Manufacturing & Service Operations Management

Publication details, including instructions for authors and subscription information:
http://pubsonline.informs.org

Newsvendor Bounds and A Heuristic for Optimal Policies in Serial Supply Chains

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Consider an N-stage serial supply system with deterministic transportation leadtimes between stages. Stationary random demand occurs at stage 1, which obtains resupply from stage 2; stage 2 obtains resupply from stage 3, and so on. Stage N replenishes its stock from an outside ample supplier. Linear ordering and inventory holding costs are assessed at all stages. Unsatisfied demands at stage 1 are backlogged and incur a linear backorder cost. This system has been studied extensively in the literature. Most notable works include Clark and Scarf (1960), Federgruen and Zipkin (1984), and Chen and Zheng (1994). We focus on minimizing the long-run average total inventory and backorder costs. It has been shown that a stationary echelon base-stock policy is optimal, and its parameters can be computed through minimizing N-nested convex functions recursively from stage 1, stage 2, . . . , to stage N.

Despite the deceivingly simple form of the optimization recursion, however, it is not easy to see insights into the key determinants of the optimal policy and cost. It is also not easy to communicate the computational procedure to managers and business-school students who have interests in learning the theory of supply chain management. For one thing, it is not easy to implement the algorithm by using simple spreadsheet calculations—a familiar tool for those students and practitioners. These challenges motivated us to look for closed-form approximations that can be easily obtained by using spreadsheets and at the same time can shed light on the effects of system parameters.

To achieve our goal, we construct an upper and a lower bound on the average echelon cost function of each stage, provided all downstream stages follow the optimal policy. These two cost-bound functions are the long-run average total costs of certain single-stage inventory systems. Specifically, assume the echelon holding cost rate and leadtime demand at stage j are h_j, and D_j, and backorder cost rate at stage 1 is b. We call the single-stage system, which forms the upper (lower) cost-bound function, as the upper bound (lower bound) system. Then, these two single-stage systems have the same backorder-cost rate $b + \sum_{i=j+1}^{N} h_i$ and leadtime demand $\sum_{i=1}^{j} D_i$, but they have different holding cost rates: $\sum_{i=1}^{j} h_i$ for the upper-bound system and h_j for the lower-bound system. Because both cost-bound functions are convex, it is fairly simple to obtain their minimizers. Note that we use single-stage and newsvendor systems interchangeably in this paper. This is because the long-run average total cost for continuous-review, single-stage systems has the same format as the total cost for single-period newsvendor models (with slightly different definitions of system parameters).

The minimizers obtained from cost-bound functions, in turn, are a lower and an upper bound for the optimal echelon base-stock level at each stage. Let the lower bound and the upper bound be s_j^{ℓ} and s_j^{u}, respectively. Then we have

$$s_j^{\ell} = F_j^{-1}\left(\frac{b + \sum_{i=j+1}^{N} h_i}{b + \sum_{i=1}^{N} h_i}\right) \text{ and } s_j^{u} = F_j^{-1}\left(\frac{b + \sum_{i=j+1}^{N} h_i}{b + \sum_{i=j}^{N} h_i}\right),$$

where F_j is the cdf of accumulated leadtime demand $\sum_{i=1}^{j} D_i$. The simple average of these bounds forms the heuristic solution for the optimal echelon base-stock level. This averaged approximation is a closed-form solution involving the original system parameters only. Therefore, the bounds and the heuristic, which can be easily obtained by simple spreadsheet calculations, enhance the accessibility and implementability of the multiechelon inventory theory. Also, they provide an analytical tool to gain insights into issues such as effects of system parameters, resource allocation, and stock positioning.

