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Early-stage entrepreneurial financing: A signaling perspective ☆

Jin-Hyuk Kim [a,*], Liad Wagman [b,1]

[a] Department of Economics, University of Colorado at Boulder, 256 UCB, Boulder, CO 80309, USA
[b] Stuart School of Business, Illinois Institute of Technology, 565 W Adams Rd, Suite 452, Chicago, IL 60661, USA

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ABSTRACT

We analyze an entrepreneur's choice between angel and venture capital (VC) financing in a competitive investment market, where the entrepreneur seeks to maintain his ownership share as well as equity value. The key to our analysis is the idea that a negative signal is inferred by the market if an inside investor chooses not to follow on a subsequent investment. We first show that when ventures are ex-ante identical, entrepreneurs retain higher ownership shares by financing with angel investors who commit to not participate in a future round. When entrepreneurs are ex-ante heterogeneous, there is a separating equilibrium where entrepreneurs with higher (lower) likelihoods of success choose VC financing (angel financing) in the first round.

1. Introduction

The market for early-stage investments has grown dramatically in the past decade. Angel investing, which used to be a boutique practice partaken by some successful entrepreneurs, has become a mass market with hundreds of angel groups and platforms dedicated to make small investments in early-stage startups.[2] In contrast, traditional VCs have moved to larger and later stages of financing and tend not to invest in deals that seek less than three or four million dollars (OECD, 2011; Sohl, 2011). Therefore, it is commonly understood that angel financing is chosen by entrepreneurs because VC financing is simply unavailable in the early stages of the firm (e.g., Hellmann and Thiele, 2014).

This explanation, however, is incomplete because VC firms have put in place dedicated funds to make small, seed investments, viewing them as sources for potential follow-on investments. In fact, there are now hundreds of VC funds (such as those run by Andreessen Horowitz) that behave like angels in the early stages of investment. Hence, if one seeks to understand the choice between angel and VC financing in the early stages of a venture, it is crucial to gain insight into the different dynamics that play out in a model of staged financing. In particular, our focus in this paper is on the idea that a first-stage financier who decides not to re-invest at the next stage conveys a negative signal to outsiders.

We take explicit account of an entrepreneur's reluctance to cede ownership shares to financiers in exchange for financing. That is, our model incorporates entrepreneurs' tradeoffs in financing on favorable terms and sharing the ownership of a business.[3] This is consistent with the pecking-order theory (e.g., Myers and Majluf, 1984), according to which entrepreneurs seek financing in an order that minimizes ownership dilution. For instance, entrepreneurs would prefer to borrow from banks rather than to sell equity stakes, everything else being equal. As is commonly the case, however, for a

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* Corresponding author. Tel.: +1 (303) 492 3827.
E-mail addresses: jinhyuk.kim@colorado.edu (J.-H. Kim), lwagman@stuart.iit.edu (L. Wagman).
1 Tel.: +1 (773) 980 9883.

2 A taxonomy for these early-stage investors includes 'super angels' (e.g., Ron Conway, Peter Thiel, etc.), 'micro-VCs' (e.g., First Round Capital, True Ventures, SoftTech VC, etc.), and 'startup accelerators' (Y Combinator, TechStars, Seedcamp, etc.).

3 Anecdotes abound in the popular press about the importance of share dilution in a founder's decision to raise capital. There is also growing evidence that there are indeed substantial non-pecuniary benefits for founder-CEOs (e.g., Hamilton, 2000; Moskowitz and Vissing-Jørgensen, 2002).

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penniless entrepreneur or an early-stage venture, debt financing is often unavailable.

We thus focus on how an entrepreneur's ownership incentives influence his choice of a financing source in the early stages of the venture, assuming that both angels and VCs provide competitive equity financing. Angel investors, who are characterized by a liquidity constraint (due to, e.g., a lack of larger follow-on funds), cannot participate in the subsequent round of investment, whereas VCs are characterized by having the option to decide whether or not to re-invest.[4] Importantly, a VC firm's second-round participation decision is observed by outside investors (e.g., through interviews and due diligence), which enables the market to partially update its posterior beliefs about a venture's likelihood of success.

More specifically, if a seed investor decides to re-invest, then a positive signal is sent to the market, which ramps up outside investors' valuation of the venture; if the seed investor decides to stay out of a subsequent round, then a negative signal is sent and the venture's valuation decreases. Because the re-investment decision leads to more competitive outside offers, a decision to re-invest ratchets up the expected value of the marginal venture, above which the seed investor is willing to re-invest. This endogenously creates an 'up' round with a higher valuation (or a 'down' round with a lower valuation) than what would have been observed without such strategic incentives.

We first show that given a competitive capital market, when ventures are ex-ante identical, entrepreneurs can retain higher ownership stakes by financing early rounds with angel investors. Intuitively, the signaling problem that follows VC financing in first-round investments creates uncertainty associated with the inside VC's participation decision in the second round. This uncertainty leads to an overall smaller expected ownership share for the entrepreneur compared to angel financing. This benchmark result (i.e., ownership advantage with angel financing) holds regardless of whether outside investors are willing to invest or the venture is simply liquidated in a 'down' round (where inside VCs do not follow through).

When entrepreneurs are heterogeneous with respect to ex-ante private information about their ventures' success probabilities as well as financing opportunities, a separating equilibrium exists where entrepreneurs with higher likelihoods of success ('high types') choose VC financing in the first stage while 'low types' choose angel financing. That is, our model can explain the co-existence of angels and VCs in the early financing stage and gives rise to the prediction that those ventures who finance with angel investors are on average of lower quality than those who finance with VCs.

The remainder of the paper is organized as follows. Section 2 presents the model, and Section 3 analyzes VC financing. Section 4 compares VC and angel financing in the benchmark case. Section 5 considers ex-ante heterogeneous entrepreneurs and characterizes the separating equilibrium. Section 6 examines robustness to interim liquidation. Section 7 discusses related works and our model's empirical implications. Section 8 concludes. All formal proofs are in the Appendix A.

2. The model

Penniless entrepreneurs seek to finance their ventures. Entrepreneurs are assumed to be risk neutral; and ventures are ex-ante identical (we later relax this assumption). The representative entrepreneur has preferences defined over his ownership share of the venture and the value of his equity. Specifically, his utility increases in both his expected ownership share and the value of his retained equity. This means that the entrepreneur is willing to trade off a (small) decrease in equity value for an increase in his ownership share of the venture. We do not need to assume anything specific about the functional form of the utility function or the degree of substitutability between its two components. All of our results hold as long as there is some (even arbitrarily small but positive) substitutability between ownership share and equity value.[5]

At the beginning of the game, nature draws a venture idea for each entrepreneur, which is characterized by a probability of success p. We assume that p is uniformly distributed between 0 and 1 in order to derive closed-form solutions. Venture development consists of two stages. In the first stage, an entrepreneur raises capital K to turn his idea into a prototype. No revenue is generated at this stage. Hence, if the venture is liquidated at the end of the first round, then the firm's equity is worthless. In the second stage, the entrepreneur raises growth capital F. At the end of the second stage, the venture either succeeds with probability p generating a revenue R, or fails with probability $1 - p$ yielding zero revenue. We assume that ventures have ex-ante positive net values, that is, $R/2 \geqslant (K + F)$.

To raise capital in each stage, entrepreneurs approach risk-neutral investors. The investment market is perfectly competitive at both stages of financing, so that VCs or angel investors can only expect to earn a zero rate of return at the time of investment. This assumption reflects the recent evidence that VC funds do not significantly outperform small-cap traded securities, particularly when taking into account selection bias (e.g., Cochrane, 2005; Harris et al., 2014). Direct evidence on the performance of angel investments is rarer, but existing evidence suggests that angel groups and the venture capital industry seem to perform similarly (e.g., Kerr et al., 2014).[6] Therefore, the perfect competition assumption can serve as a useful benchmark which allows us to present our results in a clear manner.

For tractability, we assume that the entrepreneur is unable to secure debt financing due to the nature of the venture or the lack of collateral. There are two types of equity investors, 'VCs' and 'angels.' The difference between the two is that angel investors are those who commit not to re-invest in follow-on rounds for reasons that are exogenous to the model (e.g., due to portfolio policies or lack of funds) while VCs have the option of deciding whether or not to re-invest. This distinction largely matches how the inside-signaling problem with VCs is perceived by practitioners. Although the investor type can be convexified and endogenized by assuming an ex-ante probability of a liquidity constraint, this does not change any of the results in our model, but it can make closed-form analysis intractable.[7]

A venture's success likelihood, p, is initially unknown to both the entrepreneur and investors. If an investor invests in the first round, this 'seed' investor as well as the entrepreneur learn the venture's success likelihood, p. Information asymmetry arises because p is not observed by outside investors. However, before second-round bidding begins, outside investors can observe

4 Angels typically do not follow on when a subsequent financing round involves VC participation (Wong et al., 2009). In a 2010 survey of angel investors in Europe, the number of deals made by respondents were 331, 222, and 127 in France, Italy, and the UK, respectively, of which the number of follow-on rounds were only 2, 7, and 4 (European Business Angels Network (EBAN), 2010).

