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Design and Control of Public-Service Contracts and an Application to Public Transportation Systems

Andrea Lodi, Enrico Malaguti

Dipartimento di Ingegneria dell'Energia Elettrica e dell'Informazione "Guglielmo Marconi",
Università di Bologna, 40126 Bologna, Italy
{andrea.lodi@unibo.it, enrico.malaguti@unibo.it}

Nicolás E. Stier-Moses

Graduate School of Business, Columbia University, New York, New York 10027; and
Business School, Universidad Torcuato Di Tella and CONICET, 1428 Buenos Aires, Argentina,
stier@gsb.columbia.edu

Tommaso Bonino

SRM—Reti e Mobilità Srl, 40128 Bologna, Italy, tommaso.bonino@srm-bologna.it

Until the end of the 20th century it was common that cities organized, financed, and managed their own public transportation systems. More recently (in Italy mainly during the last decade), many countries liberalized the service provision of public transportation. Although offering local public transportation is a political and financial duty of governments, the service provision is outsourced to a private operator, while the government retains a control position, given to a regulatory agency. This is a common scheme not only in transportation. In close collaboration with the public transportation agency of the city of Bologna, Italy, we designed a methodology to optimize some aspects of the contractual relationship between that agency and the bus operator. We focus on the fines specified by the contract when the operator fails to comply with the service level it has agreed to provide, and on a procedure to measure that service level. Our approach has the advantage that it aligns the incentives of both parties to the benefit of bus riders. We model the agency–operator relationship as a multistage game and find its equilibrium to establish the best operating regime. The game-theoretical approach provides expressions for the fines that the operator should be charged if it does not satisfy the contracted services, and for the optimal agency's budget to devote to control activities. Second, to check the compliance of the operator with the schedule specified by the contract in a resource-efficient way, we compute how to position the agency's employees to verify if buses are running according to the specifications of the contract. This is achieved by counting bus services and by checking other quality indicators. We formulate this NP-hard problem as a mixed integer linear program and propose an algorithm to solve it that is effective in providing itineraries for the controllers working for the agency.

Keywords: integer programming application; public transportation; transit systems; vehicle routing; economics; game theory and bargaining theory

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1. Introduction

In all modern economies, it is common for governments and local authorities to regularly outsource services to the market. Depending on the economical organization of the government and the nature of the object or service to be purchased, the transaction can be performed as a one-time activity or it can give rise to a long-term relationship with the provider. This paper is about the latter case, where a public entity outsources a service to be provided to the citizens through a contract with a provider.

Many, if not all, public-service contracts can be naturally fit to the following scheme. First, the contract is designed by the central/local government, often with

the help of a regulatory agency, and a public auction takes place. Second, companies interested in the contract participate in the auction, the "best" offer is selected, and the contract is signed. Third, the service is provided by the company that won the auction. Fourth, the central/local government controls quality of service and compliance with the contract specifications. This is done very often with the help of the same regulatory agency. Depending on the type, duration, and sector of the contract, the control activity can be performed while the contract is in effect or after it is completed.

Commonly, these contracts involve penalties of different types. They depend on the kind of mismatches

between specifications and provisions, on the reasons for which the mismatches occurred, and, of course, on how severe the mismatches have been. Although the fines charged by the government can be extremely high, it is crucial to stress that the aim of the central/local government is the overall benefit of the citizens, and, consequently, there is no incentive to make money from the fines. In other words, putting the service provider out of business by imposing fines is not a desirable option because that would result in a severe disservice to society. This implies that the design of the contract (e.g., in terms of its obligations and fines) must consider trade-offs carefully: a fine that is clearly too high will not discourage the service provider from missing contractual obligations because the provider knows that enforcing the contract has a political cost that is not affordable for the government.

For the fines specified in the contract to work, the agency must perform independent verifications of the service level. Indeed, knowing that the agency measures the quality of service accurately provides incentives to the provider to increase the effort to comply with the contract. Furthermore, contracts of this kind are often periodic (i.e., a similar contract for the same public service has to be signed every few years), which provides some opportunities to tune details, generate better contracts, and improve the relationship between both parties. This further supports the idea that the control procedure performed by the agency, whose goal is to be as informed as possible regarding the working conditions of the system, is valuable in the long term. At the same time, control activities have a cost. The issue at hand is getting the most out of the (typically small) budget that the government allocates to these control activities, i.e., distilling enough information to provide the maximum incentive for the provider to align with the contract. This is essentially a resource allocation problem where capacity constraints are typically tight.

One of the best examples of public–private contracting and of the problems related to the lack of an effective control during contract execution is the so-called “Big Dig” project in Boston (see, e.g., Wikipedia 2013). That was a megaproject that rerouted Interstate 93, which used to go through downtown Boston into a 3.5-mile tunnel. Concurrently, another tunnel was constructed to extend Interstate 90 to Logan International Airport. The planning started in 1982 and the actual construction was done between 1991 and 2006. The project was managed by the Massachusetts Turnpike Authority, and the design and construction were supervised by a private joint venture and executed by dozens of smaller subcontractors because of scale. The cost of the Big Dig was on the order of 20 billion dollars, making it the most expensive highway project

in the United States. At that price, it was plagued by problems of all possible kinds (i.e., design, scheduling, and quality).

Other meaningful examples of general areas in which public-service contracts arise are as follows:

- roadwork maintenance, where part of the road network must be renovated, and the amount of material and its quality are big discriminators for compliance with the contract;
- garbage collection, where the quality of service must be assessed in terms of number and frequency of the visits to the garbage bins; and
- public transportation systems, where contracts specify the frequencies at which different bus lines must be operated.

The present paper focuses on the last example and presents a case study in the city of Bologna, Italy. Specifically, until the end of the 20th century in many countries of western Europe, including Italy, public transportation has been organized, financed, and managed by central or local governments. More recently, mainly during the last decade, many countries liberalized their public transportation services. Currently in Italy, although offering local public transportation is a political and financial duty of regional governments, the operation of the service is outsourced to a private entity, henceforth the “operator.” The new approach represented a big discontinuity with respect to the past when local public transportation was managed and operated directly by the government. As counterparts to operators, local governments created regulatory agencies whose tasks include the design of the local transportation service, the definition of the service contract that describes the duties of the operator, and the organization of a public auction to choose a private operator that will run the service.

Although the implementation details change from country to country and often from region to region in the same country, this system has been adopted in several cities and regions in the world. Just to provide some examples, we mention Syndicat Mixte des Transports en Commun de l’Agglomération Grenobloise (<http://www.smtc-grenoble.org>) in Grenoble, France; Hamburger Verkehrsverbund (<http://www.hvv.de>) in Hamburg, Germany; Consorcio Regional de Transportes (<http://www.ctm-madrid.es>) in Madrid, Spain; Veolia Transport (<http://www.veolia-transport.com>) in Nassau County, New York; and Société de Transport de Montréal (<http://www.stm.info>) in Montreal, Canada.

Paper’s Contribution. The article is divided in two parts that are deeply connected. We look at the relationship between the agency and the operator and design a procedure to be used by the agency to control the performance of the operator regarding

satisfying its contractual obligations. We use game-theoretical techniques to model the contract between the agency and the operator, and we characterize the equilibrium of the game to shed light on its optimal structure. The control procedure we propose is, in turn, an input needed for the computation of the equilibria of the game because the operator decides to what extent it is going to report missed services based on the effectiveness of the control procedure. Clearly, an optimized control procedure allows the agency to do its job with the highest possible precision within its budget, which results in an increase of welfare for society. It also leads to an equilibrium in which the operator has a high incentive to satisfy its contractual obligations (concerning service and reporting activity).

We solve the game and show that it can be used to align the incentives of both parties. Then we solve the problem of how to optimally oversee the behavior of the operator. Closing the loop, we use the output of the optimal control procedure to compute the fines to be charged to the operator at equilibrium. The game is solved under specific assumptions about the investment structure of the operator and the effectiveness of checking if services took place.

The contracting model relies on the principal–agent problem, a traditional framework to analyze this class of phenomena. The novel feature of our model is that it adds the possibility of cheating on top of the classic moral hazard found in traditional principal–agent models. Indeed, besides choosing a level of effort, the agent also chooses a level of honesty t that captures the probability that it misreports the quality of service. The agent trades off paying a small fine f when truthfully reporting a service that was skipped and a lottery. In the lottery, the agent either gets away without paying anything if the principal does not realize that there was a problem or pays a big fine F if the principal becomes aware of the breach of contract. Note that the trade-off is conditional to the level of effort that the agent invests; with enough investment there is no trade-off since the probability that a service does not run is minimal. The possibilities of bankruptcy and cheating prevent the agency from implementing a first-best contract, which then goes with the best contract possible for the regulatory setting in which it is immersed.

The planning of control activities is obtained through the use of combinatorial optimization techniques applied to a mixed integer linear program (MIP) that captures the main aspects of the control procedure. Although the optimization problem we propose is NP-hard, we can solve it for problem instances taken from Bologna. The computational study we perform shows that we can efficiently find solutions that are useful to our purposes.

The use of the methodology we developed for the agency in Bologna led to the enforcement of new control procedures at the operational level, to an exhaustive redesign of the next public transportation bus contract to be auctioned in 2016, and to an intensive redesign of the evaluation framework in the context of the European project EPTA, “European model for Public Transport Authority as a key factor leading to transport sustainability.”

Public-service contracts in other contexts than transportation essentially follow the same scheme, and, although the optimization problem associated with the control component of the scheme will of course be different, we believe that the methodology applied in the present paper is sufficiently general to be used in public service at large.

Paper’s Organization. This introductory section is completed below by a discussion of the details of our case study (§1.1) and by a short literature overview of the general public-service contracts and the specific transportation context (§1.2). In §2 we analyze the contract between the agency and the operator from a game-theoretical perspective, and we compute the equilibrium of the corresponding game. To monitor the execution of the contract, in §3 we design an optimized control procedure to oversee the provision of service. Section 4 describes the algorithm we implemented to find the optimal control procedure while in §5, we apply the methodology to the case study of the city of Bologna, Italy. Section 6 closes the loop and applies the optimal control procedure to the optimal contract computed earlier. Finally, §7 presents our conclusions.

1.1. Public Transportation Service in Bologna, Italy

As explained above, this article conducts a case study of the service provision of local bus transportation in Bologna, Italy. The public bus operator of the city of Bologna has to run 2.2 million services every year, which cover a total of 18 million kilometers and transport 92 million passengers. The regional government entrusts the supervision of the local transportation to a government-owned agency called SRM (<http://www.srmbologna.it>). This agency has direct responsibility for the following:

- the design, organization, and promotion of local public transportation;
- the definition of a service contract, including the minimum requirements of the operator such as performed services, reliability, and punctuality, and information collection and provisions concerning actual performance;
- the setup of an auction for selecting the operator, which includes the preselection of participants based on technical and legal qualifications; and

- the supervision of the satisfaction of the operator's contractual obligations.

The contract is renewed every six years with a possible three-year extension. In 2010, the agency organized the first auction in Bologna. The winner was the operator that had been providing the service previously, which had become a private company during this process. The total revenue of the operator is approximately €150 million per year, which is used to provide service according to a schedule provided by the agency. This requires over 2,100 drivers. Since the operator is still owned by the regional and local governments, the relationship between operator and the agency is resolutely friendly at the moment. However, some large private transport operators that have a strong presence in the European market (e.g., Veolia, Transdev, Arriva, etc.) are likely to participate in future auctions, competing for a contract of more than €1 billion. As a consequence, small local operators are starting to merge to grow to a regional size to compete more efficiently in the liberalized markets of today. The winner of the auction has the obligation of running the buses in exchange for receiving the bus fare and a subsidy from the regional government.

