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Split-Award Auctions for Supplier Retention

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To stay abreast of current supply-market pricing, it is common for procurement managers to frequently organize auctions among a pool of qualified suppliers (the *supply base*). Sole awards can alienate losing suppliers and cause them to defect from the supply base. To maintain the supply base and thereby control the high costs of finding and qualifying new suppliers, buyers often employ split awards, which in turn inflate purchase costs. This results in a trade-off that we investigate in an infinite-horizon stationary setting in which the relative cost position of each supplier is randomly drawn in every period. We characterize the optimal split award that minimizes long-run costs (purchasing and qualification) and show that maintaining a constant supply-base size—using a "qualify-up-to" policy—is optimal for the buyer. We find that neither the extent of multisourcing nor the buyer's value of split awards compared with winner-take-all auctions are monotonic in the qualification cost and that split-award auctions can increase ex ante system, buyer, and supplier benefits simultaneously. To our knowledge, this is the first paper studying split-award auctions for supply-base maintenance, and it will hopefully galvanize further research on this important topic.

Keywords: auctions; split awards; multisourcing; supply base
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1. Introduction

In rapidly changing industries, the frequent evolution of products and technologies makes it difficult for a procurement manager to discover the production costs of its suppliers. In such settings, buyers often deploy reverse auctions (competitive bidding) as a vehicle for pricing supply contracts (Beall et al. 2003). This is especially true in the procurement of commodity-type components in high-tech industries, where buyers regularly organize auctions to stay abreast of the market. For example, large electronics manufacturers often run quarterly auctions for procurement of commodity electronics components.

Procuring from low-cost suppliers is vital, but it is equally important to procure from qualified suppliers because supply failures can be devastating for a firm. For example, in 2007 pet food maker Menu Foods recalled over 60 million packages of dog and cat food as a result of unauthorized, toxic chemical additives introduced by one of its suppliers (Myers 2007). To reduce the likelihood of such problems, buyers typically use stringent prequalifying procedures to verify that a supplier is indeed capable of fulfilling the contract to the buyer's satisfaction. Indeed, it is standard practice to allow only qualified suppliers to bid in auctions (Beall et al. 2003). Qualifying suppliers is a time-consuming and costly process, involving the collection of supplier information, factory audits, supplier development, and evaluation. Furthermore, many elements of qualifying a supplier (e.g., product testing and face-to-face meetings with supplier engineers) typically fall on the technical workforce of the buyer firm. The buyer firm's engineers can be (and often are) reluctant to perform these tasks, viewing them as an unwelcome distraction from their primary responsibilities of engineering design. From the perspective of the procurement manager, this further introduces additional, albeit intangible, political costs in qualifying new suppliers.

To avoid the high costs associated with supplier qualification, a buyer establishes a group of suppliers, called the *supply base*, which it utilizes when awarding business. Thus, the buyer has the possibility of inviting the prequalified suppliers from its supply base when organizing an auction, rather than spending time and money tracking down and qualifying new ones. For example, we interacted with an electronics firm in which a typical supply base for a commodity input consisted of around two to four suppliers.

Unfortunately, once established, a supply base is not something that the buyer can necessarily take for granted. At the electronics manufacturer mentioned above, procurement managers noted that suppliers do disengage from the supply base. One of these procurement managers, who ran auctions for short-term contracts, pointed out that suppliers were shopping around for other customers and several had locked in contracts with these other customers, making them unavailable for the firm in the future. In particular, suppliers who went away from an auction empty-handed were most likely to become unavailable in the future because they had payrolls and bills to pay, and without business from the buyer, they would more urgently look elsewhere for revenue. The same manager recalled a former PC industry supplier who eventually, after winning little business, decided to switch gears and start focusing on customers in a different industry, defense contracting.

Thus, a critical tension exists in the buyer's procurement strategy. On one hand, the buyer wants to procure from the cheapest supplier. This can be achieved by organizing a winner-take-all auction. On the other hand, this might alienate some suppliers from the supply base (those that did not win the auction). If the buyer wishes to maintain healthy competition in future auctions, then it has to find and qualify new suppliers to replace the suppliers that defect from the supply base, which can saddle the buyer with a cost of qualifying new suppliers for these auctions. A trade-off hence exists between getting a low purchase price today and avoiding the costs of qualifying new suppliers for future auctions.

Splitting the business among multiple suppliers can help the buyer resolve this trade-off. By giving business to a supplier, the buyer increases the chance that the supplier will be available for future competitive bidding events. It will be less likely, for example, to leave the supply base and seek greener pastures. Such supplier behavior can be captured by relating the likelihood of suppliers leaving the supply base to the amount of business awarded to them. Hence, splitting the auction volume among a few suppliers gives the buyer a lever to maintain the supply base. In fact, we spoke with some experienced procurement managers who often used split awards, considering this approach especially attractive in cases where keeping suppliers in the supply base was a central concern. For example, some auctions involved splits among up to four suppliers. However, because splitting the award involved purchasing from more expensive suppliers so as to retain them in the supply base, it was unclear when and how to best do this.

The objective of this paper is to provide an analytical framing of these important sourcing issues. We develop a model to help a buyer navigate the trade-off between paying lower purchasing costs by buying from the cheapest supplier and maintaining lower supply-base maintenance costs by splitting the award among more expensive suppliers. As we explain in the paper, optimally balancing long-run purchase cost and supply-base maintenance cost requires prior knowledge about the size of the supply base that needs to be maintained, which in turn means that the buyer's optimal auction design (how to allocate its business) and supply-base sizing (how many suppliers to include in the supply base) problems need to be jointly solved. This paper answers this need by embedding the mechanism design into a dynamic program. Thus, this paper contributes a new model to the literature that endogenizes supplier retention into a sourcing decision, built on a novel approach that combines mechanism design with dynamic programming.

Our analysis produces several key insights for practitioners and the procurement literature. First, we show that in each procurement cycle the buyer should qualify new suppliers into the supply base until a certain size is reached. This qualify-up-to policy is as appealing and practicable as the order-up-to policies of classical inventory theory. Using this result on the optimal qualification policy, we design the optimal mechanism that balances the long-run purchase cost and the supply-base maintenance cost, and we show that not only does it generally result in purchasing from both low- and high-cost suppliers, but the exact allocations to the suppliers depend on their cost spread and not just the cost ordering. Thus, unlike winner-take-all auctions where the only thing that matters is being cheapest, with split-award auctions the question of "how much cheaper" is a critical issue in determining business award quantities. Intuitively, this means that suppliers have to compete harder to win a bigger share of the business. Interestingly, we find that such an auction is not only ex ante better for the buyer, but it can even be ex ante better for the entire supply chain *and* the suppliers. Compared with the traditional winner-take-all auction, the system (and buyer) gains by curtailing unnecessary supplier qualification expenses, while suppliers gain by winning business more often than they would under a winner-take-all scenario.

We also explore how the buyer's policies change with the underlying business environment, and we find that the answers are not always obvious. For example, one might expect that the extent of multisourcing, measured as the proportion of its total business that the buyer procures from all but the lowest-cost supplier, always increases with the cost of qualifying suppliers. However, this intuition ignores the fact that, along with its multisourcing decisions, the buyer solves a concomitant problem—strategically sizing the supply base. In solving this joint problem, an intriguing trade-off exists for the buyer: On one hand, the buyer can cast a wider net by having more suppliers and hence making bidding more competitive; on the other hand, maintaining this larger pool of suppliers encourages splitting the award among more suppliers, which effectively makes the bidding less competitive. We find that the marginal savings in the long-run procurement costs from adding an additional supplier are decreasing in the supply-base size. Moreover, even though split awards are used to mitigate the cost of qualifying new suppliers, increasing the qualification cost may actually *reduce* the buyer's usage of multisourcing. This is because the buyer may respond to a higher qualification cost by strategically reducing the size of its supply base, thereby diminishing the opportunities to multisource.

Our model also allows us to compare the supply-base-size decision of a buyer using optimal split-award auctions against the supply-base-size decision of a myopic buyer that organizes winner-take-all auctions and takes for granted the future availability of its suppliers. We find that a myopic buyer would always maintain a higher supply-base size as compared with a buyer that organizes optimal split-award auctions. This larger supply base and greater need for qualifying new suppliers should weigh down even more heavily on its long-run procurement costs when the cost to qualify suppliers increases. Thus, it is tempting to conclude that the savings from optimal split-award auctions relative to myopic winner-take-all auctions should increase with the cost of qualifying suppliers. However, we find that the relative savings are nonmonotonic in the cost of qualifying suppliers. This is because a small increase in the cost of qualifying suppliers might result in a myopic buyer reducing its supply-base size and thus unintentionally reducing its supply base maintenance cost. However, a fully rational buyer would have better control over its procurement costs, and hence any small increase in the cost of qualifying suppliers would result in only a gradual increase in the long-term procurement costs.

Overall, our model provides a new framework to integrate supplier retention issues into the design of procurement strategies. In particular, we offer a novel combination of modeling and methodologies that allows us to turn an intrinsically dynamic problem into a static problem where both supply-base size and mechanism design decisions can be combined. Moreover, we extract several insights on the use of split-award auctions. Included are insights that are surprising (neither the extent of multisourcing nor the value of split awards compared with winner-take-all awards are monotonic in the qualification cost); encouraging (split-award auctions can increase ex ante system, buyer, and supplier benefits, resulting in a win-win-win that could help procurement managers promote the approach in practice); practicable (the buyer's optimal supply-base maintenance strategy follows a qualify-up-to policy); and novel (this is the first paper studying split-award auctions for supply-base maintenance, and it can hopefully galvanize further research on this important topic). The remainder of this paper explains the above as well as other findings. Section 2 reviews related literature and §3 describes the model. We then solve the buyer's mechanism design problem in §4 and characterize the optimal supply-base size in §5. Section 6 concludes. Proofs of the results and extensions to our model are included in the appendix.

2. Literature Review

Our work is related to the auction literature, particularly works featuring multiple periods, qualification costs, and multiple sourcing.

In the auction literature, Myerson (1981) was seminal in analyzing optimal auctions. We closely follow his approach in characterizing the incentives for agents (suppliers) to participate in auctions. Whereas most optimal auction literature focuses on single-period auctions, we consider a multiperiod setting. The dynamics of multiperiod auctions have seldom been explored. One exception is Klotz and Chatterjee (1995a), which considers a two-period model for defense systems procurement where suppliers face entry costs for bidding and exhibit production learning (cost reduction from one period to the next). They argue that splitting the contract award has a twofold advantage. First, it increases market participation (in line with Klotz and Chatterjee 1995b), and second, it allows the buyer to maintain second-period cost symmetry among its suppliers. Elmaghraby and Oh (2004) also investigate procurement auctions in the presence of learning-by-doing. For a two-period model, they perform a comparative analysis between sequential independent auctions in each period and an eroding price contract in which the buyer awards a multiperiod contract and the payment declines over time at a prespecified rate. Li and Debo (2009) examine whether a buyer should strike a multiperiod contract with a sole supplier or instead permit an entrant supplier option for the second period. All three papers study production learning in a multiperiod, noncommodity procurement environment. We focus on standard commodity procurement, where suppliers already know how to make the simple commodity (e.g., printed circuit boards). In our model, qualification cost and supply-base maintenance are the key issues, and relative production costs are taken to be independently drawn over time.

Another paper that considers sourcing decisions in a dynamic setting is Held et al. (2008). They investigate a buyer's trade-off between the cost of switching to an entrant supplier (because the product is not sufficiently standardized) and the cost of reduced competition due to bidder pool depletion resulting from awarding business to the incumbent. In each period, before seeing the suppliers' bids, the buyer chooses between entirely awarding to the incumbent or to an entrant. By contrast, our focus is on split-award auctions, where in each period the buyer decides how much each supplier wins after seeing their bids, and each supplier's stay or leave decision is a function of how much business they specifically were awarded. We focus on using split awards to manage supplier qualification costs, neither of which are present in Held et al. (2008).

In a different stream of work, the cost of qualifying suppliers has been studied only for a single-period auction setting. Wan and Beil (2009) propose post-auction qualification screening to reduce the cost of qualifying losing suppliers. Unlike our paper, there is no multiperiod aspect and thus no need to establish a base of qualified suppliers. In contrast to the buyer incurring the cost of including bidders (qualification cost), other papers have studied cases where instead bidders are the ones who incur cost to enter the auction. In such a setting, McAfee and McMillan (1987) determine the equilibrium number of bidders who participate in the auction. Klotz and Chatterjee (1995b) and Seshadri et al. (1991) investigate the performance of procurement auction mechanisms when suppliers incur entry costs, finding that multisourcing outperforms single sourcing only when suppliers face entry costs. In contrast, we find that multisourcing is optimal when the buyer incurs costs to recruit bidders, even when the suppliers do not face entry costs.

Finally, our paper is related to the broader procurement literature on multiple sourcing. Multisourcing has typically been analyzed in the procurement literature as a means to manage situations where suppliers may fail to deliver the units they are assigned to produce. For example, Kleindorfer and Saad (2005), Tomlin (2006), and Federgruen and Yang (2009) discuss supply risk mitigation through multisourcing; Babich et al. (2007), Yang et al. (2012), and Chaturvedi and Martínez-de-Albéniz (2011) consider supply diversification to mitigate supply disruption risk under information asymmetry. In our setting, we also multisource, but for an entirely different reason. In our paper, suppliers always fulfill their delivery obligations but do not necessarily stay in the supply base over time. We multisource to avoid the cost of qualifying new suppliers into the supply base rather than to ensure delivery. Multisourcing can also arise when suppliers face diseconomies of scale in production; see Dasgupta and Spulber (1990), Anton and Yao (1992), or Anton et al. (2010). By contrast, we investigate multisourcing as a tool for retaining suppliers and hence managing the costs of supply-base maintenance.

