Management Science

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MANAGEMENT SCIENCE
Vol. 58, No. 3, March 2012, pp. 476–492
ISSN 0025-1909 (print) | ISSN 1526-5501 (online)

http://dx.doi.org/10.1287/mnsc.1110.1433

Constant Proportion Debt Obligations: A Postmortem Analysis of Rating Models

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In its complexity and its vulnerability to market volatility, the constant proportion debt obligation (CPDO) might be viewed as the poster child for the excesses of financial engineering in the credit market. This paper examines the CPDO as a case study in model risk in the rating of complex structured products. We demonstrate that the models used by Standard and Poor's (S&P) and Moody's fail in-sample specification tests even during the precrisis period and in particular understate the kurtosis of spread changes. Because stochastic volatility is the most natural explanation for the excess kurtosis, we estimate an extended version of the S&P model with stochastic volatility and find that the volatility-of-volatility is large and significant. An implication is that agency model-implied probabilities of attaining high spread levels were biased downward, which in turn biased the rating upward. We conclude with larger lessons for the rating of complex products and for modeling credit risk in general.

Key words: credit risk; securitization; structured credit; rating agencies; stochastic volatility
History: Received September 25, 2009; accepted June 17, 2011, by Wei Xiong, finance. Published online in *Articles in Advance* October 14, 2011.

1. Introduction

The constant proportion debt obligation (CPDO) appeared at the peak of the market for structured credit products. The first CPDO issue, ABN AMRO's Surf, was arranged in the summer of 2006 and closed in November of the same year. The Surf notes were rated AAA by Standard and Poor's (S&P) yet offered a coupon 200 basis points over London Interbank Offered Rate (LIBOR). Because the AAA corporate spread in this period hovered close to LIBOR, the Surf deal earned considerable attention (not all sanguine) in the financial press and industry awards such as *Risk* magazine's "Deal of the Year" in February 2007.[1] When credit markets came under stress in 2007, CPDOs were among the first to unravel. The first CPDO default, on a financial-only CPDO issued by UBS, arrived in late November 2007. The defaulted notes had been rated Aaa by Moody's at issuance in March 2007.[2] Since 1920, no Aaa rated corporate bond has ever defaulted within a *two-year* horizon (Emery et al. 2008, Exhibit 26). Subsequently, it appears that most CPDOs were unwound voluntarily by investors, at significant loss, or forcibly unwound. Our calculations suggest that most or all of any remaining CPDO notes would likely have defaulted by late November 2008.

In its complexity and its vulnerability to market volatility, the CPDO might be viewed as the poster child for the excesses of financial engineering in the credit market. This paper offers a case study of the CPDO. Our focus is on model risk in the methodologies used by S&P and Moody's in assigning CPDO ratings. We show that these models would have assigned very low probability to the spread levels realized in the investment grade corporate credit default swap (CDS) market in late 2007. The spread levels realized in the first quarter of 2008 would have been assigned minuscule probabilities. Had the models put nonnegligible likelihood on attaining these high spread levels, the CPDO notes could never have achieved investment grade status. One can debate whether a model calibrated to available data in 2006 should have been expected to allow for the crisis conditions of 2008. However, the spread levels realized in late 2007 are qualitatively comparable to the levels seen in 2002, so they ought not to have been taken as extreme events.

[1] The Surf deal also won the "Innovation of the Year" at the 2006 International Financing Review (IFR) Awards and was featured as one of six "Deals of the Year 2006" in *Euromoney*, February 2007.

[2] S&P and Moody's use slightly different nomenclature for their ratings. Despite some important differences in methodology (Peretyatkin and Perraudin 2002), investors typically view the rating scales as parallel and interchangeable. We will use Moody's notation (Aaa, Aa, A, Baa, etc.) when specifically discussing Moody's ratings or spreads tied to Moody's ratings. Otherwise, we will follow the S&P convention (AAA, AA, A, BBB, etc.).

As a more rigorous form of assessment, we demonstrate that the models used by S&P and Moody's fail in-sample specification tests even during the precrisis period and in particular understate the kurtosis of spread changes. Because stochastic volatility is the most natural explanation for the excess kurtosis, we estimate an extended version of the S&P model with stochastic volatility, and find that the volatility-of-volatility is large and significant. An implication is that agency model-implied probabilities of attaining high spread levels were biased downward, which in turn biased the rating upward.

Our paper complements a number of recent studies that have emphasized potential pitfalls in financial innovation. Gennaioli et al. (2012) consider the possibility that investors may neglect unlikely risks in assessing nearly riskless securities. This neglect leads to excessive issuance of securities engineered to be exposed to the neglected risks. If we interpret our results through the lens of their model, the rating agencies neglected the possibility of an extended period of high and volatile spreads. Because the agencies serve as investors' delegated monitors, this led to the design and issuance of a security that proved particularly vulnerable to that risk. In a case study of a structured equity product sold to retail investors, Henderson and Pearson (2011) conclude that complexity in security design can be exploited to overprice financial products. For the CPDO, the relevant concern is not mispricing but rather misrating. Its complexity (specifically as embedded in its dynamic leverage rule) magnifies any flaws or omissions in the models employed in the rating process. Coval et al. (2009) and Heitfield (2010) show that the rating of senior tranches of collateralized debt obligations (CDOs) is highly sensitive to default probabilities and other model inputs. Even small variations in these inputs can result in large changes in the model-implied default probability of a tranche. Our findings are consistent with these warnings and extend the critique beyond parameter uncertainty to misspecification of essential elements of the model itself.

The academic literature on CPDOs as a specific subject is sparse. Cont and Jessen (2009) develop a parsimonious top-down stochastic intensity model for rating CPDOs. They show that the model-implied rating can be highly sensitive to small changes in certain parameters governing the evolution of spreads. Our study serves a complementary purpose, in that we take the agency models as given and test whether the model-implied forecast densities are consistent with the observed path of spreads prior to and during the credit crisis.

In May 2008, the *Financial Times* reported that Moody's CPDO ratings may have been affected by software coding errors. If it were the case that the poor performance of the agency ratings could be attributed to coding errors or subsequent model modifications, then our analyses would be of little interest. As we will describe in detail in §4.2, our conclusions appear to be robust. Furthermore, there have been no reports that coding errors played a significant role in the S&P rating of CPDOs.[3]

We begin in §2 with a review of the mechanics of the CPDO strategy. We compare the CPDO to earlier structured products, consider the rationale behind the strategy, and summarize the risk characteristics as understood in late 2006 and early 2007. In §3, we document the movement of spreads in the corporate CDS market in 2007 and 2008 and the impact of these spread movements on CPDO performance. The models used by the rating agencies to rate CPDOs are set out and evaluated in §4. We extend the model of S&P in §5 to allow for stochastic volatility. Section 6 offers some larger lessons for the rating of complex products and for modeling credit risk in general.

2. The CPDO Design: Rationale and Risk Characteristics

The CPDO is a fully funded structured credit product.[4] A special purpose vehicle (SPV) issues floating rate notes and receives par from the investors. The proceeds are held in a cash account as collateral for a long position (i.e., selling protection) in a portfolio of credit default swap (CDSs).[5] For the "plain-vanilla" index CPDO, the portfolio is composed in equal shares of the CDX North American and iTraxx European investment grade five-year CDS indices.[6] This portfolio is sometimes known as the Globoxx index, although the CDX and iTraxx indices are traded separately. The notional size of the long position is a multiple of the size of the cash account, and in this sense a CPDO is *leveraged*. The maturity of a CPDO is typically 10 years.

Leverage is adjusted dynamically over the lifetime of the CPDO. Each day, the manager of the SPV calculates the *shortfall*, which is the gap between the current net asset value (NAV) of the SPV holdings (i.e.,

[3] In June 2008, S&P reported the discovery of one error in a trial version of one of its CPDO-related models but stated that the correction of the error did not alter the rating of the affected CPDOs (Jones 2008c).

[4] More expansive descriptions of CPDO mechanics are provided by Saltuk et al. (2006) and Lucas et al. (2007).

[5] A credit default swap is an agreement between two counterparties to swap a fixed premium in return for a payout in case of a credit event (generally bankruptcy or debt restructuring), where the payout is linked to the actual economic loss incurred on the bonds of the reference entity.

[6] The two indices consist of portfolios of 125 single-name CDS contracts referencing investment grade corporates and financials in North America and Europe, respectively. Beyond investment grade status, inclusion in the indices is subject to general criteria, such as sufficient single-name CDS liquidity.

the sum of the cash account and the mark-to-market value of the CDS index portfolio) and the present value of all future contractual payments, inclusive of management fees. The target leverage is given by an increasing function of the shortfall. Details of the specification vary across issuers, but a typical formula is

$$\text{multiplier} \times \frac{\text{PV(future liabilities)} - \text{NAV}}{\text{index spread} \times \text{remaining maturity}}.$$

Leverage is subject to an upper bound (set at 15 in the first CPDO) and is typically at the upper bound at issuance.

If the shortfall decreases to zero, the CDS portfolio is unwound and the proceeds held as cash to fund remaining contractual payments. This is referred to as a *cash-in* event. On the other hand, if NAV falls to a predetermined lower threshold (usually 10% of par), the CDS portfolio is unwound and remaining funds are paid out to the noteholders. This *cash-out* event is equivalent to a default, where the recovery rate for the noteholders is (at best) the cash-out threshold level.

The set of names referenced by the CDS portfolio is not static. In an index CPDO, changes in the portfolio are determined by changes in the composition of the CDS indices themselves. On predetermined biannual dates known as *index roll dates*, the CDS indices are reconstituted as new *series*. Reference entities that have fallen below investment grade or for which CDS trading is no longer liquid are excluded from the new series and replaced with new names. The new index is referred to as the *on-the-run series*, whereas the previous index is referred to as the *off-the-run series*. CDS index contracts refer to a specified series of the index and continue to reference the names of the specified series even after a new series has appeared. Indeed, contracts on the off-the-run indices are still traded, but liquidity tends to drop quickly after the index roll date. The CDS portfolio of an index CPDO is kept in the on-the-run indices. Consequently, upon or shortly after each index roll date, the SPV must purchase protection on the off-the-run index and sell protection on the on-the-run index.

