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# Efficient Implementation of Collective Extended Producer Responsibility Legislation

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Extended producer responsibility (EPR) is a policy tool that holds producers financially responsible for the post-use collection, recycling, and disposal of their products. Many EPR implementations are collective—a large collection and recycling network (CRN) handles multiple producers' products in order to benefit from scale and scope economies. The total cost is then allocated to producers based on metrics such as their return shares by weight. Such weight-based proportional allocation mechanisms are criticized in practice for not taking into account the heterogeneity in the costs imposed by different producers' products. The consequence is cost allocations that impose higher costs on certain producer groups than they can achieve independently. This may lead some producers to break away from collective systems, resulting in fragmented systems with higher total cost. Yet cost efficiency is a key legislative and producer concern. To address this concern, this paper develops cost allocation mechanisms that induce participation in collective systems and maximize cost efficiency. The cost allocation mechanisms we propose consist of adjustments to the widely used return share method and include the weighing of return shares based on processing costs and the rewarding of capacity contributions to collective systems. We validate our theoretical results using Washington state EPR implementation data and provide insights into how these mechanisms can be implemented in practice.

**Keywords:** environment; regulation; extended producer responsibility; recycling; cost allocation

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## 1. Introduction

Extended producer responsibility (EPR) is a policy tool that holds producers financially responsible for the post-use collection, transportation, and processing (i.e., dismantling, shredding, and/or recycling) of their products (Lifset 1993, Lindhqvist 2000). Following its adoption in Europe through the Waste Electrical and Electronic Equipment (WEEE) Directive (European Union 2003), EPR has rapidly become the main policy tool used in the United States for managing electronic waste ("e-waste"); 24 states have passed EPR-based e-waste bills (Electronics Take-Back Coalition 2014). EPR has been adopted for other product categories as well (Product Stewardship Institute 2014).

The proper handling of e-waste is costly for most products, and EPR introduces a significant cost burden

on the electronics industry, the main stakeholder group it affects. Hence, in the phase where EPR legislation is operationalized, the focus—of not only the producers but also the architects, enforcers, and operators of these systems—typically turns to establishing a well-functioning system and minimizing the implementation cost subject to imposed regulatory standards (Washington Materials Management & Financing Authority (WMMFA) 2012, European Recycling Platform 2012, Department for Business, Innovation and Skills 2013).<sup>1</sup> This focus on cost efficiency has resulted in the promotion of collective implementations, where e-waste belonging to many producers

<sup>1</sup> Implementations where cost effectiveness is not a key concern (such as early monopolistic implementations of WEEE in the European Union (EU)) are not within the scope of this paper.

is collected and processed. This is because collective systems benefit from economies of scale by pooling return volumes and by leveraging the same to obtain lower prices from competing service providers (Lorch 2010, WEEE Forum 2014). Furthermore, collective systems leverage network synergies (i.e., more efficient return volume routing when sharing capacitated resources) better and reduce compliance monitoring costs.

However, maintaining producers' participation in collective systems can be challenging because of concerns associated with the allocation of systemwide costs. The most prevalent cost allocation mechanism in practice is a linear rule based on return share, where the average cost incurred is multiplied with the return volume corresponding to each producer. This mechanism does not differentiate between producers even if their products impose heterogeneous costs on the system. Consequently, some producers argue that they can lower their EPR compliance (i.e., collection and recycling) costs by establishing *independent plans*, i.e., collection and recycling networks (CRNs) they develop to handle their own products. In the EU, some producers have left or stated their interest in leaving existing collective systems and establishing independent plans (Shao and Lee 2009, IPR Works 2012). Similar action is taking place in the United States. In Washington, two independent plan proposals were filed in 2009 by two producer groups who believed their stand-alone costs would be lower than their cost allocation under the collective system run by the state authority. Although these proposals were declined on compliance grounds, such efforts are continuing (Electronic Manufacturers Recycling Management Company 2013), and the emergence of independent plans is a real possibility. In Oregon, three producer groups have already chosen not to participate in the state's default collective system.

Fragmentation in collective EPR implementations results in systemwide cost inefficiency. For the defecting producers, defection is inferior to remaining in a collective system that allocates them a lower cost than what they can achieve independently. Moreover, producers remaining in the collective system may incur higher costs. Consequently, producers and policy makers alike (Dempsey et al. 2010, IPR Working Group 2012) have long searched for alternative cost allocation mechanisms to induce voluntary participation in collective systems. Examples include the ad hoc adjustments to return share proposed by Mayers et al. (2013), and the heuristics-based modifications to return share experimented in the Washington state and in the United Kingdom (IPR Working Group 2012). However, finding an effective mechanism remains an open question. This is the gap this paper aims to fill.

One fundamental problem with existing cost allocation approaches is that they are unable to address the network-based operational nature of a heterogeneous collective system. A key principle in current discussions is to find mechanisms that "relate to the costs of dealing with producers' own products" (IPR Working Group 2012, p. 4). However, what constitutes one's actual cost is undefined in a collective system because it highly depends on the underlying network effects, i.e., the availability of resources (collection, transportation, and processing capacities) and the optimal product routing. We capture these network effects by adopting a network model of a collective system and focus on the incentives that drive producers' defection from that system either individually or with others. A natural benchmark to evaluate such incentives is the *stand-alone* cost, i.e., the cost that a producer or a producer group can achieve in an independent plan. Hence, a desirable property of an allocation that induces voluntary participation is to charge each producer or producer group no more than its stand-alone cost. We call this requirement *group incentive compatibility*. This is a fundamental property studied in the literature, most prominently as the key condition that defines the classic allocation notion of the *core* in cooperative game theory (Young 1994). On the other hand, we note that proportional cost allocation models such as return and market share remain widely used in practice because of their intuitive nature and simplicity. Thus, we focus on identifying group incentive-compatible mechanisms that can be presented as improvements to the return share model.<sup>2</sup> In particular, we look for improvements based on measures that capture the cost and resource heterogeneities in the network.

Our analysis focuses on a cooperative network flow game with shared resources. We show that cost allocation by return share is typically not group incentive compatible under cost and resource heterogeneity, except under restrictive conditions. We demonstrate that two types of adjustments to return share can greatly reduce or eliminate its *incentive compatibility gap* (the cost increase experienced by producers in the collective system compared to their stand-alone costs): (i) a network-based adjustment/weighting of return shares to reflect the processing cost differentials between products; and (ii) capacity-based side payments to producers who own or have contracts with cheaper CRN resources. These adjustments provide succinct and intuitive ways of achieving group incentive compatibility while remaining true to the

<sup>2</sup> We do not take market share as the basis for cost allocation because it does not take into account return share differentials between producers and is considered to be more arbitrary than return share (IPR Works 2012).

return share concept. We also propose a framework to implement the cost allocation mechanisms we developed based on the current practice of collective EPR implementations.

In addition, we address three important issues. (i) Economies of scale are one of the main reasons why producers and policy makers argue for collective systems. Our analysis, however, uncovers a surprisingly negative side effect: under allocations that are not group incentive compatible (e.g., return share), scale economies may increase the incentive compatibility gap and introduce a stronger incentive for producers to break away. (ii) Some EPR bills mandate a penalty for producers who operate independent plans but do not fulfill their obligation. We show how the adjustments we propose can be implemented to operationalize this concept while ensuring a group incentive-compatible allocation. (iii) An implementation requirement for the proposed adjustments is information on the operational costs and volumes of each producer's e-waste flow, which can be obtained by separating waste at the collection stage and thus can be costly. Another practical contribution of our work is to identify when separation at the source is valuable.

To show how our results would translate into practice, we develop a sample network with a representative cost structure and provide proof-of-concept based on the Washington state implementation. A set of numerical experiments illustrates the adjustments developed in the paper and yields insights regarding their implementation. We observe that the incentive compatibility gap of the allocation by return share, and the associated efficiency loss from fragmentation, can be substantial. Economies of scale are shown to reduce the incentive compatibility gap of return share (without guaranteeing a core allocation) while greatly accentuating efficiency loss from fragmentation. To the best of our knowledge, this is the first work that provides a rigorous treatment of cost allocation for the effective management of EPR systems, an important practical problem, made all the more so by the broadening reach and scope of EPR regulation. Our findings have already influenced policy recommendations: with reference to this paper, the UK Department for Business, Innovation and Skills (DBIS) recommends a cost correction approach as the "preferred policy option" for the revision of the WEEE Directive implementation in the United Kingdom (IPR Working Group 2012).

## 2. Contribution to the Literature

A stream of research in the environmental economics literature studies the economics of regulated collection and recycling of postconsumer products (e.g.,

Palmer and Walls 1997, Fullerton and Wu 1998, Walls and Palmer 2001, Calcott and Walls 2005, Walls 2003). These papers use stylized economic models and show that a deposit-refund policy maximizes social welfare. Yet the practice of e-waste legislation around the globe has converged to EPR implementations. The recent operations management literature recognizes that implementing EPR is essentially an operational problem and investigates how the principles of EPR can be effectively translated into working systems (e.g., Toyasaki et al. 2011, Plambeck and Wang 2009, Atasu et al. 2009, Atasu and Subramanian 2012) but using stylized models of operational decisions that do not capture network effects and group incentive compatibility problems in collective implementations. Our work contributes to this growing literature by explicitly capturing the network effects in a collective EPR implementation, without which group incentive-compatible cost allocations cannot be designed.

Another relevant stream in the operations literature (e.g., Fleischmann et al. 2001, Jayaraman et al. 2003, Wojanowski et al. 2007, Srivastava 2008) focuses on designing a reverse logistics network when product returns are valuable. Nagurney and Toyasaki (2005) study the design of e-waste networks, considering a competitive market for e-waste recycling. However, these papers do not investigate the design and operation of regulated CRNs. We contribute to this literature by (i) explicitly modeling practical issues in designing collective CRNs when recycling is carried out because of legislation and (ii) identifying cost allocation mechanisms that guarantee voluntary participation of producers in collective EPR implementations to maximize cost efficiency.

Cooperative game theory has been widely applied in the operations management literature; one prominent application is to analyze profit sharing in supply chain alliances (see Nagarajan and Sošić 2008 for a survey). However, most of these papers focus on traditional supply chain problems and do not discuss product recovery issues. One paper that studies recycling activities is Tian et al. (2014). The authors investigate the structure of coalitions that would emerge given exogenous unit recycling costs that depend on the size and product diversity of the coalitions; this cost structure defines an implicit cost allocation. In this paper, we endogenously determine a coalition's stand-alone cost by solving a network flow problem that captures the cost heterogeneity and the network effects, and we focus on developing cost allocations that induce participation in the collective system.

From a methodological perspective, we contribute to the literature on cooperative network games (Kalai and Zemel 1982b, a; Granot and Granot 1992; Derks and Kuipers 1997; Owen 1975; Samet and Zemel 1984,



to name a few). A classic result in this area is that a dual-based cost allocation is guaranteed to be in the core. Ease of implementation, on the other hand, favors the use of proportional allocations based on simple metrics, such as return share, which are prevalent in the EPR context. Thus, an important practical consideration is identifying core allocations that can be presented as derived from the return share concept. In this work, we show that adjustments to the return share model can achieve a core cost allocation under general network conditions. More importantly, we characterize the adjustments that matter, i.e., weighing producers' return shares to account for the heterogeneity in their cost burdens and resource-based side payments.

We also introduce a new perspective on the use of a mixture of player-owned and exogenous resources in collaboration. Existing analyses of flow games (Kalai and Zemel 1982b, Reijnierse et al. 1996) assume that the exogenous capacity is a public resource available free of charge, and they show that the core of such games may be empty. Granot (1994) analyzes an extended linear production game where anyone may purchase the exogenous capacity in bundles and gives bundle prices to guarantee the existence of the core. We extend this stream of research by studying a hybrid model in §4.4.2, where a public capacity is utilized at no additional cost to handle the products of the producers in the centrally operated collective system while independent subcoalitions are allowed to utilize it for an additional unit (nonmember) "access" fee. Such user differentiation reflects the shortfall fee policy in some EPR bills. We derive tight lower bounds for the unit access fees under which the core must exist.

### 3. Model Description

As described in the introduction, our analysis focuses on a cooperative network flow game with shared collection, transportation, and processing resources. In this section, we formulate the cooperative *collection and recycling flow* (CRF) game. In this game, each producer and producer subcoalition chooses between (i) being part of the centrally operated collective system that is managed by a system operator appointed by the regulatory body overseeing EPR regulation and (ii) operating an independent plan. We first develop a multicommodity CRN model that represents the practice of e-waste collection and recycling, and that forms the basis for the CRF game. We then describe the nature of the resources and capacities involved and the role of the system operator. Finally, we formulate a cooperative game model based on which we mathematically define the notion of a group incentive-compatible cost allocation that guarantees all producers voluntarily choose option (i).

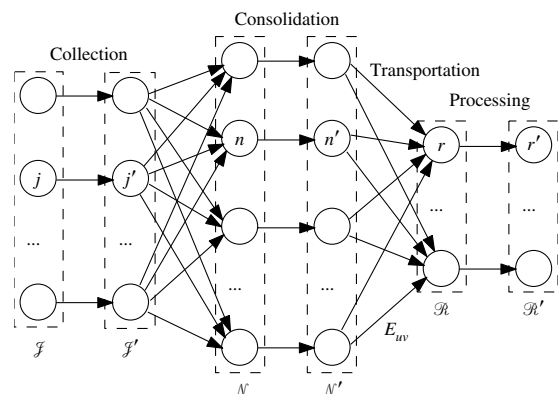
### 3.1. Network Model Formulation

A typical EPR implementation for e-waste relies on collection, consolidation, transportation, and processing (dismantling, shredding, and/or recycling) resources. These resources can be owned by producers (e.g., distribution fleet's backhaul miles, collection points at retail stores such as the Best Buy stores in Minnesota, and self-owned recycling capacities such as Hewlett-Packard's facility in California) or by service providers (e.g., third-party collectors, consolidators, recyclers, or producer responsibility organizations). Once a set of resources are identified and incorporated in a plan, the plan's operations can be modelled as a multicommodity network (Figure 1), where post-use products from participating producers are handled (see Table 1 for a summary of the notation used).

**3.1.1. The Network.** We construct three sets of nodes in a CRN:  $\mathcal{J} = \{j: j \text{ is a collection point}\}$ ,  $\mathcal{N} = \{n: n \text{ is a consolidator}\}$ , and  $\mathcal{R} = \{r: r \text{ is a processor}\}$ . To represent the capacity restriction at each entity as edge capacity, we duplicate node sets  $\mathcal{J}$ ,  $\mathcal{N}$ , and  $\mathcal{R}$  by generating  $\mathcal{J}' = \{j': j \text{ is a collection point}\}$ ,  $\mathcal{N}' = \{n': n \text{ is a consolidator}\}$ , and  $\mathcal{R}' = \{r': r \text{ is a processor}\}$ , respectively, and we link each original node with its counterpart in the duplicated sets. The capacities at collector  $j$ , consolidator  $n$ , and processor  $r$  are then modeled as edge capacities on edges  $(j, j')$ ,  $(n, n')$ , and  $(r, r')$ , respectively. Transportation capacities on edges between any pair of nodes  $(u, v)$  in  $\mathcal{J}'$  and  $\mathcal{N}$ , and in  $\mathcal{N}'$  and  $\mathcal{R}$ , are modeled by a set of capacitated parallel edges  $E_{uv}$  where each edge corresponds to a different transporter. We denote the set of all nodes and edges as  $V$  and  $E$ , respectively.

**3.1.2. Products, Costs, and Environmental Requirements.** We denote a producer by  $i$  and the set of producers by  $M$ . Producer  $i$  makes the set of products  $\Pi^i$ ; these sets are mutually exclusive. The set of all products is denoted by  $\Pi = \bigcup_i \Pi^i$ . On each edge  $e$ , each product  $\pi \in \Pi$  incurs a unit operational cost  $c_e^\pi$ .

Figure 1 The Structure of a Generic Collection and Recycling Network



**Table 1** Notation

CRN structure	
$j, n, r$	A collection point, a consolidator, and a processor
$j', n', r'$	Duplicate of a collection point, a consolidator, and a processor
$\mathcal{J}, \mathcal{N}, \mathcal{R}$	The set of collection points, consolidators, and processors
$\mathcal{J}', \mathcal{N}', \mathcal{R}'$	Duplicate of the set of collection points, consolidators, and processors,
$u, v \in V$	Two different nodes in the CRN and the set of all nodes
$e$ and $E$	An edge in the CRN and the set of all edges
$E_{uv}$	The set of transportation edges between two nodes $u$ and $v$
Product, capacity, and cost parameters	
$i$ and $M$	A producer and the set of producers
$\pi, \Pi^i, \Pi$	A product, the set of producer $i$ 's products, and the set of all products
$d_j^\pi$	Return volume of product $\pi$ at collection point $j$
$c_e^\pi$	Unit operational cost to process product $\pi$ on edge $e$
$\sigma_r^\pi, \rho_r^\pi, \hat{c}_r^\pi$	Unit downstream cost, unit downstream revenue, and the net unit Processing cost of product $\pi$ at processor $r$
$\tau$	Legislative stringency parameter
$k_e^i$ and $k_e^p$	Independent capacity of producer $i$ and operator-contracted capacity on $e$
$\eta^S$ and $\zeta^S$	Scale economies-related parameters in a coalition $S \subseteq M$ (introduced in §4)
Variables in the min-cost flow problem and the CRF game	
$f_e^\pi$	Flow variable of the amount of product $\pi$ routed on edge $e$
$[v_j^\pi, \beta_j^\pi, \alpha_e]$	Dual variables in the centralized problem (C)
$Z(f^*), v(M)$	The minimum total cost of the grand coalition $M$ in the centralized problem (C) and in the CRF game, respectively
$v(S)$	Stand-alone cost of a subcoalition $S \subseteq M$
$v^r(S)$ and $v^r(M)$	The total processing revenue in $v(S)$ and $v(M)$ , respectively (introduced in §4)
$x^i$	Cost allocation variable to producer $i$ that satisfies $\sum_{i \in M} x^i = v(M)$
$\mu^i$	Cost-corrected return share of producer $i$ (introduced in §4)
$p_e$	Unit capacity reward price on edge $e$ (introduced in §4)
$x_r, x_\mu^p, x_r^p$	Cost allocation by return share, by return share with capacity rewards, and by cost-corrected return share with capacity rewards (introduced in §4)
$\phi_e$	Nonmember access fee on edge $e$ (introduced in §4)

for collection, transportation, or processing. Moreover, each processor  $r$  also pays a *downstream cost*  $\sigma_r^\pi$  for or obtains a *downstream revenue*  $\rho_r^\pi$  (which is modeled as a *negative cost*) from sending the parts and materials extracted from a unit of processed product  $\pi$  to downstream vendors, brokers, and/or recyclers. For example, TVs typically incur downstream processing costs, whereas computers generate processing revenues. Hence, the *net unit processing cost* for product  $\pi$  at processor  $r$  is  $\hat{c}_r^\pi = c_{(r,r)}^\pi + \sigma_r^\pi + \rho_r^\pi$ . In our main model, we assume that the unit costs do not depend on the volumes handled; i.e., there are no scale economies. We relax this assumption in §4.4.1.

