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Management of new product introductions is critical for nearly all firms, and one of its most important dimensions is the management of demand during the introduction. Research analyzing this area predominantly uses versions of the diffusion model to capture the demand trajectory of a new product with a fixed potential market. The classic Bass model assumes that demand for innovative products is influenced both by "external" media influence and "internal" word-of-mouth effect, but it excludes price and assumes that capacity is unlimited. In reality, both factors critically influence firms' strategies. Price fluctuations for a new product are common, and price is often a critical lever that helps to shape the demand. Also, firms often have significant capacity constraints, which influence the feasibility of their strategies. In this paper, we consider how a capacity-constrained firm prices products during new product introductions. Thus, the demand rate is influenced by price and, when capacity is insufficient, we allow some customers to be either lost or backlogged, which slows down the word-of-mouth effect. To understand the effect of both pricing and capacity, we consider the integrated optimal pricing, production, and inventory decisions, using control-theory framework (a generalization of the classic Bass model). Most of our results are fairly robust and apply under the assumption of lost sales and partial backlogging, as well as make-to-order and make-to-stock environments. We show that in most cases, the optimal trajectory of demand is unimodal, as in the Bass model, but the optimal price trajectory and the corresponding pricing policy are more complicated when capacity is limited. Using a numerical study, we explore when pricing flexibility is most valuable and whether simple pricing policies may be effective. We find that benefits of pricing flexibility are highest when capacity is neither unlimited (very large) nor very small and when word-of-mouth effect dominates direct impact from media. The ability to adjust prices is significantly more important than the option of producing in advance and holding inventory. We also find that simple pricing policies, appropriately chosen for given capacity, perform very well. In a numerical study, we show that demand uncertainty and increases in capacity over time do not affect our main insights.

Key words: OM-marketing interface; retailing; pricing; Bass diffusion model
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1. Introduction

We are interested in the joint pricing, production, and inventory decisions made by a manufacturer introducing a new product to the market and facing capacity constraints. Pricing of new products is a critical decision that firms make, and often firms face capacity constraints that affect their capability to satisfy demand during new product introductions. We are thus interested in the following set of questions: (1) How does a firm manage pricing, production, and inventory decisions over time during new product introductions? What is the structure of optimal pricing, production, and inventory policies? (2) What is the effect of the firm's capability to adjust prices? How much does a firm lose if it has to maintain constant prices during a product's lifetime? (3) What effect does the firm's capability to make to stock prior to product introduction have on its profitability? (4) Is pricing or building inventory ahead of new product introductions a more effective tool to maximize profit and satisfy demand during product introductions?

Examples of supply shortages for new products are often observed. In December 2005, the Apple Store noted that the 1 GB iPod shuffle was "Currently Unavailable. Sold out for holiday. Expected availability mid-January." Similarly, in 2006, Sony announced a shortage of PlayStation 3's because of insufficient manufacturing capacity of its key component, blue-laser diode. In 2010, the launch of the white iPhone 4 was delayed because the capacity of its supplier, Lens Technology, covered only half of the demand for white phones. It is worth noting that capacity constraints, which are not captured in the classic Bass model, have a more significant impact on innovative products than they do on mature products. For an innovative product, not only does capacity shortage result in backlogged demand or lost sales, it may also slow down the product diffusion process, if customers who have not received the product are not able to generate positive word-of-mouth effect. (The opposite effect, not studied in this paper, is the increase of desirability of product driven by perceived unavailability of products; see Stock and Balachander 2005.) Firms clearly need to consider manufacturing capacity and resulting product availability as critical elements of new product introductions.

In the presence of capacity constraints, firms' pricing, production, and inventory decisions have great practical importance, and we are thus interested in both the characteristics of the optimal demand and price trajectories as well as in the choice of appropriate tools (levers) that can be used to manage this process. A number of levers can be applied to manage new products under limited capacity. Ho et al. (2002, 2011) and Kumar and Swaminathan (2003) have considered two managerial levers, respectively: producing in advance of product launch to build early supply and deliberately backlogging customers (not shipping available product) to slow down diffusion. Both papers have considered price as exogenous. The third lever often considered in the marketing literature is dynamic pricing.

Although prices of all products can be adjusted over time, dynamic changes in prices have potentially significant consequences for new products. It is common, especially for manufacturers of electronic products, to adjust prices of products several times during their lifetime. Clearly, how a product is priced is one of the most fundamental marketing decisions made during a product's launch. Setting prices too high and not coordinating pricing with production can lead firms to end up with too much inventory, and setting prices too low and not taking into account the word-of-mouth effect and its effect on demand can lead to shortages. Furthermore, even when the list price of a product is fixed, it is common for firms to seed initial demand and the word-of-mouth effect by offering coupons or targeted discounts to certain segments of consumers and then desist once demand has picked up (resulting in an "effective" price increase). Clearly, insufficient capacity provides an incentive to increase prices. Recently, Renault introduced a new model of its SUV Renault Duster in India. The product became so popular that the vehicle had a waiting period of four to five months as of September 2012, despite Renault increase in production capacity to three shifts in its Chennai factory. On September 29, a number of publications, e.g., Carshore.com (2012), announced that Renault would be increasing the prices of that model by RS30K to RS40K. Interestingly, the Renault spokesperson blamed instability in foreign exchange for this increase, even though, as Carshore.com pointed out, no other models from Renault had any price revision announced. It was also noted that many other manufacturers were decreasing prices at the same time. Finally, the ease of changing prices online has led, especially firms selling on the Web, to change prices much more often. Angwin and Mattioli (2012) reported that "the fast-moving Internet pricing games used by airlines and hotels are now moving deeper into the most mundane nooks of the consumer economy. Deploying a new generation of algorithms, retailers are changing the price of products from toilet paper to bicycles on an hour-by-hour and sometimes minute-by-minute basis." The spreading practice of changing prices dynamically and the acceptance of dynamic pricing by consumers allow manufacturers to also change prices more often and better match capacity to demand. Some direct-selling manufacturers, such as Dell Inc. and Lenovo Ltd., adjust prices so often that consumers use price-tracking websites (e.g., http://www.pricegrabber.com) to track their prices.

Unlike previous studies that focus on a single lever, our paper studies the full effect of the three decisions—pricing, sales, and production—and examines their relative importance. This paper makes three major contributions: (1) characterizing the structure of the optimal policies, (2) finding that dynamic pricing is a very useful lever compared to holding inventory and allowing backorders for a new product, and (3) comparing the effectiveness of three major tools for a capacity-constrained new product: dynamic pricing, deliberate backordering, and delaying launch. We find that dynamic pricing is the most effective tool.

The problem we study is within the context of new product demand diffusion. The Bass model (1969) is a classic model that characterizes demand diffusion dynamics for an innovative product. It assumes that (i) products can be produced in any quantities and are always available for purchase, and (ii) price is fixed or has no impact on the diffusion process. We modify the classical Bass model by imposing capacity constraint and incorporating price into the model through an exponential multiplier. We analyze the simultaneous optimal pricing, production, and inventory decisions, and characterize the structure of optimal policies. Qualitatively, capacity makes the pricing, production, and inventory decisions more complicated because there is a potential benefit in influencing demand dynamically with pricing to match demand with available capacity both immediately and in future. We show that analyzing production and inventory decisions with pricing as a lever can lead to results that fundamentally differ from those when the pricing lever is not available.

Assuming limited capacity, we first consider the case where partial backlogging is allowed and then consider the complete lost sales case. The optimal demand trajectory is relatively simple, being at most bimodal in the partial backlogging case and unimodal in the lost sales case. However, the optimal pricing structure is more complicated. This leads us to consider how simpler pricing and production policies perform compared with optimal policies. In a numerical study, we show that a simple unimodal pricing structure behaves extremely well in all cases, with optimality gaps of around 0.1% or below. Thus, our first significant conclusion is that relatively simple pricing policies can work well but only if they are integrated with the production decision. Another interesting issue is how much a firm loses by not being able to change prices during a product's lifecycle, not being able to build inventory of the product or backorder it. Our results indicate that pricing is a much more effective tool than inventory or backordering, and losing the capability to adjust prices affects profits much more severely than losing the capability to hold inventories or backorder customers. We specifically compare the effectiveness of dynamic pricing with the two policies analyzed in the literature, early build-up of inventory and purposeful backlogging of customers (even though inventory is available), and show that dynamic pricing significantly outperforms the other two. Thus, our second important contribution is to show that the ability to dynamically change prices can be a much more effective lever than operations-focused levers alone. We believe this is significant given the recent trend in price mutability.

While in some industries firms can increase or decrease prices multiple times, other firms may only be able to reduce prices and only at certain times. We also consider situations where firms may only be able to reduce prices and characterize the optimal pricing structure over time. Our numerical study indicates, however, that the ability to be able to increase prices during the life cycle of a product can be crucial in capacity-constrained situations to better match supply and demand.

Our basic model assumes the firm's capacity is constant over time. We conducted numerical studies and observed the same policy structures as in our model when capacity is increasing over time (rather than constant). Even though our basic model is deterministic, we consider a natural stochastic extension numerically. Our results indicate that though each sample path may vary, the average demand patterns exhibit very similar structure to our deterministic model. Finally, we test a heuristic based on the solution of the deterministic model for the stochastic case, and our numerical results indicate that the solution to the deterministic problem can be used as a very effective solution for the stochastic problem when appropriately used.

The rest of this paper is organized as follows. In §2 we review the relevant literature. The Bass model is modified to incorporate capacity constraint and pricing in §3. Section 4 analyzes the optimal decisions for limited capacity with partial backlogging, and §5 analyzes the case with lost sales. In §6 we present a numerical study to gain insights into the effectiveness of the pricing, inventory, and backordering levers in new product introduction. A number of extensions are discussed in §7. Finally, we provide conclusions in §8.

2. Literature Review

Our paper is related to the stream of literature on pricing and production control under capacity constraints. There is a rich body of research in both capacitated production control and capacitated dynamic pricing. The earliest production control model with limited capacity is the classic economic production quantity model (Taft 1918). Later work focuses on periodic-review stochastic demand and analyzes the optimal inventory ordering policy structure (e.g., Federgruen and Zipkin 1986a, b; Kapuscinski and Tayur 1998). Unlike most inventory models in operations management, in our paper, demand is interdependent across periods through word-of-mouth effect. Also, the above inventory control papers aim at cost minimization and do not factor in price changes. Dynamic pricing literature, on the other hand, assumes fixed inventory but allows price changes over time. For example, Bitran and Mondschein (1997) study a seller adjusting prices for seasonal products under both continuous-time and periodic-review settings. Su (2007) assumes customers are strategic and finds that the optimal pricing structure depends on customer heterogeneity. Optimal price is decreasing over time when high-value customers are proportionately less patient and increasing otherwise. In our model, the optimal pricing trajectory may also be affected by the relative impacts of mass media and word-of-mouth effect on customers' behavior.

