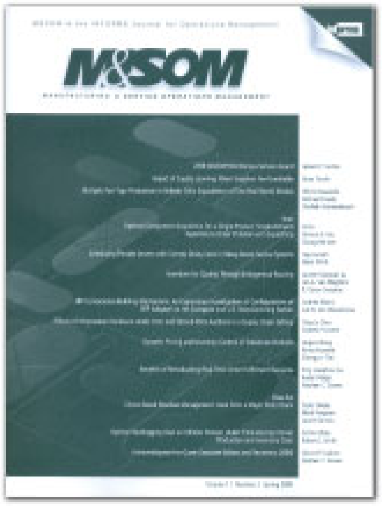

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Responsibility Tokens in Supply Chain Management

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The decentralized supply chain management scheme of Lee and Whang (1999) can be viewed as operationalizing the decentralized management scheme implicit in Clark and Scarf (1960). This paper proposes the use of what are called responsibility tokens (RTs) to further facilitate that operationalization. The proposal assumes that a management information system, presumably electronic, is established to monitor inventories and shipment quantities, and to carry out transfer payments between players. As in Lee and Whang (1999), the incentives of the system are aligned, so if each player is brilliantly self-serving, the system optimal solution will result. While the system administrator need not know how the system should be managed, the most upstream player must know how to manage the system optimally for the system optimal solution to be achieved. RTs endow the system with an attractive self-correcting property: An example illustrates that upstream players are given a mechanism and the incentive to correct for downstream overordering. The downstream players who overorder are penalized, but system performance is not degraded much. Extensions and further research are also discussed.

(*Supply Chain Management; Responsibility Tokens; MultiEchelon; Inventory Management; Management Information System, Incentives; Stochastic Demand; Cost Accounting; Decentralization*)

1. Introduction

One of the precepts of supply chain management is to first determine how a chain should be operated to achieve maximal system performance and, recognizing that the decision makers in the chain are apt to be different people in different organizations with different interests, to determine a way to design the incentives for the decision makers to act in the system's interest, and to share the gains appropriately.

Clark and Scarf (1960) pioneered the study of single-product multistage supply chains under full backlogging of customer demand. They established the concept of echelon stocks and provided conditions under which the optimal inventory policy (for the entire chain) can be determined by solving a sequence of single-variable problems, one problem defined in terms of the echelon position for each player/location in the chain. Lee and Whang (1999), among other things, interpreted Clark and Scarf's results as suggesting a way in which the system optimal solution can be achieved by revising the performance measurement system and encouraging the players to act in their self-interest.

Clark and Scarf's (1960) results derive from two important concepts, *echelon costing* and *upstream responsibility*, which are reviewed in § 3 and 4. This paper focuses on upstream responsibility and, in §5, proposes a mechanism, *responsibility tokens (RTs)*, to facilitate the decentralized implementation. Section 6 discusses their use in chains in which installation stocks are used instead of echelon stocks. Section 7 summarizes the results, places them in the context of the existing literature, and discusses extensions.

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2. Review of the Basic Supply Chain Model

The basic supply chain model of this paper is a reformulation of the Clark and Scarf (1960) model in which the lead time between adjacent players (echelons, locations) is one period. Section 7 shows how the more general case of multiple period lead times between players can be encompassed by this model. The notation and approach of Lawson and Porteus (2000) are used.

Index the players (one for each location) by $i = 0, 1, \ldots, m$, so that there are $m + 1$ different players in the chain. For convenience, the time period is taken to be a week, Player 0 is called the *retailer (R)*, player m is called the *factory (F)*, and player m receives shipments from an imaginary upstream player ($i = m + 1$) who has an unlimited supply. Orders (requests for replenishment) placed by player i are immediately transmitted to (and received by) player $i + 1$, and shipments (from player $i + 1$ to player i) take exactly one week.

For convenience, the problem has a finite horizon, with T weeks. All of the following symbols are functions of t, the week. To keep the exposition simple, however, that dependence is suppressed whenever possible. The sequence of events within an arbitrary week t is as follows:

(1) Each player receives any incoming shipments, which had been shipped from her immediate upstream player the previous week, and adds them to her current stock. The total, X_i, denotes all units physically on-hand by player i, called *installation stock* at player i. The backlog $B(t - 1)$ from the previous week represents any customer demands that could not be met then. It is possible at this point for both X_0 and $B(t - 1)$ to be strictly positive: X_0 includes units just received, and if some or all of them need to be allocated to the backlog carried over from the previous week, that will be done later in the week.

(2) Player 0 places an order for Z_0, which is transmitted immediately to Player 1.

(3) Each player $i = 1, 2, \ldots, m$, in that sequence, meets the order by player $i - 1$ to the extent that it can (by shipping $z_{i-1} = \min(Z_{i-1}, X_i)$ to be received by player $i - 1$ at the beginning of the following week) and places an order for Z_i that is transmitted immediately to player $i + 1$. (The order by player m is always fully met.) Because this paper focuses on the incentives of the various players, it is useful to distinguish between the amount ordered Z_i and the amount shipped, z_i, also called the *due-in* to player i. The *echelon stock* of player i, denoted by x_i, consists of all physical stock at player i or further downstream, less the backlog at this point in the week:

$$x_i = \sum_{j=0}^{i} X_j - B(t - 1).$$

The *echelon position* at player i, denoted by y_i, consists of the echelon stock plus the amount due-in: $y_i = x_i + z_i$. The *requested echelon position (REP)* of player i, denoted by Y_i, consists of the echelon stock plus the amount ordered: $Y_i = x_i + Z_i$.

(4) Customer demand for the week, $D(t)$, is observed by Player 0, added to the old backlog, and met to the extent possible, with any unmet demands newly backlogged: $B(t) = [D(t) + B(t - 1) - X_0]^+ = [D(t) - x_0]^+ = \max(0, D(t) - x_0)$.

(5) Costs are assessed. Player i is assessed the cost $k_i z_i$ for any processing and transportation required. Player 0 is assessed the backlog penalty of pB. Each player i is also assessed a holding cost, amounting to the unit holding cost of h_i charged on positive ending installation stock plus any due-ins, namely $(x_0 - D)^+ + z_0$ for the retailer and $X_i - z_{i-1} + z_i = y_i - y_{i-1}$ for the remaining players $i \geq 1$. For example, for the latter locations, the holding cost of h_i is charged on stock that is left unshipped at i plus what is being shipped to that player.

At the end of week T, considered to be the beginning of week $T + 1$, a terminal cost (salvage value if negative), assumed to be a convex function of $x_i(T + 1)$, is assessed for each i. Using a discount factor of α per week, the integrated system objective is to minimize the expected present value of the net costs incurred over the finite horizon. The decentralized objective for each manager is to minimize the expected present value of all the costs assigned to that manager over the finite horizon.

A management information system, presumably electronic, exists and can perfectly observe, without delay, all relevant supply chain information at all times. The administrator of the system will carry out any accounting required, but will make no managerial decisions.

3. Review of Echelon Costing

Clark and Scarf (1960) assume that holding costs are assessed against echelon stocks instead of installation stocks, and show that the problem in each week can be analyzed as a sequence of one-dimensional problems of the echelon stocks. It is instructive to reallocate the total costs in terms of the echelon stocks before doing any optimization. Lemma 1 below shows the results for the specific conventions of this paper. For convenience, h_{m+1} is assumed to exist and equal zero.

