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Capacity Investments in Supply Chains: Sharing the Gain Rather Than Sharing the Pain

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In this paper, we investigate price-only contracts in supply chain capacity procurement games. For a two-party supply chain, comprising a manufacturer and a supplier that both invest in capacity, we prove the existence of a class of coordinating price-only contracts that arbitrarily allocate the supply chain profit. Moreover, if the supplier's reservation profit is below a certain threshold, the manufacturer's optimal contract is a quantity-premium price-only schedule, that is, the average wholesale price per unit increases in the order size. We prove that the manufacturer prefers simple piecewise-linear quantity-premium contracts to linear contracts and show numerically that such contracts are highly efficient. We extend our results for piecewise-linear price schedules to N -supplier assembly systems. We also enrich the voluntary compliance regime of Cachon and Lariviere (2001). With this enrichment, we prove that share-the-pain contracts, such as firm commitment and options contracts, increase supplier capacity in the full information case, a result that contrasts with that of Cachon and Lariviere. Finally, we investigate when a manufacturer prefers single-breakpoint quantity premiums to firm commitments.

(Supply Chain; Capacity; Pricing; Coordination)

1. Introduction

Outsourcing of key-component production has increased the reliance of manufacturers on their supply base. This reliance amplifies the complexity of capacity decisions for the manufacturer, as illustrated in this excerpt from Palm Inc.'s Quarterly Report (April 2000, p. 15):

As we introduce and support multiple handheld device products and as competition in the market for our products intensifies, we expect that it will become more difficult to forecast demand.... There is the risk that even if we are able to procure enough components, our third-party manufacturers might not be able to produce enough of our devices to meet the market demand for our

products.... Lower than forecasted demand could also result in excess manufacturing capacity at our third-party manufacturers.... Our reliance on third parties exposes us to the following risks outside our control...potential lack of adequate capacity and potential inability to secure adequate volumes of components.

As suppliers risk the burden of over-capacity if realized demand is low, they may invest in less capacity than desired by the manufacturer, with the result that if realized demand is high the manufacturer may be constrained by supplier capacities. A manufacturer may therefore wish to induce a higher supplier capacity. "Share-the-pain agreements, requiring both parties to share equally the burden of future over-capacity"

(Arensman 2000) are becoming more common. For instance, manufacturers enter into supply contracts in which they make a firm commitment regarding the level of orders to be placed with a supplier and thus they share the supplier's potential pain. According to the president of Advanced Micro Devices' (AMD) Memory Group, "supply agreements are important to chip makers because they guarantee that the billions of dollars invested in new production facilities will actually be used" (Associated Press 2001). Manufacturers may also prepay suppliers for guaranteed production capacity.

In this paper, we consider contracts in which the parties share the benefits of high demand rather than the pain of low demand. We term such contracts "share the gain" contracts. Subject to the mild assumption that the sum of the parties' reservation profits is no greater than the coordinated supply chain profit, we prove the existence of a class of coordinating price-only contracts that arbitrarily allocate the supply chain profit. Moreover, if the supplier's reservation profit is below a certain threshold, the manufacturer's optimal contract is a share-the-gain type.

As noted by Cachon and Lariviere (2001), capacity investments may be difficult to verify and thus unenforceable. They show that in the case of full information and voluntary compliance, neither firm commitment nor option contracts increase supplier capacity. We enrich the voluntary compliance regime of Cachon and Lariviere and demonstrate that with this enrichment, firm commitment and options contracts can indeed increase supplier capacity.

The remainder of the paper is structured as follows. Section 2 introduces the basic model and §3 discusses the related literature. Section 4 briefly covers linear price-only contracts. Nonlinear price-only contracts are considered in §5. Extensions to the basic model are covered in §6. Share-the-pain and share-the-gain contracts are compared in §7. Conclusions are presented in §8. All proofs are contained in the Appendix.

2. The Basic Model

The supply chain comprises a manufacturer and a supplier. The supplier provides a key component to

the manufacturer who in turn processes this component into an end product and sells it to the market. Both the supplier and the manufacturer must invest in capacities before demand for the end product is known (lead times for capacity installation prohibit periodic investments over the life of the product). Capacities are expressed in units of the end product. Both parties are risk neutral. The manufacturer (supplier) participates if his (her) expected profit is at least Π_M^R (Π_S^R). The demand distribution, $F_X[x]$, is continuous and twice differentiable with support of the form $[a, b]$ for $0 \leq a < b \leq \infty$.

The manufacturer (supplier) incurs a constant marginal capacity cost of c_M (c_S) and a constant marginal processing cost of p_M (p_S). After demand is realized, the manufacturer orders and pays for a quantity of supplier components. Each component is processed and sold to the market at a fixed retail price of r with $r > c_M + c_S + p_M + p_S$. Unused capacity can be salvaged by the manufacturer (supplier) at a constant marginal value of v_M (v_S).

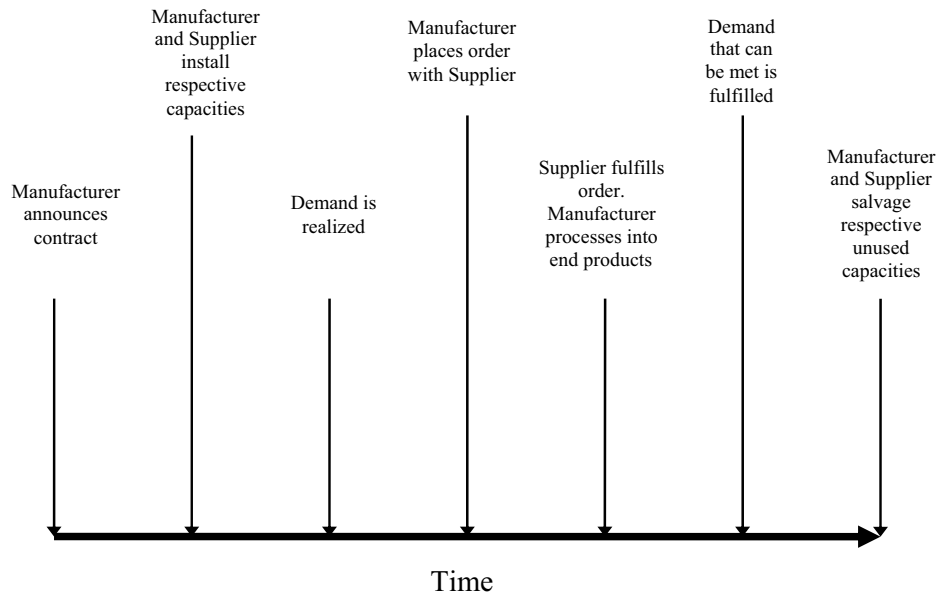
The manufacturer presents a contract, Ω , to the supplier. The contract specifies the price schedule, $W(Q)$, paid by the manufacturer for Q units of supplier components. It also specifies any additional transfer payments. Without loss of generality, $W(0) = 0$ as any fixed fees are assumed to be part of the additional transfer. $W(Q)$ is assumed to be nondecreasing in the order size Q . Furthermore, $(r - p_M)Q - W(Q)$ is assumed to be nondecreasing in Q . In other words, the manufacturer's marginal revenue earned from the Q th unit exceeds his marginal costs of procurement and processing. This mild condition ensures the manufacturer orders and sells $\min\{y_{SC}, x\}$ units where y_{SC} is the supply chain capacity.

A central decision maker optimizing the supply chain would choose a capacity, y_{SC}^l , to maximize the supply chain expected profit, $\Pi_{SC}(y)$:

$$\begin{aligned} \Pi_{SC}(y) = & -(c_S + c_M)y + \int_a^y (r - p_S - p_M)x f_X(x) dx \\ & + (r - p_S - p_M)y \bar{F}_X[y] \\ & + (v_S + v_M) \int_a^y (y - x) f_X(x) dx. \end{aligned}$$

There is, however, no central decision maker. Rather, we have two parties facing their own individual

Figure 1 Sequence of Events



optimization problems. The sequence of events is depicted in Figure 1. While capacities are built ahead of demand, the end product is built after demand is realized. We note that one could view this game as one in which two components (capacities) are built-to-stock and the end product is assembled-to-order by the manufacturer. We postpone the formalization of the relevant optimization problems for the manufacturer and the supplier until §4.

An extensive numerical study comprising 405 separate supply chain instances was carried out to enrich the theoretical results developed in the paper. The 405 instances can be grouped into 5 distinct sets. Each set is identical except for the coefficient of variation of demand (COV). Five different COVs were used—0.2, 0.4, 0.6, 0.8, 1.0. In each set the capacity and processing cost parameters (c_M , p_M , c_S , and p_S) took on low, medium, and high values (2, 5, and 8, respectively). Thus, each set had $3^4 = 81$ instances. In all instances a value of 35 was used for the revenue (r) and the salvage values (v_M and v_S) were 20% of the relevant capacity costs. A normal distribution (truncated at zero) with a mean demand of 200 was used for all instances.

3. Literature Review

This paper addresses capacity procurement in a decentralized supply chain. We focus our literature review on positioning the contribution relative to other papers in the supply chain contracting domain and do not attempt to provide a comprehensive review of the supply chain contracting literature. Reviews can be found in Tsay et al. (1998), Lariviere (1998), and Cachon (2002).

