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Process Flexibility Design in Unbalanced Networks

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Several design guidelines and flexibility indices have been developed in the literature to inform the design of flexible production networks. In this paper, we propose additional flexibility design guidelines for unbalanced networks, where the numbers of plants and products are not equal, by refining the well-known Chaining Guidelines. We study symmetric networks, where all plants have the same capacity and product demands are independent and identically distributed, and focus mainly on the case where each product is built at two plants. We also briefly discuss cases where (1) each product is built at three plants and (2) some products are built at only one plant. An extensive computational study suggests that our refinements work very well for finding flexible configurations with minimum shortfall in unbalanced networks.

Key words: process flexibility design; unbalanced networks; Chaining Guidelines

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1. Introduction

The year 2008 was tough for most automakers in the United States. The slumping economy depressed overall vehicle sales, and high gasoline prices spurred American consumers to buy small cars instead of pickup trucks and SUVs. As a result, many Detroit automakers were forced to shut down their truck and SUV productions and cut employees. On the other hand, Honda and Toyota were able to shuffle production among different plants as well as make different vehicle models at one plant (e.g., LeBeau 2008, Kent 2009, LeBeau 2010). In this regard, flexibility has widespread influence.

Many forms of flexibility have been studied in various contexts. We follow the Jordan and Graves (1995) model setting, where “process flexibility” is both the ability of each plant to produce a variety of products and the ability to produce products at multiple plants in a general multiproduct multiplant manufacturing system. Consider a bipartite graph representation of process flexibility $G = (V, V', E)$, where V is the set of plants ($V = \{1, 2, \dots, m\}$), V' is the set of products ($V' = \{1, 2, \dots, k\}$), and every edge in E connects a plant node $i \in V$ to a product node $j \in V'$, indicating that plant i can produce product j . The capacity of plant i is C_i and D_j is the realized demand for product j . The total shortfall is the optimal value of the following problem:

$$\begin{aligned} \text{Min} \quad & \sum_{j \in V'} s_j \\ \text{s.t.} \quad & \sum_{j: (i, j) \in E} f_{i,j} \leq C_i, \quad \forall i \in V, \\ & f_{i,j} + s_j = D_j, \quad \forall j \in V', \\ & f_{i,j} \geq 0, \quad \forall (i, j) \in E; \quad s_j \geq 0, \quad \forall j \in V', \end{aligned}$$

$$\begin{aligned} \sum_{i: (i, j) \in E} f_{i,j} + s_j &= D_j, \quad \forall j \in V', \\ f_{i,j} &\geq 0, \quad \forall (i, j) \in E; \quad s_j \geq 0, \quad \forall j \in V', \end{aligned}$$

where f_{ij} , $\forall (i, j) \in E$ are the amounts of product j produced by plant i , and s_j is the shortfall for product j . All of these variables are nonnegative. Because demand is random, the expected total shortfall is the average optimal value over all possible demand realizations D_j , $j \in V'$.

Jordan and Graves (1995) developed the following well-known Chaining Guidelines:

- (a) try to equalize the number of plants (measured in total units of capacity) to which each product is directly connected;
- (b) try to equalize the number of products (measured in total units of expected demand) to which each plant is directly connected; and
- (c) try to create a circuit(s) that encompasses as many plants and products as possible.

They show that the Chaining Guidelines can lead to flexible configurations that perform almost as well as the total-flexibility configuration where every plant is linked to all products.

In this paper, we focus on minimizing expected total shortfall in unbalanced and symmetric networks that have $2k$ edges. The term *balanced networks* refers to networks where the numbers of plants and products are equal. The term *symmetric networks*, by contrast, describes networks where all plants have the same capacity and different products' demands are independent and identically distributed (i.i.d.). Because of the complex combinatorial and stochastic

character of the flexibility design problem, finding an optimal solution (i.e., which links to create) under an arbitrary resource constraint (i.e., the number of links $|E|$) is challenging. We instead focus on configurations with $2k$ edges. For configurations with fewer than $2k$ edges, at least one product can only be produced by one plant, and the corresponding flexibility level can be notably enhanced by connecting one more edge to that product. On the other hand, configurations with more than $2k$ edges are not systematically considered in this paper for two reasons. First, it is accepted that chaining configuration (with $2k$ total edges) already achieves most of the benefits of total-flexibility configuration. Some pertinent arguments are available in Jordan and Graves (1995), Akşın and Karaesmen (2007), and Bassamboo et al. (2010b). Second, even in balanced and symmetric networks, the question of the optimality of the l -chain configuration ($l > 2$) among all configurations with total number of $k \cdot l$ edges is still open (e.g., Chou et al. 2011). An l -chain is an m by m bipartite graph where each product j is linked to plants $j, j+1, \dots, j+l-1$ (subtract m if exceeding m). The extension to configurations where $|E| = 3k$ or $|E| < 2k$ is briefly touched at the end of §3.

Our analysis reveals that the Chaining Guidelines may not be sufficient in designing flexible configurations in unbalanced networks. Consider a symmetric network with $m = 12$ plants. The number of products k varies from 13 to 24. We use only configurations where each product is directly connected to two plants. Figure 1 shows that the Chaining Guidelines could lead to a configuration that has a considerable (e.g., 200%) increase from the expected shortfall of total-flexibility configuration. The simulation parameters are as follows. There are $m = 12$ plants, each of

which has capacity $C_i = 20, \forall 1 \leq i \leq 12$. Each product has normally distributed demand with mean $\mathbb{E}[D_j] = (C \cdot m/1.1)/k$ and variance $\text{Var}(D_j) = (\mathbb{E}[D_j] \cdot 0.4)^2, 1 \leq j \leq k$. Negative demand is replaced with 0 when generating random demand. For each k , we randomly sampled more than 100 configurations. We use only one set of parameters, so 200% may not be a typical percentage increase. Nonetheless, this example hints at the potential room for improvement for the Chaining Guidelines. Because we cannot sample all configurations that satisfy the Chaining Guidelines, the actual worst case should be worse than the one obtained by simulation, and the numbers in Figure 1 underestimate the actual percentage increase.