Following the introduction of the first-generation CPDO, several variants were introduced. In a *bespoke* CPDO, the SPV takes a long position in a portfolio of selected single-name CDS rather than of standard CDS indices. Individual names in the porfolio would typically be replaced upon downgrade below a prespecified rating. CPDO deals referencing bespoke portfolios appeared by March 2007.

Relative to the first Surf issue, subsequent CPDOs issued typically sought a more conservative risk/return profile. Coupons on CPDO notes and arranger fees fell roughly by half to reduce the need for high leverage. Maximum leverage was sometimes reduced, say to 10.

To provide intuition on the essential characteristics of CPDOs, we compare and contrast them to earlier structured credit products. The most familiar structured product is the CDO, in which the SPV pools together assets such as bonds, loans, or residential mortgage backed securities and issues multiple classes of notes that differ in the seniority of claim to the cashflows of the SPV assets. This *tranching* of claims allows for the creation, from a single pool of assets, of a number of distinct securities (called *tranches*) with varying levels of risk (and rating) and hence promised coupons.

In the form of CDO most directly comparable to a CPDO, the CDO asset pool would consist of an unleveraged long position in a pool of CDS. The essential difference is that the structuring of a CDO is on the liability side of the SPV balance sheet, whereas the structuring of a CPDO (which issues only a single class of notes) is on the asset side of the SPV balance sheet (i.e., in the form of the time-varying leverage rule). The tranching of a CDO is the source of the sensitivity of tranche valuation to heterogeneity among and dependence between positions in the asset pool. Heterogeneity and dependence do not materially influence the valuation of CPDO notes, which is therefore relatively simple and transparent. The investors in CPDO notes hold pro-rata shares in the NAV of the SPV less the present discounted value of the management fees.[7] Because the SPV is invested exclusively in liquid CDS positions and in cash, the NAV is easily calculated on a daily basis.

The most direct antecedent to the CPDO is the leveraged super senior (LSS) product. The investor in an LSS note sells leveraged protection on a super senior tranche of a CDO. For example, with note proceeds of \$100 million as collateral, the LSS sells \$1 billion notional of protection on a 20%–100% tranche of a reference portfolio.[8] In this case the leverage is 10. Unlike the CPDO, the notional leverage in the LSS structure is constant over the lifetime of the deal, so the LSS has a lesser degree of path dependence on spreads. Another difference is that the portfolio held by the LSS SPV is fixed (i.e., it does not refresh to eliminate firms that have fallen below investment grade). Therefore, relative to the CPDO, the LSS note

[7] The present discounted value of the management fees depends on the risk-neutral probability of a cash-out event, so valuation of the CPDO notes has (slight) sensitivity to pool heterogeneity and dependence and to contractual terms such as the notes' coupon rate.

[8] In this tranche, the investor absorbs losses only when default losses in the underlying portfolio exceed 20%.

has greater exposure to the risk of defaults in the reference pool and especially to the systematic component of default risk.

The CPDO structure exploits several empirical regularities in investment grade corporate credit markets. First, credit spreads appear to embed a high risk premium. Put another way, the empirical default frequency of investment grade corporate credits explains only a small fraction of the observed risk-neutral default probability embedded in market prices (e.g., Elton et al. 2001). This would suggest that an investment grade CDS portfolio could be leveraged to achieve high returns with relatively modest default risk.

Second, investment grade default intensities appear to be mean-reverting in time series.[9] Consequently, CDS spreads on investment grade names tend to mean-revert. This underpins the CPDO's "double or nothing" strategy. If spreads balloon and the SPV loses value, the CPDO increases leverage (provided that the current leverage is not already at the maximum leverage allowed). Mean-reversion implies that future spread movements are more likely to be negative than positive and so more likely to earn gains than losses to NAV. Similarly, by decreasing leverage when spreads tighten, the CPDO avoids losses in the likely event that spreads subsequently increase.

Third, the term structure of credit spreads is typically upward sloping for investment grade issuers. If the index composition is unchanged, the SPV realizes a gain on the biannual index roll when it buys protection on off-the-run index and sells protection on the year on-the-run index. However, if the index composition changes because of credit deterioration among some names in the off-the-run index, the roll-down benefit will be reduced or possibly negative.[10]

Fourth, default times of investment grade names tend to be backloaded, i.e., fixing a default probability over a given horizon, so the default is more likely to occur later rather than earlier. As the CPDO mechanism trades out of names that have been downgraded, the only outright default risk being taken in the CPDO is short-term jump-to-default risk.

These arguments lend plausibility to the CPDO as a *trading strategy*. Many hedge funds and asset managers are active in the CDS markets, and it would not be surprising if some of these institutions were pursuing a strategy along these lines. What is less obvious is the rationale for structuring CPDO liabilities as debt rather than as equity. The sensitivity of the CPDO strategy to market volatility makes the structure inherently fragile when backed by debt. Market commentary in 2007 suggested that the demand for this debt was driven by some of the same factors that drove demand for CDOs. In particular, there are pension funds and other institutions that are restricted by law or by mandate to invest primarily in investment grade debt. In the environment of tight credit spreads that prevailed until June 2007, the ability to offer 200 basis points on an AAA-rated security made for easy marketing.

Accounting treatment may also play a role. CPDO debt is marketed primarily to buy-and-hold investors who report on an accrual basis. For these investors, high mark-to-market volatility of the SPV's NAV can be ignored so long as cashflows are stable. The caveat here is that volatility in NAV ought to imply high ratings volatility, which is a problem for many buy-and-hold investors. It is not clear whether this was well understood by investors before the credit crisis.

In contrast to the transparency of CPDO pricing, robust modeling of the future performance of a CPDO is far from straightforward. The dynamic leverage rule induces strong path dependency on spreads for the reference portfolio. For this reason, it is not easy to characterize in a simple manner the set of scenarios that would trigger cash-out or failure to return par at maturity. Isla et al. (2007) distill a large set of analyses into the following rules of thumb:

- The lower the volatility in CDS spreads and the higher the rate of mean-reversion in spreads, the less likely is the CPDO to default on contractual payments.
- The lower the coupon on the CPDO notes and the lower the arranger fees, the lower the required leverage, and so the lower the likelihood of default.
- Spread widening is generally bad news for the CPDO notes, but not necessarily. If spreads widen early in the life of the CPDO and then hold steady, the higher carry on future index positions can outweigh the initial loss of NAV. Furthermore, if spreads subsequently mean-revert, there will be a gain in NAV. However, if spreads widen late in the life of the CPDO, there is less time to make up for the loss before maturity. In this case, the CPDO is more likely to fail to repay full principal at maturity.
- A severe spike in spreads can trigger a cash-out event. One example in the Isla et al. (2007) analysis is based on a scenario in which a steady increase in spreads from 25 basis points (bp) at inception to 75 bp after 4.75 years is followed by a sudden spike to 300 bp.

[9] Pan and Singleton (2008) estimate positive mean-reversion in default intensities under the physical measure for all three sovereign issuers they examine. Huang and Zhou (2008) provide evidence of mean-reversion in corporate leverage ratios in support of the Collin-Dufresne and Goldstein (2001) structural model.

[10] Roll-down benefit will also be reduced if the term structure of CDS spreads flattens (or inverts). Isla et al. (2007) show that flattening of the spread curve can have significant implications for performance of the CPDO.

• Default events generate large losses to NAV, and in this respect the CPDO behaves similarly to a thick equity tranche of a CDO. The index roll limits but does not eliminate the CPDO's exposure to defaults. In the downturn of 2001 to 2002, Enron and WorldCom went from investment grade to default within six months (Linden et al. 2007).

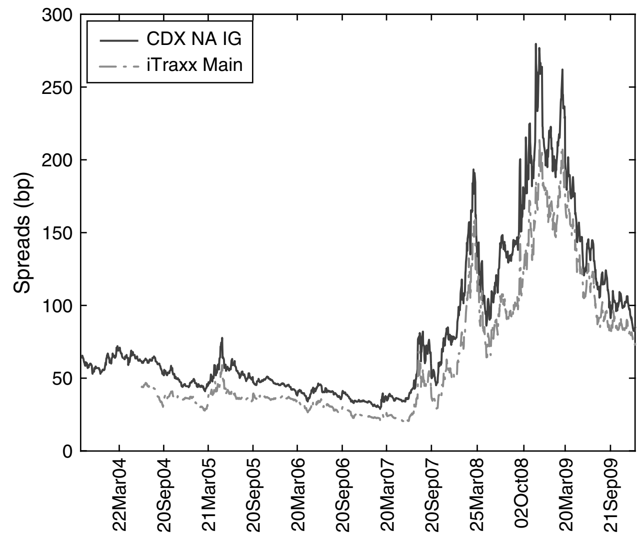
The CPDO avoids the liability structuring that makes senior CDO tranches highly sensitive to correlation among the reference names, and so it is regarded as less vulnerable to systematic credit risk than are other structured credit products of similar rating. However, published analyses have devoted little or no attention to sensitivity to a focused sectoral credit problem. If downgrades within a major sector cause several names in the index to widen significantly and then be dropped at the next roll date, the loss in NAV could be material because the CPDO will unwind the position on the old index, thereby crystalizing mark-to-market losses. If spreads are unchanged on names outside the affected sector, the CPDO structure will not receive the benefit of higher carry to compensate. This apparent gap in the analyses is surprising because it was a sectoral event in the U.S. auto industry that caused large losses for some CDO investors in the spring of 2005. Even more obviously, the greater volatility of a single-sector portfolio increases the vulnerability of any industry-specific CPDO. The early default by a financial-only CPDO bears witness to this point.

