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Option Methods for Incorporating Risk into Linear Capacity Planning Models

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Manufacturing and service operations decisions depend critically on capacity and resource limits. These limits directly affect the risk inherent in those decisions. While risk consideration is well developed in finance through efficient market theory and the capital asset pricing model, operations management models do not generally adopt these principles. One reason for this apparent inconsistency may be that analysis of an operational model does not reveal the level of risk until the model is solved. Using results from option pricing theory, we show that this inconsistency can be avoided in a wide range of planning models. By assuming the availability of market hedges, we show that risk can be incorporated into planning models by adjusting capacity and resource levels. The result resolves some possible inconsistencies between finance and operations and provides a financial basis for many planning problems. We illustrate the proposed approach using a capacity-planning example.
(*Real Options*; *Stochastic Programming*; *Financial Optimization*)

1. Introduction

Large operations management models often involve a variety of limited resources and uncertain parameters that may all affect decisions based on these models. The combination of limited resources and uncertainty makes attitudes toward risk an important consideration. Utility functions can capture these attitudes (see, for example, Keeney and Raiffa 1976), but the functions may be difficult to evaluate in the most general cases.

For profit-making corporations, investor considerations can often be used for general attitudes toward risk. In this case, the capital asset pricing model (see, for example, Sharpe 1964, and Merton 1973) provides a foundation for a utility tradeoff between risk and return that can be observed in the market. With this relationship defined, adjustments for risk can be incorporated into discount rates on future returns. The result is a project evaluation based on discounted expected cash flows.

Problems may occur if cash flows are skewed in a way that might make investors prefer risks of one nature ("upside" versus "downside") over another. This problem is often avoided by assuming sufficient diversification that total returns appear symmetric. The question for an individual project is only the fraction of risk that it might contribute to the overall portfolio of investments.

Measuring this risk contribution can often be done using β values for common stocks and other market instruments. The difficulty in these assessments with constrained operational problems is that the risk contribution varies with the capacity level and often cannot be determined at the outset. A major contribution of option theory has been to avoid this difficulty through the risk-neutral valuation method (see, for example, Cox and Ross 1976, and Duffie 1992), in which option values can be found by treating all investors as if they were risk neutral.

This foundation of option pricing theory has broad implications for decisions on projects with limited capacity (see, for example, Andreou 1990, Triantis and Hodder 1990, and Myers 1984). Option pricing methods can also give consistent results for other operational decisions such as setting inventory levels (see

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Singhal 1988). In this paper, we apply the basic principle of risk-neutral valuation to general forms of constrained resource problems, such as capacity planning. The application of these results for linear problems leads to a modification of the constraints while other characteristics of the problem remain the same. In this way, linear models can still incorporate risk without changing the linear functional form in the objective.

In the next section, we give the basic option approach. Section 3 presents the application of option pricing methods for simple capacity evaluation. In § 4, we extend this approach to constrained linear models. Section 5 applies this result to a capacity planning model for a manufacturing firm. Section 6 presents a summary and conclusions.

2. Basic Model for Call Option Valuation

This section is provided as background for readers who are not familiar with financial option pricing. Others should skip to § 3.

In general, an option has value that depends on an underlying asset's value when certain conditions on the underlying asset value are met. In a production context, the underlying asset is the market demand. If a firm had the capability of meeting all demand, no matter how great, the firm would have all the value in this market. Capacity, however, limits the firm's ability to hold the entire market.

A plant has value that increases with the demand as long as that demand does not exceed the plant's capacity. The plant, therefore, has an option on the demand market as long as its capacity is not exceeded. Another way to view this situation is that the owner of the finite-capacity plant holds all of the demand but then sells off an option to other producers to capture any demand beyond the plant's capacity. We will describe this specific model in greater detail in the next section. First, we review some basics from option pricing theory.

To introduce the option concept, we start with the most basic form of an option, a European call option on a non-dividend paying common stock. This option is the right (but not the obligation) to buy one share of the stock at a fixed *strike price*, K, at a given time, T. We assume the current time is t when the share price is S_t. The question is how to value the call option given that we know the stock's volatility (or annual standard deviation on return, assumed constant), σ, and a constant riskfree rate, r_f. The other assumptions are a frictionless market (i.e., no transactions fees) and that the share prices follow an Ito process[1] (see Black and Scholes 1973).

With these assumptions, it is always possible to hold some amount of shares with written (i.e., sold) calls to form a perfect hedge, a riskless portfolio that earns the riskfree rate. To illustrate how these ideas are used, we first consider a binomial approximation (random walk) of the stock price. In this model, prices go up or down relative to their current value and the riskfree rate of return. The probability of going up is the same for each period. This approximation is well known to approach the continuous time model as the number of intervals used in the approximation increases (see, e.g., Cox, Ross, and Rubinstein 1979, and Jarrow and Rudd 1983).

In the binomial model, prices then follow a random walk, increasing from $S(=S_t)$ to Se^{u_f} with probability p or decreasing to Se^{d_f} with probability $1 - p$. We can choose these parameters in a variety of ways. If we assume the price's expected value increases at the same rate as a riskfree asset, then we use the *risk-neutral probability* p_f so that

$$p_f Se^{u_f} + (1 - p_f)Se^{d_f} = Se^{r_f}. \tag{1}$$

Following Jarrow and Rudd (1983), we can divide the interval to expiration, $\tau = T - t$, into I equal subintervals. By choosing

$$u_f = (r_f - \sigma^2/2)\tau/I + \sigma\sqrt{\tau/I},$$

$$d_f = (r_f - \sigma^2/2)\tau/I - \sigma\sqrt{\tau/I},\ p_f = \frac{1}{2}, \tag{2}$$

we achieve a distribution with the same first two moments as the continuous time (lognormal) distribution

[1] An Ito process is a stochastic process that depends on both time and a Brownian motion or Wiener process. The Brownian motion component applies to limits of random walks which fit the efficient market theory that prices respond to new information which arrives in independent increments.

