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The Value of Operational Flexibility in the Presence of Input and Output Price Uncertainties with Oil Refining Applications

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Refining is indispensable to almost every natural-resource-based commodity industry. It involves a series of complex processes that transform inputs with a wide range of quality characteristics into refined finished products sold to end markets. In this paper, we take the perspective of a profit-maximizing refiner that considers upgrading its existing simple refinery to include intermediate-conversion flexibility, i.e., the capability of converting heavy intermediate components to light ones. We present a stylized two-stage stochastic programming model of a petroleum refinery to investigate the value drivers of conversion flexibility and the impact of input and output market conditions on its economic potential. Conversion flexibility adds value to refineries by either transforming a nonprofitable situation into a profitable one (referred to as purchase benefit) or improving profitability of an already profitable situation (referred to as unit revenue benefit). In a real-data-calibrated numerical study, we find the value of conversion flexibility (VoC) to be significant, accounting for 40% of the expected profit with conversion, and the purchase benefit and unit revenue benefit are equally important. Contrary to the intuition that, as a recourse action, conversion offers higher value for greater input price volatility, we find that VoC may decrease in input price volatility as a result of the differential impacts of increasing price volatility on the purchase benefit and the unit revenue benefit. Refineries also vary in their range flexibility, i.e., the ability to accommodate a narrow or wide range of inputs of different quality levels. Whether the range flexibility increases or decreases the value of conversion flexibility is affected by the direction in which the refinery expands its processing range and the heaviness of crude oils.

Keywords: operational flexibility; oil refining; petroleum industry; spot markets; stochastic models
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1. Introduction

Most natural-resource-based commodities share two common features: First, the quality of the commodity varies by the geographic location where it is produced or harvested. Second, the economic value of the commodity comes largely from the application of the derivative products that originate from it, not from the direct use of the commodity itself. Refining is a key link in the supply chain for these commodities; it is where raw materials from various origins and of different qualities are transformed into refined derivative products of precise specifications, some of which serve as feedstocks to downstream industries while others are consumed by the end users.

Take the petroleum refining industry, for example, which accounted for 2.55% of the world's gross domestic product in 2004 (Dixon et al. 2007). The input, crude oil, is a mixture of molecules made of carbon and hydrogen atoms. There are as many different crude oils as there are different oil fields, and each field yields its own particular quality. A rough form of characterization is the classification of "light" and "heavy" crude oils. Compared with a heavy crude oil, a light crude oil contains a higher proportion of the smaller hydrocarbon molecules with shorter carbon chains. Petroleum products obtained from crude oil can be grouped into light (e.g., gasoline), medium (e.g., diesel), and heavy (e.g., lubricating oil) products based on the composition of the light and heavy distillates in the product.

Most petroleum products are not direct distillation outputs of crude oil. A refinery contains a complex series of interconnected processing steps that separate crude oil into different fractions according to their boiling ranges (in *distillation* units), alter the hydrocarbon molecular structure and hence the quality of some of the lighter fractions (in *reforming* units), convert or crack heavy fractions into lighter fractions (in *conversion* units), and finally blend intermediate fractions (in *blending* units) to make acceptable products for sale to customers.

Refineries vary greatly in their abilities to convert heavy fractions to light fractions. Simple refineries only separate the crude oil into their constituent petroleum products by distillation, whereas complex refineries are able to convert heavy fractions to light fractions. Very complex refineries have both the traditional type of conversion units and the deep conversion units that can convert very heavy fractions to light fractions. We refer to the capability of converting heavy fractions to light fractions as *intermediate-conversion flexibility* ("conversion flexibility" hereafter). The rising price of gasoline has been the main reason for refineries to upgrade existing simple refineries to complex ones with conversion technologies, which give them the capability to derive more light fractions and improve crude oil's yield of light petroleum products. Operational flexibility in general is an important asset to oil refineries. Although it is obvious that the value of conversion flexibility to a refinery is influenced by conditions in crude oil and refined petroleum product markets, it is less obvious *how* such value is affected by volatile price movement in these markets.

Conversion flexibility also interacts with other types of operational flexibilities in the refinery. One such important flexibility is *range flexibility*: refineries vary in the range of crude oils they are capable of processing. Because light and heavy crude oils have very different composition and contaminant levels (e.g., sulfur, nitrogen, metals), a refinery that aims to accommodate a range of crude oils requires a suite of processing technologies (e.g., hydrotreating, emission control technologies) to comply with federal and state environmental protection regulations.[1] As such, some refineries are designed to efficiently process a particular type of crude oil or close substitutes, but they lack the flexibility to process other crude oils; others are able to process a wider range of heavy to light crude oils. Because range flexibility enables a refinery to adjust its crude oil purchase portfolio within its processing range, it offers an alternative means to affect the amount of heavy and light fractions available for refining, which conversion flexibility achieves through thermal and chemical processes. Is then conversion flexibility of less value to a refinery with wider crude oil processing range? This is a natural question faced by refinery executives.

Existing studies on operational flexibility have primarily focused on flexibility on the input side (e.g., the ability to process inputs of different qualities) or flexibility on the output side (e.g., the ability to satisfy demand for one product with another). Conversion flexibility in oil refining, on the other hand, operates on intermediates/works-in-process, and it affects the refiner's ability to blend in response to the output markets. The stage at which conversion takes place determines that conversion flexibility plays a unique role in linking the input procurement decision and the output blending decision, and hence it is different from those flexibilities studied in the existing literature.

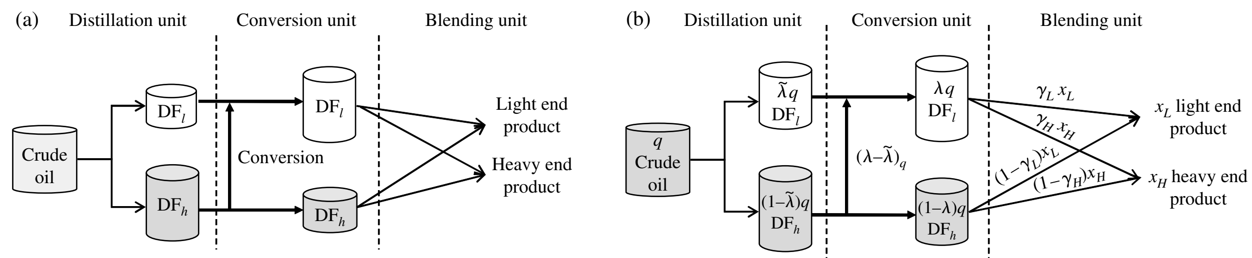
In this paper we explore the value drivers of conversion flexibility and its interplay with range flexibility. We address the following questions: First, how does conversion flexibility create value for a refinery? How significant is this value? Second, how is the value of conversion flexibility influenced by the price volatilities of crude oils and petroleum products? Finally, does a refinery with range flexibility benefit more or less from having conversion flexibility than an otherwise identical refinery that does not have range flexibility? In other words, are the two types of operational flexibility strategic complements or substitutes? We study these questions within a short-term planning horizon (weeks to several months) over which the main management objective is to maximize the profit margin over variable costs.[2] We make the following simplifying assumptions to obtain a stylized two-stage stochastic programming model while capturing key features of oil refining processes. First, we assume that any crude oil in the market is a mixture of two fractions, a light fraction (DF_l) and a heavy fraction (DF_h), and crude oils differ in the relative proportions of the two fractions. Second, as shown in Figure 1, panel (a), the refining process we consider comprises three distinct parts: (i) a distillation unit that separates the crude oil into DF_l and DF_h, (ii) a conversion unit that converts DF_h into DF_l, and (iii) a blending unit that blends the two fractions to make end petroleum products. In this stylized model, a refinery has conversion flexibility only if it is equipped with a conversion unit; a refinery with range flexibility can process a range of heavy to light crude oils, whereas a refinery without range flexibility processes a single crude oil.

We provide a preview of the main findings below. First, conversion flexibility adds value to the refinery by increasing the unit revenue potential of crude oil via altering its composition of heavy and light fractions. This benefit manifests itself in two forms: one is to enhance a refinery's already positive profit margin (unit revenue benefit), and the other is to change a negative margin to be positive, enabling the refinery to afford the crude oil when the crude oil price is high (purchase benefit). Both benefits contribute to the

[1] In addition to crude oil processing equipment, there are also added requirements for crude oil pipelines.

[2] When planning for a longer term (several years), the structure of the refinery and its operating costs are variables that must be decided in light of the company's objectives.

Figure 1 **A Stylized Representation of a Refinery Process**



market value of the refinery. Our real-data-calibrated numerical study finds the two benefits are of similar magnitude and together account for 20%–65% of the profit with conversion for a wide range of refining capacities. Second, the crude oil and petroleum product price volatilities have different impacts on the value of conversion flexibility: the value of conversion flexibility first increases and then decreases as crude oil price variability increases, but it is not sensitive to petroleum product price variability. Finally, whether the range flexibility increases or decreases the value of conversion flexibility is affected by the direction in which the refinery expands its range flexibility and the heaviness of crude oils.

The organization of the rest of this paper is as follows. Section 2 reviews related literature. We present the model for the single crude oil case in §3 and characterize the optimal policies in §4. The significance of the value of conversion flexibility and the impact of the crude oil and petroleum product market conditions on it are explored in §5. We extend the model to include range flexibility and study its impact on the value of conversion flexibility in §6. We provide concluding remarks in §7.

2. Literature Review

Our work is related to several areas of literature, including coproduction systems, operational flexibility, and applications of operations research in process industries. In this section, we review the most relevant papers from each area and state our contribution to the literature.

Petroleum oil refining can be viewed as a coproduction system, where a single production run yields multiple end products. Although coproduction systems vary greatly across industries (semiconductor, chemical, pulp and paper, agriculture, etc.), the existing literature focuses on features that are common to a few industries, where the end products are vertically differentiated on quality levels and the yields for them are random; the manufacturer may use high-quality products to satisfy demand for low-quality products (a form of output downward substitution). Main decisions studied for such coproduction systems include the ex ante production quantity decision and the ex post downward substitution and inventory allocation decisions. Papers that assume deterministic demand include Bitran and Leong (1992) and Bitran and Gilbert (1994), whereas Hsu and Bassok (1999) assume stochastic demand. Tomlin and Wang (2008) extend the literature by endogenizing the end product pricing decision and considering downconversion (converting high-speed chips into low-speed chips) and allocation flexibilities. They examine the necessity of downconversion in the presence or absence of optimal pricing. Kazaz (2008) considers the agricultural industry, where harvest and yield are uncertain. The study takes the perspective of a farming firm that grows and sells an agricultural commodity and is focused on methods of battling supply uncertainties. Motivated by the beef industry practice of meat packers sourcing inputs (fed cattle) using long-term contracts and from spot markets, Boyabatlı et al. (2011) study the optimal sourcing portfolio and the impacts of upstream and downstream market conditions and processing characteristics on the sourcing decision, with a focus on the value of using different procurement sources: long-term contracts and spot markets.

In contrast to these papers, we capture some unique features of oil refining that, to our knowledge, have not been previously studied in the coproduction literature. First, although output downward substitution is assumed in the coproduction literature, many refined petroleum products have different applications, and substitution among them is impossible; for example, a diesel-fueled car cannot use gasoline as fuel. Conversion can be viewed as a form of substitution, however, at the intermediate level in the form of converting *low-value* distillates into *high-value* distillates through chemical/thermal reactions, which incurs nonnegligible costs. Second, the input procurement decision in the oil refining industry is more complex than the total production quantity decision common in the coproduction literature (e.g., Bitran and Gilbert 1994). This is because inputs such as crude oils differ in their quality and prices. Procurement managers not only need to decide the purchase quantity but also need to choose the best crude oil or a portfolio of crude oils. Third, unlike most coproduction studies where random product yield is a key uncertainty considered, the distillation yield of a crude oil characterized by the distillation curve is usually available to refineries before the purchase of crude oil, and hence it is not a random factor of concern.[3] Uncertainties in crude oil price and end product price, on the other hand, are main concerns because refiners make procurement and intermediate processing decisions in the face of output price uncertainty, and a refiner's profitability and the value of its operational flexibility are affected by the price movement over time. Hence, input and output price uncertainty, as opposed to quantity uncertainty on the output side, draws another distinction between this paper and the aforementioned coproduction research.

The value of operational flexibility and its sensitivity to input/output market uncertainties are among the main focuses of the operational flexibility literature. Many forms of operational flexibility (e.g., component commonality, postponement, substitution, responsive pricing) can be viewed as options/recourses that enable the firm to wisely utilize its resource *after* uncertainties are resolved. Real options theory establishes that the value of options increases as the volatility of the underlying uncertainty increases (Dixit and Pindyck 1994). The operational flexibility literature finds that this conventional wisdom holds in some settings (e.g., flexible capacity in Chod and Rudi 2005) but may fail to hold in others (e.g., postponing resource commitment in Chod et al. 2010a), the latter as a result of the intricate interplay of operational factors and market uncertainty. Our paper shows that in the context of oil refining, increasing input price volatility has differential impacts on the value drivers of conversion flexibility, leading to case-dependent comparative statics of the value of conversion flexibility.

A few papers in the operational flexibility literature consider the relationship among different types of flexibility, for example, between downconversion and pricing flexibility as in Tomlin and Wang (2008); between volume flexibility and product flexibility as in Goyal and Netessine (2011); and among product mix flexibility, postponed resource commitment, and the ability to delay capacity commitment as in Chod et al. (2012). We extend the literature by investigating the relationship between two common types of flexibility, conversion flexibility and range flexibility, in the oil refining industry.

