This article was downloaded by: [155.246.103.35] On: 25 March 2017, At: 14:57
Publisher: Institute for Operations Research and the Management Sciences (INFORMS)
INFORMS is located in Maryland, USA

Manufacturing & Service Operations Management

Publication details, including instructions for authors and subscription information:
http://pubsonline.informs.org

Advance Demand Information in a Multiproduct System

Fernando Bernstein, Gregory A. DeCroix

To cite this article:
Fernando Bernstein, Gregory A. DeCroix (2015) Advance Demand Information in a Multiproduct System. Manufacturing & Service Operations Management 17(1):52-65. http://dx.doi.org/10.1287/msom.2014.0502

Full terms and conditions of use: http://pubsonline.informs.org/page/terms-and-conditions

This article may be used only for the purposes of research, teaching, and/or private study. Commercial use or systematic downloading (by robots or other automatic processes) is prohibited without explicit Publisher approval, unless otherwise noted. For more information, contact permissions@informs.org.

The Publisher does not warrant or guarantee the article's accuracy, completeness, merchantability, fitness for a particular purpose, or non-infringement. Descriptions of, or references to, products or publications, or inclusion of an advertisement in this article, neither constitutes nor implies a guarantee, endorsement, or support of claims made of that product, publication, or service.

Copyright © 2015, INFORMS

Please scroll down for article—it is on subsequent pages

INFORMS is the largest professional society in the world for professionals in the fields of operations research, management science, and analytics.
For more information on INFORMS, its publications, membership, or meetings visit http://www.informs.org

MANUFACTURING & SERVICE OPERATIONS MANAGEMENT
Vol. 17, No. 1, Winter 2015, pp. 52–65
ISSN 1523-4614 (print) | ISSN 1526-5498 (online)

http://dx.doi.org/10.1287/msom.2014.0502
© 2015 INFORMS

Downloaded from informs.org by [155.246.103.35] on 25 March 2017, at 14:57 . For personal use only, all rights reserved.

Advance Demand Information in a Multiproduct System

Fernando Bernstein
Fuqua School of Business, Duke University, Durham, North Carolina 27708, fernando@duke.edu

Gregory A. DeCroix
Wisconsin School of Business, University of Wisconsin–Madison, Madison, Wisconsin 53706, gdecroix@bus.wisc.edu

In this paper we examine the impact of different types of advance demand information on firm profit and on the benefits of resource flexibility. Specifically, we consider a firm that must choose capacities of resources that will be used to satisfy stochastic demand for multiple products, where demands follow a multivariate normal distribution. Prior to the capacity decision, the firm receives information revealing either the total volume of demand across products or the mix of demand between products. We examine two different scenarios: a *dedicated resource* setting with product-specific resources and a *common resource* scenario with one flexible resource. For both scenarios we derive the distribution of the (possibly imperfect) volume or mix demand signal, as well as the conditional distributions of demand given the particular signal. We explore the impact of either type of information on optimal capacities and profit. We find that commonality and volume information are strategic complements—so that it is more valuable to obtain volume information in settings with a common resource. On the other hand, commonality and mix information are strategic substitutes. Moreover, we find that mix and volume information themselves are complements in systems with dedicated resources. Having either type of information is valuable in reducing uncertainty for each individual product demand, but having both of them together provides information on two different dimensions, allowing for a much greater reduction in demand uncertainty. In systems with a common resource, however, the two types of information are substitutes. Because volume information is well aligned with commonality (both focus on total demand), such information already provides much of the value that can be obtained—having mix information adds limited additional value.

Keywords: advance demand information; capacity planning and investment; multiproduct system; flexible resources
History: Received: August 27, 2012; accepted: July 27, 2014. Published online in *Articles in Advance* November 3, 2014.

1. Introduction

To better manage purchasing and production decisions, supply chain managers are constantly seeking better information about future product demand. Such information can come from a variety of sources and take a variety of forms. In some settings, customers place orders in advance of their actual needs, automatically providing partial information about specific demand for individual products in future periods. For example, Boeing's current multiyear backlog allows the company to alter production capacities in time to meet that future demand (Bachman 2014). In the absence of advance orders, traditional marketing research tools such as conjoint analysis can be used to translate consumer responses in choice experiments into predictions of preferences among products (see, for example, Bakken and Frazier 2006). With the growth of online searching and purchasing activity, firms and researchers have explored ways of using customers' electronic behavior to predict demand. Individual firms can gather data about which products customers view on their websites and use these data to make predictions about future demand. Moreover, those choices may help companies assess relative preferences of customers for different product variants. Consumer trending services such as Google Trends provide vast amounts of data that have the potential to aid in demand forecasting. For example, Du and Kamakura (2012) provide a methodology to identify and interpret trends from such services.

The bulk of operations management research addressing the use of advance demand information focuses on settings with advance orders and in many cases restricts attention to a single product. Here, we explicitly explore a multiresource, multiproduct setting and consider more aggregated types of information that are less specific than advance orders. When there are multiple products, demand information has multiple dimensions, and firms may be able to obtain better information about some dimensions than others. We consider two dimensions that arise naturally in a variety of settings: the total demand volume across a family of related products (or across different geographical regions) and the mix of demand across products within such a family (or among geographic regions).

RIGHTSLINK

Downloaded from informs.org by [155.246.103.35] on 25 March 2017, at 14:57 . For personal use only, all rights reserved.

As an example of volume information, consider the auto manufacturers who were forced to change some of their paint colors because of pigment supply disruptions arising from the 2011 earthquake and tsunami in Japan (Dawson 2011). The mix of demands across these new colors may be rather uncertain, but if demand for a model is reasonably stable year to year, the previous year's sales may provide a good—though still imperfect—signal of total demand volume across the color options. In some cases, the choice of customer contract terms and/or production capabilities may make volume information available. For example, by allowing retailers to change the color mix of their initial product orders, and by incorporating postponement concepts into its production capabilities, Benetton has been able to obtain advance information about total demand volume while (color) mix information is still uncertain (Signorelli and Heskett 1984).

Availability of demand mix information is prevalent in other settings. For example, firms may conduct surveys to assess customer preferences among different product variants (see, e.g., Arora et al. 1998). In many cases purchasing quantities reported by customers in such surveys can be somewhat inflated, limiting the amount of volume information that can be obtained, but the relative preferences revealed by the surveys (mix information) tend to be more reliable. The founder of a company that sells belts and other accessories online communicated another example to one of the authors. The company obtains clickstream data on the number of times each product variant is viewed, providing some sense of the relative popularity of the various belts designs (mix information). Since conversion rates in online commerce tend to be low and volatile, however, such data may not be that useful in predicting demand volume. For example, the daily conversion data gathered by the belt online retailer (over a seven-month period in 2010) averaged 1.10%, but that rate ranged from 0% to 5%. Finally, a company choosing to enter a new product market (e.g., tablet computers) may be able to analyze incumbent companies' sales data to estimate customer preferences among different variants (e.g., different memory levels), but this information might not provide much guidance as to the overall strength of customer response to the new company's products (volume information).

To explore the impact of different information types, we model a firm that sells multiple products and fills demand using either dedicated resources or one common (flexible) resource. This allows us to explore the interaction between advance information and an alternate strategy for dealing with uncertain demand in a multiproduct, multiresource setting—namely, resource flexibility. The model is general enough to accommodate multiple interpretations. We focus on that of a production system, where the resources represent production capacities. A product requires either one unit of that product's dedicated resource or one unit of the common resource. (This model is equivalent to one where demands represent distinct geographical markets and the resources represent units held at local warehouses or at a centralized warehouse.) Demand follows a multivariate normal distribution, and the firm receives a (possibly imperfect) signal revealing either total demand volume or demand mix prior to choosing capacities. We model imperfect advance demand information signals using the Martingale Model of Forecast Evolution (MMFE). Most of the paper focuses on the case of independent and identically distributed demands for simplicity, but the methodology to derive the updated demand distributions for a given signal is applicable in general settings. (We extend the derivations to the case of asymmetric and correlated demands in Online Appendix B, available as supplemental material at http://dx.doi.org/10.1287/msom.2014.0502.) After the capacity decision, the firm observes actual demand and satisfies it to the extent possible.

We first consider the case in which the firm obtains demand volume information. Based on the density of the demand volume signal, we derive the conditional density of each product's demand given the signal. We show that obtaining any amount of demand volume information causes optimal capacities to move closer to mean demand, as would be expected given the variance-reduction effect of demand information, and that this effect becomes stronger as the accuracy of the demand signal increases.

Next we consider the case where the firm obtains advance information about the mix of demand between products, which we model as a signal revealing (a slightly modified version of) each product's market share. We use a transformation of the demand space from rectangular coordinates to polar coordinates, where the ratio of demands is reinterpreted as the angle of rotation around the origin, and observe that this (random) angle has an offset normal distribution. Using this fact, we then derive the conditional density of each product's demand given a particular demand mix signal for the case of perfect mix information. Both the offset normal density and the conditional density of product demand are complex in general. However, we show that the conditional demand density is approximately normal, as long as the standard deviation of the underlying (multivariate normal) demand distribution is not too large relative to the mean. Since the original multivariate normal demand assumption requires a standard deviation that is not too large relative to the mean

RIGHTSLINK

Downloaded from informs.org by [155.246.103.35] on 25 March 2017, at 14:57 . For personal use only, all rights reserved.

