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Can Noise Create the Size and Value Effects?

Robert D. Arnott
Research Affiliates, LLC, Newport Beach, California 92660, arnott@rallc.com

Jason C. Hsu
Research Affiliates, LLC, Newport Beach, California 92660; and University of California, Los Angeles, Los Angeles, California 90095, hsu@rallc.com

Jun Liu
University of California, San Diego, La Jolla, California 92093; and Shanghai Advanced Institute of Finance (SAIF), Shanghai Jiao Tong University, Xuhui, Shanghai 200030, China, junliu@ucsd.edu

Harry Markowitz
University of California, San Diego, La Jolla, California 92093, hmarkowitz@ucsd.edu

If the price of a stock differs from its intrinsic value by a random noise, then value stocks are more likely to have negative noise; they are thus more likely undervalued and have higher expected return than justified by risk. The same intuition applies to small capitalization stocks. We formally verify and explore this intuition by using a standard noise-in-price model. This intuition is different from the Jensen's inequality effect studied by Blume and Stambaugh [Blume ME, Stambaugh RF (1983) Biases in computed returns: An application to the size effect. *J. Financial Econom.* 12(3):387–404]. Our model is parsimonious: the value premium as well as size premium are computed in closed form and depend on only four parameters: mean of stock return, volatility of stock return, volatility of the price-to-dividend ratio, and noise volatility. We emphasize that only a moderate volatility of price noise is needed to generate the observed value premium. However, the model cannot generate the observed size premium.

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1. Introduction

In this paper, we show that noise in prices can create a rich pattern in the cross section of expected stock returns. In particular, we show that noise in prices can generate the value effect observed in U.S. market data. However, the size effect in our model diminishes over time.

For a given stock, we show that in noise-in-price models, the expected return decreases with the market cap and price-to-dividend ratio,[1] assuming the expected intrinsic value return is constant. This result is intuitive. For example, the low price of a small-cap stock may be the result of a low intrinsic value, a negative price noise, or a combination of the two. A low intrinsic value yields a fair return, but a negative noise in price implies that the stock is undervalued, leading to a superior return that is not related to risk. The same logic applies to value stocks, whereas the reciprocal logic applies to large-capitalization and growth stocks. Lewellen (1999) documents the value effect in time series by showing that book-to-market ratios predict stock returns.

For the cross section of expected stock returns, we deliberately make an unrealistic assumption that all stocks have the same ex ante return distribution. This assumption is made in other theoretical studies on size and value effects and implies that time-series draws of any given stock are the same as cross-sectional draws of different stocks. Furthermore, in our model, because there is no cross-sectional variation in parameters, the cross-sectional variation in expected stock returns is *solely* caused by variations in realizations of the price noise.

We compute the cross-sectional expected returns for different size and value deciles. There is only one parameter that is not directly measurable from U.S. market data—namely, the variance of the noise in price. The other parameters are expected stock return, the volatility of stock return, and the volatility of the log-price-to-dividend ratio, each of which can be directly estimated.

[1] We used the price-to-dividend ratio instead of the book-to-market ratio to reduce the number of parameters. To compute returns, we already have to specify the dividend process. If we used the book-to-market ratio, we would have to additionally specify the dividend-to-value process, which would necessarily introduce more parameters into the model.

We emphasize that only a moderate amount of noise is needed to generate a realistic value effect. The ratio of the variance of noise to that of total stock return is only 10%. This ratio is consistent with those found in empirical studies of market efficiency. Furthermore, the cross-sectional pattern of expected stock returns is produced essentially with only one adjustable parameter, which is the noise variance.

Since the size and value effects are documented for monthly and quarterly horizons, the noise in our model needs to persist over the monthly and then quarterly horizon. A potential cause for persistent and slow-moving errors can be driven by fluctuations in investors' wealth shares. Yan (2010) shows that investors' misperceptions can have a long-term impact on an asset's valuation.

In noise-in-price models, prices are not derived from market-clearing conditions; instead, they are exogenously specified. These models are tractable reduced-form models to capture deviations from standard asset-pricing model predictions. Thus, they should be viewed as econometric models. Noise-in-price models have been applied widely in finance and have produced important insights. For example, they have been applied to study inefficient markets (Black 1986, Summers 1986, Fama and French 1988, Poterba and Summers 1988), market microstructure (Blume and Stambaugh 1983), liquidity (Bao et al. 2010), and asset-pricing model errors (Brennan and Wang 2010).

Our noise-in-price model is similar to that of Blume and Stambaugh (1983). However, the mechanism that generates the size effect in their paper is completely different from ours. Blume and Stambuagh rely on a key extra assumption: that small-cap stocks have higher noise volatilities and thus have higher expected returns because of the Jensen effect. The variation in expected stock returns in Blume and Stambaugh is driven by the variation in ex ante distribution, that is, parameter variation. Furthermore, the premium from Blume and Stambaugh's mechanism is too small to explain size and value effects in lower-frequency data, such as quarterly or longer.

Because we assume that all stocks have the same ex ante distributions, the Blume and Stambaugh (1983) mechanism and intuition for generating the size premium is shut off completely. Instead, small-cap stocks in our paper are defined to be ones with low market caps, and as a consequence of experiencing predominantly negative price noise, they have higher expected returns. In other words, the variation in cross-sectional expected returns is completely driven by the variation in realizations of the random price noise. This mechanism is different from that in Blume and Stambaugh.

Parameter variations should lead to additional cross-sectional variations in expected returns, as shown by Blume and Stambaugh (1983). Certainly, we *believe* that both parameter variation and random noise realization contribute to the cross-sectional size and value effects. However, we choose to shut off the parameter variation channel in order to highlight that noise *alone* can generate the size and value premiums documented in the literature, with a plausible noise parameter.

In equilibrium models, the price always equals the intrinsic value, and thus the noise in price is always zero. This is true even in noisy rational expectation equilibrium models where the random noise is in supply rather than price. In equilibrium models of size and/or value effect,[2] the expected return in excess of the risk-free return is the compensation for risk. In noise-in-price models, the prices are exogenously specified; they are not derived from market-clearing conditions or any other conditions. Consequently, the expected return in excess of the risk-free return has a component that is not accounted for by risk.