Process industries (including petroleum, coal, chemical) were among the first to adopt applied operations research methodologies (e.g., Symonds 1955). The blending problem in the petrochemical industry has received extensive research attention (e.g., Martin and Lubin 1985, Candler 1991, Ashayeri et al. 1994, Karmarkar and Rajaram 2001). More complex planning and scheduling decisions over multiple periods and for multiple connected production stages appear in Majozi and Zhu (2001), Neumann et al. (2003), Maravelias and Grossmann (2003), and Gaglioppa et al. (2008). Most of these works assume a deterministic environment and resort to mathematical programming for large-scale optimization or developing effective heuristic algorithms. The focus of our work is not on offering an exact decision support tool but on offering insights that reveal the interconnection of a number of sequential planning decisions under output price uncertainty and on illustrating the impact of the stochastic input and output environments on the value of conversion flexibility.

Recent works involving some integrated decision planning with applications in a process industry are of a more stylized nature (e.g., Goel and Gutierrez 2006, Secomandi 2010, Wu and Chen 2010). These papers, although characterizing optimal (sometimes multistage) operational policies in the petroleum and natural gas industry, all consider production/inventory systems for a single product and therefore do not consider issues and challenges associated with coproduction and inputs of various quality levels such as in our paper. Devalkar et al. (2011) consider optimal procurement, processing, and trade decisions for a firm dealing with one input commodity and one or multiple outputs, with a focus on investigating the benefit of the integrated decision making rather than the value of a particular process flexibility.

3. Model

We tailor our model to petroleum refining, where the inputs of the refinery are crude oils and the outputs are petroleum products. We simplify crude oil's multiple-fraction composition down to two distillation fractions, a heavy distillation fraction DF_h (made of large hydrocarbon molecules) and a light distillation fraction DF_l (made of small hydrocarbon molecules). For any given input, let $\tilde{\lambda}$ represent the proportion of DF_l, and hence, the proportion of DF_h is $1-\tilde{\lambda}$. Petroleum products can be derived by blending DF_h and DF_l in certain proportions. We assume there are two end products traded in the petroleum spot market, a heavy end product (H) and a light end product (L); blending a unit of light end product requires more of light fraction DF_l than blending a unit of heavy end product. The assumption of two end products is not restrictive but simplifies the exposition and discussion significantly. The main insights of the two-product case hold in the general case of N (>2) end products

[3] The U.S. Bureau of Mines carried out distillation on thousands of crude oil samples from wells in all major producing fields. The results can be used without correction (Gary and Handwerk 1984). As discussed in Boyabatlı (2015), this is also the case in many agricultural industries.

(see Appendix E). For ease of exposition, hereafter we use "input" and "output" in lieu of crude oil and end product, respectively.

We consider a refinery that purchases inputs from an input spot market and sells outputs to an output spot market. The refinery can process input with a DF_l proportion of $\tilde{\lambda} \in [\lambda_h, \lambda_l]$, $\lambda_h \le \lambda_l$, where input λ_h (λ_l) is the heaviest (lightest) input within its processing range. The refinery has range flexibility over $[\lambda_h, \lambda_l]$ if $\lambda_h < \lambda_l$. Focusing on conversion flexibility first, we suppress the range flexibility by letting $\lambda_h = \lambda_l = \tilde{\lambda}$. We will explore the impact of range flexibility ($\lambda_h < \lambda_l$) in §6.

We model the refinery's input purchase, intermediate conversion, and output blending decisions in a two-stage stochastic program. The refinery comprises three process units (see Figure 1, panel (b)): a distillation unit, a conversion unit, and a blending unit. We assume the refinery's effective processing capacity in one period is K.[4] At time 0, given input spot price s, the refinery purchases $q \le K$ units of input. The purchased input is first fed to the distillation unit where DF_h and DF_l are separated, yielding $(1-\tilde{\lambda})q$ units of DF_h and $\tilde{\lambda}q$ units of DF_l. Then, the refinery decides the amount of DF_h to convert into DF_l.[5] The conversion cost $c(\cdot)$ is a convex increasing function of conversion quantity, reflecting the fact that more conversion pushes equipment to higher severity of use and hence is more costly. Let λ ($\ge \tilde{\lambda}$) be the target level of the DF_l proportion for conversion; then the corresponding conversion quantity is $(\lambda - \tilde{\lambda})q$. After further processing of the two fractions, at time 1, the refinery is ready to blend the two fractions to produce the two outputs and sell them at spot price $p = (p_H, p_L)$.[6]

We first model the time 1 output blending decision. Starting from the blending recipe that meets the quality requirement of the outputs, let $\gamma_i \in [0, 1]$ be the proportion of DF_l used in blending one unit of output i, $i = H, L$. The bill of material (BOM) of the two outputs can be represented by a BOM matrix:

$$\begin{array}{c c} & \begin{array}{cc} \text{Output } H & \text{Output } L \end{array} \\ \begin{array}{c} \mathrm{DF}_h \\ \mathrm{DF}_l \end{array} & \begin{bmatrix} 1-\gamma_H & 1-\gamma_L \\ \gamma_H & \gamma_L \end{bmatrix}. \end{array}$$

[4] The capacities of the processing units and connecting pipes are designed to ensure compatibility of these units and to enable the entire refining process to work efficiently. For a one-period model, the relevant notion of capacity is the processing quantity of the entire refining process within the period.

[5] We assume there is no information updating between the purchase decision and the conversion decision. This is a reasonable assumption for the case of local or domestic purchase, because pipeline shipping and distillation of the crude oil take a relatively shorter time in comparison to the conversion and further processing steps.

[6] Blending is a relatively quick operation, and the refinery makes blending decisions close to the output selling time.

The heavy output H requires less light fraction DF_l than the light output L, so $\gamma_H < \gamma_L$. The refinery, subject to the availability of the two fractions, decides the quantities of the two outputs (x_H, x_L) to maximize the time 1 blending revenue $r(p, q, \lambda)$, where p is the output spot price, q is the total quantity of input, and λ is the DF_l proportion at time 1. The blending decision can be formulated as a linear program:

$$r(p, q, \lambda) = \max_{x_H, x_L} \{p_H x_H + p_L x_L\} \tag{1}$$

$$\text{s.t. } \mathrm{DF}_l\colon\ \gamma_H x_H + \gamma_L x_L \le \lambda q, \tag{1a}$$

$$\mathrm{DF}_h\colon\ (1-\gamma_H)x_H + (1-\gamma_L)x_L \le (1-\lambda)q, \tag{1b}$$

$$x_H, x_L \ge 0, \tag{1c}$$

where (1a)–(1c) are the availability constraints of the two fractions and the nonnegative production constraint, respectively. We assume that unused distillation fractions have zero salvage value.

At time 0, the refinery purchases q units of crude oil with DF_l proportion $\tilde{\lambda}$ at a realized input price s and then makes the fraction conversion decision to maximize the expected profit:

$$\Pi(\tilde{\lambda}, s) = \max_{q \in [0, K],\, \lambda \in [\tilde{\lambda}, 1]} \{\pi(q, \tilde{\lambda}, \lambda) - sq\}, \tag{2}$$

$$\text{where} \quad \pi(q, \tilde{\lambda}, \lambda) = \mathbb{E}_p[r(p, q, \lambda)] - c((\lambda - \tilde{\lambda})q). \tag{2a}$$

The term sq in (2) represents the purchase cost of q units of input;[7] $\pi(q, \tilde{\lambda}, \lambda)$ is the expected revenue less conversion cost. In Expression (2a), bringing the DF_l proportion from the initial level $\tilde{\lambda}$ to the target level λ requires conversion of $(\lambda - \tilde{\lambda})q$ units from DF_h into DF_l. The corresponding conversion cost is $c((\lambda - \tilde{\lambda})q)$; $\mathbb{E}_p[r(p, q, \lambda)]$ is the expected time 1 blending revenue, where $\mathbb{E}_p$ represents taking expectation over time 1 output prices. Note that if the intertemporal correlation between s and p is positive, then the distribution of p is a function of s. For a refinery without conversion flexibility, the constraint $\lambda \in [\tilde{\lambda}, 1]$ in (2) is replaced by $\lambda = \tilde{\lambda}$.

We take the perspective of a refiner who considers upgrading an existing simple refinery to a complex one with conversion units. Let $\Pi_c(\tilde{\lambda}, s)$ and $\Pi_o(\tilde{\lambda}, s)$ represent the profits of a refinery at input price s with and without conversion flexibility, respectively, both at a given capacity level K, the one-period processing capacity of the refinery without conversion flexibility.[8] The difference of these two profits, $\Pi_c(\tilde{\lambda}, s) - \Pi_o(\tilde{\lambda}, s)$,

[7] To simplify exposition, we normalize the unit processing cost (including power, chemicals, fuel costs, etc., excluding conversion cost) to 0, which does not have an impact on the qualitative insights.

[8] It is not uncommon in practice that the refinery installs the new units to be compatible with the existing capacity of the refinery

Table 1 **Time 1 Optimal Blending Decision**

	Scenario (a): $\lambda \in (0, \gamma_H]$	Scenario (b): $\lambda \in (\gamma_H, \gamma_L]$	Scenario (c): $\lambda \in (\gamma_L, 1)$
$(x_H, x_L) =$	$\begin{cases} \left(0, \dfrac{\lambda}{\gamma_L} q\right) & \text{if } p \in \Omega_1, \\ \left(\dfrac{\lambda}{\gamma_H} q, 0\right) & \text{if } p \in \Omega_{2\cup 3}; \end{cases}$	$\begin{cases} \left(0, \dfrac{\lambda}{\gamma_L} q\right) & \text{if } p \in \Omega_1, \\ \left(\dfrac{\gamma_L-\lambda}{\gamma_L-\gamma_H} q, \dfrac{\lambda-\gamma_H}{\gamma_L-\gamma_H} q\right) & \text{if } p \in \Omega_2, \\ \left(\dfrac{1-\lambda}{1-\gamma_H} q, 0\right) & \text{if } p \in \Omega_3; \end{cases}$	$\begin{cases} \left(0, \dfrac{1-\lambda}{1-\gamma_L} q\right) & \text{if } p \in \Omega_{1\cup 2}, \\ \left(\dfrac{1-\lambda}{1-\gamma_H} q, 0\right) & \text{if } p \in \Omega_3; \end{cases}$

where $\Omega_1 = \left\{p: \dfrac{p_H}{\gamma_H} - \dfrac{p_L}{\gamma_L} \le 0\right\}$, $\Omega_2 = \left\{p: \dfrac{p_L}{\gamma_L} - \dfrac{p_H}{\gamma_H} < 0 \le \dfrac{p_L}{1-\gamma_L} - \dfrac{p_H}{1-\gamma_H}\right\}$, and $\Omega_3 = \left\{p: \dfrac{p_L}{1-\gamma_L} - \dfrac{p_H}{1-\gamma_H} < 0\right\}$.

represents the ex post value of conversion flexibility at input spot price s. To calculate the short-term impact of conversion flexibility on the refinery's profitability, we define the *value of conversion flexibility* (VoC) for input $\tilde{\lambda}$ as the expectation of this profit difference, given by $\mathrm{VoC}(\tilde{\lambda}) = \mathbb{E}_s[\Pi_c(\tilde{\lambda}, s) - \Pi_o(\tilde{\lambda}, s)]$, where $\mathbb{E}_s$ represents taking expectation over the time 0 distribution of input price s. We will investigate the property of VoC analytically under the assumption that input price s and output price p are uncorrelated and then study the implications of the input–output price correlation numerically using a calibrated model, both in §5. These studies shed light on the value drivers and economic potential of conversion flexibility in various market environments. Before that, in §4, we characterize the optimal operational decisions for general price distributions and arbitrary intertemporal input–output price correlation.

4. Optimal Operational Decisions

We first characterize the time 1 optimal blending decision and then derive the time 0 optimal conversion and purchase decisions.

4.1. Time 1 Blending Decision

The optimal time 1 blending decision is determined by the realization of the output price p and the relative availability of the two fractions (see Table 1). When prices clearly favor the light/heavy output ($p \in \Omega_1 / p \in \Omega_3$), the refinery produces the maximal possible amount of light/heavy output, leaving no resource to produce the other output. When prices of the two outputs are close ($p \in \Omega_2$), the refinery produces both outputs if both fractions are of moderate availability (i.e., Scenario (b) in Table 1) but produces only heavy/light output if the light fraction DF_l is of low/high availability (i.e., Scenario (a)/(c) in Table 1).

The expected revenue $\mathbb{E}_p[r(p, q, \lambda)]$ can be written as a linear function of purchase quantity q:

$$\mathbb{E}_p[r(p, q, \lambda)] = U(\lambda) q, \tag{3}$$

where $U(\lambda)$ is continuous and piecewise linear in λ with slope $u(\lambda)$. The expression of $U(\lambda)$ is given by

$$U(\lambda) = \begin{cases} U_a(\lambda) \equiv u_a \lambda, & \text{if } \lambda \in (0, \gamma_H], \\ U_b(\lambda) \equiv u_b \lambda + (u_a - u_b)\gamma_H & \text{if } \lambda \in (\gamma_H, \gamma_L], \\ U_c(\lambda) \equiv u_c \lambda + (u_b - u_c)\gamma_L + (u_a - u_b)\gamma_H & \\ & \text{if } \lambda \in (\gamma_L, 1), \end{cases}$$

where constants u_a, u_b, and u_c are values of slope $u(\lambda)$ for λ in $(0, \gamma_H]$, $(\gamma_H, \gamma_L]$, and $(\gamma_L, 1)$, respectively (detailed expressions are provided in Appendix A). Functions $U(\lambda)$ and $u(\lambda)$ have clear interpretations that $U(\lambda)$ represents the unit revenue of the input with DF_l proportion λ, and $u(\lambda)$ represents the marginal revenue of DF_l proportion λ per unit of input. For ease of exposition, we hereafter refer to $U(\lambda)$ as the unit revenue of input and $u(\lambda)$ as the marginal revenue of λ. Note that both $U(\lambda)$ and $u(\lambda)$ are functions of the distribution of time 1 output price p (which in turn is a function of s if s and p are correlated); we suppress this dependence in notation for expositional convenience. We will explore the impact of changes in the distribution of p in §5.

