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Advance Selling: Effects of Interdependent Consumer Valuations and Seller's Capacity

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We examine the impact of consumer valuation interdependence and capacity on a firm's optimal selling strategies. We consider a seller who can offer a single product to consumers twice, in advance and in spot. Consumers choose whether and when to buy, but if they buy in advance, they are uncertain about their own valuations. Whether they buy in advance or in spot, consumers' valuations are realized in the spot period and they may range from fully independent to perfectly correlated, creating markets with different characteristics of aggregate demand for the seller. Facing these consumers, the seller chooses a portion of the total capacity to offer in advance and prices in both periods. We describe how the optimal strategy and benefits of advance selling depend on the interdependence of consumer valuation, as well as capacity level and other market parameters. We find that a change in valuation interdependence can lead to dramatically different policies for the seller. For example, when individual valuations are highly diverse and the consumer population is large, the seller must offer a discount during advance selling but may limit the advance sales. On the other hand, when valuations are highly correlated, the seller can charge a premium price during advance selling. For the same valuation interdependence, the qualitative nature of the optimal strategy changes with available capacity.

Keywords: advance selling; interdependent valuation; capacity rationing
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1. Introduction

Advance selling has become a standard practice in the service industry (e.g., airlines and travel packages) and in the retail industry (e.g., toys, books, and some electronics). Through advance selling, a seller offers consumers an opportunity to purchase products or services considerably before the time they consume the product. Advance selling can benefit both sellers and consumers: It can help sellers to plan ahead and reduce demand variability (Tang et al. 2004, Song and Zipkin 2012, Li and Zhang 2013), and it benefits the consumers by reducing their risk of not getting the product and/or lowering the price they pay (Png 1989).

Another significant reason for sellers to offer advance selling is to exploit consumers' uncertainty about their valuations (see DeGraba 1995, Xie and Shugan 2001, Gallego and Şahin 2010, Png and Wang 2010). Valuation uncertainty is the focus of our paper. We show that, in addition to individual valuation uncertainty, intercorrelation among consumers' valuations also critically affects a seller's optimal advance-selling strategy and resultant profit.

Valuation uncertainty prevails during advance selling because an individual consumer's valuation at the time of consumption is influenced by many "state (situational) variables," which are fully realized only at, or close to, the time of consumption (Belk 1975). Some state variables are consumer dependent (e.g., taste, mood, health, and scheduling conflict), with values that are idiosyncratic and determined by an individual consumer's consumption states (Shugan and Xie 2000, 2004; Xie and Shugan 2001). For example, a nonrefundable ticket for a flight may become obsolete if a scheduling conflict or a family emergency forces a passenger to cancel the scheduled trip (Gallego and Şahin 2010). The realizations of consumer-dependent variables tend to be uncorrelated across consumers. In contrast, some state variables are environment dependent and are determined by the state of nature or exogenous events at the time of consumption. For example, weather and month affect consumer valuation and, hence, total demand of tickets to baseball games for a home team (Phillips 2005). Similarly, the appearance of famous players, star actors, or celebrities can significantly increase the appeal of a basketball game (Talluri and van Ryzin 2004), a Broadway show (Healy 2010), or a club (Ng 2007). Because the environment-dependent variables affect all consumers' valuations in a similar way, one consumer's valuation is likely to be *correlated* with the others'.

The level of valuation interdependence varies widely across products and industries, depending on whether consumer-dependent or environment-dependent variables play a more significant role. For some products, such as prepaid dinner buffets, conferences, and travel-related services (Xie and Shugan 2001, Shugan and Xie 2004), individual factors dominate. For others, such as tickets for certain sports events (Phillips 2005, Talluri and van Ryzin 2004) or Broadway shows (Healy 2010), environmental factors dominate. It is also possible that state variables can influence the valuation of a group of consumers. For example, a family emergency (e.g., an unexpected visitor or family illness) affects the valuation of a family vacation package (Shugan and Xie 2000).

When faced with valuation uncertainty and its interdependence, consumers can buy the product during advance sales or defer their decisions until uncertainties are resolved or significantly reduced. Basketball fans can purchase tickets on a game day and passengers can buy tickets once their schedules are finalized. To induce consumers to buy in advance, the seller may need to compensate for consumers' uncertainty with a better price and/or guaranteed availability. One of the primary aims of this paper is to illustrate that the interdependence of consumer valuations creates very different markets for the seller even when the valuation distribution of an individual consumer remains the same.

We explore the effect of valuation interdependence on both the behavior of strategic consumers and on the seller's strategy. We are interested in describing the situations when the seller benefits from advance selling (as a function of valuation interdependence) and examining how the seller should set the advance price (selling at a discount or at a premium) and the amount of capacity available in advance (limiting the sales during advance selling or offering all capacity). As described below, this paper is different from the existing work in several ways.

1.1. Relevant Literature

Our work belongs to the literature on advance selling. Papers in this stream identify a number of reasons explaining why a seller may offer advance selling. Tang et al. (2004) and Prasad et al. (2011) show that demand signal obtained from advance selling can help a seller to plan inventory better. When customers are averse to risk or loss, or experience regret, advance selling allows a seller to exploit customers' desire for guaranteed availability (Png 1989, Zhao and Stecke 2010, Nasiry and Popescu 2012). Desiraju and Shugan (1999), Gale and Holmes (1993), and Dana (1998) note that advance selling serves as a tool of price discrimination when customers are heterogeneous. Advance selling may arise as a response to competition (McCardle et al. 2004). Guo (2009), Gallego and Şahin (2010), and Swinney (2011) examine the interaction of advance selling with other strategies like partial refund and quick response.

Two papers are most closely related to our work. DeGraba (1995) finds that advance selling can be profitable for the seller without any of the above reasons, as long as customers are uncertain about their valuations. He shows that the seller prefers selling to uninformed consumers in advance because their valuations are more homogeneous than those of spot customers who buy after they learn their valuations. In particular, DeGraba proves that a seller with infinite production capacity may intentionally limit quantity to induce uninformed consumers to buy in advance. Xie and Shugan (2001) consider a model similar to that in DeGraba (1995), but they allow for rationing of capacity, while treating the capacity as exogenous. They show that advance selling may be profitable to the seller as long as the capacity is not too tight. Furthermore, they describe the mechanism through which advance selling should be implemented. Specifically, a premium price in advance selling is optimal when capacity is very high, whereas capacity rationing should be used with intermediate levels of capacity. Although our paper examines the effects of seller's capacity as DeGraba (1995) and Xie and Shugan (2001) do, we focus on a different factor that significantly changes the seller's strategy: interdependence among consumer valuations. DeGraba (1995) and Xie and Shugan (2001) assume that customer valuations are completely independent, but our primary focus is on interdependent valuations. We show that, even with the same valuation distribution, the seller's strategy when customers have independent valuations is significantly different from the strategy when valuations are highly correlated. Furthermore, we demonstrate that interaction between capacity and valuation interdependence is strong. We also show that distribution of customer valuations may critically affect the seller's optimal policy. Although Xie and Shugan (2001) assume that customers can only have two discrete levels of valuation (i.e., high and low), we allow customer valuation to follow a continuous distribution and find that some of the key results in Xie and Shugan (2001), related to pricing strategy, significantly change.

In addition to DeGraba (1995) and Xie and Shugan (2001), other papers that consider capacity decision in advance selling include Png (1989), Gale and Holmes (1993), Guo (2009), and Gallego and Şahin (2010). The main difference of our work from these papers is that we evaluate the impact of limited capacity on the dynamic pricing and capacity rationing strategies and focus on its interplay with valuation interdependence.

Our work is also related to papers that consider customers' valuation uncertainty, but not in the context of advance selling (e.g., Su 2009, Akçay et al. 2013, Png and Wang 2010). The focus of these papers has been on various selling mechanisms to mitigate consumers' uncertainty. In all of these papers, however, customers do not have the option of delaying purchasing decisions until after the valuation uncertainty is resolved, which is one of the key features we consider. More importantly, a primary distinction between our paper and all the existing work on valuation uncertainty is that we explicitly model the interdependence among consumer valuations.

From a broader perspective, our paper is connected to the research stream on dual channels with posted price and auctions,[1] (e.g., Etzion et al. 2006, Caldentey and Vulcano 2007, Sun 2008, Etzion and Moore 2013). In these papers, customers can either immediately purchase at a buy-it-now price or join an auction, postponing their purchase but having a chance of getting the product at a lower price. The dual channel, in many cases, outperforms the single channel with a posted price because the seller can segment the market. Although advance selling serves a similar purpose of price discrimination, it differs from the dual channel in the pricing mechanism. In advance selling, only the seller sets the price(s). In the dual-channel literature, the seller controls the price of only one channel while the market controls the price of the second channel (auction). In addition, none of the papers in the dual-channel literature considers valuation uncertainty or its interdependence.

1.2. Contributions

This paper is, to our knowledge, the first that studies the effect of the interdependence of consumers' valuations on advance selling. We propose a unified framework to model different levels of interdependence and fully characterize the seller's optimal strategy for two important cases: ∞-group and one-group models. We also numerically evaluate the impact of valuation interdependence on the seller's profit and gain from advance selling.

Our paper aims to substantiate and extend the existing results of advance selling established by Xie and Shugan (2001) and the other advance-selling papers, all of which assume that consumer valuations are independent. Our analytical and numerical results show that, even when the distribution of individual valuation remains the same, the seller's optimal strategy can be significantly different from those in the literature as valuation interdependence and the resultant overall demand change. Furthermore, we show that the seller's capacity plays a pivotal role and interacts with the valuation interdependence. Without a capacity constraint, a seller always offers a discount if advance selling is offered, and the seller's strategy is completely independent of valuation interdependence. However, when the seller's capacity is limited, the valuation interdependence significantly affects not only the pricing policy, but also the seller's decision on how much of the capacity (if any) to offer in advance. Specifically, when the degree of valuation interdependence is extremely low (as in our ∞-group model), advance selling will be deployed only when the seller's capacity is not very small and it is always offered with a discount; also, the seller may offer only a fraction of capacity in advance. On the other hand, when the degree of valuation interdependence is high (as in our one-group model), all of these results change: advance selling can be optimal even when the capacity is very tight, the seller could charge a premium when selling in advance, and selling a fraction of capacity in advance is never optimal. Between these two extreme cases, the seller's policy gradually changes as the degree of valuation interdependence changes. We examine the seller's profit and show that, when capacity is limited, valuation interdependence always hurts the seller's overall profitability but enhances average gain from advance selling. In such cases, the seller who fails to recognize the valuation interdependence when choosing selling strategies suffers a significant loss of profit. Since the degree of interdependence varies significantly across products and industries, the major findings from our paper suggest that the seller with limited capacity should explicitly consider how the valuations are derived for the seller's product (e.g., largely from environmental factors or idiosyncratic factors) before deciding on an advance-selling strategy.

2. Model and Assumptions

We consider a seller who can sell a single product over two periods, referring to the first period as the advance period and the second as the spot period.[2] Consumers are strategic and choose whether and when to buy, but if they buy in advance, they are uncertain about their own valuations. Consumers' valuations are realized in spot and they may range

[1] We thank an associate editor for pointing us to this literature.