Contrary to what one might expect, existing models of underreaction and overreaction by economic agents, such as Barberis et al. (1998) and Daniel et al. (1998), actually do not have noise in price, at least not in the form presented in this paper. These models offer economic explanations of the size or value effects.

Berk (1995, 1997) points out that in any model in which the cross-sectional covariance between expected payoff and expected return is zero, the cross-sectional correlation between price and expected return has to be negative. Berk's critique has wide applicability, but it explains neither the source nor the magnitude of the size and value effects, which have been the focus of many asset pricing studies, such as Fama and French (1992), Lakonishok et al. (1994), Barberis and Huang (2001), Daniel et al. (2001), Gomes et al. (2003), Bansal and Yaron (2004), Zhang (2005), and Yogo (2006), to name a few.

Noise in price can arise from many economic settings. The market inefficiency model of Summers (1986), Poterba and Summers (1988), and Fama and French (1988); nonsynchonous trading (Blume and Stambaugh 1983); and fixed-income empirical modeling, to name just a few, all lead to noise in price.

We do not make any attempt to favor any such mechanisms. Our main focus is to examine whether price noise can lead to size and value effects and to quantify the magnitude of these effects. If a rational or a behavioral model can generate 8% noise volatility, it can then lead to empirically observed value spread. In particular, this may not require an unreasonable amount of irrationality.

We remark that our conditional value variable needs to be price based. However, Lakonishok et al. (1994) show that the value effect is robust to using a non-price

[2] See, for example, Barberis and Huang (2001), Daniel et al. (2001), Gomes et al. (2003), Zhang (2005), and Yogo (2006).

measure of value growth. This empirical finding cannot be generated in noise-in-price models.

Our paper proposes that noise is a contributing source of the size effect and is perhaps a dominant source of the value effect. We demonstrate that this view is plausible by computing size and value premiums that match the empirical data. The premiums associated with small-cap and value stocks in our model are driven solely by a reasonable noise parameter and are not attributed to risk.

Brennan and Wang (2010) have a similar noise-in-price model and study the effect of noise in price on unconditional expected returns attributable to Jensen's inequality. Thus, they explore the same channel as Blume and Stambaugh (1983). The effect of this channel is completely switched off in our paper.

The rest of our paper is organized as follows. In §2, we formally introduce our model of noise. In §3, we solve *in closed form* the expected returns conditional on price and the price-to-dividend ratio. We then compute the expected returns conditional on 10 price and price-to-dividend ratio deciles. The value premium computed from our model with plausible parameters is the same order of magnitude as documented in previous empirical studies. The size premium goes to zero over time. In §5, we discuss related literature. We make concluding remarks in §6.

2. Noise-in-Price Model

We present in this section the classic noise-in-price model and discuss the key economic assumptions and technical assumptions underlying the model.

Assumption 1 (Noise-in-Price Model). *There are N stocks. The observed market price P_{nt} of stock n at time t deviates from its intrinsic value V_{nt}, with $n = 1, \ldots, N$, by a noise $e^{\Delta_{nt}}$. However, the average deviation is zero. Specifically,*

$$P_{nt} = V_{nt} e^{\Delta_{nt}}, \tag{1}$$

where noise Δ_{nt} is independent of $\Delta_{n't'}$ for $n \neq n'$ and $t \neq t'$. The unconditional expectation $\mathrm{E}[e^{\Delta_{nt}}]$ of $e^{\Delta_{nt}}$ satisfies

$$\mathrm{E}[e^{\Delta_{nt}}] = 1, \quad n = 1, 2, \ldots, N. \tag{2}$$

Equation (1) states that market price P_{nt} of the stock n at time t deviates from its intrinsic value V_{nt} by a random noise $e^{\Delta_{nt}}$. Equation (2) states that the deviation, on average, is zero.

The noise-in-price model described above is widely used in the literature. For example, it is used by Summers (1986), Fama and French (1988), and Poterba and Summers (1988) to study market inefficiency. It is also used by Blume and Stambaugh (1983) to model bid-ask bounces and explore their implications for the size effect. We note that similar specifications of the noise-in-price model are also found in Aboody et al. (2002), Arnott (2005a, b), Hsu (2006), Arnott and Hsu (2008), and Brennan and Wang (2007).

The additive form of noise-in-price model assumes that the price equals the sum of the intrinsic value and a noise term. An additive stationary price noise becomes negligible if V_t grows over time. Campbell and Kyle (1993) overcome this problem by using an additive form with detrended dividends. This problem does not arise with the multiplicative form.

Assumption 2 (Technical Assumptions). *Let D_{nt} denote the dividend at time t of stock n. Let $v_{nt} = \ln V_{nt}$, $p_{nt} = \ln P_{nt}$, and $d_{nt} = \ln D_{nt}$. Then*

$$v_{nt+1} = \mu_v + v_{nt} + \sigma_v \epsilon^{v}_{nt+1}, \tag{3}$$

$$\Delta_{nt+1} = -\tfrac{1}{2}\sigma^2_{\Delta} + \sigma_{\Delta}\epsilon^{\Delta}_{nt+1}, \tag{4}$$

$$v_{nt+1} - d_{nt+1} = \bar{x}_v + \sigma_{vd}\epsilon^{vd}_{nt+1}, \tag{5}$$

where σ_{Δ}, μ_v, σ, $\bar{x}_v$, and σ_{vd} are all constants. All shocks are independent across stocks and time.

