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Collusion in Dynamic Buyer-Determined Reverse Auctions

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Although binding reverse auctions have attracted a good deal of interest in the academic literature, in practice, dynamic nonbinding reverse auctions are the norm in procurement. In those, suppliers submit price quotes and can respond to quotes of their competitors during a live auction event. However, the lowest quote does not necessarily determine the winner. The buyer decides after the contest, taking further supplier information into account, on who will be awarded the contract. We show, both theoretically and empirically, that this bidding format enables suppliers to collude, thus leading to noncompetitive prices.

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1. Introduction

In nonbinding reverse auctions, bidders compete against each other like in a standard reverse auction, but the winner is not necessarily the supplier with the lowest bid. Rather, buyers decide, based on the final quotes and further information about the suppliers, who will be awarded the contract. These *buyer-determined reverse auctions* (BDRAs) are virtually the norm in competitive procurement today. Ariba, a major commercial provider of online reverse auctions and other sourcing solutions, uses nonbinding reverse auctions almost exclusively. In a recent survey, Elmaghraby (2007, p. 411) noted that "The exact manner in which the buyer makes her final selection still remains unclear. With either an online auction or a RFP [request for proposal], the buyer may still leave some terms of trade unspecified."[1]

In the context of multiattribute auction events, the advantage of a nonbinding format from the buyer's perspective seems evident. The winner should not be the supplier with the lowest quote, but further attributes, such as quality, reliability, capacity, reputation, incumbent status, and other suppliers' capabilities, should be taken into account. However, we show in this paper that there is a serious disadvantage to such dynamic nonbinding reverse auctions: If bidders are uncertain about the exact way different criteria affect the final decision by the buyer, then, in equilibrium, a nonbinding reverse auction enables them to implicitly coordinate on high prices.

The collusive arrangement in the nonbinding reverse auction works as follows: The suppliers begin the contest with a relatively high quote. These offers are such that if the process were to stop at this point, all have a positive probability of winning, given the uncertain criteria of the buyer's award decision. In equilibrium, no supplier makes an improvement on his offer, so the bidding stops at a high price. If one supplier were to lower the offer, it would trigger a response by the other suppliers, who would also lower their quotes. Thus, the deviating supplier has to reduce his price even further, which makes it unattractive to lower the price in the first place. Note that the stabilizing element in this collusion is that suppliers do not know how the buyer will ultimately determine the winner. Thus, with their initial offers, all suppliers have a positive chance of winning.

Binding reverse auctions, where the final decision rule is known in advance, do not allow for this form of collusion. In a (reverse) English auction, for example, at any moment during the auction firms do

[1] SAP (2006, p. 9) notes in a document on best practice in reverse auctions: "Often, you may find that the lowest bidder is not meeting quality and service grades and thus may select the second-lowest bidder."

not have any uncertainty about whether they would receive the contract or not if the auction were to stop at that point. Thus, suppliers who know that they will not be awarded the contract at the current price have to improve their offer, which in turn puts pressure on their competitors. Therefore, collusion cannot be sustainable in binding reverse auctions.

Buyer-determined reverse auction mechanisms have not been widely studied and are not well understood, especially theoretically. Jap (2002) was the first to point out that most reverse auctions that are conducted in industry do not determine winners—i.e., they are nonbinding. Jap (2003, 2007) shows that dynamic nonbinding reverse auctions often have a more detrimental effect on buyer-supplier relationships than do sealed-bid reverse auctions. In another study, Engelbrecht-Wiggans et al. (2007) examine sealed-bid first price reverse auctions. They compare price-based and buyer-determined mechanisms, both theoretically and using laboratory experiments, and find that buyer-determined mechanisms generate higher buyer surplus only as long as there are enough suppliers competing for the contract. Haruvy and Katok (2013) investigate the effect of information transparency on sealed-bid and dynamic nonbinding auctions and find that sealed-bid formats are generally better for buyers, especially when suppliers are aware of their competitors' nonprice attributes. In both of these studies, suppliers know the value the buyer attaches to their own nonprice attributes.[2, 3]

In contrast, in the present paper, we investigate the effect of having this information on the performance of dynamic nonbinding reverse auctions. We show that it is precisely the combination of the dynamic nature of the bidding process, which allows bidders to react to their competitors' bids, and the lack of knowledge about the valuation of the nonprice attributes by the buyer, which ensures that each bidder has some probability of winning even at a high price, that enables bidders to collude.

The way collusion works in our model has some similarity to the collusive behavior in the context of strategic demand reduction (Brusco and Lopomo 2002, Ausubel et al. 2014) and to the industrial organization literature on price clauses (see, e.g., Salop 1986, Schnitzer 1994, and references therein). Strategic demand reduction describes the phenomenon that, in a multiunit reverse auction, bidders might prefer to win a smaller number of units at a higher price than a larger number of units at a lower price. Our paper analyzes a single unit situation in which bidders are content with a small probability of winning at a higher price.

Price clauses such as a "meet-the-competition" clause or a price-matching clause might be used to sustain collusion in a market similarly to the present analysis, where suppliers refrain from lowering their quotes because this will trigger lower prices by their competitors. The literature on price clauses differs from this paper in two respects, however. First, in the pricing literature, it is either assumed that trade takes place in several periods (e.g., Schnitzer 1994) or that contingent contracts can be written in which the price depends on the prices of the competitors (e.g., Doyle 1988, Logan and Lutter 1989). In the present case, trade only takes place once, and contingent bidding is not possible. Second, the main argument why collusion is feasible, namely, the remaining uncertainty about the final decision the buyer will take, has to our knowledge not been investigated so far.

Several authors have analyzed collusion in the context of auctions (see, e.g., Robinson 1985, Graham and Marshall 1987; for an overview, see Klemperer 1999, Kwasnica and Sherstyuk 2013). Sherstyuk (1999, 2002) shows in an experimental study that the bid improvement rule has an influence on the bidders' ability to collude in repeated auctions. Usually this literature assumes that before the auction takes place, a designated winner is selected. In addition, there must be some means to divide the gains of collusion between the participating bidders. This is different from the form of collusion described here. First, all participating firms have a chance of winning the contract; thus there is no predetermined winner and no preplay communication required. Second, during the contest, all firms have a positive expected profit, even if after the decision by the buyer only one firm receives the contract. This makes it unnecessary to divide the gains of collusion after the contest.

This paper is structured as follows: In the next section we develop the model and analyze the collusive behavior in a dynamic buyer-determined reverse auction. In §3 we describe our experimental setting and present the results. In §4 we conclude this paper with a discussion of ways for overcoming the problem of collusion.

2. Analytical Results

2.1. Model Setup

The auction format we consider is one in which suppliers bid on price, but different suppliers may provide different value to the buyer. This value can be

[2] Thomas and Wilson (2005) compare experimentally multilateral negotiations and auctions. They explicitly assume that during the negotiations offers are observable, so this case resembles our buyer-determined bidding mechanism. However, everyone knew preferences of the parties in advance. So the effect we analyze here could not have occurred.

[3] Stoll and Zöttl (2014) use field data to make a counterfactual analysis that estimates the consequences of a reduction of nonprice information available to bidders.

viewed as exogenous attributes of suppliers themselves, rather than a part of their bids, and we will refer to it as *quality*. Our modeling approach is similar to that of Engelbrecht-Wiggans et al. (2007) and Haruvy and Katok (2013). There are n potential suppliers, competing to provide a single unit to a buyer. Suppliers are heterogeneous in costs and quality. In particular, supplier i has cost c_i, which is only known to the supplier i. Each c_i is taken from a common distribution $F(c)$ on $[\underline{c}, \bar{c}]$. The quality component does not enter the profit function of the supplier, so the profit of supplier i if he wins the contract at price p is given by

$$\pi_i(p, c_i) = p - c_i.$$

There are different ways to model quality differences among suppliers. For example, it may be reasonable to assume that there is some commonly known (vertical) quality component for each supplier. For example, in the procurement of a customer designed application-specific circuit, all suppliers satisfy the necessary technical requirements, but some suppliers might have a superior technology that is commonly known and that provides additional value to the buyer. But there may also be a quality component that is only known to the buyer—a horizontal quality component. This horizontal quality is the focus of our model, so we will assume in the remainder of the analysis that there are no vertical quality differences among suppliers.[4]

Let α_i be the buyer's incremental cost of dealing with supplier i relative to dealing with her most preferred supplier, and let all the α_i's be the private information of the buyer. Then the vector α containing all α_i's represents the buyer's preferences and is distributed independent of the costs of the suppliers according to a commonly known distribution with finite support $[0, \bar{\alpha}]^n$. The utility of the buyer, if she awards the contract at price p to supplier i, is

$$u(p, \alpha_i) = v - p - \alpha_i,$$

where v is the value to the buyer from the project, and the parameter α_i measures the extent to which the private preferences of the buyer about dealing with supplier i enter her surplus. Parameter $\bar{\alpha}$ can be quite small: Consider, for example, the sourcing of a display for a new mobile phone. The overall value of the contract might be several hundreds of millions in U.S. dollars, which is captured by the term v. There may be some individual observable differences between the suppliers—e.g., one firm is known to be the technology leader—that are in the range of 10 million U.S. dollars (that we omit from the model). Unobservable preferences by the buyer, i.e., a preference for a particular provider, whose engineers speak English fluently, might differ in the size of several hundred thousand U.S. dollars. These are captured by the term $\bar{\alpha}$.

