



## Manufacturing & Service Operations Management

Publication details, including instructions for authors and subscription information:  
<http://pubsonline.informs.org>

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To cite this article:

Wei-yu Kevin Chiang, (2012) Supply Chain Dynamics and Channel Efficiency in Durable Product Pricing and Distribution. Manufacturing & Service Operations Management 14(2):327-343. <http://dx.doi.org/10.1287/msom.1110.0370>

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# Supply Chain Dynamics and Channel Efficiency in Durable Product Pricing and Distribution

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This study extends the single-period vertical price interaction in a manufacturer–retailer dyad to a multi-period setting. A manufacturer distributes a durable product through an exclusive retailer to an exhaustible population of consumers with heterogeneous reservation prices. In each period, the manufacturer and retailer in turn set wholesale and retail prices, respectively, and customers with valuation above the retail price adopt the product at a constant (hazard) rate. We derive the open-loop, feedback, and myopic equilibria for this dynamic pricing game and compare it to the centralized solution. Although in an integrated supply chain a forward-looking dynamic pricing strategy is always desirable, we show that this is not the case in a decentralized setting, because of vertical competition. Our main result is that both supply chain entities are better off in the long run when they ignore the impact of current prices on future demand and focus on immediate-term profits. A numerical study confirms that this insight is robust under various supply- and demand-side effects. We use the channel efficiency corresponding to various pricing rules to further derive insights into decisions on decentralization and disintermediation.

*Key words:* supply chain management; dynamic pricing; differential game; channels of distribution; new product diffusion; operations–marketing interface

*History:* Received: October 17, 2010; accepted: November 6, 2011. Published online in *Articles in Advance* March 28, 2012.

## 1. Introduction

Retailers such as Best Buy must decide periodically how to set prices for durable goods, such as a new Sony 3D TV, in order to extract the most profit from the potential market for this product. When making pricing decisions over time, Best Buy might want to account for the opportunity of offering subsequent markdowns. Indeed, the forward-looking paradigm predicts that the retailer would be better off considering future revenue streams in its pricing decisions as opposed to focusing on short-term profits. At the same time, Best Buy must also consider that the manufacturer, Sony in this example, may also dynamically adjust the wholesale price at its own best interest. In this setting, how should Best Buy and Sony make pricing decisions over time? In particular, are both parties still better off pricing dynamically, in equilibrium, or are there advantages to focusing on short-term profits in a decentralized setting?

This paper analyzes the pricing problem in a distribution channel with intertemporal demand. This problem is challenging because channel members have to deal with the dynamic pricing problem and double marginalization at the same time. Double marginalization (Spengler 1950) occurs when an upstream firm, as a result of charging a wholesale price above the marginal cost, induces its intermediary to set a price

above the optimal level. The joint effect of these two problems creates both current and future competition, making the price decisions perplexing. Because of analytical intricacies, prior work typically examines these two problems separately. This study combines them into a single model with an objective of unveiling the impact of different pricing rules on the distribution efficiency. Specifically, based on diffusion theory, we develop a dynamic pricing game in a setting of a manufacturer–retailer dyad facing an exhaustible population of customers who differ in their valuations of purchasing. The two independent channel parties sequentially set wholesale and retail prices over time to maximize their own benefits. We analytically derive the open-loop, feedback, and myopic equilibrium prices for such a game. Our result indicates that although a forward-looking dynamic pricing maximizes the net discounted profit for a monopolist, it is not an efficient one in a decentralized supply chain. Regardless of which forward-looking equilibrium concept is applied, the manufacturer and the retailer register a higher profit when they both act myopically, basing their price decisions on immediate-term profitability. Ironically, the overpricing result induced by double marginalization can be mitigated by the myopic behavior because of its ignorance of using time to discriminate heterogeneous customers.

Within the context of a distribution channel, a recent empirical study by Che et al. (2007) reveals that simpler “shorter-horizon” games explain the behavior of channel members better than more complex “longer-horizon” games. Managerial actions that discount the future and emphasize short-term performance are usually condemned as economic short-termism or myopia, the existence of which has been extensively observed by numerous studies (e.g., Mizik and Jacobson 2007). As reflected by the remark below, concerns over economic short-termism have gained serious attention from academics and practitioners:

All employees (managers, product designers, service providers, production workers, etc.) allocate their effort between actions that influence current period sales and actions that influence sales in future periods. Unfortunately employees are generally more focused on the short term than the firm would like.

(Hauser et al. 1994, p. 328)

There are various explanations for the existence of economic short-termism. For example, behavioral research suggests that managers frequently find it difficult to accommodate intertemporal effects correctly in their decision making (e.g., Chakravarti et al. 1979, Meyer and Hutchinson 2001). In addition, employees have incentives to behave myopically when their performance evaluation depends on a current-term outcome measure, when they feel pressured to meet earnings expectations, and when their compensation and job security are tied to market reactions (Stein 1989). Although economic short-termism is typically considered harmful, our results paradoxically indicate that firms in a supply chain may benefit from such behavior. In a particular sense, the employees’ inability or failure to look ahead could turn out to be a blessing in disguise.

The explicit solutions of our analysis allow us to further explore relevant insights and implications for managing a supply chain. For example, supply chain decentralization is typically considered inefficient because it does not allow for rational allocation of resources based on a central plan. In contrast, we show that when managers are myopic in pricing, decentralization can possibly improve channel efficiency by alleviating intertemporal competition. Another notable contribution of this study is its investigation of the interaction between the pricing rule and the intermediation decision. In selling through an intermediary over time, it is often challenging to align the best interests of independent channel members. We establish conditions under which it is more profitable for the manufacturer to eliminate the retailer.

The remainder of this paper is organized as follows. After a review of the relevant literature in §2, §3 describes the process of demand dynamics and investigates the optimal monopolist pricing to establish a

performance benchmark. Section 4 analyzes the equilibrium results of forward-looking and myopic pricing games. Section 5 explores the impact of myopic pricing on channel efficiency. Section 6 provides insights into the conditions for disintermediation. Section 7 numerically tests the robustness of the major finding. The paper is concluded in §8, where the results are summarized with a discussion on limitations and future research directions.

## 2. Literature Review

The primary research streams underlying our work are studies of diffusion-based pricing of durable goods, intertemporal pricing with heterogeneous customers, supply chain coordination, and channel dynamics. The relevant work and knowledge are reviewed below to clarify our contribution.

The Bass diffusion model (Bass 1969) has proven to be seminal for characterizing the adoption process of a new durable product among a group of potential buyers. This epidemic model describes the adoption rate over time as the product of the likelihood of adoption, determined by the innovation and the imitation effects, and the remaining market potential, specified as the difference between a fixed market potential and the time-dependent installed base. Two approaches have been used to incorporate the price effect into the Bass model: one that multiplies the adoption rate by a decreasing function of price, and another where the fixed market potential is a price-dependent function. Postulating that the likelihood of adoption is negatively correlated to price, the first approach is initiated by Robinson and Lakhani (1975) and followed by numerous pricing models in both monopolistic (e.g., Dolan and Jeuland 1981, Bass and Bultez 1982, Krishnan et al. 1999) and oligopolistic settings (e.g., Thompson and Teng 1984, Eliashberg and Jeuland 1986). Although this approach is empirically supported by Jain and Rao (1990), it is potentially problematic for modeling durable goods because the resulting demand elasticity is independent of the installed base, as pointed out by Kalish (1983).

Recognized as more appropriate for studying durable goods pricing, and thus adopted in this study, the second approach follows Mahajan and Peterson (1978), who argue with data that the market potential over time is more likely to be dynamically influenced by marketing mix variables. With the rationale that customers’ decision to purchase depends on their reservation price, this approach replaces the fixed market potential in the Bass model with a price-dependent function so that whenever the price goes down, the potential goes up, and vice versa. If the price goes up, some of those who have purchased may find out that the price is above their valuation. In this case, it is

presumed that the utility maximization principle will lead them to resell to those with higher valuations in the secondary market (see Kalish 1983, 1985; Xie and Sirbu 1995). Besides rendering the analysis tractable, as will be illustrated in §3, this stylization is appealing in the sense that it captures a crucial market behavior: price increase induces arbitrage/resale behaviors that in turn decelerate diffusion. With this stylization, the formulation of price-dependent market potential has been empirically validated by multiple studies (e.g., Kalish 1985, Jain and Rao 1990, Horsky 1990, Mesak and Berg 1995) for a variety of durable goods, such as televisions, air conditioners, and dishwashers.

Relevant studies examining diffusion-based pricing with the price-dependent market potential include those of Feichtinger (1982), Jørgensen (1983), and Kalish (1983), who, respectively, apply concave, linear, and general price functions in their monopolistic models. In addition, Kalish (1985) and Horsky (1990) derive more complicated price functions to generate descriptive and normative implications of optimal pricing. Using linear price functions similar to that of Jørgensen (1983), Raman and Chatterjee (1995) investigate the effect of uncertainty on monopolist pricing, and Xie and Sirbu (1995) examine the impact of demand externalities on the open-loop equilibrium of duopolistic competition. The price function of our model, in which the rationale is more explicitly elucidated, also appears in a linear form. Although the models and the contexts vary across studies, unless under some special circumstances where the imitation effect dominates the innovation effect, the resulting pricing patterns typically follow a decreasing path, known as price skimming.

The fundamental issue of price skimming can be traced back to the economics and marketing literature (e.g., Stokey 1979, Conlisk et al. 1984, Sobel 1984, Besanko and Winston 1990) regarding ways to intertemporarily price discriminate consumers. This issue continues to motivate recent studies to develop various dynamic models from a joint perspective of operations and marketing (e.g., Klastorin and Tsai 2004, Su 2007, Aviv and Pazgal 2008, Elmaghraby et al. 2008, Ahn et al. 2009). None of the studies along this line has considered diffusion-based pricing in a supply chain context, which concerns our work. When supply chain entities vertically compete for profit margins in selling to reactive customers, we show that forward-looking dynamic pricing is undesirable. This result resonates with that of Liu and Zhang (2011), who extend the model by Besanko and Winston (1990) to a duopoly setting and discover that dynamic pricing is often an inferior strategy when firms horizontally compete for strategic customers with quality-differentiated products. Our finding also complements to other explanations for adverse effects of dynamic pricing, including

the presence of price-adjustment costs (e.g., Gallego and van Ryzin 1994, Çelik et al. 2009) and behavioral regularities of consumer learning (e.g., Nasiry and Popescu 2011).