[2] The "spot period" simply denotes the second period. The same term is used in other papers (e.g., Xie and Shugan 2001, Gallego and Şahin 2010). We assume that the seller has full control over the prices charged in both periods, i.e., there does not exist a "spot market" where the price is determined through an exogenous market mechanism (e.g., an auction).

from fully independent to perfectly correlated, creating different markets (different characteristics of aggregate demand) for the seller. The seller has total capacity T and incurs a variable cost c for each unit sold. The seller decides the price at the beginning of each period: in advance, p_1, and in spot, p_2. Also, the seller can choose to sell only a portion of capacity in advance, which is denoted by S.

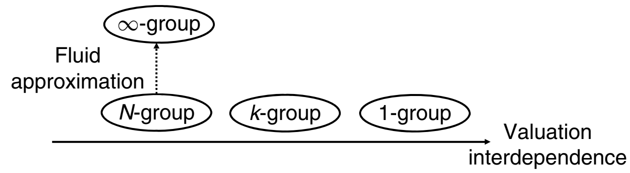
We assume that N_1 consumers become aware of the product and arrive in the advance period and N_2 arrive in the spot period. Hence, $N = N_1 + N_2$ represents the total market size, or all potential consumers. In other words, N_1 and N_2 can be interpreted as the numbers of consumers who want the product if the product is offered for free in each period. The consumers that arrive in advance may attempt to buy the product in advance or wait until spot.

Consumers arriving in advance know the distribution of valuation but not their valuations, which will be revealed only to the consumers in the spot period. All consumers' valuations are drawn from a distribution with cumulative distribution function $G(\cdot)$ and probability density function $g(\cdot)$. If a consumer's valuation is α, the utility that the consumer realizes in spot when the product is bought at price p is $U(\alpha, p) = \alpha - p$. If the consumer does not purchase a product, utility is 0. Consumers are risk neutral and strategic; thus, each consumer chooses an action that maximizes expected utility. In advance, consumers do not know their valuation; thus, consumers compare the expected utility of purchasing at price p_1 with the expected utility of deferring the decision to the spot period. In spot, all remaining consumers (remaining consumers from the advance period plus new consumers in the spot period) decide whether to buy the product or not, after they learn their valuations and the spot price, p_2.

To model different levels of valuation interdependence in a unified framework, we consider a consumer-group valuation model. We assume that N consumers comprise k groups of almost equal sizes.[3] That is, the size of group i, n_i, is equal to either $\lfloor N/k \rfloor$ or $\lfloor N/k \rfloor + 1$, and $\sum_{i=1}^{k} n_i = N$. Consumers in each group will share the same realized valuation in spot. Hence, the number of groups, k, will represent the degree of interdependence. In particular, we examine the following special cases in detail (illustrated in Figure 1).

[3] We assume that the groups are (almost) equal in size to eliminate the effect of dominating groups. Our numerical study shows that our main results are robust to this assumption. In particular, asymmetry in group sizes can amplify the effects of valuation interdependence.

Figure 1 **The Valuation-Interdependence Models**



One-Group Model, $k = 1$**.** In this case, all consumers belong to the same group and thus have the same valuation in spot. This model represents the highest level of valuation interdependence and is appropriate when environment-dependent factors (e.g., weather, hype, star presence) dominate consumer valuation.

N**-Group Model,** $k = N$**.** Each consumer forms a group, thus the consumer's individual valuation is independent from others. This model corresponds to the lowest level of valuation interdependence and is appropriate when consumer valuations are primarily idiosyncratic and determined by consumer-dependent factors (e.g., health, mood, or schedule). Under this model, the demand in the spot period (i.e., the number of consumers who will buy at the spot price, p_2) follows a binomial distribution.

∞**-Group Model,** $k = N \to \infty$**.** Instead of modeling the demand by separately considering N discrete consumers, fluid models are often used as an alternative (Gallego and Şahin 2010, Xie and Shugan 2001, Cachon and Swinney 2009, Gallego and Hu 2014). In a fluid model, the random variable that represents demand is replaced by its mean. In our model, the random spot demand, or the number of consumers buying at price p_2, is replaced by its expected value. This fluid model can be considered as an asymptotic version of the N-group model: when the size of the consumer population is sufficiently large, the coefficient of variation of the spot demand in the N-group model approaches 0. Hence, we label the fluid model as the ∞-group model.[4]

Throughout the paper, we assume that the valuation distribution, $G(\cdot)$, and its density, $g(\cdot)$, satisfy the following conditions.

(1) $G(\cdot)$ (defined on $[l, h]$) is twice continuously differentiable, and $G'(\cdot) = g(\cdot) > 0$ on (l, h).

(2) $g(\cdot)$ is log concave.

(3) $(G(x)\bar{G}(x)/g(x))' + \bar{G}(x) - k$ is *positive-negative* for any $k \in [0, 1]$. A real-valued function $a(\cdot)$ is positive-negative if $a(x_0) < 0$ for some x_0 implies that $a(x) < 0$ for all $x > x_0$. In other words, the function crosses zero at most once and from above (Butler 1979).

[4] Our numerical results show that, in our framework of advance selling, the ∞-group model is a good approximation of the N-group model, when the problem scale becomes very large.

Many distributions and their truncated versions satisfy these conditions (e.g., uniform, exponential, logistic, normal, and extreme value, as well as power, Weibull, beta, and gamma distributions). Truncation of the above distributions to $[l, h]$ is defined as a conditional distribution. Note that condition (3) implies that both $G(\cdot)$ and $\bar{G} = 1 - G(\cdot)$ are log concave, which then implies that $G(\cdot)$ has an increasing failure rate (Bergstrom and Bagnoli 2005). For ease of presentation, we assume a finite support, but all of the results that we derive hold also when $l = -\infty$ or $h = \infty$, as long as the first two moments are finite. Also, to avoid trivial cases, we assume $c < h$.

Our setup of k groups represents varying levels of valuation interdependence. Although many products are sold in advance, the nature of consumers' valuations and their interdependence differ from market to market. Among examples used in several papers (Phillips 2005, Xie and Shugan 2001, Shugan and Xie 2004), consumers of sports events or Broadway shows tend to have similar valuations (which depend on weather or star actors), thus conceptually corresponding to the one-group model. On the other hand, the valuation of a Chinese dinner buffet or prepaid spa service is largely individually determined, thus fitting closely to our N-group or ∞-group model. As the valuation interdependence weakens (k increases), the variance of the spot demand decreases. We show that the changes in the seller's optimal strategy are not abrupt. Instead, the policy changes gradually as the degree of valuation interdependence changes. Such gradual changes allow for a unified explanation of how valuation interdependence influences the seller's strategy. Consequently, the explanations and insights are applicable to the whole spectrum of scenarios/industries with varying levels of valuation interdependence.

Our consumer-group valuation model allows different levels of interdependence to exist for the same valuation distribution. Consequently, the changes in the optimal strategy are solely driven by valuation interdependence, which we control by the number of consumer groups k. An alternative way to model valuation interdependence could be the one used in common-value auctions, where an individual's value for the object being auctioned is the sum of a common value and an idiosyncratic value.[5] However, under this framework, it would be more difficult to isolate the effect of valuation interdependence.

To study the effects of valuation uncertainty and its interdependence across consumers free from other complicating factors, such as the seller's or the consumers' psychological attributes including risk aversion (Liu and van Ryzin 2008), loss aversion (Zhao and Stecke 2010), disappointment aversion (Liu and Shum 2013), and regret aversion (Nasiry and Popescu 2012), we assume that both the seller and consumers are rational (i.e., their utility functions are linear) and they want to maximize their *expected* utilities.

We assume that not all consumers are aware of the product in advance. Advance consumers either realize their needs for the product earlier than the spot consumers (Stock and Balachander 2005), or are better informed about the advance purchasing opportunity (Prasad et al. 2011). Our two-period model captures a key difference between consumers in the advance and spot periods: in advance, consumers make a decision to buy in the presence of uncertainty, but in spot consumers make the decision after such uncertainty is resolved. The main friction is reflected in the case when all consumers arrive in advance. In order to incorporate more general situations (e.g., not all consumers are aware of the advance selling), our two-period model allows for any values of N_1 and N_2.

In the following section, we start with problem formulation for the k-group model.

3. Problem Formulation and General Results

Following backward induction, we first examine the seller's decision in the spot period and then in advance.

3.1. Spot Period

Consider a subgame where the seller sold S units in the advance period. Since any advance consumer is equally likely to purchase the good in advance, S consumers who bought in advance are randomly distributed across k groups. Specifically, let $S = S_1 + S_2 + \cdots + S_k$, where S_i represents the number of group i consumers who purchased in advance. Consequently, the number of remaining consumers in each group, $n_i - S_i$, is a random variable. For any realization $(s_1, s_2, \ldots, s_k)$ such that $s_i \in \{0, 1, \ldots, n_i\}$ and $\sum_i s_i = S$, $\Pr[S_1 = s_1,\ S_2 = s_2, \ldots, S_k = s_k] = (\Pi_{I=1}^{k} \binom{n_i}{s_i}) / \binom{N}{S}$.

Let α_i denote the valuation of consumers in group i. For a given spot price p_2, all remaining consumers in group i want to buy the product if and only if $\alpha_i \geq p_2$. Thus, the spot demand is $\sum_{i=1}^{k} (n_i - S_i) 1_{\alpha_i \geq p_2}$, where $1_{\alpha_i \geq p_2}$ is an indicator variable. Because the sales quantity in the spot period is the smaller of the seller's remaining capacity $T - S$ and the spot demand, the seller's expected spot-period profit is

$$\pi_2^k(S, p_2) = (p_2 - c) \cdot \mathrm{E}_{(\alpha_1, \ldots, \alpha_k, S_1, \ldots, S_k)} \left[\min\left(T - S, \sum_{i=1}^{k} (n_i - S_i) 1_{\alpha_i \geq p_2} \right) \right], \quad (1)$$

where superscript k denotes the k-group model.

[5] We thank two anonymous referees for suggesting this.

In the special case where $k=\infty$, spot demand is equal to its expected value, which is often labeled as a fluid model. In this case,

$$\begin{aligned}&\pi_2^\infty(S,p_2)\\&=(p_2-c)\min\left(T-S,\mathrm{E}_{(\alpha_1,\dots,\alpha_k,S_1,\dots,S_k)}\left[\sum_{i=1}^{k}(n_i-S_i)1_{\alpha_i\geq p_2}\right]\right)\\&=(p_2-c)\min\big(T-S,(N-S)\Pr[1_{\alpha_i\geq p_2}]\big).\end{aligned}\tag{2}$$

For a given S, the seller chooses a spot price p_2 to maximize the spot profit $\pi_2^k(S,p_2)$. Let $p_2^k(S)$ denote the optimal spot price. Note that, if the capacity is limited, not all consumers may obtain the product in the spot period. Let $\lambda_2(S,p_2)$ be the probability that a consumer who wants to buy the product in the spot period actually obtains it, given that the remaining capacity is $T-S$ and the seller's spot price is p_2:

$$\begin{aligned}&\lambda_2^k(S,p_2)\\&\quad=\mathrm{E}_{(\alpha_1,\dots,\alpha_k,S_1,\dots,S_k)}\min\left[\frac{T-S}{\sum_{i=1}^{k}(n_i-S_i)1_{\alpha_i\geq p_2}},1\right].\end{aligned}\tag{3}$$

Specifically,

$$\lambda_2^\infty(S,p_2)=\min\left[\frac{T-S}{(N-S)\Pr[1_{\alpha_i\geq p_2}]},1\right].\tag{4}$$

This probability will play a role in the consumers' decisions in the advance period.

