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On the Effectiveness of Uniform Subsidies in Increasing Market Consumption

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Abstract. We study the problem faced by a central planner trying to increase the consumption of a good, such as new malaria drugs in Africa. The central planner allocates subsidies to its producers, subject to a budget constraint and endogenous market response. The policy most commonly implemented in practical applications of this problem is *uniform*, in the sense that it allocates the same per-unit subsidy to every firm, primarily because of its simplicity and perceived fairness. Surprisingly, we identify sufficient conditions of the firms' marginal costs such that uniform subsidies are optimal, even if the firms' efficiency levels are arbitrarily different. Moreover, this insight is usually preserved even if the central planner is uncertain about the specific market conditions. Further in many cases, uniform subsidies simultaneously attain the best social welfare solution. Additionally, simulation results in relevant settings where uniform subsidies are not optimal suggest that they induce a nearly optimal market consumption. On the other hand, if the firms face a fixed cost of entry to the market, then the performance of uniform subsidies can be significantly worse, suggesting the need for an alternative policy in this setup.

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1. Introduction

In this work, we study the important setting in which a central planner aims to impact a given market. Specifically, the central planner's goal is to increase the aggregate market consumption of a good by providing copayment subsidies, which are paid for each unit that is produced to competing (profit-maximizing) heterogeneous firms. The motivation to provide such subsidies stems from the positive societal externalities that can be obtained by increasing the market consumption and from the fact that, left alone, the resulting market equilibrium induced by the selfish competing producers might not be socially optimal. A current example is the recent efforts around the production of infectious disease treatments to the developing world, such as antimalarial drugs (e.g., Arrow et al. 2004), and vaccines (e.g., Snyder et al. 2011).

Furthermore, typically the central planner makes its subsidy allocation in the presence of a budget constraint, which is often determined before the actual subsidy allocation decision. For example, in some cases the central planner could be a foundation that raised a certain amount of money to address a related issue, and it is then facing the challenge of how to allocate the budget toward copayment subsidies. Another challenge typically faced by the central planner is that the intervention in the market through the allocation of subsidies will likely change the market equilibrium induced by the competing producers. Hence, to optimally allocate the subsidies, the central planner has to take into account these complex dynamics.

In this paper, we propose a novel modeling framework to study strategic and operational issues related to copayment subsidies allocation. The models that we develop explicitly capture the setting of a central planner aiming to maximize the market consumption of a good, in the presence of a budget constraint, and market competition between heterogeneous profit-maximizing firms. The firms are heterogeneous in terms of their respective efficiency and cost structure. This is modeled through firm-specific marginal cost functions. The models that we develop fall into the class of mathematical programs with equilibrium constraints (MPECs). They are relatively general and capture different cost structures and inverse demand functions, as well as a range of market dynamics of quantity competition typical of the settings being studied. For example, the models capture as special cases Cournot competition with linear demand, as well as Cournot competition under yield uncertainty with linear demand and linear marginal cost functions. MPEC models are typically computationally challenging, both to solve optimally and to analyze; see, e.g., Luo et al. (1996). However, by reformulating these problems, we are able to develop tractable mathematical programs that provide upper bounds on the optimal objective value and allow the development of efficient algorithms. Even more importantly, they allow analysis of the effectiveness of practical policies. In particular, this paper focuses attention on the effectiveness of commonly used *uniform copayments*, in which the per-unit copayment is the same for all competing firms in the market. The common use of uniform copayments, in spite of the existence of heterogeneous firms, each with potentially different efficiency levels, is primarily driven by the simplicity of implementation, as well as some notion of fairness. This paper addresses the important question of to what extent uniform copayments are effective in increasing market consumption, compared to potentially more sophisticated policies that could allow the copayment to be firm specific. Through the mathematical programming upper bound relaxation that we develop, this paper provides some surprising insights. First, we can show that, for a large class of firm-specific cost structures, uniform copayments are in fact *optimal*. That is, there is no loss of efficiency in using uniform copayments in these settings compared to any other possible copayment allocation. Second, this insight is maintained even if one considers the case in which there exists uncertainty about the future market state, and the central planner has to set up the subsidies before the realization of the market condition. Third, in many cases uniform subsidies do not only obtain the optimal (maximal) market consumption but at the same time obtain the best social welfare solution. Fourth, in other relevant settings where uniform subsidies are not optimal, extensive computational experiments suggest that they still perform, on average, very close to optimal. Finally, we also identify conditions where the performance of uniform subsidies is not as good.

To demonstrate the applicability of the model and the relevance of the issues studied in the paper, we next discuss in detail the case of antimalarial drugs.

1.1. Application: Global Subsidy for Antimalarial Drugs

A motivating example, where the setting modeled in this paper is observed in practice, is the global fight against malaria. This has been a long-standing challenge for the healthcare industry. It is estimated that in 2012 approximately 200 million cases occurred worldwide and more than 600,000 people died of malaria; see World Health Organization (2013). To make matters worse, recently chloroquine, the traditional drug for treating malaria, has become less effective because of growing resistance to this medication. Artemisinin combination therapies (ACTs) have been identified as the successor drugs to chloroquine for the treatment of malaria; however, they are at least 10 times more costly; see White (2008).

In 2004 the Institute of Medicine (IoM) reviewed the economics of antimalarial drugs. It identified that several manufacturers compete in an unregulated market and concluded that the most effective way of ensuring access to ACTs for the greatest number of patients would be to provide a centralized subsidy to the producers. The goal would be achieving a high overall coverage of ACTs. Moreover, the IoM recognized that firms had not invested in producing ACTs on the scale needed to supply Africa because there had been no assured market; therefore, the global capacity to produce ACTs was quite limited; see Arrow et al. (2004).

In this context, in 2007 the Roll Back Malaria Partnership and the World Bank developed the Affordable Medicines Facility for malaria (AMFm), a concrete initiative to improve access to safe, effective, and affordable antimalarial medicines. In 2008, the Global Fund started hosting the AMFm, which began operations in July 2010. By July 2012, the AMFm had managed a budget of US$336 million—pledged by UNITAID, the governments of the United Kingdom and Canada, and the Bill & Melinda Gates Foundation—to pursue its main objective: increasing the consumption of ACTs, as detailed in the evaluation report by the AMFm Independent Evaluation Team (2012).

The policy proposed by AMFm is consistent with the common practice of giving a uniform copayment. Specifically, each firm receives the same copayment for each unit sold, regardless of their individual characteristics. Moreover, there are 11 firms participating in the AMFm program, ranging from large pharmaceuticals like Novartis (having manufacturing plants in the United States and China) and Sanofi (having manufacturing plants in Germany and Morocco) to smaller firms with manufacturing plants in Uganda, India, and Korea (for more details see A2S2 2014, the market intelligence aggregator funded by UNITAID). Note that the firms receiving the uniform copayments are highly heterogeneous, both in their market size and location-wise. One additional relevant characteristic of the AMFm program is that all the ACT manufacturers that receive copayments commit to supply antimalarial drugs on a no-profit/no-loss basis; see the report by Boulton (2011). Giving the right incentives to anti-malarials producers can increase access to them, hence, it has the potential to have a significant impact on this global problem, see Arrow et al. (2004).

1.1.1. Results and Contributions. The main contributions of this paper are the following.

New Modeling Framework for a Subsidy Allocation Problem. We introduce a general optimization framework to analyze subsidy allocation problems with endogenous market response, under a budget constraint on the total amount of subsidies the central planner can pay. The central planner's objective is to maximize the market consumption of a good. Our models allow general inverse demand and marginal cost functions, assuming only that the inverse demand function is decreasing in the market consumption and that the firms' marginal costs are increasing. These are standard assumptions in the literature, and even more general than assumptions usually considered.

Sufficient Conditions for the Optimality of Uniform Copayments. We compare uniform copayments to the optimal, and potentially differentiated, copayment allocation, which provides more flexibility but is potentially harder to implement. The main result of this paper shows that uniform copayments are in fact optimal for a large family of marginal cost functions. This result is surprising, considering that firms are heterogeneous, and particularly since the assumptions on the inverse demand function are very general (essentially only monotonicity and continuity). More importantly, it establishes sufficient conditions such that the policy that is frequently being used in practice is actually optimal. Additionally, we provide sufficient conditions for uniform copayments to simultaneously maximize the social welfare.

Incorporation of Market State Uncertainty. We extend the models by assuming that the central planner does not know the exact market state with certainty (i.e., the specific inverse demand function is uncertain), but it has a set of possible scenarios and beliefs on the likelihood that each scenario will materialize. We model this setting as a stochastic MPEC, where the central planner decides its copayment allocation policy with the objective of maximizing the expected market consumption. This model is considerably harder to analyze; see Patriksson and Wynter (1999). However, we show that uniform copayments are still optimal in this setting, for a fairly large family of firms' marginal cost functions. Moreover, the analysis suggests that the central planner only needs to consider the scenario with the highest market consumption at equilibrium, regardless of the exact distribution over the different market states.

Tractable Upper Bound Problems (UBPs). Based on an innovative mathematical programming reformulation of our model, we develop tractable upper bound problems. These are used extensively in the analysis mentioned above. In addition, we use them to conduct a numerical study of the performance of uniform copayments in relevant settings where they are not optimal. Specifically, we consider Cournot competition with linear demand and constant marginal costs, as well as more general settings with nonlinear demand, and nonlinear marginal cost. The results obtained on data generated at random suggest that the market consumption induced by uniform subsidies is on average very close to optimal. We believe that the innovative reformulation of the model, and the resulting upper bounds, would be useful in the study of additional important research questions.

Limitations of Uniform Copayments. We identify setups where the performance of uniform subsidies is not as good. The most important one of those is the case where the firms face a fixed cost of entry to the market. In this case, we find that uniform subsidies are not optimal even in the simple setup where firms have linear marginal costs and face a linear demand. Specifically, their relative performance in maximizing market consumption can be as low as 63%. This suggests the need for an alternative policy in this setup.

2. Literature Review

The subject of taxes and subsidy allocation and incidence has a vast literature in the economics community. Fullerton and Metcalf (2002) present a thorough review of classical and recent results in this area. This paper is closely related to the study of subsidies in imperfect competition models. However, the traditional approach in this literature assumes homogeneous firms and focuses on studying the impact of taxes, or subsidies, on the number of firms participating in the market in a symmetric equilibrium, because it is directly related to the ability to pass taxes forward to the consumer; see Fullerton and Metcalf (2002). More generally, when doing comparative statics analysis in oligopoly models, it is fairly common to focus on symmetric equilibria with homogeneous firms in order to obtain more precise insights; see, e.g., Vives (2001). In contrast, in our model we take an operational view: we assume heterogeneous firms that produce a commodity, and we focus on the specific subsidy allocation among them. Additionally, we consider a budget constraint in the total amount of funding that can be allocated to these subsidies.