Cachon and Lariviere (2001) consider a closely related model in which the manufacturer is simply a costless reseller and as such has no capacity investment decision. The authors show that advanced contracts (such as firm commitments and options) can be used to signal demand types in the presence of asymmetric information but that firm commitments and options contracts do not induce higher supplier capacity in the full information case if supplier compliance is voluntary. Our paper differs from Cachon and Lariviere on three key dimensions. First, we enrich their compliance regimes and prove that firm commitments and options do indeed increase supplier capacity in the full information case even in the absence of forced compliance. This enrichment (see §7) can be thought of as an intermediate regime between the extreme cases of forced and voluntary

compliance. We allow for partial compliance by recognizing that the manufacturer does actually have some ability to identify lack of compliance (or shirking) on the part of the supplier. We note that Erkoc and Wu (2002) consider an extension of voluntary compliance in which there is a per unit penalty cost for noncompliance but the authors implicitly assume that the supplier capacity is readily verifiable. We make no such assumption. Second, the advanced contracts considered by Cachon and Lariviere all place some minimum value on the supplier's revenue irrespective of market demand. We focus attention on nonlinear price-only contracts that offer no such floor. Third, we extend our analysis of nonlinear pricing to allow for assembly systems (by showing that they can be collapsed to single-supplier systems). We note that Gerchak and Wang (2003) study a two-supplier system but focus on linear pricing to investigate the benefit of balancing wholesale prices in relation to supplier costs.

Many contracts in the literature compensate the follower (e.g., the retailer in the selling-to-the-newsvendor context or the supplier in the capacity-procurement context) for uncertain demand by placing a minimum value, or floor, on its revenue irrespective of demand. Quantity-flexible contracts (e.g., Lariviere 1998, Tsay 1999), options contracts (e.g., Barnes-Schuster et al. 2002, Cachon and Lariviere 2001), and firm-commitment contracts (e.g., Anupindi and Bassok 1998, Cachon and Lariviere 2001) all guarantee some minimum level of revenue to the follower irrespective of demand. Similarly, returns (or buy-back) contracts in produce-to-stock systems (Pasternack 1985) guarantee the retailer a revenue of at least yb if he stocks y units, where b is the per unit buy-back price. Instead of sharing the pain of low demand (by providing a floor on revenue), we consider nonlinear price-only contracts that offer no such floor, but that share the benefits of high demands by providing increased average unit margins. We provide a mechanism (for capacity procurement games) to classify contracts as share-the-pain or share-the-gain types and compare them to see in what circumstances one type is preferable over the other.

It is well known in the inventory literature that a supplier facing fixed costs can induce larger customer

orders by offering quantity discounts. The supplier can benefit as larger orders offset fixed costs. See, for example, Monahan (1984), Lal and Staelin (1984), Lee and Rosenblatt (1986), and Corbett and de Groote (2000). Jeuland and Shugan (1983) and Moorthy (1987) consider pricing decisions in a two-party supply chain facing both fixed costs and deterministic price-sensitive demand (but in the absence of inventory dynamics), although Moorthy argues in favour of a two-part tariff. They demonstrate that nonlinear price schedules achieve channel coordination. Weng (1995) and Chen et al. (2001) bridge the above sets of work to consider deterministic price-sensitive demand and inventory dynamics. The authors demonstrate that quantity discounts again aid in channel coordination. Barnes-Schuster et al. (2002) consider quantity discounts in a two-party supply chain facing uncertain demand in two periods in which production in the second period is more costly than in the first. This difference in production costs introduces a nonlinearity to the supplier's marginal processing costs, a key element in explaining the benefit of quantity discounts. In this paper, we consider nonlinear pricing in the absence of any fixed costs or nonlinear marginal processing costs. We note that in a two-market newsvendor context, Kouvelis and Gutierrez (1997) show that nonlinear contracts are useful even though fixed costs are zero and processing costs are linear. However, their result requires the presence of a "middle manager" that is not measured financially. We do not require a middle manager.

4. Linear Price-Only Contracts

In this section, we assume the reservation profits $\Pi_M^R = \Pi_S^R = 0$. A linear price-only contract, denoted $\Omega_{\text{linear}}(w)$, is one in which the manufacturer pays a constant price per unit of w . There are no other transfer payments. For a given supplier capacity, the manufacturer faces a newsvendor-type problem and chooses his optimal capacity, $y_M^*(y_S, w)$, to maximize his expected profit, $\Pi_M(y_M, y_S, w)$:

$$\begin{aligned}\Pi_M(y_M, y_S, w) \\ = -c_M y_M + \int_a^{y_{SC}(y_S, y_M)} (r - w - p_M) x f_X(x) dx\end{aligned}$$

$$+ (r - w - p_M) \bar{F}_X[y_{SC}(y_S, y_M)] \\ + v_M \int_a^{y_{SC}(y_S, y_M)} (y_M - x) f_X(x) dx,$$

where $y_{SC}(y_M, y_S) = \min\{y_M, y_S\}$. For a given manufacturer capacity, y_M , the supplier also faces a newsvendor-type problem and chooses her optimal capacity, $y_S^*(y_M, w)$, to maximize her expected profit, $\Pi_S(y_S, y_M, w)$:

$$\begin{aligned} \Pi_S(y_S, y_M, w) = & -c_S y_S + \int_a^{y_{SC}(y_M, y_S)} (w - p_S) x f_X(x) dx \\ & + (w - p_S) \bar{F}_X[y_{SC}(y_S, y_M)] \\ & + v_S \int_a^{y_{SC}(y_M, y_S)} (y_S - x) f_X(x) dx. \end{aligned}$$

The supplier's capacity choice is concave in w if and only if $f'_X(x)[1 - F_X(x)]/[f_X(x)]^2 \geq -2 \forall x \in [a, b]$. See Tomlin (2000) for the proof. This condition is assumed to hold. Increasing failure rate (IFR) distributions satisfy the condition. Cachon and Lariviere (2001) assume that the supplier's capacity choice is concave in the wholesale price and note that an IFR distribution is sufficient but not necessary for concavity. They remark that the distribution $F_X[x] = 1 - x^{-k}$ for $x \geq 1$ and $k \geq 1$ is not IFR but does induce concavity. We note that this distribution does satisfy the concavity condition above. For a given value of w , the players' optimal capacities both equal $y_{SC}(w)$:

$$\begin{aligned} y_{SC}(w) &= \min\{y_M(w), y_S(w)\} \\ &= \min\left\{F_X^{-1}\left(\frac{r - w - p_M - c_M}{r - w - p_M - v_M}\right), F_X^{-1}\left(\frac{w - p_S - c_S}{w - p_S - v_S}\right)\right\}, \end{aligned}$$

where $y_M(w) = y_M^*(y_S = \infty, w)$ and $y_S(w) = y_S^*(y_M = \infty, w)$, i.e., the manufacturer and supplier capacity choices if the other party had infinite capacity. See Proposition 1 in the Appendix for the proof.

The manufacturer is faced with a game in which he optimizes the constant wholesale price to maximize his expected profit, $\Pi_M(w)$, knowing how the supplier will respond (i.e., what capacity she will build) to any given wholesale price announced:

$$\begin{aligned} \Pi_M(w) = & -c_M y_M + \int_a^{y_{SC}(w)} (r - w - p_M) x f_X(x) dx \\ & + (r - w - p_M) \bar{F}_X[y_{SC}(w)] \\ & + v_M \int_a^{y_{SC}(w)} (y_M - x) f_X(x) dx. \end{aligned}$$

THEOREM 1. (i) *There is a unique linear price schedule $W(Q) = w_{\text{crit}}Q$ that coordinates the supply chain*

$$w_{\text{crit}} = \frac{(r - p_M)(c_S - v_S) + p_S(c_M - v_M) + c_M v_S - c_S v_M}{(c_S - v_S + c_M - v_M)}.$$

(ii) *The manufacturer's optimal linear price schedule $W(Q) = w_M^*Q$ is such that $w_M^* < w_{\text{crit}}$.*

So, the manufacturer chooses a wholesale price such that the supplier limits the supply chain capacity and the supply chain is not coordinated. In this supply chain, a linear price-only contract is equivalent to a revenue-sharing contract in which w/r of the revenue is allocated to the supplier and $1 - w/r$ of the revenue is allocated to the manufacturer. As such, the above result also applies to revenue-sharing contracts.

We note that $c_S = 0 \Rightarrow w_{\text{crit}} = p_S$ and $c_M = 0 \Rightarrow w_{\text{crit}} = r - p_M$. In other words, if one party has capacity cost of zero, then the only linear price schedule that coordinates the chain is one that yields a marginal processing profit of zero for this party. Thus, the party with positive capacity cost must have a marginal expected processing profit equal to the supply chain's marginal expected processing profit (processing profit excludes capacity costs and salvage values). To see this, consider that a central decision maker chooses capacities by balancing the marginal expected supply chain capacity cost (including marginal salvage value) with the marginal expected supply chain processing profit. If one party has a capacity cost of zero, then the other party's marginal expected capacity cost equals that of the supply chain. This other party chooses capacity by balancing marginal capacity costs with marginal expected processing profit and so, for linear price schedules, chooses the coordinating capacity only if the marginal expected processing profit equals the supply chain's marginal expected processing profit. Thus, only a price schedule that yields zero marginal processing profit to the party with zero capacity cost coordinates the chain. This is reminiscent of the selling-to-the-newsvendor result.

When both parties have positive capacity costs, each party faces a marginal expected capacity cost that is a fraction of the supply chain marginal expected capacity cost. Therefore, the required marginal expected processing profit at the coordi-

Table 1 Inefficiencies and Optimal Wholesale Price for Linear Price Schedules

	Coefficient of Variation of Demand					Manufacturer Capacity Cost			Supplier Capacity Cost		
	0.2	0.4	0.6	0.8	1.0	2	5	8	2	5	8
Inefficiency (%)	5.91	7.39	8.13	8.78	9.67	9.15	7.91	6.87	4.87	8.35	10.59
Optimal price, w	10.66	11.21	11.65	12.04	12.39	11.95	11.59	11.23	8.37	11.69	14.62

nating capacity need only be a fraction of the supply chain expected processing profit at this capacity. Because of this, “zero-profit” pricing is not required to coordinate the supply chain in the case of linear schedules, i.e., $p_S < w_{\text{crit}} < r - p_M$.

