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The informational content of the embedded deflation option in TIPS ☆

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ABSTRACT

We estimate the value of the embedded option in U.S. Treasury Inflation-Protected Securities (TIPS). The embedded option value exhibits time variation that is correlated with periods of deflationary expectations. We construct embedded option explanatory variables that are statistically and economically significant for explaining future inflation, even in the presence of traditional inflation variables such as lagged inflation, the gold return, the crude oil return, the VIX return, liquidity, surveys, and the yield spread between nominal Treasuries and TIPS. After conducting robustness tests, we conclude that the TIPS embedded option contains useful information for future inflation.

1. Introduction

The market for U. S. Treasury Inflation-Protected Securities (TIPS) has experienced significant growth since its inception in 1997. As of May 2010, the face amount of outstanding TIPS was about $563 billion, which was roughly 8% of the size of the nominal U. S. Treasury market. The TIPS market has averaged about $47 billion in new issuances each year and has about $10.6 billion of average daily turnover.[1] The main advantage of TIPS over nominal Treasuries is that an investor who holds TIPS is hedged against inflation risk.[2] Although there are costs to issuing TIPS (Roush, 2008), there appears to be widespread agreement that the benefits of TIPS outweigh the costs. Campbell et al. (2003), Kothari and Shanken (2004), Roll (2004), Mamun and Visaltanachoti (2006), Dudley et al. (2009), Barnes et al. (2010), Huang and Zhong (2013) and Bekaert and Wang (2010) all conclude that TIPS offer significant diversification and hedging benefits to risk averse investors.

The main contribution of our paper is to point out an informational benefit of TIPS that has been ignored in the literature. Specifically, we uncover an informational content of the embedded

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[1] Sources: U.S. Treasury and the Federal Reserve Board.
[2] The coupon payments and the principal amount of a TIPS are indexed to inflation using the Consumer Price Index (CPI), which protects an investor's purchasing power.

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deflation option in TIPS. We develop a model to value the embedded option and we show that the time variation in the option's value is correlated with periods of deflationary expectations. We also show that the embedded option return is economically important and statistically significant for explaining future inflation, even in the presence of common inflation variables such as the yield spread, the return on gold, the return on crude oil, liquidity, surveys, and lagged inflation. We argue that our results should be useful to anyone who is interested in assessing future inflation.

At the maturity date of a TIPS, the TIPS owner receives the greater of the original principal or the inflation adjusted principal. This contractual feature is an embedded put option since a TIPS investor can force the U.S. Treasury to redeem the TIPS at par if the cumulative inflation over the life of the TIPS is negative (i.e., deflation). The first TIPS auction in 1997 was for a 10-year note. Prior to the auction, Roll (1996) dismissed the importance of the embedded option since the United States had not experienced a decade of deflation for more than 100 years. Our paper directly examines the embedded deflation option in TIPS. Using a sample of 10-year TIPS from 1997 to 2010, we estimate that the value of the embedded option does not exceed \$0.0704 per \$100 principal amount. If we amortize \$0.0704 over the 10-year life of a TIPS, the impact on the TIPS yield is very small, which appears to justify the Roll (1996) comment. However, when we add 5-year TIPS to our sample, we find that the estimated embedded option value is much larger, up to \$1.4498 per \$100 principal amount. If we amortize \$1.4498 over the 5-year life of a TIPS, the impact on the yield is about 29 basis points.[3] We also find significant time variation in the embedded option values for both 5-year and 10-year TIPS. We show that this time variation is useful for explaining future inflation, even in the presence of widely used inflation variables such as the return on gold, lagged inflation, the return on crude oil, and so forth. We call this the informational content of the embedded option in TIPS.

To value the embedded option in TIPS, we use a continuous-time term structure model that has two factors, the nominal interest rate and the inflation rate. Since our two factors are jointly Gaussian, we obtain a closed-form solution for the price of a TIPS. Using our closed-form solution, we decompose the price of each TIPS into two parts, a part that corresponds to the embedded option value and a part that corresponds to the inflation-adjusted coupons and the inflation-adjusted principal. This makes our approach different from what is found in Sun (1992), Bakshi and Chen (1996), Jarrow and Yildirim (2003), Buraschi and Jiltsov (2005), Lioui and Poncet (2005), Chen et al. (2010), Ang et al. (2008) and Haubrich et al. (2012). These papers show how to value real bonds, but they ignore the embedded deflation option in TIPS. To the best of our knowledge, we are the first to price the embedded option in TIPS and to use its time variation to explain future inflation. Christensen et al. (2015) estimate the value of the embedded option in TIPS, but unlike our paper they do not use the option's return to explain future inflation. Kitsul and Wright (2013) study options-implied inflation probabilities, but they use CPI caps and floors instead of TIPS to fit their model.

When we fit our model to the data, we find that prior to 2002 the embedded option values are close to zero. During 2002–2004, the option values have considerable time variation. The overall trend during this time period is increasing option values followed by decreasing option values, with a peak around November 2003. From 2005 through the first half of 2008, there is some variation in option values, but mostly the values are close to zero. Finally, during the second half of 2008 and all of 2009, there is a surge in option values, which outstrips the previous peak value from 2003. We argue that the time variation in option values is capturing the deflation scare of 2003–2004 and the deflationary expectations that were associated with the financial crisis in 2008–2009. Our results are consistent with those in Campbell et al. (2009), Wright (2009) and Christensen et al. (2010).

To quantify the informational content of the embedded option in TIPS, we use our estimated option values from 5-year and 10-year TIPS to construct explanatory variables that we use in a regression analysis. We construct two value-weighted indices, one for the embedded option monthly return and one for the embedded option annual return. We show that the embedded option monthly (annual) return index is statistically significant for explaining the one-month (one-year) ahead inflation rate (Table 5). The embedded option return indices remain significant when we include control variables such as lagged inflation, the return on gold, the VIX index, market-wide liquidity, and the yield spread. By itself, the embedded option monthly return index explains up to 25% of the variation in the one-month ahead inflation rate (Table 5). When we include our control variables, this number increases to more than 35%. Using our regression point estimate for 10-year TIPS, we find that a 100% embedded option return (which is less than one standard deviation) is consistent with a 0.63% decrease in the one-month ahead annualized inflation rate. In terms of the standard deviations, an embedded option return equal to one standard deviation is consistent with about a one-half standard deviation change in the one-month ahead annualized inflation rate. Thus our results are economically significant as well as statistically significant. We find similar results when we exclude the period of the financial crisis. Thus our results are not being driven solely by the events of 2008–2009. We find a slightly lower level of significance when we examine the one-year ahead inflation rate. However, for most of our regressions, at least one of our embedded option indices is significant while more common variables, such as the return on gold and the yield spread, are often insignificant.

We verify our results by conducting several robustness checks. First, we argue that liquidity is not a likely explanation for our results. To investigate this, we eliminate the off-the-run securities from our sample (Section 4.6.1) and we re-construct our embedded option indices using only the on-the-run securities, which are the most liquid TIPS. We show that all of our previous regression results continue to hold with on-the-run TIPS (Table 6). Thus our results are not being driven by possible illiquidity that surrounds off-the-run TIPS (see Fleming and Krishnan, 2012). Second, we construct two additional explanatory variables, $ORF_{t-1,t}$ and $ORF_{t-12,t}$, that capture the fraction of embedded options in each month (resp. each year) that has a positive return. These variables are less sensitive to model specification since any other pricing model that produces the same sign for the embedded option returns will produce the same explanatory variables. We find that $ORF_{t-1,t}$ and $ORF_{t-12,t}$ are statistically significant for the full sample of TIPS and for the on-the-run TIPS (Table 7). Thus even if we ignore the magnitude of the option returns and focus solely on the sign of those returns, we find that the embedded option in TIPS contains useful information for explaining future inflation. Third, we examine the ability of our embedded option indices to explain the future inflation rate when we use alternative weights for our model fitting (Table 8), when we use additional control variables in the regressions (Table 9), when we use the return on the option price index or log changes in option values as explanatory variables (Table 10), when we fit our model using liquidity-adjusted TIPS prices (Table 11), and when we use a rolling window parameter estimation (Table 12). We also analyze a variety of additional robustness tests, which are described in the appendix. After conducting all of these robustness checks, we find that our main

[3] These numbers rely on parameter estimates from an equally-weighted sum of squared errors. For robustness, we also estimated our model using inverse duration weights, as in Gürkaynak et al. (2007). We provide details in Table 2.

conclusion is not altered – the embedded option in TIPS contains relevant information for explaining the future inflation rate, out to a horizon of at least one year.

Explaining future inflation has received a considerable amount of attention in the literature. Many explanatory variables have been proposed, such as the interest rate level and lagged inflation (Fama and Gibbons, 1984), the unemployment rate (Stock and Watson, 1999), the money supply (Stock and Watson, 1999; Stockton and Glassman, 1987), surveys (Mehra, 2002; Ang et al., 2007; Chernov and Mueller, 2012; Chun, 2011), the price of gold (Bekaert and Wang, 2010), and the yield spread between nominal Treasury securities and TIPS (Stock and Watson, 1999; Shen and Corning, 2001; Roll, 2004; Christensen et al., 2010; Gürkaynak et al., 2010; D'Amico et al., 2014; Pflueger and Pflueger, 2011). Our paper is different since we focus on the embedded option in TIPS. However, we include some of these traditional variables as control variables in our regressions. This allows us to analyze the marginal contribution of the variables.

The remainder of our paper is organized as follows. Section 2 introduces our model and derives a closed form solution for TIPS and for nominal Treasury securities. Section 3 describes the data. Section 4 presents our empirical methodology, our model estimation results, our regression results, and our robustness checks. Section 5 gives our concluding remarks. The technical details of our pricing model can be found in the appendix.

2. The model

We use a continuous-time model with two state variables, the nominal interest rate r_t and the inflation rate i_t. The evolution of r_t and i_t is described by the Gaussian processes

$$dr_t = (a_1 + A_{11}r_t + A_{12}i_t)dt + B_{11}dz^Q_{1t}, \quad (1)$$

$$di_t = (a_2 + A_{21}r_t + A_{22}i_t)dt + B_{21}dz^Q_{1t} + B_{22}dz^Q_{2t}, \quad (2)$$

where Q is a risk adjusted probability measure, z^Q_{1t} and z^Q_{2t} are independent Brownian motions under Q, and $a_1, a_2, A_{11}, A_{12}, A_{21}, A_{22}, B_{11}, B_{21}$, and B_{22} are parameters. Ang and Piazzesi (2003) show that the inflation rate impacts the mean of the short term nominal interest rate. We use their result as motivation for including the parameters A_{12} and A_{21} in Eqs. (1) and (2). This makes each of the processes in (1) and (2) more complex than the Vasicek (1977) process, but it allows for a richer set of dynamics between r_t and i_t.

In our empirical estimation below, we use both TIPS and nominal Treasury Notes (T-Notes). By including nominal T-Notes in our analysis, we increase the overall size of our sample. As a side benefit, we also avoid overfitting the TIPS market, which may help to control for the issues of TIPS mispricing and illiquidity that are raised by Fleckenstein et al. (2013) and Fleming and Krishnan (2012).

Our bond pricing models for TIPS and nominal T-Notes are derived under the Q probability measure, which eliminates the need to be specific about the functional form of the risk premia. For example, the inflation risk premium may be time varying, as shown in Evans (1998) and Grishchenko and Huang (2013), for the U.K. Gilt and U.S. Treasury markets, respectively. Furthermore, if the risk premia happen to be affine functions of r_t and i_t, then (1) and (2) are consistent with Barr and Campbell (1997), who show that the expected real interest rate in the UK is highly variable at short horizons, but it is comparatively stable at long horizons. Our model can support many functional forms for the risk premia since we can always describe r_t and i_t under the true probability measure and then use a prudent change of measure to arrive at (1) and (2). Thus the risk premia are subsumed by Q.

One advantage of specifying the model under Q is that the number of parameters is reduced, which makes our model parsimonious. Since the volatility matrix in (1) and (2) is lower triangular, as in Chun (2011), our model has only 9 parameters. In contrast, Sun (1992) uses a model with 13 parameters, Lioui and Poncet (2005) use 17 parameters, and Christensen et al. (2010) use 28–40 parameters. Given the limited data for TIPS, we want to keep the number of parameters as small as possible.

2.1. TIPS pricing

Consider a TIPS that is issued at time u and matures at time T. We want to determine the price P_t of the TIPS at time t, where $u < t < T$. The principal amount is F and the coupon rate is c. Suppose there are n coupons yet to be paid, where the coupon payments occur at $t_1, t_2, \dots, t_n$. If we let $u < t < t_1 < t_2 < \cdots < t_{n-1} < t_n = T$, we can write the TIPS price as

$$P_t = \mathbb{E}^Q_t\left[\sum_{k=1}^{n} cFe^{\int_u^{t_k} i_s ds}e^{-\int_t^{t_k} r_s ds} + e^{-\int_t^{t_n} r_s ds}\left[Fe^{\int_u^{t_n} i_s ds} + \max\left(0, F - Fe^{\int_u^{t_n} i_s ds}\right)\right]\right] \quad (3)$$

where $\mathbb{E}^Q_t[\cdot]$ denotes expectation at time t under Q. The right-hand side of (3) has three terms. The first term is the value of the inflation-adjusted coupon payments, the second term is the value of the inflation-adjusted principal, and the third term is the value of the embedded option. The inflation adjustment in (3) is captured by the exponential term

$$e^{\int_u^{t_k} i_s ds} \quad (4)$$

for $k = 1, 2, \dots, n$. In our empirical specification, we use the U.S. Treasury's CPI index ratio to capture the known part of the inflation adjustment.[4] The unknown inflation adjustment depends on the stochastic process in (2). Using (1) and (2), the random variables $\int_t^{t_k} r_s ds$ and $\int_t^{t_k} i_s ds$ for $k = 1, 2, \dots, n$ have a joint Gaussian distribution. Thus we can evaluate the expectation in (3) to get a closed-form solution for the TIPS price. Our solution can be written in terms of the moments $\mathbb{E}^Q_t[\int_t^{t_k} r_s ds], \mathbb{E}^Q_t[\int_t^{t_k} i_s ds], Var^Q_t[\int_t^{t_k} r_s ds], Var^Q_t[\int_t^{t_k} i_s ds]$, and $Cov^Q_t[\int_t^{t_k} r_s ds, \int_t^{t_k} i_s ds]$ for $k = 1, 2, \dots, n$, which are also available in closed-form. We give complete details in Appendix A.

In Section 4.2 we describe how the nine parameters in (1) and (2) are estimated. Once the parameters are estimated, we use the third term in (3) to calculate the value of the embedded deflation option. We show in Appendix A that the value of the embedded option is

$$\mathbb{E}^Q_t\left[e^{-\int_t^{t_n} r_s ds}\max\left(0, F - Fe^{\int_u^{t_n} i_s ds}\right)\right] = Fe^{\mathbb{E}^Q_t(Z_1) + \frac{1}{2}Var^Q_t(Z_1)}N(x_1) - Fe^{\int_u^t i_s ds}e^{\mathbb{E}^Q_t(Z_3) + \frac{1}{2}Var^Q_t(Z_3)}N(x_2),$$

$$x_1 = \frac{d - \mathbb{E}^Q_t(Z_2) - Cov^Q_t(Z_1, Z_2)}{\sqrt{Var^Q_t(Z_2)}},$$

$$x_2 = \frac{d - \mathbb{E}^Q_t(Z_2) - Cov^Q_t(Z_3, Z_2)}{\sqrt{Var^Q_t(Z_2)}},$$

where $N(\cdot)$ is the standard normal cumulative distribution function, $d = -\int_u^t i_s ds$, $Z_1 = -\int_t^{t_n} r_s ds$, $Z_2 = \int_t^{t_n} i_s ds$, and $Z_3 = Z_1 + Z_2$. Using this formula, we can calculate the embedded option value month

[4] The U.S. Treasury constructs the CPI index ratio using the lagged CPI. The impact of the index lag is small economically. Grishchenko and Huang (2013) estimate that it does not exceed four basis points in the TIPS real yield.

by month for every TIPS in our sample. We then use these option values to construct explanatory variables for our regression analysis (see Section 4.4).

2.2. Nominal Treasury Notes pricing

Consider a nominal T-Note that is issued at time u and matures at time T. We want to determine the T-Note's price $\overline{P}_t$ at time t, where $u < t < T$. The principal amount is F, the coupon rate is $\bar{c}$, and there are n coupon payments yet to be paid, at times $t_1, t_2, \ldots, t_n$. As before, we let $u < t < t_1 < t_2 < \cdots < t_{n-1} < t_n = T$ and thus we can write the T-Note's price as

$$\overline{P}_t = \mathbb{E}_t^Q \left[\sum_{k=1}^{n} \bar{c} F e^{-\int_t^{t_k} r_s ds} + F e^{-\int_t^{t_n} r_s ds} \right]. \tag{5}$$

The price in (5) contains two terms. The first term is the value of the nominal coupon payments, while the second term is the value of the principal amount. Since we are pricing a nominal T-Note, there is no explicit inflation adjustment in (5). However, since A_{12} in (1) might not be zero, the price $\bar{P}_t$ depends not only on r_t and the parameters in (1), but also on i_t and the parameters in (2). This sets our model apart from Vasicek (1977).

Like Eq. (3), our closed-form solution for Eq. (5) depends on the moments $\mathbb{E}_t^Q[\int_t^{t_k} r_s ds], \mathbb{E}_t^Q[\int_t^{t_k} i_s ds], Var_t^Q[\int_t^{t_k} r_s ds], Var_t^Q[\int_t^{t_k} i_s ds]$, and $Cov_t^Q[\int_t^{t_k} r_s ds, \int_t^{t_k} i_s ds]$ for $k = 1, 2, \ldots, n$. We give details in Appendix B.

3. The data

To estimate our model, we first construct a monthly time series for the nominal interest rate and for the inflation rate. We obtain our data from the Federal Reserve Economic Database (FRED) at the Federal Reserve Bank of St. Louis. For the nominal interest rate we use the 3-month Treasury Bill rate. Other short-term Treasury Bill rates give similar results. We start with daily observations of the 3-month Treasury Bill rate and we extract the month-end observations to get a monthly time series. To construct a monthly time series for the inflation rate, we use the non-seasonally adjusted Consumer Price Index for All Urban Consumers (CPI-U), which is released monthly by the U.S. Bureau of Labor Statistics. This is the same index that is used for inflation adjustments to TIPS. We let Π_t denote the value of the CPI-U that corresponds to month t. We define the inflation rate from month t to month $t + \tau$ as

$$i_{t,t+\tau} = \frac{12}{\tau} \ln \left[\frac{\Pi_{t+\tau}}{\Pi_t} \right], \tag{6}$$

where $12/\tau$ is an annualization factor. Setting $\tau = 1$ in (6) gives the one-month ahead inflation rate, while $\tau = 12$ gives the one-year ahead inflation rate. The inflation rate in (6) is the annualized log change in the price level, which is consistent with (4).

We use Datastream to obtain daily price data for all of the 5-year and 10-year TIPS that have been auctioned by the U.S. Treasury through May 2010. We use this daily data to construct the gross market price for each available TIPS on the last business day of each month. We use 10-year TIPS since it gives us the longest possible sample period, from January 1997 (the first ever TIPS auction) through May 2010. However, we include 5-year TIPS since the embedded option values for these TIPS are larger due to the lower cumulative inflation. Each TIPS in Datastream is identified by its International Securities Identification Number (ISIN). To verify the ISIN, we match it with the corresponding CUSIP in Treasury Direct. We use abbreviations to simplify the exposition. For example, the ISIN for the 10-year TIPS that was auctioned in January 1997 is US9128272M3. Since US9128 is common to all of the TIPS, we drop these characters and use the abbreviation 272M3. For each TIPS, we obtain from Datastream the clean price, the settlement date, the coupon rate, the issue date, and the maturity date. At the end of each month, we identify the previous and the next coupon dates, and we count the number of coupons remaining. We construct the gross market price of a TIPS as

$$\text{Gross Market Price} = (\text{Clean Price} + \text{Accrued Interest}) \times \text{Index Ratio}. \tag{7}$$

In (7), the accrued interest is calculated using the coupon rate, the settlement date, the previous coupon date, and the next coupon date, while the index ratio is the CPI-U inflation adjustment term that is reported on Treasury Direct.

In addition to our sample of 5-year and 10-year TIPS, our estimation uses data on 5-year and 10-year nominal T-Notes. There are 21 ten-year TIPS and 7 five-year TIPS in our sample. For each TIPS, we search for a nominal T-Note with approximately the same issue and maturity dates. We are able to match all but one of our TIPS (the exception is January 1999, for which we cannot identify a matching 10-year nominal T-Note). Thus our sample includes 21 ten-year TIPS and 7 five-year TIPS, plus 20 ten-year matching nominal T-Notes and 7 five-year matching nominal T-Notes. For the matching nominal T-Notes, we obtain our data from Datastream.

We include nominal T-Notes in our sample for several reasons. First, nominal Treasury securities are an important input to any term structure model that is used to assess inflationary expectations. For example, see Campbell and Viceira (2001), Brennan and Xia (2002), Ang and Piazzesi (2003), Sangvinatsos and Wachter (2005) and Kim (2009), to name just a few. Second, by including nominal T-Notes in our estimation, we effectively double our sample size in each month, which helps to estimate the model parameters more precisely. Lastly, since the TIPS market is only about 8% of the size of the nominal Treasury market, we avoid overfitting the TIPS market by including nominal Treasury securities. This helps to control for the trading differences between TIPS and nominal Treasuries (Fleming and Krishnan, 2012) and it helps to address, but does not completely resolve, the issue of relative overpricing in the TIPS market (Fleckenstein et al., 2013). Since we include nominal Treasuries in our sample, it is less likely that our fitted parameters are capturing TIPS market imperfections that are present in the data.

To summarize, our data set includes monthly interest rates, monthly inflation rates, and monthly gross prices for TIPS and matching nominal T-Notes. Table 1 shows the TIPS and the nominal T-Notes that are included in our sample. There are 1,405 monthly observations for 10-year TIPS (Panel A), 1,268 monthly observations for 10-year nominal T-Notes (Panel B), 256 monthly observations for 5-year TIPS (Panel C), and 250 monthly observations for 5-year nominal T-Notes (Panel D).