But $\bar{\alpha}$ might also be large relative to the overall project value: Consider a company recruiting a marketing agency. An optimal marketing campaign would provide value v for the company. The decision, which agency to hire, will be strongly influenced by the specific preference parameter—the extent to which the board of the firm prefers one marketing agency over the others, which includes preferences about their people, their ideas, and their creativity. This is expressed by $\bar{\alpha}$, which might be similar in size to v.

We are assuming that the bidders do not know the buyer's preferences α. If the buyer already knows her preferences α before the auction, then she can simply announce them and conduct a binding auction in which the lowest α_i-adjusted bid wins.[5]

The main focus of our paper is what we believe to be a more realistic setting, in which the buyer does not know α before the auction. This may be because bidders have not been fully vetted prior to the auction, or because determining the α_i's is a group decision that cannot be done in the abstract. In this case, announcing α before the auction and adjusting bids by α_i is not feasible, and the buyer has to choose between two formats. The buyer can conduct a *binding price-based reverse auction* (PBRA), which we analyze in §2.2. In this auction, the bidder who submits the lowest bid is guaranteed to win, but the buyer may incur additional cost due to misfit, from dealing with this bidder. The buyer can also conduct a nonbinding, or *dynamic*, *buyer-determined reverse auction*, which we analyze in §2.3. In this auction, bidders submit bids, and the buyer selects the bidder when all final bids are on the table. The buyer will then choose the bidder with the lowest quality-adjusted bid, which we also call *total cost*. Thus, the lowest bid in the BDRA is not guaranteed to win.

2.2. Binding Price-Based Reverse Auction

The rules of the binding price-based reverse auction are standard. Each bidder i submits a price

[4] Extending the model to include commonly known vertical quality is relatively straightforward and does not change our results qualitatively.

[5] If the buyer communicates the horizontal qualities α_i to all bidders and monetizes the horizontal quality differences, she can conduct a binding auction in which the bidder with the lowest quality-adjusted bid wins and is paid the amount of the second lowest quality-adjusted cost, $(c_i + \alpha_i)^{(n-1)}$. A commonly used way to monetize α_i is to set up a bonus/handicap system, which quantifies differences between suppliers with respect to the different dimensions, e.g., quality, payment terms, technical criteria, and so on. It is important to note that whether revealing private preferences α (if that is possible at all) is beneficial to the buyer is an interesting question that is beyond the scope of our paper. We refer the reader to Che (1993), who finds that the optimal revenue-maximizing mechanism discriminates against nonprice attributes to make price competition tougher.

bid b_i. The highest allowable bid is the reservation price R. During the auction, bidders observe full price feedback—they see all b_i's that have been submitted. They can place new bids that must be lower than the lowest current standing bid by some predetermined minimum bid decrement to become the leading bid. The bidder with the lowest bid is the leading bidder in the auction and would win the auction if it were to stop at this point. The auction ends when there are no new bids placed for a certain amount of time. The price the buyer pays is equal to the lowest price bid b_i.

Under this rule, it is a dominant strategy for each supplier to keep lowering his bid as long as he is not currently winning the auction, until $b_i = c_i$.[6] Thus, the auction ends when the bidder with the second lowest cost exits the auction. The bidder with the lowest cost wins the auction. If bidder i with horizontal quality α_i wins, and bidder j, with the second lowest bid, exited at c_j, then the price the buyer pays is equal to c_j. The utility of the buyer is then

$$u = v - c_j - \alpha_i,$$

where c_j is the second lowest bid, which we denote by $(c_i)^{(n-1)}$. Because the distribution of α_i is independent of costs and quality realization (by assumption), the expected buyer surplus is

$$v - E[(c_i)^{(n-1)}] - E[\alpha_i].$$

The buyer pays, in expectation, the second lowest cost and the expected value of the horizontal quality parameter.

2.3. Dynamic Buyer-Determined Reverse Auction

2.3.1. General Framework. Now consider a nonbinding reverse auction, which, as we noted in the introduction, is commonly used in procurement practice. The auction works exactly the same way as the binding price-based reverse auction in terms of the bids that bidders observe during the auction and the ending rule. The main difference is that after the auction ends, the buyer is not obligated to award the contract to the bidder with the lowest bid b_i, but may instead award the contract to a different bidder, taking her preferences α_i into account.

The fundamental difference between the nonbinding auction and its binding counterpart is that bidders might not know if at current bids they would win or lose in the nonbinding auction. A bidder j only knows for certain that he is losing when his bid b_j is more than $\bar{\alpha}$ above the current lowest bid. On the other hand, a bidder i only knows for certain that he is winning when his bid b_i is more than $\bar{\alpha}$ below the next lowest bid. Although it is optimal for a bidder who knows that he is winning not to lower his bid further, it is optimal to lower his bid for a bidder who knows that he is losing as long as the bid is still larger than his costs.

Let us call the lowest standing bid $B = \min\{b_1, b_2, \ldots, b_n\}$ and the lowest bid of the competitors B_{-i}. A bidder i whose bid is within $\bar{\alpha}$ of B_{-i}, $B_{-i} - \bar{\alpha} \le b_i \le B_{-i} + \bar{\alpha}$, does not know his winning status, and thus there is no obvious best action for him. In general, his strategy will depend on his beliefs about the other suppliers' future actions. A bidder who believes that lowering the bid would lead to an outright bidding war is less likely to lower his bid than a bidder who merely expects competitors to lower their bids by a small amount.

In the collusive equilibrium we analyze, all suppliers initially bid very high in such a way that the probability of winning for every supplier is the same. When one supplier lowers his bid to increase his probability of winning, those suppliers whose probability of winning is decreased will follow suit and lower their bids as well. This makes the initial deviation Unattractive, and thus collusion can be sustained.

In the most general formulation, the bidding behavior off the equilibrium path, i.e., bidders deviating from colluding on high prices, is complex. To facilitate the analysis, we set the information structure such that if someone lowers his bid to increase his probability of winning, the probability of winning for at least one other supplier falls to zero.[7] Thus, it is dominant for this supplier to lower his bid as well as long as he bids above costs.

2.3.2. Specific Bidding Model. We now consider a special case in which we can characterize the conditions for a collusive equilibrium to exist. The buyer has one preferred supplier, but the suppliers do not know the identity of this supplier. Let $\bar{\alpha} > 0$ be the additional cost the buyer incurs when she has to deal with a nonpreferred supplier. Let $\alpha_i = 0$ if i is the buyer's preferred supplier, and $\alpha_i = \bar{\alpha}$ otherwise. As before, the α_i's are not known by the suppliers. Since suppliers are ex ante symmetric, each supplier i believes that the probability that $\alpha_i = 0$ is $1/n$.[8]

[6] The binding price-based reverse auction has also several other equilibria, however, these are ruled out if one eliminates weakly dominated strategies or requires subgame perfection.

[7] This can be achieved by assuming that the horizontal quality of each buyer is taken from a discrete set, i.e., $\alpha_i \in \{0, \bar{\alpha}\}$, which is the approach we take in the remainder of this paper.

[8] The situation is thus like in the spokes model of horizontal product differentiation (Chen and Riordan 2007). All suppliers are located at the end of different spokes of a wheel. The buyer is located at the end of one spoke. Thus, the "distance" to one supplier is zero, whereas the distance to all other suppliers is the same given by twice the length of a spoke, here modeled by $\bar{\alpha}$.

We assume that bids must be in multiples of the minimum bid decrement ϵ, where ϵ is sufficiently small. Discreteness of prices is used to ensure that there are no ties. This is achieved by assuming that $\bar{\alpha}$ is not a multiple of ϵ.

Before specifying the equilibrium formally, one definition is necessary. Let b_{-i} be a vector of bids of all suppliers apart from supplier i. If supplier i were to bid b_i, and the bidding would stop at this point, then the probability for supplier i of obtaining the contract is given by

$$P_i(b_i^t, b_{-i}^t) = \mathrm{Prob}(\alpha_i + b_i < \alpha_j + b_j \ \forall\, j \neq i).$$

Note that from the point of view of supplier i, both α_i and all α_j are random variables.

