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## Balancing Acquisition and Retention Spending for Firms with Limited Capacity

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 $\Gamma$ his paper discusses the interaction between revenue management and customer relationship management f I for a firm that operates in a customer retention situation but faces limited capacity. We present a dynamic programming model for how the firm balances investments in customer acquisition and retention, as well as retention across multiple customer types. We characterize the optimal policy and discuss how the policy changes depending on capacity limitations. We then contrast the modeling results with those of a behavioral experiment in which subjects acted as managers making acquisition and retention decisions. In the modeling part of the paper, we introduce a concept of the value of an incremental customer (VIC), and show that when capacity is unlimited, VIC equals customer lifetime value (CLV), but when capacity is limited, VIC is much smaller and changes dynamically depending on the number of customers and their mix. As a result, the optimal spending is constant and depends on CLV for the firms with unlimited capacity, but changes dynamically and is generally unrelated to CLV when capacity is limited. In the experimental part, we introduce a concept of conditional optimality for the analysis of state-dependent decisions. Applying this concept to our data, we document a number of decision biases, specifically the subjects' tendency to overspend on retaining high-value customers and underspend on lower-value customers retention and acquisition. We show that providing CLV information exacerbates these biases and leads to a loss of net revenue when capacity is limited, but providing information about the marginal costs of acquisition and retention eliminated these biases and increases net revenue.

Data, as supplemental material, are available at http://dx.doi.org/10.1287/mnsc.2013.1842.

Keywords: behavioral operations; revenue management; limited capacity; customer relationship management; customer lifetime value

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## Introduction

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Dynamic pricing and revenue management (RM) techniques have been a notable success in many industries. These techniques vary prices/availability over time to maximize the revenue a firm obtains from a fixed inventory of perishable items. RM focuses on maximizing short-term revenue, often ignoring the potential negative impact on the firm's relationships with customers. In contrast, customer relationship management (CRM) focuses on maximizing the value of the firm's current and future customer relationships, at the expense, frequently, of short-term profitability. And although many firms that practice RM are also active in CRM (e.g., airlines have loyalty programs), the interaction between the two is not well understood in both theory and practice.

Integration of RM and CRM is most critical for firms that operate in so-called customer retention situations<sup>1</sup> with limited (and perishable) capacity<sup>2</sup>—performing arts, sports and entertainment companies selling

<sup>1</sup>Customer retention situations are those in which a service or product is contracted for over a defined period of time with the understanding that the customer has the option to end the relationship for good when the contract expires. Situations that are not retention relationships are those in which customers transact with the firm sporadically and in which a period of inactivity does not necessarily signal the end of the relationship.

<sup>2</sup> By limited capacity we mean a situation in which the firm cannot serve more than some specified number of customers. The limit may be physical, as in the case of an apartment building or performing arts venue. It may also be strategic; for example, when the principals of a boutique consulting (legal, medical, etc.) practice purposefully limit the number of clients to ensure adequate personal attention to each client.



season-ticket subscriptions, apartment complexes selling renewable leases, and the like. Such firms face a unique set of challenges: limited capacity, perishable products, and heterogeneous consumers make them well suited for RM, but the customer retention nature of the business is ideally suited for CRM. Managers of such firms must therefore combine RM and CRM approaches: consider trade-offs between retaining existing customers and acquiring new ones, while realizing that the firm may not be able to serve all customers (now or in the future) due to capacity limitations.

The goal of our paper is to provide insights for determining acquisition and retention spending for such firms. To do so, we first present a stylized dynamic programming model for selecting acquisition and retention spending, characterize the optimal policy, and discuss how it changes depending on capacity limitations. We then contrast the modeling results with those of a behavioral experiment in which subjects made acquisition and retention decisions for firms with limited or unlimited capacity and with or without explicit information about customer lifetime value (CLV). A comparison between the optimal solution and laboratory behavior is useful because often these decisions are made by managers rather than computer algorithms.

## 1.1. Mini-Case Study: Opera Company C

To illustrate the issues this paper addresses, consider the opera company C (name disguised for confidentiality). Opera company C stages approximately seven operas per season (October to May each year), with up to 10 airings of each opera during the season. As are most performing arts companies, C is a not-for-profit organization, but ticket sales are extremely important to C and account for nearly 50% of operating revenue. Another 30% comes from fundraising, and approximately 20% comes from government grants.

Opera company C's venue has 2,070 seats in 14 seating areas with six price classes with prices varying from \$60 to over \$300, depending on the location of the seat. Full-season subscriptions range from \$250 to over \$2,000, and several part-season subscription options are also available. A subscriber purchases a specific seat (usually the same) for a bundle of performances on specific dates.<sup>3</sup> Avid opera lovers get used to enjoying performances from a certain angle and place significant value on being in the same seat every season—renewing a subscription is a way to keep one's seat.

<sup>3</sup> The same is also true for a major sports company that we spoke with while working on this paper. This situation is different, however, from that described in Tereyagoglu et al. (2012), where a subscriber commits to a certain number of concerts and has flexibility to choose the seats and dates at any time during the season.

Even though a subscription is priced lower than the sum of individual tickets (e.g., \$250 < 7 performances  $\times$  \$60), C believes subscribers are much more valuable than single-ticket buyers. This happens because the renewal rate for full-season ticket subscribers is approximately 90%. In contrast, only 4% of single-ticket holders purchase in the following season. Thus the lifetime value<sup>4</sup> (CLV) of a \$250 subscriber is approximately \$2,500, whereas the value of a \$60 single-ticket buyer is only slightly above the ticket price. Differences of similar magnitude have been reported for other performing arts companies as well; e.g., Oliver Wyman, a consultancy, estimates<sup>5</sup> the average CLV of a symphony orchestra's subscriber in the United States at \$4,896 as opposed to the single-ticket buyer's CLV of \$199. For this reason, the companies like C are unlikely to benefit from "protecting" some seats for higher-paying singleticket buyers, an approach practiced by airlines' or hotels' RM systems; see §2. As one of C's managers put it (name disguised for confidentiality), "... you wonder why no one donates to airlines? Opera house is not an airplane with live music, we'd rather sell everything on season."

Opera company C's selling process is as follows. First, C approaches its current subscribers (via direct mail, emails, and call center) with offers to renew. Such offers can include both price and nonprice activities (e.g., C sent free DVDs to a subset of its current season subscribers who have not yet renewed), but for simplicity we will not make the price/nonprice distinction. At this stage, retention decisions are customer specific: In principle, each customer could be getting an offer different from everybody else. In practice, C has three to four differentiated treatments for different customer types. At some later point during the selling season, but when retention efforts are still underway, C opens up the sales of subscriptions to the general public and advertises (mostly via mass media and to a lesser extent via direct mail) to attract new subscribers. At this point, customized pricing is not possible because all prospects receive identical offers as per the advertisement. Finally, at some later date, C starts selling part-season subscriptions and single tickets. By then, the best seating locations are typically sold out to subscribers (e.g., several months before the start of the 2010 season, 9 of 14 seating areas were sold out), and C uses single-ticket buyers'



<sup>&</sup>lt;sup>4</sup> This number  $(250+0.9*250+0.9^2*250+\cdots)$  is only an estimate: it excludes the cost of soliciting the renewal and does not account for the time value of money; see §4.1.

<sup>&</sup>lt;sup>5</sup> See http://www.oliverwyman.com/1574.htm (accessed May 1, 2012). Oliver Wyman also estimates that on average for the orchestras in their study subscribers contribute 83% of ticket sales and 89% of donations.

demand to fill lower-quality seating areas where there is often insufficient demand from subscribers.

Decisions about the retention and acquisition activities are made by a single manager and effectively "by hand" based on his expertise and intuition, sales results to date, and from the previous years. No RM system is used, although a CRM system is in place that tracks past customers' purchases, donations, offers sent to customers, and their response.

In sum, C's problem combines the elements of RM and CRM. As is typical in RM, C manages an inventory of perishable items and faces customers with heterogeneous valuations. But as is typical in CRM, C's business calls for establishing long-term relationships with customers and as a result its selling process consists of reaching out to existing customers with personalized retention/renewal offers and also reaching out to the general public with nonpersonalized acquisition/advertising offers. These characteristics of C's problem, however, do not fit with the existing RM or CRM models and call for deeper analysis, which our paper presents.

## 2. Literature Review

Although there are many excellent RM and CRM works, little of that literature provides direct guidance in C's context. Traditional RM works take a shortterm view of maximizing revenue from the inventory on hand. A stream of recent works on strategic consumer behavior in RM considers (either directly, or indirectly, by employing the rational expectations equilibrium assumption) the problem of maximizing the long-term revenue in a situation where firm's policy today affects consumers purchasing behavior tomorrow; see Shen and Su (2007) for a review. This literature, however, does not capture the selling process for a business in a retention situation, where personalized retention offers are made to individual consumers with whom the firm seeks to establish long-term relationships. A number of RM authors mention the importance of incorporating lifetime value considerations in revenue management, e.g., Cross et al. (2009, p. 75) call for "customer-centric revenue management" and suggest that it will be part of the future of RM.