5 As previously mentioned, the main advantage of angel financing in our model is that entrepreneurs retain larger ownership shares. If entrepreneurs only cared about the value of their equity, then it can be shown that VC financing yields the same expected ex-ante equity value as angel financing.

6 Cochrane (2005) and Cumming and Walz (2010) estimate the mean returns to VC investments to be around 66% using US and international data, respectively. DeGennaro and Dwyer (2014) estimate that the expected returns to angel investments is around 70%, which is the same order of magnitude as those from VC investments.

7 See Supplementary Material which can be accessed online.

whether the seed investor decides to make a follow-on investment, and investors update their beliefs before finalizing their offers. We later allow the entrepreneur to have private information before the first round begins, but the information propagation via the interim signal that is observed by outside investors in the second stage remains the same.

Both angel investors and VCs offer equity financing. Specifically, in each stage, investors offer a contract to the entrepreneur, (γ_X, X), which gives the investor a share $\gamma_X \in (0,1)$ of the venture's equity in return for investing X, where $X \in \{K, F\}$. The entrepreneur accepts the best financing offer. As a tie-breaking rule, we assume that the entrepreneur stays with the inside VC when indifferent.[8] The tie-breaking rule does not affect our analysis, but it captures the fact that the seed-round contract may incorporate a right of first refusal. That is, seed investors have the right to make follow-on investments as long as they can match the financing terms offered by the market.

We abstract from convertible securities because the cash flows would be equivalent to those of common equity in our model as long as the revenue R is sufficiently large. That is, the zero-profit condition would ensure that any discount or advantage applicable to conversion also yields a zero rate of return due to competition among investors. However, it must be acknowledged that, more broadly, in some countries such as the U.S., convertibles are more frequently used in VC financing for various reasons (e.g., Kaplan and Strömberg, 2003; Cumming, 2005), in which case it will make the comparison of simple ownership shares more complex.

Finally, we assume that the seed investor's follow-on decision is irreversible. That is, seed investors cannot initially decline to participate in the follow-on round in an attempt to later beat the terms of (undervalued) outside offers. Anecdotes suggest that such maneuvers can lead to reputational damages for VC firms in the investor community; and the literature often makes this assumption (e.g., Admati and Pfleiderer, 1994). This assumption is also consistent with the evidence that VCs do not create down rounds in order to dilute an entrepreneur's ownership shares (e.g., Broughman and Fried, 2012). The timing of the game is summarized as follows:

1. Entrepreneur raises a first financing round of size K.
2. Seed investor learns the venture's success probability, p.
3. Seed investor announces its second-round participation decision.
4. Outside investors update their valuations and make financing offers.
5. Entrepreneur raises a second financing round of size F, and profits are realized.

3. VC financing

The solution concept we employ is pure-strategy Perfect-Bayesian Equilibria. As is standard in the signaling literature, given that the venture quality distribution is atomless, we focus attention on equilibria characterized by binary threshold rules.[9] Specifically, we consider the following strategy for inside VCs in the second round: make the follow-on investments if and only if the realization of the venture's success likelihood is above a certain threshold p^*. Because the VC's profit increases continuously in the success probability p, by construction this strategy yields a unique equilibrium.

Given such a strategy, outside investors update their beliefs about the venture's success likelihood based on the inside VC's follow-on decision. Conditional on the VC's follow-up participation, the venture's expected probability of success is given by $\frac{1+p^*}{2}$, while it is $\frac{p^*}{2}$ when the inside VC does not participate in the second round. Hence, for $p^* < 1$, we have $\frac{p^*}{2} < \frac{1}{2} < \frac{1+p^*}{2}$, whereby the venture's second-round valuation either increases (an 'up' round) if the VC follows through, or decreases (a 'down' round) relative to its first-round valuation if the VC decides not to re-invest.

Thus, there are two types of second-round subgames depending on the model's parameterization. In the second stage, a venture has a non-negative expected value if and only if $pR \geqslant F$. If the realization of success probabilities were perfectly observed, then no venture with success probability below $\frac{F}{R}$ would be financed in the second round. Under asymmetric information, however, the expected probability of success conditional on a down round is $\frac{p^*}{2}$. Hence, if $\frac{p^*}{2}$ is greater than $\frac{F}{R}$, then outside investors would find it worthwhile to invest in a down round even if the inside VC decided not to re-invest.

On the other hand, the inside VC may not find it worthwhile to re-invest in ventures that turn out to be just above $\frac{F}{R}$ because its participation sends a positive signal to the market, which leads to more competitive outside offers. Thus, if the venture turns out to be only moderately promising, the inside VC lets outside investors finance the venture's follow-on round. This helps the expected value of a venture in a down round remain positive so that the venture can still attract additional funding F. To be precise, the following condition ensures that ventures in a down round are financed by outside investors:

Assumption 1. $4KR \leqslant (R - 2F)^2$.

Because the left-hand side is a linear function of R and the right-hand side is a quadratic function of R, holding the capital requirements (K and F) constant, Assumption 1 is trivially satisfied for a large payoff, R. Assumption 1 also guarantees that the equilibrium threshold value p^* is in the interior $(0,1)$, so that both up and down rounds can be observed. We assume Assumption 1 in the proceeding analysis, but we also demonstrate in Section 6 that our results continue to hold when ventures are instead liquidated in a down round.

Working backwards, the financing terms in an 'up' round are characterized by the zero-profit condition for investors, $\frac{1+p^*}{2}\gamma_F R - F = 0$, which means that the investor's share will be bid down to $\gamma_F^+ = \frac{2F}{(1+p^*)R}$, and the entrepreneur stays with the inside VC. Similarly, the financing terms in a 'down' round will be bid down to $\gamma_F^- = \frac{2F}{p^* R}$. Notice that the inside VC's first-round shares are diluted in both cases. Specifically, if a VC owns a share γ_K of equity in the first round, then this share will become $\gamma_K(1-\gamma_F)$ after percentage dilution in the second round. Hence, the inside VC will finance the second round if and only if p satisfies

$$p(\gamma_K(1-\gamma_F^+) + \gamma_F^+)R - F \geqslant p(\gamma_K(1-\gamma_F^-))R. \tag{1}$$

The left-hand side of (1) is the expected profit from making the follow-on investment. The inside VC's share is the sum of the first-stage share γ_K diluted by a factor of $(1-\gamma_F^+)$ and the newly acquired share γ_F^+ in the second round. The right-hand side is the inside VC's expected profit if it does not follow through and instead lets outside investors finance the second round, in which case the inside VC's diluted share is $\gamma_K(1-\gamma_F^-)$.

[8] One could in principle consider this as a syndication that includes the inside VC. As Cumming and Dai (2013) show, lead VCs can change in subsequent rounds (in around 20% of the sample they consider) but the companies that attract more reputable lead VCs in the follow-on rounds are those with upwardly revised perceived qualities, which can be consistent with our signaling mechanism.

[9] In the context of threshold equilibria, restricting attention to pure strategies is without loss of generality (see, e.g., Milgrom and Weber, 1985).

Proposition 1. *There is a threshold value p^*, above which the inside VC re-invests in the second round in exchange for an equity share of $\gamma_F^+ = \frac{2F}{(1+p^*)R}$, and below which an outside investor finances the venture in exchange for an equity share of $\gamma_F^- = \frac{2F}{p^*R}$.*

The equilibrium displays properties that are consistent with what is known as the 'signaling hypothesis' of VC seed investments among practitioners. That is, if the early-stage inside VCs do (not) follow through, then it sends a positive (negative) signal to the market. Interestingly, the second-round financing terms only depend on the seed investor's participation, despite the fact that the probability of success, p, is known to the inside VC and the entrepreneur. This is because outside investors can only observe the insider's participation decision.

We now derive the first-stage equity share γ_K^* of the inside VC. From the above proposition, the seed investor expects to participate in the second round with probability $1-p^*$ and stay out of it with probability p^*. Notice that the fallback position (i.e., a down round) yields a positive expected profit under Assumption 1. Hence, given the competitive investment market, the first-round financing terms (γ_K^*, K) are characterized by the following zero expected-profit condition (note, however, that the seed investor's ex-post realized returns will be positive if the venture succeeds):

$$-K+(1-p^*)\left(\frac{1+p^*}{2}(\gamma_K(1-\gamma_F^+)+\gamma_F^+)R-F\right) + p^*\left(\frac{p^*}{2}(\gamma_K(1-\gamma_F^-))R\right) = 0. \tag{2}$$

Substituting in for γ_F^+, γ_F^-, and p^*, and simplifying, yields our next result.