To gain intuition for the limitations of the Chaining Guidelines in unbalanced networks, consider a symmetric network with $m = 12$ plants and $k = 15$ products (see configuration (i) in Figure 2). Each product is built at two plants. The Chaining Guideline (c) suggests building a circuit connecting all plants with the first m products (see configuration (ii) in Figure 2). The Chaining Guidelines (a) and (b) suggest that 2 of the 12 plants should be directly connected to product 13, that another two plants should be directly connected to product 14 and that another two plants should be directly connected to product 15. Because the six plants that are directly connected to products 13, 14, and 15 are connected to one more link than the other plants, they are more heavily utilized. Therefore, we should not place these more highly utilized plants around each other and should evenly mix the more used products with the rest of the plants. The proposed Unbalanced Network Flexibility Design Guidelines refine the Chaining Guidelines in this respect.

As our main contribution, we lay out the Unbalanced Network Flexibility Design Guidelines. Given a symmetric network with m plants and k products, the proposed guidelines work as follows:

(1) Build a chaining configuration between all the m plants and the first m products. Rearrange the chaining configuration as a circle. As an illustration, see Figure 2 where the chain in configuration (ii) is rearranged as a circle in configuration (iii).

(2) For products $m+1$ to k , the Unbalanced Network Flexibility Design Guidelines recommend:

(a) connecting each of these products to plants on diametrically opposite ends of the circle;

(b) connecting these products to plants that are evenly spaced on the circle; and

(c) avoiding connecting the same pair of plants to more than one product. To illustrate, see Figure 2.

The rest of this paper is organized as follows: We mention some relevant literature in §2; §3 develops the Unbalanced Network Flexibility Design Guidelines via symmetric unbalanced networks; §4 offers some concluding remarks.

Figure 1 Maximum Percentage Increase in Expected Shortfall over Total-Flexibility Configuration

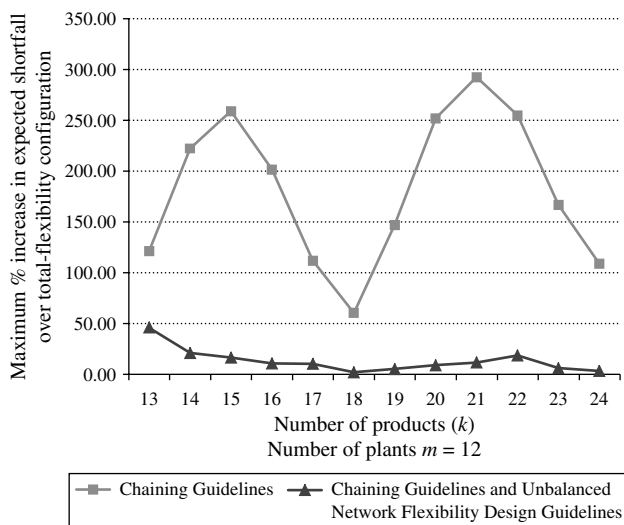
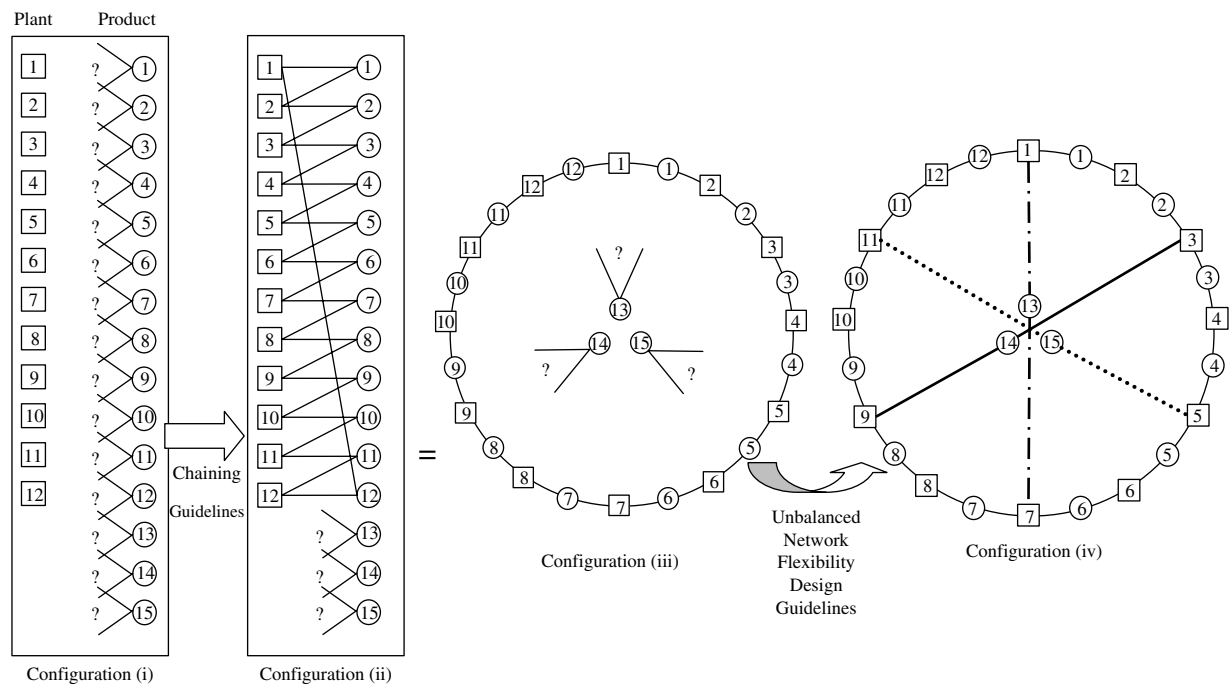