In the numeric study, the average inefficiency over the 405 instances was 7.98% with a maximum of 15.98% and a minimum of 3.04%. The impact of the COV, the supplier’s capacity cost, c_S , and the manufacturer’s capacity cost, c_M , on the inefficiency and the optimal wholesale price can be seen in Table 1. The inefficiency is increasing in the COV, increasing in c_S , and decreasing in c_M . Likewise, the optimal wholesale price is increasing in the COV, increasing in c_S , and decreasing in c_M . The impact of the COV on the wholesale price is reminiscent of Theorem 4 in Cachon and Lariviere (2001).

Rather than present 405 different results for each metric of interest, we instead report averages. We note that the behaviour reflected by the averages also held at the level of individual instances. We also note that the averages are comparable in the sense that they correspond to the same set of instances. For example, the average inefficiency for a COV of 0.4 is taken over the exact same set of 81 instances as is the average inefficiency for a COV of 0.6 and the average inefficiency for a supplier capacity cost of 2 is taken over the exact same set of 135 instances as is the average inefficiency for a supplier capacity cost of 8.

The inefficiency associated with linear price schedules might be eliminated by an alternative schedule, but the manufacturer is only interested in such a schedule if his expected profit is at least as large as with a linear price schedule.

5. Quantity-Premium Price-Only Contracts

Consider a game in which the manufacturer can choose any nonlinear price-only contract. This

game will be denoted Game B. A continuous and twice-differentiable price schedule, $W(Q)$, is a quantity-premium schedule if the marginal price is nondecreasing for all order sizes and increasing over some range. The price schedule

$$\frac{dW(Q)}{dQ} = \frac{c_S - v_S F_X[Q]}{1 - F_X[Q]} + p_S \quad (1)$$

is a quantity-premium schedule as $dW(Q)/dQ > 0$ and $d^2W(Q)/dQ^2 = (c_S - v_S f_X(Q))/(\bar{F}_X[Q])^2 > 0$. We note that the manufacturer will order the minimum of demand and supply chain capacity for this price schedule.

THEOREM 2. *If the supplier’s reservation profit, Π_S^R , is zero, then (1) is an optimal schedule for the manufacturer and it coordinates the supply chain. In addition, the manufacturer captures all of the supply chain expected profit.*

The supplier’s marginal expected revenue for the Q th unit is $[1 - F_X[Q]](dW(Q)/dQ)$ as revenue is generated only if demand for the Q th unit is realized. The marginal expected capacity cost for the Q th unit is $c_S - v_S F_X[Q]$, on accounting for the potential salvage revenue generated. The marginal expected processing cost for the Q th unit is $p_S[1 - F_X[Q]]$ as the cost is incurred only if demand for the Q th unit is realized. Setting the marginal expected revenue equal to the marginal expected capacity and processing cost yields a marginal unit price for the Q th unit of $dW(Q)/dQ = (c_S - v_S F_X[Q])/(1 - F_X[Q]) + p_S$. Therefore, (1) leaves the supplier with an expected profit of zero regardless of her capacity choice. The manufacturer captures all of the expected supply chain profit and so chooses a capacity that maximizes the supply chain profit.

The coordination property is interesting as the manufacturer’s goal is to maximize his profit and not the supply chain profit. However, while the supply chain is coordinated, the supplier is left with zero expected profit. A question arises as to whether

there are coordinating price-only contracts that yield a positive expected profit for the supplier. Consider the following price schedule with $\beta \in [0, 1]$:

$$\frac{dW^\beta(Q)}{dQ} = \beta \left(r - p_M - \frac{c_M - v_M F_X[Q]}{1 - F_X[Q]} \right) + (1 - \beta) \left(\frac{c_S - v_S F_X[Q]}{1 - F_X[Q]} + p_S \right). \quad (2)$$

THEOREM 3. (i) If (2) is used, then the manufacturer and supplier's expected profits are $(1 - \beta)\Pi_{SC}(y)$ and $\beta\Pi_{SC}(y)$, respectively, for a supply chain capacity of y .

(ii) If (2) is used, then the supply chain is coordinated.

(iii) Let $\beta_T = (c_S - v_S)/(c_S - v_S + c_M - v_M)$. If $\beta < \beta_T$, then (2) is a quantity-premium schedule; if $\beta = \beta_T$, then (2) is a linear schedule; and if $\beta > \beta_T$, then (2) is a quantity-discount schedule.

So, there exists a class of nonlinear contracts that coordinates the supply chain and provides for an arbitrary division of the expected profits between the parties. If the supplier's reservation profit (Π_S^R) is strictly positive, then the manufacturer's optimal contract is to implement (2) while choosing $\beta = \Pi_S^R/\Pi_{SC}^I$, where Π_{SC}^I is the coordinated supply chain profit. The quantity-premium schedule (1) can then be viewed as a special case of (2) with $\Pi_S^R = 0$. At $\beta = \beta_T$, the price schedule is a linear one and equals w_{crit} from Theorem 1. We note that at the coordinating capacity, y_{SC}^I , $dW^\beta(Q)/dQ = w_{crit}$ for all $\beta \in [0, 1]$.

If $c_S = 0$, then the manufacturer's and the supply chain's marginal expected capacity costs both equal $c_M - v_M F_X[Q]$. The manufacturer's marginal expected processing profit equals $(r - p_M - dW^\beta(Q)/dQ)\bar{F}_X[Q]$ while the supply chain's equals $(r - p_M - p_S)\bar{F}_X[Q]$. At the coordinating capacity, y_{SC}^I , $dW^\beta(Q)/dQ$ must equal p_S if coordination is to occur. Consider a quantity-premium schedule with $dW^\beta(Q)/dQ = p_S$ at y_{SC}^I . This implies that $dW^\beta(Q)/dQ < p_S$ for all $Q < y_{SC}^I$, and so the supplier would lose money on every unit up until y_{SC}^I . Clearly, the supplier would not sell these units to the manufacturer and thus a quantity premium cannot coordinate the supply chain if $c_S = 0$. Likewise, a quantity discount cannot coordinate the supply chain if $c_M = 0$. This is reflected in Theorem 3, Part (iii) by the fact that $c_S = 0 \Rightarrow \beta_T = 0$ and $c_M = 0 \Rightarrow \beta_T = 1$. To recap, if the supplier has

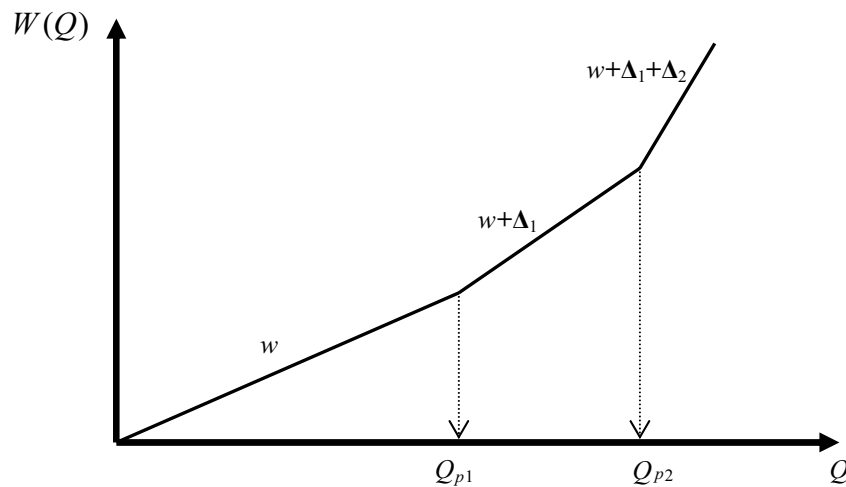
a zero capacity cost, then quantity premiums cannot coordinate and if the manufacturer has a zero capacity cost, then quantity discounts cannot coordinate. If both have positive capacity costs, then both types can coordinate. Recall from the discussion after Theorem 1 that "zero-profit" pricing was not required for a linear schedule to coordinate the chain and that the wholesale price that coordinated the chain was $p_S < w_{crit} < r - p_M$. Because $dW^\beta(Q)/dQ = w_{crit}$ at y_{SC}^I for all $\beta \in [0, 1]$, both quantity premiums and discounts can provide positive processing profits to both parties while still achieving coordination.

While schedule (2) acts like a profit-sharing contract in the expectation, it does not do so in the realization. To see this, consider what occurs in a profit-sharing contract (for a capacity investment of y) if demand is zero. If the supplier's realized profit is to equal β of the realized supply chain profit (which is negative), i.e., $\pi_s(y) = \beta\pi_{SC}(y) = -\beta(c_S y + c_M y)$, a transfer of $(1 - \beta)c_S - \beta c_M$ between the manufacturer and the supplier is required. In a price-only contract, such as (2), there is no transfer if demand is zero and so the supplier's realized profit would not equal $\beta\Pi_{SC}(y)$.

In what follows, we assume $\Pi_S^R = 0$ and focus on price schedule (1). Such a continuous quantity-premium price schedule may be optimal for the manufacturer but may be difficult to implement in practice. Piecewise-linear quantity-premium schedules with one or two breakpoints (as illustrated in Figure 2) are more easily implemented.