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for every I and convergence in distribution of this binomial model to the lognormal as $I \rightarrow \infty$.

We can also use this model to show how the risk-neutral assumption arises. (For further information on risk-neutral valuation, see, for example, Duffie 1992.) Suppose just a single interval ($I = 1$) and a call exercise price between Se^{u_f} and Se^{d_f} ($Se^{d_f} \leq K \leq Se^{u_f}$). We next show how to construct a risk-free hedge for the call we have sold by purchasing some amount of the share.

The written call requires payment of 0 if Se^{d_f} occurs and $Se^{u_f} - K$ if Se^{u_f} occurs. The return on δ shares is δSe^{d_f} if Se^{d_f} occurs and δSe^{u_f} if Se^{u_f} occurs. The future payoff from δ shares and a single written call is then:

$$FV = \begin{cases} -Se^{u_f} + K + \delta Se^{u_f} & \text{if } Se^{u_f} \text{ occurs,} \\ \delta Se^{d_f} & \text{if } Se^{d_f} \text{ occurs.} \end{cases} \tag{3}$$

A perfect hedge provides the same payoff in the future regardless of the change in stock price. We can find this hedge by choosing δ to equate the payoffs in (3), i.e., $\delta = (Se^{u_f} - K)/(Se^{u_f} - Se^{d_f})$. Since the FV in (3) is the same in each outcome, we have eliminated risk in the future payoff and can discount using the riskfree rate to achieve an equivalent present value of $PV = e^{-r_f\tau}((Se^{u_f} - K)(Se^{d_f})/Se^{u_f} - Se^{d_f}) = \delta S - C_t$, the value of δ shares and a written call. We then have the call value,

$$C_t = (S - e^{-r_f\tau}Se^{d_f})\left(\frac{Se^{u_f} - K}{Se^{u_f} - Se^{d_f}}\right). \tag{4}$$

Note that we were able to construct a hedge in (3) by choosing δ independently from the probability of rise or fall in the price, S_t. We can choose any positive probabilities for p and $1 - p$ without changing the option value. In particular, we can use the risk-neutral probabilities p_f and $1 - p_f$. This means that if we assume the risk-neutral situation and value options with that assumption, we obtain the same result as we would with risk aversion. In a single period, we should now have an equivalent call value given by the simple expectation,

$$C_t = e^{-r_f\tau}p_f(Se^{u_f} - K). \tag{5}$$

To see the equivalence of (4) and (5), note that $p_f = (Se^{r_f} - Se^{d_f})/(Se^{u_f} - Se^{d_f})$ from (1). Substitution in (5) gives (4).

We can continue to find these hedges (recursively) with increasing numbers of intervals I by maintaining different levels of shares δ in each period. The result is that we can again choose a probability consistent with any risk attitude over the intervals I as long as we can change the share levels without penalty or transaction fee. By increasing I, we approach the continuous time situation and can proceed with risk neutral pricing in that framework as well. (A direct approach to eliminate risk in the continuous time case is also possible.) Given these observations, the calculation of the value of a call option does not depend on investors' attitudes toward risk. We can act as if the probabilities were determined in a risk-neutral world.

A result of the risk-neutral approach is the Black-Scholes formula (see Black and Scholes 1973) and various extensions. In the simple European call option we have described, this formula amounts to assuming that stock price returns are lognormally distributed with logarithmic annual mean return, $\mu = E[log(S_1/S_0)]$, and annual standard deviation on the logarithm of returns, $\sigma = \sqrt{\mathrm{Var}[\log(S_1/S_0)]}$. This implies $E[S_1] = S_0e^{\mu+\sigma^2/2}$, where we assume $r_f = \mu + \sigma^2/2$ under the risk-neutral condition. Denoting the resulting distribution function on prices, S_T, at time T by F_f, the call evaluation at time t is to find:

$$C_t = e^{-r_f(T-t)} \int_K^\infty (S_T - K)dF_f(S_T). \tag{6}$$

Note that the distribution function in (6) assumes investors are indifferent toward risk. In reality, expected annual returns should show a premium for risk, but we can still use (6) to evaluate the call option because of the risk neutral equivalence.

3. Options in Operations Management: Example in Capacity Valuation

The key characteristic in the option presented above is that a return depends on the value of an asset from the market (the share price) plus some characteristic unique to the option (the strike price). By holding varying amounts of the asset, the risk-neutral pricing mechanism applies (with the assumptions about the frictionless market and independent price changes).

Options have been applied to investments in many

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areas. When fixed assets are considered, these claims are called *real options.* They include contracts, physical plant and equipment, new ventures, and general R&D decisions. For various examples, see Trigeorgis 1995, 1996, Huchzermeier and Cohen 1996, Kamrad and Lele 1998, and Kamrad and Richken 1994.

Since operational decisions for a competitive firm have a goal of maximizing value, their evaluation fits the general options framework. For example, sales are limited by demand from the market plus capacity that is unique to the firm and includes labor, flow time, reliability, capital, and inventory. Purchases depend on market supply plus unique relationships such as contracts, logistic support, and additional supplier relationships. The decisions of what, where, when, and how to produce, sell, distribute, and buy all affect overall value with the option trait of market influence coupled with individual limitations.

As a simple motivating example, we consider a capacity problem. We will expand this example to general problems but the basic approach is most easily seen in a single product context.

