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Milking the Quality Test: Improving the Milk Supply Chain Under Competing Collection Intermediaries

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We examine operational and incentive issues that conspire to reduce the quality of milk—via deliberate adulteration by milk farmers—acquired by competing collection intermediaries in developing countries. Broadly speaking, three main forces in the milk supply chain lead to the low quality of milk: high testing costs, harmful competition among stations, and free-riding among farmers. The goal of this study is to provide recommendations that address the quality problem with minimal testing. Interestingly, some intuitive interventions—such as providing stations with better infrastructure (e.g., storage and refrigeration facilities) or subsidizing testing costs—could hurt the quality of milk in the presence of competition. To save testing costs we utilize mixed testing, where the milk combined from multiple farmers is tested once. However, mixed testing makes the system vulnerable to free-riding among farmers. We counter free-riding by applying a credible threat of individual testing (although not its actual use in equilibrium). We then propose two interventions to combat the harmful competition among stations. The novelty of our proposals lies in utilizing the force of competition to solve a problem created by competition. The incentives in our proposals provide a new tool for the stations to compete and convert the harmful effect of competition (quality reduction) into a beneficial one (quality improvement), resulting in a socially desirable equilibrium outcome: all the farmers provide high-quality milk and each competing station conducts only one mixed test and no further testing.

Keywords: milk supply chain; adulteration; competition; inspection; moral hazard
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1. Introduction

Since the development of agriculture, humans have regularly consumed the milk of dairy animals (primarily cattle, sheep, and goats). This practice has strengthened in the last few decades, when both the demand for milk and its price have increased substantially. The increase in population and income levels in developing countries such as India and China, as well as the aggressive promotion of milk, has led to a rise in milk consumption in recent years (Delgado 2003, Food and Agriculture Organization of the UN 2014). Nevertheless, in most developing countries, the production of milk continues to be a cottage industry and operates on a small scale (Hemme et al. 2003, Kumar et al. 2011). In the milk supply chain, milk produced in relatively small quantities by farmers is sold to local (rural) milk collection centers that then sell the mixed milk to large urban dairies (Deshmukh 2011, Kumar et al. 2011).

Milk collection centers (or stations) are intermediaries that buy milk from farmers (typically, small farmers) and sell it to large dairy firms that process the raw milk to produce and distribute milk and related products for mass consumption. Our study is mainly focused on the quality of milk that results from the actions of the first two players in the milk supply chain, i.e., the small farmers who produce milk to earn a livelihood and the stations that are independent business entities between the farmers and the dairy firm.

For a variety of reasons, the milk collected at stations is only subjected to a simple test (based on simple visual inspection, smell, or taste) that ensures its quality above a certain baseline requirement. As we will discuss shortly, the reasons for not performing more-detailed quality tests (e.g., the UN-recommended resazurin test) are, in part, the limited resources available for testing and certain operational constraints that arise in the production and collection of milk. The other major reason for simple testing has to do with the competitive nature of the collection channel. Because of demand pressures and the relatively unorganized nature of the production sector (especially in developing economies), the supply of milk is usually not plentiful in any given location (Gale and Hu 2009, Njarui et al. 2010). Stations, therefore, compete with one another to acquire the milk from farmers. Because the wholesale price of milk (the price paid to farmers) is often regulated, the major basis for differentiation between stations is testing: stations attract farmers by being more lenient on testing. As a consequence, quality testing by stations is usually at a bare minimum.

Since the current practice by stations is to only perform a simple quality test, it is not surprising that most of the milk collected at stations is of low quality, i.e., at or just above the minimum requirement. Worse, anticipating the station's simple test, farmers are widely known to adulterate milk with certain ingredients so that it passes the simple test and is cheaper to produce (Omore et al. 2005, Khan 2008, Moran 2009, *The Hindu* 2013). These adulterants include relatively benign ingredients like water (although, sometimes the water itself may be polluted), whey (to increase protein content), starch (to make milk that is diluted with water look more viscous), vegetable fat (to increase fat content), soda bicarbonate and urea (to increase the shelf life of milk), and so on; see, e.g., Omore et al. (2005), Kasemsumran et al. (2007), Khan (2008), Gale and Hu (2009), dos Santos et al. (2012), and Stancati (2012). Except in rare cases, the adulterants used are not poisonous substances but are added to the milk only so that it passes the simple test at the station. Our paper focuses on such nonpoisonous adulteration, where the quality of milk decreases as the amount of adulterants increases. Poisonous adulteration, where even a small amount of poison can ruin the entire milk, is not considered in this study.

The broad goal of this study is to provide recommendations to alleviate milk adulteration using minimal testing, in the presence of competing milk stations.[1] We begin by examining the operations at a milk collection station. Then, Table 1 summarizes the key modeling features of our analysis and maps their origin to the real-world logistics aspects.

1.1. Logistics at a Milk Station

In developing countries, most of the milk acquired by stations is collected daily from smallholder dairy farmers in rural areas. A typical smallholder farmer owns a few (usually between one and three) heads of cattle (cows and/or buffalo). Thus, there is no significant heterogeneity in the amount of milk produced by individual farmers. We now provide a representative description of the milk collection process, based on the documented practice in several developing countries, including China, India, Kenya, Pakistan, Sri Lanka, Thailand, Uganda, Vietnam, and Zambia.

Farmers typically have a choice of two or three competing milk stations (intermediaries), which collect milk and sell it to large dairy processing firms. The popularity of milk in most developing countries, combined with a large population, leads to demand significantly exceeding supply, resulting in intense competition among milk stations to attract farmers; see, e.g., Gale and Hu (2009) on China and Njarui et al. (2010) on Kenya. An operational reason for the limited choice of milk stations for farmers is the rudimentary mode of transport they possess: most of the milk is delivered by traveling a few kilometers either on foot, bullock cart, or bicycle (Njarui et al. 2010 [Kenya], Pandey and Voskuil 2011 [Zambia]).

A typical station receives milk from approximately 100–200 farmers. Most of the milk collected at a station is delivered in the morning between 6 A.M. and 11 A.M. Evening delivery exists in some cases but accounts for a negligible fraction of the total milk collected. Usually, spoilt or unacceptable milk is identified by simple visual inspection, smell, or an alcohol test and is immediately rejected. The milk delivered by a farmer is first weighed, and then a sample is extracted for testing. In practice, tests of different intensities are available to assess the quality of milk. Some simple tests, such as the Gerber test, which measures only fat content and has therefore led to adulteration by artificial fattening agents, are relatively cheap. However, careful testing of milk—for example, using the UN-recommended resazurin test to evaluate the hygiene and potential keeping quality of raw milk—is expensive (Food and Agriculture Organization of the UN 2009). The "test" in this paper refers to a comprehensive test that can accurately test the quality of the milk along the important dimensions.

There are two ways to test the quality of milk: In an individual test, the milk from a specific farmer is tested. In a mixed test, the milk from multiple farmers is combined and tested once (Draaiyer 2002, Indian National Dairy Development Board 2012, Kenya Dairy Sector Competitiveness Program 2013, OSI Consulting 2014). There is an operational reason that some stations limit themselves to conducting only individual testing of milk. For proper preservation, until it is pasteurized later at an urban dairy firm, milk has to be stored at a temperature of 4 degrees Celsius or below. If a milk station has limited or no bulk refrigeration facility, then the collected milk must be transported to the dairy firm (that buys milk from the milk stations) at short intervals (typically one or two hours). However, as mentioned above, farmers arrive sporadically over a much larger

[1] An underlying assumption here is that social welfare improves with a reduction in the adulteration of milk and with a reduction in total testing cost.

Table 1 **Real-World Logistics Aspects and Corresponding Modeling Features**

Logistics aspects	Modeling features
Smallholder dairy farmers in rural areas with a few heads of cattle	Homogeneity in farmers' production quantity (heterogeneity is addressed in §5)
Limited choice of milk stations for farmers due to rudimentary modes of transport	A duopoly game between stations (an oligopoly is discussed in §5)
Farmers transport milk to nearby milk stations	Negligible transportation cost incurred by a farmer (heterogeneous transportation costs are discussed in §5)
Poorly resourced milk stations: (i) Lack of refrigeration facilities and (ii) Sporadic arrival of farmers to deliver milk, over a large time interval	Poorly resourced stations can only perform individual testing
Well-resourced milk stations	Well-resourced stations can perform both mixed and individual testing
Deliberate adulteration by farmers as the main cause of low milk quality	Farmers know the quality of their milk
The identification of spoilt or unacceptable milk by simple visual inspection, smell, or an alcohol test	A common lower bound (used by milk stations) on the quality of acceptable milk

time interval. Consequently, the station cannot wait to collect the milk from all farmers and then conduct a mixed test on the entire collection. In our analysis, we refer to a station that can only conduct individual testing as a "poorly resourced" station. In contrast, a station that can conduct both individual and mixed testing is referred to as a "well-resourced" station.

It is important to note that deliberate adulteration by farmers has been recognized as the main reason for low milk quality in practice (see, e.g., Moran 2009, *The Hindu* 2013 [India], Omore et al. 2005 [Kenya], Khan 2008 [Pakistan], Souza et al. 2011, dos Santos et al. 2012 [Brazil], and Kasemsumran et al. 2007 [Thailand]). Thus, in the context of the challenge to improve quality, uncertainty in farmers' assessment of the quality of their milk is not a major concern. Accordingly, it is reasonable to assume that farmers know the quality of their milk. Table 1 summarizes the real-world logistics aspects discussed above and the corresponding modeling features used in our analysis.

1.2. Review of Related Literature

In this section, our aim is to briefly discuss the connection of our work with some broad domains of research and explain the novelty of our setting. Stations face a moral hazard problem: a farmer may supply low-quality milk and claim it to be of high quality. In this regard, our paper is related to the literature on moral hazard. With inspection (as in our paper), the quality of milk can be detected, but at a cost. Our paper adds to this literature by examining two questions: (i) What is the impact of different inspection schemes on the consequent quality of the milk? (ii) Is it possible to achieve high-quality milk with minimal testing?

A notable characteristic of milk is that the milk from an individual farmer and the mixed milk from a collection of farmers are both complete products. This is different from a typical assembled product, where several distinct components come together to make a complete product; see, e.g., Wang and Gerchak (2003), Zhang (2006), Bernstein and DeCroix (2006), and Jiang and Wang (2010). When a product fails, the cause of the failure can be often narrowed down to a particular component or a subset of components. However, once milk from different farmers is mixed, the source of quality problems in the mixed milk cannot be directly linked to a subset of farmers. In a broader context, this feature is similar to "interdependent security" issues in the literature, where a low level of social security, whose magnitude depends on the joint protection actions of multiple individuals, cannot be traced to individual security investments; see, e.g., Heal and Kunreuther (2003), Kunreuther and Heal (2003), and Baiman et al. (2004).