For a single-breakpoint schedule, denoted (w, Δ_1) , with an initial unit price of w , we set the breakpoint, Q_p , equal to $y_S(w)$, the capacity the supplier is willing to invest in for a linear price schedule specified by w . The manufacturer is only interested in offering a quantity premium when it wants the supplier to invest in more capacity. For a given unit price of w , the supplier's profit is maximized at a capacity of $y_S(w)$. Setting the breakpoint Q_p to be lower than $y_S(w)$ rewards the supplier unnecessarily—there is no need to increase the marginal value of capacity at levels below $y_S(w)$. Setting the breakpoint above $y_S(w)$ may be beneficial but causes the supplier's profit to be a bimodal function of its capacity, thus introducing a technical wrinkle into the problem.

Figure 2 Piecewise-Linear Quantity Premium



For the sake of brevity and simplicity of presentation, we therefore assume Q_p equals $y_s(w)$ and note that this restriction does not alter the fundamental insight that piecewise-linear quantity premiums are a valuable tool. For a two-breakpoint schedule, denoted (w, Δ_1, Δ_2) , the first breakpoint, Q_p^1 , is set equal to $y_s(w)$, and the second, Q_p^2 , is set equal to $y_s(w, \Delta_1)$, where Δ_1 is the price premium paid on units between Q_p^1 and Q_p^2 and $y_s(w, \Delta_1)$ is the supplier capacity induced by a single-breakpoint schedule (w, Δ_1) .

THEOREM 4. (i) *The manufacturer's optimal single-breakpoint quantity-premium schedule delivers a strictly greater expected profit than his optimal linear schedule.*

(ii) *The supply chain's expected profit is also strictly greater but the supply chain is not coordinated.*

For each of the 405 instances in the numerical study the manufacturer's optimal single-breakpoint and his optimal two-breakpoint schedules were found and the resulting expected profits calculated. The average inefficiency for the single-breakpoint schedule was

2.18% with a maximum of 6.22% and a minimum of 0.32%. The average inefficiency for the two-breakpoint schedule was 0.98% with a maximum of 3.31% and a minimum of 0.12%. This implies that the performance of simple piecewise-linear quantity-premium schedules approaches that of the continuous quantity-premium schedule that coordinates the chain. We also calculated the relative change in expected profit for each of the parties obtained using the quantity-premium schedules instead of the linear schedule. As can be seen from Table 2, the manufacturer can benefit significantly by moving to a quantity-premium schedule while the supplier is significantly worse off. Again, the simple piecewise-linear schedules deliver much of the benefit that is delivered by the continuous quantity-premium schedule which was shown earlier to be optimal for the manufacturer.

The impact of the COV and the capacity costs on the inefficiencies, percentage change in the supply chain, the manufacturer's and the supplier's expected

Table 2 Inefficiencies and % Changes in Expected Profits

Wholesale Price Schedule	Inefficiency (%)	Increase in Supply Chain Profit Over Linear Schedule (%)	Increase in Manufacturer Profit Over Linear Schedule (%)	Increase in Supplier Profit Over Linear Schedule (%)
Linear	7.98	—	—	—
Single-breakpoint quantity premium	2.18	6.36	10.75	-36.68
Two-breakpoint quantity premium	0.98	7.70	14.21	-55.37
Continuous quantity premium	0.00	8.78	20.53	-100.00

Table 3 Average Inefficiencies for Linear and Quantity-Premium Price Schedules

	Coefficient of Variation of Demand (%)					Manufacturer Capacity Cost (%)			Supplier Capacity Cost (%)		
	0.2	0.4	0.6	0.8	1.0	2	5	8	2	5	8
Linear	5.91	7.39	8.13	8.78	9.67	9.15	7.91	6.87	4.87	8.35	10.59
Single-breakpoint	1.19	1.82	2.21	2.57	3.11	2.78	2.13	1.64	1.06	2.28	3.14
Two-breakpoint	0.43	0.74	0.96	1.19	1.55	1.30	0.93	0.70	0.42	1.01	1.46
Continuous	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

profits for the single-breakpoint, two-breakpoint, and continuous quantity-premium schedules can be found in Tables 3, 4, 5, and 6. As with the linear price schedule, the inefficiency is increasing in the COV, increasing in c_s , and decreasing in c_M for the piecewise-linear quantity-premium schedules. Higher inefficiencies mean that a larger fraction of the potential profit is lost. Therefore, in circumstances that lead to high inefficiencies for linear schedules (high coefficient of variation, high supplier capacity cost, low manufacturer capacity cost), one might expect quantity-premium schedules to be particularly beneficial. This is reflected in the numerical results. The manufacturer's benefit is increasing in the COV, increasing in c_s , and decreasing in c_M while the supplier's loss is decreasing in the COV, increasing in c_s , and decreasing in c_M .

The supplier can be significantly disadvantaged when the manufacturer moves from his optimal linear schedule to his optimal single-breakpoint schedule. Indeed, the manufacturer's optimal quantity-premium schedule leaves the supplier with an expected profit of zero. What if the manufacturer either cannot impose a price schedule or does not wish to reduce the supplier's expected profit? The following theorem shows that he can offer a single-breakpoint schedule that benefits both parties.

THEOREM 5. *If the manufacturer keeps the per unit wholesale price at the optimum linear schedule value w_M^* but chooses its optimal single-breakpoint quantity-premium schedule $(w_M^*, \Delta^*(w_M^*))$ for this w_M^* , then both parties' expected profits increase over the linear schedule.*

6. Extensions to the Model

In this section, we extend the results to assembly systems, to systems with nonlinear processing costs, and to situations in which the supplier is the leader.

Assembly Supply Chains. Let there be N suppliers with each supplier providing a different component to the manufacturer. The manufacturer assembles the N components into the end product. For piecewise-linear price schedules, this N supplier system can be collapsed to an equivalent single-supplier system. Specifically, if the supplier processing costs and salvage values are zero, then a piecewise-linear schedule (w, Δ_1, Δ_2) in the single-supplier system is equivalent to a piecewise-linear schedule for the n th supplier of $(\alpha_n w, \alpha_n \Delta_1, \alpha_n \Delta_2)$, where α_n is a constant given by $\alpha_n = (c_{S_n} / \sum_{n=1}^N c_{S_n})$ and c_{S_n} is the capacity cost of the n th supplier. See Tomlin (2000) for details. Therefore, the theoretical results for linear and piecewise-linear schedules carry over directly to single-tier assembly systems.

Table 4 % Increase in Supply Chain Profit for Linear and Quantity-Premium Price Schedules

	Coefficient of Variation of Demand (%)					Manufacturer Capacity Cost (%)			Supplier Capacity Cost (%)		
	0.2	0.4	0.6	0.8	1.0	2	5	8	2	5	8
Linear	—	—	—	—	—	—	—	—	—	—	—
Single-breakpoint	5.04	6.06	6.50	6.87	7.35	7.08	6.34	5.67	4.01	6.65	8.36
Two-breakpoint	5.85	7.24	7.87	8.41	9.10	8.74	7.66	6.69	4.69	8.04	10.26
Continuous	6.31	8.05	8.94	9.74	10.85	10.19	8.69	7.46	5.13	9.15	11.91

Table 5 % Increase in Manufacturer Profit for Linear and Quantity-Premium Price Schedules

	Coefficient of Variation of Demand (%)					Manufacturer Capacity Cost (%)			Supplier Capacity Cost (%)		
	0.2	0.4	0.6	0.8	1.0	2	5	8	2	5	8
Linear	—	—	—	—	—	—	—	—	—	—	—
Single-breakpoint	7.07	9.74	11.27	12.36	13.32	11.31	10.76	10.20	7.58	11.12	13.18
Two-breakpoint	8.81	12.62	14.92	16.61	18.08	15.07	14.19	13.36	9.68	14.67	17.70
Continuous	11.10	17.02	21.24	24.80	28.51	22.20	20.43	18.97	12.90	21.05	26.56

Nonlinear Processing Costs. Define $P_S(Q)$ ($P_M(Q)$) as the supplier's (manufacturer's) total processing cost for processing Q units. Let $P_S(Q)$ and $P_M(Q)$ be twice differentiable and let the sum of the marginal processing costs be less than the retail price for all order sizes within the demand range. Define a price schedule (3) by $dW(Q)/dQ = (c_s - v_s F_X[Q]) / (1 - F_X[Q]) + dP_S(Q)/dQ$. This is a generalization of schedule (1). (3) is an optimal schedule for the manufacturer (if $\Pi_S^R = 0$). It enables the manufacturer to capture all of the supply chain's expected profit. While the price schedule is not necessarily a quantity-premium schedule, the price schedule ensures that the supplier's marginal processing profit increases in the order size. The proof is available from the author upon request.

Supplier Is the Leader. Define the price-difference schedule, $PD(Q)$, for either party as the total revenue received from selling Q units less the total price paid for the Q units. For the manufacturer, $PD_M(Q) = rQ - W(Q)$. For the supplier, $PD_S(Q) = W(Q)$ as the supplier's purchase cost is zero. For a given retail price, the wholesale price schedule is specified by the price-difference schedule of either party. Note also that $PD_M(Q) = rQ - PD_S(Q)$ and $PD_S(Q) = rQ - PD_M(Q)$, so the price-difference schedule of one party is completely specified by the price-difference schedule of the other party.