The literature in supply chain coordination has documented various mechanisms through which the incentives of independent channel members can be aligned to prevent pricing breakdowns caused by double marginalization. These mechanisms include profit sharing (Jeuland and Shugan 1983), quantity discounts (Monahan 1984, Weng 1995), and quantity flexibility (Tsay and Lovejoy 1999). For a thorough review of the relevant models, refer to Cachon (1998). Most studies on this subject hold a static view of pricing. As our result suggests, failing to consider dynamic effects may leave such short-term static analyses unable to provide effective guidance. Among the first to examine manufacturer–retailer interactions in dynamic settings was Shugan (1985), who shows that implicit understandings from repeated interactions can lead to increased channel profitability. Subsequently, Jørgensen (1986) derives the open-loop equilibrium for a dynamic production, purchasing, and pricing game between a manufacturer and its retailer. Eliashberg and Steinberg (1987) take an integrated view of pricing and operations decisions to explore the dynamic nature of coordination in an unstable demand pattern. These studies, however, do not consider the diffusion-based pricing effect investigated in this paper. Other studies on channel dynamics (e.g., Chintagunta and Jain 1992) are not directly related to our work, as their models involve different control variables, such as advertising.

### 3. Model Formulation and the Integrated Supply Chain

Consider a supply chain in which a manufacturer distributes a new durable product through a retailer over an infinite time horizon. By durability, we mean that each consumer adopts one unit of the product at most. The product, for which there are no substitutes or complements, is available to a population of consumers who are price takers and whose product valuations are heterogeneous. A price must be set for each period, within which an untapped customer whose valuation is weakly higher than the price will adopt with a given likelihood. To concentrate our analysis on dynamic price interactions, we ignore capacity constraints and inventory-related costs. In particular, we assume that the manufacturer's production quantity accords with the retailer's order quantity, which follows the demand rate. This stylization applies directly to those products with short production/delivery lead times or with insignificant unit production costs (e.g., books, software, or other digital products). It is



also justifiable in make-to-order settings where the product is built once an order with payment is confirmed, or in “chasing demand” settings where the production plan matches the demand rate as closely as possible to minimize inventory holding costs.

### 3.1. Demand Dynamics

To better explain the demand dynamics, we start with a discrete-time fluid model in which the firm sets prices at the time epochs  $t_i = i\Delta t$ , where  $i = 1, 2, 3, \dots$ , and  $\Delta t \in (0, 1]$  is a fixed length of time for each period. Let  $d_i$  be the demand, or the amount of adopters, in period  $i$  (during time interval  $[t_i, t_{i+1})$ ). As explicated below, the demand  $d_i$  is affected not only by the price in period  $i$ , denoted by  $p_i$ , but also by the previous prices if  $i > 1$ . Following a conventional approach to capturing heterogeneity in consumer tastes and preferences, we assume that the customers' product valuations are spread uniformly between 0 and  $N$ , where the market density is normalized to unity without loss of generality. Let  $y_i^{[a,b]}$  be the amount of customers with valuations between  $a$  and  $b$  who have adopted by the end of period  $i$ ;  $[a, b] \subseteq [0, N]$ . Denote the cumulative demand by the end of period  $i$  by  $x_i$ ;  $x_i = y_i^{[0,N]}$ . As  $y_i^{[a,b]}$  is a subset of  $x_i$ , we call it the segmented installed base. There is no adopter prior to the initial period; i.e.,  $x_0 = y_0^{[0,N]} = 0$ .

During any period in the selling horizon, say period  $n$ ,  $n \geq 1$ , all customers with a valuation in  $[p_n, N]$  who have not yet adopted may possibly adopt at the price  $p_n$ . Those customers are referred to as likely adopters. With the segmented installed base defined above, the amount of likely adopters in period  $n$ , denoted by  $L_n$ , can be specified as  $L_n = N - p_n - y_{n-1}^{[p_n, N]}$ . Let  $\alpha \in (0, 1]$  be the likelihood per time unit that a likely adopter will adopt. This hazard rate, also known as the trial rate (Fourt and Woodlock 1960), reflects the speed of adoption. It applies at any valuation level and is assumed to be constant over time. This modeling assumption is empirically supported by Horsky (1990), who finds that whereas the price effect on the market potential is significant, the imitation effect in the Bass model does not coexist (or is very weak) with the price effect for all product classes in his study. Because the period length is  $\Delta t$ , the percentage of likely adopters who will actually purchase in each period is  $\alpha\Delta t \in (0, 1]$ . Multiplying this percentage by  $L_n$  yields the demand in period  $n$ ; that is,

$$d_n = \alpha\Delta t(N - p_n - y_{n-1}^{[p_n, N]}). \quad (1)$$

After  $d_n$  amount of customers have adopted in period  $n$ , the diffusion process proceeds to period  $n + 1$ , within which the price can be lower than, equal to, or higher than  $p_n$ . To see how price change affects the demand dynamics, according to the relationship

between the prices for the two consecutive periods, we can specify the segmented installed base pertinent to deriving  $d_{n+1}$  as

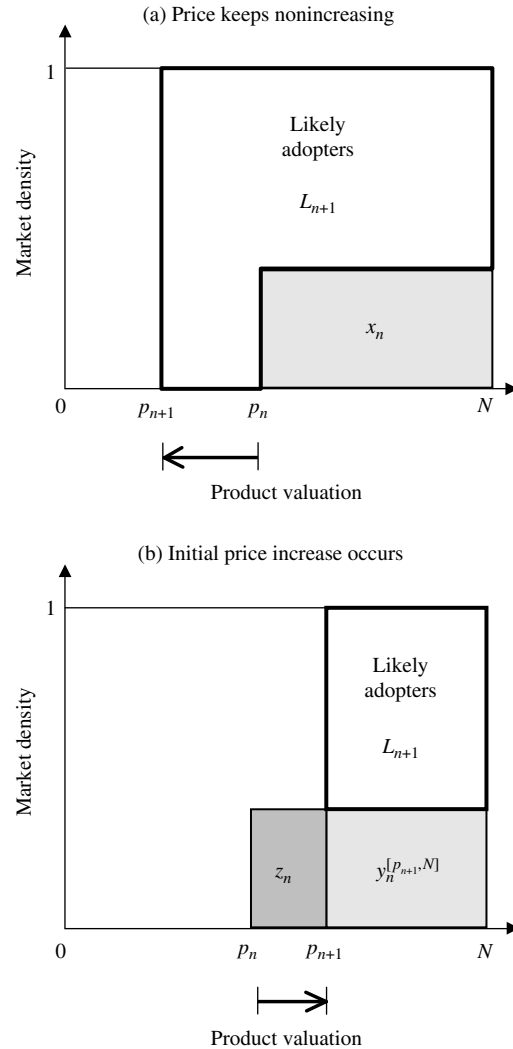
$$y_n^{[p_{n+1}, N]} = d_n + \begin{cases} y_{n-1}^{[p_{n+1}, N]} & \text{if } p_{n+1} \leq p_n, \\ y_{n-1}^{[p_n, N]} - z_n & \text{otherwise,} \end{cases} \quad (2)$$

where  $z_n = y_n^{[p_n, p_{n+1}]} > 0$ . If the price is nonincreasing over time, the segmented installed base on the left-hand side of Equation (2) recursively converges to the cumulative demand, i.e.,  $y_n^{[p_{n+1}, N]} = \sum_{i=1}^n d_i = x_n$ . In this case, as depicted in Figure 1(a), the amount of likely adopters in period  $n + 1$  can be characterized by

$$L_{n+1} = N - p_{n+1} - x_n. \quad (3)$$

We next argue that this equation remains valid even if the retailer considers the possibility of increasing prices at a given time. We justify this fact below based on the existence of efficient secondary markets.

Figure 1 Likely Adopters in Period  $n + 1$



An alternative justification, which does not involve resale markets, is provided in Appendix A, where we show that optimal prices must follow a decreasing path, so (3) does indeed describe the relevant dynamics. We focus the presentation on the model with resale, because this approach is consistent with related literature (e.g., Kalish 1983, Xie and Sirbu 1995) and it makes it easier to accommodate additional extensions (such as the imitation effect, see §7) where an analytical proof of decreasing prices is difficult. Nevertheless, all our analytical results in this paper hold without resale markets.

So, what happens if the price increases, and resale markets are allowed? Suppose that the initial increase occurs in period  $n + 1$  (i.e.,  $p_{n+1} > p_n$ , and  $p_{i+1} \leq p_i$  if  $1 \leq i < n$ ). Equation (2) then implies that  $y_n^{[p_{n+1}, N]} = x_n - z_n$ , as illustrated in Figure 1(b). Because  $z_n$  is the amount of prior adopters with valuations below the current price, the utility maximization principle will rationalize them to resell to those with higher valuations at a competitive price. As reviewed previously, the literature on durable goods pricing argues that resale is behaviorally and technically justifiable in the secondary market. As a result of resale, the eligible amount of likely adopters diminishes by  $z_n$  and turns out to be

$$L_{n+1} = (N - p_{n+1} - y_n^{[p_{n+1}, N]}) - z_n, \quad (4)$$

which, after substituting  $y_n^{[p_{n+1}, N]}$  with  $x_n - z_n$ , is identical to (3). A similar rationale continues to apply whenever the price increases thereafter. The model thus captures a sensible phenomenon that arbitrage behaviors induced by a price increase decelerate diffusion, and is meanwhile rendered tractable as it reduces to having only one state variable,  $x_i$ .

Now let  $x(t_i) \triangleq x_i$  and  $p(t_i) \triangleq p_i$ . Then the cumulative demand in period  $n + 1$  can be expressed as  $x(t_{n+1}) = x(t_n) + \alpha \Delta t L_{n+1}$ , where, based on Equation (3),  $L_{n+1} = N - p(t_{n+1}) - x(t_n)$ . Accordingly, after denoting  $t_{n+1}$  by  $t$ , we can specify the rate at which demand increases during time interval  $[t - \Delta t, t]$  as

$$\frac{x(t) - x(t - \Delta t)}{\Delta t} = \alpha(N - p(t) - x(t - \Delta t)). \quad (5)$$

This allows a continuous approach to characterizing the diffusion process with an infinitesimal length of time for each period. Specifically, as  $\Delta t \rightarrow 0$ , the demand rate on the left-hand side of Equation (5) becomes the time derivative of  $x(t)$ . Rewriting it with the dot notation for the time derivative leads to the following differential equation of demand dynamics:

$$\dot{x}(t) = \alpha(N - p(t) - x(t)). \quad (6)$$

Obviously, the model captures the saturation effect (the shrinkage in potential demand with increasing

market penetration) and is endogenous with respect to price. It corresponds to a traditional diffusion model with linear price-dependent market potential, as reviewed in §2.

### 3.2. Monopoly Benchmark: The Optimal Pricing Strategy

We first establish a performance benchmark by analyzing the problem of a vertically integrated supply chain to clarify the intuition. Assume that the supply chain incurs a constant marginal production cost  $c$ . Let  $\delta$  be the discount rate, which is exogenously determined by the cost of capital. Given the dynamic demand process in (6), the objective for an integrated supply chain (monopolistic seller) who follows a forward-looking pricing rule is to find  $p(t)$  that will maximize the net discounted profit, specified below, over an infinite horizon:

$$\pi^F(p) = \int_0^\infty e^{-\delta t} (p(t) - c) \dot{x}(t) dt. \quad (7)$$

Applying standard control theory (see Kamien and Schwartz 1991), we define the current value Hamiltonian for the dynamic optimization problem as

$$H(x, p) = (p(t) - c + \lambda(t)) \dot{x}(t), \quad (8)$$

where  $\lambda(t)$  is the shadow price (also known as the adjoint or costate variable) associated with the state variable  $x(t)$ , which can be interpreted as the impact of selling an additional unit on future profits. Ceteris paribus, a positive shadow price implies lowering the current price to sacrifice profit now for future benefit, and vice versa. The following proposition presents the optimal pricing strategy for the integrated supply chain.