4. Empirical results

Our empirical approach involves several steps. First, we estimate the parameters in (1) and (2) by minimizing the sum of the squared errors (SSE) for the full sample of 5-year and 10-year TIPS and matching nominal T-Notes (see Table 1). For completeness, we solve similar minimization problems using only 10-year TIPS and matching T-Notes (Panels A and B of Table 1) and using only 5-year TIPS and matching T-Notes (Panels C and D of Table 1). We solve each minimization problem twice, once with equally-weighted SSE and once with inverse duration weighted SSE (Gürkaynak et al., 2007). Both weighting schemes produce nearly identical model fit, as measured by the mean absolute yield error, and very similar inflation regression results. Since the results are

Table 1
Summary of treasury securities data.

ISIN	Issue date	Maturity date	Coupon	Obs.	ISIN	Issue date	Maturity date	Coupon	Obs.
Panel A: 10-year treasury inflation protected securities					*Panel B: 10-year matching nominal Treasury Notes*				
272M3	1/15/1997	1/15/2007	3.375	120	272J0	2/15/1997	2/15/2007	6.25	120
273T7	1/15/1998	1/15/2008	3.625	120	273X8	2/15/1998	2/15/2008	5.5	120
274Y5	1/15/1999	1/15/2009	3.875	120	N/A				
275W8	1/15/2000	1/15/2010	4.25	120	275Z1	2/15/2000	2/15/2010	6.5	120
276R8	1/15/2001	1/15/2011	3.5	113	276T4	2/15/2001	2/15/2011	5	112
277J5	1/15/2002	1/15/2012	3.375	101	277L0	2/15/2002	2/15/2012	4.875	100
28AF7	7/15/2002	7/15/2012	3	95	28AJ9	8/15/2002	8/15/2012	4.375	94
28BD1	7/15/2003	7/15/2013	1.875	83	28BH2	8/15/2003	8/15/2013	4.25	82
28BW9	1/15/2004	1/15/2014	2	77	28CA6	2/15/2004	2/15/2014	4	76
28CP3	7/15/2004	7/15/2014	2	71	28CT5	8/15/2004	8/15/2014	4.25	70
28DH0	1/15/2005	1/15/2015	1.625	65	28DM9	2/15/2005	2/15/2015	4	64
28EA4	7/15/2005	7/15/2015	1.875	59	28EE6	8/15/2005	8/15/2015	4.25	58
28ET3	1/15/2006	1/15/2016	2	53	28EW6	2/15/2006	2/15/2016	4.5	52
28FL9	7/15/2006	7/15/2016	2.5	47	28FQ8	8/15/2006	8/15/2016	4.875	46
28GD6	1/15/2007	1/15/2017	2.375	41	28GH7	2/15/2007	2/15/2017	4.625	40
28GX2	7/15/2007	7/15/2017	2.625	35	28HA1	8/15/2007	8/15/2017	4.75	34
28HN3	1/15/2008	1/15/2018	1.625	29	28HR4	2/15/2008	2/15/2018	3.5	28
28JE1	7/15/2008	7/15/2018	1.375	23	28JH4	8/15/2008	8/15/2018	4	22
28JX9	1/15/2009	1/15/2019	2.125	17	28KD1	2/15/2009	2/15/2019	2.75	16
28LA6	7/15/2009	7/15/2019	1.875	11	28LJ7	8/15/2009	8/15/2019	3.625	10
28MF4	1/15/2010	1/15/2020	1.375	5	28MP2	2/15/2010	2/15/2020	3.625	4
Panel C: 5-year treasury inflation protected securities					*Panel D: 5-year matching nominal Treasury Notes*				
273A8	7/15/1997	7/15/2002	3.625	60	273C4	7/31/1997	7/31/2002	6	61
28CZ1	10/15/2004	4/15/2010	0.875	66	28CX6	10/15/2004	10/15/2009	3.375	60
28FB1	4/15/2006	4/15/2011	2.375	50	28FD7	4/30/2006	4/30/2011	4.875	49
28GN4	4/15/2007	4/15/2012	2	38	28GQ7	4/30/2007	4/30/2012	4.5	38
28HW3	4/15/2008	4/15/2013	0.625	26	28HY9	4/30/2008	4/30/2013	3.125	26
28KM1	4/15/2009	4/15/2014	1.25	14	28KN9	4/30/2009	4/30/2014	1.875	14
28MY3	4/15/2010	4/15/2015	0.5	2	28MZ0	4/30/2010	4/30/2015	2.5	2

The table shows our sample of 10-year TIPS (Panel A), 10-year matching T-Notes (Panel B), 5-year TIPS (Panel C), and 5-year matching T-Notes (Panel D). The ISIN numbers are abbreviated. The full ISIN coding is preceded by US9128. "N/A" refers to "not available". Sample period is January 1997 - May 2010, monthly frequency. There are 1,405 (1,268) observations for 10-year TIPS (T-Notes) and 256 (250) observations for 5-year TIPS (T-Notes).
Source: Datastream Advance 4.0.

similar, we use the estimated parameters from the equally-weighted SSE for the main part of our empirical analysis, but we include results based on inverse duration weighted SSE as one of our robustness checks.

Second, we use our estimated parameters and our formula for the TIPS embedded option (see Section 2.1) to calculate a set of times series of embedded option values for each TIPS in our sample. We use these time series to construct value-weighted embedded option return indices. Our option return indices, along with various controls, are then used as explanatory variables for a variety of inflation regressions. In most of our regressions, our embedded option return indices are statistically significant for explaining the one-month ahead and the one-year ahead inflation rate. We then analyze several robustness checks, such as alternative weighting schemes, alternative variable specifications, additional control variables, and so forth.

4.1. Identification of r_t and i_t

Since the discrete-time inflation rate and interest rate are not equal to their continuous-time counterparts, we jointly identify the continuous-time rates by solving a system of equations. At the end of month t, we set the 3-month T-bill rate equal to $-4\ln\mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^{t+1/4} r_s ds}\right]$ and we set the annualized inflation rate for month t equal to $12\ln\mathbb{E}_t^{\mathbb{Q}}\left[e^{\int_t^{t+1/12} i_s ds}\right]$. Given our Gaussian assumptions, this produces a linear system of equations for the continuous-time variables r_t and i_t. For each month t, we solve this system of equations to identify r_t and i_t in terms of the discrete-time rates. The identified values of r_t and i_t are then used as inputs for bond pricing in (3) and (5). Since the system of equations depends on the estimated model parameters, the identification of r_t and i_t is incorporated into our model fitting process, which we describe in the next section. We also tried identifying r_t and i_t by simply equating the continuous-time variables to their discrete-time counterparts. This simpler procedure did not have a material impact on the significance of our results.

4.2. Parameter estimation

We estimate the parameters in (1) and (2) by minimizing the weighted sum of the squared errors between our model prices and the observed market prices. A similar technique is used in Bakshi et al. (1997) and Huang and Wu (2004). Specifically, we solve the problem

$$\min_{\Theta} SSE(\Theta) = \sum_{t=1}^{T}\left[\sum_{n=1}^{N_t} w_{nt}\left(P_{nt}^* - P_{nt}\right)^2 + \sum_{n=1}^{\overline{N}_t}\bar{w}_{nt}\left(\overline{P}_{nt}^* - \overline{P}_{nt}\right)^2\right], \tag{8}$$

subject to the identification conditions in Section 4.1. In (8), T is the total number of months in the sample, N_t is the number of TIPS in the sample for month t, $\overline{N}_t$ is the number of nominal T-Notes in the sample for month t, P_{nt}^* is the gross market price of the nth TIPS for month t, $\overline{P}_{nt}^*$ is the gross market price of the nth nominal T-Note for month t, P_{nt} is the model price of the nth TIPS for month t, and $\overline{P}_{nt}$ is the model price of the nth nominal T-Note for month t. The model prices P_{nt} and $\overline{P}_{nt}$ are given by (3) and (5), respectively, and the parameter vector is $\Theta = (a_1, a_2, A_{11}, A_{12}, A_{21}, A_{22}, B_{11}, B_{21}, B_{22})^\top$. For the equally-weighted SSE, we set $w_{nt} = \bar{w}_{nt} = 1$. For the inverse duration weighted SSE, we set $w_{nt} = 1/Dur_{nt}$ and $\bar{w}_{nt} = 1/\overline{Dur}_{nt}$,

Table 2
Two-factor model estimation results.

	Obs.	$\hat{\pi}_r$	$\hat{\pi}_i$	*meY*	*maeY*	Option value range
Panel A: equal weights						
5-year:	506	0.0828 (0.0099)	0.0314 (0.0041)	0.0006	0.0054	\$0.0000–\$1.4498
10-year:	2,673	0.0534 (0.0007)	0.0232 (0.0007)	−0.0002	0.0051	\$0.0000–\$0.0704
5&10-year:	3,179	0.0536 (0.0006)	0.0232 (0.0007)	−0.0001	0.0052	\$0.0000–\$1.4770
Panel B: Inverse duration weights						
5-year:	506	0.0865 (0.0105)	0.0340 (0.0052)	0.0003	0.0054	\$0.0000–\$1.7488
10-year:	2,673	0.0541 (0.0008)	0.0242 (0.0007)	−0.0002	0.0051	\$0.0000–\$0.2526
5&10-year:	3,179	0.0544 (0.0007)	0.0243 (0.0007)	−0.0001	0.0051	\$0.0000–\$1.1858

The table reports estimation errors and other statistics for the two-factor term structure model used to price TIPS and nominal T-Notes. We estimate our model using three different samples of securities: 5-year TIPS and matching T-Notes, 10-year TIPS and matching T-Notes, and 5&10-year TIPS and matching T-Notes. Nine-parameter vector Θ is estimated using Newton's method that minimizes the weighted sum of squared errors (SSE) between model prices and observed monthly market prices at the end of month t. We minimize

$$SSE(\Theta) = \sum_{t=1}^{T}\left[\sum_{n=1}^{N_t} w_{nt}\left(P^*_{nt} - P_{nt}\right)^2 + \sum_{n=1}^{\overline{N}_t} \bar{w}_{nt}\left(\overline{P}^*_{nt} - \overline{P}_{nt}\right)^2\right],$$

where T is the total number of months in our sample, N_t is the number of TIPS in our sample at the end of month t, $\overline{N}_t$ is the number of nominal T-Notes in our sample at the end of month t, P^*_{nt} is the gross market price of the nth TIPS at the end of month t, $\overline{P}^*_{nt}$ is the gross market price of the nth nominal T-Note at the end of month t, P_{nt} is the model price of the nth TIPS at the end of month t, $\overline{P}_{nt}$ is the model price of the nth nominal T-Note at the end of month t, w_{nt} is the weight assigned to the nth TIPS at the end of month t, and $\bar{w}_{nt}$ is the weight assigned to the nth nominal T-Note at the end of month t. Panel A reports estimation results based on equal weights ($w_{nt} = \bar{w}_{nt} = 1$). Panel B reports estimation results based on inverse duration weights ($w_{nt} = 1/Dur_{nt}, \bar{w}_{nt} = 1/\overline{Dur}_{nt}$, where Dur_{nt} is the duration of the nth TIPS at the end of month t, and $\overline{Dur}_{nt}$ is the duration of the nth nominal T-Note at the end of month t). $\hat{\pi}_r$ is an estimate of the long-run mean of interest rate; $\hat{\pi}_i$ is an estimate of the long-run mean of inflation rate; *meY* is the mean yield error between actual and fitted bond yields; *maeY* is the mean value of the absolute yield error; Option Value Range shows the minimum and maximum option values among all TIPS-month observations. Sample period is from January 1997 to May 2010, monthly frequency. Standard errors are calculated using Delta method and given in parentheses.

where Dur_{nt} is the duration of the nth TIPS for month t and $\overline{Dur}_{nt}$ is the duration of the nth nominal T-Note for month t.[5]

To solve (8), we use Newton's method in the nonlinear least squares (NLIN) routine in SAS. Since (8) is sensitive to the choice of initial conditions, we double check our results by re-solving the problem using the Marquardt method, which is known to be less sensitive to the choice of initial values. In particular, we use a two-step procedure, first using the Marquardt method and then polishing the estimated parameter values using Newton's method. This robustness check provides the same result as using Newton's method alone. For our reported estimates, we verify a global minimum for (8) by checking that the first-order derivatives are zero and all eigenvalues of the Hessian are positive, which implies a positive definite Hessian.

Table 2 summarizes our estimation results. When we estimate our model using all of the TIPS and matching T-Notes from Table 1, we find that the mean absolute yield error (*maeY*) is 52 basis points per annum for the equally-weighted SSE and 51 basis points for the inverse duration weighted SSE. If we use only 10-year TIPS and matching T-Notes, the *maeY* is 51 basis points for both the equally-weighted SSE and the inverse duration weighted SSE. In all of these estimations, our *maeY* is comparable in magnitude to the RMSE of 74 basis points reported by Chen et al. (2010). This is true even though our sample period is longer than theirs and our model is fit to a wider variation in economic conditions. More broadly, our yield errors are similar to other models in the literature. For example, the magnitude of our *maeY* is similar to the average pricing errors reported in Dai and Singleton (2000) for the interest rate swaps market using their $\mathbb{A}_2(3)_{DS}$ model. Our errors appear to be reasonable given that we are using a parsimonious model that is fit simultaneously to two markets, TIPS and nominal T-Notes.

We also estimated our model using only 5-year TIPS and 5-year matching nominal T-Notes. As shown in Table 1, the number of 5-year TIPS during our sample period is one-third the number of 10-year TIPS. Furthermore, we see in Table 2 that the number of monthly observations for 5-year TIPS and matching nominal T-Notes is about one-fifth the number of monthly observations for 10-year TIPS and matching nominal T-Notes. There is also a gap in the data using 5-year TIPS. There are no outstanding 5-year TIPS from August 2002 to September 2004. There is one outstanding 5-year TIPS from July 1997 to July 2002, and the next auction of 5-year TIPS occurred in October 2004. It is not until April 2006 that we have more than one outstanding 5-year TIPS in the same month. In spite of these issues, we went ahead and estimated our model using the available monthly 5-year TIPS data from July 1997–May 2010. As shown in Table 2, for both the equally-weighted SSE and the inverse duration weighted SSE, the *maeY* is 54 basis points. Although this is similar to the *maeY* from the other two estimations, it should be interpreted with caution due to the small sample of 5-year TIPS.

To check the economics of our estimations, we estimate the long-run means of r_t and i_t under Q, which we denote by $\hat{\pi}_r$ and $\hat{\pi}_i$, respectively. In Appendix C we show how to derive the formulas for the long-run means. As Table 2 shows, our estimates $\hat{\pi}_r$ and $\hat{\pi}_i$ are economically reasonable and are statistically different than zero. For example, using all of the TIPS and matching T-Notes from Table 1, and using equally-weighted SSE, we estimate the long-run mean interest rate is 5.36% and the long-run mean inflation rate is 2.32%. This implies a long-run mean real rate of 3.04%. Using inverse duration weights, the implied long-run mean real rate is 3.01%.

[5] The inverse duration weights are calculated on the last business day of each month. This is consistent with the prices in (8).

4.3. Time variation in embedded option values

The far right column of Table 2 shows the range of estimated values for the embedded deflation option in TIPS. The minimum estimated option value is close to zero. For the equally-weighted (inverse duration weighted) SSE, if we use 10-year TIPS and matching nominal T-Notes, the maximum option value across all TIPS-month observations is \$0.0704 per \$100 face amount (\$0.2526 per \$100 face amount). If we amortize \$0.0704 (\$0.2526) using semi-annual compounding over the 10-year life of a TIPS, we get about 0.7 basis points (2.5 basis points). Thus on average, ignoring the embedded option on any given trading day has only a small impact on the yield of a 10-year TIPS. This may help to explain why most of the existing TIPS literature does not focus on the embedded option.

For our estimations using 5-year TIPS and matching nominal T-Notes, the maximum option value across all TIPS-month observations is \$1.4498 per \$100 face amount for the equally-weighted SSE and \$1.7488 per \$100 face amount for the inverse duration weighted SSE. This is much higher than what we found for 10-year TIPS, but it makes sense because most of the 5-year TIPS were outstanding during the deflationary period in the second half of 2008. In addition, the probability of experiencing cumulative deflation over a 5-year period is higher than the probability of experiencing cumulative deflation over a 10-year period. This likely contributes to a higher embedded option value in 5-year TIPS relative to 10-year TIPS. If we amortize \$1.4498 (\$1.7488) over the life of a 5-year TIPS, we find that the embedded option value accounts for up to 29 basis points (35 basis points) of the TIPS yield. This is comparable to what is reported in Christensen et al. (2015), who find that the average value of the TIPS embedded option during 2009 is about 41 basis points.

We find that the estimated value of the embedded deflation option exhibits substantial time variation. We plot our results in Fig. 1, which uses the parameter estimates from the equally-weighted SSE. Panel A of Fig. 1 shows the time series of estimated option values for all 21 ten-year TIPS in our sample. We find a large spike in option values at the end of 2008 and the beginning of 2009. This corresponds to the period of the financial crisis, which was marked by deflationary expectations and negative changes in the CPI index for the second half of 2008. We also find a smaller spike in option values during the 2003–2004 period, which was also marked by deflationary pressure (Ip, 2004). The variation during 2003–2004 is difficult to see in Panel A, but it is more evident in Panel C, which is a zoomed version of Panel A. During most other time periods, the embedded option values are closer to zero. This is intuitive since if cumulative inflation is high, the embedded option will be further out-of-the-money and thus its value should be low.

We find similar results when we estimate our model using the combined sample of 5-year and 10-year TIPS and matching nominal T-Notes. We plot these results in Fig. 2, which uses the parameter estimates from the equally-weighted SSE. Panel A of Fig. 2 shows the estimated option values for all 7 five-year TIPS in our sample, while Panel B of Fig. 2 shows the estimated option values for all 21 ten-year TIPS.[6] We again find a large spike in option values during the financial crisis (both Panels A and B) and we also find a second increase during the 2003–2004 period (Panel B). Thus including 5-year TIPS does not alter the time variation in the option values.

Our results in Figs. 1 and 2 are consistent with the existing literature. Wright (2009), Christensen (2009) and Christensen et al. (2012) use TIPS to infer the probability of deflation. During the later part of 2008, Wright (2009, Fig. 2) shows that the probability of deflation was greater than one-half, which is confirmed by the results in Christensen (2009, Fig. 3). Christensen et al. (2012, Fig. 1) provide an estimate of the one-year ahead deflation probability from 1997–2010. Their Fig. 1 is strikingly similar to our Fig. 1, even though the two figures illustrate different quantities. In particular, their Fig. 1 shows the probability that the price level will decrease, while our Fig. 1 shows the value of the embedded option in TIPS. We also examine the Wright (2009) conjecture that the yield difference between on-the-run and off-the-run TIPS with similar maturity dates is mostly due to differences in the embedded deflation option values. The details are given in Appendix D.

4.4. Option-based explanatory variables

We use our estimated option values to construct explanatory variables for our regression analysis. For the nth TIPS in month t, let O_{nt} denote the estimated value of the embedded option. Thus the option return in month t for the nth TIPS is $R_{nt} = O_{nt}/O_{n,t-1} - 1$. For each of the estimations in Table 2, we construct a value-weighted index for the embedded option price level, a value-weighted index for the embedded option monthly return, and a value-weighted index for the embedded option annual return. The weight W_{nt} for the nth TIPS in month t is $W_{nt} = O_{n,t-1}/\sum_{n=1}^{N_t} O_{n,t-1}$, where N_t is the number of TIPS in the sample for month t. Note that we use the lagged value $O_{n,t-1}$ when constructing the weight W_{nt} for month t.[7] Thus the value-weighted embedded option price index in month t is

$$OP_t = \sum_{n=1}^{N_t} W_{nt} O_{nt}. \tag{9}$$

Panels B and D of Fig. 1 show (9) when the model is estimated using 10-year TIPS, 10-year matching nominal T-Notes, and an equally-weighted SSE. Likewise, Panel C of Fig. 2 shows (9) for 5-year and 10-year TIPS when the model is estimated using all of the bonds in Table 1 and an equally-weighted SSE.

In month t, our value-weighted index for the embedded option monthly return is

$$OR_{t-1,t} = \sum_{n=1}^{N_t} W_{nt} R_{nt}, \tag{10}$$

where W_{nt} and R_{nt} were defined earlier. Similarly, in month t our value-weighted index for the embedded option annual return is defined as

$$OR_{t-12,t} = \sum_{n=1}^{N_t} \widehat{W}_{nt} \widehat{R}_{nt}, \tag{11}$$

where the annual option return for the nth TIPS is $\widehat{R}_{nt} = O_{nt}/O_{n,t-12} - 1$ and the weight is $\widehat{W}_{nt} = O_{n,t-12}/\sum_{n=1}^{N_t} O_{n,t-12}$. To be included in (11), the nth TIPS must be outstanding during the time period $t-12$ to t. Panel D of Fig. 2 shows $OR_{t-12,t}$ for 5-year and 10-year TIPS when the model is estimated using all of the bonds in Table 1 and an equally-weighted SSE. The annual option return index in Panel D is winsorized above 90% to limit the extreme values.

When we construct our embedded option return indices, we also filter the data to remove outlier returns. Although the estimated option values are sometimes small (see Figs. 1 and 2), the option returns can be much larger economically. For example, in

[6] In Panel A of Fig. 2, the time series has a gap since there were no outstanding 5-year TIPS from August 2002 through September 2004.

[7] In addition to value weights, we examined other weighting schemes, but there was no impact on our results. Details are in Appendix E.