3.2. Advance Period

Without loss of generality, we can interpret the amount offered for sale in advance, S, as the amount sold in advance. Since there are only N_1 consumers in the advance period, it suffices to consider $S\in[0,\min(T,N_1)]$, because any capacity exceeding N_1 will not be used in advance and will be available again in spot. Furthermore, all consumers have the same valuation distribution, either all want to buy or all prefer to wait. Consequently, the seller (offering capacity $S\in[0,\min(T,N_1)]$ in advance) will always sell either all of S units or zero. Note that any policy that leads to zero sales in advance can be replicated by $S=0$.

In the advance period, the seller must decide the advance price p_1 and capacity ration S. When determining prices and capacity ration, the seller takes account of consumers' strategic behavior and the fact that consumers may compete for the limited capacity. (Thus our setting is a game among consumers and a game between consumers and the seller.) Although S and p_1^k are chosen jointly by the seller, for mathematical tractability, we first find the optimal advance price for a given ration S, and then we determine the amount of capacity that should be rationed for the advance period.

If the capacity ration S is less than N_1, only a portion of advance consumers can buy the product in advance. When making the buy-now-or-wait decision in advance, the consumer considers other consumers' actions because they will affect the probability of her getting the product in either period. Let $\lambda_1(S,p_1)$ be the probability that a consumer who wants to buy the product in advance obtains it. If a consumer in group i obtains the good in advance, the expected utility is $\mathrm{E}[\alpha_i]-p_1$. Otherwise, the consumer will have to wait until spot and try to buy if the consumer's valuation α_i exceeds the spot price $p_2^k(S)$, which results in the expected utility of $\mathrm{E}[\lambda_2^k(S,p_2^k(S))\max(\alpha-p_2^k(S),0)]$. Hence, the expected utility of a consumer who attempts to buy in the advance period is

$$\begin{aligned}&U_A(S)\\&=\lambda_1^k(S,p_1)\mathrm{E}[\alpha_i^k-p_1]\\&\quad+(1-\lambda_1(S,p_1))\mathrm{E}\big[\lambda_2^k(S,p_2^k(S))\max(\alpha_i-p_2^k(S),0)\big],\end{aligned}$$

whereas the expected utility from deferring the purchasing decision until the spot period is

$$U_D(S)=\mathrm{E}\big[\lambda_2^k(S,p_2^k(S))\max(\alpha_i-p_2^k(S),0)\big].$$

Since $U_D\geq 0$, buying in the advance period is optimal for a consumer if and only if $U_A(S)\geq U_D(S)$,

$$\begin{aligned}&\lambda_1^k(S,p_1)\mathrm{E}[\alpha_i-p_1]\\&\quad+(1-\lambda_1^k(S,p_1))\mathrm{E}\big[\lambda_2^k(S,p_2^k(S))\max(\alpha_i-p_2^k(S),0)\big]\\&\quad\quad\geq\mathrm{E}\big[\lambda_2^k(S,p_2^k(S))\max(\alpha_i-p_2^k(S),0)\big].\end{aligned}$$

Simplifying the inequality, we get

$$\begin{aligned}&p_1\leq p_1^{\max,k}(S)\\&\quad=\mathrm{E}\big[\alpha_i-\lambda_2^k(S,p_2^k(S))\max(\alpha_i-p_2^k(S),0)\big].\end{aligned}\tag{5}$$

Since $\alpha_1,\dots,\alpha_k$ are independently and identically distributed, the right-hand side of Equation (5) is independent of the group number i; regardless of to which group consumers belong, they have the same maximum willingness to pay in advance, $p_1^{\max,k}(S)$. It further implies that, if the seller offers advance selling, the optimal advance price must be $p_1^{\max,k}(S)$.

With the optimal spot and advance prices characterized as functions of S, the seller will choose a capacity ration S as the maximizer of the total expected profit:

$$\begin{aligned}&\max_{0\leq S\leq\min(T,N_1)}\pi_{AS}^k(S)\\&\quad=\big(p_1^{\max,k}(S)-c\big)S+\big(p_2^k(S)-c\big)\\&\quad\quad\cdot\mathrm{E}\left[\min\left(T-S,\sum_{i=1}^{k}(n_i-S_i)1_{\alpha_i\geq p_2^k(S)}\right)\right].\end{aligned}\tag{6}$$

If S^k denotes the optimal capacity ration, the seller's optimal prices in the two periods are $p_1^{\max,k}(S^k)$ and $p_2^k(S^k)$, respectively.

Note that the profit function in Equation (6) implicitly assumes that all consumers who have bought in advance will consume the product. The same assumption has been made in several advance-selling papers, starting with Xie and Shugan (2001). If some of the advance buyers do not consume the product, the seller may incur different marginal costs for them.[6] In such a case, the seller's variable cost can be divided into a portion that is incurred independently of the consumption (preconsumption variable cost) and a portion that is incurred when the product is consumed (postconsumption variable cost). With a redefinition of terms related to cost and valuation, we show that our model can capture this case and all results and insights continue to hold.

In the following section, we characterize the seller's optimal pricing and rationing strategy under different valuation models (i.e., different k values). The capacity offered in advance S will play a crucial role in characterizing the optimal policy. If $S=0$ the seller offers the product only in the spot period (*no advance selling*). If $S=\min(T, N_1)$, the seller sells as much as she can in advance (*full advance selling*). Finally, if $0<S<\min(T, N_1)$, the seller offers the product in advance but purposefully limits the sales (*limited advance selling*). In addition to the rationing strategy, we will also examine whether the seller should charge a discounted or premium advance price.

4. Base Case: Advance Selling with Unlimited Capacity

We first examine the case where the seller has sufficient capacity to meet all consumers, $T \geq N$, representing cases in which any reasonable demand can be satisfied and not bound by capacity (e.g., conference registration or advance book sales; see Xie and Shugan 2001). The next theorem shows that the seller's optimal strategy and expected profit are actually independent of valuation interdependence.

Theorem 1. *If $T \geq N$, the seller's optimal strategy and expected profit in both periods are independent of the number of consumer groups k.*

Proof. All proofs are in the appendices. □

At first this result seems surprising: a seller with ample capacity does not need to care about whether consumers' preferences are uniform or idiosyncratic. The key is that all consumers draw valuations from $G(\cdot)$. Thus, for a given spot price p_2, the probability that a consumer wants to buy the product is $\bar{G}(p_2)$, regardless of how one consumer's valuation is related to that of another. With sufficient capacity, when serving as many consumers as desired, the seller has expected spot sales that alway equal the expected demand, which in all cases is the remaining market size $N-S$ multiplied by the expected proportion of buying consumers $\bar{G}(p_2)$. The expected spot profit in Equation (1) becomes

$$\pi_2^U(S, p_2) = (p_2 - c)(N-S)\bar{G}(p_2), \tag{7}$$

where superscript U denotes an unlimited-capacity case. Consequently, the optimal spot price and spot profit are independent of the number of groups k. Applying this to Equations (5) and (6), the optimal advance price, p_1^* and optimal capacity ration S^* are independent of k. We characterize the seller's optimal pricing and rationing strategies in the next theorem.

Theorem 2 (The Seller's Optimal Strategies in the Unlimited-Capacity Model). (i) *The optimal spot price, denoted by p_2^U, is unique: $p_2^U = l$ for $c \leq \underline{c} = l - 1/g(l)$; additionally,*

$$l < p_2^U < h \quad \text{and } p_2^U \text{ is a solution to}$$

$$p_2 = c + \frac{\bar{G}(p_2)}{g(p_2)} \quad \text{for } \underline{c} < c < h. \tag{8}$$

(ii) *The optimal advance price that induces consumers to buy in advance, denoted by $p_1^{\max,U}$, never exceeds the optimal spot price: $p_1^{\max,U} = \mathrm{E}[\min(p_2^U, \alpha)] \leq p_2^U$.*

(iii) *The seller should either sell only in spot or use the full advance selling: $S^U = 0$ or N_1.*

When $T \geq N$, product availability is always guaranteed. Hence, to induce consumers to buy in advance, the seller can never charge a premium advance price (Theorem 2(ii)). To see why $S^U = 0$ or N_1, note that neither the spot nor the advance price depends on the ration S: see (i) and (ii) of Theorem 2. This implies that the seller's total expected profit, given by Equation (6), is linear in S:

$$\begin{aligned}\pi_{AS}^U(S) &= (p_1^{\max,U} - c)S + (N-S)(p_2^U - c)\bar{G}(p_2^U)\\ &= [p_1^{\max,U} - c - (p_2^U - c)\bar{G}(p_2^U)]S\\ &\quad + N(p_2^U - c)\bar{G}(p_2^U).\end{aligned} \tag{9}$$

Thus, the seller with sufficient capacity never offers limited advance sales, leading to Theorem 2(iii).

To characterize when the full advance selling is optimal, it suffices to determine when $\pi_{AS}^U(N_1) \geq \pi_{AS}^U(0)$.

Theorem 3 (Advance Selling and the Marginal Cost in the Unlimited-Capacity Model). *There exists $\bar{c} \in [\underline{c}, h)$ such that the full advance selling ($S^U = N_1$) is optimal for $c \leq \bar{c}$ and the spot-only selling ($S^U = 0$) is strictly optimal for $c > \bar{c}$ (see Figure 2).*

[6] We thank an associate editor for this observation.

Figure 2 Illustration of $\pi^U_{AS}(N_1) - \pi^U_{AS}(0)$ as a Function of Marginal Cost, c

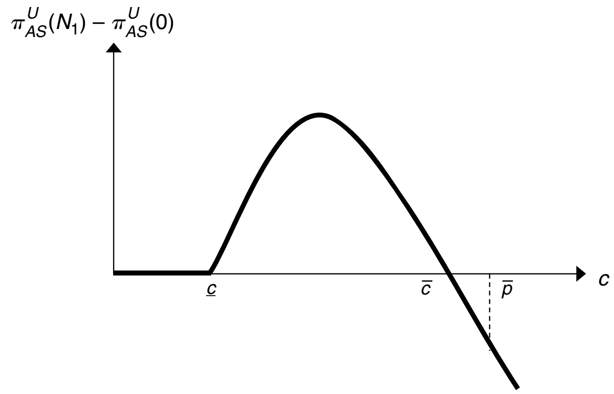


Figure 2 and Theorem 3 both illustrate that, when the marginal cost is very high, selling at a very high price only in spot and targeting a small portion of consumers with very high valuations is better. Advance selling could increase sales, but it would require a deep discount and the increased sales volume would not make up for the decrease in margin. On the other hand, when the marginal cost is very low, both full advance selling and spot-only strategies result in the same price and profit: all consumers will eventually buy (thus, the sales quantities are N) at the same price. In the case between these two extremes, the full advance selling is strictly optimal: an appropriate discount can induce all N_1 consumers to buy in advance, which improves both the sales quantity and the profit.

5. Advance Selling with Limited Capacity

We now consider the limited-capacity case $T < N$, which is at the core of the paper. In contrast to the unlimited-capacity case where the seller's optimal policy is independent of valuation interdependence, we show that interdependence significantly changes the seller's strategy. We first analytically characterize the seller's optimal strategies under the two canonical models of valuation interdependence: the ∞-group and one-group.

5.1. ∞-Group Valuation Model

Recall that the ∞-group valuation model is an asymptotic version of the N-group model, where valuations are completely independent of each other and the randomness in the spot demand is washed away as the size of the consumer population becomes very large.

