Manufacturing & Service Operations Management

Publication details, including instructions for authors and subscription information:
http://pubsonline.informs.org

Dynamic Pricing of Substitutable Products in the Presence of Capacity Flexibility

Oben Ceryan, Ozge Sahin, Izak Duenyas,

MANUFACTURING & SERVICE OPERATIONS MANAGEMENT
Vol. 15, No. 1, Winter 2013, pp. 86–101
ISSN 1523-4614 (print) | ISSN 1526-5498 (online)
http://dx.doi.org/10.1287/msom.1120.0404

Dynamic Pricing of Substitutable Products in the Presence of Capacity Flexibility

Oben Ceryan
Stephen M. Ross School of Business, University of Michigan, Ann Arbor, Michigan 48109, oceryan@umich.edu

Ozge Sahin
The Johns Hopkins Carey Business School, Baltimore, Maryland 21202, ozge.sahin@jhu.edu

Izak Duenyas
Stephen M. Ross School of Business, University of Michigan, Ann Arbor, Michigan 48109, duenyas@umich.edu

Firms that offer multiple products are often susceptible to periods of inventory mismatches where one product may face shortages while the other has excess inventories. In this paper, we study a joint implementation of price- and capacity-based substitution mechanisms to alleviate the level of such inventory disparities. We consider a firm producing substitutable products via a capacity portfolio consisting of both product-dedicated and flexible resources and characterize the structure of the optimal production and pricing decisions. We then explore how changes in various problem parameters affect the optimal policy structure. We show that the availability of a flexible resource helps maintain stable price differences across products over time even though the price of each product may fluctuate over time. This result has favorable ramifications from a marketing standpoint because it suggests that even when a firm applies a dynamic pricing strategy, it may still establish consistent price positioning among multiple products if it can employ a flexible replenishment resource. We provide numerical examples for the price stabilization effect and discuss extensions of our results to a more general multiple product setting.

Key words: pricing and revenue management; dynamic pricing; capacity flexibility; inventory control; substitutable products
History: Received: April 6, 2011; accepted: April 28, 2012. Published online in *Articles in Advance* September 4, 2012.

1. Introduction

Virtually all manufacturing and service industries are susceptible to periods of supply and demand mismatches. Because of capacity limitations and demand uncertainties, firms producing multiple products can frequently encounter instances where one of their products faces shortages while the other has excess inventories. To alleviate the level of such inventory mismatches, firms can utilize several tools to alter supply or demand. Our focus in this paper is a joint analysis of two of these mechanisms, namely, dynamic pricing and capacity flexibility.

On the demand side, through price discounts and price surcharges that shift demand from one product to another as well as stimulate or reduce the overall demand, dynamic pricing is an effective tool to better match demand with supply. As an example, consider a firm offering two substitutable products during a period in which one of the products faces low demand while the other experiences high demand leading to backlogs. A reduction in the price of the product with low demand can induce more customers to prefer that product, thereby reducing its inventory while relieving the firm from excessive backlogs on the other product. Hence dynamic pricing may reduce inventories for one product while simultaneously reducing backlogs for the other. Dynamic pricing has long been used in airline revenue management where prices respond strongly to the availability of various seat classes. More recently, with the advent of e-commerce and the ability to frequently change and advertise prices, dynamic pricing has also been increasingly used in many other industries such as electronics and automobiles. As an example from the automotive industry, Copeland et al. (2005) provided empirical observations on whether vehicle prices are correlated with inventory fluctuations, and they concluded that a significant negative relationship exists between inventories and prices.

On the supply side, flexible manufacturing systems can also be utilized to align supply with demand. In the last decade, firms in many industries have invested in flexible manufacturing systems that enable the production of multiple variations of products in the same factory. This enables the firm to easily alter its product mix if demand for one product increases while demand for another decreases. Flexible systems may be considered to have advantages over dynamic pricing as they operate to match demand and supply without sacrificing product revenues (increasing the price to lower backlogs by suppressing sales can lead to a reduction of overall revenue for the firm). Although flexible systems are beneficial if product demands are negatively correlated, the ability to change production from one item to another has limited benefits if both products have a surge in their demand or if both products experience low demand.

It is interesting that many firms use both flexible manufacturing and dynamic pricing simultaneously. For example, in the auto industry, demand for SUVs and sedans fluctuate depending on gas prices and other economic factors. In a recent paper, Moreno and Terwiesch (2010) empirically investigated how different auto manufacturers reacted to shifts in demand. They identified that companies (such as Honda) that invested in more flexible plants were able to decrease production levels of SUVs when demand for SUVs dropped, and increase production of other vehicles made in the same assembly line. On the other hand, companies that had not invested in flexible plants during the same period (such as Ford) reacted to decreases in SUV demand by significantly increasing incentives on their SUVs (i.e., reducing prices; for example, Moreno and Terwiesch (2010) estimated that deploying flexibility enables manufacturers to reduce the use of incentives typically by between \$200 and \$700 per vehicle). Similarly, during the "Great Recession" of 2009, demand for larger sized (42 inches and above) liquid crystal display (LCD) TVs slowed down in the United States as consumers trimmed their budgets and preferred smaller sized and lower priced models, according to the market research firm DisplaySearch (2009). Thus, an LCD TV manufacturer that produces multiple models of different sizes had the following choices to respond to this change in demand: (1) it could decrease the price of larger sized models to stimulate more demand, (2) it could switch more of its production to smaller sized models (e.g., 32, 37, and 40 inch), or it could choose a combination of the two policies.

Moreno and Terwiesch (2010) demonstrated that even firms that deploy flexibility still offer incentives (i.e., adjust prices) when demand fluctuates, but do so less than firms with less flexible production capabilities. This raises the following interesting questions, which constitute the main objective of our study: how does a firm offering multiple substitutable products decide (i) on the price charged for each product, (ii) how much of each product it should produce, and (iii) how the flexible resource should be allocated among products in a given period? We are also interested in understanding how operational flexibility influences a firm's pricing strategy.

Our first contribution in this paper is to provide a full characterization of joint optimal production and pricing decisions for substitutable products with limited production capacities in the form of product-dedicated and flexible resources. We show that the optimal production policy can be characterized by modified base-stock levels. Regarding the optimal pricing policy, we find that the optimal price policy consists of a list price region for each product in addition to regions where price markups and markdowns are given depending on product inventory levels. Interestingly, when both products are understocked and share the flexible capacity, the optimal pricing scheme maintains a constant price difference between products. Hence, our second major finding is that the availability of a flexible resource helps maintain stable price differences across products over time even though the price of each product may fluctuate over time. This result has favorable ramifications from a marketing standpoint because it suggests that even when a firm applies a dynamic pricing strategy, it may still establish consistent price positioning among multiple products if it can employ a flexible replenishment resource. Several studies in marketing and economics show that firms often use price to signal quality differences among products, and that consumers also use price as an indicator of quality or benefits (Monroe 1973) especially when they do not have sufficient knowledge to judge quality (Rao and Monroe 1989). Therefore, keeping a consistent price gap between different quality products is very important for retaining product positioning and brand equity. Indeed, Mela et al. (1997, 1998), and Jedidi et al. (1999) found that deep and frequent price promotions may have long-term negative effects on brand equity. Thus, we find that flexible production capacity may have benefits that go beyond operational cost savings and can help firms retain their product positioning and brand equity by maintaining consistent price differences and resorting to less frequent price promotions. Our finding is consistent with the empirical findings of Moreno and Terwiesch (2010) that flexibility provides price stabilization.

2. Literature Review

There exists a vast literature on dynamic pricing. We will restrict our attention to only those studying joint pricing and replenishment decisions. Extensive reviews on the interplay of pricing and production decisions have been provided by Elmaghraby and Keskinocak (2002), Bitran and Caldentey (2003), Chan et al. (2004), and, more recently, Chen and Simchi-Levi (2012). Single product settings have been the focus of much of the earlier work in this area. Whitin (1955) was among the first to consider joint pricing and inventory control for single period problems under both deterministic and stochastic demand models. For a finite horizon, periodic review model, Federgruen and Heching (1999) showed that the optimal policy is of a base-stock, list price type. When it is optimal to order, the inventory is brought to a base-stock level and a list price is charged. For inventory levels where no ordering takes place, the optimal policy assigns a discounted price. In a subsequent work, Li and Zheng (2006) extended the setting studied by Federgruen and Heching (1999) to include yield uncertainty for replenishments. Chen and Simchi-Levi (2004) further extended the results of Federgruen and Heching (1999) to include fixed ordering costs and showed that a stationary (s, S, p) policy is optimal for both the discounted and average profit models with general demand functions. In such a policy, the period inventory is managed based on the classical (s, S) policy, and price is determined based on the inventory position at the beginning of each period.

Recently, settings consisting of multiple substitutable products have received more attention. Aydin and Porteus (2008) studied a single period inventory and pricing problem for an assortment consisting of multiple products. They investigated various demand models and showed that a price vector accompanied by corresponding inventory stocking levels constitutes the unique solution to the profit maximization problem, although the profit function may not necessarily be quasi-concave in product prices. Tang and Yin (2007) examined a retailer's pricing and quantity decisions in a single period setting for two substitutable products with deterministic demand that share a common resource limiting the total order quantity. Zhu and Thonemann (2009) studied a periodic review, infinite capacity, joint production and pricing problem with two substitutable products assuming a linear additive demand model. They showed that production for each product follows a base-stock policy that is nonincreasing in the inventory level of the other product. They also showed that the optimal pricing decisions do not necessarily exhibit monotonicities with respect to inventory levels except for settings where the demand processes for both products are influenced by identical cross-price elasticities. They found that a list price is optimal whenever an order is placed for a product, regardless of the inventory level of the other product, and a discount is given for any product that is not ordered. Ye (2008) extended their results to an assortment of more than two products and showed that under a similar linear additive demand model and identical cross-price elasticities, a base-stock, list price policy extends to an arbitrary number of products. Song and Xue (2007) also considered dynamic pricing and inventory decisions for a set of substitutable products with price-dependent random demand. They studied more general additive and multiplicative demand models for multiple products and provided characterizations for the optimal pricing and inventory policy structure as well as algorithms to compute the optimal policy. Aside from the single period model of Tang and Yin (2007), all of these papers assume infinite production capacity. In this paper, we consider a general capacity portfolio consisting of both finite dedicated resources, and more importantly, a shared finite flexible resource. If production capacity is limited, charging list prices for a product whenever an order is placed for that product is no longer optimal. Intuitively, one would expect to charge a higher price when the desired production quantity is restricted by a limited capacity. We show that this expectation is indeed true. Consequently, as opposed to the results for the infinite capacity setting, whenever an order is placed for a product, its price is no longer independent of the inventory level of the other product. In addition, our main results highlight the impact of the flexible resource on the firm's optimal pricing policy.

