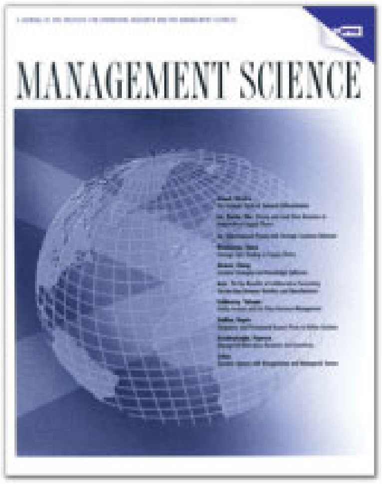

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Keyword Search Advertising and First-Page Bid Estimates: A Strategic Analysis

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In using the generalized second-price (GSP) auction to sell advertising slots, a search engine faces several challenges. Advertisers do not truthfully bid their valuations, and the valuations are uncertain. Furthermore, advertisers are budget constrained. In this paper we analyze a stylized model of the first-page bid estimate (FPBE) mechanism first developed by Google and demonstrate its advantages in dealing with these challenges. We show why and when the FPBE mechanism yields higher profits for the search engine compared with the traditional GSP auction and the GSP auction with advertiser-specific minimum bid. In the event that a high-valuation advertiser is budget constrained, the search engine can use the FPBE mechanism to alter the listing order with the intent of keeping the high-valuation advertiser in the auction for a longer time. The resulting increase in the search engine's profits is not necessarily at the expense of the advertisers because the combined profits of the advertisers and the search engine increase.

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1. Introduction

Keyword search advertising is an important form of online advertising, with annual revenues in the United States exceeding $18 billion in 2013 (Interactive Advertising Bureau 2014). When consumers search for information using keywords, the search engine displays a list of relevant information along with a slate of advertisements related to the keywords. Typically, the search engine sells the slots for these keyword advertisements through a generalized second-price (GSP) auction where the kth-highest bidder wins the kth slot in the slate of advertisements but pays the $(k+1)$th-highest bid as its price per click (often adjusted for bidders' quality scores). Because a multitude of searches occur on a continuous basis for a variety of keywords, it is easy to see that an auction is a suitable mechanism to sell slots for keyword advertising. In addition, the GSP auction results in higher-valuation advertisers occupying higher positions in the auction (Varian 2007) and is also more stable than the first-price auction (Edeleman and Ostrovsky 2007).

The GSP auction, however, is not free from limitations for the search engine (Edelman et al. 2007). Therefore, it is important for search engines and academic researchers to explore mechanisms that can improve the GSP auction. In this paper, we investigate one such mechanism: the first-page bid estimate (FPBE), which was implemented by Google as a replacement for minimum bids. The FPBE is a keyword- and advertiser-specific estimate of the bid that the advertiser would need to submit to get its sponsored link for the specific keyword to appear on the first page of search results. Since a featured sponsored link on the first page is very valuable to advertisers, the FPBE may induce advertisers to adhere to the estimate. However, unlike a minimum bid, the FPBE is not a requirement but only an estimate. In other words, it is possible that even when an advertiser does not adhere to the FPBE, its advertisement may appear on the first page. We examine how and when the FPBE can improve profitability for search engines.

To appreciate the limitations of the GSP auction, first note that the GSP auction does not induce truthful bidding, and thus advertisers may bid and pay below their valuations (Edelman et al. 2007). This could become particularly significant when there is uncertainty about advertisers' valuations (Gomes and Sweeney 2014). Although many theoretical models assume common knowledge of advertisers' valuations, in practice, advertisers may be uncertain about the valuations of other advertisers, and the search engine may also be uncertain about advertisers' valuations. These uncertainties affect bidding strategies and, in turn, auction outcome, especially the per-click payments to the search engine.

Valuation uncertainties further complicate the problems arising from advertisers' limited campaign budgets. Typically, advertisers specify a budget for their campaign, and the search engine drops advertisers from the auction if their budget is exhausted. On dropping a budget-constrained but high-valuation advertiser, the search engine loses revenue not only from the dropped advertiser but also from the other advertisers who have been allocated slots (by virtue of being ranked above the dropped advertiser).

Over the years, search engines have tried to augment the GSP auction to overcome some of these limitations and increase their profits. Both Google and Yahoo! used to stipulate an advertiser-specific minimum bid (ASMB). This minimum bid motivates an advertiser to bid more than what it would naturally do under the generalized second-price auction because the advertiser will be dropped from the auction if it fails to conform to the stipulation. Although the ASMB could motivate advertisers to bid truthfully, it comes with a downside. If there is uncertainty in advertisers' valuations, it could hurt the search engine's profits and also lead to inefficient allocation of slots. Google has moved away from the ASMB and now pursues the FPBE. Unlike advertiser-specific minimum bid, as noted earlier, the first-page bid estimate is not always enforced. In particular, as indicated on Google's AdWords Help page, an advertiser who bids lower than the estimate could still feature in the slate of advertisements.[1]

The first-page bid estimate can serve several functions. For example, the bid estimate can inform advertisers about past successful bids and serve as a benchmark while selecting bids. The search engine can use the bid estimate to coordinate advertising traffic to different keywords as well. It is possible that the set of advertisers bidding for a given keyword (and hence the set of quality scores) may vary from time to time. Therefore, another possible strategic role of the FPBE is that it can allow the search engine to adjust the bidding outcomes as the set of advertisers changes. Because the FPBE is only an estimate, it sometimes allows advertisers who do not conform to the bid estimate to participate in the auctions. Advertisers appreciate this important flexibility in the FPBE and advise others not to bid up to the estimate.[2] Some advertisers view the FPBE as the minimum bid required to participate in the auction. In developing our model, we focus on a particular aspect of the FPBE and seek an answer to the question of how probabilistic enforcement of the bid estimate affects auction outcomes and addresses some of the common challenges with the traditional GSP auction. Furthermore, we compare the profitability of the FPBE mechanism against the traditional GSP auction and a variant of the GSP auction where the search engine stipulates advertiser-specific minimum bids. To facilitate comparative analysis, we use a framework of keyword search advertising, based on Varian (2007) but enriched with two salient features:

- *Uncertain valuations*. In contrast to prior literature but consistent with market reality, we allow for uncertainty about advertisers' valuations. The uncertainty reflects the fact that an advertiser's valuation depends on what the advertiser earns in the product market, which in turn is contingent on private information such as the type of customer who buys its product, selling cost, and inventory level. Thus, sometimes the search engine and other advertisers only imperfectly infer the valuation of an advertiser. However, each advertiser knows its own valuation of a click. To facilitate exposition, we consider a simple setting where a search engine caters to three advertisers with the valuation of one advertiser being uncertain.
- *Nested structure*. Our formulation nests the GSP auction, ASMB mechanism, and FPBE mechanism as special cases. In essence, in the GSP auction the search engine does not stipulate any advertiser-specific requirement. In the ASMB mechanism the search engine always enforces the minimum bid, whereas in the FPBE mechanism the bid estimate is not always enforced.

Using our model, we compare the profitability of the FPBE mechanism with that of the traditional GSP auction and ASMB mechanism, derive the optimal level of enforcement of the FPBE, and assess the ability of the FPBE mechanism to handle budget-constrained advertisers. We begin our analysis of the theoretical model by comparing the ASMB and FPBE mechanisms. In the ASMB mechanism, an advertiser will always be dropped from the auction if its bid is lower than the minimum amount stipulated by the search engine. In the FPBE mechanism, even if an advertiser does not conform to the bid estimate, it might be allowed to participate in the auction. Because the threat of being dropped from the auction is lower in FPBE mechanism, one might expect advertisers to bid lower and the search engine to earn lower profits. Contrary to this naïve view, our analysis shows that the FPBE yields higher profits for the search engine.

To understand the intuition for this result, note that the search engine is uncertain about the valuations of advertisers. If the search engine sets the advertiser-specific minimum bid higher than an advertiser's actual valuation, then the advertiser will not conform to the ASMB. In such instances, either an advertiser is replaced by a less profitable advertiser or the slot goes vacant, thereby hurting the search engine's profits. If the advertiser-specific minimum bid is less than an advertiser's actual valuation, then the advertiser will conform to the ASMB, but the search engine

[1] See https://support.google.com/adwords/answer/105665?hl=en (accessed December 23, 2014).

