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Efficient Distribution of Water Between Head-Reach and Tail-End Farms in Developing Countries

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The necessity of surface water for irrigation and its increasing scarcity in developing economies motivate the need for its efficient distribution. The inequity in the distribution of surface water arises because of the relative physical locations of the farms. Head-reach (primary) farms are close to the source, whereas tail-end (secondary) farms are relatively farther. The lack of physical infrastructure implies that water allocated to secondary farms must pass through primary farms. Left to their individual incentives, primary farmers use more than their fair share of water by denying its release to secondary farmers. Such an inequitable sharing results in significantly suboptimal productivity of the farming community as a whole. We propose decentralized, individually rational mechanisms to achieve socially optimal distribution of surface water for a farming community under uncertainty in rainfall, choice of multiple crops, and differing risk-bearing abilities of primary and secondary farmers. We show that the mechanisms can be efficiently computed and highlight the impact of the improved sharing of surface water. We also study the movement of the price of water with its scarcity. Ideas that can help administer the mechanisms in practice are briefly discussed.

Key words: logistics and transportation; nonprofit management; incentives and contracts
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> Water allocation in an irrigation system should be done with due regard to equity and social justice. Disparities in the availability of water between head-reach and tail-end farms and between large and small farms should be obviated.
>
> —Ministry of Water Resources, India (2002, p. 5).

1. Introduction

Given the central role of agriculture in the growth of developing countries, it is no surprise that governments of these countries have, over the years, devoted special attention to the agricultural sector (World Bank 2008). Agricultural productivity can be enhanced by a variety of means, including focused investments in technology, development of policies to select high value/demand crops, education of farmers on better management of their farmlands, and the efficient utilization of water resources for farming. Although many of these methods may require years to implement, the issue of water management and allocation is most amenable to immediate resolution. We first discuss the importance of water for improving agriculture productivity and then outline the typical challenges encountered in the developing world for managing the efficient allocation of surface water for irrigation.

1.1. Water: A Critical Resource Influencing Agriculture Productivity

Water is a necessary but increasingly scarce resource for agriculture (Food and Agriculture Organization of the United Nations 2007). The resources of water for farmers include surface water, ground water, and rainfall. Surface water is one that comes through lakes, rivers, and streams and is largely a result of rainfall received in the past. The extraction of ground water requires creating and maintaining borewells, which can be an expensive option. Rainfall is typically unpredictable. Therefore, farmers rely mostly on the availability of surface water for deciding their crops. In developing economies (often with huge populations), the scarcity of surface water is further exacerbated. In developed countries, methods to alleviate the scarcity of water—rain water harvesting, waste water treatment, desalination, to name a few—have gained popularity over the years. However, the lack of such modern options and technologies (which can be expensive investments) in developing countries has increased the importance of better managing the usage of surface water for agriculture.

To discuss further, we consider India as a canonical example of a developing economy. According to the World Bank, India has inadequate infrastructure and services to support agricultural development (World Bank 2009). A typical Indian farm is quite small (averaging 1.18 hectares; Effland 2010), thus preventing investment in infrastructure by the individual farmer. Rainfall is uneven, with the southwest monsoon (June to September) accounting for most of it. Thus, surface water is arguably the only reliable source of water for irrigation. However, inefficient distribution of this scarce resource often results in lower production levels. For example, India's average yield of rice (wheat) is 3,034 (2,688) kilograms per hectare compared to 6,233 (4,155) for China (*The Hindu* 2007). Although some of this difference has been attributed to the use of advanced farming methods, e.g., genetically modified crops and consolidated farming, the inefficient distribution of available water remains a concern. We now discuss one main reason for this inefficiency.

1.2. Challenges in Water Allocation: Head-Reach and Tail-End Inequity

In our context, canals typically refer to surface pathways for water that are manually dug across farmlands. Surface water (from a source, e.g., a lake) is carried by canals that pass through farmlands and distributed to individual farms over a large physical area (Mollinga and Bolding 1996). Consider the following simple yet typical situation that arises in the distribution of water over a set of farms. A governmental office allocates to each farm a certain amount of surface water. The lack of (a) the requisite physical infrastructure and (b) human and technological resources makes it impossible for the government to monitor the actual distribution of water at a micro level i.e., ensure that each farm gets its fair share. Not surprisingly, the physical location (head-reach versus tail-end) of the farms in relation to the water source is the primary determinant of water allocation. Head-reach (primary) farms are closer to the source and have first access to the water. Tail-end (secondary) farms are farther from the water source. Water allocated to secondary farms *must* pass through (canals that are dug through) the primary farms. Left to their individual incentives, primary farmers (i.e., those with head-reach/primary farms) often use more than their fair share of water by denying its release to their secondary counterparts. Thus, secondary farmers have no independence or authority and must abide by this inequity in surface-water distribution (Bardhan and Johnson 2002, Bhattarai et al. 2002, Hussain et al. 2004, Gaur et al. 2008, Kumari et al. 2010, Meher 2011, Shah and Lele 2011). Disputes (referred to as water riots) between primary and secondary farmers from such an inequitable sharing of water are routine and often languish in local courts without any resolution. The decisions of the primary farmers naturally benefit them but are detrimental to the optimal productivity of the community as a whole.

It is important to note that the government does charge farmers a nominal price for water (Sur et al. 2002, Jalota et al. 2007). However, the lack of a proper infrastructure in developing countries makes it difficult to monitor the actual usages at individual farms. Therefore, two types of pricing methods are typically used: (i) area-based pricing, where a flat price per unit of farm area is charged; and (ii) volumetric pricing, where a price is computed for the total water delivered to a set of farmers and this price is then simply shared by the farmers in proportion to their areas (Cornish et al. 2004, Hussain 2007). Clearly, such schemes do not require verification or monitoring of water usage at individual farms and are, therefore, easy to administer. Consequently, these pricing methods have no influence on the aforementioned behavior of the primary farmers. Given this situation, the World Bank has rightly recognized the need for incentives, policies, and regulatory bodies for an efficient, fair, and sustainable allocation of water (World Bank 2009).

We now briefly state the setting of our analysis and our contributions.

1.3. Summary of Contributions and Review of Related Literature

In an attempt to study the socially optimal distribution of water, we consider a farming community—a group of farmers with their farmlands adjacent to each other. The community is divided into *lanes*, with each lane consisting of one primary and multiple secondary farmers. In each lane, the secondary farmers receive surface water through their primary counterpart in that lane. We first consider primary and secondary farmers to be risk averse when deciding on the quantity of water to share. Later, we also analyze the special case when farmers are risk neutral.

In a harvesting season, each farmer decides on planting a crop from an available set of crops. The revenue function for a crop is assumed to be an increasing and concave function of the water available for that crop (Christensen and McElyea 1988, Rao et al. 1990, Palma 2004). As mentioned above, farmers are charged a negligible fee for their use of surface water. Thus, it is reasonable to ignore the cost to the farmers for using the surface water.

We model the problem of obtaining a socially optimal distribution of the available surface water to this community under uncertainty in rainfall. This solution is used only to benchmark the decentralized solutions developed later. Under a naive decentralized scheme, which is typically realized in practice, the overall productivity of the farming community is significantly suboptimal. This motivates us to develop mechanisms that can induce farmers to self-select the socially optimal allocation of water. Our solutions depend on two factors that emerge from the realities of the problem. First, physical infrastructure solutions to resolve the inequities have not been successful, indicating the need for a financial solution. Second, head-reach farmers will resist sacrificing their first-use advantage, indicating the need for individually rational mechanisms in the sense that all farmers should be better off by participating.[1] For single-lane, single-crop systems, we design several socially optimal mechanisms that differ in the consideration of the risk-bearing ability of farmers: (i) a "reward and water-guarantee scheme," administered by a third party (e.g., the government), that has a reward for a risk-averse primary farmer; and a water guarantee for a risk-averse secondary farmer and (ii) an "up-front internal payment" scheme, which does not require any third-party intervention, when primary and secondary farmers are risk neutral. We generalize these mechanisms for multiple-lane, multiple-crop systems. We also present an alternative, socially optimal "rainfall-contingent payment" scheme.

In its general form, the problem considered in this paper is that of the decentralized distribution of a scarce resource (water) among multiple users (farmers) who have varying needs (farm areas) and can choose from a common pool of productivity functions (crop revenue functions). An institutional fact that is specific to our problem is that users have asymmetric access to the resource. To our knowledge, there is no quantitative analysis of this general problem in the literature. Although not very close to our setting, a few sequential allocation/access models have been studied in the operations management literature. For instance, Bassok and Ernst (1995) consider the problem of allocating multiple products by a distributor with limited capacity to a fixed sequence of customers with unknown demands. Another example is Cachon and Lariviere (1999), who consider a setting where a supplier's decision problem is to use an allocation mechanism to partition its limited capacity to several independent retailers. The literature on supply contracts includes some schemes that have a similar flavor to the mechanisms developed in our paper. Although there are significant differences in the settings and the treatment of the resulting models, a brief discussion may be helpful to the reader. Kouvelis and Lariviere (2000) discuss an incentive scheme based on linear transfer prices (contingent on the output level) for a decentralized cross-functional setting. The scheme is implemented through internal markets, where the price an agent receives to generate an output differs from the price another agent pays to consume the output. Rudi et al. (2001) discuss transshipment prices (to share extra inventory between the two sellers) that induce the sellers (at two different locations) to decide inventory orders to maximize joint profit. Caldentey and Wein (2003) study a linear transfer payment that induces cost sharing between the players to coordinate a decentralized production-inventory system. Cachon and Lariviere (2005) study a revenue-sharing contract to coordinate a supply chain consisting of a single supplier and competing retailers.

A variety of issues surrounding the allocation and management of water have been addressed in the extensive literature on agricultural economics. Rao et al. (1990) address the problem of determining the weekly irrigation schedule of several crops grown during the same season. Dinar et al. (1997) provide a qualitative review of existing mechanisms, including marginal cost pricing, public (administrative) water allocation, water markets, user-based distribution, etc., for water allocation among different sectors such as household consumption, irrigation, etc. Marques et al. (2005) analyze a farmer's decision of selecting permanent and annual crop production based on uncertainty in water availability and a variety of irrigation technologies. The authors study the effects of water availability, price, and reliability on economic performance, annual and long-run cropping patterns, and irrigation technology decisions. Maatman et al. (2002) present a stochastic programming model that analyzes farmers' strategies for dealing with the risk of uncertain rainfall. Georgiou and Papamichail (2008) develop a nonlinear programming model to obtain (i) the optimal allocation of water from a reservoir for various crops and (ii) the optimal cropping pattern, considering soil-water balance. Kramm and Wirkus (2010) discuss the negotiation of conflicts during the extraction of irrigation water from canals in Tanzania. Farmers' relative social position and power are identified as the key determinants of their ability to access water and their influence in the negotiation process. In the context of water resource development, Suzuki and Nakayama (1976) apply cooperative game theory for a fair allocation of costs (of exploiting a scarce resource) and benefits (by cooperation) to the members of a cooperative venture.