As the above discussion hints, the practice of testing mixed milk (together with making individual payments based on the quality of the mixed milk) can encourage free-riding (Gale and Hu 2009, Deshmukh 2012, Orregard 2013), a behavior usually detrimental to the welfare of an economic system (Balachandran and Radhakrishnan 2005 and Chao et al. 2009). At the same time, a scheme that is based on mixed testing can potentially lower testing costs.

The existing literature on inspection addresses two broad issues: (i) how to inspect efficiently (see, e.g., Herer and Raz 2000, Mayer et al. 2004) and (ii) how to use inspection to incentivize suppliers to improve quality (see, e.g., Reyniers and Tapiero 1995, Babich and Tang 2012). More recently, Mu et al. (2014) consider a single, monopolistic milk station to devise an attractive inspection scheme (see the discussion at the end of §3.2.1). However, in the presence of competition for supply, inspection plays an additional and potentially damaging role: competing milk stations can lower their inspection standards to attract more farmers. In this supply-constrained environment, milk stations can lower quality standards and still remain profitable (while causing much social harm). Our analysis reveals that, under competition, the incentive for the stations to use inspection as a competitive tool can dominate their incentive to improve quality through inspection.

In addition to prices, firms can compete using a variety of other tools: e.g., quality (Gans 2002, Banker et al. 1998), restocking fee (Shulman et al. 2011), and rationing (Liu and Ryzin 2008). To our knowledge, our study is the first to consider lowering product inspection standards as a basis for competition. The two types of inspection methods we analyze—an individual test on a farmer's milk and a mixed test on the combined milk from many farmers—differ in their impact on farmers: in the presence of a mixed test, a farmer's quality decision is influenced by the qualities of the other farmers who choose to supply the same milk station.

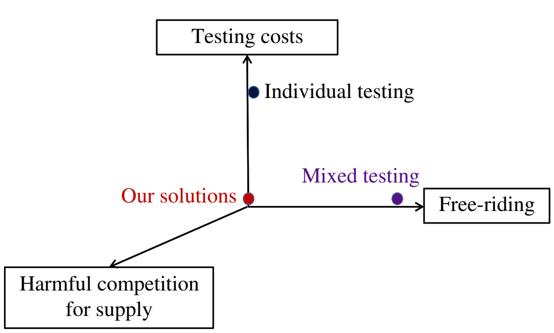
We now summarize the contribution of our paper.

1.3. Contribution of Our Analysis and Proposed Solutions

Broadly speaking, there are three main forces in the milk supply chain that lead to the low quality of milk: high testing costs, harmful competition among stations over supply, and free-riding among farmers (see Figure 1). The use of mixed testing can reduce testing cost but can potentially introduce free-riding among farmers. Furthermore, competition (among stations) over supply can lower testing standards. These three forces, acting together, drive down the quality of milk.

Intuition suggests that a possible governmental intervention to improve quality could be to provide stations with better infrastructure (e.g., storage and refrigeration facilities). This allows a station to perform a mixed test (test the mixed milk from multiple farmers once) and thereby reduce the testing costs. Such an intervention has a positive impact on quality in a monopoly. However, in the presence of competition for supply, the same intervention could possibly lead to a nondesirable equilibrium outcome where the competing stations do not test individually and the farmers supply low-quality milk. Another intuitive intervention could be to subsidize the testing costs for the stations. We show that this, too, could lead to lower quality of milk in the presence of competition

Figure 1 (Color online) Main Forces of the Milk Supply Chain That Lead to Low Quality of Milk



for supply. Thus, policy interventions to improve milk quality under competition have to be handled with care or they could do more harm than good.

Our recommendations address the milk quality problem by achieving an attractive equilibrium outcome. We elaborate below. To lower testing costs, we utilize mixed testing. However, this introduces free-riding among farmers. We counter free-riding by instituting a credible threat of individual testing (but interestingly, not its actual use in equilibrium). The harmful force of competition is tackled by two alternative recommendations: (i) bonus-based scheme, in which only the milk station with a higher testing standard is allowed to offer a bonus—proportional to the testing-standard differential between the stations—to each farmer who supplies to it; and (ii) mixed-pricing-based scheme, in which the announced payment to the farmers is based on the quality of the mixed milk. Both our recommendations achieve a socially desirable equilibrium outcome: all the farmers provide high-quality milk, and each competing station conducts only one mixed test and no further testing.

Our solutions are novel because we use the force of competition to solve a problem created by competition. To illustrate, consider our bonus-based recommendation. Without any intervention, we know that competition between the stations drives down the quality of milk. Our solution is also based on competition: the bonus provides an additional tool for the stations to compete, which reverses the outcome of competition. More specifically, under our bonus-based recommendation, the stations compete by increasing their testing standards (so that a bonus can be offered) to attract more farmers, leading to a high equilibrium quality.

Our recommendations are designed carefully to ensure high quality and yet use a minimal amount of testing and intervention. Again, consider the bonus-based recommendation as an example. An important feature of our bonus structure is that it is relative, not absolute. If it were absolute (e.g., a station pays a bonus to all the farmers who supply milk to it), then a significant amount of bonus must be paid in equilibrium. In contrast, no bonus is paid in equilibrium under our recommendation. Another important feature is that the bonus is based on testing standards, not outcomes. If it were based on testing results (e.g., a station pays a bonus to the farmers who are proven to supply high-quality milk), then a sufficient amount of individual testing must be conducted in equilibrium to incentivize high-quality milk. Under our recommendation, however, no individual testing is conducted in equilibrium and yet all farmers supply high-quality milk.

2. Basic Setup, Assumptions, and Common Notation

Based on our discussion in §1.1, our analysis in §§3 and 4 assumes that farmers are homogeneous in their supply quantity and cost. Later, in §5, we extend our main results for heterogeneous supply quantities as well.

The farmers are paid (by the station) based on the quality of the milk, which is an assessment of the various nutrients in the milk (Draaiyer et al. 2009). The sale of milk is a significant source of income for farmers, who constitute a large percentage of the population in most developing countries. Therefore, it is common to see a government-regulated, fixed procurement-price menu (i.e., prices corresponding to different qualities) for public and private companies to buy raw milk from farmers; see, e.g., Moran (2009, Chap. 6, Tables 6.2 and 6.3, pp. 77–78) for instances of milk grading schemes. In India, for instance, the price menu for the procurement of milk is set by the individual states. There are several states where both the quality-based procurement prices (for farmers) and selling prices (to consumers) are fixed (Dairy Development Department, Maharashtra State, India 2013, Rajendran and Mohanty 2004). In Vietnam, the two main milk firms, Vinamilk® and Dutch Lady®, procure milk from farmers at fixed quality-based prices (Phong 2013).

We consider the nonpoisonous contamination of milk, where quality reduces linearly with the amount of adulterants. Accordingly, we assume that the unit production cost $c(q)$ for the farmers and the unit buying price $w(q)$ and selling price $p(q)$ for the stations (respectively, to buy milk from farmers and to sell to an upstream dairy firm) are linear increasing functions of the quality q; see Draaiyer (2002) and International Livestock Research Institute (2013) for examples of quality-based linear pricing methods for milk. Let $c(q) = c_a + c_b q$, $w(q) = w_a + w_b q$, and $p(q) = p_a + p_b q$. The validity of our main results under nonlinear payoffs is discussed later in §5.

As mentioned earlier in §1.1, a farmer's milk undergoes a simple visual inspection or smell test on arrival. Let q_L denote the minimum quality required by this initial examination and q_H denote the (high) quality of milk without any adulteration. Thus, each farmer provides a quality between q_L and q_H, i.e., $q_L \le q \le q_H$.

To understand the impact of competition, our analysis in §§3 and 4 considers a setting where two competing stations serve the same population of farmers and use the same (government-regulated) unit buying-price menu $w(q)$ to buy milk from the farmers. The two stations sell their milk to the same upstream firm. Therefore, the two stations share the same unit selling-price menu $p(q)$. A station's objective is to maximize the expected profit (revenue minus testing cost and payment to the farmers) from selling the milk to the firm. A farmer's objective is to maximize her expected profit (revenue minus production cost); in §5, we briefly comment on the situation where farmers also incur significant transportation costs in traveling to the milk stations. Our main results easily extend for multiple competing stations, as will be discussed in §5.

Table 2 **Parameters Used in the Analysis**

Parameter	Definition
m	The total number of milk stations serving the population of milk farmers. For most of our analysis, $m \in \{1, 2\}$. The extension to $m \ge 3$ is considered in §5.
n	The total number of farmers supplying milk to the stations.
Q_j	The quantity of milk supplied by farmer j, $j = 1, 2, \ldots, n$. For most of our analysis, we consider a homogeneous supply quantity (i.e., $Q_j = Q$, $\forall j$). Heterogeneous supply quantities are considered in §5.
q_H	The quality of milk without any adulteration. This also defines an upper bound on the quality of milk provided by a farmer.
q_L	The (acceptable) lower bound on the quality of milk provided by a farmer.
$c(q)$	The unit production cost as a function of quality q.
$w(q)$	The unit buying price (that a station pays to a farmer) as a function of quality q.
$p(q)$	The unit selling price (that a station receives from the dairy firm) as a function of quality q.
t	The cost of testing an individual sample of milk.
t_M	The cost of testing a mixed sample of milk.

Let Q_j be the quantity of milk supplied by farmer j, $j = 1, 2, \ldots, n$. For most of our analysis, we consider a homogeneous quantity (i.e., $Q_j = Q$, $\forall j$). Let t (t_M) be the cost of testing an individual (mixed) sample of milk. The parameters used in our analysis are summarized in Table 2.

Reflecting the current reality of the milk supply chain in developing countries (stations collect and sell adulterated milk but are still profitable), we make the following assumptions. (i) The profit for a station from selling high-quality milk exceeds that from selling low-quality milk; i.e., $p(q_H) - w(q_H) > p(q_L) - w(q_L)$. (ii) If a farmer is tested, then her profit from supplying high-quality milk is higher than that from supplying low-quality milk; i.e., $w(q_H) - c(q_H) > w(q_L) - c(q_L)$. (iii) A station earns a profit by selling low-quality milk (without incurring any testing cost); i.e., $p(q_L) - w(q_H) > 0$.

The following notation is used throughout our analysis. The equilibrium profit of farmer j is denoted by g_j, $j = 1, 2, \ldots, n$. When analyzing a game between a single station and a population of farmers (§§3.1.1 and 3.2.1), the equilibrium profit of the station is denoted by f. When analyzing a game between two competing stations and a population of farmers (§§3.1.2, 3.2.2, and 4), we use (i) n_i to denote the number of farmers who choose station i and (ii) f_i to denote the equilibrium profit of station i, $i = 1, 2$.

Next, we investigate the impact of competition on the quality of milk.

3. Impact of Competition

We first examine the impact of competition when only individual testing is possible (e.g., for poorly resourced stations) and then investigate the impact of competition under both individual and mixed testing (e.g., for well-resourced stations).

