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Combinatorial Auctions for Procurement: An Empirical Study of the Chilean School Meals Auction

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In this paper we conduct an empirical investigation of a large-scale combinatorial auction (CA)—the Chilean auction for school meals in which the government procures half a billion dollars worth of meal services every year. Our empirical study is motivated by two fundamental aspects in the design of CAs: (1) which packages should bidders be allowed to bid on and (2) diversifying the supplier base to promote competition. We use bidding data to uncover important aspects of the firms' cost structure and their strategic behavior, both of which are not directly observed by the auctioneer; these estimates inform the auction design. Our results indicate that package bidding that allows firms to express their cost synergies due to economies of scale and density seems appropriate. However, we also found evidence that firms can take advantage of this flexibility by discounting package bids for strategic reasons and not driven by cost synergies. Because this behavior can lead to inefficiencies, it may be worth evaluating whether to prohibit certain specific combinations in the bidding process. Our results also suggest that market share restrictions and running sequential auctions seem to promote competition in the long run, without significantly increasing the short-run cost for the government due to unrealized cost synergies. Our results highlight that the simultaneous consideration of the firms' operational cost structure and their strategic behavior is key to the successful design of a CA. More broadly, our paper is the first to provide an econometric study of a large-scale CA, providing novel and substantive insights regarding bidding behavior in this type of auction.

Key words: combinatorial auctions; procurement; auction design; empirical; public sector applications

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1. Introduction

Auctions are increasingly becoming a common mechanism for supply chain procurement. Corporations and governments use auctions to procure billions of dollars in inputs and services (Rothkopf and Whinston 2007, Elmaghraby 2000). Auction mechanisms are also becoming more sophisticated. In particular, combinatorial auctions (CAs), multiunit auctions in which suppliers can bid for packages of items, have generated much interest in procurement applications.¹ The use of CAs has also been a widely debated topic in other contexts as well, such as the allocation of spectrum by the Federal Communications Commission (FCC). The main advantage of CAs is that

they allow suppliers (who we also refer to as firms or bidders) to express cost synergies among units in the bidding process, which often results in a lower procurement cost for the buyer (who we also refer to as the auctioneer).

Practical experience and academic research have shown that the design of an auction may have an important impact on its outcome; good designs can result in large savings, whereas poor designs may lead to large losses for the auctioneer (Milgrom 2004). Typically, the performance of an auction is critically determined by the firms' cost structure and their strategic behavior, both of which are usually not directly observed by the auctioneer. The main objective of this work is to conduct an empirical study of bidding data to uncover important aspects of the firms' cost structure and their strategic behavior in a large-scale procurement CA. Based on this

¹ See Elmaghraby and Keskinocak (2003), Hohner et al. (2003), Metty et al. (2005), Caplice and Sheffi (2006), Bichler et al. (2006), and Sandholm et al. (2006) for several recent applications.

information we evaluate potential changes to the current auction design. Although we study a particular CA—the Chilean auction for school meals in which the government procures half a billion dollars worth of meal services every year—our method and analysis could be used more broadly in other CAs.

Our empirical work is motivated by two fundamental aspects of the design of procurement CAs. The first design issue is which combinations should be allowed in the bidding process. The second design issue is whether and how to promote competition to diversify the supplier base. Both of these design decisions can influence the performance of a CA in terms of its (1) efficiency, that is, maximizing social welfare by assigning units to the set of suppliers that achieves the most cost efficient allocation; and (2) optimality, that is, minimizing the total expected payments to the suppliers. We describe how our empirical study can be useful to inform the two design issues mentioned above along both of these objectives.

When choosing which packages are allowed in the bidding process, the auctioneer should provide enough flexibility so that bidders can express their synergies in the bids. This mitigates the so called *exposure problem*: If package bidding is not allowed, a firm that exhibits synergies among two units may not bid below the costs for the individual items because of the risk of not getting the package. Indeed, under suitable conditions, allowing package bidding is a necessary condition for the efficiency (Rosenthal and Wang 1996, Rothkopf et al. 1998, Bykowsky et al. 2000) and optimality (Levin 1997) of the auction.

However, allowing too much flexibility on the package bids can also hurt the efficiency and optimality of the auction, while complicating the bidding process unnecessarily. A bidder in a CA may have incentives to submit package bids and offer discounts even in the absence of synergies, a phenomenon we refer to as *strategic bundling*. In this way a bidder may win a combination of units for which it is not the most cost efficient provider. In addition, because of the so called *threshold problem*, in which local bidders free ride on each other to outbid a global bidder in a combination, package bidding can also result in a more expensive allocation (Milgrom 2000).

The previous discussion suggests that to minimize the negative impact of strategic behavior, the auctioneer should only allow package bidding among units for which synergies are sufficiently large. Because the precise cost structure of firms is often unknown to the auctioneer, we seek to estimate the magnitude of cost synergies among different units using bidding data. (Cantillon and Pesendorfer 2006b analyze a similar problem in an auction for public bus routes and provide further details on this issue).

In terms of the second design issue, promoting diversification and competition among bidders, the auctioneer faces the following trade-off: If cost synergies are significant, it may be efficient and optimal in the short run to allocate all units to one or few firms. On the other hand, this could depress competition in the bidders' market for future auctions, as inactive firms may find it hard to compete head-to-head with incumbents, increasing expected payments in the long run. We investigate two mechanisms that together help to intensify competition: (1) imposing market share restrictions for bidders and (2) awarding the units in multiple sequential auctions. The latter could intensify competition if incumbent firms that won units in previous auctions bid more aggressively due to cost advantages given by their installed base in nearby units. If cost synergies are important, these mechanisms can hurt the efficiency and optimality of the auction, though, because they may prevent bidders to submit package bids containing a large number of units and fully expressing those synergies. Therefore, to study the effectiveness of these measures it is important to compare the intensity of cost synergies with the impact that competition has on bid prices.

To inform the two design issues discussed above, we conduct an empirical analysis to measure the intensity of cost synergies as well as the effect of incumbency and competition. Although CAs have received considerable attention from different academic communities including management science/operations research, economics, and computer science,² to the best of our knowledge there are no other empirical studies that use field data on CAs, except for the notable work of Cantillon and Pesendorfer (2006b). However, the methodology developed by Cantillon and Pesendorfer (2006b) can only be applied in settings with small package bids (three units) and their estimates suggest no synergies are present. Therefore, as far as we know, our paper is the first to provide an econometric study of a real-world large-scale CA that exhibits an important amount of package bidding and synergies, providing novel and substantive insights into bidding behavior in CAs. In that sense, we believe our work constitutes an important contribution to the literature.

The auction we study is the Chilean auction for school meals (for a detailed description, see Epstein et al. 2002, 2004). The Chilean government provides breakfast and lunch for 2.5 million children daily in primary and secondary public schools during the

²For surveys on work in CAs, see Pekeč and Rothkopf (2003), de Vries and Vohra (2003), Hoffman (2006), and Blumrosen and Nisan (2007), and the recently edited volume by Cramton et al. (2006).

school year. In a developing country where about 14% of children under the age of 18 live below the poverty line, many students depend on these free meals as a key source of nutrition. Since 1999 the contracts are awarded through a single-round, sealed-bid, first-price CA. Meal services are standardized and firms compete in prices. Chile is divided into territorial units and firms can submit bids on any package of units defining the combinatorial character of this auction. Approximately 20 firms participate in each auction; each firm submits many bids (hundreds or even thousands) ranging from just one to several units. The CA has been used every year since its inception awarding more than \$3 billion of contracts (US\$577 million were awarded in 2008).

Our data set contains bids for packages of different sizes that contain units in different locations. Our empirical strategy is based on using variation in bid prices for different combinations to quantify the magnitude of two types of *package discounts*: (1) scale discounts, which depend on the size of the package; and (2) density discounts, which arise from packaging nearby units. We decompose discounts in this form because we seek to separately identify the suppliers' economies of scale (generated by volume discounts in their input purchases) from economies of density (arising from common logistics infrastructure used to supply nearby units). One significant challenge, though, is that package discounts cannot be entirely attributed to cost synergies, because they could also be explained by markup adjustments due to strategic bundling. For this reason, we also conduct a detailed analysis to empirically measure the incentives that lead to strategic bundling and how firms respond to them. This analysis allows to quantify the fraction of the estimated discounts that is explained by pure strategic behavior. In addition, our econometric analysis exploits the panel structure of the data to estimate the effect of local competition on prices, which relates to the supplier diversification issue discussed above.

Our analysis of the Chilean auction indicates that scale and density discounts are important, and together they can be as high as 8% of the average bid price. Interestingly, our estimates indicate that these discounts get practically exhausted after combining seven or more units. Although we do find evidence of strategic bundling behavior, on average, they only explain a small fraction of the discounts in our data. Hence, most of the discounts can be attributed to cost synergies. These results suggest that package bidding that allows firms to express their cost synergies due to economies of scale and density seems appropriate. However, we do identify specific units for which strategic bundling is more severe and could be leading to inefficient allocations. Therefore, it may

be worth evaluating whether to prohibit certain combinations in the bidding process.

We also find that incumbent firms bid more aggressively for units in which they have nearby operations and, as a consequence, all firms bid more aggressively as local competition intensifies. The effect of competition is comparable in magnitude to the discounts from cost synergies. Overall, these results suggest that market share restrictions and running sequential auctions seem to foster supplier base diversification and promote competition in the long run, without significantly increasing the short-run cost for the government due to unrealized cost synergies.

The Chilean government found these empirical results to be informative of the two design issues discussed above, and they are being used to evaluate changes to the current auction design. More broadly, our results highlight that the simultaneous consideration of the firms' operational cost structure and their strategic behavior is key to the successful design of a CA.

2. Related Literature

Our paper is related to streams of literature in operations management, empirical industrial organization, and experimental economics. Recent literature in operations management studies procurement and online auctions.³ Most of these papers develop models that introduce novel and important operational aspects to traditional auction settings, providing a theoretical or computational analysis. Among these papers the most related to our work are Elmaghraby (2005), which studies the effect of economies of scale and bidders' heterogeneity in production capacity on auctions' performance, and Chen et al. (2005), which studies auctions in a supply chain network explicitly considering transportation costs. None of these papers, however, provide an empirical analysis as we do here.

Our work is related to the extensive empirical auctions' literature in economics (for several surveys, see Hendricks and Porter 2007, Athey and Haile 2006, Paarsch and Hong 2006). There are several papers that study multiunit auctions to learn about cost synergies. Ausubel et al. (1997) and Moreton and Spiller (1998) estimate synergies for wireless providers in spectrum auctions by the FCC. Gandal (1997) estimates synergies in an auction for cable television licenses. Marshall et al. (2006) estimate synergies on bidding for the Georgia school milk market using a structural model of simultaneous first-price auctions for multiple homogeneous units. However, in all these

³ See, for example, Tunca and Zenios (2006), Cachon and Zhang (2006), Gallien and Gupta (2007), and Kostamis et al. (2009).

papers the auction format does not allow for package bidding. The paper most related to ours is by Cantillon and Pesendorfer (2006b), who estimate a model for the CA for London bus routes. Whereas our approach is reduced form, their approach is structural. Because they study small auctions with a maximum of three units each, a structural approach similar to approaches previously used in single-unit auctions is computationally feasible.

Our work is also related to other articles that use field data to test theoretical predictions of equilibrium behavior in multiunit auctions, such as Hortacsu and Puller (2008). List and Lucking-Reiley (2000) study multiunit auctions in field experiments. These articles, however, do not study combinatorial package bidding.

There is also previous work studying bidding behavior in multiunit auctions using controlled laboratory experiments. Katok and Roth (2004) compare different formats of multiround auctions and, consistent with theoretical predictions, their results suggest that auction formats that permit package bidding tend to mitigate the exposure problem at the expense of reinforcing the threshold problem. Kwasnica et al. (2005) conduct experiments to compare two ascending auction formats used by the FCC to allocate spectrum, which also involve a trade-off between the exposure and threshold problems. Kagel and Levin (2005) conduct experiments to compare bidding behavior in sealed-bid versus ascending-bid uniform price multiunit auctions. Note that the school meals auction we analyze is a single-round, sealed-bid, first-price CA, which is different from the auction formats studied in these papers. In that sense, our work is more directly related to the work of Chernomaz and Levin (2012), who study a similar auction format for two homogeneous units with and without package bidding. They show examples for which if cost synergies are small, allowing for package bidding can lead to a less efficient outcome.