[1] Dr. C. Suvarna, special commissioner (watersheds), Government of the State of Andhra Pradesh, India; Dr. Tushaar Shah, senior fellow, International Water Management Institute, Gujarat, India; and Dr. N. H. Rao, joint director, National Academy of Agricultural Research Management, Andhra Pradesh, India, in private communications with the authors, June 2, 2012.

Tomkins and Weber (2010) develop a bilateral option contracting model for the water market in California. The contract is developed for a buyer and a seller, where the buyer faces uncertain demand for water and incurs a shortage cost. The seller faces uncertainty in the future spot-market price for the consumption good, i.e., the crop. A seller-optimal contract is developed and then compared with a socially optimal contract. Johansson et al. (2002) offer a broad review of the theory and practice of pricing irrigation water.

2. A Surface-Water Allocation Problem for a Farming Community

Consider a farming community with $m \geq 1$ lanes, with each lane consisting of one primary farmer and its $v \geq 1$ associated secondary farmers. Thus, we have a total of $(v+1)m$ farmers. Figure 1 illustrates the special case when each primary farmer has one associated secondary farmer. For lane i, let P_i denote the primary farmer and S_{i_t} denote the secondary farmer, $t = 1, 2, \ldots, v$, and $i = 1, 2, \ldots, m$. Let A_{P_i} (respectively, $A_{S_{i_t}}$) be the area of farmer P_i (respectively, S_{i_t}).

We consider a single harvesting season, in which a total of W units of surface water is available to the community. The amount of rainfall is (i) (*scenario* 1, representing low) R_1 units per acre with probability q_1 and (ii) (*scenario* 2, representing high) R_2 units per acre with probability $q_2 = 1 - q_1$. The generalization of our analysis to a finite number of rainfall scenarios is straightforward and is briefly discussed later in Remark 3 (§5). There are a total of $n \geq 1$ available crops and each farmer plants one crop for the season. Let $f^j(u)$ denote the increasing and concave per-acre revenue function for crop j (see Figure 2(a) for an illustration), where u is the amount of water available per acre. Based on the literature (see, e.g., Christensen and McElyea 1988), we assume the following functional form for this revenue: $f^j(u) = b^j(1-\exp(-c^j u))$, where $b^j > 0$ is the revenue when the amount of water $u = \infty$ and $c^j > 0$ is a constant for crop j. In practice, a crop typically has a maximum per-acre water requirement beyond which the marginal increase in revenue is negligible. We denote this per-acre threshold for crop j by $w^j_{\max}$. Thus, as an approximation, we assume that the revenue from crop j remains constant if the available per-acre water exceeds $w^j_{\max}$. Table 1 summarizes our main notation. Other notation is introduced later, as required.

Figure 1 A Schematic of a Community of Primary and Secondary Farmers (with Two Farmers per Lane for Illustration Purposes Only)

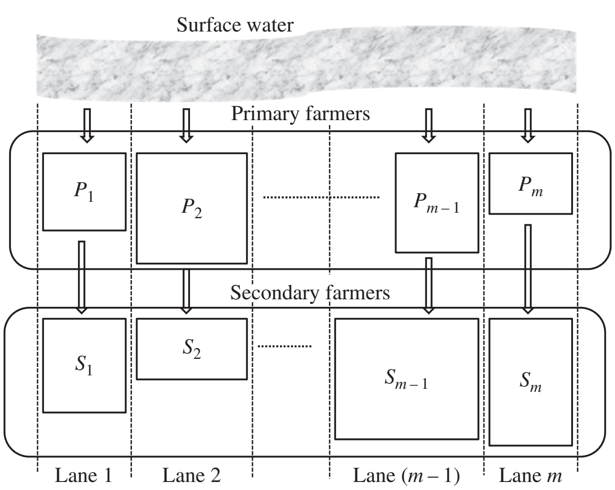
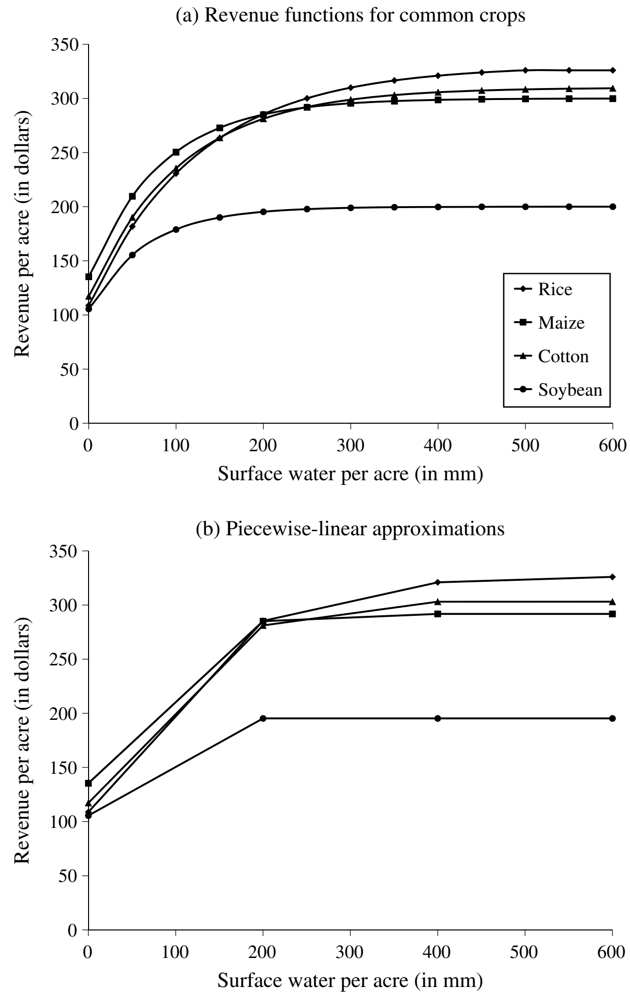


Figure 2 An Illustration of Revenue Functions and Their Piecewise-Linear Approximations



Note. The revenue figures and the maximum per-acre water requirements for the various crops have been estimated using the real data reported in Jalota et al. (2007).

2.1. A Social Planner's Problem: Problem S

To resolve the issue of efficient surface-water allocation, we first consider a social planner that allocates

Table 1 **Notation and Model Parameters**

m	Number of lanes in the farming community
I	$\{1, 2, \ldots, m\}$; set for lanes (indexed by i)
K	$\{P_1, S_{1_1}, \ldots, S_{1_v}, P_2, S_{2_1}, \ldots, S_{2_v}, \ldots, P_m, S_{m_1}, \ldots, S_{m_v}\}$; the set for farmers (indexed by k)
P	$\{P_1, P_2, \ldots, P_m\}$; set of primary farmers
J	$\{1, 2, \ldots, n\}$; set for crops (indexed by j)
A_k	Area (in acres) of farmland under the control of farmer k, $k \in K$
$w^j_{\max}$	Units of water per acre required for crop j to generate its maximum revenue; $j \in J$
W	Total available surface water for the community
R_θ	Per-acre rainfall amount in scenario θ; $\theta = 1, 2$
q_θ	Probability of scenario θ; $\theta = 1, 2$

Table 2 **Decision Variables**

X^j_k	1 if crop j is assigned to farmer k; 0 otherwise, $k \in K$, $j \in J$
y^j_k	The quantity of water allocated to crop j for farmer k, $k \in K$, $j \in J$

the surface water and decides the crop for the farmers to maximize the total revenue of the farming community (i.e., of all the farmers). Table 2 summarizes the decision variables for the social planner.

The social planner's objective is to maximize the total expected revenue of the farming community:

$$\max_{y^j_k,\, X^j_k} \sum_{k\in K} \sum_{j\in J} \sum_{\theta=1}^{2} q_\theta \left[A_k f^j \left(\frac{y^j_k}{A_k} + R_\theta X^j_k \right) \right],$$

subject to the following set of constraints:

• Exactly one crop is assigned to each farmer: $\sum_{j\in J} X^j_k = 1 \;\forall\, k$.

• Maximum available surface water to the community is W: $\sum_{k\in K} \sum_{j\in J} y^j_k \le W$.

• Farmer k receives y^j_k amount of surface water if crop j is assigned: $y^j_k \le (A_k w^j_{\max}) X^j_k \;\forall\, j, \forall\, k$.

• Nonnegativity and integrality constraints: $X^j_k \in \{0, 1\}$, $y^j_k \ge 0 \;\forall\, j, \forall\, k$.

There are two primary concerns about solving Problem S:

Solvability. Being a nonlinear mixed-integer program, Problem S is challenging to solve to optimality. We address this difficulty by considering piecewise-linear approximations of the nonlinear revenue functions for the crops. Figure 2(b) illustrates such approximations of the revenue functions presented in Figure 2(a). The piecewise-linear revenue functions allow us to approximate Problem S as a linear integer program (IP). Because this technique is well known in the literature (see, e.g., Nemhauser and Wolsey 1988), we avoid presenting the details. The solution of the IP reasonably addresses the solvability concern of Problem S. At this point, it is important to emphasize that our use of Problem S is only as a benchmark for evaluating the decentralized schemes developed later in §§4 and 5.

Need for Decentralized Mechanisms. Given the lack of resources in developing countries to monitor the centralized allocation of water, implementing the solution of Problem S is difficult. Furthermore, an authoritarian solution (such as the one resulting from Problem S) would typically be unacceptable to primary farmers, given their naturally dominant position with respect to the distribution of surface water. Therefore, the need is for a decentralized mechanism that can incentivize farmers to self-select (or closely approximate) the social optimum. The development of such mechanisms is the main focus of this paper.

