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Sequential Search and Learning from Rank Feedback: Theory and Experimental Evidence

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This paper studies the effect of limited information in a sequential search setting where a single selection is to be made from a set of random potential options. We consider both a *full-information* problem, where the decision maker observes the exact value of each option as she searches, and a *partial-information* problem, in which the decision maker only learns the *rank* of the current option relative to the options that have already been observed. We develop a model that allows for a sharp contrast between search behavior in the two information settings, both theoretically and empirically. We present the results of an experiment that tests, and supports, the key prediction of our model analysis—limited information induces longer search. Our data further suggest systematic deviations from the theoretical benchmarks in both informational settings. Importantly, subjects in our partial-information conditions are prone to stop prematurely during early stages of the search process and to suboptimally continue the search during late stages. We propose a simple model that succinctly captures the interplay of two symmetric choice and judgment biases that have asymmetric (but opposing) effects on the length of search.

Data, as supplemental material, are available at <http://dx.doi.org/10.1287/mnsc.2014.1902>.

Keywords: sequential search; optimal stopping; behavioral decision making; secretary problem

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1. Introduction

Many decision makers (DMs) face what is known as a *sequential search problem*: they can carefully evaluate a number of options, one after another, but must eventually make a single binding selection. Examples include an unmarried person looking to find a spouse, a young couple looking to buy a new home, a worker looking for a new job, or a financial firm evaluating investment proposals. Individual consumers and firms face many variations of these sorts of sequential search problems, and they have been well studied in the existing literature on labor economics (see Lippman and McCall 1976, Rogerson et al. 2005), marketing science (Moorthy et al. 1997, Zwick et al. 2003), finance (Prastacos 1983), and economic theory (Stigler 1961). Sequential search situations present a difficult task for a DM, who throughout her search process needs to balance the dynamic trade-off between the costs (potentially both explicit costs and opportunity costs) of skipping an option that is currently available and the uncertain benefit of potential future options.

Many models have been proposed in the existing literature to examine the optimal search strategy in the face of this uncertainty. These can be roughly divided into two major modeling paradigms that differ regarding how much information the decision maker

possesses prior to and during the search process. In *full-information* problems, the DM typically (a) has full knowledge of the stochastic process that generates option values and (b) observes the precise values of each inspected option as she searches. In this sense, full-information models view the search process in the absence of learning considerations. Although defensible in some contexts, such assumptions regarding knowledge and information seem restrictive in others. For example, a DM may find herself in a new search environment (e.g., if she is looking at houses for the first time, if she is searching for her first job in a new field) where she has little or no experience evaluating her available options. It may be very difficult for her to assign precise values to a particular option in such a situation. Instead, a DM may only be able to make comparisons between options that she has already observed and decide which she prefers. *No-information* problems, often referred to as “secretary problems,” address such situations. In these models, options can only be ranked relative to one another when they are inspected and assume a specific value to the DM only once the search has ended. Importantly, in such settings, a DM can gain useful context and experience by passing up earlier options, which allows her to better evaluate later options based on their relative rank. For

example, a DM may find it easier to assess the expected value of the best out of 10 options than the best of 2. In other words, in contrast to full-information settings, *learning from experience matters*.¹ These arguments suggest that, in the optimal search policy, learning from rank information will increase the DM's length of search. How this prediction extends to actual search behavior is less clear, however, particularly because choices are directly linked to how DMs interpret rank information. For example, would a decision maker prefer the best out of two options or the third-best of eight?² Our studies are designed to shed light on this issue.

How does limited information affect search behavior? To answer this question, we develop and test a model that preserves the key learning-related characteristic of no-information models—that the DM observes only the rank of the current option relative to all other alternatives observed so far. However, in contrast to no-information models, we maintain the appealing assumption of full-information models that every option ultimately possess some (random) value for the DM (and that the DM knows the distribution from which this value is sampled). It is in this sense that we refer to our model as a *partial-information* problem. Crucially, our model framework allows for a direct comparison to its full-information equivalent and, thus, for a sharp test of the effect of limited information on search behavior. We present the results from a set of experiments designed to test the key prediction of our model analysis—that limited information induces longer search. Our results support this prediction. *Ceteris paribus*, subjects search longer in partial-information settings than in full-information settings.

Do DMs search optimally? Although our results directionally support the predicted effects of limited information, the data also point to systematic deviations from the theoretical predictions *within* each of the two informational settings. In the full-information setting, subjects tend to stop the search too early, in line with previous experimental evidence (Rapoport and Tversky 1970, Cox and Oaxaca 1989, Sonnemans 2000). In contrast, subjects in the partial-information conditions do not stop too early. Although they appear to search the “right amount” on average, this aggregate view masks that they are prone to suboptimally stop during early stages of the search process and to suboptimally continue the search during late stages.

¹ It is important to distinguish this notion of learning from ranks *within* a given search process from the kind of learning we would expect (and will investigate) in the context of a DM engaged in multiple successive search processes.

² In the specific setting of our experimental studies, these two options will, in fact, have the same *expected* value.

To shed further light on this issue, we design a second experiment that directly elicits subjects' beliefs (about the value of the current option) based on the rank feedback of a partial-information setting. Our results show that subjects' beliefs about the current option's value are systematically *mean biased*—at every step of the search process, alternatives with low (high) relative rank are viewed to be better (worse) than the true expectations of a rational Bayesian DM. We illustrate how such biased beliefs, even though symmetric in nature (subjects do not over- or underestimate values on average), have an asymmetric effect on behavior in our partial-information setting, increasing the length of search. Simply stated, the underestimation of relatively attractive options causes the DM to (suboptimally) continue her search, whereas the overestimation of relatively unattractive options fails to cause her to (suboptimally) terminate the search.

To organize the primary behavioral patterns in our data, we propose a simple model that embeds this judgment bias in a stochastic choice model that, by itself, has the opposing effect on search lengths. In the stochastic model, any random erroneous early stopping decision precludes the possibility of an incorrect continuation decision in later stages of the search. The opposite is not true of a random erroneous continuation decision, since the search process remains active, so the net effect of this bias is early stopping on average (see Bearden et al. 2006 for a similar argument). Overall, we provide evidence on how DMs learn from and (mis)interpret rank information and how these judgments affect search behavior in partial-information settings.

2. Literature Review

Our paper is related to an extensive body of literature on optimal and empirical decision making in sequential search contexts. We focus here on previous theoretical and experimental results in two settings: (a) search with full information and (b) search with limited information.

Moser (1956), Sakaguchi (1961), and Karlin (1962) introduce and solve the classical formulation of the full-information problem, in which option values are drawn independently from a known distribution, the DM observes this exact value as each option is considered, and the DM seeks to maximize the expected value of the option that she selects. These models capture the dynamic trade-offs between the opportunity cost of skipping a current option and the uncertain future benefit of continuing the search that derive purely from randomness in the option values. Many variations of this modeling paradigm, where information and option realizations take cardinal values, have been explored in the existing theoretical literature

(Lippman and McCall 1976). In the present paper, we use Karlin's (1962) model as a benchmark that allows for comparison to optimal policies and behavior in our partial-information problem.

No-information, or ordinal, models, exemplified by the classical (Gilbert and Mosteller 1966) and generalized (Mucci 1973) secretary problems,³ make up another main modeling paradigm in the sequential search literature. In these settings, the DM can only make ordinal comparisons between options as she observes them and seeks to maximize a monotonic payoff function of the overall rank of the options selected. This rank-based information structure offers a natural model of learning during the search, where the DM gains information through her search experience (since previously rejected options help her to better evaluate later options). The partial-information problem that we present in this paper builds on a model introduced by Bearden (2006) and extended by Samuel-Cahn (2007), Szajowski (2009), and Ferenstein and Krasnosielska (2010) that preserves this notion of rank-based learning while allowing option values to be sampled from some known distribution. This allows us to use a more natural and graded objective function of maximizing expected discounted value rather than the fixed all-or-nothing-type payoffs in the secretary problem. In turn, and perhaps more importantly, this also keeps the payoff structure consistent with the full-information problem.

A number of experimental studies test stopping behavior empirically. Examples in full-information settings include Rapoport and Tversky (1970), Schotter and Braunstein (1981), Braunstein and Schotter (1982), Hey (1987), Cox and Oaxaca (1989, 2000), and Sonnemans (1998, 2000). Examples in no-information settings include Seale and Rapoport (1997, 2000), who assess stopping times under the classical secretary problem, and Zwick et al. (2003), who test decisions in a sequential search problem with recall. Although most of these studies find that subjects in many instances behave in a manner that is structurally consistent with the optimal policy, empirical results persistently reveal that subjects tend to stop too soon on average relative to the theoretically optimal benchmark. Bearden et al. (2006) investigate experimental decision making in a generalized secretary problem (GSP) and also find a bias toward stopping too early. They suggest that this early stopping can be partially accounted for by systematic overestimation of the probability that an option will ultimately rank highly.

Previous papers also examine the different factors that drive the length of search. The cost of searching is an obvious factor in this regard, with higher search costs decreasing the expected stopping time. In addition, the finite search horizon N also affects search lengths,

with a lower N resulting in earlier stopping times. Both of these effects hold across information settings and have been demonstrated experimentally by Rapoport and Tversky (1970), Schotter and Braunstein (1981), and Cox and Oaxaca (1989) in full-information settings and by Zwick et al. (2003) in a partial-information problem. In the present paper, we consider how the information setting itself affects search behavior.

3. Theory

To examine learning (from ranks) in a sequential search context, we consider two different models—a full-information problem where the DM possesses a full ability to evaluate option values throughout the search and a partial-information problem where the agent learns as the search progresses—by comparing the relative rankings of options she has previously observed. Each of these models is motivated by the cardinal and ordinal information modeling paradigms, respectively, in the existing literature. To allow for direct comparisons between these two information settings, we unify the two frameworks by making several common assumptions, keeping the data-generating process for options and the objective function the same.

In both models, we consider a DM who observes a sequence of N random options, where N is fixed and known by the DM ahead of time. The true value X_m of the m th option is an independent and identically distributed (i.i.d.) continuous random variable with probability density function $f(x)$ and cumulative distribution function (cdf) $F(x)$. After each option is presented, she must decide whether to end the search and accept the most recent option or continue the search and reject it. If an option is rejected, it cannot be recalled and accepted later. The search ends when an option has been accepted or all N options have been rejected, in which case she receives a payoff of zero. Let S denote the index of the option that is selected. If she decides to end her search by selecting the option S , she receives a discounted payoff of $(1/(1+r))^S X_S$, where $r \geq 0$ represents the discount rate for each time period elapsed. The DM's objective is to maximize the expected discounted value of the option she ultimately selects.