The operator is under contractual obligation to monitor its own quality of service, which has to be reported to the agency monthly. If the reported performance does not meet the contractual obligations, the operator receives a fine from the agency. As with any complex operation, sometimes it happens that some services are canceled or interrupted because, e.g., the driver does not show up, the bus breaks down, there is a road accident, etc. In some cases the failure may be out of the operator's control and in others the operator has a direct influence on the root of the problem. For example, although it cannot make sure that no bus will break down during service, it can drastically reduce the probability of this event by spending more resources on maintenance; similarly, it can implement a policy to replace unavailable drivers. To align the incentives, the contract indicates that the agency must charge fines if the operator does not comply with the agreed service level. For every service that is skipped or canceled, the operator has to pay a small fine to discourage the operator from canceling services, with the understanding that sometimes unintended operational problems arise. Instead, when the actual performance is much lower than the minimum requirement, the operator is charged expensive fines to provide a much stronger incentive to do as it should.

A major issue in this scheme is the asymmetry of information between the operator and the agency. The operator has complete knowledge of its operations, its real service level, which services are performed and which are not, etc. Instead, the agency does not have direct access to this information; the agency receives

it from the operator. The problem is that the latter might purposely alter the reports to prevent fines by declaring a level of service that is higher than the actual one. For this reason, the agency must confirm the information reported by the operator by doing an independent assessment. If the agency discovers that the operator's reports misrepresented reality, the latter receives a hefty fine. However, this fine cannot be too large; otherwise, the threat would lose credibility because neither side wants to cancel the contract under normal circumstances. It is important to highlight the fact that the agency does not have a special interest in the revenue arising from fines. Although fines are part of the agency's payoff, relatively much more importance is placed on the impact on welfare that occurs when services are not performed according to the planned schedule. The main objective of the agency is to have the level of service specified in the contract and to receive reliable information about service provision from the operator. In the absence of a control system and the associated fines, the operator would not be compelled to provide reliable information, not necessarily in bad faith, but simply because it is not cost effective to perform the necessary effort. Anticipating this, the agency optimizes the control procedure to streamline its daily operations by minimizing the associated cost and maximizing the impact of the taxpayer money spent in overseeing the contract signed with the operator.

Currently the agency performs some supervision of the service provision, but these controls are not systematic and are mainly based on complaints received from customers. In these cases, the agency sends an employee to visit the corresponding stops along an itinerary that is arranged in an informal way. The agency is interested in standardizing the checking procedure with the goal of assessing the performance of the operator and validating the current contract. This activity will help the agency to refine the contract structure if needed and to improve the structure of fines, which will be used to modify the contract at the time the concession of the bus service ends and the selection process restarts.

It is important to observe that the information asymmetry between the agency and the operator cannot be fixed by simply employing an intelligent transportation system (ITS) that provides vehicle tracking. In fact, buses are already equipped with GPS devices. Actually, the most common service quality defect in Bologna is that buses do not stop long enough to allow passengers to get on (because the bus is running late); that event cannot be identified by a tracking system. In addition, drivers could be tempted to switch off the GPS to thwart the control, claiming that the device is malfunctioning, which would be hard to assess at a later stage, leading to unpleasant

controversies. Finally, the control activities performed by employees of the agency at stops and at buses are much more accurate because they involve many aspects of quality such as cleanness, courtesy, etc. As we point out in the next section, this type of control is indeed generally preferred in many places throughout the world.

To summarize, our work describes the development of a complete control framework and analyzes its impact on the design of the contractual relationship. The agency will direct employees, referred to as controllers, to strategic intersections in Bologna to detect missing services so that the operator will refrain from not complying with the contract on purpose. The agency faces the decision of how much it should spend on deterrence, and where and what kind of controls it must put in place to use its budget wisely and maximize the likelihood of detecting problems. The operator may fail to report skipped services only when and where it is unlikely to be detected.

Although the agency can perform these controls in different ways, it is most convenient to observe services in different locations. The problem to be solved consists of deciding what locations and times need to be checked, together with the itinerary (a sequence of stops) the controllers must follow. The problem in question is hard from an optimization standpoint, and to evaluate the proposed control procedure, we compare it to a greedy heuristic that mimics how controllers build their itineraries today.

1.2. Literature Overview

The paper essentially contains three viewpoints and perspectives: the game that encodes the contract, the case study about the public transportation system, and the mathematical programming approach used to optimize the control problem. We split the literature review accordingly.

1.2.1. Contract Regulation (in Public Service).

This work is closely related to the literature on the principal–agent model with moral hazard. This literature, exemplified by the classical work of Grossman and Hart (1983) and Rogerson (1985), analyzes how a principal can delegate the execution of a job to an agent. The principal and the agent agree on a contract that provides the necessary incentives so the latter performs the job in an effective way, under the assumption that the principal cannot perfectly monitor the effort that the agent puts into the job. This model has been the workhorse in understanding many interesting economic phenomena where incentives play a crucial role.

In the contracts for public transportation described above, the regulator gives the right to the operator to run the bus services. Because of information asymmetries, the regulator is less informed than the operator

regarding the cost of providing service and the quality of the public transportation network. This implies that generally a first-best solution is not achieved. Indeed, Gagnepain and Ivaldi (2002) focus on the principal–agent framework and study the effect on welfare caused by different types of contracts between the regulator and the public transit operator. Their experiments suggest that cost-plus contracts damage welfare, whereas fixed-price mechanisms are more likely to curb the inflation of operating costs. Socorro and de Rus (2010) also consider a principal–agent model and study how to best align the incentives of both parties by comparing different financing schemes that have been used in Spain. Hansson (2010) discusses the relations between different government bodies at the local, regional, and country levels and their relations to regulating public transport procurement.

In terms of the details of the contract discussed in §2, we chose a simple model of a sequential game where both parties first choose the long-term decisions that will govern the relationship and then play a one-shot game that determines the interaction. In the literature of contract theory, some articles take on the more complex task of modeling the interaction as a repeated game (see, e.g., Green and Porter 1984, Chassang 2010, Yared 2010). These papers present how private information and imperfect monitoring can lead to punishment situations that are inappropriate. Enriching our model along those lines is a possible line for future research.

Although not the center of our focus, an important discussion related to our work is how these contracts are awarded and how to subsidize the difference between ticket prices and operating costs. Jansson and Wallin (1991) report the effect of deregulation of public transport in Sweden and how different local transport agencies reacted to the new policy. Hensher and Stanley (2008) review the recent debate over the performance of auctions (i.e., competitive tendering) versus performance-based negotiated contracts when the service to be provided is complex. A key example of this tension is provided by the case of a service contract for public transportation that quite often has very large transaction costs. The high costs arise from the difficulty of specifying and then correctly monitoring and influencing the key performance measures. Hensher and Stanley (2008) argue that the focus on auctions may have led governments to place too much emphasis on cost reduction instead of the social obligation to maximize welfare. They conclude that negotiated performance-based contracts, aimed at the development of trusting partnership based on incentive and penalties structures, are preferable to competitive tendering. Along these lines, Hensher and Stanley (2003) discuss performance-based contracting

as a more effective alternative to competitive tendering for obtaining the largest social benefit from public subsidies. Hensher et al. (2003) identify a set of important dimensions that must be considered for evaluating the service perceived by bus service passengers. Rizzi (2014) discusses different mechanisms to help defray the capital and operating expenses of public transportation and analyzes which are better in aligning incentives between parties.

1.2.2. Best Practices in and Strategic Views of Public Transportation Systems. As previously mentioned, the public transportation service currently is mostly outsourced to private companies and regulated by independent agencies that are intermediaries between the operator and the local government. The European Union (EU) has recommended the structuring of public transportation in this manner (see, e.g., European Parliament 2007); however, as mentioned in the introduction, this structure has been widely adopted in the rest of the world. In addition, the EU has enforced political objectives and rules in the attempt to make the Public Transportation System more sustainable (see, e.g., Polis, European Cities and Regions Networking for Innovative Transport Solutions, at <http://www.polisnetwork.eu>), and FP7-funded EU projects like EPTA aim at defining the regulatory foundation of the seven key functions that characterize the governance of public transportation: namely, regulation, planning, tendering and contract awarding procedures, integration, promotion, management, and control and monitoring. It is easy to see that the last two of these functions precisely define the application context of the present study, and it is worth noting that such a context is much broader than running the buses in a medium-size city like Bologna. In fact, the methodology can be applied to virtually any public transportation service that is outsourced, e.g., metro or local trains for commuters (see, e.g., Caprara et al. 2010), home-healthcare services (see, e.g., Hiermann et al. 2015), transportation of handicapped people (see, e.g., Toth and Vigo 1997), etc.

Although the central government, at least at the EU level, explicitly puts local authorities and, as a consequence, ad hoc agencies in charge of the control, the way the control is implemented depends obviously on local policies. A human-based control like that implemented in Bologna instead of (or at least in conjunction with) one achieved through an ITS has been found to have more acceptance in Italy, as pointed out in the previous section. However, this applies elsewhere, too. The Comisión Nacional de Regulación del Transporte (<http://www.cnrt.gov.ar>) in Buenos Aires in a series of documents explicitly states (AGN 2005, §1.1.1; CNRT 2007, pp. 25–47) that the local authority controls each line three times a year and takes a sample of 2,000 bus drivers per year to check their

behavior (which includes courtesy, cleanness of the bus, etc.). The authority measures frequency through controllers who stand for at least three hours in one place while they count the number of services.

A similar situation can be encountered, e.g., in London, where the bus service is provided by operators who compete for contracts to provide specified services for up to seven years and are rewarded for exceeding defined targets so as to improve the service to the passengers. The level of service, including the ability of the operator to schedule, control, and adjust services, is evaluated by a system that integrates human observations and tracking technology (see TFL 2009).

Finally, it is clear that a human-based control is more appropriate and likely to work in developing countries where buses are sometimes operated by informal and heterogeneous organizations (e.g., it is typical that some buses are owned by individuals, often the drivers, and not by companies). In those cases, the investment associated with installing a GPS device on each bus and delegating the measurements of quality of service might be infeasible or too expensive.

1.2.3. Mathematical Programming. With regard to the methodology to control the quality of service discussed in this article, we propose to solve a routing problem that collects prizes associated with the visit of specific nodes or subsets of nodes of the bus network. Prizes correspond to the value of the information that can be collected in each location. We construct routes that can be traversed during the work-day of a controller. We find one route per controller, maximizing the total prize. To model and solve this routing problem we rely on results from the routing literature. For a general introduction to vehicle routing problems (VRPs), we refer the reader to, e.g., Toth and Vigo (2001).

We now comment on some problems that are specifically related to the control problem addressed by this article. First, the orienteering problem is a problem defined on a graph where arcs have a cost, nodes have a prize, and one wants to construct a simple cycle that maximizes the prize that can be collected such that the cycle does not exceed a given length (see, e.g., Fischetti et al. 1998). This resembles the problem we study although in our case we need to construct several cycles, one for each controller. Our problem also resembles the prize-collecting Steiner tree problem, which consists of finding a connected subgraph of a graph that has prizes associated with the nodes (see, e.g., Ljubic et al. 2006). In that problem, the objective value is the sum of the prizes of the nodes minus the costs of the edges included in the subgraph. Our problem also has elements of the distance-constrained vehicle routing problem, a VRP

with constraints on the total distance traveled by each vehicle (see, e.g., Li et al. 1992). In our problem, however, some nodes may remain unvisited. Traditionally, VRPs do not have prizes; they call for minimizing the cost associated with the routes. Finally, our problem also has elements of the generalized traveling salesman problem, where, given a set of cities partitioned into (nonintersecting) clusters, one has to visit at least one city in each cluster (see, e.g., Fischetti et al. 1997). This shares with our problem the condition that nodes that represent stops of a bus line are grouped into a cluster (one that can intersect), and a prize is earned if the cluster is visited because that particular bus stop is checked. The most common solution method in all of the papers mentioned above is to formulate the problems as MIPs and design a branch-and-cut algorithm, which is the framework we adopt as well.

Finally, in most of the literature, the control phase is not modeled and solved as an optimization problem. Usually, the quality of service measurement is estimated using surveys sent to costumers. A notable exception is the work of Borndörfer et al. (2013), who write about a highway with tolls. In their case, the control decision is how often one should check whether drivers had paid the toll in a highway to minimize evasion. They use an MIP to compute the optimal control strategy.