Our work is more closely related to the joint inventory and pricing problems. One of the earliest papers is Thomas (1970), which studies a joint pricing and production problem with deterministic demand function and provides an efficient solution algorithm. Federgruen and Heching (1999) examine a periodic-review system with uncertain demand. Later, Chen and Simchi-Levi (2004a, b) prove the optimal policy structure to be an (s, S, p) policy when there is a fixed ordering cost and demand is additive. Deng and Yano (2006) study the deterministic problem with the existence of capacity constraint, and also focus on solution procedures. In addition to investigating the optimal policy structure, the benefit of dynamic pricing is examined in a number of papers. Feng (2010) studies the joint inventory and pricing problem when capacity is both constrained and uncertain. She characterizes the optimal policy structure and finds that dynamic pricing can have significant benefit under either supply limit or supply uncertainty, or both. Allon and Zeevi (2011) consider the simultaneous determination of pricing, production, and capacity investment. For a special case of their model (no inventories allowed, and prices are allowed to decrease only over time), they show that capacity and pricing are strategic substitutes. Instead of allowing prices to change freely, some papers only allow limited number of price changes. Çelik et al. (2009, p. 1206) acknowledge the "complex and counterintuitive nature of optimal price-adjustment policies" and suggest a number of heuristics that limit the number of price changes. Netessine (2006) analyzes the dynamic pricing of a fixed capacity with infrequent price changes. His numerical results indicate that the benefits of additional price changes are convex decreasing. Although our model assumes full flexibility of price adjustments, we also numerically explore the effectiveness of pricing policies where prices are changed less often. Elmaghraby and Keskinocak (2003) provide a comprehensive literature review on intertemporal pricing problems in the presence of inventory considerations.

In all of the papers cited above, demand in each period is independent and the benefit of dynamic pricing mostly stems from the ability to deplete unused inventory. A key factor that distinguishes our paper from the above-described literature is that we study new products for which the demand over time is interdependent. A sale today can generate new sales tomorrow through word of mouth. Therefore, in our paper, dynamic pricing not only helps match supply with demand in the current period, but it also helps shape demand in the future to better fit with supply. Our paper confirms previous insights that dynamic pricing improves profit, and it also demonstrates another benefit of dynamic pricing for new product, which is to influence diffusion process. In addition, in our model, pricing, production, and sales policies exhibit time-dependent structures different from mature products.

Our modeling of the new product diffusion process is built on the well-recognized Bass model (1969). The Bass model describes the instantaneous demand rate at time t as

$$d(t) = \left[p + \frac{q}{m}D(t)\right][m - D(t)],$$

where $D(t)$ is the cumulative demand up to time t, m is the market size (the remaining potential market size is $m - D(t)$), and p and q are labeled as "innovation coefficient" and "imitation coefficient" to represent the effects of direct communication and word of mouth, respectively. Since the Bass diffusion model was introduced, it has been used to describe and forecast demand in various industries. In addition to the original consumer durable goods markets, these include, e.g., technology (Bass et al. 2001), pharmaceutical firms (Vakratsas and Kolsarici 2008), service (Libai et al. 2009), tourist facility management (Hsiao et al. 2009), and media services (Seol et al. 2012). Bass (2004, p. 1833) comments that "applications of the model have been shown to apply to a much wider class of products and services and it has become especially significant in forecasting B2B products and services of many categories including telecom services and equipment, component products such as semiconductor chips, medical products, and many other technology-based products and services." The Bass model has been refined and extended in multiple directions; see Mahajan et al. (1990) and Peres et al. (2010) for thorough surveys. Later literature, e.g., Mahajan et al. (1990), assumes the same functional form but interprets p and q as external and internal factors. External factors might have the form of direct communications from mass media, such as advertisement. Internal factors refer to the internal communications between potential customers and customers who have already purchased. In the rest of the paper, we adopt this terminology and refer to p and q as external and internal influences.

A dimension particularly relevant to our work is the effect of pricing in diffusion models, which is studied primarily in the marketing literature. Robinson and Lakhani (1975) are among the first to incorporate price into a diffusion model. They assume that the diffusion rate is modified by $e^{-kp(t)}$, where $p(t)$ is price at time t and k is a constant, and production cost decreases because of learning-curve effects. The paper examines a number of intuitive pricing strategies, including constant price, constant-margin pricing, and constant-return-rate pricing. Through numerical examples, it is shown that all of these strategies can be far from optimal. The exponential form of price dependence is among the most common forms of incorporating price in diffusion models, and we use it in our model. There exist, however, some variations in how price has been used in diffusion models. Some of the papers assume the size of the potential market to be affected by price, $m = m(p)$, or assume adoption rate to be affected by price, such as Robinson and Lakhani (1975), Dolan and Jeuland (1981), and so on. Another alternative is to model customer individual behavior as maximizing individual utility functions (Kalish 1985). Krishnan et al. (1999) use the generalized Bass Model to characterize the optimal price trajectory. Sethi et al. (2008) use an alternative "Sethi model" to study the optimal pricing and advertising decisions for new products, and Krishnamoorthy et al. (2010) extend it to a duopoly setting. Dockner and Jorgensen (1988) consider the pricing decisions for new products in oligopoly setting. Lehmann and Esteban-Bravo (2006) identify situations when offering customers incentives can jumpstart the diffusion process.

All of the above models assume sufficient availability of products; i.e., production capacity is never a constraint. We show that coexistence of pricing flexibility and capacity constraint influences both the production and pricing decisions and that the resulting policies may have a different structure than policies described in the existing literature. There have been several papers, primarily in the operations literature, that consider the capacity-constrained diffusion problem. Jain et al. (1991) consider a diffusion model under capacity constraint. Through numerical comparisons they find the demand pattern with limited capacity to be negatively skewed and empirically verify such behavior using new telephone market data from Israel. Kumar and Swaminathan (2003) and Ho et al. (2002, 2011) propose models that include capacity constraint in a diffusion model and analyze the optimal operational and marketing decisions. Unlike the papers in the marketing literature, these papers assume price is fixed and focus on optimal production and backlogging policies. To the best of our knowledge, Shen et al. (2011) is the only paper that studies a new product with both capacity constraint and pricing, but its focus is different. Shen et al. (2011) focus on only the sales policy for a new product with capacity constraint and show that a myopic sales policy is suboptimal when price is constant but is optimal when price can be dynamically adjusted. On the other hand, our paper studies the joint effect of pricing, sales, and production decisions. To reiterate, we focus on a diffusion model incorporating both realistic features—capacity constraints and pricing flexibility—and evaluate appropriate policies that should be used by managers during product introductions.

3. Model

Consider a diffusion process of a new product with limited capacity. We use a modification of the Bass diffusion model (1969), where price, production, and shipping are decision variables that need to be determined during the whole product life cycle. Inventory dynamics are determined by the difference of production and shipping. The Bass model assumes that a new durable product is introduced to a potential market of size m. Given cumulative demand $D(t)$ at time t, demand rate $d(t)$ is given by

$$d(t) = \left[p + \frac{q}{m}D(t)\right][m - D(t)],$$

where p is the external influence and q is the internal influence.

Capacity shortage not only results in immediate unsatisfied demand, but it also influences the future diffusion process of demand because unsatisfied customers cannot spread product information without having experienced it. Let $S(t)$ denote the cumulative deliveries (or shipments; we use shipments and deliveries to customers interchangeably throughout the paper) at time t. Thus, in the case when backlogs are allowed, we assume that backlogged customers start generating word of mouth only when the product is delivered to them.

We make three assumptions about customer behavior:

1. We do not model detailed customer strategic purchasing behavior. Some customers may act strategically and wait for lower prices and some customers may not. We assume that $d(t)$ represents the overall demand we face as a function of the price we set.

2. If the firm is not able to fully satisfy demand at time t, a fraction ξ of the unsatisfied customers is backlogged with backlogging cost b per customer per unit time. Backlogged customers wait until they obtain the product, while the rest of the customers leave and never return. The lost sales case is captured by $\xi = 0$ and the complete backlogging case by $\xi = 1$. For backlogged customers, it is critical to specify which price is charged. We assume that customers pay the price offered when they place orders and the firm collects the payment immediately, but backlogged customers are not included in the total number of shipped units, $S(t)$, that generate the word-of-mouth effect. We discuss alternatives to modeling the time for payment collection and spreading word-of-mouth effect in §7.

3. When the firm produces new units, backlogged customers are given priority over new customers in receiving the product. All backorders are satisfied at the end of horizon $t = T$ at production cost. All remaining inventory has a salvage value of $c_s < c$ per unit.

During a finite horizon $[0, T]$, a firm with fixed capacity K determines price $\pi(t)$, shipping rate $s(t)$, and instantaneous production rate $x(t)$, to maximize total discounted profit with discount factor $r \geq 0$. Production incurs linear cost c, and excess inventory $I(t)$ is stored with linear holding cost h.

We assume that the price for the product at time t, $\pi(t)$, affects both the external influence p and the internal influence q in the same fashion, i.e., $p(\pi) = pe^{-\theta\pi}$ and $q(\pi) = qe^{-\theta\pi}$, as is typically assumed in the marketing literature (e.g., Dolan and Jeuland 1981, Robinson and Lakhani 1975). To make the proofs notationally simpler, we use $\theta = 1$, although $\theta \neq 1$ does not change the structure of the optimal demand and price. Thus, the production rate is constrained by capacity and price is an additional decision variable:

$$d(t) = \left[p + \frac{q}{m}S(t)\right][m - D(t)]e^{-\pi(t)}.$$

We define $d_B(t) = [p + (q/m)S(t)][m - D(t)]$, and therefore $d(t) = d_B(t)e^{-\pi(t)}$. Clearly, $\pi(t) = \log d_B(t) - \log d(t)$.

A discrete analog of our model would have the following interpretation. In period t, the firm's starting inventory is $I(t)$ and backlog is $W(t)$. The firm decides to produce quantity $x(t)$ (subject to capacity constraint K), sets price $\pi(t)$, and observes demand $d(t)$. It decides that it will ship a quantity $s(t)$ that cannot exceed $x(t) + I(t)$. Any remaining product becomes inventory in the next period $I(t+1)$. If $W(t) > s(t)$, then the new backlog $W(t+1)$ for period $t+1$ becomes $W(t) - s(t) + \xi d(t)$. If $W(t) \leq s(t)$, then the new backlog becomes $\xi(d(t) - (s(t) - W(t))^+$.

