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Sourcing Strategies and Supplier Incentives for Short-Life-Cycle Goods

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Multiple sourcing with quick response has been recognized as a useful tool to manage demand risk for short-life-cycle goods. However, general wisdom has traditionally ignored the effect of these practices on supplier incentives. In this paper we find that, when suppliers make pricing decisions, dual sourcing does not always lead to higher supply chain efficiency or buyer profits as compared to single sourcing. This loss takes place when suppliers commit to prices up front, before any possible forecast change, but not when they delay the price quotes after demand forecasts have been updated. Specifically, with up-front price commitment, dual sourcing leads to inflation of supplier prices because expensive suppliers will still receive part of the business if they are sufficiently quick. Thus, when supplier prices are endogenous, double marginalization may offset the additional buyer profit enabled by higher ordering flexibility. In contrast, with delayed price quotes, a buyer will find dual sourcing beneficial because single sourcing locks it into a monopolistic supplier that extracts most of the available rent.

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1. Introduction

In many industries such as apparel or electronics, the duration of a product in the point of sales has been reduced significantly in recent times. For example, the collection-based cycle in apparel, where firms launched fall-winter and spring-summer product catalogues, has evolved into a continuous release of products in the store that do not stay in the assortment for long. Such a trend is driven by fast fashion retailers such as Inditex or H&M; see Caro and Martínez-de-Albéniz (2009). Electronics is suffering from a similar evolution, although in this case the renewal of products is driven by technology innovation.

Unfortunately, when products have a short life cycle, it becomes difficult to manage them operationally for many reasons: first, forecasting tends to be quite uncertain because little historical data are available; second, the salvage value of unsold inventory is low because a newer, more recent, or technologically superior product is being sold at that time; finally, replenishment opportunities are rare due to the short sales window. These products fall into the category of "innovative" products, see Fisher (1997), for which a "responsive" supply chain should be used. Indeed, firms have typically set up flexible supply systems to react to demand uncertainty. In particular, quick response through the use of short-lead-time suppliers has been identified as the most appropriate strategy in the apparel market; see Hammond and Kelly (1990). The practical implementations of quick response vary. In apparel, some firms such as H&M or Desigual choose to work with a single supplier and request quick delivery when needed. As cited in H&M (2007, p. 35), "The time from an order being placed until the items are in the store may be anything from a few weeks up to six months. The best lead time will vary." In contrast, firms such as Zara or Bershka of the Inditex group may choose to work with two suppliers at the same time: a base order is placed at a low cost, long-lead-time supplier, and a second order (internally termed "reprise") for a nearly identical item is issued at a local supplier if the first order sells faster than expected. Similar approaches are used in other industries, such as toy manufacturing, where a company like Famosa receives approximately 80% of production from China, and the rest from local European sources only when the Chinese production is insufficient (Lago Esteban 2007).

The existing academic literature suggests that, in the absence of set-up costs, firms should seek as much flexibility and responsiveness as possible. This implies that dual sourcing is a superior supply strategy; see, e.g., Martínez-de-Albéniz (2011). However, one should keep in mind that all these studies focus on managing demand uncertainty only; they ignore some of the practical implications of multiple sourcing, namely, that suppliers will not act in the same way when they know that they are only one of many supply sources. In particular, the lack of exclusivity should affect their pricing strategies, and it is likely that the reduction in volume awarded should lead to an increase in prices. In other words, when supplier pricing decisions are endogenous and contingent on the buyer's choice of sourcing strategy, it is no longer clear that more flexibility is necessarily better for the buyer.

In this paper, we propose a simple model of a buyer and two suppliers (one slow, the other fast, with different costs) where single- and dual-sourcing strategies are compared, taking into account endogenous supplier pricing. Demand is stochastic and the quality of the forecast improves over time, which provides the fast supplier with a lead-time advantage: its price can be higher and yet it receives buyer orders. Under single sourcing, the buyer notifies up front who the winning supplier is; orders are placed later with the latest available forecast. In contrast, under dual sourcing, the buyer does not commit volume to any supplier and orders are placed sequentially using the latest available information, i.e., how much the buyer has bought until that moment, and how the demand forecast has varied. We study these two situations under two settings: we let suppliers simultaneously submit a price per unit up front or allow the fast supplier to delay its price quote to immediately after the forecast update has been received. As a result, we obtain four scenarios: we compare single sourcing with up-front price commitment versus dual sourcing with up-front price commitment and single sourcing with delayed price commitment versus dual sourcing with delayed price commitment. These two comparisons allow us to separate the direct effect of speed from the indirect effect of informational advantage due to the timing of price decisions.

We study under each scenario how the buyer decides on its purchase quantity in response to supplier prices and how the suppliers set prices in equilibrium, of which we show existence and uniqueness under some technical assumptions. For tractability, we focus on pure-strategy equilibrium in our model and analysis. When prices are set up front, we find that dual sourcing usually softens competitive pressures and suppliers end up pricing higher (when equilibrium exists) than in single sourcing. Indeed, the fast supplier may price high and still receive orders while, under single sourcing, it would have been excluded completely. This raises the fast supplier's price and, in turn, makes the slow supplier's price increase as well. This may be detrimental for the buyer and also for the supply chain: higher prices reduce the purchased quantities and supply chain profits may be reduced. We thus show that dual sourcing leads to stronger double marginalization as compared to single sourcing, a phenomenon that is prevalent in buyer/supplier relationships (Spengler 1950). When prices are set on the spot, with knowledge of the forecast update, this effect is not present. Indeed, since the fast supplier no longer commits to prices, single sourcing is quite ineffective because the fast supplier's monopoly power at the time of ordering makes the buyer reluctant to select it. Dual sourcing mitigates this effect through the early purchases at the slow supplier that force the fast supplier to reduce prices in order to sell.

We thus demonstrate that although dual sourcing is always superior when supplier prices are exogenous, it is sometimes inferior to single sourcing when suppliers take pricing decisions. The trade-off between sourcing efficiency (higher with dual sourcing) and double marginalization losses (higher with dual sourcing too) hence determines what is better for the buyer and the supply chain. We find that, when prices are set up front, the buyer generally prefers single sourcing. Surprisingly, the supply chain also prefers single sourcing unless suppliers have very different cost structures or the reduction in demand uncertainty due to quick response is low. Our work hence shows that a seemingly inefficient operational policy can be superior when supplier/buyer incentives are affected. This contributes to the literature on sourcing and complements similar effects in inventory management (Anand et al. 2008), production flexibility (Goyal and Netessine 2007), or disruption management (Babich et al. 2007).

The rest of this paper is organized as follows. Section 2 reviews the literature. Section 3 presents the model with the single- and dual-sourcing scenarios. Section 4 discusses one representative example of the outcomes of the model and then the general analysis is provided in §5. Managerial insights are discussed in §6. We conclude the paper in §7. All the proofs and some additional materials are included in the appendix.

2. Literature Review

One of the main questions in our model is how the buyer should use demand updates to place orders at the slow and fast supplier. When prices are fixed, this is a standard multiperiod inventory problem with nonstationary demand; see Zipkin (2000) for a review. To solve it, an important modeling element is the demand forecast process: Graves et al. (1986) and Heath and Jackson (1994) propose dynamic models of forecast updates. The impact of quick-response ordering on the system is studied in Iyer and Bergen (1997). Fisher and Raman (1996) optimize the two-order problem, with an application to the Sport Obermeyer case (Hammond and Raman 1996).

Fisher et al. (2001) develop a heuristic and apply it at a catalog retailer. A more general analysis of multiple sourcing is provided in Song and Zipkin (2012). It is worth pointing out that in all these cases, finding the optimal orders is tractable because demand appears after all orders have been placed. When this is not so, the problem is notoriously difficult to solve because base-stock policies are generally not optimal anymore; see, Fukuda (1964), Whittemore and Saunders (1977), and Feng et al. (2006). Heuristics are then applied; see, e.g., Veeraraghavan and Scheller-Wolf (2008) and Allon and van Mieghem (2010).

The literature on supplier/buyer competition is perhaps the most relevant to this work. There are two interaction types that must be differentiated: vertical and horizontal competition.

First, the vertical relationship between buyer and supplier has been studied since Spengler (1950). When a supplier sets a wholesale price and the buyer responds setting a retail price or an order quantity, in equilibrium, prices tend to be too high and quantities too low with respect to what would be best for the supply chain. This constitutes the double marginalization phenomenon. A model that is particularly close to our paper is Lariviere and Porteus (2001), who identify regularity conditions on the demand so that the problem is well behaved. Similar conditions are provided in van den Berg (2007). The single-period model has been extended to multiple periods in Anand et al. (2008), Erhun et al. (2008), and Martínez-de Albéniz and Simchi-Levi (2013): In a dynamic setting, the buyer carries positive inventory to put a pressure on the supplier to reduce prices. This makes inventory "strategic" because it is a lever for the buyer to reduce supplier pricing power. We find a similar effect in one of our dual-sourcing scenarios.

Second, suppliers compete horizontally for buyer orders. Pricing games often exhibit a Bertrand-type behavior, with a "race to the bottom" where one supplier prices at cost and drops out from the competition, as in one of our single-sourcing scenarios. Vives (2001) provides a good textbook overview of this broad literature. When multiple sourcing is allowed, the competitive equilibrium becomes more complex because one must decide how to split orders between the suppliers. Some papers study the pricing equilibrium for the simultaneous game. Babich et al. (2007) consider pricing in the presence of supplier default risk and identify default correlation as a major driver of competitive intensity. Martínez-de-Albéniz and Simchi-Levi (2009) study pricing when suppliers offer price and flexibility contracts and find an equilibrium structure in which suppliers are clustered. Martínez-de-Albéniz (2005) analyzes a similar problem to one of our dual-sourcing scenarios, but for an infinite horizon with repeated procurement from both suppliers; he characterizes the equilibrium when it exists. Furthermore, there is also some literature for the sequential game. Mantin et al. (2011) discuss the sequential pricing equilibrium in a price-skimming situation, where buyers visit firms one after the other. Li and Debo (2009) study the value of adding a second supplier in a two-period auction setting, where the incumbent is alone in the first period and faces competition from an entrant in the second period. They find that the incumbent increases its initial price compared to sole sourcing. We reach a similar conclusion although our model is very different: in our case, competition may be sequential rather than simultaneous due to the lead-time difference, orders are placed instead of capacity installations, and the buyer accepts supplier prices rather than screening supplier types, as occurs in their case.

3. The Model

3.1. Setting

Consider a firm, the *buyer*, that brings a product to market at a unit price $r=1$ fixed before procurement prices are realized. That is, we focus on a firm whose pricing process is market driven rather than cost plus and hence does not translate variations in procurement costs to retail prices. This is a realistic assumption in most retail settings with short sales windows, e.g., at Zara: there, prices are set at the product design stage and will not be changed until the discount season, where the salvage value is recovered. Note, however, that, as discussed at the end of §4.2, our insights continue to hold qualitatively when demand is price dependent and the buyer can change r after procurement prices are set.

The sales window is short and, after it is finished, the firm will have to sell the item at a loss, at a salvage value set to $s=0$ (this includes any excess inventory charges), although any other salvage can be easily incorporated into our model. The number of customers willing to purchase the product is unknown prior to the selling season. Forecasts are available and become more reliable as the time at which they are generated approaches the beginning of the sales window. These are used by the buyer to place production orders to *two suppliers*.

The suppliers have different characteristics. Each supplier offers a lead time L_i and quotes a unit wholesale price p_i; it also incurs a different unit production cost c_i, $i=1,2$. Without loss of generality, we assume that supplier 1 is *slower* than supplier 2, i.e., $L_1 \geq L_2$. Costs and prices can be ordered arbitrarily, but in practice shorter lead times are usually associated with higher production costs and higher wholesale prices.

For simplicity, we assume that the duration of the sales season is negligible compared to the lead times

L_1, L_2, and hence it is not possible to place replenishment orders after demand has been realized. The buyer thus faces the trade-off between sourcing from an off-shore, low-price supplier and another local, more costly supplier. Specifically, longer lead time results in less accurate demand forecasts and hence higher chances of demand and supply mismatches. The buyer thus needs to determine whether it is worth waiting for better demand forecasts despite the higher prices quoted by the fast supplier.

We are interested in determining how buyer and suppliers will interact in this setting. Specifically, suppliers will set unit wholesale prices $p_i \geq c_i$, and the buyer will place production orders of $q_i \geq 0$ units to supplier i, $i = 1, 2$. Each player will take its decision to maximize its expected profit. Clearly, the buyer's decisions will depend on the prices quoted by the suppliers. In turn, the suppliers will anticipate these decisions to set their prices in their best interest. Our objective is to establish what sourcing and pricing decisions will arise in equilibrium (if one exists), i.e., when neither the buyer nor the suppliers have an incentive to unilaterally deviate from their strategies. We will hence make use of game-theoretical tools and in particular focus on pure strategies only, i.e., when suppliers set deterministic wholesale prices. Note that in the appendix we explore numerically the structure of a mixed strategy equilibrium in an instance where no pure-strategy equilibrium exists.

3.2. Sourcing Strategies, Pricing Schemes, and Information Sharing

3.2.1. Sourcing Strategies. We first describe how the buyer takes sourcing decisions. In particular, it is important to distinguish between two types of sourcing strategies:

- *Single sourcing* (SS). The buyer concentrates its allocation into one unique supplier; i.e., *either* q_1 *or* q_2 can be positive.
- *Dual sourcing* (DS). The buyer divides its allocation among both suppliers; i.e., *both* q_1 *and* q_2 can be positive.