In the area of flexible capacity allocation, Evans (1967) studied a periodic review problem with two products produced by a single shared resource and characterized the optimal allocation policy for the flexible resource. DeCroix and Arreola-Risa (1998) studied extensions to multiple products. For an infinite horizon problem with homogeneous products where all products have identical cost parameters and resource requirements, they derived structural results regarding the optimal allocation of the flexible capacity. Bish et al. (2005) studied the impact of flexibility and various capacity allocation policies on supply chain performance with a focus on production swings and variability. Besides these periodic review models, continuous time formulations and corresponding results can also be found in works such as those by Glasserman (1996) and Ha (1997). These papers on flexible capacity allocation treat the demand process as exogenous, whereas our focus is to simultaneously study dynamic pricing decisions that influence the demand for each product.

There have also been other studies that investigate investments in capacity flexibility in the context of substitutable products. Chod and Rudi (2005) studied the effects of resource flexibility and price setting in a single period model in which the firm first decides on the capacity investments prior to demand realizations. Following the realization of demand, capacity allocations and product pricing decisions are given.

They showed that investment in flexible capacity increases in both demand variability and correlation. Lus and Muriel (2009) analyzed the impact of product substitution on the optimal mix of dedicated and flexible capacities the firm should invest in. They compared alternative metrics of product substitutability that are commonly used in the economics literature and showed that investment in manufacturing flexibility tends to decrease as the products become more substitutable. Bish and Suwandechochai (2010) studied the flexibility investment problem by considering the postponement strategies regarding whether the quantity decisions are given before or after prices are set and demand is realized. These works complement our study in the sense that our model takes the capacity investment decisions as given and focuses on the implications of capacity flexibility on a firm's optimal dynamic pricing strategy.

3. Problem Formulation

We consider a firm that produces two substitutable products through a capacity portfolio consisting of product-dedicated and flexible resources. Prices and replenishment quantities for both products are dynamically set at the beginning of each period over a finite planning horizon of length T. At the beginning of period t, the firm reviews the current inventory levels (x_1^t, x_2^t) and decides on (i) the optimal order-up-to levels (y_1^t, y_2^t) and (ii) the prices (p_1^t, p_2^t) to charge during the period, which will influence the demands (d_1^t, d_2^t) observed within the period. We assume the demands for both products are correlated by the following linear additive price demand model:

$$\begin{aligned} d_1^t(p_1^t, p_2^t, \epsilon_1^t) &= b_1^t - a_{11}^t p_1^t - a_{12}^t p_2^t + \epsilon_1^t, \\ d_2^t(p_1^t, p_2^t, \epsilon_2^t) &= b_2^t - a_{21}^t p_1^t - a_{22}^t p_2^t + \epsilon_2^t. \end{aligned} \tag{1}$$

In (1), b_i denotes the demand intercept, whereas $a_{ii}^t > 0$ and $a_{ij}^t \leq 0$, for $i, j = \{1, 2\}$, $j \neq i$, refer to the individual and cross-price elasticities for product type i, respectively. The assumption on the signs of elasticity terms reflects the substitutable nature of the products where the demand for a product is decreasing in its own price and increasing with the price of the other product. We assume strict diagonal dominance on price sensitivities, i.e., $a_{11}^t > |a_{12}^t|$ and $a_{22}^t > |a_{21}^t|$. This implies that a price change on a product influences its demand more than the demand for the other product. Furthermore, we also assume $a_{ij}^t = a_{ji}^t$. This symmetric relationship implies settings where demands for both products can be influenced by different individual price elasticities, but they experience identical cross-price elasticities. In other words, the change of the expected demand for a product with respect to the price of the other product is equivalent for both products. This same property is also inherently present in multinomial logit (MNL)-type demand models that we describe in §5. The property is a common assumption in works analyzing multiple product settings such as in Ye (2008) and in the monotonicity results of Zhu and Thonemann (2009). This assumption enables several desired structural properties of the objective function (described further in the next section), which in turn enables us to provide a characterization of the optimal production and pricing policy structure when there is a flexible resource present in the firm's capacity portfolio. Although our results and main insights rely on the assumption of symmetric cross-price elasticities, in §5 we also numerically explore and show that the insights indeed extend to more general demand functions that violate this assumption. Finally, in (1), we let ϵ_1^t and ϵ_2^t refer to independent random variables having continuous probability distributions with zero mean and nonnegative support on the product demands.

We adapt a general capacity portfolio that allows the firm to utilize any combination of dedicated capacities $K_1, K_2 \geq 0$ for the production of each product exclusively, as well as a limited flexible resource, $K_0 \geq 0$, that can be assigned partially or entirely for the production of either product. We assume that a unit of flexible resource can be used toward producing a unit of each type of product. In each period, the optimal production quantities are bounded by the corresponding available flexible and product-dedicated capacity levels. We let c_i^t denote the unit production cost for product type i in period t and assume that this unit cost is applicable to both dedicated and flexible production systems when producing the same product. This is especially applicable when the production cost for a product consists mostly of the raw materials or when the processing costs differ across products yet remain constant across types of resources. (In §6 we consider instances where production on a flexible resource is more expensive compared to production on a dedicated resource.) At the end of period t, the firm incurs holding and backorder costs of $h_i^t(x_i^t) = h_i^{t+} x_i^{t+} + h_i^{t-} x_i^{t-}$, where $x_i^{t+} := \max(0, x_i^t)$, $x_i^{t-} := \max(0, -x_i^t)$, and h_i^{t+} and h_i^{t-} refer to holding and backorders costs per unit, respectively. In §6, we also consider the setting where the firm does not backorder any demand missed in the current period but uses a more expensive expedited delivery option for any units in shortage. To simplify the notation throughout the subsequent analysis, we suppress the superscript t on demand and cost parameters a_{ij}^t, b_i^t, c_i^t, h_i^{t+}, and h_i^{t-}.

Letting $V^t(x_1^t, x_2^t)$ denote the expected discounted profit-to-go function under the optimal policy starting at state (x_1^t, x_2^t) with t periods remaining until the end of the horizon, the problem can be expressed as a stochastic dynamic program satisfying the following recursive relation:

$$\begin{aligned} V^t(x_1^t, x_2^t) &= \max_{\substack{y_1^t, y_2^t \in \mathcal{F}(x_1^t, x_2^t) \\ p_1^t, p_2^t}} \Big\{ R(p_1^t, p_2^t) - \sum_i c_i(y_i^t - x_i^t) \\ &\quad + \mathrm{E}_{\epsilon_1^t, \epsilon_2^t}\Big(-\sum_i h_i(y_i^t - \bar{d}_i^t - \epsilon_i^t) \\ &\quad + \beta V^{t-1}(y_1^t - \bar{d}_1^t - \epsilon_1^t, y_2^t - \bar{d}_2^t - \epsilon_2^t)\Big)\Big\}, \end{aligned} \quad (2)$$

where $\mathcal{F}(x_1^t, x_2^t)$ denotes the set of admissable values for the order-up-to decisions and is defined as $\mathcal{F}(x_1^t, x_2^t) := \{(y_1^t, y_2^t) \mid x_i^t \le y_i^t \le x_i^t + K_0 + K_i \ \forall\, i = 1, 2$, and $y_1^t + y_2^t \le x_1^t + x_2^t + K_0 + K_1 + K_2\}$. The term $R(p_1^t, p_2^t) := \sum_i p_i^t \bar{d}_i^t(p_1^t, p_2^t)$ represents the expected revenue in period t, where the mean demands $\bar{d}_1^t$ and $\bar{d}_2^t$ are given by $\bar{d}_1^t(p_1^t, p_2^t) = b_1 - a_{11}p_1^t - a_{12}p_2^t$ and $\bar{d}_2^t(p_1^t, p_2^t) = b_2 - a_{21}p_1^t - a_{22}p_2^t$. We define $V^0(x_1^t, x_2^t) = 0$ to be the terminal value function and let β denote the discount factor.

We define a new set of variables, (z_1^t, z_2^t), such that $z_i^t := y_i^t - \bar{d}_i^t$. An economic interpretation of z_i^t is that it represents the target safety-stock level for product i after the current inventory position is augmented by the replenishment quantity and depleted by the expected demand for that product. Then, the dynamic programming formulation given in (2) can be rewritten as

$$\begin{aligned} V^t(x_1^t, x_2^t) &= c_1x_1^t + c_2x_2^t + \max_{\substack{z_1^t, z_2^t \in \mathcal{F}'(x_1^t, x_2^t, p_1^t, p_2^t) \\ p_1^t, p_2^t}} J^t(z_1^t, z_2^t, p_1^t, p_2^t), \end{aligned} \quad (3)$$

where

$$\begin{aligned} J^t(z_1^t, z_2^t, p_1^t, p_2^t) &= R'(p_1^t, p_2^t) - c_1z_1^t - c_2z_2^t \\ &\quad + \mathrm{E}_{\epsilon_1^t, \epsilon_2^t}\Big(-\sum_i h_i(z_i^t - \epsilon_i^t) + \beta V^{t-1}(z_1^t - \epsilon_1^t, z_2^t - \epsilon_2^t)\Big). \end{aligned} \quad (4)$$

In (3), $\mathcal{F}'(x_1^t, x_2^t, p_1^t, p_2^t)$ represents the set of admissable decisions for (z_1^t, z_2^t) with $\mathcal{F}'(x_1^t, x_2^t, p_1^t, p_2^t) := \{(z_1^t, z_2^t) \mid x_i^t \le z_i^t + b_i - a_{i1}p_1^t - a_{i2}p_2^t \le x_i^t + K_0 + K_i \ \forall\, i = 1, 2$ and $z_1^t + z_2^t + b_1 + b_2 - (a_{11} + a_{21})p_1^t - (a_{12} + a_{22})p_2^t \le x_1^t + x_2^t + K_0 + K_1 + K_2\}$. The term $R'(p_1^t, p_2^t)$ in (4) denotes a modified expected revenue function with $R'(p_1^t, p_2^t) := \sum_i (p_i^t - c_i)\bar{d}_i^t(p_1^t, p_2^t)$. In this reconstructed formulation, it can be observed that the objective function, $J^t(z_1^t, z_2^t, p_1^t, p_2^t)$, is separable in the decision variables (z_1^t, z_2^t) and (p_1^t, p_2^t). In addition, the profit-to-go function, $V^{t-1}(z_1^t - \epsilon_1^t, z_2^t - \epsilon_2^t)$, now only depends on the set of variables (z_1^t, z_2^t), which facilitates the derivation of several structural results on the value function that we require in the analysis of the optimal policy.