3. A Naive Decentralization Approach: Imitating Practice

The simple decentralization approach of this section is an effort to mimic a solution that is often realized in practice. The structure of the allocation of irrigation water naturally varies across developing countries. In India, for instance, the Ministry of Water Resources (http://wrmin.nic.in/) is responsible for the overall planning and distribution of water. Thereafter, several governmental offices are involved at various stages of distribution. It is, therefore, difficult to adopt a single distribution structure for the purpose of analysis. However, the common reality across these countries is that governments typically restrict themselves to macrolevel allocations for the entire season because of complications and expenses involved in verifying water usage at a micro level, e.g., daily use by each farm (Cornish et al. 2004, Namboodiri and Gandhi 2009). We assume that a governmental office first allocates the surface water W (available for the season to our farming community) to the lanes, instead of individual farmers. Let W_i be the amount allocated to lane i. Because primary farmers have a naturally dominant position (as compared to secondary farmers), the further sharing of the allocated water within a lane is *sequential* in the naive approach. That is, in lane i, primary farmer P_i first decides on its water usage from the available surface water W_i. The remaining water is then passed on to its associated secondary farmers. This sequential sharing of surface water is well documented in the literature (Bardhan and Johnson 2002, Bhattarai et al. 2002, Hussain et al. 2004, Oza 2007, Gaur et al. 2008, Kumari et al. 2010, Meher 2011, Shah and Lele 2011). Below, we describe this approach more formally.

We consider primary and secondary farmers to be risk averse. Unlike the social planner (who maximizes the expected revenue of the community), a risk-averse farmer maximizes the expected utility from its wealth (i.e., sum of initial wealth and revenue from the crop). Let Π_i (respectively, Π_{i_t}) be the initial wealth of primary farmer P_i (respectively, secondary farmer S_{i_t}),

Table 3 Values of the Parameters for the Instance in Example 1

Parameters	Values
Primary farmers' areas (acres)	10, 7, 9, 3, 5
Secondary farmers' areas (acres)	20, 17, 15, 10, 11
Maximum water amounts ($w^j_{\max}$) required by crops (mm)	547, 144, 193, 117
Parameter b of revenue functions (dollars)	330, 210, 320, 300
Parameter c of revenue functions	0.006, 0.03, 0.008, 0.0075
Amount and probability of high (respectively, low) rainfall (mm)	100, 0.5, 50, 0.5
Amount of surface water available to the community (mm)	15,953

Table 4 The Solution from the Naive Decentralized Scheme for Example 1

Decision variables	Optimal values
Water allocated to lanes (mm)	4,290, 4,181, 3,335, 1,859, 2,288
Water used by primary farmers (mm)	4,290, 3,129, 3,335, 1,341, 2,235
Crop selection of primary farmers	1, 1, 1, 1, 1
Water received and used by secondary farmers (mm)	0, 1,052, 0, 518, 53
Crop selection of secondary farmers	4, 3, 4, 3, 4
Revenue of the community (dollars)	21,124.39

where $i = 1, 2, \ldots, m$ and $t = 1, 2, \ldots, v$. Because a farmer is risk averse, we consider its utility function U as a concave function of wealth. The sequential approach results in the following solution for lane i: Given the amount W_i of available surface water, primary farmer P_i selects its best crop (i.e., one that maximizes its expected utility), say $j°$. It releases an amount $W_i - \min\{W_i, A_{P_i}(w^{j°}_{\max} - R_1)\}$ to its associated secondary farmers. The further distribution of this water among the secondary farmers occurs in a similar fashion and depends on their incentives as well as their relative physical positioning. It is not difficult to see that this naive solution can be significantly suboptimal for the farming community as a whole. We illustrate this with a simple example.

Example 1. Consider five lanes, with one primary farmer and one secondary farmer in each lane. Each farmer has a choice of 4 crops. Table 3 lists the required data. Let the water allocated to lane i (W_i) be the corresponding optimal allocation in Problem S. Also, let the utility function U be the identity function I. That is, $U(\chi_k) = I(\chi_k)$, where χ_k is the wealth of farmer k. The solution of the naive decentralization is shown in Table 4 and its revenue is about 22% below that of the socially optimal solution (Table 5).

The reason for the significant suboptimality of the naive solution is the self-serving behavior of the primary farmer. In the absence of an incentive to share water with their secondary counterparts, the primary farmers naturally exploit their first-use advantage. As is clear from Table 4, each primary farmer uses the maximum amount of water that is required for the highest-revenue crop (crop 1). Consequently, there is insufficient water for the secondary farmers and, as a whole, the productivity of the community suffers.

Table 5 The Socially Optimal Solution for Example 1

Decision variables	Optimal values
Water allocated to lanes (mm)	4,290, 4,181, 3,335, 1,859, 2,288
Water used by primary farmers (mm)	1,430, 1,750, 1,190, 429, 715
Crop selection of primary farmers	3, 1, 3, 3, 3
Water received and used by secondary farmers (mm)	2,860, 2,431, 2,145, 1,430, 1,573
Crop selection of secondary farmers	3, 3, 3, 3, 3
Revenue of the community (dollars)	27,101.47

4. Improved Decentralized Mechanisms: Pricing of Water

It is clear that improving the situation from the naive approach above would (typically) require a primary farmer to share additional water with its secondary counterparts. This obviously would increase the revenue (hence, utility) of the secondary farmers but decrease that of the primary farmer. Because the revenue functions are concave, the benefit to the secondary farmers from an increased sharing exceeds the loss of the primary farmer, resulting in a net improvement in social revenue. However, as stated in Bhattarai et al. (2002, p. 21), "there will be differential wealth impacts between head-reach farmers and tail-end farmers. Such a policy will be strongly opposed by the head-reach farmers, unless there is a proper mechanism for compensating their loss caused by the reallocation of water." Our goal in this section is to develop such mechanisms.

To get an insight into the structure of a socially optimal pricing scheme, we first examine a (simpler) single-lane, single-crop system (§4.1). Then, §4.2 discusses two schemes that achieve socially optimal behavior from the farmers. The first scheme is a rainfall-contingent payment scheme (§4.2.1). The second scheme is a reward (for primary farmers) and water-guarantee (for secondary farmers) scheme (§4.2.2). Finally, in §4.3, the reward and water-guarantee scheme is generalized for the entire farming community via a "reward chart" (for primary farmers) and a "premium chart" (for secondary farmers).

4.1. A Single-Lane, Single-Crop System

Consider a single lane, consisting of a primary farmer (say, P) and its v associated secondary farmers S_t, $t = 1, 2, \ldots, v$, with a total of W^L amount of surface water available for the lane. Let A_p (respectively, A_{s_t}) be the area of the primary farmer (respectively, secondary farmer S_t). We assume that there is only one crop available to the farmers; let $w_{\max}$ be the maximum per-acre water requirement for this crop. Let $f(\cdot)$ be the corresponding revenue function (as defined in §2). To ensure a sufficiently high threshold of this maximum per-acre water requirement, we assume that $w_{\max} > W^L/(A_p + \sum_{t=1}^{v} A_{s_t}) + R_2$. The other notation used below is as defined in §2. We refer to this special case of Problem S as Problem C. Because we have a single crop, this problem is essentially one of efficiently allocating the surface water between the farmers to maximize the revenue for the lane. Let $y_{s_t} \geq 0$ (respectively, $y_p \geq 0$) be the amount of surface water used by secondary farmer S_t, $t = 1, 2, \ldots, v$ (respectively, primary farmer P). Without loss of generality, we can assume that $y_p + \sum_{t=1}^{v} y_{s_t} = W^L$.

Problem C. The social planner's objective is to maximize the total expected revenue of the farmers in the lane. That is,

$$\max \sum_{\theta=1}^{2} q_\theta \left(A_p f\left(\frac{W^L - \sum_{t=1}^{v} y_{s_t}}{A_p} + R_\theta \right) + \sum_{t=1}^{v} A_{s_t} f\left(\frac{y_{s_t}}{A_{s_t}} + R_\theta \right) \right) \tag{1}$$

subject to the following set of constraints:

• The surface water received by a farmer is bounded from above by both the available amount and by the maximum requirement for the crop. That is, $0 \leq W^L - \sum_{t=1}^{v} y_{s_t} \leq \min\{W^L, A_p(w_{\max} - R_1)\}$, $0 \leq y_{s_t} \leq \min\{W^L, A_{s_t}(w_{\max} - R_1)\}$, $t = 1, 2, \ldots, v$.

Let y_p^* (respectively, $y_{s_t}^*$, $t = 1, 2, \ldots, v$) be the quantity of water allocated to the primary farmer (respectively, secondary farmers) in an optimal solution to the social planner's problem.

Theorem 1. *The optimal solution of the social planner's problem is*

$$y_p^* = \frac{W^L A_p}{A_p + \sum_{t=1}^{v} A_{s_t}}, \quad y_{s_t}^* = \frac{W^L A_{s_t}}{A_p + \sum_{t=1}^{v} A_{s_t}}, \quad t = 1, 2, \ldots, v.$$

The proofs of all the technical results are in Online Appendix A (the online appendix is available at http://dx.doi.org/10.1287/msom.1120.0414).

To obtain closed-form solutions, for a risk-averse farmer, we assume the following utility function: $U(\chi) = \ln(\chi)$, where $\chi > 0$ is the wealth. However, the entire analysis can be performed without assuming a specific functional form for the utility function. Without any pricing scheme, there is no incentive for the primary farmer to follow the socially optimal solution of Theorem 1. Under the naive solution, let y_p° (respectively, $y_{s_t}^\circ$) be the amount of surface water used by the primary farmer (respectively, secondary farmer S_t).

4.2. Socially Optimal Schemes

Our discussion in this section assumes that the secondary farmers in a lane are arranged in a parallel manner and transact simultaneously either with their primary counterpart (§4.2.1) or with a third party (§4.2.2). The case when the farmers are arranged sequentially after the primary farmer is discussed in Online Appendix B. We analyze two different socially optimal schemes: (a) a rainfall-contingent payment scheme and (b) a reward and water-guarantee scheme, which is a combination of a reward for the primary farmer and a water guarantee for secondary farmers. To ensure that a lane is self-sustained, the following two properties are desirable, in addition to social optimality: (i) water sustenance (i.e, no need for surface water from external sources) and (ii) budget balance (i.e., the total amount paid by the secondary farmers to purchase water is equal to that received by the primary farmer). We first present a rainfall-contingent payment scheme that achieves these two desirable properties. Under this scheme, to purchase surface water, a secondary farmer pays the primary farmer a price that depends on the rainfall scenario.