LEMMA 1. *(a) The costs in an arbitrary week t (for $1 \le t \le T$) can be expressed as*

$$(p + h_0)B - h_0 D + \sum_{i=0}^{m} (k_i + h_i - h_{i+1})y_i - \sum_{i=0}^{m} k_i x_i.$$

(b) The costs can be reallocated so that week t (for $1 \le t \le T$) is assigned

$$(p + h_0)B - \left[h_0 - \alpha \sum_{i=0}^{m} k_i\right]D + \sum_{i=0}^{m} c_i y_i, \qquad (1)$$

where

$$c_i := k_i(1 - \alpha) + h_i - h_{i+1}.$$

(c) The costs that are affected by decisions can be further reallocated so that week t (for $1 \le t \le T - 1$) is assigned

$$\alpha(p + h_0)[D(t) + D(t + 1) - y_0]^+ + \sum_{i=0}^{m} c_i y_i,$$

and week T is simply assigned $\sum_{i=0}^{m} c_i y_i$.
(d) In addition to there being costs associated with the initial conditions, the redefined terminal cost function, defined on the terminal echelon stock levels, is additively convex.

All proofs are in the Appendix. This lemma can be interpreted as a new performance measurement system for the various managers. It is fruitful to interpret Part (a) as it confirms three points made by Lee and Whang (1999). (1) A new effective unit holding cost is assessed on echelon position, rather than installation stock levels. (2) This effective unit holding cost consists of the additional unit holding cost $(h_i - h_{i+1})$ incurred at a player's respective location (compared to her immediate upstream location). (Note that this additional hold cost will be negative if $h_i < h_{i+1}$, which is allowed.) (3) The backlog penalty is augmented by the original holding cost at the retailer level, to become $\pi := p + h_0$.

Part (b) can be interpreted as eliminating the unit transaction cost k_i as in Veinott (1965), with $k_i(1 - \alpha)$ being added to the effective unit holding cost. The middle term in (1) does not involve any decision variables and can either be allocated arbitrarily to the players or captured in the overhead, which is what is assumed here, so that it can be ignored in (c).

Feasibility amounts to $x_i \le y_i \le x_{i+1}$ for every i, where, for convenience, x_{m+1} is assumed to exist and equal plus infinity.

4. Review of Upstream Responsibility

Clark and Scarf (1960) show how the problem during a week can be analyzed as a sequence of single-variable problems, one for each manager. To understand the nature of their solution to the problem and the concept of upstream responsibility that goes with it, it is useful to review the analysis, tailored to the assumptions of this paper.

Let $f_t(x_0, \ldots, x_m)$ denote the minimum expected system cost over the remainder of the horizon, starting week t with echelon stock level x_i for each i. By Lemma 1, the terminal value function $f_{T+1}(x_0, x_1, \ldots, x_m)$ is additively convex. Assume inductively that, for a given $t \in \{1, 2, \ldots, T\}$, f_{t+1} is additively convex, namely of the form $\sum_{i=0}^{m} f_{i,t+1}(x_i)$, where each $f_{i,t+1}$ is convex.

The optimality equation for week t is therefore

$$f_t(x_0, x_1, \ldots, x_m) = \min_{\substack{x_i \le y_i \le x_i+1 \\ 1 \le i \le m}} \left\{\sum_{i=0}^{m} c_i y_i + L_t(y_0) + \alpha \sum_{i=0}^{m} E f_{i,t+1}(y_i - D)\right\},$$

where

$$L_t(y) := \begin{cases} \alpha(p + h_0)E(D(t) + D(t + 1) - y)^+ \\ \quad \text{if } 1 \le t \le T - 1, \\ 0 \quad \text{if } t = T, \end{cases}$$

is the expected present value of the revised backlog cost to be incurred in week $t + 1$ as a function of the retailer's echelon inventory position y in week t.

The optimality equation is separable into the sum of m convex optimization problems of the form

$$\min_{x_i \le y \le x_{i+1}} f_{it}^R(y)$$

for each i, where

$$f_{it}^R(y) = \begin{cases} c_0 y + L_t(y) + Ef_{0,t+1}(y - D) \\ \quad \text{if } i = 0; \\ c_i y + \alpha E f_{i,t+1}(y - D) \\ \quad \text{otherwise.} \end{cases}$$

By a result of Karush (1958) (see also Lawson and Porteus 2000), the resulting optimal value is additively convex, taking the form $a_{it} + g_{it}(x_i) + L_{it}(x_{i+1})$. Assume, for expositional convenience, until the end of this section, that the convex function f_{it}^R has a unique minimizer, S_{it}. If S_{it} is finite, then $a_{it} = f_{it}^R(S_{it})$,

$$g_{it}(x) = \begin{cases} f_{it}^R(x) - f_{it}^R(S_{it}) & \text{if } x \ge S_{it}; \\ 0 & \text{otherwise,} \end{cases}$$

and

$$L_{it}(x) = \begin{cases} f_{it}^R(x) - f_{it}^R(S_{it}) & \text{if } x \le S_{it}; \\ 0 & \text{otherwise.} \end{cases}$$

Thus, the inductive step is completed because $f_t(x_0, x_1, \ldots, x_m)$ is additively convex of the form $\Sigma_{i=0}^m f_{i,t}(x_i)$, where

$$f_{i,t}(x) = \begin{cases} a_{0t} + g_{0t}(x) & \text{if } i = 0; \\ a_{it} + g_{it}(x) + L_{i-1,t}(x) & \text{otherwise.} \end{cases}$$

The managerial insights come from further analysis. The optimal policy is a base stock policy in echelon positions: The optimal echelon position for player i is the closest point in the interval $[x_i, x_{i+1}]$ to S_{it}. A way to decentralize the decision making is to assign f_{it}^R as the performance measure for manager i to minimize. Thus, she wishes to implement S_{it}, the unconstrained minimizer of f_{it}^R. Because feasibility requires $y_i(t) \ge x_i$, that initial request gets increased to $x_i(t)$ if $S_{it} < x_i(t)$ and the system incurs the penalty cost $g_{it}(x_i)$, which can therefore can be interpreted as an induced overage cost, as it is strictly positive only when $x_i(t) > S_{it}$. Since $x_i(t) = y_i(t - 1) - D(t - 1)$, this penalty is a direct consequence of that player's echelon position in the previous week. That is, player i should be responsible for that penalty. Indeed she is, as g_{it} gets assigned to $f_{i,t}$, which, in turn gets assigned to $f_{i,t-1}^R$, her objective function in week $t - 1$. That is, manager i is responsible for the lower bound on her echelon position and therefore selects $Y_i(t) = \max(S_{it}, x_i(t))$ as her REP.

To satisfy the feasibility constraint $y_i(t) \le x_{i+1}(t)$, that request gets reduced to $y_i(t) = \min(Y_i(t), x_{i+1}(t))$ and the system incurs the additional cost $L_{it}(x_{i+1}(t))$, which can be interpreted as an induced shortage cost, because it kicks in only when $x_{i+1}(t) < S_{it}$. Since $x_{i+1}(t) = y_{i+1}(t - 1) - D(t - 1)$, this cost should be the responsibility of manager $i + 1$, which it is: L_{it} gets assigned to $f_{i+1,t}$, which, in turn, gets assigned to $f_{i+1,t-1}^R$, the objective function of the next upstream manager for week $t - 1$.

