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Supply Auctions and Relational Contracts for Procurement

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We examine the competition between procurement auctions and long-term relational contracts that emerges from the increased usage of electronic marketplaces. Procurement auctions create supply chain efficiencies by selecting the least costly bidder, and long-term relational contracts ensure the quality of the procured products or services when these have nonverifiable attributes. The following two-layer supply chain model is analyzed: Suppliers of an industrial part with nonverifiable quality trade with several manufacturers that utilize that part in the production of an end product. A price-based reverse auction is used for the procurement of low-quality parts, and a relational long-term contract is used for high-quality parts. A formal model of the competition between the two procurement models identifies conditions under which the two coexist, and conditions under which one will push the other out of the market. Market and product parameters that increase the relative economic appeal of the relational contract are determined. It is demonstrated that the competition from the auction market can either facilitate or undermine the relational contract compared with a benchmark case of a monopolist supplier. This implies that competition between the two procurement models plays an important role in the supply of high-quality products.

Key words: supply chain management; procurement auctions; relational contracting; quality differentiation
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1. Introduction

Electronic marketplaces provide an attractive procurement venue in many industries. They reduce search and negotiation costs for all parties by utilizing new communication technologies that support low-cost connectivity on both the supply side and demand side. Furthermore, the competitive market mechanisms, such as auctions, employed in these marketplaces lead to the selection of the most efficient suppliers as the providers in the market. This creates efficiencies in the supply chain that can be shared by both buyers and sellers. Indeed, as of 2003, one-third of large companies used Web-based marketplaces for some form of procurement (Bartels et al. 2003).

However, business-to-business transactions are, in general, complex and have many different dimensions that influence their nature. These include coordination costs and asset specificity (Williamson 1981a, b); time specificity and complexity of product description (Malone et al. 1987); as well as adaptation and contract costs (Levi et al. 2003). Furthermore, product and transactional complexity inhibit contractual verifiability of attributes, such as quality; performance and reliability of the parts; materials; and services (Baker et al. 2002). Transactions involving such issues are typically carried out via informal relational agreements (the relational contracts) with long-term trading partners. In fact, most of the procurement is dominated by such relationships. For example, manufacturers rely on getting high-quality products and services from a set of preferred suppliers who may in turn rely on the word of their own suppliers for the quality of the products they are purchasing. In this context, a manufacturer trusts a long-term supplier to provide her with high-quality products or services. If the product or service provided is not of the expected quality, the relationship can be damaged and the manufacturer may refuse to transact with that supplier again. This implicit threat may be sufficient to enforce the quality promise in equilibrium; this threat also provides the foundation for long-term business relationships that sustain the provision of complex nonverifiable attributes in products or services (cf. Williamson 1985, Milgrom and Roberts 1992).

On the other hand, in price electronic markets where the trading relationship between the transacting parties is usually ephemeral and based on price only, it is harder to support the delivery of products and services with complex attributes. Such markets are used primarily for cost savings rather than procurement for quality (Grey et al. 2005), and any cost efficiencies they achieve are at the expense of efficiencies that could be gained from the delivery of products or services with complex attributes that would be valued more in the marketplace. Therefore, one would expect electronic markets to prevail in procurement markets where nonverifiable product attributes are rare (such as common chemicals), whereas relational contracts would be more common in the markets for more complex goods with nonverifiable value-added components (such as business or consumer electronics). Yet, in other markets that show intermediate characteristics, one might expect the two forms of procurement to coexist. Indeed these expectations are empirically supported (Bajari et al. 2002). Industry reports also provide examples of companies, such as Best Buy, that utilize auctions for the procurement of standard operational supplies and materials, but rely on long-term contracts and relationships for the purchase of goods whose quality is critical to the company's competitive differentiation (Buss 2002). These observations motivate a formal study of the competition between auction-based procurement and long-term relational contracting.

Hewlett-Packard's (HP's) printer division provides a case example that motivated this study.[1] In competing for business with its low-cost competitors, HP has two alternative procurement options: (i) procurement of high-quality parts from suppliers that have earned HP's trust through a long history of transactions; or (ii) procurement of lower-quality parts from the open market. HP rarely purchases parts from the open market and instead relies on its network of trusted vendors. By this, HP ensures that certain technology specifications are satisfied, and unverifiable (or hard-to-verify) product qualities such as performance and reliability are provided by known suppliers. These suppliers will monitor and meet HP's quality requirements even if these are not written explicitly in contracts because meeting these requirements will lead to continued business. HP may also use warranties in the purchases from its select suppliers to ensure quality, but it does not view such warranties as a major tool to maintain quality. Enforcing warranties that cover all contingencies is costly. Furthermore, even if HP is compensated for a failure, the tarnishing of HP's reputation in the marketplace is undesirable. Therefore, relationship-based procurement provides HP with the most effective way to ensure quality. On the other hand, many of HP's lower-cost competitors utilize the open market, switch suppliers more frequently, and choose their vendors primarily based on price. They may also use one or more of the online markets available for computer hardware procurement.[2] It is natural to expect that centralization of low-cost procurement through electronic marketplaces creates cost pressures that can affect the existence and efficiency of relationship-based procurement (see Grey et al. 2005). HP's long-term relationship with Canon exemplifies some of these issues. Specifically, HP relies exclusively on Canon for the procurement of LaserJet engines for its printers, because Canon owns the patents for the underlying technology. HP can obtain lower-performance engines from other suppliers, but these engines do not meet HP's performance standards. Therefore, HP accepts the role of the downstream party who has to rely on a long-term trust-based relationship with Canon, conceding market power for performance. Furthermore, the relationship is sustained not through formal enforcements such as inspections and warranties, but through the promise of future business. In light of this example, our paper attempts to explain why manufacturers such as HP are willing to concede market power in dealing with a single high-quality supplier and why their reliance on the promise of future business can be sufficient to support quality, as opposed to more formal (and costly) mechanisms such as inspections and legal enforcement of warranties.

[1] We thank Thomas Olavson of Hewlett-Packard for providing us with the information for this example.

[2] For example, www.dramexchange.com or www.converge.com.

We develop a formal model to analyze the interplay between relational contracting and auctions in a repeated trading environment. Our model considers a two-layer supply chain where multiple suppliers of a part transact with a set of manufacturers that utilize that part in the production of an end product. This supply chain accommodates two distinct procurement models: An auction-based electronic market is used for the procurement of low-quality parts, and long-term relational contracts are used for the procurement of high-quality parts (throughout the paper we will be using the terms auction and electronic market interchangeably). The auction mechanism used in the market is equivalent to a second-price reverse auction in which the winner is the least-costly supplier. The manufacturers participating in the auction capture a surplus that is equal to the highest surplus possible in the low-quality portion of the supply chain under the second-lowest supplier cost.[3] Part quality is nonverifiable and the quality of the end product depends on the quality of the input parts. The low-quality and high-quality products are substitutable for consumers, with high-quality products commanding a price premium in the consumer market. Although all suppliers can provide parts of low quality, there is only a single supplier that can provide parts of high quality at an extra cost (if she chooses to do so). That is, the high-quality supplier is the only one with the know-how for the high-quality parts, yet she needs to spend a nonverifiable (or not easily verifiable) costly effort to produce it. This is also reflective of the HP-Canon example: A single supplier has the exclusive rights or a patent to the production technology for high-quality parts, and verification of the quality of the parts is expensive, hence the manufacturer relies on relationships as opposed to inspections for maintaining performance. In each period, the manufacturers specify the number of parts they will procure from their suppliers, and the consumer market clears with the end product that the manufacturers produce. Parts and the end products are assumed to be perishable (or to depreciate fast) and no inventory is kept. Although auctions are not the only mechanism used in electronic markets, our goal in this paper is to capture the effect that the competitive environment of the auctions has on relational contracts. Therefore, we focus on auctions as opposed to other mechanisms used in practice (e.g., requests for quotes or electronic tenders).

This setting puts the auction and relational contracting in head-to-head competition. We solve for the resulting equilibria and endogenously determine conditions under which each procurement mechanism will prevail and push the other out of the market, as well as conditions under which they coexist. Furthermore, we isolate the effect of the electronic market by comparing the equilibrium outcome to a benchmark setting of a high-quality monopolist supplier who is the sole provider of the intermediate part.

By the common nature of repeated games (Friedman 1971, Fudenberg and Maskin 1986), the relational contract we analyze is sustainable when the parties transact frequently enough or, equivalently, when the high-quality supplier is sufficiently patient. In addition, we find that the relational contract and the auction coexist when the quality premium is modest, but that as the quality premium increases, the auction for the product under consideration becomes nonviable even though it can bring high-cost efficiencies. However, the competition from the auction may facilitate the existence of the relational contract. Compared with the benchmark case of a monopolist, a relational contract may be easier to sustain when the consumer market is large. On the other hand, when there is limited downstream fragmentation and when the quality premium is low, competition from the auction market might undermine the existence of the relational contract and reduce overall product quality in the consumer market.

There is a growing literature that focuses on the interaction between contracts and electronic markets for procurement. Lee and Whang (2002) examine a secondary market for excess inventory with a large number of buyers to demonstrate that a secondary market increases allocative efficiency but does not necessarily increase a monopolistic supplier's profits. Mendelson and Tunca (2005, 2006) examine the effect of asymmetric information and liquidity in industrial spot markets in an environment where long-term

[3] This is a standard auction mechanism frequently used both in practice and theoretical analysis. Interested readers are referred to Klemperer (1999) for a comprehensive literature survey on auction theory.

contracts can precede spot trading. They find that liquidity is substantially influenced by the information structure in the supply chain and that complementing spot trading with long-term contracting can benefit the supply chain as well as the consumers. Peleg et al. (2002) consider a multiperiod setting with both long-term and spot purchases where unmet demand is carried to the next period, and they identify the regions in the parameter space where each procurement model will be optimal. Wu et al. (2002) show that capacity option contracts and spot markets can complement each other and achieve efficiency. In fact, even though our analysis focuses on the interaction between auctions and relational contracts, it can be equivalently viewed as a comparison of spot markets with relational contracts where the auction serves the purpose of the spot market clearing mechanism. A literature survey that covers more papers on the interaction of advance contracts and exchanges can be found in Kleindorfer and Wu (2003).

One branch of the literature that is more closely related to our paper explicitly recognizes the differences between purchases from relation-based contracts and spot-market purchases. Grey et al. (2005) provide a discussion of the importance of relation-based contracts when considering the viability of online exchanges for procurement. Cohen and Agrarwal (1999) build a multiperiod model that considers the trade-off between spot-price risk and costs of relationship-specific investments, deriving conditions under which spot markets would be preferable to long-term contracting. Levi et al. (2003) focus on the other side of the cost difference between long-term and spot purchases by looking at the additional costs that parties have to bear if they transact on the spot market on top of long-term contracts, finding that higher cost differences lead to higher relationship-based investments. Building on Levin (2002), Taylor and Plambeck (2003) examine how relational contracts can be used to sustain optimal upstream capacity investment in a supply chain.

There is a long tradition of papers in economic theory that examine relational contracts and their impact on supply transactions. Bull (1987) provides the conceptualization of relational contracts in an intrafirm setting. Baker et al. (1994) examine the interplay between explicit contracts based on verifiable characteristics and implicit relational contracts. Taylor and Wiggins (1997) argue that relational contracts provide the basis for the Japanese subcontracting system and contrast them to the competitive bidding model used by American manufacturers. Baker et al. (2001) investigate the interplay between relational contracting and spot contracting to revisit the boundaries of the firm. Our work follows the paradigm of these papers: It compares two distinct contracting mechanisms and determines endogenously conditions under which each mechanism prevails, and conditions under which they coexist. However, in contrast to these papers, our work examines the competition that arises between the two procurement mechanisms and how the mechanisms affect each other.

The rest of the paper is organized as follows: Section 2 describes the model and discusses the assumptions. Section 3 presents three cornerstone models that underlie the analysis: a monopoly model with nonverifiable quality, a duopoly model with verifiable quality, and an oligopoly model with verifiable quality and auction-based procurement. Section 4 uses the cornerstone models to build the main model and to examine the sustainability of a relational contracting equilibrium. It explores the factors that affect this equilibrium, as well as factors that determine the interplay between the auction market and the relational contract. Section 5 discusses our key assumptions and offers concluding remarks. All proofs are in the appendix.

2. The Model

Consider $M+1$ suppliers (referred to as "she") and N manufacturers (referred to as "he") that trade repeatedly in discrete periods over an infinite horizon. Suppliers are indexed $\{0, 1, \ldots, M\}$. Supplier 0 (the high-quality supplier) can decide to produce parts of high or low quality in the beginning of each period. The remaining suppliers (the low-quality suppliers) can only produce parts of low quality. The quality of the end product produced by the manufacturers depends on the quality of the part used in its manufacture: High-quality parts yield to high-quality products, and low-quality parts yield to low-quality products.

The end product is sold in the consumer market. There is a continuum of K consumers indexed by their reservation values, r, and in each period each consumer purchases either one or no units of the product. If the quality of the product is low (high), the consumer will purchase it if the product's market clearing price p_l (p_h) does not exceed the consumer's reservation value r ($r+v$), where $v>0$ is the quality premium consumers are willing to pay for a high-quality product.[4] Reservation values are uniform and i.i.d. on $[0, K]$. Therefore, the mass of consumers with a reservation value below any given $r \in [0, K]$ is r. In the remainder, K will be referred to as the market size, keeping in mind that it also denotes the maximum reservation value K. This consumer demand model implies a linear demand curve: When only low-quality products are available, then the market clearing price, p_l, and demand, Q, satisfy the linear demand curve $Q = K - p_l$, for $0 \leq p_l \leq K$. Similarly, if only the high-quality product is available in the market and the market clearing price is p_h, the demand curve will be $Q = K + v - p_h$, for $v \leq p_h \leq K + v$. When both products are in the market, their equilibrium market clearing prices will satisfy the indifference relation: $p_h = p_l + v$. Otherwise, all purchasing consumers will either choose the low-quality product (if $p_h > p_l + v$) or the high-quality product (if $p_h < p_l + v$). This result is established rigorously in the proof of Proposition 2 in the appendix.