3.1. Impact of Competition Under Individual Testing

Section 3.1.1 studies a sequential game between one milk station and a population of farmers, and §3.1.2 studies a similar game in the presence of two competing stations. The main result of this section is that testing is a double-edged sword: in a monopoly, high-enough testing ensures good milk quality. However, in the presence of competition, a "testing war" ensues that drives quality down.

3.1.1. Monopoly. In a monopoly, the (single) station collects milk from the farmers, mixes the milk, and sells the mixed milk to a firm. We first describe the sequential game between the station and the population of farmers and then derive the equilibrium.

Description of the Game:

Strategy of the Station. The station tests an x fraction of farmers individually and applies a penalty $b(q_H - q)Q$ on a farmer with milk of quality $q < q_H$ and quantity Q.

Strategies of the Farmers. Each farmer[2] supplies a quality q, $q_L \le q \le q_H$.

Order of Play. First, the station announces $\{x, b\}$. Then, each farmer determines her quality q.

Penalty Structure. A penalty of $b(q_H - q)Q$, $0 \le b \le \bar{b}$, is charged to a farmer who supplies quantity Q and has been proven to provide milk of quality $q < q_H$. The upper bound on the penalty for low quality is motivated by practical guidelines on the limit of such a penalty, based on fairness criteria (Balachandran and Radhakrishnan 2005).

Payoff Structure. If a farmer is tested individually, then a price corresponding to that individual quality is paid. If a farmer is not tested, then a price $w(q_H)$ corresponding to the quality of milk without any adulteration (i.e., q_H) is paid.

[2] In many developing countries, a fixed-quantity contract (which specifies the quantity of milk supplied by a farmer) is commonly used. Accordingly, our model assumes that a farmer supplies a fixed quantity and makes a decision only on the quality of milk (i.e., to what extent to adulterate the milk).

Equilibrium Solution: Given any testing probability x and penalty coefficient b charged by the station, let $g(q \mid x, b)$ be the expected profit for any farmer from selling milk of quality q to the station. We have

$$g(q \mid x, b) = x\{w(q)Q - b(q_H - q)Q\} + (1 - x)w(q_H)Q - c(q)Q. \tag{1}$$

Given a quality q from each farmer, let $f(x, b)$ be the station's expected profit from testing an x fraction of farmers individually and applying a penalty coefficient b. We have

$$f(x, b) = n\{p(q)Q - x\{w(q)Q - b(q_H - q)Q + t\} - (1 - x)w(q_H)Q\}. \tag{2}$$

The following result provides the equilibrium solution under monopoly: If the testing cost is low (relative to the maximum possible penalty), then the station employs the maximum possible penalty and a significant amount of testing, and all farmers provide high-quality milk. On the other hand, if the testing cost is high, then the station does not test, and all farmers provide low-quality milk.

Theorem 1. *Consider the game defined above between a poorly resourced station and a population of farmers, under penalties on farmers who have been proven to supply inferior-quality milk. We have the following:*[3]

- *If $t < (p_b(w_b + \bar{b})(q_H - q_L)Q)/c_b$, then the unique equilibrium solution*[4] *is $b = \bar{b}$, $x = c_b/(w_b + \bar{b})$, $q = q_H$, $f = np(q_H)Q - nw(q_H)Q - ntc_b/(w_b + \bar{b})$, and $g_j = w(q_H) - c(q_H)$, $j = 1, 2, \ldots, n$.*
- *If $t > (p_b(w_b + \bar{b})(q_H - q_L)Q)/c_b$, then the unique equilibrium solution is $x = 0$, $q = q_L$, $f = np(q_L)Q - nw(q_H)Q$, and $g_j = w(q_H)Q - c(q_L)Q$, $j = 1, 2, \ldots, n$, where f and g_j, $j = 1, 2, \ldots, n$, are as defined in §2.*

3.1.2. Competition. We now analyze a game between two competing, poorly resourced stations and a population of farmers. Recall from §2 that the two stations (i) use the same (government-regulated) unit buying-price menu $w(q)$ to buy milk from the farmers and (ii) sell their milk to the same upstream dairy firm and, therefore, share the same unit selling-price menu $p(q)$.

[3] When $t = (p_b(w_b + \bar{b})(q_H - q_L)Q)/c_b$, both the solutions are possible equilibria. For brevity, we avoid presenting this special case explicitly in the statement of the theorem. We follow the same convention (wherever applicable) throughout the remainder of the paper.

[4] When the testing probability $x = c_b/(w_b + \bar{b})$, the farmers are indifferent between supplying high quality and supplying low quality. In this case, we assume that all farmers provide high quality. This is not a significant assumption: a testing probability of $x = c_b/(w_b + \bar{b}) + \epsilon$, where ϵ is infinitesimally small and positive, guarantees that all farmers provide high-quality milk.

The strategies of the stations and the farmers and the order of play are as defined below. The penalty and payoff structures are identical to those in §3.1.1.

Description of the Game:

Strategies of the Stations. Station i, $i = 1, 2$, applies a penalty $b_i(q_H - q)Q$, $b_i \le \bar{b}$, on a farmer with milk of quality $q < q_H$ and quantity Q and tests an x_i fraction of farmers individually.

Strategies of the Farmers. Each farmer selects a station and determines the quality (q_1 if station 1 is selected and q_2 if station 2 is selected) to supply.

Order of Play. First, the two stations announce $\{b_1, x_1\}$ and $\{b_2, x_2\}$ simultaneously. Then, each farmer selects a station and determines the quality.

We now provide the equilibrium solution.

Equilibrium Solution: Given $\{x_i, b_i\}$, $i = 1, 2$, from station i, let $g(q_i \mid x_i, b_i)$ be the expected profit for a farmer from selling milk of quality q_i to station i. We have

$$g(q_i \mid x_i, b_i) = x_i\{w(q_i)Q - b_i(q_H - q_i)Q\} + (1 - x_i)w(q_H)Q - c(q_i)Q, \quad i = 1, 2. \tag{3}$$

Given n_i farmers who supply to station i and a quality q_i from each farmer, let $f(x_i, b_i)$ be the expected profit of station i from applying a testing probability x_i and a penalty coefficient b_i. We have

$$f(x_i, b_i) = n_i\{p(q_i)Q - x_i\{w(q_i)Q - b_i(q_H - q_i)Q + t\} - (1 - x_i)w(q_H)Q\}, \quad i = 1, 2. \tag{4}$$

Define

$$\underline{t} = \frac{w_b + \bar{b}}{c_b}\{2c_b(q_H - q_L) + 2p(q_L) - p(q_H) - w(q_H)\}Q,$$

$$\bar{t} = \frac{w_b + \bar{b}}{c_b}\{p(q_H) + w(q_H) - 2p(q_L)\}Q.$$

The following result shows that, under competition, no testing from both stations and low-quality milk from all farmers is always a possible equilibrium outcome. When the farmers are indifferent between selling to the two stations, our assumption is that half of them sell to station 1 and the other half to station 2.

Theorem 2. *Consider the sequential game defined above between two poorly resourced stations and a population of farmers, under penalties on farmers who have been proven to supply inferior-quality milk. We have the following*:

- *If $t < \underline{t}$, or $t > \bar{t}$, or $[\underline{t} \le t \le \bar{t}$ and $p(q_L) - w(q_H) > (p_b - c_b)(q_H - q_L)]$, then the unique equilibrium solution is $x_1 = x_2 = 0$, $n_1 = n_2 = n/2$, $q_1 = q_2 = q_L$, $f_1 = f_2 = (n/2)\{p(q_L) - w(q_H)Q\}$, and $g_j = w(q_H)Q - c(q_L)Q$, $j = 1, 2, \ldots, n$.*
- *If $\underline{t} \le t \le \bar{t}$ and $p(q_L) - w(q_H) \le (p_b - c_b)(q_H - q_L)$, then the possible equilibria are*:

  (i) $x_1 = x_2 = 0$, $n_1 = n_2 = n/2$, $q_1 = q_2 = q_L$, $f_1 = f_2 = (n/2)\{p(q_L)Q - w(q_H)Q\}$, *and* $g_j = w(q_H)Q - c(q_L)Q$, $j = 1, 2, \ldots, n$; *and*

  (ii)[5] $x_1 = x_2 = c_b/(w_b + \bar{b})$, $b_1 = b_2 = \bar{b}$, $n_1 = n_2 = n/2$, $q_1 = q_2 = q_H$, $f_1 = f_2 = (n/2)\{p(q_H)Q - w(q_H)Q - c_b t/(w_b + \bar{b})\}$, *and* $g_j = w(q_H)Q - c(q_H)Q$, $j = 1, 2, \ldots, n$, *where* f_i, n_i, $i = 1, 2$, *and* g_j, $j = 1, 2, \ldots, n$, *are as defined in* §2.

A comparison of Theorems 1 and 2 helps us to derive the following insight.

Insight 1. When stations are poorly resourced (and therefore only individual testing is possible), then we have the following cases.

- Under a monopoly, the station conducts a significant amount of testing to ensure that all farmers provide high-quality milk in equilibrium (provided the testing cost is not prohibitively high).
- Under competition, however, no testing from either station and low-quality milk from all farmers is always a possible equilibrium outcome.

The above insight—that competition can lead to lower quality of milk supply—is consistent with our observations from practice: scarcity of milk supply (which results in competition over supply) usually leads to lower quality of milk. For example, it is more likely for quality problems to arise in countries where supply is limited, e.g., India (Mann 2014, Squicciarini and Vandeplas 2010) and China (Gale and Hu 2009).

Since the stations are paid based on the quality of the mixed milk, it is intuitive that they have a natural incentive to collect high-quality milk. Under a monopoly, the station combines the use of the maximum possible penalty and a high-enough testing probability (if testing is not very expensive) to ensure that all farmers provide high-quality milk.

Under competition, the farmers have a choice among the stations. There are two possible ways for a station to attract more (inferior-quality) farmers: reduce the testing probability or reduce the penalty on low-quality farmers. Compared to a reduction in penalty, a reduction in the testing probability is more beneficial to a station: in addition to attracting more farmers, a reduction in the testing probability can also save some testing cost. Thus, the stations prefer a reduction in the testing probability over that in the penalty. Consequently, the stations first compete by lowering their testing probabilities and, as a result, both stations do not test in equilibrium. Note that, when there is no testing, a reduction in the penalty (regardless of its magnitude) cannot help the stations to attract more farmers.

Theorem 2 helps to uncover the following insight.

Insight 2. Under competition, if a station prefers (i.e., earns a higher profit from) collecting low-quality milk from the entire market instead of collecting high-quality milk but sharing the market with its competitor, then a reduction in testing cost can lead to lower equilibrium quality.