2. Contract Between the Agency and Operator

To model the contract between the agency and the operator in the city of Bologna, we rely on a principal-agent model. The contribution of this analysis lies in formalizing the interrelations of this contract with the service control operation that must be implemented by the operator. In addition, this analysis allows the agency to validate the fines that it initially considered for the contract. We suggest a model for this contract and study it in detail under realistic assumptions that fit the needs of the agency. We defer some details until later because §5 discusses the case study and it is better to discuss the implementation in the city of Bologna after the specifics have been introduced. Although we customized our analysis for Bologna, most elements of this contract are of broader appeal and apply to many other places and situations, as described in more detail in the introduction. The novel feature of our model is the inclusion of the possibility of underreporting by the operator, which is not traditional in the principal-agent literature.

The main purpose of the contract is to create a framework under which the operator runs the bus services in exchange for a payment. The operator has to provide service according to a fixed schedule for the bus lines. The contract follows a net cost

paradigm: the agency defines a service and gives the operator a subsidy to cover part of its costs. The subsidy is currently paid according to the services planned, not according to the services really performed or to the number of passengers. The revenue generated by users paying bus fare goes directly from the passengers to the operator. Because there may be situations that prevent the operator from running a service, sometimes beyond its control, the contract specifies that it has to report to the agency what services were provided successfully and other performance indicators. If the operator does not reach a specified level of service, it gets a fine. Finally, the role of the agency is to make sure that the reports provided by the operator are accurate. Because fines are levied on the operator when not performing the required services, it has the incentive to underreport that information. For that reason, the agency spends part of its budget controlling that the reports are truthful. If the agency finds that the reported quality of service and the measures differ, a much larger fine is imposed on the operator.

This section proposes a model that allows the agency to tune the details of the contract to minimize the problems of moral hazard and cheating. Using a game-theoretical model, we find the amounts of fines that are best to minimize those problems and provide incentives to the operator to act honestly and efficiently. To model the control procedure to be implemented by the agency, we postulate that the budget allocated to those activities and the effectiveness of the control are described by polynomials, which are convenient and capture well the intricacies of the resulting control procedure, as discussed in detail in the computational study of §5. Hence, it is important to highlight that, although the control is performed after designing the contract and signing it with the operator, the optimal itineraries used for checking the quality of service are taken as inputs in the contract design phase addressed by this section.

We conducted interviews with the agency and agreed on the key aspects of the contract. The primary objective of the agency is to maximize commuter welfare by enforcing the provision of the planned services. The secondary objective is to create a regulatory framework so the operator is honest and provides reliable information about its performance. We model the contract as a sequential, single-shot game between the agency and the operator. First, the agency establishes the details of the contract followed by the operator's decision of how much to invest in service provision. Then, at the same time the operator chooses to what extent to report truthfully while the agency chooses the effort to be incurred in controlling activities. The stages of the game mimic the timescale of the different decisions. We provide more details on

each stage in the list below. Although we assume that all aspects of the game are static and effects are uniform across the network, more general assumptions could also be incorporated at the expense of complicating the game. Because the contract does not specify provisions for those differences we agreed with the agency to model these decisions using as simple a game as possible.

To quantify the impact on welfare when a service is planned but not provided, we take the additional total waiting time of the passengers that would have taken that bus.

1. *Agency chooses fines.* For each planned service that is not performed but is reported truthfully as such by the operator, the operator receives a small fine f . For each service that is not performed and not reported, if the agency finds out it gives the operator a big fine of F . One caveat is that if the agency sets F too large, it would be prohibitive for the operator to pay it and it would go bankrupt, in which case the agency would have to organize a new auction to select a new operator. Such a consequence is deemed extreme and unrealistic, hence undesirable. Terminating a contract early may have negative reputation effects for the city. Furthermore, there is a (small) possibility that a service that was reported as performed by the operator and that was checked and detected missing by the agency actually took place but was not observed for various reasons (e.g., it was late, it took a different route, the controller missed it, etc.). The possibility of imperfect detection also imposes a limit to the value of F . Hence, we assume that F cannot be larger than an upper bound fixed a priori (e.g., the cost of tendering the contract again).

2. *Operator chooses investment in service provision.* In this stage the operator decides how much to invest to obtain and maintain buses, hire and train drivers, and all the other aspects of service provision. We call this budget decision b_{op} , which represents the invested money per service. We define the function $q(b_{op})$ that maps investment to the achieved quality of service, represented by the percentage of the services that are provided as planned. This relies on the observation that, the more that is invested in bus provision, the more likely a given service will be provided successfully. The percentage $q(b_{op})$ is to be interpreted as the probability that the operator provides a given service. We assume that $q(0) = 0$, $q'(b_{op}) > 0$, $q''(b_{op}) < 0$, and $q(b_{op}) \rightarrow 1$ when $b_{op} \rightarrow \infty$. A finite investment cannot guarantee that $q(b_{op}) = 1$ because there are always factors outside the operator's control.

3. *Operator chooses honesty level; agency chooses effort to control.* In this stage both players compete in a single-shot noncooperative game. The operator decides what fraction of nonperformed services t to

report honestly. Hence, a fraction $1 - t$ of missed services are not reported as missing. Note that whether to report a missed service or not is not stochastic but a decision of the operator. The operator may manage to save some money by withholding the information, but if discovered it must pay a much heavier fine. Concurrently, the agency decides the budget per service b_a allocated to controlling activities. Since the control is done by placing employees on the street, it is labor intensive, so b_a mainly consists of wages paid to those employees. The control procedure defines a function $p(b_a)$ that maps the budget devoted to control to the probability that a service is checked. The function satisfies that $p(0) = 0$, $p'(b_a) > 0$, and $p''(b_a) \leq 0$. It is theoretically possible (albeit impractical) to check all services with a finite budget, hence there exists \bar{b}_a such that $p(\bar{b}_a) = 1$.

For convenience, we summarize the variables and parameters of the game in Table 1. Now that we presented the decisions, let us specify the objectives of the players. The utility functions of the operator and the Agent are respectively defined by

$$\pi_{op} := -(1 - q(b_{op}))(tf + (1 - t)p(b_a)F) - b_{op} + R, \quad (1)$$

$$\pi_a := (1 - q(b_{op}))(tf + (1 - t)p(b_a)(F - r(F)C)) + \eta - \eta^-(1 - q(b_{op})) - b_a - R. \quad (2)$$

The first term of the operator's payoff in (1) represents the expected fines, the second term subtracts the investment incurred to attain the desired service level, and the last term is the total revenue. The expected fines paid by the operator are composed of two parts that are multiplied by $(1 - q(b_{op}))$, which represents the probability of not performing a service. The first is for the fraction t of services that the operator truthfully reports as missed, and the second is for services that the operator does not declare but that the

Table 1 Glossary of Symbols Used by the Game

Decisions	
f, F	Fine charged when operator does not perform a service and reports it (f) or is discovered not reporting it (F)
b_{op}	Investment of operator
t	Probability that operator reports a missed service
b_a	Budget for control of agency
Inputs	
$q(b_{op})$	Function that describes the fraction of services performed by the operator
$p(b_a)$	Function that describes the fraction of services checked by the agency
R	Total subsidies paid to the operator corresponding to planned services
η	Total welfare perceived by society from transportation service
η^-	Welfare loss incurred by users for a missed service
C	Cost incurred by agency if operator goes bankrupt
$r(F)$	Probability that operator goes bankrupt if it pays a fine F

agency discovers (with probability $p(b_a)$ representing the effort of the agency). Note that we assume that all probabilities are independent, so it is enough to multiply the probabilities of different events. The revenue R is assumed to be constant because the agency pays subsidies regardless of whether the service took place or not, and users pay fares as well because when a service is missed they wait for the next.

Regarding the agency's payoff in (2), the first term is the amount in fines received by the agency, adjusted by the probability of bankruptcy as discussed below; the second term is the total user welfare; the third is the reduction of user welfare caused by missed services; the fourth is the cost of performing controls; and the last is the revenue. (Note that in fact π_a is the social welfare of both the agency and society in general since it includes payments and the utility that commuters derive from transportation.) The agency receives the fines paid by the operator. There is an extra term $r(F)C$ in (2) that represents the possibility that the operator goes bankrupt when a fine F is imposed that is too high. In that case, the agency must restart the process of tendering to find a new private company that will operate the services. This has a huge cost, which we represent by C . The probability of this happening is encoded by the function $r(F)$, which is increasing. To disregard unrealistic situations, we impose the conditions (i) $F < C$, (ii) $r(C) = 1$, and (iii) $r(F)C \leq F$. These conditions impose fines smaller than the cost of finding a new operator. The constant η is the total user welfare in the system when all services are run according to the contract. This is an additive constant that represents the maximum total surplus (economic value of traveling minus time spent traveling) that society would get from transportation. Since it is fixed, it does not play a role in this model. The important coefficient is η^- , which represents the welfare loss that is caused by every service that the operator fails to provide. It captures the additional total minutes lost by users that have to wait for the next service, converted to money. This coefficient can also be regarded as a tradeoff between the contribution to payoff arising from the fines versus welfare maximization. When the coefficient is large, the agency values welfare more than the income generated by fines, and vice versa.

As usual for this type of game, we solve it by starting from the end and going to the earlier stages progressively. We work to characterize the subgame perfect equilibria of the game, describing the resulting utility functions along the way. The third stage is a one-shot game where the operator and the agency choose t and b_a , respectively. We consider the decisions of the first two stages as fixed and take the derivative of both utility functions with respect to the decisions at this stage. It is necessary that both vanish

at an interior Nash equilibrium. Hence, at the third stage of the game,

$$\partial \pi_{op}^3 / \partial t = -(1 - q(b_{op}))(f - Fp(b_a)) = 0,$$

$$\partial \pi_a^3 / \partial b_a = (1 - q(b_{op}))(1 - t)p'(b_a)(F - r(F)C) - 1 = 0.$$

Solving them, we get that the unique Nash equilibrium (b_a^{NE}, t^{NE}) of the third stage of the game is

$$(b_a^{NE}, t^{NE}) = \left(p^{-1}\left(\frac{f}{F}\right), 1 - \frac{1}{(1 - q(b_{op}))p'(p^{-1}(f/F))(F - r(F)C)} \right). \quad (3)$$

This game does not admit equilibria at extreme solutions. At equilibrium, the utilities of both players are, respectively,

$$\pi_{op}^3 = -(1 - q(b_{op}))f - b_{op} + R,$$

$$\pi_a^3 = (1 - q(b_{op}))(f - \eta^-) - \frac{f}{Fp'(p^{-1}(f/F))(F/(r(F)C) - 1)} + \eta - p^{-1}(f/F).$$

Furthermore, analyzing the best response function of the operator to the agency strategy, we see that it is optimal for the operator to be fully honest ($t = 1$) when $b_a > p^{-1}(f/F)$ and to hide all missed services ($t = 0$) when $b_a < p^{-1}(f/F)$.

In the second stage, the operator chooses the effort b_{op}^* that maximizes its total payoff π_{op} . Taking the derivative of the payoff, anticipating the outcome of the third stage, b_{op}^* satisfies that $q'(b_{op}^*) = 1/f$, which has a unique solution because $q'(\cdot)$ is a decreasing function. The solution b_{op}^* equals $(q')^{-1}(1/f)$ if $q'(0) \geq 1/f$ and 0 otherwise. Going to the first stage, the agency can anticipate the outcome of the last two stages of the game and compute the payoff for any choice of fines f and F using

$$\pi_a = \left(1 - q\left((q')^{-1}\left(\frac{1}{f}\right)\right) \right)(f - \eta^-) - \frac{f}{Fp'(p^{-1}(f/F))(F/(r(F)C) - 1)} + \eta - p^{-1}\left(\frac{f}{F}\right).$$

With the assumptions on the primitives that we required, the function π_a need not be unimodal, so there might be cases in which more than one value of f globally maximizes the utility for the agency. The possible multiplicity of solutions, however, is not as much of a concern as it would be in a one-shot game because equilibrium selection is not a problem. The agency can freely use other factors to choose among the different maximizers. The operator does not need to guess what fines the agency would impose at equilibrium; that is publicly available in the contract. We summarize the discussion above in the following proposition.

