



Management Science

Publication details, including instructions for authors and subscription information:
<http://pubsonline.informs.org>

Volatility Risks and Growth Options

Hengjie Ai, Dana Kiku

To cite this article:

Hengjie Ai, Dana Kiku (2016) Volatility Risks and Growth Options. Management Science 62(3):741-763. <http://dx.doi.org/10.1287/mnsc.2014.2129>

Full terms and conditions of use: <http://pubsonline.informs.org/page/terms-and-conditions>

This article may be used only for the purposes of research, teaching, and/or private study. Commercial use or systematic downloading (by robots or other automatic processes) is prohibited without explicit Publisher approval, unless otherwise noted. For more information, contact permissions@informs.org.

The Publisher does not warrant or guarantee the article's accuracy, completeness, merchantability, fitness for a particular purpose, or non-infringement. Descriptions of, or references to, products or publications, or inclusion of an advertisement in this article, neither constitutes nor implies a guarantee, endorsement, or support of claims made of that product, publication, or service.

Copyright © 2016, INFORMS

Please scroll down for article—it is on subsequent pages



INFORMS is the largest professional society in the world for professionals in the fields of operations research, management science, and analytics.

For more information on INFORMS, its publications, membership, or meetings visit <http://www.informs.org>

Volatility Risks and Growth Options

Hengjie Ai

Carlson School of Management, University of Minnesota, Minneapolis, Minnesota 55455, hengjie@umn.edu

Dana Kiku

University of Illinois at Urbana–Champaign, Champaign, Illinois 61820, dka@illinois.edu

We propose to measure growth opportunities by firms' exposure to idiosyncratic volatility news. Theoretically, we show that the value of a growth option increases in idiosyncratic volatility but its response to volatility of aggregate shocks can be either positive or negative depending on option moneyness. Empirically, we show that price sensitivity to variation in idiosyncratic volatility carries significant information about firms' future investment and growth even after controlling for conventional proxies of growth options such as book-to-market and other relevant firm characteristics. Consistent with our theoretical arguments, we also find that firm' exposure to aggregate volatility, while priced, does not help predict their future growth. Option-intensive firms identified using our idiosyncratic volatility-based measure earn a lower premium than do firms that rely more heavily on assets in place.

Keywords: growth options; idiosyncratic volatility; finance; asset pricing

History: Received February 5, 2014; accepted October 20, 2014, by Jerome Detemple, finance. Published online in *Articles in Advance* June 26, 2015.

1. Introduction

We propose a volatility-based measure of growth options owned by firms. Our idea originates from the conventional wisdom that option payoffs increase in volatility of the underlying cash flows. According to the standard real-options models that typically make no distinction between aggregate and idiosyncratic risks, price exposure to volatility news should contain information about growth opportunities available to firms. We argue that the source of variation does matter for this result to hold. We show that the value of a growth option increases with idiosyncratic volatility but its response to volatility of aggregate shocks is ambiguous. Guided by our theoretical analysis, we propose to measure options by exposure to idiosyncratic volatility news. We show that, empirically, it carries significant information about cross-sectional differences in firms' future investment and growth even after controlling for conventional proxies of growth options. We also find that option-intensive firms, identified by high sensitivity to idiosyncratic volatility, earn a lower premium compared with low-exposure asset-in-place intensive firms.

Unobservable growth opportunities are typically proxied by various valuation ratios such as market to book, price to earnings, etc. Valuation-based proxies, however, have significant limitations because they may vary across firms for many different reasons. For example, any heterogeneity in firms' current and/or future productivity in the presence of adjustment

costs or differences in riskiness of assets in place will generally lead to cross-sectional differences in market-to-book ratios even in the absence of any growth options. Our volatility-based measure is not subject to these types of biases and, as we show, is supplementary to the traditional proxies.

In the empirical literature and among practitioners, high valuation ratios are commonly associated with high growth-option intensity. Theoretical asset pricing models, however, have different implications for the sign of this relationship. For example, in the models of Berk et al. (1999), Gomes et al. (2003), and Carlson et al. (2004), market-to-book ratios and growth-option intensity are negatively correlated. Growth options in these models are riskier than assets in places, and to account for the value premium, low market-to-book firms (i.e., value firms) are required to be option intensive. In Kogan and Papanikolaou (2010, 2014), growth options are also riskier than assets in place because of higher exposure to investment-specific shocks. However, because investment-specific risks in their model carry a negative premium, growth-option intensive firms have high valuations. In Ai and Kiku (2013) and Ai et al. (2013), growth options are less risky than assets in place, and market-to-book ratios and growth options are positively related. Given the very different predictions as to what market-to-book proxies for, a nonvaluation-based measure of growth options could help us better understand the economic mechanism of the value premium.

Our measure of growth options is motivated by the insight that option payoffs respond positively to volatility news. We argue, however, that the standard intuition applies only to idiosyncratic volatility and cannot be generalized to volatility of aggregate risks. To gain intuition, consider a real option with a fixed strike price. An increase in volatility raises the option value—this is the traditional (partial equilibrium) effect that pertains to all types of volatility. Aggregate uncertainty, however, has an additional discount rate effect. High aggregate volatility lowers the risk-free rate and, hence, the discount rate applied to the strike asset. In a broad class of economic models, high aggregate uncertainty also raises risk premia and, therefore, the discount rate applied to the underlying cash flow. These general equilibrium implications lower the option payoff, working in the opposite direction to the traditional partial equilibrium effect. The overall response of option payoffs to aggregate volatility risks is, therefore, ambiguous and, as we show, depends on moneyness. For options close to the exercise threshold, the general equilibrium effect dominates and option payoffs respond negatively to aggregate volatility shocks. For deep out-of-the-money options, the partial equilibrium effect prevails and option payoffs increase with aggregate volatility.

Our theoretical analysis suggests that the amount of growth options owned by firms is best measured by price sensitivity to idiosyncratic volatility news. Other volatility-based measures (based on aggregate or total volatility) may be contaminated by the discount rate effect. We, therefore, distinguish between two types of volatility in our empirical work. We measure time variation in idiosyncratic volatility by variation in firm-level volatility that is orthogonal to fluctuations in aggregate uncertainty. Firm-level volatility and aggregate uncertainty are measured by realized variances of equity returns and returns of the aggregate market portfolio, respectively.¹

We first show that in the data, firm exposure to idiosyncratic volatility shocks (denoted by β^{ID}) is largely positive, whereas exposure to aggregate volatility risks (β^A) is mostly negative. That is, equity prices tend to increase on positive news about idiosyncratic volatility and tend to fall when aggregate uncertainty in the economy is high. We then show that, controlling for book-to-market and other firm characteristics, firms that are highly sensitive to variation in idiosyncratic volatility are expected to grow and invest at a high rate as they exercise their growth

options. In contrast, equity response to aggregate volatility does not appear to be informative about cross-sectional differences in growth opportunities.

Specifically, we document a steep monotonically increasing pattern in sales and investment growth rates across portfolios sorted on exposure to idiosyncratic volatility. The average annual growth in sales almost doubles and average investment growth changes from -0.7% to 8.1% from the bottom to the top quintile portfolios. In addition, firms with high β^{ID} are characterized by high R&D spending and high Tobin's Q, low leverage, and low dividend yields, all of which are characteristic of growing firms. For example, the average dividend yield of firms in the top quintile is only 1.5% , whereas it is about 3% for firms in the bottom quintile. Similarly to the value premium, we find that firms with high exposure to idiosyncratic volatility (i.e., option-intensive firms) carry lower premia relative to low- β^{ID} firms.

In a regression setting, we show that exposure to idiosyncratic volatility, by itself, is a significant predictor of future investment. Importantly, it provides additional information about firms' future investment decisions (hence, available growth options) over and beyond the conventional predictors. The effect of idiosyncratic-volatility exposure is both statistically and economically significant after we control for book-to-market, Tobin's Q, past investment, size, and other relevant characteristics. Quantitatively, a one standard deviation increase in β^{ID} results in a significant 4% increase in one-year ahead investment growth. Put differently, firms in the top β^{ID} -quintile invest by 10% more than otherwise identical firms in the bottom portfolio do. We also show that, controlling for book to market and size, idiosyncratic volatility exposure has a negative effect on expected returns. A one standard deviation increase in ID-volatility (idiosyncratic volatility) beta lowers the next year return by about 2.2% , on average.

Exposure to aggregate volatility news is also negatively related to expected returns. Firms with large negative β^A carry a substantially higher premium compared with zero-exposure firms. Thus, consistent with the long-run risk literature, aggregate volatility risks carry a negative price (Bansal and Yaron 2004). The difference in average returns between the bottom and top quintile portfolios ranked by β^A is about 4% per annum. When it comes to growth options, we find no significant evidence that exposure to aggregate volatility helps predict firms' future investment. These findings confirm our theoretical argument. The partial-equilibrium effect (that pushes option prices up) and the discount rate effect (that pushes prices down) work against each other, which makes it hard to learn about available growth options

¹ Time variation in volatility has already been well established. Bollerslev et al. (1992) provide a survey of the early literature and evidence on aggregate volatility, and Andersen et al. (2001), Campbell et al. (2001), and Brandt et al. (2010) discuss time-series dynamics of firm-level volatility.

by looking at equity exposure to aggregate volatility shocks.

Our theoretical analysis is related to a large literature on (real) option pricing. Although a positive effect of idiosyncratic volatility on option payoffs is well established (see Black and Scholes 1973, Merton 1973, McDonald and Siegel 1986, and Dixit and Pindyck 1994), our paper formalizes this result in a stochastic-volatility model and emphasizes the difference between idiosyncratic and aggregate volatility. We model the option exercise problem in a Markovian setup similar to recent models in the capital structure literature, for example, Hackbarth et al. (2006), Chen (2010), and Bhamra et al. (2010).

The effect of aggregate volatility on asset prices has been emphasized in the macrofinance literature. Bansal et al. (2005, 2014) and Boguth and Kuehn (2013) show that aggregate volatility risks carry significant premia in equity markets. Bollerslev et al. (2009) and Drechsler and Yaron (2011) explore implications of aggregate volatility for exchange-traded index options. Bloom (2009) and Bloom et al. (2011) consider equilibrium implications of volatility shocks in frameworks with irreversible investment. Duarte et al. (2012) study the relationship between aggregate volatility shocks and aggregate investment in a general equilibrium model with production.

Our paper is also related to the literature that explores the implications of option exercise and investment decisions for the cross section of asset returns, for example, Berk et al. (1999), Gomes et al. (2003), Carlson et al. (2004), Zhang (2005), Cooper (2006), Novy-Marx (2007), Garlappi and Yan (2011), Papanikolaou (2011), and Bhamra and Shim (2011) among others. Different from these papers, our focus is on providing a measure of growth options that can be used to test theory. Kogan and Papanikolaou (2010, 2014) also propose a theoretically motivated measure of growth opportunities based on return sensitivity to investment-specific shocks that empirically are proxied by returns of a long/short portfolio of investment producers and consumption-good producers. Our papers complement each other and present corroborative evidence that option-intensive firms carry lower premia than do firms with abundant assets in place.

More generally, our paper contributes to the broader literature that studies option valuation and investment decisions. Detemple and Sundaresan (1999) develop a framework for option valuation in the presence of trading restrictions. Henderson (2007) and Hugonnier and Morellec (2007) analyze the implications of market incompleteness on investment and investment timing decisions. Miao and Wang (2007) extend the real-option approach to an environment where agents make joint decisions on consumption,

investment, and portfolio selection, and emphasize the different effects of systematic and undiversifiable idiosyncratic risks on option exercise decisions.

The rest of the paper is organized as follows. Section 2 provides a theoretical analysis of growth-option exposure to volatility news. We consider two economies that feature variation in either idiosyncratic or aggregate volatility and characterize the dynamics of growth options in each of them. In §3 we present evidence that in the data, firms' exposure to variation in idiosyncratic volatility is informative about firms' future investment decisions and growth. We also show that, empirically, firms' exposure to aggregate volatility news does not help identify option-intensive firms. Section 4 provides concluding remarks.

2. Volatility Shocks and Option Returns

In this section, we provide a theoretical analysis of the relationship between option payoffs and volatility shocks. We distinguish between two types of volatility—volatility of aggregate shocks and volatility of idiosyncratic shocks—and characterize the response of growth options to each type of volatility news. We also highlight differences in volatility exposure between growth options and assets in place.

2.1. Setup of the Model

Consider an economy where a representative agent has intertemporal preferences described by the Kreps and Porteus (1978) utility with a constant relative risk aversion parameter, γ , and a constant intertemporal elasticity of substitution (IES), ψ . Time is continuous and infinite. We follow Duffie and Epstein (1992a) and represent preferences as a stochastic differential utility.

We assume that the dynamics of aggregate consumption are described by the following stochastic process:

$$dC_t = C_t[\mu_C dt + \sigma_C(\theta_t) dB_t], \quad (1)$$

where $\{B_t\}_{t \geq 0}$ is a one-dimensional standard Brownian motion, and $\{\theta_t\}_{t \geq 0}$ is a two-state Markov process with state space $\Theta = \{\theta_H, \theta_L\}$, where $\theta_H > \theta_L$. The transition probability of θ_t over an infinitesimal time interval Δ is given by

$$\begin{bmatrix} 1 - \lambda_H \Delta & \lambda_H \Delta \\ \lambda_L \Delta & 1 - \lambda_L \Delta \end{bmatrix}. \quad (2)$$

An asset in place is a project that generates cash flows, D_t , that follow

$$dD_t = D_t[\mu_D dt + \rho\{\sigma_C(\theta_t) dB_t + \sigma_D(\theta_t) dB_t^i\}], \quad (3)$$

where Brownian motion B_t is the aggregate shock that affects consumption and dividends growth simultaneously, and B_t^i is an idiosyncratic shock. The parameter ρ allows the model to account for leverage and differences in volatility of dividend and consumption growth rates. We assume that the project is subject to random termination that arrives at a Poisson rate κ per unit of time.

A growth option is the right to obtain the above project by making an irreversible investment of one unit of consumption goods. That is, the owner of the option has the right to adopt the project at any time at a fixed cost before it is terminated.

We consider two special cases of the above general setup:

Case 1. Economy with idiosyncratic volatility shocks:

$$\sigma_C(\theta) = \sigma, \quad \sigma_D(\theta) = \theta, \quad \text{for } \theta = \theta_H, \theta_L. \quad (4)$$

Case 2. Economy with aggregate volatility shocks:

$$\sigma_C(\theta) = \theta, \quad \sigma_D(\theta) = \sigma, \quad \text{for } \theta = \theta_H, \theta_L. \quad (5)$$

In the economy with idiosyncratic volatility, aggregate volatility is constant and changes in volatility of dividend growth are purely idiosyncratic. In the economy with aggregate volatility shocks, fluctuations in cash-flow volatility are perfectly correlated with changes in volatility of aggregate consumption. The latter setup allows us to capture the general equilibrium effect of variation in volatility of aggregate shocks. We use the two economies to highlight the equilibrium effect of volatility news on option values.

We make the following assumptions on the parameters of the model.

Assumptions: The parameters of the model satisfy

$$\beta + \frac{1}{2}\gamma\left(1 - \frac{1}{\psi}\right)\sigma_C^2(\theta) - \left(1 - \frac{1}{\psi}\right)\mu_C > 0, \quad \text{for } \theta = \theta_H, \theta_L \quad (6)$$

and

$$\kappa + \beta + \frac{1}{\psi}\mu_C - \mu_D + \frac{1}{2}\gamma\left(2\rho - 1 - \frac{1}{\psi}\right)\sigma_C^2(\theta) > 0, \quad \text{for } \theta = \theta_H, \theta_L. \quad (7)$$

As we show in the appendix, condition (6) ensures that the lifetime utility of the agent is finite, and assumption (7) guarantees that the present value of cash flows is finite.

Under the above assumptions, in both economies, the value of assets in place, denoted by $V_A(\theta, D)$, is a linear function of dividends. That is,

$$V_A(\theta, D) = a(\theta)D, \quad (8)$$

where $a(\theta)$ is the price-dividend ratio. As we show in the appendix, in the economy with idiosyncratic volatility shocks, the price-dividend ratio is constant: $a(\theta_H) = a(\theta_L)$. In the economy with time-varying aggregate volatility, the price-dividend ratio depends on the current state, θ , and is provided in the appendix.

We use $V_O(\theta, D)$ to denote the value of a growth option in state θ with current level of dividend D and use $\hat{D}(\theta)$ to denote the optimal option exercise threshold in state θ . The value function $V_O(\theta, D)$ in both economies permits closed-form solutions and is detailed in Proposition 1. For simplicity, here we assume $\hat{D}(\theta_L) < \hat{D}(\theta_H)$.²

PROPOSITION 1. On $[0, \hat{D}(\theta_L)]$, the value of growth options is given by

$$\begin{aligned} V_O(\theta_H, D) &= K_1 D^{\alpha_1} + K_2 D^{\alpha_2}, \\ V_O(\theta_L, D) &= K_1 e_1 D^{\alpha_1} + K_2 e_2 D^{\alpha_2}, \end{aligned} \quad (9)$$

and on $(\hat{D}(\theta_L), \hat{D}(\theta_H)]$,

$$\begin{aligned} V_O(\theta_H, D) &= a_0 D + b_0 + A_1 D^{\eta_1} + A_2 D^{\eta_2}, \\ V_O(\theta_L, D) &= a(\theta_L)D - 1, \end{aligned} \quad (10)$$

where the constants e_1 , e_2 , and α_1 , α_2 , η_1 , η_2 are given in Equations (41), (46), and (47), respectively, in §A.1 of the appendix.