4. Structure of the Optimal Production and Pricing Policy

In this section, we first present the structure of the optimal production decisions, which also consists of the allocation of the flexible resource. We then present the characterization of the pricing decisions and show the impact of capacity flexibility on optimal pricing. Under the assumptions outlined in the preceding section, our first result establishes several structural properties on the objective function that are preserved throughout the planning horizon.

Lemma 1. *For all $t = 1, 2, \ldots T$,*
(a) *$J^t(z_1^t, z_2^t, p_1^t, p_2^t)$ is strictly concave,*
(b) *$J^t(z_1^t, z_2^t, p_1^t, p_2^t)$ is submodular in (z_1^t, z_2^t), and*
(c) *$J^t(z_1^t, z_2^t, p_1^t, p_2^t)$ possesses the following strict diagonal dominance property $\forall\, i, j;\ i \ne j$: $\partial^2 J^t/(\partial z_i^t \partial z_i^t) < \partial^2 J^t/(\partial z_i^t \partial z_j^t)$.*

Proof. The proof of Lemma 1 and all subsequent results are provided in the online supplement (available at http://dx.doi.org/10.1287/msom.1120.0404).

Strict concavity of the objective function $J^t(z_1^t, z_2^t, p_1^t, p_2^t)$ implies the uniqueness of an optimal solution and that the production policy is of a modified base-stock type. In addition, the submodularity and diagonal dominance properties allow for the characterization of the optimal allocation of the flexible resource and the monotonicity of the optimal production and pricing decisions with respect to starting inventory levels.

4.1. Optimal Production Policy and Resource Allocation

To convey the structure of the optimal production policy and how the flexible resource is allocated, we segment the state space into two broad regions based on the initial inventory levels of the products. The first region, denoted by Region A, corresponds to states where there remains some resource (either dedicated or flexible) that is not fully utilized. The second, denoted as Region B, corresponds to initial inventory levels for which all resources are fully utilized. The boundaries of these two regions are described by two monotone functions, $\gamma_1^t(x_2^t)$, and $\gamma_2^t(x_1^t)$, that are specified in Theorem 1. Region A is further divided into several subregions with respect to the inventory level of each product according to the following definition.

Definition 1. Consider initial inventory levels (x_1^t, x_2^t) and the functions $\gamma_1^t(x_2^t)$ and $\gamma_2^t(x_1^t)$, and define $\bar{x}_1^t$ and $\bar{x}_2^t$ such that $\bar{x}_1^t = \gamma_1^t(\bar{x}_2^t)$ and $\bar{x}_2^t = \gamma_2^t(\bar{x}_1^t)$. Furthermore, let $\hat{\gamma}_1^t(x_2^t)$ (and $\hat{\gamma}_2^t(x_1^t)$ in a similar fashion) be given by

$$\hat{\gamma}_1^t(x_2^t) := \begin{cases} \gamma_1^t(x_2^t) - K_1 & \text{for } x_2^t \le \bar{x}_2^t - K_0 - K_2, \\ \gamma_1^t(x_2^t) + \bar{x}_2^t - x_2^t - K_0 - K_1 - K_2 & \\ \qquad \text{for } \bar{x}_2^t - K_0 - K_2 < x_2^t \le \bar{x}_2^t - K_2, & \\ \gamma_1^t(x_2^t) - K_0 - K_1 & \text{for } \bar{x}_2^t - K_2 < x_2^t. \end{cases}$$

Then, product 1 (and product 2) is classified as (a) "overstocked" if $x_1^t > \gamma_1^t(x_2^t)$, (b) "moderately understocked" if $\gamma_1^t(x_2^t) \geq x_1^t > \hat{\gamma}_1^t(x_2^t)$, and (c) "critically understocked" if $\hat{\gamma}_1^t(x_2^t) \geq x_1^t$.

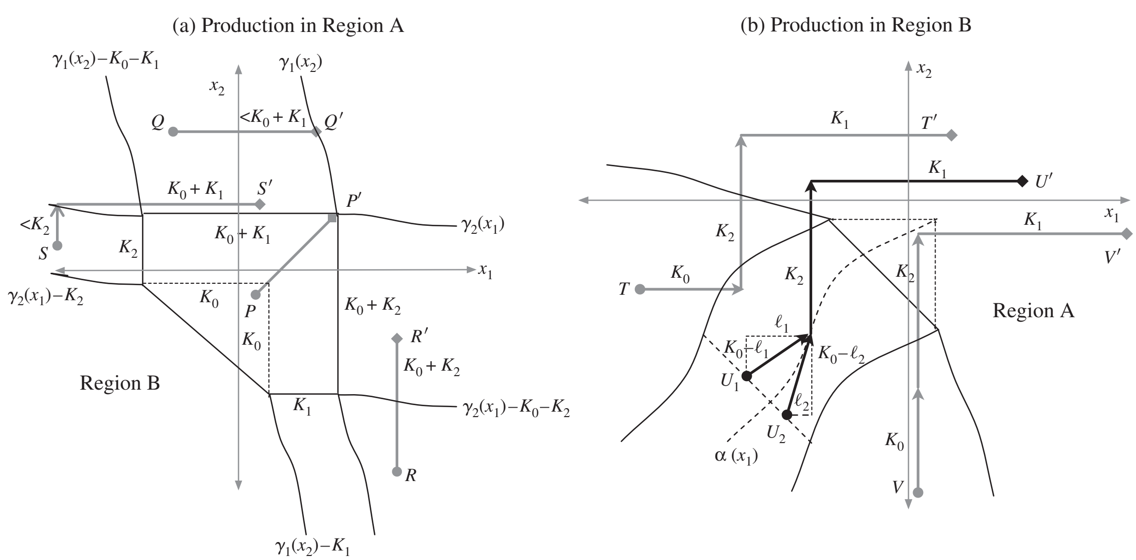
Defining a product as overstocked means the product requires no further replenishment. A moderately understocked product requires production for which the available capacity is adequate to reach the desired base-stock level, whereas a critically understocked product can not be brought to the desired base-stock level because of capacity restrictions. Region A represents all states in which at most one product is critically understocked, whereas Region B corresponds to initial inventory levels for which both products are critically understocked. The segmentation of the state space is illustrated in Figure 1 and formally derived within the proof of Theorem 1, which describes the optimal production policy.

Theorem 1. *The optimal production policy is a state-dependent modified base-stock policy characterized by three monotone functions,* $\gamma_1^t(x_2^t)$, $\gamma_2^t(x_1^t)$, *and* $\alpha^t(x_1^t)$, *such that the following hold*:

1. *In states corresponding to initial inventory levels for which at most one product is critically understocked* (*i.e., in Region A*),
   (a) *the optimal production policy for product* i ($i = 1, 2$) *is to produce up to the modified base-stock level* $\min(x_i + K_0 + K_i, \gamma_i^t(x_{3-i}^t))$, *and*
   (b) *the modified base-stock level for product* i *is nondecreasing with* x_i^t *and nonincreasing with* x_j^t, $j \neq i$.
2. *In states corresponding to initial inventory levels for which both products are critically understocked* (*i.e., in Region B*),
   (a) *the optimal production policies for product* 1 *and product* 2 *are to produce up to the modified base-stock levels* $x_1^t + K_1 + l^t(x_1^t, x_2^t)$ *and* $x_2^t + K_2 + K_0 - l^t(x_1^t, x_2^t)$, *respectively, where* $l^t(x_1^t, x_2^t)$ *denotes the amount of flexible capacity allocated to product* 1;
   (b) $l^t(x_1^t, x_2^t) = 0$ *if* $x_2^t \leq \alpha^t(x_1^t) - K_0$, *and* $l^t(x_1^t, x_2^t) = K_0$ *if* $x_2^t \geq \alpha^t(x_1^t + K_0)$, *and otherwise* $l^t(x_1^t, x_2^t)$ *satisfies* $l^t(x_1^t, x_2^t) + \alpha^t(x_1^t + l^t(x_1^t, x_2^t)) = x_2^t + K_0$, *and the modified base-stock levels for either product are a function of the starting inventory levels through their sum;*
   (c) $l^t(x_1^t, x_2^t)$ *is decreasing with* x_1^t *and increasing with* x_2^t;
   (d) *the modified base-stock levels for product* i *are nondecreasing with either product's inventory level.*

As Theorem 1 suggests, the optimal production policy has a number of properties depending on the inventory state at the beginning of a period. Figure 1(a) illustrates the optimal production policy in Region A. When both products are moderately understocked, as shown by the starting inventory level P on Figure 1(a), it is optimal to produce both products up to the point $(\bar{x}_1^t, \bar{x}_2^t)$, denoted by P', and the optimal base-stock levels in this region are independent of initial inventories. Initial inventory levels Q and R in Figure 1(a) are examples of states where one product is overstocked and the other is understocked. Starting at Q, with a base-stock level of $\gamma_1^t(x_2^t)$ for product 1 and no production for product 2, the optimal policy is to move to point Q'. Note that point Q' refers to a base-stock level for product 1 which is lower than the one suggested by P' (Theorem 1, part 1(b)). The reason is twofold. First, the overstocked product 2 results in a price decrease for that product, which in turn increases its demand and decreases the demand for product 1, which further decreases the base-stock for product 1. Second, the overstocked product 2 reduces the potential workload on the flexible resource for that product and increases

Figure 1 Optimal Production Policy



the availability of the flexible capacity for product 1 in future periods. This allows for fewer units of product 1 to be produced in the current period. An initial inventory state R shows an instance for which no production takes place for overstocked product 1, and all available capacity is used to produce a critically understocked product 2. Point S refers to a state where product 1 is critically understocked and product 2 is moderately understocked. In this case, Theorem 1 states that it is optimal to produce $K_0 + K_1$ units of product 1 and to bring the inventory of product 2 to $\gamma_2^t(x_1^t)$, as shown by point S'. Note that point S' corresponds to a base-stock level higher than the one implied by P' with similar but reverse dynamics as discussed previously.

