



# Credit and liquidity in interbank rates: A quadratic approach <sup>☆</sup>



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## ABSTRACT

A bank that lends on the unsecured market requires compensations for facing the default risk of the borrowing bank (credit risk) and the risk associated to its own future funding needs (liquidity risk). In this paper, we propose a quadratic term-structure model of the spreads between unsecured and risk-free interbank rates. Our no-arbitrage econometric framework allows us to decompose the term structure of spreads into credit and liquidity components and to identify risk premia associated with each of these two risks. Our results suggest that, over the period 2012–2013, most of the reduction in interbank spreads comes from a decrease in liquidity-related risk components.

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## 1. Introduction

Since the beginning of the financial crisis, the interbank market has been carefully scrutinized by commentators and policy-makers, both in Europe and in the US. This paper focuses on the spreads between the *Euro Interbank Offered Rates* (EURIBORs) and their risk-free counterparts, proxied by the *Overnight Indexed Swap rates* (OIS). These spreads are considered as crucial stress indicators during financial crises: they reveal not only banks' concerns regarding the credit risk of their counterparts, but also regarding

their own liquidity needs. Accordingly, the impact of the monetary-policy measures taken after the emergence of money-market tensions in mid-2007 has often been gauged through their influence on the EURIBOR-OIS spreads (e.g. [Lenza et al., 2010](#)).

Disentangling credit and liquidity effects has essential policy implications. On the one hand, if a rise in spreads reflects poor liquidity, policy measures should aim at improving funding facilities. On the other hand credit concerns should be addressed by enhancing debtors' solvency (see [Codogno et al., 2003](#)). This question has been of utmost importance in the euro area over the last few years, where most of the unconventional monetary operations conducted by the European Central Bank were designed to reduce interbank market stress (see [Gonzalez-Paramo, 2011](#)).<sup>1</sup> Several attempts have been made to provide a credit/liquidity decomposition of interbank risk (see next Section), but whereas most studies reckon that liquidity risk has been an important driver of interbank yields during the last 5 years, there is no consensus on the precise size of these effects: [Schwarz \(2009\)](#) estimates that one third of the EURIBOR-OIS

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<sup>1</sup> The particularly important role of bank-based financing in the eurozone contributes to the ECB's focus on improving banks' liquidity and funding (see e.g. [Fahr et al., 2013](#)).

1-month spread in January 2008 is linked to liquidity, whereas Filipovic and Trolle (2013) find that nearly all the spread is liquidity-related at that date.

In this paper, we identify credit and liquidity components in interbank-market spreads. Our method involves a term-structure model of the EURIBOR-OIS spreads. Credit and liquidity intensities constitute the basic ingredients of the model. These intensities reflect the two risks to which a bank that lends to another bank is exposed. The first risk corresponds to the default of the borrowing bank. The second risk pertains to the difficulty to meet potential liquidity needs that the lending bank may face over the loan period.

The credit and liquidity intensities are unobserved. An important contribution of the paper lies in the methodology employed to identify them. Our identification strategy uses two observable proxies that are known to be related to credit and liquidity risks. The approach consists in estimating a state-space model that features two types of measurement equations: a first set of equations, stemming from the term structure model, specifies the links between the interbank spreads and the two latent factors; a second set of equations formalizes the close relationship between the latent factors and the respective proxies. While the credit proxy is based on a set of 36 euro area banks' CDS premia, the liquidity proxy is computed as a combination of variables capturing different aspects of liquidity pricing, namely market and funding liquidity.<sup>2</sup> A first liquidity-pricing factor is the KfW-Bund spread. KfW is a public German agency, whose bonds are guaranteed by the Federal Republic of Germany. KfW's bonds hence possess the same credit quality as their sovereign counterparts, the Bunds. The former bonds being less liquid than the latter, the KfW-Bund spread essentially reflects liquidity-pricing effects. This spread has also been exploited by Monfort and Renne (2014a) in order to identify credit and liquidity components in euro-area sovereign spreads.<sup>3</sup> A second liquidity factor is the Tbill-repo spread, computed as the yield differential between the 3-month German T-bill and the 3-month general collateral repurchase agreement rate (repo). A third factor is based on the replies of banks to a liquidity-related question of the Bank Lending Survey (BLS) conducted by the ECB.

In addition, the term structure dimension of our analysis is exploited to identify the component of the spreads that corresponds to expected excess returns, thereby extending the existing literature on interbank risks. These expected excess returns, which we also refer to as term premia or risk premia, are the components of spreads that would not exist if economic agents – in our case, banks – were risk-neutral, or if the risks involved were not systematic, i.e. if they could be diversified away (see e.g. Longstaff et al., 2011). To remain as general as possible, we authorize these risk premia to be different from zero and time-varying, risk-neutrality being a particular case. These risk premia are not the only spreads' components: even if agents were risk-neutral, the EURIBOR-OIS spread would not be zero. In that case, the spreads would just be equal to the expected losses stemming from the total amount of risk – credit- and liquidity-related – that a bank faces when lending to another bank. In other words, the spreads would then be the ones predicted by the expectation hypothesis.

To ensure that the credit and liquidity intensities are always positive, we specify them as quadratic functions of credit and

liquidity Gaussian factors, respectively. This allows us to formulate probabilities of a default or of a liquidity event that are always constrained between zero and one, an absent feature in models where the intensities are affine functions of Gaussian factors. The quadratic form also produces time-varying volatilities for the credit and liquidity intensities, consistently with the spreads' behavior.

The model is estimated over a 6-year period, between August 2007 and September 2013. Both credit and liquidity components account for the fluctuations of the spreads over the sample period, with a higher average contribution of the latter. Our results also suggest that both kinds of risk command substantial risk premia, pointing towards the systematic nature of credit and liquidity interbank risks. We illustrate that the existence of credit-risk premia translates into substantial differences between model-implied physical and risk-neutral probabilities of default.

The spreads' decomposition allows us to explore the consequences of unconventional monetary policies conducted by the ECB during this period. Our findings support the claim that the 3-year ECB loans to euro commercial banks – the *Very Long-Term Refinancing Operations*, or VLTROs – and the announcement of the (still-unused) ECB sovereign-bond purchase program – the *Outright Monetary Transactions*, or OMTs – have helped to reduce the perception of liquidity risk and its related risk premium. However, we find little evidence that the ECB large-scale asset-purchase programs of 2010 and 2011 – the *Securities Market Programs*, or SMP 1 & 2 – have had any significant impact on interbank risk. On the whole, our results indicate that unconventional monetary policies have had very modest, if any, effects on the credit components of the spreads.

The remainder of the paper is organized as follows. Section 2 reviews the related literature. Section 3 develops the model. Section 4 describes the strategy employed to identify the credit and liquidity factors. Section 5 presents the data. The estimation results are shown in Section 6 and the last section concludes. Proofs are gathered in the [Appendices](#).

## 2. Literature review

Our paper relates to the interbank spreads literature. A wide range of studies deal with the determinants of interbank spreads: Taylor and Williams (2009) claim that counterparty risk is the main driver of the LIBOR-OIS spread, Michaud and Upper (2008) and Gyntelberg and Wooldridge (2008) find that credit and liquidity factors both play a role, while the results of Schwarz (2009) and Filipovic and Trolle (2013) suggest that liquidity risk has accounted for most of the LIBOR-OIS and EURIBOR-OIS spreads variations over the period 2007–2009. In comparison, Smith (2010) emphasizes that most of the variation in interbank spreads risk premia is explained by credit risk. Angelini et al. (2011) highlight the role of macro-factors to account for the dynamics of unsecured/secured money-market spreads.

The measured impact of unconventional monetary policies is ambiguous: Taylor and Williams (2009) find no effects of the Fed's intervention in 2008, contrary to Christensen et al. (2014). According to the latter, Fed's liquidity injections (TAF, for *Term Auction Facility*) reduced the 3-month maturity interbank spread by about 70 basis points. Angelini et al. (2011) measure a modest impact of ECB exceptional 3-month refinancing operations, in contradiction with Abbassi and Linzert (2012).

The literature on unconventional monetary policies is not limited to the study of money-market rates. In particular, Ghysels et al. (2013), Trebesch and Zettelmeyer (2014), Krishnamurthy et al. (2014), De Pooter et al. (2015), and Schwaab and Eser (2016) examine the effect of the SMP – the ECB large-scale bond-purchase program – on bond yields. All these

<sup>2</sup> While market liquidity is reflected by the difference between the market value and the fundamental value of an asset, funding liquidity relates to the scarcity of capital. These two concepts of liquidity risk intimately interact with each other (see Brunnermeier and Pedersen, 2009; Brunnermeier, 2009 or Drehmann and Nikolaou, 2009).

<sup>3</sup> In Monfort and Renne (2014a), the identification of a liquidity factor is based on the modeling of the term structure of the KfW-Bund spreads. Here, by contrast, a single representative maturity is chosen and the 5-year KfW-Bund spread is combined with other observed variables to construct a liquidity-pricing proxy.

studies find significant bond yield declines in response to ECB actions.<sup>4</sup> More generally, different contributions suggest that non-standard monetary policies have significantly contributed to sustain bank lending activity and, ultimately, to preserve price stability in the euro area (Peersman, 2011; Lenza et al., 2010; Giannone et al., 2012; Cour-Thimann and Winkler, 2012; Fahr et al., 2013; Carpenter et al., 2014; Crosignani et al., 2015).

Our identification scheme builds on several studies that rely on reduced-form no-arbitrage models to identify credit and liquidity components in the term structures of yields or spreads (e.g. Liu et al., 2006; Feldhutter and Lando, 2008; Longstaff et al., 2005). At the heart of these studies are credit/liquidity intensities whose fluctuations affect the whole term structure of spreads. The present paper closely relates to Monfort and Renne (2014a). However, whereas the latter study focuses on the sovereign bond market, the interbank money markets is targeted by the present one. While the econometric specifications differ in the two papers,<sup>5</sup> both of them allow for some dynamic interactions between credit and liquidity risks, consistently with the theoretical predictions of, among others, Goldstein and Pauzner (2005), Wagner (2007), Acharya et al. (2011), Acharya and Viswanathan (2011), and He and Xiong (2012).

In most term structure models, the authors assume that the default intensity and/or the short-term rate are affine functions of the underlying factors. A quadratic specification however possesses several advantages over the standard Gaussian affine case. Constantinides (1992) shows that a standard QTSM with a specific quadratic short-term interest rate can generate positive yields for all maturities and more flexibility in the term structure to fit bond data. Leippold and Wu (2002) generalize the quadratic term structure models and show that this specification provides convenient closed-form or semi closed-form formulas for bond pricing. Ahn et al. (2002) provide further empirical evidence that QTSM often outperforms the standard Gaussian affine term structure specification (ATSM). Leippold and Wu (2007) study the joint behavior of exchanges rates and bond yields using QTSM models for Japan and the US. Andreasen and Meldrum (2011) and Kim and Singleton (2012) exploit the QTSM framework to model the term structure of interest rates in a context of extremely low monetary-policy rates. Turning to the credit literature, Hordahl and Tristani (2012) use a quadratic specification to model euro-area sovereign spreads, and Doshi et al. (2013) consider a quadratic intensity to price corporate credit default swaps. Our paper also adopts a quadratic approach to impose positivity of the risk intensities and spreads, and it takes advantage of the quadratic Kalman filter, a well-suited technique to estimate state-space models where measurement equations are quadratic (see Monfort et al., 2015).

### 3. The model

In this section, we propose a reduced-form asset pricing model of the term structure of spreads between risky and risk-free yields. In our application, the former will correspond to the EURIBOR and the latter to the OIS rates. In this model, when a bank lends to another bank, it is exposed to credit and liquidity risks.

#### 3.1. Notations

Let us consider a pool of  $N$  banks.<sup>6</sup> At date  $t$ , market participants receive the new information  $w_t = \{r_t, X'_t, d'_t, \ell'_t\}'$ , where  $r_t$  is the

short-term risk-free rate between  $t$  and  $t + 1$ ,  $X_t = (x_{c,t}, x_{l,t})'$  is a  $(2 \times 1)$  vector whose components are respectively credit- and liquidity-related factors, and  $d_t$  and  $\ell_t$  are two  $N$ -dimensional vectors of binary variables  $d_t^{(i)}$  and  $\ell_t^{(i)}$ , with  $i \in \{1, \dots, N\}$ .  $d_t^{(i)}$  defines the credit state of bank  $i$  at date  $t$ ,  $\ell_t^{(i)}$  defines its liquidity state. The implications of defaults ( $d_t^{(i)} = 1$ ) or liquidity shocks ( $\ell_t^{(i)} = 1$ ) for interbank-loan payments will be made more precise below. For any random vector  $z_t$ , we will use the notation  $\underline{z}_t = (z_t, z_{t-1}, \dots)$ .

#### 3.2. Historical and risk-neutral dynamics

Let us first define the historical dynamics of  $w_t$ . Following Berndt et al. (2005), Pan and Singleton (2008) or Longstaff et al. (2011), we assume that the processes  $r_t$  and  $(X'_t, d'_t, \ell'_t)'$  are independent.<sup>7</sup> We further assume that  $(d'_t, \ell'_t)'$  does not cause  $X_t$  and that  $X_t$  follows a VAR(1) process, i.e.:

$$X_t = \mu + \Phi X_{t-1} + \epsilon_t, \quad (1)$$

where  $\epsilon_t = (\epsilon_{c,t}, \epsilon_{l,t})'$  is a Gaussian white noise such that  $\mathbb{E}(\epsilon_t) = 0$  and  $\mathbb{V}(\epsilon_t) = I$ .

Finally, we assume that, conditionally on  $(X_t, d_{t-1}, \ell_{t-1})$ , the vectors  $d_t$  and  $\ell_t$  are independent. The conditional dynamics of these vectors are defined by:

$$\mathbb{P}(d_t^{(j)} = 0 | X_t, d_{t-1}, \ell_{t-1}) = \begin{cases} \exp[-\lambda_c(x_{c,t})] & \text{if } d_{t-1}^{(j)} = 0 \\ 0 & \text{if } d_{t-1}^{(j)} = 1 \end{cases} \quad (2)$$

$$\mathbb{P}(\ell_t^{(j)} = 0 | X_t, d_{t-1}, \ell_{t-1}) = \begin{cases} \exp[-\lambda_\ell(x_{l,t})] & \text{if } \ell_{t-1}^{(j)} = 0 \\ 0 & \text{if } \ell_{t-1}^{(j)} = 1. \end{cases} \quad (3)$$

The intensity functions  $\lambda_c(x_{c,t})$  and  $\lambda_\ell(x_{l,t})$  will be specified in Section 3.5. Importantly, while  $d_t$  and  $\ell_t$  are conditionally independent, they are not marginally independent because  $x_{c,t}$  and  $x_{l,t}$  may cause each other through matrix  $\Phi$  (see Eq. (1)).