5.1.1. Spot Period. From Equation (1), the seller's expected spot profit in the ∞-group becomes

$$\begin{aligned}\pi_2^\infty(S, p_2) &= (p_2 - c)\min\left(T - S, \mathrm{E}_{\substack{(\alpha_1,\dots,\alpha_k)\\(S_1,\dots,S_k)}}\left[\sum_{i=1}^{k}(n_i - S_i)1_{\alpha_i \ge p_2}\right]\right)\\ &= (p_2 - c)\min[T - S, (N - S)\bar{G}(p_2)],\end{aligned} \tag{10}$$

where superscript ∞ denotes the ∞-group. Unlike the unlimited-capacity case, the optimal spot price $p_2^\infty(S)$ depends on the remaining capacity. If the remaining capacity is sufficiently large, it will not bind the sales quantity in the spot period, and the optimal spot price will be p_2^U, the optimal price in the unlimited capacity case. However, if the seller's remaining capacity is tight, the seller will raise the price to clear the market and sell only to consumers with high valuations.

Let $p_2^B(S)$ be the market-clearing (*capacity-binding*) price for the seller with the remaining capacity, $T - S$. Since $T < N$, $p_2^B(S)$ always exists and is a solution to $\bar{G}(p_2) = (T - S)/(N - S)$. The following lemma shows that the optimal spot price is the maximum of p_2^U and $p_2^B(S)$.

Lemma 1 (The Optimal Spot Price in the ∞-Group Valuation Model). *The optimal spot price $p_2^\infty(S)$ is as follows*:

$$\begin{aligned}p_2^\infty(S) &= \max[p_2^U, p_2^B(S)]\\ &= \begin{cases} p_2^U & \text{if } T - S \ge (N - S)\bar{G}(p_2^U),\\ p_2^B(S) & \text{otherwise},\end{cases}\end{aligned} \tag{11}$$

where p_2^U is defined in Theorem 2 and $p_2^B(S)$ is a solution to $\bar{G}(p_2) = (T - S)/(N - S)$.

Lemma 1 implies that the spot price is nonincreasing in the remaining capacity. Furthermore, the larger the initial capacity T, the larger the region where the unrestricted spot price, p_2^U, is optimal. To determine the likelihood that a consumer can obtain the product in the spot period, we substitute the optimal spot price to the expression of $\lambda_2(S, p_2)$ (i.e., Equation (3)).

Corollary 1. *Supply shortage in the spot period never occurs in the ∞-group valuation model.*

$$\begin{aligned}\lambda_2(S, p_2^\infty(S)) &= \min\left[1, \frac{T - S}{(N - S)\bar{G}(p_2^\infty(S))}\right]\\ &= 1 \text{ for any } S \le T.\end{aligned} \tag{12}$$

The result implies that the shortage in supply will never occur in the spot period, no matter how tight the remaining capacity is. Instead, the seller chooses to set the price so that no shortage takes place. If any shortage took place, the corresponding policy could not be optimal because the seller could do better by increasing the spot price without decreasing the sales.

5.1.2. Advance Period. In the advance period, the seller must decide the advance price, p_1, and the portion of capacity rationed, S. Since shortage never occurs in spot, substituting $\lambda_2(S, p_2^\infty(S)) = 1$ in the expression for the advance price, Equation (5), the next result immediately follows (proof omitted).

Theorem 4 (The Optimal Advance Price in the ∞-Group Model). *The optimal advance price $p_1^{\max,\infty}(S)$ is as follows*:

$$p_1^{\max,\infty}(S) = \mathrm{E}[\alpha] - \mathrm{E}[\max(\alpha - p_2^\infty(S), 0)] = \mathrm{E}[\min(p_2^\infty(S), \alpha)] \le p_2^\infty(S).$$

Thus, the seller never offers a premium price in the advance period.

A conventional intuition suggests that, when capacity is very tight, the seller should charge a premium price in the advance period. However, a premium advance price can be justified from the consumers' point of view only if there is a threat of shortage in spot. As shown in Theorem 4, in the ∞-group valuation model, the seller will always raise the spot price and clear the market. Thus, tight capacity leads to an increase in the spot price rather than a premium in the advance price.

It is interesting to note that our result differs from the findings of Xie and Shugan (2001), in which the premium advance pricing may be optimal. Their model also assumes (similarly to our ∞-group model) that the spot demand is a deterministic function of the spot price. However, they assume that there are only two discrete levels of valuation in the spot period: high (h) or low (l). Thus, the spot demand changes (jumps) at the two discrete prices, $p_2 = l$ and $p_2 = h$, and is constant otherwise. As a result, the seller chooses between selling to all N consumers at price l or to a portion of them $NP(\alpha = h)$ at price h. Due to this discontinuity of demand in price, a premium advance price is possible in Xie and Shugan (2001), and some consumers cannot buy the product in spot. Although some minor discontinuities in demand function may exist in reality, we believe that demand from a large population of consumers with diverse valuation should be responsive to any gradual change in price in most cases. Hence, the insights from Xie and Shugan (2001) are limited, because they hinge on their assumption about demand: they suggest that a premium price may exist. This result, however, is driven by their two-point valuation model. In our ∞-group model, the spot demand continuously responds to a price change. As a result, our seller will increase the spot price to match demand with remaining capacity instead of creating a shortage, and thus a premium advance price can never be optimal. We show later that premium price may be optimal, but not when demand is highly predictable. Instead, a combination of unpredictability and limited capacity make the premium price a possibility.

5.1.3. Capacity Rationing. We now analyze the seller's choices of whether to offer advance selling and how much (if any) of the total capacity to allocate for advance selling, that is, the choice of ration S:

$$\max_{0\le S\le \min(T,N_1)} \pi_{AS}^\infty(S) = (p_1^{\max,\infty}(S) - c)S + (p_2^\infty(S) - c)\min\big(T - S, (N-S)\bar{G}(p_2^\infty(S))\big).$$

The amount of capacity that the seller rations for advance sales does influence the profit: note that, if the seller sells a larger capacity in advance, total sales increase. Furthermore, the amount of capacity available in spot will decrease, which then increases the spot price. On the other hand, selling more in advance implies that a larger portion of consumers pay a lower (discounted) price. Thus, it is not clear when the net benefit is positive. To understand this tradeoff, we characterize the optimal rationing policy as a function of the seller's capacity T and marginal cost c.

Theorem 5 (The Optimal Capacity Rationing in ∞-Group Valuation Model). *There exist two thresholds, T_1 and T^∞, where $0 \le T_1 \le T^\infty \le N$, and a switching curve $c^\infty(T)$, defined for $T > T_1$, such that*

(i) *if $T \ge T^\infty$ and $c \le c^\infty(T)$, then $S^\infty = \min(T, N_1)$* [full advance selling];

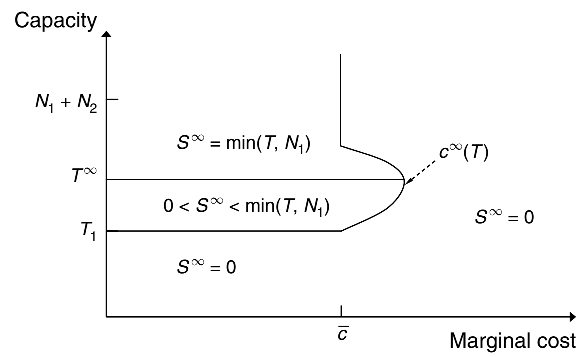
(ii) *if $T \in (T_1, T^\infty)$ and $c \le c^\infty(T)$, then $0 < S^\infty < \min(T, N_1)$* [limited advance selling]; *and*

(iii) *otherwise, $S^\infty = 0$* [no advance selling].

Further, for a given $T < N$, if $S^\infty > 0$, then $p_1^{\max,\infty}(S^\infty) < p_2^\infty(S^\infty)$. That is, a seller with limited capacity always offers a discount in the advance period if selling in advance.

As Theorem 5 and Figure 3 illustrate, advance selling is not optimal if the capacity is tight, $T \le T_1$, or the marginal cost is high, $c > c^\infty(T)$. With tight capacity, the seller charges a high spot price and sells only to consumers with high valuations rather than offering advance sales at a discounted price. Likewise, advance selling is not optimal when the cost is very high. Although advance selling can increase the sales, the required discount eats up most of the margin.

Figure 3 Seller's Optimal Strategy with Limited Capacity and ∞-Group Valuation Model



Everywhere else, advance selling is optimal, but the type of advance selling depends on the seller's capacity. At large capacity (above T^∞), the seller will not limit the quantity sold in advance. At moderate capacity level (between T_1 and T^∞), advance selling is optimal but the seller limits the sales (*limited advance selling*). To see why selling only a portion of capacity is beneficial, compare this strategy to two other extremes: spot-only and full advance selling. If the seller does not offer the good in advance, all consumers wait until the spot period and only consumers with valuations exceeding the spot price will buy the product. At moderate capacity level, this leads to leftover capacity at the end of the spot period. In contrast, offering some product in advance at a properly discounted price can induce advance consumers to buy it and, thus, increase the total sales. Furthermore, selling some capacity in advance reduces the quantity available in spot, which then increases the spot price. With higher sales and higher spot price, advance selling may be better than selling only in spot. However, if the seller offers the full advance selling, all of the capacity can be sold across two periods but a significant portion is sold at a heavily discounted price in advance. The limited advance selling is better than the other two extreme strategies, because it increases the total sales (compared to the spot only) and margin (compared to the full advance selling) at the same time. Interestingly, the practice of limited advance selling bears some resemblance to the use of booking limits in airlines. Although there may be other reasons for using the booking limit, our result indicates that one of the benefits of offering a limited number of discounted tickets is that limiting the advance sales will raise the spot price for the remaining seats, while keeping discounted sales modest.

5.2. One-Group Valuation Model

In the one-group model, consumers' valuations are determined by environment-dependent factors such as weather or hype. We solve the seller's problem starting from the spot period.

5.2.1. Spot Period. With $k=1$, all consumers will realize an identical valuation drawn from a distribution $G(\cdot)$. Hence, the seller's spot profit, Equation (1), becomes

$$\pi_2^1(S,p_2)=(p_2-c)\mathrm{E}_{\alpha_1}\big[\min(T-S,(N-S)1_{\alpha_1\ge p_2})\big]. \quad (13)$$

Lemma 2 (The Optimal Spot Price in the One-Group Valuation Model). *The optimal spot price $p_2^1(S)=p_2^U$ for all S. Furthermore, the probability of a supply shortage in spot is always positive.*

At first, it is surprising that the seller's optimal spot price is independent of the remaining capacity and the same as the price in the unlimited-capacity case. To see why, note that all consumers will have the same valuation in the spot period and, as a result, the spot demand is either 0 or $N-S$ for any spot price p_2 and any remaining capacity. Thus, the expected profit for the spot period can be rewritten as

$$\pi_2^1(S,p_2)=(T-S)\bar{G}(p_2)(p_2-c).$$

Thus, the optimal spot price is p_2^U, the same price the seller with sufficient capacity would charge. As a result, it can be easily shown that the spot price in the one-group case cannot exceed the spot price in the ∞-group case. Lemma 2 also implies that a shortage is always possible in the spot period. When all consumers want to buy (i.e., $\alpha > p_2^U$), the probability that a consumer obtains the product is $\lambda_2^1(S,p_2)=(T-S)/(N-S)$. In other words, the chance that a consumer experiences a shortage is simply $1-\lambda_2^1(S,p_2)$.