The optimal option exercise rule is given by a pair of option exercise thresholds, $\hat{D}(\theta)$ for $\theta = \theta_H, \theta_L$, such that it is optimal to exercise the growth option in state θ if and only if $D \geq \hat{D}(\theta)$, for $\theta = \theta_H, \theta_L$. The coefficients, K_1 , K_2 , A_1 , and A_2 along with the optimal option exercise threshold are jointly determined by the value-matching and smooth-pasting conditions:

$$\begin{aligned} V_O(\theta, \hat{D}(\theta)) &= a(\theta)\hat{D}(\theta) - 1, \\ \frac{\partial}{\partial D} V_O(\theta, \hat{D}(\theta)) &= a(\theta), \end{aligned} \quad \text{for } \theta = \theta_H, \theta_L, \quad (11)$$

and

$$\begin{aligned} V_O(\theta_H, \hat{D}^+(\theta_H)) &= V_O(\theta_H, \hat{D}^-(\theta_H)), \\ \frac{\partial}{\partial D} V_O(\theta_H, \hat{D}^+(\theta_H)) &= \frac{\partial}{\partial D} V_O(\theta_H, \hat{D}^-(\theta_H)). \end{aligned} \quad (12)$$

PROOF. See §A.1 of the appendix.

² As we show in the next subsection of the paper, the condition $\hat{D}(\theta_L) < \hat{D}(\theta_H)$ is always satisfied in the economy with idiosyncratic shocks. In §2.3, we provide a sufficient condition under which this condition holds in the economy with aggregate shocks. Closed-form solutions can still be obtained whenever the above condition is violated.

2.2. The Effect of Shocks to Idiosyncratic Volatility

We first consider the economy with time-varying idiosyncratic volatility that corresponds to Equation (4). As we argue in Proposition 2, high volatility of idiosyncratic shocks is always associated with a high option value and a delay in option exercise. This result is consistent with the intuition in the standard real-options theory. The real-options literature typically considers a constant volatility setup, offering comparative statics for options with different but constant volatilities. Our model incorporates stochastic volatility and allows us to explore the effect of volatility news on option returns.

PROPOSITION 2. *In the economy with time-varying idiosyncratic volatility, an increase in volatility raises option values and the option exercise threshold, that is,*

$$V_O(\theta_H, D) > V_O(\theta_L, D), \text{ for all } D,$$

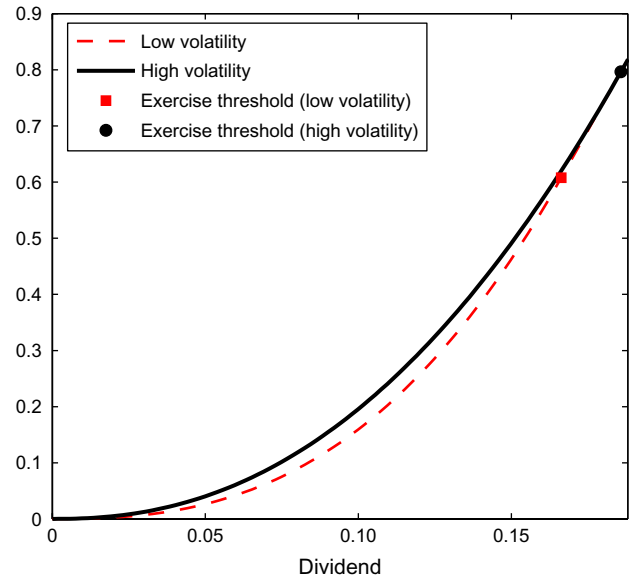
and

$$\hat{D}(\theta_H) > \hat{D}(\theta_L).$$

PROOF. See §A.3 of the appendix.

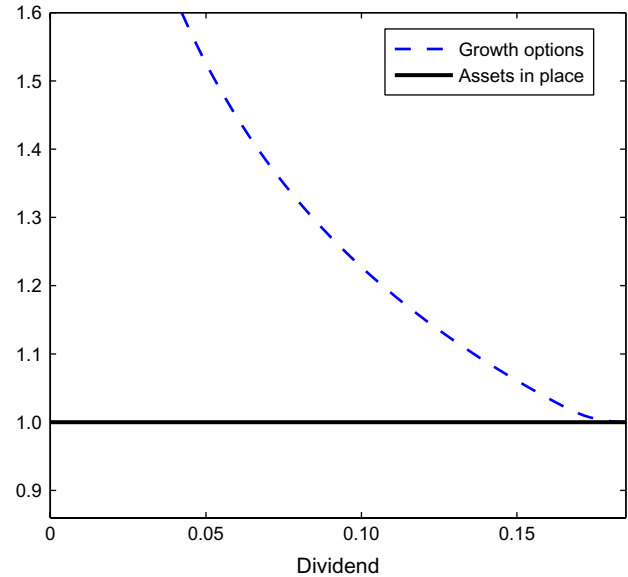
We illustrate the result of the above proposition in Figures 1 and 2. The time-series parameters used in constructing this example are chosen to match the first two moments of annual U.S. consumption and dividend growth rates, and our preference configuration implies preference for early resolution of

Figure 1 (Color online) Value of Growth Options: The Case of Idiosyncratic Volatility



Notes. This figure plots the value of growth options as a function of the current level of dividends in the low idiosyncratic volatility state (dashed line) and that in the high idiosyncratic volatility state (solid line). The figure corresponds to the economy with time-varying idiosyncratic volatility.

Figure 2 (Color online) Exposure to Idiosyncratic Volatility of Assets in Place and Growth Options



Notes. This figure plots the change in the value of assets in place (solid line) and growth options (dashed line) as the economy shifts from the low to the high volatility state. The figure corresponds to the economy with time-varying idiosyncratic volatility.

uncertainty. The full list of parameter values is presented in Table 1. The solid line in Figure 1 shows the value of growth options in the high volatility state (θ_H); the dashed line represents option values in the low volatility state (θ_L). The option exercise threshold in the low and high volatility states is depicted as a square and a circle, respectively. As the figure shows, the value of growth options is always higher in the high volatility state than in the state when idiosyncratic volatility is low. High volatility is also

Table 1 Parameter Configuration of the Example Economies

Preference parameters	
β	0.02
γ	2.0
ψ	2.0
Consumption/cash flow	
μ_C	0.02
λ	0.01
θ_H	0.03
θ_L	0.01
σ	0.02
μ_D	0.05
κ	0.12
ρ	3.00

Note. This table presents the parameter configuration used in numerical examples of the economies with time-varying idiosyncratic volatility and time-varying aggregate volatility.

associated with a high option exercise threshold due to the option value of waiting.

Figure 2 plots exposure of the two types of assets to idiosyncratic volatility risks. The solid line is the ratio of the value of assets in place in two volatility states, $V_A(\theta_H, D)/V_A(\theta_L, D)$; the dashed line is the corresponding ratio of the value of growth options, $V_O(\theta_H, D)/V_O(\theta_L, D)$. Notice that the value of growth options always responds positively to volatility news, whereas the value of assets in place does not. That is, $V_O(\theta_H, D)/V_O(\theta_L, D) > 1$ and $V_A(\theta_H, D)/V_A(\theta_L, D) = 1$. As the figure also shows, sensitivity of option returns to idiosyncratic volatility shocks is higher the further the option is from the option exercise threshold. Thus, asset exposure to variations in idiosyncratic volatility is informative about (i) the type of the asset (option versus asset in place) and (ii) moneyness of growth options.

2.3. The Effect of Shocks to Aggregate Volatility

In contrast to idiosyncratic volatility, in a general equilibrium setting, aggregate volatility risks affect the stochastic discount factor and, therefore, asset prices. Qualitatively, general equilibrium implications of aggregate volatility for option values depend on preference parameters. We assume that preference parameters satisfy $\gamma > 1 > 1/\psi$. As shown in Bansal and Yaron (2004), these preferences ensure a rise in discount rates at times of high aggregate uncertainty and are consistent with the empirical evidence of countercyclical dynamics of risk premia.

Consider the economy with time-varying aggregate volatility described in Equation (5). Let $\hat{D} = \max\{\hat{D}(\theta_H), \hat{D}(\theta_L)\}$ denote the maximum of the option exercise thresholds. The following proposition characterizes the response growth options to aggregate volatility shocks.

PROPOSITION 3. *Consider the economy with time-varying aggregate volatility and assume that $\rho = 1$.³ Suppose that*

$$\kappa + \beta + \frac{1}{\psi}\mu_C - \frac{1}{2}\gamma\left(1 + \frac{1}{\psi}\right)\theta^2 > (1 + 2\gamma)(\mu_D + \gamma\sigma^2),$$

for $\theta = \theta_H, \theta_L$, (13)

then

(1) *there exists a unique $D^* \in (0, \hat{D})$ such that*

$$V_O(\theta_H, D) > V_O(\theta_L, D), \quad \text{for all } D \in (0, D^*),$$

and

$$V_O(\theta_H, D) < V_O(\theta_L, D), \quad \text{for all } D > D^*;$$

³ The assumption of $\rho = 1$ is not critical. We impose it only to simplify the sufficient condition and to facilitate its interpretation. The more general condition that allows for arbitrary values of ρ is given in inequality (57) in §A.4 of the appendix.

(2) *The option exercise thresholds satisfy $\hat{D}(\theta_H) > \hat{D}(\theta_L)$.*

PROOF. See §A.4 of the appendix.

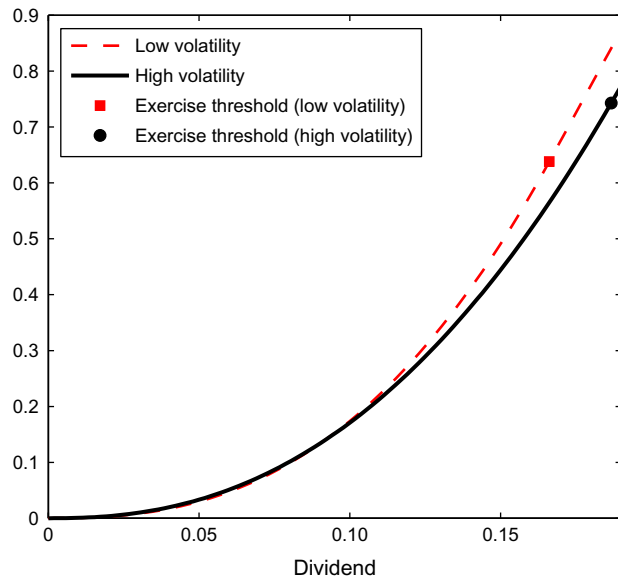
Fluctuations in aggregate volatility have two effects on option values: the standard volatility (or partial equilibrium) effect and the discount rate (or general equilibrium) effect. Whereas the first effect raises options' payoffs, the second one causes growth-option values to decline. Depending on moneyness of growth options, one or the other dominates and determines the sign of option exposure to aggregate volatility risks.

Consider first options that are close to the option exercise threshold. Note that under our assumptions on preferences, prices of asset in place are depressed in states of high aggregate uncertainty: $a(\theta_H) < a(\theta_L)$. Intuitively, high aggregate volatility states are associated with high risk premia and, therefore, low price-to-dividend ratios. Hence, at-the-money growth options also respond negatively to aggregate volatility risks: $a(\theta_H)\hat{D} - 1 = V_O(\theta_H, \hat{D}) < V_O(\theta_L, \hat{D}) = a(\theta_L)\hat{D} - 1$. Moreover, given that options are levered positions on assets in place, their values decline more on positive news about aggregate uncertainty compared with assets in place. By continuity, options that are sufficiently close to the option exercise threshold will also have negative exposure to aggregate volatility. That is, for options that are soon or about to be exercised, discount rate effect dominates and an increase in aggregate volatility leads to a decline in option values.

As options move further away from the option exercise threshold, the partial equilibrium effect becomes more important and, under the condition provided in Proposition 3, eventually dominates the negative discount rate effect. Consider an option that is deep out of the money. An increase in aggregate volatility has a twofold effect. The general equilibrium effect is still present, pushing the option value down. At the same time, an increase in aggregate volatility raises the probability that the option will eventually end up in the money, pushing its value up. Deep out of the money, the partial equilibrium effect outweighs the discount rate effect. As a result, sufficiently out-of-the-money options have positive exposure to aggregate volatility risks. Note that, in contrast to the case of idiosyncratic volatility, the relationship between option values and aggregate volatility is not uniform: option values increase with volatility when they are deep out of the money but decline when they are close to the option exercise threshold.

Depending on parameter values, the general equilibrium effect may or may not dominate over the entire domain of option moneyness. Proposition 3 provides a sufficient condition for the discount

Figure 3 (Color online) Value of Growth Options: The Case of Aggregate Volatility

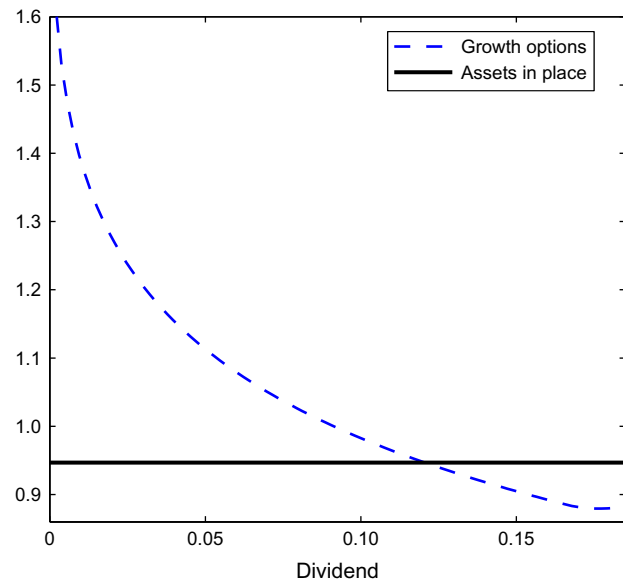


Notes. This figure plots the value of growth options as a function of the current level of dividends in the low aggregate volatility state (dashed line) and that in the high aggregate volatility state (solid line). The figure corresponds to the economy with time-varying aggregate volatility.

rate effect to get ultimately overrun by the standard volatility effect.⁴ Intuitively, inequality in Equation (13) requires effective depreciation to be high enough relative to the risk premium. The left-hand side of Equation (13) is the sum of the depreciation rate of assets in place and the risk-free interest rate.⁵ Note that the general equilibrium channel affects the value of growth options through their terminal (on exercise) payoff. Higher risk aversion enhances the general equilibrium effect through an increase in risk premia. But, if the interest rate or depreciation rate are relatively high, then the general equilibrium effect dies off fast enough and the partial equilibrium effect becomes dominant for sufficiently out-of-the-money options. The proposition also shows that, provided Equation (13) holds, the option exercise threshold is governed by the partial equilibrium effect of volatility and it is optimal to delay option exercise in the high volatility state.

We illustrate Proposition 3 in Figures 3 and 4 using the same set of parameter values as in Table 1. Figure 3 plots the value of growth options in the high aggregate volatility state (solid line) and that in the low aggregate volatility state (dashed line). In the

Figure 4 (Color online) Exposure to Aggregate Volatility of Assets in Place and Growth Options



Notes. This figure plots the change in the value of assets in place (solid line) and growth options (dashed line) as the economy shifts from the low to the high volatility state. The figure corresponds to the economy with time-varying aggregate volatility.

region close to the option exercise threshold, the general equilibrium effect dominates and option values are higher when aggregate volatility is low. In contrast, for deep out-of-the-money options, aggregate volatility has a positive effect on option values. In this region, the general equilibrium effect is dominated by the conventional volatility channel because of a higher likelihood that options end up in the money before they disappear.

Figure 4 presents exposure of assets in place and growth options to aggregate volatility risks. Notice that $V_A(\theta_H, D)/V_A(\theta_L, D) = a(\theta_H)/a(\theta_L) < 1$, i.e., the value of assets in place declines on positive volatility news because of an increase in risk premia. The response of growth options, $V_O(\theta_H, D)/V_O(\theta_L, D)$, can be higher or lower than the response of assets in place. When options are considerably out of the money, their values increase in volatility. These options are less risky than assets in place and, in fact, deep out-of-the-money options provide insurance against aggregate volatility shocks. Options that are close to the option exercise threshold are more sensitive to discount rate risks and, therefore, respond more negatively to volatility innovations than do assets in place.

There are three important implications of our theoretical analysis. First, the value of growth options always increases with idiosyncratic volatility. Hence, price exposure to idiosyncratic volatility news should certainly be informative about differences in growth opportunities across firms. Second, the response of

⁴ As we show in §A.4 of the appendix, the conclusion of the proposition holds under a more general condition, provided in Equation (57). We find it to be satisfied for a wide range of plausible parameter values.