Figure 2 Illustration of the Unbalanced Network Flexibility Design Guidelines

2. Literature Review

The Chaining Guidelines have been studied in many settings. For instance, Hopp et al. (2004) showed that the chaining strategy has strong capacity balancing and variability buffering properties in serial production systems with flexible workers. Iravani et al. (2007) studied flexibility in a call-center labor cross-training setting. Sheikhzadeh et al. (1998) analyzed the chaining configuration for equipment flexibility problems.

Jordan and Graves (1995) developed a single-period model for the flexibility design problem in a single-stage network. This work has been extended to (i) analyze the flexibility of multistage supply chains, and (ii) examine flexibility from a multiperiod, long planning horizon perspective. The following two paragraphs discuss these two extensions.

Graves and Tomlin (2003) extended the results in Jordan and Graves (1995) to multitier supply chains and discovered stage-spanning bottlenecks and floating bottlenecks. Hopp et al. (2010) considered vertical flexibility across multiple stages in a supply chain. They examined logistic flexibility and process flexibility.

Bassamboo et al. (2010a, p. 1) investigated the long-term asymptotic performance of tailored chaining and pairing in queuing systems. They proved that “a little flexibility is all you need.” Gurusurthi and Benjaafar (2004) examined the throughput in queuing systems and provided evidence for the effectiveness of chaining configurations. Muriel et al. (2006) used

simulation to study flexibility in a general multiproduct multiplant make-to-order manufacturing system. They found that partial flexibility leads to a considerable increase in production variability.

The Jordan and Graves (1995) formulation of the flexibility design problem is prevalent in the literature. Chou et al. (2010b) used the concept of a generalized random walk to compare the asymptotic performance of chaining configuration to total-flexibility configuration. Simchi-Levi and Wei (2012) proved the optimality of the chaining configuration in a symmetric and balanced network where each product and plant is incident to exactly two arcs. Akşın and Karaesmen (2007) showed that the expected throughput is concave in the degree of flexibility in a network flow model. Furthermore, they provided results on the interaction between flexibility and capacity. Chou et al. (2011) analyzed the worst-case performance. Chou et al. (2010a) studied networks where each plant produces two products at different costs.

3. Unbalanced Network Flexibility Design Guidelines

The ideal symmetric network is rarely encountered in practice. Yet, we find symmetric networks attractive for two reasons. First, the analysis for symmetric networks yields insights on flexibility for asymmetric networks. Second, as stressed by Iravani et al. (2005), accurate data and forecasts are usually unavailable for strategic and long-term decisions.

We use simulation to identify the most flexible configurations in this paper. We use expected total shortfall as the sole gauge of a configuration's flexibility. Low expected total shortfall is preferred. The expected shortfall is approximated by the average shortfall of 10,000 randomly replicated demand scenarios. Negative demand is replaced with 0 when generating random demand. In each scenario, we solve the capacity allocation problem using the randomly generated demand. We assume Normal demand distributions and fix coefficient of variation at 0.4. By definition of a symmetric network all plants have the same capacity and all products face i.i.d. demand; therefore, we arbitrarily label the plants 1 to m and label the products 1 to k . Define $\Gamma(\Lambda) = \{i \in V \mid \exists j \in \Lambda \text{ s.t. } (i, j) \in E\}$, $\Lambda \subseteq V'$. Let $\Gamma(\Lambda)$ be the set of plants that are directly connected to product set $\Lambda \subseteq V'$. There is a one-to-one correspondence between E and flexibility configurations. For instance, $E = \{(i, j) \mid i \in V, j \in V'\}$ denotes the total-flexibility configuration. There is also a one-to-one correspondence between E and $\Gamma(j)$, $\forall j \in V'$.

Chaining Guideline (c) advocates building a chaining configuration between all plants and m arbitrary products. By network symmetry, we assume without loss of generality that the chaining configuration is built out of the first m products. That is,

$$\forall 1 \leq j < m, \quad \Gamma(j) = \{j, j+1\}; \quad \Gamma(m) = \{1, m\}.$$

Chaining Guideline (c) reduces the number of undetermined edges by $2m$ and is the foundation of the Unbalance Network Flexibility Design Guidelines.

