, we have

$$h(t; \gamma, r)$$
$$= \lambda_t \delta \Pr(C_t \geq g(t; \gamma, r)) \cdot (E[C_t \mid C_t \geq g(t; \gamma, r)] + E[R(t)])$$
$$+ (1 - \lambda_t \delta) h(t - \delta; \gamma, r) + o(\delta).$$

Rearranging the above equation and taking the limit as δ goes to zero, we derive the differential equation for $h(t; \gamma, r)$ as follows:

$$h'(t; \gamma, r) = \lambda_t \Pr(C_t \geq g(t; \gamma, r))\{E[C_t \mid C_t \geq g(t; \gamma, r)]$$
$$- (h(t; \gamma, r) - E[R(t)])\}. \tag{4}$$

The boundary condition is $h(0; \gamma, r) = r$.

As mentioned, the literature on warranties shows that the main driver of the warranty profits is risk aversion; i.e., risk-averse customers loathe risk and are willing to pay higher premiums for the warranty coverage than the expected out-of-pocket repair cost. We first characterize the customer's willingness to pay for the TW, which will be used as a benchmark for investigating the profitability of the RVW.

4.1. Willingness to Pay for Traditional Warranty

The willingness to pay (WTP) of a customer with risk attitude γ for the TW covering t units of time is denoted by $w_{tw}(t; \gamma)$, which is the quantity such that the customer is indifferent between buying and not buying a TW in terms of utility, i.e., $E[-e^{-\gamma(v_t - R(t))}] = -e^{-\gamma(v_t - w_{tw}(t; \gamma))}$, where v_t is the customer's wealth level at time t, including the utility of possessing a functional product. Simple algebra leads to

$$w_{tw}(t; \gamma) = \ln(E[e^{\gamma R(t)}])/\gamma. \tag{5}$$

PROPOSITION 2 (MONOTONICITY). *The willingness to pay for the TW is increasing with respect to the risk attitude γ. Moreover, $w_{tw}(t; \gamma) \to E[R(t)]$ as $\gamma \to 0$.*

Notice that the willingness to pay for the risk-neutral customer is equal to the expected out-of-pocket repair cost. Risk-averse customers are willing to pay higher premiums for the warranty coverage, and risk-seeking customers prefer to take the risk and are willing to buy the TW only if its price is significantly below the expected out-of-pocket repair cost. The willingness to pay $w_{tw}(t; \gamma)$ can be further simplified for stationary repair cost.

PROPOSITION 3 (CLOSED-FORM EXPRESSION OF WTP FOR TW). *If C_t for any time t is i.i.d., denoted by C, the willingness to pay for the TW can be further simplified to*

$$w_{tw}(t; \gamma) = \Lambda(t)(M_C(\gamma) - 1)/\gamma, \tag{6}$$

where $M_C(\gamma) = E[e^{\gamma C}]$ is the moment generating function for the repair cost C. We assume that $M_C(\gamma)$ is finite. Moreover,

(a) Constant Repair Cost: If $C_t = c$ for any time t, the willingness to pay for the TW is

$$w_{tw}(t; \gamma) = \Lambda(t)(e^{\gamma c} - 1)/\gamma. \tag{7}$$

(b) Exponential Repair Cost: If C_t follows an exponential distribution with mean $1/\mu$ for any time t,

$$w_{tw}(t; \gamma) = \Lambda(t)/(\mu - \gamma), \tag{8}$$

assuming $\mu > \gamma$; otherwise, $w_{tw}(t; \gamma) = \infty$.

Without loss of generality, we assume that customers will buy the warranties if priced at their willingness to pay. After characterizing the willingness to pay for the TW, we are ready to investigate further the RVW, including customers' purchase behavior and its profitability to the provider.

4.2. Residual Value Warranty

A customer's willingness to pay for an RVW is equal to the quantity such that she is indifferent between buying and not buying an RVW, taking into account the possible refund, out-of-pocket repair cost, and her risk attitude. We assume a zero opportunity cost for money, mainly for convenience, since it is easy to reproduce results for the present value of the expected refunds.

Let $w_{\mathrm{rvw}}(t; \gamma, r)$ denote the willingness to pay of a customer with risk attitude γ for an RVW covering t amount of time ahead with a possible refund r. Then the optimal price is equal to $w_{\mathrm{rvw}}(T; \gamma, r)$, and its profit per customer can be expressed as follows:

$$w_{\mathrm{rvw}}(T; \gamma, r) - h(T; \gamma, r), \tag{9}$$

where $h(T; \gamma, r)$ is the support cost to the provider. To investigate the profitability of the RVW, we first study the relationship between $w_{\mathrm{rvw}}(T; \gamma, r)$ and $h(T; \gamma, r)$.

PROPOSITION 4. *The willingness to pay of a customer with risk attitude γ for the RVW is equal to the sum of the willingness to pay for the TW and the net value of the RVW; i.e.,*

$$w_{\mathrm{rvw}}(t; \gamma, r) = w_{\mathrm{tw}}(t; \gamma) + g(t; \gamma, r). \tag{10}$$

Moreover, the following structural results hold for $w_{\mathrm{rvw}}(t; \gamma, r)$, $g(t; \gamma, r)$, and $h(t; \gamma, r)$.

