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Jump and variance risk premia in the S&P 500 ☆

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ABSTRACT

We analyze the risk premia embedded in the S&P 500 spot index and option markets. We use a long time-series of spot prices and a large panel of option prices to jointly estimate the diffusive stock risk premium, the price jump risk premium, the diffusive variance risk premium and the variance jump risk premium. The risk premia are statistically and economically significant and move over time. Investigating the economic drivers of the risk premia, we are able to explain up to 63% of these variations.

1. Introduction

It is well known that asset price processes exhibit both smooth and discontinuous components. A large literature, including Merton (1976), Heston (1993), Duffie et al. (2000), Eraker et al. (2003) and Eraker (2004), makes a compelling case for models of asset prices that include stochastic volatility as well as jumps in prices and variance. This paper aims to shed more light on the compensation that investors demand for their exposure to these risks.

We contribute to extant literature in two directions. First, we use a long time-series of spot data and a large panel of option prices to estimate a stochastic volatility model with contemporaneous jumps in returns and variance (SVCJ). We first apply the Markov Chain Monte Carlo (MCMC) algorithm to the time-series of spot returns in order to estimate the latent variance process and the parameters that govern the dynamics of the S&P 500 index returns under the physical measure ($\mathbb{P}$). We then use the calibrated instantaneous variance and our option data to extract the parameters under the risk-neutral measure ($\mathbb{Q}$). In performing our estimation, we are particularly careful to impose the theoretical restrictions discussed in Bates (2000) and Broadie et al. (2007).[1] We find strong evidence of stochastic volatility and jumps, raising questions as to whether these sources of risks are priced.

Second, we study the equity and variance risk premia embedded in the spot index and index option markets. We decompose the equity risk premium into the diffusive stock risk premium (*DSRP*) and the price jump risk premium (*PJRP*). Similarly, we dissect the variance risk premium into the diffusive variance risk premium (*DVRP*) and the variance jump risk premium (*VJRP*). Generally, we find that the equity and variance risk premia are

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[1] Bates (2000) and Broadie et al. (2007) stress that the volatility of the variance process, as well as the correlation between the Brownians of the price and variance process, should be consistent across both probability measures.

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mainly driven by the compensation for jumps. Our analysis reveals important variations in the time-series of the risk premia. Using a large dataset of macroeconomic forecasts, we construct empirical proxies of macroeconomic expectations and uncertainty. We complement these variables with the default spread (DFSPD), the term spread (TSPD) and Corwin and Schultz (2012)'s illiquidity proxy (ILLIQ). We regress the individual risk premia on these variables and obtain adjusted R^2 of up to 63%. Our analysis reveals that macroeconomic uncertainty has substantially more explanatory power than macroeconomic expectations, suggesting that time varying uncertainty has a first-order impact on the variations in the risk premia, and thus on asset prices.

Naturally, our parametric approach may be subject to model misspecification risk. Especially, one might wonder whether two jump components — one in the return process and one in the variance process — are indeed necessary or whether the model is overspecified. To assuage these concerns, we compare the SVCJ model to two other model specifications often employed in the literature, namely the simple stochastic volatility model (SV) and the stochastic volatility model with jumps in returns (SVJ). We use the deviance information criterion (DIC) and the root mean squared errors (RMSE) of option prices to compare the three models. This analysis shows that the SVCJ model outperforms its rivals, lending more credence to our modeling choice. We also consider alternative ways in obtaining the latent variance and show that our findings are robust to different approaches. Finally, we assess the explanatory power of the Baker and Wurgler (2006) sentiment index for the risk premia and show that sentiment has a significantly negative impact on the price jump risk premium.

Our study is linked to the financial modeling literature that seeks to capture the dynamics of asset prices in parsimonious models. Bates (1996), Bakshi et al. (1997), Chernov and Ghysels (2000), Eraker et al. (2003), Jones (2003), Eraker (2004) and Kaeck (2013), among others, propose and test different models that feature stochastic volatility, jumps in returns or jumps in both returns and variance. Overall, these studies document the presence of stochastic volatility and jumps in both the return and variance processes. Building on this literature, we estimate a popular continuous-time model, the SVCJ model, to jointly study the dynamics of the equity and variance risk premia.

Our paper also links with the literature on the variance risk premium. Carr and Wu (2009) and Driessen et al. (2009) investigate the market price of variance risk of short-maturity in the equity market. Amengual (2009), Egloff et al. (2010) and Amengual and Xiu (2014) explore the term-structure of variance risk premia. Similar to Todorov (2010) and Bollerslev and Todorov (2011), we show that jumps play an important role in the dynamics of the equity risk premium.

Our study also carries interesting implications for the literature that focuses on theoretical models of asset prices. For instance, our analysis indicates that the price jump risk premium is time-varying and makes up a large proportion of the equity risk premium. An upshot of this result is that jumps should be incorporated in theoretical models of asset prices. This is because, a model without jumps would counterfactually imply that all of the equity risk premium is due to the diffusive component of the return process.

The works of Pan (2002) and Broadie et al. (2007) are most closely related to our study. They analyze the equity and variance risk premia in the S&P 500 option market. These studies focus on the unconditional risk premia estimated using relatively short sample periods. We improve on these papers in several respects. First, we analyze a longer sample that includes the recent financial crisis period which started around the collapse of Lehman Brothers. Obtaining a longer sample period is important in order to draw robust inferences about the time-variations of risk premia.[2] Second, we decompose the equity and variance risk premia into their continuous and discontinuous components and explore their interconnections. Third, we study the economic drivers of the variations in the risk premia.

Finally, our work adds to the literature on option returns. Bondarenko (2003) reports that average put returns are too high to be reconciled with standard factor models such as the capital asset pricing model (CAPM). Coval and Shumway (2001), Bakshi and Kapadia (2003) and Bakshi and Kapadia (2003) show that simple volatility trades such as short straddles earn as much as 3% per week. We estimate the distinct components of the variance risk premium and connect them to the macroeconomy, thus offering a risk-based explanation for these large option returns.

The remainder of this paper proceeds as follows. Section 2 describes our dataset and empirical methodology. Section 3 discusses our parameter estimates and analyzes the risk premia. Section 4 investigates the economic drivers of the risk premia. Section 5 discusses our robustness checks. Finally, Section 6 concludes.

2. Data and methodology

This section presents our data and methodology. We begin by describing our spot and options dataset. We then outline the econometric methodology used to estimate the model parameters and associated risk premia.

2.1. Data

We obtain the price-series of the S&P 500 index for the period between April 1990 and December 2010 from Bloomberg. Table 1 provides descriptive statistics of the daily percentage returns. We can see that the mean daily percentage return is positive (0.026). The mean daily volatility is 1.167. The skewness of daily returns is small and negative (−0.185). However, the kurtosis (12.168) is fairly high, indicating (not surprisingly) that S&P 500 spot returns are not normally distributed. These summary statistics are suggestive of the presence of stochastic volatility and/or jumps in the stock index market.

Our dataset of S&P 500 futures options contains daily settlement prices for the period from April 1990 to December 2010. S&P 500 futures options trade on the Chicago Mercantile Exchange (CME) and follow a quarterly expiration cycle, i.e. they expire in March, June, September and December. We process the option dataset as follows. We discard all option contracts that mature in less than 8 days, since they are typically associated with infrequent trading. In a similar vein, we expunge all options with maturity greater than a year. We also discard all option prices that are lower than five times the minimum tick size of 0.01 index points. S&P 500 futures options are of the American type. Thus, we follow Trolle and Schwartz (2009) and convert the American option prices into European option prices using the approach of Barone-Adesi and Whaley (1987).

Table 2 summarizes our final options dataset. We present the number of observations organized by moneyness, defined as the ratio of the strike price over the underlying's price. We also split our options data into three maturity groups: short (less than 60 days), medium (60–180 days) and long (more than 180 days) maturity options. This table reveals that most of our dataset contains option contracts of maturity up to 180 days.

[2] In comparison to our long sample period (1990–2010), Pan (2002) covers the period ranging from 1989 to 1996.

Table 1
Descriptive statistics. This table presents the summary statistics of the S&P 500 daily percentage returns. "Mean" reports the average return. "Std Dev" displays the standard volatility. "Min" and "Max" show the minimum and maximum percentage return, respectively. Finally, "Skew" and "Kurt" show the skewness and kurtosis of returns, respectively.