The marginal revenue of λ is positive when DF_l is scarce and negative when DF_l is abundant. That is, $u_a > 0$ in Scenario (a) and $u_c < 0$ in Scenario (c). When DF_l is of moderate availability (i.e., Scenario (b)), the sign of the marginal revenue of λ, u_b, depends on the distribution of output price p. We will see in the next subsection that these values have direct implications for time 0 conversion decisions. Following convention, we let $u_b^+ = u_b$ if $u_b > 0$ and $u_b^+ = 0$ if $u_b \le 0$. Also, the terms "increasing" and "decreasing" are used in the weak sense.

instead of adjusting the overall capacity of the plant to accommodate the new units, because capacity adjustment would require a radical overhaul of the refinery plant (equipment, pipes, control units) that could take years to complete and would have to take into consideration (often inaccurate) demand and price trend forecasts over a long planning horizon (10–20 years or more).

4.2. Time 0 Purchase Quantity and Conversion Decisions

The time 0 purchase quantity and conversion decisions tie closely together, as shown by the following proposition.

Proposition 1. *Given $\tilde{\lambda}$ and s, the optimal purchase decision q^* and the optimal conversion decision λ^* are summarized in the following table*:

	$\tilde{\lambda} \in (0, \gamma_H]$, $u(\tilde{\lambda}) = u_a$	$\tilde{\lambda} \in (\gamma_H, \gamma_L]$, $u(\tilde{\lambda}) = u_b$	$\tilde{\lambda} \in (\gamma_L, 1)$, $u(\tilde{\lambda}) = u_c$
$c'(0) \in [u_a, \infty)$	(0)	(0)	(0)
$c'(0) \in [u_b^+, u_a)$	(1)	(0)	(0)
$c'(0) \in [0, u_b^+)$	(3)	(2)	(0)

where

Case	q^*	λ^*
(0)	$\begin{cases} K & \text{if } s \in (0, U(\tilde{\lambda})), \\ 0 & \text{if } s \in [U(\tilde{\lambda}), \infty); \end{cases}$	$\tilde{\lambda}$
(1)	$\begin{cases} K & \text{if } s \in (0, U_a(\tilde{\lambda})), \\ \max[0, \min[Q_a(\tilde{\lambda}, s), K]] & \text{if } s \in [U_a(\tilde{\lambda}), \infty); \end{cases}$	*Expression* (4)
(2)	$\begin{cases} K & \text{if } s \in (0, U_b(\tilde{\lambda})), \\ \max[0, \min[Q_b(\tilde{\lambda}, s), K]] & \text{if } s \in [U_b(\tilde{\lambda}), \infty); \end{cases}$	*Expression* (4)
(3)	$\begin{cases} K & \text{if } s \in (0, U_a(\tilde{\lambda})), \\ \min[Q_a(\tilde{\lambda}, s), K] & \text{if } s \in [U_a(\tilde{\lambda}), U_b(\tilde{\lambda})), \\ \max[0, \min[Q_b(\tilde{\lambda}, s), K]] & \text{if } s \in [U_b(\tilde{\lambda}), \infty), \end{cases}$	*Expression* (4)

and in Cases (1)–(3), $\lambda^* > \tilde{\lambda}$ *is obtained from*[9]

$$\max\{\lambda: \lambda \in [\tilde{\lambda}, 1] \text{ and } u(\lambda) - c'((\lambda - \tilde{\lambda})q) \geq 0\}, \qquad (4)$$

$Q_a(\tilde{\lambda}, s)$ *is the solution to* $s = U_a(\gamma_H) - (\gamma_H - \tilde{\lambda}) \cdot c'((\gamma_H - \tilde{\lambda})Q_a)$, *and* $Q_b(\tilde{\lambda}, s)$ *is the solution to* $s = U_b(\gamma_L) - (\gamma_L - \tilde{\lambda})c'((\gamma_L - \tilde{\lambda})Q_b)$.

Cases (1)–(3) in Proposition 1 represent the cases where increasing the DF_l proportion from the initial level of $\tilde{\lambda}$ to a higher-level λ^* via conversion is desirable. Conversion is used when a higher DF_l proportion enables the refinery to better respond to the output market in the blending stage; i.e., when the marginal revenue of $\tilde{\lambda}$, $u(\tilde{\lambda})$, is higher than the initial marginal conversion cost $c'(0)$. Effectively, by changing the relative proportion of the heavy and light fractions, conversion increases the unit revenue of the input from $U(\tilde{\lambda})$ to $U(\lambda^*)$. Conversion does not take place, however, if the initial DF_l proportion is already high or the initial marginal conversion cost is high, i.e., Case (0) in Proposition 1.

Conversion plays a critical role in linking the output market and the input market. First, changes in the output market condition directly affect the marginal revenue of λ, $u(\lambda)$. For example, when a shift in the output price distribution increases the value of marginal revenue u_b from a negative value to a positive one, the time 0 decision may change from no use of conversion to use of conversion, i.e., from Case (0) to Case (2).[10]

Second, because the optimal target DF_l proportion λ^* balances the marginal revenue $u(\lambda^*)$ and the marginal conversion cost $c'((\lambda^* - \tilde{\lambda})q)$ (see (4)), λ^* is a function of both the initial DF_l proportion $\tilde{\lambda}$ and the purchase quantity q. We can show that the optimal target level λ^* decreases convexly in quantity q (shown in the proof of Proposition 1). That is, the larger the quantity the refinery purchases, the lower target DF_l proportion λ^* it can reach up from $\tilde{\lambda}$ using conversion. Thus, conversion flexibility affects the refinery's input purchase quantity decision.

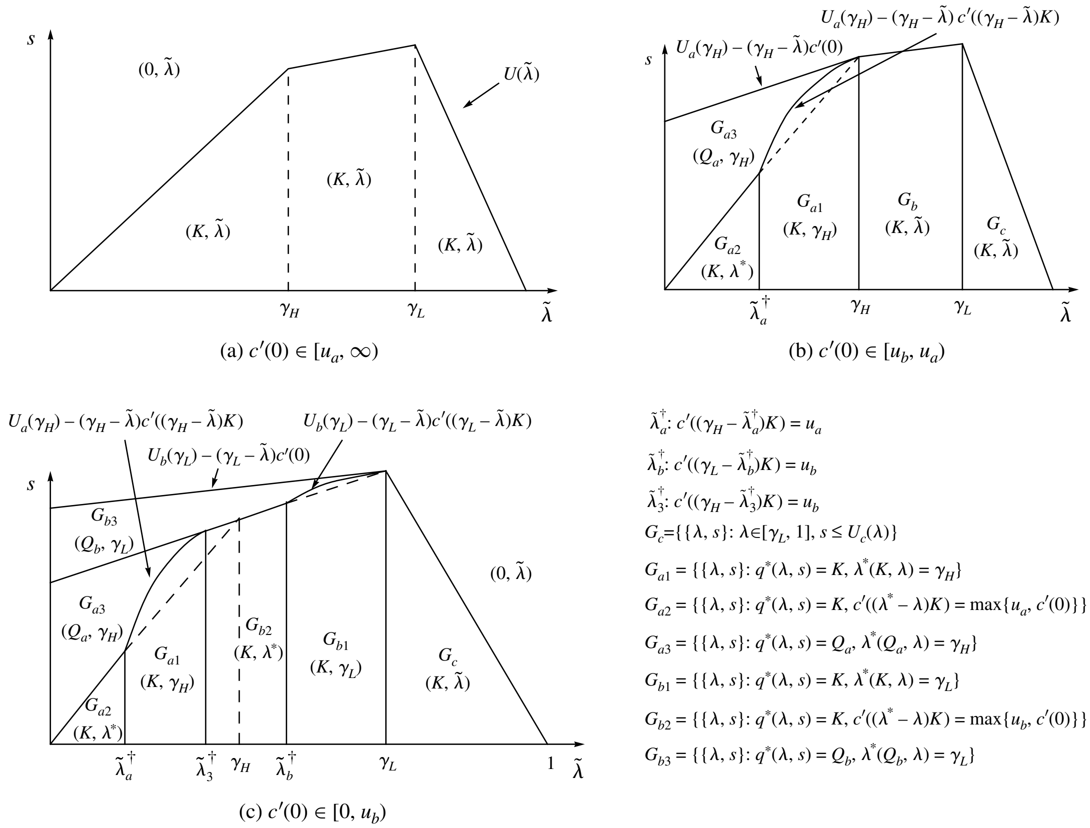
Figure 2 provides a graphical illustration of Proposition 1 in the $(\tilde{\lambda}, s)$ space, partitioning the space into subregions based on the optimal purchase quantity and conversion decisions (q^*, λ^*). When conversion is not desirable (Case (0) in Proposition 1; see also Figure 2, panel (a)), the refinery's optimal purchase decision is a simple zero-or-full-capacity policy: if the spot price is lower than the unit revenue of input, $s < U(\tilde{\lambda})$, then the refinery purchases up to full capacity K; otherwise, there is no purchase. Represented in the $(\tilde{\lambda}, s)$ space, the feasible purchase region is bounded by the unit revenue curve $U(\tilde{\lambda})$.

When conversion is desirable (Cases (1)–(3) in Proposition 1; see also Figure 2, panels (b) and (c)), the benefit of conversion flexibility manifests in two forms. When $s < U(\tilde{\lambda})$, the refinery purchases input to full capacity K and is able to extract a higher revenue than that without conversion (i.e., $U(\lambda^*)K > U(\tilde{\lambda})K$). We refer to this form of benefit as the *unit revenue benefit*. When $s \geq U(\tilde{\lambda})$, a refinery without conversion flexibility would not purchase the input, whereas a refinery with conversion flexibility would purchase quantity Q_a or Q_b to balance the effective unit revenue of input and the sum of the marginal conversion cost and the unit purchase cost, shown as the vertical expansion of the purchase region in the $(\tilde{\lambda}, s)$ space. We refer to this

[9] The detailed expression of λ^* is provided in the proof of Proposition 1.

[10] It is straightforward to show that if $p_H < p_L$ ($p_H > p_L$), almost surely, then $u_b > 0$ ($u_b < 0$).

Figure 2 Illustration of Optimal Purchase Quantity and Conversion Decisions (q^*, λ^*) **as Functions of** $\tilde{\lambda}$ **and** s **for** $u_b > 0$



form of benefit as the *purchase benefit* of conversion flexibility.

We now characterize the unit revenue benefit and the purchase benefit of conversion flexibility rigorously. Recall that the value of conversion flexibility is $\text{VoC}(\tilde{\lambda}) = \mathbb{E}_s[\Pi_c(\tilde{\lambda}, s) - \Pi_o(\tilde{\lambda}, s)]$, where $\tilde{\lambda}$ is the initial DF_l proportion, and λ^* is the target DF_l proportion for conversion. Let q_c and q_o denote the optimal purchase quantities with and without conversion flexibility, respectively. Using (2), (2a), and (3), the ex post value of conversion $\Pi_c(\tilde{\lambda}, s) - \Pi_o(\tilde{\lambda}, s)$ at a realized input price s can be written as

$$\begin{aligned}\Pi_c(\tilde{\lambda}, s) - \Pi_o(\tilde{\lambda}, s) &= (U(\lambda^*) - U(\tilde{\lambda}))q_o + (U(\lambda^*) - s)(q_c - q_o)\\ &\quad - c((\lambda^* - \tilde{\lambda})q_c) \qquad (5)\\ &= \begin{cases} (U(\lambda^*) - U(\tilde{\lambda}))q_o - c((\lambda^* - \tilde{\lambda})q_o), \\ \quad \text{if } s < U(\tilde{\lambda}), \ \text{[unit revenue benefit]} \\ (U(\lambda^*) - s)q_c - c((\lambda^* - \tilde{\lambda})q_c), \\ \quad \text{if } s \ge U(\tilde{\lambda}). \ \text{[purchase benefit]} \end{cases} \qquad (6)\end{aligned}$$

Observe from (5) that the ex post value of conversion comprises three terms. The first term, $(U(\lambda^*) - U(\tilde{\lambda}))q_o$, represents the value increase directly from the increase of unit revenue; the second term, $(U(\lambda^*) - s)(q_c - q_o)$, represents the value increase as a result of the increase of purchase quantity; and the third term, $c((\lambda^* - \tilde{\lambda})q_c)$, is the conversion cost. The ex post value of conversion is the value increases less the conversion cost. When the input price is low (i.e., $s < U(\tilde{\lambda})$), $q_c = q_o = K$, and the refinery enjoys the unit revenue benefit only; when the input price is high (i.e., $s \ge U(\tilde{\lambda})$), $q_o = 0$, and the refinery enjoys the purchase benefit (see (6)). The VoC can be written as the sum of the expected unit revenue benefit (URB) and the expected purchase benefit (PB):

$$\text{VoC}(\tilde{\lambda}) = \underbrace{\mathbb{E}_{s<U(\tilde{\lambda})}[\Pi_c(\tilde{\lambda}, s) - \Pi_o(\tilde{\lambda}, s)]}_{\text{URB}} + \underbrace{\mathbb{E}_{s\ge U(\tilde{\lambda})}[\Pi_c(\tilde{\lambda}, s) - \Pi_o(\tilde{\lambda}, s)]}_{\text{PB}}.$$

5. Sensitivity Analysis

The purpose of this section is twofold. First, we explore the significance of the value of conversion flexibility and show that it is an important form of operational flexibility with great value potential. We measure separately URB and PB to understand which benefit is more important in a real-data-calibrated parameter setting. Second, we study the impact of capacity and the input and output market conditions (i.e., changes in the variance of price distributions) on the value of conversion.