We now describe the following collusive bidding strategy β^c:

- $b_i^1 = R$: all bidders start bidding at the reservation price of R.
- For bidder i, if $P_i(b_i^t, b_{-i}^t) \geq 1/n$, then $b_i^{t+1} = b_i^t$.
- If $P_i(b_i^t, b_{-i}^t) < 1/n$, then $b_i^{t+1} = \max\{c_i, b^*(b_{-i}^t)\}$, where $b^*(b_{-i}^t)$ is the maximum bid b, which satisfies $P_i(b, b_{-i}^t) \geq 1/n$.[9]

If bidding starts at $t = 1$ with all bidders bidding R, then all bidders have the same winning probability $1/n$ and bidders will stop bidding. However, if (out of equilibrium) bids differ, and for some bidder i the probability of winning is below $1/n$, then in the next round bidder i sets his bid $b^*(b_{-i}^{t-1})$ so as to barely outbid the bidder with the lowest current bid in the event that i turns out to be the preferred supplier. Since the bidding is done in increments of ϵ, this implies, for the bid of bidder i (recalling that B^{t-1} is defined as the lowest standing bid),

$$b^*(b_{-i}^{t-1}) \in (B^{t-1} + \bar{\alpha} - \epsilon, B^{t-1} + \bar{\alpha}).$$

2.3.3. Equilibrium Analysis. We claim that the bidding strategy β^c as defined above constitutes an equilibrium, depending on the reservation price R, the size of the buyer preference term $\bar{\alpha}$, and the distribution of costs $F(c_i)$.

We start the formal analysis by considering two bidders. Proposition 1 develops a sufficient condition for collusion to occur.

Proposition 1. *Assume there are two bidders and $R \geq \bar{c}$. The bidding strategy β^c describes a collusive equilibrium if*

$$\frac{R - \underline{c}}{2} \geq \max_{p \in [0,\, \bar{c} - \bar{\alpha}]} \left\{ \int_p^{\bar{c} - \bar{\alpha}} (x - \underline{c}) \cdot f(x + \bar{\alpha})\, dx + \frac{p - \underline{c}}{2} F(p + \bar{\alpha}) \right\}. \quad (1)$$

The proof is relegated to the appendix. In the following, we provide the intuition. First, note that a supplier with the lowest costs has the strongest incentive to deviate; i.e., we need to check whether he prefers to collude or not. If both suppliers follow the collusive bidding strategy β^c, they both bid R and win with a probability of $\frac{1}{2}$ each. The resulting profit for a supplier with lowest costs is displayed in the left-hand side of inequality (1). If one supplier deviates by placing a bid of b_i, the other will respond by bidding $b_i + \bar{\alpha}$ as long as this bid is above his costs. If the deviator succeeds in outbidding his competitor, he wins and he is paid a price equal to the costs of his competitor minus $\bar{\alpha}$. However, it might also be that at some point p he stops trying to underbid the competitor, if he has not been successful so far. In that case, both suppliers still have a winning probability of one-half. The right-hand side of inequality (1) describes the profit of a deviator with costs $c = \underline{c}$ who attempts to undercut his competitor and stops lowering the price at some level p.

Corollary 1. *A sufficient condition for a collusive perfect Bayesian equilibrium to exist is that the cost distribution function is concave.*

As we show in the appendix, a concave cost distribution function guarantees that the right-hand side of inequality (1) is maximized at $p = \bar{c} - \bar{\alpha}$; thus a deviator would stop lowering the price immediately. Sticking to the collusive outcome is then preferred. Furthermore, with a concave cost distribution function, bidders always prefer collusion at current prices to lowering their bid even outside the equilibrium path. This ensures that the collusive bidding strategy β^c is sequentially rational.

Proposition 1 and Corollary 1 have interesting implications for the existence of a collusive equilibrium. Collusion is more likely if

- the reservation price R is large, because this makes collusion profitable[10]
- the probability of facing a high cost competitor is low (which is implied by a concave cost distribution function), because this makes deviation unattractive;
- the individual preference component $\bar{\alpha}$ is not too small, because this implies that anyone trying to undercut his competitor to gain a higher probability of winning must lower the price sufficiently, which makes this behavior unattractive. Additionally, if $\bar{\alpha}$ is very small, the buyer has little reason not to simply run a PBRA.

Next, consider a buyer-determined reverse auction with $n > 2$ bidders. Increasing the number of bidders has two opposing implications for the stability

[9] $b^*(b_{-i}^t)$ exists, as the optimization is done over a finite set of possible bids.

[10] Although a large reserve price R makes collusion more likely, collusion can also occur if R is small, depending on the distribution of costs;

of collusion. On the one hand, having more bidders decreases the gain from sticking to high prices, as the probability of winning (which is equal to $1/n$) is lowered. On the other hand, more bidders make it less likely that by lowering the price one will succeed in pricing the others out of the market. The analysis becomes difficult as the dynamics outside the equilibrium path can become very complex. If one of the bidders is "outbid," i.e., if his cost is more than $\bar{\alpha}$ larger than the minimum bid, then an active bidder who, according to his collusive strategy β^c, stays within $\bar{\alpha}$ of the minimum bid still has a chance of winning of $1/n$. However, by lowering his bid just below the minimum bid, he can increase his chance of winning to $2/n$. The probability $2/n$ can be obtained by conditioning on whether the bidder who was outbid was the preferred supplier: If that is the case (with probability $1/n$), the supplier who placed the lowest bid wins the auction with probability 1; otherwise (with probability $1-1/n$), the deviating supplier wins the auction if he is the preferred supplier (with probability $1/(n-1)$).

We will provide two examples where we determine the equilibrium explicitly. Example 1 has an interesting dynamic and shows some complexities, which arise in the general case. Example 2 deals with the parameterizations we used in our experiment.

Example 1. In this example some bidders will, in equilibrium, lower their bids somewhat below the reservation price and then start to collude. Suppose all bidders have costs of either 0 or 10, each with probability $\frac{1}{2}$; the reserve price R is equal to 10; and $\bar{\alpha}=0.5$. The minimum bid decrement is $\epsilon=1$. Then a bidder with costs 0 might lower the price to 9 and stop there.[11] By doing this, he will avoid the competition of those bidders with costs of 10, but he will still collude with those with costs of 0. For example, in the case of four bidders, collusion at 10 would give a profit of $\frac{10}{4}=2.5$. If a bidder with cost 0 lowers the price to 9, his expected profit is given by

$$\left(\frac{1}{2}\right)^3\cdot\frac{9}{4}+3\cdot\left(\frac{1}{2}\right)^3\cdot\frac{9}{3}+3\cdot\left(\frac{1}{2}\right)^3\cdot\frac{9}{2}+\left(\frac{1}{2}\right)^3\cdot 9=\frac{135}{32}\geq 2.5.$$

Example 2. Table 1 lists six combinations for the number of bidders (n), the size of individual buyer preference ($\bar{\alpha}$), and the reserve price (R). These six combinations correspond to the six BDRA experimental treatments we conducted (see §3.1 for more details).

In all treatments costs are uniformly distributed on $[0,100]$. The $n=2$ cases (Treatments 1, 2, and 3) are dealt with in Proposition 1. (Note that the uniform distribution is (weakly) concave, and thus Corollary 1 applies.) The $n=4$ cases (Treatments 4, 5, and 6) are analyzed in Proposition 2.

[11] If the reserve price were set at 9 or lower (i.e., $R<\bar{c}$), collusion would start immediately without further bidding.

Table 1 **Parameters for BDRA Experimental Treatments**

Treatment number	Number of bidders (n)	Individual buyer preference ($\bar{\alpha}$)	Reserve price (R)
1	2	10	100
2	2	30	100
3	2	10	150
4	4	30	100
5	4	10	150
6	4	30	150

Proposition 2. *Assume there are more than two bidders, costs are uniformly distributed on* $[0,100]$, *and* $R\geq 100$. *The bidding strategy* β^c *describes a collusive equilibrium if* $\bar{\alpha}\geq 100\cdot(n-4)/(n-2)$ *and*

$$\frac{R}{n}\geq\int_{\bar{\alpha}/(n-2)}^{100-\bar{\alpha}} x\cdot\frac{2}{n}\cdot\frac{n-1}{100}\cdot\left(\frac{x+\bar{\alpha}}{100}\right)^{n-2}dx + \frac{\bar{\alpha}}{(n-2)}\cdot\frac{1}{n}\cdot\left(\frac{\bar{\alpha}\cdot(n-1)}{100\cdot(n-2)}\right)^{n-1}. \tag{2}$$

The proof is relegated to the appendix. Proposition 2 implies that collusion is an equilibrium for Treatments 4, 5, and 6.

