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Mechanism Design for Capacity Planning Under Dynamic Evolutions of Asymmetric Demand Forecasts

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This paper investigates the role of time in forecast information sharing and decision making under uncertainty. To do so, we provide a general framework to model the evolutions of forecasts generated by multiple decision makers who forecast demand for the same product. We also model the evolutions of forecasts when decision makers have asymmetric demand information and refer to it as the *Martingale Model of Asymmetric Forecast Evolutions*. This model helps us study mechanism design problems in a dynamic environment. In particular, we consider a supplier's (principal's) problem of eliciting credible forecast information from a manufacturer (agent) when both firms obtain asymmetric demand information for the end product over multiple periods. The supplier uses demand information to better plan for a capacity investment decision. When the supplier postpones building capacity and screening the manufacturer's private information, the supplier and the manufacturer can obtain more information and update their forecasts. This delay, however, may increase (respectively, decrease) the degree of information asymmetry between the two firms, resulting in a higher (respectively, lower) cost of screening. The capacity building cost may also increase because of a tighter deadline for building capacity. Considering all such trade-offs, the supplier has to determine (i) when to stop obtaining new demand information and build capacity, (ii) whether to offer a screening contract to credibly elicit private forecast information or to determine the capacity level without information sharing, (iii) how much capacity to build, and (iv) how to design the overall mechanism so that both firms benefit from this mechanism. This paper provides an answer to these questions. In doing so, we develop a new solution approach for a class of dynamic mechanism design problems. In addition, this paper provides a framework to quantify the option value of time for a strategic investment decision under the dynamic evolutions of asymmetric forecasts.

Key words: Martingale Model of Forecast Evolution; mechanism design; mechanism-dependent reservation profit; capacity planning; information sharing; optimal stopping; real options

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1. Introduction

This paper studies a supplier's problem of eliciting credible forecast information from a manufacturer when both firms obtain forecast information over time. The supplier relies on the demand forecast information for a capacity investment decision. However, the manufacturer often has other forward-looking information because of her superior relationship with the market and expert opinion about her own product. Hence, firms have asymmetric information that changes over time. In such a dynamic environment, what is an effective dynamic mechanism/contract that improves the supply chain profit by enabling credible forecast information sharing? Is such a mechanism implementable? What is the right time for the supplier to elicit credible information and make the capacity investment decision? Does time play an important role? If so, how? This paper addresses these questions. In doing so, the paper characterizes the supplier's optimal mechanism/contract and shows

how this mechanism changes over time as a function of forecast updates. The paper also rigorously models the information evolution process for multiple decision makers who forecast the same object.

This paper first provides a general framework to model the evolutions of forecasts made by multiple decision makers in a consistent and rigorous manner. We propose a framework that can be used to model several plausible scenarios of forecasts evolutions, such as collaborative forecasting and delayed information among forecasters. Consider, for example, multiple forecasters who have different capability and speed in learning information about the demand for a product. From each forecaster's perspective, a good forecast would follow the *Martingale Model of Forecast Evolution* (MMFE), which successfully describes the evolution of forecasts made by a single forecaster who uses statistical and/or judgment-based forecasting methods. Note, however, that forecasters may obtain different demand information at different points in

time. Hence, they have asymmetric demand information that would change over time. Depending on when and how much information that each forecaster obtains, information asymmetry among forecasters may get larger or smaller. We model this scenario using our framework and refer to it as the *Martingale Model of Asymmetric Forecast Evolutions* (MMAFE). During the last decade, researchers and practitioners alike have increasingly focused on coordination and contracting problems among multiple decision makers in various contexts. We believe the proposed framework will enable researchers to consider and revisit, for example, the performance of supply chain contracts in dynamic environments. Next, this paper also provides one such study.

Using the MMAFE framework, we revisit the well-known incentive problem observed in forecast information sharing when two firms need to share this information for better planning. In particular, we consider a supplier who sells his product to a manufacturer with superior forecast information. The two firms establish a supply chain by agreeing that the manufacturer will purchase the product from the supplier. To deliver on time, the supplier needs to invest in capacity before receiving a firm order from the manufacturer. The amount of capacity built defines the upper bound on how much the supplier can produce and satisfy the manufacturer's demand during the sales season. Both firms are uncertain about the demand for the final product, and both obtain forecast updates as they get closer to the time when the supplier invests in capacity. Yet the manufacturer often has more information than the supplier because of, for example, her proximity to the market. Without a proper incentive mechanism, when asked for the forecast information, the manufacturer may inflate her forecast so that the supplier invests and builds more capacity. Being aware of this situation, the supplier may not find the information *credible* and may "correct" the forecast information, thinking that the manufacturer inflated her forecast. This interaction leads to lack of credible forecast information sharing, which reduces both firms' profits.

The aforementioned forecast sharing process, manipulation, and its adverse affects are widely documented across several industries such as telecommunication, automotive, and commercial aircraft (Files 2001, Özer and Wei 2006, *The Economist* 2012). For example, in the semiconductor industry, the manufacturer (such as Intel) and its equipment supplier establish a supply relationship by agreeing on the equipment price as well as the equipment's specifications. Long after this agreement, over time the manufacturer shares her demand forecast via soft orders that can be canceled before placing a firm equipment order. The supplier has to figure out how to use the

manufacturer's soft order information and his own forecast information; he also has to determine a point in time to invest in capacity. Cohen et al. (2003) discuss this process in detail and empirically show that the manufacturer orders on average 30% less than her shared forecast information over time.

Here, we consider capacity investments, such as construction of semiconductor fabrication, defense systems, or heavy equipment. Such investments share four important characteristics. First, capacity investments are often very expensive and irreversible. Second, demand for the capacity is uncertain at the time of the capacity decision. Yet more information becomes available as the sales season approaches. Third, adjusting capacity during the sales and production stages is often difficult or impossible. Hence, the amount of capacity built defines how much can be produced and sold during the sales season. Fourth, the supplier often has some flexibility in determining when to invest in capacity during a planning horizon. Yet there are always lead times associated with obtaining sites, equipment, and other resources when the supplier builds new capacity. Hence, the supplier often has one opportunity to invest in capacity before the sales season starts. When these decisions are delayed, the supplier may face a tighter deadline or be late in building capacity. Many strategic investments share these common characteristics (e.g., Dixit and Pindyck 1994).

When the supplier postpones building capacity, both firms can obtain more demand information and update their forecasts, which reduces demand uncertainty and increases expected profits. This delay, however, may increase (respectively, decrease) the degree of information asymmetry between the two decision makers, resulting in a higher (respectively, lower) cost of screening private information. The capacity cost may also increase as the supplier delays the capacity decision because of a tighter deadline for building capacity. Considering all such trade-offs, to maximize his expected profit, the supplier has to determine (i) when to build capacity, (ii) whether to design a contract to elicit credible forecast information from the manufacturer, (iii) how much capacity to build, and (iv) how to design the overall mechanism.

We use the MMAFE framework to model the firms' forecasting processes and characterize a policy that determines an optimal time for the supplier to invest and build capacity. We also develop a mechanism that enables credible forecast information sharing and a capacity investment decision that almost coordinates (is optimal for) the supply chain. Under this mechanism, the supplier has the option of offering a menu of contracts (screening contract) to the manufacturer before building capacity. If the supplier does not offer the contract, he decides how much capacity to build

based on his forecast information and the belief about the manufacturer's private forecast information at the time. Not exercising this option represents the status quo observed in practice, as in the semiconductor industry discussed above. If the supplier offers the menu of contracts, then the manufacturer has the option to accept or reject the menu. If the manufacturer rejects it, then the supplier updates his belief about the manufacturer's private forecast information and determines how much capacity to build based on this information. We refer to the resulting capacity level as the *base capacity level*. Otherwise, the manufacturer accepts the menu, decides how much capacity to reserve, and makes a payment in exchange. The supplier then builds capacity accordingly.

The aforementioned timing, contract design, and capacity-related decisions are closely linked because they depend on the supplier's forecast information and the information asymmetry, both of which change over time. Hence, we formulate this problem as a three-stage stochastic dynamic decision process. The first stage is an optimal stopping problem that determines the optimal time to build capacity. The solution of this problem depends on the solution of the second stage, in which the supplier decides whether to offer a menu of contracts to the manufacturer. The solution of this problem depends on the third stage, the solution of which defines an optimal menu of contracts. This third-stage problem is a dynamic game with incomplete information and three substages.

The rest of the paper is organized as follows. In §2, we review the literature. In §3, we develop a model that describes the evolutions of forecasts for multiple decision makers and introduce the MMAFE. In §4, we formulate the capacity planning problem under dynamic evolutions of asymmetric forecasts. In §5, we characterize the optimal stopping policy and develop a contract that enables credible forecast information sharing. In §6, we provide the integrated firm solution. In §7, we present numerical studies. In §8, we conclude the paper. We defer some proofs and extensions to an addendum to the present paper, available from the authors.

2. Literature Review

The MMFE is a model that successfully describes the evolution of forecasts generated by a single forecaster who uses statistical and/or judgment-based forecasting methods. Hausman (1969) initiated the development of the multiplicative MMFE and verifies the model using actual data obtained from several independent forecast-revision processes. Hausman and Peterson (1972) extend the model to a multiple products system. Graves et al. (1986) develop the additive

MMFE, and Heath and Jackson (1994) generalize both variants of the MMFE by allowing the correlation of demands across different time periods. Sapra and Jackson (2013) develop the continuous-time analog of the MMFE. Because of its descriptive power and generality, researchers have used the MMFE in several studies to develop effective production, inventory, and capacity management methods that are responsive to forecast updates (e.g., Heath and Jackson 1994, Graves et al. 1998, Gallego and Özer 2001, Toktay and Wein 2001, Schoenmeyr and Graves 2009, Altug and Muharremoglu 2011). This model is also used to understand and quantify the value of information sharing (Chen and Lee 2009, Iida and Zipkin 2010) and collaborative forecasting (Aviv 2001, 2002, 2007). The study of multiple forecasters and modeling the evolutions of their forecasts are rare. Aviv (2001) and Iida and Zipkin (2010) are the two exceptions. They model a specific forecasting scenario in which the two forecasters' forecast revisions are assumed intertemporally independent. Some important forecast scenarios do not follow this assumption, such as delayed information and asymmetric forecasts among forecasters. We discuss this issue further in §3.3. The present paper provides a general framework that can be used to model the outcome of various forecasting scenarios, including those encountered in the literature.

The incentive problem arising in forecast information sharing has been widely observed in industry and its adverse effects are also well documented. To address this problem, researchers have provided some analytical remedies. For example, Cachon and Lariviere (2001) provide some properties of an incentive mechanism to enable credible information sharing and show the existence of separating equilibria in a signaling game. Özer and Wei (2006) determine the role of contracts and incentives in forecast information sharing. In particular, they show that wholesale price contract is a reason for distorted forecast. They then design two contracts—advance purchase and capacity reservation contracts—that enable credible forecast information sharing. Many researchers have since extended the literature that studies strategic issues in forecast information sharing in various interesting directions, such as the effect of competition (e.g., Ha and Tong 2008, Shin and Tunca 2010) and the role of forecasting capability (e.g., Taylor and Xiao 2010, Kostamis and Duenyas 2011). Recently, Özer et al. (2011) determine the role of trust and trustworthiness in forecast information sharing (both in single and repeated interaction settings). In this literature, however, the information is static and not updated over time. Our MMAFE framework helps address this forecast sharing problem in a *dynamic* environment when both firms update their forecasts over time.

Our work also contributes to the broad adverse selection literature from political science, economics, and marketing to operations. The mechanism-dependent reservation profit is an important distinguishing feature of our formulation, which require a new solution method. In other words, the agent's outside option is a function of what the principal offers as a menu of contracts. We provide a solution approach to solve such problems. The adverse-selection literature assumes that the agent has an *exogenous* and *constant* reservation profit. Maggi and Rodriguez-Clare (1995) and Jullien (2000) are among the few who study the cases in which the reservation profit is not constant but a function of the agent's private information. However, these studies also consider an *exogenously* given reservation profit that is independent of the mechanism. In our case, even if the manufacturer rejects the supplier's offer to reserve capacity, the supplier builds some capacity to sell his products to the manufacturer at the wholesale price when it is profitable to do so. In addition, the manufacturer's rejection decision may also provide some information about her private demand forecast to the supplier, and thus it affects the supplier's capacity decision. The manufacturer's reservation profit is determined by this base capacity level. Hence, the manufacturer's reservation profit depends on her overall participation strategy, which in turn depends on the supplier's offered mechanism. Two exceptions that consider adverse selection problems with a mechanism-dependent reservation profit are the recent works by Philippon and Skreta (2012) and Tirole (2012). Yet our objective functions, structural properties of the optimal mechanisms, and solution approaches are different from theirs.

The consideration of the time to offer a contract is another distinguishing feature of our problem. The literature on mechanism design in a dynamic framework is sparse. Plambeck and Zenios (2000), Zhang and Zenios (2008), Lutze and Özer (2008), and Akan et al. (2009) are among the few exceptions. These authors study a principal's problem of designing a contract when the agent takes actions over multiple periods. At the beginning of the planning horizon, the principal offers a single contract to an agent who dynamically makes decisions after she accepts the contract. In our case, the principal determines when to offer a contract (if at all) during the planning horizon, and then both the principal and agent take various actions to maximize their respective profits. When the system is managed by an integrated firm, the problem is reduced to determining the optimal capacity level and the time when the firm should make the capacity decision under information updates (as in, e.g., Wang and Tomlin 2009, Boyacı and Özer 2010). Although it is not a central part of our study, the present paper also solves the integrated firm's problem.

3. Modeling Evolutions of Forecasts for Multiple Forecasters

A descriptive model of forecasting for multiple decision makers (who either employ or are forecasters themselves) should successfully capture all possible interactions among decision makers' forecast sequences. When several decision makers forecast demand for the same product, their forecast revisions are likely to be correlated. The correlation may occur intertemporally because the decision makers may obtain demand information with a time delay. For example, the supplier and the retailer of a product can use past sales data to update demand forecasts, but the supplier may obtain this information later than the retailer (Lee et al. 2000). In §§3.1 and 3.2, we develop a descriptive framework that characterizes the dynamics of general forecasting processes across multiple decision makers in a consistent manner. In §3.3, we illustrate how the model can be used to describe some forecasting scenarios discussed in the literature. In §3.4, we develop the MMAFE. We focus on the model with multiplicative forecast revisions because multiplicative models often fit empirical data better than the additive models do (in particular when the forecasts are made long before the sales season). We remark that all results presented in this paper hold for the additive case and defer the corresponding proofs to the addendum of the present paper.

3.1. The General Martingale Model of Forecast Evolutions

Consider N periods during which each decision maker independently forecasts demand for a single product. We denote the sales season by period $N + 1$. Demand for the product is X_{N+1} , which is a random variable prior to the sales season. At the beginning of each period $n \in \{1, \dots, N\}$, demand information available to decision maker i is given by the set \mathcal{F}_n^i , which is a σ -field. The demand forecast of decision maker i at the beginning of period n is $X_n^i \equiv E[X_{N+1} | \mathcal{F}_n^i]$; i.e., the expected demand given information \mathcal{F}_n^i . We denote the differences between subsequent forecasts by $\Delta_n^i \equiv X_{n+1}^i - X_n^i$. In Appendix A, we provide a glossary of notation for an easy reference.

DEFINITION 1. The forecast evolution X_n^i constructed by $(X_{N+1}, \mathcal{F}_n^i)$ is an MMFE if it satisfies the following properties: (i) X_{N+1} is square-integrable, (ii) $\mathcal{F}_n^i \subseteq \mathcal{F}_{n+1}^i$ for every n , and (iii) $\sigma(X_{N+1}) \subseteq \mathcal{F}_{N+1}^i$.

This definition provides the properties of good forecasting processes. Condition (i) is identical to the condition that X_{N+1} has a finite variance; i.e., the firm should not expect the forecaster to predict an entity that is unpredictable and has infinite variance. Condition (ii) implies that forecasters do not lose information over time. Condition (iii) implies that demand is

revealed to forecaster i at the end of the sales period. Given these properties, the following theorem shows how good forecasts should evolve over time.

THEOREM 1. *If X_n^i constructed by $(X_{N+1}, \mathcal{F}_n^i)$ is an MMFE, then we have the following properties for every n : (1) X_n^i is a Martingale adapted to \mathcal{F}_n^i ; (2) $E[X_{N+1} | \mathcal{F}_n^i] = E[X_{N+1} | X_n^i] = X_n^i$; (3) $E[X_{n+1}^i | \mathcal{F}_n^i] = E[X_{n+1}^i | X_n^i] = X_n^i$ for every $l \geq 0$; and (4) $E[\Delta_n^i] = 0$, and Δ_n^i is uncorrelated with \mathcal{F}_n^i for every $l \geq n$.*

These properties were first discussed in Heath and Jackson (1994) without a formal proof. Part (1) verifies that the MMFE is indeed a Martingale. Part (2) implies that in forecasting X_{N+1} , the value of X_n^i is sufficient information for forecaster i . Part (3) implies that the forecast is unbiased. Finally, part (4) implies that X_n^i is indeed the best forecast for forecaster i . Note that if Δ_n^i were correlated with any past information, a good forecaster could have used the correlation to improve the forecast.¹

3.2. The Multiplicative Martingale Model of Forecast Evolutions

We denote the ratios of successive forecasts by $\delta_n^i \equiv X_{n+1}^i / X_n^i$ for $n < N$ and $\delta_N^i \equiv X_{N+1} / X_N^i$ for each $i \in \{s, m\}$. We consider only two forecasters for the sake of brevity and without loss of generality. We refer to these forecasters (who are also the decision makers in our case) as the supplier (s) and the manufacturer (m) because we apply the model to a forecast sharing problem between these two firms in §4.

