Management Science

How Do Firms Become Different? A Dynamic Model

Matthew Selove

Please scroll down for article—it is on subsequent pages

How Do Firms Become Different? A Dynamic Model

Matthew Selove

Marshall School of Business, University of Southern California, Los Angeles, California 90089, selove@marshall.usc.edu

This paper presents a dynamic investment game in which firms that are initially identical develop assets that are specialized to different market segments. The model assumes that there are increasing returns to investment in a segment, for example, as a result of word-of-mouth or learning curve effects. I derive three key results: (1) Under certain conditions there is a unique equilibrium in which firms that are only slightly different focus all of their investment in different segments, causing small random differences to expand into large permanent differences. (2) If, on the other hand, sufficiently large random shocks are possible, firms over time repeatedly change their strategies, switching focus from one segment to another. (3) A firm might want to reduce its own assets in the smaller segment in order to entice its competitor to shift focus to this segment.

Keywords: marketing; new products; product policy; competitive strategy
History: Received July 29, 2010; accepted July 6, 2013, by J. Miguel Villas-Boas, marketing. Published online in *Articles in Advance* October 28, 2013.

1. Introduction

Casual observation reveals that even firms in the same industry are different from each other. Each firm has unique assets such as its reputation, relationships, and production skills (Wernerfelt 1984, Prahalad and Hamel 1990, Teece et al. 1997, Dutta et al. 1999). Standard models of product design such as the Hotelling model imply that firms should differentiate to soften price competition (D'Aspremont et al. 1979, Shaked and Sutton 1982). However, such models do not answer two important questions: What initially determines which customer segment each firm serves? And once firms start serving different segments, under what conditions will those differences persist over time, and under what conditions will firms eventually switch back and forth between different segments?

This paper develops a dynamic investment game that proposes answers to these questions. The model assumes that two competing firms each allocate investment across two customer segments of unequal size and that there are increasing returns to investment in a segment, for example, as a result of word-of-mouth effects (Rob and Fishman 2005) or learning curves (Argote and Epple 1990). Small early differences arise between the firms as a result of random shocks such as differences in founders' previous experiences or fortuitous discoveries of better production processes. I derive three key results.

First, I derive conditions that ensure that there is a unique equilibrium in which early random differences cause firms to focus their investment in different segments, eventually leading to permanent differences. This result implies that small random events can have large permanent consequences for

a company's strategy. For example, ice cream maker Ben & Jerry's initially experimented with making ice cream flavors with large chunks of chocolate, cookies, or other sweets because cofounder Ben Cohen's sinus problems made it hard for him to taste normal flavors (Lager 1994). To the surprise of the company's founders, their ice cream with large chunks turned out to be extremely popular with customers. Over the next five years, they modified the machines that dispensed their ice cream to help prevent the chunks from getting stuck and also experimented with different cookie dough mixes until they found one that would not clog their machinery. In part as a result of the production expertise they developed, Ben & Jerry's eventually became the dominant producer of chunky ice cream flavors (Collis and Conrad 2005). In this example, increasing returns arguably arose as a result of asset complementarity. Once Ben & Jerry's acquired (partly through good luck) some skill in *designing* chunky ice cream flavors, it was then profitable for the company to invest in the skills and equipment needed for *producing* these flavors. We can think of such asset complementarity as a source of increasing returns to a firm's overall level of assets devoted to a particular customer segment.[1]

---

[1] Dell Computer is another example of a company that focused on an area of early, arguably random, success. Because college student Michael Dell had only $1,000 in capital when he started assembling and selling personal computers from his college dorm room, he could not maintain a large stock of inventory. Instead, he and three employees custom assembled a computer for each order and then shipped it directly to the customer (Dell 1999). As the company grew, they continued to focus on this build-to-order model, which helped Dell achieve a cost advantage through fast inventory turnover rates (Dell 1999).

My second result is that if sufficiently large random shocks can occur, then over time firms can make major strategic changes, totally shifting focus from one segment to another. For example, Sony successfully drove Nintendo out of the segment of hard-core video game players following the invention (the large shock in this example) of CD-ROMs, an efficient information storage technology in which Sony had expertise and which it used in its PlayStation video game system (*Edge* 2009).[2]

My third result is that a firm might sometimes want to reduce its own assets in the smaller segment in order to entice its competitor to focus investments in this segment. For example, throughout the 1980s, Häagen-Dazs explicitly avoided investing in the modifications to production equipment needed to produce chunky ice cream flavors (Lager 1994). My model implies that even if Häagen-Dazs had equipment or other assets that could have helped it make chunky ice cream, it might have benefited from divesting these assets, so that Ben & Jerry's would find it profitable to continue investing in these niche flavors rather than in the more mainstream flavors that were Häagen-Dazs's strength.

The first two results, which state conditions in which this model leads to either permanent differences or major strategic changes, are consistent with what we would expect given the model setup. These results propose a new explanation for how firms develop (and potentially change) their marketing strategies and derive precise conditions in which this explanation holds. A common alternative explanation is that first movers acquire more attractive locations (Prescott and Visscher 1977, Moorthy 1988) or more assets (Sutton 1991). Although there is some empirical evidence for first-mover advantages (Urban et al. 1986), in many cases, the most successful firm in a market was not the first entrant (Golder and Tellis 1993), suggesting that factors other than order of entry must play a role in determining firm performance. The first two results in this paper provide an explanation for such factors.[3]

My third result, which shows that a firm can sometimes benefit from reducing its own assets, directly contradicts previous theoretical results from dynamic investment games in which firms compete along a *single* dimension with one state variable for each firm (e.g., Villas-Boas 1993, Ericson and Pakes 1995, Ofek and Sarvary 2003, Besanko and Doraszelski 2004). Thus, this paper demonstrates an important new implication of allowing dynamic competition along *multiple* dimensions.

2. Model Setup

Assume firms i and j compete in two segments (mainstream and niche) over an infinite number of time periods. Firms start at $t = 0$ with zero assets and assets evolve as follows:

$$M_{i,t} = \gamma M_{i,t-1} + m_{i,t} + \epsilon_{m,i,t}, \quad (1)$$

$$N_{i,t} = \gamma N_{i,t-1} + n_{i,t} + \epsilon_{n,i,t}, \quad (2)$$

where $M_{i,t}$ and $N_{i,t}$ denote firm i's mainstream and niche assets and $m_{i,t}$ and $n_{i,t}$ represent its respective investment levels at time t. The term $\gamma \in [0,1)$ reflects depreciation of assets, for example, as a result of employees leaving the firm or loss of organizational knowledge. This constant depreciation rate guarantees that assets are bounded and implies that a firm must constantly reinvest in a segment to maintain its asset level.[4] In each period, each firm allocates a maximum of one unit of investment between the two segments, $m_{i,t} + n_{i,t} \leq 1$. This investment constraint could hold, for example, because of capital market imperfections (Myers and Majluf 1984) or employees' limited time and attention.[5] The $\epsilon_{.,i,t}$ are independent and identically distributed (iid) random variables that represent random shocks to assets, for example, due to fortunate discoveries of better production processes, results of random experimentation with new product designs, or unpaid endorsements by celebrities. These variables follow a distribution with no mass points and with support on a finite range $[0, \epsilon_{max}]$, where $\epsilon_{max} < 1$.[6] In each period, firms

---

[2] As another example of a major strategic change, competition from manufacturing plants in China has eroded Dell's cost advantage, leading Dell to start selling through retail stores and to reorganize its supply chain to move away from the build-to-order model (Shah 2008). We can think of the rise of computer manufacturing in China as an extremely unfavorable shock to the value of Dell's build-to-order expertise.