We analyze the continuous-time model. In this model, we determine at all points in time t, the price level $\pi(t)$, the production rate $x(t)$, and the shipping rate $s(t)$. The price level determines the demand rate $d(t)$ we observe. We first focus on the instantaneous revenue collected at time t. Because backlogging has higher priority than new demand, there are no shipments to new customers as long as backlog remains positive, $W(t) > 0$; i.e., each unit of production is immediately used to satisfy a backlogged customer, and thus revenue is collected at a rate of $\xi\pi(t)d(t)$. (This is because we assume backlogged customers had already paid and we only collect revenues from new customers who agree to be backlogged.) If $W(t) = 0$, then $s(t) + \xi[d(t) - s(t)]$ customers will purchase, among whom $\xi[d(t) - s(t)]$ will be backlogged, and the collected revenue is $\pi(t)[s(t) + \xi(d(t) - s(t))]$. Once again, we assume that at time T, all backlogs are satisfied and any remaining inventory has a salvage value of $c_s < c$ per unit. Thus, our objective function can be stated as follows:

$$\Pi = \max_{\pi(t),\, s(t),\, x(t)} \int_0^T \Big\{\pi(t)[\xi d(t) + (1-\xi)s(t)1_{\{W(t)=0\}}] - cx(t) - hI(t) - bW(t)\Big\}e^{-rt}\,dt - cW(T)e^{-rT} + c_s I(T)e^{-rT} \tag{1}$$

$$\text{s.t.} \quad d(t) = \left[p + \frac{q}{m}S(t)\right][m - D(t)]e^{-\pi(t)}, \tag{2}$$

$$\mathrm{d}D/\mathrm{d}t = d(t), \tag{3}$$

$$\mathrm{d}S/\mathrm{d}t = s(t), \tag{4}$$

$$\mathrm{d}I/\mathrm{d}t = x(t) - s(t), \tag{5}$$

$$\mathrm{d}W/\mathrm{d}t = \xi d(t) - s(t) + (1-\xi)s(t)1_{\{W(t)=0\}}, \tag{6}$$

$$I(t) \geq 0, \quad W(t) \geq 0, \quad 0 \leq x(t) \leq K, \quad 0 \leq s(t)1_{\{W(t)=0\}} \leq d(t), \tag{7}$$

$$D(0) = S(0) = I(0) = W(0) = 0. \tag{8}$$

The objective function (1) includes the revenue rate for new customers minus production cost, inventory holding cost, and backlogging cost. Equation (2) describes the diffusion process. Equations (3)–(6) define cumulative demand, cumulative shipments, inventory, and backlogging dynamics, respectively. When backlog $W(t) = 0$, it (potentially) increases at a rate of $\xi[d(t) - s(t)]$; otherwise, with positive backlog, fraction ξ of new demand is backlogged, whereas new production is used for previously backlogged demand. Constraints (7) require nonnegative inventory, nonnegative backlogging, production not to exceed available capacity, and shipping rate smaller than demand rate in the case of no backlog. The constraints for $s(t)$, when backlog occurs, are implicitly given by Equations (5)–(7). Equation (8) provides the initial conditions. All models we consider are a special case of this general formulation.

We will first examine the optimal demand and price trajectories when unsatisfied demand can be partially backlogged. We show that the price and demand trajectories may have a complicated structure. Then, we examine the optimal policies when all unsatisfied demand is lost. We will show that the optimal demand has a unimodal trajectory whereas price may be bimodal. Two special cases are discussed: when capacity is unlimited and when holding inventory is not possible, as optimal policies possess a simpler structure in both cases.

4. Policy Structure Under Partial Backlogging

First, we consider the case of partial or full backlog, $0 < \xi \leq 1$. As $\pi(t) = \log d_B(t) - \log d(t) = \log[m - D(t)][p + (q/m)D(t)] - \log d(t)$, choosing optimal $\pi(t)$ is equivalent to choosing optimal $d(t)$; therefore, we use $d(t)$, $s(t)$, and $x(t)$ as decision variables. The Hamiltonian depends on whether the backlog is positive:

1. If $W(t) = 0$,

$$H(D, S, I, W, d, x, s, \lambda_1, \lambda_2, \lambda_3, \lambda_4, t) = (\log d_B(t) - \log d(t))(\xi d(t) + (1-\xi)s(t)) - cx(t) - hI(t) - bW(t) + \lambda_1(t)d(t) + \lambda_2(t)s(t) + \lambda_3(t)(x(t) - s(t)) + \lambda_4(t)\xi(d(t) - s(t)). \tag{9}$$

2. If $W(t) > 0$,

$$\begin{aligned} &H(D, S, I, W, d, x, s, \lambda_1, \lambda_2, \lambda_3, \lambda_4, t)\\ &\quad = (\log d_B(t) - \log d(t))\xi d(t) - cx(t) - hI(t) - bW(t)\\ &\qquad + \lambda_1(t)d(t) + \lambda_2(t)s(t) + \lambda_3(t)(x(t) - s(t))\\ &\qquad + \lambda_4(t)(\xi d(t) - s(t)). \end{aligned} \tag{10}$$

It is possible to interpret λ_1, λ_2, λ_3, and λ_4, respectively, as the discounted shadow prices of cumulative demand, cumulative sales, inventory, and backorders. To avoid trivialities, we assume that $b > rc$, which implies that the backlogging cost is higher than what the firm can save by postponing production a unit of time; otherwise, the firm may have an incentive to intentionally delay all production till the end of the horizon. By Pontryagin's maximum principle, the optimal demand, sales, and production $(d^*(t), s^*(t), x^*(t))$ maximize H subject to constraints $I(t) \geq 0$, $W(t) \geq 0$ and $0 \leq x(t) \leq K$. Because of the constraints on the state variables $I(t)$ and $W(t)$, the analysis is complicated in two aspects. First, the terminal value $\lambda_3(T)$ is not necessarily zero, but can take any nonnegative value because of the salvage value. The same applies to $\lambda_4(T)$. Second, $\lambda_3(t)$ and $\lambda_4(t)$ may be discontinuous. When the state constraint becomes binding, i.e., when $I(t)$ or $W(t)$ transitions from positive to zero, $\lambda_3(t)$ (or $\lambda_4(t)$) can have downward jumps.

First we focus on the production strategy. The production strategy depends on the discounted shadow price of inventory λ_3. When λ_3 is greater than the production cost, the firm produces as much as possible, which is full capacity. If λ_3 is smaller than the production cost, it is not beneficial to fully utilize capacity. In that case, the firm's production perfectly matches its current demand. In Lemma 1, we show that inventory, if available, is used to immediately clear any backorders and, by the same token, if no inventory is available, the firm uses all capacity to satisfy the backorders rather than build any inventory or leave any capacity unused.

Lemma 1. *For any open interval,* (a) *if $I^*(t) > 0$, then almost everywhere $x^*(t) = K$ and $\lambda_3(t) \geq c$; and* (b) *if $I^*(t) = 0$, then almost everywhere $x^*(t) = s(t)$.*

All proofs can be found in the online appendix (available as supplemental material at http://dx.doi.org/10.1287/msom.2013.0447).

Intuitively, Lemma 1(a) states that a firm cannot hold inventory and produce less than its capacity at the same time. If this were the case, then the firm would have been better off by shifting some production from a previous time to the present. By doing so, current demand can still be satisfied while production cost is delayed and inventory cost is decreased. For Lemma 1(b), note that if inventory remains zero for any open interval, production and shipping must be equal. Using similar logic, it is not optimal to have positive inventory at the end of the horizon since the excess inventory at the end of the horizon has a salvage value lower than production cost and incurs extra holding cost.

Lemma 2. *There is no inventory left at the end of the horizon; i.e., $I^*(T) = 0$.*

Therefore, the optimal decisions can be characterized as a function of inventory and backorder levels as well as the discounted shadow prices.

Lemma 3. *If $b > rc$, then the optimal demand is given by the following*:

(a) *If $W(t) > 0$ and $I(t) = 0$, then $\log d^*(t) = \log d_B(t) - 1 + (1/\xi)\lambda_1(t) + \lambda_4(t)$.*

(b) *If $W(t) = 0$ and $I(t) > 0$, then $\log d^*(t) = \log d_B(t) - 1 + \lambda_1(t) + \lambda_2(t) - \lambda_3(t)$.*

(c) *If $W(t) = 0$ and $I(t) = 0$, then $\log d^*(t) = \min(\log K, \log d_B(t) - 1 + \lambda_1(t) + \lambda_2(t) - c)$.*

We are now ready to characterize the optimal demand and price trajectories. We show that there is at most one backlogging interval and one inventory interval, while the optimal price trajectory is more complicated (at most three modes).

Theorem 1. *In a make-to-stock environment with partial backlogging, if $b > rc$ and if demand is continuous, then there is at most one backordering interval, and the optimal price has at most three local maxima. There exist time thresholds $0 \leq t_1^A \leq t_2^A \leq t_3^A \leq t_4^A \leq T$, such that*

(a) *optimal demand $d^*(t) \leq K$ and increases on $(0, t_1^A)$, $d^*(t)$ is unimodal on (t_1^A, t_2^A), $d^*(t) = K$ on (t_2^A, t_3^A), $d^*(t)$ is unimodal on (t_3^A, t_4^A), and $d^*(t) \leq K$ and decreases on (t_4^A, T);*

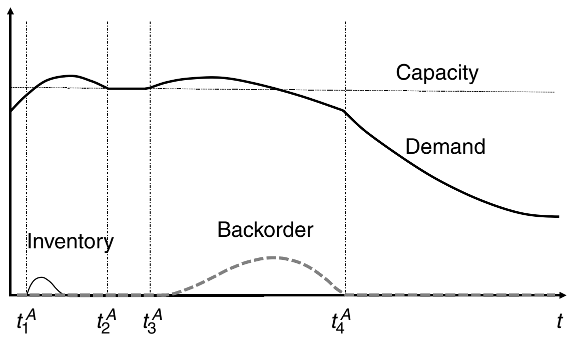
(b) *optimal price $\pi^*(t)$ is unimodal on $(0, t_3^A)$, (t_3^A, t_4^A), and (t_4^A, T);*

(c) *optimal production $x^*(t) = d^*(t)$ on $(0, t_1^A) \cup (t_4^A, T)$, and $x^*(t) = K$ on (t_1^A, t_4^A), and inventory $I(t) > 0$ and is unimodal on (t_1^A, t_2^A);*