3. Model Description

3.1. Repeated Auctions

We model a buyer that needs to purchase a fixed, divisible quantity Q (normalized to $Q = 1$) of a homogeneous product. It can buy this quantity from the pool of suppliers, or *supply base*, that the buyer has at its disposal. The buyer admits only qualified suppliers (those that have survived qualification screening) into its supply base. Finding a qualified supplier involves locating a potential supplier, screening its qualifications (e.g., through product testing, site visits, and audits), and repeating the process (if needed) until finding a supplier who survives this screening. Potential suppliers can be identified via numerous sources (e.g., supplier lists, industry contacts, and receiving cold calls from suppliers), but the qualifications still need to be ascertained through costly screening. We assume that the buyer can qualify suppliers into its supply base from a large enough group of suppliers—that is, the buyer does not run out of suppliers to qualify.[1] The buyer's expected cost to find a qualified supplier is given by k. Intuitively, k's size is related to the amount of qualification screening; for example, a buyer might deploy extensive and costly qualification screening when buying a critical direct input (large k) but be satisfied with lighter, less-costly screening when buying a noncritical indirect good (low k). We assume that k does not change from period to period.

In addition, the buyer does not know the cost of any of the suppliers either before or after qualifying them. Namely, we interpret the qualification process as a way to avoid contracting with incapable or unreliable suppliers, rather than a cost discovery attempt. We let c_i denote the per-unit production cost of supplier i (its type), so supplier i's cost to produce q_i units is $c_i \cdot q_i$. Cost c_i is supplier i's private information, and the buyer is only informed of its cumulative distribution function (c.d.f.) $F(c)$ and its probability density function (p.d.f.) $f(c)$. The cost distribution is assumed to have a finite mean. In addition, to ensure typical auction theoretic properties, we assume that $F(c)/f(c)$ is nondecreasing; for example, this assumption is satisfied when F is log-concave, including uniform, normal, exponential, and logistic distributions (Bagnoli and Bergstrom 2005). Moreover, also consistent with the auction design literature, we assume that the suppliers' costs are independent and identically distributed (i.i.d.). The vector of unit costs of n suppliers is denoted by $\mathbf{c} = (c_1, \ldots, c_n)$.

[1] For the commodity-type components that we have in mind, the number of potential suppliers in the market can be astonishing. For example, in March 2013 a search on the website Alibaba.com yielded more than 3,000 potential suppliers for USB connectors that could eventually become part of a supply base if a buyer required it.

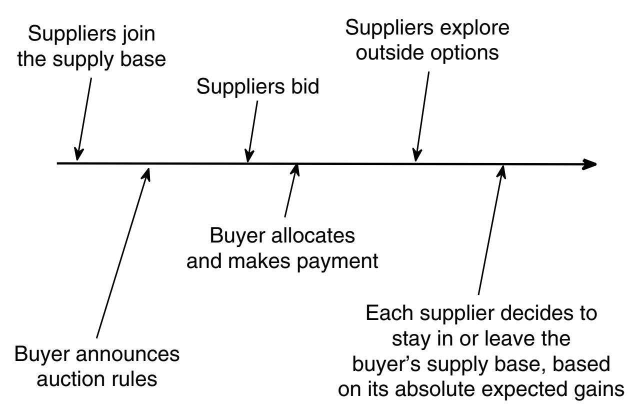
The buyer employs short-term supply contracts that are periodically reallocated among the supply base. Short-term contracts are often employed, for example, in fast-moving industries such as electronics where products and technology evolve quickly and the cheapest suppliers in the buyer's supply base for the latest product generation may be different from the ones used for the previous product generation. We model this as a multiperiod, infinite horizon setting where the buyer purchases a fixed quantity $Q = 1$ of supply in each period and does not hold inventory across periods (due to rapid product obsolescence).[2] In each period, we assume that supplier i's unit cost is drawn from a distribution with c.d.f. F. This assumption implies that costs are independent across periods. In fact, what really matters is that F captures suppliers' *relative* costs; hence, the model easily extends to cost correlations across periods when, for example, in period t, supplier i's total cost equals a publicly observed common term c_t plus the private signal on relative costs c_i governed by F. The common term c_t drops out of the analysis (because only the relative costs matter) so our results would be robust to c_t being period-dependent and correlated across periods. Our paper's structural results even extend if suppliers' costs are correlated across suppliers within a single period. The assumption of relative costs being independent across periods is appropriate for cases where technology and production capabilities/efficiencies of suppliers change rapidly and the buyer therefore wishes to use auctions to stay abreast of the current best market pricing. This is the case in the electronics industry, for example, where buyers use auctions to find which suppliers currently offer the best pricing. There may be industries where product life cycles are long, new technology does not often get introduced, and suppliers' relative costs remain very sticky across periods. Our model would not apply to such cases because, when costs are very sticky across periods, there is no need to auction in each period, and the buyer is better off just giving a long-term contract to the lowest-cost supplier in the first period (because this supplier's cost advantage will be sustained in future periods). Accordingly, there is not the same need for a supply base and supply-base maintenance in such settings.

In this dynamic context, the buyer's procurement decision depends on how the availability of suppliers evolves over time. If the suppliers, once qualified, remain in the supply base forever then the buyer would have to qualify the suppliers only at the beginning of the horizon and can organize a winner-take-all auction in every period. However, it is not uncommon that suppliers who lose in the bidding process decide not to participate in future auctions. For example, as described in the Introduction, a supplier might switch to another industry after obtaining little business from the buyer, the business reason being that such a supplier is saddled with excess capacity and therefore is more receptive to opportunities outside of the buyer's supply base. The higher the excess capacity that the supplier has, the higher the likelihood that the supplier finds it beneficial to switch from the buyer's supply base to another supply base. The next subsection presents a model that captures the interaction between the buyer's present allocation to a supplier and its future availability in the buyer's supply base. This model yields a supplier total cost function and a future availability function. Figure 1 summarizes the model timeline.

3.2. Effect of Allocation on Supplier Future Availability

Assume that supplier i commits a capacity $W_i \geq 1$ for the buyer's supply base. This assumption implies that $W_i \geq q_i$, i.e., the supplier always has sufficient capacity to fulfill the buyer's order. Thus, if the buyer chooses to multisource, it is due to a strategic desire to manage future supplier qualification costs, not because suppliers are capacity-constrained. Suppose that before the auction (ex ante), in expectation, any other industry offers the supplier the same payoff for its capacity W_i (after it has left the buyer's supply base) as the supplier would get by being in the buyer's supply base. Hence, before the auction (ex ante), the supplier has no incentive to switch over. However, after getting q_i amount of business from the buyer's auction in a given period, the supplier would explore how profitable it is to utilize its remaining capacity $W_i - q_i$ for that period by investigating other options, e.g., responding to exploratory request for quotes (RFQs) from other

Figure 1 Timeline of Events in a Typical Auction Period



[2] The analysis can easily be extended to incorporate a random demand, Q, faced by the buyer, provided that suppliers remain capable of meeting the buyer's full demand and that the demand is realized after the buyer qualifies suppliers into its supply base. However, if the demand realization occurs before the buyer qualifies suppliers into its supply base, then each period's supply-base decision would depend on the realized demand in that period.

potential customers. (We assume the supplier cannot back out of its contract to supply q_i units to the buyer.) In doing so, it formulates an estimate of the per-unit margin $p_{\text{out},\,i} - \gamma c_i$ that it expects to get in that period if it retools its capacity for another customer. Parameter γ moderates the impact of the supplier's cost on the margin it can make outside the supply base; for example, the dependence is nonexistent if $\gamma = 0$, and negative if $\gamma > 0$. Switching over to another supply base usually involves some costs: We assume that the supplier incurs a fixed retooling cost of T_i when leaving the supply base (T_i can also incorporate a profit requirement below which switching is not considered worthwhile). With all these elements, the supplier decides to leave the supply base if and only if the net expected benefit in that period is positive. Hence, the supplier leaves the supply base when $(p_{\text{out},\,i} - \gamma c_i) \cdot (W_i - q_i) - T_i > 0$, and stays in it otherwise. Thus, a supplier's future availability can be described by a random variable $A_i(q_i, c_i)$ that equals zero if the supplier leaves the supply base and equals one otherwise.

We assume that ex ante (before the supplier participates in the buyer's auction and explores other RFQs) neither the buyer nor the supplier knows $p_{\text{out},\,i}$, but both share the same prior over the c.d.f. $F_{\text{out},\,i}$ of $P_{\text{out},\,i}$. One can then characterize the likelihood of the supplier leaving the buyer's supply base as $\bar{\alpha}_i(q_i, c_i) = \mathbb{P}[(P_{\text{out},\,i} - \gamma c_i) \cdot (W_i - q_i) - T_i \geq 0]$ and the likelihood of the supplier staying as $\alpha_i = 1 - \bar{\alpha}_i$. Note that $\bar{\alpha}_i(q_i, c_i)$ is nonincreasing in the amount of business q_i that the supplier gets from the buyer. To keep the model parsimonious, we assume that $W_i = W$, $T_i = T$ for all i, and $P_{\text{out},\,i} = P_{\text{out}}$ is identically distributed although not necessarily independent across suppliers. (Our paper's structural results extend even if we relax these assumptions.) Therefore, the suppliers' future availability, namely the $A(q_i, c_i)$'s, are identically distributed with probability of staying $\alpha(q_i, c_i)$. The $A(q_i, c_i)$'s need not be independent because P_{out} can be correlated across suppliers.

Moreover, we denote by $\Omega(q_i, c_i)$ the expected value of the maximum of zero and the additional surplus that supplier i makes by switching over to another supply base, i.e., $\Omega(q_i, c_i) = \mathbb{E}_{P_{\text{out}}} \max\{0, (P_{\text{out}} - \gamma c_i) \cdot (W - q_i) - T\}$. Hence, $\Omega(q_i, c_i)$ represents the value of the supplier's outside option if it gets allocated q_i amount of business by the buyer. One can therefore write $\Omega(0, c_i) - \Omega(q_i, c_i)$ as the supplier's opportunity cost of producing q_i for the buyer.

Finally, we let $S(q, c) \equiv c \cdot q + \Omega(0, c) - \Omega(q, c)$ denote the total cost of a supplier with allocation q and cost c, which is the sum of its production and opportunity costs. We also let $B(q, c) \equiv (\partial S(q, c)/\partial c) \cdot (F(c)/f(c)) + S(q, c) + k\beta\bar{\alpha}(q, c)$. We will see later that $B(q, c)$ is the virtual cost that the buyer incurs by giving an allocation q to a supplier with a per-unit cost of c. (Note that this virtual cost includes the expected cost to qualify a new supplier if the supplier leaves.) We make three assumptions on the cost structure of the suppliers: (a) $S(q, c)$ is nondecreasing in c, (b) $\partial^2 S(q, c)/\partial q \partial c \geq 0$, and (c) $\partial^2 B(q, c)/\partial q \partial c \geq 0$. Assumptions (a) and (b) imply, respectively, that a supplier's total cost and marginal cost (derivative with respect to q) are nondecreasing in its type. Assumption (c) implies that the buyer's cost (including the cost of maintaining the supplier in the supply base) of giving a marginal unit to a supplier with a higher type is higher than giving that unit to a supplier with a lower type. Under these assumptions, the supplier's total cost and the buyer's virtual cost have single crossing differences, which is a widely used assumption in the mechanism design literature that allows the buyer to perfectly discriminate suppliers having different types and hence avoids bunching. Moreover, assumptions (a)–(c) are satisfied for example when $\gamma = 0$, in which case α is independent of c, or when $\gamma \leq 1$ and P_{out} is exponentially distributed, Pareto distributed, or uniformly distributed.

Thus far, we have presented a model capturing the interaction between the buyer's present allocation to a supplier and the supplier's future availability in the buyer's supply base. The model yielded a supplier total cost function $S(q, c)$ and availability function $\alpha(q, c)$. We conduct our theoretical analysis using a structure that relies solely on S and α (of course satisfying assumptions (a)–(c)) rather than the specifics of the underlying model. In other words, the above model can be viewed as a convenient expositional tool to quickly elucidate S and α. (We will also use it for generating numerical experiments in later sections.) However, one may fit other models of supplier total cost $S(q, c)$ and availability $\alpha(q, c)$ for different situations, and—as long as assumptions (a)–(c) hold—still use our subsequent analysis.

With suppliers' availability being dependent on the buyer's allocation, the buyer faces potential depletion of its supply base if it does not provide enough business to all the suppliers. This implies that using a winner-take-all auction may not be a good strategy. The buyer thus needs to devise an allocation rule—that does not necessarily result in single sourcing—to minimize its present and discounted future procurement cost (qualifying and purchasing cost). Finding such an allocation rule requires characterizing the future procurement costs. Because these costs depend on the size of the supply base that will be maintained in the future, the buyer must solve its problem of designing its procurement mechanism jointly with its supply-base sizing problem. In §4, we formulate this joint problem and show that the buyer should maintain a stationary supply-base size over the infinite horizon. We then find the optimal mechanism that the buyer should use for a given stationary size of its supply base. Subsequently, §5 characterizes the optimal size of the supply base that will minimize the long-run procurement costs for the buyer, under the optimal mechanism.