(to avoid negative demand realizations), this approximation is accurate under reasonable parameter values. We build on the results for perfect information to develop numerical methods for approximating the conditional demand distribution given an imperfect mix signal.

Using the fundamental results for both types of demand information, we obtain insights about how the number of products affects the value of advance demand information. We find that the impact depends on the type of information as well as the type of resource (dedicated or common). Specifically, as the number of products grows, volume information becomes less valuable (as measured by the increase in total profit) with dedicated resources but more valuable with a common resource. The net impact depends on the interaction of two effects: the impact of the number of products on the value of information for each product and the ability to apply that value to a larger number of products. With more products, total volume information yields less information about each individual product's demand, and this (negative) effect dominates when resources are dedicated. This information dilution is not a factor when a common resource is used, so the ability to leverage the information across a larger number of products dominates. Mix information yields the opposite behavior—it becomes more valuable when resources are dedicated and less valuable when a common resource is used. The former is because mix information does not become diluted (for the case of dedicated resources) as the number of products grows. The latter is because the value provided by mix information in this scenario is actually due to volume information embedded in the mix signal—and this information becomes weaker as the number of products grows. In addition, we study how product asymmetries—in terms of either selling prices or demand distributions—and demand correlation affect the value provided by either type of information.

We also explore the substitute/complement nature of multiproduct information. Specifically, we find that commonality and volume information are complements—so that it is more valuable to obtain volume information in settings with a common resource (or, equivalently, moving toward commonality is more valuable if volume information is available). This stems from the fact that, although commonality alone yields pooling benefits, there is still significant uncertainty remaining. Having volume information greatly reduces this uncertainty, by providing (at least a signal of) the exact kind of information that is needed to manage a common resource serving total demand. On the other hand, commonality and mix information are substitutes for just the opposite reason. Mix information alone helps reduce uncertainty for individual product demands (for the dedicated resource case), but in the presence of commonality, what matters is total demand—how it is split among different products (mix information) is irrelevant—so mix information is of little additional value if a common resource is used.

We find that mix and volume information themselves are complements in systems with dedicated resources. Here, having either type of information is valuable in reducing uncertainty for each individual product demand, but having both of them together provides information on two different dimensions, allowing for a much greater reduction in demand uncertainty. In systems with a common resource, however, the two types of information are substitutes. Here, since volume information is well aligned with commonality (both focus on total demand), such information already provides much of the value that can be obtained. Having mix information adds limited additional value.

The rest of the paper is organized as follows. Section 2 discusses the relevant literature. Section 3 introduces the basic model and notation. Section 4 presents fundamental results for the case of volume information, whereas §5 deals with the case of mix demand information. In §6, we study the value of advance demand information and explore substitute/complement relationships between type of information and product structure as well as between the two types of information (under both types of product structures). Section 7 provides concluding remarks. The online supplement contains three appendices: all proofs are relegated to Online Appendix A; in Online Appendix B, we extend the derivation of the conditional demand distributions to nonidentical and correlated demands; and Online Appendix C contains arguments supporting an approximation used for some parts of the analysis of mix information.

2. Literature Review

This paper is related to three streams of work. The first of these consists of operations management papers that study the use of advance demand information (ADI) in inventory and capacity decisions. In most of these papers, ADI is obtained through early customer orders, which may be induced, e.g., by early-order price discounts. We review some of the relevant papers here. Hariharan and Zipkin (1995) extend the analysis of some inventory models to a setting in which customers provide advance information on their demands. For a single-product system, DeCroix and Mookerjee (1997) study the question of when it is optimal to acquire costly advance information about stochastic demand. Chen

RIGHTSLINK

Downloaded from informs.org by [155.246.103.35] on 25 March 2017, at 14:57 . For personal use only, all rights reserved.

(2001) studies the balance between the costs associated with price discounts and the benefits obtained through ADI. Gallego and Özer (2001) characterize the optimal inventory policy in a finite-horizon setting in which customers place advance orders. The authors explore the value of ADI. Özer and Wei (2004) extend that paper by analyzing a capacitated inventory system in which the manufacturer has the ability to obtain ADI. Boyacı and Özer (2010) consider the problem of optimal installation of capacity when the manufacturer can control the availability of ADI through the choice of prices. Özer (2003) examines the value of ADI in a distribution system. The system can be interpreted as a multi-item production system with a common intermediate product, but in this case, ADI is obtained for each individual product. Karaesmen et al. (2002) investigate the structure of the optimal policy in a make-to-stock queue with ADI. The authors also propose a heuristic that is easy to implement. Thonemann (2002) analyzes the benefits of sharing ADI in a supply chain. Tan et al. (2007) model imperfect advanced demand information in a multiperiod setting and show that the optimal ordering policy is a function of the amount of advance demand information obtained. Tan et al. (2009) model imperfect demand information in a setting with two demand classes and inventory rationing. Wang and Toktay (2008) consider an inventory model with advance demand information and flexible deliveries. Demand information is obtained through customer orders and early shipments are allowed. In Gayon et al. (2009), a make-to-stock supplier serves multiple customer classes and receives imperfect advance demand information in the form of early customer orders. The objective is to determine the optimal joint production and inventory-allocation policy. Benjaafar et al. (2011) study a production-control problem in which customers provide ADI to the supplier by announcing orders ahead of their due dates. This information is imperfect because customers may later request an earlier or a later fulfillment due date. In contrast to these papers, which consider single-product problems, we examine a setting with multiple products and explore the value of aggregate demand information across products. Huang and Van Mieghem (2013) conduct an empirical study that provides statistical evidence that clickstream data can be used as advance demand information for procurement, production, and inventory management decisions. Advance demand information is also central to quick and accurate response strategies. Under these strategies, a firm can adjust its production and ordering decisions based on demand information obtained early in the selling season. The papers by Fisher and Raman (1996) and Iyer and Bergen (1997) are among the first to study the value of quick response strategies in supply chains. Both of these papers use product-specific information to update demand, whereas our focus is on aggregate information across products.

A second related research stream involves the analysis of the multiproduct newsvendor problem and the study of component commonality and resource flexibility. Papers exploring multiproduct newsvendor models tend to focus either on the role of capacity constraints (see, e.g., Nahmias and Schmidt 1984, Lau and Lau 1996) or on the substitution effect across products (e.g., Bassok et al. 1999, Smith and Agrawal 2000, Mahajan and van Ryzin 2001). We refer to Turken et al. (2012) for a review of the literature on multiproduct newsvendor problems. Along these lines, Van Mieghem and Rudi (2002) consider newsvendor networks that allow for multiple products and multiple processing and storage points. Relevant research on component commonality includes Collier (1982), Baker et al. (1986), Gerchak et al. (1988), and Gerchak and Henig (1989). These papers explore the benefits of component commonality under centralized decision making.

The two papers most related to our work are Zhu and Thonemann (2004) and Casimir (2002). Zhu and Thonemann study a setting in which customers may provide their demand forecasts at a cost. All customer demands are for a single product. The customers' advance information is used to improve the overall demand forecast for that product, modeled as an additive MMFE. The paper investigates the optimal number of customers to engage in demand information sharing. In contrast, we consider a setting with multiple products and a signal of either total demand volume or the relative mix (but not magnitudes) of different product demands. Although both models involve partial information about demand, the types of information obtained—and the analysis required to make use of that information—are fundamentally different. As in Zhu and Thonemann, we adopt the MMFE framework to capture imperfect demand information in our setting. Casimir considers a multiproduct problem but in a restricted setting with only dedicated products, perfect demand information, and symmetric cost/revenue parameters. The symmetry assumption plays a central role in that paper, as it implies that the ratios of the optimal product order quantities match the ratios of the product market shares.

The availability of volume and mix demand information is related to the marketing literature concerned with the estimation of primary and secondary demand. Primary demand for a product is the aggregate demand for all brands in a product category, whereas secondary demand is the demand for a specific brand within a category of products. A number

RIGHTSLINK

Downloaded from informs.org by [155.246.103.35] on 25 March 2017, at 14:57 . For personal use only, all rights reserved.

of papers in the marketing literature study estimation models that account for primary and secondary demand. For example, Arora et al. (1998) propose a hierarchical Bayesian model to estimate primary and secondary demand. Using data obtained from a field study, the authors statistically estimate the parameters of a logit model to assess the impact of price and other product attributes on product demand. Nair et al. (2005) present a methodology for estimating demand that can accommodate multiunit purchases.

3. Model

A firm sells n products ($i = 1, \ldots, n$) that are produced either using dedicated resources, i.e., a resource i for each product i, or using one common flexible resource. Let D_i denote random demand for product $i = 1, \ldots, n$ during a single selling season, and assume that $(D_1, \ldots, D_n)$ follows a multivariate normal distribution. For clarity of exposition, we assume that the random variables are independent, each with mean μ and standard deviation σ. (In Online Appendix B we show how our analysis extends to the case of general asymmetric and/or correlated demands.) Let $f(\cdot)$ denote the joint density of $(D_1, \ldots, D_n)$. Let $f_i(\cdot)$ and $F_i(\cdot)$ be the marginal density and cumulative distribution function (cdf) for D_i, respectively. Also, let $E[N]$ denote the expected value of N, and let $\phi(x)$ and $\Phi(x)$ denote the density and cdf of the standard normal distribution, respectively. When we wish to emphasize that the expectation is with respect to a random variable K, we denote this as $E_K[N]$.