⁵ To be precise, $\beta + (1/\psi)\mu_C - \frac{1}{2}\gamma(1+1/\psi)\theta^2$ is the risk-free interest in the economy with constant aggregate volatility θ .

growth options to volatility of aggregate shocks is ambiguous: it can be either positive or negative depending on option moneyness. Third, growth options may have either higher or lower exposure to aggregate volatility news compared with assets in place. The latter two implications suggest that exposure to aggregate volatility is unlikely to reveal differences in option intensity across firms. To summarize, from the theoretical point of view, the amount of growth options owned by firms is best measured by price sensitivity to variation in idiosyncratic volatility. Measures based on aggregate (hence, total) volatility may be contaminated by the discount rate effect and may be powerless to distinguish between growth and asset-in-place intensive firms.

3. Empirical Evidence

Motivated by our theoretical results, we propose to measure growth options by firms' sensitivity to idiosyncratic volatility news, and in this section, we explore its ability to identify option-intensive firms. In particular, we examine if equity exposure to variation in idiosyncratic volatility contains information about firms' future investment, growth, and expected returns. We also test the predictive content of exposure to aggregate volatility ("aggregate volatility beta") for future investment decisions of firms.

3.1. Firm Data

All the cross-sectional data come from Compustat and the Center for Research in Securities Prices. We focus on nonfinancial firms whose common shares are traded on NYSE, AMEX, and Nasdaq. We collect return and price series, the number of outstanding shares, capital expenditure (to measure investment), property, plant and equipment (to measure the amount of capital), expenditure on research and development (R&D), book value of assets, sales, and cash holdings. For each firm in the sample, we compute its book-to-market ratio as in Fama and French (1993), its financial leverage as a ratio of short- and long-term debt to the sum of debt and market value of equity, and Tobin's Q as a ratio of the sum of market capitalization, book value of preferred equity and long-term debt less inventories and deferred taxes to the sum of book value of common and preferred equity and long-term debt. In addition, for every firm we construct a measure of operating leverage using "market operating leverage" of Novy-Marx (2011), defined as a ratio of operating costs to market value of assets. Operating costs consist of costs of goods sold and selling, general, and administrative expenses; the market value of assets is computed as book value of assets plus market capitalization minus book value of equity. We use data sampled on daily, monthly, and annual frequency. Monthly and annual data are

converted to real using the consumer price index from the Bureau of Labor Statistics. The overall coverage of the data is from 1964 to 2012.

3.2. Aggregate and Idiosyncratic Volatility Measures

We measure aggregate and firm-level volatility by realized variance of equity returns.⁶ Monthly series of aggregate variance are constructed by summing up squared daily returns of the aggregate market portfolio. Since our focus is on volatility news and their effect on prices, we work with innovations rather than levels. Aggregate volatility news is extracted by applying an AR(1) filter to the logarithm of the market variance.⁷ We choose to measure aggregate volatility using market equity rather than consumption data since the latter are not available at high frequencies.

Idiosyncratic volatility is constructed in two steps. First, we estimate firm-level volatility as in Campbell et al. (2001) and Brandt et al. (2010) using industry-adjusted daily returns. In particular, for firm i that belongs to industry J , its variance in month t is measured as

$$V_{i,t}^{FL} = \sum_{d \in t} (R_{i,d} - \bar{R}_{J,d})^2, \quad (14)$$

where $R_{i,d}$ and $\bar{R}_{J,d}$ are daily returns of firm i and industry J , respectively.⁸ We find that the average correlation between firm-level and aggregate volatility is fairly high, of about 28%. Thus, the industry adjustment alone is not sufficient to remove all systematic variation. To further isolate purely idiosyncratic movements in volatility, we orthogonalize firm-level variance with respect to aggregate variation. Specifically, innovation in idiosyncratic volatility of firm i in month t (denoted by $\sigma_{i,t}^{ID}$) is measured by the residual in the following regression:

$$v_{i,t}^{FL} = k_{i,0} + k_{i,1}v_t^M + k_{i,2}v_{i,t-1}^{FL} + \sigma_{i,t}^{ID}, \quad (15)$$

where $v_{i,t}^{FL} \equiv \log(V_{i,t}^{FL})$, and v_t^M is the logarithm of the aggregate market variance. An autoregressive term is included to filter out any remaining persistence. Note that our estimation procedure aims to identify time variation in firm-specific volatility without taking a strong stand on the asset pricing model that governs equity returns.

3.3. Exposure to Idiosyncratic Volatility

To explore the predictive ability of ID-volatility betas for future investment, we sort firms on their exposure

⁶ We use the terms "volatility" and "variance" interchangeably.

⁷ Strong time dependence in volatility series has been well recognized in the literature (e.g., Bollerslev et al. 1992, Andersen et al. 2001).

⁸ We use the 30-industry classification available at Kenneth French's data library. Allowing for nonunit industry betas does not affect our empirical evidence. We provide a more detailed discussion of the robustness of our findings below.

Table 2 Sorting on Exposure to Idiosyncratic Volatility

Panel A: Portfolio characteristics at formation			
Portfolio	β^{ID}	Tobin's Q	$R\&D/Assets$
Low	−0.030 (0.004)	1.11 [1.11, 1.13]	3.62 [3.17, 3.43]
2	0.004 (0.004)	1.10 [1.11, 1.13]	2.56 [3.16, 3.43]
3	0.028 (0.004)	1.08 [1.11, 1.13]	2.55 [3.16, 3.43]
4	0.054 (0.004)	1.10 [1.11, 1.13]	2.96 [3.16, 3.43]
High	0.102 (0.005)	1.32 [1.11, 1.13]	5.47 [3.16, 3.43]
High – Low	0.132 (0.006)	0.21 [−0.02, 0.02]	1.86 [−0.21, 0.20]

Panel B: Portfolio characteristics over the holding period			
Portfolio	Return	$\Delta Sales$	ΔI
Low	7.48 [5.27, 7.66]	2.57 [2.92, 4.19]	−0.65 [0.87, 3.56]
2	6.04 [5.28, 7.67]	3.59 [2.92, 4.19]	0.89 [0.88, 3.57]
3	7.63 [5.28, 7.67]	3.93 [2.92, 4.20]	4.35 [0.93, 3.59]
4	7.13 [5.28, 7.66]	4.22 [2.92, 4.19]	5.06 [0.88, 3.57]
High	5.29 [5.27, 7.64]	4.67 [2.91, 4.18]	8.05 [0.87, 3.60]
High – Low	−2.18 [−1.88, 1.88]	2.10 [−1.00, 1.00]	8.71 [−2.14, 2.14]

Notes. This table presents characteristics of portfolios sorted on exposure to idiosyncratic volatility. In panel A, idiosyncratic volatility beta (β^{ID}), Tobin's Q , and the ratio of R&D expenditure to lagged assets ($R\&D/Assets$) are computed at the time when portfolios are formed. In panel B, average return (*Return*), sales growth ($\Delta Sales$), and investment growth (ΔI) are computed over the one-year holding period. The standard errors of portfolio β^{ID} 's based on the Newey and West (1987) estimator with four lags are reported in parentheses. Numbers in brackets represent 95% confidence intervals of the corresponding moments under the null that β^{ID} 's contain no information about firms' real assets. Portfolios are value weighted and rebalanced annually; the entries correspond to time-series averages over the 1967–2012 sample period; $R\&D/Assets$, *Return*, $\Delta Sales$, and ΔI are expressed in percentages.

to idiosyncratic volatility and compare growth-related characteristics of the resulting portfolios. We measure idiosyncratic volatility exposure, which we denote by β^{ID} , using three-year rolling window regressions. In particular, at the end of a given year, we regress the logarithm of firm returns on innovations in its idiosyncratic volatility using monthly data over the previous three years. We then sort firms on their ID-volatility exposure into five value-weighted portfolios and hold them for one year. The next December, we reestimate volatility betas by rolling the estimation window one year forward, and repeat the sorting procedure. Table 2 provides a description of the sorted portfolios. We show two sets of statistics: in panel A we report characteristics at the time when portfolios are formed and in panel B we present portfolio characteristics over the holding period.

Consistent with the theoretical prediction, we find that equity returns have mostly positive exposure to news in idiosyncratic volatility—with the exception of the bottom quintile, the estimated betas are positive.⁹

⁹ The estimates reported in Table 2 correspond to the time-series averages of value-weighted portfolio β^{ID} 's. The time-series averages of the cross-sectional medians are very similar. Note that the estimate of β^{ID} could potentially be downward biased because of the leverage effect discussed in Black (1976). This is likely to be the case because $\hat{\beta}^{ID}$'s tend to decline with both financial and operating leverage as we show in §3.4. Hence, the true exposure to idiosyncratic volatility is likely to be higher (i.e., more positive) relative to the reported estimates.

That is, equity prices tend to increase in response to a rise in idiosyncratic volatility. As we argued earlier, a positive response is likely to be driven by growth options and its magnitude depends on the amount and moneyness of options available to firms. The cross-sectional heterogeneity in β^{ID} 's is substantial, in particular, the difference in average betas of the top and bottom quintile portfolios is around 0.13 with a t -statistic of 21.7. Table 3 further shows that firms' exposure to idiosyncratic volatility is fairly persistent. The average probability of a firm remaining in the same portfolio from one year to the next is around 45% and the probability of staying in the same bin or transition to the nearest portfolio is 78%, on average.

Table 3 Transition Probabilities Across β^{ID} -Sorted Portfolios

	To				
	Low	2	3	4	High
From					
Low	0.56	0.25	0.11	0.06	0.03
2	0.24	0.37	0.24	0.11	0.04
3	0.11	0.25	0.34	0.23	0.08
4	0.06	0.12	0.24	0.37	0.22
High	0.03	0.04	0.09	0.23	0.61

Notes. This table reports frequencies of transition across quintile portfolios sorted on exposure to idiosyncratic volatility (β^{ID}). Transition probabilities are measured by the fraction of firms that migrate from one bin in year t to another bin in year $t + 1$. The entries correspond to time-series averages over the 1967–2012 sample period.

We find that firms with high exposure to idiosyncratic volatility are characterized by high Tobin's Q , high cash holdings, low amount of capital, and low financial leverage. High- β^{ID} firms account for only 4% of the total capital, whereas low- β^{ID} firms contribute a much larger share, of about 31%, to the total capital stock. Firms in the top portfolio, on average, have a 12% ratio of cash to assets and a 20% ratio of debt to firm value. The corresponding statistics of firms in the bottom portfolio are 7% and 25%, respectively. In addition, firms with high exposure to idiosyncratic volatility tend to spend more on research and development compared with the others. The ratio of R&D expense to lagged assets of the top quintile portfolio is 5.5%, which is about 2% larger than that of the rest of the market.

As panel B shows, forward-looking characteristics also vary substantially across portfolios sorted on idiosyncratic volatility exposure. Firms with high β^{ID} feature higher growth in sales and investment in the year following the portfolio formation compared with low-exposure firms. As β^{ID} increases, the average growth in sales increases monotonically from 2.6% to 4.7%. The difference in investment growth rates is more striking: the average growth of investment changes from -0.7% to 8.1% from the bottom to the top quintile portfolios.

To confirm that the observed cross-sectional differences in investment rates and other growth characteristics are significant and not simply due to a lucky draw, we run a Monte Carlo simulation under the null that β^{ID} 's contain no information about firms' real assets. In particular, for every sample year t , we draw β^{ID} 's from their time- t cross-sectional distribution and match them randomly with time- t firms. We then construct five portfolios by sorting firms on the randomly assigned exposure, repeat simulations 10,000 times, and construct confidence intervals of portfolio characteristics under the null. The 2.5- and 97.5-percentiles of Monte Carlo distributions are reported in Table 2 in brackets next to the corresponding sample moments. Our simulations show that under the null, all portfolios feature almost identical distributions of average investment and sales growth rates. That is, if idiosyncratic volatility exposure were not informative about firms' real assets, it would be extremely unlikely to detect differences in average growth rates across β^{ID} -sorted portfolios of the magnitude that we observe in the data. For example, under the null, with 95% confidence, the difference in average investment growth rates of the top and bottom portfolios would lie in between -2.14% and 2.14% , whereas in the data the spread in mean growth rates is equal to 8.71% . Likewise, the observed difference in average sales growth rates between low- and high-exposure portfolios is strongly significant.

Firms with higher exposure to idiosyncratic volatility tend to be more volatile relative to firms with low β^{ID} 's, although the relationship between exposure to idiosyncratic volatility and total variation is not monotone. The standard deviation of portfolio returns changes from 19% for the bottom quintile to 16% for the middle quintile and to 27% for the top quintile. We find a similar J -shaped pattern in the level of idiosyncratic volatility across β^{ID} -sorted portfolios. That is, firms with high exposure to idiosyncratic volatility that are characterized by high future investment and growth tend to feature high idiosyncratic variation of equity returns. This evidence is consistent with Cao et al. (2008) and Kogan and Papanikolaou (2013), who establish a positive relationship between the level of idiosyncratic risk and firms' growth opportunities. We will further examine the link between the level of firm-specific variation, price exposure to idiosyncratic volatility, and future investment in §3.7.

We also find that high- β^{ID} firms are characterized by high prices, low dividends, and low expected returns. The sample mean of the price-dividend ratio of the top quintile portfolio is almost 159, whereas it is only 42 for the bottom quintile. As idiosyncratic volatility exposure goes up, the average return declines from about 7.5% to 5.3%. This evidence should not be interpreted as a puzzle. Firm-specific volatility is purely idiosyncratic. Hence, exposure to innovations in idiosyncratic volatility should not be priced. However, if, as we argue, ID-volatility exposure provides a signal about relative composition of firms' assets, and growth options and assets in place have different risk characteristics, then sorting on idiosyncratic volatility beta would reveal differences in systematic risks and risk premia of growth and value assets. Our findings suggest that growth-option intensive firms carry smaller premia compared with asset-in-place intensive firms, which is consistent with theoretical predictions in Ai and Kiku (2013), Kogan and Papanikolaou (2014), and Ai et al. (2013, 2014).

The observed cross-sectional differences in risk premia are hard to reconcile within the capital asset pricing model (CAPM). In the data, contrary to the CAPM prediction, it is the portfolio that earns the lowest premium that features the highest covariation with the market. The top quintile portfolio that on average earns only 5.3% has the CAPM beta of 1.35. For comparison, the bottom portfolio that yields 7.5% average rate of return has a significantly lower market beta of 0.99, and the CAPM alpha of the high-minus-low strategy is -4.1% per annum. The failure of the CAPM to explain the cross-sectional risk-return trade-off in this context is similar to its mispricing of book-to-market sorted portfolios. Growth-intensive firms in the data, identified by either low book-to-market ratio

or high ID-volatility beta, have high CAPM betas but low premia, whereas asset-in-place intensive firms, those with high book-to-market or low exposure to ID-volatility, have low market betas but carry high premia.

3.4. Controlling for Book-to-Market and Leverage

The empirical evidence presented in Table 2 shows that, unconditionally, idiosyncratic volatility exposure is able to predict firms' future investment and growth. The one-dimensional sort, however, cannot reveal if idiosyncratic volatility betas provide additional information beyond what we can learn from the conventional, book-to-market-based classification of growth and value. To address this issue, we consider a double sort on book to market and β^{ID} . We first sort all firms into three book-to-market portfolios, then we split each BM (book-to-market ratio) bin into three portfolios with low, medium, and high ID-volatility beta. Portfolios are value weighted and rebalanced on an annual frequency.¹⁰ Table 4 presents some of the key characteristics of the resulting portfolios. To conserve space, we report statistics only for corner portfolios, those at the intersection of low and high book-to-market and low and high ID-volatility beta.

The evidence in Table 4 confirms the well-known ability of book-to-market ratio to identify option-intensive firms. Portfolios with a low ratio of book to market feature significantly higher R&D expenditure and invest at a much higher rate relative to firms with high book-to-market characteristic. In fact, whereas firms in the low book-to-market portfolio have strongly positive investment growth rates, high-BM firms undergo a decline in investment, on average.

Importantly, we find that after controlling for book-to-market characteristic, exposure to idiosyncratic volatility helps further separate out high and low expected growth firms, especially across firms with low BM ratio. The average sales growth of low-BM and low- β^{ID} firms is about 6.05%. Keeping BM ratio fixed, the growth rate increases to 11.1% for firms with high idiosyncratic volatility exposure. The increase in investment growth is more pronounced. The average investment growth of firms with low book-to-market ratio more than triples from 4.9% to 16.2% as firms' exposure to idiosyncratic volatility changes from low to high. The increase in both average sales and investment growth rates is strongly statistically significant.

