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Product Market Competition, Managerial Compensation, and Firm Size in Market Equilibrium

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We develop a tractable equilibrium model of competing firms in an industry to show how the distribution of firm qualities, moral hazard, and product market characteristics interact to affect firm size, managerial compensation, and market structure. Different determinants of product market competition have contrasting effects on firm size and managerial compensation. Although both firm size and managerial compensation increase with the entry cost, they increase with the elasticity of substitution if and only if firm size exceeds a high threshold but decrease if it is below a low threshold. Aggregate shocks to the firm productivity distribution affect incentives in our equilibrium framework. We show statistically and economically significant empirical support for several hypotheses derived from the theory that relates product market characteristics to managerial compensation, firm size, and the number of firms in the industry. Different determinants of competition indeed have contrasting effects, as predicted by the theory.

Key words: product market competition; managerial compensation; firm size; market equilibrium
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1. Introduction

A growing body of literature in industrial organization and finance examines how product market characteristics affect firm profitability and managerial compensation. Although there is broad consensus that product market competition influences managerial decisions and firm profitability, the manner in which it does so is still vigorously debated. In particular, it is an open question how different dimensions of product market competition such as the entry cost and the elasticity of substitution between *heterogeneous* products affect firm size, managerial compensation, and market structure. We build a tractable, market equilibrium model that incorporates imperfect competition among heterogeneous firms and agency conflicts due to moral hazard. We show that different determinants of product market competition—the entry cost and the elasticity of substitution between differentiated products—have sharply contrasting effects on firm size, managerial compensation, and market structure. Equilibrium effects play a central role in generating the differing impacts. We show significant support for the predicted effects of the entry cost and the elasticity of substitution in our empirical analysis. Our results suggest that in analyses of the effects of competition, it is crucial to distinguish between its different dimensions.

Our model incorporates moral hazard in a framework with a continuum of monopolistically competitive firms in an industry. Production is driven by labor provided by workers and specialized human capital (effort) provided by managers. A group of entrepreneurs establishes a firm by supplying sunk labor, after which the *quality* of the firm, which affects its productivity, is realized. Firm quality is drawn from a known distribution. The risk-neutral entrepreneurs then hire a manager to operate the firm and offer the risk-averse manager an incentive contract to induce costly effort. The firm subsequently experiences an idiosyncratic productivity shock that depends on the managers' effort and the firm's quality. Higher effort increases the likelihood of a favorable productivity shock, and there is complementarity between effort and the firm's quality. Firms are monopolistically competitive in that they take the *aggregate price index*—the weighted average of product prices—as given when they make their output and pricing decisions (Dixit and Stiglitz 1977).

We build a general equilibrium model in which the aggregate revenue equals the aggregate payoffs to all agents: entrepreneurs, managers, and workers (e.g., Melitz 2003). With the labor wage rate as the numeraire, the aggregate revenue is, therefore, determined by the total population of agents. The distribution of firms' productivities affects the endogenous mass of firms (therefore, the variety of products) and the allocation of the aggregate revenue across firms. Equilibrium is characterized by a mass of firms and incentive contracts for managers. There exists a unique equilibrium in which the mass of firms is determined by the condition that the aggregate revenue equals the aggregate payoffs to entrepreneurs, managers, and workers. Furthermore, with the free entry of firms, the net present value of the profit of each firm must equal the entry cost. The aggregate price index, which is endogenously determined by the equilibrium conditions, influences managerial compensation because it affects the marginal product of managerial effort. Our analysis, therefore, highlights an important *equilibrium* channel through which firm and product market characteristics influence managerial compensation by affecting the aggregate price index.

Firm size and managerial compensation are endogenously determined by the firm's realized quality. The complementarity between firm quality and effort ensures that firm size (measured in terms of expected revenue, gross/net profit), effort, and expected managerial compensation increase with firm quality. A manager with a higher-quality match has a *lower* pay-performance sensitivity—the sensitivity of the manager's dollar compensation to the firm's gross profit—*provided* the manager's utility function satisfies an additional elasticity condition. The condition ensures that the elasticity with respect to effort of the manager's compensation in the high state is increasing.

We investigate the effects of different determinants of the intensity of product market competition—the entry cost and the elasticity of substitution—on market structure, firm size, and managerial compensation. An increase in the intensity of competition due to a decline in the entry cost decreases the expected size of a firm of given quality, lowers its manager's expected compensation, and increases the mass of competing firms. The entry cost influences managerial compensation *purely* through the equilibrium channel by affecting the aggregate price index. A decrease in the entry cost increases competition and correspondingly lowers the aggregate price index. The decline in the aggregate price index decreases the marginal product of managerial effort so that effort, expected managerial compensation, and firm size decrease. Because the average expected revenue of firms decreases and aggregate revenue is fixed, the endogenous mass of producing firms increases.

The elasticity of substitution, however, has markedly different effects from the entry cost. An increase in the elasticity of substitution causes expected managerial compensation and firm size to decrease if firm quality is below a low threshold but increase if it is above a high threshold. Because firm size increases with firm quality, our results imply that a more elastic product market benefits large firms but is detrimental to small firms. The intuition for these results hinges on an increase in the elasticity of substitution affecting managerial effort and output differentially. It increases the marginal product of managerial effort relatively more at higher-quality firms than lower-quality firms. Because of the differential effects of the elasticity of substitution on the outputs of high- and low-quality firms, it has ambiguous effects on average firm size and the mass of firms.

Empirical studies often use the entry cost and the elasticity of substitution as alternate measures of product market competition. Our results show that they have strikingly different effects on market structure, firm size, and managerial compensation. The contrasting effects arise because the entry cost influences these variables *indirectly* by affecting the aggregate price index, that is, *solely* through the equilibrium channel. In contrast, the elasticity of substitution affects these distributions both directly through its effects on managerial effort and output as well as indirectly by influencing the aggregate price index. The contrasting effects suggest that empirical analyses of the effects of competition should appropriately account for different dimensions of competition.

We also derive additional implications for the effects of productivity risk and the firm-quality distribution. Insofar as shocks to the firm-quality distribution could be viewed as aggregate shocks, our results show that managerial incentives are affected by aggregate shocks through their effects on the aggregate price index, that is, via the equilibrium channel. This prediction contrasts sharply with that of traditional partial equilibrium principal–agent models in which incentives are only affected by idiosyncratic shocks.

We develop testable hypotheses that relate the entry cost and the elasticity of substitution to managerial compensation levels, firm size, and the number of firms in the industry. We show significant support for the hypotheses in our empirical analysis. The effects of the entry cost and the elasticity of substitution are also economically significant. For example, a one-standard-deviation increase in the (log of the) entry cost increases managerial compensation by 19% and firm size by 56%. A one-standard-deviation increase in our empirical measure of the elasticity of substitution (the negative price-cost margin) *increases* the compensation of managers of large firms by 4% and firm size by 11% but *decreases* the compensation of managers of small firms by 7% and firm size by 15%. The economically significant effects of the entry cost and the elasticity of substitution on managerial compensation and firm size suggest that they are important determinants of the substantial cross-industry variations in managerial compensation and firm size.

We contribute to the literature by developing a tractable, market equilibrium model that incorporates imperfect (specifically, monopolistic) competition among heterogeneous firms and manager-firm agency conflicts due to moral hazard. A number of studies use different *partial equilibrium* models to examine the link between product market competition, managerial slack, and pay-performance sensitivity. Although Hart (1983) finds that competition reduces managerial slack and increases incentives, Scharfstein (1988) shows that Hart's result can be reversed when the manager's utility function takes a more general form. Hermalin (1992) finds that the effect of competition on managerial slack and incentives is ambiguous using a hidden action model. Raith (2003) makes the important point that the above studies obtain ambiguous effects of competition on incentives because they develop partial equilibrium frameworks in which the market structure is exogenous. Baranchuk et al. (2011) build a framework with heterogeneous, perfectly competitive firms and show that managerial compensation increases with firm size, whereas the pay-performance sensitivity (PPS) declines.

We complement the aforementioned studies in several respects. In Raith (2003), all firms are identical so that there is no intraindustry variation in firm size or managerial compensation. We complement Baranchuk et al. (2011) by examining the effects of *imperfect* product market competition arising from nonzero entry costs and imperfect product substitutability. In Raith's (2003) model, managerial PPS and average managerial compensation increase with the entry cost and product substitutability. In contrast, in our model, expected managerial compensation increases with the entry cost, but PPS decreases, which is consistent with the empirical evidence in Karuna (2007). The effects of product substitutability, however, crucially depend on firm size because, in our model, heterogeneous firms engage in monopolistic competition. Consequently, product substitutability has both direct and indirect effects, whereas it only has indirect effects in Raith's (2003) model.

Gabaix and Landier (2008) and Tervio (2008) quantitatively explain the positive association between firm size and managerial compensation using competitive assignment models with an *exogenous* mass of firms. We can extend our model to allow for endogenous matching between heterogeneous managers and firms (details available). Although the extended model can quantitatively account for managerial compensation levels, the key implications of our study for the effects of product market characteristics on the distributions of firm size and managerial compensation are unaffected. Edmans et al. (2009) extend the model of Gabaix and Landier (2008) and show a negative relation between PPS and firm size. Edmans and Gabaix (2011) build on the model of Gabaix and Landier (2008) by incorporating risk aversion and moral hazard. They show that talent assignment is distorted by agency conflicts. The above studies also differ from ours in that they analyze partial equilibrium frameworks that abstract away from the effects of product market competition that is the main focus of our study.

2. The Model

We model an industry with a continuum of monopolistically competitive firms (Dixit and Stiglitz 1977). There are three time periods with four dates 0, 1, 2, and 3. At date 0, a group of entrepreneurs establishes a firm. The quality of the firm, which determines its productivity, is realized at date 1. At date 1, after the quality of the firm is observed, the entrepreneurs hire a manager to operate the firm. They offer the manager an incentive contract to induce costly effort by the manager. An idiosyncratic productivity shock whose distribution depends on the firm quality and the manager's effort is realized at date 2. The firm makes output and pricing decisions at date 2 after its productivity is realized. All payoffs occur at the terminal date 3. Figure 1 shows the timeline of events in the model. We now describe the various elements of the model in detail.

2.1. Preference

There is a continuum of ex ante identical agents in the economy. The representative consumer has preferences for consumption at date 3 defined over a continuum of industry goods (indexed by ω) that are described by

$$U = \left[\int_{\Omega} q(\omega)^{\rho}\, d\omega\right]^{1/\rho}, \quad 0 < \rho < 1, \tag{1}$$

where Ω is the set of available industry goods and ω is a finite measure on the Borel σ-algebra of Ω. If $p(\omega)$ is the price of good ω then, as shown by Dixit and Stiglitz (1977), the optimal consumption and expenditure decisions for individual goods are

$$q(\omega) = U\left[\frac{p(\omega)}{P}\right]^{-\sigma}, \quad r(\omega) = R\left[\frac{p(\omega)}{P}\right]^{1-\sigma}, \tag{2}$$

where

$$R = PU \tag{3}$$

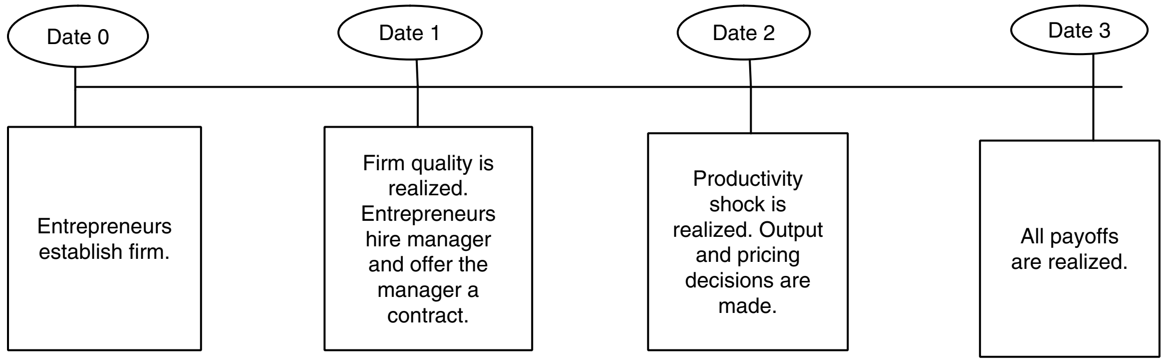
is the aggregate expenditure of the representative consumer and

$$P = \left[\int_{\Omega} p(\omega)^{1-\sigma}\, d\omega\right]^{1/(1-\sigma)} \tag{4}$$

is the aggregate price index (see Dixit and Stiglitz 1977). In (4), the elasticity of substitution is

$$\sigma = \frac{1}{1-\rho} > 1. \tag{5}$$

Figure 1 **Model Timeline**



Firms compete monopolistically in the sense that each firm produces a single product in which it enjoys a monopoly, but firms take the aggregate price index P as given in making the output and pricing decisions for their individual products. Each firm faces the price elasticity of demand σ. Given that there is a continuum of firms, no single firm perceives itself as having an impact on aggregate equilibrium outcomes.