We first suppose that the present value of demand (sales) forecasts for a single product at time T follows an Ito process as in the stock example above. We can also assume a random walk form as well. The fundamental assumption is that our forecast reflects all the information to date; new information arrives periodically with independent increments that can change the forecast in either direction in each time period.

This model fits a situation where the forecast depends on current demand data, customer surveys, other economic factors, etc., that are constantly changing over time as prices do in the market. Of course, demand forecasts are not revised continuously in this way in practice but the model just needs to assume that a continuously modified forecast would behave in this way. For entire markets, this behavior seems reasonable since the change in share prices for the firms in the market reflect changing revenue forecasts (as well as expense forecasts) and are often assumed to follow an Ito process. The result implies that the demand in the future time period follows a lognormal distribution. Other distributions are also possible but involve more complicated transformations to create an equivalent riskfree distribution.

In the full market case, we assume that all demand can be met and that revenues are linear in demand with some margin, c_T, that is fixed earlier (for example, due to competition on price and previous purchasing and labor agreements on costs). Assuming that investors require a return, r, based on the contribution to market risk (measured by firm β, for example) from these revenues, the present value at t for revenues at time T should be c_T multiplied by:

$$\bar{S}_T^t \equiv (e^{-r(T-t)}) \int_{S_T} S_T dF(S_T), \tag{7}$$

where F is now the distribution function on demand S_T at T.

Actual revenues for a firm are, however, limited by capacity. If we assume that capacity limits actual sales to at most K units, then the actual present value of revenues is $c_T(\bar{S}_T^t - C_t)$ where C_t is the value of sales in excess of the capacity K. We can interpret C_t as sales that are lost to a competitor because of inability to meet demand. In effect, the capacity has the effect of selling (writing) a call to a competitor for any demand above the capacity K.

To apply the risk-neutral pricing strategy, we now need to assume that we can hold varying amounts δ of the underlying asset of market demand while we maintain selling the capacity C_t. In other words, we would like to be in position $\delta\bar{S}_T^t - C_t$ while our plant puts us in position $\bar{S}_T^t - C_t$. We, therefore, need to sell $(1 - \delta)$ of the entire market with current value $\bar{S}_T^t$. If our assets as well as our competitors all trade in the market and can be combined so that only the effect of this particular market remains, then there exists a market portfolio with value $\bar{S}_T^t$. By selling $(1 - \delta)$ of this portfolio and revising continuously (without paying transaction fees), the producer can earn a risk-free return.

For decisions on the capacity, we do not need to construct the hedge, we only need to assume that some investor can do so. This completeness of the market and the assumption of no transaction fees may not be entirely justified in practice. An investor might only be able to remove part of the market risk and then have some uncontrollable portion that still remains. This remainder would cause a limit to the extent that a market

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can value our decision. In this case, the price of the option would follow within an interval that contains the price in the unconstrained or complete market case (see, e.g., Karatzas and Kou 1996). For the operational decisions we consider, we assume the explicit calculation of this interval is not critical and that we can assume the value corresponding to the complete market is valid. Any error cannot be too great since we show below that, in extreme cases, this model is consistent regardless of the market assumptions.

When the complete market assumptions apply, then any utility form should yield the same values and decisions as in the complete-market case. The option pricing framework and a utility function approach are still consistent without the complete market assumptions, but option pricing only produces an interval of values or decisions, while defining a specific utility function can produce a single value and decision. Defining the form of the utility function may, however, be difficult especially for a firm with many investors and other stakeholders. Smith and Nau (1995) provide a discussion of the relative merits of the two approaches.

With the complete market assumptions, we can use the formula in (6) for C_t but must first convert from the original distribution in (7) to the risk-neutral distribution in (6). We do this by transforming S_T (with distribution function F) to S_{Tf} (with distribution function F_f) where $E[S_{Tf}]$ discounted with rate r_f is equivalent to $E[S_T]$ discounted with rate r. Defining this equivalent risk-neutral distribution is not, however, necessary as shown in the following theorem, assuming our distribution model.

Theorem 1. *Assuming that sales forecasts follow an Ito process as given above, if F_f is the equivalent risk-neutral distribution function on S_T and r is the rate of return expected on all sales in the case with risk premium, then C_t in (6) is equivalent to*

$$C_t = e^{-r\tau} \int_{Ke^{(r-r_f)\tau}}^{\infty} (S_T - Ke^{(r-r_f)\tau}))dF(S_T). \quad (8)$$

Proof. Since we often will deal with discrete distributions (that seem to correspond more naturally to our sales forecast interpretation), we use the binomial approximation to establish this result. From the binomial derivation (see, e.g., Jarrow and Rudd 1983), we have that

$$C_t = \lim_{I\to\infty} e^{-r_f\tau} \sum_{l=0}^{I} q_I(l)(S_t(e^{u_f})^l(e^{d_f})^{I-l} - K)^+, \quad (9)$$

where $q_I(l)$ is the binomial probability that $S_T = S_t(e^{u_f})^l(e^{d_f})^{I-l}$.