This insight comes directly from the following observation: when the testing cost is moderate ($\underline{t} \le t \le \bar{t}$), both high milk quality and low milk quality are possible in equilibrium. However, when the testing cost is low ($t < \underline{t}$), the unique equilibrium outcome is low milk quality. Let us now examine why high quality is no longer an equilibrium outcome when the testing cost is low. Consider a situation where both stations set a high-enough individual testing probability, all farmers provide high-quality milk, and the supply is evenly split between the two stations. If one station applies an infinitesimally lower testing probability, then all farmers will sell low-quality milk to this station. The additional benefit of this latter strategy (i.e., $n\{p(q_L) - w(q_H) + c_b(q_H - q_L)\} - (n/2)\{p(q_H) - w(q_H)\}$) is that the station can earn a higher profit by attracting more low-quality farmers. The additional cost (which is infinitesimally lower than $(nc_b/(w_b + \bar{b}))t - (nc_b/2(w_b + \bar{b})t)$) is the increase in testing cost due to the increase in the number of farmers. When the testing cost is low, the additional benefit of the latter strategy outweighs its additional cost, making it more attractive. As a result, the assumed high-quality equilibrium cannot be sustained.

Section A of the appendix examines the impact of competition in the presence of rewards to farmers who are proven to supply high-quality milk. The main message of this analysis is that rewards for high quality fail to resolve the quality problem in a satisfactory manner: in the presence of competition, the stations receive zero profit in equilibrium as a result of harmful competition based on both testing and rewards.

To understand whether some of the conditions required for particular equilibria are likely to hold in practice, we obtained the required data from publicly available documents on real-world milk procurement in India: Food and Agriculture Organization of the UN (2010), Chakravarty (2013), Shah (2011), Kumar et al. (2011), Shah (2012), and Hemme et al. (2003). A summary of the relevant data, based on the information in these documents, is as follows: $c(q_L) = \$0.16/\text{kg}$, $c(q_H) = \$0.24/\text{kg}$, $w(q_L) = \$0.192/\text{kg}$, $w(q_H) = \$0.288/\text{kg}$, $p(q_L) = \$0.312/\text{kg}$, $p(q_H) = \$0.468/\text{kg}$, $Q = 10$ kg, and $t = \$1.8$. Using these data, it is easy to verify that indeed all the assumptions of our analysis (i.e., $p(q_H) - w(q_H) > p(q_L) - w(q_L)$, $w(q_H) - c(q_H) > w(q_L) - c(q_L)$, and $p(q_L) - w(q_H) > 0$; see §2) are satisfied. The data also allow us to identify the equilibrium that is more likely to hold in case there are multiple equilibria. By substituting the above data in the conditions of Theorems 1 and 2, we see that the following (unique) equilibrium outcomes are achieved: (a) Under monopoly, the station applies a significant amount of testing, and the farmers supply high-quality milk. (b) Under competition, the competing stations do not test, and the farmers supply low-quality milk. These observations are also robust to reasonable variations (approximately ±10%) in the above data. Thus, the data highlight the main message we want to convey via these results: competition among stations can reduce the equilibrium quality of milk.

One possible infrastructural enhancement is to enable the stations to conduct mixed testing in addition to individual testing, i.e., to convert poorly resourced stations to well-resourced ones. In addition to expanding the strategy space of the stations, the introduction of mixed testing can also potentially reduce testing costs. However, as we will see in the next section, this alone does not solve the quality problem in the presence of competing milk stations. Subsequently, in §4, we provide two solutions that counter the harmful effect of competition and resolve the quality issue. Furthermore, these two solutions are efficient in the sense that they require a minimal amount of testing in equilibrium.

3.2. Impact of Competition Under Individual and Mixed Testing

We now examine the impact of competition among well-resourced stations that can conduct both individual and mixed testing. First, for a monopoly, we propose a test policy (that uses both mixed and individual testing) and a mild governmental intervention (only one mixed test is sponsored by the government) that lead to a socially desirable result. However, in the presence of two competing stations, we will see that an unsatisfactory outcome occurs under the same governmental intervention. The main takeaway of this section is that, although the introduction of mixed testing benefits milk quality in a monopoly, it becomes the reason for low milk quality under competition.

3.2.1. Monopoly. In a monopoly, the well-resourced station has the following two possible strategies:

Strategy $S^M(x, \underline{q})$. The station draws samples from the milk of each farmer and mixes these samples. The station then performs a mixed test on this mixed milk. If the quality of the mixed milk exceeds a threshold $\underline{q}$, then the price $w(q_H)$ is paid to each farmer. Otherwise, the station conducts individual tests on an x fraction of farmers. If a farmer is tested individually, then that farmer is paid based on her individual quality. If a farmer is not tested individually, then a price $w(q_H)$ corresponding to the quality of milk without any adulteration (i.e., q_H) is paid.

Strategy $S^I(y)$. The station conducts individual tests on a y fraction of farmers. Again, if a farmer is tested individually, then that farmer is paid based on her individual quality. Otherwise, that farmer is paid $w(q_H)$.

We first describe the sequential game between the station and the farmers and then provide the equilibrium solution.

Description of the Generalized Game:

Strategy of the Station. The station selects a strategy between strategy $S^M(x, \underline{q})$ and strategy $S^I(y)$ and announces $\{x, \underline{q}\}$ (if strategy $S^M(x, \underline{q})$ is selected) or y (if strategy $S^I(y)$ is selected).

Strategy of the Farmers. Each farmer supplies a quality q, $q_L \le q \le q_H$.

Order of Play. First, the station selects its strategy. Then, each farmer determines her quality q.

Governmental Intervention. The government sponsors the single mixed test if a station applies strategy $S^M(x, \underline{q})$.

For brevity, we assume that no penalty is applied by the station. The case when penalty is incorporated can be analyzed in a similar manner, and it can be shown that our main conclusions remain valid.

Equilibrium Solution: First, we calculate the profit of farmer j, $j = 1, 2, \ldots, n$, when the station applies strategy $S^M(x, \underline{q})$. Denote q_{-j} as the average quality of milk from the farmers other than farmer j. Let $g(q_j \mid q_{-j}, x, \underline{q})$, $j = 1, 2, \ldots, n$, be the expected profit of farmer j from supplying milk of quality q_j. If $(n\underline{q} - q_H)/(n-1) \le q_{-j} \le q_H$, then the quality of the mixed milk can potentially exceed the quality threshold $\underline{q}$. We have

$$\begin{aligned} g(q_j \mid q_{-j}, x, \underline{q}) &= w(q_H)Q - c(q_j)Q, \\ &\qquad \text{if } q_j \ge n\underline{q} - (n-1)q_{-j}. \\ &= xw(q_j)Q + (1-x)w(q_H)Q - c(q_j)Q, \\ &\qquad \text{if } q_j < n\underline{q} - (n-1)q_{-j}. \end{aligned} \tag{5}$$

If $q_L \le q_{-j} < (n\underline{q} - q_H)/(n-1)$, then the quality of the mixed milk is less than the threshold $\underline{q}$, for any q_j. We have

$$g(q_j \mid q_{-j}, x, \underline{q}) = xw(q_j)Q + (1-x)w(q_H)Q - c(q_j)Q. \tag{6}$$

Next, consider the case when the station applies strategy $S^I(y)$. Let $g(q_j \mid y)$ be the expected profit for farmer j from supplying milk of quality q_j to the station. We have

$$g(q_j \mid y) = yw(q_j)Q + (1-y)w(q_H)Q - c(q_j)Q. \tag{7}$$

Given a quality q from each farmer, let $f^M(x, \underline{q})$ be the expected profit for the station from applying strategy $S^M(x, \underline{q})$. We have

$$\begin{aligned} f^M(x, \underline{q}) &= n\{p(q)Q - w(q_H)Q\}, \qquad \text{if } q \ge \underline{q}. \\ &= n\{p(q)Q - x[w(q)Q + t] - (1-x)w(q_H)Q\}, \\ &\qquad \text{if } q < \underline{q}. \end{aligned} \tag{8}$$

Let $f^I(y)$ be the expected profit for the station from applying strategy $S^I(y)$. We have

$$f^I(y) = n\{p(q)Q - y[w(q)Q + t] - (1-y)w(q_H)Q\}. \tag{9}$$

The following result shows that a socially desirable outcome—all farmers supply high-quality milk and the station conducts only one mixed test—is achieved in a monopoly under the governmental intervention stated above.

Theorem 3. *Under the governmental intervention stated above, the equilibrium solution for the game between the well-resourced station and the farmers is as follows: the station applies strategy* $S^M(x, \underline{q})$ *with* $x > c_b/w_b$ *and* $\underline{q} = q_H$, $q = q_H$, $f = np(q_H)Q - nw(q_H)Q$, *and* $g_j = w(q_H)Q - c(q_H)Q$, $j = 1, 2, \ldots, n$.

At this juncture, it is useful to point out that Mu et al. (2014) study a simultaneous game between one station and a population of farmers to resolve the quality issue in a monopoly using a similar mixing-based test policy. However, as we will soon see, such a solution does not work in the presence of competing stations. Worse, the monopoly solution can hurt the equilibrium quality of milk under competition. Because of the high demand for milk and a relatively limited supply, it is common to see (privately owned) milk collection stations competing for supply. Thus, from a practical viewpoint, the need for (i) understanding the different forces that together result in poor milk quality and (ii) recommendations to improve this situation, under the setting of competing collection stations, becomes important. Our solutions work by reversing the outcome of competition between the stations, i.e., by converting harmful competition (quality reduction) into beneficial competition (quality improvement), resulting in a socially desirable equilibrium outcome. We elaborate on the novelty and other attractive properties of our solutions later in §§4.1 and 4.2.

3.2.2. Competition. Under competition, the definitions of strategies $S^M(x, \underline{q})$ and $S^I(y)$ and the payoff structure are the same as those in §3.2.1. The game between the competing stations and the farmers is described below.

Description of the Game:

Strategies of the Stations. Station i, $i=1,2$, selects a strategy from strategy $S^M(x_i, \underline{q}_i)$ and strategy $S^I(y_i)$ and sets $\{x_i, \underline{q}_i\}$ (if strategy $S^M(x_i, \underline{q}_i)$ is selected) or y_i (if strategy $S^I(y_i)$ is selected).

Strategies of the Farmers. Each farmer selects a station (station 1 or station 2) and determines her quality (q_1 for station 1 or q_2 for station 2).

Order of Play. First, the stations select their strategies simultaneously. Then, the farmers select a station and a quality to supply.

Governmental Intervention. The government sponsors the single mixed test if a station applies strategy $S^M(x, \underline{q})$.

If station i applies strategy $S^M(x_i, \underline{q}_i)$ (strategy $S^I(y_i)$) and the number of farmers in station i is n_i, then (5), (6), and (7) provide the farmers' profits (by substituting $n=n_i$, $x=x_i$, $\underline{q}=\underline{q}_i$, and $y=y_i$).

