Manufacturing & Service Operations Management

Publication details, including instructions for authors and subscription information:
http://pubsonline.informs.org

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Consider a manufacturer or wholesaler who supplies some item to retailers facing demand rates that depend on the shelf or display space that is devoted to that product by themselves and their competitors. The manufacturer, via the use of financial levers at her disposal, wishes to coordinate this decentralized chain while making a profit. We model the physical scenario as one of constant displayed inventory level (on which demand rate depends positively) and continuous replenishment. With a single retailer, we show that to coordinate the channel and make a profit the manufacturer needs to augment the wholesale price lever by another—an inventory holding costs subsidy offered to the retailer. When multiple retailers compete in that product's market, there are two ways to envision and model the demand and market split. One assumes that market demand depends on aggregate inventory displayed, and then splits according to individual display levels. The other ''assigns'' customers to retailers according to their display levels, and then assumes that purchases are a function of the display level at the retailer selected. We characterize retailers' Nash equilibria in these models, and we explore whether the manufacturer can coordinate such channels.
(*Coordination; Shelf-Space-Dependent Demand; Retail Competition; Nash Equilibrium*)

1. Introduction

It it is well known that retailers can often affect sales volume of a product by increasing the shelf space allocated to it. Because increased shelf space or displays often require the retailer to keep higher inventories, choosing an item's shelf space is part of choosing its inventory level. Thus, some operations management researchers have incorporated inventory-level dependence of demand into various inventory control models (Gerchak and Wang 1994, Balakrishnan et al. 2000, and references therein). This literature dealt exclusively with a single decision maker (the retailer). It is our goal to analyze a decentralized manufacturer-retailer(s) supply chain, which, though physically greatly simplified, recognizes the positive dependence of demand on the quantity displayed.

We first analyze a stylized model by assuming that a supplier (manufacturer) supplies some product to a single retailer. The retailer decides to allocate some shelf space to display S units of a product. Once allocated, the shelf is replenished via the $(S - 1, S)$ policy, so the displayed inventory will be kept at level S all the time. This mimics store replenishment situations when replenishments are very frequent. The demand rate depends on S via a general increasing concave function. The manufacturer's decisions in a decentralized setting pertain to the financial arrangement with the retailer. To coordinate the channel as well as to make a profit, the manufacturer needs a second financial lever in addition to the wholesale price. We suggest a holding-cost subsidy offered to the retailer, which will cause the retailer to stock more. With the two levers—the price plus inventory-subsidy contract—the manufacturer can achieve not

Manufacturing & Service Operations Management © 2001 INFORMS
Vol. 3, No. 1, Winter 2001, pp. 82–87
1523-4614/01/0301/0082$05.00
1526-5498 electronic ISSN

only channel coordination but any desired allocation of the channel profit between himself and the retailer.

We then extend our analysis to two retailers who share and compete in the same market at the downstream of the supply chain. We propose two ways to model the customer demand and retailers' competition processes. The first model envisions a situation where a customer selects her demand quantity based on the *aggregate* inventory of both retailers, and then chooses which retailer to buy from based on their relative inventory levels. In the second model, a customer will first choose the retailer to buy from based on the relative amounts of inventories displayed by both retailers, and then decide on the purchase quantity based on *individual* inventory level at the chosen retailer. We assume for both models that the split of customers between the two retailers is proportional to their relative inventory.

Concrete insights into duopoly equilibrium and supply-chain coordination are then obtained by assuming that each customer's demand is a concave power function of the displayed inventory. For the first model of competition (where demand is a function of aggregate inventory), we show that there exists a unique Nash equilibrium of displayed inventories for any given contract offered by the upstream manufacturer. Surprisingly, the manufacturer, by using the price plus inventory-subsidy contract, can still achieve channel coordination for this competitive supply chain! In the second model of competitive retailers (where demand is a function of individual inventory) we characterize one symmetric equilibrium solution of the problem. One key insight we obtain here is that with demand function of individual inventory, the manufacturer cannot coordinate a competitive supply chain.

A much more extensive motivation, literature review, all mathematical proofs, and additional discussion can be found in Wang and Gerchak (2000).