[2] See, for example, Roubtsov (2009) and Hanapin (2011).

will leave some money on the table. Thus, in the ASMB mechanism any misalignment between the required minimum bid and an advertiser's actual valuation reduces the search engine's profits. In the FPBE mechanism, the search engine does not always enforce the bid estimate. As such, when an advertiser does not conform to the FPBE, the search engine will not always replace the advertiser with a less profitable one. The search engine yet needs to probabilistically enforce the FPBE such that all the advertisers are motivated to conform to the FPBE rather than not conform and risk being dropped from the auction. By optimally enforcing the FPBE, the search engine can enjoy the benefits of a higher bid without incurring too much loss from potential misalignment between an advertiser's valuation and the bid estimate. We further show how the optimal probability of enforcing FPBE systematically varies with the relative attractiveness of individual advertising slots and relative valuations of advertisers. In essence, this analysis highlights how uncertainty about advertisers' valuations can make it more profitable for the search engine to adopt the FPBE rather than the ASMB mechanism.

Although the preceding discussion shows that the FPBE mechanism can be more profitable than the ASMB mechanism, it does not clarify whether or not the FPBE mechanism is more profitable than the traditional GSP auction. An important advantage of the GSP auction is that it does not require the search engine to know advertisers' valuations. Furthermore, in equilibrium the advertisers' sponsored links will be listed in the same order as the valuations of those advertisers. The downside to the traditional GSP auction is that advertisers will not truthfully bid their valuations, and as such, the search engine leaves some money on the table. Our analysis shows that the FPBE mechanism can help the search engine to earn more profits than the GSP auction. In particular, when the uncertainty in the valuations of individual advertisers is low, the profits lost by the search engine because of misalignment between its high first-page bid estimate and the actual valuation of an advertiser turn out to be low. In such instances, it is more profitable for the search engine to adopt the FPBE mechanism rather than the traditional GSP auction. This analysis helps us to understand why Google chose to provide the first-page bid estimate rather than revert to the GSP auction.

Finally, we examine how the FPBE can help the search engine to handle budget-constrained advertisers. We find that when a high-valuation advertiser is budget constrained, the search engine can alter the listing order to keep the high-valuation advertiser in the auction for a longer period, thereby improving the search engine's profits (see also Katona and Sarvary 2009 and Jerath et al. 2011). One may wonder whether the search engine's gains are at the expense of the advertisers. We show that the search engine could improve total welfare by altering the listing order of advertisers, implying that the search engine's gains are not necessarily at the expense of the advertisers.

1.1. Related Literature and Theoretical Contribution

The early attempt to model keyword search advertising framed the search engine's problem as one of auctioning n asymmetric goods using a unidimensional bid (Edelman et al. 2007, Varian 2007). In this auction, advertisers do not always truthfully bid their valuations. Moreover, if the search engine imposes a universal minimum bid, it increases the equilibrium bid of not only the advertiser taking the last slot but also that of other advertisers. Interestingly, the increase seen in search engine's profits is mostly due to the higher bid of advertisers on higher slots rather than the one in the last slot (Edelman and Schwarz 2010). However, the universal minimum bid can impair efficiency (Liu et al. 2010, Athey and Ellison 2011). Our work builds on this body of literature about the generalized second-price auction and universal minimum bid but examines a different set of issues. We model the advertiser-specific minimum bid and first-page bid estimate and examine their impacts on search engine profits. In contrast to prior literature (Edelman et al. 2007, Varian 2007, Katona and Sarvary 2010), in our formulation each advertiser knows its valuation but may be uncertain about the valuation of other advertisers. The search engine is also uncertain about the valuation of an individual advertiser. Furthermore, prior literature views the search engine as a passive player that merely allocates the advertising slots according to the results of the GSP auction. It is important to note that we treat the search engine as a strategic player that sets the enforcement probability as well as the bid estimates or minimum bids and, in the process, changes the bidding behavior of advertisers.

In a more recent paper, Jerath et al. (2011) show that a high-quality advertiser may bid lower than a low-quality advertiser and secure a lower position in the slate of advertisements and yet obtain more clicks. A potential explanation for this "position paradox" is that the high-quality advertiser has less incentive to bid for the higher slot because some consumers will search for the high-quality advertiser even when it features in a lower slot. Katona and Sarvary (2009) provide another explanation. Specifically, a more relevant advertiser is listed higher in the organic search results, and hence it draws enough clicks through its organic link. Thus the more relevant advertiser might opt for a lower slot in the slate of advertisements, pay a lower price, and also draw fewer clicks. Thus, both of these papers offer an advertiser-based explanation for the paradox. Although we investigate a new mechanism and address a different set of issues, we offer a search-engine-based explanation for the position paradox. Specifically, we show that when the high-valuation advertiser is budget constrained, the search engine may modify the listing order such that the high-valuation advertiser takes a lower position.

Our paper is also related to the literature on common values auction and procurement auction. In common values auction, a secret reserve price can induce more bidders to participate in the auction and increase the expected revenue of the seller (Vincent 1995). Bajari and Hortaçsu (2003) confirm this theoretical prediction in a simulation study based on the structural parameter estimates of a coin auction (see also Katkar and Reiley 2006). The idea of a search engine imperfectly enforcing the FPBE has some resemblance to the notion of secret reserve price. However, in our model, the enforcement probability is common knowledge and is enforced as announced. Similarly, a search engine's attempt to alter the listing order of advertisers may be viewed as a form of favoritism (see Branco 2002). For example, when it is too costly for inefficient domestic suppliers to adopt a new technology, these suppliers may not adopt the new technology although the domestic government would prefer them to do so. Hence, in procurement auctions, the government may optimally favor these inefficient suppliers and encourage them to adopt the new technology. In a similar vein, to maximize its profits and improve overall efficiency, the search engine can strategically alter the listing order of a high-valuation budget-constrained advertiser.

1.2. Managerial Significance

Both Google and Yahoo! used to impose advertiser-specific minimum bids for keywords. Before making its bid for a given keyword, each advertiser could see on the administrative page the minimum bid set for it by the search engine. If the advertiser chose not to conform to the required minimum bid for a keyword, then its advertisements would not be displayed along with that keyword. Our analysis provides a rationale for why a search engine may choose to transition from the traditional GSP auction to the ASMB. In August 2008, Google replaced advertiser-specific minimum bids with advertiser-specific first-page bid estimates for keywords.[3] By focusing on a key facet of the FPBE, we offer an insight into why the FPBE can improve a search engine's profits when advertisers' valuations are uncertain. This insight helps us to better understand Google's motivation to adopt the FPBE mechanism. Finally, our analysis shows how a search engine can handle budget-constrained advertisers in a manner that raises its profits and yet increases total welfare.

The rest of the paper is organized as follows. In the following section, we propose a model of keyword search advertising. In §3, we examine the optimality of the FPBE mechanism by contrasting the profitability of the FPBE, ASMB, and GSP auction and by probing into the optimal enforcement probability. In §4, we explore how the FPBE mechanism can help to handle budget-constrained advertisers. Finally, in §5, we conclude the paper by summarizing the findings and outlining opportunities for further research.

2. Model

In this section, we present a model of sponsored search advertising that captures the essence of the FPBE, GSP auction, and ASMB. Consider a market for online advertising initiated by keyword search. When a consumer enters a keyword, the search engine provides a list of relevant information that is available online. In addition to this information, the search engine presents a set of sponsored links (advertisements) that are related to the keyword. These advertising links are displayed one below the other, and the click-through rate for advertiser k in slot j is given by $s_j r_k$, where s_j is the slot-specific effect and r_k is the advertiser-specific effect. The slot-specific effect depends on the rank order of the link in the slate of advertisements with $s_j > s_{j+1}$. The advertiser-specific effect reflects the relevance of an advertiser to the specific keyword, and it is also referred to as an advertiser's quality score. The size of r_k could depend on factors such as semantic relevance of a firm's advertisement to the keyword, quality of landing page, and historical click-through rate.