3. Performance During the Credit Crisis

The great bulk of CPDO issuance took place in the fall of 2006 and spring of 2007.[11] In this period, corporate credit spreads were at a cyclical low point, and volatility in credit spreads had been subdued for nearly three years. As shown in Figure 1, the CDX was trading around 35 bp during the period of issuance and had not exceeded 60 bp since July 2005. The iTraxx spread was typically about 10 bp lower during this period and had not exceeded 50 bp since May 2005.

The first tremors of the credit crisis reached the corporate CDS market in the summer of 2007. The CDX index spiked to 81 bp in early August, fell to 45 bp in early October, reached 85 bp in late November, and thereafter increased steadily through mid-March 2008, when it reached 193 bp. Spreads then fell sharply to 86 bp in early May but soon were increasing again.

Figure 1 **Investment Grade Index Spreads**



Source. MarkIt.
Notes. Spreads for on-the-run five-year investment grade indices. CDX trading began October 21, 2003, and iTraxx trading began June 22, 2004. Biannual index roll dates are marked on the time axis.

The CDX broke 279 bp on November 20 and finished the year at 195 bp. The iTraxx spread followed a very similar pattern.

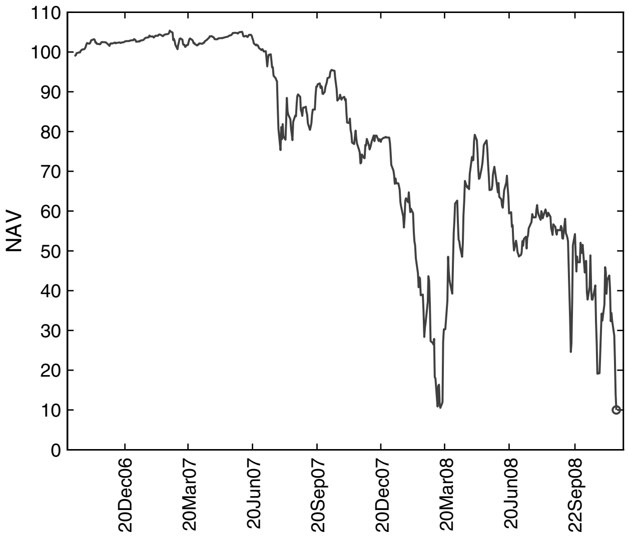
The volatility of the CDS markets induced high volatility for the NAV of CPDOs. To illustrate the effect on the performance of CPDOs, we calculate the NAV for a hypothetical CPDO that very closely emulates the first Surf note.[12] Our hypothetical CPDO is issued on August 25, 2006, with a maturity of 10 years. It references a 15x leveraged portfolio of the five-year iTraxx Main and CDX investment grade indexes. We assume the CPDO note earns 200 bp above three-month LIBOR and the SPV pays running fees of 60 bp. Arrangement fees reduce NAV at inception to 99. The cash-out barrier is 10. As depicted in Figure 2, the NAV increased steadily in the low spread period that prevailed until June 2007 and peaked at 107.5 on June 5. The NAV dropped precipitously in the first three months of 2008 and nearly triggered a cash-out event on March 14 (NAV of 14). The sharp fall in spreads over the subsequent two months brought an equally rapid recovery, and NAV reached 82.9 on May 2. The steady increase in spreads thereafter implied a steady deterioration in the NAV, and the CPDO hit its cash-out barrier on November 20, 2008.

Little of this volatility was reflected in the rating history of the Surf notes, which is shown in Table 1. The downgrade to BBB- on March 10, 2008,

[11] Sources at CreditFlux reported roughly €5.2 billion of CPDOs outstanding as of the index roll on March 20, 2007. Hard information on issuance is scant and perhaps unreliable. For example, Jobst et al. (2007) report under $2 billion of issuance by March 2007. As of February 2008, the total notional volume of CPDOs rated by Moody's was roughly €2.6 billion.

[12] Specifically, we emulate the SURF CPDO floating-rate note series 24, issued by Chess II Limited and arranged by ABN AMRO.

Figure 2 **NAV for a Hypothetical Index CPDO**



Source. Authors' calculations.
Notes. Biannual index roll dates and their midpoints are marked on the time axis. Cash-out event marked with a circle.

allowed the CPDO to retain its investment grade status (though barely, and it was on the watch list for downgrade). On the same date, the NAV for our proxy CPDO was 14.3, so the Surf deal must have been close to a cash-out default. The rating was withdrawn on October 23, less than one month prior to the cash-out of our proxy CPDO.

The credit crisis also demonstrated the particular vulnerability of a CPDO with a more concentrated reference pool. As widely reported in the financial press, a financial-only CPDO deal arranged by UBS in March 2007 hit its cash-out trigger on November 21, 2007.[13] This is the first default by a CPDO. Moody's had rated the CPDO notes Aaa at inception only eight months earlier. The UBS pool had significant concentrations of exposure to mortgage issuers and monoline financial guarantors, including FGIC, Radian, PMI, and XL Capital Assurance. By early November 2007, spreads on these four names had widened to between 400 and 800 bp. Leeming et al. (2007) study a hypothetical financial-only CPDO designed to emulate the defaulted UBS deal. They show that the lower leverage ceiling on the financial-only CPDO (relative to index CPDOs of the same vintage) could not compensate for the pool's poor diversification. Their hypothetical CPDO hit its cash-out trigger on the same date as did the UBS deal, and it is believed that other financial-only CPDOs of this vintage were equally vulnerable.

[13] Specifically, the defaulted notes were the ELM B.V. Series 103 Financial Basket TYGER Notes 2017.

Table 1 **S&P Rating Actions on First CPDO**

August 29, 2006	AAA
December 20, 2007	AAA/Watch negative
February 20, 2008	BBB+
March 10, 2008	BBB–/Watch negative
September 19, 2008	B/Watch negative
October 23, 2008	B
October 23, 2008	Not rated

Note. S&P rating on Surf CPDO floating-rate note series 24, issued by Chess II Limited and arranged by ABN AMRO.

4. Model Risk in CPDO Ratings

Agency rating of structured credit products is far more model driven than a traditional rating of corporate and other single-name obligations. Credit rating of corporates is based on the rating analyst's views on future company performance using industry and sector analysis and comparisons to peers. The process is intentionally infused with qualitative judgement and is best interpreted as a relative ranking of the credit quality of the borrower (Cantor and Mann 2003). By contrast, the rating of structured credit products, including CDOs and CPDOs, is grounded almost entirely on a probabilistic assessment. The rating analysts develop a model to describe the key risk features of a product and calibrate the model to available data. The model-implied probability of nonpayment of interest or principal (or the expected loss in the case of Moody's) is mapped to a rating in accordance with predefined criteria. For example, for a 10-year note to qualify for AAA status by S&P criteria, the cumulative 10-year default probability must be less than 72.8 basis points.

The AAA rating assigned to the first CPDOs generated significant controversy in the financial press. Rating agencies that did not participate in the first issues published reports strongly critical of the AAA rating. Jobst et al. (2007) of DBRS find that variations in model specification, and in data source and sample period for calibration, can lead to dramatic changes in estimated rating. They demonstrate particular sensitivity to assumptions governing roll-down benefits and to assumptions on CDS market liquidity (i.e., bid-ask spreads). They acknowledge that one can justify the AAA rating on the basis of reasonable models calibrated to available data but suggest that a BBB rating would be equally justifiable. Linden et al. (2007) of Fitch conduct similar exercises and obtain similar results. They put special emphasis on stress tests showing that the CPDO structure is less robust to extreme events than AA and AAA rated CDO tranches are. They conclude, and state in blunt terms, that the first-generation CPDO notes do not achieve a rating of AA, much less AAA. As noted earlier, Cont and Jessen (2009) demonstrate that the rating of CPDOs can be highly sensitive to small changes in parameters governing the evolution of spreads, in line with the conclusions of Coval et al. (2009) and Heitfield (2010) for CDO ratings.

The distress experienced in credit markets since mid-2007 provides an opportunity to evaluate ex post the modeling assumptions that underpinned the rating of CPDOs. Because we have the benefit of hindsight, our proper purpose is not to criticize rating opinions assigned ex ante but only to demonstrate how a model that may look reasonable at first glance can dramatically understate the risk of a complex structure such as a CPDO. We focus specifically on the performance of the rating agency models for credit spread dynamics.

4.1. S&P Model

During the period of CPDO issuance, the S&P approach to rating index CPDOs combined a Gaussian copula model for default losses in the pool with a stochastic model for the index spread (see Wong and Chandler 2007 for details). As the biannual index roll removes names that have drifted below investment grade, the default module simulates for the remaining life of the transaction a sequence of six-month exposures to default in a pool of investment grade names.[14] The index spread is assumed to follow a Black–Karasinski process, which is equivalent to assuming that the log of the spread follows a Vasicek process.[15] This is given by the stochastic differential equation

$$d\log(S_t) = \kappa(\theta - \log(S_t))dt + \sigma dW_t, \tag{1}$$

where W_t is a Brownian motion, θ is the long-run mean log spread, κ controls the rate of mean-reversion, and σ is the volatility parameter. Baseline parameter values (kindly supplied to us by S&P) for the Globoxx spread are $\kappa = 0.4$, $\sigma = 0.25$, and $\theta = \log(0.004) - \sigma^2/(4\kappa)$.

The Black–Karasinski specification has been applied to default intensity models, most notably by Pan and Singleton (2008), and Berndt (2007) shows that this specification generally outperforms CIR and jump CIR specifications for the default intensity. The distinction between credit spreads and default intensities notwithstanding, the Black–Karasinski specification would appear to be a plausible choice for this application. Even if the model is well specified, however, its empirical performance obviously depends on parameter calibration.

