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Product Line Design with Seller-Induced Learning

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In practice, some well-established service providers (sellers) offer one-time experiences or product demonstrations for the services that have been introduced to the market for years. Such activities, labeled as seller-induced learning, not only help the consumers learn more about themselves but also exploit the consumers by elaborating on the consumer heterogeneity. When the seller-induced learning completely resolves the consumers' valuation uncertainty, it can facilitate a more sophisticated price discrimination scheme and may give rise to a relatively more efficient allocation. Nevertheless, if there is residual valuation uncertainty, the seller may abandon the seller-induced learning to avoid the exacerbated ex post cannibalization. We show that an exploding offer shall sometimes be offered in conjunction with the seller-induced learning to encourage immediate purchases when uncertainty arises in only some consumers. We identify regimes under which the seller-induced learning is charged at a strictly positive price. Under these regimes, the seller need not sacrifice the ex post efficiency upon inducing consumer learning. Therefore, our result indicates that the seller-induced learning may eliminate the conflict between rent extraction and efficiency initiatives. However, quality distortion prevails when the seller provides an identical menu for all the consumers or the free seller-induced learning.

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1. Introduction

Not everybody finds it simple to choose her favorite product or service among a set of options. Some consumers are aware of their needs but lose their ways in searching the matching products/services. In such circumstances, a natural way to resolve the uncertainty is through some experiments or the use of trial versions. The activities to facilitate learning and resolve uncertainty are defined as seller-induced learning by Jing (2011). As a common practice, some service providers are offering the option of one-time experiences to help the consumers to learn more about their own preferences. Examples abound as we elaborate in the following.

The first example that involves the consumer uncertainty and the one-time experience is the gyms, such as 24 Hour Fitness. In its clubs, there are several membership options for the consumers to choose. These options are distinguished by the prices and the facilities or clubs that the members can access. Some consumers may not be aware of their body statuses, their preferred exercises, and the exercises that fit their body statuses. For these uncertain consumers, gyms typically allow them to experience the service once before joining the memberships. The consumers can take the seven-day trial by subscribing to the company's database. During the trial, these consumers are better informed about their bodies and the facilities. Meanwhile, other consumers negotiate with the service provider up front and purchase the membership package immediately. As another example, Rancho La Puerta also offers multiple options to the consumers. Some consumers sign up for the package with several treatments immediately and reserve the spa treatments prior to their arrival, whereas others may first purchase the one-treatment package and sign up for the package with several treatments later.[1] As the third example, some educational institutions (e.g., New Oriental School) allow the new students to audit a limited number of classes by paying a moderate fee.

All the aforementioned examples share some common features. First, for those consumers who are uncertain about their own preferences, the service providers offer the consumers some limited-time experiences to help them make their decisions. Traditionally, offering one-time experiences is regarded as

[1] The latter ones are exposed to the risk that the treatments she desires are not available after the one-time experience.

a signaling device to indicate the private quality of the products (services). Nevertheless, in the aforementioned examples, 24 Hour Fitness, Rancho La Puerta, and New Oriental School are all *leading, established* service providers in the corresponding regions and industries.[2] There are numerous chances for the consumers to learn the qualities of their services by word of mouth, online feedback, and experience sharing by past consumers. In these cases, uncertainty, if it exists, seems to arise from consumers' valuations rather than the products/services themselves.

Second, while offering these one-time experiences, the service providers may apply sales techniques to induce some consumers to buy immediately, such as informing consumers that they have no chance to purchase an identical product with the quality afterward. This corresponds to the "exploding offer" commonly adopted in various industries (for more elaborations and examples, see Armstrong and Zhou 2011). Third, after the one-time experiences, the consumers probably have more precise expectations about their preferences and decide whether to make purchases or simply walk away. Service providers can recognize the consumers who have taken the one-time experiences and provide special offers for them, according to the consumers' information stored in the database.[3]