3. The Chilean Auction for School Meals

In this section, we describe the Chilean auction for school meals, specify the design issues raised in §1 in this context, and describe our data set. We also describe some interesting patterns observed in package bids.

3.1. Description of the Chilean Auction for School Meals

We present a brief description of the auction process (for a more detailed description, see Epstein et al. 2002, 2004). Junta Nacional de Auxilio Escolar y Becas (JUNAEB) is a government agency in Chile that provides breakfast and lunch for 2.5 million children

daily in primary and secondary public schools during the school year.

Since 1999 JUNAEB assigns its school meals service contracts through a single-round, sealed-bid, first-price CA, that was fully implemented for the first time that year and has been used ever since. For the purposes of the auction, Chile is divided into approximately 100 school districts or territorial units (TUs). Firms can submit bids on various groupings of TUs defining the combinatorial character of this auction. This mechanism is motivated by the belief that firms are subject to cost synergies that arise from operational advantages when serving multiple TUs. JUNAEB holds auctions in one-third of the TUs every year, awarding three-year contracts. Figure 1 presents a map of Chile with the TUs auctioned each year.

The auction process begins when JUNAEB contacts and registers potential vendors. The agency then evaluates the companies from a managerial, technical, and financial point of view, and eliminates those that do not meet minimum reliability standards. Qualifying vendors are classified according to two characteristics: their financial capacity (based on data from the firms' balance sheets), and their managerial competence. Usually, firms below a minimum level of managerial competence are not allowed to participate in the auction. Potential vendors then submit their bids through an online system. Meal plans are standardized and firms compete in prices. Upon winning a contract, the firm is responsible for managing the entire supply chain associated to all meal services in the corresponding TUs, starting from sourcing food inputs going all the way to cooking and serving the meals in the schools.

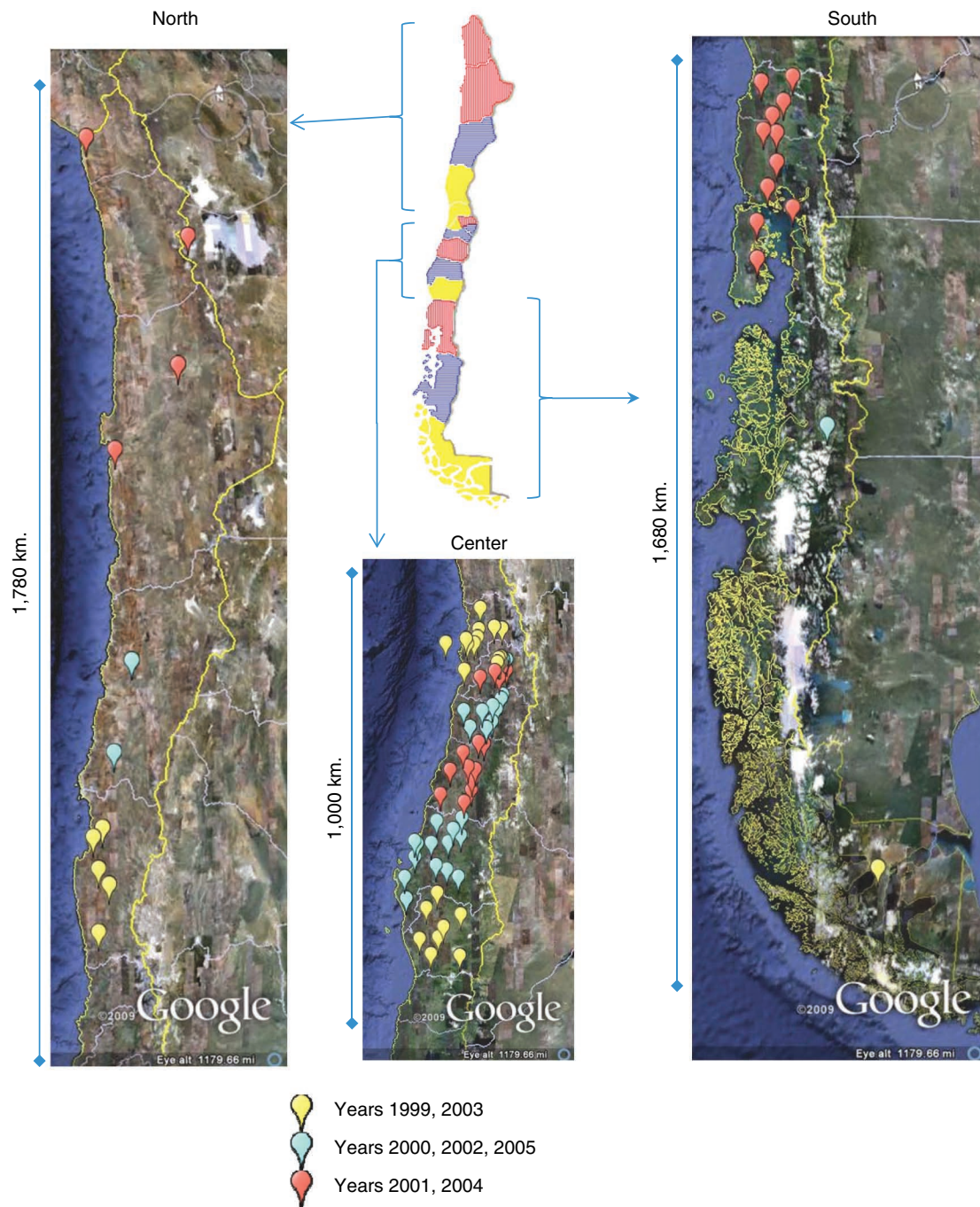
A bid can cover any combination from one to eight TUs and specifies the price for which the firm would serve all meals included in the TUs in the combination. Vendors can submit many bids and each package bid is either fully accepted or rejected (i.e., the mechanism does not allocate a fraction of a bid); most firms submit hundreds or even thousands of bids.

The allocation is chosen by selecting the combination of bids that supply all of the TUs at a minimum cost. The problem is formulated as an integer linear program that incorporates other considerations and side constraints. An important set of constraints puts limits on the maximum number of meals that can be assigned to any given firm, both nationally and in specified geographical regions (to encourage competition in the suppliers' market).

3.2. Design Issues in the Chilean Auction

In this section, we discuss the two design issues presented in §1 in the context of the school meals procurement auction: (1) which package bids should be allowed and (2) how to diversify the supplier base

Figure 1 Map of Chile



Note. Points indicate the centers of the territorial units (TUs); colors indicate the year in which they are auctioned.

to promote competition. We do it in the context of a single-round, sealed-bid, first-price CA, because this is the format used in our application and several CAs in practice share the same format.

3.2.1. Package Bidding. Package bidding should be sufficiently flexible to let bidders express cost synergies among units. Indeed, in the current design all possible combinations are allowed in the bidding process. However, too much flexibility can hurt

efficiency and increase expected total payments if bidders strategically take advantage of package bidding. Therefore, this suggests that the auction mechanism should only allow package bidding among units for which cost synergies are sufficiently large. Consequently, and given the cost structure associated to serving a typical set of TUs, we identify two types of cost synergies:

1. *Economies of Scale.* Approximately 50% of the total cost of serving meals is related to food inputs

that include perishable and nonperishable items. Most of the food is purchased centrally and firms can get important volume discounts from their providers. These discounts result in economies of scale when serving multiple TUs. Note that these synergies are only a function of the total volume of meals served and are independent of the proximity between the units served.

2. *Economies of Density*. Logistical costs associated to transportation and administration amount to approximately 9% of total cost. Some of these costs are fixed and can be shared by TUs that are close to each other (e.g., by sharing a local warehouse and a distribution network), resulting in economies of density. Note that these synergies are a function of the proximity of the units served.⁴

The distinction between these two types of cost synergies is important from an auction design perspective. If economies of density are predominant, then it could be preferable to restrict combinations to units that are closely located. If economies of scale are predominant, then there are simpler auction mechanisms that would allow bidders to express them in their bids (e.g., providing prices for each unit and a discount curve). We therefore seek to identify separately these two sources of cost synergies using bidding data.⁵

3.2.2. Promoting Competition. The school meals auction exhibits two specific characteristics that help to diversify the supplier base and promote long-term competition among bidders:

1. The current mechanism imposes strict market share restrictions in the allocation of units to bidders. There are several such restrictions: (1) a maximum number of TUs that each firm can be allocated in any given auction (this maximum is based on the financial evaluation conducted by JUNAEB every year and therefore can be different across firms and auctions, ranging from two to eight TUs); (2) at any point in time, the total standing contracts of any firm cannot exceed 16% of the total number of meals included in all TUs in the country; (3) a local market share constraint that limits the number of TUs a firm can be awarded with in preestablished geographical regions (this limit varies across regions); and (4) a constraint that enforces a minimum number of firms (around 10) included in the allocation of each auction.

2. The TUs are split into multiple sequential auctions that are conducted in consecutive years. Figure 1 shows how the different TUs across the country

are grouped on multiple auctions. In general, TUs in adjacent geographic regions are awarded in different years, so that each auction awards units scattered all along the country rather than concentrated in a specific area.

The latter generates *local incumbents*—for most of the TUs in a given auction (except the first one in 1999), there are firms with ongoing contracts for nearby TUs awarded in *previous* auctions. Because these local incumbents already have an installed base in the proximity of some TUs, they may have cost advantages and therefore bid more aggressively. If other firms anticipate this, auction theory predicts that a nonincumbent firm should also bid more aggressively (Krishna 2002). Hence, an increase in the intensity of *local competition*, measured as the number of local incumbents, is likely to reduce bid prices from all participant firms. The market share restrictions would reinforce this local competition effect as it tends to increase the number of local incumbents as well as the pool of firms that can actively compete against the local incumbents.

These benefits notwithstanding, there are also potential disadvantages of the current design. In particular, the market share restrictions together with running multiple auctions may result in many firms providing service in narrowly defined geographical areas, precluding them from fully realizing their cost synergies. It is therefore useful to compare the magnitude of the local competition effects relative to the cost synergies in order to evaluate the overall effectiveness of the current auction mechanism.⁶

3.3. Description of the Data

Our data set contains all bids presented by all firms in all auctions between 1999 and 2005. We also collected information about the firms and the TUs on each auction. In this section we describe these data and provide some summary statistics.

For each auction, we know the identity of all participant firms, which are around 20 each year.⁷ We have data on all the bids presented by each firm. Each bid specifies a set of TUs and the price per meal for which the firm would serve all units in the combination.⁸ Table 1 provides summary statistics regarding the number of participant firms and the average number of submitted bids *per firm*. Firms submit from tens

⁴ Approximately 25% of the operating costs is related to labor dedicated to the preparation of the meals at the schools. The number of servers per school and their salaries are heavily regulated and not subject to cost synergies, so we ignore this cost in our analysis.

⁵ In this sense our approach is related to Caves et al. (1984), which measures and distinguishes between economies of scale and density for the airline industry.

⁶ Our analysis is focused on the effects of *local* competition. Although more global competitive effects could be important, our application does not have variation in this dimension because the number of bidders on auctions that award the same set of TUs is relatively stable over time.

⁷ The average number of firms entering and exiting the market each year is close to three.

⁸ The current allocation mechanism uses some criteria to eliminate unrealistically low bids. In our analysis, we do not eliminate bids.

Table 1 Number of Firms and Bids on Each Auction

Auction	Firms	Bids	Classification L		Classification LM		Classification M		Classification S	
			Firms	Bids	Firms	Bids	Firms	Bids	Firms	Bids
1999	9	443	4	803	2	304	1	106	2	31
2000	21	572	4	781	4	126	11	754	2	40
2001	22	1,037	6	1,288	5	757	10	1,126	1	44
2002	16	1,466	6	1,145	2	5,298	3	1,485	5	306
2003	20	2,157	6	3,532	3	4,312	4	1,276	7	557
2004	24	7,894	5	16,543	4	11,925	3	9,127	12	2,639
2005	16	7,188	5	10,888	3	11,286	2	12,511	6	280

Notes. The second and third column contain the total number of participant firms and the average number of submitted bids *per firm* in each auction. The following columns contain the same information disaggregated by firms' classification.

to several thousand bids. The total number of bids submitted by all firms was approximately 4,000 in 1999 and increased substantially since then; in 2003 it was 43,000. In 2004 the process of submitting bids was completely digitalized for the first time; before, bids were submitted in paper. That year the total number of submitted bids jumped to 190,000 and it was 115,000 in 2005.