Consider the rating of a CPDO on the index roll date of March 20, 2007, when the Globoxx spread was 31.6 bp and the CPDO market was at its most active. Under the S&P specification, how high might spreads reasonably have been anticipated to climb? We let $S_0 = 0.00316$ and let M_t be the maximum spread on $[0, t]$. Our methodology for generating a sample of M_t is described in an online technical annex to this paper.[16] We find that $\Pr(M_{6\,\text{mo}} > 70\text{ bp})$ is well below 10^{-5} (that is, one tenth of one basis point). Looking to the one-year horizon, we find that $\Pr(M_{1\,\text{yr}} > 90\text{ bp})$ is also well below 10^{-5}. In 10 million simulated paths for $M_{1\,\text{yr}}$, the model never generated a spread above 102 bp. Yet the Globoxx reached 73.2 bp on July 30, 2007, well within a six-month horizon, and reached 144.5 bp on February 20, 2008. Simply put, the model assigns very low probability to the modestly high spreads realized in the first phase of the credit crisis in the summer of 2007 and assigns effectively zero probability to the spreads realized in the spring of 2008 (which themselves were greatly surpassed by the end of 2008).

Specification tests can be applied to the S&P model using the realized path for the Globoxx. Say we observe the path at discrete intervals $t_1, \ldots, t_n$. At each point in time, the exact transition density for the log spread at t_{i+1} is Gaussian with mean $\theta(1 - e^{-\kappa(t_{i+1}-t_i)}) + e^{-\kappa(t_{i+1}-t_i)}\log(S(t_i))$ and variance $\sigma^2(1 - e^{-2\kappa(t_{i+1}-t_i)})/(2\kappa)$ (Glasserman 2004, Equation (3.46)). Therefore, we can extract standardized innovations

$$Z_i = \frac{\log(S(t_{i+1})) - \theta(1 - e^{-\kappa(t_{i+1}-t_i)}) - e^{-\kappa(t_{i+1}-t_i)}\log(S(t_i))}{\sigma\sqrt{(1 - e^{-2\kappa(t_{i+1}-t_i)})/(2\kappa)}},$$

$i = 1, \ldots, n-1$, that are independent and identically distributed (i.i.d.) $\mathcal{N}(0,1)$ if the model is specified correctly.

We examine the empirical distribution of the daily innovations Z_i for the sample period April 4, 2003, through the end of 2009.[17] In the results reported below, we follow calendar time in defining $t_{i+1} - t_i$. Results are qualitatively robust to treating trading days as equally spaced and to sampling at weekly (rather than daily) intervals. We divide the sample into a precrisis period (April 4, 2003, to June 13, 2007) and crisis period (June 14, 2007, to the end of 2009). The breakpoint, June 14, is chosen as the date on which the troubles of two Bear Stearns hedge funds

[14] The default module is built on the CDO Evaluator tool that S&P uses for rating corporate CDOs.

[15] For bespoke pools, the model allows for a jump component in spreads and for multiple factors.

[16] The technical annex is available at http://michael.marginalq.com/docs/cpdoMSannex.pdf.

[17] CDX trading began October 21, 2003, and iTraxx trading began June 22, 2004. To maximize the length of the available time series, we use proxy indices for the CDX and iTraxx constructed by JP Morgan to backfill the data. These proxies are based on earlier indices, such as the iBoxx and TRAC-X, that were superceded by the CDX and iTraxx. All reported results are qualitatively robust to dropping the backfilled period.

Table 2 **Moments of the Innovations (S&P Model)**

Period	Days	Mean	Variance	Skewness	Kurtosis
Precrisis	1,040	−0.07*	1.84***	0.17	8.35***
Crisis	635	0.24***	8.59***	−0.07	5.82***

Notes. Tests for mean zero, variance one, and mesokurtosis are two-sided. Skewness is not tested.

*, **, and *** denote significance at the 5%, 1%, and 0.1% levels, respectively.

became public news. It is not obvious how index roll dates should be treated because the jump in the spread induced by the reconstitution of the index is outside the model framework. We choose to exclude innovations for intervals containing roll dates because this biases our analysis in favor of the S&P model. Descriptive statistics are shown in Table 2.

In the precrisis period, the null hypothesis of mean zero is rejected at the 5% level in a two-sided test. The null hypothesis of variance one is rejected even at the 0.1% level. For the crisis period, both tests reject at the 0.1% level. For the null hypothesis of mesokurtosis (i.e., kurtosis of three), the test of Anscombe and Glynn (1983) rejects at the 0.1% level in both periods. To test the distributional assumption (standard normal) rather than just the moments, we use the Cramér–von Mises test and reject at the 0.1% level in each period.[18] Excess kurtosis drives the strong rejection of normality. Say, for example, that we increase σ in the Black–Karasinski process to 0.33. The tests of mean zero and variance one would then fail to reject in the precrisis period, but the Anscombe–Glynn and Cramér–von Mises tests still would reject strongly for both periods.

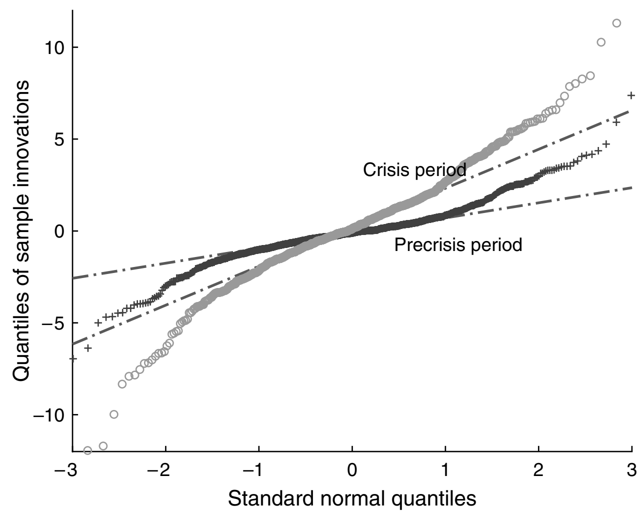
In Figure 3, we plot quantiles of the innovations in these two periods against the quantiles of the normal distribution. The slopes of the dashed lines demonstrate the excess variance, whereas the departures of the marked points from the dashed lines demonstrate the excess kurtosis.

4.2. Moody's Model

Like the S&P approach, the Moody's approach to rating CPDOs joins together a model for correlated changes in obligor status with a stochastic model for spreads. However, these component models are more elaborate than are their S&P counterparts. The obligor status model captures rating migrations as well as defaults, and spreads are represented as a vector of ratings-based constant-maturity spreads with correlated innovations. Instead of modeling the spread on the pool in a top-down fashion, Moody's

[18] The Cramér–von Mises ω^2 statistic is 1.70 in the precrisis period, 11.19 in the crisis period. Exact percentiles for the statistic under the null are given by (Csörgő and Faraway 1996, Table 1).

Figure 3 **QQ Plot for Innovations in S&P Model**



Note. Quantiles for the precrisis period are marked with plus signs, and quantiles for the crisis period are marked with circles.

constructs the pool spread bottom up from the ratings of the individual names in the pool and the spreads that correspond to those ratings. It is assumed that when an obligor changes rating grade, its CDS spread jumps to the level associated with its new grade.[19]

The engine for simulating correlated migrations and defaults is a variant on the basic Gaussian copula framework, known as CDOROM, that underpins most or all of Moody's structured credit analysis. It is similar in spirit and structure to a multi period version of the popular CreditMetrics model of portfolio credit risk.[20] Spreads are assumed to evolve over time as "constant elasticity of variance" (CEV) processes with capped variances. More precisely, the spread $S_k(t)$ for grade k follows the stochastic differential equation

$$dS_k(t) = \kappa_k(\theta_k - S_k(t))dt + \min(\bar{v}_k, \eta_k + \sigma_k S_k(t)^{\gamma_k})dW_k(t), \qquad (2)$$

where θ_k is the long-run mean spread for the grade, κ_k controls the rate of mean-reversion, $\bar{v}_k$ is the cap on volatility, η_k and σ_k are volatility parameters, and γ_k

[19] Recognizing the greater flexibility and internal consistency of a bottom-up approach, S&P adopted a similar CPDO model late in 2008. By that time, of course, issuance had ceased.

[20] The main CDOROM is a time-to-default copula model that is similar to standard models for CDO pricing. This model does not capture ratings migration short of default, so it would be poorly suited to rating LSS and CPDO instruments. The LSS module, in contrast, simulates iterated one-year correlated migrations over the remaining maturity of the instrument. This module is not covered in publicly available documentation on CDOROM. See Finger (2000) for a comparative study of iterated CreditMetrics and Gaussian copula models.

Table 3 Parameters for Moody's CEV Spread Processes

	θ (%)	κ	σ (%)	γ	η	$\bar{v}$	$S(0)$ (%)
Aaa	0.25	8.7926	306.95	1.1835	0	0.00329	0.28
Aa	0.21	3.3786	1,822.67	1.5831	0	0.00359	0.26
A	0.33	3.0298	220.36	1.3376	0	0.00325	0.30
Baa	1.01	7.3130	216.67	1.3383	0	0.01395	0.65

Source. Moody's.