	S&P 500
Mean	0.026
Std Dev	1.167
Min	−9.470
Max	10.957
Skew	−0.185
Kurt	12.168

Table 2
Options data. This table summarizes the options dataset. "Moneyness" refers to whether the option is in-the-money (ITM), at-the-money (ATM) or out-of-the-money (OTM). "Type" indicates whether the option is a put or call. "Range" denotes the moneyness range, computed as the ratio of the strike price over the underlying's price. "Short", "Medium" and "Long" refer to options that mature in less than 60 days, between 60 and 180 days and in more than 180 days, respectively. The last column reports the sum of all entries in each row. Similarly, the last row shows the sum of all entries in each column.

Moneyness	Type	Range	Short	Medium	Long	Total
ITM	Call	<0.94	14,566	11,612	5,432	31,610
	Call	0.94–0.97	7,519	6,277	2,924	16,720
	Put	>1.06	9,441	10,039	5,041	24,521
	Put	1.03–1.06	5,862	4,360	1,543	11,765
ATM	Call	0.97–1.00	9,622	8,699	3,289	21,610
	Call	1.00–1.03	10,234	9,244	3,173	22,651
	Put	1.00–1.03	8,894	6,663	2,292	17,849
	Put	0.97–1.00	10,102	9,054	3,232	22,388
OTM	Call	1.03–1.06	9,698	8,184	2,731	20,613
	Call	>1.06	21,832	30,282	15,061	67,175
	Put	0.94–0.97	9,878	9,109	3,432	22,419
	Put	<0.94	45,970	51,711	25,183	122,864
Total			163,618	165,234	73,333	402,185

2.2. Model set-up

2.2.1. Model dynamics

We consider the stochastic volatility model with contemporaneous jumps in returns and volatility (SVCJ) of Duffie et al. (2000).[3] Eqs. (1) and (2) present the dynamics of the stock price under the $\mathbb{P}$ measure:

$$dS_t = S_t(r_t - \delta_t + \gamma_t - \bar{\mu}^{\mathbb{P},s}\lambda^{\mathbb{P}})dt + S_t\sqrt{V_t}dW_t^{\mathbb{P},s} + d\left(\sum_{j=1}^{N_t} S_{\tau_{j-}}(e^{Z_j^s} - 1)\right) \tag{1}$$

$$dV_t = \kappa^{\mathbb{P}}(\theta^{\mathbb{P}} - V_t)dt + \sigma^{\mathbb{P},v}\sqrt{V_t}dW_t^{\mathbb{P},v} + d\left(\sum_{j=1}^{N_t} Z_j^v\right) \tag{2}$$

where S_t is the equity index price at time t. r_t denotes the riskless rate at time t. δ_t is the dividend yield at t. γ_t is the time-varying equity risk premium. As Broadie et al. (2007) show, γ_t depends, among other, on the product of the latent variance V_t and η (see Eq. (10) below). Intuitively, we expect a positive estimate of η so that there is a positive risk-return trade-off. $\bar{\mu}^{\mathbb{P},s}$ is defined as: $\bar{\mu}^{\mathbb{P},s} = e^{\mu^{\mathbb{P},s} + \frac{(\sigma^{\mathbb{P},s})^2}{2}} - 1$. Jumps occur with constant intensity $\lambda^{\mathbb{P}}$ under the $\mathbb{P}$ measure. Throughout this paper, the superscript $\mathbb{P}$ indicates that we are working under the physical probability measure ($\mathbb{P}$). V_t is the instantaneous variance. $W_t^{\mathbb{P},s}$ refers to the Brownian motion that drives the stock return process. It shares a correlation, $\rho^{\mathbb{P}}$, with the Brownian of the variance process $W_t^{\mathbb{P},v}$. Z_j^s is the normally distributed jump size in the stock return process, $Z_j^s \sim N(\mu^{\mathbb{P},s}, \sigma^{\mathbb{P},s})$. N_t is the Poisson process that determines the presence of jumps in the return and variance processes under the physical measure.[4] $\kappa^{\mathbb{P}}$ denotes the speed of mean reversion of the variance process under the $\mathbb{P}$ measure. $\theta^{\mathbb{P}}$ is the mean-reversion level of variance under the $\mathbb{P}$ measure. $\sigma^{\mathbb{P},v}$ denotes the volatility of the variance process. Z_j^v is the exponential jump size in the variance process, $Z_j^v \sim \exp(\mu^{\mathbb{P},v})$.[5] The jump sizes in returns and variance are assumed to be independent.[6]

The dynamics under the $\mathbb{Q}$ measure are given by the following set of equations:

$$dS_t = S_t(r_t - \delta_t - \bar{\mu}^{\mathbb{Q},s}\lambda^{\mathbb{Q}})dt + S_t\sqrt{V_t}dW_t^{\mathbb{Q},s} + d\left(\sum_{j=1}^{N_t} S_{\tau_{j-}}(e^{Z_j^s} - 1)\right) \tag{3}$$

$$dV_t = \kappa^{\mathbb{Q}}(\theta^{\mathbb{Q}} - V_t)dt + \sigma^{\mathbb{Q},v}\sqrt{V_t}dW_t^{\mathbb{Q},v} + d\left(\sum_{j=1}^{N_t} Z_j^v\right) \tag{4}$$

where all parameters are defined as before, with the only difference that the superscript $\mathbb{Q}$ replaces $\mathbb{P}$, indicating that the parameters relate to the $\mathbb{Q}$ measure.[7]

As is standard in the literature, we let the jump intensity take the same value under both probability measures: $\lambda^{\mathbb{P}} = \lambda^{\mathbb{Q}} = \lambda$.[8] Additionally, the volatility of the return jump size is the same across both measures: $\sigma^{\mathbb{P},s} = \sigma^{\mathbb{Q},s}$.[9]

2.2.2. Theoretical restrictions

The change of measure imposes the following theoretical restrictions: (i) the product of the speed of mean reversion and the long-run variance (Pan, 2002), (ii) the correlation between the Brownians of the return and variance processes (Bates, 2000) and (iii) the volatility of the variance process, should be equal under both measures (Broadie et al., 2007). Consequently, we impose these restrictions in our estimation procedure:

[3] As a robustness check, we analyze the SV and SVJ models. Section 5 clearly shows that the SVCJ model outperforms both the SV and SVJ models.

[4] As in Duffie et al. (2000), the Poisson process characterizing jumps is assumed to be identical for both the price and the variance processes. Alternatively, one could assume two independent Poisson processes. However, the empirical results of Eraker et al. (2003) and Eraker (2004) show that the former is a better approach.

[5] As discussed in Duffie et al. (2000), an exponentially distributed variance jump size has the advantage of guaranteeing the positiveness of the variance process while still being analytically tractable.

[6] The independence of jump sizes is consistent with the results of previous studies. For example, Eraker et al. (2003) and Eraker (2004) report statistically insignificant correlations between the two jump sizes.

[7] In the empirical part of the paper, we use options data related to the futures contract (rather than the spot index). Under the risk-neutral measure, the futures return follows a process similar to that of Eq. (3), except that the drift term has to be chosen such that the expected return on the futures contract equals zero.

[8] See Pan (2002) and Eraker (2004) for similar restrictions.

[9] We impose this restriction because it is empirically difficult to precisely estimate all parameters in the SVCJ model. See Kaeck (2013) for a similar point. In Section 5, we examine other models of the dynamics of the stock index, including the SVJ model, where we allow the volatility of jump returns to differ across probability measures. We find that our baseline model (SVCJ) outperforms other models.

$$\kappa^{\mathbb{P}}\theta^{\mathbb{P}} = \kappa^{\mathbb{Q}}\theta^{\mathbb{Q}} \tag{5}$$

$$\rho^{\mathbb{P}} = \rho^{\mathbb{Q}} \tag{6}$$

$$\sigma^{\mathbb{P},v} = \sigma^{\mathbb{Q},v} \tag{7}$$

2.3. Methodology

We now describe our two-step estimation methodology, which broadly follows that of Broadie et al. (2007). We begin by estimating the $\mathbb{P}$ parameters. We then estimate the risk-neutral parameters.