In this case, manager i will not be responsible for the consequences of not receiving units that she requested/ordered. From her perspective, she receives perfect upstream supply: She either gets all that she orders or, if not, the upstream manager takes responsibility for the financial consequences of the shortfall. For an upstream manager $i \ge 1$,

$$f_{it}^R(y) = c_i y + \alpha a_{i,t+1} + \alpha E g_{i,t+1}(y - D) + \alpha E L_{i-1,t+1}(y - D). \tag{2}$$

That is, the things that influence her request are: (a) the direct cost she incurs this week, (b) the discounted induced overage cost of having too much echelon stock next week to implement her ideal request then, and (c) the discounted induced shortage cost of having too little echelon stock next week to meet the request of the next downstream manager.

It is important to note that the shortage penalty function L_{it} incorporates additional costs that arise as part of f_{it}^R, the objective function of player i. For clarity, assume that $i \ge 1$ and that $c_j \ge 0$ for all $j \le i$. By receiving less than she requested, player i will save c_i for each unit of the shortfall. However, she will have less available at the beginning of the next period, so the term $\alpha E g_{i,t+1}(y - D)$ can only decrease and the term $\alpha E L_{i-1,t+1}(y - D)$ can only increase. Continuing to expand the terms of the penalty function through all downstream echelons, the consequences to player i of not receiving all of her order in period t are all beneficial to her except for the possibility that the shortfall in meeting all of manager i's request can be traced to a customer shortage i periods later at the retailer.

Lee and Whang (1999) call this process of assigning responsibility "shortage reimbursement," as each player upstream from the retailer is passed along a penalty cost function that incorporates the consequences of not fully meeting the request of its immediate downstream player. It is worth noting that this penalty cost includes not only reimbursement for possible shortages at the retailer but credits for all the possible beneficial effects as well. This reimbursement makes the downstream player indifferent (in expectation) to: (1) receiving the amount requested, and (2) receiving the lesser amount shipped plus the cash credit to compensate for the additional costs that the downstream player will subsequently incur. The theoretical properties are powerful in a decentralized setting in which players are out only for themselves: Assume inductively that there is no credible reason why any player in any future week would select anything but the optimal REP for that week, as no other decision can make her better off. Thus, the system optimal policy will be implemented in all future weeks. In particular, player i can identify the optimal base stock levels, and therefore the optimal REPs, for all downstream players for the remaining weeks of the horizon. She can then compute and optimize f_{it}^{R} and has no credible reason to select a base stock level different from the minimizer of that function and a REP that differs from what leads to the system optimal solution. That is, the resulting equilibrium is subgame perfect (e.g., see Fudenberg and Tirole 1991) and is first-best (achieves the system optimal solution).

We now drop the assumption that the optimal base stock level S_{it} is unique for all i and t. It is then possible that there is a nondegenerate interval of minimizers of f_{it}^{R}. It is straightforward to show that, regardless of which of the optimal base stock levels is selected each time a choice arises, an optimal policy is followed and the system performs in exactly the way described above. Indeed, the incentives of a manager are not affected by the actions selected by the other managers, as long as each downstream manager acts optimally in her self-interest. Thus, there may be many subgame perfect equilibria, all leading to a system optimal solution. The managers need not know which equilibrium is in effect, because their incentives are the same in each. Henceforth, we revert to the assumption that the optimal policy is unique, again for expositional convenience.

5. Responsibility Tokens

This paper proposes the use of responsibility tokens (RTs) as a mechanism for administering the transfer payments required to implement upstream responsibility. The idea is to base reimbursement on actual consequences of processing/delivering/shipping less than what was requested, rather than predicting the consequences in advance. In essence, the proposal is to implement an empirical reimbursement scheme rather than a theoretical one. Discussion of the differences between the two approaches appears in §7.

Whenever an upstream player cannot meet the entire order placed by her downstream player, she will substitute RTs in place of the missing units. The downstream players will treat these tokens as physical units and the financial consequences of their not being real units are assigned to the issuing player. Thus, rather than computing a reimbursement amount that is transferred to the downstream player at the time of the shipment, the upstream player ships physical units plus RTs, so that the sum equals the original amount requested. That is, every order is fully satisfied, either with physical units or with RTs.

Formal Representation of Echelon Tokens

Recall that Y_i denotes the requested echelon position of player i in a week, and that the (actual) echelon position is given by $y_i = \min(Y_i, x_{i+1})$. The difference, $Y_i - y_i$, will be strictly positive when there is not enough echelon stock at the next player upstream to meet the full request. That difference will be made up by RTs. Not all of these will be made up by RTs issued by the next upstream player, because she may already be holding some RTs issued by players even further upstream, and, from her perspective, such RTs are as good as physical units and should be allocated as if they were. Thus, let $Y_{im} := Y_i - y_i$ denote the amount of the REP of player i that must be made up by tokens (by some combination of upstream players). Let $x_{i+1,m}$

denote the number of tokens issued by player m (in the past) that are included in the echelon stock of player $i + 1$. Thus, the number of those tokens that are included in the echelon position of player i can be computed as $y_{im} = \min(Y_{im}, x_{i+1,m})$. That is, to the extent that player $i + 1$ must meet player i's requested echelon position by use of tokens, available tokens issued by player m are used first.

In general, for each i and $j > i$, let x_{ij} denote the number of (echelon) tokens issued by player j that are included in the echelon stock of player i at the time she receives her order that week, Y_{ij} the number of units in the echelon position of player i that must be made up by tokens issued by players $i + 1$ through j, and y_{ij} the echelon position of player i that consists of tokens issued by player j. Lemma 2 below shows how to compute Y_{ij} and y_{ij} for each i and j, given each player's REP and available echelon stock and echelon tokens.

Note that $x_{ij}(t) = y_{ij}(t - 1)$ for all pertinent cases. That is, a player's echelon token position in one week becomes her echelon token stock in the following week.

Let C_i^{B} denote the cost transferred (charged) to player i due to backlogging and C_{ij}^{H} denote the cost transferred (charged) to player i and credited to player j due to player i's echelon position, during the week. For convenience, let $B_{m+1} := B = (D - x_0)^+$, the physical backlog at the end of the week. Finally, for $i = 1, 2, \ldots, , m$, let

$$B_i := \left(D - x_0 - \sum_{j=i}^{m} x_{0j}\right)^+,$$

which is the backlog remaining, if any, after accounting for tokens held by Player 0 and issued by players m down through i. Recall that $\pi := p + h_0$.

LEMMA 2. *(a) The echelon token positions can be computed recursively as follows: First,* $Y_{im} := Y_i - y_i$. *Then, for* $j = m, m - 1, \ldots, i + 2$, $y_{ij} = \min(Y_{ij}, x_{i+1,j})$ *and* $Y_{i,j-1} := Y_{ij} - y_{ij}$. *Finally,* $y_{i,i+1} = Y_{i,i+1}$.
(b) The cost transfers due to backlogging during a week are given as follows, $C_i^{\mathrm{B}} = \pi \min(B_{i+1}, x_{0i})$ *for each* $i > 0$ *and* $C_0^{\mathrm{B}} = -\sum_{i=1}^{m} C_i^{\mathrm{B}}$.
(c) The cost transfers due to echelon position (holding) costs are: $C_{ij}^{\mathrm{H}} = c_i y_{ij}$ *for each* i *and* $j > i$.

