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Evaluating the robustness of UK term structure decompositions using linear regression methods ☆

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ABSTRACT

Dynamic no-arbitrage affine term structure models (ATSMs) have become the standard framework for monetary policy-makers to decompose long-term bond yields into expectations of future short-term risk-free interest rates and the term premia that compensate investors in long-term bonds for risk. This paper presents estimates of ATSMs for the UK and explores how much weight users of these models can place on point estimates of term premia. Over much of the period since the early 1990s, broad movements in estimated premia are robust across a wide range of reasonable specifications. But there is substantial model and parameter uncertainty associated with these models and estimates of the time-series dynamics of yields may be biased in short samples. This model uncertainty is greater towards the end of our sample period, when bond yields have been well below historically normal levels.

1. Introduction

Understanding movements in the term structure of government bond yields is of considerable importance to financial market practitioners and public policy-makers. Long-term bond yields reflect both expectations of average future short-term interest rates and a term premium - the additional expected return required by investors in long-term bonds relative to rolling over a series of short-term bonds. Both components provide useful information: expected short rates reflect investors' views about the outlook for monetary policy, while the term premium reflects (among other things) uncertainty around future short-term interest rates and investors' aversion to bearing risk. Unfortunately, however, the two components cannot be observed separately.

The workhorse model used by central banks for decomposing bond yields over the last decade has been the Gaussian no-arbitrage essentially affine term structure model (ATSM) of Duffee (2002). The long list of studies published by central bankers using these models includes Kim and Wright (2005) and Adrian et al. (2013) for the US; and Joyce et al. (2010) and Guimarães (2014) for the UK. In this paper, we find that point estimates of UK term premia from a four-factor ATSM estimated over the period October 1992 to December 2013 appear reasonable by a number of metrics. They are countercyclical (consistent with findings by Bauer et al. (2012) for the US) and positively related to the uncertainty around future inflation (consistent with findings by Wright, 2011 for a panel of countries, including the UK). The model matches the 'linear projections of yields' (LPY) specification tests proposed by Dai and Singleton (2002). And broad movements in premia appear plausible: they fell in the late 1990s, which may reflect improvements in the credibility of monetary policy and an increased demand for the safety of government bonds following the Asian crisis; they were relatively low through much of the 2000s, including the 'Greenspan conundrum' period; and they rose sharply but temporarily during the financial crisis of 2008/09.

A general difficulty, however, is that estimation of ATSMs is fraught with problems associated with weak identification and computationally intensive optimization steps.[2] This has two conse-

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[2] For example, previous studies that have estimated ATSMs using UK data have typically relied on maximum likelihood estimation (e.g. Lildholdt et al., 2007; Joyce et al., 2010; Kaminska, 2013; D'Amico et al., 2014; Guimarães, 2014). This involves a high-dimensional non-linear optimization over a likelihood surface that has many local optima and undefined regions. Hamilton and Wu (2012) discuss these issues in greater detail.

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quences. First, estimates of term premia from a single model have wide confidence bounds; and second, the challenges associated with estimating just a single model mean that establishing which properties of estimated term premia are robust across models has inevitably proved somewhat challenging. An obvious question is therefore just how much weight can policy-makers and other users place on estimates of term premia obtained from these models? To address this question, we apply the estimation approach proposed by Adrian et al. (2013) (ACM henceforth), who split the estimation into a series of linear regressions, which greatly reduces the time taken to estimate the model.[3]

In some respects, estimates of term premia for the UK are extremely robust. We highlight in particular that it makes little difference to estimates of term premia if we include additional macroeconomic variables as unspanned factors, as proposed by Joslin et al. (2014); if we extend the model to allow for a lower bound on nominal interest rates, as suggested by Black (1995); or if we vary the number of pricing factors between three and six. There is nevertheless substantial model and parameter uncertainty associated with estimating the time-series dynamics of term premia, which suggests that we should be particularly cautious about drawing strong inferences based on dynamic term structure models at times when interest rates are a long way from normal levels, as has been the case in recent years. We highlight two issues in particular. First, as discussed by Bauer et al. (2012), OLS estimates of the dynamics of the pricing factors driving yields will be biased in small samples; and correcting for this small-sample bias can have a substantial impact on estimated US term premia. Encouragingly, we find that applying a similar bias correction to the UK does not result in a materially different interpretation of past movements in bond yields for the majority of our sample period. But estimates of term premia towards the end of the sample from a bias-corrected version of the model are higher than from our benchmark model. The bias correction increases the estimated persistence of short-term interest rates, which means that it takes substantially longer for model-implied expected short rates to rise from the very low levels experienced since 2009; and with the expected path of short rates lower, the term premium is correspondingly higher.

Second, it is hard to estimate the persistence of yields precisely given the sample of yields available. A popular approach for improving the identification of dynamic term structure models is to include additional information in the form of survey expectations of future short-term interest rates (first proposed by Kim and Orphanides, 2012 and applied to UK data by Joyce et al., 2010; Guimarães, 2014). We adapt the ACM method to allow the inclusion of survey expectations of future short-term interest rates. Encouragingly, we again find that the broad pattern of movements in term premia from our 'survey model' is similar to that from our benchmark model. In contrast to the bias-corrected model, however, the main difference is that term premia from the survey model are slightly *lower* towards the end of the sample.

In the face of this model uncertainty, what practical advice can we offer policy-makers and others users of these models when seeking to estimate term premia for the UK? While it may be tempting to prefer a model in which the time-series dynamics have been bias-corrected, there are reasons to be cautious about this. In particular, the bias-corrected model performs relatively poorly against the Dai and Singleton (2002) LPY specification tests and the model-implied path of expected short rates towards the end of the sample is extremely low – perhaps implausibly so. There are also reasons to be cautious about the potential gains from including surveys in the model for the UK, since this also results in substantially inferior performance against the Dai and Singleton (2002) LPY specification tests. One possible explanation may be that UK interest rate survey expectations are only available for relatively short forecast horizons, so may be less informative about the persistence of interest rates than in the US.

The remainder of this paper is organized as follows. Section 2 sets out the standard no-arbitrage ATSM and the estimation technique of ACM. Section 3 introduces the data set and discusses the estimates of term premia from our benchmark model. Section 4 discusses the most important sources of uncertainty in estimates of UK term premia, related to the difficulty in specifying and precisely estimating the time-series dynamics of bond yields. Section 5 evaluates the robustness of term premium estimates to the inclusion of unspanned macroeconomic factors (as proposed by Joslin et al., 2014) and to the imposition of a zero lower bound on nominal interest rates. Section 6 concludes. The appendices to the paper provide a number of additional robustness checks.

2. Gaussian affine term structure models

2.1. Excess returns

This section sets out the key equations of a standard Gaussian ATSM, following the exposition from ACM. A $K \times 1$ vector of pricing factors, $\mathbf{x}_t$, evolves according to a Gaussian VAR(1):

$$\mathbf{x}_{t+1} = \boldsymbol{\mu} + \boldsymbol{\Phi}\mathbf{x}_t + \mathbf{v}_{t+1}, \tag{1}$$

where the shocks $\mathbf{v}_{t+1} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma})$ are conditionally Gaussian, homoskedastic and independent across time. We denote the time-t price of a zero-coupon bond with a maturity of n by $P_t^{(n)}$. The assumption of no-arbitrage implies the existence of a pricing kernel M_{t+1} such that

$$P_t^{(n)} = E_t\left[M_{t+1}P_{t+1}^{(n-1)}\right]. \tag{2}$$

The pricing kernel is assumed to be exponentially affine in the factors:

$$M_{t+1} = \exp\left(-r_t - \frac{1}{2}\boldsymbol{\lambda}_t'\boldsymbol{\lambda}_t - \boldsymbol{\lambda}_t'\boldsymbol{\Sigma}^{-1/2}\mathbf{v}_{t+1}\right), \tag{3}$$

where $r_t = -\ln P_t^{(1)}$ denotes the continuously compounded one-period risk-free rate, which is affine in the factors:

$$r_t = \delta_0 + \boldsymbol{\delta}_1'\mathbf{x}_t, \tag{4}$$

and the market prices of risk ($\boldsymbol{\lambda}_t$) are affine in the factors, as in Duffee (2002):

$$\boldsymbol{\lambda}_t = \boldsymbol{\Sigma}^{-1/2}(\boldsymbol{\lambda}_0 + \boldsymbol{\lambda}_1\mathbf{x}_t). \tag{5}$$

The log excess one-period holding return of a bond maturing in n periods is defined as

$$rx_{t+1}^{(n-1)} = \ln P_{t+1}^{(n-1)} - \ln P_t^{(n)} - r_t. \tag{6}$$

Using (3) and (6) in (2), ACM show that

$$E_t\left[\exp\left(rx_{t+1}^{(n-1)} - \frac{1}{2}\boldsymbol{\lambda}_t'\boldsymbol{\lambda}_t - \boldsymbol{\lambda}_t'\boldsymbol{\Sigma}^{-1/2}\mathbf{v}_{t+1}\right)\right] = 1, \tag{7}$$

and, under the assumption of joint normality of $\{rx_{t+1}^{n-1}, \mathbf{v}_{t+1}\}$, they demonstrate that

[3] A number of other studies have applied multi-step methods to reduce the numerical challenges associated with the estimation of ATSMs. Examples include Moench (2008), Joslin et al. (2011), Kaminska (2013) and Andreasen and Meldrum (2015a). All of these methods involve some non-linear optimization, which ACM avoid entirely because they do not impose the no-arbitrage restrictions inside the estimation procedure. They show, however, that the factor loadings implied by their approach satisfy these restrictions to a high degree of precision. In Appendix A we show that our benchmark estimates of term premia are almost identical to those obtained using more standard maximum likelihood techniques to estimate the cross-sectional dynamics of the yield curve while also imposing no-arbitrage.

$$E_t\left[rx_{t+1}^{(n-1)}\right] = Cov_t\left[rx_{t+1}^{(n-1)}, \mathbf{v}_{t+1}'\mathbf{\Sigma}^{-1/2}\lambda_t\right] - \frac{1}{2}Var_t\left[rx_{t+1}^{(n-1)}\right]. \quad (8)$$

By denoting $\boldsymbol{\beta}^{(n-1)} = Cov_t[rx_{t+1}^{(n-1)}, \mathbf{v}_{t+1}']\mathbf{\Sigma}^{-1}$ and using (5), (8) can be rewritten as

$$E_t\left[rx_{t+1}^{(n-1)}\right] = \boldsymbol{\beta}^{(n-1)\prime}[\boldsymbol{\lambda}_0 + \boldsymbol{\lambda}_1\mathbf{x}_t] - \frac{1}{2}Var_t\left[rx_{t+1}^{(n-1)}\right]. \quad (9)$$

This in turn can be used to decompose the unexpected excess return into a component that is correlated with $\mathbf{v}_{t+1}$ and another which is conditionally orthogonal:

$$rx_{t+1}^{(n-1)} - E_t\left[rx_{t+1}^{(n-1)}\right] = \boldsymbol{\beta}^{(n-1)\prime}\mathbf{v}_{t+1} + e_{t+1}^{(n-1)}, \quad (10)$$

where $e_{t+1}^{(n-1)} \backsim iid(0, \sigma^2)$. The return generating process for log excess returns can be written as

$$\begin{aligned} rx_{t+1}^{(n-1)} &= \boldsymbol{\beta}^{(n-1)\prime}(\boldsymbol{\lambda}_0 + \boldsymbol{\lambda}_1\mathbf{x}_t) - \frac{1}{2}(\boldsymbol{\beta}^{(n-1)\prime}\mathbf{\Sigma}\beta^{(n-1)} + \sigma^2) \\ &\quad + \boldsymbol{\beta}^{(n-1)\prime}\mathbf{v}_{t+1} + e_{t+1}^{(n-1)}. \end{aligned} \quad (11)$$

We observe returns for $t = 1, 2, \ldots, T$ and maturities $n = n_1, n_2, \ldots, n_N$. Stacking (11) across n and t, ACM construct the expression

$$\mathbf{rx} = \boldsymbol{\beta}^{(n-1)\prime}(\boldsymbol{\lambda}_0\boldsymbol{\iota}_T' + \boldsymbol{\lambda}_1\mathbf{X}_\_) - \frac{1}{2}(\mathbf{B}^* vec(\mathbf{\Sigma}) + \sigma^2\boldsymbol{\iota}_N)\boldsymbol{\iota}_T' + \boldsymbol{\beta}'\mathbf{V} + \mathbf{E}, \quad (12)$$

where $\mathbf{rx}$ is an $N \times T$ matrix of excess returns; $\boldsymbol{\beta} = [\boldsymbol{\beta}^{(1)}, \boldsymbol{\beta}^{(2)}, \ldots, \boldsymbol{\beta}^{(n)}]$ is a $K \times N$ matrix of factor loadings; $\boldsymbol{\iota}_N$ and $\boldsymbol{\iota}_T$ are $N \times 1$ and $T \times 1$ vectors of ones respectively; $\mathbf{X}_\_$ is a $K \times T$ matrix of lagged pricing factors; $\mathbf{B}^* = [vec(\boldsymbol{\beta}^{(1)}\boldsymbol{\beta}^{(1)\prime}) \ldots vec(\boldsymbol{\beta}^{(N)}\boldsymbol{\beta}^{(N)\prime})]'$ is an $N \times K^2$ matrix; and $\mathbf{V}$ and $\mathbf{E}$ are matrices of dimensions $K \times T$ and $N \times T$ respectively.