The classical MMFE assumes that the multiplicative forecast update for each decision maker; i.e., δ_n^i , is log-normally distributed for every n . Parts (3) and (4) of Theorem 1 imply that δ_n^i is independent of X_n^i and has a mean value of 1. Hence, the initial forecast X_1^i and the variances of $\log(\delta_n^i)$ fully characterize the evolution of X_n^i for each decision maker i .² However, the variance of $\log(\delta_n^i)$ is not sufficient to characterize the interaction between the two forecast processes. One may determine the correlation coefficient between $\log(\delta_{n_s}^s)$ and $\log(\delta_{n_m}^m)$ for every n_s and n_m to characterize the interaction between the two forecast sequences. However, this approach can lead to inconsistency. For example, suppose that the correlation coefficient between $\log(\delta_1^s)$ and $\log(\delta_1^m)$ is 1 and the correlation coefficient between $\log(\delta_2^s)$ and $\log(\delta_1^m)$

is also 1. Then, by obtaining the value of δ_1^s , the supplier obtains the full information of δ_1^m , which also contains the full information of δ_2^s . Hence, δ_1^s and δ_2^s are no longer independent; i.e., an important property of a good forecast does not hold anymore.

We propose a different approach to model the interaction between X_n^s and X_n^m , which does not suffer from any inconsistency such as the one discussed above. Decision makers update their forecasts by obtaining information about events that affect demand. Suppose there are in total K such events, and let e_j be the random variable that models the impact of event j (as in Hausman 1969). According to the theory of proportional effect (Aitchison and Brown 1957), the change in the forecast by each event is proportional to the size of the current forecast. In other words, after obtaining the information of event j , decision maker i updates the forecast from X_n^i to $X_n^i e_j$. Hence, one can express demand by $X_{N+1} = \prod_{j=1}^K e_j$. Next, we divide the set of all events into $(N+1) \times (N+1)$ sets by the time at which the information is obtained by each decision maker. More specifically, we define E_{n_s, n_m} as the set of events whose information is obtained by the supplier during period n_s and by the manufacturer during period n_m .

We define $\delta_{n_s, n_m} \equiv \prod_{j \in E_{n_s, n_m}} e_j$, which indicates the total information obtained by the supplier at period n_s and by the manufacturer at period n_m . Each δ_{n_s, n_m} is log-normally distributed and has a mean value of 1 except $\delta_{0,0}$.³ When E_{n_s, n_m} is an empty set—i.e., when no information is obtained by the supplier at period n_s and by the manufacturer at period n_m —then $\delta_{n_s, n_m} = 1$. By construction a distinct piece of information is contained in one event set; hence, δ_{n_s, n_m} forms an independent set of random variables.

Given this construction, we can express demand as $X_{N+1} = \prod_{n_s=0}^N \prod_{n_m=0}^N \delta_{n_s, n_m}$. The supplier's information set at the beginning of period n is $\mathcal{F}_n^s \equiv \sigma([\delta_{0,0}, \dots, \delta_{0,N}], \dots, [\delta_{n-1,0}, \dots, \delta_{n-1,N}])$. Then the supplier's demand forecast is $X_n^s = E[X_{N+1} | \mathcal{F}_n^s] = \prod_{n_s=0}^{n-1} \prod_{n_m=0}^N \delta_{n_s, n_m}$, and the ratio of successive forecasts is $\delta_n^s = \prod_{n_m=0}^N \delta_{n, n_m}$. Because the multiplication of log-normal random variables is also a log-normal random variable, δ_n^s is also log-normally distributed. Therefore, from the supplier's perspective, his forecast evolution is consistent with the classical MMFE. The manufacturer's forecast can be expressed in a

¹ From the tower property of conditional expectation, $E[X_{N+1} | \mathcal{F}_n^i] = E[X_{N+1} | X_n^i] = E[E[X_{N+1} | \mathcal{F}_{n+1}^i] | X_n^i] = E[X_{n+1}^i | X_n^i] = X_n^i + E[\Delta_n^i | X_n^i]$. If Δ_n^i is correlated to X_n^i , the value of $E[\Delta_n^i | X_n^i]$ may not be 0. In this case, decision maker i 's best demand forecast at period t would be $X_n^i + E[\Delta_n^i | X_n^i]$ rather than X_n^i .

² For a log-normal random variable σ_n^i , $E[\delta_n^i] = 1$ implies $E[\log(\delta_n^i)] = -\text{Var}(\log(\delta_n^i))/2$. Therefore, the variance of $\log(\delta_n^i)$ is sufficient to characterize δ_n^i .

³ By taking the logarithm of δ_{n_s, n_m} , we get $\log(\delta_{n_s, n_m}) = \sum_{j \in E_{n_s, n_m}} \log(e_j)$. When the number of events in E_{n_s, n_m} becomes large, $\sum_{j \in E_{n_s, n_m}} \log(e_j)$ will be asymptotically normal due to the Central Limit Theorem. The information $\delta_{0,0}$ is a deterministic value because both decision makers know $\delta_{0,0}$ before the beginning of the forecast horizon. When $E[\delta_{n_s, n_m}] \neq 1$, for some (n_s, n_m) , we can push this information to $\delta_{0,0}$ and normalize δ_{n_s, n_m} by $\delta_{n_s, n_m} / E[\delta_{n_s, n_m}]$; hence, the assumption $E[\delta_{n_s, n_m}] = 1$ is without loss of generality.

Figure 1 Information Structure Representing the Evolutions of Forecasts

n	0		1		\dots		N	(s)
0	$\delta_{0,0}$	\times	$\delta_{0,1}$	\times	\dots	\times	$\delta_{0,N}$	X_1^s
	\times		\times		\times		\times	\times
1	$\delta_{1,0}$	\times	$\delta_{1,1}$	\times	\dots	\times	$\delta_{1,N}$	δ_1^s
	\times		\times		\times		\times	\times
\vdots	\vdots		\vdots		\vdots			\vdots
	\times		\times		\times		\times	\times
N	$\delta_{N,0}$	\times	$\delta_{N,1}$	\times	\dots	\times	$\delta_{N,N}$	δ_N^s
(m)	X_1^m	\times	X_1^m	\times	\dots	\times	δ_N^m	X_{N+1}

similar way. Figure 1 illustrates the information structure of two forecast evolutions generated by two decision makers. During each period, the supplier obtains all information given in the row corresponding to that period, whereas the manufacturer obtains all information given in the corresponding column. From this construction, we can fully characterize the evolutions of X_n^s and X_n^m by determining the value of $\delta_{0,0}$ and the variances of $\log(\delta_{n_s, n_m})$.

The demand and the forecast revisions have the following relationship: $X_{N+1} = X_1^i \prod_{k=1}^N \delta_k^i$ for each decision maker i . At the beginning of period n , decision maker i has the information $X_1^i, \delta_1^i, \dots, \delta_{n-1}^i$ but does not have the information $\delta_n^i, \delta_{n+1}^i, \dots, \delta_N^i$. Hence, the multiplication $\prod_{k=n}^N \delta_k^i$ represents the demand uncertainty faced by decision maker i . All discussions and results extend to $J > 2$ decision makers. To do so, we divide the information sets into $(N+1)^J$ groups such that each δ_{n_1, \dots, n_J} represents demand information that is obtained by J decision makers at the specified periods. By determining the standard deviations of every $\log(\delta_{n_1, \dots, n_J})$, one can similarly describe several forecast evolutions for J decision makers in a consistent manner.

3.3. Collaborative Forecasting and Delayed Information

Consider the collaborative forecasting process discussed in Aviv (2001, 2002, 2007) and Kurtuluş et al. (2012). When the two decision makers collaborate to forecast demand, they share all available information. Based on Definition 1, we define the collaborative information set as $\mathcal{F}_n^{cf} \equiv \mathcal{F}_n^s \vee \mathcal{F}_n^m$. Because the join of two sub- σ -fields is also a σ -field, \mathcal{F}_n^{cf} is a well-defined information set. Then the collaborative forecast (CF) of the two decision makers is $X_n^{cf} \equiv E[X_{N+1} | \mathcal{F}_n^{cf}]$. Next we derive the most important property of the CF.

THEOREM 2. *The CF has a smaller mean-squared-error than the forecast of a single decision maker; i.e., $E[(X_{N+1} - X_n^{cf})^2] \leq E[(X_{N+1} - X_n^i)^2]$ for each $i \in \{s, m\}$.*

This result states that two decision makers who collaborate can predict demand more accurately.

Figure 2 Information Structure

(a) Collaborative forecasting

n	0	1	...	N	(s)			
0	$\delta_{0,0}$	\times	$\delta_{0,1}$	\times	\dots	\times	$\delta_{0,N}$	X_1^s
1	\times	$\delta_{1,0}$	\times	$\delta_{1,1}$	\times	\dots	\times	$\delta_{1,N}$
\vdots	\vdots	\times	\vdots	\times	\ddots	\times	\vdots	\vdots
N	\times	$\delta_{N,0}$	\times	$\delta_{N,1}$	\times	\dots	\times	$\delta_{N,N}$
(m)	X_1^m	\times	δ_1^m	\times	\dots	\times	δ_N^m	X_{N+1}
	X_1^{cf}		δ_1^{cf}				δ_N^{cf}	

(b) Asymmetric forecast evolutions

n	0	1	\dots	N	(s)	
0	$\delta_{0,0}$	\times	1	\times	1	X_1^s
	\times	\times		\times	\times	\times
1	$\delta_{1,0}$	\times	$\delta_{1,1}$	\times	1	δ_1^s
	\times	\times		\times	\times	\times
\vdots	\vdots	\times	\vdots	\times	1	\vdots
	\times	\times		\times	\times	\times
N	$\delta_{N,0}$	\times	$\delta_{N,1}$	\times	$\delta_{N,N}$	δ_N^s
(m)	X_1^m	\times	δ_1^m	\times	δ_N^m	X_{N+1}

\downarrow
 ξ_2

\downarrow
 X_2^s

\downarrow
 ϵ_2

The collaborative information set \mathcal{F}_n^{cf} includes all δ_{n_s, n_m} such that $n_s \leq n$ or $n_m \leq n$. Hence, the initial forecast is $X_1^{cf} = \delta_{0,0} \prod_{n_s=1}^N \delta_{n_s,0} \prod_{n_m=1}^N \delta_{0,n_m}$, and the ratio of successive forecasts is $\delta_n^{cf} = \delta_{n,n} \prod_{n_s=n+1}^N \delta_{n_s,n} \prod_{n_m=n+1}^N \delta_{n,n_m}$. Figure 2(a) illustrates the information structure available to decision makers under a collaborative forecasting scheme. Because each δ_n^{cf} is a log-normal random variable with the mean value of 1, the collaborative forecast is also a multiplicative MMFE.

We remark that Aviv (2001) also provides a model to describe forecasts of two decision makers. However, his model is only applicable to the case in which the two decision makers' forecast revisions are inter-temporally independent.⁴ Some important forecasting scenarios do not satisfy this assumption.

⁴ The intertemporal independence means that $\delta_{n_s, n_m} = 1$ unless $n_s = n_m$ or $n_i = N$ for $i \in \{s, m\}$. Hence, the information obtained by the two decision makers at period n consists of three parts, $\delta_{n,n}$, δ_{n,n_s} , and δ_{n,n_m} . They correspond to the three corners of a single block in Figure 2(a). The correlated part of δ_n^s and δ_n^m arises from $\delta_{n,n}$, and hence we can set $\text{Var}(\log(\delta_{n,n})) = \rho \eta^s \eta^m \sigma_n^2$, where η^s and η^m represent the supplier's and the manufacturer's forecasting

We will discuss two such examples (delayed information and asymmetric forecast evolutions). In contrast, our model of forecast evolutions for multiple decision makers can describe more general cases. In addition to being more general, our approach also does not suffer from inconsistency, which can arise in Aviv's top-down approach. Aviv (2001) constructs the forecast sequence of each decision maker separately and then models the interaction between the two forecast sequences. This approach may lead to inconsistency.⁵ In contrast, we construct an approach that models when and how much information is obtained by each decision maker. Hence, our approach automatically constructs the forecast sequence of each decision maker in a consistent manner and does not require one to posit any condition for consistency. Ours could be considered a bottom-up approach.

Next consider the delayed information scenario discussed by Chen (1999). In this case, the manufacturer observes demand of a product at each period and makes a replenishment order to the supplier. The supplier of the product receives the manufacturer's order with the delay of l periods. The decision makers use the demand history to update forecasts. Therefore, the information sets of the two decision makers are identical with l periods of delay; i.e., we have $\mathcal{F}_{n+l}^s = \mathcal{F}_n^m$ for every n . Then the supplier and the manufacturer have the same sequence of forecasts with a delay of l periods. In our model, the delayed information can be represented by $\delta_{n,k} = 1$ for every $k + l \neq n$.

3.4. The Martingale Model of Asymmetric Forecast Evolutions

Consider the scenario in which the manufacturer has all information that the supplier has at each period; i.e., $\mathcal{F}_n^s \subseteq \mathcal{F}_n^m$ for every n . Both decision makers obtain new demand information and update their forecasts over time, but the manufacturer may have additional information because of her superior information about her own product. The manufacturer has *private* demand information that is not available to the other decision maker. We model and refer to such evolutions of forecasts as the MMAFE. We denote the difference between the two decision makers' forecasts by $A_n \equiv X_n^m - X_n^s$. Then we have the following properties that describe how, in this context, two good related forecasts should evolve over time:

strength in Aviv's (2001) model, respectively. Similarly, the uncorrelated parts of δ_n^s and δ_n^m correspond to $\delta_{n,N}$ and $\delta_{N,n}$, so we can set $\text{Var}(\log(\delta_{n,N})) = (\eta^s \sigma_n)^2 - \rho \eta^s \eta^m \sigma_n^2$ and $\text{Var}(\log(\delta_{N,n})) = (\eta^m \sigma_n)^2 - \rho \eta^s \eta^m \sigma_n^2$ and obtain Aviv's model. Iida and Zipkin (2010) also follow Aviv's (2001) approach.

⁵ For example, when $\eta^s = \eta^m = 1$ and $\sigma_{N-1} = \sigma$, both decision makers obtain all demand information during period $N - 1$; hence, the correlation coefficient ρ should be exactly 1 for the demand models to be consistent. For this reason, Aviv (2001) provides a sufficient condition on (ρ, η^s, η^m) that guarantees consistency.

THEOREM 3. If (X_n^s, X_n^m) constructed by $(X_{N+1}, \mathcal{F}_n^s, \mathcal{F}_n^m)$ is an MMAFE, then we have the following properties for every n : (1) $E[X_{n+1}^m | \mathcal{F}_n^s] = E[X_{n+1}^m | X_n^s] = X_n^s$ for every $l \geq 0$; (2) $E[X_{N+1} | X_n^s, A_n] = E[X_{N+1} | \mathcal{F}_n^m] = X_n^m$; and (3) $E[A_n] = 0$ and A_n is uncorrelated with \mathcal{F}_n^s .

Part (1) implies that the supplier's estimate of the manufacturer's forecast is the same as his own forecast. Part (2) implies that by knowing the value of A_n , the supplier can obtain the best forecast. Part (3) implies that A_n is uncorrelated with the supplier's information set, \mathcal{F}_n^s . That is, the supplier cannot infer the difference between the two demand forecasts from his own demand forecast.

We can model the asymmetric information scenario by setting $\delta_{n_s, n_m} = 1$ for every $n_m > n_s$; i.e., the supplier obtains no information earlier than the manufacturer. Hence, the information the supplier obtains at period n consists of $\delta_{n,0}, \delta_{n,1}, \dots, \delta_{n,n}$, where each δ_{n, n_m} has already been obtained or is being obtained at the same time by the manufacturer. We refer to this case as the *multiplicative Martingale model of asymmetric forecast evolutions* (m -MMAFE). Figure 2(b) summarizes the information structure of the m -MMAFE.