(a) *Bounds of Support Cost: For any positive time-to-go t and refund r, $w_{\mathrm{rvw}}(t; \gamma, r) > h(t; \gamma, r) > g(t; \gamma, r) + E[R(t)]$ for any $\gamma > 0$, $w_{\mathrm{rvw}}(t; \gamma, r) = h(t; \gamma, r) = g(t; \gamma, r) + E[R(t)]$ for $\gamma = 0$, and $w_{\mathrm{rvw}}(t; \gamma, r) < h(t; \gamma, r) < g(t; \gamma, r) + E[R(t)]$ for any $\gamma < 0$.*

(b) *Monotonicity: For $\gamma > 0$, $h(t; \gamma, r) - g(t; \gamma, r)$ is increasing with respect to the refund r; for $\gamma < 0$, $h(t; \gamma, r) - g(t; \gamma, r)$ is decreasing in r. Moreover, $h(t; \gamma, r) - g(t; \gamma, r) \rightarrow w_{tw}(t; \gamma)$ as $r \rightarrow \infty$ for any γ.*

The support cost of the RVW to the provider is bounded from above and below. For risk-averse customers, it is lower than the willingness to pay but greater than the sum of the net value of the RVW and the total expected out-of-pocket repair cost. The monotonic properties with respect to the refund size give insights for the design and pricing of the RVW.

Risk-seeking customers may not buy a TW; the provider may lower the price further to attract them, but it may not be profitable to do so. Can an RVW earn a nontrivial profit or alleviate the loss in a market with risk-seeking customers?

THEOREM 2 (PROFITABILITY OF AN RVW IN A HOMOGENEOUS MARKET). *In a homogeneous market, if all the customers are risk averse with constant risk attitude $\gamma > 0$, then the optimal RVW degenerates to a TW; if all the customers are risk seeking with constant risk attitude $\gamma < 0$, then the RVW with a sufficiently large refund*

can balance the revenue and the support cost, whereas the TW loses money.

An RVW with a possible refund gives customers flexibility in their post-purchase failure claim behavior, and that makes the RVW more attractive. Customers will claim the failures strategically to maximize their total utility during the warranty coverage, taking into account their risk attitudes. With a single segment of risk-averse customers, the RVW does not outperform the TW. However, somewhat surprisingly, the strategic claim behavior of risk-seeking customers under the RVW may benefit the warranty provider compared with the situation under the TW.

Risk-seeking customers' willingness to pay for the TW can be significantly below the support cost, and that makes a TW a money-losing proposition. In contrast, the RVW can alleviate the loss in a homogeneous market, because its willingness to pay and support cost tend to be equal for a sufficiently large refund (see Proposition 4, part (b)). The risk-seeking customers have an incentive to obtain the large refund and therefore make few failure claims. As a result, the RVW can almost break even for those risk-seeking customers. This result assumes a 100% redemption rate and consistent risk attitudes before and after the warranty is bought. In practice, it is not uncommon that the service provider requires the customers to redeem the refund within a short period and customers may be inconsistent in their risk attitudes over time (see §4.3); that makes the RVW able to earn profits even with those risk-seeking customers. Moreover, although the RVW does not outperform the TW for a single segment of risk-averse customers, the RVW can indeed earn more profits over the TW in a market with heterogeneous risk attitudes (see §5.1).

4.3. Risk-Attitude Inconsistency

Behavior inconsistency is a common phenomenon in practice, which is well studied in the behavior literature. In a behavioral economics study of U.S. health clubs, DellaVigna and Malmendier (2006) find that people visit the gym much less often than they thought they would when signing up for membership.

Likewise, customers may not be consistently risk averse or risk seeking. Their risk attitude may be different before and after buying the RVW. Let γ_b (respectively, γ_a) represent the risk attitude before (respectively, after) buying the warranty. Then, the willingness to pay for the RVW is $w_{\mathrm{rvw}}(T; \gamma_b, r)$, and the support cost to the provider is $h(T; \gamma_a, r)$. The provider's profit can be expressed as follows:

$$w_{\mathrm{rvw}}(T; \gamma_b, r) - h(T; \gamma_a, r). \tag{11}$$

However, the TW does not capture the risk-attitude inconsistency because the purchase decision depends

only on the risk attitude before buying the TW and the TW customers will claim all the failures. Next, we will compare the profits of the RVW and the TW under the situation of risk-attitude inconsistency.

THEOREM 3 (RISK-ATTITUDE INCONSISTENCY). *For any given before-purchase risk attitude, there exist thresholds for the after-purchase risk attitude denoted by $\underline{\gamma}_a$ and $\bar{\gamma}_a$ such that the RVW is strictly more profitable than the TW if and only if $\gamma_a < \underline{\gamma}_a$ or $\gamma_a > \bar{\gamma}_a$.*

Under the RVW, customers face a trade-off between claiming failures and receiving refunds. This feature of the RVW can give the service provider an opportunity to capitalize on customers' risk-attitude inconsistency, whereas the TW cannot, because under the TW, customers would always claim all the failures. After buying the warranty, if customers become more risk averse (respectively, risk seeking), they will value the RVW less (respectively, more) (see Theorem 1). As a result, they will claim more (respectively, fewer) failures than they would claim if their attitude to risk were the same as when they bought the warranty; that could benefit the risk-neutral warranty provider, who pays out less in refunds (respectively, saves more in repair costs).

5. Heterogeneous Market

Customers may be heterogeneous along various dimensions. In this section, we will illustrate the profitability of the RVW through analytical results and numerical examples for heterogeneous markets. For simplicity, assume that there are two market segments denoted by the type L and type H customers. Market segment n has risk attitude γ^n, failure rate λ_t^n, and random repair cost C_t^n at time t and proportion α^n, $n \in \{L, H\}$ with $\alpha^L + \alpha^H = 1$.

Customers can simultaneously differ in numerous dimensions, including risk attitude, failure rate, and repair cost. To isolate various effects, we investigate the RVW's profitability in a heterogeneous market with customers differing in one of the factors each time.

5.1. Heterogeneity in Risk Attitude
We first study the heterogeneity in the risk attitude, assuming that all other factors, including the failure rate and repair cost, are the same for both type L and type H customers. Without loss of generality, assume that type H customers are more risk averse; i.e., $\gamma^H > \gamma^L$.