4. Controlling Qualification Costs Through Split Awards

4.1. Optimal Mechanism Design

To find the optimal auction rules, we use mechanism design theory; see Myerson (1981). Because costs are independent across periods, we can focus on the mechanism design problem for each period separately. We consider only truth-revealing mechanisms that, by the revelation principle, include an optimal mechanism, if an optimal mechanism exists. Specifically, we consider sealed-bid mechanisms in which suppliers bid their true marginal costs $\mathbf{c} = (c_1, \ldots, c_n)$. A mechanism is described by the rule, announced by the buyer prior to the bidding, that maps suppliers' bids to their allocations and payments. Let such a mechanism be denoted by $(\mathbf{z}(\mathbf{c}), \mathbf{q}(\mathbf{c}))$, where $\mathbf{z}(\mathbf{c})$ is the vector of payments made to suppliers and $\mathbf{q}(\mathbf{c})$ is the vector of allocations made to suppliers, as functions of their bids $\mathbf{c}$. Because we focus on truth-revealing mechanisms, we require that the buyer designs the payment and allocation rule such that suppliers have the incentive to reveal their true costs.

For the mechanism to be optimal, it should minimize the buyer's present and future cost. We denote this cost by $J(n_a)$, where n_a is the number of suppliers available at the beginning of the period. The buyer's optimal cost-to-go can be represented by Bellman's equation as

$$J(n_a) = \min_{n \geq n_a} \left[\min_{\mathbf{z},\, \mathbf{q} \mid \sum_{i=1}^n q_i = 1} \mathbb{E}_{\mathbf{c}} \left\{ \sum_{i=1}^n z_i(\mathbf{c}) + k(n - n_a) + \beta \mathbb{E}_{\mathbf{A}} J\left(\sum_{i=1}^n A(q_i(\mathbf{c}), c_i) \right) \right\} \right] \quad (1)$$

$$\text{s.t.} \quad (\mathbf{z}, \mathbf{q}) \text{ are truth-revealing.}$$

We next present the conditions that the mechanism must satisfy to induce truth revelation. As in the literature (Myerson 1981), we assume that each supplier is risk neutral; consistent with the model introduced in §3 the utility U_i of supplier i participating in the auction can be written as

$$U_i(\mathbf{c}) = z_i(\mathbf{c}) - S(q_i(\mathbf{c}), c_i). \quad (2)$$

Denote by $\mathbf{c}_{-i}$ the vector of per-unit cost of all suppliers except supplier i. For the mechanism $(\mathbf{z}, \mathbf{q})$ to be truth-revealing, it should satisfy individual rationality (IR) and incentive compatibility (IC) constraints:

$$\text{(IR)} \quad U_i(\mathbf{c}) \geq 0 \quad \text{for all } \mathbf{c},$$

$$\text{(IC)} \quad U_i(\mathbf{c}) \geq z_i(\hat{c}_i, \mathbf{c}_{-i}) - S(q_i(\hat{c}_i, \mathbf{c}_{-i}), c_i) \quad \text{for all } c_i,\ \hat{c}_i,\ \mathbf{c}_{-i}.$$

As one can see, we consider the IR and IC constraints in dominant strategy equilibrium. In other words, a supplier truthfully reveals its cost irrespective of other suppliers' costs. Alternatively one could also formulate the IR and IC constraints in Bayesian-Nash equilibrium—that is, a supplier's truthful revelation is its best strategy only in expectation over other suppliers' costs. Indeed, compared with the Bayesian-Nash equilibrium, a dominant strategy equilibrium is a more stringent condition on the mechanism design problem. However, using the results of Mookherjee and Reichelstein (1992, specifically Proposition 3), one can show that for our problem an optimal mechanism in the dominant strategy equilibrium gives the same expected surplus to the agents and the principal as would an optimal mechanism in the Bayesian-Nash equilibrium.[3] Therefore, there is no loss of optimality in assuming that suppliers' truthful revelation is a dominant strategy in equilibrium. The following lemma provides a simpler characterization of truth-revealing mechanisms (an analogous approach is used in Dasgupta and Spulber 1990).

Lemma 1. *The mechanism $(\mathbf{z}, \mathbf{q})$ is truth-revealing if and only if*
1. $U_i(\mathbf{c}) = U_i(\infty, \mathbf{c}_{-i}) + \int_{t=c_i}^{\infty} \partial S(q_i(t, \mathbf{c}_{-i}), c)/\partial c|_{c=t}\, dt$;
2. $q_i(c_i, \mathbf{c}_{-i})$ *is nonincreasing in* c_i *for all* $c_i, \mathbf{c}_{-i}$;
3. $U_i(\mathbf{c}) = z_i(\mathbf{c}) - S(q_i(\mathbf{c}), c_i)$;
4. $U_i(\infty, \mathbf{c}_{-i}) \geq 0$.

We now reformulate the buyer's problem in Equation (1) by substituting the value of z_i from point (3) and the value of U_i from point (1) of Lemma 1 and taking $U_i(\infty, \mathbf{c}_{-i}) = 0$ (which does satisfy point (4) of Lemma 1). We can then express the buyer's mechanism design problem as the optimization of the objective function with respect to the allocation $\mathbf{q}$ alone, subject to point (2) in Lemma 1. Given an optimal $\mathbf{q}$, the payment scheme $\mathbf{z}$ can then be found through point (3) of Lemma 1, which guarantees that the IC and IR constraints are satisfied. Taking this approach, the buyer's problem from Equation (1) can be rewritten as

$$J(n_a) = \min_{n \geq n_a} \left\langle \min_{\mathbf{q} \mid \sum_{i=1}^n q_i = 1} \mathbb{E}_{\mathbf{c}} \left\{ \sum_{i=1}^n \left\langle \int_{t=c_i}^{\infty} \frac{\partial S}{\partial c}(q_i(t, \mathbf{c}_{-i}), c) \bigg|_{c=t} dt + S(q_i(\mathbf{c}), c_i) \right\rangle + \beta \mathbb{E}_{\mathbf{A}} J\left(\sum_{i=1}^n A(q_i(\mathbf{c}), c_i) \right) + k(n - n_a) \right\} \right\rangle, \quad (3a)$$

$$\text{s.t.} \quad \mathbf{q} \text{ satisifies condition (2) of Lemma 1.} \quad (3b)$$

[3] Proposition 3 in Mookherjee and Reichelstein (1992) states that an allocation rule $\mathbf{q}$ would give the same expected utility to suppliers in both Bayesian-Nash equilibrium and dominant-strategy equilibrium as long as each supplier's cost function satisfies the weak single crossing property and the allocation function is nonincreasing in the supplier's type. From assumption (b), we know that suppliers' costs satisfy the weak single crossing property, and we will see in Theorem 1 that the allocation rule $\mathbf{q}$ is nonincreasing in supplier type.

To solve math program (3), we first relax the associated constraint (3b) (which we verify later). The resulting formulation allows us to establish that a stationary policy is optimal.

Lemma 2. *If the buyer starts the horizon with* 0 *suppliers, then a stationary policy in which it qualifies up to n^* suppliers in each period is optimal.*

Hence, assuming that the buyer starts the process with an empty supply base (0 suppliers), we can apply the stationary policy to Equation (3a). Furthermore, integrating by parts

$$\mathbb{E}_{\mathbf{c}}\left[\sum_{i=1}^{n}\int_{t=c_i}^{\infty}\frac{\partial S}{\partial c}(q_i(t,\mathbf{c}_{-i}),c)\bigg|_{c=t}dt\right]$$

allows us to rewrite Equation (3a) as

$$\begin{aligned}(1-\beta)J(0)&\\ =\min_{n}\bigg\langle\min_{\mathbf{q}\mid\sum_{i=1}^{n}q_i=1}\mathbb{E}_{\mathbf{c}}\bigg[\sum_{i=1}^{n}\bigg\{\frac{\partial S}{\partial c}(q_i(\mathbf{c}),c)\bigg|_{c=c_i}\frac{F(c_i)}{f(c_i)}+S(q_i(\mathbf{c}),c_i)&\\ +k\beta\bar{\alpha}(q_i(\mathbf{c}),c_i)\bigg\}\bigg]+kn(1-\beta)\bigg\rangle.&\qquad(4)\end{aligned}$$

Hence, given an n (qualify-up-to level), the buyer's mechanism design problem can be written as

$$\begin{aligned}\min_{\mathbf{q}\mid\sum_{i=1}^{n}q_i=1}\mathbb{E}_{\mathbf{c}}\bigg[\sum_{i=1}^{n}\bigg\{\frac{\partial S}{\partial c}(q_i(\mathbf{c}),c)\bigg|_{c=c_i}\frac{F(c_i)}{f(c_i)}+S(q_i(\mathbf{c}),c_i)&\\ +k\beta\bar{\alpha}(q_i(\mathbf{c}),c_i)\bigg\}\bigg].&\qquad(5)\end{aligned}$$

Optimizing Equation (5) over each cost realization $\mathbf{c}$ reduces the mechanism problem to

$$\min_{\mathbf{q}\mid\sum_{i=1}^{n}q_i=1}\sum_{i=1}^{n}B(q_i,c_i).\qquad(6)$$

For problem (6) to represent the optimal mechanism, we need to verify its feasibility, which we do next. That is, we show that the allocations q_i satisfy constraints (3b).

Theorem 1. *Given a qualify-up-to level n in an infinite horizon problem, $(\mathbf{z},\mathbf{q})$ represents an optimal mechanism in dominant strategy equilibrium if*

$$\mathbf{q}(\mathbf{c})=\arg\min_{\mathbf{q}\text{ s.t. }\sum q_i=1}\sum_{i=1}^{n}B(q_i,c_i),\qquad(7)$$

$$z_i(\mathbf{c})=S(q_i(\mathbf{c}),c_i)+\int_{t=c_i}^{\infty}\frac{\partial S}{\partial c}(q_i(t,\mathbf{c}_{-i}),c)\bigg|_{c=t}dt.\qquad(8)$$

Moreover, the optimal allocation, q_i, to any supplier i is nonincreasing in c_i.

Optimal mechanism uses split awards. Note that $B(q,c)$ in the buyer's optimization program in Equation (7) is comprised of two terms, namely $\partial S(q,c)/\partial c\cdot(F(c)/f(c))+S(q,c)$ and $k\beta\bar{\alpha}(q,c)$. The former term is nondecreasing in (q,c), but the latter term, $k\beta\bar{\alpha}(q,c)$, is nonincreasing in q and not necessarily concave in allocation q and therefore can impose a penalty on winner-take-all allocations. Moreover, this penalty is nondecreasing in the cost of qualifying suppliers, k, implying that the buyer would diversify its purchases more often (more often because the actual purchase would depend on the realized costs) as k increases. The optimization program in (7) also suggests that the buyer is more likely to split its purchase as the number of suppliers, n, increases, because a higher n would impose a greater cost of maintaining the suppliers. In §4.3, we further investigate how the cost of qualifying suppliers and the supply-base size affect the extent to which the buyer diversifies its purchases across the supply base.

Allocations depend on cost spread, not just cost ordering. Note that in the optimal mechanism, the allocation, and payment rules depend on the absolute position of the bids and not just on the rank of the bids. To see this, consider an open descending auction implementation of the optimal mechanism in Theorem 1. (This implementation is possible because the optimal mechanism is designed for a dominant-strategy equilibrium.) For example, one can use a clock auction whereby the auction begins at a high calling price and descends according to a price clock. At each price level, suppliers decide whether to stay in the auction or permanently drop out. After all suppliers have dropped out, the buyer applies the allocation rule in Equation (7) and the corresponding payment in Equation (8) according to the drop-out bids (which play the role of $\mathbf{c}$). In a traditional winner-take-all descending-clock auction (or reverse English auction), it is a dominant strategy for suppliers to drop out at their true cost, unless every other supplier has dropped out, at which point the lowest-cost supplier should also drop out. By contrast, with our proposed open auction implementation of the optimal mechanism, each supplier—including the lowest-cost supplier—has the incentive to stay in the auction exactly until its true cost is reached. This is because the quantity and profits for a supplier not only depend on its rank in the auction, but also on the magnitude of the cost difference with other suppliers.[4] The reason suppliers do not get a full order even if they have the cheapest cost is that the buyer has an

[4] As one would intuitively expect, if the buyer promised to sole source the contract, in the suggested implementation the lowest-cost bidder would have no incentive to continue lowering its bid once all its competitors have dropped out, just like in a traditional sole-source reverse English auction.

intrinsic benefit from multisourcing, so for example, if two suppliers offer nearly the same bid price, the buyer prefers splitting the award between them. This is why in our setting the amount of *gap* between bids matters to the buyer's allocation decision. Of course, in many settings the absolute size of the winning bid matters. For example, in Chen (2007) and Duenyas et al. (2013) the buyer is a newsvendor and because of the buyer's nonlinear utility function the quantity it awards to the winning supplier depends on the supplier's absolute cost; however, because of the suppliers' cost structures, the buyer always sole sources and the quantity awarded to the winner does not depend, for example, on the size of the highest bid, whereas this does matter in our setting.

4.2. Comparison to Myopic Winner-Take-All Policy

Here we compare the performance of the optimal split-award auction against that of a winner-take-all policy. The latter can readily arise when the buyer ignores the impact of allocations on suppliers' future availability in the supply base.