2. A Manufacturer and Single-Retailer Supply Chain

2.1. The Model and Centralized Control

A single product is produced by a manufacturer and then sold to consumers through (for now) a single retailer. The marginal production cost and retail price are constant at c dollars/unit and p dollars/unit, respectively, where $p > c$. The demand rate for the item will depend on the amount of inventory displayed on the retailer's shelf. Specifically, a constant inventory level of I units generates a demand of $D(I)$ units/year. In general, $D(I)$ can be assumed to be an increasing and concave function (i.e., $D'(I) > 0$, and $D''(I) < 0$) to reflect the motivational effect of inventory on demand and the "diminishing returns." Displaying inventory at the retailer is costly. Assume that a constant inventory cost of h dollars/unit/year is charged. So, the key decision here is to choose the displayed inventory level I to trade-off increased sales against inventory costs. Note that once I is chosen, the system is assumed to keep the inventory at level I all the time by continuously replenishing it.

Now, if this supply chain is centrally owned and controlled, the objective is to maximize the long-run average channel profit (i.e., the profit rate):

$$\max_{I \geq 0} \Pi^c(I) = (p - c)D(I) - hI, \tag{1}$$

where the first term is the sales revenue net of production cost and the second term the inventory-holding costs. $\Pi^c(I)$ is concave, and thus the unique solution I^c is given by

$$D'(I^c) = h/(p - c), \quad \text{i.e., } I^c = D'^{-1}(h/(p - c)). \tag{2}$$

We next consider a decentralized system where the manufacturer, through contractual arrangements, wholesales the product to the retailer who then chooses its displayed inventory level and, hence, the demand rate.

2.2. Price Plus Inventory-Subsidy Contract and Channel Coordination

Suppose that the manufacturer offers the retailer a wholesale price of w dollars/unit and an inventory-holding subsidy of s dollars/unit/year towards any inventory the retailer chooses to hold on the shelf. The retailer then chooses an inventory display level I to maximize her own profit (rate):

$$\max_{I \geq 0} \Pi^r(I) = (p - w)D(I) - (h - s)I, \tag{3}$$

and her unique optimal inventory level I^r is, thus, given by

$$D'(I^r) = (h - s)/(p - w), \quad \text{i.e.,}$$
$$I^r = D'^{-1}((h - s)/(p - w)). \tag{4}$$

Comparing (4) with (2), we have the following important observation:

PROPOSITION 1. *For any contract (w, s) such that*

$$(h - s)/(p - w) = h/(p - c), \quad \text{i.e.,}$$
$$s = h - h(p - w)/(p - c), \tag{5}$$

we have $I^r = I^c$.

In other words, for *any* wholesale price w, $c < w < p$, offered by the manufacturer, if he accordingly chooses an inventory subsidy $s = h - h(p - w)/(p - c)$, then the retailer will always be induced to choose the centralized system-optimal inventory level I^c and, hence, such a contract *coordinates* the decentralized supply chain. Thus, there exists a continuum of (w, s) contracts that coordinate the supply chain.

The following argument (Pasternack 1985) illustrates how the manufacturer can find his best strategy: Focus on the set of contracts (w, s) that satisfy (5) with $c \le w \le p$. We know from Proposition 1 that any contract within this set will coordinate the channel and, hence, achieve the maximum channel profit. But, as we will show next, different contracts within this set (represented by different values of w) provide the retailer with a different amount of profit—the rest of the maximal channel profit will go to the manufacturer. As a consequence, the manufacturer needs simply choose a value of w so as to allocate any amount of profit required by the retailer (so that she will accept the contract), and thus to extract as much profit out of the supply chain as he can! Now, with $s = h - h(p - w)/(p - c)$ and $I = I^c$, after some algebra, the retailer's profit in (3) can be written as

$$\Pi^r(I^c) = [D(I^c) - hI^c/(p - c)](p - w), \tag{6}$$

which is linearly decreasing in w.

To summarize, a properly designed price plus inventory-subsidy contractual arrangement (w, s) can achieve: (1) coordination of the supply-chain channel, (2) any desired allocation of channel profit between the manufacturer and the retailer. For an extension of this model to nonlinear holding costs, see Wang and Gerchak (2000).

Suppose that the manufacturer does not use the inventory-subsidy lever in the contract (i.e., set $s = 0$). Then, because $D'(I)$ is a decreasing function, by simply comparing (4) with (2), we see that as long as the manufacturer charges a wholesale price w above his marginal cost c, the inventory level I^r in a price-only contractual arrangement will always be lower than the inventory level I^c in the centralized system. Thus, a price-only contract, in general, cannot coordinate the supply chain.