By Proposition 2, the type H customer's willingness to pay for the TW is higher; i.e., $w_{\text{tw}}(T; \gamma^H) > w_{\text{tw}}(T; \gamma^L)$. The optimal price of the TW should be the willingness to pay of either type L or type H customers. If the TW charges $w_{\text{tw}}(T; \gamma^H)$, only type H customers buy it, and the profit can be expressed

by $\alpha^H(w_{\text{tw}}(T; \gamma^H) - E[R(T)])$; if it charges $w_{\text{tw}}(T; \gamma^L)$, both segments will buy it, and the profit is equal to $w_{\text{tw}}(T; \gamma^L) - E[R(T)]$. Then, the provider's optimal decision, when offering the TW, is to solve $\max\{\alpha^H(w_{\text{tw}}(T; \gamma^H) - E[R(T)]), w_{\text{tw}}(T; \gamma^L) - E[R(T)]\}$.

PROPOSITION 5. *In the heterogeneous market with two market segments differing only in the risk attitude, for any given γ^L, there exists a threshold $\hat{\gamma}^H$ such that the optimal TW captures both type L and type H customers if and only if $\gamma^L < \gamma^H < \hat{\gamma}^H$.*

For the RVW with a possible refund r, a type n customer's willingness to pay $w_{\text{rvw}}(T; \gamma^n, r)$ is equal to $w_{\text{tw}}(T; \gamma^n) + g(T; \gamma^n, r)$, $n \in \{H, L\}$ by Proposition 4. The problem faced by the RVW provider is to determine the price p and the refund size r to maximize his total expected profit over the two market segments:

$$\max_{p, r} \big\{ \alpha^L(p - h(T; \gamma^L, r)) \cdot \mathbf{1}(p \leq w_{\text{rvw}}(T; \gamma^L, r))$$

$$+ \alpha^H(p - h(T; \gamma^H, r)) \cdot \mathbf{1}(p \leq w_{\text{rvw}}(T; \gamma^H, r)) \big\}, \quad (12)$$

where $\mathbf{1}(X)$ is the indicator function, i.e., $\mathbf{1}(X) = 1$ if X is true; otherwise, $\mathbf{1}(X) = 0$. Problem (12) is not easy to solve in general because it involves the differential equations that govern $w_{\text{rvw}}(\cdot; \cdot, \cdot)$ and $h(\cdot; \cdot, \cdot)$.

Theorem 1 states that $g(t; \gamma, r)$ is decreasing in γ for any given refund r, whereas $w_{\text{tw}}(t; \gamma)$ is increasing in γ by Proposition 2. The following result further summarizes structural properties of the customer's willingness to pay and the support cost to the provider for the RVW, which will be useful in simplifying the above problem (12).

THEOREM 4. *For any given refund r, the following structural results for the RVW hold.*

(a) Monotonicity: The willingness to pay $w_{\text{rvw}}(t; \gamma, r)$ for the RVW is strictly increasing in the risk attitude γ, but at a lower rate than the TW.

(b) Unimodality: The support cost $h(t; \gamma, r)$ of the RVW to the provider is unimodal with respect to the risk attitude γ. Specifically, it is decreasing (respectively, increasing) in γ for risk-averse (respectively, risk-seeking) customers.

We remark that $w_{\text{rvw}}(t; \gamma, r)$ is increasing with respect to γ no matter whether customers are risk averse or risk seeking. The support cost for risk-neutral customers is the highest. If a risk-neutral customer changes her risk attitude after buying the RVW, e.g., she becomes more risk averse or risk seeking, the support cost to the RVW provider is lower. The risk-attitude inconsistency may benefit the RVW provider though it has no effect on the TW, as shown in Theorem 3.

To investigate the profitability of the RVW over the TW in the heterogeneous market with different risk

preferences, the RVW must capture both segments by charging the type L customers' willingness to pay. Then, the above two-dimensional optimization problem (12) becomes a single-dimensional problem of the refund r:

$$\max_r \left\{ w_{\mathrm{tw}}(T; \gamma^L) + g(T; \gamma^L, r) - \alpha^L h(T; \gamma^L, r) - \alpha^H h(T; \gamma^H, r) \right\}. \tag{13}$$

THEOREM 5 (PROFITABILITY OF RVW). *In the heterogeneous market of two segments with different risk attitudes, for any given γ^L, there exists a threshold denoted by $\overline{\gamma}^H$ such that the RVW is strictly more profitable than the TW for any $\gamma^H \in (\underline{\gamma}^H, \hat{\gamma}^H]$.*

The RVW outperforms the TW in the heterogeneous market where customers differ in their risk attitudes in an intermediate range. If the two segments are almost the same in risk preference, the optimal RVW degenerates to the TW; if they are significantly different, the TW with a high optimal price may serve only the type H customers, but the RVW that captures both segments may result in a higher profit; and even if both the TW and RVW serve the two types of customers, the RVW may still be strictly more profitable because the support cost for the type H customers is lower to the provider, as shown in Theorem 4.

EXAMPLE 1. Suppose that there are two segments: type L with proportion $\alpha^L = 30\%$ and type H with proportion $\alpha^H = 70\%$. Customers are heterogeneous only in their risk attitudes: we fix $\gamma^L = 0.01$ and vary γ^H between 0.03 and 0.04. The warranty duration is $T = 5$. The failure process follows a Poisson process with a stationary rate $\lambda = 1$. Assume that the repair cost follows the exponential distribution with mean \$20. Then, the out-of-pocket repair cost is expected to be \$100.