5.2.2. Advance Period. From Equation (5), the optimal price that the seller can charge to sell S units in the advance period is

$$p_1^{\max,1}(S)=\mathrm{E}[\alpha]-\frac{T-S}{N-S}\mathrm{E}\big[\max(\alpha-p_2^U,0)\big]. \quad (14)$$

Since $T<N$, we note that $p_1^{\max,1}(S)$ is increasing and convex in S. To find the optimal ration, S, the seller maximizes the total expected profit:

$$\max_{0\le S\le\min(N_1,T)}\pi_{AS}^1(S)=(p_1^{\max,1}(S)-c)S + (T-S)\bar{G}(p_2^U)(p_2^U-c). \quad (15)$$

The next two results characterize the optimal capacity-rationing strategy for the one-group model.

Lemma 3 (No Limited Advance Sales in the One-Group Model). *The optimal advance sales $S^1=0$ or $\min(T,N_1)$.*

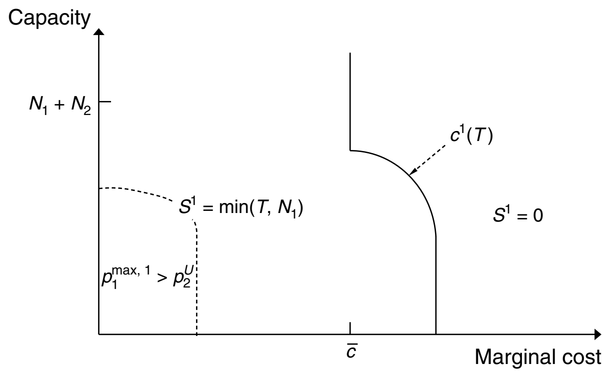
Theorem 6 (Optimal Capacity Rationing in the One-Group Model). *There exists a nonincreasing function $c^1(T)$ for all T, $0<T<N$, such that*

(i) *if $c\le c^1(T)$, $S^1=\min(T,N_1)$ [full advance selling]; and*

(ii) *if $c>c^1(T)$, $S^1=0$ [no advance selling].*

Theorem 6 implies that the seller's optimal strategy (Figure 4) when facing the one-group is significantly different from that when facing the ∞-group (Figure 3) in several ways. In the ∞-group case, advance selling is optimal only when capacity is not too tight, $T>T_1$; however, in the one-group case, advance selling is always optimal even when capacity is tight, as long as the marginal cost is not too high. Also, unlike in the ∞-group case, limited advance selling can never be optimal in the one-group case. Another significant difference is in pricing strategy. In the one-group model, the seller can charge a premium on top of the spot price in the advance period, contrasting with the result of the ∞-group model: the seller always

Figure 4 Seller's Optimal Strategy with Limited Capacity in the One-Group Model



offers a discount in advance. To see why these differences arise, first note that a shortage never occurs in the ∞-group case, whereas the opposite happens in the one-group case. Thus, consumers anticipating a possible shortage in spot are not only more likely to buy in advance but are also willing to pay a higher advance price, explaining the premium price and full advance selling. Consequently, advance selling is optimal even when capacity is very tight. Thus, when valuations are highly correlated, the seller exploits the correlation and (resultant) shortage in spot by offering an advance price that is higher than the spot price (*a premium advance price*). The next result explains when the premium advance pricing actually occurs.

Proposition 1 (The Optimality of a Premium Advance Price in the One-Group Model). *The optimal advance price $p_1^{\max,1}(S^1) > p_2^1(S^1)$ for $S^1 = \min(T, N_1)$ if and only if both the cost is low $(\mathrm{E}[\alpha] - p_2^U > 0)$ and the capacity is tight $(T < N_1 + ((\mathrm{E}[\alpha] - p_2^U)/(\mathrm{E}[\max(\alpha - p_2^U, 0)]))N_2)$.*

Proposition 1 highlights two conditions for a premium advance price: low cost and tight capacity. When the cost is low, the spot price is also low and the chance that a consumer will face shortage is high. Also, the chance of shortage increases as the capacity becomes tighter. In such a case, tight capacity (directly) and low cost (indirectly) lead to a significant increase in the chance of shortage, which then induces consumers to buy at a premium advance price. The region where a premium advance price is optimal expands as N increases and the shortage becomes more likely. Thus, for products with fairly homogeneous valuations, we expect that the seller will not ration the product and will not offer a discount: for example, for the most popular Broadway shows, the tickets will typically sell out during preorder and will not be available at the ticket counter on the day of the show.[7]

[7] http://broadwayshowsforkids.com/category/shows/wicked/ (accessed February 1, 2015).

5.3. k-Group Valuation Models

The two pivotal models examined above illustrate the effects of valuation interdependence on the seller's advance-selling strategies. For intermediate cases, with k groups where $1 < k < N$, we observe gradual changes across k, as illustrated in Figure 5.

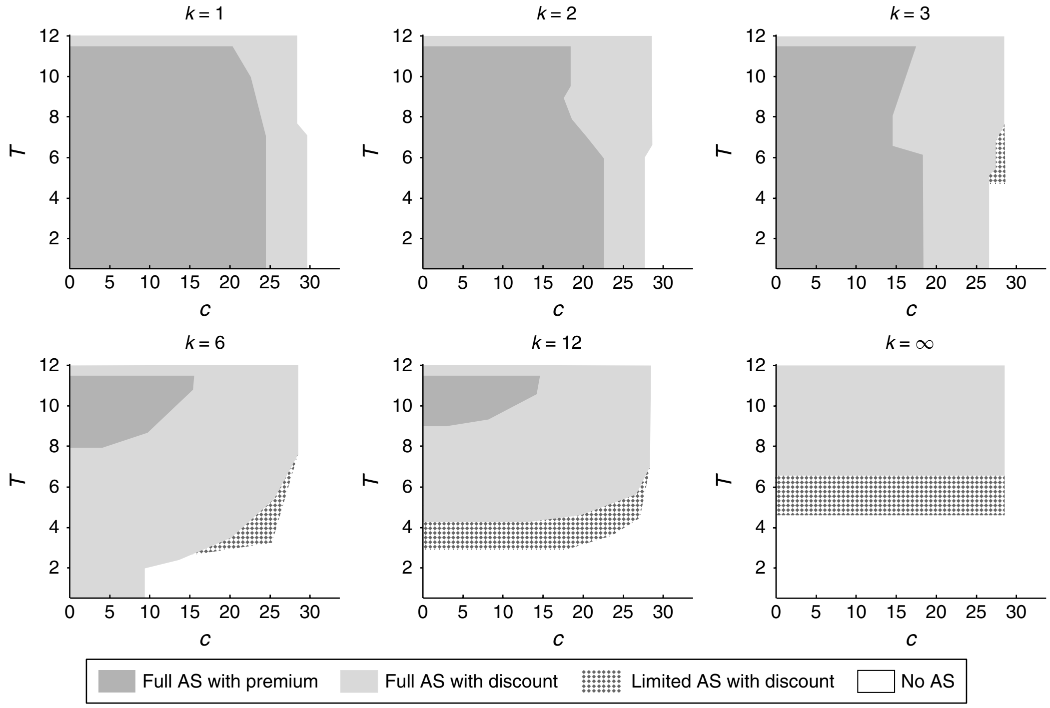
As the valuation interdependence is weakened (increasing the number of groups k), the regions where advance selling is optimal and premium pricing is used both shrink. On the other hand, the region wherelimited advance selling or no advance selling is used expands. This observation can be intuitively justified: when consumer valuations are homogeneous, either all or none of the consumers want to buy the product, and thus the aggregate spot demand is highly variable and lumpy in price. As consumer valuations become less interdependent, the variance of spot demand decreases. As a result, the seller can charge a higher spot price without worrying about losing a lot of sales. This makes selling in advance less attractive, especially when the capacity is tight. At the same time, from the consumers' point of view, the chance of supply shortage is the highest in the one-group case and gradually decreases as the number of consumer groups increases. Thus, when moving from the one-group to the ∞-group case, a premium advance price is less likely deployed.

Although the total region of advance selling decreases in the number of groups, the region where the limited advance selling is used increases, since rationing is especially beneficial to the seller when the variance of spot demand is small. To see why, recall that rationing means refusing to sell to some consumers in advance even if they are willing to buy. This may result in a loss of revenue to the seller who fails to recapture these sales in spot. As k increases, the chance of this loss becomes small, making rationing more attractive.

6. Numerical Study

We showed that the interdependence of consumer valuation and the seller's capacity can significantly change the seller's optimal policy: when the seller should offer advance sales, how much the seller should sell, and what price the seller should charge. We now examine numerically how the seller's gain from advance selling (compared to selling only in spot) changes as the degree of interdependence changes. We also examine how much is lost if the seller ignores the valuation interdependence. The results from the first question help us to understand in what environments the seller gains most (or least) by offering advance sales. The answers to the second question quantify how important it is for the seller to correctly understand how consumers form valuations and what factors (e.g., environmental versus

Figure 5 Changes in Optimal Policy in Valuation Interdependence When $N_1 = 6$, $N_2 = 6$, $\alpha \sim U[25, 35]$



Note. AS, advance selling.

individual) primarily drive their valuations. For both questions, we also examine whether the effect of valuation interdependence is amplified or diminished in the seller's capacity.

We fix total market size $N = 12$ and examine six cases indexed by the number of consumer groups $k \in \{1, 2, 3, 4, 6, 12\}$. For each case, we vary the following parameters: (i) the number of consumers during the advance sales, $N_1 \in \{1, 3, 5, 7, 9, 11\}$; (ii) the seller's capacity, $T \in \{1, 3, 5, 7, 9, 11, 12\}$; (iii) the marginal cost, $c \in \{0, 5, 10, 15, 20, 25, 30\}$; and (iv) the valuation distribution, $\alpha \sim U[30 - b, 30 + b]$, $b \in \{1, 5, 10, 15, 20\}$, resulting in 8,820 instances ($= 1{,}470 \times 6$ group numbers) in total. Although we choose a small N because of computational complexity, our analytical results and additional numerical tests suggest that the results are not sensitive to the problem scale. Specifically, consider the case where the market size N is scaled up, while the capacity-to-market ratio T/N and the percentage of customers arriving in advance N_1/N are kept fixed. For the two valuation models, the one-group and ∞-group, we can prove that the firm's gain from advance selling (compared to selling only in spot) is independent of the market size N. For intermediate levels of valuation interdependence ($k \in (1, N)$), we numerically verify that the firm's gain from advance selling is insensitive to the market size N. Furthermore, the insights we obtain from the small-scale study continue to hold for the large-scale problems.

6.1. Seller's Gain from Advance Selling

First, we evaluate the seller's gain from advance selling as a function of interdependence and capacity. To do this, we define the percentage improvement in the seller's profit under the optimal policy over the spot-only strategy when the consumers form k-distinct groups, δ^k_{AS}, as follows:

$$\delta^k_{AS} = \frac{\pi^k_{AS}(S^k) - \pi^k_{AS}(0)}{\pi^k_{AS}(0)} \cdot 100\%.$$

Figure 6 presents the seller's average gain from advance selling. Each line represents how the average gain changes as a function of capacity level, T.