Table 4 Correlations for Moody's CEV Spread Processes

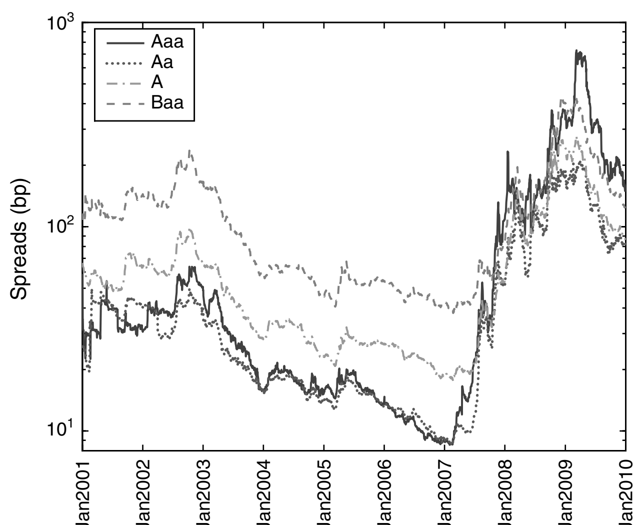
	Aaa	Aa	A	Baa
Aaa	1.0000	−0.1697	0.0953	0.2716
Aa	−0.1697	1.0000	0.0922	0.0693
A	0.0953	0.0922	1.0000	0.0763
Baa	0.2716	0.0693	0.0763	1.0000

Source. Moody's.

is the CEV elasticity parameter. $W_k(t)$ is a Brownian motion, and the correlation of increments $dW_k(t)$ and $dW_j(t)$ for grades (k, j) is ρ_{kj}. Baseline parameter values (kindly supplied to us by Moody's) for investment grade spreads are presented in Tables 3 and 4.

To permit assessment of model predictions, we construct time series of average five-year corporate CDS spreads by rating category. Our methodology is described in our online technical annex. We plot investment grade spreads for 2001 through 2009 in Figure 4, and display spreads on select dates in Table 5. For the precrisis and crisis periods, the behavior of the rating-based spread series is similar to that of the index spreads in Figure 1. Given the composition of the Globoxx, one expects the index to be above A spreads and below Baa spreads. This is generally the case in our data except in the first half of 2007,

Figure 4 Investment Grade Spreads, 2001–2009



Note. CDS spreads in basis points on log scale.

Table 5 Rating-Based Spreads on Select Dates

	Aaa	Aa	A	Baa	Globoxx
October 10, 2002	63.8	49.3	99.2	235.88	—
February 20, 2007	9.0	8.7	17.8	38.8	26.2
March 20, 2007	12.3	10.9	20.3	42.6	31.6
July 30, 2007	38.1	29.1	35.3	69.4	73.2
November 20, 2007	122.5	62.5	54.2	74.9	69.3
February 20, 2008	161.1	94.9	112.7	151.9	144.5
March 20, 2008	137.1	122.4	139.7	178.5	145.7
December 15, 2008	360.3	186.9	265.6	429.5	235.3

when the Globoxx ran a few basis points higher than did our Baa composite. Our sample of firms is much wider than the set of names on which the indexes are based, so the relationship between our ratings-based composites and the Globoxx is not fixed.

A clear feature of the data is high variability in Aaa spreads. Before the crisis, the Aaa spread would often exceed the Aa spread. During the crisis, the Aaa spread would sometimes exceed even the Baa spread. The Aaa spread is based on a much smaller sample of firms than are the other rating composite spreads (typically less than 40 firms for Aaa versus 150 to 900 for the other grades), and the Aaa category is dominated by financial firms (such as monoline insurers) that were in many cases severely affected by the crisis. To avoid issues with data quality, we omit the Aaa category from our analyses below. This biases our analysis in favor of the Moody's model.

Comparing the spreads of the crisis period to spreads during the previous recession, we see that the period of late 2007 to early 2008 appears roughly similar to that of October 2002. The earlier peak was less severe for the highest grades and more severe for Baa. Only in March 2008 and again in late 2008 do prevailing spreads appear to surpass clearly the earlier peak. Volatility, however, is unambiguously higher in the crisis period than in the earlier recession.

Our analysis begins with experiments similar in spirit to the assessment of the peak spread distribution in the S&P model. We fix an inception date of March 20, 2007 (an index roll date), and calculate the probability of reaching within eight months the spread levels seen on November 20, 2007 (the date of the cash-out of the UBS financial-only CPDO). We also calculate the probability of reaching within one year the spread levels of March 20, 2008, and of reaching within two years the spread levels of December 15, 2008. These spreads are given in Table 5.

We calculate the probabilities, both univariate (for each grade) and joint, by simulating the spread

Table 6 **Exceedance Probabilities for Moody's CEV Spread Processes**

Test	Aa	A	Baa	Joint
Panel A: Original specification				
8 month	0.2	2.4	10,000	0
12 month	0	0	10.2	0
24 month	0	0	0	0
Panel B: Remove volatility caps				
8 month	0.4	2.4	10,000	0
12 month	0.03	0	10.2	0
24 month	0.08	0	0	0
Panel C: Remove volatility caps and set $\rho=0.6$				
8 month	0.4	2.4	10,000	0
12 month	0.04	0	10.2	0
24 month	0.09	0	0	0
Panel D: Remove volatility caps and reduce κ				
8 month	0.4	1.5	7,915	0
12 month	0.3	0	72.1	0
24 month	4.8	0	2.1	0
Panel E: Remove volatility caps, set $\rho=0.6$, and reduce κ				
8 month	0.3	1.5	7,914	0
12 month	0.3	0	71.9	0
24 month	4.8	0	2.0	0

Notes. Probabilities in basis points. Based on 20 million simulated paths with 1,000 timesteps per year. Values of 0 indicate that the barrier was not breached along any sample path in the simulation. Reduced κ values equal to baseline values divided by five.

processes as specified by Moody's using an Euler scheme, i.e.,

$$\Delta S_k(t) = \kappa_k(\theta_k - S_k(t))\delta + \min(\bar{v}_k, \eta_k + \sigma_k S_k(t)^{\gamma_k})\sqrt{\delta} Z_k(t) \quad (3)$$

at time intervals $t=\delta, 2\delta, \ldots$. The innovations $Z_k(t)$ are i.i.d. $\mathcal{N}(0, R)$, where the correlations in matrix R are taken from Table 4. We fix δ to 1/1,000 of a year. In our online technical annex, we show that the discretization bias associated with the Euler scheme in Equation (3) is negligible.

Results are shown in panel A of Table 6. For example, starting at 10.9 bp, the probability that the Aa spread would break above 62.5 bp within eight-months is less than 2×10^{-5} (that is, 0.2 bp). The corresponding probability for the A spread is 2.4 bp. In contrast, we see that Baa broke its eight-month target barrier in each of the 20 million simulated paths. This is because the crisis initially had a modest effect on grades Baa and below, so the barrier for this exercise was well below the long-run mean θ_{Baa}. In none of the simulated paths did we observe the joint event, i.e., that all three spread processes simultaneously exceed barrier levels on a date within the eight-month horizon. Results for the 12-month and 24-month horizon tests are even more striking. In no simulated path did the Aa spread or A spread reach its barrier at either horizon. Even for the relatively modest 12-month barrier of 178.5 bp for Baa, the exceedance probability was only 10.2 bp. In summary, the model did not assign any significant probability to the spread levels attained early in the crisis period and assigned essentially zero probability to the spread levels reached in the midst of the crisis.

In a series of articles beginning in May 2008, the *Financial Times* blog *FT Alphaville* (hereafter, *FT*) reports that internal documents leaked from Moody's show that in early 2007, the senior staff at the rating agency learned of a coding error in the implementation of the model used to rate CPDOs (Jones 2008a, b). *FT* reports that had the error been corrected, the Aaa rated notes would have been rated up to four notches lower. Even so, *FT* reports that CPDOs rated after the discovery of the bug still achieved the Aaa rating because Moody's, although correcting the error, made two offsetting modifications to the rating methodology.[21] Of the two, the most "notable" (in the phrase of the *FT* article) was the imposition of volatility caps in the spread processes.

Although we cannot identify the model modifications with confidence, it is nonetheless important to assess the potential impact of possible modifications on our test results. Removing the volatility caps has a very modest effect on the estimated exceedance probabilities, as seen in panel B. This is largely because the volatility caps were not binding for Baa in the 8-month and 12-month tests and were not binding for A spreads in the eight month test.[22]

To generate large changes in the Globoxx, there must be large *simultaneous* changes in the Aa, A, and Baa spreads. As will be discussed below, the assumed correlations in Table 4 appear distinctly lower than can be justified empirically. Therefore, assuming that *FT* allegations are correct, it is reasonable to suppose that the correlation assumptions were the second "fix" to the model. We report in panel C an experiment in which we remove volatility caps and set cross-grade correlations to $\rho=0.6$. Marginal distributions are unaffected (relative to the experiment in panel B) by changes in the correlations. As before, at all three horizons, barriers were never jointly breached in any simulated path.

[21] As reported by *FT*, the leaked documents say of the methodology changes that "the impact of our code issue after these improvements is then reduced." According to *FT*, a third methodological change was proposed but not implemented because, according to leaked documents, it "did not help the rating." In September 2008, Moody's reported the discovery of one more (unspecified) error in the implementation of its CPDO rating model (Weill and Marjolin 2008). The correction of the error, according to *Bloomberg News*, would likely result in a downgrade of the affected CPDOs by one or two notches (Unmack and Glover 2008).

[22] The volatility caps ($\bar{v}$) for the Aa, A, and Baa processes bind at spread levels of 45.6 bp, 76.4 bp, and 230.5 bp, respectively.

Table 7 Moments of the Innovations (Moody's Model)

Rating	Mean	Variance	Skewness	Kurtosis	CvM
		Early period ($n=781$ observations)			
Aa	0.09*	1.57***	17.73	412.39***	19.46***
A	0.13***	0.38***	0.71	7.11***	11.79***
Baa	0.09**	0.46***	2.01	20.55***	11.22***
		Precrisis period ($n=897$ observations)			
Aa	−0.35***	0.36***	0.47	8.52***	27.58***
A	−0.25***	0.34***	1.41	17.05***	21.18***
Baa	−1.26***	0.49***	0.26	6.36***	141.84***
		Crisis period ($n=666$ observations)			
Aa	0.58***	6.65***	−0.55	17.77***	14.58***
A	0.63***	9.61***	−4.52	80.92***	13.44***
Baa	0.25***	0.91	−0.09	4.31***	5.83***

Notes. Tests for mean zero, variance one, and mesokurtosis are two-sided. The Cramér–von Mises test is one-sided. Skewness is not tested.
*, **, and *** denote significance at the 5%, 1%, and 0.1% levels, respectively.

