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Repurchasing Debt

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In this paper we build a theoretical model of a firm repurchasing its corporate debt. We find that firm creditors as a group sell debt to the firm only at face value. However, because of the cross-creditor externalities, buying back debt is cheaper and easier when there are many creditors, e.g., when debt is traded on the open market. We further show that repurchases contribute to flexibility in firms' capital structure and can increase ex ante firm value. The value of repurchases to the shareholders increases with the firm's ability to save cash and delay the repurchase.

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1. Introduction

The low price of corporate debt in the secondary securities market and recent tax incentives arising from the postcrisis American Recovery and Reinvestment Act of 2009 presented an opportunity to many firms to restructure or reduce outstanding debt on more favorable terms. By repurchasing their debt with cash or assets, companies were able to reduce their existing indebtedness (which carries no tax advantage net of cash) at less than the original face value, reduce their interest costs, and remove restrictive covenants. In 2010, for example, companies initiated 126 cash repurchases of publicly traded bonds, with an aggregate amount of \$26.3 billion, compared with just 49 transactions during the period 1986–1996 (see Figure 1).[1] Despite the increasing number of debt repurchases, the academic literature on this topic is virtually nonexistent. In this paper, we provide a formal theoretical framework for corporate debt buybacks, with the goal of understanding when a repurchase is optimal and what the implications are for shareholders and creditors. The framework lends itself to a number of applications and empirical predictions.

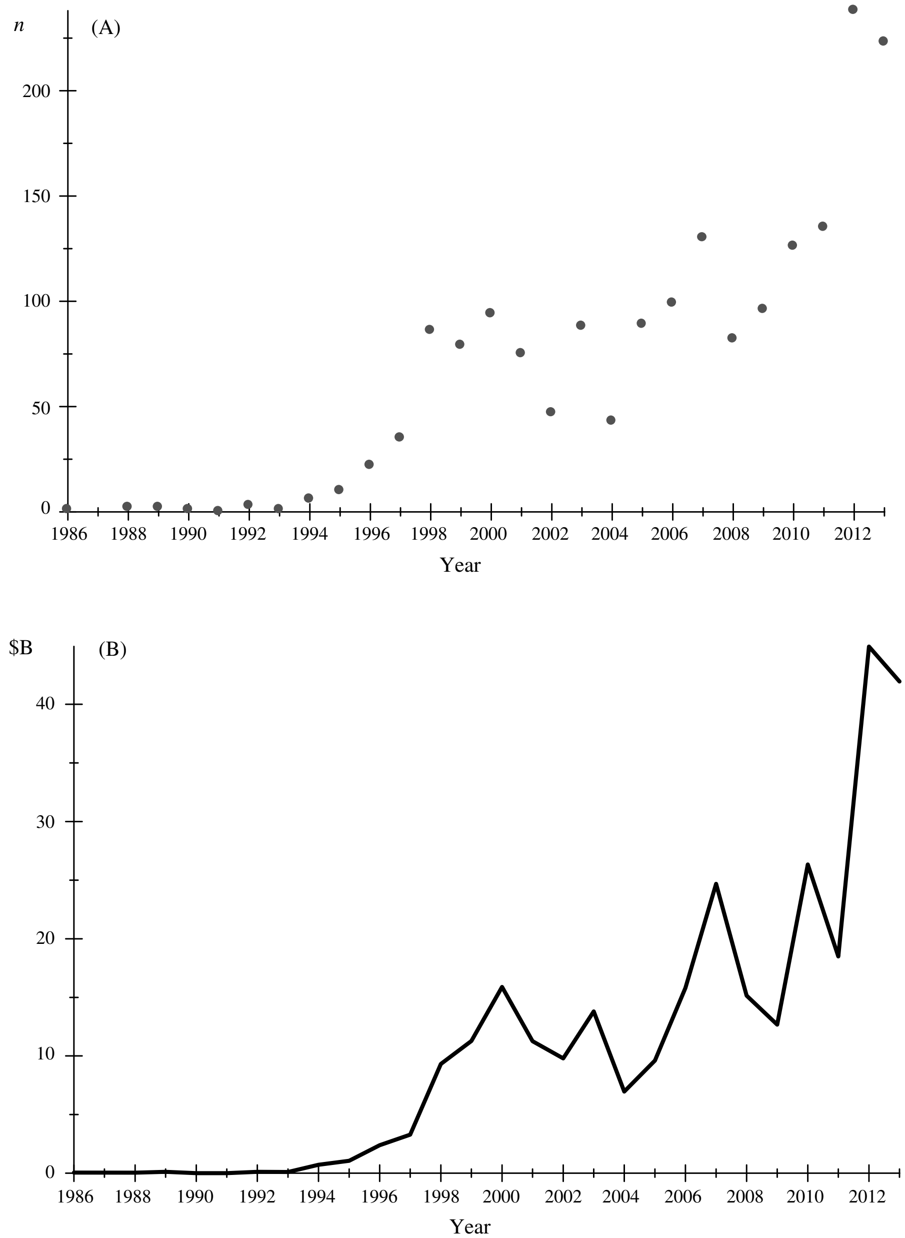
To set a benchmark, we build a simple one-period model of the corporate repurchase and show that creditors as a group should not sell risky debt back to the company at less than its face value, even if the market price at which the creditors would be willing to trade with each other or with third parties is much lower. To show this, we provide in the model an example of a firm with a sole lender or a group of coordinated lenders, under Modigliani–Miller conditions (Modigliani and Miller 1958). In this frictionless setting, the minimum price at which the lender agrees to sell the marginal unit of risky debt back to the firm is equal to the face value of the debt, above the market value. All additional bonds are also repurchased at the face value.[2] The firm has to pay a premium because using cash or any safe asset for repurchase adversely affects the value of the remaining debt claims. Because the debt is secured by cash and assets inside the firm, repurchasing the debt essentially amounts to paying the lender with his own money.

However, debt that is held by many bondholders can be repurchased at a significantly lower price because of cross-creditor externalities. For example, a firm can repurchase its bonds on the open market as long as there are small investors willing to sell their entire stake. The difference from the single-creditor case is that the sellers do not internalize a decline in the value of the remaining debt because it is held by other investors. In fact, the tender offer repurchase may even be successful for prices slightly below the market price if investors are optimistic about the repurchase completion. Those

[1] These estimates are conservative because many repurchases are not recorded in the Fixed Income Securities Database (FISD). They are omitted if they were negotiated privately or structured as exchanges for cash, or if they were bundled with assets, common shares, or senior debt. For example, the IMAX Corporation exchanged \$90 million in notes at less than 24% of their face value; however, that transaction does not appear in our data.

[2] Note, however, that it is impossible to buy back all debt at the face value, since if there were enough cash to do this, the debt would not be risky.

Figure 1 **U.S. Debt Repurchases, 1986–2013**



Notes. Panel (A) shows the annual number of repurchases *n* for the years 1986–2013. Panel (B) shows the annual volume of repurchases for 1986–2013. We include only transactions coded in the database as "T" (tender offer) or "IRP" (issue repurchase) with corporate bonds. Repeated repurchases by the same company are treated as separate. The total volume of repurchases is computed as the repurchase price, which is equal to the averaged-over-year "action price" in FISD, multiplied by the number of shares repurchased in this transaction, and summed over all transactions for this year. We dropped three observations for which the action price likely contains a recording error (e.g., equal to zero).

investors that do not tender or exchange their bonds are exposed to greater risk and lower value.[3]

Our insight—that debt held by multiple bondholders can be repurchased at a lower cost—contrasts sharply with the predictions of the literature on debt renegotiation and strategic debt service (e.g., Hart and Moore 1998, Mella-Barral and Perraudin 1997). As in this literature, shareholders in our model are able to force concessions from debtholders. However, the strategic debt service literature deals with bargaining after default, when cash effectively already belongs to creditors, and the goal of negotiations is only to reduce bankruptcy costs. For this reason, small bondholders in this literature, who can either abstain from negotiations or demand a premium by free riding on large bondholders, make debt renegotiation impractical.