2.4. Revenue Comparison

In cases where a BDRA leads to collusion and the buyer cannot fully reveal her preferences prior to the auction, there is a trade-off between using a PBRA or a BDRA. In the former case, price competition will be stronger, whereas in the latter case, the preferences can be better accommodated in the selection of the supplier. Formally, the total expected cost, including the horizontal quality component, of the buyer in a PBRA is given by

$$E[(c_i)^{(n-1)}]+E[\alpha_i]=E[(c_i)^{(n-1)}]+\frac{n-1}{n}\bar{\alpha}. \tag{3}$$

In the PBRA, bidders follow their dominant strategy. Consequently, the bidder with the lowest cost wins and is paid the second lowest cost. Because of the quality mismatch, the buyer looses on average $((n-1)/n)\bar{\alpha}$. In the BDRA, all bidders bid R, and the bidder for whom $\alpha_i=0$ wins. Thus the expected cost for the buyer in a BDRA is R. Therefore, the expected difference between the two mechanisms is given by

$$\frac{n-1}{n}\bar{\alpha}-(R-E[(c_i)^{(n-1)}]). \tag{4}$$

A BDRA has the negative effect of higher prices that amounts to an average $R-E[(c_i)^{(n-1)}]$, but at the same time leads to better accommodating buyers' preferences, worth $((n-1)/n)\bar{\alpha}$.

3. Experimental Evidence

3.1. Design of the Experiment

Like in the previous section, we work with binary individual buyer preferences, i.e., $\alpha_i \in \{0, \bar{\alpha}\}$, such that in each auction exactly one of the bidders is preferred. We vary $\bar{\alpha}$ so that in some treatments $\bar{\alpha} = 10$ and in other treatments $\bar{\alpha} = 30$. This variation captures the idea that the supplier-specific buyer preferences can differ in importance compared to the overall project size. We also vary the reserve price at $R = 100$ and $R = 150$ as well as the number of bidders at $n = 2$ and $n = 4$. In all treatments, $c_i \sim U[0, 100]$ for all suppliers i.

The focus of our design is on the influence of the buyer preferences, the reserve price, and the number of bidders on the performance of the BDRA. The six BDRA treatments we conducted are listed in Table 1. Additionally, we conducted price-based auctions with two and four bidders ($n = 2$ and $n = 4$), which we use to calculate the buyer's total cost if the buyer does not take her supplier-specific preferences (α_i) into account. If bidders follow their dominant strategy, the reserve price does not matter in PBRAs, so we used the reserve price of $R = 150$.

Comparing Treatments 1 and 3 as well as Treatments 4 and 6 allows us to test the prediction of the theory that collusion exists regardless of the reserve price. Comparing Treatments 3 and 5 as well as Treatments 2 and 4 allows us to test the prediction that collusion exists regardless of the number of bidders. Finally, comparing Treatments 1 and 2 as well as Treatments 5 and 6 tests the prediction of the theory that collusion is independent of $\bar{\alpha}$.

Expression (4) implies that in all six treatments, the expected buyer cost from the BDRA will be higher than the expected buyer cost in the PBRA. We will test this prediction by comparing the total expected cost of the buyer in each of our BDRA treatments to a corresponding expected total cost of the buyer in the PBRA treatment with the same number of bidders.

For each number of bidders ($n = 2$ and $n = 4$), we conducted each treatment with the same realizations of c_i and the same matching protocol, which we pregenerated prior to the start of the experiments.[12] This ensures that any differences in behavior we observe between the treatments with the same number of bidders are due to the factor we vary and not to different realizations of the parameters.

We used the between subjects design. Each BDRA treatment included five or six independent cohorts, and both PBRA treatments had three cohorts. Each cohort included 6 participants in the $n = 2$ treatments and 12 participants in the $n = 4$ treatments. In total, 372 participants, all in the role of supplier, were included in our study. We randomly assigned participants to one of the treatments. Each person participated only one time. We conducted all experimental sessions at a major university in the European Union. We recruited participants using the online recruitment system ORSEE (Greiner 2015). Earning cash was the only incentive offered.

Upon arrival at the laboratory, the participants were seated at computer terminals. We handed out written instructions to them, and they read the instructions on their own. When all participants finished reading the instructions, we read the instructions to them aloud, to ensure public knowledge about the rules of the game.

After we finished reading the instructions, we started the game. In each session, each participant bid in a sequence of 28 auctions. The first three auctions were practice periods to help participants better understand the setting. We used random matching that we kept the same within each cohort. At the beginning of each round, the participants in a cohort were divided into three groups of bidders according to the prespecified profile matching protocol. Each group of bidders competed for the right to sell a single unit to a computerized buyer.

We programmed the experimental interface using the z-Tree system (Fischbacher 2007). The screen included information about the subject's cost c_i, the horizontal quality $\bar{\alpha}$, and the reserve price R. Bidders could also observe all bids placed in real time.

At the end of each round, we revealed the same information in all treatments. This information included the bids of all bidders, the α_i's, and the winner in that period's auction. The history of past winning prices and quality adjustment α_i in the session were also provided.

For each auction in each period, the auction winners earned the difference between their price bids and their costs c_i, whereas the other bidders earned zero. We computed cash earnings for each participant by multiplying the total earnings from all rounds by a predetermined exchange rate and adding it to a 2.50€ participation fee. Participants were paid their earnings from the auctions they won, in private and in cash, at the end of the session.

3.2. Results: Average Buyer's Cost

Table 2 displays the buyer's average total cost and standard errors for the six conditions in our study under the BDRA and the PBRA. We also provide three theoretical benchmarks—collusion, the price-based

[12] Inadvertently, cost realizations in Treatment 4 were also pregenerated, but differed slightly from cost realizations in other four-bidder treatments. This had no effect on any of the analysis.

Table 2 Summary of Average Buyer's Total Cost Compared to Theoretical Predictions

		Buyer's total cost (observed)		Theoretical prediction		
Treatment	Description	BDRA	PBRA	BDRA (collusive benchmark)	PBRA	Binding auction with α included
1	$n=2$, $\bar{\alpha}=10$, $R=100$	74.05** (1.08)	68.52†† (3.75)	100	68.19	68.13
2	$n=2$, $\bar{\alpha}=30$, $R=100$	89.30** (1.16)	78.52††† (3.75)	100	78.19	81.98
3	$n=2$, $\bar{\alpha}=10$, $R=150$	126.85** (5.68)	68.52††† (3.75)	150	68.19	68.56
4	$n=4$, $\bar{\alpha}=30$, $R=100$	71.49** (3.96)	57.34†† (2.13)	100	59.71	59.86
5	$n=4$, $\bar{\alpha}=10$, $R=150$	56.38** (5.10)	42.34† (2.13)	150	44.70	45.43
6	$n=4$, $\bar{\alpha}=30$, $R=150$	100.63** (6.50)	57.34††† (2.13)	150	59.71	59.86

**$p \le 0.01$ (comparison between observed and theoretical; for the PBRA format none of the differences are significant); †$p \le 0.1$, ††$p \le 0.05$, †††$p \le 0.01$ (comparisons between BDRA and PBRA).

auction, and the binding auction with α included;[13] all statistics are based on cohort averages. In the PBRA, this cost is given by the lowest price bid plus the average misfit cost of supplier-specific misfit $((n-1)/n)\bar{\alpha}$.

We summarize the analysis in Table 2 as the following results:

Result 1. Average buyer's total cost is significantly below the collusive benchmark under the BDRA format (all p-values are below 0.001).

Result 2. Under the PBRA format, the average buyer's total cost is not significantly different from either the theoretical PBRA prediction or the binding auction with α included (none of the p-values are below 0.1).

Result 3. The average buyer's total cost is significantly higher under the BDRA format than under the PBRA format in all six conditions.

We also report the effect of our treatment variables on the buyer's total cost.

Result 4. If bidders were able to perfectly collude, the buyer's total cost would not have been affected by the number of bidders, but comparing Treatments 3 and 5 as well as Treatments 2 and 4 tells us that for $\bar{\alpha}=10$, the average cost decreased by 70.68 (over 50%) when the number of bidders increased from two to four ($p<0.001$). The difference (17.8, which is still nearly 20%) is smaller, but still highly significant when $\bar{\alpha}=30$.[14]

Result 5. Collusion implies that bidders should bid at the reserve, so the buyer's total cost should decrease by 50 between treatments with $R=150$ and $R=100$. For the case of $n=2$, we compare Treatments 1 and 3 and observe that the cost decreased by 52.66, which is not significantly different from 50 ($p=0.586$). But for the case of $n=4$, we compare Treatments 4 and 6 and observe that the cost decreased by only 29.14, which is significantly below 50 ($p<0.001$).

