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When Smaller Menus Are Better: Variability in Menu-Setting Ability

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Are large menus better than small menus? Recent literature argues that individuals' apparent preference for smaller menus can be explained by their behavioral biases or informational limitations. These explanations imply that absent behavioral or informational effects, larger menus would be objectively better. However, in an important economic context—401(k) pension plans—we find that larger menus are objectively worse than smaller menus, as measured by the maximum Sharpe ratio achievable. We propose a model in which menu setters differ in their ability to preselect the menu. We show that when the cost of increasing the menu size is sufficiently small, a lower-ability menu setter optimally offers more items in the menu than a higher-ability menu setter. Nevertheless, the menu optimally offered by a higher-ability menu setter remains superior. This results in a negative relation between menu size and menu quality: smaller menus are better than larger menus.

Key words: menu; menu setting; choice; pension plans; 401(k)

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1. Introduction

In many settings, people choose from a menu designed by someone else. This can be a literal menu at a restaurant, a choice of products on a supermarket shelf, or a list of assets available for investment in a 401(k) retirement plan. There is a growing literature showing how choices are influenced by the composition of the menu and that choosers often prefer smaller menus to larger menus. Explanations of this phenomenon often appeal to behavioral biases (e.g., choice overload) or to informational limitations of choosers that are alleviated when the menu size is reduced. However, the existing explanations imply that fully rational and fully informed choosers would still prefer larger menus.

Although the approach in the existing literature is plausible and applies in some contexts, we propose a model in which larger menus can be objectively worse, even for fully rational and informed choosers. Our model is based on the observation that the size and the composition of the menu are themselves the result of a prior selection by another agent, whom we refer to as the *menu setter*. When menu setters differ in ability, which is likely in practice, there can be a negative relation between the menu size and the objective quality of the menu.

Our argument is not just a theoretical abstraction. We show empirically that larger menus are objectively worse than smaller menus, on average, in an important economic context—401(k) pension plans, where a plan is a menu of investment choices. This empirical finding cannot be explained by existing theories.

When an employee is offered a menu of potential investments in a firm's 401(k) pension plan, ideally he would like to choose a portfolio that achieves a high expected return while at the same time having low risk. Low risk is achieved by diversification, that is, by choosing assets that have low correlations with each other. An investment choice adds to the quality of a plan by having a high expected return and a low correlation with other assets in the menu. However, some investment choices do not increase the quality of the plan, for example, if they have lower expected returns or are highly correlated with other assets in the menu. We measure the overall quality of each plan as the highest Sharpe ratio achievable from portfolios that can be constructed from the assets in that plan.¹ This is a measure of the objective quality of a plan, achievable by fully rational and informed investors. We find

¹ The Sharpe ratio is a common measure of portfolio quality that incorporates both risk and return. In §4 we detail how we estimate the Sharpe ratio.

a negative relation between the number of investment choices and the plan quality. Under existing theories, this negative relation would be puzzling. However, it can be rationalized by our model, in which menu setters differ in ability.

We model menu setters as selecting a menu of items from a large universe. An item may be *valuable* or *useless*. Only valuable items improve the quality of the menu. However, the marginal benefit of each additional valuable item declines. High-ability (or *expert*) menu setters are those who can always identify valuable items and for any given menu size offer a menu of the highest possible quality; lower-ability menu setters may inadvertently offer some useless items. In the absence of any cost of adding items to the menu, a menu setter of either ability could include the whole universe of items, and individuals would be able to find their most favored choice.² However, if there is a cost to offering more items on the menu, for example, a fixed cost of stocking or managing each item, then the menu setter would limit the number of choices offered.

We show that for a given cost, a lower-ability menu setter may optimally offer a longer menu than an expert menu setter. For the lower-ability menu setter, the marginal benefit of adding the n th item to the menu may be lower or higher than the marginal benefit of adding the n th item for the expert. On one hand, for any additional item, the expected marginal benefit to the lower-ability menu setter is reduced because there is a certain probability that the item is useless. On the other hand, if previous items in the lower-ability menu setter's menu were useless, the marginal benefit of the n th item, if it is a valuable item, is higher than the value of the n th item for the expert. We show that, in general, when the cost of adding an item to the menu is large (and thus the menu size is relatively small), the lower-ability menu setter optimally offers fewer items in his menu than the expert. Conversely, when the cost is small (and the menu size is large), we obtain the opposite effect: the lower-ability menu setter offers more items. Nevertheless, for any cost, the quality of the menu offered by the expert is superior to the quality of the menu offered by the lower-ability menu setter. Thus, for a low cost, we obtain a seemingly paradoxical result that a smaller menu is of higher quality than a larger menu.

1.1. Related Literature

The fact that individuals often prefer small menus over large menus is not new to the economics and management literature. For example, the reluctance to choose from large menus is documented

in Iyengar and Lepper (2000) for supermarket purchases, Bertrand et al. (2010) for consumer credit, and Iyengar et al. (2004) for 401(k) pension plans.³ The most frequent explanation refers to choice overload, based on behavioral biases or bounded rationality. Moreover, a number of recent books have argued that psychology (e.g., Schwartz 2004, Iyengar 2010) and neurology (Lehrer 2009) can affect individuals' choices. Our study differs from the literature that focuses on individual behavior and psychology, because we focus on the menu-setting decision in an environment without behavioral biases. We assume that menu setters are rational, and to evaluate the quality of menus, we assume that the individuals choosing from the menu are also rational.

Closely related to our paper is a small but growing theoretical literature that addresses the decision problem of the menu setter, and rational agents' choices in response to optimal menu setting. In Kamenica (2008), informationally disadvantaged consumers cannot identify which menu items most closely match their preferences. Thus, the menu setter limits the menu to the most popular options to increase the probability of a valuable match. In Villas-Boas (2009), informationally disadvantaged consumers must pay to evaluate menu items. If the menu size is too large, a seller (i.e., a menu setter) can extract so much surplus—after the evaluation cost is paid—that the consumer will choose ex ante not to pay the evaluation cost, that is, not to participate. Therefore, the menu setter limits the menu size as a commitment not to extract the entire surplus, thus encouraging the consumer to participate. Similarly, Kuksov and Villas-Boas (2010) argue that the menu setter limits the number of choices to make it worthwhile for choosers to evaluate the alternatives. This literature provides a compelling rationale for why choosers who lack full information may prefer a limited menu that is a subset of a larger menu. However, in these models, fully informed choosers are better served by larger menus.

In our paper, in the context of 401(k) pension plans, we show empirically that smaller menus, on average, are objectively better than larger menus. In other words, a fully informed and fully rational investor would prefer a smaller menu. An important part of this result is that when menu setters differ in ability, small menus are not subsets of larger menus, but include more valuable items than larger menus. Of course, the relevance of each model depends on the empirical setting. Our model does not explain the preference for subsets, and Kamenica (2008), for example, does not explain our empirical finding

² As explained above, to abstract from behavioral and informational effects, we assume that individuals are fully rational and costlessly identify their most desired choices.

³ In an experimental study, Salgado (2006) shows that individuals' preferences for smaller menus depend on their perceptions of the skill of the menu setter.

that smaller 401(k) menus are objectively superior to larger menus.

There is a fairly large literature on 401(k) plans documenting the investment decisions made by employees and the behavioral biases that often drive those decisions.⁴ Our paper differs from this literature by focusing on the quality of the menus in 401(k) plans rather than the decisions of individuals; it shows that smaller menus often have better risk-return characteristics than larger menus. An empirical paper that helps motivate our study is by Elton et al. (2006). They find that approximately half of all 401(k) plans are inadequate; that is, they do not span a particular set of indices. Their result opens the question of the role of the menu setter's ability, which we directly address in our paper.

2. Motivating Empirical Regularity

The motivating example that leads us to develop our model is a regularity in firms' 401(k) plans offered to their employees. The key feature of a 401(k) plan is that it offers a menu of investment choices. Employees construct a retirement portfolio by choosing from the options in the 401(k) plan.

We analyze a sample of 401(k) plan menus. For each plan, we identify the optimal portfolio that can be constructed from among the investment choices. We use the Sharpe ratio as an objective measure of a portfolio's quality, and we define the optimal portfolio as the one that achieves the highest possible Sharpe ratio. We find that, on average, the optimal portfolios constructed from smaller (i.e., shorter) menus are objectively better than the optimal portfolios constructed from larger (i.e., longer) menus. We describe the data and empirical results in detail in §§4 and 5.

Our finding is intriguing because it runs counter to the traditional model of choice. In the traditional model, longer menus are better than shorter menus for fully rational agents, because longer menus expand the choice space and allow choosers to obtain more preferred outcomes. In the context of investment portfolios, longer menus generally expand the efficient frontier, allowing for the construction of portfolios with higher returns and/or lower risk.

The fact that we observe smaller menus that are better than larger menus without any reference to psychological biases suggests that these smaller menus are not subsets of the larger menus. If smaller menus were subsets of larger menus, then it would not be possible for a smaller menu to be objectively better

than a larger menu. Our empirical regularity suggests that a theory is needed to explain why smaller menus are composed of different options than larger menus, and specifically how smaller menus can be objectively better than larger menus.

In §3 we develop a model that shows the economic forces that lead menu setters to choose different sizes and compositions for their menus. In particular, we show conditions under which smaller menus are likely to be better than larger menus. The economic forces identified in the model are general and not confined to the context of 401(k)s. Therefore, even though the empirical motivation comes from 401(k) plans, we present the model without imposing any specific context.

3. Model

Suppose that individuals choose a good or a bundle of goods from a menu that is comprised of a number of items. The menu itself is preselected from a large universe of goods by a menu setter. The menu setter selects a subset of n items to be included in the menu offered to individuals.

The universe of possible menu items includes both goods that are valuable as well as goods that have no value, that is, useless goods.⁵ We assume that the universe includes an infinite number of valuable items and an infinite number of useless items. Individuals who ultimately choose from among the menu items prefer a menu with many valuable items, either because each requires a bundle with a variety of valuable items (e.g., when choosing an investment portfolio) or because different individuals have different preferences across items (e.g., when choosing ice cream flavors).⁶ Thus, the menu is improved by including more valuable items in it, but it is not improved by including useless items. Otherwise, each valuable item is a priori the same, and the quality of the menu can be summarized by the number of valuable items included in the menu.