We assume that returned products have been sorted and separated at each collection point  $j$  so that the waste volume of every product  $\pi$  brought to  $j$  is known, which is denoted by  $d_j^\pi$ . We discuss the implications of sorting and separation costs in §5. We also assume that any plan has to meet a recycling standard, denoted by  $\tau$ , which is the minimum fraction of the return volume of each product that should be recycled (including downstream stages). Since land-filling mainly occurs after the processing/downstream recycling stage, and is outside the boundary of the CRN that is relevant for cost allocation purposes, we capture the influence of the stringency of such

a requirement by modeling  $c_{(r,r)}^\pi$ ,  $\sigma_r^\pi$ , and  $\rho_r^\pi$  and thus the net processing cost  $\hat{c}_r^\pi$  at each processor  $r$ , as functions of  $\tau$ ; i.e.,  $\hat{c}_r^\pi(\tau) = c_{(r,r)}^\pi(\tau) + \sigma_r^\pi(\tau) + \rho_r^\pi(\tau)$ . This relationship represents how the stringency of mandated recycling standards influences processing operations as well as the destination of the resulting output. For example, when the mandated recycling standard is stringent, processors should not only refrain from landfills but also contract with downstream recyclers that can thoroughly recycle the lead and glass in cathode ray tube (CRT) TVs/monitors, and as both of these increase processing costs.

**3.1.3. Capacities.** All resources are capacitated. The portion of capacity that an individual producer  $i$  can utilize on edge  $e$  if it were to operate independently is called the producer's *independent capacity* and is denoted as  $k_e^i$ . To qualify as independent capacity, producers must either own the capacity or have credible evidence of being able to use it (e.g., a contract with that service provider). In particular, independent plans utilizing service provider capacity can be approved only if producers have such evidence. In implementation, this is ensured by stringent verification and certification requirements on independent plan proposals. For example, in Washington, producers need to provide contractual agreements with the

service providers they propose to work with and have these agreements verified by the state authority (Washington State Department of Ecology 2007).

If a producer or producer group develops an independent plan, the plan uses only the independent capacities of that subcoalition  $S \subsetneq M$ . We call the network associated with such a plan an *independent CRN*. The edge capacities in an independent CRN are equal to the sum of independent edge capacities of its members ( $\sum_{i \in S} k_e^i$ ).

Alternatively, producers may choose to participate in the centrally operated collective system. The network associated with this system pools their independent capacities with *operator-contracted capacity*, denoted by  $k_e^p$ . Note that  $\{k_e^i, i \in M\}$  and  $k_e^p$  are mutually exclusive (the service provider apportions its capacity among different customers), and their sum is bounded by the total provider capacity. When all producers are part of the centrally operated collective system, the *centrally operated grand coalition* is formed. The edge capacity in the corresponding CRN is given by  $\sum_{i \in M} k_e^i + k_e^p$ .

### 3.2. Collective Operations of a CRN Under EPR

Mimicking many practical implementations, we consider a system operator (e.g., WMMFA) appointed by a state authority (e.g., the Washington State Department of Ecology). The system operator's role is to establish a centrally operated collective system to collect and process the e-waste volumes of participating producers in a cost-effective manner. To achieve this goal, we model the system operator as a non-profit, impartial control tower with three fundamental responsibilities.

1. *Establishing resource base*: The system operator contracts for operator-contracted capacity and privately obtains independent capacity information from participating producers and verifies them (e.g., through a certification process used by the Washington Department of Ecology, which certifies collection, transportation, and processing capacities that producers own or have contracted). The system operator ensures that the collective system has sufficient capacity to process all participating producers' volumes.

2. *Product routing*: The system operator routes the observed return volumes to minimize the total cost through the CRN consisting of independent capacities from participating producers and the operator-contracted capacity. The operator then pays the service providers (collectors, transporters, and processors) based on the volumes they handled at the previously contracted unit costs.

3. *Cost allocation*: The system operator allocates the total cost incurred among participating producers ex post.

Given a certain stringency level  $\tau$ , the minimum total system cost can be attained on the CRN that pools all available capacities (i.e., the CRN that corresponds to the centrally operated grand coalition). We model the system operator's optimal product routing decision under the grand coalition as the solution to the following minimum cost flow problem, which we call the *centralized problem* (C). Note that we assume sufficient capacity in the collective system; thus (C) is feasible.

$$\begin{aligned} \text{(C)} \quad \min \quad & Z(f) = \sum_{e \in E \setminus \{(r, r')\}} \sum_{\pi \in \Pi} c_e^\pi \cdot f_e^\pi \\ & + \sum_{r \in \mathcal{R}} \sum_{\pi \in \Pi} \hat{c}_r^\pi(\tau) \cdot f_{(r, r')}^\pi \quad (1) \\ \text{s.t.} \quad & \sum_{e=(u, v) \in E} f_e^\pi - \sum_{e=(v, w) \in E} f_e^\pi = 0, \\ & \forall v \in V \setminus \{\mathcal{J}, \mathcal{R}'\}, \forall \pi \in \Pi \quad [v_v^\pi]; \quad (2) \\ & f_{(j, j')}^\pi = d_j^\pi \quad \forall \pi \in \Pi, \forall j \in \mathcal{J} \quad [\beta_j^\pi]; \quad (3) \\ & \sum_{\pi \in \Pi} f_e^\pi \leq \sum_{i \in M} k_e^i + k_e^p, \forall e \in E \quad [\alpha_e]; \quad (4) \\ & \text{nonnegativity constraints.} \quad (5) \end{aligned}$$

In this program,  $f_e^\pi$  is the flow variable that denotes the amount of product  $\pi$  routed on edge  $e$ . Constraint (2) represents the flow conservation constraint at every node except for the source and terminal nodes, (3) guarantees that all collected units are processed, and (4) represents the capacity constraint on every edge of the collective CRN. The variable listed beside each constraint is its corresponding dual variable. The objective function (1) minimizes the systemwide total cost incurred by the flow under the mandated recycling requirement  $\tau$ . The optimal solution to (C) is denoted by  $f^*$ , which we call the *socially optimal routing*, and the minimum total cost is denoted by  $Z(f^*)$ .

### 3.3. Producers' Perspectives—A Cooperative Game on CRNs

Attaining  $Z(f^*)$  requires the participation of all producers in the centrally operated collective system. This is not guaranteed to happen because of the potential defection of producers to independent plans. Thus, modeling the producer perspective is an important part of the problem. Consider a subcoalition  $S \subsetneq M$  of producers. The minimum total cost to operate an independent plan for subcoalition  $S$  can be computed by a network problem ( $C^S$ ) similar to (C) except that (i) the product set is restricted to  $\Pi^S = \bigcup_{i \in S} \Pi^i$ , and (ii) constraint (4) is replaced with  $\sum_{\pi \in \Pi^S} f_e^\pi \leq \sum_{i \in S} k_e^i, \forall e \in E$ . This minimum cost is defined as the *stand-alone* cost of  $S$ , denoted by  $v(S)$ , i.e., the value of the characteristic function of the CRF game associated with  $S$ .



We define the value of the characteristic function of the CRF game associated with the grand coalition,  $v(M)$ , to be the minimum total cost on the grand coalition's CRN, i.e.,  $Z(f^*)$ . We define an allocation of  $v(M)$  by  $x = \{x^i, \forall i \in M\}$ , where  $x^i$  represents the cost allocated to producer  $i$ , such that  $\sum_{i \in M} x^i = v(M)$ ; i.e., all cost is allocated. Hence, by participating in the grand coalition, the subcoalition  $S$  pays a cost of  $\sum_{i \in S} x^i$ .

To induce all producers to voluntarily participate in the centrally operated collective system, the allocation  $x$  must satisfy  $\sum_{i \in S} x^i \leq v(S)$ ,  $\forall S \subsetneq M$ ; i.e., no subcoalition of producers is allocated a higher cost within the grand coalition compared with their stand-alone costs. We refer to such cost allocations as *group incentive-compatible* cost allocations. According to the classic game theory terminology, they can be equivalently referred to as allocations that reside in the *core* of the CRF game (Gillies 1959). In the next section, we discuss the design and the implementation of such allocations based on the widely used return share mechanism.

## 4. Designing Group Incentive-Compatible Cost Allocations

### 4.1. Cost Allocation by Return Share

A producer's *return share* is defined as the ratio of the producer's products returned to the total amount of electronic products returned by weight (Washington State Department of Ecology 2014). We denote the return volume belonging to subcoalition  $S$  as  $R^S \doteq \sum_{j \in \mathcal{J}} \sum_{\pi \in \Pi^S} d_j^\pi$ , and we let  $R \doteq \sum_{i \in M} R^i$  be the total return volume. We assume  $R^i > 0$ ,  $\forall i \in M$ . The cost allocation by *return share*,  $x_r$ , is defined such that the cost allocated to producer  $i$  is computed as

$$(x^i)_r \doteq v(M) \cdot \frac{R^i}{R}. \quad (6)$$

Note that  $\bar{v}^M \doteq v(M)/R$  can be interpreted as a flat rate charge equal to the average cost within the centrally operated grand coalition. Let  $\bar{v}^S \doteq v(S)/R^S$  be the average cost within a subcoalition  $S \subsetneq M$  operating an independent CRN. We first evaluate the maximum cost increase experienced by a subcoalition  $S \subsetneq M$  compared with its stand-alone cost  $v(S)$  under the allocation by return share. We call this measure the *incentive compatibility gap* of the allocation and denote it by  $\mathcal{G}(x_r)$ , where

$$\mathcal{G}(x_r) \doteq \max_{S \subsetneq M} \left\{ \sum_{i \in S} (x^i)_r - v(S) \right\} = \max_{S \subsetneq M} \{ R^S \cdot (\bar{v}^M - \bar{v}^S) \}. \quad (7)$$

The last term in (7) indicates that the incentive compatibility gap is determined by the change in the

average unit cost for  $S$ ,  $\bar{v}^M - \bar{v}^S$ , which is influenced by network effects. Intuitively, a subcoalition of producers who make cheaper-to-recycle electronics and have a cost-effective independent plan with sufficient capacity tends to have a smaller average unit cost when operating its independent CRN, and thus it is more likely to suffer a cost increase inside the centrally operated grand coalition when cost is allocated by return share. This intuition is substantiated by the following analysis. First, we present a sufficiency condition in Proposition 1.

**PROPOSITION 1.** *Given a CRN,  $x_r$  is in the core of the CRF game if (i) for any edge  $e \in E$  and any processor  $r \in \mathcal{R}$ , the operational cost and net processing cost of all products are identical, i.e.,  $c_e^\pi = c_e$  and  $\hat{c}_r^\pi = \hat{c}_r$ ,  $\forall \pi \in \Pi$ ; (ii) for each product  $\pi$ , its return share is the same at all collection points and is equal to its return share in the entire CRN, i.e.,  $d_j^\pi / \sum_{\pi \in \Pi} d_j^\pi$  is equal  $\forall j \in \mathcal{J}$ ; and (iii) there exists excess capacity on every edge under the socially optimal routing  $f^*$ , i.e.,  $\sum_{\pi \in \Pi} f_e^{*\pi} < \sum_{i \in M} k_e^i + k_e^p$ ,  $\forall e \in E$ .*

**PROOF.** All proofs and technical details are presented in Appendix A.

The sufficiency conditions presented in Proposition 1 imply that cost allocation by return share is in the core if the CRN is entirely homogeneous and essentially uncapacitated under the optimal routing. Furthermore, such restrictive conditions can be necessary for return share to be a core allocation even under simple CRN settings, which we demonstrate using a two-producer example (see Proposition 3 in Appendix A.2 for the detailed analysis). These observations suggest that the incentive compatibility gap of the allocation by return share is driven by cost heterogeneity in the network and the existence of critical (i.e., low-cost, fully utilized) independent capacities.

### 4.2. A Cost Allocation in the Core: Cost-Corrected Return Share with Capacity Rewards

Based on the analysis above, we propose two adjustments to the cost allocation by return share that account for the cost heterogeneity among products and for producers' independent capacities. Recall that  $\beta_j^{*\pi}$  and  $\alpha_e^*$  denote the dual optimal solutions of the centralized problem (C) with respect to constraints (3) and (4), respectively. Hence, we can interpret the term  $\beta_j^{*\pi}$  as the marginal cost to process one additional unit of product  $\pi$  returned to collection point  $j$  in the centrally operated grand coalition, which captures the network effects under a cost-minimization objective. We first weigh the return volumes of products at each collection point by their marginal costs to obtain a *cost-corrected return share* for each producer  $i$ , denoted by  $\mu^i$ :

$$\mu^i = \frac{\sum_{j \in \mathcal{J}} \sum_{\pi \in \Pi^i} d_j^\pi \beta_j^{*\pi}}{\sum_{j \in \mathcal{J}} \sum_{\pi \in \Pi} d_j^\pi \beta_j^{*\pi}}. \quad (8)$$



Second, we define  $p_e$  as the unit reward price on edge  $e$ . The system operator ex post calculates  $p_e$  on each edge  $e \in E$  based on the optimal routing implemented and distributes a total capacity reward to each producer  $i$  that equals  $\sum_{e \in E} p_e k_e^i$ . These monetary rewards increase the total cost to be allocated to  $v(M) + \sum_{i \in M} \sum_{e \in E} p_e k_e^i$ .

Consider the following allocation, denoted by  $x_\mu^p$ :

$$(x^i)_\mu^p = \left[ v(M) + \sum_{e \in E} p_e \sum_{i \in M} k_e^i \right] \cdot \mu^i - \sum_{e \in E} p_e k_e^i, \quad \forall i \in M. \quad (9)$$

We call this allocation method *cost-corrected return share with capacity rewards* and prove that it guarantees group incentive compatibility.

**THEOREM 1.** *Given any CRN, there exist capacity reward prices  $p_e \geq 0, \forall e \in E$  such that  $x_\mu^p$  is in the core of the CRF game.*

Theorem 1 states that the proposed adjustments to return share guarantee a core allocation. The proof proceeds as follows: When  $\mu^i \geq 0, \forall i \in M$  (i.e., all producers exert a nonnegative cost burden on the system), it can be shown that if the capacity reward price  $p_e$  is set equal to  $|\alpha_e^*|, \forall e \in E$ , then  $x_\mu^p$  resides in the set of dual-based allocations, and therefore, according to Theorem 2 in Granot (1986), it must be contained in the core of the CRF game. In other words, the capacity rewards are as simple as the marginal values of capacities for the grand coalition. The situation becomes more complicated when  $\mu^i < 0$  for some producer  $i$ . This case occurs if there exists a large heterogeneity among products and some producer  $i$  makes a net revenue contribution so that its cost-corrected return share  $\mu^i$  becomes negative. In that case, we propose capacity reward prices as closed-form functions of the marginal values  $|\alpha_e^*|$  with adjustments based on producers' independent capacity levels (see Equation (A8) in Appendix A.1 for details). We show that these prices also lead to an allocation equivalent to a dual-based one and thus in the core of the CRF game.

The premise of this mechanism is that it can be presented as an allocation based on return share, a prevalent model in practice, with adjustments for operational cost heterogeneities and cost-efficient resources that producers provide to the grand coalition. From a practical perspective, this mechanism achieves group incentive compatibility and allows for pooling of all capacities and the formation of the grand coalition, which, in turn, guarantees the minimum total cost to the system. An important feature of this allocation model is the distinction between the mechanism and the parameter values that operationalize it. Specifically, the cost allocation mechanism is announced before the implementation starts and is

known to all parties. In a given accounting period (e.g., quarterly), specific parameters of the mechanism such as the unit reward prices and the cost-corrected return shares are calculated ex post based on the optimal product routing and the total cost incurred, which determines each producer's cost allocation for that period. This allows the system operator to dynamically update the costs allocated to producers to account for any changes in their independent capacity profile, return volume, and product characteristics. The ex post nature of the allocation also creates a significant barrier against any producer wanting to manipulate its cost allocation by (ex ante) over- or underprovision of independent capacity.

We conceptualize the implementation of this cost allocation model as follows. An impartial nonprofit entity acts as the system operator with the following responsibilities: (i) contracting with a number of (competing) service providers to create a statewide collection and recycling network, (ii) communicating the cost allocation mechanism to producers (e.g., cost-corrected return share with capacity rewards), (iii) maintaining resource capacity and cost information, (iv) pooling capacities and routing return volumes from collection points to processors in the most cost-effective manner, (v) periodically calculating the cost allocation parameters (e.g., producers' cost-corrected return shares and the unit capacity rewards) ex post, and (vi) charging participating producers. The WMMFA operates the state collective system in a similar fashion (see §5 for details). In introducing a proposed set of new guidelines regarding the WEEE implementation, the UK DBIS similarly specifies that "an appropriate authority" should determine the allocation mechanism and administer it (Department for Business, Innovation and Skills 2013, IPR Working Group 2012). The use of such entities is also common in other network settings, such as shipper collaborations (Ergun et al. 2010).

This implementation framework involves an independent plan provision and verification procedure before operations start, where independent resource information is reported by the producers and verified by the operator (e.g., using a regulatory compliance process). This procedure can be dynamic, and it essentially take two forms. As in Washington and many EU implementations, a collective plan may be established first and independent plans submitted and verified later. Alternatively, parallel independent plans, whose capacity availability is verified, may coexist with the default collective system, e.g., as in Oregon. After the costs are allocated, if in the former case, the producers can readily verify that their allocated costs are below what their stand-alone costs would be, incentivizing them to remain in the collective system. If in the latter case, an analysis based on historical data can be used

as a proof-of-concept to demonstrate to the producers how they would have fared under the collective system with the proposed allocation.

### 4.3. An Alternative Cost Allocation Model

In this subsection, we analyze an alternative cost allocation model, *return share with capacity rewards* ( $x_r^p$ ), which uses simple return shares instead of cost-corrected return shares compared with formula (9), but continues to rely on capacity rewards:

$$(x_r^i)^p \doteq \left[ v(M) + \sum_{i \in M} \sum_{e \in E} p_e k_e^i \right] \cdot \frac{R^i}{R} - \sum_{e \in E} p_e k_e^i, \quad \forall i \in M. \quad (10)$$

This allocation has two differences from the return share model: each producer is paid a capacity reward based on its independent capacities ( $-\sum_{e \in E} p_e k_e^i$ ), and it also pays for the total capacity rewards distributed to all producers based on its return share ( $[\sum_{i \in M} \sum_{e \in E} p_e k_e^i] \cdot R^i/R$ ). Therefore, whether return share with capacity rewards yields a lower cost allocation for a given subcoalition than return share depends on the relative magnitude of these two terms. Intuitively, we would expect that a subcoalition with high independent capacity levels combined with a low return volume would receive a smaller cost allocation under return share with capacity rewards. To support this intuition, we can calculate the incentive compatibility gap of  $x_r^p$  and compare it with  $\mathcal{G}(x_r)$  in Equation (7):

$$\begin{aligned} \mathcal{G}(x_r^p) &\doteq \max_{S \subseteq M} \left\{ \sum_{i \in S} (x_r^i)^p - v(S) \right\} \\ &= \max_{S \subseteq M} \left\{ R^S \cdot \left[ \bar{v}^M - \bar{v}^S - \sum_{e \in E} p_e (\bar{k}_e^S - \bar{k}_e^M) \right] \right\}. \quad (11) \end{aligned}$$

Here,  $\bar{k}_e^S = \sum_{i \in S} k_e^i / R^S$  denotes the ratio of the independent capacity availability on edge  $e$  to the total return volume of producers in  $S$ . With reference to the last term in (11), we observe that only subcoalitions with  $\bar{k}_e^S > \bar{k}_e^M$  on at least one edge, i.e., coalitions that have a larger independent capacity to return volume ratio than the full set of producers on at least one edge, can potentially benefit from the introduction of capacity rewards to the return share mechanism.