The strategic trade policy literature, particularly the "third market model," studies a related problem; see Brander (1995). In this model, n home firms and n^* foreign firms export a commodity to a third market, where the market price is set through Cournot competition with constant marginal costs. The government can allocate subsidies to the home firms, increasing their profit at the expense of the foreign competitors. The government's utility is equal to the profit earned by the home firms, minus the cost of the subsidy payments. We focus here on the case with heterogeneous firms. In this setting, Collie (1993) and Long and Soubeyran (1997) assume a uniform subsidy and study its effect in the market shares of the firms. Later, Leahy and Montagna (2001) assume a linear demand function and derive closed-form expressions for the optimal subsidies. They conclude that the optimal subsidy policy is generally not uniform, and that the government should allocate higher subsidies to more efficient firms. This result is consistent with our numerical study in §7, where uniform subsidies are not optimal for Cournot competition with linear demand and constant marginal costs. Nonetheless, we find computational evidence that the relative performance of uniform subsidies is very good. In contrast, our model assumes more general increasing marginal costs, and the analysis obtains conditions under which uniform subsidies are optimal.

More recently, Cohen et al. (2014) show in a randomized controlled trial in Kenya that a high subsidy for ACT antimalarial drugs dramatically increases access to them. This is an important empirical insight indicating that subsidies for ACTs, as the one considered as a motivation in §1.1, are actually effective in practice. Similarly, Dupas (2014) also show that short-run subsidies for an antimalarial bed net had a positive impact on the willingness to pay for it a year later. This result suggests that short-run subsidy programs, such as the one also considered toincrease the consumption of ACTs, are expected to be beneficial in the long run as well.

MPECs are very hard to solve and analyze. In particular, even the simplest case with linear objective and linear constraints is NP-hard; see Luo et al. (1996). Moreover, stochastic mathematical programs with equilibrium constraints (SMPECs) can be even harder to solve in practice; see Patriksson and Wynter (1999). In this context, the best that we can hope for is to identify interesting structure in particular cases that might lead to structural or algorithmic results. The copayment allocation problem (CAP) in §§3 and 4 and its variant with market uncertainty (stochastic copayment allocation problem (SCAP)) in §5 are particular cases of an MPEC and an SMPEC, respectively. In both cases, our main methodological contribution is the identification of a special structure that allows us to prove surprising structural results, such as the optimality of uniform subsidies. Examples in the literature that study similar models include DeMiguel and Xu (2009) and Adida and DeMiguel (2011). Recently, Correa et al. (2014) find sufficient conditions for the existence of markup equilibria for marginal cost functions very similar to the ones where uniform copayments are optimal in our model. On the other hand, the problem of controlling and reducing the contagion of infectious diseases has been studied in the operations management literature as mainly focusing on the analysis of a vaccine's markets, particularly the influenza vaccine, its supply chain coordination (e.g., Chick et al. 2008, Mamani et al. 2012), and the market competition under yield uncertainty (e.g., Deo and Corbett 2009, Arifoglu et al. 2012), as opposed to our interest in subsidy allocation. In particular, we consider the case of allocating subsidies to Cournot competitors under yield uncertainty, showing that if the demand and the marginal costs are linear, then uniform copayments are optimal.

The problem of allocating subsidies to increase the market consumption of new antimalarial drugs is also studied by Taylor and Xiao (2014). However, they consider the case of one manufacturer selling to multiple heterogeneous retailers facing stochastic demand. Their analysis focuses on the placement of the subsidy by the central planner in the supply chain, comparing the possibility of subsidizing either sales or purchases (from the retailer's point of view). They conclude that the central planner should only subsidize purchases, which is equivalent to subsidizing the manufacturer in our setting. Furthermore, they characterize the order up to the level of the retailers, which is decreasing in the wholesale price. Therefore, this model can be characterized by an arbitrary decreasing inverse demand function from the manufacturer's point of view. Our paper complements this work by focusing on the effectiveness of uniform subsidies. The combined message of these two papers to the policy makers is that not only allocating copayments to manufacturers makes sense as a strategy to maximize the market consumption but, even if the manufacturers are heterogeneous, the very simple and practical policy of allocating the same copayments to each firm will most likely obtain most of the potential benefits. Another growing stream of work in the operations management literature is one that studies the problem of a central planner deciding rebates that are directed to the consumers, with the goal of incentivizing the adoption of green technology (see Aydin and Porteus 2009, Lobel and Perakis 2012, Krass et al. 2013, Cohen et al. 2016). In contrast to that stream of work, this paper is motivated by a different set of practical applications and focuses on copayments that are allocated to the producers.

3. Model

In this section, we introduce a mathematical programming formulation of the subsidy allocation problem. We then use this formulation to obtain a relaxation of the problem, which provides an upper bound on the largest market consumption that can be induced with the available budget.

We consider a market for a commodity composed by $n \geq 2$ heterogeneous competing firms. Each firm $i \in \{1, \ldots, n\}$ decides its output q_i independently, with the goal of maximizing its own profit. We assume that the introduction of subsidies in the market will induce an increase in the market consumption, and that the firms do not have the installed capacity to provide all of it. This implies that capacity is scarce in the market. We model this effect by assuming that the marginal cost of each firm is increasing. Specifically, we assume that the firms have a firm-specific, nonnegative, increasing, and differentiable marginal cost function on their production quantity, denoted by $h_i(q_i)$. Consumers are described by an inverse demand function $P(Q)$, where $Q \equiv \sum_{i=1}^{n} q_i$ is the market consumption. We assume that $P(Q)$ is nonnegative, decreasing, and differentiable in $[0, \bar{Q}]$, where $\bar{Q}$ is the smallest value such that $P(\bar{Q}) = 0$.

The assumption on the market equilibrium dynamics is that each firm participating in the market equilibrium produces up to the point where its marginal cost equals the market price; firms that do not participate in the market equilibrium must have a marginal cost of producing any positive amount that is larger than the market price. This can be expressed in the following condition:

$$\text{For each } i, j, \quad \text{if } q_i > 0, \quad \text{then} \quad h_i(q_i) = P(Q) \leq h_j(q_j). \tag{1}$$

Assuming a decreasing inverse demand function, and increasing firms' marginal cost functions, ensure that there exists a unique market equilibrium; see Marcotte and Patriksson (2006). Some special cases of the market equilibrium condition (1) include imperfect market competition models, such as Cournot competition with linear demand and Cournot competition under yield uncertainty, with linear demand and linear marginal costs. Another special case of condition (1) is the model where firms act as price takers and compete in quantity, for any decreasing inverse demand function. In particular, the latter special case captures the behavior of ACT manufacturers discussed in the motivation in §1.1, where all the firms receiving copayments operate on a no-profit/no-loss basis. More generally, the price-taking assumption is a reasonable approximation whenever the firms in the market have little market power, e.g., when there are many firms competing in the market or when firms face a threat of entry to the market; see Tirole (1988).

The generality of an arbitrary decreasing inverse demand function allows the modeling of complex demand mechanisms that have been considered in the operations management literature. One such example is the case of multiple competing retailers under demand uncertainty. Specifically, Bernstein and Federgruen (2005) have shown that, in a model where each retailer chooses its retail price and its order quantity and faces multiplicative random demand (the distribution of which may depend on its own retail price as well as those of the other retailers), there exists a unique Nash equilibrium in which all the retailer prices decrease when the wholesale price is reduced. Moreover, under additional mild assumptions, this leads to each equilibrium order quantity decreasing in the wholesale price, resulting in a decreasing inverse demand function. In other words, the general assumptions in our model about the inverse demand function could potentially capture relatively complex models of how the price to the final consumer is reduced when the central planner allocates copayments to the manufacturers. Specifically, as long as the model induces a demand to the suppliers that is decreasing in the wholesale price, then it can be considered as a special case of a general decreasing demand function; therefore our results apply.

3.1. Copayment Allocation Problem

We will refer to the problem faced by the central planner as the copayment allocation problem, and we will denote its mathematical programming formulation by (CAP). Problem (CAP) is a particular case of a Stackelberg game, or a bilevel optimization problem. In the first stage, the central planner allocates a given budget $B > 0$, in the form of copayments $y_i \geq 0$, to each firm $i \in \{1, \ldots, n\}$, per each unit provided in the market. Moreover, the central planner anticipates that, in the second stage, the equilibrium output of each firm will satisfy a modified version of the equilibrium condition. The difference in the market equilibrium condition is given by the fact that, from firm i's perspective, the effective price for each unit sold is now $P(Q) + y_i$ or, equivalently, that its marginal cost is reduced by y_i.

The central planner's objective is to maximize the market consumption. Note that, in the setting we study, this is equivalent to maximizing the consumer surplus, which is equal to $\int_0^Q P(x)\,dx - P(Q)Q$. Specifically, the derivative on the consumer surplus with respect to the equilibrium market consumption is $-P'(Q)Q > 0$, which is positive for any decreasing inverse demand function. This is the appropriate objective in many applications, where the central planner is a supranational authority, like the World Bank, whose main interest is effectively maximizing the market consumption, say, of an infectious disease treatment, without explicitly taking into account the additional surplus obtained by local producers (see Arrow et al. 2004 for further discussion on this topic). Additionally, in §6, we will analyze the case where the central planner's objective is to maximize social welfare, including both the consumer and the producer surplus.

Finally, let us emphasize that the central planner can only allocate copayments and can never charge taxes for the units produced in the market. In other words, the copayments being allocated have to be nonnegative. Next is a formulation of the copayment allocation problem:

$$\max_{\mathbf{y}, \mathbf{q}, Q} \quad Q$$

$$\text{s.t.} \quad \sum_{i=1}^{n} q_i y_i \leq B \tag{2}$$

$$y_i \geq 0, \quad \text{for each } i \in \{1, \ldots, n\}; \tag{3}$$
$$\sum_{i=1}^{n} q_i = Q; \tag{4}$$
$$q_i \geq 0, \quad \text{for each } i \in \{1, \ldots, n\}; \tag{5}$$
$$P(Q) + y_i = h_i(q_i), \quad \text{for each } i \in \{1, \ldots, n\}. \tag{6}$$

This is a valid formulation even if there are firms that have a positive marginal cost of producing any positive amount, which prevents them from participating in the market equilibrium. Namely, if for some firm i we have $h_i(0) \geq P(Q)$, then we can just set $q_i = 0$ and $y_i = h_i(0) - P(Q) \geq 0$. This is without loss of generality, because setting $q_i = 0$ ensures that firm i does not have any impact on the budget constraint (2), and the nonnegativity constraint on the copayment y_i ensures that the market equilibrium condition is satisfied. In other words, constraint (6) does not imply that every firm has to participate in the market equilibrium.

From the equilibrium condition given in constraint (6), it follows that we can replace all the copayment variables y_i by $h_i(q_i) - P(Q)$. Namely, we can reformulate the copayment allocation problem as if the central planner were deciding the output of each firm, as long as there exist feasible copayments that can sustain the outputs chosen as the market equilibrium. The feasibility of the copayments will be given by both the budget constraint (2) and the nonnegativity of the copayments (3). We summarize this observation in the following proposition.