Denote the quality of a menu with n valuable items as $Q(n)$, which can be interpreted as the total utility achieved by the group of individuals optimally choosing from the menu. The marginal benefit of the n th valuable item in the menu is the increase in the quality of the menu due to the n th valuable item; that is,

⁵ Of course, in reality, goods can be of many different values. However, to starkly illustrate the driving forces, we take the clearest case in which goods are either valuable or useless.

⁶ To focus on the effects of variation in menu-setter ability, we abstract away from any behavioral biases or informational limitations faced by the chooser. See Kamenica (2008) for a model with fully informed menu setters and informationally disadvantaged choosers.

⁴ This literature includes Agnew (2006), Agnew et al. (2003), Benartzi and Thaler (2001), Iyengar et al. (2004), Huberman and Jiang (2006), Iyengar and Kamenica (2010), Mottola and Utkus (2003), and Tang et al. (2010).

$q(n) = Q(n) - Q(n-1)$. Adding a useless item to the menu has no marginal benefit. Adding each valuable item to the menu always increases the quality of the menu, but not to the same degree. We assume that the quality improvement with each valuable item is decreasing in the number of items already included in the menu. That is, we assume that $Q(n)$ is strictly increasing but concave in n , or equivalently, that $q(n)$ is strictly positive for any n and strictly decreasing in n .

The marginal benefit in our model can be interpreted in different ways depending on the setting. If individuals have heterogeneous preferences and each chooses one item from the menu, then the marginal benefit of the n th valuable item is derived from offering a menu that can better satisfy the disparate preferences of the customers. Alternatively, if each individual chooses a bundle from the menu (e.g., a set of mutual funds in a 401(k) plan), then the marginal benefit of the n th valuable item can be interpreted as improving the bundle that will be chosen, even if choosers are homogeneous. In either case, it is reasonable to assume that the marginal benefit is positive and declining.⁷

ASSUMPTION OF DECLINING MARGINAL BENEFIT. *The marginal benefit of an additional valuable item in a menu is always positive, but strictly declining in the number of valuable items already in the menu; that is, $q(n) > 0$ and $q(n+1) < q(n)$ for all n .*

At this stage, the assumption of declining marginal benefit is very general. Later we propose a stronger assumption that imposes more structure on $q(n)$.

We consider two types of menu setters: high-ability menu setters, whom we refer to as *experts*, and lower-ability menu setters. We assume that each individual is exogenously assigned to a menu setter, and we do not consider competition between menu setters.⁸ Expert menu setters can always identify valuable items. Thus, when an expert offers a menu with n items, they are all valuable and the quality of his menu is equal to $Q(n)$. Lower-ability menu setters recognize with error whether an item is valuable. We assume that each item offered by a lower-ability menu setter is valuable with probability p and is use-

less with probability $1-p$.⁹ The ability of a menu setter is characterized by p .

When a lower-ability menu setter selects n items for the menu, it is not certain how many valuable items are included. Therefore, for any number of items n , the quality of the menu is a random variable, and its expectation depends on the menu setter's skill, p . Let $\mathbb{E} Q^p(n)$ denote the expected quality of a menu of size n chosen by a lower-ability menu setter with ability p . Similar to $q(n)$, we define $\mathbb{E} q^p(n)$ as the expected marginal benefit of the n th item in a menu chosen by a lower-ability menu setter; that is, $\mathbb{E} q^p(n) = \mathbb{E} Q^p(n) - \mathbb{E} Q^p(n-1)$. All parties are risk neutral.

We assume that a menu setter earns a rent equal to a fraction $\alpha < 1$ of the quality of the menu offered. For an expert, the rent is $\alpha Q(n)$, and for a lower-ability menu setter, it is $\alpha \mathbb{E} Q^p(n)$.

For every item included in the menu, the menu setter bears a cost. We assume that the marginal cost of including an additional item in the menu (whether or not it is valuable) is a constant $c > 0$.¹⁰ Because we investigate the effect of the menu setter's ability, we assume that both c and α are common to all menu setters.

The objective of a menu setter is to select a number of items from the universe into his menu, so that he maximizes his expected rent net of costs. Given p , the menu setter's problem is

$$\max_n \{\alpha \mathbb{E} Q^p(n) - c \cdot n\}. \quad (1)$$

Because the expected marginal benefit of the n th item is $\mathbb{E} q^p(n)$ —or $q(n)$ for the expert—menu setters increase the size of the menu as long as α times this marginal benefit exceeds the marginal cost, c . Because menu setters differ in their ability to recognize valuable items, and thus differ in the marginal rent they expect to receive, menus will differ in quality and size.

3.1. Ability and Menu Size

Clearly, for a given menu size, n , an expert's menu is of higher quality than the menu of a lower-ability menu setter. On the face of it, it might appear that the expert should have a higher marginal benefit than the lower-ability menu setter from adding another item, since the expert always identifies valuable items. However, in this section we present the conditions

⁷ In this paper, for expositional clarity and for transparency of the economic forces, we use a simplified model. However, in unreported results, we develop two context-specific—and considerably more complex—models that lead to the same properties and predictions. One model considers choosers who have heterogeneous preferences around a Salop circle, for example, preferences over ice cream flavors. The other model considers investors choosing portfolios of assets. Details are available upon request.

⁸ At the end of §3.3, we offer thoughts on situations where menu setters may compete.

⁹ This constant probability p is consistent with our assumption of an infinite number of goods, where p is the ratio of the mass of valuable goods to the total mass of possible goods. Our results are robust to an alternative assumption of a finite number of possible goods, but at the expense of considerable complications in notation, since p will no longer be constant.

¹⁰ If $c = 0$, all menu setters would offer menus of infinite size with all available valuable items.

under which the lower-ability menu setter has a higher expected marginal benefit to increasing the menu size than does the expert.

The key point of our argument is that as n increases, the marginal benefit for the expert declines more steeply than that for the lower-ability menu setter. Moreover, there exists a number, n^* , such that the two marginal benefits are equal. For menu sizes larger than n^* , the expected marginal benefit achieved by the lower-ability menu setter is *larger* than the marginal benefit achieved by the expert. Thus, for low enough costs the lower-ability menu setter optimally offers a larger menu than the expert.

To formalize the argument, we must first prove a series of lemmas characterizing $\mathbb{E} q^p(n)$ and comparing it to $q(n)$. For the expert, the optimal n is chosen such that $\alpha q(n) \geq c$ and $\alpha q(n+1) < c$. Similarly, for the lower-ability menu setter, the optimal n is chosen such that $\alpha \mathbb{E} q^p(n) \geq c$ and $\alpha \mathbb{E} q^p(n+1) < c$.

In Lemma 1 we show that $\mathbb{E} q^p(n)$ is decreasing in n . This result is obtained by directly applying the Assumption of Declining Marginal Benefit.

LEMMA 1. *For any $0 < p < 1$, the expected marginal benefit for the lower-ability menu setter of adding the n th item to a menu, $\mathbb{E} q^p(n)$, is decreasing in n .*

PROOF. See Appendix A.

Lemma 1 states that similar to $q(n)$, $\mathbb{E} q^p(n)$ is decreasing in n . Although the menu of a lower-ability menu setter has an unknown number of valuable items, when the menu setter adds the n th item to the menu, he either successfully increases the number of valuable items or he does not. If he successfully improves the menu, then the next improvement faces a declining marginal benefit regardless of the current number of items because $q(n)$ is declining for all n . If he did not improve the menu, the expected marginal benefit of trying again is the same. So, on average, the $(n+1)$ st item has a lower expected marginal benefit than the n th item.

Of course, the realized marginal benefit when the lower-ability menu setter increases the size of his menu may be nonmonotonic. When the menu setter mistakenly includes a useless item, the marginal benefit is zero. When he later adds a valuable item, the marginal benefit is positive. Nevertheless, in expectation, the marginal benefit is monotonically declining.¹¹

Now that we have established that the marginal benefit of increasing the menu size is declining for both types of menu setters, in what follows we show that the expert has a higher marginal benefit when the

menu is small, and that the lower-ability menu setter has a higher marginal benefit when the menu is large.

To see this, consider the first item put on the menu. When an expert offers one item, the menu is of quality $Q(1) = q(1)$. When a lower-ability menu setter offers one item, the expected marginal benefit is lower, because it is only valuable with probability p ; and the expected quality of the menu offered by the lower-ability menu setter is $\mathbb{E} Q^p(1) = p \cdot q(1)$. Thus, his marginal benefit from increasing the menu size from zero to one is less than the marginal benefit for the expert.

For the second (and later) items, the comparison between marginal benefits for the two types of menu setters is not as straightforward. For the expert, the marginal benefit of the second item declines to $q(2)$. However, for the lower-ability menu setter, the marginal benefit of the second item depends on whether the first item was valuable or not. If the first item was valuable, then the expected marginal benefit of the second item would be $p \cdot q(2)$; but if the first item was useless, then the expected marginal benefit of the second item would be $p \cdot q(1) > p \cdot q(2)$. In expectation, the marginal benefit of the second item is

$$\mathbb{E} q^p(2) = p(p \cdot q(2) + (1-p) \cdot q(1)) > p \cdot q(2). \quad (2)$$

Thus, between the first item and the second item, $\mathbb{E} q^p$ declines at the slower rate than does q . The general form of this result will be given in Lemma 2.

Note that $\mathbb{E} q^p(2)$ may even be larger than $q(2)$. This occurs when

$$\begin{aligned} \mathbb{E} q^p(2) > q(2) &\iff q(2) < q(1) \frac{p}{1+p} \\ &\iff p > \frac{q(2)}{q(1) - q(2)}. \end{aligned} \quad (3)$$

The marginal benefit of each new valuable item must be declining rapidly for this inequality to hold. If $q(2)$ is much smaller than $q(1)$, then the marginal benefit of the expert is very small, but the marginal benefit of the lower-ability menu setter is higher in expectation because he is possibly adding $q(1)$. Conversely, if $q(2) > \frac{1}{2}q(1)$, then there is no p that would satisfy inequality (3), and $q(2)$ would remain above $\mathbb{E} q^p(2)$.¹²

For larger n , the condition under which $\mathbb{E} q^p(n) > q(n)$ is easier to satisfy. That is, the trade-off tends to shift in favor of the expected marginal benefit of the lower-ability menu setter. Given any n , the lower-ability menu setter always finds a valuable item with probability p . But as n grows, it is more

¹¹ The relevant measure is the expected marginal benefit and not the realized marginal benefit. Therefore, from now on we abbreviate “expected marginal benefit” simply as “marginal benefit.”