3.1. The Full-Information Model

In the full-information setting, the DM observes the exact value of each option as it arrives. Similar versions of this model and the result of Proposition 1 can be found in Moser (1956) and Karlin (1962), but we restate it here to make clear the comparison to our partial-information model and to keep notation consistent.

In forming an optimal policy, the DM uses a sequence of values, denoted by V_m^{FI} , that capture the expected discounted value of rejecting the m th (current) option and continuing the search, excluding the discount factor

³ See Freeman (1983) and Ferguson (1989) for reviews.

$(1/(1+r))^m$ for the time that has already elapsed and assuming that the DM continues to follow an optimal policy. Let $\{c_m\}_{m=1}^N$ be the collection of policy functions $c_m: \text{supp}(X) \rightarrow \{0, 1\}$, which map option m 's value X_m to a decision about whether to continue the search ($c_m(X_m) = 0$) or stop and accept the option ($c_m(X_m) = 1$), and let $S = \arg \min_{1 \leq m \leq N} \{m \mid c_m(X_m) = 1\}$; define

$$V_m^{FI} \equiv \max_{\{c_j\}_{j=m+1}^N} \mathbb{E} \left[\left(\frac{1}{1+r} \right)^{S-m} X_S \mid S \geq m+1 \right]. \quad (1)$$

Note that the continuation value strips out the sunk-cost discount factor $(1/(1+r))^m$, which will be applied to the eventual payoff regardless of the decision.

PROPOSITION 1 (KARLIN 1962). *An optimal policy in the full-information problem is given by stopping if and only if option m 's value $X_m > V_m^{FI}$, where V_m^{FI} satisfies the recursion*

$$V_m^{FI} = \frac{1}{1+r} \left(\int_{V_{m+1}^{FI}}^{\infty} x f(x) dx + V_{m+1}^{FI} \cdot F(V_{m+1}^{FI}) \right) \\ \text{for all } m < N \text{ and } V_N^{FI} = 0.$$

3.2. The Partial-Information Model

In our partial-information setting, the DM observes only the relative rankings of the options that have been observed so far and must decide whether to accept or reject the option based on this ordinal information. This means, for example, that after observing three options with true values $X_1 = 0.6$, $X_2 = 0.5$, and $X_3 = 0.75$, the DM would only be able to tell that the third option was the best, the first option was in the middle, and the second option was the worst of the three. Given her limited information about X_m , the DM's objective is to adopt a strategy to maximize the expected discounted value of the option selected: $\max_{\{s_m\}_{m=1}^N} \mathbb{E}[(1/(1+r))^S X_S]$, where $\{s_m\}_{m=1}^N$ is a collection of policy functions $s_m: \{1, \dots, m\} \rightarrow \{0, 1\}$, which map option m 's rank K_m to a decision about whether to continue the search ($s_m(K_m) = 0$) or stop and accept the option ($s_m(K_m) = 1$), and $S = \arg \min_{1 \leq m \leq N} \{m \mid s_m(K_m) = 1\}$.

Since the DM does not observe the true value of X_m , she must make her decision based on the rank K_m of the current option relative to the m total options that have been observed. In this paper we will be following the order statistic convention $X_{(1)} \leq X_{(2)} \leq \dots \leq X_{(m)}$, meaning that higher values of k correspond to higher values of $X_{(k)}$. The k th order statistic $X_{(k)}$ of m i.i.d. random variables with continuous distribution $f(x)$ has expected value (see, e.g., David and Nagaraja 2003)

$$\mathbb{E}[X_{(k)}] = \frac{m!}{(k-1)!(m-k)!} \cdot \int_{-\infty}^{\infty} x F^{k-1}(x) [1-F(x)]^{m-k} f(x) dx. \quad (2)$$

We are particularly interested in the case where option values are uniformly distributed because this model is quite common in the existing literature and provides an intuitive basis for experimental tests. This also yields an appealing interpretation for how the DM's information structure evolves through the search. The k th order statistic of m i.i.d. $U[0, 1]$ random variables has a beta distribution $B(k, m-k+1)$, so its expected value is simply $\mathbb{E}[X_{(k)} \mid K_m = k] = k/(m+1)$. This means that as m grows larger, the relative rankings provide a finer and finer mesh across $[0, 1]$, which allows the DM to more precisely estimate each option's value, as shown in Figure 1. In other words, added search experience by the DM further fills out her ability to evaluate future options and assess their location within the distribution from which they are sampled.

Define V_m as the expected discounted value of rejecting the current option and optimally continuing the search after the m th option is observed in the partial-information problem:

$$V_m \equiv \max_{\{s_j\}_{j=m+1}^N} \mathbb{E} \left[\left(\frac{1}{1+r} \right)^{S-m} X_S \mid S \geq m+1 \right]. \quad (3)$$

V_m can be thought of as the residual value of the search process to the DM in the partial-information model when she continues to follow an optimal decision rule.

Figure 1 Location of the Expected Values of the Order Statistics $X_{(1)}$ Through $X_{(m)}$ in the Partial-Information Problem for Various Options m When $X \sim U[0, 1]$

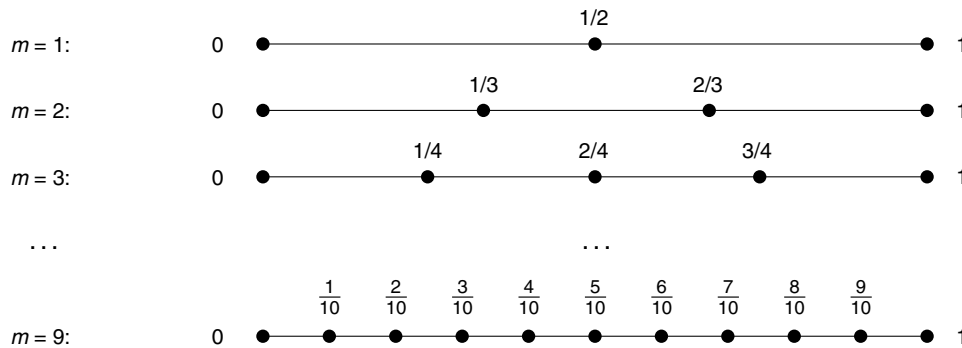
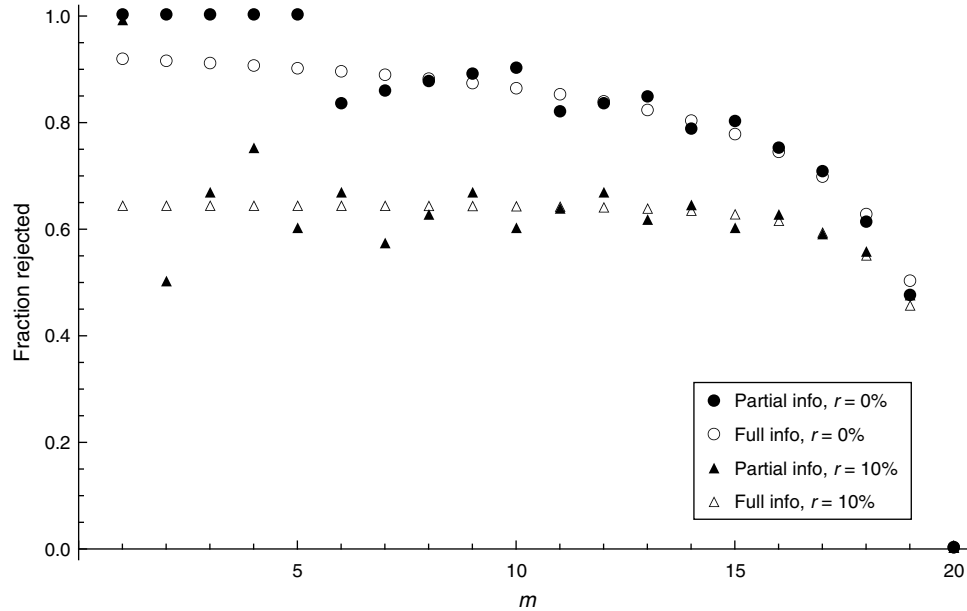


Figure 2 Selectivity (Proportions V_m^{FI} and κ_m^*/m of Options Rejected After the m th Observation) for Full- and Partial-Information Problems, Respectively, for Two Discount Rates r When $X \sim U[0, 1]$



PROPOSITION 2. An optimal policy in the partial-information problem is obtained by stopping after observing the m th option if and only if its rank K_m is strictly greater than κ_m^* , where

$$\begin{aligned} \kappa_m^* &\equiv 0 \quad \text{if } \frac{m!}{(k-1)!(m-k)!} \int_{-\infty}^{\infty} x F^{k-1}(x) [1-F(x)]^{m-k} \\ &\quad \cdot f(x) dx > V_m \quad \text{for all } k \in \{1, \dots, m\} \\ &\equiv \max \left\{ k \left| \frac{m!}{(k-1)!(m-k)!} \int_{-\infty}^{\infty} x F^{k-1}(x) [1-F(x)]^{m-k} \right. \right. \\ &\quad \cdot f(x) dx \leq V_m \left. \right\} \quad \text{otherwise;} \\ V_m &= \frac{1}{(1+r)(m+1)} \left[\sum_{k=\kappa_{m+1}^*+1}^{m+1} \frac{(m+1)!}{(k-1)!(m+1-k)!} \right. \\ &\quad \cdot \left. \int_{-\infty}^{\infty} x F^{k-1}(x) [1-F(x)]^{m+1-k} f(x) dx + \kappa_{m+1}^* \cdot V_{m+1} \right], \\ &\quad \text{and } V_N = 0. \end{aligned}$$

Together, the recursion and boundary condition allow computation of the full sequence of N continuation values V_m as well as the ex ante expected payoff V_0 for any finite partial-information problem. Using the simple stopping criteria, this also simultaneously determines the critical ranks κ_m^* and optimal strategy for the DM.

3.3. The Effect of Limited Information

The key advantage of our modeling framework is that it allows for explicit comparisons between search strategies with full-information and with rank-based learning under limited information. Because the specific structure of the randomness in the option values

and the payoff function remains the same across both models, we can control for the aspects of behavior that are inherent to the random and sequential nature of the search versus the additional learning considerations that ordinal information models seek to capture. We are particularly interested in the effect of limited information on the length of search and how this affects expected payoffs.