Proposition 1. *The copayment allocation problem faced by the central planner can be formulated as follows*:

$$\text{(CAP)} \quad \max_{\mathbf{q}, Q} \; Q$$
$$\text{s.t.} \quad \sum_{j=1}^{n} q_j h_j(q_j) - P(Q)Q \leq B \tag{7}$$
$$h_i(q_i) \geq P(Q), \quad \textit{for each } i \in \{1, \ldots, n\}; \tag{8}$$
$$\sum_{j=1}^{n} q_j = Q; \tag{9}$$
$$q_i \geq 0, \quad \textit{for each } i \in \{1, \ldots, n\}. \tag{10}$$

The copayments that the central planner must allocate to induce outputs $\mathbf{q}$ *are* $y_i(\mathbf{q}) = h_i(q_i) - P(Q)$, *for each* i.

Constraint (7) is equivalent to budget constraint (2). Note that it has a budget balance interpretation; i.e., the total cost in the market minus the total revenue in the market has to be less or equal than the budget introduced by the central planner. Constraint (8) is equivalent to the nonnegativity of the copayments (3).

3.2. Special Cases

Our model is fairly general. In particular, in this section we discuss some well-known imperfect competition models that are captured as special cases.

3.2.1. Cournot Competition with Linear Demand. The classical oligopoly model proposed by Cournot is defined in a very similar setting. The only difference is that, given all the other firms' production levels, each firm sets its output q_i at a level such that it maximizes its profit Π_i, where $\Pi_i = P(Q)q_i - \int_0^{q_i} h_i(x_i)\,dx_i$. If we assume that $P(Q)$ is decreasing and $h_i(q_i)$ are increasing, for each i, as well as $P'(Q) + q_i P''(Q) \leq 0$, then there exists a unique market equilibrium defined by the solution to the first-order conditions of the firms' profit maximization problem; see Vives (2001). Namely, at equilibrium, each firm sets its output at a level such that

$$\text{For each } i, \quad \text{if } q_i > 0, \quad \text{then} \quad \frac{\partial \Pi_i}{\partial q_i} = 0,$$
$$\text{or equivalently,} \quad P(Q) = h_i(q_i) - P'(Q)q_i. \tag{11}$$

In the equilibrium condition (11), the marginal cost must be equal to the marginal revenue, whereas, in the equilibrium condition (1), the marginal cost must be equal to the market price. Moreover, the term $P'(Q)q_i$ is not independent for each firm.

Now, for the commonly assumed special case where the inverse demand function is linear, i.e., $P(Q) = a - bQ$, it follows that $P'(Q) = -b$. Define $\tilde{h}_i(q_i) \equiv h_i(q_i) + bq_i$, for each i; then we can rewrite the equilibrium condition as follows. For each i, if $q_i > 0$ then $P(Q) = \tilde{h}_i(q_i)$. This equilibrium condition is a special case of condition (1) but for a modified cost function $\tilde{h}_i(q_i)$.

3.2.2. Cournot Competition Under Yield Uncertainty with Linear Demand and Linear Marginal Costs. We consider the Cournot competition under yield uncertainty model proposed in Deo and Corbett (2009). We assume that each firm $i \in \{1, \ldots, n\}$ decides its production target $\bar{q}_i$, although the actual output is uncertain and given by $q_i = \alpha_i \bar{q}_i$, where α_i is a random variable reflecting the random yield for firm i. We assume that the random variables α_i are identically and independently distributed (i.i.d.) for all firms, with $\mathbb{E}[\alpha_i] = \mu$, and $\text{Var}[\alpha_i] = \sigma^2$. Additionally, we assume a linear inverse demand function $P(Q) = a - bQ$, where $Q = \sum_{i=1}^{n} q_i$ is again the market consumption.

We consider two marginal costs: (i) $\bar{h}(\bar{q}_i)$ per unit of production target and (ii) $h(q_i)$ per unit actually produced. The first cost is driven by the amount of raw materials needed for production, whereas the second cost corresponds to the cost of packaging the actual output. Finally, we assume Cournot competition among the firms. Namely, given the production target of all the other firms, each firm sets its production target $\bar{q}_i$ to the level that maximizes its expected profit. We generalize the model used in Deo and Corbett (2009) in two ways. First, we consider heterogeneous firms whereas Deo and Corbett consider homogeneous firms. Second, Deo and Corbett assume a constant marginal cost function and a fixed cost to enter the market, whereas we assume more general marginal cost functions. Moreover, we extend the model to include a central planner allocating subsidies to the competing firms, anticipating the market reaction to the subsidy allocation, and facing a budget constraint. To do so, we assume that both marginal cost functions are linear. Namely, we assume that $\bar{h}(\bar{q}_i) = \bar{g}_i\bar{q}_i$ and $h(q_i) = g_i q_i$. Note that we consider heterogeneous firms, where some of them may be more efficient than the others, depending on the values of the firm-specific parameters $\bar{g}_i$ and g_i.

Let us start by considering the second-stage problem. Assume that the central planner allocates a copayment $y_i \geq 0$ to each firm $i \in \{1, \ldots, n\}$. Each firm sets its production target $\bar{q}_i$ to the level that maximizes its expected profit, given by

$$\mathbb{E}\left[P(Q)q_i + y_i q_i - \int_0^{\bar{q}_i} \bar{h}(x_i)\,dx_i - \int_0^{q_i} h(x_i)\,dx_i\right]$$
$$= \mathbb{E}\left[\left(a - b\sum_{i=1}^n \alpha_i\bar{q}_i\right)\alpha_i\bar{q}_i + y_i\alpha_i\bar{q}_i - \bar{g}_i\frac{\bar{q}_i^2}{2} - g_i\frac{\alpha_i^2\bar{q}_i^2}{2}\right].$$

The expectation is taken with respect to the random variables α_i. This is a concave maximization problem in $\bar{q}_i$; therefore, the first-order condition is sufficient for optimality. To write the first-order condition in a compact form, define $\tilde{g}_i \equiv \bar{g}_i/\mu + ((\sigma^2+\mu^2)/\mu)g_i + b\mu + 2b(\sigma^2/\mu)$, $\tilde{h}_i(\bar{q}_i) \equiv \tilde{g}_i\bar{q}_i$, $\bar{Q} = \sum_{j=1}^n \bar{q}_j$, $\bar{P}(\bar{Q}) = a - \mu b\bar{Q}$. Additionally, note that the expected market price has the following closed-form expression, $\mathbb{E}[P(Q)] = a - \mu b\sum_{i=1}^n \bar{q}_i = a - \mu b\bar{Q} = \bar{P}(\bar{Q})$. Hence, we can write the first-order condition of the firms' profit-maximization problem as $\bar{P}(\bar{Q}) = \tilde{h}_i(\bar{q}_i) - y_i$.

To define the copayment allocation problem in this setting, it remains for us to address how the yield uncertainty will be considered in the budget constraint. We consider two possible approaches that will lead to optimization problems with similar structure. First, assume that the central planner would like to find a copayment allocation, such that it satisfies the budget constraint in expectation; then we can write the budget constraint as $\mathbb{E}[\sum_{i=1}^n q_i y_i] = \mu\sum_{i=1}^n \bar{q}_i y_i \leq B$. Alternatively, assume that the central planner takes a robust approach. Namely, the central planner would like to satisfy the budget constraint in each possible yield uncertainty realization. We will assume, for simplicity, that the i.i.d. random yields for each firm have a bounded support; i.e., $\alpha_i \in [\underline{\alpha}, \bar{\alpha}]$, for each i. Then, we can write the budget constraint as $\bar{\alpha}\sum_{i=1}^n \bar{q}_i y_i \leq B$.

Finally, noting that the objective that the central planner is trying to maximize is $\mathbb{E}[Q] = \mu\bar{Q}$, we conclude that this is a special case of problem (CAP) with variables $(\bar{\mathbf{q}}, \bar{Q})$ and functions $\tilde{h}_i(\bar{q}_i) = \tilde{g}_i\bar{q}_i$, $\bar{P}(\bar{Q}) = a - \mu b\bar{Q}$.

3.3. An Upper Bound Problem

Note that under our assumptions, problem (CAP) is not necessarily a convex optimization problem. In fact, we have only assumed that the marginal cost functions $h_i(q_i)$ are increasing, for each i, and that the inverse demand function $P(Q)$ is decreasing. To gain some insights into the structure of the optimal solution, we ignore the nonnegativity of the copayments and analyze the following relaxation, which provides an upper bound on the market consumption that can be induced with the available budget B.

$$\text{(UBP)} \quad \max_{\mathbf{q}, Q} \; Q$$
$$\text{s.t.} \quad \sum_{j=1}^n q_j h_j(q_j) - P(Q)Q \leq B \tag{12}$$
$$\sum_{j=1}^n q_j = Q; \tag{13}$$
$$q_i \geq 0, \quad \text{for each } i \in \{1, \ldots, n\}. \tag{14}$$

This upper bound problem may still be nonconvex, because of the budget constraint (12). However, Lemma 1 below asserts that at optimality the budget constraint is tight, and each of the active firms i must have a value of $(h_i(q_i)q_i)'$ equal to each other, and no larger than any inactive firm. This property will have a central role in proving the main result of the next section.

Lemma 1. *Assume that the marginal cost functions $h_i(q_i)$ are nonnegative and continuously differentiable in $[0, \bar{Q})$ and that the inverse demand function $P(Q)$ is nonnegative, decreasing, and differentiable in $[0, \bar{Q}]$. Then, any optimal solution to the upper bound problem* (UBP) *must satisfy the budget constraint* (12) *with equality and also satisfy the following condition*:

$$\textit{If } q_i > 0, \textit{ then} \quad (h_i(q_i)q_i)' \leq (h_j(q_j)q_j)', \quad \textit{for each } i, j \in \{1, \ldots, n\}.$$

The proof of Lemma 1 is in the online appendix (available as supplemental material at https://doi.org/10.1287/mnsc.2015.2329). Note that the assumption that the marginal cost functions are increasing is not necessary for Lemma 1 to hold.

4. Optimality of Uniform Copayments

The result obtained in this section asserts that uniform copayments are optimal for problem (CAP), for a large class of marginal cost functions $h_i(q_i)$. Specifically, we show that if the marginal cost functions satisfy Property 1 below, then uniform subsidies are optimal.

Property 1. *For each i, j and each $q_i, q_j \geq 0$, if $h_i(q_i) > h_j(q_j)$, then $(h_i(q_i)q_i)' \neq (h_j(q_j)q_j)'$.*

Next, we show that there exists a large class of marginal cost functions that satisfy Property 1. Consider the case in which $h_i(q_i) = h(g_i q_i)$, where $h(x)$ is nonnegative, increasing, and differentiable over $x \geq 0$ and $g_i > 0$ is a firm-specific parameter. This captures the setting where all firms use a similar technology but can differ in their efficiency. Specifically, $h(x)$ models the industry-specific marginal cost function, whereas $g_i > 0$ models the efficiency of firm i.