Yet it is not straightforward to determine whether there exists a set of prices to make  $\mathcal{G}(x_r^p) = 0$ . The challenge is that the cost allocation uses only one lever, capacity reward prices, to address both cost and independent capacity heterogeneity in the system. Theorem 3 in Appendix A.3 identifies a sufficient condition for such a set to exist: the throughput of the network consisting of all independent capacities cannot be increased by an additional unit arriving to any collection point. This condition implies that there is a set of “critical” independent resources where

$\bar{k}_e^S > \bar{k}_e^M$  holds for any subcoalition  $S$  that can potentially break away. Theorem 3 shows that by associating reward prices with these capacities, both cost and resource heterogeneity are accounted for without the need for a direct cost-focused lever (such as cost correction), and group incentive compatibility is attained. The practical value of this finding is that when volume burdens originating at different collection points are not highly differentiated, the widely adopted return share approach can be retained and group incentive compatibility can be guaranteed by a capacity-rewarding mechanism without resorting to cost-based adjustments.

### 4.4. Extensions

We close this section by extending our results in two practically important directions: incorporating economies of scale and nonmember access fees.

**4.4.1. Economies of Scale.** Economies of scale are one of the frequently mentioned advantages of collective implementations of EPR. To gain insights into how economies of scale affect group incentive compatibility of cost allocations, we consider a model where a global discount (increment) factor is applied to the unit operational or downstream cost (downstream revenue) over the entire CRN. The discount and increment factors are modeled as a decreasing function  $\eta \in (0, 1]$  and an increasing function  $\zeta \in [1, \infty)$  in the total return volume, respectively. In particular, we denote the factors associated with a subcoalition  $S$  by  $\eta^S \doteq \eta(R^S)$  and  $\zeta^S \doteq \zeta(R^S)$ , respectively. We let the operational cost of product  $\pi$  on each edge  $e$  be  $\eta^S \cdot c_e^\pi$  when  $S$  operates independently. The downstream cost/revenue that processor  $r$  pays for/obtains from downstream recycling changes to  $\eta^S \cdot \sigma_r^\pi$  and  $\zeta^S \cdot \rho_r^\pi$ , respectively. Thus, the net processing cost becomes  $\eta^S \cdot c_{(r,r)}^\pi + \eta^S \cdot \sigma_r^\pi + \zeta^S \cdot \rho_r^\pi$ . By replacing the cost vector in the objective function in the program ( $C^S$ ) introduced in §3 with the unit costs defined above, we obtain a new program ( $C^{S(\eta, \zeta)}$ ) for each coalition  $S \subseteq M$ , and we denote its optimal objective function value by  $v^{(\eta, \zeta)}(S)$ . We define the core of the CRF game under scale economies and the incentive compatibility gap of a cost allocation in this setting accordingly.

We calculate the three cost allocations  $x_r^{(\eta, \zeta)}$  (return share),  $x_r^{p(\eta, \zeta)}$  (return share with capacity rewards), and  $x_\mu^{p(\eta, \zeta)}$  (cost-corrected return share with capacity rewards) by replacing  $v(M)$  with  $v^{(\eta, \zeta)}(M)$  in formulas (6) and (10) and by adjusting both  $v(M)$  and  $\mu^i$  in (9) based on ( $C^{M(\eta, \zeta)}$ ). We note that the incentive compatibility gap of an allocation can be decomposed into two components, one as a result of the cost increase and the other as a result of the revenue loss that a producer or producer group experiences from

joining the grand coalition. To present the results, we define  $v^{(\eta, \zeta)r}(M)$  as the total processing revenue obtained under the optimal routing within the grand coalition under scale economies. We define  $v^{(\eta, \zeta)r}(S)$  in the same way for each subcoalition  $S \subsetneq M$  with adequate independent capacity to fulfill the recycling obligation of its members (i.e.,  $v(S) < \infty$ ).

**PROPOSITION 2.** (1) *The allocation  $x_\mu^{p(\eta, \zeta)}$  is in the core of the CRF game under scale economies.*

(2) *Let  $\rho_r^\pi = \rho^\pi$ ,  $\forall r \in \mathcal{R}$  and  $\forall \pi \in \Pi$ . If*

$$\left| \frac{v^{(\eta, \zeta)r}(M)}{R} \right| \geq \left| \frac{v^{(\eta, \zeta)r}(S)}{R^S} \right|$$

*holds  $\forall S \subsetneq M$  such that  $v(S) < \infty$ , then the incentive compatibility gaps  $\mathcal{G}(x_r^{(\eta, \zeta)})$  and  $\min_{p_e \geq 0} \mathcal{G}(x_r^{p(\eta, \zeta)})$  satisfy  $\mathcal{G}(x_r^{(\eta, \zeta)}) \leq \eta^M \cdot \mathcal{G}(x_r)$  and  $\min_{p_e \geq 0} \mathcal{G}(x_r^{p(\eta, \zeta)}) \leq \eta^M \cdot \min_{p_e \geq 0} \mathcal{G}(x_r^p)$ .*

The first result in Proposition 2 states that cost-corrected return share with capacity rewards continues to guarantee an allocation in the core of the CRF game under scale economies. To understand the second result, first consider a case where  $\rho_r^\pi = 0$ ,  $\forall r, \forall \pi$  (no processing revenues), and the incentive compatibility gap derives only from the cost component. Since economies of scale decrease the unit costs on all edges by the same discount factor, the optimal routing remains the same, and it can be calculated that  $v^{(\eta, \zeta)}(S) = \eta^S \cdot v(S)$  and  $v^{(\eta, \zeta)}(M) = \eta^M \cdot v(M)$  in this case. Since  $\eta^M \leq \eta^S$ , producers in  $S$  enjoy more cost reduction as a result of economies of scale in the grand coalition than operating alone, and the result in Proposition 2 follows. The existence of processing revenues introduces a complication, however: the net processing costs do not scale uniformly because they depend on both  $\eta$  and  $\zeta$  factors on edges with processing revenues. Thus, the optimal routing may change under scale economies. The first condition,  $\rho_r^\pi = \rho^\pi$ ,  $\forall r \in \mathcal{R}$ , means that the revenue obtained from a given product is the same at any processor, ensuring that the optimal routing remains the same under economies of scale even when there is a processing revenue on some edges. The second condition indicates that under scale economies, the average processing revenue allocated to  $S$  in the grand coalition is no smaller than what  $S$  obtains operating independently. Hence,  $S$  suffers no processing revenue loss from participating in the grand coalition under either return share or return share with capacity rewards. When this holds for all subcoalitions that can break away, the incentive compatibility gap of each model under scale economies is at most equal to the incentive compatibility gap deriving from the cost component, and the result follows.

In Proposition 2, to achieve a scaling of the incentive compatibility gap that matches the scale factor,

we impose rather strong conditions. This indicates that scale may not be effective in reducing the incentive compatibility gap. In fact, we can further show that under high revenue heterogeneity, scale can even lead to an increased incentive compatibility gap. In particular, using a simple CRN with two producers, we can show that the incentive compatibility gap may become larger in the presence of scale economies (i.e.,  $\mathcal{G}(x_r^{(\eta, \zeta)}) > \mathcal{G}(x_r)$ ) if the revenue heterogeneity is above a threshold (see Proposition 4 in Appendix A.2 for the detailed analysis). The basic intuition behind this result follows the differentiated scale impact between the cost and revenue components of the incentive compatibility gap. Intuitively, scale economies increase the revenue component of the incentive compatibility gap of return share (through the increment factor  $\zeta$ ), and this increase is more significant when there is a larger revenue heterogeneity across different producers' products. As such, when the revenue heterogeneity is sufficiently large, the scale-driven increase on the revenue component of the incentive compatibility gap can dominate the scale-driven reduction in the cost component and lead to an increased incentive compatibility gap. The practical implication of this observation is that revenue heterogeneity among products or processors may limit the economies of scale benefit in reducing the incentive compatibility gap. This is particularly the case if a cost allocation mechanism that is not group incentive compatible, such as return share, is used. In other words, the scale advantage of collective systems may be undermined as a result of the incentive incompatibility of the prevalent return share model.

**4.4.2. Nonmember Access Fees.** In practice, some EPR bills are designed with flexibility provisions so that the capacity in an independent plan can be complemented by available operator-contracted capacity at a surcharge. For example, in Washington, a subcoalition operating an independent plan will be charged a unit fee by the system operator for the amount that it fails to process according to its mandated share of the total return volume. This fee often includes a surcharge on top of the unit processing cost, and it is equivalent to allowing independent plans to use operator-contracted capacity at a surcharge. We call these surcharges *nonmember access fees*, and we show that with properly designed nonmember access fees, the allocation by cost-corrected return share with capacity rewards continues to guarantee the voluntary participation of all producers in the collective system.

Let the unit nonmember access fee for operator-contracted capacity on edge  $e$  be  $\phi_e$ . We construct a CRF game under nonmember access fees where a subcoalition  $S$  operating independently is allowed to



use the operator-contracted capacity at a unit cost that includes both the operational cost of the capacity and the nonmember access fee (see Appendix A.1 for technical details of the game setup). By differentiating between how “members” and “nonmembers” of the centrally operated collective system can use the operator-contracted capacity, the CRF game under nonmember access fees generalizes features of existing cooperative games in the literature that assume the available exogenous capacity either is free (e.g., the pseudo-flow game by Kalai and Zemel 1982b) or can be purchased at a common price (e.g., the extended linear production game by Granot 1994) for all players and coalitions.

When  $\phi_e = \infty, \forall e \in E$ , the CRF game with nonmember access fees is equivalent to the main model, and thus  $x_\mu^p$  must be in the core by Theorem 1. The opposite extreme occurs when  $\phi_e = 0, \forall e \in E$ , resulting in a special case of the pseudo-flow game, which is not guaranteed to have a nonempty core (Kalai and Zemel 1982b). Our analysis identifies a tight threshold for  $\{\phi_e\}$  in closed form such that all values of  $\phi_e$  above this threshold guarantee a core allocation by cost-corrected return share with capacity rewards in this generalized CRF game. This result contributes to the game theory literature by showing that the core of cooperative games with exogenous resources is guaranteed to exist even under nonmember access fees that are not set to artificially high values. This implies that nonmember access fees can be effectively utilized to induce participation in a collective system. We also show that the identified thresholds are tight; i.e., they can be necessary for  $x_\mu^p$  to be in the core under nonmember access fees in certain networks (see Appendix A.2).

**THEOREM 2.** *Given any CRN where independent capacities exist,<sup>3</sup> there exist capacity reward prices  $p_e \geq 0, \forall e \in E$  such that  $x_\mu^p$  is guaranteed to be in the core of the CRF game under nonmember access fees  $\{\phi_e\}$  if*

1.  $\phi_e \geq \max_{i \in M} \{(1 - \mu^i) \cdot |\alpha_e^*|, \forall e \in E \text{ when } \mu^i \geq 0, \forall i \in M; \text{ and}$
2.  $\phi_e \geq [\min_{e^0 \in E: \sum_{i \in M} k_{e^0}^i > 0} \max_{i \in M} \{1 - k_{e^0}^i / \sum_{i \in M} k_{e^0}^i\}] \cdot |\alpha_e^*|, \forall e \in E \text{ when } \exists i \in M \text{ such that } \mu^i < 0.$

Theorem 2 states that in the presence of independent capacities, thresholds on nonmember access fees are proportional to the marginal value of the capacity on each edge. Furthermore, the proportionality constant depends on the cost-corrected return shares of producers and their independent capacity levels. Specifically, when all producers exert a cost burden on the collective system (i.e.,  $\mu^i \geq 0, \forall i \in M$ ), the proportion equals the maximum value of  $1 - \mu^i$ , i.e., the

cost-corrected return share of the subcoalition  $M \setminus \{i\}$ . Otherwise, i.e., when  $\mu^i < 0$ , the proportion is related to the maximum capacity ownership share of the subcoalition  $M \setminus \{i\}$  on an edge  $e^0$ , i.e.,  $\max_{i \in M} \{1 - k_{e^0}^i / \sum_{i \in M} k_{e^0}^i\}$ . In particular, it equals the minimum of this value among all edges. This threshold characterization in Theorem 2 allows us to provide insights as to how nonmember access fees should relate to cost heterogeneity in the CRN and producers' independent capacity profiles. The thresholds on nonmember access fees will be smaller if the cost-corrected return shares ( $\mu^i$ 's) or the percentage independent capacity ownerships ( $k_{e^0}^i / \sum_{i \in M} k_{e^0}^i$ 's) are similar among producers. In fact, they attain their lowest value when  $\mu^i$  or  $k_{e^0}^i / \sum_{i \in M} k_{e^0}^i$  equals  $1/|M|, \forall i \in M$ . The practical implication of this observation is that lower nonmember access fees can guarantee a core allocation under cost-corrected return share with capacity rewards when there is a higher level of homogeneity in cost/revenue among products and in the independent capacity ownerships among producers. The intuitive reason is that applying cost correction and capacity rewards is already more effective in ensuring group incentive compatibility of the allocation when the network is more homogeneous.

## 5. Implications for Practice

This section uses the Washington state implementation—one of the first and better-documented U.S. implementations—as a test bed to investigate the practicality, economic value added, and implications of the results developed in the previous section. The information used includes 2009 public e-waste data from Washington state (Sepanski et al. 2010, WMMFA 2010, Washington State Department of Ecology 2009) and input from interviews with collectors, processors, transporters, and NGOs.

In the state of Washington, the 2006 e-waste bill (Washington State Legislature 2006) mandated the formation of the WMMFA, whose job is to create a statewide, collective “standard plan” to process the allowable e-waste (TVs, monitors, computers, and laptops) brought to its collection points by the consumers. The authority contracts with collectors, transporters, and processors approved by the Department of Ecology; centrally manages the routing of all e-waste flows; pays the collectors, transporters, and processors; and allocates the total operational and administrative cost of these activities to producers participating in the standard plan based on return share. The bill also allows producers to opt out of the standard plan (subject to Department of Ecology approval) and operate their own CRNs. Two such independent plans were filed in 2009 by two producer groups who believed their stand-alone costs would

<sup>3</sup> A similar result can be shown for CRNs without independent capacities.



be lower than their cost allocation under the standard plan. Although these plans were rejected on the grounds that they were not sufficiently developed, they are expected to be resubmitted (Electronic Manufacturers Recycling Management Company 2013), as the adjustments made by the authority to the return share model based on market share have not fully addressed their concerns. Consequently, a fragmented statewide recycling system in Washington is a real possibility, and our proposed cost allocation mechanism and its implementation framework discussed in §4 can serve to resolve these issues.

To explore the implications of our proposed solution in the Washington example, we construct a highly representative version of the Washington state collection and recycling network, including a sample of 50 collection points, 8 consolidation points, 8 processors, and 17 producers who produce 2 product categories (TVs/monitors and computers) with a waste volume equal to the entire 2009 volume in Washington. The detailed construction of the sample CRN is provided in Appendix B. The distinction between TVs/monitors and computers reflects the primary difference in processing costs and postuse values of products at present. Specifically, TVs/monitors are expensive to process as a result of the hazardous materials contained in them, although computers generate revenues for processors as their components and/or materials have high reuse value. The example also captures the processor heterogeneity in Washington by distinguishing between high-tech and low-tech processors, as well as TV/monitor-specialized and information technology (IT)-specialized ones. We adopt a processing cost structure for each product at each processor based on disguised but structurally representative estimates (see Table 2). Motivated by the existing independent recycling capacity of producers around the Washington region and the independent plans submitted in the state, we model two producers as having the ability to establish independent CRNs and focus on their defections from the collective system either individually or as a subcoalition. Specifically, we consider a TV producer  $A$  and a computer producer  $B$  who have high-tech independent recycling capacities

at two out-of-state high-tech processors that are modeled to be specialized in recycling TVs/monitors and IT products (e.g., computers), respectively. Processing capacity contracted by the authority at in-state processors is also incorporated in the sample CRN. Collection and transportation are assumed to be uncappeditated for simplicity of exposition. In experiments not reported here, we observed similar results as discussed below when such capacities were included.

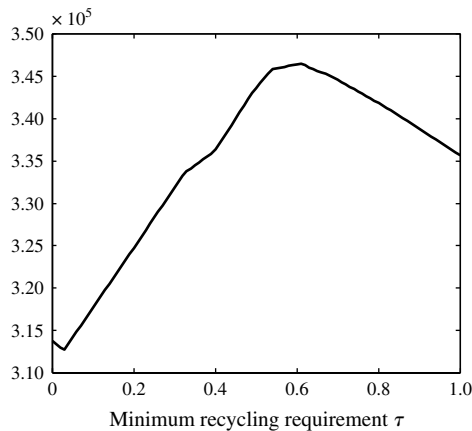
We first investigate the economic advantages of the proposed solution over the return share model. We consider the fragmented system where producers  $A$  and/or  $B$  operate independently. We observe that in Washington, the incentive compatibility gap of the cost allocation by return share and the associated efficiency loss (i.e., increase in systemwide cost) from fragmentation are substantial. Based on our sample CRN, return share is not group incentive compatible regardless of the stringency of the mandated recycling requirement  $\tau$ ; at worst (when  $\tau = 1$ ), producer  $B$  is allocated about \$0.6 million more within the centrally operated grand coalition, which is almost 60% more than what it could achieve on its own. This is because the cost allocated to producer  $B$  according to its return share reflects neither the value of  $B$ 's independent capacity in reducing the total operating cost within the grand coalition nor the much smaller cost burden (even a positive revenue) associated with computers, which are dominant in  $B$ 's waste stream. Thus,  $B$  has an incentive to defect. While  $A$  alone would not defect, if  $B$  defects,  $A$  also finds it preferable to do so provided the  $A$ - $B$  subcoalition adopts a core allocation within themselves. This would result in a 4.5%–6.5% systemwide efficiency loss as a result of the reduction in network synergies in the fragmented system, which represents a lower bound on the actual efficiency loss from fragmentation as it is calculated in the absence of scale economies.

