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When Should Firms Expose Themselves to Risk?

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I analyze a firm making a decision whether to expose itself to risk in an exogenous parameter when the firm can change a choice variable after observing the realization of the exogenous parameter. For example, the firm decides whether to choose an advertising campaign with a less certain outcome (conditional on the same expected outcome) when it can adjust the product's price after seeing the effects of the campaign. I show that in many cases, the firm wants to expose itself to risk, and I outline general conditions that need to be satisfied for this result. I then analyze the strategic version of this setup with two competing firms, provide a general characterization, and show that in many cases both firms want to expose themselves to risk, as long as the risks are not too positively correlated. This is the case for many linear demand and constant marginal cost settings (monopolies, differentiated Bertrand firms, or differentiated Cournot firms selling substitutes) where the exogenous parameter is a demand, or a marginal cost shifter results in the monopoly (or both of the competitors if the risks are not too positively correlated) voluntarily exposing itself to risk.

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1. Introduction

Modern managers receive conflicting advice about managing risk (I use "risk" and "uncertainty" interchangeably throughout the paper). On one hand, different kinds of diversification are common business practices, and hedging commodity costs and currency fluctuations are a part of every firm's risk management arsenal. On the other hand, riskier behavior is often encouraged, and the fact that a good outcome can catapult a firm into an industry-leading position is acknowledged. Popular culture and proverbs are not of much help in this case, either: should a manager be "better safe than sorry," or should she realize that "nothing ventured is nothing gained"?[1]

I shed some light on this seeming contradiction. I consider the following setting: a risk-neutral firm finds out an exogenous parameter value, and then the firm chooses the optimal value of a choice variable. The exogenous parameter could be, among other things, the realization of marginal cost or inflation this period, the degree of success (or failure) of an advertising campaign or a research and development (R&D) expense, or the current exchange rate that affects the firm's marginal cost and/or the demand that the firm is facing. The choice variable could be, among other things, the price of the product, the quantity of the product produced, or the amount spent on (potentially, the next round of) advertising or R&D.

The main question is whether a firm wants to expose itself to risk in the exogenous parameter: in other words, does the firm want to receive for certain the expected value of the parameter or the actual realization, whatever the realization might be? There are two effects of exposure to this type of risk. The first effect is effectively an option value. If the advertising campaign goes above and beyond expectations, the firm can increase its retail price, resulting in an even higher profit than otherwise. If the advertising campaign is unsuccessful, the firm can decrease the retail price to make up for some of the adverse effects, resulting in a higher profit than the one without the price decrease. In either case, the flexibility in price results in an option value of having a stochastic advertising outcome as opposed to a certain one.

The second effect depends on the shape of the profit function. The profit function might be really concave in advertising outcome, and just as that would make a consumer risk averse, that makes even a risk-neutral firm effectively risk averse. Thus, the firm needs to decide whether the option value is sufficiently large to

[1] See, for example, Fisher and Kumar (2010) for a discussion of different hedging strategies and Buehler and Pritsch (2003) for the encouragement of risky behavior. Interestingly, both discussions were published in the same journal (the *McKinsey Quarterly*), but the more recent article does not even mention the themes of the older one. Note that the two idioms can be traced back to, respectively, the 19th-century Irish novelist Samuel Lover and the 14th-century poet Geoffrey Chaucer. The coexistence of both kinds of idioms is not particular to English.

offset the concavity. Overall, I find that if the determinant of the Hessian matrix of the profit function—as a function of the parameter and the choice variable—is negative, then, and only then, should the firm prefer riskier choices.

To extend my results to strategic competition, I analyze a two-stage game between two firms. In the first stage, both firms decide whether to expose themselves to risk. In the second stage, having observed the first-stage decisions and the random draws (which might not matter at that point), both firms choose their endogenous variables. For example, consider General Motors (GM) and Toyota. Exchange rate fluctuations affect both companies' effective marginal costs (I abstract from all other aspects—for example, the effect of exchange rates on demand and any supply chain concerns). To minimize their exposure to risk, both firms can locate in both countries. Thus, the game is as follows: In stage 1, each firm decides whether its factories will be in one country or in both countries. In stage 2, given stage 1 decisions and the exchange rate realization, firms decide the prices that they charge or quantities that they produce.