PROPOSITION 1. *The subgame perfect equilibrium of the game between the operator and the agency as described earlier is given by*

$$(f^*, F^*) \in \operatorname{argmax}_{0 \leq f \leq F \leq C} \left(1 - q \left((q')^{-1} \left(\frac{1}{f} \right) \right) \right) (f - \eta^-) - \frac{f}{F p' (p^{-1}(f/F)) (F/(r(F)C) - 1)} - p^{-1} \left(\frac{f}{F} \right),$$

$$b_{\text{op}}^* = (q')^{-1}(1/f^*),$$

$$b_a^{\text{NE}} = p^{-1}(f^*/F^*),$$

$$t^{\text{NE}} = 1 - \frac{1}{(1 - q((q')^{-1}(1/f^*))) p' (p^{-1}(f^*/F^*)) (F^* - r(F^*)C)}.$$

To specialize the model, we consider simple functions that verify all assumptions we have made earlier: $p(b_a) = kb_a^\alpha$ with $k > 0$ and $0 < \alpha < 1$, $q(b_{\text{op}}) = 1 - e^{-\beta b_{\text{op}}}$ with $\beta > 0$, and $r(F) = (F/C)^2$. The level of control that we can ensure with a given budget of b_a per service determines the function that captures the probability $p(b_a)$ that a service is checked. By improving the efficiency of controls, we change the equilibrium of the game because, given the same budget allocated to the control activities, we have a larger probability of checking a given service. Section 3 provides a procedure to compute $p(\cdot)$ by optimizing how the budget b_a is allocated to performing controls in an efficient way. As we will see then, the monomial approximation of $p(\cdot)$ made earlier is reasonable given the computational studies we shall conduct.

Evaluating the formulas described in Proposition 1, the Agent chooses f and F to maximize

$$\pi_a = \frac{1}{\beta} - \frac{\eta^-}{\beta f} - \left(\frac{f}{kF} \right)^{1/\alpha} \left[1 + \frac{1}{\alpha(C/F - 1)} \right] + \eta.$$

From the previous objective function, we derive first-order optimality conditions. Indeed, we need that

$$0 = \frac{\partial \pi_a}{\partial f} = \frac{1}{f} \left\{ \frac{\eta^-}{\beta f} - \frac{1}{\alpha} \left(\frac{f}{kF} \right)^{1/\alpha} \left[1 + \frac{1}{\alpha(C/F - 1)} \right] \right\}, \quad (4)$$

$$0 = \frac{\partial \pi_a}{\partial F} = \frac{1}{\alpha} \left(\frac{f}{kF} \right)^{1/\alpha} \left\{ \frac{1}{F} + \frac{1}{\alpha(C - F)} - \frac{C}{(C - F)^2} \right\}. \quad (5)$$

Condition (5) requires that F is a root of $(1 - \alpha^{-1})F^2 + (\alpha^{-1} - 3)CF + C^2$. After some algebra,

$$F^* = C \frac{3 - \alpha^{-1} - \sqrt{\alpha^{-2} + 5 - 2\alpha^{-1}}}{2 - 2\alpha^{-1}},$$

which satisfies $C/2 < F^* < C$. Now, solving (4) and plugging the results into the formulas of Proposition 1, we complete the solution of the game as

$$f^* = \left\{ \frac{\beta}{\eta^- \alpha (kF^*)^{1/\alpha}} \left[1 + \frac{1}{\alpha(C/F^* - 1)} \right] \right\}^{-\alpha/(\alpha+1)},$$

$$b_{\text{op}}^* = \ln(\beta f^*)/\beta,$$

$$b_a^{\text{NE}} = (f^*/(kF^*))^{1/\alpha},$$

$$t^{\text{NE}} = 1 - \frac{\beta(f^*/(kF^*))^{1/\alpha}}{\alpha(1 - F^*/C)}.$$

The resulting level of service is $q(b_{\text{op}}^*) = 1 - 1/(\beta f^*)$, and the resulting probability of checking a service is $p(b_a^{\text{NE}}) = f^*/F^*$. From those formulas we verify that the budget spent by the agency on overseeing the contract, as well as the probability that a given service is checked, increases when the impact of service provision to society increases. Similarly, the level of service at equilibrium and the honesty of the operator increase with the impact of service provision. To support that, the optimal fine f must get larger with the impact of not having service.

In §5, we show how the constants required by this model can capture the details of the situation given by the case study in Bologna. Afterward, we close the loop and provide an implementation of the contract for the city of Bologna in §6.

3. Control Design

In this section, we propose a model to compute a sequence of stops that controllers must visit. That itinerary indicates where and when they should check that services took place as expected. We rely on an integer program that has elements of various combinatorial optimization problems such as covering, routing, and prize-collecting problems, as described in §1.2.3.

Let us start with the basic modeling elements. We refer to the route in the transportation network that buses take as a “line” and to a bus run on a specified line at a specified time of the day, as it appears in the bus schedule, as a “service.” For example, in Bologna, Italy, line 13 that goes from Ponte Savena to Normandia has services every 15 minutes during the day (see Figure 4 in §5).

The control procedure is performed by a limited number of *controllers* (typically three or four) who are on the payroll of the agency. These employees are assigned to the control of buses only during part of their working day, and, at that time, they check the compliance with the contract in the street and on the buses. As was already mentioned, among other things the controllers record the punctuality and frequency of services that will be used to estimate the quality of service provided by the operator. The main idea of the control procedure is to locate controllers at bus stops so they can check if a particular service took place or, if they stay longer, they can contrast the empirical frequency of service to the scheduled one. For this approach to work, controllers must have the capability of distinguishing among services and among

lines. In other words, controllers must know which bus (identified by, e.g., the license plate or bus ID) they are seeing and (possibly later) with which service it is associated. When frequency is low, it is easier to tell apart different services and contrast them to the schedule. However, when frequency is high, controllers can just determine frequencies and ignore identifying one particular service or another. In the latter case, it is enough to just count the number of buses seen per line during the specified time slot.

A control procedure can be either static or dynamic, depending on what is specifically observed at a stop. A static control procedure does not incorporate time explicitly; i.e., it does not consider the time when a given stop of the network must be visited. Static controls specify that a given stop must be visited for a given duration of time at any time during that day. This is reasonable for lines with high and relatively constant frequency. In contrast, a dynamic control procedure considers time explicitly by choosing at what exact time the controller must be present at a given stop to check on a particular service. This choice is better suited for lines with low frequency, for which being in a stop at the wrong time would be a waste of time. For example, there are some longer-distance services that are only offered during the morning or afternoon rush hour.

For the reasons mentioned above, designing a control procedure for high- or low-frequency lines leads to significantly different strategies. We focus on high-frequency lines per request of the agency because those lines carry many more passengers overall. This motivated us to use a static control procedure that records frequencies and potential problems at bus stops. We do not specifically plan for the time needed by controllers to check if a service visited a stop properly; they just do it whenever possible. In those cases, they check if the bus stopped at the request of passengers, if the bus approached the curb, if all the passengers had time to get on, etc. For example, one common problem is that bus drivers skip a stop when they are running late. Although the procedure is designed considering high-frequency lines exclusively, controllers are nevertheless asked to record services of low-frequency lines when they see one.

For a given budget for control put forward by the agency (mainly used to pay controllers), the design goal is to maximize the information collected. We select the locations to be visited by controllers daily, depending on the time window allocated to control. Another design goal consists of constructing different routes every day. The agency values diversifying the control locations because that provides a more complete picture of the bus operations. To capture this, we define the value of information collected at a specified stop relative to when the

stop was last visited. Precisely, a stop may be desirable because many buses go near it but nevertheless should be penalized today if it was visited yesterday. For this reason, we reduce the “prize” associated with recently checked stops and lines to balance the trade-off between visiting central stops, which allow us to collect information very efficiently, and visiting less-central stops, which are less desirable but where services are more likely to be skipped. As a result, we tend to cover the entire network over successive days as had been requested of us. Furthermore, this makes it harder for the operator to guess where controllers are going to be located during any given day. That was another concern of the agency, because if the operator could learn the locations of the controllers, it could adapt its strategy to do everything as it should (knowing it is being observed). Finally, our method has the flexibility of imposing that some particular lines or stops must be checked, which was another requirement of the agency when we were discussing the approach. This is used to address specific customer complaints. In the rest of this section we present the general method, in §4 we discuss how to solve the related optimization model, and we give implementation details for the case study of Bologna in §5.

3.1. The Model

From the actual transportation network, we extract an underlying oriented graph $G(S \cup \{0\}, A)$. The set of nodes includes the stops S and the agency’s office, denoted as node 0, where the data-collecting activities of the controllers start and end. We associate one or more weights with each stop $s \in S$. They represent the possible durations that a controller may spend at the node. Each stopping time comes with a prize, which is earned if a controller visits the stop for exactly that time. The prize captures the value of information that can be collected during that time and basically encodes the number of lines visiting the stop. We associate weights with arcs as well, which represent the travel time needed to traverse them. Note that A only includes existing network connections; i.e., two stops s and t are connected by an arc if they are subsequent stops for some line, or if the distance between them is below a given threshold, and thus it is possible to walk from s to t . Therefore, G is not a complete graph, as opposed to many routing problems commonly studied in the literature. Given a stop $s \in S$, $\delta^+(s)$ ($\delta^-(s)$) denotes the set of arcs leaving (entering) s . Similarly, given a set of stops $\Sigma \subseteq S$, $\delta^+(\Sigma)$ ($\delta^-(\Sigma)$) denotes the set of arcs connecting a node $s \in \Sigma$ ($s \notin \Sigma$) with a stop $\sigma \notin \Sigma$ ($\sigma \in \Sigma$).

In addition to the general prizes mentioned earlier, which capture the lines that can be observed from a stop, stops have intrinsic values. This is used to model

preferences for visiting particular stops. For example, the agency is interested in checking the endpoints of a line because it happens relatively more often (e.g., when late) that drivers cut services short and turn around before getting to the terminal of a line.

The problem has a structure similar to a classic prize-collecting routing problem. However, in addition to prizes associated with stops, we also consider a prize to be earned each time a different line is checked. This means that visiting two stops of the same line does not provide an additional reward associated with that line. Because of that, our computations are also related to the set-covering problem because a stop “covers” each line visiting the stop.

Our goal is to design as many routes on graph G as there are available controllers in such a way that the total prize collected at the visited nodes is maximized and the time along routes is bounded by the time dedicated to control. All routes must start and end at the controllers’ office, node $0 \in G$.

Let us now introduce the notation for the sets and parameters involved in the problem described above. We denote the units of measurement by $[\cdot]$ (we represent the units of information collected by the controls as [info]). Later, in §5, we provide the concrete sources and calculations we used for each of these elements in the application of the problem to the city of Bologna.

- Let K be the set of controllers; each controller $k \in K$ has an associated maximum working time W_k [min].
- Let L be the set of lines; each line $l \in L$ has an associated prize π_l [info], which is earned (at most once) if the line is checked.
- Given the set of stops S , let $S_l \subset S$ be the subset of stops visited by line l .
- Let τ_l [min] be the time that a controller must spend at a stop of line l to check the line. Such a τ_l is proportional to the inverse of the frequency of the line.
- Each arc $a \in A$ has an associated travel time w_a [min].
- Let $T_s = \{1, \dots, q_s\}$ be the set of possible durations of time to spend at stop s . For each possibility, τ_s^t [min] denotes the time spent at the stop, and π_s^t [info] denotes the associated prize that is collected if the controller stays at stop s for at least τ_s^t units of time. The prizes include preferences for particular stops.

