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Interactive Multicriteria Optimization for Multiple-Response Product and Process Design

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We consider product and process design problems (hereafter collectively called process design problems) that address issues associated with the assessment of optimum levels for process inputs that influence multiple-process performance measures. While this problem context encompasses many possible applications, we focus primarily on multiple-response design problems that have been widely studied in the quality improvement and quality management literature. For such problems, several optimization criteria have been proposed, including maximization of process yield, maximization of process capability, minimization of process costs, etc. In this research, we propose a method that accounts for many of these criteria via a procedure that interacts with and relies on the preferences of a decision maker (DM). The interactive procedure evolves from the convergence of three areas of research: notably, the research in multiple-response design, the research in multicriteria optimization, and recent developments in global optimization. The proposed interactive method is illustrated and comparatively assessed via two well-known problems in multiple-response design. Although the interactive procedure is developed for application in multiple-response design, it is not limited to this problem context. The concepts and methods developed in this research have applicability to problems that can be characterized by process inputs and process performance, such as supply chain management and multidisciplinary design optimization.
(*Multiple-Response Design; Multicriteria Optimization; Global Optimization; Interactive Decision Making; Process Capability*)

1. Introduction

Since the early work of Harrington (1965) and Director and Hachtel (1977), researchers have considered the problem of an "optimal" setting of design factors that influence multiple quality performance characteristics, where optimal design-factor settings for one characteristic are not necessarily optimal for others. Decision issues (i.e., trade-offs, etc.) associated with multiple-response problems are manifested on a regular basis in product and process design. The importance and complexity of such problems, in the glare of differing opinions about appropriate trade-offs, have provided sustained incentive for research in this area. In this regard, there have been a number of approaches proffered that have considered several different objective criteria for the joint optimization of performance characteristics. These approaches broadly include (i) maximizing

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process yield or the joint likelihood that performance characteristics are within design specifications (Director and Hachtel 1977, Brayton et al. 1981, Barton and Tsui 1991, Plante 1999), (ii) maximizing desirability and/or capability of performance characteristics with respect to tolerances and nominal targets (Harrington 1965, Derringer and Suich 1980, Khuri and Conlon 1981, DelCastillo et al. 1996, Kim and Lin 2000, Plante 2001), (iii) minimizing, usually via a Taguchi (1986) quadratic loss function, the cost associated with a process (Kacker 1985, Askin and Goldberg 1988, Pignatiello 1993), (iv) minimizing the variance of a process performance characteristic, subject to the achievement of a nominal target for the average response, which is called a dual-response (variance and mean) approach (Vining and Myers 1990, DelCastillo and Montgomery 1993, Kim and Lin 1998), (v) various partitioning procedures that attempt to partition the design factors into those that influence the mean and those that influence the variance (Hunter 1985, Leon et al. 1987, Elsayed and Chen 1993), and (vi) approaches for Multidisciplinary Design Optimization (MDO) via the Design and Analysis of Computer Experiments (DACE) (Sacks et al. 1989, Simpson et al. 1998, Liu and Batill 2002).

A decision maker (DM) has several possible criteria and approaches. In this paper we consider a method that attempts to generalize many approaches for multiple-response design problems and permits a DM to identify indirectly (without specific knowledge of the preference function) a preferred solution via an interactive/feedback procedure. We will combine research in multiple-response design (Plante 1999, 2001), interactive Multicriteria Optimization or MCO (Miettinen 1999), and global optimization (Tawarmalani and Sahinidis 2002).

We develop an approach that imposes an "unknown" DM preference function on the "efficient frontier" of available, nondominated solutions. Solutions achieved via many of the current approaches could be viewed as throwing directed darts on the efficient frontier, leaving many potentially salient but undetected possibilities. The methods proposed in this paper emanate from the premise that it is the DM who ultimately controls the flight of the dart and who eventually must stand by the mark.

Many interactive MCO approaches have been developed over the last three decades (Miettinen 1999). For this research we adapt an interactive approach proposed by Korhonen and Laakso (1986). Their approach starts from any point (design point) and determines a trajectory of solutions on the efficient frontier. This is accomplished interactively with a DM by projecting parametrically changing reference points onto the efficient frontier in directions implied by the preferences of the DM. At each iteration, the DM is presented with a two-dimensional graph that depicts efficient solution choices. The DM picks a preferred point among the obtained efficient solutions as a new starting point, and the procedure is repeated until the DM is satisfied with a generated efficient solution.

In our implementation, we modify the procedure for projecting solutions onto the efficient frontier as suggested by Steuer (1986) to guarantee generating efficient solutions. All implementations of the approach of Korhonen and Laakso (1986) of which we are aware have been done for polyhedral solution spaces. Because our solution spaces are highly nonlinear, we cannot use parametric optimization; rather, we utilize discrete incremental changes for the reference points to obtain a similar effect. A principle contribution of this research is the development and assessment of the use of parametric MCO in the optimization of multiple-response designs.

In this paper, we first review and discuss the formulation of modeling approaches for the multiple-response design problem as presented in Plante (2001). This discussion provides a mathematical basis in preparation for revised formulations, enabling the use of parametric MCO for multiple-response design problems. Parametric MCO (Korhonen and Laakso 1986) is discussed in §3, with particular emphasis on multiple-response design problems. In §4, two multiple response problems from Myers and Montgomery (1995), which include several performance characteristics and design factors, are used to illustrate the concepts and implementation of the proposed interactive/feedback procedure. Finally, in §5 we present a summary of the research and opportunities for further research.