**PROPOSITION 1 (OPTIMAL FORWARD-LOOKING PRICING).** *The optimal pricing strategy and the corresponding accumulated sales over time, respectively, are given by*

$$\begin{aligned} p^F(t) &= c + (N - c)(1 - \gamma/\alpha)e^{-\gamma t} \quad \text{and} \\ x^F(t) &= (N - c)(1 - e^{-\gamma t}); \end{aligned} \quad (9)$$

and the shadow price can be expressed as

$$\lambda(t) = -(N - c)(1 - 2\gamma/\alpha)e^{-\gamma t}, \quad (10)$$

where

$$\gamma = (\sqrt{\delta^2 + 2\alpha\delta} - \delta)/2 > 0. \quad (11)$$

As expected, the optimal strategy is that of price skimming, which consists of a monotonically decreasing price over time. One observation that makes intuitive sense is that a higher speed of adoption causes the initial price to be higher and to decrease more rapidly early in the selling process. Clearly, when the speed of adoption  $\alpha$  is low, a high price results in a

slow selling speed, preventing the firm from reaping more cash in early periods. As money now is preferred with a positive discount rate, a lower  $\alpha$  pressures the seller to lower the initial price to speed up sales and increase early cash flow, as it is too costly to start with a higher price and wait for more high-valuation customers to adopt. On the other hand, when the speed of adoption is high, most customers are willing to adopt at the outset as long as the price is right for them. In this case, because the price can always be dropped in the near future before discounting makes future revenues less valuable, the firm will set higher prices initially and then cut prices over time to sell to the low-valuation consumers remaining in the market.

After plugging (9) into (6) and then into (7), we can specify the optimal discounted profit as

$$\pi^F = \frac{\alpha + \delta - \sqrt{\delta^2 + 2\alpha\delta}}{2\alpha} (N - c)^2. \quad (12)$$

It is straightforward to show that the profit increases with the speed of adoption  $\alpha$  but decreases with the discount rate  $\delta$ . Now if the integrated supply chain is myopic when setting prices, it then disregards the evolution of cumulative sales and maximizes the instantaneous profit given by  $(p(t) - c)\dot{x}(t)$ . With the same dynamic demand process in (6), it can be verified that the myopic prices over time and the resulting net discounted profit, respectively, are

$$p^M = c + \frac{N - c}{2} e^{-(\alpha/2)t} \quad \text{and} \quad \pi^M = \frac{\alpha}{4(\alpha + \delta)} (N - c)^2. \quad (13)$$

Consistent with the result that the shadow price  $\lambda(t)$  in (10) is uniformly negative, comparing the two distinct pricing rules reveals that the myopic pricing in (13) is lower than the forward-looking pricing in (9) at any time. This conforms to our intuition that a forward-looking firm will sacrifice current profits for future benefits by setting higher current prices to price discriminate high-valuation customers across time. Not surprisingly, from a long-term perspective, the resulting net discounted profit in (12) is higher than that in (13) with myopic pricing.

#### 4. Dynamic Pricing in the Decentralized Supply Chain

When the supply chain is decentralized, the manufacturer and the retailer independently set wholesale and retail prices, ignoring the collective impact of their decisions on the supply chain profit as a whole. With the manufacturer being the Stackelberg leader, we consider three separate games to analyze the strategic price interactions over time: the open-loop, the feedback, and the myopic pricing games. In what follows, we detail the equilibrium concept and result of each game.

##### 4.1. Open-Loop Equilibrium

With the open-loop equilibrium concept, both channel members are forward looking and plan ex ante their price decisions, which depend only on time. Specifically, at the outset of the selling horizon, the manufacturer announces a schedule of wholesale prices, denoted by  $w(t)$ , before the retailer makes the retail price decision  $p(t)$  for each time instance  $t$ . Note that wherever there is no confusion, we will omit the function argument  $t$  for brevity. To obtain the equilibrium prices, we start by solving the retailer's problem and then recursively solve that of the manufacturer, taking into account the retailer's rational decision behavior. Subject to the dynamic demand process in (6), the retailer reacts to the manufacturer's wholesale price decision by maximizing its net discounted profit, which can be specified as

$$\pi_r(p) = \int_0^\infty e^{-\delta t} (p - w) \dot{x} dt. \quad (14)$$

Anticipating the retailer's price choice, the manufacturer determines a wholesale price path that maximizes its net discounted profit given by

$$\pi_m(w) = \int_0^\infty e^{-\delta t} (w - c) \dot{x} dt. \quad (15)$$

The open-loop equilibrium of the pricing game is presented in Proposition 2 below, with the proof relegated to Appendix B.

**PROPOSITION 2 (OPEN-LOOP EQUILIBRIUM).** *When the manufacturer and retailer are both forward looking, the open-loop wholesale and retail prices, respectively, are given by*

$$w^{OL}(t) = \frac{N + c}{2} \quad \text{and} \quad p^{OL}(t) = w^{OL}(t) + \frac{N - c}{2} (1 - \gamma) e^{-\gamma t}; \quad (16)$$

*the resulting cumulative sales over time can then be characterized by*

$$x^{OL}(t) = \frac{N - c}{2} (1 - e^{-\gamma t}), \quad (17)$$

where  $\gamma$  is specified in (11).

Unlike the integrated supply chain wherein the retail price continues to drop until driven down to the unit production cost, the result in Proposition 2 indicates that the decentralized supply chain stops exploiting the remaining demand when the price is slashed to the stationary wholesale price. Whereas the retail price decreases over time, the wholesale price is time invariant. One immediate interpretation of this result is that if the wholesale price decreases over time, the retailer, anticipating its future reduction, will then slow down

sales by setting higher prices early on to profit from the lower wholesale price in the future. This in turn hurts the manufacturer's profitability. By endeavoring to set a constant wholesale price to induce the retailer to sell more rapidly, the manufacturer, who faces the discounting effect, can siphon off a more profitable revenue stream in the early periods. Another way to look at it is that the manufacturer is unable to directly price discriminate customers because it must stick to the preannounced wholesale price schedule in the open-loop case. Specifically, if the retailer knows that the wholesale price will drop tomorrow, it will hold some sales today (when the discount rate is negligible, sales will halt). Compared with maintaining the same wholesale price, this implies that a chunk of revenue that could have been generated by the manufacturer today will be deferred to tomorrow, when it will become less valuable on account of the lower wholesale price and the time cost of money.

Note that the open-loop equilibrium is time-inconsistent (i.e., the original best decision for some future period is inconsistent with what is preferred when that future period arrives). Thus, to sustain the equilibrium, an implicit assumption is that the manufacturer is able to commit creditably to its preannounced wholesale price schedule. In reality, this can be achieved practically through a contract when the legal system is effective in remedying a breach of contractual obligations. Nevertheless, because periodically revisiting the wholesale price helps to exhaust the residual market, modifying the initial agreement is likely to be ex post mutually beneficial. A question arises in this context: Would the manufacturer be tempted to deviate from the original schedule as time evolves and decrease the wholesale price over time until it reaches the level of unit cost? Besides the physical costs of recontracting or renegotiation, this behavior is undesirable for two reasons. First, when the retailer anticipates such behavior and does not consider the manufacturer's price "threat" ex ante to be credible, the latter's profitability may erode. Second, as mentioned above, implementing a constant wholesale price prevents the retailer from behaving opportunistically so that the manufacturer can profit earlier. Thus, even if recontracting in the future brings in additional revenue, by the time the remaining demand is exploited, that revenue may lose its value because of the time cost of money. In general, when the manufacturer has no incentive to deviate from the original plan and has a reputation for refraining from renegotiation, the open-loop equilibrium is still sustainable even without resorting to a contract.

#### 4.2. Feedback Equilibrium

Although the open-loop equilibrium reflects some observations of actual channel practice likely due to

its relatively easier tractability in deriving strategic actions, it may unravel if the manufacturer cannot credibly commit to its decisions. We now examine another strategic concept, the feedback equilibrium, which is known to be time consistent and is thus renegotiation proof. Unlike the open-loop concept in which the manufacturer commits to the entire sequence of price decisions through time, when the feedback concept is applied, the pricing decision at each point in time is made on the basis of the status of cumulative demand at that particular time. Because of its intricacy and complexity, the derivation of the feedback equilibrium is generally intractable, even numerically. Yet, with the quadratic profit functions specified in (14) and (15) for this particular problem, the equilibrium can be analytically derived through solving two partial differential equations simultaneously; this is detailed in Appendix C.

**PROPOSITION 3 (FEEDBACK EQUILIBRIUM).** *When the manufacturer and retailer are both forward looking, the feedback wholesale and retail prices, respectively, are characterized by*

$$\begin{aligned} w^{FB}(t) &= c + \frac{2}{3}(N - c)\left(1 - \frac{\varphi}{\alpha}\right)e^{-\varphi t} \quad \text{and} \\ p^{FB}(t) &= c + (N - c)\left(1 - \frac{\varphi}{\alpha}\right)e^{-\varphi t}; \end{aligned} \quad (18)$$

and the corresponding sales volume over time is given by

$$x^{FB}(t) = (N - c)(1 - e^{-\varphi t}), \quad (19)$$

where  $\varphi = (\sqrt{6\alpha\delta + 4\delta^2} - 2\delta)/6$  and  $0 < \varphi < \gamma$ .

In contrast to the static wholesale price in the open-loop equilibrium, the feedback wholesale price  $w^{FB}(t)$  decreases over time with a relatively higher initial level and converges to the unit cost of production  $c$ . Accordingly, the corresponding retail pricing  $p^{FB}(t)$  also demonstrates a falling path over time and converges to  $c$ , which is similar to the monopolist pricing. This result is not surprising because the feedback equilibrium, which takes into account strategic interactions between the channel members through the evolution of cumulative demand over time, is subgame perfect and time consistent.