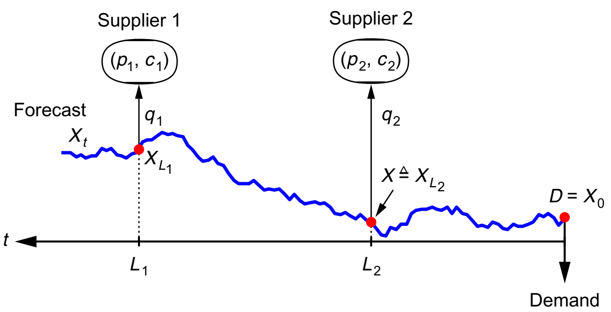
In the single-sourcing regime, the buyer commits all the production to only one supplier. In contrast, under dual sourcing, the buyer can place orders to both suppliers. Such an approach allows the buyer to reduce demand risk and mismatch costs by having the possibility of placing some late orders in case demand forecasts are revised upward. This is the case for some companies in apparel, where supplier setups can be simple and inexpensive. There, a retailer might source the bulk of its purchases at a low-price, slow supplier and, after improving its demand forecast, complement the first order with additional purchasing at a second supplier with higher price but quick delivery. A well-known application of this practice was conducted at Sport Obermeyer (see Hammond and Raman 1996). Another illustration can be found at the fast-fashion retailer Zara. For products where sales expectations increase significantly, Zara usually places additional purchase orders with quick-response suppliers; these are new orders for quasi-identical designs, manufactured by new suppliers. More generally, in industries where dealing with more than one supplier is simple, multiple sourcing may help a buyer achieve higher service levels (due to additional supply when demand forecast is revised upward) and lower excess inventory to be sold at a loss (due to lower early purchases in anticipation of later orders).

The focus in this paper is on firms that can freely choose between single and dual sourcing, i.e., in industries where the economic burden of managing multiple suppliers for the same product type is small compared to the cost of production, e.g., in apparel retailing. Our objective is to study the impact of the sourcing strategy on supplier incentives and, in particular, to demonstrate that, in some circumstances, having more flexibility is not necessarily better for the buyer and the supply chain.

In our model, the choice between single and dual sourcing must be planned up front because of the need of prior activities or investments (e.g., molds, production quality certifications). Thus, a buyer cannot decide to use dual sourcing unless it has committed to doing so before interacting with the suppliers. For example, at the toy producer Famosa (see Lago Esteban 2007), many new products are introduced every year. For each one, an investment in extrusion molds is necessary at each supplier. These molds may be expensive and need to be prepared before production takes place. Hence, Famosa decides up front whether to make one or two molds before negotiating with suppliers. After this decision is made, molds are sent to one or two suppliers and production quantities are determined. Finally, note that, if no credible commitment to single sourcing can be made, from a supplier's standpoint the situation is equivalent to having the buyer opting for dual sourcing.

Regardless of the sourcing strategy adopted by the buyer, the actual orders q_1, q_2 can be placed at different moments in time, as shown in Figure 1. By defining t as the time left before the start of the selling season and using X_t to denote the demand forecast available at that instant, the order to supplier 1, q_1, can be placed as late as $t = L_1$, when X_{L_1} is available. In contrast, the order to supplier 2, q_2, can be placed later, at $t = L_2$, after the better demand forecast X_{L_2} is incorporated. Finally, customer demand, denoted $D \geq 0$, is revealed at $t = 0$.

Figure 1 (Color online) The Sourcing Model: Supply Opportunities, Forecast Evolution, and Demand Realization



3.2.2. Pricing Schemes. Because the timing of orders differs, the timing of endogenous price quotes may impact significantly suppliers' behavior and the corresponding competitive dynamics. To isolate the sole effect of the number of suppliers on the buyer's profit, we compare SS versus DS under the following two pricing schemes:

- *Up-front price commitment* (UPC). Both suppliers quote prices p_1 and p_2 up front, at $t = L_1$, and honor them throughout. This requires supplier 2 to commit to a price and not renegotiate it at $t = L_2$. If this was not the case, the buyer should realize that suppliers will behave as in the *delayed price commitment* scheme.
- *Delayed price commitment* (DPC). Supplier 1 quotes p_1 at $t = L_1$, whereas supplier 2 quotes p_2 at $t = L_2$.

Sometimes, singular buyer-supplier bonds give rise to enduring relationships. As stated in H&M (2009, p. 26), "We strive for long-term relationships with our suppliers. When it comes to production rate, delivery reliability, efficiency and price, we work with our suppliers to create production plans spanning several years." In these situations, the buyer's promise of long-term business is often matched with supplier quotes that are rather stable irrespective of prospective volumes: this form of integration is what we model using the *up-front price commitment* scheme.

In contrast, other buyers may choose not to commit at all with suppliers over the long term. In this case, the (possibly changing) supply base is managed on an opportunistic basis and, as a result, suppliers quote prices on the spot leveraging the latest supply and demand information available. This lack of integration is what we model using the *delayed price commitment* scheme.

3.2.3. Information Sharing. To enable rational pricing by the suppliers, the buyer shares the following information with them:

- $t = L_1$. The buyer observes X_{L_1} and shares its prior demand knowledge, i.e., the distribution of $D \mid X_{L_1}$, with both suppliers.
- $t = L_2$. The buyer observes X_{L_2} and updates its demand knowledge, i.e., the distribution of $D \mid X_{L_2}$. In case of DPC, the updated distribution of excess demand, $\max\{D - q_1, 0\} \mid X_{L_2}$, is shared with supplier 2. This is equivalent to revealing both the forecast X_{L_2} and the volume of early purchases, q_1, if any have been made.[1]
- $t = 0$. Customer demand D is revealed, which coincides with the (assumed unbiased) available forecast X_0.

Note that we assume that the buyer truthfully shares the demand forecasts with the suppliers. In particular, the latest forecast available is shared with supplier 2 at $t = L_2$. If this was not the case and no additional information was given to this supplier, then the pricing decision under DPC would boil down to that of UPC. In this sense, full forecast sharing allows us to capture the different dynamics that arise in *spot* pricing as opposed to *ex ante* pricing. Whether the buyer could benefit from manipulating the information shared with suppliers to reduce their prices is out of the scope of this paper.

Since all decisions are taken knowing X_{L_1}, we omit it from the formulation in the remainder of the paper and define $X \triangleq X_{L_2}$ to simplify notation. We hence denote the cumulative distribution function (c.d.f.) of the demand prior, the conditional demand, and the forecast at $t = L_2$ by F_D, $F_{D\mid X}$, and F_X, respectively. Similarly, we shall use f and $\bar{F}$ with the right subscript to denote the corresponding probability density function (p.d.f.) and complementary c.d.f. Finally, $(x)^+$ denotes $\max\{x, 0\}$.

3.3. Scenarios

Given the sourcing strategies and pricing schemes described in §3.2, we will study the buyer and suppliers' behavior in the following four scenarios whose sequence of events is described in Table 1:

- Scenario I: Single sourcing with up-front price commitment.
- Scenario II: Single sourcing with delayed price commitment.
- Scenario III: Dual sourcing with up-front price commitment.
- Scenario IV: Dual sourcing with delayed price commitment.

In Scenarios I and III prices are committed up front, at $t = L_1$, whereas any order placed to supplier 2 can be delayed until $t = L_2$. Since the forecast X provides better information about final demand, this quantity postponement allows the buyer to reduce mismatch costs, whereas UPC prevents supplier 2 from pricing

[1] This assumption is usually adopted in the quick-response literature possibly because, in practice, in the negotiation with a faster supplier, buyers typically share the history of the item to be purchased (design by the slow supplier, purchased quantities, sales to date, etc.).

Table 1 Sequence of Events for Each Scenario

	I. SS + UPC	II. SS + DPC	III. DS + UPC	IV. DS + DPC
$t = L_1$				
(1)	p_1, p_2 are quoted	p_1 is quoted	p_1, p_2 are quoted	p_1 is quoted
(2)	Buyer chooses supplier	Buyer chooses supplier		
(3)	Buyer orders q_1 if supplier 1 is chosen	Buyer orders q_1 if supplier 1 is chosen	Buyer orders q_1	Buyer orders q_1
$t = L_2$				
(4)		X and q_1 are shared		X and q_1 are shared
(5)		p_2 is quoted		p_2 is quoted
(6)	Buyer orders q_2 if supplier 2 is chosen	Buyer orders q_2 if supplier 2 is chosen	Buyer orders q_2	Buyer orders q_2
$t = 0$				
(7)	Demand D is realized	Demand D is realized	Demand D is realized	Demand D is realized

opportunistically once X is revealed. However, the fast supplier can still leverage its lead-time advantage by charging a relatively higher price $p_2 > p_1$ while keeping business. As a consequence, the slow supplier is forced to set a lower p_1 to offset this advantage if it does not want to lose too much volume. In Scenario I, where SS is enforced, only one supplier is selected and thus both suppliers have an incentive to reduce prices. This results in a Bertrand-like pricing equilibrium, as we demonstrate later. In contrast, in Scenario III, where DS is allowed, competition intensity is relaxed because the threat to supplier 2 of losing all the business is not credible anymore. This generally leads to higher prices that, because of double marginalization, may end up reducing the buyer and supply chain's profits.

In Scenarios II and IV, prices are quoted on the spot using the latest supply and demand information. This allows the suppliers to extract a higher rent because now supplier 2 can behave monopolistically at $t = L_2$—the buyer does not have any other supply opportunity left—and, anticipating this, supplier 1 can relax its price quote and still get volume. Thus, the slow supplier becomes the leader (first mover) in the pricing game, and the fast supplier acts as a follower. In Scenario IV, where DS is allowed, supplier 2 will set p_2 knowing how much was already purchased at supplier 1, q_1, and the updated forecast X. As a result, the fast supplier can either make zero profits when $D \mid X$ is likely to be low with respect to q_1 or capture a high share of them, at the expense of the buyer, when $D \mid X$ is expected to be high. The buyer will try to use these dynamics in its favor by placing orders at the slow supplier, even at a high price, so as to reduce p_2. In some sense, these price-inflating interactions are similar to the use of *strategic inventory*, where a buyer might find optimal to carry inventory, even when it is not needed, to force suppliers to reduce their prices (see Anand et al. 2008). Note that Scenario II is a particular case of Scenario IV where either q_1 is zero—and supplier 2 exerts full monopoly power—or supplier 1 takes all the volume by cutting down p_1 significantly.

Pursuing flexibility, the buyer may be tempted to choose dual sourcing. Indeed, a quick response opportunity enables a reduction of mismatch costs when prices are exogenous. However, if the suppliers are allowed to take pricing decisions, their strategic interactions may result in inflated purchasing costs for the buyer that can make dual sourcing an inferior alternative. We next illustrate how these trade-offs depend on the timing of commitment using a simple example.

4. An Example

We are now ready to analyze the buyer and suppliers' behavior under the single- and dual-sourcing regimes. To illustrate the competitive tensions in both settings, we consider first a simple example where, for some $L_1 > 0$, the demand prior is uniform, i.e., $D \sim \mathcal{U}[0, 2]$ and $\mathbb{E}\{D\} = 1$. We study the case of $L_2 = 0$, i.e., when the lead-time advantage of supplier 2 is the strongest possible. We define $p_1^{scen}, p_2^{scen}, q_1^{scen}$, and q_2^{scen} as the equilibrium prices and quantities in Scenarios $scen =$ I, II, III, and IV; B^{scen}, S_1^{scen}, and S_2^{scen} denote the corresponding buyer, and supplier 1 and supplier 2's expected profit; and $\Pi^{scen} = B^{scen} + S_1^{scen} + S_2^{scen}$ is the total supply chain (system) expected profit.

In what comes next, most of the analysis has been omitted to streamline the presentation of the results. The interested reader is referred to the appendix.

4.1. Analysis

Because $L_2 = 0$, supplier 2 is able to respond in real time at $t = 0$ upon observation of $X = D$. Therefore, there is no uncertainty at all about demand once X is revealed at $t = L_2 = 0$.

Scenario I: *SS* + *UPC*. Given p_1, p_2 at $t = L_1$, the buyer chooses the supplier yielding the largest average profit in a winner-takes-all manner. Each supplier may be tempted to quote its monopoly price p_i^m, i.e., the price that would maximize S_i^{I} in the absence of competition (here $p_1^m = (1 + c_1)/2$, $p_2^m = 1$). However, that may be too risky because an aggressive price quote from the competitor could throw it out of business. Thus, the best strategy of supplier i is to offer the price p_i that maximizes S_i^{I} subject to $q_i^{\mathrm{I}} > 0$, which will result in capping p_i^m sometimes. This incentive to reduce prices is reminiscent of Bertrand competition, but cost is not the only differentiator here: the fast supplier enjoys a lead-time advantage, and hence when $p_2 = p_1$ supplier 2 is always chosen. The equilibrium prices can be written as follows:

- When $(1-c_1)^2 \geq (1-c_2)$, supplier 1 is chosen and $p_1^{\mathrm{I}} = \min\{p_1^m, 1-\sqrt{1-c_2}\}$.
- Otherwise, supplier 2 is chosen and $p_2^{\mathrm{I}} = 1 - (1-c_1)^2$.

Note that the lead-time advantage of supplier 2 can offset a larger cost because this supplier is preferred as long as $c_2 \leq c_1 + c_1(1-c_1)$. The intuition behind equilibrium prices is that, if the buyer chooses supplier i when both suppliers price at cost, this supplier should quote p_i^m only when it is below $p_i^{\max}$ where $(p_1^{\max}, p_2^{\max}) = (1-\sqrt{1-c_2}, 1-(1-c_1)^2)$, the price above which the buyer would switch to a competitor pricing at cost.

Scenario II: *SS* + *DPC*. The analysis of this scenario is much simpler: supplier 2 is never chosen because it would always quote $p_2^m = 1$ at $t = L_2$. Anticipating this, supplier 1 sets $p_1^{\mathrm{II}} = p_1^m$ to maximize profits (p_2^{II} is hence irrelevant).