2.1. Advertisers and the Search Engine

To facilitate exposition, we assume that there are three advertisers, labeled A, B, and C, and a single search engine. The search engine can accommodate two sponsored advertisements in the first page of keyword search results with $s_1 > s_2 > 0$. Advertisers vary in their valuation of a click as in Edelman et al. (2007) and Varian (2007). Advertiser k, where $k \in \{A, B, C\}$, submits a bid b_k based on its valuation v_k for a click. The search engine ranks the advertisers according to their potential profitability, which is given by the product of their bid b_k and their relevance r_k. According to the generalized second-price auction, each advertiser's payment per click is the minimum price that guarantees the advertising slot it wins. Specifically, when advertiser k wins slot j, its payment per click is $p_{kj} = (r_{(j+1)} b_{(j+1)})/r_k$, where $(j+1)$ refers to the advertiser ranked $(j+1)$ in the set $\{r_k b_k\}$. Therefore, the profits earned by advertiser k on winning slot j are given by

$$\Pi_k = N s_j r_k (v_k - p_{kj}) = N s_j (r_k v_k - r_{(j+1)} b_{(j+1)}), \qquad (1)$$

where N denotes the number of users searching for the keyword. For simplicity, we normalize the value of N to be 1. Sometimes the $(j+1)$th slot becomes vacant

[3] For details, refer to Claiborne (2008).

because some advertisers have failed to conform to the required universal minimum bid, and in such a case the search engine collects the default payment, namely V_M, instead of $r_{(j+1)}b_{(j+1)}$ from the winner of the jth slot. As in Athey and Nekipelov (2011) and Liu et al. (2010), the default payment can be viewed as the universal minimum bid (in units of relevance-weighted bid). Further, advertiser k's profits reduce to zero if it fails to win a slot in the auction. This merely suggests that the incremental profits from keyword search advertising will be zero but does not imply that the advertiser has no other source of obtaining customers. As $p_{kj} = r_{(j+1)}b_{(j+1)}/r_k$, the search engine's profits in a given period can be expressed as

$$\Pi_{SE} = \sum_{j\in\{1,2\}} s_j r_k p_{kj} = \sum_{j\in\{1,2\}} s_j r_{(j+1)} b_{(j+1)}. \tag{2}$$

Advertisers are often uncertain about the valuations of competing advertisers. Furthermore, the search engine is also uncertain about each advertiser's valuation. This is because an individual advertiser's valuation could be affected by factors that are not observed by the search engine, such as heterogeneity in consumer valuation, selling cost, and inventory level. For expositional reasons, we allow for uncertainty only in the valuation of advertiser B.[4] In particular, we assume that advertiser B knows its own valuation, but the valuation is not perfectly revealed to either the search engine or the competing advertisers; advertiser B's valuation is v_B^L with probability β and v_B^H with probability $1-\beta$, where $v_B^H > v_B^L$ and $0 < \beta < 1$. Regardless of the realized valuation of advertiser B, the rank of advertisers is such that $r_A v_A > r_B v_B > r_C v_C$, implying that $r_A v_A > r_B v_B^H > r_B v_B^L > r_C v_C$. To simplify the notation, define $V_k \equiv r_k v_k$. Now we proceed to incorporate the idea of the first-page bid estimate.

2.2. FPBE and ASMB Mechanisms

As the name suggests, the FPBE is an estimate of the bid that, if conformed to, would help an advertiser's link to appear on the first page of keyword related information. Because most advertisers want their advertisements to be displayed on the first page of search results, conforming to the FPBE will help achieve this goal. According to empirical research, slots in the second page are of no significant value to the advertisers (e.g., Ruby 2010), and our model reflects this reality. Under the ASMB, if an advertiser does not conform to the minimum bid for a keyword, its advertisement is not featured in the slate of advertisements displayed along with the keyword. Unlike the ASMB, the first-page bid estimate is not always enforced. That is, if an advertiser does not conform to its first-page bid estimate, there is still some chance that its advertisement will be displayed along with keyword-related information on the first page.[5] The search engine can choose to use the ASMB or FPBE. If the search engine adopts neither the ASMB nor FPBE mechanism, the game reduces to the standard GSP auction.

2.3. Decision Sequence

If the search engine chooses to implement the ASMB mechanism, the game proceeds in two stages: In stage 1, the search engine chooses a minimum bid for each advertiser—namely, m_k^{ASMB}. In stage 2, each advertiser chooses its bid amount b_k. The search engine entertains advertiser k's bid for the auction only if $b_k \geq m_k^{\text{ASMB}}$, suggesting that if $b_k < m_k^{\text{ASMB}}$, the advertiser is automatically eliminated from the auction.

If the search engine adopts the FPBE mechanism, the game proceeds in a similar fashion except for the following differences. Now the search engine does not perfectly enforce the first-page bid estimates but does so with probability $\alpha \in (0, 1)$. Thus, even when $b_k < m_k^{\text{FPBE}}$, the advertiser will be considered for the auction with probability $(1-\alpha)$. In stage 1, the search engine sets an FPBE for each advertiser, m_k^{FPBE}, and decides on the enforcement probability, α. In stage 2, as before, each advertiser chooses its bid amount b_k. Although the enforcement probability is known to advertisers, whether or not the FPBE will be actually enforced is not known to advertisers when they decide on b_k. If an advertiser bids lower than its FPBE ($b_k < m_k^{\text{FPBE}}$), the advertiser has chosen not to conform to the FPBE, and as such, it will be dropped from the auction with probability α. Otherwise, the advertiser stays in the auction with probability 1. Here, we assume that the search engine faithfully implements the enforcement probability. Using a repeated game, Amaldoss et al. (2013) have shown that the search engine will implement the FPBE as announced out of concern for its reputation. Furthermore, in our one-shot game the search engine sets both the FPBE and the enforcement probability, whereas in a repeated game the enforcement probability depends on the FPBE and advertisers' beliefs.

Prior research on keyword search advertising has modeled the competition among advertisers as a one-shot game (Edelman et al. 2007, Varian 2007). A one-shot bidding game captures the essence of the decision problem, abstracts away from some dynamic aspects of the institution, and keeps the formulation analytically

[4] We thank the department editor for this helpful suggestion. A model with uncertainty in the valuations of all advertisers generates qualitatively similar results (see Amaldoss et al. 2013 for further details).

[5] Google's official AdWords Help page notes that "your ad can still appear if your bid does not meet this estimate, but it's more likely not to appear on the first page of search results"; see https://support.google.com/adwords/answer/105665?hl=en (accessed December 23, 2014).

tractable. As in prior literature, we also model the bidding subgame among advertisers as a one-shot decision and derive the symmetric Nash equilibrium where each advertiser makes the lowest possible bid. Note that the lower-bound solution has been suggested in the literature as the compelling equilibrium (Varian 2007). A key difference between our model and prior literature is that we view the search engine as a strategic player that first chooses the enforcement probability and the bid estimates or minimum bids. After observing the decision of the search engine, advertisers play the bidding game in the second stage. We examine the subgame perfect equilibrium of this two-stage game to understand normative behavior.

3. Optimality of the FPBE

Using the model outlined in the previous section, we next analyze the optimality of the FPBE mechanism. To begin with, in §§3.1 and 3.2, we contrast the profitability of the FPBE mechanism with that of the ASMB mechanism and traditional GSP auction. Then in §3.3, we analyze the factors that influence the optimal level of enforcement of the FPBE mechanism.

3.1. Relative Profitability of the FPBE and ASMB

Since in a GSP auction advertisers do not bid truthfully, the search engine could curb this tendency by imposing an ASMB. However, in the presence of the uncertainty on advertisers' valuation, it is not clear whether the ASMB provides the best profits to the search engine. In this section, we examine whether the FPBE mechanism can yield higher profits than the ASMB mechanism. Note that a fundamental difference between these two mechanisms is that the search engine always enforces the advertiser-specific minimum bids but probabilistically enforces the first-page bid estimates. The more stringent enforcement of the ASMB may lead some to believe that the ASMB mechanism will yield higher profits than the FPBE mechanism. On examining the equilibrium profits of the search engine under these two mechanisms, we have the following result.

Proposition 1. *Compared with the case when it imposes the ASMB, the search engine's profits are weakly higher when it provides the FPBE and enforces it with probability* $\alpha \geq \alpha^{\mathrm{FPBE}}$, *where* $\alpha^{\mathrm{FPBE}} \equiv 1 - s_1(V_A - V_B^H)/\{s_2(V_A - V_C)\}$.