Moreover, if the follower's price-difference schedule is specified by $PD_F(Q)$, then the leader's price schedule is given by $rQ - PD_F(Q)$ irrespective of which party is the leader. Let p_L and p_F be the constant marginal processing cost of the leader and follower, respectively. The leader's total profit from fulfilling an order of Q units is then given by $rQ - PD_F(Q) - p_L Q$ and the follower's by $PD_F(Q) - p_F Q$. One can carry out the entire analysis of earlier sections in terms of a leader and a follower's price-difference schedule rather than a manufacturer and a supplier's price schedule. Therefore, all the results carry through directly to the case where the supplier is the leader. There is, however, one important caveat. All references to a price schedule should instead be interpreted as the follower's price-difference schedule. If the supplier is the leader, she offers the manufacturer a quantity discount. We note that similar to sales rebates in the selling-to-the-newsvendor context (Taylor 2002), a single-breakpoint quantity-discount schedule increases the marginal sales profit for the downstream party.

7. Share-the-Pain Contracts

For any contract, Ω , define $T[Q, y_S, \Omega]$ as the transfer payment from manufacturer to supplier if the manufacturer orders Q units and the supplier capacity is y_S .

Table 6 % Decrease in Supplier Profit for Linear and Quantity-Premium Price Schedules

	Coefficient of Variation of Demand (%)					Manufacturer Capacity Cost (%)			Supplier Capacity Cost (%)		
	0.2	0.4	0.6	0.8	1.0	2	5	8	2	5	8
Linear	—	—	—	—	—	—	—	—	—	—	—
Single-breakpoint	40.78	39.48	37.05	34.53	31.56	34.01	36.96	39.06	45.78	35.55	29.01
Two-breakpoint	60.74	58.67	55.88	52.77	48.78	51.90	55.54	58.67	64.48	54.15	47.78
Continuous	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

The transfer payment may include side payments that are a function of the manufacturer's order size. We assume the manufacturer never presents the supplier with a contract such that the supply chain capacity is limited by the manufacturer's capacity choice. As in §4, such a contract can always be improved upon from the manufacturer's perspective as the terms can be modified so that the supply chain capacity remains the same (but with the supplier capacity choice being limiting) while the transfer payment decreases. We also assume that all contract parameters are restricted to those that ensure that the manufacturer orders $Q(x) = \min\{x, y_s\}$, where x is the demand. A linear price-only contract, $\Omega_{\text{linear}}(w)$, results in a transfer payment of $T[Q, y_s, \Omega_{\text{linear}}(w)] = w \min\{y_s, Q\}$. Define $I[x, y_s, \Omega, w] = T[Q(x), y_s, \Omega] - T[Q(x), y_s, \Omega_{\text{linear}}(w)]$ as the difference (positive or negative) in transfer payments between contract Ω and the linear price-only contract when the market demand is x .

Consider a quantity-premium contract specified by $W(Q)$. $T[Q, y_s, \Omega_{\text{premium}}] = W(\min\{y_s, Q\})$ and $I[x, y_s, \Omega_{\text{premium}}, w] = W(\min\{y_s, x\}) - w \min\{y_s, x\}$. Let ε be an arbitrarily small positive number. If $W(\varepsilon) \geq w\varepsilon \Rightarrow I[x, y_s, \Omega_{\text{premium}}, w] \geq 0 \forall x$ (i.e., for all demands, the manufacturer transfers at least as much money to the supplier as in the linear contract). Additionally, $I[x, y_s, \Omega_{\text{premium}}, w]$ increases in the demand and so the supplier is increasingly better off (relative to the linear contract) as demand increases.

We define "share the gain" contracts as ones in which $I[x, y_s, \Omega, w] \geq 0 \forall x$ and $I[x_1, y_s, \Omega, w] \leq I[x_2, y_s, \Omega, w] \forall x_1 \leq x_2$. We define "share the pain" contracts as ones in which $I[x, y_s, \Omega, w] \geq 0 \forall x$ and $I[x_1, y_s, \Omega, w] \geq I[x_2, y_s, \Omega, w] \forall x_1 \leq x_2$ as the degree to which the supplier is better off relative to the linear contract decreases in the demand.

Define a reimbursement contract, $\Omega_{\text{reimburse}}(w, b)$, as one in which the manufacturer pays w per unit delivered and b per unit of unused supplier capacity. Enforcement of this contract assumes that the manufacturer can verify the capacity built by the supplier. This contract is closely related to the buy-back contract in a produce-to-stock supply chain. A reimbursement contract results in a transfer payment of $T[Q, y_s, \Omega_{\text{reimburse}}(w, b)] = w \min\{y_s, Q\} + b[y_s - Q]^+$.

Define a firm-commitment contract, $\Omega_{\text{firm}}(w, y_{\text{firm}})$, as one in which the manufacturer pays w per unit ordered (and delivered) and also $(w - p_s - v_s)[y_{\text{firm}} - Q]^+$ if the order $Q < y_{\text{firm}}$. In essence, this is equivalent to the typical firm commitment in which the manufacturer commits to order at least y_{firm} . The supplier is guaranteed to earn a (post-capacity investment) profit of at least $(w - p_s)y_{\text{firm}}$ irrespective of market demand. However, it provides an incentive for the supplier to produce-to-order rather than produce-to-stock, as the supplier may not recoup processing costs if she produces-to-stock. Consider a case where the supplier does not build sufficient capacity to cover the firm commitments, y_{firm} . The supplier can fulfill at most $y_s < y_{\text{firm}}$ units. If the market demand is $x \leq y_s$, then the supplier's lack of capacity is not revealed. In this case, the manufacturer pays the supplier for the full amount of firm commitments. If $x > y_s$, then the supplier's lack of capacity is revealed and we assume that the manufacturer does not pay for the undelivered firm commitments. This differs from Cachon and Lariviere (2001), where the firm-commitment payment is made irrespective of whether the supplier can deliver y_{firm} . We thus enrich the voluntary compliance regime of Cachon and Lariviere by noting that while capacity may not be verifiable before market demand is realized, the supplier's failure to build the agreed capacity may be exposed after demand is realized. One could think of this as partial compliance with the manufacturer having some ability to identify shirking on the part of the supplier. The transfer payment function is

$$T[Q, y_s, \Omega_{\text{firm}}(w, y_{\text{firm}})] = \begin{cases} w \min\{y_s, Q\} + (w - p_s - v_s)[y_{\text{firm}} - Q]^+, & y_{\text{firm}} \leq y_s, \\ wQ + (w - p_s - v_s)[y_{\text{firm}} - Q]^+, & y_{\text{firm}} > y_s, Q \leq y_s, \\ wy_s, & y_{\text{firm}} > y_s, Q > y_s. \end{cases}$$

Define an option contract, $\Omega_{\text{options}}(w, w_o, y_{\text{options}})$, as one in which the manufacturer buys y_{options} options from the supplier at a unit price of w_o and pays w per option exercised (and w per any additional unit above y_{options} ordered and delivered). As above, if

the supplier builds insufficient capacity to cover the options, the failure may be revealed to the manufacturer depending on the order size. If the failure is revealed, the manufacturer could either demand return of all option payments or just the portion for options not fulfilled. We assume the latter. Again, this differs from Cachon and Lariviere (2001) in which the total option payment is always made. An options contract results in a transfer payment function of

$$T[Q, y_S, \Omega_{\text{options}}(w, w_o, y_{\text{options}})] = \begin{cases} w \min\{y_S, Q\} + w_o y_{\text{options}}, & y_{\text{options}} \leq y_S, \\ w \min\{y_S, Q\} + w_o y_{\text{options}}, & y_{\text{options}} > y_S, Q \leq y_S, \\ w \min\{y_S, Q\} + w_o y_S, & y_{\text{options}} > y_S, Q > y_S. \end{cases}$$

THEOREM 6. *For a given price, w , the reimbursement, firm-commitment, and option contracts are all share-the-pain contracts and all induce a higher supplier capacity than the linear price-only contract.*

We note that in the construction of the transfer payment function, we assumed it was never optimal for the manufacturer to order more than the market demand, x . We also note that it might be optimal for the manufacturer to order more than y_S if demand is $x < y_S$ because while he would be buying units he cannot sell he would identify supplier shirking and thus reduce his overall payment because he no longer has to pay up to y_{firm} (or y_{options}). If one allows for this behaviour on the part of the manufacturer, the supplier's optimal capacity cannot be lower than in the absence of this behaviour. To see this, recall that because of full information, each party knows the optimal response of the opposite party to any action they take. As such, if it is optimal for the manufacturer to order more than x if demand is $x < y_S$, then the supplier will simply build a larger capacity as she will get a profit from fulfilling extra units and also lowers the probability that the manufacturer will identify her shirking. Therefore, if one allows for this behaviour, the resulting optimal supplier capacity cannot be lower than for the case where the manufacturer only ever orders market demand.

This capacity increase result differs from Cachon and Lariviere (2001), where firm commitments and options were shown never to increase the supplier

capacity in the voluntary compliance regime. Their result hinged on the fact that the supplier receives the full floor revenue associated with such contracts irrespective of the capacity she builds. By allowing for the fact that the supplier's capacity investment is in some sense verifiable after market demand is realized, we show that such contracts can indeed increase capacity. Note that the supplier does not necessarily build y_{firm} units of capacity as the marginal value of capacity may become negative for $y_S \leq y_{\text{firm}}$ and, therefore, firm-commitment contracts may not coordinate the supply chain. One could envisage some form of supplier penalty payment if failure to build sufficient capacity to cover the firm commitment is revealed. Such a penalty could be used to induce the supplier to build y_{firm} units.

Consider a situation in which the manufacturer has already agreed to pay a minimum wholesale price of $w < w_{\text{crit}}$. For a price-only linear contract the supplier's capacity choice, $y_S(\Omega_{\text{linear}}(w))$, is less than the manufacturer wants. If he wishes to induce a higher supplier capacity, say y_{target} , where $y_{\text{target}} \leq y_{\text{SC}}^l$ (the supply chain optimal capacity), then he needs to consider an alternative contract. Both firm-commitment and single-breakpoint quantity-premium contracts can be used to induce a higher supplier capacity. Which should he choose?