3.1. Unbalanced Network Flexibility Design

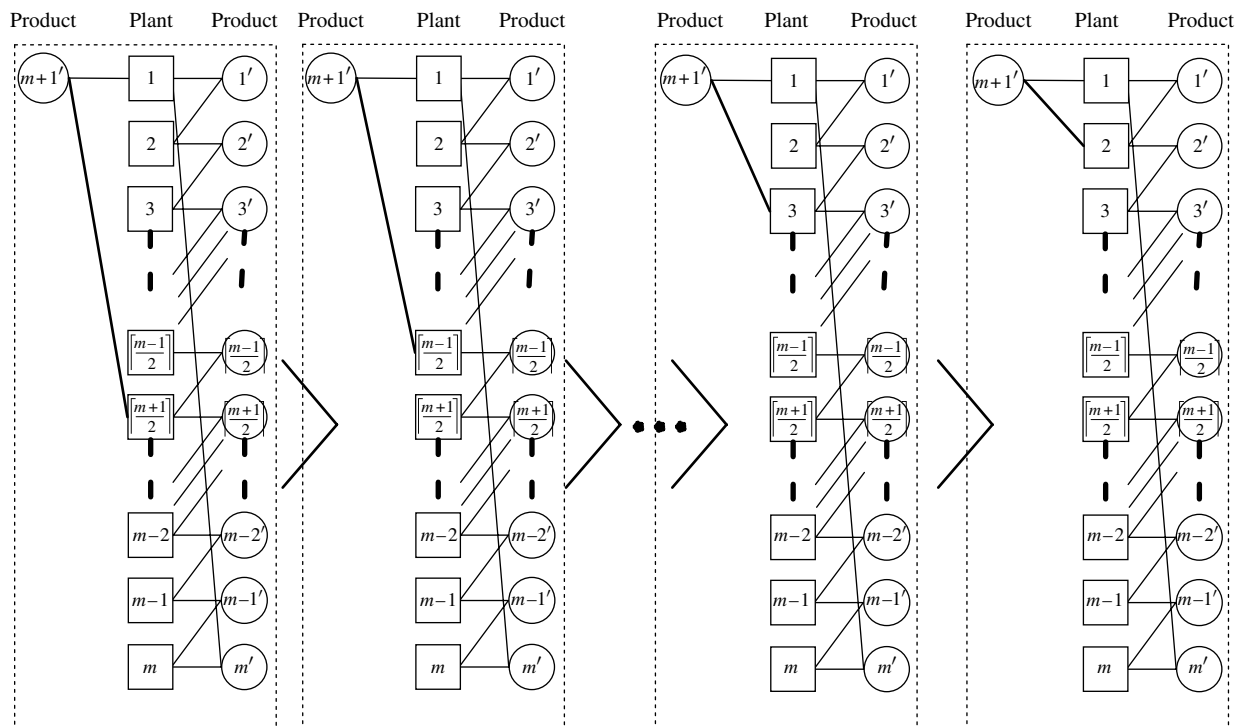
Guidelines (a) and (b): $m < k \leq m + \lfloor m/2 \rfloor$

According to the Chaining Guidelines (a) and (b), $m - \text{mod}(k, m)$ plants should all be directly connected to $\lfloor k/m \rfloor$ products, and $\text{mod}(k, m)$ plants should all be directly connected to $\lceil k/m \rceil$ products. When k/m is not an integer, some plants are directly connected to one more product than other plants, showing that some plants are more utilized than others. Intuitively, plants that are more utilized should not be close to each other. In other words, we want to evenly mix the more utilized plants with the less utilized plants. How to define the distance between plants?

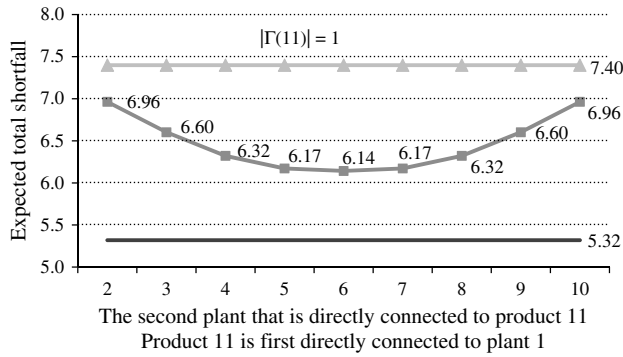
Recall that the chaining configuration can be rearranged as a circle. The circular representation of the chaining configuration provides a natural measure of the distance between two plants. For example, in Figure 2, plants 1 and 2 are closer to each other than plants 1 and 3. We define the *distance* between two plants to be the minimum number of links needed to connect them in the chaining configuration.

For the simplest case where $k = m + 1$, Figure 3 lists all the distinct configurations that satisfy the Chaining Guidelines. Figure 4 reveals that the far left configuration in Figure 3 performs the best in terms of expected total shortfall. In addition, the network's expected

Figure 3 Symmetric Configurations Where $k = m + 1$



Note. "Configuration A > configuration B" means that configuration A is more flexible than configuration B.

Figure 4 Expected Total Shortfalls of Configurations in Figure 3

Notes. The network parameters are $m = 10$, $k = 11$, $C_i = 11$, and $\mathbb{E}[D_j] = 10$. The horizontal line above the curve is the expected total shortfall when product 11 is directly connected to only one plant. The horizontal line below the curve is the expected total shortfall of the total-flexibility configuration.

total shortfall decreases from 7.4 to 6.14 when a link is properly added to product 11 ($m + 1$). We suggest that all products should have some level of flexibility, which is one of the many reasons why we compare configurations with $2k$ edges. Figure 4 indicates that the gap between the total-flexibility configuration and a configuration with $2k$ edges is significant even when the $2k$ edges are properly added. Because the system capacity distributed over 10 plants is equal to the average total demand across 11 products, each plant has extra capacity that needs to be shared to satisfy demand of the last product, $m + 1$.

Let i_{1j} and i_{2j} be the two plants that are directly connected to product j , $\forall m < j \leq k$. Unbalanced Network Flexibility Design Guideline (a):

$$\forall m < j \leq k,$$

$$\Gamma(j) = \{i_{1j}, i_{2j}\} \text{ s.t. } \left\lfloor \frac{m+1}{2} \right\rfloor \geq |i_{2j} - i_{1j}| \geq \left\lceil \frac{m-1}{2} \right\rceil.$$

Unbalanced Network Flexibility Design Guideline (a) suggests connecting product j ($m < j \leq k$) to a pair of plants that are far from each other. For example, in configuration (iv) of Figure 2, product 13 is directly connected to plants 1 and 7. Unbalanced Network Flexibility Design Guideline (a) now virtually brings those segments of the circle that were far away close together, which is beneficial not only for that additional product being linked, but also for the ones in the circle that can now easily reach capacity at the other end of the circle.