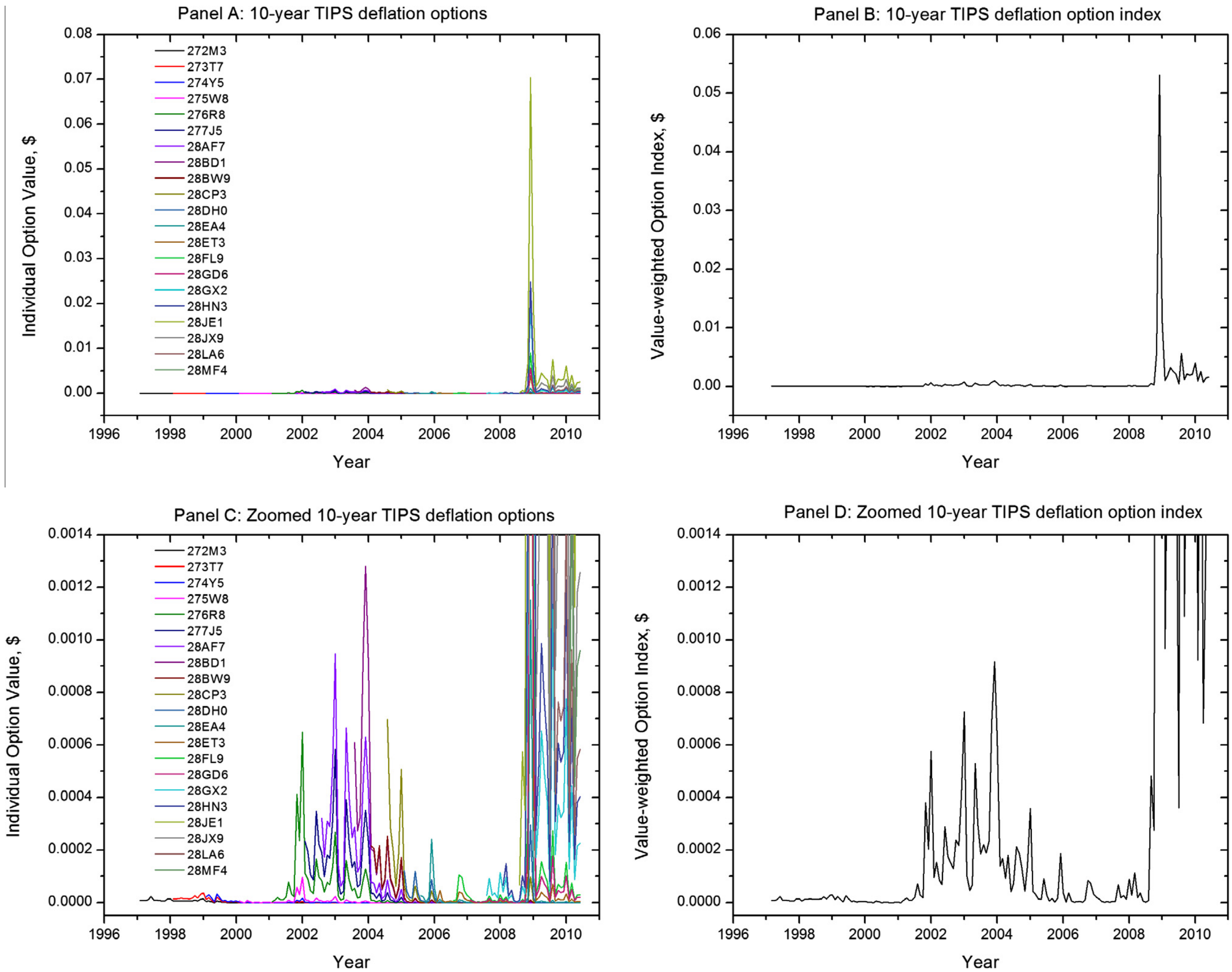

Fig. 1. Estimated deflation option values for 10-year TIPS. Panels A and B show the individual embedded deflation option values for 10-year TIPS and the corresponding value-weighted option index, respectively. Panels C and D show the individual embedded deflation option values for 10-year TIPS and the corresponding value-weighted option index, respectively, on the zoomed scale with a maximum value of \$0.0014. The model parameters are estimated by minimizing the equally-weighted sum of squared errors for both 10-year TIPS and 10-year matching T-Notes. Sample period is January 1997–May 2010, monthly frequency.

Panel A of Fig. 1, when the embedded option value increases from \$0.01 to \$0.06 during the second half of 2008, the return is 500%.[8] Our interpretation is that when the estimated value of the embedded deflation option increases sharply like this, it is an indication of deflationary expectations. However, we sometimes find abnormally high returns even when the estimated option value is economically unchanged. These abnormal returns originate in months where the beginning and ending option values have different orders of magnitude, yet both values are small. For example, if an option value moves from 10^{-12} to 10^{-10}, the monthly return is very large, but both of the option values are approximately zero. To control for this effect, we discard option values that are smaller than 10^{-8}. We tried other cutoff values, such as 10^{-6} and 10^{-10}, but it does not impact our regression results that are shown below in Sections 4.6–4.8. We use a cutoff of 10^{-8} since it maintains a relatively large sample size while having the effect of trimming outlier returns. Thus our regression results in the sequel are not driven by outliers.

4.5. Summary statistics

We use linear regressions to examine if our variables $OR_{t-1,t}$ and $OR_{t-12,t}$ are relevant for explaining the future inflation rate. We use (6) as the dependent variable in our regression analysis. In addition to our variables in (10) and (11), our explanatory variables include: (i) the yield spread YS_t, which is the difference between the average yields of the nominal T-Notes and the TIPS in our sample; (ii) the one-month and 12-month lagged inflation rates, $i_{t-1,t}$ and $i_{t-12,t}$; (iii) the monthly and annual return on gold, $GoldRet_{t-1,t}$ and $GoldRet_{t-12,t}$, which we calculate using gold prices from the London Bullion Market; (iv) the monthly and annual return on VIX, $VIXRet_{t-1,t}$ and $VIXRet_{t-12,t}$, which is the return on the S&P 500 implied volatility VIX index; (v) the value-weighted average monthly and annual returns on the TIPS in our sample, $BondRet_{t-1,t}$ and $BondRet_{t-12,t}$; (vi) the nominal yield curve fitting error, Liq_t, which serves as a market-wide liquidity measure (Hu et al., 2013); (vii) the one-month ahead and 12-month ahead interpolated inflation forecasts from Blue Chip Economic Indicators, $BlueSurvey1_t$ and $BlueSurvey12_t$; and (viii) the one-month ahead interpolated and 12-month ahead actual inflation forecasts from the University of Michigan survey of households, $MichSurvey1_t$ and $MichSurvey12_t$. In Section 4.7.4 we explain the interpolation method that we use for the survey variables.

[8] We are not suggesting that a trader could actually earn such a large return. TIPS qualify for the U.S. Treasury's STRIPS program, but given the illiquidity of the STRIPS market and the issues associated with hedging the consumer price index, a trading strategy might not be feasible.

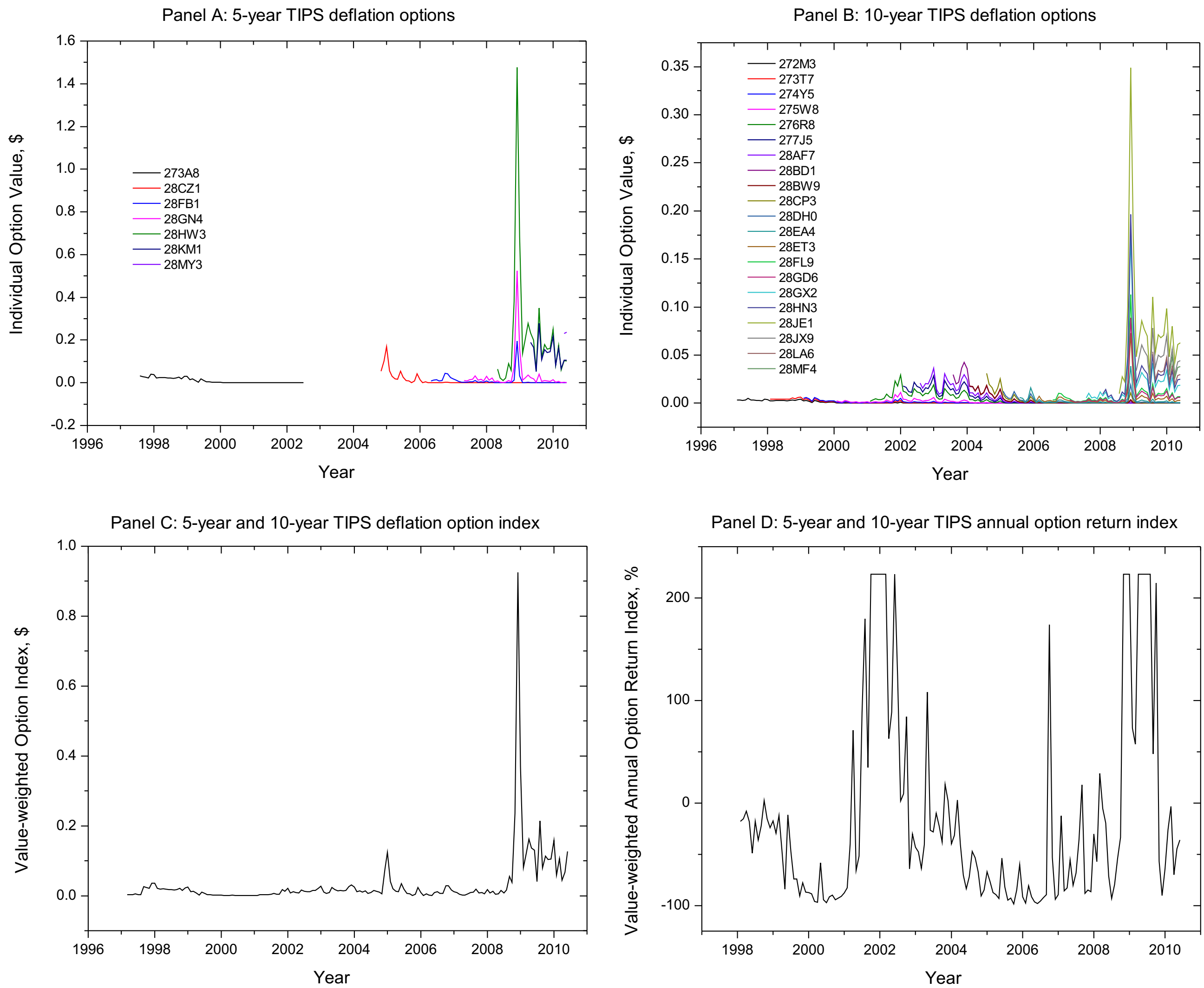

Fig. 2. Jointly estimated deflation option values for 5-year and 10-year TIPS. Panels A and B show the individual embedded deflation option values for 5-year and 10-year TIPS, respectively. Panel C shows the value-weighted option index constructed from both 5-year and 10-year options. Panel D shows the annual option return index constructed as the value-weighted average of all annual option returns, where an option value cutoff of 1E-8 is imposed and the annual option return index is winsorized above 90%. The model parameters are estimated by minimizing the equally-weighted sum of squared errors for both 5-year and 10-year TIPS and matching T-Notes. Sample period is January 1997–May 2010, monthly frequency. Note: There were no outstanding 5-year TIPS from August 2002 through September 2004.

We include YS_t as an explanatory variable since it is a common measure of inflation expectations. Hunter and Simon (2005) have also shown that the yield spread is correlated with TIPS returns. We include the gold return since the fluctuation in the price of gold has long been associated with inflationary expectations. Bekaert and Wang (2010) show that the inflation beta for gold in North America is about 1.45. We include the VIX return since its time variation captures the uncertainty associated with macroeconomic activity, as described in Bloom (2009) and David and Veronesi (2011). We include Liq_t as a control variable because Christensen and Gillan (2014) have shown that this variable has significant explanatory power for the TIPS liquidity premium. We include $BondRet_{t-1,t}$ and $BondRet_{t-12,t}$ as control variables to see if the TIPS total return has incremental explanatory power beyond that of the embedded option and our other variables. This allows us to compare the informational content of the embedded option, which is the focus of our study, to that of the TIPS itself, which is examined by Chu et al. (2007), D'Amico et al. (2014) and Chu et al. (2011). Lastly, we include the survey variables because Ang et al. (2007) have shown that surveys are useful variables for forecasting future inflation.

Table 3 shows summary statistics for our monthly explanatory variables. For our sample of 5-year TIPS and matching nominal T-Notes, the mean of the embedded option return index is about 0.306, which is a 30.6% monthly average return. The standard deviation of the 5-year embedded option return index is about 1.445, or 144.5%. For our sample of 10-year TIPS and matching nominal T-Notes, the mean and standard deviation of the option return index are about 111% and 373%, respectively. The fact that the standard deviations are big coincides with our earlier statement that there is substantial time variation in the option returns. This is also apparent by examining the minimum and maximum values for the option return indices, as shown in the last two columns of Table 3.

Table 4 shows the sample correlation matrix for a subset of our monthly explanatory variables. Panel A (Panel B) shows the matrix for 5-year (10-year) TIPS, while Panel C shows the matrix for the combined sample of 5-year and 10-year TIPS. The number in parentheses below each correlation is the p-value for a test of

Table 3
Summary statistics.

Variable	Obs.	Mean	Median	Std. Dev.	Minimum	Maximum
Option Val Index, 5-year	121	0.0653	0.0169	0.1422	3.7449E−8	1.1834
Option Val Index, 10-year	160	0.0007	2.7427E−05	0.0043	1.8741E−7	0.0531
Option Val Index, 5&10-year	160	0.0325	0.0120	0.0856	0.0003	0.9248
Option Ret Index, 5-year	121	0.3061	−0.0783	1.4447	−0.9989	9.6079
Option Ret Index, 10-year	160	1.1091	−0.0843	3.7287	−0.9805	22.0054
Option Ret Index, 5&10-year	160	0.2318	−0.0613	1.0823	−0.8986	5.5709
Yield Spread	160	0.0188	0.0204	0.0086	−0.0288	0.0333
Liquidity	160	3.0205	2.3440	2.6272	1.1300	17.3901
Inflation, lag1	160	0.0237	0.0236	0.0468	−0.2321	0.1458
Gold Ret	160	0.0089	0.0067	0.0471	−0.1698	0.1797
VIX Ret	160	0.0187	−0.0115	0.1918	−0.3150	0.9075
Bond Ret	160	0.0027	0.0033	0.0142	−0.0798	0.0449
Oil Ret	160	0.0142	0.0200	0.1181	−0.3555	0.4187
BlueSurvey1	160	0.0147	0.0151	0.0652	−0.3097	0.5321
MichSurvey1	160	0.0250	0.0330	0.0785	−0.3481	0.2846

The table shows descriptive statistics of monthly variables, using three different samples: 5-year TIPS and matching T-Notes, 10-year TIPS and matching T-Notes, and 5&10-year TIPS and matching T-Notes. *Option Val* (*Ret*) *Index* is the monthly option value (return) index constructed as the value-weighted average of all option values (returns) available in each month, *Yield Spread* is the difference between the cross-sectional averages of nominal yields and TIPS yields, *Liquidity* is Svensson nominal yield curve fitting error (in basis points), *Inflation, lag1* is the one-month lagged seasonally-unadjusted CPI-based annualized log inflation rate, *Gold Ret* is the return on gold from the London Bullion Market, *VIX Ret* is the return on the S&P500 implied volatility index, *Bond Ret* is the value-weighted average of TIPS gross price returns, *Oil Ret* is the return on Brent Crude Oil spot prices, *BlueSurvey1* is one-month ahead interpolated inflation forecast obtained from the respondents' consensus of the Blue Chip Economic Indicators survey, and *MichSurvey1* is one-month ahead interpolated inflation forecast obtained from survey of consumers by University of Michigan. An option value cutoff of $1E-8$ is imposed. Sample period is from January 1997 to May 2010, monthly frequency.

the null hypothesis that the correlation coefficient is equal to zero. If we examine the column for the option return index, we see that the return index in all three panels has a negative sample correlation with the yield spread, lagged inflation, and the return on gold. This is intuitive since the option return index is more likely to be high (low) during periods of deflationary (inflationary) expectations. We also see that the correlation between the option return index and the TIPS total return is negative. During periods of deflationary expectations, investors tend to shun TIPS in favor of nominal bonds, which helps to explain this result. For the combined sample in Panel C, upon examining the p-values, we cannot reject the null hypothesis that the sample correlation between the VIX return and the option return index is zero. In the same panel, for the yield spread, lagged inflation, the return on gold, and the TIPS total return, the p-values are small and we reject the null that the correlations are zero. However, even for these variables, the magnitude of the coefficients is relatively small. The numbers vary across Panels A, B, and C, but the yield spread, the gold return, and the TIPS total return each have a correlation coefficient with the option return that lies between about −0.13 and −0.28, while lagged inflation has a correlation coefficient with the option return of about −0.5. Thus it appears that our option return index might be useful for explaining future inflation, even in the presence of these traditional explanatory variables. Lastly, we see that the option return index is not highly correlated with the market-wide liquidity variable. The biggest correlation coefficient is about 0.23. We believe that our option return index is picking up deflationary expectations, not illiquidity associated with the bond market or the financial crisis.

4.6. In-sample inflation regressions

Our first regression is

$$i_{t,t+\tau} = \beta_0 + \beta_1 OR_{t-\tau,t} + \beta_2 YS_t + \beta_3 Liq_t + \beta_4 i_{t-\tau,t} + \beta_5 GoldRet_{t-\tau,t} + \beta_6 VIXRet_{t-\tau,t} + \beta_7 BondRet_{t-\tau,t} + \epsilon_{t+\tau}, \tag{12}$$

which is shown in Table 5. Panel A uses $\tau = 1$ (one-month ahead) while Panel B uses $\tau = 12$ (one-year ahead). In Panel A, when we include the control variables that are known to capture future inflation, our variable $OR_{t-1,t}$ is statistically significant at the 10% level for the sample of 5-year TIPS and is statistically significant at the 1% level for the other two samples.[9] In Panel B, when we include the control variables, $OR_{t-12,t}$ is statistically significant at the 5% level or better for the sample of 5-year TIPS, the sample of 10-year TIPS, and the combined sample of 5-year and 10-year TIPS.

In Panel A of Table 5, note that lagged inflation and the VIX return are statistically significant for all three samples. However, the significance of these variables is greatly reduced in Panel B. With the exception of the yield spread and lagged inflation for the 5-year sample, and the liquidity variable for the other two samples, the main significant variables in Panel B are $OR_{t-12,t}$ and $BondRet_{t-12,t}$. The control variables that are significant for explaining the one-month ahead inflation rate in Panel A mostly fail to be significant for the one-year ahead inflation rate in Panel B, and vice versa. In contrast, our option return index variable is important over both horizons.[10]

For the combined sample of 5-year and 10-year TIPS, if we examine the adjusted-R^2 values in Panel A, we find that $OR_{t-1,t}$ alone explains 24.8% of the variation in the one-month ahead inflation rate. Once we add all of our control variables, the adjusted-R^2 increases to 36%. In Panel B, $OR_{t-12,t}$ alone explains 3.9% of the variation in the one-year ahead inflation rate, but this increases to 41.2% when we include the full set of control variables. Furthermore, for all of our regressions in Table 5, the sign of the coefficient on $OR_{t-1,t}$ and $OR_{t-12,t}$ is negative. This is consistent with our economic intuition. Since the embedded TIPS option is a deflation option, a higher option return should be associated with a lower future inflation rate.

We find that our results are not only statistically significant, but also economically significant. For example, for the sample of 5-year TIPS in Panel A of Table 5, the coefficient on $OR_{t-1,t}$ is −0.0073 when the control variables are included. Thus a 100% embedded option return, which is less than one standard deviation, predicts a decrease of 73 basis points in the one-month ahead annualized rate of inflation. If we compare this result to the other variables in the same regression, we find that $OR_{t-1,t}$ is about as important economically as the yield spread (coefficient of 0.99 for the

[9] For all of our regressions, Newey and West (1987) t-statistics with four lags are reported. We also calculated standard errors using 3, 5, and 6 lags, but this had no impact on our results.

[10] We also verified that $OR_{t-1,t}$ is significant for explaining the one-month forward inflation rate, $i_{t+1,t+2}$. Thus our results in Panel A are not driven by short-term timing differences between measuring and reporting inflation (i.e., CPI-U announcements).

Table 4
Correlations.

	Option Ret	Option Val	Yield Spread	Liquidity	Inflation, Lag1	Gold Ret	VIX Ret	Bond Ret
Panel A: Correlations for the 5-year sample								
Option Ret	1.0000							
Option Val	0.2123	1.0000						
	(0.0194)							
Yield Spread	−0.1316	−0.7036	1.0000					
	(0.1502)	(<0.0001)						
Liquidity	0.1364	0.6600	−0.8019	1.0000				
	(0.1358)	(<0.0001)	(<0.0001)					
Inflation, lag1	−0.4736	−0.5550	0.5073	−0.3422	1.0000			
	(<0.0001)	(<0.0001)	(<0.0001)	(0.0001)				
Gold Ret	−0.2782	0.1264	−0.0446	0.0501	0.1004	1.0000		
	(0.0020)	(0.1671)	(0.6273)	(0.5849)	(0.2731)			
VIX Ret	0.0720	−0.0653	−0.0074	0.0615	0.0537	−0.0916	1.0000	
	(0.4327)	(0.4765)	(0.9355)	(0.5031)	(0.5584)	(0.3176)		
Bond Ret	−0.1947	−0.0375	0.1878	−0.1157	0.1911	0.3641	−0.0729	1.0000
	(0.0323)	(0.6826)	(0.0392)	(0.2063)	(0.0357)	(<0.0001)	(0.4271)	
Panel B: Correlations for the 10-year sample								
Option Ret	1.0000							
Option Val	0.2894	1.0000						
	(0.0002)							
Yield Spread	−0.2155	−0.5934	1.0000					
	(0.0062)	(<0.0001)						
Liquidity	0.2307	0.5590	−0.7759	1.0000				
	(0.0033)	(<0.0001)	(<0.0001)					
Inflation, lag1	−0.5034	−0.5205	0.5116	−0.3208	1.0000			
	(<0.0001)	(<0.0001)	(<0.0001)	(<0.0001)				
Gold Ret	−0.2461	0.1667	−0.0631	0.0434	0.0637	1.0000		
	(0.0017)	(0.0351)	(0.4281)	(0.5858)	(0.4237)			
VIX Ret	0.0598	−0.0568	−0.0060	0.0690	0.0626	−0.0697	1.0000	
	(0.4525)	(0.4757)	(0.9399)	(0.3857)	(0.4319)	(0.3812)		
Bond Ret	−0.2500	−0.0401	0.0265	−0.0822	0.1532	0.3369	−0.0293	1.0000
	(0.0014)	(0.6150)	(0.7391)	(0.3012)	(0.0531)	(<0.0001)	(0.7131)	
Panel C: Correlations for the 5&10-year sample								
Option Ret	1.0000							
Option Val	0.3397	1.0000						
	(<0.0001)							
Yield Spread	−0.1946	−0.6947	1.0000					
	(0.0137)	(<0.0001)						
Liquidity	0.2087	0.6625	−0.7823	1.0000				
	(0.0081)	(<0.0001)	(<0.0001)					
Inflation, lag1	−0.5587	−0.5760	0.5086	−0.3208	1.0000			
	(<0.0001)	(<0.0001)	(<0.0001)	(<0.0001)				
Gold Ret	−0.2619	0.1247	−0.0555	0.0434	0.0637	1.0000		
	(0.0008)	(0.1162)	(0.4861)	(0.5858)	(0.4237)			
VIX Ret	0.0340	−0.0580	−0.0101	0.0690	0.0626	−0.0697	1.0000	
	(0.6700)	(0.4665)	(0.8993)	(0.3857)	(0.4319)	(0.3812)		
Bond Ret	−0.2245	−0.0454	0.0567	−0.0847	0.1526	0.3444	−0.0277	1.0000
	(0.0043)	(0.5687)	(0.4761)	(0.2870)	(0.0541)	(<0.0001)	(0.7285)	

The table shows the correlations between our monthly variables, using three different samples: 5-year TIPS and matching T-Notes (Panel A), 10-year TIPS and matching T-Notes (Panel B), and 5&10-year TIPS and matching T-Notes (Panel C). *Option Val* (*Ret*) is the monthly option value (return) index constructed as the value-weighted average of all option values (returns) available in each month, *Yield Spread* is the difference between the cross-sectional averages of nominal yields and TIPS yields, *Liquidity* is Svensson nominal yield curve fitting error, *Inflation, lag1* is the one-month lagged seasonally-unadjusted CPI-based annualized log inflation rate, *Gold Ret* is the return on gold from the London Bullion Market, *VIX Ret* is the return on the S&P500 implied volatility index, and *Bond Ret* is the value-weighted average of TIPS gross price returns. The *p*-values for the null hypothesis that the correlation is zero are reported in parentheses. An option value cutoff of $1E-8$ is imposed. Sample period is from January 1997 to May 2010, monthly frequency.