The quality of the parts is not verifiable; it is observed by the manufacturer only after he takes possession (during or after his production) and the manufacturer cannot return products of inferior quality to the suppliers, nor can he get a refund from the supplier. Future payoffs are discounted and the one-period discount factor for the high-quality supplier is α, where $0 < \alpha < 1$. The discount factor for the low-quality suppliers and for the manufacturers will not be relevant in our analysis. The discount factor α also reflects the market-clearing frequency, that is, how frequently transactions occur in this marketplace. In high-turnover marketplaces the discount factor will be close to one, whereas in low-turnover situations the discount factor will be lower. It can also be used to model products with random product life cycle where the probability that the product becomes obsolete is $(1-\alpha)$ in each period. For future reference, define $\beta \triangleq \alpha/(1-\alpha)$. Notice that β is increasing in α and onto $(0, \infty)$.

The analysis will focus on the equilibrium structure of this supply chain under two procurement models: Procurement in an electronic market in which the manufacturers purchase the desired number of parts through a mechanism equivalent to a second-price reverse auction, and relational procurement in which the high-quality supplier and a group of high-quality manufacturers engage in a relational contract where the supplier promises to deliver high-quality parts to the manufacturers in that group in each period. The high-quality supplier will be assumed to be the Stackelberg leader in this setting: She will move first to offer the contractual terms to the manufacturers. Furthermore, the manufacturers' reservation utility is assumed to be zero for simplicity.

In the beginning of each period t, all suppliers $j = 0, \ldots, M$ privately observe their per-unit production cost $c_j(t)$ for that period. The production costs are i.i.d. across time and suppliers, with mean μ, variance σ^2, cumulative distribution function $F(y)$, probability density function $f(y)$, and support $S \subset [\underline{c}, \infty)$, where, without loss of generality, $\underline{c} = \inf\{y \mid f(y) > 0\}$ and $0 < \underline{c} < K$. Let $\bar{c} = \sup\{c \mid c \in S\}$. When S is bounded, $\bar{c}$ will be finite. The high-quality supplier then decides whether to produce high-quality or low-quality parts and the form of procurement she will use in that period. If she decides to produce high-quality parts, she makes a two-part tariff offer, consisting of a wholesale price and a fixed fee, to a group of high-quality manufacturers. She also makes a promise that the delivered parts will be of high quality. The two-part tariff offer form is well known to coordinate the supply chain and commonly used in practice and to model vertical contractual agreements (see, e.g., Schmalensee 1981). Upon receipt of this offer, each manufacturer determines the size of his order. The total production cost for the high-quality parts is $c_0(t) + \delta$, where the cost of quality δ satisfies $0 \leq \delta < v$. The manufacturers receive their orders, detect the quality of the parts, and complete production. Simultaneously, all remaining manufacturers procure low-quality parts from the auction

[4] In reality, this premium can be influenced by other factors such as brand, word-of-mouth effects, prior experience with the product, and so on.

market and subsequently complete their own production. Next, all manufacturers sell their products in the consumer market, which then clears. If the manufacturers who expected high-quality parts detect low quality instead, they can decide to punish the supplier by not believing in any high-quality promises in the future. Equilibria based on this trigger style strategies are standard in the repeated games literature (Friedman 1971).

For brevity's sake and to streamline the analysis, it is convenient to define several quantities relating to the order statistics for the supplier costs. Consider a sample of k cost realizations from the cumulative density function $F(y)$, and let $c^k_{(i)}$ be the ith order statistic. Let $F^k_{(i)}(y)$ denote the cumulative density function for the ith order statistic, $f^k_{(i)}(y)$ denote the probability density function, $\mu^k_{(i)}$ denote its mean, and $\sigma^k_{(i)}$ denote its standard deviation. Define $G(x) = E[(x-c)^+]$, $G^k_{(i)}(x) = E[(x-c^k_{(i)})^+]$, and $H(x) = E[((x-c)^+)^2]$ and $H^k_{(i)}(x) = E[((x-c^k_{(i)})^+)^2]$. Note that $F(x)$, $G(x)$, and $H(x)$ trivially equal $F^1_{(1)}$, $G^1_{(1)}$, and $H^1_{(1)}$. The following lemma about the characteristics of these functions will be used frequently in our analysis and is provided here without proof.

Lemma 1. *For any k and i, the functions $G^k_{(i)}(x)$ and $H^k_{(i)}(x)$ are increasing and convex in x with $dG^k_{(i)}(x)/dx = F^k_{(i)}(x)$ and $dH^k_{(i)}(x)/dx = 2G^k_{(i)}(x)$. Furthermore,*

$$\lim_{x\to\infty}\{G^k_{(i)}(x) - (x-\mu^k_{(i)})\} = 0 \quad \text{and}$$
$$\lim_{x\to\infty}\{H^k_{(i)}(x) - ((x-\mu^k_{(i)})^2 + (\sigma^k_{(i)})^2)\} = 0. \tag{1}$$

If S is bounded, then $G^k_{(i)}(x) = x-\mu^k_{(i)}$ and $H^k_{(i)}(x) = (x-\mu^k_{(i)})^2 + (\sigma^k_{(i)})^2$ for $x \geq \bar{c}$.

For technical reasons it is convenient to assume that $\underline{c} \geq \mu/3$ and $K + \underline{c} \geq v - \delta$. These conditions imply that the manufacturers cannot capture the entire consumer market even under the lowest possible realization of the production costs.

2.1.1. Overview of the Analysis. The analysis uses three cornerstone models described in §§3.1, 3.2, and 3.3 that serve as building blocks for the main model, which integrates them in §4.

The first model considers a single high-quality supplier and a single manufacturer. The supplier promises to deliver parts of high quality, and the manufacturer places orders according to that promise. If in a given period the promise is kept, then subsequent transactions are based on the expectation that the supplier will continue to honor the promise. Violation of the promise triggers a severe punishment: The manufacturer will not trust the supplier anymore, and he will assume low quality for all his subsequent orders. This arrangement gives rise to a relational contract if the high-quality promise is self-enforceable: The supplier's profit from cheating does not exceed her profit from honoring the quality promise in perpetuity. Therefore, using this repeated game formulation, conditions will be derived under which the relational contract equilibrium for high-quality procurement is sustainable. This analysis illustrates the relational contract equilibrium in an environment isolated from the auction market, and provides a useful benchmark.

The second model incrementally incorporates the complexity generated by the competition from the auction market to investigate the interplay between the relational contract and the auction market. Rather than jumping directly to a model that captures the full complexity, we start with a duopoly model with a single supplier of high-quality parts, a single supplier of low-quality parts, and verifiable product quality. This temporarily shifts the focus away from the relational contract and onto the consumer market equilibrium with product quality differentiation. Specifically, because in the economic environment described in this paper low-quality and high-quality products can be available in the market simultaneously, a model that characterizes the effect of product differentiation on both the consumer market equilibrium and the supply chain equilibrium provides a desirable stepping stone.

Building on the duopoly model, we next consider an oligopoly model (the third cornerstone model) where the single low-quality supplier is replaced by multiple low-quality suppliers who compete in an auction market. In the oligopoly setting, the auction collapses the multiple low-quality suppliers into the single winning supplier with the lowest cost. Hence, the emerging equilibrium parallels the equilibrium for the duopoly model but with the cost structure of the single low-quality supplier modified to reflect the cost of the winning supplier in the auction. This analysis provides a complete characterization of the market equilibrium with the auction market when product quality is verifiable.

The third model provides the last piece of the puzzle. Specifically relational contracting introduces three constraints into the oligopoly model that reflect the nonverifiability of product quality: (a) that the high-quality supplier will not gain by misleading the manufacturers; (b) that the expected profit for the manufacturers that procure from the high-quality supplier is at least equal to the profit each of them would make in the auction market; and (c) that the high-quality supplier will not gain by changing the number of manufacturers in her trading set. The first constraint is identical to that developed in the monopoly cornerstone model, but the other two reflect the expanded options available to all parties in the presence of competition from the auction market. This analysis provides conditions under which a relational contracting equilibrium can be sustained, conditions under which it can compete with the auction market, and conditions under which it can shut off the auction market and so become the only procurement model.

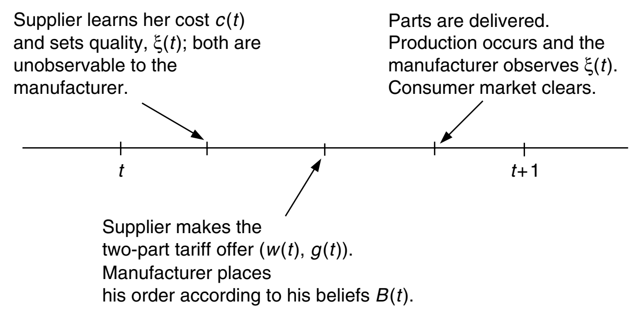
The model uses several assumptions to be discussed in depth in §5. These assumptions are listed here for the sake of completeness: (i) There is only one high-quality supplier. (ii) The high-quality supplier is the Stackelberg Leader. (iii) Product quality is nonverifiable. (iv) The high-quality supplier and all manufacturers trading with that supplier do not participate in the low-quality supply chain as long as they wish to continue trading with each other. (v) The analysis is focused on the trigger-strategy equilibria with infinite punishment.

3. The Cornerstone Models

3.1. Monopoly with Nonverifiable Product Quality

The analysis of the first cornerstone model aims to determine whether high-quality parts will be produced in equilibrium. The timeline is shown in Figure 1: In each period t the high-quality supplier learns her production cost $c(t)$ and sets the quality, $\xi(t) \in \{L, H\}$, of production for the period. Next, she makes a two-part tariff offer to the manufacturer

Figure 1 Timeline Within a Period



with wholesale price $w(t)$ and constant tariff $g(t)$. The manufacturer observes the offer, and given his beliefs about the supplier's behavior, decides the quantity he wants to order. The parts are delivered, the manufacturer produces for the consumer market, learns the quality of the products, and the consumer market clears.

The generic form of a two-party supply chain has been studied extensively without the issue of nonverifiability of product quality (see, e.g., Spengler 1950, Pasternack 1985). Because the manufacturer has zero reservation utility, the supplier sets the wholesale price for each period t, $w(t)$, equal to the marginal cost for that period, and uses the fixed tariff, $g(t)$, to extract all the surplus. For parts of low quality, the optimal production quantity in period t is $Q(t) = \frac{1}{2}(K - c(t))^+$, the two-part tariff is $g(t) = \frac{1}{4}((K - c(t))^+)^2$, $w = c(t)$, and the supplier's expected total discounted profit is $\frac{1}{4}(((K - c(t))^+)^2 + \beta H(K))$. Similarly, for high-quality parts the total production quantity will be $Q(t) = \frac{1}{2}(K + v - \delta - c(t))^+$, the two-part tariff is $g(t) = \frac{1}{4}((K + v - \delta - c)^+)^2$, $w(t) = c + \delta$, and the supplier's expected total discounted profit will be $\frac{1}{4}(((K + v - \delta - c(t))^+)^2 + \beta H(K + v - \delta))$.

Comparing the infinite horizon profit expressions of the two alternatives described in the paragraph above, one concludes that if product quality were verifiable, then the supplier's best strategy would be to deliver high-quality parts in perpetuity. However, because quality is nonverifiable, it is possible that the supplier may promise to deliver high-quality parts but then cheat and provide parts of low quality, instead. This generates a short-term increase in profit because the manufacturer will have naively assumed that the part will be of high quality and will have placed an order and paid accordingly. The quality promise will be self-enforceable if the short-term increase in profits from cheating does not exceed the long-term benefits from honoring the promise in perpetuity. To formalize this logic, we examine an equilibrium in which the manufacturer plays a trigger strategy driven by his beliefs and his ex post observations of quality ξ. Specifically, define the manufacturer's beliefs on ξ, $B = \{B(t) \mid B(t) \in \{L, H\},\ t \geq 0\}$ as follows:

$$B(t) = \begin{cases} L & \text{if } w(t) < \underline{c} + \delta \text{ or } \xi(\tau) = L \\ & \quad \text{for any } 0 \leq \tau < t \text{ and } t > 0, \\ H & \text{otherwise.} \end{cases} \qquad (2)$$

This states that when the manufacturer detects the violation of the promise, he absorbs the loss for that period but punishes the supplier by assuming that all subsequent deliveries from her will be of low quality. Furthermore, (2) also states that if the supplier's wholesale price is less than the minimum marginal production cost for high-quality parts, i.e., $w(t) < \underline{c} + \delta$, then the manufacturer's belief is that the part is low quality. The supplier should counter these beliefs with a quality-setting behavior $\xi(t) = B(t)$. The intuition is as follows: Once the manufacturer believes that the supplier will not produce high-quality parts and hence orders and pays as if she is producing low-quality parts, it becomes suboptimal for the supplier to set the quality to high and bear the higher costs. For the sake of brevity, henceforth, whenever we refer to the relational contract for this cornerstone model we will be referring to the (Perfect Bayesian) equilibrium supported by the beliefs and the strategies above.

The following proposition gives the characterization of a Perfect Bayesian Equilibrium of the game under this manufacturer belief system and supplier behavior.

Proposition 1.