Strategic competition adds several wrinkles to the relatively straightforward option value monopoly model. Now, exposing a firm to risk also implies that my rival's strategy is going to change to best reply to my choices of the endogenous variable (and possibly the exogenous shock directly as well). In addition, when both rivals are deciding whether to expose themselves to risk, the familiar Hotelling (1929) and Motta (1993) dynamics come into play: being as far apart from one's rival as possible generally means higher profits for the industry, and drawing different shocks of the exogenous parameter might help with endogenous differentiation. On a more abstract level, all these concerns manifest themselves in the relative convexity/concavity of the profit function, with respect both to the realization of my own exogenous shock and to the realization of my competitor's exogenous shock, while accounting for the strategic incentives when both firms choose their endogenous responses to these shocks.

I find that, in a simplified model where the second-stage competition is differentiated Bertrand or differentiated Cournot with linear demand and linear cost, a setup such as this with negative covariance (e.g., a shock that is good for GM tends to coincide with a shock that is bad for Toyota, and vice versa) results in both firms exposing themselves to risk in equilibrium. In addition to the same incentives to expose themselves to risk as in the monopoly case, the firms want to be differentiated, even if this differentiation results in a given firm losing some market share and profits in a given bad draw—the firm more than makes up for it in expectation. Exposing themselves to negatively correlated shocks makes it more likely that firms will be differentiated.

On the other hand, if the shocks are positively correlated and both firms are exposed, that makes differentiation harder. If the shocks are sufficiently close to being perfectly positively correlated, then this strategic incentive might overcome the monopoly option value incentive, and the firms play an equilibrium where either one of the competitors exposes itself to risk or both competitors play a mixed strategy. In addition, there will be market settings where firms do not want to differentiate themselves, and they would prefer to get similar draws. For example, if the game is a Stackelberg manufacturer–retailer interaction, then firms would prefer the shocks to be positively correlated, which reverses the intuition described above. I analyze these trade-offs and, using the results of Vives (2009) on strategic substitutes and strategic complements in Markov equilibria, provide conditions for when firms prefer to be differentiated and when they do not.

Another interesting application is the ubiquitous contracts that lock in a particular price for a period of time—for example, an airline securing fuel supply for the upcoming year. Although one would think that these contracts are written with a particular quantity specified (making it not the relevant marginal cost), this is actually not always the case: there is often no quantity specified in the contract, not even a maximum quantity (that would make the contract a call option). At least in the United States, the seller is protected by state laws, largely U.C.C. §§2-103(1)(b) and 2-306 of the relevant state's Uniform Commercial Code, against the buyer who would use this contract not in good faith.[2]

The closest existing treatment of the subject is Oi (1961) and the stream of literature that followed. Oi finds that if a firm is in a perfectly competitive market and has an increasing marginal cost, then random fluctuations in the market price increase the firm's profit. Mas-Colell et al. (1995) extend the result of Oi to the netput vector but still require perfect competition and a convex production set and thus nonincreasing returns to scale (see their Proposition 5.C.1). Farrell et al. (2003) document the fact that a monopolist firm's profit is convex in its (constant) marginal cost and use this to figure out the optimal portfolio of the firm's R&D investments and to compare the social planner's risk-lovingness to the monopolist's risk-lovingness. (Other research about R&D shows a

[2] "Not in good faith," in this case, would include purchasing fuel for resale when the price on the market is much higher than the locked-in contract price. See, for example, *Eastern Air Lines, Inc. v. Gulf Oil Corp.*, 415 F. Supp. 429 (S.D. Fla. 1975).

similar risk-seeking incentive in a duopoly setting for the firm that is behind; see, for example, Cabral 2003 and Hörner 2004.)

Profit is convex in all the papers above as a result of exactly the same mechanism as in §2 of this paper: the optimal profit (after plugging in the optimal quantity) is convex in the market price, and the firm can adjust the quantity. These results are also reminiscent of the consumer theory results and consumers' indirect utility being convex in prices. This is natural, since both lines of results are applications of the envelope theorem (either Hotelling's lemma or Shephard's lemma). Similar to the results of Oi (1961), consumer theory results concentrate on prices/quantities and quasilinear preferences.

In §2, I extend the proposition of Mas-Colell et al. (1995) from perfect competition to monopoly, and I extend the work of Farrell et al. (2003) to any parameter (not just marginal cost), and a general shape of the profit function (not just linear cost), while showing that exposing oneself to risk is not always better. In §3, I allow for strategic competition where both firms decide whether to expose themselves and the competitor to possibly correlated cost shocks, bringing in intuition and techniques from the strategic substitutes and complements intuition of Bulow et al. (1985) as well as multivariate risk aversion literature following Karni (1979).