3.3.1. Selectivity and Length of Search. Figure 2 displays the selectivity of a DM, expressed by the fraction of options that will be rejected at each stage of the search if she follows the optimal strategy. When the discount rate is zero, indicating that the DM is patient, she can afford to be more selective. In the full-information problem, this means that her continuation values are initially high and only begin to drop as she nears the option limit N . In the partial-information problem, the optimal policy prescribes that she always skip a number of initial options in order to gain information (e.g., when $N = 20$ and $r = 0\%$, she should skip the first five options). She can then use this database of rejected options to better evaluate later options, which leads to better decisions and improvements in the residual expected outcome.

Selectivity in the partial-information problem is not monotonic in m because rank observations are limited to integer values in $\{1, \dots, m\}$, which restricts the fraction rejected to rational numbers $\{0, 1/m, 2/m, \dots, 1\}$. As a result, the selectivity at stage m of the search corresponds to the highest rational fraction $(k/m) \leq 1$ such that $(k/(m+1)) < V_m$, which need not be monotonic in m . Observe that the DM starts to lower her selectivity as she nears the option limit N , illustrating that the

Table 1 Expected Optimal Search Length S in the Partial- and Full-Information Problems with $N = 20$ and $N = \infty$ for Various Discount Rates When $X \sim U[0, 1]$

| | $r = 0\%$ | $r = 1\%$ | $r = 5\%$ | $r = 10\%$ | $r = 20\%$ |
|-------------------------------------|-----------|-----------|-----------|------------|------------|
| Partial-information problem | | | | | |
| Finite ($N = 20$) | 10.99 | 9.50 | 5.53 | 3.49 | 1 |
| Infinite ($N \rightarrow \infty$) | ∞ | 11.03 | 5.56 | 3.49 | 1 |
| Full-information problem | | | | | |
| Finite ($N = 20$) | 8.14 | 6.23 | 3.67 | 2.79 | 2.16 |
| Infinite ($N \rightarrow \infty$) | ∞ | 7.59 | 3.70 | 2.79 | 2.16 |

DM is more willing to compromise when she knows that she only has the opportunity to choose among a handful of remaining options. When there is a positive discount rate, selectivity also gradually decreases as m increases because of the declining marginal benefits of additional experience. When m is low, the DM can afford to be more selective because she knows that rejecting options will help her make a better choice later on. When m is high, rejecting more options adds little to the DM's ability to evaluate future options, so there is less incentive for her to reject the current option.

These selectivity levels can also be used to calculate the distribution of search lengths in different settings, and Table 1 lists some expected search lengths when $X \sim U[0, 1]$.⁴ Unless the discount rate is prohibitively high,⁵ average search lengths in the partial-information setting are higher than with full information, reflecting the informational benefits of rejecting early options when the DM has limited information.

3.3.2. Expected Payoffs. As might be expected, payoffs are always higher in the full-information problem than in the partial-information setting.

PROPOSITION 3. For any fixed point in the search $m \geq 0$ and search limit N , $V_m^{FI} \geq V_m$.

Table 2 compares these expected payoffs at the beginning of the search, before the DM has examined any options. The ex ante negative impact of this limited information can then be computed by comparing V_0 with the corresponding V_0^{FI} from the full-information problem. This difference is highest when the discount rate is high because the DM tends to examine very few options before ending her search (in both information

settings), meaning that she does not have time to build up much experience to help her make a decision in the partial-information setting. As a result, having full information from the beginning of the search yields a large improvement in her ability to make a good decision. Full information still provides benefits to a DM with a low discount rate, but her patience means that she can afford to build up a large database of rejected options that help her make a better decision later on. Accordingly, her expected search payoff does not show as large an increase in the full-information problem relative to the partial-information problem.

4. Study 1

We designed controlled laboratory experiments to test the key predictions from the preceding analysis. Our main interest concerns the effect of information—do decision makers search longer under partial information, as suggested by our analysis above? Relative to the optimal policy, do they stop searching too soon, as suggested by previous empirical evidence for both full and partial information settings? Are the main effects of information robust across environments characterized by different search costs? To answer these questions, we use a 2 (information: full versus partial) \times 2 (discount: none versus 10%) between-subjects experimental design (see Table 3). The number of options, N , was set to 20.

4.1. Task Description

In each of a total of 50 independent rounds ("trials") of the experiment, subjects were presented with a search task described in the previous section. Subjects were instructed to imagine that they were a property agent searching for an apartment for a client and that the housing market was very tight, so available apartments did not remain on the market for long. They were told that they would have enough time to view up to 20 apartments (in each trial). Each time they viewed an apartment, they had to decide whether to secure the apartment or not. If they secured it, they would stop searching; if they did not secure it, they would move on, with the understanding that they would not be able to return to secure an apartment once it was rejected. Subjects were told that the true value of each apartment they viewed was a value drawn at random from the numbers between \$0 and \$20. For the two partial-information conditions, subjects were told that they would not learn the true value until they decided to select an apartment; until then they would only see the rank of the apartment relative to those they had seen so far. Appendix B provides sample instructions from the experiment.

4.2. Software, Recruitment, and Payment

The experiment was implemented in the experimental software z-Tree (Fischbacher 2007). Throughout the

⁴ Note that the finite search horizon N can be relaxed to ∞ in both our full- and partial-information problems. A detailed mathematical treatment of these results can be found in Propositions 4 and 5 and Corollary 1 of Appendix A.

⁵ For example, observe in Table 1 that if $r = 20\%$, then the DM finds the time required to examine options and build up information to be too costly. Instead, she opts to stop on the first option without searching at all (knowing only that its expected value equals 0.5) since the time needed to gather information is more costly than any benefits it can offer.

Table 2 Ex Ante Optimal Expected Search Payoffs V_0 and V_0^{FI} in the Partial- and Full-Information Problems with $N = 20$ and $N = \infty$ for Various Discount Rates r When $X \sim U[0, 1]$

| | $r = 0\%$ | $r = 1\%$ | $r = 5\%$ | $r = 10\%$ | $r = 20\%$ |
|---|-----------|-----------|-----------|------------|------------|
| Partial-information problem | | | | | |
| Finite ($N = 20$), V_0 | 0.857 | 0.776 | 0.608 | 0.518 | 0.401 |
| Infinite ($N \rightarrow \infty$), V_0 | 1 | 0.790 | 0.608 | 0.518 | 0.401 |
| Full-information problem | | | | | |
| Finite ($N = 20$), V_0^{FI} | 0.920 | 0.860 | 0.730 | 0.642 | 0.537 |
| Infinite ($N \rightarrow \infty$), V_0^{FI} | 1 | 0.868 | 0.730 | 0.642 | 0.537 |

Table 3 Experimental Design and Number of Subjects in Each Condition

| Information | Discount | |
|-------------|-----------|------------|
| | $r = 0\%$ | $r = 10\%$ |
| Partial | 33 | 32 |
| Full | 33 | 23 |

experiment, the computer screen displayed all relevant task information. In addition, subjects had access to a history box containing all relevant information from past trials. We recruited subjects from an experimental subject pool at the Pennsylvania State University and conducted all sessions at the Laboratory for Economic Management and Auctions at the Smeal College of Business. After arriving at the laboratory facilities, participants were given the instructions to read and then watched a brief presentation with screenshots of the experimental interface. Subjects then played two training rounds to familiarize themselves with the task environment and the computer interface, with the option to clarify any comprehension issues before proceeding to the actual experiment. A typical session lasted about 35–45 minutes. At the end of the session, subjects were independently paid the average amount that they earned in two trials randomly selected by the computer, in addition to a fixed fee that was adjusted by condition in order to level between-condition differences in expected profits. Compensation (including the fixed fee) varied from \$12 to \$28, averaging \$21, and participants were paid in private at the end of the session; cash was the only incentive offered.

4.3. Results

4.3.1. Preliminaries. For each subject, our data comprise decisions from 50 consecutive trials.⁶ Our analysis focuses on two key outcomes: the stopping decision and the resulting payoffs. Let S_{it} denote the stopping decision of subject i in trial t , with $\pi_{it} \equiv \pi(S_{it})$ denoting the corresponding payoff. Because we only used a small sample from the infinite set of

possible problem instances,⁷ the relevant comparison benchmarks are those predicted by the application of the optimal policy on the same sample of problem instances. When needed, we will denote these as S_{it}^* and π_{it}^* , respectively. For ease of reference, we also provide these benchmarks “in the limit” (i.e., the expected payoffs and search lengths over the entire infinite set of possible problem instances).

4.3.2. Earnings. Table 4 presents observed and optimal payoffs. In all conditions, the realized payoffs were significantly lower than the payoffs predicted by the application of the optimal strategy. As predicted by our model analysis, optimal payoffs are lower in the partial-information setting than in the full-information setting (by \$1.14 when $r = 0\%$ and by \$2.17 when $r = 10\%$). Interestingly, these gaps are larger empirically (\$1.66 when $r = 0\%$ and \$2.57 when $r = 10\%$).

To test for learning across trials, we calculate the difference $\delta_{it}^\pi = \pi_{it} - \pi_{it}^*$ for each subject i and trial t . We fit a simple two-level mixed model, $\delta_{itc}^\pi = \beta_0 + \beta_1 t + u_i + v_c + \epsilon_{it}$, which includes random intercepts at both the condition (c) and the subject-within-condition levels. Although subjects make suboptimal decisions ($\beta_0 = -0.69$, $p < 0.01$), our estimates suggests that subjects learn to make better stopping decisions as they complete more trials, although rather slowly ($\beta_1 = 0.008$, $p < 0.01$).

4.3.3. Stopping Times. Figure 3 displays the cumulative distributions of stopping times (i.e., the probability of stopping at the m th apartment or sooner) under the optimal decision rule (solid lines) compared with the observed cumulative distribution (dashed lines) in each condition. For example, in the full-information condition without discounting, the cumulative probability of stopping after inspection of apartment 6 (or earlier) is about 18% for both the optimal policy and the empirical data.

⁷ The apartment values were randomly and independently sampled from $X \sim U[0, 20]$. To generate a representative set of trials from the general population, we sampled seven sets of 50 trials (with 20 apartment values each), resulting in a total of $50 \times 7 = 350$ independent trials. To reduce noise between conditions, we used the same seven sets of trials in each of the four conditions of our study, and subjects were randomly assigned to one of the seven sets.

⁶ For four subjects in our sample, the data from the last round (i.e., trial 50) are missing as a result of a software issue.