Given n_i farmers who supply to station i and a quality q_i from each farmer, let $f^M(x_i, \underline{q}_i)$ ($f^I(y_i)$) be the expected profit of station i from applying strategy $S^M(x_i, \underline{q}_i)$ (strategy $S^I(y_i)$). We have

$$\begin{aligned} f^M(x_i, \underline{q}_i) &= n_i\{p(q_i)Q - w(q_H)Q\}, \quad \text{if } q_i \geq \underline{q}_i. \\ &= n_i\{p(q_i)Q - x_i[w(q_i)Q + t] \\ &\qquad - (1-x_i)w(q_H)Q\}, \quad \text{if } q_i < \underline{q}_i. \\ f^I(y_i) &= n_i\{p(q_i)Q - y_i[w(q_i)Q + t] \\ &\qquad - (1-y_i)w(q_H)Q\}. \end{aligned} \tag{10}$$

The following result shows that, under the same governmental intervention as that in a monopoly, low-quality milk is the unique equilibrium outcome under competition. The proof is similar to that of Theorem 2 and is therefore omitted for brevity.

Theorem 4. *Under the governmental intervention stated above, the equilibrium solution for the game between two well-resourced stations and a population of farmers is as follows: station i, $i=1,2$, applies strategy $S^M(x_i, \underline{q}_i)$ with $\underline{q}_i = q_L$ or strategy $S^I(y_i)$ with $y_i = 0$, $n_1 = n_2 = n/2$, $q_1 = q_2 = q_L$, $f_1 = f_2 = (n/2)\{p(q_L)Q - w(q_H)Q\}$, and $g_j = w(q_H)Q - c(q_L)Q$, $j = 1, 2, \ldots, n$.*

A comparison of the equilibrium outcome in a monopoly with that under competition helps us to derive the following insight.

Insight 3. When the stations are well-resourced (and therefore mixed testing is possible), then we have the following cases.

- In a monopoly, a socially desirable result, where the station performs only one mixed test and all farmers supply high quality in equilibrium, can be achieved through a mild governmental intervention.
- In the presence of two competing stations, however, an unsatisfactory outcome—both stations perform no individual testing and all farmers supply low quality in equilibrium—occurs under the same governmental intervention.

We examine this insight further. In a monopoly, from the station's viewpoint, conducting the mixed test, setting q_H as the quality threshold, and using the threat of high individual testing if the quality of the mixed milk is less than the quality threshold, together serve two purposes: (i) all farmers supply high-quality milk and (ii) no individual testing is actually conducted in equilibrium, since the equilibrium quality equals the quality threshold. Thus, the mixing-based test policy achieves the desired result in a monopoly.

Under competition, however, the stations compete for the limited supply. In general, there are two ways for a station to attract more farmers: (i) perform the free mixed test and reduce the quality threshold and (ii) do not perform the mixed test and reduce the individual testing probability. The former leads to an equilibrium where both stations set the minimum-possible quality threshold, and the latter leads to an equilibrium where both stations perform no individual testing. In both cases, the farmers deliver low-quality milk.

Together, Theorem 2 (two competing, poorly resourced stations) and Theorem 4 (two competing, well-resourced stations) result in the following conclusion.

Insight 4. Under competition, an improvement in the infrastructure of the stations (to enable mixed testing of milk) and a sponsorship of one mixed test for each station might lead to a worse equilibrium outcome. Before the improvement, in some cases, high-quality milk from the farmers (albeit through a significant amount of testing) is one possible equilibrium outcome. After the improvement, however, low-quality milk is the unique equilibrium outcome.

We now explain why the use of mixed testing alone (i.e., not in combination with any other intervention) cannot result in a high-quality equilibrium in the presence of competing stations. In general, the introduction of mixed testing offers the stations a new tool for competition: reducing the quality threshold. To see this, consider a situation where both stations set q_H as the quality threshold (and announce the threat of high individual testing if quality of the mixed milk is less than the threshold), all farmers provide high-quality milk, and the supply is evenly split between the two stations. If one station reduces its quality threshold slightly, then it can attract all the farmers of quality close to q_H (i.e., at the new quality threshold). This creates intense pressure for the stations to undercut each other's quality threshold. The end result is that both stations adopt the lowest possible threshold (i.e., quality q_L) and, therefore, all farmers supply low-quality milk.

To summarize, although the introduction of mixed testing resolves the quality issue in a monopoly, it fails under competition. In the next section, we provide two policies that achieve a socially desirable result under competition.

4. Two Recommendations

We first provide a recommendation based on mixed pricing and then provide a recommendation based on the testing-standard differential between stations.

4.1. A Recommendation Based on Mixed Pricing

We saw in the previous section that the availability of mixed testing can do more harm than good. However, together with a pricing intervention, the government can use mixed testing to achieve good effect.[6] We propose the following governmental intervention.

Governmental Intervention. (i) The government sponsors one mixed test for each station and requires the stations to perform this free mixed test (on all farmers) before any testing.

(ii) The stations are required (by the government) to announce mixed pricing: if a subset of farmers are not tested individually, then these farmers are paid based on the quality of their mixed milk.[7]

We first describe the sequential game under this intervention and then present the equilibrium solution.

Description of the Game:

Strategies of the Stations. Station i, $i=1,2$, draws equal-sized samples from the milk of each farmer (who supplies to that station) and mixes these samples. The station then conducts a mixed test on this mixed milk. If the quality of the mixed milk in station i is greater than or equal to a threshold $\underline{q}_i$, then the station pays each farmer based on the quality of the mixed milk. Otherwise, station i tests an x_i fraction of farmers individually. The farmers who are tested individually are paid based on their respective individual qualities. The remaining farmers are paid based on the average quality of their mixed milk.

Strategies of the Farmers. Each farmer selects a station (station 1 or station 2) and determines her quality (q_1 for station 1 or q_2 for station 2) to provide.

[6] For poorly resourced stations, we advocate improving their basic infrastructure (such as refrigeration and storage facilities) so that mixed testing is feasible.

[7] If a fraction of farmers are tested individually after the mixed test, then the average quality of the farmers that are not individually tested can be derived based on the results of the initial mixed test and the subsequent individual tests.

Order of Play. First, the two stations announce $\{x_1, \underline{q}_1\}$ and $\{x_2, \underline{q}_2\}$ simultaneously. Then, each farmer selects a station and determines her quality.

Equilibrium Solution: Suppose station i, $i=1,2$, applies a testing probability x_i and a quality threshold $\underline{q}_i$. Let n_i be the number of farmers providing milk to station i. For farmer j who supplies milk to station i, let $q_{i,-j}$ be the average quality of milk samples drawn from the other farmers (i.e., other than farmer j) supplying to station i. Let $g(q_{ij} \mid x_i, \underline{q}_i, n_i, q_{i,-j})$ be the expected profit for farmer j, $j=1,2,\ldots,n$, from selling milk of quality q_{ij} to station i, $i=1,2$. If $(n_i\underline{q}_i - q_H)/(n_i-1) \le q_{i,-j} \le q_H$, then the quality of the mixed milk can potentially exceed the quality threshold $\underline{q}_i$. We have

$$
\begin{aligned}
&g(q_{ij}\mid x_i,\underline{q}_i,n_i,q_{i,-j})\\
&=\left\{\frac{(n_i-1)w(q_{i,-j})+w(q_{ij})}{n_i}-c(q_{ij})\right\}Q,\\
&\qquad\qquad \text{if } q_{ij}\ge n_i\underline{q}_i-(n_i-1)q_{i,-j}.\\
&=\left\{x_iw(q_{ij})+\frac{\{n_i(1-x_i)-1\}w(q_{i,-j})+w(q_{ij})}{n_i}-c(q_{ij})\right\}Q,\\
&\qquad\qquad \text{if } q_{ij}< n_i\underline{q}_i-(n_i-1)q_{i,-j} \text{ and } x_i<1.\\
&=\{w(q_{ij})-c(q_{ij})\}Q,\\
&\qquad\qquad \text{if } q_{ij}< n_i\underline{q}_i-(n_i-1)q_{i,-j} \text{ and } x_i=1. \qquad (11)
\end{aligned}
$$

If $q_L \le q_{i,-j} < (n_i\underline{q}_i - q_H)/(n_i-1)$, then the quality of the mixed milk is less than the threshold $\underline{q}_i$ for any quality q_{ij}. We have

$$
\begin{aligned}
&g(q_{ij}\mid x_i,\underline{q}_i,n_i,q_{i,-j})\\
&=\left\{x_iw(q_{ij})+\frac{\{n_i(1-x_i)-1\}w(q_{i,-j})+w(q_{ij})}{n_i}-c(q_{ij})\right\}Q,\\
&\qquad\qquad \text{if } x_i<1.\\
&=\{w(q_{ij})-c(q_{ij})\}Q, \quad \text{if } x_i=1. \qquad (12)
\end{aligned}
$$

Given n_i farmers who supply to station i and a quality q_i from each farmer, let $f(x_i, \underline{q}_i)$ be the expected profit for station i from applying testing probability x_i and quality threshold $\underline{q}_i$, $i=1,2$. We have

$$
\begin{aligned}
f(x_i,\underline{q}_i) &= n_i\{p(q_i)Q - w(q_i)Q\}, && \text{if } q_i \ge \underline{q}_i.\\
&= n_i\{p(q_i)Q - w(q_i)Q - x_it\}, && \text{if } q_i < \underline{q}_i. \quad (13)
\end{aligned}
$$

The following result shows that, under the above governmental intervention, all farmers supply high-quality milk and each station conducts only one mixed test (and no further testing) in equilibrium.

Theorem 5. *The equilibrium solution under the above governmental intervention is as follows*: $n_1 = n_2 = n/2$,

$\underline{q}_1 = \underline{q}_2 = q_H$, $f_1 = f_2 = n/2\{p(q_H)Q - w(q_H)Q\}$, and $g_i = w(q_H)Q - c(q_H)Q$, $i = 1, 2, \ldots, n$. If $n > (2w_b/c_b)$, then $x_i > (c_b - 2w_b/n)/w_b$ and $\underline{q}_i = q_H$, $i = 1, 2$. If $n < (2w_b/c_b)$, then any $\{x_i, \underline{q}_i\}$ is an equilibrium strategy for station i, $i = 1, 2$.

Note that the values of x_1 and x_2 in the equilibrium solution above are just an announcement by the stations. No individual testing is conducted in equilibrium.

We now discuss the intuition behind this equilibrium in Theorem 5. Recall from §3.2 that the root cause of the low-quality equilibrium in Theorem 4 was the farmers' preference for the station that has a lower quality threshold (or a lower individual-testing probability): by choosing such a station, a farmer gains by serving low-quality milk and still receiving a high payment. Our proposed mixed pricing directly eliminates such an incentive for the farmers by conditioning the payment (for untested farmers) on the quality of the mixed milk.

Given the fact that the farmers now prefer a station with a relatively high testing standard, from the viewpoint of the stations, quality improvement is aligned with profit improvement. Thus, in equilibrium, both stations apply the maximum possible quality threshold and a high-enough testing probability. As a result, all farmers supply high-quality milk and the stations perform only one mixed test and no further testing in equilibrium.

The above recommendation has some attractive properties, with respect to its implementation. We discuss these below.