Note also that the type of exposure to risk that I describe in this paper does not preclude the firm from financially hedging, since financial hedging generally has a fixed quantity written into the contract (whether of a commodity or of a foreign currency), and the firm's incentives at the margin are unchanged. It is often pointed out in the finance literature that managers should not be risk averse on behalf of their shareholders—shareholders can themselves diversify their holdings to mitigate risk.[3] However, there are still many reasons for a manager to smooth a firm's cash flows—for example, tax convexities or liquidity concerns. In this paper, I abstract from the reasons for a firm to make a financial hedge that does not affect the marginal incentives. Because I am interested in the marginal effects on the current profit, my theory is a complement to the financial hedging literature, since managers can easily accomplish both: financial hedging does not change marginal incentives.

Another way that firms reduce their risk exposure is operational hedging. For example, a firm can organize its supply chain so that the decision of which color the product will be is made last, and thus the firm is able to respond to any color-related demand shocks. See Chod et al. (2010) for an analysis of when operational and financial hedging are substitutes or complements. In this paper, I effectively assume that the supply chain is already perfectly flexible: it does not cost anything for a firm to adjust, say, its price and production after observing the marginal cost realization. Thus, this paper is a complement and, effectively, a third way to look at hedging. Of course, at least some degree of operational flexibility is a necessary condition to even consider my problem. Operational hedging literature also examines the strategic incentives of firms, but those are generally of the Stackelberg commitment variety; see Anand and Girotra (2007). A firm that is not operationally flexible is committed to produce a given quantity regardless of the shock realization, effectively making the flexible competitor a Stackelberg follower. On the other end of the spectrum is the literature following Leland (1972) examining the question of demand and cost uncertainty when the firm has to set the prices before the uncertainty is resolved or, in other words, the firm that has no operational flexibility at all.

2. Monopoly

2.1. Example

Consider a monopolist firm that faces a demand of $D(p) = a - bp$, where a and b are positive. Suppose that the firm's marginal cost is distributed according to some distribution, $c \sim f(\cdot)$. For expositional ease, assume that $c < a/b$ or, in other words, that some production is always efficient. The question is whether the firm prefers marginal cost to fluctuate according to this distribution or whether it is more profitable for the firm to get the expected value of the marginal cost (define $\bar{c} \equiv E(c)$) for sure instead. For any marginal cost c, the firm's profit is $\pi(p, c) = (p - c)D(p)$. Note that, since the optimal price that the firm charges depends on its marginal cost, $E[\pi(p, c)] \neq \pi(p, \bar{c})$. The question thus becomes, when is $E[\pi(p, c)] > \pi(p, \bar{c})$?

If the firm chooses to expose itself to risk, then for any realization of marginal cost, say, c, the firm charges $p^*(c) = (a + bc)/(2b)$, and the maximum profit as a function of c is then (after some algebra) $\Pi(c) = (a - bc)^2/(4b)$, where $\Pi(c) \equiv \pi(p^*(c), c)$.

Calculating the expected value,

$$E[\Pi(c)] = E\left[\frac{(a-bc)^2}{4b}\right] = \frac{a^2 - 2b\bar{c} + b^2E[c^2]}{4b} \geq \frac{a^2 - 2b\bar{c} + b^2\bar{c}^2}{4b} = \Pi(\bar{c}), \tag{1}$$

where the last inequality is strict as long as c has a nonzero variance.

[3] Smith and Stulz (1985, p. 391) state, "Although this literature [about hedging because of risk aversion] provides a useful basis for the analysis of hedging in closely-held corporations, partnerships, or individual proprietorships, it is not as applicable to large, widely-held corporations whose owners, the stockholders and bondholders, have the ability to hold diversified portfolios of securities."

Thus, a monopoly with linear cost prefers to expose itself to risk in marginal cost because of the option value of price adjustment. Note that this setup is equivalent to a setup where the uncertainty is in the demand intercept: $\pi(p, \epsilon) = (p - c)D(p + \epsilon)$, and ϵ is the random variable that would correspond to stochastic parallel shifts in demand. The demand shocks could be due to the random changes in the effectiveness of advertising, different outcomes of R&D, or exchange rate fluctuations, to list a few reasons.

Note that if the assumption of $c < a/b$ (some production is always efficient) is not satisfied, then exposing the firm to risk becomes even more profitable in expectation. Consider a limiting case: $\bar{c} = a/b$. In this case, the mean of the distribution is a marginal cost such that the optimal production is zero. Thus, if the monopoly hedges, then it earns a profit of zero every period. However, if the monopoly exposes itself to risk, then it earns a positive profit whenever the realization of marginal cost is below the mean and zero profit when the realization of marginal cost is above the mean (by not producing at all).