[3] There are legal restrictions applicable to the tender offer repurchases of publicly traded debt that prohibit changing the debt principal without the debtholders' unanimous consent. We discuss this later on.

The distinction must be made, because the existing literature on the topic often draws conclusions on the basis of debt renegotiation theory. For example, Mann and Powers (2007) argue that tender offers are easier to complete in firms with more concentrated debt ownership.

Using the benchmark model, we explore in detail how debt repurchases work in different situations. First, we discuss the cases that include bankruptcy costs, tax, and transaction costs, and we show that (1) costly bankruptcy encourages repurchases, and (2) taxation and transaction costs discourage repurchases. Intuitively, fixed and proportional bankruptcy costs decrease the value a lender can recover after a firm defaults and therefore encourage bondholders to make concessions. The surplus from the lower bankruptcy costs is then split between shareholders and bondholders according to the relative bargaining power. Expected bankruptcy costs in repurchases are reduced in two distinct ways. Cash or assets transferred to creditors before bankruptcy reduce the proportional bankruptcy costs. In addition, a repurchase at a price below face value reduces the probability of bankruptcy. Taxation is shown to affect the repurchase incentive primarily through cancellation of indebtedness (COD) tax, proportional to the repurchase discount.

Second, we argue that, from the ex ante perspective, the ability to repurchase debt at a price below face value is beneficial to the firm. The option to repurchase increases the firm's ex ante value and the firm's initial debt capacity. Although bondholders may be exploited ex post, the overall effect on firm value is positive because the firm has greater financial flexibility. Therefore, we conclude that publicly traded debt or multiple investor ownership that facilitates future repurchases increases the ex ante firm value. In contrast, a conversion option, which contains an equity component and makes it harder to repurchase debt because it requires additional U.S. Securities and Exchange Commission (SEC) approval, decreases firm value.[4]

Third, although a repurchase reduces firm indebtedness, it does not always reduce firm risk and the probability of bankruptcy. For example, when the repurchase price is equal to face value, the firm's liabilities and assets are reduced by the same amount, and therefore firm risk remains unchanged. Therefore, repurchases at a high price will have only limited success in mitigating agency conflicts that originate from risky debt, such as the underinvestment problem. A low-price repurchase, which would be successful in decreasing the asset risk, conflicts with the bondholders' holdout problem in anticipation of a higher price following the investment.

Fourth, our framework provides insights on debt repurchase timing. Because the expected gain from a repurchase increases with the risk of default, and the payout function is convex in the repurchase price, managers have an incentive to postpone the repurchase until a date closer to debt maturity. Therefore, the option to buy back debt must be kept "alive" by increasing cash reserves instead of immediately reducing debt. It is therefore important, going forward, to recognize that shareholders may intentionally engage in simultaneous borrowing and saving to increase the value of the repurchase option.

This paper is related to the literature on debt restructuring and debt exchanges. Since the seminal contributions of Froot (1989), Bulow and Rogoff (1991), Bulow et al. (1988), and Gertner and Scharfstein (1991), who focused on the implications of the sovereign and corporate debt exchanges that were prevalent in the 1980s, ours is the first formal study to address the current phenomenon of corporate debt repurchases. Gertner and Scharfstein (1991) show, in particular, that offering new senior securities (cash paid to debtholders is one example of such a security) in exchange for distressed junior debt is beneficial to shareholders and that offering new junior securities, such as equity, can reduce shareholder value. However, Gertner and Scharfstein do not discuss the price, timing, and other determinants of debt repurchases, which are the focus of our investigation. Froot (1989), Bulow and Rogoff (1991), Bulow et al. (1988), and others study open-market *sovereign* debt repurchases in the presence of the debt overhang problem. The major difference between corporate and sovereign debt buybacks is that, in the latter, cash and assets cannot be meaningfully pledged (see Bulow 1992 for details).

Our hypothesis that the firm benefits from saving cash for a future repurchase contributes to the literature on the determinants of cash holdings. Several recent studies examine the optimal cash holding policy. For example, Foley et al. (2007), Opler et al. (1999), and Faulkender and Wang (2006) point to a variety of problems that originate from holding excessive cash. From a theoretical standpoint, DeAngelo et al. (2011) argue that carrying cash is costly, whereas Dasgupta et al. (2009) posit that cash holdings have a beneficial effect by relaxing intertemporal financing constraints.

Our results have connections to the literature that investigates investment, debt, and the propensity to save in financially constrained firms. In related empirical studies that focus on debt exchanges and repurchases, the propensity for debt reduction has been linked to the proportion of public and bank debt, debt seniority, maturity, and the value of growth options.

[4] Contrary to the intuition that a call option is similar to a repurchase option, they are significantly different for the following reason. The call option is in the money when the debt value is high, and the repurchase option is in the money when the debt value is low.

James (1996) offers a comprehensive overview of this literature. Kruse et al. (2009) provide recent evidence that shareholders benefit when a firm repurchases debt. We contribute to these results by laying out the conditions that determine the repurchase price. Finally, our study is related to the growing literature that examines the role of debt and savings using structural dynamic models.[5]

2. Institutional Background

There are three main mechanisms for buying back corporate debt: an open-market repurchase, a tender offer, and a privately negotiated repurchase.[6] An open-market repurchase, which includes repurchases in private markets by institutional buyers, is executed over a period of time and allows for potentially different prices for each bond sold back to the firm. A tender offer is typically conducted by offering a single price to all bondholders. Repurchases are conducted using cash savings, proceeds from the sale of assets, or proceeds from senior security issuance collateralized by these assets (Gertner and Scharfstein 1991). In this paper, we do not discuss debt-for-equity exchanges or debt-for-debt exchanges, which have different implications.

An open-market repurchase is an easy way for an issuer to buy back relatively small amounts of debt. Other than complying with the antifraud provisions of the federal security laws, these transactions are not normally subject to review by the SEC.[7] However, it is difficult to repurchase large amounts in a limited time on the open market. Also, this mechanism does not permit the issuer to amend the covenants of the bonds because the issuer or the affiliates are not entitled to vote for the purpose of giving consents under the indenture.

Tender offers can include a fixed premium over the current trading price and allow the repurchase of larger amounts. Importantly, tender offers may include additional incentives for bond investors, which all but guarantee a successful repurchase. To motivate the holders of bonds to tender without offering a large premium, and to avoid the need to comply with all of the existing contractual requirements, companies also solicit "exit consents" with their offer, in which case the holders of the securities are asked to consent to amendments to the security as a condition of their acceptance of the offer (Kaplan and Truesdell 2008). If the consent solicitation is successful, any holders who refuse to accept the offer would continue to hold their old securities, which are stripped of protective covenants and made effectively junior to the new security.

An additional advantage of conducting a tender offer with exit consents is the ability to remove existing covenants that restrict the borrower's future actions (Mann and Powers 2007). Having removed these covenants, the company may gain more flexibility in making investment and financing decisions. For example, a firm may be able to increase capital expenditures, make an acquisition, increase dividends, liquidate assets, transfer money to subsidiaries, change the financial reporting procedure, alter collateral, consolidate assets, merge with another company, change lines of business, or modify its bylaws (King and Mauer 2000, Roberts and Sufi 2009).

However, there are two difficulties that companies must overcome. First, tender offers for publicly traded debt require compliance with the Trust Indenture Act of 1939, §316(b), which prohibits debtholders from changing the principal of debt without the debtholders' unanimous consent. It is designed, in particular, to prevent the company from exploiting minority bondholders. Managers can (and do) avoid this restriction by buying back a portion of debt on the open market or by combining cash repurchases with exchanges for other securities (see, e.g., Brudney 1992, Gertner and Scharfstein 1991, Shuster 2007).[8] They can also avoid having their repurchase classified as a tender offer by soliciting a limited number of holders, repurchasing over a fairly long period of time, and/or purchasing on different terms from each holder.