A revealing property of the efficiency loss from fragmentation in this example is its nonmonotonicity with respect to  $\tau$  (see Figure 2, which plots the absolute systemwide efficiency loss after  $A$  and  $B$  defect). This phenomenon derives from the change in cost heterogeneity in the CRN and the consequent network synergies as the recycling requirement becomes more stringent. Specifically, there is a change in the relative cost saving from rerouting product flow from a local low-tech processor to a high-tech one between TVs/monitors and computers when  $\tau$  reaches 0.5. This gives rise to a change in the optimal flow pattern: in the fragmented system, it is optimal to process the computer volume of producers other than  $A$  and  $B$  at the local high-tech (low-tech) processors when  $\tau$  is below (above) 0.5. Therefore, how e-waste flows are rerouted relative to this baseline when a centrally operated grand coalition is formed changes

**Table 2** Representative Net Processing Cost Structure (Cents per Pound) by Facility and Product as a Function of  $\tau$ , the Mandated Recycling Requirement

	High-tech processor			Low-tech processor ( $r_5 - r_6$ )
	Local ( $r_1 - r_2$ )	Out-of-state TV/monitor- specialized ( $r_3$ )	Out-of-state IT-specialized ( $r_4$ )	
TVs/monitors	$5 + 9\tau$	$5 + 5\tau$	$5 + 11\tau$	$7 + 13\tau$
Computers	$-10$	$-7$	$-20$	$-6$

Note. The negative numbers indicate revenues.

**Figure 2** Efficiency Loss (in Dollars) When the *A-B* Subcoalition Does Not Join the Centrally Operated Plan

with  $\tau$ , yielding a nonmonotone efficiency loss function. Another way to see this is to note that network synergies depend not only on the absolute magnitude of costs but also on their relative values. Hence, although one expects the efficiency loss to increase as  $\tau$  rises, the reverse may be observed. This discussion highlights the strong influence of network synergies in determining the implications of legislative choices.

We now turn to the effect of scale economies. We model the discount factor  $\eta^S$  of a coalition  $S$  as a convex quadratic decreasing function of  $R^S$  (the parameters of the function are calculated based on input from WMMFA; see Appendix B for details); the increment  $\zeta$  is assumed to be 1 for any return volume. We find that in this case study, the presence of economies of scale reduces the incentive compatibility gap of the allocation by return share (yet cannot guarantee a core allocation), and it can greatly accentuate the efficiency loss from fragmentation to about \$1.42 million (equivalent to a 20% increase in system cost) when  $\tau = 1$ , which is more than four times that without scale economies (about \$0.35 million). Note that proponents of collective systems tend to fall back on the economies of scale argument when faced with criticisms regarding the incentive incompatibility of return share. Our analysis underlines that economies of scale may not be sufficient to ensure group incentive compatibility of return share and thus voluntary participation of all producers in a collective system. Meanwhile, scale makes it all the more important to resolve incentive compatibility issues as it multiplies the efficiency loss that results from fragmentation.

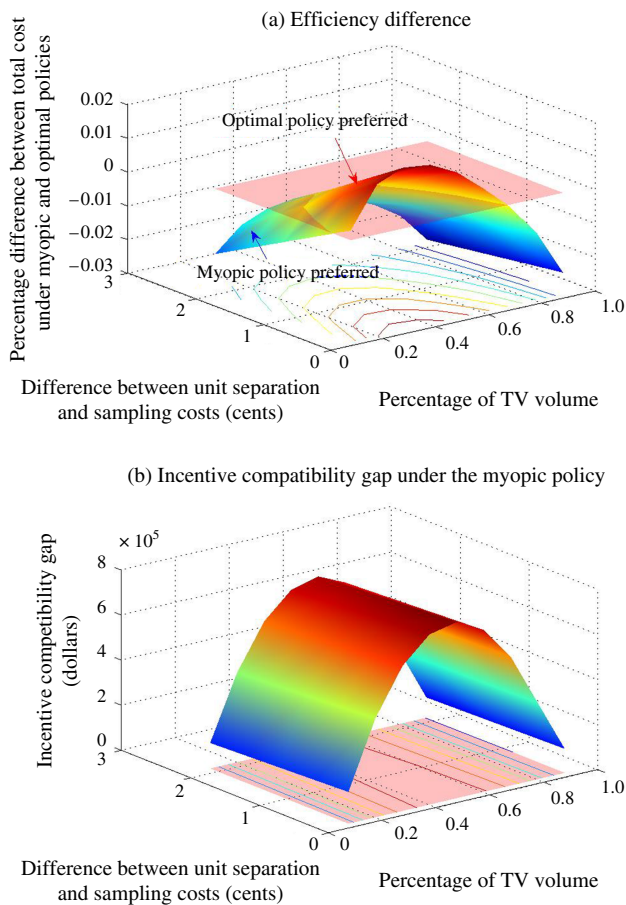
We also use the sample CRN to validate our theoretical results regarding the group incentive compatibility of the proposed allocation mechanisms. In particular, we find that the nonmember access fees to be charged at the local processors (where capacity is centrally contracted) that ensure a core allocation under

the cost-corrected return share model with capacity rewards have a lower bound of 2–6 cents/lb. (implying a minimum processing fee of 26–30 cents/lb. for nonmembers) depending on the stringency of the recycling standard. This suggests that the 50 cents/lb. shortfall fee currently charged in Washington (Electronics TakeBack Coalition 2014) would induce producers' participation in the collective system under a flexibility provision.

Finally, recall that our model assumes counting and separation (by product type and producer) at collection points. The value of separation at the source in product recovery has been discussed in the literature from various perspectives such as collection structure, producer competition, and decision making regarding recovery options (e.g., Parlikad and McFarlane 2010, Toyasaki et al. 2013). In the context of our problem, separation at source is required to implement the optimal routing and the proposed cost allocations, and it can have a cost efficiency impact of its own. In particular, in practice, e-waste is often routed to the processors without separation and return shares are calculated by sampling. Thus, the CRN gains by avoiding the separation cost, but it loses from not routing the e-waste optimally through the CRN. Moreover, a group incentive-compatible cost allocation as proposed here cannot be implemented. In the rest of this section, we investigate this issue using the Washington example. First, to capture the routing inefficiency under sampling, we develop a myopic routing policy inspired by the current practice in Washington (as opposed to the optimal routing) where the e-waste is shipped to the processors without separation to minimize the total consolidation and transportation cost subject to capacity constraints at each processor. Sampling is carried out at each processor so as to achieve a desired accuracy level, based on which the total processing cost is calculated (please refer to Appendix B.4 for the detailed specification of the myopic policy). To reflect the cost differential between separation and sampling, we incorporate a unit separation cost and a sampling cost. We also model product heterogeneity as the fraction  $p$  of the TVs in the total volume; a  $p$ -value close to 0.5 means a higher heterogeneity level.

Figure 3(a) compares the centralized CRN's cost under the optimal routing with separation at source and the myopic routing with sampling. It shows that the myopic routing has a total cost advantage over the optimal one only under a high cost difference between separation and sampling, and a low level of product heterogeneity. Intuitively, the former condition makes it relatively too expensive to implement the optimal routing, whereas the latter one requires a small sample size to achieve the desired sampling precision and thus further reduces

**Figure 3** (Color online) Percentage Difference in the Centralized System's Total Cost (Including Separation or Sampling Cost) Between the Myopic Policy and the Optimal Policy (a) and the Incentive Compatibility Gap Under the Myopic Policy (b)



the cost of the myopic policy. Note that the Washington instance is essentially a special case of the numerical example used in this section with  $p = 70\%$  (see Appendix B.4 for details), and for this particular instance, the myopic policy dominates the optimal one. However, an important caveat is in order: the centralized cost of the myopic policy can only be achieved if return share produces a core allocation, since return share is the only cost allocation that can be implemented with sampling at the processors. We find that return share has a positive incentive compatibility gap for the entire range of the difference between the unit separation cost and sampling cost values (see Figure 3(b)). Therefore, although the myopic routing with return share may seem attractive because it saves on separation costs, it will continue to raise producer concerns that may culminate in a fragmented system in Washington and ultimately result in efficiency loss from fragmentation.

## 6. Conclusions

In this paper, we contribute to the ongoing debate regarding the implementation efficiency of EPR-based

take-back legislation. The choice in practice is often framed as one between an efficient collective system, where producers share a lower total cost but may subsidize others and incur a higher cost individually, and an individual system, where a producer is only responsible for its own cost but is unable to benefit from network synergies. In this paper, we propose an alternative paradigm that is capable of resolving this trade-off by identifying group incentive-compatible adjustments to the return share method, which is prevalent in practice because of its simplicity. We first show that cost allocation by return share is generally not group incentive compatible as a result of its inability to account for processing cost heterogeneity among products or for network effects that arise in the CRN. Then, we show that these shortcomings can be alleviated by intuitive adjustments such as correcting return shares to account for differences in processing costs and rewarding independent capacities based on their value to the collective system.

These results can influence the practice of EPR, as already proven by their impact on the recently proposed preferred policy option for the revision of the WEEE Directive implementation in the United Kingdom (IPR Working Group 2012). They can also help different stakeholders influence EPR implementation strategies. For example, producers lobbying for bills and regulations that reflect their cost burdens more accurately can focus their efforts on promoting these two easy-to-communicate concepts. Similarly, state or producer-operated systems that aim to achieve scale economies by drawing as many producers as possible to their system can implement these concepts. As we show, scale by itself is not a guarantor of stability for collective systems, and it can even exacerbate their incentive compatibility gap, so these notions continue to be valuable for any size system. Finally, collective systems that wish to institute flexibility provisions can institute nonmember access fees developed here and maintain group incentive compatibility in cost allocation.

The producer dynamics regarding independent plans in the state of Washington provide the opportunity to assess the practical significance of the concepts we develop. With a simple population-based projection of this scenario-based analysis to the United States, our findings suggest that the collective system with cost allocation by return share could charge these defecting producers up to \$30 million more than their actual end-of-life cost burdens, providing strong incentives for breaking away. Assuming the same competitive recycling market infrastructure and the need for product separation before and after the system fragments, the proposed allocation models, by ensuring group incentive compatibility, can



retain these producers in the statewide collective system and prevent an efficiency loss from fragmentation of 5%–20% for Washington, which translates to \$0.35–\$1.42 million of opportunity cost. This cost efficiency improvement would amount to approximately \$16–\$65 million for the electronics industry in the United States. Note that this projection is based on the Washington collection rate of 5.78 pounds per capita, which is much lower than its European counterpart that reaches 18.2 pounds per capita because of its broader scope (WEEE Forum 2010). If similar collection volumes are attained (e.g., via scope expansion) in the United States, the predicted efficiency improvement by a group incentive-compatible cost allocation can go up to \$50–\$200 million for the electronics industry. Although this is a case-based analysis, it underlines the significant economic potential of achieving group incentive compatibility in collective implementation. The analysis also highlights the sensitivity of the outcomes with respect to legislative choices and network characteristics.

One practical concern in implementing group incentive-compatible cost allocations is that it requires separation at source, which can be costly. Nevertheless, our analysis with the Washington data shows that the gain in routing efficiency in a collective system would justify performing separation at source if the per-unit separation cost is modest and product heterogeneity is high. This appears to be the case in the near future: the Japanese implementations of EPR (Tojo 2004) show that separation at source can be achieved at low cost by a simple barcode technology. Recent technological improvements have also been reported to yield higher radio-frequency identification read rates and lower tag costs (O'Connell et al. 2013). Moreover, the e-waste composition will presumably change into a variety of different TV sets (e.g., CRTs, LCDs with mercury backlights, LCDs without mercury backlights, LEDs) and IT products, instead of being dominated by CRT TVs as it is at present. The increased product heterogeneity would require larger sample sizes, and more variation in the value and processing requirements would increase the attractiveness of separation at source. Hence, our analysis suggests that collective systems such as the WMMFA should seriously consider separation at source.

Finally, an important policy goal of EPR is to make producers internalize the end-of-life burden of their products and encourage them to design better products. Although our focus in this paper is cost efficiency given an existing set of products, it is interesting to investigate whether the cost allocations we develop are effective in providing design incentives. Preliminary analysis shows that cost-corrected return share with capacity rewards can give producers a higher or lower return on their design investment

than if they were to operate independently, depending on how the design improvement impacts network synergies. This observation indicates a strong connection between design incentives, cost efficiency, and group incentive compatibility in the context of EPR. Understanding the interactions among these three dimensions and developing mechanisms that provably increase design investments would be an interesting direction for future research.

## Acknowledgments

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## Appendix A. Technical Results and Proofs

### A.1. Proofs

**PROOF OF PROPOSITION 1.** According to Granot (1986), the following *dual-based cost allocation*, which we denote as  $x_d^v$ , is guaranteed to be in the core of the CRF game:

$$(x_d^v)^i = \sum_{j \in \mathcal{J}} \sum_{\pi \in \Pi^i} \beta_j^{*\pi} d_j^\pi + \sum_{e \in E} \alpha_e^* k_e^i + \sum_{e \in E} \alpha_e^* \nu_e^i, \quad \forall i \in M, \quad (\text{A1})$$

where  $\nu_e^i \geq 0$ ,  $\forall i \in M$ ,  $\forall e \in E$ , and  $\sum_{i \in M} \nu_e^i = k_e^p$ ,  $\forall e \in E$ .

In this paper, the centralized problem (C) is assumed to be feasible. Moreover, (C) is also lower bounded, e.g., by  $\sum_{\pi \in \Pi} \min_{r \in \mathcal{R}} \rho_r^\pi \cdot (\sum_{j \in \mathcal{J}} d_j^\pi)$ . Thus strong duality must hold for (C); i.e.,

$$v(M) = \sum_{j \in \mathcal{J}} \sum_{\pi \in \Pi} \beta_j^{*\pi} d_j^\pi + \sum_{e \in E} \alpha_e^* \cdot \left( \sum_{i \in M} k_e^i + k_e^p \right), \quad (\text{A2})$$

where  $\beta_j^{*\pi}$  and  $\alpha_e^*$  are the optimal dual solutions associated with the demand and the capacity constraints in (C), respectively. According to condition (i) in Proposition 1,  $\beta_j^{*\pi}$  are identical for all  $\pi$  given any  $j \in \mathcal{J}$ ; thus we denote it as  $\beta_j^*$  in the rest of this proof for convenience. Moreover, according to condition (iii),  $\alpha_e^* = 0$ ,  $\forall e \in E$ . In this case, (A2) reduces to  $v(M) = \sum_{j \in \mathcal{J}} \beta_j^* \cdot [\sum_{\pi \in \Pi} d_j^\pi]$ , and thus for each producer  $i$ ,

$$(x_d^v)^i_r = \frac{v(M)}{R} \cdot R^i = \frac{\sum_{j \in \mathcal{J}} \beta_j^* \cdot [\sum_{\pi \in \Pi} d_j^\pi]}{R} \cdot R^i. \quad (\text{A3})$$

As a result of condition (ii) in Proposition 1, for each product  $\pi$ , the ratio  $d_j^\pi / \sum_{\pi \in \Pi} d_j^\pi$  is identical at any collection point  $j$ , and thus we can denote it as  $l_\pi \doteq d_j^\pi / \sum_{\pi \in \Pi} d_j^\pi$ . Hence, we can write  $R^i \doteq \sum_{\pi \in \Pi^i} \sum_{j \in \mathcal{J}} d_j^\pi = \sum_{\pi \in \Pi^i} \sum_{j \in \mathcal{J}} l_\pi \cdot \sum_{\pi \in \Pi} d_j^\pi = \sum_{\pi \in \Pi^i} l_\pi \cdot \sum_{j \in \mathcal{J}} \sum_{\pi \in \Pi} d_j^\pi = \sum_{\pi \in \Pi^i} l_\pi \cdot R$ . This indicates that  $R^i / R = \sum_{\pi \in \Pi^i} l_\pi$ , and substituting in (A3) yields

$$\frac{\sum_{j \in \mathcal{J}} \beta_j^* \cdot [\sum_{\pi \in \Pi} d_j^\pi]}{R} \cdot R^i = \sum_{j \in \mathcal{J}} \beta_j^* \cdot \sum_{\pi \in \Pi} d_j^\pi \cdot \sum_{\pi \in \Pi^i} l_\pi$$



$$\begin{aligned}
 &= \sum_{j \in \mathcal{J}} \beta_j^* \cdot \sum_{\pi \in \Pi^i} \left( \sum_{\pi \in \Pi} d_j^\pi \right) \cdot l_\pi \\
 &= \sum_{j \in \mathcal{J}} \beta_j^* \cdot \sum_{\pi \in \Pi^i} \left( \sum_{\pi \in \Pi} d_j^\pi \right) \cdot \frac{d_j^\pi}{\sum_{\pi \in \Pi} d_j^\pi} \\
 &= \sum_{j \in \mathcal{J}} \beta_j^* \cdot \sum_{\pi \in \Pi^i} d_j^\pi. \quad (\text{A4})
 \end{aligned}$$

Combining (A3) and (A4), we conclude that the allocation by return share  $(x^i)_r = \sum_{j \in \mathcal{J}} \beta_j^* \cdot \sum_{\pi \in \Pi^i} d_j^\pi$ ,  $\forall i \in M$ . Therefore, under the conditions (i)–(iii) in the proposition,  $x_r$  is equivalent to the dual-based allocation defined in (A1) and thus is in the core of the CRF game.  $\square$

**PROOF OF THEOREM 1.** First, in CRNs without independent capacity, no producers are able to operate an independent plan and thus no defection is feasible; i.e.,  $v(S) = \infty$ ,  $\forall S \subsetneq M$ . In this case, the allocation by cost-corrected return share with capacity rewards  $x_\mu^p$  boils down to the allocation by cost-corrected return share, and it remains well defined and in the core. In the rest of this proof, we assume independent capacity exists, and we discuss two cases to prove this theorem. Note that in this problem, the dual optimal solution satisfies  $\alpha_e^* \leq 0$ ,  $\forall e \in E$ .

*Case 1.* When  $\mu^i \geq 0$ ,  $\forall i \in M$ , we set  $p_e = |\alpha_e^*|$ ,  $\forall e \in E$ . Recall that strong duality is shown to hold for the centralized problem (C) in the proof of Proposition 1. Hence, using (A2) and noting that  $|\alpha_e^*| = -\alpha_e^*$ ,  $\forall e$ , we conclude that  $v(M) + \sum_{e \in E} p_e \sum_{i \in M} k_e^i = \sum_{j \in \mathcal{J}} \sum_{\pi \in \Pi} \beta_j^* d_j^\pi + \sum_{e \in E} \alpha_e^* k_e^p$ . Thus by the definition of the allocation by cost-corrected return share with capacity rewards in formula (9), we can calculate that

$$\begin{aligned}
 (x^i)_\mu^p &= \left[ \sum_{j \in \mathcal{J}} \sum_{\pi \in \Pi} \beta_j^* d_j^\pi + \sum_{e \in E} \alpha_e^* k_e^p \right] \cdot \frac{\sum_{j \in \mathcal{J}} \sum_{\pi \in \Pi^i} \beta_j^* d_j^\pi}{\sum_{j \in \mathcal{J}} \sum_{\pi \in \Pi} \beta_j^* d_j^\pi} + \sum_{e \in E} \alpha_e^* k_e^i \\
 &= \sum_{j \in \mathcal{J}} \sum_{\pi \in \Pi^i} \beta_j^* d_j^\pi + \sum_{e \in E} \alpha_e^* k_e^i + \sum_{e \in E} \alpha_e^* \mu^i k_e^p. \quad (\text{A5})
 \end{aligned}$$

Hence,  $x_\mu^p$  is equivalent to the cost allocation  $x_d^\nu$  defined in (A1) if we set  $\nu_e^i = \mu^i k_e^p$ ,  $\forall e \in E$ ,  $\forall i \in M$ . Since  $\sum_{i \in M} \mu^i = 1$  and  $\mu^i \geq 0$ ,  $\forall i \in M$  in this case, we have  $\nu_e^i \geq 0$ ,  $\forall e \in E$ ,  $\forall i \in M$ , and  $\sum_{i \in M} \nu_e^i = k_e^p$ ,  $\forall e \in E$ . Hence, this cost allocation is equivalent to a dual-based cost allocation and thus is in the core of the CRF game.