The technical assumptions expressed in Equations (3)–(5) are made for tractability. A general case of noise following an autoregressive model 1 (AR(1)) process can be similarly studied.[3] Equation (3) says that the log intrinsic value is a Gaussian random walk, which is a standard assumption in finance literature. Note that the return of the intrinsic value is

$$\frac{V_{nt+1} + D_{nt+1}}{V_{nt}} = e^{v_{nt+1} - v_{nt}}(1 + e^{-\bar{x}_v - \sigma_{vd}\epsilon^{vd}_{nt+1}}),$$

and the expected return of the intrinsic value is

$$\mathrm{E}\left[\frac{V_{nt+1} + D_{nt+1}}{V_{nt}}\right] = e^{\mu_v + (1/2)\sigma^2_v}(1 + e^{-\bar{x}_v + (1/2)\sigma^2_{vd}}) \equiv \bar{R}_v. \tag{6}$$

Thus, in our model, the intrinsic value return is a constant mean that is the same for all stocks, so we are assuming no size and value effects in the absence of noise. Equation (4) says that the log noise is normal and identically and independently distributed (i.i.d.) over time. Although this assumption implies that the pricing error will disappear after one period, the assumption can be relaxed without losing tractability. Equation (5) says that the price-to-dividend ratio is mean reverting over time in the sense that it is an AR(1) process with a zero AR(1) coefficient.

The assumption on value dividend ratio $v_{nt} - d_{nt}$ is necessary for computing returns since dividend D_{nt+1} is part of the cashflow for $t+1$, in addition to the value V_{nt+1}. Equation (5) is used in the literature on predictive regressions (see, for example, Stambaugh 1999, Torous and Valkanov 2000). There is no price noise

[3] See the extended online appendix to this paper at http://rady.ucsd.edu/docs/faculty/online_appendix.pdf.

in these studies; the value-dividend ratio is the price-dividend ratio. We can make both noise process Δ_{nt} and the value-dividend ratio $v_{nt} - d_{nt}$ AR(1) processes without losing tractability.

Assuming lognormality in our random variables gives us tractability to compute conditional expected returns. With non-Gaussian specifications, tractability is lost, but all the insights and intuitions remain. We note also that the price noise is specified in multiplicative form instead of additive form, again for tractability.

We have made our assumptions as simple as possible to make it clear that noise realization *alone* is sufficient to match the desired empirical moments. Extending the model to incorporate other realistic features is often straightforward and is discussed in §6.

In the return dynamics described in this section, the theory that determines intrinsic value V_{nt} is unspecified; it can be the consumption-based asset-pricing models, the capital asset-pricing model (CAPM), the arbitrage pricing theory, or any other model. The exact choice will not affect the results of this paper. For convenience, we can think of V_{nt} as the discounted present value of expected future cash flows, where the discount rates are determined by the return covariance with systematic risk.

3. Expected Returns

We first compute the unconditional expected stock return. Then we derive the closed-form solution for the expected return of a stock conditional on its market price and price-to-dividend ratio.

3.1. Unconditional Expected Returns

We first present the unconditional expected return for stocks in our noise-in-price model.

The expected return of stock n satisfies

$$\mathrm{E}\left[\frac{P_{nt+1}+D_{nt+1}}{P_{nt}}\right] = \mathrm{E}\left[\frac{V_{nt+1}+D_{nt+1}}{P_{nt}}\right] = \mathrm{E}\left[\frac{V_{nt+1}+D_{nt+1}}{V_{kt}}\right]\mathrm{E}\left[\frac{V_{nt}}{P_{nt}}\right], \quad (7)$$

where the first equality is due to $P_{nt+1} = V_{nt+1} \cdot e^{-(1/2)\sigma_\Delta^2+\sigma_\Delta\epsilon_{t+1}^\Delta}$. From $P_{nt+1}/V_{nt} = e^{-(1/2)\sigma_\Delta^2+\sigma_\Delta\epsilon_{t+1}^\Delta}$, we then have

$$\begin{aligned}\mathrm{E}\left[\frac{P_{nt+1}+D_{nt+1}}{P_{nt}}\right] &= \mathrm{E}\left[\frac{V_{nt+1}+D_{nt+1}}{V_{kt}}\right]\mathrm{E}\left[\exp\left(\frac{1}{2}\sigma_\Delta^2-\sigma_\Delta\epsilon_t^\Delta\right)\right]\\ &= \mathrm{E}\left[\frac{V_{nt+1}+D_{nt+1}}{V_{kt}}\right]\exp(\sigma_\Delta^2) = \bar{R}_v\exp(\sigma_\Delta^2).\end{aligned}$$

Therefore, we have the following proposition.

Proposition 1 (Unconditional Expected Return). *If Assumptions* 1 *and* 2 *hold, then unconditional expected price return* $\bar{R}_p$ *is*

$$\begin{aligned}\bar{R}_p &\equiv \mathrm{E}\left[\frac{P_{nt+1}+D_{nt+1}}{P_{nt}}\right]\\ &= \bar{R}_v e^{\sigma_\Delta^2} = e^{\mu_v+(1/2)\sigma_v^2}(1+e^{-\bar{x}_v+\sigma_{vd}^2/2})e^{\sigma_\Delta^2}. \quad (8)\end{aligned}$$

Note that the unconditional expected return is independent of n; therefore, it is the same for all stocks. The cross section of unconditional expected returns in our model is trivial. The difference between the expected return and the expected intrinsic value return increases with σ_Δ^2 and is a result of Jensen's inequality, which is driven by the variance of the random noise.

Proposition 1 is an analytic closed-form counterpart to Blume and Stambaugh's Equation (6). Blume and Stambaugh compute unconditional expected returns for stocks and show that price noise resulting from bid-ask bounce increases the unconditional expected return. Additionally, the increase in the unconditional expected return rises with the noise variance. They find that the higher bid-ask spread for smaller stocks explains 50% of the size effect documented in daily data. The premium from the Blume and Stambaugh mechanism is too small in magnitude in our model.

Following Blume and Stambaugh's (1983) intuition, we could generate the cross-sectional variation in expected returns documented in Fama and French (1992) if we assumed variations in the exogenously specified noise volatility for stocks. Although we believe that a portion of the observed variation must be driven by variations in parameters, it is not very satisfying from a modeling perspective that the cross-sectional variation is essentially exogenously assumed. We demonstrate that the size and value effects can be generated with cross-sectional variation in parameters.

In our model, we wish to examine a different aspect of the noise-in-price model. We focus exclusively on the effect of the random noise realization on the cross section of stock returns. We deliberately make the extreme assumption that all stocks are ex ante identical in return distribution. With our model, no pattern in cross-sectional expected return variations is driven by model parameter variations.