Note that firms with low book-to-market ratios are typically firms with relatively large market capitalization. Hence, the evidence of a significant β^{ID} -effect

Table 4 Double Sort on BM and Exposure to Idiosyncratic Volatility

Panel A: Portfolio characteristics at formation						
Portfolio	BM		β^{ID}		R&D/Assets	
	Low BM	High BM	Low BM	High BM	Low BM	High BM
Low β^{ID}	0.36	1.43	−0.016	−0.019	5.3	1.8
High β^{ID}	0.34	1.41	0.084	0.069	7.8	1.9
Panel B: Portfolio characteristics over the holding period						
Portfolio	Return		$\Delta Sales$		ΔI	
	Low BM	High BM	Low BM	High BM	Low BM	High BM
Low β^{ID}	6.24	10.73	6.05	−0.77	4.91	−5.57
High β^{ID}	3.76	10.86	11.13	0.46	16.15	−0.82

Notes. This table shows characteristics of portfolios sorted on BM and exposure to idiosyncratic volatility (β^{ID}). Portfolios are constructed by first sorting firms into three book-to-market portfolios, and then dividing each BM bin into three idiosyncratic volatility beta portfolios. The table presents four portfolios with opposite characteristics: low and high BM, and low and high β^{ID} . In panel A, book-to-market ratio (BM), β^{ID} , and R&D to lagged assets (R&D/Assets) are computed at the time when portfolios are formed. In panel B, average return (Return), sales growth ($\Delta Sales$), and growth in investment (ΔI) are computed over the one-year holding period. Portfolios are value weighted and rebalanced annually; the entries correspond to time-series averages over the 1967–2012 period; R&D/Assets, return, $\Delta Sales$, and ΔI are expressed in percentages.

among firms with low BM ratios suggests that it is not driven by very small firms. Indeed, controlling for size, we find that the effect is especially strong for medium and large firms. In particular, across medium-sized firms, those with low ID-volatility exposure have, on average, a 6.3% decline in investment, whereas those with high exposure to idiosyncratic volatility tend to increase investment by 6%.

In Table 5 we examine the interaction between exposure to idiosyncratic volatility and leverage. Sample characteristics of portfolios double sorted on either financial or operating leverage and β^{ID} 's are presented in panels A and B, respectively. First, notice that the cross-sectional dispersion in β^{ID} 's varies little with leverage. For example, for firms with low degree of financial leverage, the spread in β^{ID} 's for the top and bottom portfolios is 0.10, and among firms with high financial leverage, the corresponding difference is a similar 0.09 (both spreads are highly significant). Thus, the cross-sectional variation in β^{ID} 's is not simply driven by differences in firms' leverage.

Second, similar to book to market, firm leverage is strongly negatively correlated with future investment and growth—firms with low degree of financial or operating leverage invest more and grow faster compared with highly levered firms. Yet, leverage does not account for all cross-sectional variation in growth rates. A substantial fraction of variation in future investment growth is explained by the cross-sectional

¹⁰ Empirical evidence based on an independent two-dimensional sort is very similar to the one presented here.

Table 5 Double Sort on Leverage and Exposure to Idiosyncratic Volatility

Panel A: Financial leverage						
Portfolio	Financial leverage		β^{ID}		ΔI	
	Low FL	High FL	Low FL	High FL	Low FL	High FL
Low β^{ID}	0.04	0.54	−0.018	−0.019	7.94	−4.86
High β^{ID}	0.04	0.53	0.084	0.074	20.12	−0.51

Panel B: Operating leverage						
Portfolio	Operating leverage		β^{ID}		ΔI	
	Low OL	High OL	Low OL	High OL	Low OL	High OL
Low β^{ID}	0.38	1.70	−0.017	−0.018	2.57	−2.14
High β^{ID}	0.35	1.68	0.080	0.078	8.22	4.72

Notes. This table shows characteristics of portfolios sorted on leverage and exposure to idiosyncratic volatility (β^{ID}). Panels A and B correspond to financial leverage (FL) and operating leverage (OL), respectively. Portfolios are constructed by first sorting firms into three leverage portfolios, and then dividing each bin into three idiosyncratic volatility beta portfolios. The table presents four portfolios with opposite characteristics: low and high leverage, and low and high β^{ID} . Financial/operating leverage and β^{ID} are computed at the time when portfolios are formed, investment growth (ΔI) is computed over the one-year holding period. Portfolios are value weighted and rebalanced annually; the entries correspond to time-series averages over the 1967–2012 period; ΔI is expressed in percentages.

differences in firms' exposure to idiosyncratic volatility. For example, as β^{ID} changes from low to high, average investment growth rates increase from 7.9% to 20.1%, from 1.1% to 9.1%, and from −4.9% to −0.5% for low, median, and high financial leverage firms, respectively. All the differences are statistically significant, with t -statistics varying between 3.1 and 5.9. Similarly, average sales growth rates are also significantly higher for firms with high exposure compared with low- β^{ID} firms.

3.5. Idiosyncratic Volatility Exposure and Future Investment

To formally quantify the extent to which idiosyncratic volatility exposure is able to account for unobservable growth opportunities, we test its ability to forecast firms' future investment. We consider two specifications. In the first specification, we use ID-volatility betas to run the following predictive regression:

$$\log \frac{\bar{I}_{i,t+k}}{I_{i,t}} = \phi_0 + \phi \beta_{i,t}^{ID} + \varphi X_{i,t} + u_{i,t+1}, \quad (16)$$

where the left-hand side variable is the logarithm of cumulative (annualized) investment growth of firm i (i.e., $\bar{I}_{i,t+k} \equiv (1/k) \sum_{j=1}^k I_{i,t+j}$), $\beta_{i,t}^{ID}$ is firm- i exposure to idiosyncratic volatility at time t , and $X_{i,t}$ is a vector of controls. Our focus here is on the magnitude and significance of the slope coefficient ϕ .

In our second specification, instead of using volatility exposure directly, we use dummy variables that represent the location of each firm within ID-volatility beta-sorted portfolios. That is, we estimate

$$\log \frac{\bar{I}_{i,t+k}}{I_{i,t}} = \phi_0 + \sum_{p=2}^5 \phi_j D_{i,t}^{(p)} + \varphi X_{i,t} + u_{i,t+1}, \quad (17)$$

where $D_{i,t}^{(p)}$ is a dummy variable that equals one if firm i belongs to portfolio p at time t , and all other variables are defined as in Equation (16). One potential advantage of the second specification is that it might help reduce firm-specific noise coming from the estimated betas.

We consider several variations of each specification: with and without firm fixed effects (FE), and with and without controls. In regression specifications with control variables, we include firm characteristics that are known to predict future investment. We use ratios of sales to assets, cash to capital, book to market, and investment to capital, as well as firm market share, return, and Tobin's Q . Predictability of the one-year ahead investment growth (i.e., $k = 1$) is presented in Table 6. The four columns ("Model I"–"Model IV") correspond to different regression specifications. Panel A reports the estimate of ϕ in Equation (16), panel B shows the estimates of ϕ_j in specification (17), and t -statistics are reported in parentheses. To ensure robustness of our inference to both cross-sectional dependence in errors and residual correlation across time, we cluster standard errors by firm and time.

Table 6 Idiosyncratic Volatility Exposure and Future Investment Growth

	Model I	Model II	Model III	Model IV
Panel A: Using β^{ID}				
ϕ	0.87 (4.52)	1.49 (7.59)	0.55 (4.90)	0.73 (6.58)
Panel B: Using portfolio position				
ϕ_2	0.06 (6.85)	0.06 (6.92)	0.02 (3.57)	0.02 (3.58)
ϕ_3	0.09 (10.69)	0.10 (11.21)	0.04 (5.36)	0.05 (5.76)
ϕ_4	0.11 (9.75)	0.14 (13.29)	0.06 (6.80)	0.06 (6.69)
ϕ_5	0.14 (7.71)	0.22 (16.44)	0.09 (7.23)	0.10 (8.20)
Firm FE	No	Yes	No	Yes
Controls	No	No	Yes	Yes

Notes. This table documents predictability of firms' investment growth rate, $\log(I_{t+1}/I_t)$, by their exposure to idiosyncratic volatility. In panel A, ϕ is the regression coefficient on the idiosyncratic volatility beta (β^{ID}); in panel B, ϕ_j is the slope coefficient on a dummy variable that equals one if a firm belongs to the j -quintile portfolio of firms sorted on β^{ID} . The four columns, "Model I"–"Model IV," correspond to different regression specifications with and without firm fixed effects, and with and without controls. The set of controls comprises the ratios of sales to assets, cash to capital, book to market, investment to capital, Tobin's Q , market share, and firm return. Numbers in parentheses are t -statistics based on standard errors clustered by firm and time. The data span the 1967–2012 period.

The first two columns show that exposure to idiosyncratic volatility, by itself, is a strongly significant predictor of future investment growth. Firms with higher ID-volatility betas invest at a much higher rate than do firms that are less sensitive to idiosyncratic volatility news. This evidence is consistent across the two specifications in Equations (16) and (17) and is robust to the inclusion of firm fixed effects. Controlling for firm characteristics makes the magnitude of the β^{ID} -effect decline, and yet it remains strongly significant. The estimates of ϕ in the last two columns are all positive, and the estimates on portfolio dummies, $\hat{\phi}_j$, are monotonically increasing across quintiles. Quantitatively, controlling for firm characteristics and fixed effects, a one standard deviation increase in idiosyncratic volatility exposure results in a significant 4% increase in investment growth. Put differently, firms in the top β^{ID} -sorted portfolio invest by 10% more than do otherwise identical firms in the bottom portfolio. This evidence reveals that firms' exposure to idiosyncratic volatility contains significant independent information about firms' future investment decisions (hence, available growth options) over and beyond what is already captured by such powerful predictors as cash balances, book-to-market ratio, past investment, and returns.¹¹

We should add that similar to the evidence based on double-sorted portfolios discussed above, in a regression setting, β^{ID} and the conventional measures of growth options seem to complement rather than compete with each other in predicting investment growth. For example, in our most comprehensive specification (Model IV reported in panel A of Table 6), all three proxies, β^{ID} , Tobin's Q, and book to market, are significant predictors of firms' future investment with t -statistics of 6.58, 2.88, and -2.94 , respectively. Earlier, Grullon et al. (2012) found that sensitivity of stock returns to total firm volatility is correlated with conventional measures of growth options. Unlike them, we discriminate between idiosyncratic and aggregate components of volatility, and show that exposure to idiosyncratic volatility is not simply related to the traditional proxies but provides independent information about firms' future investment. We also highlight the importance of separating idiosyncratic and aggregate

Table 7 Idiosyncratic Volatility Exposure, Future Investment, and Returns

	ΔI		I/K		Return	
	Estimate	t -stat.	Estimate	t -stat.	Estimate	t -stat.
One year	0.73	6.58	0.61	5.76	−0.41	−1.89
Two year	0.62	5.19	0.51	4.33	−0.21	−1.80
Three year	0.61	5.74	0.48	4.55	−0.12	−2.14

Notes. This table documents predictability of the (annualized) cumulative one-, two-, and three-year ahead investment growth (ΔI), investment rate (I/K), and return (return) by firms' exposure to idiosyncratic volatility. Regression specifications include firm fixed effects and control for firm characteristics. The return regression controls for firms' sales-to-assets and book-to-market ratios, investment rate, and market capitalization. In investment regressions, the set of controls is augmented by the ratio of cash to capital, Tobin's Q, and firm return. The t -statistics are based on standard errors clustered by firm and time. The data span the 1967–2012 period.

volatility movements for both understanding price reaction to different volatility news and for accurate measurement of growth options.

As discussed earlier, price sensitivity to idiosyncratic volatility captures not only the relative amount of growth options but also their moneyness. Therefore, we expect the effect of ID-volatility exposure on future investment to be persistent because options that are sufficiently out of the money may not be able to reach the option exercise threshold that soon and may spur investment only several years later. We test this hypothesis in Table 7. It shows the response of one-, two- and three-year ahead cumulative investment growth and investment rate to ID-volatility betas, controlling for firm fixed effects and firm characteristics. Investment-to-capital ratio of firm i for horizon k is defined as $\log(\bar{I}_{i,t+k}/K_{i,t})$, where $\bar{I}_{i,t+k}$ is the cumulative (annualized) investment and $K_{i,t}$ is the book value of firm capital at time t . Consistent with our hypothesis, we find that firms with higher sensitivity to idiosyncratic volatility feature higher rates of investment relative to firms with low exposure even three years out. The magnitude of the estimated slope tends to decline with the horizon but it remains statistically significant. For example, for investment rate, the estimated coefficients are 0.61, 0.51, and 0.48 for horizons of one-, two- and three-years, respectively, with the corresponding t -statistics of 5.76, 4.33, and 4.55.

The last set of columns in Table 7 characterizes the effect of idiosyncratic volatility exposure on future firm returns. The evidence is based on the regression specification (16) where the left-hand side variable is replaced by the cumulative (annualized) return. We narrow a set of controls to firm characteristics that are known to capture variation in expected returns and remain statistically significant. In particular, we control for sales-to-assets and book-to-market ratios,

¹¹ Consistent with a large body of literature that followed Fazzari et al. (1988), we find that investment is highly sensitive to cash flows even after controlling for investment opportunities using Tobin's Q, book-to-market, and β^{ID} . In a frictionless environment, a significant cash-flow effect may arise because of measurement errors or identification issues (e.g., Erickson and Whited 2000, Gomes 2001, Altı 2003). The existence of financial constraints may amplify the effect of cash flows on investment and, like some argue, accounts for the observed investment-cash-flow sensitivity (e.g., Moyen 2004, Almeida and Campello 2007).

investment rate, and market capitalization, and we include firm fixed effects. We find that idiosyncratic volatility exposure has a negative effect on expected returns. Quantitatively, a one standard deviation increase in ID-volatility beta lowers next-year return by about 2.2%. The effect is significant at the 10% level at the one- and two-year horizons, and at the 5% level at the three-year horizon. This evidence is consistent with the well-known value premium. In the data, growth firms identified by low book-to-market ratio on average have lower rates of returns compared to value or high book-to-market firms. We identify growth firms by their high exposure to idiosyncratic volatility and also find that they carry low expected returns even after controlling for book-to-market and size.

3.6. Robustness

To confirm our findings we conduct a series of robustness checks. First, we construct idiosyncratic volatility by adjusting returns using the Fama and French (1993) three-factor model instead of industry portfolios. Second, we use our benchmark volatility series but vary the length of the estimation window over which ID-volatility exposure is measured. We consider two alternatives: two- and five-year windows that are not too short or too long. Table 8 presents average growth rates in sales and investment of quintile portfolios ranked on alternative ID-volatility betas. For brevity, we report only two characteristics, because other cross-sectional patterns are both qualitatively and quantitatively consistent with our benchmark findings. As the table shows, our empirical evidence is robust—across the alternative measures, exposure to idiosyncratic volatility is highly informative about future investment and growth. Sorting on alternative ID-volatility betas results in a monotonic increase in future sales growth and generates a sizable spread in average investment growth rates between high- and low-exposure firms. For example, with the Fama–French adjustment, the difference in investment growth between the top and the bottom portfolios exceeds 10%. In regression settings, alternative ID-volatility exposure remains a significant predictor of firms' future investment after controlling for firm characteristics. To conserve space, this evidence is not presented and is available upon request.

We focus on quintile portfolios to ensure that they are not too thin. In a decile sort, the cross-sectional dispersion in investment growth and returns is further amplified. In particular, the difference in average sales growth between the top and the bottom decile portfolios is 3.2% (5.5% vs. 2.3%), the spread in average investment growth is 13% (10.2% vs. −2.8%), and the dispersion in average returns is 6.2% (3.5% of the

Table 8 Robustness

Portfolio	FF-adjusted		Two-year window		Five-year window	
	$\Delta Sales$	ΔI	$\Delta Sales$	ΔI	$\Delta Sales$	ΔI
Low	2.48	−0.51	2.45	−0.75	3.08	0.82
2	3.61	1.56	3.60	1.19	3.03	1.27
3	3.73	2.72	3.86	3.10	3.72	3.10
4	4.80	6.05	4.86	5.50	4.07	5.17
High	5.31	9.89	5.21	8.02	5.53	7.52

Notes. This table reports average sales growth ($\Delta Sales$) and investment growth (ΔI) of portfolios sorted on exposure to idiosyncratic volatility. In the first set of columns (FF-adjusted), idiosyncratic volatility is constructed by adjusting firm returns using the Fama–French three-factor model. In the other columns, idiosyncratic volatility betas are measures using either a two-year or a five-year estimation window. Portfolios are value weighted and rebalanced annually; the entries correspond to time-series averages over the 1967–2012 sample period and are expressed in percentages.

high ID-volatility beta portfolio versus 9.7% of the bottom decile).¹²

3.7. Idiosyncratic Volatility: Exposure vs. Level

Idiosyncratic volatility has recently attracted a lot of attention in the asset pricing literature. Campbell et al. (2001), Cao et al. (2008), and Brandt et al. (2010) focus on understanding time-series dynamics of aggregate idiosyncratic volatility. Ang et al. (2006, 2009) analyze the cross-sectional relationship between the level of idiosyncratic volatility and average returns. Kogan and Papanikolaou (2013) and Guo and Savickas (2008) explore the relationship between idiosyncratic volatility level and firm' growth opportunities. Our paper aims to understand if price sensitivity to idiosyncratic volatility news reveals information about growth options owned by firms.