Table 4 Average Optimal and Observed Payoffs

| Payoff | $r = 0\%$ | | $r = 10\%$ | |
|--------------------|------------------|---------------------|------------------|---------------------|
| | Full information | Partial information | Full information | Partial information |
| Optimal (limiting) | 18.40 | 17.14 | 12.83 | 10.36 |
| Observed | 18.05 (0.06) >* | 16.39 (0.11) | 12.33 (0.12) >* | 9.76 (0.12) |
| Optimal (sample) | 18.27 (0.05) >* | 17.13 (0.07) | 12.56 (0.11) >* | 10.38 (0.12) |

Note. Standard errors are clustered at the subject level.

* $p \leq 0.05$; ** $p \leq 0.01$.

As expected, stopping times under partial-information are first-order stochastically higher than stopping times with full information both under the optimal search policy and in the empirical stopping behavior. To formally test this observation, we compare the mean stopping times under the two informational settings in Table 5. Our results confirm that subjects do (and should) search significantly longer in partial-information settings than in full-information settings, indicating that subjects respond to limited information as prescribed by searching further and learning more about the options (through the rank-based information) before stopping.

We next compare empirical stopping behavior relative to the optimal policy. As the unit of statistical analysis, we calculate the difference between observed and optimal stopping time, $\delta_{it}^S = S_{it} - S_{it}^*$, such that positive values for δ_{it}^S indicate late stopping. In the full-information conditions, subjects stop earlier than prescribed by the optimal policy, but this difference is significant only in the discounted condition (2.74 versus 3.03, $p = 0.03$). In the partial-information conditions, subjects on average stop too late, but the difference between observed and optimal is not significant in

either condition. Although this suggests that subjects are mostly “right on average,” Figure 3 shows that these simple averages do not tell the whole story—in the partial-information conditions, we observe suboptimal early stopping early in the search process and suboptimal late stopping late in the search process. The tendency to stop too early in early stages is reminiscent of the pattern commonly detected in experimental tests of the GSP (Bearden et al. 2006) and may well be an artifact of the optimal policy that dictates that a fixed number of apartments is skipped regardless of rank. On the other hand, in contrast to most evidence from previous research on related problems, subjects tend to stop too late once past a certain point in the search process. We will later argue, and explicitly test in Study 2, that this stopping pattern is likely driven at least partially by systematically biased beliefs about apartment values.

To test for learning in stopping behavior across trials, we compute the mean relative stopping times, $\delta_{it}^S = S_{it} - S_{it}^*$, for the first half (block 1: trials 1–25) and second half (block 2: trials 26–50) of the data in Table 6. For the full-information conditions, subjects on average

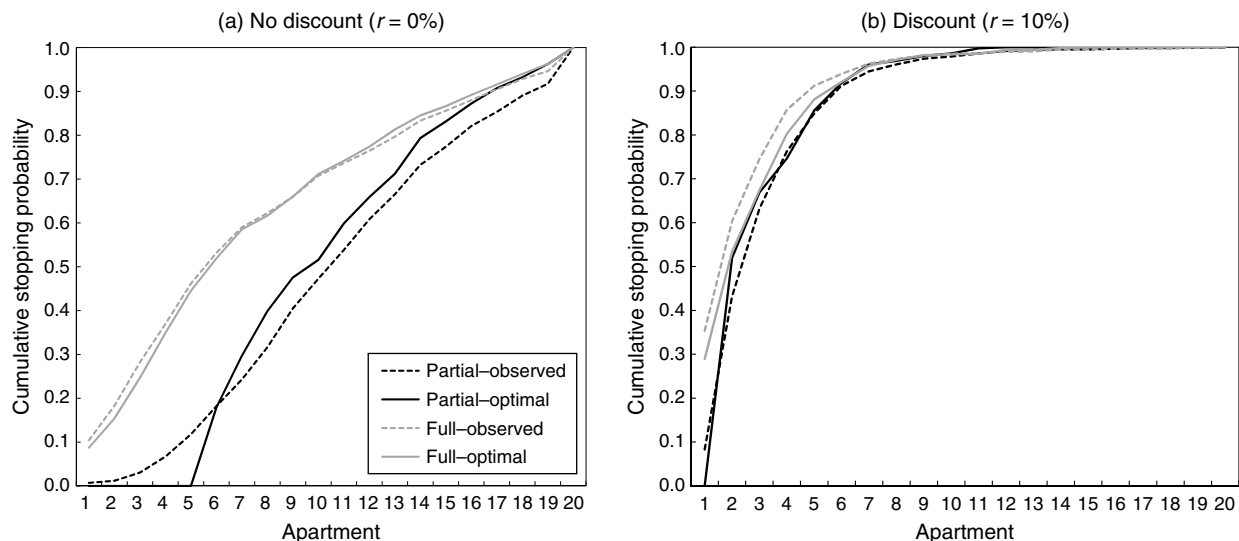
Figure 3 Cumulative Stopping Probabilities

Table 5 Average Optimal and Observed Stopping Times

| Stopping time | $r = 0\%$ | | | $r = 10\%$ | | |
|--------------------|------------------|-----|---------------------|------------------|-----|---------------------|
| | Full information | | Partial information | Full information | | Partial information |
| Optimal (limiting) | 8.14 | | 10.99 | 2.79 | | 3.49 |
| Observed | 7.85 (0.23) | <** | 11.46 (0.41) | 2.74 (0.12) | <** | 3.52 (0.11) |
| | ^ | | ∇ | ^ | | ∇ |
| Optimal (sample) | 8.01 (0.13) | <** | 10.86 (0.09) | 3.03 (0.09) | <** | 3.40 (0.04) |

Note. Standard errors are clustered at the subject level.

* $p \leq 0.05$; ** $p \leq 0.01$.

Table 6 Relative Stopping Times δ_{it}^S by Study Block

| Study block | $r = 0\%$ | | | $r = 10\%$ | | |
|-------------------------|------------------|-----|---------------------|------------------|-----|---------------------|
| | Full information | | Partial information | Full information | | Partial information |
| Block 1 ($t = 1-25$) | 0.20 (0.24) | <** | 0.51 (0.49) | -0.08 (0.12) | <** | 0.36* (0.16) |
| | ∇** | | ^ | ∇** | | ∇** |
| Block 2 ($t = 26-50$) | -0.51* (0.22) | <** | 0.69 (0.43) | -0.50** (0.17) | <** | -0.12 (0.12) |

Note. Standard errors are clustered at the subject level.

* $p \leq 0.05$; ** $p \leq 0.01$.

appear to stop close to the optimal stopping time in the first half of the experiment. However, masked by the pooled analysis of Table 5, subjects stop too soon on average in the second half of the experiment, consistent with previous experimental evidence for the full-information problem (Rapoport and Tversky 1970, Cox and Oaxaca 1989, Sonnemans 2000). In the partial-information conditions, the results are less conclusive. Although subjects in the discount condition seem to move from searching too much to searching too little (0.39 versus -0.12, $p < 0.01$), we do not see much of an effect in the condition without discount.

4.4. A Stochastic Choice Model

Many decision accounts have been proposed to organize observed choices in optimal stopping problems (e.g., in Seale and Rapoport 1997). In the following, we analyze our data through the lens of a simple stochastic choice model that captures the idea that, because of limited cognitive abilities, decision makers are prone to errors (Loomes et al. 2002). Such errors in the choice process can occur for a variety of reasons. For example, the decision maker may make mistakes when assessing the expected values of each available action (stop or continue), when comparing these expected values, or when executing her preferences—i.e., when making a choice.

4.4.1. Model. Recall that under the optimal policy, the DM stops on an apartment m if and only if the apartment's observed value (under full information) or expected value (under partial information) exceeds

a continuation value V_m .⁸ For the sake of notational brevity, let

$$C_m = \begin{cases} X_m & \text{under full information,} \\ \mathbb{E}[X_{(k)} | K_m = k] = k/(m+1) & \text{under partial information,} \end{cases} \quad (4)$$

denote the (expected) value of the current apartment m . To model randomness in choices, we add a random element to the choice rule (similar to Bearden and Murphy 2007). In particular, we assume the decision maker stops the search when $C_m - V_m + \beta\epsilon > 0$, where ϵ is a random variable with standard normal distribution $\Phi(\cdot)$ and $\beta \geq 0$ parameterizes the degree of noise in the decision. After observing apartment m , the DM thus stops her search with probability

$$\begin{aligned} &\mathbb{P}(\text{Stop on apartment } m | S \geq m) \\ &= \mathbb{P}(C_m - V_m + \beta\epsilon > 0) = \Phi\left(\frac{C_m - V_m}{\beta}\right). \end{aligned} \quad (5)$$

As β grows to infinity, the stopping probability approaches 1/2; this implies that the decision maker randomizes between stopping and continuing with equal probability. On the other hand, $\beta \rightarrow 0^+$ models a rational decision maker who makes choices strictly based on the sign of $C_m - V_m$. This simple stochastic choice model has intuitive predictions in the optimal stopping problem.

⁸ For notational ease, we suppress the superscript that distinguishes between partial-information continuation values V_m and full-information continuation values V_m^{FI} in Equations (5) and (6) below. The reader may assume that the term V_m corresponds to the appropriate continuation value for that particular problem (these values will also vary based on the discount rate in that condition).

4.4.2. Intuition. The presence of noise in a DM's stopping decisions reduces the length of search S in both the full- and partial-information settings. When the decision between stopping on apartment m and continuing is close, a positive noise term may cause the DM to erroneously end the search rather than continuing. This results in a decrease in the stopping time that cannot be averaged out by a negative noise term later because the search has already ended. Likewise, when the decision between stopping and continuing is close and the DM observes a negative noise term, she may erroneously continue the search rather than stopping. This increases the stopping time, but because the DM is still searching, there remains a possibility that she observes a positive noise term later that ends the search earlier than expected. Late stopping caused by noise can thereby be moderated to some extent by subsequent noise terms, whereas early stopping cannot. As a result, even though the noise terms are symmetric around zero, their net effect is asymmetric in that they reduce the expected length of search.