[3] My model bears some resemblance to the model by Levinthal (1997) in which firms search across a "rugged landscape" for an optimal organizational form. One major difference is that in this earlier model, each firm tries to optimize its own fitness, without regard for competition. By contrast, the current paper uses a game-theoretic approach in which each firm's investment decisions are influenced by its competitor's assets.

[4] For example, even today, Ben & Jerry's must continuously reinvest in training its employees and maintaining its equipment to retain its production expertise at creating chunky ice cream pints. Despite these efforts, it still occasionally produces "bad batches" of ice cream, in which all the chunks sink to the bottom. This information was provided to me during a tour of the Ben & Jerry's factory in Vermont.

[5] In principle, we could allow a firm's investment constraint to grow as its assets grow; this would not qualitatively change the key results, but the conditions in §3.1 would need to be adjusted accordingly.

[6] Note that although these random shocks are assumed to be non-negative, the model does allow for a reduction in a firm's assets as a result of the depreciation term. Asset depreciation combined with an ϵ term that is close to zero for the period effectively results in a negative shock to the firm's assets in a segment.

simultaneously choose investment levels, and then random shocks occur and profits are realized.

Firm i's profits in each segment in period t are

$$\pi_{i,M,t} = F(M_{i,t}) - G(M_{j,t}) - \psi M_{i,t} M_{j,t} - Cm_{i,t}, \quad (3)$$

$$\pi_{i,N,t} = \alpha[F(N_{i,t}) - G(N_{j,t}) - \psi N_{i,t} N_{j,t}] - Cn_{i,t}, \quad (4)$$

where $F' > 0$, $F'' > 0$, $G' > 0$, $\alpha \in [0,1]$, and $\psi \geq 0$. The second derivative of G can be either positive or negative, but we need $G'' \geq -\psi$.[7] The assumption $F'' > 0$ implies increasing returns, which could be true as a result of demand-side effects such as word-of-mouth effects (Rob and Fishman 2005) and network effects or supply-side effects such as division of labor benefits and learning curves (Argote and Epple 1990). More generally, having an existing set of assets devoted to a segment often makes it more valuable for a firm to acquire additional assets devoted to this segment.[8] All of my results would also hold for S-shaped (increasing-then-decreasing) returns as long as marginal returns do not decrease to the point that firms want to diversify. The negative interaction terms, $-\psi M_{i,t} M_{j,t}$ and $-\psi N_{i,t} N_{j,t}$, imply that it is more profitable for a firm to invest in a segment where its competitor is weak. These negative competitive interactions tend to be strong (ψ is large) if consumers' search costs are low (Kuksov 2004) or if firms are similar in dimensions other than those represented by the assets in which firms are investing (Bronnenberg 2008). Such conditions imply that price competition is intense in segments where both firms have large assets. The parameter C is a scaling factor that represents the unit cost of investment. I assume that this term is small enough such that each firm can always increase its profits by investing in at least one segment, so neither firm ever drops out of the market. A sufficient condition for this is

$$C < F'(0) - \psi \left[\frac{1 + \epsilon_{\max}}{1 - \gamma} \right]. \quad (5)$$

As an example of a microfoundation for profit functions (3) and (4), assume each firm sells a separate product in each segment, and customers (who have masses 1 and α in the mainstream and niche segments, respectively) will only purchase a product targeted toward their segment. Firms are exogenously located at opposite ends of a Hotelling line of

length 1. Assets improve quality on a vertical dimension orthogonal to the Hotelling line, so that a mainstream customer located a distance d from firm i receives utility $U - kd + M_{i,t} - P_{i,m,t}$ from purchasing firm i's product, where k is customers' transportation cost and $P_{i,m,t}$ is firm i's price in the mainstream segment. This implies that firm i's demand in the mainstream segment is $\frac{1}{2} + (1/(2k))[M_{i,t} - M_{j,t} - P_{i,m,t} + P_{j,m,t}]$. Simultaneously solving each firm's first-order condition for its optimal price, we find that firm i's equilibrium mainstream segment profits are[9]

$$\pi^*_{i,M,t} = \frac{k}{2} + \frac{M_{i,t}}{3} + \frac{M_{i,t}^2}{18k} - \frac{M_{j,t}}{3} + \frac{M_{j,t}^2}{18k} - \frac{M_{i,t}M_{j,t}}{9k}. \quad (6)$$

Equilibrium profits in the niche segment are analogous, except that they are scaled by α. Note this profit function is consistent with the profit functions (3) and (4).[10]

Finally, I focus on Markov perfect equilibria and make the simplifying assumption that firms are *myopic*, meaning that each firm just maximizes its expected profits in the current period given the previous period's asset levels.[11] Analyzing the myopic case is common in the dynamic investment games literature (e.g., Athey and Schmutzler 2001). One justification for this approach is that strategies followed by myopic firms approximate those followed by impatient firms, which place small positive weight on future profits and allocate investment primarily based on where their marginal returns are currently the highest. On the other hand, if firms were sufficiently forward looking, then arguments similar to the folk theorem (Fudenberg and Maskin 1986) would apply, and regardless of the game's history, there would be an equilibrium in which the firm that is currently stronger in the niche segment launches a prolonged attack on the mainstream segment until its competitor eventually retreats from this segment.[12] The intuition

---

[7] This restriction on the second derivative of G ensures that a firm that is leading in the mainstream segment and trailing in the niche segment would prefer its competitor to focus on the niche segment; this condition is needed for the asset destruction result in §3.3.

[8] In several of these previous models, having a high asset level makes it *easier* (but not necessarily more valuable) to acquire additional assets. With some modifications the current model could accommodate this alternative type of increasing return.

[9] I assume parameter values are in a range where the market is covered and both firms have positive demand in equilibrium. To be precise, we need the constant $U > 3k/2$, and we also need for each firm's assets in each segment to lie in the range $[0, 3k]$, which will always be true if the depreciation factor satisfies $\gamma < 1 - (1 + \epsilon_{\max})/(3k)$. Without loss of generality, I assume marginal production costs are zero.