(i) *A Perfect Bayesian Equilibrium of the game described above is characterized by the beliefs* (2), *supplier's quality strategy* $\xi(t) = B(t)$ *for all* $t \geq 0$ *and the outcome* $\xi(t) = B(t) = H$ *for each* $t \geq 1$ *if and only if*

$$\beta > \frac{2\delta(K + v - \delta - \underline{c})}{H(K + v - \delta) - H(K)} \triangleq \beta_0. \qquad (3)$$

(ii) *The critical threshold* β_0 *is decreasing in the market size* K. *It is also decreasing in the quality premium* v, *and increasing in the marginal cost of quality* δ.

(iii) $\lim_{K\to\infty} \beta_0 = \delta/(v - \delta)$.

Proposition 1 states that the relational contract is self-enforceable if (a) The discount factor α exceeds the critical value, i.e., the α value corresponding to β_0, $\alpha_0 \triangleq \beta_0/(1 + \beta_0)$ (by Part (i)). (b) The quality premium v is sufficiently high (by Parts (i) and (ii)). (c) The cost of quality δ is sufficiently small (by Parts (i) and (ii)). (d) The market size K is sufficiently large and the discount factor α is greater than δ/v (by Parts (i), (ii), and (iii)). In all four cases, the long-term benefits from fulfilling the quality promise in each period exceed the short-term gains from violating the promise. The impact of the quality cost on the supplier's incentives is worth highlighting: Even if the high-quality supplier can pass the full cost of quality to the manufacturer (and ultimately to the consumers), the cost itself may be sufficient to make the relational contract nonsustainable. Another important observation is that as the market size K increases, it becomes easier to enforce the relational contract. As K increases, both the relative appeal of high-quality parts compared with low quality and the one-time profit from cheating increase. Part (ii) of Proposition 1 implies that the former increases at a faster pace and hence it becomes easier to support the delivery of high-quality parts using self-enforceable contracts. However, as stated in Part (iii), a very large market guarantees the sustainability of a relational contract as long as the discount factor α is sufficiently high. For future reference, it is worth noting that the statistic β_0 summarizes the key economic and technical parameters that affect self-enforceability. Notice that because $v > \delta$, β_0 is always finite, but when δ is bounded away from zero, $\beta_0 \to \infty$ as $v \to \delta$ (also see §4.2).

3.2. Duopoly with Verifiable Product Quality

In the second cornerstone model, there is a single high-quality supplier (Supplier 0), a single low-quality supplier (Supplier 1), and verifiable part quality. The analysis generalizes to multiple low-quality suppliers but that case is not relevant for our purposes and it is omitted. The high-quality supplier provides high-quality parts and trades with n manufacturers, and the low-quality supplier trades with $N - n$ manufacturers. Both suppliers make offers that consist of a two-part tariff. The low-quality supplier's production cost at period t, denoted by $c_1(t)$, is from a distribution F_1 with support on S. The mean and variance of $c_1(t)$ are denoted by μ_1 and σ_1^2, respectively, and $G_1(c) = E[(c - c_1)^+]$. The high-quality supplier's production cost at period t, $c_0(t)$, is drawn from distribution F with support on S. The following proposition characterizes the market equilibrium under this oligopolistic competition.

Proposition 2.
(i) *If $K - \underline{c} - \frac{1}{2}G(K + v - \delta) \le 0$, then the low-quality supplier is out of the market, and the high-quality supplier produces in each period.*
(ii) *If $K - \underline{c} - \frac{1}{2}G(K + v - \delta) > 0$, then there exists a unique equilibrium in quantities produced and the strategies are defined as follows: Let c_l^* denote the unique solution to the equation*

$$c_l^* - K + \frac{1}{2}G\left(K + v - \delta - \frac{1}{2}G_1(c_l^*)\right) = 0, \qquad (4)$$

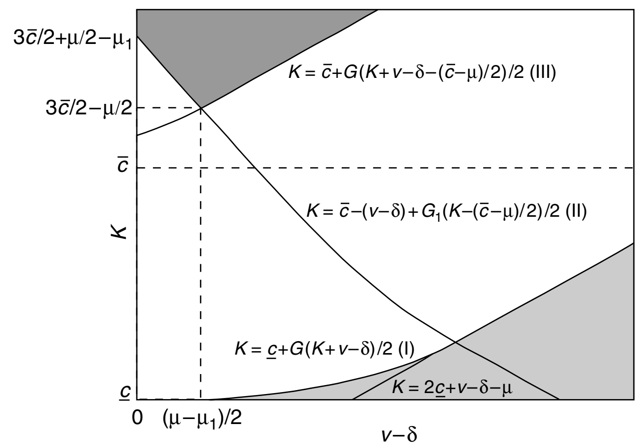
and let $c_h^* = K + v - \delta - \frac{1}{2}G_1(c_l^*)$. Then the high-quality supplier produces only when her marginal production cost, $c_0(t)$, is less than or equal to the critical value c_h^*, and the low-quality supplier produces when her marginal production cost $c_1(t)$ is similarly less than or equal to the critical value c_l^*.
(iii) *The high-quality supplier is never fully out of the market no matter how efficient the low-quality supplier is.*

Example (Bounded Costs). To illustrate this result, we now focus on the special case when S is bounded, i.e., when $\bar{c} < \infty$. The following corollary summarizes conditions under which the high- and low-quality suppliers produce in each period, and conditions under which they produce only when their costs are below a critical value.

Corollary 1. *Suppose $\bar{c} < \infty$. Then $\underline{c} \le c_l^* \le \bar{c}$ if and only if $\underline{c} + \frac{1}{2}G(K + v - \delta) \le K \le \frac{1}{2}G((K + v - \delta - \frac{1}{2}(\bar{c} - \mu)) + \bar{c}$ and $\underline{c} \le c_h^* \le \bar{c}$ if and only if $K \le \frac{1}{2}G_1(K - \frac{1}{2}(\bar{c} - \mu)) + \bar{c} - (v - \delta)$.*

Figure 2 illustrates Proposition 2 and Corollary 1 under the additional assumption that the low-quality supplier is more efficient than the high-quality one (i.e., $\mu_1 < \mu$). There are six regions depending on the relative magnitudes of K and $v - \delta$. These six

Figure 2 Borders for Different Regions of Participation by Low- and High-Quality Suppliers in the Market



regions correspond to different strategies for the two types of suppliers; they are defined by three distinct lines, denoted (I), (II), and (III) in the figure. Line (II) refers to the high-quality supplier. In the area above Line (II), this supplier produces in all periods because the quality premium is sufficiently high. Below that line, the high-quality supplier produces only in periods where her costs are sufficiently low. Lines (I) and (III) refer to the low-quality supplier. In the shaded area below Line (I), the low-quality supplier is out of the market because the quality premium is too large relative to the market size. Between Lines (I) and (III), the low-quality supplier produces only when her costs are sufficiently low. Above Line (III), this supplier always produces. In the shaded area above both Lines (II) and (III), both suppliers produce in each period.

3.3. Oligopoly with the Auction Market

In the third cornerstone model, product quality is still assumed verifiable and the number of manufacturers that transact with the high-quality supplier (Supplier 0) is n (≥ 1). There are now M (≥ 2) suppliers (Suppliers $1, \ldots, M$) of low-quality parts that sell their parts in the auction market, and $N - n$ manufacturers that procure these parts from the electronic market ($N \ge 2$). The unit production cost for the low-quality product for all low-quality suppliers and for the high-quality supplier are i.i.d. from a distribution with cumulative density function $F(y)$. All other assumptions and notation are the same as before.

The electronic market operates through a standard reverse auction mechanism: Each of the M low-quality suppliers participating in the auction submit an offer that consists of a wholesale price and a fixed tariff. Bidding continues in rounds. If at any point some supplier basing on the realization of her marginal costs for that period cannot match the current-best (winning) offer, then that supplier drops out of the bidding. The process continues until a single winning bid remains in the market.[5] Each manufacturer then places an order, the orders are delivered, and each manufacturer pays the total wholesale cost as well as the fixed tariff.

In equilibrium, the winning fixed tariff is equal to the difference between the total supply chain profits under the lowest and the second-lowest bid divided equally among the $N-n$ manufacturers (see the proof of Proposition 3 in the appendix). This leaves each manufacturer with a profit equal to his share of the total supply chain profits under the second lowest bid, it achieves truthful revelation of the suppliers' production costs, and it obtains the most efficient outcome at each period for the low-quality product supply chain. It would be desirable to interpret this in the context of prices paid by the manufacturers to the suppliers for these products. Unfortunately, because of the two-part tariff nature, a simple interpretation based on wholesale prices alone is not possible. What is possible to state is that the total procurement cost paid by the manufacturers to the suppliers is equal to the difference in the total supply chain profit under the lowest- and second-lowest production costs and the total supply chain surplus is equal to the surplus under the second-lowest production cost. Note also that although this outcome can be implemented via the auction described above, it is also the Nash equilibrium of an invisible-hand-clearing based open market game in which the suppliers openly compete by pricing their products through two-part tariffs.

Although the auction market changes the market equilibrium compared with the benchmark monopoly case, the two are closely related. Specifically, since the winning supplier in the auction is the one with the lowest cost, it follows that the M low-quality suppliers collapse into a single supplier with cost $c_{(1)}$. Thus, the market equilibrium is equivalent to the equilibrium in a duopoly in which the high-quality supplier's production cost has cumulative density function $F(y)$, and the low-quality supplier's cost has cumulative density function $F^M_{(1)}(y)$. Therefore, the equilibrium production quantities for both high- and low-quality parts and the wholesale prices are obtained by an application of Proposition 2. However, the tariff structure needs to be modified to reflect the auction mechanism, and the option that the auction provides to those manufacturers' that trade with the high-quality supplier. The following proposition summarizes the results.

Proposition 3. *Define c_l^* and c_h^* as in Proposition 2 with G_1 replaced by $G^M_{(1)}$, and define*

$$J(n,t)=\begin{cases}\frac{1}{4}\left([(c_h^*-c_0(t))^+]^2-\frac{1}{N}H^M_{(2)}(K)\right) & \text{if } n=1;\\[2ex] \frac{1}{4}\left([(c_h^*-c_0(t))^+]^2-\frac{n}{(N-n+1)}H^M_{(2)}(c_l^*)\right) & \\ \quad \text{if } 2\le n\le N-1; & \\[2ex] \frac{1}{4}\left([(K+v-\delta-c_0(t))^+]^2-NH^M_{(2)}(c_l^*)\right) & \\ \quad \text{if } n=N. & \end{cases} \tag{5}$$

The electronic market will be operational if and only if $K-\underline{c}-\frac{1}{2}G(K+v-\delta)>0$ and only if $n<N$. The high-quality supplier will be in the market only if $E[J(n,t)]\ge 0$. In that case, there exists a unique symmetric equilibrium in which the strategies are defined as follows: The high-quality supplier produces only when her costs are less than or equal to the critical value c_h^*, and the lowest-cost low-quality supplier produces only when her costs are less than or equal to the critical value c_l^*.

Equation (5) highlights how auction competition affects this market. Specifically, this expression gives the single-period profit for the high-quality supplier, and it reveals three distinct cases: (a) When there is only one high-quality manufacturer, if he switches to the electronic market he becomes part of a monopolistic low-quality (electronic market–based) supply chain. Therefore, the high-quality supplier must leave

[5] The process can terminate with a tie among multiple winning bidders. In that case, the manufacturers' orders are awarded equally to all winning bidders. This, however, is a zero probability event in equilibrium under the assumption of continuous cost distributions.

that manufacturer with a surplus equal to what she would make as a part of that supply chain. (b) When n is between 1 and N, if a manufacturer switches to the electronic market he becomes part of a low-quality supply chain that engages in a duopolistic competition with the high-quality supply chain. Hence, each high-quality manufacturer has to receive at least as much as he would receive in the auction. (c) When $n = N$, the high-quality supplier is a monopolist and a switch by one of the manufacturers she serves to the electronic market makes it operational and leaves the manufacturer with a large share of the surplus created within the low-quality supply chain. Therefore, the high-quality supplier must pay each high-quality manufacturer an amount $H^M_{(2)}(c^*_l)$ to prevent him from switching to the electronic market.

4. The Interplay Between Relational Contracts and the Auction Market

We now integrate the cornerstone models developed in §3 to examine the interplay between the auctions and relation-based contracting when product quality is nonverifiable. A Perfect Bayesian Equilibrium analogous to the one presented in §3.1 is developed. The main result mirrors Proposition 1 and identifies a lower bound on the discount factor such that a relational contracting equilibrium with the trigger strategies introduced in §3.1 can be sustained. Analysis of this lower bound determines the main factors that affect the self-enforceability of such relational contracts.

4.1. The Relational Contracting Equilibrium Under Competition from the Auction Market

The scenario considered now is identical to that in the third cornerstone model but with part quality assumed nonverifiable. The timeline within a period is based on that described in Figure 1 but expanded as follows to accommodate the new setting: (a) All low-quality suppliers learn their production costs at the same instance as the high-quality supplier. (b) The auction clears in the end of each period, and the high-quality transactions for that period settle at the same time.

Assuming that the high-quality offer is extended to manufacturers $i = 1, \ldots, n$, the belief system for manufacturer i is defined as follows:

$$B_i(t) = \begin{cases} L & \text{if } w(t) < \underline{c} + \delta \text{ or } \xi(\tau) = L \\ & \quad \text{for any } 0 \le \tau < t \text{ and } t > 0 \text{ and} \\ & \quad 1 \le i \le n; \text{ or } \xi(\tau) = L \text{ for any} \\ & \quad 0 \le \tau < t \text{ and } t > 0 \text{ and } n+1 \le i \le N; \\ H & \text{otherwise.} \end{cases} \tag{6}$$

This states that the high-quality manufacturers, i.e., for $1 \le i \le n$, believe that the part is of high quality unless they observe cheating in the past behavior of the high-quality supplier or they observe a wholesale price that is too low to support a high-quality belief. Note that in the end of each period the consumer market outcome is observed by each player and thus the true quality of the products for that period becomes known to all parties. Consequently, all low-quality manufacturers can also deduct whether the high-quality supplier kept her quality promise at each period. If the high-quality supplier betrays the trust of the manufacturers, she loses credibility with all manufacturers and has to switch to low-quality parts from that period onward.