In this setting there is no loss of generality in assuming that $h(0) = 0$. Specifically, any positive value for $h(0)$ will affect each firm in the same way; therefore, it will only shift the market price by a constant that can be rescaled to zero. This assumption implies that all firms have a positive output in the market equilibrium, for any positive market price. Therefore, the underlying assumption in order to show the optimality of uniform subsidies is that all firms have already entered the market before the subsidy is decided, and there is no subsequent entry into or exit of firms from the market. This assumption is reasonable in our setting, where the subsidy is not permanent (it only applies until the budget is exhausted), and it is paid ex post to the firms, for each unit already sold.

In this setting, any function $h(x)$ such that $h(x) + h'(x)x$ is monotonic will satisfy Property 1. Specifically, for each such function we would have that $h_i(q_i) > h_j(q_j)$ is equivalent, by definition, to $h(g_i q_i) > h(g_j q_j)$. However, $h(x)$ increasing implies $g_i q_i > g_j q_j$. Moreover, $h(x) + h'(x)x$ monotonic implies $h(g_i q_i) + h'(g_i q_i) g_i q_i \neq h(g_j q_j) + h'(g_j q_j) g_j q_j$, which is, again by definition, equivalent to $(h_i(q_i)q_i)' \neq (h_j(q_j)q_j)'$. Some functions that satisfy this condition, and the respective marginal cost functions associated with them, are as follow:

- $h(x) = e^x - 1$, $h_i(q_i) = e^{g_i q_i} - 1$.
- $h(x) = x^u$, for $u > 0$, $h_i(q_i) = g_i q_i^u$.
- $h(x) = \ln(x+1)$, $h_i(q_i) = \ln(g_i q_i + 1)$.
- Any polynomial with positive coefficients.

Specifically, all these functions have the property that $h(x)x$ is convex over $x \geq 0$; therefore, $h(x) + h'(x)x$ is increasing. Note that the marginal cost functions $h(x)$ are allowed to be concave; e.g., $h(x) = x^u$ for $0 < u < 1$, and $h(x) = \ln(x+1)$. Moreover, note that $h_i(q_i) = g_i q_i^u$ corresponds to the unique homogeneous function of degree $u > 0$ in one variable.

4.1. Sufficient Optimality Condition

The following theorem is the main result in this section.

Theorem 1. *Assume that the marginal cost functions $h_i(q_i)$ are nonnegative, increasing, and continuously differentiable in $[0, \bar{Q})$; the inverse demand function $P(Q)$ is nonnegative, decreasing, and differentiable in $[0, \bar{Q}]$. If the marginal cost functions satisfy Property 1, then uniform copayments are optimal for problem* (CAP).

Proof. The existence of an optimal solution to problem (UBP) was shown in Lemma 1. Let $(\mathbf{q}, Q)$ be an optimal solution to problem (UBP). We will show that if the marginal cost functions satisfy Property 1, then $(\mathbf{q}, Q)$ induces uniform copayments for every firm with a positive output $q_i > 0$. Moreover, $(\mathbf{q}, Q)$ is also a feasible solution of problem (CAP), therefore optimal.

From Lemma 1 it follows that $(\mathbf{q}, Q)$ is such that the budget constraint is binding, and for each i, j with $q_i > 0$ and $q_j > 0$, we must have $(h_i(q_i)q_i)' = (h_j(q_j)q_j)'$. The assumption that the marginal cost functions satisfy Property 1 implies that $h_i(q_i) = h_j(q_j)$, which implies that uniform subsidies are optimal. Specifically, because the budget constraint is tight, it follows that $h_i(q_i) - P(Q) = B/Q > 0$ for every i such that $q_i > 0$.

It remains to show that the firms that do not participate in the market equilibrium effectively have a marginal cost of producing zero units, which is larger than the induced market price. To do so, note that $(\mathbf{q}, Q)$ is such that, for each i, j with $q_i > 0$ and $q_j = 0$, we have $h_j(0) - P(Q) \geq h_i(q_i) + h_i'(q_i)q_i - P(Q) \geq h_i(q_i) - P(Q) = B/Q > 0$. The first inequality follows from Lemma 1, and the second inequality follows from $h_i(q_i)$ increasing. The equality follows from the fact that the budget constraint is tight, and $q_i > 0$.

Hence, $(\mathbf{q}, Q)$ is also feasible for problem (CAP), therefore optimal. Moreover, $(\mathbf{q}, Q)$ induces uniform copayments. Therefore, uniform copayments are optimal for problem (CAP). □

This result is surprising, considering that the assumptions on the inverse demand function are very general, particularly since firms can be heterogeneous and the central planner has the freedom to allocate differentiated copayments to each firm. The intuition behind it comes from the market equilibrium condition and the budget constraint. Essentially, if the central planner allocates a larger copayment to a firm, then its resulting market share will increase, which is exactly the rate at which it will consume budget. This will in turn make less budget available to the rest of the firms; therefore, their copayments would have to decrease. Theorem 1 shows that if the marginal cost functions satisfy Property 1, then the net effect of this change will never be positive.

In particular, Theorem 1 applies for the special cases we considered in §3.2. For Cournot competition with linear demand, uniform copayments are optimal for any marginal cost functions $h_i(q_i)$, such that the functions $\tilde{h}_i(q_i) = h_i(q_i) + bq_i$ satisfy Property 1. Specifically, if the marginal cost functions are linear, i.e., $h_i(q_i) = g_i q_i$, for each i, then uniform copayments are optimal. Similarly, for Cournot competition under yield uncertainty with linear demand, if both marginal costs are linear, then uniform subsidies are optimal. Note that in both cases we allow for heterogeneous firms, where some can be significantly more efficient than others.

5. Incorporating Market State Uncertainty

A natural extension of the model discussed in §3 is to consider the setting where the central planner does not know the market state (defined by the inverse demand function) with certainty but generally will have a set of possible market state scenarios and beliefs on the likelihood that each scenario will materialize.

Specifically, we assume that the central planner has a discrete description of the market state uncertainty, where each scenario $s \in \{1, \ldots, m\}$ is realized with probability p_s. Each scenario s is characterized by a scenario-dependent inverse demand function $P^s(Q^s)$. For each scenario $s \in \{1, \ldots, m\}$, we make assumptions as in §3. Namely, we assume that each inverse demand function $P^s(Q)$ is nonnegative, decreasing, and differentiable in $[0, \bar{Q}^s]$, where $\bar{Q}^s$ is the smallest value such that $P^s(\bar{Q}^s) = 0$. Similarly, for the market equilibrium condition we assume that if scenario s is realized, then firms set their output q_i^s at a level such that, for each i, j, if $q_i^s > 0$, then $h_i(q_i^s) = P^s(Q^s) \le h_j(q_j^s)$.

Similar to §3, a formulation of the copayment allocation problem under market state uncertainty can be written as follows:

$$\max_{(\mathbf{q}^s, Q^s)_{s\in\{1,\ldots,m\}}, \mathbf{y}} \quad \sum_{s=1}^{m} Q^s p_s$$

$$\text{s.t.} \quad \sum_{j=1}^{n} q_j^s y_j \le B, \quad \text{for each } s \in \{1, \ldots, m\}; \tag{15}$$

$$y_i \ge 0, \quad \text{for each } i \in \{1, \ldots, n\}; \tag{16}$$

$$\sum_{j=1}^{n} q_j^s = Q^s, \quad \text{for each } s \in \{1, \ldots, m\}; \tag{17}$$

$$q_i^s \ge 0, \quad \text{for each } i \in \{1, \ldots, n\}, \; s \in \{1, \ldots, m\}; \tag{18}$$

$$h_i(q_i^s) - P^s(Q^s) - y_i \ge 0, \quad \text{for each } i \in \{1, \ldots, n\}, \; s \in \{1, \ldots, m\}; \tag{19}$$

$$q_i^s(h_i(q_i^s) - P^s(Q^s) - y_i) = 0, \quad \text{for each } i \in \{1, \ldots, n\}, \; s \in \{1, \ldots, m\}. \tag{20}$$

The objective is to maximize the expected market consumption. Constraint (15) is the budget constraint for each market state scenario. Constraint (16) corresponds to the nonnegativity of the copayments. Constraint (17) defines the market consumption for each scenario. Finally, constraints (18)–(20) are the complementarity constraints, which tie together the different scenarios. They state that, in each scenario, each firm must satisfy one of the following two cases: either it participates in the market equilibrium (in which case it produces the quantity that equates its marginal cost with the market price plus the copayment), or it is inactive (in which case its marginal cost of producing any positive amount is strictly larger than the market equilibrium price plus the copayment). Naturally, each firm must get the same copayment in each possible scenario.

In other words, constraints (18)–(20) correspond to the "nonanticipativity" constraints, and they state that copayments are a first-stage decision made by the central planner before the uncertainty is realized. This is precisely what prevents us from using the copayments to eliminate the complementarity constraints from the model formulation, similarly to Proposition 1. This makes the problem significantly harder to analyze. To somewhat simplify this formulation, we make the additional assumption that producing zero units has a marginal cost of zero.

Proposition 2. *If we additionally assume $h_i(0) = 0$, for each $i \in \{1, \ldots, n\}$, then the copayment allocation problem under market state uncertainty faced by the central planner can be rewritten as follows:*

(SCAP)

$$\max_{(\mathbf{q}^s, Q^s)_{s=1,\ldots,m}} \quad \sum_{s=1}^{m} Q^s p_s$$

$$\text{s.t.} \quad \sum_{j=1}^{n} q_j^s h_j(q_j^s) - P^s(Q^s) Q^s \le B, \quad \textit{for each } s \in \{1, \ldots, m\}; \tag{21}$$

$$h_i(q_i^s) \ge P^s(Q^s), \quad \textit{for each } i \in \{1, \ldots, n\}, \; s \in \{1, \ldots, m\}; \tag{22}$$

$$\sum_{j=1}^{n} q_j^s = Q^s, \quad \textit{for each } s \in \{1, \ldots, m\}; \tag{23}$$

$$q_i^s \ge 0, \quad \textit{for each } i \in \{1, \ldots, n\}, \; s \in \{1, \ldots, m\}; \tag{24}$$

$$h_i(q_i^s) - P^s(Q^s) = h_i(q_i^{s'}) - P^{s'}(Q^{s'}), \quad \textit{for each } i \in \{1, \ldots, n\}, \; s, s' \in \{1, \ldots, m\}. \tag{25}$$

The copayments that the central planner must allocate to induce outputs $\{\mathbf{q}^s\}_{s\in\{1,\ldots,m\}}$ are

$$y_i = h_i(q_i^s) - P^s(Q^s), \quad \textit{for each } i \in \{1, \ldots, n\}, \; s \in \{1, \ldots, m\}. \tag{26}$$

Proposition 2 states that, if the marginal cost of producing zero units is zero, then every firm will participate in the market equilibrium for any nonnegative market price. Therefore, Equation (26) holds, and we can eliminate the variables y_i from the problem formulation.