THEOREM 7. *If the demand distribution is IFR, then*

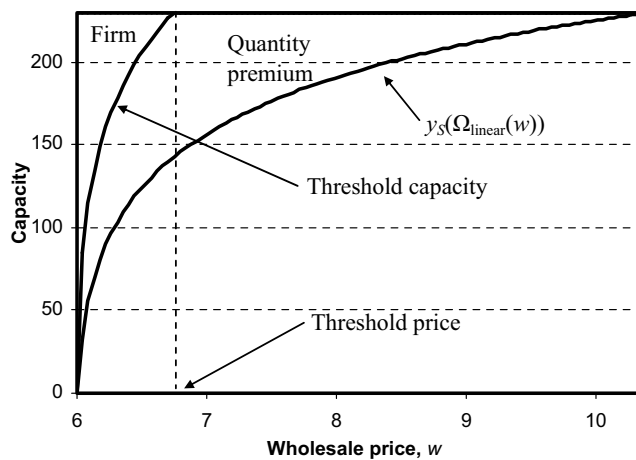
(i) *There exists a threshold price $w_{\text{threshold}}$ such that for $w \geq w_{\text{threshold}}$ the manufacturer prefers the premium contract for all $y_S(\Omega_{\text{linear}}(w)) < y_{\text{target}} \leq y_{\text{SC}}^*$.*

(ii) *For prices $w < w_{\text{threshold}}$, there exists a threshold target capacity such that the manufacturer prefers the price-premium contract for all $y_{\text{target}} \leq y_{\text{threshold}}(w)$ but prefers the firm-commitment contract for all $y_{\text{SC}}^l \geq y_{\text{target}} > y_{\text{threshold}}(w)$.*

(iii) *$y_{\text{threshold}}(w)$ increases in the price w .*

Figure 3 illustrates the theorem. There exists a per-unit price threshold above which quantity-premium contracts are always preferred and below which the firm-commitment contract is preferred only when the desired capacity is sufficiently high. This second point may seem counter-intuitive as for low capacities the floor payment associated with firm commitments is low. However, even though the floor payment may

Figure 3 Regions of Preference for Contracts



be low, the quantity premium is preferred as neither the per-unit quantity premium nor the expected quantity to which the premium applies is large. As the desired capacity becomes high, the quantity premium becomes relatively more expensive as both the per-unit quantity premium and the expected quantity to which the premium applies both become increasingly large.

In practice, firm-commitment contracts are a relatively common paradigm, whereas quantity-premium contracts are not readily observed. Why might this be? One reason might lie in the risk-neutrality assumption in this paper, and allowing for supplier risk aversion might lead to a preference for firm commitments. To see this, consider a supplier that faces some fixed cost if the capacity investment fails to yield some minimum revenue, for example, insufficient revenue might lead to a need for debt restructuring. A sufficiently large firm commitment that guarantees the minimum revenue might induce a higher supplier capacity than a quantity-premium contract that offers no such revenue guarantee.

8. Conclusion

In this paper, we introduce an alternative to share-the-pain contracts in which the parties share the gain of high demand rather than the pain of low demand. If the supplier's reservation profit is below a certain threshold, the manufacturer's optimal contract is a share-the-gain type. Specifically, it is a

quantity-premium price schedule. Simple piecewise-linear (one or two breakpoints) quantity-premium schedules are shown to be highly efficient. The intuition for such quantity-premium contracts is that the manufacturer can motivate a supplier to invest in more capacity by transferring a portion of his marginal profit to the supplier in high demand states. The supplier benefits from increased marginal profit in these high demand states and is, therefore, willing to build more capacity. The manufacturer's marginal profit decreases, but only in those high demand states that he is prevented from accessing anyway if the capacity is not increased. Quantity-premium schedules may seem counter to accepted practice where large orders are associated with discounts. However, the notion that a manufacturer may pay higher marginal prices when demand is high is not without precedent. Manufacturers sometimes negotiate with suppliers to obtain surge capacity that may only be used in times of high demand. A supplier providing uncertain surge capacity might reasonably expect a higher marginal wholesale price than a supplier engaged to provide capacity for "normal" production.

We also enrich the voluntary compliance regime of Cachon and Lariviere (2001) to allow for the fact that a supplier's failure to build an agreed capacity is revealed in certain demand states. With this, firm-commitment and options contracts are proven to increase supplier capacity, contrary to the result of Cachon and Lariviere. We identify certain circumstances that lead a risk-neutral manufacturer to prefer quantity premiums to firm commitments and other circumstances that reverse this preference. Future work explicitly considering how risk tolerance impacts contract preference would be of interest.

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Appendix

PROPOSITION 1. If $W(Q) = wQ$, the supply chain capacity $y_{sc}(w) = \min\{y_M(w), y_S(w)\}$.

PROOF. The supplier will not choose a capacity larger than the manufacturer would, as her sales are limited by the manufacturer's capacity. The supplier's profit, $\Pi_S(y)$, is concave with the maximum achieved at $y_S(w)$. For $y < y_S(w)$, $\Pi_S(y)$ is increasing in y . Therefore, if the manufacturer were to choose a capacity of y , the supplier would choose $\min\{y, y_S(w)\}$. Likewise, the manufacturer never announces a capacity larger than the supplier would choose, as his sales are limited by the supplier's capacity choice. The manufacturer's profit, $\Pi_M(y)$, is concave with the maximum achieved at $y_M(w)$. For $y < y_M(w)$, $\Pi_M(y)$ is increasing in y . Therefore, the manufacturer and supplier both choose a capacity, $y_{SC}(w) = \min\{y_M(w), y_S(w)\}$. \square

PROOF OF THEOREM 1. (i) From Proposition 1, $y_{SC}(w) = \min\{y_S(w), y_M(w)\}$. Now,

$$\frac{dy_M(w)}{dw} = \left(\frac{1}{f_X(y_M(w))} \right) \left(\frac{v_M - c_M}{(r - w - p_M - v_M)^2} \right) < 0 \quad \text{and}$$

$$\frac{dy_S(w)}{dw} = \left(\frac{1}{f_X(y_S(w))} \right) \left(\frac{c_S - v_S}{(w - p_S - v_S)^2} \right) > 0.$$

Therefore, $y_M(w)$ and $y_S(w)$ can cross each other at most once and $y_M(w) = y_S(w)$ iff

$$\left(\frac{r - w - p_M - c_M}{r - w - p_M - v_M} \right) = \left(\frac{w - p_S - c_S}{w - p_S - v_S} \right) \quad \text{or}$$

$$w = \frac{(r - p_M)(c_S - v_S) + p_S(c_M - v_M) + c_M v_S - c_S v_M}{(c_S - v_S + c_M - v_M)} = w_{\text{crit}}.$$

Now, $y_S(w) = y_{SC}^l$, the optimal supply chain capacity, iff

$$\left(\frac{r - p_M - p_S - c_M - c_S}{r - p_M - p_S - v_M - v_S} \right) = \left(\frac{w - p_S - c_S}{w - p_S - v_S} \right).$$

Therefore, $y_S(w) = y_{SC}^l$ iff

$$w = \frac{(r - p_M)(c_S - v_S) + p_S(c_M - v_M) + c_M v_S - c_S v_M}{(c_S - v_S + c_M - v_M)} = w_{\text{crit}},$$

which is the same condition for $y_M(w) = y_S(w)$. So, $y_{SC}(w) = \min\{y_S(w), y_M(w)\} = y_{SC}^l \Leftrightarrow w = w_{\text{crit}}$.

(ii) Let $\Pi_M(w)$ be the manufacturer's expected profit as a function of w . From Tomlin (2000), $p_S + c_S \leq w_M^* \leq w_{\text{crit}}$, where w_M^* is the manufacturer's optimal wholesale price. Therefore,

$$\begin{aligned} \Pi_M(w) &= -c_M y_S(w) + (r - p_M - w) \int_a^{y_S(w)} x f_X(x) dx \\ &\quad + (r - p_M - w) y_S(w) [1 - F_X(y_S(w))] \\ &\quad + v_M \int_a^{y_S(w)} [y_S(w) - x] f_X(x) dx. \end{aligned}$$

$\Pi_M(w_{\text{crit}}) > 0$ and $\Pi_M(w)$ is concave (as $F_X[x]$ satisfies the concavity condition). Therefore,

$$\left. \frac{d\Pi_M(w)}{dw} \right|_{w=p_S+c_S} > 0.$$

In addition,

$$\left. \frac{d\Pi_M(w)}{dw} \right|_{w=w_{\text{crit}}} = - \int_a^{y_S(w_{\text{crit}})} x f_X(x) dx - y_S(w_{\text{crit}}) [1 - F_X(y_S(w_{\text{crit}}))] < 0.$$

Therefore, $p_S + c_S < w_M^* < w_{\text{crit}} \Rightarrow y_{SC}(w_M^*) < y_{SC}^l$ and $\Rightarrow \Pi_{SC}(w_M^*) < \Pi_{SC}^l$. \square

PROOF OF THEOREM 2. This is a special case of Theorem 3, Parts (i) and (ii), therefore the proof is left to the general case. \square

PROOF OF THEOREM 3. (i) Denote price schedule (2) by $W^\beta(Q)$. Let $\Pi_M(y, W^\beta(Q))$ and $\Pi_S(y, W^\beta(Q))$ denote the manufacturer and supplier's expected profit for a capacity y (if the other party has infinite capacity).