To the beliefs given in (6), the high-quality supplier counters with the quality setting

$$\xi(t) = \begin{cases} H & \text{if } B_i(t) = H \text{ for any } i, 1 \le i \le n, t \ge 0; \\ L & \text{otherwise.} \end{cases} \tag{7}$$

The intuition is that, if a betrayal of trust happens and the manufacturers do not believe that Supplier 0 will provide high-quality parts, they will switch to the auction market. However, under this scenario it is suboptimal for the supplier to produce high-quality parts, either. Then the high-quality supplier's best option is to switch to the electronic market as well and become one of the competing suppliers there for all subsequent periods. As with the first cornerstone model, for the sake of brevity, whenever we refer to the relational contract for this (main) model, we will be referring to the Perfect Bayesian Equilibrium supported by the beliefs and the strategies above.

The following proposition states the characterization of a Perfect Bayesian Equilibrium under this manufacturer's belief system and supplier behavior.[6]

Proposition 4.

(i) *If* $K - \underline{c} - \frac{1}{2}G(K+v-\delta) \leq 0$ *and if* $\beta > \beta_0$ *given in Proposition* 1, *then the electronic market shuts down and only the high-quality supplier is in the market.*

(ii) *If* $K - \underline{c} - \frac{1}{2}G(K+v-\delta) > 0$ *a Perfect Bayesian Equilibrium exists with the manufacturer beliefs satisfying* (6), *the high-quality supplier setting the quality according to* (7), *and with an outcome of* $\xi(t) = B_i(t) = H$ *for* $1 \leq i \leq n$ *for all* $t \geq 0$ *only if*

$$\max_{1\leq n\leq N} E[J(n,t)] - \frac{1}{M+1}\left[H_{(1)}^{M+1}(K) - H_{(2)}^{M+1}(K)\right] > 0; \qquad (8)$$

and

$$\beta \geq \frac{2(c_h^* - \underline{c})\delta}{\max_{1\leq n\leq N} E[J(n,t)] - \dfrac{[H_{(1)}^{M+1}(K) - H_{(2)}^{M+1}(K)]}{M+1}} \triangleq \beta^*; \qquad (9)$$

where $E[J(n,t)]$ *is derived from* (5), c_h^* *is defined in Proposition* 2 *with* $G_1 = G_{(1)}^M$.

This proposition has implications about the equilibrium number of high-quality manufacturers, the auction's ability to compete with the relational contract, and about the sustainability of the relational contract. These issues are now discussed in that order.

4.1.1. Equilibrium Number of High-Quality Manufacturers. Another important observation is that (8) implies that the number of manufacturers trading with the high-quality supplier in equilibrium maximizes that supplier's expected total profit. The following result further characterizes the exact equilibrium number.

Proposition 5.

(i) *The equilibrium number of high-quality manufacturers takes values in the set* $\{1, 2, N\}$.

(ii) *For any given number of manufacturers* N, *there exists a subset of the parameter space defined as* $\{(K, v-\delta)$: $\frac{1}{2}G(K+v-\delta) + \underline{c} \leq K \leq \frac{1}{2}G(K+v-\delta) + \underline{c} + \kappa\}$, *where* $\kappa > 0$, *such that the equilibrium number of high-quality manufacturers for the high-quality supplier is* N.

(iii) *For a fixed parameter set there exists an* $\underline{N} \geq 2$ *such that if the number of manufacturers N exceeds* $\underline{N}$, *then the equilibrium number of manufacturers for the high-quality supplier is* $n < N$.

[6] Under the caveat that the continuation in equilibrium path is self-enforceable.

The rationale behind (i) is as follows: Equation (5) implies that the high-quality supplier's profits are decreasing in n for $2 \leq n \leq N-1$. As long as n is in this interior, increasing the number of high-quality manufacturers does not affect the total supply chain profit for either the high-quality product or the low-quality product, but it increases the total amount of money the supplier must pay to prevent the increased number of high-quality manufacturers from joining the auction market. Hence, if n is in the interior it must be $n = 2$. In the two extreme cases ($n = 1$ or $n = N$) the market structure changes. If $n = N$, the total high-quality supply chain profit is equivalent to the monopoly profit, but the payments made to each of the manufacturers are equivalent to the profit they would make if they unilaterally became the sole buyer in the electronic market. At the other extreme, if $n = 1$ the supplier must offer the single manufacturer the profit he would make if he would abandon the high-quality supplier, in which case the electronic market would become the sole source of parts in the marketplace as discussed in §3.3. This implies that, in equilibrium, there is a push toward the extreme in the determination of the number of high-quality manufacturers. Other factors such as transaction and distribution costs will affect the choice of the number of manufacturers in reality, but our results imply that there is a prevailing force toward the extreme. As we discuss next, this force is driven by the costs necessary to keep the manufacturers from abandoning the long-term relational contract and joining the auction market.

Part (ii) of Proposition 5 states that on a strip above the curve $\frac{1}{2}G(K+v-\delta) + \underline{c} = K$, each manufacturer's expected profit from the auction is slim and hence the high-quality supplier does not have to pay much to each manufacturer to keep him in the contract. Therefore, in equilibrium, the high-quality supplier can serve all available manufacturers and becomes a monopolist. However, as one moves away from the strip in the neighborhood of the curve $\frac{1}{2}G(K+v-\delta) + \underline{c} = K$, the profits for the manufacturers that participate in the auction market increase. It becomes too expensive to pay all manufacturers enough to prevent them from joining the electronic market, and the high-quality supplier sets $n = 1$ or $n = 2$, and allows the emergence of competition from the electronic market.

Similarly, when the number of manufacturers is sufficiently large, it becomes prohibitively expensive to contract with all of them, hence it is optimal to contract with either one or two.

4.1.2. Ability of the Auction to Compete with the Relational Contract. One important implication of Proposition 4 is that when the quality premium $(v-\delta)$ and the discount factor (α) are large enough, relational contracting will push the auction out of the market (Part (i)). This can occur either because the quality premium is so high that the electronic market cannot compete no matter how efficient it is (Proposition 2), or because the quality premium is moderate but the downstream fragmentation is small so that all manufacturers produce high quality (i.e., $n = N$; see Proposition 5). In either case, this result states that (auction-based) electronic markets may not be viable procurement venues for products with important nonverifiable quality attributes. This is consistent with the empirical and industry observations (see, e.g., Bajari et al. 2002) that, whereas auctions that purely provide cost efficiencies can be successful for more commodity-like products, long-term contractual relationships become the dominant choice for procurement when product characteristics get complicated, yielding high costs of verifiability or nonverifiability in the extreme.

4.1.3. Sustainability of the Relational Contract. Returning to Proposition 4, the critical threshold β^* given in Part (ii) summarizes how easy or difficult it is to have a self-enforceable relational contract, similar to the threshold quantity defined in Proposition 1. It is, therefore, important to understand how the critical threshold β^* changes with market characteristics. The main results are summarized below.

Proposition 6. (i) *β^* is decreasing in v.* (ii) *β^* is increasing in δ.* (iii) *If either K is sufficiently large or both N and M are sufficiently large, then β^* is decreasing in K.* (iv) *If M is sufficiently large, then β^* is decreasing with M.* (v) *β^* is decreasing with N if and only if $n < N$.*

Most of these results mirror the observations made in §3.1. However, now the number of suppliers and the number of manufacturers emerge as important factors. As the number of low-quality suppliers M increases, the auction market becomes increasingly more competitive and hence increasingly unattractive for the high-quality supplier. Thus the relational contract becomes easier to sustain—the threat that keeps the relation together is more severe. Similarly, as the number of manufacturers increases, their bargaining power is reduced and the payment made by the high-quality supplier to the manufacturers is also reduced (provided she is only trading with one or two manufacturers). This also makes it easier to sustain the relational contract. On the other hand, if the high-quality supplier trades with all manufacturers and the number of manufacturers increases, then the cost of keeping all these manufacturers in the relationship also increases, and hence it becomes more difficult to sustain the quality promise. Part (v) says that if for the current number of manufacturers N the equilibrium number of high-quality manufacturers n is strictly less than N (i.e., $n = 1$ or 2), increasing N will decrease β^*. On the other hand, if in equilibrium all manufacturers are producing high-quality products (i.e., if $n = N$) then increasing the number of manufacturers will increase β^*.

Finally, we examine the sustainability of the relational contract under competition from the auction for certain limits of interest. The following proposition will be useful in the next section where we will discuss the emerging market structure.

Proposition 7. *Let β^* be as defined in Proposition* 4.

(i) $\lim_{K\to\infty} \beta^* = 0$.

(ii) *There exist cost distributions for which there exists an $\overline{N} \geq 2$ and $\underline{\Theta} > 0$, such that when $N \leq \overline{N}$ and when δ is bounded away from zero, $\lim_{(v-\delta)\to\underline{\Theta}^+} \beta^* = \infty$.*

When there is an electronic market that competes with relational contracting, the relational contract is self-enforceable as long as the market size K is sufficiently large as stated in Part (i) of Proposition 7. As the market size gets bigger, the expected profits from switching to the electronic market and the one-time gain from violating the high-quality promise become relatively small compared with the total expected supply chain profits from staying in the relational contract in perpetuity.[7] This pushes the cutoff β (and hence the cutoff α) to zero in the limit, easing the sustainability of a relational contract even when there

[7] See the proof of Proposition 7 in the appendix for details.

is heavy discounting or when the trading frequency is low. On the other hand, if downstream fragmentation is limited, the manufacturers' bargaining power in the auction is sizeable and can negate any gains from high-quality production if the quality premium $v-\delta$ is small. This can make the relational contract unattractive for the high-quality supplier because the premium she will have to pay the manufacturers to prevent them from joining the electronic market will be sizeable. As a result, the relational contract cannot be self-enforceable below a critical value of the quality premium, $\underline{\Theta}$, as stated in Part (ii).

We complete this section with an examination of the impact of model parameters on expected production quantities and on consumer prices.

Corollary 2. *For all t:*

(i) *The expected sales volume of high-quality and low-quality products sold in each period and the corresponding expected clearing prices are increasing in the market size K.*

(ii) *The per-period expected sales volume and expected clearing price for high-quality (low-quality) products are increasing (decreasing) in the premium differential $v-\delta$.*

(iii) *As the number of low-quality suppliers M increases, the expected (per-period) sales volume for low-quality products increases, while the expected clearing prices for both high-quality and low-quality products and the expected sales volume for high-quality products decrease.*

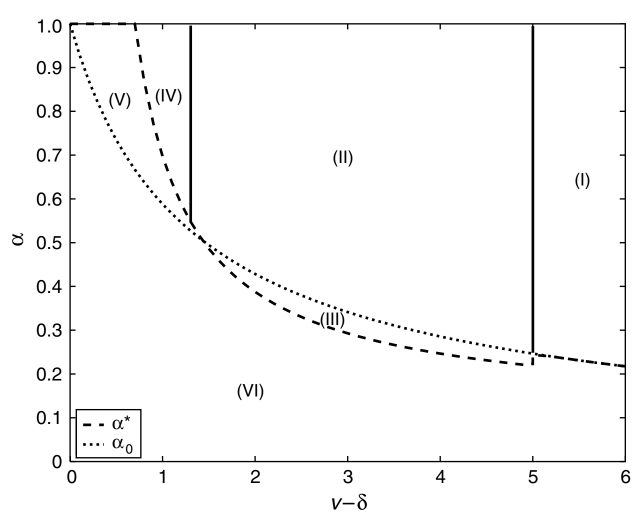
4.2. Comparison with the Benchmark Monopoly Case

We now contrast our findings from the environment where the electronic market competes with the relational contract to the monopolist case reflected in the cornerstone model of §3.1. The monopoly model provides a useful benchmark because the threat that sustains the relationship in that model is that the two parties will stop trading high-quality parts and trade low-quality parts instead. By contrast, in the case with the electronic market, the threat is augmented by the competitive forces it introduces. Therefore, the comparison provides insights about the effect of electronic market–based competitive forces on relational contracting. We will examine how this competition affects the relational contracting equilibrium, the market structure that emerges, the supply chain performance, and product quality by contrasting all relevant statistics from the two cases.

4.2.1. Emerging Market Structure. Consider first the big picture that emerges from Propositions 1–7. By Proposition 7, when relational contracting is under competition from an electronic market, the relational contracting is always self-enforceable if the market size K is sufficiently large. By contrast, in the benchmark case (see Proposition 1) the relational contract is not self-enforceable when $\alpha<\delta/v$, no matter how large the consumer market is. On the flip side, Proposition 7 states that when quality premium $v-\delta$ is small, there exist conditions such that the relational contract will not be sustained. Again this contrasts to the result implied by Proposition 1 in the benchmark monopoly case where a relational contract can be possible even if the quality premium is arbitrarily small.

Figure 3 highlights these observations. The figure identifies six regions in the plane $(v-\delta,\alpha)$ with K kept constant. The dashed line refers to the discount factor cutoff $\alpha^*=\beta^*/(1+\beta^*)$ (where β^* is as described in Proposition 4) for the case of competition between relational contracting and the electronic market. The thin dotted line refers to the discount factor cutoff (α_0) in the monopoly cornerstone model of §3.1 as implied by Proposition 1. That is, under competition from the auctions, the relational contract is sustainable only above the α^* line (Regions (I), (II), (III), (IV)). Similarly, the relational contract is sustainable in the

Figure 3 Regions of Different Emerging Market Structures Under Oligopoly with the Electronic Market and the Benchmark Monopoly Case (Proposition 1), with Cost Distribution $U[1,3]$, Large M (=20), $N=2$, $K=5$, and $\delta=1$



benchmark monopoly case only above the α_0 line (Regions (I), (II), (IV), (V)). In Region (I), there is no room for the low-quality product in the market so only the relational contract exists under both cases. In Regions (II) and (III), there is room for the low-quality product but the high-quality supplier finds it advantageous to trade with all manufacturers and this shuts down the electronic market. In Region (IV), the electronic market and the relational contract coexist. In Regions (V) and (VI), the high-quality supplier is not patient enough to sustain the relational contract under competition from the electronic market and only the latter is operational. Note that in Region (III) the relational contract can be sustained with but not without the electronic market. The existence of such a region is guaranteed by Propositions 1 and 7. The opposite is true in Region (V), the existence of which is, again, guaranteed by the same propositions.