Now, if we suppose an environment with an annual expected rate risk premium of $\alpha = r - r_f$, then the binomial model in the risk averse case would replace u_f and d_f in (2) with

$$u = (r - \sigma^2/2)\tau/I + \sigma\sqrt{\tau/I},$$

$$d = (r - \sigma^2/2)\tau/I - \sigma\sqrt{\tau/I},\ p = \frac{1}{2}.$$

Thus, we would have a binomial distribution with $Se^u = Se^{((r-r_f)\tau/I)+u_f}$ in place of Se^{u_f} and $Se^d = Se^{((r-r_f)\tau/I)+d_f}$ in place of Se^{d_f}. The result is then that $q_I(l)$ is the binomial probability in the risk averse case for $S_T = S_t(e^u)^l(e^d)^{I-l} = e^{\alpha\tau}S_t(e^{u_f})^l(e^{d_f})^{I-l}$. Substituting in (9), we obtain:

$$\begin{aligned} C_t &= \lim_{I\to\infty} e^{-r_f\tau}e^{-\alpha\tau} \sum_{l=0}^{I} q_I(l)(S_t e^{\alpha\tau}(e^{u_f})^l \\ &\quad (e^{d_f})^{I-l} - Ke^{\alpha\tau})^+, \\ &= \lim_{I\to\infty} e^{-r\tau} \sum_{l=0}^{I} q_I(l)(S_t(e^u)^l(e^d)^{I-l} - Ke^{\alpha\tau})^+, \\ &= e^{-r\tau}\int_{Ke^{(r-r_f)\tau}}^{\infty} (S_T - Ke^{(r-r_f)\tau})dF(S_T), \end{aligned} \quad (10)$$

where we use the binomial approximation result again in the last step to obtain the desired result. □

From the result in Theorem 1, we have that the present value of a finite capacity plant is given by:

$$\begin{aligned} PV &= e^{-r\tau}c_T\left(\int_0^{Ke^{(r-r_f)\tau}} (S_T)dF(S_T) + Ke^{(r-r_f)\tau}\right. \\ &\quad \left.(1 - F(Ke^{(r-r_f)\tau}))\right), \end{aligned} \quad (11)$$

which involves the discount rate r that applies to the uncapacitated case, the original distribution function on S_T, and an adjusted capacity level from K to $Ke^{(r-r_f)\tau}$. Note that when capacity is tight so that $F(Ke^{(r-r_f)\tau}) = 0$, then the first term in (11) disappears yielding:

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$$PV_{tight} = e^{-r_f\tau} c_T K, \tag{12}$$

which is indeed the present value for a fixed, riskless future value K at $\tau = T - t$ periods into the future. If the capacity is loose ($F(Ke^{(r-r_f)\tau}) = 1$), then we obtain

$$PV_{loose} = e^{-r\tau} c_T \left(\int_0^{\infty} S_T dF(S_T) \right), \tag{13}$$

which is again the uncapacitated present value.

While the assumption of a sufficiently complete market to enable a risk-free hedge may not always hold, these two extreme condition results should hold under quite broad conditions. In this way, the formula in (11) is at worst an approximation that interpolates between values that are correct at their endpoints and that is exactly the value in complete markets that enable risk-neutral pricing.

The advantage of (11) over manipulation of the distribution function into (6) is that the only necessary additions to reflect risk attitude are the uncapacitated discount factor, $e^{-r\tau}$, and an adjustment of the overall capacity. In optimization, this is especially useful because the actual distribution on S_T is often determined within the problem. The transformation in Theorem 1 allows the modeler to proceed without determining a risk-neutral equivalent at the outset of an optimization procedure.

This result is also used in a slightly different way in McDonald and Siegel (1985) where they consider an option to shut down. The option there depends on price so that when market price does not exceed operating cost, the firm has the option not to produce. Our model uses demand as the underlying stochastic element and essentially subtracts the option not to produce as the competitors' option to buy above the producer's capacity.

The general approach of considering risk-neutral equivalents also appears in Constantinides (1978). Extensions appear in Triantis and Hodder (1990), who consider multiple products and varying constraints, and Andreou (1990), who considers random prices as well as demand. In each of these considerations and the general approach taken in Dixit and Pindyk (1994), the emphasis is on valuation of a given plan or the timing of investment decisions (as in Majd and Pindyck 1987). Our emphasis, however, is on choosing an optimal operational plan that is consistent with this form of evaluation. The next section describes our generalization to this operational optimization model.

4. Generalized Problem and Multistage Stochastic Linear Program

To generalize the result in the previous section, suppose instead of a single dimensional S_T that S_T is a random vector with support, $\Sigma_T \subset \Re^n$, and a probability P_T (so that formally there is an associated probability space, $(\Sigma_T, \mathcal{B}_T, P_T)$). Suppose a linear revenue vector c_T so that the uncapacitated present value at time $t = T - \tau$ of the period T revenue is:

$$PV = e^{-r\tau} \left(\int_{\Sigma_T} c_T'(S_T) P_T(dS_T) \right), \tag{14}$$

where $'$ denotes a transpose. The result in (14) may also be expressed in an optimization problem. The main requirement is that the returns have a symmetric form that enables determination and use of the appropriate discount factor with rate r.

Now, suppose that actual revenues, $x_T \in \Re^n$, are restricted so that x_T must satisfy both $x_T \le S_T$, and $Ax_T \le h_T$. The expected future revenues become:

$$FV = c_T' \left(\int_{\Sigma_T} (S_T) P_T(dS_T) - \int_{\Sigma_T} \max_{Ax_T \le h_T} (S_T - x_T)^+ P_T(dS_T) \right), \tag{15}$$

which has the same form as the simple capacity evaluation with the second term being analogous to (6).

To evaluate the present value of the contingent claim term, $\int_{\Sigma_T} \max_{Ax_T \le h_T} (S_T - x_T)^+ P_T(dS_T)$, in (15), we can make assumptions similar to the single product case. We suppose a frictionless (no transaction fee) market with no riskless arbitrage (positive profits without initial investment), but with short selling, continuous trading, and having each component of S_T follow an Ito process. We also need to assume n instruments other than the contingent claim term that also depend on the components of S_T in order to be able to create a risk-free hedge. These instruments may, for example,

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represent other products that can depend on the same demand factors as S_T and that would be available in the market in some way. With a wide range of products, this assumption becomes quite realistic.