Ang et al. (2006, 2009) show that firms with a high level of idiosyncratic volatility have puzzlingly low average returns. As we show, firms with high exposure to idiosyncratic volatility also carry low premia and, in addition, are expected to invest at a high rate. It is possible that because the level of idiosyncratic volatility and exposure to firm-specific volatility are positively correlated our findings are just a restatement of the idiosyncratic volatility puzzle. We argue, however, that it is not the case. Table 9 provides evidence in support of our argument. It shows average returns and investment growth rates of 3×3 portfolios constructed by double-sorting firms according to the level of their idiosyncratic volatility and ID-volatility exposure. We present evidence based on two sorts. In panel A, we first rank firms on the level of idiosyncratic volatility, and then divide the resulting portfolios into three β^{ID} bins. In panel B, we

¹² These moments are based on our benchmark measure of β^{ID} . Alternative measures produce similar evidence.

Table 9 Double Sort on ID-Volatility Level and ID-Volatility Exposure

Portfolio	Return			ΔI		
	Low ID	Med ID	High ID	Low ID	Med ID	High ID
Panel A. Conditional sort						
Low β^{ID}	6.58	8.97	7.03	0.45	−1.94	−2.82
Med β^{ID}	6.76	8.04	5.47	2.63	2.67	6.01
High β^{ID}	6.87	7.34	2.05	4.26	6.99	9.95
Panel B. Independent sort						
Low β^{ID}	6.54	8.61	5.75	0.66	−1.78	−6.30
Med β^{ID}	7.07	7.87	6.87	3.05	2.75	3.60
High β^{ID}	7.10	7.51	3.23	5.74	6.96	8.56

Notes. This table shows characteristics of 3×3 portfolios sorted on the level of ID and exposure to idiosyncratic volatility (β^{ID}). In panel A, we first rank firms on the level of idiosyncratic volatility, and then divide the resulting portfolios into three β^{ID} bins. In panel B, portfolios are the intersection of two independent sorts. Average return (return) and growth in investment (ΔI) are computed over the one-year holding period and are expressed in percentages. Portfolios are value weighted and rebalanced annually; the entries correspond to time-series averages over the 1967–2012 period.

present the intersections of portfolios sorted independently on each characteristic.¹³

Is high idiosyncratic volatility informative about future investment? Sometimes but not always. For firms with relatively high ID-volatility betas, average investment growth overall increases with the level of firm-specific volatility. However, among firms with low sensitivity to idiosyncratic volatility, the pattern in investment growth is completely the opposite—in this cohort, highly volatile firms tend to cut down their future investment. For example, based on an independent sort, investment growth of firms with high β^{ID} increases from 5.7% to 8.6% when idiosyncratic volatility changes from low to high but across firms with low β^{ID} it declines from 0.7% to −6.3%, respectively. In contrast, high exposure to ID-volatility consistently signals high investment growth for any level of idiosyncratic volatility. Controlling for the level of firm-specific volatility, average investment growth monotonically increases in β^{ID} in both sorts. The spread in average growth rates between high and low ID-volatility beta portfolios varies between 5% and 15%.

Similar to our earlier evidence, expected returns tend to decline in ID-volatility exposure (except for firms with a low level of idiosyncratic volatility that feature no discernible dispersion in mean returns). Under the two sorting schemes, as β^{ID} changes from low to high, the average return declines by about 1.4% and 3.8% for firms with median and high level of idiosyncratic volatility, respectively. Consistent with

idiosyncratic volatility puzzle literature, we find that firms with a high level of idiosyncratic volatility have low rates of return. What is interesting is that this result appears to be strong only among firms with high β^{ID} . For them, the average return declines by about 4.3%, on average, as the level of firm-specific volatility changes from low to high. For firms with low and median ID-volatility exposure, the corresponding spread is only 0.5%.

3.8. Exposure to Aggregate Volatility and Future Growth

As we showed both theoretically and empirically, exposure to idiosyncratic volatility is informative about cross-sectional differences in growth options. In this section we examine if price sensitivity to aggregate volatility risks is also able to reveal differences in growth characteristics across firms. Table 10 presents summary statistics of portfolios sorted on exposure to news about aggregate volatility. Aggregate volatility exposure, which we denote by β^A , is estimated similarly to ID-volatility betas by running three-year rolling window regressions of log equity returns on innovations in aggregate volatility.

We find exposure to aggregate volatility to be largely negative. That is, equity prices tend to fall during times of high economic uncertainty, which is consistent with price reaction of portfolio-level returns documented in Bansal et al. (2005, 2014). In our sample, four out of five firms experience low returns when market volatility goes up. This is a manifestation of the discount rate effect—high aggregate uncertainty rises risk premia and lowers asset prices. Aggregate volatility risks carry a negative price. Firms with high exposure to aggregate uncertainty (i.e., firms with highly negative β^A) earn high risk premia. The average return is monotonically declining in β^A , with firms in the bottom quintile earning about 9%

Table 10 Sorting on Exposure to Aggregate Volatility

Portfolio	Formation period			Holding period		
	β^A	Tobin's Q	R&D/Assets	Return	$\Delta Sales$	ΔI
Low	−0.093	1.22	5.14	8.87	4.01	0.67
2	−0.056	1.11	3.44	8.80	3.47	2.77
3	−0.036	1.10	2.71	7.84	3.07	1.83
4	−0.018	1.11	2.65	6.20	3.38	2.26
High	0.006	1.15	3.15	4.95	4.47	2.58

Notes. This table presents characteristics of portfolios sorted on exposure to aggregate volatility. Aggregate volatility beta (β^A), Tobin's Q, and the ratio of R&D expenditure to lagged assets ($R\&D/Assets$) are computed at the time when portfolios are formed. Average return (*Return*), sales growth ($\Delta Sales$), and investment growth (ΔI) are computed over the one-year holding period. Portfolios are value weighted and rebalanced annually; the entries correspond to time-series averages over the 1967–2012 sample period; $R\&D/Assets$, *Return*, $\Delta Sales$, and ΔI are expressed in percentages.

¹³ In Table 9, we continue to rely on our benchmark measure of idiosyncratic volatility and exposure. The evidence based on alternative measures discussed in the robustness section above is very similar.

Table 11 Aggregate Volatility Exposure and Future Investment Growth

	Model I	Model II	Model III	Model IV
Panel A: Using β^A				
ϕ	0.415 (1.25)	0.482 (1.79)	0.304 (1.77)	0.256 (1.50)
Panel B: Using portfolio position				
ϕ_2	0.021 (1.60)	−0.003 (−0.28)	−0.001 (−0.13)	0.002 (0.32)
ϕ_3	0.030 (1.69)	−0.002 (−0.13)	−0.004 (−0.49)	0.011 (1.40)
ϕ_4	0.035 (1.81)	0.007 (0.51)	−0.009 (−1.03)	0.008 (0.88)
ϕ_5	0.026 (1.10)	0.016 (0.90)	0.003 (0.30)	0.013 (1.24)
Firm FE	No	Yes	No	Yes
Controls	No	No	Yes	Yes

Notes. This table documents predictability of firms' investment growth rate, $\log(I_{t+1}/I_t)$, by their exposure to aggregate volatility. In panel A, ϕ is the regression coefficient on the aggregate volatility beta (β^A); in panel B, ϕ_j is the slope coefficient on a dummy variable that equals one if a firm belongs to the j -quintile portfolio of firms sorted on β^A . The four columns, "Model I"–"Model IV," correspond to different regression specifications with and without firm fixed effects, and with and without controls. The set of controls comprises the ratios of sales to assets, cash to capital, book to market and investment to capital, Tobin's Q, market share, and firm return. Numbers in parentheses are t -statistics based on standard errors clustered by firm and time. The data span the 1967–2012 period.

and firms in the top quintile earning only 5% per annum.

Although we find a strong and robust dispersion in risk premia, we observe no discernible pattern in investment- and growth-related characteristics across portfolios sorted on exposure to aggregate volatility. Along some characteristics, such as Tobin's Q, leverage, and R&D spending, firms with low aggregate volatility betas appear marginally more option-intensive relative to firms with high betas. However, the average investment growth of the top quintile portfolio turns out to be higher than that of the bottom portfolio (2.6% vs. 0.7%), and the correlation between aggregate volatility betas and future growth in sales is virtually zero. We also fail to find any conclusive variation in future investment and sales across portfolios sorted on β^A after we control for either book to market or size.

Table 11 shows more rigorously that sensitivity of equity prices to aggregate volatility news does not help predict future firm-level investment in a significant way. The table presents the estimates of the slope coefficients in panel regressions of one-year ahead investment growth on aggregate volatility beta. We exploit the same specifications as in Equations (16) and (17) replacing ID-volatility betas with β^A s. As the table shows, the β^A -effect is positive but lacks significance even if firm controls are excluded.

To further highlight differences between exposure to aggregate and idiosyncratic volatility news, we consider a double sort on the two betas. Table 12 presents characteristics of 3×3 portfolios constructed by raking firms independently on β^A and β^{ID} . As the

Table 12 Double Sort on Exposure to Volatility

Panel A: Portfolio characteristics at formation						
Portfolio	β^A		β^{ID}		R&D/Assets	
	Low β^A	High β^A	Low β^A	High β^A	Low β^A	High β^A
Low β^{ID}	−0.072	−0.003	−0.023	−0.017	4.1	3.0
High β^{ID}	−0.080	−0.003	0.085	0.073	5.3	4.0
Panel B: Portfolio characteristics over the holding period						
Portfolio	Return		Δ Sales		ΔI	
	Low β^A	High β^A	Low β^A	High β^A	Low β^A	High β^A
Low β^{ID}	10.08	5.04	3.20	3.03	−1.95	0.56
High β^{ID}	7.83	4.22	3.94	5.93	6.23	8.74

Notes. This table shows characteristics of portfolios sorted on exposure to aggregate and idiosyncratic volatility (β^A and β^{ID} , respectively). The four portfolios presented in the table correspond to portfolios with opposite characteristics in the 3×3 sort. In panel A, β^A , β^{ID} , and R&D to lagged assets ($R\&D/Assets$) are computed at the time when portfolios are formed. In panel B, average return ($Return$), sales growth ($\Delta Sales$), and growth in investment (ΔI) are computed over the one-year holding period. Portfolios are value weighted and rebalanced annually; the entries correspond to time-series averages over the 1967–2012 period; $R\&D/Assets$, $Return$, $\Delta Sales$, and ΔI are expressed in percentages.

table shows, cross-sectional differences in investment characteristics are primarily driven by differences in exposure to idiosyncratic volatility. Controlling for β^A , average R&D spending, sales, and investment growth increase by about 1%, 2%, and 8%, respectively, from low- to high- β^{ID} bins. The dispersion across β^A -terciles is conflicting and, if anything, is quite weak. To summarize, we find no convincing evidence that exposure to aggregate volatility helps differentiate between option- and asset-in-place-intensive firms.

4. Conclusion

We argue, both theoretically and empirically, that price exposure to idiosyncratic volatility news is informative about cross-sectional differences in growth opportunities. We introduce two types of volatility risks in a standard real-option setting: risks due to variation in volatility of idiosyncratic shocks, and risks due to variation in volatility of aggregate shocks. Because variation in idiosyncratic volatility has no effect on marginal utility, growth-option values always rise on positive news in idiosyncratic volatility. In contrast, because of the discount-rate effect, the response of growth options to aggregate volatility news is ambiguous. Under fairly general and empirically plausible conditions, an increase in aggregate volatility raises discount rates and lowers valuations. Hence, the discount rate and the conventional volatility channels work against each other. We show that, in general, the discount-rate effect dominates for options

that are close to the option exercise threshold, and the traditional volatility effect dominates when options are deep out of the money. We also show that the response of growth options to aggregate volatility news may exceed or be lower than the corresponding exposure of assets in place.

Guided by our theoretical results, we propose to measure growth-option intensity by firms' exposure to idiosyncratic volatility news. We show that in the data, equity exposure to innovations in idiosyncratic volatility is a significant predictor of firms' future investment and growth. Importantly, information carried by idiosyncratic volatility exposure is not subsumed by the conventional measures of growth such as book-to-market ratio or Tobin's Q, and helps identify option-intensive firms even after controlling for relevant firm characteristics. We also show that firms with high exposure to idiosyncratic volatility, on average, have lower premia compared with firms that show low response. This evidence suggests that growth options are less risky than assets in place and confirms the well-known differences in risk premia in the cross section of book-to-market sorted portfolios. Further, we find no significant evidence that exposure to aggregate volatility is informative about cross-sectional differences in the relative composition of firms' assets. In all, our paper emphasizes the importance of distinguishing between idiosyncratic and aggregate volatility shocks in understanding firms' growth opportunities.

Acknowledgments

The authors thank Ravi Bansal, Andres Donangelo, Laurent Fresard, Pete Kyle, Howard Kung, Bradley Paye, and Yajun Wang, as well as seminar participants at Duke University, Temple University, the University of Texas at Dallas, the University of Maryland, the University of Illinois at Urbana-Champaign, the University of Utah, the University of Houston, INSEAD, the 2013 Society for Financial Studies Finance Cavalcade, the 2014 Arizona State University Sonoran Winter Finance Conference, the 2014 Finance Down Under Conference, and the 2014 Western Finance Association meetings for their helpful comments. The authors also thank Jerome Detemple (department editor), an associate editor, and a referee for their insightful comments. The usual disclaimer applies.

Appendix

A.1. Proof of Proposition 1

We first derive the pricing kernel of the economies with aggregate and idiosyncratic volatility shocks. The representative agent's intertemporal preference is represented by the [Kreps and Porteus \(1978\)](#) utility with constant relative risk aversion parameter γ and constant intertemporal elasticity of substitution parameter ψ . We use $\{V_t\}_{t \geq 0}$ to denote the

utility process of the representative agent. The representative consumer's preference is specified by a pair of aggregators (f, \mathcal{A}) such that

$$dV_t = [-f(C_t, V_t) - \frac{1}{2}\mathcal{A}(V_t)\|\sigma_V(t)\|^2]dt + \sigma_V(t)dB_t. \quad (18)$$

We adopt the convenient normalization $\mathcal{A}(v) = 0$ ([Duffie and Epstein 1992b](#)), and we denote the normalized aggregator by \tilde{f} . In this case, $\tilde{f}(C, V)$ is

$$\tilde{f}(C, V) = \frac{\beta}{1-1/\psi} \frac{C^{1-1/\psi} - ((1-\gamma)V)^{(1-1/\psi)/(1-\gamma)}}{((1-\gamma)V)^{(1-1/\psi)/(1-\gamma)-1}}. \quad (19)$$

Because of the homogeneity of the utility function, it can be represented as

$$V(\theta, C) = \frac{1}{1-\gamma} H(\theta) C^{1-\gamma} \quad (20)$$

for some function $H(\theta)$. Under assumption (6), $H(\theta)$ is given by

$$\begin{aligned} H(\theta_H) &= \left\{ \frac{1}{\beta} \left[\beta + \frac{1}{2}\gamma \left(1 - \frac{1}{\psi}\right) \sigma_C^2(\theta_H) - \left(1 - \frac{1}{\psi}\right) \mu_C \right. \right. \\ &\quad \left. \left. - \frac{1-1/\psi}{1-\gamma} \lambda(\omega-1) \right] \right\}^{-(1-\gamma)/(1-1/\psi)} \\ H(\theta_L) &= \left\{ \frac{1}{\beta} \left[\beta + \frac{1}{2}\gamma \left(1 - \frac{1}{\psi}\right) \sigma_C^2(\theta_L) - \left(1 - \frac{1}{\psi}\right) \mu_C \right. \right. \\ &\quad \left. \left. - \frac{1-1/\psi}{1-\gamma} \lambda(\omega^{-1}-1) \right] \right\}^{-(1-\gamma)/(1-1/\psi)}, \end{aligned}$$

where $\omega \geq 1$ is the unique solution to the following equation on $(0, \infty)$:

$$\begin{aligned} \omega^{-(1-1/\psi)/(1-\gamma)} &= \left[\beta + \frac{1}{2}\gamma \left(1 - \frac{1}{\psi}\right) \sigma_C^2(\theta_L) - \left(1 - \frac{1}{\psi}\right) \mu_C \right. \\ &\quad \left. - \frac{1-1/\psi}{1-\gamma} \lambda(\omega^{-1}-1) \right] \\ &\quad \cdot \left[\beta + \frac{1}{2}\gamma \left(1 - \frac{1}{\psi}\right) \sigma_C^2(\theta_H) - \left(1 - \frac{1}{\psi}\right) \mu_C \right. \\ &\quad \left. - \frac{1-1/\psi}{1-\gamma} \lambda(\omega-1) \right]^{-1}. \quad (21) \end{aligned}$$

The above results apply to both the economy with idiosyncratic volatility shocks and the economy with aggregate volatility shocks as special cases. In the economy with idiosyncratic volatility shocks, $\sigma_C(\theta_H) = \sigma_C(\theta_L) = \sigma$ and $\omega = 1$. In this case, $H(\theta_H) = H(\theta_L)$. In the economy with aggregate volatility shocks, $\sigma_C(\theta_H) > \sigma_C(\theta_L)$ and $\omega > 1$. Given our assumption of $\gamma > 1$, high aggregate volatility is associated with lower utility of the representative agent.