(d) *optimal sales $s^*(t) = d^*(t)$ on $(0, t_3^A) \cup (t_4^A, T)$, and $s^*(t) = K$ on (t_3^A, t_4^A), and backlog $W(t) > 0$ on (t_3^A, t_4^A). In the special case where the product is made to order (holding cost $h = +\infty$), $t_1^A = t_2^A$, the optimal demand trajectory is unimodal over time on $[0, T]$, and the optimal price is bimodal.*

Theorem 1 illustrates that when capacity is insufficient, the optimal demand trajectory has a "truncated-bimodal" shape whereas the optimal price trajectory has at most three maxima. A typical situation is shown in Figures 1 and 2, where the selling season includes a positive inventory period and a positive backorder period. We can thus divide the product life cycle into five phases. In Phase I ($[0, t_1^A]$), the

Figure 1 **Optimal Demand, Inventory, and Backorder Trajectories with Make to Stock and Partial Backordering**

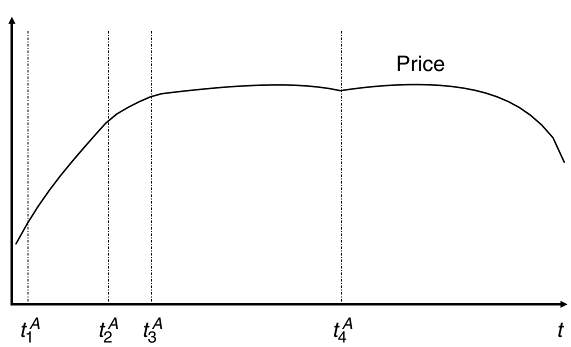


product has just been introduced with no or minimal word of mouth. Thus, demand is low and a low promotion price is offered to boost demand. The firm's production and shipments match demand. In Phase II ($[t_1^A, t_2^A]$), demand is still below capacity, but the firm starts producing at full capacity and accumulates inventory. Demand at some time exceeds the firm's capacity and inventory is naturally depleted. In Phase III ($[t_2^A, t_3^A]$), with no inventory, the firm continues selling and shipping at capacity rate. To ensure demand is not too high, and does not exceed capacity, the firm may need to raise prices. In Phase IV($[t_3^A, t_4^A]$), demand is more closely matched to capacity; therefore, the firm prefers to first backlog customers for a short duration (instead of increasing prices to discourage demand) and continues producing at capacity level even after demand decreases and the firm can satisfy backorders. Finally, in Phase V ($[t_4^A, T]$), the market is nearly saturated and demand continues to drop.

We see above that the ability to hold inventory allows the firm to start producing larger amounts than demand early on, so that it can sell more than what it is producing at a later time. Thus, pricing does not negate the need for inventory build-up. However, in this case, with pricing, we do not have to resort to reducing shipments and turning away customers even though we have inventory, which would be the case with constant pricing (Kumar and Swaminathan 2003). On the other hand, the ability to take backorders allows the firm to secure more customer demand when capacity is tight.

Theorem 1 indicates that the optimal pricing policy has a complicated structure. However, in practice, it is relatively rare to observe companies that intentionally design multimode pricing plans during a product's life cycle. In §6 we evaluate use of simpler (unimodal) pricing. Our numerical results indicate that its performance is very close to optimal when used in conjunction with optimal inventory and production policies. However, first we characterize the policy in a few special cases where the policy simplifies considerably.

Figure 2 **Optimal Price Trajectories with Make to Stock and Partial Backordering**



5. Policy Structure Under Lost Sales

In this section, we examine the case when all unsatisfied demand is lost and first describe the optimal shipping policy. When price is exogenous, it may be optimal not to satisfy all demand for the lost sales case. However, we show below that this never happens with dynamic pricing. Price will be adjusted so that demand never exceeds availability of product and all demanded units are shipped. Consequently, lost sales never occur. By simplifying the previous formulation, and in a similar way to the proof of Theorem 1, we can obtain the structure of the optimal demand and price trajectory.

Theorem 2. *Under the lost sales assumption, $s^*(t) = d^*(t)$ for all t, and there exist time thresholds $0 \le t_1^B \le t_2^B \le t_3^B \le T$, such that*

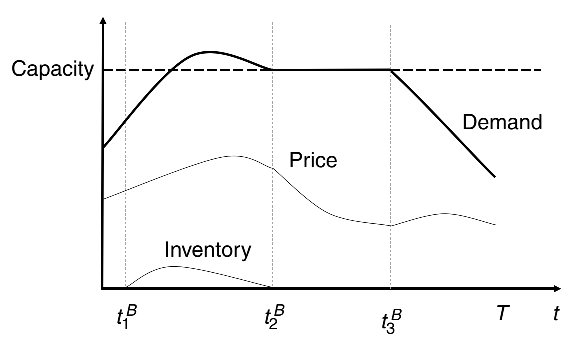
(a) *optimal demand $d^*(t)$ increases on $(0, t_1^B)$, is unimodal on (t_1^B, t_2^B), $d^*(t) = K$ on (t_2^B, t_3^B), and decreases on (t_3^B, T);*

(b) *optimal price $\pi^*(t)$ increases on $(0, t_1^B)$, is unimodal on (t_1^B, t_3^B), and is unimodal on (t_3^B, T);*

(c) *optimal production $x^*(t) = d^*(t)$ on $(0, t_1^B) \cup (t_3^B, T)$, and $x^*(t) = K$ on (t_1^B, t_3^B), and inventory $I^*(t) > 0$ and is unimodal on (t_1^B, t_2^B) and 0 otherwise.*

Theorem 2 is illustrated in Figure 3. The demand and inventory trajectories appear similar to Figure 1

Figure 3 **Optimal Policies with Lost Sales in a Make-to-Stock Environment**



except that demand never exceeds capacity. The backorder phase disappears and price trajectory has at most two local maxima. Until t_1^B, the firm's production matches demand. At t_1^B, demand is still below capacity, but the firm starts producing at full capacity rate and accumulates inventory. Demand at some time exceeds the firm's capacity and inventory is naturally depleted. At t_2^B, inventory is sold out and the firm continues selling and shipping at capacity rate in time interval (t_2^B, t_3^B). Finally, demand drops below capacity after t_3^B.

We next consider two special cases, first when the firm produces to order (no inventory is held) and then the case when capacity is not a limiting factor.

5.1. Special Case: Make to Order and Lost Sales

When the firm produces to order and unsatisfied demand is lost, the optimal demand and price trajectories have an even simpler form. As inventory cannot be held, the firm will not start producing in anticipation of demand and will not be able to satisfy any demand that is higher than capacity rate. We summarize the results below.

Corollary 1. *Assume that products are made to order and unsatisfied demand is lost. There exist time thresholds $0 \le t_1^C \le t_2^C \le T$, such that*

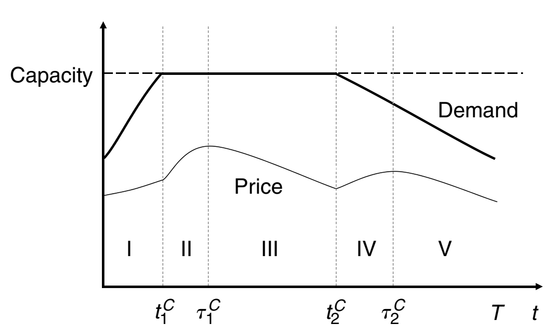
(a) *optimal demand $d^*(t)$ increases on $(0, t_1^C)$, $d^*(t) = K$ on (t_1^C, t_2^C), and decreases on (t_2^C, T);*

(b) *optimal price $\pi^*(t)$ increases on $(0, t_1^C)$, is unimodal on (t_1^C, t_2^C), and unimodal on (t_2^C, T);*

(c) *there exists $\bar{K}$ such that $t_1^C = t_2^C$ for all $K \ge \bar{K}$.*

Corollary 1 and Figure 4 show that in a make-to-order and complete lost sales environment, both the inventory phase and backorder phase in Figure 1 disappear and demand becomes "truncated unimodal" (there still exists only one demand peak, but when this demand peak is equal to the firm's capacity, demand may equal capacity for an interval), whereas optimal price can be bimodal. When capacity is sufficiently large, part (c) indicates that the constrained phase (t_1^C, t_2^C) disappears, and the price and demand trajectories are identical to those in the infinite-capacity case illustrated in the next subsection.

Figure 4 Optimal Policy with Lost Sales in a Make-to-Order Environment



5.2. Special Case: Infinite Capacity

It is interesting to examine the special case when capacity is sufficiently large, so that it effectively does not constrain the firm's production. Even though holding inventory is allowed, inventory's only purpose is to hedge against future capacity shortage. Therefore, under infinite capacity, we will not hold any inventory. Also, strategically backlogging some demand slows down the diffusion process and would only be useful when capacity is insufficient. Using price adjustments, we can decrease the sales (slow down the diffusion) and achieve higher profit. Thus, myopic shipments and production policies are optimal; i.e., $s^*(t) = x^*(t) = d^*(t)$. In this case, both demand and price are unimodal (we omit the proof for brevity). Let τ_d and τ_π denote the time at which we reach the maximum (peak) of demand and of price, respectively. We show that they satisfy the following relationship.

Corollary 2. (a) *When discount factor $r > 0$ and capacity $K = +\infty$, the peaks of optimal demand, and price are ordered as follows: $\tau_d \le \tau_\pi$. Further, assume that the firm does not have pricing flexibility and is forced to set the same fixed price π_F for all times $t \le T$. Consider any constant price, and let the time at which the peak demand is reached under the constant price be τ_F. Then we have that $\tau_d \le \tau_F \le \tau_\pi$.*

(b) *If discount factor $r = 0$ and capacity $K = +\infty$, then for all $t \in [0, T]$, demand rate is constant, while price is correspondingly adjusted over time:*

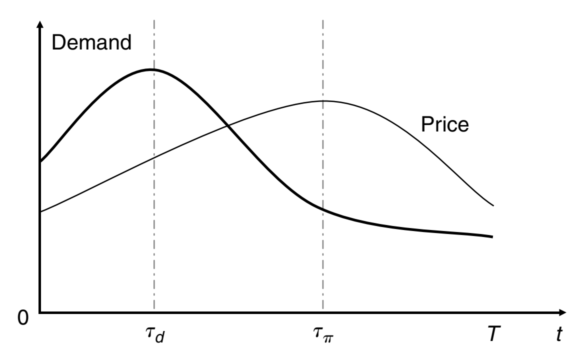
$$d(t) = \bar{d} = \frac{1}{2qT^2/m}\Big\{-(p-q)T - e^{1+c} + \sqrt{((p-q)T + e^{1+c})^2 + 4pqT^2}\Big\}$$

$$\pi(t) = 1 + c + \log\frac{(m - \bar{d}t)(p + q\bar{d}t/m)}{(m - \bar{d}T)(p + q\bar{d}T/m)} \qquad (11)$$

Further, price is decreasing on $[0, T]$ if and only if $q < p$.