[10] Intuitively, when a firm has higher equilibrium demand (because its own assets are high or its competitor's assets are low in a particular segment), incremental improvements in its product quality allow it to raise its price and collect incremental profits from a larger number of customers. This implies that marginal returns to quality are higher.

[11] The focus on Markov strategies is for notational convenience and does not affect the results. The paper derives conditions in which there is a unique Nash equilibrium at each time t.

[12] A previous version of this paper derived conditions in which such an equilibrium always exists if we consider all subgame perfect equilibria (and do not restrict firms to Markov perfect equilibria).

behind the results in the current paper apply in cases where firms are not patient enough to launch such attacks.

3. Results

A pure Markov strategy for firm i is a mapping $S_i(A_{t-1}) \rightarrow (m_{i,t}, n_{i,t})$ from asset levels to investment decisions, where $A_t \equiv (M_{i,t}, N_{i,t}, M_{j,t}, N_{j,t})$. To simplify notation, I also define $S_{i,t} \equiv S_i(A_{t-1})$. The appendix proves the following preliminary result.

LEMMA 1. *A pure strategy equilibrium exists, and in any pure strategy equilibrium, at each time t, firm i sets $S_{i,t}$ equal to either $(1, 0)$ or $(0, 1)$, and firm j sets $S_{j,t}$ equal to either $(1, 0)$ or $(0, 1)$.*

It is intuitive that, given that each firm's profit function is convex in its own assets, in any given period a firm's optimal strategy is always to focus all of its investment in a single segment.

3.1. Permanent Differences

I now derive a set of conditions that ensures that there is a unique equilibrium in which firms initially focus on the mainstream segment ($S_{i,1} = S_{j,1} = (1, 0)$) but later become permanently focused on different segments. I first define the following operators:[13]

$$B(X) \equiv E[F(X+1+\epsilon) - F(X+\epsilon)], \quad (7)$$

$$L(X) \equiv \psi E[X + \epsilon]. \quad (8)$$

Intuitively, $B(X)$ gives the expected profit increase from investing one unit in the mainstream segment if a firm's asset level in this segment is X, not accounting for competitive preemption effects. $L(X)$ gives the marginal expected loss as a result of preemption effects from investing one unit in the mainstream segment if the competitor has an asset level of X in this segment, accounting for the competitor's expected random shock in the segment. For the niche segment, these values need to be scaled by α.

The first key condition states that for all $X \in [0, 1/(1-\gamma)]$ and all $Y \leq (X\epsilon_{\max})$, the following holds.

CONDITION 1. $B(\gamma X) - L(\gamma X(1 + \epsilon_{\max}) + 1) > \alpha[B(\gamma Y) - L(\gamma Y)]$.

Condition 1 ensures that the niche segment (α) is *small* enough and preemption effects (ψ) are *weak* enough that in every period at least one firm has a dominant strategy of investing in the mainstream segment (see the proof of Proposition 1 in the appendix for more details).

The second key condition needed for random shocks to cause permanent differences is as follows.

Figure 1   Condition 1 Ensures That Firms Initially Race for the Mainstream Segment
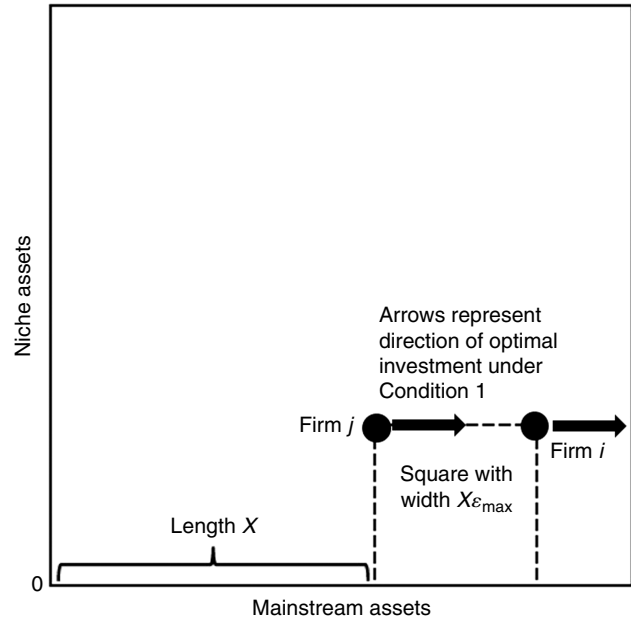
Note. Illustrated is the condition for one particular value of X.

CONDITION 2. $B(\gamma X_M) - L(\gamma X_H + 1) < \alpha[B(\gamma X_L) - L(0)]$, *where* $X_L = \epsilon_{\max}(1/(1-\gamma))$, $X_M = (1/(1-\gamma))$, *and* $X_H = (1 + \epsilon_{\max})(1/(1-\gamma))$.

This condition ensures the niche segment is *large* enough and preemption effects are *strong* enough that once a large enough gap opens between the firms, one firm switches focus to the niche segment. This condition also ensures that the upper bound on random shocks is large enough that such a gap eventually occurs.

The third key condition is as follows.

CONDITION 3. $B(\gamma X_L) - L(\gamma X_M + 1) < \alpha[B(\gamma X_M) - L(\gamma X_L)]$, *where* X_L *and* X_M *are defined as above.*

This condition ensures that the upper bound on random shocks is small enough that once firms have focused on different segments long enough, random shocks will never cause them to switch their focus. Figures 1–3 illustrate these conditions.[14]

PROPOSITION 1. *If Conditions 1–3 hold, then with probability one, firms have unique equilibrium strategies at every time t given asset levels A_{t-1}. In equilibrium, both firms initially focus their investment in the mainstream segment, but the firms eventually become permanently focused on different segments. Stated formally, given starting point $A_0 = (0, 0, 0, 0)$, in equilibrium*

---

[13] For simplicity of notation, I have suppressed the subscripts on the ϵ terms; recall that these terms are assumed to be iid.

[14] For the Hotelling model example in §2, all three conditions hold if $k = 4$, $\alpha = 0.6$, $\gamma = 0.85$, and $\epsilon_{\max} = 0.5$. If we then increase $\epsilon_{\max}$ to 0.6, Conditions 1 and 2 still hold, but Condition 3 does not.