2.2. General Model and Solution

A risk-neutral firm is a monopoly and has a profit function $\pi(x, a)$. The firm observes a, an exogenously given parameter, and then chooses x to maximize its profit. I assume that the firm's profit function is twice differentiable in both x and a and is concave in x so that the unique maximum could be found by using the first-order condition. Suppose that $x^*(a)$ is the profit-maximizing action for a given outcome a. Define $\Pi(a) \equiv \pi(x^*(a), a)$.

Extending the example above to this much more general setup is the same as asking when $E[\Pi(a)] > \Pi(E[a])$. For a smooth $\Pi(\cdot)$, which I assume from now on, the answer is straightforward: when $\Pi(\cdot)$ is convex.

Proposition 1. *The firm chooses to expose itself to risk in a if and only if $\Pi(a)$ is convex, and $\Pi(a)$ is convex if and only if the determinant of the Hessian of $\pi(x, a)$ is negative.*

Proof. The first claim is trivial. For the second claim, by the envelope theorem, $d\Pi/da = \partial\pi/\partial a$.

Differentiating with respect to a again, and then plugging in the pass-through rate $\partial x^*(a)/\partial a$, derived by totally differentiating the firm's first-order condition, results in

$$\frac{d^2\Pi}{da^2} = \frac{\partial^2\pi}{\partial a^2} + \frac{\partial^2\pi}{\partial x\partial a}\frac{\partial^2\pi/(\partial x\partial a)}{-\partial^2\pi/\partial x^2}$$
$$= \frac{-(\partial^2\pi/\partial x^2)(\partial^2\pi/\partial a^2) + (\partial^2\pi/\partial x\partial a)^2}{-\partial^2\pi/\partial x^2}. \quad \square \tag{2}$$

Consider adding noise to x—say, $x + \epsilon$, where ϵ is random with mean zero. This noise can be either artificial (e.g., manager intentionally miscommunicating to employees) or inadvertent (e.g., genuine communication difficulties). In either case, given that profit is concave in x, the determinant of the Hessian will not be negative and $\Pi(\epsilon)$ is not convex. Thus, this noise is guaranteed to backfire: the firm does not want to introduce any artificial noise and wants to stop any inadvertent noise that is added to the choice variable. This intuition also extends the work of Kirstein (2009), who makes a similar point, to a general profit function with any shock and choice variable from his linear demand and constant marginal cost, shock in quantity, and quantity as the choice variable setting.

I have assumed that both the exogenous variable a and the endogenous variable x are scalars. Extending the proposition to a vector of endogenous variables is straightforward. However, extending the proposition to a vector of exogenous variables is more complicated. To do that, one would need to account for covariances between different dimensions of vector a and deal with multivariate risk aversion, as in Karni (1979). We now elaborate on the multivariable approach.

3. Duopoly

3.1. Model and Solution

Consider duopolist firms, firm 1 and firm 2, that choose x_1 and x_2, respectively. For simplicity, assume that both x variables are real scalars. Also, suppose that there exist two stochastic (real scalar) parameters, a_1 and a_2. Denote the parameters' means, standard deviations, covariance, and correlation as $\bar{a}_i$, σ_i, σ_{12}, and ρ_{12}, respectively. The firms' profits can be expressed as $\pi^i(a_i, a_j, x_i, x_j)$.

The firms play a two-stage game. In stage 1, firms simultaneously and noncooperatively decide whether firm i prefers to expose itself to risk in the exogenous parameter a_i. In stage 2, firms observe each other's decisions and the realizations of parameters (note that these will not matter if each firm decided not to expose itself to risk). Then, each firm simultaneously chooses its x. Note, for example, that while firm 1 chooses whether to expose itself to risk in a_1, it also chooses whether to expose firm 2 to risk in a_1, either directly, if a_1 directly enters firm 2's profit function, or indirectly, through x_1.

Assume that, given stage 1, there exist, unique for each player, optimal strategies $x_i^*(a_i, a_j)$. Define $\Pi^i(a_i, a_j) \equiv \pi^i(a_i, a_j, x_i^*(a_i, a_j), x_j^*(a_i, a_j))$. Then, the game can be expressed as a standard 2×2 game with the strategies and payoffs as presented in Table 1.