Proposition 2. *The equilibrium equity share of the VC firm investing in the first round is $\gamma_K^* = \frac{2K}{R-2F}$. The equilibrium threshold likelihood of success, p^*, above which the VC firm will re-invest in the second round is given by $p^* = 1 - \frac{4K}{R-2F}$.*

4. Angel financing

We now consider a class of investors, to which we refer as angels, that specializes in early stage investments — whether due to a lack of sufficient funds to follow through or because of a stated policy of exclusively investing in early-stage ventures. A consequence of financing the first stage with an angel is that the entrepreneur must raise capital in the second round from outside investors. Suppose that, in the first stage, an entrepreneur receives offers from angels as well as from VCs. There are two possible outcomes. First, the entrepreneur may accept an offer from a VC firm, in which case the subsequent equilibrium is described by Propositions 1 and 2. Second, the entrepreneur may instead accept a financing offer from an angel in the first round.

In the latter case, consider the second-stage subgame. Since angels do not re-invest, outside investors do not glean any new information from the angel's non-participation. Hence, the same ex-ante (uniform) informational environment as in the first round applies to outside investors, who would then bid down the equity share until the expected profit from the investment is driven down to zero, that is, $\frac{1}{2}\gamma_F R - F = 0$. Thus, an outside investor's second-round equity share will be given by $\gamma_F^0 = \frac{2F}{R}$. Similarly to the preceding section, the angel investor's initial share, $\hat{\gamma}_K$, is then determined by a zero expected-profit condition which takes into account the subsequent percentage dilution by a factor of $(1-\gamma_F^0)$.

To summarize, If an entrepreneur chooses angel financing, then $(\hat{\gamma}_K, K)$ specify the first-round terms offered by angels, and the second-round terms are given by (γ_F^0, F) above. If he chooses VC financing, then from the previous section (γ_K^*, K) are the first-round terms offered by VCs, and the second-round terms are given by (γ_F^-, F) with probability p^* and (γ_F^+, F) with probability $1-p^*$. Thus, the entrepreneur's as well as financiers' ownership shares are different depending on the entrepreneur's choice of a seed investor. On the other hand, given a competitive investment market, the expected *value* of the entrepreneur's equity is the same with either angel and VC financing. This leads to the following benchmark result.

Proposition 3. *An entrepreneur's ex-ante expected ownership share is larger with angel financing than with VC financing. Entrepreneurs prefer to finance the first round with angel investors.*

The intuition for this result is as follows. Notice that the equity required by financiers is inversely related to (and hence convex in) the venture's valuation. Since VC financing creates two possible valuations – stepped up or stepped down – in the second stage, each is associated with different equity demands from financiers. A direct consequence is that under VC financing, the entrepreneur has to cede a potentially very large ownership stake when a down round takes place. Given the convex relationship between equity shares and valuations, this downside risk is greater than the gain in up round (where the entrepreneur cedes a smaller equity share in exchange for funding). This is in direct contrast to angel financing, where, due angels' commitment not to re-invest, second-stage valuations are unchanged relative to the first stage. Thus, overall, the entrepreneur's expected retained equity share is higher with angel financing than with VC financing.

On the other hand, note that the ex-ante expected total surplus created by a venture is the same regardless of angel or VC financing. Indeed, the equity shares given up in exchange for the first-round investment are the same for VCs and angels (i.e., $\hat{\gamma}_K = \gamma_K^*$). It is thus straightforward that the entrepreneur's expected equity *value* is the same regardless of his first-round financing choice because (i) the expected venture value is fixed ex ante, and (ii) given a perfectly competitive investment market, the expected cost of financing each round is also the same for angels and VCs. That is, the value of equity given up in the second round following angel investment is equal to the investment F; and the value of equity given up following VC investment is also equal to F. [10] Similarly, the value of equity given up in the first round is equal to the investment K.

The observation that the entrepreneur's expected retained equity *value* is the same across angels and VCs suggests that the inside VC's signaling possibility is not necessarily a concern for entrepreneurs if they only care about retained equity values. However, as Proposition 3 indicates, the entrepreneur's first-round financing decision may still be affected, because his expected ownership *share* is smaller with VCs than with angels – due to the possibility of a down round triggered by the inside VC's non-participation, which results in severe ownership dilution. Recalling that entrepreneurs' utility encompasses both equity value and equity ownership, with a tradeoff between the two, Proposition 3 then implies that entrepreneurs in this benchmark case would strictly prefer to finance the first round with angels.

4.1. Contingent contracts

One may wonder whether the above result is due to the fact that the VC firm's first-round terms, (γ_K^*, K), did not depend on the second-round contingencies (i.e., up or down rounds). In the

[10] In the former case, the equity given up in the second round is $E[\gamma_F pR] = \gamma_F^0 E[p]R = F$; and in the latter case, it is $E[\gamma_F pR] = p^*\gamma_F^- E[p|p<p^*]R + (1-p^*)\gamma_F^+ E[p|p>p^*]R = 2F\frac{p^*}{2} + (1-p^*)F = F$.

following, we show that the above result is robust to the introduction of contingent contracts. First, notice that restricting attention to straight equity financing in the second round is without loss of generality because various forms of claims are indistinguishable given a cash flow realization, R. Second, in the case of angels, financing terms are in fact not contingent on any event, since angels do not create up or down rounds in the second stage.

A VC firm's first-round financing terms can be made contingent on the creation of an up or down round in the second stage. That is, the first-round financing terms can be $(\gamma_K^+, \gamma_K^-, K)$, where γ_K^+ and γ_K^- are the VC's shares contingent upon an up or down round, respectively. In this case, the subgame as well as overall equilibria are characterized by equations analogous to (1) and (2) above. The only change is that now there is a range of financing contracts consistent with these two conditions (rather than the unique solution we characterized above). Formally, the VC firm will re-invest when the success likelihood exceeds a threshold p^*, which satisfies

$$p^*(\gamma_K^+(1-\gamma_F^+)+\gamma_F^+)R - F = p^*(\gamma_K^-(1-\gamma_F^-))R, \tag{3}$$

and the expected profit from the first-round investment is zero,

$$-K + (1-p^*)\left(\frac{1+p^*}{2}(\gamma_K^+(1-\gamma_F^+)+\gamma_F^+)R - F\right) + p^*\left(\frac{p^*}{2}(\gamma_K^-(1-\gamma_F^-))R\right) = 0. \tag{4}$$

Given the financing terms in the second-round subgames (i.e., γ_F^+ and γ_F^-), the above expressions in (3) and (4) define three equilibrium quantities $(\gamma_K^+, \gamma_K^-, p^*)$ in the first round, leaving one degree of freedom. VC firms are, however, indifferent among the set of contingent-claims contracts that are consistent with (3) and (4) due to the zero expected-profit condition. The remaining question is then whether there are any contingent-claims contracts that VCs can offer in the first round which leave entrepreneurs with larger expected shares, so that the entrepreneurs would prefer to finance with VCs. The following says there are not.

Proposition 4. *Suppose VC firms can offer contingent-claims contracts $(\gamma_K^+, \gamma_K^-, K)$ in the first stage. Proposition 3 continues to hold.*

The intuitive reason is that the VCs cannot eliminate the signaling problem by merely using a contingent contract. That is, the only way in which VCs can avoid creating a signaling problem in the interm stage is by committing ex ante to a second round policy of investment — to always re-invest or to always not re-invest. Under either policy, the investment instrument becomes essentially equivalent to first-stage angel financing in our framework.

We note that the contingent-claims contracts considered above incorporate the possibility of financing with anti-dilution protections in a down round. Given angel-financing offers on the same competitive terms, however, entrepreneurs in our model would not accept VC contracts with anti-dilution protections.[11] Of course, entrepreneurs may take VC financing (even with anti-dilution protections) for reasons that are not captured by this model. Thus, the inside signaling problem studied in this paper is complementary to other considerations in the literature.

5. Ex-ante heterogeneity

Thus far, we have assumed that ventures are ex-ante identical; that is, a venture's probability of success is initially unknown by all parties. Signaling occurs at the second (interim) stage, after the entrepreneur and the inside financier learn about this probability. Moreover, we have assumed that entrepreneurs have equal access opportunities to potential financiers. In this section, we relax these assumptions. First, we incorporate the possibility that entrepreneurs have ex-ante private information about their ventures' likelihoods of success. Second, we incorporate a heterogeneous cost of accessing financiers, which may also depend on the supply of angel and VC financing. Our objective is to establish the existence of a separating equilibrium where angel and VC financing co-exist, and then conduct a comparative statics exercise.