2.3.1. Physical measure

Following Eraker et al. (2003), we implement the MCMC estimation approach on the time-series of index returns to estimate the $\mathbb{P}$ parameters.[10] The key advantage of the MCMC algorithm over other approaches, e.g. efficient method of moments, general method of moments or maximum likelihood, is that it allows the econometrician to extract not only the model parameters but also the latent variables, e.g. the latent variance which we also need for the second step of our estimation procedure as variance is a key determinant of option prices. It also accounts for model risk and works well in high-dimensional settings including several state variables and many parameters.

2.3.2. Risk-neutral measure

In order to estimate the risk-neutral parameters, we exploit our options data and minimize the squared distance between the model and market implied volatilities. Minimizing the difference between implied volatilities (rather than option prices) presents a distinct advantage. From a purely theoretical point of view, implied volatilities (as opposed to option prices) should not exhibit a monotonic relationship with strike prices. Hence, the optimization does not overweight a specific range of option contracts. Our objective function reads as follows:

$$\Theta^{\mathbb{Q}} = \arg\min \sum_{t=1}^{T}\sum_{n=1}^{O_t}[IV_t(K_n,\tau_n,S_t,r_t,V_t) - IV_t(\Theta^{\mathbb{Q}}|\Theta^{\mathbb{P}},K_n,\tau_n,S_t,r_t,V_t)]^2 \tag{8}$$

where K_n is the strike of the n^{th} option. τ_n is the time to maturity of the n^{th} option. $\Theta^{\mathbb{P}}$ and $\Theta^{\mathbb{Q}}$ are the sets of physical and risk-neutral parameters, respectively. O_t is the number of options on day t and $IV_t(K_n,\tau_n,S_t,r_t)$ is the annualized Black (1976) market implied volatility. $IV_t(\Theta^{\mathbb{Q}}|\Theta^{\mathbb{P}},K_n,\tau_n,S_t,r_t,V_t)$ is the annualized model implied volatility. To obtain this quantity, we first implement the option pricing formula provided by Duffie et al. (2000) to obtain the option's value. We then use the option price to recover the corresponding Black (1976) implied volatility.[11]

For each maturity and observation date, we fit a piece-wise quadratic function to all implied volatilities:

$$y = \mathbf{1}_{[x\leqslant x_0]}[a_2(x-x_0)^2 + a_1(x-x_0) + a_0] + \mathbf{1}_{[x>x_0]}[b_2(x-x_0)^2 + a_1(x-x_0) + a_0] + \epsilon \tag{9}$$

where y is a vector that contains the implied volatilities. x relates to the moneyness, defined as the strike price over the spot price. x_0 is the knot point of the two quadratic functions. If there are fewer than 10 traded option contracts, we fit a linear function to the implied volatilities.[12] We then take 9 equidistant implied volatilities in the moneyness interval ranging from 0.8 to 1.2, which we use as input to the optimization problem described by Eq. (8).[13] We repeat our estimation each month, yielding a time-series of monthly risk-neutral parameters.[14,15]

2.3.3. Risk Premia

Having recovered the $\mathbb{P}$ and $\mathbb{Q}$ parameters, we then focus on the task of estimating the risk premia.[16] Similar to Broadie et al. (2007), we define the equity risk premium as the difference between the physical and risk-neutral expectations of the stock return:

$$E_t^{\mathbb{P}}\left(\frac{dS_t}{S_t}\right) - E_t^{\mathbb{Q}}\left(\frac{dS_t}{S_t}\right) = \gamma_t$$

$$\underbrace{E_t^{\mathbb{P}}\left(\frac{dS_t}{S_t}\right) - E_t^{\mathbb{Q}}\left(\frac{dS_t}{S_t}\right)}_{ERP_t} = \underbrace{\eta V_t}_{DSRP_t} + \underbrace{(\bar{\mu}^{\mathbb{P},s} - \bar{\mu}^{\mathbb{Q},s})\lambda}_{PJRP_t} \tag{10}$$

$$ERP_t \equiv DSRP_t + PJRP_t \tag{11}$$

where ERP_t denotes the equity risk premium at time t. The equity risk premium is the sum of two components. The first component $DSRP_t$ is the diffusive stock risk premium at time t. This is the part of the equity risk premium due to the diffusive component of the return process. The second component, $PJRP_t$ is the price jump risk premium at time t. It reflects the compensation related to the discontinuous component of the return process.

The variance risk premium is obtained as follows:

$$\underbrace{E_t^{\mathbb{Q}}[dV_t] - E_t^{\mathbb{P}}[dV_t]}_{VRP_t} = \underbrace{(\kappa^{\mathbb{P}} - \kappa^{\mathbb{Q}})V_t\,dt}_{DVRP_t} + \underbrace{(\mu^{\mathbb{Q},v} - \mu^{\mathbb{P},v})\lambda\,dt}_{VJRP_t} \tag{12}$$

$$VRP_t \equiv DVRP_t + VJRP_t \tag{13}$$

Eq. (13) shows that the variance risk premium is the sum of two components: the diffusive variance risk premium (*DVRP*) and the variance jump risk premium (*VJRP*). They relate to compensations for diffusive and discontinuous movements in the variance process, respectively.

3. The dynamics of the risk premia

This section discusses the dynamics of the risk premia. We first present and discuss our parameter estimates under both probability measures. We then analyze the sign, magnitude, and dynamics of the risk premia.

[10] We closely follow the steps outlined in Eraker et al. (2003) and use the same priors and hyperparameters. Details are available upon request.

[11] Note that our objective function differs slightly from that of Broadie et al. (2007), who jointly estimate the variance (V_t) and the risk-neutral ($\mathbb{Q}$) parameters. This approach is computationally demanding. The computational burden is particularly serious if the optimization is performed on a monthly basis (as we do). By directly using the spot variance estimated under $\mathbb{P}$, we are able to eschew this difficulty and estimate the monthly $\mathbb{Q}$ parameters that we use to study time-variations in the risk premia. In Section 5, we perform several robustness tests, which confirm that our findings are robust to alternative research designs.

[12] Our curve fitting approach closely follows that of Broadie et al. (2007). They experiment with several other methods, e.g. piece-wise cubic, linear and piece-wise functions and find the piece-wise quadratic (linear) interpolation to be the best when there are more (less) than 10 option prices.

[13] The choice of this interval is broad enough to cover a large range of options without being vulnerable to lightly traded deep OTM option prices, which may significantly distort the results for the jump risk premia. We also use different moneyness ranges and obtain broadly similar results.

[14] In Section 5, we consider different estimation frequencies and find similar results.

[15] It is important to point out that, from a strictly theoretical point of view, the model parameters should be constant. As we shall see, our empirical results reveal that the parameter estimates vary from one month to the next. This likely indicates that, although widely popular in the empirical option pricing literature, the benchmark SVCJ model is misspecified. We thank the referees for this remark.

[16] We are very grateful to our referees for several suggestions that helped to improve this section of the paper.

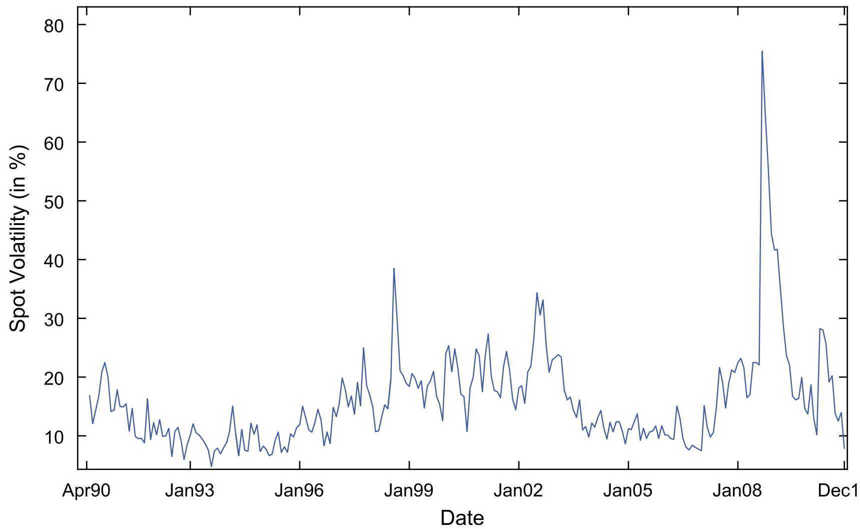

Fig. 1. Latent volatility. This figure displays the time-series of the annualized latent instantaneous volatility estimated under the physical measure. We express the volatility in percentage points per annum.