2. Multiple-Response Design Problems

The multiple-response design problem consists of m performance characteristics that are influenced by a common set of n design factors. There are several approaches for approximating the relationship between performance characteristics and design factors. These approaches include response surface (polynomial) models and interpolating (Kriging) models. While response surface models have a rich history and broad applicability (Myers and Montgomery 1995), the interpolating models were developed in the 1980s primarily for the design and analysis of computer experiments (Sacks et al. 1989). Simpson et al. (1998) and other researchers have conducted comparative assessments of these two approaches and found, via several simulation studies, that neither approach dominates with respect to prediction error. For our research, we choose the more predominant response surface approach for approximating the response relationship between the ith performance characteristic and the design factors via a quadratic response function as follows,

$$Y_i = \beta_{i0} + \sum_{j=1}^{n} \beta_{ij} X_j + \sum_{j=1}^{n} \beta_{ijj} X_j^2 + \sum_{k=1}^{n} \sum_{l>k}^{n} \beta_{ikl} X_k X_l + \varepsilon_i, \qquad i = 1, \ldots, m, \tag{1}$$

where

Y_i = The performance characteristic,

ε_i = An error term that is independently distributed with variance $\sigma^2_{\varepsilon_i}$ and mean zero,

β_{ij} = Coefficients for the linear terms of the ith performance characteristic,

β_{ikl} = Coefficients for the squared and interaction terms of the ith performance characteristic,

X_j = The jth design factor.

Via response surface methodology (Myers and Montgomery 1995), the mean of the performance characteristic is then estimated as a function of the design factors and is designated as $\mu_i(x)$, such that

$$\mu_i(x) = a_{i0} + \sum_{j=1}^{n} a_{ij} X_j + \sum_{j=1}^{n} a_{ijj} X_j^2 + \sum_{k=1}^{n} \sum_{l>k}^{n} a_{ikl} X_k X_l, \qquad i = 1, \ldots, m, \tag{2}$$

where

a_{ij} = Estimated coefficients for the linear terms of the ith performance characteristic and

a_{ikl} = Estimated coefficients for the squared and interaction terms of the ith performance characteristic.

The ith performance characteristic has engineering specification limits that are designated LS_i for the lower specification limit and US_i for the upper specification limit.

The error variance, $\sigma^2_{\varepsilon_i}$, may also be a function of the design factors. To account for this possibility, consistent with methodological approaches used to assess heteroscedastic systems, Plante (2001) proposed the use of estimation techniques associated with reweighted least squares (Neter et al. 1990). The impact of design factors on the error variance can then be approximated as a function of the design factors as

$$\sigma_i^2(x) \cong |\hat{e}_i|^2 + (1 - R^2_{e_i})\mathrm{MSE}_i, \tag{3}$$

where

$\sigma_i^2(x)$ = An approximate partitioning of the impact of design factors on the error variance for the ith performance characteristic into that part explained by the design factors and that part left unexplained,

MSE_i = The mean square error resulting from the response surface estimate for the ith performance characteristic in (2),

$|\hat{e}_i|$ = The predicted standard deviation for the ith performance characteristic, estimated as follows,

$$|\hat{e}_i| = b_{i0} + \sum_{j=1}^{n} b_{ij} X_j + \sum_{j=1}^{n} b_{ijj} X_j^2 + \sum_{k=1}^{n} \sum_{l>k}^{n} b_{ikl} X_k X_l, \tag{4}$$

where

b_{ij} = Response surface estimate of the main-effect coefficient,

b_{ikl} = Response surface estimate of the square and interaction effect coefficients,

and

$$R^2_{e_i} = \frac{\sum_{k=1}^{N} |\hat{e}_k|^2}{\sum_{k=1}^{N} e_k^2}, \tag{5}$$

where

$R^2_{e_i}$ = The coefficient of multiple determination resulting from response surface estimation of (4),

e_k = The actual residual for the kth design point resulting from response surface estimation of (4).

The multiple-response design problem is the determination of the design-factor settings that optimize some stated objective criterion, presumably representing a DM's true preference. For this problem, using concepts of process capability, Plante (2001) proposed a general nonlinear programming formulation for determining the levels of the design factors that maximize a system capability as follows:

$$\underset{X}{\text{maximize}}: \prod_{i=1}^{m} C_i \tag{6a}$$

subject to

$$\frac{\mu_i(x) - LS_i}{3\sigma_i(x)} \geq C_i, \qquad i = 1, \ldots, m \tag{6b}$$

$$\frac{US_i - \mu_i(x)}{3\sigma_i(x)} \geq C_i, \qquad i = 1, \ldots, m \tag{6c}$$

$$C_i \geq 0, \qquad i = 1, \ldots, m \tag{6d}$$

and

$$X = \{x\colon LR_j \leq X_j \leq UR_j,\ j = 1, 2, \ldots, n\}, \tag{6e}$$

where the range limits for the design factors are

LR_j = The lower range limit for the jth design factor that was used for the response surface estimation of (2) and (4),

UR_j = The upper range limit for the jth design factor that was used for the response surface estimation of (2) and (4).

The mathematical model described by (6) provides a flexible framework for multiple-response design, lending itself to the mathematical formulation of several special cases, including maximization of desirability originated by Derringer and Suich (1980) and a max min approach proposed by Barton and Tsui (1991). In particular, the max min approach is defined as follows:

$$\text{SP} = \underset{X}{\text{maximize}}\left\{\underset{i=1,\ldots,m}{\text{minimum}}\,(C_i)\right\}, \tag{7a}$$

where SP stands for the standardized performance measure and

$$C_i = \text{minimum}\left[\frac{\mu_i(x) - LS_i}{\sigma_i(x)}, \frac{US_i - \mu_i(x)}{\sigma_i(x)}\right]. \tag{7b}$$

The standardized performance measure may be formulated as a special case of Problem (6), wherein it seeks a design that strengthens the weakest "link in the chain" of capability among the performance characteristics. It should be noted that this and other max min- or min max-type models may yield weakly efficient solutions that are inefficient (solutions that are not on the efficient frontier). Later, we shall demonstrate this circumstance via an example. In addition, during our discussion of interactive MCO, we will also present a revision of (7), whereby the generation of inefficient solutions is voided.