The manufacturer's private demand information represents the information asymmetry between the two decision makers. The manufacturer's demand uncertainty is also the demand uncertainty that the system faces. Recall that the multiplication of $\delta_n^m, \delta_{n+1}^m, \dots, \delta_N^m$ represents the demand uncertainty the manufacturer faces at the beginning of period n , and we denote it by $\epsilon_n \equiv \prod_{k=n}^N \delta_k^m$. From the manufacturer's perspective, demand is $X_{N+1} = X_n^m \epsilon_n$, where X_n^m is her current forecast, which is deterministically known to her. The remaining market uncertainty ϵ_n is resolved over periods n to N as the manufacturer obtains information, i.e., as the forecast updates.

The supplier's demand uncertainty at the beginning of period n is $\prod_{k=n}^N \delta_k^s$. The manufacturer has already obtained part of this information. To distinguish the known part, we rewrite

$$\begin{aligned} \prod_{k=n}^N \delta_k^s &= \prod_{k=n}^N \left(\prod_{n_m=0}^{n-1} \delta_{k, n_m} \prod_{n_m=n}^N \delta_{k, n_m} \right) \\ &= \underbrace{\left(\prod_{k=n}^N \prod_{n_m=0}^{n-1} \delta_{k, n_m} \right)}_{\xi_n} \underbrace{\left(\prod_{k=n}^N \prod_{n_m=n}^N \delta_{k, n_m} \right)}_{\epsilon_n}. \end{aligned}$$

The first part of the last equation represents the demand information that is already obtained by the manufacturer. Hence, it is the manufacturer's private information, and we denote it by ξ_n . The second part represents the demand information that the manufacturer has not yet obtained. Because $\delta_{n_s, n_m} = 1$ for $n_m > n_s$, the second part $\prod_{k=n}^N \prod_{n_m=n}^N \delta_{k, n_m} = \prod_{n_m=n}^N \prod_{k=n}^N \delta_{k, n_m} = \prod_{n_m=n}^N \prod_{k=0}^N \delta_{k, n_m} = \prod_{n_m=n}^N \delta_{n_m}^m$ is equal to ϵ_n . Then demand can be represented as

$X_{N+1} = X_n^s \xi_n \epsilon_n$. From the supplier's perspective, X_n^s is deterministic, and ξ_n and ϵ_n are uncertain. By construction, X_n^s , ξ_n , and ϵ_n are independent. Note also that $X_n^m = X_n^s \xi_n$. The supplier obtains only part of the information of ϵ_n and ξ_n during period n and he obtains the full information of ϵ_n and ξ_n over periods n to N .

For notational simplicity, we denote the standard deviation of $\log(Z)$ of a log-normal random variable Z as σ_Z throughout this paper. The value of σ_{ϵ_n} represents the degree of demand uncertainty of the system at period n , and the value of σ_{ξ_n} represents the degree of information asymmetry between the supplier and the manufacturer at period n . By construction, σ_{ϵ_n} always decreases⁶ in n . In contrast, σ_{ξ_n} can either increase or decrease in n , depending on the values of $\sigma_{\delta_n^s}$ and $\sigma_{\delta_n^m}$. When the supplier obtains more information than the manufacturer during period n , i.e., $\sigma_{\delta_n^s} > \sigma_{\delta_n^m}$, we have $\sigma_{\xi_{n+1}} < \sigma_{\xi_n}$, and vice versa.

4. Planning Under Dynamic Evolutions of Asymmetric Forecasts

A supplier and a manufacturer establish a supply chain by agreeing that the manufacturer will purchase the supplier's product at the wholesale price w and the supplier will deliver the product based on the agreed-upon product specifications. To deliver on time, the supplier secures production capacity before the sales season. The supplier has some flexibility in determining when to build the capacity. We refer to the time window during which the supplier decides and builds capacity as the *capacity planning horizon*, which consists of N periods. Periods $n \in \{1, \dots, N\}$ represent the planning periods and period $N+1$ represents the sales season. If the supplier decides to build capacity at period n , he incurs both a unit capacity cost c_n and a fixed cost C_n . During the sales season, the manufacturer observes the demand and places an order. The supplier produces at a unit production cost c and fulfills the order to the extent possible, given the secured capacity. The manufacturer sells the final product to the market at a unit retail price r . Unmet demand is lost, and unsold products have zero salvage value.

Both the supplier and the manufacturer are uncertain about demand X_{N+1} for the final product prior to the sales season. Hence, the supplier relies on the demand forecast for the capacity decision. During the capacity planning horizon, the supplier and the manufacturer obtain new demand information and update their forecasts over time. The manufacturer, however, may have additional information because of her superior relationship with her customers and/or expert

opinion about her product. These two firms' unbiased forecast processes; i.e., (X_n^s, X_n^m) , can be described by the m -MMAFE defined in §3.4. Recall that $X_{N+1} = X_n^s \xi_n \epsilon_n = X_n^m \epsilon_n$. At the beginning of period n , ξ_n is the private demand information revealed to the manufacturer, whereas it is a random variable to the supplier. Its c.d.f. is $F_n(\cdot)$ and its p.d.f. is $f_n(\cdot)$. Yet ϵ_n is a random variable with c.d.f. $G_n(\cdot)$ and p.d.f. $g_n(\cdot)$ to both firms, and it represents the market uncertainty.

Suppose that the supplier sets the capacity level to K at the beginning of period n based on his forecast information X_n^s . Suppose also that he knows the manufacturer's private information ξ_n . The expected profits of the supplier, manufacturer, and the total supply chain are, respectively,

$$\Pi_n^s(K, X_n^s, \xi_n) \equiv (w - c)E_{\epsilon_n}[\min(X_n^s \xi_n \epsilon_n, K)] - c_n K - C_n,$$

$$\Pi_n^m(K, X_n^s, \xi_n) \equiv (r - w)E_{\epsilon_n}[\min(X_n^s \xi_n \epsilon_n, K)],$$

$$\Pi_n^{\text{tot}}(K, X_n^s, \xi_n) \equiv (r - c)E_{\epsilon_n}[\min(X_n^s \xi_n \epsilon_n, K)] - c_n K - C_n.$$

To maximize his profit, the supplier would have set the capacity to $K_n^{ws}(\xi_n) \equiv X_n^s \xi_n G_n^{-1}((w - c - c_n)/(w - c))$. The system optimal capacity that maximizes the total supply chain profit at period n is given as

$$K_n^{so}(\xi_n) \equiv X_n^s \xi_n G_n^{-1}\left(\frac{r - c - c_n}{r - c}\right). \quad (1)$$

The supplier, however, does not know the manufacturer's private forecast information. To facilitate information sharing, the manufacturer can report her forecast prior to the capacity decision. However, the manufacturer's forecast report is nonbinding (it is not a firm order and can be canceled or revised), nonverifiable (after demand is realized, a third party cannot verify that the manufacturer disclosed her actual forecast), and costless (the manufacturer does not incur any cost when sharing her forecast). This kind of forecast reporting strategy leads to what is known as "cheap talk" communication. In such an interaction, the manufacturer can follow various forecast reporting strategies (for example, always exaggerate her forecast update when reporting, occasionally tell the truth, always tell the truth, or any other strategy). Özer et al. (2011) show that any such forecast reporting strategy leads to an uninformative information update for the supplier. As a result, the supplier resorts to his belief about what this information might be and sets the capacity to maximize his expected profit. The resulting capacity level and his optimal expected profit are

$$K_n^w \equiv X_n^s (F_n * G_n)^{-1}\left(\frac{w - c - c_n}{w - c}\right), \quad (2)$$

$$\pi_n^w(X_n^s) \equiv E_{\xi_n}[\Pi_n^s(K_n^w, X_n^s, \xi_n)],$$

where $F_n * G_n$ is the c.d.f. of $\xi_n \epsilon_n$.

⁶ We use the terms increasing and decreasing in the weak sense; i.e., increasing means nondecreasing.

To ensure credible forecast information sharing, we develop a mechanism that aligns incentives to share information voluntarily and credibly. Under this mechanism, the sequence of events is as follows. At the beginning of period $n \in \{1, 2, \dots, N\}$, the supplier decides whether to build the capacity, depending on his demand forecast, X_n^s . If he delays building capacity, the supplier and the manufacturer obtain their forecast updates δ_n^s and δ_n^m during period n , and the problem proceeds to period $n + 1$. Otherwise, the supplier chooses between the two options: (i) determine the capacity level by himself based on his own forecast information or (ii) offer a menu of contracts $\{(K(\xi), P(\xi)): \xi \in [0, \infty)\}$ to the manufacturer. If the supplier offers a menu and the manufacturer decides to participate, the manufacturer chooses a contract $(K(\hat{\xi}), P(\hat{\xi}))$ that maximizes her profit. The supplier reserves $K(\hat{\xi})$ units of capacity in exchange for a monetary payment $P(\hat{\xi})$. If the manufacturer decides not to participate, then the supplier updates his belief about the manufacturer's private information ξ_n and builds capacity to maximize his expected profit.⁷ Next, the manufacturer observes demand, X_{n+1} , and places an order. The supplier produces and fulfills the order to the extent possible, given the available capacity.

4.1. The First- and Second-Stage Problems

At the beginning of period n , the supplier decides whether to start building capacity or to delay this decision to the next period, given his forecast information X_n^s ; that is,

$$u_n(X_n^s) = \begin{cases} u^s & \text{start building capacity,} \\ u^d & \text{delay building capacity} \end{cases}$$

for $n < N$ and $u_N(\cdot) = u^s$. If the supplier decides to build capacity at period n , then the state is updated as t to indicate that the capacity decision is made. If the supplier defers building capacity, he obtains demand information δ_n^s during period n . Hence, the state transition is

$$X_{n+1}^s = \begin{cases} t & \text{if } X_n^s = t, \text{ or } X_n^s \neq t \text{ and } u_n(X_n^s) = u^s, \\ X_n^s \delta_n^s & \text{otherwise.} \end{cases}$$

⁷ When an offer is rejected, the supplier neither offers another contract nor defers the capacity decision to a later period. The supplier can credibly commit to this strategy for three reasons. First, Riley and Zeckhauser (1983) show that making such a commitment is optimal. Second, many other processes and functional departments within supplier's organization are involved in costly and irreversible capacity decision. Once the decision to build capacity is made, the firm acts on it. Hence, it will not wait another period to get more information (which could be several months to a few quarters). Third, the manufacturer does not know the time window in which the supplier has to secure capacity because the time required to build the capacity and the planning horizon are the supplier's private information. Hence, the manufacturer would not decline the offer in anticipation of receiving another offer later (as in Tirole 2012).

We denote the supplier's optimal expected profit when he decides to build capacity at period n by $\pi_n(X_n^s)$. This profit depends on the supplier's decision made in the second stage, that is, $\pi_n(X_n^s) \equiv \max\{\pi_n^w(X_n^s), \pi_n^{cr}(X_n^s)\}$, where $\pi_n^w(\cdot)$ is defined in (2) and $\pi_n^{cr}(\cdot)$ denotes the supplier's expected profit under the optimal menu of contracts. We will define this function in the next subsection. Then the reward function is given by

$$h_n(X_n^s) = \begin{cases} \pi_n(X_n^s) & \text{if } X_n^s \neq t \text{ and } u_n(X_n^s) = u^s, \\ 0 & \text{if } X_n^s = t \text{ or } u_n(X_n^s) = u^d. \end{cases}$$

Let $\mathcal{P} = \{u_1(X_1^s), \dots, u_N(X_N^s)\}$ represent a policy that determines when to stop and build capacity. Then the optimal stopping problem is given as $\max_{\mathcal{P}} E[\sum_{n=1}^N h_n(X_n^s)]$, where the maximization is taken over all admissible policies. The following dynamic program provides the solution:

$$V_n(X_n^s) = \max\{\pi_n(X_n^s), E[V_{n+1}(X_{n+1}^s) | X_n^s]\}$$

if $X_n^s \neq t$ and otherwise $V_n(X_n^s) = 0$ for $n < N$. For the final period, $V_N(X_N^s) = \pi_N(X_N^s)$ if $X_N^s \neq t$ and 0 otherwise.⁸ Notice that it is optimal for the supplier to start building the capacity at period n if $\pi_n(X_n^s) \geq E[V_{n+1}(X_{n+1}^s) | X_n^s]$; otherwise, it is optimal to delay building capacity.

4.2. The Third-Stage Problem

The supplier's third-stage mechanism design problem is a dynamic game with incomplete information consisting of three substages. In the first substage, the supplier's objective is to maximize his expected profit by designing and offering a menu of contracts $\{(K(\xi), P(\xi)): \xi \in [0, \infty)\}$ to the manufacturer. In the second substage, the manufacturer either accepts a contract from the menu or declines to do so. If the manufacturer accepts the contract $(K(\hat{\xi}), P(\hat{\xi}))$, then the supplier builds $K(\hat{\xi})$ units of capacity and receives the payment $P(\hat{\xi})$ from the manufacturer. If the manufacturer rejects the supplier's offer, then the problem proceeds to the third substage. In the third substage, the supplier updates his information about the manufacturer's private forecast and determines the capacity level, based on his updated information.

To determine an optimal menu of contracts, the supplier can limit his search to the class of direct-revealing contracts, because of the revelation principle (Myerson 1979). This principle applies in the

⁸ To study the case in which the supplier may renege and not build any capacity during the planning horizon, one can include an additional period $N + 1$ and a corresponding reward function $h_{N+1}(X_{N+1}^s) = 0$ to the stopping formulation. In this case, delaying the capacity decision at period N means that the supplier decides not to build any capacity. Our results continue to hold for this case as well.

present context because the supplier offers a menu of contracts and can commit to it (Skreta 2006). If the manufacturer rejects the offer, the supplier builds capacity based on his own information without offering another contract (as in Philippon and Skreta 2012, Tirole 2012). Hence, the supplier can limit his search to menus that satisfy the following constraint:

$$(IC) \quad \Pi_n^m(K(\xi), X_n^s, \xi) - P(\xi) \geq \Pi_n^m(K(\hat{\xi}), X_n^s, \xi) - P(\hat{\xi})$$

for every $\hat{\xi}$ and ξ .

This set of incentive compatibility constraints (ICs) ensures that the manufacturer credibly shares her private forecast information if she decides to accept a contract from the menu. Let $\mathcal{R} \subseteq [0, \infty)$ be the set of private forecast levels at which the manufacturer rejects the menu. Given (IC), the set \mathcal{R} fully characterizes the manufacturer's decision at the second substage. If she accepts the menu, i.e., $\xi_n \notin \mathcal{R}$, then the manufacturer chooses the contract $(K(\xi_n), P(\xi_n))$. Otherwise, she rejects the menu. Hence, we interpret the set \mathcal{R} as the manufacturer's overall strategy.

Now we consider the third substage problem. If the manufacturer rejects the menu at the second substage, then the supplier obtains the information that $\xi_n \in \mathcal{R}$ and determines the capacity as

$$K_n^b(\mathcal{R}) \equiv \arg \max_K E_{\xi_n}[\Pi_n^s(K, X_n^s, \xi_n) | \xi_n \in \mathcal{R}]. \quad (3)$$

The manufacturer can indirectly reserve $K_n^b(\mathcal{R})$ without making a payment (i.e., by rejecting the menu). Hence, we refer to this capacity level as the *base capacity level*. To avoid any ambiguity, when \mathcal{R} is an empty set, we define $K_n^b(\mathcal{R}) \equiv 0$.

Next we consider the second substage problem, in which the manufacturer determines her strategy \mathcal{R} . Note that for \mathcal{R} to be the manufacturer's optimal strategy, it should satisfy

$$(PC1) \quad \Pi_n^m(K(\xi), X_n^s, \xi) - P(\xi) \geq \Pi_n^m(K_n^b(\mathcal{R}), X_n^s, \xi)$$

for every $\xi \notin \mathcal{R}$,

$$(PC2) \quad \Pi_n^m(K_n^b(\mathcal{R}), X_n^s, \xi) \geq \Pi_n^m(K(\xi), X_n^s, \xi) - P(\xi)$$

for every $\xi \in \mathcal{R}$.

These two sets of constraints are necessary for the manufacturer to participate in the game designed by the supplier. Hence, they are called participation constraints. Although (PC1) and (PC2) are necessary for \mathcal{R} to be an optimal strategy for the manufacturer, they are not sufficient. For a given menu, there can be multiple \mathcal{R} that satisfy both (PC1) and (PC2). This multiplicity can be resolved by considering the dominance between different strategies. The manufacturer's optimal expected profit increases with the capacity for

every ξ_n . Hence, the manufacturer's optimal strategy is the set \mathcal{R} that satisfies

$$(OPT) \quad K_n^b(\mathcal{R}) \geq K_n^b(\hat{\mathcal{R}})$$

for every $\hat{\mathcal{R}}$ that satisfies (PC1) and (PC2).

We define $\mathcal{R}_n^m(K(\cdot), P(\cdot))$ as the rejection set $\mathcal{R} \subseteq [0, \infty)$ that satisfies (PC1), (PC2), and (OPT) for a given menu of contracts $\{K(\cdot), P(\cdot)\}$. This set represents the manufacturer's optimal strategy. We refer to this set as \mathcal{R}_n^m when its dependency on $\{K(\cdot), P(\cdot)\}$ is clear.