¹² Note that regardless of the difference between the marginal benefits, the total quality of a menu with two items is always greater for the expert than for the lower-ability menu setter; that is, $Q(2) > \mathbb{E} Q^p(2)$. We generalize this in Proposition 2.

and more likely that the lower-ability menu setter's menu includes fewer than n valuable items, and the marginal benefit conditional on the $(n + 1)$ st item being valuable is likely to be substantially greater than the expert's marginal benefit of the next item, $q(n + 1)$. If this difference is large enough, then the *unconditional* expected marginal benefit of the lower-ability menu setter is larger than $q(n + 1)$.

Of course, the exact nature of the trade-off between the lower probability of identifying a valuable item and the higher marginal benefit of an additional valuable item for the lower-ability menu setter depends on the marginal benefit function $q(n)$. Therefore, we now impose a specific structure on $q(n)$ with the stronger assumption that the marginal benefit of each additional valuable item in a menu declines at a fixed rate; that is, $q(n + 1) = k \cdot q(n)$, where $0 < k < 1$.¹³ We maintain this stronger assumption for all remaining results in this section.

STRONG ASSUMPTION OF DECLINING MARGINAL BENEFIT. For any n , the marginal benefit of an additional valuable item in a menu is strictly positive (i.e., $q(n) > 0$) and declines at a rate such that $q(n + 1) = k \cdot q(n)$ for some $k < 1$.

Lemmas 2 and 3 along with Corollary 1 establish a *single crossing* property between the marginal benefits of the two types of menu setters: For any n , the expert's marginal benefit declines quicker than the marginal benefit of the lower-ability menu setter. There exists a unique number n^* where the two marginal benefits are equal. Thus, for all menu sizes larger than n^* , the marginal benefit of the lower-ability menu setter is larger than the marginal benefit of the expert.

LEMMA 2. The expected marginal benefit of an additional menu item for a lower-ability menu setter is $\mathbb{E} q^p(n + 1) = [(1 - p) + pk] \mathbb{E} q^p(n)$. Therefore, the expected marginal benefit of an additional menu item for the lower-ability menu setter decreases more slowly than the marginal benefit of an additional menu item for the expert; that is, $\mathbb{E} q^p(n + 1)/\mathbb{E} q^p(n) > q(n + 1)/q(n)$ for any $0 < p < 1$.

PROOF. See Appendix A.

Lemma 2 arises because for the expert the marginal benefit of adding another item to the menu declines at a rate $q(n + 1)/q(n) = k$. In contrast, when the lower-ability menu setter adds the n th item, he may or may not have been successful in adding a valuable item.

¹³ The assumption that the marginal benefit of additional valuable items declines at a fixed rate is made for modeling tractability and expositional simplicity. However, such a restrictive assumption is not necessary to obtain our results. In Appendix C, we discuss weaker assumptions that are sufficient for obtaining the results in Propositions 1 and 2.

If he was successful, then the expected marginal benefit for the $(n + 1)$ st item declines at a rate k —just like the expert; but if he was unsuccessful, then the expected marginal benefit remains unchanged. Thus, the expected marginal benefit declines with a factor $p \cdot k + (1 - p) \cdot 1 > k$, and the overall expected decline in the marginal benefit is not as steep as it is for the expert.

Corollary 1 of Lemma 2 states that if for some n the marginal benefit for the lower-ability menu setter is higher than the marginal benefit for the expert, then it remains higher for any size larger than n .

COROLLARY 1. For any $0 < p < 1$, if $q(n) < \mathbb{E} q^p(n)$, then $q(n + 1) < \mathbb{E} q^p(n + 1)$.

PROOF. See Appendix A.

Lemma 3 completes the single-crossing property argument by proving that there always exists a finite menu size n such that $q(n) < \mathbb{E} q^p(n)$; that is, the marginal benefit for the lower-ability menu setter is larger than the marginal benefit for the expert. Moreover, Lemma 3 characterizes the crossing point.

LEMMA 3. For any $0 < p < 1$, let

$$n^* = 1 + \frac{\ln(p)}{\ln(k) - \ln((1 - p) + pk)}. \quad (4)$$

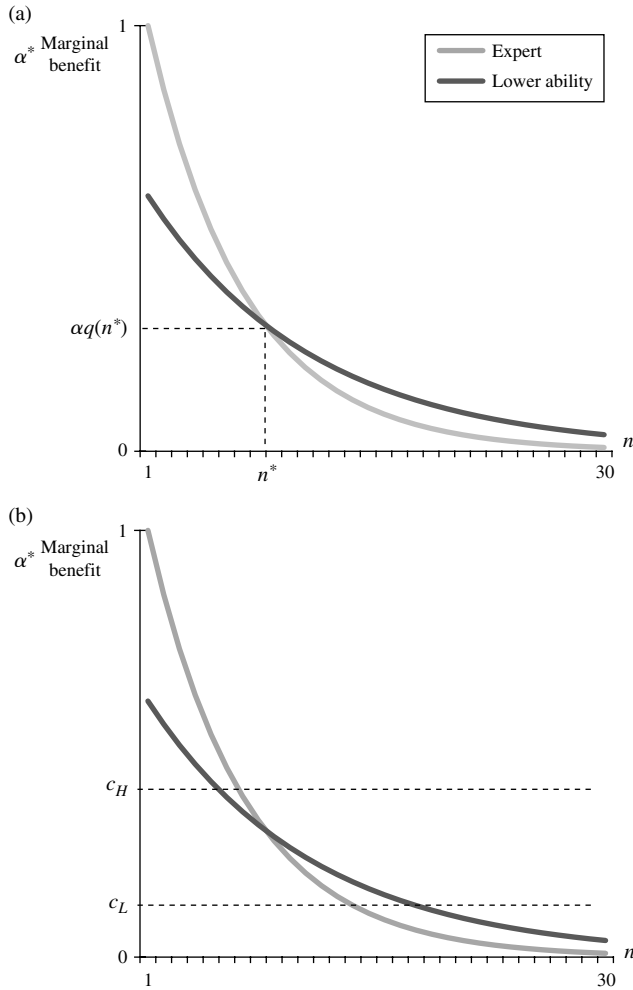
(i) The expected marginal benefit of an additional menu item is the same for the expert and the lower-ability menu setter; that is, $\mathbb{E} q^p(n) = q(n)$ if and only if $n = n^*$.

(ii) For n larger (smaller) than n^* , the expected marginal benefit of an additional menu item for the lower-ability menu setter is larger (smaller) than the marginal benefit for the expert; that is, $n > n^* \Rightarrow \mathbb{E} q^p(n) > q(n)$, and $n < n^* \Rightarrow \mathbb{E} q^p(n) < q(n)$.

PROOF. See Appendix A.

Lemma 3 characterizes n^* as the point at which the marginal benefit for the expert and the marginal benefit for the lower-ability menu setter are equal. It should not be surprising that $n^* > 1$, as we have previously shown that $\mathbb{E} q^p(1) = p \cdot q(1)$. Of course, if n^* is not an integer, there is no menu size at which the two types of menu setters have exactly the same marginal benefit. Nevertheless, n^* delineates where the marginal benefit of an additional menu item is higher for one type of menu setter than the other: for menu sizes larger than n^* , the marginal benefit for the lower-ability menu setter is larger than the marginal benefit of the expert, and for menu sizes smaller than n^* , the marginal benefit for the expert is larger. Figure 1(a) displays an example of the comparison between the expert's and lower-ability menu setter's marginal benefits.

Recall that menu setters increase the menu size as long as their marginal rent exceeds the marginal

Figure 1 Marginal Benefit as a Function of the Menu Size for the Expert and the Lower-Ability Menu Setter

Notes. Costs c_H and c_L are chosen so that $c_L < \alpha q(n^*) < c_H$. This example assumes that $p = 0.6$, $k = 0.85$, and $\alpha q(1) = 1$.

cost of including an additional item. As we can see in Figure 1(b), for any cost above $\alpha q(n^*)$, the lower-ability menu setter includes fewer items in his menu than the expert.¹⁴ However, for any cost below $\alpha q(n^*)$, the expert includes fewer items in his menu than the lower-ability menu setter. This property is formally stated in Proposition 1. For clarity, we abuse the notation somewhat by allowing n to be a continuous variable and $q(\cdot)$ to be defined over that continuous variable.¹⁵

¹⁴ This inequality is strict when we ignore the complication that n must be an integer. However, if the menu size must be an integer, it becomes a weak inequality—both menu setters offer the same menu size when they both round down to the same integer. Menu setters never round up, because that would result in a marginal benefit less than the marginal cost.

¹⁵ Ellison et al. (2004) refer to this as a quasi equilibrium when they take a similar approach in an auction problem and ignore the integer constraint on the number of participants.

PROPOSITION 1. *The relative sizes of menus optimally offered by expert and lower-ability menu setters depend on the marginal cost of increasing the menu size, c .*

(i) *If $c < \alpha q(n^*)$, the lower-ability menu setter optimally includes more items in his menu than the expert.*

(ii) *If $c > \alpha q(n^*)$, the expert optimally includes more items in his menu than the lower-ability menu setter.*

PROOF. See Appendix A.

When the cost of including more items is high, we are in the conventional situation, in which the expert uses his ability to find a larger number of valuable items, whereas the lower-ability menu setter does not find it worthwhile to include as many items (see Figure 1(b) for cost c_H). In such a case, a larger menu would suggest an expert menu setter. Moreover, a larger menu would be associated with higher quality—both because the larger menu is designed by a menu setter with a higher ability, and more simply because a larger menu offers more choices to individuals.

However, Proposition 1 shows that when the cost of including more items is low, the lower-ability menu setter offers a larger menu than the expert (as in Figure 1(b) for cost c_L). This is because when c is small, both menu setters include more than n^* items in their menus. By the single-crossing property in Lemma 3, when $n > n^*$, the marginal benefit for the lower-ability menu setter is above the expert's. Thus, when $c < \alpha q(n^*)$, the lower-ability menu setter optimally offers more items in his menu, leading to a negative relation between the number of items offered in the menu and the menu setter's ability.

Now that we have established the situations when the lower-ability menu setter offers a larger menu than the expert, it is not immediately clear which menu is of higher quality—the larger menu offered by the lower-ability menu setter or the smaller menu offered by the expert. We explore this question in the following two sections.