To derive the risk-neutral  $\mathbb{Q}$ -dynamics of  $w_t$ , we introduce a stochastic discount factor between  $t - 1$  and  $t$  of the form (see Duffee, 2002):

$$M_{t-1,t} = \exp \left[ \Gamma'_{t-1} \epsilon_t - \frac{1}{2} \Gamma'_{t-1} \Gamma_{t-1} - r_{t-1} + g(r_t) \right],$$

where  $g(r_t)$  is any function such that  $\mathbb{E}_{t-1}[M_{t-1,t}] = \exp(-r_t)$ , i.e.  $\mathbb{E}_{t-1}[\exp(g(r_t))] = 1$ , and

$$\Gamma_{t-1} = \Gamma_0 + \Gamma X_{t-1},$$

where  $\Gamma_0$  is a  $(2 \times 1)$  vector and  $\Gamma$  is a  $(2 \times 2)$  matrix. We show in Appendix A.1 that three main properties can be obtained with respect to  $w_t$  risk-neutral dynamics. First,  $X_t$  also follows a VAR(1) under the risk-neutral measure, that is:

$$X_t = \mu^* + \Phi^* X_{t-1} + \epsilon_t^*, \quad (4)$$

where  $\mu^* = \mu + \Gamma_0$ ,  $\Phi^* = \Phi + \Gamma$  and  $\epsilon_t^*$  is a  $\mathbb{Q}$ -standardized Gaussian white noise. In particular,  $(d'_t, \ell'_t)$  does not cause  $X_t$  under the  $\mathbb{Q}$ -measure. Second,  $r_t$  and  $(X'_t, d'_t, \ell'_t)$  are also independent in the risk-neutral world. Third, the  $\mathbb{Q}$ -conditional distribution of  $(d'_t, \ell'_t)$  given  $(X_t, d_{t-1}, \ell_{t-1})$  is the same as in the historical world. More precisely, the risk-neutral intensities  $\lambda_c^{\mathbb{Q}}(x_{c,t})$  and  $\lambda_\ell^{\mathbb{Q}}(x_{l,t})$  are the same

<sup>4</sup> For the U.S., studies on the effects on large-scale purchase programs include, among others, Gagnon et al. (2011), Krishnamurthy and Vissing-Jorgensen (2011), Bauer and Rudebusch (2014), D'Amico and King (2013), and Hamilton and Wu (2012).

<sup>5</sup> Monfort and Renne (2014a) use a regime-switching term-structure model.

<sup>6</sup> These banks would be those participating to the EURIBOR panel.

<sup>7</sup> The article 127 of the Treaty on the Functioning of the European Union states that the primary objective of the euro-area monetary policy is to maintain price stability. To the extent that the interbank market conditions affect the inflation outlook, it is likely that the interest rate decisions taken by the ECB monetary policy committee are influenced by the interbank market conditions. Modeling the link between an implicit monetary policy rule and the interbank market conditions is yet beyond the scope of this paper.

functions of  $X_t$  in the risk-neutral and in the historical world. However, since the dynamics of  $X_t$  are different under  $\mathbb{P}$  and  $\mathbb{Q}$ , the dynamics of  $\lambda_c(x_{c,t})$  and  $\lambda_\ell(x_{\ell,t})$  are different under the two measures.

### 3.3. Intensities and interbank rates

We assume that the panel of banks is homogeneous: conditionally on  $(X_t, d_{t-1}, \ell_{t-1})$ , the default probabilities and the probabilities of being affected by a liquidity shock are the same for all the banks of the panel (as reflected by Eqs. (2) and (3)). This assumption implies in particular that at each date  $t$ , there is a single rate prevailing for interbank unsecured loans between  $t$  and a future date  $t+h$ . This interest rate is denoted by  $R_{t,h}^{IB}$ . By definition of this rate, an interbank loan of unit face value between dates  $t$  and  $t+h$  provides the borrower with the amount  $B(t, h) = \exp(-hR_{t,h}^{IB})$  at date  $t$ .<sup>8</sup>

Suppose that, at date  $t$ , bank  $i$  lends  $B(t, h)$  to bank  $j$  for a period of length  $h$ . The maturity date is therefore  $t+h$  and, assuming no premature termination of the loan, the repayment is EUR1. Now, consider an intermediary date  $t^*$  (i.e.  $t < t^* \leq t+h$ ). At date  $t^*$ , if bank  $j$  defaults or if bank  $i$  is hit by a liquidity shock, this terminates the interbank loan and the resulting payoffs are as follows. On the one hand, if bank  $j$  defaults at date  $t^*$  ( $d_{t^*}^{(j)} = 1$ ), then bank  $i$  will not obtain full repayment at  $t+h$ . Instead, at date  $t^*$ , it recovers a fraction  $\theta_c < 1$  of the loan value that would have prevailed at date  $t^*$  in the absence of default. This counterfactual value corresponds to the face value of the loan discounted by the interbank risky rate  $R_{t^*,t+h}^{IB}$ . This set up builds on the “recovery at market value” assumption of Duffie and Singleton (1999). On the other hand, if bank  $i$  is hit by a liquidity shock at date  $t^*$  (i.e.  $\ell_{t^*}^{(i)} = 1$ ), bank  $i$  has to find some cash in a limited period of time to meet an unexpected liquidity need. It may do so by negotiating a premature termination of the loan with bank  $j$ . The latter agrees at a discount: the repayment at date  $t^*$  is expressed as a fraction  $\theta_\ell < 1$  of the aforementioned counterfactual value of the loan. Such a mechanism of costly liquidation is in the spirit of Ericsson and Renault (2006) or He and Xiong (2012).

In that context, the value of the loan at date  $t+1$  writes:

$$B(t+1, h-1) \left\{ (1 - d_{t+1}^{(j)} + \theta_c d_{t+1}^{(j)}) (1 - \ell_{t+1}^{(i)} + \theta_\ell \ell_{t+1}^{(i)}) \right\},$$

where  $B(t+1, h-1)$  is the loan price prevailing in the absence of credit or liquidity event at date  $t+1$ . At date  $t$ , if  $d_t^{(j)} = \ell_t^{(i)} = 0$ , the loan price is:

$$B(t, h) = \exp(-r_t) \mathbb{E}^{\mathbb{Q}} \left[ B(t+1, h-1) \left\{ (1 - d_{t+1}^{(j)} + \theta_c d_{t+1}^{(j)}) (1 - \ell_{t+1}^{(i)} + \theta_\ell \ell_{t+1}^{(i)}) \right\} \middle| \mathcal{W}_t \right], \quad (5)$$

where  $\mathbb{E}^{\mathbb{Q}}(\cdot)$  denotes the expectation under the  $\mathbb{Q}$ -measure.

Given the definitions of  $\lambda_c(x_{c,t})$  and  $\lambda_\ell(x_{\ell,t})$ , we have:

$$\begin{cases} \mathbb{E}^{\mathbb{Q}}(d_{t+1}^{(j)} | \mathcal{W}_t, X_{t+1}, r_{t+1}, d_t^{(j)} = 0, \ell_t^{(i)} = 0) = 1 - \exp[-\lambda_c(x_{c,t+1})] \\ \mathbb{E}^{\mathbb{Q}}(\ell_{t+1}^{(i)} | \mathcal{W}_t, X_{t+1}, r_{t+1}, d_t^{(j)} = 0, \ell_t^{(i)} = 0) = 1 - \exp[-\lambda_\ell(x_{\ell,t+1})]. \end{cases} \quad (6)$$

$\exp[-\lambda_c(x_{c,t+1})]$  and  $\exp[-\lambda_\ell(x_{\ell,t+1})]$  are probabilities and, therefore,  $\lambda_c(x_{c,t+1})$  and  $\lambda_\ell(x_{\ell,t+1})$  must be positive at all times. When these intensities are small, they are close to the default probabilities

and to the probabilities of being hit by the liquidity shock, respectively. First-order approximations yield:

$$\begin{cases} \mathbb{E}^{\mathbb{Q}}((1 - \theta_c) d_{t+1}^{(j)} | \mathcal{W}_t, X_{t+1}) = 1 - \exp[-(1 - \theta_c) \lambda_c(x_{c,t+1})] \\ \mathbb{E}^{\mathbb{Q}}((1 - \theta_\ell) \ell_{t+1}^{(i)} | \mathcal{W}_t, X_{t+1}) = 1 - \exp[-(1 - \theta_\ell) \lambda_\ell(x_{\ell,t+1})]. \end{cases} \quad (7)$$

Introducing the total intensity  $\lambda_t = (1 - \theta_c) \lambda_c(x_{c,t}) + (1 - \theta_\ell) \lambda_\ell(x_{\ell,t})$  in Eq. (5) implies (see Appendix A.2):

$$B(t, h) = \mathbb{E}_t^{\mathbb{Q}}[\exp(-r_t - \lambda_{t+1} - \dots - r_{t+h-1} - \lambda_{t+h})]. \quad (8)$$

Since  $B(t, h) = \exp(-hR_{t,h}^{IB})$ , we have:

$$R_{t,h}^{IB} = -\frac{1}{h} \ln \left\{ \mathbb{E}_t^{\mathbb{Q}}[\exp(-r_t - \lambda_{t+1} - \dots - r_{t+h-1} - \lambda_{t+h})] \right\}. \quad (9)$$

### 3.4. Spreads between risky and risk-free rates

At date  $t$ , the risk-free rate of maturity  $h$  is defined as the continuously-compounded yield of a zero-coupon bond paying off EUR1 at date  $t+h$ . This yield is given by

$$-\frac{1}{h} \log \mathbb{E}_t^{\mathbb{Q}}[\exp\{-r_t - \dots - r_{t+h-1}\}].$$

It corresponds to the rate of a maturity- $h$  overnight indexed swap (OIS) whose floating leg is indexed on the risk-free short-term rate  $r_t$  (see Appendix A.3). Denoting by  $R_{t,h}^{OIS}$  the rate of an OIS swap of maturity  $h$ , we have:

$$R_{t,h}^{OIS} = -\frac{1}{h} \log \mathbb{E}_t^{\mathbb{Q}}[\exp\{-r_t - \dots - r_{t+h-1}\}]. \quad (10)$$

Because  $r_t$  and  $\lambda_t$  are independent under  $\mathbb{Q}$  (see Section 3.2 and Appendix A.1), the EURIBOR-OIS spread of maturity  $h$ , denoted by  $S(t, h)$ , is given by:

$$S(t, h) = R_{t,h}^{IB} - R_{t,h}^{OIS} = -\frac{1}{h} \log \left( \mathbb{E}_t^{\mathbb{Q}} \left[ \exp \left( \sum_{i=1}^h -\lambda_{t+i} \right) \right] \right). \quad (11)$$

Eq. (11) shows that the study of the spreads between unsecured and risk-free yields does not require the modeling of the short-term risk-free interest rate  $r_t$ .

### 3.5. Intensity specification

It remains now to specify the intensity functions  $\lambda_c$  and  $\lambda_\ell$  and the factors  $(x_{c,t}, x_{\ell,t})'$ . In a preliminary analysis (results are not reported here for sake of brevity), we postulated a linear relationship between the intensities and the factors, within a standard Gaussian affine term-structure model. In such a model the distribution of model-implied spreads is Gaussian. The results were not satisfying, the model clearly violating the non-negativity of spreads. The model-implied frequencies of generating negative spreads – considering their marginal densities – were close to 50% for all maturities. In addition, authorizing the intensities to go in the negative territory leads to an erroneous probabilistic construction since the conditional survival and default probabilities can go below zero or above 1. This failure illustrates the inappropriateness of Gaussian ATSM to model such spreads. Therefore, following Doshi et al. (2013) or Gouriéroux and Monfort (2008), we set a quadratic relationship between the intensities and the associated factors:

$$\lambda_c(x_{c,t}) = \Lambda_c x_{c,t}^2 \quad \text{and} \quad \lambda_\ell(x_{\ell,t}) = \Lambda_\ell x_{\ell,t}^2. \quad (12)$$

This ensures that the underlying probabilities of liquidity and default events are constrained between 0 and 1, both  $\lambda_c(x_{c,t})$  and  $\lambda_\ell(x_{\ell,t})$  being positive (see Eq. (6)). In turn, this implies that the spreads at any maturity are positive, which can be seen from

<sup>8</sup> Note that the pricing formulas derived in this paper feature continuously-compounded interest rates: denoting by  $z$  a market-quoted interest rate and applying the money-market day-count convention (ACT/360), the corresponding continuously-compounded rate is given by  $\ln(1 + d \times z/360) \times 365/d$  where  $d$  is the residual maturity of the considered instrument, expressed in days.



Eq. (11). An additional advantage of this modeling is that it allows to accommodate heteroskedasticity in the spreads (see Ahn et al., 2002).

### 3.6. Recursive pricing formulas

Putting together the risk-neutral dynamics of  $X_t$  given by Eq. (4) and the intensities specification of Eq. (12), we see that our model belongs to the class of Quadratic Term Structure Models (QTSM). We show in Appendix A.4 that the spreads  $S(t, h)$  of Eq. (11) can be expressed as quadratic combinations of  $x_{c,t}$  and  $x_{\ell,t}$ . Indeed the conditional Laplace transform of the augmented vector of factors  $[X'_{t+1}, \text{Vec}(X_{t+1}X'_{t+1})']'$  given  $X_t$  is exponential-affine in  $[X'_t, \text{Vec}(X_tX'_t)]'$  and the process  $[X'_{t+1}, \text{Vec}(X_{t+1}X'_{t+1})']'$  is hence affine (see Gouriéroux and Sufana (2011) or Cheng and Scaillet (2007)). We have:

$$S(t, h) = -\frac{1}{h} \left( \Theta_{0,h} + \Theta'_{1,h}X_t + X'_t\Theta_{2,h}X_t \right) \triangleq \theta_{0,h} + \theta'_{1,h}X_t + X'_t\theta_{2,h}X_t. \quad (13)$$

The factor loadings  $\theta_{0,h}$ ,  $\theta_{1,h}$  and  $\theta_{2,h}$  are maturity-dependent and are functions of the risk-neutral parameters and of  $\Lambda$ , which is the  $(2 \times 2)$ -dimensional diagonal matrix containing  $(1 - \theta_c)\Lambda_c$  and  $(1 - \theta_\ell)\Lambda_\ell$  on its diagonal. The loadings  $\Theta_{0,h}$ ,  $\Theta_{1,h}$  and  $\Theta_{2,h}$  can be computed recursively as (see Appendix A.4):

$$\begin{cases} \Theta_{0,h} = \Theta_{0,h-1} + \Theta'_{1,h-1} [I_n - 2(\Theta_{2,h-1} - \Lambda)]^{-1} (\mu^* + \frac{1}{2}\Theta_{1,h-1}) \\ \quad + \mu^{*'} (\Theta_{2,h-1} - \Lambda) [I_n - 2(\Theta_{2,h-1} - \Lambda)]^{-1} \mu^* \\ \quad - \frac{1}{2} \log |I_n - 2(\Theta_{2,h-1} - \Lambda)| \\ \Theta_{1,h} = \Phi^{*'} \left\{ [I_n - 2(\Theta_{2,h-1} - \Lambda)]^{-1} [\Theta_{1,h-1} + 2(\Theta_{2,h-1} - \Lambda)\mu^*] \right\} \\ \Theta_{2,h} = \Phi^{*'} (\Theta_{2,h-1} - \Lambda) [I_n - 2(\Theta_{2,h-1} - \Lambda)]^{-1} \Phi^*, \end{cases} \quad (14)$$

where initial conditions are given by  $\Theta_{0,0} = 0$ ,  $\Theta_{1,0} = (0, 0)'$ , and  $\Theta_{2,0} = [0]_{i,j \in \{1,2\}}$ . One of our main objectives is to decompose spreads into a credit and a liquidity component. A necessary condition to obtain such a twofold decomposition is that  $\Theta_{2,h}$  is diagonal for all maturities  $h$ . This condition constrains  $\Phi^*$  to be diagonal.