Result 6. Collusion should not be affected by the magnitude of $\bar{\alpha}$; however, the average buyer's total cost increased by 15.15 with two bidders when $\bar{\alpha}$ increased from 10 to 30 ($p<0.001$ when comparing Treatments 1 and 2) and by 45.34 with four bidders ($p=0.0002$ when comparing Treatments 4 and 5).

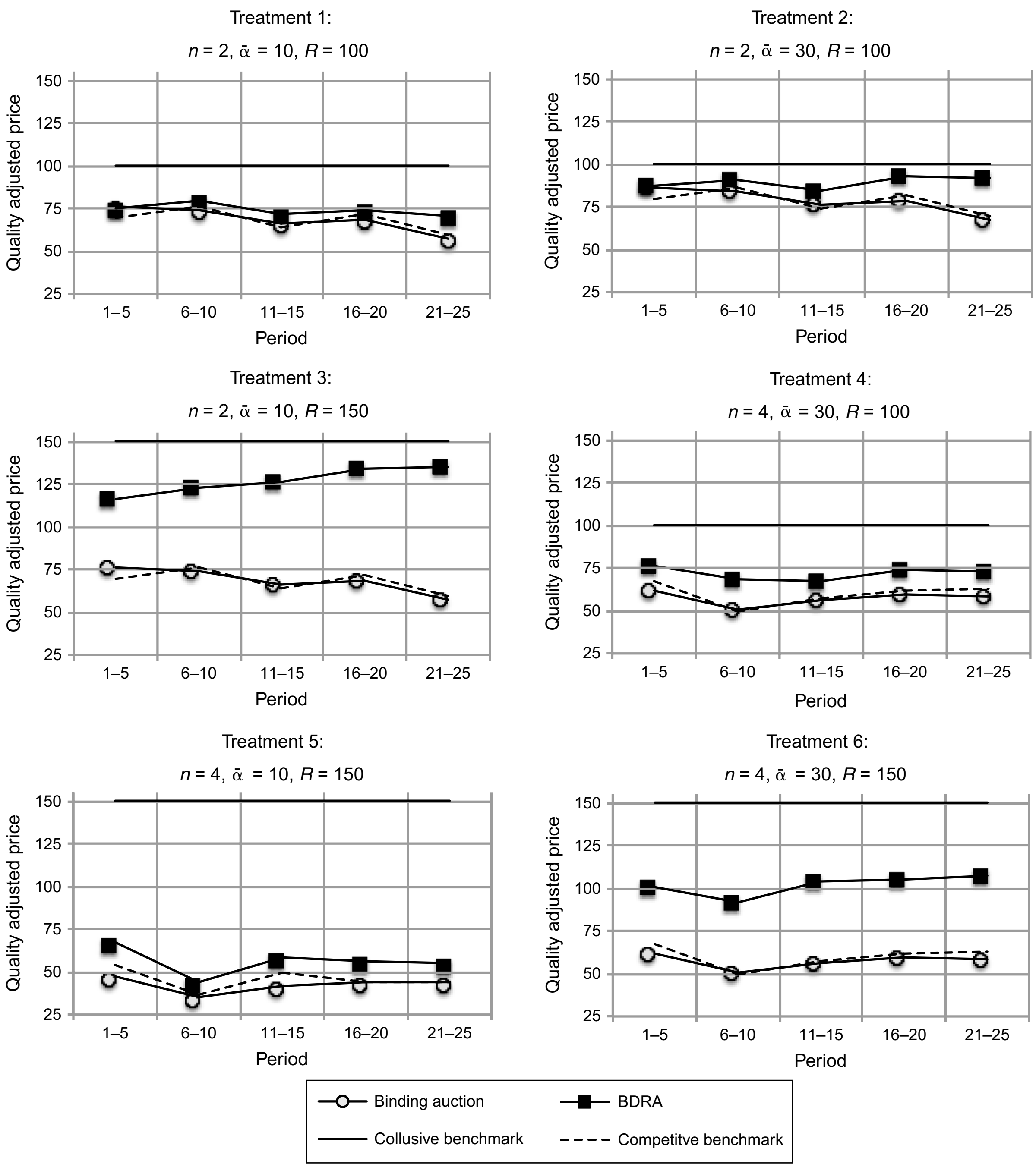
Additionally, a t-test based on cohort averages tells us that the lowest bid increased by 14.79 with two bidders when $\bar{\alpha}$ increased from 10 to 30 ($p<0.0001$), and by 36.93 with four bidders ($p=0.0027$); that is, a higher $\bar{\alpha}$ harms the buyer in two ways: it weakens competition and also sometimes results in a larger misfit.

In Figure 1 we plot, for each of the six conditions, the average buyer's total cost over time (aggregated into five-period blocks) under the BDRA and PBRA formats. Also, for comparison, we plot theoretical predictions: the collusive benchmark (R) is the benchmark for the BRDA format, and the competitive

[13] The binding auction with α included provides a benchmark for average buyer cost in the case in which the buyer is able to communicate the α information before the auction. It is a reasonable benchmark because we know enough about open-bid auctions to know that in such auctions people would bid approximately as theory predicts, and we include this for the purpose of providing a benchmark as to how much of a benefit providing α would. Note that the revenue maximizing auction would underweigh the alpha component (see Footnotes 5 and 18). Just including alpha might be worse than a price-based auction (Engelbrecht-Wiggans et al. 2007).

[14] The fact that collusion decreases with the number of bidders has also been pointed out in other contexts (see, for example, Huck et al. 2004).

Figure 1 Average Buyer's Total Cost Over Time and Theoretical Benchmarks



benchmark is the expected buyer's total cost when the price ends up at the second lowest cost.

To formally analyze how the buyer's total cost in BDRA treatments is affected by the treatment variables (the number of bidders, the reserve price, the size of the $\bar{\alpha}$ parameter) as well as the bidder experience, we estimate a regression model (with random effects) in which the dependent variable is the buyer's total cost, and independent variables, along with estimated coefficients, are listed in Table 3. This regression uses data from BDRA treatments only.

The coefficients for the three treatment dummy variables echo Results 4–6. Coefficients of the *Period* variable and of the interaction variables between *Period* and the treatment variables tell us how the buyer's total cost is affected by bidder experience.

Result 7. When the reserve price is low (100) and $\bar{\alpha}$ is low (10), buyer's cost decreases with experience (not significantly), but the decrease becomes strongly significant when the number of bidders is large. There is some collusion that is occurring even in treatments

Table 3 Regression Estimates for the Effect of Treatment Variables and Bidder Experience on the Expected Cost of the Buyer

Dependent variable: *Buyer's total cost*	Description	Coefficient (standard errors)
β_o	Constant	74.86** (2.934)
n-Dummy	1 when $n = 4$, 0 otherwise	−34.19** (3.548)
R-Dummy	1 when $R = 150$, 0 otherwise	28.18** (3.503)
$\bar{\alpha}$*-Dummy*	1 when $\bar{\alpha} = 30$, 0 otherwise	26.78** (3.425)
Period	Period number 1–25	−0.21 (0.141)
Period × (*R-Dummy*)	Interaction variables between treatment variables and the period number	0.74** (0.178)
Period × (*n-Dummy*)		−0.723** (0.177)
Period × ($\bar{\alpha}$*-Dummy*)		0.596** (0.173)
R^2	0.289	
Observations (groups)	2,625 (318)	

** $p < 0.001$

with low $\bar{\alpha}$ (Treatment 1 and Treatment 5) because buyer's cost is still significantly higher under the BDRA format than under the PBRA format; collusion may be decreasing over time.

Result 8. Higher reserve price reverses this learning trend, making collusion easier to sustain, as is evidenced by the positive and significant coefficient of *Period* × (*R-Dummy*).

Result 9. Higher $\bar{\alpha}$ also makes collusion easier to sustain, as is evidenced by the positive and significant coefficient of *Period* × ($\bar{\alpha}$*-Dummy*).

3.3. Results: Bidding Behavior

In this section we focus on the individual bidding behavior. First, we briefly describe bidding behavior in PBRAs.

We plot bids as a function of cost (for losing bidders only) in Figure 2(a) for two bidders and in Figure 2(b) for four bidders. We also estimate a regression model (with random effects) using losing bids in PBRAs with the dependent variable *Bid* and independent variable *Cost*. The coefficient of *Cost* is 0.964 (std. err. = 0.025), which is not different from 1 at the 5% level of significance. There is also a small but significant constant term (9.52, std. err. = 1.78).[15]

Result 10. The bidding in PBRAs is close to behavior implied by the dominant strategy; almost 80% of losing bidders drop out within 10 ECU of their cost, and the cost coefficient in regression is not significantly different from 1.