A plausible alternative is to suppose that the second fix to the model affected the mean-reversion parameters. A lower speed of mean-reversion increases the likelihood of extreme excursions away from long-run mean spreads. In panel D, we report an experiment in which we remove volatility caps and set each κ_k to one-fifth of its baseline value. The experiment reported in panel E combines all three modifications. There is a modest effect on exceedance probabilities for the Aa and Baa paths. As before, however, barriers were never jointly breached in our simulations.[23]

Specification tests can be constructed as in our analysis of the S&P model. Using Equation (3), we extract the implied vector of innovations $(Z_{\mathrm{Aa},t}, Z_{\mathrm{A},t}, Z_{\mathrm{Baa},t})$ from the observed spread changes at times $t_1, \ldots, t_n$. If the model is correctly specified, this vector is distributed standard multivariate normal with correlation matrix R given by Table 4. For ease of comparison against our results for S&P, we divide our longer sample into three periods: an "early" period (January 1, 2001, to December 31, 2003), a precrisis period (January 1, 2004, to June 13, 2007), and a crisis period (June 14, 2007, to the end of 2009).

Univariate descriptive statistics for the innovations by rating grade and by period are shown in Table 7. In nearly all cases, the null hypotheses of mean zero and variance one are each rejected in two-sided tests at the 0.1% level. The Anscombe and Glynn (1983) test for mesokurtosis and the Cramér–von Mises test are rejected at the 0.1% level (one-sided) in all cases. Thus, for each of the three rating grades, the data firmly reject Moody's capped CEV specification.

[23] We have not explored the sensitivity of the Moody's model to the elasticity parameter γ. Davydov and Linetsky (1992) show that CEV option pricing models are not terribly sensitive to this parameter for vanilla options, but for barrier options the sensitivity can be quite material.

Table 8 Empirical Correlations

Period	Aa/A	Aa/Baa	A/Baa
Early	0.15	0.13	0.39
Precrisis	0.55	0.56	0.66
Crisis	0.48	0.45	0.51

A rejection of the mean and variance tests is not necessarily undesirable from the perspective of the rating agencies, so long as the direction of the misestimation leads to a more conservative rating. In the precrisis period, the mean innovation is negative and the variance of the innovations too small, which suggest a conservative parameterization. However, the excess kurtosis undoes the conservative bias in the model-implied mean and variance because the rating ultimately rests only on the "bad tail" of the distribution. QQ plots (not reported) are qualitatively similar to Figure 3 in tail divergence for all three rating grades in each of the three periods.

Bivariate correlations between spread process innovations are presented in Table 8. In the early period, correlations between Aa and A and between Aa and Baa were relatively modest, though still larger than the corresponding correlations in Moody's calibration, as shown in Table 4. The correlation between A and Baa spreads in this period is nearly 40% and more than five times as large as the assumed correlation. Because of the predominance of A and Baa issuers in the Globoxx, this has the largest impact of the three correlation parameters. In the precrisis and crisis period, all estimated correlations are above 45%. Thus, even if the univariate processes had been well specified and calibrated, the model would have dramatically understated the probability of *joint* exceedances under the baseline calibration. Joint tests of model fit can be constructed. Under the null hypothesis, the quadratic form $Q_t \equiv Z_t R^{-1} Z_t'$ is distributed $\chi^2(3)$. Cramér–von Mises tests (not reported) reject the null hypothesis at the 0.1% level in each of the three periods.

The rejections of Moody's specification cannot be satisfactorily attributed to the parameter "fixes" suggested by the *FT* report. Lifting the volatility caps reduces the variance of innovations in the crisis period to 0.70, 1.41, and 0.70 for the Aa, A, and Baa spreads, respectively. If the speed of mean-reversion is reduced as well, the mean innovation in the crisis period is greatly reduced (though the tests of mean zero still reject at conventional levels of significance). However, even if we change correlations to $\rho=0.6$, the Cramér–von Mises tests for the quadratic form Q_t continue to reject. Most importantly, excessive kurtosis is entirely robust to these parameter changes. Thus, even under the most conservative alternative specification (i.e., that of panel E in Table 6), the model would still have understated the probability of large shocks to the spread processes both before and during the crisis.

Finally, from the perspective of Moody's model, another anomaly of 2007 is the sectoral variation in ratings-based spreads. Sectoral concentration in the Moody's model is addressed via the ratings migration correlation structure of the CDOROM/LSS module. That is, there may be sectoral risk in ratings migration but, conditional on the realized ratings of the obligors, there is no sectoral or idiosyncratic risk in spread movements. For financial obligors in 2007, this stylized characterization of risk proved inadequate. Of the four identifiable obligors included in the failed UBS financial-only CPDO, none suffered any downgrade up to the cash-out date, and all remained A rated or better. However, the market assessment of these obligors differed sharply from the assessment of the rating agencies.

5. Stochastic Volatility in Index Spreads

Stochastic volatility is the most natural way to explain the excess kurtosis observed in the specification tests and to allow for the dramatic increase in volatility at the onset of the crisis. We have long had empirical evidence on the ubiquity of stochastic volatility in financial markets (see Bollerslev et al. 1992 for a survey of the early literature), so this is not surprising. Exploration of stochastic volatility in the credit risk literature has gained momentum only recently (Fouque et al. 2006, Gouriéroux 2006, Jacobs and Li 2008, Zhang et al. 2009). In this section, we obtain direct evidence of stochastic volatility (SV) in index spreads and assess its impact on the tail of the model-implied distribution of peak spreads.

We extend the S&P model for log spreads to include a latent volatility factor.[24] The S&P model is, in our view, the most parsimonious plausible model for index spreads, and our model is intended to be the most parsimonious way to introduce SV. We emphasize parsimony to keep the focus on the effect of relaxing the assumption of constant volatility. The model is estimated using Bayesian MCMC methods. We closely follow the methodology of Jacquier et al. (1994, 2004), who estimate a similar model for stochastic volatility in equity returns and exchange rates. Our application to the calculation of exceedance probabilities serves to illustrate how the Bayesian approach facilitates recognition of parameter uncertainty in rating exercises, as suggested earlier by Coval et al. (2009).

Recall that the S&P model specifies $X_t = \log(S_t)$ as a Vasicek process with constant volatility σ. We replace σ with the time-varying $\sigma_t = \exp(h_t/2)$, where the latent log volatility h_t is specified as a Vasicek process. The naïve specification would be

$$dX_t = \kappa_x(\theta_x - X_t)\,dt + \exp(h_t/2)\,dW_t^x,$$
$$dh_t = \kappa_h(\theta_h - h_t)\,dt + \xi\,dW_t^h.$$

The correlation between increments to the Brownian motions W_t^x and W_t^h is ρ. The log-volatility specification has been widely used in the SV literature (early examples include Jacquier et al. 1994, Andersen and Lund 1997).

This naïve specification would have difficulty capturing the jumps in the spread associated with the biannual rollover in the index. Under this specification, the rollover jump would appear as a short burst of higher volatility and so bias upward our estimate of the volatility-of-volatility (ξ). We assume that the rollover jumps are anticipated in both timing and magnitude. The rollover date is indeed known in advance, so the timing of the jump is obviously predictable. Perhaps less obvious, it is reasonable to assume that market participants can anticipate the magnitude of the jumps as well. The composition of the new index is known prior to the rollover date, so the intrinsic (theoretical) spread can be calculated using knowledge of the single-name CDS spreads of the new index. Even if the movement in market spread cannot be forecast, the *difference* in spreads between the current and pending index spreads is likely to be stable over short horizons. Therefore, to account for the index roll, we introduce a jump process J_t with zero increments on nonrollover dates and assume that J_{t+1} is observable at date t. Thus, our model can be written as

$$dX_t = dJ_t + \kappa_x(\theta_x - (X_t + dJ_t))\,dt + \exp(h_t/2)\,dW_t^x, \tag{4}$$
$$dh_t = \kappa_h(\theta_h - h_t)\,dt + \xi\,dW_t^h. \tag{5}$$

Empirically, we measure the jump as the difference on the rollover date between the log spread on the new on-the-run Globoxx and the log spread on the newly off-the-run Globoxx. To the extent that the magnitude of the rollover jump is uncertain ex ante, it is an additional source of risk for the investor. Thus, by taking it as anticipated, we bias our exercise in favor of the S&P model.

We discretize the model as

$$X_{t+1} - X_t = (J_{t+1} - J_t) + \kappa_x(\theta_x - X_t - (J_{t+1} - J_t))\delta + \exp(h_t/2)\sqrt{\delta}\epsilon_{t+1}^x, \tag{6}$$
$$h_{t+1} - h_t = \kappa_h(\theta_h - h_t)\delta + \xi\sqrt{\delta}\epsilon_{t+1}^h, \tag{7}$$

[24] In principle, stochastic volatility could be introduced to the Moody's model in much the same way. Because of the relative complexity of the model, the extension would be somewhat tedious to implement.

Table 9 Prior Distributions for Model Parameters

	κ_x	θ_x	κ_h	θ_h	ω_h
Distribution	$\mathcal{N}(\mu_1, 1/\tau_1)$	$\mathcal{N}(\mu_2, 1/\tau_2)$	$\mathcal{N}(\mu_3, 1/\tau_3)$	$\mathcal{N}(\mu_4, 1/\tau_4)$	$\mathcal{IG}(\mu_5, \tau_5)$
μ^0	0.4	3.65	5	−2.77	3
τ^0	1/5	1/15	1/15	1/15	1/20

where the time-step $\delta = 1/250$ reflects the approximate number of trading dates per year. The $(\epsilon_t^x, \epsilon_t^h)$ are serially independent, bivariate standard normal innovations with correlation ρ. The parameter vector to be estimated is $(\kappa_x, \theta_x, \kappa_h, \theta_h, \xi, \rho)$. Indexing this vector by j, we specify for each element a marginal prior distribution with two parameters (μ_j^0, τ_j^0). For the first four parameters, the marginal prior distribution is normal with mean μ_j^0 and variance $1/\tau_j^0$. We call τ_j the *precision* of parameter j.