Corollary 2 is demonstrated in Figure 5. Without discounting, the price structure is similar to that in

Figure 5 Optimal Demand and Price with Unlimited Capacity and Discounting



Dolan and Jeuland (1981). Interestingly, Dolan and Jeuland never analyzed the trajectory of demand. Surprisingly, the optimal demand remains flat over the time horizon. That is, the firm should modify the price, according to (11), to maintain constant demand. This may contradict the intuitive expectation that the firm might want to set a constant price over the whole time horizon when profit is not discounted. However, since the firm's myopic profit rate at time t is given by $(\pi(t)-c)d(t)=(\pi(t)-c)(m-D(t))(p+qD(t)/m)e^{-\pi(t)}$, constant price $\pi(t)=1+c$ maximizes only the firm's *myopic* profit, while ignoring the indirect effect of pricing on future product diffusion. This indirect effect takes place through cumulative demand. For example, a low price drives a faster increase in cumulative demand. Fast increase in cumulative demand is desirable at the beginning of the time horizon (because it translates into needed word-of-mouth effect), not desirable in the middle phase because of market saturation, and eventually not critical at the end of the time horizon. Therefore, if the time horizon is long enough, the firm first stimulates word-of-mouth effect through a low price, then charges a higher price, and a low price at the end of the life cycle to clear the market. Consequently, the demand should be much flatter than the case where the firm is forced to set a constant price for the whole time horizon.

It is easy to verify that $\pi(t)$ peaks at $m(q-p)/2\bar{d}q$. Therefore, price is decreasing throughout the whole life cycle if and only if $q<p$, which is easy to understand in the extreme case of $q=0$: Since the remaining market size decreases, the price correspondingly decreases. (The same logic applies to the general $q<p$ case.) Note that the corollary above has immediate implications for the interactions of marketing and operations in new product markets with fairly small discount rates and sufficient capacity. Given that the "optimal" diffusion implies constant demand, a direct lesson is that the firm would build a facility that would serve a fairly constant demand over time, whereas all potential nonstationarities are absorbed by the pricing policy. The interesting fact here is that although it is generally recommended in the operations literature to level demand to help operations, in this case, it turns out to be optimal even when operations do not create a constraint. If discounting is significant, however, the firm is more impatient to satisfy customers and stimulates demand more aggressively in the early phase of the product life cycle.

6. Numerical Study

In the last two sections, we have characterized the optimal demand, price, inventory, and backorder trajectories under different situations. Although the firm can use price, inventory, or backorders to hedge against capacity shortage, it is not clear how effective these tools are. We conduct a numerical study to compare the benefit of these alternative tools. We have shown that the optimal price trajectory has a complicated bimodal (or potentially tri-modal) structure. The interesting question is how much benefit the firm has gained from using this complicated structure as opposed to simpler structures, which we also investigate in this section.

For our numerical study, we solve the discrete-time version of the continuous-time model. Assuming there are T periods, and using D_t, S_t, I_t, and W_t to represent states at the end of each period, our decision variables are s_t, π_t, and x_t. Demand, inventory, and backorder have the following dynamics: $D_0=S_0=I_0=W_0=0$, $d_t=(m-D_{t-1})(p+q*S_{t-1}/m)e^{-\theta\pi_t}$, $D_t=D_{t-1}+d_t$, $S_t=S_{t-1}+s_t$, $I_t=I_{t-1}+x_t-s_t$, and $W_{t+1}=\xi[d_t-(s_t-W_t)^+]1_{W(t)\leq s(t)}+[W(t)-s(t)+\xi d(t)]1_{W(t)>s(t)}$. Constraints include $x_t\leq K$, $I_t\geq 0$, $W_t\geq 0$, for all $t=1,2,\ldots,T$. A discrete-time discount factor β is used to replace r.

Policy structures derived in §§4 and 5 are critical for numerical studies. The trend of demand trajectories is used to reduce the search space during optimization. The fact that inventory and backlogging each take place in only one time interval is also used to limit search space. Lemmas 1 and 2 are used to directly compute optimal production decisions. Finally, in special cases such as lost sales, sales decision can be eliminated using Theorem 2.

Jeuland (1994) reports that q is often between 0.3 and 0.5; Sultan et al. (1990) report that the external factor p has an average value 0.03 and average $q=0.38$. Therefore, we consider $p\in\{0.01,0.02,0.03,0.04,0.05\}$, and $q\in\{0.1,0.2,0.3,0.4,0.5,0.6,0.7\}$. We consider 50 periods, for a typical product with two-year selling horizon; this implies that each period roughly corresponds to two weeks, so we use holding cost per period equal to 1% of the unit cost, corresponding to 25% annual holding cost. Backorder cost is selected as 2, 6, and 10 times of holding cost. Capacity K is selected from 20 to 150 with an increment of 10, where the optimal demand is constrained for almost the entire selling horizon when $K=20$ and never constrained when $K=150$. Other parameters are $m=3{,}000$, $c=10$, $\xi\in\{0,0.3,0.6,1\}$, $\beta\in\{0.9,0.99\}$, and $\theta=0.01$.

Value of Pricing Flexibility. We define the value of pricing flexibility as

$$\begin{aligned}&(\text{Optimal profit with dynamic pricing}\\&\quad-\text{Profit with the optimal fixed price})\\&\cdot(\text{Profit with the optimal fixed price})^{-1}\times 100\%.\end{aligned}$$

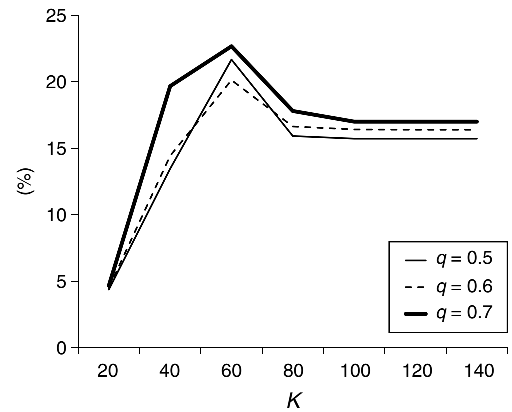
An example shown in Figure 6 illustrates that as capacity increases, the value of pricing flexibility first increases and then slowly decreases, and thus it is most beneficial when the firm's capacity is in the intermediate range and fairly beneficial for high capacity. Intuitively, if the firm has sufficient capacity, demand is seldom constrained, and thus price, although actively used, has moderate variation, driven solely by revenue maximization. When the firm has low capacity, demand is always constrained, so use of price is very important. However, since price is used solely to shape demand to fit available capacity, price does not vary dramatically. When capacity is intermediate, both of these reasons exist and the seller more actively accelerates and decelerates demand to maximize revenue and take advantage of slack capacity in the beginning and the end of the product life cycle, while constraining the sales in the middle of the product life cycle.

In a recent paper, Allon and Zeevi (2011) consider a model with production capacity and pricing decisions. They show that pricing and capacity are strategic substitutes in a special case where the firm cannot carry any inventory and price can only be adjusted downward. This result indicates that the effect of a price change is smaller on profits when capacity is higher. We note that their model assumes independent demand from one period to the next. Thus, pricing in our model plays a much more significant role because it also is a tool to influence demand diffusion over time. Therefore, a parallel question that one can ask in our model is whether having ample capacity obviates the need for pricing flexibility. As the discussion above indicates, this is not so in our model because even in the case of ample capacity, the diffusion process has to be managed through pricing, and in the case of moderate capacity the firm has to worry about diffusion in the long run and matching supply and demand in the short run. Thus, with these additional considerations, in our model's excess capacity does not lead to pricing's completely diminished importance (although there is a small decline as seen in Figure 6).

Figure 6 also shows that the value of pricing flexibility roughly increases in the internal influence q. As the customers are more easily influenced by other people's purchases, a low promotion price can create a significant word-of-mouth effect and thus is more helpful.

Value of Holding Inventory. Figure 7 provides an example for the value of holding inventory. The value of holding inventory is highest for intermediate range of capacity. However, compared to value of pricing flexibility (20%–50%), value of holding inventory is very small (0–0.76%), which implies that carrying inventory does not help much when price adjustments are a viable tool, even for a firm with potential capacity constraints.

Figure 6 **Value of Pricing Flexibility as a Function of Capacity K and Internal Influence q**

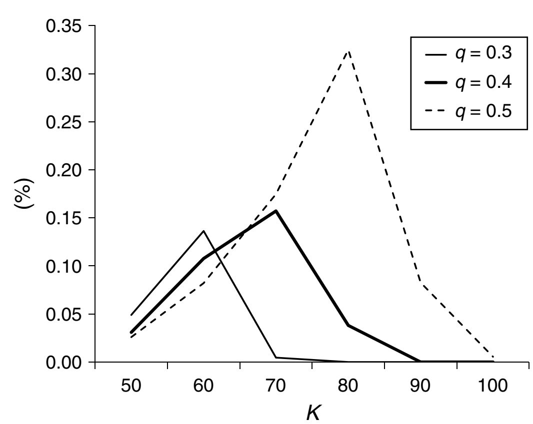


Note. Parameters: $m = 3{,}000$, $p = 0.01$, $c = 10$, $\beta = 0.99$, $h = 0.1$, $b = 0.2$, $\xi = 1$, $\theta = 0.01$, and $T = 50$.

Value of Allowing Backorders. Figure 8 illustrates the value of allowing backorders, defined as the percentage profit increase when backlogging is allowed as compared to the lost sales case. The value of allowing backorders is typically larger than that of holding inventory, ranging from 0 to 8.84%, and it decreases in capacity. Clearly, when capacity is large, backorders and lost sales do not occur very often. When capacity is small, it may, however, be significant, as shown in Figure 8. Similarly, it is higher for larger internal influence, q.