3.2. Ability and Menu Quality

We now consider the relation between the ability of the menu setter and the quality of the menu. In the cases when the expert offers a larger menu than the lower-ability menu setter (i.e., when c is high), it is immediately clear that the expert's menu is of higher quality—both because of his ability as an expert menu setter and because the menu offers more choices. However, when the lower-ability menu setter offers a larger menu (i.e., when c is low), there is a trade-off. On one hand, the larger menu has more items to choose from. On the other hand, those items are pre-selected by a menu setter with lower ability and are more likely to include useless items.

In this section, we show that the trade-off always favors menu-setter ability. Specifically, in Proposition 2, we establish that when both menu setters select their menu sizes optimally, the menu offered by the expert is, in expectation, of higher total quality than the menu offered by the lower-ability menu setter, even when the lower-ability menu setter offers more items. Of course, in any one instance, the lower-ability menu setter may have more valuable items in his menu than the expert, but the expectation is that the expert offers a higher-quality menu.¹⁶

PROPOSITION 2. *Suppose that for a given cost, c , an expert and a lower-ability menu setter set their optimal menu sizes. Then the quality of the menu offered by the expert is always higher than the expected quality of the menu offered by the lower-ability menu setter.*

PROOF. See Appendix A.

The main force driving this result is Lemma 2, which states that the marginal benefit of an additional menu item declines more rapidly for the expert than for the lower-ability menu setter. The intuition is as follows: When each type of menu setter stops at the optimal menu size, each is at the same marginal benefit. If they were each to increase the menu size by one more item (which, in the presence of costs they would choose not to do), by Lemma 2 the extra marginal benefit to the lower-ability menu setter would be greater than the extra marginal benefit to the expert. This inequality would continue to hold for hypothetical increases in the menu size ad infinitum. Thus, the total benefit foregone due to $c > 0$ is larger for the lower-ability menu setter than for the expert. In the absence of costs, the total menu quality would be the same for the two menu setters (i.e., the quality of a menu including all possible valuable items). Thus, in the presence of costs, the total quality of the optimal menu set by the expert must be higher than the total quality of the lower-ability menu setter's optimal menu.

3.3. Menu Size and Menu Quality

Because lower-ability menu setters sometimes offer larger menus than the experts (Proposition 1), but at the same time the expert's menu always has higher quality (Proposition 2), it follows that larger menus may be of lower quality. More specifically, if the menus are larger than n^* (i.e., if $c < \alpha q(n^*)$), then there is a negative relation between the menu size and expected quality. This seemingly counterintuitive result follows from a difference in composition: A smaller menu offered by an expert is not merely a

subset of the larger menu offered by a lower-ability menu setter. Instead, the smaller menu is likely comprised of a larger number of valuable items and consequently is expected to be of higher quality.

A negative relation between menu size and menu quality is possible because of differences in the ability of menu setters. Most importantly, this prediction would not hold in an alternative model with menu setters of the same ability but who differ in the cost each incurs when increasing the size of the menu. If the menu setters differ in cost but not in ability, we would only ever observe a positive relation between menu size and menu quality.

Different costs would lead menu setters to offer menus of different sizes. Specifically, a menu setter with a low cost would offer a larger menu than a menu setter with a high cost. Thus, for a menu setter of a fixed ability (or two menu setters of the same ability), a larger menu would always have higher expected quality. Because menu setters of the same ability identify valuable items with the same probability, a larger menu would be expected to include more valuable items than the smaller menu. The total benefit from the larger menu would be higher than the total benefit from the smaller menu.

Conversely, if menu setters differ in ability but incur the same cost, c , the menu of the expert is likely to include a larger number of valuable items than the menu of the lower-ability menu setter. Therefore, when c is small and the lower-ability menu setter offers a larger menu than the expert, we would observe a negative relation between the menu size and menu quality.

Until now, we have been comparing an expert who always successfully identifies valuable menu items with a lower-ability menu setter. However, in most settings, it is unlikely that any menu setter will be perfect. Instead, it may be more fitting to compare two nonexpert menu setters, each with a different ability. In such a case, all the results in this section can be generalized, albeit with slightly more complicated notation. If two menu setters had abilities p' and p , where $p' > p$, the crossing point of marginal benefits would be

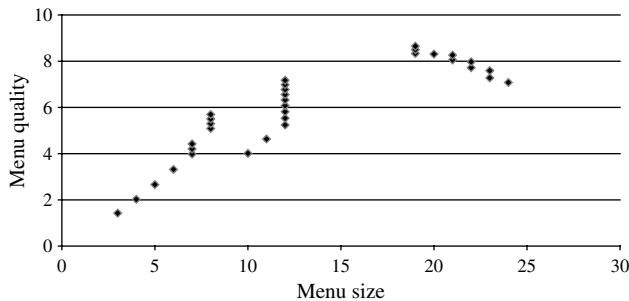
$$n^{**} = 1 + \frac{\ln(p) - \ln(p')}{\ln[(1 - p') + p'k] - \ln[(1 - p) + pk]}.$$

When costs are sufficiently low, that is, when $c < \alpha \mathbb{E} q^p(n^{**}) = \alpha \mathbb{E} q^{p'}(n^{**})$, the lower-ability menu setter offers more menu items than the higher-ability menu setter, and again there is a negative relation between menu size and menu quality.

The model compares menu setters of different ability for a fixed cost c , but in reality, different menu setters also face different costs. To illustrate the model with variation in both ability and costs, we simulate it as follows: Consider 33 menu setters, with each menu

¹⁶ Proposition 2 allows n to be a continuous variable. In Appendix B, we show the condition that ensures that the proposition holds when n is limited to being an integer.

Figure 2 Model Predictions for 33 Simulated Menu Setters
 $p = \{0.5, 0.55, 0.60, \dots, 1.0\}$, $c = \{0.15, 0.30, 0.45\}$,
 and $k = 0.9$



setter characterized by his ability, p , and his cost, c , where there are 11 different values for $p = \{0.5, 0.55, 0.60, \dots, 1.0\}$ and 3 values for $c = \{0.15, 0.30, 0.45\}$. Figure 2 displays the resulting menu sizes and menu qualities.

The left third of the plot illustrates the relation between menu size and menu quality for high cost, and an upward relation holds. The middle of the plot is almost vertical. There is little relation between menu size and quality corresponding to menus close to the crossing point of Figure 1. The rightmost third of the plot represents a negative relation between menu size and quality because menu setters have low c over this range.

The overall relation between menu size and menu quality is not simple, but it resembles an inverted U. If a linear relation is fitted across all the points, the slope may be positive or negative, depending on the composition of the high-cost and low-cost menu setters. In this particular simulation, a linear relation fitted across all the points in Figure 2 would be positive. Of course, a linear fit would mask the more subtle ability-driven relation between menu size and menu quality.

Our model focuses on a menu setter who may be an expert or of lower ability and does not explore competition between menu setters. We show that an expert menu setter may offer a shorter menu than a lower-ability menu setter would. In many environments, including 401(k) plans and local monopolies, it is reasonable to assume that choosers are restricted to a single menu. However, in other environments, menu setters may be competing for choosers. If menu setters compete for business, our results open the question of whether the lower-ability menu setter can mimic the expert by shortening his menu. To answer this question, suppose that the lower-ability menu setter has both a set of existing captive customers as well as a set of potential customers who will only come to him if he can mimic the expert. Then a trade-off exists between attracting the new customers and offering the best possible menu to existing customers. It is relatively straightforward to show the conditions under

which a separating equilibrium can be sustained and mimicking would not occur.¹⁷

Even if lower-ability menu setters do not mimic experts, the coexistence of menu setters of differing ability raises the issue of why a customer would choose a lower-ability menu setter in the first place. This could occur if customers select menu setters for reasons other than menu quality—perhaps because of other services provided by the menu setter or simply because of personal relationships. This may not be satisfying from a theory perspective, but there is a well-established empirical literature that struggles with the question of why coexisting superior and inferior financial products are frequently observed.¹⁸ In many cases documented in the literature, the link between observable characteristics and quality is more direct than it is for menu size, yet high-quality products apparently do not always drive out lower-quality products. Like the empirical literature, we must appeal to arguments such as intangible benefits or agency conflicts if we want to allow for multiple menu setters with different abilities to coexist in the market.

4. Empirical Setting and Data

Although this theory is written to be general and applies to many settings, we focus on 401(k) pension plans as a particular example. The unique empirical implication of the theory is that under certain circumstances (i.e., low c) there is a negative relation between menu size and menu quality. 401(k) plans are especially suitable to test this empirical implication because the number of options in plan menus varies and the objective quality of 401(k) plans can be estimated. Indeed, we show a negative relation between the number of investment choices offered by a 401(k) plan and the quality of the plan.

When a company provides a 401(k) plan for its employees, it typically appoints an outside trustee to design and manage the plan. Each plan is a menu of investment choices for employees' retirement savings, mostly mutual funds and similar investments, as well as a money market fund. Frequently the company's own stock is one of the possible investments. Each employee allocates his 401(k) savings across the various choices in the plan.

¹⁷ Details of the separating equilibrium are available upon request.

¹⁸ Gruber (1996) addresses the growth of actively managed mutual funds that regularly underperform index funds. Christoffersen and Musto (2002) discuss high-fee money market funds that are dominated by low-fee money market funds. Similarly, Elton et al. (2004) show that high-fee S&P500 index funds coexist with low-fee S&P500 funds. Bergstresser et al. (2009) compare broker-sold and direct-sold mutual funds; see also Gil-Bazo and Ruiz-Verdu (2009) and Choi et al. (2010).

Because the employer appoints the trustee, and because the employer, as the plan sponsor, has influence over the list of possible investments, one could view menu setting as designed by two separate agents or as having two stages (as described in Cohen and Schmidt 2009). Instead, for our purpose, it is sufficient to think of the menu setter as a single party that is a combination of the sponsor and the trustee.

Our data are comprised of 401(k) plans of companies that file SEC form 11-K.¹⁹ This form must be filed if the company offers its own stock as one of the choices in the 401(k) plan and includes the full menu of investment choices offered to employees. We collected the 11-K data of 300 randomly selected company plans offered in 2007 to examine how menu setters affect the portfolios that investors can achieve. After eliminating unusable data (see §4.1 for details) we are left with 191 plans. Among the plans in our data set, the number of funds offered (aside from a company's own stock and any money market funds) ranges from 4 to 28.

We study how the number of fund choices in a plan menu is related to the quality of the plan. We define a plan's quality as the highest Sharpe ratio achievable within the plan. The portfolio of investment choices that achieves this maximum Sharpe ratio is referred to as the optimal portfolio of the plan.