For a line l that stops at stop s , there is enough time to check that line for a controller who selects that stop and stays τ_s^t minutes only if $\tau_s^t \geq \tau_l$. By distinguishing between line prizes π_l and stop prizes π_s , we obtain a flexible model that allows us to place prizes on lines to be obtained at most once per line, and on stops of the line to be obtained every time a stop of the line is visited.

Table 2 Glossary of Symbols Used by the Control Model

K, k, W_k	Set of controllers, one particular controller, maximum time devoted to control for a controller
L, l, π_l	Set of bus lines, one particular line, and prize associated with line
S_l, τ_l	Set of stops of line, minimum time to spend at stop to check line
w_a	Travel time of arc a
T_s, τ_s^t, π_s^t	Set of possible times to spend at stop s , and one possible duration of time and prize for the duration of index t and for the stop s
x_l	Line l is checked
$y_s^{t,k}$	Stop s is checked for time τ_s^t by controller k
z_a^k	Number of times controller k travels along arc a

To implement the problem as an MIP, we associate with each line l a binary variable x_l taking value of 1 if line l is checked, and 0 otherwise. We also define binary variables $y_s^{t,k}$ that take a value of 1 if stop s is checked for time τ_s^t by controller k , and 0 otherwise. We say that a node is visited if one of the associated y variables is set to 1. Finally, integer variables z_a^k denote the number of times controller k travels along arc a . For convenience we summarize the characteristics of the model in Table 2.

The integer programming (IP) formulation of the problem can be stated as follows:

$$\max \left\{ \sum_{l \in L} \pi_l x_l + \sum_{k \in K} \sum_{s \in S} \sum_{t \in T_s} \pi_s^t y_s^{t,k} \right\} \quad (6)$$

$$\text{subject to } \sum_{k \in K} \sum_{s \in S_l} \sum_{t \in T_s: \tau_s^t \geq \tau_l} y_s^{t,k} \geq x_l, \quad l \in L; \quad (7)$$

$$\sum_{a \in A} w_a z_a^k + \sum_{s \in S} \sum_{t \in T_s} \tau_s^t y_s^{t,k} \leq W_k, \quad k \in K; \quad (8)$$

$$\sum_{a \in \delta^+(0)} z_a^k \geq 1, \quad k \in K; \quad (9)$$

$$\sum_{k \in K} \sum_{t \in T_s} y_s^{t,k} \leq 1, \quad s \in S; \quad (10)$$

$$\sum_{a \in \delta^+(s)} z_a^k = \sum_{a \in \delta^-(s)} z_a^k \geq \sum_{t \in T_s} y_s^{t,k}, \quad s \in S, k \in K; \quad (11)$$

$$\sum_{a \in \delta^+(\Sigma)} z_a^k \geq \sum_{t \in T_s} y_s^{t,k}, \quad k \in K, \Sigma \subset S, s \in \Sigma; \quad (12)$$

$$y_s^{t,k} \in \{0, 1\}, \quad s \in S, t \in T, k \in K; \quad (13)$$

$$x_l \in \{0, 1\}, \quad l \in L; \quad (14)$$

$$z_a^k \in \mathbb{Z}_+, \quad a \in A, k \in K. \quad (15)$$

The objective function (6) captures the prize arising from checking lines (first term) and visiting stops (second term). Constraints (7) impose the condition that, to earn the line prize, the line must be checked by at least one controller during an interval of sufficient length. Constraints (8) impose an upper bound on

the working time of each controller. Note that different controllers can have different working times. Constraints (9) impose the condition that each controller visits the office during the working day (because time is not explicit, this visit can be thought of as the start of the route). Constraints (10) impose the condition that at most one controller visits a stop and at most one duration of time in T_s is selected. Constraints (11) guarantee flow conservation and impose the condition that if a node s is visited, at some point a controller must have arrived at it and left from it. Note that a route can go by a node without stopping to visit it. To guarantee the feasibility of itineraries, we add subtour elimination constraints in (12) (note that $0 \notin \Sigma \subset S$ by definition of S). Finally, constraints (13) and (14) impose the conditions that variables are binary, whereas constraints (15) require arc variables z to be (nonnegative) integers. The latter constraints allow arcs to be used more than once, which is necessary because the graph is not complete.

3.2. Building Solutions That Exhibit Less Clustering

When building a route, we would not like to visit a set of stops that are very close to each other and serve the same lines because the overall aim is to collect information as complete as possible in terms of checked lines and geographic locations. However, model (6)–(15) naturally produces solutions that visit stops close to each other and are visited by the same line, because this is a very efficient way of spending the available time. To make solutions of this type infeasible, we impose incompatibility constraints among stops and prevent the selection of stops that are “too close” according to a given distance function. Specifically, we say that two stops are incompatible if their distance is lower than 1 km or if they are connected by an arc (in A) with travel time lower than 10 minutes and are served by the same lines. To avoid a scenario where a pair of incompatible stops, say s_1 and s_2 , are checked during the same day, we can trivially impose the constraints

$$\sum_{k \in K} \sum_{t \in T_{s_1}} y_{s_1}^{t,k} + \sum_{k \in K} \sum_{t \in T_{s_2}} y_{s_2}^{t,k} \leq 1. \quad (16)$$

However, it is well known that incompatibility constraints expressed by constraints of the form of (16) lead to weak formulations because their continuous relaxations are loose.

A much more effective way to exclude tours that are highly clustered is by defining an incompatibility graph $I = (S, E)$ whose edges connect pairs of stops that are incompatible (i.e., close and served by the same lines). Then, we compute a cover of maximal cliques of I and include a clique constraint of the form

$$\sum_{s \in C} \sum_{k \in K} \sum_{t \in T_s} y_s^{t,k} \leq 1 \quad (17)$$

for each clique $C \subseteq S$ in the cover. The clique cover of I is computed through a greedy heuristic procedure that iteratively constructs a maximal clique (see, e.g., Malaguti et al. 2011).

3.3. Additional Constraints

In some situations, mainly where there are complaints, the agency might require to control a specified set of lines, or a specified number of stops (usually, terminals where bus lines end). These requirements can be translated into constraints. Let us consider the following additional sets and parameters.

- Let $\bar{L} \subseteq L$ be the set of lines that must be compulsorily checked.
- Let $\bar{S} \subseteq S$ be the set of stops that must be compulsorily visited. For each stop we can additionally impose a minimum stopping time $\bar{\tau}_s$.

The possible additional constraints consist of

$$x_l = 1, \quad l \in \bar{L}, \quad (18)$$

$$\sum_{k \in K} \sum_{t \in T_s: \tau_s^t \geq \bar{\tau}_s} y_s^{t,k} \geq 1, \quad s \in \bar{S}. \quad (19)$$

4. The Algorithm

We refer to the IP model described in §3 as the “control design problem,” (CDP). In this section we describe the components of an algorithm that solves it. We use a branch-and-cut (B&C) algorithm embedded in the IBM ILOG CPLEX MIP solver. The solver takes care of linear programming (LP) relaxations and of the branching tree. Because we cannot include all subtour elimination constraints (12) in the model, when a new (fractional or integer) solution is discovered, the control is passed back to our code via callbacks to generate valid inequalities. In particular, we generate subtour elimination constraints to iteratively obtain valid formulations.

Since there are exponentially many subtour elimination constraints (12), to solve the CDP we must introduce them as needed. We implement the following separation procedure. Given a fractional solution $(\tilde{x}, \tilde{y}, \tilde{z})$ to the CDP, we solve maximum-flow problems from the office location 0 to s , for every controller $k \in K$, and for every stop $s \in S$ using the algorithm by Goldberg and Tarjan (1988). In each problem we set arc capacities to be \tilde{z}_a^k for all arcs and get a minimum-cost flow f_s^k and a maximum cut $\Sigma_s^k \ni s$. If we have

$$\sum_{t \in T_s} \tilde{y}_s^{t,k} > f_s^k,$$

we add the corresponding violated constraint (12)

$$\sum_{a \in \delta^+(\Sigma)} z_a^k \geq \sum_{t \in T_s} y_s^{t,k}.$$

After the addition of some violated constraints (details in §5), we reoptimize the problem and iterate

until a solution that does not violate any of the subtour elimination constraints is obtained. The separation described above can also be used for cutting off integer solutions with subtours.

Instances with up to approximately 100 nodes can be solved through the B&C solver of CPLEX. For large instances like those of the city of Bologna, instead, a textbook implementation of the B&C method highlighted in the previous paragraph is ineffective. In our preliminary experiments the continuous relaxation of the CDP provided good bounds, but the solver had difficulties producing feasible solutions without ad hoc procedures and heuristics. This is because, when we find a subtour and add the corresponding subtour elimination constraint (12), the solver easily finds a similar solution by rerouting on some node having all the associated y variables at 0.

For this reason, the implementation of a heuristic that returns good solutions has proven to be a key factor in solving this problem successfully. This heuristic, presented in §4.1, is based on selecting a subset of nodes and reducing the original sparse graph to a smaller but complete graph, where we can solve a slightly different version of the CDP, so as to produce good feasible solutions.

4.1. Primal Heuristics and Suboptimal Solutions

We use a primal heuristic algorithm based on an integration of large neighborhood search (LNS) (see Shaw 2008) and (Pisinger and Ropke 2010) and rounding to produce feasible solutions during the execution of the B&C algorithm. The idea is to use information from the current infeasible solution to define a smaller problem on a complete network. We solve the small problem with the CPLEX MIP solver for a limited amount of time, possibly obtaining integer solutions of good quality, which are then mapped back to the original network. This algorithm, which we refer to as “MIP-heuristic,” is executed with a specified frequency at the nodes of the B&C tree.

To run the MIP-heuristic, we select a subset $\tilde{S} \subset S$ of nodes and define the distance network graph \tilde{G} by computing the shortest path in G between any pair of nodes in \tilde{S} . We precompute all-pair shortest paths offline; for large graphs, the computation can be efficiently performed by means of the algorithm proposed by Mehlhorn (1988). Having defined \tilde{G} , we solve the CDP on that graph using B&C but with the subtle difference that constraints (11) are set to equality. We can do this because \tilde{G} is a complete graph, and thus it is not necessary to allow a path to traverse a node without visiting it since there is always the option to go straight (triangle inequalities are satisfied since distances are given by shortest paths in the original graph). Another simplification is that variables z_a^k are binary, since again we can always go

from any node to any other node directly. These simplifications improve efficiency because they prevent the solver from producing similar infeasible solutions after adding subtour elimination constraints, in contrast to what could happen when working with sparse graphs. Finally, as soon as we become aware of a feasible solution, we can use its value, say z_G^* , as a cutoff for the CDP when solved on \tilde{G} .

In general, CPLEX managed to produce solutions to the reduced problems effectively. These solutions are then mapped back to the original network G . This process generates feasible solutions to G which can be used to prune the B&C tree for the original problem. Similarly, the best feasible solution to the problem on G found at any stage can be used to prune the B&C tree used to compute the solution to the reduced problem, thus accelerating the computation.

To select the nodes \tilde{S} to include in the small graph, we use the information associated with the fractional and integer solutions explored by the B&C tree since the last time the heuristic was called. One possibility is to include nodes corresponding to an integer solution containing cycles. A second possibility is to include nodes corresponding to a fractional acyclic solution. A third possibility, which combines the previous two, is to choose the solution with the smallest number of fractional variables generated since the last call. According to our computational experience, we did not find that one strategy uniformly dominated the others, thus we keep the simplest one, which is a fractional rounding method. We select the nodes s such that in the last generated fractional solution at least one of the associated variables $y_s^{t,k}$ has a value larger than a threshold η . Algorithm MIP-heuristic is given in Figure 1.