Unbalanced Network Flexibility Design Guideline (b) suggests connecting the last $k - m$ products to plants that are evenly spaced on the circle denoting the chaining configuration. Unbalanced Network Flexibility Design Guideline (b) is comprised of two parts. We rearrange plant indices i_{1j} and i_{2j} , $\forall m < j \leq k$ in ascending order, $i_{(1)} \leq i_{(2)} \leq \dots \leq i_{(2k-2m-1)} \leq i_{(2k-2m)}$. Then, the conditions $i_{(l+1)} - i_{(l)} \geq \lfloor m/2(k-m) \rfloor$,

$\forall 1 \leq l < 2k - 2m$ and $i_{(1)} + m - i_{(2k-2m)} \geq \lfloor m/2(k-m) \rfloor$ guarantee that plants i_{1j} and i_{2j} , $\forall m < j \leq k$ are evenly spaced on the chain built by the Chaining Guidelines. The above idea is formally represented as the first part of Unbalanced Network Flexibility Design Guideline (b):

$$\Gamma(j) = \{i_{1j}, i_{2j}\}, \quad \forall m < j \leq k,$$

such that $i_{(1)} \leq i_{(2)} \leq \dots \leq i_{(2k-2m-1)} \leq i_{(2k-2m)}$ —the rearrangement of i_{1j} and i_{2j} , $\forall m < j \leq k$ in ascending order—satisfy

$$i_{(l+1)} - i_{(l)} \geq \left\lfloor \frac{m}{2(k-m)} \right\rfloor \quad \forall 1 \leq l < 2k - 2m$$

and

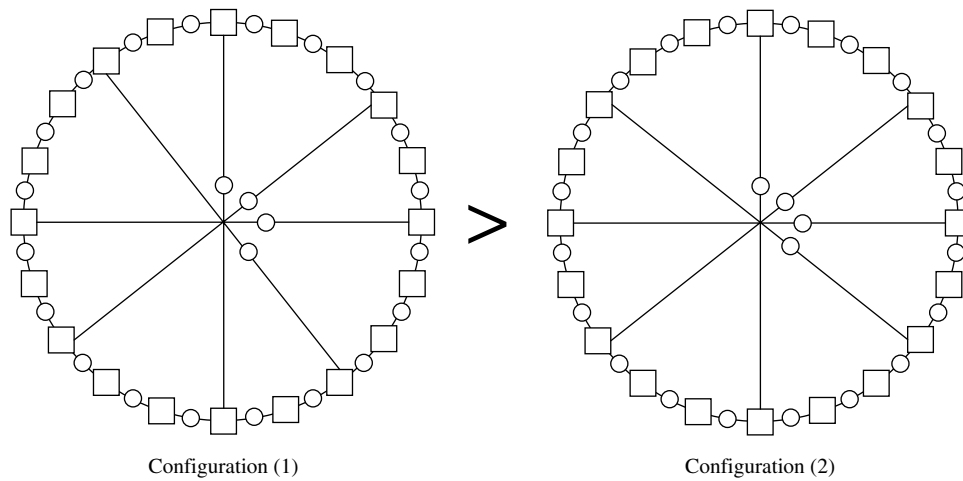
$$i_{(1)} + m - i_{(2k-2m)} \geq \left\lfloor \frac{m}{2(k-m)} \right\rfloor.$$

For example, in configuration (iv) of Figure 2, we connect plants 1, 3, 5, 7, 9, and 11 to products 13, 14, and 15. Note that plants 1, 3, 5, 7, 9, and 11 are evenly spaced on the chaining configuration.

When $k \leq m + \lfloor m/2 \rfloor$, the circle will be cut into $2(k-m)$ arcs after applying the Unbalanced Network Flexibility Design Guideline (b). When $m/(2(k-m))$ is not an integer, $\text{mod}(m, 2(k-m))$ of these arcs each will contain $\lceil m/(2(k-m)) \rceil$ products, and other arcs each will contain $\lfloor m/(2(k-m)) \rfloor$ products. How to array these arcs in the circle? The second part of Unbalanced Network Flexibility Design Guideline (b) advises against placing the arcs that contain $\lceil m/(2(k-m)) \rceil$ products adjacent to each other.

To see why this arrangement makes sense, let us compare the two configurations in Figure 5. Both configurations satisfy Unbalanced Network Flexibility Design Guideline (a) and the first part of Guideline (b). Both configurations have $m = 20$ plants and $k = 24$ products. All 20 plants and the first 20 products form a chaining configuration, which transforms to a circle in circular representation. The circle is cut into eight circle arcs by the eight links connected to the last four products. Because $20/8$ is not an integer, the first part of Unbalanced Network Flexibility Design Guideline (b) suggests that four of these eight circle arcs each contain three products, and other circle arcs each contain two products. The only difference between configurations (1) and (2) in Figure 5 is that circle arcs containing two products are not adjacent to each other in configuration (1). Simulation results indicate that configuration (1) performs notably better than configuration (2). When each plant has capacity $C_i = 20$ and each product has average demand $\mathbb{E}[D_j] = (m \cdot C/1.1)/k$, the expected total shortfalls of configurations (1) and (2) are 2.7592 and 2.8085, respectively.