With these assumptions, we can again (see, for example, Hull 1997) assume that a riskless hedge is possible with these securities and that, therefore, we can again employ the risk-neutral valuation method.

We continue to make the assumptions consistent with risk-neutral valuation. We can then find a present value for FV in (15) by using a probability measure, P_{Tf}, that includes no risk premium on the rate of increase in the present value of the expectation of S_T. The present value of the contingent claim portion of (15) becomes:

$$C_t = e^{-r_f\tau} c_T' \int_{\Sigma_T} \max_{Ax_T \le h_T} (S_T - x_T)^+ P_{Tf}(dS_T). \quad (16)$$

By applying an annual risk premium, e^{r-r_f}, to each component of the present value of the expectation of S_T as in Theorem 1, we obtain a probability measure, P_T, such that $P_T(A) = P_{Tf}(e^{(r_f-r)\tau} A)$ for any set $A \subset \mathcal{B}$ and thus obtain:

$$\begin{aligned} C_t &= e^{-r_f\tau} e^{-\alpha\tau} c_T' \int_{\Sigma_T} \max_{Ax_T \le h_T} e^{\alpha\tau}(S_T - x_T)^+ P_{Tf}(dS_T) \\ &= e^{-r\tau} c_T' \int_{\Sigma_T} \max_{Ax_T \le h_T} (S_T - e^{\alpha\tau}x_T)^+ P_T(dS_T) \\ &= e^{-r\tau} c_T' \int_{\Sigma_T} \max_{Ax_T' \le e^{\alpha\tau} h_T} (S_T - x_T')^+ P_T(dS_T), \end{aligned}$$

where in the last step we have substituted x_T' for $e^{\alpha\tau}x_T$.

The result is the following corollary to Theorem 1.

COROLLARY 1. *Assuming the conditions for a riskless hedge given above, the present value of the result in (15) is equivalent to*

$$e^{-r\tau} c_T' \left(\int_{\Sigma_T} (S_T) P_T(dS_T) - \int_{\Sigma_T} \max_{Ax_T \le e^{\alpha\tau} h_T} (S_T - x_T)^+ P_T(dS_T) \right). \quad (17)$$

From Corollary 1, we can derive present value equivalents for a broad range of stochastic optimization problems with linear constraints. We consider the general form of a multistage stochastic program with fixed linear recourse to maximize expected utility. We express this model as

$$\max_{x_0, \boldsymbol{x}^1, \ldots, \boldsymbol{x}^H} cx_0 + E_\xi[U^1(\boldsymbol{c}^1\boldsymbol{x}^1) + \ldots U^H(\boldsymbol{c}^H\boldsymbol{x}^H)] \quad (18)$$

$$\begin{array}{llll} \text{s.t.} & Ax^0 & = b, & \\ & \boldsymbol{T}^1 x^0 + \boldsymbol{W}^1 \boldsymbol{x}^1 & = \boldsymbol{h}^1, & a.s., \\ & \vdots & \vdots & \\ & \boldsymbol{T}^H x^{H-1} + \boldsymbol{W}^H \boldsymbol{x}^H & = \boldsymbol{h}^H, & a.s., \\ & 0 \le x^0,\ 0 \le \boldsymbol{x}^t, & t = 1, \ldots, H, & a.s., \\ & \boldsymbol{x}^1, \ldots, \boldsymbol{x}^H \text{ nonanticipative,} & & \end{array}$$

where bold face indicates random quantities, the decisions, $x^t \in \mathcal{R}^{n_t}$, and the right-hand side parameters, $b \in \mathcal{R}^{m_0}$ and $h^t \in \mathcal{R}^{m_t}$, with other matrices defined accordingly. The functions, U^t, represent the present utility of objective outcomes in time t. The term *nonanticipative* indicates that $\boldsymbol{x}^t$ can only depend on outcomes up to time t (see Wets 1980 or Dempster 1988, for details). (Formally, we assume the stochastic elements are defined over a canonical probability space $(\Xi, \sigma(\Xi), P)$, where $\Xi = \Xi_1 \otimes \ldots \otimes \Xi_H$, and the elements of Ξ_t are $\{\xi^t = (T^t, W^t, h^t, c^t)\}$.)

The outcomes in Ξ represent possible future realizations of the uncertain quantities. They are called *scenarios*. If a finite number of these scenarios is considered, the expected value can be written as a finite sum of the objective values of these decisions. The requirement that each stochastic constraint must hold almost surely may be enforced by defining the same set of constraints for each realization. We can use the result in Corollary 1 now to construct an equivalent deterministic linear program to (18).

First, we assume a discrete distribution with N_t possible outcomes for ξ_t in stage t yielding $\bar{N}_t = N_1 \times \ldots \times N_t$ total scenarios at time t. This distribution may, for example, be generated by a generalization of the binomial model used for a single product. We also assume a financial objective so that, if we knew the contribution of the current project to overall portfolio risk, we could find a discount factor to represent the utility and reduce (18) to a linear program. This is consistent with financial portfolio theory.

Using Corollary 1, we do not need to determine this contribution completely. We assume only that we know an appropriate discount factor for the revenues

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without explicit capacity constraints. This assumption is met, for example, if every product considered has equivalent correlation to the market overall. In general, this might be true, for example, for a single automotive manufacturer or a single telecommunications provider but not perhaps for a conglomerate that has a choice of which markets to enter.