2.2. Firm Formation and Production

There is an unbounded pool of prospective entrants and firms are identical prior to entry. Production is driven by labor supplied by workers and specialized human capital (effort) supplied by managers. If an agent is hired as a worker, he or she inelastically supplies one unit of labor. The labor wage rate is w; that is, a worker receives a wage of w if she is hired by a firm and 0 otherwise.

At date 0, firms are created by groups of entrepreneurs who supply sunk "investment labor" $f_e > 0$ (see Melitz 2003). The quality α of the firm, which determines its productivity, is realized at date 1. Firm qualities are drawn from a known distribution $g(\cdot)$ that has positive support over $(0, \infty)$ and a continuous cumulative distribution $G(\cdot)$. After the firm's quality is realized at date 1, the entrepreneurs hire a manager. Because agents are ex ante identical, every agent has the same probability of becoming a manager. Agents who are not hired as managers could either be hired as production workers or remain unemployed.

To simplify the analysis, we assume that the manager of a firm also supplies one unit of labor in addition to specialized human capital or "effort." Ours is a general equilibrium model in which the aggregate revenue equals the aggregate payoffs to all agents, that is, entrepreneurs, managers, and workers (e.g., Melitz 2003). We normalize the labor wage rate to 1; that is, the labor wage rate is the numeraire with respect to which all payoffs are measured. Consequently, the aggregate revenue is determined by the total population of agents, which could be interpreted as the size of the market. Firms' qualities and productivity shocks determine the endogenous mass of active firms (therefore, the variety of products) and the allocation of the aggregate revenue across the active firms. As is typical in models of monopolistic competition, consumer welfare increases with the *variety* of products in the economy.

The manager of each firm receives a contract from the firm's owners (the entrepreneurs) at date 1 and then exerts effort over the second period. The firm's productivity is realized at date 2. The firm's realized productivity, which determines its marginal cost of production, depends stochastically on the firm's quality and the manager's effort. If the manager exerts effort $e \in [0, 1]$ in the second period, then the firm's realized productivity ϕ at date 2 is a random variable that takes on two possible values: αh and αl with probabilities e and $1 - e$, respectively, where $h > 1$, $l < 1$. Productivity shocks are independent across firms; that is, they are idiosyncratic.

Production decisions are made *after* the productivity ϕ is realized. Each firm has a constant marginal cost (measured in units of labor). The labor used by a firm is therefore a linear function of output and is given by

$$x = \frac{q}{\phi}. \tag{6}$$

Because the labor wage rate is the numeraire, the optimal price $p(\phi)$ set by the firm is[1]

$$p(\phi) = \frac{1}{\rho\phi}. \tag{7}$$

Furthermore, the firm's revenue after its realized productivity ϕ is given by

$$r(\phi) = R(P\rho\phi)^{\sigma-1}. \tag{8}$$

[1] In equilibrium of our model, the respective payoffs of the entrepreneurs (hereafter, alternately referred to as the *firm*) and the manager both increase with the firm's gross profits (inclusive of managerial compensation). Consequently, the firm's optimal output and/or pricing decisions maximize its gross profits.

The firm's gross profit is

$$\pi(\phi) = \frac{r(\phi)}{\sigma} = \frac{R(P\rho\phi)^{\sigma-1}}{\sigma}. \tag{9}$$

2.3. Managerial Preferences and Contracts

Because the ex post productivity ϕ is observable and is the only source of randomness, we can, without loss of generality, assume that each manager's contractual compensation is contingent on the realized productivity. Each manager is risk averse with respect to her monetary payoffs in the "high" and "low" productivity states. Recall that the manager supplies one unit of labor, for which she receives a wage of 1, in addition to exerting effort. We denote the manager's compensation *in excess* of the labor wage by $t(\phi)$; that is, the manager receives a total payoff of $1+t(\phi)$ if the firm's realized productivity is ϕ.

Because managers and firms do not internalize the effects of their decisions on the aggregate economy, a manager's compensation contract is only contingent on the realized productivity of her firm. For generality, we allow for the manager to be risk averse with respect to *productivity risk* described by a von Neumann–Morgenstern utility function $\tilde{u}$. More precisely, from an ex ante perspective, if the manager exerts effort e, her total expected utility from a given compensation contract $t(\cdot)$ is

$$\begin{aligned} &E_{\alpha,e}[\tilde{u}(1+t(\phi))] - \kappa(e) \\ &\quad = e\tilde{u}(1+t(\alpha h)) + (1-e)\tilde{u}(1+t(\alpha l)) - \kappa(e). \end{aligned} \tag{10}$$

Define

$$u(x) = \tilde{u}(1+x) - \tilde{u}(1). \tag{11}$$

Hereafter, we simply refer to u as the manager's utility function that is strictly increasing, concave, and twice continuously differentiable, whereas $\kappa(\cdot)$ is strictly increasing, convex, and thrice continuously differentiable with $\kappa(0) = \kappa'(0) = 0$. In addition, we assume that

$$u(\infty) = \infty, \quad \kappa'(1) = \infty, \quad \kappa'''(\cdot) > 0. \tag{12}$$

The first two conditions above ensure that the manager's effort takes interior values, which simplifies the statements of our results. As we see later, the third condition is sufficient to ensure that managerial compensation increases with firm size, which is a very well documented empirical regularity. Let

$$v(\cdot) \equiv u^{-1}(\cdot) \tag{13}$$

denote the inverse of the manager's utility function.

As in the traditional principal–agent literature, it is convenient to augment the definition of a manager's contract to also include her effort. A contract $(t(\cdot), e)$ is incentive compatible for the manager if and only if

$$e = \arg\max_{\tilde{e}} E_{\alpha,\tilde{e}}[u[t(\phi)]] - \kappa(\tilde{e}). \tag{14}$$

A contract $(t(\cdot), e)$ is *feasible* if and only if it satisfies the following constraints for the manager that ensure that the manager receives at least the labor wage of 1 in each state:[2]

$$t(\alpha h) \geq 0, \quad t(\alpha l) \geq 0. \tag{15}$$

The firm chooses an incentive feasible contract for the manager to maximize its expected *net* profit, that is, its expected gross profit less the manager's compensation. The manager's optimal contract $(t_\alpha(\cdot), e_\alpha)$ (the subscript denotes dependence on the firm quality) therefore solves

$$\begin{aligned} (t_\alpha(\cdot),\, e_\alpha) &= \arg\max_{(t(\cdot),\, e)} E_{\alpha,e}[\pi(\phi) - t(\phi)] \\ &= \arg\max_{(t(\cdot),\, e)} E_{\alpha,e}\left[\frac{R(P\rho\phi)^{\sigma-1}}{\sigma} - t(\phi)\right], \end{aligned} \tag{16}$$

subject to the implementability constraint (14) and the constraints (15). The second equality in (16) follows from the expression (9) for gross profit.[3]

2.4. Equilibrium Characterization

An equilibrium is characterized by a mass M of producing firms (and hence M products), a distribution $\mu(\alpha)$ of firm qualities over $(0, \infty)$, an aggregate price index P^*, and a contract $(t^*_\alpha(\cdot), e^*_\alpha)$ for the manager with firm quality α. Note that there is a continuum of firms and the productivity shocks are independent across firms. The aggregate price index defined by (4) is

$$\begin{aligned} P^* &= \left[\int_0^\infty E_{\alpha,e^*_\alpha}(p(\phi)^{1-\sigma})\, M\mu(\alpha)\,d\alpha\right]^{1/(1-\sigma)} \\ &= \left[\int_0^\infty (e^*_\alpha p(\alpha h)^{1-\sigma} + (1-e^*_\alpha)p(\alpha l)^{1-\sigma}) \right. \\ &\qquad \left. \cdot M\mu(\alpha)\,d\alpha\right]^{1/(1-\sigma)}. \end{aligned} \tag{17}$$

[2] It follows from (11) and (15) that a feasible contract guarantees the manager a reservation expected utility payoff of zero. We can modify the model to allow for managers to have nonzero reservation utilities without altering our main implications. Managerial compensation would simply have a lower bound that depends on the reservation utility.

[3] In the basic model described above, managers are ex ante homogeneous and heterogeneity is generated by the realized firm qualities. We can extend the model to incorporate ex ante two-sided heterogeneity in managers and firms and *endogenous matching* between firms and managers. In the extended model, the firm quality is replaced by the manager-firm "match quality" that is the composition of manager-specific and firm-specific characteristics. The main implications of the basic model remain robust to the extended model.

In (17), the expectation appears inside the integral by the (generalized) law of large numbers for a continuum of firms. The aggregate revenue (or expenditure) and profit are given by

$$R = \int_0^\infty E_{\alpha,\, e_\alpha^*}(r(\phi))\, M\mu(\alpha)\, d\alpha, \tag{18}$$

$$\Pi = \int_0^\infty E_{\alpha,\, e_\alpha^*}(\pi(\phi))\, M\mu(\alpha)\, d\alpha, \tag{19}$$

where $r(\cdot)$ and $\pi(\cdot)$ are given by (8) and (9), respectively. From (18), we see that the firms' productivity distribution affects the endogenous mass M of active firms (hence, products) and the intraindustry allocation of the aggregate revenue R across the active firms.

A firm with quality α earns expected gross profit of $E_{\alpha,e^*}[\pi(\phi)]$, where e^* is the effort exerted by the manager. The firm produces if and only if its value is nonnegative. Firm value is therefore

$$\begin{aligned} b(\alpha) &= \max\{0, E_{\alpha,\, e_\alpha^*}[\pi(\phi) - t_\alpha^*(\phi)]\} \\ &= \max\{0, \hat{\xi}(\alpha)\}, \end{aligned} \tag{20}$$

where

$$\begin{aligned} \hat{\xi}(\alpha) &= E_{\alpha,\, e_\alpha^*}[\pi(\phi) - t_\alpha^*(\phi)] \\ &= e_\alpha^*[\pi(\alpha h) - t_\alpha^*(\alpha h)] + (1 - e_\alpha^*)[\pi(\alpha l) - t_\alpha^*(\alpha l)] \end{aligned} \tag{21}$$

is the firm's *expected net profit* (net of the manager's compensation). As will be clear shortly, a firm's expected net profit is nonnegative so that a firm produces regardless of its realized quality. Consequently,

$$\mu(\alpha) = g(\alpha). \tag{22}$$

3. Equilibrium

In equilibrium with free entry of firms, the expected net profit earned by entering firms must equal the entry cost f_e:

$$\int_0^\infty b(\alpha) g(\alpha)\, d\alpha = \int_0^\infty \hat{\xi}(\alpha) g(\alpha)\, d\alpha = f_e. \tag{23}$$

Define the equilibrium *expected* compensation of a manager whose firm's quality is α

$$\hat{t}(\alpha) = e_\alpha^* t_\alpha^*(\alpha h) + (1 - e_\alpha^*) t_\alpha^*(\alpha l), \tag{24}$$

the equilibrium expected revenue of the firm

$$\hat{r}(\alpha) = e_\alpha^* r(\alpha h) + (1 - e_\alpha^*) r(\alpha l), \tag{25}$$

and the equilibrium *expected* gross profit of the firm

$$\begin{aligned} \hat{\pi}(\alpha) &= e_\alpha^* \pi(\alpha h) + (1 - e_\alpha^*) \pi(\alpha l) \\ &= \frac{e_\alpha^* r(\alpha h) + (1 - e_\alpha^*) r(\alpha l)}{\sigma} = \frac{\hat{r}(\alpha)}{\sigma}, \end{aligned} \tag{26}$$

where the second equality follows from (9). By (21) and (23), we have

$$\int_0^\infty \frac{\hat{r}(\alpha)}{\sigma} g(\alpha)\, d\alpha - \int_0^\infty \hat{t}(\alpha) g(\alpha)\, d\alpha = f_e. \tag{27}$$

By (9), (21), (24), and (26), the equilibrium expected net profit of a firm with firm quality α is

$$\hat{\xi}(\alpha) = \hat{\pi}(\alpha) - \hat{t}(\alpha). \tag{28}$$

We first derive each manager's optimal contract for a *given* aggregate price index P in the following lemma, whose proof we omit for brevity.