Part 2 of Theorem 1 corresponds to the states in Region B where both products are critically understocked. As illustrated in Figure 1(b), Theorem 1, part 2, states that when the initial inventory levels for both products fall within a "band" defined by $\{(x_1^t, x_2^t)$, s.t. $(x_1^t, x_2^t) \in$ Region B, and $\alpha(x_1^t + K_0) > x_2^t > \alpha(x_1^t) - K_0\}$, the optimal policy allocates $l^t(x_1^t, x_2^t) > 0$ units of the flexible resource to product 1 and the remaining $K_0 - l^t(x_1^t, x_2^t) > 0$ units to product 2. Moreover, for any two inventory states corresponding to the same total inventory (points U_1 and U_2 in Figure 1(b)), the intermediate inventory levels after the flexible resource is utilized are identical. From this point on, additional units of each product are produced to the full extent of their dedicated resources. For initial inventory levels that fall outside this band, the flexible resource is fully assigned to the product that experiences the most severe shortage. For example, in Figure 1(b), points T and V refer to instances where all flexible capacity is used toward product 1 and product 2, respectively. Part 2(c) of Theorem 1 states that the share of flexible resource a product receives is decreasing with its own inventory and increasing with the other product's inventory. Referring to Figure 1(b), because the initial inventory level of product 1 corresponding to point U_1 is less than that of corresponding to U_2, the amount of flexible capacity allocated to product 1 when starting at U_1 is larger than the one starting at U_2. As we will discuss next, we find that the optimal prices charged for each product have a specific relationship within this band.

4.2. Optimal Pricing Policy

When making pricing decisions, it is often helpful to think in terms of markdowns and markups where a markdown (markup) corresponds to a price discount (surcharge) relative to a current-period list price. Earlier results in the literature on pricing of substitutable products focused on infinite capacity settings; hence, the optimal pricing policy was characterized by a list price and markdown policy. In the presence of capacity limitations, however, the characterization of the optimal pricing policy relies on a third component, namely, price markups. We let $m_i^t(x_1^t, x_2^t)$ denote the price markup/markdown for product i in period t, with $m_i^t < 0$ corresponding to markdowns and $m_i^t > 0$ corresponding to markups in reference to a current-period list price p_{iL}^t. Thus, in period t we have

$$p_i^t(x_1^t, x_2^t) = p_{iL}^t + m_i^t(x_1^t, x_2^t). \tag{5}$$

The following theorem defines the optimal pricing policy.

Theorem 2. *For all $i = 1, 2$, in period t, we have the following*:

(a) *In Region A, if a product i is moderately understocked, then $m_i^t(x_1^t, x_2^t) = 0$, and it is optimal to charge a list price, p_{iL}^t, for that product, where $p_{i,L}^t = (a_{3-i,3-i}b_i - a_{12}b_{3-i})/(2(a_{11}a_{22} - a_{12}{}^2)) + c_i/2$. If a product i is overstocked, then $m_i^t(x_1^t, x_2^t) < 0$, i.e., it is optimal to give a price discount to that product. If, on the other hand, product i is critically understocked, then $m_i^t(x_1^t, x_2^t) > 0$, indicating that it is optimal to give a price markup to that product.*

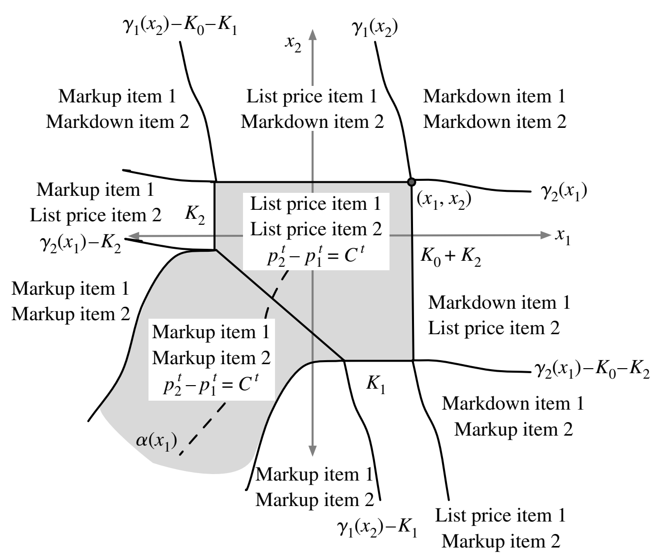
(b) *In Region B, $m_i^t(x_1^t, x_2^t) > 0$; hence the optimal policy marks up the price of both products. Furthermore, if (x_1^t, x_2^t) is such that $0 < l^t(x_1^t, x_2^t) < K_0$, then $m_1^t(x_1^t, x_2^t) = m_2^t(x_1^t, x_2^t)$, resulting in*

$$p_2^t(x_1^t, x_2^t) = p_1^t(x_1^t, x_2^t) + C^t, \quad \text{where } C^t = p_{2L}^t - p_{1L}^t.$$

(c) *The optimal price $p_i^t(x_1^t, x_2^t)$, $i = 1, 2$ is decreasing with respect to x_1^t and x_2^t.*

Figure 2 illustrates the optimal pricing policy in terms of markups and markdowns for each product. It is optimal to give discounts on a product if it is overstocked, apply the list price on the product if it is

Figure 2 **Optimal Pricing Policy, with the Shaded Area Indicating Constant Price Difference Between Products**



moderately understocked, and mark up the price of the product if it is critically understocked. Part (b) of Theorem 2 suggests an interesting fact about the pricing policy when the inventory level falls within the band in Region B where both products use a positive share of the flexible capacity. In states corresponding to this region, both products are marked up by exactly the same amount. This results in the price difference between products remaining identical to the difference between their list prices.

This special structure of the optimal price policy has favorable ramifications from a marketing standpoint. Capacity flexibility may be viewed as a significantly beneficial tool when firms use dynamic pricing and are sensitive to maintaining consistent price differences among products to preserve price positioning across products. In §5, we will demonstrate through a numerical study that this insight can be extended to other demand models such as the multinomial logit model (i.e., flexibility still enables consistent price differences among products when we consider different demand distributions).

4.3. Sensitivity of the Optimal Policy

Having characterized the optimal production and pricing policy, we are also able to explore the sensitivity of the optimal policy structure with respect to changes in various problem parameters. Specifically, we analytically explore the sensitivity of the optimal policy to (i) cost parameters including the production, holding, and backorder costs; (ii) capacity parameters; and (iii) demand parameters including demand intercepts and individual price elasticities. The sensitivity results provided in Table 1 have been obtained analytically by studying the effects of an increase in the current-period value of the corresponding parameter to optimal pricing and production decisions. (For brevity, we omit the proofs, which are available from the authors.) Where applicable, we report the effects of changes in the parameters corresponding to product 1 only as the results for the parameters for product 2 are symmetric. In Table 1, the symbols ↓ and ↑ denote nonincreasing and nondecreasing, respectively. In addition, † denotes the states for which product 1 is critically understocked, whereas †′ refers to all remaining states; ‡ refers to states where both products are critically understocked and the flexible resource is shared between the products, whereas ‡′ denotes the remaining states.

Table 1 **Sensitivity of the Optimal Policy to Various Problem Parameters**

	Price of prod. 1	Price of prod. 2	Base-stock for prod. 1	Base-stock for prod. 2
Production cost (c_1)	↑	$\downarrow^{\ddagger}$ $\uparrow^{\ddagger'}$	↓	↑
Holding cost (h_1^+)	↓	↓ ↑	↓	↓ ↑
Backorder cost (h_1^-)	↑	↓ ↑	↑	↓ ↑
Dedicated capacity (K_1)	↓	↓	↑	$\downarrow^{\ddagger'}$ $\uparrow^{\ddagger}$
Flexible capacity (K_0)	↓	↓	$\downarrow^{\dagger'}$ $\uparrow^{\dagger}$	↓ ↑
Demand intercept (b_1)	↑	↑	↑	$\downarrow^{\ddagger}$ $\uparrow^{\ddagger'}$
Price sensitivity (a_{11})	↓	↓ ↑	↓	↓ ↑

Note. ↑, increasing; ↓, decreasing.
†Product 1 is critically understocked; †′all remaining states.
‡Both products are critically understocked; ‡′all remaining states.

As a particular case, we would like to highlight the results corresponding to an increase in the dedicated or flexible capacity levels. When capacity increases, one would expect that the price for both products would decrease. As shown in Table 1, we note that this expectation is true. A capacity increase in either the dedicated resource or the flexible resource helps reduce instances where products are critically understocked, which limits price markups and hence reduces prices. Regarding the modified base-stock levels, an increase in the current-period dedicated capacity for product 1 leads to an increase in the modified base-stock level for product 1. When both products share the flexible resource and are critically understocked, an increase in the current-period dedicated capacity for product 1 allows more flexible capacity to be allocated to product 2, increasing product 2's modified base-stock level. In all other instances, the modified base-stock level for product 2 decreases. The logic for the results corresponding to an increase in the flexible capacity is similar.

5. Implications of Capacity Flexibility on Optimal Pricing

We showed previously in Theorem 2 that the existence of flexible capacity in a firm's portfolio results in an extended region where the price differences between products remain constant. We now further explore numerically how capacity flexibility influences a firm's optimal pricing strategy. Specifically, we compare the optimal prices charged over a planning horizon for several problem instances (e.g., different demand models and parameters, different processing times) where the share of the flexible resource in the capacity portfolio is gradually increased.

The numerical results presented below were obtained in two steps. As a first step, we solved the finite horizon stochastic dynamic program by using a series of fine discretization and value function approximations for all initial inventory states at each period. We then recorded the optimal production and pricing policy over the entire planning horizon. For the second step, we initialized the starting inventory levels at state $(0, 0)$ and ran 500 randomly generated sample paths that result from the optimal policies for the corresponding state at each period. Our first numerical study considers the following demand model:

$$d_1^t(p_1^t, p_2^t, \epsilon_1^t) = 35 - 0.75p_1^t + 0.25p_2^t + \epsilon_1^t,$$
$$d_2^t(p_1^t, p_2^t, \epsilon_2^t) = 30 + 0.25p_1^t - 0.5p_2^t + \epsilon_2^t. \tag{6}$$

The remaining problem parameters are set as $c_1 = 15$, $c_2 = 20$, $h_1^+ = 3$, $h_2^+ = 4$, $h_1^- = 20$, $h_2^- = 25$, and $\beta = 0.8$. We let ϵ_1^t and ϵ_2^t be randomly drawn from a uniform distribution over the interval $[-10, 10]$ with a positive support on the realized demand. We selected the parameters corresponding to demand intercepts, cross-price elasticities, and production costs to construct a setting where the two products have a reasonable list price difference. Additionally, as demand for lower priced products generally exhibits higher sensitivity to price, we let the product with the lower list price (product 1) be more sensitive to changes in its own price and the product with the higher list price (product 2) be less sensitive to changes in its own price. The model yields list prices of $p_{1,L}^t = 47.5$ and $p_{2,L}^t = 60.0$. In the first setting, we consider a firm with dedicated production capacities, $K_1 = K_2 = 15$ and no flexible capacity, $K_0 = 0$. In the second setting, the firm employs a "hybrid" portfolio of dedicated and flexible resources where $K_0 = K_1 = K_2 = 10$. Finally, in the third setting, the firm utilizes full flexibility with $K_0 = 30$ and $K_1 = K_2 = 0$. These parameters correspond to utilizations of approximately 95% and 80% for product 1 and product 2, respectively, in the dedicated capacity only setting and approximately 90% overall utilization in the flexible capacity only setting.