Figure 6 (Color online) Average Percentage Gain from Advance Selling (AS) over Spot-Only in Group (k) and Capacity (T)

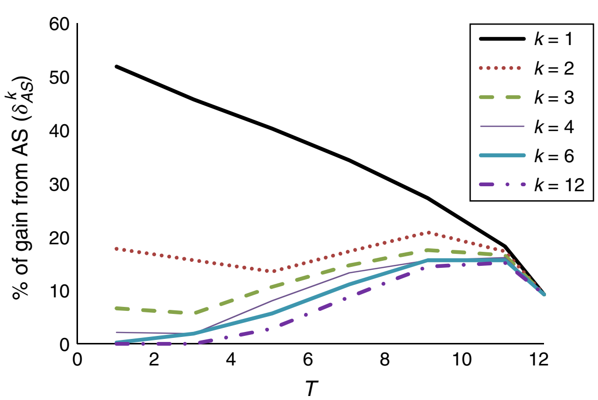
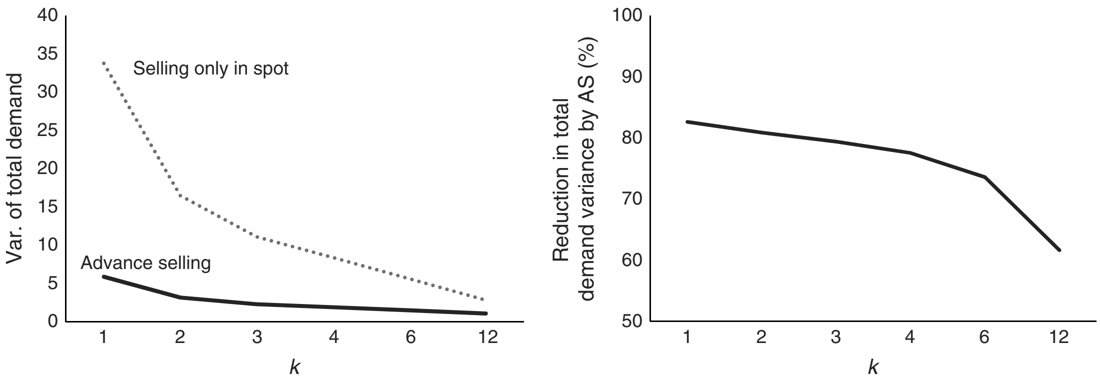


Figure 7 The Variance of Total Demand and Its Percentage Reduction Due to Advance Selling, as Functions of Number of Consumer Groups k, with $N = 12$, $N_1 = 7$, $T = 9$, $c = 20$, $\alpha \sim U[10, 50]$



Note that, for any T, the seller's gain is the largest in the one-group model ($k = 1$), and then it is decreasing in k. Recall that the seller can benefit from advance selling, as compared to spot-only selling, in two ways: increased sales quantity and/or premium advance price. As valuations become more interdependent, the spot demand becomes more variable and its realizations get lumpier: for instance, in the one-group model, either all remaining consumers want to buy or no one does. Consequently, the chance of shortage in the spot period decreases in k. The threat of potential shortage induces more people facing such a situation to buy early; thus, advance selling increases the total quantity sold. In addition, this shortage increases the advance price: the seller may even add a premium on top of the spot price. Combining these two effects together, the seller's gain from advance selling increases when valuations are highly interdependent (i.e., k is small). On the other hand, as consumer valuations become more diverse, the spot demand becomes a more predictable function of the spot price. In such a case, the seller must offer a discount to induce consumers during advance sales; thus, the gain from advance selling decreases. Note that the change of the seller's gain with respect to a change in the valuation interdependence becomes less sensitive as the seller's capacity (T) increases. In the case where the seller has sufficient capacity, the gain becomes totally insensitive to valuation interdependence. To see why this is the case, note from Theorem 2 that, if the seller has unlimited capacity, the seller's pricing and rationing policy is independent from the number of groups, resulting in the same profit for any level of valuation interdependence.

Figure 7 suggests that advance selling has further operational benefits. Specifically, by securing the advance sales, the seller lowers the amount of capacity exposed to random demand in the spot period, thus reducing the variance of the total demand seen by the seller while increasing the total sales. As illustrated in Figure 7,[8] the decrease in total demand variance is most pronounced when valuations are highly intercorrelated (i.e., low k). Note that the operational advantage of advance selling captured in our model is different from that considered in literature (e.g., Tang et al. 2004, Song and Zipkin 2012) where advance sales information helps the seller to better forecast spot demand and plan inventory.

6.2. Profit Loss from Ignoring Valuation Interdependence

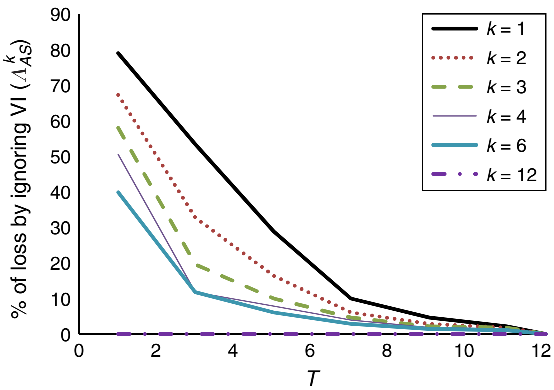
To measure how much the seller would lose for ignoring the valuation interdependence, for each number of groups k, we compare the profit that the seller earns under the policy that is optimal for the independent valuation (N-group) case (denoted by $\pi^k_{AS}(\{p_1, p_2, S\}^N)$) to the profit under the optimal policy (denoted by $\pi^k_{AS}(S^k)$). Then, the percentage loss of ignoring the valuation interdependence, Λ^k_{AS}, is defined as follows:

$$\Lambda^k_{AS} = \frac{\pi^k_{AS}(S^k) - \pi^k_{AS}(\{p_1, p_2, S\}^N)}{\pi^k_{AS}(S^k)} \cdot 100\%.$$

Each line in Figure 8 represents how the seller's loss changes as a function of capacity T. For a given capacity level, the loss from ignoring interdependence decreases as consumer valuations become less interdependent and the realized valuations become more diverse. Note from Theorems 5 and 6 that, when the consumer valuations are highly interdependent, the seller uses a significantly different policy than when valuations are independent. For instance, in the one-group model, the seller uses the full advance sales and may charge a premium price. However, in the N-group model, the seller limits the advance sales and always uses a discount to induce consumers to

[8] The figure shows a typical pattern observed in the 1,470 problem instances that we have examined.

Figure 8 (Color online) Average Loss for Ignoring Valuation Interdependence (VI) in the Number of Groups, k, and Capacity, T



buy in advance. Hence, the loss from ignoring the valuation interdependence is the largest when $k = 1$ and decreases in k. Note also that the percentage loss decreases as the seller's capacity increases. When the seller has tight capacity, the difference between the optimal policy and the policy ignoring interdependence is pronounced, resulting in a large loss. On the other hand, when the seller has sufficient capacity (i.e., $T = 12$), the seller's profit is flat independently from the number of groups, confirming Theorem 1.

7. Conclusion

This paper examines how valuation interdependence and capacity influence the seller's advance selling strategy, characterized by price and quantity available for advance selling.

We evaluate when and what type of advance selling should be deployed as a function of valuation interdependence and capacity. As opposed to other papers that consider independent valuations, we show that the benefits, as well as the policy, dramatically differ as valuation interdependence changes. We also show that capacity plays a critical role in the choice of the seller's policy. Specifically, we find that, if the seller has unlimited capacity, the seller need not be concerned about how valuations are interdependent at all. In this case, the seller offers a discount if advance selling is offered. The discounted advance price is compensated by increased sales. The depth of discount and quantity available for advance selling depend on marginal cost and ex ante distribution of valuation, but not on how valuations are correlated.

However, these insights change drastically when the seller has limited capacity. All aspects of the seller's decision—whether to sell in advance, how much to offer in advance, and what price to offer in advance—depend on valuation interdependence. Specifically, when the degree of valuation interdependence is extremely low (as in the ∞-group model), advance selling will be deployed only when the seller's capacity is not very small, and it is always offered with a discount. In addition, the seller may partially ration the quantity sold during advance sales. On the other hand, when the degree of valuation interdependence is high (as in the one-group model), almost the opposite results: advance selling is optimal across the whole range of capacities, even when the capacity is very tight, the seller could charge a premium when selling in advance, and the seller should make the entire capacity available in advance. Between these two extreme cases, the seller's policy gradually changes as the degree of valuation interdependence changes.

Depending on the nature of the product, consumers' valuations can be highly independent (when personal or idiosyncratic factors dominate) or closely correlated (when common environmental factors prevail). Our model explains how the seller should take these factors into account: when individual differences largely determine the consumer's valuation (e.g., the value of prepurchased meal tickets in Disneyland or the value of an airline ticket) and the valuations are diverse, the seller can set the spot price to avoid major shortages in the spot market. Consequently, consumers are not willing to pay a premium price in advance (as typically observed for hotel, airline, and rental car reservations). In such cases, we find that a seller with limited capacity benefits from allocating only a portion of the capacity to advance selling (e.g., hotels and airlines may set booking limits at lower rates) so as to raise prices in both periods and reserve some capacity for the spot period to take advantage of the high spot price.

On the other hand, when consumers' valuations are highly interdependent (highly correlated), advance selling can be beneficial to the seller at any capacity level, even when the capacity is tight. The seller can exploit consumers' uncertainty about product availability in the spot period and charge a premium price in advance. This matches much-hyped or award-winning Broadway shows and concerts, or sports events, for which the advance price can be much higher (e.g., 40% higher for Broadway shows (Healy 2011)) than spot price. We also find that the seller never limits the advance sales. Interestingly, between these two extreme cases, the seller's policy gradually changes in the valuation interdependence. Thus, our models and resulting insights allow us to provide a directional explanation consistent with the phenomena we observe in various applications. Although we acknowledge that our stylized model does not cover all aspects of advance selling, we believe that the insights derived from the simple model highlight the importance of recognizing valuation interdependence in decisions regarding advance selling.

We show that our main results and insights are robust when several assumptions are relaxed: first, the seller may have the flexibility to choose the capacity before the decisions on advance selling; second, valuation distributions may be different in the advance and spot periods; and third, consumers may incur an inconvenience cost for spot purchases. The details of these extensions are provided in a supplementary document (available at http://dx.doi.org/10.1287/mnsc.2014.2047). We find that all of the extensions (endogenous capacity, multiple valuation distributions, and convenience of advance selling) make advance selling even more desirable and preserve the insights related to valuation interdependence.

Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/mnsc.2014.2047.

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Appendix A. Proof of Theorem 1

To show that the optimal spot price and spot profit are independent of k, it suffices to show that, given $N-S$ consumers in spot and spot price p_2, the expected spot profit function in Equation (1) is independent of k. Note that with unlimited capacity $T \geq N$, expected spot sales are equal to expected spot demand. Also, the group sizes in spot $S_1, \ldots, S_k$ and group valuations $\alpha_1, \ldots, \alpha_k$ are independent. Hence, Equation (1) becomes $\pi_2^U(S, p_2) = (p_2 - c) \cdot \mathrm{E}_{(\alpha_1,\ldots,\alpha_k,S_1,\ldots,S_k)}[\sum_{i=1}^{k}(n_i - S_i)1_{\alpha_i \geq p_2}] = (p_2 - c)(N-S)\bar{G}(p_2)$, independent of k. Hence, the optimal spot price and spot profit are independent of k. Furthermore, since $\lambda_2^U(S, p_2) \equiv 1$ as a result of unlimited capacity, from Equation (5), the optimal advance price is also independent of k. This result, together with Equation (6), further implies that the optimal capacity ration is also independent of k.