LEMMA 1 (MANAGERIAL CONTRACTS). *Consider a firm with quality α. Suppose the aggregate price index is P. The optimal contract for the firm's manager is described as follows.* (i) *The incentive compensation of the manager in the low and high states is*

$$t_\alpha(\alpha l) = 0, \quad t_\alpha(\alpha h) = v[\kappa'(e_\alpha)], \tag{29}$$

where v is the manager's inverse utility function defined in (13). (ii) *The manager's effort e_α solves*

$$\begin{aligned} &\alpha^{\sigma-1}\left[\frac{R(P\rho)^{\sigma-1}}{\sigma}(h^{\sigma-1} - l^{\sigma-1})\right] \\ &\quad = v[\kappa'(e_\alpha)] + e_\alpha v'[\kappa'(e_\alpha)]\kappa''(e_\alpha). \end{aligned} \tag{30}$$

(iii) *The manager's dollar pay-performance sensitivity is*

$$\begin{aligned} \mathrm{PPS}(\alpha) &= \frac{t_\alpha(\alpha h) - t_\alpha(\alpha l)}{\pi(\alpha h) - \pi(\alpha l)} \\ &= \frac{v[\kappa'(e_\alpha)]}{v[\kappa'(e_\alpha)] + e_\alpha v'[\kappa'(e_\alpha)]\kappa''(e_\alpha)}. \end{aligned} \tag{31}$$

As in standard moral hazard models, the manager's optimal incentive compensation and effort depend on the difference between gross profit in the high and low states. The manager's effort and compensation depend on the aggregate price index P because the revenue and profit in each productivity state depend on it by (8) and (9); that is, the marginal product of managerial effort is affected by the aggregate price index. The effect of the aggregate price index on managerial effort leads to an equilibrium link between firm and product market characteristics and managerial compensation. By (31), we define the PPS as the ratio of the *dollar difference* in the manager's compensation in the high and low states to the *dollar difference* in the firm's output. The following lemma establishes the existence and uniqueness of equilibrium.

LEMMA 2 (EXISTENCE AND UNIQUENESS OF EQUILIBRIUM). *There exists a unique equilibrium where the aggregate price P^* satisfies the following equation*:

$$\begin{aligned} \int_0^\infty \bigg[&\frac{R(P^*\rho\alpha)^{\sigma-1}}{\sigma}[e_\alpha(P^*)h^{\sigma-1} + (1 - e_\alpha(P^*))l^{\sigma-1}] \\ &- e_\alpha(P^*) v[\kappa'(e_\alpha(P^*))]\bigg] g(\alpha)\, d\alpha = f_e. \end{aligned} \tag{32}$$

In the above, the argument of the manager's effort explicitly indicates its dependence on the aggregate price. The manager's equilibrium contracts $(t_\alpha^*(\cdot), e_\alpha^*)$ *are described by Lemma* 1 *where the aggregate price index is* P^*.

The equilibrium condition (32) ensures that each entering firm's expected future profit, which rationally incorporates each manager's contract and effort, is equal to the entry cost.

The mass M of firms is endogenously determined by the condition that the aggregate revenue of producing firms must equal the aggregate expenditure R of the representative consumer. By (18), the mass of firms M is given by

$$M = \frac{R}{\int_0^\infty E_{\alpha, e_\alpha^*}(r(\phi))\, g(\alpha)\, d\alpha} = \frac{R}{\int_0^\infty \hat{r}(\alpha)\, g(\alpha)\, d\alpha} = \frac{R}{\bar{r}}, \tag{33}$$

where we recall that $\hat{r}(\alpha)$ is the expected revenue of a firm with firm quality α and $\bar{r}$ is the average expected revenue of firms. The mass of firms therefore declines with the average expected revenue of firms.

4. Results and Implications

We now derive our main results by analyzing the properties of the equilibrium. To highlight the key mechanisms driving the results, we compare them with the corresponding results obtained from two benchmark models: (i) a *partial equilibrium* model in which the aggregate price index P and the mass of firms M are both exogenous so that the free entry condition (27) and the market clearing condition (33) are not satisfied in general and (ii) a *pure I/O* model in which we shut down the role of managerial effort so that output is entirely driven by production labor; that is, there is no distinction between managers and workers. We use the following terminology. The *average value* of a firm-specific variable refers to the average across all firms computed with respect to the firm-quality distribution $g(\cdot)$. The *expected* value of a variable that is contingent on the realized value of a firm's productivity shock is its expectation computed with respect to the distribution of the firm's productivity shocks. The *average expected value* of a firm-specific variable that is contingent on a firm's realized productivity is computed by taking its expected value for each firm and then determining its average across all firms.

4.1. Distributions of Firm Size and Managerial Compensation

There are two factors that contribute toward firm heterogeneity, which in turn generates a firm-size distribution and a distribution of managerial compensation (levels and incentives): firm quality and the productivity shocks. Productivity shocks affect the distributions indirectly through their effects on managerial effort.

Proposition 1 (Firm Quality, Firm Size, and Compensation). (i) *Managerial effort, expected managerial compensation, expected revenue, expected gross and net profit, and the expected labor force increase monotonically with firm quality* α. (ii) *Pay-performance sensitivity declines with firm quality if and only if*

$$F(e) = \frac{e v'[\kappa'(e)]\kappa''(e)}{v[\kappa'(e)]} \text{ is monotonically increasing in effort } e. \tag{34}$$

Because firm quality and managerial effort are complements, managers of higher-quality firms exert greater effort; generate greater expected revenues, gross, and net profits; and receive greater expected compensation. Consequently, firm size (measured in terms of expected revenue, gross/net profits, or the labor force) increases with firm quality. The function $F(e)$ is the elasticity with respect to effort of the agent's compensation in the high state. Because effort increases with firm quality, a greater elasticity with respect to effort for a manager of a higher-quality firm ensures that she has a *lower* pay-performance sensitivity. It is easy to show that the condition (34) holds if the utility function $u(\cdot)$ is constant relative risk aversion and the effort cost function $\kappa(\cdot)$ has a power or exponential form.

The predicted positive relation between the managerial compensation level and firm size and negative relation between managerial PPS and firm size are consistent with empirical evidence (Schaefer 1998, Baker and Hall 2004). The relations are, however, not causal because firm size and managerial compensation are endogenously determined by firm quality. Recall from (31) that, consistent with the empirical measures used in these studies, we define the PPS as the ratio of the dollar difference in managerial compensation in the high and low states to the dollar difference in the firm's gross profits.[4] The implications discussed above are by no means unique to our theory and are not the focus of our study (e.g., see Edmans et al. 2009, Baranchuk et al. 2011). Proposition 1 only serves to establish that these basic predictions of the model are consistent with empirical evidence and with the predictions of previous theories.

[4] Edmans et al. (2009) build a model in which PPS is the sensitivity of the percentage change in managerial pay to the percentage change in profits. They discuss the fact that the PPS is not consistently measured across empirical studies; some studies measure it as the "dollar-dollar" sensitivity, whereas others measure it as the "percentage-percentage" sensitivity.

The results of Proposition 1 also hold qualitatively in the benchmark partial equilibrium model. Consequently, from a qualitative standpoint, the effects described by the proposition are essentially partial equilibrium effects. Given that managerial effort has no role to play in the benchmark pure I/O model, the only implication that holds in this model is the increase of firm size with firm quality.

4.2. The Entry Cost and the Elasticity of Substitution

We now explore the effects of the entry cost and the elasticity of substitution on the distributions of firm size and managerial compensation. The following lemma shows the intuitive result that the aggregate price increases with the entry cost.

Lemma 3 (Entry Cost and the Aggregate Price). *The aggregate price index increases with the entry cost* f_e.

An increase in the entry cost increases the expected net profit that an entering firm must earn by the free entry equilibrium condition (32). Because an increase in the aggregate price has a positive effect on managerial effort and expected net profit by (30), the aggregate price index must increase with the entry cost to satisfy the equilibrium condition (32). The following proposition shows the effects of the entry cost on firm size and managerial compensation.

Proposition 2 (Entry Cost, Firm Size, and Compensation). *Managerial effort, expected managerial compensation, and firm size (measured in terms of the expected revenue, expected gross and net profit, or the expected labor force) increase with the entry cost. Managerial pay-performance sensitivities decline provided condition* (34) *holds.*

An increase in the entry cost raises the aggregate price index that, in turn, increases the return on managerial effort. Because managers exert greater effort, they receive greater expected compensation. The increase in managerial effort raises the expected productivity of a firm that enhances its expected revenue and gross profit. The increase in expected gross profit is sufficiently large to offset the increase in expected managerial compensation so that expected net profit also increases. Because the mass of workers hired by the firm in each productivity state is proportional to its revenue by (6), the firm's expected labor force also increases. Therefore, regardless of whether firm size is measured in terms of expected revenue, gross or net profit, or the labor force, an increase in the entry cost leads to larger firms. Condition (34) ensures that, because effort increases with the entry cost, managerial PPS declines.

The predicted negative relation between managerial PPS and the entry cost is consistent with the empirical findings of Karuna (2007) and contrasts with Raith (2003), who predicts a positive relation. The differing predictions stem from Raith's model, where managers have constant absolute risk-aversion preferences and effort additively affects output. In contrast, we allow for a more general utility function for the manager and effort affects both the mean and variance of output. The following corollary shows the effect of the entry cost on the mass of firms.

Corollary 1 (Entry Cost and the Mass of Firms). *The mass of firms declines with the entry cost.*

By Proposition 2, an increase in the entry cost dampens competition and increases the expected revenue of each firm and therefore the average expected revenue of all firms. Because aggregate revenue is fixed at R, the equilibrium mass of firms declines by (33).

None of the implications regarding the effects of the entry cost holds in the benchmark partial equilibrium model, which shows that the entry cost affects managerial compensation, firm size, and the mass of firms purely through the equilibrium channel in the model. As expected, the implications regarding managerial compensation and PPS do not hold in the benchmark pure I/O model because the manager has no role to play in this model.

We now examine the effects of another dimension of competition among firms: the elasticity of substitution. The elasticity of substitution σ between any pair of products is the price elasticity of demand faced by each monopolistic firm in equilibrium. The greater is σ, the more substitutable (and thereby price elastic) are the products being offered by the monopolists. The following proposition describes the effects of (marginal) changes in the elasticity of substitution on firm size, managerial compensation, and the mass of firms.