Let us suppose that at the beginning of the game, the entrepreneur privately learns his venture's likelihood of success p with probability $\theta \in (0,1)$, and he learns nothing with probability $1-\theta$. (Alternatively, a fraction θ of entrepreneurs are *informed* and a fraction $1-\theta$ are *uninformed*.) It is useful to first discuss another benchmark case where all entrepreneurs are privately informed about their ventures' likelihoods of success (i.e., $\theta = 1$). Notice that this setup entails its own signaling problem for entrepreneurs *in the first stage*. That is, entrepreneurs can try to signal their ventures' ex-ante types with their choice of financiers.

As before, let p^* denote the marginal type above which inside VC firms would re-invest in the second stage, and consider the case where all informed entrepreneur types pool on VC financing. It is well known that there are multiple pooling equilibria in signaling games due to the fact that equilibrium consistency imposes no restrictions on off-the-equilibrium beliefs, so an equilibrium refinement is required to eliminate less desirable equilibria. We thus refine our set of equilibria by imposing the Intuitive Criterion (Cho and Kreps, 1987) for those deviations that occur with probability zero in equilibrium.[12]

To be more precise, off-equilibrium beliefs are specified by a type distribution on $[0, p^*)$ for an entrepreneur who deviates to angel financing. This entails that the deviation comes from the type that would benefit the most from doing so. That is, if an entrepreneur deviates from VC pooling to angel financing, those entrepreneur types who stand most to benefit are those seeking to avoid the VC's down round, that is, those who have $p < p^*$. This refinement eliminates the angel pooling equilibrium, which can be sustained by less plausible off-equilibrium beliefs (e.g., entrepreneurs who deviate to VC financing have $p = 0$). The following proposition shows that the VC pooling equilibrium survives this refinement criterion.

Proposition 5. *Suppose all entrepreneurs privately learn their ventures' types with probability $\theta = 1$. Then entrepreneurs prefer VC financing in the first round.*

Proposition 5 demonstrates that angel financing can unravel in equilibrium when entrepreneurs are ex-ante perfectly informed about their ventures' types. To see this, consider a threshold equilibrium, with a cutoff value $\tilde{p} \in (0,1)$, such that entrepreneurs with $p < \tilde{p}$ finance the first round with angels, and those with $p \geqslant \tilde{p}$ finance with VCs (the opposite case can also be considered, but it trivially cannot be sustained). Then entrepreneur types with $p < \tilde{p}$ can strictly gain by deviating from angel financing to VC financing, even if they know that they will face a down round, because the second-round terms would be based on more favorable beliefs (i.e., $\tilde{p} \leqslant p < p^*$); in comparison, angel financing entails $p < \tilde{p}$. Consequently, when $\theta = 1$, entrepreneurs would pool on VC financing, in complete contrast to the result in Proposition 3.

Intuitively, the difference from the previous section is the presence of adverse selection, whereby there is no mechanism for the

[11] This implication can be consistent with the empirical evidence that full ratchet anti-dilution is only found in less than 10% of VC contracts (Bengtsson and Sensoy, 2011).

[12] Technically, the refinement requires the set of entrepreneur types to be finite. We can take the set of entrepreneur types to be an arbitrarily large set of finite numbers in the interval $[0,1]$.

high types to sustain separation from the low types in the first stage. To distinguish themselves, high types would always have an incentive to deviate from angel to VC financing, because they benefit from separation in the first stage and also face an up round in the second stage. The equilibrium involves complete unraveling because those who remain with angels would then clearly reveal that they are low types. Without any mass of uninformed types ($\theta = 1$), the separation is stark — this is because a deviation to angel financing is not perceived as coming from an uninformed type but rather from a low type. Hence, while entrepreneurs would retain larger equity shares if *all* types finance with angels (e.g., if ex-ante commitment were an option), angel pooling is not sustainable with fully informed entrepreneurs.

5.1. Separating equilibrium

Consider now the extended model where $\theta \in (0, 1)$. Notice that the VC pooling equilibrium characterized in Proposition 5 continues to hold, given the aforementioned off-the-equilibrium beliefs. The only additional requirement to establish the VC pooling equilibrium is to verify that the uninformed entrepreneurs (with mass $1 - \theta$) would not benefit by deviating to angel financing. This is indeed the case because investors cannot directly observe whether an entrepreneur is informed or not, whereby investors' off-equilibrium beliefs are the same (i.e., $p < p^*$) following any deviation to angel financing in the VC pooling equilibrium. Given this specification of off-equilibrium beliefs, uninformed types also possess a strict incentive to pool on VC financing.

However, the VC pooling equilibrium characterized above is, of course, inconsistent with real-world observations, where angel investments have grown substantially and represent a considerable segment of the market in the early stages of financing. It is also inconsistent with our earlier benchmark result that demonstrates the benefit to entrepreneurs from financing with angels. Here, we show that there are in fact separating equilibria where, among the informed group of entrepreneurs, relatively low-type ventures finance the first stage with angel investors and relatively high types choose VCs. For ease of exposition, suppose initially that all uninformed entrepreneurs choose angel financing in the first stage. (This can be endogenized in the next subsection, but the main mechanism behind our result involves separating the informed entrepreneurs.)

To gain some intuition for why a separating equilibrium exists, suppose that all informed entrepreneurs (mass θ) finance with VCs. Then for any given θ, the first-stage beliefs are the uniform prior (i.e., there is no belief updating) for both informed and uninformed groups of entrepreneurs. Given that VCs and angels finance the same distribution of venture types, an informed entrepreneur with a type p arbitrarily close to zero would strictly benefit by switching to angel financing — because, by doing so, the entrepreneur can avoid a down round in the second stage, while angel investors' prior beliefs are minimally affected. That is, by financing with angels, a low-type entrepreneur can avoid the signaling problem at the interim stage, whereas he would surely face a down round had he chosen VC financing.

Building on the above, a deviation by a small mass of low-type ventures to angel financing would "weigh down" the ex-ante beliefs of angel investors from the uniform prior, but improve the ex-ante beliefs of VC investors. That is, as more informed entrepreneurs with low types switch to angels, the first-stage beliefs of VC investors are increasingly more favorable than those of angels. Then, there exists a marginal type, $\tilde{p}(\theta)$, who is just indifferent between angel and VC financing in the first round; that is, for this threshold type, the benefit from avoiding a down round with VC financing in the second stage exactly offsets the loss due to facing less favorable first-stage beliefs from angel investors. Formally, this yields the following result.

Proposition 6. *Suppose uninformed entrepreneurs finance the first round with angels. For any $\theta \in (0, 1)$, there exists a threshold value $\tilde{p}(\theta)$ such that informed entrepreneurs for whom $p < \tilde{p}(\theta)$ finance the first round with angels and those with $p \geqslant \tilde{p}(\theta)$ finance with VCs.*

Proposition 6 establishes that angels and VCs may draw different sets of entrepreneurs in the early stage. The threshold value $\tilde{p}(\theta)$ would depend on the model's parameter specification, but the qualitative features of the separating equilibria are essentially the same. That is, in this type of separating equilibria, angel investors, on average, finance lower-quality ventures than VCs. We emphasize that this finding is obtained purely as a result of the signaling problem associated with VC financing — this is because, in our framework, the only difference in the two financing sources is that angels can commit not to participate in the second stage.

5.2. Access cost

Thus far, entrepreneurs are differentiated only by their likelihoods of success; however, entrepreneurs are often also differentiated by their abilities to approach investors. For instance, some entrepreneurs may have easier access to VCs due to their personal networks. To incorporate this possibility, we consider a continuous distribution over a range of possible costs (denoted by c) for entrepreneurs to access VCs. Let us denote this distribution over access costs as $G(c)$, with support $[\underline{c}, \overline{c}]$, where $\underline{c} < 0$ and $\overline{c} > 0$ are arbitrary bounds.[13] We could further assume an arbitrary correlation between the two type parameters c and p, but such a correlation would not affect the qualitative nature of the result that follows.

Another motivation to consider a distribution of access costs is that the (relative) supply of angel and VC financing may not be the same. Note that our analysis has to do with the types of ventures that match to angels or VCs in the first stage, and in our framework the supply of angels and VCs is exogenous. Using this setup, we can illustrate the comparative statics implications of changes in the supply side for our model's equilibrium predictions. Specifically, we maintain the assumption that the supply of financing is given outside of our model and the resultant investment market is perfectly competitive. However, the relative supply and the size of the angel and VC financing markets can vary (e.g., across different locations or over time).