In the CRM literature (see Rust and Chung 2006 for a review), the focus is on evaluating the value of a customer relationship and using CLV for various business problems, which range from guiding marketing spending to valuing firms. This literature, however—and this is crucial for an RM-like application—does not consider capacity constraints. A capacity constraint creates a natural dynamic in spending and dependency among customer relationships as the firm approaches capacity limit. Although most CRM

papers assume independent customer relationships, demand side dependencies (such as referrals or customer network effects) have been recognized, e.g., Pfeifer (1999). Supply side dependencies, however, received little attention. To our knowledge, Pfeifer and Ovchinnikov (2011) is the only paper that considers a capacity constraint in CRM.

Several papers consider problems that have some common elements to ours. Liu et al. (2007) consider an intertemporal capacity limitation where providing better service (allocating more effort) to a customer today implies that less effort can be allocated to the same customer tomorrow. But serving one customer in their context has no impact on the firm's ability to serve another; thus in our interpretation the capacity is unlimited. Aydin and Ziya (2009) consider personalized dynamic pricing of limited inventories, but focus on its transactional aspect: the firm decides on discounts to sell inventory on hand, without regard of the future customer behavior. Aflaki and Popescu (2014) consider the relationship between customer satisfaction and retention probability in the CRM context, but do not consider capacity constraints. Lewis (2005) considers the dynamics of spending in CRM based on the empirical observation that retention probabilities increase with customers' tenure with the firm; he does not consider capacity constraints either.

Drake et al. (2008) and de Vericourt and Lobo (2009) consider a typical "airline/hotel" RM approach to managing a nonprofit organization: the problem of protecting some capacity for higher-paying customers (single-ticket buyers at a stadium and patients at a hospital, respectively). As we discussed in the minicase study of C, such practice is not common for the firms in retention situations. As Tereyagoglu et al. (2012, p. 3) put it, "the general assumptions and pricing strategy from the airline setting...does not hold true for the theater setting." Because of bundling, donations, and the retention nature of the business, even though subscribers receive discounts, they bring more value to the firm than higher-price single-ticket buyers, but arrive first. Thus firms like C do not protect inventory from subscribers in hope for selling it later to single-ticket buyers. This does not mean that single-ticket buyers are not important: C and similar companies use their demand to fill seats in the sitting areas with insufficient subscribers' demand and also work on converting single-ticket buyers into subscribers; the latter is, in fact, the main recommendation of the Oliver Wyman report.

Last, because our paper combines a model with behavioral analysis, it is also related to the behavioral operations literature dealing with RM. Bearden et al. (2008) discuss decision biases in a standard single-selling-season RM problem. Bendoly (2011) extends the work of Bearden et al. (2008) for multiple booking



classes linking subjects performance/errors to physiological markers. Osadchiy and Bendoly (2011) discuss experimental evidence for whether customers are really strategic. Neither of these studies deal with firms in retention situations and thus are quite different from our paper. From the methodological perspective, Mak et al. (2012, p. 263) discuss behavioral "best response analysis" and Lau et al. (2014) emphasize the need to perform analysis on the individual subject level; our conditionally optimal policies and individual level analysis are in line with their approaches.

## 3. Model

In this section we formulate a model of a firm that decides on how much to spend on acquisition and retention. Our model is purposefully stylized to capture the general elements of the problem and yet isolate the effect of a capacity constraint on the otherwise "standard" firm in a retention situation. As a result, our expressions for CLV and their interpretations are effectively identical to those of other authors, e.g., Blattberg and Deighton (1996) and Gupta and Zeithaml (2006). In our model, retention spending is tailored to individual customers, but acquisition spending is not. This reflects the practice of many firms, including C, that use "mass" advertising to acquire customers and tailored spending (e.g., direct mail with personalized offers) to retain them.

We assume that there are two types of customers: "high" (H) and "low" (L). In period  $t=1,2,\ldots$ , the firm serves  $N_t^H$  H-type and  $N_t^L$  L-type customers for a total of  $N_t = N_t^H + N_t^L$ ; note that "period" here refers to the entire selling season, not a slice of time within a given season. Customers contribute  $M^H$  and  $M^L \leq M^H$ , respectively, per period. Spending  $R_t^H$  on retention per H-type customer results in a retention fraction  $r^H(R_t^H)$ , and spending  $R_t^L$  per an L-type results in fraction  $r^L(R_t^L)$ . We further assume that spending  $A_t$  on acquisition results in  $n(A_t)$  prospects who arrive in period t+1, a fraction  $\xi$  of which are of H-type; we call them prospects because they will become customers only if the firm has the capacity to serve them. Then the net revenue in period t is

$$g_t(N_t^H, N_t^L, R_t^H, R_t^L, A_t) = N_t^H \times (M^H - R_t^H) + N_t^L \times (M^L - R_t^L) - A_t.$$
 (1)

Note that M - R represents the effective price that a customer pays, hence the connection to RM.

Period t+1 demand is  $D_{t+1}=N_t^H r^H(R_t^H)+N_t^L r^L(R_t^L)+n(A_t)$ , and the number of customers served in period t+1 is  $N_{t+1}=\min[\bar{N},D_{t+1}]$ , where  $\bar{N}$  is the firm's capacity (one customer occupies one

unit of capacity per period). Therefore, the number of customers of each type in period t+1 equals

$$\begin{split} N_{t+1}^{H} &= N_{t}^{H} r^{H}(R_{t}^{H}) \\ &+ \xi \min[n(A), \bar{N} - (N_{t}^{H} r^{H}(R_{t}^{H}) + N_{t}^{L} r^{L}(R_{t}^{L}))], \\ N_{t+1}^{L} &= N_{t}^{L} r^{L}(R_{t}^{L}) \\ &+ (1 - \xi) \min[n(A), \bar{N} - (N_{t}^{H}, r^{H}(R_{t}^{H}) + N_{t}^{L} r^{L}(R_{t}^{L}))], \end{split} \tag{2}$$

reflecting proportional bumping of prospects in the case of excess/overflow demand; i.e., the firm first accommodates its renewing customers and then allocates the remaining capacity on a first-come, first-served basis to prospects who arrive in random order.

With these definitions, the objective of the firm is to maximize its discounted net revenue:

$$\pi_{t}(\bar{N}, N_{t}^{H}, N_{t}^{L}, R_{t}^{H}, R_{t}^{L}, A_{t})$$

$$= g_{t}(N_{t}^{H}, N_{t}^{L}, R_{t}^{H}, R_{t}^{L}, A_{t}) + \beta \pi_{t+1}^{*}(\bar{N}, N_{t+1}^{H}, N_{t+1}^{L}), \quad (3)$$

where  $\pi_{t+1}^*(\bar{N}, N_{t+1}^H, N_{t+1}^L) = \max_{R_{t+1}^H, R_{t+1}^L, A_{t+1}} \pi_{t+1}(\bar{N}, N_{t+1}^H, N_{t+1}^L, R_{t+1}^H, R_{t+1}^L, A_{t+1})$  is the optimal net revenue-to-go from state  $(\bar{N}, N_{t+1}^H, N_{t+1}^L)$  onward,  $\beta \in [0, 1]$  is the firm's discount factor, and state transitions  $(N_{t+1}^H, N_{t+1}^L)$  are given by (2).

To complete the specification of the model, we discuss several important assumptions:

Assumption S1. The contribution amounts  $M^H$  and  $M^L$  represent the net of the list price (rack rate, etc.) charged and the cost of goods/services sold and all other direct expenses, excluding retention spending. In reality, contribution can be a random variable; e.g., in the case of the opera company C, contributions include possible donations from season ticket holders. As long as its distribution is unaffected by retention and acquisition spending, all our results will hold.

Assumption S2. Functions  $r^H(\cdot)$  and  $r^L(\cdot)$  are increasing concave functions such that  $r^L(x) \le r^H(x)$  and  $\partial r^H/\partial x \ge \partial r^L/\partial x$  for all x > 0. These assumptions are consistent with the idea that higher-spending customers are more loyal (Reichheld and Sasser 1990) and were supported by our discussions with the management of opera company C.

Assumption S3. Function  $n(\cdot)$  is an increasing concave function. This implies that a new cohort of potential customers (prospects) arrives each period and their responsiveness to the firm's acquisition spending is identical in each period. This is also consistent with the experience of C, whose management believes that because of the population migration each year there is a number of new potential opera lovers who move into the city.

All of our structural results continue to hold when acquisition and retention processes depend on time and/or are stochastically increasing and concave random variables (as opposed to deterministic functions presented above for simplicity): e.g., in



our numerical and behavioral analysis, §§4.3 and 5, we consider binomial retention and acquisition, i.e., the number of retained H-type customers is given by  $Binomial[N_t^H, r^H(R_t^H)]$ , the number of acquired H-types is  $Binomial[n(A_t), \xi]$ , etc.

Assumption S4. Retention and acquisition activities are done simultaneously, which to a reasonable degree resembles the practice of C, where advertising in broadcast media starts only a short while after the retention offers are mailed and well before the retention activities are over.