Figure 3 displays contours of the price difference between products 1 and 2 (i.e., $p_2 - p_1$) resulting from the optimal pricing policy for each product in period 10 for the three capacity settings. The white areas in each figure indicate the regions where the price difference between products is identical to the difference between their list prices. The figures collectively show how the constant price region gets larger as the share of the flexible resource in the capacity portfolio increases.

We next consider price evolutions over multiple periods. Table 2 reports the average and standard deviation (with their 95% confidence intervals) of the prices and price differences observed along the planning horizon of 15 periods for the 500 sample paths. Average price 1 reports the mean (over all the sample paths) of the average price for that product along the 15-period horizon. Similarly, the standard deviation of price 1 reports the mean (over all the sample paths) of the standard deviation of the price for that product along the 15-period horizon. Finally, the standard deviation of the price difference is the mean of the standard deviations for the price differences between products along the planning horizon.

The most interesting aspect of the results in Table 2 is that when flexible systems are used, the difference between the prices charged for products 1 and 2 remain very stable across periods. (In Table 2, compare the standard deviations 2.47, 0.64, and 0.37 for price differences between the two products for the dedicated only, hybrid, and fully flexible capacity settings, respectively.) We actually proved in Theorem 2 that the price difference between the two products will be constant when both products are either moderately understocked or critically understocked and share the flexible resource. When following the optimal policy, we expect the inventory positions for products through the sample path to generally fall within or close to this combined constant price region (white areas in Figure 3, (b) and (c)), thus yielding the results observed in Table 2. In Table 2, we also report the standard deviations for percentage price differences. Most economics papers assume that absolute price differences affect consumer choice between products (Azar 2011). However, behavioral decision theory has shown that in some situations customers are

Figure 3 Price Gaps Between Products 1 and 2 ($p_2 - p_1$) Reflected by the Optimal Pricing Policy for Each Product in Period 10 for Systems with (a) Only Dedicated Resources, (b) a Hybrid Portfolio of Dedicated and Flexible Resources, and (c) a Fully Flexible Resource

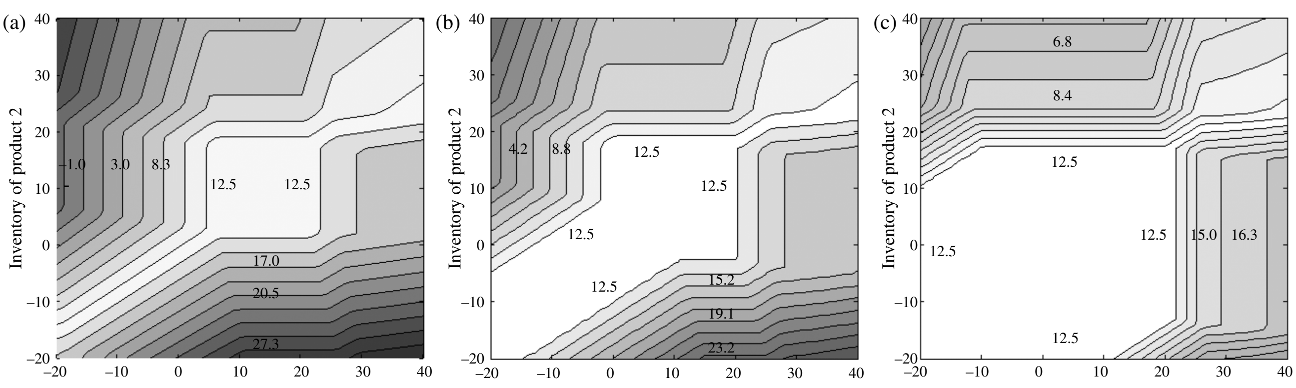


Table 2 **Price Statistics for Systems with (i) Only Dedicated Resources** ($K_0 = 0, K_1 = K_2 = 15$)**, (ii) a Hybrid Portfolio of Dedicated and Flexible Resources** ($K_0 = K_1 = K_2 = 10$)**, and (iii) a Fully Flexible Resource** ($K_0 = 30, K_1 = K_2 = 0$)

	Only dedicated	Hybrid portfolio	Only flexible
Average price 1	49.40 ± 0.003	48.64 ± 0.002	48.53 ± 0.002
Average price 2	61.27 ± 0.003	61.00 ± 0.002	60.99 ± 0.002
Std. dev. of price 1	2.13 ± 0.002	1.60 ± 0.002	1.57 ± 0.002
Std. dev. of price 2	1.99 ± 0.002	1.66 ± 0.002	1.64 ± 0.002
Std. dev. of price difference	2.47 ± 0.003	0.64 ± 0.001	0.37 ± 0.001
Std. dev. of % price difference	0.037 ± 0.00003	0.010 ± 0.00002	0.007 ± 0.00001

influenced more by percentage price differences than actual price differences (Kahneman and Tversky 1984, Darke and Freedman 1993). The recent empirical work by Azar (2011) considered relative price differences and showed that absolute price differences are important where there is a perceived and quantifiable quality gap between products and where the consumers can directly attribute a value for the increased quality. But Azar (2011) also identified certain other situations where percentage price differences can also be important. As depicted in Table 2, flexibility also significantly reduces the standard deviation of percentage price difference between the products.

To visualize the effect of flexible capacity on the optimal pricing policy demonstrated in Table 2, we next illustrate a particular sample path over the 15-period horizon. Figure 4 depicts the optimal prices at each period for the three settings for the same sample path and highlights the advantages of flexible resources. Our main observation from Figure 4 is that the availability of flexible capacity enables the price difference between the products to be fairly stable across periods. (We observed similarly that percentage price differences also are much more stable with flexible capacity.) Because the characteristics of the structure of the optimal policy do not depend on an individual parameter set, we obtain similar policy results as displayed in Figure 3 with varying parameters in Equation (6). For brevity we omit the results from additional numerical tests in which we varied the demand intercepts and individual and cross-sensitivities in Equation (6). The main insight that flexibility significantly reduces the variability of price differences holds in all tested instances of differing demand parameters.

5.1. Asymmetric Cross-Price Elasticities

Although our theoretical results on the policy structure and the above numerical study assume identical cross-price elasticities, we are also interested to see whether a similar price behavior extends to settings with asymmetric cross-price elasticities. To that end, we use the parameters discussed in the previous setting as a base case and gradually increase the cross-price elasticity differential, δ, between the products. We ran computations with the elasticity values $a_{12} = -0.25 + \delta$ and $a_{21} = -0.25 - \delta$ where we increased δ from 0 to 0.20 with increments of 0.02. Because the changes in cross-price elasticities affect the list price demand levels, we set the available capacities in each problem instance so as to maintain identical utilization levels with the base setting. Figure 5 displays the price difference between the optimal prices for product 1 and product 2 in period 10 in an instance with asymmetric cross-price elasticities ($\delta = 0.10$) and for a system with only dedicated resources and a fully flexible resource. In Figure 5(b), we observe that with asymmetric cross-price elasticities, the price differences in the critically understocked region are no longer constant. However we see that flexibility nevertheless continues to provide a less variable price difference compared to the price gap exhibited in the pure dedicated capacity case shown in Figure 5(a).

Figure 4 **Prices for and Price Differences Between Products 1 and 2 for a Particular Sample Path for Systems with (a) Only Dedicated Resources** ($K_0 = 0, K_1 = K_2 = 15$)**, (b) a Hybrid Portfolio of Dedicated and Flexible Resources** ($K_0 = K_1 = K_2 = 10$)**, and (c) a Fully Flexible Resource** ($K_0 = 30, K_1 = K_2 = 0$)

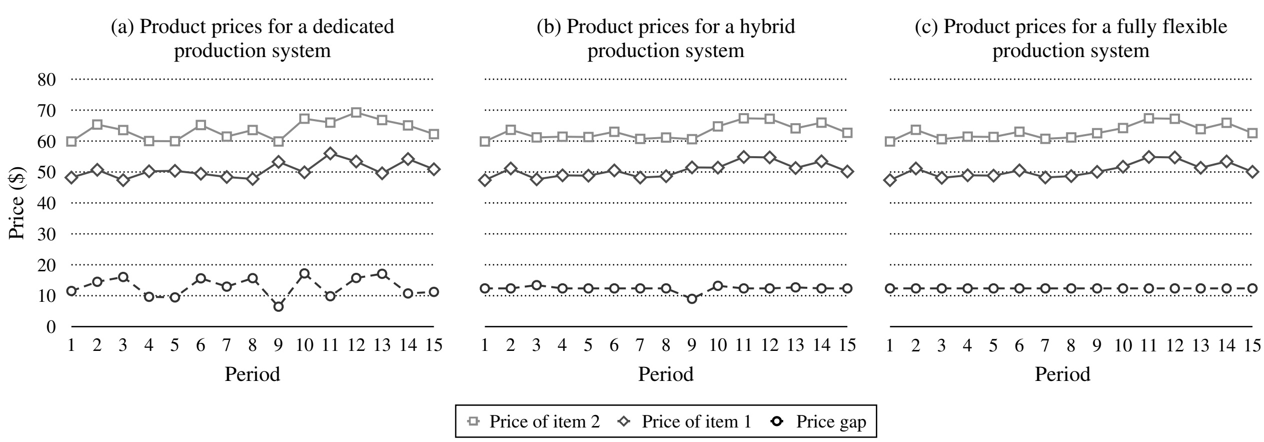
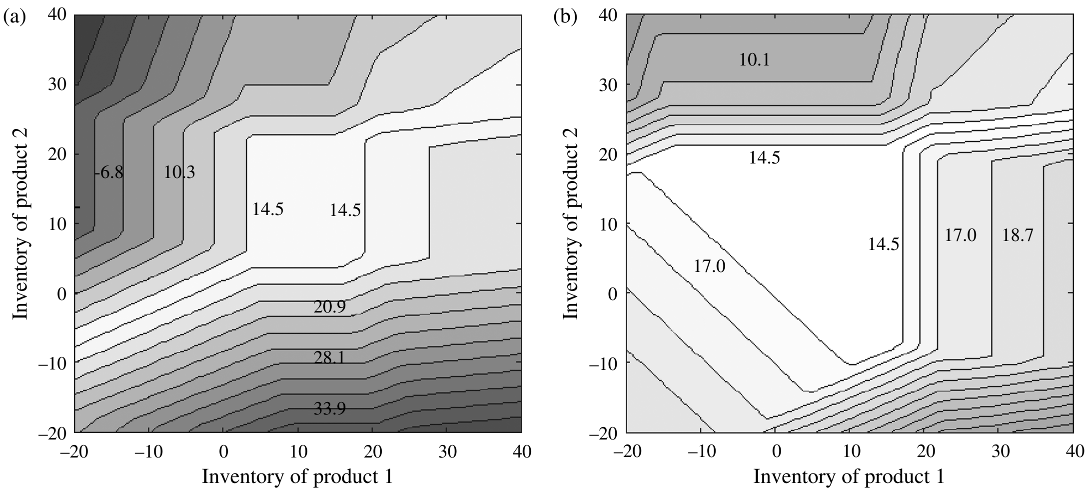