The financial capacity evaluation made by JUNAEB determines the maximum number of units a firm can be awarded with. Firms cannot submit bids that contain more units than this upper bound. We group firms in four categories according to their classification: L, LM, M, and S, which correspond, respectively, to large, medium-large, medium, and small. Roughly, L firms can win up to eight units, LM firms up to six–seven units, M firms up to four–five units, and S firms up to two–three units (there are some exceptions to this rule). In Table 1 we also provide the number of participant firms and the average number of submitted bids *per firm* disaggregated by firms' classification. Because of these restrictions, large firms submit more bids on average because they are allowed to bid on larger packages. Note that on average firms only submit a small subset of all possible package bids: the number of possible combinations for L firms is about 19.5 million; for LM firms is 3.5 million; for M firms is 165,000; and for S firms is 3,000. In the next section, we describe some patterns on how the submitted packages are selected.

In addition, we know the set of winning bids in each auction and, therefore, at every point in time we know the identity of the firm serving each TU. On average, in each auction there are 11.5 winning bids distributed among 10 winning firms. The average size of the winning packages is 2.7 units. In Table 2 we describe the number of winning bids over all auctions disaggregated by firms' classification and size of the winning package bid. Whereas large firms win more frequently, other firms, including the smallest firms win quite often (15 out of 69 bids). In addition, small packages of one or two units also win quite

often (35 out of 69 bids). Similar patterns are observed in each individual auction.

We have detailed data about TUs including the location and population of all schools in them. TUs are heterogeneous in their size and density of school population, factors that strongly affect the procurement cost. For example, there are units in urban locations that cover a small geographic area with high school population but also units in isolated parts of the country that cover large areas with a low population density. Urban TUs have an average size of 2.8 million meals per year and an average of 71 schools; TUs in rural areas have 2.2 million meals per year and 99 schools on average. The average price per meal over all bids is \$0.75, and the average unit size among all bids is 2.5 million. Typically, each TU is auctioned every three years so that each unit is auctioned at least twice in our data set, which is an important aspect for our estimation method.

3.4. Package Bidding

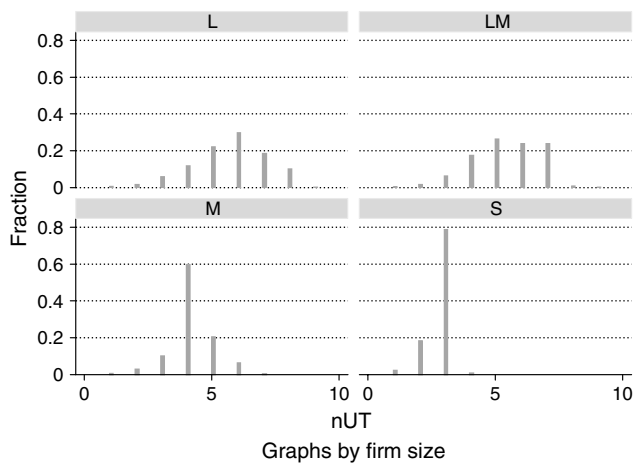
In this section we describe patterns observed in the data regarding how firms select units in package bids. Not only these patterns are interesting by itself, but they will also inform the regression analysis of bid prices that we do in the next sections.

We start by showing in Figure 2 a histogram of the number of TUs in a bid disaggregated by firms' classification. We observe a significant degree of variation

Table 2 Total Number of Winning Bids Over All Auctions as a Function of Winning Firm's Classification and Number of TUs in Winning Package

Firm classification	Number of TUs in winning package								Total
	1	2	3	4	5	6	7	8	
L	8	5	4	4	1	1	0	5	28
LM	4	1	5	1	2	0	0	0	13
M	5	1	1	6	0	0	0	0	13
S	10	1	4	0	0	0	0	0	15
Total	27	8	14	11	3	0	0	5	69

Figure 2 Histogram of Number of Units in a Bid, by Firm Size



Note. L denotes large, LM denotes medium-large, M denotes medium, and S denotes small.

in bid sizes and an important presence of combinatorial bidding. Firms submit bids as small as one TU to large ones with eight TUs. As expected, larger firms tend to submit larger packages. Note, however, that the mode of the package size for large firms is six; this is below the maximum package size these firms can win. In addition, Figure 3 shows a scatter plot of bid prices (per meal, in US\$) versus the size of the package bid (measured in million meals per year). The figure suggests that bidders use volume discounts in their package bids.

The previous analysis suggests how firms select the size of their packages. The rest of this section provides a series of analysis to study how firms select the units in these packages. First, we note that bids from large firms essentially include all units being auctioned in at least one package. On average, the fraction of units covered is 95%. For the rest of the firms, this number is somewhat smaller, approximately 80%. The TUs for

Figure 3 Bid Price as a Function of the Size of the Bid; Includes All Bids in Auctions 1999, 2000, and 2001

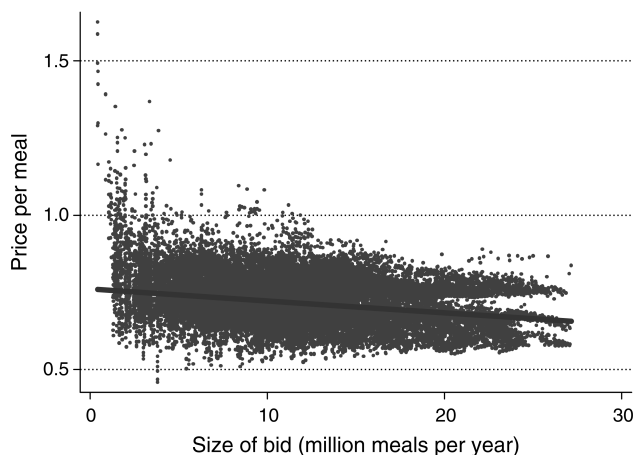
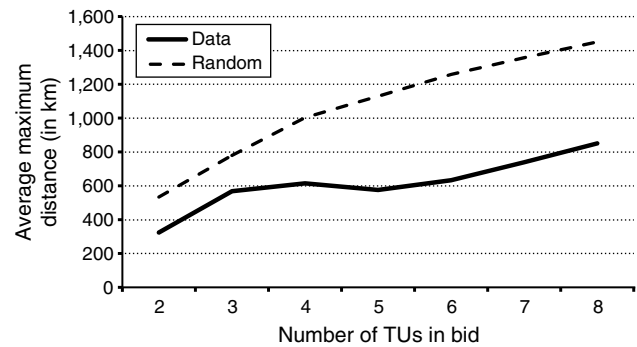


Figure 4 Average Maximum Distance Among TUs Contained in a Bid, for Bids with Different Number of Units



Note. Dashed line indicates the average maximum distance when the units in the package are sampled at random.

which these firms do not bid tend to be co-located, suggesting that some smaller firms select specific geographical regions in which they do not compete.

Second, Figure 4 shows the average maximum distance among the TUs contained in a bid, for bids of different sizes.⁹ Firms make bids that includes TUs that are close to each other and also TUs that are far apart. The figure also shows the expected average maximum distance if the packages were selected randomly. We observe that in the actual bids firms tend to select combinations of TUs that are closer to each other when compared to a random pattern.

Motivated by the previous observation and that we observe significant heterogeneity with respect to how many times a unit appears in a package, we explore in more detail how firms select units in package bids. For this purpose, for each firm-auction pair we constructed a graph where each node represents a different TU and has a value equal to the number of package bids the TU is included on. Each edge represents a pair of TUs and has a value equal to the number of package bids the pair is included on (if the value is zero there is no edge between those units). We call these values the *popularity of a TU* and the *popularity of a pair of TUs*, respectively. We define the *degree of a TU* as the number of different units it is packaged with (or equivalently, the degree of the node representing the TU in the graph).

We separately regress the popularity and degree of firm-auction-TU nodes against several characteristics of the firm and unit. The results are shown in Table 3, reporting standardized coefficients (normalizing all the variables to have a standard deviation of one so that the coefficients can be interpreted as

⁹ The distance between two TUs is the bird-fly distance between the weighted geographic centers of the respective TUs. The weighted geographic center of a TU is calculated as the weighted average latitude and longitude of the schools it contains, weighted by the school populations.

Table 3 Regression Analysis of Unit Popularity (Measured as the Number of Packages a Unit Is Included on) and Degree of a Unit (Measured as the Number of Different TUs That Are Packaged with a Unit)

	(1) Unit popularity	(2) Degree
Size	0.108** (6.10)	0.025* (2.50)
Meals/School	0.176** (10.06)	0.115** (11.80)
Special meals	0.087** (5.89)	0.031** (3.80)
Nearby unit	0.098** (5.49)	0.008 (0.84)
Renew	0.092** (6.39)	0.028** (3.42)
Firm won unit	0.105** (7.44)	0.034** (4.33)
Observations	3,328	3,328
R-square	0.1360	0.0921

Note. Coefficients are standardized, and *t*-statistics are shown in parentheses.

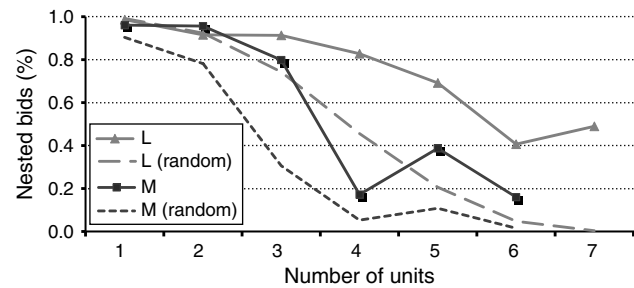
* and ** indicate significance at the 5% and 1% levels, respectively.

the changes in the dependent variable—measured in standard deviations—when the covariate changes one standard deviation). Both regressions include firm-auction fixed effects, and the *R*-square reports the explained variation within a firm auction. The results suggest that larger and more dense units (in terms of meals per school) are more popular and have a higher degree; these units tend to be cheaper to serve. In particular, meals per school is among the factors that explain most variation in the popularity and degree of a unit. In addition, a TU is more popular and connected for firms that are incumbents, in the sense that they are trying to renew the contract in that TU or that they have ongoing operations in neighboring TUs.

We also estimated a regression where the dependent variable is the popularity of a pair of TUs for a given firm in a particular auction. An important factor in this regression is distance: increasing distance by two standard deviations decreases the popularity of a pair of TUs by 0.13 standard deviations. Also, the popularity increases by 0.5 standard deviations for two units in the same region. A pair of units for which the firms is looking to renew contracts tends to be more popular (0.5 standard deviations higher); a smaller effect is also observed when the firm operates nearby TUs in both units.¹⁰

The previous analysis suggests that firms focus their bidding efforts in certain units and certain

¹⁰ This regression includes firm-auction fixed effects. Because of limited space, we do not report the results in this paper; however, they can be obtained from the authors upon request.

Figure 5 Percentage of “Nested” Bids for Large and Medium Firms in the Data

Note. Dashed lines show percentage of nested bids for a set of randomly generated bids that is consistent with the total number and sizes of bids submitted by each firm in the data.

combinations of them. Moreover, the following analysis confirms that package bids are built systematically. We say that a bid on a package a by firm f in auction t is “nested” if there exists at least one package bid from the same firm in the same auction for a package a' such that $a \subset a'$. Small packages (say of one or two units) are likely to be nested. However, given the large number of possible combinations and the relatively small fraction of this set that firms bid on, it becomes more likely that larger packages are not nested. In Figure 5 we show the fraction of nested bids observed in the data as a function of their size and compared them with the fractions induced by a set of randomly generated bids that is consistent with the total number and sizes of bids submitted by each firm. We observe that firms tend to nest bids much more significantly than in a random sample of bids, suggesting they systematically use a “bottom-up” approach to construct the package bids.

The cost discovery and bid submission process of small firms—who bid on packages of up to two units—is relatively simpler, because the total number of packages they can bid on is manageable (around 500 hundred). In contrast, for large firms that can bid on packages of up to eight units, this number is around 20 million and it is unrealistic to expect that firms will evaluate all possible combinations. In fact, firms only submit a small fraction of these combinations. Moreover, our analysis suggests that firms are systematic when selecting package bids and focus their efforts on certain type of packages. Perhaps this is not surprising as we have learned anecdotally from conversations with JUNAEB that many of the firms use computer programs to generate the bids (e.g., spreadsheet macros). This process help firms to methodically evaluate the costs of serving the different packages. On top of these costs, firms add a markup that can depend on the package. This provides further support that the bidding data that we use in the next sections conveys useful information regarding firms’ economic costs and their strategic behavior.