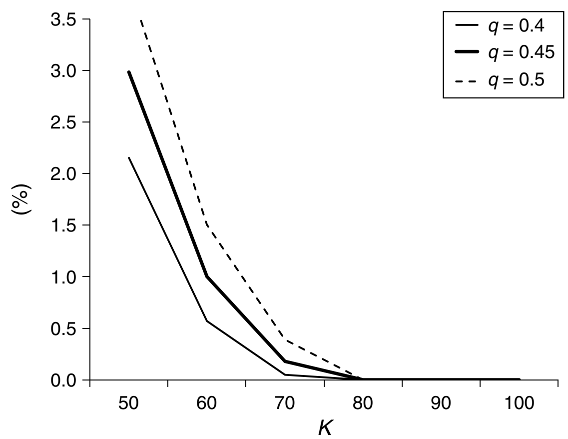
Comparison of the Optimal Pricing Strategy with Simple Pricing Strategies. Although the marketing literature has suggested that the optimal price should have either a decreasing or an increasing-decreasing structure, they have assumed that the firm has infinite capacity. We have shown that when the firm has capacity constraints, the optimal pricing strategies may have a complicated structure. However,

Figure 7 **Value of Holding Inventory as a Function of Capacity K and Internal Influence q**



Note. Parameters: $m = 3{,}000$, $p = 0.03$, $c = 10$, $\beta = 0.9$, $h = 0.1$, $b = 0.2$, $\xi = 1$, $\theta = 0.01$, and $T = 50$.

Figure 8 Value of Allowing Backorder as a Function of Capacity K and Internal Influence q



Note. Parameters: $m = 3{,}000$, $p = 0.03$, $c = 10$, $\beta = 0.9$, $h = 0.1$, $b = 0.2$, $\xi = 1$, $\theta = 0.01$, and $T = 50$.

implementing such a complicated dynamic pricing structure may be difficult, and explaining it to customers may be even harder. Therefore, we compare the optimal pricing strategy with the simple pricing strategies suggested in the marketing literature (when those simple policies are used in conjunction with optimal production/inventory policies) and examine the profit implications.

We define a DCR pricing strategy as the price decreasing over time, and an INCR-DCR pricing strategy as the price increasing and then decreasing over time; thus, the INCR-DCR pricing trajectory is unimodal. We define the optimality gap for DCR price as

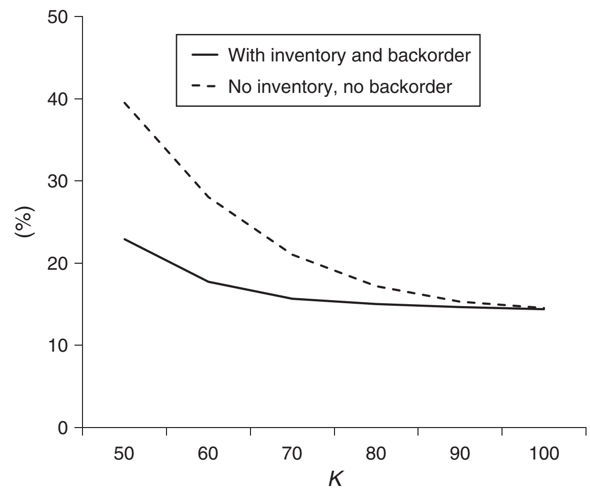
$$\begin{aligned}&(\text{Profit with the optimal price}\\&\quad - \text{Profit with the optimal DCR price})\\&\cdot (\text{Profit with the optimal DCR price})^{-1}.\end{aligned}$$

Similarly, we define the optimality gap for INCR-DCR price as

$$\begin{aligned}&(\text{Profit with the optimal price}\\&\quad - \text{Profit with the optimal INCR-DCR price})\\&\cdot (\text{Profit with the optimal INCR-DCR price})^{-1}.\end{aligned}$$

Clearly, for some cases the INCR-DCR policy is a DCR policy and the gap disappears. This is, however, infrequent, and most of the time the gap is very significant. Figures 9 and 10 provide examples of the optimality gaps for DCR and INCR-DCR price policies, respectively, and show that the gap can easily be 10% to 40% for DCR, while it is below 0.1% for INCR-DCR. This is very encouraging since we have shown that even though the optimal price trajectory may have multiple modes, the simple unimodal strategy, suggested in marketing literature without capacity constraints, actually works well in the situation of

Figure 9 Optimality Gap of Decreasing (DCR) Pricing Strategies

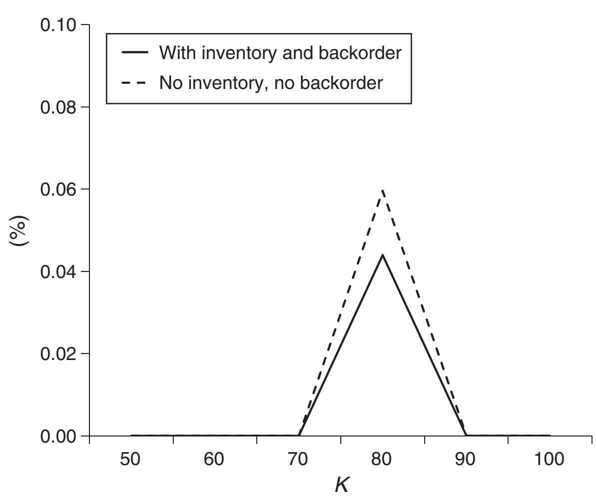


Note. Parameters: $m = 3{,}000$, $p = 0.01$, $q = 0.3$, $c = 10$, $\beta = 0.9$, $h = 0.1$, $b = 0.2$, $\xi = 1$, $\theta = 0.01$, and $T = 50$.

capacity constraints. Therefore, managers can avoid the frustration of using a complicated pricing policy as long as they focus on coordinating optimal production/inventory policies with the right INCR-DCR pricing.

Comparison of Strategies to Hedge Against Limited Capacity. When the limited capacity for a new product is hard to increase in a short period, a firm may choose different strategies to mitigate its impact on profit. Kumar and Swaminathan (2003) consider the strategy of deliberately backlogging customers even if inventory is available (hereafter referred to as the *BO* strategy). Those customers who do not receive the products immediately cannot generate word-of-mouth effect; therefore, the diffusion process

Figure 10 Optimality Gap of Increasing-Decreasing (INCR-DCR) Pricing Strategies



Note. Parameters: $m = 3{,}000$, $p = 0.03$, $q = 0.4$, $c = 10$, $\beta = 0.9$, $h = 0.1$, $b = 0.2$, $\xi = 1$, $\theta = 0.01$, and $T = 50$.

can be slowed down, reducing the sales loss during peak time. Ho et al. (2002, 2011) consider myopic sales policy but allow delay of launch time (hereafter referred to as the *DL* strategy). Delaying launch allows the firm to produce in advance and build inventory prior to launch. In our paper, we consider the strategy of dynamically adjusting prices (hereafter referred to as the *DP* strategy). We compare these three strategies. As a benchmark case, we calculate the optimal profit when none of these strategies is used; i.e., when the product is launched at time 0, myopic sales strategy is used, and an optimal static price is determined. This case is labeled as the *CP* strategy (constant pricing). With the *BO* strategy, product is launched at time 0, and with the *DL* strategy, myopic sales strategy is used so backlog is not allowed if inventory is available. Define percentage profit difference $\Delta_i = (\text{Profit}_i - \text{Profit}_{CP})/\text{Profit}_{CP} \times 100\%$, for $i = \{DP, BO, DL\}$. Then Δ_i's measure the effectiveness of each strategy compared to constant pricing.

Intuitively, the *BO* strategy should be more beneficial when the fraction of customers backordered is high and backorder cost is low, whereas the *DL* strategy has more benefit when future profit is not heavily discounted and holding costs are low. We intentionally choose parameters that favor *BO* and *DL* strategies and compare the profit gain of the *DP* strategy under these situations, by using discount factor $\beta = 0.99$, the fraction of customers backlogged $\xi = 1$, and backorder cost $b = 0.2$. We consider a total of 175 cases for $p = \{0.01, 0.02, 0.03, 0.04, 0.05\}$, $q = \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7\}$, and $K = \{20, 40, 60, 80, 100\}$. Other parameters are the same as in previous sections; i.e., $m = 3{,}000$, $c = 10$, $h = 0.1$, $\xi = 1$, $\theta = 0.01$, and $T = 50$.

The second, third, and fourth columns in Table 1 provide the average Δ_i's where, for example, the average Δ_{DP} in the 35 cases when $p = 0.01$ is 11.01%. The first observation is that Δ_{DP} is higher than both Δ_{BO} and Δ_{DL} in all cases, implying that the *DP* strategy is very effective. This is not surprising as the *DL* strategy can be considered a special case of the *DP* strategy where the prices in the before-launch periods are set extremely high, and prices in later periods are set as a constant. Similarly, the *BO* strategy can also be considered a feasible policy under the *DP* strategy where prices throughout the selling horizon are set as a constant.

In the 175 cases, Δ_{DP} ranges between 0.22% and 29.91%, Δ_{BO} ranges between 0.0 and 20.57%, whereas the *DL* strategy has a very limited impact with Δ_{DL} ranging between 0.0 and 0.31%. The external influence p appears to have a significant impact on both Δ_{DP} and Δ_{BO}. As p increases, both percentage profit differences becomes small very quickly. On the other hand, both percentage profit differences in general have an increasing or increasing-decreasing trend with respect to internal influence q and capacity limit K, but the effect is not very significant.

Table 1 **Average Percentage Profit Increase from the *CP* Strategy to the *DP*, *BO*, and *DL* Strategies**

	Avg. Δ_{DP} (%)	Avg. Δ_{BO} (%)	Avg. Δ_{DL} (%)	Avg. R_{BO} (%)	Avg. R_{DL} (%)
p					
0.01	11.01	0.81	0.03	6.29	0.27
0.02	8.37	2.21	0.01	19.29	0.18
0.03	5.78	0.67	0.05	8.65	0.86
0.04	3.93	0.52	0.01	7.25	0.25
0.05	3.33	0.67	0.01	7.98	0.30
q					
0.1	1.66	0.07	0.01	4.01	0.60
0.2	4.01	0.39	0.02	8.54	0.50
0.3	7.95	1.73	0.01	16.21	0.13
0.4	6.70	0.87	0.01	8.23	0.15
0.5	7.59	1.02	0.01	9.52	0.13
0.6	8.39	1.17	0.02	8.07	0.24
0.7	9.11	1.31	0.02	10.10	0.22
K					
20	2.49	0.42	0.02	14.05	0.80
40	8.64	2.99	0.01	25.64	0.13
60	7.82	0.87	0.02	4.42	0.26
80	6.77	0.22	0.01	1.19	0.15
100	6.71	0.19	0.01	0.89	0.15

Since the *DP* strategy always provides the most benefit, it is also interesting to examine the relative effectiveness of the *BO* and *DL* strategies compared with the *DP* strategy. We define two ratios $R_i = \Delta_i/\Delta_{DP}$ for $i = \{BO, DL\}$. This number measures the percentage of benefits from the *DP* strategy that can be captured by the *BO* and *DL* strategies. The fifth and sixth columns in Table 1 show the average values for R_i's. In general, intentionally backordering demand even when inventory is available can capture 20%–40% of the benefits from dynamic pricing, whereas delaying launch time is not very effective. Our numerical results suggest that compared with the *BO* and *DL* strategies, dynamic pricing is always very effective, but it is most beneficial when the market has a very small external influence, a not-too-small internal influence, and capacity constraint is not very tight.