3. Two Competitive Retailers with Demand a Function of Aggregate Inventory

Here a manufacturer faces two competitive retailers (R1 and R2) who share a market with a constant selling price p. The total demand will depend on their aggregate inventory, i.e., $D(I_1 + I_2)$, where I_1 and I_2 are the inventory-stocking levels of R1 and R2, respectively. The two retailers compete for this total demand based on their inventory levels. Say R1 secures a portion $A(I_1, I_2)$ of $D(I_1 + I_2)$, where the fraction $A(I_1, I_2)$ is a function of I_1 and I_2. Therefore, R2 has the rest, i.e., $1 - A(I_1, I_2)$ of $D(I_1 + I_2)$. $A(I_1, I_2)$ is increasing in I_1 and decreasing in I_2, capturing the shelf space exposure competition between the retailers.

The manufacturer offers both retailers an *identical* price plus inventory-subsidy contract (w, s). Then, each retailer chooses her own inventory-stocking level so as to maximize her own profit (knowing her profit also depends on the other retailer's action). That is,

$$\max_{I_1 \ge 0} \Pi_1(I_1, I_2) = (p - w)A(I_1, I_2)D(I_1 + I_2) - (h_1 - s)I_1, \quad \text{for R1;} \tag{7}$$

$$\max_{I_2 \ge 0} \Pi_2(I_1, I_2) = (p - w)[1 - A(I_1, I_2)]D(I_1 + I_2) - (h_2 - s)I_2, \quad \text{for R2.} \tag{8}$$

h_1 and h_2 are the inventory holding costs for R1 and R2, respectively.

Before analyzing the behaviour of the two competitive retailers, we want to comment on a scenario where the two retailers (but not the manufacturer) are *centrally* controlled, so as to maximize the total profit $\Pi_1(I_1, I_2) + \Pi_2(I_1, I_2)$ by choosing I_1 and I_2 jointly. This centralized scenario may not be interesting by itself; but the results obtained here will provide interesting insights into our later analysis of competitive/decentralized retailers. From (7)–(8), we have

$$\Pi_1(I_1, I_2) + \Pi_2(I_1, I_2)$$
$$= (p - w)D(I_1 + I_2) - (h_1 - s)I_1 - (h_2 - s)I_2.$$

Note that I_1 and I_2 appear here in the first term (i.e., total revenue net of wholesale costs) in an aggregate form, i.e., $I_1 + I_2$, but they appear separately in the holding-cost terms. Thus, without loss of generality, if R1 has a lower holding cost (i.e., $h_1 < h_2$), the best centralized decision will be to stock *only* at R1, and the optimal quantity will thus be the same as when R1 alone owns the whole market. Furthermore, we have

PROPOSITION 2. *If the two retailers are identical (i.e.,* $h_1 = h_2$*) and are centrally controlled, their total profit* $\Pi_1(I_1, I_2) + \Pi_2(I_1, I_2)$ *will depend only on their total inventory level* $I_1 + I_2$ *(i.e., it does not matter how any given total inventory is allocated between the retailers). The optimal total inventory will be the same as when one of the retailers is the only one operating on the given market place.*

Let us now return to the competitive retailers setting. For this supply chain, we are interested in the following questions: For any given manufacturer contract (w, s), what are the retailers' equilibrium inventory decisions and their properties? Is the equilibrium unique? Can this two-parameter contractual arrangement coordinate the supply chain?

The general problem in (7)–(8) is too complex to analyze. To gain some concrete insights, we will consider the following specific demand function:

$$D(I_1 + I_2) = a(I_1 + I_2)^b, \qquad a > 0, \qquad 1 > b > 0.$$

Note that $[dD(I)/dI]/[D(I)/I]$, the demand's inventory elasticity, equals to b. Second, we will use the proportional demand allocation model, which was motivated in the Introduction. That is,

$$A(I_1, I_2) = I_1/(I_1 + I_2), \quad \text{and so}$$
$$1 - A(I_1, I_2) = I_2/(I_1 + I_2).$$

Finally, we consider the case of two identical retailers; so we let $h_1 = h_2 = h$. With these specifications, Problem (7)–(8) now reduces to

$$\max_{I_1 \geq 0} \Pi_1(I_1, I_2) = (p - w)\cdot I_1/(I_1 + I_2)\cdot a(I_1 + I_2)^b - (h - s)I_1, \quad \text{for R1;} \tag{9}$$

$$\max_{I_2 \geq 0} \Pi_2(I_1, I_2) = (p - w)\cdot I_2/(I_1 + I_2)\cdot a(I_1 + I_2)^b - (h - s)I_2, \quad \text{for R2.} \tag{10}$$