Suppose that the TW captures both market segments, so its price is equal to the type L customers' willingness to pay; i.e., $w_{\mathrm{tw}}(T; \gamma^L) = \Lambda(T)/(\mu - \gamma^L) = \$125$. Then, the TW earns a profit of \$25, which is independent of the risk attitude of the type H customers. Table 1 summarizes the performance of the RVW in this heterogeneous market. For $0.03 \le \gamma^H \le 0.032$, the optimal RVW degenerates to the TW and earns the same profit. For $\gamma^H > 0.032$, the optimal RVW offers almost a full refund and significantly improves its profitability beyond the TW. For $\gamma^H =$

0.037, the optimal RVW price is \$220.6, and its refund is equal to \$220. The RVW earns profit \$37.2, which is an improvement of 48.8% over the TW.

5.2. Heterogeneity in the Failure Rate

We study the heterogeneity in the failure rate, assuming that all other factors, including the risk attitudes and repair cost distributions, are the same for both type L and type H customers. Without loss of generality, assume that the type H customers have higher failure rate; i.e., $\lambda_t^H > \lambda_t^L$ for any $0 \le t \le T$. To simplify the exposition, we further assume that the failure processes are stationary, i.e., $\lambda_t^n = \lambda^n$ and $C_t = C$ for any time t, and $\lambda^H > \lambda^L$. Let $c = E[C]$. Similarly, for any given time to go t and refund r, let $g(t; \lambda^n, r)$, $w_{\mathrm{rvw}}(t; \lambda^n, r)$, and $h(t; \lambda^n, r)$ denote the net value of the RVW, the type n customer's willingness to pay for the RVW, and the support cost to the provider under the strategic claim behavior, $n \in \{L, H\}$, respectively.

PROPOSITION 6 (MONOTONICITY). *The functions $g(t; \lambda, r)$ and $w_{\mathrm{rvw}}(t; \lambda, r)$ are monotone in λ for any fixed r.*

(a) *The net value of the RVW is decreasing with respect to the failure rate; i.e., $g(t; \lambda^H, r) \le g(t; \lambda^L, r)$.*

(b) *The willingness to pay for the RVW is increasing with respect to the failure rate; i.e., $w_{\mathrm{rvw}}(T; \lambda^H, r) \ge w_{\mathrm{rvw}}(T; \lambda^L, r)$.*

Similar to Proposition 5, in the heterogeneous market with two segments differing in failure rates, the TW will capture both segments if and only if $\lambda^H \in (\lambda^L, \hat{\lambda}^H)$, where

$$\hat{\lambda}^H = \lambda^L \cdot \frac{(M_C(\gamma) - 1)/\gamma - \alpha^L c}{\alpha^H (M_C(\gamma) - 1)/\gamma}, \tag{14}$$

derived from solving $w_{\mathrm{tw}}(T; \lambda^L) - (\alpha^L \lambda^L + \alpha^H \lambda^H) Tc = \alpha^H (w_{\mathrm{tw}}(T; \lambda^H) - \lambda^H Tc)$ for λ^H.

To investigate the profit advantage of the RVW over the TW, the RVW must capture both segments. By Proposition 6, the willingness to pay is monotonically increasing in the failure rate. Then, given any refund r, the optimal price of the RVW must be equal to the type L customers' willingness to pay. As a result, the two-dimensional optimization becomes a single-dimensional problem of the refund r as follows: $\max_r \{ w_{\mathrm{tw}}(T; \lambda^L) + g(T; \lambda^L, r) - \alpha^L h(T; \lambda^L, r) - \alpha^H h(T; \lambda^H, r) \}$.

Table 1 Performance of RVW in a Heterogeneous Market

γ^H	0.03 (\$)	0.031 (\$)	0.032 (\$)	0.033 (\$)	0.034 (\$)	0.035 (\$)	0.036 (\$)	0.037 (\$)	0.038 (\$)	0.039 (\$)	0.04 (\$)
Optimal price of RVW	125	125	125	182.7	191.8	200.3	209.9	220.6	231.4	244.3	258.2
Optimal refund of RVW	0	0	0	180	190	199	209	220	231	244	258
Optimal profit of RVW	25	25	25	26	28.2	30.7	33.7	37.2	41.2	45.7	51.0

Insights similar to the scenario of the heterogeneity in risk attitude (e.g., Theorem 5) can be derived. Consider the following heuristic claim policy, which may be adopted by more bounded-rational customers and can be near optimal when customers are very risk averse, the refunds are small, or the out-of-pocket repair costs are very high.

CLAIM-ALL POLICY. Suppose that a customer adopts the *claim-all* policy; i.e., she claims all the failures no matter how much the out-of-pocket repair cost is. Similarly, let $\tilde{g}(t; \lambda, r)$ be the net value of an RVW that is *certainty-equivalent* to the random benefit of holding an RVW with possible refund r, as a customer who has not claimed any failure and will adopt the claim-all policy.

Similar to Equation (1), $\tilde{g}(t; \lambda, r)$ is governed by the following differential equation

$$\tilde{g}'(t; \lambda, r) = -\frac{\lambda_t}{\gamma}(e^{\gamma \tilde{g}(t; \lambda, r)} - 1), \qquad (15)$$

with the boundary condition $\tilde{g}(0; \lambda, r) = r$.