$$\begin{aligned} \Pi_M(y, W^\beta(Q)) &= -c_M y + \int_a^y ((r - p_M)x - W^\beta(x)) f_X(x) dx \\ &\quad + ((r - p_M)y - W^\beta(y)) \bar{F}_X[y] \\ &\quad + v_M \int_a^y (y - x) f_X(x) dx, \end{aligned}$$

$$\begin{aligned} \Pi_S(y, W^\beta(Q)) &= -c_S y + \int_a^y (W^\beta(x) - p_S x) f_X(x) dx \\ &\quad + (W^\beta(y) - p_S y) \bar{F}_X[y] + v_S \int_a^y (y - x) f_X(x) dx, \end{aligned}$$

$$\begin{aligned} \frac{d\Pi_M(y, W^\beta(Q))}{dy} &= (1 - \beta) \left(-(c_S + c_M)y + (r - p_S - p_M) \bar{F}_X[y] \right. \\ &\quad \left. + (v_S + v_M) F_X[y] \right) \\ &= (1 - \beta) \frac{d\Pi_{SC}(y)}{dy}, \end{aligned}$$

$$\begin{aligned} \frac{d\Pi_S(y, W^\beta(Q))}{dy} &= \beta \left(-(c_S + c_M)y + (r - p_S - p_M) \bar{F}_X[y] \right. \\ &\quad \left. + (v_S + v_M) F_X[y] \right) \\ &= \beta \frac{d\Pi_{SC}(y)}{dy}. \end{aligned}$$

Now,

$$\begin{aligned} \Pi_S(0, W^\beta(Q)) &= -c_S(0) + \int_a^0 (W^\beta(x) - p_S x) f_X(x) dx \\ &\quad + (W^\beta(0) - p_S(0)) \bar{F}_X[0] + \int_a^0 (v_S(0 - x)) f_X(x) dx. \end{aligned}$$

Because $a \geq 0$ (i.e., demand is nonnegative), we have $\Pi_S(0, W^\beta(Q)) = W^\beta(0) = 0$. In other words, the supplier's expected profit at a capacity of zero equals zero.

Using the fact that if $f(0) = 0$, then $\int_0^y f'(x) dx = f(y) - f(0) = f(y)$, and we have

$$\begin{aligned} \int_a^y \left(\frac{d\Pi_S(x, W^\beta(Q))}{dx} \right) dx &= \Pi_S(y, W^\beta(Q)) - \Pi_S(0, W^\beta(Q)) \\ &= \Pi_S(y, W^\beta(Q)). \end{aligned}$$

So,

$$\Pi_S(y, W^\beta(Q)) = \int_a^y \left(\frac{d\Pi_S(x, W^\beta(Q))}{dx} \right) dx.$$

It has already been proven above that

$$\frac{d\Pi_S(x, W^\beta(Q))}{dx} = \beta \frac{d\Pi_{SC}(x)}{dx}.$$

Therefore,

$$\begin{aligned}\Pi_S(y, W^\beta(Q)) &= \beta \int_a^y \left(\frac{d\Pi_{SC}(x)}{dx} \right) dx = \beta (\Pi_{SC}(y) - \Pi_{SC}(0)) \\ &= \beta \Pi_{SC}(y) \quad \text{as } \Pi_{SC}(0) = 0.\end{aligned}$$

So, we have proven that $\Pi_S(y, W^\beta(Q)) = \beta \Pi_{SC}(y)$. Similarly, it can be shown that

$$\begin{aligned}\Pi_M(y, W^\beta(Q)) &= (1 - \beta) \Pi_{SC}(y) \quad \text{as } W^\beta(0) = 0 \quad \text{and} \\ \frac{d\Pi_M(y, W^\beta(Q))}{dy} &= (1 - \beta) \frac{d\Pi_{SC}(y)}{dy}.\end{aligned}$$

(ii) The expected profits are concave functions and the first-order conditions are satisfied at the same capacity as for the supply chain expected profit, i.e., at y_{SC}^l . Therefore, the supply chain is coordinated.

(iii) Taking the derivative of the price schedule (2) with respect to Q , we get

$$\frac{d^2 W^\beta(Q)}{dQ^2} = \left(\frac{f_X[Q]}{(1 - F_X[Q])^2} \right) (c_S - v_S - \beta(c_M - v_M + c_S - v_S)).$$

Therefore,

$$\begin{aligned}\frac{d^2 W^\beta(Q)}{dQ^2} &< 0 \quad \text{if } \beta < \beta_T, \\ \frac{d^2 W^\beta(Q)}{dQ^2} &= 0 \quad \text{if } \beta = \beta_T, \quad \text{and} \\ \frac{d^2 W^\beta(Q)}{dQ^2} &> 0 \quad \text{if } \beta > \beta_T, \quad \text{where} \\ \beta_T &= \frac{c_S - v_S}{c_M - v_M + c_S - v_S}. \quad \square\end{aligned}$$

PROOF OF THEOREM 4. (i) Let $\Pi_M(w)$ be the manufacturer's profit when a linear price schedule w is chosen and let $\Pi_M(w, \Delta)$ be the manufacturer's profit when a single-breakpoint quantity-premium schedule (w, Δ) is chosen. Define $y_S(w, \Delta)$ as the supplier's optimal capacity for the single-breakpoint schedule (w, Δ) if the manufacturer has infinite capacity,

$$\begin{aligned}y_S(w) &= F_X^{-1} \left(\frac{w - p_S - c_S}{w - p_S - v_S} \right) \quad \text{and} \\ y_S(w, \Delta) &= F_X^{-1} \left(\frac{w + \Delta - p_S - c_S}{w + \Delta - p_S - v_S} \right).\end{aligned}$$

Consider any wholesale price w such that $p_S + c_S < w < w_{\text{crit}}$ and any quantity premium in the range $p_S + c_S < w + \Delta < w_{\text{crit}}$. The supply chain capacity will be $y_S(w, \Delta)$ (Tomlin 2000). Recall that the price breakpoint occurs at $y_S(w)$,

$$\begin{aligned}\Pi_M(w, \Delta) &= -c_M y_S(w, \Delta) + m_M \int_a^{y_S(w)} x f_X(x) dx + m_M y_S(w) \bar{F}_X[y_S(w)] \\ &\quad + (m_M - \Delta) \int_{y_S(w)}^{y_S(w, \Delta)} (x - y_S(w)) f_X(x) dx\end{aligned}$$

$$\begin{aligned}&+ (m_M - \Delta)(y_S(w, \Delta) - y_S(w)) \bar{F}_X[y_S(w, \Delta)] \\ &+ v_M \int_a^{y_S(w, \Delta)} [y_S(w, \Delta) - x] f_X(x) dx,\end{aligned}$$

where $m_M = r - w - p_M$. The demand distribution satisfies the concavity condition, so $\Pi_M(w, \Delta)$ is a concave function of Δ (Tomlin 2000). For a fixed w ,

$$\begin{aligned}\frac{d\Pi_M(w, \Delta)}{d\Delta} \Big|_{\Delta=0} &= \left(\frac{dy_S(w, \Delta)}{d\Delta} \Big|_{\Delta=0} \right) [(m_M - c_M) - (m_M - v_M) F_X(y_S(w))] \\ &> 0 \quad \text{as } \frac{dy_S(w, \Delta)}{d\Delta} > 0,\end{aligned}$$

$c_M > v_M$ and $F_X(y_S(w)) \leq 1$. Therefore, $\Delta^*(w) > 0$, $\Pi_M(w, \Delta^*(w)) > \Pi_M(w, 0) = \Pi_M(w)$ (a quantity-premium price schedule with $\Delta = 0$ is a linear price schedule). This is true for all $p_S + c_S < w < w_{\text{crit}}$ and so is true for w_M^* .

(ii) Define $y_{SC}(w_M^{**}, \Delta_M^{**})$ and $y_{SC}(w_M^*)$ as the supply chain capacities induced by the manufacturer's optimal single-breakpoint schedule and optimal linear price schedule, respectively. Π_{SC} is concave in y with a maximum at y_{SC}^l , the supply chain optimal capacity, which is independent of the contract. So,

$$y_{SC}^l \geq y_{SC}(w_M^{**}, \Delta_M^{**}) > y_{SC}(w_M^*) \Rightarrow \Pi_{SC}(w_M^{**}, \Delta_M^{**}) > \Pi_{SC}(w_M^*).$$

This sufficient condition is equivalent to $w_{\text{crit}} \geq w_M^{**} + \Delta_M^{**} > w_M^* \Rightarrow \Pi_{SC}(w_M^{**}, \Delta_M^{**}) > \Pi_{SC}(w_M^*)$.