3. Interactive Multicriteria Optimization (MCO) for Multiple-Response Design

In this section we first briefly discuss interactive MCO. We then describe interactive parametric MCO, paying particular attention to definitions and mathematical presentations directly influencing the development for multiple-response design. We adapt this procedure for assisting a DM in the assessment of preferred designs, combining and revising the mathematical models in (6) and (7) with those of parametric MCO.

3.1. Interactive MCO

Many theoretical developments have been made in the area of MCO, and most of these have been published in the mainstream operations research journals over the last 40 years. In particular, the shortcomings of using linear utility functions and the fact that every efficient solution can be reached using appropriate "achievement-scalarizing functions" regardless of the shape of the solution space have long been known (see, for example, Wierzbicki 1980 and Steuer 1986) by MCO researchers. More recently, researchers in different areas such as combinatorial optimization, process design, and design optimization have started addressing multiple-criteria issues in their applications.

Many interactive MCO approaches have been developed since the early 1970s, and many of the more recent approaches are utilizing achievement-scalarizing functions to generate efficient solutions (see Miettinen 1999). Korhonen and Yu (1997) developed an extension of the approach of Korhonen and

Laakso (1986) for quadratic objective functions. Tappeta et al. (2000) developed an interactive approach for design optimization. Of the many interactive MCO approaches that have been developed, it is difficult to single out a specific one and claim that it works best under all circumstances. Different approaches are better suited for different situations and decision-making styles. The ability of an approach to converge to solutions that are desirable to the DM is very important. The type and amount of preference information required from the DM are also important aspects of choosing a specific method. If the required preference information is not easy to provide for the DM, then the reliability of the results would be questionable. The ease of understanding, the ease of use, and the availability of visual aids provided to the DM are factors that should be considered in evaluating interactive MCO approaches. With a user-friendly software (Korhonen and Wallenius 1988), the approach developed by Korhonen and Laakso (1986), and its variation, implemented for linear problems, perform well in all these aspects. Various comparative studies have demonstrated favorable performance of these approaches (see, for example, Korhonen and Wallenius 1989 and Stewart 1997). These approaches have also been successfully applied in various real-life problems (see, for example, Kananen et al. 1990 and Karpak et al. 1999). Furthermore, Korhonen and Laakso (1986) establish sufficient conditions for optimality of the solution at termination under reasonable assumptions (i.e., under a pseudoconcave utility function and a convex, compact solution space).

Based on our experience with many interactive MCO approaches and the above discussion, we selected the approach of Korhonen and Laakso (1986) as very appropriate for the multiple-response design problem.

3.2. Interactive Parametric MCO

We initiate the development of parametric MCO by first providing a description of an efficient frontier. Let $f_i(x)$ represent the ith objective to be maximized and let X represent the feasible region or solution space. A given solution $x \in X$ is defined as an efficient solution if there does not exist any other feasible solution $x' \in X$ such that $f_i(x') \geq f_i(x)$ for all i and $f_k(x') > f_k(x)$ for at least one k. If there exists such an $x' \in X$, then x is said to be an inefficient solution. The set of all efficient solutions is referred to as the *efficient frontier*. A solution $x \in X$ is said to be weakly efficient if there does not exist any other solution $x' \in X$ such that $f_i(x') > f_i(x)$ for all i. Note that the set of weakly efficient solutions contains all efficient solutions and some inefficient solutions. The inefficient solutions in this set are of particular interest because they commonly appear in applications, and care must be taken to distinguish them from efficient solutions. This point will be further discussed through the example problems in §4.

In the absence of preference information from a DM, any efficient solution is a candidate to be the "best" solution. Different criteria or preferences between DMs and differences caused by particular decision environments would imply differences in what is considered to be the "best" efficient solution. In real applications, the size of the efficient frontier could be too large for the DM to pick a desirable solution without guidance by a methodology. The decision process would benefit from a scheme that permits the imposition of a DM's preference upon the efficient frontier, leading to the identification of the best solution.

The parametric procedure of Korhonen and Laakso (1986), starting from a reference point (feasible or not) having resulting objectives designated by $q = (q_1, q_2, \ldots, q_m)$, projects onto the efficient frontier through the use of an achievement-scalarization program (ASP) as follows:

$$\underset{X}{\text{minimize}}: \quad \alpha - \varepsilon \sum_{i=1}^{m} f_i(x) \tag{8a}$$

subject to

$$\alpha \geq \lambda_i(q_i - f_i(x)), \qquad i = 1, \ldots, m \tag{8b}$$

$$x \in X, \tag{8c}$$

where

λ_i = A weight for the ith objective. For the multiple-response design problem, this would be similar to the relative importance a DM places on each performance characteristic,

$\varepsilon =$ A sufficiently small positive constant that prevents weakly efficient solutions that are not efficient from sneaking through.

Maximizing the minimum *capability* (proposed by Barton and Tsui 1991) corresponds to a special case of the above formulation where $\varepsilon = 0$, $q_i = 0$, $\lambda_i = 1$, $i = 1, \ldots, m$.

Korhonen and Laakso (1986) proposed the following parametric ASP, where the direction vector for improved preference is designated as $d = (d_1, d_2, \ldots, d_m)$ and the magnitude of the step to take in these directions is designated by the parameter θ,

$$\underset{X}{\text{minimize}}: \alpha - \varepsilon \sum_{i=1}^{m} f_i(x) \tag{9a}$$

subject to

$$f_i(x) + \tfrac{\alpha}{\lambda_i} \geq q_i + \theta d_i, \qquad i = 1, \ldots, m \tag{9b}$$

$$x \in X, \tag{9c}$$

In (9b) $q_i + \theta d_i$ corresponds to the ith component of the q vector, and for a given d_i and a specific θ value, the above formulation is the same as the ASP.

During the interactive process, the directions are determined by the responses of a DM, and the step size, θ, is varied from 0 to ∞. A DM is then presented with a resulting two-dimensional graph of efficient solutions where values of the step size $\theta > 0$ are placed along the x-axis and the ordinate corresponds to the objective values achieved for each step size. A DM chooses a preferred solution among the efficient solutions presented on the graph, and based on further interaction with the DM the direction vector, d, is updated. The interactive process continues until the DM is satisfied with a given efficient solution. Later, we will use examples to illustrate various concepts associated with interactive MCO.