Formally, for a buyer who assumes that the suppliers will always stay in the supply base irrespective of how the buyer allocates to them, we define

$$\begin{aligned} B^{\text{myopic}}(q,c) &= S(q,c) + \frac{\partial S}{\partial c}(q,c)\frac{F(c)}{f(c)} \\ &= B(q,c) - k\beta\bar{\alpha}(q,c). \end{aligned}$$

The allocation decision of the myopic buyer can be formulated as

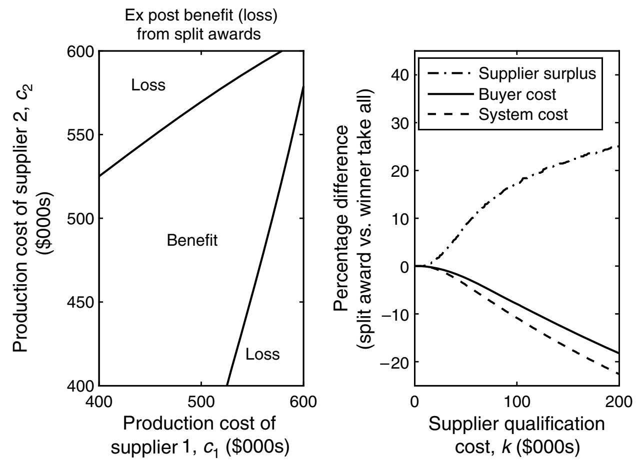
$$\mathbf{q}(\mathbf{c}) = \underset{\mathbf{q}\ \text{s.t.}\ \sum q_i = 1}{\arg\min} \sum_{i=1}^{n} B^{\text{myopic}}(q_i, c_i), \tag{9}$$

When $\partial^2 B^{\text{myopic}}/\partial q \partial c \geq 0$, the allocation decision (9) along with the payment scheme in (8) induces truthful revelation of supplier costs and consequently is the best possible allocation rule taken by the myopic buyer. Obviously, if the allocation in (9) results in sole sourcing, then a myopic buyer's optimal policy is indeed a winner-take-all policy.[5]

We would first like to compare the surplus that suppliers make in a split-award auction and in a winner-take-all auction for a given size of the supply base. In Figure 2, we show the ex post (left panel) and ex ante (right panel) total supplier surplus for each auction. In the figure, we take supplier costs to be distributed between \$400,000 and \$600,000. The retooling cost is assumed to be \$40,000, which could correspond to the cost of retrofitting machinery, training

[5] This is true, for example, when $\gamma = 0$, and we use this case for our numerical example in Figure 2. It is also true when $\gamma \cdot F(c)/f(c) \leq 1/\lambda$ for all c and P_{out} is exponentially distributed with mean $1/\lambda$. In general, it is true whenever $B^{\text{myopic}}(q,c)$ is concave in q.

Figure 2 Comparing Ex Post Total Supplier Surplus (Left Panel) and the Percentage Ex Ante Differences (in Buyer Cost, System Cost, and Total Supplier Surplus) Between the Split-Award and Winner-Take-All Auctions (Right Panel) in a Supply Base Having Two Suppliers



Notes. In both panels, P_{out} is Pareto distributed with mean \$45,000 (minimum support at \$40,000 and shape parameter 9), $T = \$40{,}000$, $\beta = 0.9$, $W = 1$, and $\gamma = 0$. Supplier production costs are uniformly distributed in the interval \$[400,000; 600,000]. In the left panel, $k = \$95{,}000$.

workers, resequencing the production line, etc. needed to enter a different product industry. The mean of P_{out} is taken to be \$45,000, making the outside industry attractive for suppliers who have a lot of spare capacity. The qualification costs range from zero—an extreme where the buyer conducts no screening on suppliers—to \$200,000, which could correspond to an opposite extreme case where the buyer needs to fly a team to the supplier site for extensive on-site investigations and interviews and conduct costly testing (such as failure analyses) on supplier parts.

In the left panel of Figure 2, for a two-supplier case, we see that the ex post supplier surplus is higher for a winner-take-all auction when the costs of the suppliers differ substantially; however, it is higher for split awards when the costs are closer together. In the former case (different costs), split awards force the lower-cost supplier to compete harder than it would have in a winner-take-all auction, which reduces the surplus. The intuition is as we described earlier: With split awards the relative sizes of costs matter, but with winner take all only the cost ranking matters. Likewise, when the suppliers' costs are already relatively similar, they still earn reasonable profits under split awards but would have to compete extremely fiercely if the allocation were winner take all. Thus, we see that split awards could increase or shrink ex post supplier surplus, depending on the supplier's cost realizations. Interestingly, the following proposition proves that there are cases where the ex ante expected total supplier surplus is provably higher under split awards as compared with winner take all.

Proposition 1. *For a given supply-base size n, the expected total surplus given to the suppliers in the optimal split-award auction is higher than in a winner-take-all auction if the following sufficient condition is satisfied for any $\sum_{i=1}^n q_i = 1$ and $c_1 \le c_2 \le \cdots \le c_n$:*

$$\sum_{i=1}^{n} \frac{\partial S}{\partial c}(q_i, c_i) \ge \frac{\partial S}{\partial c}(1, c_1).$$

Note that the sufficient condition in Proposition 1 is satisfied, for example, when $\gamma = 0$. The right panel of Figure 2 illustrates this, showing that the total ex ante supplier surplus is higher under split awards. Moreover, because the suppliers are ex ante symmetric, the ex ante expected surplus of each supplier is higher in a split-award auction as compared with a winner-take-all auction.

One should be careful, however, not to overgeneralize the conclusion that split awards are always better for suppliers. We saw that they were better when $\gamma = 0$. This corresponds to settings where a cost advantage or disadvantage does not persist in the new industry, which could be the case when entering a different industry requires substantially different production approaches and the supplier making a transition would essentially start from scratch. However, when γ is positive, we can have cases where split awards reduce the ex ante total expected supplier surplus compared with winner take all.[6] This is because supplier production cost advantages or disadvantages spill over to the different industry, magnifying the effect that the buyer's contract award allocation size has on the suppliers' opportunity costs: The total cost differences between suppliers in all-or-nothing allocations become more pronounced, leading to higher supplier surplus in winner-take-all auctions. The intuition is again what we saw earlier for the left panel of Figure 2—namely, winner take all leads to higher supplier surplus when supplier costs are more disparate.

Next we investigate the performance of the optimal split-award auction relative to a winner-take-all auction from the system's cost perspective. We define the system cost as the sum of the suppliers' cost (i.e., their production cost and opportunity cost) and the expected cost the buyer would incur to maintain the supply base at a given size (i.e., the expected cost of replacing the suppliers that leave the supply base and the cost of starting the supply base at the beginning of the horizon).

Proposition 2. *For a given supply-base size, if the sufficient conditions of Proposition* 1 *are satisfied, then the expected system cost in any period for the optimal split-award auction is less than that for a winner-take-all auction.*

Thus, we find that the optimal split-award auction not only minimizes the buyer's expected total cost but, compared with the winner-take-all actuion, can also be better for the suppliers and the entire system. The right panel of Figure 2 compares the ex ante performance from the buyer's, system's, and suppliers' perspective as k is changed. The system's savings increase with k because the supply-base maintenance costs are better controlled by the split-award auction and not controlled at all by the winner-take-all auctions. These savings are then shared by both the buyer and the suppliers, so both are better off with split awards as k increases.

4.3. Extent of Multisourcing

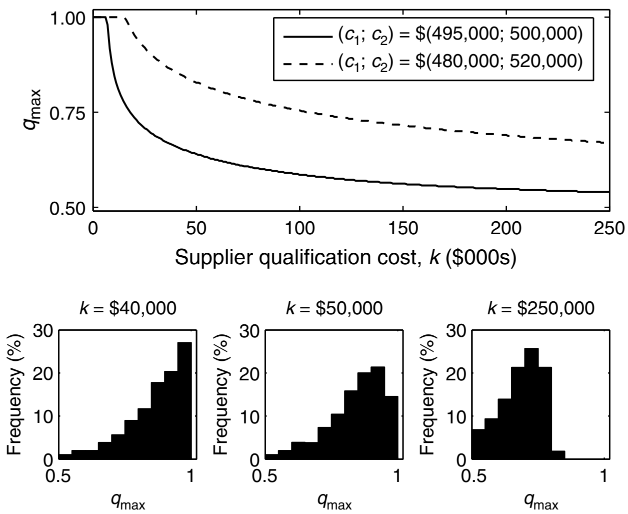
In this section, we would like to understand how often and to what extent the buyer diversifies its purchases across the supply base in any given period. For this purpose, we consider the allocation given to the lowest-cost supplier. Let $q_{\max}(\mathbf{c}) = \max_i q_i(\mathbf{c})$ denote the allocation to the cheapest supplier (which obtains the largest allocation).[7] Because $q_{\max}$ depends on the actual cost realizations of the suppliers (i.e., their bids), to measure the extent to which a buyer using split awards diversifies its purchase, we define the *concentration of allocation* as $\bar{q}_{\max} = \mathbb{E}_{\mathbf{c}}\, q_{\max}(\mathbf{c})$. Hence, $\bar{q}_{\max}$ measures the highest allocation averaged across all sample path realizations of suppliers' costs. Intuitively, $\bar{q}_{\max}$ represents the average proportion of business that the buyer gives to the cheapest supplier and $1 - \bar{q}_{\max}$ represents the average proportion of business it divides among the more expensive suppliers to better manage its supply base.

Effect of qualification cost. Here we consider the effect that the cost of qualifying suppliers, k, has on the concentration of allocation. Figure 3 depicts the change in the allocation to the minimum cost supplier, $q_{\max}$, for given supplier cost realizations as k increases. The figure illustrates the basic trade-off in the buyer's decision on splitting the award: When the relative magnitude of k is small compared with the cost of the suppliers, then the marginal savings in supply-base maintenance costs achieved from multisourcing are less than the additional cost of purchasing from a more expensive supplier. This effect becomes more pronounced as the difference between the costs of suppliers increases, and therefore we see that the buyer single sources for a wider range of k. From the figure, we can also observe

[6] For example, one can show that this happens when $\gamma = 1$, P_{out} is Pareto distributed with mean \$120,000 (minimum support at \$100,000 and shape parameter 6), $T = \$100{,}000$, $\beta = 0.9$, $W = 1$, and $F \sim \text{Uniform}[\$0; \$100{,}000]$.

[7] One could employ other concentration measures, such as the Herfindahl-Hirschman Index (HHI), $\sum_{i=1}^n q_i^2$. Comparatively, $q_{\max}$ is easier to analyze and is similar to HHI; both vary between $1/n$ and 1, $q_{\max} = 1/n$ implies perfect diversification, and $q_{\max} = 1$ implies sole sourcing.

Figure 3 **Effect of Qualification Cost on Allocations**



Note. Model parameter values are the same as in Figure 2.

that for a large enough k the mechanism tends toward an equal split between the suppliers. In fact, in the next lemma we analytically show that, as k becomes large, the concentration of allocation (which is the expectation of q_{max}) tends toward a limit that depends on γ. Recall that γ moderates the impact of supplier cost on the margin it can make outside the supply base, and therefore for $\gamma = 0$, the likelihood of the supplier leaving the supply base is independent of its cost.

Lemma 3. *For a fixed supply-base size n, as the qualification cost k increases, the concentration $\bar{q}_{max}$ tends toward a value equal to $1/n$ when $\gamma = 0$ and $\bar{\alpha}(q, c)$ is convex in q, and at least $1/n$ otherwise.*

Lemma 3 echoes the findings in Figure 3—i.e., the buyer would want to spread its allocation to retain most of its suppliers as k increases in the hope of avoiding the high costs of qualifying suppliers in future periods.

Effect of supply-base size. We now investigate the effect of supply-base size on the concentration of allocation.

Lemma 4. *The concentration of allocation $\bar{q}_{max}$ is nonincreasing in the size of the supply base, n.*

The intuition behind Lemma 4 is that, as the supply-base size increases, the average gap between the cost of two consecutive suppliers decreases. Therefore, the balance between the savings achieved in purchase price from allocating the marginal unit to the lower-cost supplier and the savings achieved in future cost of maintaining the supply base by allocating this marginal unit to the higher-cost supplier tips in the favor of the latter savings. As a result the allocation gets less concentrated toward the lower-cost supplier as the supply-base size increases. To summarize, all else being equal, buyers with a larger supply base will tend to move away from winner-take-all allocations.

We saw in Lemma 2 that it is optimal for the buyer to maintain a qualify-up-to level. Lemma 4 illustrates the trade-off in deciding the optimal qualify-up-to level, the supply-base size. On one hand, the buyer can cast a wider net by having more suppliers and hence make bidding more competitive, but on the other hand, maintaining this larger pool of suppliers encourages splitting the award among more suppliers, which effectively makes the bidding less competitive.

5. Optimal Supply-Base Size

We are now interested in characterizing the optimal qualify-up-to level, n^*, the supply-base size that the buyer wants to use in every period, under the optimal mechanism. For this purpose, we need to solve the buyer's objective in Equation (4). From this equation, let

$$G(n) = \mathbb{E}_{\mathbf{c}}\left[\min_{\mathbf{q}}\left\{\sum_{i=1}^{n} B(q_i(\mathbf{c}), c_i)\right\}\right] + k(1-\beta)n \qquad (10)$$

represent the value to be minimized in n, i.e., the buyer's cost-to-go at the beginning of the horizon. Note that $G(n)$ consists of two terms: the first term is the expected sum of suppliers' virtual costs that the buyer would incur by giving an allocation $\mathbf{q}$, and the second term is the amortized cost of starting the supply base with n suppliers at the beginning of the horizon. From Equation (4) and the definition of $G(n)$, we can characterize the marginal effect of the $(n+1)$th supplier as $G(n) - G(n+1)$. The next theorem shows that this marginal effect decreases in the number of suppliers, n.