## 4. Identification and estimation strategy

### 4.1. Overview

The credit and liquidity factors  $x_{c,t}$  and  $x_{\ell,t}$  are not observed by the econometrician. While the spreads' dynamics reveal information about these factors, this is not sufficient to identify them. That is, there exists an infinite number of pairs  $(x_{c,t}, x_{\ell,t})$  that satisfy the specification presented above and that result in the same observed spreads. To identify credit and liquidity factors, we introduce a proxy for credit risk ( $P_{c,t}$ ) and another one for liquidity risk ( $P_{\ell,t}$ ). We further impose that each of these proxies relate to the latent factors  $x_{c,t}$  and  $x_{\ell,t}$ , respectively. In the remaining of this section, we detail the computation of our proxies (Section 4.2), we show that the postulated relationship between the proxies and the latent factors uniquely identifies the latter (Section 4.3) and we introduce the resulting state-space model (SubSection 4.4).

### 4.2. Credit and liquidity proxies

The credit proxy is the first principal component of a set of 36 Euro-zone bank CDS. We use 5-year CDS denominated in USD since these are the most traded and the most liquid ones. Eight banks are German, six Italian, five Spanish, four French, four Dutch, three

Irish, three Portuguese, two Austrian, and one Belgian. Nearly 72% of the total variance is explained by the first principal component.

The liquidity proxy used in our term-structure model is the first principal component of a set of three liquidity-related variables. These variables are chosen to capture different aspects of liquidity pricing. Specifically, whereas our first two proxies are mostly related to market liquidity, the last one pertains to funding liquidity.<sup>9</sup> Nearly 60% of the total variance is explained by the first principal component.

A first liquidity-pricing factor is the *KfW-Bund spread* (5-year maturity). KfW is a public German agency. KfW bonds are guaranteed by the Federal Republic of Germany. Hence, they embed the same credit quality as their sovereign counterparts, the so-called Bunds. KfW bonds being less liquid than their sovereign counterparts, the KfW-Bund spread essentially reflect liquidity-pricing effects.<sup>10</sup>

A second liquidity factor is the *Tbill-repo spread*, computed as the yield differential between the 3-month German T-bill and the 3-month general-collateral repurchase agreement rate (repo). From an investor point of view, the credit qualities of the two instruments are comparable (as argued by Liu et al. (2006)). The differential between the two rates corresponds to the convenience yield, that can be seen as a premium that one is willing to pay when holding highly-liquid Treasury securities (see Feldhutter and Lando, 2008).

A third factor is based on the Bank Lending Survey conducted by the ECB on a quarterly basis. This survey is sent to senior loan officers of a representative sample of around 90 euro-area banks. It addresses issues such as credit standards for approving loans as well as credit terms and conditions applied to enterprises and households. Our indicator is based on the following specific question of the survey: *Over the past three months, how has your bank's liquidity position affected the credit standards as applied to the approval of loans or credit lines to enterprises?*<sup>11</sup>

### 4.3. Identification strategy: linking proxies and latent factors

We assume that – up to measurement error terms – the proxies are quadratic functions of the corresponding latent factors. Formally:

$$P_{i,t} = \pi_{i,0} + \pi_i x_{i,t}^2 + \sigma_{v_i} v_{i,t} \quad \text{for } i \in \{c, \ell\}, \quad (15)$$

where  $v_{c,t}$  and  $v_{\ell,t}$  follow independent Gaussian distributions with zero mean and unit variance. The presence of measurement errors addresses potential concerns regarding the fact that our proxies are not pure measures of credit and liquidity risks.<sup>12</sup>

It is worth stressing that, even though risk factors  $x_{c,t}$  and  $x_{\ell,t}$  are contemporaneously uncorrelated, their VAR(1) dynamics accommodate the presence of lagged Granger causality between them.<sup>13</sup> Eq. (15) therefore implies that the credit (respectively liquidity)

<sup>9</sup> See Brunnermeier and Pedersen (2009) or Brunnermeier (2009) for a description of market liquidity and funding liquidity.

<sup>10</sup> See Schwarz (2009) and Monfort and Renne (2014b) or Schuster and Uhrig-Homburg (2015). In the same spirit, Longstaff (2004) computes liquidity premia based on the spread between U.S. Treasuries and government-guaranteed bonds issued by Refcorp.

<sup>11</sup> The respondents can answer ++, +, 0, – or – – to that question. We compute the proportion of – and – – as a ratio of total answers. To obtain weekly series, we assign the same value to all weeks in a quarter (step function).

<sup>12</sup> Typically, CDS contracts may be contemporaneously affected by microstructure issues (see e.g. Fulop and Lescouret, 2008).

<sup>13</sup> The assumption according to which the factors are not contemporaneously correlated is not an identification restriction. We have estimated a version of the model with a non-zero contemporaneous correlation between  $\epsilon_{c,t}$  and  $\epsilon_{\ell,t}$ . The results were very similar to those reported below. The fact that the data frequency – that is weekly – is relatively high explains why this assumption is not key for the results.

proxy is a combination of past (resp. past and current) liquidity shocks, of past and current (resp. past) credit shocks and of an error  $v_{c,t}$  (resp.  $v_{l,t}$ ). This makes our framework consistent with banking theory predicting the prevalence of dynamic interactions between credit and liquidity risks (see Section 2).

Let us now address identification issues. Consider an affine transformation  $\tilde{X}_t$  of  $X_t$ , i.e.  $\tilde{X}_t = m + MX_t$ . Under which conditions do we have an observationally equivalent model when  $X_t$  is replaced by  $\tilde{X}_t$ ? Since each of the two proxies is a function of a single component of  $X_t$ ,  $M$  has to be diagonal. Hence the alternative factors are of the form:  $\tilde{x}_{i,t} = m_{0,i} + m_{i,i}x_{i,t}$  for  $i = \{c, \ell\}$ . Since the conditional variance of  $\tilde{x}_{i,t}$  must be equal to 1, we necessarily have  $m_{i,i} = \pm 1$ . Injecting these results in Eq. (15) leads to:  $P_{i,t} = (\pi_{i,0} + \pi_i m_{0,i}^2) \pm 2\pi_i m_{0,i}x_{i,t} + \pi_i x_{i,t}^2 + \sigma_{v_i} v_{i,t}$ . For  $\tilde{x}_{i,t}$  to satisfy Eq. (15), the linear term in the previous equation has to be zero. Therefore  $m_{0,i} = 0$ , that is  $\tilde{x}_{i,t} \equiv \pm x_{i,t}$ . The proxies equations hence identify the factors  $x_{c,t}$  and  $x_{\ell,t}$  up to their sign. Since the spreads' measurement equations gather linear and quadratic terms for maturities greater than one week, the sign of the factors is also identified.<sup>14</sup>

#### 4.4. State-space model and estimation methodology

The model parameters are estimated by maximizing the likelihood function. In order to handle the latency of factors  $x_{c,t}$  and  $x_{\ell,t}$ , this function is approximated by means of a Kalman-type algorithm. Whereas most recent articles use the unscented Kalman filter (UKF, see for instance Filipovic and Trolle (2013) or Christoffersen et al. (2014)), we rely on the quadratic Kalman filter (QKF) of Monfort et al. (2015). The latter approach is specifically fitted to quadratic measurement equations. The filtering algorithm is detailed in Appendix A.5.

The filter can be used once the model is cast into its state-space form. Eq. (1), which describes the  $\mathbb{P}$ -dynamics of the factors  $x_{c,t}$  and  $x_{\ell,t}$ , constitutes the transition equation of the state-space model. The measurement equations are composed of the spread formulas and of the proxies measurement equations (Eqs. (13) and (15), respectively). They write:

$$\begin{aligned} S(t, h) &= \theta_{0,h} + \theta'_{1,h} X_t + X'_t \theta_{2,h} X_t + \sigma_\eta \eta_{t,h} \quad \forall h \in \{13, 26, 39, 52 \text{ weeks}\} \\ P_{i,t} &= \pi_{i,0} + \pi_i x_{i,t}^2 + \sigma_{v,i} v_{i,t} \quad \forall i = \{c, \ell\}, \end{aligned} \quad (16)$$

where the components of the vector of pricing errors  $\eta_t$  and  $v_{i,t}$  are independent Gaussian white noises with unit variance. Parameters  $\pi_{i,0}$  and  $\pi_i > 0$  are not constrained by model-implied restrictions, contrary to the loadings  $\theta_{0,h}$ ,  $\theta_{1,h}$ , and  $\theta_{2,h}$  that derive from Eqs. (13) and (14).

To help pinning down the  $\mathbb{P}$ -parameters, we constrain the marginal variance of the intensity  $\lambda_t$  to be equal to the sample variance of the shortest-maturity spread we consider (3 month).<sup>15</sup> Without this constraint, the estimated  $\mathbb{P}$  dynamics of the latent factors is nearly non-stationary, which implies, in particular, unrealistic marginal distributions of spreads. The rationale for using the sample variance of a short-term spread to calibrate the variance of  $\lambda_t$  is as follows. Neglecting Jensen's inequality in Eq. (11), short-term spreads  $S(t, h)$  appear to be close to  $\mathbb{E}_t^Q(1/h \sum_{i=1}^h \lambda_{t+i})$ . If the intensity

is persistent under  $\mathbb{Q}$ , it follows that short-term spreads can be seen as rough approximations of the total intensity  $\lambda_t$ .<sup>16</sup>

Finally, we control the accuracy of the fit of the proxies in the estimation by imposing that both  $\sigma_{v_c}^2$  and  $\sigma_{v_\ell}^2$  are equal to 0.1. This corresponds to a tenth of the proxies' variance. The results are fairly robust to this choice.

## 5. Data

The estimation data cover the period from August 31 2007 to September 13 2013 at the weekly frequency (end-of-week data). Interest rates and CDS data are extracted from Bloomberg. The EURIBOR-OIS spreads of the following maturities enter the measurement equations: 3, 6, 9, and 12 months. The remaining of this section provides details on the EURIBOR rates (Section 5.1) and on the OIS rates (SubSection 5.2).

### 5.1. The unsecured interbank rates

The Euro Interbank Offered Rate (EURIBOR) provides a daily measure of the interest rates at which banks can raise unsecured funds from other financial institutions in the euro wholesale money market, for maturities ranging from one week to twelve months. A daily survey is sent to a panel of 30–50 creditworthy banks in the euro area. The question of the survey is: *what are the rates at which euro interbank term deposits are being offered within the Eurozone by one prime bank to another?* The EURIBORs are trimmed means of the contributed rates, the 15% of each tail being erased.<sup>17</sup>

While there are no publicly available data on volumes in term money markets, anecdotal evidence suggests that the financial crisis has resulted in a sharp decline in unsecured term money market volumes (see Eisenschmidt and Tapking, 2009). In spite of this, there is evidence that EURIBOR rates remain reliable proxies for bank funding costs. Typically, data collected from the ECB Short Term European Papers (STEP) database suggest that EURIBORs are very close to quotations of certificates of deposits issued by banks.<sup>18</sup> Moreover, using U.S. data, Kuo et al. (2012) find that public interbank yield data beyond LIBOR are moderately informative about bank funding costs.

### 5.2. The interbank risk-free rate

The risk-free rates are proxied by the Overnight Indexed Swap (OIS) rates. An OIS is a fixed-for-floating interest rate swap with a floating rate leg indexed on overnight interbank rates. In the euro area, the reference overnight rate is the EONIA (Euro OverNight Index Average). OIS have become especially popular hedging and positioning vehicles in euro financial markets and grew significantly in importance during the financial turmoil of the last few years. The OIS curve is more and more seen by market participants

<sup>14</sup> In spite of  $\Phi^*$  being diagonal, the factor loadings associated to the linear term in  $X_t$ , i.e.  $\theta_{1,h}$  (see Eq. (13)), are not null when  $\mu^* \neq 0$ .

<sup>15</sup> Recall that the total intensity is  $\lambda_t = (1 - \theta_c)\Lambda_c x_{c,t}^2 + (1 - \theta_\ell)\Lambda_\ell x_{\ell,t}^2$ . Hence, it is a linear combination of  $\text{Vec}(X_t X'_t)$ , whose marginal variance is given in closed-form by Monfort et al. (2015). The marginal variance of  $\lambda_t$  is a quadratic function of  $(1 - \theta_c)\Lambda_c$  and  $(1 - \theta_\ell)\Lambda_\ell$ . In the numerical optimization procedure, for each considered set of parameter estimates, we solve the quadratic equation  $\mathbb{V}ar(\lambda_t) = \mathbb{V}ar(S(t, 3\text{months}))$  for  $(1 - \theta_c)\Lambda_c$ .

<sup>16</sup> The fact that the intensity is extremely persistent under  $\mathbb{Q}$  is a common result in the credit-risk literature and is obtained in the unconstrained estimation of the model. Besides, as will be shown below in Table 1, the term structure of the sample variances of the EURIBOR-OIS spreads is relatively flat; as a consequence, our results do not strongly depend on the choice of the spread used to calibrate the marginal variance of  $\lambda_t$ .

<sup>17</sup> As banks do not possess the same characteristics and underlying risks, there is no uniqueness of interbank rates. Disaggregated rates are however not publicly available. Therefore, in order to conduct an analysis on interbank risks, a more aggregated measure must be considered.

<sup>18</sup> Indeed, it appears that the average of the spreads between (a) the issuance yields for certificates of deposits with an initial maturity comprised between 101 and 200 days and (b) the 5-month EURIBOR rate was lower than 3 basis points over the 2008–2012 period.

as a proxy for the risk-free interbank yield curve.<sup>19</sup> As no principal is exchanged, an OIS requires nearly no immobilization of capital. Further, due to netting and credit-enhancement mechanisms (including call margins), the counterparty risk is limited in the case of a swap contract (Bomfim, 2003).