For the case of dedicated resources, let c_i be the unit capacity cost, let p_i be the exogenous product market price (net of any production costs), and let Q_i be the production capacity installed for product i. In the system with a common resource, let c_C be the unit capacity cost, let Q_C be the flexible production capacity installed, and let p_i be the exogenous market price (net of any production costs) for product i. Assume that profit margins are positive, i.e., $p_i > c_i$ in the system with dedicated resources and $p_i > c_C$ in the system with a common resource. In the absence of additional demand information, the firm first chooses resource capacity levels (Q_i or Q_C), then actual demand is observed, and sales are generated by filling demand to the extent possible based on the installed capacity. In the system with dedicated resources, product i sales are equal to $\min\{Q_i, D_i\}$. As a result, the resource capacity decisions correspond to newsvendor problems with solutions $Q_i = \mu + z_i\sigma$, where $z_i = \Phi^{-1}(1 - c_i/p_i)$. In the system with a common resource, the total sales quantity is $\min\{Q_C, \sum_{i=1}^n D_i\}$. When market prices differ across products, however, demand is filled in a nested fashion—demands for the highest-priced products are satisfied first, followed by the second highest, etc. As a result, choosing the optimal capacity still has a newsvendor flavor but is somewhat more complicated. When all market prices are identical with $p_i = p$, the capacity decision corresponds to a newsvendor problem with solution $Q = n\mu + z_C\sqrt{n}\sigma$, where $z_C = \Phi^{-1}(1 - c_C/p)$.

4. Volume Information

Now suppose that, prior to making capacity decisions, the firm is able to observe an imperfect signal of total aggregate demand volume $\sum_{i=1}^n D_i$; otherwise, the sequence of events is the same as for the no-information case. To make use of and assess the value of advance demand information, we need to characterize the distribution of the signal itself and also the distribution of demands given the signal. We begin with the distribution of the demand volume signal.

We model this imperfect signal using the Martingale Model of Forecast Evolution, which was independently introduced by Graves et al. (1986) and Heath and Jackson (1994), and was also used by Zhu and Thonemann (2004). Specifically, let $\hat{X}$ be the random volume signal (with a specific value denoted $\hat{x}$), and let Δ be the error (with a specific value denoted δ) so that the true total demand volume $X = \hat{X} + \Delta$. From the assumptions on the underlying demand distributions, $X \sim \mathrm{N}(n\mu, n\sigma^2)$. The MMFE approach assumes that the imperfect signal is unbiased, and it splits the variance of total demand between the signal and the error term using a parameter β, with $0 \le \beta \le 1$. Specifically, we have the imperfect signal $\hat{X} \sim \mathrm{N}(n\mu, \beta n\sigma^2)$ and the error term $\Delta \sim \mathrm{N}(0, (1-\beta)n\sigma^2)$. The parameter β can be interpreted as a measure of signal accuracy, with $\beta = 0$ corresponding to no information (the error term contains all demand variation) and $\beta = 1$ corresponding to perfect information (the error term vanishes).

We next derive the joint distribution of the D_i given an observed (imperfect) volume signal $\hat{X} = \hat{x}$. We begin by deriving the joint distribution of the conditional demand vector given observation of a perfect demand volume signal; i.e., $(D_1, \ldots, D_{n-1}) \mid X = x$. (Note that, with a perfect signal, demand for the nth product D_n is just $x - (D_1 + \cdots + D_{n-1})$.)

Proposition 1. *The conditional demand vector $(D_1, \ldots, D_{n-1}) \mid X = x$ has an $(n-1)$-dimensional multivariate normal distribution with identical marginal means x/n, identical marginal standard deviations $\sigma\sqrt{(n-1)/n}$, and identical pairwise correlation coefficients $-1/(n-1)$ for $i \neq j$.*

The preceding result can be used to obtain the conditional joint distribution of the product demands given an *imperfect* signal $\hat{X} = \hat{x}$ with signal strength β. The following result characterizes this distribution.

RIGHTSLINK

Downloaded from informs.org by [155.246.103.35] on 25 March 2017, at 14:57 . For personal use only, all rights reserved.

Proposition 2. *The vector of random variables* $(D_1, \ldots, D_n) \mid \hat{X} = \hat{x}$ *follows a multivariate normal distribution with identical marginal means* $\hat{x}/n$, *identical marginal standard deviations* $\sigma\sqrt{(n-\beta)/n}$, *and identical pairwise correlation coefficients* $-\beta/(n-\beta)$ *for* $i \neq j$.

5. Mix Information

We now explore a setting in which the firm receives a signal revealing information about the market shares for each product $\{(D_i/\sum_{j=1}^n D_j)\colon i = 1, \ldots, n\}$. Observing the market shares is equivalent to observing the quantities

$$\left\{A_i \stackrel{\text{def}}{\equiv} \frac{\sum_{j\neq i} D_j}{(n-1)D_i}\colon i = 1, \ldots, n\right\},$$

since there is a one-to-one correspondence between the two, as

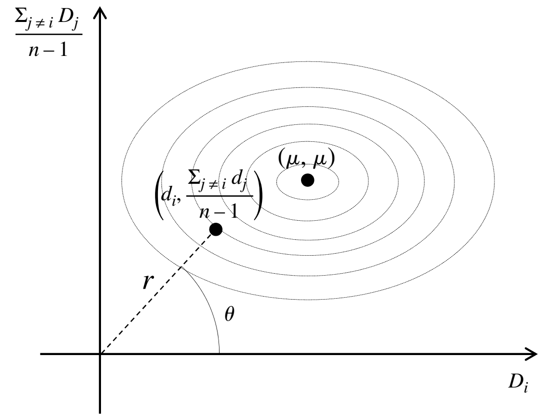
$$\frac{D_i}{\sum_{j=1}^n D_j} = \frac{1}{1+(n-1)A_i}.$$

For analytical convenience, we will work with the quantity A_i. We first assume that the signal reveals *perfect* information about the mix of demands for the products and later extend our findings to the case of an imperfect signal. The same sequence of events as in the case of volume information applies here, except that in the first step the firm learns the realization a_i of A_i, $i = 1, \ldots, n$ instead of total demand volume. (For ease of exposition, we again begin by assuming identically distributed and independent demands; the extension to asymmetric and/or correlated demands is given in Online Appendix B.)

As with volume information, we need to characterize the distribution of the A_i, as well as the conditional distribution of demands given the mix signal. Before performing the detailed analysis, we first note that, given perfect mix signals, the updated demand random variables $D_i \mid \{A_j = a_j\colon j = 1, \ldots, n\}$ are perfectly positively correlated, since having information on $(1+(n-1)a_i)/(1+(n-1)a_j) = D_j/D_i$ implies that $D_j = ((1+(n-1)a_i)/(1+(n-1)a_j))D_i$. As a result, we can focus on deriving the marginal distribution of $D_i \mid \{A_j = a_j\colon j = 1, \ldots, n\}$.

Unfortunately, obtaining the joint distribution of $(A_1, \ldots, A_n)$ for general n appears to be analytically intractable, so we develop a simplified approach. The basis for this approach is the observation that the signal A_i alone (without the A_j, $j \neq i$) contains the bulk of the relevant information about demand D_i; i.e., product i's demand is influenced much more heavily by its own market share than how market shares are distributed among other products given a particular market share for product i. (See Online Appendix C for further justification of this claim.) In light of this, we use a restricted information set to derive the conditional distribution of D_i given a mix signal by

Figure 1 **Angle θ and Radius r of a Realization** $(d_i, (\sum_{j\neq i} d_j)/(n-1))$ **of the Bivariate Normal** $(D_i, (\sum_{j\neq i} D_j)/(n-1))$



replacing $D_i \mid \{A_j = a_j\colon j = 1, \ldots, n\}$ with $D_i \mid \{A_i = a_i\}$. Our task now becomes one of obtaining the marginal distribution of each $A_i = \sum_{j\neq i} D_j/[(n-1)D_i]$ and then the conditional distribution of demand given the signal, $D_i \mid \{A_i = a_i\}$.

To obtain these distributions, it will be more convenient to perform the analysis using polar coordinates. To that end, define random variables $\Theta_i \equiv \tan^{-1}(A_i)$ and $R_i \equiv \sqrt{D_i^2 + (\sum_{j\neq i} D_j/(n-1))^2}$ with realizations θ and r, respectively. (We omit the subindices to simplify the notation.) The radius R_i is the length of a line segment from the origin to $(D_i, \sum_{j\neq i} D_j/(n-1))$, and Θ_i is the angle between the positive horizontal axis and that line segment (see Figure 1). Since $A_i = \tan(\Theta_i)$ and $D_i = R_i \cos(\Theta_i)$, our goal of obtaining the distributions of A_i and $D_i \mid A_i$ is equivalent to obtaining the distributions of Θ_i and $R_i \mid \Theta_i$, which we turn to now.