*Case 2.* When  $\exists i \in M$  such that  $\mu^i < 0$ , we have  $\mu^i k_e^p < 0$  in (A5). Thus the allocation by cost-corrected return share with capacity reward  $p_e$  set equal to  $|\alpha_e^*|$  is no longer a dual-based cost allocation and may reside outside the core of the CRF game. In this case, we propose another set of reward prices that gives rise to an allocation  $x_\mu^p$  in the core. We first show that any set of prices  $\{p_e\}$  that satisfies the two conditions in (A6) is able to do so:

$$\begin{cases} p_e \geq |\alpha_e^*| \quad \forall e \in E & (\text{a}) \\ \sum_{e \in E} p_e \cdot \sum_{i \in M} k_e^i = \sum_{e \in E} |\alpha_e^*| \cdot \left( \sum_{i \in M} k_e^i + k_e^p \right) & (\text{b}). \end{cases} \quad (\text{A6})$$

To show that  $x_\mu^p$  is a core allocation given the above conditions, consider an arbitrary subcoalition  $S \subset M$ . Because

of strong duality with respect to (C) (formula (A2)), condition (A6(b)), and the fact that  $|\alpha_e^*| = -\alpha_e^*$ , we have

$$\begin{aligned}
 v(M) + \sum_{e \in E} p_e \sum_{i \in M} k_e^i &= \sum_{j \in \mathcal{J}} \sum_{\pi \in \Pi} \beta_j^* d_j^\pi + \sum_{e \in E} \alpha_e^* \cdot \left( \sum_{i \in M} k_e^i + k_e^p \right) \\
 &\quad + \sum_{e \in E} |\alpha_e^*| \cdot \left( \sum_{i \in M} k_e^i + k_e^p \right) \\
 &= \sum_{j \in \mathcal{J}} \sum_{\pi \in \Pi} \beta_j^* d_j^\pi.
 \end{aligned}$$

Then by formula (9), the cost allocated to  $S$  satisfies

$$\begin{aligned}
 \sum_{i \in S} (x^i)_\mu^p &= \sum_{i \in S} \left\{ \left[ \sum_{j \in \mathcal{J}} \sum_{\pi \in \Pi} \beta_j^* d_j^\pi \right] \cdot \frac{\sum_{j \in \mathcal{J}} \sum_{\pi \in \Pi^i} \beta_j^* d_j^\pi}{\sum_{j \in \mathcal{J}} \sum_{\pi \in \Pi} \beta_j^* d_j^\pi} - \sum_{e \in E} p_e k_e^i \right\} \\
 &= \sum_{i \in S} \left\{ \sum_{j \in \mathcal{J}} \sum_{\pi \in \Pi^i} \beta_j^* d_j^\pi - \sum_{e \in E} p_e k_e^i \right\} \\
 &\leq \sum_{i \in S} \left\{ \sum_{j \in \mathcal{J}} \sum_{\pi \in \Pi^i} \beta_j^* d_j^\pi + \sum_{e \in E} \alpha_e^* k_e^i \right\} \leq v(S). \quad (\text{A7})
 \end{aligned}$$

The inequality  $\leq^1$  is due to condition (A6(a)). To validate the second inequality  $\leq^2$ , note that the optimal dual solution associated with (C),  $[\beta^*, \alpha^*]$ , is a feasible dual solution to the centralized problem within  $S$ ,  $(C^S)$ . Hence inequality  $\leq^2$  follows as a result of weak duality with respect to the dual problem associated with  $(C^S)$ . Since we can show that (A7) holds for any subcoalition  $S$ , the allocation  $x_\mu^p$  is in the core of the CRF game by definition.

Second, we show that there exist prices that satisfy both conditions in (A6). To do this, we construct a set of reward prices by adjusting the marginal capacity value  $|\alpha_e^*|$  based on producers' independent capacity levels: Pick one edge  $e^0$  such that  $\sum_{i \in M} k_{e^0}^i > 0$  (such an edge always exists under the assumption that independent capacity exists). Define a set of reward prices such that

$$p_{e^0} \doteq |\alpha_{e^0}^*| + \frac{\sum_{e \in E} |\alpha_e^*| \cdot k_e^p}{\sum_{i \in M} k_{e^0}^i} \quad \text{and} \quad p_e \doteq |\alpha_e^*| \quad \forall e \neq e^0. \quad (\text{A8})$$

It is easy to see that the prices defined in (A8) satisfy  $p_e \geq |\alpha_e^*|$  on each edge  $e$ ; i.e., condition (A6(a)) is met. Moreover, we can calculate that this set of prices also satisfies

$$\begin{aligned}
 \sum_{e \in E} p_e \cdot \sum_{i \in M} k_e^i &= \sum_{e \in E} |\alpha_e^*| \cdot \sum_{i \in M} k_e^i + \frac{\sum_{e \in E} |\alpha_e^*| \cdot k_e^p}{\sum_{i \in M} k_{e^0}^i} \cdot \sum_{i \in M} k_{e^0}^i \\
 &= \sum_{e \in E} |\alpha_e^*| \cdot \left( \sum_{i \in M} k_e^i + k_e^p \right). \quad (\text{A9})
 \end{aligned}$$

That is, condition (A6(b)) is also met. Therefore, based on our analysis above, the prices defined in (A8) give rise to a core allocation under cost-corrected return share with capacity rewards. In fact, when these prices are adopted, we can calculate that the cost allocated to producer  $i$  equals

$$\begin{aligned}
 (x^i)_\mu^p &= \sum_{j \in \mathcal{J}} \sum_{\pi \in \Pi^i} \beta_j^* d_j^\pi - \sum_{e \in E} p_e k_e^i \\
 &= \sum_{j \in \mathcal{J}} \sum_{\pi \in \Pi^i} \beta_j^* d_j^\pi + \sum_{e \in E} \alpha_e^* k_e^i + \sum_{e \in E} \alpha_e^* k_e^p \cdot \frac{k_{e^0}^i}{\sum_{j \in \mathcal{J}} \sum_{\pi \in \Pi^i} \beta_j^* d_j^\pi}. \quad (\text{A10})
 \end{aligned}$$

Hence the cost allocation  $x_\mu^p$  is equivalent to the dual-based cost allocation  $x_d^\nu$  defined in (A1) if we set

$v_e^i = k_{e0}^i / \sum_{i \in M} k_{e0}^i \cdot k_e^p, \forall e \in E, \forall i \in M$  and thus is in the core of the CRF game.  $\square$

**PROOF OF PROPOSITION 2.** We first prove result (1) in the proposition, i.e.,  $\min_{p_e \geq 0} \mathcal{G}(x_\mu^{p(\eta, \zeta)}) = 0$ . Consider any CRN. We modify its unit cost and revenue by the discount factors  $\eta^M$  and increment factor  $\zeta^M$  associated with the grand coalition  $M$ , respectively. We then define a CRF game  $(M, v^{(\eta^M, \zeta^M)})$  based on the modified CRN by calculating the stand-alone cost of any coalition  $S$  operating on this CRN, which we denote by  $v^{(\eta^M, \zeta^M)}(S)$ . According to Theorem 1, we know that there exists a set of prices  $\{p_e\}$  such that the allocation by cost-corrected return share with capacity rewards  $x_\mu^{p(\eta, \zeta)}$  is inside the core of the CRF game  $(M, v^{(\eta^M, \zeta^M)})$ . In other words,  $\sum_{i \in S} (x_i^p)_\mu^{p(\eta, \zeta)} \leq v^{(\eta^M, \zeta^M)}(S)$  holds for any subcoalition  $S \subset M$ . Next we show that  $v^{(\eta^M, \zeta^M)}(S) \leq v^{(\eta, \zeta)}(S), \forall S \subset M$ . Recall that  $v^{(\eta, \zeta)}(S)$  is defined as the stand-alone cost of  $S$  calculated based on the CRN where the unit cost and revenue modified by the discount factor  $\eta^S$  and increment factor  $\zeta^S$ . Since  $\eta$  and  $\zeta$  decrease and increase, respectively, as the return volume in the CRN grows,  $\eta^M \leq \eta^S$  and  $\zeta^M \geq \zeta^S$  hold for each subcoalition  $S \subsetneq M$ . Hence, we conclude that on each edge  $e \notin E \setminus \{(r, r')\}$ , the unit operational cost satisfies  $\eta^M \cdot c_e^\pi \leq \eta^S \cdot c_e^\pi$ , and at each processor  $r$ , the net processing cost satisfies  $\eta^M \cdot c_{(r, r')}^\pi + \eta^M \cdot \sigma_r^\pi + \zeta^M \cdot \rho_r^\pi \leq \eta^S \cdot c_{(r, r')}^\pi + \eta^S \cdot \sigma_r^\pi + \zeta^S \cdot \rho_r^\pi$  (recall that the revenue  $\rho_r^\pi$  is modeled as negative cost). Thus we can show that  $v^{(\eta^M, \zeta^M)}(S) \leq v^{(\eta, \zeta)}(S)$  and  $\sum_{i \in S} (x_i^p)_\mu^{p(\eta, \zeta)} \leq v^{(\eta^M, \zeta^M)}(S) \leq v^{(\eta, \zeta)}(S), \forall S \subsetneq M$ . By definition, the allocation  $x_\mu^{p(\eta, \zeta)}$  remains in the core of the CRF game under scale economies.

Next we prove result (2) in the proposition, i.e.,  $\mathcal{G}(x_r^{(\eta, \zeta)}) \leq \eta^M \cdot \mathcal{G}(x_r)$  and  $\min_{p_e \geq 0} \mathcal{G}(x_r^{p(\eta, \zeta)}) \leq \eta^M \cdot \min_{p_e \geq 0} \mathcal{G}(x_r^p)$  hold under the conditions specified. First notice that since the unit processing revenue of each product  $\pi$  is identical among the processors, the total revenue obtained within a coalition  $S \subseteq M$  under any feasible product routing equals  $\sum_{\pi \in \Pi^S} \rho^\pi \cdot \sum_{j \in J} d_j^\pi$ . We denote such a revenue component when  $S$  operates independently without scale economies as  $v^r(S)$ . It is easy to see that the total processing revenue that  $S$  obtains operating independently under scale economies,  $v^{(\eta, \zeta)r}(S)$ , equals  $\zeta^S \cdot v^r(S)$ . Moreover, note that under this condition, the optimal routing is not affected by scale economies (since the unit cost is uniformly scaled throughout the network and the revenue part in the objective function is a constant under any feasible routing). Hence, if we define  $v^c(S) \doteq v(S) - v^r(S)$  as the cost component of the stand-alone cost of  $S$  without scale economies, we can calculate that for all  $S \subseteq M$ , this cost component is scaled by  $\eta^S$  when scale economies are factored in, i.e.,  $v^{(\eta, \zeta)c}(S) = \eta^S \cdot v^c(S)$ . Thus we have  $v^{(\eta, \zeta)}(S) = \eta^S \cdot v^c(S) + \zeta^S \cdot v^r(S)$ .

Now consider an arbitrary subcoalition  $S$  such that  $v(S) < \infty$ . Since  $\eta^M \leq \eta^S$  and  $v^c(S) \geq 0$ , we obtain the following inequality:

$$\begin{aligned} & \bar{v}^{(\eta, \zeta)M} - \bar{v}^{(\eta, \zeta)S} \\ &= \left( \frac{\eta^M \cdot v^c(M)}{R} - \frac{\eta^S \cdot v^c(S)}{R^S} \right) + \left( \frac{\zeta^M \cdot v^r(M)}{R} - \frac{\zeta^S \cdot v^r(S)}{R^S} \right) \\ &\leq \eta^M \cdot \left( \frac{v^c(M)}{R} - \frac{v^c(S)}{R^S} \right) + \left( \frac{\zeta^M \cdot v^r(M)}{R} - \frac{\zeta^S \cdot v^r(S)}{R^S} \right). \quad (\text{A11}) \end{aligned}$$

We further derive the following inequalities considering the following two cases.

**Case I.** When  $|v^r(M)/R| \geq |v^r(S)/R^S|$ , then since  $\zeta^M \geq \zeta^S$  and both  $v^r(M)$  and  $v^r(S)$  are nonpositive, and  $\eta^M \leq 1 \leq \eta^S$ , we calculate that

$$\begin{aligned} & \eta^M \cdot \left( \frac{v^c(M)}{R} - \frac{v^c(S)}{R^S} \right) + \left( \frac{\zeta^M \cdot v^r(M)}{R} - \frac{\zeta^S \cdot v^r(S)}{R^S} \right) \\ &\leq \eta^M \cdot \left( \frac{v^c(M)}{R} - \frac{v^c(S)}{R^S} \right) + \zeta^M \cdot \left( \frac{v^r(M)}{R} - \frac{v^r(S)}{R^S} \right) \\ &\leq \eta^M \cdot \left( \frac{v^c(M)}{R} - \frac{v^c(S)}{R^S} + \frac{v^r(M)}{R} - \frac{v^r(S)}{R^S} \right) \\ &= \eta^M (\bar{v}^M - \bar{v}^S). \quad (\text{A12}) \end{aligned}$$

**Case II.** When  $|v^r(M)/R| < |v^r(S)/R^S|$ , then since  $\zeta^M \cdot |v^r(M)/R| \geq \zeta^S \cdot |v^r(S)/R^S|$ , we calculate that

$$\begin{aligned} & \eta^M \cdot \left( \frac{v^c(M)}{R} - \frac{v^c(S)}{R^S} \right) + \left( \frac{\zeta^M \cdot v^r(M)}{R} - \frac{\zeta^S \cdot v^r(S)}{R^S} \right) \\ &\leq \eta^M \cdot \left( \frac{v^c(M)}{R} - \frac{v^c(S)}{R^S} \right) \\ &\leq \eta^M \cdot \left( \frac{v^c(M)}{R} - \frac{v^c(S)}{R^S} + \frac{v^r(M)}{R} - \frac{v^r(S)}{R^S} \right) \\ &= \eta^M (\bar{v}^M - \bar{v}^S). \quad (\text{A13}) \end{aligned}$$

Hence, combining (A11) with both (A12) and (A13), we conclude that  $\bar{v}^{(\eta, \zeta)M} - \bar{v}^{(\eta, \zeta)S} \leq \eta^M \cdot (\bar{v}^M - \bar{v}^S)$ . By formula (7), we know that  $\mathcal{G}(x_r^{(\eta, \zeta)}) = \max_{S \subseteq M} \{R^S \cdot (\bar{v}^{(\eta, \zeta)M} - \bar{v}^{(\eta, \zeta)S})\} \leq \eta^M \cdot \max_{S \subseteq M} \{R^S \cdot (\bar{v}^M - \bar{v}^S)\} = \eta^M \cdot \mathcal{G}(x_r)$ . To prove  $\min_{p_e \geq 0} \mathcal{G}(x_r^{p(\eta, \zeta)}) \leq \eta^M \cdot \min_{p_e \geq 0} \mathcal{G}(x_r^p)$  under the model of return share with capacity rewards, we first prove the following lemma where the incentive compatibility gap  $\min_{p_e \geq 0} \mathcal{G}(x_r^p)$  is calculated as the optimal objective value of a linear program.

**LEMMA 1.** Let  $\Psi = \{S \subseteq M: v(S) < \infty\}$  be the set of subcoalitions whose members have sufficient independent capacity to process their own return volumes in their individual CRNs. Then

$$\min_{p_e \geq 0} \mathcal{G}(x_r^p) = \max \left\{ \sum_{S \in \Psi} [\bar{v}^M - \bar{v}^S] \cdot y^S \right\} \quad (\text{A14})$$

$$\text{s.t. } \sum_{S \in \Psi} [\bar{k}_e^S - \bar{k}_e^M] \cdot y^S \leq 0, \quad \forall e \in E, \quad (\text{A15})$$

$$\sum_{S \in \Psi} \frac{1}{R^S} \cdot y^S \leq 1, \quad (\text{A16})$$

$$\text{nonnegativity constraints.} \quad (\text{A17})$$

**PROOF OF LEMMA 1.** By formula (11), the minimum incentive compatibility gap under return share with capacity rewards  $\min_{p_e \geq 0} \mathcal{G}(x_r^p)$  is calculated by solving the problem  $\min_{p_e \geq 0} \max_{S \subseteq M} \{R^S \cdot [\bar{v}^M - \bar{v}^S - \sum_{e \in E} p_e (\bar{k}_e^S - \bar{k}_e^M)]\}$ . This problem is equivalent to the following linear program:

$$\begin{aligned} & \text{minimize } z \\ & \text{s.t. } z \geq R^S \cdot \left[ \bar{v}^M - \bar{v}^S - \sum_{e \in E} p_e (\bar{k}_e^S - \bar{k}_e^M) \right], \quad \forall S \in \Psi, \\ & \text{nonnegativity constraints,} \quad (\text{A18}) \end{aligned}$$

where the variables are  $\{p_e, \forall e \in E\}$  and  $z$ . Rearranging the constraint in the above program, we obtain  $(1/R^S) \cdot z + \sum_{e \in E} p_e (\bar{k}_e^S - \bar{k}_e^M) \geq \bar{v}^M - \bar{v}^S, \forall S \in \Psi$ . To derive the

dual of this linear program, we associate with each of these constraints a dual variable  $y^S$  and obtain the program (A14)–(A17).  $\square$

Continuing the proof of Proposition 2, if we denote the feasible region of the above linear program (A14)–(A17) as  $Y = \{y^S, S \subset \Psi\}$ : (A15)–(A17), then  $\min_{p_e \geq 0} \mathcal{G}(x_r^{p(\eta, \zeta)}) = \max_Y \sum_{S \in \Psi} [\bar{v}(\eta, \zeta)^M - \bar{v}(\eta, \zeta)^S] \cdot y^S \leq \eta^M \cdot \max_Y \sum_{S \in \Psi} [\bar{v}^M - \bar{v}^S] \cdot y^S = \eta^M \cdot \min_{p_e \geq 0} \mathcal{G}(x_r^p)$ . This completes the proof of Proposition 2.  $\square$

**PROOF OF THEOREM 2.** To prove Theorem 2, we first construct a CRF game under nonmember access fees as follows. We modify the program (C<sup>S</sup>) defined in §3.2 that computes the stand-alone cost of subcoalition  $S \subseteq M$ . Specifically, we add the term  $\sum_{e \in E} \phi_e \cdot \max\{0; \sum_{\pi \in \Pi^S} f_e^\pi - \sum_{i \in S} k_e^i\}$  to the objective function to account for the nonmember access fees paid. We then increase the right-hand side of the capacity constraint on each edge  $e$  by the amount of operator-contracted capacity  $k_e^p$ . We define the optimal value of the modified program as  $v_\phi(S)$  and call the game  $(M, v_\phi(S))$  a CRF game under nonmember access fees  $\{\phi_e\}$ . We present the following lemma that provides lower bounds on the nonmember access fees  $\{\phi_e\}$  under which a given dual-based allocation as defined in (A1) is guaranteed to be in the core of the game  $(M, v_\phi(S))$ .