Illustration of Echelon Tokens: Example 1

It is instructive to illustrate echelon tokens in the context of a numerical example: There are three players, Player 0 is (R), the Retailer, Player 1 is (W), the Wholesaler, and Player 2 is (F), the Factory. There are no transactions costs ($k_i = 0$ for all i) and the discount factor is $\alpha = 1$. The original holding costs are $h_0 = 6$, $h_1 = 5$, and $h_2 = 3$, and the unit backlog penalty is $p = 24$. Thus, the cost parameters for echelon costing are $\pi = 30$, $c_0 = 1$, $c_1 = 2$, and $c_2 = 3$.

Exhibit 1 presents a numerical example in which the players do not necessarily order optimally. Customer demand in each week equals 20 units. In each week, each player has a current inventory box and two immediate upstream boxes, the upper one showing her requested echelon position and the lower one showing the breakout of that position into physical echelon stock and echelon tokens.

The players order each week to bring their echelon positions up to their requested echelon positions (REP), which are 42, 64, and 86, respectively.

Lemma 2 is applied in each week to determine how each REP is met. For example, in week 1, R asks for an REP of 42, but W only has $x_1 = 24$ physical units available, so W must have 18 of her RTs cover the difference. That is, $y_{01} = 18$. The usual dynamics apply in going from one week to the next: $x_i(t + 1) = y_i(t) - D(t)$ for physical echelon units and $x_{ij}(t + 1) = y_{ij}(t)$ for echelon tokens.

The costs for each player are shown at the bottom of the exhibit. The "Original Costs" line gives the costs based on installation stocks. The "Echelon Costs" line gives the results for each player using echelon costing. Next come the cost transfers based on RTs. The final line gives the net echelon costs after all transfers have been made.

In this example, the bulk of the costs were borne by R (the Retailer) in the original system, but they are borne by F (the Factory) under the new system. Such a radical reallocation of costs would pose significant implementation problems in practice. By Lemma 1(b), the credit of $h_0 D(t)$ is incurred in week t, for each t, and the total, \$480, is shown, for convenience, in the overhead column. This amount could be fully allocated to F to lessen the impact of the new system on F, without

Exhibit 1 **Example 1, Echelon Tokens**

	R (0)		W (1)		F (2)	
W	$x_0 = 12$	$Y_0 = 42$	$x_1 = 24$	$Y_1 = 64$	$x_2 = 36$	$Y_2 = 86$
E	$x_{02} = 0$		$x_{12} = 0$			
E	$x_{01} = 0$	$y_0 = 24$		$y_1 = 36$		$y_2 = 86$
K	$D = 20$	$y_{02} = 0$		$y_{12} = 28$		
1	$B = 8$	$y_{01} = 18$				

	R (0)		W (1)		F (2)	
W	$x_0 = 4$	$Y_0 = 42$	$x_1 = 16$	$Y_1 = 64$	$x_2 = 66$	$Y_2 = 86$
E	$x_{02} = 0$		$x_{12} = 28$			
E	$x_{01} = 18$	$y_0 = 16$		$y_1 = 64$		$y_2 = 86$
K	$D = 20$	$y_{02} = 26$		$y_{12} = 0$		
2	$B = 16$	$y_{01} = 0$				

	R (0)		W (1)		F (2)	
W	$x_0 = -4$	$Y_0 = 42$	$x_1 = 44$	$Y_1 = 64$	$x_2 = 66$	$Y_2 = 86$
E	$x_{02} = 26$		$x_{12} = 0$			
E	$x_{01} = 0$	$y_0 = 42$		$y_1 = 64$		$y_2 = 86$
K	$D = 20$	$y_{02} = 0$		$y_{12} = 0$		
3	$B = 24$	$y_{01} = 0$				

	R (0)		W (1)		F (2)	
W	$x_0 = 22$	$Y_0 = 42$	$x_1 = 44$	$Y_1 = 64$	$x_2 = 66$	$Y_2 = 86$
E	$x_{02} = 0$		$x_{12} = 0$			
E	$x_{01} = 0$	$y_0 = 42$		$y_1 = 64$		$y_2 = 86$
K	$D = 20$	$y_{02} = 0$		$y_{12} = 0$		
4	$B = 0$	$y_{01} = 0$				

	R (0)	W (1)	F (2)	Overhead	Total
Original Costs	1704	520	348	0	2572
Echelon Costs	1564	456	1032	-480	2572
Cost Transfers					
Backlogs	-1200	480	720	0	0
Holding	44	38	-82	0	0
Net E-Costs	408	974	1670	-480	2572

affecting F's incentives. However, F would still be bearing a much greater share of the costs than under the original system.

The main effect of RTs in this example is to transfer responsibility for shortages in Weeks 2 and 3 to W and F, respectively. Before the demand arises in Week 1, R requests a position of 42, which is met by 24 physical units and 18 tokens issued by W. The entire backlog of

8 in that week is charged to R, because she has no upstream tokens that would transfer responsibility for them.

At the beginning of Week 2, R's echelon stock is 4 ($y_0 - D(1) = 24 - 20$) and she has 18 tokens issued by W ($x_{01} = 18$). She again requests a position of 42, which is met this week by 16 physical units and 26 tokens issued by F. The backlog of 16 at the end of that week is not the responsibility of R, because if she had received her entire request in physical units, she would have had 22 units available, more than enough to meet the 20 units demanded. She only has 4 units available, which are used, and the backlog penalty for the 16 unmet demands is transferred to W, because of the tokens available.

At the beginning of Week 3, R's echelon stock is -4 because even after receiving the incoming delivery, a backlog still exists. She also has 26 tokens issued by F ($x_{02} = 26$): A substantial portion of *W*'s requested position in Week 1 was met by F using tokens, which arrived in Week 2 and were used by W then to meet a substantial portion of R's request then. These tokens have finally arrived at R and, because a backlog arises in the week in which the tokens arrive, cost transfers take place.

There are also holding cost transfers that slightly benefit the upstream players who have issued tokens. For example, W requests a position of $Y_1 = 64$ in Week 1, which is met by F using 36 physical units and 28 tokens. In echelon costing, W only gets charged holding costs for the $y_1 = 36$ physical units. However, she must get charged for her full request, as if it had been fully met by physical units, so W gets charged an additional $c_1 y_1 = 2(28) = 56$ and F gets credited with that amount.

It is clear that each of the upstream players now have the incentive to carry stock to hedge against the risk of being charged for backlogs. Indeed, as implied by Clark and Scarf (1960) and clarified by Lee and Whang (1999), the decentralized system is designed to produce the centralized optimal solution. In particular, R will minimize the costs charged against her account by picking the centralized optimal requested echelon position for R. Then, recursively, given that the players downstream from her select the optimal REPs, each player will minimize the expected costs charged against her account by picking the centralized optimal REP for her installation.

Example 2

While the primary effect of the RTs in Example 1 was to shift responsibility for backlog costs, the shifting of holding costs also can play a useful role. Example 2, presented in Exhibit 2, shows that RTs have an interesting self-correcting property: Overordering by a downstream player need not hurt system performance much, if at all. Rather, it offers the opportunity for an upstream player to line her account with excess costs charged to the downstream player. That is, the upstream player has the incentive to save the system from needless extra costs by claiming them for her own account. The only change from Example 1 is that R changes her REP from 42 to 70, and W changes her REP from 64 to 74.