5-year sample) or the lagged inflation (coefficient of 0.27 for the 5-year sample). A one percentage point increase in the yield spread (lagged inflation rate) predicts a 99 basis point (27 basis point) increase in the one-month ahead annualized rate of inflation.

For the sample of 10-year TIPS in Panel A of Table 5, the coefficient on $OR_{t-1,t}$ is −0.0039 when the control variables are included. This is a lower magnitude than the coefficient of −0.0073 for 5-year TIPS. However, using Table 3, we see that $OR_{t-1,t}$ for 5-year TIPS has a lower mean and standard deviation than $OR_{t-1,t}$ for 10-year TIPS. If we multiply the regression coefficient for $OR_{t-1,t}$ times the expected option index return, we get 22 basis points (43 basis points) for the sample of 5-year (10-year) TIPS. Likewise, if we multiply the regression coefficient for $OR_{t-1,t}$ times the standard deviation of the option index return, we get 105 basis points (145 basis points) for 5-year (10-year) TIPS. The economic significance tends to be slightly higher when we estimate our model using 10-year TIPS.

In Panel B of Table 5, the coefficients on $OR_{t-12,t}$ are lower than the coefficients for $OR_{t-1,t}$ in Panel A. For example, using the 5-year (10-year) sample of TIPS, a 100% embedded option return predicts a decrease of 71 basis points (11 basis points) in the one-year ahead inflation rate when the control variables are included. If we multiply the regression coefficient for $OR_{t-12,t}$ times the standard deviation of the option return index, we get 58 basis points (31 basis points) for the sample of 5-year (10-year) TIPS. In both cases, the economic significance is lower than what we find in Panel A. In summary, it appears that $OR_{t-1,t}$ and $OR_{t-12,t}$ contain relevant information for future inflation out to a horizon of at least 12 months. Given the evidence from Table 5, we conclude that the embedded option in TIPS contains useful information about future inflation.

Christensen and Gillan (2014) show that the market-wide liquidity measure of Hu et al. (2013), which is our Liq_t variable, is important for explaining the TIPS liquidity premium. In Panel A of Table 5, Liq_t is not significant in any of our regressions. In Panel B of Table 5, Liq_t is only significant for the regressions that include 10-year TIPS. Since our variables $OR_{t-1,t}$ and $OR_{t-12,t}$ are significant when Liq_t is included as a control variable, we do not believe that TIPS liquidity is driving our results. We investigate this further in Section 4.7.6.

4.6.1. Regressions with on-the-run TIPS

To investigate whether illiquidity associated with off-the-run TIPS is a contributing factor in our results, we reconstruct the option return indices in (10) and (11) using only on-the-run TIPS for each sample. Typically, the on-the-run TIPS is more liquid than any of the off-the-run TIPS. For example, Fleming and Krishnan (2012) show that trading volume is substantially higher for on-the-run TIPS as compared to off-the-run TIPS. In addition, Fleming and Krishnan (2012) report that about 85% of the time, the off-the-run 10-year TIPS has only a one-sided price quote (a bid or an ask, but not both) or no price quote at all. The quote incidence for off-the-run TIPS is lower than that of on-the-run TIPS. Since off-the-runs are not as liquid, we eliminate these bonds from each sample when we reconstruct the indices in (10) and (11).

Our regression results using only on-the-run TIPS are shown in Table 6. In Panel A of Table 6, the statistical significance of $OR_{t-1,t}$ is the same as that in Panel A of Table 5. We continue to find that lagged inflation and the VIX return are also significant. In Panel B of Table 6, $OR_{t-12,t}$ is significant at the 1% level for the sample of 10-year TIPS and for the combined sample of 5-year and 10-year TIPS. However, $OR_{t-12,t}$ is significant at the 10% level for the sample of 5-year on-the-run TIPS, even when the control variables are included.

In Panel B of Table 6, the coefficient on $OR_{t-12,t}$ for the sample of 10-year TIPS is -0.00033. Although this is smaller than its counterpart from Table 5, the economic significance is slightly higher. When only on-the-run TIPS are used to construct $OR_{t-12,t}$, the mean and standard deviation of $OR_{t-12,t}$ are relatively big since cumulative inflation is low and the embedded deflation option is close to the money. If we multiply the regression coefficient of -0.00033 times the standard deviation of $OR_{t-12,t}$, we get about 39 basis points for the sample of 10-year on-the-run TIPS. This is comparable to the 31 basis points that we found for the full sample of 10-year TIPS. Overall, our analysis suggests that illiquidity is not driving our results. Even after discarding the most illiquid TIPS in each sample (i.e., the off-the-run TIPS), we still find that the embedded option return indices $OR_{t-1,t}$ and $OR_{t-12,t}$ are useful variables for explaining the one-month ahead and one-year ahead inflation rates.

4.7. Robustness

We now investigate whether our results are robust to changes in our modeling assumptions, our variable definitions, and our empirical approach. In addition to what we include here, Appendix F examines other robustness tests.

4.7.1. Alternative measure of option returns

In the previous sections, we used (10) and (11) to construct $OR_{t-1,t}$ and $OR_{t-12,t}$, where the individual embedded option values were obtained from our TIPS pricing model that uses (1) and (2). In this section, we explore an alternative explanatory variable that is less sensitive to model specification. We use the embedded option returns to compute a new variable, $ORF_{t-1,t}$, which we define as the fraction of options that has a positive return from $t-1$ to t. To calculate $ORF_{t-1,t}$ in a given month, we divide the number of embedded options with a positive return by the total number of embedded options. Using $ORF_{t-1,t}$ instead of $OR_{t-1,t}$ allows us to investigate the robustness of our modeling assumptions. Any other model that produces positive (negative) embedded option returns when our model produces positive (negative) embedded option returns will give the same time series for $ORF_{t-1,t}$ and thus the same regression results. To check our annual regression results we construct $ORF_{t-12,t}$, which is the fraction of options that has a positive return from $t-12$ to t.

Table 7 shows our regression results when $ORF_{t-1,t}$ and $ORF_{t-12,t}$ are used in place of $OR_{t-1,t}$ and $OR_{t-12,t}$, respectively. The first two columns of Table 7 use the combined sample of 5-year and 10-year TIPS, while the last two columns use the subsample that includes only on-the-run 5-year and 10-year TIPS. The results in Table 7 show that $ORF_{t-1,t}$ and $ORF_{t-12,t}$ are statistically significant. In Panel A of Table 7, lagged inflation and the VIX return are also significant for explaining the one-month ahead inflation rate, which is also true in Panel A of Tables 5 and 6. In Panel B of Table 7, the TIPS total return, $BondRet_{t-12,t}$, is statistically significant, which mirrors our results from Panel B of Tables 5 and 6.

The regressions in Table 7 show that our modeling assumptions in (1) and (2) are not critical to our results. If we were to alter (1) and (2) in such a way that the sign of each monthly (annual) option return did not change, we would get the same variable $ORF_{t-1,t}$ ($ORF_{t-12,t}$) and thus the same results in Panel A (Panel B) of Table 7. Tables 5 and 6 show that our embedded option return indices are informationally relevant for explaining the one-month ahead and the one-year ahead inflation rate. When we ignore the magnitude of the option returns and focus only on the sign of those returns, we get explanatory variables (namely, $ORF_{t-1,t}$ and $ORF_{t-12,t}$) that are also informationally relevant. However, if we compare the adjusted-R^2 values in Table 7 to those in Tables 5 and 6, we see that the values in Table 7 are mostly smaller. This is exactly what we would expect to find given that $ORF_{t-1,t}$ and $ORF_{t-12,t}$ capture only the sign of the option returns and not the magnitude. Overall, Table 7 helps to reduce concerns about model specification.

4.7.2. Parameter estimation with inverse duration weighted SSE

Table 8 shows our regression results when we construct our option-based explanatory variables using the parameter estimates from the inverse duration weighted SSE (see Panel B of Table 2). Table 8 uses the combined sample of 5-year and 10-year TIPS and matching nominal bonds. In Panel A of Table 8, when we include the control variables, $OR_{t-1,t}$ is significant at the 1% level, which mirrors our result from Table 5, and $ORF_{t-1,t}$ is significant at the 5% level, which is better than Table 7. In addition, like our earlier results, lagged inflation and the VIX return are important for explaining the one-month ahead inflation rate. In Panel B of Table 8, if we again focus on the regressions with the control variables, $OR_{t-12,t}$ and $ORF_{t-12,t}$ are significant at the 1% level and 5% level, respectively. In addition, like Tables 5 and 7, $BondRet_{t-12,t}$ is also significant. For both panels A and B, the adjusted-R^2 values are close to their counterparts from Tables 5 and 7.

Although Table 8 shows the results for only the combined sample of 5-year and 10-year TIPS and matching nominal bonds, we also checked the results using the sample of 5-year TIPS and matching nominal bonds, and again using the sample of 10-year TIPS and matching nominal bonds. For both of these samples, our regression results are statistically similar to their counterparts in Tables 5 and 7. Thus using inverse duration weights in the SSE, as opposed to equal weights, does not alter our conclusions.

To better frame our results with inverse duration weights, consider the sample of 5-year TIPS and matching nominal bonds

Table 5
Full sample inflation regressions.

	5-year	5-year	10-year	10-year	5&10-year	5&10-year
Panel A: Dependent variable is one-month ahead inflation, $i_{t,t+1}$						
$OR_{t-1,t}$	-0.014^{**}	-0.0073^{*}	-0.0063^{***}	-0.0039^{***}	-0.022^{***}	-0.014^{***}
	(−2.23)	(−1.85)	(−3.51)	(−3.01)	(−3.67)	(−2.95)
YS_t		0.99^{*}		0.86		1.01^{*}
		(1.76)		(1.39)		(1.70)
Liq_t		0.00093		0.00074		0.00098
		(0.42)		(0.37)		(0.49)
$i_{t-1,t}$		0.27^{***}		0.24^{***}		0.20^{**}
		(3.09)		(3.08)		(2.39)
$GoldRet_{t-1,t}$		0.0060		0.026		0.015
		(0.07)		(0.37)		(0.19)
$VIXRet_{t-1,t}$		-0.064^{***}		-0.054^{**}		-0.056^{**}
		(−2.87)		(−2.46)		(−2.48)
$BondRet_{t-1,t}$		0.84^{*}		0.34		0.39
		(1.68)		(1.15)		(1.24)
Constant	0.027^{***}	−0.0016	0.030^{***}	0.0032	0.028^{***}	−0.00027
	(5.67)	(−0.12)	(8.42)	(0.22)	(8.41)	(−0.02)
Obs.	121	121	160	160	160	160
Adj-R^2	0.162	0.371	0.246	0.358	0.248	0.360
Panel B: Dependent variable is one-year ahead inflation, $i_{t,t+12}$						
$OR_{t-12,t}$	-0.0026^{*}	-0.0071^{**}	-0.00093^{***}	-0.0011^{**}	-0.0028^{***}	-0.0039^{***}
	(−1.69)	(−2.44)	(−2.77)	(−2.36)	(−2.66)	(−2.88)
YS_t		0.48^{**}		−0.32		−0.25
		(2.42)		(−1.20)		(−0.99)
Liq_t		−0.00048		-0.0024^{**}		-0.0022^{**}
		(−0.49)		(−2.15)		(−2.07)
$i_{t-12,t}$		-0.38^{*}		−0.12		−0.19
		(−1.72)		(−0.62)		(−0.87)
$GoldRet_{t-12,t}$		−0.012		−0.0047		−0.0063
		(−1.47)		(−0.56)		(−0.73)
$VIXRet_{t-12,t}$		0.0041		−0.0025		−0.0019
		(0.78)		(−0.57)		(−0.45)
$BondRet_{t-12,t}$		-0.23^{***}		-0.13^{***}		-0.13^{***}
		(−4.09)		(−4.55)		(−3.99)
Constant	0.022^{***}	0.032^{***}	0.025^{***}	0.046^{***}	0.024^{***}	0.045^{***}
	(7.53)	(4.62)	(11.15)	(5.29)	(11.89)	(5.14)
Obs.	99	99	149	149	149	149
Adj-R^2	0.010	0.490	0.035	0.412	0.039	0.412

The table shows results for the in-sample regression:

$$i_{t,t+\tau} = \beta_0 + \beta_1 OR_{t-\tau,t} + \beta_2 YS_t + \beta_3 Liq_t + \beta_4 i_{t-\tau,t} + \beta_5 GoldRet_{t-\tau,t} + \beta_6 VIXRet_{t-\tau,t} + \beta_7 BondRet_{t-\tau,t} + \epsilon_{t+\tau},$$

where $i_{t,t+\tau}$ is the τ-month ahead seasonally-unadjusted CPI-based annualized log inflation rate, $OR_{t-\tau,t}$ is the option return index constructed as the value-weighted average of all option returns from month $t-\tau$ to month t, YS_t is the difference between the cross-sectional averages of nominal yields and TIPS yields, Liq_t is Svensson nominal yield curve fitting error (in basis points), $GoldRet_{t-\tau,t}$ is the return on gold bullion, $VIXRet_{t-\tau,t}$ is the return on the S&P500 implied volatility index, and $BondRet_{t-\tau,t}$ is the value-weighted average of TIPS gross price returns. Panel A(B) reports results of one-month (one-year) ahead inflation regressions for three different samples: 5-year TIPS and matching T-Notes (Columns 2 and 3), 10-year TIPS and matching T-Notes (Columns 4 and 5), and 5&10-year TIPS and matching T-Notes (Columns 6 and 7). The model parameters are estimated by minimizing the equally-weighted sum of squared errors. An option value cutoff of $1E-8$ is imposed and $OR_{t-12,t}$ is winsorized above 90%. The t-statistics based on four-lag Newey-West adjusted standard errors are reported in parentheses below the estimated coefficients. * stat. sign. at 10% level; ** stat. sign. at 5% level; *** stat. sign. at 1% level. Sample period is from January 1997 to May 2010, monthly frequency.

or the sample of 10-year TIPS and matching nominal bonds. For either of these samples, when we use inverse duration weights instead of equal weights in (8), relatively higher weights are assigned to more seasoned securities, which have shorter remaining times to maturity, and relatively lower weights are assigned to newly issued securities, which have longer remaining times to maturity. Thus the inverse duration weighting scheme is biased more towards the off-the-run securities in a given sample of 5-year or 10-year TIPS. In addition, for the combined sample of 5-year and 10-year TIPS and matching nominal bonds, there is a second effect since the sample includes bonds with different original maturities. When using inverse duration weights instead of equal weights, newly issued 5-year securities get larger weights than newly issued 10-year securities. Thus when we use inverse duration weights in (8) to solve for the model parameters in the combined sample, we are overweighting newly issued shorter-term securities and overweighting more seasoned securities, relative to the case of using equal weights. Our results in Table 8 show that even when these features are incorporated into the parameter estimation problem, we still find that our option-based explanatory variables are significant for explaining the one-month ahead and the 12-month ahead rate of inflation.

Our results in Table 8 are consistent with the earlier results in Tables 5 and 6, which use parameter estimates from an equally-weighted SSE. Table 6 uses only the on-the-run TIPS and the matching nominal T-Note, while Table 5 uses both on-the-run and off-the-run TIPS and matching nominal T-Notes. Thus relative to Table 6, Table 5 is biased towards off-the-run (i.e., seasoned) securities. Since the parameters are the same for both tables, the bias is introduced when we construct $OR_{t-1,t}$ and $OR_{t-12,t}$. In contrast, when we use inverse duration weights as in Table 8, the bias is introduced in (8), which impacts the parameter estimates. Overall, we find that our results are not sensitive to whether the bias shows up in the parameter estimates or in the construction of the option return variables.

Table 6
On-the-run securities' inflation regressions.

	5-year	5-year	10-year	10-year	5&10-year	5&10-year
Panel A: Dependent variable is one-month ahead inflation, $i_{t,t+1}$						
$OR_{t-1,t}$	-0.014^{**}	-0.0073^{*}	-0.0068^{***}	-0.0041^{***}	-0.022^{***}	-0.014^{***}
	(−2.19)	(−1.69)	(−3.32)	(−2.80)	(−3.62)	(−2.97)
YS_t		0.98		0.38		0.57
		(1.62)		(0.61)		(1.00)
Liq_t		−0.00032		−0.00092		−0.00076
		(−0.15)		(−0.53)		(−0.42)
$i_{t-1,t}$		0.32^{***}		0.30^{***}		0.27^{***}
		(3.28)		(3.89)		(3.17)
$GoldRet_{t-1,t}$		0.022		0.037		0.023
		(0.26)		(0.50)		(0.30)
$VIXRet_{t-1,t}$		-0.067^{***}		-0.055^{**}		-0.057^{**}
		(−2.75)		(−2.36)		(−2.37)
$BondRet_{t-1,t}$		0.53		0.15		0.20
		(1.15)		(0.59)		(0.72)
Constant	0.027^{***}	0.00034	0.030^{***}	0.015	0.028^{***}	0.011
	(5.41)	(0.02)	(8.34)	(1.05)	(8.24)	(0.81)
Obs.	121	121	160	160	160	160
Adj-R^2	0.146	0.347	0.232	0.341	0.234	0.343
Panel B: Dependent variable is one-year ahead inflation, $i_{t,t+12}$						
$OR_{t-12,t}$	-0.0013^{*}	-0.0019^{*}	-0.00035^{***}	-0.00033^{***}	-0.0018^{***}	-0.0018^{***}
	(−1.74)	(−1.71)	(−3.73)	(−3.55)	(−3.49)	(−3.00)
YS_t		0.46^{**}		0.62		0.29
		(2.09)		(1.53)		(1.14)
Liq_t		−0.0020		−0.0012		−0.0014
		(−1.56)		(−1.16)		(−1.24)
$i_{t-12,t}$		-0.46^{*}		−0.39		−0.37
		(−1.66)		(−1.64)		(−1.61)
$GoldRet_{t-12,t}$		−0.011		-0.019^{*}		−0.014
		(−1.41)		(−1.89)		(−1.48)
$VIXRet_{t-12,t}$		0.00044		0.00033		−0.000014
		(0.10)		(0.07)		(−0.00)
$BondRet_{t-12,t}$		-0.20^{***}		-0.044^{**}		-0.070^{**}
		(−4.80)		(−2.13)		(−2.54)
Constant	0.024^{***}	0.040^{***}	0.026^{***}	0.029^{***}	0.026^{***}	0.036^{***}
	(9.07)	(5.68)	(12.67)	(3.69)	(13.16)	(4.60)
Obs.	99	99	149	149	149	149
Adj-R^2	0.048	0.444	0.091	0.309	0.084	0.316

The table shows results for the in-sample regression:

$$i_{t,t+\tau} = \beta_0 + \beta_1 OR_{t-\tau,t} + \beta_2 YS_t + \beta_3 Liq_t + \beta_4 i_{t-\tau,t} + \beta_5 GoldRet_{t-\tau,t} + \beta_6 VIXRet_{t-\tau,t} + \beta_7 BondRet_{t-\tau,t} + \epsilon_{t+\tau},$$

where $i_{t,t+\tau}$ is the τ-month ahead seasonally-unadjusted CPI-based annualized log inflation rate, $OR_{t-\tau,t}$ is the option return index constructed as the value-weighted average of all option returns from month $t-\tau$ to month t, YS_t is the difference between the cross-sectional averages of nominal yields and TIPS yields, Liq_t is Svensson nominal yield curve fitting error (in basis points), $GoldRet_{t-\tau,t}$ is the return on gold bullion, $VIXRet_{t-\tau,t}$ is the return on the S&P500 implied volatility index, and $BondRet_{t-\tau,t}$ is the value-weighted average of TIPS gross price returns. Panel A(B) reports results of one-month (one-year) ahead inflation regressions for three different on-the-run samples: on-the-run 5-year TIPS and matching T-Notes (Columns 2 and 3), on-the-run 10-year TIPS and matching T-Notes (Columns 4 and 5), and on-the-run 5&10-year TIPS and matching T-Notes (Columns 6 and 7). The model parameters are estimated by minimizing the equally-weighted sum of squared errors. An option value cutoff of $1E-8$ is imposed and $OR_{t-12,t}$ is winsorized above 90%. The t-statistics based on four-lag Newey-West adjusted standard errors are reported in parentheses below the estimated coefficients. * stat. sign. at 10% level; ** stat. sign. at 5% level; *** stat. sign. at 1% level. Sample period is from January 1997 to May 2010, monthly frequency.

4.7.3. Crude oil return as an additional control variable

In this section we examine the ability of $OR_{t-1,t}$ and $OR_{t-12,t}$ to explain the future rate of inflation in the presence of an additional control variable, the return on crude oil. The price of crude oil is impacted by many factors, such as pricing policies in the OPEC cartel, supply disruptions due to weather or political instability, and speculative demand. The relationship between inflation and the price of crude oil is not necessarily stable over time, a point of view that is supported by Bekaert and Wang (2010) and Hamilton (2009). Because of this, we treat crude oil separately so as to better gauge the marginal impact of including the crude oil return as a control variable in our regressions.