Assumption S5. Newly acquired prospects are served in the first-come, first-served manner. That is, if the number of customers exceeds capacity, then the firm bumps both H- and L-type customers in their arrival proportion,  $\xi$ . This can happen if the firm cannot differentiate between types of prospects upon acquisition (but can, once retained), or, equivalently, if the firm is not willing to selectively bump prospects based on their type for fairness or other reasons.

Assumption S6. Customer behaviors are independent. That is, customer retention depends only on retention spending directed toward that customer and nothing else; likewise, the number of acquired prospects depends only on the acquisition spending. As mentioned earlier, the purpose of this assumption is to isolate the impact of capacity constraint from other possible dependencies that could be creating dynamics in spending, as well as to make our model (in the unlimited capacity case), comparable to CLV models used by other researchers.

Next we use the above model to characterize the optimal spending and test it behaviorally.

# 4. Analysis of the Model: Optimal Spending

Proposition 1. The optimal spending,  $R_t^{*H}$ ,  $R_t^{*L}$ , and  $A_t^*$  satisfy the following conditions:

$$\frac{1}{\partial r^{H}/\partial R^{H}|_{R=R_{t}^{*H}}} = \beta VIC_{t+1}^{H}(N_{t+1}^{H}, N_{t+1}^{L}, \bar{N}),$$

$$\frac{1}{\partial r^{L}/\partial R^{L}|_{R=R_{t}^{*L}}} = \beta VIC_{t+1}^{L}(N_{t+1}^{H}, N_{t+1}^{L}, \bar{N}),$$

$$\frac{1}{\partial n/\partial A|_{A=A_{t}^{*}}} = \beta \left(\xi VIC_{t+1}^{H}(N_{t+1}^{H}, N_{t+1}^{L}, \bar{N}) + (1 - \xi)VIC_{t+1}^{L}(N_{t+1}^{H}, N_{t+1}^{L}, \bar{N})\right),$$
(4)

where

$$\begin{split} VIC_{t+1}^{H}(N_{t+1}^{H}, N_{t+1}^{L}, \bar{N}) &= \frac{\partial \pi_{t+1}^{*}}{\partial N_{t+1}^{H}} \bigg|_{N_{t+1}^{H}, N_{t+1}^{L}} \\ &- \frac{\partial \pi_{t+1}^{*}}{\partial N_{t+1}^{L}} \bigg|_{N_{t+1}^{H}, N_{t+1}^{L}} \times \mathbf{1}_{N_{t+1}^{H} + N_{t+1}^{L} \geq \bar{N}} \end{split}$$

and

$$\begin{split} VIC_{t+1}^{L}(N_{t+1}^{H},N_{t+1}^{L},\bar{N}) &= \frac{\partial \pi_{t+1}^{*}}{\partial N_{t+1}^{L}} \bigg|_{N_{t+1}^{H},N_{t+1}^{L}} \\ &- \frac{\partial \pi_{t+1}^{*}}{\partial N_{t+1}^{H}} \bigg|_{N_{t+1}^{H},N_{t+1}^{L}} \times \mathbf{1}_{N_{t+1}^{H}+N_{t+1}^{L} \geq \bar{N}}. \end{split}$$

The proofs of all results in the paper follow directly from the known structural properties of dynamic programs, e.g., Smith and McCardle (2002), and are therefore omitted for conciseness. Following Proposition 1, the firm determines the *value of an incremental customer*, VIC, and spends up to a point where the marginal cost of retaining or acquiring such a customer equals his or her incremental value. This value, however, depends on the capacity limitations, as we discuss next.

## 4.1. Firm with Unlimited Capacity: CLV-Maximizing Spending

Defined very generally, customer lifetime value is the expected net present value of the future cash flows attributed to the customer relationship (Pfeifer et al. 2005). Thus for the firm with unlimited capacity and otherwise independent customer relationships, Assumption S6, CLV is equivalent to VIC by definition. For such a firm, H-type CLV depends only on the H-type retention, and likewise for L-types. Further, since the retention process is memoryless, it is sufficient to consider the case when  $R_t = R_{t+1} = \cdots = R$  (we temporarily omit the *H*- and L-type indices). Then in the first period of relationship with a customer, the firm will generate cash-flow M-R. In the second period, the customer returns with probability r(R), in which case the firm will generate cash-flow M-R again, which has a present value  $\beta \times (M - R_{t+1})$ , and so on. Therefore,

$$CLV(R) = (M - R)(1 + \beta r(R) + (\beta r(R))^{2} + \cdots)$$

$$= \frac{M - R}{1 - \beta r(R)},$$
(5)

implying that  $CLV^H(R^H) = (M^H - R^H)/(1 - \beta r^H(R^H))$  and  $CLV^L(R^L) = (M^L - R^L)/(1 - \beta r^L(R^L))$ . Note that these expressions are effectively identical to some existing formulations in the literature, e.g., in Blattberg and Deighton (1996) and Gupta and Zeithaml (2006). Subject to Assumption S2, CLV(R) is quasi-concave and so the optimal CLV-maximizing retention spending  $R_{CLV}^*$  satisfies

$$\frac{1}{\partial r/\partial R|_{R=R_{\text{CLV}}^*}} = \beta CLV(R_{\text{CLV}}^*), \tag{6}$$

where the H and L indices are omitted for convenience. From the properties of the inverse functions, the inverse of the derivative on the left-hand side can be interpreted as the marginal cost of ensuring the



desired retention rate/probability. Thus this equation also has a succinct managerial interpretation that CLV is "the limit on retention spending."

Likewise, substituting the expression for the CLV into the corresponding condition in (4), and subject to Assumption S3, the optimal *CLV-maximizing* acquisition spending  $A_{\text{CLV}}^*$  satisfies

$$\begin{split} \frac{1}{\partial n/\partial A|_{A=A_{\text{CLV}}^*}} \\ &= \beta [\xi CLV^H(R_{\text{CLV}}^{*H}) + (1-\xi)CLV^L(R_{\text{CLV}}^{*L})], \end{split} \tag{7}$$

which, same as above, can be interpreted as "the limit on acquisition spending."

That is, the decision process for the firm with unlimited capacity is as follows. Once a customer of a given type is acquired, the firm spends according to (6) so as to maximize the present value from the relationship with that specific customer, his or her CLV equals VIC. The firm then infers how valuable, on average, a newly acquired customer is and spends up to the point when the marginal cost of acquiring a customer equals the average CLV. This logic became popular with both marketing practitioners and academics, because it provides simple, managerially relevant guidance of how much the firm should be willing to spend to acquire a new customer and what retention spending it should provide to retain him or her; see Katzenstein and Sachs (1992), Blattberg et al. (2008), and Pfeifer (1999). The separation of acquisition and retention processes is also very convenient from the organizational perspective (Blattberg and Deighton 1996). A direct application of this logic to a firm with limited capacity, however, is obviously problematic: the above and other existing models of CLV do not consider a firm that may not be able to serve all its customers, i.e., that its customers' CLV and VIC differ, as we discuss next.

# 4.2. Firm with Limited Capacity: Dynamics of Spending

PROPOSITION 2. Both  $VIC_t^H$  and  $VIC_t^L$  are increasing in  $\bar{N}$  and decreasing in  $N_t^H$  and  $N_t^L$ .

Driving these results are the three properties of the revenue-to-go function: supermodularity in  $(\bar{N}, N_{t+1}^H)$  and  $(\bar{N}, N_{t+1}^L)$ ; concavity in  $N_{t+1}^H$  and  $N_{t+1}^L$ , i.e., within customer type; and submodularity in  $(N_{t+1}^H, N_{t+1}^L)$ , i.e., across customer types.

From a managerial perspective, supermodularity and concavity are natural because a firm with a potentially binding capacity constraint will not be able to fully reap the benefits from additional acquisition or retention spending. Aggressive spending in the face of a capacity limit is likely to result in more customers showing up than the firm can serve. Since these

excess/overflow customers are bumped, the profitability of spending is therefore less than it would otherwise be with a larger capacity limit. The more capacity the firm has, the fewer customers it will bump, and thus an increase in spending is more beneficial, hence supermodularity. Getting an additional customer of a given type is less beneficial because the firm may not be able to serve him or her as profitably as existing customers, hence concavity within types. But losing a customer of a different type is beneficial because it allows the firm to serve other customers better, hence submodularity across types.

Combining the results of Propositions 1 and 2, we obtain the following corollary:

**COROLLARY 1.** The optimal decisions  $R_t^{*H}$ ,  $R_t^{*L}$ , and  $A_t^*$  possess the following properties:

- 1. Given  $N_t^H$  and  $N_t^L$ , all decisions are increasing in capacity limit,  $\bar{N}$ .
- 2. Given  $N_t^L$  ( $N_t^H$ ), the optimal H- (L-)type retention spending is increasing in  $N_t^H$  ( $N_t^L$ ), whereas the optimal L-(H-)type retention and acquisition spending are decreasing in  $N_t^H$  ( $N_t^L$ ).