Finally, we observe that the noise volatility in Blume and Stambaugh (1983) required to match the observed size premium is large. Even at a 10% noise volatility for small-cap stocks and a 0% volatility for large-cap stocks, the predicted difference in expected return would be only 1%. This is small relative to the documented size premium.

Brennan and Wang (2010) also study the effect of noise in prices on unconditional expected stock returns. They show that the mispricing-induced unconditional return premium, either estimated using a Kalman filter or proxied by the volatility and variance ratio of residual returns, is significantly associated with realized risk-adjusted returns.

3.2. Expected Stock Return Conditional on Size and Value

We now compute the expected stock return conditioned on price and price-to-dividend ratio of a given stock. These results will be used to compute the expected return conditional on price decile and price-to-dividend ratio decile in the next section. In our model, a stock with a low price and low price-to-dividend ratio has a higher expected return because such a stock is more likely than others to be undervalued. This mechanism is different from that of Blume and Stambaugh (1983).

The mechanism in our model is straightforwardly intuitive. If Δ_{nt} is negative, the market price of stock n is lower than its intrinsic value and the expected return is high. In reality, we do not observe the noise Δ_{nt}. However, the price P_{nt} or a price-to-dividend ratio provides information about Δ_{nt}. The lower the price or the price-to-dividend ratio, the more likely Δ_{nt} is to be negative and the stock undervalued.

The expected return conditional on the information P_{nt} can be written as

$$\mathrm{E}\left[\frac{P_{nt+1}+D_{nt+1}}{P_{nt}}\,\middle|\,P_{nt}\right]=\mathrm{E}\left[\frac{V_{nt+1}+D_{nt+1}}{P_{nt}}\,\middle|\,P_{nt}\right]=\mathrm{E}\left[\frac{V_{nt+1}+D_{nt+1}}{V_{kt}}\right]\mathrm{E}\left[\frac{V_{nt}}{P_{nt}}\,\middle|\,P_{nt}\right]. \quad (9)$$

From Assumption 1, we then have

$$\mathrm{E}\left[\frac{P_{nt+1}+D_{nt+1}}{P_{nt}}\,\middle|\,P_{nt}\right]=\mathrm{E}\left[\frac{V_{nt+1}+D_{nt+1}}{V_{kt}}\right]\mathrm{E}\left[\exp\left(\frac{1}{2}\sigma_\Delta^2-\sigma_\Delta\epsilon_t^\Delta\right)\,\middle|\,P_{nt}\right]. \quad (10)$$

In Equation (10), the first factor is the expected return without noise. The effect of noise is in the second factor. The positive noise ϵ_t^Δ implies that the market price p_{nt} is higher than the intrinsic value v_{nt}. Thus it would lead to a lower expected return.

Furthermore, we can write

$$\begin{aligned}\mathrm{E}\left[\frac{P_{nt+1}+D_{nt+1}}{P_{nt}}\,\middle|\,P_{nt}\right]&=\mathrm{E}\left[\frac{V_{nt+1}+D_{nt+1}}{V_{kt}}\right]\exp(\sigma_\Delta^2)\cdot\mathrm{E}\left[\exp\left(-\frac{1}{2}\sigma_\Delta^2-\sigma_\Delta\epsilon_t^\Delta\right)\,\middle|\,P_{nt}\right]\\&=\bar{R}_p\mathrm{E}\left[\exp\left(-\frac{1}{2}\sigma_\Delta^2-\sigma_\Delta\epsilon_t^\Delta\right)\,\middle|\,P_{nt}\right].\end{aligned}$$

Thus, the expected return conditional on price is decomposed into two components. The first one is the unconditional expected price return studied in Blume and Stambaugh (1983), and the second one, the focus of this paper, is the expected value of inverse noise conditional on price.

Let $p_{nt}=\ln P_{nt}$ and $v_{nt}=\ln V_{nt}$. Then

$$p_{nt}=v_{nt}-\tfrac{1}{2}\sigma_\Delta^2+\sigma_\Delta\epsilon_t^\Delta.$$

The price can be viewed as a signal on the noise ϵ_t^Δ. From Assumption 2, v_{nt} and ϵ_t^Δ are independent normal random variables with means $\mu_v t$ and 0 and variances $\sigma_v^2 t$ and σ_Δ^2, respectively. Thus, we have

$$\mathrm{E}[\epsilon_t^\Delta\mid p_{nt}]=\frac{\sigma_\Delta^2}{\sigma_\Delta^2+\sigma_v^2 t}(p_{nt}-\bar{p}_t),$$

and

$$\mathrm{var}[\epsilon_t^\Delta\mid p_{nt}]=\frac{1}{1/\sigma_\Delta^2+(1/\sigma_v^2)t}.$$

Thus

$$\mathrm{E}[e^{-\epsilon_t^\Delta}\mid p_{nt}]=\exp\left(-\frac{\sigma_\Delta^2}{\sigma_\Delta^2+\sigma_v^2 t}(p_{nt}-\bar{p}_t)+\frac{1}{2(1/\sigma_\Delta^2+(1/\sigma_v^2)t)}\right),$$

where $\bar{p}_t=\mu_v t-\frac{1}{2}\sigma_\Delta^2$. The expected return conditional on p_{nt} can be written as

$$\mathrm{E}\left[\frac{P_{nt+1}+D_{nt+1}}{P_{nt}}\,\middle|\,p_{nt}\right]=\bar{R}_p\exp\left(-\frac{\sigma_\Delta^2}{\sigma_\Delta^2+\sigma_v^2 t}(p_{nt}-\bar{p}_t)-\frac{\sigma_\Delta^4}{\sigma_\Delta^2+\sigma_v^2 t}\right).$$

Noting that $\mathrm{E}[\exp(-(\sigma_\Delta^2/(\sigma_\Delta^2+\sigma_v^2 t))(p_{nt}-\bar{p}_t))]=\exp(\sigma_\Delta^4/(\sigma_\Delta^2+\sigma_v^2 t))$, we have the following proposition.