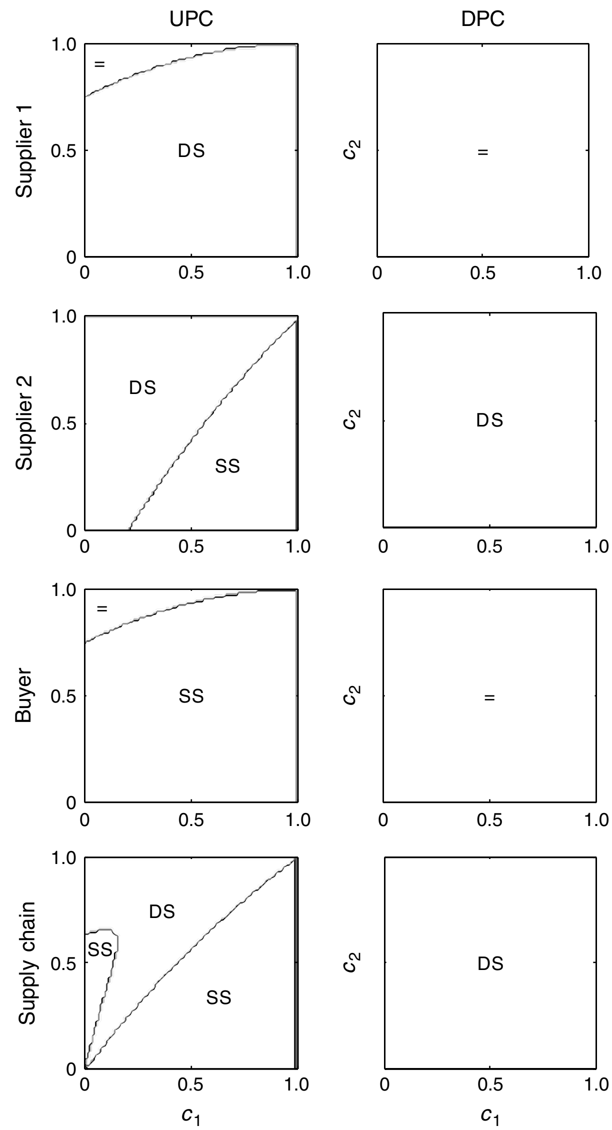
Scenario III: *DS* + *UPC*. At $t = L_2 = 0$, the buyer will use supplier 2 to top up demand when $X > q_1$; i.e., $q_2(X) = (X-q_1)^+$ regardless of p_2. After some analysis, one can realize that the best response of supplier 2 consists of two alternatives: (i) quote $p_2^m = 1$ to cash in future excess demand or (ii) quote p_1 to kick supplier 1 out of the market. It can be shown that, in this example, quoting p_2^m at $t = L_1$ always provides supplier 2 with larger expected profits at the end of the season and therefore $p_2^{\mathrm{III}} = 1$, which drives supplier 1 to quote $p_1^{\mathrm{III}} = p_1^m$.

Scenario IV: *DS* + *DPC*. As in Scenario III, the buyer uses supplier 2 to top up demand when $X > q_1$. Since q_2 does not depend on p_2, supplier 2 always quotes $p_2^{\mathrm{IV}} = 1$ at $t = L_2$ and, anticipating this, $p_1^{\mathrm{IV}} = p_1^m$ as well.

4.2. Comparison

We explore the different scenarios to understand how the choice of single- versus dual-sourcing impacts the profits of all the players and the supply chain. We consider different values of c_1 and c_2 between 0 and 1. For each possible combination of (c_1, c_2), we compare S_1^{scen}, S_2^{scen}, B^{scen}, and Π^{scen}. Specifically, we compare the profits of $scen = \mathrm{III}$ versus those of I (UPC) in the left-hand plots of Figure 2; we compare those of $scen = \mathrm{IV}$ versus those of II (DPC) in the right-hand plots of Figure 2. Note that the discussion in this section follows from the analytical characterization of prices, quantities, and profits in equilibrium, derived in §4.1.

Figure 2 **Comparison of Suppliers, Buyer, and Supply Chain Profits for Different Values of c_1, c_2**



Notes. The left-hand figures refer to UPC and the right-hand figures refer to DPC. In each plot, we show the regions where dual sourcing (DS) and single sourcing (SS) are preferred by each of the players. We use "=" to denote indifference between both sourcing strategies.

UPC. In this case, single sourcing (Scenario I) is better for the buyer than dual sourcing (Scenario III), unless c_2 is very high. When c_2 is not very high and the buyer enforces single sourcing, supplier 1 is forced to reduce prices if it wants to stay in business. If this is not possible because supplier 1 reaches its cost, it will quote c_1 and thus limit the price quoted by supplier 2. Essentially, when both suppliers *are in the game*, the Bertrand-like competition of single sourcing moderates supplier prices, which boosts the buyer's profit. On the other hand, a buyer that dual sources here suffers from inflated prices (both suppliers quote their monopoly prices) because of the softer competition intensity of this strategy. As a result, the buyer's profit is hurt. When c_2 is very high, the buyer is indifferent between both sourcing strategies. This is because c_2 is so high that supplier 1 can afford to quote its monopoly price in single sourcing and still keep business.

As per the supply chain profit, the effect of dual sourcing may be positive or negative. In particular, when c_2 is lower than c_1 the supply chain is worse off with dual sourcing because a slower supplier with higher cost (supplier 1) is added to the mix.

DPC. Here, the buyer and supplier 1 are indifferent between both sourcing strategies. In single sourcing with spot pricing, the buyer ends up locking itself in with supplier 1, who anticipates the intention of supplier 2 of usurping all profits at $t = L_2$ and hence it is able to set its monopoly price and stay in business. Thus, as supplier prices coincide both in single and dual sourcing, the profits of the buyer and supplier 1 stay the same. As per supplier 2 and the supply chain, they both benefit from dual sourcing because, with high demand, the buyer will place late orders (even at zero profit) in order not to lose unit sales.

This example shows that even when there are no set-up costs associated with a second supplier, a seemingly beneficial strategy that provides sourcing flexibility (dual sourcing) may have negative effects for the buyer and the supply chain under UPC. These results are robust to the buyer being able to vary retail prices *after* revelation of suppliers' prices in a setup with price elasticity of demand. With fixed retail prices, the double marginalization that arises in dual sourcing causes thinner unit margins for the buyer. With variable retail prices, it causes lower unit sales as r increases to preserve margins. If demand is highly elastic, this is not possible and thus single sourcing emerges again as dominant.

In the remainder of the paper, we examine how these insights extend under general conditions of cost, lead time, demand distribution, and forecast quality.

5. General Results

We generalize in this section the results obtained in §4: we consider arbitrary lead times and demand/forecast distributions. We shall subsequently study the implications of these factors on the optimal purchasing policy of the buyer and the existence (and uniqueness) of equilibria in the pricing game of the suppliers.

5.1. Scenario I: SS + UPC

In this scenario, production is committed only to the supplier yielding the largest expected profit to the buyer. Hence, the profit of the buyer amounts to $B^{\mathrm{I}} = \max\{B_1^{\mathrm{I}}, B_2^{\mathrm{I}}\}$, where

$$B_1^{\mathrm{I}} = \max_{q_1 \geq 0} \mathbb{E}_D\{\min\{D, q_1\}\} - p_1^{\mathrm{I}} q_1 \tag{1}$$

is the profit achieved if the slow supplier is selected, in which case the optimal quantity should satisfy $\bar{F}_D(q_1^{\mathrm{I}}) = p_1^{\mathrm{I}}$ and $B_2^{\mathrm{I}} = \mathbb{E}_X\{B_2^{\mathrm{I}}(X)\}$, where

$$B_2^{\mathrm{I}}(x) = \max_{q_2 \geq 0} \mathbb{E}_D\{\min\{D, q_2\} \mid X = x\} - p_2^{\mathrm{I}} q_2 \tag{2}$$

is the profit achieved if the fast supplier is selected, in which case the optimal quantity should satisfy $\bar{F}_{D\mid X}(q_2^{\mathrm{I}}(x) \mid x) = p_2^{\mathrm{I}}$. Note that because of the up-front commitment, prices are quoted at $t = L_1$, and orders to the fast supplier, if any, are placed at $t = L_2$. This is why the future forecast evolution must be averaged out in B_2^{I}.

Given that a supplier is selected, the price quoted by this supplier maximizes its profit $S_i^{\mathrm{I}} = (p_i^{\mathrm{I}} - c_i) q_i^{\mathrm{I}}$, $i = 1, 2$, where $q_2^{\mathrm{I}} = \mathbb{E}_X\{q_2^{\mathrm{I}}(X)\}$. In the absence of competition, suppliers quote their monopoly prices, which are defined as

$$p_1^m = \arg\max_{c_1 \leq p_1 \leq 1} (p_1 - c_1) \bar{F}_D^{-1}(p_1), \tag{3}$$

$$p_2^m = \arg\max_{c_2 \leq p_2 \leq 1} (p_2 - c_2) \mathbb{E}_X\{\bar{F}_{D\mid X}^{-1}(p_2 \mid X)\}. \tag{4}$$

The pricing problem of the monopolist suppliers is that of selling to the newsvendor (see Lariviere and Porteus 2001) with a subtlety: supplier 2 is forced to quote at $t = L_1$ a price that works *on average* at $t = L_2$ (i.e., a price that works well once X is revealed at $t = L_2$). For this reason, while the maximization in (3) is well-behaved (unimodal objective and unique solution) for all demand priors with increasing generalized failure rate (IGFR),[2] the maximization involved in (4) requires that the density of $D \mid X$ is log-concave for all values of $X = x$ to be well-behaved.[3] This covers a broad range of demand distributions such as normal, log-normal, uniform, gamma with shape parameter larger than one, etc.

We are interested in determining the Nash equilibrium of the game: the prices $(p_1^{\mathrm{I}}, p_2^{\mathrm{I}})$ such that no supplier has an incentive to unilaterally deviate from its strategy, i.e., to change its price away from p_i^{I}.

Theorem 1. *When D is IGFR and $f_{D\mid X}$ is log-concave for all values of $X = x$, the pricing game of Scenario* I *has a unique equilibrium. In equilibrium:*

- *If*

$$\int_{c_1}^{1} \bar{F}_D^{-1}(y)\, dy \geq \mathbb{E}_X\left\{\int_{c_2}^{1} \bar{F}_{D\mid X}^{-1}(y \mid X)\, dy\right\}, \tag{5}$$

supplier 1 is selected and $q_2^{\mathrm{I}} = 0$; $p_1^{\mathrm{I}} = \min\{p_1^m, p_1^{\max, \mathrm{I}}\}$ and $q_1^{\mathrm{I}} = \bar{F}_D^{-1}(p_1^{\mathrm{I}})$, where $\int_{p_1^{\max,\mathrm{I}}}^{1} \bar{F}_D^{-1}(y)\, dy = \mathbb{E}_X\{\int_{c_2}^{1} \bar{F}_{D\mid X}^{-1} \cdot (y \mid X)\, dy\}$.

[2] A distribution with p.d.f. f and complementary c.d.f. $\bar{F}$ is said to be IGFR when $xf/\bar{F}$ is nondecreasing.

[3] A distribution with p.d.f. f is said to be log-concave when $\log(f)$ is concave.

- *Otherwise, supplier* 2 *is selected and* $q_1^{\mathrm{I}} = 0$; $p_2^{\mathrm{I}} = \min\{p_2^m, p_2^{\max,\mathrm{I}}\}$ *and* $q_2^{\mathrm{I}}(x) = \bar{F}_{D|X}^{-1}(p_2^{\mathrm{I}} \mid x)$ *where* $\int_{c_1}^1 \bar{F}_D^{-1}(y)\,dy = \mathbb{E}_X\{\int_{p_2^{\max,\mathrm{I}}}^1 \bar{F}_{D|X}^{-1}(y \mid X)\,dy\}$.

The structure of the equilibrium prices is such that, regardless of the underlying dynamics that govern forecast evolution, an *expensive* supplier is always pushed out of the market. Expensive here means that the unit production cost of a supplier is larger than the buyer's indifference price when its competitor prices at cost, defined as $p_i^{\max,\mathrm{I}}$. Thus, an expensive supplier will reduce its price until it reaches cost, which will enable the competitor to quote the monopoly price p_i^{m}, if that is not *expensive*, or stick to $p_i^{\max,\mathrm{I}}$ otherwise. As a consequence, $p_i^{\mathrm{I}} \le p_i^{\mathrm{m}}$, $i = 1, 2$. Furthermore, if $c_2 \le c_1$, the slow supplier is always expensive (i.e., $p_1^{\max,\mathrm{I}} < c_1$) and the fast supplier is always selected. Note that the competitive dynamics have a Bertrand flavor because suppliers always have an incentive to reduce prices until one of them reaches its cost and exits competition. However, the winner of such race to the bottom is not necessarily the supplier with the lowest production cost, but the one that offers the best balance between cost and lead time.

It is worth pointing out that as long as the information structure remains the same (i.e., the suppliers share the same demand prior and quote prices simultaneously), all the results can be extended to the case of n suppliers. Essentially, price competition still remains Bertrand-like but, again, both cost and lead time influence equilibrium prices.

5.2. Scenario II: SS + DPC

As in Scenario I, in Scenario II production is committed only to one supplier. However, prices are no longer committed up front. Thus, the buyer's profit changes to $B^{\mathrm{II}} = \max\{B_1^{\mathrm{II}}, B_2^{\mathrm{II}}\}$, where B_1^{II} has the same functional form as B_1^{I} defined in Equation (1), $B_2^{\mathrm{II}} = \mathbb{E}_X\{B_2^{\mathrm{II}}(X)\}$ and

$$B_2^{\mathrm{II}}(x) = \max_{q_2 \ge 0} \mathbb{E}_D\{\min\{D, q_2\} \mid X = x\} - p_2^{\mathrm{II}}(x) q_2. \tag{6}$$

Note that, unlike Equation (2), the pricing decision of supplier 2 is now delayed until $t = L_2$ and hence depends on $X = x$.

Suppliers take pricing decisions to maximize their profits but submit their quotes at different time instants. At $t = L_1$, supplier 1 quotes p_1^{II} to maximize $S_1^{\mathrm{II}} = (p_1^{\mathrm{II}} - c_1) q_1^{\mathrm{II}}$. At $t = L_2$, upon observing $X = x$ and only if $q_1^{\mathrm{II}} = 0$, supplier 2 quotes $p_2^{\mathrm{II}}(x)$ realizing an expected profit of $S_2^{\mathrm{II}} = \mathbb{E}_X\{(p_2^{\mathrm{II}}(X) - c_2) q_2^{\mathrm{II}}(X)\}$. The optimal price of supplier 2 can thus be defined as

$$p_2^{\mathrm{II}}(x) = \arg\max_{c_2 \le p_2 \le 1} (p_2 - c_2) \bar{F}_{D|X}^{-1}(p_2 \mid x). \tag{7}$$

The next theorem characterizes the pricing strategies in equilibrium.