We first derive the distribution of the angle Θ_i. To that end, note that because the pair $(D_i, \sum_{j\neq i} D_j/(n-1))$ is obtained through a linear transformation of a multivariate normal random vector, it has a bivariate normal distribution. Also, because of the original independence of demands, D_i and $\sum_{j\neq i} D_j/(n-1)$ are also independent. (See, for example, Theorem 3.3.3 in Tong 1990.) The distribution of Θ_i is obtained by expressing the density of the bivariate normal distribution of $(D_i, \sum_{j\neq i} D_j/(n-1))$ in polar coordinates (using the relationship $d_i = r\cos\theta$ and $\sum_{j\neq i} d_j/(n-1) = r\sin\theta$) and integrating with respect to the radius, yielding

$$\begin{aligned} q(\theta) &= \int_0^\infty r f(r\cos\theta, r\sin\theta)\, dr \\ &= \int_0^\infty \frac{r\sqrt{n-1}}{2\pi\sigma^2} e^{[-\frac{1}{2\sigma^2}[(r\cos\theta-\mu)^2+(n-1)(r\sin\theta-\mu)^2]]}\, dr \\ &= \frac{\sqrt{n-1}s_\theta}{\sqrt{2\pi}\sigma} e^{[-\frac{1}{2}\left(\frac{\mu^2}{\sigma^2}\right)[(n-1)s_\theta^2(\sin\theta-\cos\theta)^2]]} \\ &\quad \cdot \int_0^\infty \frac{r}{\sqrt{2\pi}s_\theta\sigma} e^{-\frac{1}{2}\left(\frac{r-m_\theta\mu}{s_\theta\sigma}\right)^2}\, dr, \end{aligned} \tag{1}$$

Downloaded from informs.org by [155.246.103.35] on 25 March 2017, at 14:57 . For personal use only, all rights reserved.

where $m_\theta \stackrel{\text{def}}{=} ((n-1)\sin\theta+\cos\theta)/((n-1)\sin^2\theta+\cos^2\theta)$ and $s_\theta \stackrel{\text{def}}{=} 1/\sqrt{(n-1)\sin^2\theta+\cos^2\theta}$. After some algebra, we get

$$q(\theta)=\frac{\sqrt{n-1}s_\theta^2}{2\pi}e^{-\frac{1}{2}\left(\frac{\mu^2 n}{\sigma^2}\right)}+\sqrt{n-1}\frac{m_\theta s_\theta \mu}{\sigma}\Phi\left(\frac{m_\theta\mu}{s_\theta\sigma}\right)$$
$$\cdot\phi\left(\sqrt{n-1}\frac{s_\theta\mu}{\sigma}(\sin\theta-\cos\theta)\right),\qquad \theta\in[0,2\pi].$$

The distribution of the angle Θ_i is called the *offset normal distribution*, and it has density $q(\theta)$. (Mardia 1972 derives the offset normal distribution as part of a broader study on directional distributions.)

In developing (and verifying the accuracy of) an approximation for the distribution of $R_i \mid \Theta_i$, it will be convenient to use an approximation $\tilde{q}(\theta)$ to the offset normal density $q(\theta)$, which is accurate when the standard deviation of the underlying demand distribution is not too large relative to the mean. Since the original multivariate normal distribution requires the same type of condition to avoid negative demand realizations, this approximation (and subsequent results utilizing it) are fairly accurate under reasonable parameter values. Proposition 3 states the approximation and establishes some theoretical results regarding its accuracy in approximating $q(\theta)$.

Proposition 3. *Define*

$$\tilde{q}(\theta)=\frac{\sqrt{n-1}s_\theta m_\theta\mu}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2}\left(\frac{\mu^2}{\sigma^2}\right)[(n-1)s_\theta^2(\sin\theta-\cos\theta)^2]}.$$

Then,

$$\int_0^{\pi/2}\tilde{q}(\theta)\,d\theta=\int_{-\sqrt{n-1}\mu/\sigma}^{\mu/\sigma}\phi(z)\,dz\to 1\qquad \text{as } \mu/\sigma\to\infty,$$

and $\lim_{\mu/\sigma\to\infty}[\tilde{q}(\theta)-q(\theta)]=0$ *for* $\theta\in[0,\pi/2]$.

The first part of Proposition 3 shows that the support of $\tilde{q}(\theta)$ approaches the interval $[0,\pi/2]$ as μ/σ increases. The result then shows that $\tilde{q}(\theta)$ approximates the offset normal density function arbitrarily closely in the interval $[0,\pi/2]$ as μ/σ becomes large.

We use the approximate distribution of Θ_i from Proposition 3 to obtain an approximation for the conditional distribution of the radius R_i given a realization of the angle Θ_i, denoted by $R_\theta \stackrel{\text{def}}{=} R_i \mid (\Theta_i=\theta)$. The following result shows that R_θ converges in distribution to a normal random variable.

Proposition 4. *Let*

$$\tilde{m}_\theta \stackrel{\text{def}}{=} \frac{m_\theta^2\mu^2+s_\theta^2\sigma^2}{m_\theta\mu}\quad \text{and}\quad \tilde{s}_\theta \stackrel{\text{def}}{=} \frac{s_\theta\sigma}{m_\theta\mu}\sqrt{m_\theta^2\mu^2-s_\theta^2\sigma^2}.$$

Define $Q_\theta=(R_\theta-\tilde{m}_\theta)/\tilde{s}_\theta$, *and let* Z *denote the standard normal random variable. Then,* $Q_\theta \xrightarrow{d} Z$ *as* $\mu/\sigma\to\infty$.

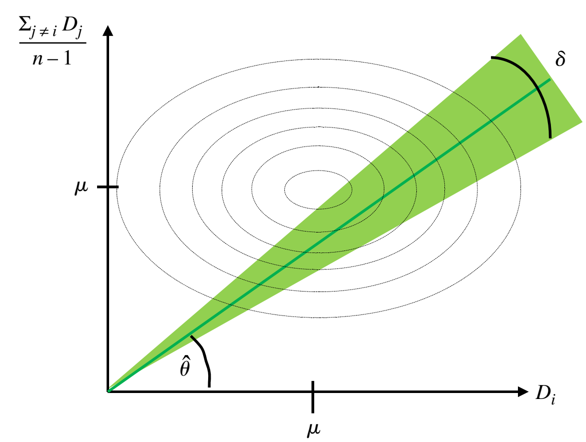
Thus, the distribution of R_θ is approximately normal with mean $\tilde{m}_\theta$ and standard deviation $\tilde{s}_\theta$, as long as μ/σ is sufficiently large. It turns out that this approximation is extremely accurate even for moderate values of μ/σ. Let $h(r\mid\theta)=rf(r\cos\theta, r\sin\theta)/q(\theta)$ be the exact density of R_θ, and define $g(r\mid\theta)$ as a normal density function with mean $\tilde{m}_\theta$ and standard deviation $\tilde{s}_\theta$. To measure the accuracy of the approximation, we calculate the distance $\int_0^{\pi/2}\int_0^\infty(h(r\mid\theta)-g(r\mid\theta))^2\,dr\,d\theta$. The table below reports an upper bound for this distance for any n, given specific values of μ and σ. In all cases, the approximation becomes more accurate as μ/σ increases or as n increases.

$\mu=2,\sigma=2$	$\mu=3,\sigma=2$	$\mu=4,\sigma=2$	$\mu=5,\sigma=2$
3×10^{-2}	7×10^{-4}	3×10^{-4}	7×10^{-5}

Translating this back to the world of product demands, recall that the conditional demand distribution $D_i \mid (A_i=a)$ is distributed as $(\cos\theta)R_\theta$ with $\theta=\tan^{-1}a$. It follows from Proposition 4 that $D_i \mid (\tan^{-1}A_i=\theta)$ is approximately normally distributed with mean $\mu(\theta)\stackrel{\text{def}}{=}(\cos\theta)\tilde{m}_\theta$ and standard deviation $\sigma(\theta)\stackrel{\text{def}}{=}(\cos\theta)\tilde{s}_\theta$. Because $m_\theta^2\mu^2-s_\theta^2\sigma^2<m_\theta^2\mu^2$, we have that $\tilde{s}_\theta<s_\theta\sigma$. This implies that $\sigma(\theta)=(\cos\theta)\tilde{s}_\theta<(\cos\theta)s_\theta\sigma<\sigma$. That is, mix information reduces the standard deviation of demand relative to the case without information.

We now examine the case of imperfect mix information. Define $\hat{\Theta}_i$ to be a random variable (with realization $\hat{\theta}$) representing an imperfect signal corresponding to the angle Θ_i. Let Δ_i be the random error (with realization δ), so that the actual angle is given by $\theta=\hat{\theta}+\delta$. (See Figure 2.) As demonstrated earlier, Θ_i has an offset normal distribution with density $q(\theta)$. Let $\hat{m}_i$ and $\hat{s}_i$ be the mean and standard deviation, respectively, of Θ_i (i.e., associated with the density $q(\theta)$).

Figure 2 (Color online) Imperfect Mix Signal



Downloaded from informs.org by [155.246.103.35] on 25 March 2017, at 14:57 . For personal use only, all rights reserved.