2.2. Estimation

In this section we provide a brief outline of the estimation technique of ACM, which involves four linear regressions. We refer the reader to ACM for further details. Even though the technique does not impose no-arbitrage restrictions on the cross-section of yields during the estimation, in Appendix A we show that this has almost no impact on estimated term premia.

1. In the first step, we estimate (1) by OLS, giving estimates $\widehat{\mathbf{\Phi}}$ and $\widehat{\mathbf{\Sigma}}$. Following ACM, we calibrate $\boldsymbol{\mu} = \mathbf{0}$, which ensures that the means of the factors (mean-zero principal components) are equal to their sample averages.
2. In the second step, we estimate the reduced-form of (12), $\mathbf{rx} = \mathbf{a}\boldsymbol{\iota}_T' + \boldsymbol{\beta}'\widehat{\mathbf{V}} + \mathbf{c}\mathbf{X}_\_ + \mathbf{E}$, giving estimates $\widehat{\mathbf{a}}, \widehat{\boldsymbol{\beta}}$ and $\widehat{\mathbf{c}}$, as well as $\widehat{\sigma}^2 = \text{ tr } (\widehat{\mathbf{E}}\widehat{\mathbf{E}}')/NT$.
3. Using $\widehat{\mathbf{B}}^*$ constructed from $\widehat{\boldsymbol{\beta}}$ and noting that $\mathbf{a} = \boldsymbol{\beta}'\boldsymbol{\lambda}_0 - \frac{1}{2}(\mathbf{B}^* vec(\mathbf{\Sigma}) + \sigma^2\boldsymbol{\iota}_N)$ and $\mathbf{c} = \boldsymbol{\beta}'\boldsymbol{\lambda}_1$, we can estimate the parameters of the price of risk using cross-sectional regressions:

$$\widehat{\boldsymbol{\lambda}}_0 = (\widehat{\boldsymbol{\beta}}\widehat{\boldsymbol{\beta}}')^{-1}\widehat{\boldsymbol{\beta}}\left(\widehat{\mathbf{a}} + \frac{1}{2}\left(\widehat{\mathbf{B}}^* vec(\widehat{\mathbf{\Sigma}}) + \widehat{\sigma}^2\boldsymbol{\iota}_N\right)\right), \quad (13)$$

$$\widehat{\boldsymbol{\lambda}}_1 = (\widehat{\boldsymbol{\beta}}\widehat{\boldsymbol{\beta}}')^{-1}\widehat{\boldsymbol{\beta}}\widehat{\mathbf{c}}. \quad (14)$$

4. The short-term interest rate is assumed to be measured with error $\varepsilon_t \sim iid(0, \sigma_\varepsilon^2)$:

$$r_t = \delta_0 + \boldsymbol{\delta}_1'\mathbf{x}_t + \varepsilon_t. \quad (15)$$

We can estimate (15) by OLS to obtain estimates $\widehat{\delta}_0$ and $\widehat{\boldsymbol{\delta}}_1$.

2.3. Model-implied zero-coupon yields and term premia

Given the above assumptions, it is straightforward to show (e.g. Piazzesi, 2003) that zero-coupon bond yields are affine functions of the factors:

$$y_t^{(n)} = -\frac{1}{n}\left[a_n + \mathbf{b}_n'\mathbf{x}_t\right]. \quad (16)$$

where the coefficients a_n and $\mathbf{b}_n$ follow the recursive equations

$$a_n = a_{n-1} + \mathbf{b}_{n-1}'(\boldsymbol{\mu} - \boldsymbol{\lambda}_0) + \frac{1}{2}(\mathbf{b}_{n-1}'\mathbf{\Sigma}b_{n-1} + \sigma^2) - \delta_0 \quad (17)$$

$$\mathbf{b}_n' = \mathbf{b}_{n-1}'(\mathbf{\Phi} - \boldsymbol{\lambda}_1) - \boldsymbol{\delta}_1' \quad (18)$$

and where we have the initial conditions $a_0 = 0$ and $\mathbf{b}_0 = \mathbf{0}$.

Following the definition of Dai and Singleton (2002), an n-period bond yield can be decomposed as

$$y_t^{(n)} = \frac{1}{n}\sum_{i=0}^{n-1}E_t r_{t+i} + TP_t^{(n)}, \quad (19)$$

where the first-term on the right-hand side of (19) is the average expected short rate over the current and $n-1$ subsequent periods; and $TP_t^{(n)}$ is the term premium – the additional compensation investors require for investing in a long-term bond rather than rolling over a series of investments in short-term bonds. The expectations term can be computed using (1) and (15) and the term premium can therefore be computed as a residual from (19).

2.4. Confidence intervals for term premia

One way of assessing the effect of changing a model specification on term premia is to view it in the light of the uncertainty around term premia associated with *parameter* uncertainty within a benchmark model. In the following sections, we therefore report 95% confidence intervals for term premia, computed using a bootstrap procedure. The algorithm we use to bootstrap the confidence interval has the following steps.

1. Estimate the model using the procedure explained above.
2. At iteration h of the bootstrap, generate a sample of the factors for periods $t = 1, 2, \ldots, T$ (where the sample length T is same as for the data). For $t = 1$ randomly select $\mathbf{x}_1^{(h)}$ from the original sample. For $t > 1$ randomly resample (with replacement) $\mathbf{v}_t^{(h)}$ from the estimated residuals from (1) and compute $\mathbf{x}_t^{(h)} = \widehat{\mathbf{\Phi}}\mathbf{x}_{t-1}^{(h)} + \mathbf{v}_t^{(h)}$.
3. For periods $t = 1, 2, \ldots, T$ and maturities $n = n_1, \ldots, n_N$, randomly resample (with replacement) residuals $e_{t+1}^{(n-1)(h)}$ from (11) and compute $\mathbf{rx}^{(h)} = \widehat{\mathbf{a}}\boldsymbol{\iota}_T' + \widehat{\boldsymbol{\beta}}'\mathbf{V}^{(h)} + \widehat{\mathbf{c}}\mathbf{X}_\_^{(h)} + \mathbf{E}^{(h)}$.
4. For periods $t = 1, 2, \ldots, T$, randomly resample (with replacement) residuals $\varepsilon_t^{(h)}$ from (15) and compute $r_t^{(h)} = \widehat{\delta}_0 + \widehat{\boldsymbol{\delta}}_1'\mathbf{x}_t^{(h)} + \varepsilon_t^{(h)}$.
5. Re-estimate the model using the h^{th} bootstrapped sample, saving the term premium estimates implied by the parameters obtained at the h^{th} bootstrap draw.
6. While $h < 10,000$ increase h by one and return to step 2.
7. Compute the percentiles of the bootstrapped term premium estimates.

3. Benchmark model

3.1. Data

We estimate the model using estimated month-end zero-coupon yields. The pricing factors are principal components extracted from yields with maturities of $1, 6, 12, 18, \ldots, 120$ months; and we use excess returns for $n = 12, 18, 24, 30, \ldots, 120$ months, giving a cross-section of $N = 19$ maturities. Yields with maturities of 6 months and longer are constructed using the smoothed cubic spline method of Anderson and Sleath (2001) (selected maturities from which are published by the Bank of England).[4] We use Bank Rate, the UK policy interest rate, as a proxy for the 1-month yield (aside from being used as one of the set of yields to compute principal components, we also use the one-month yield for computing excess returns, as defined in (6)).[5] In our benchmark specification we use a sample starting in October 1992 (when the UK adopted an inflation target framework for monetary policy) and ending in December 2013, giving a time series of $T = 255$ observations. Issues of sensitivity to the sample period are discussed further in Section 4.1.

Our benchmark specification uses the first four principal components of bond yields as spanned pricing factors. While much of the previous literature on nominal bond yields has used models with just three pricing factors, in Appendix B we show that a more standard three-factor model provides an inferior fit to the short end of the UK yield curve and performs less well against the specification tests of Dai and Singleton (2002) relative to models with at least four factors. This is consistent with some recent studies using US data, which find evidence in favour of more than three factors (e.g. Cochrane and Piazzesi, 2005; Duffee, 2011b and ACM).

3.2. Term premia in the benchmark model

Fig. 1 shows estimates of term premia from the benchmark model at maturities of two, five and ten years. We highlight the following features of our estimates. Term premia at different maturities are positively correlated but are generally larger and more volatile at longer maturities. They were relatively high in the early 1990s, fell in the late 1990s and were relatively low during the 2000s, including the 'Greenspan conundrum' period of the mid 2000s when US long-term rates were falling despite increases in short-term rates. This broad pattern in long-maturity premia is consistent with the findings of previous studies using UK data, such as Joyce et al. (2010) and Wright (2011). Premia at longer maturities then rose sharply but temporarily during the financial crisis of 2008/09.

Dai and Singleton (2002) propose a formal approach for assessing how well dynamic term structure models capture the time-series and risk-neutral dynamics of bond yields using two 'linear projections of yields' (LPY) specification tests. The first test, LPY (i), considers whether the model parameters imply the same pattern of slope coefficients obtained by estimating Campbell and Shiller (1991) regressions

$$y_{t+1}^{(n-1)} - y_t^{(n)} = \alpha^{(n)} + \phi^{(n)}(y_t^{(n)} - r_t)/(n-1) + \zeta_t^{(n)} \tag{20}$$

using the observed data. This is a test of the capacity of the model to replicate the time-series dynamics of yields. The second test, LPY(ii), considers a risk-adjusted version of (20):

$$y_{t+1}^{(n-1)} - y_t^{(n)} - e_t^{(n)}/(n-1) = \alpha^{*(n)} + \phi^{*(n)}(y_t^{(n)} - r_t)/(n-1) + \zeta_t^{*(n)}, \tag{21}$$

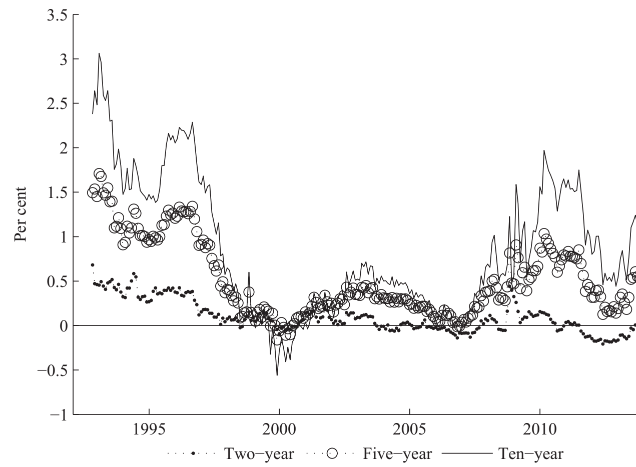

Fig. 1. Estimates of term premia from the benchmark model.

where $e_t^{(n)} \equiv E_t[\ln P_{t+1}^{(n-1)} - \ln P_t^{(n)} - r_t]$ is the expected excess one-period return on an n-period bond. If the model is able to capture the dynamics of term premia, then we should recover a coefficient $\phi^{*(n)} = 1$, i.e. yields adjusted for term premia should satisfy the expectations hypothesis of the term structure.