4.4.3. Estimation. To test this idea formally in the context of our study, we estimate the single parameter β of the stochastic choice model (5) using the observed stopping decisions in our experimental data. For a particular trial t , let a_{it}^m denote the choice made by subject i when facing apartment m , where $a_{it}^m = 1$ if the DM stops on apartment m . Furthermore, let all decisions for a trial be represented by a vector $\mathbf{A}_{it} = (a_{it}^1, \dots, a_{it}^S) = (0, 0, \dots, 1)$, with $S = S_{it}$ denoting the position of the selected apartment. In all of our analyses of stochastic choice models, we omit the responses from $m = N$ because the DM's decision is already determined when she reaches the last apartment.⁹ As shorthand, let $\mathbf{A} = \{\mathbf{A}_{it} \mid i = 1, 2, \dots, I; t = 1, 2, \dots, T\}$ denote all observed choices in our experimental data. We can then write the likelihood of our data as

$$L(\beta | \mathbf{A}) = \prod_{i=1}^I \prod_{t=1}^T \left[\prod_{m=1}^{S_{it}-1} \bar{\Phi} \left(\frac{C_m - V_m}{\beta} \right) \right] \Phi \left(\frac{C_{S_{it}} - V_{S_{it}}}{\beta} \right), \quad (6)$$

where $\bar{\Phi} = 1 - \Phi$. We maximize the likelihood function over the parameter β , using standard techniques implemented in Stata 12. To account for the repeated-measures nature of our data, we cluster standard errors at the subject level.

⁹ DMs had no choice other than to accept the last apartment since all apartment values were nonnegative. In our formal notation, in the relatively few cases where subject i ended up with apartment N in trial t , we would have $S_{it} = N$, $\mathbf{A}_{it} = (a_{it}^1, \dots, a_{it}^{N-1}) = (0, 0, \dots, 0)$, and the $\Phi((C_{S_{it}} - V_{S_{it}})/\beta)$ term was omitted from the likelihood equation.

Table 7 Estimation Results ($t = 1-50$)

| | $r = 0\%$ | | $r = 10\%$ | |
|--------------------------------|---------------|---------------|---------------|---------------|
| | Full | Partial | Full | Partial |
| β | 1.44** (0.18) | 1.88** (0.17) | 2.32** (0.21) | 2.40** (0.29) |
| LL | -1,360.69 | -2,486.14 | -689.82 | -2,033.30 |
| No. of trials ($T \times I$) | 1,650 | 1,650 | 1,150 | 1,600 |
| No. of observations | 12,840 | 18,767 | 3,153 | 5,629 |

Note. Standard errors are clustered at the subject level.

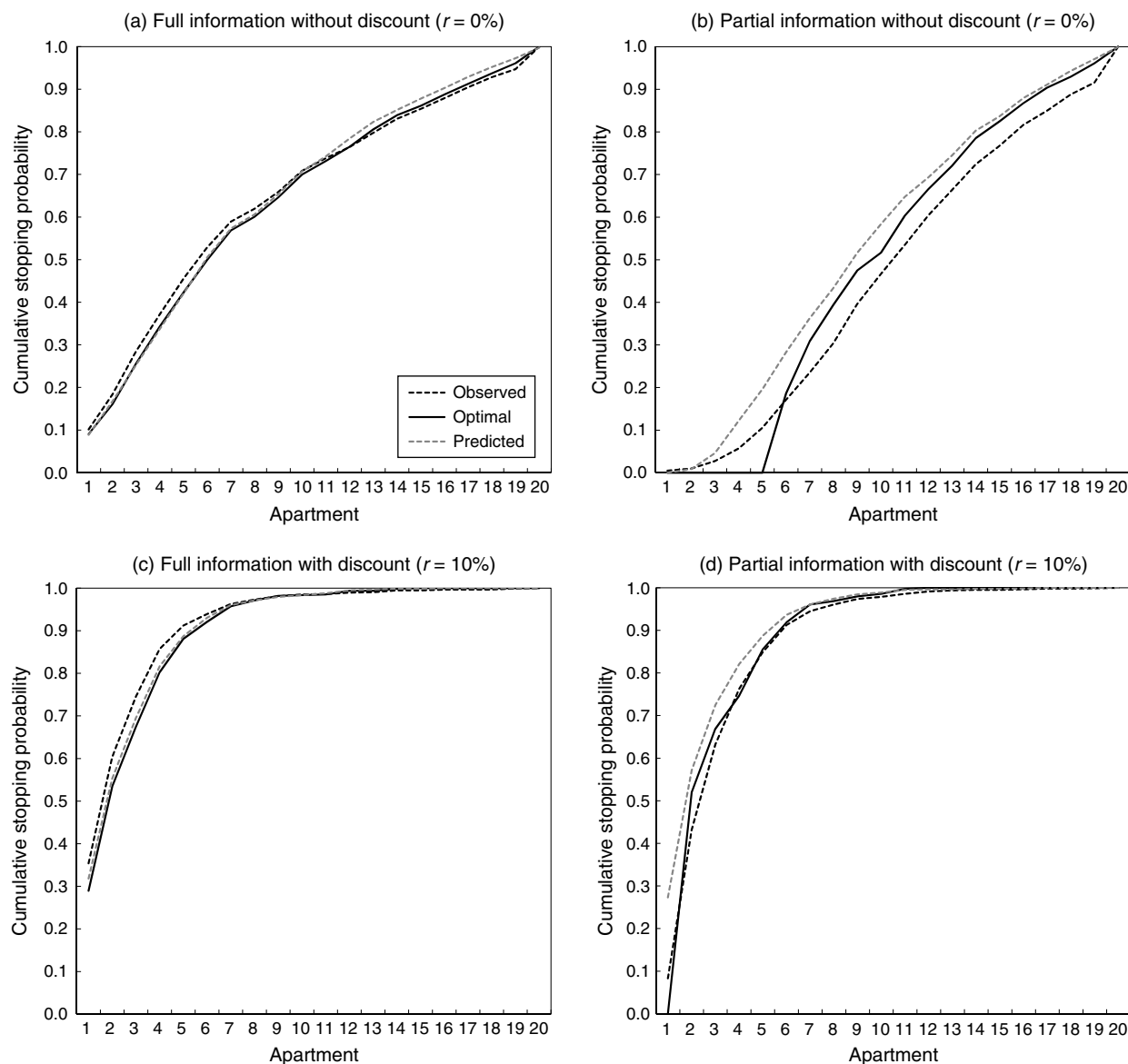
* $p \leq 0.05$; ** $p \leq 0.01$.

4.4.4. Prediction. The estimation results are presented in Table 7. Our main interest at this point of the analysis concerns the stopping behavior predicted by the stochastic choice model *in sample*. To this end, we use the stopping criteria stated in Equation (5) (calibrated with the β estimates in Table 7) to simulate stopping decisions on our input data (i.e., the apartment values, or ranks) for each condition. Figure 4 illustrates that, on the level of aggregation of our analysis, the stochastic model provides a reasonably good in-sample fit with the data from the full-information conditions. However, the model also predicts early stopping under partial information, which evidently is at odds with the observed data from these conditions. These observations leave us with two possible conclusions: (1) It may be that the stochastic choice model simply lacks descriptive validity in optimal stopping contexts. (2) Alternatively, the notion of random choice errors may be a valid one, but the partial-information setting creates systematic biases that (partially) offset the early stopping predicted by stochastic choice model alone. In the following study, we search for evidence on the latter hypothesis.

5. Study 2: Estimation Task

The key observation from Study 1 is that decision makers stopped later under partial-information conditions than when given full information. This result falls in line with the theoretical optimal policy, which predicts that DMs with limited information should be more selective early in the search so that they can build up enough information to make more precise and careful decisions later. Furthermore, we showed that a simple model of stochastic choice organizes much of the data from the full-information conditions but conflicts with the observed stopping behavior in the partial-information conditions. Because we continue to find the idea of random error as a driver for early stopping appealing, we posit that subjects stop *too late*, relative to the prediction of the stochastic choice model, because of systematic bias on the "belief stage" of the stopping task. Study 2 is designed to shed more light on this idea.

Figure 4 In-Sample Predictions



Recall that, to estimate an apartment's expected value, the decision maker has knowledge of three relevant pieces of information: (a) the underlying distribution of apartment values, (b) the number of apartments m she has looked at, and (c) the rank k of the current apartment relative to those m apartments. The estimation task is succinctly captured by the expected value $\mathbb{E}[X_{(k)} | K_m = k] = k/(m+1)$. Ceteris paribus, the expected value of an apartment increases in the rank k and decreases in m (e.g., the second-best of 20 apartments is likely to be better than the second-best of 3 apartments). Although these mechanics are intuitive, human judgment in probability-related tasks is known to be biased for a number of reasons. For example, in an experimental test of the GSP, Bearden et al. (2006) cautiously suggest that decision makers

stop early because of overconfidence—they overestimate the probability that an option that is the best out of a few options is also the best overall.¹⁰ Of course, the overconfidence explanation has to be interpreted with great caution in the context of our study because estimating probabilities (as in Bearden et al. 2006) is quite different from estimating expected values (as in our study) and may introduce significantly different types of bias. For these reasons, we choose to be agnostic before we see the data and do not go beyond the (admittedly, broad) hypothesis that beliefs may be systematically biased.

Testing this hypothesis is empirically challenging because, unlike choices, beliefs are difficult to observe.

¹⁰ The term "optimism" may be more accurate in describing this idea.

One empirical strategy is to infer beliefs from choice data, but this is problematic because of various identification issues that could be overcome only with additional assumptions (Heath and Tversky 1991, Wang 2011). Because reliable direct measurements of beliefs allow for sharper tests of behavioral theories than beliefs inferred from choice data (Nyarko and Schotter 2002), we choose to elicit beliefs directly in Study 2.

5.1. Experimental Design and Implementation

We elicit rank-based beliefs about expected apartment values in the nondiscounted version of the partial information condition from Study 1. This choice was made because we observed the sharpest behavioral discrepancies between the full-information and partial-information conditions. Furthermore, the discount has no bearing on the estimation part of the decision. Eighteen subjects participated in Study 2. The general procedure was the same as in Study 1, with the following exceptions: For the task description, we used the same instructions, but subjects were told that “rather than decide to stop or continue each time you view an apartment, you will be asked to give your *best estimate* for what you believe the *true value* of each encountered apartment is based on its rank. For instance, suppose that you’re viewing the 10th apartment and its rank is 3. In this case, you’d be asked to give your best guess for the true value of the apartment (which you know is somewhere between \$0 and \$20).” Because subjects only submitted estimates, we needed to exogenously fix when to terminate the estimation task within each trial. To do this, we randomly sampled a termination point from the set $\{1, \dots, 20\}$ for each trial. This meant that subjects provided on average approximately 10.5 estimates per trial, which is close to the observed average search length in the corresponding condition in Study 1.