To gain insight into how participants bid in our BDRA treatments, we show distributions of bids for the six BDRA treatments in Figure 3. Figure 3 indicates that some, but not all, of the bidders in all of the BDRA treatments attempt to collude, because in all six treatments the modal bid is at the reserve. However, the proportion of collusive bids varies with our treatment variables. To formally analyze how bids are affected by treatment variables, as well as by the bidders' cost and experience, we estimate a Tobit model (because, as is clear from Figure 3, bids are censored by the reserve) with random effects, with the dependent variable *Bid* and independent variables listed in Table 4.

To show robustness, we estimate four models, starting with *Cost* only (Model 1) and then adding *Period* to control for bidder experience (Model 2), adding treatment variables (Model 3), and finally adding interaction effects between the treatment variables and *Cost* as well as the treatment variables and *Period* (Model 4).

Result 11. Contrary to theoretical predictions, BDRA bids are affected by cost. This relationship is weaker for high reserve and high $\bar{\alpha}$ (positive and significant *Cost* × (*R-Dummy*) and *Cost* × ($\bar{\alpha}$*-Dummy*)), and stronger for more bidders (positive and significant *Cost* × (*n-Dummy*)).

Result 12. Bids slightly increase with experience in two-bidder auctions (positive *Period* variable). This increase is higher for high reserve and high $\bar{\alpha}$ (positive and significant *Period* × (*R-Dummy*) and *Period* × ($\bar{\alpha}$*-Dummy*)) and lower for four-bidder auctions (negative and significant *Period* × (*n-Dummy*)). Interestingly, this slight increase in average bids does not translate into higher buyer's total cost (Result 7).

We can also see (Models 3 and 4) that the effect of treatment variables on bids is similar to the effect of treatment variables on the buyer's total cost.

Figure 4 displays bid–cost pairs of bidders that did not win in the six BDRA treatments. In contrast to the PBRA treatments, the correlations between cost and bid are weaker, which indicates less competition. We also observe, in all six BDRA treatments, a fair number of bids at the reserve.

[15] As is typical with open-bid auctions, we observe jump bidding in all our treatments. A consequence of jump bidding is that prices might drop quite fast, not giving high-cost bidders an opportunity to lower their bid. Jump bidding explains some of the observations in the upper right corner of Figure 2(b). If we use only the second lowest bids in the $n = 4$ treatments in the regressions, the constant term is significantly lower (3.442, std. err. = 0.052), and the cost coefficient remains almost unchanged.

Figure 2 Losers' Bidding Behavior in PBRAs

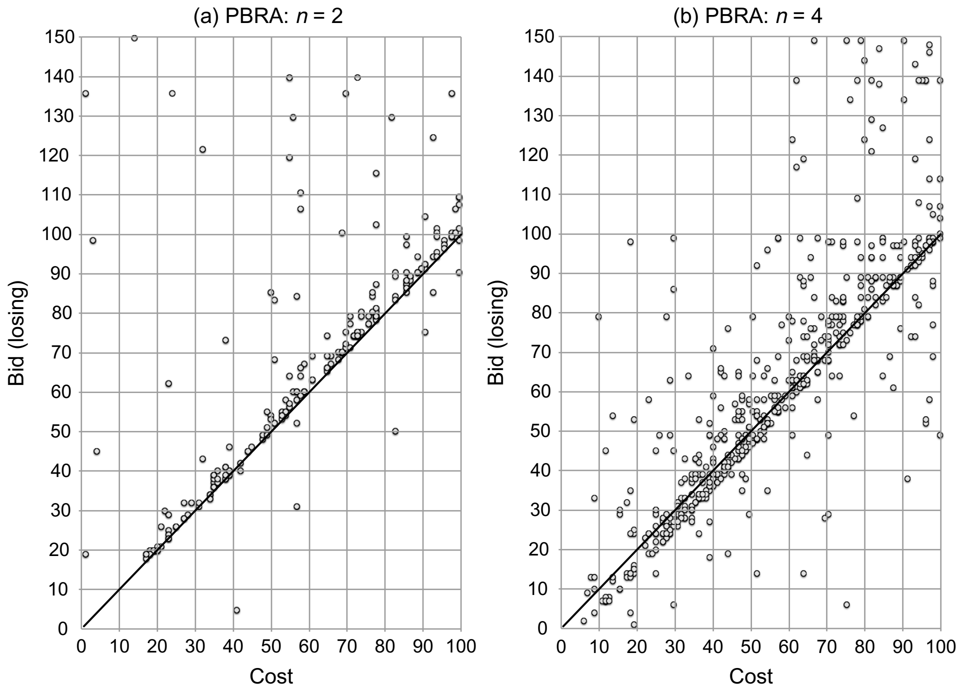
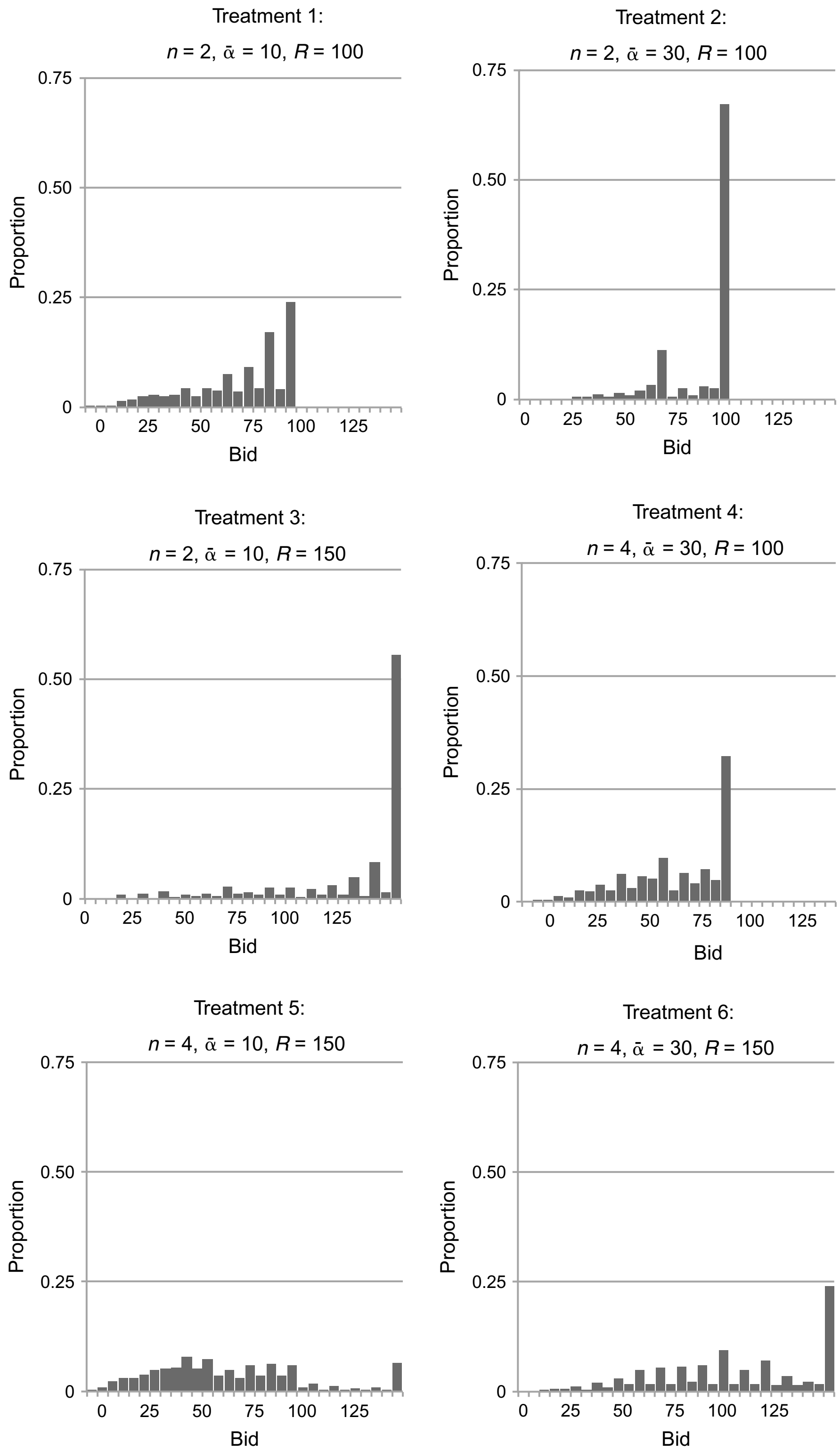


Table 4 Estimates for the Effect of Treatment Variables and Bidders' Experience on Bids in the BDRA Treatments

Dependent variable: *Bid*	Model 1	Model 2	Model 3	Model 4
β_o	65.75**	60.47**	50.96**	46.61**
	(1.674)	(1.807)	(2.615)	(3.203)
Cost	0.56**	0.56**	0.56**	0.61**
	(0.013)	(0.013)	(0.013)	(0.028)
Period		0.39**	0.39**	0.52**
		(0.050)	(0.050)	(0.110)
n-Dummy			−45.69**	−42.80**
			(2.867)	(3.741)
R-Dummy			37.47**	35.41**
			(2.712)	(3.46)
$\bar{\alpha}$*-Dummy*			35.49**	41.38**
			(2.672)	(3.422)
Cost × (*R-Dummy*)				−0.19**
				(0.031)
Cost × (*n-Dummy*)				0.31**
				(0.034)
Cost × ($\bar{\alpha}$*-Dummy*)				−0.30**
				(0.031)
Period × (*R-Dummy*)				0.85**
				(0.120)
Period × (*n-Dummy*)				−1.38**
				(0.132)
Period × (α*-Dummy*)				0.62**
				(0.117)
Log likelihood	−30,848.699	−30,817.496	−30,706.390	−30,580.866
Observations (groups)	7,924 (318)			

$^{**}p < 0.001$.