We are aware of other heuristic algorithms for solving routing problems based on the solution of a restricted instance of the problem. For example, De Franceschi et al. (2006) proposed a refinement method for VRP-related problems, where a feasible solution is improved by removing some nodes and sequences of nodes, which are then reinserted in the solution in an optimal way, by solving an optimization problem modeled as an MIP. More generally, restricting the solution space using local information on the current solution is at the core of two methods for general MIPs that are very effective: i.e., local branching (Fischetti and Lodi 2003) and relaxation-induced neighborhood search (Danna et al. 2005). However, in those methods the idea is to improve a given feasible solution, whereas our method is actually designed to produce a good solution starting from a currently fractional, thus infeasible, one. In this spirit, our method is also inspired by the feasibility pump method (Fischetti et al. 2005), where the idea is to start from a fractional solution and then solve a

Figure 1 Summary of the MIP-Heuristic Algorithm

algorithm *MIP-heuristic*
input: CDP data, fractional solution $(\tilde{x}, \tilde{y}, \tilde{z})$, best feasible solution value z_G^*
output: integer feasible solution
initialization:
 select a subset of nodes $\tilde{S} = \{s \in S: \tilde{y}_s^{t,k} \geq \eta \text{ for some } t, k\}$
 define \tilde{G} by computing shortest paths among all nodes in \tilde{S}
 set constraints (11) to equality in the CDP
 set a cutoff value of z_G^*
algorithm:
 solve the CDP on \tilde{G} with a specified time limit
 return the best feasible solution found
end

related problem to find a feasible one close to it (with respect to some metric). More generally, our algorithm can be seen as a rather sophisticated primal heuristic that repairs a solution of the continuous relaxation of the model at hand.

In addition to solutions generated by the MIP-heuristic, we obtain other feasible solutions to the CDP with the general-purpose primal heuristics implemented within CPLEX. However, these solutions, which are computed for the original graph G , may include subtours and detours so they are not really feasible. A subtour may occur if the solution includes a tour that does not visit any nodes; i.e., if it only involves arc variables z without a corresponding y variable set to 1. This is because there is no violated subtour elimination constraint when all the corresponding y variables are at 0. In a detour, two subsequent nodes that are visited are connected by a path that is not shortest. A solution containing a detour is depicted in Figure 2 (left), where the circles represent visited stops. Although subtours and detours cannot be present in optimal solutions to the CDP (and cannot appear in feasible solutions when constraints (11) are set to equality), they consistently appear in intermediate nodes of the B&C tree.

To remove subtours and detours from feasible solutions, we use an iterative algorithm that enumerates all possible paths that are contained in the current integral solution and selects the shortest one that traverses all nodes visited in the solution and comes back to the office. In Figure 2 (right), we show the resulting solution after the removal of the detour, and the iterative algorithm is described in Figure 3.

Figure 2 (Color online) Solution with a Detour (Left) and After Its Removal (Right)

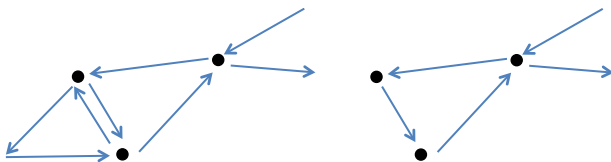


Figure 3 Summary of the Algorithm Used to Remove Subtours and Detours

algorithm *subtour-detour removal*
input: CDP data, integer feasible solution $(\tilde{x}, \tilde{y}, \tilde{z})$
output: k cycles without subtours or detours, each one crossing the same nodes visited by the input solution
algorithm:
 for each controller k ,
 $T(i, j) = 0$ for all $(i, j) \in A$
 $path = \{0\}$, $last = 0$
 explore($path$, $last$, k)
 return best k -th path
 endfor
end

procedure *explore*
input: current path $path$, last explored node $last$, controller k
procedure:
 if ($last = 0$ and $|path| > 1$)
 if (path crossed all nodes visited by k)
 update best k -th path
 return
 for each $j \in \delta^+(last)$, $T(last, j) < \tilde{z}_{last, j}^k$
 $path = path \cup \{j\}$, $last = j$, $T(last, j) = T(last, j) + 1$
 if ($path$ shorter than best path)
 explore($path$, $last$, k)
 endfor

5. Case Study: Definition of the Control Methodology and the Contract for Bologna, Italy

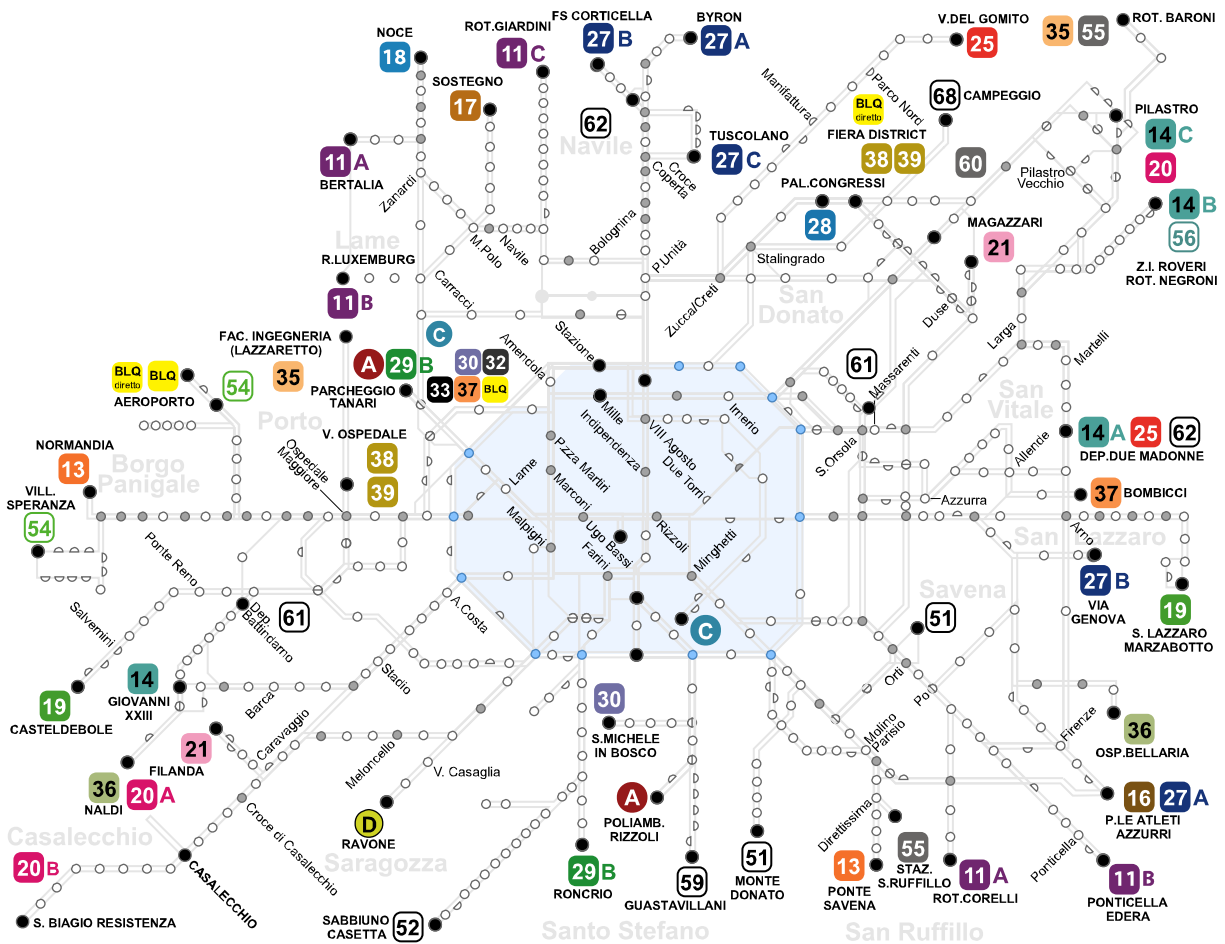
In this section, we tune the details of the control model and the contract for the city of Bologna, Italy. We defined these details in close cooperation with the agency that oversees public transportation there. We start with the control procedure and then use the output of the corresponding computational experiments to define the contract.

5.1. Setting Up the Data

Figure 4 shows a diagram with the network of buses in Bologna. Following the wish of the agency, we focused on high-frequency lines and thus considered the 29 urban lines with an average time between two consecutive services of at most 30 minutes. These lines serve 1,104 stops with 6,000 services per day, which accounts for 81.9% of the passengers served by the operator in Bologna.

We derive the graph $G = (S \cup \{0\}, A)$ from the real network. This graph contains two kinds of arcs: (i) bus connections and (ii) pedestrian connections. For bus connections, we connect stops along a bus line by defining an arc $a = (i, j)$ for each pair of two subsequent stops i and j . The travel time w_a of an arc a served by more than one bus line is set to the maximum travel time. This is a conservative choice because a faster bus may not be the first to come when the controller needs it. Figure 5 shows all bus connections among stops as black lines. We use pedestrian connections to connect all pairs of stops that are not

Figure 4 (Color online) Bus Network in Bologna



already connected and that are within walking distance. We set the maximum distance to correspond to 10 minutes. We show the full network corresponding to Bologna in Figure 5, where gray lines show pedestrian connections.

The values of the model parameters were set following the recommendations of the agency. (We refer

the reader to Agresti 2007 for details.) For each of the 1,104 stops, we consider that controllers can spend 15, 20, or 30 minutes at the stop counting services.

- The score of a line p_i to be collected potentially multiple times when observing a stop is defined as the sum of three scores $w_1^i + w_2^i + w_3^i$. The score w_1^i is the main component and depends on the number of passengers per year on the line. This captures the impact on commuters when services are missed or delayed. Table 3 reports scores w_1^i associated with each line. These scores were constructed from the categorization used internally by the agency. For instance, the agency follows this categorization when constructing statistics and evaluating bus performance. Scores w_1^i provide levers to prioritize the control tasks. The score w_2^i is a corrective term proportional to the traveled kilometers per year for that line. We use it because lines covering more distance usually have a larger probability of incurring a delay or a cancellation. The score w_3^i is a corrective term proportional to the inverse of the line's number of services per year. We use it in the opposite direction compared to the previous measures. Indeed, checking

Figure 5 Graph of Bus Connections (Black) and Overall Graph (Gray)

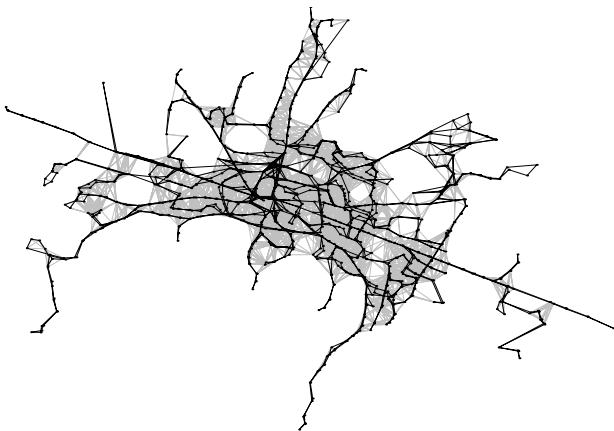


Table 3 Scores and Prizes Associated with Bus Lines

Passengers per year	w_l	π_l
<20,000	1	15
20,001–100,000	2	30
100,001–500,001	3	45
500,001–1,000,000	4	60
1,000,001–5,000,000	5	75
>5,000,000	6	90

a particular service is less important when the frequency of a route is high, actually: a delay or missing service for a high-frequency line would have a minor impact on the service quality perceived by customers. Although one could also say that having more services makes a line more important to check, we have already partially captured this effect in the other two scores.

- The prize π_s^t associated with a stop s where the controller stays for τ_s^t minutes is defined as the sum of the prizes p_l associated with the lines that can be observed during that time. Concretely, $\pi_s^t := \sum_{l: \tau_l > \tau_s^t} p_l$.

- Finally, the prize π_l , which is earned at most once daily, is set as a function of the passengers the line serves. Table 3 indicates the values that, again, describe the internal ranking used by the agency.