The optimal Sharpe ratio is a measure of the objective quality of the plan. It corresponds to the portfolio that a fully rational and fully informed investor would choose. This is in line with §3, where we model the objective quality of the menu. We do not measure the actual choices of investors in 401(k) plans, as they are subject to a number of well-known behavioral biases (e.g., Benartzi and Thaler 2001, Iyengar and Kamenica 2010).

The Sharpe ratio of a portfolio is the expected return of the portfolio above the risk-free rate divided by the standard deviation of the portfolio's returns. Thus, the Sharpe ratio increases in the expected return and decreases in the risk of the portfolio. In the absence of any differences in menu setters' abilities, larger menus should be more likely to include high expected return funds and/or provide more scope for diversification of risk.

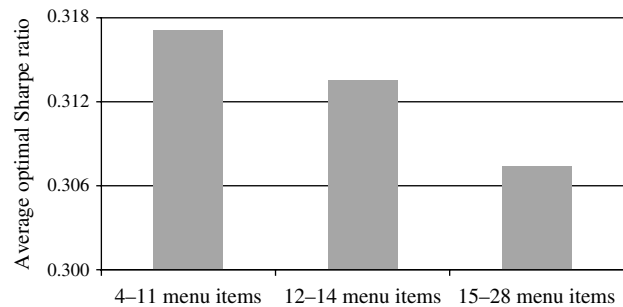
In §4.1 we provide further details on determining the number of choices in each plan. In §4.2 we

Table 1 Summary Statistics

	Average	Std. dev.	Min	Max
No. of funds	13.10	3.82	4	28
No. of stocks	1.07	0.25	1	2
No. of money market funds	1.32	0.57	0	4
Plan assets (\$million)	301.10	614.00	0.1	4,392.4
Optimal Sharpe ratio	0.313	0.059	0.101	0.432

Note. The optimal Sharpe ratio is based on the world CAPM, assuming an equity risk premium of 0.05.

Figure 3 Average Optimal Sharpe Ratio by Menu Size



Notes. Ranges are chosen so that each has as close to the same number of plans as possible. The number of plans in each range is 64, 66, and 61, respectively.

explain the methodology for estimating the maximum Sharpe ratio.

Table 1 summarizes the number of choices and Sharpe ratios for the plans in our data set.²⁰ Before starting the formal analysis, in Figure 3 we give a preview of the relation between the number of choices and the optimal Sharpe ratio. The plans are divided into three groups by the number of menu items in each plan, with each group having approximately the same number of plans. For each group, Figure 3 presents the optimal Sharpe ratio averaged across the plans in the group. On average, plans with a larger number of funds have a smaller optimal Sharpe ratio than plans with fewer funds. In §5 we consider the relation between the number of choices and the optimal Sharpe ratio in detail.

4.1. Number of Funds

Our original data are comprised of 300 11-K filings. The 11-K forms include a list of all investment choices available for employees' 401(k) pension savings and the total amount invested in each.²¹ Most of the

¹⁹ In Elton et al. (2007) and Cohen and Schmidt (2009), 11-K filings are also used as the main data source. The nature of the 11-K filing requirements means that our selection is limited to publicly traded companies. Being publicly traded, these companies and their pension plans are likely larger than average. Elton et al. (2006) show that pension plans that offer company stock as a choice have more assets under management. We see no reason why this selection should bias our results.

²⁰ The average number of funds is almost exactly the same as that in Huberman and Jiang (2006). However, it is significantly larger than the average number of funds in Elton et al. (2006), who study 401(k) plans from a survey that includes smaller nonpublicly listed firms.

²¹ The total amount invested in each asset is given at the aggregate level for the plan; thus, it is not appropriate for studying individual investors' choices.

investment choices in each plan are mutual funds. In addition, every plan in our sample includes the company's own stock, because offering the company's own stock to employees is the trigger for the requirement to file an 11-K. Almost every plan also includes one or more money market funds. Finally, some of the plans offer nonstandard investment choices, such as insurance contracts, warrants, as well as self-directed accounts. Whenever possible, we use historical prices for the mutual funds in the data from the Center for Research in Security Prices (CRSP). However, for a significant subset of the funds within the plans, we were unable to obtain a CRSP identifier. We exclude plans if more than 6% of the amount invested is in funds for which we were unable to obtain price data.²² This reduced our sample size from 300 pension plans to 191.

Within the 191 plans, there are a total of 1,131 unique mutual funds. More than 60% of funds are offered by only one of the 401(k) plans in our sample. More than 90% of the funds are offered by five or fewer plans. The large number of funds in our sample, and the relatively small overlap of funds used in multiple plans, indicates significant differences in the composition of 401(k) plans. These differences in menu composition make it unlikely that 401(k) menu setters are likely to mimic the menus in other plans. Furthermore, it gives scope to the variation in skill that underpins our modeling assumptions.

For each plan, we count the number of investment choices available to participants. We exclude nonstandard investments, as well as any choices that are not open to new investments. For example, after a merger, there may be investments that remain in funds that were previously available to the employees of the acquired company, but are no longer open to new investments. Our count of investment choices does not include money market funds or the company's own stock. However, excluding money market funds and own stock from the count makes little difference because there is very little variation across plans. A number of plans include a set of "lifecycle" funds. Each lifecycle fund targets a subset of employees based on their expected retirement date. For example, a plan may include a set of lifecycle funds aimed at employees who will retire in the years 2020, 2030, 2040, and 2050. In such cases, because each employee is targeted by one of these funds, we count the entire set of lifecycle funds as one choice.

To find the optimal Sharpe ratio, we include all mutual funds available for investment, as well as the company's own stock. The exclusion of money market funds does not materially affect the Sharpe

ratio because it affects the numerator and denominator essentially in the same way.

4.2. Sharpe Ratios

For almost all of our analysis, we determine the expected return of each fund using the world capital asset pricing model (CAPM).²³ To obtain expected returns, we regress the returns of each investment choice against the MSCI World Index returns using weekly data over the five year period 2003–2007. The estimated coefficient on the world index, that is, the *beta*, is used within the CAPM to estimate the expected return.²⁴ The expected return of a portfolio of funds within a plan is the weighted average of the expected returns of the component funds. The standard deviation of returns for a portfolio of funds is estimated based on the historical variance–covariance matrix of weekly returns of the funds within each plan. For each 401(k) plan, we identify the optimal portfolio as the weights on each fund within the plan that give the highest Sharpe ratio. Because short sales are not possible in 401(k) accounts, the optimization routine assumes that the amount of money invested in any choice is nonnegative.

We use the world index for the beta estimation, as opposed to a U.S. index such as the S&P 500, because international equity funds are an important area of potential diversification. We use weekly data, as opposed to daily data, to minimize the lead-lag effects in international beta estimation that can be caused by time zone differences. Although some lead-lag issues surely remain in our data, we find that weekly data are an effective compromise with the need for a larger sample. Note that we use a model-based estimate of expected returns rather than historical returns in the numerator of the Sharpe ratio to avoid the well-known problems associated with backward-looking returns. In contrast, because variances and covariances tend to be persistent, we use historical data to estimate the variance–covariance matrix.

When estimating variances and covariances, we run into the problem that newer funds do not have a full five-year return history. In these cases, we necessarily estimate the variance based on the shorter time series,

²³ We assume that in expectation, fund managers do not earn any excess return (often referred to as *alpha*) above that predicted by the CAPM. Of course, the question of why each plan is not comprised solely of the world market index remains open. This is part of a broader question in the investments literature and is far beyond the scope of this paper.

²⁴ We assume an equity risk premium of 5%, but because the Sharpe ratio is proportional to the equity risk premium, it does not have any effect on the statistical significance in our cross-sectional analysis.

²² We chose 6% as our cutoff because it coincides with a natural gap in the data.

Table 2 Regression of Optimal Sharpe Ratio on Number of Funds in Menu

	CAPM		Four-factor (domestic)		Three-factor (global)	
	(1) All data	(2) $n \geq 10$	(3) All data	(4) $n \geq 10$	(5) All data	(6) $n \geq 10$
No. of menu items	−0.075 (−0.61)	−0.358*** (−2.74)	−0.325* (−1.80)	−0.418** (−1.96)	−0.403** (−2.07)	−0.531** (−2.43)
R^2 (%)	0.24	3.96	2.09	2.72	1.95	2.48
N	191	166	191	166	191	166

Notes. The optimal Sharpe ratio uses model-based expected returns and historical variance–covariance matrices. The expected return is determined either by CAPM, a four-domestic-factor model, or a three-global-factor model. For readability, coefficients are multiplied by 100 and intercepts are suppressed. The t -statistics are calculated using robust standard errors clustered at the trustee level and are reported in parentheses.

*, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively.

and we estimate the covariance based on the shorter of the two time series for each pair of funds.²⁵

To check the robustness of our results (see Table 2), we also use alternatives to the world CAPM to estimate expected returns in the numerator of the optimal Sharpe ratio. We use a four-factor expected return model that includes the U.S. market index, the Fama–French size factor (SMB), the Fama–French value factor (HML), and a momentum factor. As a further alternative, to capture the international aspects of the funds in many plans, we use a three-factor model of expected returns using global factors: the world market index, global SMB, and global HML. A global momentum factor was not available.²⁶

5. Empirical Analysis

The empirical implication unique to our theory is that when the cost of including an additional menu item is low, there is a negative relation between the number of choices and the menu quality.²⁷ We compare the optimal (annualized) Sharpe ratio—a measure of plan quality—and the number of funds available for investment in the plan. The regression is

$$\text{optimal Sharpe ratio}_i = a + b \cdot \text{number of funds}_i,$$

where the optimal Sharpe ratio for a plan and the number of funds in the plan menu are defined in the previous section. The results are displayed in Table 2.

²⁵ We exclude a very small number of funds for which we have fewer than 20 weekly observations.

²⁶ These data were taken from Ken French’s website (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html). We used an equity risk premium of 5%, an SMB premium of 2%, an HML premium of 4%, and a momentum premium of 8%. These risk premia are similar to those often used in the literature and are close to the average excess returns in the factor data since 1926.

²⁷ We do not directly observe costs, but under our model of differing ability, low costs would rationalize such a result. The alternative model, with equal ability but differing costs, cannot explain a negative relation.

Regressions (1) and (2) use optimal Sharpe ratios calculated using CAPM for expected returns, whereas regressions (3) and (4) use the four-factor (domestic) expected return model to calculate optimal Sharpe ratios, and regressions (5) and (6) use the three-factor (global) model. For readability, all coefficients in the table are multiplied by 100.