When $f_i(x)$ is linear and $X = \{Ax \leq b,\ x \geq 0\}$ (i.e., polyhedral), then the parametric ASP in (9) can be solved in a straightforward manner at each iteration of the interactive process. However, this is not the case for the multiple-response design problem in (6), requiring an adaptation of this procedure, which we now discuss.

3.3. Adaptation for Multiple-Response Design Problems

Using the formulation of the parametric MCO presented in (9), the equivalent representation for the multiple-response design problem, where C_i depicts the capability of the process in terms of the ith performance measure (that is, number of 3σ-deviations of the mean value of the performance measure from the closest specification limit of that performance measure), would be

$$\underset{X}{\text{minimize}}: \alpha - \varepsilon \sum_{i=1}^{m} C_i \tag{10a}$$

subject to

$$\frac{\mu_i(x) - LS_i}{3\sigma_i(x)} \geq C_i, \qquad i = 1, \ldots, m \tag{10b}$$

$$\frac{US_i - \mu_i(x)}{3\sigma_i(x)} \geq C_i, \qquad i = 1, \ldots, m \tag{10c}$$

$$C_i + \frac{\alpha}{\lambda_i} \geq q_i + \theta d_i, \qquad i = 1, \ldots, m \tag{10d}$$

$$C_i \geq 0, \qquad i = 1, \ldots, m \tag{10e}$$

and

$$X = \{x\colon LR_j \leq X_j \leq UR_j,\ j = 1, 2, \ldots, n\}. \tag{10f}$$

Notice that $f_i(x)$ of Formulations (8) and (9) is replaced by

$$C_i = \text{minimum}\left[\frac{\mu_i(x) - LS_i}{3\sigma_i(x)}, \frac{US_i - \mu_i(x)}{3\sigma_i(x)}\right]$$

in Formulation (10). This formulation of the multiple-response design problem essentially establishes, from a DM's perspective, a promising portion of the efficient frontier for design capabilities that a DM then navigates in search of a most preferred set of design capabilities. However, from the general formulation of $\mu_i(x)$ and $\sigma_i(x)$ provided in (2), (3), and (4), it is clear that the solution space defined by (10b)–(10f) is not polyhedral. Thus, the use of a straightforward parametric programming approach proposed by Korhonen and Laakso (1986) is not feasible.

The nonlinear structure of the above problem indicates that there could be local optimal solutions. Recent developments in global optimization address such problems. In particular, Tawarmalani and Sahinidis (2002) develop a branch-and-bound-based approach and software (which they call BARON). Their approach is guaranteed to find the global optimal solution to a large class of mixed-integer nonlinear programs. The fact that the ASP we develop consists of factorable nonlinear functions (in (10b) and (10c)) guarantees that BARON will solve it to global optimality. BARON is currently available from GAMS Corporation in Beta form (see http://www.gams.com/mccarl/newsletter/news7.pdf).

In the examples that follow, we used BARON to solve the ASPs. Because the problem structure does not allow employing parametric programming, we solved the problems consecutively for small increments in the θ value until we achieved the point beyond which the solution did not change. In terms of time and effort, it should be noted that BARON does not provide a practical impediment. Its availability has been noted above and, for each run of the example problems, BARON found the optimal solution within several seconds.

4. Examples

In this section we provide two examples from Myers and Montgomery (1995) to illustrate the concepts and an implementation of the proposed procedure for solving multiple-response design problems. The first example consists of two performance characteristics and serves as a reasonably straightforward vehicle to illustrate various concepts associated with the interactive procedure. The second example, originated by Derringer and Suich (1980), consists of four performance characteristics and serves to illustrate a somewhat more complex and well-known application. Consistent with empirical studies for interactive approaches in MCO, we also use this example to simulate a DM's preference (via an implicit utility function) and to assess the convergence of the proposed interactive procedure to the DM's most preferred solution.

4.1. Example for Two Performance Characteristics

The first example considers a chemical process that consists of two performance characteristics (percent conversion with specification $LS_1 = 80.0$; and thermal activity with specifications $LS_2 = 55.0$ and $US_2 = 60.0$) that are influenced by three design factors (reaction time, X_1; temperature, X_2; and percent catalyst, X_3). A central composite design is used to estimate a quadratic response function for each performance characteristic. The resulting response functions for mean performance are given in Appendix A.

Using (10), the mathematical formulation for this problem is:

$$\underset{X}{\text{minimize}}: \alpha - \varepsilon(C_1 + C_2) \tag{11a}$$

subject to

$$\frac{\mu_1(x) - 80}{3\sigma_1(x)} \geq C_1 \tag{11b}$$

$$\frac{\mu_2(x) - 55}{3\sigma_2(x)} \geq C_2 \tag{11c}$$

$$\frac{60 - \mu_2(x)}{3\sigma_2(x)} \geq C_2 \tag{11d}$$

$$C_1 + \frac{\alpha}{\lambda_1} \geq q_1 + \theta d_1 \tag{11e}$$

$$C_2 + \frac{\alpha}{\lambda_2} \geq q_2 + \theta d_2 \tag{11f}$$

$$\{C_i \geq 0,\ i = 1, 2\} \tag{11g}$$

and

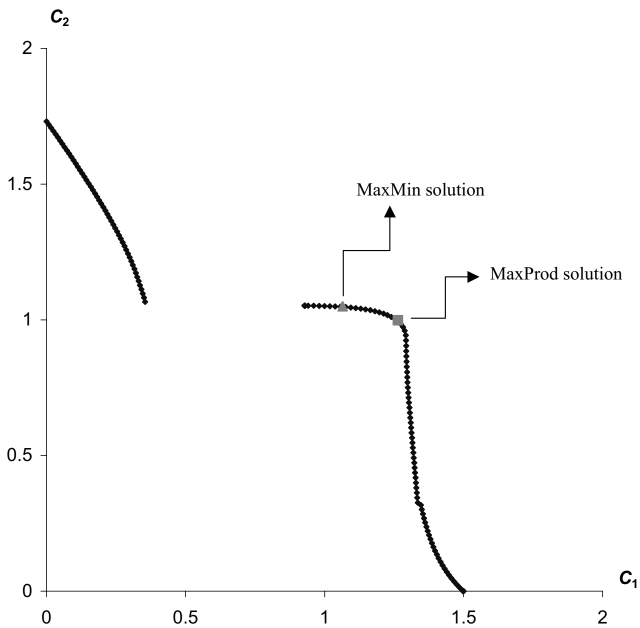
$$X = \{x\colon -1.682 \leq X_j \leq 1.682,\ j = 1, 2, 3\}. \tag{11h}$$