We use Brent crude oil spot prices to construct the monthly crude oil return, $OilRet_{t-1,t}$, and the annual crude oil return, $OilRet_{t-12,t}$. Our regression results with crude oil are shown in Table 9. The second column of Panel A (Panel B) shows our results for the one-month (one-year) ahead inflation rate. In both of these columns, we see that the crude oil return is statistically significant. To see how our previous results are impacted by $OilRet_{t-1,t}$ and $OilRet_{t-12,t}$, we can compare Table 9 to Table 5. In the last column of Table 5, $OR_{t-1,t}$ and $OR_{t-12,t}$ are significant at the 1% level in Panels A and B, respectively. In the second column of Panel A (Panel B) in Table 9, $OR_{t-1,t}$ ($OR_{t-12,t}$) is significant at the 5% level in the presence of $OilRet_{t-1,t}$ ($OilRet_{t-12,t}$). Thus the return on crude oil reduces, but does not drive out, the statistical significance of $OR_{t-1,t}$ and $OR_{t-12,t}$. As we move from Table 5 to Table 9, the coefficient on $OR_{t-1,t}$ changes from −0.014 to −0.010 and the coefficient on $OR_{t-12,t}$ changes from −0.0039 to −0.0030. Thus the crude oil return reduces slightly the economic significance of our embedded option indices. Overall, even in the presence of $OilRet_{t-1,t}$ and $OilRet_{t-12,t}$, the embedded option in TIPS contains useful information for explaining the future inflation rate.

Table 7
Alternative Measure of Option Returns.

	5&10-year	5&10-year	5&10-year, on-the-run	5&10-year, on-the-run
Panel A: Dependent variable is one-month ahead inflation, $i_{t,t+1}$				
$ORF_{t-1,t}$	−0.029***	−0.014*	−0.030***	−0.013*
	(−3.72)	(−1.96)	(−3.85)	(−1.83)
YS_t		0.71		0.18
		(1.21)		(0.31)
Liq_t		−0.00012		−0.0015
		(−0.05)		(−0.72)
$i_{t-1,t}$		0.33***		0.38***
		(4.13)		(4.21)
$GoldRet_{t-1,t}$		0.054		0.071
		(0.75)		(0.95)
$VIXRet_{t-1,t}$		−0.059**		−0.060**
		(−2.50)		(−2.33)
$BondRet_{t-1,t}$		0.47		0.24
		(1.22)		(0.75)
Constant	0.036***	0.0083	0.037***	0.021
	(8.29)	(0.52)	(8.42)	(1.31)
Obs.	160	160	160	160
Adj-R^2	0.089	0.311	0.094	0.297
Panel B: Dependent variable is one-year ahead inflation, $i_{t,t+12}$				
$ORF_{t-12,t}$	−0.0071***	−0.0066**	−0.0091***	−0.011***
	(−2.68)	(−2.06)	(−2.70)	(−3.33)
YS_t		−0.17		0.28
		(−0.72)		(1.24)
Liq_t		−0.0022*		−0.0018*
		(−1.95)		(−1.72)
$i_{t-12,t}$		−0.12		−0.41*
		(−0.58)		(−1.71)
$GoldRet_{t-12,t}$		−0.0067		−0.013
		(−0.78)		(−1.57)
$VIXRet_{t-12,t}$		−0.0022		0.00052
		(−0.48)		(0.11)
$BondRet_{t-12,t}$		−0.13***		−0.078***
		(−4.11)		(−2.62)
Constant	0.026***	0.044***	0.029***	0.042***
	(10.49)	(5.01)	(15.18)	(5.39)
Obs.	149	149	149	149
Adj-R^2	0.034	0.392	0.094	0.374

The table shows results for the in-sample regression:

$$i_{t,t+\tau} = \beta_0 + \beta_1 ORF_{t-\tau,t} + \beta_2 YS_t + \beta_3 Liq_t + \beta_4 i_{t-\tau,t} + \beta_5 GoldRet_{t-\tau,t} + \beta_6 VIXRet_{t-\tau,t} + \beta_7 BondRet_{t-\tau,t} + \epsilon_{t+\tau},$$

where $i_{t,t+\tau}$ is the τ-month ahead seasonally-unadjusted CPI-based annualized log inflation rate, $ORF_{t-\tau,t}$ is a fraction calculated as the number of positive option returns divided by the total number of available option returns from month $t-\tau$ to month t, YS_t is the difference between the cross-sectional averages of nominal yields and TIPS yields, Liq_t is Svensson nominal yield curve fitting error (in basis points), $GoldRet_{t-\tau,t}$ is the return on gold bullion, $VIXRet_{t-\tau,t}$ is the return on the S&P500 implied volatility index, and $BondRet_{t-\tau,t}$ is the value-weighted average of TIPS gross price returns. Panel A(B) reports results of one-month (one-year) ahead inflation regressions for the 5&10-year full and on-the-run samples: 5&10-year TIPS and matching T-Notes (Columns 2 and 3), and on-the-run 5&10-year TIPS and matching T-Notes (Columns 4 and 5). The model parameters are estimated by minimizing the equally-weighted sum of squared errors. An option value cutoff of $1E-8$ is imposed. The t-statistics based on four-lag Newey-West adjusted standard errors are reported in parentheses below the estimated coefficients. * stat. sign. at 10% level; ** stat. sign. at 5% level; *** stat. sign. at 1% level. Sample period is from January 1997 to May 2010, monthly frequency.

Table 8
Alternative inverse duration weighted model estimation.

	5&10-year	5&10-year	5&10-year	5&10-year
Panel A: Dependent variable is one-month ahead inflation, $i_{t,t+1}$				
$OR_{t-1,t}$	−0.039***	−0.025***		
	(−3.81)	(−3.04)		
$ORF_{t-1,t}$			−0.030***	−0.016**
			(−3.59)	(−2.13)
YS_t		0.92		0.71
		(1.61)		(1.22)
Liq_t		0.00083		−0.000076
		(0.42)		(−0.03)
$i_{t-1,t}$		0.19**		0.33***
		(2.39)		(4.10)
$GoldRet_{t-1,t}$		0.014		0.055
		(0.18)		(0.77)
$VIXRet_{t-1,t}$		−0.055**		−0.060**
		(−2.48)		(−2.51)
$BondRet_{t-1,t}$		0.41		0.47
		(1.33)		(1.21)
Constant	0.027***	0.00079	0.036***	0.0088
	(8.05)	(0.06)	(8.48)	(0.56)
Obs.	160	160	160	160
Adj-R^2	0.259	0.357	0.092	0.316
Panel B: Dependent variable is one-year ahead inflation, $i_{t,t+12}$				
$OR_{t-12,t}$	−0.0038***	−0.0052***		
	(−3.22)	(−2.78)		
$ORF_{t-12,t}$			−0.0085***	−0.0084**
			(−2.95)	(−2.17)
YS_t		−0.24		−0.19
		(−0.95)		(−0.80)
Liq_t		−0.0021**		−0.0022**
		(−1.98)		(−1.98)
$i_{t-12,t}$		−0.25		−0.15
		(−1.15)		(−0.70)
$GoldRet_{t-12,t}$		−0.0062		−0.0067
		(−0.73)		(−0.78)
$VIXRet_{t-12,t}$		−0.0014		−0.0016
		(−0.31)		(−0.35)
$BondRet_{t-12,t}$		−0.12***		−0.13***
		(−3.76)		(−4.14)
Constant	0.024***	0.045***	0.026***	0.045***
	(11.89)	(5.32)	(10.87)	(5.11)
Obs.	149	149	149	149
Adj-R^2	0.065	0.430	0.053	0.404

The table shows results for the in-sample regression:

$$i_{t,t+\tau} = \beta_0 + \beta_1 ORX_{t-\tau,t} + \beta_2 YS_t + \beta_3 Liq_t + \beta_4 i_{t-\tau,t} + \beta_5 GoldRet_{t-\tau,t} + \beta_6 VIXRet_{t-\tau,t} + \beta_7 BondRet_{t-\tau,t} + \epsilon_{t+\tau},$$

where $i_{t,t+\tau}$ is the τ-month ahead seasonally-unadjusted CPI-based annualized log inflation rate. $ORX_{t-\tau,t}$ represents $OR_{t-\tau,t}$ (Columns 2 and 3) or $ORF_{t-\tau,t}$ (Columns 4 and 5), where $OR_{t-\tau,t}$ is the option return index constructed as the value-weighted average of all option returns from month $t-\tau$ to month t, and $ORF_{t-\tau,t}$ is a fraction calculated as the number of positive option returns divided by the total number of available option returns. YS_t is the difference between the cross-sectional averages of nominal yields and TIPS yields, Liq_t is Svensson nominal yield curve fitting error (in basis points), $GoldRet_{t-\tau,t}$ is the return on gold bullion, $VIXRet_{t-\tau,t}$ is the return on the S&P500 implied volatility index, and $BondRet_{t-\tau,t}$ is the value-weighted average of TIPS gross price returns. Panel A(B) reports results of one-month (one-year) ahead inflation regressions for the 5&10-year TIPS and matching T-Notes. The model parameters are estimated by minimizing the inverse duration weighted sum of squared errors. An option value cutoff of $1E-8$ is imposed and $OR_{t-12,t}$ is winsorized above 90%. The t-statistics based on four-lag Newey-West adjusted standard errors are reported in parentheses below the estimated coefficients. * stat. sign. at 10% level; ** stat. sign. at 5% level; *** stat. sign. at 1% level. Sample period is from January 1997 to May 2010, monthly frequency.

4.7.4. Surveys as additional control variables

Table 9 also shows our results when we add inflation surveys as explanatory variables. Ang et al. (2007) have shown that surveys are important for forecasting future inflation. The first survey that we examine is the Blue Chip Economic Indicators survey of expected inflation. For each month in our sample period, we obtain the Blue Chip survey's consensus (i.e., average) inflation forecast for the current calendar year and for the following calendar year (January–December). In each month, we use these two numbers to calculate a one-month ahead and a 12-month ahead inflation forecast. For the one-month ahead forecast, we take the survey's current year forecast, we subtract the cumulative inflation that has been experienced so far during the year, and then we divide by the number of months remaining in the current year. We then annualize this number to get our one-month ahead forecast

Table 9
Full sample inflation regressions with additional control variables.

	5&10-year	5&10-year	5&10-year		5&10-year	5&10-year	5&10-year
Panel A: Dependent variable is one-month ahead inflation, $i_{t,t+1}$				*Panel B: Dependent variable is one-year ahead inflation,* $i_{t,t+12}$			
$OR_{t-1,t}$	−0.010**	−0.013***	−0.014***	$OR_{t-12,t}$	−0.0030**	−0.0047***	−0.0046***
	(−2.19)	(−2.69)	(−2.90)		(−2.00)	(−3.65)	(−5.17)
$OilRet_{t-1,t}$	0.14***			$OilRet_{t-12,t}$	0.0061*		
	(3.95)				(1.95)		
$BlueSurvey1_t$		0.065*		$BlueSurvey12_t$		0.31**	
		(1.74)				(2.27)	
$MichSurvey1_t$			0.048	$MichSurvey12_t$			−0.94***
			(1.06)				(−3.56)
YS_t	0.76	1.03*	0.93	YS_t	−0.36	0.17	0.25
	(1.46)	(1.70)	(1.58)		(−1.37)	(0.55)	(0.98)
Liq_t	0.00075	0.00063	0.00047	Liq_t	−0.0025**	−0.0014	−0.00093
	(0.43)	(0.30)	(0.23)		(−2.19)	(−1.19)	(−1.33)
$i_{t-1,t}$	0.17**	0.24**	0.18**	$i_{t-12,t}$	−0.22	−0.50**	−0.078
	(2.29)	(2.43)	(2.08)		(−0.99)	(−2.49)	(−0.47)
$GoldRet_{t-1,t}$	−0.016	0.020	0.029	$GoldRet_{t-12,t}$	−0.0075	−0.0054	0.0000019
	(−0.25)	(0.26)	(0.39)		(−0.98)	(−0.65)	(0.00)
$VIXRet_{t-1,t}$	−0.043**	−0.055**	−0.055**	$VIXRet_{t-12,t}$	−0.00092	−0.00086	0.0010
	(−2.10)	(−2.54)	(−2.46)		(−0.21)	(−0.20)	(0.25)
$BondRet_{t-1,t}$	0.33	0.38	0.39	$BondRet_{t-12,t}$	−0.14***	−0.12***	−0.13***
	(1.01)	(1.28)	(1.24)		(−4.42)	(−3.52)	(−4.54)
Constant	0.0033	−0.0018	0.0019	*Constant*	0.048***	0.036***	0.056***
	(0.26)	(−0.12)	(0.13)		(5.35)	(3.69)	(7.91)
Obs.	160	160	160	*Obs.*	149	149	149
Adj-R^2	0.457	0.362	0.361	Adj-R^2	0.429	0.448	0.509

The table shows results for the in-sample regression:

$$i_{t,t+\tau} = \beta_0 + \beta_1 OR_{t-\tau,t} + \beta_2 YS_t + \beta_3 Liq_t + \beta_4 i_{t-\tau,t} + \beta_5 GoldRet_{t-\tau,t} + \beta_6 VIXRet_{t-\tau,t} + \beta_7 BondRet_{t-\tau,t} + \beta_8 AddControl_{t-\tau,t} + \epsilon_{t+\tau},$$

where $i_{t,t+\tau}$ is the τ-month ahead seasonally-unadjusted CPI-based annualized log inflation rate, $OR_{t-\tau,t}$ is the option return index constructed as the value-weighted average of all option returns from month $t-\tau$ to month t, YS_t is the difference between the cross-sectional averages of nominal yields and TIPS yields, Liq_t is Svensson nominal yield curve fitting error (in basis points), $GoldRet_{t-\tau,t}$ is the return on gold bullion, $VIXRet_{t-\tau,t}$ is the return on the S&P500 implied volatility index, and $BondRet_{t-\tau,t}$ is the value-weighted average of TIPS gross price returns. $AddControl_{t-\tau,t}$ represents additional control variables: $OilRet_{t-\tau,t}$ (Columns 2 and 6), $BlueSurvey1_t$ (Column 3), $MichSurvey1_t$ (Column 4), $BlueSurvey12_t$ (Column 7), and $MichSurvey12_t$ (Column 8), where $OilRet_{t-\tau,t}$ is the return on Brent Crude Oil spot prices from month $t-\tau$ to month t, $BlueSurvey1_t$ ($BlueSurvey12_t$) is one-month (one-year) ahead interpolated inflation forecast obtained from the respondents' consensus of the Blue Chip Economic Indicators survey, and $MichSurvey1_t$ ($MichSurvey12_t$) is one-month (one-year) ahead inflation forecast obtained from surveys of consumers by University of Michigan. Panel A(B) reports results of one-month (one-year) ahead inflation regressions for the 5&10-year TIPS and matching T-Notes. The model parameters are estimated by minimizing the equally-weighted sum of squared errors. An option value cutoff of $1E-8$ is imposed and $OR_{t-12,t}$ is winsorized above 90%. The t-statistics based on four-lag Newey-West adjusted standard errors are reported in parentheses below the estimated coefficients. * stat. sign. at 10% level; ** stat. sign. at 5% level; *** stat. sign. at 1% level. Sample period is from January 1997 to May 2010, monthly frequency.

variable, $BlueSurvey1_t$. For the 12-month ahead forecast, we again start with the Blue Chip survey's current year forecast and we subtract the cumulative inflation that has been experienced so far during the year. We then add a prorated fraction of the survey's forecast for the following calendar year. For example, if there are x months remaining in the current year and the forecast for the following year is Z, we add $Z(12-x)/12$. We do this month by month to obtain our 12-month ahead forecast variable, $BlueSurvey12_t$.

For example, suppose there are two months remaining in the year and the Blue Chip consensus is 10% inflation for the current year and 12% inflation for the following year. Further suppose that the cumulative inflation through the first ten months of the year is 11%. Since the forecast is 10% for the current year, this implies 1% deflation over the last two months of the year (an average of 0.5% deflation per month). Thus the one-month ahead forecast is -0.5%, which we annualize to get $BlueSurvey1_t$. Since the forecast for the next calendar year is 12%, our forecast for the upcoming 12 months is 9%. We get this number by using -1% inflation for the last two months of the current year and adding $(12\%)(12-2)/12 = 10\%$ for the first ten months of the following year. Thus the value of $BlueSurvey12_t$ is 9%.

For each month in our sample period, we also obtain the survey of expected inflation from the University of Michigan's Survey Research Center. This monthly survey asks households what percent they expect prices to go up or down over the next 12 months. Unlike the Blue Chip survey, it does not rely on calendar year forecasts. Thus we use the reported number from the University of Michigan survey as our 12-month ahead forecast variable, $MichSurvey12_t$. The University of Michigan survey does not report a one-month ahead forecast, which we need for our monthly inflation regressions. To calculate a one-month ahead forecast, we use the CPI level and the reported number from the University of Michigan survey to calculate a 12-month ahead forecasted CPI level. We do this month by month, which creates a forecasted time series for the CPI. We then use this forecasted CPI time series to calculate a time series of monthly inflation rates. We annualize these rates to get our one-month ahead forecast variable, $MichSurvey1_t$.

Table 9 shows our regression results when we include the inflation surveys as control variables. For the Blue Chip survey, we find that $BlueSurvey1_t$ is significant at the 10% level (Panel A of Table 9) and $BlueSurvey12_t$ is significant at the 5% level (Panel B of Table 9). Our variables $OR_{t-1,t}$ and $OR_{t-12,t}$ are both significant at the 1% level. We also find that lagged inflation and the VIX return are significant in Panel A of Table 9, while $BondRet_{t-12,t}$ is significant in Panel B of Table 9. This mirrors many of our earlier results. Thus including $BlueSurvey1_t$ and $BlueSurvey12_t$ does not alter our prior conclusions.

The results with the University of Michigan survey are remarkably different. In Panel A of Table 9, $MichSurvey1_t$ is not statistically significant. Our variable $OR_{t-1,t}$ is significant at the 1% level. Lagged inflation and the VIX return are also significant. In Panel B of Table 9, our variable $OR_{t-12,t}$ is significant at the 1% level and $BondRet_{t-12,t}$ is also significant. Although $MichSurvey12_t$ is significant in Panel B, the coefficient has the wrong sign. We verified that the sample correlation between the Michigan survey and annual inflation was negative during our sample period. The Michigan survey completely missed the deflationary periods that show up in our Figs. 1 and 2. Thus $MichSurvey1_t$ and $MichSurvey12_t$, at least during our sample period, are not useful variables for explaining future inflation.

4.7.5. Option price index returns and log changes in option values

Table 10 shows regression results for several alternative option-based explanatory variables. First, instead of using $OR_{t-1,t}$ and $OR_{t-12,t}$ in (10) and (11) as explanatory variables, we calculate the option return using the price index OP_t from (9). Specifically, we construct two new variables $OPR_{t-1,t} = OP_t/OP_{t-1} - 1$ and $OPR_{t-12,t} = OP_t/OP_{t-12} - 1$. Second, to take care of some of the asymmetries in the return distribution, we create two new variables that rely on log changes in option values. Specifically, we let $OPL_{t-1,t} = \sum_{n=1}^{N_t} W_{nt}[\log(O_{nt}) - \log(O_{n,t-1})]$ and $OPL_{t-12,t} = \sum_{n=1}^{N_t} W_{nt}[\log(O_{nt}) - \log(O_{n,t-12})]$, where the option values O_{nt}, the number of TIPS N_t, and the weights W_{nt} are the same as in (9).

Table 10 shows that these alternative explanatory variables are statistically significant whether we include the control variables in the regression (columns 3 and 5) or not (columns 2 and 4). The variables are also significant whether we use equal weights to estimate the model parameters (panels A and B) or inverse duration weights (panels C and D). The results in Table 10 help to show that our conclusions are robust with respect to how we define the option index return variables.

4.7.6. Liquidity-adjusted TIPS prices

In Tables 5–9, the control variable Liq_t is statistically significant in only 7 out of 26 regressions. This raises the possibility that Liq_t, which is designed to capture market-wide liquidity, does not fully capture liquidity in the TIPS market. To investigate this issue, we use the methods in Pflueger and Viceira (2015) and Christensen and Gillan (2011), which we describe below, to construct two liquidity adjustments that are specific to the TIPS market. Using the Pflueger and Viceira (2015) method, if we regress the liquidity adjustment onto Liq_t we get an adjusted-R^2 of about 48% for 10-year TIPS. Using the Christensen and Gillan (2011) method, a similar exercise gives an adjusted-R^2 of about 38%. For both methods the regression coefficient on Liq_t is statistically significant at the 1% level. However, given the adjusted-R^2 values, there is some concern that maybe we have not adequately controlled for TIPS market liquidity in Tables 5–9. We address this issue by adjusting the TIPS yields (and therefore the TIPS prices) for liquidity prior to estimating our model parameters. Our regression results from this approach are in Table 11.

In Panels A and B of Table 11, we use the Pflueger and Viceira (2015) method to adjust the TIPS yields before estimating the model parameters. To obtain the liquidity adjustment for 10-year TIPS, we regress the 10-year breakeven inflation rate onto the 10-year nominal Treasury off-the-run spread (i.e., the off-the-run par yield minus the on-the-run par yield).[11] We control for inflation expectations by including our variable $BlueSurvey12_t$ in the regression. Following Pflueger and Viceira (2015), we use the regression coefficient on the off-the-run spread and the off-the-run spread itself to obtain a monthly TIPS liquidity adjustment. We subtract the liquidity adjustment from the observed 10-year TIPS yield to get the liquidity-adjusted 10-year TIPS yield. The liquidity-adjusted TIPS yields are then used to calculate liquidity-adjusted TIPS prices, which we use in our model fitting. We follow the same procedure for 5-year TIPS by using the 5-year breakeven inflation rate and the 5-year off-the-run spread. Since the data for the breakeven inflation rate starts in January 1999, only the TIPS yields after this date

Table 10
Alternative returns of option index or log changes in option values.