Although mixed pricing may be deemed an unfair pricing scheme by high-quality farmers, all farmers supply high-quality milk in equilibrium. Therefore, in equilibrium, the farmers are paid based on their true qualities.

The stations use the threat that a certain fraction of the farmers will be individually tested if the quality of the mixed milk is less than the quality threshold. However, note from Theorem 5 that this individual testing is never conducted in equilibrium, since the equilibrium quality equals the equilibrium quality threshold (both are equal to q_H). If an off-equilibrium situation is a concern, then conducting significant individual testing might be a financial burden on the stations. To make this threat (of individual testing) more credible in an off-equilibrium situation, the stations could ask all the farmers who have been proven to provide inferior-quality milk to share the total (or partial) individual testing cost. It can be easily shown that such a "shared" penalty does not change the equilibrium.

4.2. A Recommendation Based on the Testing-Standard Differential

Compared to the proposal in the previous section, the recommendation here involves less governmental intervention and offers more freedom to the stations. The idea is to use the force of competition to solve a problem created by competition: we propose a bonus scheme, based on relative testing standards, that converts harmful competition between stations to a positive competition for high-quality milk. Specifically, the station with a higher testing standard is allowed to offer a bonus to each farmer who supplies milk to that station. This bonus depends on the testing-standard differential between the two stations. The values of the bonus coefficients are decided by the two stations collectively (the choice of zero bonus is allowed but, interestingly, the stations do not choose this value in equilibrium).

The high-level insight from our analysis in this section is as follows: although rewarding individual farmers for high quality is not a good idea, rewarding farmers for choosing a station that has more rigorous testing standards is socially beneficial. Viewed differently, the bonus provides a tool for the station with higher testing standards to protect itself; however, the bonus is never paid in equilibrium.

Let us use station $-i$ to denote the station other than station i. Consider the following policy:

A Bonus-Based Policy:

- *Bonus Structure*. The following relative bonus structure (but not the value of the bonus) is imposed by the government:
  - If the quality threshold announced by one station is higher than that of the other station ($\underline{q}_i \geq \underline{q}_{-i}$), then the former station (i.e., station i) offers a per-unit bonus $a(\underline{q}_i - \underline{q}_{-i})$ to each farmer who supplies to it, where $a \geq 0$.
  - If the individual testing probability announced by one station is higher than that of the other station (e.g., $x_i > x_{-i}$, if both stations choose strategy $S^M(x, \underline{q})$), then the former station (e.g., station i) offers a per-unit bonus $d(x_i - x_{-i})$ to each farmer who supplies to it, where $d \geq 0$. Similar bonuses, namely $d(x_i - y_{-i})$, $d(y_i - x_{-i})$, and $d(y_i - y_{-i})$, are offered for the other choices of the stations' strategies.

The policy does not mandate specific values of the bonus coefficients a and d. Instead, these values are left for the stations to decide.[8] The stations have flexibility in deciding how they interact in setting the common values of the bonus coefficients a and d. One

[8] Allowing the stations to choose the bonus coefficients a and d could offer more freedom to the stations and therefore make this bonus-based policy easier to implement in practice. Alternatively, the government could also fix the values of the bonus coefficients if the possibility of violating price regulation is a concern.

simple, two-step process is as follows. First, one station (say, station 1) proposes values for a and d. Second, the other station either accepts them or rejects them. It is important to note that we allow zero values for both a and d. In other words, the stations are allowed to offer a bonus, but not forced to. We assume that if the stations fail to agree on a common pair of a and d, then the bonus-based scheme will not be implemented. However, as we will soon see, the stations will always agree in equilibrium.

• *Limited Support*. The government sponsors the single mixed test if a station applies strategy $S^M(x, \underline{q})$.

The description of the game under the bonus-based policy above is identical to that in §3.2.2. The payoff structure is also the same except that, under the bonus-based policy, a station with the higher testing standard pays an additional bonus to the farmers who supply milk to it. We now derive the equilibrium solution.

Equilibrium Solution: Denote by r_i the per-unit bonus received by a farmer for choosing station i. The expression of r_i depends on the comparison of the quality thresholds and the individual testing probabilities of the stations and is provided in §C of the appendix. We now calculate the profit of farmer j, $j = 1, 2, \ldots, n$, from selling milk to station i. First, consider the case when station i applies strategy $S^M(x_i, \underline{q}_i)$, $i = 1, 2$. Let n_i be the number of farmers providing milk to station i. For farmer j who supplies milk to station i, let $q_{i,-j}$ be the average quality of milk samples drawn from the other farmers (i.e., other than farmer j) supplying to station i. Let $g(q_{ij} \mid x_i, \underline{q}_i, n_i, q_{i,-j})$ be the expected profit for farmer j, $j = 1, 2, \ldots, n$, from selling milk of quality q_{ij} to station i. If $(n_i \underline{q}_i - q_H)/(n_i - 1) \le q_{i,-j} \le q_H$, then the quality of the mixed milk can potentially exceed the quality threshold $\underline{q}_i$. We have

$$\begin{aligned} g(q_{ij} \mid x_i, \underline{q}_i, n_i, q_{i,-j}) &= \{w(q_H) - c(q_{ij}) + r_i\}Q, \quad \text{if } q_{ij} \ge n_i \underline{q}_i - (n_i - 1)q_{i,-j}. \\ &= \{x_i w(q_{ij}) + (1 - x_i)w(q_H) - c(q_{ij}) + r_i\}Q, \\ &\qquad \text{if } q_{ij} < n_i \underline{q}_i - (n_i - 1)q_{i,-j}. \end{aligned}$$

If $q_L \le q_{i,-j} < (n_i \underline{q}_i - q_H)/(n_i - 1)$, then the quality of the mixed milk is less than the threshold $\underline{q}_i$, for any quality q_{ij}. We have

$$\begin{aligned} g(q_{ij} \mid x_i, \underline{q}_i, n_i, q_{i,-j}) \\ = \{x_i w(q_{ij}) + (1 - x_i)w(q_H) - c(q_{ij}) + r_i\}Q. \end{aligned}$$

Next, consider the case when station i, $i = 1, 2$, applies strategy $S^I(y_i)$. Let $g(q_{ij} \mid y_i)$ be the expected profit for farmer j, $j = 1, 2, \ldots, n$, from selling milk of quality q_{ij} to station i. We have

$$g(q_{ij} \mid y_i) = \{y_i w(q_{ij}) + (1 - y_i)w(q_H) - c(q_{ij}) + r_i\}Q.$$

Given n_i farmers who supply to station i and a quality q_i from each farmer, let $f^M(x_i, \underline{q}_i)$ be the expected profit of station i from applying strategy $S^M(x_i, \underline{q}_i)$. We have

$$\begin{aligned} f^M(x_i, \underline{q}_i) &= n_i\{p(q_i) - w(q_H) - r_i\}Q, \qquad \text{if } q_i \ge \underline{q}_i. \\ &= n_i\{p(q_i)Q - x_i[w(q_i)Q + t] \\ &\qquad - (1 - x_i)w(q_H)Q - r_i Q\}, \quad \text{if } q_i < \underline{q}_i. \end{aligned}$$

Let $f^I(y_i)$ be the expected profit of station i from applying strategy $S^I(y_i)$. We have

$$\begin{aligned} f^I(y_i) = n_i\{p(q_i)Q - y_i[w(q_i)Q + t] \\ - (1 - y_i)w(q_H)Q - r_i Q\}. \end{aligned}$$

The following result shows that, under the above bonus-based policy, all farmers supply high-quality milk and each station conducts only one mixed test (and no further testing) in equilibrium.

Theorem 6. *The equilibrium solution under the bonus-based policy is as follows: station i, $i = 1, 2$, applies strategy $S^M(x_i, \underline{q}_i)$ with $x_i = 1$ and $\underline{q}_i = q_H$, $a > c_b$, $d > c_b(q_H - q_L)$, $n_1 = n_2 = n/2$, $q_1 = q_2 = q_H$, $f_1 = f_2 = (n/2)\{p(q_H)Q - w(q_H)Q\}$, and $g_j = w(q_H)Q - c(q_H)Q$, $j = 1, 2, \ldots, n$.*

We again note that the individual testing probabilities x_i, $i = 1, 2$, in the equilibrium above are only announced by the stations, but no individual testing is actually executed in equilibrium.

We now provide an intuitive explanation of the ability of the policy to resolve the quality issue under competition.

4.2.1. The Intuition Behind the Bonus-Based Policy. Recall from §3.2.2, where mixed testing was introduced in the presence of competing stations, that the stations perform no individual testing and all farmers supply low quality in equilibrium. Let us see why the new idea, a bonus based on the testing-standard differential, resolves the quality issue.

In the setting considered in §3.2.2, there are two ways for the stations to compete: (i) perform the mixed test and reduce the quality thresholds and (ii) do not perform the mixed test and reduce the individual testing probability. In our new policy, the (high) bonus, which is based on the quality-threshold differential, reverses the farmers' choice (as compared to the scenario in §3.2.2) between the higher quality-threshold station and the lower quality-threshold one. In §3.2.2, the farmers prefer the station with a lower quality threshold because doing so saves cost: they can earn the high-quality price by supplying inferior quality (i.e., quality equal to the threshold). Under the bonus-based policy, however, the farmers benefit by choosing the station with the higher quality threshold. If this benefit exceeds the cost saving from choosing the lower quality-threshold station, then the farmers will instead choose the station with the higher quality threshold. In equilibrium, the stations indeed choose a large-enough bonus to enable such a reversal. In a similar manner, a bonus based on the testing-probability differential makes the farmers prefer the station with the higher testing probability. Thus, in essence, the bonus provides a way for the station with higher testing standards to protect itself in competing for the limited supply of milk.

The structure of the bonus here has two properties that make it an effective tool to incentivize quality: (i) it is relative, rather than absolute and (ii) it is based on testing standards, not outcomes. We discuss these further below.

(i) If a station attempts to attract farmers using an absolute bonus (e.g., a station simply pays each farmer for supplying milk to it), then a significant amount of bonus money must be paid in equilibrium. Thus, an absolute bonus scheme places an economic burden on the stations. Under our relative bonus structure, however, no bonus payments are paid in equilibrium.

(ii) On the other hand, rewards based upon testing outcomes can spur quality improvements only if the stations incur a sufficient amount of individual testing in equilibrium. An advantage of our bonus structure here is that it is based on testing standards, rather than testing outcomes. Under our bonus scheme, in equilibrium, no individual testing is actually conducted, yet all farmers provide high-quality milk.

As with the proposal in the previous section, the bonus-based policy also has some attractive features from an implementation viewpoint. We briefly discuss these below.