Figure 5 Unbalanced Network Flexibility Design Guideline (b)



Note. "Configuration A > configuration B" means that configuration A is more flexible than configuration B.

CONJECTURE 1. In a symmetric network with m plants, k products, $m < k \leq m + \lfloor m/2 \rfloor$, and $|E| \leq 2k$, the most flexible configuration obeys the Chaining Guidelines and the Unbalanced Network Flexibility Design Guidelines (a) and (b).

Intuitively, the optimal configuration for symmetric networks should be symmetric. Perfect symmetry is one of the many reasons that Unbalanced Network Flexibility Design Guidelines (a) and (b) lead to flexible configurations. Figure 1 suggests that the combination of the Chaining Guidelines and Unbalanced Network Flexibility Design Guidelines performs very well when $m < k \leq m + \lfloor m/2 \rfloor$.

Figure 1 displays a cyclical pattern in the Chaining Guidelines maximum percentage increase in expected shortfall relative to the total-flexibility configuration. For example, the Chaining Guidelines' percentage increase is much lower for 18 products than for 15 products. When $k = 18$, configurations that satisfy the Chaining Guidelines must also satisfy Unbalanced Network Flexibility Design Guideline (b). However, when $k = 15$, configurations that satisfy the Chaining Guidelines may violate Unbalanced Network Flexibility Design Guideline (b). Hence, the Chaining Guidelines perform better for 18 products than for 15 products.

3.2. Unbalanced Network Flexibility Design

Guideline (c): $m < k \leq m(m-1)/2$

For $k > m + \lfloor m/2 \rfloor$, the diametrically opposed, evenly spaced assignment of products to plants cannot be continued for the last $k - (m + \lfloor m/2 \rfloor)$ products, because there is no pair of diametrically opposed plants that do not have a third product assigned to each of them. Thus, we need a new guideline, (c), to

guide the assignment of those last $k - (m + \lfloor m/2 \rfloor)$ products.

Unbalanced Network Flexibility Design Guideline (c) advises against assigning two products to the same pair of plants. The key idea is to spread risk as much as possible. Assigning products to different pairs of plants provides a better hedge against the uncertain demand. Figure 1 indicates that when $k \geq m + \lfloor m/2 \rfloor$, the combination of Chaining Guidelines and Unbalanced Network Flexibility Design Guidelines guarantees that deviation from expected shortfall of total-flexibility configuration is small. We hypothesize that Unbalanced Network Flexibility Design Guideline (c) and the Chaining Guidelines adequately manage the last $k - (m + \lfloor m/2 \rfloor)$ products.

3.3. Guidelines for Certain Configurations with $|E| \neq 2k$

The key to our analysis is the circular representation of flexibility configuration. By this circular representation, the distance between any two plants is well defined. This circular representation is meaningful as long as there exists a chaining configuration among all plants. It also has implications beyond networks where each product is directly connected to two plants.

For symmetric and balanced networks where $|E| = 3k$, it is tempting to say that the three-chain configuration is optimal; e.g., see Theorem 1 in Iravani et al. (2005). Figure 6 plots the three-chain configuration and its circular representation. Recall that the intuition behind Unbalanced Network Flexibility Design Guideline (a) is to connect diametrically opposite nodes. We follow this idea and create a configuration that has $3k$ edges in Figure 7. Numerical examples suggest that the configuration in Figure 7 consistently outperforms the configuration in

Figure 6 Three-Chain Configuration and Its Circular Representation

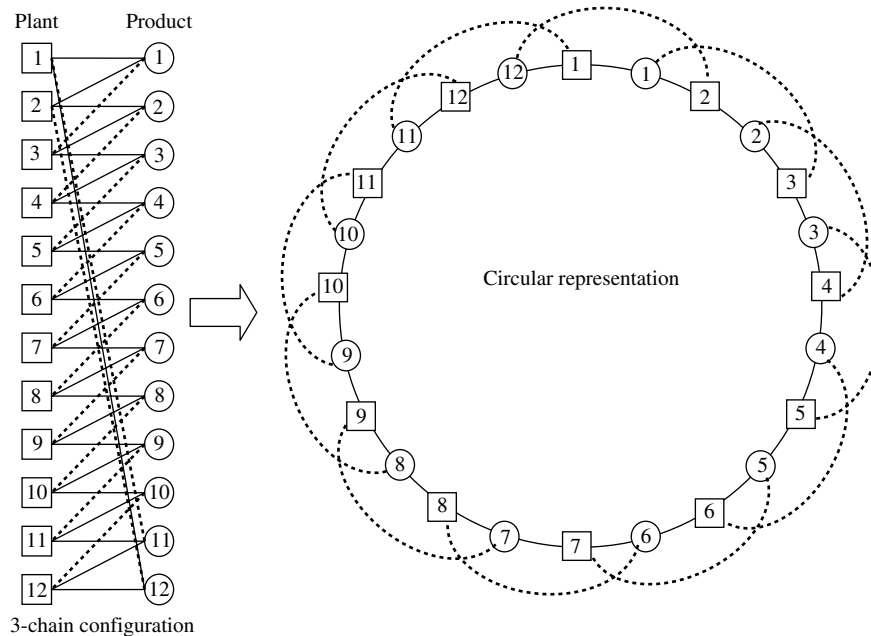
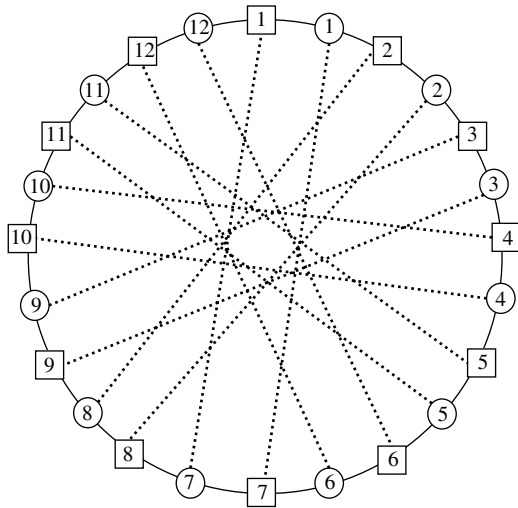
Figure 7 A Configuration with $3k$ Edges