Figure 5 **Price Differences Between Products 1 and 2 Reflected by the Optimal Pricing Policy in Period 10 Corresponding to Asymmetric Cross-Price Elasticities ($\delta = 0.10$) and for Systems with (a) Only Dedicated Resources and (b) a Fully Flexible Resource**



To consider the price evolution over multiple periods, we again ran simulations varying the cross price differential δ from 0 to 0.20. For brevity, in Table 3, we only report the price statistics for $\delta = 0.10$, i.e., where $a_{12} = -0.15$ and $a_{21} = -0.35$. We found that the average standard deviations of the price difference between products 1 and 2 over the 15-period horizon are 2.95 ± 0.003 and 0.50 ± 0.001 for the dedicated only and fully flexible capacity settings, respectively. Thus, flexible capacity continues to reduce the variation in price differences.

5.2. Asymmetric Processing Times

Next, we explore the effects of different processing times for the two products on the flexible resource. We let κ denote the processing time differential such that a unit of product 1 requires $1/\kappa$ units of the flexible resource, whereas a unit of product 2 requires κ units of the flexible resource. Hence, as κ increases, the same amount of flexible capacity can produce more of product 1 compared to product 2. We vary κ from 1 to 2 by increments of 0.2, where $\kappa = 1$ corresponds to the base case studied earlier. We adjust the capacity levels such that the utilization of the flexible resource in each setting is identical to that of the base setting.

We observe that different processing times on the flexible resource do not result in an extended constant price difference region and yield to higher markups for the item with the longer processing time. To see why, consider a deviation from identical processing times that reduces the processing time of product 1 on the flexible resource. When both products are understocked, offering a relatively higher markup for product 2 enables the firm to suppress product 2 demand as opposed to the demand for product 1 with the shorter processing time. This in turn shifts more of the backlog to product 1, which can be quickly produced, helping the firm better recover from understocked inventory levels. Considering the price evolution over multiple periods, we again ran 500 sample paths using the demand and cost parameters of the base setting and for each instance of κ. We found that the average standard deviation of price difference for the entire range of κ is 1.52. Compared to the standard deviation of 2.47 for the pure dedicated capacity setting, this suggests that capacity flexibility continues to provide a smoothing effect for the price difference between products when there is processing time discrepancies for the products that share the flexible resource. We find that a higher κ value yields a higher standard deviation of the price difference between products. One would expect to see this since increasing the deviation of the processing time results in even higher relative markups for the product with the slower processing time, thereby increasing the price difference variability.

Table 3 **Asymmetric Cross-Price Elasticity: Price Statistics for the Instance with Cross-Price Elasticity Differential $\delta = 0.10$ for Systems with (i) Only Dedicated Resources and (ii) a Fully Flexible Resource**

	Only dedicated	Only flexible
Average price 1	47.46 ± 0.003	47.56 ± 0.002
Average price 2	63.67 ± 0.004	62.39 ± 0.003
Std. dev. of price 1	1.85 ± 0.002	1.69 ± 0.002
Std. dev. of price 2	2.82 ± 0.003	2.05 ± 0.003
Std. dev. of price difference	2.95 ± 0.003	0.50 ± 0.001

5.3. MNL Demand Models

Finally, we also explore whether the main insight that flexibility provides stability in the price difference between products extends to other demand models. For this purpose, we consider the MNL demand model. For a detailed discussion of MNL demand models in this context, we refer the reader to Aydin and Porteus (2008). Following Aydin and

Table 4 MNL Demand Model: Price Statistics for Systems with (i) Only Dedicated Resources ($K_0 = 0, K_1 = K_2 = 15$) and (ii) a Fully Flexible Resource ($K_0 = 30, K_1 = K_2 = 0$)

	$K_0 = 0$, $K_1 = K_2 = 15$	$K_0 = 30$, $K_1 = K_2 = 0$
Average price 1	7.84 ± 0.0054	7.81 ± 0.0050
Average price 2	9.84 ± 0.0056	9.81 ± 0.0050
Std. dev. of price 1	0.40 ± 0.0003	0.35 ± 0.0003
Std. dev. of price 2	0.45 ± 0.0003	0.35 ± 0.0003
Std. dev. of price difference	0.48 ± 0.0004	0.01 ± 0.0001

Porteus (2008), we let $u_i^t - p_i^t + \zeta_i$ denote the surplus utility of a customer who purchases product i where ζ_i is a Gumbel error term with shape parameter μ. The demand for product i is then given by $\Theta(\exp((u_i^t - p_i^t)/\mu))/(1+\sum_j \exp((u_j^t - p_j^t)/\mu)) + \epsilon_i^t$ where Θ denotes the market size and ϵ_i^t is an additional additive demand uncertainty term. Table 4 similarly displays the results for 500 sample paths for a setting where $u_1 = 8$, $u_2 = 10$, $\mu = 1$, $\Theta = 30$ and with $c_1 = 3$, $c_2 = 5$, $h_1 = 1.5$, $h_2 = 2.5$, $\pi_1 = 6$, $\pi_2 = 10$, and $\beta = 0.8$. We find that the insights we have gained by the linear demand model regarding the price difference stabilizing effects of flexible capacity continue to strongly hold under the MNL demand model. (For an illustration of a particular sample path, see Figure 6.)

6. Extensions

6.1. Expedited Delivery Option

First we consider a setting where the firm does not backorder any demand missed in the current period. When revenue is only collected from the current-period satisfied demand with any unmet demand considered as lost sales, even a single product case with a linear demand function leads to an objective function that is not concave (see, for example, Federgruen and Heching 1999). Hence, we instead consider a different case where the firm has an expedited delivery option for any units in shortage and still collects revenue from all incoming demand during the period. The expedited delivery may correspond to a more expensive outsourcing option or the possibility of producing products during overtime. In such a setting, the problem can then be reformulated with a slight modification as

$$V^t(x_1^t, x_2^t) = c_1 x_1^t + c_2 x_2^t + \max_{\substack{z_1^t, z_2^t \in \mathcal{F}'(x_1^t, x_2^t, p_1^t, p_2^t) \\ p_1^t, p_2^t}} J^t(z_1^t, z_2^t, p_1^t, p_2^t)$$

where

$$J^t(z_1^t, z_2^t, p_1^t, p_2^t) = R'(p_1^t, p_2^t) - c_1 z_1^t - c_2 z_2^t + \mathrm{E}_{\epsilon_1^t, \epsilon_2^t}\left(-\sum_i h_i'(z_i^t - \epsilon_i^t) + \beta V^{t-1}((z_1^t - \epsilon_1^t)^+, (z_2^t - \epsilon_2^t)^+)\right),$$

and the term $h_i'(z_i^t - \epsilon_i^t)$ is defined as $h_i'(z_i^t - \epsilon_i^t) := h_i^+(z_i^t - \epsilon_i^t)^+ + s_i(z_i^t - \epsilon_i^t)^-$, where $s_i > c_i$ denotes the unit expedited production cost. Under this formulation, it can be shown that the optimal policy structure outlined in Theorems 1 and 2 is preserved.

6.2. Costly Flexible Production

We next study a problem that takes into account production cost differences between a flexible resource and a dedicated resource, where the production cost for the former may be more expensive than that for the latter. Such a setting may arise in a labor-intensive production environment where operating a flexible resource requires additional skills and training. Let $\delta c_1, \delta c_2 \geq 0$ be the incremental cost of producing product 1 and product 2 on the flexible resource, i.e., $c_i^F = c_i^D + \delta c_i$, where c_i^D and c_i^F denote the cost to produce product i on its corresponding dedicated and flexible resources, respectively. Introducing two new

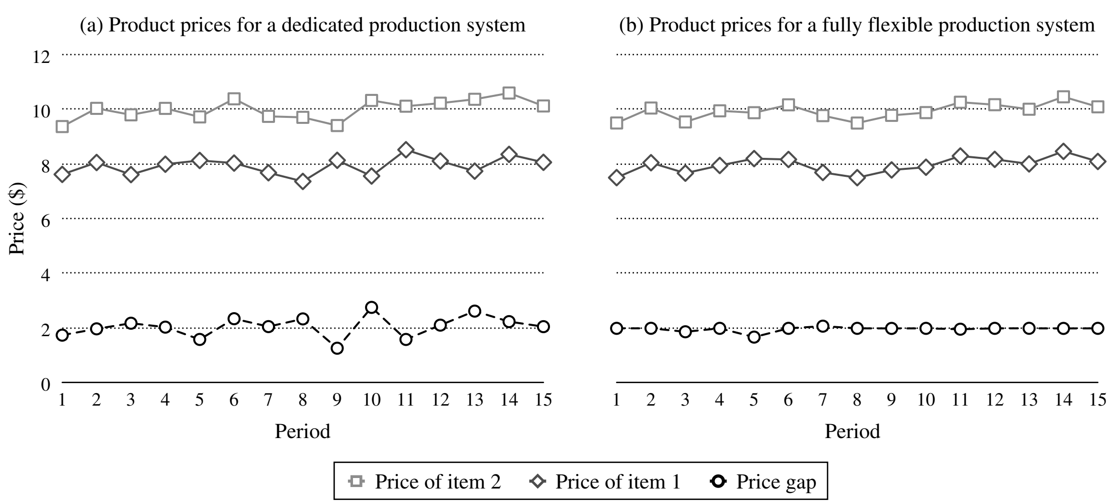


Figure 6 MNL Demand Model: Prices for and Price Gaps Between Products 1 and 2 for Systems with (a) Only Dedicated Resources and (b) a Fully Flexible Resource

variables, w_1^t and w_2^t, to represent the amount of product 1 and product 2, respectively, produced on the flexible resource, we can rewrite the problem formulation as follows:

$$V^t(x_1^t, x_2^t) = \max_{\substack{z_1^t, z_2^t, w_1^t, w_2^t \in \mathcal{F}'(x_1^t, x_2^t, p_1^t, p_2^t)\\ p_1^t, p_2^t}} \{c_1^D x_1^t + c_2^D x_2^t + J^t(z_1^t, z_2^t, w_1^t, w_2^t, p_1^t, p_2^t)\}$$

where

$$J^t(z_1^t, z_2^t, w_1^t, w_2^t, p_1^t, p_2^t) = R'(p_1^t, p_2^t) - \sum_i (c_i^D z_i^t + \delta c_i w_i^t) + \mathrm{E}_{\epsilon_1^t, \epsilon_2^t}\bigg(-\sum_i h_i(z_i^t - \epsilon_i^t) + \beta V^{t-1}(z_1^t - \epsilon_1^t, z_2^t - \epsilon_2^t)\bigg),$$