Theorem 2. *The marginal value of an extra supplier, $G(n) - G(n+1)$, is nonincreasing in n and the optimal qualify-up-to level n^* can be characterized as*

$$n^* = \min(n \mid G(n) - G(n+1) \le 0). \qquad (11)$$

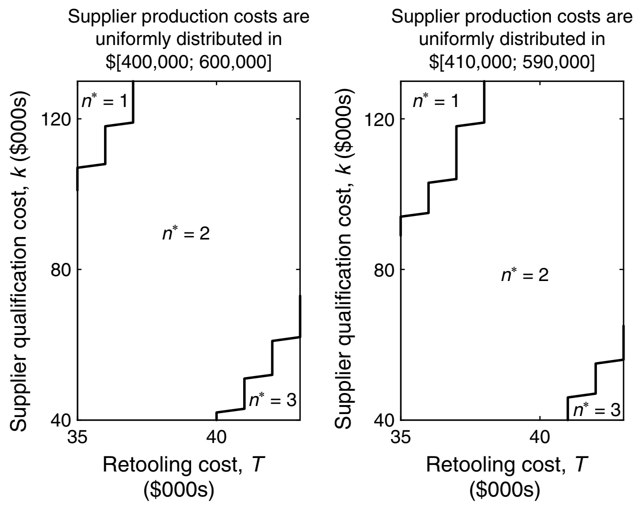
Theorem 2 helps us perform sensitivity analyses on the optimal supply-base size as model parameters change. Below, we highlight some sensitivity results that give interesting insights on using both split-award auctions and supply-base size as levers for managing long-run procurement costs.

5.1. Sensitivity of the Supply-Base Size

We now study how the optimal supply-base size, as characterized in Equation (11), changes with model parameters. To do so we study the change in $G(n) - G(n+1)$; if the marginal savings in the buyer's cost-to-go upon adding an extra supplier are increasing with the model parameters, then the optimal size of the supply base increases.

Lemma 5. *The optimal supply-base size n^* is nonincreasing in the qualification cost k.*

Figure 4 Sensitivity of the Optimal Supply-Base Size (n^*) as k, T, and the Supplier Production Costs Distribution Are Changed



Note. Parameter values for P_{out}, β, W, and γ are the same as in Figure 2.

Indeed, a higher k would imply a higher cost of maintaining an additional supplier in the supply base, and therefore the marginal benefit of an additional supplier will decrease as k increases.

Proceeding with our investigation, Figure 4 numerically depicts how the supply-base size changes as one or more parameters are changed.[8] From the figure, we can see that, all else being equal, the supply base shrinks as the buyer's cost of maintaining the suppliers in the supply base increases—i.e., the supply base shrinks in k and grows in the cost of retooling capacity, T. Indeed, with higher retooling costs, the suppliers are less likely to leave the supply base, which reduces the supply-base maintenance cost and hence increases the marginal value of an extra supplier. However, note that even when suppliers do not ever leave (e.g., T is infinity or P_{out} is zero), the buyer will not wish to form an infinitely large supply base because of the initial qualification costs of forming the supply base. So, as one would expect $n^*(k=0) \geq n^*(P_{\text{out}}=0) \geq n^*(P_{\text{out}}=\infty) = n^*(k=\infty) = 1$. In the same figure we also observe how the supply-base size is affected when the spread of the distribution of supplier production costs is changed, keeping the mean constant. Note that the supply-base size grows as the cost spread is increased. This observation is not obvious up front because, on one hand, a wider range of supplier production costs increases the probability of getting a low-cost supplier, but on the other hand, a wider range of supplier production costs makes multisourcing more expensive compared with single sourcing (by worsening the disparity between suppliers' costs), thereby increasing the supply-base maintenance cost. However, we find that typically the former effect outweighs the latter, and therefore for a wider range of supplier production costs, the buyer increases its supply-base size in hopes of accessing even lower-cost suppliers.

5.2. Comparison with Myopic Winner-Take-All Policy

Section 4.2 compared the performance of the split-award auction with the winner-take-all auction (myopic policy) for a fixed supply-base size. Here we revisit this question while considering optimal decisions on supply-base size. To do so, we first characterize the optimal supply-base-size decision of the myopic buyer with an optimal award policy that is taken as winner take all. By denoting $G^{\text{myopic}}(n) = \mathbb{E}_{\mathbf{c}} \min_{\mathbf{q}} \sum_{i=1}^{n} B^{\text{myopic}}(q_i(\mathbf{c}), c_i) + k(1-\beta)n$, the optimal supply-base-size decision of the myopic buyer can be characterized as

$$n^{\text{myopic}} = \min\big(n \mid G^{\text{myopic}}(n) - G^{\text{myopic}}(n+1) \leq 0\big). \quad (12)$$

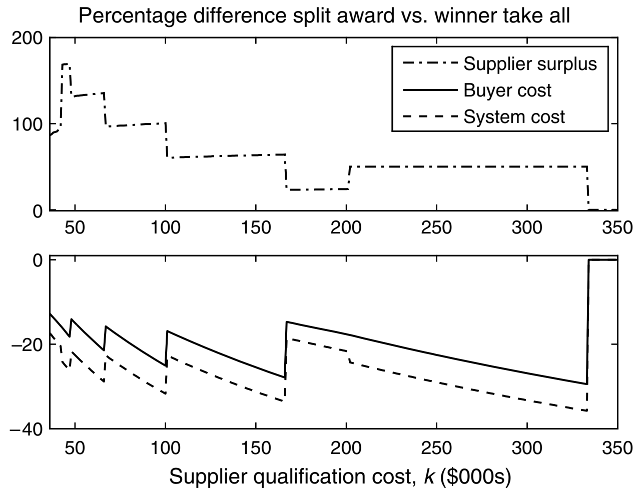
Theorem 2 characterized the optimal supply-base-size decision of the buyer using split-award auctions. Our next result compares the optimal supply-base-size decision of the myopic buyer using winner take all and the optimal buyer using split awards. We compare the savings in the buyers' cost-to-go by adding an extra supplier in both the cases. Which case results in a larger marginal benefit of an additional supplier is unclear upfront. On one hand, the drop in the purchase price from adding an extra supplier could be higher for the buyer using split-award auctions because the drop in the higher-order statistics might be higher than the drop in the first-order statistics. On the other hand, the ex ante cost of maintaining a supplier in the supply base is higher for the split-award auction as compared with the myopic (winner-take-all) auction because the cost of maintaining the supplier is ignored by the myopic policy and is always zero in that case. Interestingly, the following result shows that the marginal savings from an extra supplier evaluated by the myopic policy are always higher than those evaluated by the split-award policy.

Lemma 6. *Let n^{myopic} and n^* represent the optimal supply-base sizes from Equations* (12) *and* (11), *respectively. Then, $n^{\text{myopic}} \geq n^*$.*

Lemma 6 implies that any manager that switches to split-award auctions from a winner-take-all auction, upon realizing the significant cost of maintaining the supply base, should never increase its supply-base size. Clearly, for a given supply-base size, a buyer using the myopic sourcing policy would incur a higher expected cost of maintaining its supply base as compared with a buyer using optimal split awards. This

[8] As a robustness check, we also ran experiments with different distributional assumptions, e.g., P_{out} following an exponential distribution or supplier costs following a power function distribution; the insights were as in Figure 4.

Figure 5 Ex Ante Comparison Between Split-Award and Winner-Take-All Auctions



Note. Supply-base size equals n^* for the optimal policy and n^{myopic} for the myopic policy. All other parameter values are as in Figure 2.

follows from the fact that a myopic buyer is optimizing $\sum_{i=1}^{n} B^{\text{myopic}}(q_i, c_i)$ and an optimal buyer is optimizing $\sum_{i=1}^{n}(B^{\text{myopic}}(q_i, c_i) + k\beta\bar{\alpha}(q_i, c_i))$. Moreover, from Lemma 6, a larger supply-base size would further increase the supply-base maintenance costs for the myopic buyer. Thus, as a direct consequence of nonoptimal allocations and a bigger supply-base size, one would expect that the difference between the long-run procurement costs of a myopic buyer (including its eventual cost of maintaining the supply base) and that of an optimal buyer would increase with k.

In fact, the cost difference is not monotonic in the supplier qualification cost, k. Figure 5 illustrates the percentage difference in the long-run procurement cost as k changes. In particular, we see a jump in this percentage difference whenever the myopic buyer reduces its supply-base size to control the initial cost of starting the supply base and thus unintentionally reduces its supply-base maintenance cost, thus narrowing the gap between its total cost and that of the buyer using the optimal sourcing policy (e.g., n^{myopic} changes from 6 to 5 at $k = \$48{,}000$, from 5 to 4 at $k = \$67{,}000$, from 4 to 3 at $k = \$101{,}000$, etc.).[9] Although no buyer would ever wish for a higher supplier qualification cost, ironically it could wind up benefiting a myopic buyer by reducing their tendency to use large supply bases that are expensive to maintain in the long run.

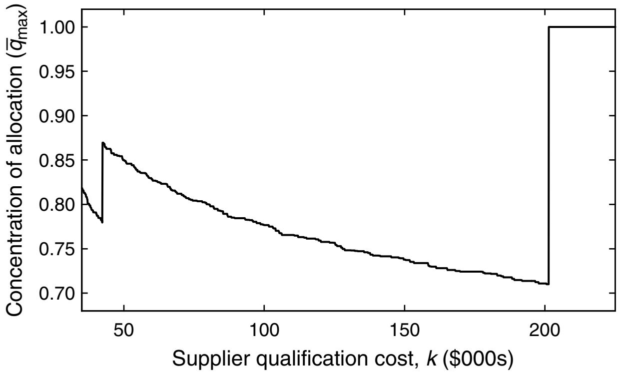
Comparing the surplus of participating suppliers across the two auction types (split award and winner take all) also reveals nonmonotonicity in the qualification cost k. We observe that the percentage difference in the suppliers' surplus jumps up whenever the optimal buyer reduces its supply-base size because the suppliers do not face as many competitors in the split-award auctions (n^* changes from 3 to 2 at $k = \$43{,}000$ and from 2 to 1 at $k = \$202{,}000$). Thus, compared with winner take all, the suppliers can get two types of benefits from the optimal buyer's use of split awards—they win business more often, and they face fewer competitors. Similar intuition explains the downward jumps, which arise whenever the myopic buyer shrinks its supply base. In between the jumps (i.e., for constant supply-base sizes), we see a gradual increase in the percentage difference of surplus, with exactly the same intuition as we explained for Figure 2. Finally, with fewer suppliers the cost of maintaining the supply-base decreases. We therefore see the percentage difference in the system cost jumping down (up) whenever the optimal (myopic) buyer shrinks its supply base.

5.3. Extent of Multisourcing

In §4.3 we explored the buyer's extent of multisourcing when the supply-base size, n, was exogenous. Here we explore the same when the buyer's supply-base size is set optimally according to Equation (11). When supplier qualification cost k increases, what happens to the extent of multisourcing? We saw in Lemma 5 that the supply-base size decreases with k, which in conjunction with Lemma 4, would imply that the concentration of allocation should increase in k. However, the supply-base size takes integer values. This implies that in between integer changes of supply-base size, increasing k typically decreases the concentration of allocation. At values where a marginal increase in k decreases the supply-base size, the concentration of allocation jumps up. Figure 6 illustrates exactly this phenomenon (the supply-base size changes from 3 to 2 at $k = \$43{,}000$ and from 2 to 1 at $k = \$202{,}000$). Indeed, for large enough values of k, the concentration of allocation converges to 1. To summarize, more expensive qualification pushes the buyer away from winner-take-all allocations, until qualification costs become so onerous that the buyer responds by shrinking its supply base. Hence, there are two effects at play: The direct effect typically reduces concentration as k increases, and the indirect effect reduces the supply-base size, which triggers an increase in the concentration of allocation. The combination of these two effects results in a concentration of allocation that is nonmonotonic in k. The takeaway is that, although splits awards are a powerful vehicle for reducing supply-base maintenance costs, one should not assume that the concentration of awards will always decrease as a result of higher supplier qualification costs.

[9] The buyer using the optimal sourcing policy also shrinks its supply-base size as k increases, but this does not alter the percentage difference of buyer cost because the optimal buyer can better control its cost through the optimal splits and the supply-base size, and therefore its cost would be continuous in k.

Figure 6 Effect of k on $\bar{q}_{\max}$



Notes. Supply-base size equals n^*. All other parameter values are as in Figure 2.

6. Conclusion

In procurement practice, many buyer firms re-auction their business regularly to keep abreast of the current best supply-market pricing. To have suppliers bid in these auctions, the buyer incurs a cost of qualifying suppliers and therefore maintains a supply base to avoid qualifying new suppliers for each auction. However, suppliers tend to fall out of this pool if they are not given adequate business, and therefore the buyer faces a trade-off between giving adequate business to each supplier or being forced to qualify new suppliers in consequent periods. We analyze this trade-off and characterize the optimal split-award mechanism that the buyer should use in each period. We then characterize the optimal size of the supply base that the buyer should maintain over the long run.