Similar to Proposition 4, the willingness to pay for an RVW with refund r under the claim-all policy can be expressed by $\tilde{w}_{\text{rvw}}(T; \lambda, r) = w_{\text{tw}}(T; \lambda) + \tilde{g}(T; \lambda, r)$. Although $\tilde{w}_{\text{rvw}}(t; r, \lambda)$ may fail to be monotonic with respect to the failure rate in general, the monotonicity holds under certain mild conditions, as shown in Theorem 6. Then, the optimization problem in price and refund can be simplified to maximizing the following profit function with respect to a single variable r:

$$\tilde{w}_{\text{rvw}}(T; \lambda^L, r) - \left[\alpha^L \tilde{h}_{\text{rvw}}(T; \lambda^L, r) \right. \\ \left. + \alpha^H \tilde{h}_{\text{rvw}}(T; \lambda^H, r)\right], \qquad (16)$$

where $\tilde{h}_{\text{rvw}}(T; \lambda, r)$ represents the support cost to the provider under the claim-all policy. It is apparent that $\tilde{h}_{\text{rvw}}(T; \lambda, r) = E[R(T)] + e^{-\Lambda(T)}r$, where the first term is the expected repair cost and the second is the expected refund under the claim-all policy.

THEOREM 6. *Assume that all the customers follow the claim-all policy. In a heterogeneous market, with risk-averse customers differing only in the failure rates, the following results hold.*

(a) *The willingness to pay for the RVW is strictly increasing in the failure rate for any given refund if $M_C(\gamma) \geq 1/(1 - e^{-\Lambda(T)})$.*

(b) *The profit function (16) is strictly concave in r, so there exists a unique optimal refund.*

(c) *For any given λ^L, there exists a threshold denoted by $\underline{\lambda}^H$ such that the RVW is strictly more profitable than the TW for any $\lambda^H \in (\underline{\lambda}^H, \hat{\lambda}^H)$.*

The willingness to pay for the RVW is equal to the sum of the willingness to pay for the TW and the net value of the RVW. The former is increasing in the failure rate, whereas the latter is decreasing. The condition $M_C(\gamma) \geq 1/(1 - e^{-\Lambda(T)})$ guarantees that the former dominates the latter, so the willingness to pay for the RVW is increasing in the failure rate. The closed-form expression of the optimal refund is provided in the proof of Theorem 6 in the online appendix. We note that Theorem 6 also holds in a heterogeneous market with more than two segments.

EXAMPLE 2. Suppose that there are two segments: type L with proportion $\alpha^L = 80\%$ and type H with proportion $\alpha^H = 20\%$. Customers are all risk averse, and their risk-attitude parameter is $\gamma = 0.005$. The warranty duration is $T = 1.5$. The failure processes are stationary: the failure rate is 0.25 for type L customers, i.e., $\lambda^L = 0.25$; the repair cost follows the exponential distribution with parameter $\mu = 0.01$, i.e., $E[C] = \$100$.

We compare the profits of the TW and the RVW for $\lambda^H \in [0.5, 0.85]$. Notice that $M_C(\gamma) = E[e^{\gamma C}] = \mu/(\mu - \gamma) = 2$ and $1/(1 - e^{-\lambda^H T}) < 1.90$ for any $\lambda^H \in [0.5, 0.85]$, so the condition in part (a) of Theorem 6 is satisfied. Equation (14) yields the threshold $\hat{\lambda}^H = 0.75$, so the TW captures both segments if and only if $\lambda^H < 0.75$. Figure 1 demonstrates that the RVW is strictly more profitable and captures both segments in a wider range: for $0.75 \leq \lambda^H \leq 0.79$ the TW only attracts the type H customers, but the RVW always captures both segments and earns a higher profit.

Table 2 shows the comparison between the TW and the RVW under the claim-all policy in this heterogeneous market. The profit improvement can be significant for certain ranges of λ^H. When $\gamma^H = 0.75$, the TW price is equal to $75, and it earns a profit of $22.5; the RVW price is $119.1, and it offers a possible refund

Figure 1 (Color online) Optimal Profit per Customer of RVW vs. TW Under the Claim-All Policy
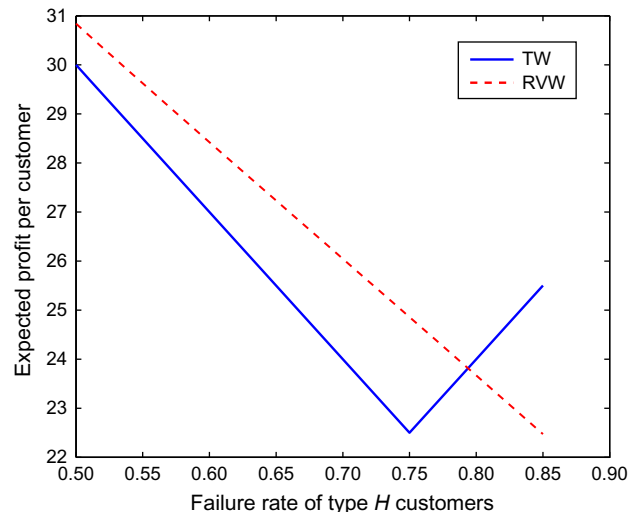
Table 2   Comparison of TW and RVW Under a Claim-All Policy

λ^H	0.5 ($)	0.55 ($)	0.6 ($)	0.65 ($)	0.7 ($)	0.75 ($)	0.8 ($)	0.85 ($)
Optimal price of TW	75	75	75	75	75	75	240	255
Optimal profit of TW	30	28.5	27	25.5	24.0	22.5	24.0	25.5
Optimal price of RVW	100.8	104.6	108.0	111.2	114.1	116.7	119.1	121.4
Optimal refund	38.68	44.61	50.07	55.10	59.74	64.02	67.97	71.63
Optimal profit of RVW	30.84	29.62	28.42	27.23	26.05	24.86	23.67	22.47

of \$64.02, which is 54% of the up-front price; and the RVW earns a profit of \$24.86, an improvement of 10.5% over the TW.