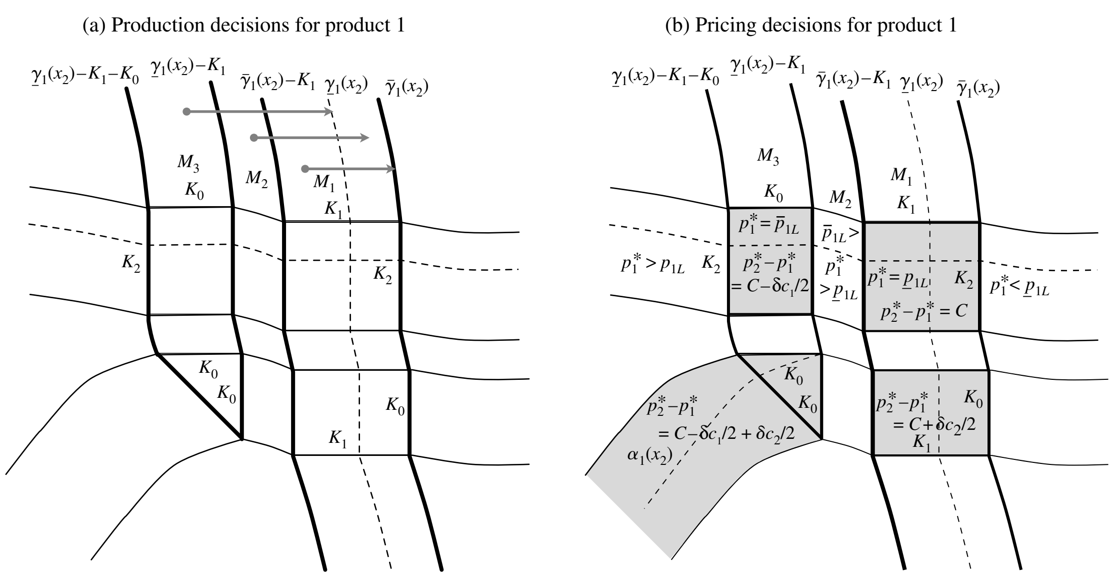
and the term $\mathcal{F}'(x_1^t, x_2^t, p_1^t, p_2^t)$ corresponding to the feasible region is given by $\{(z_1^t, z_2^t, w_1^t, w_2^t) \mid x_i^t \le z_i^t + b_i - a_{i1}p_1^t - a_{i2}p_2^t - w_i^t \le x_i^t + K_i \ \forall i = 1, 2, \ w_1^t + w_2^t \le K_0$, and $w_i^t \ge 0 \ \forall i = 1, 2\}$. The optimal policy for the case where production via flexible resources is more costly exhibits characteristics identical to those of the optimal policy structure given in Theorem 1 when both products are critically understocked. However, the production cost differential between the flexible and the dedicated resources leads to a "two-tier" modified base-stock level denoted by $\bar{\gamma}_i(x_{3-i})$ and $\underline{\gamma}_i(x_{3-i})$, which segment the moderately understocked region into three subregions for each product (depicted by M_1, M_2, and M_3 in Figure 7). The following theorem summarizes the changes in the optimal policy structure.

Theorem 3. (a) *When at most one product is critically understocked, the optimal production policy for product i is a state-dependent modified base-stock policy consisting of two tiers. If $\underline{\gamma}_i(x_{3-i}^t) - K_i < x_i^t < \bar{\gamma}_i(x_{3-i}^t)$, it is optimal to bring the inventory of product i to $\min(x_i^t + K_i^t, \bar{\gamma}_i(x_{3-i}^t))$. If $x_i^t < \underline{\gamma}_i(x_{3-i}^t) - K_i$, then it is optimal to produce up to $\min(x_i^t + K_i^t + K_0^t, \underline{\gamma}_i(x_{3-i}^t))$. Moreover, $\underline{\gamma}_i(x_{3-i}^t)$ and $\bar{\gamma}_i(x_{3-i}^t)$ are nonincreasing with the inventory level of product j, $j \neq i$.*

(b) *The optimal pricing policy for product i is defined by dual list prices $\bar{p}_{i,L}$ and $\underline{p}_{i,L}$ (where $\bar{p}_{i,L} = \underline{p}_{i,L} + \delta c_i/2$) as well as markups and markdowns. When $\bar{\gamma}_i(x_{3-i}^t) - K_i < x_i^t < \bar{\gamma}_i(x_{3-i}^t)$, it is optimal to charge the lower list price, $\underline{p}_{i,L}$. If $\underline{\gamma}_i(x_{3-i}^t) - K_i < x_i^t < \bar{\gamma}_i(x_{3-i}^t) - K_i$, it is optimal to mark up the price of product i such that $\bar{p}_{i,L} > p_i^t > \underline{p}_{i,L}$. For $\underline{\gamma}_i(x_{3-i}^t) - K_i - K_0 < x_i^t < \underline{\gamma}_i(x_{3-i}^t) - K_i$, it is optimal to charge the higher list price, $\bar{p}_{i,L}$. When both products are critically understocked and share the flexible capacity, the optimal policy marks up the prices of both products such that the price difference is equivalent to the difference between the high list prices. The price of each product is decreasing with the inventory level of either product.*

For expositional clarity, in Figure 7, we only present the differences in the optimal policy from our earlier main results and only for product 1. As can be observed in Figure 7(a), the production policy consists of a two-tier base-stock level arising due to the production cost difference between the dedicated and the flexible resources. If the initial inventory level of product 1 falls in region M_1, only the dedicated resource for this product is used to bring the inventory level to the upper base-stock level. When the initial inventory level is lower such that the dedicated capacity is not adequate to bring the inventory level to the desired upper base-stock level, the use of the flexible resource is not immediately justified because of its higher cost. Starting in a state within region M_2, it is optimal to fully use the dedicated resource and none

Figure 7 **Changes in the Optimal Policy When It Is More Costly to Produce via Flexible Resources**



of the flexible resource. Region M_3 corresponds to the states where the use of the flexible resource is required. Starting in this region, the inventory is brought up to the lower base-stock level for the product.

Consequently, the production cost difference also yields a two-tier list price for each product, where a lower list price is applied when the initial inventory level is within region M_1, and a higher list price is applied when the starting inventory level is within region M_3. When starting in region M_2, the optimal price decreases with the initial inventory level and is between the higher and the lower list prices. These results are analogous to our findings in the original model in the sense that list prices are charged when there is adequate capacity to bring the inventory up to a desired level, and prices decrease in the starting inventory level. In this modified case, the main difference is that we have two sets of desired base-stock levels for the two types of capacity being utilized, and hence we have two list prices corresponding to the regions where capacity is adequate to reach the desired base-stock level. When also considering the pricing policy of product 2, the superposition of the two-tier list price policies for both products collectively yield four separate regions where the price differences between the products remain constant individually in each of these regions (shown in Figure 7(b)). Finally, we note that $\delta c_1 = 0$ implies that there is no cost surcharge to use the flexible resource, and hence the problem reduces to the original setting. Specifically, the region depicted by M_2 collapses, and regions M_1 and M_3 merge to construct the moderately understocked region of the original problem.

6.3. Higher Number of Products

We also consider a general N-product setting. Extending the previously studied demand model in §3, we can represent the demand for product i by $d_i^t(p_1^t, \ldots, p_N^t, \epsilon_i^t) = b_i - \sum_{n=1:N} a_{in}^t p_n^t + \epsilon_i^t$. We let the square matrix $\mathbf{A}^t$ with elements a_{ij}^t for $i, j \in \mathcal{N} = \{1, \ldots, N\}$ denote the price elasticity matrix, and we let $\mathbf{p}^t = (p_1^t, \ldots, p_N^t)$ denote the vector of product prices. Hence, we can write the expected demand vector as a function of the product prices as $\bar{\mathbf{d}}^t = \mathbf{b}^t - \mathbf{A}^t\mathbf{p}^t$. We again assume that $\mathbf{A}^t$ has positive diagonal elements and nonpositive off-diagonal elements, that is, $a_{ii}^t > 0$ and $a_{ij}^t \le 0$ for $i \ne j$, to reflect the substitutability of the products. We further assume that $\mathbf{A}^t$ possesses strict diagonally dominance property, i.e., $a_{ii}^t > \sum_{j \ne i} |a_{ij}^t|$, and that $\mathbf{A}^t$ is symmetric. Following analogous assumptions as given in §3, we can present the problem formulation as

$$V^t(\mathbf{x}^t) = \mathbf{c}\mathbf{x}^t + \max_{\mathbf{z}^t \in \mathcal{F}'(\mathbf{x}^t, \mathbf{p}^t), \mathbf{p}^t} J^t(\mathbf{z}^t, \mathbf{p}^t), \tag{7}$$

$$J^t(\mathbf{z}^t, \mathbf{p}^t) = R'(\mathbf{p}^t) - \mathbf{c}\mathbf{z}^t + \mathrm{E}_{\boldsymbol{\epsilon}}^t(-\mathbf{h}(\mathbf{z}^t - \boldsymbol{\epsilon}^t) + \beta V^{t-1}(\mathbf{z}^t - \boldsymbol{\epsilon}^t)). \tag{8}$$

In (7), $V^t(\mathbf{x}^t)$ denotes the expected discounted profit starting at state $\mathbf{x}^t$ with t periods remaining until the end of the planning horizon. In (8), the term $R'(\mathbf{p}^t)$ stands for the modified expected revenue function, with $R'(\mathbf{p}^t) := \mathbf{p}^{t'}\mathbf{b} + \mathbf{c}\mathbf{A}\mathbf{p}^t - \mathbf{p}^{t'}\mathbf{A}\mathbf{p}^t - \mathbf{c}\mathbf{b}$. Because of the combinatorial nature of the product-capacity assignments, we first consider a special case where the firm uses dedicated capacities for each product. We then partially extend the results to a setting consisting of a particular portfolio of dedicated and flexible resources. We denote the subset of products requiring production in the current period by $\mathcal{P}$ and the products that are overstocked by $\mathcal{N} \backslash \mathcal{P}$.