We prove this claim in the appendix. To see the intuition for this result, first note that if the search engine knows every advertiser's valuation, it could set the minimum bid at the individual advertiser's valuation and earn more profits. However, the search engine is uncertain about advertiser B's valuation: advertiser B's valuation could be v_B^H with probability β, but v_B^L with probability $(1-\beta)$. This uncertainty has a bearing on the search engine's profits. On the one hand, the search engine could set $m_B^{\mathrm{ASMB}} = v_B^H$ for advertiser B. If the realized valuation of the advertiser is indeed high, the search engine earns more profits. But with probability β, the realized valuation will be v_B^L, and in such instances advertiser B will not conform to the minimum bid and will be dropped from the auction. When advertiser B is dropped from the auction, it is replaced with a less profitable advertiser, advertiser C, and this hurts the search engine's profits. On the other hand, if the search engine sets $m_B^{\mathrm{ASMB}} = v_B^L$, advertiser B always conforms to the minimum bid. However, by setting a low minimum bid, the search engine passes up an opportunity to earn more when the realized valuation of the advertiser is high. Therefore, while deciding m_B^{ASMB}, the search engine carefully weighs the benefits of a higher minimum bid against the prospect of losing a more profitable advertiser. More generally, in the ASMB mechanism it is profitable for the search engine to set a high minimum bid for advertiser B, that is, $m_B^{\mathrm{ASMB}} = v_B^H$, only if the probability of low valuation is below a threshold. We see a similar pattern in the FPBE mechanism: it is profitable to provide the first-page bid estimate $m_B^{\mathrm{FPBE}} = v_B^H$ if β is sufficiently low. For a detailed analysis on the profitability of the FPBE and ASMB mechanisms, see Lemmas A2 and A6 in the online appendix (available as supplemental material at http://dx.doi.org/10.1287/mnsc.2014.2033). Next, we clarify why it can be more profitable for the search engine to provide the FPBE rather than implement the ASMB.

The FPBE mechanism turns out to be more profitable than ASMB mechanism for the search engine, as long as it enforces the first-page bid estimate with probability $\alpha \geq \alpha^{\mathrm{FPBE}}$ but not always (perfectly). Recall that in the ASMB mechanism, advertiser B will conform to the higher minimum bid $m_B^{\mathrm{ASMB}} = v_B^H$ when its realized valuation is high. However, if its realized valuation is low, advertiser B will not conform to m_B^{ASMB}; hence, the search engine drops the advertiser from the auction and offers the slot to advertiser C. In contrast to the ASMB mechanism, under the FPBE mechanism the search engine does not always enforce $m_B^{\mathrm{FPBE}} = v_B^H$. By not always enforcing the FPBE, the search engine reduces the need to replace advertiser B with the less profitable advertiser when the realized valuation is lower than m_B^{FPBE}. Thus, although the lower enforcement of the FPBE in comparison to the ASMB may seem like a weakness of the FPBE mechanism, it is precisely the reason for the higher profitability of the FPBE mechanism.

While enforcing the FPBE mechanism with a lower probability, the search engine needs to ensure that every advertiser remains motivated to conform to the FPBE and does not take the chance of bidding an amount lower than m_k^{FPBE} when its valuation is greater than or equal to m_k^{FPBE}. In fact, the search engine does so by enforcing the FPBE with a sufficiently high probability; that is, $\alpha \geq \alpha^{\text{FPBE}}$. To see this, denote the profits of advertiser k on conforming to the FPBE by $X_k > 0$ and the profits on not conforming to the FPBE by $(1-\alpha)Y_k$. If the enforcement probability α is such that $X_k \geq (1-\alpha)Y_k$, then advertiser k will find it profitable to conform to the FPBE. It is always possible to obtain such an enforcement probability for each advertiser (see for proof Lemma A3 in the online appendix). To ensure that all advertisers conform to the FPBE, the search engine should enforce the FPBE with probability $\alpha \geq \max\{1 - X_k/Y_k\}$. Thus a well-managed FPBE gives the search engine more revenue and at the same time reduces the potential loss due to misalignment between m_B^{FPBE} and the realized valuation v_B.

3.2. Relative Profitability of the FPBE and Traditional GSP Auction

So far, we analyzed the FPBE mechanism and contrasted it with the ASMB mechanism. Although the FPBE reduces the potential loss as a result of the misalignment problem, it does not completely eliminate the misalignment possibility. Thus, in establishing the optimality of the FPBE mechanism, we still need to examine whether the FPBE mechanism is more profitable than the traditional generalized second-price auction.

To understand the answer to this question, note that in the symmetric Nash equilibrium of the GSP auction, each advertiser makes a bid based on its actual valuation, such that the advertiser cannot improve its profits by switching its position with any other advertiser (Varian 2007). As such, advertiser A takes the first slot and advertiser B occupies the second slot. More generally, the kth slot winner will be the advertiser with the kth rank in the set $\{r_k v_k\}$. We prove this claim in the online appendix (see Lemma A1 in the appendix). In the GSP auction, however, advertisers do not truthfully bid their valuations, as we show below. Hence, the bids obtained through a GSP auction will be lower than those under an FPBE mechanism.

However, the major problem in adopting the FPBE mechanism is that it could lead to misalignment between an advertiser's realized valuation and the FPBE imposed by the search engine. When the FPBE is above an advertiser's realized valuation, the advertiser fails to conform to the FPBE, and with some probability the advertiser is replaced by a lower-valuation advertiser, or the slot goes vacant. This hurts the search engine's profits. However, if the within-advertiser heterogeneity in valuation is small, the negative impact as a result of the misalignment between v_B and m_B^{FPBE} is attenuated. Thus, in this case, the search engine earns more on adopting the FPBE rather than the GSP auction. Since in our model only the valuation of advertiser B is uncertain, the within-advertiser heterogeneity in valuation is given by $V_B^H - V_B^L$. Our analysis shows that if this heterogeneity is less than a measure of between-advertiser heterogeneity—namely, $[s_2/\{(1-\beta)(s_1-s_2)\}](V_B^L - V_C)$—the FPBE turns out to be more profitable than the GSP auction. Hence we have the following finding.

Proposition 2. *The search engine's profits under the FPBE mechanism are higher than those under the GSP auction if the within-advertiser heterogeneity in realized valuations is below a critical threshold.*

This finding may go against the grain of some of our intuitions. Specifically, in contrast to the GSP auction, the search engine can leverage two additional levers to augment its profits under the FPBE mechanism—namely, the bid estimates and the enforcement probability. Given these two additional levers, one might expect the FPBE mechanism to weakly dominate the GSP auction at the very least. Yet our analysis suggests that, if within-advertiser heterogeneity in valuation is high, the FPBE could be less profitable. This is because in such circumstances the misalignment between the first-page bid estimates and the realized valuations of the advertisers may become too costly. This highlights the need to consider the fit between the FPBE mechanism and the market context when the search engine decides whether or not to adopt the new mechanism.

To facilitate a more detailed understanding of the proposition, below we discuss the profitability of the GSP auction given the uncertainty in valuations of all advertisers, and then we compare the GSP profits with the FPBE profits. In a GSP auction, when $V_A > V_B > V_C$, the equilibrium listing order will be A-B-C, as we show in the online appendix. The listing order has strong implications for each advertiser's bid. First, for the listing order to hold, advertiser A should find it (weakly) profitable to take the first slot rather than the second slot. Consequently, we have

$$s_1(r_A v_A - r_B b_B) \geq s_2(r_A v_A - r_C b_C). \tag{3}$$

An implication of this condition is that $b_B \leq (1 - s_2/s_1)(r_A v_A/r_B) + (s_2/s_1)(r_C b_C/r_B)$. Furthermore, advertiser B cannot improve its payoff by exchanging positions with advertiser A, which is ranked one slot above. Hence, we have the following condition:

$$s_2(r_B v_B - r_C b_C) \geq s_1(r_B v_B - r_B b_B). \tag{4}$$

The above condition, in turn, gives the lowest possible bid for advertiser B: $b_B = (1 - s_2/s_1)v_B + (s_2/s_1) r_C b_C / r_B$.

Second, we know that in equilibrium $r_A b_A \geq r_B b_B$, and hence $b_A \geq r_B b_B / r_A$. Because advertiser A does not know advertiser B's valuation, advertiser A needs to make its bid based on the highest possible valuation of advertiser B, which is v_B^H, in order to guarantee itself the top slot. Hence, the lowest bid that will guarantee advertiser A the first slot is $(1 - s_2/s_1)(r_B v_B^H/r_A) + (s_2/s_1)(r_C b_C/r_A)$.