With the equilibrium prices in Propositions 2 and 3, the net discounted profits for the manufacturer with the open-loop and feedback pricing strategies, respectively, can be spelled out as

$$\begin{aligned} \pi_m^{OL} &= \frac{1}{2}\left(\frac{1}{2} - \frac{\gamma}{\alpha}\right)(N - c)^2 \quad \text{and} \\ \pi_m^{FB} &= \frac{1}{3}\left(1 - \frac{4\varphi}{\alpha}\right)(N - c)^2. \end{aligned} \quad (20)$$



Moreover, it is straightforward to verify that the retailer appropriates only half of what the manufacturer makes in either equilibrium; that is,  $\pi_r^{OL} = \pi_m^{OL}/2$  and  $\pi_r^{FB} = \pi_m^{FB}/2$ , which coincides with the results in the typical static analysis of vertical price competition. Comparing the net discounted profits with the two forward-looking equilibrium concepts, we find that both supply chain members are better off with the feedback than with the open-loop equilibrium; that is,  $\pi_m^{FB} > \pi_m^{OL}$  and  $\pi_r^{FB} > \pi_r^{OL}$ . Consistent with the traditional analysis of the static supply chain interaction, the total net discounted profit generated by the two independent channel members is lower than that of the integrated supply chain (i.e.,  $\pi_m^{OL} + \pi_r^{OL} < \pi^F$  and  $\pi_m^{FB} + \pi_r^{FB} < \pi^F$ ), regardless of which equilibrium concept is applied. The reason behind the loss in gains is that the equilibrium prices specified in (16) and (18) are always higher than the optimal pricing, given in (9), over time. That is, the inefficiency of double marginalization with the static analysis also exists with the dynamic analysis in a multiple-period manner.

#### 4.3. Myopic Equilibrium

Now we explore the pricing behaviors in the myopic supply chain, wherein the manufacturer and the retailer emphasize on immediate-term profits when setting prices, ignoring the future impact of their decisions on the dynamics describing the demand evolution. By adopting a myopic pricing rule, both channel members act as if the planning horizon is reduced to one period; therefore, they solve a static optimization problem for each time instance. It is straightforward to show that the retailer's best reaction to the manufacturer's wholesale price decision is

$$p^* = (N + w - x)/2. \quad (21)$$

Backward substitution implies that the equilibrium of the myopic pricing game corresponds to the solution of the manufacturer's control problem specified as follows:

$$\max_w (w - c)\alpha(N - p - x), \quad \text{subject to (6) and (21).} \quad (22)$$

Following the standard optimization approach, we obtain the equilibrium result below.

**PROPOSITION 4 (MYOPIC EQUILIBRIUM).** *When the manufacturer and retailer are both myopic, the equilibrium wholesale and retail prices, respectively, can be specified as*

$$\begin{aligned} \tilde{w}^M(t) &= c + \frac{N - c}{2} e^{-(\alpha/4)t} \quad \text{and} \\ \tilde{p}^M(t) &= c + 3 \frac{N - c}{4} e^{-(\alpha/4)t}, \end{aligned} \quad (23)$$

with the following accumulated sales over time:

$$\tilde{x}^M(t) = (N - c)(1 - e^{-(\alpha/4)t}). \quad (24)$$

We can make a few observations after comparing the myopic equilibrium pricings in Proposition 4 with the forward-looking ones in Propositions 2 and 3. First, the wholesale price decreases over time in the myopic supply chain, with an initial level equal to the static wholesale price in the open-loop equilibrium. Second, regardless of which forward-looking equilibrium concept is applied, over time the myopic prices are uniformly lower than the forward-looking prices. Next, although a higher initial price is associated with a higher speed of adoption in the forward-looking supply chain, the speed of adoption has no impact on the initial price in the myopic supply chain. Finally, unlike the forward-looking prices that decrease in the discount rate regardless of which equilibrium concept is adopted, the myopic prices are insensitive to the discount rate.

### 5. Myopic Pricing and Channel Efficiency

In this section, we investigate the impact of myopic pricing on supply chain profitability and channel efficiency. In line with the myopic equilibrium in Proposition 4, it can be verified that the net discounted profits for the manufacturer and the retailer, respectively, are

$$\pi_m^M = \frac{\alpha(N - c)^2}{4(\alpha + 2\delta)} \quad \text{and} \quad \pi_r^M = \frac{1}{2} \pi_m^M. \quad (25)$$

Comparing these profits to those with the open-loop and feedback forward-looking pricing rules leads to the following proposition.

**PROPOSITION 5 (BENEFIT FROM MYOPIC PRICING).** *In terms of long-run profitability, both the manufacturer and the retailer are better off when they are myopic instead of forward looking in deciding prices. Moreover, with myopic pricing, the decentralized supply chain performs at the optimal level (i.e.,  $\pi_m^M + \pi_r^M = \pi^F$ ) when  $\alpha = 4\delta$ .*

To explain the intuition underlying this surprising result, Figure 2 plots the pricing trajectories under different scenarios. On one hand, because of the saturation effect, the future marginal benefit of selling one more unit is negative. As Figure 2(a) shows, in failing to accommodate this effect, myopic behavior places a downward pressure on prices and results in the equilibrium wholesale price  $\tilde{w}^M$  being lower than the optimal pricing path. On the other hand, Figure 2(b) illustrates that the effect of double marginalization forces the myopic price path  $\tilde{p}^M$  to move upward above  $\tilde{w}^M$ . The two effects, counteracting each other, cause  $\tilde{p}^M$  to remain in the vicinity of the optimal pricing path in equilibrium. Consequently, the myopic decentralized supply chain is economically more efficient than the forward-looking one. Figure 3 illustrates

Figure 2 Pricing Trajectories

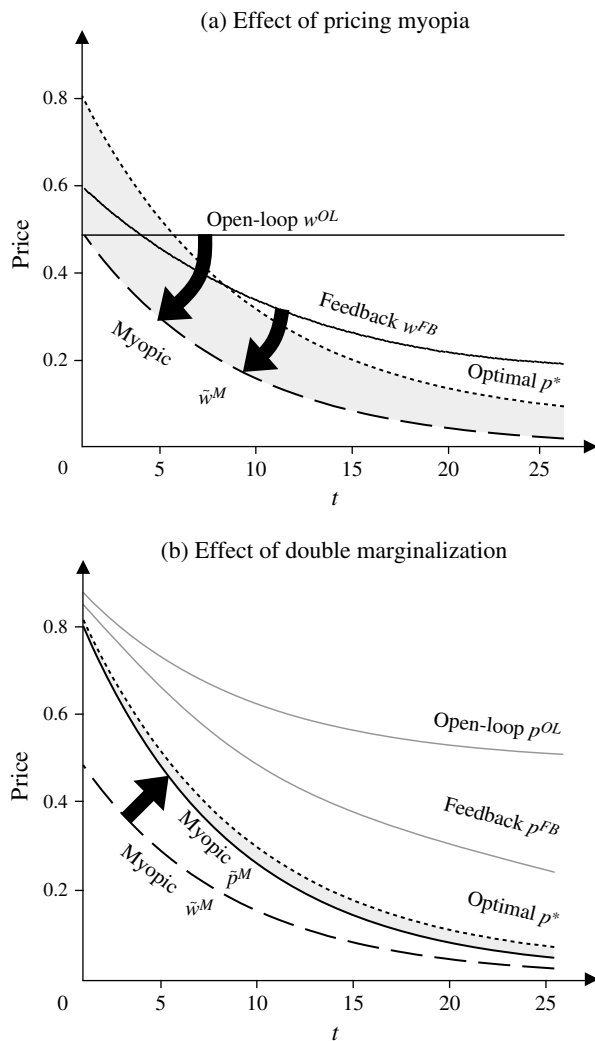
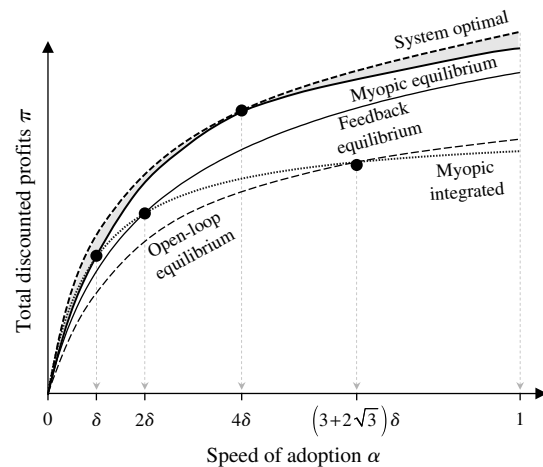


Figure 3 Impact of Speed of Adoption on Net Discounted Profits



As mentioned in §1, in many situations managers are compelled to engage in myopic management. Proposition 6 suggests that when managers are more focused on short-term profits than the firm would like, strategic decentralization (by establishing a transfer pricing mechanism in the context of a multidivisional organization) may enhance supply chain profitability. If focusing on the short term is a reactive behavior, one may conjecture that after the supply chain is strategically decentralized, managers might not have incentives to remain short-term focused. Our results show that even if the decision makers become long-term focused after decentralization, the myopic integrated supply chain can still benefit from decentralization. As Figure 3 illustrates, this occurs when  $\alpha > (3+2\sqrt{3})\delta$  in the open-loop equilibrium, and when  $\alpha > 2\delta$  in the feedback equilibrium. In general, past research has shown how strategic decentralization can mitigate interbrand (McGuire and Staelin 1983) and intrabrand (Arya and Mittendorf 2006) competition. We add another upside of decentralization by showing that it can also mitigate intertemporal competition.

## 6. To Intermediate or Disintermediate?

The primary role of an intermediary is to facilitate product diffusion by offering specific sales skills (including brand building and product promotion), providing service, gathering market information, making access available to customers with special locations, and so forth. For example, as a major retailer of consumer electronics, Best Buy has the marketing expertise and service competency to help Sony better demonstrate and promote the unique features of its 3D TV. Best Buy can also enhance customers' perceived value of the TV by integrating it into a home theater system. Nevertheless, in selling through an

the impact of the speed of adoption  $\alpha$  on supply chain performance under different supply chain structures and pricing rules. Interestingly, in some special situations where  $\alpha = 4\delta$ , the myopic decentralized supply chain performs at the system's optimal level in terms of long-run profitability.

There is another interesting result we can explore by comparing performance under different supply chain structures: If the speed of adoption is higher than the discount rate, the myopic decentralized supply chain is economically more efficient than the myopic integrated supply chain. This result yields an important managerial implication highlighted in the proposition below.

**PROPOSITION 6 (STRATEGIC DECENTRALIZATION).** *With myopic pricing, the net discounted profit of the decentralized supply chain is higher than that of the integrated one when the speed of adoption is higher than the discount rate (i.e.,  $\pi_m^M + \pi_r^M > \pi^M$  if  $\alpha > \delta$ ). That is, decentralization might improve profitability if the integrated supply chain acts myopically.*

intermediary, besides the fact that the intermediary appropriates a certain portion of market revenue, it is typically challenging to align the best interests of independent channel members. Whether it would be more lucrative for a manufacturer to disintermediate is a pertinent question. In this section, we address this question by providing insights into the conditions for disintermediation based on the speed of adoption associated with the manufacturer's competence to engage in direct sales.

Indeed, manufacturers may bypass intermediaries and engage in direct sales through their own outlet stores or via Internet marketing. However, if they cannot reach a reasonable speed of sales, removing intermediaries will lead to an erosion of profits. Recall that in our model, the speed of adoption  $\alpha$  is the likelihood that a customer will purchase the product in a given period. Essentially this parameter reflects the competence of a given channel to distribute a particular product to a market. Given that the value of  $\alpha$  may vary depending on which sales channel the product is distributed through, we now distinctly define  $\alpha_m$  and  $\alpha_r$  as the respective speeds of adoption when the product is distributed through the manufacturer's direct sales channel and the retail store. In terms of  $\alpha_m$  and  $\alpha_r$ , the conditions under which the manufacturer is better off eliminating the retailer can be analytically established.