Fig. 1 (Panel (a)) presents the evolution of the 3-month EURIBOR from August 2007 to September 2013. Panel (a) of Fig. 1 also displays the 3-month OIS rate during the same period. While this chart shows that EURIBOR and OIS rates present strong common fluctuations, Panel (b) also highlights that the spread between the two rates has undergone substantial variations over the last five years. In the next subsection, we discuss the term structure of the EURIBOR-OIS spreads.

### 5.3. Preliminary analysis of the EURIBOR-OIS spreads

Being mostly stable before August 2008, the spreads abruptly increased during Lehman crisis until December 2008, the 3-month spread peaking at 207 basis points, where a slow decay begins (see Fig. 1, Panel b). For sake of comparison, before the summer of 2007, the 3-months EURIBOR-OIS spread was around ten basis points and part of this deviation was accounted for by the fact that the EURIBOR is an offer rate while the OIS rate is a mid rate (average of bid and ask yields). Then, following a long stabilization period between August 2009 and 2010, a sharp increase occurred in mid-2011. Since the beginning of 2012, the EURIBOR-OIS spreads have decreased, alternating between a linear decreasing trend and stable phases.

Standard descriptive statistics of spreads are provided in Table 1. The spreads' average increases with respect to maturity, from 48 basis points (3-month maturity) to 82 basis points (12-month maturity). This indicates a positive slope in the term structure of spreads, which is graphically illustrated by the Panel (b) in Fig. 1: except at the very beginning of the sample, the 12-month spread is always larger than the 3-month spread, up to around 50 basis points in late 2011.

Whereas the standard deviations of OIS and EURIBOR rates are respectively stable and decreasing with respect to maturity, the standard deviations of spreads slightly increase with maturity. Regarding higher-order moments, Table 1 indicates that spreads are more positively skewed than the rates in level. Also, contrary to the latter, spreads are heavy-tailed (positive excess kurtosis). The heavy-tail behavior is typically illustrated during Lehman's crisis on Fig. 1, where both 3-month and 12-month spreads peak to 207 and 239 basis points, respectively. These levels are about 4 standard-deviation far from their respective sample means.

A principal component analysis performed on the four EURIBOR-OIS spreads proves that the first two principal components captures most of spread fluctuations, explaining 99.7% of the whole variance of the spreads (96.4% and 3.3% for the first and second principal components respectively). This suggests that two factors are sufficient to capture the bulk of the spreads' fluctuations.

## 6. Results

### 6.1. Estimation results

Table 2 reports the estimates of the physical and risk-neutral parameters of  $x_{c,t}$  and  $x_{\ell,t}$ . Both processes are highly persistent, especially under the risk-neutral measure. Preliminary estimations suggested that the risk-neutral auto-regressive parameters were indistinguishable from one. In the final specification, to reduce

the computational burden of optimization, they are not estimated but are set to one. Nonstationarity under  $\mathbb{Q}$  is a relatively common feature of intensities in the empirical credit-risk literature.<sup>20</sup>

In a preliminary estimation, we found that the Granger causality from credit to liquidity was not significantly different from zero. It has therefore been imposed to zero. By contrast, we find that the liquidity factor significantly Granger causes the credit factor, which implies some liquidity feedback to credit risk. Table 2 also reports the market prices of risk parameters. These parameters are deduced from physical and risk-neutral parameters (see Eq. (4)).

The remaining parameter estimates are gathered in Table 3. Both intensities loadings are significantly different from zero, and we observe that  $(1 - \theta_\ell)\Lambda_\ell > (1 - \theta_c)\Lambda_c$  (last row of Table 3). This means that liquidity shocks are the main drivers of the short-term fluctuations of the total intensity since the innovations ( $\epsilon_{c,t}$ ,  $\epsilon_{\ell,t}$ ) of factors ( $x_{c,t}$ ,  $x_{\ell,t}$ ) are of unit variance.

Fig. 2 presents the filtered time-series of the squared factors  $x_{c,t}^2$  and  $x_{\ell,t}^2$ , together with  $\pm 2$ -standard-deviation bands. By construction, these factors possess roughly the same patterns as the credit and liquidity proxies.

The variance estimate  $\hat{\sigma}_\eta^2$  associated with the error terms in the spread equation is 0.006, which translates into an average pricing error of 8 basis points for all maturities. This implies that the model captures 95% of the variation of the spreads.

Besides, the estimated model proves to be able to capture part of the heteroskedasticity in spreads. Indeed, unreported results suggest that the model-implied conditional volatility of spreads exhibits a 80% correlation with volatility series extracted from ARMA(1,0)-GARCH(1,1) models on the spreads. Note that this is due to our quadratic framework, a standard Gaussian model being unable to generate time-variation in conditional yields' variance. Fig. 3 presents the empirical and model-implied distributions of the spreads with respect to maturity. The model-implied distributions are obtained by simulations. For all maturities, the model-implied marginal distribution is hump-shaped. Its mode is close to that of the empirical distribution.

### 6.2. Decomposing EURIBOR-OIS spreads

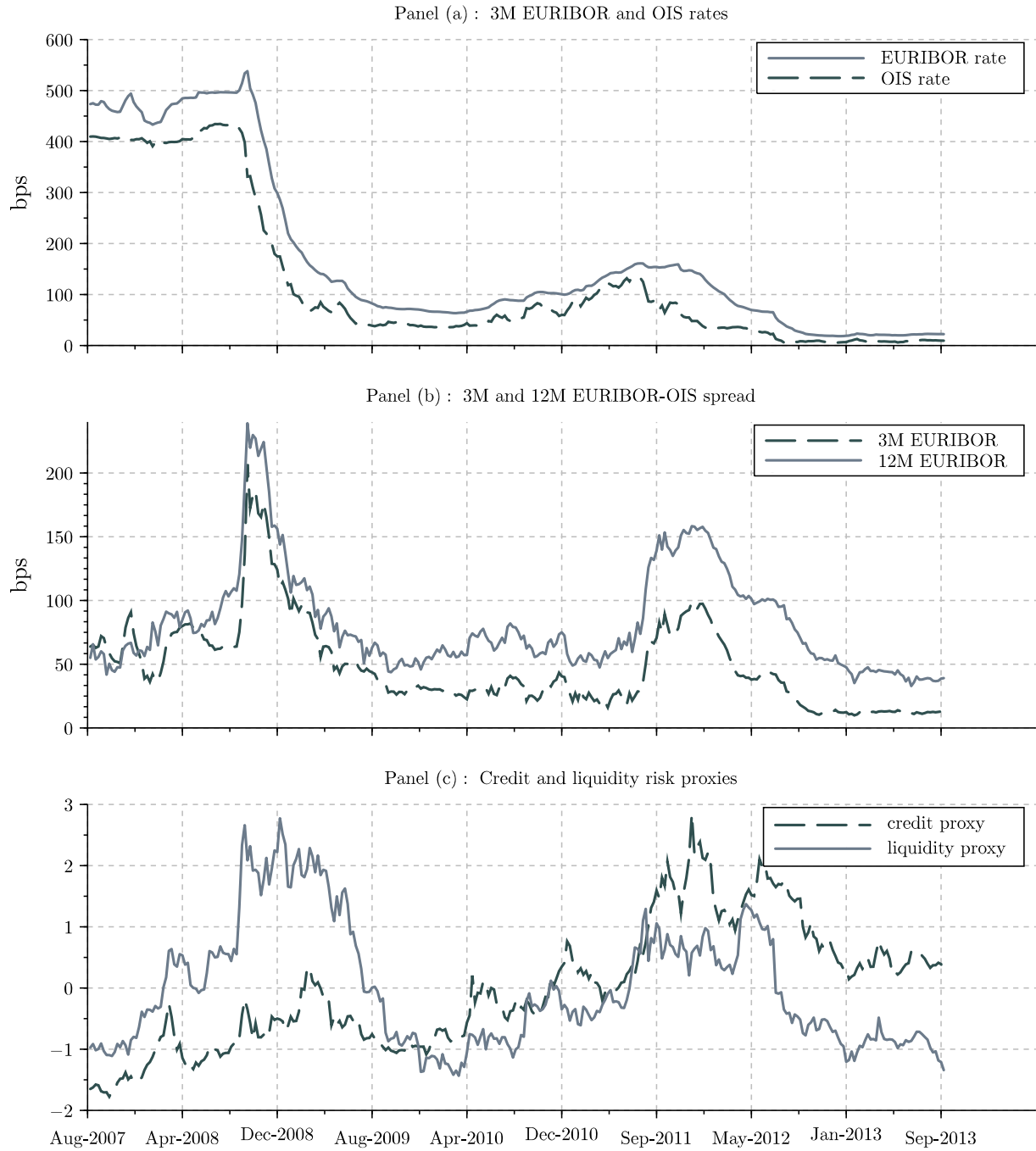
In this section, we present the model-implied decomposition of EURIBOR-OIS spreads for all maturities. We can perform our spread decomposition along two dimensions: credit vs. liquidity on the one hand (as in e.g. Filipovic and Trolle (2013)) and risk premia (or term premia) vs. parts of the spreads predicted by the expectation hypothesis on the other hand (as in e.g. Pan and Singleton (2008)). In the following, we refer to the latter component of the spreads by "expected component".

#### 6.2.1. The decomposition method

First, we decompose observed spreads into credit and liquidity components. Recall from Eq. (11) that the spread of maturity  $h$  involves the conditional  $\mathbb{Q}$ -expectations of the next  $h$  credit and liquidity intensities. To obtain the effects on credit only (say), we simply put  $\Lambda_\ell = 0$  and recompute the counterfactual spread implied by this restriction. Formally, if we denote by  $S_c(t, h)$  and  $S_\ell(t, h)$  the respective credit and liquidity components of the maturity- $h$  modeled spread, we have:

<sup>20</sup> In one-factor credit-risk models, Longstaff et al. (2011) (Table 4), Pan and Singleton (2008) (Table III) and Doshi et al. (2013) (Table 3) even find, for some of the entities they consider, autoregressive parameters slightly above 1 (or, equivalently, negative speeds of adjustment in CIR processes).

<sup>19</sup> E.g. Joyce et al. (2011) and BIS (2013) or Duffie and Stein (2014).



**Fig. 1.** Level of 3M rates and spreads. *Notes:* Panel (a): plot of the 3M EURIBOR (light gray) and 3M OIS (dashed dark gray). Panel (b): plot of the 3M (dashed dark gray) and 12M (lighter gray) EURIBOR-OIS spreads. Units are in basis points. Panel (c): credit (dashed dark gray) and liquidity (light gray) proxies. These proxies are demeaned and standardized. Time ranges from August 31 2007 to September 13 2013.

$$S_c(t, h) = -\frac{1}{h} \log \left( \mathbb{E}_t^Q \left[ \exp \left\{ \sum_{i=1}^h - (1 - \theta_c) \lambda_c(x_{c,t+i}) \right\} \right] \right) \triangleq \theta_{0,h}^{(c)} + \theta_{1,h}^{(c)} x_{c,t} + \theta_{2,h}^{(c)} x_{c,t}^2 \quad (17)$$

$$S_\ell(t, h) = -\frac{1}{h} \log \left( \mathbb{E}_t^Q \left[ \exp \left\{ \sum_{i=1}^h - (1 - \theta_\ell) \lambda_\ell(x_{\ell,t+i}) \right\} \right] \right) \triangleq \theta_{0,h}^{(\ell)} + \theta_{1,h}^{(\ell)} x_{\ell,t} + \theta_{2,h}^{(\ell)} x_{\ell,t}^2, \quad (18)$$

where  $\theta_{0,h}^{(c)}$ ,  $\theta_{1,h}^{(c)}$ ,  $\theta_{2,h}^{(c)}$ , and  $\theta_{0,h}^{(\ell)}$ ,  $\theta_{1,h}^{(\ell)}$ , and  $\theta_{2,h}^{(\ell)}$  are the elements of  $\theta_{0,h}$ ,  $\theta_{1,h}$  and  $\theta_{2,h}$  that correspond to credit and liquidity risks,

respectively. Since both credit and liquidity factors are independent under the risk-neutral measure, we have an exact decomposition of the modeled spread and, for the observed spread, we get:

$$S(t, h) = S_c(t, h) + S_\ell(t, h) + \sigma_\eta \eta_{t,h}, \quad (19)$$

where  $\sigma_\eta \eta_{t,h}$  exactly matches the measurement errors included in the measurement equations (Eq. (16)). Given their relative small size and following the usual approach, we neglect these measurement errors in the analysis and consider only the decomposition of the modeled spread  $S(t, h) - \sigma_\eta \eta_{t,h}$ .

Each of the credit and liquidity components of the spreads can further be decomposed into a risk premia component and an



**Table 1**

Descriptive statistics of EURIBOR and OIS rates.

	Min	Max	Amplitude	Mean	Std	Skewness	Excess kurtosis
	<i>bps</i>						
EURIBOR 3M	18.4	538.1	519.7	172.0	165.1	1.12	−0.35
EURIBOR 6M	29.4	543.1	513.7	190.8	158.9	1.10	−0.32
EURIBOR 9M	38.8	546.3	507.5	202.3	155.1	1.07	−0.31
EURIBOR 12M	47.4	549.3	501.9	213.0	152.1	1.06	−0.29
OIS 3M	4.5	434.6	430.1	123.6	145.7	1.29	−0.03
OIS 6M	2.35	442.9	440.5	125.1	144.9	1.30	0.03
OIS 9M	−0.5	453.5	454.0	127.9	143.7	1.29	0.05
OIS 12M	−1.1	465.3	466.4	131.2	142.2	1.27	0.07
Spread 3M	9.9	206.9	197.0	48.4	34.9	1.61	3.37
Spread 6M	19.6	222.5	202.9	65.7	36.5	1.62	3.44
Spread 9M	26.8	227.9	201.1	74.4	38.1	1.63	3.05
Spread 12M	32.9	239.0	206.1	81.8	40.0	1.54	2.38

Note: Those figures are computed with weekly data ranging from August, 31 2007 to September, 13 2013.

**Table 2**

Factor parameter estimates.