This result has an appealing managerial interpretation: the firm should decrease its spending as it approaches capacity and should increase its spending if more capacity becomes available, but also reallocate spending from H- to L-types if it faces a state with a disproportional number of *H*-types, and vice versa. Therefore, when the firm has ample available capacity, VIC is approximately equal to CLV, and hence it spends close to the CLV-maximizing amounts. But as the firm grows, VICs decrease. In fact, for a firm at capacity, VICH could be as low as approximately  $CLV^{H}(R_{cap}^{H}) - CLV^{L}(R_{cap}^{L})$  because to get an additional H-type customer, the firm must displace an L-type customer (here  $R_{\text{cap}}^H$  and  $R_{\text{cap}}^L$  denote the optimal spendings for the firm at capacity). In this case,  $VIC^H$  is obviously much smaller than  $CLV^H(R_{CLV}^*)$ , and therefore the associated retention and acquisition spending is also much smaller than the CLVmaximizing spending.

## 4.3. Optimal Spending: Numerical Example

In this section we illustrate the phenomena discussed earlier using an example patterned after that presented in Blattberg and Deighton (1996); the parameters and functional forms of this example will also be used in the behavioral experiments in the next section. Suppose that currently the firm spends A = \$2,500 on acquisition and  $R^H = R^L = \$150$  on retention, which results in n(A) = 15,  $r^H = 80\%$ , and  $r^L = 60\%$ . Its data show that a new customer is of type H with probability  $\xi = 50\%$ , and each existing customer contributes  $M^H = M^L = \$1,000$  per period. The firm's research further indicates



Table 1 Comparison Between the Firm at Capacity and the Uncapacitated Firm

	Current	Unlimited capacity	At capacity $(\bar{N} = 150)$
A (\$)	2,500.00	20,143	14,265.92
$R^{H}$ (\$)	150.00	252.56	147.65
$R^{L}$ (\$)	150.00	177.40	63.88
n(A)	15.00	46.74	42.81
$r^{H}()$	0.800	0.852	0.798
$r^{L}()$	0.600	0.618	0.512
$CLV_H$ (\$)	3,036	3,215	3,027.18
$CLV_{L}$ (\$)	1,848	1,854	1,736.41
No. of customers	56.25	220.00	150

that  $r^H(\cdot) \in [0.6, 0.9]$ ,  $r^L(\cdot) \in [0.4, 0.7]$ , and  $n(\cdot) \in [1, 50]$ , and that in all three cases the relationship between spending and the outcome is described by an exponential function. This implies that  $n(A) = 1 + 49 \times [1 - \exp(-0.00013459A)]$ ,  $r^H(R^H) = 0.6 + 0.3[1 - \exp(-0.007324R^H)]$ , and  $r^L(R^L) = 0.4 + 0.3[1 - \exp(-0.007324R^L)]$ . The discount rate is  $\beta = 0.9$ .

With these parameters, Table 1 summarizes three cases. If the firm continues what it is currently doing, it will converge to having 38 H-type and 18 L-type customers, which from (5) have CLVs \$3,036 and \$1,848, respectively. Without capacity limitations, as per (6), the firm should somewhat increase its retention spending to \$252 and \$177, but as per (7) it should dramatically increase its acquisition spending by about 10 times to \$20,143, converging to 220 customers (with CLVs \$3,215 and \$1,854); note that here CLV equals VIC. With capacity limits, VIC is less than CLV; thus the firm spends less on all three components. Even though L-type customers are generally less valuable, the firm actively spends on them (\$63) because of the decreasing effectiveness of retention: for high-enough  $R^H$ , the cost of retaining an incremental H-type customer  $(1/(\partial r^H/\partial R^H))$  will exceed  $VIC^{H}$ ; at the same time, for low enough  $R^{L}$  the cost of retaining an incremental L-type customer will be lower than VIC<sup>L</sup>. The firm thus has some fraction of Ls, which is more profitable than aiming to have the "full house" of Hs; the latter is just too costly. Likewise, even when the firm is at capacity, it actively spends on acquisition (\$14,265): customers can be either retained or acquired, and the firm balances the cost of acquisition and retention with the customers' incremental values.

## 5. Behavioral Analysis

Our goal in this section is to understand how managers of firms in retention situations with limited capacity handle acquisition and retention spending trade-offs. Arguably, if managers can intuitively arrive at policies that are nearly optimal, then the model we presented above is of little use. In contrast,

if managers systematically and significantly deviate from the optimal policies, then the insights generated by our model can be quite helpful.

## 5.1. Design of the Experiment

To perform behavioral analysis, we designed an online simulation platform, which we call the "opera house simulation." The simulation asked subjects to decide on retention spending for two types of customers, A and B (we thought using "high" and "low" labels would be too suggestive), and provided a number of decision support tools. The simulation used the same response functions and parameters as the example in §4.3. Because the goal of the experiment is to understand the impact of limited capacity and the role of CLV, we used a two-factor design with four treatments: with limited or unlimited capacity, and with or without a CLV calculation and graph in the subjects' instructions; see the appendix.

Subjects (graduate and undergraduate business students, 20 in each of the four treatments) were recruited via the subject pool system in place at the authors' university. Upon showing up, they were given a brief overview of the experiment and were asked to sign a consent form. Then, the detailed instructions were distributed along with the link and password to the trial session of the simulation and the practice questionnaire. The trial and practice questionnaire asked subjects to construct an example themselves (see the appendix) and served two purposes: (i) to ensure that the subjects understood the interface and economics behind the simulation, and (ii) to avoid biasing subjects with specific examples given by the authors.6 Upon completion of the practice questionnaire, the subjects saw the experimenters who checked the questionnaire for correctness. Subjects were then given the logins and passwords to the main simulation from which the data were collected. That is, each subject played the simulation twice: once as a trial, and a second time for data collection. Subjects played for 40 periods/seasons in the data collection phase. Upon finishing the simulation, the subjects were asked to complete the postsimulation questionnaire. They were then paid in cash (earnings ranged from \$11 to \$21 depending on the subject's performance with the average of \$17) and were offered a light meal.

Next we analyze the impact of CLV and capacity limit on the overall performance of the subjects (i.e., on the net revenue they generated), and then perform detailed analysis of the decisions made during the

<sup>6</sup> The latter turned out to be an important feature in the experiment design: in the pilot that was run prior to data collection, we gave instructions that contained specific numbers (e.g., "suppose you spent \$4,000 on acquisition"), and 9 out of 10 subjects in the pilot started the simulation with a level of acquisition spending equal to \$4,000.



Limited capacity Unlimited capacity CLV 0.10.15 Mean CLV (\$1,219,637) Mean CLV (\$904,774) 0.2 0.00 0.1 No CLV 0.3 Mean No CLV (\$1,143,817) 0 0.05 0.1 0. Mean No CLV (\$950,170) 0.2 0.1 Optimal Optimal 5 Mean optimal (\$1,489,593) 0.10.1 Mean optimal (\$1,109,778) 0.2 0.1 0.05 1,000,000 1,200,000 800,000 1,000,000 1,400,000 1,600,000 800,000 900,000 1,100,000 1.200.000 Net revenue Net revenue

Figure 1 Histograms of Observed and Optimal Net Revenues

experiment. We summarize our findings in a set of managerially relevant observations.

## 5.2. Analysis of the Overall Performance

Figure 1 presents histograms of the observed and optimal net revenues. The mean percentage differences between the optimal and observed net revenue (revenue shortfall) in the limited capacity cases were 18.4% and 14.3% (CLV and No CLV treatments, respectively), and in the unlimited capacity cases, they were 18.1% and 23.2%; all four differences are significant at p < 0.01. This leads to the following observation:

Observation 1. Subjects fell far short of performing optimally.

Note that the revenue shortfall in the CLV treatment is larger than in the No CLV treatment when capacity is unlimited, but smaller when capacity is limited. This leads to the following observation:

OBSERVATION 2. Exposing decision makers to the information about CLV increased net revenue in the unlimited capacity cases, but decreased net revenue in the limited capacity cases.

Statistically, the significance of the differences in net revenues between the CLV and No CLV treatments is rather low (Wilcoxon p=0.085 for limited and p=0.114 for unlimited capacity) because discounting gives higher weight to subjects' performance in the beginning of the experiment, when their performance was in fact not that different. In the second half of the experiment, the difference between net revenues in CLV and No CLV treatments was significant at p<0.01.

Managerially, our results suggest that firms should be cautious when using CLV to guide spending decisions in cases where capacity is limited. When near capacity, it is better to ignore CLV and focus instead on maximizing the net revenue per period while keeping the firm at capacity with the optimal customer mix. The mechanisms, however, through which CLV information biased subjects decisions warrants deeper analysis, which we present next.