The one-time experiences provided by the service providers (sellers hereafter) facilitate the consumers in *endogenous dynamic learning*. Before the learning stage, the consumers endogenously choose whether or not to take the one-time experiences. If the consumers take the one-time experiences, they observe their valuations privately. Therefore, we label the activity of offering the one-time experiences as seller-induced learning (SIL). In this paper, we seek to provide a unified framework to investigate the role of SIL in the service/product line design with consumer uncertainty. We build a stylized model in which a monopolistic seller faces two segments of consumers that may exhibit valuation uncertainties, and we allow the seller to charge for the SIL and make product line offering contingent on whether the SIL is taken. In our basic model, the SIL leads to a precise revelation of the consumer valuation. We find that offering the SIL enables the seller to obtain higher profits than the non-SIL strategy. Thus, offering the SIL always emerges as an optimal solution to this adverse selection problem. It also gives rise to a relatively more efficient allocation, and the seller can implement ex post "selective targeting" to exploit the consumer segmentation. Our analysis indicates that for the revenue maximization purpose, the seller shall typically charge a strictly positive price for the SIL. This allows the seller to facilitate ex ante screening and also appropriate the efficiency gain from the ex post finer segmentation.

Notably, the above model characteristics are included to best illustrate our messages; there are certainly some important situations that they do not fit well. To this end, we discuss the scenarios in which the SIL is offered *for free* or the seller provides an *identical* menu to all the consumers, respectively. We find that the seller deliberately distorts the qualities of all the low-valuation consumers. This finding implies that the SIL can no longer counterbalance the inefficiency, and it functions only as an instrument to facilitate learning of the uncertain consumers. Furthermore, in these two scenarios, the seller offers identical products (with identical qualities and prices) for the consumers of the same types from either segment. This suggests that the ability of charging for the SIL and targeted offering are both necessary for eliminating the disincentive, and it highlights the importance of coordinating the ex ante pricing and ex post offering of tailored products. Finally, we investigate the possibility that the SIL may lead to only an imprecise (noisy) valuation revelation. We find that the more informative the SIL is, the more profit the seller obtains. Thus, the seller should make the SIL as informative as possible if the cost allows. Nevertheless, with an imprecise information revelation, sometimes he is better off by prohibiting consumer learning.

Our research is built upon the monopoly pricing literature within economics originated from the seminal work of Mussa and Rosen (1978). In the majority of this literature, the consumers are assumed to know their preferences precisely, whereas our primary goal is to investigate the seller's optimal selling strategy in the presence of consumer uncertainty. Since the consumers in our model may obtain updated and covert information after SIL, our paper is also related to the literature on "sequential screening" (see Courty and Li 2000, Pavan et al. 2013). In all these papers, the learning by consumers regarding their own valuations is driven by nature (time); in contrast, we allow the seller to actively determine whether to help the consumers learn their valuations.

There have been some studies on the consumers' valuation uncertainty in the product line design context (e.g., Guo and Zhang 2012, Kuksov and Villas-Boas 2010, Villas-Boas 2009). In these studies, the

[2] 24 Hour Fitness is the largest privately owned U.S. fitness center chain, with more than 400 athletic clubs; Rancho La Puerta is one of the spa-goers' favorite spas in North America, voted by the readers of SpaFinder publications (including SpaFinder.com) in 2008; and New Oriental School is the most dominant English cram school in China.

[3] This type of behavior-based sales technique can be facilitated if either the cookies of consumers' online activities are recorded or the past transaction histories are kept in records; see Armstrong and Zhou (2011), Fudenberg and Villas-Boas (2007), and Shin and Sudhir (2010). In reality, many companies (e.g., Comcast) offer different pricing and bundles to new versus old customers.

seller can affect the consumer learning process passively only by offering fewer or more options; thus, the central theme is that sometimes the seller is unwilling to induce consumer learning even if learning completely resolves the uncertainty. In contrast, in our context the seller should always facilitate consumer learning as long as it completely resolves the consumers' uncertainty. In the industry of health clubs, DellaVigna and Malmendier (2006) find empirical evidence that the consumers of health clubs may overestimate their attendance frequencies. We assume away this psychological factor and focus on the seller-induced endogenous learning.