Theorem 2. *When* D *and* $D \mid X$ (*for all values of* $X = x$) *are IGFR, the pricing game of Scenario* II *has a unique equilibrium. In equilibrium*:

- *If*

$$\int_{c_1}^1 \bar{F}_D^{-1}(y)\,dy \ge \mathbb{E}_X\left\{\int_{p_2^{\mathrm{II}}(X)}^1 \bar{F}_{D|X}^{-1}(y \mid X)\,dy\right\}, \tag{8}$$

supplier 1 *is selected and* $q_2^{\mathrm{II}} = 0$; $p_1^{\mathrm{II}} = \min\{p_1^m, p_1^{\max,\mathrm{II}}\}$ *is quoted at* $t = L_1$ *and* $q_1^{\mathrm{II}} = \bar{F}_D^{-1}(p_1^{\mathrm{II}})$, *where* $\int_{p_1^{\max,\mathrm{II}}}^1 \bar{F}_D^{-1}(y)\,dy = \mathbb{E}_X\{\int_{p_2^{\mathrm{II}}(X)}^1 \bar{F}_{D|X}^{-1}(y \mid X)\}\,dy$.
- *Otherwise, supplier* 2 *is selected and* $q_1^{\mathrm{II}} = 0$; $p_2^{\mathrm{II}}(x)$ *is quoted at* $t = L_2$ *and* $q_2^{\mathrm{II}}(x) = \bar{F}_{D|X}^{-1}(p_2^{\mathrm{II}}(x) \mid x)$.

Although the pricing strategy of supplier 1 is very similar to that of Scenario I, the competitive dynamics are radically different here. At $t = L_2$ and in case $q_1^{\mathrm{II}} = 0$, supplier 2 is entitled with the monopoly of late purchases and quotes $p_2^{\mathrm{II}}(x)$ from Equation (7) to extract the highest possible rent given $X = x$. Since now the price of supplier 2 has to behave well *given* X and not *on average*, the regularity constraints imposed on $D \mid X$ in Theorem 2 relax to IGFR instead of log-concave, as it was in Theorem 1. Intuitively, $p_2^{\mathrm{II}}(x)$ will be high when $D \mid X$ is likely to be large (because the buyer is willing to buy a larger quantity) and low in the opposite case (because the risk of holding stock is larger for the buyer). Hence, the pricing power of supplier 2 reduces the buyers' profit in large demand scenarios and mitigates the buyer's losses in low demand realizations. All in all, this generally translates into lower expected profits for the buyer who, anticipating this, usually prefers the stable price of supplier 1.

The monopolistic behavior of supplier 2 can be anticipated by supplier 1, which, enjoying a first-mover advantage in the pricing game, sets a price p_1^{II} to keep business whenever possible. This is why the supplier selection rule in Equation (8) is very similar to Equation (5), except that now supplier 2 prices opportunistically once X has been revealed. In particular, supplier 1 will try to quote its monopoly price when it is not expensive. Note, however, that the maximum price $p_1^{\max,\mathrm{II}}$ now takes into account the different timing of the pricing of supplier 2.

The results of Theorem 2 can be readily extended to the case of n suppliers. Intuitively, suppliers would calculate backward what their maximum prices are and decide either to stick to them, to quote their monopoly prices, or to exit competition.

5.3. Scenario III: DS + UPC

In Scenario III, the dual-sourcing approach allows orders to be placed to different suppliers. Thus, instead of a winner-takes-all competition, suppliers fight for their share of the total pie. At $t = L_1$, the suppliers quote $(p_1^{\mathrm{III}}, p_2^{\mathrm{III}})$ and the buyer determines q_1^{III} by solving

$$B^{\mathrm{III}} = \max_{q_1 \ge 0} \mathbb{E}_X \Big\{ \max_{q_2 \ge 0} \mathbb{E}_D\{\min\{D, q_1 + q_2\} \mid X\} - p_2^{\mathrm{III}} q_2 \Big\} - p_1^{\mathrm{III}} q_1. \tag{9}$$

At the later stage, at $t = L_2$, $q_2^{\mathrm{III}}(x)$ is determined given q_1^{III} and $X = x$ by solving the inner maximization in Equation (9). Thus, the profit of supplier 1 amounts to $S_1^{\mathrm{III}} = (p_1^{\mathrm{III}} - c_1) q_1^{\mathrm{III}}$, while the expected profit of supplier 2 is $S_2^{\mathrm{III}} = (p_2^{\mathrm{III}} - c_2)\, \mathbb{E}_X\{q_2^{\mathrm{III}}(X)\}$.

Theorem 3. *Given* $(p_1^{\mathrm{III}}, p_2^{\mathrm{III}})$, *the purchasing policy* $(q_1^{\mathrm{III}}, q_2^{\mathrm{III}}(x))$ *that maximizes the buyer's profit is given by*

$$p_1^{\mathrm{III}} = \mathbb{E}_X \big\{ \min\{ \bar{F}_{D\mid X}(q_1^{\mathrm{III}} \mid X), p_2^{\mathrm{III}} \} \big\} \tag{10}$$

and $q_2^{\mathrm{III}}(x) = (\bar{F}_{D\mid X}^{-1}(p_2^{\mathrm{III}} \mid x) - q_1^{\mathrm{III}})^+$.

The pricing game of Scenario III *has an equilibrium when* $1/q_1^{\mathrm{III}}$ *is concave in* p_1^{III} *for all* p_2^{III} *and* $1/\mathbb{E}_X\{q_2^{\mathrm{III}}(X)\}$ *is concave in* p_2^{III} *for all* p_1^{III}.

In this scenario, as in Scenario I, there is a mismatch between the timing of order placement and the price commitment. In single sourcing, the behavior of each supplier could be worked out independently assuming that they were in business. Unfortunately, the fact that the buyer applies dual sourcing couples suppliers' strategies and complicates the analysis and existence of equilibria. Intuitively, the uncertain impact of the future forecast update $X = x$ on the suppliers' profits sometimes prevents them from reaching a stable behavior.

Specifically, if demand forecasts were known to be revised strongly upward, supplier 2 could charge an up-front premium to realize expensive future sales (and large profits). However, demand forecasts could be revealed mediocre in light of the early purchases q_1^{III} already realized at $t = L_1$, and this would kick supplier 2 out of the market. The only way supplier 2 can hedge this risk is by being cheaper than supplier 1. Thus, the best response of the fast supplier is in general one of the following: either sell highly profitable excess demand (i.e., demand exceeding q_1) by quoting a high price or capture the whole market by kicking out supplier 1 with a more modest price quote. This ambiguity can be resolved differently depending on the price of supplier 1. In other words, the best response of supplier 2 can be discontinuous, which may prevent the existence of a Nash equilibrium in pure strategies. This can be explained because of the incentive misalignment present in this scenario: while the buyer places spot orders that depend on an updated forecast, suppliers are not allowed to quote spot prices. The mismatch between the timing of purchases and price commitment makes rational suppliers behave discontinuously.

Furthermore, the large power that the buyer enjoys in this scenario turns out to be a doubled-edged sword: surprisingly, as shown in the numerical results provided in §6, when a Nash equilibrium in pure strategies exists, the equilibrium prices tend to be higher than those of Scenario I. This is because the fast supplier usually opts for realizing expensive sales of excess demand and this reduces competition intensity in the sense that supplier 1 often quotes its monopoly price. The end result is that the average purchasing price as seen by the buyer increases with respect to Scenario I.

Finally, because there is no general guarantee of existence and uniqueness of equilibrium in pure strategies, we explore, in §6, the relationship between the existence of an equilibrium in pure strategies and the level of demand uncertainty present in the supply chain. In particular we identify under which parameter ranges we can obtain a unique equilibrium. Note in addition that an equilibrium in mixed strategies will always exist because suppliers' profit functions are continuous and their support sets (i.e., $c_i \le p_i \le 1$, $i = 1, 2$) are compact (Glicksberg 1952). One example of equilibrium in mixed strategies is discussed in §6 and presented in the appendix.

5.4. Scenario IV: DS + DPC

Scenario IV is characterized by dual sourcing and delayed price commitment, which implies that the fast supplier will not commit to a price before $t = L_2$. Thus, the pricing of supplier 2 will depend on the volume of early purchases. The buyer will hence have to anticipate future price quotes when deciding the order quantity to be placed to the slow supplier. Let

$$B_2^{\mathrm{IV}}(q_1 \mid x) = \max_{q_2 \ge 0} \mathbb{E}_D \big\{ \min\{D, q_1 + q_2\} \mid x \big\} - p_2^{\mathrm{IV}}(x) q_2, \tag{11}$$

denote the buyer's expected profit at $t = L_2$ when $X = x$ is revealed and q_1 units have been already ordered to supplier 1 at $t = L_1$. Note that, as in Scenario II, supplier 2 has the ability to quote its price upon observing $X = x$. Analyzing the decision making sequence backward, the optimal volume of early purchases q_1^{IV} is hence determined at $t = L_1$ by solving

$$B^{\mathrm{IV}} = \max_{q_1 \ge 0} \mathbb{E}_X\{B_2^{\mathrm{IV}}(q_1 \mid X)\} - p_1^{\mathrm{IV}} q_1. \tag{12}$$

To set prices, at $t = L_1$ supplier 1 quotes p_1^{IV} to maximize $S_1^{\mathrm{IV}} = (p_1^{\mathrm{IV}} - c_1) q_1^{\mathrm{IV}}$; at $t = L_2$, upon observing $X = x$ and q_1^{IV}, supplier 2 quotes $p_2^{\mathrm{IV}}(x)$ resulting in an expected profit of $S_2^{\mathrm{IV}} = \mathbb{E}_X\{(p_2^{\mathrm{IV}}(X) - c_2) q_2^{\mathrm{IV}}(X)\}$. We next consider what equilibrium strategies emerge in this sequential pricing game.

Theorem 4. *At* $t = L_2$, *for some given* q_1, *the optimal equilibrium purchasing-pricing strategy with supplier* 2 *is characterized as follows.*

- *If $\bar{F}_{D\mid X}(q_1 \mid x) \le c_2$, the buyer is overstocked and hence $q_2^{\mathrm{IV}}(x) = 0$.*
- *If $\bar{F}_{D\mid X}(q_1 \mid x) > c_2$, supplier 2 is able to set a price such that $q_2^{\mathrm{IV}}(x) > 0$. When $D \mid X$ is IGFR for all $X = x$, the equilibrium price is $p_2^{\mathrm{IV}}(x) = \bar{F}_{D\mid X}(q_1 + q_2^{\mathrm{IV}}(x) \mid x)$ and $q_2^{\mathrm{IV}}(x)$ is uniquely determined by*

$$f_{D\mid X}(q_1 + q_2^{\mathrm{IV}}(x) \mid x) q_2^{\mathrm{IV}}(x) = \bar{F}_{D\mid X}(q_1 + q_2^{\mathrm{IV}}(x) \mid x) - c_2. \tag{13}$$

At $t = L_1$, when $D \mid X$ is IGFR for all $X = x$ and

$$h'_{D\mid X}(y \mid x) + 4h_{D\mid X}(y \mid x)\frac{f_{D\mid X}(y \mid x)}{\bar{F}_{D\mid X}(y \mid x) - c_2} + 6\frac{f^2_{D\mid X}(y \mid x)}{(\bar{F}_{D\mid X}(y \mid x) - c_2)^2} \ge 0 \quad \forall\, y \ge y_0(x, c_2), \tag{14}$$

where $h_{D|X} \triangleq f'_{D|X}/f_{D|X}$ and $y_0(x, c_2) = \min\{y \ge 0 \mid f_{D\mid X} \cdot (y \mid x) y = \bar{F}_{D\mid X}(y \mid x) - c_2\}$, there is a one-to-one mapping between the price p_1 and the purchased quantity q_1 given by

$$\mathbb{E}_X\left\{\frac{\partial B_2^{\mathrm{IV}}}{\partial q_1}(q_1 \mid X)\right\} = p_1. \tag{15}$$

When

$$\mathbb{E}_X\left\{\frac{\partial^3 B_2^{\mathrm{IV}}}{\partial q_1^3} + 2\frac{\partial^2 B_2^{\mathrm{IV}}}{\partial q_1^2}\right\} \le 0, \tag{16}$$

the optimal equilibrium order q_1^{IV} is uniquely determined by

$$q_1^{\mathrm{IV}} = \frac{\mathbb{E}_X\{(\partial B_2^{\mathrm{IV}}/\partial q_1)(q_1^{\mathrm{IV}} \mid X)\} - c_1}{-\mathbb{E}_X\{(\partial^2 B_2^{\mathrm{IV}}/\partial q_1^2)(q_1^{\mathrm{IV}} \mid X)\}}, \tag{17}$$

and the optimal equilibrium price p_1^{IV} can be found using Equation (15).

The equilibrium prices and quantities of Theorem 4 are intricate but, as happened in the example studied in §4, show the central role that the volume of early purchases plays in determining the profits of supplier 2. When the buyer does not foresee a sufficient level of excess demand with respect to q_1^{IV} at $t = L_2$, it enters into the *overstocked* regime and it does not place any additional order to supplier 2. Otherwise, the buyer is willing to buy from supplier 2, which then tries to extract the highest rent out of potential late purchases. Intuition is more complicated as we move backward on the decision-making sequence because the equilibrium results for supplier 1 lack an intuitive structure.

Regarding the regularity conditions for existence and uniqueness of a Nash equilibrium, Equation (14) is relatively easy to satisfy. In particular, it is satisfied by all uniform, exponential, and normal $D \mid X$. It is also satisfied by all log-normal distributions with $\sigma > 1.8$ and by all gamma distributions with shape parameter between 1 and 3. For log-normal and gamma distributions with parameters out of these ranges, Equation (14) can still hold if c_2 is large enough. Indeed, Equation (14) is a sufficient but not necessary condition to determine the price/quantity relation of supplier 1 using Equation (15). Pursuing simplicity, Equation (14) is expressed in terms of the distribution of $D \mid X$, regardless of the distribution of X. As a result, we expect the buyer's problem at $t = L_1$ to be well-behaved for a larger range of parameters and a larger variety of distributions. As for Equation (16), when it holds, the computation of the optimal price set by the slow supplier is a well-behaved problem. In general, our numerical simulations show that for normal and gamma-distributed X, there is a unique price/quantity that maximizes the slow supplier's profit.