To evaluate the solution returned by the model, we use a greedy algorithm that generates solutions that resemble the current behavior of the agency's controllers. For each controller available, the algorithm initially selects a random node in the network to start the route. Then, it selects the next node to visit among those that can be directly reached from the current one and are neither incompatible with some previously visited node nor already visited by the route under construction or by a previously constructed route. To choose the next node, the algorithm selects the node, stopping-time pair that maximizes the ratio between the time needed to visit the node (including the travel time from the current node) and the earned prize. If all neighbor nodes are already visited (by the current or previously constructed routes), the algorithm moves to the closest node that was not traversed by the current route (choosing an already traversed node would make the algorithm cycle). If no such a node exists, the algorithm closes the route, directing the controller back to the office using a shortest path. The same happens if visiting the next node and traveling back to the office requires the controller to exceed the allocated time to perform controls. Figure 6 provides a sketch of the greedy algorithm we implemented.

We run the greedy route-generation procedure 30 times and select the best solution as a reference point to evaluate our algorithm. This is done to capture the facts that controllers know the city well and

Figure 6 Summary of the Greedy Algorithm

```

algorithm Greedy
input: CDP data
output: feasible routing
initialization: mark all nodes as nonvisited
algorithm:
  for each controller  $k$ 
     $W = W_k$ , mark all nodes as nontraversed
    randomly select a nonvisited node  $s$ 
    visit  $s$  for the stopping time  $t$  maximizing ratio  $\pi_s^t/\tau_t$ ,
    update  $W = W - \tau_t$ 
  do
    for each node  $j$ ,  $(s, j) \in A$ ,  $j$  nonvisited and
      compatible with a visited node, and each
      possible stopping time  $t \in T_j$  such that
       $W - w_{sj} - \tau_t - w_{j0} \geq 0$ ,
      compute the ratio  $r_j^t = \pi_j^t/(w_{sj} + \tau_t)$ 
    endfor
    choose node  $j$  and stopping time  $t \in T_j$  maximizing
      ratio  $r_j^t$ , mark  $j$  as visited, update
       $W = W - w_{sj} - \tau_t$ , move on  $j$  by setting  $s = j$ 
    if no move was done, choose the closest non
      traversed node  $j$  such that  $W - w_{sj} - w_{j0} \geq 0$ ,
      mark  $j$  as traversed, update  $W = W - w_{sj}$ , move
      on  $j$  by setting  $s = j$ 
    while a move is done
      close the cycle by going back to node 0
    endfor
  end.

```

use judgment in selecting their routes, without necessarily taking a holistic approach to construct the whole route at the same time.

5.2. Computational Experiments

All algorithms were coded in C and tested on a single core of a Linux-based PC equipped with an i5-650 processor at 3.20 GHz and 8 GB of memory. The algorithm is implemented under the IBM ILOG CPLEX MIP solver platform, version 12.2. We implemented user-defined cuts and other customizations using callbacks to enhance the standard solution procedures.

We tuned our solution algorithm and the parameters of the solver engine through extensive computational experiments. This required a careful calibration of the frequency of invoking the cut-generation routine. We had to balance the trade-off between obtaining strong bounds, which would suggest generating many cuts, and not overwhelming the solver with too many cuts, which would slow down the solution of LP relaxations. According to our experiments, we obtained good results from the following methodology.

- We call the cut-generating procedure only for separating integer solutions, and we generate at most five cuts per controller. Although in principle we do not need to perform a maximum-flow computation for separating integer solutions, and we could instead use a more efficient problem-specific procedure, in

Table 4 Computational Study of the Algorithm to Solve the CDP

Total working hours	Working pattern	Greedy gap %	Optimality gap %	Nodes tree	Cuts tree	Gap with no MIP-heur. %	Gap MIP-heur. rand. selection %
6	3–3	19.54	3.66	20,600	6,222	94.54	22.47
9	3–6	16.27	1.33	4,205	2,107	94.19	8.31
9	3–3–3	22.98	3.67	6,100	2,702	97.36	25.82
12	3–3–3–3	22.54	4.42	3,500	1,370	98.24	27.21
12	6–6	19.46	2.22	6,700	5,398	98.69	26.27
15	3–6–6	13.77	1.22	3,401	1,806	94.82	21.29
18	6–6–6	21.69	2.83	3,870	2,756	98.93	32.61
18	3–3–6–6	17.09	2.70	3,200	1,908	94.95	25.05
24	6–6–6–6	20.96	2.39	2,900	2,093	99.11	35.77

practice the cut-generating procedure accounts for only 0.32% of the overall computing time.

- We run the primal heuristic of §4.1 on every 100 nodes of the B&C tree with a time limit of 200 seconds. In that case, because graphs are small and complete, we call the cut-generating procedure every 100 nodes of the B&C tree, in addition to separating all integer solutions.

- We give the overall algorithm a time limit of two hours.

In Table 4, we report computational results for a variety of cases. We considered nine different scenarios with two, three, and four controllers working for different combinations of three and six hours, as indicated in the working pattern column. For every scenario, we report the optimality gap of the greedy algorithm described in §5.1 with respect to the best lower bound, the optimality gap, the number of nodes explored in the B&C tree, and the number of cuts added to the tree by the separation procedure. These two values regarding the tree are for the main problem; they do not include the B&C trees generated when solving the heuristic.

The optimality gaps for all scenarios are very low. Although in the time allocated we could not achieve optimality, for the purposes of the use of the controls by the agency, gaps lower than 5% are considered more than sufficient. In any case, in §6 we compare solutions according to the objective introduced by the game presented in §2. Note that the greedy procedure used as a reference point achieves gaps that are many times larger, validating the use of our model. The size of the tree explored decreases with the complexity of the instance because LP relaxations become harder so there is less time to explore nodes within the time allocated.

To show the importance of the MIP-heuristic, Table 4 also reports the optimality gaps with the heuristic disabled. It can be seen that gaps are always larger than 94%. This happens because the solver cannot produce feasible solutions of good quality. Finally, to validate the choice of nodes made by the MIP-heuristic, we compared our solutions, obtained

using the strategy described in §4.1, to those computed using a random selection of 100 nodes. The last column of Table 4 contains the corresponding optimality gap. It clearly shows that a strategy of choosing 100 nodes randomly does not work, providing evidence that the fractional solution returned by the LP solver contains crucial information that can be exploited by the heuristic when constructing the smaller graph \tilde{G} .

5.3. Scalability of the Algorithm

To investigate the scalability of the proposed algorithm, we tested it on some artificial instances obtained by modifying the case-study network. We considered two sets of instances, obtained by modifying either of the following elements.

1. Number of nodes. We generated new nodes in the proximity of each existing node and connected them to neighboring nodes. New nodes have the same probability of being served by some bus line as the original ones. We considered instances with two and four copies of each of the original nodes.

2. Geographical extension of the network. Keeping the topology fixed, we modified distances by a factor of 0.5 and 2. That way we trade off the balance between how long it takes to collect useful information and how long it takes to move to other locations in the network. This case is interesting because, if the algorithm is in trouble when solving networks with a large geographical extension and a large number of stops (nodes), one possible strategy is to preselect a subset of promising nodes and solve a smaller problem. We already know that a smart selection of nodes can lead to very good solutions (this is the rationale of the MIP-heuristic).

Table 5 reports the optimality gaps achieved by our algorithm after two hours of computing time. The first two columns of the table indicate the total number of nodes and the geographic extension of the network instance compared to the case study. Quite surprisingly, the algorithm obtains very small optimality gaps compared to those in Table 4, when the number of nodes is doubled to 2,208 and the extension of

Table 5 Test of the Scalability of Our Algorithm with Artificial Instances

Network nodes	Distance factor	Total working hours	Working pattern	Optimality gap %
2,208	1.0	6	3–3	0.70
2,208	1.0	9	3–3–3	1.43
2,208	1.0	12	3–3–3–3	2.44
2,208	1.0	12	6–6	0.61
2,208	1.0	18	6–6–6	1.39
2,208	1.0	24	6–6–6–6	3.20
1,104	0.5	6	3–3	1.73
1,104	0.5	9	3–3–3	1.57
1,104	0.5	12	3–3–3–3	1.85
1,104	0.5	12	6–6	0.52
1,104	0.5	18	6–6–6	0.53
1,104	0.5	24	6–6–6–6	1.06
1,104	2.0	6	3–3	3.17
1,104	2.0	9	3–3–3	4.32
1,104	2.0	12	3–3–3–3	5.06
1,104	2.0	12	6–6	2.44
1,104	2.0	18	6–6–6	2.99
1,104	2.0	24	6–6–6–6	3.89

the network is maintained. Our interpretation of this effect is that, when more nodes are present in the network, routing takes a less important role, thus making the instance easier. That said, when we quadruple the nodes (the case of 4,416 nodes is not reported in the table), the problem becomes too large and the algorithm struggles to find good feasible solutions. Consequently, the optimality gaps obtained for two, three, and four controllers working three hours become very large, and no feasible solution is found for two, three, and four controllers working six hours.

The intuition that the routing aspect of the problem is the most difficult one is confirmed by the gaps obtained by our algorithm when the network is shrunk or enlarged. The center and bottom parts of Table 5 display gaps when distances are multiplied by a fixed factor. Taking these gaps together with those in Table 4, we see that gaps are strongly positively correlated with the multiplicative factor. This confirms that difficulty grows when distances are larger, which we attribute to the routing aspect of the problem.

5.4. Dynamics: Generation of Schedules over Successive Days

As mentioned before, the controllers' daily schedule should vary so as to collect different information at different times. To address this, one possibility would have been to run our model including additional controllers, representing the routes to be followed in the successive days. Although the model becomes much larger, it could be solved by giving the algorithm

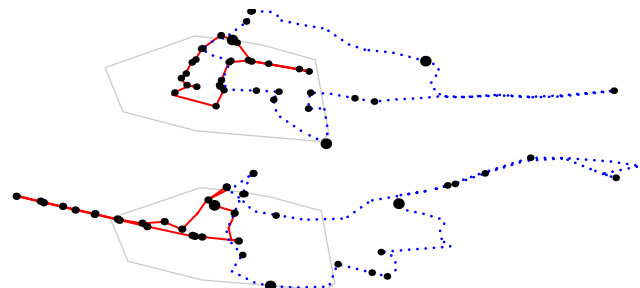
more CPU time. In that case, different routes for different days are subject to the constraints of the model. The agency opted for adopting soft constraints by penalizing the objective. Hence, we discount the value of the information observable at the recently checked stops. Prizes associated with lines and stops are modified as follows: $\pi_s^{\text{int}} = \pi_s(1 - (1 + \text{int})^{-1})$, where int is the interval of time (in days) since the last visit to the stop. A similar idea is implemented to discount the value of information associated with lines checked in the previous days, by modifying the prizes of lines π_l in the same fashion.

Figure 7 shows the output of the algorithm for two consecutive working days in the case of two controllers working six hours per day. The gray hexagons in the figure represent the ancient walls around the perimeter of the city (currently, one of the main traffic avenues). The two cycles represent the routes followed by each controller, and the black dots show where controllers stop to check bus services. The size of these dots is proportional to the stopping time. Because controllers follow different schedules daily, in a medium-term period they cover most of the network.

As explained earlier, the CDP allows the agency to specify a set of stops or lines that will be checked for sure. This is very important in practice because the agency wishes to integrate the day-by-day optimized control with the specific control of some stops, like some terminals or specific lines, based on customer complaints or other needs of the agency. Concerning the constraints added to reduce clustering in the solutions, they are active in the solutions depicted in Figure 7. As soon as these constraints are removed, the model would tend to select clusters of stops close to each other.

Finally, although we designed the model focusing on high-frequency lines, in practice controllers also check low-frequency lines that can be seen from their control locations.

Figure 7 (Color online) Depiction of Two Consecutive Working Days for Two Controllers



Note. Each controller's route is shown respectively by a dotted or solid line, and the hexagon represents the ancient city walls.