Figure 2 Condition 2 Ensures That a Sufficient "Gap" Will Eventually Open Between the Two Firms Such That One Firm Will Start Investing in the Niche Segment
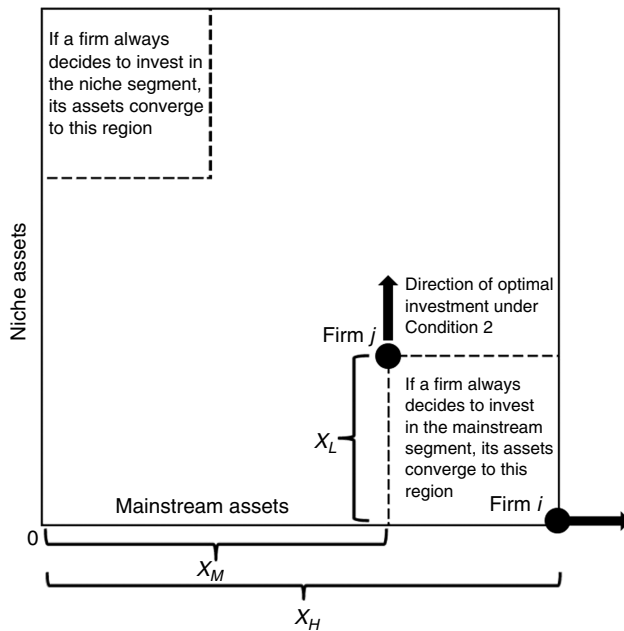

$S_i(A_0) = S_j(A_0) = (1, 0)$, *and there exists a finite T for which one of the following is true*:

(1) $S_i(A_{t-1}) = (1, 0)$ *and* $S_j(A_{t-1}) = (0, 1)$ *for all* $t > T$, *or*

(2) $S_i(A_{t-1}) = (0, 1)$ *and* $S_j(A_{t-1}) = (1, 0)$ *for all* $t > T$.

The proof of this result (see the appendix) depends on both increasing returns and preemption effects.

Figure 3 Condition 3 Ensures That Once Firms Have Focused in Different Segments Long Enough, the Firm in the Niche Segment Will Never Want to Attack the Mainstream Segment
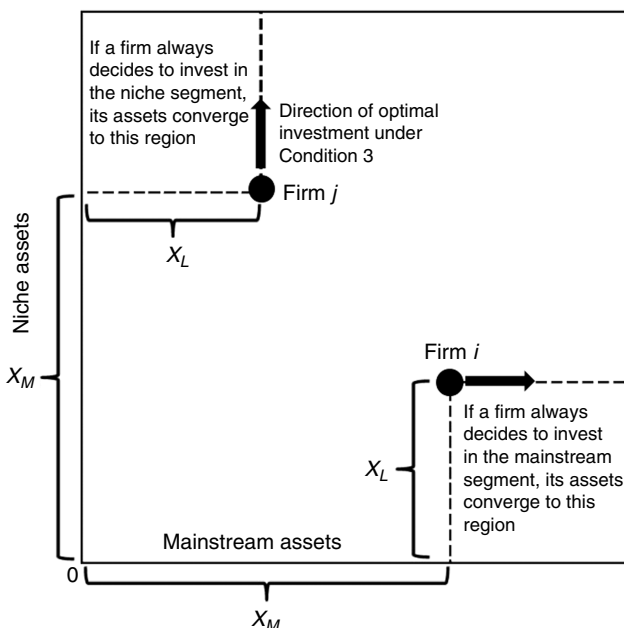
Intuitively, Conditions 1 and 2 imply that a firm is only willing to switch focus to the niche segment after its own recent shocks have been stronger in the *niche* segment (moving it up the increasing returns curve in that segment) and that its competitor's recent shocks have been stronger in the *mainstream* segment (generating strong preemption effects in that segment). This implies that, in any given period, at most one firm can be in a position where it is willing to invest in the niche segment (if its competitor invests in the mainstream segment in the current period). Therefore, there are two possibilities. If neither firm is in such a position, then they both invest in the mainstream segment. If only one firm is in such a position, then it invests in the niche segment and its competitor invests in the mainstream segment. In either case, the equilibrium outcome is unique.

3.2. Major Strategic Changes

The previous section derived conditions that guarantee that permanent differences arise. On the other hand, as the size of the largest possible random shock grows, the boxed regions in Figure 3 grow until they become arbitrarily close to intersecting, which ensures that a firm focusing on the niche segment will eventually come close enough to its rival that its best strategy is to attack the mainstream segment.[15]

PROPOSITION 2. *If Conditions 1 and 2 hold, but Condition 3 does not hold, then with probability one, firms have unique equilibrium strategies at every time t given asset levels* A_{t-1}. *In equilibrium, both firms initially focus their investment in the mainstream segment for at least one period, and one firm eventually switches focus to the niche segment; however, this firm later switches focus back to the mainstream segment.*

See the appendix for a proof of this result. Under the conditions of this proposition, over time both firms continually shift their focus as a result of random fluctuations.

3.3. Model Extension: Asset Destruction

Another implication of this model is that a firm can sometimes benefit from a reduction of its own assets

---

[15] Note that the term X_L is the level to which a firm's assets converge if it *never* invests in a segment and repeatedly has the *best* possible shock in that segment, whereas X_M is the analogous asset level for a firm that *always* invests in a segment but has the *worst* possible shock in that segment. Thus, once a firm has repeatedly invested in the niche segment and its competitor has repeatedly invested in the mainstream segment, the left side of Condition 3 represents the greatest possible return to the focal firm from investing in the mainstream segment, whereas the right side represents the worst possible return to the focal firm from investing in the niche segment. If sufficiently large shocks are possible, then the left side of this condition exceeds the right side, and the firms will eventually experience a series of shocks such that the firm that has been focusing on the niche segment changes its strategy and attacks the mainstream segment.

in the niche segment. For example, imagine that a firm has a lead in the mainstream segment, but the lead is not large enough to cause the firm's competitor to switch its focus to the niche segment. The firm might benefit from *reducing* its own niche assets so as to entice its competitor to focus on the niche segment.

To formalize this intuition, I now extend the model to allow each firm to destroy some of its own assets. At the beginning of each period t, firm i chooses a mainstream asset level anywhere in the range $[0, M_{i, t-1}]$ and chooses a niche asset level in the range $[0, N_{i, t-1}]$; firm j makes analogous choices. Choosing an asset level below the top of these ranges represents partial asset destruction. The game timing at each period is now as follows: (1) Firms simultaneously make asset destruction decisions in both segments. (2) Firms simultaneously make investment decisions. (3) Random shocks occur, and profits are realized.

PROPOSITION 3. *If Conditions* 1 *and* 2 *hold, there exist asset levels for which there is an equilibrium in which one firm chooses to destroy some of its niche assets.*

See the appendix for a proof of this result. For this proposition to hold, it is important for asset destruction to be publicly observable and irreversible. Otherwise, a firm might want to temporarily reduce its niche assets at the start of the period in order to influence its competitor's investment decision but then restore those "destroyed" assets before the end of the period.

4. Conclusion

This paper has developed a model in which firms dynamically compete for different market segments. Under certain conditions, each firm becomes permanently focused on the segment where it has the best initial luck relative to its competitor. When these conditions hold, the model does not depend on (as many models of product design do) an arbitrary selection among various possible equilibria to determine where each firm allocates investment. Rather, a rational firm must always follow the optimal path determined by its previous history.