7. Extensions

7.1. Timings of Word-of-Mouth Effect Generation and Payment Collection

We assumed that payment is collected immediately when a customer decides to buy a product, whereas the word of mouth starts taking place only when product is received. In practice, the timing may be different. Customers may start generating word-of-mouth effect immediately when they order, or they may make a payment only when the product is received. In this section, we examine the sensitivity of our results to the timing of word-of-mouth effect and the timing of the payment.

First, we assume that the word-of-mouth effect is generated immediately. The payment, as before, is collected immediately. Customers may or may not be able to obtain the product immediately; however, they are able to tell their family and friends about their excitement about the product (e.g., a lot of customers preorder iPhones when a new generation is realized, and there is a lot of excitement and word of mouth even before customers receive the product), which generates word-of-mouth effect. Jain et al. (1991) study this behavior and characterize the diffusion process without including price, with two different internal influence factors q_2 for backordered customers, and q_1 for customers who did not have to wait. For analytical tractability, we assume both groups of customers generate the same word-of-mouth effect. In this case, deliberately holding back sales, as in Kumar and Swaminathan (2003), does not have the benefit of slowing down the diffusion process. Myopic shipment policy is also optimal: $s^*(t) = \min(d(t), K)$ if there is no backorder, and $s^*(t) = K$ if there is backorder.

The formulation is similar to that in §3, except that (2) is changed into $d(t) = [p + qD(t)/m][m - D(t)]e^{-\pi(t)}$, indicating that all customers generate word-of-mouth effect immediately upon purchase. It is shown below that if customers can generate word-of-mouth effect immediately, the optimal policy structure is not affected:

Theorem 3. *When payment is collected and word of mouth is distributed immediately after adoption decision is made, if $b > rc$ and demand is continuous, then the optimal decisions have structures described in Theorem* 1.

Next, we examine the effect of the timing of collecting payment. We continue to assume that word-of-mouth effect is generated only by customers who receive the product but now assume that customers pay when the product is shipped. The diffusion process will remain the same as in §3. If there is positive backorder, then payment for new demand is collected after $W(t)/K$ unit of time. Thus, the instantaneous profit from the new demand is $[\pi(t) - c]d(t)e^{-rW(t)/K}$. Therefore, Equation (1) is changed into the following:

$$\begin{aligned}\Pi = \max_{d(t),\,s(t),\,x(t)} \int_0^T &\{[\log d_B(t) - \log d(t)]\\ &\cdot [\xi d(t) + (1-\xi)s(t)1_{\{W(t)=0\}}]e^{-rW(t)/K}\\ &- cx(t) - hI(t) - bW(t)\}e^{-rt}\,dt\\ &- cW(T)e^{-rT} + c_s I(T)e^{-rT}. \qquad (12)\end{aligned}$$

Because of the exponential term of $W(t)$, the shape of the optimal policy trajectory is difficult to obtain analytically. However, through numerical studies we continue to observe similar shape of optimal decisions as in the original model. When the firm does not discount the profit, the optimal decision is the same as in our original model:

Corollary 3. *If discount factor $r = 0$, then the optimal demand is a constant in* $[0, T]$. *The optimal demand and price are given by Equation* (11) *in Corollary* 2.

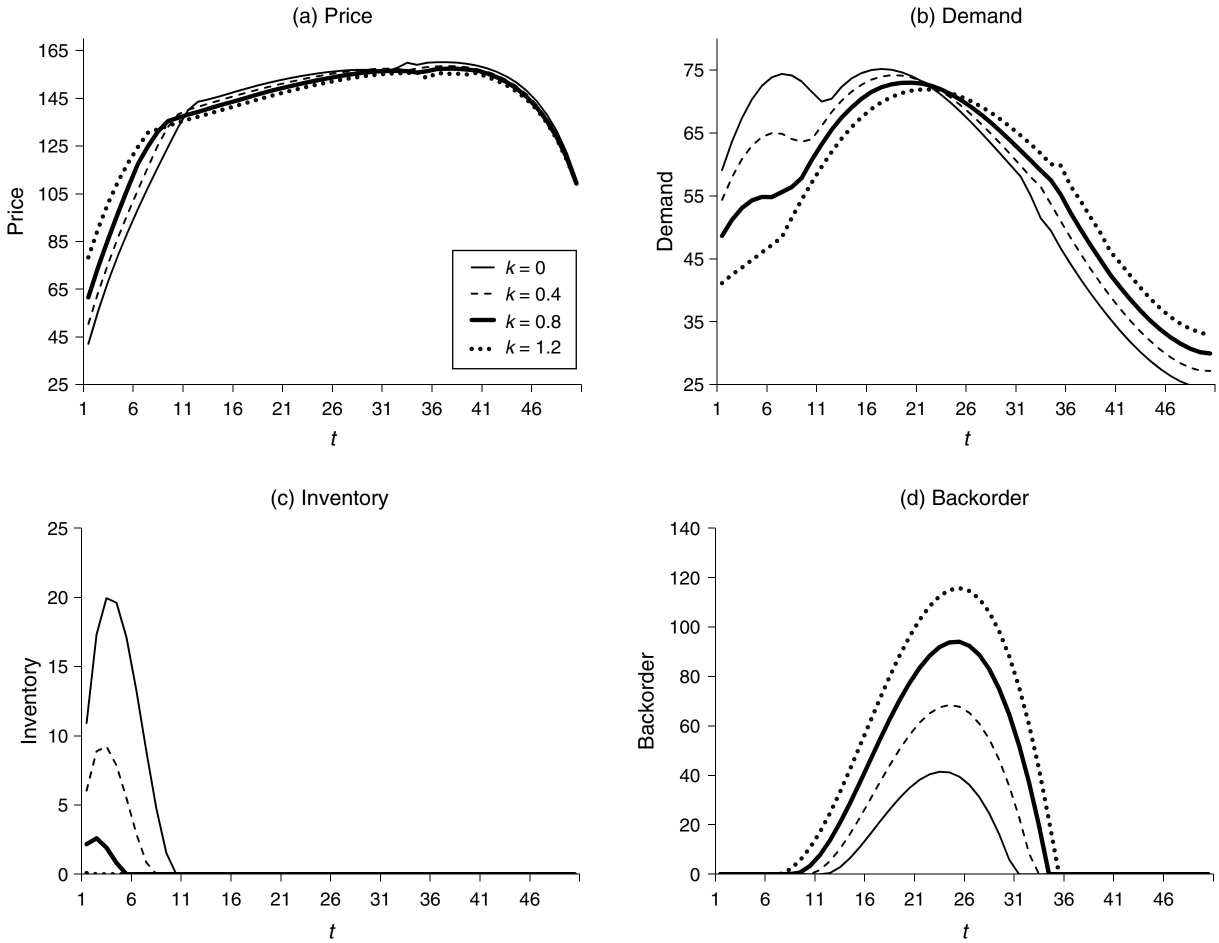
7.2. Nonincreasing Price

We assumed that price can either increase or decrease at any time and show that optimal price may have multiple local maxima. Price increases for a product during its life cycle are observed in practice. In addition, promotional instruments such as coupons and other monetary incentives are commonly offered during new product launch (e.g., Song and Parry 2009). According to the Most Memorable New Product Launch Survey in 2010, receiving a coupon ranks among the top three reasons to purchase a new product (with 82% of surveyed customers agreeing) (Celentano 2011). A marketing information supplier, Kantar Media, studies 214 new product introductions during the first nine months of 2011 and reports that digital coupons were used in 83 introductions, or 38.7% of them (Kantar Media 2011). Even though the listed prices may be kept at a constant level, these promotions effectively reduce the real prices consumers pay; therefore, customers may enjoy a lower effective price in the early phase of product introduction. However, if price is constrained to be nonincreasing in time in our model, then without the flexibility of raising price, a myopic sales policy may no longer be optimal, and the strategy of deliberately holding sales may be needed. The strategy of deliberately refusing customer requests is not common in practice, and it is hard to convince managers to do so. We assume that a myopic sales policy is used and characterize the policy structure for nonincreasing price.

For the most general case with partial backlogging and inventory, we can characterize the policy structure, which is somewhat more complex. Optimal price trajectory can have at most three constant-decreasing phases. There are only one inventory interval and one backorder interval as before.

Theorem 4. *If price can only be nonincreasing and a myopic sales policy is used, then there may exist $0 \le t_1 \le t_2 \le t_3 \le t_4 \le t_5 \le t_6 \le t_7 \le T$, such that price is constant on $[0, t_2]$, $[t_4, t_5]$ and $[t_6, t_7]$, and is decreasing in other phases. Demand is unimodal on $[0, t_3]$ and $[t_4, T]$, and $d(t) = K$ on $[t_3, t_4]$. Inventory $I(t) > 0$ on $[t_1, t_3]$, and $W(t) > 0$ on $[t_4, t_6]$.*

Figure 11 Optimal Price, Demand, Inventory, and Backorder when Capacity $K(t) = a - (kT/2) + kt$



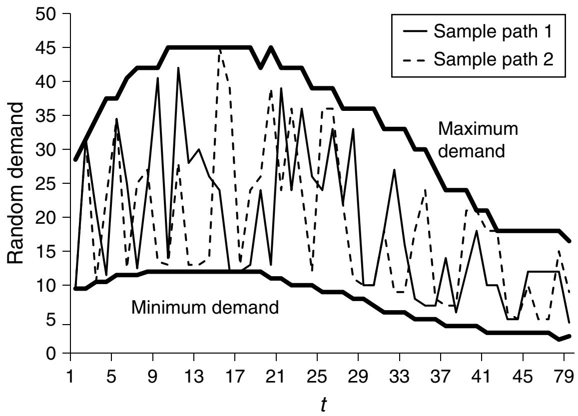
Note. Parameters: $m = 3{,}000$, $p = 0.03$, $q = 0.4$, $c = 10$, $\beta = 0.9$, $h = 0.1$, $a = 70$, $b = 0.2$, $\xi = 1$, $\theta = 0.01$, and $T = 50$.