The following theorem characterizes the competitive equilibrium for this retailers' game:

THEOREM 1. *The unique Nash equilibrium is for each of the two retailers to stock*

$$I^N = [a(1 + b)(p - w)2^{b-2}/(h - s)]^{1/(1-b)}. \tag{11}$$

At the equilibrium, the system-wide inventory of both retailers will be

$$2I^N = [a(1 + b)(p - w)/2(h - s)]^{1/(1-b)}. \tag{12}$$

Now, had one of the retailers occupied the entire market (and faced the same manufacturer contract), her optimal inventory would have been (specializing (4) to our particular demand function):

$$I^r = [ab(p - w)/(h - s)]^{1/(1-b)}. \tag{13}$$

Comparing this with (12), because $\infty > (1 + b)/2b > 1$ for $0 < b < 1$, we have $2I^N > I^r$. That is,

COROLLARY 1. *For any given manufacturer contract* (w, s)*, the total inventory displayed by two competitive retailers is always higher than that of a single retailer (or two centrally controlled retailers). Thus, competition generates* ***inefficiency*** *at the retail stage of the supply chain.*

Can a (w, s) contractual arrangement coordinate this manufacturer-competitive retailers supply chain? The answer is yes! To see that we need only to show that such a contract can induce the two retailers to choose the centralized or system-optimal inventory levels. However, from Proposition 2 we know that the *total* inventory of two retailers (independent of inventory allocation between them) maximizing the *system-wide* performance will be the same as that maximizing a system with a single retailer. Thus, to achieve channel coordination, one only needs to design (w, s), so that the decentralized total inventory given in (12) is equal to the centralized optimal inventory, which can be derived from (2) for a single-retailer system (specializing to our specific demand function) as

$$I^c = [ab(p - c)/h]^{1/(1-b)}. \qquad (14)$$

Thus, one can easily show,

PROPOSITION 3. *If* (w, s) *is offered such that*

$$s = h - (1 + b)(p - w)h/2b(p - c), \qquad (15)$$

then $2I^N = I^c$.

In light of Corollary 1, this coordinating property of a (w, s) contract is particularly valuable: *It can actually eliminate the inefficiency generated by the presence of competing retailers within the supply chain!*

When coordination is achieved, each retailer's inventory is $I^N = I^c/2$. Substituting (15) into either (9) or (10), we can show that the retailers' profits will be

$$\Pi_1 = \Pi_2 = [a(I^c)^b/2 - (1 + b)hI^c/4b(p - c)] \times (p - w). \qquad (16)$$

Thus, again, not only can a (w, s) contract coordinate the supply chain, but it can also achieve, by varying w, any desired allocation of the total channel profit between the manufacturer and the retailers.

4. Two Competing Retailers with Demand a Function of Individual Inventory

In this section we model a situation where a customer chooses between R1 and R2 based on their relative displayed-inventory levels, but her demand quantity then depends solely on the inventory of the chosen retailer. Specifically, the competition process of the two retailers can be thought of as follows: When their displayed inventories are I_1 and I_2 units, respectively, a portion $A(I_1, I_2) = I_1/(I_1 + I_2)$ of the total a customers will choose R1, *each* with a demand quantity of $D(I_1) = I_1^b$, $0 < b < 1$; and $1 - A(I_1, I_2) = I_2/(I_1 + I_2)$ of the a customers will choose R2, *each* with a demand quantity of $D(I_2) = I_2^b$. Consider two identical retailers, i.e., $h_1 = h_2 = h$. With a manufacturer's (w, s) contract, the retailers face the following decisions:

$$\max_{I_1 \geq 0} \Pi_1(I_1, I_2) = (p - w) \cdot I_1/(I_1 + I_2) \cdot a \cdot I_1^b - (h - s) \cdot I_1, \quad \text{for R1}; \qquad (17)$$

$$\max_{I_2 \geq 0} \Pi_2(I_1, I_2) = (p - w) \cdot I_2/(I_1 + I_2) \cdot a \cdot I_2^b - (h - s) \cdot I_2, \quad \text{for R2}. \qquad (18)$$

We note that here the two retailers essentially face the same market (i.e., the same a customers) as that in (9)–(10). The difference is that a customer who chooses, say, R1 will here contribute a demand of I_1^b, while in (9)–(10) it was $(I_1 + I_2)^b$. Thus, the total demands are different in the two scenarios even when the inventory levels are the same.