- No bonus is actually paid in equilibrium—it only serves as an effective off-equilibrium threat that prevents either station from undercutting the quality threshold or the testing probability. Thus, in equilibrium, the policy does not impose an additional burden on the stations.
- The bonus is a credible threat even in an off-equilibrium situation: since it is based on the testing-standard differential between the two stations, if one station applies a slightly higher testing standard than the other, then it only needs to pay a small bonus to the farmers.
- Individual testing is never used in equilibrium. To make the threat of high individual testing more credible in an off-equilibrium scenario, the stations could ask all the farmers who have been proven to provide inferior-quality milk to share the total (or partial) individual testing cost. Such a requirement does not change the equilibrium outcome.

5. Extensions

Our recommendations can be extended to (i) multiple stations and supply quantity heterogeneity, (ii) nonlinear production costs for the farmers, and (iii) transportation costs incurred by the farmers. We now discuss these extensions.

5.1. Multiple Stations and Supply Quantity Heterogeneity

Both our recommendations—the one based on mixed pricing in §4.1 as well as the one in §4.2 based on the testing-standard differential—can be easily extended to achieve the same socially desirable outcome when (i) there are m, $m \geq 3$, competing stations and (ii) the farmers supply different quantities of milk; i.e., the quantity supplied by farmer j is Q_j.

5.1.1. The Mixed-Pricing Recommendation of §4.1. The same governmental intervention achieves the socially desirable outcome (high quality from all farmers and one mixed test from each station) in the presence of multiple competing stations and different supply quantities, using the following obvious clarification in the notion of mixed pricing.

As before, at any station the mixed test involves drawing equal-sized samples from the milk of the farmers, mixing them, and then testing the mixed milk. A farmer who is not tested individually receives a unit price based on quality of this mixed milk. The farmers who are tested individually receive a unit price based on their individual qualities.

5.1.2. The Recommendation in §4.2 Based on the Testing-Standard Differential. The same bonus-based policy achieves the socially desirable outcome in the presence of multiple stations and supply quantity heterogeneity, using the following natural generalization of the bonus to this more general setting: the station that announces the highest quality threshold (individual testing probability) offers a per-unit bonus that is based on the differential between the highest and lowest quality thresholds (individual testing probabilities) over all the stations, to the farmers who supply milk to it.

5.2. Nonlinear Production Costs

To understand the impact of nonlinear production costs for the farmers, consider a per-unit quadratic cost form: $c(q) = cq^2$, where q is the quality of milk. Our main insight in §3.1, i.e., that competition reduces the quality of milk, continues to hold under quadratic production costs. Our mixed-pricing-based recommendation continues to achieve the socially desirable equilibrium outcome in the presence of competing stations and nonlinear production costs. Below we briefly discuss the intuition behind this result. The intuition behind the bonus-based recommendation is similar and therefore omitted for brevity.

Under linear production costs (and linear prices), the farmers' quality response is a discontinuous function of the individual testing probability: they supply high quality if this probability is above a threshold and supply low quality otherwise. In contrast, under nonlinear production costs, the farmers' quality response becomes a continuously increasing function of the individual testing probability. However, regardless of the form (linear or nonlinear) of the production cost, the quality of milk supplied by the farmers increases as the testing standard from a station increases. As long as this basic property holds, the reasoning in §4.1—the farmers prefer the station with a higher testing standard under the mixed-pricing-based scheme, thereby incentivizing the stations to increase their testing standards—remains valid. Consequently, the socially desirable equilibrium outcome is again achieved under the above nonlinear production costs.

5.3. Transportation Costs

To incorporate transportation costs for the farmers, assume that n farmers are uniformly distributed on a linear city and incur transportation costs when visiting the milk stations located at the opposite ends of the city. Assume further that transportation cost is an increasing function of distance. Under this new setting, our mixed-pricing-based recommendation (§4.1) still achieves the same socially desirable outcome. A modification of our bonus-based recommendation (§4.2) also achieves the socially desirable outcome. This modification is to have the government sponsor the bonus for the stations; the change is mild in that no bonus is actually paid in equilibrium. Therefore, this addition of governmental sponsorship of the bonus does not impose any burden on the government.

To understand the impact of transportation costs, consider, e.g., the mixed-pricing-based recommendation. Without transportation costs, all farmers prefer the station with a higher testing standard under the mixed-pricing scheme. Let us now take transportation costs into consideration. A farmer's choice of a station is now driven by the trade-off between two factors: Her preference for the station with a higher testing standard and her preference for the station that is closer. Note that the distance of a farmer from a station is fixed; thus the aforementioned second factor is a constant for each farmer. Fixing the other station's strategy, a station can attract more farmers by marginally increasing its testing standard. The presence of transportation costs only affects the magnitude of the number of farmers a station can attract by increasing its testing standard: a station attracts all the farmers (by increasing its testing standard above that of the competing station) without transportation costs, whereas it attracts only a limited number of farmers in the presence of transportation costs. Nevertheless, this limited increase in supply is enough to incentivize the station to increase its testing standard. This leads to an equilibrium where the stations apply high testing standards and the farmers supply high-quality milk in the presence of transportation costs.

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Appendix[9]

A. Poorly Resourced Stations: Impact of Competition in the Presence of Rewards for Proven High Quality

We now examine the impact of competition in the presence of a reward for high quality. Section A.1 studies a sequential game between one station and a population of farmers and §A.2 studies a similar game in the presence of two competing stations.

A.1. Monopoly

We first describe the game and then provide the equilibrium solution. The payoff structure is identical to that in §3.1.1.

Description of the Game:

- *Strategy of the Station*. The station tests an x fraction of farmers individually and offers a reward r, $0 \le r \le \bar{r}$, to each farmer who has been proven to provide high-quality (i.e., quality q_H) milk.
- *Strategies of the Farmers*. Each farmer selects a quality q to supply, $q_L \le q \le q_H$.
- *Order of Play*. First, the station announces $\{r, x\}$. Then, each farmer selects a quality q.

Equilibrium Solution: The following result states the equilibrium solution in a monopoly. The proof is similar to that of Theorem 1 and is therefore omitted for brevity.

Theorem 7. *Consider the sequential game defined above between a poorly resourced station and a population of farmers, under rewards to farmers who have been proven to supply high-quality milk. We have the following cases.*

- *If $t < w_b(q_H - q_L)Q$, then the unique equilibrium solution is $x = c_b/w_b$, $r = 0$, $q = q_H$, $f = n\{p(q_H)Q - w(q_H)Q - c_b t/w_b\}$, and $g_j = w(q_H)Q - c(q_H)Q$, $j = 1, 2, \ldots, n$.*
- *If $t > w_b(q_H - q_L)Q$, then the unique equilibrium solution is*

$$x = \frac{c_b(q_H - q_L)Q}{w_b(q_H - q_L)Q + \bar{r}}, \quad r = \bar{r}, \; q = q_H,$$

$$f = n\left\{p(q_H)Q - w(q_H)Q - \frac{c_b(q_H - q_L)Q(t + \bar{r})}{w_b(q_H - q_L)Q + \bar{r}}\right\}, \text{ and}$$

$$g_j = w(q_H)Q - c(q_H)Q + \frac{c_b(q_H - q_L)Q\bar{r}}{w_b(q_H - q_L)Q + \bar{r}}, \quad j = 1, 2, \ldots, n.$$

[9] The proofs of some of the technical results cannot be provided here due to space limitation. These proofs are available from the authors upon request.

A.2. Competition

We first describe the game and then provide the equilibrium solution. The payoff and reward structures are identical to those in §A.1.

Description of the Game:

- *Strategies of the Stations.* Station i tests an x_i fraction of farmers individually, and offers a reward r_i, $r_i \le \bar{r}$, to each farmer who has been proven to provide milk of quality q_H.
- *Strategies of the Farmers.* Each farmer selects a station and determines her quality (q_1 if station 1 is selected and q_2 if station 2 is selected) to provide.
- *Order of Play.* First, the two stations announce $\{r_1, x_1\}$ and $\{r_2, x_2\}$ simultaneously. Then, each farmer selects a station and determines her quality.

Equilibrium Solution: The following result states the equilibrium solution under competition. The proof is similar to that of Theorem 2 and is therefore omitted for brevity.

Theorem 8. *Consider the sequential game defined above between two poorly resourced stations and a population of farmers, under rewards to farmers who have been proven to provide high-quality milk. Then, the unique equilibrium solution is $x_1 = x_2 = (p(q_H)Q - w(q_H)Q)/(t + \bar{r})$, $r_1 = r_2 = \bar{r}$, $n_1 = n_2 = n/2$, $q_1 = q_2 = q_H$, $f_1 = f_2 = 0$, and $g_j = w(q_H)Q - c(q_H)Q + \{p(q_H)Q - w(q_H)Q\}\bar{r}/(t + \bar{r})$, $j = 1, 2, \ldots, n$.*

The following insight is a consequence of Theorems 7 and 8.

Insight 5. When stations are poorly resourced and a reward for high quality is offered, then we have the following cases.

- Under a monopoly, the station either chooses a high-enough testing probability (when testing cost is low) or offers a high-enough reward (when testing cost is high) to ensure that all farmers provide high-quality milk. In both cases, the station receives a positive profit in equilibrium.
- Under competition, zero profits for both stations and high-quality milk from all farmers is the unique equilibrium outcome. This is a result of the stations competing on the reward to attract more farmers (until their profits reach zero).

B. Proof of Theorem 5

We first examine the best response of the farmers and then derive the equilibrium solution.

Lemma 1 below provides the farmers' best response for a given station and Lemma 2 provides the overall best response of the farmers under competing stations.

Lemma 1. *If station i, $i = 1, 2$, applies a testing probability x_i and a threshold $\underline{q}_i$, and there are n_i farmers supplying milk to station i, then the best response of the farmers in station i is as follows:*

- *If $n_i < w_b/c_b$, then $q_i = q_H$ is the unique equilibrium outcome.*
- *If $n_i > w_b/c_b$ and $x_i > (c_b - w_b/n_i)/w_b$, then $q_i = \underline{q}_i$ is the unique equilibrium outcome.*
- *If $n_i > w_b/c_b$ and $x_i < (c_b - w_b/n_i)/w_b$, then $q_i = q_L$ is the unique equilibrium outcome.*

Lemma 2. *If station i, $i = 1, 2$, applies a testing probability x_i and a threshold $\underline{q}_i$, then the best response of the farmers is as follows:*

- *If $n < 2w_b/c_b$, then $n_1 = n_2 = n/2$ and $q_1 = q_2 = q_H$ in equilibrium.*
- *If $n > 2w_b/c_b$, then,*
  - *If $\underline{q}_1 = \underline{q}_2 = q_H$, $x_1 > (c_b - 2w_b/n)/w_b$, and $x_2 > (c_b - 2w_b/n)/w_b$, then $n_1 = n_2 = n/2$ and $q_1 = q_2 = q_H$ in equilibrium.*
  - *If station i applies $\underline{q}_i = q_H$ and $x_i > (c_b - 2w_b/n)/w_b$, and the other station $-i$ applies $\underline{q}_{-i} < q_H$ or $x_{-i} < (c_b - 2w_b/n)/w_b$, then $n_{-i} < w_b/c_b$, $n_i > n_{-i}$, and $q_i = q_{-i} = q_H$ in equilibrium.*

From Lemmas 1 and 2 and (11)–(13), it can be shown that (i) if $n < 2w_b/c_b$, then any $\{x_1, \underline{q}_1\}$ and $\{x_2, \underline{q}_2\}$ is an equilibrium outcome and (ii) if $n > 2w_b/c_b$, then any $\underline{q}_1 = \underline{q}_2 = q_H$, $x_1 > (c_b - 2w_b/n)/w_b$, and $x_2 > (c_b - 2w_b/n)/w_b$ is an equilibrium outcome. The equilibrium qualities and profits can be calculated by using Lemma 2, (11), and (13). The following result states the equilibrium solution.