Third, as advertiser B should find it profitable to participate in the auction, we have

$$s_2(r_B v_B - r_C b_C) \geq 0. \tag{5}$$

This equation provides the upper bound on advertiser C's bid. That is, $b_C \leq r_B v_B/r_C$. In equilibrium, advertiser C should not be able to improve its profits by exchanging its position with that of advertiser B. Hence, we have

$$0 \geq s_2(r_C v_C - r_C b_C). \tag{6}$$

From the above condition, we can see that the lowest possible equilibrium bid for advertiser C is $b_C = v_C$. Thus, the lowest possible equilibrium bids for advertisers A and B are given by $b_A = (1 - s_2/s_1)(r_B v_B^H/r_A) + (s_2/s_1)(r_C v_C/r_A)$ and $b_B = (1 - s_2/s_1)v_B + (s_2/s_1)(r_C v_C/r_B)$, respectively.

As the search engine's revenue from the advertiser in slot k is given by $s_k r_{(k+1)} b_{(k+1)}$, the total payment from advertiser A is $(s_1 - s_2) r_B v_B + s_2 r_C v_C$ and that from advertiser B is $s_2 r_C v_C$. Since the search engine is uncertain about the valuation of advertiser B, the expected profits of the search engine are as follows:

$$\begin{aligned}\Pi_{SE}^{\text{GSP}} &= (s_1 - s_2) r_B v_B + 2 s_2 r_C v_C \\ &= (s_1 - s_2)\{\beta V_B^L + (1-\beta) V_B^H\} + 2 s_2 V_C.\end{aligned} \tag{7}$$

In the online appendix (see Lemma A6), we also show that the search engine's profits under the FPBE are at least greater than

$$\Pi_{SE}^{\text{FPBE}(1)} \equiv s_1 V_B^L + s_2 V_C^L. \tag{8}$$

Recall that $(V_B^H - V_B^L)$ reflects the within-advertiser heterogeneity in the relevance-weighted valuation of advertiser B and that $[s_2/\{(1-\beta)(s_1 - s_2)\}](V_B^L - V_C)$ is a measure of the between-advertiser heterogeneity in valuations. On comparing (7) and (8), we obtain $\Pi_{SE}^{\text{FPBE}(1)} \geq \Pi_{SE}^{\text{GSP}}$ if the within-advertiser heterogeneity is below the critical threshold of the between-advertiser heterogeneity. Therefore, under this condition, it follows that $\Pi_{SE}^{\text{FPBE}} \geq \Pi_{SE}^{\text{GSP}}$.[6] Finally, note that when β is very close to 0 or 1 (i.e., when the uncertainty in valuations is almost resolved), the FPBE mechanism dominates the GSP auction without condition. This is because, in this case, the misalignment problem is negligible.

3.3. Optimal Probability of Enforcing the FPBE

The preceding analysis has shown that the FPBE mechanism can be more profitable than the traditional GSP auction as well as the GSP auction with advertiser-specific minimum bids if the search engine enforces the first-page bid estimates with probability $\alpha \geq \alpha^{\text{FPBE}}$. The threshold α^{FPBE} is chosen such that all advertisers are motivated to conform to the FPBE, rather than taking the risk of not conforming to the FPBE and being dropped from the auction. It is useful to note that raising the enforcement probability beyond α^{FPBE} does not increase any further advertisers' motivation to conform to the FPBE but comes to hurt the search engine's profits. Specifically, when the enforcement probability increases, the probability of an advertiser being dropped from the auction increases if the advertiser's realized valuation is lower than the high first-page bid estimate. Recognizing the negative impact of increasing α on its profits, the search engine chooses α^{FPBE} as the optimal probability of enforcing the FPBE.

Recall that, in our formulation, the relevance-weighted valuation of the three advertisers is such that $r_A v_A > r_B v_B^H > r_B v_B^L > r_C v_C$. As such, the desired listing order of advertisers in the FPBE mechanism is A-B-C, with advertisers A and B winning the two available slots and advertiser C losing the auction. Yet, occasionally, the actual listing order may be different because of misalignment between realized valuations and the bid estimates. To facilitate exposition, we refer to advertisers A and B as the likely winners and advertiser C as the likely loser. On analyzing the optimal probability of enforcing the FPBE, we have the following result.

Proposition 3. *The optimal level of the FPBE enforcement weakly increases as*

1. *the slot-specific effects become similar,*
2. *the relevance-weighted valuations of the two likely winners become similar, and*
3. *the relevance-weighted valuations of the likely loser and of a likely winner become dissimilar.*

To understand the intuition for this proposition, note that the probability of enforcing the FPBE drives advertiser A's incentive to conform to the FPBE. First, when the slot-specific effects s_1 and s_2 become similar, advertiser A is less motivated to seek the first slot because its expected profits from the second slot come closer to those from the first slot. To ensure that advertiser A conforms to its first-page bid estimate in this case, the search engine increases the enforcement probability and thereby increases the threat of being dropped from the auction altogether if advertiser A fails to conform to its first-page bid estimate. Second, when the relevance-weighted valuations of the likely winners become similar, advertiser A's profits from conforming to the FPBE decrease.[7] Therefore, the search engine

[6] A detailed proof can be found in the appendix.

[7] This is because, when advertiser B conforms to the FPBE, the conforming profits of advertiser A are given as $s_1(r_A v_A - r_B v_B)$ based on Equation (1).

raises the enforcement probability with the intention of penalizing advertiser A if it does not comply with the first-page bid estimate. Third, note that when the relevance-weighted valuation of advertiser C shifts farther down from that of advertiser A (or advertiser B), it becomes more attractive for advertiser A to seek the second slot and thus not conform to its first-page bid estimate. To curb the attractiveness of this option, the search engine increases the enforcement probability and thereby increases the risk of being dropped from the auction.

It is useful to highlight that the optimal level of enforcement of the FPBE is independent of β. However, some might naïvely think that when the probability of low valuation increases, the search engine should increase the enforcement probability. The reasoning for this line of thinking could be that, as β increases, the risk of an advertiser not conforming to the FPBE increases, and the search engine should clamp down on such behavior by enforcing the FPBE with a higher probability. Our analysis shows that this argument is not valid. Note that the search engine announces α^{FPBE} such that every advertiser conforms to the FPBE even when advertiser B's realized valuation is low. As such, the enforcement level does not depend on the size of β but only depends on the valuations and relevance of advertisers as well as slot-specific effects. In general, the optimal enforcement probability is strictly smaller than 1. This is in contrast with the perfect enforcement of the ASMB.

4. Budget-Constrained Advertisers and the FPBE

Because keyword search advertising constitutes only a part of the promotional mix of an advertiser, the budget allocated for keyword advertising can be limited. In practice, when an advertiser's sponsored link is displayed in a slot, the advertiser continually incurs advertising expense with every click to its link. Soon the advertiser could well exhaust its advertising budget. On exhausting its budget, the advertiser will become inactive in the auction, and its sponsored link will no longer be displayed by the search engine. Indeed, search engines such as Google ask advertisers to specify their budgets. In this section, we extend our model to explore how the search engine can use the FPBE mechanism to deal with budget-constrained advertisers.

To understand the effect of budget constraint, consider the case where advertiser A has a limited budget. Specifically, if the search engine implements the FPBE mechanism, advertiser A does not have sufficient funds for its sponsored link to be displayed till the end of the period. However, if the search engine were to implement the GSP auction, then advertiser A has sufficient funds to last the entire duration. We face this situation because advertiser A ends up paying more for the first slot under the FPBE mechanism than under the GSP auction. To capture such a situation, we assume that advertiser A's budget E_A is bounded as follows:

$$(s_1 - s_2)V_B^H + s_2 V_C \le E_A \le s_1 V_B^L. \tag{9}$$

More generally, denote advertiser k's budget by E_k. Further, let the price paid by advertiser k for slot j be p_{kj} and the duration for which the advertiser occupies slot j be $D(p_{kj})$. In this setting, each advertiser maximizes its profits:

$$\Pi_k = \sum_{j\in\{1,2\}} D(p_{kj}) s_j r_k (v_k - p_{kj}) \tag{10}$$

subject to the constraints

$$\sum_{j\in\{1,2\}} D(p_{kj}) s_j r_k p_{kj} \le E_k \quad \text{and} \quad \sum_{j\in\{1,2\}} D(p_{kj}) \le 1. \tag{11}$$

In our formulation, advertisers B and C are not budget constrained, implying that E_B and E_C are large enough that both of these advertisers can be in the auction for the entire period.[8] On analyzing this model extension, we have the following result.