The range for the design factors shown in (11h) are the coded values for the axial points used in the central composite design for estimating (A1) through (A4) in Appendix A.

To find the efficient solutions corresponding to maximum possible values of C_1 and C_2, we first solved the above problem by replacing (11a) with the objective functions: minimize $-C_1 - \varepsilon C_2$ and minimize $-C_2 - \varepsilon C_1$, letting $q_i + \theta d_i = 0$, $\lambda_i = 1$ for $i = 1, 2$. Using the first objective function we obtained $C_1 = 1.51$ and $C_2 = 0$, and using the second objective function we obtained $C_1 = 0$ and $C_2 = 1.73$.

Because this problem has only two criteria, it is possible to approximate and plot the entire efficient frontier. To do this, we set $q_1 = 0$ and $q_2 = 1.73$, the values of the efficient solution having the maximum possible

Figure 1 The Efficient Frontier



C_2 value. We set $d_1 = 1$, $d_2 = 0$, $\lambda_1 = \lambda_2 = 1$. Hence, the right-hand sides (RHSs) of (11e) and (11f) become θ and 1.73, respectively. Solving for increments of 0.02 in θ, we obtain the efficient frontier given in Figure 1. We also solved the problem by maximizing the minimum C_i (proposed by Barton and Tsui 1991) and maximizing the product of C_i (proposed by Plante 2001). As discussed before, the former does not guarantee producing an efficient solution. Both these solutions are also given in Figure 1. It should be noted that the discontinuity in the efficient frontier shown in Figure 1 is not atypical. It simply means that all solutions on this interval of discontinuity are inefficient (i.e., dominated) and thus do not appear on the efficient frontier.

4.2. Example for Four Performance Characteristics

This example was first introduced by Derringer and Suich (1980) as part of their development of the desirability index. This example considers part of the fabrication process for a tire tread compound which consists of four performance characteristics (PICO abrasion index with lower specification $LS_1 = 120.0$; 200% modulus with lower specification $LS_2 = 1000.0$; elongation at break with specifications $LS_3 = 400.0$ and $US_3 = 600.0$; and hardness with specifications $LS_4 = 60.0$ and $US_4 = 75.0$) that are influenced by three design factors (hydrated silica, X_1; silane coupling agent, X_2; and sulfur, X_3). A central composite design is used to estimate a quadratic response function for each performance characteristic. The resulting response functions for mean performance are given in Appendix B.

Using (10), the mathematical formulation for this problem is:

$$\underset{X}{\text{minimize}}: \ \alpha - \varepsilon(C_1 + C_2 + C_3 + C_4) \tag{12a}$$

subject to

$$\frac{\mu_1(x) - 120}{3\sigma_1(x)} \geq C_1 \tag{12b}$$

$$\frac{\mu_2(x) - 1000}{3\sigma_2(x)} \geq C_2 \tag{12c}$$

$$\frac{\mu_3(x) - 400}{3\sigma_3(x)} \geq C_3 \tag{12d}$$

$$\frac{600 - \mu_3(x)}{3\sigma_3(x)} \geq C_3 \tag{12e}$$

$$\frac{\mu_4(x) - 60}{3\sigma_4(x)} \geq C_4 \tag{12f}$$

$$\frac{75 - \mu_4(x)}{3\sigma_4(x)} \geq C_4 \tag{12g}$$

$$C_1 + \frac{\alpha}{\lambda_1} \geq q_1 + \theta d_1 \tag{12h}$$

$$C_2 + \frac{\alpha}{\lambda_2} \geq q_2 + \theta d_2 \tag{12i}$$

$$C_3 + \frac{\alpha}{\lambda_3} \geq q_3 + \theta d_3 \tag{12j}$$

$$C_4 + \frac{\alpha}{\lambda_4} \geq q_4 + \theta d_4 \tag{12k}$$

$$\{C_i \geq 0, \ i = 1, 2, 3, 4\} \tag{12l}$$

and

$$X = \{x: -1.682 \leq X_j \leq 1.682, \ j = 1, 2, 3\}. \tag{12m}$$

The range for the design factors shown in (12m) are the coded values for the axial points used in the central composite design for estimating (B1) through (B8) in Appendix B.

We demonstrate the details of the approach and discuss its implications to management through this example problem. To facilitate an empirical assessment of the interactive procedure, we assume that the DM has an underlying utility function of the form:

$$U = \text{maximize}\left(-\sum_{i=1}^{4} w_i (z_i^* - C_i)^t\right),$$

where w_i is the weight and z_i^* is an ideal point associated with criterion i. $t = 1$ corresponds to a linear utility function and larger values of t put more emphasis on the criteria that are farther from their respective ideal points. Such utility functions have been used in the multicriteria literature (see, for example, Koksalan and Sagala 1995 and Korhonen et al. 1984).