Second, whenever debt is repurchased below its face value, the firm is subject to a tax on the COD income. Unless an exception applies, such as insolvency or bankruptcy at the time of the repurchase, shareholders must recognize the COD income upon satisfaction of its indebtedness for less than the amount due under the obligation. The COD income is usually the difference between the amount due under contract and the amount paid.[9] Firms facing COD tax may find that the additional tax partially offsets the benefits of

[5] For example, Morellec and Nikolov (2009) and Hugonnier et al. (2011) link cash holdings to investment, competition, and a desire for liquidity. In the model of Riddick and Whited (2009), a savings policy presents a trade-off between tax penalties and the reduction in expected future financing costs.

[6] Debt repurchases may also be conducted as auctions. For example, Hovnanian Enterprises, Inc., used a modified Dutch auction with base bid prices ranging from \$480 to \$750 per \$1,000 of the face value. The company eventually paid \$223 million to buy back \$578 million of debt in February and April of 2009.

[7] However, issuers may face greater regulation by the SEC if they proceed with very large repurchases through these transactions. See, for example, Demont (2009).

[8] Shuster (2007) gives examples of the provisions that were originally designed to remove small percentages of abstaining bondholders in otherwise fully consensual agreements but can be used to satisfy the requirements of §316(b) without agreement of the majority of bondholders.

[9] This difference and the associated COD tax can be nontrivial. For example, Harrah's Entertainment paid about 48 cents on the dollar to repurchase \$788 million of its debt in the second quarter of 2009.

buying back debt at a low price. However, the 2009 American Recovery and Reinvestment Act allows deferring the COD tax costs for up to 11 years, effectively making debt repurchases more attractive.[10]

3. Model of Debt Repurchase

In this section, we lay out the benchmark single-period model in the frictionless case with a single bondholder and then with multiple bondholders. We are interested in the repurchase price, the creditors' incentives to tender/sell debt, and the implications of the repurchases for the firm value. Later we relax some of the assumptions of this framework and analyze how different financing frictions affect debt repurchases.

3.1. Model Setup

Suppose that the firm has cash C, or a liquid riskless asset of an equivalent value, or proceeds from a senior security collateralized by this asset. The existing assets of the firm generate a cash flow x, distributed continuously according to the cumulative distribution function (CDF) $F(x)$ on the nonnegative bounded support $[\underline{X}, \bar{X}]$. If cash flows can be negative, $\underline{X} < 0$, we redefine $C' = C + \min(\underline{X})$; if the firm can spend only part of available cash on debt repurchase, then C contains only this part. Cash or safe asset C is given and cannot be increased by, e.g., issuing new equity or warrants.[11] Therefore, the objective of the manager is to maximize the value of equity with respect to two alternative strategies: saving amount C or using cash to repurchase an amount of debt ΔD from the lenders. Note that the average repurchase price is $P_R = C/\Delta D$; for example, $P_R = 1$ means that repurchase is made at face value.

The firm's cash flows are independent of the repurchase; that is, the repurchase does not generate any synergies that can increase the value of the assets and therefore lead to the bondholder holdout problem (similar to, e.g., Shleifer and Vishny 1986). We assume that all of the firm's debt D (including accumulated interest at rate r) matures shortly after realization of x. Since the problem is trivial in the case of riskless debt, we require that the firm defaults in at least some states of the world; i.e.,

$$C + \underline{X} < D \leq C + \bar{X}. \tag{1}$$

If the firm becomes bankrupt, the priority rule is observed, and debtholders have first claim on the firm's assets. In the frictionless model, we assume that there are no costs associated with bankruptcy. Additionally, since our objective is to determine the impact of a firm's financial position on the incentive to increase or decrease leverage, we assume that the firm "inherits" debt and postpone the discussion of optimal leverage until later. Lenders assume equal seniority; however, future debt issues are restricted to subordinate claims only and do not affect the recovered amount of the senior lender in the event of default.

We assume that the firm is restricted from paying dividends or conducting share repurchases because such a distribution of cash would result in the value transfer from the lenders to the shareholders.[12] Provisions limiting distributions affecting debt repayments are commonly included in debt covenants (Smith and Warner 1979). Obviously, if unlimited dividends or share repurchases are allowed before the principle amount of debt comes due, shareholders' first-best strategy entails selling all assets to maximize the payout. Shareholder–debtholder conflicts are trivially resolved in this case (see, e.g., Jensen and Meckling 1976). Finally, there are other uses for the firm's cash that we do not allow in a simple model, such as investment considered in the later sections of this paper, compensation to employees, or perks to the management.

3.2. Single-Creditor Case

We first consider debt held by a sole lender, such as a private investor or, alternatively, several large lenders, who collude when negotiating the sale price of debt. The repurchase price restrictions can be derived from the participation conditions for equity and debt holders. Define the equity value, S_0, as follows:

$$S_0 = \int_{D-C}^{\bar{X}} (x + C - D)\, dF(x). \tag{2}$$

Define the equity value if the firm buys back ΔD of outstanding debt using all available cash C as S_R; i.e.,

$$S_R = \int_{D-\Delta D}^{\bar{X}} (x - D + \Delta D)\, dF(x). \tag{3}$$

If not for the 2009 American Recovery and Reinvestment Act tax deferral, Harrah's would face an immediate COD tax levied on the discount of about $400 million.

[10] The act does not alter how COD income arises, but rather it affects when the debtor pays tax on the income. Usually, for repurchases after December 31, 2008 and before January 1, 2011, bondholders can elect to apply the COD over a five-year period beginning in 2014. Therefore, a firm that repurchased in 2008 will finish paying the COD tax in 2019. The interested reader can find details in, for example, Bortnick and Leska (2009).

[11] If new equity is raised to repurchase or exchange risky debt, the shareholder value is almost always affected negatively. For example, Gertner and Sharfstein (1991) show that raising new equity to finance debt repurchases (debt-for-equity swaps) decreases shareholder value.

[12] Clearly, the assumptions of "no dividends" and "no new equity" are closely related and make the study of repurchases (or any study of risky debt) nontrivial.

Similarly, define the market values of debt as, respectively, d_0 and d_R, where

$$d_0 = \int_{\underline{X}}^{D-C} (x+C)\,dF(x) + D\int_{D-C}^{\bar{X}} dF(x), \qquad (4)$$

$$d_R = \int_{\underline{X}}^{D-\Delta D} x\,dF(x) + (D-\Delta D)\int_{D-\Delta D}^{\bar{X}} dF(x). \qquad (5)$$

Note that, because of assumption (1), the initial price of debt is below face value; $P_0 = d_0/D < 1$. The following proposition links equity and debt values to the price of the repurchase.

Proposition 1 (The Fair Repurchase Price Is the Face Value). *Under the assumptions of the benchmark model,*

$$\text{(i) if } P_R < 1, \text{ then } \quad S_R > S_0 \quad \text{and} \quad d_R + C < d_0, \qquad (6)$$
$$\text{(ii) if } P_R > 1, \text{ then } \quad S_R < S_0 \quad \text{and} \quad d_R + C > d_0; \qquad (7)$$

that is, regardless of the current market price, repurchases by the firm at the price below face value benefit shareholders, and repurchases at the price above face value benefit bondholders.

Proof. All proofs can be found in the appendix. □

It follows that the face value is the only price at which both sides agree to buy and sell debt. Note that the repurchase price is unique because in the frictionless case debt repurchase does not change the total value of the firm. However, we show in later sections that there may be a range of acceptable prices in cases when the firm's assets increase after the repurchase due to, e.g., reduction in bankruptcy costs, smaller tax, or more efficient investment. For example, we show that the prospect of reducing bankruptcy costs makes room for negotiations between shareholders and bondholders and generally leads to a lower repurchase price.

3.3. Multiple Creditors

In this section we argue that the firm can repurchase debt from multiple bondholders at the market price, or potentially at an even lower price. We show that a firm can repurchase its bonds on the open market or through the tender offer as long as there are small investors willing to sell their entire stake. The important difference from the single-creditor case is that the sellers do not internalize a decline in the value of the remaining debt because it is held by other investors.