## 5.3. Analysis of the Subjects' Decisions

Because our model and experiment involve vectors of state-dependent decisions, a direct comparison between the observed and optimal decisions may provide an incomplete understanding of subjects' behavior. Indeed, given a subject's suboptimal decisions with respect to some components, e.g., *H*- and *L*-type retention, the *conditionally* optimal remaining component (acquisition) is potentially different from the globally (or jointly) optimal one. Next we discuss how to calculate such conditionally optimal decisions, and then compare subjects' observed decisions to both global and conditional optima. Because of heterogeneity in the subjects' decisions and corresponding suboptimalities, we perform the analysis at the individual subject level.

**5.3.1.** Calculation of the Conditionally Optimal Policy. Let the "asterisk" symbol (e.g.,  $A_t^*$ ) denote the globally optimal, the "hat" symbol (e.g.,  $\hat{A}_t$ ) denote the observed, and the "tilde asterisk" (e.g.,  $\tilde{A}_t^*(\hat{R}_t^H, \hat{R}_t^L)$ ) denote the conditionally optimal decisions; all decisions are state dependent, but we omit the corresponding indices for notational convenience.

For the firm with unlimited capacity from §4.1 and Equation (6),  $\tilde{R}_t^{*H} = R_{\text{CLV}}^{*H}$  and  $\tilde{R}_t^{*L} = R_{\text{CLV}}^{*L}$ ; i.e., globally and conditionally optimal retention spending are identical. This is not true, however, for the acquisition spending: from Equation (7) to calculate  $\tilde{A}_t^*$ , we need to calculate H- and L-type customers' CLVs conditional on the observed decisions  $\hat{R}_t^H$  and  $\hat{R}_t^L$ .



There is no obvious definition of conditional CLV because given the subject's observed decisions in period t, an assumption has to be made about his or her decisions in periods t+1 and onward. One possibility is to assume that although the subject deviated from optimality in period t, the subject will act optimally in periods t+1 and onward, i.e., that  $\hat{R}_{\tau}^{H} = R_{\text{CLV}}^{*H}$ and  $\hat{R}_{\tau}^{L} = R_{\text{CLV}}^{*L}$  for all  $\tau \geq t + 1$ . An alternative is to assume that the subject will make the same decision as in period t for perpetuity, i.e., that for all  $\tau \ge t + 1$ ,  $\hat{R}_{\tau}^{H} = \hat{R}_{t}^{H}$  and  $\hat{R}_{\tau}^{L} = \hat{R}_{t}^{L}$ . Our data show that in the unlimited capacity cases, subjects did not change their decisions from the previous season 74.2% of the time. Thus we believe that the second alternative is more plausible. Hence, we input the subject's observed  $R_t^H$ and  $\hat{R}_{t}^{L}$  into Equation (5) and then input the resulting conditional CLVs into Equation (7) to obtain  $\tilde{A}_{t}^{*}$ . Note that by the optimality principle,  $CLV(\hat{R}_t^{(\cdot)}) \leq$  $CLV(R_{CLV}^{*(\cdot)})$  and thus  $\tilde{A}_t^* \leq A_{CLV}^*$ .

For the firm with limited capacity, all three conditionally optimal decisions are interdependent and thus need to be evaluated. Following the same logic as above, the decisions were left unchanged in 59.5% of the seasons; therefore, as in the unlimited capacity case, we also assumed that for all decisions  $x \sim A_t$ ,  $R_t^H$ ,  $R_t^L$  and seasons  $\tau \geq t+1$ ,  $\hat{x}_{\tau} = \hat{x}_t$ . Then to compute, e.g., the conditionally optimal  $\tilde{A}_t^*$ , we optimized the dynamic program (3) with constraints on  $R_t^H$ ,  $R_t^L$ as per the above. As a result of this dynamic optimization, the behavior of the conditionally optimal decisions in the limited capacity case is very different from unlimited. Indeed, suppose that  $A_t^* < A_t^*$ , i.e., a subject spent less than optimal on acquisition. Then from (2),  $N_{t+1}^H$  and  $N_{t+1}^L$  will be below optimal and therefore  $VIC_{t+1}^H$  and  $VIC_{t+1}^L$  will be above optimal, which from (4) implies that  $\tilde{R}_t^H > R_t^{*H}$  and  $\tilde{R}_t^L > R_t^{*H}$ . The argument works in the opposite directions as well. In sum, we have the following observation:

Observation 3. The conditionally optimal decisions satisfy the following properties:

- 1. For the firm with unlimited capacity,
- Conditionally optimal retention spending is equal to the globally optimal retention spending.
- Conditionally optimal acquisition is below the globally optimal acquisition spending. Further, if both  $\hat{R}_t^H$  and  $\hat{R}_t^L$  are above (below) optimum, then  $\tilde{A}_t^*$  is decreasing (increasing) in  $\hat{R}_t^H$  and  $\hat{R}_t^L$ .
- 2. For the firm with limited capacity, conditionally optimal decisions are interdependent in a complex way; e.g., if a subject spends more (less) on acquisition, then conditionally optimal retention spending is lower (higher) than the globally optimal retention spending, and so on.

**5.3.2.** Comparison to the Globally and Conditionally Optimal Policies. Figures 2 and 3 present globally optimal, conditionally optimal, and observed spending for the four treatments averaged across subjects; error bars correspond to one standard deviation in each direction. Observe that in Figure 2, the closed triangles are below the line and the open triangles, whereas in Figure 3, the closed triangles are below the line, but in many cases overlap with the open triangles. This leads to the following observation:

OBSERVATION 4. Subjects underspent on acquisition as compared to the global optimum. But, in the limited capacity treatments, the observed spending was in many cases conditionally optimal.

Observe further that in the upper-left charts of Figures 2 and 3, the closed squares are systematically above the open ones, whereas the closed and open circles overlap, and conversely, in the upper-right charts, the closed squares are overlapping, whereas the closed circles are below the open ones. This leads to the following observation:

Observation 5. Subjects overspent on *H*-type retention relative to *L*-type retention: in the CLV treatments, *L*-type retention is conditionally optimal but *H*-type is above optimum, whereas in the No CLV treatments, *H*-type retention is conditionally optimal but *L*-type is below conditional optimum.

Statistically (at p < 0.05), the differences  $\hat{A}_t - A_t^{*H}$  are negative in over 20 seasons except for the unlimited CLV, but the differences  $\hat{A}_t - \tilde{A}_t^{*H}$  are not significant in over 20 seasons except for the unlimited No CLV, confirming Observation 4. The differences  $\hat{R}_t^H - R_t^{*H}$  are positive in 33 seasons in the limited CLV, but only in 1 season in the limited No CLV treatment, and, in contrast, the differences  $\hat{R}_t^L - \tilde{R}_t^{*L}$  are negative in 7 seasons in the limited CLV, but in 32 seasons in the limited No CLV treatment, confirming Observation 5.

These observations also hold true on the individual subject's level. We calculated the differences between the subject's observed and globally (and conditionally) optimal levels and classified a subject as overspending or underspending if the subject's average difference is two or more standard deviations above (below) zero. Consistent with Observation 5, in the CLV treatments, 11 and 10 subjects overspent on H-type retention in the limited and unlimited treatments, respectively, with respect to both globally and conditionally optimal policies; in contrast, in the No CLV treatments, between 2 and 7 subjects overspent. Consistent with Observation 4, in the unlimited capacity cases, 11 and 16 subjects underspent as compared with the globally optimal acquisition, but in the limited capacity cases, only 9 and 8 (CLV versus No CLV) underspent as compared with conditional



Unlimited No CLV—Retention spending Unlimited CLV—Retention spending 350 350 300 300 250 200 250 100 150 200 100 150 20 10 20 30 40 10 20 Season Season Globally optimal H-type Globally optimal L-type Globally optimal H-type Globally optimal L-type Conditionally optimal Conditionally optimal Conditionally optimal Conditionally optimal H-type H-type L-type Observed H-type Observed H-type Observed L-type Observed L-type Unlimited No CLV—Acquisition spending Unlimited CLV—Acquisition spending 21,000 40 10 20 30 10 20 30 40 Season Season ----- Globally optimal △ Conditionally optimal ▲ Observed ·---- Globally optimal △ Conditionally optimal ▲ Observed

Figure 2 Globally Optimal, Conditionally Optimal, and Observed Spending in the Unlimited Capacity Treatments

optimum; for 5 subjects in either CLV or No CLV the difference was inconclusive.

Returning to Observation 2: Why did subjects who were exposed to CLV information get higher net revenue when capacity was unlimited, but lower net revenue when capacity was limited? The answer is related to how overspending on retention and underspending on acquisition affected the number of customers and the relative proportion of *H*- and *L*-types. When capacity is unlimited, subjects spent more in the CLV treatment on both retention and acquisition (compare the left and right panels in Figure 2). Therefore, they also got more customers, whom they were all able to serve, although somewhat less profitably than optimal because  $R^H$  was too high. When capacity was limited, however, getting more customers was not possible, and so to avoid excessive bumping, subjects decreased acquisition by more in the CLV treatment than in the No CLV (compare the left and right panels in Figure 3). Since retention has decreasing effectiveness, attempting to retain a disproportionably large number of H-type customers was too costly: from our data, in the limited capacity CLV treatment, the average savings from replacing a customer through acquisition as opposed to retaining him or her was \$1,430 per *H*-type and \$76 per *L*-type per season. This explains Observation 2.