	$\mathbb{P}$ -dynamics			$\mathbb{Q}$ -dynamics			Market prices of risk		
	$\mu$	$X_{c,t-1}$	$X_{\ell,t-1}$	$\mu^*$	$X_{c,t-1}$	$X_{\ell,t-1}$	$\Gamma_0$	$X_{c,t-1}$	$X_{\ell,t-1}$
$X_{c,t}$	0	0.9867 (0.0075)	0.0054 (0.0019)	−0.6087 (0.1081)	1	0	−0.6087 (0.1081)	0.0133 (0.0018)	−0.0054 (0.0019)
$X_{\ell,t}$	0.1130 (0.0404)	0	0.9946 (0.0023)	0.0815 (0.0146)	0	1	−0.0315 (0.0400)	0	0.0054 (0.0023)

Notes: Standard errors are in parentheses. The ‘−’ sign indicates that the value is calibrated (see Section 6.1), thus the parameter is not estimated and its estimator has therefore no standard deviation.

expected component. Indeed, our estimation strategy provides both the physical and the risk-neutral dynamics of the factors. This knowledge enables us to extract risk premia (or term premia) from observed spreads. Risk premia are defined as the differentials between observed (or model-implied) spreads and the ones that would prevail if investors were risk-neutral. In the latter case, which corresponds to the expectation hypothesis, spreads would be those obtained by using the physical dynamics to compute the expectation term in Eq. (11). Using the estimated  $\mathbb{P}$ -dynamics parameters and the fact that the total intensity  $\lambda_t$  is the same function of  $X_t$  under both measures (see Section 3.2), we calculate a new set of factor loadings under the expectation hypothesis. Moreover, to perform the credit/liquidity decomposition of this expected component, we use the same formulas as in System (17) and (18), replacing the  $\mathbb{Q}$  parameters by the  $\mathbb{P}$  parameters. We denote these components by  $S_c^{\mathbb{P}}(t, h)$  and  $S_{\ell}^{\mathbb{P}}(t, h)$ .<sup>21</sup>

### 6.2.2. Decomposition results

The decompositions of the 6- and 12-month maturity spreads are represented in Fig. 4 and the descriptive statistics of the decompositions are provided on Table 4. The liquidity component accounts for most of the spread averages over the sample period, representing more than 60% of spreads' levels for all maturities. The average share of spreads that is associated with credit risks is comprised between 15% and 33% and increases with respect to maturity. Panels a.1 and a.2 of Fig. 4 illustrate that the liquidity factor accounts for much of the high-frequency variations in the spreads, in particular during the distress period of late 2008 after

<sup>21</sup> Specifically, to obtain  $S_c^{\mathbb{P}}(t, h)$  and  $S_{\ell}^{\mathbb{P}}(t, h)$ , we respectively impose  $\Lambda_c = 0$  or  $\Lambda_c = 0$  and then compute associated sets of factor loadings  $\theta_{i,h}$  ( $i \in \{0, 1, 2\}$ ) by using the recursive formulas of System (14). Note that, under the physical measure,  $X_{c,t}$  and  $X_{\ell,t}$  are not independent. Therefore, since the spreads are nonlinear functions of these variables,  $S_c^{\mathbb{P}}(t, h)$  and  $S_{\ell}^{\mathbb{P}}(t, h)$  do not sum to  $S^{\mathbb{P}}(t, h)$ . The discrepancy is however extremely small.

**Table 3**

Parameter estimates of measurement equations.

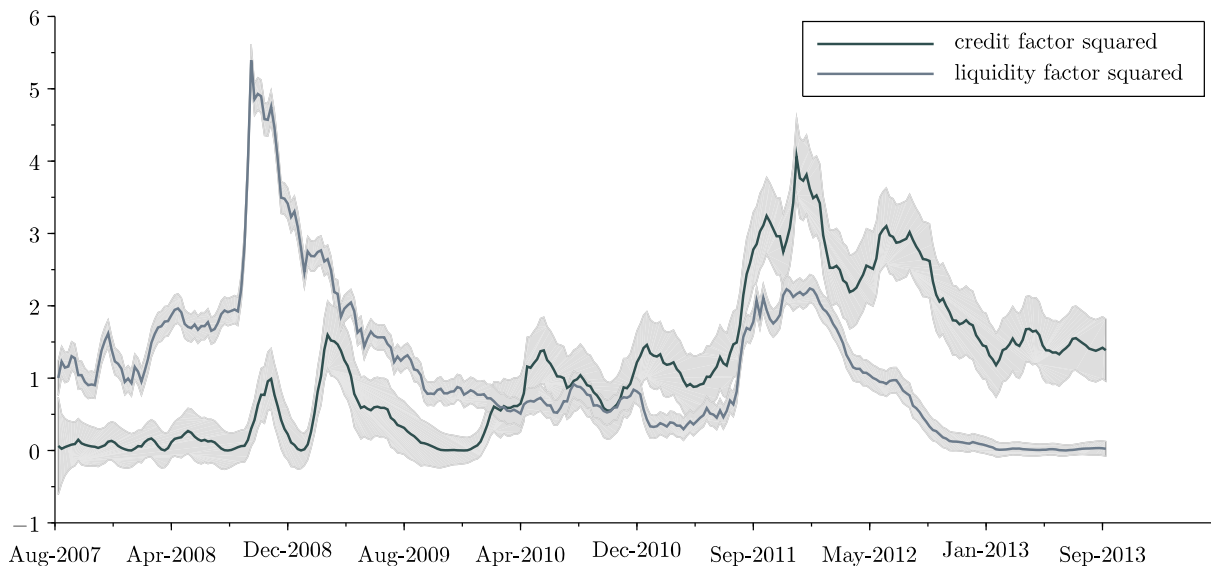
Equation	Parameter	Estimate	Parameter	Estimate
$P_{c,t}$	$\pi_{c,0}$	−0.9608 (0.0455)	$\pi_c$	0.0092 (0.0024)
$P_{\ell,t}$	$\pi_{\ell,0}$	−0.8927 (0.0234)	$\pi_{\ell}$	0.0031 (0.0002)
Noise	$\sigma_{\eta_c}^2 = \sigma_{\eta_{\ell}}^2$	0.1	$\sigma_{\eta}^2$	0.0057 (0.0002)
$\lambda_t$	$(1 - \theta_c)\Lambda_c \cdot 10^7$	1.1818 (0.2206)	$(1 - \theta_{\ell})\Lambda_{\ell} \cdot 10^7$	3.1810 (0.2037)

Notes: Standard errors are in parentheses. The ‘−’ sign indicates that the value is calibrated.

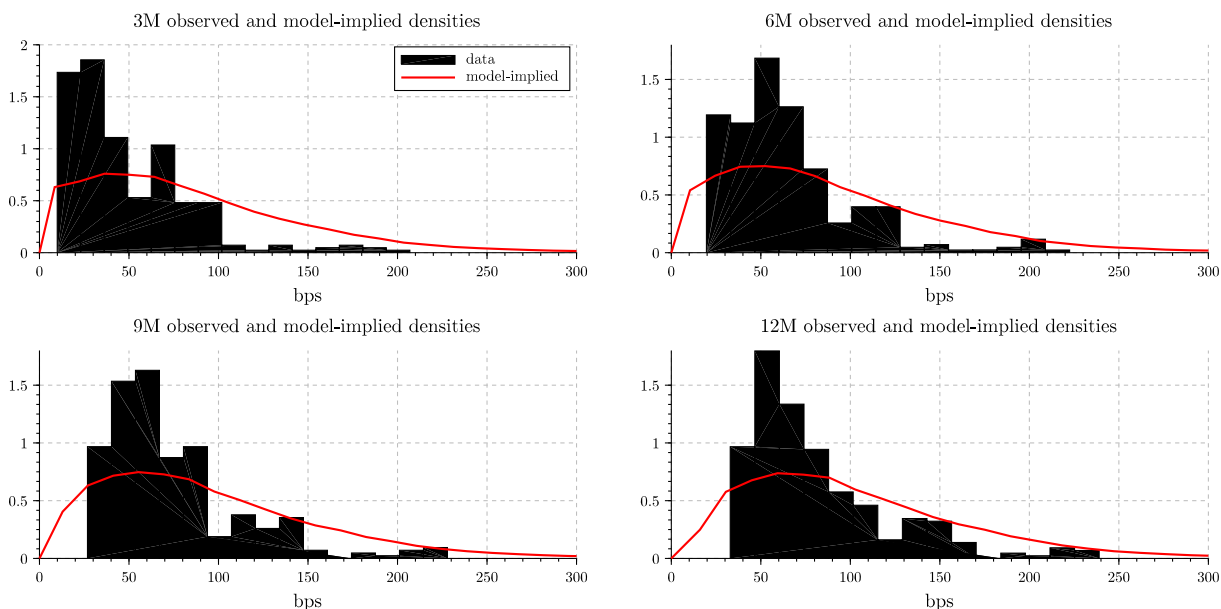
the Lehman collapse and in late 2011, a turbulent period in the European sovereign markets, no matter the maturity.

Panels b.1 and b.2 of Fig. 4 displays the decomposition of the observed spreads into the risk premium and the expected components. The two components are positively correlated. In times of distress (Lehman collapse or the European debt crisis), the level of risk premia increases for all maturities. Together with Table 4, Panels b.1–b.2 show that the share of the spreads explained by risk premia is increasing with the maturity: while the credit and liquidity risk premia account respectively for 9% and 7% of the 3-month average spread, they represent 31% and 17% of the 12-month spread (see third and fourth columns of Table 4).

Fig. 5 confirms the previous statements by presenting decompositions of the term structure of EURIBOR-OIS spreads at different dates. In particular, the second (respectively third) row of plots shows the credit/liquidity decompositions of the expected components of the spreads (resp. of the risk premia). Under the expectation hypothesis the liquidity risk term structure is flat whereas the credit component is smaller and slightly downward sloping with maturity (second row of plots). By contrast, the bottom plots of Fig. 5 show that the term structures of both credit and liquidity risk



**Fig. 2.** Filtered squared credit and liquidity factors. *Notes:* This figure shows the estimates of  $x_{c,t}^2$  and of  $x_{l,t}^2$ . Time ranges from August 31 2007 to September 13 2013. The gray shaded areas delineate the  $\pm 2$ -standard-deviation bands of the latent factors. This confidence band reflects the uncertainty associated to the filtering technique.



**Fig. 3.** Comparison between empirical and model-implied spread densities. *Notes:* Black bars represent the histograms based on the estimation data. Model-implied densities (solid lines) are obtained by simulating the estimated model on 1,000,000 periods.

premia are upward sloping and that the credit component is larger than the liquidity one. These features are not specific to the four chosen dates.

In the next section, we exploit the time series and the term structure of the spreads components to analyze the effectiveness of unconventional monetary policies in the Eurozone.

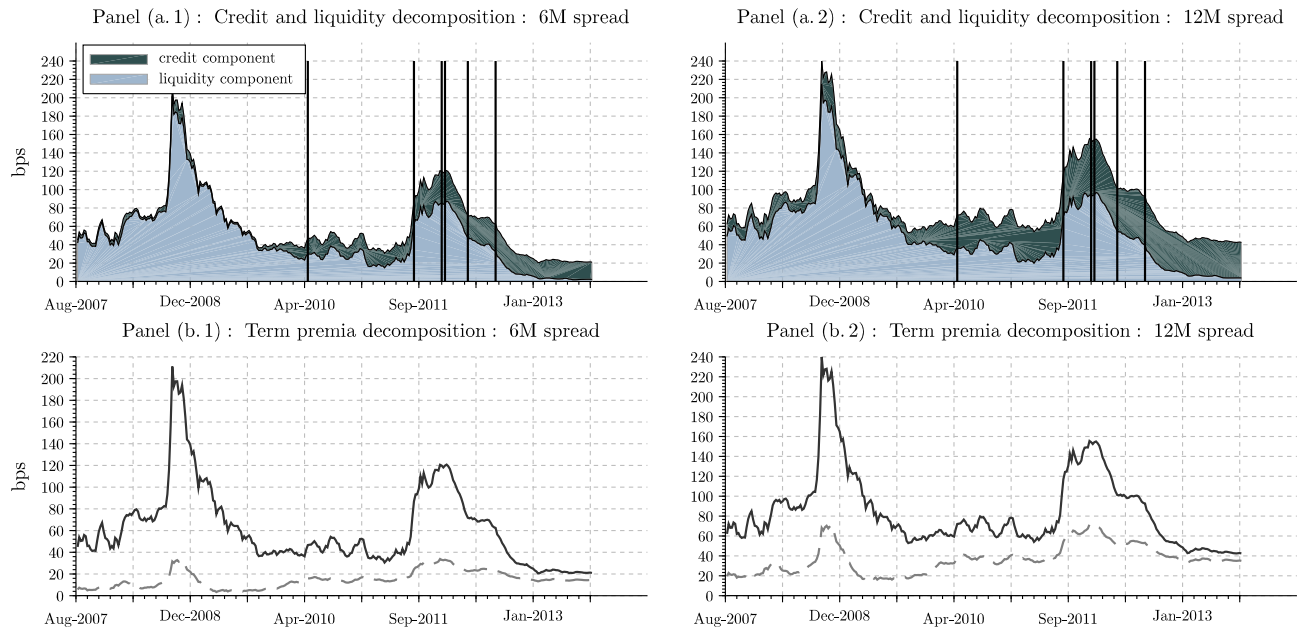
### 6.3. The impact of unconventional monetary policy on interbank risk

The main programs of unconventional monetary policies in the Eurozone can be broadly separated into three periods. The *Securities Market Program* (SMP) consisted in sterilized bond-buying in the secondary market. It was designed to “ensure depth and liquidity in [...] market segments that are dysfunctional” and

was implemented in May 2010 and August 2011.<sup>22</sup> Later, on the December 8 2011, the ECB disclosed the design of *Very Long Term Refinancing Operations* (VLTROs), whereby 3-year maturity open market operations were proposed in the form of reverse repo.<sup>23</sup> Two allotments were granted on December 21 2011 and on the February 29 2012, of respectively EUR 489bn and EUR 530bn to 523 and 800 banks. During August 2012, Mario Draghi announced the setting of *Outright Monetary Transactions* (OMTs) in his London speech. Conditionally on fiscal adjustments or precautionary programs enforcement by candidate countries, the ECB is ready to trade

<sup>22</sup> Press release available at <http://www.ecb.europa.eu/press/pr/date/2010/html/pr100510.en.html> (dated 10 May 2010).

<sup>23</sup> Press release available at [http://www.ecb.europa.eu/press/pr/date/2011/html/pr111208\\_1.en.html](http://www.ecb.europa.eu/press/pr/date/2011/html/pr111208_1.en.html) (dated December 8 2011).



**Fig. 4.** 6M EURIBOR-OIS spreads decomposition. *Notes:* Date ranges from August 31 2007 to September 13 2013. Units are in basis points. Panels (a) represent the stacked components of the spread: light gray component is the liquidity component and the dark gray corresponds to the credit component. Panels (b) represent the modeled spread (black) and its term premia (gray dashed). The black vertical axes stand from left to right for: SMP program announcements (first two axis), VLTRO announcement and allotments (next three axis), and Mario Draghi's London speech (last axis).

**Table 4**

Descriptive statistics of EURIBOR-OIS components.