To facilitate the use of the MMFE model for analyzing imperfect mix information, we will approximate the offset normal distribution by a normal distribution with mean $\hat{m}_i$ and standard deviation $\hat{s}_i$. (This approximation is nearly exact for $n = 2$ and is reasonably accurate even for large n.) Similar to the case of volume information, we then assume that $\hat{\Theta}_i \sim \mathrm{N}(\hat{m}_i, \beta\hat{s}_i^2)$ and $\Delta \sim \mathrm{N}(0, (1-\beta)\hat{s}_i^2)$, with $0 \le \beta \le 1$ again measuring the accuracy of the signal.

All that remains then is to derive the marginal distribution of D_i given an observed (imperfect) mix signal $\hat{\Theta}_i = \hat{\theta}$. Similar to our approach for volume information, we make use of the perfect information results above. The density of $D_i \mid \hat{\Theta}_i = \hat{\theta}$ is given by

$$f_{D_i|\hat{\theta}}(d_i) = \int_{-\pi}^{\pi} \frac{1}{\sqrt{2\pi}\sigma(\hat{\theta}+\delta)} \exp\left\{-\frac{1}{2}\left(\frac{d_i - \mu(\hat{\theta}+\delta)}{\sigma(\hat{\theta}+\delta)}\right)^2\right\} \cdot \frac{1}{\sqrt{2\pi}\sqrt{\beta}\hat{s}_i} \exp\left\{-\frac{1}{2}\left(\frac{\delta}{\sqrt{\beta}\hat{s}_i}\right)^2\right\} d\delta, \tag{2}$$

where, recall, $\mu(\theta) = (\cos\theta)((m_\theta^2\mu^2 + s_\theta^2\sigma^2)/(m_\theta\mu))$ and $\sigma(\theta) = (\cos\theta)(s_\theta\sigma/(m_\theta\mu))\sqrt{m_\theta^2\mu^2 - s_\theta^2\sigma^2}$. The first factor in the integral in (2) is the (approximate) normal density of $D_i \mid \Theta_i = \hat{\theta} + \delta$, i.e., the conditional density given a perfect signal of the angle given by $\hat{\theta} + \delta$, for each δ. (Recall that the conditional distribution of demand given a perfect mix signal is approximately normal following the result in Proposition 4.) The second factor in the integral in (2) is the density of the error $\Delta \sim \mathrm{N}(0, (1-\beta)\hat{s}_i^2)$. Unlike the case of volume information, it does not appear possible to obtain a closed-form expression for (2). However, numerical methods can be used to compute the density given any particular signal $\hat{\theta}$.

6. Value of Advance Demand Information

In this section, we explore the value of advance demand information and its interplay with the system configuration. In §§6.1 and 6.2, we first consider a setting in which all unit costs and product prices are the same; i.e., $c_i = c$ and $p_i = p$ for all i. In those sections, we let $z = \Phi^{-1}(1 - c/p)$ and $z_C = \Phi^{-1}(1 - c_C/p)$. In §6.3, we study the impact of product asymmetries in product profit margins and demand distributions, as well as correlation in demands, on the value of information.

6.1. Value of Volume Information

Let Q_i^V denote the optimal capacity for product i in a system with dedicated resources and advance demand volume information. It follows from Proposition 2 that the optimal capacities are given by $Q_i^V = \hat{x}/n + z_i\sqrt{(n-\beta)/n}\sigma$, so that

$$|E_{\hat{X}}[Q_i^V] - \mu| = \left|E_{\hat{X}}\left[\frac{\hat{X}}{n} + z\sqrt{\frac{n-\beta}{n}}\sigma\right] - \mu\right| = |z|\sqrt{\frac{n-\beta}{n}}\sigma$$

decreases as β increases to 1. Similarly, in a system with a common resource, the optimal capacity level is $Q_C^V = \hat{x} + z_C\sqrt{n(1-\beta)}\ \sigma$, and thus $|E_{\hat{X}}[Q_C^V] - n\mu| = |z_C|\sqrt{n(1-\beta)}\sigma$, which also decreases with β. In both cases, as the quality of the signal improves, volume information pushes expected capacity closer to mean demand.

We can similarly examine the increase in profit derived from volume information. In a system with dedicated resources and no information, the expected profit is

$$\pi^D = n(p-c)\mu - n\sigma p\phi(z), \tag{3}$$

whereas with volume information, the expected profit given a particular signal $\hat{x}$ is

$$\pi_{\hat{x}}^{VD} = (p-c)\hat{x} - \sigma\sqrt{n(n-\beta)}p\phi(z). \tag{4}$$

Letting $\pi^{VD} = E_{\hat{X}}[\pi_{\hat{X}}^{VD}]$ be the expected profit (across all signals) given volume information, the expected value of volume information is

$$\pi^{VD} - \pi^D = n\sigma p\phi(z)\left(1 - \sqrt{\frac{n-\beta}{n}}\right). \tag{5}$$

This difference is increasing and convex in β; i.e., a more accurate volume signal results in higher profit, with a higher rate of increase as β increases to 1 (i.e., perfect volume information). Somewhat less intuitive is the fact that, for a given signal quality β, the value of volume information is decreasing in the number of products n. The impact of n on this value comes in two parts. First, as n increases, the number of products benefiting from the information increases, pushing the value upward. On the other hand, since a volume signal provides just one "dimension" of information aggregated across all products, this information becomes diluted—i.e., it provides less direct information about each individual product's demand—as the number of products grows. (This can be seen clearly in (5) by dividing the expression by n, yielding the per-product profit gain.) Indeed, Proposition 2 implies that the mean of the information-updated random variable converges to μ as $n \to \infty$ while the standard deviation converges to σ; i.e., variability is the same as without information. From (5) we see that the latter effect (dilution of information) dominates so that the total profit gain drops as the number of products n increases.

In a system with a common resource, expected profit without and with a specific volume signal are, respectively,

$$\pi^C = n(p-c_C)\mu - \sqrt{n}p\sigma\phi(z_C) \quad \text{and} \tag{6}$$

$$\pi_{\hat{x}}^{VC} = (p-c_C)\hat{x} - p\sqrt{n(1-\beta)}\sigma\phi(z_C). \tag{7}$$

RIGHTSLINK

Downloaded from informs.org by [155.246.103.35] on 25 March 2017, at 14:57 . For personal use only, all rights reserved.

Letting $\pi^{VC} = E_{\hat{X}}[\pi^{VC}_{\hat{X}}]$ be the expected profit (across all signals) with a common resource given volume information, the expected value of volume information is $\pi^{VC} - \pi^{C} = p\sigma\phi(z_C)\sqrt{n}(1-\sqrt{1-\beta})$. As with dedicated resources, this value is again increasing in the signal quality β. However, in contrast with the dedicated resource case, here volume information delivers more value as the number of products n increases. The key difference is that, with a common resource, volume information is exactly what is needed to manage the capacity decision, so that no dilution occurs as the number of products increases. Because of the risk-pooling benefit a common resource provides (as n increases) in the absence of any information, the overall profit gain from information grows as the square root of n. (Dividing this gain by n again shows that the gain per product decreases in n—but this time as a result of the risk-pooling benefit of the common resource, not because of dilution in the information obtained.) Figure 3 illustrates the per-product percentage profit gain derived from volume information as a function of the number of products for both dedicated and common resource settings.

6.2. Value of Mix Information

We next explore the value of mix information in the context of a perfect mix signal. We begin by calculating $E_\theta[\mu(\theta)]$ and $E_\theta[\sigma(\theta)]$. Because $\mu(\theta) = E[D_i \mid \tan^{-1} A_i = \theta]$, we have that

$$E_\theta[\mu(\theta)] = E_\theta[E[D_i \mid \tan^{-1} A_i = \theta]] = E[D_i] = \mu.$$

We obtain the expected standard deviation by integrating with respect to the density of θ:

$$E_\theta[\sigma(\theta)] = \sigma \int_0^{\pi/2} \cos\theta \frac{\sqrt{n-1}}{\sqrt{2\pi}} s_\theta^2 \sqrt{m_\theta^2 u^2 - s_\theta^2} \cdot e^{-\frac{1}{2}u^2 s_\theta^2 (n-1)(\sin\theta - \cos\theta)^2}\, d\theta \stackrel{\text{def}}{=} \sigma l_1(n, u),$$

where $u = \mu/\sigma$. The function $l_1(n,u)$ is decreasing in n. Moreover, $1 - \sqrt{1-1/n} < l_1(n,u) < 1/\sqrt{n}$ for all $u \geq 2$ and $n \geq 2$.

Consider first a setting with dedicated resources. For each product i, given a perfect mix signal θ_i, the optimal capacity is given by $Q_i^M = \mu(\theta_i) + z\sigma(\theta_i)$. As in the case of volume information, we compute $|E_\theta[Q_i^M] - \mu| = |z| E_\theta[\sigma(\theta_i)] = |z|\sigma l_1(n, \mu/\sigma) < |z|(\sigma/\sqrt{n}) < z\sigma$. That is, mix information pushes optimal capacities closer to mean demand (relative to no information). In terms of profitability, recall that total expected profit without information is $\pi^D = n(p-c)\mu - n\sigma p\phi(z)$. Given access to mix information, taking expectation over all possible signal values yields total expected profit

$$\pi^{MD} = n(p-c)\mu - \sigma n l_1(n, \mu/\sigma) p\phi(z). \tag{8}$$

(This and the subsequent profit expressions derived under the mix information signal are approximate to the extent discussed in §5.) We thus have that $\pi^{MD} - \pi^D = np\phi(z)\sigma(1 - l_1(n, \mu/\sigma))$, which is increasing in n because $l_1(n,u)$ is decreasing in n. By dividing the expression for $\pi^{MD} - \pi^D$ by n, one can see that the value per product obtained from mix information is also increasing in n. In contrast with the case of volume information, mix information does not become diluted as the number of products increases.