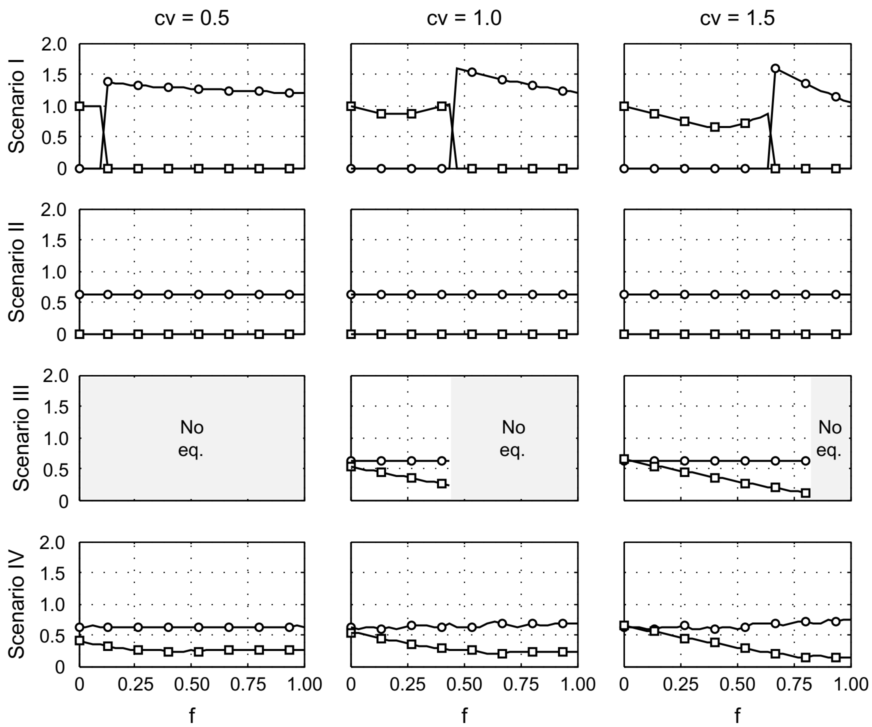
6. Managerial Insights

To illustrate the interplay between supplier incentives and sourcing strategies, we consider a practical setup where demand is sequentially revealed in advance of the selling season, when sales are effectively realized. For instance, this is the case in the toy industry, where retailers (the customers in our model) start placing orders for the Christmas campaign to toy brands (the buyers in our model) some time after observing their new products in fairs held yearly from February to October. In turn, toy brands place orders to toy manufacturers (the suppliers in our model) early enough so as to be able to deliver all the orders to retailers before Christmas. A similar situation happens to collection-driven fashion companies after releasing a new catalogue to their client retailers some months before the actual selling season takes place in the store.

To meet demand, the buyer has supply opportunities at $t = L_1$ and $t = L_2 \le L_1$ and, without loss of generality, we assume that no sales are revealed before $t = L_1$. Thus, the demand D seen by the buyer at $t = 0$ is the sum of the firm sales revealed between $t = L_1$ and $t = L_2$, denoted I_1, and the firm sales revealed after $t = L_2$, denoted I_2. Note that in this setup, $X = I_1$ and $\mathbb{E}\{D \mid X\} = X + \mathbb{E}\{I_2\}$. Both I_1 and I_2 are modeled as independent gamma random variables with scale parameter θ and shape parameters k_1 and k_2, respectively: $I_i \sim \Gamma(k_i, \theta)$ $i = 1, 2$. Since I_1, I_2 have the same scale parameter θ, $D = I_1 + I_2$ is also gamma-distributed with scale θ, shape $k_1 + k_2$, and coefficient of variation $\mathsf{cv} = (k_1 + k_2)^{-1/2}$. We set $\theta = \mathsf{cv}^2$ such that $\mathbb{E}\{D\} = 1$ and $\sigma_D = \mathsf{cv}$.

Although the scale parameter of I_1, I_2 is fixed, the shape parameter (or intensity) k_i depends on the duration of the observation period because longer periods should bring in more orders and thus resolve more demand uncertainty. The specific fraction of demand uncertainty that is resolved by revealing X depends on the sector and the type of product. Intuitively, the demand of less innovative (plain vanilla)

Figure 3 Equilibrium Quantities Ordered by the Buyer as a Function of the Fraction of Unresolved Uncertainty f



Notes. Circle markers refer to q_1^{scen} and square markers refer to q_2^{scen} (Scenarios I and III) and $\mathbb{E}_X\{q_2^{scen}(X)\}$ (Scenarios II and IV). "No eq.," no equilibrium.

products would be revealed very early (i.e., a large fraction of demand uncertainty would be removed even when L_2 is long), whereas the demand of more innovative or riskier products would be more difficult to anticipate. To provide robust insights that are independent of this sector- and product-specific attribute, we express the results as a function of the *fraction of unresolved uncertainty*, defined as $\mathsf{f} \triangleq \sigma^2_{D|X}/\sigma^2_D$. Note that $0 \le \mathsf{f} \le 1$. Moreover, when $\mathsf{f} = 0$, $X = D$ ($L_2 = 0$) and when $\mathsf{f} = 1$, $X = 0$ is irrelevant ($L_2 = L_1$). The shape parameters of I_1, I_2 are set accordingly to $k_1 = (1-\mathsf{f})\mathsf{cv}^{-2}$ and $k_2 = \mathsf{f}\mathsf{cv}^{-2}$.

Following we compare the performance of the four scenarios studied in §5 in a situation where $c_1 = 0.2$ and $c_2 = 0.3$ (these values are reasonable in apparel, for example[4]). Different values of the coefficient of variation of the demand prior will be explored ($\mathsf{cv} \in \{0.5, 1, 1.5\}$) to study the impact of f on the purchased quantities, equilibrium prices, and profits.

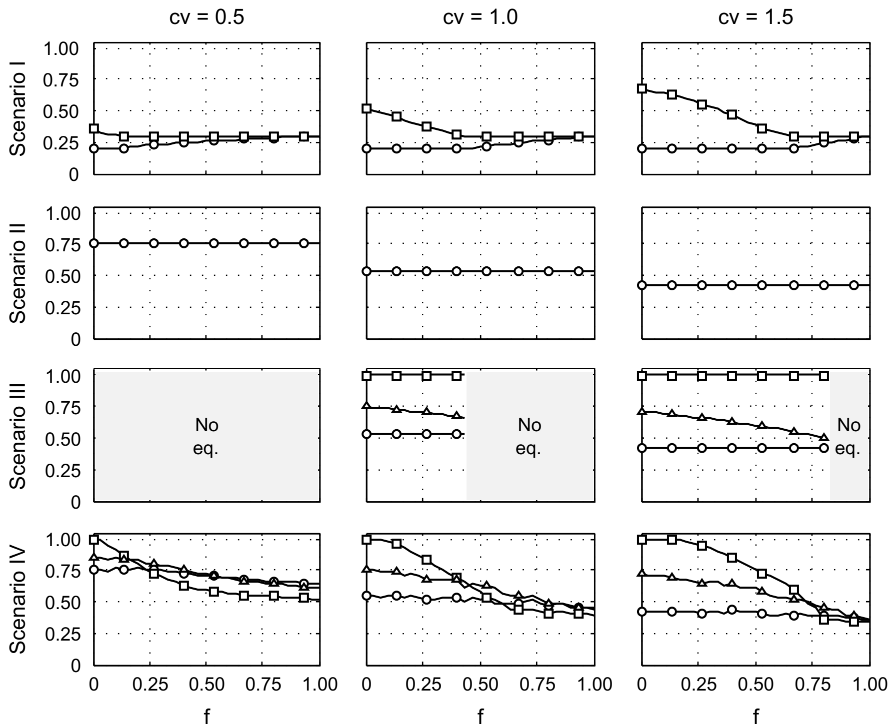
6.1. Purchased Quantities and Equilibrium Prices

Figure 3 shows the orders placed by the buyer to both suppliers in the different scenarios for several demand uncertainty levels. Similarly, Figure 4 depicts the corresponding equilibrium prices. In Scenario I (first row of Figure 4), we set $p_i^{\mathrm{I}} = c_i$ when supplier i exits competition. In Scenario II (second row of Figure 4), we only plot p_1^{II} because supplier 2 is never selected. In Scenarios III and IV (the two bottom rows of Figure 4), we report $\mathbb{E}_X\{p_2^{scen}(X)\}$ conditioned on $q_2^{scen}(X) > 0$ (otherwise, the price of supplier 2 is irrelevant) and the average procurement price as seen by the buyer, $p_{\text{avg}}^{scen} = \mathbb{E}_X\{p_1 q_1^{scen} + p_2^{scen}(X) q_2^{scen}(X)\} / \mathbb{E}_X\{q_1^{scen} + q_2^{scen}(X)\}$.

In Scenario I, supplier 2 is selected when f is low; otherwise, its lead-time advantage is too weak to offset the cost gap with respect to supplier 1, who receives all the business. Interestingly, the lead-time advantage of supplier 2 increases with prior demand uncertainty: a higher demand cv increases both the range of values of f where the buyer sources from the fast supplier and its price (see the first row of Figures 3 and 4). Interestingly, $\mathbb{E}_X\{q_2^{\mathrm{I}}(X)\}$ is first decreasing and then increasing in f. This is because when $\mathsf{f} = 0$, the buyer is able to satisfy all the demand; i.e., $\mathbb{E}_X\{q_2^{\mathrm{I}}(X)\} = \mathbb{E}\{D\}$. As f increases, the demand risk increases at $t = L_2$ and the buyer thus orders less from supplier 2. However, an increasing f also intensifies competition (the lead-time advantage of supplier 2 weakens) and substantial price cuts eventually drive the purchased quantities upward (see the first row of Figure 4).

[4] Apparel retailers such as Desigual have retail prices that multiply procurement costs by a factor between 3 and 5 (see discussion in Carroll 2012) and report cost increases of up to 50% when sourcing from local versus off-shore suppliers.

Figure 4 **Equilibrium Prices Quoted by the Suppliers as a Function of the Fraction of Unresolved Uncertainty** f



Notes. Circle markers refer to p_1^{scen} and square markers refer to p_2^{scen} (Scenarios I and III) and $\mathbb{E}_X\{p_2^{scen}(X)\}$ (Scenarios II and IV). In Scenarios III and IV, where orders to more than one supplier can be placed, triangle markers refer to the average price as seen by the buyer, $p_{avg}^{scen} = \mathbb{E}_X\{p_1 q_1^{scen} + p_2^{scen}(X) q_2^{scen}(X)\}/\mathbb{E}_X\{q_1^{scen} + q_2^{scen}(X)\}$. "No eq.," no equilibrium.

In Scenario II, the buyer orders roughly half of what it orders in Scenario I. Furthermore, the quantity ordered is constant regardless of f and cv. This is because the delay in supplier 2's price quote relaxes the competitive pressure so much so that this supplier is unable to pose a credible threat to supplier 1, who can afford to price at the monopoly level and still keep all the business. However, the ability of this supplier to raise prices diminishes as cv increases (see the second row of Figure 4): the willingness to pay of the buyer decreases with increased demand uncertainty.

Moving forward to dual sourcing we realize that, in Scenario III, the existence of an equilibrium in pure strategies is strongly related to the level of demand uncertainty. That is, the lower the cv, the lower the chances of existence of an equilibrium. Moreover, we find that, when an equilibrium exists, the resulting prices are $(p_1^{\mathrm{III}}, p_2^{\mathrm{III}}) = (p_1^m, 1)$. This is why p_1^{III} coincides with p_1^{II}. When an equilibrium in pure strategies does not exist, we can guarantee existence of an equilibrium in mixed strategies. The structure of such equilibrium is the following: the suppliers' mixed strategies are such that some probability density is placed below p_1^m and 1, respectively. Average prices are hence reduced, and this results in higher average profits for the buyer. The interested reader is referred to the appendix for more details on the structure and results achieved by mixed strategies.

In Scenario III, dual sourcing allows the buyer to meet excess demand yet at zero profit (because $p_2^{\mathrm{III}} = 1$). This result is counterintuitive in the sense that, in this scenario, the buyer has the tightest control of the supply chain: it places spot orders and forces the suppliers to commit prices up front. What does it get in exchange? The largest price quotes that a pair of rational suppliers can offer! Interestingly, these high prices are robust to supplier 2 being able to place two prices quotes at $t = L_1$: one for orders placed at $t = L_1$ (thus competing for early purchases with supplier 1) and another one for prospective orders the buyer may place later, at $t = L_2$. This is because when $c_2 \geq c_1$, it is optimal for supplier 2 to give up the business of early orders by setting a very high early price and thus concentrate on getting late orders. As a result, the extended setup where supplier 2 is able to price twice becomes identical to Scenario III.

Lastly, Scenario IV shows that dual sourcing puts pressure on suppliers: prices are generally lower and purchase quantities higher compared to Scenario II. It is worth highlighting that a stronger lead-time advantage of the fast supplier (lower f) allows it to increase prices and at the same time increase order quantities. This suggests that, in this scenario, the fast supplier is able to capture most of the benefits of better supply-demand matching.