The first column of Table 1 shows the pattern of $\phi^{(n)}$ slope coefficients from (20) observed in the data; these decrease with maturity, consistent with the pattern reported by Dai and Singleton (2002) for the US. To obtain values of $\phi^{(n)}$ for the models, we simulate time series of 100,000 periods using the point estimates of the parameters. The benchmark model (the second column) matches the pattern in the data well: although the model-implied coefficients are a little higher than those in the data at long maturities, the differences are not statistically significant. It also performs well against the LPY(ii) test, with slope coefficients from the risk-adjusted regression (21) very close to one at all maturities (Table 2).

A less formal approach for assessing estimates of term premia is to establish whether they co-vary with other variables in a way that is economically reasonable. We show that this is the case for our estimates by regressing estimated 10-year term premia ($TP_t^{(120)}$) on four macroeconomic variables:

$$TP_t^{(120)} = \alpha_0 + \alpha_1 U_t + \alpha_2 INF_t + \alpha_3 UGDP_t + \alpha_4 UINF_t + u_t, \tag{22}$$

where U_t is the seasonally adjusted Labour Force Survey measure of UK unemployment among those aged 16 and over; INF_t is the Consensus Forecasts survey-based RPI inflation expectation for the next twelve months; and $UGDP_t$ and $UINF_t$ are Consensus Forecasts survey dispersion measures for next-year real GDP growth and inflation respectively. Consensus Forecasts reports RPI inflation and GDP growth (i.e. the percentage change over the previous year) forecasts for a panel of professional forecasters. The inflation expectation measure is computed as the mean of the point forecasts across respondents; and the dispersion measures as the standard deviations of the point forecasts. Although cross-sectional dispersion in beliefs is not the same as aggregate uncertainty, Rich et al. (1992) show that measures of inflation dispersion are positively and significantly related to measures of inflation uncertainty for the US so (following Wright, 2011) it is reasonable to think of these measures as proxies for aggregate uncertainty.

Estimates of the coefficients in (22) and the associated estimated standard errors are reported for ten-year term premia in the first column of Table 3, while Fig. 2 shows bivariate plots of ten-year term premia against the four macroeconomic variables. Term premia from the benchmark model covary positively with

[4] Available at: http://www.bankofengland.co.uk/statistics/pages/yieldcurve/default.aspx. Since the 6-month yield is not always available before 1997 and after 2012, in periods when it is unavailable we use the fitted values from a regression of 6-month rates on a constant, Bank Rate and the 12-month yield estimated for the period February 1997-December 2012. In addition, the 11-month yield is not available in January 1990, so we linearly interpolate between the December 1989 and February 1990 values.

[5] Strictly speaking, Bank Rate is the rate at which the Bank of England remunerates reserves. In Appendix C we show that our results are robust to omitting yields with maturities less than 12 months from the calculation of the pricing factors.

Table 1
Model performance against LPY(i) test.

Maturity (months)	Data	Benchmark	Random walk level	Survey
12	1.19	1.14	−0.10	0.52
	(0.32)			
24	0.80	0.83	−1.18	0.47
	(0.50)			
36	0.56	0.64	−1.55	0.58
	(0.60)			
60	0.22	0.40	−1.33	0.69
	(0.75)			
84	−0.10	0.19	−1.05	0.58
	(0.89)			
120	−0.44	−0.10	−0.76	0.13
	(1.07)			

Estimated slope coefficients ($\widehat{\phi}^{(n)}$) from the regression $y_{t+1}^{(n-1)} - y_t^{(n)} = \alpha^{(n)} + \phi^{(n)}(y_t^{(n)} - r_t)/(n-1) + \zeta_t^{(n)}$. Standard errors for the data are shown in parentheses. Model-implied slope coefficients are estimated using a simulated data set with 100,000 periods using the point estimates of the model parameters. The 'benchmark' model is a four-factor model estimated using data for October 1992–December 2013. The 'random walk level' model imposes that the first principal component of the term structure follows a random walk, as described in Section 4.3. The 'survey' model includes survey expectations of future short-term interest rates, as described in Section 4.4.

Table 2
Model performance against LPY(ii) test.

Maturity (months)	Data	Benchmark	Bias-corrected	Random walk level	Survey
12	1	0.97	0.95	1.68	1.60
		(0.35)	(0.35)	(0.34)	(0.30)
24	1	0.99	0.97	1.96	1.58
		(0.54)	(0.58)	(0.54)	(0.44)
36	1	1.01	0.99	2.11	1.37
		(0.63)	(0.70)	(0.63)	(0.50)
60	1	1.02	1.08	2.28	1.01
		(0.74)	(0.90)	(0.79)	(0.57)
84	1	1.00	1.18	2.34	0.81
		(0.81)	(1.09)	(0.94)	(0.57)
120	1	1.02	1.40	2.44	0.72
		(0.90)	(1.36)	(1.12)	(0.57)

Estimated slope coefficients ($\widehat{\phi}^{*(n)}$) from the regression $y_{t+1}^{(n-1)} - y_t^{(n)} - e_t^{(n)}/(n-1) = \alpha^{*(n)} + \phi^{*(n)}(y_t^{(n)} - r_t)/(n-1) + \zeta_t^{*(n)}$. Standard errors are shown in parentheses. The 'benchmark' model is a four-factor model estimated using data for October 1992-December 2013. The 'bias-corrected' model applies the bootstrap bias correction, as described in Section 4.2. The 'random walk level' model imposes that the first principal component of the term structure follows a random walk, as described in Section 4.3. The 'survey' model includes survey expectations of future short-term interest rates, as described in Section 4.4.

Table 3
Regressions of ten-year term premia on macroeconomic variables.

	Benchmark	Bias-corrected	Random walk level	Survey
Constant	**−2.07**	**−1.72**	−0.73	**−3.40**
	(0.22)	(0.77)	(0.66)	(0.55)
Unemployment	**0.20**	**0.41**	**0.43**	0.06
	(0.06)	(0.12)	(0.11)	(0.16)
Inflation expectations	**0.38**	−0.25	**−0.71**	**1.16**
	(0.11)	(0.22)	(0.20)	(0.26)
GDP uncertainty	**0.48**	1.03	0.93	0.16
	(0.24)	(0.62)	(0.59)	(0.46)
Inflation uncertainty	**0.64**	−0.03	0.34	1.28
	(0.27)	(0.74)	(0.74)	(0.68)
R^2	0.74	0.35	0.35	0.54

Coefficient estimates and estimated standard errors (in parentheses) from a regression of 10-year term premia on the variables listed in the table: $TP_t^{(120)} = \alpha_0 + \alpha_1 U_t + \alpha_2 INF_t + \alpha_3 UGDP_t + \alpha_4 UINF_t + u_t$ where the symbols are as defined in Section 3.2. Coefficients in bold type are significant at the 5% significance level. The 'benchmark' model is a four-factor model estimated using data for October 1992–December 2013. The 'bias-corrected' model applies the bootstrap bias correction, as described in Section 4.2. The 'random walk level' model imposes that the first principal component of the term structure follows a random walk, as described in Section 4.3. The 'survey' model includes survey expectations of future short-term interest rates, as described in Section 4.4.

unemployment – i.e. they are larger during times of high unemployment – which Bauer et al. (2012) argue is a desirable feature of term premia. The finding that term premia are countercyclical is also broadly consistent with the findings of Gil-Alana and Moreno (2012), who include estimates of US term premia in a VAR alongside macroeconomic variables and show that a positive shock to unemployment is followed by a rising term premium. We also find a positive relationship between inflation expectations and the term premium and - consistent with the findings of Wright (2011) – we also find that macroeconomic uncertainty is associated with a higher term premium.

4. Robustness of the time series dynamics of bond yields

In maximally flexible ATSMs, the primary determinant of model-implied interest rate expectations is how well we specify – and how accurately we can estimate – the time-series dynamic of yields – i.e. (1).[6] Unfortunately, in common with studies using US data, it is not straightforward to discriminate between different specifications or estimate parameters precisely given the sample of UK yields available. To illustrate the difficulty, Fig. 3 shows yields at selected maturities between January 1979 and December 2013. Yields drifted downwards steadily during much of the sample, which means that the average level of yields will depend on the precise sample period considered. Yields are also highly persistent - for example, the first-order autocorrelation coefficient of the 10-year yield is 0.995 and it is not possible to reject the presence of a unit root using standard tests. This means that it is hard to estimate the persistence of yields precisely and - as discussed by Bauer et al. (2012) – autoregressive parameters may be biased in short samples. In this section, we explore the extent to which these difficulties affect the robustness of estimates of UK term premia.

4.1. Choice of sample period

As explained above, for our benchmark model we use a sample starting in October 1992. This coincides with the introduction of an inflation targeting framework for monetary policy in the UK (following the UK's exit from the European Exchange Rate Mechanism), so structural breaks are less likely to be a concern than if we had used data from the 1980s and early 1990s. This is the same as or similar to most other studies that have estimated no-arbitrage term structure models using UK data (e.g. Joyce et al., 2010 also use a sample starting in October 1992 and D'Amico et al., 2014 in January 1993). And studies using US data often use a similar rationale to justify starting samples in the late 1980s or early 1990s (e.g. Kim and Wright, 2005).

Ultimately, however, the choice of sample period does involve an element of judgement. On one hand, there is still a possibility of structural breaks since October 1992; for example, the introduction of Bank of England independence in May 1997 is a potential structural break in the conduct of monetary policy and therefore in the dynamics of nominal interest rates.[7] On the other hand, some studies of dynamic term structure models have considered much longer samples that span multiple changes in monetary policy

[6] For example, Duffee (2011a) shows that no-arbitrage restrictions do not have a material impact on the time-series dynamics of interest rates.

[7] The recent period of very low interest rates is another. In Section 5 we show that estimates of UK term premia are not materially affected if we impose a zero lower bound on nominal interest rates using the shadow rate framework of Black (1995).

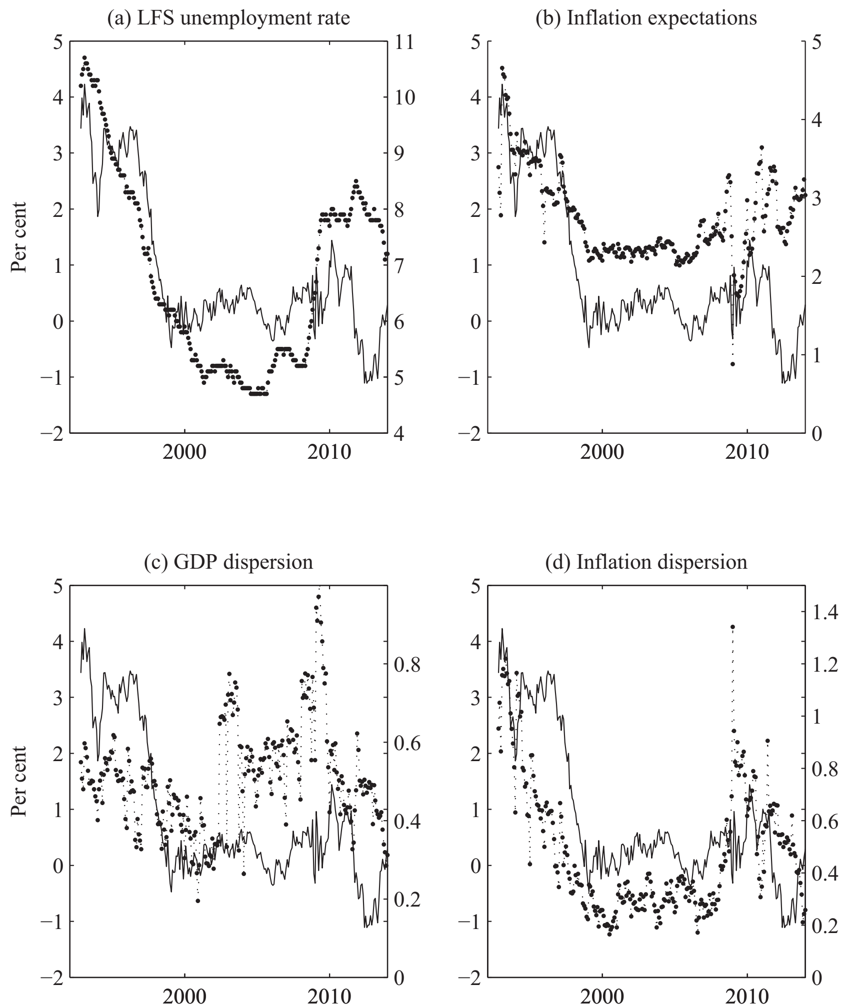

Fig. 2. UK term premia and macroeconomic variables. In each plot the solid line shows the estimated ten-year term premium from the benchmark model (left-hand axis) and the dotted line shows a different macroeconomic variable (right-hand axis). Panel (a) shows the Labour Force Survey measure of UK unemployment among those aged 16 and over. Panel (b) shows Consensus Forecasts survey-based RPI inflation expectations for the next 12 months. Panels (c) and (d) show the standard deviation of Consensus Forecasts survey-based GDP and inflation expectations for the next 12 months respectively. All y-axis units are per cent.