4.2.1. Rainfall-Contingent Payment Scheme. Consider the following scheme: Secondary farmer S_t purchases an amount, say y_{s_t}, of surface water (regardless of the rainfall scenario) from the primary farmer. The payment, however, depends on the realized rainfall scenario: if scenario θ occurs, the secondary farmer pays a price β_θ for every extra unit (over the naive decentralized solution $y_{s_t}^\circ$) of surface water received. Under this scheme, the individual optimization problems of the farmers are as follows:

A Primary Farmer's Problem Under the Rainfall-Contingent Payment Scheme:

$$\max_{y_{s_t}} \sum_{\theta=1}^{2} q_\theta \ln \left(\Pi_p + A_p f\left(\frac{W^L - \sum_{t=1}^{v} y_{s_t}}{A_p} + R_\theta \right) + \beta_\theta \sum_{t=1}^{v} (y_{s_t} - y_{s_t}^\circ) \right),$$

where $0 \leq y_{s_t} \leq \min\{W^L, A_{s_t}(w_{\max} - R_1)\}$, $0 \leq W^L - \sum_{t=1}^{v} y_{s_t} \leq \min\{W^L, A_p(w_{\max} - R_1)\}$.

Problem for Secondary Farmer S_t Under the Rainfall-Contingent Payment Scheme:

$$\max_{y_{s_t}} \sum_{\theta=1}^{2} q_\theta \ln \left(\Pi_t + A_{s_t} f\left(\frac{y_{s_t}}{A_{s_t}} + R_\theta \right) - \beta_\theta (y_{s_t} - y_{s_t}^\circ) \right),$$

where $0 \leq y_{s_t} \leq \min\{W^L, A_{s_t}(w_{\max} - R_1)\}$, $t = 1, 2, \ldots, v$.

Theorem 2. *The following values of the constants β_θ, $\theta = 1, 2$, result in a primary farmer and its secondary* *counterparts simultaneously choosing the socially optimal distribution of surface water.*

$$\beta_\theta = f'\left(\frac{W^L}{A_p + \sum_{t=1}^{v} A_{s_t}} + R_\theta\right).$$

A potential drawback of the rainfall-contingent payment scheme is that it reflects the uncertainty in rainfall into the payments to be made by a secondary farmer. In other words, a secondary (respectively, primary) farmer remains uncertain of the payment to be made (respectively, received) until rainfall has realized. To avoid setting rainfall-contingent prices and instead allow only for up-front payment by the secondary farmers, we explore the intervention of an interested third party, e.g., the government. To this end, we introduce a reward and water-guarantee scheme in which the payment is independent of the rainfall scenario and socially optimal distribution, too, is guaranteed for the single-lane, single-crop system.

4.2.2. Reward and Water-Guarantee Scheme. In this section, we envision the government as a third party that sells water to the secondary farmers. Conceptually, (a) the government first buys water from the primary farmer, (b) the secondary farmers then purchase water guarantees, and (c) the government then uses the water bought from the primary farmer to supply the secondary farmers.[2] Below, we define the scheme considered by the government.

A Reward for Primary Farmer. The government pays a price γ to the primary farmer for every extra unit (over the naive solution) of surface water purchased.

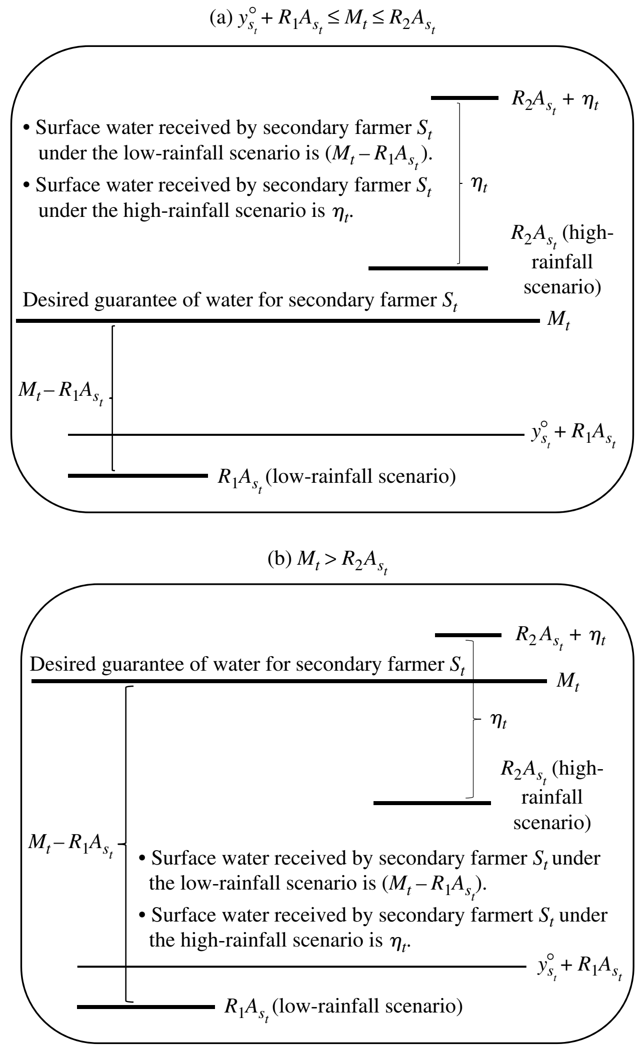
A Water Guarantee for Secondary Farmers. Secondary farmer S_t purchases up front a guaranteed level of water, say M_t. This guarantee is exercised by the farmer if the low-rainfall scenario occurs. If the high-rainfall scenario occurs, the farmer receives an amount, say η_t, of surface water that is independent of its choice of M_t but depends on the lane-specific parameters of the secondary farmer (i.e., the areas of the farmers and the total water available for their lane). Figure 3 illustrates two possible choices for M_t. Next, we analyze the primary and secondary farmers' problems under the reward and water-guarantee scheme.

A Primary Farmer's Decision Problem. Let y_p be the surface water used by the primary farmer. Let $\Lambda(y_p)$ be the payment made by the government to the primary farmer. Under this scheme, the objective of the primary farmer is

$$\max_{y_p} \sum_{\theta=1}^{2} q_\theta \ln\left(\Pi_p + A_p f\left(\frac{y_p}{A_p} + R_\theta\right) + \Lambda(y_p)\right),$$

[2] The government does not need to get involved in the storage of water. The required amount of water can be directly transferred by the primary farmer to its secondary counterparts, and the government's role can be limited to conducting the monetary transaction.

Figure 3 **An Illustration of Water Guarantee for a Risk-Averse Secondary Farmer**



where $y_p \le \min\{W^L, (A_p w_{\max} - R_1 A_p)\}$. We next define the structure of the payment scheme:

Definition. The price $\Lambda(y_p)$ paid to the primary farmer to use an amount $y_p \le y_p^\circ$ amount of surface water is

$$\Lambda(y_p) = \gamma(y_p^\circ - y_p),$$

where $\gamma \ge 0$ is the per-unit price and y_p° is the amount the primary farmer uses under naive decentralization. Thus, $\Lambda(y_p^\circ) = 0$. In Theorem 3, we obtain the value of γ so that the primary farmer's maximum utility is achieved at $y_p = y_p^*$.

Theorem 3 (Price Structure for Social Optimality). *The value of the constant γ that guarantees*

$$y_p^* = \arg\max\left[\sum_{\theta=1}^{2} q_\theta \ln\left(\Pi_p + A_p f\left(\frac{y_p}{A_p} + R_\theta\right) + \Lambda(y_p)\right)\right]$$

is as follows:

$$\gamma=\begin{cases}\dfrac{-(q_1(X_2-Y_1)+q_2(X_1-Y_2))+\sqrt{\Delta}}{2(y_p^\circ-y_p^*)} & \text{if } y_p^*<y_p^\circ,\\ 0 & \text{if } y_p^*=y_p^\circ,\end{cases}$$

where

$$\Delta=(q_1(X_2-Y_1)+q_2(X_1-Y_2))^2+4q_1Y_1X_2+4q_2Y_2X_1,$$

$$X_1=\Pi_p+A_pf\left(\frac{y_p^*}{A_p}+R_1\right),\quad X_2=\Pi_p+A_pf\left(\frac{y_p^*}{A_p}+R_2\right),$$

$$Y_1=(y_p^\circ-y_p^*)f'\left(\frac{y_p^*}{A_p}+R_1\right),\quad Y_2=(y_p^\circ-y_p^*)f'\left(\frac{y_p^*}{A_p}+R_2\right).$$

The Problem for Secondary Farmer S_t. Let M_t be the guaranteed level of water chosen by secondary farmer S_t. Without loss of generality, we assume that $M_t\ge y_{s_t}^\circ+R_1A_{s_t}$, where $y_{s_t}^\circ$ is the amount of surface water shared by the primary farmer without any incentive and R_1 is the per-acre rainfall amount under the low-rainfall scenario. The secondary farmer receives $y_{s_t}=M_t-R_1A_{s_t}$ amount of surface water under the low-rainfall scenario. The government buys $W^L-y_p^*=\sum_{t=1}^{v}y_{s_t}^*$ amount of surface water from the primary farmer. To avoid the need for water from sources external to the community, we impose $y_{s_t}\le y_{s_t}^*$, $\forall\, t$. This implies $\sum_{t=1}^{v}y_{s_t}\le\sum_{t=1}^{v}y_{s_t}^*$. Let $\eta_t\le y_{s_t}^*$ be the amount of surface water (determined by the government; the value will be obtained shortly) allocated to the secondary farmer in the case of high rainfall. Let $\Omega_t(M_t)$ be the price for the water guarantee M_t sought by the secondary farmer; the structure of $\Omega_t(M_t)$ is stated below. The payment $\Omega_t(M_t)$ is made by the farmer at the beginning of the season. Under the reward and water-guarantee scheme, the objective of secondary farmer S_t is

$$\max_{M_t}\left\{q_1\ln\left(\Pi_t+A_{s_t}f\left(\frac{M_t}{A_{s_t}}\right)-\Omega_t(M_t)\right)+q_2\ln\left(\Pi_t+A_{s_t}f\left(R_2+\frac{\eta_t}{A_{s_t}}\right)-\Omega_t(M_t)\right)\right\},$$

where $y_{s_t}^\circ+R_1A_{s_t}\le M_t\le y_{s_t}^*+R_1A_{s_t}$. We first define the structure of our pricing scheme.

Definition. The price $\Omega_t(M_t)$ paid by the secondary farmer for a water guarantee M_t is

$$\Omega_t(M_t)=\alpha_t(M_t-y_{s_t}^\circ-R_1A_{s_t}),$$

where $\alpha_t\ge 0$ can be interpreted as the "premium" per unit of guarantee sought. Thus, $\Omega_t(M_t)=0$ for $M_t=y_{s_t}^\circ+R_1A_{s_t}$ because the secondary farmer receives $y_{s_t}^\circ$ amount of surface water voluntarily from its primary counterpart.