The transfers of backlog costs are exactly the same as in Example 1. What differs is that R can be viewed as overordering, and the upstream players' accounts benefit as a result. For example, in Week 4, R's echelon stock consists of 26 physical units, 8 tokens issued by F, and 16 tokens issued by W. The 26 physical units are more than enough to meet the demand of 20. If R had received all of her request in physical units, she would have 50 physical units available, which would be excessive. By having the upstream players issue tokens instead of providing physical units, the system cost is kept under control and the players issuing the tokens are made better-off. The total cost is \$2592, which is only \$20 more than in Example 1. However, because of R's overordering, her (net) costs increase by over 25%, and F's costs decrease.

In this example, W can also be viewed as overordering: While she gains some benefits from issuing tokens to gain from R's excessive orders, she too overorders and F takes advantage, meeting at most 66 of W's requests with physical units and the rest with tokens. In the end, W's costs go up slightly compared to Example 1. In short, some of the requests by R and W will *never* be met (by physical units).

Discussion

Under the Clark and Scarf (1960) approach, it may be optimal for players to make requests that are never met. Each player only considers the cost at her location,

Exhibit 2 **Example 2, Echelon Tokens**

	R (0)		W (1)		F (2)	
W	$x_0 =$ 12	$Y_0 =$ 70	$x_1 =$ 24	$Y_1 =$ 74	$x_2 =$ 36	$Y_2 =$ 86
E	$x_{02} =$ 0		$x_{12} =$ 0			
E	$x_{01} =$ 0	$y_0 =$ 24		$y_1 =$ 36		$y_2 =$ 86
K	$D =$ 20	$y_{02} =$ 0		$y_{12} =$ 38		
1	$B =$ 8	$y_{01} =$ 46				

	R (0)		W (1)		F (2)	
W	$x_0 =$ 4	$Y_0 =$ 70	$x_1 =$ 16	$Y_1 =$ 74	$x_2 =$ 66	$Y_2 =$ 86
E	$x_{02} =$ 0		$x_{12} =$ 38			
E	$x_{01} =$ 46	$y_0 =$ 16		$y_1 =$ 66		$y_2 =$ 86
K	$D =$ 20	$y_{02} =$ 38		$y_{12} =$ 8		
2	$B =$ 16	$y_{01} =$ 16				

	R (0)		W (1)		F (2)	
W	$x_0 =$ -4	$Y_0 =$ 70	$x_1 =$ 46	$Y_1 =$ 74	$x_2 =$ 66	$Y_2 =$ 86
E	$x_{02} =$ 38		$x_{12} =$ 8			
E	$x_{01} =$ 16	$y_0 =$ 46		$y_1 =$ 66		$y_2 =$ 86
K	$D =$ 20	$y_{02} =$ 8		$y_{12} =$ 8		
3	$B =$ 24	$y_{01} =$ 16				

	R (0)		W (1)		F (2)	
W	$x_0 =$ 26	$Y_0 =$ 70	$x_1 =$ 46	$Y_1 =$ 74	$x_2 =$ 66	$Y_2 =$ 86
E	$x_{02} =$ 8		$x_{12} =$ 8			
E	$x_{01} =$ 16	$y_0 =$ 46		$y_1 =$ 66		$y_2 =$ 86
K	$D =$ 20	$y_{02} =$ 8		$y_{12} =$ 8		
4	$B =$ 0	$y_{01} =$ 16				

	R (0)	W (1)	F (2)	Overhead	Total
Original Costs	1752	510	330	0	2592
Echelon Costs	1572	468	1032	-480	2592
Cost Transfers					
Backlogs	-1200	480	720	0	0
Holding	148	30	-178	0	0
Net E-Costs	520	978	1574	-480	2592

and further downstream, in deciding how much to request. Because the upstream costs of fully meeting that request may be expensive, it may be optimal for an upstream player to refuse to fully meet a request (with physical units). In Clark and Scarf's accounting scheme, which is elaborated on by Lee and Whang (1999), a transfer payment of the expected financial consequences of not fully meeting that request is made by the refusing player. With tokens, the same effect is achieved, but without having to compute the expected consequences in advance. In particular, the incentives for the upstream players to line their pockets by refusing to meet inappropriately large downstream requests are part of the theory of Clark and Scarf (1960) and Lee and Whang (1999). The RTs simply make it easier to see this phenomenon.

An interesting phenomenon arises when comparing these examples. Both R and W have higher base costs charged to them in the realization of the process in Example 2, because of their overordering. The transfer payments are also less favorable to them. However, F has the same base costs charged to her, and prefers the transfer payments made in Example 2. Indeed, she is much better-off than before. She actually prefers that the downstream players overorder, so she can get credit for correcting their errors. If each player maximizes the expected present value of her respective account, then the centralized optimal solution will be attained. However, it appears that upstream players would still prefer that downstream players overorder. In essence, there is no potential for upstream players to complain about downstream overordering, only opportunities to gain. Of course, in general, the system is designed so that the downstream players are indifferent to whatever the upstream players do: The downstream players can approach the problem as if they will get perfect upstream supply, and they will get charged the same regardless of what the upstream players do.

It is also interesting to note that tokens are *transient* in the following sense. Once RTs arrive at R, they are used for carrying out transfer payments that week and then vaporize, without being carried forward into the next week. (R's echelon token stock next week will equal her echelon token position this week ($x_{0i}(t + 1) = y_{0i}(t)$).) Furthermore, any RTs that are delivered to any upstream player in a week must either be sent downstream that week or they expire: When player i requests a position and player $i + 1$ cannot meet it fully with physical units, then $i + 1$ uses RTs to cover the difference, and player i's echelon token position does not play a role in this determination. Thus, once issued, echelon tokens move downstream to the next player, who will send them only if they are needed to meet a request that week. Tokens not sent along essentially expire, because they have no effect on the echelon token levels the following week, or on anything else for that matter. So, tokens either expire when a player does not send them on, or they reach R in a week when they may be used to transfer backlog costs to the issuing player before they expire.

One consequence of this observation that tokens can be interpreted as being transient is that useful intuition can be generated about the optimal base stock levels (REPs) for each upstream player. (We already know the optimal REP for the retailer is the solution to a simple newsvendor problem.) If (upstream) player i reduces her REP by one unit, then, one week later, her EP will be one less. If the REP of the immediate downstream player calls for shipment of that unit (because of a surge in customer demand, say), then player i must send a token in its place. That token will accumulate credits in each week that it is sent along to the next player (and is part of an EP box). If that token stops somewhere before reaching the retailer, player i only benefits from the results. However, if the token makes it all the way to R and there is a stockout in that period, then player i will be charged π then. Because the token then expires, the player faces the tradeoff between the expected gains while the token travels downstream versus the expected cost of a shortage should the token reach the retailer. If the player knows the base stock levels that will be used by the downstream players in the appropriate periods, she faces a problem that looks like a generalization of a newsvendor problem with a variable leadtime and uncertain obsolescence during the leadtime. This problem would be simplified in a stationary, infinite-horizon problem. Details and consequences are left to future research.

6. Installation Tokens

The previous section illustrates RTs using the echelon stock representation, which, despite its theoretical applicability, is not commonly found in practice. By converting the examples into installation stock representations, it should be clearer how the issuance and allocation of tokens actually can work. There are two different representations, one using transient tokens and the other permanent tokens, that are illustrated using Example 1 in Exhibits 3 and 4, respectively.