5.3.   Heterogeneity in Repair Cost

We next consider the heterogeneity in the repair cost, assuming that all other factors are the same for both type L and type H customers. We assume that the repair cost for the type H customers is stochastically greater than that for the type L customers at any time t; i.e., $C_t^H \geq_{st} C_t^L$ for any $0 \leq t \leq T$. As in the homogeneous case, we assume that the repair cost is i.i.d. over time for each segment. Let $c^H = E[C_t^H]$ and $c^L = E[C_t^L]$. Similarly, for given time to go t and refund r, let $g(t; C^n, r)$, $w_{\text{rvw}}(t; C^n, r)$ and $h(t; C^n, r)$, $n \in \{L, H\}$, denote the net value of the RVW, the type n customer's willingness to pay for the RVW, and the support cost to the provider under the strategic claim behavior, respectively.

PROPOSITION 7 (MONOTONICITY). *The monotonic results for $g(t; C, r)$ and $w_{\text{rvw}}(t; C, r)$ with respect to C are summarized as follows.*

(a) *The net value of the RVW is decreasing in the repair cost; i.e., $g(t; C^H, r) \leq g(t; C^L, r)$ for any refund r.*

(b) *The willingness to pay for the RVW is increasing in the random repair cost; i.e., $w_{\text{rvw}}(t; C^H, r) \geq w_{\text{rvw}}(t; C^L, r)$ for any refund r.*

Since the willingness to pay is monotonically increasing in the random repair cost by Proposition 7, the optimization of the RVW that captures both segments is reduced to a single-dimensional problem of the refund r as follows:

$$\max_r \left\{ w_{\text{tw}}(T; C^L) + g(T; C^L, r) - \alpha^L h(T; C^L, r) \right.$$
$$\left. - \alpha^H h(T; C^H, r) \right\}.$$

Like the heterogeneity in the risk attitude and the failure rate, the performance advantage of the RVW can also be derived in a market with customers who differ in the repair cost, e.g., the RVW alone can be strictly more profitable than the TW alone for the random repair costs that differ in a certain range.

6.   Extensions

We consider two extensions. First, the firm may want to offer a menu instead of only a TW or an RVW. Second, the concept of a single refund can be generalized

to a refund schedule, under which the refund amount depends on the number of claims made in the duration of the warranty.

6.1.   Warranty Menu

Because the RVW offers many advantages, it may attract a broad range of customers. But for a warranty provider who also sells a TW, the introduction of an RVW will likely cannibalize some or all of the demand for the TW. Therefore, the RVW should be carefully designed and priced to avoid eroding the profits from the existing TW and, indeed, to improve profitability.

We consider a warranty menu that consists of a TW and an RVW in a heterogeneous market with customers only differing in risk attitude. Other scenarios can be investigated in a similar way. Under the assumption of individual rationality, each customer will choose the option with the lowest total certainty-equivalent cost. For the TW in particular, the total certainty-equivalent cost is simply equal to the up-front price p_{tw}; for the RVW, the total certainty-equivalent cost is equal to the up-front price minus the net value. The latter is certainty-equivalent to the random benefit of holding an RVW with a possible refund; i.e., it is equal to $p_{\text{rvw}} - g(T; \gamma, r)$ for a customer with risk attitude γ. Notice that no purchase is always one of the options. There are two options in the menu, with each type of warranty targeting each type of customer in this heterogeneous market.

First, assume that we design a warranty menu such that the type L customers prefer the TW and the type H customers prefer the RVW. By the rationality constraints, it follows that $p_{\text{rvw}} - g(T; \gamma^L, r) > p_{\text{tw}}$ and $p_{\text{rvw}} - g(T; \gamma^H, r) < p_{\text{tw}}$. So we have $p_{\text{tw}} + g(T; \gamma^L, r) < p_{\text{rvw}} < p_{\text{tw}} + g(T; \gamma^H, r)$, which is impossible because $g(T; \gamma^L, r) \geq g(T; \gamma^H, r)$ for any positive refund size r by Theorem 1. Therefore, it is infeasible to have a warranty menu with a TW targeting type L customers and an RVW targeting type H customers under the assumption of individual rationality.

Therefore, to study the performance advantage of the warranty menu over the TW or the RVW alone, we need to consider only the menu with a TW targeting type H customers and an RVW targeting type L customers. Then, the problem for the warranty provider is to determine the prices and the refund to

maximize the total profit over the two segments subject to the incentive and participation constraints:

$$\max_{p_{\text{tw}}, p_{\text{rvw}}, r} \left\{ \alpha^L (p_{\text{rvw}} - h(T; \gamma^L, r)) + \alpha^H (p_{\text{tw}} - E[R(T)]) \right\}$$

$$\text{s.t.} \quad p_{\text{rvw}} - g(T; \gamma^L, r) < p_{\text{tw}},$$
$$p_{\text{rvw}} \le w_{\text{rvw}}(T; \gamma^L, r), \qquad (17)$$
$$p_{\text{rvw}} - g(T; \gamma^H, r) \ge p_{\text{tw}},$$
$$p_{\text{tw}} \le w_{\text{tw}}(T; \gamma^H).$$

Under the constraints $p_{\text{rvw}} - g(T; \gamma^L, r) < p_{\text{tw}}$ and $p_{\text{rvw}} \le w_{\text{rvw}}(T; \gamma^L, r)$, the type L customers will buy the RVW; similarly, the type H customers will buy the TW under the constraints $p_{\text{rvw}} - g(T; \gamma^H, r) > p_{\text{tw}}$ and $p_{\text{tw}} \le w_{\text{tw}}(T; \gamma^H)$.

Recall the relationship between the willingness to pay for the TW and the RVW: $w_{\text{rvw}}(t; \gamma, r) = w_{\text{tw}}(t; \gamma) + g(t; \gamma, r)$ for a customer with risk attitude γ by Proposition 4. Then, we can obtain the optimal prices to the above optimization problem (17), given any refund size.