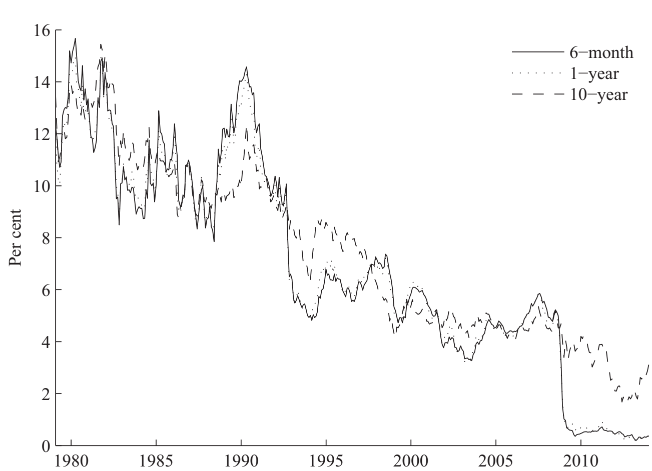

Fig. 3. UK end-month bond yields, January 1979–December 2013.

regime (e.g. Duffee, 2002 uses a US sample starting in 1952 and Guimarães, 2014 a UK sample starting in 1972). While a longer sample runs greater risk of structural breaks, it might also help in estimating the persistence of bond yields. In this section we therefore illustrate the impact that varying the sample period has on estimates of term premia and consider whether that affects our interpretation of movements in bond yields.

Fig. 4 shows term premia obtained using three different sample periods. The broad dynamics of long-maturity term premium estimates from models estimated using samples starting in January 1979, October 1992 and May 1997 are similar. And the level of term premium estimates obtained from models using the two shorter samples are also quite similar. But term premium estimates are generally lower from the model estimated using the longer sample. To understand this, note that the unconditional mean short rate estimated from the sample starting in January 1979 is 7.3%, compared with 4.2% from the sample starting in October 1992 and 3.7% from the sample starting in May 1997. If short rates are forecast to revert back to a higher mean, the expected path of short-term interest rates will be higher and the term premium component of yields correspondingly lower. Finally, we note that estimates of short-maturity term premia from the longer sample are more volatile than for the shorter samples.

Ultimately, it is difficult to be conclusive about which is the most appropriate sample period with which to estimate term structure models. In line with most previous studies, our judgement is that omitting data from the period when the UK had very different monetary policy frameworks – involving the targeting of

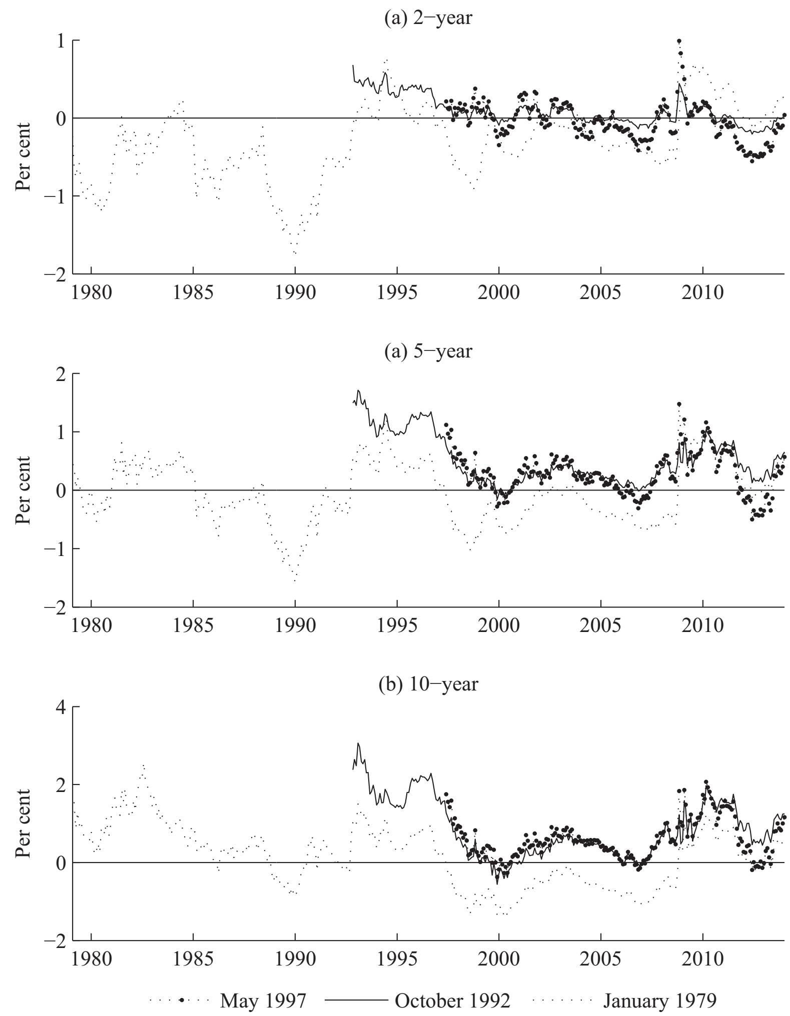

Fig. 4. Term premium estimates from four-factor models with different sample start dates.

monetary aggregates or the exchange rate – is sensible approach when estimating the dynamics of UK nominal interest rates; and that including data from before the early 1990s is likely to result in a misleadingly low estimate of the term premium.

4.2. Small sample bias

An issue recently highlighted by Bauer et al. (2012) is that estimates of the time series dynamics of the factors in term structure models may suffer materially from bias due to the high persistence of interest rates. The bias typically goes in the direction of making the estimated system less persistent than the data-generating process: future short rates are forecast to revert to their unconditional mean too quickly and, as a consequence, the component of long-term yields reflecting expected future short rates is too stable and term premia are too volatile.

A multi-step estimation procedure such as the method of ACM makes bias-correction at the stage of estimating the time series dynamics of the factors straightforward. We employ bootstrap (mean) bias correction in which data are simulated using a distribution-free residual bootstrap taking the OLS estimates as the data-generating parameters. The algorithm is set out below.

1. Estimate (1) by OLS as before, storing the residuals and the OLS estimate $\widehat{\mathbf{\Phi}}$.
2. At iteration h of the bootstrap, generate a bootstrap sample for periods $t = 1, 2, \ldots, T$ (where the sample length T is same as for the data). For $t = 1$ randomly select $\mathbf{x}_1^{(h)}$ from the original sample. For $t > 1$ randomly resample (with replacement) residuals $\mathbf{v}_t^{(h)}$ and construct $\mathbf{x}_t^{(h)} = \widehat{\mathbf{\Phi}}\mathbf{x}_{t-1}^{(h)} + \mathbf{v}_t^{(h)}$.
3. Calculate the OLS estimates $\widehat{\boldsymbol{\mu}}^{(h)}$ and $\widehat{\mathbf{\Phi}}^{(h)}$ using the h^{th} bootstrapped sample.
4. While $h < 10,000$ increase h by one and return to step 2.
5. Calculate the average over all bootstrap samples: $\overline{\mathbf{\Phi}} = \frac{1}{H}\sum_{h=1}^{H}\widehat{\mathbf{\Phi}}^{(h)}$.
6. The bootstrap bias-corrected estimate is: $\widetilde{\mathbf{\Phi}}^H = \widehat{\mathbf{\Phi}} - [\overline{\mathbf{\Phi}} - \widehat{\mathbf{\Phi}}]$.
7. If the bias-corrected estimate $\widetilde{\mathbf{\Phi}}^H$ has eigenvalues outside the unit circle, we follow Bauer et al. (2012) and apply the adjustment by Kilian (1998) to ensure stationarity. Specifically, the modified bias-corrected estimate is given by $\widetilde{\mathbf{\Phi}}^H = \widehat{\mathbf{\Phi}} - \delta_i[\overline{\mathbf{\Phi}} - \widehat{\mathbf{\Phi}}]$. We start with $\delta_1 = 1$ and iterate according to $\delta_i = \delta_{i-1} - 0.01$ until all the eigenvalues of $\widetilde{\mathbf{\Phi}}^H$ are within the unit circle.[8]
8. As before, we set the bias-corrected intercept, $\widetilde{\boldsymbol{\mu}}^H = \mathbf{0}$, equal to zero. The remaining steps of the estimation are unaffected.

[8] In practice, we find a terminal value of δ_i of 0.40 is required to ensure stationarity, i.e. relatively little of the bias correction is preserved in the final estimate.

Fig. 5 shows estimates of term premia with and without the bias correction applied. At short maturities, it makes little difference. For the ten-year yield, the differences are also small at times when interest rates are close to their average levels. In the early part of the sample – when bond yields were relatively high – term premia from the bias-corrected model are lower than from the benchmark model; whereas at the end of the sample – when bond yields were relatively low – estimates of term premia from the bias-corrected model are somewhat higher (although all remain with the 95% confidence interval from the benchmark model). This is because the bias correction has the effect of increasing the estimated persistence of the factors, which means that short rates are expected to take longer to revert back to their unconditional mean. For example, at the end of the sample the expected path of short rates is lower in the bias-corrected model and the term premium is higher. In Section 4.5, we consider possible approaches for discriminating between point estimates of premia from different models.

4.3. Imposing that the level factor follows a random walk

Aside from the issue of small-sample bias, the high persistence of bond yields means that it is difficult to distinguish between yields following a random walk or a stationary (but extremely persistent) autoregressive process. Duffee (2011a) shows that imposing that the first principal component of yields follows a random walk improves the out-of-sample forecasting performance of dynamic term structure models, while Jardet et al. (2013) suggest that forecast performance may be further improved by by averaging across a stationary model and one in which yields are non-stationary. We do not consider the latter approach in this paper but it is nevertheless instructive to compare two extreme cases: one in which the $\mathbf{\Phi}$ matrix in (1) is unrestricted and one in which the first principal component follows a random walk, i.e.:

$$\mathbf{\Phi} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \phi_{21} & \phi_{22} & \phi_{23} & \phi_{24} \\ \phi_{31} & \phi_{32} & \phi_{33} & \phi_{34} \\ \phi_{41} & \phi_{42} & \phi_{43} & \phi_{44} \end{bmatrix}.$$

Aside from the difficulty in accurately estimating the persistence of the first principal component, there are at least two reasons for considering such a specification. First, it is possible that a model with a fixed unconditional mean is inappropriate for modelling bond yields. For example, Kozicki and Tinsley (2001), Dewachter et al. (2006) and Van Dijk et al. (2014) make the case for a shifting end-point in the term structure – i.e. allowing for time-variation in the long-horizon expectation of the short-term interest rate. Other papers suggest modelling interest rates allowing for other forms of structural change: regime-switching (e.g. Bansal and Zhou, 2002; Dai et al., 2007; Monfort and Pegoraro, 2007; Ang et al., 2008; Gourieroux et al., 2014); multiple break points (Chib and Kang, 2013); or time-varying parameters (e.g. Ang et al., 2011; Bianchi et al., 2009). Rather than taking a stand on how quickly end-points should shift or on how exactly structural change should be modelled, it is again nevertheless instructive to compare two extreme cases: (i) the standard model in which all parameters are fixed (i.e. in which the long-horizon expectation is constant); and (ii) a model in which the first principal component follows a random walk (i.e. in which the long-horizon expectation of the level of the term structure is revised period-by-period to the current value).

Second, Gil-Alana and Moreno (2012) and Abbritti et al. (2015) provide an alternative interpretation of the high persistence of bond yields: they argue that there is evidence of long-memory in the term structure of US interest rates, with the order of integration of the short-term interest rate being closer to one than to zero. While distinguishing between models of structural change/shifting end-points on one hand and long-memory processes on the other is generally challenging (see e.g. Granger and Hyung, 2004) and we do not take a view on that question in this paper, it is again instructive to consider the two extreme cases of the level of the term structure following an $I(0)$ process (as in the benchmark model) and an $I(1)$ process (as in the random walk level model).[9]

Fig. 5 reports estimates of term premia from the random walk level model alongside those from the benchmark and bias-corrected models. At least at medium-long maturities, the general pattern of movements in term premia from the random-walk level model is similar to that in the bias-corrected model: in both cases the persistence of yields is increased relative to the benchmark model. At shorter maturities, term premium estimates are more volatile and are occasionally outside the 95% confidence interval.