Exhibit 3 Example 1, Installation Tokens (Transient)

	R (0)		W (1)		F (2)	
W	$X_0 =$ 12	$Z_0 =$ 30	$X_1 =$ 12	$Z_1 =$ 40	$X_2 =$ 12	$Z_2 =$ 50
E	$x_{02} =$ 0		$x_{12} =$ 0			
E	$x_{01} =$ 0	$z_0 =$ 12		$z_1 =$ 12		$z_2 =$ 50
K	$D =$ 20	$y_{02} =$ 0		$y_{12} =$ 28		
1	$B =$ 8	$y_{01} =$ 18				

	R (0)		W (1)		F (2)	
W	$X_0 =$ 4	$Z_0 =$ 38	$X_1 =$ 12	$Z_1 =$ 48	$X_2 =$ 50	$Z_2 =$ 20
E	$x_{02} =$ 0		$x_{12} =$ 28			
E	$x_{01} =$ 18	$z_0 =$ 12		$z_1 =$ 48		$z_2 =$ 20
K	$D =$ 20	$y_{02} =$ 26		$y_{12} =$ 0		
2	$B =$ 16	$y_{01} =$ 0				

	R (0)		W (1)		F (2)	
W	$X_0 =$ -4	$Z_0 =$ 46	$X_1 =$ 48	$Z_1 =$ 20	$X_2 =$ 22	$Z_2 =$ 20
E	$x_{02} =$ 26		$x_{12} =$ 0			
E	$x_{01} =$ 0	$z_0 =$ 46		$z_1 =$ 20		$z_2 =$ 20
K	$D =$ 20	$y_{02} =$ 0		$y_{12} =$ 0		
3	$B =$ 24	$y_{01} =$ 0				

	R (0)		W (1)		F (2)	
W	$X_0 =$ 22	$Z_0 =$ 20	$X_1 =$ 22	$Z_1 =$ 20	$X_2 =$ 22	$Z_2 =$ 20
E	$x_{02} =$ 0		$x_{12} =$ 0			
E	$x_{01} =$ 0	$z_0 =$ 20		$z_1 =$ 20		$z_2 =$ 20
K	$D =$ 20	$y_{02} =$ 0		$y_{12} =$ 0		
4	$B =$ 0	$y_{01} =$ 0				

Transient Tokens and No Upstream Backlogging

While the Retailer always backlogs unmet customer demands, the Clark/Scarf theory is conveniently presented when we assume, as we have, that upstream players do not backlog unmet orders placed by their downstream partners. That is, a player's REP (and the implied quantity ordered, Z_i) does not depend on the extent to which previous orders placed by this player were met in the past. In effect, unmet orders are placed again and again. Transient tokens correspond to this setting.

Recall that the tokens in the echelon stock representation can be interpreted as being transient, expiring as soon as they stop moving downstream. Exhibit 3 was generated by converting from echelon stocks and echelon positions to installation stocks and order quantities for physical units, and leaving tokens unchanged. That is, the identities $X_0 = x_0$ and $X_i = x_i - x_{i-1}$ for $i \geq 1$, and $Z_i = Y_i - x_i$ and $z_i = y_i - x_i$ for all i are used.

The result can be interpreted easily. The echelon tokens can be interpreted as transient installation tokens. They are issued as needed to meet orders that cannot be met fully through physical units, and expire as soon as they stop moving downstream or end a week at R. In this example, each player orders the amount that would bring her echelon inventory position up to the base stock level used in Exhibit 1. For example, R orders a different amount each week because her initial

Exhibit 4 **Example 1, Installation Tokens (Permanent)**

Week	R (0)		W (1)		F (2)	
W	$X_0 = 12$	$Z_0 = 30$	$X_1 = 12$	$Z_1 = 40$	$X_2 = 12$	$Z_2 = 50$
E	$X_{02} = 0$		$X_{12} = 0$			
E	$X_{01} = 0$	$z_0 = 12$		$z_1 = 12$		$z_2 = 50$
K	$D = 20$	$z_{02} = 0$		$z_{12} = 28$		
1	$B = 8$	$z_{01} = 18$				
W	$X_0 = 4$	$Z_0 = 20$	$X_1 = 12$	$Z_1 = 20$	$X_2 = 50$	$Z_2 = 20$
E	$X_{02} = 0$		$X_{12} = 28$			
E	$X_{01} = 18$	$z_0 = 12$		$z_1 = 48$		$z_2 = 20$
K	$D = 20$	$z_{02} = 26$		$z_{12} = -28$		
2	$B = 16$	$z_{01} = -18$				
W	$X_0 = -4$	$Z_0 = 20$	$X_1 = 48$	$Z_1 = 20$	$X_2 = 22$	$Z_2 = 20$
E	$X_{02} = 26$		$X_{12} = -26$			
E	$X_{01} = 0$	$z_0 = 46$		$z_1 = 20$		$z_2 = 20$
K	$D = 20$	$z_{02} = -26$		$z_{12} = 0$		
3	$B = 24$	$z_{01} = 0$				
W	$X_0 = 22$	$Z_0 = 20$	$X_1 = 22$	$Z_1 = 20$	$X_2 = 22$	$Z_2 = 20$
E	$X_{02} = 0$		$X_{12} = 0$			
E	$X_{01} = 0$	$z_0 = 20$		$z_1 = 20$		$z_2 = 20$
K	$D = 20$	$z_{02} = 0$		$z_{12} = 0$		
4	$B = 0$	$z_{01} = 0$				

installation stock levels differ. To achieve optimal incentives, echelon costing must still be carried out. The results are the same as in Exhibit 1. Thus, except for the orders placed each week, Exhibit 3 can be generated directly without reference to the echelon stock representation.

Permanent Tokens and Upstream Backlogging

It is also possible to work with permanent tokens. We shall see that permanent tokens correspond to the setting in which upstream players backlog unmet orders. In this case, downstream players only place orders for newly desired units, as they know their upstream partners are keeping track of any orders that may have been unmet in the past. When RTs are used, upstream players must meet unmet orders with tokens, so when adequate physical stock arrives, they must be able to send it along to cancel any previously issued tokens. We shall see that this is done by the use of *negative tokens.*

Exhibit 4 was generated by first converting to installation stocks and order quantities, as in Exhibit 3, and then considering echelon tokens to be analogous to echelon stocks and converting them into installation tokens. In particular, let X_{ij} denote the installation tokens held (at the beginning of a week) by player i and issued originally by player j, and let z_{ij} denote the installation tokens being sent during a week to player i (from player $i + 1$) that were issued by player j, for each i and $j > i$.

The following identities are used: $X_{0j} = x_{0j}$, $X_{ij} = x_{ij} - x_{i-1,j}$ for $i \geq 1$, and $z_{ij} = y_{ij} - x_{ij}$. These are analogous to the identities that turn echelon stocks and positions into installation stocks and order quantities. As in Exhibit 3, the identities $X_0 = x_0$ and $X_i = x_i - x_{i-1}$ for $i \geq 1$, and $z_i = y_i - x_i$ for all i are also used. However, instead of $Z_i = Y_i - x_i$, the identity $Z_i = Y_i - x_i - \Sigma_{j>i} x_{ij}$ is used. The reason for this latter identity is that tokens are now permanent, so tokens possessed by a player will continue to exist in the future, and are as good as physical units, so must be considered that way when a new order is placed.