Table 1 The Payoff Matrix in Duopoly

	Do not expose to risk in a_2	Expose to risk in a_2
Do not expose to risk in a_1	$\Pi^1(\bar{a}_1, \bar{a}_2); \Pi^2(\bar{a}_1, \bar{a}_2)$	$E[\Pi^1(\bar{a}_1, a_2)]; E[\Pi^2(\bar{a}_1, a_2)]$
Expose to risk in a_1	$E[\Pi^1(a_1, \bar{a}_2)]; E[\Pi^2(a_1, \bar{a}_2)]$	$E[\Pi^1(a_1, a_2)]; E[\Pi^2(a_1, a_2)]$

3.1.1. Should Firm i Expose Itself to Risk If Firm j Does Not Expose Itself to Risk? Conditional on firm j not exposing itself to risk, firm i chooses to expose itself to risk if and only if $E[\Pi^i(a_i, \bar{a}_j)] > \Pi^i(\bar{a}_i, \bar{a}_j)$. Similarly to Proposition 1, this condition is satisfied if and only if Π^i is convex in a_i; however, now Π^i is also a function of firm j's strategic reply and includes those interaction terms. If this condition is satisfied, I denote it by $\Pi^i_{a_i a_i} > 0$: Π^i is the profit of player i as a function of the realization of the stochastic parameters after accounting for all the strategic effects in stage 2, and variables in the subscript denote differentiation.

3.1.2. Should Firm i Expose Itself to Risk If Firm j Exposes Itself to Risk? Conditional on firm j exposing itself to risk, firm i chooses to expose itself to risk if and only if $E[\Pi^i(a_i, a_j)] > E[\Pi^i(\bar{a}_i, a_j)]$. The additional risk of exposing oneself to a stochastic a_i is not only the risk associated with the stochastic a_i but also the risk associated with the interaction between the stochastic a_i and the stochastic a_j. Thus, techniques from the multivariable risk aversion literature (see Karni 1979 and Finkelshtain and Chalfant 1993) must be utilized. Just as the Arrow–Pratt premium is a first-order approximation of the risk aversion premium, one can derive a similar premium for the bivariate version:[4]

$$\Psi_i = -\frac{1}{2}\sigma_i^2 \frac{\Pi^i_{a_i a_i}}{\Pi^i_{a_i}} - \sigma_{12}\frac{\Pi^i_{a_i a_j}}{\Pi^i_{a_i}}. \tag{3}$$

The first term is the standard Arrow–Pratt index, and the second term captures the interaction between a stochastic a_i and a stochastic a_j. Just like in the standard Arrow–Pratt index, the sign of the first derivative in the denominator is a normalization, and since we are interested only in the sign of the premium (as opposed to the level), we might as well work with

$$\hat{\Psi}_i \equiv -\frac{1}{2}\sigma_i^2 \Pi^i_{a_i a_i} - \sigma_{12}\Pi^i_{a_i a_j}. \tag{4}$$

Firm i chooses to expose itself to the additional risk if and only if (iff) $\hat{\Psi}_i < 0$. It is clear that, in the one-dimensional case, this condition is equivalent to $\Pi^i_{a_i a_i} > 0$, the condition that describes both Proposition 1 and §3.1.1. In this case, the analogous condition is that firm i chooses to expose itself to additional risk iff

$$(-\Pi^i_{a_i a_j})\rho_{12} < \frac{1}{2}\frac{\sigma_i}{\sigma_j}\Pi^i_{a_i a_i}. \tag{5}$$

Note that $\Pi^i_{a_i a_i} > 0$ is neither necessary nor sufficient for (5) to hold. Thus, it is possible for firm i to choose to expose itself to risk if firm j does but to not expose itself to risk if j does not, and vice versa, depending on the correlation between the risks.

3.1.3. Classifying the Equilibria. Bringing together all the previous results from this section, the equilibria can be classified as follows.

Proposition 2. *Several different equilibria are possible in the game above, depending on the parameters.*

- *Case* 1. *If $\hat{\Psi}_i < 0$ and $\hat{\Psi}_j < 0$, then there exists an equilibrium where both firms choose to expose themselves to risk; moreover, this equilibrium is unique if $\Pi^i_{a_i a_i} < 0$ or $\Pi^j_{a_j a_j} < 0$.*
- *Case* 2. *If $\Pi^i_{a_i a_i} \le 0$ for both firms, then there exists an equilibrium where both firms choose not to expose themselves to risk; moreover, this equilibrium is unique if $\hat{\Psi}_i > 0$ and/or $\hat{\Psi}_j > 0$.*
- *Case* 3. *If $\Pi^i_{a_i a_i} > 0$ and $\hat{\Psi}_j > 0$, then there exists an equilibrium where firm i exposes itself to risk but firm j does not; moreover, this equilibrium is unique either if $\hat{\Psi}_i < 0$ or if $\Pi^j_{a_j a_j} \le 0$.*
- *Case* 4. *In cases that do not fall under the unique equilibria clauses above, there also exist equilibria in the opposite corner of the 2×2 table and a mixed strategy equilibrium where both firms randomize between risky and safe.*