We evaluate our heuristic by examining a wide range of system parameters. We show that our heuristic can generate very good approximations to the *optimal echelon base-stock levels.* Also, we provide the percentage error between the optimal and heuristic *costs* to show the effectiveness of our heuristic. We begin with a four-stage system where demand follows a Poisson distribution. We also change the other system parameters and demand distribution so that the total number of tested cases is 64. In this numerical experiment, the average percentage error is 0.171%, with the maximum error less than 1.3%. In terms of solutions, our heuristic provides the optimal solutions in 19 out of 64 cases. For the other 45 cases, the differences between optimal and heuristic solutions are all very small. To examine the effect of N on the performance of our heuristic, we further test N from 2, 4, . . . , to 64 stages under four different holding cost forms: linear, kink, affine, and jump. Again, the results of this numerical experiment consistently demonstrate the effectiveness of our heuristic: the average percentage error among 36 tested cases is only 0.175%, with the maximum error of 1.227%. In addition, the most important observation is all bounds and heuristic solutions move in the same pattern as the optimal echelon base-stock levels. Thus, the heuristic policy is highly adaptive to the optimal one under different system parameters.

The simplicity of the bounding cost functions and the closed-form expressions of the solution bounds and the heuristic allow us to analyze the effect of system parameters easily. This is in sharp contrast to the not-so-transparent exact recursive procedure. Given the good performance of the heuristic and the fact that optimal echelon, base-stock level is ''wrapped'' by the bounds, it is reasonable to believe the parametric effects on the bounds hold for the true optimal policy as well.

Some important effects of system parameters on the optimal policy and cost can be observed. First, as echelon holding cost at a stage increases, the optimal echelon base-stock levels tend to increase at its downstream stages, but they tend to decrease at its own and upstream stages. This implies that when local holding cost increases at a stage, one should reduce the inventory stocking at that stage for achieving the optimal policy. Thus, from the system's perspective, the overall system stock tends to decrease. Similarly, as backorder cost or demand variance increases, the optimal echelon base-stock levels tend to increase at all stages. This implies that when backorder cost increases or demand is more variable, one should always increase the stock holding at stage 1. Thus, the overall system stock tends to increase. In addition, as leadtime at a stage increases, the optimal echelon, base-stock levels tend to increase at its own and upstream stages, but tend to remain unchanged at its downstream stages. This implies that when leadtime increases at a stage, one should increase inventory stocking at that stage. Hence, the overall system stock tends to increase. Finally, as echelon holding cost, backorder cost, leadtime, or demand variability increases, the optimal system cost tends to increase.

The parametric analysis on system parameters can further provide managerial insights on resource allocation and stock-positioning issues. For instance, if a company can reduce inventory holding cost by outsourcing or adopting new technology, which location should perform holding-cost reduction? Through our heuristic, we point out that reducing holding cost at an upstream stage is more effective. Hence, a manager should allocate resources to reduce holding costs at upper stages. Similarly, if a company can reduce leadtime by introducing EDI or online transactions at a location, leadtime reduction should be performed at a downstream stage. Another issue is customer demand. Suppose an owner of a supply chain predicts customer demand will become more variable, she should increase the inventory stocking. However, if the inventory supply is restricted, she should reallocate stocks to downstream stages.

To summarize, the contribution of this paper is two fold. The first one is computational and implementational. In particular, we develop a simple and surprisingly good heuristic for the optimal echelon base-stock levels, which can be obtained by solving $2N$ separate newsvendor-type problems. This closed-form heuristic can dramatically reduce the computational efforts and, therefore, facilitate the implementation of inventory control for businesses. The second main contribution, which perhaps is more important, is transparency. Specifically, the simple structures of the bounding functions and the closed-form heuristic solution help open the ''multiechelon black box.'' It enables us to study the effects of system parameters on the optimal cost and policies analytically. This, in turn, provides guidance on how to allocate critical resources to improve system performance. Finally, because the Clark and Scarf (1960) result has served as a benchmark for the increasingly active supply chain research on decentralized systems, we hope the tools developed here will help mitigate the analytical challenge and generate more insights in this line of research.

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