To summarize, the analysis in this section suggests that package bids are built systematically and exhibit some distinguishable patterns. First, combinatorial bidding is widely observed suggesting the presence of economies of scale. Second, firms tend to include TUs that are close to each other in the same bid suggesting the presence of economies of density. Third, firms tend to bid more frequently on units for which they are incumbents, suggesting incumbency advantages. In the next section, we test these hypotheses and quantify these effects with an econometric analysis of the bid prices.

4. Estimating Package Discounts

As previously discussed, a CA should ideally permit package bidding among units that exhibit sufficiently large cost synergies. To inform this design issue we follow a two-step approach. First, in this section we develop an econometric analysis to identify two types of package discounts: (1) volume or scale discounts, which depend on the size of a package; and (2) density discounts, which depend on the proximity of the units in a package.

It is important, however, to interpret these estimated discounts with caution because they do not necessarily reflect the magnitude of suppliers' cost synergies due to economies of scale and economies of density; package discounts could also arise due to strategic effects. Thus motivated, §5 provides a detailed empirical analysis to measure what portion of the estimated discounts can be explained by strategic markup adjustments as opposed to cost synergies. Then, by considering the estimated discounts and the markup adjustments simultaneously, we can assess the magnitude of cost synergies.

4.1. Econometric Model

We develop an econometric model that exploits variation in bid prices to identify different forms of package discounts. Because most of the variation in the prices is explained by the number of meals included in the package, we normalize the total bid price dividing it by the number of meals. Throughout, we use i to index territorial units (hereafter, units), a to index sets of units (also referred to as combinations or packages), t to index auctions, and f to index firms. The dependent variable in our empirical analysis is the price per meal, defined as total bid price (in 1999 Chilean pesos) divided by the number of meals of a combination a submitted by a firm f in a specific auction t , and is denoted by b_{aft} (hereafter referred to as the bid price).¹¹

There are two main factors that explain the variation in bid prices. First, there is heterogeneity in

the individual average prices of the units, which arises from differences in costs and markups associated with each unit. This heterogeneity is across units (e.g., some units are more expensive to operate than others), across firms (e.g., some firms could be more efficient in particular regions, or charge different markups), and across auctions (e.g., the price of inputs may vary from year to year). Second, different combinations are subject to different discounts and the data shows that larger combinations tend to have, on average, a lower bid price.

To account for these sources of variation, our specification decomposes the bid price into two parts: (1) the stand-alone prices of the units contained in the package, which captures heterogeneity across units; and (2) discounts for combinations, which captures *interactions* among the units contained in the package and is therefore package specific. We use the following notation to describe these sources of variation. Let δ_{ift} be the average price of a unit i for firm f in auction t (we also refer to δ_{ift} as the average unit price). Let v_i be the size of unit i (measured in million number of meals per year), and accordingly define, with a slight abuse of notation, the size of a combination a as $v_a = \sum_{i \in a} v_i$. We model the bid price as

$$b_{aft} = \sum_{i \in a} \delta_{ift} \cdot \frac{v_i}{v_a} - g_{ft}(a). \quad (1)$$

Equation (1) decomposes the bid price as a weighted average of the average unit prices δ_{ift} minus a discount function $g_{ft}(a)$. The discount function depends on the package a and could potentially be different across firms and auctions.

As mentioned earlier, there are two main forms of cost synergies in the procurement process: economies of scale, which only depend on the total size of the combination (v_a), and economies of density, which depend on the geographic location of the units in a . We are interested in analyzing how firms express these synergies in their package bids. Accordingly, we define two separate discount functions: (i) $g^{\text{scale}}(v_a, \beta^{\text{scale}})$, called the *scale discount function*, intends to capture discounts that depend on the size of a package; and (ii) $g^{\text{dens}}(a, \beta^{\text{dens}})$, called the *density discount function*, intends to capture discounts arising from the proximity of the units in a package a . Recall that the discount functions represent discounts per *meal*. These functions are parametrized by the vectors β^{scale} and β^{dens} that measure the intensity of the respective discounts. These parameters could vary across firms and auctions, but for simplicity we assume they are common to all firms and auctions (§4.3 shows results from a more flexible specification to capture firms' heterogeneity with respect to the discounts). Because we want to measure discounts

¹¹ We used the consumer price index of food to normalize prices to real 1999 Chilean pesos.

arising from the interaction among units, both discounts functions g^{scale} and g^{dens} are set to zero for bids with a single unit (i.e., when a is a singleton). We emphasize that these discounts do not directly measure economies of scale and density because they also incorporate markup adjustment due to strategic effects. We deal with this issue in §5.

In estimating g^{scale} and g^{dens} , it is important to allow a sufficiently flexible specification to capture a potential saturation effect of these discounts. Note that $g^{\text{scale}}(v_a, \beta^{\text{scale}})$ is a function of a single variable and is therefore relatively easy to approximate with a flexible, yet parsimonious, specification. In contrast, representing $g^{\text{dens}}(a, \beta^{\text{dens}})$ compactly is more challenging because the number of possible sets a is enormous. To overcome this difficulty, we make this function depend only on measures that summarize the geographic proximity of the units in a . We describe this procedure next.

For a given set a , we identified *clusters* of units located within a circular perimeter of 150 km radius.¹² Let $Cl(a)$ be the set of clusters formed by the units in a , which form a partition of a . The size of each cluster $c \in Cl(a)$ can be calculated as the sum of the sizes of its units, $v_c = \sum_{i \in c} v_i$ (with some abuse of notation). Density discounts should be nondecreasing in the size of a cluster: as more demand is concentrated within the limits of the cluster, the larger the discounts. For example, given two packages a and a' of the same size ($v_a = v_{a'}$), density discounts should be larger for the package with fewer and larger clusters. Accordingly, we define discounts at the level of a cluster as a function of its size, denoted $g^{\text{clust}}(v_c, \beta^{\text{dens}})$. Again, this discount function is set to zero when the cluster contains a single unit, so that it indeed captures discounts from combining units that are co-located. Given a specification for $g^{\text{clust}}(v_c, \beta^{\text{dens}})$, the overall density discounts (per meal) are given by $g^{\text{dens}}(a, \beta^{\text{dens}}) = \sum_{c \in Cl(a)} g^{\text{clust}}(v_c, \beta^{\text{dens}}) \cdot (v_c / v_a)$, that is, a weighted average of the discounts generated from each cluster. The following example helps to illustrate the logic behind the specification of the density discount function $g^{\text{dens}}(\cdot)$.

Consider four units, labeled 1–4. To simplify the discussion, we assume all units have the same size equal to one and have similar average unit prices. Because of their location, {1, 2, 3} form a cluster but unit 4 does not form a cluster with any of the other units. Consider two hypothetical bids: bid A for units {1, 2, 4}, and bid B for {1, 2, 3}. Both bids have the same size, so scale discounts will be similar. However, the clusters formed within each bid are

different. Bid A has two clusters, {1, 2} and {4}, one of which is a singleton and therefore does not get any discounts. Bid B has a single cluster of size 3. Let $g_v = g^{\text{clust}}(v, \beta^{\text{dens}})$ be the discount per meal for a cluster of size v . The density discount for bid A is equal to $\frac{2}{3}g_2$. For bid B, the discount is equal to g_3 , which is larger than the discount on bid A because (1) g_v is increasing in the size of the cluster; and (2) all three units in bid B benefit from the density discounts, compared to only two units in bid A. Also note that a bid for package {1, 2}—which is a subset of bid A—could have a larger discount per meal than bid A if density discounts are sufficiently large (relative to scale).

Combining the average bid prices δ_{ift} and the two discounts functions g^{scale} and g^{dens} , we estimate the follow regression to explain variation in bid prices:

$$b_{\text{aft}} = \sum_{i \in a} \delta_{\text{ift}} \frac{v_i}{v_a} - g^{\text{scale}}(v_a, \beta^{\text{scale}}) - \sum_{c \in Cl(a)} g^{\text{clust}}(v_c, \beta^{\text{dens}}) \cdot \frac{v_c}{v_a} + \varepsilon_{\text{aft}}, \quad (2)$$

where the error term ε_{aft} captures idiosyncratic discounts (or charges) for combination a . The specific parametric forms of g^{scale} and g^{clust} used in the estimation are described in §4.2. We seek to estimate the parameters $(\beta^{\text{dens}}, \beta^{\text{scale}})$ that characterize scale and density discounts.

Estimating density discounts can be challenging when firms have local cost advantages that are not observable in the data. To explain, Figure 4 suggests that bids are more likely to include units that are located in close proximity, which could be interpreted as evidence of package discounts arising from economies of density. However, an alternative explanation is that firms have local cost advantages in specific regions and are more likely to submit bids for units where they have an advantage. Because advantages are local, packages with co-located units are more likely to be submitted by firms with lower costs and therefore have a lower price. Because we do not observe the local cost advantages of firms, using variation across bids submitted by different firms will tend to overestimate the density discounts, as we would attribute the lower price of co-located units entirely to their proximity.¹³

Controlling for the average unit prices δ_{ift} eliminates this source of bias in the estimation of density discounts because these absorb firm-specific cost advantages in a unit. Hence, in our estimation, scale and density discounts are estimated using variation across different *combinations* submitted by the same firm over the same set of units in the same auction. Consequently, our estimation strategy requires a large number of combination bids submitted by the same firm in

¹² In §4.3 we discuss results with alternative cluster definitions. We describe the clustering algorithm in a separate online appendix that can be found on the authors' webpages (<http://www.columbia.edu/~mo2338/>; <http://www.columbia.edu/~gyw2105/>).

¹³ This issue was previously noted by Holmes and Lee (2012).

order to obtain consistent estimates of the parameters in (2). Note that the estimation provides estimates of the average unit prices δ_{ift} along with β^{dens} and β^{scale} . In the next section, we discuss the results regarding discounts (β^{dens} and β^{scale}). The estimated average unit prices (δ_{ift}) are later used in §§5 and 6.

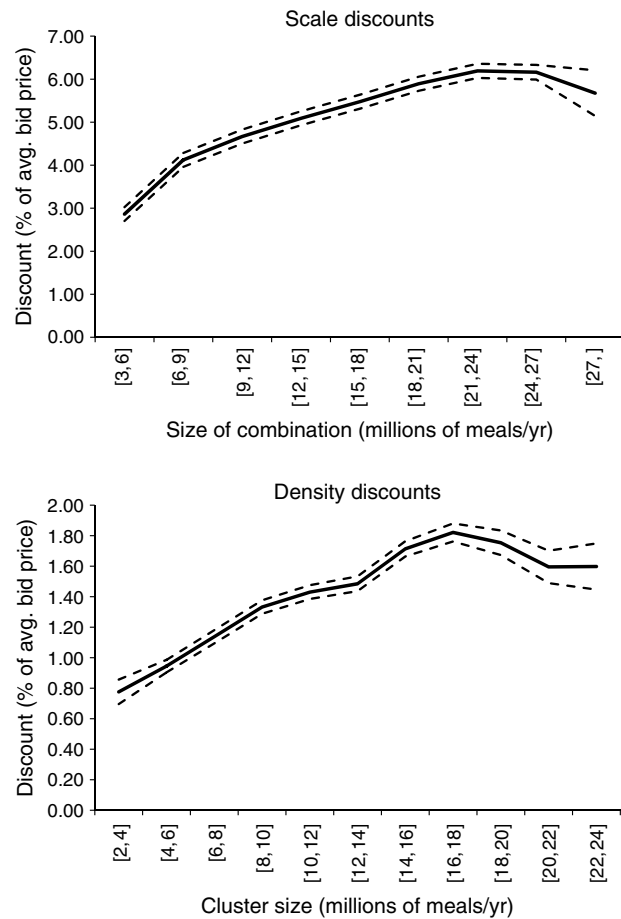
4.2. Estimation Results

We begin by providing details on the specifications for the discount functions $g^{\text{dens}}(v_c, \beta^{\text{dens}})$ and $g^{\text{scale}}(v_a, \beta^{\text{scale}})$ in the regression equation (2). Recall, the cluster size v_c and package size v_a are measured in million meals per year. The scale discount function $g^{\text{scale}}(\cdot)$ is estimated by a step function with 10 equally spaced intervals of size three that cover all the range of bids. Similarly, the density discount function $g^{\text{clust}}(\cdot)$ is estimated with a step function with 13 equally spaced intervals of size two. The step functions for g^{clust} and g^{scale} are implemented with binary variables indicating the size level, excluding the smallest level, so that the coefficient represents the average change in bid price at that level relative to the excluded level. For example, the results in Table 4, right column (under scale discounts), indicate that a combination a of size v_a in the range $[6, 9]$ is CLP\$17.49 cheaper than one of size $v_a \in [0, 3]$.