	5&10-year	5&10-year	5&10-year	5&10-year
Panel A: Equally-weighted model estimation with dependent variable $i_{t,t+1}$				
$OPR_{t-1,t}$	−0.0088**	−0.0053**		
	(−2.44)	(−2.27)		
$OPL_{t-1,t}$			−0.029***	−0.018***
			(−3.71)	(−2.67)
Controls included	no	yes	no	yes
Obs.	159	159	160	160
Adj-R^2	0.100	0.326	0.183	0.337
Panel B: Equally-weighted model estimation with dependent variable $i_{t,t+12}$				
$OPR_{t-12,t}$	−0.0010***	−0.0010**		
	(−3.15)	(−2.22)		
$OPL_{t-12,t}$			−0.0014**	−0.0016**
			(−2.38)	(−1.98)
Controls included	no	yes	no	yes
Obs.	148	148	149	149
Adj-R^2	0.041	0.414	0.025	0.389
Panel C: Inverse duration weighted model estimation with dependent variable $i_{t,t+1}$				
$OPR_{t-1,t}$	−0.014**	−0.0074*		
	(−2.20)	(−1.92)		
$OPL_{t-1,t}$			−0.043***	−0.026***
			(-3.58)	(-2.75)
Controls included	no	yes	no	yes
Obs.	159	159	160	160
Adj-R^2	0.085	0.316	0.195	0.337
Panel D: Inverse duration weighted model estimation with dependent variable $i_{t,t+12}$				
$OPR_{t-12,t}$	−0.0017***	−0.0016***		
	(−3.58)	(−2.66)		
$OPL_{t-12,t}$			−0.0022***	−0.0025**
			(−3.22)	(−2.20)
Controls included	no	yes	no	yes
Obs.	148	148	149	149
Adj-R^2	0.081	0.431	0.049	0.400

The table shows results for the in-sample regression:

$$i_{t,t+\tau} = \beta_0 + \beta_1 OPX_{t-\tau,t} + \beta_2 YS_t + \beta_3 Liq_t + \beta_4 i_{t-\tau,t} + \beta_5 GoldRet_{t-\tau,t} + \beta_6 VIXRet_{t-\tau,t} + \beta_7 BondRet_{t-\tau,t} + \epsilon_{t+\tau},$$

where $i_{t,t+\tau}$ is the τ-month ahead seasonally-unadjusted CPI-based annualized log inflation rate. $OPX_{t-\tau,t}$ represents $OPR_{t-\tau,t}$ (Columns 2 and 3) or $OPL_{t-\tau,t}$ (Columns 4 and 5), where $OPR_{t-\tau,t} = OP_t/OP_{t-\tau} - 1$ is the return of the value-weighted option value index from month $t-\tau$ to month t, and $OPL_{t-\tau,t}$ is the log option return index constructed as the value-weighted average of log change in option values from month $t-\tau$ to month t. The control variables, when included, are $YS_t, Liq_t, i_{t-\tau,t}, GoldRet_{t-\tau,t}, VIXRet_{t-\tau,t}$, and $BondRet_{t-\tau,t}$, where YS_t is the difference between the cross-sectional averages of nominal yields and TIPS yields, Liq_t is Svensson nominal yield curve fitting error, $GoldRet_{t-\tau,t}$ is the return on gold bullion, $VIXRet_{t-\tau,t}$ is the return on the S&P500 implied volatility index, and $BondRet_{t-\tau,t}$ is the value-weighted average of TIPS gross price returns. Panels A and C (B and D) report results of one-month (one-year) ahead inflation regressions for the 5&10-year TIPS and matching T-Notes. The model parameters are estimated by minimizing the equally-weighted (inverse duration weighted) sum of squared errors in Panels A and B (C and D). An option value cutoff of $1E-8$ is imposed and $OPR_{t-12,t}$ is winsorized above 90%. The t-statistics based on four-lag Newey-West adjusted standard errors are reported in parentheses below the estimated coefficients. * stat. sign. at 10% level; ** stat. sign. at 5% level; *** stat. sign. at 1% level. Sample period is from January 1997 to May 2010, monthly frequency.

[11] The breakeven inflation rate is the difference between the nominal and TIPS par yields of comparable maturities. The par yields are estimated using the Svensson (1995) functional form for the yield curve. The estimation procedure is described in Gürkaynak et al. (2010) for the nominal Treasury securities and in Gürkaynak et al. (2010) for the TIPS. The nominal Treasury and TIPS yield curve data is available at http://www.federalreserve.gov/pubs/feds/2006/200628/200628abs.html and http://www.federalreserve.gov/econresdata/researchdata/feds200805_1.html, respectively.

Table 11
Inflation regressions based on liquidity-adjusted TIPS yields.

	5&10-year	5&10-year	5&10-year	5&10-year
Panel A: Pflueger and Viceira (2015) liquidity adjustment with dependent variable $i_{t,t+1}$				
$OR_{t-1,t}$	−0.010***	−0.0090**		
	(−2.77)	(−2.42)		
$ORF_{t-1,t}$			−0.013*	−0.014*
			(−1.82)	(−1.89)
Weights	equal	inv dur	equal	inv dur
Obs.	160	160	160	160
Adj-R^2	0.339	0.334	0.297	0.298
Panel B: Pflueger and Viceira (2015) liquidity adjustment with dependent variable $i_{t,t+12}$				
$OR_{t-12,t}$	−0.0030*	−0.0016**		
	(−1.84)	(−1.98)		
$ORF_{t-12,t}$			−0.0087***	−0.0086***
			(−2.69)	(−2.63)
Weights	equal	inv dur	equal	inv dur
Obs.	149	149	149	149
Adj-R^2	0.388	0.403	0.408	0.409
Panel C: Christensen and Gillan (2011) liquidity adjustment with dependent variable $i_{t,t+1}$				
$OR_{t-1,t}$	−0.036***	−0.046***		
	(−2.71)	(−2.68)		
$ORF_{t-1,t}$			−0.013	−0.014*
			(−1.65)	(−1.78)
Weights	equal	inv dur	equal	inv dur
Obs.	160	160	160	160
Adj-R^2	0.337	0.331	0.294	0.298
Panel D: Christensen and Gillan, 2011 liquidity adjustment with dependent variable $i_{t,t+12}$				
$OR_{t-12,t}$	−0.012***	−0.014***		
	(−2.94)	(−2.83)		
$ORF_{t-12,t}$			−0.0081*	−0.010**
			(−1.82)	(−2.05)
Weights	equal	inv dur	equal	inv dur
Obs.	149	149	149	149
Adj-R^2	0.377	0.406	0.348	0.361

The table shows results for the in-sample regression:

$$i_{t,t+\tau} = \beta_0 + \beta_1 ORX_{t-\tau,t} + \beta_2 YS_t + \beta_3 Liq_t + \beta_4 i_{t-\tau,t} + \beta_5 GoldRet_{t-\tau,t} + \beta_6 VIXRet_{t-\tau,t} + \beta_7 BondRet_{t-\tau,t} + \epsilon_{t+\tau},$$

where $i_{t,t+\tau}$ is the τ-month ahead seasonally-unadjusted CPI-based annualized log inflation rate. $ORX_{t-\tau,t}$ represents $OR_{t-\tau,t}$ (Columns 2 and 3) or $ORF_{t-\tau,t}$ (Columns 4 and 5), where $OR_{t-\tau,t}$ is the option return index constructed as the value-weighted average of all option returns from month $t-\tau$ to month t, and $ORF_{t-\tau,t}$ is a fraction calculated as the number of positive option returns divided by the total number of available option returns. All regressions use an intercept and include the control variables $YS_t, Liq_t, i_{t-\tau,t}, GoldRet_{t-\tau,t}, VIXRet_{t-1,t}$, and $BondRet_{t-\tau,t}$, where YS_t is the difference between the cross-sectional averages of nominal yields and TIPS yields, Liq_t is Svensson nominal yield curve fitting error, $GoldRet_{t-\tau,t}$ is the return on gold bullion, $VIXRet_{t-\tau,t}$ is the return on the S&P500 implied volatility index, and $BondRet_{t-\tau,t}$ is the value-weighted average of TIPS gross price returns. The TIPS prices are adjusted for liquidity prior to estimating the model parameters and constructing $OR_{t-\tau,t}$ and $ORF_{t-\tau,t}$. In panels A and B the liquidity adjustment is calculated using Pflueger and Viceira (2015)'s methodology. In panels C and D the liquidity adjustment is calculated using Christensen and Gillan (2011)'s methodology. Panels A and C (B and D) reports results of one-month (one-year) ahead inflation regressions for the 5&10-year TIPS and matching T-Notes. The model parameters are estimated by minimizing the equally-weighted (inverse duration weighted) sum of squared errors in columns 2 and 4 (3 and 5). An option value cutoff of $1E-8$ is imposed and $OR_{t-12,t}$ is winsorized above 90%. The $t-$statistics based on four-lag Newey-West adjusted standard errors are reported in parentheses below the estimated coefficients. * stat. sign. at 10% level; ** stat. sign. at 5% level; *** stat. sign. at 1% level. Sample period is from January 1997 to May 2010, monthly frequency.

are adjusted for liquidity. According to Panels A and B in Table 11, our option-based explanatory variables remain statistically significant when we adjust the TIPS yields for liquidity prior to parameter estimation. This is true for both equal weights and inverse duration weights used in the SSE in (8).

In Panels C and D of Table 11, we use the Christensen and Gillan (2011) method to adjust the TIPS yields for liquidity. To obtain the liquidity adjustment in a given month, we subtract the breakeven inflation rate from the inflation swap rate using comparable maturities. As discussed in Christensen and Gillan (2011), this provides an upper bound on the TIPS liquidity premium. For each TIPS in our sample, we subtract the upper bound from the observed TIPS yield to get the liquidity-adjusted TIPS yield. We use the liquidity-adjusted TIPS yields to calculate liquidity-adjusted TIPS prices, which we use in our model fitting. The inflation swaps market was launched in 2004 and thus our dataset for inflation swaps begins in August 2004.[12] Consequently, the TIPS prices are adjusted for liquidity only after this date. In our sample, the average TIPS liquidity adjustment is about 47 basis points, which is consistent with Fig. 1 in Christensen and Gillan (2011). In Panels C and D of our Table 11, the variables $OR_{t-1,t}$ and $OR_{t-12,t}$ are significant at the 1% level, which is consistent with the results in Tables 5 and 8 for the combined sample of 5-year and 10-year TIPS. However, the significance of $ORF_{t-1,t}$ and $ORF_{t-12,t}$ in Table 11 is reduced relative to what we see in Tables 7 and 8. Thus the magnitude of the option return appears to be important once we adjust the TIPS prices for liquidity using the Christensen and Gillan (2011) method.

4.7.7. Sub-sample results

We analyzed our model using the sample period 1997–2006, which excludes the financial crisis. When we recalculated Tables 5–7 using all 5-year and 10-year TIPS for this sample period, we found that the coefficient on the option return index remained statistically significant in the presence of all of the control variables. Thus our results do not appear to be driven by the financial crisis or the accompanying decrease in market liquidity.

We also split our full sample into two discontiguous sub-samples based on deflationary expectations. We used 2002–2005 and 2008–2010 as the deflationary sub-sample since it includes the deflation scare period of 2003–2004 and the financial crisis of 2008–2009. We used 1997–2001 and 2006–2007 as the inflationary sub-sample. Although our split is somewhat arbitrary, we tried to keep the number of months about the same for the two sub-samples. Using all 5-year and 10-year TIPS and equal weights in (8), when we estimate the model parameters using the deflationary (inflationary) sub-sample, the mean absolute yield error is 47 basis points (56 basis points). In Table 2, for the full sample, the mean absolute yield error is 52 basis points. Thus the model fit is slightly better for the deflationary sub-sample. Using the parameter estimates for the two sub-samples, we construct a new set of option-based explanatory variables. In particular, we construct a monthly option return index $ORI_{t-1,t}$ ($ORD_{t-1,t}$) that uses the inflationary (deflationary) parameter estimates. Similarly, we construct an annual option return index $ORI_{t-12,t}$ ($ORD_{t-12,t}$) that uses the inflationary (deflationary) parameter estimates. We also construct variables that measure the fraction of options with positive returns, $ORFI_{t-1,t}, ORFD_{t-1,t}, ORFI_{t-12,t}$, and $ORFD_{t-12,t}$. We then reran the regressions in Tables 5–9. For each regression we include both the inflationary and deflationary option return variables, but we interact these variables with dummy variables to capture the relevant sub-period. For example, $ORI_{t-1,t}$ and $ORD_{t-1,t}$ enter the regression as $\delta^I_t ORI_{t-1,t}$ and $\delta^D_t ORD_{t-1,t}$, where δ^I_t (δ^D_t) is a dummy variable that is equal to one during the inflationary (deflationary) sub-period and is equal to zero otherwise.

To summarize our results, we focused on the 26 regressions in Tables 5–9 that include the control variables. For 25 of these 26 regressions, we found that the option return variable during the deflationary sub-period was more significant statistically than

[12] Sources: Barclays and the Federal Reserve Board.

the option return variable during the inflationary sub-period. Furthermore, for 19 of the 26 regressions, the option return variable during the deflationary (inflationary) sub-period was as significant or more (less) significant than the significance shown in Tables 5–9. Although we have not used a formal regime-switching model, the evidence suggests that our option-based explanatory variables are more relevant during deflationary periods than during inflationary periods.

4.8. Regressions with rolling window parameter estimation

In Section 4.6, we showed that $OR_{t-1,t}$ and $OR_{t-12,t}$ are significant for explaining the one-month ahead and the 12–month ahead inflation rate, respectively. Since our estimation results in Table 2 use data for the entire sample period 1997–2010, our embedded option index variables in (10) and (11) rely on parameter estimates that have a forward looking bias. Thus our results in Section 4.6 are not inflation forecasts – they are simply in-sample results. We now address this issue by using a rolling window approach to estimate the model parameters. We use all of the securities in Table 1 and we re-estimate our model using rolling subsamples. Using the parameter estimates for each subsample, we calculate the embedded option values and the embedded option returns. The option return index is then used to explain the future inflation rate in a regression analysis that is free from the forward looking bias.

More specifically, our full sample period is January 1997 through May 2010, which is 161 months. We use a 48-month rolling window, which allows us to construct 114 subsamples. The first subsample spans January 1997 through December 2000, the second subsample spans February 1997 through January 2001, and so forth. For each subsample, we seek a solution to the optimization problem in (8) using equal weights. We then use the embedded option values from the last month and from the next to the last month of each subsample to calculate $OR_{t-1,t}$ according to (10). In the subsample that spans January 1997–December 2000, we use the embedded option values from November-December 2000 to calculate $OR_{t-1,t}$ for December 2000; in the subsample that spans February 1997–January 2001, we use the embedded option values from December 2000 and January 2001 to calculate $OR_{t-1,t}$ for January 2001; and so forth. This gives us a new time series for $OR_{t-1,t}$ that does not suffer from forward looking bias. We follow a similar procedure using the last 12 months in each subsample to calculate $OR_{t-12,t}$.

Table 12 shows our regression results when the parameters are calculated using a rolling window. For Table 12 we use the combined sample of 5-year and 10-year TIPS and matching nominal bonds. In Panel A of Table 12, $OR_{t-1,t}$ is statistically significant at the 1% level, even in the presence of the control variables. The VIX return and lagged inflation are also significant, similar to the last column of Panel A in Table 5. If we examine the adjusted-R^2 value, we see that $OR_{t-1,t}$ and the control variables in Panel A of Table 12 explain 36.3% of the variation in the one-month ahead inflation rate. This number is about the same as the corresponding adjusted-R^2 value in the last column in Panel A of Table 5. We also use $ORF_{t-1,t}$ as an explanatory variable in Panel A of Table 12. We find that $ORF_{t-1,t}$ alone is significant at the 1% level, but the significance is driven out by the control variables. Thus in Panel A of Table 12, it appears that the magnitude of the option returns is important for explaining the one-month ahead inflation rate. The variable $OR_{t-1,t}$, which is significant, captures both the magnitude and the sign of the option returns. The variable $ORF_{t-1,t}$, which is insignificant, captures only the sign of the option returns.

In Panel B of Table 12, we find that the variable $OR_{t-12,t}$ is not significant even at the 10% level. Our 12-month ahead results with $ORF_{t-12,t}$ are shown in the last two columns in Panel B of Table 12.

Table 12
Inflation regressions with rolling window parameter estimates.

	5&10-year	5&10-year	5&10-year	5&10-year
Panel A: Dependent variable is one-month ahead inflation, $i_{t,t+1}$				
$OR_{t-1,t}$	-0.0041^{***}	-0.0022^{***}		
	(−18.66)	(−3.09)		
$ORF_{t-1,t}$			-0.037^{***}	−0.012
			(−2.77)	(−0.99)
YS_t		0.38		0.67
		(0.43)		(0.82)
Liq_t		0.0012		0.00014
		(0.51)		(0.06)
$i_{t-1,t}$		0.36^{***}		0.38^{***}
		(4.20)		(3.90)
$GoldRet_{t-1,t}$		0.060		0.082
		(0.70)		(0.86)
$VIXRet_{t-1,t}$		-0.057^{**}		-0.070^{**}
		(−2.11)		(−2.57)
$BondRet_{t-1,t}$		0.22		0.53
		(0.69)		(1.37)
Constant	0.027^{***}	0.0051	0.036^{***}	0.0041
	(5.40)	(0.25)	(6.54)	(0.19)
Obs.	112	112	112	112
Adj-R^2	0.265	0.363	0.084	0.332
Panel B: Dependent variable is one-year ahead inflation, $i_{t,t+12}$				
$OR_{t-12,t}$	0.00030	−0.0071		
	(0.08)	(−1.41)		
$ORF_{t-12,t}$			-0.0084^{**}	-0.028^{***}
			(−2.10)	(−3.13)
YS_t		-0.50^{*}		-0.70^{***}
		(−1.74)		(−3.05)
Liq_t		-0.0037^{***}		-0.0033^{***}
		(−3.89)		(−3.68)
$i_{t-12,t}$		−0.11		−0.22
		(−0.48)		(−1.22)
$GoldRet_{t-12,t}$		-0.027^{**}		-0.023^{**}
		(−2.05)		(−2.56)
$VIXRet_{t-12,t}$		−0.00070		0.00067
		(−0.15)		(0.17)
$BondRet_{t-12,t}$		-0.15^{***}		-0.18^{***}
		(−4.36)		(−6.15)
Constant	0.023^{***}	0.056^{***}	0.025^{***}	0.067^{***}
	(9.50)	(6.58)	(7.65)	(8.51)
Obs.	110	110	110	110
Adj-R^2	<0.001	0.482	0.024	0.578

The table shows results for the regression:

$$i_{t,t+\tau} = \beta_0 + \beta_1 ORX_{t-\tau,t} + \beta_2 YS_t + \beta_3 Liq_t + \beta_4 i_{t-\tau,t} + \beta_5 GoldRet_{t-\tau,t} + \beta_6 VIXRet_{t-\tau,t} + \beta_7 BondRet_{t-\tau,t} + \epsilon_{t+\tau},$$

where $i_{t,t+\tau}$ is the τ-month ahead seasonally-unadjusted CPI-based annualized log inflation rate. $ORX_{t-\tau,t}$ represents $OR_{t-\tau,t}$ (Columns 2 and 3) or $ORF_{t-\tau,t}$ (Columns 4 and 5), where $OR_{t-\tau,t}$ is the option return index constructed as the value-weighted average of all option returns from month $t-\tau$ to month t, and $ORF_{t-\tau,t}$ is a fraction calculated as the number of positive option returns divided by the total number of available option returns. YS_t is the difference between the cross-sectional averages of nominal yields and TIPS yields, Liq_t is Svensson nominal yield curve fitting error (in basis points), $GoldRet_{t-\tau,t}$ is the return on gold bullion, $VIXRet_{t-\tau,t}$ is the return on the S&P500 implied volatility index, and $BondRet_{t-\tau,t}$ is the value-weighted average of TIPS gross price returns. Panel A(B) reports results of one-month (one-year) ahead inflation regressions for the 5&10-year TIPS and matching T-Notes. We use a 4-year window of monthly observations, rolled monthly, from January 1997 to May 2010. Starting from January 2001, a 4-year sample preceding this month is used to estimate the model parameters by minimizing the equally-weighted sum of squared errors. We compute $ORX_{t-1,t}$ using the last two months of each rolling window. An option value cutoff of $1E-8$ is imposed and $OR_{t-12,t}$ is winsorized above 90%. The t-statistics based on four-lag Newey-West adjusted standard errors are reported in parentheses below the estimated coefficients. * stat. sign. at 10% level; ** stat. sign. at 5% level; *** stat. sign. at 1% level.

In these two regressions we find that $ORF_{t-12,t}$ is significant at the 5% level or better. This suggests the sign of the option return contains some information for explaining the 12-month ahead inflation rate.

If we compare the results in Table 12 to the results in Tables 5 and 7, we see that the Table 12 results are sometimes weaker than their counterparts in Tables 5 and 7. There are at least two contributing reasons. First, our rolling subsample is only 48 months long, which is much shorter than our full sample of 161 months. Thus our parameter estimates and our embedded option estimates are noisier in the subsamples, which makes for noisier embedded option explanatory variables. This tends to have a relatively bigger impact on the magnitude of the 12-month option returns, which helps to explain why $OR_{t-12,t}$ in Panel B of Table 12 is not significant. Second, the short length of our window decreases not only the time length of each subsample, but it can also decrease the number of securities that is included in each subsample. For example, in our early subsamples, the number of TIPS and matching nominal Treasuries is reduced since some of these securities have not yet been auctioned. The smaller number of securities implies that there are fewer observations within the subsample for estimating our model parameters, which again will lead to noisier parameter estimates. In spite of these issues, our results in Table 12 suggest that the embedded option in TIPS contains some information that is useful for explaining future inflation.

4.9. Analysis of UK inflation-indexed gilts

As a robustness check on our modeling approach, we fit our model to the UK inflation-indexed gilts market. Unlike TIPS, gilts do not contain an embedded deflation option. Thus if the fitted model in the gilts market produces option values that are similar to what we observe in the TIPS market, it is a sign that our approach is likely misspecified.

To fit our model to the gilts market, we used the same sample period as we did for the TIPS market. During this sample period we found 24 inflation-indexed gilts.[13] In order to mirror our analysis of the TIPS market, we also identified 24 matching nominal gilts. We used (5) as the model price for the nominal gilts and we used (3) as the model price for the inflation-indexed gilts.[14] To choose the model parameters, we minimized the sum of squared pricing errors between the model prices and the market prices, where the sum is over all 48 gilts and all months in our sample. After solving for the optimal parameters, we then used the option term in (3) to estimate a hypothetical value for the deflation option in each month for each of the 24 inflation-indexed gilts.