**PROPOSITION 7 (DISINTERMEDIATION CONDITIONS).** *There exists a threshold  $\theta_{(i,j)}$  such that if  $\alpha_m > \theta_{(i,j)}$ , it is better off for the manufacturer with pricing rule  $i$  to disintermediate its retailer with pricing rule  $j$ , where  $i, j \in \{F, M\}$ ;  $F$  denotes forward looking and  $M$  symbolizes myopic. The relevant thresholds are characterized by (i)  $\theta_{(M,F)}^{OL} < \theta_{(F,F)}^{OL} < \alpha_r/2$  and  $\theta_{(F,M)}^{OL} = \theta_{(M,M)}^{OL} = \alpha_r/2$  in the case of open-loop equilibrium; and (ii)  $\theta_{(F,F)}^{FB} < \theta_{(M,F)}^{FB} < \alpha_r/2$  and  $\theta_{(F,M)}^{FB} < \theta_{(M,M)}^{FB} = \alpha_r/2$  in the case of feedback equilibrium.*

This proposition can be proven by applying the results of Table 1, which summarizes all possible outcomes of net discounted profits when each channel member alternatively adopts different pricing rules (more details about the proof can be found in the online appendix, which is available at <http://msom.journal.informs.org/>). In line with Proposition 7, the intermediation conditions for various scenarios are juxtaposed in Figure 4. In the case of open-loop equilibrium, the result shows that when the retailer is myopic, the disintermediation condition is not affected by the discount rate. On the other hand, when the retailer is forward looking, it can be verified that both  $\theta_{(F,F)}^{OL}$  and  $\theta_{(M,F)}^{OL}$  increase with discount rate  $\delta$ . Moreover, whereas  $\theta_{(F,F)}^{OL}$  is always increasing in  $\alpha_r$  (the speed of adoption through the retailer), we find that  $\theta_{(M,F)}^{OL}$  may decrease with  $\alpha_r$  when  $\alpha_r > 2\delta(\sqrt{2} + 1)$ . Table 1(a) indicates that each supply chain entity can appropriate more profits at the expense of the other partner when it is forward looking and its partner is myopic. Hence, when faced with a myopic retailer, the thresholds for disintermediation are higher than when faced with a forward-looking retailer. When  $\theta_{(M,F)}^{OL} < \alpha_m < \theta_{(F,F)}^{OL}$ , only the myopic manufacturer will bypass the retailer. In such situations, rather than getting rid of the middleman, the manufacturer would be better off motivating the myopic manager to become forward looking through appropriate incentives. Different from open-loop pricing, Table 1(b) reveals that the worst possible outcome of the feedback equilibrium for the manufacturer and the retailer is when the managers of both are forward looking. Thus, regardless of the retailer's pricing behavior, the forward-looking manufacturer has a lower disintermediation threshold than the myopic manufacturer. Also, there are circumstances ( $\theta_{(F,F)}^{FB} < \alpha_m < \theta_{(M,F)}^{FB}$  or  $\theta_{(F,M)}^{FB} < \alpha_m < \theta_{(M,M)}^{FB}$ ) under which only the forward-looking manufacturer will do away with the intermediary.

**Table 1** Summary of Profit Outcomes with Alternative Pricing Rules

| Pricing rule <sup>a</sup> |          | (a) Open-loop equilibrium <sup>b, c, d</sup>                  |   | (b) Feedback equilibrium <sup>b, c, d</sup>  |                   |
|---------------------------|----------|---|---|--|-------------------|
| Manufacturer              | Retailer | Manufacturer  | Retailer  | Manufacturer   | Retailer          |
| F                         | F        | $\pi_1 \triangleq \frac{Z}{4}(1 + S - \sqrt{S^2 + 2S})$       | $\frac{\pi_1}{2}$   | $\pi_3 \triangleq \frac{Z}{3}\left(1 - \frac{4S + 2\sqrt{4S^2 + 6S}}{3}\right)$        | $\frac{\pi_3}{2}$ |
| F                         | M        | $\frac{Z}{2}(\Lambda - 2\sqrt{S^2 + S})$                      | $\frac{ZS}{2}\left(\frac{\Lambda}{2\sqrt{S^2 + S}} - 1\right)$            | $\pi_4 \triangleq \frac{Z}{3}\left(1 + S - \frac{2S^2 + 3S}{2\sqrt{S^2 + S}}\right)$   | $\frac{\pi_4}{2}$ |
| M                         | F        | $\frac{ZS}{2}\left(\frac{\Lambda}{\sqrt{S^2 + S}} - 2\right)$ | $\frac{Z}{2}\left(\Lambda + S - \frac{6S^2 + 5S}{2\sqrt{S^2 + S}}\right)$ | $\pi_5 \triangleq \frac{Z}{3}\frac{\Lambda^2 + (1 + S)(1 - \sqrt{8S\Lambda})}{3S + 2}$ | $\frac{\pi_5}{2}$ |
| M                         | M        | $\pi_2 \triangleq \frac{Z}{4\Lambda}$                         | $\frac{\pi_2}{2}$   | $\pi_2$  | $\frac{\pi_2}{2}$ |

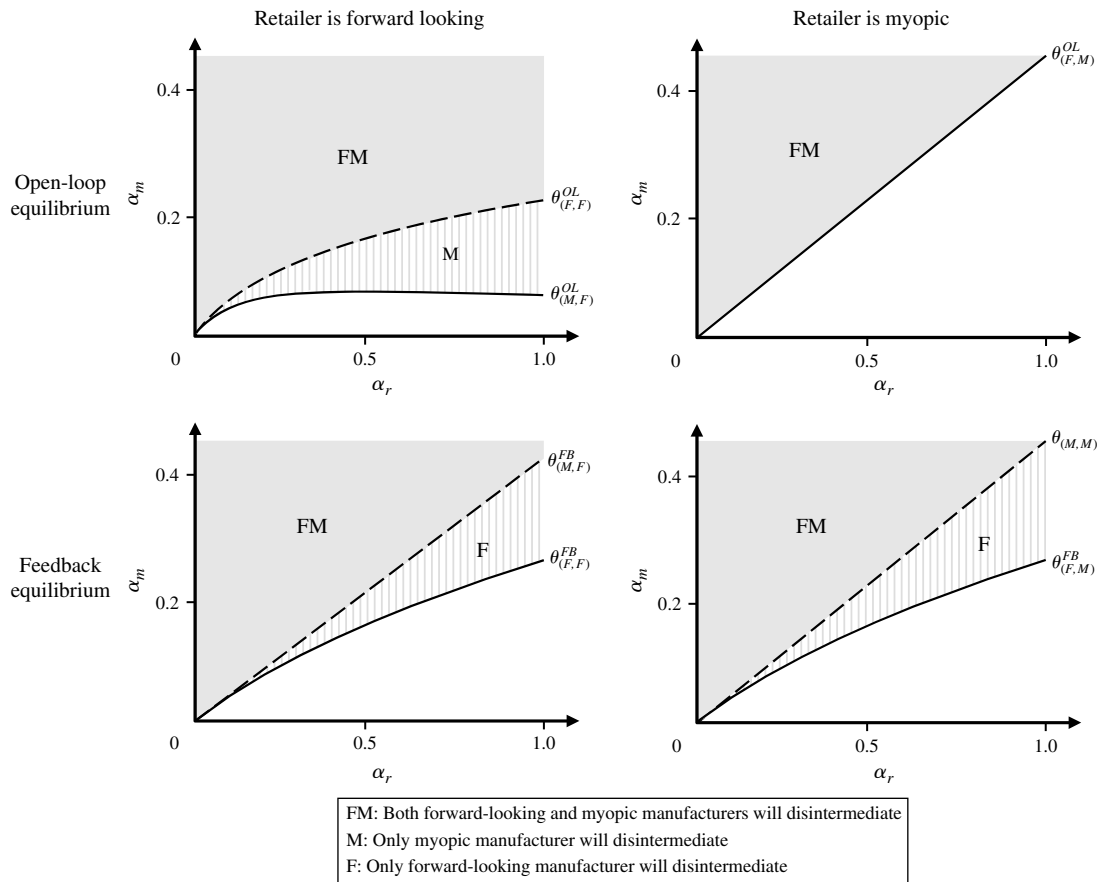
<sup>a</sup>F, forward looking; M, myopic.

<sup>b</sup> $Z = (N - c)^2$ .

<sup>c</sup> $S = \delta/\alpha$ .

<sup>d</sup> $\Lambda = 1 + 2S$ .

Figure 4 Disintermediation Conditions



Note.  $\alpha_m$  = speed of adoption through the manufacturer's direct channel;  $\alpha_r$  = speed of adoption through the retailer.

## 7. Model Extensions: A Numerical Study

One of the main findings of this study is that myopic pricing helps to improve channel efficiency. At the cost of compromising generality, this intriguing result is derived from an analytically tractable model with fundamental assumptions concerning supply and demand dynamics. These assumptions include the absence of the cost learning effect, supply capacity constraints, the imitation effect, and the reference price effect. Although it is reasonable to infer that qualitative insights will extend when these effects are not strong enough, could the applicability of this result be justified when the validity of the model assumptions trumps the tractability of the analysis? In this section, we relax model assumptions by independently and collectively incorporating additional factors and relevant effects into the analysis. To the extent possible, we seek to test the robustness of the major finding through a numerical study.

### 7.1. Supply-Side Effects

**7.1.1. Cost Learning Effect.** The cost learning effect, also known as the learning curve effect or the

experience curve effect, is evident in various industries. To accommodate this effect in our model, we adopt a well-recognized exponential learning curve to capture the essence that unit production cost declines as cumulative output increases (Spence 1981). In particular, we replace the constant unit production cost  $c$  in the original model with the following time-dependent cost function:

$$c(t) = c_0 + c_1 e^{-\Lambda x(t)}, \quad (26)$$

where  $c_0$ ,  $c_1$ , and  $\Lambda$  are nonnegative constants. Clearly, the higher the value of  $\Lambda$ , the greater the cost learning effect. The effect is absent when  $\Lambda = 0$ .

**7.1.2. Supply Capacity Constraint.** Ignoring the capacity constraint that a firm may encounter in practice, our model assumes that the supply quantity is in accordance with the demand rate, which is influenced by the retail price at each point of the diffusion process. Now we consider the case where the supply quantity is constrained by a given capacity limit  $K$  during the entire time. Specifically, the following constraint is included in the analysis:

$$\dot{x}(t) \leq K. \quad (27)$$



Note that in modeling new product diffusion under supply constraints, Ho et al. (2002) and Kumar and Swaminathan (2003) examine the inventory-related decisions of a monopolist. Unlike these two studies wherein the price effect is absent, we do not consider inventories and backlogs as the product is produced on a make-to-order basis in our setting and prices can be dynamically adjusted such that the demand rate does not exceed the capacity limit. Although we limit our analysis to the make-to-order setting, it should be noted out that holding inventory, which would otherwise not be preferable, could potentially be part of an optimal strategy under a capacity constraint.