Finally, we consider the first substage mechanism design problem. The supplier solves the following problem to design an incentive compatible menu that maximizes his expected profit:

$$\begin{aligned} \pi_n^{cr}(X_n^s) = \max_{K(\cdot), P(\cdot)} & \{E_{\xi_n}[\Pi_n^s(K(\xi_n), X_n^s, \xi_n) + P(\xi_n) | \xi_n \\ & \notin \mathcal{R}_n^m(K(\cdot), P(\cdot))] \\ & + E_{\xi_n}[\Pi_n^s(K_n^b(\mathcal{R}_n^m), X_n^s, \xi_n) | \xi_n \\ & \in \mathcal{R}_n^m(K(\cdot), P(\cdot))]\} \\ \text{s.t. (IC).} \end{aligned} \quad (4)$$

5. Analysis

To solve the first-stage optimal stopping problem, we need to solve for the second- and third-stage problems. Thus, we first present the solution of the third-stage mechanism design problem.

5.1. The Third-Stage Problem

The supplier solves the mechanism design problem outlined in §4.2 to determine an optimal menu of contracts. Recall from the optimization problem defined in (4) that the supplier's expected profit depends on the manufacturer's optimal strategy \mathcal{R}_n^m , which in turn depends on the supplier's offer of the menu of contracts $\{K(\cdot), P(\cdot)\}$. Because of this circularity, determining the set \mathcal{R}_n^m is a difficult problem. Next we provide an equivalent condition for (IC) and use this result to obtain \mathcal{R}_n^m 's structural properties and solve the supplier's problem.

THEOREM 4. (IC) holds if and only if the following two conditions hold:

$$(IC1) \quad \frac{d}{d\xi}(\Pi_n^m(K(\xi), X_n^s, \xi) - P(\xi)) = (r - w)X_n^s \int_0^{K(\xi)/X_n^s \xi} x g_n(x) dx.$$

$$(IC2) \quad K(\xi) \text{ is increasing in } \xi.$$

(IC1) is the local optimality condition under which truthful revelation is locally optimal for the manufacturer. Combined with the increasing property of $K(\cdot)$, the local optimality assures that the manufacturer shares information credibly. We use (IC1) to compute the optimal payment $P(\cdot)$ for any given $K(\cdot)$ by integrating both sides with

respect to ξ . (IC2) implies that the manufacturer will reserve more capacity when her demand forecast is larger. We define $\bar{\mathcal{R}}_n^m \equiv \{\xi: \Pi_n^m(K_n^b(\mathcal{R}_n^m), X_n^s, \xi) \geq \Pi_n^m(K(\xi), X_n^s, \xi) - P(\xi)\}$. By definition, $\mathcal{R}_n^m \subseteq \bar{\mathcal{R}}_n^m$. Note that $\bar{\mathcal{R}}_n^m$ may include forecast levels for which the manufacturer would receive the same expected profit by accepting or rejecting the menu. Thus, we call $\bar{\mathcal{R}}_n^m$ the weak rejection set and define lower and upper bounds for it; i.e., $\xi_l(\bar{\mathcal{R}}_n^m) \equiv \min(\bar{\mathcal{R}}_n^m)$ and $\xi_h(\bar{\mathcal{R}}_n^m) \equiv \max(\bar{\mathcal{R}}_n^m)$.

THEOREM 5. *If $\{K(\cdot), P(\cdot)\}$ satisfies (IC), the following properties hold:*

- (1) $\bar{\mathcal{R}}_n^m$ is a convex set, hence $\bar{\mathcal{R}}_n^m = [\xi_l(\bar{\mathcal{R}}_n^m), \xi_h(\bar{\mathcal{R}}_n^m)]$.
- (2) For all $\xi < \xi_l(\bar{\mathcal{R}}_n^m)$, $K(\xi) < K_n^b(\mathcal{R}_n^m)$ and $P(\xi) < 0$.
- (3) For all $\xi > \xi_h(\bar{\mathcal{R}}_n^m)$, $K(\xi) > K_n^b(\mathcal{R}_n^m)$ and $P(\xi) > 0$.
- (4) If $(K_n^b(\mathcal{R}_n^m), 0) \in \{K(\cdot), P(\cdot)\}$, then $K(\xi) = K_n^b(\mathcal{R}_n^m)$ and $P(\xi) = 0$ for every $\xi \in (\xi_l(\bar{\mathcal{R}}_n^m), \xi_h(\bar{\mathcal{R}}_n^m))$.
- (5) If $(K_n^b(\mathcal{R}_n^m), 0) \notin \{K(\cdot), P(\cdot)\}$, then (i) $\mathcal{R}_n^m = \bar{\mathcal{R}}_n^m$; (ii) $\Pi_n^m(K_n^b(\mathcal{R}_n^m), X_n^s, \xi) > \Pi_n^m(K(\xi), X_n^s, \xi) - P(\xi)$ for every $\xi \in (\xi_l(\bar{\mathcal{R}}_n^m), \xi_h(\bar{\mathcal{R}}_n^m))$; and (iii) if $P(\xi) > 0$ for every ξ , then $\mathcal{R}_n^m = [0, \xi_h(\bar{\mathcal{R}}_n^m)]$.

This theorem characterizes the properties of incentive compatible contracts. The supplier builds the base capacity level $K_n^b(\mathcal{R}_n^m)$ when the manufacturer rejects his offer. Thus, the manufacturer can indirectly reserve this much capacity without any payment by simply rejecting the supplier's offer. The manufacturer's profit increases with the capacity level. Hence, the manufacturer reserves less than $K_n^b(\mathcal{R}_n^m)$ only when the required payment is negative. In contrast, if the manufacturer wants to reserve more than $K_n^b(\mathcal{R}_n^m)$, then she needs to make a positive payment to the supplier. To summarize, the manufacturer takes one of the three actions: (i) reject the menu and let the supplier build $K_n^b(\mathcal{R}_n^m)$ by himself, (ii) reserve less than $K_n^b(\mathcal{R}_n^m)$ and receive a payment from the supplier, or (iii) reserve more than $K_n^b(\mathcal{R}_n^m)$ and make a payment to the supplier. A large amount of capacity is more valuable when the private demand forecast level is higher. Thus, the manufacturer reserves more than $K_n^b(\mathcal{R}_n^m)$ when ξ_n is large and reserves less than $K_n^b(\mathcal{R}_n^m)$ when ξ_n is small, which implies that the weak rejection set is a convex set. In other words, this set does not include disjoint intervals on the real line. These properties are shown in parts (1)–(3) of Theorem 5.

Parts (4) and (5) show that any incentive compatible menu belongs to either one of the two classes of menus. In the first class, the menu contains a contract consisting of the base capacity level and the corresponding reservation price, which is zero. In the second class, the menu does not include $(K_n^b(\mathcal{R}_n^m), 0)$. In this case, the weak rejection set $\bar{\mathcal{R}}_n^m$ is the same as \mathcal{R}_n^m , and the manufacturer earns a strictly higher expected profit by rejecting the supplier's offer when $\xi_n \in \mathcal{R}_n^m$ than by accepting a contract. In addition,

if the supplier requires positive payments for reserving capacity, then the rejection set has a threshold structure; i.e., if the manufacturer's forecast ξ_n is less than $\xi_h(\bar{\mathcal{R}}_n^m)$, she would reject the menu.

The manufacturer may have multiple strategies $\mathcal{R}_n^m(K(\cdot), P(\cdot))$ that satisfy (PC1), (PC2), and (OPT). Nevertheless, parts (4) and (5) show that all such strategies yield the same expected profits for both the supplier and the manufacturer. Hence, it suffices to focus on one set in designing the optimal menu of contracts. To observe this result, first note that because of (OPT), under every strategy \mathcal{R}_n^m , the supplier builds the same base capacity level $K_n^b(\mathcal{R}_n^m)$. Note also that the weak rejection set $\bar{\mathcal{R}}_n^m$ only depends on $K_n^b(\mathcal{R}_n^m)$. Hence, the base capacity level and the weak rejection set are always uniquely defined. From Theorem 5, we also know that any incentive compatible menu of contracts belongs to either one of the two classes, as specified in parts (4) and (5). Suppose $\{K(\cdot), P(\cdot)\}$ satisfies the condition in part (5). In that case, $\mathcal{R}_n^m = \bar{\mathcal{R}}_n^m$. Hence, the manufacturer's optimal strategy \mathcal{R}_n^m is also uniquely defined.

Multiplicity of \mathcal{R}_n^m arises when the menu of contracts satisfies the condition in part (4). In this case, every $\hat{\mathcal{R}} \subseteq \bar{\mathcal{R}}_n^m$ such that $K_n^b(\hat{\mathcal{R}}) = K_n^b(\mathcal{R}_n^m)$ also satisfies (PC1), (PC2), and (OPT). In other words, every subset $\hat{\mathcal{R}}$ of $\bar{\mathcal{R}}_n^m$ can be defined as the manufacturer's optimal strategy as long as $K_n^b(\hat{\mathcal{R}})$ is equal to the unique base capacity level. However, all such strategies result in the same capacity decision and the transfer payment. To observe this, first note from part (4) that when $\xi_n \in \bar{\mathcal{R}}_n^m$, the supplier always builds $K_n^b(\mathcal{R}_n^m)$ units of capacity and no payment is exchanged, either because the manufacturer rejects the supplier's offer or because the manufacturer accepts $(K_n^b(\mathcal{R}_n^m), 0)$. When $\xi_n \notin \bar{\mathcal{R}}_n^m$, the manufacturer accepts $(K(\xi_n), P(\xi_n))$. Thus, every such $\hat{\mathcal{R}}$ yields the same expected profits for both the supplier and the manufacturer. In addition, accepting the menu for every ξ_n also gives the same expected profits as \mathcal{R}_n^m .

We call the manufacturer's strategy to accept $(K(\xi_n), P(\xi_n))$ for every ξ_n the *full acceptance strategy*. When designing the optimal menu of contracts it is sufficient for the supplier to consider only the menus under which the full acceptance strategy is optimal for the manufacturer.

THEOREM 6. *Suppose that $\{K(\cdot), P(\cdot)\}$ satisfies (IC). We define $\{\hat{K}(\cdot), \hat{P}(\cdot)\}$ such that $(\hat{K}(\xi), \hat{P}(\xi)) = (K_n^b(\mathcal{R}_n^m), 0)$ if $\xi \in \mathcal{R}_n^m(K(\cdot), P(\cdot))$, and $(\hat{K}(\xi), \hat{P}(\xi)) = (K(\xi), P(\xi))$ otherwise. Then $\{\hat{K}(\cdot), \hat{P}(\cdot)\}$ satisfies (IC), and under $\{\hat{K}(\cdot), \hat{P}(\cdot)\}$ the full acceptance strategy is optimal for the manufacturer. The two menus, $\{K(\cdot), P(\cdot)\}$ and $\{\hat{K}(\cdot), \hat{P}(\cdot)\}$, yield the same expected profits for both firms.*

Note that the full acceptance strategy is also optimal for the manufacturer if and only if

$$(FA) \quad \Pi_n^m(K(\xi), X_n^s, \xi) - P(\xi) \geq \Pi_n^m(K_n^b(\mathcal{R}_n^m(K(\cdot), P(\cdot))), X_n^s, \xi) \quad \text{for every } \xi.$$

Given Theorem 6, the supplier can obtain an optimal menu of contracts by solving

$$\begin{aligned} \pi_n^{cr}(X_n^s) = \max_{K(\cdot), P(\cdot)} E_{\xi_n} [\Pi_n^s(K(\xi_n), X_n^s, \xi_n) + P(\xi_n)] \\ \text{s.t. (IC) and (FA).} \end{aligned} \quad (5)$$

This new formulation helps reduce the search for an optimal menu of contracts. Note the manufacturer participates when her expected profit is at least $\Pi_n^m(K_n^b(\mathcal{R}_n^m(K(\cdot), P(\cdot))), X_n^s, \xi)$, which is her *reservation profit*. This reservation profit is a function of the menu $\{K(\cdot), P(\cdot)\}$. In broad adverse-selection literature, the agent's reservation profit is given *exogenously*—i.e., it is independent of the offered menu (e.g., Myerson 1981, Maggi and Rodriguez-Clare 1995, Jullien 2000, Özer and Wei 2006, Taylor and Xiao 2010). Such mechanism design problems have a simple participation constraint instead of (FA). Hence, for those problems, determining the set of feasible menus that satisfy the participation constraint can be constructed following the solution approach by Mirrlees (1971) or Jullien (2000). However, a simple condition that assures (FA) does not exist in the present context because reservation profit depends *endogenously* on the offered menu of contracts. To determine $\mathcal{R}_n^m(K(\cdot), P(\cdot))$, the supplier first needs to find every set $\mathcal{R} \subseteq [0, \infty)$ that satisfies (PC1) and (PC2) under $\{K(\cdot), P(\cdot)\}$ and then determine the one that gives the largest $K_n^b(\mathcal{R})$. Because such a set is different for each menu, the problem of constructing the set of feasible menus of contracts itself is an intractable problem. Thus, instead of optimally solving (5), we provide a heuristic solution and show that it is close to optimal. To do so, we first replace (FA) in (5) with a weaker condition and solve the relaxed problem. By modifying the solution to this relaxed problem, we construct a feasible menu of contracts.

LEMMA 1. (1) If (FA) holds, then $\Pi_n^m(K(\xi), X_n^s, \xi) - P(\xi) \geq 0$ for every ξ .

(2) Let

$$\begin{aligned} \hat{\pi}_n^s(K(\cdot), P(\cdot), X_n^s) \equiv E_{\xi_n} [\Pi_n^s(K(\xi_n), X_n^s, \xi_n) + P(\xi_n)] \\ = \int_0^\infty \overbrace{(\Pi_n^{\text{tot}}(K(\xi), X_n^s, \xi) - \frac{1 - F_n(\xi)}{f_n(\xi)} (r - w) X_n^s \int_0^{K(\xi)/(X_n^s \xi)} x g_n(x) dx) f_n(\xi)}^{H(K(\xi), \xi)} d\xi. \end{aligned} \quad (6)$$

If we replace (FA) in (5) with $\Pi_n^m(K(\xi), X_n^s, \xi) - P(\xi) \geq 0$ for every ξ , the optimization problem can

equivalently be written as $\max_{K(\cdot), P(\cdot)} \hat{\pi}_n^s(K(\cdot), P(\cdot), X_n^s)$ subject to $K(\xi)$ increasing in ξ . The solution of this problem is given by $\{K_n^r(\xi), P_n^r(\xi)\}$, where $K_n^r(\xi)$ is the unique solution of $\partial H(K, \xi) / \partial \xi = 0$ and $P_n^r(\xi) \equiv \Pi_n^m(K_n^r(\xi), X_n^s, \xi) - \int_0^\xi (r - w) X_n^s \int_0^{K_n^r(y)/(X_n^s y)} x g_n(x) dx dy$.

The solution and the resulting contract for the relaxed problem in Lemma 1 are similar to those of Özer and Wei (2006), except that we solve the multiplicative and dynamic version here. The resulting expected profit is an upper bound for the supplier's optimal expected profit $\pi_n^{cr}(X_n^s)$ because it is the optimal solution for the relaxed problem. The solution in Lemma 1 assumes that the manufacturer would accept the supplier's offer as long as her expected profit is not negative. However, when offered this menu, the manufacturer may reject it at certain levels of private demand forecast in anticipation of the supplier building a large amount of capacity by himself. In other words, the menu $\{K_n^r(\cdot), P_n^r(\cdot)\}$ could result in a lower reservation profit for the manufacturer than that specified in the constraint (5). Next we construct an effective heuristic solution for (5) using $\{K_n^r(\cdot), P_n^r(\cdot)\}$. We first provide necessary and sufficient conditions for (FA).

THEOREM 7. Suppose that $\{K(\cdot), P(\cdot)\}$ satisfies (IC). Then the following properties hold:

(1) $K_n^b([\xi_l, \xi_h])$ is increasing in ξ_l and also in ξ_h . Hence, $K_n^b(\mathcal{R}_n^m) \geq K_n^b([0, \xi])$ when \mathcal{R}_n^m is a convex set containing ξ .

(2) If (FA) holds, then $\Pi_n^m(K(\xi), X_n^s, \xi) - P(\xi) \geq \Pi_n^m(K_n^b([0, \xi]), X_n^s, \xi)$ for every ξ .

(3) If $\Pi_n^m(K(\xi), X_n^s, \xi) - P(\xi) > \Pi_n^m(K_n^b([0, \xi]), X_n^s, \xi)$ and $P(\xi) > 0$ for every $\xi > 0$, then (FA) holds.