In each subsection, we first develop analytical results for the tractable case where time 0 input price is uncorrelated with the time 1 output prices. Then we resort to the numerical study to investigate how the intertemporal input–output price correlation might affect the results and insights.

To calibrate the distribution of input and output prices, we obtain an empirical data set consisting of weekly crude oil and petroleum product prices between the years 2000 and 2003 from the website of the U.S. Energy Information Administration, the statistical and analytical agency within the U.S. Department of Energy.[11] We choose Saudi Arabia Arabian Light as the crude oil and gasoline and residual fuel oil as the light end product and the heavy end product, respectively. To capture the intertemporal input–output correlation, we adopt a standard vector autoregressive (VAR) model, which implicitly assumes the prices follow normal distribution.[12] Let $\chi_t = (s, p_H, p_L)_t^T$ represent time t price vector for crude oil, heavy, and light end products, where subscript t is the time index measured by week, and superscript T is the transpose operator. We use the weekly price data to calibrate the following VAR model:

$$\chi_{t+1} - \mu = \eta \cdot B \cdot (\chi_t - \mu) + e', \tag{7}$$

where $\mu = (\mu_s, \mu_H, \mu_L)^T$ is the long-run average price vector, B is a 3×3 coefficient matrix, and $e' \sim \mathrm{N}(\mathbf{0}, \Sigma')$. Parameter η is the intertemporal input–output correlation factor. Before calibrating the model parameters, we adjust all weekly prices for U.S. Consumer Price Index inflation and represent prices in constant year 2000 dollars, and then we remove the monthly seasonality effect from the deflated price data.[13] We set μ to be the overall mean of the adjusted data. We normalize the real-world intertemporal correlation factor $\eta = 1$ and then use the package "vars" for R language to calibrate parameters B and Σ'. The R^2 is approximately 90%. A typical lead time of two weeks between time 0 (when purchase and conversion decisions are made) and time 1 (when the blending decision is made) is considered, and the two-week VAR model for our study is derived below by applying the calibrated one-week VAR model iteratively twice:

$$\chi_{t+2} - \mu = \eta \cdot A \cdot (\chi_t - \mu) + e, \tag{8}$$

where $\mu = (24.59, 23.90, 33.65)$, coefficient matrix $A = B^2$, $e = (e_s, e_H, e_L)^T \sim \mathrm{N}(\mathbf{0}, \Sigma)$, and $\Sigma = B\Sigma' B^T + \Sigma'$. Specifically,

$$A = \begin{bmatrix} 0.76 & 0.089 & 0.005 \\ 0.063 & 0.87 & -0.018 \\ 0.45 & 0.31 & 0.43 \end{bmatrix}, \quad \text{and}$$

$$\Sigma = \begin{bmatrix} \sigma_{es}^2 & \rho_{sH}\sigma_{es}\xi\sigma_{eH} & \rho_{sL}\sigma_{es}\xi\sigma_{eL} \\ \rho_{sH}\sigma_{es}\xi\sigma_{eH} & (\xi\sigma_{eH})^2 & \rho\xi^2\sigma_{eH}\sigma_{eL} \\ \rho_{sL}\sigma_{es}\xi\sigma_{eL} & \rho\xi^2\sigma_{eH}\sigma_{eL} & (\xi\sigma_{eL})^2 \end{bmatrix} = \begin{bmatrix} 2.13 & 0.89 & 0.72 \\ 0.89 & 3.34 & 0.32 \\ 0.72 & 0.32 & 3.82 \end{bmatrix}.$$

Most notation in Σ is self-explanatory except for ξ, which represents the common volatility factor of the output prices (ξ is normalized to 1 in the calibration), reflecting the fact that the price movement in the output market is often driven by a common set of economic factors.

To compute the value of conversion flexibility, VoC, we simulate time 0 prices using the stationary distribution of $(s, p_H, p_L)^T$ derived from the VAR model (8) by a standard approach (Cryer and Chan 2008) and then simulate time 1 prices according to (8).[14] Let σ_s denote the standard deviation of the stationary input price. In the following subsections, we will vary σ_s, ξ, and ρ to study the impact of the variance of input price, variance of output prices, and correlation of output prices on VoC, respectively. We vary the value of η to around 1 in the numerical study to explore the impact of the intertemporal input–output price correlation on the sensitivity analysis results.[15]

Complying with the industry convention, we use the length of a carbon chain as a quality index of distillation fractions (heavy fractions typically have longer carbon chains).[16] On the output side, gasoline

[11] See http://www.eia.doe.gov/, accessed April 2009.

[12] The VAR model is commonly used to describe the evolution of crude oil, gasoline, and other commodity prices (see, e.g., Kilian 2009, Akram 2009, and Kilian 2010 for references).

[13] Briefly, to remove the monthly seasonality effect, we let ς_t denote deflated price vector at week t, $\varsigma_t = \chi_t + \kappa_{m(t)}$, where $\kappa_{m(t)}$ is a deterministic vector that captures the monthly seasonality in spot prices with the subscript $m(t)$ indicating the month that week t falls in, and $\chi_t = (s, p_H, p_L)_t^T$ is the deseasonalized price vector. We impose the constraint that the 12 monthly seasonality vectors sum to a zero vector; i.e., $\sum_{m=1}^{m=12} \kappa_m = \mathbf{0}$.

[14] A unit root test shows our VAR model is stable. A stable VAR model implies that for any t, the prices conform to a stationary distribution. Initializing time 0 prices at the stationary distribution allows us to assess the realistic magnitude of value of conversion. The stationary distribution of $(s, p_H, p_L)^T$ is given in Appendix D.

[15] Changing η alters not only the intertemporal correlation between input and output prices but also the variability of output prices. Therefore, the impact of increasing η is a combination of both effects.

[16] Sulfur content is another common quality dimension in oil refining. However, most sulfur removal is accomplished in a hydrotreater following the distillation process but before conversion. Some residual sulfur remains in the blending stage. Heaviness is a more relevant quality dimension to conversion flexibility. Using one dimension of quality measure allows analytical tractability.

ranges from C_4 to C_{12}, and residual fuel oil ranges from C_{20} to C_{50} (CONCAWE 1998). DF_l is defined as consisting of carbon chains ranging from C_1 to C_{13} and DF_h of carbon chains from at least C_{14}. Correspondingly, Arabian Light yields 30% DF_l after the distillation, so let $\tilde{\lambda} = 0.3$ (see Summers 2006). Using the average carbon chain length as the quality index, we derive $\gamma_H = 0.2$ and $\gamma_L = 0.97$ for the BOM matrix.[17]

We consider a refinery with the capacity of 3 mmtpa (million metric tons per annum), which is equivalent to around 850,000 barrels every two weeks. Thus, we set $K = 850$ kbl (where kbl stands for thousand barrels). We model conversion cost using a quadratic function $c(x) = \tau x + \upsilon x^2$, where x is in thousand barrels and $c(x)$ is in thousand dollars. We let $\tau = 5.5$, $\upsilon = 0.035$, which corresponds to an average conversion cost of $4 per barrel when the refinery is running at its full capacity (the calibration of the conversion cost is discussed in the appendix). In the analytical study, we have normalized the unit distillation cost, denoted as z, to be 0 for notational convenience. Setting $z = 0$ does not affect the qualitative results of comparative statics but will affect the magnitude of the profit and the value of conversion. A positive z leads to a higher VoC relative to the expected profit. To reflect a realistic magnitude in the numerical study, we set $z = 1.3$, which is $1/3$ of the average conversion cost at the full capacity (Favennec 2003).

5.1. Impact of Capacity (K)

A refinery's processing capacity K influences the value potential of conversion flexibility as follows.

Proposition 2. *Assume zero intertemporal input–output price correlation and prices following general distribution. There exists a threshold $\tilde{K}$ such that* VoC *increases concavely in K for $K \in (0, \tilde{K}]$ and is constant in K for $K \in (\tilde{K}, \infty)$, where for Cases* (1) *and* (3) *in Proposition* 1, *$\tilde{K}$ satisfies $c'((\gamma_H - \tilde{\lambda})\tilde{K}) = u_a$, and for Case* (2), *$\tilde{K}$ satisfies $c'((\gamma_L - \tilde{\lambda})\tilde{K}) = u_b$.*

Recall in the discussion of Proposition 1 that the optimal conversion quantity $(\lambda^* - \tilde{\lambda})q$ should never exceed a level that results in a marginal conversion cost being higher than the marginal revenue of the DF_l proportion (see (4)). This means that the optimal conversion quantity has an upper bound that is determined by the type of input it processes (i.e., the initial DF_l proportion $\tilde{\lambda}$), the conversion cost function $c(\cdot)$, and the marginal revenue of DF_l proportion $u(\cdot)$. It implies that as the capacity of the refinery increases, the value of conversion approaches and then remains at an upper bound. Proposition 2 formalizes this intuition.

[17] In our numerical study, we vary γ_H from 0.15 to 0.25 and γ_L from 0.6 to 0.97, and we find our qualitative sensitivity results are robust against the choice of γ_H and γ_L.

Figure 3, panel (a) shows that the result of Proposition 2 holds under the positive intertemporal input–output price correlation (VoC is measured in thousand dollars and capacity in thousand barrels). An implication of this result is that adding capacity alone cannot help increase VoC when it reaches a plateau.

We first focus on the calibrated parameter setting with η normalized to 1, i.e., the thick curves in all three figures. We find that VoC is significant, accounting for 20%–65% of the profit with conversion for the range of K corresponding to 8 mmtpa−1 mmtpa in Figure 3, panel (b). At the base capacity level ($K = 3$ mmtpa $=$ 850 kbl), VoC accounts for approximately 40% of the profit with conversion.

Recall that VoC is the sum of URB and PB. We investigate which of the two benefits plays a more significant role. Figure 3, panel (c) shows that PB accounts for approximately 60% of VoC and URB accounts for 40% of VoC. It implies that both unit revenue benefit and purchase benefit are important. The significant purchase benefit also explains the high VoC relative to the expected profit (see Figure 3, panel (b)), because when the purchase benefit manifests, the profit equals the ex post value of conversion.

Figure 3 shows that as η increases, VoC increases, but the proportion of VoC in expected profit and the proportion of PB in VoC decrease. Intuitively, a stronger intertemporal input–output price correlation implies that a higher input price often results in a higher output price in the future; therefore, the gross margin of a refinery increases as η increases and so do the refinery's profitability and the value of conversion flexibility. The increase of the gross margin also implies the increase of the unit revenue benefit of conversion flexibility, which explains the decreasing proportion of PB in VoC and the decreasing proportion of VoC in expected profit.

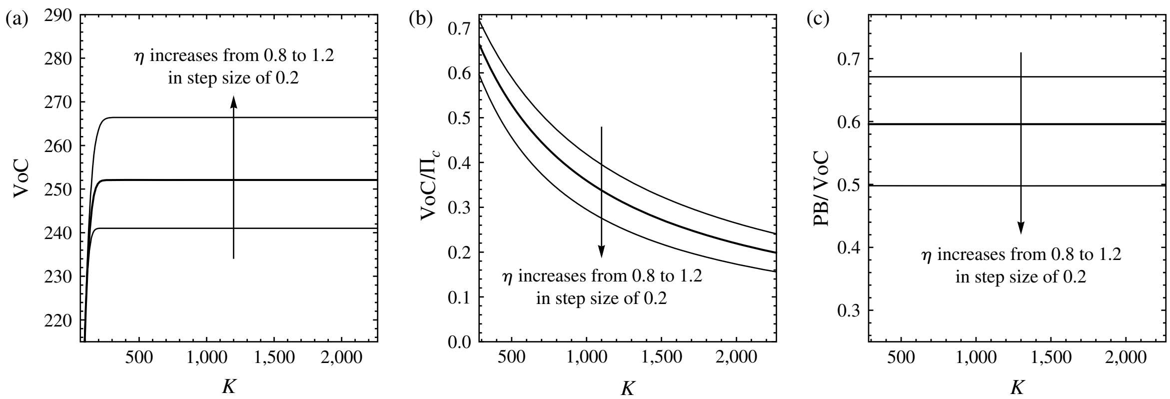
5.2. Impact of Input Price Volatility on VoC

Here and in the following subsections, we explore how the value of conversion flexibility is influenced by input and output market conditions such as changes in the variance and correlation of prices.

Because conversion is a recourse action taking place after the realization of the input spot price, one might expect that VoC increases as the input price variability σ_s increases. Proposition 3 shows that the impact of input price variability is influenced by the level of the mean input price, and VoC can decrease in σ_s.