Proof. See the previous subsections. □

3.2. Linear Demands and Constant Marginal Costs

The conditions in the proposition above would be easier to interpret if we knew the signs of $\Pi^i_{a_i a_j}$ and, to a lesser extent, $\Pi^i_{a_i a_i}$.

Proposition 3. *Suppose that the second stage in the game described above is either a differentiated Bertrand or a differentiated Cournot competition of substitute products with linear demand and linear cost. Suppose that the stochastic parameters are either each firm's demand intercept or each firm's marginal cost. Then,*

- *$\Pi^i_{a_i a_j} < 0$ and $\Pi^i_{a_i a_i} > 0$.*

[4] Just like the standard univariate risk premium, the risk premium in Finkelshtain and Chalfant (1993) is derived by using a Taylor series approximation. In the univariate case, the risk premium, Ψ, is derived from equation $E(u(w, a)) = u(w - \Psi)$, where u is the utility function, w is the initial wealth, and a is the risk realization. The derivation then utilizes a Taylor series approximation to arrive at the standard Arrow–Pratt premium. In the multivariate case, the risk premium is derived from a similar equation $E(u(w, a)) = E(u(\bar{w} - \Psi, a))$, with the exceptions that a is now a vector and w can potentially be a random variable; thus, a Taylor series expansion involves covariance terms between various dimensions of a (and, possibly, w). See Karni (1979) and the appendix of Finkelshtain and Chalfant (1993) for more.

• *If the stochastic parameters are negatively correlated or uncorrelated, both firms expose themselves to risk. If correlation is positive, but the standard deviations are sufficiently different, then one of the firms (the one whose stochastic parameter has a higher standard deviation, all else equal) chooses to expose itself to risk, but the other firm does not expose itself to risk. If correlation is sufficiently larger than zero and the standard deviations are sufficiently close to each other, then each firm plays mixed strategies. (See the appendix for the exact cutoffs for various equilibria in the differentiated Bertrand case.)*

Proof. See the appendix for the first item; the second item is a corollary of the first item and Proposition 2. □

Example 1 (Macroeconomic cost fluctuations, commodities). Suppose the firms require the same commodity, which they both procure from the global market. The firms can, potentially, avoid risk in a number of ways: for example, they could switch to a different technology that does not require that input or enter into a long-term price lock-in contract without a prespecified quantity (see §1). If firms do not avoid risk, then both firms get highly correlated cost draws each period, likely with similar standard deviations. Since correlation is positive and high, the likely result is firms playing an equilibrium where one firm exposes itself to risk and the other one does not, or both firms play mixed strategies. If both firms expose themselves to risk, then the cost difference is unlikely to fluctuate widely: it is likely that they both get either a good draw or a bad draw, resulting in the same cost difference. Of course, if both firms do not expose themselves to risk, then the cost difference does not fluctuate, either. Thus, to ensure cost difference fluctuations, and thus take advantage of the familiar Hotelling (1929) and Motta (1993) differentiation incentives, it makes sense for firms to end up with asymmetric outcomes, with one of the firms exposing itself to risk.

Example 2 (Outsourcing to the same or to different firms). Suppose the firms have a choice of outsourcing either to the same subcontractor or to different subcontractors, and suppose that a part of the cost is not contractible. I am assuming away any tariffs, transportation costs, and political and public relation issues. For example, the transportation cost depends on the regional transport prices, and the subcontractors are in different regions (e.g., China and Mexico). This example is similar to the previous one. The difference is that hedging entails that the firm chooses the same supplier (or the same marketing agency or strategy) as its competitor. In this case, outsourcing to the same subcontractor implies that the cost difference does not fluctuate. Thus, firms should outsource to different subcontractors.