4.2.2. Production Quantity and Expected Surplus. We now compare the expected total quantity (which is inversely related to consumer price), expected total supply chain profit, and expected social surplus for the two cases when the relational contracts are sustainable. The following proposition provides the results.

Proposition 8.

(i) *When the electronic market and the relational contract coexist, the expected total quantity is higher under competition with electronic market relative to the benchmark case.*

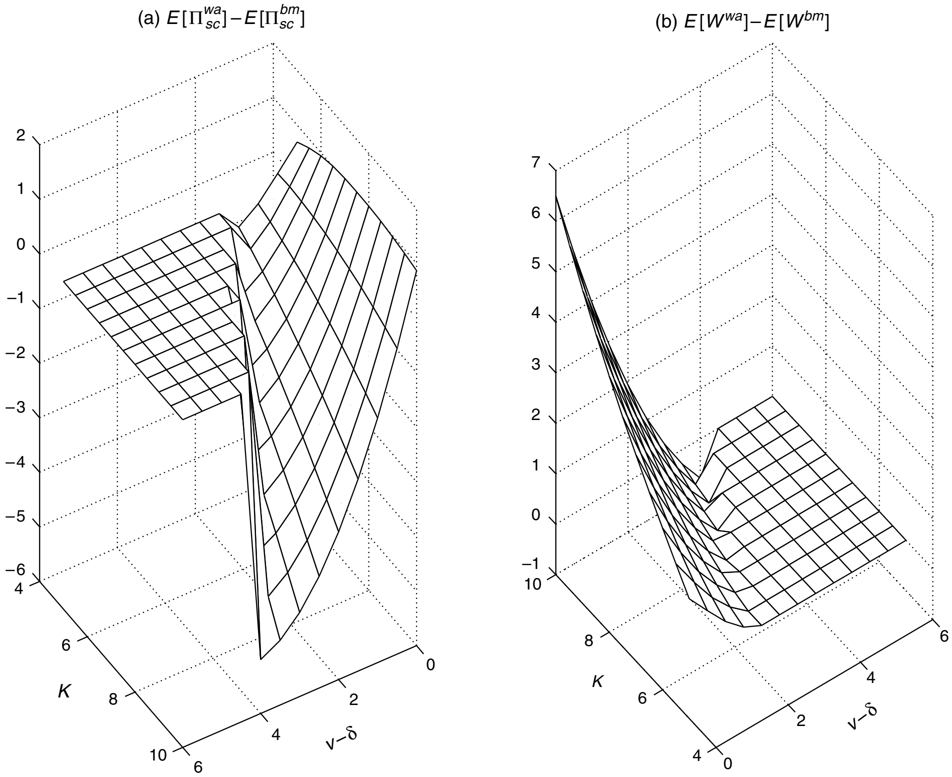
(ii) *There exist $\bar{K} > 0$ and $\bar{\Theta} > 0$ such that the expected supply chain profits are higher than the benchmark monopolist case when $K < \bar{K}$ and $v - \delta < \bar{\Theta}$. If M and N are sufficiently large, there exist $\underline{K} > 0$ and $\bar{\Theta} > 0$ such that social surplus under competition from the electronic market is higher than the benchmark monopolist case when $K > \underline{K}$ and $v - \delta < \bar{\Theta}$.*

(iii) *If N is sufficiently large, then there exist $\kappa_1 > \kappa_2 > 0$ such that the expected supply chain profits and expected social surplus under competition from the electronic market are lower than the benchmark monopolist case when $\underline{c} + \frac{1}{2}G(K + v - \delta) + \kappa_1 > K > \underline{c} + \frac{1}{2}G(K + v - \delta) + \kappa_2$.*

By the proof of Proposition 2, the total expected high-quality production volume is $\frac{1}{2}G(c_h^*)$, and the total low-quality production volume is $\frac{1}{2}G_{(1)}^M(c_l^*)$. Compared to the monopoly case where only the high-quality product is traded and the volume is $\frac{1}{2}G(K + v - \delta)$, the total high-quality production volume decreases because $\frac{1}{2}G(c_h^*) < \frac{1}{2}G(K + v - \delta)$; this follows by the fact that $c_h^* = K + v - \delta - \frac{1}{2}G(c_l^*)$ (Proposition 2) and by Lemma 1. Then the question becomes whether the total production level increases under competition from the auction, i.e., whether $\frac{1}{2}G(c_h^*) + \frac{1}{2}G_{(1)}^M(c_l^*) > \frac{1}{2}G(K + v - \delta)$; Part (i) of Proposition 8 states that it does. Hence, with the electronic market the total production volume increases but the total (and average) quality decreases.

Figure 4 illustrates Parts (ii) and (iii) of Proposition 8. Panel (a) presents the supply chain profit difference with the electronic market compared with the benchmark monopolist case. For small values of $v - \delta$ and K, the cost savings associated with the selection of the most efficient supplier in the auction easily overcome the losses to the supply chain from the reduction in the high-quality output. For large values of $v - \delta$, the high-quality supplier is able to maintain her status as the monopolist by pushing the auction out of the market for procurement. In the middle, however, high-quality production is reduced compared with the monopolist high-quality case, while at the same time the high-quality supplier pushes a sufficiently large number of low-quality products out of the market to more than cancel the cost efficiency gains from the auction. Therefore, supply chain profits are reduced compared with the benchmark case. Panel (b) illustrates the total surplus. When K is large enough and $v - \delta$ is small enough, social welfare increases with the electronic market compared with the benchmark monopoly case because the total production volume increases, whereas the quality of the production volume has a negligible effect on welfare. On the other hand, when $v - \delta$ is large enough, again the high-quality supplier pushes the low-quality suppliers out of the market and becomes a monopolist. Hence, the difference in surplus between the two cases is zero. This extends to the band on top of $K = \underline{c} + \frac{1}{2}G(K + v - \delta)$ where the high-quality supplier contracts with all manufacturers and eliminates the auction market. However, as one moves away from this band, the electronic market becomes operational and the resulting reduction in high-quality production decreases the total social welfare.

Figure 4 Sample Plots of Differences in Expected Supply Chain Profit and Expected Social Welfare



Note. Panel (a) shows the difference between total supply chain profits under competition from the auction ($E[\Pi_{SC}^{wa}]$) and under benchmark monopoly ($E[\Pi_{SC}^{bm}]$). Panel (b) shows the difference in expected social welfare under competition from the auction ($E[W^{wa}]$) and under benchmark monopoly ($E[W^{bm}]$). The cost distribution is $U[2,3]$, $M = 5$, and $N = 10$.

5. Concluding Remarks

In this paper, we consider a formal model of the interaction between relational contracts and supply auctions. Our results suggest that if the premium that the consumers pay for quality over the marginal production cost is sufficiently high and if the provider of high-quality intermediate parts is sufficiently patient, then there will be a relational contracting equilibrium that will guarantee the supply of quality parts. Furthermore, if the quality premium is sufficiently large, then an auction-based electronic market that relies purely on the price dimension will not be able to compete with the high-quality supply chain, and therefore is not viable. On the other hand, if the quality premium is moderate, then relational contracting and the electronic market can coexist as procurement venues in equilibrium. If the market size is large, competition from the auction can enhance the self-enforceability of the relational contract and hence help the provision of high-quality goods to the market. When the downstream fragmentation and the quality premium are low, however, competition from the electronic market may undermine the sustainability of the relational contracts and hurt the channels that provide high-quality goods to the market. We also find that, when competing with relational contracts, auctions may either increase or decrease the total supply chain profits and social surplus.

Our analysis relies on the following key assumptions.

i. Single High-Quality Supplier. In most procurement relationships, the manufacturer avoids relying on a single strategic supplier for quality parts. However, as the HP-Canon example presented in §1 illustrates, single-sourcing arrangements are not uncommon, especially when the underlying technology is patented. Keep in mind that the term *quality* in our model represents an attribute or collection of attributes of a product that provides higher performance and desirability. The ability to provide such a performance can be exclusive to a certain supplier due to technological know-how, yet that supplier still needs to spend a costly and noneasily verifiable effort to deliver the high quality. In such cases, relying on long-term relational contracts allows the buyers to avoid costly quality verification processes (both physical and legal). This assumption also enhances the tractability of the analysis. For example, in a model with two high-quality suppliers, three distinct equilibria need to be examined: one where both suppliers produce high-quality parts, both produce low-quality parts, or one produces high-quality parts and the second produces low-quality parts. The complexity increases with the number of suppliers. In a model with a single high-quality supplier, tractability is attained and the analysis focuses on the interaction between two procurement models: relational contracts and auction markets. With multiple high-quality suppliers, the nature of the insights will be confounded by competition within the set of high-quality suppliers and this would obstruct a pure comparison between the two procurement models. Although the qualitative insights we provide in this paper are not expected to be sensitive to this assumption, relaxing this assumption can be an interesting avenue for future research for other issues worth being investigated.

ii. High-Quality Supplier Is the Stackelberg Leader. This assumption seems to suggest that the high-quality supplier has all the market power. However, in the presence of competition from the auction, market power shifts from the high-quality supplier to the manufacturers. In fact, one of the results developed in §4.2 is that under competition from the auction market, in equilibrium, the manufacturers may retain a significant fraction of the total supply chain profits while the high-quality supplier's profits may diminish. Therefore, market power is determined endogenously in equilibrium.

iii. Nonverifiability of Quality. In our model, the quality of the part procured by the manufacturers is nonverifiable, and this implies that the manufacturer is at full risk for the quality of the part and of the end product: If he purchases the part assuming it is of high quality but then the clearing price for the end product indicates low quality, the manufacturer cannot obtain a refund from the supplier in that period.

There are two equivalent interpretations for the current mechanism: (a) The manufacturer may realize the quality of the components during production and consequently price the product accordingly in the market. (b) The manufacturer may not detect the quality but refunds the consumers if the product turns out to be of low quality (i.e., through replacements, repairs, and so on). In the case of HP, for instance, both cases can and do happen. Several other more involved approaches that get into further details of verification can be analyzed in significant expense of clarity and brevity. Note that all such approaches are equivalent for our purposes and that dwelling on the details of some of those mechanisms above would make the analysis cumbersome, and deviate focus without adding much useful insight. The main point here is that the manufacturer cannot verify the quality at the time of purchase (or verification at that point is too costly), he assumes full responsibility for the product armed mainly by a promise of future business, and he cannot obtain refunds for inferior quality because of lack of court-enforceability (see, e.g., Baker et al. 2002). Our goal is to incorporate this into the model in the simplest and clearest way. Note that relationship-based transactions here also serve a role of enhancing efficiency by avoiding or reducing costs of inspections and legal enforcements of warranties for verification of quality.

iv. Nonparticipation of High-Quality Supplier and the Manufacturers in the Electronic Market. In a given period, the high-quality supplier produces only one type of product. The main reason for this assumption is tractability. On the other hand, although no assumption is perfect, this assumption also reflects reality to a degree: First, it is not uncommon for firms to operate in a single dominant procurement mode for each part because of resource constraints. Suppliers and manufacturers that engage in long-term relational contracts devote their resources mainly to managing these contractual relationships and consequently, even if they participate in alternative procurement venues, their activities beyond their main procurement mode is relatively limited. (This is again true in the context of HP's supply chain.) Second, the decision of the quality level and the choice of the procurement mode in our model is period based, i.e., even though in the same period, each supplier and manufacturer has to choose one of the two trading options. The model does not exogenously tie the suppliers and manufacturers to a single procurement mode indefinitely. The quality and the trading mode decisions are up to each player in the beginning of each period and, if they prefer, they can choose to switch, although we show that in equilibrium there is no incentive for the participants to switch.

This assumption also states that the manufacturers trading with the high-quality supplier do not participate in the electronic market even in periods in which their supplier does not provide them with the parts. The assumption reflects the high-quality manufacturers' desire to preserve their brand by avoiding fluctuations in the quality of their products. Furthermore, many supply chain relationships, such as that of HP and Canon, are founded based solidly on quality requirements and manufacturers tend not to dilute their brand image by mixing products using inferior parts obtained from generic suppliers with the ones equipped with those procured from their high-quality suppliers in the same product line. Interpreted in the context of multiple periods, this assumption implies that a firm's long-run average production quantity is reduced in periods of high costs, whereas the product's quality is not compromised. This interpretation reflects real-life observations. This assumption is also supported by production process constraints: For example, setups may prevent the supplier from producing different qualities of parts in the same period. There are cases where a supplier may produce parts of both high and low quality in a period, but our analysis is restricted to those cases where that supplier can only produce parts of either high or low quality in each period and the insights apply only to these cases. A model without this assumption could also be tractable, but it would be more complicated.

v. Equilibrium Selection. When the high-quality supplier cheats the manufacturers and delivers low-quality parts instead of the anticipated high-quality ones, the manufacturers retaliate by never believing the quality promise again. This equilibrium has several attractive features. First, the outcome it supports is the best outcome because it maximizes the high-quality supply chain's surplus. In other words, in any other equilibrium the total high-quality supply chain surplus cannot be higher.[8] Second, the underlying punishment strategy is the most stringent one in a family of trigger strategies where successive and possibly finite periods of punishment follow a deviation from the equilibrium, and it is the one most commonly used in the literature (see, e.g., Bull 1987; Baker et al. 1994, 2001, 2002; among many others). It is commonly known that although exploring a finite period punishment strategy clutters the algebra, it does not change the main insights (except for making the relational contracts less sustainable based on the length of the punishment in the strategies examined). Therefore, the finite punishment strategy equilibria (as well as other structures of equilibria) are rarely considered in the literature, whereas equilibria based on infinite punishment strategies are dominantly used as the canonical equilibria of repeated games. Finally, the equilibrium that infinite punishment strategies support possesses the most steady cooperation and punishment phases among all nontrivial equilibria of the game examined, which is realistic barring outside shocks to the market. Another concern that may arise is what happens when the quantity β is less than the critical thresholds β_0 or β^*. Our results state that high quality cannot be sustained by the trigger strategies considered. Furthermore, because the trigger strategies considered here involve the strictest punishment for misrepresenting quality, any relief from such strict punisment would make a deviation more attractive and make quality harder to sustain. Therefore, one would expect that with β values lower than the critical thresholds high quality would not be sustainable.