6. Closing the Loop: Implementation of the Optimal Contract

This section closes the loop and goes back to the contract design problem (CDP) presented in §2. We discuss how to implement the contract, in light of the optimized control procedure presented in §§3 and 4. This completes the case study by providing an illustration of how the game can be used to find fines that are appropriate for the contract. In the current implementation of the contract, the required level of service (intended as number of services to be performed over the total scheduled) is fixed at 99.4%. For every 0.01% under this threshold the operator receives a fine of €150, but the fine for a nondeclared canceled service ranges between €5,000 and €200,000 (depending on the reason for failing to declare the missed service). However, as anticipated in the introduction, these values result from political negotiations and a long-term relationship, rather than a quantitative approach. Because service levels in recent times have been very good, fines are rarely imposed. However, it is not certain that this level of service will be maintained in the future, especially if a different operator were to be selected in the next auction.

We now estimate the optimal fines arising from the game, using the optimal control procedure. We start by estimating the function $p(b_a)$ that links both parts of the methodology because the probability that a service is checked depends on the control procedure. The argument is the investment made by the agency, which we express in Euros per service; that is, b_a is the money allocated to pay controllers, prorated to each planned service. We take this measurement unit because it is independent of the duration of the contract. To estimate $p(b_a)$, we compute the percentage of checked services for various budget levels b_a used for controls. This can be estimated using the objective function (6), after setting all rewards associated with lines π_l to zero. Indeed, the remaining part of the objective can be used to count the services when π_s^t measures how many of them can be observed by staying at stop s for τ_s^t minutes.

Relying on the computational study, Table 6 provides a comparison between the number of services checked under the greedy solution of §5.1, which is a good representation of what happens currently in practice, and the solution to the CDP. The values in the table correspond to solutions when prizes for checked lines are 0, and prizes for checked services are 1. (The reported solutions are those given after two hours of computation. Although suboptimal, optimality gaps are similar to those in Table 4, and always lower than 5%.) Interestingly, this comparison was the initial study that the agency in Bologna requested, so this was the starting point for the rest of the methodology developed in this study.

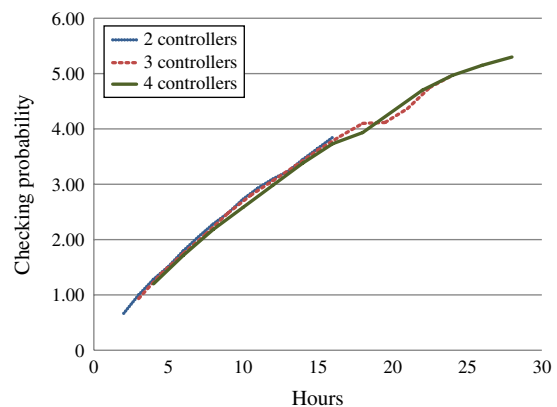
Table 6 Solutions to the CDP as a Function of the Percentage of Services Checked

Total working hours	Working pattern	Services checked	% checked	% checked by greedy	Relative difference %
6	3–3	108	1.80	1.22	47.54
9	3–3–3	146	2.43	1.27	91.33
12	3–3–3–3	181	3.02	1.82	65.93
12	6–6	185	3.08	2.27	35.68
18	6–6–6	242	4.03	3.10	30.00
24	6–6–6–6	294	4.90	3.62	35.35

The table reports, for six working pattern configurations, the number of services checked by the solution to the CDP, the corresponding percentage over the total number of services daily, the percentage of services checked by the greedy solution, and relative difference measured as the percentage improvement of the CDP over greedy. Consistent with the conclusions drawn earlier, the solution of the CDP is consistently and significantly better than the greedy heuristic.

From the total number of hours needed for each scenario (this is the main explanatory variable of the cost of performing controls), we can readily estimate the function $p(\cdot)$. In Figure 8, we add more computations to those in the table and show these curves for two, three, and four controllers. From the figure, we observe that the main driver of the probability of checking a service is the total number of working hours and not so much how these hours are allocated among controllers. This is because the central office is in Bologna, so fixed costs are low and the first and last trip of a controller are not significantly more time-consuming than the other trips. Putting it all together, we need to find the coefficients of the function $p(b_a) = kb_a^\alpha$ used in §2 empirically with our computational study. Converting the variable “working hours” to “Euros per service” using the wages paid to controllers and number of services per day,

Figure 8 (Color online) Probability of Checking a Service as a Function of Total Working Hours



Note. The curves correspond to two, three, and four controllers.

regression analysis tells us that the best fit is provided by the function $p(b_a) = 0.0733b_a^{0.7236}$ with an R^2 of 0.9985.

To find the function that captures the probability that the operator performs services, we must find the constant β . We merely estimated the cost of labor (driver) per service taking into account the hourly rate, and we added the cost of equipment (bus) prorated to the number of services that it typically lasts before it is replaced. This basic cost-of-service provision, which does not include any extra drivers or extra equipment, amounts to 26.6€/service. Furthermore, we estimate that the probability of providing a particular service is of the order of 90% since the driver may not show up or the bus may not work. Solving for β , we get the function $q(b_{op}) = 1 - e^{-0.09b_{op}}$. Finally, we set $r(F) = (F/C)^2$ as explained earlier.

Having estimated the necessary functions, we need to fit the other constants used as inputs to the model. The values R and η are additive constants so they are irrelevant to the outcome of the model; hence we do not need to specify them. The coefficient η^- that quantifies the impact on welfare of each missed service is estimated using the average number of people waiting in a stop multiplied by the average number of stops in a line and the average interarrival time between services, which is then converted to money using the average value of time of the population. It is reasonable to set these to, respectively, 1.3 people per stop, 15 stops per service, 6 services per hour, and €7 per hour. Putting all together, $\eta^- = €22.75$ per service. Finally, we set C to €200,000.

Plugging in all values in the formulas derived in §2, we get that optimal fines are $F^* = €109,464$ and $f^* = €1,104$. At equilibrium, the operator improves the quality of service $q(b_{op})$ from 90% to 99% and hence only misses approximately 1% of the planned services. It achieves that by investing €52.62 per service, approximately twice the minimum operating cost specified above. The agency invests approximately €0.06 per service to check that services run properly. At that investment level, the optimized controls can check approximately $p(b_a) = 1.01\%$ of the services. As shown in Figure 8, to be able to control that percentage of services, controllers need to work for approximately 2.7 hours daily. Of the few services that are not performed, the operator truthfully reports $t = 98.29\%$ of them; the remaining 1.71% are not reported because the probability that the agency will find out is small. The probability that the operator will go bankrupt if given fine F^* is 30%. Overall, the agency utility earned at equilibrium, not counting the constant terms R and η in (2), is 11.13.

Although we cannot observe this in real life (since currently the agency-operator relationship operates out of the equilibrium path), in Table 7 we display the

Table 7 Comparison of Different Scenarios

	Current low	Current high	Optimal
F (€)	10,000.00	150,000.00	109,463.94
f (€)	150.00	150.00	1,103.58
b_{op} (€)	29.59	29.59	52.62
$q(b_{op})$ (%)	92.31	92.31	98.95
b_a (€)	0.11	0.003	0.06
$p(b_a)$ (%)	1.500	0.100	1.008
t (%)	98.59	99.87	98.29
$1 - t$ (%)	1.41	0.13	1.71
$r(F)$ (%)	0.25	56.25	29.96
π_a	9.6680	9.7741	11.1276

model predictions for the current fines used by the agency. As we said at the beginning of this section, fines F range from €5,000 to €200,000 so we selected representative low and high values to perform these computations. The operator would provide a lower quality of service of 92% by investing €29.59 per service. The agency would invest between €0.003 and €0.11 per service to check that services run properly, checking between $p(b_a) = 0.1\%$ and 1.5% of the services, and working between 0 and 5 hours daily. Of the few services that are not performed, the operator would truthfully report between $t = 98.59\%$ and 99.87% of them. The probability that the operator will go bankrupt if given fine F^* is between 0% and 56%. Putting it all together, the agency utility, not counting the constant terms, would be between 9.67 and 9.78, which represents a reduction on the order of 10% from the optimal utility computed earlier.

7. Conclusions

We have conducted a case study in the management of public-private partnerships in bus transportation. Here, the two parties consist of (a) the agency who designs the contract and oversees the service and (b) the operator who provides the bus service. There is a strong information asymmetry between them, whereby the agency does not necessarily know if the operator provides all services as scheduled by the contract. We have addressed this asymmetry by providing an incentive to the operator to report the quality of service to the agency truthfully. In addition, we have proposed that the agency controls the validity of the information reported by the operator using the solution of a prize-collecting routing model. The solution provides itineraries of locations where controllers will go to count bus services and compare them with those reported by the operator.

The case study was conducted in close collaboration with the public transportation agency of the city of Bologna, Italy. The agency intends to apply the methodology we have developed as the basis of the contract design for the upcoming auction to be held

when the contract with the current operator expires in 2016. Although the agency has had a long-term relationship with the current operator, the tendering for service will be open to many large companies and will be very competitive. Indeed, the company that wins the contract to operate the buses may be based anywhere in Italy or even outside Italy. There is no reason to believe that the relationship of trust that exists today will continue and the agency wants to be prepared. This type of transition has already been observed in other locations and the role of the agency is to anticipate that need and be ready for it. In addition, the decision of the agency to rely on this study is motivated by the fact that it has been developed in close collaboration with it and has been validated in multiple occasions by the working groups of the EU project EPTA.

On the quantitative side, although the goal of the agency is not and will never be to make money from the auction mechanism and control strategy, the managerial impact that the agency expects is associated with the enhanced information provision brought by the system operations. More precisely, it has been estimated that the implemented control policy will lead—as a byproduct—to double the amount of information related to quality of service, including aspects that are not directly under the responsibility of the operator. The additional information is expected to be crucial to improving and promoting the service by making the communication with the users more timely and transparent. Evidently, service improvement and promotion will likely lead to an increase in the user base of the public transportation system, which will improve sustainability indicators in Bologna (e.g., reduction in the use of private cars and the ensuing reduction of pollutants, CO₂, etc). Although this points in the right direction, it is premature to quantify these benefits precisely.

Regarding the modeling methodology, we have made some assumptions about the information that is revealed by various events. When the operator reports that a service took place, the agency may or may not be sure about it, depending whether a controller witnessed that particular service. When the operator reports that a service did not take place, the agency does not doubt this and charges it a fine. When a controller witnesses that the count of bus services is smaller than expected, the operator receives a fine. The assumption of perfect information for positive detection of missed bus services implies that imposing higher fines F encourages the operator to be more honest because the threat of being fined makes being dishonest too risky. Nevertheless this is compensated by the fact that large fines are unlikely to be paid because the operator may go bankrupt. Hence, in reality higher fines make the threat less credible because

both parties know that the operator is less likely to afford them.

Some of the assumptions in our modeling methodology could be removed, thus defining potential avenues for future research:

- Our model assumes that all parameters are deterministic. In particular, we consider fixed travel times when planning control routes, although we know that in practice this is an approximation of reality. It would be interesting to explore how to make robust plans that consider that in the time allocated controllers may fall behind the planned schedule for the day.
- In many areas related to security (air marshals, counter-terrorist groups, police), models explicitly consider that both parties learn what the opponent does. For example, a counter-terrorist group plans strategies to discover activities whereas terrorist groups learn those strategies and exploit them by attacking in the location where they estimate that the probability of being discovered is minimal. There are many relations between this problem and the one we discuss in this article, making it interesting to study their interplay in the future.
- It would be interesting for the agency to conduct experiments for validating the assumptions and the resulting equilibrium. For example, the agency could, for different periods, increase or decrease the time spent checking on services and observe the response of the operator.

Concerning the algorithmic approach, we solve a prize-collecting routing model via a B&C algorithm and use an MIP-heuristic to generate feasible solutions by considering a subset of the network nodes. This approach proved effective for the bus network of Bologna with approximately 1,000 nodes, and the experiments show that the approach is still effective even when the size of the network is doubled. Nevertheless, control problems in some applied domains may require to consider up to millions of nodes. In this case it can be a viable strategy to design a mechanism for the preselection of a subset of nodes, and then to restrict the control problem to the selected subset. In the case of a problem with a prize-collecting routing structure with a budget constraint, like the one we consider, one could initially disregard the routing, preselect nodes by solving an associated knapsack problem, and finally solve the CDP for the subset of nodes.

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