The results of Proposition 3 are the same for both differentiated Bertrand and differentiated Cournot competition, raising a question of what might change the result. Either differentiated Bertrand or differentiated Cournot competition where the products are complements instead of being substitutes results in $\Pi^i_{a_i a_i} < 0$. A game, with linear demand and linear costs, where firms play a Stackelberg competition in the second stage results in $\Pi^i_{a_i a_j} > 0$. Thus, results that differ qualitatively from those in Proposition 3 can be achieved even with linear demands and costs; however, both of the counterexamples indicate that there needs to be a particular type of a game (likely not standard competition in substitutes) to achieve the opposite results.

3.3. Nonlinear Demands and Nonconstant Marginal Costs

Term $\Pi^i_{a_i a_j}$ is reminiscent of the strategic substitutes of Bulow et al. (1985). However, this is a dynamic strategic term. The concern is whether parameters a_i and a_j are strategic substitutes even after accounting for their effect in the second stage of the game, via themselves, x_i^*, and x_j^*. Vives (2009) analyzes strategic substitutability in this dynamic sense.

Proposition 4. *Suppose that for each i, Π^i is continuous, supermodular in $(a_i, -a_j, x_i, -x_j)$, and convex in x_j. Then Π^i is supermodular in $(a_i, -a_j)$.*

Proof. See Proposition 1 and Corollary 5 in Vives (2009). □

Thus, Vives (2009) provides the conditions for when $\Pi^i_{a_i a_j} < 0$, the analog of those in Proposition 1. This allows us to simplify condition (5) to

$$\rho_{12} < \frac{1}{2}\frac{\sigma_i}{\sigma_j}\frac{\Pi^i_{a_i a_i}}{-\Pi^i_{a_i a_j}}. \tag{6}$$

The results from the linear demand and linear cost differentiated Bertrand and Cournot are likely to apply in many cases, and, given Vives's (2009) results, it is easy to check whether they indeed apply or to make any necessary corrections based on Proposition 2. Given the strategic substitutes/complements interpretation, the results also become easier to explain.

In the strategic setting, firms still have the same option value incentive as in the monopoly case to expose themselves to risk. As for the additional strategic incentive, suppose that the shocks are (dynamic) strategic substitutes ($\Pi^i_{a_i a_j} < 0$), and firms would want to expose themselves to risk unilaterally ($\Pi^i_{a_i a_i} > 0$). Then, firms prefer the shocks to be different, and the strategic incentive is to get negatively correlated risks. That is naturally accomplished by both firms exposing themselves to risk in equilibrium, as long as the shocks are negatively correlated.

If the shocks are positively correlated, then the monopoly option value incentive and the strategic substitute incentive are pushing in different directions. Thus, if the shocks become close to being perfectly positively correlated, the strategic incentive can outweigh the monopoly option value, resulting in one of the firms not exposing itself to risk or both firms playing mixed strategies in equilibrium. See the previous subsection and the appendix for an example in a differentiated Bertrand competition setting.

If the firm does not want to expose itself to risk unilaterally ($\Pi^i_{a_i a_i} < 0$), then the strategic incentive has to overcome the unilateral incentive. Thus negative correlation is not sufficient: it has to be sufficiently negative. The intuition reverses when shocks are dynamic strategic complements ($\Pi^i_{a_i a_j} > 0$).

4. Conclusion

I show when firms should expose themselves to risk in their marginal cost/revenue parameters. There are numerous potential applications of this model, ranging from choosing whether to go with a risky or a safe advertising campaign, to whether to outsource to multiple suppliers or to a single supplier, to where to locate factories to potentially hedge against exchange rate fluctuations. The case of strategic competition is more intricate; however, even in that case I outline the general solution using the results from multivariate risk aversion and dynamic strategic substitutes/complements literatures, and I show that for many cases both competitors prefer to expose themselves to risk as long as the risks are not too positively correlated.

For empirical researchers, there can be several tests of whether firms prefer to be more or less risky depending on the shape of their profit functions. A potential shortcut is identifying the settings where the model definitely predicts as-if risk-loving firms (for example, near-monopolies with constant marginal costs or competitors with negatively correlated cost or demand shocks) and examining whether firms actually behave in a risk-loving manner.

When deciding whether to play it safe, or when testing the theory, both practitioners and researchers need to be careful only to consider fluctuations in marginal cost/revenue. However, for the purposes of empirical research, if the managers confuse fixed and marginal cost, then they will behave as my model predicts even when the hedge does not change the economic cost.[5]

[5] See Al-Najjar et al. (2008) for research on managers confusing fixed and marginal costs. See Bowley and Neuman (2011) for more industries where managers apparently confuse fixed and marginal costs, with a particular emphasis on hedging the exchange rates (which does not affect marginal costs, unless it is done operationally).