Consider now a setting with a common resource. In the absence of mix information, $\pi^C = n(p-c_C)\mu - \sqrt{n}p\sigma\phi(z_C)$. Using the fact that $D_i \mid \{A_i = a\}$ approximates $D_i \mid \{A_j = a_j\colon j = 1, \ldots, n\}$, we approximate the standard deviation of $\sum_{i=1}^n D_i \mid \{A_j = a_j\colon j = 1, \ldots, n\}$ by the standard deviation of $(1+(n-1)a_1)(D_1 \mid \{A_1 = a_1\})$, which represents the sum of all demands given the perfect correlation that comes when all market shares are observed. Expressed in polar coordinates, the standard deviation of aggregate demand across products, conditional on the mix information signal, is $\sigma_C(\theta) = (\sigma/s_\theta\mu)\sqrt{m_\theta^2\mu^2 - s_\theta^2\sigma^2}$. Taking expectation over the signal, we obtain

$$E_\theta[\sigma_C(\theta)] = \sigma \int_0^{\pi/2} \frac{\sqrt{n-1}}{\sqrt{2\pi}} m_\theta \sqrt{m_\theta^2 u^2 - s_\theta^2} \cdot e^{-\frac{1}{2}u^2 s_\theta^2 (n-1)(\sin\theta - \cos\theta)^2}\, d\theta \stackrel{\text{def}}{=} \sigma l_2(n,u) < \sigma\sqrt{n},$$

where $u = \mu/\sigma$. We then have that the optimal flexible capacity level under mix information is $Q_C^M = n\mu(\theta) + z_C\sigma_C(\theta)$ and $|E_\theta[Q_C^M] - n\mu| = |z_C|\sigma l_2(n,u) < |z_C|\sigma\sqrt{n}$. That is, the optimal flexible capacity level is also closer to mean demand in the presence of mix information. The corresponding expected optimal profit is

$$\pi^{MC} = n(p-c_C)\mu - p\sigma l_2(n, \mu/\sigma)\phi(z_C), \tag{9}$$

and the value of mix information with a common resource is given by

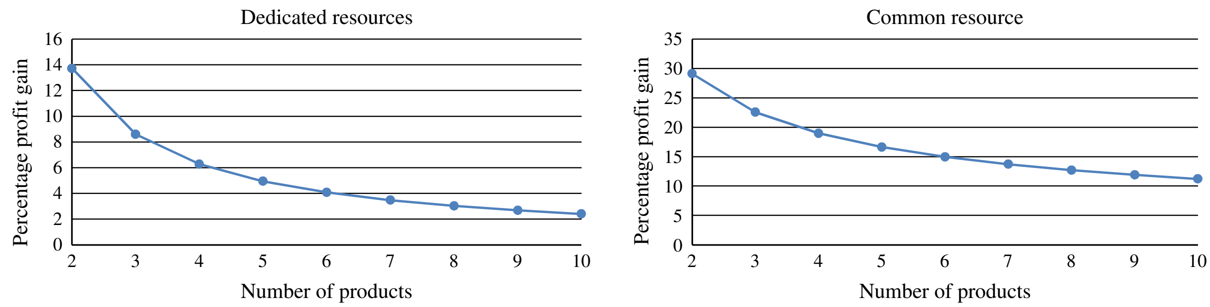
$$\pi^{MC} - \pi^C = p\sigma\phi(z_C)(\sqrt{n} - l_2(n, \mu/\sigma)).$$

In the presence of commonality, what matters is total demand—how it is split among different products (mix information) is irrelevant—so mix information does not provide value directly. Despite this, mix information does provide some value in such cases ($\pi^{MC} - \pi^C > 0$), but this value arises indirectly, because any particular mix signal actually contains some elements of volume information embedded in it. To see this, note that the mean of the sum

RIGHTSLINK

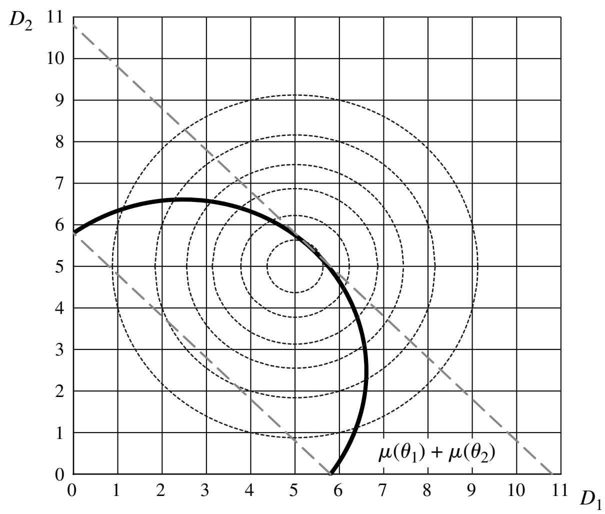
Downloaded from informs.org by [155.246.103.35] on 25 March 2017, at 14:57 . For personal use only, all rights reserved.

Figure 3 (Color online) Value of Volume Information as a Function of the Number of Products



Note. Parameters: $\mu_i = 5$, $\sigma_i = 2$, $p_i = 10$, and $c_i = 5$, $i = 1, \ldots, n$.

Figure 4 Volume Information Contained in a Mix-Demand Signal



of the mix-information-updated demands, $\sum_{i=1}^{n} \mu(\theta_i)$, changes with $\theta_i = \tan^{-1} a_i$. This is illustrated in Figure 4, where $(\mu(\theta_1), \mu(\theta_2))$, the thick curve in the graph, changes as the mix information signal $a_i = \tan\theta_i$ varies from zero to infinity (resulting in average volume values ranging between the two parallel dashed lines in Figure 4). The number of products does not appear to have a consistent impact on the total profit gain, $\pi^{MC} - \pi^{C}$, delivered by mix information given a common resource. However, in all cases the gain increases more slowly than n, so the value added per product decreases as the number of products grows.

For the case of an imperfect mix information signal, we use the conditional density derived in (2) to compute optimal resource capacities and expected profit. Similar to the case of volume information, for dedicated resources we find (numerically) that the value of mix information is convex increasing in the signal accuracy parameter β. Figure 5 illustrates the per-product percentage profit gain derived from mix information as a function of the number of products for both dedicated and common resource settings.

6.3. Asymmetric Products

Much of the preceding analysis has assumed that products are symmetric in terms of both profit margins and demand parameters and that product demands are independent. In this section we explore the impact of product asymmetry and correlation on the value of information. For ease of exposition, we focus on the case of two products and perfect information.

6.3.1. Asymmetric Prices. To evaluate the impact of asymmetric prices, we consider $p_1 = p + \epsilon$ and $p_2 = p - \epsilon$ for $\epsilon > 0$. For dedicated resources, we can show analytically that such asymmetric prices cause the value of either volume or mix information to decrease. To see this, note that $\pi^{VD} - \pi^{D}$ and $\pi^{MD} - \pi^{D}$ are both proportional to

$$(p+\epsilon)\phi\left(\Phi^{-1}\left(\frac{p+\epsilon-c}{p+\epsilon}\right)\right) + (p-\epsilon)\phi\left(\Phi^{-1}\left(\frac{p-\epsilon-c}{p-\epsilon}\right)\right), \tag{10}$$

which is decreasing in ϵ. The expression in (10) represents the incremental rate of profit gain from reducing the standard deviation of product demand. Since that quantity does not depend on the particular type of information used to reduce demand variability, the fact that asymmetric prices yield smaller profit gains from information is not a result of the type of information, but rather derives from the fundamental economics of the newsvendor problem.

The setting with a common resource is more complex to analyze because, with asymmetric prices, it is optimal to satisfy demands in a nested way—first serving demand for the product with the highest price, then the one with the next highest price, etc. This complicates the expected profit function and the calculation of the optimal resource capacity, making analytical characterization of the impact of asymmetric prices more difficult. As a result, we explore this case through numerical studies. We begin with a (symmetric) base case with $c_C = 5$; $p_i = 10$, $\mu_i = 5$,

Downloaded from informs.org by [155.246.103.35] on 25 March 2017, at 14:57 . For personal use only, all rights reserved.