## 7.2. Demand-Side Effects

**7.2.1. Imitation Effect.** The initial model of this paper assumes that the likelihood of an individual adoption per time unit is a constant  $\alpha$ , and the resulting diffusion process resembles the pure innovation curve of Fourt and Woodlock (1960). In the presence of the imitation effect, however, the likelihood of an individual adoption is correlated with the previous number of purchasers. To accommodate this effect, we now assume that the probability that an untapped customer will adopt at each time instance is a linear function of the tapped customers  $x(t)$ , instead of being a constant  $\alpha$ . Hence, the model of demand dynamics in (6) extends to

$$\dot{x}(t) = \left( \alpha + \beta \frac{x(t)}{N} \right) (N - x(t) - p(t)), \quad (28)$$

where  $\beta$  is the coefficient of imitation. Obviously, without the price effect, this demand model corresponds to the well-known Bass model.

**7.2.2. Reference Price Effect.** A reference price is a price benchmark formed by customers based on their perception of past prices. With an exponentially decaying weighting function (see Fibich et al. 2003), the reference price over time, denoted by  $r(t)$ , can be described by

$$\dot{r}(t) = \kappa(p(t) - r(t)), \quad (29)$$

where  $\kappa \geq 0$  and a higher  $\kappa$  implies that customers have a shorter term price memory. In the relevant literature, the impact of the reference price on consumer willingness to buy is typically modeled as an additive linear function of the difference between  $p(t)$  and  $r(t)$ , such that demand is discouraged if  $p(t) > r(t)$ , and vice versa. Following similar concepts to include the reference price effect, we develop the diffusion process in (28) into

$$\dot{x}(t) = \left( \alpha + \beta \frac{x(t)}{N} \right) [N - x(t) - (p(t) + \Omega(p(t) - r(t)))], \quad (30)$$

where  $\Omega$  is a nonnegative parameter that captures the reference price effect. A higher  $\Omega$  implies that consumers are more sensitive to the gap between the two prices. When customers are reactive only to the current price (i.e.,  $\Omega = 0$ ), the model degenerates to (28).

## 7.3. The Numerical Study

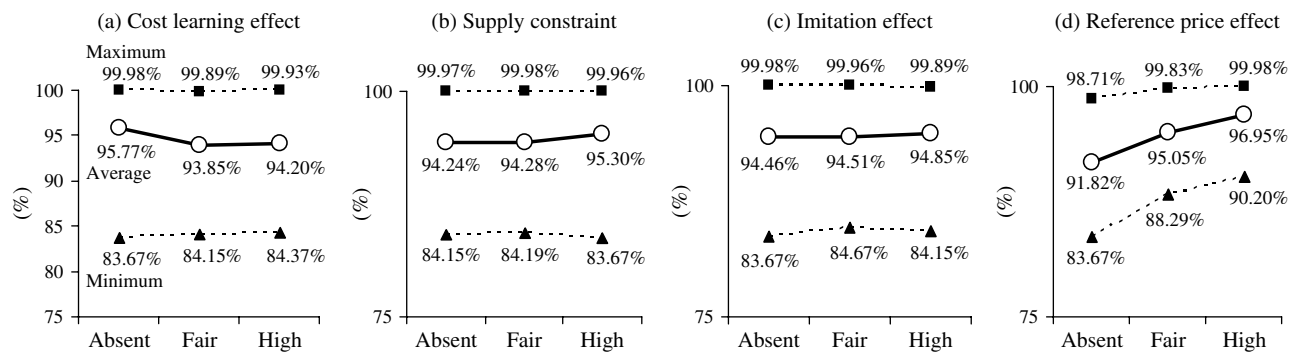
Following the extended model that accommodates additional effects, we perform the numerical study. For clarity of presentation, we generate a report from a set of parametric values that have been carefully designed to try to provide a more comprehensive and representative picture. Specifically, for those parameters reflecting a certain effect of concern, we scrutinize the sensitivity of the effect by varying the parametric values in the realistically attainable range by three distinct levels: {absent, fair, high}. To avoid distracting the focus, we assign a single default value, as specified below, to each of the remaining parameters that have little or no impact on the insights of the main analysis:  $N = 100$ ,  $c_0 = 0$ ,  $c_1 = 20$ ,  $\alpha = 0.1$ ,  $\kappa = 0.5$ . Accordingly, we end up studying 243 cases formed by all possible combinations of the following:

$$\delta = \{0.05, 0.1, 0.15\}, \quad \Lambda \in \{0, 0.05, 0.1\}, \quad K \in \{\infty, 4, 3\}, \\ \beta \in \{0, 0.1, 0.2\}, \quad \Omega \in \{0, 0.25, 0.5\}.$$

For each case, we solve both the optimal pricing of a monopolist and the equilibrium myopic pricing of a decentralized channel. We then compute the ratio of the corresponding discounted profit of the later to that of the former. This ratio, known as channel efficiency, measures the degree to which the total decentralized channel profit reaches the optimal level of a vertically integrated channel. The numerical analysis, carried out using Matlab®, involves repetitively constructing and solving systems of nonlinear differential equations (the optimality conditions of the corresponding problems and the computational result of the numerical study are detailed in the online appendix). Figure 5 summarizes the computational result.

As expected, the result indicates that the emergence of various effects may increase or decrease channel efficiency with myopic pricing. Although there is no discernible trend in the efficiency pattern, ranging from 83.67% to 99.98%, the channel efficiency of the 243 observed cases is 94.61% on average, which is conspicuously high. It should be pointed out that more than half of the observed cases achieve efficiency of 95% or above, whereas nearly one third possess an efficiency of less than 2% below the optimal level. To a certain extent, although the intricacy of the joint dynamic effects makes it difficult to generalize the impact of myopic pricing, the numerical evidence presented here extends the robustness of our major result: myopic pricing could lead to high channel efficiency in the decentralized supply chain.

Figure 5 Channel Efficiency with Pricing Myopia in the Presence of Various Effects



This computational result is valuable in the following sense. Ignoring dynamic effects, most studies in the supply chain coordination literature focus on the contracts that help to improve channel efficiency based on the one-shot (short-term focused) interaction of self-interested agents. Such a static approach in the long run corresponds to a sequence of one-shot games, where the channel members optimize their profits of each period, disregarding possible dynamic effects. The resulting channel efficiency thus corresponds to that of the myopic equilibrium in our model. In contrast to the typical result from the static analysis of double marginalization, which claims an efficiency of only 75%, our result indicates that a supply chain may already perform at or close to optimal efficiency from a long-term perspective. This poses a caveat to previous studies, where the proposed coordination contracts should be implemented with caution, as the solutions may have been carried too far with underestimated channel efficiencies.

## 8. Concluding Remarks

When a manufacturer and its retailer are involved in setting prices dynamically while distributing a durable product over time, not only do they intertemporally compete against themselves, but they also compete against each other to maximize their benefits. To investigate the impact of various dynamic pricing rules on the channel efficiency in such a context, this study develops a game-theoretical model that extends the traditional analysis of the double-marginalization problem to an intertemporal setting. Although the analysis is a priori complex, our model captures the essence of the problem and yields closed-form results, on the basis of which we analytically generate a number of relevant insights that have never been identified or formally clarified.

Prior research reveals that managers in many situations are compelled to engage in myopic management by replacing decisions that produce superior future profits with those that generate an immediate payback. Although short-term focused behavior

is typically considered detrimental from a long-term perspective, we provide a counter view. We show that firms in a supply chain can benefit from myopic pricing when distributing durable goods. The reason is that the future marginal gain of selling one additional unit is negative because of the saturation effect. Myopic pricing, failing to accommodate this effect, causes prices to be lower than the optimal level. Ironically, the “too-low” prices are counterbalanced by the “too-high” prices caused by double marginalization. The resulting equilibrium pricing path thus closely matches the optimal one. The robustness of this surprising outcome is further extended by a numerical study in the presence of various effects, including those of cost learning, capacity constraint, word-of-mouth, and reference price. The main implication here is that coordination contracts proposed by prior studies to remedy channel inefficiency should be implemented with caution. With intertemporal interactions, a supply chain may already perform at or close to its optimal efficiency because of bounded rationality.

Looking at the insights from a different slant, our further analysis indicates that strategic decentralization can possibly lead to a higher profit when price managers are more focused on the short-term profits. Past research has shown how decentralization can mitigate interbrand (McGuire and Staelin 1983) and intrabrand (Arya and Mittendorf 2006) competition. We show that decentralization can also alleviate intertemporal competition. In selling through an intermediary, it is often challenging to align the best interests of independent channel members. To eliminate the inefficiency caused by vertical price competition, the most dramatic structural change that can be made is disintermediation, wherein a manufacturer bypasses its retailer and sells directly to the customers. We demonstrate that a manufacturer’s incentive to disintermediate may vary with respect to the pricing rule adopted by its retailer.

As in most diffusion-based pricing models, customers in our model respond only to current prices. Although our further numerical study considers the

case where customers may react to past prices (reference price effect), what happens if they perceive future consumption benefits and act strategically? In reality, customers are unlikely to be exclusively reactive or utterly strategic. Thus, our model serves as an approximation when customers are more reactive. When customers are more strategic, demand is expected to be more price elastic than it is among reactive customers (Besanko and Winston 1990). In this case, we postulate that our main implications remain unchanged when the speed of adoption and/or the discount rate are relatively low. Under these circumstances, the equilibrium prices will not decrease dramatically. As it is pessimistic to expect a rapid price drop in the future, the effect of strategic behavior will not be significant. Certainly, in the absence of a formal analysis, not much can be inferred beyond these conjectures.

Our model is limited in several other respects. For example, we derive the dynamic demands based on the assumptions that the customers' reservation prices for the product are uniformly distributed and that the demand arrivals are deterministic with no repeat buying. Moreover, our model, which considers only a single product, assumes that the speed of adoption is not endogenous to any variable that a firm may have control over, such as advertising expenditures and inventory decisions. Relaxing any of these assumptions could possibly lead to changes in our findings. We hope that the groundwork developed in this study will foreshadow future research extensions that overcome our limitations and bring out additional insights and implications to contextualize this challenging subject with a more diverse spectrum of considerations.

### Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at <http://msom.journal.informs.org/>.

### Acknowledgments

The author thanks the review team, James D. Hess, and Grace Kiang for their valuable comments and constructive suggestions. The exposition of this paper, in particular, has substantially benefited from the associate editor's thoughtful guidance. This research is supported, in part, by a grant from the City University of Hong Kong [Project 7200127].

### Appendix A. Justification of Price Skimming Without Resale Markets

In this section we show that, in absence of resale markets, prices cannot increase in an optimal pricing policy, so Equation (3) remains valid even if the firm considers nonmonotone pricing strategies. Note that after backward substituting the retailer's price reaction, the demand dynamics of the manufacture's control problem in the decentralized supply chain should have an analogous structure to that of the integrated one. Thus, we infer that the

nonincreasing pricing result extends when the supply chain is not integrated, though we only focus on the proof of the proposition below.