Appendix B. Proof of Theorem 2

(i) Since $\bar{G}(l) = 1$ and $\bar{G}(h) = 0$, it suffices to consider $p_2 \in [l, h]$. We divide the proof into two cases depending on the marginal cost, c.

- ($\underline{c} = l - 1/g(l) < c < h$). Taking the derivative with respect to p_2, we have

$$\frac{\partial \pi_2^U(S, p_2)}{\partial p_2} = (N-S)\big[\bar{G}(p_2) - (p_2 - c)g(p_2)\big]$$
$$= (N-S)g(p_2)\left[\frac{\bar{G}(p_2)}{g(p_2)} - (p_2 - c)\right].$$

Evaluating at two boundary points, $p_2 = l$ and $p_2 = h^-$, we have $\partial \pi_2^U(S, p_2)/\partial p_2|_{p_2=l} = (N-S)\{1 - (l-c)g(l)\} > 0$ and $\partial \pi_2^U(S, p_2)/\partial p_2|_{p_2=h^-} = (N-S)\{0 - (h^- - c)g(h^-)\} < 0$. Note that, from log concavity, $\bar{G}(x)/g(x) - x$ is strictly decreasing. Hence, $\pi_2^U(S, p_2)$ is strictly quasiconcave in p_2. Thus, the first-order condition, $p_2^* = c + \bar{G}(p_2^*)/g(p_2^*)$, results in a unique optimal solution in (l, h). Applying implicit function theorem to this condition, we obtain $dp_2^U/dc = [1 - (\bar{G}(x)/g(x))'|_{x=p_2^U}]^{-1} > 0$, as a result of the log concavity of $G(\cdot)$.

- ($c \leq \underline{c}$). It is easy to show that $\partial \pi_2^U(p_2, S)/\partial p_2|_{p_2=l} \leq 0$ and $\partial \pi_2^U(p_2, S)/\partial p_2|_{p_2=h^-} < 0$. By the strict quasiconcavity above, $\partial \pi_2^U(p_2, S)/\partial p_2 \leq 0$ for all $p_2 \in [l, h]$. Thus, p_2^U must be l.

(ii) Since $\lambda_2^U(S, p_2) \equiv 1$, from Equation (5), $p_1^{\max, U} = \mathrm{E}[\alpha] - \mathrm{E}[\max(\alpha - p_2^U)] = \mathrm{E}[\min(p_2^U, \alpha)]$. Since $\min(p_2^U, \alpha) \leq p_2^U$ for any given α, $p_1^{\max, U} \leq p_2^U$.

(iii) Since both the spot price and the advance price are independent of S, the total expected profit function (as given in Equation (9)) is linear in S. Hence, the optimal capacity ration is equal to either 0 or $\min(T, N_1) = N_1$.

Appendix C. Proof of Theorem 3

It suffices to show that $\pi_{AS}^U(N_1) - \pi_{AS}^U(0) \geq 0$ if and only if $c \leq \bar{c}$. Applying the definition of $p_1^{\max, U}$ (given in Theorem 2(ii)) to the profit function $\pi_{AS}^U(S)$ (given by Equation (9)), we have

$$\begin{aligned}
&\pi_{AS}^U(N_1) - \pi_{AS}^U(0)\\
&\quad = N_1\big\{p_1^{\max,U} - c - (p_2^U - c)\bar{G}(p_2^U)\big\}\\
&\quad = N_1\big\{\mathrm{E}[\min(\alpha, p_2^U)] - p_2^U + p_2^U - c - (p_2^U - c)\bar{G}(p_2^U)\big\}\\
&\quad = N_1\big\{\mathrm{E}[\min(\alpha - p_2^U, 0)] + (p_2^U - c)G(p_2^U)\big\}. \qquad \text{(C1)}
\end{aligned}$$

Evaluating for the two extreme cases $c \leq \underline{c}$ and $c = h$ and noting that by Theorem 2(i), $p_2^U|_{c \leq \underline{c}} = l$ and $p_2^U|_{c=h} = h$,

$$\begin{aligned}
&\big[\pi_{AS}^U(N_1) - \pi_{AS}^U(0)\big]\big|_{c \leq \underline{c}}\\
&\qquad = N_1\big\{\mathrm{E}[\min(\alpha - l, 0)] + (l-c)G(l)\big\} = 0,\\
&\big[\pi_{AS}^U(N_1) - \pi_{AS}^U(0)\big]\big|_{c=h}\\
&\qquad = N_1\big\{\mathrm{E}[\min(\alpha - h, 0)] + (h-c)G(h)\big\} < 0.
\end{aligned}$$

Note that p_2^U is continuous in c, and so is $p_1^{\max, U}$. As a result, $\pi_{AS}^U(N_1)$ and $\pi_{AS}^U(0)$ are continuous in c, and so is the difference $\pi_{AS}^U(N_1) - \pi_{AS}^U(0)$. Thus, there must exist a threshold $\bar{c} \in [\underline{c}, h)$, such that $\pi_{AS}^U(N_1)|_{\bar{c}} = \pi_{AS}^U(0)|_{\bar{c}}$ and $\pi_{AS}^U(N_1)|_c < \pi_{AS}^U(0)|_c$ for all $c > \bar{c}$. To prove $\pi_{AS}^U(N_1) - \pi_{AS}^U(0) \geq 0$ for $c \in (\underline{c}, \bar{c})$, it then suffices to show that $d[\pi_{AS}^U(N_1) - \pi_{AS}^U(0)]/dc$ is positive-negative in $c \in (\underline{c}, h)$ (implying that once $\pi_{AS}^U(N_1) - \pi_{AS}^U(0)$ drops below zero, it remains negative afterward).

To derive $d[\pi_{AS}^U(N_1) - \pi_{AS}^U(0)]/dc$, note that

$$\mathrm{E}[\min(\alpha - p_2^U, 0)] = \int_l^{p_2^U}(y - p_2^U)\,dG(y) = -\int_l^{p_2^U} G(y)\,dy$$

(the last equality follows from integration by parts). Also, recall that p_2^U is the solution to Equation (2) (i.e., $p_2^U = c + \bar{G}(p_2^U)/g(p_2^U)$ for $c \in (\underline{c}, h)$). Applying these facts to Equation (C1), we have

$$\pi_{AS}^U(N_1) - \pi_{AS}^U(0) = N_1\left\{\frac{G(p_2^U)\bar{G}(p_2^U)}{g(p_2^U)} - \int_l^{p_2^U} G(y)\,dy\right\}. \qquad \text{(C2)}$$

Taking the derivative with respect to c,

$$\frac{d[\pi_{AS}^U(N_1)-\pi_{AS}^U(0)]}{dc} = N_1\left\{\left(\frac{G(x)\bar{G}(x)}{g(x)}\right)' - G(x)\right\}\bigg|_{x=p_2^U}\frac{dp_2^U}{dc}. \tag{C3}$$

Recall that $(G(x)\bar{G}(x)/g(x))' + \bar{G}(x) - 1$ is positive-negative and, from Theorem 2 (i), $dp_2^U/dc > 0$. Hence, the derivative $d[\pi_{AS}^U(N_1) - \pi_{AS}^U(0)]/dc$ is positive-negative in $c \in [\underline{c}, h]$. Note that, since $\pi_{AS}^U(N_1) - \pi_{AS}^U(0) < 0$ for $c = h$, we have

$$\pi_{AS}^U(N_1) - \pi_{AS}^U(0) \text{ is decreasing in } c > \bar{c}, \tag{C4}$$

which we use in later proofs.

Appendix D. Proof of Lemma 1

The proof for $p_2^\infty(S) = \max\{p_2^U, p_2^B(S)\}$ follows a standard argument and is hence omitted.

Appendix E. Proof of Theorem 5

As shown in Lemma 1, the remaining capacity influences the seller's spot pricing strategy. If S is small and sufficient capacity is left in spot, the seller charges p_2^U, the spot price when capacity is unlimited. If the remaining capacity is small, the seller charges $p_2^B(S)$, the maximum price that clears the remaining capacity.

Let $S^=$ be the *lowest* amount of capacity used in advance at which the remaining capacity will be binding in the spot period. Clearly, $S^=$ is the solution to $(T-S)/(N-S) = \bar{G}(p_2^U)$ and $S^= = (T - N\bar{G}(p_2^U))/G(p_2^U)$. Thus, $p_2^\infty = p_2^U$ for $0 \le S \le S^=$, and $p_2^\infty = p_2^B(S)$ for $S \ge S^=$.

We define two functions, $f^U(S)$ and $f^B(S)$, corresponding to the seller's total profits when the remaining capacity, $T - S$, is not binding and when it is binding, respectively. For $S \in [0, \min(T, N_1)]$, we have

$$\begin{aligned} f^U(S) &:= \big(\mathrm{E}[\min(p_2^U, \alpha)] - c\big)S + (N-S)\bar{G}(p_2^U)(p_2^U - c) \\ &= \frac{1}{N_1}[\pi_{AS}^U(N_1) - \pi_{AS}^U(0)]S + N\overline{G}(p_2^U)(p_2^U - c), \end{aligned}$$

$$f^B(S) := \big(\mathrm{E}[\min(p_2^B(S), \alpha)] - c\big)S + (T-S)(p_2^B(S) - c), \text{ and}$$

$$\pi_{AS}^\infty(S) = \begin{cases} f^U(S) & \text{for } S \le S^= \text{ and } S \in [0, \min(T, N_1)], \\ f^B(S) & \text{for } S \ge S^= \text{ and } S \in [0, \min(T, N_1)]. \end{cases} \tag{E1}$$

The property of $f^B(S)$ is characterized in the technical Lemma E.1, which uses the two-threshold capacity defined as follows: $T_1 = N\bar{G}(p_2^U(\bar{c}))$ and $T_2 = N_1 + N_2\bar{G}(p_2^U(\bar{c}))$, where $\bar{c}$ is defined in Theorem 3.

Lemma E.1 (Proof Available Upon Request). (i) *$f^B(S)$ is strictly quasiconcave in S and has a unique maximizer S^B on* $[0, \min(T, N_1)]$.

(ii) *S^B is nondecreasing in T and independent of c.*

(iii) *$S^B = 0$ for $0 < T \le T_1$.*

(iv) *There exists a critical number $T^\infty \in (T_1, T_2)$ such that $0 < S^B < \min(T, N_1)$ for $T_1 < T < T^\infty$, and $S^B = \min(T, N_1)$ for $T^\infty \le T < N$.*

By Lemma E.1, the location of maximizer S^B defines the monotonicity of $f^B(S)$. If $T_1 < T < T^\infty$, $f^B(S)$ is first increasing and then decreasing in S, but when $0 < T \le T_1$, $S^B = 0$ and $f^B(S)$ are monotonically decreasing in S; however, for $T^\infty \le T < N$, $f^B(S)$ is monotonically increasing in S. On the other hand, note that $f^U(S)$ is a linear function of S with slope equal to $1/N_1[\pi_{AS}^U(N_1) - \pi_{AS}^U(0)]$. By Theorem 3, marginal cost c determines the monotonicity of f^U: $f^U(S)$ is nondecreasing in S if $c \le \bar{c}$ and decreasing in S if $c > \bar{c}$.