Like in Proposition 1, constraint (21) corresponds to the budget constraint. Namely, for each scenario s, the total cost minus the total revenue in the market has to be less than or equal to the budget introduced by the central planner. Constraint (22) is the nonnegativity of the copayments: it ensures that the solution proposed by the central planner can be sustained as a market equilibrium by allocating only subsidies, not taxes. Like before, the only constraint that ties all the scenarios together is the nonanticipativity constraint (25), which states that each firm must get the same copayment in each possible scenario.

This problem is still hard to analyze directly, which motivates us to develop a relaxation that provides an upper bound on the expected market consumption that can be induced with the available budget, as shown below. All the proofs in this section are presented in the online appendix.

5.1. An Upper Bound Problem

We start with a simple observation that is derived from the structure of problem (SCAP).

Lemma 2. *For any feasible solution to problem* (SCAP), *without loss of generality, the scenarios can be renumbered, such that the following inequalities hold true*:

$$P^1(Q^1) \geq P^2(Q^2) \geq \cdots \geq P^m(Q^m); \tag{27}$$

$$h_i(q_i^1) \geq h_i(q_i^2) \geq \cdots \geq h_i(q_i^m), \text{ for each } i; \tag{28}$$

$$q_i^1 \geq q_i^2 \geq \cdots \geq q_i^m, \text{ for each } i; \tag{29}$$

$$Q^1 \geq Q^2 \geq \cdots \geq Q^m; \tag{30}$$

$$\sum_{j=1}^n q_j^1 y_j \geq \sum_{j=1}^n q_j^2 y_j \geq \cdots \geq \sum_{j=1}^n q_j^m y_j, \tag{31}$$

where $\sum_{j=1}^n q_j^s y_j$ is the total amount spent in copayments in scenario s.

Let $(\mathbf{q}^s, Q^s)^*_{s=1,\ldots,m}$ be an optimal solution to problem (SCAP), and assume that the scenarios are numbered such that Equations (27)–(31) hold. Then, we claim that the solution to problem (SUBP) below provides an upper bound on the expected market consumption that can be induced with the available budget. Specifically, problem (SUBP) is derived from problem (SCAP) by adding constraints (36) and (37) below and replacing the non-anticipativity constraint (25) with the relaxed version (38).

(SUBP)

$$\max_{\mathbf{q}^s, Q^s} \sum_{s=1}^m Q^s p_s$$

$$\text{s.t.} \sum_{j=1}^n q_j^s h_j(q_j^s) - P^s(Q^s)Q^s \leq B, \quad \text{for each } s \in \{1,\ldots,m\}; \tag{32}$$

$$h_i(q_i^s) \geq P^s(Q^s), \quad \text{for each } i \in \{1,\ldots,n\},\ s \in \{1,\ldots,m\}; \tag{33}$$

$$\sum_{j=1}^n q_j^s = Q^s, \quad \text{for each } s \in \{1,\ldots,m\}; \tag{34}$$

$$q_i^s \geq 0, \quad \text{for each } i \in \{1,\ldots,n\},\ s \in \{1,\ldots,m\}; \tag{35}$$

$$P^1(Q^1) \geq P^s(Q^s), \quad \text{for each } s \in \{1,\ldots,m\}; \tag{36}$$

$$Q^1 \geq Q^s, \quad \text{for each } s \in \{1,\ldots,m\}; \tag{37}$$

$$h_i(q_i^s) - P^s(Q^s) \leq h_i(q_i^1) - P^1(Q^1), \quad \text{for each } i \in \{1,\ldots,n\},\ s \in \{1,\ldots,m\}. \tag{38}$$

Problem (SUBP) is a valid relaxation of problem (SCAP). Specifically, the optimal solution of problem (SCAP), $(\mathbf{q}^s, Q^s)^*_{s=1,\ldots,m}$, is feasible for problem (SUBP) and attains the same objective value. To argue the feasibility of solution $(\mathbf{q}^s, Q^s)^*_{s=1,\ldots,m}$ for problem (SUBP), recall from Lemma 2 that $s=1$ is the scenario that attains the largest value for both the induced market price (see (27)) and the induced market consumption (see (30)) in solution $(\mathbf{q}^s, Q^s)^*_{s=1,\ldots,m}$. It follows that adding constraints (36) and (37) does not cut off solution $(\mathbf{q}^s, Q^s)^*_{s=1,\ldots,m}$. Finally, solution $(\mathbf{q}^s, Q^s)^*_{s=1,\ldots,m}$ satisfies constraint (38) with equality.

5.2. Optimality of Uniform Copayments

In this section, we consider again the setting where all firms use a similar technology but can differ in their respective efficiency, similar to the assumptions in §4. Specifically, we consider the case in which $h_i(q_i) = h(g_i q_i)$, where h(x) is nonnegative, increasing, and differentiable over $x > 0$, and $g_i > 0$ is a firm-specific parameter. The function $h(x)$ models the industry-specific marginal cost function, whereas g_i models the efficiency of firm i. Recall from §4 that we can assume, without loss of generality, that $h(0) = 0$; therefore, we will refer to problem (SCAP), and its upper bound problem (SUBP).

We now present sufficient conditions to ensure that uniform subsidies maximize the expected market consumption in this setting. Specifically, we show that if the firms' marginal cost functions satisfy Property 2 below, then uniform subsidies are optimal for problem (SCAP).

Property 2. *The function $h(x)$ is convex, such that for any $x_1 > x_2 \geq 0$ and $x_1 > x_3 > x_4 \geq 0$, if $h(x_2)/h(x_1) > h(x_4)/h(x_3)$, then $h'(x_2)/h'(x_1) > h'(x_4)/h'(x_3)$.*

Note that Property 2 implies Property 1, discussed in §4. Specifically, $h(x)$ increasing and convex implies that $h(x) + h'(x)x$ is increasing. This is a sufficient condition for Property 1 to hold.

Remark 1. The functions $h_i(q_i) = g_i q_i^m$, for $m \geq 1$, and $h_i(q_i) = e^{g_i q_i} - 1$, satisfy Property 2.

Note, from Remark 1, that from the examples of marginal cost functions that satisfy Property 1 given in §4, all the examples that are also convex satisfy Property 2 as well. In this sense, the extra requirements in Property 2, with respect to Property 1, are mainly driven by the convexity assumption. Finally, note that functions $h_i(q_i) = g_i q_i^m$, for $m \geq 1$, are the unique convex homogeneous functions in one variable.

Theorem 2 below shows that there exists an optimal solution to the upper bound problem (SUBP), such that the copayments induced in scenario $s = 1$, the one that attains the largest market consumption (see (30)), and the largest amount spent in copayments (see (31)) are uniform. This result will play a central role in proving the main result in this section.

Theorem 2. *Assume that the inverse demand function $P(Q)$ is nonnegative, decreasing, and differentiable in $[0, \bar{Q}]$. Assume that the marginal cost functions are given by $h_i(q_i) = h(g_i q_i)$ for each i, for any increasing and continuously differentiable function $h(x)$, such that $h(0) = 0$. If $h(x)$ satisfies Property 2, then there exists an optimal solution to the upper bound problem* (SUBP), $(\hat{\mathbf{q}}^s, \hat{Q}^s)_{s=1,\ldots,m}$, *such that, $h_i(\hat{q}_i^1) - P^1(\hat{Q}^1) = y^1$ for each $i \in \{1, \ldots, n\}$, for some value $y^1 > 0$.*

To prove Theorem 2, we show several lemmas in the online appendix that are useful in the analysis. Specifically, we consider the optimal solution to problem (SUBP) with the smallest difference between $(\max_{i\in\{1,\ldots,n\}}\{h_i(q_i^1)\})$ and $(\min_{i\in\{1,\ldots,n\}}\{h_i(q_i^1)\})$ (see Lemma 3 in the online appendix). Note that proving Theorem 2 is equivalent to showing that this difference is zero. We assume by contradiction that this difference is strictly positive and show that then we can construct another optimal solution with an even smaller difference, a contradiction.

When constructing the modified optimal solution, Lemma 4 in the online appendix allows us to focus only on constraint (38). On the other hand, using the convexity assumption on $h(x)$, Lemma 5 in the online appendix provides bounds on the impact that the modifications to the optimal solution have on constraint (38). These bounds allow us to complete the proof by arguing that the modified solution is feasible and optimal, while attaining a smaller difference between the maximum marginal cost in scenario $s = 1$ and the minimum marginal cost in scenario $s = 1$.

Theorem 3 below concludes this section characterizing a family of firms' marginal cost functions such that uniform copayments are optimal, even if the central planner is uncertain about the market state. This family includes convex homogeneous functions of the same degree.

Theorem 3. *Assume that the inverse demand function $P(Q)$ is nonnegative, decreasing, and differentiable in $[0, \bar{Q}]$. Assume that the marginal cost functions are given by $h_i(q_i) = h(g_i q_i)$ for each i, for any increasing and continuously differentiable function $h(x)$, such that $h(0) = 0$. If $h(x)$ satisfies Property 2, then allocating the largest feasible uniform copayment is an optimal solution for problem* (SCAP).

This result is surprising because it shows that, with some additional conditions, the optimality of uniform subsidies is preserved, even if the central planner is uncertain about the market state. Different market states induce different inverse demand functions, which can be arbitrarily different. Moreover, the assumptions on the inverse demand functions of each scenario are very mild. Specifically, we only assume that they are decreasing. This is a very relevant setup because it corresponds to a more realistic representation of the problem faced in practice, where there are large uncertainties about different characteristics of the market state, which ultimately define the effective response of the demand side to different market prices.

Moreover, the analysis suggests that the central planner only needs to consider the scenario with the highest market consumption at equilibrium (see (30)), i.e., scenario $s = 1$, regardless of the exact distribution over the different market states. Specifically, Theorem 2 shows that uniform subsidies are optimal for scenario $s = 1$ in the relaxed upper bound problem (SUBP), whereas Theorem 3 shows that the uniform subsidies induced by scenario $s = 1$ are, in fact, optimal for problem (SCAP). This insight suggests that the central planner only needs to identify the scenario with the highest market consumption at equilibrium, and implement the uniform subsidies induced by it, as opposed to taking into consideration beliefs on the likelihood that each market state will be realized and the effect that the subsidy allocation will have on each possible scenario.

6. Maximizing Social Welfare

In this section, we assume that the central planner's objective is in fact to maximize social welfare. Given some $\delta \in (0, 1]$, which represents the social cost of funds, the central planner's problem of allocating subsidies to maximize social welfare can be written as follows:

$$\max_{\mathbf{y},\mathbf{q},Q} \int_0^Q P(x)\,dx - \sum_{i=1}^n \int_0^{q_i} (h(x_i) - y_i)\,dx_i - \delta\left(\sum_{i=1}^n q_i y_i\right)$$

$$\text{s.t.}\quad y_i \geq 0, \quad \text{for each } i \in \{1, \ldots, n\} \tag{39}$$

$$\sum_{i=1}^n q_i = Q; \tag{40}$$

$$q_i \geq 0, \quad \text{for each } i \in \{1, \ldots, n\}; \tag{41}$$

$$P(Q) + y_i = h_i(q_i), \quad \text{for each } i \in \{1, \ldots, n\}. \tag{42}$$

The first two terms in the objective function correspond to the sum of the consumer and producer surplus, including the copayments y_i. The third term in the objective function corresponds to the social cost of financing the subsidies. Note that in this problem there is no budget constraint. Specifically, the social cost of funds $\delta \in (0, 1]$ will induce a total amount invested in subsidies at optimality, which can be interpreted as the implicit budget available. Constraint (39) states that the central planner is only allowed to allocate subsidies, not taxes, to the firms. Like in §3, constraint (42) does not imply that every firm has to participate in the market equilibrium.