This model implies that in industries such as consumer packaged goods, where key assets such as brands undergo relatively small random shocks in any given year, once a firm establishes dominance in a segment, it should focus on that segment forever; this is consistent with the empirical findings of Bronnenberg et al. (2009). On the other hand, in markets for high-tech products, in which assets such as up-to-date technical expertise are subject to large shocks as new technologies arrive and displace old ones, each firm should occasionally make large changes in strategy based on shocks to its assets.

Another insight from the model is that a firm might want to deliberately reduce its assets in a small niche segment. For example, a company might want to spin off a division that serves such a segment by selling this division to a firm in another market, thus enticing its competitor to invest in the niche segment.

Appendix

PROOF OF LEMMA 1. At each time t, firm i chooses investment levels $(m_{i, t}, n_{i, t})$ to maximize $E[\pi_{i, M, t}(m_{i, t}, m_{j, t}) + \pi_{i, N, t}(n_{i, t}, n_{j, t})]$ subject to $m_{i, t} + n_{i, t} \leq 1$, conditional on firm j's investment choices $(m_{j, t}, n_{j, t})$. I first show that it can never be an optimal for firm i to choose any investment levels other than $(1, 0)$ or $(0, 1)$. By differentiating firm i's profit function in each segment, we have

$$\frac{dE[\pi_{i, M, t}]}{dm_{i, t}} = E[F'(\gamma M_{i, t-1} + m_{i, t} + \epsilon_{m, i, t})]$$
$$- \psi E[\gamma M_{j, t-1} + m_{j, t} + \epsilon_{m, j, t}] - C, \quad (9)$$

$$\frac{dE[\pi_{i, N, t}]}{dn_{i, t}} = \alpha \big[E[F'(\gamma N_{i, t-1} + n_{i, t} + \epsilon_{n, i, t})]$$
$$- \psi E[\gamma N_{j, t-1} + n_{j, t} + \epsilon_{n, j, t}] \big] - C. \quad (10)$$

Inequality (5) ensures that (9) is positive for any feasible asset values. This implies that firm i would never set $m_{i, t} + n_{i, t} < 1$ because it could always increase its expected profits by increasing $m_{i, t}$ until its investment constraint is binding—that is, until $m_{i, t} + n_{i, t} = 1$. We can therefore substitute $(1 - m_{i, t})$ for $n_{i, t}$ and rewrite firm i's objective as maximizing

$$\hat{\pi}(m_{i, t}) \equiv E[\pi_{i, M, t}(m_{i, t}, m_{j, t}) + \pi_{i, N, t}(1 - m_{i, t}, n_{j, t})] \quad (11)$$

subject to $m_{i, t} \in [0, 1]$. By inserting the profit functions (3) and (4) into (11) and differentiating twice with respect to $m_{i, t}$, we have

$$\frac{d^2\hat{\pi}(m_{i, t})}{dm_{i, t}^2} = E[F''(\gamma M_{i, t-1} + m_{i, t} + \epsilon_{m, i, t})]$$
$$+ \alpha E[F''(\gamma N_{i, t-1} + (1 - m_{i, t}) + \epsilon_{n, i, t})]. \quad (12)$$

Therefore, the assumption that F'' is positive implies that the second derivative of $\hat{\pi}$ is also positive, and because this expected profit function is convex, its maximum must occur at one of the extreme points $m_{i, t} = 0$ or $m_{i, t} = 1$. Thus,

the only possible optimal levels for $(m_{i,t}, n_{i,t})$ are $(1, 0)$ or $(0, 1)$. Although there could be mixed strategy equilibria in which firms randomize between these two investment choices, I will show that a pure strategy equilibrium must exist.

Note that firm i's marginal returns to investing in the mainstream segment, given by (9), are decreasing in $m_{j,t}$, whereas its marginal returns to investing in the niche segment, given by (10), are decreasing in $n_{j,t}$. In other words, a firm's marginal investment returns in a segment decrease if its competitor invests in that segment. As a result, there are five possible scenarios at each time t: (a) both firms have a dominant strategy of investing in the *mainstream* segment; (b) both firms have a dominant strategy of investing in the *niche* segment; (c) one firm's dominant strategy is to invest in the *mainstream* segment, whereas the other firm's best response is to invest wherever its competitor does not; (d) one firm's dominant strategy is to invest in the *niche* segment, whereas the other firm's best response is to invest wherever its competitor does not; and (e) each firm's best response is to invest wherever its competitor does not. In scenarios (a) and (b), in equilibrium, both firms invest in the same segment. In scenarios (c) and (d), in equilibrium, the firm with a dominant strategy invests in the segment dictated by its dominant strategy, whereas the other firm invests in the other segment. Finally, in scenario (e), there are two pure strategy equilibria: one in which firm i invests in the mainstream segment while firm j invests in the niche segment and another in which the opposite occurs. Thus, in all five scenarios at least one pure strategy equilibrium exists.   Q.E.D.

PROOF OF PROPOSITION 1. I show that under the conditions of this proposition, (1) with probability one, at each time $t \geq 1$ there is a unique pure strategy equilibrium in which at least one firm (which we label firm i) has a dominant strategy of setting $S_{i,t} = (1, 0)$; (2) with probability one, there exists a finite $T > 0$ such that one firm (which we label firm i) has assets $M_{i,T} \in [X_M, X_H]$ and $N_{i,T} \in [0, X_L]$, and the other firm (which we label firm j) has assets $M_{j,T} \in [0, X_L]$ and $N_{j,T} \in [X_M, X_H]$; and (3) for all $t > T$, there is a unique pure strategy equilibrium in which $S_{i,t} = (1, 0)$ and $S_{j,t} = (0, 1)$. The proof proceeds in three steps.

Step 1. Note that if firm i has always invested in the mainstream segment for t periods, its assets then lie in a region that is a square with width determined by the largest possible random shock:

$$\sum_{u=1}^{t} \gamma^{u-1} \leq M_{i,t} \leq \sum_{u=1}^{t} \gamma^{u-1}(1 + \epsilon_{\max}), \qquad (13)$$

$$0 \leq N_{i,t} \leq \sum_{u=1}^{t} \gamma^{u-1} \epsilon_{\max}. \qquad (14)$$

The box enclosed by the dashed lines in Figure 1 provides an example of how this region looks after a finite number of periods. I will show that Condition 1 guarantees that for all t at least one firm always lies in the regions defined by (13) and (14). Condition 1 immediately implies that both firms have a dominant strategy of investing in the mainstream segment in period one, so both firms' assets lie in this region in the first period. I will show that if at least one

firm lies in this region at time t, then at least one firm will also lie in the region at time $t + 1$.