Proposition 2. *Suppose that Assumptions* 1 *and* 2 *hold. Then the expected return of stock n conditional on price P_{nt} is given by*

$$\mathrm{E}\left[\frac{P_{nt+1}+D_{nt+1}}{P_{nt}}\,\middle|\,p_{nt}\right]=\bar{R}_p\frac{\exp(-(\sigma_\Delta^2/(\sigma_\Delta^2+\sigma_v^2 t))p_{nt})}{\mathrm{E}[\exp(-(\sigma_\Delta^2/(\sigma_\Delta^2+\sigma_v^2 t))p_{nt})]}=\bar{R}_p\frac{\exp(-(\sigma_\Delta^2/\sigma_{pt}^2)p_{nt})}{\mathrm{E}[\exp(-(\sigma_\Delta^2/\sigma_{pt}^2)p_{nt})]},$$

where $\sigma_{pt}^2=\sigma_\Delta^2+\sigma_v^2 t$ is the variance of p_{nt}.

Note that as t increases, dependence on p_{nt} decays exponentially. This will imply that, in our model, the size effect disappears over time. The critical assumption leading to this result is that v_{nt} is a random walk; thus its variance increases with t. But Δ_{nt} is a white noise; thus its variance is constant. If we assume counterfactually that Δ_{nt} is also a random walk, then the size effect will not decay.

Next, we will derive the expected return conditional on the price-to-dividend ratio. Let $x_{nt}=p_{nt}-d_{nt}$ denote the log-price-to-dividend ratio. Then from Assumptions 1 and 2, we have

$$x_{nt} = v_{nt} - d_{nt} - \tfrac{1}{2}\sigma_\Delta^2 + \sigma_\Delta \epsilon_t^\Delta = \bar{x}_v - \tfrac{1}{2}\sigma_\Delta^2 + \sigma_v \epsilon_{nt}^v + \sigma_\Delta \epsilon_{nt}^\Delta.$$

Following exactly the same steps as before, we can obtain the following proposition.

Proposition 3 (Expected Return Conditional on Value). *Suppose Assumptions* 1 *and* 2 *hold. Then, the expected return conditional on x_{nt} is*

$$\mathrm{E}\left[\frac{P_{nt+1}+D_{nt+1}}{P_{nt}}\,\middle|\, x_{nt}\right] = \bar{R}_p \frac{\exp(-(\sigma_\Delta^2/(\sigma_\Delta^2+\sigma_{vd}^2))x_{nt})}{\mathrm{E}[\exp(-(\sigma_\Delta^2/(\sigma_\Delta^2+\sigma_{vd}^2))x_{nt})]}$$
$$= \bar{R}_p \frac{\exp(-(\sigma_\Delta^2/\sigma_{pd}^2)x_{nt})}{\mathrm{E}[\exp(-(\sigma_\Delta^2/\sigma_{pd}^2)x_{nt})]}, \quad (11)$$

where $\sigma_{pd}^2 = \sigma_\Delta^2 + \sigma_{vd}^2$ is the variance of the log-price-to-dividend ratio.

We assume that the correlation between log intrinsic value v_{nt} and log intrinsic value-to-dividend ratio $v_{nt} - d_{nt}$ is zero for notational simplicity (this follows from Equations (3)–(5)). Incorporation of a nonzero correlation is straightforward.

We compute the expected returns conditional on the price-to-dividend ratio. Many empirical studies analyze expected returns conditional on other price ratios, such as the price-to-book ratio. Conceptually, the analysis applies in the same way to any price ratio dependence. We choose the price-to-dividend ratio instead of other ratios to avoid additional parameters.

Note that the cross-sectional dependence of the conditional expectation is only through P_{nt} and X_{nt}. Accordingly, two stocks with the same price P_{nt} and price-to-dividend ratio X_{nt} have the same expected return. Naturally, if there is no noise ($\sigma_\Delta = 0$), the expected return is independent of the state variables P_{nt} and X_{nt}. In this case, the stock is fairly priced for all levels of P_{nt} and X_{nt}, and conditioning on them would produce no effects in expected returns.

4. The Cross Section of Expected Stock Returns

In this section, we show that with reasonable parameters, random noise realization can produce significant cross-sectional variations in expected returns.

It is well documented that when all stocks are used to construct 10 portfolios based on market caps or price-to-dividend ratios, portfolios with a higher market cap or price-to-dividend ratio have lower expected returns. Many models have been proposed to explain these observed regularities, the size and value effects.

In this paper, we show that these empirical patterns arise naturally with noise-in-price models. In our model, because the expected return on intrinsic value is the same for all stocks, portfolios formed based on intrinsic values or intrinsic value ratios have the same expected return. Thus there is no size or value effect for intrinsic value returns.

However, we show that with noise in prices, portfolios with higher prices or price-to-dividend ratios will have lower expected returns. This is because stocks with higher price or price-to-dividend ratios are more likely to have higher price noise leading to a lower expected return. The cross-sectional size and value effects are driven by random shocks of different stocks.

More specifically, in our model, because all stocks have the same distribution, the cross-sectional draw of different stocks is the same as different draws from the distribution of a single stock. Thus, the expected return of a portfolio formed from a price or price-to-dividend ratio decile is the expected return conditional on the decile in the space of price or price-to-dividend ratio. We compute the conditional expected return in closed form.

Quantitatively, in our model, the size effect decreases with time in the sense that the size premium decreases to zero over time. However, our model can produce the value effect with three plausible values on three parameters—namely, an average stock return of 10%, a volatility of log-price-to-dividend ratio of 40%, and a noise volatility of 10%. The value premium in our model is approximately 9%.

4.1. The Size Effect

To compute the size effect, we partition the p_t space into 10 deciles. We will drop the subscript n because we are now focusing on the distribution of a single stock. Note that

$$p_t = v_t - \tfrac{1}{2}\sigma_\Delta^2 + \sigma_\Delta \epsilon_t^\Delta = \mu_v t - \tfrac{1}{2}\sigma_\Delta^2 + \sigma_v \sum_{k=1}^{t} \epsilon_k^v + \sigma_\Delta \epsilon_t^\Delta;$$

thus, p_t is a normal random variable with mean $\bar{p}_t = \mu_v t - \frac{1}{2}\sigma_\Delta^2$ and variance $\sigma_{pt}^2 = \sigma_v^2 t + \sigma_\Delta^2$.