Table 4 shows the estimates of the first-stage regression (2), which includes covariates measuring scale and density discounts and a set of firm-unit-auction average unit prices $\{\delta_{ift}\}$ (the estimates of average unit prices are not shown). Robust standard errors are reported in parentheses. All the coefficients are estimated with precision, which is expected given the large sample size. The explanatory power of the regression is remarkably high (R -square equal to 0.98).

Figure 6 compares the estimates of the discount functions due to scale and density in terms of percent change on the average bid price (the average bid

Figure 6 Estimates of Discount Curves Due to Scale (Top) and Density (Bottom)



Note. Dotted line indicates the 95% confidence interval of the estimate.

price is 75¢). The results suggest that the magnitude of scale discounts are economically significant (see the top of Figure 6): increasing the package size to 20 million (about eight units) generates discounts of approximately 6%.¹⁴ Interestingly, most of the scale discounts are seen for volumes below 18 million (about seven units), suggesting they get exhausted after that point.

Figure 6, bottom chart, graphs the estimates for the density discounts ($g^{\text{clust}}(v_c, \beta^{\text{dens}})$). The graph shows the average discount (as a function of the cluster size), in addition to any discounts generated by scale. Density discounts appear to be economically significant, but relatively smaller than the discounts due to scale. Similarly to scale, they also get exhausted after seven units. To illustrate, consider a bid for three units of size 3 million each. Scale discounts are approximately 4.5% of the average bid price. In addition, if the three units form a cluster, the additional discount due to density is 1.3%, for a total discount of 5.8%.

Table 4 Results from First-Step Regression (Equation (2))

Density discounts			Scale discounts		
Size	Estimate	SE	Size	Estimate	SE
[2, 4]	3.29	(0.17)	[3, 6]	12.14	(0.35)
[4, 6]	4.01	(0.09)	[6, 9]	17.49	(0.35)
[6, 8]	4.82	(0.09)	[9, 12]	19.78	(0.35)
[8, 10]	5.65	(0.10)	[12, 15]	21.59	(0.35)
[10, 12]	6.06	(0.10)	[15, 18]	23.22	(0.35)
[12, 14]	6.30	(0.10)	[18, 21]	24.99	(0.35)
[14, 16]	7.27	(0.11)	[21, 24]	26.26	(0.36)
[16, 18]	7.72	(0.13)	[24, 27]	26.12	(0.37)
[18, 20]	7.44	(0.17)	[27, .]	24.07	(1.15)
[20, 22]	6.76	(0.23)			
[22, 24]	6.78	(0.33)			
[24, .]	7.03	(1.18)			

Notes. Robust standard errors are shown in parentheses. Size measured in million meals per year. Number of observations is 409,831. Centered R -square is equal to 0.98.

¹⁴ Note that typical margins in this industry are between 4% and 8%.

4.3. Sensitivity Analysis

In this section we describe some additional empirical analysis to validate the robustness of the results. To assess the validity of the estimated δ_{ijt} —which capture the *average* price for a unit after accounting for scale and density discounts—we compared them with the *actual submitted* stand-alone bids b_{ijt} . The correlation between the two measures is 0.983, and a scatter plot shows a very small dispersion along the identity line. The average ratio b_{ijt}/δ_{ijt} is 0.9994, with a standard deviation of 0.038. These results suggest that regression (2) is effectively separating average individual unit prices from package discounts.

The CA that is ran by the government eliminates some bids that fall below a given price band. This price band is calculated with the actual bids submitted by all firms, and is therefore unknown to the firms at the time of bidding. For this reason, we did not exclude bids outside the band in our analysis. To check the robustness of the estimates, we re-estimated regression (2) excluding the bids below this price band—that is, using only the bids that were considered in the actual allocation mechanism. The estimated discount curves are practically identical to those reported in Table 4 and Figure 6.

Our definition of clusters—using a 150 km radius—is a reasonable area to capture economies of density in this industry based on the information available to JUNAEB. To analyze the robustness of the estimates of density discounts, we considered alternative cluster sizes, with radiuses of 100, 200, 300, and 400 km. Density discounts appear to decrease rapidly as the radius of the cluster increases. The estimated discounts with a cluster radius of 150 km can be as much as 40% larger than the discounts estimated with a cluster radius of 400 km. The discounts estimated with cluster radiuses of 100 km and 200 km are similar to those obtained in our main results. Overall, this analysis suggests that our cluster definition of 150 km radius is reasonable at capturing density discounts.

We also estimated regression (2) with the logarithm of the bid price as the dependent variable. All the estimated effects were similar in order of magnitude and statistical significance. We also estimated a more parsimonious specification of regression (2) that replaces the average unit prices δ_{ijt} with separate indicator variables for firm, units, and auction (with no interactions between them). The estimates of density discounts in this model differ substantially from those reported in our main results. This suggests that controlling for the average unit prices $\{\delta_{ijt}\}$, which controls for firm's local advantages, are important to obtain consistent estimates of the discount functions.

We studied heterogeneity of the discount curves for firms of different sizes to see if the observed pattern in the discounts varied. We used a firm classification

used by JUNAEB based on the firm's financial capacity to group firms into three types: large, medium, and small (this is similar to the size classification introduced in §3.3). We ran regression (2) separately for the three types of firms. Whereas we found heterogeneity, the magnitude and shape of the scale discounts were quite similar across the three types. Small and medium firms tend to exhaust their scale discounts at around 15 million meals, and large firms exhaust their discounts at around 20 million. In all the regressions, the scale discounts were in the order of 4% to 7%. The estimates of density discounts were in the order of 1% to 2%, and medium firms tend to offer higher density discounts. In summary, although we do find some heterogeneity across firms of different size, our main results prevail: (1) discounts are economically significant, (2) scale discounts dominate, and (3) discounts get exhausted at around seven units.

One possible interpretation for why the scale discounts get exhausted after seven units is that cost synergies get exhausted after that point. However, there could also be other alternative explanations related to markup adjustments. In particular, it may be possible that firms adjust markups because of the side constraints in the allocation mechanism that limit the maximum number of meals that can be awarded to a single firm. This is specially relevant for large firms for which the average maximum number allowed is around 20 million meals, which is close to the number at which discounts get exhausted. To test this alternative explanation, we allowed for interactions between the maximum number of meals allowed (which may change across auctions for a given firm) and the scale discount function, so that large firms with different maximums may exhaust their discounts at different quantities. The estimates suggest that the restriction on the maximum meals allowed does not change significantly the quantity at which scale discounts get exhausted. We cannot reject the null hypothesis that a one standard deviation change in the maximum restriction does not change the quantity at which the scale discounts are maximized (p -value > 0.11).¹⁵

¹⁵ More specifically, for the purpose of this test the discount curve is specified as a quadratic polynomial including main effects for volume and volume square plus their interactions with maximum volume restriction. This analysis is limited to large firms, which are the only firm types that have variation in the maximum volume restriction. For these firms, the restriction is usually determined by the market share constraint over the total standing contracts (see point 1 in §3.2.2 for a description of the different types of market share restrictions). A given firm may have different standing contracts in different auctions. Therefore, the maximum number of meals the firm can be awarded with in a given auction changes. For other types of firms, the maximum number allowed is determined by the financial evaluation, which usually does not change across years.

To further study the magnitude of markup adjustment, the next section presents a detailed study of other potential strategic effects that could affect package discounts.

5. Strategic Behavior and Markup Adjustments

The regression analysis suggests that discounts for combinations of units are significant. It also shows that scale discounts are larger than density discounts. Although this may be suggestive that cost synergies are substantial, it is not conclusive because discounts may also arise as a result of markup adjustments. In fact, the following simple statistics regarding the discounts observed in the bidding data provide some evidence that this may be the case.

Table 5 shows a comparison of the average discounts for packages of two units ($N = 2$), where one of the units is located in Santiago—the centrally located capital with a high population density—and the other unit varies among the following locations: (1) another unit located in Santiago, (2) a unit located in the extreme south (these units are more than 2,000 kms apart from Santiago), (3) any location (including Santiago and extreme south). Here, the package discount (per meal) is defined as $(b_{ift}v_i + b_{aft}v_a - b_{i \cup a, ft}(v_i + v_a)) / (v_i + v_a)$. We only used bids submitted by firms already established and with standing awarded contracts to control for the effect of fixed costs needed to initiate the operations. The table also provides the average discounts for larger packages of five units ($N = 5$), in which four units are in Santiago and the fifth unit changes its location.

The table shows that combining two units in Santiago yields larger discounts than average, which is consistent with economies of density. What is surprising is that combining a unit in Santiago with a unit in the extreme south results in even larger average discounts. A similar pattern is observed in larger packages. This discount pattern is unlikely to be generated by cost synergies alone. Arguably, a package formed by two units in Santiago should exhibit larger cost synergies than a package formed by a unit in Santiago and another unit in the extreme south, because only in the former case economies of density

can be exploited. Moreover, because of transportation costs, some of the inputs to serve the units in the extreme south are purchased locally, so economies of scale that arise from volume discounts associated to centrally purchasing inputs cannot be exploited as much. Consequently, we conjecture that a fraction of the discounts observed for packages combining units in Santiago with units in the extreme south could be explained by strategic behavior in bidding.

The focus of this section is to formally analyze markup adjustments that arise from such strategic behavior and to measure what portion of the discounts estimated in §4 is explained by these markup adjustments vis-à-vis cost synergies. We start by revisiting the existing theory describing bidders' incentives to submit discounted package bids even in the absence of cost synergies, which we refer to as *strategic bundling*. Based on this theory, we establish testable hypotheses that predict which type of packages in a CA should be subject to more discounting due to strategic bundling (§5.2). This requires quantifying the uncertainty bidders face due to asymmetric information (§5.3). Then, §5.4 provides an empirical strategy to compare package discounts for which the incentives to make markup adjustments due to strategy bundling change as predicted by the theory. This provides an estimate of the magnitude of package discounts arising from strategic bundling. Finally, in §5.5 we discuss another potential source of strategic behavior: the threshold problem.

5.1. The Bidder's Problem and Strategic Bundling

We start by providing intuition that illustrates why bidders may have incentives to submit discounted package bids even in the absence of cost synergies. To do so, we use the observation by Cantillon and Pesendorfer (2006b) that when consumer valuations are additive (and under other fairly general conditions) the pricing problem of a multiproduct monopolist is equivalent to the bidder's problem in a private value CA. The mapping is as follows. In the auction setting, from the perspective of a firm, bids submitted by opponents are random due to asymmetric information. Similarly, the multiproduct monopolist faces random consumers' valuations. The distribution of consumer valuations for each unit maps to the distribution of the minimum of the opponents' bids for the unit. In addition, the optimal prices for the bundles in the multiproduct monopolist problem correspond to the optimal bid prices of the bidder in each package in the CA setting.

An established literature in multiproduct monopolist pricing has shown that the firm may have incentives to provide discounts for a bundle of products

Table 5 Comparison of Discounts from Combining Units in Santiago with Other Units in Different Geographic Regions

Sample	Discount	
	$N = 2$	$N = 5$
Extreme south	13.62	7.44
Santiago	11.84	7.17
Any location	11	6.62

even if consumers' valuations over units are additive and in the absence of production cost synergies (see Adams and Yellen 1976, Schmalensee 1984, McAfee et al. 1989, Fang and Norman 2006, Chu et al. 2011). This is observed when the consumers' valuations for the bundle exhibit less variance than the valuations for individual items, because of averaging. Because the bundle exhibits less variance, the monopolist can extract surplus more easily from it than from individual units, providing incentives to sell it at a discounted price or even to just offer the bundle. A typical example of this type of behavior is the selling of bundles of channels by TV cable companies (Crawford 2008). Given the parallel between the monopoly problem and the bidder's problem in a CA, these results suggest that bidders may have incentives to submit discounted package bids even in the absence of cost synergies. Cantillon and Pesendorfer (2006a) provide a simple example of this behavior, which we refer to as strategic bundling.