## 5.4. Drivers of Suboptimal Behavior

Our analysis identified two kinds of suboptimal behavior: underspending on acquisition, and overspending on *H*-type versus *L*-type retention. In this section we discuss several potential drivers of these behaviors, and we present three additional experiments designed to narrow down the list.

5.4.1. Stated Drivers: Evidence from Postexperimental Questionnaire. We text mined the responses to the postexperimental questionnaire (appendix) and noticed two common themes in subjects' stated explanations for how they decided on spending. Related to Observation 4, 36 subjects mentioned that they used acquisition only to replace lost customers, expressed by such statements as "... spend for acquisition sort of the minimum amount that would replace the



Limited No CLV—Retention spending Limited CLV—Retention spending 350 350 300 300 150 200 250 9 20 20 Season Season Globally optimal H-type Globally optimal L-type Globally optimal H-type Globally optimal L-type Conditionally optimal Conditionally optimal Conditionally optimal Conditionally optimal H-type L-type Observed H-type Observed L-type Observed H-type Observed L-type Limited CLV—Acquisition spending Limited No CLV—Acquisition spending 8 90 2 2 10 20 30 40 10 20 30 40 Season Season ----- Globally optimal △ Conditionally optimal ▲ Observed △ Conditionally optimal ▲ Observed ----- Globally optimal

Figure 3 Globally Optimal, Conditionally Optimal, and Observed Spending in the Limited Capacity Treatments

customers that left," "…acquiring new customers to replace those that I expected to lose," etc. Related to Observation 5, 53 subjects mentioned preference to keep only H-types, expressed by such statements<sup>7</sup> as "main objective was to retain Type A customers," "spend more on Type A than Type B," "…work to replace Type B with Type A," "I spent zero on retaining B customers." Spending zero on L-type retention was in fact quite common: we observed  $\hat{R}^L = 0$  in 10.3%–21.0% of subject–season combinations, as opposed to only 1.9%–7.5% for  $\hat{R}^H$ .

**5.4.2. Difference in Control.** The difference in control is relevant in our setting because retention can be targeted at *H*- and *L*-type customers separately, whereas acquisition cannot. Thus subjects might have had a preference toward retention because its outcome is more controllable. In this sense, the difference in control is related to risk aversion: subjects were averse to the risk of acquiring *L*-types and thus

 $^{7}$  Recall that in the experiment, we replaced H and L with more neutral A and B to avoid unnecessary framing.

exhibited a tendency to pay extra (on *H*-type retention) to avoid this risk.

To test the impact of the difference in control, we ran two additional experiments:

- *Exogenous-arrivals* experiment. Subjects decided on retention for *H* and *L*-type customers in a situation with exogenous customer arrivals (43/47 in the limited/unlimited cases, Table 1).
- One-type experiment. Subjects decided on acquisition and retention in a situation with only one customer type (weighted average between H and L, see §4.3).

Both additional experiments had the same four treatments as in the main experiment with 12 subjects in each of the eight treatments. The instructions were adapted to both settings. All other elements of the experimental design and analysis/estimation were identical to those of the main experiment.

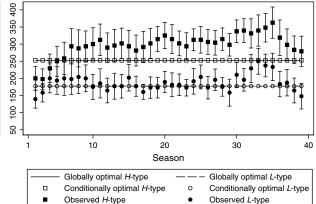
The results of these experiments show the following:

Observation 6. The suboptimal behaviors are not driven by the difference in control.



Observed L-type

Figure 4 Overspending on H-Type Retention in the Exogenous Arrivals Experiment



Exogenous arrival unlimited No CLV—Retention spending

Figure 4 presents two illustrative examples<sup>8</sup> for the exogenous arrivals experiment: the subjects' tendency to overspend on *H*-type retention versus *L*-type retention persists even when the customer arrivals (acquisition) are exogenous. Note that this tendency is present even in the unlimited CLV treatments. This provides a robustness check for Observation 5.

Observed H-type

The left panel in Figure 5 presents an illustrative example for the one-type experiment. The subjects' tendency to overspend on retention and underspend on acquisition persists even when the difference in customer types is eliminated, i.e., when there is no risk of acquiring customers that are less desirable than others. Note also that just as in the main experiment, acquisition is conditionally nearly optimal. This provides a robustness check for Observation 4.

We finally note that in both additional experiments, providing subjects with the CLV information led to an increase in the net revenue in the unlimited capacity cases but to a decrease in the net revenue in the limited capacity cases. This provides a robustness check for Observation 2.

5.4.3. Stock and Flow Biases. Stock and flow biases refer to the difficulties that people have in connecting the state of a system (stock) to the change in the state (flow) (Booth Sweeney and Sterman 2000), especially in situations when flows are nonlinear. In our setting the nonlinearity is in the relationship between the spending and the retention and acquisition outcomes and the resultant net revenue. Since the optimal policy calls for thinking in terms of flows (marginal cost equals incremental value), a difficulty that the subjects might have had in understanding the relationships between stocks and flows could

have had contributed to the suboptimality in their decisions.

To test for the impact of the stock and flow biases, we ran an intervention experiment; for simplicity, we ran it in only the limited CLV treatment:

• One-type (intervention) experiment. Subjects were provided with a spreadsheet that calculated the costs to acquire and retain an incremental customer given the current number of customers and proposed levels of acquisition and retention spending.

The right panel of Figure 5 presents the result of this intervention experiment. The intervention considers the same setting as the one-type experiment (depicted on the left panel), thus a direct comparison between the left and right panels is valid, and the result is striking:

Observation 7. Providing subjects with the ability to assess the incremental costs of acquiring and retaining a customer effectively eliminated underspending on acquisition and considerably decreased overspending on retention.

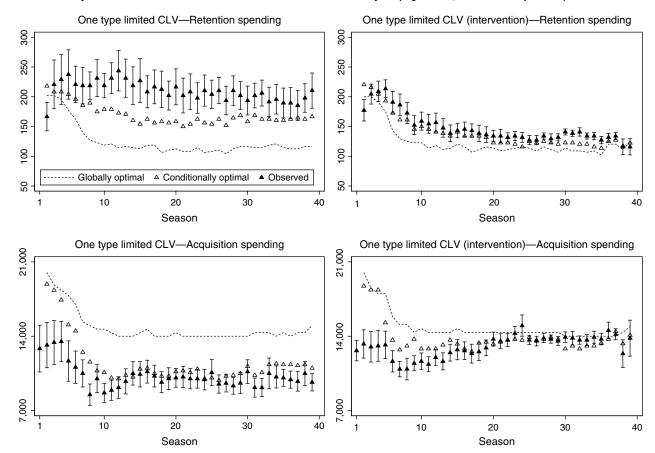
Managerially, and in conjunction with earlier results (Observation 2), this implies that firms in limited capacity situations should refocus their attention from estimating the value of customer relationships to estimating the marginal effectiveness of their acquisition and retention activities. Information about such marginal effectiveness allows managers to better keep the firm optimally at capacity and not be misled by the information about CLV, which disregards capacity limitations and as such should only be used when



<sup>&</sup>lt;sup>8</sup> Following the same rather condensed format as in Figures 2 and 3, there are in total 12 plots for the additional experiments; we do not believe it is feasible to present all of them, thus presenting for only a few illustrative examples.

<sup>&</sup>lt;sup>9</sup> The intervention also decreased the revenue shortfall. However, for the entire experiment, the mean of the observed net revenues was still significantly below optimum, which is due to suboptimal performance in the beginning of the experiment; the median net revenue was not significantly below the optimum. In the second half of the experiment both mean and median net revenues were insignificantly different from optimum.

Figure 5 Overspending on Retention (Top) and Underspending in Acquisition (Bottom) Persist with One Customer Type (Left Panel), But Are Nearly Eliminated When Incremental Cost Information Is Provided to Subjects (Right Panel, Intervention Experiment)



the firm has ample available capacity. Subjects in the intervention experiment realized this, as is evident from their responses to the postexperimental questionnaire, e.g., "[a]t first, [I] used CLV to determine the optimal retention spend level; however, when looking at alternative scenarios at full capacity, it was cheaper to keep a full house by spending less on retention and more on acquisition," and "[i]nitially I tried to maximize CLV, however, later found it more worthwhile to decrease retention spend and simply focus on acquiring new customers."