Our paper is also related to some recent attempts to investigate the consumers' dynamic learning behaviors. Matthews and Persico (2005) propose that if the seller provides generous refunds, consumers may try out the products and then return to the seller. Villas-Boas (2004) considers a two-period problem in which the consumers can observe their private valuations only by experiencing them in the first period. He shows that the forward-looking firm charges lower prices to obtain a higher market share when a large proportion of consumers is likely to be loyal to the firm in the future. Jing (2011) examines the firm's product release and pricing strategies when SIL can resolve consumers' uncertainties about valuation. His finding shows that the high-end consumers may buy early at a lower price and the low-end consumers prefer to buy at a higher price after being better informed. While the consumers in our context also exhibit valuation uncertainties and engage in learning, the product line design problem has no counterpart in Villas-Boas (2004) and Jing (2011). Sun (2011) investigates the seller's disclosure strategy regarding the product attributes (of the horizontal and/or vertical dimensions) prior to the consumers' purchase. In contrast, in our model the product attribute is publicly known, but the consumers exhibit valuation uncertainties.

The rest of this paper is organized as follows. In §2, we present the model setup. Section 3 derives the optimal selling strategy under various scenarios. The cases with free SILs, no availability restriction, and imprecise (noisy) information are introduced in §4. Section 5 concludes. All the proofs are provided in the appendix and the online supplement (available at http://ssrn.com/abstract=2289232). The appendix covers the major steps of the proofs; more details and the numerical analysis are presented in the online supplement.

2. Model

We consider a model in which a monopolistic seller faces two segments of consumers that are heterogeneous with regard to their marginal willingness to pay for quality. For example, the consumers in two distinct markets may have different expectations about their marginal willingness to pay for quality. Consumers within each segment share the same marginal willingness to pay, and the high-segment consumers (with proportion $1-v$) are willing to pay more than the low-segment ones (with proportion v). Ex ante, the seller is unable to distinguish between the two segments of consumers. As a departure from the classical setting, we assume that some consumers are uncertain regarding their own willingness to pay, which leaves room for the seller to help the consumers learn about themselves. Specifically, the ultimate willingness to pay of the consumers takes only two values: θ_1 or θ_2, where $\theta_2 > \theta_1 > 0$.

Given the two-value willingness to pay and the two-segment consumer composition, uncertainties may arise for either segment or both segments. In the most general form, all the consumers are unsure about their preferences. Specifically, the low-segment consumers are composed of θ_1 and θ_2 with proportions α and $1-\alpha$, and type θ_1 and type θ_2 consumers take proportions β and $1-\beta$ in the high segment, respectively. We assume that $0 \le \beta < \alpha \le 1$ and define $\theta_\alpha = \alpha\theta_1 + (1-\alpha)\theta_2$ and $\theta_\beta = \beta\theta_1 + (1-\beta)\theta_2$. According to our assumptions, $\theta_\beta > \theta_\alpha$. Note that when either $\beta = 0$ or $\alpha = 1$, uncertainty in one segment degenerates. In the following we distinguish between three scenarios: scenario H ($\alpha = 1$), scenario L ($\beta = 0$), and the nondegenerate scenario B ($0 < \beta < \alpha < 1$).[4] Moreover, in the appendix and the online supplement, we extend our discussions to the scenario with heterogeneous consumers in the segment without uncertainty.

We assume that the seller and the consumers are risk neutral; i.e., they intend to maximize their expected payoffs. The seller can determine to supply the product at quality q by incurring a cost $\frac{1}{2}q^2$. This quadratic functional form is adopted to facilitate closed-form expressions; in §4, we present a generalization of the cost function. Each consumer demands at most one unit of product. Given a product with quality q and the corresponding price $p(q)$, if a consumer knows her willingness to pay θ_i for sure, her payoff from obtaining the product will be $\theta_i q - p(q)$, where $\theta_i \in \{\theta_1, \theta_2\}$. The utility function is generalized in §4 as well. If a consumer is uncertain regarding her true willingness to pay, θ_i should be replaced by their expected willingness (θ_α or θ_β). We assume that any consumer from the two segments obtains a null (zero) utility if she does not purchase any product.

[4] This two-segment setup allows us to articulate the benefit of quality distortion in some scenarios and quality nondistortion in others. If we were to consider a single consumer segment, the ex ante adverse selection vanishes. In such a scenario, the seller shall always offer the SIL, offer a product line with efficient high-quality and low-quality products, and extract the consumer surplus from charging the SIL (Eso and Szentes 2003).