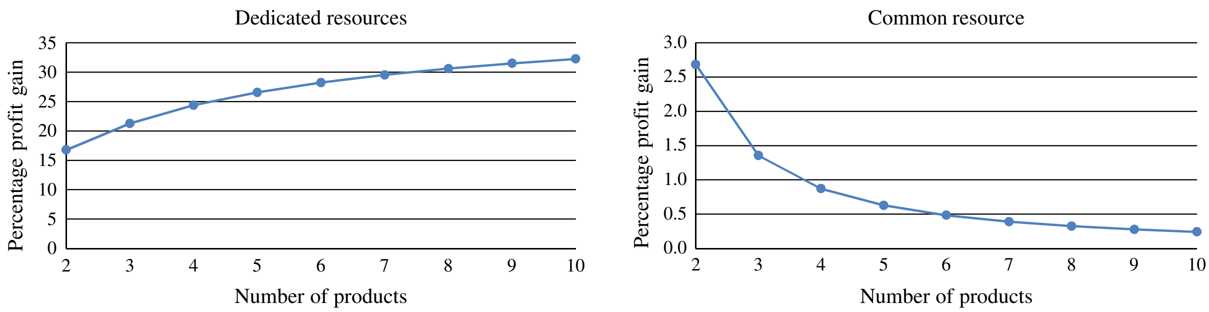


Figure 5 (Color online) Value of Mix Information as a Function of the Number of Products

Note. Parameters: $\mu_i = 5$, $\sigma_i = 2$, $p_i = 10$, and $c_i = 5$, $i = 1, \ldots, n$.

$\sigma_i = 2$ for $i = 1, 2$; and demand correlation $\rho = 0$. We then introduce asymmetric selling prices by setting $p_1 = 10 + 0.5k$ and $p_2 = 10 - 0.5k$ for $k = 0, 1, \ldots, 9$ (to preserve positive margins for both products). Then for each information setting (volume and mix) we simulate 2,000 problem instances, where in each instance we (i) randomly generate a demand pair (D_1, D_2), (ii) compute the information signal given the demands, (iii) compute the optimal resource quantity Q_C given the signal (but not using the actual demand values), and (iv) use the actual demands to compute the actual profit earned for that instance. For each setting, we also compute (once) the optimal resource capacity assuming no information signal is available, and we use the simulated demands to compute expected profit for this no-information case. We then compare the average profits (over the 2,000 instances) with and without information to compute the percentage gain in profit stemming from the use of information.

The numerical results suggest behavior similar to the dedicated resource case; i.e., for both mix and volume information, the value of information showed a decreasing trend as prices became more asymmetric. One might expect that asymmetric prices would make information more valuable in this setting—especially mix information, since it would provide not only a (weak) signal of total demand (as discussed in the previous section) but also a signal of the "effective" product profit margin (by signaling if one product's demand is likely to be larger than the other). However, as selling prices diverge, the bulk of the profit is earned on the higher-priced item. When choosing the total flexible resource capacity, there is strong incentive to order enough to cover that product's demand, but the firm is relatively indifferent about whether it stocks enough to cover the lower-priced product's demand. As a result, overall profits are relatively insensitive to the exact capacity quantity—as long as it is likely to cover product 1's demand and unlikely to go beyond total demand. As a result, more precise information about product demand becomes somewhat less valuable.

6.3.2. Asymmetric and/or Correlated Demands. To study product demand asymmetries and/or correlation, we perform a similar set of studies, but rather than varying product price, we vary product demands, with $\mu_1 = 5 + k$ and $\mu_2 = 5 - k$ for $k = 0, 1, 2$, and each product's demand standard deviation is adjusted to preserve a coefficient of variation $\sigma_i/\mu_i = 0.4$ as in the base case. For each combination of asymmetric demands, we also vary the demand correlation over the values $\rho = -0.5, 0, 0.5$.

For volume information, we find that the value of information increases moderately as product demands become more asymmetric (for both common and dedicated resources) and that the value increases significantly as product demands become more correlated. The latter observation is intuitive—since volume information is in some sense orthogonal to positive correlation, having the two together allows for a more precise estimate of demand. The former observation arises from the fact that asymmetric demands (in particular, asymmetric standard deviations) cause the conditional distribution (given x) of each demand to become less variable—a small standard deviation for one product's demand, combined with the correlation induced by information about total demand, causes both posterior standard deviations to be smaller. (Using the material on asymmetric products presented in Online Appendix B, both of these observations can be shown analytically for the case of dedicated resources.)

For mix information, the impact of demand asymmetry is less clear overall. With dedicated resources, we observe that increasing correlation leads to reduced value of mix information. This is again intuitive, since negative correlation and mix information can be viewed as orthogonal. However, for the other scenarios, the results of the numerical study do not show any clear pattern, and any changes in value (as asymmetry or correlation changes) are small.

6.4. Impact of Information Type and System Configuration

We next explore the interplay between either type of demand information and the configuration of the

Downloaded from informs.org by [155.246.103.35] on 25 March 2017, at 14:57 . For personal use only, all rights reserved.

system (based on either dedicated or common resources). We also determine whether mix and volume information are strategic complements or substitutes.

We begin by studying the interaction between volume information and system flexibility (given by the use of a common resource). To focus on the relationship between information and system structure, we assume, for $i = 1, \ldots, n$, that $c_i = c_C$ in the dedicated resource system and $p_i = p$ in both dedicated and common resource systems. We compare $\pi^C - \pi^D$ and $\pi^{VC} - \pi^{VD}$ using (3)–(7). That is,

$$\pi^C - \pi^D = (n - \sqrt{n}) p\sigma\phi(z) \quad \text{and}$$
$$\pi^{VC} - \pi^{VD} = \left(n\sqrt{\frac{n-\beta}{n}} - \sqrt{n(1-\beta)} \right) p\sigma\phi(z),$$

where $z = z_i = z_C$. It follows that $\pi^C - \pi^D \le \pi^{VC} - \pi^{VD}$ for all n and for all β. That is, commonality and volume information are strategic complements. This makes sense—having information about total demand volume is more valuable when using a common resource, since in that case, total demand is all that is relevant.

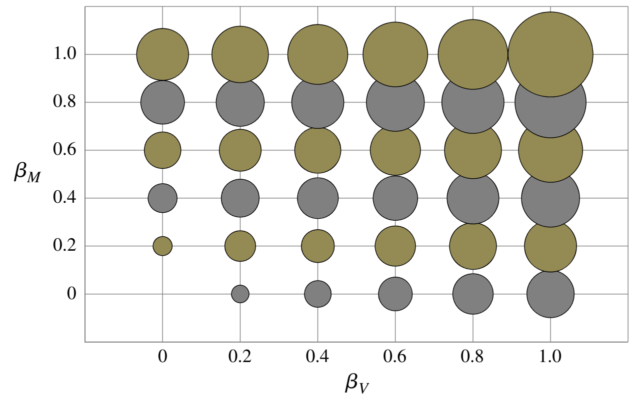
We next examine the interplay between mix information and system flexibility by comparing $\pi^C - \pi^D$ and $\pi^{MC} - \pi^{MD}$. Because of the lack of closed-form expressions for imperfect mix information, we focus on the case of perfect mix information. It follows from (8) and (9) that

$$\pi^C - \pi^D = (n - \sqrt{n}) p\sigma\phi(z) \quad \text{and}$$
$$\pi^{MC} - \pi^{MD} = (nl_1(n, \mu/\sigma) - l_2(n, \mu/\sigma)) p\sigma\phi(z) \approx 0.$$

This implies that $\pi^C - \pi^D > \pi^{MC} - \pi^{MD}$, so that in contrast to volume information, commonality and mix information are strategic substitutes. Commonality adds value by providing protection against mix uncertainty (i.e., the relative size of demand for different products), and as a result, it increases profit when no advance information is available. Mix information removes this uncertainty, and so commonality adds no further value when perfect mix information is available. The same argument applies under an imperfect signal of mix information. Under intermediate values of β, we find that commonality and mix information are (weaker) strategic substitutes (relative to the case of perfect mix information).

We now turn attention to settings involving a combination of mix and volume information. To that end, we first examine a scenario in which the firm has access to (imperfect) signals of both the volume and mix of demands for n products. Following Proposition 2, we have that the conditional distribution of demands given a volume signal $\hat{X} = \hat{x}$ is multivariate normal with identical marginal means $\hat{x}/n$, identical standard deviations $\sigma\sqrt{(n-\beta)/n}$, and identical pairwise correlation coefficients $-\beta/(n-\beta)$. Using as a starting point the multivariate normal distribution of the volume-information-updated demand random variables, we follow the results in Online Appendix B to derive the marginal distribution of demand D_i for a given mix signal θ. From there, we can numerically calculate the density of the marginal demand distributions for the case of an imperfect mix signal. For a representative example, Figure 6 illustrates the value of the combined volume and mix information signals as a percentage profit improvement from a setting without information. In that graph, β_V and β_M denote the accuracy of the volume and mix information signals, respectively. We next examine the interplay between mix and volume information.

Figure 6 **(Color online) Value of Combined Imperfect Mix and Volume Information in a Setting with Dedicated Resources**



Notes. The size of a bubble represents the relative gain of the corresponding combined imperfect mix and volume signals over the case with no information, with $\beta_V = \beta_M = 1$ corresponding to a 50.1% gain. Parameters: $n = 2$; $\mu_i = 5$, $\sigma_i = 2$, $p_i = 10$, and $c_i = 5$, $i = 1, 2$.