Figure 3 Distribution of Bids in the BDRA Treatments



In Table 5 we show the proportion of BDRAs that ended in a collusive outcome. We classify an outcome as collusive if all suppliers have a positive probability of winning and all bids are above costs. Furthermore, we display the average lowest bid given that the BDRA ended in with a collusive outcome.

Table 5 shows that there are two reasons why prices in the BDRA are lower than predicted for the collusive

Figure 4 **Bid as a Function of Cost of Losing Bidders for the BDRA Treatments**

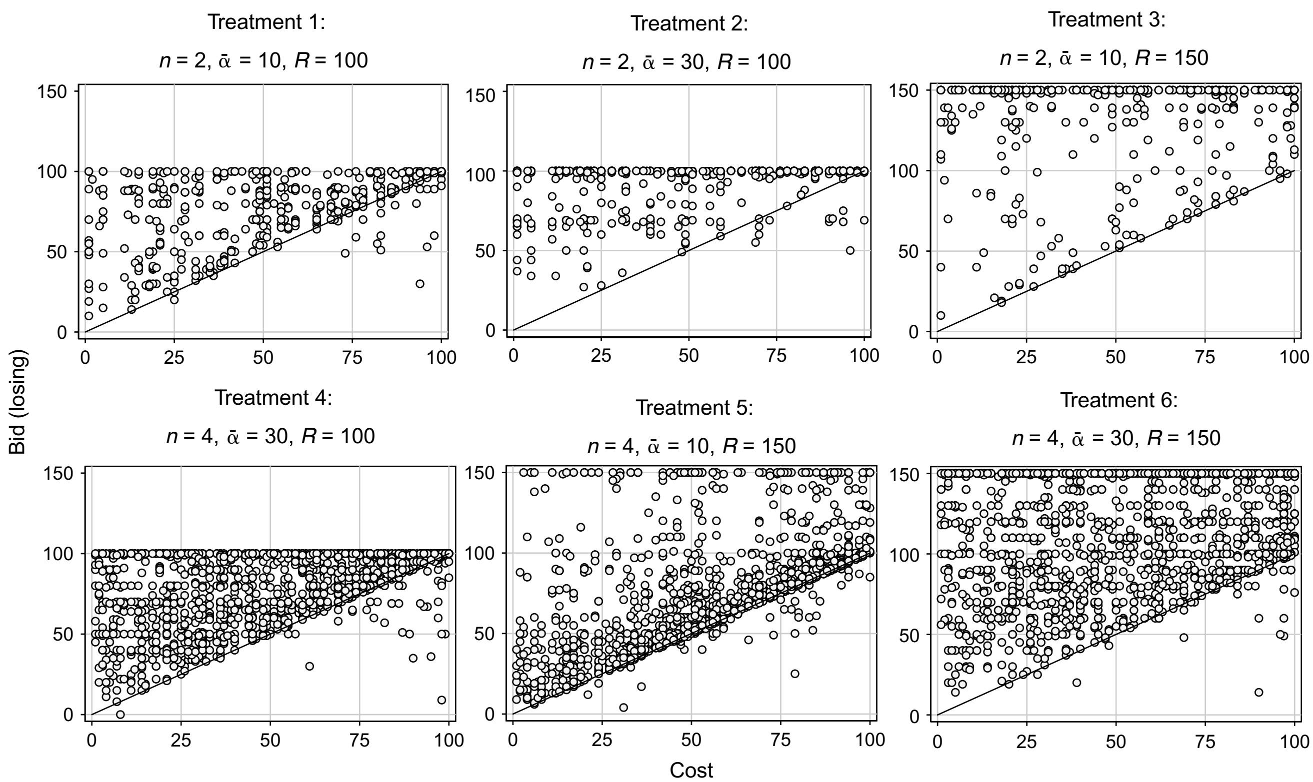


Table 5 **Proportion and Characteristics of Collusive Outcomes in BDRA Treatments**

Treatment	Proportion of collusive outcomes (%)	Average lowest bid in collusive auctions (standard error)
1: $n=2$, $\bar{\alpha}=10$, $R=100$	44.89	80.81 (1.46)
2: $n=2$, $\bar{\alpha}=30$, $R=100$	79.78	90.89 (0.83)
3: $n=2$, $\bar{\alpha}=10$, $R=150$	73.60	136.64 (1.68)
4: $n=4$, $\bar{\alpha}=30$, $R=100$	33.33	80.61 (1.76)
5: $n=4$, $\bar{\alpha}=10$, $R=150$	10.00	112.91 (6.68)
6: $n=4$, $\bar{\alpha}=30$, $R=150$	39.78	122.80 (2.40)

equilibrium. First, not all BDRAs end in a collusive outcome, and second, even if the outcome is collusive bids, are, on average, below reserve.[16]

[16] BDRAs with four bidders sometimes ended in partial collusion, meaning that at least two of the bidders stopped bidding above cost while still having a positive probability of winning. The proportion of BDRAs with four bidders that ended in partial collusion is 85% in Treatment 4, 66% in Treatment 5, and 87% in Treatment 6. The average differences between the lowest bid and lowest cost in those BDRAs are 41.83 (28.36) in Treatment 4, 35.11 (25.76) in Treatment 5, and 73.78 (53.93) in Treatment 6.

4. Discussion and Conclusions

We have shown that the common practice in procurement of using dynamic buyer-determined reverse auctions allows suppliers to collude on high prices. Collusion can be supported because of the uncertainty in the buyer's final decision-making process. Suppliers have a *chance* of winning at high prices, which might be more attractive than starting a price war and winning at a considerably lower price (with a possibly higher probability). This reasoning can be applied to other circumstances in which the uncertainty of the final decision allows firms to collude in the first place. For example, all private or public tenders in which prices and conditions are negotiated and offers are displayed are prone to the same form of collusion as described above. The reason is that participating firms can react to their opponents' offers, and, most importantly, the final decision is uncertain.

There are several ways the buyer can counteract the problem of collusive behavior. Simple ones would be to precisely communicate α before the reverse auction starts or to conduct a PBRA. Both solutions resolve the uncertainty around the decision process, and thus collusion would no longer be sustainable.[17] However, our practical experience showed us that the manager

[17] Alternatively, the buyer could announce after each round a provisional winner, such that the suppliers can deduce α_i by themselves.

in charge of the procurement does not have (at least not alone by herself) the final say on who will be awarded the contract. This is particularly true if she is using a nonbinding auction. Consequently, she has no information on the exact α and therefore cannot credibly communicate a clear decision rule. From a practical point of view, the best alternative would be to commit to a clear scoring rule that takes the non-price attributes of the different suppliers into account. However, this implies that all parties involved in the decision-making process—procurement, logistic, quality, management—have to become involved even before the auction is designed. For example, a supplier who offers a better quality such that the expected additional costs for recalls are expected to be lower by 3% should be given a price preference of 3% in the auction. If all different dimensions are adequately quantified ex ante, then a price auction will lead to the efficient outcome.[18] If the buyer, however, does not succeed in getting the uncertainty out of the process, then in a dynamic buyer-determined auction collusion can prevail.[19]

Our experimental results confirm the prediction that dynamic buyer-determined reverse auctions often result in high prices and are also more expensive in terms of buyer's total cost than binding auctions. Consistent with intuition, but in contrast to theoretical predictions, we found that collusion at high prices becomes less likely if the number of bidders increases, if the reserve prices decreases, and if the uncertainty about the decision criteria decreases.