One can show that

$$\begin{aligned}\frac{\partial \Pi_M(w, \Delta)}{\partial w} \Big|_{w=w^*, \Delta=\Delta^*} &= \Delta^* \left(\frac{dy_S(w)}{dw} \Big|_{w=w^*} \right) [1 - F_X[y_S(w^*)]] \\ &\quad + \left(\frac{d\Pi_M(w)}{dw} \Big|_{w'=w^*+\Delta^*} \right).\end{aligned}$$

Therefore,

$$\frac{d\Pi_M(w)}{dw} \Big|_{w'=w^*+\Delta^*} < 0$$

because

$$(a) \quad \frac{\partial \Pi_M(w, \Delta)}{\partial w} \Big|_{w=w^*, \Delta=\Delta^*} = 0$$

as first-order conditions are necessary and sufficient for optimality of the single-breakpoint quantity-premium price schedule (Tomlin 2000); (b) $\Delta^* > 0$ from (i); (c) $dy_S(w)/dw > 0$; and

$$(d) \quad F_X[y_S(w^*)] = \frac{w^* - p_S - c_S}{w^* - p_S - v_S} < 1.$$

So, $\Pi_M(w)$ is decreasing at $w_M^{**} + \Delta_M^{**}$ and so $w_M^{**} + \Delta_M^{**} > w_M^*$ as $\Pi_M(w)$ is concave. Tomlin (2000) shows that $w_{\text{crit}} > w_M^{**} + \Delta_M^{**}$ so the sufficient condition for $\Pi_{SC}(w_M^{**}, \Delta_M^{**}) > \Pi_{SC}(w_M^*)$ is met. \square

PROOF OF THEOREM 5. For a fixed wholesale price, $w < w_{\text{crit}}$,

$$\begin{aligned}\frac{d\Pi_S(w, \Delta)}{d\Delta} &= \left(\frac{dy_S(w, \Delta)}{d\Delta} \right) [(m_S + \Delta - c_S) - (m_S - \Delta - v_S) F_X(y_S(w, \Delta))]\end{aligned}$$

$$+ \int_{y_S(w)}^{y_S(w, \Delta)} (x - y_S(w)) f_X(x) dx + (y_S(w, \Delta) - y_S(w)) \cdot [1 - F_X(y_S(w, \Delta))] > 0.$$

As $w_M^* < w_{crit}$, the supplier's expected profit increases if the manufacturer offers a positive quantity premium. From Theorem 3, $\Delta^*(w_M^*) > 0$. \square

PROOF OF THEOREM 6. We use $y_S(w)$ to denote $y_S(\Omega_{linear}(w))$ for ease of presentation. One can show that sufficient conditions for a contract, Ω , to increase the supplier capacity, i.e., $y_S(\Omega) > y_S(w)$, are given by

$$E[I[x, y_S, \Omega, w]] \geq 0 \quad \text{and} \quad \frac{dE[I[x, y_S, \Omega, w]]}{dy_S} > 0 \Rightarrow y_S(\Omega) > y_S(w).$$

Reimbursement Contract. (i) $I[x, \Omega_{reimburse}(w, b)] = b[y_S - x]^+ \geq 0$. This is nonincreasing in x so it is a share-the-pain contract.

(ii) $E[I[x, y_S(w), \Omega_{reimburse}(w, b), w]] = b \int_0^{y_S(w)} (y_S(w) - x) f_X(x) dx \geq 0$ and

$$\left. \frac{dE[I[x, y_S, \Omega, w]]}{dy_S} \right|_{y \leq y_S(\Omega_{linear}(w))} = b F_X[y] \geq 0.$$

Firm-Commitment Contract.

$$(i) \quad I[x, \Omega_{firm}(w, y_{firm})] = \begin{cases} (w - p_S - v_S)[y_{firm} - x]^+ \geq 0, & y_{firm} \leq y_S, \\ (w - p_S - v_S)(y_{firm} - x) \geq 0, & y_{firm} > y_S, \quad x \leq y_S, \\ 0, & y_{firm} > y_S, \quad x > y_S. \end{cases}$$

This is nonincreasing in x so it is a share-the-pain contract.

$$(ii) \quad E[I[x, y_S(w), \Omega_{firm}(w, y_{firm}), w]] = \begin{cases} (w - p_S - v_S) \int_0^{y_{firm}} (y_{firm} - x) f_X(x) dx, & y_{firm} \leq y_S(w), \\ (w - p_S - v_S) \int_0^{y_S(w)} (y_{firm} - x) f_X(x) dx, & y_{firm} > y_S(w). \end{cases}$$

Therefore, $E[I[x, y_S(w), \Omega_{firm}(w, y_{firm}), w]] \geq 0$, and

$$\frac{dE[I[x, y_S, \Omega_{firm}(w, y_{firm}), w]]}{dy_S} = \begin{cases} 0, & y_{firm} \leq y_S, \\ (w - p_S - v_S)(y_{firm} - y_S) f_X(y) \geq 0, & y_{firm} > y_S. \end{cases}$$

Options Contract.

$$(i) \quad I[x, \Omega_{options}(w, y_{firm})] = \begin{cases} w_0 y_{options} \geq 0, & y_{options} \leq y_S, \\ w_0 y_{options} \geq 0, & y_{options} > y_S, \quad x \leq y_S, \\ w_0 y_S \geq 0, & y_{options} > y_S, \quad x > y_S. \end{cases}$$

This is nonincreasing in x so it is a share-the-pain contract.

$$(ii) \quad E[I[x, y_S(w), \Omega_{options}(w, y_{options}), w]] = \begin{cases} w_0 y_{options} \geq 0, & y_{options} \leq y_S(w), \\ w_0 y_{options} F_X[y_S] + w_0 y \bar{F}_X[y_S] \geq 0, & y_{options} > y_S(w), \end{cases}$$

$$\frac{dE[I[x, y_S, \Omega_{options}(w, y_{options}), w]]}{dy_S} = \begin{cases} 0, & y_{options} \leq y_S, \\ w_0((y_{options} - y_S) f_X(y) + \bar{F}_X[y]) \geq 0, & y_{options} > y_S. \quad \square \end{cases}$$

PROOF OF THEOREM 7. Let $\Pi_M(y_{target}, \Omega)$ be the manufacturer's expected profit for offering a contract Ω that induces a supplier capacity (and hence manufacturer's capacity) of y_{target} . For the premium contract,

$$F_X^{-1} \left[\frac{w + \Delta(w, y_{target}) - p_S - c_S}{w + \Delta(w, y_{target}) - p_S - v_S} \right] = y_{target} \Rightarrow \Delta(w, y_{target}) = \frac{c_S - (w - p_S) \bar{F}_X[y_{target}] - v_S F_X[y_{target}]}{\bar{F}_X[y_{target}]}.$$

For the firm-commitment contract, $y_{firm} = y_{target}$. Define

$$\Psi(y_{target}, w) = \Pi_M(y_{target}, \Omega_{premium}(w, \Delta = \Delta(w, y_{target}))) - \Pi_M(y_{target}, \Omega_{firm}(w, y_{firm} = (y_{target}))).$$

The manufacturer prefers the price-premium contract if $\Psi(y_{target}, w) \geq 0$. Now,

$$\Psi(y_{target}, w) = (w - p_S) \int_a^{y_{target}} (y_{target} - x) f_X(x) dx - \Delta(w, y_{target}) \cdot \left(\int_{y_S(w)}^{y_{target}} (x - y_S(w)) f_X(x) dx + (y_{target} - y_S(w)) \bar{F}_X[y_{target}] \right),$$

where $y_S(w)$ is shorthand for $y_S(\Omega_{linear}(w))$. So,

$$\begin{aligned} \frac{d\Psi(y_{target}, w)}{dy_{target}} &= w - c_S - c_S h_X(y_{target}) \\ &\cdot \left(\int_{y_S(w)}^{y_{target}} (x - y_S(w)) f_X(x) dx + (y_{target} - y_S(w)) \bar{F}_X[y_{target}] \right) \quad \text{and} \\ \frac{d^2\Psi(y_{target}, w)}{dy_{target}^2} &= -c_S \left(\frac{dh_X(y_{target})}{dy_{target}} \left(\int_{y_S(w)}^{y_{target}} (x - y_S(w)) f_X(x) dx \right. \right. \\ &\quad \left. \left. + (y_{target} - y_S(w)) \bar{F}_X[y_{target}] \right) + h_X(y_{target}) \bar{F}_X[y_{target}] \right) \leq 0, \end{aligned}$$

where $h_X(x)$ is the failure rate function (which is increasing for IFR distributions). Therefore, $\Psi(y_{target}, w)$ is concave in y_{target} .

$$\Psi(y_{target} = y_S(w), w) = (w - p_S) \int_a^{y_{target}} (y_{target} - x) f_X(x) dx > 0$$

So, as y_{target} increases from $y_s(w)$ to y_{SC}^l , $\Psi(y_{\text{target}}, w)$ crosses zero at most once. Let $y_{\text{threshold}}(w)$ denote the cross-over capacity if it crosses. Now, $y_{\text{threshold}}(w) > y_s(w)$ and

$$\begin{aligned} \frac{dy_{\text{threshold}}(w)}{dw} &\geq 0 \quad \text{as} \quad \frac{d\Psi[y_{\text{target}}, w]}{dw} \\ &= \int_a^{y_{\text{target}}} (y_{\text{target}} - x) f_x(x) dx + \int_{y_s(w)}^{y_{\text{target}}} (x - y_s(w)) f_x(x) dx \\ &\quad + (y_{\text{target}} - y_s(w)) \bar{F}_x[y_{\text{target}}] + \Delta(w, y_{\text{target}}) \frac{dy_s(w)}{dw} \bar{F}_x[y_{\text{target}}] \geq 0. \end{aligned}$$

Therefore, there exists a price, $w_{\text{threshold}} < w_{\text{crit}}$, such that $\Psi[y_{\text{target}}, w]$ never crosses zero (before y_{SC}^l) for any $w \geq w_{\text{threshold}}$ and so the manufacturer prefers the price-premium contract for all $y_s(w) \leq y_{\text{target}} \leq y_{\text{SC}}^l$ if $w \geq w_{\text{threshold}}$. For prices $w < w_{\text{threshold}}$ there exists a threshold target capacity $y_s(w) \leq y_{\text{threshold}}(w) < y_{\text{SC}}^l$ such that the manufacturer prefers the price-premium contract if $y_{\text{target}} < y_{\text{threshold}}(w)$ but prefers the firm-commitment contract if $y_{\text{SC}}^l \geq y_{\text{target}} > y_{\text{threshold}}(w)$. At $y_{\text{target}} = y_{\text{threshold}}(w)$, he is indifferent between the contracts. \square

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