**5.4.4. Endowment Effect and Loss Aversion.** Kahneman et al. (1991) might also explain the suboptimal behaviors to the extent that subjects were willing to pay more for retaining, i.e., not losing, existing customers as opposed to acquiring new ones. This maps directly to underspending on acquisition, the impact on the unbalance in retention spending between types is more nuanced. Loss aversion could perhaps be at work because the loss from an *H*-churn is larger than from an *L*-churn. Overspending on *H*-type retention was more pronounced in the CLV treatments, i.e., when the difference in the value of losing an *H*-type customer versus an *L*-type was

explicitly presented to subjects. The endowment effect could be impacting our subjects similarly to those in Bordalo et al. (2012), who were willing to pay more to retain something good, but were willing to suffer a loss to get rid of something bad. To the extent that our subjects perceived *L*-type customers as "bad," the endowment effect could have contributed to suboptimality in retention as well. Note, though, that treating *L*-type customers as undesirable is a mistake. With unlimited capacity, *L*-types generate additional value and do not limit the ability to generate value from *H*-types. With limited capacity, a low enough spending on *L*-types can be more profitable than excessive spending on *H*-types (see the discussion at the end of §4.3).

**5.4.5. Impact of "Popular Press."** The impact of popular press could also be a factor because the subjects in our study were business school students who, through their studies or work, may have been aware of the conventional wisdom about the balance between acquisition and retention. Keiningham et al. (2005, p. ix) present a list of "myths" in managing customer relationships, and their top three myths are the following: (1) "The number one goal of any firm should



be customer loyalty." (2) "Firms should emphasize retention efforts rather than acquisition activities." (3) "Companies should strive to make all of their customers attitudinally and/or behaviorally loyal." These myths map directly on the biases we observed and provide anecdotal support for our results.

## 6. Conclusions

This paper discusses the interaction between revenue management and customer relationship management for a firm that operates in a customer retention situation but faces limited capacity. We present a dynamic programming model for how the firm balances investments in customer acquisition and retention, as well as retention across multiple customer types. We characterize the optimal policy and discuss how the policy changes depending on capacity limitations. We then contrast the modeling results with those of a behavioral experiment: such contrasting is especially of interest because in many practical situations acquisition and retention decisions are made by people and not automated systems.

In the modeling part of the paper, we introduce a new concept, the value of an incremental customer, and we show that when capacity is unlimited, VIC equals customer lifetime value, but when capacity is limited, VIC is much smaller and changes dynamically depending on the number of customers and their mix. As a result, the optimal spending is constant and depends on CLV for the firms with unlimited capacity, but changes dynamically and is generally unrelated to CLV when capacity is limited. The reason for this dynamic is in the interplay between the firm's two types of scarce resources: customers and capacity. When the firm has ample empty capacity, customers are relatively more scarce; as a result, VIC is approximately equal to CLV, and the firm spends effectively as an unlimited capacity firm. But when the firm is at or near capacity limit, capacity, and not customers, is scarce; as a result, VIC is considerably smaller than CLV, and the firm trades off the cost of retaining a customer with that of replacing him or her with a new one effectively, regardless of the customer's CLV.

In the experimental part, we introduce a new concept, conditional optimality, for the analysis of state-dependent optimal policies with vector decisions. We apply this concept to the data we collected through an online simulation and document a number of managerially relevant biases. Specifically, we find that (i) in terms of balancing across customer types, subjects overspent on retention of high-value customers and underspent on low-value customers; (ii) in terms of balancing between acquisition and retention subjects underspent on acquisition and overspent on retention; and (iii) in terms of the role of

CLV, CLV information helped subjects when capacity was unlimited, but hurt them when capacity was limited. We show that these suboptimalities hold on an individual subject level, and we discuss a number of drivers of suboptimal behaviors. We show that suboptimal behaviors are not driven by the difference in control, but are likely related to stock and flow biases: providing subjects with the ability to assess the marginal effectiveness of acquisition and retention effectively eliminated suboptimal behaviors.

Overall, our paper suggests that managers who are responsible for acquisition and retention trade-offs need to be more cognizant of the relative costs and benefits (VIC) of each customer, and of the strategic interplay between acquisition and retention activities and their interchangeability when firms' capacity is limited. Instead of viewing low-value customers retention and acquisition as secondary, less-important activities, managers must realize that the firm can benefit more from retaining/acquiring a low-value customer if doing so is inexpensive relative to going out of its way trying to retain a higher-value customer. Managers should also be aware of their tendency to be biased by the CLV information presented in industry reports, e.g., the \$4,896 figure presented in the Oliver Wyman report: the value the firm gets from acquiring/retaining a customer could be much smaller than his or her CLV if doing so requires displacing another customer now or in the future. As such, managers should consider CLVs only when they have ample available capacity, but when capacity is scarce, they should focus instead on assessing the marginal profitability of retention and acquisition activities.

## Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/mnsc.2013.1842.

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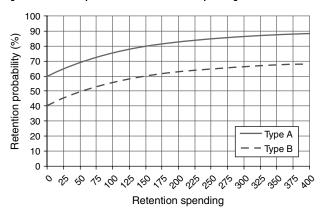
#### Appendix

#### Simulation Instructions and Practice Interface

Instructions for the limited CLV treatment are presented below. In the unlimited capacity treatments, the first line of the "capacity" section in the subjects' instructions read as follows: "Your venue has 270 seats and each customer occupies one seat. This number is larger than what you reasonably expect to fill in, however,...." No changes were made in the questionnaires. In the No CLV treatments, the changes were as follows. CLV questions were removed from



Figure A.1 Responsiveness to Retention Spending



both questionnaires, and the section titled "customer lifetime value" with the corresponding table and plot was removed from the subjects' instructions. In the exogenousarrivals experiment, the content of the section "acquisition spending" was replaced with the following paragraph: "In addition to retention, the opera company also spends money on acquiring new customers. That activity, however, is done by another manager in the company; you do not need to decide on acquisition. That manager will select the amount of acquisition spending before you will make your retention decision, and he will let you know how many customers he expects to be acquired. Such newly acquired customers, however, will represent a mixture of type A and B customers and neither he nor you will know immediately how many As or Bs were acquired. That will become apparent only next season, when they renew (or not)." In the one-type experiment, the instructions and Figure A.1 were adapted so that there is only one type of customer—labeled "a customer." For both experiments, corresponding changes were made to the preexperiment practice questions.

Background. Imagine that you are a customer relationships manager at an opera company. Your sole responsibility is to manage the profit of the opera company. The profit of the opera company equals its revenue minus its costs. Revenues are generated by selling season tickets. Costs correspond to the spending to retain or acquire customers. There is also a cost of staging the productions, but that cost is independent of your activities. Your customers, also called subscribers or members, buy season tickets to see multiple performances by your company. The list price of a season ticket is \$1,000. Your customers may be true opera lovers, and once an individual becomes a member, there is a chance that the individual will renew the membership in the following season. The likelihood of a renewal, however, can be increased by spending some money on retention activities. Such activities could include providing a member with complementary guest tickets, sending a member an opera DVD on their birthday, inviting a member to a reception with a "star" soloist, and the like.

**Retention Spending.** Your market research shows that your customers differ in their response to retention spending. In particular, they can be of two types: type A customers are generally more responsive, and type B are less.

The probability that a customer of each type will renew a subscription (i.e., purchase a season ticket next season) as a function of the spending directed toward a specific customer type this season is depicted on Figure A.1.

For example, if you spend \$0 on retaining a type A customer, then that customer will renew with a probability of about 60%. That is, if this season you have 20 type A customers, then on average 20\*60% = 12 of them will renew next season. Likewise, if you spend \$400 on retaining a type A customer, then some 90% will renew, and so on. You can choose any spending amount in the increment of \$25.

Acquisition Spending. In addition to retaining your existing customers, you can also acquire new customers by spending on publicity and advertising. Unlike the retention, however, the acquisition process is not personalized. This means that if you spend some amount this season, you will get a mixture of type A and B customers showing up to purchase season tickets for the next season, but you will not be able to immediately tell if a customer is type A or type B. That will become apparent only next season when they renew (or not).

Your market research shows that a newly arriving customer is equally likely to be of either type A or B. That is, if 10 new customers arrive, on average, 10\*50%=5 of them will be of type A and 5 will be of type B. Your market research shows that the number of customers you acquire also increases with acquisition spending. You can choose any spending amount in the increment of \$1,000. The relationship between the acquisition spending this season and the total number of new customers arriving next season is depicted in Figure A.2.

Capacity. Your venue has 150 seats, and each customer occupies one seat. However, if the number of customers exceeds the number of seats, then you first accommodate the renewing customers (their number can never exceed the number of seats: even if you had a full house last season, and all customers were retained, then your venue will be exactly full this season as well), and then you accommodate the newly arriving customers on the first-come, first-served basis. That is, if you have to decline access to a newly acquired customer ("bump" him or her), then such a customer is equally likely to be of type A or B.