3.1. Parameter estimates

3.1.1. Physical measure

We implement the MCMC algorithm to estimate the physical parameters. Fig. 1 plots the time-series of the annualized latent instantaneous volatility, expressed in percentage. As one would expect, the instantaneous volatility peaks during the crisis of 2008.[17] Looking at the period leading up to 2008, we find that the dynamics of the calibrated volatility are similar to those presented in the top left quadrant of Fig. 11 in Ignatieva et al. (2009).[18,19]

Table 3 presents the important parameter estimates under the $\mathbb{P}$ measure. The last two columns report the coefficient estimates and standard errors for the SVCJ model, respectively.[20] As we can see most parameter estimates are statistically significant. The only exception to this is η, which is imprecisely estimated. This is consistent with the empirical literature on the elusive risk-return relationship discussed in Andersen et al. (2002). The speed of mean reversion, $\kappa^{\mathbb{P}}$, equals 0.019. The positive value of this parameter implies that the variance process is stationary. The mean-reversion level of variance, $\theta^{\mathbb{P}}$ (0.888), is not only statistically significant but also broadly consistent with the unconditional volatility reported in Table 1.[21] Turning to the dynamics of jumps, we can see that jumps in returns are rare events, that occur with a low probability (0.002). However, the average return jump size (−3.600) is significantly negative.

Table 3
MCMC parameter estimates. This table reports the parameter estimates obtained under the $\mathbb{P}$ probability measure based on S&P 500 daily percentage returns. We report the results for the SV, SVJ and SVCJ models separately. The figures in brackets are the standard errors.

	SV		SVJ		SVCJ	
η	0.004	(0.016)	0.011	(0.017)	0.004	(0.016)
$\kappa^{\mathbb{P}}$	0.016	(0.003)	0.015	(0.003)	0.019	(0.003)
$\theta^{\mathbb{P}}$	1.213	(0.144)	1.192	(0.149)	0.888	(0.102)
$\sigma^{\mathbb{P},v}$	0.152	(0.011)	0.146	(0.009)	0.133	(0.010)
$\rho^{\mathbb{P}}$	−0.684	(0.031)	−0.703	(0.032)	−0.684	(0.036)
$\lambda^{\mathbb{P}}$			0.009	(0.005)	0.0025	(0.0009)
$\mu^{\mathbb{P},s}$			−1.385	(0.671)	−3.600	(0.968)
$\sigma^{\mathbb{P},s}$			1.720	(0.305)	1.985	(0.399)
$\mu^{\mathbb{P},v}$					4.231	(1.997)

3.1.2. Risk-neutral measure

Table 4 reports the average of the monthly estimates of the speed of mean reversion, the average return jump size, the volatility of jump returns and the average variance jump size under the $\mathbb{Q}$ measure, respectively. We report the standard errors in brackets.

We can see from the last two columns that these parameter estimates are statistically significant. This is evidenced by the small standard errors and the relatively large coefficient estimates. Although the signs of the parameter estimates are consistent across probability measures, there are differences in the magnitude of these estimates. For instance, the speed of mean reversion implied by the SVCJ model is higher under $\mathbb{Q}$ than $\mathbb{P}$, implying that the DVRP is negative (since variance is positive). This result is

Table 4
Risk-neutral parameters. This table reports the parameter estimates obtained under the $\mathbb{Q}$ probability measure. We report the results for the SV, SVJ and SVCJ models. The figures in brackets are the standard errors.

	SV		SVJ		SVCJ	
$\kappa^{\mathbb{Q}}$	0.008	(0.000)	0.092	(0.005)	0.051	(0.003)
$\mu^{\mathbb{Q},s}$			−5.20	(0.269)	−9.75	(0.535)
$\sigma^{\mathbb{Q},v}$			9.23	(0.188)	$=\sigma^{\mathbb{P},v}$	
$\mu^{\mathbb{Q},v}$					21.81	(1.202)

[17] The rapid movements in the instantaneous volatility observed in September and October 2008 are interesting for two reasons. First, these large movements in the volatility will result in spikes in the dynamics of the *DSRP* and the *DVRP*. This is because variance, i.e. the squared value of the instantaneous volatility, enters the computation of these risk premia (see Eqs. (10) and (12)). Second, in order to capture these sudden and rapid movements in the dynamics of variance, the model would require very large estimates of the average jump size in the variance process. To verify this, we estimate the model using all data up to (and including) December 2007. While most parameter estimates are broadly the same, we find an average jump size in the variance process of 1.232. This is clearly smaller than the 4.231 obtained when the crisis period is included (see Table 3). We thank the reviewers for suggesting this analysis.

[18] This is the working paper version of Ignatieva et al. (2015). We refer to the working paper as the plot is not included in the published version.

[19] It is important to point out that the sample period of Ignatieva et al. (2009) starts earlier than ours and includes the crash of 1987. This makes a like for like comparison difficult to carry out.

[20] All parameter estimates are based on daily percentage returns. Thus, the parameters associated with the price dynamics are in percent, whereas the parameters associated with the variance dynamics are in percent squared.

[21] Note that the unconditional volatility of the SVCJ model is given by $\sqrt{\theta^{\mathbb{P}} + \frac{\mu^{\mathbb{P},v}\lambda^{\mathbb{P}}}{\kappa^{\mathbb{P}}}}$.

consistent with the work of Broadie et al. (2007). We find that the average jump size related to the variance process is higher under $\mathbb{Q}$ and than under $\mathbb{P}$, hinting at a positive VJRP. These observations set the scene for the detailed analysis of the risk premia that follows.

3.2. Characterizing the risk premia

Using our $\mathbb{P}$ and $\mathbb{Q}$ parameter estimates, it is straightforward to obtain the different risk premia (see Eqs. (10)–(13)). Table 5 presents summary statistics of the risk premia. On average, we can see that both the *DSRP* and *PJRP* are positive. Because these two risk premia make up the equity risk premium, this result implies a positive equity risk premium of around 5.29% per year.[22] Table 5 also allows us to ascertain which of the smooth and discontinuous components of the return process makes the most important contribution to the equity risk premium. Using the mean values shown in the table, we can easily see that the discontinuous component (*PJRP*) accounts for most (71.43%) of the equity risk premium. The top Panels of Fig. 2 show that this result holds not only in an unconditional sense but also conditionally. Indeed, we observe that the *PJRP* is generally higher than the *DSRP*, confirming that it makes a sizable contribution to the equity risk premium. This result is consistent with the work of Bollerslev and Todorov (2011).

Turning our attention to the components of the variance risk premium, we notice that the *DVRP* is on average negative (−0.052). As previously discussed, this reflects the fact that the speed of mean reversion is higher under the risk-neutral measure, a finding that is consistent with the estimates of Broadie et al. (2007) among others. The *VJRP* is on average positive (0.043). Taken together, these results indicate a negative variance risk premium on average. To better understand this result, it is helpful to study the time-series behavior of the *DVRP* and *VJRP*. The bottom Panels of Fig. 2 show that the *VJRP* is generally larger (in absolute value) than the *DVRP*, suggesting that most of the variance risk premium is essentially a compensation for jumps in the variance process. The only exception occurs in September and October 2008, when the *DVRP* takes extremely large values. This is mainly due to the dramatic increase in the instantaneous volatility displayed in Fig. 1. Because the *DVRP* depends on the squared of the latent volatility, we obtain very large values of the *DVRP*. This explains why (i) on average, the *DVRP* is larger than the *VJRP*, (ii) the skewness and kurtosis of the *DVRP* are quite large (in absolute value).

Overall, this analysis shows that the risk premia are economically large and move a lot over time. We find that jumps play an important role in the dynamics of the equity and variance risk premia. This finding carries important implications for theoretical models of asset prices. A realistic model should allow the price jump risk premium and the variance jump risk premium to account for a sizable share of the equity and variance risk premia, respectively. For instance, if one posits a long-run risk model without any jumps, the model would counterfactually imply that jumps play no role in the dynamics of the equity and variance risk premia. In other words, the equity and variance risk premia in such a model are due to smooth movements in the processes only.