We let $(w_1, w_2, w_3, w_4) = (0.1, 0.2, 0.3, 0.4)$ and $t = 4$. We use $z_i^* = 4$, $i = 1, \ldots, 4$ after checking that $\text{Max}\, C_i \leq 4$, $i = 1, \ldots, 4$ (that is, z_i^* is indeed an ideal or utopia point). We use U only to simulate the responses of the DM. According to this utility function, the optimal solution of the DM is $(C_1, C_2, C_3, C_4) = (0.28, 0.33, 2.16, 3.15)$, which can be obtained by replacing the objective function (12a) by U and solving (12). The optimal solution reflects the preferences of the DM as captured by the weights of the utility function.

For each iteration of the interactive procedure, the parametric ASP in (12) is solved, requiring values for the reference point (q-vector), the direction in which the reference point is changed (d-vector), and the weights (λ-vector). We initiate the procedure by first solving the ASP, setting the RHSs of (12h)–(12k) to zero and $\lambda_i = 1$, $i = 1, \ldots, 4$. This yields an efficient solution where the minimum C_i value is maximized. The resulting solution is $(C_1, C_2, C_3, C_4) = (1.20, 0.51, 0.51, 0.51)$. We next set this solution as the reference point, q, for Iteration 1. This solution indicates that the structure of the problem (that is, the constraint set X) does not allow capabilities larger than 0.51 in all performance characteristics simultaneously.

The direction vector, d, and the weight vector, λ, both affect the direction to pursue in generating efficient solutions starting from the reference point. We let $\lambda_i = 1$, $i = 1, \ldots, 4$ and use only d as the direction to pursue. There are various ways of assessing preference information of the DM to estimate the d vector. A sophisticated approach could be to estimate the local gradient of the utility function of the DM at the current point. Geoffrion et al. (1972) suggest a way of assessing the gradient interacting with the DM. For demonstration purposes, we use the gradient vector as the direction in Iteration 1 and a much simpler information-acquisition process in Iterations 2 and 3. Hence, in Iteration 1, RHSs of (12h)–(12k) are $q + \theta d = (1.20, 0.51, 0.51, 0.51) + \theta(0.55, 2.10, 3.15, 4.20)$. The direction vector shows that the DM would like to improve C_4 most and C_1 least at the current solution.

Solving for different θ values (at increments of 0.02), we obtain the C_i values given in Figure 2a. This graph shows a set of efficient solutions that are encountered in the direction specified by the DM starting from the current solution. The DM may observe possible trade-offs between different criteria as well as a set of efficient solutions available in his/her preferred portion of the complex solution space. Figure 2a indicates, for example, how C_3 and C_4 can be improved in return for the illustrated reductions in C_1 and C_2. Corresponding to each θ value in the graph, there is an efficient solution represented by its C_i values. For example, for $\theta = 1$, we have the solution $(C_1, C_2, C_3, C_4) = (0.42, 0.36, 1.41, 3.40)$.

The graph can be presented to the DM, who then selects the most preferred of these solutions. After evaluating the C_i values of different efficient solutions, the DM may indicate which one he/she likes most. Figure 2b gives the DM's utility values corresponding to each of these solutions as simulated by the underlying utility function U. Consistent with this utility, the DM should choose solution $(C_1, C_2, C_3, C_4) = (0.20, 0.34, 2.07, 3.79)$, corresponding to $\theta = 1.64$. We use this solution as the new reference point, q, and move to Iteration 2.

In Iteration 2, we use a relatively simple procedure to determine a d-vector. Essentially, we query the DM about the criterion he/she would like to improve most at the current solution. The DM could respond C_1 or C_3. Since the marginal utility of C_1 is larger at this point, let us assume that the DM chooses C_1. We then simply set $d = (1, 0, 0, 0)$, change the RHSs of (12h)–(12k) to $(0.20, 0.34, 2.07, 3.79) + \theta(1, 0, 0, 0)$ and solve the problem for different θ values (in increments of 0.02) again. The DM, utilizing a graph similar to

Figure 2 Efficient Solutions Obtained in Iteration 1 (a) Step-Size vs. C_i Values; (b) Underlying Utility Value

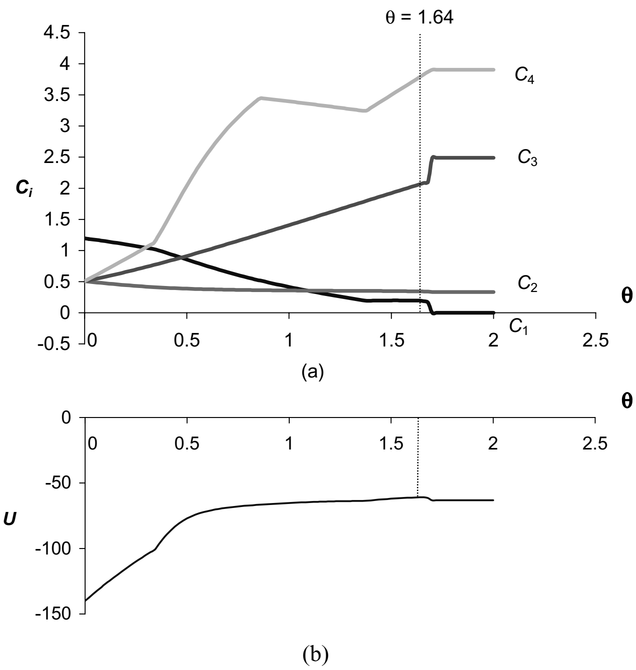


Figure 3 Efficient Solutions Obtained in Iteration 3 (a) Step-Size vs. C_i Values; (b) Underlying Utility Value