Your market research shows that such a possibility of being bumped has no impact on the behavior of the current

Figure A.2 Responsiveness to Acquisition Spending

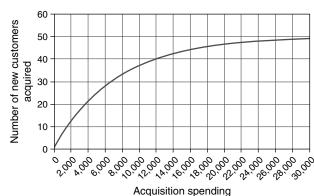




Table A.1. Details of CLV Calculation for Example in the Instructions

	Probability that customer is a member	Revenue (\$)	Cost (\$)	Discount factor (time value of money)	Expected value (\$)		
This season (0)	The customer is acquired, i.e., has purchased a season ticket for the next season						
Season 1	100% = 100.00%	1,000	100	100% = 100.00%	900.00		
Season 2	75% = 75.00%	1,000	100	90% = 90.00%	607.50		
Season 3	75% * 75% = 56.25%	1,000	100	90% * 90% = 81.00%	410.06		
Season 4	75% * 75% * 75% = 42.19%	1,000	100	90% * 90% * 90% = 72.90%	276.79		
Season 5	75% * 75% * 75% * 75% = 31.64%	1,000	100	90% * 90% * 90% * 90% = 65.61%	186.83		
				Total (CLV)	\$2,814.23		

Note. More precisely, the retention probability is 75.58%.

or future customers. Current customers are not affected by bumping since retained customers are accommodated first, and future customers lose nothing if they are bumped. Your opera company enjoys a reputation for its artistic qualities. As a consequence, there will be a group of new customers vying to subscribe for the season tickets in the foreseeable future.

Customer Lifetime Value. As we just discussed, your customers are likely to be true opera lovers and thus have the inclination to renew their season tickets subscriptions and remain members for more than one season. Thus, the immediate profitability that your company gets from a customer next season is an underestimation of the true value the customer brings to the company. A more accurate measure should account for the probability that a customer renews and purchases subscriptions in the future, as well as account for the time value of money such that to equate those future revenues with revenues and costs today. Such a measure is called customer lifetime value.

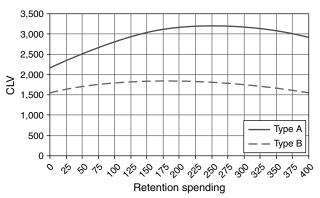
To obtain intuition behind the calculation of CLV, let us consider the example of a type A customer who purchases a season ticket at \$1,000 and renews with a 75% probability when you spend \$100 on retention activities directed toward that customer each season. Then the expected net present value (NPV) of the cash flows from relationship with that customer at the discount rate of 10% can be calculated as shown in Table A.1.

That is, the total expected net present value that your opera company gets over time from such a customer—customer lifetime value or CLV, assuming the constant \$100 spending until the customer leaves, is about \$2,814—which is significantly larger than \$1,000. Your management research tells you that the CLV of a customer changes depending on the amount of retention spending directed toward that customer. The result of such analysis is depicted on Figure A.3.

**Seasons.** You will have a chance to manage the subscriptions for the opera company for approximately 40 seasons. Your goal is to maximize the NPV of the profits over that time period. A discount rate of 10% per season will be applied to future cash flows. This reflects the time value of money and is equivalent to saying that \$100 next season is worth \$100/1.1 = (approximately) \$91 this season.

**Interface.** The interface is presented in Figure A.4. The screenshot is divided into three panes. The left-most pane contains charts with the historical data. You can click on the charts to magnify them. The middle pane contains

Figure A.3 Customer Lifetime Value as a Function of Retention Spending



information about the current season: clock on the top, customer counts in the middle, and revenues, costs, and profitability figures at the bottom. At the bottom of the middle pane, for your reference, the interface also displays the profits from the last season, as well as the NPV of the profits to date. The right-most pane contains information about the next season: your decisions and expected numbers of retained and acquired customers. Note that the decisions made this season are costs this season, but they affect customer counts, and hence revenues, next season. Each season you will have two minutes to make your decisions.

**Summary.** To summarize, each season you have three decisions to make:

- (1) how much to spend on retention per type A customer;
  - (2) how much to spend on retention per type B customer;
- (3) how much to spend (in total) on acquiring new customers.

Good luck!

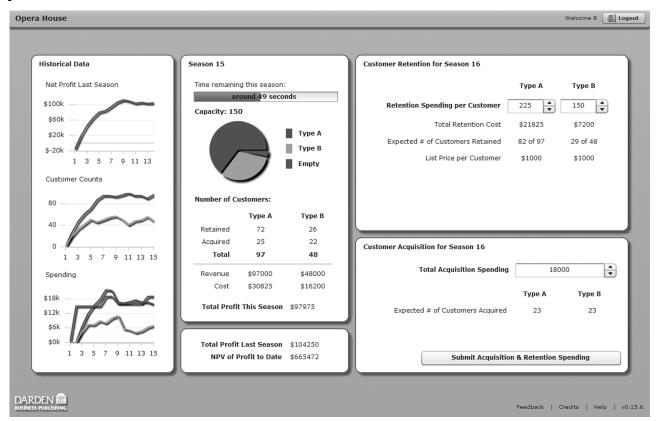
### **Preexperiment Practice Questions**

Your trial session login is [] and your experiment login is []. *Question* 1. Choose three spending levels and, referring to Figures A.1 and A.2, complete the blanks below:

- —If you spend \$[] on retention per type A customer, then that customer's retention probability is []%.
- —If you spend \$[] on retention per type B customer, then that customer's retention probability is []%.
- —If you spend \$[] on acquisition, then the total expected number of acquired customers is [].



Figure A.4 Simulation Interface



Source. Used with permission of Darden Business Publishing, Charlottesville, Virginia, USA.

*Question* 2. Suppose you have 20 type A and 20 type B customers. Then with the spending as in Question 1, you have the following:

- —Your revenue = [] (hint: it is (number of As) \* (list price) + (number of Bs) \* (list price)).
- —Your total retention cost on type A customers is \$[] (hint: it is (number of As) \* (retention spending per A)).
- —Your total retention cost on type B customers is \$[] (hint: it is (number of Bs)\*(retention spending per B).
  - —Your total acquisition cost = \$[].
  - —Your profit this season = [].

*Question* 3. Suppose that this season you have 20 type As and 20 type Bs. Then with the spending as in Question 1 you have the following:

- —Next season, the expected number of retained type A customers is [] (hint: it is (number of As) \* (retention prob A)).
- —Next season, the expected number of retained type B customers is [] (hint: it is (number of Bs) \* (retention prob B)).
- —Next season, the expected number of acquired type A customers is [] (hint: it is (number of acquired) \*50%).
- —Next season, the expected number of acquired type B customers is [] (hint: it is (number of acquired) \* 50%).
- —The total number of type A customers next season is [] (hint: it is (number of retained A) + (number of acquired A)).
- —The total number of type B customers next season is [] (hint: it is (number of retained B)+(number of acquired B)).

—The total number of customers next season is [] (hint: it is (number of A) + (number of B)).

Question 4. Choose two spending levels and, referring to Figure A.3, complete the blanks below:

- —If you spend \$[] on retention activities dedicated to a certain type A customer until that customer leaves, then the lifetime value of that customer is \$[].
- —If you spend \$[] on retention activities dedicated to a certain type B customer until that customer leaves, then the lifetime value of that customer is \$[].

Question 5. Check the correct answer(s):

- (a) If your capacity is greater or equal to the total number of customers next season, then
  - $\square$  you accommodate all customers next season;
- $\hfill\Box$  you decline access to the opera house to some customers;
- $\hfill$ you accommodate all type A customers and bump some type B customers;
- $\hfill \square$  you accommodate all type B customers and bump some type A customers;
- $\square$  you accommodate all renewing customers (of both types A and B) and bump some of the newly acquired customers not knowing which type they are.
- (b) If your capacity is smaller than your total number of customers next season, then
  - ☐ you accommodate all customers next season;
- $\square$  you decline access to the opera house to some customers;
- $\square$  you accommodate all type A customers and bump some type B customers;



- $\square$  you accommodate all type B customers and bump some type A customers;
- ☐ you accommodate all renewing customers (of both types A and B) and bump some of the newly acquired customers not knowing which type they are.

## Postexperiment Questionnaire

- (1) Please enter your password for the experiment here [].
- (2) Describe your strategy. Tell us about your main motivations [textbox].
- —Specifically when retaining Type A customers? [textbox]
- —Specifically when retaining Type B customers? [textbox]
  - —Specifically when acquiring customers? [textbox]
- (3) In your mind, what is the best customer mix? [textbox]
- —When under capacity (early in the simulation)? [textbox]
- —When closer to capacity or at capacity (later in the simulation)? [textbox]
- (4) Now that you have participated in this experiment, what would be, according to you, the optimal strategy in managing revenues of arts companies? [textbox]
  - (5) What is your gender? [drop down list]
  - (6) How old are you? [drop down list]
  - (7) Describe your work experience. [textbox]
- (8) What is your current university program and/or your last degree? [textbox]
- (9) How interested in the arts are you? [four point scale (not interested at all; somewhat interested; generally interested, but not that much; very interested)]
- (10) How interested in management are you? [four point scale (not interested at all; somewhat interested; generally interested, but not that much; very interested)]
- (11) Would you consider being a manager in an arts company? [four point scale (definitely no; unlikely; likely yes)]
- (12) Have you found the concept of customer lifetime value (CLV)] helpful in making your decisions? [four point scale (not helpful at all; somewhat helpful; generally helpful, but not that much; very helpful)]
- (13) How did you use CLV in making retention decisions? [textbox]
- (14) How did you use CLV in making acquisition decisions? [textbox]

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