More specifically, entrepreneurs who seek financing when relatively little VC financing is available would generally incur higher costs of accessing VCs in terms of opportunity cost. Normalizing the cost of accessing angel investors to zero, the cost parameter c and its corresponding distribution $G(c)$ then represent the *relative* cost of accessing VCs. If, for instance, c is very large, then entrepreneurs would not seek VC financing; in contrast, if c is a large negative number, then entrepreneurs would choose VC financing. Hence, the parameter c can both accentuate (e.g., low types with positive VC access costs prefer angels more strongly) and attenuate (e.g., high types with high VC access costs may switch to angels) entrepreneurs' financing choices.

We know from Proposition 6 that an informed type with p higher than a certain value $\tilde{p}(\theta)$ would choose VC financing in the first stage. This threshold value $\tilde{p}(\theta)$ would now depend on the VC-access cost c as well, whereby a larger mass of informed entrepreneurs would pursue angel financing as VC-access costs rise. Notice that the informed types with p just above p^* will face an up round in the second stage under VC financing and thus are

[13] We assume that the upper bound $\overline{c}$ is sufficiently high to ensure an interior equilibrium.

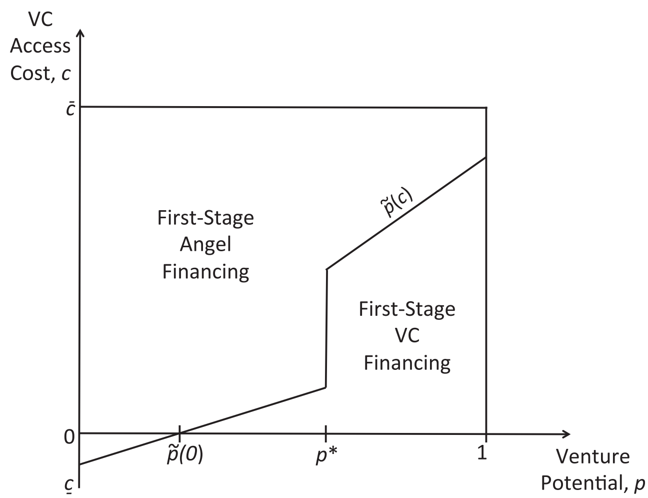

Fig. 1. First-round investor choice by informed types for a given θ.

discontinuously less sensitive to a rising VC-access cost. This implies that the access cost threshold above which the informed type $p = p^*$ would switch to angel financing exhibits a "jump" relative to the informed type p just below p^*. Fig. 1 demonstrates the access cost thresholds for a given value of θ. The following proposition formalizes our findings.

Proposition 7. *For any $\theta \in (0, 1)$, there exists a separating equilibrium where informed entrepreneurs for whom $p < \tilde{p}(c, \theta)$ finance with angels and those with $p \geqslant \tilde{p}(c, \theta)$ finance with VCs. Uninformed entrepreneurs with low (high) access costs finance with VCs (angels).*

Thus, given a distribution of c and p, the fractions of entrepreneurs who choose angels and VCs are determined from the two areas indicated in Fig. 1. Holding other factors constant, this yields a clear comparative statics implication of our model. That is, when the supply of VC financing becomes relatively scarce, so that access costs to VCs increase, a smaller fraction of entrepreneurs would receive VC financing in the first stage (because the population of entrepreneurs are now more densely distributed at higher values of c).[14] Further, our model predicts that as the fraction of entrepreneurs who seek first-stage VC financing decreases, the average success likelihood of VCs' portfolios would *increase*. This is because only higher-type entrepreneurs – those above the threshold $\tilde{p}(\theta)$ – would still find VC financing worthwhile, despite the higher VC-access costs incurred.

6. Interim liquidation

In this section, we examine the robustness of our results to changes in Assumption 1. If Assumption 1 is not satisfied, then ventures will be liquidated in a down round, generating no revenues. Notice that, at the interim stage (the second round), outside investors would only be willing to participate if a venture's expected value is non-negative, i.e., $E[pR - F|h] \geqslant 0$, where h denotes the history of the game observed by outside investors. Thus, the conditions under which outside investors are willing to participate in the second round following VC financing are as follows.

- **Up round:** $E[p|p \geqslant p^*] \geqslant \frac{F}{R}$. Simplifying yields

$$RF - F^2 + 2KR \leqslant (R - F)^2. \tag{5}$$

- **Down round:** $E[p|p < p^*] \geqslant \frac{F}{R}$. Simplifying yields

$$4KR \leqslant (R - 2F)^2. \tag{6}$$

Notice that (6) is Assumption 1. This condition can be rewritten as $2RF + 3F^2 + 4KR \leqslant (R - F)^2$; hence, Assumption 1 implies that (5) is satisfied. This is intuitive because the posterior beliefs for ventures in a down round are worse than for those in an up round. Notice that somewhere in between these two conditions lies the condition for angel investors to participate in the first round, $R \geqslant 2F + 2K$. Previously, Assumption 1 guaranteed $R \geqslant 2F + 2K$ trivially; however, if we only assume that inequality (5) is satisfied, then $R \geqslant 2F + 2K$ becomes a binding constraint.

In the following, we characterize the equilibrium of the baseline model, when inequality (5) is satisfied but (6) is not. That is, we assume $RF - F^2 + 2KR \leqslant (R - F)^2 < 2RF + 3F^2 + 4KR$, to which we refer as the Limited Viability (LV) case, and show that our main results continue to hold with a slightly more binding assumption (i.e., $R > 3F + 2K$) as a *sufficient* condition. This is indeed stronger than the necessary condition, $R \geqslant 2F + 2K$; however, it is not so restrictive if most ventures worth considering would have at least a moderately higher expected revenue than the required capital investments.

Similarly to the previous analysis, let p^*_{LV} denote the inside VC's threshold value, where the VC re-invests if and only if the probability of success is greater than p^*_{LV}; otherwise, the venture is liquidated in the second stage. Outside investors' updated beliefs are that the success likelihood of a venture conditional on the inside VC following through is uniform on $[p^*_{LV}, 1]$. Thus, the financing terms in an up round are given by $\gamma_F^+ = \frac{2F}{(1+p^*_{LV})R}$. Since ventures in a down round are worth nothing, the inside VC's outside option value is now zero. Hence, an inside VC would re-invest if and only if the following condition is satisfied:

$$p(\gamma_K(1 - \gamma_F^+) + \gamma_F^+)R - F \geqslant 0.$$

Then, the optimal threshold value, p^*_{LV}, is determined by substituting in for γ_F^+:

$$p^*_{LV}\left(\gamma_K + (1 - \gamma_K)\frac{2F}{(1 + p^*_{LV})R}\right)R - F = 0. \tag{7}$$

Proposition 8. *With limited-viability ventures, entrepreneurs' expected ownership shares are larger with angels than with VC financing if $R > 2K + 3F$.*

It follows from Proposition 8 that when entrepreneurs are ex-ante identical (as in Section 2's model), the ownership advantage associated with angel financing continues to hold even if ventures are liquidated in a down round. It can be further shown that the equilibrium characterization in the previous section (where entrepreneurs have private information) extends to the LV case under certain conditions. Specifically, while the VC pooling equilibrium can always exist, the separating equilibrium may fail to exist if the mass of uninformed entrepreneurs who pursue angel financing is too small.

To gain some intuition, consider the separating equilibrium. Given a set of parameter values, we now observe that $\tilde{p} \geqslant p^*_{LV}$ must be satisfied, where $\tilde{p}$ is the threshold type below (above) which informed entrepreneurs choose angel (VC) financing. To see this, suppose, by way of contradiction, that a candidate separating equi-

[14] To be more precise, as the supply of VC financing becomes relative scarce, the new conditional distribution function $G(c|p)$ first-order stochastically dominates the previous distribution function at all values of entrepreneur types $p \in [0, 1]$.

librium entails $\tilde{p} < p^*_{LV}$. Notice then that informed entrepreneurs in $[\tilde{p}, p^*_{LV})$ would strictly benefit by switching from VC to angel financing — because otherwise their ventures will be liquidated in a down round due to no follow-on participation by the inside VC — a contradiction.