Let $F(x)$ denote the cumulative probability distribution function of a standard normal random variable. Define δ_i by

$$F(\delta_i) = \frac{i}{10}, \quad i = 1, \ldots, 9.$$

The points

$$p_{t,i} = \bar{p}_t + \sigma_{pt}\delta_i$$

divide the p_t space into 10 intervals with equal probabilities of 10%.

The average return of the portfolio in decile i is given by

$$\mathrm{E}\left[\frac{P_{t+1}+D_{t+1}}{P_t}\,\middle|\, p_t \in \mathrm{decile}_i\right]$$

$$= \frac{\int_{p_{t,i-1}}^{p_{t,i}} \mathrm{E}[(P_{t+1}+D_{t+1})/P_t \mid p_t]\, dF((p_t-\bar{p}_t)/\sigma_{pt})}{\int_{p_{t,i-1}}^{p_{t,i}} dF((p_t-\bar{p}_t)/\sigma_{pt})}$$
$$= \frac{\bar{R}_p}{\mathrm{E}[e^{-(\sigma_\Delta^2/\sigma_{pt}^2)p_{nt}}]} \int_{p_{t,i-1}}^{p_{t,i}} e^{-(\sigma_\Delta^2/\sigma_{pt}^2)p_{nt}}\, dF\left(\frac{p_t-\bar{p}_t}{\sigma_{pt}}\right)$$
$$= \bar{R}_p \frac{F(\delta_i+\sigma_\Delta^2/\sigma_{pt}) - F(\delta_{i-1}+\sigma_\Delta^2/\sigma_{pt})}{F(\delta_1)},$$

where $\delta_0 = -\infty$, $p_{t,0} = -\infty$, $\delta_{10} = +\infty$, and $p_{t,10} = +\infty$. Note that

$$\int_{\delta_i}^{\delta_{i+1}} dF(x) = 0.1.$$

Proposition 4 (Size Effect). *Suppose that Assumptions 1 and 2 hold. Furthermore, the number of stocks tends to infinity. Then, the expected return conditional on decile i is given by*

$$\mathrm{E}\left[\frac{P_{nt+1}+D_{nt+1}}{P_{nt}} \,\middle|\, P_{nt} \in \mathrm{decile}_i\right] = \bar{R}_p \frac{F(\delta_i+\sigma_\Delta^2/\sigma_{pt}) - F(\delta_{i-1}+\sigma_\Delta^2/\sigma_{pt})}{F(\delta_1)}.$$

Note that, for each decile, when $t \to \infty$,

$$\mathrm{E}\left[\frac{P_{nt+1}+D_{nt+1}}{P_{nt}} \,\middle|\, P_{nt} \in \mathrm{decile}_i\right] \to \bar{R}_p \frac{F(\delta_i) - F(\delta_{i-1})}{F(\delta_1)} = \bar{R}_p.$$

Thus, when t is large, the size effect disappears. The intuition is due to the following logic. In the equation

$$p_t - \bar{p}_t = \sigma_v \sum_{k=1}^{t} \sigma_k^v + \sigma_\Delta \epsilon_t^\Delta,$$

the intrinsic value term $\sum_{k=1}^{t} \sigma_k^v$ has a variance of $\sigma_v^2 t$, which complete dominates the noise variance σ_Δ^2 in the large t case. Thus, the inference of noise from intrinsic value becomes less and less significant as t becomes large.

Proposition 5 (Size Premium). *Suppose that Assumptions 1 and 2 hold. Then, the size premium is given by*

$$\bar{R}_p \frac{F(\delta_1+\sigma_\Delta^2/\sigma_{pt}) - F(\delta_1-\sigma_\Delta^2/\sigma_{pt})}{F(\delta_1)} \approx \bar{R}_p \frac{2f(\delta_1)}{F(\delta_1)} \frac{\sigma_\Delta^2}{\sigma_{pt}},$$

where $f(x)$ is the probability density function of the standard normal random variable.

In our model, not surprisingly, the size premium decreases with time t. The size premium for $t = 10$ is roughly a third of the size premium for $t = 1$. Our model will not generate a size effect because, as shown, the size premium implied by our model decreases to zero with time. Therefore, in the rest of the paper, we will focus on the value effect.

4.2. The Value Effect

As we previously did with the p_t space, we partition the x_t space into 10 deciles. Once again, we will drop the subscript n because we are now focusing on the distribution of a single stock. Note that x_t is normal with mean $\bar{x}$ and variance $\sigma_{pd}^2 = \sigma_{vd}^2 + \sigma_\Delta^2$. Let $x_{t,i} = \bar{x} + \sigma_{pd}\delta_i$. These points divide the x_t space into 10 intervals with equal probabilities of 10%. Going through almost exactly the same steps as above, we have the following proposition.

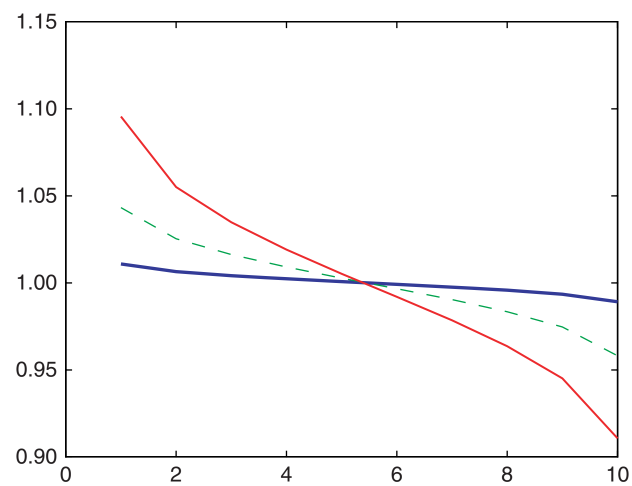
Proposition 6 (Value Effect). *Suppose that Assumptions 1 and 2 hold. Furthermore, the number of stocks tends to infinity. Then, the expected return conditional on decile i of the (p_t, x_t) space is given by*

$$\mathrm{E}\left[\frac{P_{t+1}+D_{t+1}}{P_t} \,\middle|\, x_t \in \mathrm{decile}_i\right] = \bar{R}_p \frac{F(\delta_i+\sigma_\Delta^2/\sigma_{pd}) - F(\delta_{i-1}+\sigma_\Delta^2/\sigma_{pd})}{F(\delta_1)}.$$

In contrast to the size effect we studied earlier, the average return for each value decile does not change with time. Recall that $\bar{R}_p$ is the unconditional price return. The decile variation is in the second factor. However, when $\sigma_\Delta \to 0$ or $\sigma_{vd} \to \infty$, the decile variation becomes zero, and all deciles have the same expected return of $\bar{R}_p$. The expected decile return decreases with decile monotonically.