**LEMMA 2.** The dual-based allocation  $x_d^v$  is in the core of the CRF game with nonmember access fees  $\{\phi_e\}$  if

$$\phi_e \geq \max_{i \in M} \{(1 - v_e^i/k_e^p) \cdot |\alpha_e^*|\}, \quad \forall e \in E.$$

**PROOF OF LEMMA 2.** Consider an arbitrary subcoalition  $S \subset M$ . By defining  $(k_e^p)^S \doteq \max\{0, \sum_{\pi \in \Pi^S} f_e^\pi - \sum_{i \in S} k_e^i\}$ , we can model the centralized problem within  $S$  under nonmember access fees  $\{\phi_e\}$ , i.e.,  $(C_\phi^S)$ , as follows:

$$(C_\phi^S) \quad v_\phi(S) = \min \left\{ \sum_{e \in E \setminus \{(r, r')\}} \sum_{\pi \in \Pi^S} c_e^\pi f_e^\pi + \sum_{r \in \mathcal{R}} \sum_{\pi \in \Pi} \hat{c}_r^\pi f_{r'}^\pi + \sum_{e \in E} \phi_e \cdot (k_e^p)^S \right\} \quad (\text{A19})$$

$$\begin{aligned} \text{s.t.} \quad & \sum_{e=(u, v) \in E} f_e^\pi - \sum_{e=(v, w) \in E} f_e^\pi = 0, \\ & \forall v \in V \setminus \{\mathcal{F}, \mathcal{R}'\}, \\ & \forall \pi \in \Pi^S \quad [(v^S)^\pi_v], \quad (\text{A20}) \end{aligned}$$

$$\begin{aligned} f_{(j, j')}^\pi &= d_j^\pi, \quad \forall \pi \in \Pi^S, \\ & \forall j \in \mathcal{F} \quad [(\beta^S)_j^\pi], \quad (\text{A21}) \end{aligned}$$

$$\sum_{\pi \in \Pi^S} f_e^\pi \leq \sum_{i \in S} k_e^i + k_e^p, \quad \forall e \in E \quad [\alpha_e^S], \quad (\text{A22})$$

$$\begin{aligned} \sum_{\pi \in \Pi^S} f_e^\pi - (k_e^p)^S &\leq \sum_{i \in S} k_e^i, \\ & \forall e \in E \quad [\omega_e^S], \quad (\text{A23}) \end{aligned}$$

$$\text{nonnegativity constraints}, \quad (\text{A24})$$

where  $[v, \beta, \alpha, \omega, \sigma]$  are the dual variables associated with the constraints in  $(C_\phi^S)$ . Set  $(v^S)_j^\pi = 0, \forall \pi \in \Pi^S, \forall j \in \mathcal{F}$  and  $(v^S)_{r'}^\pi = 0, \forall \pi \in \Pi^S, \forall r' \in \mathcal{R}'$ . Define the set  $E_j \doteq \{(j, j'), \forall j \in \mathcal{F}\}$ , and  $E_r \doteq \{(r, r'), \forall r \in \mathcal{R}\}$ . We can then formulate

the dual program to the above linear program  $(C_\phi^S)$  as follows:

$$(D_\phi^S) \quad \max \left\{ \sum_{j \in \mathcal{F}} \sum_{\pi \in \Pi^S} d_j^\pi (\beta^S)_j^\pi + \sum_{e \in E} \sum_{i \in S} k_e^i \cdot (\alpha_e^S + \omega_e^S) + \sum_{e \in E} k_e^p \cdot \alpha_e^S \right\} \quad (\text{A25})$$

$$\begin{aligned} \text{s.t.} \quad & (v^S)_j^\pi - (v^S)_j^\pi + \alpha_e^S + \omega_e^S + (\beta^S)_j^\pi \leq c_e^\pi, \\ & \forall \pi \in \Pi^S, \forall e \in E_j, \quad (\text{A26}) \end{aligned}$$

$$\begin{aligned} & (v^S)_v^\pi - (v^S)_u^\pi + \alpha_e^S + \omega_e^S \leq c_e^\pi, \\ & \forall \pi \in \Pi^S, \forall e = (u, v) \in E \setminus (E_j \cup E_r), \quad (\text{A27}) \end{aligned}$$

$$\begin{aligned} & (v^S)_{r'}^\pi - (v^S)_r^\pi + \alpha_e^S + \omega_e^S \leq \hat{c}_r^\pi, \\ & \forall \pi \in \Pi^S, \forall e \in E_r, \quad (\text{A28}) \end{aligned}$$

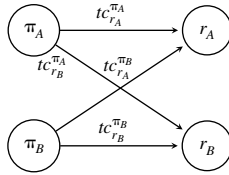
$$-\omega_e^S \leq \phi_e, \quad \alpha_e^S \leq 0, \quad \omega_e^S \leq 0, \quad \forall e \in E. \quad (\text{A29})$$

We define a solution to the above program  $(D_\phi^S)$  based on the optimal dual solutions to the centralized problem (C)  $[v_v^{*\pi}, \alpha_e^*, \beta_j^{*\pi}]$ :

$$\begin{aligned} (v^S)_v^\pi &\doteq v_v^{*\pi}, \quad \forall v \in V \setminus \{\mathcal{F}, \mathcal{R}'\}, \forall \pi \in \Pi^S, \\ \alpha_e^S &\doteq \frac{\sum_{i \in S} v_e^i}{k_e^p} \cdot \alpha_e^*, \\ \omega_e^S &\doteq \left(1 - \frac{\sum_{i \in S} v_e^i}{k_e^p}\right) \cdot \alpha_e^*, \quad \forall e \in E, \\ (\beta^S)_j^\pi &\doteq \beta_j^{*\pi}, \quad \forall \pi \in \Pi^S, \forall j \in \mathcal{F}. \end{aligned} \quad (\text{A30})$$

Notice that  $[v_v^{*\pi}, \alpha_e^*, \beta_j^{*\pi}]$  satisfies the dual constraints with respect to (C); i.e.,  $v_j^{*\pi} - v_j^{*\pi} + \alpha_e^* + \beta_j^{*\pi} \leq c_e^\pi, \forall \pi \in \Pi, \forall e \in E_j, v_v^{*\pi} - v_u^{*\pi} + \alpha_e^* \leq c_e^\pi, \forall \pi \in \Pi, \forall e \in E \setminus (E_j \cup E_r), v_{r'}^{*\pi} - v_r^{*\pi} + \alpha_e^* \leq \hat{c}_r^\pi, \forall \pi \in \Pi, \forall e \in E_r$ . Hence, it is easy to check that the solution defined in (A30) is feasible for  $(D_\phi^S)$  under the condition in the lemma that  $\phi_e \geq \max_{i \in M} \{(1 - v_e^i/k_e^p) \cdot |\alpha_e^*|\}, \forall e \in E$ . By weak duality, we conclude that the objective value of  $(D_\phi^S)$  under the above solution, which equals the cost allocated to subcoalition  $S$  under the dual-based allocation  $x_d^v$ , is no greater than  $v_\phi(S)$ . Since we can show that such a relationship holds for all subcoalitions, we prove that the dual-based allocation  $x_d^v$  lies in the core of the CRF game with nonmember access fee  $\phi_e$  when  $\phi_e \geq \max_{i \in M} \{(1 - v_e^i/k_e^p) \cdot |\alpha_e^*|\}, \forall e \in E$ .  $\square$

Continuing the proof of Theorem 2, we show that the bounds identified directly follow from Lemma 2 and the proof of Theorem 1. Specifically, when  $\mu^i \geq 0, \forall i \in M$  (i.e., Case 1 in the proof of Theorem 1), according to (A5), we can see that by adopting the prices  $p_e = |\alpha_e^*|, \forall e \in E$ , a dual-based allocation is obtained with  $v_e^i$  set equal to  $\mu^i \cdot k_e^p, \forall e \in E, \forall i \in M$ . Hence, by replacing  $v_e^i$  in Lemma 2 by  $\mu^i \cdot k_e^p$ , we obtain the first bound in Theorem 2. Similarly, when  $\exists i \in M$  such that  $\mu^i < 0$  and independent capacity exists (i.e., Case 2 in the proof of Theorem 1), we can adopt the prices given by (A8) and, according to formula (A10), obtain a dual-based allocation where  $v_e^i$  is set equal to  $k_{e^0}^i / \sum_{i \in M} k_{e^0}^i k_e^p, \forall e \in E, \forall i \in M$ . Hence, by choosing  $e^0$  to be the edge where the value of  $\max_{i \in M} \{1 - k_e^i / \sum_{i \in M} k_e^i\}$  is minimized, we obtain the second bound given in Theorem 2.  $\square$

**Figure A.1** A Two-Producer Special Case of the CRN Represented as a Transportation Network  $CRN_2$ **A.2. A Two-Producer Example of the CRN**

In this subsection, we consider a two-producer example of the CRN. We use this example to demonstrate the stringency of the sufficient conditions for (i) the allocation by return share to be in the core (i.e., Proposition 1), (ii) scale economies to reduce the incentive compatibility gap of return share (i.e., Proposition 2), and (iii) cost-corrected return share with capacity rewards to be in the core of the CRF game under nonmember access fees (i.e., Theorem 2). Specifically, let  $A$  and  $B$  denote the two producers who have products  $\pi_A$  and  $\pi_B$  with return volumes  $d^{\pi_A}$  and  $d^{\pi_B}$ , respectively. They respectively own or have contracts with independent processing resources  $r_A$  and  $r_B$ , with capacities  $k_{r_A} \geq d^{\pi_A}$  and  $k_{r_B} \geq d^{\pi_B}$ . We assume that the associated CRN is simple enough such that the cost for a product to be routed to (i.e., collected, consolidated, transported, and finally processed at) a processor can be presented as a combined unit cost. The optimal routing then can be determined based on this cost as well as the capacity availability at the processors. In other words, if for every product and processor pair  $(\pi_i, r_i)$ , we denote the associated unit total cost to route  $\pi_i$  to  $r_i$  as  $tc_{r_i}^{\pi_i}$ , then the CRN can be reduced to a two-by-two transportation network as shown in Figure A.1, which we denote by  $CRN_2$ . We further assume that in  $CRN_2$ , it is cheaper to route both products to  $r_A$  than  $r_B$ , i.e.,  $tc_{r_A}^{\pi_A} < tc_{r_B}^{\pi_A}$  and  $tc_{r_A}^{\pi_B} < tc_{r_B}^{\pi_B}$ .

**A.2.1. Group Incentive Compatibility of Return Share.**

Proposition 3 characterizes the necessary and sufficient conditions for the allocation by return share to be in the core in  $CRN_2$ .

**PROPOSITION 3.** In  $CRN_2$ , when  $tc_{r_A}^{\pi_A} \leq tc_{r_A}^{\pi_B}$ ,  $x_r$  is in the core of the CRF game if and only if (i) there is sufficient capacity at processor  $r_A$  to process both products, i.e.,  $k_{r_A} \geq d^{\pi_A} + d^{\pi_B}$ , and (ii) the unit total costs to route both products to processor  $r_A$  are identical, i.e.,  $tc_{r_A}^{\pi_A} = tc_{r_A}^{\pi_B}$ . When  $tc_{r_A}^{\pi_A} > tc_{r_A}^{\pi_B}$ , there exist two constants  $\underline{\Delta}$  and  $\bar{\Delta}$  such that  $x_r$  is in the core of the CRF game if and only if the difference between the unit total costs to route product  $\pi_A$  and  $\pi_B$  to processor  $r_A$  satisfies  $\underline{\Delta} \leq tc_{r_A}^{\pi_A} - tc_{r_A}^{\pi_B} \leq \bar{\Delta}$ .

**PROOF OF PROPOSITION 3.** We first prove the result in the situation where  $tc_{r_A}^{\pi_A} \leq tc_{r_A}^{\pi_B}$ . In  $CRN_2$ , the average cost incurred when  $A$  operates alone is  $\bar{v}^A = tc_{r_A}^{\pi_A}$ . Hence by Equation (6), if  $x_r$  is in the core, the minimum average cost within the grand coalition  $M$  (i.e.,  $\{A, B\}$ ) must satisfy  $\bar{v}^M \leq \bar{v}^A = tc_{r_A}^{\pi_A}$ . However, when  $tc_{r_A}^{\pi_A} \leq tc_{r_A}^{\pi_B}$ ,  $tc_{r_A}^{\pi_A}$  is the smallest unit total cost to route a product to a processor in  $CRN_2$ ; thus  $tc_{r_A}^{\pi_A} \leq \bar{v}^M$ . Therefore, we conclude that if  $x_r$  is a core allocation when  $tc_{r_A}^{\pi_A} \leq tc_{r_A}^{\pi_B}$ , then  $\bar{v}^M = tc_{r_A}^{\pi_A}$  must hold. This requires that all products are processed at  $r_A$  and

$tc_{r_A}^{\pi_A} = tc_{r_A}^{\pi_B}$ . Hence, the condition that (i) there is sufficient capacity at  $r_A$  to process both products (i.e.,  $k_{r_A} \geq d^{\pi_A} + d^{\pi_B}$ ) and (ii) their unit total costs to be routed to  $r_A$  are identical (i.e.,  $tc_{r_A}^{\pi_A} = tc_{r_A}^{\pi_B}$ ) is necessary for  $x_r$  to be in the core. On the other hand, when this condition holds, we can show that  $\bar{v}^M = \bar{v}^A = \bar{v}^B$ , and  $x_r$  must be in the core according to Equation (6). Hence, this condition is also sufficient to ensure the core membership of the allocation by return share.

Next, we prove the proposition in the situation where  $tc_{r_A}^{\pi_A} > tc_{r_A}^{\pi_B}$ . To do that, we consider the following four scenarios. For discussion convenience, we first define the following notation that represents the difference in the unit total costs for each product to be routed to the two processors:  $\delta^{\pi_i} \doteq tc_{r_B}^{\pi_i} - tc_{r_A}^{\pi_i}$ ,  $i = A, B$ .

**Scenario 1.**  $k_{r_A} \geq d^{\pi_A} + d^{\pi_B}$ .

**Scenario 2.**  $k_{r_A} < d^{\pi_A} + d^{\pi_B}$  and  $\delta^{\pi_A} \geq \delta^{\pi_B}$ .

**Scenario 3.**  $d^{\pi_B} \leq k_{r_A} < d^{\pi_A} + d^{\pi_B}$  and  $\delta^{\pi_A} < \delta^{\pi_B}$ .

**Scenario 4.**  $k_{r_A} < d^{\pi_B}$  and  $\delta^{\pi_A} < \delta^{\pi_B}$ .

In each scenario, we calculate the minimum average cost within the grand coalition  $\bar{v}^M$ :

$$\bar{v}^M = \frac{1}{d^{\pi_A} + d^{\pi_B}} \begin{cases} [tc_{r_A}^{\pi_A} \cdot d^{\pi_A} + tc_{r_A}^{\pi_B} \cdot d^{\pi_B}], & \text{(Scenario 1)} \\ [tc_{r_A}^{\pi_A} \cdot d^{\pi_A} + tc_{r_A}^{\pi_B} \cdot (k_{r_A} - d^{\pi_A}) \\ + tc_{r_B}^{\pi_B} \cdot (d^{\pi_B} - k_{r_A} + d^{\pi_A})], & \text{(Scenario 2)} \\ [tc_{r_A}^{\pi_B} \cdot d^{\pi_B} + tc_{r_A}^{\pi_A} \cdot (k_{r_A} - d^{\pi_B}) \\ + tc_{r_B}^{\pi_A} \cdot (d^{\pi_A} - k_{r_A} + d^{\pi_B})], & \text{(Scenario 3)} \\ [tc_{r_A}^{\pi_B} \cdot k_{r_A} + tc_{r_B}^{\pi_B} \cdot (d^{\pi_B} - k_{r_A}) \\ + tc_{r_B}^{\pi_A} \cdot d^{\pi_A}]. & \text{(Scenario 4)} \end{cases} \quad (A31)$$

By Equation (6), the allocation by return share is in the core of the CRF game if and only if the inequalities  $\bar{v}^M \leq \bar{v}^A = tc_{r_A}^{\pi_A}$  and  $\bar{v}^M \leq \bar{v}^B = tc_{r_B}^{\pi_B}$  both hold. We show that these two inequalities hold if and only if the cost differential between the two products being routed to processor  $r_A$ , i.e.,  $tc_{r_A}^{\pi_A} - tc_{r_A}^{\pi_B}$ , satisfies each condition identified in (A32) in the corresponding scenario:

$$tc_{r_A}^{\pi_A} - tc_{r_A}^{\pi_B} \in \begin{cases} \left[ 0, \frac{d^{\pi_A} + d^{\pi_B}}{d^{\pi_A}} \cdot \delta^{\pi_B} \right], & \text{(Scenario 1)} \\ \left[ \frac{d^{\pi_A} + d^{\pi_B} - k_{r_A}}{d^{\pi_B}} \cdot \delta^{\pi_B}, \frac{k_{r_A}}{d^{\pi_A}} \cdot \delta^{\pi_B} \right], & \text{(Scenario 2)} \\ \left[ \frac{d^{\pi_A} + d^{\pi_B} - k_{r_A}}{d^{\pi_B}} \cdot \delta^{\pi_A}, \frac{1}{d^{\pi_A}} \cdot ((d^{\pi_A} + d^{\pi_B}) \cdot \delta^{\pi_B} - (d^{\pi_A} + d^{\pi_B} - k_{r_A}) \cdot \delta^{\pi_A}) \right], & \text{(Scenario 3)} \\ \left[ \frac{1}{d^{\pi_B}} \cdot ((d^{\pi_B} - k_{r_A}) \cdot \delta^{\pi_B} + d^{\pi_A} \cdot \delta^{\pi_A}), \frac{1}{d^{\pi_A}} \cdot ((d^{\pi_A} + k_{r_A}) \cdot \delta^{\pi_B} - d^{\pi_A} \cdot \delta^{\pi_A}) \right]. & \text{(Scenario 4)} \end{cases} \quad (A32)$$

Hence, we conclude that in the situation where  $tc_{r_A}^{\pi_A} > tc_{r_A}^{\pi_B}$ , the allocation by return share is in the core if and only



if the cost difference  $tc_{r_A}^{\pi_A} - tc_{r_B}^{\pi_B}$  is within the range specified in (A32) for each scenario. This completes the proof of Proposition 3.  $\square$

**A.2.2. Impact of Scale Economies.** This two-producer example of the CRN also provides insights into how scale economies affect the incentive compatibility gap of return share when the condition in Proposition 2 is violated, i.e., when there exists a subcoalition  $S$  such that  $\zeta^M \cdot |v^r(M)/R| < \zeta^S \cdot |v^r(S)/R^S|$ . Specifically, we study the case of  $CRN_2$  where  $\pi_A$  creates a positive unit processing revenue  $\rho$  and zero downstream cost at both processors, while  $\pi_B$  generates no processing revenue but exerts downstream costs at both processors. We denote this network as  $CRN_2^0$ . Proposition 4 provides sufficient conditions under which scale economies increase the incentive compatibility gap of return share; i.e.,  $\mathcal{G}(x_r^{(\eta, \zeta)}) > \mathcal{G}(x_r)$ .