3.3. Commonalities across risk premia

Up to this point, we have studied each risk premium in isolation. Naturally, one may wonder about the comovements among the different risk premia. To tackle this issue, we proceed in two steps. First, we compute the unconditional pairwise correlations between the risk premia. Second, we condition our correlation analysis on the stage of the business cycle. That is, we study the commonalities in risk premia during expansions and recessions, separately. To identify expansionary and recessionary periods, we use the NBER recession dummy downloaded from the St Louis' Federal Reserve database. Panel A of Table 6 presents the unconditional correlation between pairs of risk premia. Panels B and C report the pairwise correlations during expansions and recessions, respectively.

Table 5
Summary statistics of the risk premia. This table summarizes the statistics of the monthly risk premia. Each risk premium is computed according to Eqs. (10)–(13). "Mean" reports the average. "Median" is the median. "Std Dev" displays the standard volatility. "Min" and "Max" show the minimum and maximum, respectively. Finally, "Skew", "Kurt" and "AR(1)" show the skewness, the kurtosis and the autocorrelation coefficient of first order, respectively.

	DSRP	PJRP	DVRP	VJRP
Mean	0.006	0.015	−0.052	0.043
Median	0.004	0.014	−0.014	0.028
Std Dev	0.009	0.021	0.378	0.047
Min	0.000	−0.032	−5.820	−0.010
Max	0.101	0.130	0.035	0.298
Skew	6.463	1.279	−14.536	2.528
Kurt	55.454	7.782	220.640	11.184
AR(1)	0.734	0.735	0.214	0.294

It is worth noticing the highly positive correlation between the *DSRP* and the *PJRP*. This result is surprising because our model specification does not introduce any mechanical relationship between the two quantities. For instance, if one formulates a model with a time-varying jump intensity, and assumes that the intensity of the jump depends on the latent variance (V_t), then the *DSRP* and *PJRP* will be, by construction, correlated. This is because V_t will affect both these risk premia. However, our model assumes a constant jump intensity, thus making this result somewhat unexpected. One possible explanation is that the jump size under the risk-neutral measure is time-varying and might depend on several factors, including V_t.[23] It is thus possible that by re-calibrating the model frequently, we are able to pick up such time-variations. This might also explain why the correlation between the two risk premia is broadly the same during expansions and contractions. A similar observation emerges for the correlation between the *DVRP* and *VJRP*.[24] If the risk-neutral speed of mean reversion and the jump size in the variance process (under the risk-neutral measure) are driven by some common factors, this could result in the negative correlation between the two components of the variance risk premium. A challenge for future theoretical models consists in developing a realistic model of asset returns that is able to reproduce these facts.

4. The drivers of risk premia

Having estimated and analyzed the time-series of the risk premia, we now turn to their economic drivers. We consider the following variables.

4.1. Data

We use the following variables to capture the variations in economic growth and uncertainty.

[22] To get this figure, we add together the *DSRP* and *PJRP* and multiply the result by 252 (to annualize).

[23] We rule out a one factor model because if the risk-neutral jump size has a constant exposure to the unique factor V_t, then we would observe a perfect correlation coefficient between the *DSRP* and the *PJRP*.

[24] The reader might wonder why the correlation is stable across both subsamples but substantially lower when computed using all sample information. The reason is related to the large spike observed in the time-series of the *DVRP*.

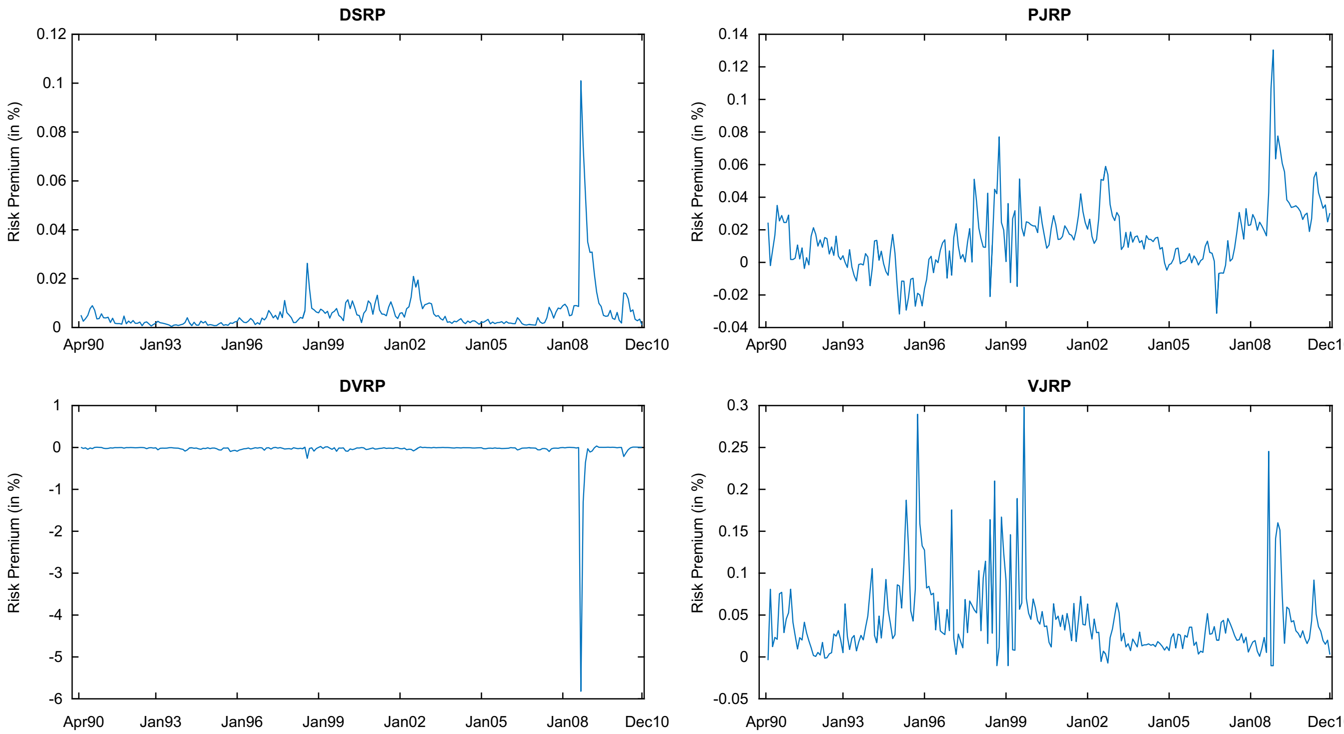

Fig. 2. The dynamics of the risk premia. This figure displays the time-series of the monthly risk premia. The top left panel displays the diffusive stock risk premium (*DSRP*). The top right panel shows the price jump risk premium (*PJRP*). The bottom left panel shows the diffusive variance risk premium (*DVRP*) and the bottom right panel shows the variance jump risk premium (*VJRP*).

Table 6
Correlations across risk premia. This table reports the pairwise correlations between risk premia. The first panel shows the results of the unconditional analysis. Panel B shows the results for the expansionary periods, as indicated by the NBER recession dummy. Panel C reports the pairwise correlations during contractions.

	DSRP	PJRP	DVRP	VJRP
Panel A: unconditional				
DSRP	1.00			
PJRP	0.63	1.00		
DVRP	0.23	−0.16	1.00	
VJRP	−0.75	−0.16	−0.29	1.00
Panel B: expansion				
DSRP	1.00			
PJRP	0.64	1.00		
DVRP	0.08	−0.30	1.00	
VJRP	−0.46	−0.01	−0.55	1.00
Panel C: contraction				
DSRP	1.00			
PJRP	0.68	1.00		
DVRP	0.48	0.13	1.00	
VJRP	−0.82	−0.19	−0.58	1.00

4.1.1. Macroeconomic expectations

We obtain macroeconomic forecasts from Blue Chip Economic Indicators (BCEI). This dataset contains a rich cross-section of macroeconomic forecasts for the current year. We focus on the growth rate of the real gross domestic product, the consumer price index and of housing starts. Together, these variables cover different facets of the economy. For each economic variable and month, we construct our proxy for macroeconomic expectations by taking the median of all forecasts. We repeat this for all months and variables to obtain a time-series of expectations of the growth rate of the real gross domestic product (RGDP), the consumer price index (CPI) and housing starts (HS).