Theorem 4 achieves several important properties. First, we show the existence of a water guarantee M_t^* and a high-rainfall-scenario allocation η_t^* that would achieve socially optimal revenue for secondary farmer S_t. Second, we obtain specific values for M_t^* and η_t^*. Third, we derive a value of the premium α_t so that the secondary farmer's maximum utility is achieved at the level M_t^*.

Let SOR denote the socially optimal revenue of secondary farmer S_t. That is,

$$SOR=\sum_{\theta=1}^{2}q_\theta\left(A_{s_t}f\left(\frac{y_{s_t}^*}{A_{s_t}}+R_\theta\right)\right),$$

where $y_{s_t}^*=W^LA_{s_t}/(A_p+\sum_{t=1}^{v}A_{s_t})$ is the socially optimal allocation of water to the secondary farmer (see §4.1).

Theorem 4. (a) *There exists a water guarantee M_t^* and an allocation η_t^* such that the revenue of secondary farmer S_t corresponding to M_t^* and η_t^* equals its socially optimal revenue. That is,*

$$q_1A_{s_t}f\left(\frac{M_t^*}{A_{s_t}}\right)+q_2A_{s_t}f\left(R_2+\frac{\eta_t^*}{A_{s_t}}\right)=SOR. \tag{2}$$

(b) *$M_t^*=R_1A_{s_t}+y_{s_t}^*$ and $\eta_t^*=y_{s_t}^*$.*

(c) *Given $\eta_t^*=y_{s_t}^*$, the value of the constant α_t in the pricing scheme Ω_t (as defined above) that guarantees*

$$M_t^*=\underset{y_{s_t}^\circ+R_1A_{s_t}\le M_t\le y_{s_t}^*+R_1A_{s_t}}{\arg\max}\left[q_1\ln\left(\Pi_t+A_{s_t}f\left(\frac{M_t}{A_{s_t}}\right)-\Omega_t(M_t)\right)+q_2\ln\left(\Pi_t+A_{s_t}f\left(R_2+\frac{y_{s_t}^*}{A_{s_t}}\right)-\Omega_t(M_t)\right)\right]$$

is as follows:

$$\alpha_t=\frac{q_1(T_{2t}+\Gamma_t)+q_2(T_{1t})-\sqrt{\Delta_t}}{2(M_t^*-y_{s_t}^\circ-R_1A_{s_t})}\quad \text{if } M_t^*>y_{s_t}^\circ+R_1A_{s_t};\ \text{otherwise } \Omega_t(M_t^*)=0,$$

where $\Delta_t=(q_1(T_{2t}+\Gamma_t)+q_2(T_{1t}))^2-4q_1\Gamma_tT_{2t}$, $T_{1t}=\Pi_t+A_{s_t}f(M_t^*/A_{s_t})$, $T_{2t}=\Pi_t+A_{s_t}f(R_2+y_{s_t}^*/A_{s_t})$, and $\Gamma_t=f'(M_t^*/A_{s_t})(M_t^*-y_{s_t}^\circ-R_1A_{s_t})$.

In addition to achieving social optimality, the reward and water-guarantee scheme improves the welfare of the farmers with respect to their naive decentralized solutions. Furthermore, the scheme satisfies the water sustenance property because the total amount allocated to the secondary farmers is equal to that purchased from the primary farmer. However, unlike the rainfall-contingent payment scheme of §4.2.1, we cannot guarantee budget balance here.

The government administers the monetary transaction by purchasing water from the primary farmer and selling it to the secondary farmers. Hence, the government can be either in deficit or in profit during this transaction. In the former case, the loss of the government can be treated as a subsidy. This lack of balance in the government's transactions is probably not a critical shortcoming in the context of irrigation because water subsidies are quite common in both developed and developing countries (see, e.g., Malik 2008). However, if a balance of both water and budget is absolutely necessary, then the rainfall-contingent payment scheme could be considered.

In the next section, we extend this pricing scheme to design a reward chart for primary farmers and a premium chart for secondary farmers in the presence of multiple lanes and multiple crops.

4.3. Generalization of the Reward and Water-Guarantee Scheme for the Entire Farming Community

The goal of the decentralized reward and premium charts is to achieve the socially optimal revenue for the entire community (i.e., the solution to Problem S). As in §3, the total surface water available to the entire community is allocated a priori to the lanes. Assuming this allocation, the reward and premium charts described below mimic the structure of the reward and water-guarantee scheme. Here, the main difference from the single-lane, single-crop system (considered in §4.1) is the availability of multiple crops for a lane. Below, we formally define these two charts.

4.3.1. A Reward Chart. An entry in the reward chart is a per-unit price $\gamma(A_p, A_{s_1}, \ldots, A_{s_v}, W^L, y_p^*)$, where A_p (respectively, A_{s_t}) is the area for the primary farmer (respectively, secondary farmer S_t), W^L is the surface water available to the lane and y_p^* is the socially optimal allocation to the primary farmer. The reward chart contains entries that correspond to all values of A_p, A_{s_t}, and W^L that are of interest across farming communities.

The per-unit price $\gamma(A_p, A_{s_1}, \ldots, A_{s_v}, W^L, y_p^*)$ can be computed by the following steps:

- *The Naive and Socially Optimal Solutions for a Primary Farmer*. Given $(A_p, A_{s_1}, \ldots, A_{s_v}, W^L)$, the naive (respectively, socially optimal) allocation of surface water is y_p° (respectively, y_p^*) and the corresponding best crop is, say, j_p° (respectively, j_p^*) for the primary farmer in a lane.
- *Calculate the Price Corresponding to Crop j_p^**. For ease of presentation, we consider $y_p^* + R_2 A_p \le w_{\max}^{j_p^*} A_p$. Using Theorem 3, the value of $\gamma(A_p, A_{s_1}, \ldots, A_{s_v}, W^L, y_p^*)$ is computed as follows:

$$\gamma(A_p, A_{s_1}, \ldots, A_{s_v}, W^L, y_p^*) = \begin{cases} \dfrac{-(q_1(X_2 - Y_1) + q_2(X_1 - Y_2)) + \sqrt{\Delta}}{2(y_p^\circ - y_p^*)} & \text{if } y_p^* < y_p^\circ, \\ 0 & \text{if } y_p^* = y_p^\circ, \end{cases}$$

where

$$\Delta = (q_1(X_2 - Y_1) + q_2(X_1 - Y_2))^2 + 4q_1 Y_1 X_2 + 4q_2 Y_2 X_1,$$

$$X_1 = \Pi_p + A_p f^{j_p^*}\left(\frac{y_p^*}{A_p} + R_1\right),$$

$$X_2 = \Pi_p + A_p f^{j_p^*}\left(\frac{y_p^*}{A_p} + R_2\right),$$

$$Y_1 = (y_p^\circ - y_p^*) f^{j_p^{*\prime}}\left(\frac{y_p^*}{A_p} + R_1\right),$$

$$Y_2 = (y_p^\circ - y_p^*) f^{j_p^{*\prime}}\left(\frac{y_p^*}{A_p} + R_2\right).$$

Given the complete reward chart, a primary farmer selects a crop and a water amount $y_p \le y_p^*$ that maximizes its expected utility.

Decision Problem for a Primary Farmer with Area A_p:

$$\max_{j \in J,\, 0 \le y_p \le y_p^*} \sum_{\theta=1}^{2} q_\theta \ln\left(\Pi_p + A_p f^{j_p}\left(\min\left\{w_{\max}^{j_p}, \left(R_\theta + \frac{y_p}{A_p}\right)\right\}\right) + \gamma(A_p, A_{s_1}, A_{s_2}, \ldots, A_{s_v}, W^L, y_p^*)(y_p^\circ - y_p)\right).$$

Note that y_p^* is the upper bound on the amount of water that can be utilized by the primary farmer. Consequently, the government ensures the purchase of at least $W^L - y_p^*$ amount of surface water from the primary farmer. The purchased water is then used to meet the demand from its secondary counterparts.

4.3.2. A Premium Chart. We begin by explaining the development of the premium chart: An entry in the premium chart is a triplet $(\alpha_t, M_t^*, \eta_t^*)$ corresponding to the lane-based parameters $A_p, A_{s_1}, \ldots, A_{s_v}$, and W^L. Given these values, the triplet $(\alpha_t, M_t^*, \eta_t^*)$ has the following interpretation: secondary farmer S_t can choose any water-level guarantee $M_t \le M_t^*$ at a per-unit price of α_t. In the low-rainfall scenario, the farmer receives an amount of surface water so that the total water (rainfall plus surface water) reaches the chosen guarantee M_t. In the high-rainfall scenario, the farmer receives an amount η_t^* that is independent of M_t but depends on $A_p, A_{s_1}, \ldots, A_{s_v}$, and W^L. The premium chart contains triplets $(\alpha_t, M_t^*, \eta_t^*)$ corresponding to all values of A_p, $A_{s_1}, \ldots, A_{s_v}$, and W^L, that are of interest across farming communities.

Using Theorem 4, the premium α_t, the corresponding water level M_t^* and the high-rainfall-scenario allocation η_t^* can be computed by following steps that are similar to those for the reward chart. Given the complete premium chart, secondary farmer S_t selects a crop and a water guarantee (less than or equal to M_t^*) that maximizes its expected utility.

Decision Problem for Secondary Farmer S_t with Parameters $(A_p, A_{s_1}, \ldots, A_{s_v}, W^L)$:

$$\max_{\substack{j_t\in J,\\ y^\circ_{s_t}+R_1A_{s_t}\le M_t\le y^*_{s_t}+R_1A_{s_t}}} q_1\ln\left(\Pi_t + A_{s_t}f^{j_t}\left(\min\left\{w^{j_t}_{\max}, \frac{M_t}{A_{s_t}}\right\}\right) - \alpha_t(M_t - y^\circ_{s_t} - R_1A_{s_t})\right)$$
$$+ q_2\ln\left(\Pi_t + A_{s_t}f^{j_t}\left(\min\left\{w^{j_t}_{\max}, \left(R_2 + \frac{\eta_t^*}{A_{s_t}}\right)\right\}\right) - \alpha_t(M_t - y^\circ_{s_t} - R_1A_{s_t})\right).$$

As noted above, the reward and premium charts are generalizations (for multiple lanes and multiple crops) of the reward and water-guarantee scheme. The reward and water-guarantee scheme is individually rational and socially optimal for a single-lane, single-crop system, where the crop selection decision does not arise. However, crop selection is a decision variable for a multiple-lane, multiple-crop system. In this case, the reward and premium charts are individually rational for the farmers. However, these charts do not necessarily guarantee socially optimal productivity from the overall farming community. The basic reason is that a revenue maximizing crop does not necessarily maximize a farmer's utility.