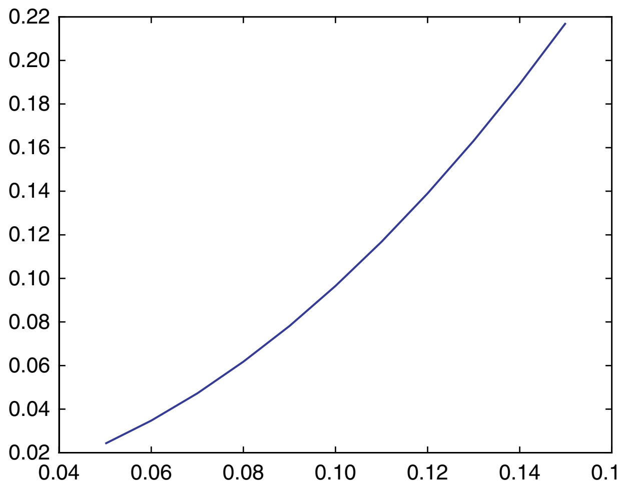
In Figure 1, we plot expected return as a function of price-to-dividend ratio decile for three parameters of noise volatility σ_Δ while keeping constant the log-price-to-dividend ratio volatility σ_{pd}, which can be directly estimated from market data. The curve is monotonically decreasing and is steeper for higher σ_Δ. The intuition

Figure 1 **Value (D/P) Decile Portfolio Returns for Three Parameters of Noise Volatility**



Notes. The x axis is the value (D/P) decile. The y axis is the expected portfolio return. The blue line corresponds to 5% noise volatility; the green, 10%; and the red, 15%.

Figure 2 Value Spread as a Function of Noise Volatility



Notes. The x axis is the noise volatility. The y axis is the value spread. At a noise volatility of 8%, the mode generates a value spread of 6%, which matches well to the empirical value spread.

is simple. The expected stock return computed for each of the 10 deciles is the expected return conditional on belonging to a price-to-dividend ratio decile. Belonging to a low price-to-dividend ratio decile i signals for a high likelihood of negative price noise and thus being undervalued.

Proposition 7 (Value Premium). *Suppose that Assumption 1 holds. Then, the value premium is given by*

$$\bar{R}_p \frac{F(\delta_1 + \sigma_\Delta^2/\sigma_{pd}) - F(\delta_1 - \sigma_\Delta^2/\sigma_{pd})}{F(\delta_1)} \approx \bar{R}_p \frac{2f(\delta_1)}{F(\delta_1)} \frac{\sigma_\Delta^2}{\sigma_{pd}}.$$

The value premium is a decreasing function of σ_{vd} and an increasing function of σ_Δ, as expected. Figure 2 plots the value premium as a function of noise volatility. The value premium goes to zero as noise volatility goes to zero, as expected.

We now use U.S. equity market data to calibrate the value effect. The parameters are summarized in Table 1. We take the gross expected price return $\bar{R}_p$ to be 110%; thus $\bar{R}_p = 1.1$. The value premium does not separately depend on parameters μ_v, $\bar{x}_v$, and σ_v. We set the volatility σ_{vd} of the log-price-to-dividend ratio to be 40%; $\sigma_{vd} = 40\%$. This is roughly what is reported in the literature.

We set $\sigma_\Delta = 10\%$. This gives a ratio of about 10% between the variance of the noise and the total variance of the stock return. French and Roll (1986, p. 6) suggest that "between 4% and 12% of the daily return variance is caused by mispricing." Fama and French (1988) estimate that the predictable variation in stock returns as a result of mean reversion is about 35% of the long-horizon variances, and they suggest, similar to Summers (1986), that mean reversion may be a result of market inefficiency. In his calibration exercises, Summers uses a value for return variance of the same order of magnitude as noise variance, σ_Δ^2. When $\sigma_\Delta = 10\%$, the value premium is

$$1.1 \times 3.4 \times 0.1^2 / 0.4 = 9.35\%.$$

As alternatives, $\sigma_\Delta = 5\%$ would lead to a value premium of 2.34%, whereas $\sigma_\Delta = 15\%$ would lead to a value premium of 21.04%.

As we pointed out earlier, we choose the price-to-dividend ratio mainly to avoid extra parameters. We would expect little or no difference if the price-to-book or price-to-earnings ratios were used instead.

In this paper, we have assumed that the distributions of all stocks are identical in order to focus on the effect of noise. In reality, different stocks have different distributions. In this case, it is obvious that, qualitatively, there still should be size and value effects. Quantitatively, we expect a higher value premium. It is reasonable to expect that small stocks have a higher noise variance. The expected return of a price-to-dividend decile portfolio is a simple average of the expected return conditional on the price-to-dividend ratio decile. Consequently, small stocks make a biased contribution to the simple average, which leads to a larger value premium.

In this section, we have shown that, qualitatively, a noise-in-price model produces the size effect and the value effect. A plausible calibration to U.S. market data shows that the model can produce the value effect of the observed magnitude.