With these assumptions (and, for simplicity, an assumed constant discount rate), we find an equivalent program to (18). We do so by first assuming that the constraints in (18) can be split into two categories. The first corresponds to variations from the entire market while the second corresponds to the unique circumstances of the firm. This decomposition of the constraints is possible when sales are limited only by demand (orders) and production capacity. In each period, then, the constraints

$$\boldsymbol{T}^t\boldsymbol{x}^{t-1} + \boldsymbol{W}^t\boldsymbol{x}^t = \boldsymbol{h}^t$$

can be written as

$$W^t x^t \le S_t \tag{19}$$

and

$$\bar{W}x^t \le K_t + \bar{T}x^{t-1}, \tag{20}$$

where $W^t = [0, I]$ with zero entries corresponding to components of x^t corresponding to capacity investments and the identity corresponding to products that can be sold up to some random order vector S_t, $\bar{W}$ might have the same form as $\bar{W}^t$ or could have different entries depending on the effective capacity of each production resource (e.g, $\bar{W} = [0, e^T]$ for $e = (1, \ldots, 1)^T$ would correspond to a single production resource capable of producing all products), K_t is the existing capacity of each resource, and $\bar{T}$ represents the capacity received in each production resource per unit of investment in previous periods (e.g., $\bar{T} = [I, 0]$ for unique resources for each product and $\bar{T} = [1, 0]$ for a single flexible production resource).

The result of this decomposition is that the constraints in (19) depend on the entire market for the products. The constraints in (20) depend on the specifics of the firm's capacity. Faced only with the first set of constraints, we could discount the period t objective with the overall rate from the CAPM for this market. (We need to assume that all of the products' demands have the same correlation to the overall market.)

The second set of constraints, however, represents characteristics of the individual firm. These are the constraints on overall capacity that the firm faces. In this case, we can use the result of Corollary 1. The return in period t has a present value at time 0 that is the maximum revenue that can be produced without exceeding either the demand S_t or the adjusted constraint as in (20),

$$\max_{x_t} e^{-rt} c_t[\mathrm{E}[x_t] \text{ s. t. } W'x_t \le S_t,$$

$$\bar{W}x^t \le e^{\alpha t} (K_t + \bar{T}x_{t-1})]. \tag{21}$$

From (22), we can accumulate the present values of all period t decisions into the following form.

$$\max cx^0 + \sum_{k=1}^{\bar{N}_1} p_k^1 e^{-r} c_k^1 x_k^1 + \cdots + \sum_{k=1}^{\bar{N}_H} p_k^H e^{-rH} c_k^H x_k^H$$

subject to

$$Ax^0 = b, \tag{22}$$

$$T_j^t x_{\gamma(j,t)}^{t-1} + W^t x_j^t = h_j^t,\ j = 1, \ldots, \bar{N}_t,\ t = 1, \ldots, H,$$

$$\bar{T}_j^t x_{\gamma(j,t)}^{t-1} + \bar{W}^t x_j^t \le e^{\alpha t}\bar{h}_j^t,\ j = 1, \ldots, \bar{N}_t,\ t = 1, \ldots, H,$$

$$0 \le x_j^t,\ j = 1, \ldots, \bar{N}_t,\ t = 1, \ldots, H,$$

where

- x_i^t = decision vector to take in stage t given outcome i;
- p_i^t = probability that scenario i in stage t occurs, $i = 1, \ldots, \bar{N}_t$;
- (c_i^t, h_i^t, T_i^t) = cost, right-hand side vectors, and technology matrix for scenario i in stage t;
- W^t = recourse matrix for stage t, $W^t \in \mathcal{R}^{m_t \times n_t}$;
- $\gamma(j,t)$ = $\lfloor (j-1)/N_t \rfloor + 1$, the *ancestor* (scenario sharing history up to time $t-1$) of node j in stage $t-1$.

In (22), we have split the period t constraints into two components, T', W', h' and $\bar{T}$, $\bar{W}$, $\bar{h}$. These could be as given in (19) and (20) or might be further generalizations. The principle is that the first set of constraints depends on overall market conditions (such as maximum possible demand) and not on specific capacity decisions. These constraints correspond to the

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limits on sales in (17) from the distribution of S_T. The second set with coefficients, T', W', and h', distinguishes specific resource restrictions that prevent realizations of full market potential. These are the $Ax_T \leq e^{\alpha T}\bar{h}_T$ constraints in (17).

The ability to make this distinction between unique constraints (equivalent to unique risk) and market constraints determines whether constraints should be adjusted or not. In general, this distinction may be difficult to determine; however, capacity planning problems often directly lead to this division as we noted above. This application to capacity planning is given in the next section.

5. A Capacity Planning Example

We consider a manufacturer with certain plants with installed capacity to produce specific products. The capacity planning question is to determine whether additional capacity should be installed at a plant where no capacity for a product currently exists. This additional capacity would allow the plant to continue production if demand for the new product is higher than existing capacity at other plants and if the demand for other products at the new plant is lower than the existing plant capacity.

In the specific example that we consider, the facilities assemble different models of the same general product. The assembly operations, however, require specific fixtures and tools for the individual models of the product. Normally, this equipment is specific to a given facility so that this facility only produces a single primary product. The capacity decision is whether to install additional tooling and fixturing to allow a facility to produce a secondary model in addition to its primary model.

In general, flexibility of this type can almost never be justified by point forecasts of demand for the various models since it is almost always cheaper to install a fixed amount of same-model capacity at an existing capacity than to install additional fixturing at a different facility. The capacity in these cases generally depends on the degree to which the assembly operations are decomposed into operations that can be performed in parallel. Additional costs for increasing capacity at a single facility do not generally have significantly greater fixed costs but can be accommodated by rebalancing the system and creating more parallel operations with shorter overall cycle time. A single-forecast deterministic model generally favors this low-capital cost solution to produce a fixed amount of capacity and uses only a single facility.