PROPOSITION 8 (OPTIMAL MENU PRICES). *For the warranty menu consisting of a TW and an RVW, the best configuration is one in which the TW targets type H customers and the RVW targets type L customers. Moreover, for any given refund r, the optimal prices are*

$$p_{\text{tw}}^* = w_{\text{rvw}}(T; \gamma^L, r) - g(T; \gamma^H, r) \quad \text{and}$$
$$p_{\text{rvw}}^* = w_{\text{rvw}}(T; \gamma^L, r).$$

Then, the three-variable constrained optimization (17) can be simplified to a single-variable unconstrained problem of the refund r as follows:

$$\max_r \left\{ w_{\text{tw}}(T; \gamma^L) + g(T; \gamma^L, r) - \alpha^L h(T; \gamma^L, r) \right.$$
$$\left. - \alpha^H (g(T; \gamma^H, r) + E[R(T)]) \right\}. \qquad (18)$$

If we compare the warranty menu to the TW or the RVW alone, it is clear that the warranty menu is significantly more profitable than either type of warranty alone.

THEOREM 7 (PROFITABILITY OF RVW MENU). *For any given γ^L, there exists a threshold γ'^H such that the warranty menu consisting of a TW and an RVW outperforms the TW or the RVW alone for any $\gamma^H \in (\gamma'^H, \hat\gamma^H]$. Moreover, $\gamma'^H < \underline{\gamma}^H$, where $\underline{\gamma}^H$ is the threshold beyond which the RVW alone is more profitable than the TW alone.*

The following example demonstrates that an RVW menu can earn significantly more profit than the TW or RVW alone in a heterogeneous market, even though the menu is not optimally designed.

EXAMPLE 3. Continue with Example 1 and consider the warranty menu consisting of a TW and an RVW for the same heterogeneous market. Table 3 illustrates the optimal menu and its total profit for γ^H varying between 0.03 and 0.04.

Under the optimal warranty menu, the type L customers will buy the RVW and the type H customers will buy the TW. The warranty menu can further significantly improve the profit compared with the RVW alone, as shown in Table 3. For $\gamma^H = 0.037$, the optimal menu earns a profit of \$221.6, which is an improvement of 496% over the RVW alone and an improvement of 786% over than the TW alone. Note that the RVW price and its refund in the optimal warranty menu can be unreasonably high. If we fix the refund at $r = \$200$, the menu still outperforms the TW or the RVW alone. For $\gamma^H = 0.037$, the corresponding TW price is \$196 and the RVW price is \$201, which is only \$1 higher than the refund. This menu is expected to earn a profit of \$67.4, which is still 81% more than the RVW alone and 170% more than the TW alone.

6.2. Refund Schedule

The simplest RVW offers a single refund if a customer has not claimed any failure during the warranty term. The refund concept can be generalized to a schedule with multiple refunds depending on the customer's failure-claim history. The RVW may have a refund schedule $\mathbf{r} := (r_0, \ldots, r_{m-1})$ for some positive integer m, where $r_0 \ge r_1 \ge \cdots \ge r_{m-1}$. A customer who makes $0 \le j \le m-1$ claims over the warranty period will receive a positive refund r_j. A customer who makes m or more claims will not receive a refund but

Table 3   **Performance of an Optimal Menu and a Menu with a Fixed Refund** $r = \$200$

γ^H	0.03 (\$)	0.031 (\$)	0.032 (\$)	0.033 (\$)	0.034 (\$)	0.035 (\$)	0.036 (\$)	0.037 (\$)	0.038 (\$)	0.039 (\$)	0.04 (\$)
Optimal price of TW	263	278	294	312	333	357	384	417	454	500	555
Optimal price of RVW	6,407	6,407	6,418	8,007	6,932	6,407	6,407	6,482	6,507	6,407	6,407
Optimal refund of RVW	6,407	6,407	6,418	8,007	6,932	6,407	6,407	6,482	6,507	6,407	6,407
Optimal total profit	114.2	124.4	135.8	148.7	163.3	180.0	199.2	221.6	248.1	279.9	318.8
Fixed refund	200	200	200	200	200	200	200	200	200	200	200
Price of TW	188	189	191	192	193	194	195	196	196	197	198
Price of RVW	201	201	201	201	201	201	201	201	201	201	201
Total profit	61.7	62.9	63.9	64.8	65.6	66.3	66.9	67.4	67.9	68.3	68.7

will still be covered for any failures that occur in the remaining warranty duration.

For convenience of exposition, we define $r_j := 0$ for all integers $j \geq m$. Notice that the RVW can be implemented by giving customers m coupons with values $\Delta r_j := r_{j-1} - r_j$ for $j = 1, \ldots, m$. To file the jth claim, a customer surrenders the jth coupon for $j = 1, 2, \ldots, m$. At the end of the warranty duration, the customer can redeem any remaining coupons at their face value. An important special case is when Δr_j is a constant for every $j = 1, 2, \ldots, m$. Once the coupons are exhausted, (rational) customers will file all claims since all the failures are covered by the RVW. Of course, the refunds can be automatically delivered without the need for physical coupons or mail-in redemptions, but the coupons may help customers conceptualize the idea behind the RVW with a refund schedule.