7.3. Increasing Capacity and Other Demand Functions

In this section, we check the sensitivity of two assumptions in our model—constant capacity and exponential demand form. In both cases it is found that the main results are not affected by relaxing these assumptions. First, our model assumes that capacity is constant over time. In practice, capacity can be expanded during the product life cycle because of investment or various improvements. We provide a numerical example with capacity linearly increasing over time. For a fair comparison, we keep the average capacity the same but change the rate of its increase; i.e., $K(t) = a - (kT/2) + kt$. Figure 11 plots the optimal demand, price, inventory, and backorder trajectories when $a = 70$ and k changes from 0 to 1.2. The pricing structure remains the same. A capacity-constrained phase typically exists as shown in (b) of the figure. As k becomes larger, capacity is smaller at the beginning of the life cycle and larger at the end, and optimal demand shows a similar pattern to our original model. Time-increasing capacity results in the firm selling less at the beginning and more later. Also, inventory is used less as capacity increases faster. However, backorder is used more often for higher rates of capacity increase. The reason is that when backorder is generated, the firm wants to sell more than capacity, hoping that extra demand will be filled when capacity becomes abundant later. When capacity increases faster, there will be more abundant capacity later, thus the firm can have more backordered customers.

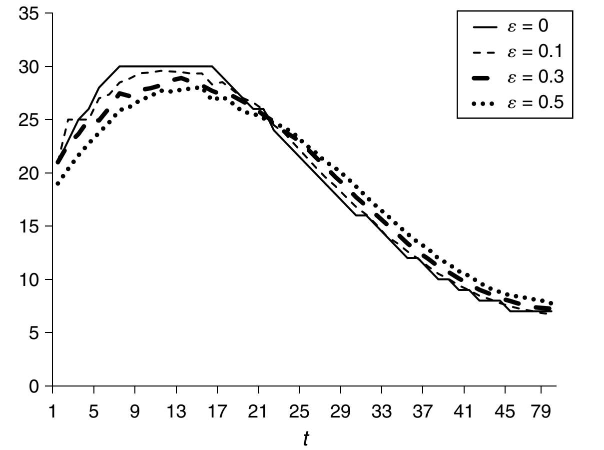
Second, our model, defined in §3, adopts the demand functions of Robinson and Lakhani (1975) and Doland and Jeuland (1981), where Bass instantaneous demand is multiplied by an exponential factor of price. We also examine two demand functions other than exponential form: (1) linear function, i.e., $d(t) = d_B(t)[a - \gamma\pi(t)]$, where a is a constant and γ is a constant parameter indicating price elasticity; and (2) the model used in Bass (1980), i.e., $d(t) = d_B(t)C\pi(t)^{-\eta}$, where C is a constant scalar and η is also a constant parameter representing price elasticity. We conducted a numerical study to evaluate the optimal policies and for both functions we continue to observe optimal policies similar to those in Figures 1 and 2.

Figure 12 Two Sample Paths and Maximum and Minimum Demand Realization $\hat{d}(t)$ Among 50 Sample Paths



Note. Parameters: $m = 1{,}000$, $p = 0.03$, $q = 0.4$, $c = 10$, $\beta = 0.9$, $b = 0.2$, $K = 30$, $\theta = 0.01$, $\xi = 1$, $T = 50$, and $\varepsilon = 0.5$.

Figure 13 The Average of Optimal Decision $\bar{d}(t)$ Among 50 Sample Paths



Note. Parameters: $m = 1{,}000$, $p = 0.03$, $q = 0.4$, $c = 10$, $\beta = 0.9$, $b = 0.2$, $K = 30$, $\theta = 0.01$, $\xi = 1$, and $T = 50$.

7.4. Stochastic Model

There are only a limited number of papers that consider uncertainty in the Bass diffusion model, including Skiadas and Giovanis (1997), Niu (2002), and Kanniainen et al. (2011). Kanniainen et al. (2011) assume that future sales $S(t)$ is given by $S(t) = g(t)e^{X(t)}$, where $g(t)$ is a continuous deterministic function of time and $X(t)$ is stochastic. We use a similar representation and assume that the instantaneous demand rate is multiplied by a random variable $X(t)$ with expectation 1; thus, $d(t) \triangleq \bar{d}(t)X(t) = [m - D(t)][p + qS(t)/m]e^{-\pi(t)}X(t)$, where $\bar{d}(t)$ is the expected demand rate. We assume $X(t)$ and $X(\tau)$ are independent for $t \neq \tau$ and $X(t) = \{1-\varepsilon, 1, 1+\varepsilon\}$ with equal probabilities.

With random demand, price would have to depend on the realized demand and sales until time t. Since maximizing over $\pi(t)$ is equivalent to maximizing over $\bar{d}(t)$, the dynamic pricing problem for the make-to-order and backorder case (in discrete time) can be written as

$$
\begin{aligned}
&\Pi_t(t, D_t, S_t)\\
&\quad = \max_{\bar{d}(t)} \; E_{d(t)}[\log d_B(t) - \log \bar{d}(t) - c]\min[d(t) + W_t, K]\\
&\qquad\qquad - bW_t + \beta E_{d(t)}\Pi_{t+1}(t+1, D_{t+1}, S_{t+1}),\\
&\quad\text{s.t. } d(t) = \bar{d}(t)X(t) = [m - D_t][p + qS_t/m]e^{-\pi(t)}X(t),\\
&\qquad W_t = D_t - S_t, \quad D_{t+1} = D_t + d(t),\\
&\qquad S_{t+1} = S_t + \min(d(t), K),\\
&\qquad D_0 = 0,\; S_0 = 0,\\
&\qquad \Pi_t(T+1, D_{T+1}, S_{T+1}) = -c(D_T - S_T),\\
&\qquad \text{for all } D_{T+1}, \quad S_{T+1} \in [0, m].
\end{aligned}
$$

To examine the behavior of the optimal pricing policy, we conduct a numerical study. We approximate the problem above by also discretizing $S_t \in \{0, 1, , \ldots, m\}$ and solve the resulting discrete deterministic dynamic program. Below we report our results. Figure 12 plots two sample paths of demand trajectories $\hat{d}(t)$, and it also plots the minimum and maximum demand among 50 sample paths, for $\varepsilon = 0.5$. Because of random disturbance, the sample paths can differ from each other significantly. Figure 13 takes the average of the optimal expected demand rate $\bar{d}(t)$ among 50 sample paths, for each different values of ε. Clearly, the average decisions are very similar to the one under deterministic setting. Even though Figures 12 and 13 show that the average demand has a very similar structure in the stochastic demand case, an interesting question is whether the deterministic solution can be used as an approximate solution in the stochastic case.

We note that the optimal price in the stochastic problem is a function of three parameters: current time t, cumulative demand D_t, and cumulative shipments S_t. We suggest a deterministic heuristic to the stochastic problem as follows. First, we solve the deterministic model to obtain T optimal prices $\{\tilde{\pi}_1, \tilde{\pi}_2, \ldots, \tilde{\pi}_T\}$, and their corresponding cumulative demands $\{\tilde{D}_1, \tilde{D}_2, \ldots, \tilde{D}_T\}$. Then for the stochastic problem, in period t, the optimal decision is obtained using the interpolation of $\tilde{D}$'s. Specifically, if the stochastic $D_t \in [\tilde{D}_i, \tilde{D}_{i+1}]$, then the heuristic price for the stochastic problem is $\pi_t = \tilde{\pi}_i + (\tilde{\pi}_{i+1} - \tilde{\pi}_i) * (D_t - \tilde{D}_i)/(\tilde{D}_{i+1} - \tilde{D}_i)$. If $D_t > \tilde{D}_T$, then the heuristic simply uses the last-period deterministic price; i.e., $\pi_t = \tilde{\pi}_T$. The corresponding demand is $\bar{d}_t = (m - D_t)(p + qS_t/m)e^{-\pi_t}$. We tried this heuristic for all the examples in Table 1. Since we had 175 examples in Table 1, and we had three values for ε, this corresponds to 525 cases. For each of these 525 cases, we ran 50 sample paths as described above. The average profit difference between the optimal stochastic problem and our heuristic was 0.77%. For cases with $\varepsilon = 0.1, 0.3, 0.5$, the percentage differences were 0.44, 0.84, and 1.04%, respectively. Thus, even with higher values of ε, corresponding to higher variability, the solution to the deterministic problem served as a very effective heuristic for the stochastic case. This may be because the deterministic heuristic incorporates the effect of randomness by conditioning the optimal decision on the realized cumulative demand.

8. Conclusions

In this paper, we analyzed a capacity-constrained firm introducing a new product and making decisions about how to price and manufacture a new product with the objective of achieving the most profitable diffusion of the product into the marketplace. Our model generalizes operations models by incorporating price decisions. Although marketing literature has considered pricing decisions, it typically ignores capacity constraints. We incorporate both. As multiple decisions about production, sales, and pricing interact with each other, we examined these interactions in partial backlogging and lost sales models.

Our first contribution was to derive the structure of integrated pricing, production, and inventory policies under capacity constraints. We demonstrated that the presence of supply constraints may result in fairly complex optimal pricing strategies as well as production and inventory policies. Thus, the increasing-decreasing and decreasing pricing policies suggested in marketing are no longer optimal.

Although the operations diffusion literature had considered capacity, it had ignored pricing. Our results suggest that pricing flexibility is a very effective lever to use when facing capacity constraints. Moreover, when dynamic pricing is used, intentionally holding sales is not necessarily optimal any longer, as pricing flexibility allows the firm to delay diffusion process without losing any profits. The potential benefits of holding inventory and allowing for backlogs are relatively small.

We examined the benefit of pricing flexibility and found that it is most effective when the firm has intermediate to high capacity and when internal influence is high. The benefits of holding inventory (produce to stock) and allowing for backorders tend to be significantly lower than benefits of pricing flexibility. We also showed that an increasing-decreasing pricing policy, although not optimal, does perform very well when used in conjunction with optimal production and inventory policies.

Our model leaves several paths for future research. For example, introductions of new generation of products are more popular than introduction of completely new products. There exist papers that examine multigeneration products. Li and Graves (2012) use a multinomial logit model to study the pricing and product transition decisions. However, they do not consider new products with word-of-mouth effect. Danaher et al. (2001) use a multigenerational Bass model with market-mix variables to empirically estimate demand. They do not study the optimal decisions and do not consider capacity constraints. How our results can be extended to multigeneration setting with diffusion effect is worth investigating.

Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/msom.2013.0447.

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