Suppose *both* firms lie in this region at time t. Without loss of generality, label the firms such that $N_{i,t} \leq N_{j,t}$. Inequalities (13) and (14) imply that $N_i < M_i \epsilon_{\max}$. Therefore, Condition 1 implies that

$$B(\gamma M_i) - L(\gamma M_i(1 + \epsilon_{\max}) + 1) > \alpha[B(\gamma N_i) - L(\gamma N_i)]. \quad (15)$$

That both firms' assets satisfy (13) and (14) implies that $M_j < M_i(1 + \epsilon_{\max})$, and the firms were labeled such that $N_j > N_i$. Therefore, because $B' > 0$ and $L' > 0$, (15) implies that

$$B(\gamma M_i) - L(\gamma M_j + 1) > \alpha[B(\gamma N_i) - L(\gamma N_j)]. \qquad (16)$$

If firm j invests one unit in the mainstream segment, then the left side of this inequality is the expected return to firm i of investing in the mainstream segment, and the right side is the expected return to firm i of investing in the niche segment. Thus, this condition guarantees that firm i has a dominant strategy of investing in the mainstream segment even if firm j invests in this segment too.

Now suppose only firm i's assets satisfy (13) and (14) at time t. If $N_i \leq N_j$, the same argument as above holds. On the other hand, even if $N_i > N_j$, I show that firm i still must have a dominant strategy of investing in the mainstream segment. In this case, we must have $M_i > M_j$, or firm j's assets would satisfy (13) and (14) too. Also, because firm i's assets satisfy (13) and (14), we have $N_i \leq (M_i \epsilon_{\max})$. These two findings imply that

$$M_i(1 + \epsilon_{\max}) > M_j + N_i. \qquad (17)$$

By substituting the right side of (17) into (15), we have

$$B(\gamma M_i) - L(\gamma M_j + \gamma N_i + 1) > \alpha[B(\gamma N_i) - L(\gamma N_i)]. \qquad (18)$$

The operator L has the property that $L(X) - L(Y) = \psi(X - Y)$ for any X and Y. Therefore, if we add $[L(\gamma M_j + \gamma N_i + 1) - L(\gamma M_j + 1)]$ to the left side of this inequality and add $\alpha[L(\gamma N_i) - L(\gamma N_j)]$ to the right side, the inequality still holds. This is true because $(\gamma M_j + \gamma N_i + 1) - (\gamma M_j + 1) = \gamma N_i \geq \gamma N_i - \gamma N_j$. This implies that

$$B(\gamma M_i) - L(\gamma M_j + 1) > \alpha[B(\gamma N_i) - L(\gamma N_j)]. \qquad (19)$$

Thus, Condition 1 still implies that firm i has a dominant strategy of investing in the mainstream segment, and it will stay in the region defined by (13) and (14) in the next period.

To summarize, I have shown that at least one firm's assets satisfy (13) and (14) in the first period and that if at least one firm is in this region in period t, then the same will hold true in period $t + 1$. This guarantees that at least one firm will always be in this region. Intuitively, if both firms are in the region, then the one with lower niche assets will invest in mainstream, and if only one firm is in the region, it will continue investing in mainstream.

We have established that in each period, one firm (call it firm i) has a dominant strategy of focusing on the mainstream segment. Thus, in each period there is a unique equilibrium in which this firm sets $S_{i,t} = (1, 0)$, and its competitor plays its best response to this strategy. (Technically

speaking, there are *two* equilibria if firm j is exactly indifferent between setting $S_{j,t} = (1,0)$ and $S_{j,t} = (0,1)$ in response to $S_{i,t} = (1,0)$; however, the strict convexity of the profit functions and the assumption that error terms have no mass points ensure that assets have zero probability of landing precisely at such a state, so the equilibrium is unique with probability one.)

Step 2. I will now show that one firm (let us call it firm j) eventually switches its focus to the niche segment, and its assets then enter a region defined by $M_j \in [0, X_L]$ and $N_j \in [X_M, X_H]$, where X_L, X_M, and X_H are defined as in §3.1.

By the results shown in Step 1, at any time t, at least one firm's assets are in the region defined by (13) and (14). If only one firm is in this region, label this firm i. If both firms are in the region, label the one with lower niche assets firm i. The results from Step 1 of the proof imply that firm i focuses on the mainstream segment in the current period.

Now suppose firm i receives shocks within a sufficiently small distance ϕ of $(\epsilon_{\max}, 0)$ and firm j receives shocks within ϕ of $(0, \epsilon_{\max})$. Note that such shocks have positive probability for any $\phi > 0$.

If we choose ϕ sufficiently small, these shocks ensure that firm i's assets remain in the region defined by (13) and (14); that if firm j's assets were not in this region, they do not enter this region in the next period; and that if firm i had lower niche assets than firm j, this continues to be the case in the next period. Therefore, the results from Step 1 guarantee that firm i focuses on the mainstream segment again in the next period.

Suppose firms receive z such shocks in a row (where z is a finite integer). Regardless of the starting point, we then have

$$M_{i,t} > \sum_{u=1}^{z} \gamma^{u-1}(1+\epsilon_{\max} - \phi)$$
$$= \frac{1+\epsilon_{\max} - \phi}{1-\gamma}(1-\gamma^z) = X_H - \frac{\phi}{1-\gamma} - \gamma^z \frac{1+\epsilon_{\max} - \phi}{1-\gamma}. \quad (20)$$

Therefore, for any $d > 0$, if we choose ϕ small enough and z large enough that $\phi/(1-\gamma) + \gamma^z((1+\epsilon_{\max}-\phi)/(1-\gamma)) < d$, then z such shocks in a row guarantee $M_{i,t} > X_H - d$. Similarly, it can be shown that such values of ϕ and z ensure that $M_{j,t} < X_M + d$, $N_{j,t} > X_L - d$, and $N_{i,t} < d$. If we have chosen d sufficiently small, then Condition 2 guarantees that firm j focuses on the niche segment at any such state. Thus, firm j focuses on the niche segment after z such shocks in a row. If the firms still continue receiving such shocks, firm j's niche assets grow while its mainstream assets shrink, so it continues to focus on the niche segment. In fact, similar analysis to the preceding shows that, if we have chosen d small enough, then an additional z such shocks guarantee that firm j's assets enter the region where $M_j \in [0, X_L]$ and $N_j \in [X_M, X_H]$.