It is also known that trigger strategy–based self-enforceable contracts are prone to renegotiations. Although the corresponding theory is well developed for finitely repeated games, serious conceptual difficulties exist for infinitely repeated games (see Bernheim and Ray 1989). Attempts have been made to develop refined equilibrium concepts that enforce renegotiation-proofness: These include employing assumptions of a predefined set of social norms (e.g.,

[8] We thank an anonymous referee for bringing this issue to our attention.

Farrell and Maskin 1989) or assumptions on bargaining powers (Abreu et al. 1993). From an alternative angle, Kreps et al. (1982), provide a basis for renegotiation-proofness relying on hidden types and reputation in finitely repeated games. It is widely agreed that this intuition applies for infinitely repeated games as well, and trigger strategy equilibria are widely used in the relevant literature.

The model also makes the additional assumption of the linearity of the consumer demand curve for analytical tractability. In fact, some of the results presented in the paper, such as the effect of market size on sustainability of the relational contract, use exact expressions based on the linear demand assumption. These results can be generalized to suitable nonlinear demand curves, but the analysis would be cumbersome and is left as a topic for future research.

In this paper we aimed to investigate the sustainability of relational contracts under competition from an auction market. Our model assumes that the delivery of quality parts is sustained by repeated interactions outside the auctions. However, repeated interactions can occur within the auction market, and in certain markets relationship management can become part of an auction-based procurement system. That is, the suppliers that participate in the auction can have a reputation for the quality of their parts. This reputation can provide the basis for enforcing the delivery of quality parts within the context of the auction. Repeated interactions can also provide incentives for low-quality suppliers to build a reputation by consistently providing high-quality parts. This can significantly benefit the supply chain in the long run. The model also assumes that the product is perishable or has high depreciation, implying that neither the suppliers nor the manufacturers can carry inventory across periods. A model without this assumption may be quite challenging in terms of analytical tractability, but is worth investigating. These issues are left as subjects for future research.

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Appendix. Proofs of Propositions

Proof of Proposition 1. First, when the game is at the beginning of a period t, given (2), $B(t)=H$ unless $w(t)<\underline{c}+\delta$. Then the strategy that maximizes that period's profit for the supplier will be given as $(\xi(t), w(t), g(t)) = (H, c(t)+\delta, \frac{1}{4}(K+v-\delta-c(t))^2)$. In this regime, the expected discounted remaining profit for the supplier is

$$\Pi(t)=\frac{1}{4}((K+v-\delta-c(t))^+)^2+\frac{\alpha}{4(1-\alpha)}H(K+v-\delta). \quad \text{(A.1)}$$

On the other hand, when the manufacturer's belief is $B(t)=L$, the supplier's strategy that maximizes her profits for that period is $(\xi(t), w(t), g(t)) = (L, c(t), \frac{1}{4}((K-c(t))^+)^2)$. Now, suppose the supplier's unit cost in any period t is $c(t)$ and suppose she chooses to cheat by not producing the high-quality product and pricing it as one, i.e., by setting $w\geq \underline{c}+\delta$. If she chooses to declare that her costs are $\hat{c}\geq\underline{c}$ by setting the wholesale price at that period to $\hat{w}=\hat{c}+\delta$, her optimal constant tariff follows to be $\hat{g}=\frac{1}{4}((K+v-\delta-\hat{c})^+)^2$ and her profits from that period will be $\pi_s(t)=\frac{1}{4}((K+v-\delta-\hat{c})^+)^2+\frac{1}{2}(K+v-\delta-\hat{c})^+(\hat{c}+\delta-c)$. However, by (2), the manufacturer will treat the input as low quality for all subsequent periods, which will make her expected profit for each future period $\tau>t$ equal to $\frac{1}{4}H(K)$ and her expected discounted profits from period t on

$$\widehat{\Pi}(t) = \frac{1}{4}((K+v-\delta-\hat{c})^+)^2+\frac{1}{2}(\hat{c}+\delta-c(t)) \cdot (K+v-\delta-\hat{c})^+ + \frac{\alpha}{4(1-\alpha)}H(K). \quad \text{(A.2)}$$

Therefore, the high-quality regime will be supportable if and only if there does not exist $c(t)\in S$, $\hat{c}\geq\underline{c}$ such that $\widehat{\Pi}(t)>\Pi(t)$. By (A.1) and (A.2) this is equivalent to

$$\frac{\alpha}{4(1-\alpha)}(H(K+v-\delta)-H(K)) \geq \sup_{c(t)\in S,\,\hat{c}\geq\underline{c}} \frac{1}{2}\left\{\frac{1}{2}\left(((\theta-\hat{c})^+)^2-((\theta-c(t))^+)^2\right) + (\hat{c}+\delta-c(t))(\theta-\hat{c})^+\right\}, \quad \text{(A.3)}$$

where $\theta=K+v-\delta$. Now for any $\theta>\underline{c}$, consider the solution to the problem

$$\sup_{c,\,\hat{c}\geq\underline{c}}\left\{\frac{1}{2}\left(((\theta-\hat{c})^+)^2-((\theta-c)^+)^2\right)+(\hat{c}+\delta-c)(\theta-\hat{c})^+\right\}. \quad \text{(A.4)}$$

First, in the optimal solution for (A.4), $\hat{c} \geq \theta$ implies $c \geq \theta$, yielding an objective of zero. Now suppose $\hat{c} < \theta$. Then, on the region $c \geq \theta$, the problem is equivalent to

$$\sup_{c\geq\theta,\,\hat{c}\geq\underline{c}} \left\{ \frac{1}{2}(\theta-\hat{c})^2 + (\hat{c}+\delta-c)(\theta-\hat{c}) \right\}$$
$$= \sup_{\hat{c}\geq\underline{c}} \left\{ \delta(\theta-\hat{c}) - \frac{1}{2}(\theta-\hat{c})^2 \right\}$$
$$= \begin{cases} (\theta-\underline{c})(\delta-\frac{1}{2}(\theta-\underline{c})) & \text{if } \delta \geq \theta-\underline{c} \\ \frac{1}{2}\delta^2 & \text{otherwise} \end{cases}$$
$$< \delta(\theta-\underline{c}). \tag{A.5}$$

But

$$\delta(\theta-\underline{c}) = \sup_{\underline{c}\leq c\leq\theta,\,\hat{c}\geq\underline{c}} \left\{ \delta(\theta-\hat{c}) - \frac{1}{2}(c-\hat{c})^2 \right\}$$
$$= \sup_{\underline{c}\leq c\leq\theta,\,\hat{c}\geq\underline{c}} \left\{ \frac{1}{2}((\theta-\hat{c})^+)^2 - ((\theta-c)^+)^2) + (\hat{c}+\delta-c)(\theta-\hat{c})^+ \right\}. \tag{A.6}$$

Therefore, we conclude that

$$\sup_{c,\,\hat{c}\geq\underline{c}} \left\{ \frac{1}{2}((\theta-\hat{c})^+)^2 - ((\theta-c)^+)^2) + (\hat{c}+\delta-c)(\theta-\hat{c})^+ \right\}$$
$$= \delta(\theta-\underline{c}), \tag{A.7}$$

and because $c(t) \in S$ implies $c(t) \geq \underline{c}$, we find that (A.3) is satisfied if and only if (3) is satisfied. Checking the converse and the consistency of off the equilibrium path beliefs with the equilibrium strategies is straightforward. This proves Part (i).

To see Part (ii), using Lemma 1 notice that

$$d\beta_0/dv < 0 \iff H(K+v-\delta) - H(K) < 2G(K+v-\delta)(K+v-\delta-\underline{c}). \tag{A.8}$$

But $2G(K+v-\delta)(K+v-\delta-\underline{c}) > 2G(K+v-\delta)(v-\delta) \geq H(K+v-\delta) - H(K)$, because G is increasing and $K > \underline{c}$. Hence, $d\beta_0/dv < 0$ follows. Second,

$$d\beta_0/d\delta > 0 \iff (K+v-\underline{c}-2\delta)(H(K+v-\delta)-H(K)) + 2G(K+v-\delta)(K+v-\delta-\underline{c})\delta > 0. \tag{A.9}$$

By Lemma 1, $H(K+v-\delta) - H(K) < 2G(K+v-\delta)(v-\delta)$, and because $v > \delta$ and $K > \underline{c}$, we conclude that $d\beta_0/d\delta > 0$. Third,

$$d\beta_0/dK < 0 \iff H(K+v-\delta) - H(K) - 2(G(K+v-\delta) - G(K)) \cdot (K+v-\delta-\underline{c}) < 0. \tag{A.10}$$

Now because $F(K+v-\delta)(K-\underline{c}) \geq F(K)(K-\underline{c}) \geq G(K)$, we have $(K-\underline{c})\int_K^{K+v-\delta} F(c)\,dc \geq (v-\delta)G(K)$. Adding $G(K)(K-\underline{c})$ to both sides, we obtain $G(K+v-\delta)(K-\underline{c}) \geq G(K)(K+v-\delta-\underline{c})$. Moreover, again by Lemma 1, $2G(K+v-\delta)(v-\delta) \geq H(K+v-\delta) - H(K)$. Combining the last two inequalities gives us $d\beta_0/dK < 0$. This completes the proof of Part (ii).

Finally, to see Part (iii), notice that by Lemma 1,

$$\lim_{K\to\infty} \beta_0 = \lim_{K\to\infty} \frac{2\delta(K+v-\delta-\underline{c})}{H(K+v-\delta)-H(K)}$$
$$= \lim_{K\to\infty} \frac{2\delta(K+v-\delta-\underline{c})}{(2(K-\mu)+v-\delta)(v-\delta)}$$
$$= \frac{\delta}{v-\delta}. \tag{A.11}$$

This completes the proof. $\square$

Proof of Proposition 2. The proof proceeds in three steps. First, assuming $\sum_{i=1}^N q_i \leq K$, we demonstrate the Nash equilibrium in the consumer market among the manufacturers who submit the market with the quantities that they purchased from their respective suppliers. Next, the duopoly driven by the two suppliers is examined, and the equilibrium production quantities are obtained. Finally, expressions are derived for the two-part tariffs that will induce the manufacturers to place the order desired by the suppliers and verify that $\sum_{i=1}^N q_i \leq K$ in equilibrium.

1^0. Denote the high-quality manufacturers with indices $1 \leq i \leq n$ and the low-quality ones with indices $n+1 \leq j \leq N$. Also denote the quantity produced by manufacturer i by q_i and price for manufacturer i by p_i. Assume $\sum_{i=1}^N q_i < K$. We first conjecture that, in the market clearing outcome $p_i = p_h$, $1 \leq i \leq n$ and $p_i = p_l$, $n+1 \leq i \leq N$, where $p_h = p_l + v$ and $p_l = K - \sum_{i=1}^N q_i$. Note that $\sum_{i=1}^N q_i \leq K$ guarantees $Q_h, Q_l > 0$. These prices indeed clear the market. To see this, notice that, first, all consumers are indifferent between the high-quality and the low-quality products. The consumers with base valuations in $[0, K - \sum_{i=1}^N q_i)$ do not choose to purchase either good, while the consumers with base valuations in $[K - \sum_{i=1}^N q_i, K]$ choose to purchase one. So the demand is exactly $\sum_{i=1}^N q_i$. Furthermore, this set of prices is the unique market clearing price set. To see this, first consider a price set such that $\min_{1\leq i\leq n} p_i > K + v - \sum_{i=1}^N q_i$ or $\min_{n+1\leq i\leq N} p_i > K - \sum_{i=1}^N q_i$. In that case, the consumer demand will be less than or equal to $K - \min\{\min_{1\leq i\leq n} p_i - v, \min_{n+1\leq i\leq N} p_i\} < K - (K - \sum_{i=1}^N q_i) = \sum_{i=1}^N q_i$, and the market will not clear. On the other hand, if we have a price set such that $\min\{\min_{1\leq i\leq n} p_i - v, \min_{n+1\leq i\leq N} p_i\} < K - \sum_{i=1}^N q_i$, then the demand is $K - \min\{\min_{1\leq i\leq n} p_i - v, \min_{n+1\leq i\leq N} p_i\} > \sum_{i=1}^N q_i$ and the market does not clear. Applying the same argument iteratively, we conclude that the only market clearing that price set satisfies the equilibrium conditions is $p_i = p_h$, $1 \leq i \leq n$ and $p_i = p_l$, $n+1 \leq i \leq N$, where $p_h = p_l + v$ and $p_l = K - \sum_{i=1}^N q_i$.