If a candidate equilibrium satisfies $\tilde{p} \geqslant p^*_{LV}$, then the informed entrepreneurs with $p < p^*_{LV}$ are already financing with angels, and no further deviation takes place. For a candidate equilibrium with $\tilde{p} \geqslant p^*_{LV}$ to exist, however, the mass of the uninformed group who chooses angels must be sufficiently large. If the mass of this uninformed group is arbitrarily small, then given the Bayesian updating of beliefs, the threshold value $\tilde{p}$ must also be arbitrarily close to zero, violating the requirement $\tilde{p} \geqslant p^*_{LV}$. Therefore, a separating equilibrium can exist in the LV case provided that the mass of uninformed entrepreneurs who seek angel financing is sufficiently large. This case is more likely when the distribution of VC access costs is weighed towards higher values.

7. Related works

In looking at the role of angel financing, Kerr et al. (2014) observe that angels engage in efficient selection and screening just as traditional VCs do. Cosh et al. (2009) show that angel investors finance ventures that exhibit similar characteristics as those in which VCs invest. However, existing theoretical models typically rely on ex-ante differences between angels and VCs regarding deal access and value added. As previously mentioned, Hellmann and Thiele (2014) consider a matching model where VCs cannot participate in the early stage of financing, and Chemmanur and Chen (2014) consider an agency model where VCs can add more value than angels can. [15]

While our paper is complementary to these works, our signaling model provides somewhat different empirical implications. In the agency model, the main prediction comes from the entrepreneur's efforts or financier's value-adding activities. For instance, to test a version of Chemmanur and Chen (2014)'s model, a researcher can measure portfolio exit rates as a proxy for the success probability, and if the researcher controls for measures of value-adding activities of VCs and angels, then there should be no difference in a VC's and an angel's portfolio exit rates. In contrast, our model predicts that VCs would have a higher exit rate than angels, even after controlling for value added and other observable characteristics.

More generally, our model's core assumption — that the inside VC observes the venture's success probability, whereas outside investors cannot except through the insider's follow-on decision — may be testable in a number of ways. For instance, holding constant other factors, the insider's follow-on decision should be a significant predictor for the financing terms offered by outsiders. This would be the case as long as the insider had the requisite liquidity (i.e., unconstrained by portfolio or investment policies), so the follow-on decision contains new information hitherto unknown to outsiders. If outsiders have the same information as insiders, then the follow-on decision should not matter.

It should also be mentioned that our model is related to the literature on competition between inside and outside banks under asymmetric information. For instance, Rajan (1992) assumes a public signaling device that is correlated with the inside bank's private information, so that outside banks can update their beliefs before bidding on re-investment terms. The difference is that in our model, there is no public signaling device; rather, the insider's private information is endogenously signaled via the inside VC's action (i.e., follow-on decision). Thus, outside investors can observe some information that insiders have but this information has to be inferred from the inside VC's action and strategic incentives in equilibrium (rather than from observing a correlated public signal).

In a similar informational environment to ours, Mahrt-Smith (2006) shows that a bank's equity participation can reduce the inside bank's bargaining power, which in turn reduces the bank's incentives to extract rents from multiple rounds of financing. Other banks consequently infer that only bad firms ask for outside funding, thus creating down rounds with worse financing terms, whereas good firms continue to obtain loans from the informed bank. Our model extends this framework to incorporate two different types of equity investors (VCs and angels) and show that similar results continue to hold under our model's assumptions, taking into account an entrepreneur's share dilution. [16]

8. Conclusion

This paper highlights informational aspects of the choice between angel and VC financing in the early stages of a venture. When outside investors evaluate a venture, they are typically at an informational disadvantage relative to inside investors; however, they may update their beliefs by observing an inside investor's re-investment decision. When ventures are ex-ante identical, we showed that an entrepreneur's expected ownership share is lower with VC than with angel financing. When entrepreneurs exhibit preferences over ownership, they would benefit from angel financing.

When entrepreneurs have ex-ante private information, another layer of signaling is introduced, pertaining to entrepreneurs' choice of first-round financiers. While a VC-pooling equilibrium can always be sustained, there is a separating equilibrium that can help explain the co-existence of angels and VCs in the early stages of financing. The separating equilibrium gives rise to the prediction that angel investors finance ventures that, on average, have lower probabilities of success, whereas VCs, on average, finance ventures with higher probabilities of success.

While the main driving force in the equilibrium we study is our finding that angel financing leaves a larger ownership share to entrepreneurs, our analysis relies on a few critical assumptions. For instance, we abstract from the moral-hazard problem, the use of convertible securities, and other sources of early-stage financing such as debt. Although we think that the VC 'signaling hypothesis' is an issue relatively independent of these other factors, future research may examine this hypothesis under alternative assumptions.

Appendix A

Proposition 1

Proof. Let the outside VC firm's strategy be to offer to finance the second round in return for acquiring a $\gamma_F^+ = \frac{2F}{(1+p^*)R}$ share in an up round and a $\gamma_F^- = \frac{2F}{p^*R}$ in a down round, respectively. We can easily see that this strategy profile satisfies the zero-profit condition. First, in an up round, the expected probability of success is $\frac{1+p^*}{2}$, so

[15] The traditional VC-financing literature (e.g., Admati and Pfleiderer, 1994; Gompers, 1995; Bergemann and Hege, 2005) also focuses on agency problems and has often neglected an entrepreneur's ownership tradeoffs across different financing sources.

[16] The choice of financier among broader types of investors, including banks, is beyond the scope of our paper (see, e.g., de Bettignies, 2008). That is, the signaling problem faced by entrepreneurs in the early-stage in our model is conditional on narrowing down the choice to angels and VCs.

that an outside firm's expected profit is $\frac{1+p^*}{2}\gamma_F^+R - F = 0$. Since the offer terms are the same, the entrepreneur stays with the inside investor by the tie-breaking assumption. Second, in a down round, the expected probability of success is $\frac{p^*}{2}$, so that an outside investor's expected profit is $\frac{p^*}{2}\gamma_F^- R - F = 0$. Since only outside investors offer to finance, the entrepreneur takes any one of such offers.

We now verify that the inside VC firm's cut-off strategy is optimal given outside firms' strategy. When the inside VC decides to follow through, outside VCs believe that the success probability is greater than p^* ; and, when it does not, the belief is $p < p^*$. It is never optimal for the inside VC to offer better terms than the market; and offering worse terms does no better because outside VCs will offer better terms, which the inside VC has to match. Given this, it is optimal for the inside VC firm to finance an entrepreneur in the second round if and only if the investor's expected profit conditional on $p > p^*$ is greater than its expected profit conditional on $p \leqslant p^*$, that is, if and only if the observed success probability p satisfies the following inequality:

$$p\left(\gamma_K + (1-\gamma_K)\frac{2F}{(1+p^*)R}\right)R - F \geqslant p\left(\gamma_K\left(1-\frac{2F}{p^*R}\right)\right)R.$$

Setting both sides equal yields the threshold value, $p^* = 1 - 2\gamma_K$. □

Proposition 3

Proof. Notice that if an angel invests in the first round, then there is no signal sent to the market at the beginning of the second round. Given our Assumption 1, outside VC firms will invest in the second round following angel investment (i.e., with no signal from an inside investor) because the expected probability of success is higher than in a down round. The second-round investor's share, $\gamma_F^0 = \frac{2F}{R}$, is pinned down by the zero-profit condition. Given this, an angel investor's expected profit from the first-round investment is $-K + \int_0^1 p(\gamma_K(1-\frac{2F}{R})R)dp$, where the second term is the expected share value. The angel investor's share is derived by the zero-profit constraint, $-K + \frac{1}{2}\gamma_K(1-\frac{2F}{R})R = 0$, which gives $\hat{\gamma}_K = \frac{2K}{R-2F}$.