4.4. Inclusion of interest rate survey expectations

An alternative approach for improving the identification of the persistence of bond yields proposed by Kim and Orphanides (2012) is to include additional information in the form of survey expectations of future short-term interest rates.[10] In this section, we consider a version of the model that incorporates information from the Bank of England's Survey of External Forecasters, which is published quarterly from November 1999 and asks respondents (who are professional economists) for their expectations of Bank Rate at horizons one, two and three years ahead, which we take as a measure of the expected value of the one-month risk-free rate.[11]

Previous studies that have incorporated interest rate survey data do so within a maximum likelihood framework, jointly estimating the pricing factors and all parameters of the model together. We instead adapt ACM's approach to allow the inclusion of forecast data by augmenting (1) to include the short rate and h-month ahead survey expectations (where as above we set $\boldsymbol{\mu} = \mathbf{0}$):

$$\begin{bmatrix} \mathbf{x}_{t+1} \\ r_t \\ E_t[r_{t+h}] \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{\Phi} \\ \delta_0 & \delta_1' \\ \delta_0 & \delta_1'\mathbf{\Phi}^h \end{bmatrix} \begin{bmatrix} 1 \\ \mathbf{x}_t \end{bmatrix} + \boldsymbol{\eta}_t \tag{23}$$

$$\boldsymbol{\eta}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_\eta)$$

$$\mathbf{R}_\eta = diag([\,\mathbf{\Sigma} \quad \sigma_\varepsilon^2 \quad \sigma_s^2\,])$$

We include survey expectations 12, 24 and 36 months ahead, assuming that the mean of the individual responses is a noisy measure of the expectation of Bank Rate at each horizon; and estimate the parameters of (23) (i.e. δ_0, $\boldsymbol{\delta}_1$, $\mathbf{\Phi}$, $\mathbf{\Sigma}$, σ_ε^2 and σ_s^2) using maximum likelihood. As in previous studies that have incorporated survey expectations into no-arbitrage term structure models, this setup allows survey data to inform the path of expectations but without constraining the model to fit the surveys exactly. We can then proceed as before with steps 2 and 3 of the standard ACM technique set out in Section 2.2.

Fig. 6 reports estimates of term premia from a four-factor model that includes surveys alongside those from our benchmark model. The broad pattern of movements is similar from the two models

[9] A related issue is the appropriate number of lags in the time-series dynamics (1). The majority of the literature on ATSMs assumes the pricing factors follow a VAR with a single lag. In Appendix D, we discuss the sensitivity of results to including additional lags (as proposed previously by e.g. Monfort and Pegoraro, 2007).

[10] This has been implemented previously using UK data by Joyce et al. (2010) (using survey expectations of inflation) and Guimarães (2014) (using survey expectations of both short-term interest rates and inflation).

[11] This is the same set of Bank Rate surveys used by Guimarães (2014).

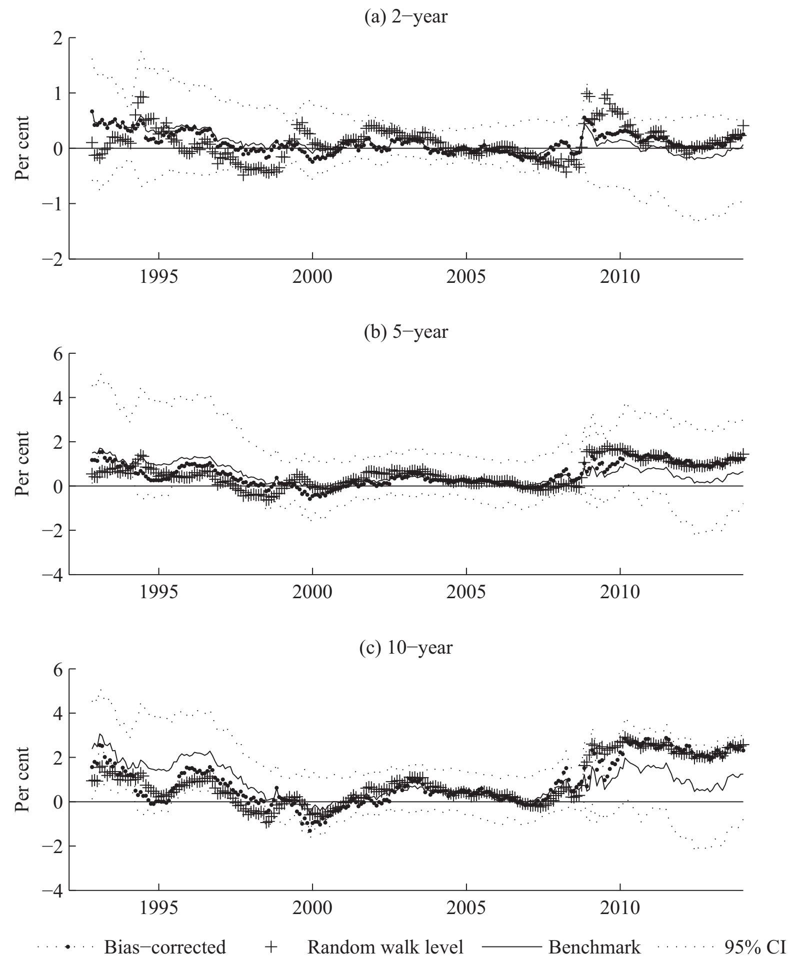

Fig. 5. Term premium estimates from benchmark model, a model with bootstrapped bias correction and a model in which the first principal component of bond yields follows a random walk. The 95% confidence interval (CI in the legend) is taken from the benchmark model.

and estimates of term premia from the survey model are well within the 95% confidence interval from the benchmark model. The impact of including surveys is roughly the opposite of applying the bias correction: term premia are a little higher from the survey model at the beginning of the sample and a little lower at the end. This may seem a little surprising given that one motivation for including surveys – as discussed by Kim and Orphanides (2012) – is that the estimated persistence may be too low in short samples. Here, however, including the surveys reduces the persistence of yields: in the benchmark model, the maximum eigenvalue of $\mathbf{\Phi}$ is 0.985, whereas in the survey model it is 0.968. One possible explanation is that the available UK surveys have shorter maturities than those used by Kim and Wright (2005) and Kim and Orphanides (2012) for the US, so may be less informative about the persistence of interest rates.

4.5. Which estimate of term premia should we prefer? Some practical advice for policy-makers

The results of Sections 4.1–4.4 demonstrate that, for the most part, the broad pattern of moves in term premia is consistent across models. That said, when interest rates a long way from their average levels the model uncertainty around term premium estimates is clearly greater. So which point estimate should we prefer from a practical perspective? Performance against specification tests involving moments not included in the estimation, i.e. the LPY tests, may be particularly helpful in this respect. The standard errors from these regressions are admittedly quite large but the random walk level model performs extremely poorly, with LPY(i) coefficients that are well below those in the data and LPY(ii) coefficients that are well above one (Tables 1 and 2 respectively). The bias-corrected model also performs relatively poorly against the LPY(ii) tests: the slope coefficients from these regressions show a general pattern of increasing with maturity, rather than being flat at one.[12] Including surveys in the model also results in substantially inferior performance against the LPY tests: the coefficients for the LPY(i) test decrease more slowly with maturity than in the data; while those for the LPY(ii) test are well above one for short maturities and below one for long maturities.

If we regress term premia from the survey model on macroeconomic variables, the results are broadly similar to those from the benchmark model (Table 3). While these regressions cannot be conclusive given the inevitable uncertainty over the appropriate set of explanatory variables to include, there is tentative evidence that the term premium estimates from the benchmark model are the most economically plausible: the considered macroeconomic

[12] Population moments for the bias-corrected model are essentially arbitrary, so we cannot use the LPY(i) test. Under the ad hoc stationarity adjustment by Kilian (1998), and as applied here and by BRW, the largest eigenvalue of $\mathbf{\Phi}$ depends on the step-size for adjusting the bias correction (the interval $\delta_i - \delta_{i-1}$ above). Unreported results show that making this step-size smaller does not have a material impact on in-sample term premium estimates at the horizons we are considering.

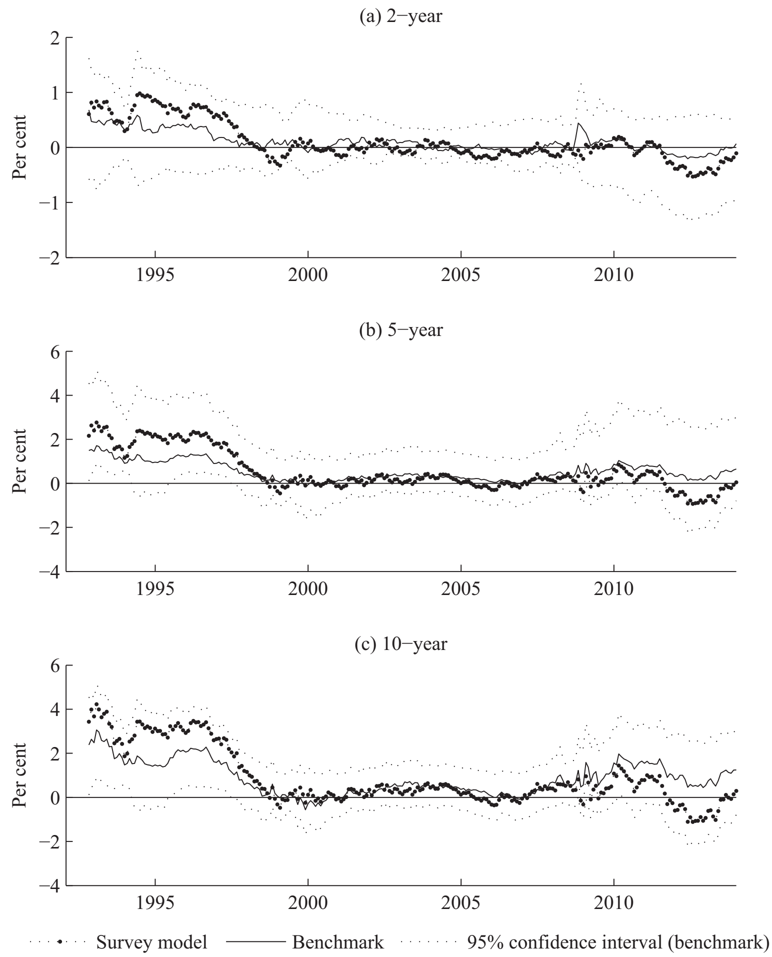

Fig. 6. Term premium estimates from benchmark model and a model that incorporates interest rate survey data.

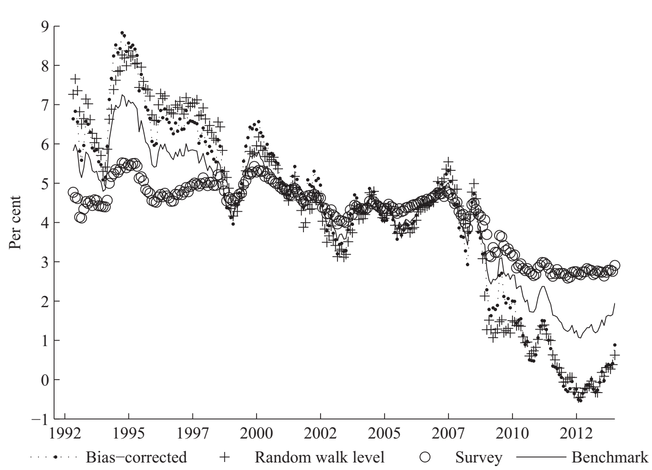

Fig. 7. Average expected short-term interest rates over a ten-year horizon.

Table 4
Regressions of macroeconomic variables on yield curve factors.