We model a single-date, same-seniority (pari passu) debt repurchase from a group of identical bondholders, each holding the same small share of debt. When the firm has outstanding debt of different seniorities, the argument extends to the most senior debt. Sometimes, in addition to the senior debt, the companies also attempt to buy back their junior debt. For example, the 2009 Royal Bank of Scotland tender debt repurchase offer included subordinated notes. However, understanding repurchase offers for junior debt is complicated because they lead to an additional conflict between the different *classes* of the bondholders. Additionally, we assume in this section that revolving credit facilities and other high-priority obligations are repaid before the price for senior debt can be negotiated, debtholders are fully rational and attentive, and there are no bankruptcy costs or other financing frictions.

Consider first a tender offer, when a fixed price is offered to everyone who sells their bonds. If all bondholders tender simultaneously, they are served sequentially in random order until the full amount allocated for this purpose is spent. There is usually no minimum subscription requirement for the offer. It is intuitive that the tender offer equilibrium is contingent on how the offer price, P, compares to the prerepurchase and the expected postrepurchase prices. For example, if the tender offer price is high, the bondholders will participate because the expected postrepurchase price, P_R, is going to be lower. If the tender offer price is low, the bondholders will all abstain because the debt price without the repurchase, P_0, is higher. As the first step in formalizing this intuition, we define a "fixed-point" price, P_F, at which the postrepurchase price remains exactly the same as the offer price.

Lemma 1. *Suppose debt is repurchased through the tender offer from multiple bondholders*:

(i) *There is a unique fixed-point tender offer price P_F, such that post-tender price remains unchanged*:

$$P_F \equiv P = P_R.$$

(ii) *The fixed-point price is lower than the prerepurchase price, $P_F < P_0$.*

According to its definition, the fixed-point price is the only tender offer repurchase price at which bondholders expect the same price after the repurchase is complete and are therefore indifferent between keeping or tendering their bonds. The significance of the lemma is in showing that the fixed-point price is lower than the market price. Therefore, repurchasing part of the debt at the market price will decrease the value of the bonds for the remaining bondholders. We next use this result to discuss the possible equilibria based on the tender offer price.

First, we consider the case when the tender offer price is high, above the prerepurchase and the fixed-point price, $P \geq P_0 > P_F$. From Lemma 1, the bondholders who do not tender receive a strictly smaller postrepurchase price, $P_R < P_0$. Therefore, there is a unique equilibrium in this case: the firm offers a price equal to or just above P_0, all bondholders tender, and a fraction $C/(PD)$ of them are served randomly until all cash C is spent.

Second, suppose that the tender offer price is between the prerepurchase price and the fixed-point price, $P_0 > P > P_F$. The equilibrium in this region depends on the beliefs about the number of bondholders participating in the repurchase.

Proposition 2. *Suppose the tender offer price $P \in (P_F, P_0)$. If every bondholder has a uniform belief j about the fraction of bondholders who will participate in the offer, then,*

1. *for $j \geq j^*$, all bondholders tender, and the tender offer is successful;*
2. *for $j < j^*$, all bondholders abstain from the tender offer, and the offer fails.*

The threshold belief $j^* \in (0, C/(PD))$ is given as a unique solution to the Equation (A8) in the appendix.

The proposition gives the threshold belief regarding the fraction of tendering bondholders, which can trigger a "bank run" (Diamond and Dybvig 1983). For example, if the belief about the success of the offer is highly optimistic, i.e., $j \to 1$, then it implies $P_R < P$, and the offer is successful as nontendering bondholders are expected to be worse off. In contrast, $j \to 0$ implies that $P_R > P$, and the offer fails. Following Diamond and Dybvig, we treat belief j as exogenous.

Third, an even lower tender offer price, which is lower than the fixed-point price, trivially leads to the repurchase failure. By the definition of the fixed point, for any $P \leq P_F$ and any belief j, the postrepurchase price is expected to increase, $P_R \geq P$, because, intuitively, too little cash is spent compared to the debt reduced. Therefore, every bondholder will abstain from tendering.

Intuition for the *open-market* debt repurchases is similar to that for the tender offer case. An important difference, however, is that bondholders may receive different prices for their holdings, depending on the relative timing of the sale. As we have argued, the price for the remaining debt will decrease with each repurchase at the price above the fixed point, including the market price. Therefore, bondholders have a strong incentive to participate, and those who sell first will receive the best deal. At first, this may appear counterintuitive because a debt reduction would seem to make the remaining debt safer. Instead, a repurchase consumes cash inside the firm, making the remaining debt riskier. We do not formally define the equilibrium for the case of open-market repurchases because it requires modeling heterogeneity among bondholders and building a sequential game for the stages of the repurchase. In sum, our model suggests that, given a continuum of dispersed creditors, the firm is always able to repurchase at the current market price.

Anecdotal evidence supports this conjecture. For example, in the year 2009, Hexion Specialty Chemicals spent \$63 million to buy back debt from the open market with face value up to \$288 million. The average price was 22% of the face value. Hovnanian Enterprises spent \$223 million to buy back debt with a face value equal to \$578 million, of which the average price was 39% of the face value. Harrah's used \$378 million cash to buy back a \$788 million face value of debt traded on the market. Beazer Homes used \$58 million to buy back \$116 million in debt (Ng 2009).

There are two other important points that we would like to bring to light in conjunction with the case of multiple bondholders. First, we have assumed throughout that each investor holds an identical small fraction of debt and sells it entirely to the firm. Such continuum of homogeneous investors is a sufficient condition for our results, but not a necessary one. For example, when the creditor composition involves both large and small investors, debt will first be repurchased from the small investors. These investors can sell their entire debt holding in response to the offer and do not need to internalize the consequences of the repurchase on the outstanding debt.

Second, the news of the incoming tender offer, including information on the size of the offer and its outcome, may alter the market prices for both debt and equity. Specifically, anticipation of the repurchase at a discount can result in the market value of debt lower than the initial price. Recall that the price, P_0, is defined as the expected payoff to bondholders if the repurchase is not anticipated or if it is not expected to be successful. The appendix provides the expression for the market price with the adjustment for the repurchase, which may be different from the initial price P_0. It is important, however, that the equilibrium does not hinge on the precise price, which incorporates anticipation of the repurchase. Instead, it depends on the relation between the (i) tender offer price (P), (ii) the price if the repurchase fails (P_0), and (iii) the price if the repurchase is successful (P_R).

Our result that debt is more easily repurchased when there are many bondholders seems to run against the conclusions of the strategic debt service literature (Hart and Moore 1998, Mella-Barral and Perraudin 1997). In that literature, firms facing financial distress can act strategically and force concessions from debtholders. However, whereas strategic debt service deals with bargaining after default, when cash effectively already belongs to creditors, we discuss repurchases by a solvent firm. Because of this difference, we find that the dispersion of debtholders that is commonly seen as an impediment to renegotiations actually helps to reduce leverage and the probability of bankruptcy in debt repurchases.

4. Model Extensions

4.1. Bankruptcy and Bankruptcy Costs

In this section we assume that in the event of default, lenders take over the firm and implement first-best policies, subject to a fraction of the firm's assets being lost during the transfer. It is clear from our discussion that the probability of bankruptcy is not affected when the repurchase is conducted at the face value. However, the prospect of having lower bankruptcy costs and a lower probability of bankruptcy after the repurchase is complete allows for a lower price even in the single-creditor case.

To show this, we assume that a firm entering bankruptcy results in a fixed cost B and proportional cost β, which are known both to shareholders and to creditors. Unlike in, e.g., Leland (1994), we recognize that safe assets may be different from risky assets and assume that cash or liquid assets are subject to cost $\beta_1 \in (0, 1)$, and other assets are subject to cost $\beta_2 \in (0, 1)$. Although not crucial for our argument, it may be reasonable to conjecture $\beta_1 < \beta_2$, meaning that safe/liquid assets are easier to transfer to new owners. Parameter β_1 can also be interpreted as the agency cost, such as the manager's ability to "burn" cash before bankruptcy. For simplicity, we only consider a single-creditor case.