4.1.2. Macroeconomic uncertainty

Building on the BCEI dataset, we construct forward looking measures of macroeconomic uncertainty in a manner analogous to that of Pasquariello and Vega (2007). For each month and economic variable, we compute the cross-sectional standard deviation of all forecasts and use these time series as proxies for macroeconomic uncertainty. We denote uncertainty about the growth rates of real gross domestic product, consumer price index and housing starts by URGDP, UCPI, and UHS, respectively.

4.1.3. Credit and funding risks

We supplement the macroeconomic variables with two financial variables: the default (DFSPD) and term (TSPD) spreads. To construct the DFSPD, we take the difference between the BAA and AAA bond yields. We construct the TSPD as the spread between the 10-year and 2-year Treasury yields. These data are obtained from the website of the St Louis' Federal Reserve.

4.1.4. Illiquidity

We also analyze the effect of illiquidity on the different risk premia. To proxy for illiquidity in the equity market, we implement the novel measure of Corwin and Schultz (2012). Each day, we use daily high, low and closing S&P 500 spot prices to obtain the illiquidity measure (ILLIQ). We adjust for overnight returns and negative 2-day spreads as in Corwin and Schultz (2012). We then average all the intra-month estimates to obtain a monthly measure.

4.1.5. Equity market conditions

We also use the historical mean return (RET) and volatility (VOL), computed over a trailing window of six months, as explanatory variables.

4.2. Empirical results

It is useful to examine the correlation structure of regressors (see Table 7). We can see that the pairwise correlations are (in absolute value) typically smaller than 0.5. We run both univariate and multivariate regressions of individual risk premia on the proposed explanatory variables.[25] Tables 8–11 separately report the results for each risk premium. All regressions are based on standardized variables and Newey–West corrected standard errors with 6 lags.[26]

4.2.1. Diffusive Stock Risk Premium

Table 8 shows that the economic variables account for 57% of the variations in this risk premium. We notice that the proxies for economic expectations have very little explanatory power (less than 3%). Once we introduce the proxy for economic uncertainty, the explanatory power rises substantially to 33%. We see that uncertainty about inflation has a positive and significant impact on the *DSRP*. The inclusion of TSPD, DFSPD and ILLIQ further enhances the model, as indicated by the high *Adj* R^2 (57%). We can also see that the coefficients associated with DFSPD and ILLIQ are significant. The positive coefficient estimates indicate that investors require a higher compensation for the diffusive stock risk when credit and default risks increase.

4.2.2. Price Jump Risk Premium

Table 9 shows that all variables collectively yield an *Adj* R^2 of 63%. Starting with the fourth column from the right, we can see that macroeconomic expectations explain around 15% of variations in the PJRP. RGDP and CPI enter the model with statistically significant estimates. An increase in RGDP or CPI decreases the risk premium. This result suggests that investors require a small compensation for bearing price jump risk when they are optimistic about the economy. This makes intuitive sense because improvements in economic conditions are typically associated with price jumps of smaller magnitude.

The importance of macroeconomic uncertainty is evidenced by the twofold rise in the *Adj* R^2, from 15% to 38%. Most of this increase stems from UCPI, which boasts a significantly positive coefficient estimate. Again, this result is economically sound. An increase in inflation uncertainty raises the premium investors require for their exposure to price jumps.

The last regression model documents a positive and significant relationship between historical volatility and price jump risk premium. To understand the intuition behind this result, it is important to bear in mind that jumps tend to be larger during volatile periods. Hence, one would expect investors to require a high compensation for their exposure to jump risk during particularly volatile episodes of the stock market.

4.2.3. Diffusive variance risk premium

Table 10 shows that we can explain close to 12% of the variations in the DVRP. The last four columns shed light on the contribution of each set of regressors to this explanatory power. We can see that the first three regressors, i.e. RGDP, CPI and HS, do not have a significant effect and account for less than 1% of variations in the DVRP. This result suggests that macroeconomic expectations alone cannot explain the diffusive variance risk premium satisfactorily. The next columns show the effect of macroeconomic uncertainty on the risk premium. The explanatory power increases meaningfully. This confirms that macroeconomic uncertainty plays a more important role than macroeconomic expectations.

4.2.4. Variance jump risk premium

Table 11 reports our results for the VJRP. Overall, our economic variables capture 15% of variations in the market price of variance jump risk. We can see that macroeconomic expectations contribute very little to this overall result. Indeed, the *Adj* R^2 of the regression model that includes only the three proxies for macroeconomic expectations is negligible (0.05%). When we include macroeconomic uncertainty in our regression analysis, we observe a sharp improvement in explanatory power to 8%, indicating that macroeconomic uncertainty significantly affects the market price of variance jump risk. It is also worth noticing that *UCPI* affects both the *PJRP* and the *VJRP*, suggesting that investors really care about uncertainty about future inflation.

5. Robustness

In this section, we investigate the robustness of our main findings. First, we address concerns of model misspecification by comparing the SVCJ model to two other commonly used models, i.e. the SV and SVJ models. Second, we assess the robustness of our estimation method. Third, we examine the explanatory power of sentiment for the risk premia.

5.1. Model misspecification

Our study may be criticized on the grounds that it suffers from model misspecification and that this misspecification may be mistaken for risk premia. To investigate this important issue, we estimate and compare the SV and SVJ models to the SVCJ model.

In particular, we employ the model of Heston (1993) as an alternative specification:

$$dS_t = S_t(r_t - \delta_t + \gamma_t)dt + S_t\sqrt{V_t}dW_t^{\mathbb{P},s} \tag{14}$$

$$dV_t = \kappa^{\mathbb{P}}(\theta^{\mathbb{P}} - V_t)dt + \sigma^{\mathbb{P},v}\sqrt{V_t}dW_t^{\mathbb{P},v} \tag{15}$$

As second alternative model, we employ a specification featuring only jumps in the return dynamics but not in the variance process:

$$dS_t = S_t(r_t - \delta_t + \gamma_t - \bar{\mu}^{\mathbb{P},s}\lambda^{\mathbb{P}})dt + S_t\sqrt{V_t}dW_t^{\mathbb{P},s} + d\left(\sum_{j=1}^{N_t} S_{\tau_j-}(e^{Z_j^s} - 1)\right) \tag{16}$$

$$dV_t = \kappa^{\mathbb{P}}(\theta^{\mathbb{P}} - V_t)dt + \sigma^{\mathbb{P},v}\sqrt{V_t}dW_t^{\mathbb{P},v} \tag{17}$$

Tables 3 and 4 report the parameter estimates. We now compare all three models based on their DIC scores and RMSE of option prices.[27]

5.1.1. DIC

Originally introduced by Spiegelhalter et al. (2002), the DIC belongs to the family of information criteria that includes the Akaike and Bayesian information criteria. Like other information criteria, the DIC takes into account the number of parameters and penalizes complex models. The lower the DIC score, the better the model.

[25] Given the AR(1) coefficient presented in Table 5, one may wonder whether the results are sensitive to the inclusion of the lagged value of the risk premium in the regression model. Our untabulated analysis reveals that adding the past value of the risk premium does not materially affect the performance of the regression model. Indeed, the explanatory power of the model is little changed and the lagged risk premium enters the regression model with a statistically insignificant estimate in nearly all cases. We thank the reviewers for suggesting this analysis.

[26] In order to assess the robustness of our regression results to the large observations recorded in September and October 2008, we repeat our analysis using all sample observations except those of September and October 2008. We obtained qualitatively similar results. These results are available upon request.

[27] Strictly speaking, we analyze the RMSE of implied volatilities.

Table 7
Correlation across regressors. This table reports the pairwise correlation between explanatory variables. RGDP, CPI and HS denote the expectations about real GDP, consumer price index and housing starts, respectively. URGDP, UCPI and UHS are disagreement proxies surrounding real GDP, consumer price index and housing starts, respectively. TSPD and DFSPD denote the term and default spreads, respectively. ILLIQ refers to the illiquidity proxy of Corwin and Schultz (2012). RET and VOL indicate the average and standard deviation of returns over a trailing window of six months, respectively.