Proposition 3 (Elasticity of Substitution, Firm Size, Compensation, and the Mass of Firms). (i) *There exist two triggers* $\alpha_{\text{low}}(\sigma), \alpha_{\text{high}}(\sigma)$ *with* $\alpha_{\text{low}}(\sigma) < \alpha_{\text{high}}(\sigma)$ *such that managerial effort, expected managerial compensation, the firm's expected gross and net profits, and the expected labor force increase with a marginal increase in the elasticity of substitution if the firm quality* $\alpha > \alpha_{\text{high}}(\sigma)$ *and decrease if* $\alpha < \alpha_{\text{low}}(\sigma)$. (ii) *If condition* (34) *holds, then the manager's pay-performance sensitivity decreases with a marginal increase in* σ *if* $\alpha > \alpha_{\text{high}}(\sigma)$ *and increases if* $\alpha < \alpha_{\text{low}}(\sigma)$. (iii) *The mass of firms may increase or decrease with a marginal increase in* σ.

An increase in the elasticity of substitution affects managerial effort and output differentially. It increases the marginal product of managerial effort relatively more for higher-quality firms compared to lower-quality firms. Specifically, the left-hand side (LHS) of (30) increases (decreases) with a marginal increase in σ when the firm quality is above (below) a threshold. Consequently, by (24), (25), (26), and (28), managerial effort, expected managerial compensation, expected firm revenue, and gross and net profits increase with a marginal increase in σ if firm quality is above a high trigger but decrease if firm quality is below a low trigger. As with our earlier results, condition (34) ensures that managerial pay-performance sensitivities move in the opposite direction relative to effort.

By Proposition 1, firm size increases with firm quality. Consequently, Proposition 3 implies that an increase in the elasticity of substitution increases firm size and managerial compensation if firm size exceeds a high threshold and decreases firm size and managerial compensation if firm size is below a low threshold. In other words, a more elastic product market benefits large firms and their managers at the expense of small firms and their managers.

In his model of an industry with *homogeneous* firms, Raith (2003) shows that an increase in product substitutability increases incentives, average managerial compensation, and firm size. In contrast, in our framework with heterogeneous firms engaged in monopolistic competition, firm quality plays a central role in determining the effects of product substitutability.

In general, the triggers, $\alpha_{\text{low}}(\sigma)$ and $\alpha_{\text{high}}(\sigma)$ in Proposition 3 are distinct from each other so that there is a nondegenerate intermediate interval, $[\alpha_{\text{low}}(\sigma), \alpha_{\text{high}}(\sigma)]$, where the effects of an increase in σ on firm revenue and profit are ambiguous. This is because a firm's revenue and profit are also affected by the aggregate price index that is endogenously determined by (32). Because managerial effort choices at high- and low-quality firms are differentially affected by a change in σ, the effects on the equilibrium aggregate price depend on the distribution of firm qualities and are, therefore, ambiguous. For similar reasons, the effects of σ on the average expected revenue and gross and net profits of firms depend on the firm-quality distribution $g(\cdot)$. Consequently, in sharp contrast with the effects of the entry cost, the elasticity of substitution could increase or decrease the mass of firms.

Note that the LHS of the free entry equilibrium condition (36) is the average expected net profit of firms in the industry. Because the right-hand side (RHS) of (36) does not depend on σ, the LHS must also be invariant. By Proposition 3 and the above discussion, it follows that, for *any* firm-quality distribution $g(\cdot)$, the effects of σ on the expected net profits of firms must offset each other for large and small firms. Hence, there must exist two thresholds such that the expected net profit of a firm increases with σ if its quality is above the high threshold and decreases if it is below the low threshold. The effects of σ on firms' expected gross profits and revenues depend on its effects on managers' expected compensation. If the average expected compensation of managers increases with σ, then we can easily see that the average expected gross profits and revenues of firms also increase so that the mass of firms decreases. Even for special choices of the firm-quality distribution, however, the effects of σ on the average expected compensation of managers depends on the managers' disutility of effort function $\kappa(\cdot)$ because it directly affects their effort choices and compensation. Furthermore, managerial effort choices are affected by the aggregate price index that endogenously depends on the firm-quality distribution. Consequently, the effects of σ on the average expected compensation of managers are ambiguous even if one makes specific assumptions on the firm-quality distribution.[5]

The following proposition sheds further light on the ambiguous effects of (marginal) changes in the elasticity of substitution on the aggregate price index.

Proposition 4 (Elasticity of Substitution and the Aggregate Price). *There exists a threshold level $f_T'(\sigma)$ of the entry cost such that the equilibrium aggregate price index increases with a marginal increase in the elasticity of substitution σ if $f_e < f_T'(\sigma)$ and decreases if $f_e > f_T'(\sigma)$.*

For a given aggregate price, if the entry cost is below a threshold, then the firm-quality trigger $\alpha_{\text{low}}(\sigma)$ defined in Proposition 3 is high enough that a relatively large proportion of managers decrease their effort and output as the elasticity of substitution increases. The equilibrium condition (32) then implies that the equilibrium aggregate price must increase. On the other hand, if the entry cost is above the threshold, then the firm-quality trigger $\alpha_{\text{high}}(\sigma)$ defined in Proposition 3 is low enough that a relatively large proportion of managers increase their effort and output. Hence, the equilibrium aggregate price declines in response to an increase in the elasticity of substitution.

The intuition for Propositions 2–4 shows that, in contrast with the entry cost, the elasticity of substitution has *direct* and *indirect* effects on the distributions of firms and managerial compensation. By Lemma 1

[5] Consider the very special case where $\kappa(\cdot)$ is linear so that the optimal effort choice of a manager is either 0 or 1. Suppose also that it is optimal for any firm to induce high effort from its manager. In this case, the average expected revenue of firms increases with σ so that the mass of firms declines. It is still, however, the case that the expected revenue, gross profit, and net profit increase for firms with qualities above a high trigger and decrease for firms with qualities below a low trigger as shown by Proposition 3.

(see Equations (29) and (30)), the elasticity of substitution *directly* affects managerial effort, output, and compensation even if the aggregate price index is kept fixed, that is, in the absence of the equilibrium channel. As in the case of the entry cost, the elasticity of substitution also influences managerial compensation and output *indirectly* by affecting the aggregate price index. The direct and indirect effects of the elasticity of substitution together contribute toward explaining its differing effects from those of the entry cost.

The direct and indirect effects of product substitutability contribute toward explaining why the variation of firm size and managerial compensation with product substitutability contrast sharply with Raith (2003). In Raith (2003), homogeneous firms compete along a Salop circle, and the parameter values are assumed to be such that each firm only competes with its nearest neighbors. In his framework, product substitutability only has indirect effects that arise when the mass of firms is endogenized. In contrast, in our model, product substitutability has direct and indirect effects. Furthermore, because firms are heterogeneous in our model, firm quality plays a key role in influencing the effects of product substitutability on firm size and managerial compensation.

The predicted effects of the elasticity of substitution on managerial compensation, PPS, and firm size hold in the benchmark partial equilibrium model, but the effect on the aggregate price index does not hold because it is exogenously fixed. As expected, the implications pertaining to managerial compensation and PPS do not hold in the benchmark pure I/O model.

The entry cost and the elasticity of substitution are both determinants of competition among firms. The results of this section show that differing facets of competition have contrasting effects on the firm-size distribution, managerial compensation, and incentives. Our analysis, therefore, suggests that empirical analyses of the effects of competition on the firm-size distribution and managerial compensation should appropriately account for different dimensions of competition.

4.3. Productivity Risk

The notion of "risk" captures the variability in a firm's output. By (9), the spread in the firm's gross profits in the high and low states increases with the quantity $h^{\sigma-1} - l^{\sigma-1}$. Accordingly, we define the firm's productivity risk as $x = h^{\sigma-1} - l^{\sigma-1}$ because it determines the spread in realized productivities in the high and low states. In the following proposition, we examine the effects of changing the productivity risk, keeping the quantity $h^{\sigma-1} + l^{\sigma-1}$ fixed. The following proposition derives the effects of productivity risk on firm size and managerial compensation.

Proposition 5 (Productivity Risk, Firm Size, and Compensation). (i) *Managerial effort and expected managerial compensation increase with productivity risk, whereas managerial pay-performance sensitivity declines if condition* (34) *holds.* (ii) *The average expected revenue and average expected gross profit of firms increase with productivity risk.*

Productivity risk has both direct and indirect effects on managerial effort and output. For a fixed aggregate price index, it follows from (30) that an increase in productivity risk raises the marginal product of managerial effort. Consequently, managerial effort increases at the optimum that, in turn, enhances the manager's expected compensation. The aggregate price index, however, is endogenously determined by the equilibrium condition (32). An increase (decrease) in the equilibrium aggregate price with productivity risk has a positive (negative) effect on managerial effort and output by (30). It turns out that the former "direct" effect of productivity risk dominates the latter "indirect" effect. An increase in productivity risk, therefore, increases managerial effort and output in equilibrium.

Note that, although the average expected revenue and average expected gross profit of firms increase with productivity risk, the effects of productivity risk on the expected revenue and profit of a *particular* firm are ambiguous in general. The ambiguity arises because for a firm with quality α, an increase in productivity risk has a negative effect on productivity (and, by extension, output) in the low state αl. Furthermore, it can be shown that productivity risk may increase or decrease the aggregate price depending on the values of the entry cost.

By Proposition 5, an increase in productivity risk raises the average expected revenue of firms. Because aggregate revenue R is fixed by the population of consumers, the equilibrium mass of firms declines with productivity risk.

Corollary 2 (Productivity Risk and the Mass of Firms). *The equilibrium mass of firms decreases with productivity risk.*

The benchmark partial equilibrium model does not generate the predicted effects on the mass of firms or the aggregate price index because these are exogenous in a partial equilibrium setting. None of the predicted effects of productivity risk arises in the benchmark pure I/O model, which suggests that the impact of productivity risk hinges on the presence of both the I/O and moral hazard aspects of the model.

4.4. Effects of the Firm-Quality Distribution

The following proposition shows the effects of a change in the firm-quality distribution g in the sense of first-order stochastic dominance (FOSD).

Proposition 6 (FOSD Change in Firm Quality Distribution). *Let g_1 and g_2 be two firm-quality distributions where g_1 first-order stochastically dominates g_2.* (i) *The aggregate price index corresponding to distribution g_1 is lower than that corresponding to g_2.* (ii) *For a firm with quality α, managerial effort, expected managerial compensation, expected revenue, expected gross and net profit, and the expected labor force are lower when the firm-quality distribution is g_1, whereas the manager's pay-performance sensitivity is greater provided condition* (34) *holds.*

As the firm-quality distribution shifts "to the right," the LHS of (32) increases for a *given* value of the aggregate price index P. Therefore, viewed as a function of P, the entire curve representing the LHS of (32) shifts upward. It immediately follows that the equilibrium aggregate price index must decrease to satisfy (32). The underlying intuition is that higher-quality firms are more efficient; that is, they require less labor to manufacture their product. Because a more efficient firm produces a cheaper product at the optimum, a more efficient distribution of firms leads to a lower aggregate price index. The implications from (29) and (30) are that for a given firm quality α, the manager exerts lower effort and receives lower expected compensation, and each firm's size (measured in terms of expected revenues, gross profit, net profit, or the labor force) is lower in response to an FOSD increase in the firm-quality distribution. If condition (34) holds, the manager's pay-performance sensitivity is greater.

The average (across all managers) expected compensation is affected by two conflicting forces. On one hand, the manager of a firm with given quality exerts lower effort and therefore receives lower expected compensation. On the other hand, an FOSD increase in the firm-quality distribution leads to a larger mass of higher-quality firms. The effect on average expected revenue and the mass of firms depends on which effect dominates. If the FOSD effect dominates, average firm size increases and the mass of firms decreases. If the managerial compensation effect dominates, average firm size decreases and the mass of firms increases.

Shocks to the firm-quality distribution could be viewed as aggregate shocks because they affect all firms. In this respect, Proposition 6 implies that in an equilibrium setting, managerial incentives are affected by aggregate shocks through their effects on the aggregate price index. This prediction contrasts sharply with the predictions of traditional principal–agent models in which incentives are only affected by idiosyncratic, firm-specific shocks.