**PROPOSITION A1.** *If the pricing policy  $\mathbf{p} = (p_1, p_2, \dots, p_n, p_{n+1}, \dots)$  maximizes the integrated supply chain profit from this customer base under the dynamics given by (1) and (2), then  $p_{n+1} \leq p_n$  for all  $n \geq 1$ .*

**PROOF.** If price increase(s) are expected, there must be some period in the selling horizon in which the initial increase occurs. We will show by contradiction that this period does not exist in optimal pricing. Let  $\Pi_n(\mathbf{p})$  be the remaining discounted profit looking from period  $n$  with the price vector  $\mathbf{p}$ . Recursively,

$$\Pi_n(\mathbf{p}) = \pi_{(p_n, p_{n+1})} + \rho^2 \Pi_{n+2}(\mathbf{p}), \quad (\text{A1})$$

where  $\pi_{(p_n, p_{n+1})}$  is the discounted profit of periods  $n$  and  $n+1$ , and  $\rho$  is the discount factor. Assuming without loss of generality that the unit cost  $c = 0$  and  $\Delta t = 1$ , we have  $\pi_{(p_n, p_{n+1})} = p_n d_n + \rho p_{n+1} d_{n+1}$ , where according to (1) and (2)

$$\begin{aligned} d_n &= \alpha(N - p_n - y_{n-1}^{[p_n, N]}) \quad \text{and} \\ d_{n+1} &= \begin{cases} \alpha(N - p_{n+1} - y_{n-1}^{[p_{n+1}, N]} - d_n) & \text{if } p_{n+1} \leq p_n, \\ \alpha(1 - \alpha)(N - p_{n+1} - y_{n-1}^{[p_{n+1}, N]}) & \text{otherwise.} \end{cases} \end{aligned} \quad (\text{A2})$$

Let  $\chi_{(p_n, p_{n+1})}^{[a, b]}$  denote the amount of customers with valuations between  $a$  and  $b$  who have adopted in periods  $n$  and  $n+1$  with  $p_n$  and  $p_{n+1}$ . The lemma below, which will facilitate the proof, is based on the argument that a price set for any given two consecutive periods cannot be optimal if there exists another price set that yields a higher two-period discounted profit with weakly fewer buyers for each valuation segment.

**LEMMA A1.** *If  $(p_n, p_{n+1}) = (p, p + \varepsilon)$  is optimal for some  $p, \varepsilon > 0$ , then, for any  $p'_n, p'_{n+1} \geq p$ ,  $\pi_{(p'_n, p'_{n+1})} > \pi_{(p, p + \varepsilon)}$  implies at least one of the followings is true: (i)  $\chi_{(p'_n, p'_{n+1})}^{[p, p + \varepsilon]} > \chi_{(p, p + \varepsilon)}^{[p, p + \varepsilon]}$ , (ii)  $\chi_{(p'_n, p'_{n+1})}^{[p + \varepsilon, N]} > \chi_{(p, p + \varepsilon)}^{[p + \varepsilon, N]}$ .*

**PROOF.** Let  $\mathbf{p}^* = (p_1^*, p_2^*, \dots, p_n^* = p, p_{n+1}^* = p + \varepsilon, p_{n+2}^*, \dots)$  be the optimal pricing. Consider a price vector  $\mathbf{p}'$  with  $p_i = p_i^*$  if  $i < n$  and  $p_i = p'_i$  if  $i \geq n$ , and

$$\mathbf{p}' \in \arg \max \Pi_{n+2}(\mathbf{p})|_{(p_1, \dots, p_{n+1}) = (p_1^*, p_2^*, \dots, p'_n, p'_{n+1})}.$$

If  $\pi_{(p'_n, p'_{n+1})} > \pi_{(p, p + \varepsilon)}$ , then from (A1) we must have  $\Pi_{n+2}(\mathbf{p}^*) > \Pi_{n+2}(\mathbf{p}')$ . The result follows with  $p'_n, p'_{n+1} \geq p$ , since  $\Pi_{n+2}(\mathbf{p}^*) \leq \Pi_{n+2}(\mathbf{p}')$  if  $\chi_{(p'_n, p'_{n+1})}^{[p, p + \varepsilon]} \leq \chi_{(p, p + \varepsilon)}^{[p, p + \varepsilon]}$  and  $\chi_{(p'_n, p'_{n+1})}^{[p + \varepsilon, N]} \leq \chi_{(p, p + \varepsilon)}^{[p + \varepsilon, N]}$ .  $\square$

Suppose that the initial price increase occurs in period  $n+1$  for some  $n \geq 1$ . We must then have  $p_n^* < p_{n+1}^*$ , which implies that  $(p_n^*, p_{n+1}^*) = (p, p + \varepsilon)$  for some  $p, \varepsilon > 0$ . To conclude the result, we now prove by contradiction that this is not possible. Based on (A2), it can be verified that

$$\pi_{(p + \varepsilon, p + \varepsilon)} - \pi_{(p, p + \varepsilon)} = \alpha(\Theta_1 - \Theta_2), \quad (\text{A3})$$

$$\pi_{(p, p)} - \pi_{(p, p + \varepsilon)} = -\rho\alpha(1 - \alpha)(\Theta_1 - \Theta_2), \quad (\text{A4})$$

where  $\Theta_1 = \varepsilon(N - (p + \varepsilon) - y_{n-1}^{[p+\varepsilon, N]}) > 0$  and  $\Theta_2 = p(\varepsilon - y_{n-1}^{[p+\varepsilon, N]}) > 0$ . Given that  $\chi_{(p+\varepsilon, p+\varepsilon)}^{[p+\varepsilon, N]} - \chi_{(p, p+\varepsilon)}^{[p+\varepsilon, N]} = 0$  and  $\chi_{(p+\varepsilon, p+\varepsilon)}^{[p+\varepsilon, N]} - \chi_{(p, p+\varepsilon)}^{[p+\varepsilon, N]} = -\alpha(\varepsilon - y_{n-1}^{[p+\varepsilon, N]}) < 0$ , by Lemma A1, in order for  $(p, p + \varepsilon)$  to be optimal, we must have  $\pi_{(p, p+\varepsilon)} > \pi_{(p+\varepsilon, p+\varepsilon)}$ . Accordingly, (A3) implies that  $\Theta_1 < \Theta_2$  and it follows from (A4) that  $\pi_{(p, p)} > \pi_{(p, p+\varepsilon)}$ . Since  $\pi_{(p, p)} > \pi_{(p, p+\varepsilon)} > \pi_{(p+\varepsilon, p+\varepsilon)}$ , there exists  $\hat{p} \in [p, p + \varepsilon]$  such that  $\pi_{(\hat{p}, \hat{p})} \geq \pi_{(p, p+\varepsilon)}$ . Based on (A2) again, we can show that

$$\pi_{(\hat{p}, \hat{p})} - \pi_{(p+\varepsilon, p+\varepsilon)} = \alpha(\rho(1 - \alpha) + 1)(\Theta_1 - \Theta_3), \quad (\text{A5})$$

$$\pi_{(p+\varepsilon, \hat{p})} - \pi_{(\hat{p}, \hat{p})} = \alpha(\Theta_1 - \Theta_3) + \rho\alpha^2\Theta_3, \quad (\text{A6})$$

where  $\Theta_3 = p(\varepsilon - y_{n-1}^{[\hat{p}, p+\varepsilon]}) > 0$ . Since  $\pi_{(\hat{p}, \hat{p})} \geq \pi_{(p, p+\varepsilon)} > \pi_{(p+\varepsilon, p+\varepsilon)}$ , (A5) implies  $\Theta_1 \geq \Theta_3$ . Accordingly, from (A6), we must have  $\pi_{(p+\varepsilon, \hat{p})} > \pi_{(\hat{p}, \hat{p})}$ , which, given that  $\pi_{(\hat{p}, \hat{p})} \geq \pi_{(p, p+\varepsilon)}$ , leads to  $\pi_{(p+\varepsilon, \hat{p})} > \pi_{(p, p+\varepsilon)}$ . Since  $\chi_{(p+\varepsilon, \hat{p})}^{[p+\varepsilon, N]} - \chi_{(p, p+\varepsilon)}^{[p+\varepsilon, N]} = 0$  and  $\chi_{(p+\varepsilon, \hat{p})}^{[p+\varepsilon, N]} - \chi_{(p, p+\varepsilon)}^{[p+\varepsilon, N]} = -\alpha((\hat{p} - p) - y_{n-1}^{[p, \hat{p}]}) < 0$ , Lemma A1 implies that  $(p_n, p_{n+1}) = (p, p + \varepsilon)$  cannot possibly be optimal.  $\square$

## Appendix B. Proof of Proposition 2 (Open-Loop Equilibrium)

Based on Pontryagin's maximum principle, we will first derive the retailer's best price response to the manufacturer's price decision. After backward substituting the retailer's response into the manufacturer's control problem, the equilibrium will then correspond to the solution of the problem. Specifically, the current value Hamiltonian for the retailer's problem is  $H_r(x, p) = (p - w + \lambda)\dot{x}$ , which yields

$$\dot{x} = -\alpha(x - \lambda - N + w)/2 \quad \text{and} \quad \dot{\lambda} = \dot{x} + \delta\lambda. \quad (\text{B1})$$

The optimality conditions in (B1), obtained with a similar approach in Online Appendix I, characterize the retailer's best reaction to the manufacturer's wholesale price decision. Treating the shadow price  $\lambda$  as a state variable, the current value Hamiltonian for the manufacturer's optimization problem is defined as

$$H_m(w, x, \lambda, \psi, \mu) = (w - c + \psi)\dot{x} + \mu\dot{\lambda}, \quad (\text{B2})$$

where  $\psi$  and  $\mu$  are the shadow prices associated with  $x$  and  $\lambda$ , respectively. Plugging (B1) into (B2), we obtain  $H_m = (w - c + \psi)(-x + \lambda + N - w)\alpha/2 + \mu(-x + (1 + 2\delta/\alpha)\lambda + N - w)\alpha/2$ , which yields

$$w = (N + c - x + \lambda - \mu - \psi)/2, \quad (\text{B3})$$

and the corresponding optimality conditions

$$\dot{\psi} = (\alpha/4 + \delta)\psi + (\alpha/4)(\mu - x + \lambda + N - c), \quad (\text{B4})$$

$$\dot{\mu} = -(\alpha/4)(\psi + \mu - x + \lambda + N - c), \quad (\text{B5})$$

$$\dot{x} = (\alpha/4)(\psi + \mu - x + \lambda + N - c), \quad (\text{B6})$$

$$\dot{\lambda} = (\alpha/4)(\psi + \mu - x + N - c) + (\delta + \alpha/4)\lambda. \quad (\text{B7})$$