A stochastic model is necessary to include the value of capacity that can be used for other purposes, such as producing the high-demand product in place of the low-demand product when these demand levels are not known a priori. The stochastic value is, therefore, necessary in justifying flexible capacity. Flexibility might even be justified when the demands are perfectly correlated (see, for example, van Mieghem 1998).

Another difficulty in traditional analyses besides the problem in single-point forecasts is that single forecasts generally ignore the limit on production or sales that the capacity forces. Using the expectation of demand without capacity restriction is, therefore, optimistic because the capacity limit effectively truncates the demand distribution.

A third problem is that the discount factor or hurdle rate for a project is not traditionally changed based on limited production capacity. (One manufacturing firm did report to the author on a practice of using a higher hurdle rate for certain products that coincidentally had larger capacities than other products, but, in this case, the higher hurdle rate was used to reflect the volatility of that product's market.) As noted above, tighter capacity projects should have lower hurdle rates while looser capacity projects should have higher rates. Using a single rate reflecting overall industry risk again may cause projects that use capacity more fully (as in flexible systems) to be undervalued.

In determining the extent of flexible capacity installation, the decision problem is to trade off the costs of adding capacity against the potential revenue from additional sales due to the extra capacity. This basic problem has been considered by Eppen et al. (1989) where they use a one-sided loss utility function and apply a mixed integer, stochastic linear programming model. Valuation procedures also appear in Triantis and Hodder (1990) and Andreou (1990) who provide consistent financial models for given decisions. We combine these results in a single model. The advantage is that we avoid somewhat ad hoc procedures such as

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downside risk and use an objective consistent with the overall performance of the firm. The risk of the decision is actually determined within the model but we are able to compensate for this without presupposing a decision.

For two-stages, the model from (22) becomes:

$$\max \sum_{i=1}^{L} \sum_{j=1}^{R} \left[c_{ij} x_{ij}^0 + e^{-r} \sum_{k=1}^{\bar{N}_1} p_k^1 c_{k,ij}^1 x_{k,ij}^1 \right]$$

subject to

$$\sum_{i=1}^{L} \sum_{j=1}^{R} c_{ij} x_{ij}^0 \leq b, \tag{23}$$

$$\sum_{i=1}^{L} x_{k,ij}^1 \leq h_{jk}^1, \quad j = 1, \ldots, R;\ k = 1, \ldots, \bar{N}_1,$$

$$T_{ij}'^1 x_{ij}^0 + x_{k,ij}^1 \leq e^{\alpha} h_{1,ij}'^1, \quad i = 1, \ldots, L;\ j = 1, \ldots, \ldots, R;\ k = 1, \ldots, \bar{N}_1,$$

$$\sum_{j=1}^{R} x_{k,ij}^1 \leq e^{\alpha} h_{2,i}'^1, \quad i = 1, \ldots, L;\ k = 1, \ldots, \bar{N}_1,$$

$$x_{ij}^0 \in X_{ij}^0,\ x_{k,ij}^1 \geq 0, \quad i = 1, \ldots, L;\ j = 1, \ldots, \ldots, R;\ k = 1, \ldots, \bar{N}_1,$$

where

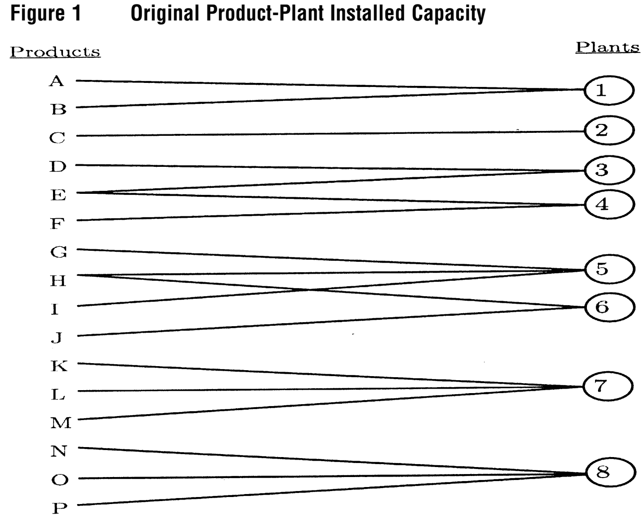
- L = number of plants,
- R = number of products,
- x_{ij}^0 = new capacity installed for product j at plant i,
- c_{ij} = cost (negative revenue) for installing each capacity unit at start,
- $x_{k,ij}^1$ = actual production of product j at plant i under scenario k,
- $c_{k,ij}^1$ = unit production margin for j at i under scenario k,
- b = initial budget constraint,
- h_{jk}^1 = maximum possible sales (demand) of product j under scenario k,
- $T_{ij}'^1$ = negative of factor for increase in production capacity for j at i,
- $h_{1,ij}'^1$ = original production capacity for j at i,
- $h_{2,i}'^1$ = total production capacity for all products at i,
- X_{ij}^0 = set of possible capacity levels (may be discrete).

The capacity installation costs again refer to additional fixturing that would be necessary to allow production of a different product. This model was used in Sims (1992) to determine optimal additional flexible capacity levels for a manufacturer with a class of products with eight plants and sixteen products. The data given are the same as used in Jordan and Graves (1991, 1995) that were modified to protect proprietary data. The costs for installing capacity were also proprietary and modified for this study but were typical for automotive demand and assembly facilities.

An initial assumption was that capacity decisions were made once and that production could not be held in inventory from one period to the next (i.e., sales were lost above the capacity limit). The original installed capacities for the products at each of the plants are indicated in Figure 1. The total capacity in these plants and the expected demands are given in Table 1.