More formally, following Equation (2), but for now assuming additive bids to draw a parallel with the multiproduct monopolist literature, we consider that a bidder's statistical model of the bids (per meal) of opponent firm f are given by

$$b_{aft} = \sum_{i \in a} \delta_{ift} \frac{v_i}{v_a}, \quad (3)$$

where for given t , $(\delta_{1ft}, \dots, \delta_{N(t)ft})$ are i.i.d. samples across f of a multivariate distribution known to the bidder ($N(t)$ is the number of units in auction t). To win a unit, the bidder needs to underbid all of its competitors; in that sense, he competes against the minimum of the bids submitted by its opponents. Accordingly, let $\underline{\delta}_i = \min_f \delta_{ift}$, where the minimum is taken over all of the bidder's opponents (to simplify notation, we omit the bidding firm under consideration and auction indexes from $\underline{\delta}_i$).

As mentioned above, the main insight of the multiproduct monopolist literature is that bidders may have incentives to submit discounted package bids in any best response if the competitors' bids for the package exhibit less variance than the bids for the individual items. This will be the case if the pairwise correlations across i among the minimum unit bids in the package, $\{\underline{\delta}_i\}_{i \in a}$, is not too high. More generally, the incentives to submit discounted package bids increase as the effect of the reduction in variance of a package becomes larger. For example, because of the law of large numbers, this will be the case if a is a large package and $\underline{\delta}_i$ are independent random variables (Bakos and Brynjolfsson 1999). Moreover, the reduction in dispersion provides stronger incentives to engage in strategic bundling for firms with relative cost advantages, because they have a higher chance to win the package.

5.2. Testable Predictions

We now elaborate on the previous insights to formulate testable predictions regarding discounts due to strategic bundling in a bidder's best response when competing against the distribution of its opponents' bids.¹⁶ Although the bidder's problem in our setting has similarities to the multiproduct monopolist, there are at least three issues that limit the applicability of the existing results in the literature in the context of our study:

1. Most of the existing results focus on the case of two units only.

2. Although a few studies analyze settings with more units (Fang and Norman 2006, Bakos and Brynjolfsson 1999), their focus is to analyze under what conditions it is preferable for a bidder to exclusively submit a package bid covering all units (referred to as pure bundling) as opposed to individual bids for each unit. In contrast, in our data firms often submit bids for individual items as well as package bids. Our focus is to analyze the *magnitude of discounts* due to strategic bundling given a mechanism that allows for both package bids and individual bids.

3. Previous work assumes that bidder's costs and competitors' bids are additive (like in Equation (3) above). In contrast, in the CA of school meals, package bids exhibit discounts (they are subadditive) and firms appear to have cost synergies.¹⁷

We perform a significant set of computational experiments to deal with these challenges and study whether the insights from the multiproduct monopolist literature carry over to a setting that resembles our CA more closely. First, to deal with the first two problems mentioned above, we analyzed the extensive numerical experiments in Chu et al. (2011).¹⁸ These results, when translated to our CA setting, provide the optimal bid prices for a firm that can bid on any package in a CA when competing against a distribution of (additive) competitors' bids. The results show how package discounts due to strategic bundling are affected by certain statistics of the distribution of competitors' bids as well as the firm's own costs. The number of packages (and hence of optimal bid price decisions) grows exponentially with the number of units, so the analysis is limited to five units. The results are provided in the separate online appendix that can be found on the authors' webpages.

¹⁶ Our approach is similar to Crawford (2008), which tests the theoretical predictions for multiproduct bundling in the cable television industry and finds evidence of strategic bundling.

¹⁷ The only exception we are aware of is Salinger (1995), which incorporates cost synergies. However, it only studies a specific numerical example.

¹⁸ We are grateful to the authors for providing us with this data.

The results in Chu et al. (2011) (like most of the literature in multiproduct monopolist) assume additive opponents' bids and the absence of cost synergies. To deal with this limitation we develop our own set of numerical experiments. The results (reported in the separate online appendix) suggest that the predictions from the first set of experiments described above extend to the case of subadditive bids and bidder's cost synergies. Because of the computational complexity involved in computing optimal bid prices, the second set of experiments is focused on two units only and does not consider the market share constraints that the actual CA includes.

Finally, note that the theoretical predictions from the multiproduct monopolist literature are based on the distribution of the minimum of the opponents' bids. Unfortunately, we do not have enough data to quantify statistical properties of the distribution of the minimum average unit prices directly. Nevertheless, numerical experiments show that, with normally distributed bids, the correlation structure and coefficient of variation of the underlying opponents' bids carries over to the distribution of the minimum of these bids. More specifically, the coefficient of variation of the random variable $\min_f \delta_{ift}$ increases with the coefficient of variation of the underlying random variable δ_{ift} ; the correlation between $\min_f \delta_{ift}$ and $\min_f \delta_{jft}$ increases with the correlation between δ_{ift} and δ_{jft} . In addition, in the second set of experiments mentioned above we explicitly consider the distribution of the minimum of the opponents' bids.

Overall our computational study confirms the insights from the multiproduct monopolist literature. Based on them we establish testable predictions for the presence of strategic bundling. From the perspective of a bidder in a multiunit auction, bids submitted by opponent firms (indexed by f) are random due to asymmetric information. We assume that the bidder's statistical model of its opponent bids is given by Equation (2) (so they are subadditive), where for given t , the bid price vector of each firm f , $(\delta_{1ft}, \dots, \delta_{N(t)ft})$, is viewed as an i.i.d. sample of a multivariate distribution.

We formulate the following testable hypotheses:

HYPOTHESIS 1 (H1). *The magnitude of the discounts due to strategic bundling when combining a unit i with a package a (possibly a singleton) increases, as the correlation among the competitors' average unit bid prices between the individual unit (δ_{ift}) and the package ($\{\delta_{jft}\}_{j \in a}$) decreases. (We define the correlation between a unit and a package as the average correlation: $\text{AvgCorr}(i, a) = \sum_{j \in a} \text{Corr}(i, j)(v_j/v_a)$.)*

HYPOTHESIS 2 (H2). *The magnitude of the discounts due to strategic bundling when combining a unit i with a package a (possibly a singleton) increases, as the coefficient*

of variation of the competitors' bid price for the individual unit, δ_{ift} , increases.

HYPOTHESIS 3 (H3). *Firms with a relative cost advantage in the combined package have stronger incentives to engage in strategic bundling. Hence, for this set of firms the effects described in H1 and H2 are more pronounced.*

In what follows we describe an empirical strategy to test H1–H3 in our data. Finding empirical support of these hypotheses is suggestive that firms' bidding behavior is consistent with strategic bundling.

5.3. Quantifying Bid Randomness and Cost Advantages

In this section, we describe how to come up with estimates of statistical measures of the joint distribution of average unit bid prices ($\delta_{1ft}, \dots, \delta_{N(t)ft}$) and of firms' cost advantages that are useful to test H1–H3.

First, we use the estimates of the average unit prices $\hat{\delta}_{ift}$ (obtained from the first-step regression (2)) to estimate statistical measures of the joint distribution of the average unit prices ($\delta_{1ft}, \dots, \delta_{N(t)ft}$). Recall that from the perspective of a bidder, the average unit prices of another firm f in auction t , $\hat{\delta}_{ft} = (\hat{\delta}_{1ft}, \dots, \hat{\delta}_{N(t)ft})$, are viewed as an i.i.d. random vector sampled from a common multivariate distribution that we seek to characterize.¹⁹ Specifically, using the estimates $\{\hat{\delta}_{ift}\}$, which are available for F_{it} firms on unit i and auction t ,²⁰ we calculate the sample average $E(i, t) = (1/F_{it}) \sum_f \hat{\delta}_{ift}$, sample standard deviation $SD(i, t) = ((1/F_{it}) \sum_f (\hat{\delta}_{ift} - E(i, t))^2)^{1/2}$, and coefficient of variation $CV(i, t) = SD(i, t)/E(i, t)$, for unit i in auction t . Similarly, we calculate the sample correlation between units. To get more precise estimates, we pooled all the auctions to estimate one correlation coefficient for each pair as $\text{Corr}(i, j) = \sum_{f, t} (\hat{\delta}_{ift} - E(i)) \cdot (\hat{\delta}_{jft} - E(j)) / (SD(i)SD(j) \sum_t F_{it})$, where $E(i)$ and $SD(i)$ are the sample average and sample standard deviation for unit i , respectively, pooling data across all

¹⁹ Variation in average unit prices is not a perfect proxy for uncertainty due to asymmetric information if firms can anticipate some of the variation in δ_{ift} . In particular, the set of local incumbents and other firm characteristics are known to all firms and they may help in anticipating some variation in bid prices across firms, as we show later in §6. As a robustness check, we also did all of the subsequent analysis using the *residuals* of the regression of δ_{ift} on the following observable firm characteristics: *LocIncumb*, *Renew*, *Performance*, *NewFirm*, *FinGrade*, and *FinGrade = 1* (these are defined in §6). All the main results that follow in this section were similar.

²⁰ Some firms did not submit bids for all units, which precluded the estimation of those average unit prices. A few average unit prices were not identified (about 1%); for example, when two units are never observed in separate package bids. We also note that we consider all firms to compute statistics of the joint distribution of bids; these provide a good approximation to statistics of the joint distribution of *opponents'* bids.

auctions,²¹ F_{ijt} is the number of firms with estimated average unit prices for both units i and j in auction t .²²

In addition, testing H3 requires a measure of firms' cost advantages. As we show later in §6, firms operating nearby TUs to a unit being auctioned tend to offer lower prices, possibly because of cost advantages from economies of density. Therefore, we define these local incumbents as firms with a relative cost advantage in a unit.

5.4. Empirical Tests of Discounts Due to Strategic Bundling

The correlation among units is a key incentive that drives discounts due to strategic bundling: the higher the correlation, the lower the benefit of combining units in terms of reducing dispersion on the distribution of competitors' bids. In the data, more than 90% of the sample correlations are positive, and the median is about 0.6. The relatively high correlations suggest that on average the incentives to do strategic bundling are perhaps not too strong. Now, we study this more rigorously.

In particular, to test H1 and H2 empirically, we seek to estimate the effect of variability (captured by CV) and correlation (measured by *AvgCorr*) on the discounts observed in the bids. To do this, we define an alternative measure of package discounts that isolates the *incremental* effect of adding a unit i to a package of units a ; this definition is consistent with the statements in H1–H3. For this, we require using “nested” bids submitted by a firm, for which we observe a stand-alone bid for i , a bid for package a (which could also be a single unit), and a bid for the package $a' = a \cup \{i\}$. This incremental discount, that we normalize per meal, is calculated as

$$D_{ft}(i, a) = \frac{b_{aft}v_a + b_{ift}v_i - b_{a'ft}(v_a + v_i)}{v_a + v_i}. \quad (4)$$

Hence, these incremental discounts can only be calculated for a subset of combinations for which we observe nesting, which is the main limitation of this

²¹ We also estimated different correlation coefficients for each auction, but some correlation pairs had few observations (as low as two) and were very noisy.

²² For robustness, we also did the following analysis using the stand-alone bid prices (b_{ift}) instead of the average unit prices (δ_{ift}) to compute all the statistics that describe the joint distribution bids. The results obtained were similar. Because some firms do not submit stand-alone bids for all units, we decided to use the average prices instead. Moreover, firms compete against all bids submitted by other firms, not just stand-alone bids. For this reason, we believe average unit prices are a better proxy to summarize the distribution of opponent firms.

Table 6 Comparison of Two Approaches to Measure Discounts

No. of units	Equation (4) ($D_{ft}(i, a)$)	Equation (5) (g^{scale})
1 to 2	8.31	10.59
2 to 3	7.61	7.6
3 to 4	6.95	6.64
4 to 5	4.1	5.83
5 to 6	5.03	5.31

approach with respect to model (2).²³ Because we want to isolate the effect of discounts generated by strategic bundling, we also limited our sample to combinations that contained units that are not co-located—units that do not form clusters and are not located in the same geographic region—to remove the effect of density discounts (we revise this point later).