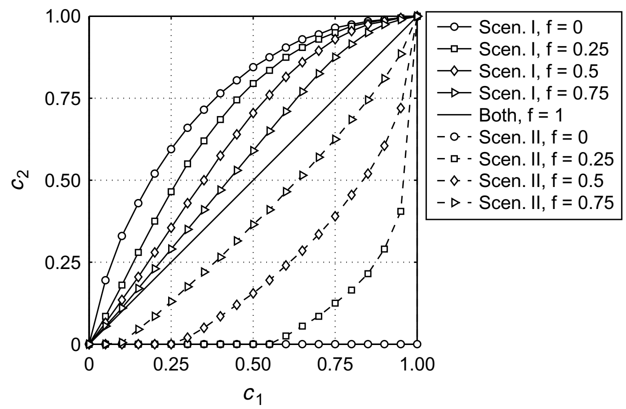
Regarding the comparison between single and dual sourcing, supplier prices are generally higher when the buyer dual sources: with UPC, the average price paid by the buyer is always higher in Scenario III than in Scenario I; with DPC, the average price paid by the buyer is higher in Scenario IV than in Scenario II for low to moderate values of f. With DPC, where an equilibrium always exists, this is because competition is tempered under DS. Supplier 2 has a natural pricing advantage. First, unless demand uncertainty is bounded, it is impossible to keep supplier 2 out of the market with probability one (it may happen that the forecast is revised sufficiently upward and this supplier is needed). Second, it acts as a monopolist because there is no other supply opportunity left after $t = L_2$. The buyer tries to mitigate this situation by placing higher early orders to force price reductions at $t = L_2$ since $\partial p_2^{\mathrm{IV}}(x)/\partial q_1^{\mathrm{IV}} \leq 0$. However, this behavior ends up being harmful for the buyer: Supplier 1 anticipates the strategic role of early purchases and charges a higher markup on its goods knowing that it will still retain high volumes. Interestingly, the strategic power of supplier 1 (as measured by its quoted price) is strongly decreasing in cv. This is because larger values of cv increase the risk of early purchases for the buyer, which makes those less effective as a tool for reducing the fast supplier's price. This forces the slow supplier to moderate prices to stay in business. As a result, the fast supplier generally quotes larger prices (contingent on receiving business) than supplier 1 with higher values of cv, unless its lead-time advantage is small (high f).

On the other hand, competition is in-or-out in single sourcing and the buyer selects the supplier with the best cost versus lead-time profile. However, as seen in Figure 5, the interplay between cost and lead time is very different in Scenario I than in Scenario II. In Scenario I, supplier 2 can offset higher costs with a strong lead-time advantage (i.e., a low value of f). In contrast, in Scenario II, supplier 2 is rarely selected because of the artificial monopoly it is entitled with under DPC, which becomes stronger as f decreases. For this reason, paradoxically, the stronger the lead-time advantage of supplier 2, the less likely it is to be selected.

6.2. Profits

We finally study the net impact of the different scenarios on the profit of each player and the entire supply chain. This is shown in Figure 6, where profits are stacked. As expected, except in Scenario II, supplier 2 always benefits from lower lead times (associated with lower values of f) and higher demand uncertainty. In Scenario I, this comes at the expense of the slow supplier, which is forced to reduce prices and hence profits; in dual sourcing (Scenarios III and IV),

Figure 5 **Supplier Selection Regions of Scenarios I and II for Different Values of the Fraction of Unresolved Uncertainty f**



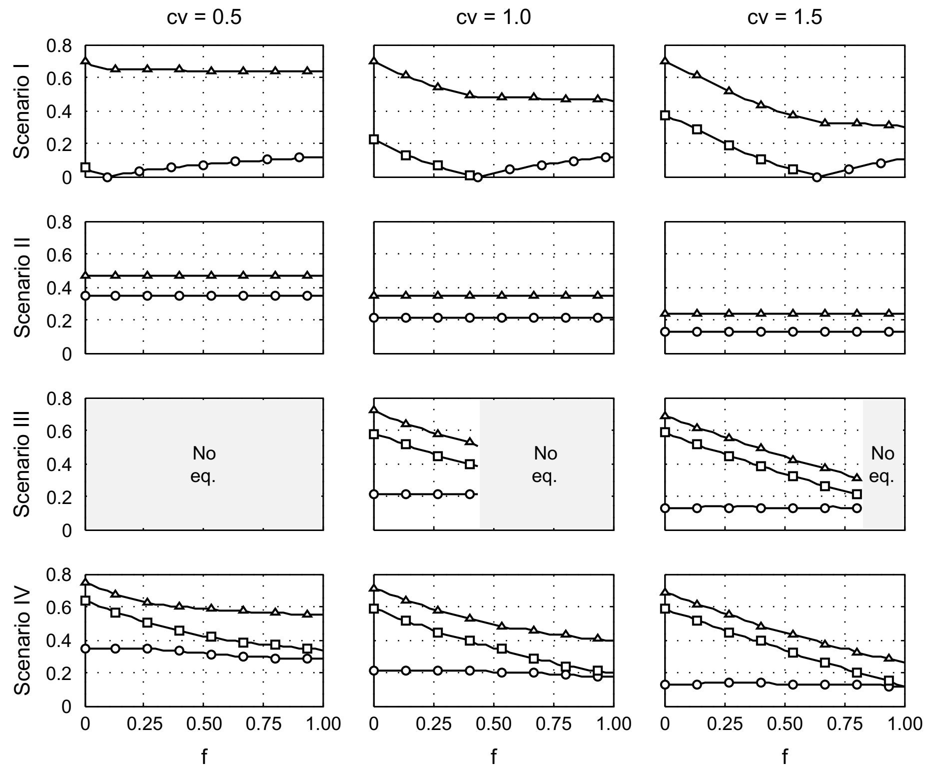
Note. For each value of c_1, the curves show the maximum c_2 such that supplier 2 is selected.

this comes at the expense of the buyer, which faces stronger double marginalization. Surprisingly, under UPC, double marginalization can jeopardize the flexibility gains that dual sourcing provides over single sourcing: clearly, the buyer is better off in Scenario I than in III (when an equilibrium in pure strategies exists). However, this is not the case in DPC, where the buyer always prefers Scenario IV to Scenario II. As already pointed out and made explicit in Figure 5, rather than this being a merit of dual sourcing, it is a consequence of the absolute lack of competition inherent in Scenario II.

We can further quantify the inefficiency created by inflated supplier prices under UPC. For this purpose, we study the ratio of the buyer's profit in Scenario III versus Scenario I (i.e., $B^{\mathrm{III}}/B^{\mathrm{I}}$). The profit loss of the buyer is generally large and stable with f: 73% and 69% of profits are lost when adopting dual sourcing with $\mathrm{cv} = 1$ and $\mathrm{cv} = 1.5$, respectively (note that, because there is no equilibrium in pure strategies for $\mathrm{cv} = 0.5$, we are unable to quantify the profit loss there). This picture is reversed under DPC, where the profit ratio $B^{\mathrm{IV}}/B^{\mathrm{II}}$ amounts to 1 when $\mathrm{f} = 0$ but increases by 82% ($\mathrm{cv} = 0.5$), 53% ($\mathrm{cv} = 1$), and 42% ($\mathrm{cv} = 1.5$) when $\mathrm{f} = 1$. In other words, with DPC, dual sourcing is especially beneficial for the buyer when the lead-time advantage of the fast supplier and the uncertainty of the demand prior are both small.

Finally, note that the double marginalization effect of dual sourcing may hurt the overall supply chain profits as well. On the positive side, dual sourcing enables the buyer to order from supplier 2 when forecasts are revised upward, which creates value for the supply chain. However, under UPC, price inflation at supplier 1 reduces the total amount purchased (even considering late purchases at $t = L_2$) and supply chain profits. This latter effect dominates when f is large, and the chain is worse off.

Figure 6 **Equilibrium Profits (Stacked) Achieved by the Suppliers and the Buyer as a Function of the Fraction of Unresolved Uncertainty f**



Notes. Circle markers refer to S_1^{scen}, square markers refer to S_2^{scen}, and triangle markers refer to B^{scen}. “No eq.,” no equilibrium.

7. Conclusions and Further Research

In this paper, we developed a simple model to compare the performance of single and dual sourcing when suppliers take pricing decisions. We find that, in contrast to models where supplier prices are exogenous, dual sourcing may be detrimental for the buyer because it may lead suppliers to inflate their price quotes. This is especially prominent when prices are committed up front, but not when they are quoted on the spot. Hence, despite the higher ordering flexibility enabled by dual sourcing, single sourcing with up-front price commitment enforces stronger competition between the suppliers; this reduces double marginalization and benefits the buyer. Our results are robust to variable retail prices due to the presence of price elasticity of demand and to the faster supplier placing multiple quotes. We also recover our conclusions when suppliers mix between (pricing) strategies under dual sourcing with up-front price commitment.

Our work opens a number of future research questions. First, although the single-sourcing model can be extended to an arbitrary number of suppliers, the analysis quickly becomes intractable under dual sourcing. It would be interesting to identify how the number of suppliers affects our results. For this purpose, one could identify a family of demand forecast updates that is amenable to analysis. Second, the incentive friction present in up-front price commitment could be mitigated if the fast supplier quoted a price-quantity schedule. To that end, the structure of such an optimal price schedule and its performance should be studied. Third, our results rely on the buyer willingly sharing its demand forecasts with the suppliers. It would be interesting to study whether the buyer could benefit from manipulating the information shared with suppliers to reduce their prices. Such behavior would certainly depend on the sourcing strategy and the timing of price quotes. Fourth, single sourcing may be risky for the buyer because of the exclusive relationship with one supplier and the possible reliability problems this creates. It would be useful to extend our analysis to a situation where there is risk of supplier failure. It is likely that the region where dual sourcing is preferred expands, but above all it is unclear how supply risk affects the strategic interaction between suppliers and buyer. Finally, supplier lead times are given in this model. Our model demonstrates that having the longest lead time is not necessarily detrimental under delayed price commitment because being the first mover in the game can confer the slow supplier with additional pricing power. If suppliers could invest in reducing their lead time, it is hence not clear that they would benefit from doing so. Identifying the conditions under which lead time reduction is beneficial would be helpful.

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Appendix

Analysis of the Example of §4

In Scenarios I and III (UPC), where the two suppliers take the pricing decision simultaneously, we seek the Nash equilibrium in pure strategies. In Scenarios II and IV (DPC), we determine the optimal decisions of the players in backward order; i.e., given all past decisions, we seek what the optimal decision in the current stage is, assuming that all future decisions are chosen optimally. In other words, we are interested in a subgame-perfect equilibrium, see Fudenberg and Tirole (1991). Note that Scenario IV is a dynamic game with three players, where the buyer plays twice (after supplier 1 first and after supplier 2 at the end).

Scenario I (*UPC* + *SS*). In this scenario, the buyer must choose one supplier upon observation of p_1, p_2. If supplier 1 is selected, the profit of the buyer can be written as $B_1^{\mathrm{I}} = \max_{q_1\geq 0} \mathbb{E}_D\{\min\{D, q_1\}\} - p_1q_1 = \max_{q_1\geq 0}(1-p_1)q_1 - q_1^2/4 = (1-p_1)^2$, where $q_1^{\mathrm{I}} = \bar{F}_D^{-1}(p_1) = 2(1-p_1)$ is the corresponding optimal order. Accordingly, the profit of the slow supplier amounts to $S_1^{\mathrm{I}} = (p_1-c_1)q_1^{\mathrm{I}} = 2(p_1-c_1)(1-p_1)$, whereas the fast supplier would be out of the market; i.e., $q_2^{\mathrm{I}}(x) = 0$ and $S_2^{\mathrm{I}} = 0$. In contrast, if supplier 2 is selected, the average profit of the buyer is $B_2^{\mathrm{I}} = (1-p_2)\mathbb{E}_X\{q_2^I(X)\} = 1-p_2$, where we have used that $q_2^I(x) = x$ because the forecast update x is equal to the actual demand. Thus, the profit of the suppliers is such that $q_1^I = 0$, $S_1^{\mathrm{I}} = 0$ and $S_2^{\mathrm{I}} = (p_2-c_2)\mathbb{E}_X\{q_2^{\mathrm{I}}(X)\} = p_2 - c_2$. Because the buyer selects the supplier yielding the largest profit, supplier 1 is selected if and only if $B_1^{\mathrm{I}} \geq B_2^{\mathrm{I}}$; i.e., $(1-p_1)^2 \geq 1-p_2$.

The suppliers' pricing game can now be analyzed. The selection mechanism described above creates a winner-takes-all effect, reminiscent of Bertrand competition. The best strategy of supplier $i = 1, 2$ is to offer the price p_i that maximizes S_i^{I}, provided that $q_i^{\mathrm{I}} > 0$. The only equilibrium of the pricing game is straightforward: in equilibrium, supplier 1 will be chosen by the buyer if and only if $(1-c_1)^2 \geq 1 - c_2$. The resulting equilibrium prices are intuitive: if by pricing at cost supplier i is selected by the buyer, it should quote a price p_i^{I} equal to the minimum between the monopoly price (i.e., the one that maximizes S_i^{I} contingent on receiving business) and the price above which the buyer selects the competitor.

Scenario II (*DPC* + *SS*). If supplier 2 is chosen, it will always quote $p_2^{\mathrm{II}} = 1$ once demand is revealed, sell $q_2^{\mathrm{II}}(X) = X$, and extract all the profits from the buyer. As a result, the buyer will never buy from supplier 2 and hence $p_1^{\mathrm{II}} = p_1^m$.

Scenario III (*UPC* + *DS*). When the buyer dual sources, supplier 2 will always sell $q_2^{\mathrm{III}}(X) = (X-q_1)^+$ regardless of the price quoted. This is equal to $1-(q_1-q_1^2/4)$ on average. However, depending on how high p_2 is, it may influence q_1 indirectly. Let us examine how q_1 depends on p_1 and q_2. The buyer solves $\max_{q_1\geq 0}\mathbb{E}_X\{X - p_1q_1 - p_2(X-q_1)^+\} = 1 - p_2 + p_2(q_1 - q_1^2/4) - p_1q_1$, which results in $q_1^{\mathrm{III}} = 2(p_2-p_1)^+$, thus only positive when $p_1 < p_2$. This implies that supplier 1 will set a price that maximizes $2(p_1-c_1)(p_2-p_1)^+$ and hence equal to $p_1 = (p_2+c_1)/2$ whenever $p_2 > c_1$ (otherwise supplier 1 exits competition and p_1 becomes irrelevant). Similarly, supplier 2 sets a price that maximizes $(p_2-c_2)[1-2(p_2-p_1)+(p_2-p_1)^2]$. It turns out that this value is always highest at $p_2 = 1$. Thus, in equilibrium, $p_1^{\mathrm{III}} = p_1^m$ and $p_2^{\mathrm{III}} = 1$.