	RGDP	CPI	HS	URGDP	UCPI	UHS	TSPD	DFSPD	ILLIQ	RET	VOL
RGDP	1.00										
CPI	0.21	1.00									
HS	0.20	0.05	1.00								
URGDP	−0.42	0.00	−0.04	1.00							
UCPI	−0.44	0.05	−0.14	0.58	1.00						
UHS	−0.44	0.01	−0.13	0.14	0.51	1.00					
TSPD	−0.42	−0.28	0.06	0.25	0.32	0.38	1.00				
DFSPD	−0.58	−0.25	−0.27	0.35	0.66	0.48	0.39	1.00			
ILLIQ	−0.17	−0.18	−0.10	0.27	0.45	0.15	0.15	0.63	1.00		
RET	0.21	0.00	0.27	−0.21	−0.31	−0.19	−0.20	−0.63	−0.52	1.00	
VOL	−0.32	−0.27	−0.08	0.43	0.26	0.03	0.18	0.49	0.51	−0.39	1.00

Table 8
The determinants of the diffusive stock risk premium (DSRP). This table reports the results of regressions of the diffusive stock risk premium on explanatory variables. RGDP, CPI and HS denote the expectations of real gross domestic product, consumer price index and housing starts, respectively. URGDP, UCPI and UHS refer to uncertainty around real gross domestic product, consumer price index and housing starts, respectively. TSPD and DFSPD denote the term and default spread variables, respectively. ILLIQ indicates the illiquidity proxy. RET and VOL are the average and volatility of historical returns computed over a trailing window of six months. To facilitate comparisons, we standardize all variables. T-statistics are provided in parentheses and computed based on adjusted standard errors following the method of Newey–West with 6 lags.

	Univariate											Multivariate			
RGDP	−0.00											−0.00	0.00	0.00	0.00
	(−1.51)											(−1.31)	(0.58)	(1.20)	(1.57)
CPI		−0.00										0.00	−0.00	0.00	0.00
		(−0.30)										(0.00)	(−0.93)	(0.96)	(1.01)
HS			−0.00									−0.00	−0.00	−0.00	−0.00
			(−1.09)									(−1.16)	(−1.85)	(−0.21)	(−0.64)
URGDP				0.00									−0.00	−0.00	−0.00
				(2.01)									(−1.10)	(−1.36)	(−1.44)
UCPI					0.01								0.01	0.00	0.00
					(2.26)								(2.34)	(2.28)	(2.63)
UHS						0.00							−0.00	−0.00	−0.00
						(1.16)							(−1.79)	(−2.35)	(−2.24)
TSPD							0.00							−0.00	−0.00
							(1.20)							(−0.22)	(−0.06)
DFSPD								0.01						0.00	0.00
								(3.40)						(2.00)	(3.22)
ILLIQ									0.01					0.00	0.00
									(3.25)					(3.16)	(2.96)
RET										−0.00					0.00
										(−2.68)					(0.82)
VOL											0.00				−0.00
											(2.93)				(−0.01)
Adj R^2	2.67%	−0.27%	1.18%	4.68%	30.20%	3.02%	1.41%	34.44%	47.67%	15.16%	10.54%	2.75%	33.04%	57.09%	57.03%

The first row of Table 12 reports the DIC scores of the models considered. The DIC scores of the SV, SVJ and SVCJ models are 14,672, 14,577 and 14,193, respectively. These scores suggest that, of all three models, the SVCJ model provides the best description of the stock index dynamics. It is followed by the SVJ model, which achieves the second lowest score. Finally, the SV model provides the worst fit to the data. Overall, these results are consistent with those reported in previous studies, e.g. Eraker et al. (2003) and Eraker (2004).

5.1.2. RMSE

We also examine the RMSE obtained after estimating the risk-neutral parameters (see Eq. (8)). Intuitively, the best model should minimize the squared distance between the market and model implied volatilities. The bottom row of Table 12 shows that the SV, SVJ and SVCJ models yield RMSE equal to 8.54%, 5.87% and 5.78%, respectively. This suggests that the SVCJ model outperforms its competitors. It is followed by the SVJ and SV models, which yield the second and third smallest RMSE, respectively.

Overall, this result echoes that of the DIC scores. The SVCJ model best describes the dynamics of the stock and options data. This result implies that our benchmark model, the SVCJ, is the least likely to suffer from model misspecification risk.

5.2. Estimation approach

To check the robustness of our results with respect to the estimation frequency, we repeat our analysis at the quarterly frequency. Each quarter, we use option prices to estimate the risk-neutral parameters. Our (unreported) results are very similar. In particular, the quarterly average parameter estimates amount to 0.05, −9.95 and 20.37 for $\kappa^{\mathbb{Q}}, \mu_s^{\mathbb{Q}}$ and $\mu_v^{\mathbb{Q}}$, respectively. This is very similar to 0.05, −9.75 and 21.81 obtained at the monthly frequency.[28]

Second, one might also wonder what would happen if we jointly estimated the latent variance V_t and the $\mathbb{Q}$ parameters in

[28] Detailed results are available upon request.

Table 9
The determinants of the price jump risk premium (PJRP). This table reports the results of regressions of the price jump risk premium on explanatory variables. RGDP, CPI and HS denote the expectations of real gross domestic product, consumer price index and housing starts, respectively. URGDP, UCPI and UHS refer to uncertainty around real gross domestic product, consumer price index and housing starts, respectively. TSPD and DFSPD denote the term and default spread variables, respectively. ILLIQ indicates the illiquidity proxy. RET and VOL are the average and volatility of historical returns computed over a trailing window of six months. To facilitate comparisons, we standardize all variables. T-statistics are provided in parentheses and computed based on adjusted standard errors following the method of Newey–West with 6 lags.

	Univariate											Multivariate			
RGDP	−0.01											−0.00	0.00	0.00	0.00
	(−2.57)											(−2.33)	(0.65)	(1.50)	(1.64)
CPI		−0.01										−0.01	−0.01	−0.00	−0.00
		(−2.74)										(−2.28)	(−4.79)	(−3.02)	(−3.31)
HS			−0.00									−0.00	−0.00	−0.00	−0.00
			(−0.94)									(−0.90)	(−1.55)	(−0.91)	(−0.81)
URGDP				0.01									0.00	0.00	−0.00
				(3.29)									(0.76)	(1.48)	(−0.02)
UCPI					0.01								0.01	0.00	0.00
					(3.96)								(2.27)	(1.17)	(2.26)
UHS						0.01							0.00	0.00	0.00
						(2.04)							(0.46)	(0.63)	(1.20)
TSPD							0.01							0.00	0.00
							(2.66)							(1.16)	(1.33)
DFSPD								0.01						0.00	0.00
								(7.69)						(3.03)	(0.50)
ILLIQ									0.01					0.01	0.01
									(8.06)					(7.61)	(6.36)
RET										−0.01					−0.00
										(−3.66)					(−1.10)
VOL											0.01				0.01
											(5.92)				(4.46)
Adj R^2	8.69%	8.62%	1.50%	13.09%	27.69%	7.54%	9.85%	42.51%	46.29%	22.71%	34.07%	14.46%	38.03%	58.74%	63.19%

Table 10
The determinants of the diffusive variance risk premium (DVRP). This table reports the results of regressions of the diffusive variance risk premium on explanatory variables. RGDP, CPI and HS denote the expectations of real gross domestic product, consumer price index and housing starts, respectively. URGDP, UCPI and UHS refer to uncertainty around real gross domestic product, consumer price index and housing starts, respectively. TSPD and DFSPD denote the term and default spread variables, respectively. ILLIQ indicates the illiquidity proxy. RET and VOL are the average and volatility of historical returns computed over a trailing window of six months. To facilitate comparisons, we standardize all variables. T-statistics are provided in parentheses and computed based on adjusted standard errors following the method of Newey–West with 6 lags.