The result has an interesting interpretation. There are now *negative tokens*. For example, in Week 2, $z_{01} = -18$. Because tokens are now permanent, negative tokens are required to effectively get previously issued tokens to expire. Thus, the shipment in Week 2 of 18 negative tokens from W to R, results in Week 3 with the cancellation of the 18 positive tokens that R had in Week 2. This exhibit clearly indicates that each player was understocked in Week 1, as each ordered much more than 20 then. Every week thereafter, each player orders the amount demanded in the previous week.

Again assuming the orders are as given, it is possible to generate this exhibit without generating the echelon stock representation first: Orders from player i are met by player $i + 1$ first with physical stock, then from tokens, starting with the upstream-most issuing player and working down. If necessary, she will issue tokens of her own. Each player also keeps track of the total number of tokens that she has issued and sent downstream. When she obtains sufficient physical stock to meet all of the order placed while still having issued tokens that are active downstream, she sends along as much physical stock as she can, along with an equal number of negative tokens, up to the amount of her outstanding active tokens. Thus, for example, in Week 2, F receives an order for 20 and has 50 physical units available. She has issued 28 outstanding tokens, so she is able to send along an additional 28 physical units and 28 negative tokens to eventually cancel her currently outstanding tokens. Thus, at each point, the order placed by a player is always met exactly, sometimes simply with physical units, sometimes with some physical units and the rest in tokens, and sometimes with excess physical units with a compensating number of negative tokens. Because the receiving player considers tokens to be equally as good as physical units, each order is satisfied exactly. Physical stocks and tokens move downstream in the usual way.

The cost transfers seem to be most easily accomplished through echelon costing.

Discussion

Thus, a system with permanent tokens requires negative tokens which are used to replace RTs when something else comes along to replace them. The concept of negative tokens raises an interesting possibility to increase the incentive stability of a supply chain. Recall from Example 2 that (positive) tokens provide a mechanism and incentive for upstream players to correct for downstream overordering. Negative tokens offer the opportunity to do the same thing for underordering.

In particular, each upstream player can be allowed to send more physical goods than what was ordered (by the next player downstream). The difference between the amount sent and the amount requested is now the responsibility of the upstream player, who sends negative tokens to cover that difference. Thus, any holding costs that are incurred due to those units being at a particular downstream location at the end of a period are charged directly to the upstream player. (A transfer payment is made from the upstream player to the downstream player to compensate for costs the downstream player was not responsible for.) Similarly, if these units arrive at R and are used to meet demand, then R is charged the shortage penalty costs for those units and the upstream player issuing the negative tokens is credited with that amount.

While this mechanism is unnecessary when each player acts optimally, it provides a way for upstream players to correct for downstream underordering mistakes. Both the chain and the upstream player benefit from the correction. This change can be interpreted as a form of vendor managed inventory (VMI) because the upstream player can decide to supplement the order made by the downstream player. In contrast to the usual implementation of VMI in which the upstream player makes all the decisions about downstream inventory levels, this proposal only calls for VMI on supplemental orders and the upstream player shoulders all financial responsibility for pushing those units downstream.

This proposal is straightforward to administer in a two-echelon supply chain. However, in general, there is more than one way to address whether a player must send along any negative tokens (and accompanying physical units and/or further upstream positive tokens) when they are received. Thus, further details are left to future research.

It is worth pointing out here that the accounting reallocation of costs that comes from Clark and Scarf (1960) and was clarified by Lee and Whang (1999) can dramatically increase the incentives of the downstream players to overorder by increasing the effective shortage penalty cost and decreasing the holding cost. Thus, the scheme stacks the deck in favor of inducing the furthest player downstream to overorder, if anything. That is, it may not be important to provide protection against downstream underordering.

7. Summary and Extensions

This paper can be viewed as proposing a way to implement the decentralized supply chain coordination scheme of Lee and Whang (1999), which in turn can be viewed as proposing a way to operationalize the decentralized management scheme implicit in Clark and Scarf (1960). Note that the Clark and Scarf (1960) results are valid in nonstationary environments (in which unit costs and demand distributions can change over time). Thus, the Lee and Whang (1999) approach and the use of responsibility tokens remain valid in such environments as well. There are two other interesting proposed implementation mechanisms that apply only in stationary environments.

Chen (1999) proposes the use of *accounting inventory* levels which are the installation stock inventory levels that would be experienced if upstream orders had been fully filled at the time they were received. The owner of the firm must first determine the stationary optimal base stock levels for every player. The owner then uses those levels to determine the unit backlog penalties to charge the players based on their accounting inventory levels. Accounting inventories and RTs are similar in that they both are based on all orders being fully filled. However, they differ in important ways. RTs apply in nonstationary settings and do not require a centralized player to determine the optimal policy first. The right incentives are built into the RTs in the first place. Chen (1999) shows that his approach can perform better than a centralized scheme when the players have more accurate information about the environment than does the owner/administrator. He also incorporates something that this paper does not, a positive order transmittal delay time, as in the Beer Game, the famous experiential supply chain exercise developed at MIT's Sloan School of Management (e.g., see Senge 1990 and Sterman 1989).

Cachon and Zipkin (1999) recognize that the upstream players may have an explicit, direct interest in customer backlogs. As is Chen (1999), they are interested in a decentralized scheme in which the players track only local information. Focusing on the two-player case, they propose an array of transfer payment contracts (between the players) that are based on what is local information to either one or the other player. They show in the stationary demand case that the system optimal solution is a Nash equilibrium for a wide range of contracts within their proposed array. They assert that the system optimal solution Pareto dominates any other Nash equilibrium that might exist, and, therefore, that it should be easy for the players to agree to follow it. Responsibility tokens have no beneficial role in helping to implement their approach.

Cachon (1999) reviews a variety of approaches to coordinating a supply chain, including both papers discussed above. The volume edited by Tayur et al. (1999) also includes a number of review articles on supply chain contracting, which aims, at least in part, to achieve efficient supply chain coordination.

The proposal in this paper assumes that a system administrator, presumably electronic, is established to monitor inventories and shipment quantities at all times, so that the proper rules for issuing, allocating, and destroying tokens, and making transfer payments, can be enforced. From a practical perspective, the RTs might be both electronically and physically distinguishable, perhaps using bar codes and distinct colors for each possible issuer. The approach of Lee and Whang (1999) also assumes the existence of some form of system administrator. Their administrator need not monitor the complexities of RTs, but theirs is assumed to know the expected present value of the consequences of not fully meeting an optimal order placed by each player, so that the proper penalty can be computed at the time of the order not being met, and transferred to the ordering player's account. That is, for their scheme to work optimally, their administrator must know how to optimally manage the chain.

While our administrator needn't have that knowledge, it is true that, for the chain to perform optimally using RTs, the most upstream player must know how to manage the chain optimally. For example, in order to evaluate the consequences of having to issue a transient RT, she must be able to anticipate its flow downstream. In particular, she must be able to anticipate the decisions that will be made by the downstream players in every period up until that token might arrive at the retailer. Nevertheless, there are some potential gains from RTs. For instance, in a stationary setting, it would be possible to signal the base stock levels that players are using to each upstream player. The downstream players will not object to sharing this information because their performance is not affected by what the upstream players do. Thus, each player could be given the optimal policy for the downstream players (without having to do any optimization) before optimizing over her base stock level. This approach would be likely to break down in a nonstationary setting because players would be unlikely to agree to commit to announcing their future base stock levels.

Another possible benefit of RTs is the potential for empirical learning. As players make decisions over time, they can observe the consequences of those decisions and move toward an optimal policy through trial and error. Further pursuit of this interesting avenue for further research is beyond the scope of this paper.