Proposition 3. *Assume zero intertemporal input–output price correlation and prices following normal distribution. There exist two thresholds T_1 and T_2, $T_1 < T_2$, such that* VoC *decreases in σ_s if $\mu_s \le T_1$ and* VoC *increases in σ_s if $\mu_s \ge T_2$.*

Figure 3 Sensitivity of (a) VoC, (b) Proportion of VoC in Expected Profit, and (c) Proportion of Purchase Benefit in VoC, to Processing Capacity K, for Three Levels of Intertemporal Input–Output Price Correlation



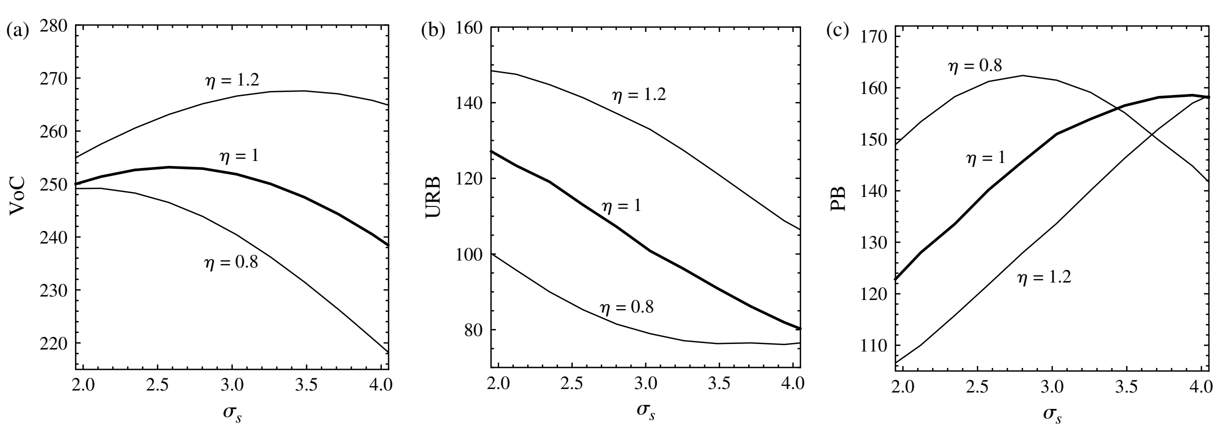
We can show that the unit revenue benefit is constant in the input price for $s < U(\tilde{\lambda})$, and the purchase benefit decreases convexly in the input price for $s \geq U(\tilde{\lambda})$ (Lemma 1; see Appendix B). For this reason, we refer to the input price domain where the unit revenue benefit is in play as the high-benefit region and where the purchase benefit is in play as the low-benefit region of conversion flexibility.

When the mean input price is low, the price realizations fall mainly in the low-price region, where the refinery enjoys the unit revenue benefit, i.e., the high-benefit region. As the price variance increases, more price realizations take place in the high-price region, where the refinery enjoys the purchase benefit, i.e., the low-benefit region; thus, the value of conversion decreases.

When the mean input price is high, the price realizations fall mainly in the high-price region, i.e., the low-benefit region. As the price variance increases, more price realizations take place in the low-price region, i.e., the high-benefit region. Hence, the value of conversion increases.

The calibrated model belongs to the case of $\mu_s \leq T_1$, so VoC decreases in σ_s for $\eta = 0$. Figure 4 shows how a positive intertemporal input–output price correlation η affects the sensitivity result. Varying σ_s within 35% of the calibrated value 3.0, i.e., $\sigma_s \in [1.95, 4.05]$, we find that when η is not very high, VoC decreases in σ_s; but when η is fairly high ($\eta = 1, 1.2$), VoC first increases and then decreases in σ_s. To understand the nonmonotonicity under high η, we examine the effect of σ_s on URB and PB separately. When μ_s is low, as σ_s increases, fewer input price realizations fall in the unit revenue benefit region (low-price region), so URB decreases (see Figure 4, panel (b)); on the other hand, more input price realizations take place in the purchase benefit region (high-price region). When the intertemporal input–output price correlation is high, a higher time 0 input price is likely to be followed by a higher time 1 output price, and hence the purchase benefit is higher. Figure 4, panel (c) shows the purchase benefit increases in σ_s for low and medium values of σ_s. But as σ_s increases further, more input price realizations fall into the region where the refinery decides not to

Figure 4 Sensitivity of (a) VoC, (b) URB, and (c) PB to Input Price Variability σ_s for Three Levels of Intertemporal Input–Output Price Correlation



purchase, and hence the purchase benefit decreases (the purchase benefit for $\eta = 1.2$ eventually decreases for larger values of σ_s outside the plot range). Because of the differential impacts of σ_s on URB and PB, VoC (i.e., the sum of these two benefits) first increases and then decreases in σ_s. The practical implication of this observation is that when the average crude oil price over a period of time is low, conversion flexibility adds less value to the refinery when the crude oil price becomes very volatile.

5.3. Impact of Output Price Volatility (ξ) and Correlation (ρ) on VoC

The output price distribution affects the magnitude of the unit revenue benefit and purchase benefit via its influence on the marginal revenue of the DF$_l$ proportion, u. Proposition 4 shows that the impacts of the output price volatility ξ and correlation ρ on the value of conversion flexibility depend on the heaviness of the input (i.e., the initial DF$_l$ proportion $\tilde{\lambda}$) and the marginal conversion cost.

Proposition 4. *Assume zero intertemporal input–output price correlation and prices following normal distribution*. (1) *For Case* (1) *in Proposition* 1, *i.e., for* $c'(0) \in [u_b^+, u_a)$ *and* $\tilde{\lambda} \in (0, \gamma_H]$, VoC *increases in* ξ *and decreases in* ρ. (2) *For Cases* (2) *and* (3) *in Proposition* 1, *i.e., for* $c'(0) \in [0, u_b^+)$ *and* $\tilde{\lambda} \in (0, \gamma_L]$, *there exist thresholds* t_1 *and* t_2 *such that* VoC *increases in* ξ *if* $f_{p_L/\gamma_L - p_H/\gamma_H}(0)/f_{p_H/(1-\gamma_H)-p_L/(1-\gamma_L)}(0) \le t_1$, *and* VoC *decreases in* ρ *if* $f_{p_L/\gamma_L - p_H/\gamma_H}(0)/f_{p_H/(1-\gamma_H)-p_L/(1-\gamma_L)}(0) \le t_2$, *where* $f_{p_L/\gamma_L - p_H/\gamma_H}$ *and* $f_{p_H/(1-\gamma_H)-p_L/(1-\gamma_L)}$ *are the probability density functions of* $p_L/\gamma_L - p_H/\gamma_H$ *and* $p_H/(1-\gamma_H) - p_L/(1-\gamma_L)$, *respectively.*

When the input is heavy and conversion cost is moderate (Proposition 4(1)), conversion can bring the DF$_l$ proportion to at most γ_H. This implies that in the blending stage, the refinery uses up its scarce DF$_l$ and produces only *one* output, either heavy or light (Scenario (a) in Table 1). In this situation, an increase of the *maximum* of the two output prices increases the marginal revenue of the DF$_l$ proportion. Increasing output price volatility or decreasing price correlation increases the probability of high realizations of the maximum of the two output prices and hence increases the marginal revenue of the DF$_l$ proportion and the value of conversion.

When the input is not very light and the conversion cost is low (Proposition 4(2)), conversion is able to significantly increase the DF$_l$ proportion, but the refinery ends up with surplus DF$_l$ if the realization of output prices favors the heavy output. If the current output price distribution favors the heavy output (conditions $f_{p_L/\gamma_L - p_H/\gamma_H}(0)/f_{p_H/(1-\gamma_H)-p_L/(1-\gamma_L)}(0) \le t_1$ and $f_{p_L/\gamma_L - p_H/\gamma_H}(0)/f_{p_H/(1-\gamma_H)-p_L/(1-\gamma_L)}(0) \le t_2$ in Proposition 4(2)), then the marginal revenue of the DF$_l$ proportion is low. Increasing price volatility or decreasing price correlation increases the likelihood that output prices favor the light output, resulting in increasing the marginal revenue of the DF$_l$ proportion and increasing the value of conversion. In other words, increasing output price volatility or decreasing output price correlation is likely to increase the value of conversion when the refinery processes a heavy input, or the current output price distribution often leaves the refinery with surplus DF$_l$. The conditions in Proposition 4(2) are sufficient conditions. If they fail, increasing price volatility or decreasing price correlation *may* decrease the marginal revenue of the DF$_l$ proportion. In the calibrated model, the output price distribution is such that the refinery in most cases uses up both distillation fractions to blend both heavy and light outputs and continues to do so when the variability or correlation of output prices changes. Therefore, VoC is not sensitive to ξ or ρ.

6. Refinery with Range Flexibility

A refinery with range flexibility needs to determine a purchase portfolio of inputs within its processing range. We formulate the input portfolio decision and characterize the optimal decision in §6.1 and examine the impact of range flexibility on the value of conversion flexibility in §6.2.

6.1. Time 0 Input Portfolio Decision

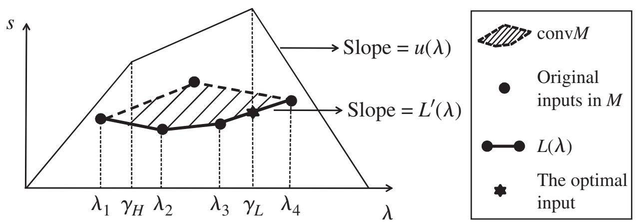
Suppose that n crude oils traded in the input spot market fall into the refinery's processing range $[\lambda_h, \lambda_l]$, $\lambda_h < \lambda_l$. Each input is represented by its DF$_l$ proportion λ_i and spot price s_i, $i = 1, \ldots, n$. Let M represent this set of inputs, $M = \{(\lambda_i, s_i)$: $i = 1, \ldots, n$, with $\lambda_h \le \lambda_1 < \cdots < \lambda_n \le \lambda_l\}$. Let I be the index set of inputs that the refinery chooses to purchase, and let $\theta = (\theta_j)_{j \in I}$ be the proportion vector of the chosen inputs in the portfolio, $\theta_j > 0$, $\sum_{j \in I} \theta_j = 1$. Because the total available DF$_l$ from distilling the portfolio of inputs is the sum of the DF$_l$ from each of the purchased inputs, equivalently, one can view the portfolio of inputs as mixing the inputs into a "new" input according to θ. The new input has a DF$_l$ proportion (spot price) equal to the linear combination of the DF$_l$ proportions (spot prices) of the inputs in the portfolio. The input portfolio decision is given by

$$\max_{I \subseteq \{1, \ldots, n\},\, \theta \in (0,1]^{|I|},\, \sum_{j \in I} \theta_j = 1} \Pi(\tilde{\lambda}, s), \tag{9}$$

where $\tilde{\lambda} = \sum_{j \in I} \theta_j \lambda_j$, $s = \sum_{j \in I} \theta_j s_j$, and $\Pi(\tilde{\lambda}, s)$ is defined in (2) and (2a). The proportion of DF$_l$ and the effective price of the new input are $\tilde{\lambda}$ and s, respectively.

Let conv$M$ denote the convex hull of M (see Figure 5), which includes all convex combinations of inputs in set M. The lower boundary of conv$M$, defined

Figure 5 **Input Selection in the Absence of Conversion Flexibility When There Are Five Original Inputs in the Market, Which Are Represented by Round Dots in the (λ, s) Space**



as $L(\lambda) = \inf\{s \mid (\lambda, s) \in \text{conv}M\}$, is a piecewise linear convex function of λ. For a given λ value, the input with the lowest s value is preferred, which means inputs lying strictly above the lower boundary $L(\lambda)$ are dominated by inputs on $L(\lambda)$. This property implies that the optimal purchase portfolio should consist of inputs represented by $L(\lambda)$ only. Without loss of generality, let λ_1 and λ_n be the first and the last original inputs on $L(\lambda)$, respectively. Formulation (9) can be rewritten as

$$\max_{\tilde{\lambda} \in [\lambda_1, \lambda_n]} \Pi(\tilde{\lambda}, L(\tilde{\lambda})). \tag{10}$$

Hence, the search for the optimal purchase portfolio in the (λ, s) space is reduced to a one-dimensional optimization problem of finding the optimal DF_l proportion $\tilde{\lambda}$ on interval $[\lambda_1, \lambda_n]$. Proposition 5 below states that the expected profit is unimodal along the lower boundary. Analytical results presented in this section hold for general price distributions and arbitrary intertemporal price correlations, unless stated otherwise.

Proposition 5. *$\Pi(\lambda, L(\lambda))$ is unimodal in λ.*

Without loss of generality, we start the search of the optimal DF_l proportion $\tilde{\lambda}$ from λ_1 and move to higher λ values along $L(\lambda)$. We first consider the simple case that the refinery does not have conversion flexibility or conversion is not desirable (e.g., $c'(0) > u_a$). In this case, determining the optimal DF_l proportion, $\tilde{\lambda}$, boils down to balancing the marginal cost of increasing λ, $L'(\lambda)$, and the marginal revenue of λ, $u(\lambda)$. Since $L'(\lambda)$ is piecewise constant and increases at λ_i (illustrated in Figure 5) and $u(\lambda)$ is piecewise constant and decreases at γ_H and γ_L, the optimal $\tilde{\lambda}$ is the point at which $L'(\lambda)$ exceeds $u(\lambda)$ for the *first time*.

When the refinery has conversion flexibility and conversion is affordable (e.g., $c'(0) < u_b^+$ and $\tilde{\lambda} < \gamma_L$), the input portfolio decision involves interesting trade-offs. Starting from λ_1 and moving to higher λ values, the search of the optimal DF_l proportion $\tilde{\lambda}$ may stop earlier than when the refinery does not have conversion flexibility. This happens when the marginal cost of increasing λ, $L'(\lambda)$, exceeds the marginal conversion cost $c'((\lambda^* - \lambda)q^*)$, where λ^* and q^* are the optimal target conversion level and optimal purchase quantity for input λ, respectively, because at that point, conversion is more cost effective than including more lighter inputs to increase the DF_l proportion. Thus, with conversion flexibility, the optimal $\tilde{\lambda}$ is the point at which $L'(\lambda)$ exceeds $u(\lambda)$ or $c'((\lambda^* - \lambda)q^*)$, whichever happens *first*. The possibility that the search can stop earlier at a heavier input implies that a refinery with conversion flexibility can make better use of heavier crude oils in its processing range than a refinery without such flexibility.

The unimodal property of $\Pi(\lambda, L(\lambda))$ greatly simplifies the input search along $L(\lambda)$, $\lambda \in [\lambda_1, \lambda_n]$: as the search moves along $L(\lambda)$ from a low λ value to a high λ value, once a local optimal input is found, it is also the global optimal one.