		Total spread		Risk premium	
		Credit	Liquidity	Credit	Liquidity
Average level (in bps)	3M	7.88 [7.45–9.76]	44.57 [42.96–45.85]	4.51 [3.51–5.26]	3.61 [1.12–4.08]
	6M	12.84 [11.63–15.42]	48.57 [45.75–50.40]	10.10 [8.19–11.84]	7.16 [2.25–8.14]
	9M	19.54 [17.24–23.73]	52.71 [48.43–55.64]	17.21 [14.10–20.56]	10.90 [3.52–12.45]
	12M	27.97 [23.35–35.14]	57.00 [51.21–60.43]	25.90 [20.58–31.78]	14.81 [4.93–17.01]
Average (% of spread avg)	3M	15 [14–18]	85 [82–86]	9 [7–10]	7 [2–8]
	6M	21 [19–25]	79 [75–81]	16 [13–19]	12 [4–13]
	9M	27 [24–33]	73 [67–77]	24 [19–29]	15 [5–17]
	12M	33 [28–41]	67 [59–73]	31 [25–37]	17 [6–20]

*Notes:* The modeled spreads are decomposed into four components, along two dimensions: credit vs. liquidity and expected part vs. risk premium. The risk premia are the parts of the spreads that would not exist if investors were risk-neutral. The table shows for instance that for the 9-month maturity, 39% of the EURIBOR-OIS spread correspond to risk premia, 60% of which ( $= 24/(24 + 15)$ ) being accounted for by aversion to credit risk. Brackets contain the 95% confidence intervals. These intervals are calculated with 1000 simulations of the parameters using their asymptotic distribution.

in secondary sovereign bond markets with “no ex ante quantitative limits”.<sup>24</sup> Whereas the OMT framework has been announced it has not been implemented yet.<sup>25</sup>

Interestingly, the EURIBOR-OIS spreads have decreased continuously since the VLTRO announcement in December 2011. This drop has led many commentators (and central bankers) to claim that the ECB unconventional refinancing operations were successful in alleviating interbank market tensions. In particular, according to ECB officials, the non-standard VLTRO operations addressed “only the liquidity side of the [interbank market] problem” (see Draghi (2012)’s interview with the Wall Street Journal, published on February 24 2012). Our results support this view since the liquidity component of the spreads has slowly decreased

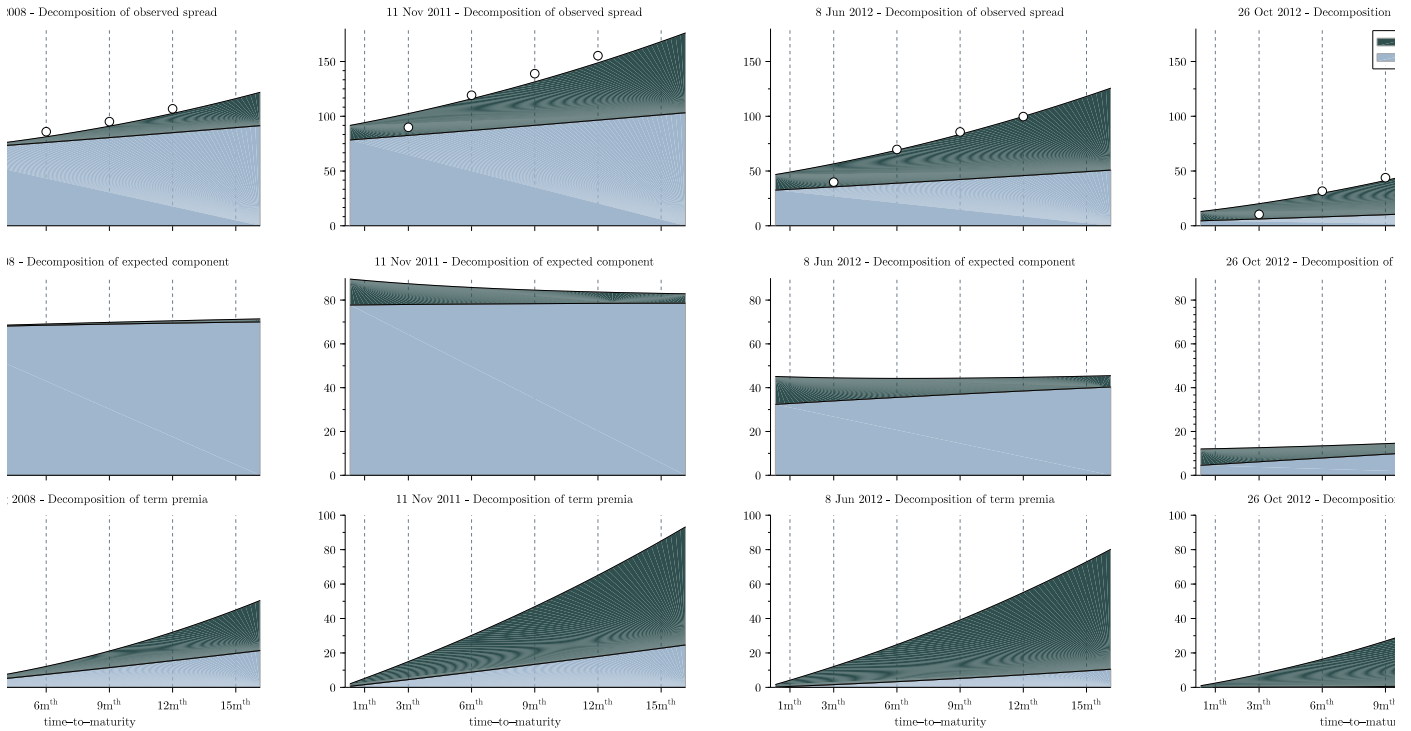
since the VLTRO announcement date (see Fig. 4, first row). A further positive liquidity-related effect can also be attributed to the OMT announcement through liquidity (see the last vertical bar in the charts).<sup>26</sup>

The same pattern can be observed in Fig. 5. After the SMP and before the VLTRO announcement (second column of plots), liquidity risk still accounts for most of the term structure of interbank spreads, with between 80 and 100 basis points depending on the maturity. However, after the VLTRO allotments, the liquidity component represents only 40–50 basis points across maturities (see third column of plots in Fig. 5) and further drops to around 5 basis points for all maturities after the OMT announcement (fourth column). In comparison, looking at Figs. 4 and 5, it appears that those policy measures only had a small impact on the credit components of the spreads: between November 2011 and October 2012, the range of this component goes from [20 bps, 60 bps] to [10 bps,

<sup>24</sup> Details about OMTs are available at [http://www.ecb.europa.eu/press/pr/date/2012/html/pr120906\\_1.en.html](http://www.ecb.europa.eu/press/pr/date/2012/html/pr120906_1.en.html) (dated September 6 2012).

<sup>25</sup> Our estimation period does not cover the announcement and implementation of the Eurosystem’s expanded asset purchase program announced on January 22 2015, which consists of combined monthly purchases of EUR 60bn in public and private sector securities.

<sup>26</sup> To some extent, these results are consistent with some of the findings reported by Acharya and Steffen (2015), who find evidence that VLTROs contributed to reduce funding risk pressure for peripheral banks in the euro area.



**Fig. 5.** Decompositions of the EURIBOR-OIS term structure. *Notes:* The first to fourth columns correspond respectively to the following dates: August 29 2008; November 11 2011; January 8 2012; and October 26 2012. The first row represents the stacked components of the term structure: the light gray component is the liquidity component and the dark one is the credit component. The white dots are the observed spreads. The second row presents the same components under the expectation hypothesis, i.e. if the agents were risk-neutral. The graphs at the bottom display the decomposition of the risk premia, that are the differences between the total modeled spreads and the expected components of the spreads. The sum of the last two rows of charts results in the first row.

50 bps]. Even though there is a small drop in the credit component, the evidence of the effectiveness of unconventional monetary policies on credit risk is far thinner than on liquidity risk. Table 5 confirms the previous statements. It reports the values of the credit and liquidity components of the spreads one week before the unconventional monetary policy events and one month after, along with 95% confidence intervals. Looking at the confidence intervals, we see that while VLTRO measures were accompanied by statistically significant decreases in the liquidity component only, the effect on the credit component is not significant. Similarly, the OMT only had a significant impact on the liquidity component.

Turning to the second and third rows of Fig. 5, it appears that the VLTROs and the OMT announcement were followed by decreases in the expected components of the spreads and, to a lesser extent, in the risk premia.

The fact that the VLTROs were followed by a reduction in the liquidity components of the money-market spreads is consistent with the objectives formulated by the ECB. The ECB monthly bulletin of October 2010 states (about previous Long Term Refinancing Operations of shorter – 6-month – maturities): “The aim of these operations was to improve banks’ liquidity position, further reduce money market spreads and contribute to keeping term money market interest rates at a low level” (ECB, 2010). Arguably, the longer maturities of the VLTROs enabled banks to attenuate the mismatch between the investment side and the funding side of their balance sheet, thereby reducing their funding risk for the years covered by the VLTROs.

The channels through which the OMT announcement has had an impact on interbank money-market spreads is less obvious because the primary targets of this measure are sovereign bonds. Several possible mechanisms could however be at play. First, the OMT has been motivated by perceived re-denomination risk associated

with the breakup of the euro area (Coeuré, 2013; Krishnamurthy et al., 2014). In mid 2012 the euro area was on the verge of a big financial instability crisis. The OMT announcement has been effective at alleviating these tail risks, preserving the stability of the euro-area financial system as a whole and, in particular, of the interbank market. Second, an increase in the price of sovereign bonds translates into gains on the banks’ government-bond portfolios, thereby improving banks’ solvency. A third channel pertains to the value of the collateral used by banks for refinancing operations. Government bonds are among the most important securities used by banks as collateral. Hence, banks’ funding conditions are expected to be enhanced when the value of these bonds increases. The last two channels relate to the so-called bank-sovereign nexus, that refers to the close financial connection between domestic banks and their respective sovereigns (see e.g. Acharya et al., 2014; Battistini et al., 2014; Acharya and Steffen, 2015; Farhi and Tirole, 2015 or Fratzscher and Rieth, 2015).

#### 6.4. Model-implied probabilities of default

Following Doshi et al. (2013), we present an additional by-product of our framework: the model-implied probabilities of default (PDs). In our model, the panel of banks is homogeneous and the probabilities of default are not bank-dependent. Formally, for any bank  $i$ , we have:

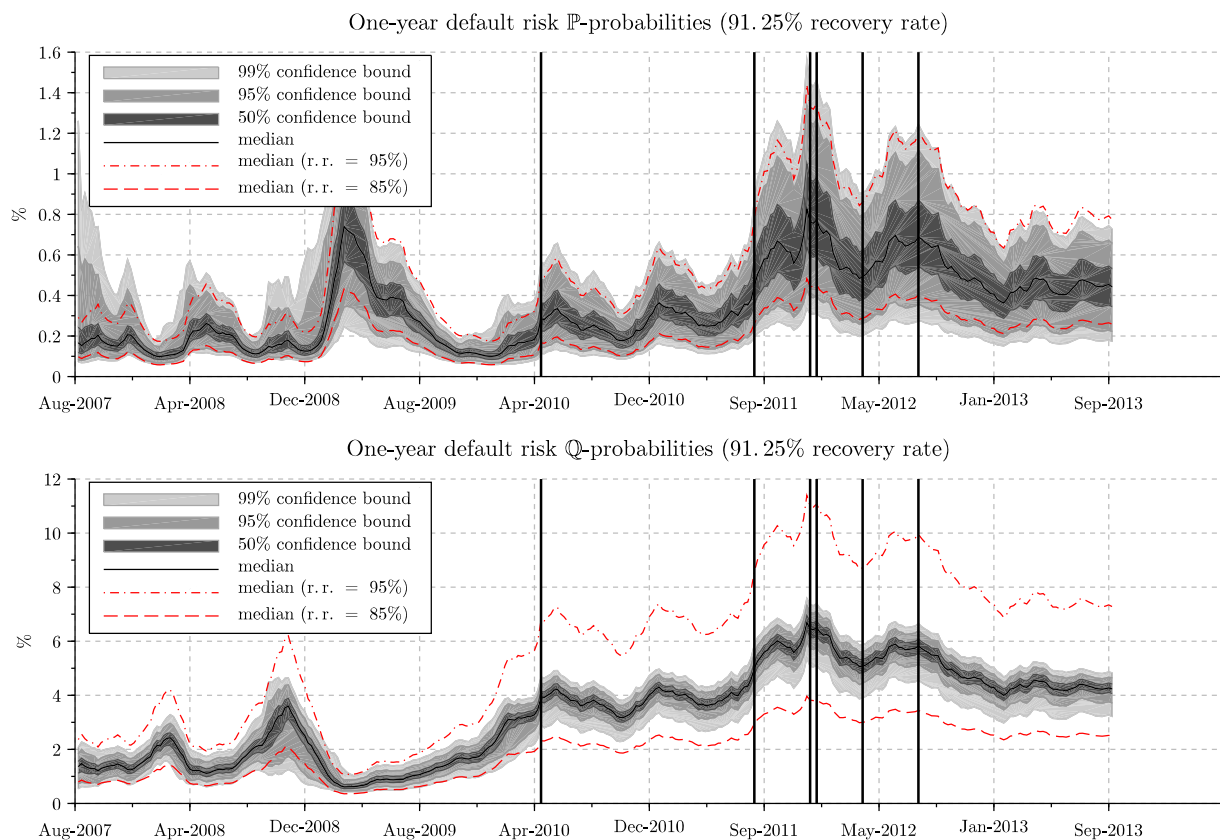
$$\begin{aligned} \mathbb{P}(d_{t+h}^{(i)} = 1 | d_t^{(i)} = 0, \underline{w}_t) &= 1 - \mathbb{P}(d_{t+1}^{(i)} = 0, \dots, d_{t+h}^{(i)} = 0 | d_t^{(i)} = 0, \underline{w}_t) \\ &= 1 - \mathbb{E}_t^{\mathbb{P}}(\exp(-\lambda_c(x_{c,t+1}) - \dots - \lambda_c(x_{c,t+h}))) \\ &= 1 - \mathbb{E}_t^{\mathbb{P}}\left(\exp\left(-\Lambda_c[x_{c,t+1}^2 + \dots + x_{c,t+h}^2]\right)\right). \end{aligned} \quad (20)$$



**Table 5**  
Unconventional monetary policy effects.

	Maturity	Liquidity		Credit	
		Week – 1	Week + 4	Week – 1	Week + 4
SMP1	3M	25.5 [23.9–28.1]	28.6 [27.0–30.9]	7.7 [6.5–8.9]	11.2 [10.1–12.5]
	6M	28.5 [26.3–32.0]	31.7 [29.7–34.9]	13.9 [11.6–15.3]	18.5 [16.3–19.9]
	9M	31.6 [28.9–36.0]	35.0 [32.4–39.2]	21.9 [18.0–24.3]	27.5 [23.5–30.1]
	12M	34.9 [31.6–40.1]	38.3 [35.3–43.8]	31.7 [25.6–35.3]	38.2 [32.3–42.0]
SMP2	3M	47.1 [44.9–49.3]	77.2 [74.6–80.1]	13.4 [12.2–14.7]	19.4 [17.6–21.5]
	6M	50.8 [48.2–53.7]	81.6 [78.7–85.7]	21.0 [19.1–22.9]	28.4 [25.5–30.8]
	9M	54.7 [51.5–58.7]	86.2 [82.7–91.5]	30.4 [26.9–33.2]	39.2 [34.2–42.5]
	12M	58.8 [55.0–63.9]	90.9 [86.7–97.7]	41.6 [35.7–45.9]	51.7 [44.1–56.6]
LTROS	3M	80.4 [77.4–83.6]	41.0 [38.9–43.5]	16.0 [14.4–17.5]	17.9 [16.4–19.6]
	6M	85.0 [81.7–89.0]	44.5 [42.3–47.3]	24.3 [21.6–26.2]	26.6 [24.2–28.6]
	9M	89.6 [85.7–94.9]	48.2 [45.6–51.9]	34.3 [29.5–37.3]	37.0 [32.9–39.9]
	12M	94.4 [89.6–100.9]	52.0 [48.9–56.6]	46.0 [39.0–50.7]	49.1 [42.5–53.6]
OMT	3M	30.9 [28.6–33.8]	18.6 [16.4–21.4]	19.9 [18.1–21.9]	18.5 [16.8–20.2]
	6M	34.1 [31.7–37.6]	21.3 [19.0–24.7]	28.8 [26.0–31.1]	27.2 [24.3–29.3]
	9M	37.3 [34.6–42.1]	24.0 [21.5–28.5]	39.9 [35.0–43.1]	37.9 [32.9–40.9]
	12M	40.9 [37.4–46.4]	27.0 [23.9–32.3]	52.3 [44.5–57.2]	50.0 [42.4–54.8]

Notes: Units are in basis points. ‘Week – 1’ and ‘week + 4’ respectively denote one week before the event and one month after. The 3 VLTRO events have been grouped in one set. For the VLTRO row, the ‘week – 1’ columns correspond to one week before the announcement, i.e. December, 2nd 2011 and the ‘week + 4’ columns correspond to one month after the second allotment, i.e. April, 27th 2012. Brackets contain the 95% confidence intervals. These intervals are based on 1000 simulations of the parameters using their asymptotic distribution.