Another benefit has already been discussed: RTs allow a natural way for upstream players to correct for downstream overordering, without unduly penalizing the performance of the chain. This benefit is therefore also shared by Lee and Whang's (1999) theoretical transfer costing scheme.

Another benefit of RTs is that their empirical transfer costing may be easier for the players to understand and accept than the theoretical scheme. There is another interesting issue surrounding the use of the theoretical scheme. In practice, an administrator may have imperfect knowledge about the supply chain environment, may merely guess at how to optimally manage the chain, and may specify that cost transfers be carried out based on that guess. Anand and Mendelson (1997) study the more general issue of the relationship between decision rights and information structure in more depth. Chen (1999) shows that excellent chain performance can be achieved (with his model) even if the administrator makes a poor guess at an optimal policy, which is used only to establish incentives for the decentralized decision-making environment. RTs offer the potential advantage in an analogous setting of allowing the benefits of local knowledge to be reaped by the chain without requiring an administrator to guess at an optimal policy.

Cachon (1999) observes that, compared to the mechanism of Lee and Whang (1999), the use of RTs shifts the risks of actual outcomes from receivers of tokens to their issuers, because issuing a token is riskier than making a one-time transfer payment for anticipated losses. In particular, risk-averse players will not view the Lee and Whang's mechanism as equivalent to one using RTs as proposed here. Of course, further discussion of this topic is highly speculative, as the theory of optimal serial echelon inventory control has not yet been developed for the risk-averse setting.

The model in this paper assumes that the delivery lead time between players is always exactly one period (week). This model can be applied when the lead times are integer multiples of the review period as follows. Introduce an additional player for each week of the lead time beyond the first. We must assure that everything that arrives at player j continues moving downstream. Suppose that, in addition to actual costs, we could impose a unit penalty cost M on all inventory retained by player j at the end of a period. If M is sufficiently large, no system optimal solution would have any such inventory retained by player j. Node j would be a flow-through node as desired. The inventory retained by player j is $X_j - z_{j-1}$, the installation stock available to be shipped out at the beginning of the week, less the amount shipped out. By replacing k_j by $k_j - M$ and h_j by $h_j + M$, and leaving all other parameters unchanged, it is straightforward to see, by Lemma 1(a), that we have added $M(X_j - z_{j-1})$ to the objective function, exactly as desired. There is an interesting consequence to this formulation: c_{j-1}, the marginal cost paid by player $j - 1$ on her echelon position, is lowered by M so that it becomes very negative, so that she will always request (essentially) an infinite REP. c_j, the marginal cost paid by player j on her echelon position, is increased by αM, which might suggest that she would always request as little as possible. However, doing so would require sending RTs to player $j - 1$, which will each earn a credit of M more than the old c_{j-1} the following period. Discounting cancels the two changes, so that player j is the one facing the real ordering decision each period. The decision making for player $j - 1$ can be mechanized and taken over by the administrator. Clearly, everything that player j orders will be shipped out to player $j - 1$ as soon as it is received.

It should be possible to directly apply the concept of responsibility tokens to the case of serial supply chains with expediting, as modeled and analyzed by Lawson and Porteus (2000). Indeed, examples were presented by Porteus (1997a).

While the concept of responsibility tokens arose from the Clark and Scarf (1960) setting, it would be interesting to explore how well other systems can perform under the concept. For example, the Clark and Scarf theory does not apply to the multiretailer, multiwholesaler, single-factory setting. Yet RTs could be implemented in such a setting. How would such a system be managed under such an implementation? Would the resulting performance of that system be close to optimal in certain situations?

Other settings in which RTs may be worth studying are serial supply chains with either random yields or stochastic lead times between players. In the latter case, one could set a deterministic shipment time that the upstream player is responsible for meeting. If the shipment is either early or late, then he takes responsibility for the consequences. One variation of this scheme, which is already being implemented by several companies, is to specify a delivery date, with a window around it, and refuse to accept the delivery if it does not take place within the window.

This paper addresses the issue of how to align the incentives of the players in a supply chain, but it does not address the issue of sharing the gains appropriately. For example, in an existing chain, successful implementation might require that all players be better-off by adopting the new incentive system.[1]

Appendix

The following recursions are useful for updating the stock levels from week to week under the conventions of this paper:

$$x_i(t + 1) = y_i(t) - D(t),$$

for each week t and player i, where $D(t)$ is the customer demand in week t, and

$$X_i(t + 1) = X_i(t) + z_i(t) - z_{i-1}(t) \text{ for } i = 1, 2, \ldots, m.$$

When the context is clear, the index t of the week is suppressed.

Proof of Lemma 1. (a) As indicated in §2, the actual costs are as follows:

$$\sum_{i=0}^{m} k_i z_i + pB + h_0[(x_0 - D)^+ + z_0] + \sum_{i=1}^{m} h_i(y_i - y_{i-1}).$$

Using the identities $z_i = y_i - x_i$ for all i, $x = x^+ - (-x)^+$ with $x = x_0 - D$, and $B = (D - x_0)^+$, and rearranging leads to the desired result.

Part (b) follows from use of $x_i(t) = y_i(t - 1) - D(t - 1)$ for $t = 2, 3, \ldots, T + 1$, and, as in (a), reassigning costs to weeks after appropriately discounting.

(c) First, the middle terms in (b) that do not depend on any decisions are dropped. Second, $B(t)$ is re-expressed as $B(t) = [D(t - 1) + D(t) - y_0(t - 1)]^+$. Because the costs of $y_0(t - 1)$ are charged in period $t - 1$, these costs are reallocated to the previous period and discounted appropriately. There is no shortage penalty cost assigned to $y_0(T)$.

Part (d) follows because only terms that are linear in the terminal echelon stock variables are added to the redefined terminal cost function. □

Proof of Lemma 2. (a) The requested echelon position is Y_i. Of that, y_i can be met by physical units. The remainder, Y_{im}, must be met by tokens. The next resource to be used consists of tokens issued by player m, because all downstream players consider those tokens to be as good as physical units. Thus, the number of such tokens to be sent along, y_{im}, is the minimum of the remaining request, Y_{im}, and the number available to be sent, $x_{i+1,m}$. Recursively, Y_{ij} is the remaining request to be met through tokens issued by player j or further downstream, and the number of tokens issued by j to be sent,

[1]This paper is an outgrowth of presentations (Porteus 1997a, 1997b) made by the author. The author wishes to thank the attendees at those presentations, Leroy Schwarz, the senior editor, the anonymous reviewers, Hau Lee, Seungjin Whang, and especially Gérard Cachon and Prashant Fuloria, for their helpful comments and suggestions.

y_{ij}, is the minimum of that remaining request and the number available, $x_{i+1,j}$. Finally, player $i + 1$ must issue tokens of her own if there is still a remaining request to be met.

(b) The backlog $B_{m+1} = B$ incurs shortage costs in the system. However, none of the players downstream from m should be responsible for any of these until all the tokens issued by m that are in Player 0's possession are first applied. That is, player m is responsible for the minimum of that backlog, B_{m+1}, and the number of her tokens on site, x_{0m}. Once those tokens are applied, the pertinent backlog becomes B_m, as defined. The argument repeats for $i = m - 1, \ldots, 1$. Finally, all the cost transfers made to upstream players must be credited to Player 0, whose account was initially charged for all the backlog B.

(c) There are no hidden subtleties in this part. □

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