The law of large numbers guarantees that such a series of shocks eventually occurs with probability one. To see why, define a sequence of random variables $z_1, z_2, \ldots,$ where z_1 equals 1 if such shocks occur in each of the first $2z$ periods of the game and equals 0 otherwise, z_2 is defined equivalently for the next $2z$ periods, and so on. Note that the expectation of each variable is positive because z is finite and there is always positive probability of this sequence

of shocks occurring. The law of large numbers guarantees that the average of these variables converges to their expectation, which can only occur if such a sequence of shocks eventually does occur. Thus, firm j's assets eventually enter the proposed region. Intuitively, because it is always possible for firms to have a string of good luck in different segments, if the game continues long enough, such a string of shocks will eventually occur. Also, note that the labeling of firm j was based on the state before this series of shocks occurred; depending on the early shocks in the game, either firm could be the one we label "firm j," so either could end up in the niche segment.

Step 3. We have established that for some $T > 0$, one firm (call it firm i) has assets that satisfy $M_{i,T} \in [X_M, X_H]$ and $N_{i,T} \in [0, X_L]$, whereas the other firm (call it firm j) has assets that satisfy $M_{j,T} \in [0, X_L]$ and $N_{j,T} \in [X_M, X_H]$.

Condition 3, along with the assumptions of increasing returns and a negative interaction with the competitor's assets, implies that whenever each firm's assets are in the regions given above, firm i focuses on the mainstream segment and firm j focuses on the niche segment. Therefore, each firm stays in its respective region in the next period. Thus, they will always remain in these regions.   Q.E.D.

PROOF OF PROPOSITION 2. Steps 1 and 2 from the proof of Proposition 1 still hold. The only difference is we are now assuming Condition 3 does not hold, so Step 3 of Proposition 1 no longer holds.

Label the firm with larger niche assets firm j. Now suppose firm j receives a series of shocks within a sufficiently small distance ϕ of $(\epsilon_{\max}, 0)$, and firm i receives a series of shocks within ϕ of $(0, \epsilon_{\max})$. By choosing ϕ sufficiently small, and allowing a long enough sequence of such shocks, we guarantee that firm i's assets will become arbitrarily close (X_M, X_L), whereas firm j's assets will become arbitrarily close to point (X_L, X_M). Because Condition 3 no longer holds, once both firms are within a small enough distance d from these points, firm j switches its investment focus to the mainstream segment. If the firms continue receiving such shocks, firm j continues focusing on the mainstream segment and eventually enters the region given by (13) and (14).

As in the previous proof, because this entire sequence of events has positive probability of occurring in a finite amount of time, the law of large numbers guarantees that it does eventually occur.   Q.E.D.

PROOF OF PROPOSITION 3. I show there exist asset values such that there is an equilibrium in which firm i destroys some of in its own niche assets and firm j does not destroy any assets. Suppose firm j has assets (X_M, X_L) and firm i has assets (X_H, N_i), where $N_i \in [0, X_L]$. Condition 1 guarantees that firm i invests in the mainstream segment at any such state. On the other hand, it also guarantees that firm j invests in the *mainstream* segment if $N_i = X_L$; Condition 2 guarantees that firm j invests in the *niche* segment if $N_i = 0$. It is intuitive that, given the negative interaction term in each firm's profit function, a reduction in firm i's niche assets makes it more attractive for firm j to invest in the niche rather than mainstream segment. Let $\hat{N}_i$ denote the cutoff value such that there is an equilibrium in which

Downloaded from informs.org by [155.246.103.35] on 06 April 2017, at 07:51 . For personal use only, all rights reserved.

firm j invests in the niche segment for $N_i = \hat{N}_i$ but invests in the mainstream segment for $N_i > \hat{N}_i$.

Consider the case when firm i's niche assets are $\hat{N}_i + d$, where $d > 0$. If neither firm destroys any assets, then both firms invest in the mainstream segment, and firm i's expected profits are

$$E\big[F(\gamma X_H + 1 + \epsilon_{m,i,t}) - G(\gamma X_M + 1 + \epsilon_{m,j,t})$$
$$- \psi(\gamma X_H + 1 + \epsilon_{m,i,t})(\gamma X_M + 1 + \epsilon_{m,j,t})$$
$$+ \alpha F(\gamma(\hat{N}_i + d) + \epsilon_{n,i,t}) - \alpha G(\gamma X_L + \epsilon_{n,j,t})$$
$$- \alpha \psi(\gamma(\hat{N}_i + d) + \epsilon_{n,i,t})(\gamma X_L + \epsilon_{n,j,t})\big] - C. \quad (21)$$

On the other hand, if firm i chooses to reduce its niche assets to $\hat{N}_i$, then firm i invests in the mainstream segment and firm j invests in the niche segment, and firm i's expected profits are

$$E\big[F(\gamma X_H + 1 + \epsilon_{m,i,t}) - G(\gamma X_M + \epsilon_{m,j,t})$$
$$- \psi(\gamma X_H + 1 + \epsilon_{m,i,t})(\gamma X_M + \epsilon_{m,j,t})$$
$$+ \alpha F(\gamma \hat{N}_i + \epsilon_{n,i,t}) - \alpha G(\gamma X_L + 1 + \epsilon_{n,j,t})$$
$$- \alpha \psi(\gamma \hat{N}_i + \epsilon_{n,i,t})(\gamma X_L + 1 + \epsilon_{n,j,t})\big] - C. \quad (22)$$

As d approaches zero, then (22) minus (21) approaches

$$E\big[G(\gamma X_M + 1 + \epsilon_{m,j,t}) - G(\gamma X_M + \epsilon_{m,j,t})$$
$$+ \alpha G(\gamma X_L + \epsilon_{n,j,t}) - \alpha G(\gamma X_L + 1 + \epsilon_{n,j,t})\big]$$
$$+ L(\gamma X_H + 1) - \alpha L(\gamma \hat{N}_i). \quad (23)$$

The four terms in the expectation operator are equal to

$$\int_0^1 G'(\gamma X_M + \Delta + \epsilon_{m,j,t})\, d\Delta$$
$$- \alpha \int_0^1 G'(\gamma X_L + \Delta + \epsilon_{n,j,t})\, d\Delta. \quad (24)$$

Since all error terms are assumed to follow the same distribution, and we are taking expectations of these terms, we can substitute $\epsilon_{m,j,t}$ for $\epsilon_{n,j,t}$. Making this substitution, and given that $G' > 0$ and $\alpha < 1$, expression (24) is greater than

$$\int_0^1 \big[G'(\gamma X_M + \Delta + \epsilon_{m,j,t}) - G'(\gamma X_L + \Delta + \epsilon_{m,j,t})\big]\, d\Delta$$
$$= \int_0^1 \int_{\gamma X_L}^{\gamma X_M} G''(\Gamma + \Delta + \epsilon_{m,j,t})\, d\Gamma\, d\Delta. \quad (25)$$

By the assumption that $G'' \geq -\psi$, the expression on the right side of this equation is greater than $-\psi\gamma(X_M - X_L)$, which implies that the sum of the terms in the expectation operator in (23) is also greater than this value. Finally, because $L(X) - L(Y) = \psi(X - Y)$ for any X and Y, the sum of the last two terms in (23) is greater than $\psi(\gamma X_H + 1 - \gamma \hat{N}_i)$. This implies that expression (23) is greater than

$$-\psi\gamma(X_M - X_L) + \psi\gamma(X_H - \hat{N}_i) + \psi$$
$$= \psi\gamma(X_H - X_M + X_L - \hat{N}_i) + \psi. \quad (26)$$

Because $X_H > X_M$ and $X_L > \hat{N}_i$, this expression is always strictly greater than zero. Thus, firm i's expected profits

increase when it reduces its assets from $\hat{N}_i + d$ to $\hat{N}_i$. Intuitively, as d becomes sufficiently small, the direct impact on firm i's profits from reducing its own niche assets approaches zero. However, the indirect effect from enticing firm j to change its investment stays strictly positive, resulting in an overall increase in firm i's profits.