From the equilibrium condition given in constraint (42), it follows again that we can replace all the copayment variables y_i by $h_i(q_i) - P(Q)$, as stated in the proposition below.

Proposition 3. *The social welfare maximization problem can be written as follows:*

(CAP-SW)

$$\max_{\mathbf{q},Q} \equiv \int_0^Q P(x)\,dx - \sum_{i=1}^n \int_0^{q_i} h(x_i)\,dx_i + (1-\delta)\left(\sum_{i=1}^n h(q_i)q_i - P(Q)Q\right)$$

$$\text{s.t.}\quad h(q_i) \geq P(Q), \quad \textit{for each } i \in \{1,\dots,n\}; \tag{43}$$

$$\sum_{i=1}^n q_i = Q; \tag{44}$$

$$q_i \geq 0, \quad \textit{for each } i \in \{1,\dots,n\}. \tag{45}$$

The copayments that the central planner must allocate to induce outputs $\mathbf{q}$ *are* $y_i(\mathbf{q}) = h_i(q_i) - P(Q)$, *for each* i.

The first two terms in the objective function correspond to the sum of the consumer and producer surplus, with no subsidies. The third term corresponds to the increase in social welfare induced by subsidies, minus the social cost of financing them. Constraint (43) states that the central planner is only allowed to allocate subsidies, not taxes, to the firms.

We will make the natural assumption that the social cost of funds, $\delta \in (0,1]$, is such that objective function of problem (CAP-SW), denoted by $SW(\mathbf{q},Q)$, is coercive;[1] therefore, there exists an optimal solution (see, e.g., Bertsekas (1999)). Then, the budget B that the central planner spends in subsidies, in order to maximize social welfare, can be written as follows: let $(\mathbf{q}^*,Q^*)$ be an optimal solution of problem (CAP-SW), then $B \equiv \sum_{i=1}^n h(q_i^*)q_i^* - P(Q^*)Q^*$.

6.1. Optimality of Uniform Copayments

We conclude this section by characterizing settings where, in addition to maximizing the market consumption, uniform copayments also maximize social welfare. Specifically, we show that if the marginal cost functions satisfy Property 3 below, then uniform subsidies are optimal for problem (CAP-SW).

Property 3. *For each* i,j *and each* $q_i, q_j \geq 0$, *if* $h_i(q_i) > h_j(q_j)$, *then* $h_i(q_i)/(h_i'(q_i)q_i) \geq h_j(q_j)/(h_j'(q_j)q_j)$.

Two examples of marginal cost functions that satisfy Property 3 are

- $h_i(q_i) = g_i q_i^u$ for $u > 0$;
- $h_i(q_i) = \ln(g_i q_i + 1)$.

Note that these marginal cost functions also satisfy Property 1. Therefore, for these two marginal cost functions, uniform subsidies maximize both the consumer surplus and social welfare.

This leads to the main result of this section.

Theorem 4. *Assume that the marginal cost functions* $h_i(q_i)$ *are nonnegative, increasing, and differentiable in* $[0,\bar{Q})$; *the inverse demand function* $P(Q)$ *is nonnegative, decreasing, and differentiable in* $[0,\bar{Q}]$; *and the social cost of funds* $\delta \in (0,1]$ *is such that it induces a finite central planner's budget* B. *If the marginal cost functions satisfy Property 3, then uniform copayments are optimal for the social welfare maximization problem* (CAP-SW).

Proof. Let $(\mathbf{q}^*,Q^*)$ be the optimal solution of problem (CAP-SW). First, we show that $(\mathbf{q}^*,Q^*)$ must satisfy

$$(1-\delta) < \frac{h_i(q_i^*)}{h_i(q_i^*) + h_i'(q_i^*)q_i^*}, \quad \text{for each } i. \tag{46}$$

Specifically, the expression in the objective function of problem (CAP-SW) related to the market consumption Q is strictly increasing in Q. Namely, $(\partial/\partial q_i)(\int_0^Q P(x)\,dx - (1-\delta)P(Q)Q) = \delta P(Q) - (1-\delta)P'(Q)Q > 0$, where the inequality follows from the inverse demand function $P(Q)$ being nonnegative and decreasing. On the other hand, the remaining expression in the objective function of problem (CAP-SW), related to firm i's output q_i, is such that $(\partial/\partial q_i)((1-\delta)\sum_{i=1}^n h(q_i)q_i - \sum_{i=1}^n \int_0^{q_i} h(x_i)\,dx_i) = (1-\delta)\cdot(h_i(q_i) + h_i'(q_i)q_i) - h_i(q_i)$. Assume for a contradiction that Equation (46) does not hold. Namely, there exists an index i such that $(1-\delta)(h_i(q_i^*) + h_i'(q_i^*)q_i^*) - h_i(q_i^*) \geq 0$. It follows that we can increase q_i^* by $\epsilon > 0$ sufficiently small and obtain a feasible solution that attains a strictly larger objective value. This is a contradiction to the optimality of $(\mathbf{q}^*,Q^*)$.

Second, assume by contradiction that there exist indexes i, j, with $q_i^* > 0$ and $q_j^* > 0$, such that $h_i(q_i^*) > h_j(q_j^*)$. The fact that the marginal cost functions satisfy Property 3 implies that $h_i(q_i^*)/(h_i'(q_i^*)q_i^*) \geq h_j(q_j^*)/(h_j'(q_j^*)q_j^*)$. From direct algebraic manipulations it follows that

$$\frac{h_i(q_i^*)}{h_i(q_i^*) + h_i'(q_i^*)q_i^*} \leq \frac{h_i(q_i^*) - h_j(q_j^*)}{h_i(q_i^*) + h_i'(q_i^*)q_i^* - h_j(q_j^*) - h_j'(q_j^*)q_j^*}. \tag{47}$$

Now, Equations (46) and (47) imply that $(1-\delta) < (h_i(q_i^*) - h_j(q_j^*))/(h_i(q_i^*) + h_i'(q_i^*)q_i^* - h_j(q_j^*) - h_j'(q_j^*)q_j^*)$. Therefore, we can transfer $\epsilon > 0$ sufficiently small from q_i^* to q_j^* and obtain the following positive marginal change in the objective function, $h_i(q_i^*) - h_j(q_j^*) - (1-\delta)\cdot(h_i(q_i) + h_i'(q_i)q_i - h_j(q_j^*) - h_j'(q_j^*)q_j^*) > 0$. Namely, there exists a feasible solution with a strictly larger objective value. This contradicts the optimality of $(\mathbf{q}^*,Q^*)$. Hence, we conclude that for each i, j, with $q_i^* > 0$ and $q_j^* > 0$, it must be the case that $h_i(q_i^*) = h_j(q_j^*)$. Therefore, uniform subsidies maximize social welfare. □

7. Numerical Results

In §4, we have identified conditions on the firms' marginal cost functions that guarantee the optimality of uniform copayments to maximize the market consumption of a good. In this section we study the performance of uniform copayments, in relevant settings where they are suboptimal. Our goal here is to study numerically the performance of uniform copayments on problems with data generated at random.

To evaluate the relative performance of uniform subsidies, we need to be able to compute the respective market consumption induced by them. Proposition 4 below addresses this issue for the special cases of our model studied next in §§7.1 and 7.2.

Proposition 4. *Assume that the marginal cost functions $h_i(q_i)$ are nonnegative, increasing, and differentiable in $[0, \bar{Q})$ and that the inverse demand function $P(Q)$ is nonnegative, decreasing, and differentiable in $[0, \bar{Q}]$. Then, the market equilibrium induced by the largest feasible uniform copayment can be computed as the solution to the following convex optimization problem,*

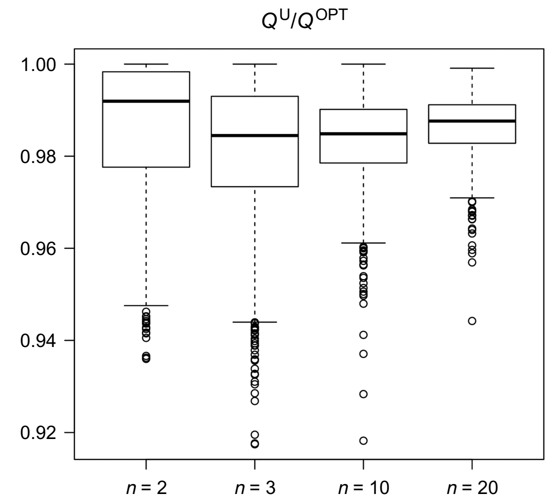
$$\begin{aligned}
\text{(UCAP)} \quad \min_{\mathbf{q}} \quad & \left\{ \sum_{j=1}^{n} \int_0^{q_j} h_j(x_j)\, dx_j \right. \\
& - \int_0^{q_{n+1}} P(\bar{Q} - x_{n+1})\, dx_{n+1} \\
& \left. - B \ln(\bar{Q} - q_{n+1}) \right\} \\
\text{s.t.} \quad & \sum_{j=1}^{n} q_j + q_{n+1} = \bar{Q} \\
& q_i \geq 0, \quad \textit{for each } i.
\end{aligned}$$

Assuming that the inverse demand function $P(Q)$ is decreasing and that the firms'marginal cost functions $h_i(q_i)$ are increasing implies that problem (UCAP) is a convex optimization problem. On the other hand, in the experimental settings we consider in this section, it will always be the case that at least the upper bound problem (UBP) is a convex optimization problem. To solve these problems we used CVX, a package for specifying and solving convex programs; see Grant and Boyd (2012). We will denote by Q^{U}, Q^{OPT}, and Q^{UB} the market consumption component of the optimal solutions to problems (UCAP), (CAP), and (UBP), respectively.

7.1. Cournot Competition with Linear Demand and Constant Marginal Costs

The model presented in §3 captures Cournot competition with linear demand and nondecreasing marginal cost functions $h_i(q_i)$. Specifically, this implies that the modified marginal cost function defined in §3.2, $\tilde{h}_i(q_i) \equiv h_i(q_i) + bq_i$, is increasing. In particular, in this section we consider constant marginal costs. Although the constant marginal costs case moves away from our scarce installed capacity assumption, it is a well-understood model where uniform copayments are not optimal. Therefore, it is interesting to study the performance of uniform copayments in this setting.