Table 1 Summary of Parameters

Parameter	Value (%)	Description
$\bar{R}_p$	110	Expected price return
σ_Δ	10	Volatility of noise
σ_{pd}	40	Volatility of log-price-to-dividend

5. Related Literature

The size and value effects have spurred spirited debates since Banz (1981) and Reinganum (1981) documented that small-cap stocks tend to outperform on a risk-adjusted basis, and Stattman (1980) and Rosenberg et al. (1985) documented that high book-to-market ratio stocks also outperform. Similarly, other ratios such as earnings-to-price (documented by Basu 1977) and dividend yield (documented by Blume 1980, Rozeff 1984, Shiller 1984, and Keim 1985) also predict future performance.

Berk (1995, 1997) points out that cross-sectional dispersion in expected returns leads to negative cross-sectional correlation between price and expected returns in most reasonable models, whether rational or behavioral. Thus, qualitatively predicting size and value effects is not a distinguishing model feature. The hard work, then, lies in identifying the mechanism for the size and value effects, matching the magnitudes to the observed levels and variations in the cross section of stock returns with reasonable parameters and generating additional intuitions and testable implications.

Fama and French (1992) show that size and value, together with market beta, capture much of the cross-sectional variation in stock returns and subsume the explanatory powers of other financial variables. They propose that the size and value premiums are compensation for risk. Gomes et al. (2003) and Zhang (2005) explore the value effects through irreversible investments. Bansal and Yaron (2004) argue that long-run risk can be used to explain cross-sectional patterns of stock return. Yogo (2006) proposes that the size and value effects can be explained by investor preferences that are nonseparable in nondurable and durable consumption.

Fama and French (2007) study the pattern of decile migration amongst small-cap and large-cap stocks and value and growth stocks, and they demonstrate that this migration is an important contributor to the size and value premiums. Chen and Zhao (2009) reach similar conclusions. Our model predicts that mean reversion in noise leads to decile migration and size and value premiums. It remains to compute the premium associated with various migration patterns in our model and compare them with the Fama–French findings.

Blume and Stambaugh (1983) point out that the unconditional expected return of a stock in the presence of price noise increases with the noise volatility because of Jensen's inequality effect. Small-cap stocks probably have a higher noise volatility, which leads to a higher expected return. Even though the Blume and Stambaugh study is motivated by bid-ask random bounce, their results are applicable to general noise-in-price models.

However, to generate the 10 decile portfolio returns in the Blume and Stambaugh (1983) model, one needs to specify noise volatilities exogenously for 10 decile portfolios, which means that we need to specify at least 9 more parameters. Finally, these noise volatilities imply an unreasonable amount of price noise. In our paper, the mechanism of Blume and Stambaugh is switched off by our assumption that all stocks have the same noise volatility, and thus the same unconditional expected returns. Indeed, if we introduce a cross-sectional variation in noise volatility, then an even smaller noise volatility can generate the observed value premium.

Additionally, we note that in term structure models, where the number of shocks is smaller than the number of independent securities, the general assumption is that the market prices for bonds are different from their fair values by a noise factor.

Although the noise-in-price framework we use is simple and stylized, it is not narrow, nor are the model implications obvious. The framework is, in fact, surprisingly rich in its applications. Specifically, Blume and Stambaugh (1983) use a noise-in-price model to study the effect of bid-ask bounce on expected returns. Campbell and Kyle (1993) use the framework to explain the high volatility and predictability of U.S. stock returns. Hughes et al. (2007) use the framework to explain why less transparent firms would have a higher cost of capital after controlling for risk. Arnott (2005a, b) suggests that the model formalized in this paper may be the key to understand the value and size effects. Hsu (2006) uses the framework to argue that a mispricing premium may exist because there are investors with liquidity needs. Hsu (2006) and Arnott and Hsu (2008) use the noise model to demonstrate that a diversified capitalization-weighted portfolio is suboptimal to any diversified nonprice-weighted portfolio. Brennan and Wang (2007), using a similar framework, study the effect of mispricing on unconditional expected returns for a larger class of pricing models.

The behavioral finance literature finds that size and value effects can also be generated from investor overreaction or underreaction, as suggested by Shiller (1981), De Bondt and Thaler (1985, 1987), Lakonishok et al. (1994), Barberis et al. (1998), and Daniel et al. (1998), among others.

In our model, abnormal returns can be earned by exploiting size and value as signals for noise. They are arguably two sides of the same coin. Summers (1986, p. 592) argues that "data in conjunction with current [econometric] methods provide no evidence against the view that financial market prices deviate widely and frequently from rational valuations." Fama and French (1988) and Poterba and Summers (1988) study mean reversion in prices and posit that one possible explanation is mean-reverting price noise. Black (1986) argues that noise should always be present because investors are risk averse and are not sure whether a free lunch is truly a free lunch. According to Black (1986, p. 534), "noise creates the opportunity to trade profitably, but at the same time makes it difficult to trade profitably." If Black is right, the size and value effects are likely to continue to persist.

6. Conclusion

We use a classic noise-in-price model to produce a new insight into the role of price noise as a source of cross-sectional variations in expected returns. Even with no variation in unconditional expected returns (i.e., effectively with the mechanism studied by Blume and Stambaugh 1983 switched off), small-cap and value stocks have higher expected returns because they are more likely to be undervalued as a result of negative price shocks.

With only one parameter that is not measured directly from the data (noise volatility, which is assumed to be 10%), we calculate expected returns conditional on value deciles, which quantitatively match the pattern documented as the value effect. Our results suggest that a modest amount of noise can explain the value premium. Qualitatively, there is a size effect in the noise-in-price model, too. However, the size premium declines to zero over time.

In this paper, we assumed that the ex ante distributions for all stocks are identical and used noise realization to generate the size and value premiums in the cross section. We deliberately made this unrealistic assumption to demonstrate that the random realization of noise alone can generate sufficient cross-sectional variation to match the data. Introducing differences in ex ante distributions would introduce more variations in the cross section of expected returns and would allow us to match the empirical evidence with even less noise volatility.

We can extend our model in some directions without losing tractability. For example, we can allow the noise process to be an AR(1) process instead of i.i.d. Empirical studies document that the size and value premiums in economic booms are different from those in recessions. In our paper, we can introduce a dependence of the conditional variance of noise on macroeconomic state variables. This condition can then generate a business cycle pattern in the size and value effects.

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