6.2. Impact of Range Flexibility on Value of Conversion Flexibility

Refinery facilities vary by the range of crude oils they are designed to process. A refinery with a processing range $[\lambda_h, \lambda_l]$ is considered as having a heavier-(lighter-) range flexibility than a refinery that processes input λ_l (λ_h) only. Does conversion flexibility offer more value to a refinery with range flexibility than to a single-input refinery? If so, a refiner should choose among its refineries the one with range flexibility to invest in conversion flexibility; otherwise, the refiner should choose the refinery without range flexibility to invest in conversion flexibility. To investigate how the value of conversion flexibility is affected by range flexibility, we compare VoC of a refinery that processes input λ_l or λ_h only with VoC of a refinery that has a processing range $[\lambda_h, \lambda_l]$. We define the latter as

$$\text{VoC}(\lambda_h, \lambda_l) = \mathbb{E}_{\mathbf{s}}\Big[\max_{\tilde{\lambda} \in [\lambda_h, \lambda_l]} \Pi_c(\tilde{\lambda}, L(\tilde{\lambda})) - \max_{\tilde{\lambda} \in [\lambda_h, \lambda_l]} \Pi_o(\tilde{\lambda}, L(\tilde{\lambda}))\Big],$$

where

$$\max_{\tilde{\lambda} \in [\lambda_h, \lambda_l]} \Pi_c(\tilde{\lambda}, L(\tilde{\lambda})) \text{ and } \max_{\tilde{\lambda} \in [\lambda_h, \lambda_l]} \Pi_o(\tilde{\lambda}, L(\tilde{\lambda})),$$

following the derivation for (10), are the profits with and without conversion flexibility, respectively, and $\mathbb{E}_{\mathbf{s}}$ denotes taking the expectation over the time 0 spot prices $\mathbf{s}$ of inputs λ_h and λ_l.

Recall that $\text{VoC}(\lambda)$ represents the value of conversion flexibility for a single input λ. Proposition 6 compares $\text{VoC}(\lambda_h, \lambda_l)$ with $\text{VoC}(\lambda_l)$ and $\text{VoC}(\lambda_h, \lambda_l)$ with $\text{VoC}(\lambda_h)$.

Proposition 6. (1) *The heavier-range flexibility enhances the value of conversion flexibility; i.e.,* $\text{VoC}(\lambda_h, \lambda_l) \geq \text{VoC}(\lambda_l)$. (2) *Assume zero intertemporal input–output price correlation. The lighter-range flexibility enhances the value of conversion flexibility; i.e.,* $\text{VoC}(\lambda_h, \lambda_l) \geq \text{VoC}(\lambda_h)$, *if $\lambda_l < \lambda_a^\dagger$, where $\lambda_a^\dagger$ is the solution to $c'((\gamma_H - \lambda_a^\dagger)K) = u_a$.*

The heavier-range flexibility allows the refinery to choose a heavier input over the light input λ_l when the heavier input offers higher profit. When this happens, the unit revenue benefit or the purchase benefit of conversion is greater for the heavier input than for the light input. This implies Proposition 6(1).

One might conclude that the lighter-range flexibility diminishes the value of conversion by conjecturing less conversion is needed by lighter inputs. This intuition, however, does not hold when both inputs are very heavy ($\lambda_h < \lambda_l < \lambda_a^\dagger$), as shown in Proposition 6(2). To understand the reason, let us consider four representative input price scenarios and compare the ex post value of conversion for the light and heavy inputs.

- Scenario (i): Spot prices for both inputs are low such that both inputs fall into unit revenue benefit region. In this case, because both inputs are quite heavy, they require the same amount of conversion if being processed and thus have the same ex post value of conversion.
- Scenario (ii): Spot prices for both inputs are high such that both inputs fall into purchase benefit region. In this case, the profit of each input is derived completely from the purchase benefit of conversion. If the lighter input has a higher profit, then it also has a higher ex post value of conversion.
- Scenario (iii): The lighter input falls into unit revenue benefit region (i.e., high-benefit region), but the heavy input falls into purchase benefit region (i.e., low-benefit region). Again, the lighter input has higher ex post value of conversion. In Scenarios (i)–(iii), the lighter input enjoys higher or at least the same level of ex post value of conversion.
- Scenario (iv): The lighter input falls into purchase benefit region (i.e., low-benefit region), but the heavy input falls into unit revenue benefit region (i.e., high-benefit region). In this case, the heavy input is chosen over the lighter input, and thus lighter-range flexibility does not affect the ex post value of conversion.

The above four scenarios together imply that lighter-range flexibility enhances the value of conversion.

When the lighter input is indeed light ($\lambda_l > \lambda_a^\dagger$), in Scenario (i), this much lighter input needs less conversion and has a lower ex post value of conversion than the heavy one. Therefore, it is inconclusive whether the lighter-range flexibility enhances or diminishes the value of conversion.

In other words, the intuition that heavier-range flexibility enhances the value of conversion flexibility holds. The intuition that lighter-range flexibility diminishes the value of conversion does not hold when the lighter input is also very heavy, in which case the lighter-range flexibility enhances the value of conversion; it may hold, however, only when the lighter input is light enough. This insight on the interplay of range flexibility and conversion flexibility holds even in the presence of intertemporal input–output price correlation; however, in this case it is hard to characterize the threshold for input heaviness, $\lambda_a^\dagger$, because the marginal revenue of the DF_l proportion $u(\cdot)$ depends on the time 0 input price realization.

To understand the magnitude of the impact of lighter- and heavier-range flexibility on VoC, we conducted two sets of numerical study. In the first study, we choose West Texas Intermediate (WTI) as the lighter input; in the second study, we choose Nigeria Bonny Light (NBL) as the lighter input. Arabian Light (AL) is used as the heavier input in both studies. The DF_l proportion for WTI is $\lambda_{WTI} = 0.43$; for NBL, it is $\lambda_{NBL} = 0.37$, and recall for AL that $\lambda_{AL} = 0.3$ (see Summers 2006). Note that WTI is lighter than NBL.

In each study, we use the following VAR model:

$$\chi_{t+2} - \mu = \eta \cdot A \cdot (\chi_t - \mu) + e, \tag{11}$$

where $\chi = (s_H, s_L, p_H, p_L)^T$, with s_H and s_L representing the spot price of the heavier input and lighter input, respectively; $\mu = (\mu_{sH}, \mu_{sL}, \mu_H, \mu_L)^T$ is the long-run average price vector of χ; A is a 4×4 coefficient matrix; and $e = (e_{sH}, e_{sL}, e_H, e_L)^T \sim \mathrm{N}(\mathbf{0}, \Sigma)$. The calibrated parameters for each study are summarized in Appendix D.

The impact of heavier-range flexibility, defined as $(\mathrm{VoC}(\lambda_h, \lambda_l) - \mathrm{VoC}(\lambda_l))/\mathrm{VoC}(\lambda_l)$, and the impact of lighter-range flexibility, defined as $(\mathrm{VoC}(\lambda_h, \lambda_l) - \mathrm{VoC}(\lambda_h))/\mathrm{VoC}(\lambda_h)$, for both studies are summarized in Table 2.

Table 2 shows that heavier-range flexibility can enhance the value of conversion flexibility significantly. For a refinery currently processing a very light input—say, WTI—expanding the capability to process the heavier input AL can enhance VoC by 337% for the calibrated parameter setting ($\eta = 1$). For a refinery currently processing a light input NBL, which is not as light as WTI, the impact of heavier-range flexibility on VoC, although lower, remains significant, enhancing VoC by 61.4%.

Compared with the impact of heavier-range flexibility, the impact of lighter-range flexibility on VoC is modest. For a refinery currently processing heavy

Table 2 **Impacts of the Heavier- and Lighter-Range Flexibility on the Value of Conversion Flexibility**

$[\lambda_h, \lambda_l]$	η	Impact of heavier-range flexibility (%)	Impact of lighter-range flexibility (%)
$[\lambda_{AL}, \lambda_{WTI}]$	0.8	+499	−3.92
	1	+337	−3.93
	1.2	+166	−4.13
$[\lambda_{AL}, \lambda_{NBL}]$	0.8	+80.4	+1.16
	1	+61.4	+0.58
	1.2	+42.4	+0.11

input AL, expanding to process the very light input WTI decreases the value of conversion by 3.93% for the calibrated parameter setting, whereas expanding its processing range to include lighter input NBL, which is heavier than WTI, can increase the value of conversion by 0.58% for the calibrated parameter setting. These numerical results echo the above discussion: lighter-range flexibility does not necessarily diminish the value of conversion flexibility; it can enhance its value when the lighter input is not very light.

7. Conclusions

Commodity industries such as petroleum oil refining face tremendous price uncertainties in both input and output markets. Refiners' survival and profitability in volatile marketplaces depend on their ability to maximally utilize the process flexibility of their refining facilities and to make prudent procurement decisions. This paper includes a number of important, basic decisions of an oil refinery (input purchase, intermediate processing, and output blending) in a stylized two-stage stochastic programming model to study the economic potential of a process flexibility for intermediate processing, namely, conversion flexibility, and the impacts of market conditions on the value of this flexibility.

Conversion flexibility in the oil refining process, converting heavy fraction to light fraction after distillation, serves to bridge the gap between the diversity of inputs available in the input market and the specific requirement of the outputs from the output market. The benefits of conversion flexibility are manifested in two forms: one is to enhance the unit revenue of the procured input (unit revenue benefit) and the other is to enable the refinery to afford the input when the input price is high (purchase benefit). Our real-data-calibrated numerical study shows that VoC is significant, accounting for approximately 40% of the profit with conversion. The unit revenue benefit and the purchase benefit are of similar importance, with the former contributing 40% and the latter contributing 60% of the value of conversion.

The unit revenue benefit and the purchase benefit are realized in different ranges of input price. Increasing input price variance can affect expected values of the two benefits in the same direction or different directions. For example, our real-data-calibrated numerical study shows that the purchase benefit increases for a small increase of input price variance but decreases together with the unit revenue benefit as the input price variance increases further. Hence, VoC first increases and then decreases as the input price variance increases. The impact of output price volatility on VoC, on the other hand, is affected by the heaviness of the input that the refinery processes and the conversion cost. The real-data-based numerical study shows that VoC is not sensitive to output market conditions. These insights are helpful to refinery executives when it is pertinent to assess the value of conversion flexibility over a long time horizon consisting of many short high-volatility and low-volatility periods.

Conversion flexibility interacts with range flexibility of the refinery in an interesting way. Expanding a refinery's input processing range toward heavier inputs can enhance VoC significantly. Surprisingly, expanding the processing range toward lighter inputs does not necessarily diminish VoC, although it does when the lighter input is indeed very light. The lighter-range flexibility can enhance VoC when the new input, although relatively lighter than the current input, is still not very light. Also interestingly, VoC increases with the processing capacity of a refinery only up to a capacity threshold, beyond which VoC remains constant. These findings are useful when refinery executives contemplate whether and how the decision on conversion flexibility investment should be jointly considered with capacity and/or range flexibility.

We conclude the paper with a brief discussion of extensions of the model and future research. First, most of the analytical results established in this paper can be extended to the case of more than two outputs and the case of light-to-heavy conversion (see details in Appendix E). Second, although the refinery considered in this paper is a price taker in the input and output markets, a realistic assumption for many petroleum refineries, some large refiners can influence prices in both markets through their purchases and sales. It is worthwhile to investigate the use of conversion flexibility in the presence of pricing power as well as the interplay of conversion and pricing. Third, our model focuses on the input spot purchase and the output spot sale. When the decision maker is risk neutral and the trading market and the capital market are frictionless, spot trading and long-term contracts are separable decisions; otherwise, long-term contracts and spot trading should be considered jointly (e.g., Dong and Liu 2007, Boyabatlı et al. 2011). In the latter case, whether the employment of operational flexibility reduces or increases the need for financial hedging of commodity price risk, although beyond the scope of this paper, is an important question for future research (see Chod et al. 2010b).

Finally, conversion is also an important form of operational flexibility in agricultural oil refining. Oil extracted from oil seeds (e.g., soybeans; sunflower, canola, and safflower seeds) is hydrogenated to generate intermediates called basestocks. A wide range of basestocks can be produced with the hydrogenation process, depending on the conditions used and the degree of saturation and/or isomerization (O'Brien 2010). Basestocks are then blended according to recipes to produce different types of end products (e.g., edible oil, shortenings, margarines). Hydrogenation plays a similar role in agricultural oil refining as conversion does in petroleum oil refining. Although great similarity exists between the oil refining processes of the two industries, the agricultural industry has limited, often illiquid spot markets for the end products, and the refineries face different challenges. Investigating the roles of operational flexibility in dealing with these challenges can be a fruitful direction for future research.

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Appendix A. Expression of $u(\lambda)$

$$u(\lambda)=\begin{cases} u_a\equiv \mathbb{E}_{\Omega_1}\left[\dfrac{p_L}{\gamma_L}\right]+\mathbb{E}_{\Omega_{2\cup 3}}\left[\dfrac{p_H}{\gamma_H}\right]\ (>0) & \text{if } \lambda\in(0,\gamma_H],\\ u_b\equiv \mathbb{E}_{\Omega_1}\left[\dfrac{p_L}{\gamma_L}\right]+\mathbb{E}_{\Omega_2}\left[\dfrac{-p_H+p_L}{\gamma_L-\gamma_H}\right]+\mathbb{E}_{\Omega_3}\left[\dfrac{-p_H}{1-\gamma_H}\right] & \text{if } \lambda\in(\gamma_H,\gamma_L],\\ u_c\equiv -\mathbb{E}_{\Omega_{1\cup 2}}\left[\dfrac{p_L}{1-\gamma_L}\right]-\mathbb{E}_{\Omega_3}\left[\dfrac{p_H}{1-\gamma_H}\right]\ (<0) & \text{if } \lambda\in(\gamma_L,1), \end{cases} \tag{A1}$$

where $\mathbb{E}_{\Omega_i}[\cdot]=\int\int_{\Omega_i}\cdot f(p_H,p_L)\,dp_H\,dp_L$, and $f(p_H,p_L)$ is the joint probability density function of p_H and p_L.