Although the effects of an FOSD change in the firm-quality distribution on incentives and firm size can be pinned down, the effects of a second-order stochastic dominance (SOSD) change are, in general, ambiguous. The integrand on the LHS of (32) could be convex, concave, or neither convex nor concave in the firm quality α. Consequently, an SOSD shift in the firm-quality distribution could increase or decrease the LHS of (32) for a given aggregate price index. The implication is that the equilibrium aggregate price index could increase or decrease so that the effects on managerial incentives, effort, and firm size are ambiguous.

5. Empirical Analysis

We use our theoretical predictions to develop testable hypotheses that relate product market/industry characteristics—the entry cost, the elasticity of substitution, and productivity risk—to managerial compensation, firm size, and the number of firms.

5.1. Testable Hypotheses

Table 1 lists the testable hypotheses predicted by the theory that we empirically examine. Hypothesis 1 follows from the result of Proposition 1 that expected managerial compensation and firm size (measured by expected revenue or sales) both increase with firm quality. Consequently, managerial compensation increases with the rank of the firm in the industry with respect to size. Hypothesis 2 follows from Proposition 2. By Proposition 3, expected managerial compensation increases with the elasticity of product substitution if the match quality is above a threshold but decreases with the elasticity of product substitution if match quality is below the threshold. By Proposition 1, firm size increases with firm quality. Combining these results, we obtain Hypothesis 3. Hypothesis 4 follows from Proposition 5.

Hypotheses 5, 6, and 7 follow from Propositions 2, 3, and 5, respectively. Hypotheses 8 and 9 follow from Corollaries 1 and 2, respectively.

Table 1 Testable Hypotheses

No.	Hypothesis
1	Managerial compensation increases with the rank of the firm within the industry with respect to size.
2	Managerial compensation increases with the entry cost of the industry.
3	Managerial compensation increases with the elasticity of substitution for large firms and decreases for small firms.
4	Managerial compensation increases with productivity risk.
5	Firm size increases with the entry cost.
6	Firm size increases with the elasticity of substitution if firm size is above a high trigger and declines if it is below a low trigger.
7	Firm size increases with productivity risk.
8	The number of firms decreases with the entry cost of the industry.
9	The number of firms decreases with productivity risk.

Note that, with the exception of Hypothesis 1, all our hypotheses relate *industry-level* variables—the entry cost, the elasticity of substitution, and productivity risk—to managerial compensation, firm size, and the number of firms.

5.2. Sample Selection and Variable Construction

We obtain CEO compensation data from ExecuComp, financial data from Compustat, and stock return data from the Center for Research in Security Prices. We delete firm-year observations in which CEOs do not work for a full fiscal year. We also eliminate firms with negative book values of total equity to ensure that market-to-book ratios are meaningful. As is standard in the literature, we exclude firms in the financial services industry. The firms in the model are conventional firms that produce goods for consumption rather than financial intermediaries.

We define an industry at the two-digit Standard Industrial Classification (SIC) level. The results are robust to grouping firms by the Fama-French 48 industry classification. Given that the majority of the variables are at the industry level, we only retain observations in industries that have at least five firms each year. (The results are robust to retaining industry observations that have at least two firms each year.) To remove the effects of outliers, we winsorize all continuous variables at the 1% and 99% levels. The final sample includes 18,963 firm-year observations with available data over the period 1992 to 2009. We convert all dollar items into 2009 dollars using the gross domestic product deflator index from the Bureau of Economic Analysis.

To test our hypotheses pertaining to managerial compensation, the dependent variable *TOTALPAY* is the logarithm of the total compensation of CEOs, including salary, bonus, stock options, restricted stocks, and other long-term incentives. We compute the value of stock options using the Black-Scholes formula. The distribution of the dollar amount of total compensation is skewed, so we take the log to obtain *TOTALPAY*. To test our hypotheses pertaining to firm size, the dependent variable *FIRM SIZE* is the logarithm of the firm's total revenue/sales. To test our hypotheses pertaining to the number of firms, the dependent variable *NFIRMS* is the number of firms in the two-digit SIC industry.

TARANK is the scaled rank of the firm in the industry by firm size. It is the proxy for firm quality because high-quality firms are larger according to Proposition 1 and in agreement with Gabaix and Landier (2008) and Tervio (2008). We use the firm's rank in the size distribution, rather than the size itself, because we interpet the model as one of competing firms in a particular industry so that the relevant variable for a firm is its position relative to others in the same industry.

Given that our main hypotheses link product market/industry variables—the entry cost, the elasticity of substitution, and productivity risk—to CEO compensation, firm size, and the number of firms, all our remaining variables are at the industry level.

For robustness, we use four different measures of the entry cost. *ENCOST_PPE*1 equals the logarithm of the weighted average of the gross value of property, plant, and equipment in the industry, where the weight is the equity value of each firm divided by the total equity value of the industry. *ENCOST_PPE*2 is the logarithm of the weighted average of the gross value of property, plant, and equipment in the industry, where the weight is the lagged asset value of each firm divided by the lagged total asset value of the industry. The second measure of the entry cost, *ENCOST_EMPL*, equals the logarithm of the total gross value of property, plant, and equipment in the industry multiplied by the ratio of the median employment in the industry to the total employment. In the model, the entry cost is incurred *before* firm quality, which determines firm size, is realized. Because the ex post firm quality could take high and low values consistent with the observed intra-industry variations in firm size, it seems appropriate to use "average" or "median" measures for the entry cost (see also Karuna 2007). Nevertheless, we use a fourth measure of the entry cost, *ENCOST_PPE*3, that is the logarithm of the minimum gross value of property, plant, and equipment in the industry.

Consistent with most previous estimates of product substitutability, which use the negative price-cost margin (e.g., Domowitz et al. 1986, Karuna 2007), our measure of product substitutability, *PSUB*, is the negative value of industry sales divided by industry operating costs, which include costs of goods sold; selling, general, and administrative expenses; and depreciation, depletion, and amortization. The theory predicts that the impact of the elasticity of substitution on compensation and firm size depends on threshold firm sizes. Therefore, we define the dummy *LOW* to equal 1 if the firm has a value of *TARANK* less than or equal to 0.33 and to equal 0 otherwise; similarly, we define the dummy *HIGH* to equal 1 if the firm has a value of *TARANK* greater than or equal to 0.66 and to equal 0 otherwise. Our results are robust to alternative cutoffs for *LOW* and *HIGH*.

For robustness, we use three different measures of the productivity risk. In the model, it can be shown that the standard deviation of the return of a firm's stock price between dates 1 and 2 increases with the productivity risk as defined in §4.3. Accordingly, we define *RISK*1 as the standard deviation of the monthly industry stock returns over the past three years. In the model, the productivity shocks affect the firm's final earnings, and the standard deviation of earnings increases with the productivity risk. Consequently, our second measure, *RISK*2, is the standard deviation of quarterly industry returns on assets (operating income before depreciation divided by total assets) over the past three years. Our third measure, *ID_RISK*, is the standard deviation of the residuals of a capital asset pricing model regression of industry returns on the market return over the past three years.

We also include additional control variables in the empirical specifications that are defined in Table 2. Finally, we include industry and year dummies in all our regressions to control, respectively, for unobserved time-invariant, industry-specific factors and time trends that are constant across industries. Table 2 lists all the variables along with their definitions. Table 3 shows the descriptive statistics. In Table 4, we report the univariate statistics, that is, the means and standard deviations of managerial compensation, firm size, and the number of firms for each quartile of observations sorted by each of the entry cost, elasticity of substitution, and productivity risk.

Table 2 Variable Definitions

TOTALPAY	Log of CEO's total compensation for the year, including salary, bonus, stock option, restricted stock, and other long-term incentives.
FIRM SIZE	Log of firm sales.
NFIRMS	Log of the number of firms in the 2-digit SIC industry.
TARANK	The fractional rank of the company's total assets within the industry. Fractional rank is the normal rank scaled by the number of firms in the industry. Range is (0, 1].
*ENCOST_PPE*1	Log of the weighted average of the gross value of property, plant, and equipment in the industry, where the weight is the equity value of each firm divided by the total equity value of the industry.
*ENCOST_PPE*2	Log of the weighted average of the gross value of property, plant, and equipment in the industry, where the weight is the lagged asset value of each firm divided by the lagged total asset value of the industry.
*ENCOST_PPE*3	Log of the minimum gross value of property, plant, and equipment in the industry.
ENCOST_EMPL	Log of [the total gross value of property, plant, and equipment in the industry $\times$ median employment/total employment].
PSUB	Product substitutability is the negative value of industry sales divided by industry operating costs in the year. Industry operating costs include costs of goods sold; selling, general, and administrative expenses; and depreciation, depletion, and amortization.
LOW	Dummy variable that equals 1 if *TARANK* ≤ 0.33; 0 otherwise.
HIGH	Dummy variable that equals 1 if *TARANK* ≥ 0.66; 0 otherwise.
ID_RISK	Time-series standard deviation of the residuals of a capital asset pricing model regression of industry returns on the market return over the past three years.
*RISK*1	Time-series standard deviation of monthly industry stock returns over the past three years.
*RISK*2	Time-series standard deviation of quarterly industry returns on assets (operating income before depreciation divided by total assets) over the past three years.
IND_MB	The weighted average of the market to book value ratios for the industry {MB $= \sum$ each firm's MB ratio $\times$ the firm's market share in the industry}, where the weight is the equity value of each firm divided by the total equity value of the industry.
IND_LEV	The weighted average of the leverage ratios for the industry where the weight is the equity value of each firm divided by the total equity value of the industry.
IND_SALES	Log of total sales in the industry for the year.

Note. This table describes the dependent and independent variables in our empirical analysis.

5.3. Results

The results reported in Table 5 support all our hypotheses pertaining to managerial compensation. Consistent with Hypothesis 1, all four measures of the entry cost (*ENCOST_PPE*1, *ENCOST_PPE*2, *ENCOST_PPE*3, and *ENCOST_EMPL*) have positive and significant coefficients. The coefficients on the two interaction terms *PSUB* ∗ *HIGH* and *PSUB* ∗ *LOW* are significantly positive and negative, respectively, providing supportive evidence for Hypothesis 2. The coefficients on all three measures of risk (*RISK*1, *RISK*2, and *ID_RISK*) are significant and positive in all the regressions, providing support for Hypothesis 3. The effects are also economically significant. For example, from the coefficients in specification (1), a one-standard-deviation increase in the log of the entry cost measure, *ENCOST_PPE*1, increases managerial compensation by approximately 7%. From the coefficients in specification (4), a one-standard-deviation increase in the log of the entry cost measure, *ENCOST_EMPL*, increases managerial compensation by 19%. A one standard deviation increase in our measure of the elasticity of substitution *increases* the compensation of managers of large firms in the industry by approximately 4% and *decreases* the compensation of managers of small firms by approximately 7%.

Table 6 shows the effects of product market characteristics on firm size. We find support for our Hypotheses 4–6 pertaining to firm size. The predicted effects are again economically significant. From the coefficients in specification (1), a one-standard-deviation increase in the log of the corresponding entry cost measure, *ENCOST_PPE*1, increases firm size by 9%. From the coefficients in specification (4), a one-standard-deviation increase in the log of the entry cost measure, *ENCOST_EMPL*, increases firm size by 56%. A one-standard-deviation increase in the elasticity of substitution increases the size of large firms by 11% and decreases the size of small firms by 15%. A one-standard-deviation increase in risk increases firm size by 2%. The results of Table 7 show that our Hypotheses 7–9 are also supported in the data.