**Fig. 6.** Default probabilities of banks under the physical and pricing measures. Notes: Time ranges from August 31 2007 to September 13 2013. The upper plot shows the model-implied one-year probability of default of a bank of the panel (banks are assumed to share the same characteristics). This probability is derived from Eq. (20). The lower chart shows the risk-neutral probability of default, which is obtained by replacing the physical dynamics parameters by the risk-neutral ones in Eq. (20). The shaded areas are 50–99% confidence bands. The dashed lines correspond to median default probabilities obtained with alternative recovery rate assumptions: 85% (dashed line) and 95% (dashed-dotted line), instead of 91.25% in the baseline case. The black vertical axes stand from left to right for: SMP program announcements (first two axes), VLTRO announcement and allotments (next three axes), and Mario Draghi's London speech (last axis).

The last term of the previous equation is a multi-horizon Laplace transform of  $x_{c,t}^2$ , which can be computed analytically by means of recursive formulas of the same kind as those presented in System

(14) (replacing  $\mu^*$  and  $\Phi^*$  by  $\mu$  and  $\Phi$ , and redefining  $\Lambda$  as the matrix with  $(\Lambda_c, 0)$  on its diagonal.). The computation requires an estimate of the default recovery rate  $\theta_c$ . To the best of our knowledge, the

existing literature presents no euro-area figure that can serve as a basis for the calibration of such a parameter. Hence, we set it to 91.25%, which is the recovery rate on unsecured deposits on U.S. banks with at least \$5bn assets (see Kuritzkes et al., 2005).<sup>27</sup>

Fig. 6 displays the physical (top plot) and risk-neutral (bottom plot) one-year PDs resulting from this computation. Confidence bands are added on the plots. These bands reflect the uncertainty regarding the model parameterization. These confidence bands are obtained by drawing 1,000 sets of model parameters from their asymptotic joint distribution. For each set of parameters, we use the quadratic Kalman filter to estimate time series of  $(x_{c,t}, x_{\ell,t})$  and compute the implied PDs. For each date, the confidence intervals are based on the percentiles of the 1000 simulated PDs. It appears that the risk neutral probabilities are very far from their physical counterparts, the deviations being accounted for by sizeable credit-risk premia. These findings are in line with those of a large body of empirical studies highlighting the substantial deviations existing between physical and risk-neutral PDs.<sup>28</sup> The existence of credit-risk premia constitutes one of the main explanations for the so-called credit-spread puzzle (see e.g. Amato and Remolona, 2003). This puzzle corresponds to the observation that observed credit spreads tend to be higher than average credit-losses (while they should be equal under some conditions, that notably include the risk-neutrality of investors).

The estimated median physical probabilities of default are roughly comprised between 0.2% and 0.6%. Though small, this order of magnitude is consistent with historical default data of investment-grade issuers. For instance, Moody's (2011) reports that, on average over the period 1983–2010, the one-year default rate of a A-rated financial institutions is of 0.1%. (The median rating of EURIBOR-panel banks is A across the three main rating agencies.) On a longer time-scale, Moody's (2013) indicates that the default rate of A-rated corporates has been of 0.10% (respectively 0.06%) over the period 1920–2013 (respectively 1970–2013).<sup>29</sup>

Fig. 6 illustrates that the VLTRO and OMT announcements were followed by decreases in the bank probabilities of default, corrected for risk premia or not. However, at the end of the sample, these probabilities remain higher than their mid-2007 value.

The dashed lines correspond to the median default probabilities obtained when setting the recovery rate to 85% and 95% (versus 91.25% in the baseline case). Unsurprisingly, the probabilities of default are particularly sensitive to the choice of the recovery rate.

## 7. Conclusion

We develop a no-arbitrage two-factor quadratic term structure model for the EURIBOR-OIS spreads across several maturities, from August 2007 to September 2013. To identify credit and liquidity components in the spreads, we exploit credit and liquidity proxies based on CDS prices, market liquidity and funding liquidity measures. Our decomposition handles potential interdependence between credit and liquidity risks and is consistent across maturities. We find that changes in the liquidity components generate most of the spreads' variance over the estimation period. The credit components are less volatile, but represents most of the spreads' levels in late 2012. Our decompositions shed new light on the effects of unconventional monetary policy of the ECB on the

interbank risk. We show that whereas the bond-purchase programs of 2010 and 2011 were not followed by decreases in any of the EURIBOR-OIS spread components, the VLTROs and the OMT announcements have had substantial impacts, mainly on the liquidity components. At the end of the sample, the latter are at their lowest since the beginning of the financial crisis.

## Appendix A

### A.1. Risk neutral distribution of $w_t$

The risk-neutral conditional distribution of  $w_t$  given  $w_{t-1}$  has a p.d.f., with respect to the same historical distribution, that is given by:

$$\exp \left[ \Gamma'_{t-1} (X_t - \mu - \Phi X_{t-1}) - \frac{1}{2} \Gamma'_{t-1} \Gamma_{t-1} + g(r_t) \right].$$

Since this p.d.f. factorizes into a function depending on process  $X_t$  and a function depending on process  $r_t$ , the independence between these processes is preserved under the  $\mathbb{Q}$ -measure. The derivation of the dynamics of  $X_t$  is standard (see e.g. Ang and Piazzesi, 2003).

Moreover, since  $(d'_t, \ell'_t)$  does not appear in the S.D.F., the conditional distribution of  $(d'_t, \ell'_t)$  given  $(X_t, d_{t-1}, \ell_{t-1})$  are the same in both worlds. Indeed, it is a consequence of the following lemma.

**Lemma A.1.** *If  $w_t = (w'_{1,t}, w'_{2,t})'$  and if the S.D.F.  $M_{t-1,t}$  is a function of  $w_{1,t}$  only, the conditional distribution of  $w_{2,t}$  given  $(w_{1,t}, w_{2,t-1})$  is the same in both worlds.*

**Proof.** We have:

$$\begin{aligned} f_t^{\mathbb{Q}}(w_{1,t} | w_{t-1}) f_t^{\mathbb{Q}}(w_{2,t} | w_{1,t}, w_{2,t-1}) \\ = M_{t-1,t}(w_{1,t}) \exp(-r_{t-1}(w_{t-1})) f_t^{\mathbb{P}}(w_{1,t} | w_{t-1}) f_t^{\mathbb{P}}(w_{2,t} | w_{1,t}, w_{2,t-1}). \end{aligned}$$

Integrating both sides w.r.t.  $w_{2,t}$  gives:

$$f_t^{\mathbb{Q}}(w_{1,t} | w_{t-1}) = M_{t-1,t}(w_{1,t}) \exp(-r_{t-1}(w_{t-1})) f_t^{\mathbb{P}}(w_{1,t} | w_{t-1})$$

and the result follows.  $\square$

### A.2. Derivation of Eq. (8)

$B(t, h)$  denotes the price of a maturity  $h$  interbank loan prevailing in the absence of credit and liquidity event at date  $t$ . For any  $\tau \in \{0, \dots, h\}$ , we have:

$$\begin{aligned} B(t + \tau, h - \tau) &= \exp(-r_{t+\tau}) \mathbb{E}^{\mathbb{Q}}[B(t + \tau + 1, h - \tau - 1) \\ &\quad \times (1 - d_{t+\tau+1}^{(j)} + \theta_c d_{t+\tau+1}^{(j)})(1 - \ell_{t+\tau+1}^{(i)} + \theta_\ell \ell_{t+\tau+1}^{(i)}) | w_{t+\tau}]. \end{aligned}$$

Suppose that  $B(t + \tau + 1, h - \tau - 1)$  is function of  $(X_{t+\tau+1}, r_{t+\tau+1})$  and not of  $(d_{t+\tau+1}^{(j)}, \ell_{t+\tau+1}^{(i)})$  (Assumption A), we get, using the law of iterated expectations:

$$\begin{aligned} B(t + \tau, h - \tau) &= \exp(-r_{t+\tau}) \mathbb{E}^{\mathbb{Q}}[B(t + \tau + 1, h - \tau - 1) \\ &\quad \times \mathbb{E}^{\mathbb{Q}}\{(1 - d_{t+\tau+1}^{(j)} + \theta_c d_{t+\tau+1}^{(j)}) \\ &\quad \times (1 - \ell_{t+\tau+1}^{(i)} + \theta_\ell \ell_{t+\tau+1}^{(i)}) | w_{t+\tau}, X_{t+\tau+1}, r_{t+\tau+1}\} | w_{t+\tau}, \\ &\quad d_{t+\tau}^{(j)} = 0, \ell_{t+\tau}^{(i)} = 0]. \end{aligned}$$

Using the conditional independence of  $d_{t+\tau+1}^{(j)}$  and  $\ell_{t+\tau+1}^{(i)}$  and the approximation of Eq. (7), we get:

$$\begin{aligned} B(t + \tau, h - \tau) &= \exp(-r_{t+\tau}) \mathbb{E}^{\mathbb{Q}}[B(t + \tau + 1, h - \tau - 1) \\ &\quad \times \exp(-\lambda_{t+\tau+1}) | w_{t+\tau}]. \end{aligned} \quad (21)$$

<sup>27</sup> Christensen et al. (2014) note that such a recovery rate is high – compared to usual corporate-bond recovery rates – because an unsecured deposit is more senior in the liability structure of a bank than senior unsecured debt.

<sup>28</sup> See for instance Monfort and Renne (2014a) in the case of sovereign issuers and Elton et al. (2004) in the case of corporate issuers.

<sup>29</sup> For lower-rated investment-grade issuers (Baa using the Moody's rating system, which is equivalent to the BBB rating of S&P), the default rates for these two periods are respectively of 0.27% and of 0.17%.

Moreover, since  $(d_t^{(j)}, \ell_t^{(i)})$  does not Granger-cause  $(r_t, X_t)', B(t + \tau, h - \tau)$  is not function of  $(d_{t+\tau}^j, \ell_{t+\tau}^i)$ , and Assumption A is confirmed.

To use Eq. (21) recursively backward, starting at  $\tau = h - 1$ , we have to check Assumption A for  $B(t + h, 0)$ , which is trivial since  $B(t + h, 0) \equiv 1$ .

The recursive use of Eq. (21), for  $\tau = h - 1, h - 2, \dots, 0$  gives:

$$B(t + \tau, h - \tau) = \mathbb{E}^Q[\exp(-r_t - \lambda_{t+1} - \dots - r_{t+h-1} - \lambda_{t+h}) | \mathcal{W}_t],$$

which depends on  $(X_t, r_t)$  (and not on  $(d_t^{(j)}, \ell_t^{(i)})$ ) and gives Eq. (8).

### A.3. The OIS rate as a risk-free rate

An OIS is an interest-rate derivative that allows for exchanges between a fixed-interest-rate cash flow and a variable-rate cash flow. In the euro area, the floating leg of an OIS is indexed on the EONIA. At maturity, the payoff received by the fixed-rate payer is the difference between (a) the notional ( $W$ , say) inflated with the date- $t$  OIS (fixed) rate (i.e.  $W \exp\{hR_{t,h}^{OIS}\}$ ) and (b) the same notional capitalized with the realized short-term rates (i.e.  $W \exp\{r_t + \dots + r_{t+h-1}\}$ ). Note that the latter expression implicitly reckons that the OIS reference rate – the EONIA rate in the euro area – corresponds to the risk-free rate  $r_t$ , thereby assuming that lending on the overnight interbank market preserves the lending bank from (i) liquidity and (ii) credit risk. The rationale behind (i) and (ii) are the following:

- (i) By rolling its cash on the overnight market (at the EONIA rate), a bank is not exposed to the risk of having to liquidate longer-term investments upon the realization of the liquidity shock.
- (ii) While the EONIA is an unsecured-transaction rate, the extremely-short maturity of these transactions substantially reduces the credit-risk exposure of the lending bank. This point is corroborated by a comparison of EURIBOR-OIS spreads with spreads between Repo rates – where credit-risk effects are kept at a minimum through collateralization schemes – and OIS rates: over 2007–2013, the mean absolute value of the 3-month Repo-OIS spread is about 10 times smaller than the one of the EURIBOR-OIS spread of the same maturity (the former being of a few basis points).