**PROPOSITION 4.** *In  $CRN_2^0$ , assume that the scale economies-related parameters satisfy  $(\zeta^A - 1)/(\zeta^M - 1) > d_{\pi_A}/(d_{\pi_A} + d_{\pi_B})$ . There exists a constant  $\bar{\rho}$  such that  $\mathcal{G}(x_r^{(\eta, \zeta)}) > \mathcal{G}(x_r)$  if the unit processing revenue  $\rho > \bar{\rho}$ .*

**PROOF OF PROPOSITION 4.** For convenience, in this proof, we use  $\bar{v}^{cS} \doteq v^c(S)/R^S$  to denote the average of the cost component of the stand-alone cost of each subcoalition  $S$ . Given any  $CRN_2^0$ , we can calculate that

$$\begin{aligned}\bar{v}^M &= \frac{v^c(M) + \rho \cdot d_{\pi_A}}{d_{\pi_A} + d_{\pi_B}} = \bar{v}^{cM} + \rho \cdot \frac{d_{\pi_A}}{d_{\pi_A} + d_{\pi_B}}, \\ \bar{v}^A &= v^c(A) + \rho \cdot d_{\pi_A} d_{\pi_A} = \bar{v}^{cA} + \rho, \quad \bar{v}^B = \bar{v}^{cB}, \\ \bar{v}^{(\eta, \zeta)M} &= \eta^M \cdot \bar{v}^{cM} + \zeta^M \cdot \rho \cdot \frac{d_{\pi_A}}{d_{\pi_A} + d_{\pi_B}}, \\ \bar{v}^{(\eta, \zeta)A} &= \eta^A \cdot \bar{v}^{cA} + \zeta^A \cdot \rho.\end{aligned}$$

Under the condition  $(\zeta^A - 1)/(\zeta^M - 1) > d_{\pi_A}/(d_{\pi_A} + d_{\pi_B})$ , we can show the following (algebraic manipulations omitted):

$$\begin{aligned}\text{If } |\rho| > \bar{\rho}_1 &\doteq \frac{\bar{v}^{cM}(1 - \eta^M) - \bar{v}^{cA}(1 - \eta^A)}{(\zeta^A - 1) - (\zeta^M - 1) \cdot d_{\pi_A}/(d_{\pi_A} + d_{\pi_B})}, \\ &\text{then } d_{\pi_A} \cdot (\bar{v}^{(\eta, \zeta)M} - \bar{v}^{(\eta, \zeta)A}) > d_{\pi_A} \cdot (\bar{v}^M - \bar{v}^A). \\ \text{If } |\rho| > \bar{\rho}_2 &\doteq \frac{\bar{v}^{cM}(d_{\pi_B}/d_{\pi_A} - \eta^M) + \bar{v}^{cA} \cdot \eta^A - \bar{v}^{cB} \cdot d_{\pi_B}/d_{\pi_A}}{\zeta^A - \zeta^M \cdot d_{\pi_A}/(d_{\pi_A} + d_{\pi_B}) + d_{\pi_B}/(d_{\pi_A} + d_{\pi_B})}, \\ &\text{then } d_{\pi_A} \cdot (\bar{v}^{(\eta, \zeta)M} - \bar{v}^{(\eta, \zeta)A}) > d_{\pi_A} \cdot (\bar{v}^M - \bar{v}^B). \\ \text{If } |\rho| > \bar{\rho}_3 &\doteq \frac{\bar{v}^{cA} \cdot \eta^A - \bar{v}^{cM} \cdot \eta^M}{\zeta^A - \zeta^M \cdot d_{\pi_A}/(d_{\pi_A} + d_{\pi_B})}, \\ &\text{then } d_{\pi_A} \cdot (\bar{v}^{(\eta, \zeta)M} - \bar{v}^{(\eta, \zeta)A}) > 0.\end{aligned}\tag{A33}$$

Hence, we can conclude that when

$$\begin{aligned}|\rho| > \bar{\rho} &\doteq \max\{\bar{\rho}_1, \bar{\rho}_2, \bar{\rho}_3\}, \\ d_{\pi_A} \cdot (\bar{v}^{(\eta, \zeta)M} - \bar{v}^{(\eta, \zeta)A}) &> \max\{d_{\pi_A} \cdot (\bar{v}^M - \bar{v}^A), d_{\pi_B} \cdot (\bar{v}^M - \bar{v}^B), 0\} \\ &= \max_{S \subseteq M} \{R^S \cdot (\bar{v}^M - \bar{v}^S)\} = \mathcal{G}(x_r),\end{aligned}$$

and therefore  $\mathcal{G}(x_r^{(\eta, \zeta)}) = \max_{S \subseteq M} \{R^S \cdot (\bar{v}^{(\eta, \zeta)M} - \bar{v}^{(\eta, \zeta)S})\} \geq d_{\pi_A} \cdot (\bar{v}^{(\eta, \zeta)M} - \bar{v}^{(\eta, \zeta)A}) > \mathcal{G}(x_r)$ .  $\square$

Here is an intuitive explanation of Proposition 4. Given the condition  $(\zeta^A - 1)/(\zeta^M - 1) > d_{\pi_A}/(d_{\pi_A} + d_{\pi_B})$ , we can derive that  $\rho \cdot (\zeta^A - 1) \geq \rho \cdot (\zeta^M - 1) \cdot d_{\pi_A}/(d_{\pi_A} + d_{\pi_B})$ , and thus  $\rho \cdot \zeta^A - \rho \cdot \zeta^M \cdot d_{\pi_A}/(d_{\pi_A} + d_{\pi_B}) \geq \rho - \rho \cdot d_{\pi_A}/(d_{\pi_A} + d_{\pi_B})$ . The first term in the difference on the left- and right-hand sides of this inequality is the unit revenue  $A$  obtains when operating independently with and without scale economies, respectively; the second term represents  $A$ 's share of the unit revenue in the grand coalition under the allocation by return share. This inequality indicates that producer  $A$  incurs a bigger loss in average revenue from participating in the collective system after scale economies are factored in. In other words, the revenue component of the incentive compatibility gap is increased by scale economies. When the unit revenue  $\rho$  is large enough, this increase dominates the reduction in the cost component of the gap as a result of scale economies; the threshold  $\bar{\rho}$  reflects the relative magnitude of the two effects.

**A.2.3. Tightness of the Lower Bounds of Nonmember Access Fees in Theorem 2.** We show that under the CRF game with nonmember access fees, there exist instances of  $CRN_2$  where the bounds of nonmember access fees identified in Theorem 2 are necessary to obtain a core allocation by cost-corrected return share with capacity rewards; i.e., the mechanism cannot generate a core allocation if the nonmember access fees are priced strictly lower than the bounds. Assume that the two producers share the same return volume that equals the amount of the capacity at each processor; i.e.,  $d^{\pi_A} = d^{\pi_B} = k_{r_A} = k_{r_B}$ . We denote such a network instance of  $CRN_2$  as  $CRN_2^c$ . Without loss of generality, we assume that  $\delta^{\pi_A} \geq \delta^{\pi_B}$ ; i.e.,  $tc_{r_B}^{\pi_A} - tc_{r_A}^{\pi_A} \geq tc_{r_B}^{\pi_B} - tc_{r_A}^{\pi_B}$ . Hence, the optimal routing on  $CRN_2^c$  is to route  $\pi_A$  and  $\pi_B$  at  $r_A$  and  $r_B$ , respectively. We can calculate a set of dual optimal solution to be  $\beta^{*\pi_A} = tc_{r_A}^{\pi_A} + \delta^{\pi_B}$ ,  $\beta^{*\pi_B} = tc_{r_B}^{\pi_B} + \delta^{\pi_A}$ ,  $\alpha_{r_A}^* = -\delta^{\pi_B}$ , and  $\alpha_{r_B}^* = 0$ . We denote the nonmember access fee charged at processor  $r_A$  and  $r_B$  by  $\phi_{r_A}$  and  $\phi_{r_B}$ . Since  $\alpha_{r_B}^* = 0$ , the bounds identified in Theorem 2 for  $\phi_{r_B}$  equal zero and thus are obviously necessary as the nonmember access fees must be nonnegative. We further show that the bounds for  $\phi_{r_A}$  are also necessary for the allocation  $x_\mu^p$  to be in the core of the CRF game under nonmember access fees.

**Case 1.** When  $\mu^i \geq 0$  for both  $i = A, B$ , consider the following scenario of  $CRN_2^c$ : The capacity at both  $r_A$  and  $r_B$  is operator-contracted and no independent capacity exists. In this network, the costs allocated by cost-corrected return share with capacity rewards to the producers are  $(x^A)_\mu = (tc_{r_A}^{\pi_A} \cdot d^{\pi_A} + tc_{r_B}^{\pi_B} \cdot d^{\pi_B}) \cdot \mu^A = (tc_{r_A}^{\pi_A} + \delta^{\pi_B}) \cdot d^{\pi_A} + \alpha_{r_A}^* \cdot k_{r_A} \cdot \mu^A$  and  $(x^B)_\mu = (tc_{r_A}^{\pi_A} \cdot d^{\pi_A} + tc_{r_B}^{\pi_B} \cdot d^{\pi_B}) \cdot \mu^B = tc_{r_B}^{\pi_B} \cdot d^{\pi_B} + \alpha_{r_A}^* \cdot k_{r_A} \cdot \mu^B$ . We can further calculate that the stand-alone cost of each producer  $i$  under the nonmember access fees equals  $v_\phi(i) = \min\{tc_{r_i}^{\pi_i} + \phi_{r_A}, tc_{r_i}^{\pi_i} + \phi_{r_B}\} \cdot d^{\pi_i}$   $i = A, B$ . Hence, for the allocation  $x_\mu^p$  to be in the core, the following inequalities need to hold:

$$\begin{aligned}(x^A)_\mu \leq v_\phi(A) &\Rightarrow \phi_{r_A} \geq \delta^{\pi_B} + \alpha_{r_A}^* \cdot \mu^A = |\alpha_{r_A}^*| \cdot (1 - \mu^A), \\ (x^B)_\mu \leq v_\phi(B) &\Rightarrow \phi_{r_A} \geq \delta^{\pi_B} + \alpha_{r_A}^* \cdot \mu^B = |\alpha_{r_A}^*| \cdot (1 - \mu^B).\end{aligned}\tag{A34}$$

Therefore, in this example, if  $x_\mu^p$  is in the core of the CRF game under nonmember access fees, then  $\phi_{r_A} \geq \max_{i=A, B} (1 - \mu^i) \cdot |\alpha_{r_A}^*|$  must hold; i.e., the first bound identified in Theorem 2 is necessary in this network scenario.

Case 2. When there is a producer  $i \in \{A, B\}$  such that  $\mu^i < 0$  and independent capacity exists in the CRN, consider another scenario of CRN<sub>2</sub>: The capacity at  $r_A$  is operator-contracted, whereas  $A$  and  $B$  share the capacity at  $r_B$  equally. Let  $p_{r_B}$  be the unit capacity reward price at  $r_B$ . We can calculate that in this network, the costs allocated to the producers under cost-corrected return share with capacity rewards are  $(x^A)_\mu^p = (tc_{r_A}^{\pi_A} \cdot d^{\pi_A} + tc_{r_B}^{\pi_B} \cdot d^{\pi_B} + p_{r_B} \cdot k_{r_B}) \cdot \mu^A - \frac{1}{2} \cdot p_{r_B} \cdot k_{r_B}$  and  $(x^B)_\mu^p = (tc_{r_A}^{\pi_A} \cdot d^{\pi_A} + tc_{r_B}^{\pi_B} \cdot d^{\pi_B} + p_{r_B} \cdot k_{r_B}) \cdot \mu^B - \frac{1}{2} \cdot p_{r_B} \cdot k_{r_B}$ . The stand-alone cost of each producer  $i$  equals  $v_\phi(i) = \min\{tc_{r_A}^{\pi_i} + \phi_{r_A}, tc_{r_B}^{\pi_i} + \frac{1}{2}\phi_{r_B}\} \cdot d^{\pi_i}$   $i = A, B$ . Thus, we conclude that if the allocation  $x_\mu^p$  is in the core of the CRF game under nonmember access fees, then the capacity reward price  $p_{r_B}$  must satisfy

$$\begin{aligned} (x^A)_\mu^p &\leq v_\phi(A) \\ &\Rightarrow (\mu^A - \frac{1}{2}) \cdot p_{r_B} \leq tc_{r_A}^{\pi_A} + \phi_{r_A} - \mu^A \cdot (tc_{r_A}^{\pi_A} + tc_{r_B}^{\pi_B}), \\ (x^B)_\mu^p &\leq v_\phi(B) \\ &\Rightarrow (\frac{1}{2} - \mu^B) \cdot p_{r_B} \geq \mu^B \cdot (tc_{r_A}^{\pi_A} + tc_{r_B}^{\pi_B}) - tc_{r_A}^{\pi_B} - \phi_{r_A}. \end{aligned} \quad (A35)$$

Notice that  $\mu^A - \frac{1}{2} = \frac{1}{2} - \mu^B$ . Hence, to ensure the existence of capacity reward prices such that  $x_\mu^p$  is in the core, the inequality  $\mu^B \cdot (tc_{r_A}^{\pi_A} + tc_{r_B}^{\pi_B}) - tc_{r_A}^{\pi_B} - \phi_{r_A} \leq tc_{r_A}^{\pi_A} + \phi_{r_A} - \mu^A \cdot (tc_{r_A}^{\pi_A} + tc_{r_B}^{\pi_B})$  must hold, which implies that  $\phi_{r_A} \geq \frac{1}{2}\delta^{\pi_B} = \frac{1}{2}|\alpha_{r_A}^*|$  is a necessary condition. It is easy to verify that  $\frac{1}{2}|\alpha_{r_A}^*|$  is exactly the second bound identified by Theorem 2 in this scenario of CRN<sub>2</sub>.

### A.3. Group Incentive Compatibility of the Allocation by Return Share with Capacity Rewards

In §4.3, one critical observation is that the capacity-rewarding mechanism can be effective in reducing the incentive compatibility gap of return share when the average independent capacity availability in the grand coalition is relatively low compared with that in the subcoalitions that have sufficient independent capacities to break away. To analyze this observation, we assume for the rest of this section that the collective independent capacity of all producers is insufficient to process the total volume of products returned, necessitating additional operator-contracted capacity. We denote a CRN formed with the independent capacity of producer  $i$  by  $G^i$ , and we analyze the factors limiting the maximum throughput in the network  $\bigcup_{i \in M} G^i$  (i.e., the grand coalition network without operator-contracted capacity) as follows. We first transform the CRN  $\bigcup_{i \in M} G^i$  into a capacitated single-commodity network by adding an artificial origin node  $o$  that is linked to each of the collection points  $j$  via a fictitious edge  $(o, j)$  with capacity equal to  $\sum_\pi d_j^\pi$ , the total return volume at  $j$ . We also add an artificial destination node  $d$  and connect every node  $r' \in \mathcal{R}'$  to  $d$  with an infinite capacity edge. Let the resulting network be called  $G_o^d$ . Then the maximum flow on  $G_o^d$  is equivalent to the maximum throughput of  $\bigcup_{i \in M} G^i$ . A cut in  $G_o^d$  is a set of nodes containing the origin  $o$  but not the sink  $d$ , and the corresponding cut set is defined by the set of edges that cross the cut. Let  $\bar{C}$  be the minimum capacity  $o$ - $d$  cut in  $G_o^d$  and let  $E(\bar{C})$  be its cut set. The max-flow min-cut theorem (Ahuja et al. 1993) indicates that the maximum flow passing from  $o$  to  $d$  in  $G_o^d$  equals the total capacity of  $E(\bar{C})$ .

Let the capacity of  $E(\bar{C})$  be  $K_{E(\bar{C})} \doteq \sum_{e \in E(\bar{C}) \setminus \{(o, j), j \in \mathcal{J}\}} \sum_{i \in M} k_e^i + \sum_{j: (o, j) \in E(\bar{C})} \sum_\pi d_j^\pi$ . Hence,  $K_{E(\bar{C})} < R$  indicates an inadequate level of independent capacity in  $G_o^d$ .

**THEOREM 3.** Assume that the minimum cut  $\bar{C}$  is unique. If  $K_{E(\bar{C})} < R$  and  $E(\bar{C}) \cap \{(o, j), j \in \mathcal{J}\} = \emptyset$ , indicating that the throughput of  $\bigcup_{i \in M} G^i$  cannot be increased by an additional unit of return volume at any collection point, then  $\min_{p_e \geq 0} \mathcal{G}(x_r^p) = 0$ ; i.e., there exist capacity reward prices  $p_e \geq 0$ ,  $\forall e \in E$  such that return share with capacity rewards generates an allocation in the core of the CRF game.