Figure 1. Relative Performance of Uniform Copayments: Cournot Constant Marginal Costs



Specifically, in this section we assume that $P(Q) = a - bQ$, and $h_i(q_i) = c_i$, for each i. Therefore, the modified marginal cost is $\tilde{h}_i(q_i) = c_i + bq_i$, for each i. Under these assumptions, problem (CAP) is a convex optimization problem. Therefore, we solve both the uniform copayments allocation problem (UCAP) and problem (CAP), and we compare their objective functions directly. We consider four cases in the number of firms participating in the market, $n \in \{2, 3, 10, 20\}$. For each one of these four cases, we solve 1,000 instances of the problem. These instances are randomly generated, with parameters sampled from the following distributions: a, b are uniformly distributed in $[0, 50]$ and c_i are independent and uniformly distributed in $[0, a]$, for each i.

Figure 1 presents a box plot of the results for the ratio $Q^{\mathrm{U}}/Q^{\mathrm{OPT}}$, and Table 1 presents some summary statistics. It is interesting that the minimum value of the ratio $Q^{\mathrm{U}}/Q^{\mathrm{OPT}}$ never went below 91% in the simulation results. Moreover, the mean and median values are above 98% for each value of the number of firms participating in the market n. This suggests that, in most cases, the market consumption induced by uniform copayments is fairly close to the market consumption induced by the optimal copayment allocation.

7.2. Price-Taking Firms with Nonlinear Demand and Nonlinear Marginal Costs

Now we consider a more general experimental setup, with nonlinear demand and nonlinear marginal costs, where the firms act as price takers. In this setting we assume that $P(Q) = a - bQ^{m_0}$ and $h_i(q_i) = c_i + g_i q_i^{m_i}$ for each i. Under these assumptions, problem (CAP) is a nonconvex optimization problem. However, the upper bound problem (UBP) is convex. Therefore, we solve both the uniform copayment allocation problem

Table 1. Summary of Relative Performance of Uniform Copayments: Cournot Constant Marginal Costs

Q^U/Q^{OPT}	$n=2$	$n=3$	$n=10$	$n=20$
Min.	0.9360	0.9175	0.9182	0.9442
1st Qu.	0.9776	0.9734	0.9785	0.9828
Median	0.9919	0.9845	0.9849	0.9876
Mean	0.9860	0.9806	0.9834	0.9866
3rd Qu.	0.9983	0.9930	0.9902	0.9912
Max.	1.0000	1.0000	1.0000	0.9991

Table 2. Summary of Relative Performance of Uniform Copayments: Price-Taking Firms

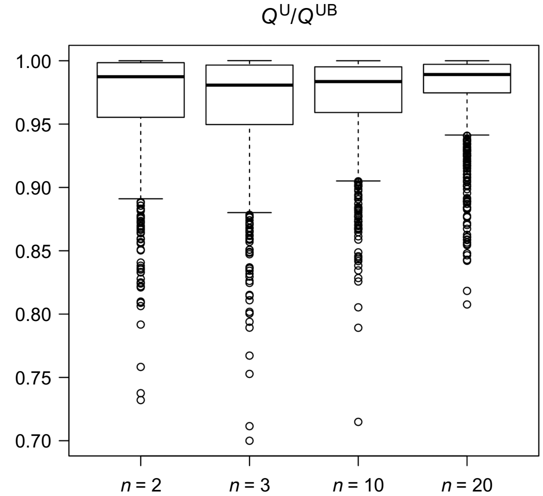
Q^U/Q^{UB}	$n=2$	$n=3$	$n=10$	$n=20$
Min.	0.7321	0.7000	0.7149	0.8076
1st Qu.	0.9554	0.9497	0.9592	0.9747
Median	0.9874	0.9808	0.9836	0.9892
Mean	0.9698	0.9661	0.9710	0.9784
3rd Qu.	0.9985	0.9966	0.9952	0.9972
Max.	1.0000	1.0000	1.0000	1.0000

(UCAP) and the upper bound problem (UBP), and we compare their objective functions.

We consider again four cases in the number of firms participating in the market, $n \in \{2, 3, 10, 20\}$. For each one of these four cases, we solve 1,000 instances of the problem. These instances are randomly generated, with parameters sampled from the following distributions: a, b are uniformly distributed in $[0, 50]$. For each i, c_i are independent and uniformly distributed in $[0, a]$, g_i are independent and uniformly distributed in $[0, 50]$, and m_i are independent and uniformly distributed in $(0, 20]$. Finally, m_0 is uniformly distributed in $(0, 3]$.

Note that $P(Q) = a - bQ^{m_0}$, $m_0 \in (0, 3]$ captures both convex and concave decreasing inverse demand functions. Similarly, $h_i(q_i) = c_i + g_i q_i^{m_i}$, $m_i \in (0, 20]$ for each i captures both convex and concave marginal cost firms. The results for the ratio Q^U/Q^{UB} are displayed in Figure 2 and in Table 2. The minimum value of the ratio Q^U/Q^{UB} never went below 70% in the simulation results. Moreover, the mean and median values are above 96% for each value of the number of firms participating in the market n, where in this case we are not comparing directly to the optimal solution, but to an upper bound. This again suggests that, in most cases, the market consumption induced by uniform copayments is fairly close to the market consumption induced by the optimal copayment allocation.

Figure 2. Relative Performance of Uniform Copayments: Price-Taking Firms



7.3. Cournot Competition with Nonlinear Demand and Nonlinear Marginal Costs

In this section, we extend the numerical study of the performance of uniform subsidies for Cournot competition with nonlinear demand. Recall from Equation (11) that this model is not one of the special cases of formulation (CAP) given in §3.2, because it would correspond to each firm having a nonseparable marginal cost function $\tilde{h}(q_i, Q) = h_i(q_i) - P'(Q)q_i$, which depends on the market output of all the other firms. Nonetheless, additional modeling techniques will allow us to numerically compute a bound on the performance of uniform subsidies in the experiments for this case.

We consider the same nonlinear setting as in §7.2: the difference is that now firms are assumed to engage in Cournot competition. Under these assumptions, the modified nonseparable marginal cost function of each firm becomes $\tilde{h}(q_i, Q) = c_i + g_i q_i^{m_i} + m_0 b Q^{m_0-1} q_i$. Moreover, we consider the natural generalizations for problem (CAP) and the upper bound problem (UBP), where we directly replace the marginal cost function $h_i(q_i)$ with the nonseparable function $\tilde{h}(q_i, Q)$; hence we skip the problem statements here.

Note that both problems (CAP) and (UBP) are nonconvex for our experimental setup; however, we will be able to solve problem (UBP) efficiently as follows. First, note that any optimal solution to problem (UBP) must satisfy the budget constraint as equality. Second, note that the function $TC(Q)$ defined below is increasing in Q.

$$\begin{aligned} TC(Q) &\equiv \min_{\mathbf{q}} \sum_{i=1}^{n} \tilde{h}(q_i, Q) q_i \\ &= \sum_{i=1}^{n} (c_i q_i + g_i q_i^{m_i+1} + m_0 b Q^{m_0-1} q_i^2) \\ &\quad \text{s.t.} \sum_{i=1}^{n} q_i = Q, q_i \geq 0 \quad \text{for each } i. \end{aligned}$$

Finally, note that the total revenue in the market $P(Q)Q = aQ - bQ^{m_0+1}$ is concave for any $m_0 > 0$. Let us denote the total revenue-maximizing market output by Q^{M}. From the first observation it follows that the optimal objective value of problem (UBP), denoted by Q^{UB}, must satisfy $P(Q^{\mathrm{UB}})Q^{\mathrm{UB}} = TC(Q^{\mathrm{UB}}) - B$. Hence, if $B \geq TC(Q^{\mathrm{M}}) - P(Q^{\mathrm{M}})Q^{\mathrm{M}}$ then the functions $P(Q)Q$ and $TC(Q) - B$ have a unique intersection. Therefore, Q^{UB} can be computed efficiently using binary search, where a convex optimization problem must be solved to evaluate $TC(Q)$ at each iteration.

On the other hand, we need to be able to compute the market consumption induced by uniform subsidies Q^{U}. However, from $\tilde{h}(q_i, Q)$ nonseparable and asymmetric, it follows that Q^{U} cannot be computed by solving a convex optimization problem; see, e.g., Correa and Stier-Moses (2011). Nonetheless, Q^{U} can be computed by solving an asymmetric variational inequality. Specifically, from Corollary 1 in Aghassi et al. (2006), it follows that $(\mathbf{q}^{\mathrm{U}}, Q^{\mathrm{U}})$ is the market equilibrium induced by uniform subsidies if and only if there exists λ^{U} such that the solution $(\mathbf{q}^{\mathrm{U}}, Q^{\mathrm{U}}, \lambda^{\mathrm{U}})$ is feasible for problem (GUCAP) below, and it attains an objective value of zero.

$$
\begin{aligned}
\text{(GUCAP)} \quad \min_{\mathbf{q}, Q, \lambda} \quad & \sum_{i=1}^{n} h_i(q_i, Q) q_i - \sum_{i=1}^{n} P'(Q) q_i^2 \\
& + \left(P(Q) + \frac{B}{Q} \right)(\bar{Q} - Q) - \lambda \bar{Q} \\
\text{s.t.} \quad & \sum_{i=1}^{n} q_i = Q \\
& q_i \geq 0, \quad \text{for each } i; \\
& \lambda \leq h_i(q_i, Q), \quad \text{for each } i; \\
& \lambda \leq P(Q) + \frac{B}{Q}.
\end{aligned}
$$

In fact, this implies that this solution is optimal for problem (GUCAP) because the objective is nonnegative for any feasible solution; see Aghassi et al. (2006).

Figure 3. Relative Performance of Uniform Copayments: Cournot Nonlinear Demand

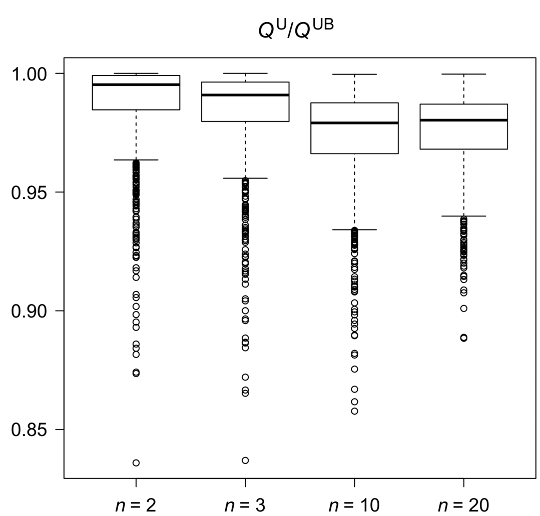


Table 3. Summary of Relative Performance of Uniform Copayments: Cournot Nonlinear Demand

$Q^{\mathrm{U}}/Q^{\mathrm{OPT}}$	$n = 2$	$n = 3$	$n = 10$	$n = 20$
Min.	0.8360	0.8370	0.8577	0.8884
1st Qu.	0.9846	0.9798	0.9662	0.9681
Median	0.9952	0.9909	0.9791	0.9803
Mean	0.9878	0.9840	0.9735	0.9756
3rd Qu.	0.9991	0.9963	0.9976	0.9870
Max.	1.0000	1.0000	0.9996	0.9997

In our setting, problem (GUCAP) is nonconvex. However, if a nonlinear solver finds a feasible solution $(\mathbf{q}^{\mathrm{U}}, Q^{\mathrm{U}}, \lambda^{\mathrm{U}})$ with objective value equal to zero, then it follows that Q^{U} is the market consumption induced by uniform subsidies. We use LOQO to solve the smooth nonconvex problem (GUCAP) (see Vanderbei 2006), and in our experiments the solver finds the optimal solution in 94% of the instances considered.