Figure 6. When each plant has capacity $C_i = 11$ and each product has average demand $\mathbb{E}[D_j] = 10$, the expected total shortfalls of configurations in Figures 5 and 6 are 1.3934 and 1.3923, respectively. Although the difference is practically negligible, it is statistically significantly different from 0 ($p = 3.357e - 006$, Wilcoxon).

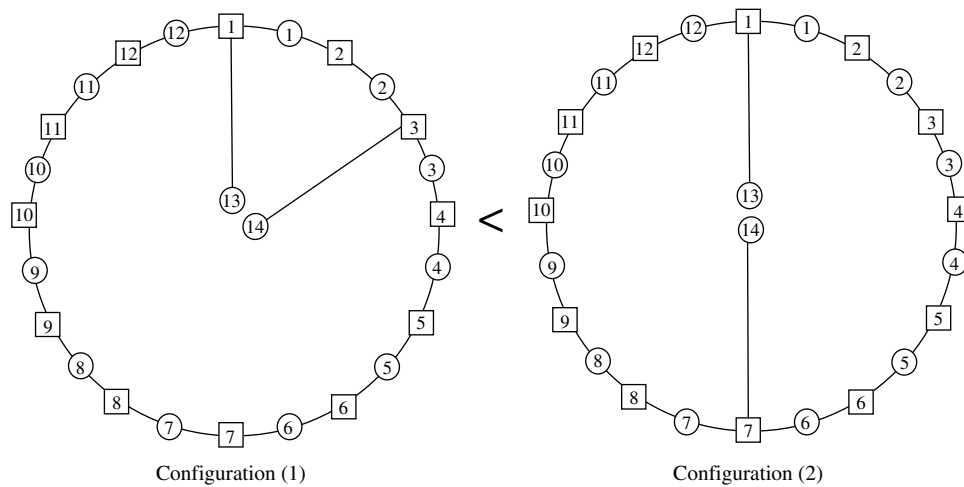
The intuition behind the Unbalanced Network Flexibility Design Guidelines also applies to networks where $|E| = m + k$. When only $m + k$ links are available, we can build a chaining configuration¹

¹ We thank Stephen C. Graves for suggesting this configuration.

connecting all m plants and m products, and have one link for each product $m + 1, m + 2, \dots, k$. Unbalanced Network Flexibility Design Guideline (b) suggests that links for products $m + 1$ to k should be evenly spread. Consider the two configurations in Figure 8. Both configurations satisfy the Chaining Guidelines, but configuration (1) violates the intuition behind Unbalanced Network Flexibility Design Guideline (b) because the circle is cut into two arcs of different lengths (in the circular representation). Simulation confirms that configuration (2) performs better than configuration (1). When each plant has capacity $C_i = 20$ and each product has average demand $\mathbb{E}[D_j] = (20 \cdot m/k)/1.1$, configurations (1) and (2) have expected total shortfalls of 9.3996 and 6.5047, respectively.

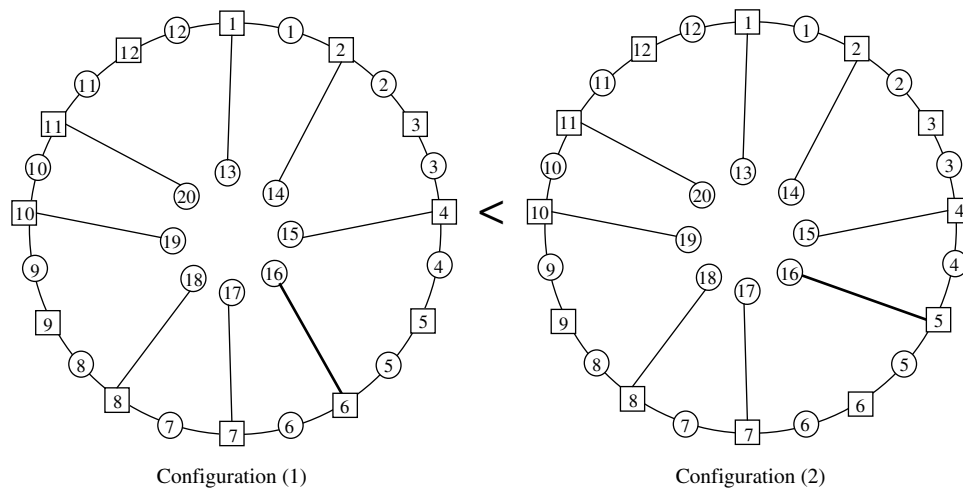
Figure 9 plots another example in networks where $|E| = m + k$. Both configurations have 12 plants and 20 products. All 12 plants and the first 12 products form a chaining configuration, which is a circle in the circular representation. The circle is cut into eight circle arcs by the eight links connected to the last eight products. In both configurations, four of the eight circle arcs each contain two plants and the remaining four circle arcs each contain one plant. The difference between these two configurations is the link for product 16. Configuration (2) is better because its links for products 13–20 are chosen to alternate the circle arcs containing two products with the groups containing one. Simulation indicates that configuration (2) performs notably better than configuration (1). When each plant has capacity $C_i = 20$ and each product has average demand $\mathbb{E}[D_j] = (m \cdot C/1.1)/k$, the expected

Figure 8 A Configuration Where $\Gamma(j) \neq 2$



Note. "Configuration A > configuration B" means that configuration A is more flexible than configuration B.