Overall, the results in Table 2 suggest a negative relation between the number of choices and the optimal Sharpe ratio. When the entire data set is included (regressions (1), (3), and (5)), the negative relation between the number of choices and the optimal Sharpe ratio is only statistically significant under the three-factor and four-factor models, but not under the CAPM. Examination of the data shows that there is a large degree of variability in the Sharpe ratio for plans with the fewest number of funds. Thus, in regressions (2), (4), and (6), we exclude those plans with fewer than 10 menu items and run the regression on the remaining 87% of the data. The results are starker when the smallest menus are excluded, with a clear negative relation between the number of funds and the optimal Sharpe ratio. This relation is significant at the 5% or 1% level, depending on which expected return model is used.²⁸

The results are economically significant. For example, when considering plans with at least 10 menu items using CAPM, the coefficient on the number of funds in a plans is −0.00358. When compared with the average annual optimal Sharpe ratio of 0.313, the coefficient suggests that each additional menu item corresponds to a 1.14% decline in the plan’s quality. A one standard deviation change in the number of

²⁸ For robustness, we also calculate Sharpe ratios with less than the full five years of historical data using weekly data for the period 2005–2007. Furthermore, because Sharpe ratios are not unambiguous measures of quality in the absence of a risk-free asset, we rerun the regressions excluding the three plans that did not offer money market accounts. For both robustness checks, we obtain similar results and similar statistical significance.

menu items (i.e., a change of 3.82 menu items) corresponds to a 4.4% change in plan quality.²⁹

We cannot definitively explain why the plans with the fewest menu items do not display the downward relation, but the difference between shorter menus and longer menus may be consistent with an extended model that allows for variation in both menu setter ability and costs. In §3 we presented a simulation showing that if menu setters differ in costs, the relation between menu size and menu quality may resemble an inverted U. Nonetheless, the fact that the bulk of the plans have a significant negative relation can be explained by our theory: Experts offer smaller but better menus, whereas lower-ability menu setters offer larger but worse menus. Moreover, whether or not we exclude any plans, we do not observe a positive relation between the number of choices and plan quality, as would be predicted by the alternative theory with homogeneous menu-setter ability and driven only by varying costs.

Because the trustee has an important influence on the offering of funds in a plan, and because companies may use the same trustee, all our regressions use robust standard errors clustered at the trustee level.

In our sample of 191 plans, Fidelity, Vanguard, Merrill Lynch, T. Rowe Price, Charles Schwab, and Wells Fargo are the most common trustees, with 43, 18, 10, 11, 10, and 7 plans, respectively. To ensure that our results are not driven by the choice of trustee, in Table 3 we include individual dummy variables for these six frequent trustees as controls. We find that our results are robust to these controls. The coefficient on the number of menu items remains negative and is statistically significant when limited to plans with at least 10 menu items.³⁰ Interestingly, in Table 3 the coefficients on the individual trustee dummies are positive and strongly significant for Fidelity and Vanguard (and to some extent for T. Rowe Price). Plans that use these trustees have higher optimal Sharpe ratios than other plans.³¹

The dependent variable in the regressions reported until now was the optimal Sharpe ratio in each plan,

²⁹ When limited to plans with at least 10 funds, the R^2 of the regressions ranges from 2.5% to 4%. Not surprisingly, there are other important factors contributing to the variation in quality across 401(k) plans.

³⁰ When we run the analysis limited to the most frequent trustees (unreported), we again find similar results. The statistical significance in those regressions is reduced, as one would expect with fewer observations.

³¹ The effect of the individual trustee dummy variables remains when we control for the total plan assets, as in Table 5. The t -statistics on the individual trustee dummies are smaller when we use standard errors that are not clustered at the trustee level. All other t -statistics in this paper are only affected by a small amount by the clustering technique.

Table 3 Regression of Optimal Sharpe Ratio on Number of Funds in Menu, Including Dummy Variables for Frequent Trustees

	(1) All data	(2) $n \geq 10$
<i>No. of menu items</i>	−0.100 (−0.81)	−0.357** (−2.47)
<i>Fidelity</i>	4.218*** (5.16)	4.433*** (4.63)
<i>Vanguard</i>	2.839*** (3.81)	2.107** (2.36)
<i>Merrill Lynch</i>	−0.574 (−0.51)	0.330 (0.25)
<i>T. Rowe Price</i>	2.083** (2.60)	1.582* (1.72)
<i>Charles Schwab</i>	−0.464 (−0.60)	−0.761 (−0.82)
<i>Wells Fargo</i>	0.387 (0.48)	−0.084 (−0.09)
R^2 (%)	9.99	14.10
N	191	166

Notes. For readability, coefficients are multiplied by 100 and intercepts are suppressed. The t -statistics are calculated using robust standard errors clustered at the trustee level and are reported in parentheses.

*, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively.

where expected returns ignoring expenses are used in the optimization routine and in the calculation of the Sharpe ratio. In regressions (1) and (2) of Table 4, we repeat the main regressions using expected returns net of expenses. For each fund, we calculate the expected return based on the world CAPM and then subtract the expense ratio for 2007 as reported in CRSP. This net-of-expenses expected return is used to calculate the optimal Sharpe ratio. The results are robust to the adjustment for expenses. The coefficients and t -statistics vary slightly, but the results are qualitatively the same as in the original regressions.³²

As is generally well known, employees who invest in 401(k) plans tend to hold a portion of their assets in the company's own stock. In our data, the percentage of assets held by employees in company stock within the 401(k) plan varies from less than 1% to 69% and averages 16.6% of assets.³³ Not surprisingly, even though company stock is one of the menu items, the optimal portfolio in our sample never includes company stock. Nevertheless, employees hold company stock, whether because of restrictions, explicit incentives, implicit incentives, or behavioral biases. As such, for robustness, we consider an alternative

³² The specification in regressions (1) and (2) of Table 4 accounts for expenses but ignores loads. In an alternative specification, for the few funds that appear to have front-end loads, we add the maximum load divided by five to the expenses. This makes almost no difference to the results.

³³ Our data include the aggregate amount held by employees in each investment. Unfortunately, we do not observe the selections of individual employees.

Table 4 Regression of Optimal Sharpe Ratio on Number of Funds in Menu with Adjustments for Expenses and Stock Holdings

	Net of expenses		Constrained by stock holdings			
	(1) All data	(2) $n \geq 10$	(3) All data	(4) $n \geq 10$	(5) All data	(6) $n \geq 10$
<i>No. of menu items</i>	−0.083 (−0.55)	−0.420** (−2.58)	−0.041 (−0.26)	−0.285* (−1.75)	−0.026 (−0.16)	−0.301* (−1.81)
<i>Weight on stock</i>			−47.42*** (−16.6)	−49.37*** (−15.6)	−47.31*** (−16.4)	−49.67*** (−14.0)
<i>No. of menu items</i> × <i>Weight on stock</i>					−0.280 (−0.58)	0.299 (0.54)
R^2 (%)	0.22	4.00	49.82	51.55	49.88	51.59
N	191	166	191	166	191	166

Notes. In regressions (1) and (2), the optimal Sharpe ratio is based on expected returns net of total fund expenses. Regressions (3)–(6) are based on the constrained optimal Sharpe ratio when the weight on the company stock is set equal to observed weight. The variables *no. of menu items* and *weight on stock* are demeaned. For readability, coefficients are multiplied by 100 and intercepts are suppressed. The t -statistics are calculated using robust standard errors clustered at the trustee level and are reported in parentheses.

*, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively.

measure of menu quality in which we assume that employees hold company stock, and we calculate the Sharpe ratio using an optimization constrained to having a portion of assets in the company stock.

The dependent variable in regressions (3)–(6) of Table 4 is the optimal Sharpe ratio, in which the optimization is constrained to have a weight on the company stock equal to that held in aggregate by employees. We find very similar results. When considering plans with at least 10 funds in the menu, there is a negative relation between the number of menu items and the constrained-optimal Sharpe ratio, although the statistical significance is not as strong.

Not surprisingly, there is an extremely strong relation between the proportion of assets held in company stock and the highest achievable Sharpe ratio in this constrained scenario. If individuals are constrained to hold a large proportion of their funds in company stock, their wealth will not be well diversified. It is unlikely that even a well-designed menu could offset such an unbalanced investment portfolio. The interaction term between the number of menu items and the weight on company stock shows no effect.

The value of assets in the plans in our sample varies widely. It is possible that larger plans have access to higher quality menus. Thus, in Table 5 we control for the value of assets in each plan. All regressions in Table 5 include the logarithm of the total assets as a control variable, whereas regressions (3) and (4) also include the interaction term between the number of menu items and the logarithm of total assets. Even with this control, for more than 10 menu items, the results indicate a negative relation between the number of funds in a pension plan and plan quality. Interestingly, we find evidence that the size of the 401(k) plans, as measured by the logarithm of total assets,

is generally positively related to the optimal Sharpe ratio, indicating that larger plans have better menus. The interaction term is not statistically significant.

Until this point, we assume that the 401(k) menu is the entire investment opportunity set for all investors and that the quality of a menu can be summarized by a single optimal Sharpe ratio. In reality, investors may prefer different portfolios because they hold risky assets outside the 401(k), have risky human capital, or face different consumption risks in the future. We cannot directly observe these risks in our data. To ensure that our results are robust to investors who also own assets outside 401(k) plans, we identify the optimal portfolios, assuming that investors hold 20% of their wealth in one of the broad Fama–French industry portfolios and that the remaining 80% is invested in the 401(k). We rerun the analysis of Table 2 using the

Table 5 Regression of Optimal Sharpe Ratio on Number of Funds in Menu, Controlling for Plan Assets

	(1) All data	(2) $n \geq 10$	(3) All data	(4) $n \geq 10$
<i>No. of menu items</i>	−0.096 (−0.82)	−0.356*** (−2.79)	−0.095 (−0.82)	−0.380*** (−2.95)
$\ln(\text{total assets})$	0.613** (2.55)	0.577** (1.96)	0.624** (2.45)	0.494 (1.64)
<i>No. of menu items</i> × $\ln(\text{total assets})$			0.018 (0.37)	0.083 (1.07)
R^2 (%)	3.41	6.50	3.47	7.09
N	191	166	191	166

Notes. This table includes robustness checks for the logarithm of total assets, as well as an interaction term. The variables *no. of menu items* and $\ln(\text{total assets})$ are demeaned. For readability, coefficients are multiplied by 100 and intercepts are suppressed. The t -statistics are calculated using robust standard errors clustered at the trustee level and are reported in parentheses.