This completes our discussion of the reward and premium charts. In the next section, we analyze the special case when primary and secondary farmers are risk neutral.

5. A Special Case: Risk-Neutral Farmers

In this special case, it is possible to design an "up-front internal payment scheme" in which (i) secondary farmers purchase surface water directly (i.e., without third-party intervention) from the primary farmers and (ii) the payment made by a secondary farmer is independent of the rainfall. Because this scheme has internal payment and involves internal exchange of water, it obviously achieves both water sustenance and budget balance. Apart from being socially optimal, this scheme is also individually rational for each farmer. As in §4.2, we assume in this section that secondary farmers are arranged in a parallel manner within a lane. The case of sequential secondary farmers is summarized in Online Appendix B.

We first present the scheme for the single-lane, single-crop system (§5.1) and then generalize it to achieve social optimality for the entire farming community (§5.2).

5.1. An Up-Front Internal Payment Scheme

We begin by defining the mechanism.

Definition. Let y_{s_t} be the amount of surface water that secondary farmer S_t receives from its primary counterpart and let $\pi_t(y_{s_t})$ be the corresponding price (paid by this secondary farmer). Recall that in the absence of any mechanism, the primary farmer voluntarily shares an amount $y^\circ_{s_t}$ with the secondary farmer. Thus, without loss of generality, we assume that $y_{s_t} \ge y^\circ_{s_t}$ and $\pi_t(y_{s_t}) = 0$ if $y_{s_t} = y^\circ_{s_t}$.

Naturally, the secondary farmers should be willing to compensate for the loss they cause to the primary farmer. Therefore, to compute the loss caused by each secondary farmer, consider v virtual farms of the primary farmer with areas in proportion to the areas of its secondary counterparts. That is, the v virtual primary farms have areas $A_{p_t} = A_pA_{s_t}/\sum_{t=1}^v A_{s_t}$ and receive surface water $y_{p_t} = y_pA_{p_t}/A_p$, $t = 1, 2, \ldots, v$. We can now imagine v virtual lanes, with one primary farmer (area A_{p_t}) and its secondary counterpart (area A_{s_t}) in each such lane. Let $W_t^L = W^L(A_{p_t} + A_{s_t})/(A_p + \sum_{t=1}^v A_{s_t})$ be the surface water available to virtual lane t, $t = 1, 2, \ldots, v$. Then, secondary farmer S_t pays the following to purchase an amount y_{s_t} of surface water:

$$\pi_t(y_{s_t}) = 0.5\sum_{\theta=1}^2 q_\theta\left[A_{p_t}f\left(\frac{W_t^L - y^\circ_{s_t}}{A_{p_t}} + R_\theta\right) - A_{p_t}f\left(\frac{W_t^L - y_{s_t}}{A_{p_t}} + R_\theta\right)\right]$$
$$+ 0.5\sum_{\theta=1}^2 q_\theta\left[A_{s_t}f\left(\frac{y_{s_t}}{A_{s_t}} + R_\theta\right) - A_{s_t}f\left(\frac{y^\circ_{s_t}}{A_{s_t}} + R_\theta\right)\right].$$

In words, the payment to the primary farmer for sharing a certain amount of water in virtual lane t is the average of the expected loss (of the primary farmer) and the expected gain (of secondary farmer S_t) from that amount over their respective decentralized revenues in the absence of any mechanism. Under the above payment scheme, we next state the decision problems for the primary and secondary farmers:

Decision Problem for the Primary Farmer:

$$\max\left\{\sum_{\theta=1}^2 q_\theta A_p f\left(\frac{W^L - \sum_{t=1}^v y_{s_t}}{A_p} + R_\theta\right) + \sum_{t=1}^v \pi_t(y_{s_t})\right\},$$

where $0 \le y_{s_t} \le \min\{W^L, A_{s_t}(w_{\max} - R_1)\}$, $0 \le W^L - \sum_{t=1}^v y_{s_t} \le \min\{W^L, A_p(w_{\max} - R_1)\}$.

Decision Problem for Secondary Farmer S_t:

$$\max \sum_{\theta=1}^2 q_\theta A_{s_t} f\left(\frac{y_{s_t}}{A_{s_t}} + R_\theta\right) - \pi_t(y_{s_t}),$$

where $0 \le y_{s_t} \le \min\{W^L, A_{s_t}(w_{\max} - R_1)\}$.

Theorem 5 (Individual Welfare Improvement and Social Optimality). *The payment scheme π_t (defined as above) is rational for the farmers and they self-select the socially optimal allocation $(y_p^*, y_{s_t}^*)$, $t = 1, 2, \ldots, v$. Furthermore, under this scheme, half of the surplus (over the total revenue from the naive decentralization) from the redistribution of water goes to the primary farmer while the remaining half is shared among the secondary farmers.*

As a consequence of the above result, if a lane consists of only one primary and one secondary farmer, the surplus is shared equally between the two farmers.

Remark 1 (A Linear Pricing Scheme). A simpler, linear pricing scheme can also achieve the socially optimal distribution but does not guarantee that the secondary farmers share half of the total surplus (over the naive solution). Such a scheme can be constructed as follows: Let $\mathscr{C}(y_{s_t})$ be the payment by secondary farmer S_t to purchase y_{s_t} amount of surface water from the primary farmer. Let the constant τ be defined as

$$\tau = \sum_{\theta=1}^{2} q_\theta f'\left(\frac{W^L}{A_p + \sum_{t=1}^{v} A_{s_t}} + R_\theta\right)$$

and let

$$\mathscr{C}(y_{s_t}) = \tau(y_{s_t} - y_{s_t}^\circ),$$

where $y_{s_t}^\circ$ is the amount of surface water received under naive decentralization by secondary farmer S_t. It is straightforward to show that under the above linear pricing scheme, the farmers self-select the socially optimal distribution of surface water.

Remark 2 (Distribution of Surplus in Any Desired Proportion). The nonlinear scheme π_t stated above distributes the surplus equally between the primary farmer and the set of its secondary counterparts. On the other hand, the distribution of the surplus under the linear pricing scheme in Remark 1 is different and depends on the areas of the farmers and the total water available to their lane. More generally, it might be preferable to have a scheme that distributes the surplus in *any desired proportion* λ; that is, the primary farmer receives a $(1-\lambda)$ fraction and the remaining λ fraction of the surplus is shared among the secondary farmers. To achieve this, the payment π_t is revised as follows:

$$\begin{aligned}
&\pi_t^\lambda(W^L - y_p)\\
&= \lambda \sum_{\theta=1}^{2} q_\theta \left[A_{p_t} f\left(\frac{W_t^L - y_{s_t}^\circ}{A_{p_t}} + R_\theta\right) - A_{p_t} f\left(\frac{W_t^L - y_{s_t}}{A_{p_t}} + R_\theta\right)\right]\\
&\quad + (1-\lambda) \sum_{\theta=1}^{2} q_\theta \left[A_{s_t} f\left(\frac{y_{s_t}}{A_{s_t}} + R_\theta\right) - A_{s_t} f\left(\frac{y_{s_t}^\circ}{A_{s_t}} + R_\theta\right)\right],
\end{aligned}$$

where $0 \le \lambda \le 1$. It is easy to see that the scheme π_t^λ has all the properties of π_t and distributes the surplus in the desired proportion λ.

We now extend the basic structure of the up-front internal payment scheme π_t to our original problem (i.e., Problem S), which involves multiple lanes and multiple crops.

5.2. A Rate Card Generated from the Up-Front Internal Payment Scheme

The rate card is formally defined as follows:

Definition of the Rate Card. An entry in the rate card is a price Υ_t corresponding to $(A_p, A_{s_1}, A_{s_2}, \ldots, A_{s_v}, W_t^L, y_{s_t}^*)$, where $W_t^L = W^L(A_{p_t} + A_{s_t})/(A_p + \sum_{t=1}^{v} A_{s_t})$ is the amount of water available to virtual lane t, $A_{p_t} = A_p A_{s_t}/\sum_{t=1}^{v} A_{s_t}$, and $y_{s_t}^*$ is the socially optimal amount of water the primary farmer shares with its secondary counterpart S_t. The rate card is assumed to have entries corresponding to all realistic values of areas and water availability that are of interest across communities.

To compute the price Υ_t corresponding to the shared amount $y_{s_t}^*$, we follow the spirit of the payment scheme π_t to calculate the gain/loss of the secondary/primary farmer over their respective revenues were they to follow the naive decentralization approach of §3. The steps in this calculation are as follows:

1. *Decentralized solution without any mechanism*: Under the naive approach, given the area A_p of the primary farmer and the available surface water (W^L), the amount of water y_p° is used by the primary farmer, the corresponding best crop is j_p°, and the revenue is $(f^{j_p^\circ}(\cdot))$. Because each farmer can only plant one crop in its field, it follows that the same crop j_p° is planted in each virtual farm of the primary farmer. Similarly, secondary farmer S_t receives $y_{s_t}^\circ$ amount of water under the naive decentralization. Let $j_{s_t}^\circ$ be the best crop for the secondary farmer and the $f^{j_{s_t}^\circ}(\cdot)$ be the corresponding revenue.

2. *Solution from sharing an amount $y_{s_t}^*$*: Obtain the socially optimal solution for a lane with parameters $(A_p, A_{s_1}, A_{s_2}, \ldots, A_{s_v}, W^L)$, which includes (i) the surface-water amount $y_{s_t}^*$, the best crop $(j_{s_t}^*)$, and the corresponding revenue $(f^{j_{s_t}^*}(\cdot))$ for secondary farmer S_t; and (ii) the surface water amount y_p^*, the best crop j_p^*, and the revenue $(f^{j_p^*}(\cdot))$ for the primary farmer. We use the same crop j_p^* for each virtual lane of the primary farm. That is, $j_{p_t}^* = j_p^*$ and $(f^{j_{p_t}^*}(\cdot) = f^{j_p^*}(\cdot))$. Also, $y_{p_t}^* = A_{s_t} y_p^*/\sum_{t=1}^{v} A_{s_t}$.