**PROOF OF THEOREM 3.** First, recall that by Lemma 1, the smallest incentive compatibility gap under the model of return share with capacity rewards,  $\min_{p_e \geq 0} \mathcal{G}(x_r^p)$ , can be calculated as the optimal value of a linear program (A14)–(A17). By adding up the constraints (A15) over the set  $E(\bar{C})$ , we obtain another constraint,  $\sum_{S \in \Psi} \{\sum_{e \in E(\bar{C})} [\bar{k}_e^S - \bar{k}_e^M]\} \cdot y^S \leq 0$ . Since  $v(S) < \infty$ ,  $\forall S \in \Psi$ , i.e., subset  $S$  has enough independent capacity to process its own return volume in its independent CRN, we can show that  $\sum_{e \in E(\bar{C})} \sum_{i \in S} k_e^i \geq R^S$ . However, since  $E(\bar{C}) \cap \{(o, j), j \in \mathcal{J}\} = \emptyset$ ,  $K_{E(\bar{C})} = \sum_{e \in E(\bar{C})} \sum_{i \in M} k_e^i < R$ . Thus  $\sum_{e \in E(\bar{C})} [\bar{k}_e^S - \bar{k}_e^M] = \sum_{e \in E(\bar{C})} \sum_{i \in S} k_e^i / R^S - \sum_{e \in E(\bar{C})} \sum_{i \in M} k_e^i / R > 0$ , as the first (second) term is greater than or equal to (strictly less than) 1. Because of the nonnegativity constraints on  $y^S$ , we conclude that the only solution that satisfies  $\sum_{S \in \Psi} \{\sum_{e \in E(\bar{C})} [\bar{k}_e^S - \bar{k}_e^M]\} \cdot y^S \leq 0$  is the zero vector. In other words, the only feasible solution to the program (A14)–(A16) is the zero vector, and thus  $\min_{p_e \geq 0} \mathcal{G}(x_r^p) = 0$ .  $\square$

The intuition behind this result is the following: The condition  $K_{E(\bar{C})} < R$  implies an inadequate independent capacity availability within the grand coalition  $M$  on the edges in the minimum cut set  $E(\bar{C})$ . Hence, for any subcoalition  $S$  that has sufficient independent capacity to operate its own independent CRN, we have  $\sum_{e \in E(\bar{C})} \bar{k}_e^S \geq 1 > \sum_{e \in E(\bar{C})} \bar{k}_e^M$ . For this inequality to hold, there must exist at least one edge in  $E(\bar{C})$  where  $\bar{k}_e^S > \bar{k}_e^M$ . Since we assume  $E(\bar{C}) \cap \{(o, j), j \in \mathcal{J}\} = \emptyset$ , this edge must be in the original CRN and essentially represents a critical resource that we can associate a capacity reward with. According to formula (11) in §4.3, doing so can potentially benefit all subcoalitions that have sufficient independent capacity to break away. Theorem 3 proves the existence of a set of prices  $\{p_e, \forall e \in E\}$  that collectively achieves this and in fact eliminates the incentive compatibility gap of return share. Theorem 3 can also be extended to cases where the min-cut is not unique.

## Appendix B. Construction of the Numerical Example in §5

### B.1. Facilities

We build a sample CRN based on the state of Washington's implementation of its EPR program in 2009. First, 50 collection points are chosen from the 244 that were registered with the WMMFA (WMMFA 2010). The sample contains at least one collection point for every county and is generated according to Table B.1. In counties where more than one collection point is to be chosen, we select one in each of the largest cities by population. In case no collection point is registered in such a city, we pick the one that is the nearest to it. The resulting sample is displayed in Table B.2.

**Table B.1** The Number of Collection Points Chosen in Each County

Number of collection points registered	Number of collection points chosen
1–5	1
6–19	2
≥20	3

We assume that there is a consolidator in each of the eight counties with more than one collection point. Each consolidator is in or is near the biggest city by population of the corresponding county and is identical to the collection

**Table B.2** The List of Collection Points in the Sample CRN

1	Experience Merchandise Thrift Store
2	CEP Recycle Asotin Co.
3	Clayton-Ward Company Kennewick
4	City of Chelan Recycle Center
5	Goodwill Port Angeles Store
6	Goodwill Vancouver Outlet
7	Goodwill Battleground Store
8	CEP Recycle Columbia Co.
9	Goodwill Longview Store
10	Goodwill East Wenatchee Store
11	Torboy Transfer Station
12	Tommy's Steel and Salvage
13	CEP Recycle Garfield Co.
14	CDSI Transfer and Recycling Center
15	Waste Connections Inc. Aberdeen Sanitation
16	Oak Harbor Drop Box Station
17	Goodwill Port Townsend Store
18	RE-PC Seattle
19	Goodwill Bellevue Store
20	Goodwill Federal Way Store
21	Bremerton St. Vincent dePaul
22	Goodwill Bainbridge Island Don. Center
23	Goodwill Ellensburg Store
24	Regional Disposal Company—Goldendale Transfer
25	Goodwill Centralia Store
26	Lincoln County Transfer Station
27	Wilson Recycling LLC
28	Methow Recycles
29	Royal Heights Transfer Station
30	Deer Valley Transfer Station
31	Green PC Recycling
32	Goodwill Lakewood Store
33	Public Recycling Center—Canyon Rd
34	Consignment Treasures LLC
35	Appliance Recycling Connection
36	Stevenson Transfer Facility
37	St. Vincent dePaul Everett
38	Goodwill Marysville
39	E-Waste, LLC
40	Earthworks Recycling, Inc.
41	Jaco Environmental
42	Goodwill Colville Store
43	MIDWAY RECOVERY INC.
44	Goodwill Lacey Store
45	Stanley's Sanitary Service
46	Walla Walla Recycling
47	Safe and Easy Recycling Bellingham
48	Pullman Disposal Shop
49	Yakima Waste Systems Yakima
50	Sunnyside Christian Thrift Shop

**Table B.3** The List of Processors in the Sample CRN

Processor	Facility location	Operation type
$r_1$	Seattle	In-state high-tech
$r_2$	Vancouver	In-state high-tech
$r_3$	Clackamas, OR	Out-of-state TV/monitor-specialized high-tech
$r_4$	Roseville, CA	Out-of-state IT-specialized high-tech
$r_5$	Mukilteo	Low-tech
$r_6$	Auburn	Low-tech
$r_7$	Lynnwood	Low-tech
$r_8$	Tukwila	Low-tech

point chosen in that city. The consolidators are assumed to only handle the return volume within their respective counties. In counties with only one collection point, we assume the return volume is directly transported from the collection point to the processors.

WMMFA contracted with eight processors in 2009, six within Washington state, which are reported to have processed 98.85% of the total volume in 2009. Within these six processors, two have high-tech facilities and the remaining four operate mainly based on manual labor (WMMFA 2010). We incorporate these six processors into our example and label them as processor  $r_1 - r_2$  (high-tech) and  $r_5 - r_8$  (low-tech), plus two out-of-state processors (labeled  $r_3$  and  $r_4$ ) associated with two potential “independent plans” (see Table B.3). Figure B.1 depicts the locations of the sampled collection points and processors in the example CRN. Note that all transportation distances between entities in the sample CRN are measured by the minimum traveling time to account for the differences in road conditions.

### B.2. Products, Return Volumes, and Capacity

The EPR bill in Washington covers TVs, computers, laptops, and monitors (WMMFA 2010). Based on their processing costs, the products are basically classified into CRT-TV/monitor (which is reported to account for 98.5% of the total TV/monitor return volume in Washington), LCD TV/monitor, desktops, laptops, and computers, because these are the only cost drivers with the current processing technology. In particular, TVs/monitors contain hazardous materials and thus are costly to recycle under certain environmental standards, although the parts and materials used in computers have high reuse value and usually generate a revenue in recycling. Hence, we distinguish in the example two product types, TVs/monitors and computers. A total volume of 38,509,563 pounds of products, among which about 30% are computers and 70% are TVs/monitors, were collected in Washington in 2009 in the form of 137 different product brands from 87 producers with return shares varying from 0.001% to 7.9% (Washington State Department of Ecology 2009). In constructing the sample CRN, we uniformly choose a set of 19 products (labeled from  $\pi_1$  to  $\pi_{19}$ ), manufactured by 17 producers (labeled from  $m_1$  to  $m_{17}$ ) from the pool to capture the heterogeneity of the actual return shares in the Washington implementation. The products are also chosen to reflect the 7:3 proportion of TVs/monitors versus computers in the total volume. We then calculate the volume of each of the 19 products in the



**Figure B.1** (Color online) Locations of the Collection Points and Processors (Except  $r_4$  Located in California) Considered in Sample CRN

Note. Note that recyclers  $r_5$  and  $r_7$  are very close to each other and overlap in panel (b).

example proportional to their relative return shares based on the actual collection volume of 38,509,563 pounds (see Table B.4).

To distribute the collective return volume of the 19 products shown in Table B.4 among individual collection points, we first calculate the total amount collected in each of the collection points in the sample CRN. Specifically, if only one collection point is chosen in a county, then we assume that the entire volume within this particular county is returned to this collection point. Otherwise, the county volumes are proportionally allocated among the sampled collection points based on the corresponding city populations. Based on these data, we calculate each collection point's share of the total volume by  $\lambda_j = \text{collective volume returned to collection point } j / 38,509,563$ . Then, the volume of each product  $\pi$  at each collection point  $j$  is calculated based on a homogeneous distribution of the product's total volume among the collection points that follows  $\{\lambda_j, j = 1, 2, \dots, 50\}$ , i.e.,  $d_j^\pi = \lambda_j \cdot (\text{return volume of } \pi)$ , shown in Table B.4.

Motivated by the existing independent recycling capacity of producers around the Washington region, we model two producers as having access to independent CRNs. In particular, a group of TV producers has contracted with a processor in Oregon that has advanced TV/monitor recycling technology in order to fulfill its recycling obligations in that state, and it has filed one of the two independent plans in

Washington. An IT producer has established a nationwide collection and recycling system including its own processing facility in northern California and is expected to apply for an independent plan in Washington. In light of this information, and based on these producers' return volumes by category, we consider a TV producer  $A$  with 5.5 million return volume, and a computer producer  $B$  with 2.1 million pounds (25%) monitor volume and 6.4 million pounds (75%) computer volume. We assume that  $A$  and  $B$  have access to 6 million and 9 million pounds of independent recycling capacities at two out-of-state high-tech processors that specialize in recycling TVs/monitors ( $r_3$ ) and IT products such as computers ( $r_4$ ), respectively. In addition, we assume that the authority can contract with in-state processors  $r_1$ ,  $r_2$ , and each processor  $r_5$ – $r_8$  for up to 10 million, 5 million, and 6 million pounds of capacity, respectively. Motivated by the independent plans submitted in Washington, we focus on three subcoalitions seceding from the grand coalition— $A$  only,  $B$  only, and  $A$  and  $B$  together.

### B.3. Cost Structure on the CRN and Economies of Scale Model

The unit costs in the sample CRN are disguised but structurally representative of costs in WA that are reported as aggregate averages within each stage of the CRN for a product type. All unit prices are in cents per weight (in pounds) except for the transportation cost, which is in cents per pound hour.

**Table B.4** List of Sampled Producers and Their Return Volume and Product Type

Producer	Product(s)	Volume (lb.)	Product type	Producer	Product(s)	Volume (lb.)	Product type
$m_1$	$\pi_1$	4,016	TV/monitor	$m_2$	$\pi_2$	6,718	TV/monitor
$m_3$	$\pi_3$	10,193	Computer	$m_4$	$\pi_4$	14,672	TV/monitor
$m_5$	$\pi_5$	25,329	TV/monitor	$m_6$	$\pi_6$	34,055	TV/monitor
$m_7$	$\pi_7$	67,724	TV/monitor	$m_8$	$\pi_8$	115,216	TV/monitor
$m_9$	$\pi_9$	140,468	TV/monitor	$m_{10}$	$\pi_{10}$	185,334	TV/monitor
$m_{11}$	$\pi_{11}$	231,127	TV/monitor	$m_{12}$	$\pi_{13}$	361,015	TV/monitor
$m_{14}$ (A)	$\pi_{16}$	1,995,040	TV/monitor	$m_{13}$	$\pi_{14}$	436,461	TV/monitor
$m_{15}$	$\pi_{17}$	5,565,725	TV/monitor	$m_{16}$ (B)	$\pi_{15}$	1,097,793	TV/monitor
$m_{17}$	$\pi_{19}$	6,965,767	TV/monitor (25%) Computer (75%)		$\pi_{18}$	8,510,292	TV/monitor (25%) Computer (75%)
		12,742,619	TV/monitor				



We assume no administrative cost in the sample CRN, as this cost is negligible in Washington. For each product type, the unit price for collection and consolidation is assumed to be identical at each such site (10 cents/lb.), although different processing cost structures are quoted by different processors (see Table B.5 for details). The unit downstream recycling cost (revenue) used in this example is a weighted average over all parts and materials according to their proportions by weight inside one unit of the corresponding product. Hence, given the mandated recycling requirement  $\tau$ , the net processing cost is calculated as a linear approximation as follows: for TVs/monitors, the net processing cost equals (operational cost) +  $\tau \cdot$  (downstream recycling cost) +  $(1 - \tau) \cdot$  (landfilling cost), whereas that for computers is simply operational cost + downstream revenue, because 100% recycling will be implemented for computers regardless of  $\tau$  as a result of the potential processing revenues.

The transportation rates are reported based on the geographic location of the route. Specifically, all processors in Washington are located along the north–south corridor between Seattle and Vancouver (which is referred to as the “west-of-the-mountain” area). Many common carriers operate busy routes along this north–south corridor and thus can provide cheap backhaul miles for the collection points within the area. By contrast, the transportation services for collection points located “east of the mountain” are more expensive. Hence, in our example, we use two different rates for the collection points sampled in these two areas (0.8 and 2 cents/pound  $\times$  hour). The calculation is done based on a 2 cents/lb. average and the assumption that the return volume at each collection point is distributed to the eight processors according to the aggregate percentages reported by WMMFA (2010) and that the cost in the west-of-the-mountain area is about 50% cheaper than that in the east.

As for our modeling of economies of scale in the example, our stakeholder interviews indicated that a 20% cost reduction can be expected in Washington when the total volume doubles. According to this information, we calculate

**Table B.5 Representative Full Processing Cost Structure (Cents per Pound) by Facility and Product**

	High-tech processor			
	Local ( $r_1$ – $r_2$ )	Out-of-state TV/monitor- specialized ( $r_3$ )	Out-of-state IT-specialized ( $r_4$ )	Low-tech processor ( $r_5$ – $r_8$ )
TVs/monitors				
Operational cost	2	2	2	4
Downstream recycling cost	12	8	14	16
Net processing cost	$5 + 9\tau$	$5 + 5\tau$	$5 + 11\tau$	$7 + 13\tau$
Computers				
Operational cost	12	10	15	4
Downstream cost (revenue)	–22	–17	–35	–10
Net processing cost	–10	–7	–20	–6

*Note.* The negative numbers indicate revenues. The landfilling cost is 3 cents/lb.

the parameters of a decreasing quadratic function  $\eta(x) = a \cdot (x - b)^2$  such that  $\eta(38,509,563) = 1$  and  $\eta(38,509,563 \cdot 2) = 0.8$ , which produces the function  $0.011146 \cdot (1/38,509,563 \cdot x - 10.47214)^2$ .

#### B.4. The Myopic Policy in Studying the Value of Source Separation

In this section, we develop the myopic policy used in the numerical analysis to study the value of source separation of products at the collection points. The assumption behind the myopic policy is that the e-waste volumes at collection centers are not separated by brand. In other words, the distribution of individual producers’ e-waste volumes is not known at the point of collection. Rather, the total volume at each collection point  $j$ ,  $d_j = \sum_{\pi \in \Pi} d_j^\pi$  is the only available volume information. Because separating each producer’s products at collection can be costly for high return volumes and a heterogeneous mixture of products, the myopic policy estimates the mixture of individual producers’ return shares by sampling at the processors. This setup is similar to many practical settings. Sampling typically takes place at processors, because return volumes from different collection points are consolidated there, which increases the statistical significance of the sampling procedure. This myopic policy can be cost effective if sampling costs in practice are significantly smaller than separation costs. Moreover, because sampling requires the handling of only a fraction of the waste volume, the return share calculation can be achieved at a very low cost under this policy.

To highlight the above effects, for the rest of the discussion, we assume an identical unit processing cost among all products on every edge of the CRN except at the processors (which is exactly the case in the sample CRN used in the numerical study). Because the myopic policy does not assume that the actual mixture of TVs/monitors and computers at collection points is readily available, the network flow problem cannot be solved optimally. Rather, the system operator needs to transport the e-waste volumes to the processors using a suboptimal procedure. We model this myopic procedure as follows: First, the e-waste is routed to the processors to minimize the total collection, consolidation, and transportation cost, which can be solved by the following program:

$$(C) \quad \min Z_1(y) = \sum_{e \in E \setminus \{(r, r')\}} \sum_{\pi \in \Pi} c_e \cdot y_e \quad (B1)$$

$$\text{s.t.} \quad \sum_{e=(u,v) \in E} y_e - \sum_{e=(v,w) \in E} y_e = 0, \quad \forall v \in V \setminus \{\mathcal{J}, \mathcal{R}'\}, \quad (B2)$$

$$y_{(j,j')} = d_j, \quad \forall j \in \mathcal{J}, \quad (B3)$$

$$y_e \leq \sum_{i \in M} k_e^i + K_e^p, \quad \forall e \in E, \quad (B4)$$

$$\text{nonnegativity constraints.} \quad (B5)$$

Let  $y^*$  denote the optimal solution to the above program. Next, we model the sampling procedure at the processors to determine the composition of the arriving volumes, i.e., the amount of each product  $\pi$  (TVs/monitors or computers), at processor  $r$ . Assume that the volume of TVs/monitors at a collection point  $j$  is generated from a binomial distribution with parameters  $(d_j, p)$ , and the products are transported to processors in a uniform mix of the two product

types. Let  $d_r(y^*) \doteq \sum_{n' \in N'} y_{(n', r)}^*$ . The required size of the product sample at each processor  $r$  is calculated by (B6) (adapted from Cochran 1963), which estimates the actual volume mixture of TVs/monitors and computers with a 1% precision rate and a 99% confidence level:

$$\text{SampleSize}_r = \frac{\frac{Z_{1-0.01/2}^2 \cdot p(1-p)}{0.012}}{1 + \frac{1}{d_r(y^*)} \left( \frac{Z_{1-0.01/2}^2 \cdot p(1-p)}{0.012} - 1 \right)}. \quad (\text{B6})$$

Based on the composition information of the arriving volume under the myopic routing  $y^*$  at each processor  $r$ , the processing cost at  $r$  can be calculated, which we denote as  $Z_2(y^*)$ . Hence, the total cost incurred under the myopic routing policy is  $Z_1(y^*) + Z_2(y^*) + c_{\text{samp}} \cdot \text{SampleSize}_r$ , where  $c_{\text{samp}}$  is the unit sampling cost. Our numerical analysis compares this cost with  $Z(f^*) + c_{\text{sep}} \cdot \sum_{\pi \in \Pi} \sum_{j \in J} d_j^\pi$ , where  $c_{\text{sep}}$  is the unit separation cost, based on the sample CRN constructed. We also analyze the potential group incentive compatibility of the cost allocations under the myopic policy. We show that using the myopic routing policy with sampling can result in an inefficient fragmented system under the assumption that producers who defect from the centrally operated CRN need to separate their own products to operate their own private CRNs. Both the cost difference and the incentive compatibility gap are plotted as a function of product heterogeneity (modeled by the percentage of TV/monitor volume,  $p$ ) and the cost difference between sampling and separation  $c_{\text{sep}} - c_{\text{samp}}$ . Note that we also assume that as  $p$  changes, the return volumes and the independent capacity of producers  $A$  and  $B$  vary in the way such that their shares of the total TV and IT volume, as well as their normalized independent capacity availability (i.e., capacity/volume), remain constant and identical to those values in the nominal sample CRN based on the Washington instance.

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