Another crucial point is that the two seemingly contradictory behaviors—playing it safe and being risky at the margin—can easily go together if playing it safe is for balance sheet purposes. If half of the profit of an American firm is due to sales made in Europe, then my model suggests that the firm should not build a factory in Europe to protect itself from demand shocks as a result of fluctuations in the dollar/euro exchange rate. However, if there are some liquidity or tax concerns, the firm should buy (or sell) some options on euros in the financial markets.

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Appendix. Analyzing Differentiated Bertrand and Differentiated Cournot

Setup
There are two firms, 1 and 2, each selling one product with marginal production costs of m_1 and m_2. Firms simultaneously set prices (quantities) in the differentiated Bertrand (Cournot) setup.

The demands are derived from maximizing the utility function of a representative consumer, $U(q_1, q_2) = \alpha_1 q_1 + \alpha_2 q_2 - (\beta_1 q_1^2 + 2\gamma q_1 q_2 + \beta_2 q_2^2)/2 - q_1 p_1 - q_2 p_2$. Parameters α are vertical intercepts of the demand function, parameters β are own demand slopes, and parameter γ measures differentiation. If $\gamma = 1$, then firms are perfectly competitive and their products are substitutes; if $\gamma = 0$, then firms are monopolistic; and if $\gamma < 0$, then firms are selling complementary products. The stability condition is $\beta_1\beta_2 - \gamma^2 > 0$.

Solution: Differentiated Bertrand
From Table 6.1 in Vives (2001), after making all the substitutions defined in that text, the equilibrium margin in the differentiated Bertrand setup is

$$p_i^* - m_i = \frac{(2\beta_i\beta_j - \gamma^2)(\alpha_i - m_i) - \beta_i\gamma(\alpha_j - m_j)}{\beta_i\beta_j - \gamma^2}. \tag{7}$$

The equilibrium quantity is $q_i^* = (\beta_j/(\beta_i\beta_j - \gamma^2)(p_i^* - m_i)$. Thus, the profit function of firm i is

$$\Pi^i = \frac{\beta_j}{(\beta_i\beta_j - \gamma^2)^5}((2\beta_i\beta_j - \gamma^2)(\alpha_i - m_i) - \beta_i\gamma(\alpha_j - m_j))^2. \tag{8}$$

It is trivial to show that the sign of $\Pi^i_{\alpha_i\alpha_j}$ is the opposite of the sign of γ (less than zero iff products are substitutes) and $\Pi^i_{\alpha_i\alpha_i} > 0$. The signs of second-order partial derivatives with respect to m variables are the same.

Plugging in the partial derivatives, it is easy to see that

$$\frac{\Pi^i_{\alpha_i\alpha_i}}{\Pi^i_{\alpha_i\alpha_j}} = \frac{2\beta_i\beta_j - \gamma^2}{\beta_i\gamma}; \tag{9}$$

thus, $\hat{\Psi}_i < 0$ iff

$$\rho_{12} < \frac{1}{2}\frac{\sigma_i}{\sigma_j}\frac{2\beta_i\beta_j - \gamma^2}{\beta_i\gamma}, \tag{10}$$

where σ values are the standard deviations of exogenous shocks of each firm, and ρ_{12} is their correlation.

Therefore, if $\hat{\Psi}_i < 0$ and $\hat{\Psi}_j < 0$, then both firms expose themselves to risk. If $\hat{\Psi}_i < 0$ but $\hat{\Psi}_j > 0$, then i exposes itself to risk, but j hedges. Note that with similar demand functions this occurs only if firm i's shock has a higher standard deviation. Finally, if $\hat{\Psi}_i > 0$ and $\hat{\Psi}_j > 0$, then both firms play mixed strategies.

Solution: Differentiated Cournot

In the case of a differentiated Cournot setup, from Table 6.1 in Vives (2001),

$$q_i^* = 2\beta_j(\alpha_i - m_i) - \gamma(\alpha_j - m_j), \tag{11}$$

and the corresponding equilibrium markup is

$$p_i^* - m_i = \beta_i q_i^* - 4\beta_i\beta_j m_i + \gamma^2 m_i. \tag{12}$$

It is trivial to show that the sign of $\Pi^i_{\alpha_i\alpha_j}$ is the opposite of the sign of γ (less than zero iff products are substitutes) and $\Pi^i_{\alpha_i\alpha_i} > 0$. The signs of second-order partial derivatives with respect to m variables are the same.

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