Scenario IV (*DPC*+*DS*). When prices are decided on the spot, supplier 2 will set a price that maximizes $(p_2-c_2)q_2^{\mathrm{IV}}(X) = (p_2-c_2)(X-q_1)^+$. In all sample paths $p_2^{\mathrm{IV}}(x) = 1$ and, as a result, the buyer interacts with supplier 1 as if supplier 2 were not available (since it is indifferent from losing sales or buying from the fast supplier). Hence, this scenario is the same as Scenario III.

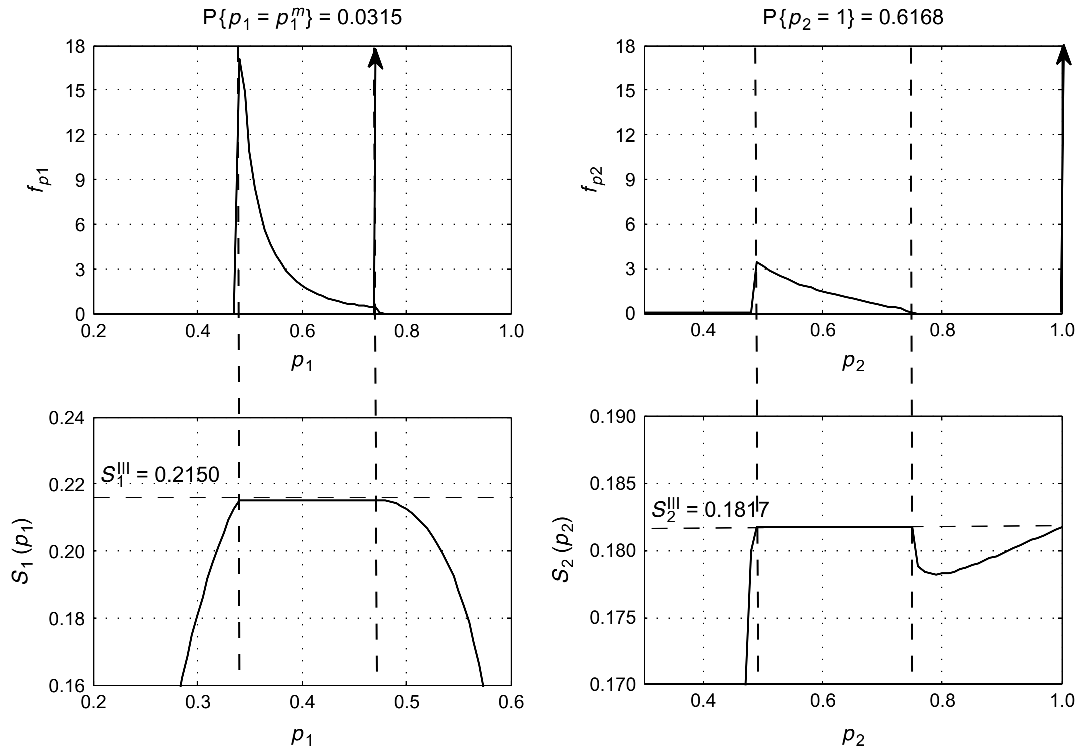
An Example of Mixed Strategy Equilibrium Distributions in Scenario III

When no equilibrium in pure strategies exists in Scenario III, there exists an equilibrium in mixed strategies because the suppliers' profit functions are continuous and their support sets are compact (Glicksberg 1952). This has several shortcomings. First, it leads, for each realization, to a different profit level for the retailer. Thus, orienting the sourcing policy by anticipating the behavior of suppliers does not make managerial sense anymore. This limits the practical value of the model in this scenario. Second, the analysis is not generally tractable. We provide here in Figure A.1 an example were we have computed numerically the mixed strategy equilibrium distributions under a given parameter set for which no equilibrium in pure strategies could be found in §6.

In this example, the best response of supplier 2 is discontinuous: it quotes 1 whenever $p_1 < 0.4388$ or p_1 otherwise. The best response of supplier 1 is increasing in p_2 and ranges from 0.2733, when $p_2 = c_2 = 0.3$, to its monopoly price $p_1^m = 0.7482$, when $p_2 = 1$. An equilibrium in pure strategies does not exist because when $p_1 = p_1^m$, the best response of supplier 2 is not to quote $p_2 = 1$ but to set $p_2 = p_1^m$ instead. Given the structure of the best-response functions, the mixed strategy equilibrium distributions of p_1 and p_2 share the same support set except that the distribution of p_2 has a mass point at $p_2 = 1$ (the distribution of p_1 has another mass point at $p_1 = p_1^m$). This is interesting because the best response of supplier 2 over the support set of p_1 is never to quote 1 but to mirror p_1. However, as the profit of supplier 2 is nondecreasing in p_1, the role of the mass point at 1 is to encourage supplier 1 to raise prices beyond 0.4388 (note that $q_1 = 0$ if $p_2 \leq p_1$). In turn, supplier 1 responds by randomizing over the support set $[0.48, p_1^m]$ and by setting up a mass point at p_1^m to take advantage of the large probability that $p_2 = 1$. However, the probability that $p_1 = p_1^m$ is much smaller as this price is risky unless $p_2 = 1$. Note that, when an equilibrium in pure strategies exists, $p_1 = p_1^m$ and $p_2 = 1$ with probability one.

As opposed to what happens when an equilibrium in pure strategies exists, the profits in Scenario III do not match those of Scenario IV (DS + DPC) in this case. The mixed strategy equilibrium distributions of

Figure A.1 Mixed Strategy Equilibrium Distributions (Top Row) of Supplier 1, f_{p_1} (Left), and Supplier 2, f_{p_2} (Right); Profit Functions (Bottom Row) of Supplier 1, $S_1(p_1)$ (Left), and Supplier 2, $S_2(p_2)$ (Right)



Note. Relevant parameter values are $c_1 = 0.2$, $c_2 = 0.3$, cv $= 0.5$, and f $= 0$ (i.e., $L_2 = 0$).

Figure A.1 yield $(S_1^{\mathrm{III}}, S_2^{\mathrm{III}}, B^{\mathrm{III}}) = (0.2150, 0.1817, 0.3354)$, whereas $(S_1^{\mathrm{IV}}, S_2^{\mathrm{IV}}, B^{\mathrm{IV}}) = (0.3483, 0.2902, 0.1123)$ (in Figure 6). This is because to achieve equilibrium, supplier 1 (2) has to place some probability density below p_1^m (1). As average prices are reduced, the buyer's profit is boosted (+199%) at the expense of the suppliers' profits (−38% and −37%, respectively). However, although the buyer is taking advantage of the nonexistence of an equilibrium in pure strategies, its profit is still far from what it would get from single sourcing ($B^{\mathrm{I}} = 0.6448$ from Figure 6).

Proofs

Proof of Theorem 1. Note that all "I" superindexes have been dropped for simplicity.

We first study the demand regularity conditions of the theorem, which follow from imposing that the computation of p_1^m, p_2^m is well behaved. An IGFR D is needed to ensure that p_1^m follows after the maximization of a unimodal function (see Lariviere and Porteus 2001). We next show that a log-concave $f_{D\mid X}$ is required to ensure the same structure in p_2^m (see Equation (4)). The monopoly price of the fast supplier maximizes

$$S_2 = (p_2 - c_2)\,\mathbb{E}_X\{\bar{F}_{D\mid X}^{-1}(p_2 \mid X)\}, \tag{18}$$

whose second derivative with respect to p_2 is

$$\begin{aligned}\frac{\partial^2 S_2}{\partial p_2^2} &= -2\,\mathbb{E}_X\left\{\frac{1}{f_{D\mid X}(\bar{F}_{D\mid X}^{-1}(p_2 \mid X) \mid X)}\right\}\\ &\quad - (p_2 - c_2)\,\mathbb{E}_X\left\{\frac{f'_{D\mid X}(\bar{F}_{D\mid X}^{-1}(p_2 \mid X) \mid X)}{f_{D\mid X}^3(\bar{F}_{D\mid X}^{-1}(p_2 \mid X) \mid X)}\right\}\\ &\le -2\,\mathbb{E}_X\left\{\frac{1}{f_{D\mid X}(\bar{F}_{D\mid X}^{-1}(p_2 \mid X) \mid X)}\right\}\\ &\quad + (p_2 - c_2)\,\mathbb{E}_X\left\{\frac{1}{p_2 f_{D\mid X}(\bar{F}_{D\mid X}^{-1}(p_2 \mid X) \mid X)}\right\}\\ &= -(1 + c_2/p_2)\,\mathbb{E}_X\left\{\frac{1}{f_{D\mid X}(\bar{F}_{D\mid X}^{-1}(p_2 \mid X) \mid X)}\right\} \le 0,\end{aligned} \tag{19}$$

where inequality (19) follows because any log-concave density f satisfies $f'/f^2 \ge -1/\bar{F}$ because $f/\bar{F}$ is increasing. Thus, when $f_{D\mid X}$ is log-concave S_2 is concave, which implies that p_2^m is unique.

We next move on to the equilibrium structure. The optimal purchasing quantities $q_1 = \bar{F}_D^{-1}(p_1)$ and $q_2 = \bar{F}_{D\mid X}^{-1}(p_2 \mid x)$ follow from the newsvendor structure of Equations (1) and (2), and simplify to

$$B_1 = \int_0^{q_1} z f_D(z)\,dz + q_1\left(\int_{q_1}^{\infty} f_D(z)\,dz - p_1\right) = \int_{p_1}^{1} \bar{F}_D^{-1}(y)\,dy, \tag{20}$$

where we have used the variable change $y = \bar{F}_D(z)$. Taking into account that

$$\begin{aligned}B_2 &= \mathbb{E}_X\{B_2(X)\}\\ &= \int_0^{\infty} B_2(x) f_X(x)\,dx \overset{(a)}{=} \int_0^{\infty}\int_{p_2}^{1} \bar{F}_{D\mid X}^{-1}(y \mid x) f_X(x)\,dy\,dx\\ &= \int_{p_2}^{1}\int_0^{\infty} \bar{F}_{D\mid X}^{-1}(y \mid x) f_X(x)\,dx\,dy\\ &= \int_{p_2}^{1} \mathbb{E}_X\{\bar{F}_{D\mid X}^{-1}(y \mid X)\}\,dy,\end{aligned} \tag{21}$$

where (a) follows from similar arguments to Equation (20), expression (5) follows from comparing Equations (20) and (21) when both suppliers price at cost. When both suppliers set equal prices $p_1 = p_2 = p$, the buyer always prefers the fast supplier because

$$\begin{aligned} B_1 &= \max_{q_1\geq 0} \mathbb{E}_D\{\min\{D, q_1\}\} - pq_1 \\ &\overset{(a)}{=} \max_{q_1\geq 0} \mathbb{E}_X\{\mathbb{E}_D\{\min\{D, q_1\} \mid X\} - pq_1\} \\ &\leq \mathbb{E}_X\left\{\max_{q_1\geq 0} \mathbb{E}_D\{\min(D, q_1) \mid X\} - pq_1\right\} \equiv \mathbb{E}_X\{B_2(X)\} = B_2, \end{aligned}$$

where (a) follows because $\mathbb{E}\{\mathbb{E}\{D \mid X\}\} = \mathbb{E}\{D\}$ for any pair of random variables X and D. Pursuing business, the slow supplier quotes a price below p_2, the fast supplier responds reducing p_2, and this is iterated until an equilibrium is eventually reached when one of the suppliers quotes its production cost and desists. Because each B_i is monotone decreasing in p_i and the remaining supplier can afford further price reductions, the winner is determined evaluating Equation (5). The selected supplier maximizes profit subject to $p_i \leq p_i^{\max}$ to keep business and thus $p_i = \min\{p_i^m, p_i^{\max}\}$. □

Proof of Theorem 2. Note that all "II" superindexes have been dropped for simplicity.

The newsvendor structure of B_1 and $B_2(x)$ implies that p_1^m and $p_2(x)$ are well behaved when D and $D \mid X$ are IGFR, respectively. Then similar arguments to the ones used in the proof of Theorem 1 can be applied to derive the supplier selection rule (Equation (8)) and the equilibrium prices. The only difference is that now the price quoted by supplier 2 depends on $X = x$. □

Proof of Theorem 3. Note that all "III" superindexes have been dropped for simplicity.

First note that the optimal $q_2(x) = (\bar{F}^{-1}_{D\mid X}(p_2 \mid X) - q_1)^+$ arises from the newsvendor structure of the inner maximization in Equation (9). Plugging $q_2(x)$ explicitly onto Equation (9) results in

$$\begin{aligned} B = \max_{q_1\geq 0} \mathbb{E}_X\bigg\{\int_0^{q_1+q_2(X)} y f_{D\mid X}(y \mid X)\, dy \\ + q_1 \bar{F}_{D\mid X}(q_1 + q_2(X) \mid X)\bigg\} - p_1 q_1, \end{aligned} \tag{22}$$

where we have used that $q_2(x)(\bar{F}_{D\mid X}(q_1 + q_2(x) \mid x) - p_2) = 0$. To determine the optimal q_1, we differentiate Equation (22) to obtain

$$\begin{aligned} \frac{\partial B}{\partial q_1} = \mathbb{E}_X\{q_2(X) f_{D\mid X}(q_1 + q_2(X))(1 + \partial q_2(X)/\partial q_1) \\ + \bar{F}_{D\mid X}(q_1 + q_2(X) \mid X)\} - p_1. \end{aligned} \tag{23}$$

When $\bar{F}^{-1}_{D\mid X}(p_2 \mid x) \leq q_1$, $q_2 = 0$ and the expression inside the expectation reduces to $\bar{F}_{D\mid X}(q_1 \mid X)$; otherwise, $\partial q_2/\partial q_1 = -1$ and the expectation applies to p_2. Thus,

$$\frac{\partial B}{\partial q_1} = \mathbb{E}_X\{\min\{\bar{F}_{D\mid X}(q_1 \mid X), p_2\}\} - p_1. \tag{24}$$

Expression (10) then follows from the first order conditions applied to Equation (22). Note that $\partial B/\partial q_1$ is decreasing and therefore the price/quantity relation is unique.