The entrepreneur's share, if he finances with an angel in the first round, is $1 - [\hat{\gamma}_K(1-\gamma_F^0) + \gamma_F^0]$. On the other hand, if the entrepreneur finances with a VC firm that can re-invest in the second round, his expected share is $1 - [\gamma_K^*(1-E\gamma_F) + E\gamma_F]$, where $E\gamma_F = p^*\gamma_F^- + (1-p^*)\gamma_F^+$, from the previous proof. The entrepreneur's ownership share is larger with an angel than with a VC firm if and only if $\hat{\gamma}_K(1-\gamma_F^0) + \gamma_F^0 < \gamma_K^*(1-E\gamma_F) + E\gamma_F$. By substitution, $\hat{\gamma}_K(1-\gamma_F^0) + \gamma_F^0 = \frac{2K+2F}{R}$. Since $E\gamma_F = p^*\gamma_F^- + (1-p^*)\gamma_F^+ = p^*\frac{2F}{p^*R} + (1-p^*)\frac{2F}{(1+p^*)R} = \frac{2F}{(1-\gamma_K^*)R}$, it follows that $\gamma_K^*(1-E\gamma_F) + E\gamma_F = \gamma_K^* + \frac{2F}{R}$. Thus, the inequality holds if and only if $\frac{2K+2F}{R} < \frac{2K}{R-2F} + \frac{2F}{R}$, or $\frac{2K}{R} < \frac{2K}{R-2F}$, which is indeed the case. □

Proposition 4

Proof. Setting $\gamma_F^+ = \frac{2F}{(1+p^*)R}$ and $\gamma_F^- = \frac{2F}{p^*R}$ and solving Eqs. (3) and (4) with equality, we obtain

$$\gamma_K^+ = \frac{F(1-p^*)p^* + 2K(1+p^*)}{(1+p^*)R - 2F}$$

and

$$\gamma_K^- = \frac{2Kp^* - F(1-p^*)^2}{p^*R - 2F}. \tag{8}$$

Proposition 3 then holds if and only if

$$\hat{\gamma}_K(1-\gamma_F^0) + \gamma_F^0 < p^*(\gamma_K^-(1-\gamma_F^-) + \gamma_F^-) + (1-p^*)(\gamma_K^+(1-\gamma_F^+) + \gamma_F^+).$$

Using (3) and substituting $\hat{\gamma}_K(1-\gamma_F^0) + \gamma_F^0 = \frac{2K+2F}{R}$, we obtain

$$\frac{2K+2F}{R} \leqslant p^*(\gamma_K^+(1-\gamma_F^+) + \gamma_F^+ + \gamma_F^-) - \frac{F}{R} + (1-p^*)(\gamma_K^+(1-\gamma_F^+) + \gamma_F^+).$$

Rearranging yields

$$\frac{2K+3F}{R} \leqslant \gamma_K^+(1-\gamma_F^+) + \gamma_F^+ + p^*\gamma_F^-.$$

Substituting for $\gamma_K^+, \gamma_F^-, \gamma_F^+$, and simplifying, Proposition 3 continues to hold when

$$\frac{2K+3F}{R} \leqslant \frac{2K+4F-Fp^*}{R},$$

which is true since $p^* \leqslant 1$. □

Proposition 5

Proof. Suppose there exists a threshold $\tilde{p} \in (0,1)$ such that entrepreneurs with ideas $p \geqslant \tilde{p}$ finance the first round with a VC firm and entrepreneurs with ideas $p < \tilde{p}$ finance with an angel. Let $p^*, p^* \geqslant \tilde{p}$, denote the threshold level above which the VC firm follows through. Then the second-stage zero profit condition, $E[p|p > p^*]\gamma_F^+R - F = 0$, determines $\gamma_F^+ = \frac{2F}{(1+p^*)R}$, and similarly $E[p|\tilde{p} < p \leqslant p^*]\gamma_F^-R - F = 0$ determines $\gamma_F^- = \frac{2F}{(\tilde{p}+p^*)R}$. In the following, let γ_K^+ denote the VC firm's share from the first round of financing given the threshold strategies. Then γ_K^+ is obtained from

$$-K + \frac{1-p^*}{1-\tilde{p}}\left(\frac{1+p^*}{2}\left(\gamma_K^+ + (1-\gamma_K^+)\frac{2F}{(1+p^*)R}\right)R - F\right) + \frac{p^*-\tilde{p}}{1-\tilde{p}}\left(\frac{\tilde{p}+p^*}{2}\gamma_K^+\left(1-\frac{2F}{(\tilde{p}+p^*)R}\right)R\right) = 0.$$

Rearranging and simplifying yields

$$\gamma_K^+ = \frac{2K}{(1+\tilde{p})R - 2F}.$$

Given the VC firm's strategy, an entrepreneur with $p \leqslant p^*$ who finances with a VC will have an ownership share of

$$(1-\gamma_K^+)(1-\gamma_F^-), \tag{9}$$

because the entrepreneur knows that he will face a down round in the second stage. Similarly, an entrepreneur with $p > p^*$ who finances with a VC knows that he will face an up round and thus has an ownership share of $(1-\gamma_K^+)(1-\gamma_F^+)$, which is larger than (9) because $\gamma_F^+ < \gamma_F^-$.

On the other hand, suppose entrepreneurs with $p < \tilde{p}$ finance with angels. In the second round, γ_F^0 is bid down such that $\frac{\tilde{p}}{2}\gamma_F^0R - F = 0$, giving $\gamma_F^0 = \frac{2F}{\tilde{p}R}$. Let γ_K^0 denote the angel's first-round equity share in exchange for K. Then γ_K^0 is obtained from the zero-profit condition, $\frac{\tilde{p}}{2}\gamma_K^0\left(1-\frac{2F}{\tilde{p}R}\right)R - K = 0$, giving $\gamma_K^0 = \frac{2K}{\tilde{p}R-2F}$.

If $\tilde{p}$ is below a certain value, then $\gamma_K^0 > 1$, which means that the angel investor cannot break even by investing in the venture given that only entrepreneurs with $p < \tilde{p}$ apply to angels. Thus, given a sufficiently low threshold value for $\tilde{p}$, all entrepreneurs have to finance with VCs, resulting in the VC pooling equilibrium.

If $\tilde{p}$ is above a certain value, then it becomes possible to have $\gamma_K^0 \leqslant 1$, so that angel investors can break even. In this case, an entrepreneur with $p < \tilde{p}$ who finances with an angel will have an ownership share of

$$(1-\gamma_K^0)(1-\gamma_F^0). \tag{10}$$

First, suppose an entrepreneur with $p < \tilde{p}$ deviates to VC financing. Comparing (9) and (10), the entrepreneur would gain from such a deviation because $\gamma_K^+ < \gamma_K^0$ and $\gamma_F^- < \gamma_F^0$. That is, the entrepreneur's share is higher under VC financing. Notice that the entrepreneur knows his success likelihood p, so his equity value is also higher given that his equity share increases. Therefore, the threshold $\tilde{p}$ cannot be positive.

Second, suppose all entrepreneurs finance with VCs, that is, $\tilde{p} = 0$. From Propositions 1 and 2, the VC firm's shares in such a case are $\gamma_F^+ = \frac{2F}{(1+p^*)R}, \gamma_F^- = \frac{2F}{p^*R}$, and $\gamma_K^* = \frac{2K}{R-2F}$. A deviation to an angel investor (a probability zero event) would lead to $\gamma_F^0 = \frac{2F}{p^*R}$ and $\gamma_K^0 = \frac{2K}{p^*R-2F}$, given that the off-equilibrium beliefs is $p \in [0, p^*)$. The deviating entrepreneur's equity share as well as value are lower because $\gamma_F^0 = \gamma_F^-$ and $\gamma_K^0 > \gamma_K^*$. Thus, no entrepreneur has an incentive to deviate.

Third, consider the case in which entrepreneurs with $p \geqslant \tilde{p}$ finance with angels and those with $p < \tilde{p}$ finance with VCs. Then the VC's shares are $\gamma_F^+ = \frac{2F}{(\tilde{p}+p^*)R}$ and $\gamma_F^- = \frac{2F}{p^*R}$. Using a zero-profit condition, the VC's first-round share is $\gamma_K^+ = \frac{2K}{\tilde{p}R-2F}$. On the other hand, an angel investor's shares are $\gamma_K^0 = \frac{2K}{(1+\tilde{p})R-2F}$ and $\gamma_F^0 = \frac{2F}{(1+\tilde{p})R}$. Following the same logic as above, an entrepreneur with $p \leqslant \tilde{p}$ possesses a profitable deviation by instead financing with an angel because $\gamma_K^+ > \gamma_K^0$ and $\gamma_F^- > \gamma_F^+ > \gamma_F^0$.