Proposition 4. *In the presence of budget-constrained advertisers, a search engine can leverage the FPBE mechanism to retain a high-valuation advertiser for a longer period and improve its profits. The increase in the search engine's profits, however, need not totally be at the cost of the advertisers, because the total profits of the search engine and the advertisers could increase.*

To understand the proposition, first note that keeping the high-valuation advertiser in the auction helps the search engine to earn more profits. This is because the high-valuation advertiser not only bids high but also induces other advertisers to bid higher. Thus, if the search engine expects advertiser A to drop out too early because of budget constraints, the search engine can retain advertiser A for a longer period by reversing the listing order via the FPBE. Clearly, letting advertiser A participate in the auction for a longer period improves advertiser A's profits in addition to improving the search engine's profits. However, advertiser B may be worse off because of the higher advertising cost. Yet the total welfare improves because now advertisers with greater potential (namely, advertisers A and B) are assigned the top two slots for a longer period.

To further appreciate the intuition, consider the case where the search engine sets the first-page bid estimates

[8] Although we assume nonbinding budgets for advertisers B and C, this assumption is not critical to the derivation of our result. We can obtain the same result when a higher slot winner is more budget constrained than a lower slot winner.

at $m_A^{\text{FPBE}} = r_B v_B^L / r_A$, $m_B^{\text{FPBE}} = v_B^L$, and $m_B^{\text{FPBE}} = v_C$. Further assume that the realized valuation of advertiser B is $v_B = v_B^H$. In this context, the search engine could induce the listing order A-B-C or B-A-C. Focusing attention on the listing order B-A-C, note that conforming to m_A^{FPBE} is the dominant strategy for advertiser A. Advertiser B earns $s_1(V_B^H - V_B^L)$ on conforming to m_B^{FPBE} but earns $(1-\alpha)s_2(V_B^H - V_C)$ if it does not conform to the FPBE. This implies that advertiser B will conform to the FPBE if it is enforced with a probability greater than $1 - s_1(V_B^H - V_B^L)/\{s_2(V_B^H - V_C)\}$. Turning attention to the A-B-C listing order, note that the search engine can motivate every advertiser to conform to the FPBE in this case as well. Now, however, the duration for which advertiser A remains active in the auction is limited because $E_A < s_1 V_B^L$. Specifically, after lasting for $E_A/(s_1 V_B^L)$ portion of the period in the auction, advertiser A becomes inactive and fails to participate in the auction, and the two slots are allocated to advertisers B and C. After advertiser A becomes inactive in the auction, the search engine earns lower profits because the current winners are, on average, of lower valuations than the winners before. Therefore, if advertiser A's budget is so tight that it might be out of the auction for an extended period, the search engine might be motivated to reverse the listing order. Then, by keeping a higher-valuation advertiser for a longer period in the auction, the search engine can extract more profits from other advertisers for its slots.

When the listing order is reversed to B-A-C, advertiser A takes the second slot and earns $s_2(V_A - V_C)$, whereas advertiser B takes the first slot and earns $s_1(V_B^H - V_B^L)$. Advertiser C, however, earns nothing because its link is never displayed in the first two slots. Hence, if $\alpha > 1 - s_1(V_B^H - V_B^L)/\{s_2(V_B^H - V_C)\}$ and the listing order induced by the search engine is B-A-C, the equilibrium profits of the advertisers are $\Pi_A(BAC) = s_2(V_A - V_C)$, $\Pi_B(BAC) = s_1(V_B^H - V_B^L)$, and $\Pi_C(BAC) = 0$, and the total profits of all players (including the search engine and the advertisers) are $\Pi_T(BAC) = s_1 V_B^H + s_2 V_A$.

However, if the search engine were to induce the listing order A-B-C, as noted above, advertiser A would remain in the first slot for only $E_A/(s_1 V_B^L)$ fraction of time. Thereafter, advertiser B would take the first slot, and advertiser C would occupy the second slot for the remaining $1 - E_A/(s_1 V_B^L)$ fraction of time. The corresponding equilibrium profits of advertisers and the total profits are $\Pi_A(ABC) = \{E_A/(s_1 V_B^L)\} s_1(V_A - V_B^L)$, $\Pi_B(ABC) = \{E_A/(s_1 V_B^L)\} s_2(V_B^H - V_C) + \{1 - E_A/(s_1 V_B^L)\} s_1(V_B^H - V_C)$, $\Pi_C(ABC) = \{1 - E_A/(s_1 V_B^L)\} s_2(V_C - V_M)$, and $\Pi_T(ABC) = \{E_A/(s_1 V_B^L)\}(s_1 V_A + s_2 V_B^H) + \{1 - E_A/(s_1 V_B^L)\}(s_1 V_B^H + s_2 V_C)$.

Therefore, if the search engine induces the listing order B-A-C instead of A-B-C, we see the following changes in the equilibrium profits of advertisers and the total profits:

$$\Delta\Pi_A \equiv \Pi_A(BAC) - \Pi_A(ABC) = s_2(V_A - V_C) - \frac{E_A}{s_1 V_B^L} s_1(V_A - V_B^L), \tag{12}$$

$$\Delta\Pi_B \equiv \Pi_B(BAC) - \Pi_B(ABC) = \left(1 - \frac{E_A}{s_1 V_B^L}\right) s_2(V_B^L - V_C) - \frac{E_A}{s_1 V_B^L} \{s_2(V_B^H - V_C) - s_1(V_B^H - V_B^L)\}, \tag{13}$$

$$\Delta\Pi_C \equiv \Pi_C(BAC) - \Pi_C(ABC) = -\left(1 - \frac{E_A}{s_1 V_B^L}\right) s_2(V_C - V_M), \tag{14}$$

$$\Delta\Pi_T \equiv \Pi_T(BAC) - \Pi_T(ABC) = s_2(V_A - V_C) - \frac{E_A}{s_1 V_B^L} \{s_1(V_A - V_B^H) + s_2(V_B^H - V_C)\}. \tag{15}$$

We establish in the appendix that $\Delta\Pi_A$ and $\Delta\Pi_T$ are positive, whereas $\Delta\Pi_B$ and $\Delta\Pi_C$ are negative. This implies that advertiser A is better off with the B-A-C listing order, although this listing order hurts the profits of advertisers B and C. Interestingly, the increase in the search engine's profits on altering the listing order to B-A-C is not totally at the cost of advertisers given that we see an improvement in total profits. This improvement in total profits is because a higher-valuation advertiser generates more surplus than a lower-valuation advertiser.

5. Conclusion

The objective of our research is to theoretically investigate the first-page bid estimate mechanism and assess its ability to deal with some challenges in keyword search advertising. To this end, we present a stylized model of the first-page bid estimate and contrast its performance with the traditional GSP auction as well as the ASMB mechanism. As summarized below, our analysis offers useful insights on a few questions of managerial significance.

1. *Is the FPBE mechanism more profitable than the ASMB mechanism when advertisers' valuations are uncertain?* The answer is yes. The fundamental difference between these two mechanisms is that the ASMB is always enforced, whereas the FPBE is not always enforced by the search engine. The lower level of enforcement of the FPBE may lead some to believe that the FPBE mechanism would yield lower profits than the ASMB mechanism. Contrary to this belief, our analysis shows that the FPBE mechanism is weakly more profitable than the ASMB mechanism. The intuition for this finding brings to the fore the strength of the FPBE

mechanism. When the search engine is uncertain about the valuations of advertisers, there is a downside risk to setting a high minimum bid for any advertiser: the advertiser will drop out of the auction if its actual valuation is lower. Clearly, replacing the dropped advertiser with a lower-valuation advertiser reduces the search engine's profits. Alternatively, the search engine could set a low minimum bid, but this also decreases the search engine's profits, as now the search engine is leaving money on the table if the actual valuation of the advertiser is higher. The FPBE mechanism is well designed to address this challenge. It gives the search engine the opportunity to set high minimum bids but probabilistically enforce them such that the costs of misalignment are reduced. Thus our analysis offers a potential explanation for Google's initiative to offer first-page bid estimates instead of stipulating minimum bids.