In the dedicated capacity case where each product is replenished by its own limited resource K_i, the term corresponding to the feasible region $\mathcal{F}'(\mathbf{x}^t, \mathbf{p}^t)$ in (7) is given by $\mathcal{F}'(\mathbf{x}^t, \mathbf{p}^t) := \{\mathbf{z}^t \mid z_i^t + b_i - \sum_{n=1:N} a_{in} p_n \le x_i^t + K_i\}$.

Theorem 4. (a) *The optimal production policy for product $i \in \mathcal{P}$ consists of a modified base-stock level where it is optimal to bring the inventory level of product i up to this level as much as capacity K_i permits. The modified base-stock level for product i is nonincreasing with x_j^t, $j \ne i$.*

(b) *For product $i \in \mathcal{P}$, it is optimal to charge a list price $(\mathbf{A}^{-1}\mathbf{b} + \mathbf{c})_i/2$ if its available capacity K_i is adequate to bring the inventory to its base-stock level. Otherwise, it is optimal to mark up the price of product i. For product $j \in \mathcal{N} \backslash \mathcal{P}$, it is optimal to give a price discount. The optimal price for each product is nonincreasing with the starting inventory level of other products.*

The results in Theorem 4 are the extensions of the results given in Theorems 1 and 2 for a portfolio consisting solely of dedicated resources.

Finally, we consider the effects of flexibility on the optimal pricing decisions for a capacity portfolio where the products are indexed such that the first k products are produced by dedicated resources and the remaining $N - k$ products are replenished by a shared flexible resource K_0. For this setting, the term corresponding to the feasible region $\mathcal{F}'(\mathbf{x}^t, \mathbf{p}^t)$ in (7) is now given by $\mathcal{F}'(\mathbf{x}^t, \mathbf{p}^t) := \{\mathbf{z}^t \mid z_i^t + b_i - \sum_{n=1:N} a_{in} p_n \le x_i^t + K_i,\ \sum_j (z_j^t + b_j - \sum_{n=1:N} a_{in} p_n) \le \sum_j x_j^t + K_0\}$, where $i \le k$, $j > k$, and $i, j \in N$. The results below summarize the impact of flexibility on optimal prices.

Theorem 5. (a) *For product $i \le k$, $i \in \mathcal{P}$, it is optimal to charge a list price $(\mathbf{A}^{-1}\mathbf{b} + \mathbf{c})_i/2$ if its available capacity K_i is adequate to bring the inventory to its base-stock level, and to mark up the price otherwise. For product $i' \le k$, $i' \in \mathcal{N} \backslash \mathcal{P}$, it is optimal to give a price discount.*

(b) *If there remains some flexible capacity that is not fully utilized, then it is optimal to set the price of each product $j > k$, $j \in \mathcal{P}$ at its list price given by $(\mathbf{A}^{-1}\mathbf{b} + \mathbf{c})_j/2$ and give a discount to a product $j' > k$, $j' \in \mathcal{N} \backslash \mathcal{P}$. If the flexible capacity is fully utilized, then it is optimal to mark up the price of each product $j > k$, $j \in \mathcal{P}$, and the optimal markup amount is identical for each product.*

Theorem 5 outlines the instances when applying list prices, charging markups, or offering discounts is optimal for each of the two product groups based on whether the product is produced by a dedicated resource or shares a flexible resource with other products. (For the case of a product portfolio consisting of three products where two products share a flexible resource and one has a dedicated resource or all three products share a flexible resource, it can be also shown that the optimal prices are decreasing with respect to the inventory levels of all products.) Among the products that share the flexible resource, Theorem 5, part (b), indicates that the constant price difference region exhibited by the availability of a shared flexible resource extends beyond two products to an arbitrary number of products for a setting with comparable demand models. The issue of choosing k optimally ultimately requires an analysis that considers several dimensions such as the investment cost for each type of resource and an evaluation of process requirements for the products. From a practical standpoint, capacity flexibility possesses a significant benefit in reducing the complexity of optimal price selection for a product portfolio consisting of a large number of products. Our result indicates that in instances where the firm has to apply price markups for any subset of products that share the flexible resource, it only needs to identify one markup level that will be applied across all products that are understocked.

7. Conclusions

In this paper, we studied a joint mechanism of dynamic pricing and capacity flexibility to mitigate demand and supply mismatches. We considered a firm producing two products with correlated demands utilizing limited product-dedicated and flexible resources and characterized the structure and sensitivity of the optimal production and pricing decisions. We found that the presence of a flexible resource may significantly reduce the fluctuations of price differences across products over time. Thus, the existence of a flexible resource in the firm's capacity portfolio helps maintain stable price differences across products over time. This enables the firm to establish consistent price positioning among multiple products even if it uses a dynamic pricing strategy. Finally, we have extended our results to a more general setting with multiple products and showed that the availability of a flexible resource continues to induce constant price differences among multiple products sharing a single flexible resource.

Electronic Companion

An electronic companion to this paper is available as part of the online version at http://dx.doi.org/10.1287/msom.1120.0404.

Acknowledgments

The authors are grateful to the associate editor and two anonymous referees for their helpful comments, which significantly helped improve the paper.

References

Aydin G, Porteus E (2008) Joint inventory and pricing decisions for an assortment. *Oper. Res.* 56(5):1247–1255.

Azar OH (2011) Relative thinking in consumer choice between differentiated goods and services and its implications for business strategy. *Judgement Decision Making* 6(1):176–185.

Bish EK, Suwandechochai R (2010) Optimal capacity for substitutable products under operational postponement. *Eur. J. Oper. Res.* 207(2):775–783.

Bish EK, Muriel A, Biller S (2005) Managing flexible capacity in a make-to-order environment. *Management Sci.* 51(2):167–180.

Bitran RG, Caldentey R (2003) Pricing models for revenue management. *Manufacturing Service Oper. Management* 5:203–229.

Chan LMA, Shen ZJM, Simchi-Levi D, Swann JL (2004) Coordination of pricing and inventory decisions: A survey and classification. Simchi-Levi D, Wu SD, Shen Z-J, eds. *Handbook of Quantitative Supply Chain Analysis: Modeling in the E-Business Era* (Kluwer Academic, Boston), 335–392.

Chen X, Simchi-Levi D (2004) Coordinating inventory control and pricing strategies with random demand and fixed ordering cost: The finite horizon case. *Math. Oper. Res.* 29(3):698–723.

Chen X, Simchi-Levi D (2012) Pricing and inventory management. Özer Ö, Phillips R eds. *The Oxford Handbook of Pricing Management* (Oxford University Press, Oxford, UK), 784–824.

Chod J, Rudi N (2005) Resource flexibility with responsive pricing. *Oper. Res.* 53(3):532–548.

Copeland A, Dunn W, Hall G (2005) Prices, production, and inventories over the automotive model year. Working paper, Board of Governors of the Federal Reserve System, Washington, DC.

Darke PR, Freedman JL (1993) Deciding whether to seek a bargain: Effects of both amount and percentage off. *J. Appl. Psych.* 78(6):960–965.

DeCroix G, Arreola-Risa A (1998) Optimal production and inventory policy for multiple products under resource constraints. *Management Sci.* 44(7):950–961.

DisplaySearch (2009) LCD TV outlook improves: CRT demand plummets. Quarterly global TV shipment and forecast report, September 29, http://www.displaysearch.com.

Elmaghraby W, Keskinocak P (2002) Dynamic pricing: Research overview, current practices and future directions. *Management Sci.* 49(10):1287–1309.

Evans R (1967) Inventory control of a multiproduct system with a limited production resource. *Naval Res. Logist.* 14(2):173–184.

Federgruen A, Heching A (1999) Combined pricing and inventory control under uncertainty. *Oper. Res.* 47(3):454–475.

Glasserman P (1996) Allocating production capacity among multiple products. *Oper. Res.* 44(5):724–734.

Ha A (1997) Optimal dynamic scheduling policy for a make-to-stock production system. *Oper. Res.* 45(1):42–53

Jedidi K, Mela CF, Gupta S (1999) Managing advertising and promotion for long-run profitability. *Marketing Sci.* 18(1):1–22.

Kahneman D, Tversky A (1984) Choices, values and frames. *Amer. Psychologist* 39(4):341–350.

Li Q, Zheng S (2006) Joint inventory replenishment and pricing control for systems with uncertain yield and demand. *Oper. Res.* 54(4):696–705.

Lus B, Muriel A (2009) Measuring the impact of increased product substitution on pricing and capacity decisions under linear demand models. *Production Oper. Management* 18(1):95–113.

Mela CF, Gupta S, Lehmann DR (1997) The long term impact of promotion and advertising on consumer brand choice. *J. Marketing Res.* 34(2):248–261.

Mela CF, Gupta S, Jedidi K (1998) Assessing long term promotional influences on market structure. *Internat. J. Res. Marketing* 15(2):89–107.

Monroe KB (1973) Buyers' subjective perceptions of price. *J. Marketing Res.* 10(1):70–80.

Moreno A, Terwiesch C (2010) Pricing and production flexibility: An empirical analysis of the U.S. automobile industry. Available at http://dx.doi.org/10.2139/ssrn.2188246.

Rao A, Monroe K (1989) The effect of price, brand name, and store name on buyers' perceptions of product quality. An integrative review. *J. Marketing Res.* 26(3):351–57.

Song JS, Xue Z (2007) Demand management and inventory control for substitutable products. Working paper, Duke University, Durham, NC.

Tang CS, Yin RR (2007) Joint ordering and pricing strategies for managing substitutable products. *Production Oper. Management* 16(1):138–153.

Whitin TM (1955) Inventory control and price theory. *Management Sci.* 2(1):61–68.

Ye Y (2008) Joint pricing and inventory decision for competitive products. Master's thesis, Massachusetts Institute of Technology, Cambridge, MA.

Zhu K, Thonemann UW (2009) Coordination of pricing and inventory control across products. *Naval Res. Logist.* 56(2):175–190.