Let us first consider the situation in which it is impossible for the consumers to resolve the uncertainty. In this case, the seller can simply design two pairs of quality-price bundles in each scenario: $\{(q_l, p_l), (q_h, p_h)\}$, where (q_l, p_l) is designed for the consumers in the low segment and (q_h, p_h) for the high segment. Then, we switch to the situation in which the seller can help the consumers learn. We assume that the SIL generates a signal that indicates the true willingness of the consumers to pay. In our basic model, after participating in the SIL, all the consumers with uncertainty know their willingness is either θ_1 or θ_2. In §4, we relax this assumption and investigate the impact of informativeness of the SIL. The cost for SIL is negligible in our context.

In the presence of the SIL, the seller can ask whether a consumer wants to learn, and then provide different menus accordingly. The sequence of events is as follows. (1) Each consumer privately observes which segment she belongs to. (2) The seller asks whether the consumer would like to take the non-SIL product (q_n, p_n) (the subscript n stands for "non-SIL") or join SIL. (3) The consumer decides whether or not to participate in SIL. If the consumer chooses not to participate in SIL, she either takes the non-SIL product (q_n, p_n) or simply walks away; on the other hand, if the consumer participates in SIL, she pays p_t to the seller. (4) After joining the SIL, the consumer realizes her willingness is θ_1 or θ_2, and the seller provides a menu $\{(q_1, p_1), (q_2, p_2)\}$ for those consumers who have learned. The consumers choose among (q_1, p_1), (q_2, p_2), or simply walk away. As such, the SIL merely allows the consumers to learn their own preferences rather than the product quality.[5]

Having introduced the consumers' preferences, the seller's objective, and the informational scenarios, we next characterize the equilibrium behaviors in the three scenarios.

3. Analysis

In this section, we start our analysis with scenario H (uncertainty in the high segment) and scenario L (uncertainty in the low segment); following this, we proceed to study the general case scenario B in which uncertainties arise in both segments.

3.1. Uncertainty in the High Segment

We first consider the scenario in which only the high-segment consumers face the uncertainty. In the setting without the SIL, the analysis is similar to the standard two-type monopoly pricing problem. Thus, we ignore the details here. In the sequel, we focus on the setting with the SIL. Since the optimal product line must induce the consumers to participate, thereby leading to the following individual rationality (IR) constraints:

$$(\text{IR}_n) \quad \theta_1 q_n - p_n \geq 0, \quad (\text{IR}_1) \quad \theta_1 q_1 - p_1 \geq 0,$$
$$(\text{IR}_2) \quad \theta_2 q_2 - p_2 \geq 0,$$
$$(\text{IR}_t) \quad \beta(\theta_1 q_1 - p_1) + (1-\beta)(\theta_2 q_2 - p_2) - p_t \geq 0,$$

(IR_n) guarantees that the low-segment consumers obtain their null payoff, and (IR_1) and (IR_2) should hold so that type θ_1 and type θ_2 consumers are willing to take the products (rather than walking away). For those who are unsure about their ultimate types, their net payoff (i.e., the expected utility net the payment p_t) should be nonnegative, giving rise to the expression (IR_t).

Moreover, the optimal product line should induce the consumers to willingly disclose their true types, as demonstrated by the following incentive compatibility (IC) constraints:

$$(\text{IC}_{tn}) \quad \beta(\theta_1 q_1 - p_1) + (1-\beta)(\theta_2 q_2 - p_2) - p_t \geq \theta_\beta q_n - p_n,$$
$$(\text{IC}_{nt}) \quad \theta_1 q_n - p_n \geq \max\{\theta_1 q_1 - p_1, \theta_1 q_2 - p_2, 0\} - p_t,$$
$$(\text{IC}_{12}) \quad \theta_1 q_1 - p_1 \geq \theta_1 q_2 - p_2,$$
$$(\text{IC}_{21}) \quad \theta_2 q_2 - p_2 \geq \theta_2 q_1 - p_1,$$

where (IC_{tn}) guarantees that a high-segment consumer is willing to take the SIL over the product (q_n, p_n), and (IC_{nt}) indicates that the low-segment consumers are better off by not participating in the SIL. Moreover, at the second stage, (IC_{12}) and (IC_{21}) simply ensure that after joining the SIL, the (high-segment) consumers are willing to reveal their own signals.