2^0. Denote the volume of high-quality products sold in the market in each period as Q_h, and the volume of low-quality products as Q_l. Considering the manufacturer behavior as described above and because the tariff structure gives the suppliers the flexibility to induce any quantity ordered by the manufacturers, the game between the suppliers becomes one of competing in quantities. The low-quality supplier chooses Q_l to solve $\max_{Q_l} E[Q_l(K - Q_h - Q_l - c_1)]$ and the high-quality supplier chooses Q_h to solve $\max_{Q_h} E[Q_h(K + v - Q_h - Q_l - c_0 - \delta)]$. These expectations are necessary because the respective production quantities depend on each supplier's random production costs, which are unobservable by the other supplier. The first-order condition for the low-quality supplier is $K - E[Q_h] - c_1 - 2Q_l = 0$. Therefore, the low-quality supplier will produce if and only if $c_1 < K - E[Q_h]$, and the production quantity will be $Q_l = \frac{1}{2}(K - E[Q_h] - c_1)^+$. Similarly, the high-quality supplier will produce if and only if $c_0 < K + v - \delta - E[Q_l]$, and her production quantity will be $Q_h = \frac{1}{2}(K + v - \delta - E[Q_l] - c_1)^+$.

We are now in a position to derive the expressions for c_l^* and c_h^*. Let $c_l^* = K - E[Q_h]$, and $c_h^* = K + v - \delta - E[Q_l]$. Then by the definition of $G(y)$ we have $E[Q_l] = \frac{1}{2}G(c_l^*)$. Plugging this in the definition of c_h^* and the first-order condition for the high-quality supplier we obtain $E[Q_h] = \frac{1}{2}G(K + v - \delta - \frac{1}{2}G(c_l^*))$. Substituting this into the definition of c_l^* implies that $c_l^* = K - \frac{1}{2}G(K + v - \delta - \frac{1}{2}G(c_l^*))$, and similarly because $E[Q_l] = \frac{1}{2}G(c_l^*)$, into the definition of c_h^* yield $c_h^* = K + v - \delta - \frac{1}{2}G(c_l^*)$. Next, notice that the left-hand side of (4) is monotonically increasing in c_l^*. Therefore if a solution to (4) exists, it will be unique and the low-quality supplier is out of the market only if

$$c_l^* - K + \frac{1}{2}G\left(K + v - \delta - \frac{1}{2}G(c_l^*)\right)\Big|_{c_l^* = \underline{c}}$$
$$= \underline{c} - K + \frac{1}{2}G(K + v - \delta) \geq 0, \tag{A.12}$$

which gives the desired condition.

3^0. The next step is to determine the wholesale prices that will induce the manufacturers' trading with the high-quality supplier to place a total order Q_h, and the manufacturers trading with the low-quality supplier to place a total order Q_l. Let w_h denote the wholesale price for high-quality goods and w_l denote the wholesale price for low-quality goods.

It follows from the first-order conditions for the high- and low-quality suppliers, from the definitions of c_l^* and c_h^*, that $Q_h = \frac{1}{2}(c_h^* - c_0)^+$, and $Q_l = \frac{1}{2}(c_l^* - c_1)^+$. Hence, the order quantity desired by each of the N manufacturers is

$$q_i = \begin{cases} \dfrac{1}{2n}(c_h^* - c_0)^+ & \text{for } i = 1, \ldots, n \\[2ex] \dfrac{1}{2(N-n)}(c_l^* - c_0)^+ & \text{for } i = n+1, \ldots, N. \end{cases} \tag{A.13}$$

Manufacturer $i = 1, \ldots, n$ determines the production quantity q_i by maximizing $E[q_i(K + v - \sum_{j=1, j\neq i}^{N} q_j - q_i - w_0)]$, which implies the first-order condition

$$K + v - E\left[\sum_{j=1, j\neq i}^{N} q_j\right] - 2q_i - w_0 = 0.$$

Because it is desired to obtain wholesale prices such that $\sum_{j=1}^{N} q_j = Q_h + Q_l$ and $q_i = (1/n)Q_h$, it follows that

$$w_h = K + v - E[Q_l] - \frac{n+1}{n}Q_h = c_h^* - \frac{n+1}{2n}(c_h^* - c_0)^+, \tag{A.14}$$

where (A.14) follows by using the substitution $c_h^* = K + v - E[Q_l]$, and $Q_h = \frac{1}{2}(c_h^* - c_0)^+$. Similar analysis leads to the expression for w_1. The tariffs g_h and g_l are then calculated so that the two suppliers can extract the full surplus from the manufacturers.

We must show one final thing in that our assumption in the beginning of the proof that $\sum_{i=1}^{N} q_i \leq K$ is indeed satisfied. To see this

$$\sum_{i=1}^{N} q_i \leq \frac{1}{2}(c_h^* - \underline{c})^+ + \frac{1}{2}(c_l^* - \underline{c})^+$$
$$= \frac{1}{2}\left(K + v - \delta - \frac{1}{2}G(c_l^*) - \underline{c}\right) + \frac{1}{2}\left(K - \frac{1}{2}G(c_h^*) - \underline{c}\right)$$
$$= K - \underline{c} + \frac{1}{2}(v - \delta) - \frac{1}{2}\left(\frac{1}{2}G(c_h^*) + \frac{1}{2}G(c_l^*)\right). \tag{A.15}$$

Now $\frac{1}{2}G(c_h^*) + \frac{1}{2}G(c_l^*)|_{c_l^* = \underline{c}} - \frac{1}{2}G(K + v - \delta) = 0$. However, plugging in $dc_h^*/d(v - \delta)$ and $dc_l^*/d(v - \delta)$ from (4), and using $c_h^* = K + v - \delta - \frac{1}{2}G(c_l^*)$ and Lemma 1, we obtain that $\frac{1}{2}G(c_h^*) + \frac{1}{2}G(c_l^*) - \frac{1}{2}G(K + v - \delta)$ is increasing in $v - \delta$ and hence $\frac{1}{2}G(c_h^*) + \frac{1}{2}G(c_l^*) \geq \frac{1}{2}G(K + v - \delta)$ for $K - \underline{c} - \frac{1}{2}G(K + v - \delta) \geq 0$. Therefore,

$$\sum_{i=1}^{N} q_i \leq K - \underline{c} + \frac{1}{2}(v - \delta) - \frac{1}{4}G(K + v - \delta)$$
$$\leq \frac{1}{2}(v - \delta) - \frac{1}{4}G(K + v - \delta) - \underline{c} + K \leq K, \tag{A.16}$$

because $v - \delta < K + \underline{c}$ and $K - \underline{c} - \frac{1}{2}G(K + v - \delta) \geq 0$. This completes the proof of Part (ii).

Finally, to see Part (iii), notice that $c_h^* = K + v - \delta - \frac{1}{2}G_1(c_l^*) \geq K + v - \delta - \frac{1}{2}G_1(K) \geq \frac{1}{2}(K + \mu_1) + v - \delta > \underline{c}$. This completes the proof of the proposition. □

Proof of Corollary 1. Define the function $\phi(c_l^*) = c_l^* - K + \frac{1}{2}G(K + v - \delta - \frac{1}{2}G_1(c_l^*))$. Notice that the solution c_l^* is in the interior $[\underline{c}, \bar{c}]$ if $\phi(\underline{c}) < 0$ and $\phi(\bar{c}) > 0$. The condition for the first inequality was given in Part (i) of Proposition 2. The second inequality can be obtained by plugging in $c_l^* = \bar{c}$ and using Lemma 1. Similarly, c_h^* is in the interior $[\underline{c}, \bar{c}]$ if $K + v - \delta - \frac{1}{2}G(c_l^*) < \bar{c}$. Plugging in $c_h^* = \bar{c}$ and again using Lemma 1 we obtain the condition stated for c_h^* of the proposition. □

Proof of Proposition 3. The proof proceeds along the lines of the proof of Proposition 2. The major difference emerges from the surplus left to the manufacturers in equilibrium. This is determined by the outcome of the auction. We start by the equilibrium outcome here and the rest follows in the lines of the arguments in Proposition 2 with modifications on sharing of the surplus.

Denote the profit for the jth lowest-cost supplier in the auction by $\Pi_{(j)}^s$ and that for manufacturer i by $\Pi_{(i)}^m$. In the auction equilibrium, the lowest-cost supplier is the winner and her wholesale price and two-part tariff are determined by the marginal cost of the second-lowest-cost supplier as well as her own costs. Her two-part tariff scheme is set so that (i) it maximizes the supply chain profits under production at her marginal cost; and (ii) it leaves the manufacturers in the auction a surplus equally divided among them, and that would maximize the supply chain profits with the second-lowest-cost supplier's marginal costs. This implies

$$w_l = c_l^* - \frac{(N-n+1)}{2(N-n)}(c_l^* - c_{(1)})^+, \quad \text{and} \qquad g_l = \frac{[(c_l^* - c_{(1)}^M)^+]^2}{4(N-n)}. \tag{A.17}$$

Now, given this, the best response for the second-lowest-cost supplier is

$$w_{(2)} = c_l^* - \frac{(N-n+1)}{2(N-n)}(c_l^* - c_{(2)})^+, \quad \text{and} \qquad g_{(2)} = \frac{[(c_l^* - c_{(2)}^M)^+]^2}{4(N-n)}. \tag{A.18}$$

This is because, under this pricing, the second-lowest-cost supplier makes zero profit and for any other $(w, g) \in R^2$ either (i) $\Pi^s_{(2)}(w, g) < 0$, or (ii) $E[\Pi^m_i(w, g)] < E[\Pi^m_i(w_l, g_l)]$ for $n+1 \le i \le N$, and therefore $\Pi^s_{(2)}(w, g) = 0$. Furthermore, the tariff scheme given in (A.17) is the best response for the lowest-cost supplier because for any other $(w, g) \in R^2$ it is either (i) $\Pi^s_{(1)}(w, g) < \Pi^s_{(1)}(w_l, g_l)$, or (ii) $E[\Pi^m_i(w, g)] < E[\Pi^m_i(w_{(2)}, g_{(2)})]$, and hence $\Pi^s_{(1)}(w, g) = 0$. $\square$

Proof of Corollary 2. For Parts (i) and (ii), we will provide the proof only for the effect of K on $E[Q_h]$ and $E[p_h]$. The proofs for $v-\delta$ will be similar. From (4) and because $c_h^* = K + v - \delta - \frac{1}{2}G(c_l^*)$, we have

$$\begin{aligned} \frac{dc_h^*}{dK} &= 1 - \frac{1}{2}F(c_l^*)\frac{dc_l^*}{dK} \\ &= 1 - \frac{1}{2}F_1(c_l^*)\frac{1-\frac{1}{2}F(c_h^*)}{1-\frac{1}{4}F(c_h^*)F_1(c_l^*)} > 0. \end{aligned} \tag{A.19}$$

Therefore, by Lemma 1, $E[Q_h] = \frac{1}{2}G(c_h^*)$ is increasing in K. Now, $E[p_h] = K + v - E[Q_h + Q_l]$. This implies

$$\begin{aligned} \frac{dE[p_h]}{dK} &= 1 - \frac{dE[Q_h]}{dK} - \frac{dE[Q_l]}{dK} \\ &= 1 - \frac{1}{2}F(c_h^*)\left(1 - \frac{1}{2}F_1(c_l^*)\frac{1-\frac{1}{2}F(c_h^*)}{1-\frac{1}{4}F(c_h^*)F_1(c_l^*)}\right) \\ &\quad - \frac{1}{2}F_1(c_l^*)\frac{1-\frac{1}{2}F(c_h^*)}{1-\frac{1}{4}F(c_h^*)F_1(c_l^*)} \\ &= 1 - \frac{1}{2(1-\frac{1}{4}F(c_h^*)F_1(c_l^*))}\left[F(c_h^*)\left(1-\frac{1}{2}F_1(c_l^*)\right)\right. \\ &\qquad \left. + F_1(c_l^*)\left(1 - \frac{1}{2}F(c_h^*)\right)\right] \\ &= \frac{2 - F(c_h^*) - F_1(c_l^*) + \frac{1}{2}F(c_h^*)F_1(c_l^*)}{2(1-\frac{1}{4}F(c_h^*)F_1(c_l^*))} > 0, \end{aligned} \tag{A.20}$$

that is, $E[p_h]$ is increasing in K. Remaining statements can be derived similarly.