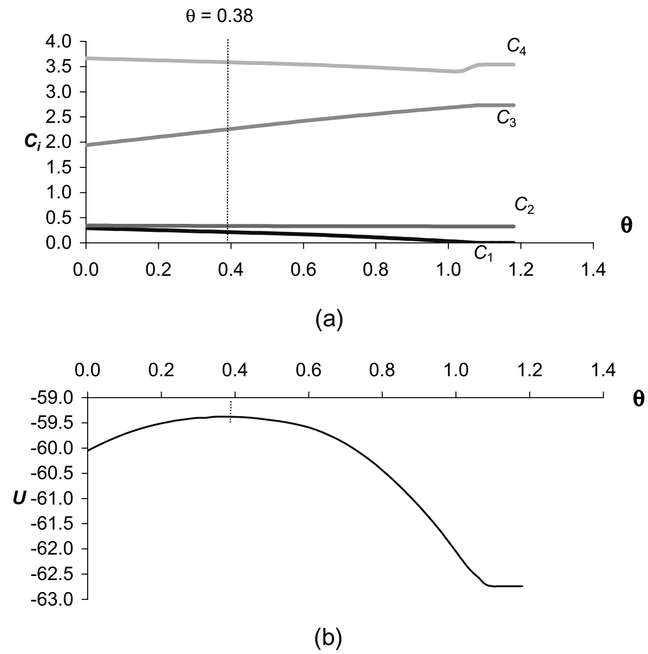


Figure 2a and consistent with the underlying utility function U depicted in Figure 2b, should choose $(C_1, C_2, C_3, C_4) = (0.29, 0.34, 1.94, 3.66)$, corresponding to $\theta = 0.22$.

The DM may wish to stop at any iteration if he/she is satisfied with the current solution. Assuming the DM wishes to continue, we assess the new d-vector. At this point, relative to the underlying simulated preference, the DM would wish to improve C_3. We let $d = (0, 0, 1, 0)$ and the RHSs of (12h)–(12k) become $(0.23, 0.34, 1.94, 3.66) + \theta(0, 0, 1, 0)$.

Using 0.02 increments for adjusting θ values, we obtain the solutions depicted in the graph in Figure 3a. Again, consistent with the underlying utility function U, Figure 3b indicates that the DM should choose solution $(C_1, C_2, C_3, C_4) = (0.22, 0.34, 2.25, 3.59)$, corresponding to $\theta = 0.38$. Assuming that the DM is satisfied with this solution, we stop. Being guided through the efficient frontier by the interactive approach, the DM has arrived at the largest capability value of 3.59 in Criterion 4 (hardness criterion), with a capability value of 2.25 in Criterion 3 (elongation at break criterion), and with relatively smaller capability values for the first two criteria. This solution reflects the DM's relative preferences implied by the assumed utility function. The DM considers the third and fourth performance characteristics relatively more important and chooses to improve them, accepting small capabilities in the first two performance characteristics. The DM is also aware of many efficient solutions along the directions of his/her preferences shown in Figures 2 and 3. The solution process has shown the DM that there are no feasible solutions where all capabilities are larger than 0.51. The DM may find this information useful and may decide to consider relaxing some of the constraints if the obtained capabilities still need improvement. For this example, the ability to achieve such limited capabilities points to broader issues of process and product design improvement. From a managerial perspective, this interactive process results in a better understanding of the various trade-offs involved in establishing and improving process capability.

Note that the obtained solution is very close to the optimal solution we obtained using the utility function U. In fact, the underlying utility value of the

Table 1 **Comparison of Different Solutions**

Approach	C_1	C_2	C_3	C_4	Utility
Max min (InEff)	0.51	0.51	0.51	0.51	−148.73
Max min (Eff)	1.20	0.51	0.51	0.51	−140.05
Max product	0.50	0.36	1.41	3.35	−63.95
Interactive	0.22	0.34	2.25	3.35	−59.38
Optimal	0.28	0.33	2.16	3.15	−59.04

obtained solution is −59.38 compared to that of the optimal solution, −59.04.

If we solve the same problem by maximizing the minimum C_i (as suggested by Barton and Tsui 1991), we obtain a solution $(C_1, C_2, C_3, C_4) = (C_1, 0.51, 0.51, 0.51)$, where C_1 can take values between 0.51 and 1.20 (all of which are alternate optimal solutions in their formulation). All these solutions are inefficient except when $C_1 = 1.20$. This efficient solution can be obtained as a special case of the ASP by setting the RHSs of (12h)–(12k) to zero and $\lambda_i = 1$, $i = 1, \ldots, 4$. If, on the other hand, we maximize the product of C_i variables (proposed by Plante 2001), we obtain $(C_1, C_2, C_3, C_4) = (0.50, 0.36, 1.41, 3.35)$. We summarize these solutions, together with the true optimal solution and the solution obtained by the proposed procedure in Table 1. We also give the corresponding utility values. It is easy to see that approaches that do not take into account the DM's preferences may perform very poorly for specific DMs.

5. Summary and Discussion

We have considered the multiple-response design problem for assessing optimal levels of process inputs that influence the joint performance of multiple-process output measures. Our proposed procedure accounts for many previously proposed optimization criteria and, importantly, actively includes the DM in the solution process. The development of this procedure relies upon and evolves from a convergence of three important areas of research: multiple-response design, interactive multicriteria optimization, and global optimization.

A discussion and development of multiple-response design models is presented. These models are then revised in a manner that facilitates the use of interactive multicriteria optimization. In so doing, model structure issues were uncovered, requiring the implementation of recent developments in global optimization, assuring that proposed optimal solutions were not local optimums.

Two well-known problems in the multiple-response design literature were used to illustrate and to empirically assess the interactive procedure with other proposed methodologies for solving multiple-response design problems. The first problem had a solution space that allowed capabilities above one in both performance characteristics. The second problem, on the other hand, demonstrated a restricted solution space where some of the performance characteristics would have to assume rather small capabilities. Such information could highlight areas for investment in process and product design improvement. It was shown that some previously proposed criteria (i.e., max min-type criteria) could result in inefficient solutions, whereas the proposed interactive procedure is guaranteed to achieve an efficient solution. Further, using a simulated preference function for a DM, it was shown that the interactive procedure converges reasonably well to a known optimal solution.

As previously mentioned, there are different ways of implementing the interactive procedure. These variations are mostly related to the type of preference information required of the DM. In the final analysis, the purpose of the interactive procedure is to sample solutions on the efficient frontier toward preferred solutions of the DM. It is the authors' belief that the versions which ask for simpler preference information may be more desirable because they place a relatively smaller cognitive burden on the DM, and the DM may be more confident with the responses and therefore with the process, leading to a higher likelihood that a solution will be implemented.