In equilibrium, the consumers choose the products intended for their types, and the corresponding expected payoff for the seller, as well as the optimization problem, are as follows:

$$\textbf{(P1)} \quad \max_{p_n, q_n, p_1, q_1, p_2, q_2} \pi, \quad \text{where}$$
$$\pi = v\left(p_n - \tfrac{1}{2}q_n^2\right) + (1-v)\beta\left(p_1 - \tfrac{1}{2}q_1^2\right) + (1-v)(1-\beta)\left(p_2 - \tfrac{1}{2}q_2^2\right) + (1-v)p_t$$

s.t.

(IR_n), (IR_1), (IR_2), (IR_t), (IC_{tn}), (IC_{nt}), (IC_{12}), (IC_{21}).

In the objective function, the term $v(p_n - \frac{1}{2}q_n^2)$ corresponds to the net payoff the seller obtains from selling to the low-segment consumers, and the remaining terms come from the high-segment consumers (with $1 - v$ being the proportion/population). It is noteworthy that there are other situations in which the

[5] In the online supplement, we build an alternative model in which product quality is uncertain and the SIL helps the consumers to resolve this uncertainty. After participating in the seller-induced learning (SIL), some consumers may perceive a higher quality than they expected and purchase the products, while others perceive a lower one and walk away. We compare the findings in different setups to highlight the SIL's role to resolve valuation uncertainty.

seller may abandon either the SIL or some consumer segments (ex ante or ex post). These alternative situations require slightly different formulations of the seller's optimization problem, and we relegate the details to the appendix and the online supplement. Despite these complicated incentive constraints, we are able to fully solve the problem, as summarized in the following proposition. Note that if the seller intends to exclude some consumers, he can equivalently offer a "null quality" to them.

Proposition 1. *With consumer uncertainty in the high segment, the seller either serves the low segment consumers with an "exploding offer" or simply discards them, as summarized in the following table*:

SIL strategy	q_1	q_2	q_n
Regular solution	θ_1	θ_2	$\theta_1-(1-v)(\theta_\beta-\theta_1)/v$
Shutdown solution	θ_1	θ_2	0

Proposition 1 shows that there are two possible optimal strategies to capitalize on the consumer uncertainty by providing the SIL. Under both strategies, the seller induces the consumers with different valuation realizations to choose different products. This strategy ensures that the consumer learning indeed leads to an improvement of production efficiency, because the updated information will be incorporated owing to the consumers' ex post self-selection. In addition, by offering a menu to the consumers ex post, this "selective targeting" strategy allows the seller to exploit the consumers' taste heterogeneity (in the high segment).

We also find that in this context, the seller need not sacrifice the ex post efficiency upon inducing learning (as $q_1=\theta_1$ and $q_2=\theta_2$), even if consumers are ex ante heterogeneous. These quality levels are called "efficient" because they coincide with those in an environment without asymmetric information. Accordingly, a quality level is "downward distorted" if it falls below the efficient level. The ex post efficiency implies that the presence of SIL allows the seller to implement effective consumer segmentation while setting the product qualities at the socially optimal levels ex post. Notably, even if the low-segment consumers are ultimately induced not to purchase (under the shutdown solution), this ex post efficiency result continues to hold. Recall that the seller downward distorts the quality for the low-end consumers in the standard two-type monopoly pricing problem to reduce the information rent. The primary difference between the classical monopoly pricing problem and ours is the seller's ability to charge for the SIL. In the absence of this, the (ex post) cannibalization is so severe that the seller needs to distort the quality level to better differentiate the consumers. When the seller charges for the SIL, the seller can give away more information rent to the consumers in the second stage and take the loss back in the form of first-stage payment. This eliminates the seller's incentive to purposely distort the product quality.

Proposition 1 also implies that when uncertainty arises in the high segment, the seller should always capitalize on the SIL and implement the two-stage screening process. Furthermore, as we verify in the appendix, the SIL should never be offered for free for the revenue maximization purpose. We hereby highlight these findings as the following corollary.