It can be inferred from (B5) and (B6) that  $\dot{\mu} = -\dot{x}$ . Thus, the solution must have  $\mu = -x + k$ , where  $k$  is an arbitrary constant. Substituting it into (B4), (B6), and (B7), we obtain the following system of differential equations whose Jacobian

matrix is nonsingular:

$$\begin{bmatrix} \dot{\psi} \\ \dot{\mu} \\ \dot{x} \\ \dot{\lambda} \end{bmatrix} = \mathbf{A} \begin{bmatrix} \psi \\ \mu \\ x \\ \lambda \end{bmatrix} + \mathbf{b}, \quad \text{where}$$

$$\mathbf{A} = \frac{\alpha}{4} \begin{bmatrix} \frac{\alpha + 4\delta}{\alpha} & 0 & -2 & 1 \\ -1 & -1 & 1 & -1 \\ 1 & 0 & -2 & 1 \\ 1 & 0 & -2 & \frac{\alpha + 4\delta}{\alpha} \end{bmatrix} \quad \text{and}$$

$$\mathbf{b} = \frac{\alpha}{4} \begin{bmatrix} k + N - c \\ -N + c \\ k + N - c \\ k + N - c \end{bmatrix}. \quad (\text{B8})$$

The eigenvalues of  $\mathbf{A}$  and the corresponding matrix of eigenvectors of  $\mathbf{A}$  are given by

$$\begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\delta - \frac{1}{2}\sqrt{\delta^2 + 2\alpha\delta} \\ \frac{1}{2}\delta + \frac{1}{2}\sqrt{\delta^2 + 2\alpha\delta} \\ \delta \\ -\frac{1}{4}\alpha \end{bmatrix} \quad \text{and}$$

$$\mathbf{H} = \begin{bmatrix} \frac{\alpha + 2r_1}{\alpha} & \frac{\alpha + 2r_2}{\alpha} & -1 & 0 \\ -1 & -1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ \frac{\alpha + 2r_1}{\alpha} & \frac{\alpha + 2r_2}{\alpha} & 1 & 0 \end{bmatrix}. \quad (\text{B9})$$

Following the same method found in Online Appendix I, we have

$$\begin{bmatrix} \psi \\ \mu \\ x \\ \lambda \end{bmatrix} = \mathbf{H} \begin{bmatrix} e^{r_1 t} & 0 & 0 & 0 \\ 0 & e^{r_2 t} & 0 & 0 \\ 0 & 0 & e^{r_3 t} & 0 \\ 0 & 0 & 0 & e^{r_4 t} \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_0 \end{bmatrix} - \mathbf{A}^{-1}\mathbf{b}$$

$$= \begin{bmatrix} k_1 \frac{\alpha + 2r_1}{\alpha} e^{tr_1} + k_2 \frac{\alpha + 2r_2}{\alpha} e^{tr_2} - k_3 e^{tr_3} \\ -k_1 e^{tr_1} - k_2 e^{tr_2} + k_0 e^{tr_4} + \frac{k}{2} - \frac{V - c}{2} \\ k_1 e^{tr_1} + k_2 e^{tr_2} + \frac{k}{2} + \frac{V - c}{2} \\ k_1 \frac{\alpha + 2r_1}{\alpha} e^{tr_1} + k_2 \frac{\alpha + 2r_2}{\alpha} e^{tr_2} + k_3 e^{tr_3} \end{bmatrix}. \quad (\text{B10})$$

Since  $\mu = -x + k$ , we must have  $k_0 = 0$ . Let  $k_4 = k/2$ , then the general solution for (B10) is given by

$$\begin{bmatrix} \psi \\ \mu \\ x \\ \lambda \end{bmatrix} = \begin{bmatrix} \frac{\alpha + 2r_1}{\alpha} e^{tr_1} & \frac{\alpha + 2r_2}{\alpha} e^{tr_2} & -e^{tr_3} & 0 \\ -e^{tr_1} & -e^{tr_2} & 0 & 1 \\ e^{tr_1} & e^{tr_2} & 0 & 1 \\ \frac{\alpha + 2r_1}{\alpha} e^{tr_1} & \frac{\alpha + 2r_2}{\alpha} e^{tr_2} & e^{tr_3} & 0 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ -(N - c) \\ 2 \\ N - c \\ 2 \\ 0 \end{bmatrix}. \quad (\text{B11})$$



The four boundary conditions  $x(0) = 0$ ,  $\mu(0) = 0$ ,  $\lim_{t \rightarrow \infty} e^{-\delta t} \psi(t)x(t) = 0$  and  $\lim_{t \rightarrow \infty} e^{-\delta t} \mu(t)\lambda(t) = 0$  imply  $k_1 = -(N - c)/2$  and  $k_2 = k_3 = k_4 = 0$ . Substituting in (B11), it follows that

$$\begin{aligned}\psi(t) &= \lambda(t) = -\frac{N-c}{2} \left(1 - \frac{2\gamma}{a}\right) e^{-\gamma t} \quad \text{and} \\ \mu(t) &= -x(t) = -\frac{N-c}{2} (1 - e^{-\gamma t}),\end{aligned}\quad (\text{B12})$$

where  $\gamma = -r_1$ . The result in (16) follows immediately after plugging (B12) into (B3) and the retailer's reaction function  $p(w) = (N + w - x - \lambda)/2$ .

### Appendix C. Proof of Proposition 3 (Feedback Equilibrium)

With the feedback equilibrium concept, the two channel members make decisions at each time instance, taking into account the status of the current installed base. Because the nature of the decisions is in the spirit of dynamic programming, as detailed below, the proof will follow the principles of optimality of dynamic programming. Let  $V_r(t, x)$  and  $V_m(t, x)$  denote the value functions for the retailer and the manufacturer, respectively. Given the manufacturer's price decision  $w$ , the retailer's Hamilton–Jacobi–Bellman (HJB) equation can be specified as

$$\delta V_r = \max_p \left[ \left( (p - w) + \frac{\partial V_r}{\partial x} \right) \alpha(N - x - p) \right]. \quad (\text{C1})$$

The maximization with respect to  $p$  yields the retailer's instantaneous reaction function:

$$p = (N + w - x - \partial V_r / \partial x) / 2. \quad (\text{C2})$$

Anticipating the retailer's response in (C2), the manufacturer's HJB equation is given by

$$\delta V_m = \max_w \left[ \left( (w - c) + \frac{\partial V_m}{\partial x} \right) \alpha(N - x - p) \right], \quad (\text{C3})$$

which yields below the optimal feedback wholesale price decision for the manufacturer:

$$w = (N - x + c + \partial V_r / \partial x - \partial V_m / \partial x) / 2. \quad (\text{C4})$$

Substituting (C4) into (C2) produces

$$p = (3(N - x) + c - \partial V_r / \partial x - \partial V_m / \partial x) / 4. \quad (\text{C5})$$

Subject to (C4) and (C5), the feedback equilibrium corresponds to the solution of the system of the partial differential equations in (C1) and (C3). Substituting (C4) and (C5) into (C1) and (C3) yields

$$\begin{aligned}\delta V_r &= \frac{\theta}{16} \left( N - c - x + \frac{\partial V_r}{\partial x} + \frac{\partial V_m}{\partial x} \right)^2 \quad \text{and} \\ \delta V_m &= \frac{\theta}{8} \left( N - c - x + \frac{\partial V_r}{\partial x} + \frac{\partial V_m}{\partial x} \right)^2.\end{aligned}\quad (\text{C6})$$

Conjecture the following quadratic functions as the solution to (C6):

$$V_i = (A_i/2)x^2 + B_i x + K_i, \quad (\text{C7})$$

where  $i \in \{r, m\}$  and the values of  $A_i$ ,  $B_i$ , and  $K_i$  are to be determined. It follows from (C7) that

$$\partial V_i / \partial x = A_i x + B_i. \quad (\text{C8})$$

After substituting (C8) into (C5) and then into (2), the state equation can be specified as

$$\dot{x} = (\alpha/4)(\Sigma A_i - 1)x + (\alpha/4)(\Sigma B_i + N - c),$$

$$\text{where } \Sigma A_i = A_r + A_m \text{ and } \Sigma B_i = B_r + B_m. \quad (\text{C9})$$

Solving the differential equation in (C9) with the initial condition  $x(0) = 0$  results in

$$x(t) = \frac{-(\Sigma B_i + N - c)}{\Sigma A_i - 1} (1 - e^{(\alpha/4)(\Sigma A_i - 1)t}). \quad (\text{C10})$$

Plugging (C7) and (C8) into (C6) yields

$$\begin{aligned}\frac{\delta}{2} A_r x^2 + \delta B_r x + \delta K_r \\ = \frac{\alpha}{16} (\Sigma A_i - 1)^2 x^2 + \frac{\alpha}{8} (\Sigma A_i - 1)(\Sigma B_i + N - c)x \\ + \frac{\alpha}{16} (\Sigma B_i + N - c)^2,\end{aligned}\quad (\text{C11})$$

$$\begin{aligned}\frac{\delta}{2} A_m x^2 + \delta B_m x + \delta K_m \\ = \frac{\alpha}{8} (\Sigma A_i - 1)^2 x^2 + \frac{\alpha}{4} (\Sigma A_i - 1)(\Sigma B_i + N - c)x \\ + \frac{\alpha}{8} (\Sigma B_i + N - c)^2.\end{aligned}\quad (\text{C12})$$

Equating the coefficients of  $x^2$  on both sides of (C11) and (C12) gives

$$\begin{aligned}\alpha(\Sigma A_i - 1)^2 - 8\delta A_r &= 0, \\ \alpha(\Sigma A_i - 1)^2 - 4\delta A_m &= 0,\end{aligned}\quad (\text{C13})$$

which leads to two possible solutions of  $A_r$  and  $A_m$ . Because  $x(t)$  specified in (C10) has to converge in  $t$ , the solution to (C13) must satisfy  $\Sigma A_i - 1 < 0$ . Only one of the two solutions is eligible. The unique one is

$$A_m = 2A_r = (2/9)(3\theta + 4\delta - 2\sqrt{6\theta\delta + 4\delta^2})/\theta. \quad (\text{C14})$$

Similarly, equating the coefficients of  $x$  on both sides of (C11) and (C12) yields a unique solution

$$\begin{aligned}B_m = 2B_r &= \frac{2\alpha(\Sigma A_i - 1)(N - c)}{8\delta - 3\alpha(\Sigma A_i - 1)} \\ &= -\frac{2}{3} \frac{2\delta - \sqrt{6\theta\delta + 4\delta^2}}{2\delta + \sqrt{6\theta\delta + 4\delta^2}} (N - c).\end{aligned}\quad (\text{C15})$$

It follows from (C14) and (C15) that

$$\Sigma A_i = 1 - 4\phi/\alpha \quad \text{and} \quad \Sigma B_i = -(1 - 4\phi/\alpha)(N - c),$$

$$\text{where } \phi = (\sqrt{6\alpha\delta + 4\delta^2} - 2\delta)/6. \quad (\text{C16})$$

Substituting (C16) into (C10) yields the result in (19). The result in (18) can be obtained by substituting (C14) and (C15) into (C8), and then plugging (C8) and (19) into (C4) and (C5).

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