We extract several insights from our model. First, buyers implementing split awards should follow a qualify-up-to supply-base maintenance policy, where suppliers that drop out of the supply base are replaced and the supply-base size remains constant. Second, split awards can increase ex ante system, buyer, and supplier benefits, resulting in a win-win-win that could help procurement managers promote the approach in practice. Third, the role of qualification cost is not obvious; neither the extent of multisourcing nor the value of split awards compared with winner take all are monotonic in the qualification cost. We hope that the model and the insights presented here will spark further research on this important topic.

The appendix provides several extensions. We discuss the optimal mechanism and the supply-base composition when suppliers are ex ante asymmetric or when their types are correlated. We find that a buyer having access to heterogeneous supplier types would never maintain a smaller supply base as compared with a buyer having access to just one type of suppliers. Our model could easily be extended to capture other aspects of the buyer's supply-base maintenance costs provided that the buyer's virtual cost function satisfies assumption (c). This could capture, for example, the buyer's costs associated with replacing poor-performing suppliers who did not get a large allocation and become inattentive to the buyer and too difficult to work with. This could also capture the buyer's costs of performing routine due diligence on suppliers who remain in the supply base; because due diligence is partly a replacement for recent familiarity with a supplier, suppliers with a smaller allocation would be less familiar and would require more requalification by the buyer before the next period.

To conclude, it is worth mentioning some future research questions related to the model developed in this paper. An interesting extension of this work would be to analyze the optimal procurement policy when suppliers have private information signals that are correlated across periods. The analysis becomes difficult because of the complex dynamic nature of the resulting game. In fact, the problem falls in a notoriously challenging area in economics for which positive results have only been obtained in limited cases, e.g., Klotz and Chatterjee (1995a). This is certainly a challenging area for future work, but we suspect that some of the additional insights would be rather straightforward: For example, to the extent that individual suppliers' costs are more positively correlated across periods, the use of split awards would become less attractive for the buyer. Another possible extension to this work could include analyzing the optimal procurement policy when the suppliers' availability not only depends on their present allocation but also on their past allocations. In this case, one would expect that a supplier's willingness to stay in the supply base would continuously decrease over time unless it receives additional business. This would greatly complicate the buyer's optimal control problem by increasing the state space, rendering it analytically challenging. Finally, in our paper the buyer faced an infinite-horizon planning problem. This could correspond to a buyer who uses the same supply base over successive generations of products. However, there may be situations where a buyer plans to discontinue its supply base and exit the business at some future date (e.g., when its patent expires). One could adapt our model to accommodate such situations. Although such embellishments would be interesting and may include additional operational details, they will not change our paper's main idea and its main contribution—namely, the use of split awards for supply-base maintenance.

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Appendix

A.1. Proofs

Proof of Lemma 1. To see the "only if" direction, note that the (IC) conditions imply

$$\begin{aligned}S(q_i(c_i,\mathbf{c}_{-i}),\hat{c}_i)-S(q_i(c_i,\mathbf{c}_{-i}),c_i)&\geq U_i(c_i,\mathbf{c}_{-i})-U_i(\hat{c}_i,\mathbf{c}_{-i})\\&\geq S(q_i(\hat{c}_i,\mathbf{c}_{-i}),\hat{c}_i)-S(q_i(\hat{c}_i,\mathbf{c}_{-i}),c_i),\quad \forall c_i,\hat{c}_i,\mathbf{c}_{-i}.\end{aligned}\tag{13}$$

For $\hat{c}_i \to c_i$ and $\hat{c}_i > c_i$, we get $\partial S(q_i(c_i,\mathbf{c}_{-i}),c_i)/\partial c_i \geq \partial S(q_i(\hat{c}_i,\mathbf{c}_{-i}),c_i)/\partial c_i$, which along with $\partial^2 S(q,c)/\partial q\partial c \geq 0$ implies that $q_i(\hat{c}_i,\mathbf{c}_{-i}) \leq q_i(c_i,\mathbf{c}_{-i})$. This gives point (2) of the lemma. Also the inequality in Equation (13) yields $-\partial U_i(c_i,\mathbf{c}_{-i})/\partial c_i = \partial S(q_i(c_i,\mathbf{c}_{-i}),t)/\partial t|_{t=c_i}$, giving point (1) of the lemma. Point (3) follows from the definition of U_i and point (4) from the IR condition. We now show the "if" direction, namely, that the conditions of the lemma give the IC and IR conditions. Let $\Xi_i(\hat{c}_i,c_i)$ represent the utility of supplier i that has a marginal cost c_i but reports $\hat{c}_i$—that is, $\Xi_i(\hat{c}_i,c_i)=z_i(\hat{c}_i,\mathbf{c}_{-i})-S(q_i(\hat{c}_i,\mathbf{c}_{-i}),c_i)$. Using condition (3) we know that $U_i(\hat{c}_i,\mathbf{c}_{-i})=z_i(\hat{c}_i,\mathbf{c}_{-i})-S(q_i(\hat{c}_i,\mathbf{c}_{-i}),\hat{c}_i)$. Hence, from condition (1), we get $z_i(\hat{c}_i,\mathbf{c}_{-i})=U_i(\infty,\mathbf{c}_{-i})+\int_{t=\hat{c}_i}^{\infty}\partial S(q_i(t,\mathbf{c}_{-i}),c)/\partial c|_{c=t}\,dt+S(q_i(\mathbf{c}_{-i},\hat{c}_i),\hat{c}_i)$. Putting this value back into the expression of $\Xi_i(\hat{c}_i,c_i)$, we get

$$\begin{aligned}\Xi_i(\hat{c}_i,c_i) &= U_i(\infty,\mathbf{c}_{-i})+\int_{t=\hat{c}_i}^{\infty}\frac{\partial S}{\partial c}(q_i(t,\mathbf{c}_{-i}),c)\bigg|_{c=t}dt\\&\quad+S(q_i(\hat{c}_i,\mathbf{c}_{-i}),\hat{c}_i)-S(q_i(\hat{c}_i,\mathbf{c}_{-i}),c_i).\end{aligned}$$

This can also be written as

$$\begin{aligned}\Xi_i(\hat{c}_i,c_i) &= U_i(c_i,\mathbf{c}_{-i})-\int_{t=c_i}^{\hat{c}_i}\frac{\partial S}{\partial c}(q_i(t,\mathbf{c}_{-i}),c)\bigg|_{c=t}dt\\&\quad+\int_{t=c_i}^{\hat{c}_i}\frac{\partial S}{\partial c}(q_i(\hat{c}_i,\mathbf{c}_{-i}),c)\bigg|_{c=t}dt.\end{aligned}$$

Finally, from condition (2) and $\partial^2 S(q,c)/\partial q\partial c \geq 0$, we get that

$$\int_{t=c_i}^{\hat{c}_i}\frac{\partial S}{\partial c}(q_i(t,\mathbf{c}_{-i}),t)\,dt \geq \int_{t=c_i}^{\hat{c}_i}\frac{\partial S}{\partial c}(q_i(\hat{c}_i,\mathbf{c}_{-i}),t)\,dt$$

for all $\mathbf{c}_{-i}$, $\hat{c}_i$, c_i and hence $\Xi_i(\hat{c}_i,c_i) \leq U_i(c_i,\mathbf{c}_{-i})$ for all $\mathbf{c}_{-i}$, $\hat{c}_i$, c_i, which is precisely the IC condition. Conditions (1) and (4) imply the IR condition. □

Proof of Lemma 2. Define

$$\begin{aligned}\Upsilon(n) = \min_{\mathbf{q}\,|\,\sum_{i=1}^{n}\mathbf{q}_i=1}\mathbb{E}_{\mathbf{c}}\Bigg\{\sum_{i=1}^{n}\bigg\langle\int_{t=c_i}^{\infty}\frac{\partial S}{\partial c}(q_i(t,\mathbf{c}_{-i}),c)\bigg|_{c=t}dt\\+S(q_i(\mathbf{c}),c_i)\bigg\rangle+\beta\mathbb{E}_{\mathbf{A}}J\bigg(\sum_{i=1}^{n}A(q_i(\mathbf{c}),c_i)\bigg)\Bigg\},\end{aligned}$$

The Bellman's equation in (3a) can then be expressed as $J(n_a)=\min_{n\geq n_a}(\Upsilon(n)+k(n-n_a))$. For any c_{n+1} and any $q_1,\ldots,q_n$ and $q_{n+1}=0$, we get

$$J\bigg(\sum_{i=1}^{n}A(q_i,c_i)\bigg)\geq J\bigg(\sum_{i=1}^{n+1}A(q_i,c_i)\bigg)$$

(because $\alpha(0,c_i)\geq 0$) and

$$\int_{t=c_{n+1}}^{\infty}\frac{\partial S}{\partial c}(0,c)\bigg|_{c=t}dt+S(0,c_{n+1})=0.$$

Hence, by the optimality of $\mathbf{q}$, we get $\Upsilon(n+1)\leq\Upsilon(n)$. Because $\Upsilon(n)$ is strictly nonnegative, it implies that a finite n^* must exist such that a policy of maintaining the supply-base size at $\max(n^*,n_a)$ is optimal. In fact, starting the horizon with 0 suppliers, the buyer would initially qualify n^* suppliers, such that $n^*=\arg\min_{n\geq 0}(\Upsilon(n)+kn)$ and would qualify-up-to n^* in all future periods. □

Proof of Theorem 1. The allocation is clearly optimal provided that q_i is nonincreasing in c_i, which we prove here. For $c_i^1\geq c_i^0$ and any two realizations of costs $(c_i^0,\mathbf{c}_{-i})$ and $(c_i^1,\mathbf{c}_{-i})$, let $q_1^0,\ldots,q_n^0$ and $q_1^1,\ldots,q_n^1$ denote the respective allocations obtained from the minimization program in Equation (7). Then, by optimality of these allocations, we get

$$\begin{aligned}&\sum_{j=1,\ldots,n,\,j\neq i}B(q_j^0,c_j)+B(q_i^0,c_i^0)\\&\quad\leq\sum_{j=1,\ldots,n,\,j\neq i}B(q_j^1,c_j)+B(q_i^1,c_i^0),\end{aligned}\tag{14}$$

and similarly,

$$\begin{aligned}&\sum_{j=1,\ldots,n,\,j\neq i}B(q_j^1,c_j)+B(q_i^1,c_i^1)\\&\quad\leq\sum_{j=1,\ldots,n,\,j\neq i}B(q_j^0,c_j)+B(q_i^0,c_i^1).\end{aligned}\tag{15}$$

Adding Equations (14) and (15) gives $B(q_i^0,c_i^0)+B(q_i^1,c_i^1)\leq B(q_i^1,c_i^0)+B(q_i^0,c_i^1)$, implying that $B(q_i^1,c_i^1)-B(q_i^1,c_i^0)\leq B(q_i^0,c_i^1)-B(q_i^0,c_i^0)$. For $\partial^2 B(q,c)/\partial q\partial c\geq 0$ and $c_i^1\geq c_i^0$, this implies $q_i^1\leq q_i^0$. The optimal payment follows from Lemma 1. □

Proof of Proposition 1. The expected total surplus of the suppliers is given by $\mathbb{E}_{\mathbf{c}}\sum_{i=1}^{n}[(\partial S(q_i,c_i)/\partial c_i)\cdot(F(c_i)/f(c_i))]$. Because $F(c)/f(c)$ is nondecreasing and $\partial S(0,c_i)/\partial c_i=0$, we have that, for any sample path of cost $c_1\leq c_2\leq\cdots\leq c_n$, any n, and $\sum_{1=i}^{n}q_i=1$, if $\sum_{i=1}^{n}\partial S(q_i,c_i)/\partial c_i\geq\partial S(1,c_1)/\partial c_1$, then the total expected supplier surplus for any split cannot be lower than in a winner-take-all auction. □

Proof of Proposition 2. Let $\mathbf{q}^*$ and $\mathbf{q}^\dagger$ denote the vector of allocations in the optimal split-award and winner-take-all auctions for any sample of cost. The system cost for the two auctions would then be $\sum_{i=1}^{n}(S(q_i^*,c_i)+k\beta\bar{\alpha}(q_i^*,c_i))$ and $\sum_{i=1}^{n}(S(q_i^\dagger,c_i)+k\beta\bar{\alpha}(q_i^\dagger,c_i))$, respectively. From the buyer's optimization program in Equation (7), we know that $\sum_{i=1}^{n}B(q_i^*,c_i)\leq\sum_{i=1}^{n}B(q_i^\dagger,c_i)$. Because $B(q,c)\equiv(\partial S(q,c)/\partial c)\cdot(F(c)/f(c))+S(q,c)+k\beta\bar{\alpha}(q,c)$, the system cost in a split-award auction would be less than in the winner-take-all auction if $\sum_{i=1}^{n}[(\partial S(q_i^*,c_i)/\partial c_i)\cdot(F(c_i)/f(c_i))]\geq\sum_{i=1}^{n}[(\partial S(q_i^\dagger,c_i)/\partial c_i)\cdot(F(c_i)/f(c_i))]$. Because $F(c)/f(c)$ is nondecreasing and $\partial S(0,c_i)/\partial c_i=0$, for any sample path of cost $c_1\leq c_2\leq\cdots\leq c_n$, any n, and $\sum_{1=i}^{n}q_i=1$, if $\sum_{i=1}^{n}\partial S(q_i,c_i)/\partial c_i\geq\partial S(1,c_1)/\partial c_1$, then the required condition is satisfied. □