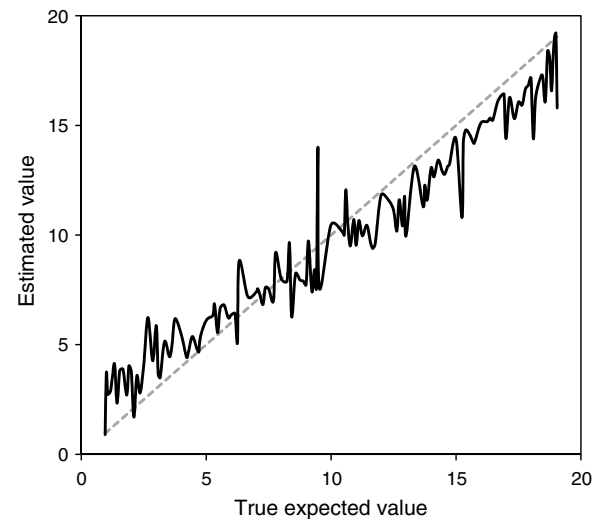
Earnings were based on the accuracy of subjects’ estimates. In particular, at the end of each trial, the computer calculated the mean absolute error (MAE) as the average of all absolute errors (i.e., the difference between estimate and actual) from that trial, and the potential earnings from a completed trial was given by $\$(20 - \text{MAE})$. At the end of the session, total earnings were calculated as the average of two randomly selected trials. Average earnings were \$16.30.

Subjects completed two practice trials to familiarize themselves with the task and the computer interface, followed by 25 trials under incentive-compatible conditions. The number of trials was reduced to 25 because, relative to Study 1, it is more time consuming to submit a value estimate (using a scrollbar) than indicating a stopping decision (using a button).

5.2. Results

5.2.1. Estimation Error. The average mean absolute error (MAE) is significantly larger than the MAE that

Figure 5 Mean Estimated Option Value vs. True Expected Option Value



would be obtained by submitting the true expected value for each apartment¹¹ (3.37 versus 2.58, $p < 0.01$). To test for learning, we fit a simple linear model, $\text{MAE}_{it} = \beta_0 + \beta_1 t + u_i + \epsilon_{it}$, which includes a random intercept at the subject level. The results show that subjects learn to make more accurate estimates as they gain experience with the task ($\beta_1 = -0.02$, $p < 0.01$).

5.2.2. Estimates. Figure 5 displays the mean estimated values as a function of the true expected value. The average estimate is not significantly different from the corresponding average of true values (10.15 versus 10.06, $p = 0.86$); hence, we do not observe a bias toward universal over- or underestimation. However, the data reveal a systematic *regression to the mean* bias. Estimates tend to be too high for apartments ranked lower than the mean (i.e., with true expected value of $\mathbb{E}[X_{(k)}] < 10$) and too low for apartments ranked higher than the mean.

To formally capture this bias, we model the estimated value as function of the observable data and a behavioral parameter γ :¹²

$$v(m, k, \gamma) = \mathbb{E}[X_m | K_m = k, \gamma] = 20 \frac{k + \gamma m}{m + 2\gamma m + 1}. \quad (7)$$

¹¹ These *objective* expected values are determined according to the order statistic in Equation (2) for each given apartment.

¹² The model that we introduce here is closely related to a simple linear regression line through the data in Figure 5, which can be reformulated as $v = (\beta_0 + \mathbb{E}[X_m]) + \beta_1 \cdot (\mathbb{E}[X_{(k)}] - \mathbb{E}[X_m])$. Here, β_0 captures a general bias toward overestimation ($\beta_0 > 0$) or underestimation ($\beta_0 < 0$). The second parameter, β_1 , measures the extent to which estimates are systematically biased toward the unconditional expected value $\mathbb{E}[X_m]$ (for $0 < \beta_1 < 1$). Beliefs are calibrated correctly when $\beta_0 = 0$ and $\beta_1 = 1$. The γ estimates we obtain from model (7) can be mapped into β_1 , but the correspondence changes slightly as m changes. The results we obtain from estimating the direct-mean bias model (not presented here) match up closely with the estimates we obtain from model (7).

In this formulation, γ parameterizes the degree of mean bias. For $\gamma = 0$, the subject possesses no mean bias and the estimated value equals the true expected value $20(k/(m+1))$. As γ increases, the degree of mean bias increases as well, with $v(m, k, \gamma)$ converging to the mean of 10 as $\gamma \rightarrow \infty$ for all periods m and observed ranks k .

5.2.3. Intuition. Consider the DM's estimate of apartment m . Before observing any information about the rank of this apartment relative to those previously evaluated, her prior beliefs are uniform, equal to the generating distribution $U[0, 20]$. After evaluating the apartment and comparing it to the previous $m-1$, she finds its rank k and her updated posterior beliefs about its value follow a $B(k, m-k+1)$ distribution (rescaled to the interval $[0, 20]$). One way to interpret this updating of the posterior distribution is to think of each of the previously observed apartments as "signals" about the value of the current apartment. The $k-1$ previous apartments that are worse than the current one provide a signal that the current apartment has a high value, whereas the $m-k$ previous apartments that are better than the current one provide a signal that the current apartment value is low. These signals are incorporated into the prior uniform distribution, which is a rescaled $B(1, 1)$ and is equivalent to having previously observed one positive signal and one negative signal. The mean bias that we observe can therefore be understood as a biased perception of these signals, where the subject also includes an additional γm positive signals and γm negative signals, which moderate the posterior beta distribution and push it closer to the prior mean of 10. Here, the size of the parameter γ measures in some sense the skepticism the subject feels toward the relative rank information and the extent to which she continues to anchor on the initially equal number of positive and negative signals.

We estimate the parameter γ on the experimental data from Study 2. Let v_{mit}^k denote, for a given trial t , the estimate of subject i for the value of apartment m that has rank k . Individual observations v_{mit}^k are assumed to follow a normal distribution $f(v_{mit}^k | \gamma, \beta) \sim N(v(m, k, \gamma), \beta^2)$. In other words, observations are centered around the estimate predicted by Equation (7) above plus some normal mean-zero error. As in Study 1, β can be thought of as a parameter that captures the magnitude of this noise. The likelihood function for our data is then given by

$$L(\gamma, \beta | \mathbf{V}) = \prod_{i=1}^I \prod_{t=1}^T \prod_{m=1}^{N_{it}} f(v_{mit}^k | \gamma, \beta), \quad (8)$$

where $\mathbf{V} = \{v_{mit}^k | m = 1, \dots, N_{it}; i = 1, 2, \dots, I; t = 1, 2, \dots, T\}$ denotes all elicited estimates in our experimental data and N_{it} denotes the total number of estimates elicited from subject i in trial t . We estimate

the parameters using standard maximum likelihood routines, clustering standard errors at the subject level. The estimation results are summarized as (standard errors in parentheses): $\beta = 2.69$ (0.33), $\gamma = 0.11$ (0.04), and log-likelihood (LL) = $-11,817.19$. Importantly, a simple Wald test against the constrained model (with $\gamma = 0$) provides support for the hypothesis that estimates are biased toward the unconditional mean of the distribution for apartment value ($\chi^2(1) = 7.18$, $p < 0.01$).

5.2.4. Do Biased Beliefs Predict Stopping Decisions? Can the observed bias toward the prior expected value of an apartment help explain, and predict, stopping decisions under partial information? As a first step toward an answer, we next study the stopping decisions that mean-biased estimates of apartment values would predict if they were embedded in an optimal stopping task. In particular, we apply the mean bias model (calibrated with the estimate of γ from the belief data from Study 2) to the sample data from Study 1. Using the resulting predicted estimates of the (expected) apartment value along each sample path, we then calculate the predicted stopping decisions in each trial. To tease out the "raw" effect of biased beliefs, we calculate predicted stopping decisions absent any other behavioral considerations (in particular, without any random error on the choice stage). Figure 6 illustrates how the mean-biased estimation of apartment values results in longer search lengths in the undiscounted partial-information problem. To understand why, note that a DM does not want to stop on apartments whose expected value falls below the mean of the distribution, and a mean bias will not change this decision. However, a mean bias may lower her estimate of an option with a high (true) expected value, causing her to continue her search rather than stopping. Mean-biased estimates therefore increase the distribution of stopping

Figure 6 Cumulative Stopping Probabilities for the Undiscounted Partial-Information Problem, Including Predictions from the Estimation Task and Predictions from the Stochastic Choice Model

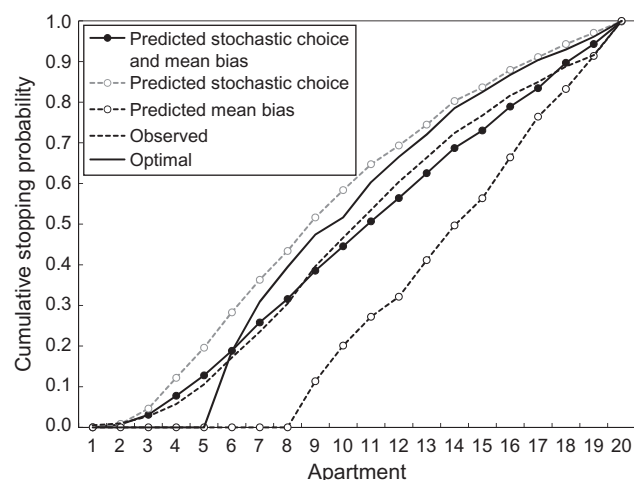


Table 8 Estimation Results with γ

| | $t = 1-25$ | | $t = 26-50$ | $t = 1-50$ |
|-------------|---------------|---------------|---------------|---------------|
| | Belief data | Choice data | Choice data | Choice data |
| β | 2.69** (0.33) | 2.13** (0.26) | 1.69** (0.16) | 1.93** (0.18) |
| γ | 0.11** (0.04) | 0.07** (0.02) | 0.04** (0.02) | 0.06** (0.02) |
| LL | -11,817.19 | -1,254.93 | -1,112.74 | -2,385.93 |
| $\chi^2(1)$ | 7.18 | 12.13 | 4.66 | 10.43 |
| p -value | 0.007 | 0.000 | 0.032 | 0.001 |
| No. of obs. | 4,903 | 9,308 | 9,459 | 18,767 |

Note. Standard errors are clustered at the subject level.

* $p \leq 0.05$; ** $p \leq 0.01$.

times, even though the aggregate effect of mean-biased estimates on apartment values is symmetric.