At the inception date of the swap, there is no cash-flow exchange between the two counterparties, that is, the discounted values of the two legs are initially the same:

$$W \mathbb{E}_t^Q[\exp(hR_{t,h}^{OIS}) \exp\{-r_t - \dots - r_{t+h-1}\}] = W,$$

or:

$$R_{t,h}^{OIS} = -\frac{1}{h} \log \mathbb{E}_t^Q[\exp\{-r_t - \dots - r_{t+h-1}\}]. \quad (22)$$

### A.4. Solving for yield/spread loadings in a QTSM

#### A.4.1. Computing the Laplace transform of $Z_t = [X_t', \text{Vec}(X_t X_t')]'$

**Lemma A.2.** If  $\epsilon_{t+1}^* \sim \mathcal{N}(0, I)$ , we have

$$\mathbb{E}_t[\exp(\theta' \epsilon_{t+1}^* + \epsilon_{t+1}'^* V \epsilon_{t+1}^*)] = \frac{1}{|I - 2V|^{1/2}} \exp\left[\frac{1}{2} \theta' (I - 2V)^{-1} \theta\right]. \quad (23)$$

**Proof.** It can be shown that

$$\forall u \in \mathbb{R}^n, \int_{\mathbb{R}^n} \exp(-u' Qu + v' u) du = \frac{\pi^{n/2}}{|Q|^{1/2}} \exp\left(\frac{1}{4} v' Q^{-1} v\right). \quad (24)$$

Therefore, we have:

$$\begin{aligned} \mathbb{E}_t[\exp(\theta' \epsilon_{t+1}^* + \epsilon_{t+1}'^* V \epsilon_{t+1}^*)] &= \int_{\mathbb{R}^n} \exp(\theta' \epsilon + \epsilon' V \epsilon) \frac{1}{(2\pi)^{n/2}} \exp\left(-\frac{1}{2} \epsilon' I \epsilon\right) d\epsilon \\ &= \frac{1}{(2\pi)^{n/2}} \int_{\mathbb{R}^n} \exp\left[-\epsilon' \left(\frac{1}{2} I - V\right) \epsilon + \theta' \epsilon\right] d\epsilon \\ &= \frac{1}{|I - 2V|^{1/2}} \exp\left[\frac{1}{2} \theta' (I - 2V)^{-1} \theta\right] \quad \square \end{aligned}$$

Let  $X_t$  be a random vector of size  $n$  following Gaussian VAR (1) dynamics:  $X_t = \mu + \Phi X_{t-1} + \Omega \epsilon_t$ , where  $\epsilon_t$  are i.i.d. normalized Gaussian vectors, and  $\Sigma = \Omega \Omega'$  is the conditional variance-covariance matrix of  $X_t$ . We define  $Z_t$  as the augmented vector of factors composed of  $X_t$  and of its vectorized outer-product, that is:  $Z_t = [X_t', \text{Vec}(X_t X_t')]'$ .

Let us consider  $u \in \mathbb{R}^n$  and  $V$  a square symmetric matrix of size  $n$ . The conditional Laplace transform of  $Z_{t+1}$  is denoted by  $\varphi_t$  and defined by:

$$\begin{aligned} \varphi_t(u, V) &= \mathbb{E}_t\{\exp[(u', \text{Vec}(V)') \times Z_{t+1}]\} \\ &= \mathbb{E}_t\{\exp[u' X_{t+1} + X_{t+1}' V X_{t+1}]\} \end{aligned}$$

In the following, we compute the explicit affine form of the conditional Laplace transform of  $Z_{t+1}$ . Let us first consider the term in the expectation. Substituting  $\mu + \Phi X_t + \Omega \epsilon_{t+1}$  for  $X_{t+1}$  leads to:

$$\begin{aligned} \exp\{u' X_{t+1} + X_{t+1}' V X_{t+1}\} &= \exp\{u'(\mu + \Phi X_t) + \mu' V \mu + 2\mu' V \Phi X_t + X_t' V \Phi X_t\} \\ &\quad \times \exp\{[u' \Omega + 2(\mu + \Phi X_t)' V \Omega] \epsilon_{t+1} + \epsilon_{t+1}' [\Omega' V \Omega] \epsilon_{t+1}\} \end{aligned}$$

Taking the conditional expectation leaves the first part of the previous expression unchanged as everything is known in  $t$ . For the second part of the previous expression, we apply Lemma 1 and algebraic computation leads to:

$$\begin{aligned} \mathbb{E}_t[\exp\{[u' \Omega + 2(\mu + \Phi X_t)' V \Omega] \epsilon_{t+1} + \epsilon_{t+1}' [\Omega' V \Omega] \epsilon_{t+1}\}] &= \exp\left\{-\frac{1}{2} \log |I_n - 2\Omega' V \Omega| + \frac{1}{2} u' \Omega (I_n - 2\Omega' V \Omega)^{-1} \Omega' u \right. \\ &\quad + 2u' \Omega (I_n - 2\Omega' V \Omega)^{-1} \Omega' V \mu + 2\mu' V \Omega (I_n - 2\Omega' V \Omega)^{-1} \Omega' V \mu \\ &\quad + [2u' \Omega (I_n - 2\Omega' V \Omega)^{-1} \Omega' V \Phi + 4\mu' V \Omega (I_n - 2\Omega' V \Omega)^{-1} \Omega' V \Phi] X_t \\ &\quad \left. + X_t' [2\Phi' V \Omega (I_n - 2\Omega' V \Omega)^{-1} \Omega' V \Phi] X_t\right\}. \end{aligned}$$

Putting together the first and the second part in the expectation, we obtain:  $\varphi_t(u, V) = \exp\{a_1(u, V) X_t + X_t' a_2(u, V) X_t + b(u, V)\}$ , where:

$$\begin{aligned} a_1(u, V) &= \Phi' \left[ u + 2V \mu + 2V \Omega (I_n - 2\Omega' V \Omega)^{-1} \Omega' u \right. \\ &\quad \left. + 4V \Omega (I_n - 2\Omega' V \Omega)^{-1} \Omega' V \mu \right] \\ a_2(u, V) &= \Phi' \left[ V + 2V \Omega (I_n - 2\Omega' V \Omega)^{-1} \Omega' V \right] \Phi \\ b(u, V) &= u' \mu + \mu' V \mu - \frac{1}{2} \log |I_n - 2\Omega' V \Omega| + \frac{1}{2} u' \Omega (I_n - 2\Omega' V \Omega)^{-1} \Omega' u \\ &\quad + 2u' \Omega (I_n - 2\Omega' V \Omega)^{-1} \Omega' V \mu + 2\mu' V \Omega (I_n - 2\Omega' V \Omega)^{-1} \Omega' V \mu. \end{aligned}$$

Then, noticing that:

$$\Omega(I_n - 2\Omega'V\Omega)^{-1}\Omega' = [\Omega^{-1}(I_n - 2\Omega'V\Omega)^{-1}\Omega^{-1}]^{-1} = [\Sigma^{-1} - 2V]^{-1},$$

we can simplify the previous expressions and obtain:

$$\begin{aligned} a_2(u, V) &= \Phi'V(I_n - 2\Sigma V)^{-1}\Phi \\ a_1(u, V) &= \Phi'[(I_n - 2\Sigma V)^{-1}(u + 2V\mu)] \\ b(u, V) &= u'(I_n - 2\Sigma V)^{-1}\left(\mu + \frac{1}{2}\Sigma u\right) + \mu'V(I_n - 2\Sigma V)^{-1}\mu \\ &\quad - \frac{1}{2}\log|I_n - 2\Sigma V|. \end{aligned}$$

#### A.4.2. Calculation of our model's loadings

Let us denote by  $\lambda_t$  the total intensity, that is:  $\lambda_t = (1 - \theta_c)\lambda_{c,t} + (1 - \theta_\ell)\lambda_{\ell,t}$ . We have:  $\lambda_t = X_t'\Lambda X_t$  where  $\Lambda = \text{diag}[(1 - \theta_c)\Lambda_c, (1 - \theta_\ell)\Lambda_\ell]$ . We can then re-express the pricing formula (11) as:

$$S(t, h) = -\frac{1}{h} \log \left( \mathbb{E}_t^Q \left[ \exp \left\{ -\sum_{i=1}^h X_{t+i}' \Lambda X_{t+i} \right\} \right] \right),$$

which is the log of the multihorizon Laplace transform of a quadratic combination of Gaussian variables. Let us postulate that:  $S(t, h) = \theta_{0,h} + \theta_{1,h}'X_t + X_t'\theta_{2,h}X_t$ . (We know that the model belongs to the class of quadratic term structure models, and that the spreads at all maturities can be expressed as a quadratic combination of  $X_t$ .) Using the law of iterated expectation, we obtain the following recursion:

$$\begin{aligned} S(t, h) &= -\frac{1}{h} \log \left( \mathbb{E}_t^Q \left[ \mathbb{E}_t^Q \left( \exp \left\{ -\sum_{i=1}^h X_{t+i}' \Lambda X_{t+i} \right\} \middle| X_{t+h-1} \right) \right] \right) \\ &= -\frac{1}{h} \log \left( \mathbb{E}_t^Q \left[ \exp \left\{ -\sum_{i=1}^{h-1} X_{t+i}' \Lambda X_{t+i} \right\} \varphi_{t+h-1}^Q(0, -\Lambda) \right] \right) \\ &= -\frac{1}{h} \log \left( \mathbb{E}_t^Q \left[ \exp \left\{ -\sum_{i=1}^{h-1} X_{t+i}' \Lambda X_{t+i} \right\} \right] [b^Q(0, -\Lambda) \right. \\ &\quad \left. + a_1^Q(0, -\Lambda)'X_{t+h-1} + X_{t+h-1}'a_2^Q(0, -\Lambda)X_{t+h-1}] \right) \\ &\triangleq -\frac{1}{h} (\Theta_{0,h} + \Theta_{1,h}'X_t + X_t'\Theta_{2,h}X_t). \end{aligned}$$

Eventually, the recursive equations of system (14) are obtained by using the closed-form coefficients of the conditional Laplace transform of the previous section, plugging the risk-neutral parameters  $\mu^*$  and  $\Phi^*$  (and recalling that we have  $\Omega = I_2$ ).

#### A.5. The quadratic Kalman filter

The QKF is based on the fact that the measurement equations are quadratic in the latent factor  $X_t = (X_{c,t}, X_{\ell,t})'$  but affine in the augmented vector  $Z_t = (X_t', \text{Vec}(X_t X_t'))'$ . This stacked vector  $Z_t$  defines a new state-space representation, and new factor dynamics. The physical dynamics of  $X_t$  writes:

$$X_{t+1} = \mu + \Phi X_t + \Omega \epsilon_{t+1}$$

where  $\Omega\Omega' = \Sigma = I_2$  and  $\epsilon_{t+1} \sim \mathcal{IIN}(0, I_2)$ . Relying on Monfort et al. (2015), we express the augmented state vector (physical) dynamics as:

$$Z_t = \tilde{\mu} + \tilde{\Phi} Z_{t-1} + \tilde{\Sigma}_{t-1}^{1/2} \tilde{\epsilon}_t$$

**Table 6**  
Quadratic Kalman Filter (QKF) algorithm.

Initialization:		$Z_{0 0} = \tilde{\mu}^u$ and $P_{0 0}^Z = \tilde{\Sigma}^u$ .
State prediction:	$Z_{t t-1}$ $P_{t t-1}^Z$	$\tilde{\mu} + \tilde{\Phi} Z_{t-1 t-1}$ $\tilde{\Phi} P_{t-1 t-1}^Z \tilde{\Phi}' + \tilde{\Sigma}(Z_{t-1 t-1})$
Measurement prediction:	$Y_{t t-1}$ $M_{t t-1}$	$A + \tilde{B} Z_{t t-1}$ $\tilde{B} P_{t t-1}^Z \tilde{B}' + V$
Gain:	$K_t$	$P_{t t-1}^Z \tilde{B}' M_{t t-1}^{-1}$
State updating:	$Z_{t t}$ $P_{t t}^Z$	$Z_{t t-1} + K_t(Y_t - Y_{t t-1})$ $P_{t t-1}^Z - K_t M_{t t-1} K_t'$

Note:  $\tilde{\mu}^u$  and  $\tilde{\Sigma}^u$  are respectively the unconditional mean and variance of process  $Z_t$ . In the filtering method, we impose consistency between the linear and the quadratic part of  $Z_t$  by constraining the filtered  $X_t$  to be equal to the square root of the filtered quadratic components of  $Z_t$ .

such that:

$$\begin{aligned} \tilde{\mu} = \begin{pmatrix} \mu \\ \text{Vec}(\mu\mu' + \Sigma) \end{pmatrix}, \tilde{\Phi} &= \begin{pmatrix} \Phi & 0 \\ \mu \otimes \Phi + \Phi \otimes \mu & \Phi \otimes \Phi \end{pmatrix} \\ \tilde{\Sigma}_{t-1} \equiv \tilde{\Sigma}(Z_{t-1}) &= \begin{pmatrix} \Sigma & \\ \Gamma_{t-1}\Sigma & \Gamma_{t-1}\Sigma\Gamma_{t-1}' + (I_{n^2} + \Lambda_n)(\Sigma \otimes \Sigma) \end{pmatrix} \end{aligned}$$

$$\Gamma_{t-1} = I_n \otimes (\mu + \Phi X_{t-1}) + (\mu + \Phi X_{t-1}) \otimes I_n$$

$\Lambda_n$  being the  $n^2 \times n^2$  matrix, partitioned in  $(n \times n)$  blocks, such that the  $(i, j)$  block is  $e_i e_j'$ , and the distribution of  $\xi_t$  is unknown. Let  $Y_t$  be the set of measured variables, thus  $Y_t = [S(t, 13), S(t, 26), S(t, 39), S(t, 52), P_{c,t}, P_{\ell,t}]'$ . The measurement equations can be transformed in affine functions of  $Z_t$ :

$$\begin{aligned} \begin{pmatrix} S(t, h) \\ P_{c,t} \\ P_{\ell,t} \end{pmatrix} &= \begin{pmatrix} \theta_{0,h} \\ \pi_{c,0} \\ \pi_{\ell,0} \end{pmatrix} + \begin{pmatrix} \theta_{1,h}^{(c)} & \theta_{1,h}^{(\ell)} & \theta_{2,h}^{(c)} & 0 & 0 & \theta_{2,h}^{(\ell)} \\ 0 & 0 & \pi_c & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \pi_\ell \end{pmatrix} Z_t \\ &\quad + \begin{pmatrix} \sigma_\eta \eta_{t,h} \\ \sigma_{v_c} v_{c,t} \\ \sigma_{v_\ell} v_{\ell,t} \end{pmatrix}, \\ &\Rightarrow Y_t \triangleq A + \tilde{B} Z_t + D \tilde{\epsilon}_t \end{aligned}$$

Approximating the conditional distribution of  $Z_{t+1}$  given  $Z_t$  by a Gaussian distribution and considering the augmented state-space model based on  $Z_t$ , a standard linear Kalman filter can be used for filtering and estimation purposes. To get the global likelihood maximum, the estimation is achieved in two steps. The *Artificial Bee Colony* stochastic algorithm (see Karaboga and Basturk (2007)) is used to find the potential maxima areas of parameters. The results are then used as starting values for a usual simplex maximization algorithm and the best estimate is selected. The full algorithm is presented in Table 6 taken from Monfort et al. (2015).

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