For the last two parameters (ξ, ρ), we use the block transformation of Jacquier et al. (2004). Let $\omega_h = \xi^2(1-\rho^2)$ and $\phi_h = \xi\rho$. The prior distribution for ω_h is inverse gamma with shape parameter μ_5^0 and scale parameter τ_5^0, which implies prior mean $\tau_5^0/(\mu_5^0 - 1)$ and variance $\tau_5^0/((\mu_5^0-1)^2(\mu_5^0-2))$. The conditional prior for ϕ_h is $\mathcal{N}(0, \omega_h/2)$.

Table 9 reports our baseline assumptions for the μ_j^0 and τ_j^0. We adopt the S&P assumptions for κ_x and θ_x as our prior means μ_1^0 and μ_2^0. For the long-run mean of the log volatility, we let $\mu_4^0 = 2\ln(\sigma)$, where $\sigma = 0.25$ is the S&P assumption for fixed volatility. Reflecting our belief that volatility of log spreads should mean-revert more quickly than the level of log spreads, we set $\mu_3^0 = 5$. In each case, we adopt a reasonably high variance for the prior. For ω_h, the prior distribution has mean 0.025, standard deviation 0.025, and infinite skewness and kurtosis. This implies that the prior distribution for volatility-of-volatility ξ has a large probability mass near zero and so accommodates the possibility of nearly deterministic volatility.

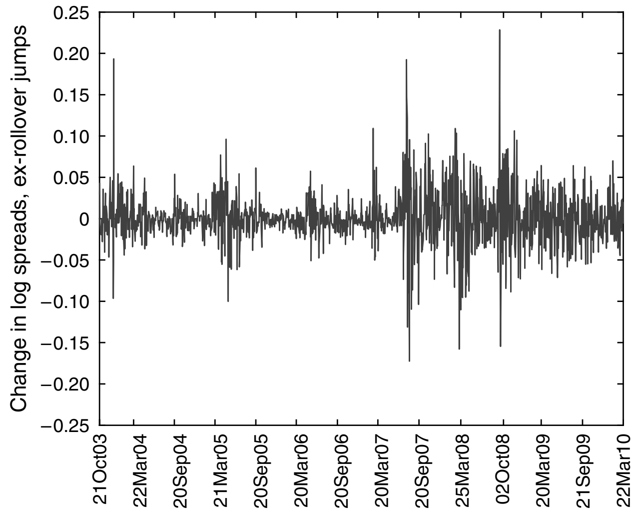
The posterior distributions of the parameters are in each case conjugate to prior. In each iteration of the Markov chain, we obtain new draws of the parameters via Gibbs sampling and then a new sample path for the log volatilities $\{h_t\}$ via Metropolis–Hastings. To draw κ_x conditional on the current draws of all other parameters and the log volatilities, we first decompose ϵ_t^x as

$$\epsilon_t^x = \rho\epsilon_t^h + \sqrt{1-\rho^2} Z_t^x,$$

where the Z_t^x are i.i.d. standard normal and independent of ϵ_t^h. We calculate the residuals $\{\epsilon_t^h\}$ using Equation (7) and then rearrange Equation (6) as

$$\begin{aligned} X_{t+1} - X_t - (J_{t+1} - J_t) - \exp(h_t/2)\sqrt{\delta}\rho\epsilon_{t+1}^h \\ = \kappa_x(\theta_x - X_t - (J_{t+1} - J_t))\delta + \exp(h_t/2)\sqrt{\delta}\sqrt{1-\rho^2} Z_{t+1}^x. \end{aligned}$$

Figure 5 Innovations in Log Spreads



Note. Daily changes in the log spread for the Globoxx index, net of rollover jumps.

In this form, it is straightforward to obtain the updated mean and precision of κ_x by Bayesian linear regression and then to draw the new κ_x as a normal random variate with the updated mean and precision. The new draws of parameters θ_x, κ_h, and θ_h are obtained in turn by parallel methods. Next, parameters (ξ, ρ) are drawn as a block by the method of Jacquier et al. (2004). Last, we draw the latent log-volatility states using the algorithm described by Szerszen (2009, §6.1).

As in the analysis of §4.1, we let S_t be the Globoxx spread. Our sample period is backfilled to April 4, 2003, and runs through the end of 2009. In Figure 5, we plot the changes in the log spreads, net of rollover jumps.[25] The visual pattern is strongly suggestive of time-varying volatility. We estimate the model on the precrisis sample (April 4, 2003, through June 13, 2007) and on the full sample through end 2009. We simulate the posterior distribution with a chain of $N = 300{,}000$ iterations and discard the first $50{,}000$ trials as a burn-in period. For each sample, Table 10 reports the mean and standard deviation of the posterior distribution and the 2.5% and 97.5% quantiles.[26] The parameter of greatest interest is volatility (ξ) for the

[25] Because X_t is a log spread, jumps are naturally expressed as a percentage change in the level of the spread. Of the 14 rolldate jumps in the sample, only five were negative. The largest jump (in absolute magnitude) was the −17.3% jump on the roll of March 20, 2009, which is similar in magnitude to the largest nonrolldate innovations seen in Figure 5. The next largest jumps were those of September 2007 (−8.7%), March 2005 (7.4%), and September 2006 (7.1%). The remaining jumps were all smaller than 6% in absolute size.

[26] We have simulated Markov chains with alternative starting values and lower values of τ^0 and find the results qualitatively unchanged.

Table 10 Posterior Distributions for Model Parameters

	κ_x	θ_x	κ_h	θ_h	ξ_h	ρ
			Precrisis sample			
Mean	−0.045	4.078	8.327	−2.973	4.680	0.041
Std. dev.	(0.338)	(3.119)	(2.431)	(0.328)	(0.613)	(0.087)
2.5%	−0.691	−2.226	3.784	−3.606	3.561	−0.130
97.5%	0.636	10.070	13.320	−2.347	5.962	0.210
			Precrisis sample (restricted)			
Mean	0.400	3.650	8.048	−2.839	4.168	0.118
Std. dev.	—	—	(2.366)	(0.307)	(0.574)	(0.088)
2.5%	0.400	3.650	3.633	−3.430	3.131	−0.055
97.5%	0.400	3.650	12.909	−2.248	5.355	0.289
			Full sample			
Mean	−0.059	4.711	4.392	−2.350	3.687	0.160
Std. dev.	(0.214)	(3.289)	(1.359)	(0.398)	(0.391)	(0.077)
2.5%	−0.447	−2.169	1.837	−3.133	2.971	0.009
97.5%	0.392	10.714	7.192	−1.615	4.500	0.312
			Full sample (restricted)			
Mean	0.400	3.650	4.937	−2.196	3.545	0.228
Std. dev.	—	—	(1.351)	(0.310)	(0.366)	(0.073)
2.5%	0.400	3.650	2.423	−2.799	2.875	0.084
97.5%	0.400	3.650	7.731	−1.591	4.314	0.369

Notes. $N = 300,000$ trials in Markov chain of which first 50,000 discarded as burn-in. Restricted models impose the S&P calibrations of κ_x and θ_x.

log-volatility process. The daily frequency of the data allows this parameter to be identified to reasonably high precision. In the precrisis sample, we find strong evidence of stochastic volatility because the 2.5% percentile value of ξ is more than 3.5. Even at the 0.5% percentile level (not reported in the table), ξ is more than 3.2.[27]

The time series behavior of volatility depends on θ_h and κ_h, as well as on ξ. Let $\sigma_t \equiv \exp(h_t/2)$ be the instantaneous volatility of the log spread at time t. The stationary distribution for h_t is $\mathcal{N}(\theta_h, \xi^2/2\kappa_h)$, which implies that σ_t is lognormal with expected value

$$\mathrm{E}[\sigma_t \mid \kappa_h, \theta_h, \xi] = \exp\left(\frac{1}{2}\theta_h + \frac{\xi^2}{16\kappa_h}\right).$$

For the precrisis sample, the median of the posterior distribution for $\mathrm{E}[\sigma_t \mid \kappa_h, \theta_h, \xi]$ is 0.267, which is quite close to the constant $\sigma = 0.25$ in the S&P calibration.[28] However, because the stationary distribution of σ_t has a large variance, the variation of σ_t over a long sample path will be substantial. Let $\bar{\kappa}_h, \bar{\theta}_h, \bar{\xi}$ be the posterior sample parameter values associated with the median value of the posterior sample of $\mathrm{E}[\sigma_t|\kappa_h, \theta_h, \xi]$. The conditional standard deviation of

[27] The 99.5% percentile value of the prior distribution of ξ is 0.51, so the chosen prior is biased against finding economically significant volatility-of-volatility.

[28] The posterior sample contains a very small number of draws of κ_h near zero. The corresponding values of σ_t are sometimes extremely large and so cause the sample mean to be extremely large.

Table 11 Quantiles of the Stationary Distribution of σ_t

	1%	5%	Median	95%	99%
			Precrisis sample		
$\bar{\alpha}_q(\sigma_t)$	0.058	0.087	0.224	0.581	0.862
$\alpha_q(\sigma_t)$	0.051	0.082	0.226	0.619	0.990
			Full sample		
$\bar{\alpha}_q(\sigma_t)$	0.067	0.105	0.303	0.880	1.368
$\alpha_q(\sigma_t)$	0.059	0.102	0.310	0.932	1.570

Note. $\bar{\alpha}_q(\sigma_t)$ is quantile of σ_t conditioned on $(\kappa_h = \bar{\kappa}_h,\ \theta_h = \bar{\theta}_h,\ \xi = \bar{\xi})$, whereas $\alpha_q(\sigma_t)$ samples over parameter posterior as well as the stationary distribution of σ_t.