The methods proposed in this research have some limitations that should be mentioned. First, we assume that the quadratic response function can be estimated and is representative of the process under study, accurately reflecting the influence of design parameters on process performance measures. Secondly, the use of the interactive procedure may become too cumbersome for a DM when the number of performance measures is large. Based on the research of Miller (1956), it is agreed by many authors that a DM may find it difficult to compare solutions directly using roughly seven or more performance measures.

The procedures developed in this research have broad applicability beyond product and process design. Essentially, any managed process can benefit from the use of these procedures where processes can be characterized by performance measures, performance standards, and inputs influencing performance. For example, processes that are endemic to supply chain management could be better understood and improved. For such studies, the required response surface estimates could be obtained via designed experiments that are conducted using simulation models for supply chain management, which already enjoys a vigorously researched and well-established literature. The investigation and study of such problems would provide interesting and potentially fertile ground for furthering the research in the area of multiple-response design and expanding the scope of its implementation.

Finally, the real proof of value of a methodology is in the using. A better assessment of our methodology awaits its implementation with real DMs on real product and process design tasks.

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Appendix A

Percent Conversion (Y_1)

$$\mu_1(x) = 81.091 + 1.028X_1 + 4.040X_2 + 6.204X_3 - 1.834X_1^2 + 2.938X_2^2 - 5.191X_3^2 + 2.125X_1X_2 + 11.375X_1X_3 - 3.875X_2X_3 \quad (A1)$$

with

$$\sigma_1^2(x) = |\hat{e}_1|^2 + (1 - 0.337)22.25$$

and

$$|\hat{e}_1| = 4.038 - 0.183X_1 - 0.322X_2 + 0.784X_3 - 0.913X_1^2 - 0.913X_2^2 - 0.571X_3^2 - 0.412X_1X_2 + 0.119X_1X_3 - 0.119X_2X_3. \quad (A2)$$

Thermal Activity (Y_2)

$$\mu_2(x) = 59.850 + 3.583X_1 + 0.255X_2 + 2.230X_3 + 0.835X_1^2 + 0.075X_2^2 + 0.057X_3^2 - 0.388X_1X_2 - 0.038X_1X_3 + 0.313X_2X_3 \quad (A3)$$

with

$$\sigma_2^2(x) = |\hat{e}_2|^2 + (1 - 0.963)3.109$$

and

$$|\hat{e}_2| = 0.773 - 0.191X_1 - 0.019X_2 - 0.071X_3 + 0.619X_1^2 - 0.207X_2^2 - 0.008X_3^2 + 0.033X_1X_2 + 0.358X_1X_3 - 0.263X_2X_3. \quad (A4)$$

Appendix B

PICO Abrasion Index (Y_1)

$$\mu_1(x) = 139.119 + 16.494X_1 + 17.881X_2 + 10.907X_3 - 4.010X_1^2 - 3.4471X_2^2 - 1.5721X_3^2 + 5.125X_1X_2 + 7.125X_1X_3 + 7.875X_2X_3 \quad (B1)$$

with

$$\sigma_1^2(x) = |\hat{e}_1|^2 + (1 - 0.538)31.49$$

and

$$|\hat{e}_1| = 3.993 + 0.105X_1 - 0.105X_2 - 0.105X_3 - 0.697X_1^2 + 0.566X_2^2 - 0.788X_3^2 - 0.282X_1X_2 - 1.756X_1X_3 + 0.381X_2X_3. \quad (B2)$$

200% Modulus (Y_2)

$$\mu_2(x) = 1261.13 + 268.151X_1 + 246.503X_2 + 139.485X_3 - 83.566X_1^2 - 124.816X_2^2 + 199.182X_3^2 + 69.375X_1X_2 + 94.125X_1X_3 + 104.375X_2X_3 \quad (B3)$$

with

$$\sigma_2^2(x) = |\hat{e}_2|^2 + (1 - 0.874)108039$$

and

$$|\hat{e}_2| = 80.921 + 5.985X_1 + 3.708X_2 - 110.881X_3 + 20.999X_1^2 + 20.999X_2^2 + 109.523X_3^2 + 68.104X_1X_2 - 8.077X_1X_3 - 8.077X_2X_3. \quad (B4)$$

Elongation at Break (Y_3)

$$\mu_3(x) = 400.385 - 99.666X_1 - 31.396X_2 - 73.919X_3 + 7.933X_1^2 + 17.308X_2^2 + 0.433X_3^2 + 8.750X_1X_2 + 6.250X_1X_3 + 1.250X_2X_3. \quad (B5)$$

with

$$\sigma_3^2(x) = |\hat{e}_3|^2 + (1 - 0.720)422.3$$

and

$$|\hat{e}_3| = 20.147 - 0.550X_1 - 2.232X_2 + 0.848X_3 - 6.319X_1^2 - 0.594X_2^2 - 5.476X_3^2 + 0.916X_1X_2 - 1.250X_1X_3 - 2.331X_2X_3. \quad (B6)$$

Hardness (Y_4)

$$\mu_4(x) = 68.910 - 1.410X_1 + 4.320X_2 + 1.635X_3 + 1.558X_1^2 + 0.058X_2^2 - 0.317X_3^2 - 1.625X_1X_2 + 0.125X_1X_3 - 0.250X_2X_3 \tag{B7}$$

with

$$\sigma_4^2(x) = |\hat{e}_4|^2 + (1 - 0.742)1.606$$

and

$$|\hat{e}_4| = 0.738 + 0.018X_1 + 0.018X_2 - 0.018X_3 + 0.025X_1^2 + 0.158X_2^2 - 0.113X_3^2 - 0.131X_1X_2 + 0.426X_1X_3 + 0.281X_2X_3. \tag{B8}$$

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