Part (1) of Theorem 7 shows that if the manufacturer finds it optimal to reject the supplier's offer when her private forecast is larger, then the supplier would find it optimal to build a larger capacity. Part (1) also provides a lower bound on the base capacity level for each rejection strategy of the manufacturer. Using this bound, part (2) determines the minimum profit that is necessary to ensure the manufacturer's participation. Suppose that the manufacturer's forecast is $\hat{\xi}$ and her optimal strategy is to reject the supplier's offer. In this case, the manufacturer's optimal strategy \mathcal{R}_n^m must be a convex set including $\hat{\xi}$ (because of parts (1) and (5) of Theorem 5). The resulting base capacity level $K_n^b(\mathcal{R}_n^m)$ would at least be equal to $K_n^b([0, \hat{\xi}])$ units. Hence, by rejecting the supplier's offer, the manufacturer can earn at least $\Pi_n^m(K_n^b([0, \hat{\xi}]), X_n^s, \hat{\xi})$. Hence, if the manufacturer's expected profit $\Pi_n^m(K(\xi), X_n^s, \xi) - P(\xi)$ is smaller than $\Pi_n^m(K_n^b([0, \hat{\xi}]), X_n^s, \hat{\xi})$ for some ξ , then the full acceptance strategy cannot be optimal for the manufacturer.

Part (2) also provides the lower bound, $\Pi_n^m(K_n^b([0, \xi]), X_n^s, \xi)$, on the manufacturer's reservation profit, $\Pi_n^m(K_n^b(R_n^m(K(\cdot), P(\cdot))), X_n^s, \xi)$, in (FA). Unlike the reservation profit, its lower bound is independent of the offered menu. Hence, the supplier can use part (2) to easily verify whether a given incentive-compatible menu of contracts violates (FA) in (5). Later, we also use this lower bound for the reservation profit to obtain an upper bound for the supplier's optimal expected profit.

Part (3) shows that when the payment for reserving capacity is positive, $\Pi_n^m(K_n^b([0, \xi]), X_n^s, \xi)$ is also sufficient to ensure the manufacturer's participation. The result also shows that an effective menu of contracts generally requires the manufacturer to make a positive payment $P(\xi)$ for every ξ . Negative reservation prices imply that the supplier pays a positive fee to the manufacturer. In addition, they imply that the supplier needs to provide an expected profit higher than $\Pi_n^m(K_n^b([0, \xi]), X_n^s, \xi)$, i.e., high information rent, to ensure the manufacturer's participation for each ξ . Hence, we consider the menus with positive payments. Next, we establish a condition on $K(\xi)$ that assures (FA).

LEMMA 2. Suppose that $\{K(\cdot), P(\cdot)\}$ satisfies (IC). We define $K_n^{fa}(\xi)$ as the unique solution for the following equation $(d(\Pi_n^m(K(\xi), X_n^s, \xi) - P(\xi)))/d\xi|_{K(\xi)=K} = (d\Pi_n^m(K_n^b([0, \xi]), X_n^s, \xi))/d\xi$; that is,

$$X_n^s \int_0^{K/(X_n^s \xi)} x g_n(x) dx = X_n^s \int_0^{(K_n^b([0, \xi])/(X_n^s \xi)} x g_n(x) dx + \frac{dK_n^b([0, \xi])}{d\xi} \left(1 - G_n \left(\frac{K_n^b([0, \xi])}{X_n^s \xi} \right) \right). \quad (7)$$

If $K(\xi) > K_n^{fa}(\xi)$ for every ξ and $P(0) = 0$, then (FA) holds.

Using $K_n^r(\xi)$ from Lemma 1 and $K_n^{fa}(\xi)$ from Lemma 2, we design a heuristic solution for (5). We modify $K_n^r(\xi)$ such that the capacity levels are always greater than $K_n^{fa}(\xi)$, so that it satisfies the condition stated in Lemma 2. Recall from Theorem 4 that for a menu of contracts to satisfy (IC), we need $K(\xi)$ to be increasing in ξ . Because $K_n^r(\xi)$ satisfies (IC), it is increasing in ξ . In contrast, $K_n^{fa}(\xi)$ may not be increasing in ξ . For a marginally small $\delta > 0$, we define $\bar{K}_n^{fa}(\xi) \equiv \max_{\hat{\xi} \leq \xi} (K_n^{fa}(\hat{\xi}) + \delta)$, which is increasing in ξ . Then we design the heuristic menu as follows:

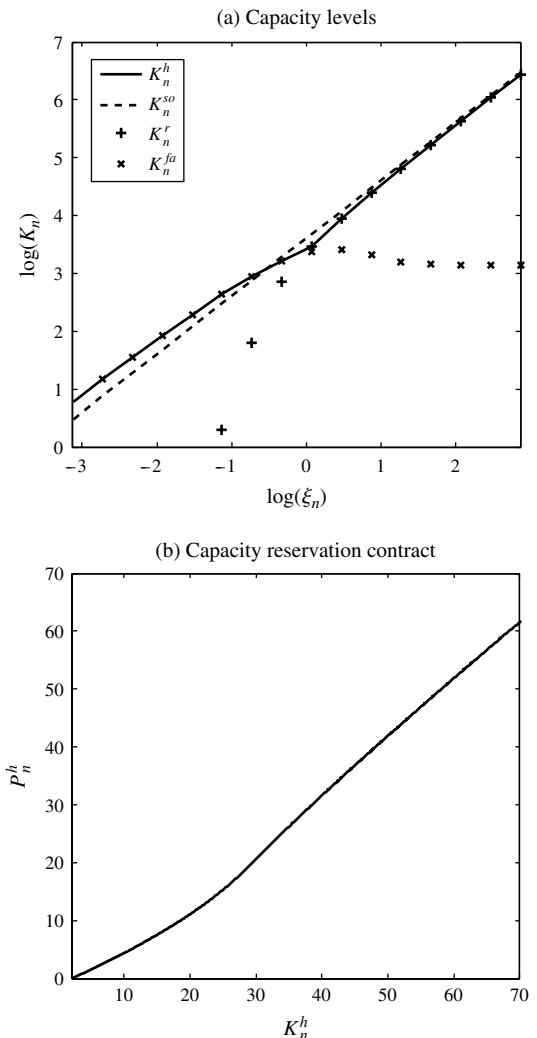
THEOREM 8. We define

$$K_n^h(\xi) \equiv \max\{\bar{K}_n^{fa}(\xi), K_n^r(\xi)\} \quad \text{and} \\ P_n^h(\xi) \equiv \Pi_n^m(K_n^h(\xi), X_n^s, \xi) - \int_0^\xi (r - w) \cdot X_n^s \int_0^{K_n^h(y)/(X_n^s y)} x g_n(x) dx dy.$$

The menu of contracts $\{K_n^h(\cdot), P_n^h(\cdot)\}$ satisfies both (IC) and (FA), i.e., is a feasible solution for (5). We denote the resulting expected profit by $\pi^h(X_n^s)$.

We provide an example of $K_n^h(\xi)$ along with other reference capacity levels in Figure 3(a). The capacity level set under the heuristic menu is consistently close to the system optimal level for every ξ . Hence, the total supply chain profit is also close to the system optimal profit. Because both $K_n^h(\xi)$ and $P_n^h(\xi)$ are increasing in ξ , we can map K_n^h to P_n^h directly without ξ . Figure 3(b) shows this mapping. The supplier can offer this single function $P_n^h(K_n^h)$ to the manufacturer without having to communicate any forecast information. The manufacturer can reserve K_n^h units of capacity by paying $P_n^h(K_n^h)$. Hence, this function can be interpreted as a capacity reservation contract. This function is neither concave nor convex in ξ . We remark that when the manufacturer has a

Figure 3 Reference Capacity Levels and Proposed Contract



Note. The parameters are $r = 15$, $c = 3$, $w = 10$, $c_n = 2$, $\sigma_{\epsilon_n} = 1$, $\sigma_{\xi_n} = 1$, and $X_n^s = 23$.

constant reservation profit, the optimal capacity reservation contract $P(K)$ is concave increasing (Özer and Wei 2006), suggesting a quantity discount scheme.

To measure the performance of the heuristic solution, we compare the resulting profit $\pi_n^h(X_n^s)$ with an upper bound for $\pi_n^{cr}(X_n^s)$ provided in the following theorem.

THEOREM 9. *We have $\pi_n^{cr}(X_n^s) \leq \bar{\pi}_n^{cr}(X_n^s) \equiv \min(\bar{\pi}_n^{cr1}(X_n^s), \bar{\pi}_n^{cr2}(X_n^s))$ for every X_n^s , where $\bar{\pi}_n^{cr1}(X_n^s) \equiv E_{\xi_n}[\Pi_n^s(K_n^r(\xi_n), X_n^s, \xi_n) + P_n^r(\xi_n)]$ and $\bar{\pi}_n^{cr2}(X_n^s) \equiv E_{\xi_n}[\Pi_n^{\text{tot}}(K_n^{\text{so}}(\xi_n), X_n^s, \xi_n) - \Pi_n^m(K_n^b([0, \xi_n]), X_n^s, \xi_n)]$.*

The theorem provides two upper bounds. The first one is obtained by solving the relaxed problem in Lemma 1. The second one is obtained by using part (2) of Theorem 7, which provides a lower bound on the manufacturer's profit (and hence an upper bound on the supplier's). Then the percentage difference $G = ((\bar{\pi}_n^{cr}(X_n^s) - \pi_n^h(X_n^s)) / \bar{\pi}_n^{cr}(X_n^s)) \times 100\%$ is an upper bound for the true optimality gap of the heuristic solution. We show that the optimality gap is very small in §7.

Next, we discuss how the capacity reservation contract and the resulting expected profit depend on the supplier's demand forecast at the time when he offers the contract. We denote the optimal capacity reservation contract, i.e., the optimal solution for (5), by $\{K_n^{cr}(\cdot), P_n^{cr}(\cdot)\}$.

THEOREM 10. *The following statements are true for each $i \in \{cr, h\}$: (1) $K_n^i(\xi)$ is proportionally increasing in X_n^s ; (2) $P_n^i(\xi)$ is proportionally increasing in X_n^s ; (3) $\pi_n^i(X_n^s) = X_n^s(\pi_n^i(1) + C_n) - C_n$.*

This result characterizes the role of time in designing an incentive mechanism and shows that the menu is state dependent. In other words, an effective menu of contracts should depend on the supplier's (principal's) forecast information and the time when the menu is offered. This result also significantly reduces the computational effort required to obtain the solutions for the overall problem.

5.2. The First- and Second-Stage Problems

In the second stage, the supplier determines whether to offer an optimal menu of contracts to the manufacturer or to build capacity by himself; i.e., $\pi_n(X_n^s) = \max\{\pi_n^w(X_n^s), \pi_n^{cr}(X_n^s)\}$. The following theorem shows that offering the menu of contracts is always optimal for the supplier.

THEOREM 11. *For every X_n^s , $\pi_n^{cr}(X_n^s) \geq \pi_n^w(X_n^s)$ holds.*

A similar result holds for the supplier's profit under the heuristic menu of contracts. In all numerical experiments, $\pi_n^h(X_n^s) > \pi_n^w(X_n^s)$, and on average it is 80% larger, as we will discuss in §7. Hence, the supplier optimally offers a menu of contracts before building capacity. The rest of the results in this subsection hold for both the heuristic and optimal

menu of contracts. If the supplier offers the heuristic menu of contracts in the third stage, then the second-stage problem is $\pi_n(X_n^s) = \max(\pi_n^w(X_n^s), \pi_n^h(X_n^s))$.

The supplier's expected profit depends primarily on the following four factors: demand forecast, cost of demand uncertainty, cost of asymmetric information (i.e., cost of screening), and cost of capacity. Intuitively, the supplier's expected profit increases as the demand uncertainty decreases and the capacity costs decrease. In addition, the supplier's expected profit is greater when the information asymmetry is smaller, because the manufacturer gets a lower information rent. The impact of these three factors depends on the supplier's demand forecast (part (3) of Theorem 10). The optimal stopping problem takes such trade-offs into consideration. The following theorem characterizes the optimal stopping policy.

THEOREM 12. *A control band policy that offers a capacity reservation contract at period n if $X_n^s \in [L_n, U_n]$ is optimal, and the optimal thresholds are given as $U_n \equiv \sup\{X_n^s: \pi_n(X_n^s) \geq E[V_{n+1}(X_{n+1}^s) | X_n^s]\}$, and $L_n \equiv \inf\{X_n^s: \pi_n(X_n^s) \geq E[V_{n+1}(X_{n+1}^s) | X_n^s]\}$.*

Under this control-band policy, after the supplier updates his information, he checks whether his demand forecast falls within a control band. If it does, then delaying the capacity decision to obtain new demand forecast information is not optimal; i.e., the supplier should offer the optimal capacity reservation contract to the manufacturer at period n . If the supplier delays the capacity decision, then the manufacturer obtains more demand information, which enables the supplier to make a better capacity decision under the capacity reservation contract. The impact of the improved demand forecast depends on the forecast level. When the demand forecast is very large, the benefit of reduced demand uncertainty is significant. Hence, the optimal policy has an upper threshold U_n above which delaying the capacity decision is optimal. When the demand forecast is very small, the reduction in the demand uncertainty, the change in the information asymmetry, and unit capacity cost have limited impact on the overall profit change. Thus, the optimal stopping decision critically depends on the changes in the fixed capacity cost. If the fixed cost is smaller at a later time period than the current period and the current demand forecast is small, the supplier should delay the capacity decision. Thus, the optimal policy also has a lower threshold L_n on X_n^s below which delaying the capacity decision is optimal. When the fixed cost never decreases over time, this benefit disappears. The following theorem establishes this case and another one.

THEOREM 13. *The following statements are true for all n :*

(1) *When $C_{n+1} > C_n$ for all n , the lower threshold, L_n , is 0 for all n . Hence, an upper threshold policy that offers*

the capacity reservation contract at period n if $X_n^s \leq U_n$ is optimal.

(2) When $C_{n+1} = C_n$ for all n , the lower and upper thresholds satisfy that $L_n = U_n = 0$ for $n \neq n^*$ and that $L_n = 0$ and $U_n = \infty$ for $n = n^*$, where $n^* \equiv \arg \max_n \pi_n(1)$. Hence, a state-independent stopping policy that offers the capacity reservation contract at period n^* is optimal.

If the supplier incurs higher fixed capacity costs by delaying the capacity decision, as the forecast level converges to 0, the benefit of reduced demand uncertainty and information asymmetry vanishes, whereas the loss in the fixed capacity cost remains constant. Hence, the supplier should delay offering the contract only when the demand forecast is large, which implies that the lower threshold is 0 and that an upper threshold policy is optimal. When the fixed capacity cost is constant over time, the optimal time to offer the contract is fully determined by the trade-off among the demand uncertainty, information asymmetry, and the unit capacity cost, whose impact on the supplier's profit is proportional to the forecast level. Hence, if the expected profit of offering the contract at period n is greater than the expected profit of offering the contract at period m , then it is true, regardless of the forecast level. Thus, the optimal policy is to always stop at n^* .

6. Centralized Supply Chain

Consider a single decision maker, an integrated firm that owns both the supplier and the manufacturer. Such a firm would have access to all forecast information X_n^m (i.e., the concept of private information is irrelevant in this case) to decide when and how much capacity to build and maximize the total supply chain profit. The firm's problem can be formulated as a two-stage stochastic dynamic program. The first stage is the optimal stopping problem that determines the time to set the capacity level. The second stage is a newsvendor problem that determines the optimal capacity level; i.e., $\pi_n^{cs}(X_n^m) \equiv \max_K \Pi_n^{\text{tot}}(K, X_n^s, \xi_n)$, where $X_n^m = X_n^s \xi_n$. The optimal solution for this problem is the system optimal capacity level $K_n^{so}(\xi_n)$. To formulate the first-stage optimal stopping problem for a centralized system, we replace X_n^s with X_n^m and $\pi_n(X_n^s)$ with $\pi_n^{cs}(X_n^m)$ in §4.1. If the central decision maker delays the capacity decision, she obtains the forecast update δ_n^m and updates the forecast as $X_{n+1}^m = X_n^m \delta_n^m$. Otherwise, she stops at period n and earns the reward $\pi_n^{cs}(X_n^m)$.

THEOREM 14. *For the centralized supply chain, the following statements are true for all n :*

(1) A control-band policy that determines the capacity level at period n if $X_n^m \in [L_n^{cs}, U_n^{cs}]$ is optimal.

(2) When $C_{n+1} > C_n$ for all n , an upper threshold policy that determines the capacity level at period n if $X_n^m \leq U_n^{cs}$ is optimal.

(3) When $C_{n+1} = C_n$ for all n , a state-independent policy that determines the capacity level at period $n^{cs} \equiv \arg \max_n \pi_n^{cs}(1)$ is optimal.

The centralized decision maker's optimal stopping policy has the same structure as the supplier's policy, discussed in the previous section. Note, however, that the centralized system makes the stopping decision based on X_n^m , whereas the decentralized system makes the decision based on X_n^s . The resulting optimal stopping thresholds and profits will also be different.