We have shown that if firm j has asset levels (X_M, X_L), firm i has asset levels $(X_H, \hat{N}_i + d)$ with d sufficiently small, and firm j does not destroy any assets, then firm i's best response is to reduce its niche assets to $\hat{N}_i$.

We now need to show that if firm i reduces its niche assets to $\hat{N}_i$, firm j's best response is not to destroy any of its assets. To demonstrate this, I will show that firm j's asset destruction cannot influence firm i's investment behavior. Condition 1 implies that

$$B(\gamma X_H) - L(\gamma X_H + 1) > \alpha[B(\gamma \hat{N}_i) - L(\gamma \hat{N}_i)]. \quad (27)$$

Note that $X_H - X_M = X_L > \hat{N}_i$, which implies that $[(\gamma X_H + 1) - (\gamma X_M + 1)] > \alpha[(\gamma \hat{N}_i) - 0]$, and so if we add $[L(\gamma X_H + 1) - L(\gamma X_M + 1)]$ to the left side of this inequality and add $\alpha[L(\gamma \hat{N}_i) - L(0)]$ to the right side, then the inequality still holds. (Note we have used the fact that $L(X) - L(Y) = \psi(X - Y)$ for any X and Y.) This implies that

$$B(\gamma X_H) - L(\gamma X_M + 1) > \alpha[B(\gamma \hat{N}_i) - L(0)]. \quad (28)$$

The left side of this inequality is the return to firm i of investing in the mainstream segment, and the right side is the return to firm i of investing in the niche segment, assuming firm j reduces its niche assets to zero and also invests in the mainstream segment in the current period. Thus, this inequality implies that even if firm j reduces its niche assets to zero, firm i still has a dominant strategy of investing in the mainstream segment. As a result, firm j reducing its assets would directly reduce its own profits without affecting firm i's investment behavior in the current period, and so firm j does not engage in asset destruction, and the proposed equilibrium holds.   Q.E.D.

References

Argote L, Epple D (1990) Learning curves in manufacturing. *Science* 247(4945):920–924.

Athey S, Schmutzler A (2001) Investment and market dominance. *RAND J. Econom.* 32(1):1–26.

Besanko D, Doraszelski U (2004) Capacity dynamics and endogenous asymmetries in firm size. *RAND J. Econom.* 35(1):23–49.

Bronnenberg BJ (2008) Brand competition in CPG industries: Sustaining large local advantages with little product differentiation. *Quant. Marketing Econom.* 6(1):79–107.

Bronnenberg BJ, Dhar SK, Dubé J-P (2009) Brand history, geography, and the persistence of brand shares. *J. Political Econom.* 117(1):87–115.

Collis DJ, Conrad MB (2005) Ben & Jerry's Homemade Ice Cream, Inc.: A period of transition. HBS Case 796-109, Harvard Business School, Boston.

D'Aspremont C, Gabszewicz JJ, Thisse J-F (1979) On Hotelling's stability in competition. *Econometrica* 47(5):1145–1150.

Dell M (1999) *Direct from Dell: Strategies that Revolutionized an Industry* (HarperCollins, New York).

Dutta S, Narasimhan O, Rajiv S (1999) Success in high-technology markets: Is marketing capability critical? *Marketing Sci.* 18(4):547–568.

Edge (2009) The making of: PlayStation. (April 24), http://www.edge-online.com/features/making-playstation/.

Ericson R, Pakes A (1995) Markov-perfect industry dynamics: A framework for empirical work. *Rev. Econom. Stud.* 62(1):53–82.

Fudenberg D, Maskin E (1986) The folk theorem in repeated games with discounting or with incomplete information. *Econometrica* 54(3):533–554.

Golder P, Tellis G (1993) Pioneer advantage: Marketing logic or marketing legend? *J. Marketing Res.* 30(2):158–170.

Kuksov D (2004) Buyer search costs and endogenous product design. *Marketing Sci.* 23(4):490–499.

Lager F (1994) *Ben & Jerry's: The Inside Scoop* (Crown, New York).

Levinthal DA (1997) Adaptation on rugged landscapes. *Management Sci.* 43(7):934–950.

Moorthy KS (1988) Product and price competition in a duopoly. *Marketing Sci.* 7(2):141–168.

Myers SC, Majluf N (1984) Corporate financing and investment decisions when firms have information that investors do not have. *J. Financial Econom.* 1(2):187–221.

Ofek E, Sarvary M (2003) R&D, marketing, and the success of next-generation products. *Marketing Sci.* 22(3):355–370.

Prahalad CK, Hamel G (1990) The core competence of the corporation. *Harvard Bus. Rev.* 68(3):79–91.

Prescott EC, Visscher M (1977) Sequential location among firms with foresight. *Bell J. Econom.* 8(2):378–393.

Rob R, Fishman A (2005) Is bigger better? Customer base expansion through word-of-mouth reputation. *J. Political Econom.* 113(5):1146–1162.

Shah A (2008) Dell ironing out kinks in supply chain, executive says. *InfoWorld* (April 3), http://www.infoworld.com/t/hardware/dell-ironing-out-kinks-in-supply-chain-executive-says-499.

Shaked A, Sutton J (1982) Relaxing price competition through product differentiation. *Rev. Econom. Stud.* 49(1):3–13.

Sutton J (1991) *Sunk Costs and Market Structure: Price Competition, Advertising, and the Evolution of Concentration* (MIT Press, Cambridge, MA).

Teece DJ, Pisano G, Shuen A (1997) Dynamic capabilities and strategic management. *Strategic Management J.* 18(7):509–533.

Urban GL, Carter T, Gaskin S, Mucha Z (1986) Market share rewards to pioneering brands: An empirical analysis and strategic implications. *Management Sci.* 32(6):645–659.

Villas-Boas JM (1993) Predicting advertising pulsing policies in an oligopoly: A model and empirical test. *Marketing Sci.* 12(1):88–102.

Wernerfelt B (1984) A resource-based view of the firm. *Strategic Management J.* 5(2):171–180.