2. *Does the FPBE mechanism yield more profits than the traditional GSP auction by resolving the challenge posed by untruthful bidding?* The answer to this question is not a categorical yes, because it is a complex issue. When the within-advertiser heterogeneity in valuations is below a threshold, it is more profitable for the search engine to implement the FPBE mechanism instead of the GSP auction. Recall that advertisers do not truthfully bid their valuations in the GSP auction. By adopting the FPBE mechanism, the search engine can motivate advertisers to bid more than what they may bid in the GSP auction. However, in its efforts to make advertisers bid more by stipulating higher bid estimates, the search engine runs the risk of dropping some high-valuation advertisers and replacing them with lower-valuation advertisers. However, if the within-advertiser heterogeneity in valuations is low, the costs of misalignment between the first-page bid estimates and advertisers' realized valuations could be relatively low. Consequently, the FPBE mechanism becomes more profitable than the GSP auction in such situations.

3. *What influences the probability of a search engine's enforcing the first-page bid estimates?* It is only natural to think that the enforcement probability will be related to the probability of advertisers' valuations being low (β). Our analysis, however, shows that the optimal level of enforcement of the FPBE is independent of β but depends on the slot-specific effects as well as advertisers' relevance-weighted valuations. Specifically, as the slot-specific effects of the top two slots become similar, the search engine increases the enforcement probability so that advertisers remain motivated to conform to its first-page bid estimate. Similarly, when the relevance-weighted valuations of the winning advertisers become similar, it reduces the highest-valuation advertiser's motivation to conform to the FPBE. Also, if the gap between the relevance-weighted valuation of the losing advertiser and that of any of the winners increases, advertisers again might be motivated to not conform to the FPBE. In both of these instances, the search engine raises the enforcement probability to increase advertisers' cost for not conforming to the FPBE. These issues have not been investigated in prior literature.

4. *Can the FPBE mechanism help to handle budget-constrained advertisers?* Although extant models of keyword search advertising assume that advertisers are not budget constrained, in reality, advertisers have limited budgets for online advertising. Typically, an advertiser specifies a budget for each campaign and when its budget is exhausted, the advertiser becomes inactive and does not participate in the auction. Therefore, an answer to the above question is of practical significance. Our analysis shows that if a high-valuation advertiser is budget constrained, the search engine can use the FPBE mechanism to keep the advertiser in the auction for a longer period, thereby improving the search engine's profits. Interestingly, the search engine's gains are not necessarily at the expense of the advertisers because the total welfare can increase.

5.1. Directions for Further Research

In developing our model, we examined a market made up of a single search engine and multiple advertisers. Future research can extend the model to consider competition among search engines. Our analysis focused on how the search engine can use first-page bid estimates to handle the uncertainty in advertisers' valuations by modeling the FPBE with threshold bids and enforcement probability. Future research can explore other strategic implications of the FPBE. In reality, the first-page bid estimate is just one piece of the body of information that the search engine provides to advertisers. For example, advertisers also have access to detailed information on position, clicks, and payment. The strategic implications of such information await further scrutiny. In keeping with prior literature, our work considered advertisers who are heterogeneous in their valuations and relevance. Advertisers can also vary in other dimensions such as brand equity. Future research can examine the implications of such factors on the strategic behavior of advertisers (Desai et al. 2014). In our analysis, we examined the implications of a budget-constrained advertiser. A more thorough analysis of the bidding game among budget-constrained advertisers will be another fruitful avenue for future research (Shin 2015; see also Sayedi et al. 2014). In addition, some important aspects of online advertising such as advertising targeting have been abstracted away in our model, but investigation of such aspects will add to the online advertising literature (Zhang and Katona 2012). More generally, search engines are continuously trying to improve the mechanism used to sell advertising slots (see Jerath and Sayedi 2012, for example). Academic literature, unfortunately, lags behind the developments in practice. In part, this is because search advertising is a multisided market, and it is difficult to get data that provide a comprehensive picture of all the strategic players. Despite this challenge, empirical researchers have strived to characterize the behavior in individual facets of the market. This challenge should stimulate more game-theoretic analysis of keyword search advertising, for it can help us to see the outcome of the strategic interactions of all the different players in the market: advertisers, consumers, and search engines. Experimental research can also be used to fill gaps in our understanding of strategic behavior in the keyword advertising market where field data are sparse.

Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/mnsc.2014.2033.

Appendix

We first state a set of lemmas and then prove the propositions using the lemmas. The detailed proofs for the lemmas can be seen in the online appendix.

Lemma A1. *In the traditional GSP auction,* (i) *advertiser A takes the first slot while advertiser B takes the second slot, and* (ii) *the resulting search engine's profits are given by*

$$\Pi_{SE}^{\mathrm{GSP}} = (s_1 - s_2)\{\beta V_B^L + (1-\beta)V_B^H\} + 2s_2 V_C. \tag{16}$$

Lemma A2. *When the search engine adopts advertiser-specific minimum bids, the search engine's optimal profits are given by*

$$\Pi_{SE}^{\mathrm{ASMB}} = \begin{cases} s_1 V_B^L + s_2 V_C (\equiv \Pi_{SE}^{ASMB(1)}) & \text{if } \beta \geq \beta^{\mathrm{ASMB}}, \\ \beta(s_1 V_C + s_2 V_M) + (1-\beta) \\ \quad \cdot (s_1 V_B^H + s_2 V_C)(\equiv \Pi_{SE}^{ASMB(2)}) & \text{otherwise}, \end{cases} \tag{17}$$

where $\beta^{\mathrm{ASMB}} \equiv s_1(V_B^H - V_B^L)/\{s_1(V_B^H - V_C) + s_2(V_C - V_M)\}$.

Lemma A3. *When the search engine provides first-page bid estimates, the search engine can induce every advertiser to conform to the FPBE, as long as each advertiser's bid estimate is not strictly higher than the advertiser's valuation and the advertiser's expected profit on conforming to the FPBE is positive.*

Lemma A4. *Under the FPBE mechanism, listing advertisers in the B-A-C order does not yield higher profits to the search engine than those obtained with the A-B-C listing order.*

Lemma A5. *The lower bound of the enforcement probability* α *at which the search engine can induce every advertiser to conform to the FPBE is given by* $\alpha^{\mathrm{FPBE}} = 1 - s_1(V_A - V_B^H)/\{s_2(V_A - V_C)\}$.

Lemma A6. *When the search engine provides first-page bid estimates and enforces them with probability* $\alpha \geq \alpha^{\mathrm{FPBE}}$*, the search engine's profits are given by*

$$\Pi_{SE}^{\mathrm{FPBE}}(\alpha) = \begin{cases} s_1 V_B^L + s_2 V_C (\equiv \Pi_{SE}^{\mathrm{FPBE}(1)}) & \text{if } \beta \geq \beta^{\mathrm{FPBE}}, \\ \beta\{\alpha(s_1 V_C + s_2 V_M) \\ \quad + (1-\alpha)((s_1 - s_2)V_B^L + 2s_2 V_C)\} \\ \quad + (1-\beta)(s_1 V_B^H + s_2 V_C)(\equiv \Pi_{SE}^{\mathrm{FPBE}(2)}) & \text{otherwise}, \end{cases} \tag{18}$$

where $\beta^{\mathrm{FPBE}} \equiv s_1(V_B^H - V_B^L) \cdot [s_1\{V_B^H - \alpha V_C - (1-\alpha)V_B^L\} + s_2\{V_C - \alpha V_M - (1-\alpha)(2V_C - V_B^L)\}]^{-1}$.