Figure 9 Unbalanced Network Flexibility Design Guideline (b)



Note. "Configuration A > configuration B" means that configuration A is more flexible than configuration B.

total shortfalls of configurations (1) and (2) are 5.5529 and 5.1234, respectively.

4. Conclusion

This paper proposes several new flexibility design guidelines for unbalanced network. When the number of products is larger than the number of plants and each product is directly connected to two plants, we suggest the following.

Build a chaining configuration between all plants and the same number of products. View this chaining configuration as a circle. Then,

- try to connect each remaining product to diametrically opposite plants on the circle;
- try to connect remaining products to plants that are evenly spaced on the circle;
- try to avoid connecting the same pair of plants to more than one product.

(c) try to avoid connecting the same pair of plants to more than one product.

The optimal configuration of a symmetric network should exhibit strong symmetry. Because the proposed Unbalanced Network Flexibility Design Guidelines are perfectly symmetric, we conjecture that the Unbalanced Network Flexibility Design Guidelines capture critical elements of the flexibility design problem in symmetric networks. Even though we primarily focus on configurations where each product is directly connected to two plants, the proposed guidelines have implications in more general settings. Furthermore, the Unbalanced Network Flexibility Design Guidelines serve as a test for future flexibility indices. Finally, our guidelines are easy to implement and can significantly reduce the number of candidate configurations in the planning stage of flexibility design.

This paper builds on the essential intuition from the Jordan and Graves (1995) Chaining Guidelines and subsequent flexibility indices, but fleshes out and extends these intuitions to develop Guidelines for unbalanced networks. Because we focus only on the Jordan and Graves (1995) formulation, whether our results could be extended in recognizable form to more general situations is a potential future research question.

Acknowledgments

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References

- Akşın OZ, Karaesmen F (2007) Characterizing the performance of process flexibility structures. *Oper. Res. Lett.* 35(4):477–484.
- Bassamboo A, Randhawa RS, Van Mieghem JA (2010a) A little flexibility is all you need: On the asymptotic value of flexibility in parallel queuing systems with linear capacity sizing costs. *Oper. Res.* 60(6):1423–1435.
- Bassamboo A, Randhawa RS, Van Mieghem JA (2010b) Optimal flexibility configurations in newsvendor networks: Going beyond chaining and pairing. *Management Sci.* 56(8):1285–1303.
- Chou MC, Chua GA, Teo CP (2010a) On range and response: Dimensions of process flexibility. *Eur. J. Oper. Res.* 207(2):711–724.
- Chou MC, Chua GA, Teo CP, Zheng H (2010b) Design for process flexibility: Efficiency of the long chain and sparse structure. *Oper. Res.* 58(1):43–58.
- Chou MC, Chua GA, Teo C-P, Zheng H (2011) Process flexibility revisited: The graph expander and its applications. *Oper. Res.* 59(5):1090–1105.
- Graves SC, Tomlin BT (2003) Process flexibility in supply chains. *Management Sci.* 49(7):907–919.
- Gurumurthi S, Benjaafar S (2004) Modeling and analysis of flexible queueing systems. *Naval Res. Logist.* 51(5):755–782.
- Hopp WJ, Iravani SMR, Xu WL (2010) Vertical flexibility in supply chains. *Management Sci.* 56(3):495–502.
- Hopp WJ, Tekin E, Van Oyen MP (2004) Benefits of skill chaining in serial production lines with cross-trained workers. *Management Sci.* 50(1):83–98.
- Iravani SMR, Kolfal B, Van Oyen MP (2007) Call-center labor cross-training: It's a small world after all. *Management Sci.* 53(7):1102–1112.
- Iravani SMR, Van Oyen MP, Sims KT (2005) Structural flexibility: A new perspective on the design of manufacturing and service operations. *Management Sci.* 51(2):155–166.
- Jordan WC, Graves SC (1995) Principles on the benefits of manufacturing process flexibility. *Management Sci.* 41(4):577–594.
- Kent D (2009) Alabama's auto plants use flexibility to deal with industry downturn. *Birmingham News* (June 10), http://blog.al.com/assembly-lines/2009/06/flexible_operations.html.
- LeBeau P (2008) Toyota shows flexibility with Prius/Highlander/Tundra moves. *CNBC News* (July 10), http://www.cnbc.com/id/25409881/Toyota_Shows_Flexibility_With_Prius_Highlander_Tundra_Moves.
- LeBeau P (2010) Ford's flexibility is the story to watch. *CNBC News* (December 15), http://www.cnbc.com/id/40678778/Ford_s_Flexibility_Is_the_Story_to_Watch.
- Muriel A, Somasundaram A, Zhang Y (2006) Impact of partial manufacturing flexibility on production variability. *Manufacturing Service Oper. Management* 8(2):192–205.
- Sheikhzadeh M, Benjaafar S, Gupta D (1998) Machine sharing in manufacturing systems: Total flexibility versus chaining. *Internat. J. Flexible Manufacturing Systems* 10(4):351–378.
- Simchi-Levi D, Wei Y (2012) Understanding the performance of the long chain and sparse designs in process flexibility. *Oper. Res.* 60(5):1125–1141.