Let $\{\pi_t\}_{t=0}^\infty$ denote the marginal utility process of the representative agent. To derive an explicit expression of the law of motion of π_t , it is convenient to represent the Markov process $\{\theta_t\}_{t=0}^\infty$ as integration with respect to counting processes:

$$d\theta_t = \Delta\eta(\theta_t^-) dN_t, \quad (22)$$

where

$$\Delta = \theta_H - \theta_L,$$

and

$$N(t) = [N_{Ht}, N_{Lt}]'$$

are independent counting processes with intensity λ (for simplicity, we assume that N_{Ht} and N_{Lt} have the same intensity parameter, λ). The function $\eta(\theta)$ in Equation (22) is given by

$$\eta(\theta) = [-I_{\{\theta_H\}}(\theta), I_{\{\theta_L\}}(\theta)]',$$

where $I_{\{x\}}$ is the indicator function

$$I_{\{x\}}(y) = \begin{cases} 1 & \text{if } y = x, \\ 0 & \text{if } y \neq x. \end{cases}$$

Using the results in Duffie and Epstein (1992b),

$$\frac{d\pi_t}{\pi_t} = \frac{d\tilde{f}_C(C_t, V_t)}{\tilde{f}_C(C_t, V_t)} + \tilde{f}_V(C_t, V_t) dt. \quad (23)$$

Using Equations (20) and (23), one can show that

$$d\pi_t = \pi_t[-\tilde{r}(\theta_t) dt - \gamma \sigma_C(\theta) dB_t - \eta_\pi(\theta_t)' dN_t],$$

where

$$\begin{aligned} \tilde{r}(\theta_L) &= \beta + \frac{1}{\psi} \mu_C - \frac{1}{2} \gamma \left(1 + \frac{1}{\psi}\right) \sigma_C^2(\theta_L) \\ &\quad + \frac{1/\psi - \gamma}{1 - \gamma} \lambda (\omega^{-1} - 1), \end{aligned} \quad (24)$$

$$\begin{aligned} \tilde{r}(\theta_H) &= \beta + \frac{1}{\psi} \mu_C - \frac{1}{2} \gamma \left(1 + \frac{1}{\psi}\right) \sigma_C^2(\theta_H) \\ &\quad + \frac{1/\psi - \gamma}{1 - \gamma} \lambda (\omega - 1), \end{aligned} \quad (25)$$

and

$$\eta_\pi(\theta) = [(1 - \hat{\omega}) I_{\{\theta_H\}}(\theta), (1 - \hat{\omega}^{-1}) I_{\{\theta_L\}}(\theta)],$$

with $\hat{\omega} = \omega^{(1/\psi - \gamma)/(1 - \gamma)}$. Using the expression for the pricing kernel, it is straightforward to show that the risk-free interest rate of the economy, denoted $r(\theta)$, is given by

$$\begin{aligned} r(\theta_L) &= \beta + \frac{1}{\psi} \mu_C - \frac{1}{2} \gamma \left(1 + \frac{1}{\psi}\right) \sigma_C^2(\theta_L) \\ &\quad + \lambda \left[1 - \hat{\omega}^{-1} + \frac{1/\psi - \gamma}{1 - \gamma} (\omega^{-1} - 1)\right], \\ r(\theta_H) &= \beta + \frac{1}{\psi} \mu_C - \frac{1}{2} \gamma \left(1 + \frac{1}{\psi}\right) \sigma_C^2(\theta_H) \\ &\quad + \lambda \left[1 - \hat{\omega} + \frac{1/\psi - \gamma}{1 - \gamma} (\omega - 1)\right]. \end{aligned} \quad (26)$$

It follows that $r(\theta_L) \geq r(\theta_H)$, that is, high volatility is associated with low interest rates.

We introduce the following notation:

$$\beta_H = \kappa + \tilde{r}(\theta_H) + \gamma \rho \sigma_C^2(\theta_H); \quad \beta_L = \kappa + \tilde{r}(\theta_L) + \gamma \rho \sigma_C^2(\theta_L).$$

The value of assets in place is summarized by the following lemma.

LEMMA 1. *The value of assets in place is linear in dividend as in (8), where*

$$a(\theta_L) = \frac{\beta_H - \mu_D + \lambda(1 + \hat{\omega}^{-1})}{(\beta_H - \mu_D + \lambda)(\beta_L - \mu_D + \lambda) - \lambda^2}, \quad (27)$$

$$a(\theta_H) = \frac{\beta_L - \mu_D + \lambda(1 + \hat{\omega})}{(\beta_H - \mu_D + \lambda)(\beta_L - \mu_D + \lambda) - \lambda^2}. \quad (28)$$

In addition, $a(\theta_L) = a(\theta_H)$ in the economy with idiosyncratic volatility shocks. Under the assumption $\gamma > 1 > 1/\psi$, $a(\theta_L) > a(\theta_H)$ in the economy with aggregate volatility shocks.

PROOF. Let $CF(\theta, D)$ be a continuous function of (θ, D) , let τ be a stopping time, and let $BD(\theta, D)$ be a twice continuously differentiable function of (θ, D) . Let $V(\theta, D)$ denote the value function of an asset that pays cash flow at rate $CF(\theta_t, D_t)$ for all $t < \tau$ and pays $B(\theta_\tau, D_\tau)$ at time τ :

$$V(\theta, D) = E \left[\int_0^\tau \frac{\pi_t}{\pi_0} CF(\theta_t, D_t) dt + \frac{\pi_\tau}{\pi_0} B(\theta_\tau, D_\tau) \middle| \theta_0 = \theta, D_0 = D \right]. \quad (29)$$

Standard results imply that $V(\theta, D)$ is twice continuously differentiable, and for all $t < \tau$, it satisfies

$$\pi_t CF(\theta_t, D_t) + \mathcal{L}[\pi_t V(\theta_t, D_t)] = 0, \quad (30)$$

where

$$\mathcal{L}[\pi_t V(\theta_t, D_t)] = \lim_{\Delta \rightarrow 0} \frac{1}{\Delta} E_t[\pi_{t+\Delta} V(\theta_{t+\Delta}, D_{t+\Delta}) - \pi_t V(\theta_t, D_t)],$$

and $V(\theta_\tau, D_\tau) = BD(\theta_\tau, D_\tau)$ at time τ .

Using generalized Ito's formula, Equation (30) can be written as

$$\begin{aligned} &[\tilde{r}(\theta) + \kappa] V(\theta, D) \\ &= CF(\theta, D) + \frac{\partial}{\partial D} V(\theta, D) D[\mu_D - \gamma \sigma_C^2(\theta)] \\ &\quad + \frac{1}{2} \frac{\partial^2}{\partial D^2} V(\theta, D) \rho^2 [\sigma_C^2(\theta) + \sigma_D^2(\theta)] \\ &\quad + I_{\{\theta_H\}}(\theta) \lambda [\hat{\omega} V(\theta_L, D) - V(\theta_H, D)] \\ &\quad + I_{\{\theta_L\}}(\theta) \lambda [\hat{\omega}^{-1} V(\theta_H, D) - V(\theta_L, D)]. \end{aligned} \quad (31)$$

The valuation of assets in place is a special case where $CF(\theta, D) = D$ and $\tau = \infty$. In this case, $V(\theta, D)$ is linear and Equation (31) implies that $a(\theta)$ is given by Equations (27) and (28). This completes the proof of Lemma 1. \square

Below, we study the solutions to a quartic equation, which characterizes the value function of the growth options. We denote

$$\tau_L^2 = \rho^2 [\sigma_C^2(\theta_L) + \sigma_D^2(\theta_L)], \quad (32)$$

$$\tau_H^2 = \rho^2 [\sigma_C^2(\theta_H) + \sigma_D^2(\theta_H)], \quad (33)$$

$$\mu_L = \mu_D - \gamma \rho \sigma_C^2(\theta_L), \quad (34)$$

$$\mu_H = \mu_D - \gamma \rho \sigma_C^2(\theta_H), \quad (35)$$

and

$$r_L = \tilde{r}(\theta_L) + \kappa + \lambda, \quad (36)$$

$$r_H = \tilde{r}(\theta_H) + \kappa + \lambda. \quad (37)$$

Let $f_L(\alpha)$ and $f_H(\alpha)$ be the following quadratic functions of α :

$$f_L(\alpha) = \frac{1}{2} \tau_L^2 \alpha^2 + (\mu_L - \frac{1}{2} \tau_L^2) \alpha - r_L, \quad (38)$$

$$f_H(\alpha) = \frac{1}{2} \tau_H^2 \alpha^2 + (\mu_H - \frac{1}{2} \tau_H^2) \alpha - r_H.$$

As we show in Lemma 4 in the next section, the quartic equation that characterizes the solution to the ODE in Equation (31) is

$$f_H(\alpha) f_L(\alpha) - \lambda^2 = 0. \quad (39)$$

The following lemma characterizes the solution to a class of quadratic functions that will be useful in analyzing the solutions to Equation (39).

LEMMA 2. Suppose $r > 0$, then the quadratic equation

$$\frac{1}{2}\tau^2\alpha^2 + (\mu - \frac{1}{2}\tau^2)\alpha - r = 0,$$

has one and only one positive solution. If $r > \mu > 0$, then the positive solution is strictly greater than 1.

Using Lemma 2, it is straightforward to verify that under the assumptions on the parameter values in Equations (6) and (7), the quadratic equation $f_L(\alpha) = 0$ has two roots, α_L^- and α_L^+ that satisfy $\alpha_L^- < 0 < 1 < \alpha_L^+$. Similarly, $f_H(\alpha) = 0$ has two roots that satisfy $\alpha_H^- < 0 < 1 < \alpha_H^+$. We define

$$\alpha_{\min} = \min\{\alpha_L^+, \alpha_H^+\}, \quad \alpha_{\max} = \max\{\alpha_L^+, \alpha_H^+\}. \quad (40)$$

LEMMA 3. Equation (39) has two strictly positive roots that satisfy $1 < \alpha_1 < \alpha_2$. In addition, $f_H(\alpha)$, $f_L(\alpha) < 0$, and $(d/d\alpha)f_H(\alpha)$, $(d/d\alpha)f_L(\alpha) > 0$ for all $\alpha \in (1, \alpha_{\min})$. Finally, $\alpha_1 \in (1, \alpha_{\min})$.

PROOF. First, we show that Equation (39) has two positive roots, which we denote as α_1 and α_2 , that satisfy $\alpha_1 \in (1, \alpha_{\min})$ and $\alpha_2 \in (\alpha_{\max}, \infty)$. To see this, note that $f_H(1) - \lambda^2 = (r_L - \mu_L)(r_H - \mu_H) - \lambda^2 > 0$ and $f_H(\alpha_{\min})f_L(\alpha_{\min}) - \lambda^2 = -\lambda^2 < 0$. Also, $f_H(\alpha_{\max})f_L(\alpha_{\max}) - \lambda^2 = -\lambda^2 < 0$ and $f_H(\alpha)f_L(\alpha) - \lambda^2 > 0$ for α large enough.

Similarly, we can verify that Equation (39) also has two negative roots. Because Equation (39) can have at most four real roots, α_1 and α_2 are the only two positive roots of Equation (39). Finally, for $\alpha > 1$, $(d/d\alpha)f_i(\alpha) = \tau_i^2\alpha + (\mu_i - \frac{1}{2}\tau_i^2) > \mu_i + \frac{1}{2}\tau_i^2 > 0$ for $i = H, L$. This completes the proof. \square

For $i = 1, 2$, we define

$$e(\alpha_i) = \frac{-f_H(\alpha_i)}{\lambda\hat{\omega}} = \frac{\lambda\hat{\omega}^{-1}}{-f_L(\alpha_i)}, \quad (41)$$

where the second equality holds because α_1 and α_2 are solutions to Equation (39). The following corollary characterizes the signs of $e(\alpha_1)$ and $e(\alpha_2)$.

COROLLARY 1. $e(\alpha_1) > 0$ and $e(\alpha_2) < 0$.

PROOF. Note that $\alpha_1 \in (1, \alpha_{\min})$ and $(d/d\alpha)f_H(\alpha) > 0$ for $\alpha > 1$ implies that $f_H(\alpha_1) < 0$. Therefore, $e(\alpha_1) > 0$. Similarly, $\alpha_2 \in (\alpha_{\max}, \infty)$ implies that $e(\alpha_2) < 0$. \square

The value of growth options is given by the following lemma.

LEMMA 4. Let $e_1 = e(\alpha_1)$ and $e_2 = e(\alpha_2)$, then the value of growth options is given by Equations (9) and (10) in Proposition 1, where the option exercise thresholds, $\hat{D}(\theta_H)$ and $\hat{D}(\theta_L)$ and the constants K_1 , K_2 , A_1 , and A_2 are determined by the value-matching and smooth pasting conditions in Equations (11) and (12). In addition, $K_1 > 0$.

PROOF. The valuation of options can be viewed as a special case of (29), where $CF(\theta, D) = 0$, $B(\theta_\tau, D_\tau) = V(\theta_\tau, D_\tau) - 1$ and the stopping time τ takes the following form:

$$\tau = \inf\{t: D_t > \hat{D}(\theta_H), \theta_t = \theta_H\} \cup \{t: D_t > \hat{D}(\theta_L), \theta_t = \theta_L\},$$

where the option exercise threshold $\hat{D}(\theta)$ is chosen optimally to maximize the value of the option. Equation (30) implies that the value of options must satisfy

$$\begin{aligned} \bar{r}(\theta_H)V_O(\theta_H, D) &= \frac{\partial}{\partial D}V_O(\theta_H, D)D[\mu_D - \gamma\rho\sigma_C^2(\theta_H)] \\ &\quad + \frac{1}{2}\frac{\partial^2}{\partial D^2}V_O(\theta_H, D)\rho^2D^2[\sigma_C^2(\theta_H) + \sigma_D^2(\theta_H)] \\ &\quad + \lambda[\hat{\omega}V_O(\theta_L, D) - V_O(\theta_H, D)] \end{aligned} \quad (42)$$

and

$$\begin{aligned} \bar{r}(\theta_L)V_O(\theta_L, D) &= \frac{\partial}{\partial D}V_O(\theta_L, D)D[\mu_D - \gamma\rho\sigma_C^2(\theta_L)] \\ &\quad + \frac{1}{2}\frac{\partial^2}{\partial D^2}V_O(\theta_L, D)\rho^2D^2[\sigma_C^2(\theta_L) + \sigma_D^2(\theta_L)] \\ &\quad + \lambda[\hat{\omega}^{-1}V(\theta_H, D) - V(\theta_L, D)]. \end{aligned} \quad (43)$$

We guess that $V_O(\theta, D)$ takes the form of $b(\theta)D^\alpha$. Then Equations (42) and (43) can be written as

$$\begin{aligned} \bar{r}(\theta_H)b(\theta_H)D^\alpha &= \alpha b(\theta_H)D^\alpha[\mu_D - \gamma\rho\sigma_C^2(\theta_H)] \\ &\quad + \frac{1}{2}b(\theta_H)\alpha(\alpha - 1)D^\alpha\rho^2[\sigma_C^2(\theta_H) + \sigma_D^2(\theta_H)] \\ &\quad + \lambda[\hat{\omega}b(\theta_L) - b(\theta_H)]D^\alpha \end{aligned}$$

and

$$\begin{aligned} \bar{r}(\theta_L)b(\theta_L)D^\alpha &= \alpha b(\theta_L)D^\alpha[\mu_D - \gamma\rho\sigma_C^2(\theta_L)] \\ &\quad + \frac{1}{2}b(\theta_L)\alpha(\alpha - 1)D^\alpha\rho^2[\sigma_C^2(\theta_L) + \sigma_D^2(\theta_L)] \\ &\quad + \lambda[\hat{\omega}^{-1}b(\theta_H) - b(\theta_L)]D^\alpha. \end{aligned}$$

Assume that $\hat{D}(\theta_L) < \hat{D}(\theta_H)$, then for $D \in (0, \hat{D}(\theta_L))$, $V_O(\theta, D) = b(\theta)D^\alpha$ for $\theta = \theta_H, \theta_L$, and the above equations imply that $b(\theta)$ and α must jointly satisfy

$$\begin{aligned} &\frac{1}{2}\rho^2[\sigma_C^2(\theta_H) + \sigma_D^2(\theta_H)]\alpha^2b(\theta_H) \\ &\quad + [\mu_D - \gamma\rho\sigma_C^2(\theta_H) - \frac{1}{2}\rho^2[\sigma_C^2(\theta_H) + \sigma_D^2(\theta_H)]]\alpha b(\theta_H) \\ &\quad - [\kappa + \lambda + \bar{r}(\theta_H)]b(\theta_H) + \lambda\hat{\omega}b(\theta_L) = 0 \end{aligned} \quad (44)$$

and

$$\begin{aligned} &\frac{1}{2}\rho^2[\sigma_C^2(\theta_L) + \sigma_D^2(\theta_L)]\alpha^2b(\theta_L) \\ &\quad + [\mu_D - \gamma\rho\sigma_C^2(\theta_L) - \frac{1}{2}\rho^2[\sigma_C^2(\theta_L) + \sigma_D^2(\theta_L)]]\alpha b(\theta_L) \\ &\quad - [\kappa + \lambda + \bar{r}(\theta_L)]b(\theta_L) + \lambda\hat{\omega}^{-1}b(\theta_H) = 0. \end{aligned} \quad (45)$$

Here, α is the eigenvalue of the quadratic eigenvalue problem in Equations (44) and (45), and $[b(\theta_H), b(\theta_L)]$ is the corresponding eigenvector. We normalize $b(\theta_H) = 1$. Using notations defined in Equations (32)–(37), Equations (44) and (45) can be written as

$$\begin{aligned} \frac{1}{2}\tau_H^2\alpha^2 + \left[\mu_H - \frac{1}{2}\tau_H^2\right]\alpha - r_H &= -b(\theta_L)\lambda\hat{\omega}, \\ \frac{1}{2}\tau_L^2\alpha^2 + \left[\mu_L - \frac{1}{2}\tau_L^2\right]\alpha - r_L &= -\frac{\lambda\hat{\omega}^{-1}}{b(\theta_L)}. \end{aligned}$$

Clearly, α must satisfy the quartic Equation (39) and the corresponding normalized eigenvector is $[1, e(\alpha)]$, where $e(\alpha)$

is defined by (41). The general solution to (42) and (43) is therefore of the form

$$V_O(\theta_H, D) = \sum_{j=1}^4 K_j D^{\alpha_j}, \quad V_O(\theta_L, D) = \sum_{j=1}^4 K_j e(\alpha_j) D^{\alpha_j}.$$

The boundary condition $V_O(\theta, 0) = 0$ implies that $K_3 = K_4 = 0$.