Figure E.1 shows several cases of the trajectories of $\Pi_{AS}^\infty(S)$ in S. Its precise behavior is driven by whether $S^= \le 0$ (f^U disappears) or $S^= \ge \min(T, N_1)$ (f^B disappears); $S^B < S^=$ (monotonicity of f^B); and whether $c > \bar{c}$. We divide the proof into five cases based on combinations of c and T as illustrated in Figure E.2(a). For each case, we characterize the profit function $\pi_{AS}^\infty(S)$ and the optimal ration S^∞. Based on the optimal ration S^∞, we then prove that the optimal advance price is always strictly less than the optimal spot price. Detailed analysis is available in the supplementary document. The optimal ration S^∞ is summarized in Figure E.2(b).

Figure E.1 **Illustration of $\pi_{AS}^\infty(S)$ for Cases (R3), (R4), and (R5)**

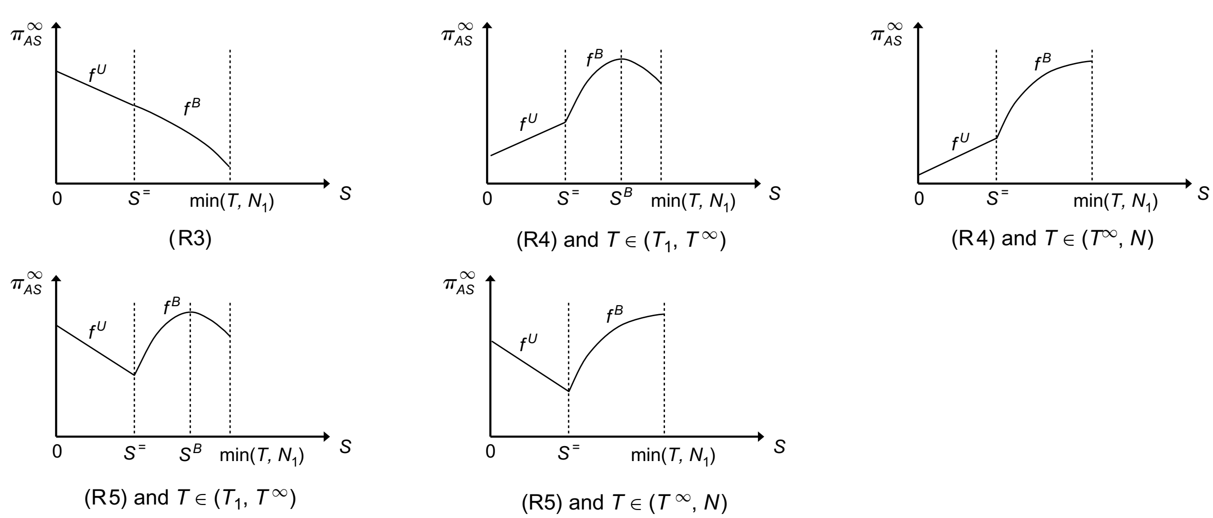
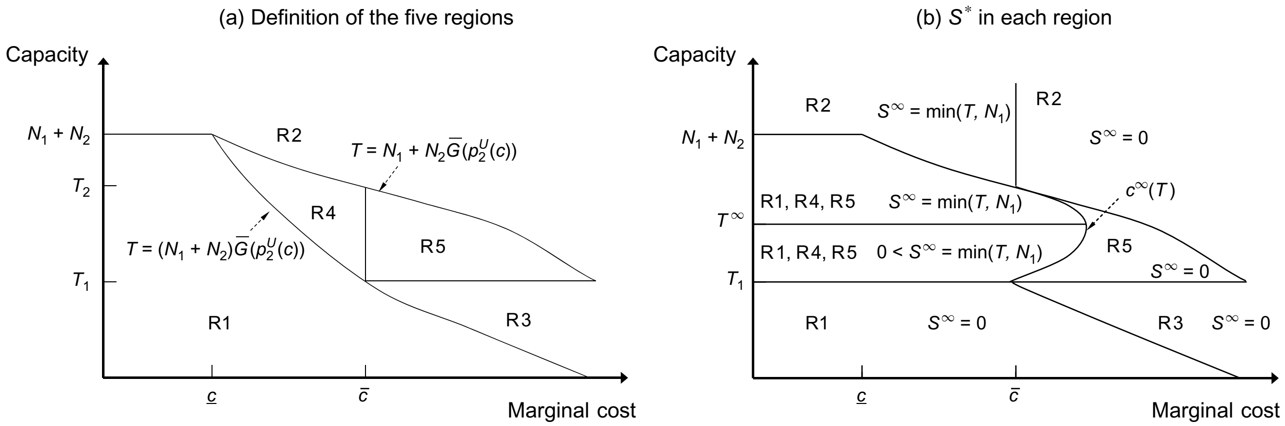


Figure E.2 **Five Regions in the Capacity-Cost Space**



Appendix F. Proof of Lemma 2

To see that $p_2^1(S) = p_2^U$, note that since $T - S < N - S$, by Equation (13),

$$\begin{aligned}\pi_2^1(S, p_2) &= (p_2 - c)(T - S)\mathrm{Prob}[\alpha \geq p_2] \\ &= (p_2 - c)(T - S)\bar{G}(p_2). \qquad \text{(F1)}\end{aligned}$$

In Equation (F1), $\pi_2^1(S, p_2)$ is the same function of price p_2 as in the unlimited-capacity case (Equation (7)) except that potential sales $N - S$ are replaced by $T - S$. Thus, the optimal spot price is equal to the optimal spot price of the unlimited-capacity model, i.e., $p_2^1(S) = p_2^U$.

Since $c < h$, recall that by Theorem 2(i), $\bar{G}(p_2^U) > 0$. Furthermore, by definition of the one-group valuation model, a supply shortage occurs in spot whenever $\alpha > p_2^1$. That is, the probability of shortage is $\bar{G}(p_2^1) = \bar{G}(p_2^U)$. Hence, the probability of shortage is always positive.

Appendix G. Proof of Lemma 3

For an increasing and convex function $f(x)$ with $x \geq 0$, clearly $xf(x)$ is convex. Since $p_1^{\max,1}(S) - c$ is convex increasing, we have that profit function is convex and, thus, the maximum is at the boundary $S = 0$ or $S = \min(T, N_1)$.

Appendix H. Proof of Theorem 6

Recall that from Lemma 3, for any $T \in (0, N)$, we have $S^1 = \min(T, N_1)$ (full advance selling) or 0 (spot only). Define the difference in profits between full advance selling and spot only as a function of marginal cost c, $\Delta(c)$:

$$\begin{aligned}\Delta(c) &= \pi_{AS}^1(\min(T, N_1)) - \pi_{AS}^1(0) \\ &= \min(T, N_1)(p_1^{\max,1}(\min(T, N_1)) - c) \\ &\quad + (T - \min(T, N_1))\bar{G}(p_2^U(c))(p_2^U(c) - c) \\ &\quad - T\bar{G}(p_2^U(c))(p_2^U(c) - c) \\ &= \min(T, N_1)\{p_1^{\max,1}(\min(T, N_1)) \\ &\qquad - c - \bar{G}(p_2^U(c))(p_2^U(c) - c)\} \\ &= \min(T, N_1)\bigg\{\mathrm{E}[\alpha] - \frac{T - \min(T, N_1)}{N - \min(T, N_1)} \\ &\qquad \cdot \mathrm{E}[\max(\alpha - p_2^U(c), 0)] - c \\ &\qquad - \bar{G}(p_2^U(c))(p_2^U(c) - c)\bigg\}, \qquad \text{(H1)}\end{aligned}$$

where the expressions of $\pi_{AS}^1(S)$ and $p_1^{\max,1}(S)$ are given in Equations (15) and (14), respectively.

We divide the proof into two cases: $c \leq \bar{c}$ and $c > \bar{c}$.

- $c \leq \bar{c}$. For any $T \in (0, N)$, $(T - \min(T, N_1))/(N - \min(T, N_1)) < 1$. Thus, from Equation (H1),

$$\begin{aligned}\Delta(c) &> \min(T, N_1)\{\mathrm{E}[\alpha] - \mathrm{E}[\max(\alpha - p_2^U(c), 0)] \\ &\qquad - c - \bar{G}(p_2^U(c))(p_2^U(c) - c)\} \\ &= \frac{\min(T, N_1)}{N_1}(\pi_{AS}^U(N_1) - \pi_{AS}^U(0)) \geq 0,\end{aligned}$$

where the last equation follows from Theorem 3. Hence, full advance selling is always optimal.

- $c > \bar{c}$. In Equation (H1), note that $\mathrm{E}[\max(\alpha - p_2^U(c), 0)]$ is nonincreasing in c and

$$\frac{T - \min(T, N_1)}{N - \min(T, N_1)} < 1.$$

Thus,

$$\begin{aligned}\frac{d\Delta(c)}{dc} &\leq \min(T, N_1)d\{\mathrm{E}[\alpha] - \mathrm{E}[\max(\alpha - p_2^U(c), 0)] \\ &\qquad - c - \bar{G}(p_2^U(c))(p_2^U(c) - c)\}/dc \\ &= \frac{\min(T, N_1)}{N_1}\frac{d[\pi_{AS}^U(N_1) - \pi_{AS}^U(0)]}{dc} < 0.\end{aligned}$$

The last inequality is due to Equation (C4). Thus, $\Delta(c)$ is monotonically decreasing in c for $c > \bar{c}$. Furthermore, $\Delta(\bar{c}) > 0$ (as shown in the first case), and $\Delta(h) = \min(T, N_1)(\mathrm{E}[\alpha] - \bar{c}) < 0$. Therefore, there must exist $c^1(T) \in (\bar{c}, h)$ such that $S^1 = \min(T, N_1)$ for $c \leq c^1(T)$ and $S^1 = 0$ otherwise.

Furthermore, since the function

$$\frac{T - \min(T, N_1)}{N - \min(T, N_1)}$$

is nondecreasing in T, $\Delta(c)$ is the product of a positive term and a term nonincreasing in T, which implies that $c^1(T)$ is nonincreasing in T.

Appendix I. Proof of Proposition 1

Substituting $S^1 = \min(T, N_1)$ into the expression of $p_1^{\max,1}(S^1)$ (given by Equation (14)) and recalling that $p_2^1(S^1) = p_2^U$ (shown in Lemma 2), the price premium is

$$p_1^{\max,1}(S^1) - p_2^1(S^1) = \mathrm{E}[\alpha] - p_2^U - \frac{T - \min(T, N_1)}{N - \min(T, N_1)} \mathrm{E}[\max(\alpha - p_2^U, 0)]. \quad \text{(I1)}$$

($\Leftarrow$) Directly follows Equation (I1).

($\Rightarrow$) In Equation (I1), note that $(T - \min(T, N_1))/(N - \min(T, N_1))\mathrm{E}[\max(\alpha - p_2^U, 0)]$ is always nonnegative. Hence, we immediately have that $p_1^{\max,1}(S^1) - p_2^1(S^1) > 0$ implies $\mathrm{E}[\alpha] - p_2^U > 0$. We prove the necessity of the capacity condition by contradiction. Suppose that $p_1^{\max,1}(S^1) - p_2^1(S^1) > 0$ and $T \geq N_1 + \mathrm{E}[\alpha] - p_2^U)/(\mathrm{E}[\max(\alpha - p_2^U, 0)])N_2$; then $\min(T, N_1) = N_1$ and (after easy rearrangement) the right-hand side of Equation (I1) is nonpositive, which contradicts the hypothesis $p_1^{\max,1}(S^1) - p_2^1(S^1) > 0$.

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