Table 3 Descriptive Statistics

Variable	N	Mean	Std. dev.	Min	25th percentile	50th percentile	75th percentile	Max
TOTALPAY	18,949	7.8270	1.0298	5.5196	7.0962	7.7873	8.5339	10.4179
FIRM SIZE	18,949	7.3284	1.4464	4.3554	6.3048	7.2117	8.2780	11.0474
NFIRMS	18,949	3.6418	0.8868	1.6094	3.0910	3.7136	4.4188	4.9273
TARANK	18,949	0.5204	0.2893	0.0075	0.2700	0.5188	0.7711	1
*ENCOST_PPE*1	18,949	8.8021	1.0874	4.9055	7.9449	9.0866	9.6114	10.6909
*ENCOST_PPE*2	18,949	8.8884	1.0728	5.0274	8.2101	9.1877	9.6812	10.6488
*ENCOST_PPE*3	18,949	3.6919	1.1778	2.5568	2.5568	3.3210	4.7611	9.7471
ENCOST_EMPL	18,949	6.7612	1.7330	1.6680	5.5116	6.7106	7.9762	11.7702
PSUB	18,949	−1.1520	0.0674	−1.6214	−1.1879	−1.1478	−1.1110	−1.0232
PSUB ∗ *HIGH*	18,949	−0.4166	0.5545	−1.6214	−1.1171	0	0	0
PSUB ∗ *LOW*	18,949	−0.3529	0.5330	−1.6214	−1.1015	0	0	0
*RISK*1	18,949	0.0501	0.0191	0.0195	0.0368	0.0462	0.0608	0.1370
*RISK*2	18,949	0.0122	0.0070	0.0018	0.0069	0.0108	0.0168	0.0576
ID_RISK	18,949	0.0333	0.0166	0.0109	0.0205	0.0284	0.0432	0.1103
IND_SALES	18,949	11.8105	1.0768	7.8889	11.1176	12.1219	12.6539	13.5388
IND_MB	18,949	2.5151	1.1108	0.8564	1.6372	2.3005	3.1680	7.1985
IND_LEV	18,949	0.2191	0.0940	0.0109	0.1399	0.2036	0.2917	0.6300

Note. This table shows the descriptive statistics of the dependent and independent variables in our regressions.

Table 4 Univariate Table

	*ENCOST_PPE*1				*ENCOST_PPE*2				*PSUB*			
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
TOTALPAY	7.7064	7.8279	7.8666	7.9050	7.7208	7.8087	7.8775	7.9006	7.9019	7.8727	7.8403	7.6924
	(0.9314)	(1.0493)	(1.0472)	(1.0734)	(0.9493)	(1.0119)	(1.0598)	(1.0836)	(1.079)	(1.0409)	(1.0093)	(0.9744)
FIRM SIZE	7.0465	7.2146	7.2669	7.7862	7.1745	7.1545	7.2142	7.7705	7.3228	7.0379	7.3278	7.6262
	(1.1863)	(1.4705)	(1.4922)	(1.5026)	(1.2012)	(1.4554)	(1.4924)	(1.5231)	(1.4331)	(1.43)	(1.4271)	(1.436)
NFIRMS	3.0871	3.7209	3.9750	3.7703	2.9621	3.8406	3.8893	3.8764	4.1715	4.0379	3.4730	2.8798
	(0.86)	(0.8601)	(0.7181)	(0.8464)	(0.7611)	(0.8183)	(0.7459)	(0.8507)	(0.7926)	(0.7107)	(0.7565)	(0.6273)
	*RISK*1				*RISK*2				*ID_RISK*			
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
TOTALPAY	7.6329	7.7753	7.9111	7.9900	7.6497	7.8025	7.8659	7.9874	7.6962	7.8055	7.8473	7.9592
	(0.9843)	(1.0038)	(1.0135)	(1.0805)	(0.9664)	(1.0039)	(1.0187)	(1.0969)	(1.0004)	(0.986)	(1.033)	(1.081)
FIRM SIZE	7.4646	7.3763	7.3425	7.1274	7.5374	7.2686	7.3072	7.2046	7.2440	7.3348	7.3627	7.3721
	(1.4057)	(1.454)	(1.4767)	(1.4271)	(1.3959)	(1.4371)	(1.4102)	(1.5176)	(1.4371)	(1.4037)	(1.4943)	(1.4473)
NFIRMS	3.7271	3.5159	3.5022	3.8268	3.9103	3.5716	3.3783	3.7126	4.0502	3.7582	3.3964	3.3598
	(0.8202)	(0.8441)	(0.9616)	(0.8704)	(0.8918)	(0.7953)	(0.7881)	(0.974)	(0.6604)	(0.7625)	(0.9624)	(0.9418)

Note. This table shows the means and standard deviations of managerial compensation, firm size, and the number of firms for each quartile of observations sorted by each of the measures of the entry cost, the elasticity of substitution, and the productivity risk.

Table 5 Managerial Compensation

Variables	Predicted sign	*TOTALPAY* (1)	*TOTALPAY* (2)	*TOTALPAY* (3)	*TOTALPAY* (4)	*TOTALPAY* (5)	*TOTALPAY* (6)
TARANK	+	2.3228***	2.3218***	2.3229***	2.0351***	2.3201***	2.3222***
		(0.0565)	(0.0565)	(0.0565)	(0.0646)	(0.0565)	(0.0565)
*ENCOST_PPE*1	+	0.0609***					
		(0.0181)					
*ENCOST_PPE*2	+		0.0916***				
			(0.0182)				
*ENCOST_PPE*3	+			0.0569***		0.0846***	0.0856***
				(0.0118)		(0.0184)	(0.0183)
ENCOST_EMPL	+				0.0643***		
					(0.007)		
PSUB	?	−0.2696*	−0.2698*	−0.3362**	−0.2691*	−0.3529**	−0.2997**
		(0.1503)	(0.1494)	(0.1487)	(0.1486)	(0.1491)	(0.1488)
PSUB ∗ *HIGH*	+	0.0637***	0.0634***	0.0637***	0.0534**	0.0629***	0.0636***
		(0.0206)	(0.0206)	(0.0206)	(0.0206)	(0.0206)	(0.0206)

Table 5 (Continued)

Variables	Predicted sign	*TOTALPAY* (1)	*TOTALPAY* (2)	*TOTALPAY* (3)	*TOTALPAY* (4)	*TOTALPAY* (5)	*TOTALPAY* (6)
PSUB ∗ *LOW*	−	−0.1184*** (0.0197)	−0.1182*** (0.0197)	−0.1187*** (0.0197)	−0.104*** (0.0198)	−0.1177*** (0.0197)	−0.1184*** (0.0197)
*RISK*1	+	2.4171*** (0.6667)	2.3559*** (0.6664)	2.3343*** (0.6668)	2.3137*** (0.6649)		
*RISK*2	+					5.1675*** (1.2621)	
ID_RISK	+						3.1616*** (0.7427)
IND_SALES	?	0.1249*** (0.0268)	0.1181*** (0.0263)	0.1533*** (0.0252)	0.1359*** (0.0253)	0.1137*** (0.0261)	0.1126*** (0.0261)
IND_LEV	?	0.0404 (0.1854)	−0.0487 (0.1868)	0.1144 (0.1832)	0.0839 (0.1829)	−0.0355 (0.1868)	−0.0639 (0.186)
IND_MB	?	0.0359*** (0.0108)	0.0326*** (0.0108)	0.0297*** (0.0108)	0.0332*** (0.0107)	0.0316*** (0.0108)	0.0311*** (0.0108)
No. of observations		18,949	18,949	18,949	18,949	18,949	18,949
R-squared		0.4463	0.4467	0.4466	0.4484	0.4468	0.4468

Notes. This table shows the effects of product market characteristics on managerial compensation. The dependent variable, *TOTALPAY*, as well as the independent and control variables are described in Table 1. In all specifications, industry and year dummies are included, and the coefficients of the intercept terms are not shown. Standard errors clustered at the firm level are reported in parentheses under the coefficient estimates.

***, **, and * indicate statistical significance at the 1%, 5%, and 10% confidence levels, respectively.

Table 6 Firm Size

Variables	Predicted sign	*FIRM SIZE* (1)	*FIRM SIZE* (2)	*FIRM SIZE* (3)	*FIRM SIZE* (4)	*FIRM SIZE* (5)	*FIRM SIZE* (6)	*FIRM SIZE* (7)
TARANK	+	4.7499*** (0.0424)	4.749*** (0.0424)	4.7499*** (0.0421)	2.5674*** (0.0361)	4.7494*** (0.0424)	4.7487*** (0.0424)	4.7495*** (0.0424)
*ENCOST_PPE*1	+	0.0726*** (0.0136)				0.0746*** (0.0136)		
*ENCOST_PPE*2	+		0.0817*** (0.0137)					
*ENCOST_PPE*3	+			0.1517*** (0.0088)			0.0828*** (0.0138)	0.0753*** (0.0137)
ENCOST_EMPL	+				0.4875*** (0.0039)			
PSUB	?	−0.2589** (0.1127)	−0.2815** (0.112)	−0.3277*** (0.1107)	−0.2165*** (0.0829)	−0.3121*** (0.1126)	−0.333*** (0.1119)	−0.309*** (0.1116)
PSUB ∗ *HIGH*	+	0.1887*** (0.0155)	0.1885*** (0.0155)	0.1886*** (0.0154)	0.1109*** (0.0115)	0.1886*** (0.0155)	0.1883*** (0.0155)	0.1886*** (0.0155)
PSUB ∗ *LOW*	−	−0.2828*** (0.0148)	−0.2826*** (0.0148)	−0.2835*** (0.0147)	−0.1737*** (0.011)	−0.2827*** (0.0148)	−0.2825*** (0.0148)	−0.2828*** (0.0148)
*RISK*1	+	2.2758*** (0.5)	2.2574*** (0.4999)	1.8785*** (0.4967)	0.7254* (0.371)			
*RISK*2	+					1.598* (0.9398)	1.1053 (0.9473)	
ID_RISK	+							3.2482*** (0.557)
IND_SALES	?	0.3971*** (0.0201)	0.4002*** (0.0197)	0.4282*** (0.0188)	0.2865*** (0.0141)	0.3858*** (0.0199)	0.389*** (0.0196)	0.3954*** (0.0195)
IND_LEV	?	0.0626 (0.139)	0.0122 (0.1401)	0.1178 (0.1364)	−0.2237** (0.1021)	0.0096 (0.1388)	−0.0465 (0.1402)	0.0023 (0.1395)
IND_MB	?	−0.0119 (0.0081)	−0.0163** (0.0081)	−0.0216*** (0.008)	−0.0020 (0.006)	−0.0103 (0.0081)	−0.0145* (0.0081)	−0.0181** (0.0081)
No. of observations		18,949	18,949	18,949	18,949	18,949	18,949	18,949
R-squared		0.8421	0.8422	0.8444	0.9129	0.842	0.842	0.8423

Notes. This table shows the effects of product market characteristics on firm size. The dependent variable, *FIRM SIZE*, as well as the independent and control variables are described in Table 1. In all specifications, industry and year dummies are included, and the coefficients of the intercept terms are not shown. Standard errors clustered at the firm level are reported in parentheses under the coefficient estimates.