By solving the maximization in Equation (22) in q_1, the profit of supplier 1, $S_1 = (p_1 - c_1)q_1$, can be shown to depend only on p_1 for some given p_2. If $1/q_1$ is concave in p_1 given p_2, $q_1'' q_1/(q_1')^2 < 2$ ($q_1' < 0$ by construction). Let p_1 satisfy the first order condition; i.e.,

$$\frac{\partial S_1}{\partial p_1} = q_1 + (p_1 - c_1)q_1'. \tag{25}$$

Then,

$$\begin{aligned} \frac{\partial^2 S_1}{\partial p_1^2} &= (p_1 - c_1)q_1'' + 2q_1' = \frac{q_1 q_1''}{-q_1'} + 2q_1' \\ &= q_1'\left(2 - \frac{q_1'' q_1}{(q_1')^2}\right) \leq 0, \end{aligned} \tag{26}$$

which implies that S_1 must be increasing and then decreasing, and hence unimodal.

Given the structure of $S_2 = (p_2 - c_2)\mathbb{E}_X\{q_2(X)\}$, similar concavity arguments can be drawn to show unimodality of S_2 with respect to p_2. Note that the unimodality results of S_1 and S_2 guarantee existence but are not sufficient to guarantee uniqueness of the Nash equilibrium. □

Proof of Theorem 4. Note that all "IV" superindexes have been dropped for simplicity.

We shall start the proof at $t = L_2$. That the buyer can eventually be overstocked is directly derived from the functional form $q_2(x) = (\bar{F}^{-1}_{D\mid X}(p_2 \mid x) - q_1)^+$, where $\bar{F}_{D\mid X}(q_1 \mid x) \leq c_2$ implies $q_2(x) = 0$ $\forall p_2$. We then assume $\bar{F}_{D\mid X}(q_1 \mid x) > c_2$ to derive the pricing strategy of the fast supplier. To avoid $q_2(x) = 0$, the rational supplier sets a price $c_2 \leq p_2 \leq \bar{F}_{D\mid X}(q_1 \mid x)$ and thus $p_2 = \bar{F}_{D\mid X}(q_1 + q_2(x))$. We can hence analyze the pricing problem indirectly, concentrating on the optimal value of $q_2(x)$ because this uniquely determines the price quoted by supplier 2; i.e.,

$$\begin{aligned} S_2 &= \max_{c_2 \leq p_2 \leq \bar{F}_{D\mid X}(q_1\mid x)} (p_2 - c_2)(\bar{F}^{-1}_{D\mid X}(p_2 \mid x) - q_1) \\ &= \max_{0 \leq q_2 \leq \bar{F}^{-1}_{D\mid X}(c_2\mid x) - q_1} (\bar{F}_{D\mid X}(q_1 + q_2(x) \mid x) - c_2) q_2(x). \end{aligned} \tag{27}$$

Since $S_2 = 0$ if $q_2(x) = 0$ or $q_2(x) = \bar{F}^{-1}_{D\mid X}(c_2 \mid x) - q_1$, $q_2(x)$ is interior and thus satisfies the first order condition

$$f_{D\mid X}(q_1 + q_2(x) \mid x) q_2(x) = \bar{F}_{D\mid X}(q_1 + q_2(x) \mid x) - c_2, \tag{28}$$

which then leads to $p_2 = \bar{F}_{D\mid X}(q_1 + q_2(x) \mid x)$. However, to be able to *uniquely* determine $q_2(x)$ with Equation (28) we require the objective in Equation (27) to be quasiconcave (unimodal). That is, the second derivative of Equation (27) evaluated at q_2,

$$-2 f_{D\mid X}(q_1 + q_2(x) \mid x) - f'_{D\mid X}(q_1 + q_2(x) \mid x) q_2(x),$$

should be nonpositive. As the density f of an IGFR random variable satisfies

$$f' \geq -f/x - f^2/\bar{F}, \tag{29}$$

quasiconvexity of Equation (27) follows readily when $D \mid X$ is IGFR:

$$\begin{aligned} &-2 f_{D\mid X}(q_1 + q_2(x) \mid x) - f'_{D\mid X}(q_1 + q_2(x) \mid x) q_2(x) \\ &\quad\leq \left(-2 + \frac{q_2(x)}{q_1 + q_2(x)}\right) f_{D\mid X}(q_1 + q_2(x) \mid x) \\ &\qquad + \frac{f^2_{D\mid X}(q_1 + q_2(x) \mid x)}{\bar{F}_{D\mid X}(q_1 + q_2(x) \mid x)} q_2(x) \end{aligned}$$

$$\begin{aligned}&\le f_{D\mid X}(q_1+q_2(x)\mid x)\left(\frac{f_{D\mid X}(q_1+q_2(x)\mid x)}{\bar F_{D\mid X}(q_1+q_2(x)\mid x)}q_2(x)-1\right)\\&=f_{D\mid X}(q_1+q_2(x)\mid x)\left(\frac{\bar F_{D\mid X}(q_1+q_2(x)\mid x)-c_2}{\bar F_{D\mid X}(q_1+q_2(x)\mid x)}-1\right)\\&=-c_2 f_{D\mid X}(q_1+q_2(x)\mid x)/\bar F_{D\mid X}(q_1+q_2(x)\mid x)\le 0,\end{aligned}\tag{30}$$

where we have used Equation (28) in Equation (30).

We next move on to $t=L_1$. The optimal order quantity to be placed to the slow supplier follows from the maximization in Equation (12). The first order condition of this problem,

$$\mathbb{E}_X\left\{\frac{\partial B_2}{\partial q_1}(q_1\mid X)\right\}-p_1=0,$$

corresponds to Equation (15) and provides a fixed point equation to compute q_1 for some given p_1. A sufficient condition for ensuring that Equation (15) *uniquely* determines q_1 is that the objective inside the maximization in Equation (12) is concave in q_1 for all $x\ge 0$; i.e., that

$$\frac{\partial^2 B_2}{\partial q_1^2}(q_1\mid X)\le 0,\quad \forall x\ge 0.\tag{31}$$

This is equivalent to ensuring that the first derivative of B_2,

$$B_2'=\bar F_{D\mid X}+(\bar F_{D\mid X}-c_2)^+(1+q_2'),\tag{32}$$

is nonincreasing in q_1. Note that we have used $B_2{}'$ to denote $\partial B_2/\partial q_1$, $\bar F_{D\mid X}$ to denote $\bar F_{D\mid X}(q_1+q_2(x)\mid x)$, and q_2' instead of $\partial q_2/\partial q_1$ to shorten notation. To this end, we first check that Equation (32) is continuous in q_1 at the potentially problematic point $q_1=\bar F^{-1}_{D\mid X}(c_2)$ (the dependence on x will be omitted for simplicity):

$$\begin{aligned}&\lim_{q_1\to \bar F^{-1}_{D\mid X}(c_2)^+} B_2{}'\\&\stackrel{(a)}{=}\lim_{q_1\to \bar F^{-1}_{D\mid X}(c_2)^+}\bar F_{D\mid X}(q_1+q_2(x))\stackrel{(b)}{=}\bar F_{D\mid X}\left(\lim_{q_1\to \bar F^{-1}_{D\mid X}(c_2)^+} q_1+q_2(x)\right)\\&=\bar F_{D\mid X}\left(\bar F^{-1}_{D\mid X}(c_2)\right)=c_2\end{aligned}\tag{33}$$

$$\begin{aligned}&\lim_{q_1\to \bar F^{-1}_{D\mid X}(c_2)^-} B_2{}'\\&\stackrel{(c)}{=}\lim_{q_1\to \bar F^{-1}_{D\mid X}(c_2)^-}\bar F_{D\mid X}(q_1+q_2(x))\\&\quad+(\bar F_{D\mid X}(q_1+q_2(x))-c_2)(1+q_2(x)')\\&\stackrel{(d)}{=}\bar F_{D\mid X}\left(\lim_{q_1\to \bar F^{-1}_{D\mid X}(c_2)^-} q_1+q_2(x)\right)\\&\quad+\left(\bar F_{D\mid X}\left(\lim_{q_1\to \bar F^{-1}_{D\mid X}(c_2)^-} q_1+q_2(x)\right)-c_2\right)\lim_{q_1\to \bar F^{-1}_{D\mid X}(c_2)^-}(1+q_2'(x))\\&\stackrel{(e)}{=}\bar F_{D\mid X}(\bar F^{-1}_{D\mid X}(c_2))\\&\quad+(\bar F_{D\mid X}(\bar F^{-1}_{D\mid X}(c_2))-c_2)\lim_{q_1\to \bar F^{-1}_{D\mid X}(c_2)^-}(1+q_2'(x))=c_2.\end{aligned}\tag{34}$$

Equalities (33) and (34) show that $B_2{}'$ is indeed continuous. Specifically, (a) follows because whenever $q_1\ge \bar F^{-1}_{D\mid X}(c_2)$ the buyer is overstocked, the second term in Equation (32) is set to zero, and $q_2(x)=0$. Analogously, the buyer is not overstocked in (c), and therefore $q_2(x)>0$ implies that we must consider also the second term of Equation (32). In (b) and (d) we have implicitly assumed that the limits we were finding existed, as it was the case. Finally, (e) follows because $q_2(x)$ must satisfy the first order condition shown in Equation (13) for some value $q_2(x)\in(0,\bar F^{-1}_{D\mid X}(c_2)-q_1)$. As $q_1\to\bar F^{-1}_{D\mid X}(c_2)^-$, the only possible value of $q_2(x)$ that satisfies Equation (13) is $q_2(x)=\epsilon$ for some arbitrarily small $\epsilon>0$. Because $B_2{}'$ is continuous, concavity of B_2 will follow from nonpositivity of $B_2{}''\equiv\partial^2 B_2/\partial q_1^2$.

When $q_1\ge \bar F^{-1}_{D\mid X}(c_2)$, i.e., when the buyer is overstocked, and $D\mid X$ is IGFR,

$$B_2{}''=-(1+q_2(x)')f_{D\mid X}\le 0$$

because $q_2(x)'\ge -1$ and thus B_2^{ds} is concave in q_1. This is because

$$q_2'(x)=\frac{1}{2+q_2(x)(f'_{D\mid X}(q_1+q_2(x)\mid x)/f_{D\mid X}(q_1+q_2(x)\mid x))}-1,\tag{35}$$

and when $D\mid X$ is IGFR we can use Equations (13) and (29) to lower bound the term $q_2(x)f'_{D\mid X}/f_{D\mid X}$ by

$$\begin{aligned}q_2(x)\frac{f'_{D\mid X}}{f_{D\mid X}}&\ge-\frac{q_2(x)}{q_1+q_2(x)}-q_2(x)\frac{f_{D\mid X}}{\bar F_{D\mid X}}\\&\ge -1-\frac{\bar F_{D\mid X}-c_2}{\bar F_{D\mid X}}=-2+\frac{c_2}{p_2}.\end{aligned}\tag{36}$$

In the nonoverstocked regime, where $q_1<\bar F^{-1}_{D\mid X}(c_2)$, it follows that

$$\begin{aligned}B_2{}''&=-(1+q_2'(x))f_{D\mid X}-(1+q_2'(x))^2 f_{D\mid X}+(\bar F_{D\mid X}-c_2)q_2''(x)\\&\stackrel{(a)}{=}(\bar F_{D\mid X}-c_2)\left(q_2''(x)-\frac{(1+q_2'(x))(2+q_2'(x))}{q_2(x)}\right),\end{aligned}$$

where (a) follows from Equation (13). Concavity of B_2, hence, follows if

$$q_2''(x)\le\frac{(1+q_2'(x))(2+q_2'(x))}{q_2(x)}.\tag{37}$$

Next, by differentiating Equation (13) implicitly with respect to q_1 and defining $h_{D\mid X}=f'_{D\mid X}/f_{D\mid X}$, we get to

$$q_2'(x)=\frac{1}{2+q_2(x)h_{D\mid X}}-1,\tag{38}$$

$$q_2'(x)=-(1+q_2'(x))^2(q_2(x)(1+q_2'(x))h'_{D\mid X}+q_2'(x)h_{D\mid X}),\tag{39}$$

which can be plugged into Equation (37) to obtain

$$q_2(x)(1+q_2'(x))h'_{D\mid X}+(1+q_2'(x))h_{D\mid X}+\frac{3}{q_2(x)}\ge 0,$$

where it has been assumed that $q_2'(x)\ge -1$, which is true for all IGFR $D\mid X$. We next replace $q_2'(x)$ by its explicit value from Equation (38) to obtain

$$h'_{D\mid X}+\frac{4}{q_2(x)}h_{D\mid X}+\frac{6}{q_2(x)^2}\ge 0,$$

which has to be satisfied for all values of c_2, q_1, and $q_2(x)$ such that Equation (13) holds. Precisely because Equation (13) holds, the previous concavity condition can be transformed into

$$h'_{D\mid X}+4h_{D\mid X}\frac{f_{D\mid X}}{\bar F_{D\mid X}-c_2}+6\frac{f^2_{D\mid X}}{(\bar F_{D\mid X}-c_2)^2}\ge 0,\tag{40}$$

where all the functions involved are evaluated in $q_1 + q_2(x)$. Because $q_2(x) \geq 0$ and $q_2'(x) > -1$, $q_1 + q_2'(x)$ will potentially span almost all the real positive semiaxis. Thus, to ensure that B_2 is concave, it is sufficient that Equation (40) holds for all real values $y \geq y_0 \triangleq (q_1 + q_2(x))|_{q_1=0}$, where

$$f_{D\,|\,X}(y_0 \mid x) y_0 = \bar{F}_{D\,|\,X}(y_0 \mid x) - c_2. \tag{41}$$

Note that this is equivalent to Equation (14).

Finally, we move on to the pricing decision of supplier 1. Given Equation (15), the profit of the slow supplier can be expressed in terms of q_1 as

$$S_1 = \max_{q_1 \geq 0} \big(\mathbb{E}_X\{B_2{}'(q_1 \mid X)\} - c_1\big) q_1, \tag{42}$$

where the optimal price is determined using $p_1 = \mathbb{E}_X\{B_2{}'(q_1|X)\}$. The optimal quantity q_1, stated in Equation (17), is the solution to the maximization in Equation (42) and follows from the first order condition applied to the objective. Such q_1 is *uniquely* characterized when the objective function in Equation (42) is unimodal, that is, when the second derivative of Equation (42) evaluated at q_1^* is negative. This is stated in Equation (16). □

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