Although irrefutable on a descriptive level for our belief data, it is evident that the stopping predictions from the mean bias model are at odds with the stopping data, assuming that any other bias is absent. To draw a sharp contrast and to remind the reader of a previous key result, Figure 6 also includes the prediction of the stochastic choice model *without* mean-biased beliefs (calibrated with the choice data from Study 1)—random choice errors tend to result in early stopping. Jointly, these observations lead to the following question: Can a simple model of mean bias (on the belief stage) with random error (on the choice stage) then predict stopping behavior under partial information? As a direct test of this hypothesis, we estimate the mean bias model using only the choice data for the undiscounted partial-information problem from Study 1. Following the general estimation procedures detailed in §4.4, we simply substitute the mean bias model from Equation (7) into the choice rule from Equations (4) and (5) and estimate β and γ jointly via maximum likelihood procedures as before, using the likelihood expression in Equation (6). The results are shown in Table 8, which for easy comparison also includes in the first column the estimation results based on the belief data from Study 2. Furthermore, we report χ^2 values for Wald tests (against the constrained model with $\gamma = 0$), as well as their associated p -values, at the bottom. The choice-based estimations (the right three columns) reproduce the structural mean bias that we had uncovered in the belief data. Although the strength of the effect is somewhat mitigated when estimated on the choice data from Study 1, mean bias seems to remain a significant driver of stopping behavior. Together, the estimation results lend support to the hypothesis that the mean bias matters in a sequential search task.

6. Discussion

To isolate the effect of rank information on search behavior, we develop a theoretical model that allows

for a direct comparison of behavior in both partial- and full-information settings. To our knowledge, a model that leverages this direct comparison between the two main streams of ordinal and full-information models in the existing literature has not yet been presented. Our unified framework allows us to demonstrate some novel results comparing stopping times and average payoffs between the two information settings, both theoretically and empirically.

How does limited information affect search behavior? We demonstrate, both in the optimal policy and experimentally, that limited information motivates a DM to search longer in order to build up information that allows her to make better decisions later in the search. As might be expected, these informational limitations also result in lower optimal and empirical expected payoffs.

Do DMs search optimally? In our full-information conditions, we find evidence that decision makers stop too soon, which is consistent with previous results in the literature. However, in the partial-information conditions, we find no evidence for early stopping. The results from the estimation task in Study 2, together with the structural estimations based on our choice data from Study 1, point to an interplay between two *symmetric* choice and judgment biases, each of which induces an *asymmetric* (but opposing) effect on search behavior under partial information. Although randomness in decision making tends to cause early stopping, mean-biased beliefs about the expected values of the current alternative tend to cause late stopping, leaving the impression that decision makers stop “just right” on average.

On the surface, the results from our partial-information conditions seem to contradict the prior empirical evidence in no-information settings (e.g., Seale and Rapoport 1997, 2000). However, it is important to note the structural difference between these problems and ours. Crucially, and contrary to most experimental implementations of the (generalized) secretary problem, DMs in our context do not run the risk of selecting an option that has zero value. Specifically, in settings where the DM derives utility only from selecting the best (few) option(s) from a relatively large set, the *overestimation* of low probabilities for relatively attractive options may indeed induce the DM to stop too early (Bearden et al. 2006). In contrast, in our setting, *overestimation* of relatively unattractive options has a negligible effect; the *underestimation* of relatively attractive options is a key driver that induces the DM to stop too late.

Our study has natural limitations that, with the extensive body of sequential search literature in mind, suggests a number of opportunities for future studies. First and foremost, for both the full- and the partial-information settings, we assumed that the DM has

full knowledge of the distribution from which the true values of the options are sampled. Although a reasonable approximation for some situations, this assumption is undoubtedly questionable for many other real-world search settings. Our study was deliberately tailored to the creation of *testable* theory, and the DM's full knowledge of the data-generating process is crucial to establish the link between the full- and partial-information settings that we wished to test. Whether relaxations of this assumption make a meaningful difference for search behavior is a remaining empirical question. Although the derivation of optimal search strategies may be overly ambitious for such a setting, we believe that a clean experimental test could provide insights even in the absence of a normative benchmark.

Furthermore, some qualifications are in order regarding the behavioral accounts underlying the stopping decisions that we observe in the four conditions of our study. We do not wish to claim that mean-biased estimates are the *only* drivers that extend the length of search, but we simply suggest that such biased beliefs offer a plausible explanation that is consistent with our data. Whether this central result of our experiment translates into a context without prior knowledge of the data-generating mechanism (i.e., without knowledge of some "mean" to anchor on) is an important open question in our view. In this sense, our study leaves plenty of room for a more nuanced investigation of decision-making and judgment biases.

Supplemental Material

Supplemental material to this paper is available at <http://dx.doi.org/10.1287/mnsc.2014.1902>.

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Appendix A. Proofs of Propositions 1–5 and Corollary 1

PROOF OF PROPOSITION 1. It is optimal to stop iff

$$\left(\frac{1}{1+r}\right)^m X_m > \max_{\{c_j\}_{j=m+1}^N} \mathbb{E}\left[\left(\frac{1}{1+r}\right)^S X_S \mid S \geq m+1\right] \iff X_m > V_m^{FI}.$$

Conditioning Equation (3) on whether the DM stops or continues after examining the next option leads to a simple first-order recursion:

$$V_m^{FI} = \mathbb{E}\left[\left(\frac{1}{1+r}\right)^{S-m} X_S \mid S = m+1\right] \mathbb{P}(S = m+1) + \max_{\{c_j\}_{j=m+2}^N} \mathbb{E}\left[\left(\frac{1}{1+r}\right)^{S-m} X_S \mid S > m+1\right] \cdot \mathbb{P}(S > m+1).$$

The decision rule implies that $S = m+1 \iff X_{m+1} > V_{m+1}^{FI}$, so $\mathbb{P}(S > m+1) = \mathbb{P}(X_{m+1} \leq V_{m+1}^{FI}) = F(V_{m+1}^{FI})$ and $\mathbb{P}(S = m+1) = 1 - F(V_{m+1}^{FI})$. Also,

$$\begin{aligned} & \max_{\{c_j\}_{j=m+2}^N} \mathbb{E}\left[\left(\frac{1}{1+r}\right)^{S-m} X_S \mid S > m+1\right] \\ &= \left(\frac{1}{1+r}\right) \max_{\{c_j\}_{j=m+2}^N} \mathbb{E}\left[\left(\frac{1}{1+r}\right)^{S-(m+1)} X_S \mid S \geq m+2\right] \\ &= \left(\frac{1}{1+r}\right) V_{m+1}^{FI} \end{aligned}$$

and

$$\begin{aligned} & \mathbb{E}\left[\left(\frac{1}{1+r}\right)^{S-m} X_S \mid S = m+1\right] \\ &= \frac{1}{(1+r)(1-F(V_{m+1}^{FI}))} \int_{V_{m+1}^{FI}}^{\infty} x f(x) dx. \end{aligned}$$

Substituting yields the desired first-order recursion. Finally, if the DM reaches option N , there are no remaining options that she can consider; she will receive a continuation value of zero from rejecting the last option: $V_N^{FI} = 0$. \square

PROOF OF PROPOSITION 2. The rank of the m th option is a random variable $K_m \in \{1, \dots, m\}$, so for any collection of policies, $s_m(K_m)$ is a random variable for all m , meaning that S is a random variable as well. After examining the m th option, $m < N$, it is optimal for the DM to stop and accept the current option if and only if the expected value of the current option exceeds V_m , the expected discounted value of rejecting him and optimally continuing the search. Since this expected value is increasing in K_m , there is some threshold policy $\kappa_m^* \in \{0, \dots, m\}$ such that it is optimal for the DM to stop if and only if $K_m > \kappa_m^*$. Using the expression for the expectation in Equation (2), this means that

$$\begin{aligned} \kappa_m^* &= 0 \quad \text{if } \frac{m!}{(k-1)!(m-k)!} \int_{-\infty}^{\infty} x F^{k-1}(x) [1-F(x)]^{m-k} f(x) dx \\ &> V_m \quad \text{for all } k \in \{1, \dots, m\} \\ &= \max \left\{ k \mid \frac{m!}{(k-1)!(m-k)!} \int_{-\infty}^{\infty} x F^{k-1}(x) [1-F(x)]^{m-k} f(x) dx \right. \\ &\quad \left. \leq V_m \right\} \quad \text{otherwise.} \end{aligned}$$

Taking Equation (2) and conditioning on whether the DM stops or continues after examining the next option leads to a first-order recursion: $V_m = \mathbb{E}[(1/(1+r))^{S-m} X_S \mid S = m+1] \cdot \mathbb{P}(S = m+1) + \mathbb{E}[(1/(1+r))^{S-m} X_S \mid S > m+1] \cdot \mathbb{P}(S > m+1)$.

Since the values X_i are independent and identically distributed, the rank K_{m+1} of the $m+1$ st option has an equal probability of taking any of the values $1, 2, \dots, m+1$. The decision rule implies that $S > m+1 \iff K_{m+1} \leq \kappa_{m+1}^*$, so $\mathbb{P}(S > m+1) = \kappa_{m+1}^*/(m+1)$ and $\mathbb{P}(S = m+1) = (m+1 - \kappa_{m+1}^*)/(m+1)$; $\mathbb{E}[(1/(1+r))^{S-m} X_{m+1} | S = m+1]$ can be calculated by conditioning over the possible ranks of the accepted option and, again using the fact that the DM is equally likely to observe each of these ranks,

$$\begin{aligned} & \mathbb{E}\left[\left(\frac{1}{1+r}\right) X_{m+1} \mid K_{m+1} > \kappa_{m+1}^*\right] \\ &= \left(\frac{1}{1+r}\right) \sum_{k=\kappa_{m+1}^*+1}^{m+1} \mathbb{E}[X_{m+1} | K_{m+1} = k] \cdot \left(\frac{1}{m+1 - \kappa_{m+1}^*}\right). \end{aligned}$$

Finally, observe that

$$\begin{aligned} & \mathbb{E}\left[\left(\frac{1}{1+r}\right)^{S-m} X_S \mid S > m+1\right] \\ &= \left(\frac{1}{1+r}\right) \mathbb{E}\left[\left(\frac{1}{1+r}\right)^{S-(m+1)} X_S \mid S \geq m+2\right] \\ &= \left(\frac{1}{1+r}\right) V_{m+1}. \end{aligned}$$

Substituting yields the desired recursion. Finally, the boundary condition states that the continuation value from rejecting the final option N is zero: $V_N = 0$. \square

PROOF OF PROPOSITION 3. Fix any integer $m \geq 0$ and consider the DM's residual search problem (recall that V_m and V_m^{FI} are defined conditional on not having already ended the search before option $m+1$). In the full-information problem, the DM can choose her policy functions $\{c_j\}_{j=m+1}^N$ to mimic exactly the optimal policy $\{s_j\}_{j=m+1}^N$ in the partial-information problem. However, the DM is maximizing her search policy over a larger set of possible strategies, and she may choose a different policy. But if she wants to do so, it must be the case that V_m^{FI} is at least as high as V_m .