***, **, and * indicate statistical significance at the 1%, 5%, and 10% confidence levels, respectively.

Table 7 **Number of Firms**

Variables	Predicted sign	*NFIRMS* (1)	*NFIRMS* (2)	*NFIRMS* (3)	*NFIRMS* (4)	*NFIRMS* (5)	*NFIRMS* (6)
*ENCOST_PPE*1	–	−0.1973*** (0.015)					
*ENCOST_PPE*2	–		−0.223*** (0.0155)				
*ENCOST_PPE*3	–			−0.0697*** (0.0108)		−0.2233*** (0.0157)	−0.2191*** (0.0156)
ENCOST_EMPL	–				−0.341*** (0.0189)		
PSUB	?	−0.2934* (0.1591)	−0.2313 (0.1549)	0.0461 (0.17)	−0.3173** (0.1456)	−0.1651 (0.1548)	−0.2010 (0.1548)
*RISK*1	–	−1.7634** (0.7229)	−2.0157*** (0.7091)	−1.1711 (0.7879)	−1.1701* (0.6653)		
*RISK*2	–					−1.4350 (1.1792)	
ID_RISK	–						−1.397* (0.7408)
IND_SALES	?	0.5098*** (0.0208)	0.5077*** (0.0201)	0.4023*** (0.0207)	0.497*** (0.0183)	0.5146*** (0.02)	0.5083*** (0.0203)
IND_LEV	?	−0.0300 (0.1312)	0.0640 (0.1295)	−0.3351** (0.1412)	−0.0642 (0.12)	0.0489 (0.1305)	0.0716 (0.1301)
IND_MB	?	0.0241*** (0.009)	0.0311*** (0.0088)	0.0315*** (0.0098)	0.0196** (0.0083)	0.0307*** (0.0089)	0.0328*** (0.0089)
Industry and year dummies		Yes	Yes	Yes	Yes	Yes	Yes
No. of observations		757	757	757	757	757	757
R-squared		0.9722	0.9733	0.9672	0.9765	0.973	0.9731

Notes. This table shows the effects of product market characteristics on firm size. The dependent variable, *NFIRMS*, as well as the independent and control variables are described in Table 1. In all specifications, the coefficients of the intercept terms are not shown. Standard errors clustered at the firm level are reported in parentheses under the coefficient estimates.

***, **, and * indicate statistical significance at the 1%, 5%, and 10% confidence levels, respectively.

6. Conclusions

Using a tractable, market equilibrium model, we show how the distribution of heterogeneous firm qualities, moral hazard, and monopolistic competition interact to affect firm size and managerial compensation. We exploit the properties of the unique equilibrium to show that different determinants of product market competition—the entry cost and the elasticity of substitution—have sharply contrasting effects on managerial compensation, firm size, and market structure. Market equilibrium effects play a central role in generating these different impacts. In contrast with the predictions of a partial equilibrium analysis, aggregate shocks to the firm-quality distribution affect incentives in our market equilibrium framework. We develop and test several hypotheses derived from the theory that relate product market/industry characteristics—the entry cost, the elasticity of substitution, and productivity risk—to managerial compensation, firm size, and the number of firms. We show statistically and economically significant support for the hypotheses in our empirical analysis. The economically significant effects of product market characteristics on CEO compensation and firm size potentially explain why CEO compensation levels and firm sizes differ so significantly across industries.

We use our model to show how imperfect product market competition and moral hazard affect CEO compensation, firm size, and market structure in this study. The relative simplicity and tractability of the model, however, suggest that it could potentially serve as a foundation for more complex models that could be used to further explore the relationship between CEO markets and product markets.

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Appendix. Proofs of Selected Results

Proof of Lemma 2. Consider an arbitrary value P of the aggregate price index. Suppose that $e_\alpha(P) < 1$, where $e_\alpha(P)$ is the optimal effort exerted by the manager of a firm with quality α when the price index is P. By (30),

$$e_\alpha(P)\alpha^{\sigma-1}\left[\frac{R(P\rho)^{\sigma-1}}{\sigma}(h^{\sigma-1}-l^{\sigma-1})\right] = e_\alpha(P)v[\kappa'(e_\alpha(P))]+(e_\alpha(P))^2v'[\kappa''(e_\alpha(P))]\kappa''(e_\alpha(P)). \quad (35)$$

Plugging (35) in (32), we have

$$\int_0^\infty\left[\frac{R(P\rho\alpha)^{\sigma-1}}{\sigma}[e_\alpha(P)h^{\sigma-1}+(1-e_\alpha(P))l^{\sigma-1}] - e_\alpha(P)v[\kappa'(e_\alpha(P))]\right]g(\alpha)\,d\alpha = \int_0^\infty\left[\frac{R(P\rho\alpha)^{\sigma-1}}{\sigma}l^{\sigma-1} + (e_\alpha(P))^2v'[\kappa'(e_\alpha(P))]\kappa''(e_\alpha(P))\right]g(\alpha)\,d\alpha. \quad (36)$$

Because v, v', κ', and κ'' are increasing, $e_\alpha(P)$ increases with P from (30). Consequently, the RHS of (36) and, therefore, the LHS of (32) are increasing in P. By (36), the LHS of (32) tends to infinity as $P\to\infty$ and tends to 0 as $P\to 0$. Hence, there exists exactly one aggregate price P^* that satisfies the Equation (32). Q.E.D.

Proof of Proposition 1. Because v, v', κ', and κ'' are increasing, it follows from (30) that the managers' equilibrium effort choices $e^*_\alpha = e^*_\alpha(P^*)$ increase with firm quality α. By (29), the expected compensation of managers also increases with their firm's quality. By (8), the expected revenue of a firm with firm quality α is

$$\alpha^{\sigma-1}R(P^*\rho)^{\sigma-1}e^*_\alpha[h^{\sigma-1}-l^{\sigma-1}]+\alpha^{\sigma-1}R(P^*\rho)^{\sigma-1}l^{\sigma-1}, \quad (37)$$

which increases with α because e^*_α increases with α. By (9), firms' expected gross profits also increase with firm quality. By (36), a firm's expected net profit is

$$\frac{R(P^*\rho\alpha)^{\sigma-1}}{\sigma}l^{\sigma-1}+(e^*_\alpha)^2v'[\kappa'(e^*_\alpha)]\kappa''(e^*_\alpha), \quad (38)$$

which also increases with firm quality α because e^*_α increases with α and v', κ' and κ'' are increasing. By (6) and (7), the total labor employed by a firm if its realized productivity is ϕ is

$$x(\phi)=\rho r(\phi). \quad (39)$$

Because expected revenue increases with firm quality, it follows that the expected amount of labor employed also increases.

By (31),

$$\mathrm{PPS}(\alpha)=\frac{1}{1+(e^*_\alpha v'[\kappa'(e^*_\alpha)]\kappa''(e^*_\alpha))/v[\kappa'(e^*_\alpha)]}.$$

By condition (34), managerial pay-performance sensitivities increase with firm quality because managerial effort increases. Q.E.D.

Proof of Lemma 3. As the entry cost f_e increases, the RHS of (32) increases. In the proof of Lemma 2, we showed that the LHS of (32), expressed as a function of the aggregate price index, is increasing. It follows that the equilibrium aggregate price must increase with f_e to satisfy (32). Q.E.D.

Proof of Proposition 2. Because the equilibrium aggregate price increases with the entry cost, it follows from (30) that each manager's effort increases with the entry cost. In the following, we explicitly indicate the dependence of the equilibrium aggregate price and managerial effort levels on the entry cost. By (29), the expected compensation of managers also increases with the entry cost because their effort choices increase. By (8), the expected revenue of a firm with firm quality α is

$$\alpha^{\sigma-1}R(P^*(f_e)\rho)^{\sigma-1}e^*_\alpha(f_e)[h^{\sigma-1}-l^{\sigma-1}]+\alpha^{\sigma-1}R(P^*(f_e)\rho)^{\sigma-1}l^{\sigma-1},$$

which increases with the entry cost because $P^*(f_e)$ and $e^*_\alpha(f_e)$ increase with f_e. By (9), firms' expected gross profit also increase with the entry cost. By (36), a firm's expected net profit is

$$\frac{R(P^*(f_e)\rho\alpha)^{\sigma-1}}{\sigma}l^{\sigma-1}+(e^*_\alpha(f_e))^2v'[\kappa'(e^*_\alpha(f_e))]\kappa''(e^*_\alpha(f_e)),$$

which also increases with the entry cost because $P^*(f_e)$ and $e^*_\alpha(f_e)$ increase, and v', κ', and κ'' are increasing. By (39), the expected labor force increases with the entry cost because the expected revenue increases. By (31),

$$\mathrm{PPS}(\alpha)=\frac{1}{1+(e^*_\alpha(f_e)v'[\kappa'(e^*_\alpha(f_e))]\kappa''(e^*_\alpha(f_e)))/v[\kappa'(e^*_\alpha(f_e))]}.$$

By condition (34), managerial pay-performance sensitivities decrease with the entry cost because managerial effort increases. Q.E.D.

Proof of Proposition 3. (i) Let $f(\alpha,\sigma)$ denote the LHS of (30) in equilibrium. We can write

$$f(\alpha,\sigma)=\alpha^{\sigma-1}(P^*(\sigma)^{\sigma-1})g(\sigma), \quad (40)$$

where

$$g(\sigma)=\left[\frac{R(\rho)^{\sigma-1}}{\sigma}(h^{\sigma-1}-l^{\sigma-1})\right], \quad (41)$$

and $P^*(\sigma)$ is the equilibrium aggregate price index when the elasticity of substitution is σ. By (40) and (41),

$$\frac{\partial}{\partial\sigma}\ln f(\alpha,\sigma)=\ln\alpha+(\sigma-1)\frac{\partial}{\partial\sigma}\ln P^*(\sigma) +\ln P^*(\sigma)+\frac{\partial}{\partial\sigma}\ln g(\sigma). \quad (42)$$

Because $\lim_{\alpha\to 0}\ln\alpha=-\infty$ and $\lim_{\alpha\to\infty}\ln\alpha=\infty$, there exists a trigger level $\alpha_1(\sigma)$ of firm quality such that $(\partial\ln f/\partial\sigma)(\alpha,\sigma)>0$ for $\alpha>\alpha_1(\sigma)$ and $(\partial\ln f/\partial\sigma)(\alpha,\sigma)<0$ for $\alpha<\alpha_1(\sigma)$. By (30) and (29), the manager's equilibrium effort and expected compensation increase with a marginal increase in σ if $\alpha>\alpha_1(\sigma)$ and decrease if $\alpha<\alpha_1(\sigma)$. Because managerial effort $e^*_\alpha(\sigma)$ increases with σ for $\alpha > \alpha_1(\sigma)$ and decreases with σ for $\alpha < \alpha_1(\sigma)$, it follows from the above and condition (34) that managerial pay-performance sensitivities decrease with a (marginal) increase in σ for $\alpha > \alpha_1(\sigma)$ and increase for $\alpha < \alpha_1(\sigma)$.

By (25), the expected revenue of a firm with quality α is

$$\hat{r}(\alpha,\sigma) = R(P^*(\sigma)\rho\alpha)^{\sigma-1}[e^*_\alpha(\sigma)h^{\sigma-1} + (1-e^*_\alpha(\sigma))l^{\sigma-1}].$$

From the above,

$$\begin{aligned}\ln\hat{r}(\alpha,\sigma) &= (\sigma-1)\ln\alpha + \ln[e^*_\alpha(\sigma)h^{\sigma-1} + (1-e^*_\alpha(\sigma))l^{\sigma-1}]\\ &\quad + \ln[R(P^*(\sigma)\rho)^{\sigma-1}].\end{aligned}$$

Because $\lim_{\alpha\to 0}\ln\alpha = -\infty$ and $\lim_{\alpha\to\infty}\ln\alpha = \infty$, it follows from the above, and the results derived earlier for the effects of a marginal increase in σ on managerial effort, that there exist triggers $\alpha_2(\sigma)$, $\alpha'_2(\sigma)$ with $\alpha_2(\sigma) < \alpha'_2(\sigma)$ such that $(\partial\ln\hat{r}(\alpha,\sigma)/\partial\sigma)(\alpha,\sigma) > 0$ for $\alpha > \alpha'_2(\sigma)$ and $(\partial\ln\hat{r}(\alpha,\sigma)/\partial\sigma)(\alpha,\sigma) < 0$ for $\alpha < \alpha_2(\sigma)$. If $\hat{\pi}(\alpha,\sigma)$ denotes the expected gross profit of a firm with quality α, we can use similar arguments to show that there exist triggers $\alpha_3(\sigma)$, $\alpha'_3(\sigma)$ with $\alpha_3(\sigma) < \alpha'_3(\sigma)$ such that $(\partial\ln\hat{\pi}(\alpha,\sigma)/\partial\sigma)(\alpha,\sigma) > 0$ for $\alpha > \alpha'_3(\sigma)$ and $(\partial\ln\hat{\pi}(\alpha,\sigma)/\partial\sigma)(\alpha,\sigma) < 0$ for $\alpha < \alpha_3(\sigma)$. By (38), the expected net profit of a firm with quality α is

$$\begin{aligned}\hat{\xi}(\alpha,\sigma) &= \frac{R(P^*(\sigma)\rho\alpha)^{\sigma-1}}{\sigma}l^{\sigma-1}\\ &\quad + (e^*_\alpha(\sigma))^2 v'[\kappa'(e^*_\alpha(\sigma))]\kappa''(e^*_\alpha(\sigma)).\end{aligned}$$