We first consider a setting with dedicated resources. The goal is to compare $\pi^{MVD} - \pi^{VD}$ and $\pi^{MD} - \pi^D$, where π^{MVD} is the expected profit associated with the case in which the firm has both volume and mix information in a setting with dedicated resources (where expectations are taken over demand as well as the volume and mix signals). For the following analytical arguments, we focus on the case with perfect volume and mix information. (For imperfect information, the analytical result is confirmed by the numerical results shown in Figure 6.) In that setting, having perfect information about $\sum_{j=1}^n D_j$ and $\sum_{j\neq i} D_j/(n-1)D_i$ removes all the uncertainty in the system, so $\pi^{MVD} = n(p-c)\mu$. Based on (3), (4), and (8), the comparison between $\pi^{MVD} - \pi^{VD}$ and $\pi^{MD} - \pi^D$ reduces to a comparison between $\sqrt{(n-1)/n}$ and $1 - l_1(n, \mu/\sigma)$. One can verify that $1 - l_1(n, \mu/\sigma) < \sqrt{(n-1)/n}$ for all $n \ge 2$. We conclude that volume and mix information are strategic complements in systems with dedicated resources. This is because neither volume nor mix

Downloaded from informs.org by [155.246.103.35] on 25 March 2017, at 14:57 . For personal use only, all rights reserved.

information are very closely aligned with the information needed to manage dedicated products, i.e., the demand of each product individually. As a result, the value provided by either type of information on its own is limited, whereas, working together, the two types of information provide exactly the information that is needed. On the other hand, since volume information provides all the information that is needed when a common resource is used, the addition of mix information does not increase profit; i.e., $\pi^{MVC} - \pi^{VC} = 0$. This implies that $\pi^{MC} - \pi^{C} > \pi^{MVC} - \pi^{VC}$. Therefore, in systems with commonality, volume and mix information are strategic substitutes.

7. Conclusion

In this paper we examine the impact of different types of demand information on profit and the benefits of flexibility in a multiproduct setting. We focus on two specific types of information—total demand volume across products and demand mix between products. For both demand types we derive the relevant probability distributions—i.e., the distribution of the information signal, and the distributions of product demands given that signal—and then use those results to obtain insights into the impact of information and its interaction with various system features. In particular, we explore the impact of the number of products in the system on the value of demand information, and we characterize how this impact depends on the specific resource configuration (dedicated versus flexible). We also examine strategic substitute/complement relationships between information type and resource configuration, finding that volume information has a complementary relationship with resource flexibility, whereas mix information acts as a substitute. Finally, we show that mix and volume information are themselves complements with dedicated resources—either type provides some value on its own but that value is amplified in the presence of the other information type—although they act as substitutes when a common resource is used.

Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/msom.2014.0502.

Acknowledgments

The authors are grateful to the editor-in-chief, associate editor, and the referees for comments that have helped to improve the paper.

References

Arora N, Allenby GM, Ginter JL (1998) A hierarchical Bayes model of primary and secondary demand. *Marketing Sci.* 17(1):29–44.

Bachman J (2014) With epic backlogs at Boeing and Airbus, can business be too good? *Bus. Week* (January 29), http://www.businessweek.com/articles/2014-01-29/with-epic-backlogs-at-boeing-and-airbus-can-business-be-too-good.

Baker KR, Magazine MJ, Nuttle HLW (1986) The effect of commonality on safety stock in a simple inventory model. *Management Sci.* 32(8):982–988.

Bakken D, Frazier CL (2006) Conjoint analysis: Understanding consumer decision making. Grover R, Vriens M, eds. *The Handbook of Marketing Research: Uses, Misuses, and Future Advances* (Sage, Thousand Oaks, CA), 288–311.

Bassok Y, Anupindi R, Akella R (1999) Single-period multiproduct inventory models with substitution. *Oper. Res.* 47(4):632–642.

Benjaafar S, Cooper WL, Mardan S (2011) Production-inventory systems with imperfect advance demand information and updating. *Naval Res. Logist.* 58(2):88–106.

Boyacı T, Özer Ö (2010) Information acquisition for capacity planning via pricing and advance selling: When to stop and act? *Oper. Res.* 58(5):1328–1349.

Casimir RJ (2002) The value of information in the multi-item newsboy problem. *Omega* 30(1):45–50.

Chen F (2001) Market segmentation, advanced demand information, and supply chain performance. *Manufacturing Service Oper. Management* 3(1):53–67.

Collier DA (1982) Aggregate safety stocks and component part commonality. *Management Sci.* 28(11):1296–1303.

Dawson C (2011) Quake still rattles suppliers. *Wall Street Journal* (September 29), http://online.wsj.com/news/articles/SB10001424053111904563904576586040856135596.

DeCroix GA, Mookerjee VS (1997) Purchasing demand information in a stochastic demand inventory system. *Eur. J. Oper. Res.* 102(1):36–57.

Du RY, Kamakura WA (2012) Quantitative trendspotting. *J. Marketing Res.* 49(4):514–536.

Fisher M, Raman A (1996) Reducing the cost of demand uncertainty through accurate response to early sales. *Oper. Res.* 44(1):87–99.

Gallego G, Özer Ö (2001) Integrating replenishment decisions with advance demand information. *Management Sci.* 47(10):1344–1360.

Gayon J-P, Benjaafar S, de Véricourt F (2009) Using imperfect advance demand information in production-inventory systems with multiple customer classes. *Manufacturing Service Oper. Management* 11(1):128–143.

Gerchak Y, Henig M (1989) Component commonality in assemble-to-order systems: Models and properties. *Naval Logist. Res.* 36(1):61–68.

Gerchak Y, Magazine MJ, Gamble AB (1988) Component commonality with service level requirements. *Management Sci.* 34(6):753–760.

Graves SC, Meal HC, Dasu S, Qiu Y (1986) Two-stage production planning in a dynamic environment. Axsäter S, Schneeweiss C, Silver E, eds. *Multi-Stage Production Planning and Control*, Lecture Notes in Economics and Mathematical Systems, Vol. 266 (Springer-Verlag, Berlin), 9–43.

Hariharan R, Zipkin P (1995) Customer-order information, leadtimes, and inventories. *Management Sci.* 41(10):1599–1607.

Heath DC, Jackson PL (1994) Modeling the evolution of demand forecasts with application to safety stock analysis in production/distribution systems. *IIE Trans.* 26(3):17–30.

Huang T, Van Mieghem JA (2013) Clickstream data and inventory management: Model and empirical analysis. *Production Oper. Management* 23(3):333–347.

Iyer AV, Bergen ME (1997) Quick response in manufacturer-retailer channels. *Management Sci.* 43(4):559–570.

Karaesmen F, Buzacott JA, Dallery Y (2002) Integrating advance order information in make-to-stock production systems. *IIE Trans.* 34(8):649–662.

Lau H, Lau A (1996) The newsstand problem: A capacitated multiple-product single-period inventory problem. *Eur. J. Oper. Res.* 94(1):29–42.

Mahajan S, van Ryzin G (2001) Stocking retail assortments under dynamic consumer substitution. *Oper. Res.* 49(3):334–351.

Mardia KV (1972) *Statistics of Directional Data* (Academic Press, New York).

RIGHTSLINK

Nahmias S, Schmidt C (1984) An efficient heuristic for the multi-item newsboy problem with a single constraint. *Naval Res. Logist. Quart.* 31(3):463–474.

Nair H, Dubé J-P, Chintagunta P (2005) Accounting for primary and secondary demand effects with aggregate data. *Marketing Sci.* 24(3):444–460.

Özer Ö (2003) Replenishment strategies for distribution systems under advance demand information. *Management Sci.* 49(3):255–272.

Özer Ö, Wei W (2004) Inventory control with limited capacity and advance demand information. *Oper. Res.* 52(6):988–1000.

Signorelli S, Heskett J (1984) Benetton (A). HBS Case 9-685-014, Harvard Business School, Boston.

Smith SA, Agrawal N (2000) Management of multi-item retail inventory systems with demand substitution. *Oper. Res.* 48(1): 50–64.

Tan T, Güllü R, Erkip N (2007) Modelling imperfect advance demand information and analysis of optimal inventory policies. *Eur. J. Oper. Res.* 177(2):897–923.

Tan T, Güllü R, Erkip N (2009) Using imperfect advance demand information in ordering and rationing decisions. *Internat. J. Production Econom.* 121(2):665–677.

Thonemann UW (2002) Improving supply-chain performance by sharing advance demand information. *Eur. J. Oper. Res.* 142(1): 81–107.

Tong YL (1990) *The Multivariate Normal Distribution* (Springer-Verlag, New York).

Turken N, Tan Y, Vakharia A, Wang L, Wang R, Yenipazarli A (2012) The multi-product newsvendor problem: Review, extensions, and directions for future research. Choi T-M, ed. *Handbook of Newsvendor Problems*, Chap. 1 (Springer, New York).

Van Mieghem JA, Rudi N (2002) Newsvendor networks: Inventory management and capacity investment with discretionary activities. *Manufacturing Service Oper. Management* 4(4):313–335.

Wang T, Toktay BL (2008) Inventory management with advance demand information and flexible delivery. *Management Sci.* 54(4):716–732.

Zhu K, Thonemann UW (2004) Modeling the benefits of sharing future demand information. *Oper. Res.* 52(1):136–147.

Downloaded from informs.org by [155.246.103.35] on 25 March 2017, at 14:57 . For personal use only, all rights reserved.