Lemma A7. *Suppose that the following inequalities hold true*:

$$s_2(V_B^H - V_C) > s_1(V_B^H - \theta V_B^L), \tag{19}$$

$$s_2(V_B^H - V_C)\{s_1(V_B^H - V_C) + s_2(V_C - V_M)\} > s_1^2 V_B^H (V_B^H - V_B^L), \tag{20}$$

where $\theta \equiv s_2(V_A - V_C)/\{s_1(V_A - V_B^H) + s_2(V_B^H - V_C)\}$. *Further, define*

$$\alpha^{(1)} \equiv 1 - \frac{s_1(V_B^H - V_B^L)}{s_2(V_B^H - V_C)}, \tag{21}$$

$$\alpha^{(2)} \equiv 1 - \frac{\beta s_1 V_B^H (V_A - V_C) + (1-\beta)E_A(V_A - V_B^H)}{\{\beta s_1 + (1-\beta)s_2\}V_B^H(V_A - V_C)}, \tag{22}$$

$$E_A^{(1)} \equiv s_1 V_B^L (1-\beta), \tag{23}$$

$$E_A^{(2)} \equiv s_1 V_B^H [\{(1-\beta)s_1 - \beta(\alpha^{(1)} - \alpha^{(2)})(s_1 - s_2)\}(V_B^L - V_C) + \{(1-\beta) - \beta(\alpha^{(1)} - \alpha^{(2)})\}s_2(V_C - V_M)] \cdot [(1-\beta)\{s_1(V_B^H - V_C) + s_2(V_C - V_M)\}]^{-1}, \tag{24}$$

$$E_A^{(3)} \equiv s_1 V_B^L \frac{s_2(V_A - V_C)}{s_1(V_A - V_B^H) + s_2(V_B^H - V_C)}, \tag{25}$$

$$E_A^* \equiv (s_1 - s_2)V_B^H + s_2 V_C, \tag{26}$$

$$E_A^{**} \equiv \min\{E_A^{(1)}, E_A^{(2)}, E_A^{(3)}\}, \tag{27}$$

and suppose that $E_A^* < E_A < E_A^{**}$. *Then the search engine retains advertiser A for a longer period by reversing the listing order of advertiser A and advertiser B by using the FPBE mechanism, if* $\beta < \beta^*$ *holds true. (The threshold* β^* *is defined in the online appendix.)*

Proof of Proposition 1. Observe from (17) and (18) that $\Pi_{SE}^{\mathrm{FPBE}(j)} \geq \Pi_{SE}^{\mathrm{ASMB}(j)} \forall j \; (j = 1,2)$, since $\Pi_{SE}^{\mathrm{FPBE}(1)} - \Pi_{SE}^{\mathrm{ASMB}(1)} = 0$, and $\Pi_{SE}^{\mathrm{FPBE}(2)} - \Pi_{SE}^{\mathrm{ASMB}(2)} = (1-\alpha)\beta\{(s_1 - s_2) \cdot (V_B^L - V_C) + s_2(V_C + V_M)\} > 0$ for any α. Since, by definition, $\Pi_{SE}^{\mathrm{FPBE}}(\alpha) = \max\{\Pi_{SE}^{\mathrm{FPBE}(1)}, \Pi_{SE}^{\mathrm{FPBE}(2)}\}$, we have $\Pi_{SE}^{\mathrm{FPBE}}(\alpha) \geq \Pi_{SE}^{\mathrm{FPBE}(j)}$ for any $j \; (j=1,2)$. Then it follows that $\Pi_{SE}^{\mathrm{FPBE}}(\alpha) \geq \Pi_{SE}^{\mathrm{ASMB}(j)} \forall j \; (j = 1,2), \forall \alpha \in (0,1)$. Furthermore, as $\Pi_{SE}^{\mathrm{ASMB}} = \max\{\Pi_{SE}^{\mathrm{ASMB}(1)}, \Pi_{SE}^{\mathrm{ASMB}(2)}\}$, we have $\Pi_{SE}^{\mathrm{FPBE}}(\alpha) \geq \Pi_{SE}^{\mathrm{ASMB}} \; \forall \alpha \in (0,1)$. $\square$

Proof of Proposition 2. Suppose $V_B^H - V_B^L \leq [s_2/\{(1-\beta)(s_1 - s_2)\}](V_B^L - V_C)$ holds. Then, from (18) and (16), we obtain $\Pi_{SE}^{\mathrm{FPBE}(1)} - \Pi_{SE}^{\mathrm{GSP}} = s_2(V_B^L - V_C) - (1-\beta)(s_1 - s_2) \cdot$

$(V_B^H - V_B^L) \geq 0$. Since, by definition, $\Pi_{SE}^{\text{FPBE}}(\alpha) \geq \Pi_{SE}^{\text{FPBE}(1)}$, we have $\Pi_{SE}^{\text{FPBE}}(\alpha) \geq \Pi_{SE}^{\text{GSP}}$ $\forall \alpha \in (0,1)$. □

Proof of Proposition 3. On taking the first derivative of profits in (18) with respect to the enforcement probability α, we have $\partial \Pi_{SE}^{\text{FPBE}(1)}/\partial\alpha = 0$ and $\partial \Pi_{SE}^{\text{FPBE}(2)}/\partial\alpha = -\beta\{(s_1 - s_2)\cdot (V_B^L - V_C) + s_2(V_C - V_M)\} < 0$, implying that $\partial \Pi_{SE}^{\text{FPBE}}(\alpha)/\partial\alpha \leq 0$. Therefore, the optimal level of α is the minimum value that α can take. Thus, the optimal probability of enforcement is given by α^{FPBE} if $\alpha^{\text{FPBE}} > 0$ and by ϵ otherwise. Then the following comparative statics complete the proof: (1) $\partial\alpha^{\text{FPBE}}/\partial s_1 < 0$ and $\partial\alpha^{\text{FPBE}}/\partial s_2 > 0$; (2) $\partial\alpha^{\text{FPBE}}/\partial(V_A - V_B^H) > 0$; and (3) $\partial\alpha^{\text{FPBE}}/\partial V_B^H > 0$, $\partial\alpha^{\text{FPBE}}/\partial V_C < 0$, and $\partial\alpha^{\text{FPBE}}/\partial(V_A - V_C) > 0$. □

Proof of Proposition 4. Suppose the conditions of Lemma A7 hold true. Then the lemma suggests that the search engine can reverse the listing order of advertisers A and B by providing ($m_A^{\text{FPBE}} = r_B m_B^{\text{FPBE}}/r_A$, $m_B^{\text{FPBE}} = v_B^L$, and $m_C^{\text{FPBE}} = v_C$). We now show the existence of a case where advertiser A is better off while advertisers B and C are made worse off by the B-A-C listing order compared with the A-B-C listing order, while focusing on the case where $v_B = v_B^H$. (Note that the condition on β specified in Lemma A7 indeed implies that $v_B = v_B^H$ happens more often.) Based on the profits derived in the main body of the paper, we have

$$\Delta\Pi_A \equiv \Pi_A(BAC) - \Pi_A(ABC) = s_2(V_A - V_C) - \frac{E_A}{s_1 V_B^L} s_1(V_A - V_B^L), \tag{28}$$

$$\Delta\Pi_B \equiv \Pi_B(BAC) - \Pi_B(ABC) = \frac{E_A}{s_1 V_B^L}\{s_1(V_B^H - V_B^L) - s_2(V_B^H - V_C)\} - \left(1 - \frac{E_A}{s_1 V_B^L}\right) s_2(V_B^L - V_C), \tag{29}$$

$$\Delta\Pi_C \equiv \Pi_C(BAC) - \Pi_C(ABC) = -\left(1 - \frac{E_A}{s_1 V_B^L}\right) s_2(V_C - V_M), \tag{30}$$

$$\Delta\Pi_T \equiv \Pi_T(BAC) - \Pi_T(ABC) = s_2(V_A - V_C) - \frac{E_A}{s_1 V_B^L}\{s_1(V_A - V_B^H) + s_2(V_B^H - V_C)\}. \tag{31}$$

First, it is easy to see that $\Delta\Pi_C < 0$. In addition, $\Delta\Pi_B < 0$, since $s_2(V_B^H - V_C) > s_1(V_B^H - V_B^L)$ by Equation (9). These results show that both advertisers B and C are worse off with the B-A-C listing order. Next, note that $\Delta\Pi_A > 0$ is equivalent to $E_A < s_1 V_B^L[s_2(V_A - V_C)/\{s_1(V_A - V_B^L)\}]$. Since $s_2(V_A - V_C)/\{s_1(V_A - V_B^L)\} > 1$ (implied by (9)), it is easy to see that $s_1 V_B^L < s_1 V_B^L s_2(V_A - V_C)/\{s_1(V_A - V_B^L)\}$. At the same time, by definition, we have $E_A^{**} < E_A^{(1)} < s_1 V_B^L$. Therefore, if $E_A < E_A^{**}$, then it follows that $\Delta\Pi_A > 0$, implying that advertiser A is always better off with the B-A-C listing order. Finally, $\Delta\Pi_T > 0$ is equivalent to $E_A < s_1 V_B^H[s_2(V_A - V_C)/\{s_1(V_A - V_B^H) + s_2(V_B^H - V_C)\}]$. Note that the right-hand side of this inequality is equivalent to $E_A^{(3)}$. Therefore, if $E_A < E_A^{**}$, then $\Delta\Pi_T > 0$. Thus when the search engine profits increase, the total profits also increase. □

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