Proof of Lemma 3. Let $\phi(q,c)\equiv S(q,c)+\partial S(q,c)/\partial c\cdot(F(c)/f(c))$. For any k, let the optimal allocation be $q_1,\ldots,q_n$. Optimality of allocation implies that for any $q_i>q_j$ a transfer of an amount $\epsilon > 0$ from q_i to q_j would result in a nonoptimal solution (a higher cost), i.e.,

$$\sum_{l=1,\,l\neq i,\,l\neq j}^{n} B(q_l, c_l) + \phi(q_i, c_i) + \phi(q_j, c_j) + k\beta\bar{\alpha}(q_i) + k\beta\bar{\alpha}(q_j)$$
$$\leq \sum_{l=1,\,l\neq i,\,l\neq j}^{n} B(q_l, c_l) + \phi(q_i-\epsilon, c_i) + \phi(q_j+\epsilon, c_j) + k\beta\bar{\alpha}(q_i-\epsilon) + k\beta\bar{\alpha}(q_j+\epsilon),$$

which implies that

$$\phi(q_i, c_i) + \phi(q_j, c_j) - \phi(q_i-\epsilon, c_i) - \phi(q_j+\epsilon, c_j) \leq k\beta\big(\bar{\alpha}(q_i-\epsilon) + \bar{\alpha}(q_j+\epsilon) - \bar{\alpha}(q_i) - \bar{\alpha}(q_j)\big).$$

However, for $\bar{\alpha}$ convex the right-hand side of the above inequality is less than 0 and for $k\to\infty$ the above inequality cannot be true if $q_i > q_j$. In fact, for $k\to\infty$, the above inequality holds only if $q_i = q_j$. Applying this argument for all the pairs of allocation results in $q_i = 1/n$ for all i as $k\to\infty$. $\square$

Proof of Lemma 4. We order the marginal cost such that $c_1 \leq c_2 \leq \cdots \leq c_n$. Also we define the maximum allocation $\chi_{\max}$ as a function of the differences between consecutive order statistics of marginal costs, $\Delta\mathbf{c} = (\Delta c_1, \Delta c_2, \ldots, \Delta c_{n-1}) = (c_2 - c_1, c_3 - c_2, \ldots, c_n - c_{n-1})$—that is, $\chi_{\max}(\Delta c_1, \Delta c_2, \ldots, \Delta c_{n-1}, \nu) = q_{\max}(c_1, c_2, \ldots, c_n)$, where ν is the cost of a supplier (could be any supplier) such that one can infer $c_1, \ldots, c_n$ from $\Delta c_1, \Delta c_2, \ldots, \Delta c_{n-1}, \nu$. (Note that ν controls for change in allocation when the costs of each supplier is shifted by the same amount.) Indeed, ν is distributed according to the cost of the suppliers, i.e., with c.d.f. f and p.d.f. F.

The remainder of the proof is organized as follows. We first show that the probability distribution function of any Δc_i can be characterized by $h(w)$. We next show that the survival function of any Δc_i is decreasing in the sample size n (i.e., the ith order statistic with sample size n stochastically dominates, in the first order, the ith order statistic with sample size $n+1$). Finally we use this result to show that the expectation of $\chi_{\max}$ is decreasing in n.

The probability density function of the difference between two consecutive order statistics $m+1$ and m of virtual cost, for sample size n, can be written as (see David and Nagaraja 2003)

$$h_{m,n}(w) = \int_{x=0}^{\infty} F^{m-1}(x)[1-F(x+w)]^{n-m-1} f(x) f(x+w)\,dx.$$

Indeed, with n suppliers, we can express $\bar{q}_{\max}(n) = \mathbb{E}_{\Delta\mathbf{c},\nu}\,\chi_{\max}(\Delta c_1, \Delta c_2, \ldots, \Delta c_{n-1}, \nu)$ and for $n+1$ suppliers $\bar{q}_{\max}(n+1) = \mathbb{E}_{\Delta\mathbf{c},\nu}\,\chi_{\max}(\Delta c_1, \Delta c_2, \ldots, \Delta c_{n-1}, \Delta c_n, \nu)$.

Adding an extra supplier will not increase the allocation of the cheapest supplier (which follows directly from the proof of Theorem 2). Thus, for any sample path $\Delta c_1, \ldots, \Delta c_{n-1}, \nu$, representing $\mathbb{E}_{\Delta c_{i,n}}$ as the expectation taken over the distribution of Δc_i when n suppliers are present, we can write

$$\chi_{\max}(\Delta c_1, \Delta c_2, \ldots, \Delta c_{n-1}, \nu) \geq \mathbb{E}_{\Delta c_{n,n+1}}\,\chi_{\max}(\Delta c_1, \Delta c_2, \ldots, \Delta c_{n-1}, \Delta c_n, \nu). \quad (16)$$

However, the probability density function of the difference between consecutive order statistics will be different for n and $n+1$ suppliers—i.e., $h_{m,n}(w)$ will be different from $h_{m,n+1}(w)$. Let $\bar{H}_{m,n}(z)$ denote the survival function of the difference between consecutive order statistics of marginal cost for n suppliers. Therefore,

$$\begin{aligned}\bar{H}_{m,n}(z) &= \int_{w=z}^{\infty} h_{m,n}(w)\,dw\\ &= \int_{x=0}^{\infty}\int_{w=z}^{\infty} F^{m-1}(x)[1-F(x+w)]^{n-m-1}\\ &\quad\cdot f(x)f(x+w)\,dw\,dx\\ &= \int_{x=0}^{\infty} F^{m-1}(x)\frac{[1-F(x+z)]^{n-m}}{n-m} f(x)\,dx.\end{aligned}$$

Indeed, $[1-F(x+z)]^{n-m}/(n-m)$ is decreasing in n and therefore $\bar{H}_{m,n}(z) \geq \bar{H}_{m,n+1}(z)$ for all $1 \leq m \leq n-1$.

From Shaked and Shanthikumar (1988), we know that for any two random variables X and Y (with c.d.f. F and G, respectively) and a function $\phi(z)$ that is nondecreasing in z the following is true: $\mathbb{E}_X\,\phi(x) \geq \mathbb{E}_Y\,\phi(y)$ if $\bar{F}(x) \geq \bar{G}(x)$.

From the feasibility of the mechanism, we know that the allocation can only decrease. Therefore, any increase in Δc_i with all the other Δc_j, for $j\neq i$, and ν held constant would imply that q_k would not increase for all $k \geq i+1$ and q_l would not decrease for all $l \leq i$ to maintain the constraint $\sum q_i = 1$. Hence, $\chi_{\max}(\Delta c_1, \Delta c_2, \ldots, \Delta c_{n-1})$ is nondecreasing in Δc_i, for all $1 \leq i \leq n-1$ when for all $j \neq i$, Δc_j are held constant. Therefore,

$$\mathbb{E}_{\Delta c_{i,n}}\,\chi_{\max}(\Delta c_1, \Delta c_2, \ldots, \Delta c_{n-1}, \nu) \geq \mathbb{E}_{\Delta c_{i,n+1}}\,\chi_{\max}(\Delta c_1, \Delta c_2, \ldots, \Delta c_{n-1}, \nu). \quad (17)$$

Adding a supplier does not change the distribution of ν, so the inequality in Equation (17) can be written as

$$\mathbb{E}_{\Delta c_{i,n},\nu}\,\chi_{\max}(\Delta c_1, \Delta c_2, \ldots, \Delta c_{n-1}, \nu) \geq \mathbb{E}_{\Delta c_{i,n+1},\nu}\,\chi_{\max}(\Delta c_1, \Delta c_2, \ldots, \Delta c_{n-1}, \nu). \quad (18)$$

Indeed, using Equation (16) along with Equation (18), it follows that for any sample path $\Delta c_1, \ldots, \Delta c_{n-2}$,

$$\begin{aligned}&\mathbb{E}_{\Delta c_{n-1,n},\nu}\,\chi_{\max}(\Delta c_1, \Delta c_2, \ldots, \Delta c_{n-1}, \nu)\\ &\quad\geq \mathbb{E}_{\Delta c_{n-1,n+1},\nu}\,\chi_{\max}(\Delta c_1, \Delta c_2, \ldots, \Delta c_{n-1}, \nu)\\ &\quad\geq \mathbb{E}_{\Delta c_{n-1,n+1},\nu}\big(\mathbb{E}_{\Delta c_{n,n+1}}\,\chi_{\max}(\Delta c_1, \Delta c_2, \ldots, \Delta c_{n-1}, \Delta c_n, \nu)\big).\end{aligned}$$

This is true for every sample path $\Delta c_1, \ldots, \Delta c_{n-2}$, so

$$\begin{aligned}&\mathbb{E}_{\Delta\mathbf{c},\nu}\,\chi_{\max}(\Delta c_1, \Delta c_2, \ldots, \Delta c_{n-1}, \nu)\\ &\quad\geq \mathbb{E}_{\Delta\mathbf{c},\nu}\big(\mathbb{E}_{\Delta c_{n,n+1}}\,\chi_{\max}(\Delta c_1, \Delta c_2, \ldots, \Delta c_{n-1}, \Delta c_n, \nu)\big),\end{aligned}$$

and hence $\bar{q}_{\max}(n+1) \leq \bar{q}_{\max}(n)$. $\square$

Proof of Theorem 2. Define

$$\Psi(c_1, \ldots, c_n) = \min_{\mathbf{q}} \sum_{i=1}^{n} B(q_i(\mathbf{c}), c_i).$$

We can then write $\Psi(c_1, \ldots, c_n, \infty) = \Psi(c_1, \ldots, c_n)$ (because $B(0, \infty) = 0$). Therefore,

$$\Psi(c_1, \ldots, c_n, c_{n+1}) = -\int_{w=c_{n+1}}^{\infty} \frac{d\Psi}{dw}(c_1, \ldots, c_n, w)\,dw + \Psi(c_1, \ldots, c_n).$$

Using the envelope theorem, we get

$$\frac{d\Psi}{dw}(c_1, \ldots, c_n, w) = \frac{\partial B}{\partial c}(q_{n+1}(\mathbf{c}, w), c)\bigg|_{c=w},$$

and therefore

$$\Psi(c_1,\dots,c_n,c_{n+1})-\Psi(c_1,\dots,c_n)$$
$$=-\int_{w=c_{n+1}}^{\infty}\frac{\partial B}{\partial c}(q_{n+1}(\mathbf{c},w),c)\Big|_{c=w}\,dw.$$

From Equation (10), we know that $G(n)=\mathbb{E}_{\mathbf{c}_n}\{\Psi(c_1,\dots,c_n)\}+k(1-\beta)n$, where $\mathbf{c}_n$ denotes an n-dimensional vector of virtual costs. Therefore,

$$G(n+1)-G(n)=-\mathbb{E}_{\mathbf{c}_{n+1}}\int_{w=c_{n+1}}^{\infty}\frac{\partial B}{\partial c}(q_{n+1}(\mathbf{c}_n,w),c)\Big|_{c=w}\,dw+k(1-\beta).$$

For symmetric suppliers (having the same cost distribution), the above equation can equivalently be written as

$$G(n+1)-G(n)=-\mathbb{E}_{\mathbf{c}_{n+1}}\int_{w=c_1}^{\infty}\frac{\partial B}{\partial c}(q_1(w,\mathbf{c}_n),c)\Big|_{c=w}\,dw+k(1-\beta).$$

Similarly,

$$G(n+2)-G(n+1)=-\mathbb{E}_{\mathbf{c}_{n+2}}\int_{w=c_1}^{\infty}\frac{\partial B}{\partial c}(q_1(w,\mathbf{c}_{n+1}),c)\Big|_{c=w}\,dw+k(1-\beta).$$

For $\partial^2B(q,c)/\partial q\partial c\ge 0$ and a constraint on total allocation, we get $q_1(w,\mathbf{c}_n)=q_1(w,\mathbf{c}_n,\infty)\ge q_1(w,\mathbf{c}_n,c_{n+2})$ for all w, $\mathbf{c}_n$, c_{n+2}. Hence,

$$\frac{\partial B}{\partial c}(q_1(w,\mathbf{c}_n),c)\ge\frac{\partial B}{\partial c}(q_1(w,\mathbf{c}_{n+1}),c)$$

for all w, $\mathbf{c}_n$, c_{n+1}, c. Therefore, $G(n+2)-G(n+1)\ge G(n+1)-G(n)$, so $G(n)-G(n+1)$ is nonincreasing in n. $\square$

Proof of Lemma 5. Differentiating $G(n)$ w.r.t. k (and applying the envelope theorem), we get

$$\frac{dG(n)}{dk}=\mathbb{E}_{\mathbf{c}_n}\left\{\sum_{i=1}^{n}\bar\alpha(q_i(\mathbf{c}_n),c_i)\right\}+(1-\beta)n,$$

where $q_i(\mathbf{c}_n)$ represents the optimal allocation to supplier i when there are n suppliers in the supply base. Similarly,

$$\frac{dG(n+1)}{dk}=\mathbb{E}_{\mathbf{c}_{n+1}}\left\{\sum_{i=1}^{n+1}\bar\alpha(\hat q_i(\mathbf{c}_{n+1}),c_i)\right\}+(1-\beta)(n+1),$$

where $\hat q_i(\mathbf{c}_{n+1})$ represents the optimal allocation to supplier i when there are $n+1$ suppliers in the supply base. Taking the difference between the above two expressions, we get

$$\frac{d(G(n)-G(n+1))}{dk}=\mathbb{E}_{\mathbf{c}_{n+1}}\left[\sum_{i=1}^{n}(\bar\alpha(q_i(\mathbf{c}_n),c_i)-\bar\alpha(\hat q_i(\mathbf{c}_{n+1}),c_i))-\bar\alpha(\hat q_{n+1}(\mathbf{c}_{n+1}),c_{n+1})\right]-(1-\beta).$$