This alternative approach to measure discounts can be directly linked to the scale discount function g^{scale} used in regression (2). Specifically, ignoring density discounts and replacing Equation (2) in Equation (4), we get the relation

$$D_{ft}(i, a) = (g^{\text{scale}}(v_{a'}, \beta^{\text{scale}})v_{a'} - g^{\text{scale}}(v_a, \beta^{\text{scale}})v_a + \bar{\epsilon}_{i, a, ft})/v_{a'}, \quad (5)$$

where $\bar{\epsilon}_{i, a, ft} = \epsilon_{ift}v_i + \epsilon_{aft}v_a - \epsilon_{a'ft}v_{a'}$. Hence, ignoring the error terms, we observe that $D_{ft}(i, a)$ is just a different way of measuring volume discounts—the marginal additional volume discount per meal when going from a package of $|a|$ units to $|a| + 1$ units—but it is directly related to the estimated discount function g^{scale} . In fact, Table 6 makes a direct comparison of the discounts, showing the average $D_{ft}(i, a)$ calculated directly from the data using Equation (4), and the estimated discounts using Equation (5) based on the estimates of g^{scale} from the first-stage regression (2), ignoring the error term $\bar{\epsilon}_{iaft}$. As seen in the table, the quantities are similar.²⁴ Given this equivalence, the empirical results that we provide in this section can be used to analyze what portion of the scale discount function estimated in regression (2) can be explained by strategic bundling effects vis-à-vis cost synergies.

We estimated the following linear regression to describe the incremental discounts $D_{ft}(i, a)$:

$$D_{ft}(i, a) = \beta_1 CV(i, t) + \beta_2 AvgCorr(i, a) + \beta_3 Size_i + \phi_f + \tau_t + \chi_a + e_{ftia}, \quad (6)$$

²³ We note, however, that the estimated scale discount function from regression (2) in the complete sample is similar to the estimate in the subsample of nested bids. In that sense, this subsample is not “special” regarding scale discounts. Another advantage of regression (2) is that it provides estimates of the average unit prices δ_{ift} that will be used later in this section and also in §6.

²⁴ The comparison is done over the subsample of nested bids, removing two outlier firms with large discounts.

Table 7 Results of the Regression Analysis on Strategic Bundling (Equation (5))

	(1) 2 units, small firms	(2) 2 units	(3) 2 units, low cost	(4) <5 units
<i>CV</i>	4.656* (2.242)	2.640* (1.262)	15.42** (5.305)	3.352** (0.443)
<i>AvgCorr</i>	−0.819** (0.306)	−0.592* (0.241)	−1.279+ (0.709)	−0.295* (0.117)
Observations	4,425	11,268	564	77,619
<i>R</i> -square	0.266	0.432	0.465	0.635

Notes. Both regressions include the control variable *Size* and dummies for firm, auction, and combination. Standard errors are in parentheses.

+, *, and ** indicate statistical significance at the 0.1, 0.05, and 0.01 confidence levels, respectively.

where e_{ftia} is an error term, and ϕ_f and τ_t are firm and auction fixed effects, respectively. The package fixed-effect χ_a controls for effects from scale and other factors associated with package a that may affect the discount. Similarly, $Size_i$, which measures the number of meals of unit i , is included to control for scale. Given the controls included in Equation (6), the estimation uses variation with respect to the different units that were combined with a given package a .

As previously observed, $D_{fi}(i, a)$ is directly related to the volume discounts estimated in the first-stage regression, g^{scale} . Therefore, we can use the regression results from (6) to measure how much of these volume discounts are driven by strategic effects. In particular, the regression will allow us to compare the magnitude of the discounts for packages in which the incentives for strategic bundling are high (when *CV* is high and *AvgCorr* is low) relative to those with low incentives for this type of strategic behavior. The smaller this difference is, the smaller the fraction of volume discounts that can be explained by strategic bundling as oppose to economies of scale.

Table 7 shows the results of regression (6). Each column in the table shows the estimates over a different sample. Column (1) reports the estimates over a sample of small firms that can only bid on packages of one or two units.²⁵ In this case, the stand-alone and two unit package bids of one of these firms do not compete against its own bids for larger packages (because they cannot submit bids for larger packages). Hence, this setting resembles more closely the multiproduct monopolist problem analyzed in the literature (which we used to motivate our hypotheses) and the second set of numerical experiments presented in

the online appendix. Here, *AvgCorr* measures the sample correlation between the two units in the package. The results suggest that discounts increase as the coefficient of variation (*CV*) increases and as the correlation between the units decreases, providing support for H1 and H2.

Column (2) shows the results in a sample including packages of two units from all firms. Here, the coefficient of *CV* and correlation are significant and their directional effect provides further support for H1 and H2, although the magnitude seems somewhat smaller than in column (1). To test H3, column (3) uses a subsample of regression in column (2) selecting those bids where the bidder is a local incumbent in both units. The coefficients of the *CV* and *AvgCorr* increase in magnitude, although the statistical significance of the correlation coefficient is smaller (p -value = 0.07). The larger magnitude of the coefficients is suggestive that the incentives for strategic bundling are larger for firms with a cost advantage.²⁶

The results show a similar pattern when the sample includes packages of up to four units, reported in column (4) of Table 7. Discounts increase with *CV* and decrease as the average correlation between a unit and the other units in the package increases. Analyzing the effect of relative cost advantages is harder in this sample because as packages become larger, it is unlikely that a firm will have a relative cost advantage in all units. We have done some analysis limiting the sample to firms that are incumbents in a significant portion of the units (say, 60%) and also found that the effects of correlation and *CV* were larger in this sample.

Our empirical analysis suggests that the firms' discounts are consistent with our H1–H3, and therefore, with strategic bundling. However, this strategic behavior explains a small fraction of the discounts on average. For packages of four units, based on the estimates of column (4), a two-standard deviation increase in *CV* and decrease in *AvgCorr*, increases average discounts by only 2.7% and 1.0%, respectively. To further quantify the portion of the average discounts that is explained by strategic bundling, we compare strategic bundling discounts predicted by the estimated model (6) for an average package (with *CV* and *AvgCorr* at their respective sample averages) with a package with the smallest incentives for strategic bundling—that is, when $CV = 0$ and $AvgCorr = 1$. This calculation suggests that only 3% of the average discounts are explained by strategic bundling.

We got similar results in the other specifications. In fact, for packages of two units, based on the estimates of column (2) of Table 7, a two standard deviation change in *CV* and *AvgCorr* increases discounts

²⁵ To increase the sample size, we also included in this subsample packages with co-located units and controlled for the distance between the units (not reported in the table). In all other specifications we restrict the sample to packages with units that are not co-located.

²⁶ Further restricting the sample to small firms that can bid on packages of one or two units only and are incumbents on both units revealed similar results.

by less than 3.5% and 4.5%, respectively, of the average discount. For packages of two units submitted by low cost firms, the effect is larger (as predicted by theory), but still reasonably low: 14.7% for CV and 10% for *AvgCorr*. We also tested other specifications that included more controls into regression (6) and the effects were similar.²⁷

Recall that to remove the effect of density discounts, the empirical analysis is limited to packages including units that are not co-located. However, the correlation between two units tends to increase as the distance between them becomes smaller.²⁸ Therefore, in light of our results, discounts due to strategic bundling between co-located units should be smaller than for the sample of packages we analyzed.

Let us summarize our findings. Section 4 estimated discounts observed in the bidding data; these discounts can be explained by cost drivers and/or strategic bundling effects. The results of this section suggest that although bidders do engage on strategic bundling, these markup adjustments explain on average only a small fraction of package discounts. Hence, we will in the remainder of the paper take the discount functions estimated in §4 as proxies for the scale and density *cost synergies* involved in these auctions. Nevertheless, whereas on average strategic bundling behavior appears to be minor, our initial motivating example and the analysis suggests that for some specific units it could be more severe and this may affect the efficiency of the allocation mechanism. We discuss this effect together with other design recommendations in §7.

5.5. The Threshold Problem

In this section we discuss another potential source of strategic behavior that could affect markups, namely, the threshold problem. In a nutshell, the threshold problem arises when two local bidders each interested in a single unit, say A and B , respectively, need to underprice a large global firm that submits a package bid for the package $A \cup B$ (the “threshold” bid). In this case, a local bidder lowering its bid benefits the other local bidder. Because local bidders do not internalize these benefits, their bids may not be as competitive as they could have been if package bidding was not allowed. A severe threshold problem could bias the auction toward large bidders to the disadvantage of small bidders, potentially creating inefficiencies and increasing the cost allocation (for a more

detailed discussion, see Milgrom (2000) and Pekeč and Rothkopf (2003); Chernomaz and Levin (2012) and Maréchal and Morand (2009) provide concrete examples of the threshold problem in equilibrium bidding strategies).

Unfortunately, the equilibrium theory of CAs is not well developed. Moreover, there is not a recognized clear mapping between the threshold problem in CAs with another well-studied economic problem, like in the case of strategic bundling. Hence, we are not able to directly test in our data whether bidders’ behavior is consistent with the threshold problem. However, we provide some evidence that the threshold problem may not be too severe in our application.

One of the negative consequences of the threshold problem is that package bidding creates a disadvantage for small bidders, making their bids less aggressive and thereby depressing competition in the auction. Our analysis of the winning bids suggests this problem is not severe in the combinatorial auction for school meals: small packages (and firms) do win frequently. About 50% of the winning bids are for packages of two units or less; and 22% of the winning bids are from small firms (firms that can bid up to or three units; they constitute about 28% of the firms), suggesting that small bidders are not at a large disadvantage in the auction (see Table 2).

One possible explanation for why small firms win is that the market share constraints force the allocation to award units to these firms. However, this could be inefficient and costly for the government if bids from small firms are not competitive, so we also study this potential negative effect. To do it, we compared the average bid prices (δ_{ift} ’s) across different types of firms. For each unit, we ranked the average prices across firms. Small firms rank in the lowest 10 percentile in units across all regions (except region two, where the cheapest small firm ranks in the lowest 12.5 percentile). Inspection of the average prices reveals that it is not just one small firm that ranks low everywhere—small firms seem to have local advantages so that different small firms are competitive in different regions. This analysis suggest that small firms are, overall, quite competitive due to local advantages.

To further study potential inefficiencies and increases in procurement cost generated by the market share constraints, we re-solved the optimal allocation problem in each auction removing these constraints. In particular, using the same submitted bids, we solved the mathematical program that finds the optimal allocation with and without the market share constraints and compared the allocations and the value of the objective function. Details of the mathematical program can be found in Epstein et al. (2002). We find that even though the optimal allocation can

²⁷ We estimated regressions that include a firm-auction fixed effect (instead of the separate fixed effects ϕ_f and τ_t). All results were similar, although in a few cases the coefficients were not statistically significant.

²⁸ Increasing the distance between two units by 400 km lowers the correlation by 0.2 on average.

change, the optimal objective function value does not; the average change across all auctions is 0.3%, confirming that small firms seem to be quite competitive.

Finally, the intuition behind the threshold problem suggests that large bidders should make steeper discounts than small firms. The idea is that large firms may inflate their single unit bids relative to the package bids to decrease the chances that a small firm wins a single unit bid by matching it with a large firm's own single unit bid and in that way beat the package. We do not observe this phenomena in our data. In fact, the average discount over all packages of size 2 is actually 20% larger for small firms (that can only win two units maximum) than for large firms (that can win up to eight units). This difference is still observed after correcting for the type of units included in the packages, which may be different on average for small and large firms.

6. Supplier Diversification and Local Competition

The second design issue that motivated our empirical study is whether to diversify the supplier base to promote competition. In this section, the analysis focuses in estimating the effect of local competition on prices, which can be used as a benchmark to evaluate the benefits of supplier diversification in reducing procurement costs.

The average unit price δ_{ift} in regression (2) captures the average price charged for supplying a unit before accounting for the interactions that may arise with other units. It is affected by the firm's cost in supplying the unit—net of any cost synergies with other units in a combination—plus a markup. The unit's cost is affected by the characteristics of the unit (e.g., size and location of the schools) and other specific costs advantages that a firm may have. An example of a firm-specific advantage is when a bidder has a lower cost of supplying a unit because it has a nearby warehouse used to serve other related businesses in the area. Consequently, the average unit price should be lower for firms that are local incumbents; recall that we defined local incumbents as firms with ongoing contracts for *nearby* TUs awarded in a previous auction (consistent with our cluster definition we say that two TUs are near if the distance between their weighted geographical centers is less than 150 km). As discussed in §3.2, the presence of local incumbents could generate additional price reductions from all firms (including nonincumbents) through a local competition effect.

For the estimation, we define a binary variable $LocIncumb_{ift}$ indicating if firm f is a local incumbent for unit i in auction t . Local competition is measured

as the number of rival firms that are nearby incumbents, defined as $LocComp_{ift} = \sum_{f' \neq f} LocIncumb_{if't}$. We seek to estimate the following regression:

$$\delta_{ift} = \beta_1 LocIncumb_{ift} + \beta_2 LocComp_{ift} + \beta_x X_{ift} + u_{ift}, \quad (7)$$

which captures the effect of local incumbents and competition on the average unit price δ_{ift} . The vector X_{ift} includes other controls (described shortly), and the error term u_{ift} captures other unobservable factors.