3. Thus, the payment for the proposed sharing $y_{s_t}^*$ is

$$\begin{aligned}
\Upsilon_t = 0.5\Bigg[\sum_{\theta=1}^{2} q_\theta A_{p_t} \Bigg(& f^{j_p^\circ}\left(\min\left\{w_{\max}^{j_p^\circ}, \frac{y_{p_t}^\circ}{A_{p_t}} + R_\theta\right\}\right)\\
& - f^{j_{p_t}^*}\left(\min\left\{w_{\max}^{j_{p_t}^*}, \frac{y_{p_t}^*}{A_{p_t}} + R_\theta\right\}\right)\Bigg)\Bigg]
\end{aligned}$$

$$+0.5\left[\sum_{\theta=1}^{2} q_\theta A_{s_t}\left(f^{j^*_{s_t}}\left(\min\left\{w^{j^*_{s_t}}_{\max}, \frac{y^*_{s_t}}{A_{s_t}}+R_\theta\right\}\right) - f^{j^\circ_{s_t}}\left(\min\left\{w^{j^\circ_{s_t}}_{\max}, \frac{y^\circ_{s_t}}{A_{s_t}}+R_\theta\right\}\right)\right)\right].$$

Given the complete rate card, a secondary farmer maximizes its objective (revenue minus the payment) by selecting the corresponding best crop:

Decision for a Secondary Farmer with Parameters (A_p, A_{s_t}, W^L) for Shared Water $y^*_{s_t}$:

$$B_t(y^*_{s_t}) = \max_j\left[\sum_{\theta=1}^{2} q_\theta\left(A_{s_t} f^j\left(\min\left\{w^j_{\max}, \frac{y^*_{s_t}}{A_{s_t}}+R_\theta\right\}\right)\right)\right] - \Upsilon_t.$$

Note that the socially optimal revenue of the secondary farmer (i.e., $\sum_{\theta=1}^{2} q_\theta A_{s_t} f^{j^*_{s_t}}(y^*_{s_t}/A_{s_t}+R_\theta)$) is the best revenue it can achieve for the amount of surface water $y^*_{s_t}$. Thus, given $y^*_{s_t}$ and the corresponding payment Υ_t, the optimal revenue of the secondary farmer's individual decision problem is also achieved for the socially optimal crop (i.e., $j^* = j^*_{s_t}$). Also, by the construction of the price Υ_t, it is easy to see that $B_t(y^*_{s_t}) \geq B_t(y^\circ_{s_t})$ where $B_t(y^\circ_{s_t})$ is the value of the secondary farmer's objective under naive decentralization. Hence, it is individually rational for secondary farmer S_t to offer the price Υ_t to the primary farmer. Similarly, it is also individually rational for the primary farmer to accept the offer and share $y^*_{s_t}$ amount of surface water. Along with being socially optimal within a virtual lane and therefore in a lane (because the same crop is planted in each virtual farm of the primary farmer), the rate card guarantees the socially optimal solution for the entire farming community if the a priori allocation of surface water W^L to a lane is the corresponding optimal allocation in Problem S.

Table 6 summarizes the main results of our analysis thus far. For brevity, the table avoids mentioning some auxiliary schemes such as the rainfall-contingent payment scheme (§4.2.1) and the linear pricing scheme (Remark 1, §5.1).

Remark 3 (Multiple Rainfall Scenarios). Our discussion thus far assumed two rainfall scenarios, corresponding to high and low amounts of rainfall. To generalize the rainfall distribution, consider $l \geq 2$ scenarios: $R_\theta, \theta = 1, 2, \ldots, l$. Without loss of generality, we assume that $R_1 \leq R_2 \leq \cdots \leq R_l$. It is straightforward to observe that considering l rainfall scenarios does not affect our discussion in §§2, 3, 4.1, and 5. For risk-averse farmers, our discussion on the rainfall-contingent payment scheme (§4.2.1) also remains the same, except that there are l payments (instead of two) based on the rainfall scenarios. The consideration of more than two scenarios does not impact the reward for primary farmers under the reward and water-guarantee scheme (§4.2.2); however, it affects the design of the water guarantee for the secondary farmers. We can generalize this scheme as follows: A risk-averse secondary farmer, say S_t, can purchase a guaranteed level of water, say M_t, that can be exercised if the lowest rainfall scenario R_1 is realized. If a scenario other than R_1 occurs, the farmer receives an amount, say η_t, of surface water that is independent of its chosen guarantee but depends on the areas of the farmers and on the total water available to their lane. Further analysis of this updated reward and water-guarantee scheme is similar to that in §4.2.2 and is therefore avoided here.

Table 6 Our Main Schemes

	Single-lane, single-crop system	Multiple-lane, multiple-crop system
Risk-averse farmers	Reward and Water-Guarantee Scheme	Reward and Premium Charts
	Socially optimal, individually rational, water sustenance, third-party intervention	Individually rational, water sustenance, third-party intervention
Risk-neutral farmers	Up-front Internal Payment Scheme	Rate Card
	Socially optimal, individually rational, water sustenance, budget balance	Socially optimal, individually rational, water sustenance, budget balance

6. Computational Experience

On a test bed that is informed by realistic data, our aim in this section is twofold: (i) contrast the performance of our decentralized mechanisms with that of naive decentralization, with respect to surface-water scarcity and the relative farm areas of primary and secondary farmers; and (ii) study the movement in the price of water with the probabilities of rainfall scenarios. We begin by describing the test bed considered to address these objectives.

6.1. The Test Bed

To anchor our experiments in reality, we use information about real-world crop-related data and farm sizes provided in Jalota et al. (2007). To represent a reasonable-sized farming community, we consider $m = 10$ lanes. For ease of exposition, we consider the following simplified setting: (i) risk-neutral farmers; (ii) one primary farmer and one secondary farmer per lane (denoted as P_i and S_i, respectively, $i = 1, 2, \ldots, 10$); and (iii) two rainfall scenarios. Thus, we have 10 primary farmers and 10 secondary farmers. Recall from §1 that agricultural farms in developing countries such as India are typically quite small (Effland 2010). However, farm sizes vary significantly across individual states/regions. For instance, the relatively richer states (e.g., Punjab and Haryana in India) have a larger average farm size. Farm sizes vary from less than 1 hectare (referred to as a *marginal farm*; 1 hectare = 2.47 acres) to more than 10 hectares (known as a *large farm*; Organisation for Economic Co-operation and Development 1999). To represent typical farming communities, the farm areas (in acres) were randomly drawn from $U[1, 20]$. A farmer has to choose one crop from a set of $n = 6$ crops. The following crop-related parameters are based on the field data from Jalota et al. (2007): For crop j, the maximum per-acre water requirement ($w^j_{\max}$, in mm) is randomly chosen from $U[100, 1{,}000]$. To decide the parameters (b^j, c^j) of the revenue function f^j, we estimate the maximum possible revenue per acre (i.e., the revenue corresponding to $w^j_{\max}$), denoted by $f^j_{\max}$. To this end and to maintain consistency among crops (i.e., to avoid a crop that needs less water relative to another crop but generates higher revenue), we fix the value of the ratio $w^j_{\max}/f^j_{\max}$. Based on data for maize, cotton, and soybeans in Jalota et al. (2007), we use the average value of 0.66 for this ratio. Thus, given this value and the maximum per-acre water requirement for each crop, we generate its maximum possible revenue per acre. The parameters (b^j, c^j) are then estimated to achieve this maximum revenue per acre at $w^j_{\max}$. Let $W_{\max} = \max_j w^j_{\max}$. The baseline value of the surface water available to the community is set at $W = 0.5 \sum_{i\in I}(A_{P_i} + A_{S_i})W_{\max}$; this value is later varied in our experiments. The baseline probability of the high-rainfall scenario is set at 0.5 and is varied for individual experiments. We consider 10 different settings for the areas of the farmers and 10 different settings for the maximum per-acre water requirement to obtain 100 base instances of Problem S. We now extend this test bed in three different directions for our specific goals.

(T1) *Increasing the Scarcity of Surface Water*. The issue of efficient sharing of water becomes relatively more important when water is scarce. Clearly, the available amount of surface water impacts the performance of the naive decentralization as well as that of our mechanisms. For each of the 100 base instances, we consider 8 instances by varying the water amount from 0 to $1.4W$, in steps of $0.2W$. Thus, T1 consists of a total of $8 \times 100 = 800$ instances.

(T2) *Increasing the Relative Areas of Secondary Farmers*. In Example 1 (§3), the naive decentralization suffered because of the relatively larger areas of the secondary farmers. We use the following setup to understand the impact of the relative areas of the farmers: For each of the 100 base instances generated above, we keep the total area in each lane fixed. From each instance, we then generate another instance by considering a secondary farmer's area as t% of the total area of its lane and that of its primary counterpart as $(100 - t)$%, where $t \in [0, 100]$. Thus, although the total area in each lane remains fixed at its value in the base instance, the relative proportions of the farmers change. We consider five values of t, for a total of $5 \times 100 = 500$ instances.

(T3) *Variation in the Probability of a Rainfall Scenario*. To understand the impact of uncertainty in rainfall, we vary the probabilities of the rainfall scenarios. We retain the entire construction of the 100 base instances, except for the probabilities of the rainfall scenarios. For each of these instances, the probability of the high-rainfall scenario is varied—starting with 0—in steps of 0.2, until it reaches a value of 1. Thus, we have a total of $100 \times 6 = 600$ instances.

Whenever required, the various optimization problems (e.g., Problem S and the decentralized problems of the farmers) were solved to optimality by using CPLEX (version 11.1.1).