The latter issue implies that buyers who use buyer-determined reverse auctions could reduce collusion by reducing the uncertainty surrounding the decision-making process. This includes providing the seller with information on the attributes, which enter the decision, such as quality, reliability, capacity, and reputation. To reduce the uncertainty further, buyers might also communicate to the suppliers the organizational procedure of the decision-taking process, e.g., whether a committee or the top management will take the final decision.

[18] There exists an extensive literature on the optimal mechanism and auction design in a multidimensional framework (Che 1993, Branco 1997, Morand and Thomas 2006, Rezende 2009, among others), once these different dimensions are quantified. As a general result, it is advisable for the auctioneer to use a scoring rule where nonprice attributes are underrepresented, because this fosters price competition between the suppliers.

[19] Collusion can also be prevented by the use of a static mechanism, e.g., a sealed-bid auction, or a dynamic contest with a hard ending rule. If in the latter case the last-second bids are accepted for sure, then this mechanism becomes similar to a sealed-bid format, i.e., a static auction. If the acceptance of last second bids is uncertain, high price equilibria can occur (Ockenfels and Roth 2006).

Acknowledgments

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Appendix

Proof of Proposition 1. If both bidders bid according to the collusive strategy β^c, then bidding ends after the first round, and the expected profit of supplier i is given by

$$\pi_i^c(c_i) = \frac{R - c_i}{2}. \tag{5}$$

Now consider a deviation from the equilibrium strategy. Undercutting the opponent's bid by less than $\bar{\alpha}$ cannot be optimal because this reduces the profit in case of winning without affecting the probability of winning. Thus, the deviator has to lower his bid by more than $\bar{\alpha}$. If by doing so a deviator increases his probability of winning, this immediately implies that the other supplier has a zero probability of winning if the BDRA were to stop at this point. This supplier will, according to the collusive bidding strategy β^c, lower his bid as well. Consequently, a deviator can only increase his probability of winning if his bid is so low that the other will not follow suit anymore. This is the case if the bid b_i is smaller than $c_j - \bar{\alpha}$. The expected profit of a deviator that lowers his bid until his opponent drops out or his bid is equal to some stopping price p is given by

$$\pi_i^d = \int_p^{\bar{c}-\bar{\alpha}} (x - c_i) \cdot f(x + \bar{\alpha})\, dx + \frac{p - c_i}{2} \cdot F(p + \bar{\alpha}). \tag{6}$$

Comparing (5) and (6) shows that the incentive to deviate is largest for a supplier with lowest costs ($c_i = \underline{c}$). This leads to expression (1) in Proposition 1. □

Proof of Corollary 1. The first derivative of the expected deviation profit (6) with respect to the stopping price p is given by

$$\frac{\partial \pi_i^d}{\partial p} = \frac{F(p + \bar{\alpha}) - (p - c_i) \cdot f(p + \bar{\alpha})}{2}. \tag{7}$$

Hence, the deviator wants to stop as early as possible if (7) is positive. This requirement is always fulfilled if F is concave, as then $F(x) \geq x \cdot f(x)$ holds. Note that in this case the collusive bidding strategies β^c indeed constitute a perfect Bayesian equilibrium. Even outside the equilibrium path, if someone is undercut, it is optimal to place the highest bid that is still in the range of $\bar{\alpha}$ of the other bid. Any higher bid would lead to a zero probability of winning. Any lower bid that is still in the range of $\bar{\alpha}$ of the other bid would also result in a winning probability of one-half if the auction were to stop at this point, but with a lower price in case of winning. Last, trying to outbid the deviator cannot be optimal since (7) is positive. □

Proof of Proposition 2. We show that if bidders behave according to the collusive strategy β^c, then no one can make himself better off in the BDRA by deviating. Again we concentrate on a bidder with lowest costs. The expected profit of such a bidder from collusion is given by

$$\pi_i^c = \frac{R}{n}. \tag{8}$$

If he instead tries to outbid one competitor by lowering his bid at most to p and then colludes with the remaining $n-1$ competitors, his profit can be written as

$$\int_p^{100-\bar{\alpha}} x \cdot \frac{2}{n} \cdot \overbrace{\frac{n-1}{100} \cdot \left(\frac{x+\bar{\alpha}}{100}\right)^{n-2}}^{(n-1)\cdot f(x+\bar{\alpha})\cdot F(x+\bar{\alpha})^{n-2}} dx + p \cdot \frac{1}{n} \cdot \overbrace{\left(\frac{p+\bar{\alpha}}{100}\right)^{n-1}}^{F(p+\bar{\alpha})^{n-1}}. \tag{9}$$

Note that by outbidding one competitor the winning probability of the deviator increases to $2/n$, because he then wins not only if he is preferred, but also if the outbid competitor is the preferred supplier. Optimizing expression (9) with respect to the stopping price p yields $p^* = \bar{\alpha}/(n-2)$ for $n > 2$. Hence, the profit from trying to outbid one of the $n-1$ competitors optimally is given by

$$\pi^d = \int_{\bar{\alpha}/(n-2)}^{100-\bar{\alpha}} x \cdot \frac{2}{n} \cdot \frac{n-1}{100} \cdot \left(\frac{x+\bar{\alpha}}{100}\right)^{n-2} dx + \frac{\bar{\alpha}}{(n-2)} \cdot \frac{1}{n} \cdot \left(\frac{\bar{\alpha}\cdot(n-1)}{100\cdot(n-2)}\right)^{n-1}. \tag{10}$$

Now it remains to be shown that the deviating bidder has no incentive to lower his bid further when the first competitor has dropped out. To see this suppose that $m < n$ bidders are still active when the deviator reduces his bid to p_m, i.e., one (or more) competitors has already dropped out. Then, the expected profit from trying to outbid a further competitor by reducing the own bid at most to p_{m-1} can be expressed as

$$\int_{p_{m-1}}^{p_m} x \cdot \frac{(n-m+2)}{n} \cdot \overbrace{\frac{m-1}{100} \cdot \left(\frac{x+\bar{\alpha}}{100}\right)^{m-2}}^{(m-1)\cdot f(x+\bar{\alpha})\cdot F(x+\bar{\alpha})^{m-2}} dx + p_{m-1} \cdot \frac{(n-m+1)}{n} \cdot \overbrace{\left(\frac{p_{m-1}+\bar{\alpha}}{100}\right)^{m-1}}^{F(p_{m-1}+\bar{\alpha})^{m-1}}. \tag{11}$$

Observe that by outbidding a further competitor the winning probability of the deviator increases to $(1-(m-2)/n)$, as he then wins as long as none of the surviving competitors is the preferred supplier. The first derivative of (11) with respect to the stopping price p_{m-1} is given by

$$\frac{(p_{m-1}+\bar{\alpha})^{m-2}}{n} \cdot [p_{m-1}\cdot(n-2m+2) + \bar{\alpha}\cdot(n-m+1)]. \tag{12}$$

A bidder has no incentive to lower his bid as long as expression (12) is positive for all p_{m-1}. Because expression (12) is decreasing in m, it suffices to show that it is positive for $m = n-1$ to prove that it is positive for all $m \le n-1$. At this point it is easy to see that it can never be optimal to outbid more than half of the competitors, as expression (12) is always positive if $m \le (n+2)/2$.

If we plug in $m = n-1$ in expression (12), we get a condition that guarantees that no bidder has an incentive to lower his bid further once a bidder has dropped out:

$$\frac{(p_{n-2}+\bar{\alpha})^{n-3}}{n} \cdot [p_{n-2}\cdot(4-n) + 2\cdot\bar{\alpha}] \ge 0. \tag{13}$$

For $n \le 4$, this condition is always fulfilled. For $n > 4$, the term on the left reaches its minimum when the price reaches its maximum. Because the price is bounded at $100-\bar{\alpha}$, we can state a sufficient condition as

$$\bar{\alpha} \ge 100 \cdot \frac{n-4}{n-2}. \tag{14}$$

For these parameter values, the collusive bidding strategies β^c constitute an equilibrium.

To prove that also in a perfect Bayesian equilibrium collusion is possible, we next show that the way bidders react to a deviating competitor as defined in β^c determines an upper bound for the deviation incentive for any sequentially rational strategy. Not following suit if a deviator bids more than $\bar{\alpha}$ below the own bid cannot be optimal, because this leads to a zero probability of winning. Hence, no bid higher than defined by our collusive bidding strategy can be a best response to a deviation. As a consequence, the winning probability of a deviator and thereby also the expected profit from deviating cannot be larger when competitors behave sequentially rationally than when they behave according to the collusive bidding strategy β^c. Thus, in any perfect Bayesian equilibrium, collusion remains to be an equilibrium outcome if it is an equilibrium given our collusive bidding strategy β^c. □

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