For Part (iii), we will only give the proof for $E[Q_h]$. The rest of the statements will be similar. First, clearly $c_{(1)}^{(M)}$ is first-order stochastically dominated by $c_{(1)}^{(M')}$ when $M > M'$ and hence $G^M_{(1)}(c) = E[(c - c^M_{(1)})^+] > E[(c - c^{M'}_{(1)})^+] = G^{M'}_{(1)}(c)$. Therefore, plugging $G_1 = G^M_{(1)}$ in (4), when M increases, for any fixed c, the left-hand side decreases. Since there is still a unique solution to the equation, it follows that, when M increases, c_l^* increases. Combining this with $c_h^* = K + v - \delta - \frac{1}{2}G(c_l^*)$, and again with Lemma 1, we conclude that $E[Q_h]$ is decreasing with M. The rest of the statements follow by similar arguments. $\square$

Proof of Proposition 4. Part (i) follows from the fact that when $\frac{1}{2}G(K+v-\delta) + \underline{c} > K$, the low-quality product is not viable in the market as found in Proposition 2. To see Part (ii), notice that when she stays in relational contracting, the high-quality supplier has an expected surplus of $\max_{\{1\le n\le N\}} E[J(n,t)]$. Alternatively, if she switches to the electronic market, her expected payoff at each period will be $1/(M+1)[H^{M+1}_{(1)}(K) - H^{M+1}_{(2)}(K)]$. Hence, she will choose to provide high-quality output only if

$$\max_{\{1\le n\le N\}} E[J(n,t)] \ge \frac{1}{M+1}\left[H^{M+1}_{(1)}(K) - H^{M+1}_{(2)}(K)\right],$$

and in the case of equality, she will always choose to cheat. Therefore, an equilibrium as described can be sustained only when (8) is satisfied. Furthermore, replicating the arguments in Proposition 1 with $\theta = c_h^*$ gives (9). This completes the proof. $\square$

Proof of Proposition 5. To see Part (i), notice that the optimal number of high-quality manufacturers is N if and only if

$$\begin{aligned} &H(K+v-\delta) - NH^M_{(2)}(c_l^*) \\ &\ge \max\left\{H(c_h^*) - \frac{1}{N}H^M_{(2)}(K), H(c_h^*) - \frac{2}{N-1}H^M_{(2)}(c_l^*)\right\}. \end{aligned} \tag{A.21}$$

Consider first the inequality $H(K+v-\delta) - NH^M_{(2)}(c_l^*) \ge H(c_h^*) - 2/(N-1)H^M_{(2)}(c_l^*)$. This is satisfied if and only if

$$\begin{aligned} \varphi &\triangleq H(K+v-\delta) - H\left(K+v-\delta - \frac{1}{2}G^M_{(1)}(c_l^*)\right) \\ &\quad - \frac{N^2-N-2}{N-1}H_{(2,M)}(c_l^*) \ge 0; \end{aligned} \tag{A.22}$$

this follows by using $c_h^* = K + v - \delta - \frac{1}{2}G^M_{(1)}(c_l^*)$. This inequality is satisfied with equality at $c_l^* = \underline{c}$, that is, when $K = \frac{1}{2}G(K+v-\delta) + \underline{c}$. Therefore, it can be shown that this inequality is satisfied in the neighborhood of $K = \frac{1}{2}G(K+v-\delta) + \underline{c}$ by demonstrating that the first derivative of (A.22) with respect to K is positive. First,

$$\frac{d\varphi}{dK} = \frac{\partial\varphi}{\partial K} + \frac{\partial\varphi}{\partial c_l^*}\frac{dc_l^*}{dK}. \tag{A.23}$$

Now by Lemma 1, $\partial\varphi/\partial K = 2(G(K+v-\delta)-G(c_h^*)) > 0$. Furthermore, at $K=\frac{1}{2}G(K+v-\delta)+\underline{c}$, $c_l^*=\underline{c}$ and

$$\left.\frac{\partial\varphi}{\partial c_l^*}\right|_{c_l^*=\underline{c}} = G\Big(K+v-\delta-\frac{1}{2}G_{(1)}^M(\underline{c})\Big)F_{(1)}^M(\underline{c}) - \frac{N^2-N-2}{N-1}G_{(2)}^M(\underline{c}) = 0. \quad \text{(A.24)}$$

It follows by the continuity of φ that there exists a $\kappa>0$ such that if $\frac{1}{2}G(K+v-\delta)+\underline{c}\le K\le \frac{1}{2}G(K+v-\delta)+\underline{c}+\kappa$ then (A.22) will be satisfied. A similar argument applies to the case $H(K+v-\delta)-NH_{(2)}^M(c_l^*)\ge H(c_h^*)-(1/N)H_{(2)}^M(c_l^*)$. This proves Part (i). Part (ii) follows by letting $N\to\infty$ in (5). □

Proof of Proposition 6. Again we will give the proof only for the first statement and the rest will be similar. To see that β^* is decreasing in v, first we have

$$\frac{d\beta^*}{dv} = \frac{\partial\beta^*}{\partial c_h^*}\frac{dc_h^*}{dv} + \frac{\partial\beta^*}{\partial c_l^*}\frac{dc_l^*}{dv}. \quad \text{(A.25)}$$

Now, using (4) and $c_h^* = K+v-\delta-\frac{1}{2}G(c_l^*)$ and substituting G_1 with $G_{(1)}^M$, we have

$$\frac{dc_l^*}{dv} = -\frac{\frac{1}{2}F(c_h^*)}{1-\frac{1}{4}F(c_h^*)F_{(1)}^M(c_l^*)} \quad \text{and} \quad \frac{dc_h^*}{dv} = \frac{1}{1-\frac{1}{4}F(c_h^*)F_{(1)}^M(c_l^*)}. \quad \text{(A.26)}$$

Using (9) to find $\partial\beta^*/\partial c_l^*$ and $\partial\beta^*/\partial c_h^*$ and plugging in (A.26), we find that $d\beta^*/dv$ is negative if

$$2\delta\Big(H(c_h^*)-2G(c_h^*)(c_h^*-\underline{c}) - \frac{1}{M+1}\big[H_{(1)}^{M+1}(K)-H_{(2)}^{M+1}(K)\big]\Big)<0. \quad \text{(A.27)}$$

But this is satisfied because, by Lemma 1, $2G(c_h^*)(c_h^*-\underline{c}) > H(c_h^*)$ and $H_{(1)}^{M+1}(K) > H_{(2)}^{M+1}(K)$. Therefore $d\beta^*/dv<0$. □

Proof of Proposition 7. For Part (i), we first have to examine the behavior of c_l^* and c_h^* as $K\to\infty$. To see that, first, by Proposition 3, as $K\to\infty$, $K\ge c_l^*>\underline{c}$ holds and the electronic market is operational. By dividing each side in (4), we obtain

$$\frac{c_l^*}{K} = 1-\frac{1}{2K}G\Big(K+v-\delta-\frac{1}{2}G(c_l^*)\Big). \quad \text{(A.28)}$$

Now, clearly $0<\lim_{K\to\infty}\{(K+v+\delta-\frac{1}{2}G(c_l^*))/K\}\le 1$. Suppose that

$$\lim_{K\to\infty}\big\{(K+v+\delta-\tfrac{1}{2}G(c_l^*))/K\big\}=1. \quad \text{(A.29)}$$

That would imply, first,

$$\lim_{K\to\infty}\big\{(\tfrac{1}{2}G(c_l^*))/K\big\} = \lim_{K\to\infty}\{c_l^*/K\}=0,$$

and second, by Lemma 1,

$$\lim_{K\to\infty}\{G(K+v-\delta-\tfrac{1}{2}G(c_l^*))/(2K)\}=0,$$

which, by (A.28), implies that

$$\lim_{K\to\infty}\{c_l^*/K\}=1, \quad \text{(A.30)}$$

i.e., a contradiction. Therefore, $0<\lim_{K\to\infty}\{(K+v+\delta-\frac{1}{2}G(c_l^*))/K\}<1$ and hence, by the fact that $K\ge c_l^*$ and again by Lemma 1, it follows that $0<\lim_{K\to\infty}(\frac{1}{2}G(c_l^*))/K<1$ and $0<\lim_{K\to\infty}\{c_l^*/K\}<1$. Plugging this in (A.28), once again, using Lemma 1, and because by Proposition 3 $c_h^*=K+v-\delta-\frac{1}{2}G(c_l^*)$, we obtain

$$\lim_{K\to\infty}\{c_l^*/K\} = \lim_{K\to\infty}\{c_h^*/K\}=2/3. \quad \text{(A.31)}$$

Equation (A.31) and Lemma 1 imply that

$$\lim_{K\to\infty}\frac{\max_{\{1\le n\le N\}}E[J(n,t)]}{K^2} = \varpi$$

where $0<\varpi<1$ and $\lim_{K\to\infty}\{(H_{(1)}^{M+1}(K)-H_{(2)}^{M+1}(K))/K^2\}=0$. Combining these with the fact that $\lim_{K\to\infty}\{c_h^*/K^2\}=0$ and taking the limit as $K\to\infty$ in (9), we obtain $\lim_{K\to\infty}\beta^*=0$.

To see Part (ii) first, suppose $\bar{c}<\infty$ and notice that when $K>\bar{c}$, and as $(v-\delta)\to 0$, in simultaneous solution to (4) and $c_h^*=K+v-\delta-\frac{1}{2}G(c_l^*)$, with $c_l^*>0$, we have $c_h^*\le(2K+\mu)/3\le c_l^*$. Therefore, for $N=2$ and for sufficiently large M, to have $\max_{\{1\le n\le N\}}E[J(n,t)]\le 0$, it is sufficient to have

$$H\Big(\frac{2K+\mu}{3}\Big)\le\frac{1}{2}(K-\underline{c})^2, \quad \text{and} \quad \text{(A.32)}$$

$$H(K)\le 2\Big(\frac{2K+\mu}{3}-\underline{c}\Big)^2. \quad \text{(A.33)}$$

Choosing $\bar{c}<\mu+\varepsilon$ where $\varepsilon>0$ and σ are sufficiently small and using Lemma 1, it is straightforward to see that (A.32) and (A.33) hold if

$$\mu>\frac{3}{2\sqrt{2}}\underline{c} \quad \text{and} \quad \mu<K<\frac{3+\sqrt{2}}{3-2\sqrt{2}}\mu-\frac{3\sqrt{2}}{3-2\sqrt{2}}\underline{c}, \quad \text{(A.34)}$$

respectively. Therefore, there exists an $\bar{N}\ge 2$ such that when $N\le\bar{N}$, there exists distributions such that for a critical value $\underline{\Theta}>0$, $\lim_{(v-\delta)\to\underline{\Theta}^+}\max_{\{1\le n\le N\}}E[J(n,t)]=0$, making the denominator of β^* will converge to zero, and when δ is bounded away from zero, providing the desired result. □

Proof of Proposition 8. Part (i) follows from noting $E[Q_h]+E[Q_l]=\frac{1}{2}G(c_h^*)+\frac{1}{2}G_{(1)}^M(c_l^*)$ and the expected total quantity in the benchmark monopoly case being $\frac{1}{2}G(K+v-\delta)$ and following the lines of the argument given in the end of phase 2^0 of the proof of Part (ii) of Proposition 2.

To see the first statement of Part (ii), notice that as $v-\delta\to 0$, and $K\to\underline{c}$, the expected supply chain surplus for both cases converges to zero. Also notice that by

Equation (4), and because $c_h^* = K + v - \delta - \frac{1}{2}G(c_l^*)$, it follows that

$$\left.\frac{dc_l^*}{dK}\right|_{K=\underline{c},\, v-\delta=0} = \left.\frac{dc_h^*}{dK}\right|_{K=\underline{c},\, v-\delta=0} = 1.$$

Using this result, taking the derivatives of the total supply chain surplus difference with respect to K sequentially and using Lemma 1 we find that the supply chain surplus under competition from the auction is larger than that of the benchmark case in a neighborhood of $K = \underline{c}$ if and only if $f_{(1)}^M(\underline{c}) > 0$ or $f_{(1)}^M(\underline{c}) = 0$ and $\lim_{c\to\underline{c}^+} f_{(1)}^M(c) > 0$, which is satisfied by the definition of $\underline{c}$. To see the second statement, the assumption that N is large implies that the optimal number of suppliers in the relational contract is less than N. Now, let $Q_T = Q_h + Q_l$. Then, the total expected social surplus with the electronic market is

$$\begin{aligned} &E[W^{wa}] \\ &= E\left[\frac{1}{2}Q_T^2 + (K + v - \delta - Q_T - c_0)Q_h + \big(K - Q_T - c_{(1)}^M\big)Q_l\right] \\ &= \frac{1}{8}\big(H(c_h^*) + H_{(1)}^M(c_l^*) + 2G(c_h^*)G_{(1)}^M(c_l^*)\big) + \frac{1}{4}H(c_h^*) + \frac{1}{4}H_{(1)}^M(c_l^*) \\ &= \frac{1}{2}\bigg[G_{(1)}^M(c_l^*)(K - c_l^*) + \frac{3}{4}H\Big(K + v - \delta - \frac{1}{2}G_{(1)}^M(c_l^*)\Big) \\ &\qquad + \frac{3}{4}H_{(1)}^M(c_l^*)\bigg]. \end{aligned} \tag{A.35}$$

The expected social surplus with the electronic market when $c_l^* = \underline{c}$ is equal to the expected social surplus of the monopolist because setting $c_l^* = \underline{c}$ implies that the electronic market shuts down and only the high-quality supplier may trade in each period. Therefore, to establish the result we first show that $E[W^{we}]$ is decreasing in c_l^* in the neighborhood of $c_l^* = \underline{c}$. To see this, taking the derivative with respect to $v - \delta$,

$$\frac{dE[W^{wa}]}{d(v-\delta)} = \frac{\partial E[W^{wa}]}{\partial(v-\delta)} + \frac{\partial E[W^{wa}]}{\partial c_l^*}\frac{dc_l^*}{d(v-\delta)}. \tag{A.36}$$

Now, $\partial E[W^e]/\partial(v-\delta) = 3G(c_h^*)/4 > 0$. Moreover, once again, at $K = \underline{c} + \frac{1}{2}G(K + v - \delta)$, $c_l^* = \underline{c}$ and

$$\begin{aligned} \left.\frac{\partial E[W^{wa}]}{\partial c_l^*}\right|_{c_l^*=\underline{c}} &= \frac{1}{2}\bigg(F_{(1)}^M(c_l^*)(K - c_l^*) - G_{(1)}^M(c_l^*) \\ &\qquad \left. - \frac{3}{4}G(c_h^*)F_{(1)}^M(c_l^*) + \frac{3}{2}G_{(1)}^M(c_l^*)\bigg)\right|_{c_l^*=\underline{c}} = 0. \end{aligned} \tag{A.37}$$

Therefore, $E[W^a]$ is strictly increasing in a neighborhood of $K = \underline{c} + \frac{1}{2}G(K + v - \delta)$.

By Proposition 5, when K is in the neighborhood of the $K = \underline{c} + \frac{1}{2}G(K + v - \delta)$ and when N is large enough, both the auction and relational contracting are active venues for procurement. Combining this with $dE[W^a]/d(v-\delta) > 0$ establishes that the social surplus with the electronic market is strictly less than the social surplus with the monopoly in that neighborhood as desired. A similar argument applies for expected total supply chain profit. This completes the proof. □

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