Let $g(t, k; \gamma, \mathbf{r})$ be the net value for a customer with risk attitude γ at state (t, k) (i.e., k claims have already been made by time t) under an RVW with refund schedule $\mathbf{r}$. Under the refund schedule, the differential Equation (1) that characterizes the customers' strategic claim behavior needs to be modified as follows:

$$\frac{\partial g(t, k; \gamma, \mathbf{r})}{\partial t} = -\frac{\lambda_t}{\gamma}(E[e^{\gamma \cdot \min(C_t, \Delta g(t, k; \gamma, \mathbf{r}))}] - 1), \quad (19)$$

where $\Delta g(t, k; \gamma, \mathbf{r}) = g(t, k; \gamma, \mathbf{r}) - g(t, k+1; \gamma, \mathbf{r})$. The boundary conditions are $g(0, k; \gamma, \mathbf{r}) = r_k$ for each $k = 0, 1, \ldots, m-1$ and $g(t, k; \gamma, \mathbf{r}) = 0$ for any t and $k \geq m$. Similar to Theorem 1, the customers' strategic claim behavior also has a threshold structure; i.e., it is optimal for a customer with risk attitude γ to claim a failure with repair cost C_t at state (t, k) if and only if $C_t \geq \Delta g(t, k; \gamma, \mathbf{r})$.

For the provider of an RVW with refund schedule $\mathbf{r}$, let $h(t, k; \gamma, \mathbf{r})$ be the expected support cost of covering a customer with risk attitude γ at state (t, k). Under the customers' strategic claim behavior described above, $h(t, k; \gamma, \mathbf{r})$ satisfies the following differential equation:

$$\frac{\partial h(t, k; \gamma, \mathbf{r})}{\partial t}$$
$$= \lambda_t \Pr(C_t > \Delta g(t, k; \gamma, \mathbf{r}))$$
$$\cdot \{E[C_t \mid C_t > \Delta g(t, k; \gamma, \mathbf{r})] - \Delta h(t, k; \gamma, \mathbf{r})\}, \quad (20)$$

where $\Delta h(t, k; \gamma, \mathbf{r}) = h(t, k; \gamma, \mathbf{r}) - h(t, k+1; \gamma, \mathbf{r})$. The boundary conditions are $h(0, k; \gamma, \mathbf{r}) = r_k$ for each $k = 0, 1, \ldots, m-1$ and $h(t, k; \gamma, \mathbf{r}) = E[R(t)]$ for any t and $k \geq m$.

The RVW with a refund schedule gives the warranty provider more instruments for differentiating the heterogeneous markets than the RVW with a single refund, so it can be expected that the refund schedule can further help the RVW to capture more customers and earn a higher profit.

7. Concluding Remarks

We have developed a general framework for the RVW that offers a certain amount of refunds to customers with few or no claims. We have shown that the strategic claim behavior has a threshold structure. We have investigated the provider's problem taking into account customers' risk attitudes in both homogeneous and heterogeneous markets. We have identified explicit conditions under which RVWs outperform TWs given customers' rational purchase and strategic claim behavior. Numerical experiments have further demonstrated that RVWs can be significantly more profitable than TWs under a wide range of parameters.

When correctly designed, RVWs are especially helpful in dealing with heterogeneous market segments because they are a mechanism for attracting market segments that would be priced out by TWs. This mechanism does not require the monitoring of usage or of the environment where the product is used, since customers can influence their net cost by taking into account their risk attitudes, failure rates, and the given refund schedule. In addition, RVWs help overcome the moral hazard, a problem that is not captured in the model. To receive refunds at the end of the warranty duration, customers may use the products more cautiously. As a result, customers may experience fewer failures and may be willing to buy the same brand again in the future.

We have assumed that the warranty provider will automatically pay all refunds due to the customers. We think that this maximizes customers' valuation of RVW and the lifetime value of customers since automatic refunds will encourage them to buy the base products and the RVWs again. In practice, it is not uncommon for providers to observe low redemption rates that result from customers not claiming eligible refunds. This happens when eligible refunds are relatively small and the process of claiming them is time consuming. Customers who anticipate not claiming eligible refunds may discount the up-front value of RVWs. Modeling redemption rates and how such customers value RVWs can be a topic of future research.

A service provider can design a profitable RVW such that strategic claim behavior results in customers' bringing all repairs to the provider. In particular, if the incremental refund sizes (coupon sizes) are lower than the single repair cost offered by third parties, then customers are induced to make claims for every failure, and their strategic claim behavior coincides with claim-all behavior.

On the basis of the RVW concept, we introduce several novel extended warranty (EW) services as variants of the RVW for future research.

• *Claim-limited RVW:* The *claim-limited RVW* is similar to RVWs except that it allows only a predetermined finite number of claims. Once a customer

makes more than n claims, she will neither receive a refund nor be eligible for future coverage under the warranty.

• *Flexible RVW:* We call an EW a *flexible RVW* if a customer pays an up-front price p for a service "debit card" with an initial balance mr; she pays $\min(C_t, r)$ for a failure with cost C_t from the card; once debit card funds are exhausted, all repairs are covered; any positive balance remaining on the card at the end of warranty coverage is refunded to the customer.

• *Capped copayment EW:* We call an EW a *capped copayment EW* if a customer pays an up-front price p; she pays a copayment r each time for the first m repairs brought to the service provider, regardless of repair costs; all repairs beyond the first m repairs are completely covered.

• *Capped deductible EW:* We call an EW a *capped deductible EW* if a customer pays an up-front price p; she pays a minimum value $\min(C_t, r)$ between cost C_t for a failure and a deductible r; all repairs beyond the cap mr on the total out-of-pocket expenses are completely covered.

In addition to further studying innovative EW services, a future research direction is to investigate an oligopolistic market where competing service providers offer RVWs. In particular, it would be interesting to see whether the benefits of RVWs will prevail under competition.

Finally, this paper assumes that the failures follow a nonhomogeneous Poisson process, but there may be a certain correlation among failures and repairs. For instance, products may experience fewer failures after recent repairs, so another future research topic is to investigate whether the profit advantage of the RVW still holds under other failure processes, e.g., the renewal process.

Supplemental Material

Acknowledgments

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