** and *** denote statistical significance at the 5% and 1% levels, respectively.

resulting (constrained) optimal Sharpe ratios and find that for menus with at least 10 funds, the relation between the number of menu items and the optimal Sharpe ratio remains very strong for three of the four industry portfolios.³⁴

As described previously, we explain the negative relation between the number of menu items and the optimal Sharpe ratio as stemming from differences in menu-setter ability. The possibility that some menu setters have imperfect ability is also explored in Elton et al. (2006), who show that about half of all 401(k) plans in their sample do not span a set of eight indices. Although the relation between the number of choices and menu quality is not the focus of that paper, looking at the results, there does not appear to be a negative relation between the number of choices and whether a plan spans the indices—indeed, they find a positive relation, or no relation at all when the tails are excluded. The difference between our results and those in Elton et al. (2006) comes from differences in the sample analyzed, as well as differences in the method of measuring menu quality. The two samples differ in their spectrum of plan sizes. Elton et al. (2006) study a sample drawn from a survey that includes smaller plans and fewer menu items than our sample. To identify the effect of the difference in the sample on the results, one needs to use the same method of analysis on both samples. Without access to data used in Elton et al. (2006), we could not analyze their sample using Sharpe ratios. Instead, we use the spanning methodology of Elton et al. (2006) on our data.³⁵ We do not find the positive relation reported in Elton et al. (2006). But we also do not find the negative relation we report here obtained using Sharpe ratios as a measure of menu quality. We believe that the continuous Sharpe ratio can better distinguish between suboptimal menus of varying quality. In contrast, the spanning criteria are appropriate for Elton et al. (2006), who classify menus as either adequate or inadequate.

6. Conclusion

In this paper, we study the relation between menu size and menu quality. There exists a growing literature showing that individuals often prefer to choose from a smaller set. This preference is often ascribed to choosers' behavioral biases or informational limitations. Although the existing literature can rationalize a preference for smaller menus, it predicts that larger menus will still be better for fully rational, fully informed choosers.

³⁴ Detailed results are available upon request.

³⁵ Details of this analysis are available upon request.

Empirically, we study the relation between the number of investment choices offered by 401(k) pension plans and the objective quality of those plans. We measure plan quality by the maximum Sharpe ratio achievable given the investment choices in the plan. Excluding the funds with the fewest investment choices, we find a statistically significant negative relation between the number of investment choices and plan quality.

Motivated by the empirical finding that larger 401(k) menus are objectively worse, we take a different approach to the theoretical model. We recognize that menus are generally preselected by menu setters from a larger universe of items and that menu setters may differ in their ability to construct menus. Our results show that when the marginal cost of increasing a menu is low, menu setters with lower ability offer larger menus than experts. At the same time, the menu of the expert is of higher quality, in the sense that it offers a larger number of valuable items. Together, these two results lead to smaller menus that are of higher objective quality than larger menus.

The empirical application in our paper addresses only investment portfolios; the central insight that menus are preselected by a menu setter is applicable to many scenarios. Of course, other forces also come into play when evaluating menus. Certainly the behavioral and informational effects recently addressed in the literature affect choices made by agents facing menus of different sizes. Nevertheless, we argue that one must be aware of the role played by the ability of menu setters in designing the menu offered to individuals.

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Appendix A. Proofs

PROOF OF LEMMA 1. Fix arbitrary $p \in (0, 1)$. The general formula for $\mathbb{E} q^p(n)$ is

$$\mathbb{E} q^p(n) = \sum_{i=0}^{n-1} \frac{(n-1)!}{(n-1-i)!i!} p^{n-i} (1-p)^i q(n-i). \quad (\text{A1})$$

By applying this formula to $\mathbb{E} q^p(n+1)$ and algebraical manipulation, we get

$$\begin{aligned} \mathbb{E} q^p(n+1) &= (1-p) \sum_{i=0}^{n-1} \frac{(n-1)!}{(n-1-i)!i!} p^{n-i} (1-p)^i q(n-i) \\ &\quad + p \sum_{i=0}^{n-1} \frac{(n-1)!}{(n-1-i)!i!} p^{n-i} (1-p)^i q(n+1-i). \end{aligned} \quad (\text{A2})$$

By the Assumption of Declining Marginal Benefit, $q(n)$ is decreasing; that is, $q(n+1) < q(n)$. Therefore,

$$\mathbb{E}q^p(n+1) < \sum_{i=0}^{n-1} \frac{(n-1)!}{(n-1-i)!i!} p^{n-i}(1-p)^i q(n-i) = \mathbb{E}q^p(n). \quad \square$$

PROOF OF LEMMA 2. We can write $\mathbb{E}q^p(n)$ as $\mathbb{E}q^p(n) = p \cdot \mathbb{E}(q \mid \text{success at } n)$, where $\mathbb{E}(q \mid \text{success at } n)$ is the expected marginal benefit conditional on getting a valuable item.³⁶ Then³⁷

$$\begin{aligned} \mathbb{E}q^p(n+1) &= [(1-p) + pk]p \mathbb{E}(q \mid \text{success at } n) \\ &= [(1-p) + pk] \mathbb{E}q^p(n) \\ \implies \frac{\mathbb{E}q^p(n+1)}{\mathbb{E}q^p(n)} &= (1-p) + pk. \end{aligned}$$

For $p \in (0, 1)$, this ratio is a convex combination between k and 1. Hence, $k < \mathbb{E}q^p(n+1)/\mathbb{E}q^p(n) < 1$. By the Strong Assumption of Declining Marginal Benefit, $q(n+1)/q(n) = k$. Therefore, $q(n+1)/q(n) < \mathbb{E}q^p(n+1)/\mathbb{E}q^p(n)$. \square

PROOF OF COROLLARY 1. By the Strong Assumption of Declining Marginal Benefit, $q(n+1) = kq(n)$ for some $k < 1$. Suppose also that $q(n) < \mathbb{E}q^p(n)$. From the proof of Lemma 2, $\mathbb{E}q^p(n+1) = [(1-p) + pk]\mathbb{E}q^p(n) > k\mathbb{E}q^p(n)$. Then $\mathbb{E}q^p(n+1) > k\mathbb{E}q^p(n) > kq(n) = q(n+1) \Leftrightarrow \mathbb{E}q^p(n+1) > q(n+1)$. \square

PROOF OF LEMMA 3. By the Strong Assumption of Declining Marginal Benefit, $q(n) = k^{n-1}q(1)$. By Lemma 2, $\mathbb{E}q^p(n) = \mathbb{E}q^p(1)[(1-p) + p \cdot k]^{n-1} = pq(1)[(1-p) + p \cdot k]^{n-1}$. Moreover, $\mathbb{E}q^p(1) = pq(1)$. Let $n^* = 1 + \ln(p)/(\ln(k) - \ln((1-p) + p \cdot k))$. By applying those formulas, and some algebraic manipulation, we obtain

$$\begin{aligned} \mathbb{E}q^p(n) > q(n) &\iff p^{1/(n-1)} > \frac{k}{(1-p) + p \cdot k} \\ &\iff n > 1 + \frac{\ln(p)}{\ln(k) - \ln((1-p) + p \cdot k)} = n^*. \end{aligned}$$

Therefore, $\mathbb{E}q^p(n) > q(n) \Leftrightarrow n > n^*$. By similar calculations, we obtain $\mathbb{E}q^p(n) < q(n) \Leftrightarrow n < n^*$, and $\mathbb{E}q^p(n) = q(n) \Leftrightarrow n = n^*$. \square

PROOF OF PROPOSITION 1. The objective of the menu setter is to $\max_n \{\alpha \mathbb{E}Q^p(n) - c \cdot n\}$. By the first-order condition, the optimal menu size for the lower-ability menu setter, \hat{m} , is characterized by $\alpha \mathbb{E}q^p(\hat{m}) = c$. And the optimal menu size for the expert, \hat{x} , is characterized by $\alpha q(\hat{x}) = c$.

(i) If $c < \alpha q(n^*)$, then $c = \alpha q(\hat{x})$ for $\hat{x} > n^*$ (by decreasing $q(n)$). Then, by Lemma 3(ii), $\mathbb{E}q^p(\hat{x}) > q(\hat{x})$. So $c < \alpha \mathbb{E}q^p(\hat{x})$. Since $\mathbb{E}q^p(n)$ is also decreasing, $c = \alpha \mathbb{E}q^p(\hat{m})$ for $\hat{m} > \hat{x}$. Therefore, the menu of the lower-ability menu setter is larger than the menu of the expert.

³⁶ Since $\mathbb{E}q^p(n)$ is given by (A1) and $q(n) = k^{n-1}q(1)$, $\mathbb{E}(q \mid \text{success at } n) = \sum_{i=0}^{n-1} ((n-1)!/((n-1-i)!i!))p^{n-i}(1-p)^i k^{n-i-1}q(1)$.

³⁷ By substituting $q(n) = k^{n-1}q(1)$ into (A1) and (A2), we also obtain $\mathbb{E}q^p(n+1) = [(1-p) + pk]\mathbb{E}q^p(n)$.

(ii) By analogous reasoning, we obtain that for $c > \alpha q(n^*)$ the expert offers larger menu than the lower-ability menu setter. \square

PROOF OF PROPOSITION 2. Suppose that for a given c , \hat{x} is the optimal size of a menu selected by the expert; that is, $\alpha q(\hat{x}) = c$. And \hat{m} is the optimal size of a menu selected by the lower-ability menu setter; that is, $\alpha \mathbb{E}q^p(\hat{m}) = c$. Because the cost is the same for both menu setters, it must be that the respective optimal menu sizes satisfy $q(\hat{x}) = \mathbb{E}q^p(\hat{m})$.