7. Numerical Study

The purpose of this section is fourfold. First, we evaluate the performance of the heuristic menu of contracts. Second, we provide additional insights regarding the capacity reservation contract. Third, we examine the impact of the capacity reservation contract on channel efficiency. Fourth, we discuss comparative statistics of the optimal stopping thresholds to characterize when the supplier should offer the contract—early or late. Unless otherwise noted, we set $r = 15$, $w = 10$, $c = 3$, $c_n = 2$, $C_n = 10$, $\sigma_{\xi_n} = 1$, $\sigma_{\xi_n} = 0.5$, and $X_n^s = 23$.

7.1. Performance of the Capacity Reservation Contract $P_n^h(K_n^h)$

To test the performance of the heuristic solution, we compute $I = (\pi_n^h(X_n^s) - \pi_n^w(X_n^s)) / \pi_n^w(X_n^s)$ and $G = (\bar{\pi}_n^{cr}(X_n^s) - \pi_n^h(X_n^s)) / \bar{\pi}_n^{cr}(X_n^s)$ under 3,328 different test settings. The percentage difference I quantifies the supplier's expected profit improvement as a result of using the capacity reservation contract to elicit credible information from the manufacturer. The percentage difference G quantifies the optimality gap. Together, they measure the effectiveness of the heuristic menu of contracts. We report the two measures under every combination of the following parameters except for the cases in which $r < w$ or $w - c - c_n < 0$: $r \in \{11, 13, 15, 17\}$, $w \in \{8, 10, 12, 14\}$, $c \in \{1, 2, 3, 4\}$, $c_n \in \{1, 1.5, 2, 2.5\}$, $\sigma_{\xi_n} \in \{0.3, 0.5, 0.7, 0.9\}$, and $\sigma_{\epsilon_n} \in \{0.6, 0.8, 1, 1.2\}$. The average value of the profit improvement I is 80.75% and the median value is 44.57%, which shows that the supplier can dramatically improve his expected profit by offering the capacity reservation contract $P_n^h(K_n^h)$. The optimality gap G is 7.52% on average, and the median value is 5.40%. Given that the difference between the optimal expected profit $\pi_n^{cr}(X_n^s)$ and its upper bound $\bar{\pi}_n^{cr}(X_n^s)$ also contributes to G , the result shows that the proposed contract $P_n^h(K_n^h)$ performs very well. Table 1 provides additional statistics for each r and σ_{ξ_n} . This table illustrates that the capacity reservation contract improves the supplier's profit substantially when r (respectively, σ_{ξ_n}) is large, i.e., when the adverse effects of double marginalization (respectively, information asymmetry between the firms) are large.

Table 1 Effectiveness and Optimality Gap

r	Average I (%)	Median I (%)	Average G (%)	Median G (%)	σ_{ξ_n}	Average I (%)	Median I (%)	Average G (%)	Median G (%)
11	61.2	40.5	5.5	4.1	0.3	32.4	13.4	6.6	4.7
13	71.0	41.2	6.5	4.8	0.5	60.0	30.6	5.9	3.9
15	78.2	41.2	7.3	5.1	0.7	93.0	52.5	7.8	5.9
17	100.4	52.4	9.5	7.4	0.9	137.6	81.7	9.8	8.7

7.2. Capacity Reservation Contracts

Figure 4(a) shows the capacity reservation contract for three different wholesale prices. For a given K , the reservation price $P_n^h(K)$ decreases as the wholesale price increases. When the wholesale price is high, the manufacturer's expected profit is small. Thus, to ensure the manufacturer accepts the contract, the supplier needs to charge a small reservation price. In Figure 4(b) we hold the supplier's demand uncertainty $\sigma_{\xi_n}^2 + \sigma_{\epsilon_n}^2$ constant and increase σ_{ξ_n} . The larger σ_{ξ_n} means that the manufacturer has more accurate

demand information. When the manufacturer has more accurate information and when she shares her private information credibly via the capacity reservation contract, both firms can earn higher expected profits. Hence, the supplier can ask, and the manufacturer is willing to pay, a higher price to reserve capacity.

7.3. Supply Chain Efficiency

Table 2 reports the impact of the capacity reservation contract on the supply chain efficiency. We define $CE^h \equiv E[\Pi_n^{\text{tot}}(K_n^h(\xi_n), X_n^s, \xi_n)]/E[\Pi_n^{\text{tot}}(K_n^{so}(\xi_n), X_n^s, \xi_n)]$ and $CE^w \equiv E[\Pi_n^{\text{tot}}(K_n^w, X_n^s, \xi_n)]/E[\Pi_n^{\text{tot}}(K_n^{so}(\xi_n), X_n^s, \xi_n)]$, which measure the channel efficiency with and without the capacity reservation contract. The supply chain efficiency without a capacity reservation contract, (i.e., with only the wholesale price contract) CE^w is low when (i) w is small and (ii) the manufacturer has better (i.e., more accurate) forecast information. In contrast, with the capacity reservation contract, the supply chain efficiency is consistently close to the optimal level. The contract $P^h(K^h)$ mitigates the adverse affects of both double marginalization and information asymmetry by enabling credible information sharing. The table also shows that the capacity reservation contract almost coordinates the channel regardless of how the wholesale price is negotiated or the resulting wholesale price.

7.4. Optimal Time to Offer the Contract

Finally, we examine whether the supplier should offer the capacity reservation contract early or late. We set $N = 6$ and $C_n = 10 + (n-1)\Delta_C$, where Δ_C indicates the rate of increase in the fixed capacity cost. We also set $\sigma_{\delta_N^s} = \sqrt{1.6}$, $\sigma_{\delta_n^s} = 0.25$, $\sigma_{\delta_N^m} = 1$, and $\sigma_{\delta_n^m} = \sigma^m$ for $n = 1, 2, \dots, N-1$. The value of $\sigma_{\delta_n^i}$ quantifies the information that decision maker i obtains at each period of the capacity planning horizon, and $\sigma_{\delta_N^i}$ indicates the degree of residual demand uncertainty of decision maker i at the end of the capacity planning horizon. Hence, when $\sigma^m < 0.25$, the supplier obtains more information in each period than the manufacturer, and the opposite is true when $\sigma^m > 0.25$.

Table 3 reports the stopping thresholds U_n for different values of σ^m and Δ_C . Recall from Theorem 13 that when C_n is increasing in n , U_n describes the optimal stopping policy. High threshold levels indicate

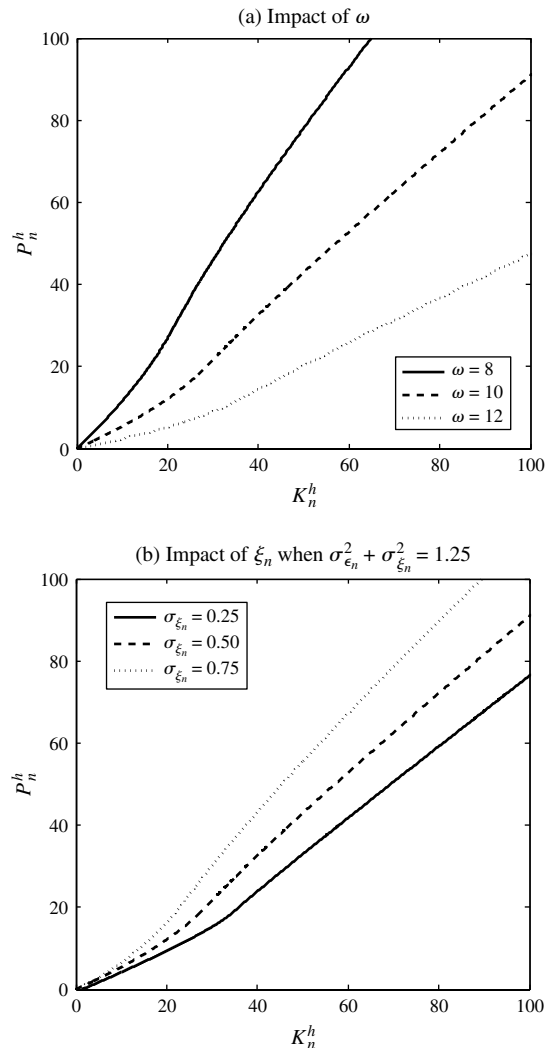
Figure 4 Optimal Capacity Reservation Contracts

Table 2 Channel Efficiency

w	Π_n^s	Π_n^m	CE^h	CE^w	$\sigma_{\varepsilon_n}^2$	Π_n^s	Π_n^m	CE^h	CE^w
7	61.6	66.9	0.955	0.637	0.03	57.6	64.9	0.996	0.921
8	63.0	68.0	0.974	0.738	0.23	74.1	58.2	0.993	0.850
9	67.9	64.6	0.985	0.802	0.43	84.8	57.9	0.985	0.782
10	75.2	58.1	0.992	0.843	0.63	95.5	58.7	0.978	0.718
11	84.4	49.3	0.995	0.870	0.83	107.4	59.9	0.970	0.656
12	94.9	38.8	0.995	0.887	1.03	122.4	60.5	0.959	0.594
$\sigma_{\varepsilon_n} = 0.5$					$w = 10, \sigma_{\varepsilon_n}^2 + \sigma_{\varepsilon_n}^2 = 1.25$				

Table 3 Optimal Stopping Thresholds

σ^m	Δ_c	U_1	U_3	U_5	U_1^{cs}	U_3^{cs}	U_5^{cs}
(a) 0.25	4	40.3	41.9	45.1	24.1	25.2	27.2
(b) 0.35	4	18.4	19.0	20.0	12.4	12.9	14.1
(c) 0.15	4	199.6	234.1	294.2	68.5	70.7	74.7
(d) 0.25	2	20.1	20.9	22.6	12.1	12.6	13.6

that the decision maker tends to offer a contract early. In case (a), the two decision makers obtain the same amount of demand information over time. In case (b), the manufacturer obtains more demand information than she does in case (a), and thus the degree of information asymmetry between her and the supplier increases over time. In case (b) the supplier waits longer than in case (a), so that the manufacturer obtains more information. The result implies that the manufacturer's improved demand forecast benefits the supplier. In case (c) the information asymmetry decreases over time. In this case, the manufacturer has an accurate demand forecast early on, and thus it is beneficial for the supplier and the supply chain to stop early, before the capacity cost increases too much. Finally, the result of case (d) implies that the supplier should offer the contract early when the capacity cost increases rapidly. Next, we discuss the differences between U_n and U_n^{cs} . This difference shows how much the supplier's timing decision deviates from the socially optimal timing decision. In all four cases, the optimal thresholds for the supplier are higher than the optimal thresholds for the centralized supply chain. The result implies that the supplier tends to determine the capacity level earlier than the socially optimal time.

8. Discussion and Conclusion

This paper provides a framework to model evolutions of forecasts made by multiple decision makers who forecast demand for the same product. We show that the framework can be used to consistently model several scenarios for evolutions of forecasts. For example, we model the scenario in which forecasters have asymmetric demand information that

changes over time; refer to the model as the *Martingale Model of Asymmetric Forecast Evolutions* (MMAFE). This framework also helps study an adverse selection problem in a dynamic environment. We consider a supplier's (principal's) problem of eliciting credible forecast information from a manufacturer (agent) for a capacity investment decision when both firms obtain asymmetric demand information for the end product over multiple periods. We determine the optimal time for the supplier to make the capacity decision and establish the optimality of a control-band policy. Our research also highlights that the firms' alternative option of not using (or rejecting) the mechanism should be endogenous to the overall sequence of events in a dynamic environment. We show how this consideration affects the form and the outcome of screening contracts. To do so, we also provide a new approach to solve adverse-selection problems with mechanism-dependent reservation profits. Our main results can be extended to account for a variety of other supply chain scenarios. In the addendum to the present paper, we provide the complete analysis of the following plausible scenarios: when the forecast updates are additive; when the supplier incurs a cost in updating the forecast; and when the supplier sets the wholesale price (in addition to the capacity reservation contract). We conclude the paper with a brief discussions of the additive case.

The Additive Case. We construct the additive model of forecast evolutions for multiple decision makers by defining the difference between successive forecasts as $\delta_n^i \equiv X_{n+1}^i - X_n^i$, for $n < N$ and $\delta_N^i \equiv X_{N+1}^i - X_N^i$. We replace the ratio of successive forecasts with this new definition, all multiplication operators with additions, and all product operators with summations. Unlike in the multiplicative case, now we assume that the change in the forecast caused by each event is independent of the current forecast. By invoking the Central Limit Theorem, δ_{n_s, n_m} is normally distributed with $E[\delta_{n_s, n_m}] = 0$ except for $\delta_{0,0}$.⁹ This model satisfies

⁹ Both decision makers have the information $\delta_{0,0}$ before the beginning of the forecast horizon. Hence, $\delta_{0,0}$ is a deterministic value. Note also that when $E[\delta_{n_s, n_m}] \neq 0$ for some (n_s, n_m) , we can push this information to $\delta_{0,0}$ and normalize δ_{n_s, n_m} by $\delta_{n_s, n_m} - E[\delta_{n_s, n_m}]$. Hence, $E[\delta_{n_s, n_m}] = 0$ is without loss of generality.

all properties of Theorem 1. This construction fully characterizes the evolutions of X_n^s and X_n^m by determining the value of $\delta_{0,0}$ and the variances of δ_{n_s, n_m} . The *additive Martingale model of asymmetric forecast evolutions* (a-MMAFE) is also similar to the multiplicative case. At the beginning of period n , the manufacturer's demand uncertainty is given by $\epsilon_n \equiv \sum_{k=n}^N \delta_k^m$. Similarly, the manufacturer's private information is given by $\xi_n \equiv \sum_{k=n}^N \delta_k^s - \epsilon_n = \sum_{k=n}^N \sum_{n_m=0}^{n-1} \delta_{k, n_m}$. Then we have $X_{N+1} = X_n^m + \epsilon_n = X_n^s + \xi_n + \epsilon_n$. In the additive case, the supplier's problem is the same as before except that now the forecast model is an a-MMAFE. The supplier updates the demand forecast as $X_{n+1}^s = X_n^s + \delta_n^s$ if he delays to offer the contract at period n . All of our results continue to hold for the additive case. The results are deferred to the addendum, available from the authors.

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Appendix A. Summary of Notation

We denote the supplier by (s), the manufacturer by (m), and the centralized decision maker by (cs).

Demand and Forecast

- X_{N+1} : demand during the sales season
- e_j : random variable representing the impact of event j on demand
- E_{n_s, n_m} : set of events whose information is obtained by (s) at period n_s and by (m) at period n_m
- $\delta_{n_s, n_m} = \prod_{j \in E_{n_s, n_m}} e_j$: random variable representing the total information obtained by (s) at period n_s and by (m) at period n_m
- X_n^i : (i)'s demand forecast for each $i \in \{s, m\}$
- $\Delta_n^i = X_{n+1}^i - X_n^i$: difference between (i)'s subsequent forecasts for each $i \in \{s, m\}$
- $A_n = X_n^m - X_n^s$: difference between (s) and (m)'s forecasts
- $\delta_n^i = X_{n+1}^i / X_n^i$: random variable representing the ratio of (i)'s successive forecasts for each $i \in \{s, m\}$
- $\epsilon_n = \prod_{k=n}^N \delta_k^m$: random variable representing demand uncertainty at period n
- $\xi_n = \prod_{k=n}^N \delta_k^s / \epsilon_n$: random variable representing information asymmetry at period n
- $X_{N+1} = X_n^m \epsilon_n = X_n^s \xi_n \epsilon_n$
- $G_n(\cdot), g_n(\cdot)$: c.d.f. and p.d.f. of ϵ_n
- $F_n(\cdot), f_n(\cdot)$: c.d.f. and p.d.f. of ξ_n
- σ_Z : standard deviation of $\log(Z)$ when Z is a log-normal random variable

Cost Parameters

- r : unit retail price
- c : unit production cost
- w : unit wholesale price
- C_n : fixed capacity cost at period n
- c_n : unit capacity cost at period n

Decision Variables

- u_n : (s)'s stopping decision
- $K_n^{so}(\xi)$: system optimal capacity level
- $\mathcal{R}_n^m(K(\cdot), P(\cdot))$: (m)'s optimal strategy
- \mathcal{R}_n^m : weak rejection set
- $\xi_l(\mathcal{R}_n^m), \xi_h(\mathcal{R}_n^m)$: boundaries of \mathcal{R}_n^m
- K_n^w : optimal capacity under wholesale price contract
- $K_n^{ws}(\xi)$: optimal capacity under symmetric information
- $K_n^b(\mathcal{R}_n^m)$: base capacity level defined in Equation (3)
- $K_n^r(\xi)$: solution for $\partial H(K, \xi) / \partial \xi$ defined in Lemma 1
- $K_n^{fa}(\xi)$: solution for Equation (7)
- $\{(K_n^{cr}(\cdot), P_n^{cr}(\cdot))\}$: optimal menu of contracts
- $\{(K_n^h(\cdot), P_n^h(\cdot))\}$: heuristic solution
- $[L_n, U_n]$: (s)'s optimal control-band
- $[L_n^{cs}, U_n^{cs}]$: (cs)'s optimal control-band

Profit Functions

- $\Pi_n^s(K, X_n^s, \xi)$: (s)'s profit during sales season
- $\Pi_n^m(K, X_n^s, \xi)$: (m)'s profit during sales season
- $\Pi_n^{\text{tot}}(K, X_n^s, \xi)$: total supply chain profit
- $\pi_n^{cr}(X_n^s) = E_{\xi_n}[\Pi_n^s(K_n^{cr}(\xi_n), X_n^s, \xi_n) + P_n^{cr}(\xi_n)]$
- $\pi_n^h(X_n^s) = E_{\xi_n}[\Pi_n^s(K_n^h(\xi_n), X_n^s, \xi_n) + P_n^h(\xi_n)]$
- $\pi_n^{cs}(X_n^m)$: (cs)'s optimal expected profit
- $\pi_n^w(X_n^s) = E_{\xi_n}[\Pi_n^s(K_n^w, X_n^s, \xi_n)]$
- $\pi_n(X_n^s) = \max\{\pi_n^{cr}(X_n^s), \pi_n^w(X_n^s)\}$
- $V_n(X_n^s)$: (s)'s optimal value-to-go function
- $V_n^{cs}(X_n^m)$: (cs)'s optimal value-to-go function

Appendix B. Proofs

We defer the proofs of Theorems 1, 2, 3, 4, and 14, Lemma 1, and also more detailed version of the remaining proofs to the addendum, available from the authors. We also remark that we suppress X_n^s in profit functions when doing so does not cause any ambiguity for two reasons: (i) notational convenience and (ii) to highlight that the proof and the result holds for general scenarios that does not involve forecast updates.