To summarize our findings, the overall model fit for the gilts market is about the same as for the TIPS market. The mean absolute yield error for the gilts sample is 0.0058, which is comparable to the TIPS sample in Table 2. However, in contrast to the TIPS market, all of the estimated option values for the gilts are very small. The largest estimated option value across all months and all inflation-indexed gilts is about 10^{-9}, which is smaller than the cutoff we used in our TIPS analysis. This value occurred in 1997, near the beginning of our sample. During the financial crisis period, the estimated option values for all inflation-indexed gilts were on the order of 10^{-12} or smaller. Thus the estimated option values are negligible throughout the sample period relative to what we found in the TIPS market. The largest estimated option value for gilts occurs in 1997, not during the financial crisis, as in the TIPS market. Thus we cannot reject our model based on this evidence from the UK gilts market.

As a final point, note that even if the gilts had embedded deflation options, the value of these options would very likely be small relative to the embedded options in TIPS. The reason is that the maturity structure of the gilts is much longer than that of the TIPS. For the TIPS, our sample includes only 5-year and 10-year TIPS. In contrast, for the 24 inflation-indexed gilts in our sample, the average maturity is 27.5 years. There is only one inflation-indexed gilt with a maturity less than 10 years. Five gilts have maturities between 10 and 19 years; six have maturities between 20 and 29 years; nine have maturities between 30 and 39 years; two have maturities of 40 years; and one has a 50 year maturity. The longer average maturity is probably one of the reasons why the UK Debt Management Office did not design the inflation-indexed gilt with an embedded deflation option. Over a long time period such as 30 years or 40 years, the likelihood of cumulative deflation is extremely low. Thus the option would not be valuable to an investor. This is certainly what we found above. The hypothetical option values are negligible.

5. Concluding remarks

Our paper uncovers the informational content of the embedded option in TIPS. We show that the embedded option returns contain relevant information for explaining the one-month ahead and the 12–month ahead inflation rates, even in the presence of standard inflation variables. In almost all of our regressions, the embedded option return index is statistically and economically important. Our results suggest that the time variation in the embedded option return is a valuable tool for anyone who is interested in assessing future inflation.

Our paper contains several new findings. First, using 5-year (10-year) TIPS, our results suggest that a 100% embedded option return, which is less than one standard deviation, is consistent with a 73 basis point (39 basis point) decrease in the one-month ahead annualized rate of inflation. For most of our regressions, the traditional inflation variables such as the return on gold are insignificant in the presence of our embedded option return index. However, the lagged inflation rate and the return on the VIX index continue to be important variables. Presumably, these variables capture additional uncertainty beyond what is contained in the embedded option return. Second, our main conclusions are not altered when we discard off-the-run TIPS, when we use alternative weighting schemes, when we add additional control variables (the crude oil return and inflation surveys), when we analyze inflation using rolling window parameter estimates, or when we use our variables $ORF_{t-1,t}$ and $ORF_{t-12,t}$, which are less sensitive to model specification. In summary, our paper shows that the embedded deflation option in TIPS is informationally relevant for explaining future inflation out to a horizon of twelve months.

There are several areas for future research. First, our TIPS pricing model is a traditional asset pricing model in the sense that we do not model liquidity directly. This is one of the reasons that we discard the off-the-run TIPS (Table 6) and we adjust the TIPS prices for liquidity (Table 11). A more complicated approach would be to derive a TIPS pricing model that accommodates liquidity explicitly. This type of pricing model could be estimated using both on-the-run and off-the-run TIPS, with the understanding that liquidity is captured by the model itself. Second, although we conduct robustness checks using our variables $ORF_{t-1,t}$ and $ORF_{t-12,t}$, which are significant in Tables 7 and 8, we do not claim that our model in (1) and (2) is the best way to price a TIPS. Our motivation for using (1) and (2) is twofold – the model is parsimonious and we can solve the model in closed-form. Thus one avenue for future research is to explore other pricing models and perhaps run a horse race between them to find the best pricing model. In the context of our paper, the best pricing model would be the one that provides the most information for forecasting future inflation. Lastly, we have shown that $OR_{t-1,t}, OR_{t-12,t}, ORF_{t-1,t}$, and

[13] Some of the inflation-indexed gilts in our sample had 3-month inflation lags and some had 8-month inflation lags. We accommodated these features in our analysis.

[14] Since the inflation-indexed gilts do not contain an embedded deflation option, we also tried omitting the option term in (3) when estimating the model parameters. We found identical results using this alternative method.

$ORF_{t-12,t}$ are informationally relevant variables for explaining the inflation rate. However, we have not examined higher-order moments of these variables, nor have we examined how the inflation probability density evolves over time. This latter topic is complicated since we estimate our model under the risk-adjusted probabilities. We leave these areas as ideas for future research.

Appendix A. Pricing model for TIPS

We stack the nominal interest rate r_t and the inflation rate i_t into a vector $X_t = [r_t \;\; i_t]^\top$, where $\top$ denotes the transpose. Thus we can rewrite (1) and (2) as

$$dX_t = (a + AX_t)dt + Bdz_t^Q, \tag{13}$$

where $a = [a_1 \;\; a_2]^\top, z_t^Q = [z_{1t}^Q \;\; z_{2t}^Q]^\top$, and A and B are the matrices

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad B = \begin{bmatrix} B_{11} & 0 \\ B_{21} & B_{22} \end{bmatrix}.$$

Since A is not a diagonal matrix, (13) is a coupled system of equations. Changes in r_t depend on both r_t and i_t, while changes in i_t depend on both i_t and r_t. Instead of working with X_t directly, we work with a decoupled system that is related to (13). Define Λ as

$$\Lambda = \begin{bmatrix} 1 & \frac{A_{12}}{\lambda_2 - A_{11}} \\ \frac{A_{21}}{\lambda_1 - A_{22}} & 1 \end{bmatrix},$$

where λ_1 and λ_2 are

$$\lambda_1 = \frac{1}{2}(A_{11} + A_{22}) + \frac{1}{2}\sqrt{(A_{11} - A_{22})^2 + 4A_{12}A_{21}},$$
$$\lambda_2 = \frac{1}{2}(A_{11} + A_{22}) - \frac{1}{2}\sqrt{(A_{11} - A_{22})^2 + 4A_{12}A_{21}}.$$

The constants λ_1 and λ_2 are the eigenvalues of A, while the columns of Λ are the associated eigenvectors. It is easily verified that $\Lambda^{-1}A\Lambda = D$, where D is the diagonal matrix

$$D = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}.$$

We now define a new set of variables $Y_t = \Lambda^{-1}X_t$, where $Y_t = [Y_{1t} \;\; Y_{2t}]^\top$. Also define $b = \Lambda^{-1}a$ and $\Sigma = \Lambda^{-1}B$, where $b = [b_1 \;\; b_2]^\top$ and where

$$\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}.$$

Using Itô's lemma, the process for Y_t is

$$dY_t = (b + DY_t)dt + \Sigma dz_t^Q, \tag{14}$$

which is an uncoupled system since D is diagonal. We solve (3) using the variables Y_{1t} and Y_{2t}. We then recover the TIPS price in terms of r_t and i_t by noting that $X_t = \Lambda Y_t$, i.e.,

$$\begin{bmatrix} r_t \\ i_t \end{bmatrix} = \begin{bmatrix} 1 & \frac{A_{12}}{\lambda_2 - A_{11}} \\ \frac{A_{21}}{\lambda_1 - A_{22}} & 1 \end{bmatrix} \begin{bmatrix} Y_{1t} \\ Y_{2t} \end{bmatrix} = \begin{bmatrix} Y_{1t} + \left(\frac{A_{12}}{\lambda_2 - A_{11}}\right)Y_{2t} \\ \left(\frac{A_{21}}{\lambda_1 - A_{22}}\right)Y_{1t} + Y_{2t} \end{bmatrix}. \tag{15}$$

To get the moments for Y_{1t} and Y_{2t}, we solve (14) to get

$$Y_{1s} = e^{\lambda_1(s-t)}Y_{1t} + \frac{b_1}{\lambda_1}\left[e^{\lambda_1(s-t)} - 1\right] + e^{\lambda_1 s}\int_t^s e^{-\lambda_1 u}\left(\sigma_{11}dz_{1u}^Q + \sigma_{12}dz_{2u}^Q\right), \tag{16}$$

$$Y_{2s} = e^{\lambda_2(s-t)}Y_{2t} + \frac{b_2}{\lambda_2}\left[e^{\lambda_2(s-t)} - 1\right] + e^{\lambda_2 s}\int_t^s e^{-\lambda_2 u}\left(\sigma_{21}dz_{1u}^Q + \sigma_{22}dz_{2u}^Q\right), \tag{17}$$

for $s \geqslant t$. Taking expectations of (16) and (17) gives

$$\mathbb{E}_t^Q[Y_{1s}] = e^{\lambda_1(s-t)}Y_{1t} + \frac{b_1}{\lambda_1}\left[e^{\lambda_1(s-t)} - 1\right], \tag{18}$$

$$\mathbb{E}_t^Q[Y_{2s}] = e^{\lambda_2(s-t)}Y_{2t} + \frac{b_2}{\lambda_2}\left[e^{\lambda_2(s-t)} - 1\right]. \tag{19}$$

To get the variance of Y_{1s}, note that

$$Var_t^Q[Y_{1s}] = \mathbb{E}_t^Q\left[\left(Y_{1s} - \mathbb{E}_t^Q[Y_{1s}]\right)^2\right] = e^{2\lambda_1 s}\int_t^s e^{-2\lambda_1 u}(\sigma_{11}^2 + \sigma_{12}^2)du = \frac{\sigma_{11}^2 + \sigma_{12}^2}{2\lambda_1}\left[e^{2\lambda_1(s-t)} - 1\right]. \tag{20}$$

A similar calculation gives

$$Var_t^Q[Y_{2s}] = \frac{\sigma_{21}^2 + \sigma_{22}^2}{2\lambda_2}\left[e^{2\lambda_2(s-t)} - 1\right]. \tag{21}$$

To get the covariance between Y_{1t} and Y_{2t}, note that

$$\begin{aligned} Cov_t^Q[Y_{1s}, Y_{2s}] &= \mathbb{E}_t^Q\left[\left(Y_{1s} - \mathbb{E}_t^Q[Y_{1s}]\right)\left(Y_{2s} - \mathbb{E}_t^Q[Y_{2s}]\right)\right] \\ &= e^{(\lambda_1+\lambda_2)s}(\sigma_{11}\sigma_{21} + \sigma_{12}\sigma_{22})\int_t^s e^{-(\lambda_1+\lambda_2)u}du \\ &= \frac{\sigma_{11}\sigma_{21} + \sigma_{12}\sigma_{22}}{\lambda_1 + \lambda_2}\left[e^{(\lambda_1+\lambda_2)(s-t)} - 1\right]. \end{aligned} \tag{22}$$

Given (14), Y_{1s} and Y_{2s} are bivariate normal with conditional moments (18)–(22). To evaluate the TIPS price, we need to know the joint distribution of $\int_t^{t_k} r_s ds$ and $\int_t^{t_k} i_s ds$ for $k = 1, 2, \ldots, n$. Using (15), note that

$$\int_t^{t_k} r_s ds = \int_t^{t_k} Y_{1s}ds + \left(\frac{A_{12}}{\lambda_2 - A_{11}}\right)\int_t^{t_k} Y_{2s}ds,$$
$$\int_t^{t_k} i_s ds = \left(\frac{A_{21}}{\lambda_1 - A_{22}}\right)\int_t^{t_k} Y_{1s}ds + \int_t^{t_k} Y_{2s}ds.$$

Thus to get the joint distribution of $\int_t^{t_k} r_s ds$ and $\int_t^{t_k} i_s ds$, it is sufficient to characterize the joint distribution of $\int_t^{t_k} Y_{1s}ds$ and $\int_t^{t_k} Y_{2s}ds$. Since Y_{1s} and Y_{2s} are jointly normal, $\int_t^{t_k} Y_{1s}ds$ and $\int_t^{t_k} Y_{2s}ds$ are also jointly normal. This follows since the sum of normally distributed random variables is also normally distributed. Thus we only need to characterize the first two moments of $\int_t^{t_k} Y_{1s}ds$ and $\int_t^{t_k} Y_{2s}ds$.

Suppose $k = n$ and recall that $t_n = T$. We focus on the case of time T, but our results apply for any t_k in the upper limit of integration. Using (16) and (17), we have

$$\int_t^T Y_{1s}ds = \int_t^T e^{\lambda_1(s-t)}Y_{1t}ds + \frac{b_1}{\lambda_1}\int_t^T \left[e^{\lambda_1(s-t)} - 1\right]ds + \int_t^T e^{\lambda_1 s}\int_t^s e^{-\lambda_1 u}\left(\sigma_{11}dz_{1u}^Q + \sigma_{12}dz_{2u}^Q\right)ds \tag{23}$$

and

$$\int_t^T Y_{2s}ds = \int_t^T e^{\lambda_2(s-t)}Y_{2t}ds + \frac{b_2}{\lambda_2}\int_t^T \left[e^{\lambda_2(s-t)} - 1\right]ds + \int_t^T e^{\lambda_2 s}\int_t^s e^{-\lambda_2 u}\left(\sigma_{21}dz_{1u}^Q + \sigma_{22}dz_{2u}^Q\right)ds. \tag{24}$$

Thus

$$\mathbb{E}_t^Q\left[\int_t^T Y_{1s}ds\right] = \left(Y_{1t} + \frac{b_1}{\lambda_1}\right)\frac{1}{\lambda_1}\left[e^{\lambda_1(T-t)} - 1\right] - \frac{b_1}{\lambda_1}(T-t), \tag{25}$$

$$\mathbb{E}_t^Q\left[\int_t^T Y_{2s}ds\right] = \left(Y_{2t} + \frac{b_2}{\lambda_2}\right)\frac{1}{\lambda_2}\left[e^{\lambda_2(T-t)} - 1\right] - \frac{b_2}{\lambda_2}(T-t). \tag{26}$$

To get the variance of $\int_t^T Y_{1s}ds$ note that

$$\begin{aligned} Var_t^Q\left[\int_t^T Y_{1s}ds\right] &= Cov_t^Q\left[\int_t^T Y_{1s}ds, \int_t^T Y_{1u}du\right] \\ &= \int_t^T Cov_t^Q\left[Y_{1s}, \int_t^s Y_{1u}du\right]ds + \int_t^T Cov_t^Q\left[Y_{1s}, \int_s^T Y_{1u}du\right]ds. \end{aligned} \tag{27}$$

The last line of (27) includes two terms. The first term is

$$\int_t^T Cov_t^Q\left[Y_{1s}, \int_t^s Y_{1u}du\right]ds = \int_t^T \left(\int_t^s Cov_t^Q[Y_{1s}, Y_{1u}]du\right)ds. \tag{28}$$

We need to calculate $Cov_t^Q[Y_{1s}, Y_{1u}]$ which is

$$\begin{aligned} Cov_t^Q[Y_{1s}, Y_{1u}] &= \mathbb{E}_t^Q\left[\left(Y_{1s} - \mathbb{E}_t^Q[Y_{1s}]\right)\left(Y_{1u} - \mathbb{E}_t^Q[Y_{1u}]\right)\right] \\ &= e^{\lambda_1 s}e^{\lambda_1 u}\int_t^u e^{-2\lambda_1 v}(\sigma_{11}^2 + \sigma_{12}^2)dv \\ &= e^{\lambda_1 s}e^{\lambda_1 u}\frac{\sigma_{11}^2 + \sigma_{12}^2}{2\lambda_1}\left[e^{-2\lambda_1 t} - e^{-2\lambda_1 u}\right]. \end{aligned} \tag{29}$$

Substituting (29) into the right-hand side of (28), we get

$$\frac{\sigma_{11}^2 + \sigma_{12}^2}{2\lambda_1}\int_t^T \left(\int_t^s e^{\lambda_1 s}e^{\lambda_1 u}\left[e^{-2\lambda_1 t} - e^{-2\lambda_1 u}\right]du\right)ds \tag{30}$$

which is easy to evaluate. The second term in the last line of (27) is

$$\int_t^T Cov_t^Q\left[Y_{1s}, \int_s^T Y_{1u}du\right]ds.$$

Using (23), note that

$$\int_s^T Y_{1u}du = \int_s^T e^{\lambda_1(u-s)}Y_{1s}du + \frac{b_1}{\lambda_1}\int_s^T \left[e^{\lambda_1(u-s)} - 1\right]du + \int_s^T e^{\lambda_1 u}\int_s^u e^{-\lambda_1 v}\left(\sigma_{11}dz_{1v}^Q + \sigma_{12}dz_{2v}^Q\right)du.$$

The right hand side of the above expression has three terms, but only the first term on the right hand side has non-zero correlation with Y_{1s}. Thus

$$\begin{aligned} \int_t^T Cov_t^Q\left[Y_{1s}, \int_s^T Y_{1u}du\right]ds &= \int_t^T Cov_t^Q\left[Y_{1s}, \int_s^T e^{\lambda_1(u-s)}Y_{1s}du\right]ds \\ &= \int_t^T Var_t^Q[Y_{1s}]\left[\int_s^T e^{\lambda_1(u-s)}du\right]ds \end{aligned} \tag{31}$$

which can be evaluated using (20). Combining (30) and (31) gives the result

$$\begin{aligned} Var_t^Q\left[\int_t^T Y_{1s}ds\right] &= \frac{\sigma_{11}^2 + \sigma_{12}^2}{\lambda_1^2}(T-t) + \frac{\sigma_{11}^2 + \sigma_{12}^2}{2\lambda_1^3}\left[e^{2\lambda_1(T-t)} - 1\right] \\ &\quad + \frac{\sigma_{11}^2 + \sigma_{12}^2}{\lambda_1^3}\left[2 - 2e^{\lambda_1(T-t)}\right]. \end{aligned}$$

A similar calculation gives

$$\begin{aligned} Var_t^Q\left[\int_t^T Y_{2s}ds\right] &= \frac{\sigma_{21}^2 + \sigma_{22}^2}{\lambda_2^2}(T-t) + \frac{\sigma_{21}^2 + \sigma_{22}^2}{2\lambda_2^3}\left[e^{2\lambda_2(T-t)} - 1\right] \\ &\quad + \frac{\sigma_{21}^2 + \sigma_{22}^2}{\lambda_2^3}\left[2 - 2e^{\lambda_2(T-t)}\right]. \end{aligned}$$

To get the covariance between $\int_t^T Y_{1s}ds$ and $\int_t^T Y_{2s}ds$, note that

$$\begin{aligned} Cov_t^Q\left[\int_t^T Y_{1s}ds, \int_t^T Y_{2u}du\right] &= \int_t^T Cov_t^Q\left[Y_{1s}, \int_t^T Y_{2u}du\right]ds \\ &= \int_t^T Cov_t^Q\left[Y_{1s}, \int_t^s Y_{2u}du\right]ds \\ &\quad + \int_t^T Cov_t^Q\left[Y_{1s}, \int_s^T Y_{2u}du\right]ds. \end{aligned} \tag{32}$$

Like Eq. (27), there are two terms in (32) that must be evaluated. The first term is

$$\int_t^T Cov_t^Q\left[Y_{1s}, \int_t^s Y_{2u}du\right]ds = \int_t^T \left[\int_t^s Cov_t^Q[Y_{1s}, Y_{2u}]du\right]ds. \tag{33}$$

Since $u \leqslant s$ we have,

$$\begin{aligned} &Cov_t^Q[Y_{1s}, Y_{2u}] = \mathbb{E}_t^Q\left[\left(Y_{1s} - \mathbb{E}_t^Q[Y_{1s}]\right)\left(Y_{2u} - \mathbb{E}_t^Q[Y_{2u}]\right)\right] \\ &= e^{\lambda_1 s}e^{\lambda_1 u}\frac{\sigma_{11}\sigma_{21} + \sigma_{12}\sigma_{22}}{\lambda_1 + \lambda_2}\left[e^{-(\lambda_1+\lambda_2)t} - e^{-(\lambda_1+\lambda_2)u}\right], \end{aligned}$$

and thus the right-hand side of (33) is easy to evaluate. The second term in (32) is

$$\int_t^T Cov_t^Q\left[Y_{1s}, \int_s^T Y_{2u}du\right]ds. \tag{34}$$

Using (24), we have

$$\int_s^T Y_{2u}du = \int_s^T e^{\lambda_2(u-s)}Y_{2s}du + \frac{b_2}{\lambda_2}\int_s^T \left[e^{\lambda_2(u-s)} - 1\right]du + \int_s^T e^{\lambda_2 u}\int_s^u e^{-\lambda_2 v}\left(\sigma_{21}dz_{1v}^Q + \sigma_{22}dz_{2v}^Q\right)du.$$

The right hand side of the above expression has three terms, but only the first term on the right hand side has non-zero correlation with Y_{1s}. Thus (34) is

$$\int_t^T Cov_t^Q\left[Y_{1s}, \int_s^T Y_{2u}du\right]ds = \int_t^T Cov_t^Q[Y_{1s}, Y_{2s}]\left[\int_s^T e^{\lambda_2(u-s)}du\right]ds \tag{35}$$

which can be evaluated using (22). Combining (33) and (35) gives the result

$$Cov_t^Q\left[\int_t^T Y_{1s}ds, \int_t^T Y_{2u}du\right] = \frac{\sigma_{11}\sigma_{21} + \sigma_{12}\sigma_{22}}{\lambda_1 + \lambda_2}\left\{\begin{array}{c} \left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2}\right)(T-t) \\ +\frac{1}{\lambda_1}\left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2}\right)\left[1 - e^{\lambda_1(T-t)}\right] \\ +\frac{1}{\lambda_2}\left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2}\right)\left[1 - e^{\lambda_2(T-t)}\right] \\ +\frac{1}{\lambda_1\lambda_2}\left[e^{(\lambda_1+\lambda_2)(T-t)} - 1\right] \end{array}\right\}.$$

We now return to (3) to evaluate the TIPS price. The first term in (3) is

$$\sum_{k=1}^n cF\,\mathbb{E}_t^Q\left[e^{\int_u^{t_k} i_s ds}e^{-\int_t^{t_k} r_s ds}\right].$$