Appendix B. Proofs of Propositions

Proof of Proposition 1. The proof of this proposition is provided in the supplementary document.[18] We provide the expressions of λ^* in Cases (1)–(3) as follows:

(1) $\lambda^*=\gamma_H$ if $q\in[0,q_a(\tilde{\lambda})]$; otherwise, $\lambda^*\in(\tilde{\lambda},\gamma_H)$ is the solution to $u_a-c'((\lambda^*-\tilde{\lambda})q)=0$.

(2) $\lambda^*=\gamma_L$ if $q\in[0,q_b(\tilde{\lambda},\gamma_L)]$; otherwise, $\lambda^*\in(\tilde{\lambda},\gamma_L)$ is the solution to $u_b-c'((\lambda^*-\tilde{\lambda})q)=0$.

$$(3)\quad \lambda^*\begin{cases} =\gamma_L, & q\in[0,q_b(\tilde{\lambda},\gamma_L)];\\ \in(\gamma_H,\gamma_L) \text{ is the solution to } u_b-c'((\lambda^*-\tilde{\lambda})q)=0, & q\in(q_b(\tilde{\lambda},,\gamma_L),q_b(\tilde{\lambda},\gamma_H));\\ =\gamma_H, & q\in[q_b(\tilde{\lambda},\gamma_H),q_a(\tilde{\lambda})];\\ \in(\tilde{\lambda},\gamma_H) \text{ is the solution to } u_a-c'((\lambda^*-\tilde{\lambda})q)=0, & q\in(q_a(\tilde{\lambda}),\infty); \end{cases}$$

and $q_a(\tilde{\lambda})$ is the solution to $u_a-c'((\gamma_H-\tilde{\lambda})q_a)=0$, and $q_b(\tilde{\lambda},\gamma)$ is the solution to $u_b-c'((\gamma-\tilde{\lambda})q_b)=0$. □

[18] The supplementary document is available at http://apps.olin.wustl.edu/faculty/dong/Supplements/DongKouvelisWu-OperationalFlexibilityOilRefining.pdf.

Let voc(s) denote the ex post value of conversion; voc$(s)=\Pi_c(\tilde{\lambda},s)-\Pi_o(\tilde{\lambda},s)$.

Lemma 1. *Assume zero intertemporal input–output price correlation. For a given input with a DF_l proportion $\tilde{\lambda}$,* voc(s) *is constant in s for $s\in(0,U(\tilde{\lambda}))$ and decreases convexly in s for $s\in[U(\tilde{\lambda}),\infty)$.*

Proof of Proposition 3. We need to prove for Cases (1)–(3) in Proposition 1. Here, we illustrate in detail only for Case (1) with $\tilde{\lambda}\in(\tilde{\lambda}_a^{\dagger},\gamma_H]$. The other cases can be shown similarly. Assume the input price follows normal distribution $N(\mu_s,\sigma_s^2)$. For $c'(0)\in[u_b^{+},u_a)$ and $\tilde{\lambda}\in(\tilde{\lambda}_a^{\dagger},\gamma_H]$,

$$\begin{aligned}\text{VoC}=&\int_{-\infty}^{U_a(\tilde{\lambda})}(U_a(\gamma_H)K-c((\gamma_H-\tilde{\lambda})K)-U_a(\tilde{\lambda})K)f(s)\,ds\\ &+\int_{U_a(\tilde{\lambda})}^{U_a(\gamma_H)-(\gamma_H-\tilde{\lambda})c'((\gamma_H-\tilde{\lambda})K)}\big(U_a(\gamma_H)K-c((\gamma_H-\tilde{\lambda})K)-sK\big)f(s)\,ds\\ &+\int_{U_a(\gamma_H)-(\gamma_H-\tilde{\lambda})c'((\gamma_H-\tilde{\lambda})K)}^{U_a(\gamma_H)-(\gamma_H-\tilde{\lambda})c'(0)}\big(Q_aU_a(\gamma_H)-c((\gamma_H-\tilde{\lambda})Q_a)-sQ_a\big)f(s)\,ds.\end{aligned}$$

Let $t=(s-\mu_s)/\sigma_s$. Let $\phi(\cdot)$ and $\Phi(\cdot)$ denote the standard normal probability density function (pdf) and cumulative distribution function, respectively. We have

$$\begin{aligned}\text{VoC}=&\int_{-\infty}^{(U_a(\tilde{\lambda})-\mu_s)/\sigma_s}(U_a(\gamma_H)K-c((\gamma_H-\tilde{\lambda})K)-U_a(\tilde{\lambda})K)\phi(t)\,dt\\ &+\int_{(U_a(\tilde{\lambda})-\mu_s)/\sigma_s}^{(U_a(\gamma_H)-(\gamma_H-\tilde{\lambda})c'((\gamma_H-\tilde{\lambda})K)-\mu_s)/\sigma_s}\big(U_a(\gamma_H)K-c((\gamma_H-\tilde{\lambda})K)-(t\sigma_s+\mu_s)K\big)\phi(t)\,dt\\ &+\int_{(U_a(\gamma_H)-(\gamma_H-\tilde{\lambda})c'((\gamma_H-\tilde{\lambda})K)-\mu_s)/\sigma_s}^{(U_a(\gamma_H)-(\gamma_H-\tilde{\lambda})c'(0)-\mu_s)/\sigma_s}\big(Q_aU_a(\gamma_H)-c((\gamma_H-\tilde{\lambda})Q_a)-(t\sigma_s+\mu_s)Q_a\big)\phi(t)\,dt,\end{aligned}$$

where Q_a satisfies $t\sigma_s+\mu_s=U_a(\gamma_H)-(\gamma_H-\tilde{\lambda})c'((\gamma_H-\tilde{\lambda})Q_a)$. Taking the first-order derivative of VoC with respect to σ_s using the Leibniz rule leads to

$$\begin{aligned}\frac{d\text{VoC}}{d\sigma_s}=&\int_{(U_a(\tilde{\lambda})-\mu_s)/\sigma_s}^{(U_a(\gamma_H)-(\gamma_H-\tilde{\lambda})c'((\gamma_H-\tilde{\lambda})K)-\mu_s)/\sigma_s}-tK\phi(t)dt\\ &+\int_{(U_a(\gamma_H)-(\gamma_H-\tilde{\lambda})c'((\gamma_H-\tilde{\lambda})K)-\mu_s)/\sigma_s}^{(U_a(\gamma_H)-(\gamma_H-\tilde{\lambda})c'(0)-\mu_s)/\sigma_s}-tQ_a\phi(t)\,dt.\end{aligned}$$

Note that all terms from differentiating the limits of integration cancel out by applying the results that $t=(U_a(\gamma_H)-(\gamma_H-\tilde{\lambda})c'(0)-\mu_s)/\sigma_s\Rightarrow Q_a=0$ and $t=(U_a(\gamma_H)-(\gamma_H-\tilde{\lambda})\cdot c'((\gamma_H-\tilde{\lambda})K)-\mu_s)/\sigma_s\Rightarrow Q_a=K$. By the integral limits,

$$(U_a(\tilde{\lambda})-\mu_s)/\sigma_s\le t\le(U_a(\gamma_H)-(\gamma_H-\tilde{\lambda})c'(0)-\mu_s)/\sigma_s.$$

If $(U_a(\gamma_H)-(\gamma_H-\tilde{\lambda})c'(0)-\mu_s)/\sigma_s\le 0$, i.e., $\mu_s\ge U_a(\gamma_H)-(\gamma_H-\tilde{\lambda})c'(0)=T_2$, then $t\le 0$, which implies $d\text{VoC}/d\sigma_s>0$. If $(U_a(\tilde{\lambda})-\mu_s)/\sigma_s\ge 0$, i.e., $\mu_s\le U_a(\tilde{\lambda})=T_1$, then $t\ge 0$, which implies $d\text{VoC}/d\sigma_s<0$. Similarly, we can show for Case (1) with $\tilde{\lambda}\in(0,\tilde{\lambda}_a^{\dagger}]$, $T_2=U_a(\gamma_H)-(\gamma_H-\tilde{\lambda})c'(0)$ and $T_1=U_a(\tilde{\lambda})$ as well; for Case (2), $T_1=U_b(\tilde{\lambda})$ and $T_2=U_b(\gamma_L)-(\gamma_L-\tilde{\lambda})c'(0)$; and for Case (3), $T_1=U_a(\tilde{\lambda})$ and $T_2=U_b(\gamma_L)-(\gamma_L-\tilde{\lambda})c'(0)$. □

The following lemma facilitates the proof of Proposition 4.

Lemma 2. *Assume zero intertemporal input–output price correlation and prices following normal distribution.* (a) $U(\lambda)$ *increases in* ξ *and decreases in* ρ. (b) u_a *increases in* ξ *and decreases in* ρ. (c) *There exist thresholds* t_1 *and* t_2 *such that* u_b *increases in* ξ *if and only if (iff)* $f_{p_L/\gamma_L - p_H/\gamma_H}(0)/f_{p_H/(1-\gamma_H)-p_L/(1-\gamma_L)}(0) \le t_1$, *and* u_b *decreases in* ρ *iff* $f_{p_L/\gamma_L - p_H/\gamma_H}(0)/f_{p_H/(1-\gamma_H)-p_L/(1-\gamma_L)}(0) \le t_2$.

Proof of Proposition 4. $\tilde{\lambda}$ is the input's DF$_l$ proportion. $f(\cdot)$ denotes the pdf of input price s. $\mathrm{VoC} = \mathbb{E}_{\mathbf{s}}[\mathrm{voc}(\mathbf{s})]$. If $s \ge U(\tilde{\lambda})$, $\Pi_o(\tilde{\lambda}, s) = 0$, and thus $\mathrm{voc}(s) = \Pi_c(\tilde{\lambda}, s)$, which increases in ξ but decreases in ρ by Lemma 2(a). If $s < U(\tilde{\lambda})$, then $\mathrm{voc}(s) = \Pi_c(\tilde{\lambda}, s) - \Pi_o(\tilde{\lambda}, s)$, and $q^* = K$ always. Let λ^* be the target level of the DF$_l$ proportion after conversion ($\lambda^* \le \gamma_L$). Let e denote ξ or ρ. There are three cases of $d\mathrm{voc}(s)/de$ depending on the relationship between γ_H, $\tilde{\lambda}$, and λ^*:

$$\frac{d\mathrm{voc}(s)}{de} = \begin{cases} K(\lambda^* - \tilde{\lambda})\dfrac{du_a}{de} & \tilde{\lambda} \le \lambda^* \le \gamma_H, \\ K(\lambda^* - \tilde{\lambda})\dfrac{du_b}{de} & \gamma_H \le \tilde{\lambda} \le \lambda^*, \\ K(\lambda^* - \gamma_H)\dfrac{du_b}{de} + K(\gamma_H - \tilde{\lambda})\dfrac{du_a}{de} & \tilde{\lambda} \le \gamma_H \le \lambda^*. \end{cases} \tag{B1}$$

Combining the two intervals of s, we have $d\mathrm{VoC}/de = \int_{\underline{s}}^{U(\tilde{\lambda})} (d\mathrm{voc}(s)/de) f(s)\,ds + \int_{U(\tilde{\lambda})}^{\bar{s}} (d\mathrm{voc}(s)/de) f(s)\,ds$. (The terms from differentiating the limits of integration cancel out.) As discussed above, in the second term, $d\mathrm{voc}(s)/d\xi > 0$ and $d\mathrm{voc}(s)/d\rho < 0$; $d\mathrm{voc}(s)/de$ in the first term can involve du_a/de or du_b/de, or both du_b/de and du_a/de. If $\tilde{\lambda} \le \lambda^* \le \gamma_H$, then $d\mathrm{voc}(s)/de$ in the first term involves only du_a/de. By Lemma 2(b), $du_a/d\xi > 0$ and $du_a/d\rho < 0$. Thus, we also have $d\mathrm{voc}(s)/d\xi > 0$ and $d\mathrm{voc}(s)/d\rho < 0$ in the first term. This leads to $d\mathrm{VoC}/d\xi > 0$ and $d\mathrm{VoC}/d\rho < 0$. Since $c'(0) \in [u_b, u_a)$ and $\tilde{\lambda} \in (0, \gamma_H]$ imply $\tilde{\lambda} \le \lambda^* \le \gamma_H$, (1) is proved. If $\lambda^* > \gamma_H$, $d\mathrm{voc}(s)/de$ in the first term involves u_b. Let $X = p_L/\gamma_L - p_H/\gamma_H$ and $Y = p_H/(1-\gamma_H) - p_L/(1-\gamma_L)$. The sign of du_b/de depends on the magnitude of $f_X(0)/f_Y(0)$ by Lemma 2(c). When $f_X(0)/f_Y(0) \le t_1$, $du_b/d\xi \ge 0$. Then $d\mathrm{voc}(s)/d\xi \ge 0$ in the first term. Together with $d\mathrm{voc}(s)/d\xi > 0$ in the second term, $d\mathrm{VoC}/d\xi > 0$. When $f_X(0)/f_Y(0) > t_1$, $du_b/d\xi < 0$. Then $d\mathrm{voc}(s)/d\xi < 0$ must (may) hold in the first term for $\gamma_H \le \tilde{\lambda} \le \lambda^*$ ($\tilde{\lambda} \le \gamma_H \le \lambda^*$). Since $d\mathrm{voc}(s)/d\xi > 0$ in the second term, $d\mathrm{VoC}/d\xi$ can be either positive or negative, depending on the weight the input price distribution puts over the two integral domains in the two terms. When $f_X(0)/f_Y(0) \le t_2$, $du_b/d\rho \le 0$. Then $d\mathrm{voc}(s)/d\rho \le 0$ in the first term. Together with $d\mathrm{voc}(s)/d\rho < 0$ in the second term, $d\mathrm{VoC}/d\rho < 0$. When $f_X(0)/f_Y(0) > t_2$, $du_b/d\rho > 0$. Then $d\mathrm{voc}(s)/d\rho > 0$ must (may) hold in the first term for $\gamma_H \le \tilde{\lambda} \le \lambda^*$ ($\tilde{\lambda} \le \gamma_H \le \lambda^*$). Since $d\mathrm{voc}(s)/d\rho < 0$ in the second term, $d\mathrm{VoC}/d\rho$ can be either positive or negative, depending on the weight the input price distribution puts over the two integral domains in the two terms. $c'(0) \in [0, u_b)$ and $\tilde{\lambda} \in (0, \gamma_L]$ imply that conversion takes place, and either $\lambda^* > \gamma_H$ or $\lambda^* \le \gamma_H$. For both cases, the conditions in (2) are sufficient. □