σ_t is $\sqrt{\mathrm{V}[\sigma_t \mid \bar{\kappa}_h, \bar{\theta}_h, \bar{\xi}]} = 0.17$. Let $\bar{\alpha}_q(h_t)$ be the qth percentile value of the stationary distribution for h_t given $\bar{\kappa}_h, \bar{\theta}_h, \bar{\xi}$. Because h_t is Gaussian, we have

$$\bar{\alpha}_q(h_t) = \bar{\theta}_h + \frac{\bar{\xi}}{\sqrt{2\bar{\kappa}_h}}\Phi^{-1}(q).$$

The monotonic relationship between h_t and σ_t implies $\bar{\alpha}_q(\sigma_t) = \exp(\bar{\alpha}_q(h_t)/2)$. For $q = 95\%$, $\bar{\alpha}_q(\sigma_t) = 0.58$, which is more than double the constant in the S&P calibration.

The quantiles $\bar{\alpha}_q(\sigma_t)$ condition on a fixed vector of parameters $(\bar{\kappa}_h, \bar{\theta}_h, \bar{\xi})$. To introduce parameter uncertainty, we define $\alpha_q(h_t)$ as the qth percentile value of the distribution of $h_t = \theta_h + (\xi/\sqrt{2\kappa_h})Z$, where Z is standard normal and the vector $(\kappa_h, \theta_h, \xi)$ is drawn from the posterior distribution. The quantiles $\alpha_q(h_t)$ are estimated via simulation and then transformed as $\alpha_q(\sigma_t) = \exp(\alpha_q(h_t)/2)$. We find that the "unconditional" upper tail quantiles are only modestly larger than the corresponding conditional tail quantiles $\bar{\alpha}_q(\sigma_t)$. That is, the "within" variation due to volatility in σ_t dominates the "between" variation due to uncertainty in the posterior distribution of the model parameters. These quantiles are displayed in Table 11.

As shown in Table 10, the posterior distribution for the full sample is not dramatically different from that of the precrisis sample. The inclusion of the crisis period of high spreads and high volatility shifts the posterior distributions of the long-run means θ_x and θ_h to the right. Similarly, as volatility was sustained during the crisis period, the posterior distributions of the mean-reversion parameter κ_h and volatility-of-volatility ξ are shifted to the left. In Table 11, we see that the net effect on the posterior distribution of σ_t is that upper tail quantiles are increased by 50%. This is arguably a modest increase relative to the impact of the credit crisis on perceptions of market risk and does not change the qualitative implications of the estimates.

We see that the speed of mean-reversion in X_t is poorly estimated. This is to be expected. In estimation

Table 12 Exceedance Probabilities Under Stochastic Volatility

Sample period	Sample from posterior?	$\Pr(M_{6\text{ mo}} >$ 70 bp)	$\Pr(M_{1\text{ yr}} >$ 90 bp)	$\Pr(M_{1\text{ yr}} >$ 150 bp)
Precrisis	No	42.1	46.2	2.7
	Yes	25.9	23.8	5.7
Full sample	No	178.5	279.2	59.5
	Yes	121.9	172.2	53.1

Note. Probabilities in basis points.

of a CIR model by maximum likelihood, Chapman and Pearson (2000) and Phillips and Yu (2005) show that $\hat{\kappa}_x$ is biased and has large standard error in reasonable sample sizes and that these problems are most severe when κ_x is near zero. We find that the mean of the posterior distribution of κ_x is small and negative in both the precrisis and full samples, which implies that X_t follows an explosive process. Given that the confidence interval for κ_x is large and roughly symmetric around zero, a more reasonable interpretation is that κ_x is near zero and therefore that the rate of mean-reversion is too slow to be estimated with any precision given only a few years of observations. This runs counter to the "stylized fact" of mean-reversion as a motivation for the CPDO (see §2) but is consistent with the estimates of Jacobs and Li (2008) for a similar model.[29] Because κ_x is difficult to estimate, the confidence interval for θ_x is quite wide as well. To test the robustness of our estimates of the parameters of the log-volatility process to uncertainty in (κ_x, θ_x), we also estimate the model with κ_x and θ_x fixed at the values imposed by S&P. As can be observed in Table 10, the posterior distribution for $(\kappa_h, \theta_h, \xi)$ is qualitatively unaffected by our assumptions on (κ_x, θ_x).

Finally, we evaluate exceedance probabilities implied by our SV model. As in §4.1, we consider the probability of breaching 70 bp within six months and the probability of breaching 90 bp within one year. We estimate the exceedance probabilities via simulation using the precrisis sample. When parameters are held fixed at the posterior mean, the exceedance probabilities are each more than 40 basis points. When parameters are sampled from the posterior distribution, the exceedance probabilities are around 24 basis points. By comparison, in the exercises of §4.1, the comparable exceedance probabilities for the S&P specification were under 0.1 basis points. As shown in Table 12, the probability of breaching 150 bp—virtually impossible under the S&P specification—is in the range of two to six basis points. We also examine the exceedance probabilities implied by the full sample posterior and find they are magnified several times.

[29] In their estimation of a stochastic volatility model for spreads on corporate bonds, Jacobs and Li (2008) find an interquartile range of [0.035, 0.093] for the speed of mean-reversion in spreads.

6. Discussion

For both S&P and Moody's, the rating of CPDOs was driven by models that assigned effectively zero probability (of order 10^{-7} or less) to reaching the levels of spreads actually realized in the CDS markets in late 2008. This spike in spreads would, to the best of our knowledge, have triggered default and large losses for every CPDO note that had managed to survive to that point. All the issues in question had been rated AAA at origination in late 2006 or early 2007.

Perhaps more importantly, we find that the agency models failed under more mundane conditions. The models assigned very small probabilities (well under one basis point) to reaching the spread levels seen in the second half of 2007. Credit market conditions at the time were qualitatively comparable to those of the previous downturn (2001–2003) and therefore should not have been relegated to the extreme tail of possible outcomes. Even during the quiet precrisis period, the models significantly understated kurtosis. Thus, even if the crisis is viewed as an unforeseeable regime shift, the model underpredicted the likelihood of realizing high spreads within the precrisis regime.

It seems highly unlikely that there will be any new issuance of CPDO in the foreseeable future. Ratings presumably would be based on less sanguine assumptions, and this would greatly reduce the maximal coupons consistent with an investment grade rating.[30] The CPDO is implicitly an arbitrage of ratings-based investment mandates. Because it merely repackages existing instruments and provides no structuring of liabilities, its raison d'être is to accommodate fixed-income investors who seek higher yield but are constrained by limits on noninvestment grade holdings. If CPDO notes cannot offer a substantial yield premium over comparably-rated debt, the costs of the arbitrage (i.e., the management fees) cannot be covered. As the CPDO itself may no longer be viable, the proper aim of this case study is to extract larger lessons for the rating of complex products and for modeling credit risk in general.

We identify four broad lessons from the CPDO experience. First, there appears to be scope for mitigating model risk in rating methodologies through application of existing econometric tools. Formal specification tests on the distribution of model residuals are straightforward to implement. In the case of CPDO rating models, these tests would have revealed the models' failure to capture salient characteristics of spread processes in precrisis data. Similarly, the impact of parameter uncertainty on ratings is amenable to rigorous treatment. Following the suggestion of Coval et al. (2009), we demonstate the

[30] Rating agency treatment of CPDOs has already become more conservative as well as less model driven (Marjolin and Toutain 2008).

utility of Bayesian methods in assessing exceedance probabilities and other summary statistics of the distribution of future spreads.

Second, stochastic volatility is a salient characteristic of credit spreads, at least under the physical measure, even in normal market conditions. Our estimates in a model with stochastic volatility imply much higher probabilities on reaching high spread levels than can be obtained in the rating agency specifications. For structured products with sensitivity to market spreads, stochastic volatility can significantly raise the hurdle for high investment grade status.

Third, model implementation should not be neglected as a form of model risk. Complex products demand complex software. Unless model specifications and calibrations and pool compositions are disclosed in full detail, which heretofore has not been the case, it is impossible for third parties to verify independently the results of model simulations. Software issues seem to have affected the rating efforts of both agencies. Our results suggest that the bugs and Moody's ex post model modifications (as reported by the *Financial Times*) did not play a fundamental role in the failure of the agency rating of CPDOs. Nonetheless, the episode underscores the possibility that misrating of other complex instruments could arise because of software errors.

Finally, it should be recognized that specification and calibration of a tractable yet robust process for market spreads is far from trivial. It is an adage among quants that *all* models break down under severe distress in the markets, so a test rooted in the experience of late 2008 might seem unfair. However, the AAA designation is intended to convey that a security is nearly riskless, so the evaluation exercise is inextricably linked to performance under extreme situations.[31] Indeed, capturing the dynamics under "once per century" distress situations is more important for a rating model than fidelity to market behavior under normal conditions. The manifest difficulty of the task suggests that model risk may pose an insurmountable barrier to reliable rating of complex and market-sensitive instruments.

Acknowledgments

The opinions expressed here are the authors' own and do not reflect the views of the Board of Governors or its staff or of Barclays Capital. This article is academic research; it is not investment advice or an invitation to make an investment. The authors are grateful to Anne Le Henaff, Mehdi Kheloufi-Trabaud, and Michael Mueller-Heumann of Moody's and to William Morokoff and Cristina Polizu of Standard & Poor's for providing model details. The authors thank Pawel Szerszen for software and advice on MCMC estimation. The authors also benefitted from helpful discussion with Michael Gibson, David Jones, David Lynch, Pat Parkinson, Yasemin Saltuk, and Mark Van Der Weide. Paul Reverdy and Jim Marrone provided outstanding research assistance.

[31] Danielsson (2008) builds upon this observation to argue that model-based risk management cannot be a robust foundation for macroprudential regulation of financial institutions.

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