It turns out that Problem (17)–(18) is much more complex to analyze than Problem (9)–(10). Instead of trying to fully characterize the response curves and equilibrium point(s), in the following we simply identify one specific equilibrium—the symmetric equilibrium, where the two retailers display the same amount of inventory. Intuitively, because the two retailers are identical, the most likely equilibrium, if any, should be symmetric.

From (17), a symmetric equilibrium, *if any,* can be found by substituting $I_1 = I_2 = I^N$ into

$$\partial\Pi_1(I_1, I_2)/\partial I_1 = F_1(I_1, I_2) = a(p - w)I_1^b[bI_1 + (1 + b)I_2] \div (I_1 + I_2)^2 - (h - s) = 0 \qquad (19)$$

and solving for I^N. We thus obtain the *unique* solution

$$I^N = [a(1 + 2b)(p - w)/4(h - s)]^{1/(1-b)}. \qquad (20)$$

The next theorem states that, if $b < 0.5$, $I_1 = I_2 = I^N$ is indeed an equilibrium point.

THEOREM 2. (I^N, I^N) *constitutes a Nash equilibrium point of (21)–(22) if* b < 0.5.

Now suppose that the two retailers are centrally controlled, so as to maximize their total profit by choosing jointly how much inventory each should stock (i.e., I_1 and I_2).

PROPOSITION 4. *If the two retailers are centrally controlled, then for any given total inventory the best policy is to stock at only one of the retailers (and, hence, to close down the other).*

The intuition here is as follows. Because each customer's demand depends (increases) only on (in) the size of ONE pile, and as the total number of customers is constant in this model, then for any given total inventory, stocking all of it at one location will induce more demand than splitting it into two piles. While the demand function itself is concave, the profit function it gives rise to is convex (see proof in Wang and Gerchak 2000) and, hence, the boundary solution.

Can the manufacturer coordinate such a supply chain? It depends on how the retail stage operates. If the two retail locations are centrally controlled, Proposition 4 shows that one should close down one of the locations and, thus, the retail stage acts just like the single retailer. Then, as we have showed earlier, a (w, s) contractual arrangement offered by the manufacturer can coordinate the supply chain. But when two retail locations coexist through competition, Proposition 4 states that inefficiency/waste will occur within the retail stage. As a result, with a (w, s) contract arrangement, the manufacturer will not be able to coordinate the supply chain. Any attempt of coordination here must have (among other arrangements) the contingency of *physically* pooling inventory. In contrast, as we have shown in §3, supply-chain coordination can be achieved with two competing retailers when customer demand depends on the ''aggregate'' inventory of both retailers. The fundamental difference is that there the allocation of inventory between the retailers does not in itself cause inefficiency (e.g., Proposition 2).

5. Concluding Remarks

As argued by Moorthy (1993, p. 182), ''The interesting issues in channel competition arise from the effect of downstream (retail) competition on relations between the manufacturer and the retailers.'' Our model indeed attempted to capture such interactions within a concrete setting. We did so (in the analysis of two competitive retailers) by viewing the system as that of a Stackleberg leader (the manufacturer) who considers the effect of its actions on the resulting Nash equilibrium of the competing retailers.

Finally, we want to point out that a retailer's inventory-holding cost may partially depend on the wholesale price w she pays. For example, we can model retailer's unit-holding cost (rate) as $h + h_1 w$, where h represents, say, the shelf space cost, and h_1 the opportunity cost of investment in inventory. With a single retailer, one can easily show that the manufacturer can achieve channel coordination with a (w, s) contract, so that $s = (h + h_1 p)(w - c)/(p - c)$.

Acknowledgments
The authors would like to thank the Editor-in-Chief, Leroy B. Schwarz, the Special Issue/Senior Editor, Hau L. Lee, two anonymous referees, and Saibal Ray for their valuable comments and suggestions which have improved the paper significantly.

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Wang, Y., Y. Gerchak. 2000. Supply chain coordination when demand is shelf-space dependent. Working paper. Dept. of Operations, Weatherhead School of Management, Case Western Reserve University, Cleveland, OH.

The consulting Senior Editor for this manuscript was Hau Lee. This manuscript was received on August 18, 1999, and was with the authors 118 days for 4 revisions. The average review cycle time was 82.75 days.