	Unemployment	Inflation expectations	GDP uncertainty	Inflation uncertainty
Constant	**6.80**	**2.73**	0.51	0.48
	(0.07)	(0.17)	(0.47)	(0.43)
$x_{1,t}$	0.02	0.02	−0.00	0.00
	(0.01)	(0.02)	(0.05)	(0.05)
$x_{2,t}$	**−0.63**	−0.14	−0.01	−0.09
	(0.03)	(0.09)	(0.24)	(0.21)
$x_{3,t}$	**0.98**	0.36	−0.00	0.14
	(0.12)	(0.31)	(0.85)	(0.77)
$x_{4,t}$	**−0.81**	−0.71	0.06	0.13
	(0.30)	(0.78)	(2.11)	(1.91)
R^2	0.70	0.58	0.05	0.66

Table provides coefficient estimates and standard errors (in parentheses) from the regression $z_t = \alpha_0 + \alpha_1 x_{1,t} + \alpha_2 x_{2,t} + \alpha_3 x_{3,t} + \alpha_4 x_{4,t} + u_t$, where z_t denotes one of the variables in the column headings. Bold figures indicate significance at the 5% level.

variables explain a greater part of the variation in term premia; and several of the variables have statistically insignificant coefficients from models other than the benchmark model.

A final (and also informal) way of evaluating the plausibility of term premium estimates is to consider what they imply for the path of the expected short-term interest rate. Fig. 7 shows the average expected short-term interest rate over a ten-year horizon from the four models. Over much of the sample these are broadly similar (unsurprisingly, given the similar movements in term premia). Towards the end of the sample, however, the extreme persistence implied by the bias-corrected and random walk level models implies that the short-term interest rate was expected to remain very close to zero – and occasionally even slightly below zero – for a very protracted period (Bank Rate, the UK short-term policy interest rate, was 0.5% over this period). While we cannot of course

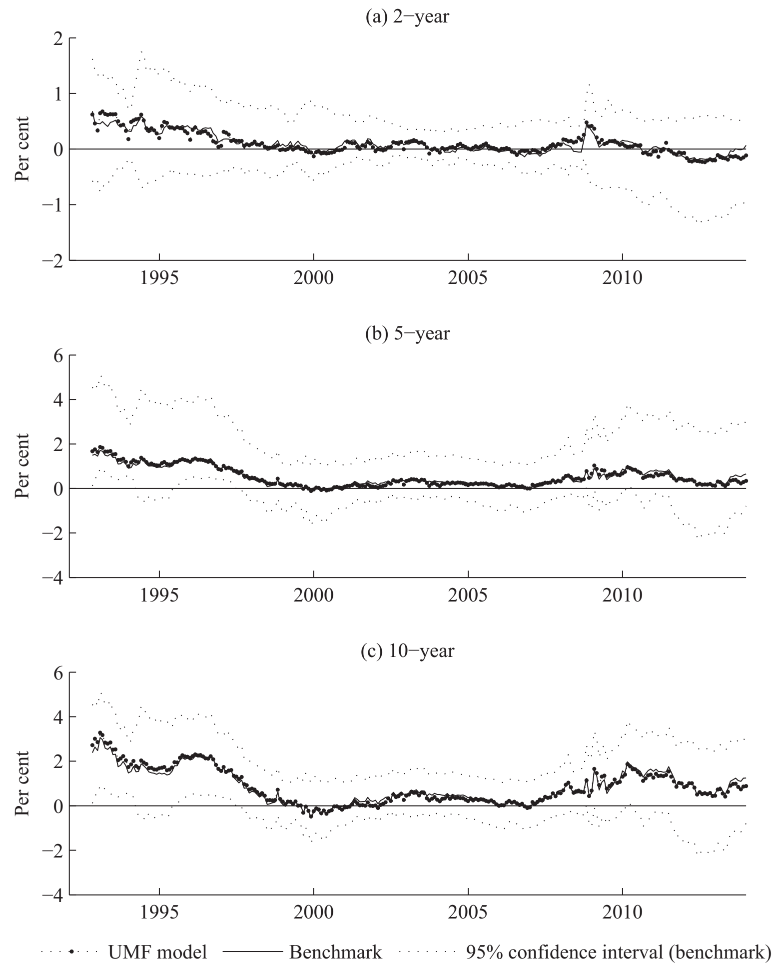

Fig. 8. Term premium estimates from benchmark model and unspanned macro factor (UMF) model.

rule out that these truly were investors' expectations, our prior is that this would be extremely unlikely, which would lead us to place less weight on these models.

5. Further robustness checks

5.1. Robustness to the inclusion of unspanned macro factors

The fact that estimated term premia are countercyclical and related to the uncertainty about inflation motivates an exploration of whether the inclusion of macroeconomic variables as factors within the model affects term premium estimates. The inclusion of macroeconomic variables in ATSMs has become increasingly popular in recent years, following Ang and Piazzesi (2003) among others.[13] More recently, Joslin et al. (2014) find evidence that macroeconomic variables are 'unspanned' by the yield curve, yet contain information about future excess returns on bonds beyond that which is contained in bond yields – and are therefore important variables for the time series dynamics of yield curve factors.

To check the robustness of our results to the inclusion of additional macroeconomic variables as factors, we consider a model that includes the four macroeconomic variables considered previously (unemployment, inflation expectations and the dispersion of GDP and inflation expectations) as unspanned factors. Following Joslin et al. (2014), evidence that these variables are unspanned by UK bond yields is provided by regressing those variables on the first four principal components of yields (i.e. the spanned yield curve factors):

$$z_t = \alpha_0 + \alpha_1 x_{1,t} + \alpha_2 x_{2,t} + \alpha_3 x_{3,t} + \alpha_4 x_{4,t} + u_t \qquad (24)$$

where z_t denotes one of the considered macroeconomic variables. Results are reported in Table 4. The yield curve factors can explain a little more than half of the variation in the unemployment rate, inflation expectations and the inflation dispersion measure; and much less for the GDP dispersion measure, with an R^2 of 0.05. Consistent with the findings of Joslin et al. (2014) for the US, therefore the macroeconomic variables are not (linearly) spanned by the yield curve.

In our 'unspanned macro factor' model, the vector of pricing factors can be written as

$$\mathbf{x}_t = [\mathbf{x}_t^{s\prime} \quad \mathbf{x}_t^{u\prime}]',$$

where $\mathbf{x}_t^s$ is a vector of the four spanned factors and $\mathbf{x}_t^u$ contains the four unspanned macro factors. The requirement that yields do not load on the unspanned factors contemporaneously requires that:

$$\delta_1 = \left[\delta'_{1,s} \quad \mathbf{0}'_{(4\times1)}\right]',$$

$$\Phi^{\mathbb{Q}} = \Phi - \lambda_1 = \begin{bmatrix} \Phi^{\mathbb{Q}}_{ss} & \mathbf{0}_{4\times4} \\ \Phi^{\mathbb{Q}}_{us} & \Phi^{\mathbb{Q}}_{uu} \end{bmatrix}.$$

[13] Lildholdt et al. (2007) and Kaminska (2013) estimate macro-factor ATSMs for the UK.

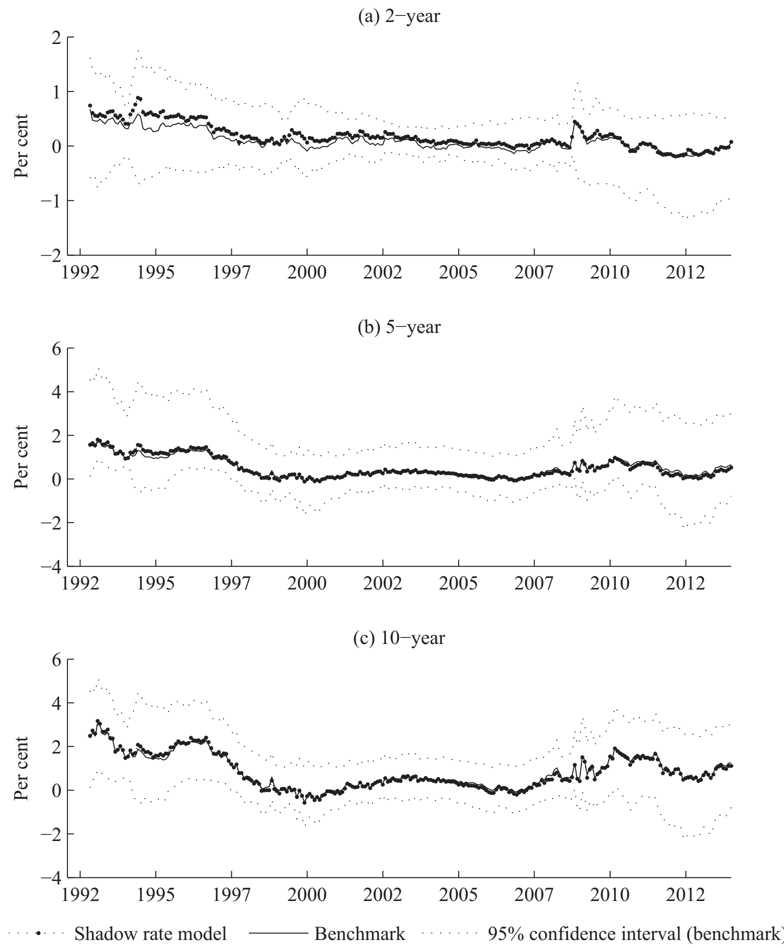

Fig. 9. Term premia from benchmark model and UK shadow rate model of Andreasen and Meldrum (2015).

We also follow the convention of setting $\mathbf{\Phi}_{us}^{\mathbb{Q}} = \mathbf{\Phi}_{us}$ and $\mathbf{\Phi}_{uu}^{\mathbb{Q}} = \mathbf{\Phi}_{uu}$, since these parameters are not separately identified.[14] Fig. 8 shows estimates of term premia from this model. These are practically identical to those from the benchmark model, so we do not consider the issue of unspanned macroeconomic factors further.

5.2. Robustness to the zero lower bound

One drawback of Gaussian ATSMs is that they do not impose the zero lower bound on nominal interest rates. If bond yields remain well above zero, this is likely to be of trivial concern, since the model-implied probability of negative rates will be small. As a number of other studies have documented, however, as nominal interest rates fall close to zero, as in recent years, Gaussian ATSMs can imply a substantial probability of negative rates (e.g. Andreasen and Meldrum, 2013; Bauer and Rudebusch, 2014). A number of recent studies have therefore used alternative no-arbitrage models of yields, such as the shadow rate framework proposed by Black (1995). In multi-factor shadow rate models, the law of motion for the factors (1) and specification of the price of risk (5) are the same as in an ATSM. However, rather than being affine in the factors, the short-term interest rate is assumed to be the maximum of zero and a 'shadow rate' (s_t) that is affine in the factors and can therefore be negative:

[14] We refer the reader to ACM for details of how the estimation procedure is modified to allow for unspanned factors.

$$r_t = \max\{0, s_t\}, \tag{25}$$
$$s_t = \delta_0 + \boldsymbol{\delta}_1' \mathbf{x}_t. \tag{26}$$

To explore the robustness of term premium estimates to the zero lower bound, we compare estimates of term premia from our benchmark model with the estimates from Andreasen and Meldrum (2015b), who estimate a four-factor shadow rate model for the UK using the 'sequential regression' method of Andreasen and Christensen (2015). These estimates are shown in Fig. 9. For the five- and ten-year yields, estimates of term premia are nearly identical from the affine and shadow rate models, which is consistent with the findings of Kim and Priebsch (2013) for the US. There are some smaller differences at shorter maturities, although these are not of great consequence given the parameter uncertainty involved.

6. Conclusion

This paper applies the method for estimating affine term structure models recently proposed in Adrian et al. (2013) to UK nominal government bond yields. Term premia from our benchmark four-factor model display countercyclical behaviour (consistent with Bauer et al., 2012 for the US) and are positively related to uncertainty about future GDP and inflation (consistent with Wright, 2011 for a panel of countries, including the UK).