Under the assumption $\hat{D}(\theta_L) < \hat{D}(\theta_H)$, for $D \in (\hat{D}(\theta_L), \hat{D}(\theta_H))$, $V_O(\theta_L, D) = a(\theta_L)D - 1$. Therefore, $V_O(\theta_H, D)$ must satisfy

$$\begin{aligned} \bar{r}(\theta_H)V_O(\theta_H, D) &= \frac{\partial}{\partial D} V_O(\theta_H, D) D [\mu_D - \gamma \rho \sigma_C^2(\theta_H)] \\ &\quad + \frac{1}{2} \frac{\partial^2}{\partial D^2} V_O(\theta_H, D) \rho^2 D^2 [\sigma_C^2(\theta_H) + \sigma_D^2(\theta_H)] \\ &\quad + \lambda [\hat{\omega}(a_L D - 1) - V_O(\theta_H, D)]. \end{aligned}$$

This implies that in this region, $V_O(\theta_H, D)$ is of the form

$$V_O(\theta_H, D) = a_0 D + b_0 + A_1 D^{\eta_1} + A_2 D^{\eta_2},$$

where

$$\begin{aligned} a_0 &= \frac{\lambda \hat{\omega} a_L}{[\kappa + \lambda + \bar{r}(\theta_H)] - [\mu_D - \gamma \rho \sigma_C(\theta_H)]}, \\ b_0 &= -\frac{\lambda \hat{\omega}}{\kappa + \lambda + \bar{r}(\theta_H)}, \end{aligned} \quad (46)$$

and $\eta_1 > 0 > \eta_2$ are given by

$$\begin{aligned} \eta_{1,2} &= \pm \sqrt{\left[\frac{\mu_D - \gamma \rho \sigma_C(\theta_H)}{\rho^2 [\sigma_C^2(\theta_H) + \sigma_D^2(\theta_H)]} - \frac{1}{2} \right] + \frac{2[\kappa + \lambda + \bar{r}(\theta_H)]}{\rho^2 [\sigma_C^2(\theta_H) + \sigma_D^2(\theta_H)]}} \\ &\quad - \left[\frac{\mu_D - \gamma \rho \sigma_C(\theta_H)}{\rho^2 [\sigma_C^2(\theta_H) + \sigma_D^2(\theta_H)]} - \frac{1}{2} \right]}. \end{aligned} \quad (47)$$

Finally, the constants, K_1, K_2, A_1, A_2 , and the option exercise thresholds $\hat{D}(\theta_L)$ and $\hat{D}(\theta_H)$ are determined by the following value-matching and smooth pasting conditions:

$$\begin{aligned} V_O(\theta_L, \hat{D}(\theta_L)) &= a(\theta_L)D - 1, \\ \frac{\partial}{\partial D} V_O(\theta_L, \hat{D}(\theta_L)) &= a(\theta_L); \end{aligned} \quad (48)$$

$$\begin{aligned} V_O(\theta_H, \hat{D}(\theta_H)) &= a(\theta_H)D - 1, \\ \frac{\partial}{\partial D} V_O(\theta_H, \hat{D}(\theta_H)) &= a(\theta_H); \end{aligned} \quad (49)$$

$$\begin{aligned} V_O(\theta_H, \hat{D}^+(\theta_H)) &= V_O(\theta_H, \hat{D}^-(\theta_H)), \\ \frac{\partial}{\partial D} V_O(\theta_H, \hat{D}^+(\theta_H)) &= \frac{\partial}{\partial D} V_O(\theta_H, \hat{D}^-(\theta_H)). \end{aligned} \quad (50)$$

To prove the last part of the lemma, note that because $\alpha_1 < \alpha_2$, as $D \rightarrow 0$, the value of the option is dominated by the first term, that is,

$$\begin{aligned} V_O(\theta_L, D) &= K_1 e(\alpha_1) D^{\alpha_1} + o(D^{\alpha_1}), \\ V_O(\theta_H, D) &= K_1 D^{\alpha_1} + o(D^{\alpha_1}). \end{aligned}$$

Because option value must be positive, $K_1 > 0$ must hold. This completes the proof of the lemma and therefore Proposition 1. \square

A.2. Several Useful Results

Here we establish several useful results for the general model specified in Equations (1)–(3). This will prepare us for the proofs of Propositions 2 and 3.

LEMMA 5. Let $\hat{\alpha}_H$ denote the unique solution to the quadratic equation

$$f_H(\alpha) + \lambda \hat{\omega} = 0 \quad (51)$$

on $(1, \alpha_{\max})$, and $\hat{\alpha}_L$ be the unique solution to

$$f_L(\alpha) + \lambda \hat{\omega}^{-1} = 0 \quad (52)$$

on the same interval, where α_{\max} is defined in Equation (40). Then $\hat{\alpha}_H < \hat{\alpha}_L$ implies that $\hat{\alpha}_H < \alpha_1 < \hat{\alpha}_L$ and $e(\alpha_1) < 1$.

PROOF. By Lemma 3, $(d/d\alpha)f_H(\alpha)$, $(d/d\alpha)f_L(\alpha) > 0$ for $\alpha > 1$. This implies that $\hat{\alpha}_H < \alpha_H^+$ and $\hat{\alpha}_L < \alpha_L^+$, where α_H^+ and α_L^+ are the positive solutions to $f_H(\alpha) = 0$ and $f_L(\alpha) = 0$, respectively. In addition, by Lemma 2, both Equations (51) and (52) have a unique positive root that is strictly greater than 1. Therefore, $\hat{\alpha}_H, \hat{\alpha}_L \in (1, \alpha_{\max})$.

Note that α_1 is the unique solution to (39) on $(1, \alpha_{\max})$. Suppose $\hat{\alpha}_H < \hat{\alpha}_L$. To see that $\hat{\alpha}_H < \alpha_1 < \hat{\alpha}_L$, note that because both $f_H(\alpha)$ and $f_L(\alpha)$ are strictly increasing on $(\hat{\alpha}_H, \hat{\alpha}_L)$, we have

$$f_H(\hat{\alpha}_H) = -\lambda \hat{\omega}, \quad f_H(\hat{\alpha}_L) > -\lambda \hat{\omega},$$

and

$$f_L(\hat{\alpha}_H) < -\lambda \hat{\omega}^{-1}, \quad f_L(\hat{\alpha}_L) = -\lambda \hat{\omega}^{-1}.$$

These imply

$$f_H(\hat{\alpha}_H)f_L(\hat{\alpha}_H) - \lambda^2 > 0, \quad f_H(\hat{\alpha}_L)f_L(\hat{\alpha}_L) - \lambda^2 < 0.$$

Therefore, α_1 must lie in the open interval $(\hat{\alpha}_H, \hat{\alpha}_L)$. Because on $(\hat{\alpha}_H, \hat{\alpha}_L)$, $f_H(\alpha) > -\lambda \hat{\omega}$ and $f_L(\alpha) < -\lambda \hat{\omega}^{-1}$, we must have $e(\alpha_1) < 1$ by the definition of $e(\cdot)$ in Equation (41). \square

COROLLARY 2. A sufficient condition for $\hat{\alpha}_H < \hat{\alpha}_L$ is

$$\begin{aligned} \frac{1}{2} \rho^2 (\theta_H^2 - \theta_L^2) \hat{\alpha}_L \\ - \left\{ \frac{1}{2} \rho^2 (\theta_H^2 - \theta_L^2) + \gamma \rho [\sigma_C^2(\theta_H) - \sigma_C^2(\theta_L)] \right\} > 0. \end{aligned} \quad (53)$$

PROOF. Note that $f_H(\hat{\alpha}_H) + \lambda \hat{\omega} = 0$ by the definition of $\hat{\alpha}_H$. Because $f_H(\alpha)$ is a strictly increasing function of α on $(1, \alpha_{\max})$, to prove $\hat{\alpha}_H < \hat{\alpha}_L$, it is enough to show that

$$f_H(\hat{\alpha}_L) + \lambda \hat{\omega} > 0. \quad (54)$$

Because $f_L(\hat{\alpha}_L) - \lambda \hat{\omega}^{-1} = 0$ by the definition of $\hat{\alpha}_L$, inequality (54) is equivalent to $f_H(\hat{\alpha}_L) - \lambda \hat{\omega} > f_L(\hat{\alpha}_L) + \lambda \hat{\omega}^{-1}$. Using the definition of $f_H(\alpha)$ and $f_L(\alpha)$ in (38), we have

$$\begin{aligned} [f_H(\alpha) - \lambda \hat{\omega}] - [f_L(\alpha) + \lambda \hat{\omega}^{-1}] \\ = \frac{1}{2} \rho^2 (\theta_H^2 - \theta_L^2) \alpha^2 - \left\{ \frac{1}{2} \rho^2 (\theta_H^2 - \theta_L^2) + \gamma \rho [\sigma_C^2(\theta_H) - \sigma_C^2(\theta_L)] \right\} \alpha \\ - [r(\theta_H) - r(\theta_L)], \end{aligned} \quad (55)$$

where $r(\theta)$ is the risk-free interest rate given in Equation (27). Because $r(\theta_H) \leq r(\theta_L)$, and $\hat{\alpha}_L > 0$, (53) is a sufficient condition for $[f_H(\hat{\alpha}_L) - \lambda \hat{\omega}] - [f_L(\hat{\alpha}_L) + \lambda \hat{\omega}^{-1}] > 0$. \square

A.3. Proof of Proposition 2

Note that in the economy with idiosyncratic volatility shocks, $\sigma_C(\theta) = \sigma$ is a constant, and $\sigma_D(\theta) = \theta$ for $\theta = \theta_H, \theta_L$. Following the notation in Lemma 4, we denote $e_1 = e(\alpha_1)$ and $e_2 = e(\alpha_2)$. We first prove the following lemma.

LEMMA 6. In the economy with idiosyncratic shocks, $0 < e_1 < 1$.

PROOF. Because $\sigma_C(\theta_H) = \sigma(\theta_L) = \sigma$, condition (53) in Corollary 2 becomes

$$\frac{1}{2}\rho^2(\theta_H^2 - \theta_L^2)\hat{\alpha}_L - \frac{1}{2}\rho^2(\theta_H^2 - \theta_L^2) > 0,$$

which is clearly true because $\hat{\alpha}_L > 1$. \square

In the case of idiosyncratic volatility, the value function of assets in place can be simplified to $V_A(\theta, D) = aD$, where

$$a = a(\theta_L) = a(\theta_H) = \frac{1}{\kappa + \beta + (1/\psi)\mu_C + \frac{1}{2}\gamma(2\rho - 1 - 1/\psi)\sigma^2 - \mu_D}.$$

Let $\hat{D}_{\max} = \max\{\hat{D}(\theta_H), \hat{D}(\theta_L)\}$. We first prove the following lemma.

LEMMA 7. In the economy with idiosyncratic volatility shocks, $V_O(\theta_H, D) > V_O(\theta_L, D)$ on $(0, \hat{D}_{\max})$.

PROOF. We first show that one of the following two cases must be true. Case 1: $V_O(\theta_H, D) > V_O(\theta_L, D)$ for all $D \in (0, \hat{D}_{\max})$; Case 2: $V_O(\theta_H, D) < V_O(\theta_L, D)$ for all $D \in (0, \hat{D}_{\max})$. To see this, note that $V_O(\theta_H, 0) = V_O(\theta_L, 0) = 0$. Also, value matching implies $V_O(\theta_H, \hat{D}_{\max}) = V_O(\theta_L, \hat{D}_{\max}) = a\hat{D}_{\max} - 1$. Because both value functions are continuously differentiable, $(\partial/\partial D)V_O(\theta_H, D)$ and $(\partial/\partial D)V_O(\theta_L, D)$ must cross at least once on $(0, \hat{D}_{\max})$. To establish that either Case 1 or Case 2 must be true, it is enough to show that the derivatives cross each other at most once on $(0, \hat{D}_{\max})$.

First, let $\hat{D}_{\min} = \min\{\hat{D}(\theta_H), \hat{D}(\theta_L)\}$. $(\partial/\partial D)V_O(\theta_H, D)$ and $(\partial/\partial D)V_O(\theta_L, D)$ can not cross each other on $(\hat{D}_{\min}, \hat{D}_{\max})$. Assume without loss of generality that $\hat{D}_{\min} = \hat{D}(\theta_H)$ and $\hat{D}_{\max} = \hat{D}(\theta_L)$. Then $(\partial/\partial D)V_O(\theta_H, D) = a$ on $(\hat{D}_{\min}, \hat{D}_{\max})$. Also, $(\partial/\partial D)V_O(\theta_L, D) < a$ because $V(\theta_L, D)$ is strictly convex and $(\partial/\partial D)V_O(\theta_L, \hat{D}_{\max}) = a$. Therefore, $(\partial/\partial D)V_O(\theta_H, D) = (\partial/\partial D)V_O(\theta_L, D)$ cannot happen on $(\hat{D}_{\min}, \hat{D}_{\max})$. Similarly, we can rule out the possibility of $(\partial/\partial D)V_O(\theta_H, D) = (\partial/\partial D)V_O(\theta_L, D)$ on $(\hat{D}_{\min}, \hat{D}_{\max})$ for the case of $\hat{D}(\theta_H) > \hat{D}(\theta_L)$.

On $(0, \hat{D}_{\min})$,

$$\begin{aligned} \frac{\partial}{\partial D}V_O(\theta_L, D) &= \alpha_1 K_1 e_1 D^{\alpha_1-1} + \alpha_2 K_2 e_2 D^{\alpha_2-1}, \\ \frac{\partial}{\partial D}V_O(\theta_H, D) &= \alpha_1 K_1 D^{\alpha_1-1} + \alpha_2 K_2 D^{\alpha_2-1}. \end{aligned}$$

The solution to $(\partial/\partial D)V_O(\theta_L, D) = (\partial/\partial D)V_O(\theta_H, D)$ is given by

$$D = \left[\frac{(1 - e_1)\alpha_1 K_1}{(e_2 - 1)\alpha_2 K_2} \right]^{1/(\alpha_2 - \alpha_1)}, \quad (56)$$

which is unique.

Next, we show that Case 2 cannot be true. It is enough to show that $V_O(\theta_H, D) > V_O(\theta_L, D)$ for some $D \in (0, \hat{D}_{\min})$. Consider

$$\frac{V_O(\theta_H, \varepsilon)}{V_O(\theta_D, \varepsilon)} = \frac{K_1 \varepsilon^{\alpha_1} + K_2 \varepsilon^{\alpha_2}}{K_1 e_1 \varepsilon^{\alpha_1} + K_2 e_2 \varepsilon^{\alpha_2}}.$$

For $\varepsilon \rightarrow 0$, the first terms in the numerator and denominator dominate because $1 < \alpha_1 < \alpha_2$. We have

$$\frac{V_O(\theta_H, \varepsilon)}{V_O(\theta_D, \varepsilon)} \rightarrow \frac{K_1 \varepsilon^{\alpha_1}}{K_1 e_1 \varepsilon^{\alpha_1}} = \frac{1}{e_1} > 1$$

by Lemma 6. This completes the proof. \square

Finally, note that $V_O(\theta_H, D) > V_O(\theta_L, D)$ for all D on $(0, \hat{D}_{\max})$ and $V_O(\theta_H, \hat{D}_{\max}) = V_O(\theta_L, \hat{D}_{\max}) = a\hat{D}_{\max} - 1$ imply $\hat{D}(\theta_H) > \hat{D}(\theta_L)$.