The expected bankruptcy costs are given by

$$BC_0 = \int_{\underline{X}}^{D-C} (\beta_2 x + \beta_1 C + B)\, dF(x). \tag{8}$$

The following proposition shows that the repurchase price is generally lower when there are fixed and proportional bankruptcy costs and gives the range of possible prices.

Proposition 3. *Assume $B > 0$, $\beta_1 > 0$, $\beta_2 > 0$, and that otherwise the assumptions from the frictionless case hold. Then the repurchase price is*

$$P_R \in [P_R^{\min}, 1],$$

where the lower bound on the repurchase price, $P_R^{\min} < 1$, is the solution to Equation (A12) in the appendix.

The proposition gives the upper and lower bounds for the repurchase price according to how the surplus from lower bankruptcy costs is split between shareholders and bondholders. If shareholders have all bargaining power, the lowest price, $P_R^{\min}$, is obtained. If, instead, bondholders have all bargaining power, then debt is repurchased at the face value, as in the frictionless case.

Because the bankruptcy costs are lower in the expectation, firm value increases after the repurchase. Using expression (8), we find that firm value increases by

$$\Delta(BC) = C\beta_1 \int_{\underline{X}}^{D-C} dF(x) + \int_{D-\Delta D}^{D-C} (B + \beta_2 x)\, dF(x). \tag{9}$$

Bankruptcy costs decrease, intuitively, for two reasons. First, during the repurchase, cash or safe asset C is transferred directly to bondholders in exchange for lower debt. It matters because if the firm subsequently defaults or becomes bankrupt, this cash or asset, which is inside the firm, would be subject to the proportional cost β_1.[13] Therefore, expected bankruptcy costs are reduced in debt repurchases even if the probability of bankruptcy is fixed, as captured in the first term in (9).

The second effect arises because repurchases generally lead to a lower probability of bankruptcy. Because of the expected reduction in proportional bankruptcy costs, debtholders agree to tender at a lower price. As a result, there is an additional benefit in the form of lower bankruptcy risk (the second term in (9)).

Overall, we predict that the average debt repurchase price is lower when expected bankruptcy costs are higher. Additionally, keeping bankruptcy cost parameters fixed, the repurchase price increases with the relative bargaining power of bondholders.

4.2. Optimal Leverage and Firm Value

As discussed earlier, discounted debt repurchases, at a price below the face value, result in ex post (relative to the debt issuance date) wealth transfers from bondholders to shareholders. In this section, we argue that repurchases positively affect the ex ante total firm value—the sum of initial equity and debt values. We are concerned with the implications of debt repurchases on optimal leverage, expected tax, and optimal debt structure.

We cast the classical trade-off intuition in our model. Following previous work on capital structure (e.g., Leland 1994), we assume that the firm trades tax benefits of debt with bankruptcy costs. Since we know from the previous sections that buying back debt at face value leaves the total firm value unchanged, we focus only on the market-price repurchases. The following proposition demonstrates, using for simplicity the uniform distribution for the profit x, that both optimal leverage and firm value increase with repurchases.

Proposition 4. *Suppose x is distributed uniformly on $[\underline{X}, \bar{X}]$, the corporate income tax is T_C, and shareholders have an option to repurchase debt with cash C at the market price. Then the optimal amount of debt issued at $t = 0$ is*

$$D^* = \frac{rT_C}{\beta_2}(\bar{X} - \underline{X})\frac{d_0}{d_0 - C} - B\left(\frac{d_0}{d_0 - C}\right)^2. \tag{10}$$

The ex ante firm value is given by

$$V^* = \underbrace{\int_{\underline{X}}^{\bar{X}} x(1 - T_C)\, dF(x)}_{\text{after-tax asset value}} + \underbrace{D^* r\left(1 - \frac{C}{d_0}\right)T_C}_{\text{tax shield}} + C - \underbrace{\beta_2 \int_{\underline{X}}^{D^* - CD^*/d_0} x\, dF(x)}_{\text{bankruptcy cost on assets}}$$

[13] Cash is subject to bankruptcy costs, even if the firm can eventually restructure and exit the bankruptcy. For example, LoPucki and Doherty (2011) estimate that only direct legal fees on all assets including cash can be as high as 2%.

$$\underbrace{-B\int_{\underline{X}}^{D^*-CD^*/d_0} dF(x)}_{\text{fixed costs of bankruptcy}}. \tag{11}$$

Both D^* and V^* increase with the amount of repurchase.

Proof. See the appendix. □

The firm value and leverage are higher when repurchases are allowed because the bankruptcy cost and the probability of bankruptcy are lower. Note that (10) must be treated as an implicit equation, because d_0 can also depend on the optimal debt. The proposition only determines optimal leverage given cash holdings; that is, cash C is not jointly determined with the optimal debt, D^*.

Our result is directly comparable to the classic dynamic capital structure literature (Goldstein et al. 2001). In our model, the firm has an option to lower the leverage ratio in the future and is more aggressive initially in order to increase current debt benefits. We conclude that discounted debt repurchases are beneficial to shareholders. Contrary to the initial intuition that exploiting bondholders during the process leads to an agency problem, allowing debt repurchases can actually increase the ex ante firm value. This is because the shareholders gain more than bondholders lose. The option to repurchase reduces the instances of defaults and increases debt capacity.

The result that the option to repurchase debt is valuable to the firm has implications on debt design and the optimal creditor structure. In particular, some features of debt can interfere with the firm's ability to repurchase. For example, consider conversion options. They contain an equity part and therefore necessitate an additional SEC approval prior to the repurchase. Furthermore, the option to repurchase debt is directly affected by seniority structure. Our base model gives results for same seniority for all bondholders, based on the observation that the repurchase offer is typically made for a single class of senior debt. However, firms commonly carry several tranches of debt with a slightly different seniority for each separately sold debt fraction, making repurchases more difficult and reducing shareholder value. Finally, the optimal creditor structure—in particular, distribution of debt among creditors—can also affect repurchases. The more dispersed debt ownership is, the easier it is to restructure through a tender offer or an open-market debt repurchase.

Overall, we find that bankruptcy costs and dispersed debt ownership, two assumptions that are common to U.S. firms, result in a lower repurchase price. With moderate transaction costs, debt repurchases are therefore beneficial to equity. However, despite the potential advantage of immediate repurchase, we show in the next section that treating the repurchase as an option and delaying its exercise results in even higher expected profits.

4.3. Repurchase Timing

We previously adopted the assumption that debt must be repurchased on a single date. This section extends the previous analysis by studying the intertemporal debt/cash policy in a two-period model. Such a model allows us to understand what determines the optimal timing of debt repurchase and, in particular, the shareholder's incentives to delay the repurchase.

Assume there are three dates, $t=0$, 1, 2, and values are denominated in date $t=0$ dollars. The firm's total profit at the end date $t=2$ is equal to the sum of the independently distributed profits from the first and second periods, x_1+x_2, where $x_{1,2}\in[\underline{X},\bar{X}]$. At $t=1$, the information about x_1 becomes available, and at date $t=2$, the information about x_2 becomes available.

Since the model now extends beyond a single period, we need to adjust the subscript notation accordingly. Assume that the initial face value of debt is D_0 and that it can be reduced to D_1 (before profit x_1 is revealed). At the next date, D_1 can be further reduced to D_2 (before x_2 is revealed). Similarly, we denote the cash changes due to the first and second repurchases as C_0-C_1 and C_1-C_2. We already know that at $t=2$, shareholders benefit from buying the maximum amount of debt using all remaining cash; therefore we set $C_2=0$.

In the absence of intermediate dividends, the objective function of the shareholders is therefore the expected value of the payoff at the last date $t=2$:

$$\max_{(C_1)} V_0 = \int_{\underline{X}}^{\bar{X}}\left[\int_{D_2(C_1)-x_1}^{\bar{X}}(x_1+x_2-D_2(C_1))\,dF(x_2)\right]dF(x_1), \tag{12}$$

where $F(x_1)$ and $F(x_2)$ are the cumulative distribution functions for x_1 and x_2. With a minor abuse of terms, the derivative of this function, $\partial V_0/\partial C_1$, can be interpreted as the "propensity to save" or "propensity not to reduce debt." Using the fact that the manager, maximizing equity value, equivalently minimizes the final-period debt value, we derive the following results for the repurchase timing.