In many respects, our term premium estimates are remarkably robust. We highlight in particular that including unspanned

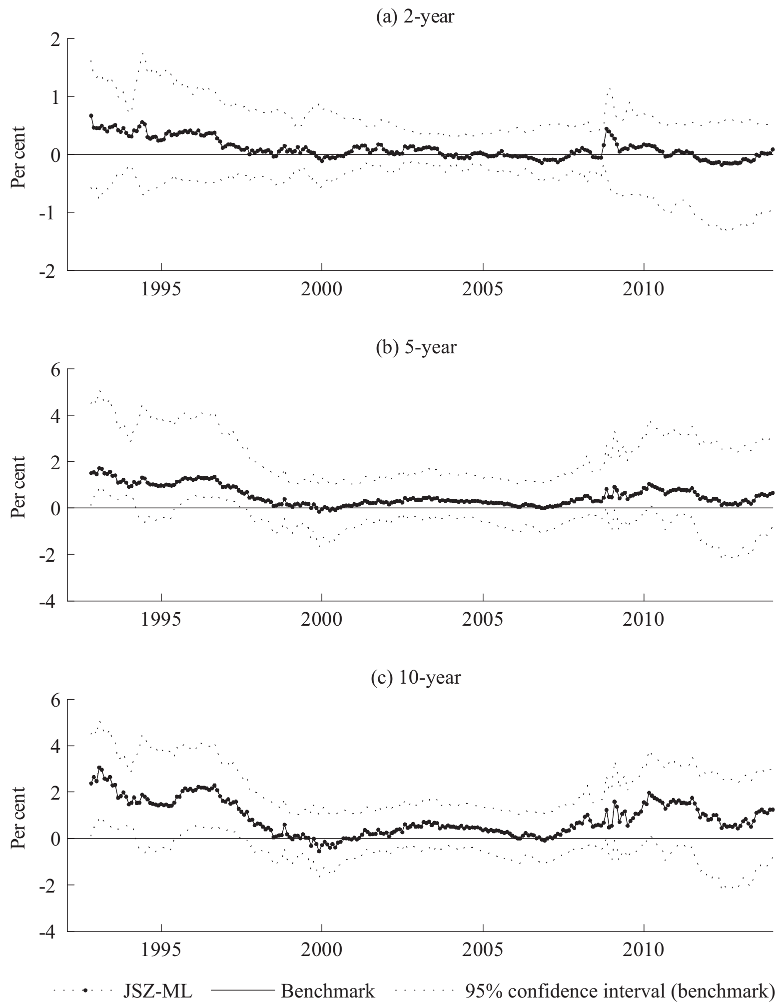

Fig. 10. Term premium estimates from benchmark model estimated using the ACM technique and using the JSZ normalisation and maximum likelihood (JSZ-ML).

Table 5
Principal component analysis of UK bond yields.

Principal component	Proportion of total variance explained (%)	Cumulative proportion (%)
1	95.0	95.0
2	4.6	99.6
3	0.4	99.9
4	0.1	100.0
5	0.0	100.0

Estimated using zero-coupon yields with maturities of 1,6,12,18,...,120 months using a sample period of October 1992–December 2013.

Table 6
Root mean squared fitting errors from models with different numbers of factors.

Maturity (months)	Yield RMSE (percentage points)			
	3 factors	4 factors	5 factors	6 factors
6	0.21	0.08	0.12	0.15
12	0.19	0.04	0.05	0.08
24	0.10	0.03	0.01	0.05
36	0.07	0.02	0.02	0.04
60	0.09	0.02	0.01	0.02
84	0.07	0.02	0.01	0.02
120	0.07	0.03	0.02	0.01

In-sample root mean squared fitting errors (RMSE) for yields of selected maturities from models with different numbers of factors. All models are estimated using a sample period of October 1992–December 2013.

Table 7
Performance against LPY(i) test by models with different numbers of factors.

Maturity (months)	Data	3 factors	4 factors	5 factors	6 factors
12	1.19 (0.32)	0.87	1.15	1.11	1.09
24	0.80 (0.50)	0.68	0.83	0.79	0.79
36	0.56 (0.60)	0.52	0.64	0.62	0.61
60	0.22 (0.75)	0.35	0.40	0.39	0.33
84	−0.10 (0.89)	0.29	0.19	0.19	0.08
120	−0.44 (1.07)	0.25	−0.10	−0.12	−0.14

Estimated slope coefficients $(\widehat{\phi}^{(n)})$ from the regression $y_{t+1}^{(n-1)} - y_t^{(n)} = \alpha^{(n)} + \phi^{(n)}(y_t^{(n)} - r_t)/(n-1) + \zeta_t^{(n)}$. Standard errors for the data are shown in parentheses. Model-implied slope coefficients are estimated using a simulated data set with 100,000 periods using the point estimates of the model parameters. All models are estimated using a sample period of October 1992–December 2013.

macroeconomic variables as factors – as proposed by Joslin et al. (2014) – or imposing a zero lower bound on nominal interest rates using the shadow rate framework proposed by Black (1995) makes little difference to estimated premia. On the other hand, there is substantial model and parameter uncertainty associated with

Table 8
Performance against LPY(ii) test by models with different numbers of factors.

Maturity (months)	Data	3 factors	4 factors	5 factors	6 factors
12	1	1.11	0.97	1.01	0.96
		(0.33)	(0.35)	(0.34)	(0.34)
24	1	1.07	0.99	1.01	1.00
		(0.51)	(0.54)	(0.53)	(0.54)
36	1	1.06	1.01	1.01	1.00
		(0.61)	(0.63)	(0.62)	(0.62)
60	1	1.00	1.02	0.99	1.00
		(0.77)	(0.74)	(0.74)	(0.73)
84	1	0.85	1.00	0.97	1.00
		(0.87)	(0.81)	(0.81)	(0.81)
120	1	0.65	1.02	1.04	1.00
		(0.93)	(0.90)	(0.89)	(0.88)

Estimated slope coefficients ($\widehat{\phi}^{*(n)}$) from the regression $y_{t+1}^{(n-1)} - y_t^{(n)} - e_t^{(n)}/(n-1) = \alpha^{*(n)} + \phi^{*(n)}(y_t^{(n)} - r_t)/(n-1) + \zeta_t^{*(n)}$. Standard errors are shown in parentheses. All models are estimated using a sample period of October 1992–December 2013.

pinning down the time-series dynamics of the pricing factors driving yields. While the broad dynamics of premia are robust across a range of reasonable specifications, at times when interest rates are a long way from their average levels – such as during the period of low rates at the end of our sample – the model uncertainty is greater. In particular, we highlight that using the bias correction technique of Bauer and Rudebusch (2014) results in term premia that are higher than from the benchmark model towards the end of the sample, whereas including surveys of interest rate expectations in the estimation of the model (as proposed by Kim and Orphanides, 2012) results in estimates of term premia that are lower. In both cases, however, we believe that there are reasons to favour the interpretation from the benchmark model: most notably, that the bias-corrected and survey models perform poorly against the LPY specification tests of the Dai and Singleton (2002).

Appendix A. Robustness of the estimation technique

Gaussian ATSMs have more typically been estimated using maximum likelihood techniques. These exploit distributional assumptions and the linear state-space representation of the model to estimate the parameters of the model. Maximum likelihood could be considered a natural way to estimate these models given they provide a complete characterization of the joint distribution of yields. However, in practice the large number of parameters to be estimated combined with a likelihood function that has many global optima and undefined regions makes convergence to a global maximum computationally challenging, which ACM's multi-step procedure avoids at the cost of not imposing no-arbitrage in the estimation.

To illustrate the robustness of the ACM estimation technique for UK term premia, we compare the term premium estimates from our benchmark model with those obtained using maximum

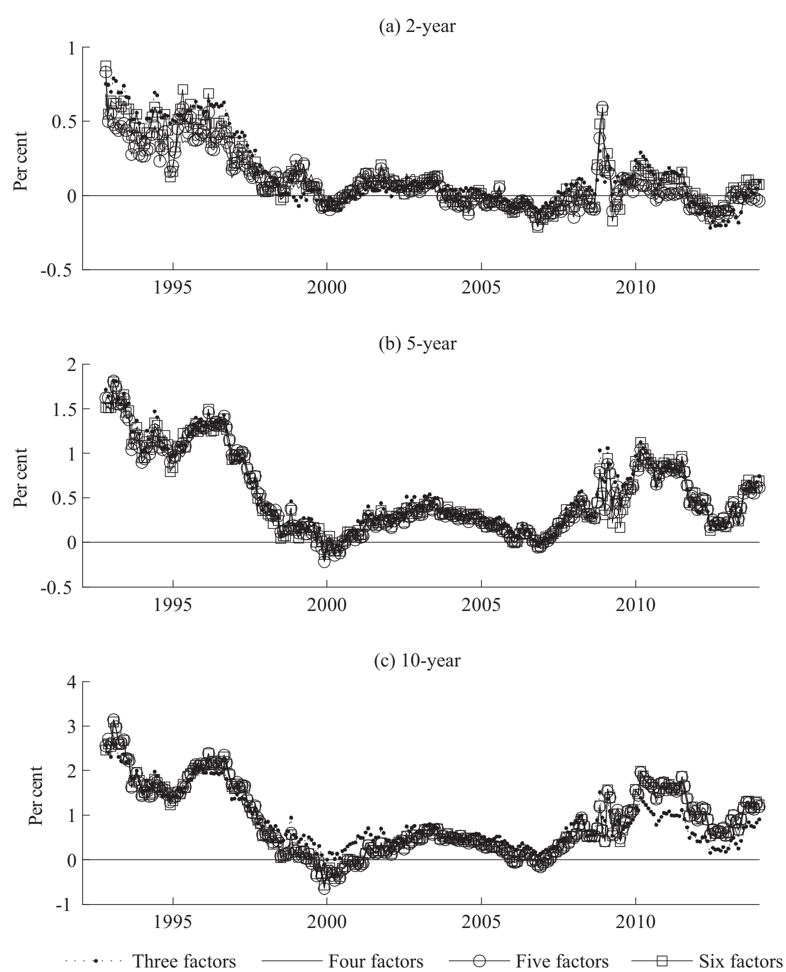

Fig. 11. Term premium estimates from models with different numbers of factors.

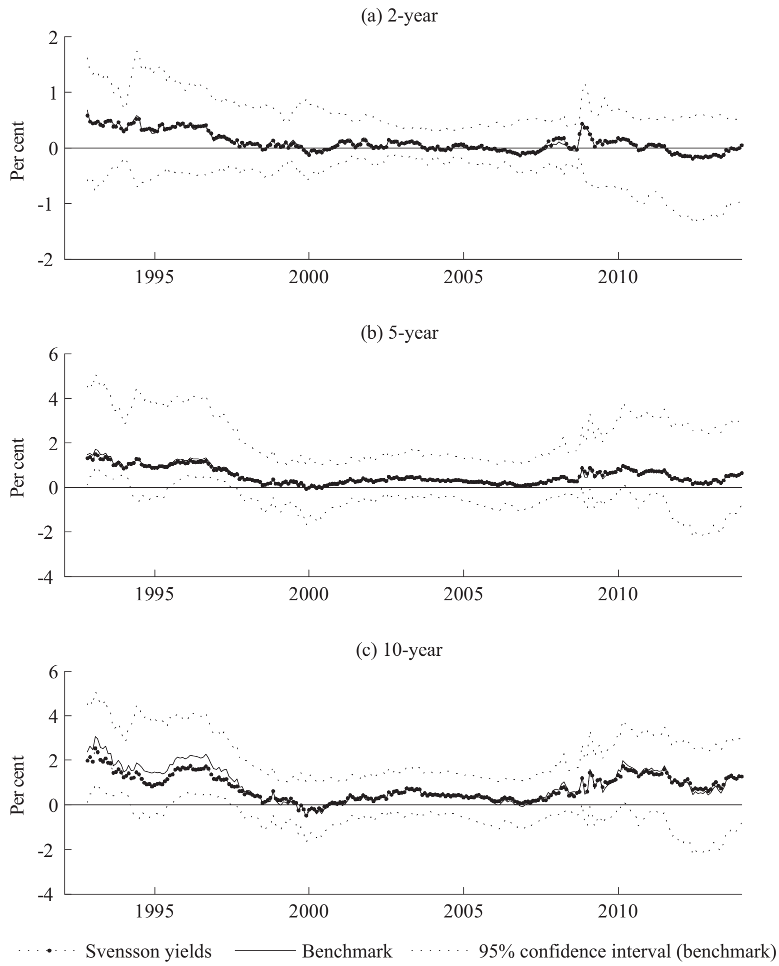

Fig. 12. Term premium estimates from benchmark model estimated using the Anderson and Sleath (2001) data set and using zero-coupon yields constructed using the Svensson (1994) parametric functional form.

likelihood imposing the no-arbitrage restrictions, again using the first four principal components as observed pricing factors. We apply the normalization by Joslin et al. (2011) (JSZ), which re-parameterizes the model in terms of $\{\boldsymbol{\mu}, \boldsymbol{\Phi}, \boldsymbol{\Sigma}, r^{\mathbb{Q}}_{\infty}, \boldsymbol{\lambda}^{\mathbb{Q}}\}$, where $r^{\mathbb{Q}}_{\infty}$ is the unconditional mean of the short rate and $\boldsymbol{\lambda}^{\mathbb{Q}}$ is a vector of the eigenvalues of $\boldsymbol{\Phi} - \lambda_1$. The estimation of the model involves the following steps:

1. Estimate (1) by OLS to obtain estimates $\widehat{\boldsymbol{\Phi}}$ and $\widehat{\boldsymbol{\Sigma}}$. We again set $\boldsymbol{\mu} = \mathbf{0}$.
2. Conditional on the parameter estimates at the first step, estimate $r^{\mathbb{Q}}_{\infty}$ and $\boldsymbol{\lambda}^{\mathbb{Q}}$ by maximum likelihood. We fit the model to yields with maturities of 1, 3, 6, 12, 24, 36, 60, 84 and 120 months, assuming that all yields are measured with error:

$$y_t^{(n)} = -\frac{1}{n}\left(A_n + \mathbf{B}_n'\mathbf{x}_t\right) + w_t^{(n)}, \tag{27}$$

where $w_t^{(n)} \sim \mathcal{N}(0, \sigma_w^2)$.