Given this network characterization, the model determines where to install additional capacity to maximize the value added to the firm by these capacity decisions. Demand forecasting in this situation is difficult due to limited distribution information. Historical data may exist on some older products but newer product forecasts must be based on other techniques. In general, the level of information is at most an expected

Figure 1 Original Product-Plant Installed Capacity



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value for demand on each product, an estimate of a variance, and some general guidelines for correlation coefficients.

For this model, the general variance information also followed Jordan and Graves (1991, 1995) in assuming 40% of the expected demand as a standard deviation for a single product demand. Correlation coefficients were assumed to be 0.3 for products in groups, A–F, G–M, and N–P. For products from different groups, a 0.0 correlation coefficient was assumed. To accommodate this limited distribution information, approximations (as in Birge and Wets 1986, and Birge and Louveaux 1997) were created to find a distribution with few samples that matched the given distribution information as closely as possible.

For computing the solution to (23), the basic procedure used the integer programming capability of IBM's OSL (IBM 1991) on an IBM RS6000 workstation. This software enabled solutions of relatively large capacity expansion models, although most analyses considered the six-scenario problem in order to have relatively quick turnaround times for varying parameter combinations on the RS6000 processors.

The linear programming relaxations of the basic model in (24) were also solved using the nested decomposition method, ND-UM (Birge et al. 1996), which uses the OSL solver for each subproblem. As en example of the efficiency of these procedures and their capability for solving realistically sized problems, we present the linear programming times for varying numbers of scenarios in Table 2.

The model in (23) only includes a single year's production. When only one year of production is considered, a single additional capacity (J at 7) is added. The single-year model would correspond to a situation where every product has only one year remaining in its life cycle and all fixturing must be replaced at that time for the next generation product. The relative lack of flexibility in this situation is due to the high cost of installing duplicate fixtures at the different plants. In another industry, e.g., apparel, the cost of additional flexibility might be much less so that even single-season fashion products would include multiple flexible capacity installations.

The model in (23) was extended to multiple product years by assuming that all capacity investments only occur in the first period as x^0_{ij}. In this way, each constraint with $x^1_{k,ij}$ is duplicated for period t with $x^t_{k,ij}$) replacing $x^1_{k,ij}$ and $e^{\alpha t}$ replacing e^{α}. The discount factor in the objective is then e^{-rt} instead of e^{-r} for the period t terms. This restriction of capacity investment to the first period corresponds to the situation that was reported in this industry of no serial correlation between demand in one period and demand in the next period, the same expectations and correlations among product demands in each period, and no possibilities for inventory or backordering from one period to the next. This property called *block-separable recourse* (see Louveaux 1986) ensures that all capacity investment decisions occur in the first period. The two-stage structure of (23) can then be used to model multiple periods.

When two years of production are considered, two additional capabilities are added (P at 5 and G at 3). As more production years are included, more capacity is added. When five years are assumed a total of nine capacities are added. Including ten years yields only

Table 1 **Total Plant Capacities and Expected Demands**

Plant	Capacity (1000s)	Product	Mean Demand (1000s)	Product	Mean Demand (1000s)
1	380	A	320	I	140
2	230	B	150	J	160
3	250	C	270	K	60
4	230	D	110	L	35
5	240	E	220	M	40
6	230	F	110	N	35
7	230	G	120	O	30
8	240	H	80	P	180

Table 2 **Capacity Planning Solution Times**

Stages	Scenarios	Rows	Columns	OSL (CPUs)	ND-UM (CPUs)
2	4	478	966	4.7	1.7
2	8	894	1.8E03	11.9	2.5
2	16	1.7E03	3.5E03	35.4	3.6
3	36	4.4E03	9.0E03	230.7	15.2
3	256	2.8E04	5.7E04	12361	140.5
4	4096	4.5E05	9.2E05	Failed	5024

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two additional flexible capacity installations over the five-year decision. Little additional flexibility is justified at that point because the network of production facilities is now able to absorb virtually all of the demand (as described in Jordan and Graves 1995).

Qualitatively, a traditional model for capacity investment based on single-product demand expectations would produce no flexibly capacity investments as explained earlier. Incorporating demand scenarios and the capacity constraint produces some flexible capacity investments but using the overall market discount factor without adjusting capacity as in (21) leads to less investment than is optimal. In this case, value is lost relative to the optimal decision.

The order of the new installations as the years increase also gives qualitative insight by providing a ranking of investment decisions. Strategic decisions on the timing of new product introductions (hence the number of years for existing models) can then be made with greater information about the effect on operational resources (capacity).

The expected number of lost sales and expected utilizations can also be considered. These values give the decision makers more information about the value of flexible capacity. In general, flexible capacity increases both values. In this model, utilizations increased to 98% and lost sales declined by 80% as flexibility was added up to the ten-year model lifetimes.

This model shows that capacity planning models with varying possible risk outcomes can be incorporated into a single linear model that is solvable with current techniques. The actual model was provided to a manufacturer for use in evaluating the effect of flexible capacity. Some other extensions, such as intermediate-stage capacity additions and variable product lifetimes, could represent the actual situation more fully, but this simple model gives much of the information used for actual decision making.

6. Conclusions

Measuring risk attitudes in operations management problems often causes difficulties for modelers as well as decision makers. We have shown that application of option pricing can readily incorporate financial risk attitudes into linear models even before the levels of risk have been determined. Our capacity planning example illustrates an especially important case. In general, however, a modeler must distinguish constraints as related to market or project-specific restrictions. Finding this distinction is a valuable topic for future research.

Other issues of importance to this development include the assumptions of the completeness of the market and the absence of transaction costs. These issues are considered within a utility function approach in Smith and Nau (1995). Specific evidence may be necessary to justify these assumptions completely in practice, but the model matches general portfolio conditions in the extreme cases of tight and loose constraints. Further studies should explore the decision-making significance of these additional assumptions.[2]

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