Full Information with Infinite Options. When the support of the distribution of option values is bounded (this only requires us to assume that there is some "best" possible option and some "worst" possible option), the option limit N can be relaxed so that the DM is allowed to observe an arbitrarily large number of options. Define the infinite full-information problem to be the limit of the finite full-information problem as $N \rightarrow \infty$.

PROPOSITION A1 (KARLIN 1962). Suppose the support of X is bounded and $r > 0$. Let $V_m^{FI}(N)$ be the expected value of continuing the search after observing the m th option when the maximum number of options is $N \geq m$ in the full-information problem. Then for all fixed $m \geq 0$, $V_m^{FI}(N)$ converges as $N \rightarrow \infty$. Furthermore, $V_m^{FI}(\infty) \equiv \lim_{N \rightarrow \infty} V_m^{FI}(N)$ equals the same constant V_∞^{FI} for all m .

PROOF OF PROPOSITION 4. Fix any integer $m \geq 0$; then $V_m^{FI}(N)$ is easily calculated via backward induction for any finite N . Starting at $N = m$ (this is the lowest N for which $V_m^{FI}(N)$ is defined), increment N upward and consider the sequence $\{V_m^{FI}(N)\}_{N=m}^\infty$. The first term is the boundary condition $V_m^{FI}(m) = 0$. Note that the recursion

from Proposition 1 can be written as $V_m^{FI}(N) = g(V_{m+1}^{FI}(N))$, where $g(y) = (1/(1+r))(\int_y^\infty xf(x)dx + yF(y))$. Observe that for fixed boundaries N and $N+1$, backward induction yields

$$\begin{aligned} V_m^{FI}(N) &= \underbrace{g(g(\dots g(0)\dots))}_{N-m} \quad \text{and} \\ V_m^{FI}(N+1) &= \underbrace{g(g(g(\dots g(0)\dots)))}_{N-m+1}, \end{aligned}$$

so $V_m^{FI}(N+1) = g(V_m^{FI}(N))$. In addition, g is continuous with $g'(y) = (1/(1+r))F(y)$. Then, by the mean value theorem, for all $N \geq m$, there exists $c \in (V_m^{FI}(N), V_m^{FI}(N+1))$ such that $g'(c) = (g(V_m^{FI}(N+1)) - g(V_m^{FI}(N)))/(V_m^{FI}(N+1) - V_m^{FI}(N))$, meaning that $(1/(1+r))F(c) = (V_m^{FI}(N+2) - V_m^{FI}(N+1))/(V_m^{FI}(N+1) - V_m^{FI}(N))$. Since it is a cdf, $0 \leq 1/(1+r) < 1$ and $0 \leq F(c) \leq 1$, so $0 \leq (1/(1+r))F(c) < 1$. In particular, this satisfies $|V_m^{FI}(N+2) - V_m^{FI}(N+1)| \leq (1/(1+r))F(c)|V_m^{FI}(N+1) - V_m^{FI}(N)|$. Since X is bounded between some $[\underline{x}, \bar{x}]$, $V_N^{FI}(m) \in [\min\{0, \underline{x}\}, \bar{x}] \forall m$ and $g(y)$ is a contraction mapping from the compact set $[\min\{0, \underline{x}\}, \bar{x}]$ to $[\min\{0, \underline{x}\}, \bar{x}]$. By the contraction mapping theorem, as $N \rightarrow \infty$, the sequence $\{V_m^{FI}(N)\}_{N=m}^\infty$ converges to a unique fixed point $V_m^{FI}(\infty)$ for all m . Then

$$\begin{aligned} V_m^{FI}(\infty) &= \lim_{N \rightarrow \infty} V_m^{FI}(N) = \lim_{N \rightarrow \infty} \underbrace{g(g(\dots g(0)\dots))}_{N-m} \\ &= \lim_{N \rightarrow \infty} \underbrace{g(g(\dots g(0)\dots))}_N \equiv V_\infty^{FI} \quad \text{for all } m. \quad \square \end{aligned}$$

COROLLARY 1. If $X \sim U[0, 1]$, then $V_\infty^{FI} = 1 + r - \sqrt{r^2 + 2r}$.

PROOF OF COROLLARY 1. As noted in the proof of Proposition 4, $V_m^{FI}(\infty) = g(V_{m+1}^{FI}(\infty))$ and $V_m^{FI}(\infty) = V_{m+1}^{FI}(\infty) \equiv V_\infty^{FI}$. For $X \sim U[0, 1]$, $F(x) = x$ and $f(x) = 1$ with support $[0, 1]$, so

$$\begin{aligned} g(y) &= \frac{1}{1+r} \left(\int_y^\infty xf(x)dx + yF(y) \right) \\ &= \frac{1}{1+r} \left(\int_y^1 x dx + y^2 \right) \\ &= \frac{1+y^2}{2(1+r)}. \end{aligned}$$

Then $V_\infty^{FI} = g(V_\infty^{FI}) = (1 + (V_\infty^{FI})^2)/(2(1+r))$ and $(1/(2(1+r)) \cdot (V_\infty^{FI})^2 - V_\infty^{FI} + 1/(2(1+r))) = 0$. Applying the quadratic formula,

$$\begin{aligned} V_\infty^{FI} &= (1 \pm \sqrt{1 - (1/(1+r))^2})/(1/(1+r)) \\ &= (1+r) \pm \sqrt{(1+r)^2 - 1}. \end{aligned}$$

Note that the first term is greater than or equal to 1, the second term is greater than or equal to 0, and $0 \leq X \leq 1$, so the only attainable solution is $V_\infty^{FI} = 1 + r - \sqrt{r^2 + 2r}$. \square

Partial Information with Infinite Options. The recursion in Proposition 2 is too complex to derive an explicit solution for this infinite problem, but the sequence of values converges as this boundary condition is eased.

PROPOSITION 5. Suppose the support of X is bounded, and let $V_m(N)$ be the expected value of continuing the search after observing the m th option when the maximum number of options is $N \geq m$. Then for all fixed $m \geq 0$, $V_m(N)$ converges as $N \rightarrow \infty$. Define $V_m(\infty) \equiv \lim_{N \rightarrow \infty} V_m(N)$ for all m .

Then $\{V_m(\infty)\}_{m=0}^\infty$ and the corresponding critical ranks $\{\kappa_m^*(\infty)\}_{m=0}^\infty$ determine the optimal policy and expected outcomes in the infinite partial-information problem where the DM does not face an explicit search limit.

PROOF OF PROPOSITION 5. Fix any integer $m \geq 0$; then $V_m(N)$ can be calculated via backward induction for any finite $N \geq m$ (the lowest N for which $V_m(N)$ is defined). Recall that $V_m(N)$ is the expected value of rejecting the current option and continuing the search after observing the m th of N total options and consider the sequence $\{V_m(N)\}_{N=m}^\infty$; $V_m(N+1) \geq V_m(N)$ because an additional option (whose true value is independent of any previous options) is simply added on to the end of the list of remaining selections for the DM. So the DM can follow the exact same stopping strategy as before, when there were N total options, and simply accept the N th with certainty if she reaches that point in the search. This mimics the search process and expected payoffs exactly. However, the DM is maximizing her search policy over a larger set of possible strategies than before, and she may choose a different policy. But if she wants to do so, it must be the case that $V_m(N+1)$ is at least as high as $V_m(N)$. This holds for all N , so $V_m(N)$ is monotonically nondecreasing in N . Also note that $V_m(N)$ is bounded since the support of X is bounded. So letting $N \rightarrow \infty$, $V_m(N)$ converges to a limit $V_m(\infty)$ by the monotonic convergence theorem. Since this holds for all nonnegative integers m , there exists a limiting sequence $\{V_m(\infty)\}_{m=1}^\infty$ that dictates the optimal policy in the infinite partial-information problem. \square

Appendix B. Sample Instructions (Partial Information, No Discount)

You are about to participate in an experiment on judgment. Please note that the tasks and questions in this experiment are designed not to test your knowledge, but to learn about your personal judgments. Moreover, all individual responses are completely confidential and anonymous. If you have any questions, feel free to raise your hand.

Task Description

Imagine that you are a property agent searching for an apartment for a client, and that the housing market is very tight; that is, available apartments do not remain on the market for long. You are not sure about what apartments will be available in the near future, but know that you have enough time to view up to 20 apartments. Each apartment has a *true value* to your client, but you cannot observe this value. Each time you view an apartment, you will learn how good it is (with respect to your client's criteria) relative to those apartments you have already seen. The first apartment will be the best (and worst) apartment you've seen. The second one will either be the best or the second best, and so on. For the sake of this experiment, the true value of an apartment to your client is a number between \$0 and \$20. The true value of each apartment you view is a value drawn at random from the numbers between \$0 and \$20, where all dollar amounts are equally likely to be drawn. Importantly, you do not learn the true value until you decide to select an apartment—until then you only see the rank of the apartment relative to those you have seen so far.

Suppose that the true value of the first apartment you view is \$8.50. You will not see this value; instead, you will

only see that this is the best apartment you have seen: it has a rank of 1. Now, suppose you reject the apartment and continue your search, and that the true value of the second apartment is \$15.30. You will learn that the second apartment has a rank of 1 (because \$15.30 is more than the true value of the first apartment (\$8.50)). Continuing, suppose that you turn down the second apartment and continue to the third one, which has a true value of \$4.90. You will learn that the third apartment has a rank of 3 (it's the worst one you've seen.) You go on to the fourth apartment and it has a true value (which, again, you cannot observe) of \$18.10. You will see that this apartment has rank of 1 (relative to those you have viewed, it's the best so far). Let us imagine that you stop and accept the fourth apartment. In this case, you will earn the true value of the apartment, \$18.10.

Crucially, each time you view an apartment, you have to decide whether to secure the apartment for your client or not. If you secure it, you stop searching; if you do not secure it, you move on, and that apartment will no longer be available. You cannot return to secure an apartment once you have rejected it. And, again, you will only learn the true value of an apartment once you have selected it; until then, you only see the relative rank of an apartment (relative to what you have viewed so far).

Payment

You will complete search rounds for 50 clients. Then, at the end of the experiment, the computer will select two search rounds at random. You will receive the amount you earned on average in the two rounds in cash, in addition to a \$1 show-up fee. Since the trials you'll be paid for are decided entirely randomly, your best strategy is to do as well as you can for each of your clients: each one matters.

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