Lemma 2. *In the frictionless case, timing of the repurchase is irrelevant.*

It is straightforward to see from (12) why in the absence of frictions equity value is independent of repurchase timing. In this case, the price equals the face value, regardless of the time of the repurchase, and cash simply cancels an equal amount of debt, $D_2=D_0-C_0$. The irrelevancy result exists in the frictionless case because the firm never "regrets" undertaking repurchases earlier. Below we show that the timing matters outside of frictionless case.

Suppose the firm can repurchase on the open market at the current market price. We show that the shareholders are better off repurchasing later. This is because the future price is uncertain, and the value function is convex in the repurchase price. Therefore, by invoking Jensen's inequality and using the fact that the price at the first date is equal to the expectation of the price at the second date, we immediately obtain the following result.

Proposition 5. *Suppose debt is repurchased at price P_M^1 at date $t=1$ or/and at price $P_M^2(x_1)$ at date $t=2$ such that*

$$P_M^1 = \int_{x_1} P_M^2(x_1)\, dF(x_1). \tag{13}$$

Then it is optimal to delay repurchase.

If debt repurchase is associated with additional transaction costs, it may become optimal to abandon the repurchase when the debt price becomes too high. Transaction costs effectively increase the cost of the repurchase, and therefore the firm repurchases selectively. We delegate details for this case to the appendix. We show, in particular, that a proportional linear fee levied on the total transaction amount forces the firm to repurchase only if the first-date profit does not exceed a particular trigger value, x_1^*. Otherwise, the firm will optimally abandon the repurchase and avoid paying the transaction fee. Therefore, waiting until $t=2$ to learn about the realization of the first-period profitability leads to a higher firm value. A similar intuition applies to the fixed costs, with the exception that the optimal strategy depends on the volume of the repurchase.

Based on the two-date model in this section, we conclude that companies, including those that would benefit from buying back debt using the first opportunity, are better off delaying the repurchase. At the end, we may not observe as many repurchases in the data as predicted by the simple one-period model because the option to buy back debt may expire unexercised.

4.4. Net Tax Benefit

Here, we discuss the tax implications of repurchasing debt versus saving cash. The purpose of this exercise is twofold. First, it clarifies the tax implications for the debt net of cash or negative debt, extensively discussed in the literature, but allows for a general price of debt instead of the dollar-for-dollar price. Second, it shows the trade-off between the value of a discounted repurchase and the cost of a lower tax shield and the COD tax.

As is standard in the literature (see, e.g., Auerbach 2001), we track the after-tax payoff to shareholders under the two alternatives. If the firm saves C for one period, the after-tax dividend to shareholders is

$$\text{Payoff}_{\text{Save}} = C(1+r(1-T_c))(1-T_d), \tag{14}$$

assuming that tax T_c is levied on corporate income and cash distributions are subject to further tax at the rate T_d. Alternatively, if the firm repurchases a portion of its debt, ΔD, the after-tax dividend is

$$\begin{aligned}\text{Payoff}_{\text{Rep}} &= \Delta D(1+r(1-T_c))(1-T_d)\\ &\quad - T_{\text{COD}}\max(\Delta D - C, 0)(1-T_d),\end{aligned} \tag{15}$$

where the second term is an additional tax on the COD income if debt is repurchased at a discount. From (14) and (15), we compute the debt repurchase tax advantage over saving as

$$\begin{aligned}\text{Adv}_{\text{Rep}} &= (\Delta D - C)(1+r) - (\Delta D - C) r T_C\\ &\quad - T_{\text{COD}}\max(\Delta D - C, 0).\end{aligned} \tag{16}$$

The direct benefit of repurchasing debt at a discount (first term) is reduced by a higher corporate tax because of the lower debt net of cash (second term) and also a higher COD tax (third term). We compare this expression to our base model and conclude that corporate and COD tax reduce the ex post benefits from the repurchase.

4.5. Debt Repurchase and Investment

It is well known that excessive and risky debt suppresses new investment.[14] Therefore, it seems natural that restructuring through a debt repurchase would increase investment efficiency. However, our previous analysis demonstrates that debt repurchases reduce firm risk only to the extent that the repurchase price is lower than face value. We therefore anticipate that the ability to mitigate the debt overhang problem also hinges on negotiating a repurchase discount. To formalize and extend this idea, we introduce capital investment into the existing model of risky debt and study how buying back debt affects investment incentives.

To model investment in a simple form, we assume that shareholders can invest amount I, expecting the payoff $x(I)$. The effect of investment on the cash flows is modeled through the cumulative distribution function $G(x \mid I)$ on the domain $[\underline{X}, \bar{X}]$. Specifically, since investment must positively affect future profits, we assume $G_I(x \mid I) < 0$, that is, the payoff from larger investment first-order stochastically dominates the payoff from the smaller investment. A simple example for this investment is the linear shift in the probability distribution of the payoff, corresponding to a constant positive return $R > 1$,

$$G(x \mid I) = F(x - RI), \tag{17}$$

where $F(x)$ is the CDF of the payoff distribution without investment. Finally, we assume that investment must be financed externally, and the financing is subject to cost $\phi(\cdot)$.

[14] The debt overhang problem manifests itself in the prohibitively high cost of external equity for firms with risky debt, leading to insufficient capital expenditures and high postinvestment "marginal q" (Hennessy 2004, Myers 1977, and others).

Proposition 6. *The optimal investment I^* is monotonically decreasing with the repurchase price and is given by*

$$\int_{D-C/P}^{\bar{X}} -G_I(x \mid I^*)\, dx = \phi_I(I^*). \tag{18}$$

Expression (18) can be interpreted as the marginal value of investment equal to the marginal cost of external financing. The proposition shows that optimal investment is a decreasing function of debt net of cash. Therefore, as we conjectured, investment incentives are unchanged with the dollar-for-dollar repurchases.

Repurchases at the price below the face value would positively affect the optimal investment, but because the anticipation of investment raises debt price, negotiating such a low price may be difficult. Intuitively, the prospect of valuable investment increases the repurchase price. This is because the market will internalize the benefits of the investment, and bondholders demand the premium. Overall, our results in this section extend and complement the analysis in Bulow and Rogoff (1991). They show that the buyback of sovereign debt is a giveaway to creditors because the relief from debt overhang is expected to increase the market value of debt. We discuss corporate debt and link the effect on debt overhang to the repurchase price. Our contribution is to show that the ability of the corporate debt buyback to mitigate the agency problems such as debt overhang crucially relies on the low repurchase price. Finally, it follows from the analyses both in our paper and in Bulow and Rogoff (1991) that debt overhang can be mitigated if cash is alternatively used to cover a part of investment cost directly, instead of repurchasing debt first.

5. Conclusion

When managers are confronted with a choice between saving cash and repurchasing debt, they face a tradeoff between the costs and benefits of the repurchase. This paper provides a theoretical guidance for these decisions. We find that firms that can buy back debt at a discounted price benefit from the repurchase and also benefit more if they delay the repurchase. Simultaneous saving and borrowing creates an opportunity to buy back debt conditional on a lower price in the future, or scrap the repurchase plan otherwise.

Importantly, we show that the dispersion of debtholders that is commonly seen as an impediment to renegotiations actually helps to reduce leverage and the probability of bankruptcy in debt repurchases. Our findings have implications for security design and the pricing of debt contracts. In particular, public debt dominates private debt held by larger shareholders because the former is easier to repurchase as the firm profitability decreases.

Our theory produces novel empirical hypotheses. First, discounted debt repurchases result in a value transfer from bondholders to shareholders and therefore should increase the value of equity and decrease the value of debt. The size of the value transfer, and therefore the magnitude of the price reaction, is expected to be larger with the repurchase discount. A similar contrasting prediction for the bond and share prices was developed and verified in the stock share repurchase literature; see, e.g., Maxwell and Stephens (2003). Second, the repurchase price must be lower when the expected bankruptcy costs are higher or when debt is dispersedly held and can be repurchased in the open market. Third, we expect firms to simultaneously carry cash and risky debt. This hypothesis finds some support in the existing studies. For example, Bates et al. (2009, p. 1985) state that "the average firm can retire all debt obligations with its cash holdings." Finally, we predict lower market values for the firms that are unable to utilize debt buybacks for restructuring, for example, if debt agreements prohibit repurchases.

Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/mnsc.2014.1965.

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Appendix A. Repurchase Price Derivation

Proof of Proposition 1. To show that $S_R > S_0 \Leftrightarrow P_R < 1$, we define the function of $G(y)$ as

$$G(y) = \int_{D-y}^{\bar{X}} [x + y - D]\, dF(x), \quad y \in [0, D]. \tag{A1}$$

Function $G(y)$ increases in the argument

$$\begin{aligned} G'(y) &= (D-y) f(D-y) + \int_{D-y}^{\bar{X}} dF(x) - (D-y) f(D-y) \\ &= \int_{D-y}^{\bar{X}} dF(x) > 0, \end{aligned} \tag{A2}$$

and therefore $P_R < 1$, or alternatively $C < \Delta D$, implies $G(C) < G(\Delta D)$,

$$\int_{D-C}^{\bar{X}} (x + C - D)\, dF(x) < \int_{D-\Delta D}^{\bar{X}} [x + \Delta D - D]\, dF(x), \quad \text{(A3)}$$

which is the same as $S_R > S_0$, using notations (2) and (3) in the main text. The claim in the proposition for the debt value can be easily checked using expressions (4)–(6) in the main text.

Proof of Lemma 1 (Fixed-Point Price). Suppose the tender offer repurchase price is equal to the postrepurchase price, $P_F \equiv P = P_R$. Note that, in this case, the face value of the debt after repurchase is reduced to $(D - C/P_F)$. Therefore, using (5), P_F can be solved from the following equation:

$$\frac{d_R}{D - C/P_F} \equiv \int_{\underline{X}}^{D-C/P_F} \frac{x}{D - C/P_F}\, dF(x) + \int_{D-C/P_F}^{\bar{X}} dF(x) = P_F. \quad \text{(A4)}$$

This equation has a unique solution for P_F, which is between C/D and the prerepurchase price, $P_0 > C/D$. It follows from considering $P_F = P_0$ and $P_F \to (C/D)+$ and using the fact that function (A4) is continuous in between.

Specifically, suppose $P_F = P_0$; we show that the left-hand side of (A4) is smaller than the right-hand side. This is because, from Proposition 1, we have

$$d_R < d_0 - C, \quad \text{(A5)}$$

or

$$\frac{d_R}{D - C/P_0} < \frac{d_0 - C}{D - C/P_0} = P_0. \quad \text{(A6)}$$

Now suppose $P_F \to (C/D)+$; then the left-hand side of (A4) approaches 1, which is higher than P_F.

Proof of Proposition 2 (Tender Offer Equilibria). The threshold belief j^* is defined as the fraction of participating bondholders, at which the postrepurchase price is exactly equal to the tender offer price:

$$P_R(j^*) = P, \quad \text{(A7)}$$

which is

$$\frac{1}{D - j^* D} \int_{\underline{X}}^{D-j^*D} x\, dF(x) + \int_{D-j^*D}^{\bar{X}} dF(x) = P. \quad \text{(A8)}$$

The left-hand side is monotonically decreasing in j^*; therefore the solution for j^* is unique for all $P \in (P_F, P_0)$. In particular, $j^* = 0$ for $P = P_0$, and $j^* = C/(PD)$ for $P = P_F$. Therefore, for $j > j^*$, $P_R(j) < P$, and the offer is successful. For $j < j^*$, $P_R(j) > P$, the bondholders will abstain.

Derivation of the Market Price After the Repurchase Announcement

To support the discussion of the multicreditor case, we prove the following: (i) the market price of debt reacts negatively to the news of the discounted repurchase, and (ii) the market price is lower when the tender offer price is lower.

The market price is the weighted average of the price paid in the tender offer and the price of the bonds after the repurchase. Since (C/DP) of the bonds are repurchased and $(1 - C/DP)$ remain outstanding, we have

$$P_{EX}(P) = \frac{C}{DP} P + \left[1 - \frac{C}{DP}\right] P_R(P). \quad \text{(A9)}$$

Note that for $P = 1$ (repurchase at the face value), the market price is unaffected by the repurchase announcement,

$$P_{EX}(1) = P_0. \quad \text{(A10)}$$

Finally, note that

$$\frac{dP_{EX}(P)}{dP} = \frac{\partial d_R}{\partial D_R} \frac{dD_R}{dP} > 0,$$

and therefore the market price decreases more if the offer price is lower.

Proof of Proposition 3 (Bankruptcy Costs). The lower bound on the repurchase price obtains when the bondholders' participation condition binds:

$$d_0 \le d_0^R + C. \quad \text{(A11)}$$

Setting it to equality and using (4) and (5), with $\beta_{1,2} > 0$, we obtain the implicit expression for the minimum repurchase price $P_R^{\min}$:

$$\int_{D-C/P_R^{\min}}^{D-C} (x - D + C/P_R^{\min})\, dF(x) + \int_{D-C}^{\bar{X}} (C/P_R^{\min} - C)\, dF(x) = \left[\int_{D-C/P_R^{\min}}^{D-C} (B + \beta_2 x)\, dF(x) + \beta_1 \int_{\underline{X}}^{D-C} C\, dF(x)\right]. \quad \text{(A12)}$$

Since $F(x)$ is continuous on $[\underline{X}, \bar{X}]$, the left-hand side of this equation is a continuously decreasing function for $P_R^{\min} \in [C/D, 1]$ and has a minimum of zero at $P_R^{\min} = 1$. Additionally, since the right-hand side of the equation is strictly positive, there is a *unique* $P_R^{\min} < 1$; that is, the lower bound on the repurchase price is below face value. The upper bound to the repurchase price obtains when the shareholders' participation constraint binds. From Proposition 1, $P_R^{\max} = 1$. Finally, note that after the repurchase, the bankruptcy costs decrease to

$$BC_R = \int_{\underline{X}}^{D-\Delta D} (B + \beta_2 x)\, dF(x), \quad \text{(A13)}$$

which is used to derive (9) in the text.

Appendix B. Extensions

Proof of Proposition 4 (Optimal Leverage). Omitting the distribution tax, we can write the value of equity as

$$S_0 = \int_{D-C}^{\bar{X}} (x + C - D)\, dF(x) - \int_{\underline{X}}^{\bar{X}} (x + r(C - D)) T_C\, dF(x), \quad \text{(B1)}$$

where the second term is the expected value of tax payments. The market value of debt is

$$d_0 = \int_{\underline{X}}^{D-C} (x + C)\, dF(x) + D \int_{D-C}^{\bar{X}} dF(x) - \int_{\underline{X}}^{D-C} (\beta_2 x + \beta_1 C - B)\, dF(x), \quad \text{(B2)}$$

where the last term is the expected value of bankruptcy costs. Summing (B1) and (B2) produces firm value without repurchases:

$$V = \underbrace{\int_{\underline{X}}^{\bar{X}} (x+C)(1-T_C)\,dF(x)}_{\text{after-tax asset value}} + \underbrace{r(D-C)T_C}_{\text{tax shield}} - \underbrace{\beta_1 C \int_{\underline{X}}^{D-C} dF(x)}_{\text{bankruptcy costs (on cash)}} - \underbrace{\beta_2 \int_{\underline{X}}^{D-C} (x+B)\,dF(x)}_{\text{bankruptcy costs}}. \tag{B3}$$

The optimal debt D^* is directly obtained from the first-order condition. For example, if x is distributed uniformly on $[\underline{X}, \bar{X}]$, then we have

$$D^* = \frac{r}{\beta_2} T_C(\bar{X} - \underline{X}) + \frac{\beta_2 - \beta_1}{\beta_2} C - \frac{B}{\beta_2}. \tag{B4}$$

Now consider the case with discounted debt repurchases. Suppose that debt is repurchased at the price $d_0 < D$. Bondholders compute the expected value of debt taking into account the anticipated repurchase,

$$d_R = C + (1-\beta_2)\int_{\underline{X}}^{D_R} x\,dF(x) - B\int_{\underline{X}}^{D_R} dF(x) + D_R \int_{D_R}^{\bar{X}} dF(x),$$

where, from the budget condition, the remaining debt after the repurchase is

$$D_R = D - \frac{CD}{d_0}, \tag{B5}$$

and d_0 is given by (B2).