	Univariate											Multivariate			
RGDP	0.01											0.02	−0.01	0.01	0.00
	(0.55)											(0.68)	(−0.81)	(0.40)	(0.05)
CPI		−0.04										−0.05	−0.03	−0.05	−0.06
		(−1.04)										(−1.00)	(−1.03)	(−1.39)	(−1.42)
HS			0.01									0.01	−0.00	−0.00	0.01
			(1.29)									(0.99)	(−0.33)	(−0.09)	(0.50)
URGDP				−0.01									0.06	0.07	0.05
				(−0.81)									(1.71)	(1.82)	(1.90)
UCPI					−0.09								−0.15	−0.11	−0.08
					(−1.53)								(−1.94)	(−2.31)	(−2.89)
UHS						−0.03							0.03	0.02	0.02
						(−1.00)							(1.74)	(1.29)	(1.61)
TSPD							−0.01							−0.02	−0.02
							(−0.74)							(−0.81)	(−0.92)
DFSPD								−0.06						0.05	−0.01
								(−1.28)						(0.70)	(−0.17)
ILLIQ									−0.11					−0.11	−0.13
									(−1.40)					(−1.28)	(−1.33)
RET										0.02					−0.06
										(1.15)					(−1.28)
VOL											−0.01				0.02
											(−0.72)				(1.12)
Adj R^2	−0.36%	0.83%	−0.38%	−0.27%	5.61%	0.33%	−0.25%	2.27%	7.57%	−0.04%	−0.32%	0.26%	6.90%	10.96%	11.57%

the second step, rather than directly using the latent variance estimates provided by the MCMC algorithm.[29] We address this question by following two distinct empirical methodologies. Our first approach (Method 1) resembles the baseline optimization approach used in the main body of our paper. We use our MCMC spot variance to repeat the calibration described by Eq. (8) each year. We then average the yearly parameters. The second approach (Method 2) mirrors that of Kaeck (2013). Each year, we use the options data to *jointly* estimate the $\mathbb{Q}$ parameters and latent spot variance. We then compute the average, across all years, of the parameter estimates. If our approach is robust, Methods 1 and 2 should yield similar estimates.

Table 13 shows that there is very little to distinguish between the two sets of results. This demonstrates that our main findings are robust to the estimation methodology.

[29] Theoretically, the values should, of course, be identical. Practically, however, V_t is estimated so different estimation approaches may yield slightly different results.

Table 11
The determinants of the variance jump risk premium (VJRP). This table reports the results of regressions of the variance jump risk premium on explanatory variables. RGDP, CPI and HS denote the expectations of real gross domestic product, consumer price index and housing starts, respectively. URGDP, UCPI and UHS refer to uncertainty around real gross domestic product, consumer price index and housing starts, respectively. TSPD and DFSPD denote the term and default spread variables, respectively. ILLIQ indicates the illiquidity proxy. RET and VOL are the average and volatility of historical returns computed over a trailing window of six months. To facilitate comparisons, we standardize all variables. T-statistics are provided in parentheses and computed based on adjusted standard errors following the method of Newey–West with 6 lags.

	Univariate											Multivariate			
RGDP	0.00											0.00	0.00	0.00	0.00
	(0.18)											(0.62)	(0.95)	(0.30)	(0.35)
CPI		−0.00										−0.00	−0.00	−0.01	−0.01
		(−1.23)										(−1.33)	(−1.69)	(−3.85)	(−3.59)
HS			−0.00									−0.00	−0.00	−0.00	−0.00
			(−0.90)									(−1.18)	(−1.36)	(−0.28)	(−0.46)
URGDP				0.01									0.01	0.01	0.01
				(2.28)									(2.07)	(2.77)	(2.60)
UCPI					0.00								0.01	0.01	0.01
					(1.05)								(1.97)	(2.15)	(1.45)
UHS						−0.01							−0.01	−0.01	−0.01
						(−2.19)							(−3.54)	(−3.06)	(−3.25)
TSPD							−0.01							−0.02	−0.02
							(−2.97)							(−5.07)	(−4.99)
DFSPD								0.00						0.00	0.01
								(0.12)						(0.67)	(1.18)
ILLIQ									0.00					−0.01	−0.00
									(0.28)					(−1.03)	(−0.64)
RET										0.00					0.00
										(0.33)					(0.68)
VOL											0.00				−0.00
											(0.51)				(−0.75)
Adj R^2	−0.38%	0.39%	−0.10%	2.44%	0.20%	3.13%	5.92%	−0.38%	−0.31%	−0.24%	−0.18%	0.05%	7.72%	15.53%	15.29%

Table 12
Model selection. This table reports the DIC scores and RMSE for the SV, SVJ and SVCJ models, respectively.

	SV	SVJ	SVCJ
DIC	14,672	14,577	14,193
RMSE	8.54%	5.87%	5.78%

Table 13
Sensitivity to the spot variance estimates. This table reports the parameter estimates obtained following two distinct strategies. Method 1 uses the MCMC spot variance to estimate the $\mathbb{Q}$ parameters. Method 2 uses the options data to jointly estimate the $\mathbb{Q}$ parameters and spot variance. We estimate the $\mathbb{Q}$ parameters each year. We then compute the average of the yearly estimates across all years to obtain the results presented below.

	Method 1	Method 2
$\kappa^{\mathbb{Q}}$	0.044	0.048
$\mu^{\mathbb{Q},s}$	−8.839	−8.851
$\mu^{\mathbb{Q},v}$	18.860	17.504

5.3. The role of sentiment

Han (2008) documents a significant relationship between investor sentiment and option prices. This study motivates us to investigate the role of market sentiment on the risk premia. We exploit the sentiment index of Baker and Wurgler (2006). More specifically, we obtain the time-series of the change in the sentiment index from Wurgler's website.

We regress individual risk premia on our macroeconomic variables as well as the sentiment variable of Baker and Wurgler (2006). Table 14 shows that the change in the sentiment proxy has a statistically significant impact on the price jump risk premium only. The sign of the coefficient estimate is also very intuitive. We can see that an improvement in sentiment has a negative impact on the price jump risk premium and thus results in a lower equity risk premium. This result is consistent with the empirical evidence of Schmeling (2009) and suggests that sentiment affects the equity risk premium mainly through the jump channel.

Table 14
Controlling for sentiment. This table reports the results of regressions of the risk premia on a constant, the economic variables and a proxy for sentiment (*BW*). To facilitate comparisons, we standardize all variables. All standard errors are adjusted following the method of Newey–West with 6 lags.

	DSRP	PJRP	DVRP	VJRP
RGDP	0.00	0.00	0.00	0.00
	(1.57)	(1.63)	(0.03)	(0.39)
CPI	0.00	−0.00	−0.07	−0.01
	(1.00)	(−3.38)	(−1.43)	(−3.59)
HS	−0.00	−0.00	0.01	−0.00
	(−0.64)	(−0.91)	(0.48)	(−0.37)
URGDP	−0.00	0.00	0.05	0.01
	(−1.43)	(0.04)	(1.96)	(2.53)
UCPI	0.00	0.00	−0.08	0.01
	(2.64)	(2.21)	(−3.02)	(1.43)
UHS	−0.00	0.00	0.02	−0.01
	(−2.28)	(1.14)	(1.58)	(−3.19)
TSPD	−0.00	0.00	−0.02	−0.02
	(−0.05)	(1.26)	(−0.94)	(−4.96)
DFSPD	0.00	0.00	−0.01	0.01
	(3.21)	(0.44)	(−0.21)	(1.19)
ILLIQ	0.00	0.01	−0.13	−0.00
	(3.00)	(6.43)	(−1.36)	(−0.68)
RET	0.00	−0.00	−0.06	0.00
	(0.80)	(−1.19)	(−1.30)	(0.80)
VOL	0.00	0.01	0.02	−0.00
	(0.03)	(4.60)	(1.06)	(−0.68)
BW	0.00	−0.00	−0.04	0.00
	(0.33)	(−2.21)	(−1.17)	(1.63)
Adj R^2	56.88%	64.41%	12.17%	15.96%

6. Conclusion

In this paper, we study the risk premia embedded in the S&P 500 spot and option markets. We find that the market prices of risks are significant and economically large. We document substantial time-variations in the risk premia. We decompose the equity and variance risk premia into their smooth and discontinuous components. We find that jumps play an important role in the dynamics of these risk premia.

Using several economic variables, we investigate the drivers of these time-variations. We are able to explain a sizable share of variations in the risk premia. Our analysis reveals that proxies of macroeconomic uncertainty capture much more variations in the risk premia than macroeconomic expectations.

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