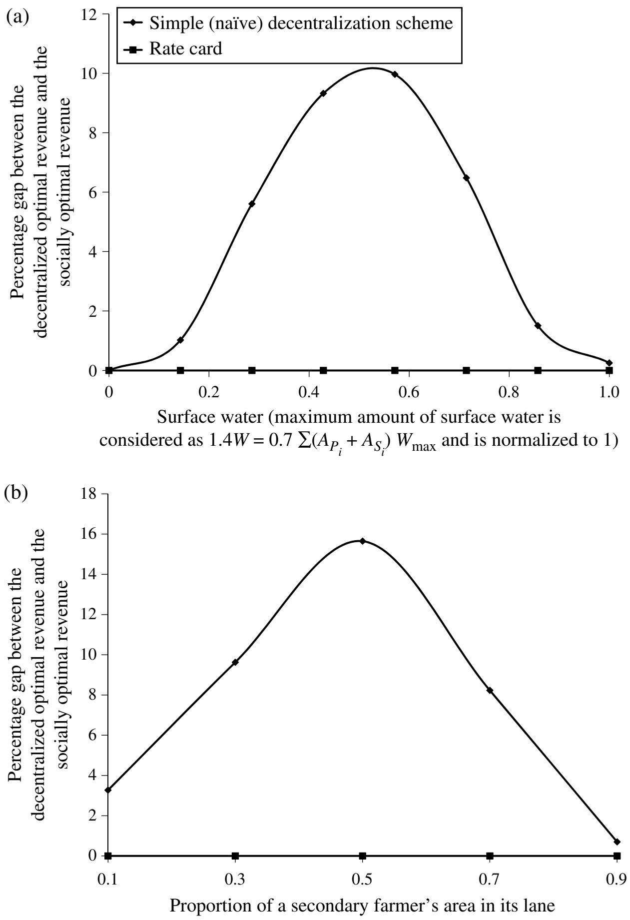
6.2. Effectiveness of the Decentralized Mechanisms

We divide our discussion into two parts:

1. Figure 4(a) illustrates the comparison between the revenues under the naive decentralization and the rate card. As expected, the rate card achieves the socially optimal solution. The percentage gap corresponding to naive decentralization (difference between the socially optimal and naive revenues divided by the socially optimal revenue) is shown in the figure. Each of the eight data points (on each of the two curves) in the figure represents the average percentage gap over the corresponding 100 instances, as described in T1 above.

For naive decentralization (Figure 4(a)), the gap first increases with water scarcity. The gap is at its highest when approximately half of the total water W is available and then starts decreasing. Consider two extreme situations. On the one hand, when water is very scarce, the primary farmers themselves do not have enough water to reach the relatively flat portions of their respective revenue functions. Consequently, the sharing of water with the secondary farmer does not provide a significant social benefit. Hence, naive decentralization is close to the social optimum. On the other hand, when water is abundant, then enough of it remains even after the primary farmers use their maximum required amounts. Therefore, primary farmers need not be incentivized to share water and naive decentralization again closely approximates the social optimum. The need to incentivize the primary farmers is highlighted when surface water is neither very scarce nor abundant. This can be seen in Figure 4(a) (when the normalized surface water ranges between 0.2 to 0.8). In this figure, when the normalized surface water is about 0.5, the effective sharing

Figure 4 Performance of the Decentralized Mechanisms with an Increase in (a) Surface Water Availability and (b) the Relative Areas of Secondary Farmers



can enable both the primary and secondary farmers to be on the increasing portions of their respective revenue functions. Therefore, the performance of the naive decentralization (which lacks such an efficient sharing) is at its worst in this region. Accordingly, the power of our decentralized mechanism is also best illustrated in this region.

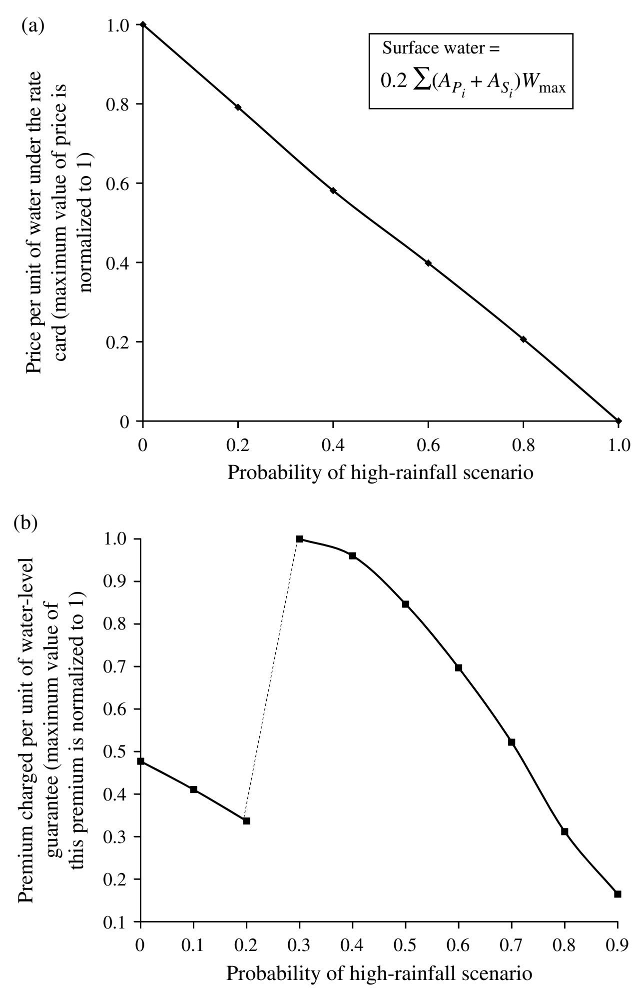
2. Figure 4(b) shows the percentage gap from the social optimum with an increase in the proportion of the secondary farmers' areas. The gap under naive decentralization first increases as this proportion increases. The gap is highest when the areas of both types of farmers are approximately equal. Beyond this point, the gap decreases as the proportion of a secondary farmer's area further increases. Here, the primary farmer—because it occupies a relatively smaller proportion of area—requires less water to maximize its revenue and voluntarily passes on the remainder to the secondary farmer. Thus, the usefulness of incentivizing the primary farmers is highlighted in the region when the proportion of secondary farmers' areas is neither too small nor too large.

In the case of risk-averse farmers, the comparison between the solution under naive decentralization and that under the reward and premium charts is similar.

6.3. The Movement of Price of Surface Water

Figures 5(a) and 5(b) highlight two possibilities of the movement of price, paid by a secondary farmer, as the probability of the high-rainfall scenario decreases. The price of water can increase as this probability decreases (Figure 5(a)), or it can first increase, then drop and increase again (Figure 5(b)). For a fixed set of crop parameters, we consider 10 base instances for Figure 5(b). For each of these base instances, we consider 10 values of the probability of high-rainfall scenario—starting with 0—in steps of 0.1. Hence, we have a total of $10 \times 10 = 100$ instances for Figure 5(b). Note that a decrease in the probability of high rainfall implies an increase in the scarcity of water. Thus,

Figure 5 Illustrations of the Movement of Price: (a) Increase in Price as the Probability of High-Rainfall Scenario Decreases; (b) Drop in Price (Due to Change in Crop) as the Probability of High-Rainfall Scenario Decreases



Note. Here, the total available surface water is 40% of W.

intuitively, the price of surface water should increase as this probability decreases. However, this effect is guaranteed only as long as the farmer's choice of crop and the surface-water allocation to the farmer remains the same as scarcity increases. Theorem 6 establishes this for the single-lane, single-crop system consisting of one primary farmer and v secondary farmers $S_1, S_2, \ldots, S_v$ (i.e, the same geometry as in §4.2). However, under both the rate card and the premium chart, our decentralized problems involve a single lane but multiple crops. Together, the scarcity of surface water and the low value of high-rainfall probability both influence the secondary farmer's choice of crop. When surface water is scarce, the crop (say, crop A) that maximizes the expected utility for high values of the probability of high-rainfall scenario differs from the crop (say, crop B) that maximizes the expected utility for smaller values of this probability. Because crop B utilizes the scarce amount of surface water more efficiently as compared to crop A, the price can decrease as the probability decreases.

Theorem 6. *For a single-lane, single-crop system, let g^*_{1t} (respectively, g^*_{2t}) be the optimal surface water received by secondary farmer S_t under the up-front internal payment scheme (respectively, under the reward and water-guarantee scheme). Then, the respective prices per unit of water ($\pi_t(g^*_{1t})/g^*_{1t}$ and $\Omega_t(g^*_{2t}+R_1A_{s_t})/g^*_{2t}$) increase as the probability of high rainfall decreases.*

7. Implementation Issues and Recommendations for Future Research

There are two properties that would help the implementation of our mechanisms. One, they are individually rational for the farmers, in the sense that primary and secondary farmers have an incentive to participate. Two, under the mechanisms, the government would need to manage financial transactions instead of micromanaging the distribution (or actual usage) of water to individual farms. This is relatively easier because the need for significant human resources is avoided. For instance, as discussed below, a single governmental office can advise a population of farmers on the amount of water to be shared and the corresponding payment. The following ideas may be of further help.

• *Government as an Optimization Consultant*. Although the rate card (respectively, the reward and premium charts) corresponding to the up-front internal payment scheme (respectively, the reward and water-guarantee scheme) could be physically printed, it is not necessary to do so. Instead, a primary farmer and its secondary counterparts can collectively communicate with a governmental office that plays the role of an optimization consultant by creating a computerized tool based on our mechanisms. Given the areas of the farmers and the surface water available in their lane, this office can then provide advice on the amount of water (or the water guarantee) to be purchased and the corresponding payment.

• *Verification of Water Shared*. Recall that the up-front internal payment scheme is a bilateral agreement and does not require intervention from a third party. Nevertheless, there are several ways in which an interested third party can assist with the implementation of the scheme. Although the commitment to purchase surface water is made up front by a secondary farmer, water is released over the season. This leaves the secondary farmer somewhat vulnerable. Thus, there is a need to ensure that the secondary farmer receives the water corresponding to its payment. Instead of releasing the payment up front to the primary farmer, the secondary farmer could deposit it with a governmental entity (e.g., a designated account at a nationalized bank) at the beginning of the season. At the end of the season, the primary and secondary farmers can provide information on the revenue generated, or the government can set up an appropriate verification mechanism. Upon satisfactory verification, the payment can be released to the primary farmer. Another option is to seek active supervision of the sharing of water (by making the sharing agreements public) from cooperative bodies, such as the Pani Panchayats (Water Councils) in India (http://panipanchayat.org; Sahu 2008).
The government can further help the secondary farmers in the socially beneficial sharing of water by providing them with zero-interest loans for purchasing water, using their crop outputs as collateral. Such loans have been routinely offered to farmers in developing countries for buying seeds and fertilizers (Press Trust of India 2011).

In our analysis, primary farmers are incentivized to improve the sharing of water through an explicit payment for the shared amount. Instead, an interesting alternative could be the possibility of improved sharing through the various forms of subsidies that governments typically offer. For instance, primary farmers could be offered low-interest loans to adopt new irrigation technologies such as drip irrigation (Bhaskar 2010) in return for an improved sharing of surface water. Our analysis in this paper considers a deterministic revenue function for a given amount of water. More generally, one can imagine a situation where the revenue function is stochastic, depending on farmer-specific knowledge, likelihood of pest attacks, etc. Given this additional risk from yield uncertainty, the up-front payment schemes may be less attractive to the secondary farmers. Revenue sharing schemes could be investigated to address the sharing of water in such situations.

Electronic Companion

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