A.4. Proof of Proposition 3

Note that in the economy with aggregate volatility shocks, $\sigma_C(\theta) = \theta$ for $\theta = \theta_H, \theta_L$, and $\sigma_D(\theta) = \sigma$ is a constant. We continue to denote $e_1 = e(\alpha_1)$ and $e_2 = e(\alpha_2)$. We first prove that $e_1 < 1$.

LEMMA 8. Suppose that in the economy with aggregate volatility shocks,

$$\kappa + \beta + \frac{1}{\psi}\mu_C - \frac{1}{2}\gamma\left(1 + \frac{1}{\psi}\right)\theta^2 > \left(1 + \frac{2\gamma}{\rho}\right)(\mu_D + \gamma\rho\sigma^2), \quad \text{for } \theta = \theta_H, \theta_L, \quad (57)$$

then $0 < e_1 < 1$.

PROOF. Because $\sigma_C(\theta) = \theta$ for $\theta = \theta_H, \theta_L$, condition (53) in Corollary 2 becomes

$$\frac{1}{2}\rho^2(\theta_H^2 - \theta_L^2)\hat{\alpha}_L - \left\{\frac{1}{2}\rho^2(\theta_H^2 - \theta_L^2) + \gamma\rho[\theta_H^2 - \theta_L^2]\right\} > 0,$$

which is equivalent to $\hat{\alpha}_L > 1 + (2\gamma)/\rho$.

Note that

$$\begin{aligned} \hat{\alpha}_L^+ &= \left[\frac{1}{2} - \frac{\mu_D - \gamma\rho\sigma_C^2(\theta_L)}{\rho^2[\sigma_C^2(\theta_L) + \sigma^2]} \right] \\ &\quad + \sqrt{\left[\frac{1}{2} - \frac{\mu_D - \gamma\rho\sigma_C^2(\theta_L)}{\rho^2[\sigma_C^2(\theta_L) + \sigma^2]} \right]^2 + \frac{2[\kappa + r(\theta_L)]}{\rho^2[\sigma_C^2(\theta_L) + \sigma^2]}}. \end{aligned} \quad (58)$$

Using Equation (58), $\hat{\alpha}_L > 1 + (2\gamma)/\rho$ can be written as

$$\frac{2[\kappa + r(\theta_L)]}{\rho^2[\sigma_C^2(\theta_L) + \sigma^2]} > \left(1 + \frac{2\gamma}{\rho}\right)^2 + 2\left[1 + \frac{2\gamma}{\rho}\right]\left[\frac{\mu_D - \gamma\rho\sigma_C^2(\theta_L)}{\rho^2[\sigma_C^2(\theta_L) + \sigma^2]} - \frac{1}{2}\right],$$

which can be simplified to

$$\kappa + r(\theta_L) > [\mu_D + \gamma\rho\sigma^2]\left(1 + \frac{2\gamma}{\rho}\right). \quad (59)$$

Using the expression for the real interest rate in Equation (27), we can show that inequality (57) is sufficient for condition (59). \square

Now we show that the conclusions of Proposition 3 are true under the condition that $e_1 \in (0, 1)$. We continue to denote $\hat{D}_{\max} = \max\{\hat{D}(\theta_H), \hat{D}(\theta_L)\}$ and $\hat{D}_{\min} = \min\{\hat{D}(\theta_H), \hat{D}(\theta_L)\}$.

We first prove the existence of D^* . Note that at \hat{D}_{\max} , $V_O(\theta_L, D) = a(\theta_L) - 1 > V_O(\theta_H, D) = a(\theta_H) - 1$. We now show that for $\varepsilon \in (0, \hat{D}_{\max})$ and ε small, $V_O(\theta_L, \varepsilon) < V_O(\theta_H, \varepsilon)$. This is true because

$$\frac{V_O(\theta_H, \varepsilon)}{V_O(\theta_D, \varepsilon)} = \frac{K_1 \varepsilon^{\alpha_1} + K_2 \varepsilon^{\alpha_2}}{K_1 e_1 \varepsilon^{\alpha_1} + K_2 e_2 \varepsilon^{\alpha_2}} \rightarrow \frac{1}{e_1} > 1 \quad \text{as } \varepsilon \rightarrow 0.$$

Because both $V_O(\theta_L, D)$ and $V_O(\theta_H, D)$ are continuous, there must be a $D^* \in (0, \hat{D}_{\max})$ such that $V_O(\theta_L, D^*) = V_O(\theta_H, D^*)$.

Next, we prove that $\hat{D}(\theta_L) < \hat{D}(\theta_H)$ by contradiction. Suppose that this is not true, that is, $\hat{D}(\theta_L) \geq \hat{D}(\theta_H)$. Note that $V_O(\theta_H, D)$ must satisfy the value-matching and smooth pasting conditions at $\hat{D}(\theta_H)$, that is,

$$K_1 \hat{D}(\theta_H)^{\alpha_1} + K_2 \hat{D}(\theta_H)^{\alpha_2} = a(\theta_H) \hat{D}(\theta_H) - 1, \quad (60)$$

$$\alpha_1 K_1 \hat{D}(\theta_H)^{\alpha_1-1} + \alpha_2 K_2 \hat{D}(\theta_H)^{\alpha_2-1} = a(\theta_H). \quad (61)$$

The above equations imply that K_1 must satisfy

$$K_1 = \frac{\alpha_2 - 1}{\hat{D}(\theta_H)^{\alpha_1}(\alpha_2 - \alpha_1)} \left[a(\theta_H) \hat{D}(\theta_H) - \frac{\alpha_2}{\alpha_2 - 1} \right]. \quad (62)$$

Note also, because $\hat{D}(\theta_L) \geq \hat{D}(\theta_H)$, at $\hat{D}(\theta_H)$, $V_O(\theta_L, D)$ must satisfy

$$K_1 e_1 \hat{D}(\theta_H)^{\alpha_1} + K_2 e_2 \hat{D}(\theta_H)^{\alpha_2} = a(\theta_H) \hat{D}(\theta_H) - 1 + \varepsilon_0, \quad (63)$$

$$\alpha_1 K_1 e_1 \hat{D}(\theta_H)^{\alpha_1-1} + \alpha_2 K_2 e_2 \hat{D}(\theta_H)^{\alpha_2-1} = a(\theta_H) \varepsilon_1, \quad (64)$$

for some $\varepsilon_0 \geq 0$ and $\varepsilon_1 \in (0, 1)$. Equation (63) is true because $V_O(\theta_L, D) \geq a_L D - 1$ for $D \leq \hat{D}(\theta_L)$. The second equality is due to the convexity of the value function $(\partial/\partial D)V_O(\theta_L, D) \leq a_L$ for $D \leq \hat{D}(\theta_L)$. Jointly, Equations (60) and (61) imply

$$K_1 = \frac{\alpha_2 - 1}{\hat{D}(\theta_H)^{\alpha_1}(\alpha_2 - \alpha_1)} \frac{1}{e_1} \cdot \left[\frac{\alpha_2 - \varepsilon_1}{\alpha_2 - 1} a(\theta_L) \hat{D}(\theta_H) - \frac{\alpha_2}{\alpha_2 - 1} (1 - \varepsilon_0) \right]. \quad (65)$$

Given that $\varepsilon_0 \geq 0$, $\varepsilon_1 \in (0, 1)$ and $e_1 \in (0, 1)$, the expression in Equation (65) is strictly larger than the expression in Equation (62), which is a contradiction. This establishes that $\hat{D}(\theta_L) < \hat{D}(\theta_H)$.

Lastly, we prove that the intersection of $V_O(\theta_H, D)$ and $V_O(\theta_L, D)$, D^* , must be unique. First, consider the case where there exists a $D^* \in (0, \hat{D}(\theta_L)]$. In this case, $V_O(\theta_L, D) = V_O(\theta_H, D)$ can only happen on $(0, \hat{D}(\theta_L)]$ and can happen at most once. To see that $V_O(\theta_L, D) = V_O(\theta_H, D)$ can only happen on $(0, \hat{D}(\theta_L)]$, note that on $(\hat{D}(\theta_L), \hat{D}(\theta_H)]$, $V_O(\theta_L, D) \geq a(\theta_L)D - 1 > a(\theta_H)D - 1 = V_O(\theta_H, D)$. To see that $V_O(\theta_L, D) = V_O(\theta_H, D)$ at most once on $(0, \hat{D}(\theta_L)]$, note that in this region, $V_O(\theta, D)$ is of the form in Equation (9); therefore, the solution to $V_O(\theta_L, D) = V_O(\theta_H, D)$ on $(0, \hat{D}(\theta_L)]$ must satisfy

$$D = \left[\frac{(1 - e_1)K_1}{(e_2 - 1)K_2} \right]^{1/(\alpha_2 - \alpha_1)}, \quad (66)$$

and must be unique.

Second, consider the case where there exists a $D^* \in (\hat{D}(\theta_L), \hat{D}(\theta_H)]$. In this case, $V_O(\theta_L, D)$ and $V_O(\theta_H, D)$ can only cross once on $(\hat{D}(\theta_L), \hat{D}(\theta_H)]$, because in this region, $(\partial/\partial D)V_O(\theta_L, D) = a(\theta_L) > a(\theta_H) \geq (\partial/\partial D)V_O(\theta_H, D)$. This completes the proof.

References

Ai H, Kiku D (2013) Growth to value: Option exercise and the cross section of equity returns. *J. Financial Econom.* 107:325–349.

- Ai H, Croce MM, Li K (2013) Toward a quantitative general equilibrium asset pricing model with intangible capital. *Rev. Financial Stud.* 26:491–530.
- Ai H, Croce MM, Diercks AM, Li K (2014) Production-based term structure of equity returns. Working paper, University of Minnesota, Minneapolis.
- Almeida H, Campello M (2007) Financial constraints, asset tangibility and corporate investment. *Rev. Financial Stud.* 20:1429–1460.
- Alti A (2003) How sensitive is investment to cash flow when financing is frictionless? *J. Finance* 58:707–722.
- Andersen TG, Bollerslev T, Diebold FX, Ebens H (2001) The distribution of realized stock return volatility. *J. Financial Econom.* 61:43–76.
- Ang A, Hodrick R, Xing Y, Zhang X (2006) The cross section of volatility and expected returns. *J. Finance* 61:259–299.
- Ang A, Hodrick R, Xing Y, Zhang X (2009) High idiosyncratic volatility and low returns: International and further U.S. evidence. *J. Financial Econom.* 91:1–23.
- Bansal R, Yaron A (2004) Risks for the long run: A potential resolution of asset pricing puzzles. *J. Finance* 59:1481–1509.
- Bansal R, Khatchatrian V, Yaron A (2005) Interpretable asset markets? *Eur. Econom. Rev.* 49:531–560.
- Bansal R, Kiku D, Shaliastovich I, Yaron A (2014) Volatility, the macroeconomy and asset prices. *J. Finance* 69:2471–2511.
- Berk J, Green RC, Naik V (1999) Optimal investment, growth options, and security returns. *J. Finance* 54:1553–1608.
- Bhamra HS, Shim KH (2011) Stochastic idiosyncratic operating risk and real options: Implications for stock returns. Working paper, Imperial College Business School, London.
- Bhamra HS, Kuehn L-A, Strebulaev IA (2010) The levered equity risk premium and credit spreads: A unified framework. *Rev. Financial Stud.* 23:645–703.
- Black F (1976) Studies of stock price volatility changes. *Proc. 1976 Meetings Amer. Statist. Assoc.*, 171–181.
- Black F, Scholes M (1973) The pricing of options and corporate liabilities. *J. Political Econom.* 81:637–654.
- Bloom N (2009) The impact of uncertainty shocks. *Econometrica* 77:623–685.
- Bloom N, Floetotto M, Jaimovich N, Saporta-Eksten I, Terry S (2011) Really uncertain business cycles. Working paper, Stanford University, Stanford, CA.
- Boguth O, Kuehn L-A (2013) Consumption volatility risk. *J. Finance* 68:2589–2615.
- Bollerslev T, Chou R, Kroner K (1992) ARCH modeling in finance: A review of the theory and empirical evidence. *J. Econometrics* 52:5–59.
- Bollerslev T, Tauchen G, Zhou H (2009) Expected stock returns and variance premia. *Rev. Financial Stud.* 22:4463–4492.
- Brandt MW, Brav A, Graham JR, Kumar A (2010) The idiosyncratic volatility puzzle: Time trend or speculative episodes? *Rev. Financial Stud.* 23:863–899.
- Campbell JY, Lettau M, Malkiel BG, Xu Y (2001) Have individual stocks become more volatile? An empirical exploration of idiosyncratic risk. *J. Finance* 56:1–43.
- Cao C, Simin T, Zhao J (2008) Can growth options explain the trend in idiosyncratic risk? *Rev. Financial Stud.* 21:2599–2633.
- Carlson M, Fisher A, Giammarino R (2004) Corporate investment and asset price dynamics: Implications for the cross-section of returns. *J. Finance* 59:2577–2603.
- Chen H (2010) Macroeconomic conditions and the puzzles of credit spreads and capital structure. *J. Finance* 65:2171–2212.
- Cooper I (2006) Asset pricing implications of nonconvex adjustment costs and irreversibility of investment. *J. Finance* 61:139–170.
- Detemple J, Sundaresan S (1999) Non-traded asset valuation with portfolio constraints: A binomial approach. *Rev. Financial Studies* 12:835–872.
- Dixit A, Pindyck R (1994) *Investment Under Uncertainty* (Princeton University Press, Princeton, NJ).
- Drechsler I, Yaron A (2011) What's vol got to do with it. *Rev. Financial Stud.* 24:1–45.

- Duarte F, Kogan L, Livdan D (2012) Aggregate investment and stock returns. Working paper, Federal Reserve Bank of New York, New York.
- Duffie D, Epstein LG (1992a) Asset pricing with stochastic differential utility. *Rev. Financial Stud.* 5:411–436.
- Duffie D, Epstein LG (1992b) Stochastic differential utility. *Econometrica* 60:353–394.
- Erickson T, Whited T (2000) Measurement error and the relationship between investment and Q. *J. Political Econom.* 108: 1027–1057.
- Fama E, French K (1993) Common risk factors in the returns on stocks and bonds. *J. Financial Econom.* 33:3–56.
- Fazzari SM, Hubbard RG, Petersen BC (1988) Financing constraints and corporate investment. *Brookings Paper Econom. Activity* 1:141–195.
- Garlappi L, Yan H (2011) Financial distress and the cross-section of equity returns. *J. Finance* 66:789–822.
- Gomes JF (2001) Financing investment. *Amer. Econom. Rev.* 91: 1263–1285.
- Gomes J, Kogan L, Zhang L (2003) Euqilibrium cross section of returns. *J. Political Econom.* 111:693–732.
- Grullon G, Lyandres E, Zhdanov A (2012) Real options, volatility, and stock returns. *J. Finance* 67:1499–1537.
- Guo H, Savickas R (2008) Average idiosyncratic volatility in G7 countries. *Rev. Financial Stud.* 21:1259–1296.
- Hackbarth D, Miao J, Morellec E (2006) Capital structure, credit risk, and macroeconomic conditions. *J. Financial Econom.* 82: 519–550.
- Henderson V (2007) Valuing the option to invest in an incomplete market. *Math. Financial Econom.* 1:103–128.
- Hugonnier J, Morellec E (2007) Corporate control and real investment in incomplete markets. *J. Econom. Dynam. Control* 31: 1781–1800.
- Kogan L, Papanikolaou D (2010) Growth opportunities and technology shocks. *Amer. Econom. Rev. Papers Proc.* 100:532–536.
- Kogan L, Papanikolaou D (2013) Firm characteristics and stock returns: The role of investment-specific shocks. *Rev. Financial Stud.* 26:2718–2759.
- Kogan L, Papanikolaou D (2014) Growth opportunities, technology shocks, and asset prices. *J. Finance* 69:675–718.
- Kreps DM, Porteus EL (1978) Temporal resolution of uncertainty and dynamic choice theory. *Econometrica* 46:185–200.
- McDonald R, Siegel D (1986) The value of waiting to invest. *Quart. J. Econom.* 101:707–727.
- Merton RC (1973) Theory of rational option pricing. *Bell J. Econom. Management Sci.* 4:143–183.
- Miao J, Wang N (2007) Investment, consumption, and hedging under incomplete markets. *J. Financial Econom.* 86:608–642.
- Moyen N (2004) Investment-cash flow sensitivities: Constrained versus unconstrained firms. *J. Finance* 59:2061–2092.
- Newey W, West K (1987) A simple, positive semi-definite, heteroscedasticity and autocorrelation consistent covariance matrix. *Econometrica* 55:703–708.
- Novy-Marx R (2007) An equilibrium model of investment under uncertainty. *Rev. Financial Stud.* 20:1461–1502.
- Novy-Marx R (2011) Operating leverage. *Rev. Finance* 15:103–134.
- Papanikolaou D (2011) Investment shocks and asset prices. *J. Political Econom.* 119:639–685.
- Zhang L (2005) The value premium. *J. Finance* 60:67–103.