Proof of Proposition 6. (1) First we consider the impact of heavier-range flexibility on the value of conversion. Without heavier-range flexibility, the refinery can purchase only input λ_l; with heavier-range flexibility, the refinery can purchase any mix of inputs λ_h and λ_l. We consider two cases: In the first case, heavier-range flexibility does not affect the purchase decision in the absence of conversion flexibility, i.e., $\max_{\lambda \in [\lambda_h, \lambda_l]} \Pi_o(\lambda, L(\lambda)) = \Pi_o(\lambda_l, L(\lambda_l))$. Since $\max_{\lambda \in [\lambda_h, \lambda_l]} \Pi_c(\lambda, L(\lambda)) \ge \Pi_c(\lambda_l, L(\lambda_l))$, $\mathrm{voc}_r \ge \mathrm{voc}_o$, where voc_r (voc_o) denotes the ex post value of conversion with (without) range flexibility. In the second case, $\max_{\lambda \in [\lambda_h, \lambda_l]} \Pi_o(\lambda, L(\lambda)) > \Pi_o(\lambda_l, L(\lambda_l))$. For the second case, we use a matrix to represent all possible purchase decisions under the four flexibility scenarios. Each entry corresponds to one scenario, as shown below:

$$\begin{bmatrix} \text{with both conversion and range flexibility,} & \text{with only range flexibility} \\ \text{with only conversion flexibility,} & \text{with neither flexibility} \end{bmatrix}.$$

All possible purchase decisions in the second case are summarized in the following six matrices, where λ^* is the optimal crude oil to purchase with both flexibilities, and 0 stands for no purchasing:

$$\underset{\text{(A)}}{\begin{bmatrix} \lambda^* & \lambda_h \\ 0 & 0 \end{bmatrix}} \quad \underset{\text{(B)}}{\begin{bmatrix} \lambda^* & \gamma_i \\ 0 & 0 \end{bmatrix}} \quad \underset{\text{(C)}}{\begin{bmatrix} \lambda^* & \lambda_h \\ \lambda_l & 0 \end{bmatrix}}$$

$$\underset{\text{(D)}}{\begin{bmatrix} \lambda^* & \gamma_H \\ \lambda_l & 0 \end{bmatrix}} \quad \underset{\text{(E)}}{\begin{bmatrix} \lambda^* & \lambda_h \\ \lambda_l & \lambda_l \end{bmatrix}} \quad \underset{\text{(F)}}{\begin{bmatrix} \lambda & \gamma_H \\ \lambda_l & \lambda_l \end{bmatrix}}$$

When it is optimal not to purchase without range flexibility, as matrices (A) *and* (B) show, $\mathrm{voc}_o = 0$, which implies $\mathrm{voc}_r \ge \mathrm{voc}_o$. For matrix (C), for the case with neither conversion nor range flexibility, no purchasing implies $s_l > U(\lambda_l)$. Purchasing input λ_l if with conversion but no range flexibility implies that s_l is in G_{a3}(or G_{b3}), or in G_{a1}(or G_{b1}) but above $U(\lambda_l)$. All these cases can be shown to lead to $\mathrm{voc}_r \ge \mathrm{voc}_o$, but we illustrate only the case of s_l in G_{a3} as follows: $\mathrm{voc}_o = \Pi_c(\lambda_l, s_l) = U_a(\gamma_H)Q_a - c((\gamma_H - \lambda_l)Q_a) - s_l Q_a$; $\mathrm{voc}_r = \Pi_c(\lambda^*, s^*) - \Pi_o(\lambda_h, s_h) \ge \Pi_c(\lambda_h, s_h) - \Pi_o(\lambda_h, s_h)$; Assume s_h is in G_{a1}(the case of s_h in G_{a2} can be shown similarly), then

$$\begin{aligned} \mathrm{voc}_r - \mathrm{voc}_o &\ge \Pi_c(\lambda_h, s_h) - \Pi_o(\lambda_h, s_h) - \Pi_c(\lambda_l, s_l) \\ &= u_a(\gamma_H - \lambda_h)K - c((\gamma_H - \lambda_h)K) \\ &\quad - (\gamma_H - \lambda_l)Q_a c'((\gamma_H - \lambda_l)Q_a) - c((\gamma_H - \lambda_l)Q_a) \\ &> u_a(\gamma_H - \lambda_h)K - c'((\gamma_H - \lambda_h)K)\big[(\gamma_H - \lambda_h)K - (\gamma_H - \lambda_l)Q_a\big] \\ &\quad - (\gamma_H - \lambda_l)Q_a c'((\gamma_H - \lambda_l)Q_a) \\ &= (u_a - c'((\gamma_H - \lambda_h)K))(\gamma_H - \lambda_h)K \\ &\quad + (\gamma_H - \lambda_l)Q_a\big[c'((\gamma_H - \lambda_h)K) - c'((\gamma_H - \lambda_l)Q_a)\big] \\ &> 0. \end{aligned}$$

Note that the first equality is by the definition of Q_a. The last inequality is by $c'((\gamma_H - \lambda_h)K) \ge c'((\gamma_H - \lambda_l)Q_a)$ and $u_a \ge c'((\gamma_H - \lambda_h)K)$ (from the definition of G_{a1}). For matrices (D)–(F), we can similarly show that $\mathrm{voc}_r - \mathrm{voc}_o \ge 0$. (2) For $\lambda_l < \lambda_a^\dagger$, we need to consider three cases of marginal conversion cost $c'(0)$: $\in [u_a, \infty)$, $\in [u_b, u_a)$ and $\in [0, u_b)$. For

$c'(0) \in [u_a, \infty)$, conversion is too costly to take place. Thus, the value of conversion is 0 both with and without range flexibility. For $c'(0) \in [u_b, u_a)$, we refer to Figure 2, panel (b). There are four cases of realized s_h and s_l: both locate in G_{a2}, both locate in G_{a3}, both locate in no purchase region, and one locates in G_{a3} and the other in G_{a2} or no purchase region. If $L'(\lambda) = (s_l - s_h)/(\lambda_l - \lambda_h) \geq u_a$, in all cases, input λ_h is weakly preferred no matter with or without lighter-range flexibility, and thus, lighter-range flexibility does not affect the value of conversion. If $L'(\lambda) < u_a$, we can show that for all the four cases, lighter-range flexibility enhances the value of conversion. Here, we illustrate only the case of s_h in G_{a3} and s_l in G_{a2}: $\text{voc}_o = \Pi_c(\lambda_h, s_h) - \Pi_o(\lambda_h, s_h) = U_a(\gamma_H)Q_a - c((\gamma_H - \lambda_h)Q_a) - s_h Q_a < u_a(\gamma_H - \lambda_h)Q_a - c((\gamma_H - \lambda_h)Q_a)$, where the inequality is because $s_h > U_a(\lambda_h)$; $\text{voc}_r = \Pi_c(\lambda_l, s_l) - \Pi_o(\lambda_l, s_l) = U_a(\lambda_l^*)K - s_l K - c((\lambda_l^* - \lambda_l)K) - (U_a(\lambda_l)K - s_l K) = u_a(\lambda_l^* - \lambda_l)K - c((\lambda_l^* - \lambda_l)K)$. To compare voc_o and voc_r, we examine $u_a\epsilon - c(\epsilon)$ as a function of ϵ and compare $(\gamma_H - \lambda_h)Q_a$ and $(\lambda_l^* - \lambda_l)K$. It is found that $u_a\epsilon - c(\epsilon)$ increases in ϵ for $u_a \geq c'(\epsilon)$, and $c'((\lambda_l^* - \lambda_l)K) = u_a > c'((\gamma_H - \lambda_h)Q_a)$; thus, $\text{voc}_r > \text{voc}_o$. For $c'(0) \in [0, u_b)$, the proof is similar to that for the case of $c'(0) \in [u_b, u_a)$. $\square$

Appendix C. Conversion Cost Calibration

If the refinery is running at its full capacity (3 mmtpa), then K $(=850)$ kbl crude oil is fed to the distillation unit every two weeks, and $(1-\tilde{\lambda})K$ $(=595)$ kbl heavy fraction is fed to the conversion unit. We derive the average conversion cost for a typical refinery with a 50% yield of gasoline, which corresponds to a $\lambda^* = 0.5\gamma_L + 0.5\gamma_H$ portion of DF_l. Then, the conversion quantity is $(\lambda^* - \tilde{\lambda})K$ $(=242.25)$ kbl. We model conversion cost using a quadratic function $c(x) = \tau x + \upsilon x^2$. An average conversion cost of \$4 per barrel of crude oil when the refinery is running at its full capacity leads to

$$\frac{c((\lambda^* - \tilde{\lambda})K)}{K} = 4 \implies \tau + 242.25\upsilon = 14. \tag{C1}$$

Assume the average conversion cost decreases by \$0.5 per barrel if the refinery is running at 80% of full capacity. Then,

$$\frac{c((\lambda^* - \tilde{\lambda})0.8K)}{0.8K} = 3.5 \implies \tau + 193.8\upsilon = 12.3. \tag{C2}$$

Solving Equations (C1) and (C2) leads to $\tau = 5.5$ and $\upsilon = 0.035$.

Appendix D. Parameter Calibration for the Price Model

We calibrate parameters by setting $\eta = 1$.

When using Arabian Light as the input and the residual fuel oil and gasoline as the heavy and light end products, respectively, the stationary distribution of $(s, p_H, p_L)^T$ has mean $\mu = (24.59, 23.90, 33.65)^T$ and stationary covariance matrix Γ satisfying $\text{vec}(\Gamma) = A \otimes A\ \text{vec}(\Gamma) + \text{vec}(\Sigma)$, where $\otimes$ is the Kronecker product operator, vec() is the vectorization operator, and

$$\Gamma = \begin{bmatrix} 8.96 & 9.81 & 12.73 \\ 9.81 & 16.33 & 15.55 \\ 12.73 & 15.55 & 24.14 \end{bmatrix}.$$

When using Arabian Light as the heavier input and WTI as the lighter input, the calibrated parameters are $\mu = (24.59, 28.35, 23.90, 33.65)^T$,

$$A = \begin{bmatrix} 0.61 & 0.24 & 0.011 & -0.019 \\ -0.025 & 0.73 & 0.14 & 0.023 \\ -0.16 & 0.35 & 0.75 & -0.054 \\ 0.12 & 0.52 & 0.14 & 0.38 \end{bmatrix} \quad \text{and}$$

$$\Sigma = \begin{bmatrix} 2.22 & 2.15 & 1.69 & 1.58 \\ 2.15 & 2.98 & 2.14 & 1.69 \\ 1.69 & 2.14 & 3.33 & 0.96 \\ 1.58 & 1.69 & 0.96 & 4.26 \end{bmatrix}.$$

When using Arabian Light as the heavier input and NBL as the lighter input, the calibrated parameters are $\mu = (24.59, 26.55, 23.90, 33.65)^T$,

$$A = \begin{bmatrix} 0.81 & -0.063 & 0.094 & 0.013 \\ 0.28 & 0.35 & 0.19 & 0.043 \\ 0.16 & -0.11 & 0.88 & -0.012 \\ 0.25 & 0.25 & 0.27 & 0.41 \end{bmatrix} \quad \text{and}$$

$$\Sigma = \begin{bmatrix} 2.23 & 2.29 & 1.72 & 1.66 \\ 2.29 & 2.98 & 1.92 & 1.84 \\ 1.72 & 1.92 & 3.40 & 1.10 \\ 1.66 & 1.84 & 1.10 & 4.43 \end{bmatrix}.$$

Appendix E. Extensions

Most of the analytical results established in this paper can be extended to the case of more than two outputs and the case of light-to-heavy conversion. First, suppose we have N $(N \geq 3)$ outputs. Output i is characterized by its BOM γ_i and spot price p_i. Production quantity decision x_i is made at time 1. The time 1 linear programming formulation is given by

$$\begin{aligned} r(p, q, \lambda) = \max_{x_i} \ & \sum_{i=1}^{N} p_i x_i \\ \text{s.t.} \ & \sum_{i=1}^{N} \gamma_i x_i \leq \lambda q, \quad \sum_{i=1}^{N} (1-\gamma_i) x_i \leq (1-\lambda) q, \\ & x_i \geq 0, \quad i = 1, \ldots, N. \end{aligned}$$

One can solve the dual of the above linear program and show that at most two outputs should be produced in the optimal blending, a property shared by the two-output case. The expression of unit revenue $U(\lambda)$ should account for $N + C_N^2$ possible output combinations (N cases of one output, C_N^2 cases of two outputs) based on the price realization of the N outputs. Despite the complexity, the main results and insights from the two-output case can be extended to the N-output case. Although our model focuses on the operational flexibility of converting heavy fraction to light fraction, it can be easily adapted to study light-to-heavy conversion by viewing the light-to-heavy conversion as a mirror image of heavy-to-light conversion.

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