Fourth, suppose all entrepreneurs finance with angels. Then the angel investor's shares are $\gamma_K^0 = \frac{2K}{R-2F}$ and $\gamma_F^0 = \frac{2F}{R}$. A deviation to VC financing leads to $\gamma_K^+ = \frac{2K}{(1+\tilde{p})R-2F}$ and $\gamma_F^+ = \frac{2F}{(1+p^*)R}$, given the off-equilibrium beliefs $p \in [p^*, 1]$. An entrepreneur with $p > p^*$ knows that he will face an up round in the second stage with a VC. Such an entrepreneur is indeed better off by deviating to VC financing because $\gamma_F^0 > \gamma_F^+$ and $\gamma_K^0 > \gamma_K^+$, that is, because his equity share as well as share value increase. $\square$

Proposition 6

Proof. Suppose all uninformed entrepreneurs finance with angels. For a given $\theta \in (0,1)$, we will consider a threshold $\tilde{p}(\theta)$ such that informed entrepreneurs with $p < \tilde{p}(\theta)$ finance with angels and those with $p \geqslant \tilde{p}(\theta)$ finance with VCs. Then the expected type of an entrepreneur who finances with an angel is given by

$$p_0(\theta) = \frac{\theta\tilde{p}(\theta)}{1-\theta+\theta\tilde{p}(\theta)}\frac{\tilde{p}(\theta)}{2} + \frac{1-\theta}{1-\theta+\theta\tilde{p}(\theta)}\frac{1}{2} = \frac{1-\theta+\theta\tilde{p}(\theta)^2}{2(1-\theta+\theta\tilde{p}(\theta))}.$$

With a slight abuse of notation, it follows that the angel financing terms are $\gamma_F^0 = \frac{F}{p_0(\theta)R}$ and $\gamma_K^0 = \frac{K}{p_0(\theta)R-F}$ as previously shown. Thus, an entrepreneur's equity share under angel financing at the end of the second round is given by $(1-\gamma_K^0)(1-\gamma_F^0)$. Similarly, a VC's financing terms are $\gamma_F^+ = \frac{2F}{(1+p^*)R}, \gamma_F^- = \frac{2F}{(\tilde{p}(\theta)+p^*)R}, \gamma_K^+ = \frac{2K}{(1+\tilde{p}(\theta))R-2F}$, where p^* is defined by the VC's indifference condition for re-investment in the second round.

It suffices to show that there exists a threshold type $\tilde{p}(\theta)$ such that informed entrepreneurs with $p < \tilde{p}(\theta)$ finance with angels and those with $p \geqslant \tilde{p}(\theta)$ finance with VCs, all uninformed entrepreneurs finance with angels, and no entrepreneur has an incentive to deviate.

First, suppose that $\tilde{p}(\theta) = 0$, whereby all informed entrepreneurs finance with VCs and all uninformed entrepreneurs finance with angels. Consider informed entrepreneurs whose types fall below the threshold p^* for an up round under VC financing. Because first-round investors learn nothing from the entrepreneurs' choice of first-round financiers, it follows from Proposition 3 that $(1-\gamma_K^0)(1-\gamma_F^0) > (1-\gamma_K^+)(1-\gamma_F^-)$, i.e., such informed entrepreneurs would prefer to deviate to angel financing.

Second, suppose that $\tilde{p}(\theta) = 1 - \frac{1-\sqrt{1-\theta}}{\theta} \leqslant \frac{1}{2}$. Then by substitution above $p_0(\theta) = 1 - \frac{1-\sqrt{1-\theta}}{\theta} = \tilde{p}(\theta)$ as well as $\frac{\partial\tilde{p}(\theta)}{\partial\theta} < 0$. Consider an informed entrepreneur with $p < 1 - \frac{1-\sqrt{1-\theta}}{\theta}$. His share when financing with an angel is $(1-\gamma_K^0)(1-\gamma_F^0)$, where $\gamma_F^0 = \frac{F}{\tilde{p}(\theta)R}$ and $\gamma_K^0 = \frac{K}{\tilde{p}(\theta)R-F}$. If he finances with a VC, his share is $(1-\gamma_K^+)(1-\gamma_F^-)$, where $\gamma_F^- = \frac{2F}{(\tilde{p}(\theta)+p^*)R}$ and $\gamma_K^+ = \frac{2K}{(1+\tilde{p}(\theta))R-2F}$. Because $\gamma_F^0 > \gamma_F^-$ and $\gamma_K^0 > \gamma_K^+$, the entrepreneur will deviate to VC financing.

Thus, the entrepreneur prefers to deviate to angel financing when $\tilde{p}(\theta) = 0$, and prefers to deviate to VC financing when $\tilde{p}(\theta) = 1 - \frac{1-\sqrt{1-\theta}}{\theta}$. Since the entrepreneur's shares from angel and VC financing are continuous functions of $\tilde{p}(\theta)$, it follows from the Intermediate Value Theorem that there exists a value $\tilde{p}(\theta) \in (0, 1 - \frac{1-\sqrt{1-\theta}}{\theta})$ such that informed entrepreneurs with $p < \tilde{p}(\theta)$ prefer angel financing while those with $p \geqslant \tilde{p}(\theta)$ prefer VC financing. $\square$

Proposition 7

Proof. To establish a separating equilibrium, it suffices to show that no entrepreneur has an incentive to deviate given a set of strategies and beliefs derived from Bayesian updating. Proposition 6 shows that for any given mass of uninformed entrepreneurs, there is a threshold type $\tilde{p}(\theta) > 0$ above which informed entrepreneurs choose VCs when $c = 0$. For any given realization of c, it is straightforward to see that the threshold value $\tilde{p}(c,\theta)$ is increasing in c. Moreover, there is a range of c values where the threshold stays the same at $\tilde{p}(c,\theta) = p^*$ because the entrepreneur would surely face an up round if they choose VC financing instead of angel financing. Given the threshold strategy $\tilde{p}(c,\theta)$, no informed entrepreneur has an incentive to deviate.

Let us now take as given the above threshold strategy of informed entrepreneurs, and let us consider a similar threshold strategy for the uninformed types; that is, an uninformed type choose VCs if his cost parameter c is below a certain threshold $\tilde{c}$, and alternatively chooses angels when $c \geqslant \tilde{c}$. If $\tilde{c}$ is sufficiently close to $\underline{c}$, then almost all uninformed entrepreneurs choose angel financing. This means that angel investors' prior beliefs are also arbitrarily close to those specified in Proposition 6. However, this in turn implies that uninformed entrepreneurs with c just above $\tilde{c}$ would be better off by switching to VC financing because, unlike the informed threshold type, these uninformed entrepreneurs are not guaranteed a down round with VCs. Given angel investors' Bayesian belief updating, an (informed and uninformed) entrepreneur's payoff from angel financing is monotone decreasing in the threshold $\tilde{c}$. Thus, for $\bar{c}$ sufficiently high, there exists a unique threshold $\tilde{c} \in (\underline{c}, \bar{c})$ at which an uninformed entrepreneur of type $\tilde{c}$ is indifferent between VC and angel financing. $\square$

Proposition 8

Proof. Ex ante, the second-round terms are $\gamma_F^+ = \frac{2F}{(1+p_{LV}^*)R}$ with probability $1 - p_{LV}^*$ and non-investment with probability p_{LV}^*. Thus, the expected profit of a first-round VC can be written as

$$-K + (1 - p_{LV}^*)\left(\frac{1 + p_{LV}^*}{2}\left(\gamma_K + (1 - \gamma_K)\frac{2F}{(1 + p_{LV}^*)R}\right)R - F\right) \geqslant 0.$$

Setting the expected profit equal to zero and substituting in for (7) yield

$$F(1 - p_{LV}^*)^2 = 2Kp_{LV}^*.$$

Solving for p_{LV}^*, the only root satisfying $p_{LV}^* \leqslant 1$ is given by

$$p_{LV}^* = \frac{F + K - \sqrt{2FK + K^2}}{F}.$$

Substituting p_{LV}^* in for (7) and rearranging yield the first-round share,

$$\gamma_K = \frac{F^2(\sqrt{K(2F + K)} - K)}{(\sqrt{K(2F + K)} - F - K)\left(2F^2 + R\sqrt{K(2F + K)} - 2FR - KR\right)}.$$

Since ventures are liquidated in a down round with probability p_{LV}^*, the entrepreneur's expected equity share is given by

$$(1 - p_{LV}^*)(1 - \gamma_F^+)(1 - \gamma_K).$$

Substituting in for $\gamma_F^+ = \frac{2F}{(1+p_{LV}^*)R}$, we have

$$(1 - p_{LV}^*)\left(1 - \frac{2F}{(1 + p_{LV}^*)R}\right)(1 - \gamma_K).$$

From Proposition 3, the entrepreneur's expected share under angel investing is given by $1 - \frac{2K+2F}{R} = \frac{R-2(K+F)}{R}$. Thus, the entrepreneur's shares are larger with angels if

$$\frac{R - 2(K + F)}{R} > (1 - p_{LV}^*)\left(1 - \frac{2F}{(1 + p_{LV}^*)R}\right)(1 - \gamma_K).$$

Substituting in for p_{LV}^* and simplifying, this inequality can be rewritten as

$$\frac{R-2(K+F)}{R} > \frac{\sqrt{2FK+K^2}-K}{F}\left(1 - \frac{2F^2}{(2F+K-\sqrt{2FK+K^2})R}\right)(1-\gamma_K).$$

Substituting in for γ_K and simplifying, this further reduces to

$$F^2 + (R - F)(R - 2K - 3F) > 0.$$

A sufficient condition is thus given by $R > 2K + 3F$. □

Appendix B. Supplementary Material

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.jbankfin.2016.03.004.

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