Note that the dependent variable in regression (7), δ_{ift} , is not directly observed in our data. Hence, we replace δ_{ift} in (7) by the estimates $\hat{\delta}_{ift}$ from regression (2). If $\hat{\delta}_{ift}$ is a consistent estimator of δ_{ift} and assuming the usual orthogonality conditions ($E(u_{ift} | LocIncumb, LocComp, X) = 0$), $(\beta_1, \beta_2, \beta_x)$ can be estimated consistently through ordinary least squares. However, the covariates $LocIncumb$ and $LocComp$ are endogenous and its effect could be confounded by other factors not captured in the regression. To mitigate endogeneity bias, we include several controls in X_{ift} , which we discuss in detail in what follows.

$LocComp$ measures the number of rival firms that are incumbent to a unit and is therefore affected by the unit's location. Units located in urban areas tend to be smaller and have more “neighboring” units, and thereby tend to have more incumbent firms. Because these territories are also more densely populated and have better transportation infrastructure, they tend to be cheaper to supply. Hence, unobservable unit costs could confound the effect of competition, generating a negative bias on the coefficient on $LocComp$. To mitigate this source of bias, we include unit fixed effects that control for time-invariant characteristics of a unit. Note that the panel structure of our data set—with two or more auctions observed for each unit—is essential for the identification of local competition effects. We also include the following controls to capture costs associated to a unit that can vary across auctions: (1) the size of the unit (*Size*, measured as the number of meals), which reduces the cost of serving the unit due to scale economies; and (2) the fraction of “*Special Meals*” supplied, which increase the costs of serving the unit.

The $LocIncumb$ covariate could also lead to endogeneity bias. Similar to the bias regarding density discounts described earlier, a local incumbent firm that was previously awarded units in the “neighborhood” of unit i is a potential indication that it had a local cost advantage in the vicinity. If this advantage is persistent over time, the firm would have a lower cost of serving unit i in the *current* auction and therefore could submit a lower bid. In this alternative explanation, $LocIncumb$ is a proxy for permanent local cost advantages, which would bias the estimate of

the *LocIncumb* coefficient. Note that unit fixed effects do not control for this source of bias because local advantages are firm specific. In the absence of good instrumental variables for *LocIncumb*, we introduce controls into the regression that capture permanent firm local advantages. The idea is to use predefined regions, which can contain one or more units, and introduce firm regions specific fixed effects that capture unobservable firm advantages on each area.²⁹ With these controls, the identification of *LocIncumb* relies in the variation across units' average unit prices from the same firm within the predefined areas and the variation across auctions. Defining these predefined areas is somewhat subjective. We used the political regions of Chile as our areas, which contain on average 7.7 units (see Figure 1 for a map describing the regions). We also did some analysis with other prespecified areas to assess the robustness of the results. In addition, we also include the following firm characteristics that can change over time as additional controls: (1) an indicator variable on whether a firm is attempting to renew a previously awarded contract for the unit, to capture possible sunk costs in the service provision (*Renew*); (2) a firm performance measure assigned by JUNAEB each year, which captures the managerial competence of firms (*Performance*); (3) a financial grade assigned by JUNAEB based on firms' financial classification in the range 1 to 7, 1 being the best grade (*FinGrade*); and (4) an indicator for firms that are participating in the auction for the first time (*NewFirm*). Finally, we include auction fixed effects to capture temporal price trends.

6.1. Estimation Results

Table 8 shows the results from the second-stage regression (Equation (7)), replacing δ_{ift} with its estimate from the first stage.³⁰ As is common with panel data, we decompose the error term u_{ift} into a random effect ξ_{if} plus an idiosyncratic error, and account for this heteroskedasticity in the estimation (typically known as "random-effect" estimator, see Wooldridge 2002). Specification (1) in Table 8 includes firm, auction, and unit dummy variables as controls, plus all the control variables described in §6 (*FinGrade* is included with a linear effect plus a dummy variable

Table 8 Results from Second-Stage Regression (Equation (6))

	(1)	(2)
<i>LocIncumb</i>	−13.29** (3.876)	−10.14* (4.555)
<i>LocComp</i>	−3.407* (1.504)	−3.578* (1.545)
<i>Renew</i>	−18.02** (6.412)	−8.839 (6.734)
<i>Size</i>	−21.09 (11.53)	−19.06 (11.08)
<i>Special meals</i>	497.8** (36.21)	497.7** (34.77)
<i>Performance</i>	−18.73* (7.925)	−32.83** (9.986)
<i>NewFirm</i>	−19.67** (4.394)	−15.94** (5.261)
<i>FinGrade</i>	−5.175** (1.809)	−7.362** (2.074)
<i>FinGrade = 1</i>	−54.26** (9.086)	−58.97** (11.36)
Observations	2,839	2,839
R-square	0.683	0.741

Notes. Both specifications include dummy variables for firm, auction, unit, and the controls specified in §4.2. Specification (2) also includes dummy variables for firm region (which absorb the firm dummies). Standard errors are in parentheses.

* and ** indicate statistical significance at the 0.05 and 0.01 confidence levels, respectively.

indicating firms with the highest financial grade, *FinGrade* = 1). The coefficient on *LocIncumb* is negative and significant, revealing that firms that operate in nearby units (awarded in previous auctions) tend to bid, on average, 2.3% lower. This is similar in order of magnitude to the discounts generated by density. The effect of local competition (*LocComp*) is also negative and significant.³¹ Increasing the number of local incumbents by four (about one standard deviation) reduces average unit prices by 3%. The controls *Renew*, *Size*, and *SpecialMeals* all have the expected signs.³²

Column (2) in Table 8 includes firm-region fixed effects as additional controls (together with auction and unit fixed effects and all the other controls). The coefficient of *LocComp* and *LocIncumb* are similar to those obtained in column (2), although the statistical significance of *LocIncumb* is smaller. Given the small difference across the estimates, it appears that the

²⁹ Because the objective is to control for *permanent* cost advantages, these fixed effects do not vary across auctions.

³⁰ The 1999 auction was the first year in which the CA was fully implemented. Hence the measures of local incumbency (*LocIncumb*) and local competition (*LocComp*) are not well defined. For this reason, we chose to exclude this auction from the second-stage estimation. We also note that on average, each δ_{ift} is estimated with 600 observations (which corresponds to the number of packages containing that unit for that firm auction) and the average relative precision of the estimates—the standard error divided by the point estimate—is 0.7%, which is quite high.

³¹ We also estimated additional regressions to test a possible non-linear effect of *LocComp*. Perhaps surprisingly, all of these models suggest a linear effect of *LocComp*.

³² Although the other controls are statistically significant, we do not have theory to predict their sign a priori.

potential bias on *LocIncumb* due to permanent local firm advantages is small.³³

As a robustness check, we also estimated (7) with the stand-alone bid prices b_{ift} as the dependent variable (instead of the average unit price δ_{ift}). Because some firms do not submit stand-alone bids for all units, the sample size in this regression is smaller. Nevertheless, the results were similar.

7. Conclusions

We have empirically analyzed the procurement combinatorial Chilean auction for school meals. Our analysis is motivated by important auction design issues and the empirical results presented are useful to inform the design of CAs. In fact, the results of our work are being considered by the Chilean government to evaluate potential changes to the auction mechanism. Our analysis highlights the importance of considering together the firms' operational cost structure and their strategic behavior when evaluating the effectiveness of CAs.

First, our results show that scale and density discounts are important; together they can be as high as 8% of the average bid price. Moreover, the results suggest that, on average, only a small fraction of the discounts can be explained by strategic bundling. In addition, other strategic effects that may arise in package bidding—in particular, the threshold problem—appear not to be too severe in the context of this CA. Therefore, most of the discounts seem to be explained by cost synergies, suggesting that allowing for package bids is appropriate. Furthermore, firms take advantage of the flexibility of the current mechanism that allows them to express cost synergies due to scale and density, both of which seem to be present.

Second, although on average the discounts due to strategic bundling seem to be relatively small, our data analysis suggests that for specific combinations the effect could be relevant. These are packages that include units in isolated regions of the country (regions 1 and 2 in the extreme north, and regions 11 and 12 in the extreme south) that exhibit discounts that tend to be above average when combined with other units located far away. Given the location of

these isolated units, we believe it is implausible that all of these discounts can be explained by cost synergies. Moreover, consistent with strategic bundling theory, these isolated units tend to have higher coefficients of variation of average unit prices and lower correlations with centrally located units. As a consequence, strategic bundling behavior may be particularly severe in combinations involving these isolated units, creating inefficiencies. Actually, based on intuition and anecdotal evidence, JUNAEB's officials share this belief (Martínez 2010). Hence, although package bidding appears to be generally desirable, we propose to evaluate the prohibition of some *specific combinations* in the auction.

Third, our analysis suggests that market share restrictions together with splitting the TUs into multiple sequential auctions may be a useful design element to diversify the supplier base and promote local competition through the presence of local incumbent firms. In fact, the effect of local competition is comparable in magnitude to the discounts from cost synergies. This suggests that it may be worth revising the years at which the TUs are auctioned in order to further increase the benefits from local incumbency in the outcome of the auction.

Fourth, at the time of writing this paper, the government was evaluating whether to relax the restrictions on bidders' market shares in order to enable them to further exploit cost synergies. In particular, they were evaluating to eliminate the constraint that enforces the total standing contracts to be no larger than 16%. This constraint imposes a maximum of 19 million meals per year on average (equivalent to approximately seven units), mainly affecting the large firms. Our analysis suggests, however, that at this point most cost synergies have been exhausted, so relaxing the market share constraint may not be convenient from a cost efficiency perspective. Along these lines, it is also suggestive that when we re-solved the optimal allocation problem removing the market share constraints (but keeping the same bids), we found that the reduction in the total procurement costs was very small (a 0.3% decrease). Hence, the potential downside of removing market share constraints—reducing the intensity of local competition through a concentration of the supplier base—could outweigh the benefit of further exploiting cost synergies.

Although our results are useful and suggestive, they are not totally conclusive. First, a significant challenge in our approach was to distinguish whether discounts were determined by cost synergies or markup adjustments. We empirically tested how much of the measured discounts can be explained by a specific mechanism identified in the literature that affects markups, namely, strategic bundling. It could be interesting to explore whether there are other important

³³ A Hausman test cannot reject that the coefficients of *LocIncumb* in models (1) and (2) are similar (t -statistic = 0.87). We also ran a regression that treats δ_{if} as fixed effects. The standard errors in this regression are much larger and the coefficients on *LocIncumb* and *LocComp* were no longer statistically significant. However, their magnitude and sign were similar to those obtained in Table 8, and a Hausman test cannot reject that the estimates are equivalent. Finally, we also estimated a regression with firm-auction fixed effects (instead of firm fixed effects) to account for unobservable firm characteristics that change over time beyond the ones we control for as described in §6; the results were also similar (although the statistical significance for *LocIncumb* was smaller).

mechanisms that affect markups for package bids and test whether they are present in the data. Second, our reduced form approach does not allow us to perform counterfactuals to study more directly the consequences of alternative auction designs (e.g., removing the market share constraints). An alternative approach to the reduced form analysis presented in this paper is to use a structural estimation method that poses a model of bidders' behavior, imposing restrictions on how markups are determined and thereby identifying costs. Once costs are identified, one could potentially perform some useful counterfactuals. The main drawback of the structural approach is that identification relies on nontestable assumptions on bidders' behavior and that it is computationally demanding. However, we believe it is a useful complement to the work we have presented in this paper, and we have developed it in a separate paper (Kim et al. 2012).

In addition, our analysis was focused on single-round, sealed-bid, first-price CAs, because the Chilean government has used this format for several years and there are many other CAs that have the same format. However, it could be interesting to study the impact of other auction formats (e.g., a multi-round ascending auction) in future research.

Finally, although our focus has been in the Chilean auction for school meals, we believe our method of analysis can be used broadly for many other CAs, enabling the study of a wide range of issues. More generally, our empirical analysis provides substantive insight into bidding behavior in CAs, suggesting that firms are sophisticated in their bidding and that they take advantage of the flexibility of the bidding mechanism by expressing cost synergies and by adjusting their markups. We hope that our study together with future similar studies in other CAs enhances the understanding of bidders' behavior and, as a consequence, gives us insights to improve the design of this type of auction.

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