Fig. 10 compares term premium estimates from our benchmark model with those obtained from the model estimated using this alternative technique. The two estimates are almost identical, which suggests that not imposing the no-arbitrage restrictions in estimation has negligible impact on the results.

Table 9
Model selection criteria for models with different numbers of lags (bold numbers indicate the preferred model according to the relevant criterion).

Number of lags	AIC	SIC
1	0.707	**0.934**
2	0.589	1.044
3	**0.466**	1.148
4	0.494	1.403
5	0.506	1.643
6	0.540	1.904
7	0.559	2.150
8	0.496	2.314

Table provides Schwarz Information Criterion (SIC) and Akaike Information Criterion (AIC) for VAR models of the form $x_t = \Phi_1 x_{t-1} + \cdots + \Phi_p x_{t-p} + v_{t+1}$ with different numbers of lags (p). All models include the first four principal components of yields and are estimated for the period October 1992–December 2013.

Appendix B. How many pricing factors are required to model UK yields?

The standard approach when modelling the term structure of US nominal interest rates has been to assume three pricing factors, since three principal components are sufficient to explain the large majority of the cross-sectional variation in yields (Litterman and Scheinkman, 1991). Table 5 illustrates this for the UK. It shows the cumulative proportion of the sample variance of yields with maturities of $1, 6, 12, 18, \ldots, 120$ months for the period October 1992–December 2013 explained by the first five principal

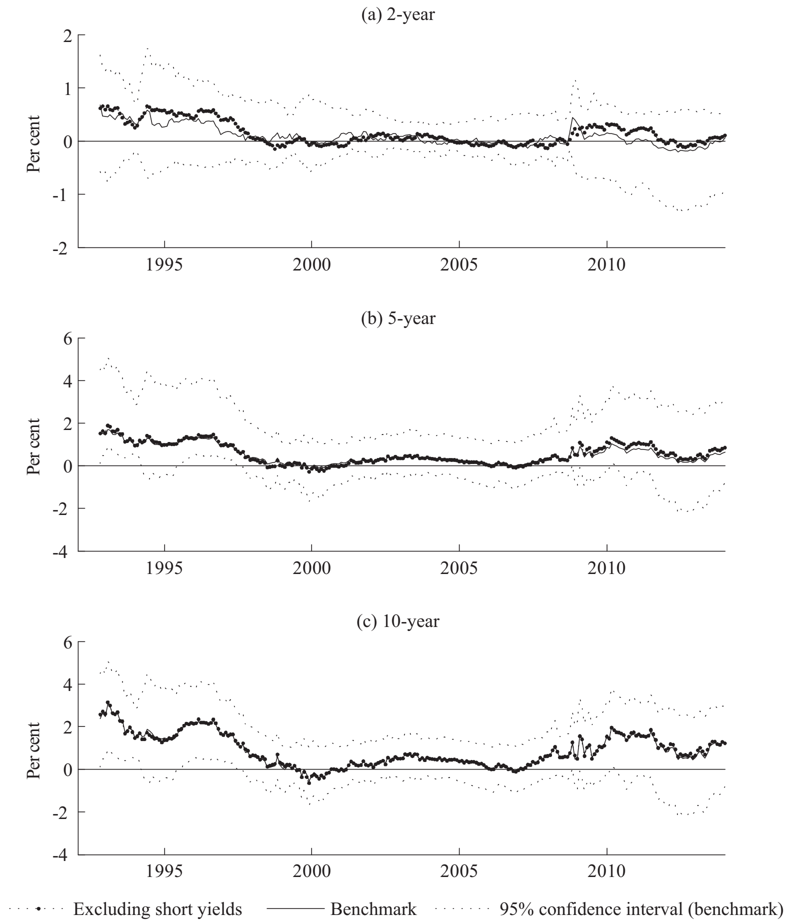

Fig. 13. Term premium estimates from benchmark model and an alternative model in which the pricing factors are constructed without using yields with less than 12 months to maturity.

components of those yields. The first principal component explains about 95% of the sample variation. Adding a second principal component takes the cumulative proportion explained to over 99% and a third to 99.9%.

ACM propose that one way for testing for redundant factors is to use a Wald test of whether factor loadings are zero. Unreported results show that we cannot reject a six-factor specification (i.e. more than the maximum of five factors ACM consider when modelling the US term structure) on this basis. While it may not be possible to reject a six-factor model, however, the marginal improvement in the in-sample fit to bond yields gained from using such a large number of factors is economically trivial. Table 6 shows annualised percentage point root mean squared error statistics for bond yields at maturities of 6, 12, 24, 36, 60, 84 and 120 months. In a three-factor model, the RMSE of pricing errors is more than 7 basis points for all the considered maturities and is over 20 basis points for the six-month yield. In a four-factor model, the RMSE is reduced below 10 basis points for all maturities. Adding further factors actually results in *larger* pricing errors at short maturities (recall that using the ACM estimation technique the model is not fitted directly to yields but to excess returns, so an increase in the number of factors does not necessarily imply a better fit to yields).

Moving beyond four factors also does not significantly affect the ability of the model to match the Dai and Singleton (2002) LPY tests explained in the main text. Tables 7 and 8 report results from these two tests applied to models with different numbers of factors. The estimated standard errors for the coefficients obtained using the data are large, which means that it is difficult to distinguish between the models conclusively, but looking at the pattern across maturities, the three-factor model struggles relative to models with four or more factors to match the extent to which $\phi^{(n)}$ decreases with maturity in the data. The LPY(ii) test suggests a similar conclusion: in the risk-adjusted regressions, the estimated slope coefficients are generally very close to the desired value of one, with the possible exception of long-maturity yields in the three-factor model. While there is again considerable estimation uncertainty involved with the tests, the general pattern is that values of $\phi^{*(n)}$ from the three-factor model fall further below one as maturity increases, unlike for models with four or more factors.

The fact that models with four or more factors perform very similarly against the LPY(i) and (ii) tests suggests that they should deliver very similar model-implied term premia. Fig. 11 confirms this, showing that estimates of term premia in the two-, five- and ten-year bond yields from these models are very similar. There are some modest differences at longer maturities if we restrict the model to three pricing factors. Taken together, the evidence in this section (yield fitting errors, LPY tests and estimates of term premia) suggests that there is a reasonable case for adopting more than three pricing factors in the UK data but that the advantages of moving beyond four factors are small at best.

Appendix C. Robustness of the data set

A further potential concern is that – in common with many previous studies of dynamic term structure models – our models are estimated using zero-coupon yields that are themselves estimated

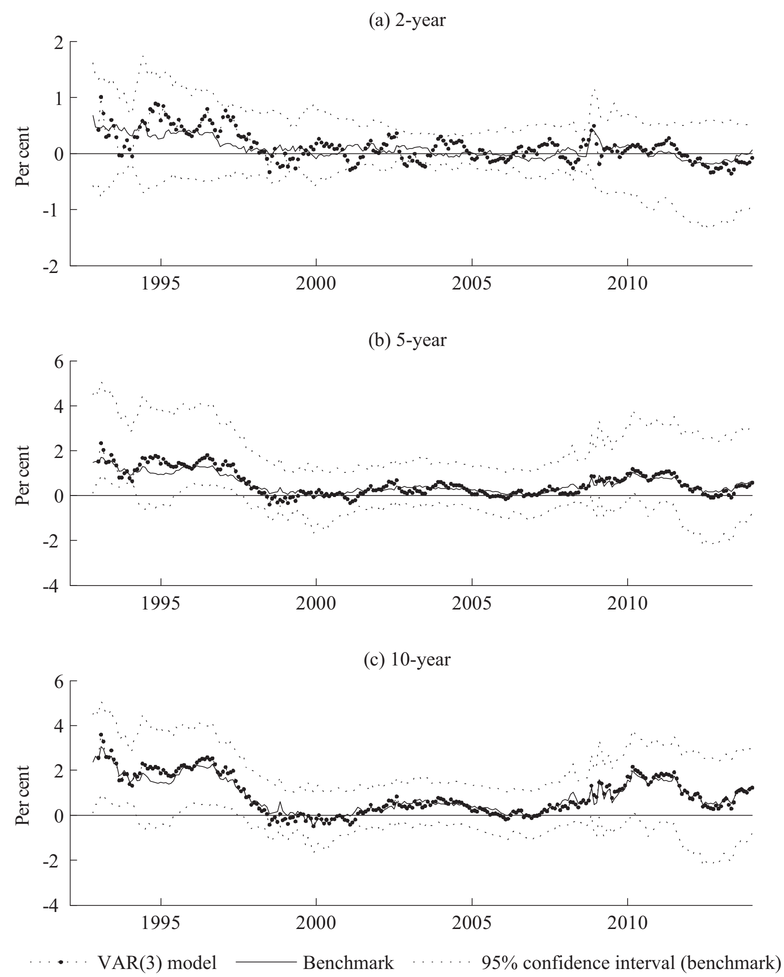

Fig. 14. Term premium estimates from benchmark model and an alternative model in which the pricing factors follow a VAR(3).

from the prices of coupon-bearing bonds. These techniques typically involve fitting a smoothed curve to the term structure of yields, forward rates or the discount function that minimizes fitting errors to observed bond prices or yields.[15] In common with many of these techniques, the smoothed cubic spline of Anderson and Sleath (2001) involves some loss of information in the underlying bond prices. This will in turn affect the pricing factors, which are constructed as principal components of estimated zero-coupon yields.

In principle, one could fit a dynamic no-arbitrage term structure model directly to the observed prices of coupon-bearing bonds, although the non-linear mapping between pricing factors and prices would substantially complicate the estimation. We can, however, evaluate whether there are any particular concerns involved with the non-parametric method of Anderson and Sleath (2001) by comparing term premium from our benchmark model with those obtained using zero-coupon yields obtained estimated using the parametric technique of Svensson (1994). Fig. 12 shows that the estimates are almost identical.

Finally, our benchmark model involves the mixing of data from different sources, including Bank Rate as a proxy for the one-month rate and a partly interpolated series for the six-month rate. This will affect the covariance matrix of yields and therefore the principal components used as pricing factors. Unreported results show that omitting the short yields from the dataset results in inferior performance of the models against the LPY tests, so we prefer to incorporate them in our benchmark data set. But to illustrate the robustness of our specification, Fig. 13 demonstrates that estimates of term premia from a model in which the one- and six-month yields are omitted from the set of yields used to construct the factors are practically identical to those from our benchmark model.

Appendix D. Robustness to alternative lag specifications

Our VAR(1) specification for the time-series dynamics of the pricing factors (1) is standard in the literature on dynamic term structure models. Standard specification tests do not provide a clear answer to the question of the appropriate number of lags, however. For example, Table 9 shows that the Schwarz Information Criterion (SIC) favours a specification with a single lag, whereas the Akaike Information Criterion (AIC) favours a specification with three lags. But the difference this makes for estimates of term premia is fairly small (Fig. 14). The differences are a little more substantial at short maturities, where term premia from a VAR(3) model are more volatile than from the benchmark model, but still well within the 95% confidence interval in almost all time periods. At long maturities the difference increasing the number of lags makes to estimates of term premia is negligible.

[15] Bank for International Settlements (2005) provide an overview of several popular yield curve estimation techniques

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