Note that

$$\begin{aligned} \mathbb{E}_t^Q\left[e^{\int_u^{t_k} i_s ds}e^{-\int_t^{t_k} r_s ds}\right] &= e^{\int_u^t i_s ds}\mathbb{E}_t^Q\left[e^{\left(\frac{A_{21}}{\lambda_1 - A_{22}} - 1\right)\int_t^{t_k} Y_{1s}ds + \left(1 - \frac{A_{12}}{\lambda_2 - A_{11}}\right)\int_t^{t_k} Y_{2s}ds}\right] \\ &= e^{\int_u^t i_s ds}e^{G\left(Y_{1t}, Y_{2t}, t, t_k\right)}, \end{aligned} \tag{36}$$

where $G = G(Y_{1t}, Y_{2t}, t, t_k)$ is

$$G = \left(\frac{A_{21}}{\lambda_1 - A_{22}} - 1\right)\mathbb{E}_t^Q\left[\int_t^{t_k} Y_{1s}ds\right] + \left(1 - \frac{A_{12}}{\lambda_2 - A_{11}}\right)\mathbb{E}_t^Q\left[\int_t^{t_k} Y_{2s}ds\right]$$
$$+\frac{1}{2}\left(\frac{A_{21}}{\lambda_1 - A_{22}} - 1\right)^2 Var_t^Q\left[\int_t^{t_k} Y_{1s}ds\right] + \frac{1}{2}\left(1 - \frac{A_{12}}{\lambda_2 - A_{11}}\right)^2 Var_t^Q\left[\int_t^{t_k} Y_{2s}ds\right]$$
$$+\left(\frac{A_{21}}{\lambda_1 - A_{22}} - 1\right)\left(1 - \frac{A_{12}}{\lambda_2 - A_{11}}\right)Cov_t^Q\left[\int_t^{t_k} Y_{1s}ds, \int_t^{t_k} Y_{2s}ds\right]. \tag{37}$$

In (36), we have used the property that for any normally distributed random variable $Z, \mathbb{E}[e^Z] = e^{\mathbb{E}(Z)+0.5Var(Z)}$. The second term in (3) is

$$\mathbb{E}_t^Q\left[Fe^{\int_u^{t_n} i_s ds} e^{-\int_t^{t_n} r_s ds}\right] = Fe^{\int_u^t i_s ds} e^{G(Y_{1t}, Y_{2t}, t, t_n)},$$

where G is given in (37). The third term in (3) is

$$\mathbb{E}_t^Q\left[e^{-\int_t^{t_n} r_s ds}\max\left(0, F - Fe^{\int_u^{t_n} i_s ds}\right)\right]$$
$$= Fe^{\int_u^t i_s ds}\mathbb{E}_t^Q\left[e^{-\int_t^{t_n} r_s ds}\left(e^{-\int_u^t i_s ds} - e^{\int_t^{t_n} i_s ds}\right)1_{\left\{-\int_u^t i_s ds > \int_t^{t_n} i_s ds\right\}}\right] \tag{38}$$

where $1_{\{\cdot\}}$ is the indicator function for the event in curly brackets. Eq. (38) involves two expectations, where each expectation is of the form

$$\mathbb{E}\left[e^{Z_1}1_{\{d>Z_2\}}\right], \tag{39}$$

where Z_1 and Z_2 are bivariate normal random variables and d is a constant. The joint distribution of Z_1 and Z_2 is characterized by $\mathbb{E}(Z_1), \mathbb{E}(Z_2), Var(Z_1), Var(Z_2)$, and $Cov(Z_1, Z_2)$. A direct calculation reveals that (39) is equal to

$$\mathbb{E}\left[e^{Z_1}1_{\{d>Z_2\}}\right] = e^{\mathbb{E}(Z_1)+\frac{1}{2}Var(Z_1)}N\left(\frac{d - \mathbb{E}(Z_2) - Cov(Z_1, Z_2)}{\sqrt{Var(Z_2)}}\right), \tag{40}$$

where $N(\cdot)$ is the standard normal cumulative distribution function. To analyze the first expectation in (38), we use (40) and we let

$$Z_1 = -\int_t^{t_n} r_s ds = -\int_t^{t_n} Y_{1s}ds - \left(\frac{A_{12}}{\lambda_2 - A_{11}}\right)\int_t^{t_n} Y_{2s}ds, \tag{41}$$

$$Z_2 = \int_t^{t_n} i_s ds = \left(\frac{A_{21}}{\lambda_1 - A_{22}}\right)\int_t^{t_n} Y_{1s}ds + \int_t^{t_n} Y_{2s}ds, \tag{42}$$

$$d = -\int_u^t i_s ds. \tag{43}$$

To analyze the second expectation in (38), we use (40) and we let

$$Z_1 = -\int_t^{t_n} r_s ds + \int_t^{t_n} i_s ds$$
$$= \left(\frac{A_{21}}{\lambda_1 - A_{22}} - 1\right)\int_t^{t_n} Y_{1s}ds + \left(1 - \frac{A_{12}}{\lambda_2 - A_{11}}\right)\int_t^{t_n} Y_{2s}ds,$$

where Z_2 and d are given by (42) and (43), respectively. Thus (38) depends on $\mathbb{E}_t^Q[\int_t^{t_n} Y_{1s}ds], \mathbb{E}_t^Q[\int_t^{t_n} Y_{2s}ds], Var_t^Q[\int_t^{t_n} Y_{1s}ds], Var_t^Q[\int_t^{t_n} Y_{2s}ds]$, and $Cov_t^Q[\int_t^{t_n} Y_{1s}ds, \int_t^{t_n} Y_{2s}ds]$, which are given above. This completes the derivation of the TIPS price in (3).

Appendix B. Pricing model for nominal Treasuries

We now derive the price of a nominal Treasury Note. Using Eq. (15), the first term in (5) can be written as

$$\sum_{k=1}^{n}\bar{c}F\,\mathbb{E}_t^Q\left[e^{-\int_t^{t_k} r_s ds}\right] = \sum_{k=1}^{n}\bar{c}F\,\mathbb{E}_t^Q\left[e^{-\int_t^{t_k} Y_{1s}ds - \left(\frac{A_{12}}{\lambda_2 - A_{11}}\right)\int_t^{t_k} Y_{2s}ds}\right]$$

Note that

$$\mathbb{E}_t^Q\left[e^{-\int_t^{t_k} Y_{1s}ds - \left(\frac{A_{12}}{\lambda_2 - A_{11}}\right)\int_t^{t_k} Y_{2s}ds}\right] = e^{H(Y_{1t}, Y_{2t}, t, t_k)},$$

where $H = H(Y_{1t}, Y_{2t}, t, t_k)$ is

$$H = -\mathbb{E}_t^Q\left[\int_t^{t_k} Y_{1s}ds\right] - \left(\frac{A_{12}}{\lambda_2 - A_{11}}\right)\mathbb{E}_t^Q\left[\int_t^{t_k} Y_{2s}ds\right]$$
$$+\frac{1}{2}Var_t^Q\left[\int_t^{t_k} Y_{1s}ds\right] + \frac{1}{2}\left(\frac{A_{12}}{\lambda_2 - A_{11}}\right)^2 Var_t^Q\left[\int_t^{t_k} Y_{2s}ds\right]$$
$$+\left(\frac{A_{12}}{\lambda_2 - A_{11}}\right)Cov_t^Q\left[\int_t^{t_k} Y_{1s}ds, \int_t^{t_k} Y_{2s}ds\right]. \tag{44}$$

Like Eq. (37), Eq. (44) uses the property that for any normally distributed random variable $Z, \mathbb{E}[e^Z] = e^{\mathbb{E}(Z)+0.5Var(Z)}$. Similarly, the second term in (5) is

$$\mathbb{E}_t^Q\left[Fe^{-\int_t^{t_n} r_s ds}\right] = Fe^{H(Y_{1t}, Y_{2t}, t, t_n)},$$

where the function $H(Y_{1t}, Y_{2t}, t, t_n)$ is obtained by substituting t_n for t_k in (44). This completes the derivation of the nominal Treasury Note price in (5).

Appendix C. Long-run means

In this section we show how to derive the long-run means and the speeds of mean reversion for r_t and i_t. We can rewrite (13) as $dX_t = -A\left(-A^{-1}a - X_t\right)dt + Bdz_t^Q$, where we define $\kappa = -A$ and $\pi = -A^{-1}a = [\pi_r\ \pi_i]^\top$. Upon substituting we get $dX_t = \kappa(\pi - X_t)dt + Bdz_t^Q$, which is a more traditional form. The long-run means are

$$\pi_r = \frac{a_2A_{12} - a_1A_{22}}{A_{11}A_{22} - A_{12}A_{21}}, \tag{45}$$

$$\pi_i = \frac{a_1A_{21} - a_2A_{11}}{A_{11}A_{22} - A_{12}A_{21}}. \tag{46}$$

Our empirical estimates for (45) and (46) are shown in Table 2.

Appendix D. Comparison to Wright (2009)

We can compare our results to Fig. 1 in Wright (2009), which shows the yields on two TIPS that have similar maturity dates but different issue dates. The two TIPS are the 1.875% 10-year TIPS with ISIN ending in 28BD1 and the 0.625% 5-year TIPS with ISIN ending in 28HW3. Wright's Fig. 1 shows that the 10-year TIPS yield is higher than the 5-year TIPS yield during the last few months of 2008 and the first half of 2009. Wright (2009) argues that the yield difference between these two TIPS is mostly due to differences in the deflation option value and not due to liquidity. In other words, the embedded deflation option in the 5-year TIPS is worth more than the embedded deflation option in the 10-year TIPS, which coincides with our summary statistics in Table 3. We verify the Wright (2009) conclusions for this pair of bonds by using our TIPS option pricing model. The results are shown in our Fig. A1. Panel A of Fig. A1 verifies Wright's Fig. 1, while Panel B of Fig. A1 shows the yield difference, which is the 10-year TIPS yield minus the 5-year TIPS yield. Panel C of Fig. A1 plots our estimated option values for these two TIPS, while Panel D of Fig. A1 shows the option value difference, which is the 5-year TIPS option value minus the 10-year TIPS option value. If we compare Panels B and D, we find that the option value difference closely tracks the yield difference. The biggest difference in yields and option values occurs in the Fall of 2008, which was a deflationary period. When we regress the yield difference in Panel B onto the option value difference in Panel D, we get an adjusted-R^2 of 81%. Thus for this pair of TIPS, we are able to confirm the Wright (2009) conjecture that the yield difference between the on-the-run and off-the-run TIPS is mostly due to different embedded option values.

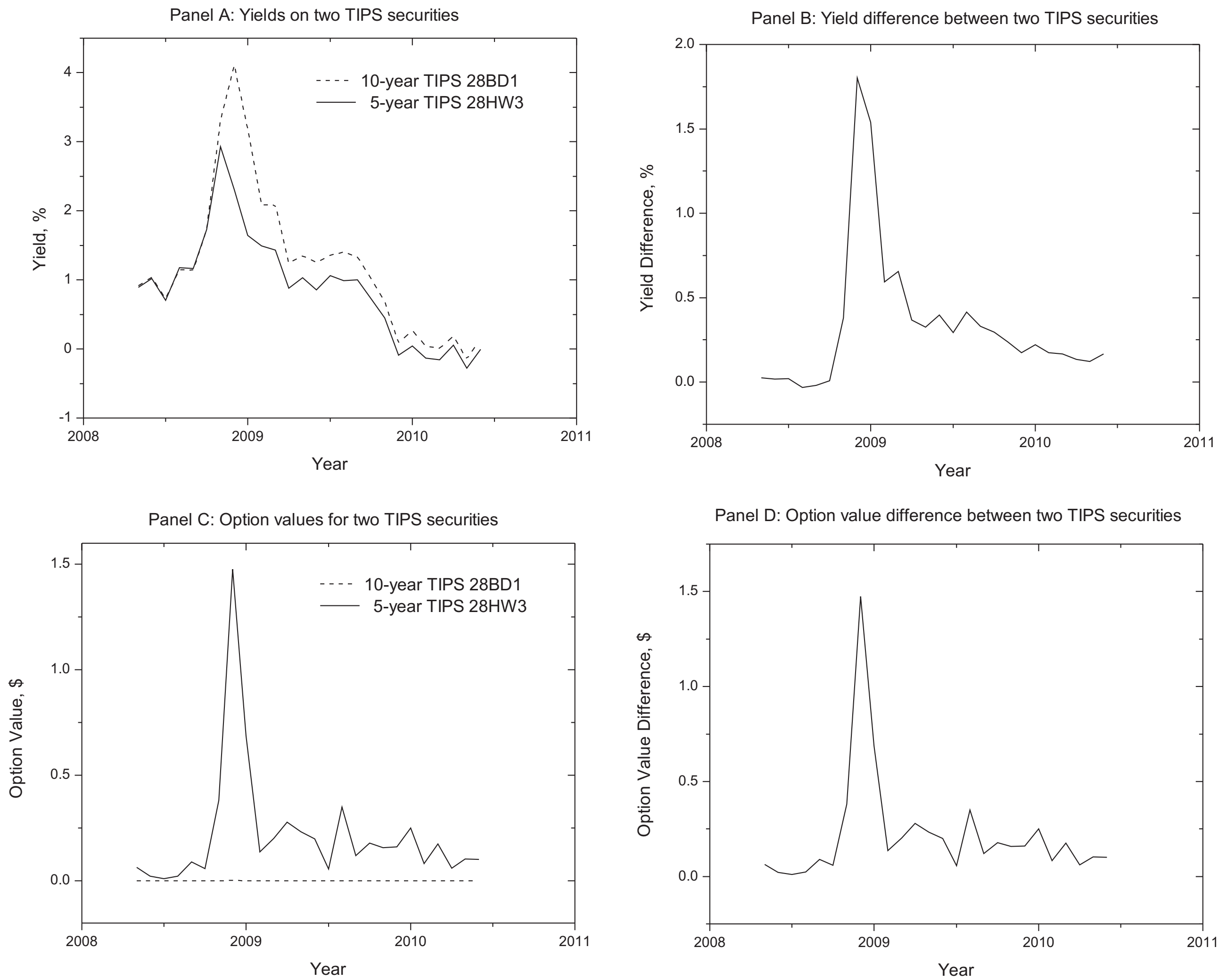

Fig. A1. Yield Difference versus option value difference. The figure shows the yield difference and the option value difference between two TIPS securities, the 10-year TIPS 28BD1 maturing on July 15, 2013 and the 5-year TIPS 28HW3 maturing on April 15, 2013. The yields and the yield difference are plotted in Panels A and B, respectively. The estimated option values and the option value difference are plotted in Panels C and D, respectively. The OLS regression of the yield difference on the option value difference generates an adjusted-R^2 of 81%. The model parameters are estimated by minimizing the equally-weighted sum of squared errors for both 5-year and 10-year TIPS and matching T-Notes. Sample period is January 1997–May 2010, monthly frequency.

We also examined the other TIPS in Panel C of Table 1. For 273A8 we were unable to find a matching 10-year TIPS. In addition, we discarded 28MY3 since we have only two observations. For the remaining four 5-year TIPS, we identified a 10-year TIPS with approximately five years remaining. This gave us four additional pairs of bonds. For each pair, we calculated the yield difference, as in Panel B of Fig. A1, and we calculated the option value difference, as in Panel D of Fig. A1. For each pair, we regressed the yield difference onto the option value difference. The average adjusted-R^2 of the four regressions was about 26%. Thus the yield difference is not always a good proxy for the option value difference. This is the reason why we prefer to estimate the embedded option values directly and then use these estimated values to construct our variables $OR_{t-1,t}$ and $OR_{t-12,t}$.

Appendix E. Alternative weighting schemes

In (10) and (11), we used value weights to construct the variables $OR_{t-1,t}$ and $OR_{t-12,t}$. To see if our results are sensitive to the choice of weights, we reconstruct these variables using other weighting schemes. We first construct weights based on maturity. Following Section 4.4, let N_t denote the number of TIPS in the sample in month t. Suppose the nth TIPS in month t has a remaining time to maturity V_{nt}, which is measured in years. We use V_{nt} to construct a set of maturity weights, where the weight assigned to the nth TIPS in month t is $W_{nt} = V_{nt}/\sum_{n=1}^{N_t} V_{nt}$. When we use these weights in (10) and (11), longer term options are assigned higher weights. We also construct weights that favor shorter term options. To do this, the weight assigned to the nth TIPS in month t is $W_{nt} = (K_n - V_{nt})/\sum_{n=1}^{N_t}(K_n - V_{nt})$, where K_n is the original maturity of the nth TIPS.

We also construct weights based on moneyness. Using (38) in Appendix A, the embedded option's strike price divided by the inflation-adjusted face value for the nth TIPS in month t is

$$M_{nt} = \frac{F}{Fe^{\int_u^t i_s ds}}, \tag{47}$$

where the exponential term in (47) is the inflation adjustment factor. As discussed in Section 2.1, we substitute the U.S. Treasury's CPI-U index ratio for the inflation adjustment factor. Thus M_{nt} in (47) describes the moneyness of the embedded option. The inflation rate in our sample is usually positive, so almost all of the embedded options are out-of-the-money. However, we use M_{nt} to construct weighting schemes that depend on the level of option moneyness. To favor nearer-to-the-money (NTM) options, the weight assigned to the nth TIPS in month t is $W_{nt} = M_{nt}/\sum_{n=1}^{N_t} M_{nt}$. Alternatively, to favor deeper out-of-the-money (OTM) options, the weight assigned to the nth TIPS in month t is $W_{nt} = (1 - M_{nt})/\sum_{n=1}^{N_t}(1 - M_{nt})$, where the number 1 represents an at-the-money option.

When we re-run the regressions in Panel A of Table 5 using each of the above weighting schemes, we find that our results are not altered. Our variable $OR_{t-1,t}$ maintains its significance in every regression. In addition, lagged inflation and the VIX return are also significant variables. When we re-run the regressions in Panel B of Table 5 using each of above weighting schemes, our results are unaltered for the sample of 10-year TIPS and for the combined sample of 5-year and 10-year TIPS. However, for each alternative weighting scheme, $OR_{t-12,t}$ is insignificant for the sample of 5-year TIPS. As we mentioned in Section 4.2, our sample of 5-year TIPS is relatively small and it appears that this is impacting our results. With the exception of the 12-month ahead regression using 5-year TIPS, it appears that our results in Table 5 are robust to the type of weighting scheme that is used to construct the variables $OR_{t-1,t}$ and $OR_{t-12,t}$.

Appendix F. Additional robustness tests

To check the validity of our results, we performed several additional robustness tests. First, we examined the impact of other explanatory variables, such as the interest rate, the lagged interest rate, and an additional lag of the inflation rate. When we added each of these variables to panels A and B in Table 5, we found that none were statistically significant. For example, when we added the current interest rate to Panel A of Table 5, the t-statistic for the coefficient was less than 0.48 and our option return variable was not impacted.

We also added the option price index OP_t as an explanatory variable in our tables. In the presence of $OR_{t-1,t}$ ($OR_{t-12,t}$), the option price index OP_t was not statistically significant for explaining the one-month (one-year) ahead inflation rate. Furthermore, the inclusion of OP_t did not alter the statistical significance of $OR_{t-1,t}$ and $OR_{t-12,t}$ relative to what is shown in our tables.

We also performed joint tests to determine which subset of variables is most important for explaining future inflation. For the multivariate regressions in Panel A of Table 5, we rejected the null hypothesis that all of the betas are zero. The p-values for this test using 5-year TIPS, 10-year TIPS, or the combined sample of 5-year and 10-year TIPS were all less than 0.0001. We next tested the null hypothesis that all of the betas except the beta on lagged inflation are zero. For this test, the p-values using 10-year TIPS or the combined sample of 5-year and 10-year TIPS were both less than 0.01, while the p-value using 5-year TIPS was 0.008. Thus it appears that lagged inflation by itself does not tell the whole story. Additional joint tests revealed that three explanatory variables at most are needed to describe one-month ahead inflation. We cannot reject the null that one-month ahead inflation is explained by lagged inflation, the VIX return, and the option return index $OR_{t-1,t}$.

We also performed joint tests for Panel B of Table 5. For the sample of 5-year TIPS and matching nominal bonds, we cannot reject the null hypothesis that 12-month ahead inflation is explained by at most three variables. The variables are the yield spread YS_t, the TIPS return $BondRet_{t-12,t}$, and the option return index $OR_{t-12,t}$. For the sample of 10-year TIPS and matching nominal bonds, and for the combined sample of 5-year and 10-year TIPS and matching nominal bonds, we also cannot reject the null hypothesis that 12-month ahead inflation is explained by at most three variables. For these samples, the variables are the market-wide liquidity Liq_t, the TIPS return $BondRet_{t-12,t}$, and the option return index $OR_{t-12,t}$.

We also used our embedded option pricing model to calculate the Greeks for each embedded option. We estimated the partial derivative of the option value with respect to each of the nine model parameters and we also estimated the partial derivative with respect to the interest rate and the inflation rate. We found that the magnitudes of the Greeks increase during periods of deflationary expectations. This fact coincides with our option pricing intuition. For example, in the Black–Scholes model, the magnitudes of delta and vega are small when the option is deep out-of-the-money, but these Greeks are larger when the option is only slightly out-of-the-money. Our embedded option pricing model has the same feature. During periods of deflationary expectations, such as the price spike periods in Fig. 1, the embedded options are closer-to-the-money and the Greeks tend to increase in magnitude.

Our additional robustness tests suggest that the embedded option's nonlinearity is a contributing factor for explaining the future inflation rate. The embedded option return is negatively related to future inflation and the magnitude of this relationship increases during deflationary periods. This type of nonlinearity is not captured in our tables by using only linear variables, such as lagged inflation, the interest rate level, or the yield spread. The nonlinearity is most pronounced for those TIPS that have lower cumulative inflation. In particular, during the price spike periods in Fig. 1, we verified that the on-the-run TIPS have bigger Greeks than the off-the-run TIPS. However, even when we use a weighting scheme that emphasizes higher cumulative inflation, such as the shorter time-to-maturity options in Appendix E, we find that our embedded option return index is useful for explaining inflation. As a final check, we recalculated Table 6 using only off-the-run securities. Even though the off-the-run securities have smaller Greeks and lower liquidity than the on-the-run securities, our conclusions still hold.

Lastly, we investigated how the prices of inflation floors correlate with our value for the TIPS embedded option. Data for inflation floors are available on Bloomberg starting in October 2009. Although the overlap with our full sample period is short, we found that the prices of inflation floors tend to track our embedded option values in Fig. 1.

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