PROOF OF THEOREM 5. To prove part (1), define $q(\xi, \hat{K}) \equiv \Pi_n^m(K(\xi), \xi) - P(\xi) - \Pi_n^m(\hat{K}, \xi)$. From (IC1), we have

$$\frac{dq(\xi, K_n^b(\mathcal{R}_n^m))}{d\xi} = (r - w)X_n^s \int_{K_n^b(\mathcal{R}_n^m)/(X_n^s \xi)}^{K(\xi)/(X_n^s \xi)} x g_n(x) dx. \quad (B1)$$

Then (IC2) implies that $q(\xi, K_n^b(\mathcal{R}_n^m))$ is a quasiconvex function, where its first-order derivative is 0 if and only if $K(\xi) = K_n^b(\mathcal{R}_n^m)$. The quasiconvexity implies that \mathcal{R}_n^m is a convex set.

We prove part (2) by contradiction. Suppose that $K(\tilde{\xi}) \geq K_n^b(\mathcal{R}_n^m)$ for some $\tilde{\xi} < \xi_l(\mathcal{R}_n^m)$. By the definition of \mathcal{R}_n^m , we have $\Pi_n^m(K(\tilde{\xi}), \tilde{\xi}) - P(\tilde{\xi}) > \Pi_n^m(K_n^b(\mathcal{R}_n^m), \tilde{\xi})$. Because $K(\xi)$ is increasing in ξ , we have $K(\xi) \geq K_n^b(\mathcal{R}_n^m)$ for every $\xi \geq \tilde{\xi}$. Then, from (B1), we have $\Pi_n^m(K(\xi), \xi) - P(\xi) > \Pi_n^m(K_n^b(\mathcal{R}_n^m), \xi)$ for every $\xi \geq \tilde{\xi}$, which contradicts the definition of \mathcal{R}_n^m . Thus, $K(\xi) < K_n^b(\mathcal{R}_n^m)$ holds for every $\xi < \xi_l(\mathcal{R}_n^m)$. When $K(\xi) < K_n^b(\mathcal{R}_n^m)$, $\Pi_n^m(K(\xi), \xi) - P(\xi) \geq \Pi_n^m(K_n^b(\mathcal{R}_n^m), \xi)$

holds only when $P(\xi) < 0$, which concludes the proof for part (2). We can prove part (3) in a similar way.

We next prove part (4). If $(K_n^b(\mathcal{R}_n^m), 0) \in \{K(\cdot), P(\cdot)\}$, we have from (IC) that $\Pi_n^m(K(\xi), \xi) - P(\xi) \geq \Pi_n^m(K_n^b(\mathcal{R}_n^m), \xi) - 0$ for every ξ . Hence, we must have $\Pi_n^m(K(\xi), \xi) - P(\xi) = \Pi_n^m(K_n^b(\mathcal{R}_n^m), \xi)$ for every $\xi \in \mathcal{R}_n^m$. Then, for $\Pi_n^m(K(\xi), \xi) - P(\xi) - \Pi_n^m(K_n^b(\mathcal{R}_n^m), \xi) = 0$ to hold for every $\xi \in \mathcal{R}_n^m$, we need $dq(\xi, K_n^b(\mathcal{R}_n^m))/d\xi = 0$ for every $\xi_l(\mathcal{R}_n^m) < \xi < \xi_h(\mathcal{R}_n^m)$. From (B1), $dq(\xi, K_n^b(\mathcal{R}_n^m))/d\xi = 0$ only when $K(\xi) = K_n^b(\mathcal{R}_n^m)$. Thus, $K(\xi) = K_n^b(\mathcal{R}_n^m)$ for every $\xi_l(\mathcal{R}_n^m) < \xi < \xi_h(\mathcal{R}_n^m)$. Because $(K_n^b(\mathcal{R}_n^m), 0) \in \{K(\cdot), P(\cdot)\}$, (IC) implies that $P(\xi) = 0$ when $K(\xi) = K_n^b(\mathcal{R}_n^m)$.

Finally, we prove part (5). We first prove that if $K(\xi) = K_n^b(\mathcal{R}_n^m)$ for some ξ , we have $P(\xi) > 0$. Suppose that $K(\hat{\xi}) = K_n^b(\mathcal{R}_n^m)$ and $P(\hat{\xi}) < 0$ for some $\hat{\xi}$. Then (IC) implies that $\Pi_n^m(K(\xi), \xi) - P(\xi) \geq \Pi_n^m(K_n^b(\mathcal{R}_n^m), \xi) - P(\hat{\xi}) > \Pi_n^m(K_n^b(\mathcal{R}_n^m), \xi)$ for every ξ , which contradicts the definition of \mathcal{R}_n^m . From (B1), $q(\xi, K_n^b(\mathcal{R}_n^m))$ is a quasiconvex function of ξ , which stays constant only when $K(\xi) = K_n^b(\mathcal{R}_n^m)$. Because $P(\xi) > 0$ when $K(\xi) = K_n^b(\mathcal{R}_n^m)$, we have $q(\xi, K_n^b(\mathcal{R}_n^m)) = \Pi_n^m(K(\xi), \xi) - P(\xi) - \Pi_n^m(K_n^b(\mathcal{R}_n^m), \xi) < 0$ when $(d/d\xi)q(\xi, K_n^b(\mathcal{R}_n^m)) = 0$. Hence, $\Pi_n^m(K(\xi), \xi) - P(\xi) = \Pi_n^m(K_n^b(\mathcal{R}_n^m), \xi)$ can hold at two points, which implies that \mathcal{R}_n^m and $\hat{\mathcal{R}}_n^m$ coincide and that $\Pi_n^m(K(\xi), \xi) - P(\xi) < \Pi_n^m(K_n^b(\mathcal{R}_n^m), \xi)$ for every $\xi_l(\mathcal{R}_n^m) < \xi < \xi_h(\mathcal{R}_n^m)$. When $P(\xi) > 0$ for every ξ , part (2) implies that $\xi_l(\mathcal{R}_n^m) = 0$. In this case, $\mathcal{R}_n^m = \hat{\mathcal{R}}_n^m = [0, \xi_h(\mathcal{R}_n^m)]$. \square

PROOF OF THEOREM 6. We denote $\{K(\cdot), P(\cdot)\}$ by \mathcal{M} and $\{\hat{K}(\cdot), \hat{P}(\cdot)\}$ by $\hat{\mathcal{M}}$. If $\mathcal{R}_n^m = \phi$, $\hat{\mathcal{M}} = \mathcal{M}$, and thus the theorem holds trivially. If $(K_n^b(\mathcal{R}_n^m), 0) \in \mathcal{M}$, part (4) of Theorem 5 implies that $\hat{\mathcal{M}} = \mathcal{M}$, and thus the theorem also holds. Next we consider the case in which $\mathcal{R}_n^m \neq \phi$ and $(K_n^b(\mathcal{R}_n^m), 0) \notin \mathcal{M}$. We first show that $\hat{\mathcal{M}}$ satisfies (IC). Parts (2) and (3) of Theorem 5 imply that $\hat{K}(\xi) < K_n^b(\mathcal{R}_n^m)$ for every $\xi < \inf(\mathcal{R}_n^m)$ and $\hat{K}(\xi) > K_n^b(\mathcal{R}_n^m)$ for every $\xi > \sup(\mathcal{R}_n^m)$. Hence, $\hat{\mathcal{M}}$ satisfies (IC2). Because $\hat{P}(\xi) = 0$ for every $\xi \in \mathcal{R}_n^m$, we also have $(d/d\xi)(\Pi_n^m(\hat{K}(\xi), \xi) - \hat{P}(\xi)) = (r - w)X_n^s \int_0^{K_n^b(\mathcal{R}_n^m)/(X_n^s \xi)} x g_n(x) dx$ for every $\xi \in \mathcal{R}_n^m$, which implies (IC1). Hence, $\hat{\mathcal{M}}$ satisfies (IC).

Next we show that the full acceptance strategy is optimal for the manufacturer under $\hat{\mathcal{M}}$ by a contradiction argument. Suppose that under $\hat{\mathcal{M}}$ the full acceptance strategy is not optimal for the manufacturer. We denote the manufacturer's optimal strategy under $\hat{\mathcal{M}}$ by $\hat{\mathcal{R}}_n^m$. Because the full acceptance strategy is not optimal, $\Pi_n^m(K_n^b(\hat{\mathcal{R}}_n^m), \xi) > \Pi_n^m(\hat{K}(\xi), \xi) - \hat{P}(\xi)$ for some ξ . By construction, $(K_n^b(\mathcal{R}_n^m), 0) \in \mathcal{M}$, and thus (IC) implies that $\Pi_n^m(\hat{K}(\xi), \xi) - \hat{P}(\xi) \geq \Pi_n^m(K_n^b(\mathcal{R}_n^m), \xi)$ for every ξ . The two inequalities imply that $\Pi_n^m(K_n^b(\mathcal{R}_n^m), \xi) > \Pi_n^m(K_n^b(\hat{\mathcal{R}}_n^m), \xi)$ holds for some ξ , which implies that $K_n^b(\hat{\mathcal{R}}_n^m) > K_n^b(\mathcal{R}_n^m)$. Because $\hat{\mathcal{R}}_n^m$ satisfies (PC2) under $\hat{\mathcal{M}}$, $\Pi_n^m(K_n^b(\hat{\mathcal{R}}_n^m), \xi) \geq \Pi_n^m(\hat{K}(\xi), \xi) - \hat{P}(\xi)$ for $\xi \in \hat{\mathcal{R}}_n^m$. When $\xi \notin \mathcal{R}_n^m$ and $\xi \in \hat{\mathcal{R}}_n^m$, $\hat{K}(\xi) = K(\xi)$ and $\hat{P}(\xi) = P(\xi)$, and thus $\Pi_n^m(K(\xi), \xi) - P(\xi) = \Pi_n^m(\hat{K}(\xi), \xi) - \hat{P}(\xi) \leq \Pi_n^m(K_n^b(\hat{\mathcal{R}}_n^m), \xi)$. When $\xi \in \mathcal{R}_n^m$ and $\xi \in \hat{\mathcal{R}}_n^m$, $\Pi_n^m(K(\xi), \xi) - P(\xi) \leq \Pi_n^m(K_n^b(\mathcal{R}_n^m), \xi) < \Pi_n^m(K_n^b(\hat{\mathcal{R}}_n^m), \xi)$. Thus, \mathcal{R}_n^m satisfies (PC2) under \mathcal{M} . When $\xi \notin \mathcal{R}_n^m$, we have $\Pi_n^m(\hat{K}(\xi), \xi) - \hat{P}(\xi) \geq \Pi_n^m(K_n^b(\hat{\mathcal{R}}_n^m), \xi) > \Pi_n^m(K_n^b(\mathcal{R}_n^m), \xi)$, which implies that $(\hat{K}(\xi), \hat{P}(\xi)) \neq (K_n^b(\mathcal{R}_n^m), 0)$, and thus $(\hat{K}(\xi), \hat{P}(\xi)) = (K(\xi), P(\xi))$. The result implies that when

$\xi \notin \hat{\mathcal{R}}_n^m$, $\Pi_n^m(K(\xi), \xi) - P(\xi) = \Pi_n^m(\hat{K}(\xi), \xi) - \hat{P}(\xi) \geq \Pi_n^m(K_n^b(\hat{\mathcal{R}}_n^m), \xi)$, i.e., $\hat{\mathcal{R}}_n^m$ satisfies (PC1) under \mathcal{M} . Because $\hat{\mathcal{R}}_n^m$ satisfies (PC1) and (PC2) under \mathcal{M} , the fact that $K_n^b(\hat{\mathcal{R}}_n^m) > K_n^b(\mathcal{R}_n^m)$ contradicts the fact that \mathcal{R}_n^m is the manufacturer's optimal strategy under \mathcal{M} .

Finally, we prove that the two menus \mathcal{M} and $\hat{\mathcal{M}}$ result in the same outcome in terms of the capacity decision and the transfer payment. When $\xi_n \notin \mathcal{R}_n^m$, the manufacturer accepts $(K(\xi_n), P(\xi_n))$ under both \mathcal{M} and $\hat{\mathcal{M}}$. When $\xi_n \in \mathcal{R}_n^m$, the manufacturer accepts $(\hat{K}(\xi_n), \hat{P}(\xi_n)) = (K_n^b(\mathcal{R}_n^m), 0)$ under $\hat{\mathcal{M}}$ and rejects the menu and lets the supplier build $K_n^b(\mathcal{R}_n^m)$ under \mathcal{M} . The two cases are identical in terms of final capacity decision and the transfer payment. Thus, \mathcal{M} and $\hat{\mathcal{M}}$ yield the same expected profits for both the supplier and the manufacturer, which concludes the proof. \square

PROOF OF THEOREM 7. We first prove part (1). Note that $K_n^b([\xi_l, \xi_h])$ is the maximizer of $E_{\xi_n}[\Pi_n^s(K, \xi_n) \mid \xi_n \in [\xi_l, \xi_h]]$ whose first-order condition after normalization is given as $\int_{\xi_l}^{\xi_h} [(w - c)(1 - G_n(K/(X_n^s \xi))) - c_n] f_n(\xi) d\xi = 0$. Thus, we have

$$\int_{\xi_l}^{\xi_h} \left[(w - c) \left(1 - G_n \left(\frac{K_n^b([\xi_l, \xi_h])}{X_n^s \xi} \right) \right) - c_n \right] f_n(\xi) d\xi = 0. \quad (\text{B2})$$

Because $[(w - c)(1 - G_n(K/(X_n^s \xi))) - c_n]$ increases as ξ increases, we have $(w - c)(1 - G_n(K_n^b([\xi_l, \xi_h])/(X_n^s \xi_l))) - c_n \leq 0$ and $(w - c)(1 - G_n(K_n^b([\xi_l, \xi_h])/(X_n^s \xi_h))) - c_n \geq 0$. By taking derivative on (B2) with respect to ξ_h , we have $[(w - c)(1 - G_n(K_n^b([\xi_l, \xi_h])/(X_n^s \xi_h))) - c_n] f_n(\xi_h) - (dK_n^b([\xi_l, \xi_h])/d\xi_h) \int_{\xi_l}^{\xi_h} ((w - c)/(X_n^s \xi)) g_n(K_n^b([\xi_l, \xi_h])/(X_n^s \xi)) \cdot f_n(\xi) d\xi = 0$, which implies that $(dK_n^b([\xi_l, \xi_h])/d\xi_h) \geq 0$. We can prove that $(dK_n^b([\xi_l, \xi_h])/d\xi_l) \geq 0$ in a similar way.