Corollary 1. *With consumer uncertainty in the high segment, the seller should always induce the consumers to learn and should charge a strictly positive price for it.*

As articulated by Proposition 1, inducing the consumers to learn allows the seller to increase his payoff by ex ante screening the unsure consumers and ex post exploiting the taste heterogeneity in the high segment. Conventional wisdom suggests that the ex post unobservable type heterogeneity may hinder the seller from helping the consumers learn their true valuations. Nevertheless, our results demonstrate that this disincentive may be counterbalanced by the seller's ability to charge for the SIL, because it allows the seller to appropriate the efficiency gain from the ex post finer segmentation. A legitimate question is whether these results go through in other scenarios. To this end, we proceed to study the other extreme case in which valuation uncertainty arises in the low segment.

3.2. Uncertainty in the Low Segment

The scenario in which only the low-segment consumers face the uncertainty (scenario L), as a mirror image of the scenario discussed before, will be considered in this section. In this scenario, the seller's optimization problem is formulated as follows.

$$\textbf{(P2)}\qquad \max_{p_n,\,q_n,\,p_1,\,q_1,\,p_2,\,q_2} \pi, \quad \text{where}$$

$$\pi = v\alpha\left(p_1-\tfrac{1}{2}q_1^2\right)+v(1-\alpha)\left(p_2-\tfrac{1}{2}q_2^2\right)+(1-v)\left(p_n-\tfrac{1}{2}q_n^2\right)+vp_t$$

s.t.

$$(\mathrm{IR}_1)\quad \theta_1 q_1 - p_1 \ge 0,\quad (\mathrm{IR}_2)\quad \theta_2 q_2 - p_2 \ge 0,$$

$$(\mathrm{IR}_n)\quad \theta_2 q_n - p_n \ge 0,$$

$$(\mathrm{IR}_t)\quad \alpha(\theta_1 q_1 - p_1)+(1-\alpha)(\theta_2 q_2 - p_2) - p_t \ge 0,$$

$$(\mathrm{IC}_{tn})\quad \alpha(\theta_1 q_1 - p_1)+(1-\alpha)(\theta_2 q_2 - p_2) - p_t \ge \theta_\alpha q_n - p_n,$$

$$(\mathrm{IC}_{nt})\quad \theta_2 q_n - p_n \ge \max\{\theta_2 q_1 - p_1,\, \theta_2 q_2 - p_2\} - p_t,$$

$$(\mathrm{IC}_{12})\quad \theta_1 q_1 - p_1 \ge \theta_1 q_2 - p_2,$$

$$(\mathrm{IC}_{21})\quad \theta_2 q_2 - p_2 \ge \theta_2 q_1 - p_1.$$

We can apply the same technique to solve (**P2**), and the results are summarized in the following proposition.

Proposition 2. *With consumer uncertainty in the low segment, the seller either serves the ex post low-valuation consumers in the low segment with seriously downward distorted quality or simply discards them. The corresponding quality levels are summarized below:*

SIL strategy	q_1	q_2	q_n
Regular solution	$\theta_1-(1-v)(\theta_2-\theta_1)/v$	θ_2	θ_2
Shutdown solution	0	θ_2	θ_2

In strict contrast with the results in Proposition 1, here we find that the seller may intentionally distort the quality levels offered to the consumers who have joined the SIL. Specifically, due to the ex ante information asymmetry, the seller reduces $\alpha(\theta_2-\theta_1)q_1$ from the efficient amount of price for the high-segment consumers (i.e., $\theta_2 q_n$). Thus, to minimize the information rent, the seller downward distorts the quality for type θ_1 consumers. This is reminiscent of the classical cannibalization problem: the seller is more concerned about the possibility that the high segment mimics the low segment, and it leads to the distortion of type θ_1 consumers' quality (i.e., q_1). However, when the uncertainty arises in the high segment, the primary concern of cannibalization is taken care of in the first stage; it therefore does not require the seller to distort the quality levels in the second stage.

Moreover, we find that the ex post distortion occurs only when the seller serves the low-valuation (type θ_1) consumers in the low segment. Furthermore, although the conventional wisdom may imply that the qualities offered to the low-segment consumers are unambiguously lower than the efficient levels, our results show that this is not applicable to the consumers whose ex post valuation realizations are high (type θ_2). Proposition 2 also suggests that the seller should always capitalize on the SIL and implement the two-stage screening process. This is because offering the SIL enables the seller to obtain higher profits than the non-SIL strategy.

Having discussed the qualities with SIL, we now articulate the pricing strategy for SIL. The next corollary shows that, in contrast with the previous section, sometimes the seller should induce consumer learning for free.