Taking the derivative with respect to σ above, using the results derived earlier for the variation of the manager's effort with σ, and the fact that v', κ', and κ'' are all increasing, there exist triggers $\alpha_4(\sigma)$, $\alpha'_4(\sigma)$ with $\alpha_4(\sigma) < \alpha'_4(\sigma)$ such that $(\partial\hat{\xi}(\alpha,\sigma)/\partial\sigma)(\alpha,\sigma) > 0$ for $\alpha > \alpha'_4(\sigma)$ and $(\partial\hat{\xi}(\alpha,\sigma)/\partial\sigma)(\alpha,\sigma) < 0$ for $\alpha < \alpha_4(\sigma)$. If we define

$$\alpha_{\text{high}}(\sigma) = \max(\alpha_1(\sigma), \alpha'_2(\sigma), \alpha'_3(\sigma), \alpha'_4(\sigma))$$

and

$$\alpha_{\text{low}}(\sigma) = \min(\alpha_1(\sigma), \alpha_2(\sigma), \alpha_3(\sigma), \alpha_4(\sigma)),$$

the above arguments imply that the triggers $\alpha_{\text{low}}(\sigma)$ and $\alpha_{\text{high}}(\sigma)$ satisfy the statements of the proposition.

(ii) The effects of σ on the average expected revenue and average expected gross profit of all firms depend on the relative proportions of managers whose firms have qualities that are greater than $\alpha_{\text{high}}(\sigma)$ or less than $\alpha_{\text{low}}(\sigma)$; that is, they are affected by the firm-quality distribution g. Because this distribution is arbitrary, the effects are ambiguous. The equilibrium mass of firms is equal to $R/\bar{r}$ where R is the aggregate revenue and $\bar{r}$ is the average revenue. Because $\bar{r}$ could increase or decrease with σ depending on the firm-quality distribution, the equilibrium mass of firms could also decrease or increase. Q.E.D.

Proof of Proposition 4. Define

$$\begin{aligned}f(\sigma,P) &= \int_0^\infty \bigg[\frac{R(P\rho\alpha)^{\sigma-1}}{\sigma}[e_\alpha(\sigma,P)h^{\sigma-1} + (1-e_\alpha(\sigma,P))l^{\sigma-1}]\\ &\quad - e_\alpha(\sigma,P)v[\kappa'(e_\alpha(\sigma,P))]\bigg]g(\alpha)\,d\alpha, \qquad (43)\end{aligned}$$

where $e_\alpha(\sigma,P)$ is the optimal effort choice of the manager with firm quality α when the aggregate price is P and the elasticity of substitution is σ. If $P^*(\sigma)$ denotes the equilibrium aggregate price when the elasticity of substitution is σ, then it follows from (32) and (43) that

$$f(\sigma, P^*(\sigma)) = f_e. \qquad (44)$$

We note that

$$\begin{aligned}\frac{\partial f}{\partial\sigma} &= \int_0^\infty\bigg[\frac{\partial}{\partial\sigma}\bigg(\frac{R(P\rho\alpha h)^{\sigma-1}}{\sigma}\bigg)e_\alpha(\sigma,P)\\ &\quad + \frac{\partial}{\partial\sigma}\bigg(\frac{R(P\rho\alpha l)^{\sigma-1}}{\sigma}\bigg)(1-e_\alpha(\sigma,P))\\ &\quad + \bigg(\frac{R(P\rho\alpha h)^{\sigma-1}}{\sigma} - \frac{R(P\rho\alpha l)^{\sigma-1}}{\sigma}\bigg)\frac{\partial e_\alpha(\sigma,P)}{\partial\sigma}\\ &\quad - v[\kappa'(e_\alpha(\sigma,P))]\frac{\partial e_\alpha(\sigma,P)}{\partial\sigma}\\ &\quad - e_\alpha(\sigma,P)v'[\kappa'(e_\alpha(\sigma,P))]\\ &\quad \cdot\kappa''(e_\alpha(\sigma,P))\frac{\partial e_\alpha(\sigma,P)}{\partial\sigma}\bigg]g(\alpha)\,d\alpha. \qquad (45)\end{aligned}$$

By (30),

$$\begin{aligned}&\frac{R(P\rho\alpha h)^{\sigma-1}}{\sigma} - \frac{R(P\rho\alpha l)^{\sigma-1}}{\sigma}\\ &\quad = v[\kappa'(e_\alpha(\sigma,P))]\\ &\quad\quad + e_\alpha(\sigma,P)v'[\kappa'(e_\alpha(\sigma,P))]\kappa''(e_\alpha(\sigma,P)). \qquad (46)\end{aligned}$$

Substituting (46) in (45), we have

$$\begin{aligned}\frac{\partial f}{\partial\sigma} &= \int_0^\infty\bigg[\frac{\partial}{\partial\sigma}\bigg(\frac{R(P\rho\alpha h)^{\sigma-1}}{\sigma} - \frac{R(P\rho\alpha l)^{\sigma-1}}{\sigma}\bigg)e_\alpha(\sigma,P)\\ &\quad + \frac{\partial}{\partial\sigma}\bigg(\frac{R(P\rho\alpha l)^{\sigma-1}}{\sigma}\bigg)\bigg]g(\alpha)\,d\alpha. \qquad (47)\end{aligned}$$

After calculating the derivatives in the integrand above, we can show that $\partial f/\partial\sigma > 0$ if P exceeds a threshold $P_T(\sigma)$ and is less than 0 otherwise.

Next, by (46), we can rewrite (43) as follows:

$$\begin{aligned}f(\sigma,P) &= \int_0^\infty\bigg[\frac{R(P\rho\alpha)^{\sigma-1}l^{\sigma-1}}{\sigma}\\ &\quad + e_\alpha(\sigma,P)^2 v'[\kappa'(e_\alpha(\sigma,P))]\kappa''(e_\alpha(\sigma,P))\bigg]g(\alpha)\,d\alpha. \qquad (48)\end{aligned}$$

By (30) and the fact that κ', κ'', v, and v' are all increasing, the effort $e_\alpha(P,\sigma)$ increases with the aggregate price index P. It follows from (48) that $\partial f/\partial P > 0$.

By (44), the equilibrium price $P^*(\sigma)$ increases with f_e. Moreover, its support is $(0,\infty)$. Therefore, there exists a threshold level $f'_T(\sigma)$ of the entry cost such that $P^*(\sigma) > P_T(\sigma)$ if $f_e > f'_T(\sigma)$ and $P^*(\sigma) < P_T(\sigma)$ if $f_e < f'_T(\sigma)$. By (44) and the implicit function theorem,

$$\frac{dP^*(\sigma)}{d\sigma} = -\frac{\partial f/\partial\sigma}{\partial f/\partial P}\bigg|_{P=P^*(\sigma)}. \qquad (49)$$

By the above, it follows that $dP^*(\sigma)/d\sigma > 0$ if $f_e(\sigma) < f'_T(\sigma)$ and $dP^*(\sigma)/d\sigma < 0$ if $f_e(\sigma) > f'_T(\sigma)$. Q.E.D.

Proof of Proposition 5. Let $x = h^{\sigma-1} - l^{\sigma-1}$ and $y = h^{\sigma-1} + l^{\sigma-1}$. Define

$$f(x,P) = \int_0^\infty \left[\frac{R(P\rho\alpha)^{\sigma-1}}{\sigma}[e_\alpha(x,P)h^{\sigma-1} + (1-e_\alpha(x,P))l^{\sigma-1}] - e_\alpha(x,P)v[\kappa'(e_\alpha(x,P))]\right] g(\alpha)\,d\alpha, \quad (50)$$

where $e_\alpha(x,P)$ is the optimal effort choice of the manager with firm quality α when the aggregate price is P and the productivity risk is x. If $P^*(x)$ denotes the equilibrium aggregate price when the productivity risk is x, then it follows from (32) and (50) that

$$f(x, P^*(x)) = f_e. \quad (51)$$

By (30), the equilibrium effort of a manager with firm quality α solves

$$\alpha^{\sigma-1}\left[\frac{R(P^*(x)\rho)^{\sigma-1}}{\sigma}x\right] = v[\kappa'(e^*_\alpha(x))] + e^*_\alpha(x)v'[\kappa'(e^*_\alpha(x))]\kappa''(e^*_\alpha(x)), \quad (52)$$

where we explicitly indicate the dependence of the equilibrium effort on the productivity risk x. If $P^*(x)$ increases with x, then because κ', κ'', v, and v' are all increasing, $e^*_\alpha(x)$ increases.

Suppose that $P^*(x)$ decreases with x. We show that the LHS of (52) is still increasing with x. Suppose that the contrary holds. Because κ', κ'', v, and v' are all increasing, $e^*_\alpha(x)$ decreases. By (52), we can rewrite (50) as follows:

$$f(x,P) = \int_0^\infty \left[(e_\alpha(x,P))^2 v'[\kappa'(e_\alpha(x,P))]\kappa''(e_\alpha(x,P)) + \frac{R(P\rho\alpha)^{\sigma-1}}{\sigma}l^{\sigma-1}\right] g(\alpha)\,d\alpha. \quad (53)$$

Because $l^{\sigma-1}$ decreases with x (because $y = h^{\sigma-1} + l^{\sigma-1}$ is fixed), it follows from (53) that if $P^*(x)$ decreases with x, $f(x, P^*(x))$ is decreasing, which contradicts the equilibrium condition (51). Hence, the LHS of (52) is increasing with x.

It follows that managerial effort increases with the productivity risk. By (29), expected managerial compensation, $e^*_\alpha(x)v[\kappa'(e^*_\alpha(x))]$, also increases with the productivity risk. By (31) and condition (34), managerial pay-performance sensitivities decline with productivity risk. Because the average managerial compensation increases, it follows from the equilibrium condition (51) that the average gross profit of all firms increases. By (8) and (9), the gross profit of each firm is a constant proportion of its revenue. Hence, the average revenue of all firms also increases. Q.E.D.

Proof of Proposition 6. By (30),

$$\left[\frac{R(P\rho\alpha)^{\sigma-1}}{\sigma}[e_\alpha(P)h^{\sigma-1} + (1-e_\alpha(P))l^{\sigma-1}] - e_\alpha(P)v'[\kappa'(e_\alpha(P))]\right] = \frac{R(P\rho\alpha)^{\sigma-1}}{\sigma}l^{\sigma-1} + (e_\alpha(P))^2 v'[\kappa'(e_\alpha(P))]\kappa''(e_\alpha(P)),$$

where we explicitly indicate the dependence of the optimal effort on the aggregate price P. Because $e_\alpha(P)$ increases with α by Proposition 1, and v', κ', and κ'' are increasing, the integrand on the LHS of (32) increases with α. It follows from the properties of first-order stochastic dominance that, *for a given P*, the LHS of (32) is greater when the firm-quality distribution is g_1. Hence, the equilibrium aggregate price index must be lower when the firm-quality distribution is g_1 to satisfy the equilibrium condition (32).

By (30) and (34), for a given firm quality α, the manager's effort is lower while her pay-performance sensitivity is higher. By (37), the expected revenue of a firm with firm quality α is lower when the firm-quality distribution is g_1 because the aggregate price index and the manager's effort are lower. By (9) and (38), the firm's expected gross and net profits are also lower. Q.E.D.

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