We consider the same experimental setup as in §§7.1 and 7.2. The results for the ratio $Q^{\mathrm{U}}/Q^{\mathrm{UB}}$ are displayed in Figure 3 and in Table 3. The minimum value of the ratio $Q^{\mathrm{U}}/Q^{\mathrm{UB}}$ never went below 83% in the simulation results. Moreover, the mean and median values are above 97% for each value of the number of firms participating in the market n. These are relatively better results than the ones we obtained for price-taking firms in the same setting, in §7.2. This suggests that, in most cases, the market consumption induced by uniform copayments is fairly close to the market consumption induced by the optimal copayment allocation, even for the setting of Cournot competition with nonlinear demand and nonlinear marginal costs considered in this section.

8. Extensions and Limitations

In this section, we show that there are instances of problem (CAP) where the market consumption induced by uniform subsidies can be significantly lower than the one induced by optimal subsidies. Additionally, we explore further extensions to our basic model, described in §3. We conclude that uniform subsidies can be a good policy even if the firms have economies of scale. However, this does not extend to the case where each firm has a fixed cost of entry to the market, suggesting the need of an alternative policy in this case.

8.1. Relatively Low Performance of Uniform Subsidies

Proposition 6 in the online appendix shows that there are instances of problem (CAP) such that the performance of uniform subsidies can be as bad as inducing only half of the optimal market consumption. The details of these carefully constructed instances are described in the proof in the online appendix. They consist of only one efficient firm, and $(n-1)$ homogeneous inefficient firms, facing a linear inverse demand function. The budget is exactly balanced such that the uniform subsidies policy subsidizes the efficient firm only (the inefficient firms are on the boundary of joining the market equilibrium), although it is optimal to subsidize the inefficient firms only. The structure of this type of instances is very particular, which partially explains the good performance of uniform subsidies in the numerical experiments in §7. Furthermore, the instances where uniform subsidies have a low performance in §7 share a similar structure. Surprisingly, this suggests that when all the firms are very different from each other, as opposed to having two clusters of firms, then the relative performance of uniform subsidies can be expected to be better.

The relative performance of one-half can be approximated arbitrarily closely as the number of firms in the market n increases and as the inverse demand function approaches a constant. Moreover, we conjecture that this corresponds to the worst performance of uniform subsidies for a large family of instances, including arbitrary decreasing inverse demand functions and any affine increasing marginal cost functions.

8.2. Economies of Scale

We consider the case where the firms have economies of scale in their production. We model it by assuming decreasing marginal costs. Note that then the uniqueness of the market equilibrium induced by any subsidy allocation is no longer guaranteed. Nonetheless, we can show the following result in the spirit of Theorem 1.

Proposition 5. *Assume that the marginal cost functions $h_i(q_i)$ are nonnegative, decreasing, and continuously differentiable in $[0,\bar{Q})$, and that $(h_i(q_i)q_i)'$ are monotonic in $[0,\bar{Q})$ as well. Assume that the inverse demand function $P(Q)$ is nonnegative, decreasing, and differentiable in $[0,\bar{Q}]$. If the marginal cost functions satisfy Property 1, then uniform copayments are optimal for problem* (CAP).

Proof. From the assumption that $h_i(q_i)$ are decreasing and continuously differentiable, $(h_i(q_i)q_i)'$ are monotonic, and Property 1, it follows that $h_i(0)=h_j(0)$ for each $i,j\in\{1,\dots,n\}$. Specifically, assume that $(h_j(q_j)q_j)'$ is decreasing and $h_i(0)<h_j(0)$ for some $i,j\in\{1,\dots,n\}$. Then there exists some $\bar{q}_j>0$ such that $h_i(0)=h_j(\bar{q}_j)+h_j'(\bar{q}_j)\bar{q}_j>h_j(\bar{q}_j)$. Hence, for $q_i=0$ we have $h_i(q_i)>h_j(\bar{q}_j)$ and $(h_i(q_i)q_i)'=(h_j(\bar{q}_j)\bar{q}_j)'$, a contradiction of Property 1. The case where $(h_j(q_j)q_j)'$ is increasing is analogous.

The rest of the proof is exactly as in Theorem 1, except that from the previous discussion there are no firms that do not participate in the market equilibrium. □

From Proposition 5 we conclude that the largest possible market consumption can only be induced by uniform subsidies in this model, for any decreasing inverse demand function, and for any set of marginal cost functions that satisfy Property 1 and such that $(h_i(q_i)q_i)'$ are monotonic. The analogous examples to the ones discussed in §4 are $h_i(q_i)=c-g_iq_i^u$, $h_i(q_i)=c-e^{g_iq_i}-1$, $h_i(q_i)=c-\ln(g_iq_i+1)$, for any $c>0$, and any polynomial with negative coefficients. More generally, any positive and decreasing function $h_i(q_i)$, that can be written as $h_i(q_i)=h(g_iq_i)$ for some function $h(x)$ such that $h(x)x$ is concave, will satisfy the conditions in Proposition 5.

However, as opposed to the model in §3, note that these uniform subsidies may also induce alternative market equilibria, potentially having market consumption levels that are smaller than the ones that can be induced by nonuniform subsidy allocations. Consider the following example with $n=2$ homogeneous firms such that $h_i(q_i)=5-q_i$ for $i\in\{1,2\}$, $P(Q)=9-2Q$, and $B=1$. Then, the optimal uniform subsidy is $y^{\mathrm{U}}=0.236$, and the largest possible market consumption is attained when only one firm participates in the market equilibrium and the budget is exhausted; e.g., $Q^{\mathrm{U}}=q_1^{\mathrm{U}}=4.236$, $q_2^{\mathrm{U}}=0$. However, the same uniform subsidy also supports an alternative market equilibrium where both firms participate, i.e., $q_1^{\mathrm{U}'}=q_2^{\mathrm{U}'}=1.412$ and $Q^{\mathrm{U}'}=2.824$, where the total amount spend on subsidies is $y^{\mathrm{U}}Q^{\mathrm{U}'}=2/3<1$. The latter market consumption can be increased by allocating a larger subsidy to any of the two firms, showing that uniform subsidies may lead to a market consumption that is dominated by nonuniform subsidy allocations. This characteristic only gets worse when considering markets with a larger number of heterogeneous firms. In summary, if each subsidy allocation is evaluated by the largest market consumption it can induce, then Proposition 5 implies that uniform subsidies are the best possible policy.

8.3. Fixed Cost of Entry

We additionally consider the case where each firm $i\in\{1,\dots,n\}$ must pay a fixed cost $K_i\geq 0$ to enter the market; thus we include an entry game stage after the central planner (the leader) has allocated the subsidies, and before the market competition between the firms. This type of entry game is usually analyzed under a symmetric equilibrium for homogeneous firms (see, e.g., Deo and Corbett (2009)), and, not surprisingly, uniform subsidies are optimal in that case. However, for heterogeneous firms, uniform subsidies may be suboptimal.

Consider the following example: let $n=2$ and $h_1(q_1)=q_1$, $K_1=0$, $h_2(q_2)=0.35q_2$, $K_2=6.3$ (i.e., one relatively inefficient firm with no fixed cost of entry and one efficient firm with a positive fixed cost of entry).

Let $P(Q) = 10 - Q$, and $B = 1$. From Theorem 1, if we ignore the fixed cost of entry, then the optimal subsidy allocation is uniform. Both firms should participate in the induced market equilibrium; however, this solution is *unattainable* because it leads to the profits of firm 2 being $\Pi_2 = 6.207 < 6.3 = K_2$; hence, firm 2 does not enter the market. The best *attainable* market equilibrium induced by uniform subsidies is $y^{\mathrm{U}} = 0.1962$, where only firm 1 participates in the market equilibrium and the budget is exhausted, leading to $Q^{\mathrm{U}} = q_1^{\mathrm{U}} = 5.098$, $q_2^{\mathrm{U}} = 0$. (If firm 2 enters the market with this uniform subsidy, its profits at the market equilibrium are $\Pi_2 = 6.295 < 6.3 = K_2$.) However, the subsidies $y_1 = 0.079$ and $y_2 = 0.14$ support a market equilibrium where both firms participate and the budget is exhausted, i.e., $q_1 = 2.04$, $q_2 = 6$, and $Q = 8.04 > Q^{\mathrm{U}}$, where the firms' profits are $\Pi_1 = 2.08$ and $\Pi_2 = 6.3$. Moreover, the relative performance of uniform subsidies in this instance is only 63.4%, in contrast to the good results in the extensive numerical examples from §7. Hence, if a fixed cost of entry is added to the model, then uniform subsidies are not optimal even in the simple setting with linear demand and linear marginal costs.

9. Conclusions

We provide a new modeling framework to analyze the problem of a central planner injecting a budget of subsidies into a competitive market, with the objective of maximizing the market consumption of a good. The copayment allocation policy that is usually implemented in practice is uniform, in the sense that every firm gets the same copayment. A central question in this paper is how efficient uniform copayments are compared to the optimal subsidy allocation, assuming that some firms could be significantly more efficient than others.

Using our framework, we show that uniform copayments are in fact optimal for a large family of marginal cost functions. Moreover, we show that the optimality of uniform copayments is preserved, under less general conditions, in the case where the central planner is uncertain about the market state. Furthermore, we show that uniform copayments also maximize the social welfare for a large family of marginal cost functions. Additionally, we study the performance of uniform copayments in relevant settings where they are not optimal. Our simulation results suggest that the market consumption induced by uniform copayments is relatively close to the market consumption induced by the optimal copayment allocation. It is an interesting research question to explore whether there exist theoretical bounds on the effectiveness of uniform subsidies in these settings.

On the other hand, we also identify settings where the performance of uniform subsidies is not as good. In particular, if the firms face a fixed cost of entry to the market, then uniform subsidies are not optimal even if the firms have linear marginal costs and they face a linear demand. This suggests the need for an alternative policy in this setup, an interesting direction for future research.

In summary, we present evidence that gives theoretical support to the use of uniform copayments under some conditions. Specifically, if the central planner's intervention is concentrated in a short period of time, when the entry of new firms into the market that need to build their whole infrastructure is less relevant, then it is very likely that the very simple uniform subsidy policy will attain most of the potential benefits of more sophisticated allocation policies.

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Endnote

[1] $SW(\mathbf{q}, Q)$ is coercive if $SW(\mathbf{q}^k, Q^k) \longrightarrow -\infty$ for any feasible sequence such that $\|(\mathbf{q}^k, Q^k)\| \longrightarrow \infty$.

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