Lemma 3. *Under the governmental intervention, the equilibria for the game between two well-resourced stations and a population of farmers are as follows:*

- *If $n > 2w_b/c_b$, then any strategy $\{x_i > (c_b - 2w_b/n)/w_b, \underline{q}_i = q_H\}$ is an equilibrium strategy for station i, $i = 1, 2$. In each of these equilibria, we have $n_1 = n_2 = n/2$, $q_1 = q_2 = q_H$, $f_1 = f_2 = (n/2)\{p(q_H)Q - w(q_H)Q\}$, and $g_j = w(q_H)Q - c(q_H)Q$, $j = 1, 2, \ldots, n$.*
- *If $n < 2w_b/c_b$, then any strategy $\{x_i, \underline{q}_i\}$ is an equilibrium strategy for station i, $i = 1, 2$. In each of these equilibria, we have $n_1 = n_2 = n/2$, $q_1 = q_2 = q_H$, $f_1 = f_2 = (n/2)\{p(q_H)Q - w(q_H)Q\}$, and $g_j = w(q_H)Q - c(q_H)Q$, $j = 1, 2, \ldots, n$.*

This completes the proof of Theorem 5. □

C. Proof of Theorem 6

Under competition, the highest possible profit for a station, in a symmetric equilibrium (in which, therefore, the supply is evenly split), is achieved through collecting high-quality milk from the farmers without paying for any testing or bonus. Therefore, if there exist bonus coefficients a and d such that, in equilibrium, both the stations receive this highest possible profit, then both will agree to choose such bonus coefficients. As a result, the socially desirable outcome is achieved in equilibrium. We now show that any values of a and d satisfying $a > c_b$ and $d > c_b(q_H - q_L)$ are such coefficients.

First, we calculate the per-unit bonus received by a farmer, denoted as r_i, for choosing station i. Then, we derive the best response of the farmers and the equilibrium solution under $a > c_b$ and $d > c_b(q_H - q_L)$.

Denote station $-i$ as the station other than station i, $i = 1, 2$. If station i applies strategy $S^M(x_i, \underline{q}_i)$ and station $-i$ applies strategy $S^M(x_{-i}, \underline{q}_{-i})$, then we have

$$\begin{aligned} r_i &= a(\underline{q}_i - \underline{q}_{-i}) + d(x_i - x_{-i}), && \text{if } \underline{q}_i \ge \underline{q}_{-i} \text{ and } x_i \ge x_{-i}. \\ &= a(\underline{q}_i - \underline{q}_{-i}), && \text{if } \underline{q}_i \ge \underline{q}_{-i} \text{ and } x_i < x_{-i}. \\ &= d(x_i - x_{-i}), && \text{if } \underline{q}_i < \underline{q}_{-i} \text{ and } x_i \ge x_{-i}. \\ &= 0, && \text{if } \underline{q}_i < \underline{q}_{-i} \text{ and } x_i < x_{-i}. \end{aligned} \tag{14}$$

If station i applies strategy $S^M(x_i, \underline{q}_i)$ and station $-i$ applies strategy $S^I(y_{-i})$, then we have

$$\begin{aligned} r_i &= d(x_i - y_{-i}), && \text{if } x_i \ge y_{-i}. \\ &= 0, && \text{if } x_i < y_{-i}. \end{aligned} \tag{15}$$

If station i applies strategy $S^I(y_i)$ and station $-i$ applies strategy $S^M(x_{-i}, \underline{q}_{-i})$, then we have

$$r_i = d(y_i - x_{-i}), \quad \text{if } y_i \geq x_{-i}.$$
$$= 0, \quad \text{if } y_i < x_{-i}. \tag{16}$$

If station i applies strategy $S^I(y_i)$ and station $-i$ applies strategy $S^I(y_{-i})$, then we have

$$r_i = d(y_i - y_{-i}), \quad \text{if } y_i \geq y_{-i}.$$
$$= 0, \quad \text{if } y_i < y_{-i}. \tag{17}$$

C.1. Best Response of the Farmers

Lemma 4 below provides the overall best response of the farmers under competing stations in the presence of the bonus.

Lemma 4. *When $a > c_b$ and $d > c_b(q_H - q_L)$, the best response of the farmers is as follows:*

- *If station i applies strategy $S^M(x_i = 1, \underline{q}_i = q_H)$ or strategy $S^I(y_i = 1)$, and station $-i$ applies strategy $S^M(x_{-i} = 1, \underline{q}_{-i} = q_H)$ or strategy $S^I(y_{-i} = 1)$, then $n_i = n_{-i} = n/2$ and $q_i = q_{-i} = q_H$ in equilibrium.*
- *If station i applies strategy $S^M(x_i = 1, \underline{q}_i = q_H)$ or strategy $S^I(y_i = 1)$, and station $-i$ applies a strategy other than strategy $S^M(x_{-i} = 1, \underline{q}_{-i} = q_H)$ and strategy $S^I(y_{-i} = 1)$, then $n_i = n$, $n_{-i} = 0$, and $q_i = q_H$ in equilibrium.*

Proof of Lemma 4. For brevity, we prove the result for the case when station i applies strategy $S^M(x_i = 1, \underline{q}_i = q_H)$ and station $-i$ applies strategy $S^M(x_{-i} = 1, \underline{q}_{-i} < q_H)$. The result for the other cases can be established in a similar manner.

Given $x_i = 1$, $\underline{q}_i = q_H$, $x_{-i} = 1$, and $\underline{q}_{-i} < q_H$, from (14), we know that the farmers who choose station i receive a bonus $r_i = a(q_H - \underline{q}_{-i})$ and those who choose station $-i$ receive no bonus. If a farmer chooses station i, then the best response for this farmer is to supply quality q_H, and the corresponding profit is $g_i = \{w(q_H) - c(q_H) + a(q_H - \underline{q}_{-i})\}Q$. If a farmer chooses station $-i$, then the best response for this farmer is to supply quality $\underline{q}_{-i}$, and the corresponding profit is $g_{-i} = \{w(q_H) - c(\underline{q}_{-i})\}Q$. Therefore, $g_i - g_{-i} = (a - c_b) \cdot (q_H - \underline{q}_{-i})Q$. Thus, if $a > c_b$, then $n_i = n$, $n_{-i} = 0$, and $q_i = q_H$ in equilibrium. In words, all farmers sell milk of quality q_H to station i. The best response of the farmers under the other cases can be derived in a similar manner and therefore is omitted. □

C.2. Equilibrium Solution

The result below states the equilibrium solution when $a > c_b$ and $d > c_b(q_H - q_L)$.

Lemma 5. *When $a > c_b$ and $d > c_b(q_H - q_L)$, the unique equilibrium solution is as follows: station i, $i = 1, 2$, applies strategy $S^M(x_i = 1, \underline{q}_i = q_H)$, $n_1 = n_2 = n/2$, $q_1 = q_2 = q_H$, $f_1 = f_2 = (n/2)\{p(q_H) - w(q_H)\}Q$, and $g_j = \{w(q_H) - c(q_H)\}Q$, $j = 1, 2, \ldots, n$.*

Proof of Lemma 5. By comparing all possible strategies (i.e., strategy $S^M(x, \underline{q})$ with any $x \in [0, 1]$ and $\underline{q} \in [q_L, q_H]$ and strategy $S^I(y)$ with any $y \in [0, 1]$) for the stations, it can be shown that, in equilibrium, the strategy used by the stations must be one of the following two: (i) strategy $S^M(x = 1, \underline{q} = q_H)$ and (ii) strategy $S^M(x = 1, \underline{q} = q_L)$.

We first show that, if $a > c_b$ and $d > c_b(q_H - q_L)$, then strategy $S^M(x_i = 1, \underline{q}_i = q_H)$, $i = 1, 2$, is an equilibrium outcome. Consider the case when station i applies strategy $S^M(x_i = 1, \underline{q}_i = q_H)$. If station $-i$ applies strategy $S^M(x_{-i} = 1, \underline{q}_{-i} = q_H)$, then from Lemma 4, $a > c_b$, and $d > c_b(q_H - q_L)$, we have $n_i = n_{-i} = n/2$ and $q_i = q_{-i} = q_H$. Thus, the profit of station $-i$ is

$$f_{-i} = \frac{n}{2}\{p(q_H) - w(q_H)\}Q. \tag{18}$$

If station $-i$ applies strategy $S^I(y_{-i} = 1)$, then from Lemma 4, $a > c_b$, and $d > c_b(q_H - q_L)$, we have $n_i = n_{-i} = n/2$ and $q_i = q_{-i} = q_H$. Thus, the profit of station $-i$ is

$$f_{-i} = \frac{n}{2}\{p(q_H) - w(q_H) - t\}Q. \tag{19}$$

If station $-i$ applies a strategy other than strategy $S^M(x_{-i} = 1, \underline{q}_{-i} = q_H)$ and strategy $S^I(y_{-i} = 1)$, then from Lemma 4, $a > c_b$, and $d > c_b(q_H - q_L)$, we have $n_i = n$, $n_{-i} = 0$, and $q_i = q_H$. Thus, the profit of station $-i$ is

$$f_{-i} = 0. \tag{20}$$

Comparing (18)–(20), we know that strategy $S^M(x_i = 1, \underline{q}_i = q_H)$, $i = 1, 2$, is an equilibrium outcome when $a > c_b$ and $d > c_b(q_H - q_L)$.

Using a similar argument, it can be shown that, if $a > c_b$ and $d > c_b(q_H - q_L)$, then strategy $S^M(x_i = 1, \underline{q}_i = q_L)$, $i = 1, 2$, is not an equilibrium outcome. Thus, when $a > c_b$ and $d > c_b(q_H - q_L)$, the unique equilibrium is as follows: station i, $i = 1, 2$, applies strategy $S^M(x_i = 1, \underline{q}_i = q_H)$, $n_1 = n_2 = n/2$, $q_1 = q_2 = q_H$, $f_1 = f_2 = (n/2)\{p(q_H) - w(q_H)\}Q$, and $g_j = \{w(q_H) - c(q_H)\}Q$, $j = 1, 2, \ldots, n$. This completes the proof of Lemma 5. □

This completes the proof of Theorem 6. □

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