Next, we prove part (2). We define $Q(K) \equiv \{\xi: \Pi_n^m(K, \xi) \geq \Pi_n^m(K(\xi), \xi) - P(\xi)\}$, $\xi_l(K) \equiv \inf Q(K)$, $\xi_h(K) \equiv \sup Q(K)$, and $q(K) \equiv K - K_n^b([\xi_l(K), \xi_h(K)])$. Suppose for a contradiction argument that (FA) holds and $\Pi_n^m(K(\tilde{\xi}), \tilde{\xi}) - P(\tilde{\xi}) < \Pi_n^m(K_n^b([0, \tilde{\xi}]), \tilde{\xi})$ for some $\tilde{\xi}$. We show that there exists a $\tilde{K} \geq K_n^b([0, \tilde{\xi}])$ such that $q(\tilde{K}) = 0$. By the definition of $Q(K)$, $\tilde{\xi} \in Q(K_n^b([0, \tilde{\xi}]))$, which implies that $\xi_h(K_n^b([0, \tilde{\xi}])) \geq \tilde{\xi}$. Trivially, $\xi_l(K_n^b([0, \tilde{\xi}])) \geq 0$. Because $K_n^b([\xi_l, \xi_h])$ is increasing in ξ_l and ξ_h , these inequalities imply that $q(K_n^b([0, \tilde{\xi}])) = K_n^b([0, \tilde{\xi}]) - K_n^b([\xi_l(K_n^b([0, \tilde{\xi}]), \xi_h(K_n^b([0, \tilde{\xi}])))] \leq K_n^b([0, \tilde{\xi}]) - K_n^b([0, \tilde{\xi}]) = 0$. Next we show that $\lim_{K \rightarrow \infty} q(K) > 0$. Because $Q(K_1) \subseteq Q(K_2)$ when $K_2 \geq K_1$, $\xi_l(K)$ is decreasing in K . Hence, for every $K \geq K_n^b([0, \tilde{\xi}])$, $K_n^b([\xi_l(K), \xi_h(K)]) \leq K_n^b([\xi_l(K_n^b([0, \tilde{\xi}]), \xi_h(K)]) \leq K_n^b([\xi_l(K_n^b([0, \tilde{\xi}]), \infty)])$. Note that $K_n^b([\xi_l(K_n^b([0, \tilde{\xi}]), \infty)])$ is the solution for $\int_{\xi_l(K_n^b([0, \tilde{\xi}]))}^{\infty} [(w - c) \cdot (1 - G_n(K/(X_n^s \xi))) - c_n] f_n(\xi) d\xi = 0$, which is finite because $\xi_l(K_n^b([0, \tilde{\xi}])) \leq \tilde{\xi}$. Thus, $\lim_{K \rightarrow \infty} q(K) \rightarrow \infty$. Because $q(K_n^b([0, \tilde{\xi}])) \leq 0$ and $\lim_{K \rightarrow \infty} q(K) \rightarrow \infty$, there exists a $\tilde{K} \geq K_n^b([0, \tilde{\xi}])$ such that $q(\tilde{K}) = 0$. We define $\tilde{\mathcal{R}} \equiv [\xi_l(\tilde{K}), \xi_h(\tilde{K})]$ for such \tilde{K} . Because $q(\tilde{K}) = 0$, we have $\tilde{K} = K_n^b([\xi_l(\tilde{K}), \xi_h(\tilde{K})]) = K_n^b(\tilde{\mathcal{R}})$. By the definition of $\xi_l(K)$ and $\xi_h(K)$, $Q(\tilde{K}) = [\xi_l(\tilde{K}), \xi_h(\tilde{K})] = \tilde{\mathcal{R}}$, which implies that $\tilde{\mathcal{R}}$ satisfies (PC1) and (PC2). Finally, because $K_n^b(\tilde{\mathcal{R}}) = \tilde{K} \geq K_n^b([0, \tilde{\xi}])$ and $\Pi_n^m(K(\tilde{\xi}), \tilde{\xi}) - P(\tilde{\xi}) < \Pi_n^m(K_n^b([0, \tilde{\xi}]), \tilde{\xi})$, we have $\Pi_n^m(K(\tilde{\xi}), \tilde{\xi}) - P(\tilde{\xi}) < \Pi_n^m(K_n^b(\tilde{\mathcal{R}}), \tilde{\xi})$, which contradicts the assumption that (FA) holds. Thus, if (FA) holds, $\Pi_n^m(K(\tilde{\xi}), \tilde{\xi}) - P(\tilde{\xi}) \geq \Pi_n^m(K_n^b([0, \tilde{\xi}]), \tilde{\xi})$ for every $\tilde{\xi}$.

Next we prove part (3). We consider a menu $\{K(\cdot), P(\cdot)\}$ that satisfies $\Pi_n^m(K(\xi), \xi) - P(\xi) > \Pi_n^m(K_n^b([0, \xi]), \xi)$ and

$P(\xi) > 0$ for every $\xi > 0$. Suppose for a contradiction argument that under this menu the full acceptance strategy is not optimal for the manufacturer. In this case, part (5) of Theorem 5 implies that $\mathcal{R}_n^m = [0, \xi]$ for some $\xi > 0$. Then, for \mathcal{R}_n^m to satisfy (PC2), which is a necessary condition for the manufacturer's optimal strategy, we must have $\Pi_n^m(K_n^b(\mathcal{R}_n^m), \xi) = \Pi_n^m(K_n^b([0, \xi]), \xi) \geq \Pi_n^m(K(\xi), \xi) - P(\xi)$, which is a contradiction. \square

PROOF OF LEMMA 2. From (IC1),

$$\frac{1}{r-w} \frac{d(\Pi_n^m(K(\xi), \xi) - P(\xi))}{d\xi} = X_n^s \int_0^{K(\xi)/X_n^s} x g_n(x) dx,$$

and

$$\begin{aligned} & \frac{1}{r-w} \frac{d(\Pi_n^m(K_n^b([0, \xi]), \xi) - P(\xi))}{d\xi} \\ &= X_n^s \int_0^{K_n^b([0, \xi])/X_n^s} x g_n(x) dx + \frac{dK_n^b([0, \xi])}{d\xi} \left(1 - G_n\left(\frac{K_n^b([0, \xi])}{X_n^s \xi}\right)\right). \end{aligned}$$

Because $d(\Pi_n^m(K(\xi), \xi) - P(\xi))/d\xi$ is strictly increasing in $K(\xi)$ and $(\Pi_n^m(K_n^b([0, \xi]), \xi))/d\xi$ is independent of it, $K_n^b(\xi)$ is uniquely defined. Because $\Pi_n^m(K(0), 0) - P(0) = \Pi_n^m(K_n^b(0, 0), 0) = 0$ when $P(0) = 0$, the fact that $K(\xi) > K_n^b(\xi)$ for every ξ implies that $\Pi_n^m(K(\xi), \xi) - P(\xi) = \int_0^\xi (d(\Pi_n^m(K(x), x) - P(x))/dx) dx > \int_0^\xi (d(\Pi_n^m(K_n^b([0, x]), x)/dx) dx = \Pi_n^m(K_n^b([0, \xi]), \xi)$. Then, part (3) of Theorem 7 implies that (FA) holds. \square

PROOF OF THEOREM 8. By construction, $K_n^h(\xi)$ is increasing in ξ ; i.e., it satisfies (IC2). Because the reservation price $P_n^h(\xi)$ is constructed by integrating (IC1) with $K(\xi) = K_n^h(\xi)$, the menu satisfies (IC1). Because $K_n^h(\xi) \geq \bar{K}_n^{fa}(\xi) > K_n^{fa}(\xi)$ holds for every ξ and $P_n^h(0) = 0$, Lemma 2 implies that this menu satisfies (FA). Hence, $\{K_n^h(\cdot), P_n^h(\cdot)\}$ is a feasible solution for (5). \square

PROOF OF THEOREM 9. The fact that $\pi_n^{cr}(X_n^s) \leq \bar{\pi}_n^{cr1}(X_n^s)$ stems directly from the fact that $\bar{\pi}_n^{cr1}(X_n^s)$ is the resulting expected profit of a relaxed problem of (5). By the definition of $K_n^{so}(\xi)$, we have $\Pi_n^{\text{tot}}(K, \xi) \leq \Pi_n^{\text{tot}}(K_n^{so}(\xi), \xi)$ for every K and ξ . Then, part (2) of Theorem 7 implies that if (FA) holds, $\Pi_n^m(K(\xi), \xi) - P(\xi) \geq \Pi_n^m(K_n^b([0, \xi]), \xi)$ for every ξ . Thus, if (FA) holds, we have $\Pi_n^s(K(\xi), \xi) + P(\xi) = \Pi_n^{\text{tot}}(K(\xi), \xi) - \Pi_n^m(K(\xi), \xi) + P(\xi) \leq \Pi_n^{\text{tot}}(K_n^{so}(\xi), \xi) - \Pi_n^m(K_n^b([0, \xi]), \xi)$, for every ξ , which implies that $\pi_n^{cr}(X_n^s) \leq \bar{\pi}_n^{cr2}(X_n^s)$. \square

PROOF OF THEOREM 10. We first define $\hat{K}(\cdot) \equiv K(\cdot)/X_n^s$, $\hat{P}(\cdot) \equiv P(\cdot)/X_n^s$, $\hat{K}_n^b(\mathcal{R}) \equiv K_n^b(\mathcal{R})/X_n^s$, and $\hat{\mathcal{R}}_n^m(\hat{K}(\cdot), \hat{P}(\cdot)) \equiv \mathcal{R}_n^m(K(\cdot), P(\cdot))$, which are the normalized decision variables. If we replace $K(\cdot)$ and $P(\cdot)$ with $\hat{K}(\cdot)$ and $\hat{P}(\cdot)$ in (IC), we have $\Pi_n^m(\hat{K}(\xi), 1, \xi) - \hat{P}(\xi) \geq \Pi_n^m(\hat{K}_n^b(\xi), 1, \xi) - \hat{P}(\xi)$ for every ξ and ξ . This new constraint is independent of X_n^s . Similarly, if we replace decision variables in (FA) with the normalized ones, we have $\Pi_n^m(\hat{K}(\xi), 1, \xi) - \hat{P}(\xi) \geq \Pi_n^m(\hat{K}_n^b(\hat{\mathcal{R}}_n^m(\hat{K}(\cdot), \hat{P}(\cdot)), 1, \xi) - \hat{P}(\xi)$ for every ξ , which is also independent of X_n^s . Note that the supplier's expected profit is given as $\Pi_n^s(K(\xi), X_n^s, \xi) + P(\xi) = X_n^s[(w-c)\min(\xi\epsilon_n, \hat{K}(\xi)) - c_n\hat{K}(\xi) + \hat{P}(\xi)] - C_n$. The optimal $\{\hat{K}(\cdot), \hat{P}(\cdot)\}$ that maximizes the supplier's profit is thus the maximizer of $E_{\xi_n}[(w-c)E_{\epsilon_n}[\min(\xi_n\epsilon_n, \hat{K}(\xi_n))] - c_n\hat{K}(\xi_n) +$

$\hat{P}(\xi_n)]$ under the two normalized constraints. Because this objective function and the two constraints are independent of X_n^s , the optimal $\hat{K}(\cdot)$ and $\hat{P}(\cdot)$ are independent of X_n^s . Because $K(\cdot) = X_n^s\hat{K}(\cdot)$ and $P(\cdot) = X_n^s\hat{P}(\cdot)$, we can conclude that the optimal $K_n^{cr}(\xi)$ and $P_n^{cr}(\xi)$ are proportional to X_n^s . Because $\pi_n^{cr}(1) = E_{\xi_n}[(w-c)E_{\epsilon_n}[\min(\xi_n\epsilon_n, K_n^{cr}(\xi_n))] - c_nK_n^{cr}(\xi_n) + P_n^{cr}(\xi_n)] - C_n$, we have $\pi_n^{cr}(X_n^s) = X_n^s \cdot (\pi_n^{cr}(1) + C_n) - C_n$, which concludes the proof for $i = cr$. We can prove the case for $i = h$ in a similar way using the normalized decision variables. \square

PROOF OF THEOREM 11. We first prove that the menu $\{(K(\xi) = K_n^w, P(\xi) = 0): \xi \in [0, \infty)\}$ is a feasible solution for (5). Because only one contract is offered, (IC) holds trivially. Let \mathcal{R}_n^m be the manufacturer's optimal strategy under this menu. If $K_n^b(\mathcal{R}_n^m) > K_n^w$, then (PC1) implies that $\mathcal{R}_n^m = [0, \infty)$, which contradicts the fact that $K_n^b(\mathcal{R}_n^m) > K_n^w$ because $K_n^b([0, \xi]) = K_n^w$. Because $\Pi_n^m(K(\xi), \xi) - P(\xi) = \Pi_n^m(K_n^w, \xi)$ and $\Pi_n^m(K_n^w, \xi) \geq \Pi_n^m(\mathcal{R}_n^m, \xi)$, (FA) holds. Because this menu is feasible for (5), we have $\pi_n^{cr}(X_n^s) \geq E_{\xi_n}[\Pi_n^s(K_n^w, \xi_n)] = \pi_n^w(X_n^s)$, which concludes the proof. \square

PROOF OF THEOREM 12. From Theorems 10 and 11, we have that $\pi_n(X_n^s) = X_n^s(\pi_n(1) + C_n) - C_n$. We first show that this structure remains the same when the supplier offers the heuristic menu of contracts instead of the optimal menu. Because

$$K_n^w = X_n^s(F_n * G_n)^{-1}\left(\frac{(w-c-c_n)}{(w-c)}\right),$$

we have

$$\begin{aligned} \pi_n^w(X_n^s) &= (w-c)E_{\xi_n}\left[\min(X_n^s\xi_n\epsilon_n, X_n^s(F_n * G_n)^{-1}\left(\frac{w-c-c_n}{w-c}\right))\right] \\ &\quad - c_nX_n^s(F_n * G_n)^{-1}\left(\frac{w-c-c_n}{w-c}\right) - C_n \\ &= X_n^s(\pi_n^w(1) + C_n) - C_n. \end{aligned}$$

From Theorem 10, we have $\pi_n(X_n^s) = \max\{\pi_n^h(X_n^s), \pi_n^w(X_n^s)\} = X_n^s(\max\{\pi_n^h(1), \pi_n^w(1)\} + C_n) - C_n = X_n^s(\pi_n(1) + C_n) - C_n$. The proofs for Theorems 12 and 13 are based on this structure. Hence, these results hold for both the optimal and the heuristic menus of contracts. The result follows from Proposition 2 in Oh and Özer (2009) after defining $M_n(X_n^s) \equiv E[\pi_{n+1}(X_{n+1}^s) | X_n^s] - \pi_n(X_n^s)$ and $B_n(X_n^s) \equiv E[V_{n+1}(X_{n+1}^s) | X_n^s] - \pi_n(X_n^s)$, which are convex in X_n^s . \square

PROOF OF THEOREM 13. We define $\hat{\pi}_n \equiv \pi_n(1) + C_n$ for notational convenience. For part (1), we first note that $B_n(X_n^s)$ is convex from the proof of Theorem 12. We prove that $B_n(0) \leq 0$ when C_n is increasing. When $X_n^s = 0$, $X_l^s = 0$ almost surely for every $l \geq n$. Therefore, $V_n(0) = -C_n$, which implies that $B_n(0) = -(C_{n+1} - C_n) \leq 0$. If a convex function satisfies $B_n(0) \leq 0$, then $B_n(X_n^s)$ can cross 0 at most once from below to above in $(0, \infty)$. Therefore, the lower threshold, L_n , is 0, and the upper threshold policy is optimal.

For part (2), we first define $\eta_n \equiv \max_{m>n} \hat{\pi}_m$. We prove by induction that $B_n(X_n^s) = (\eta_n - \hat{\pi}_n)X_n^s$ for all n . For period $n = N - 1$, we have $B_n(X_n^s) = M_n(X_n^s) = X_n^s(\hat{\pi}_{n+1} - \hat{\pi}_n) - (C_{n+1} - C_n) = (\hat{\pi}_{n+1} - \hat{\pi}_n)X_n^s$, where $\eta_n = \hat{\pi}_{n+1}$ by definition. Next assume for an induction argument that $B_{n+1}(X) =$

$(\eta_{n+1} - \hat{\pi}_{n+1})X_n^s$. If $\eta_{n+1} \geq \hat{\pi}_{n+1}$, then $\eta_n = \eta_{n+1}$ and $B_n(X_n^s) = E[\max\{0, B_{n+1}(X_{n+1}^s)\} | X_n^s] + M_n(X_n^s) = (\eta_{n+1} - \hat{\pi}_{n+1})X_n^s + M_n(X_n^s) = (\eta_{n+1} - \hat{\pi}_{n+1})X_n^s + (\hat{\pi}_{n+1} - \hat{\pi}_n)X_n^s = (\eta_n - \hat{\pi}_n)X_n^s$. In contrast, if $\eta_{n+1} < \hat{\pi}_{n+1}$, then $\eta_n = \hat{\pi}_{n+1}$ and $B_n(X_n^s) = E[\max\{0, B_{n+1}(X_{n+1}^s)\} | X_n^s] + M_n(X_n^s) = 0 + M_n(X_n^s) = (\hat{\pi}_{n+1} - \hat{\pi}_n)X_n^s = (\eta_n - \hat{\pi}_n)X_n^s$, which concludes the induction argument.

We next prove that the optimal policy always stops at period n^* . For $n < n^*$, $\eta_n = \hat{\pi}_{n^*}$; hence, $\eta_n > \hat{\pi}_n$ by the definition of n^* . In this case, $B_n(X_n^s) \geq 0$ for all X_n^s , and it is always optimal to continue the process. For $n = n^*$, we have $\eta_{n^*} \leq \hat{\pi}_{n^*}$, which implies that $B_{n^*}(X_{n^*}^s) \leq 0$. Hence, the optimal policy always stops at period n^* . \square

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