The value of equity is

$$S_R = \int_{D_R}^{\bar{X}} (x - D_R)\,dF(x) - \int_{\underline{X}}^{\bar{X}} (x - rD_R)T_C\,dF(x). \tag{B6}$$

The sum of the value of debt and equity values produces (11) in the main text. The first-order condition of (11) with respect to D^* yields the optimal level of debt in (10). It then follows directly that D^* and the firm value increase with the amount of the repurchase.

Proof of Lemma 2 (Multiperiod Extension). This lemma concerns repurchase timing in the *frictionless case*. Using $P_R^{\max} = 1$ from Proposition 1, and therefore letting

$$C_1 = C_0 + D_1 - D_0 \quad \text{and} \quad D_2 = D_1 - C_1 \tag{B7}$$

in (12), we find that (12) is independent of C_1, and therefore the timing of the repurchase is irrelevant in the frictionless case.

Proof of Proposition 5 (Repurchase Timing). The proposition assumes that bonds are sold at the *market price* $P_M^1 \equiv d_0/D_0$ at $t=1$ and $P_M^2(x_1) \equiv d_1(x_1)/D_1$ at $t=2$, where

$$d_1(x_1) = \int_{x_1+\tilde{x}_2+C_1 \ge D_1} D_1\,dF(\tilde{x}_2) + \int_{x_1+\tilde{x}_2+C_1 < D_1} (x_1+\tilde{x}_2+C_1)\,dF(\tilde{x}_2). \tag{B8}$$

Then the budget conditions are

$$C_1 = C_0 + P_M^1(D_0 - D_1), \quad D_2 = D_1 - C_1/P_M^2(x_1). \tag{B9}$$

To show that it is optimal to repurchase at $t=2$, we compare shareholders' value at date $t=0$, $S(C_1=0)$ and $S(C_1=C_0)$, under two cases: $C_1=0$ (use all cash to repurchase at $t=0$) and $C_1=C_0$ (use all cash to repurchase at $t=1$). We show that $S(C_1=0) < S(C_1=C_0)$ by applying Jensen's inequality twice to get

$$\begin{aligned} &S(C_1=0)\\ &= \int_{\underline{X}}^{\bar{X}} \int_{D_0 - C_0/P_M^1 - x_1}^{\bar{X}} (x_1 + x_2 - (D_0 - C_0/P_M^1 - x_1))\,dF(x_2)\,dF(x_1)\\ &\le \int_{\underline{X}}^{\bar{X}} \left[\int_{E_{x_1}[D_0 - C_0/P_M^2] - x_1}^{\bar{X}} (x_1 + x_2 - E_{x_1}[D_0 - C_0/P_M^2])\,dF(x_2)\right] \cdot dF(x_1)\\ &\le \int_{\underline{X}}^{\bar{X}} \left[\int_{D_0 - C_0/P_M^2 - x_1}^{\bar{X}} (x_1 + x_2 - D_0 - C_0/P_M^2)\,dF(x_2)\right] dF(x_1)\\ &= S(C_1 = C_0), \end{aligned} \tag{B10}$$

and the result follows.

Optimal Timing with Transaction Costs. The model introduces transaction costs as a proportional fee γ, levied on the total transaction amount. We consider only open-market repurchases and prove the following claims: (i) the firm repurchases at $t=2$ only if $x_1 < x_1^*$ for some threshold $x_1^* \in [\underline{X}, \bar{X}]$, and (ii) the propensity to delay the repurchase increases in γ.

Note that, from Proposition 1, shareholders benefit from a repurchase when $D_1 - D_2 < C_1 - C_2$. Therefore, from (B14),

$$(1-\gamma)D_1 = d_1(x_1^*), \tag{B11}$$

which proves our first claim.[15]

Second, for an interior x_1^*, we can rewrite (12) as a sum of two separate terms reflecting value when the repurchase is optimal (the first term) and when it is not (the second term):

$$\begin{aligned} &\max_{(C_1, C_2)} S\\ &\quad = \int_{\underline{X}}^{x_1^*} \int_{D_2 - x_1}^{\bar{X}} (x_1 + x_2 - D_2)\,dF(x_2)\,dF(x_1)\\ &\qquad + \int_{x_1^*}^{\bar{X}} \int_{D_1 - C_1 - x_1}^{\bar{X}} (x_1 + x_2 + C_1 - D_1)\,dF(x_2)\,dF(x_1). \end{aligned} \tag{B12}$$

Equity maximization is subject to the budget constraints for the repurchase at $t=1$,

$$(1-\gamma)(C_0 - C_1) = P_M^1(D_0 - D_1), \tag{B13}$$

and at $t=2$,

$$(1-\gamma)(C_1) = P_M^2(x_1)(D_1 - D_2), \tag{B14}$$

[15] Note that benefits per dollar used in the repurchase are measured by the difference between the face value and the market value, $1 - d_1(x_1^*)/D_1$, and that the cost of the repurchase per dollar is given by γ; therefore the threshold x_1^* defines the point at which the benefit exactly offsets the cost. Similar intuition applies to the case with nonlinear transaction costs.

where we used $C_2 = 0$, by Proposition 1, since $t = 2$ is the final date. The first derivative of (B12) with respect to C_1 (the propensity to save) produces

$$\frac{\partial S}{\partial C_1} = \int_{x_1^*}^{\bar{X}} \left(1 - \frac{\partial D_1}{\partial C_1}\right) \int_{D_1 - C_1 - x_1}^{\bar{X}} dF(x_2)\, dF(x_1) - \int_{\underline{X}}^{x_1^*} \frac{\partial D_2}{\partial C_1} \int_{D_2 - x_1}^{\bar{X}} dF(x_2)\, dF(x_1), \tag{B15}$$

which increases in γ because $\partial D_1/\partial C_1$ decreases in γ from (B13), and because $\partial D_2/\partial C_1$ increases in γ from (B14). Therefore,

$$\left.\frac{\partial S}{\partial C_1}\right|_{\gamma > 0} > \left.\frac{\partial S}{\partial C_1}\right|_{\gamma = 0} = 0, \tag{B16}$$

where the last equality follows from Lemma 1. This proves the second claim.

Proof of Proposition 6 (Optimal Investment). The optimal investment maximizes firm value net of costs of investment:

$$\max_I \left[\int_{D-C}^{\bar{X}} (x + C - D)\, dG(x \mid I) - \phi(I) \right]. \tag{B17}$$

It can be simplified with integration by parts as

$$\max_I \left[(\bar{X} + C - D) - \int_{D-C}^{\bar{X}} G(x \mid I)\, dx - \phi(I) \right]. \tag{B18}$$

The optimal investment, without repurchase, is determined by the following first-order condition (we assume the second-order condition holds):

$$\int_{D-C/P}^{\bar{X}} -G_I(x \mid I^*) = \phi_I(I^*), \tag{B19}$$

where $P = 1$ corresponds to the original debt level before the repurchase.

Note that debt overhang can also be mitigated if cash or asset C is simply used to cover a part of investment cost instead of repurchasing debt. To illustrate this, consider a firm deciding to allocate one dollar to the cost of repurchase or to the cost of investment. The condition for this trade-off is that the marginal value from the repurchase is equal to the marginal cost of financing investment; that is,

$$\int_D^{\bar{X}} \left(-\frac{dG(x \mid I)}{dI} \right) dx = \phi'(I - C). \tag{B20}$$

Obviously, for the firms with high marginal costs of external financing, the optimal solution is to allocate at least some of the cash directly to investment.

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