For any sample path, that is, for any $c_1,\dots,c_{n+1}$, from the proof of Theorem 2, we get $q_i\ge\hat q_i$, for all $i=1,\dots,n$. Because $\bar\alpha(q,c)$ is nonincreasing in q, $d(G(n)-G(n+1))/dk\le 0$. $\square$

Proof of Lemma 6. Parameterize $\bar\alpha(q,c)$ with a parameter $0<a\le 1$ such that $\bar\alpha(q,c,a)=a\cdot\bar\alpha(q,c)$. Also denote $C(n,a)=\mathbb{E}_{\mathbf{c}}\min_{\mathbf{q}}\sum_{i=1}^{n}B(q_i(\mathbf{c}),c_i,a)+k(1-\beta)n$, where $B(q_i(\mathbf{c}),c_i,a)=S(q,c)+\partial S(q,c)/\partial c\cdot(F(c)/f(c))+a\cdot k\beta\bar\alpha(q,c)$. One can then express $G^{\text{myopic}}(n)=C(n,0)$ and $G(n)=C(n,1)$. Differentiating $C(n,a)$ w.r.t. a (and applying the envelope theorem), we get

$$\frac{dC(n,a)}{da}=k\beta\,\mathbb{E}_{\mathbf{c}_n}\sum_{i=1}^{n}\bar\alpha(q_i(\mathbf{c}_n),c_i),$$

where $q_i(\mathbf{c}_n)$ represents the optimal allocation to supplier i when there are n suppliers in the supply base. Similarly,

$$\frac{dC(n+1,a)}{da}=k\beta\,\mathbb{E}_{\mathbf{c}_{n+1}}\sum_{i=1}^{n+1}\bar\alpha(\hat q_i(\mathbf{c}_{n+1}),c_i).$$

where $\hat q_i(\mathbf{c}_{n+1})$ represents the optimal allocation to supplier i when there are $n+1$ suppliers in the supply base. Taking the difference between the above two expressions, we get

$$\frac{d(C(n+1,a)-C(n,a))}{da}=k\beta\,\mathbb{E}_{\mathbf{c}_{n+1}}\left\langle\sum_{i=1}^{n}(\bar\alpha(\hat q_i(\mathbf{c}_{n+1}),c_i)-\bar\alpha(q_i(\mathbf{c}_n),c_i))+\bar\alpha(\hat q_{n+1}(\mathbf{c}_{n+1}),c_{n+1})\right\rangle.$$

For any sample path, that is, for any $c_1,\dots,c_{n+1}$, from the proof of Theorem 2, we get $q_i\ge\hat q_i$, for all $i=1,\dots,n$. Because $\bar\alpha(q,c)$ is nonincreasing in q,

$$\frac{d(C(n+1,a)-C(n,a))}{da}\ge 0.$$

Therefore, $G(n)-G(n+1)\le G^{\text{myopic}}(n)-G^{\text{myopic}}(n+1)$, and hence $n^{\dagger}\ge n^{*}$. $\square$

A.2. Extensions

In this section, we illustrate two directions in which the model presented in §3 can be extended. Namely, we first consider the case when the suppliers are ex ante asymmetric and then the case when suppliers' per-unit costs are correlated.

A.2.1. Supply-Base Composition with Heterogeneous Supplier Types. We explore here what recommendations can be made when supplier types are heterogeneous. For this purpose, we consider r type of suppliers and use subscript $j=1,\dots,r$ to depict the type of supplier. For a supplier belonging to a type j, its per-unit production cost is distributed according to c.d.f. F_j, its availability equals $\alpha_j(q,c)$, and its expected profit from switching over equals $\Omega_j(q,c)$. Moreover, suppliers can have different qualification costs k_j. In the subsequent analysis, we use two subscripts on some of our parameters: The first subscript represents the type of the supplier, and the second subscript is the supplier index; e.g., $q_{j,i}$ represents the allocation of the ith supplier of type j.

We are interested in understanding how the buyer should design its supply base in this case—namely, the number (including 0) of each type of suppliers it qualifies (or does not qualify) for the supply base. Again, we focus on the infinite-horizon setting, where the optimization problem can be written, similar to Equation (3a) (by relaxing condition (2) of Lemma 1), as

$$J(n_{1,a},\dots,n_{r,a})=\min_{n_1\ge n_{1,a},\dots,n_r\ge n_{r,a}}\left\{\min_{\substack{\mathbf{q}_1,\dots,\mathbf{q}_r\\ \text{s.t. }\sum_{j=1}^{r}\sum_{i=1}^{n_j}q_{j,i}=1}}\mathbb{E}_{\mathbf{c}_1,\dots,\mathbf{c}_r}\right.$$

$$\cdot\left\{\sum_{j=1}^{r}\sum_{i=1}^{n_j}\left\langle\int_{t=c_{j,i}}^{\infty}\frac{\partial S_j}{\partial c}(q_{j,i}(t,\mathbf{c}_{-i}),c)\Big|_{c=t}dt+S_j(q_{j,i}(\mathbf{c}),c_{j,i})\right\rangle\right.$$
$$\left.+\beta\mathbb{E}_{\mathbf{A}_1,\ldots,\mathbf{A}_r}J\left(\sum_{i=1}^{n_1}A_1(q_{1,i}),\ldots,\sum_{i=1}^{n_r}A_r(q_{r,i})\right)\right\}$$
$$\left.+\sum_{j=1}^{r}k_j(n_j-n_{j,a})\right\},$$

Lemma 7. *If the buyer starts the horizon with 0 suppliers, then a stationary policy in which it qualifies up-to n_j^* of type j suppliers for $j=1,\ldots,r$ is optimal.*

Proof. Similar to the proof of Lemma 2, finite optimal supply-base levels $n_1^*,\ldots,n_r^*$ exist for each type of suppliers that minimize the cost-to-go at the beginning of the horizon (when the buyer has an empty supply base). Hence, starting any consequent period with $n_{j,a}\le n_j^*$ (for $j=1,\ldots,r$) suppliers, a stationary policy of qualify-up-to n_j^* for $j=1,\ldots,r$ is optimal. □

Therefore, similar to Equation (6), the buyer's mechanism problem can be written as

$$\min_{\substack{\mathbf{q}_1,\ldots,\mathbf{q}_r\\ \text{s.t. }\sum_{j=1}^{r}\sum_{i=1}^{n_j}q_{j,i}=1}}\mathbb{E}_{\mathbf{c}_1,\ldots,\mathbf{c}_r}\left[\sum_{j=1}^{r}\sum_{i=1}^{n_j}B_j(q_{j,i},c_{j,i})\right].$$

Similar to Theorem 1, it can be verified that the above allocations indeed satisfy condition (2) of Lemma 1. The optimal mechanism can then be characterized by the corollary stated below.

Corollary 1. *In an infinite-horizon problem* $\mathbf{z},\mathbf{q}$ *represents an optimal mechanism in dominant strategy if the payment* z_i *made to each supplier is given by Equation* (8) *where*

$$\mathbf{q}(\mathbf{c})=\min_{\substack{\mathbf{q}_1,\ldots,\mathbf{q}_r\\ \text{s.t. }\sum_{j=1}^{r}\sum_{i=1}^{n_j}q_{j,i}=1}}\left[\sum_{j=1}^{r}\sum_{i=1}^{n_j}B_j(q_{j,i},c_{j,i})\right].$$

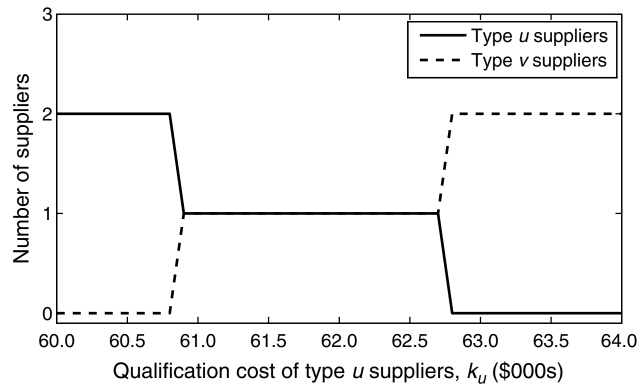
To solve the buyer's optimal supply-base size problem, we follow the approach of §5. Denote

$$G(n_1,\ldots,n_r)=\mathbb{E}_{\mathbf{c}_1,\ldots,\mathbf{c}_r}\left\{\min_{\substack{\mathbf{q}_1,\ldots,\mathbf{q}_r\\ \text{s.t. }\sum_{j=1}^{r}\sum_{i=1}^{n_j}q_{j,i}=1}}\left[\sum_{j=1}^{r}\sum_{i=1}^{n_j}B_j(q_{j,i},c_{j,i})\right]\right\}+\sum_{j=1}^{r}k_j(1-\beta)n_j.$$

Then the buyer's optimal supply-base-size decision can be characterized as $\min_{n_1,\ldots,n_r}G(n_1,\ldots,n_r)$. Let $n^*_{\text{hetero},j}$ represent the optimal number of suppliers of type j in an heterogeneous supply base. Also let n_j^* denote the optimal number of suppliers when the supply base consists of only type j suppliers, i.e., n_j^* can be found from the solution of Equation (11). Indeed, the number of suppliers in a homogeneous supply base, consisting of only type j suppliers, would never be less than the number of type j suppliers in an heterogeneous supply base.

Figure A.1 illustrates the change in the supply-base composition (the change in supply-base size) when $r=2$ as the cost of qualifying a given type (type u) of suppliers is changed. We find that the buyer maintains a homogeneous supply base (consisting of a singletype of suppliers) if the attributes

Figure A.1 **Optimal Composition of Supply Base with Type u and Type v Suppliers as k_u Is Changed**



Notes. Here, $k_v=\$70{,}000$, $T_u=\$40{,}000$, and $T_v=\$42{,}000$. Costs are uniformly distributed for type u and type v suppliers in the intervals \$[400,000; 600,000] and \$[410,000; 590,000], respectively. All other parameter values are as in Figure 2.

of a particular type of suppliers move to the extreme relative to the attributes of the other type of suppliers. Specifically, we see that the proportion of a given type of suppliers in the composition of the supply base is nonincreasing in their qualification cost.

A.2.2. Correlated Cost. We now relax the assumption on the independence of costs across suppliers, in the manner of Wan and Beil (2009). We consider that each supplier observes a private signal θ_i that partially informs its cost estimate. The signals $\boldsymbol{\theta}=(\theta_1,\ldots,\theta_n)$ are i.i.d. across suppliers and are distributed with p.d.f. $f(\theta)$ and c.d.f. $F(\theta)$. Supplier cost can then be represented as $c_i(\boldsymbol{\theta},\xi)$, where ξ is an observable common factor across all suppliers. For example, if all the suppliers are located in the same country (different from the buyer's), ξ could represent the price of energy in that country. Because the costs depend on $\boldsymbol{\theta}$ and not only on θ_i, this formulation allows for cost correlation. With this cost model, we denote the total cost of a supplier getting an allocation q as $S(q,\boldsymbol{\theta},\xi)$. We assume that $\partial S(q,\boldsymbol{\theta},\xi)/\partial\theta_i\ge 0$ and $\partial^2 S(q,\boldsymbol{\theta},\xi)/\partial\theta_i\partial q\ge 0$. The feasibility conditions of Lemma 1 (in dominant strategy equilibria) can then be written as follows.

Corollary 2. *Given the observed* ξ, $(\mathbf{z},\mathbf{q})$ *represents a truth-revealing mechanism if and only if*

1. $U_i(\boldsymbol{\theta})=U_i(\infty,\boldsymbol{\theta}_{-i})+\int_{t=\theta_i}^{\infty}\partial S(q_i(t,\boldsymbol{\theta}_{-i}),\theta,\boldsymbol{\theta}_{-i},\xi)/\partial\theta|_{\theta=t}\,dt$;
2. $q_i(\theta_i,\boldsymbol{\theta}_{-i})$ is nonincreasing in θ_i for all $\theta_i,\boldsymbol{\theta}_{-i}$;
3. $U_i(\boldsymbol{\theta})=z_i(\boldsymbol{\theta})-S(q_i(\boldsymbol{\theta}),\boldsymbol{\theta},\xi)$;
4. $U_i(\infty,\boldsymbol{\theta}_{-i})\ge 0$.

The buyer's virtual cost function for a supplier can then defined as

$$B(q,\boldsymbol{\theta},\xi)\equiv\frac{\partial S}{\partial\theta}(q,\boldsymbol{\theta},\xi)\cdot\frac{F(\theta_i)}{f(\theta_i)}+S(q,\boldsymbol{\theta},\xi)+k\beta\bar{\alpha}(q_i,\boldsymbol{\theta}).$$

If $\partial^2 B(q,\boldsymbol{\theta},\xi)/\partial q\partial\theta_i\ge 0$ for all i and if

$$\frac{\partial^2 B}{\partial q\partial\theta_i}(q,\theta_i,\boldsymbol{\theta}_{-i},\xi)\ge\frac{\partial^2 B}{\partial q\partial\theta_i}(q,\theta_j,\boldsymbol{\theta}_{-j},\xi)$$

for any i,j, then the allocation q_i obtained in Equation (7) is nonincreasing in θ_i, and therefore conditions of Corollary 2 are satisfied. Hence, the mechanism in §4.1 would remain optimal with the above characterization of virtual costs. We can then proceed similar to §5 to characterize the optimal size of the supply base.

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