By the Strong Assumption of Declining Marginal Benefit, $q(\hat{x}) = k^{\hat{x}-1}q(1)$. By Lemma 2,

$$\mathbb{E}q^p(\hat{m}) = \mathbb{E}q^p(1)[(1-p) + pk]^{\hat{m}-1}.$$

Moreover, $\mathbb{E}q^p(1) = pq(1)$. Therefore,

$$\begin{aligned} q(\hat{x}) = \mathbb{E}q^p(\hat{m}) &\iff k^{\hat{x}-1}q(1) = pq(1)[(1-p) + pk]^{\hat{m}-1} \\ &\iff k^{\hat{x}-1} = p[(1-p) + pk]^{\hat{m}-1}. \end{aligned} \quad (\text{A3})$$

The respective qualities of the menus are given by³⁸

$$\begin{aligned} Q(\hat{x}) &= q(1) \frac{k}{1-k} (1 - k^{\hat{x}}) \quad \text{and} \\ \mathbb{E}Q^p(\hat{m}) &= q(1) \frac{k}{1-k} (1 - [(1-p) + pk]^{\hat{m}}). \end{aligned}$$

Then $Q(\hat{x}) > \mathbb{E}Q^p(\hat{m}) \Leftrightarrow_{\text{by (A3)}} kp < [(1-p) + pk] \Leftrightarrow p < 1$. Therefore, for all $p < 1$, $Q(\hat{x}) > \mathbb{E}Q^p(\hat{m})$. \square

Appendix B. Proposition 2 with Integer Constraint

The proof of Proposition 2 ignores the constraint that the menu size, n , must be an integer. With the integer constraint, both the lower-ability menu setters and the experts round down the number of menu items offered. For the purpose of Proposition 2, there is concern in cases where the expert rounds down the number of menu items by more than the lower-ability menu setter, as this results in a possibility that the lower-ability menu setter's menu would be of higher quality. We derive the condition that ensures that even in the most severe case of rounding, the expert offers the superior menu.

To account for the integer constraint, we approach the problem as follows: Suppose that x' and m' are the largest integers less than or equal to \hat{x} and \hat{m} . (Note that (A3) still holds for \hat{x} and \hat{m} .) The *worst case scenario* is when \hat{m} is an integer and \hat{x} is just below an integer. Then $m' = \hat{m}$ and we can approximate $x' = \hat{x} - 1$. Then $Q(\hat{x} - 1) > \mathbb{E}Q^p(\hat{m})$ would ensure that the expert offers a better menu than the lower-ability menu setter, even under the worst case scenario.

By similar calculations as in the proof of Proposition 2, we find that

$$Q(\hat{x} - 1) = q(1)k \sum_{l=0}^{\hat{x}-2} k^l = q(1) \frac{k}{1-k} (1 - k^{\hat{x}-1}).$$

Then

$$\begin{aligned} Q(\hat{x} - 1) > \mathbb{E}Q^p(\hat{m}) &\iff k^{\hat{x}-1} < [(1-p) + pk]^{\hat{m}} \\ &\iff p < 1 - p + pk \iff p(2 - k) < 1. \end{aligned}$$

³⁸ We directly use the formula for the finite geometric series: $\sum_{k=0}^{N-1} g^k = (1 - g^N)/(1 - g)$.

That is, for sufficiently small p and large k the condition will be satisfied.

Note that this is a strong sufficient condition designed to hold under the worst case scenario for x' and m' . In most cases, the inequality $Q(x') - \mathbb{E}Q^p(m') > 0$ is satisfied for a weaker condition.

Appendix C. Role of k in Propositions 1 and 2

To obtain Propositions 1 and 2, we introduced the Strong Assumption of Declining Marginal Benefit, stating that the marginal benefit of additional valuable items declines at a rate k , such that $q(n+1) = k \cdot q(n)$. This assumption of fixed k allows for modeling tractability and expositional simplicity, but far weaker assumptions are sufficient for the results in Propositions 1 and 2.

The necessary feature that leads to the single-crossing property in Proposition 1 is that the marginal benefit for the expert declines more rapidly than the marginal benefit for the lower-ability menu setter (as in Lemma 2). For a fixed k , the marginal benefit for the lower-ability menu setter is $(1-p) + pk$, which is greater than k , thus leading to our result (see the proof of Lemma 2). However, if k varies depending on the number of valuable items already in the menu—and we will denote this as k_n , where $q(n+1) = k_n \cdot q(n)$ —then the comparison of slopes is less clear. This is because after including n items in his menu, the lower-ability menu setter does not know how many of those n items are valuable. Thus, the marginal benefit for the lower-ability menu setter depends on all past k 's; that is, $\mathbb{E}q^p(n+1) = [(1-p) + p\bar{k}_n] \mathbb{E}q^p(n)$, where \bar{k}_n denotes the combination of past k 's:

$$\bar{k}_n = \frac{\sum_{i=0}^{n-1} \frac{(n-1)!}{(n-1-i)!i!} p^i (1-p)^{n-1-i} \prod_{l=1}^{i+1} k_l}{\sum_{i=0}^{n-1} \frac{(n-1)!}{(n-1-i)!i!} p^i (1-p)^{n-1-i} \prod_{l=0}^i k_l}, \quad \text{with } k_0 = 1.$$

A sufficient condition for the single-crossing property is that for all n

$$(1-p) + p\bar{k}_n > k_n. \quad (\text{C1})$$

The left-hand side of this inequality is a convex combination of \bar{k}_n and 1. So (C1) is violated only if previous k_i 's ($i = 1, \dots, n-1$) are much below k_n . If k is constant or if k_i is decreasing, the condition is satisfied. If k_i is increasing at a small rate, the condition still is satisfied. However, if k_i is increasing sufficiently to violate condition (C1), then there are areas for which the marginal benefit for the lower-ability menu setter will be declining more rapidly than the marginal benefit for the expert, and it will be possible for the single-crossing condition to be violated.

A similar argument applies for Proposition 2, which states that the expert's menu is always of higher total quality than that of the lower-ability menu setter. Using the same notation as in the proof to Proposition 2, \hat{x} and \hat{m} , respectively, are the optimal menu sizes for the high-ability menu setter and the lower-ability menu setter, for a given c . The argument behind the proposition is that the slope of the marginal benefit at \hat{x} for the expert is more negative than

the slope of the marginal benefit for the lower-ability menu setter \hat{m} . Moreover, this inequality must continue for each additional item beyond \hat{x} and \hat{m} .

Thus, the sufficient condition for Proposition 2 is that for all optimal menu sizes for the two types of menu setters, \hat{x} and \hat{m} , inequality $(1-p) + p\bar{k}_{\hat{m}+i} > k_{\hat{x}+i}$ holds for all $i \geq 0$.

Again, the left-hand side is a convex combination of $\bar{k}_{\hat{m}+i}$ and 1, so it is violated only if $k_{\hat{x}+i}$ is much higher than the previous k 's. In other words, Proposition 2 will hold as long as k_i does not vary excessively.

References

- Agnew J (2006) Do behavioral biases vary across individuals: Evidence from individual level data. *J. Financial Quant. Anal.* 41(4):939–961.
- Agnew J, Balduzzi P, Sunden A (2003) Portfolio choice and trading in a large 401(k) plan. *Amer. Econom. Rev.* 93(1):193–215.
- Benartzi S, Thaler RH (2001) Naive diversification strategies in defined contribution saving plans. *Amer. Econom. Rev.* 91(1):79–98.
- Bergstresser D, Chalmers J, Tufano P (2009) Assessing the costs and benefits of brokers in the mutual fund industry. *Rev. Financial Stud.* 22(10):4129–4156.
- Bertrand M, Karlan D, Mullainathan S, Shafir E, Zinman J (2010) What's advertising content worth? Evidence from a consumer credit marketing field experiment. *Quart. J. Econom.* 125(1):263–305.
- Choi JJ, Laibson D, Madrian BC (2010) Why does the law of one price fail? An experiment on index mutual funds. *Rev. Financial Stud.* 23(4):1405–1432.
- Christoffersen S, Musto D (2002) Demand curves and the pricing of money management. *Rev. Financial Stud.* 15(5):1499–1524.
- Cohen L, Schmidt B (2009) Attracting flows by attracting big clients. *J. Finance* 64(5):2125–2151.
- Ellison G, Fudenberg D, Mobius M (2004) Competing auctions. *J. Eur. Econom. Assoc.* 2(1):30–66.
- Elton E, Gruber M, Blake C (2006) The adequacy of investment choices offered by 401(k) plans. *J. Public Econom.* 90(6):1299–1314.
- Elton E, Gruber M, Blake C (2007) Participant reaction and the performance of funds offered by 401(k) plans. *J. Financial Intermediation* 16(2):249–271.
- Elton E, Gruber M, Busse J (2004) Are investors rational? Choices among index funds. *J. Finance* 59(1):261–288.
- Gil-Bazo J, Ruiz-Verdu P (2009) The relation between price and performance in the mutual fund industry. *J. Finance* 64(5):2153–2183.
- Gruber M (1996) Another puzzle: The growth in actively managed mutual funds. *J. Finance* 51(3):783–810.
- Huberman G, Jiang W (2006) Offering versus choice in 401(k) plans: Equity exposure and the number of funds. *J. Finance* 61(2):763–801.
- Iyengar S (2010) *The Art of Choosing* (Twelve, New York).
- Iyengar S, Kamenica E (2010) Choice proliferation, simplicity seeking, and asset allocation. *J. Public Econom.* 94(7–8):530–539.
- Iyengar S, Lepper M (2000) When choice is demotivating: Can one desire too much of a good thing? *J. Personality Soc. Psych.* 76(6):995–1006.
- Iyengar S, Huberman G, Jiang W (2004) How much choice is too much: Determinants of individual contributions in 401(k) retirement plans. Mitchell OS, Utkus S, eds. *Pension Design and Structure: New Lessons from Behavioral Finance* (Oxford University Press, Oxford, UK), 83–95.

- Kamenica E (2008) Contextual inference in markets: On the informational content of product lines. *Amer. Econom. Rev.* 98(5): 2127–2149.
- Kuksov D, Villas-Boas JM (2010) When more alternatives lead to less choice. *Marketing Sci.* 29(3):507–524.
- Lehrer J (2009) *How We Decide* (Houghton Mifflin, New York).
- Mottola GR, Utkus S (2003) Can there be too much choice in a retirement savings plan? Report, Vanguard Center for Retirement Research, The Vanguard Group, Valley Forge, PA.
- Salgado M (2006) Choosing to have less choice. FEEM Working Paper 37.2006, Fondazione Eni Enrico Mattei, Milano, Italy.
- Schwartz B (2004) *The Paradox of Choice* (Ecco, New York).
- Tang N, Mitchell OS, Mottola GR, Utkus SP (2010) The efficiency of sponsor and participant portfolio choices in 401(k) plans. *J. Public Econom.* 94(11–12):1073–1085.
- Villas-Boas JM (2009) Product variety and endogenous pricing with evaluation costs. *Management Sci.* 55(8):1338–1346.