Corollary 2. *With consumer uncertainty in the low segment, if the ex post low-valuation consumers in the low segment are excluded, the SIL is offered for free.*

Corollary 2 provides an exact characterization of the regime for free SIL. Specifically, this happens when the seller intends to target the high-valuation consumers (and exclude the low-valuation ones). In this case, the seller is primarily concerned about the high-valuation (type θ_2) consumers, regardless of whether they know their true valuations ex ante or ex post (after joining the SIL). Consequently, it is optimal for the seller to induce consumer learning, and target only those consumers whose valuation realizations are high. In essence, the SIL only serves as a simple (and natural) screening instrument that helps identify unprofitable consumers with low valuations.

3.3. Uncertainty in Both Segments

Finally, let us now proceed to the nondegenerate case in which every consumer faces valuation uncertainty (scenario B). Recollect from our model section that consumers of both segments are a mixture of type θ_1 and type θ_2 consumers, whose average willingness to pay are θ_α and θ_β, respectively. As in the previous subsections, we can analyze the consumers' behavior and then formulate the seller's optimization problem as follows:

$$(\mathbf{P3})\qquad \max_{p_1,\,q_1,\,p_2,\,q_2}\ \pi,\quad \text{where}$$

$$\begin{aligned}\pi={}&v\alpha\left(p_1-\tfrac{1}{2}q_1^2\right)+v(1-\alpha)\left(p_2-\tfrac{1}{2}q_2^2\right)\\&+(1-v)\beta\left(p_1-\tfrac{1}{2}q_1^2\right)\\&+(1-v)(1-\beta)\left(p_2-\tfrac{1}{2}q_2^2\right)+p_t\end{aligned}$$

$$\begin{aligned}\text{s.t.}\quad &(\mathrm{IR}_1)\quad \theta_1q_1-p_1\ge 0,\qquad (\mathrm{IR}_2)\quad \theta_2q_2-p_2\ge 0,\\&(\mathrm{IR}_{t\alpha})\quad \alpha(\theta_1q_1-p_1)+(1-\alpha)(\theta_2q_2-p_2)-p_t\ge 0,\\&(\mathrm{IR}_{t\beta})\quad \beta(\theta_1q_1-p_1)+(1-\beta)(\theta_2q_2-p_2)-p_t\ge 0,\\&(\mathrm{IC}_{12})\quad \theta_1q_1-p_1\ge\theta_1q_2-p_2,\\&(\mathrm{IC}_{21})\quad \theta_2q_2-p_2\ge\theta_2q_1-p_1,\end{aligned}$$

where $(\mathrm{IR}_{t\alpha})$ and $(\mathrm{IR}_{t\beta})$ induce consumers in both segments to learn. After learning, all the consumers recognize themselves as either type θ_1 or type θ_2 consumers, thereby leading to (IC_{12}) and (IC_{21}). Likewise, we can also write the alternative formulations if the seller abandons either the SIL or some consumer segments. We obtain the optimal product line design in the following proposition.

PROPOSITION 3. *With consumer uncertainty in both segments, the seller always induces consumer learning and may serve either one or two segments. The corresponding quality levels are summarized below:*

SIL strategy	q_1	q_2	q_n
Regular solution	$\theta_1 - \dfrac{(1-v)(\theta_\beta - \theta_\alpha)}{v\alpha + (1-v)\beta}$	θ_2	—
SIL only for the high segment	θ_1	θ_2	$\theta_\alpha - \dfrac{(1-v)(\theta_\beta - \theta_\alpha)}{v}$
SIL only for the low segment	$\theta_1 - \dfrac{(1-v)(\theta_\beta - \theta_\alpha)}{v\alpha}$	θ_2	θ_β
Shutdown 1 (i.e., abandon the low segment)	θ_1	θ_2	0
Shutdown 2 (i.e., abandon type θ_1 consumers)	0	θ_2	—
Shutdown 3 (i.e., abandon type θ_1 in the low segment)	0	θ_2	θ_β

As indicated by Proposition 3, this nondegenerate scenario simply summarizes our findings and the possible strategies in the aforementioned two extreme cases (scenarios H and L); thus, the insights obtained from the previous subsections can be applied directly to the more general setting. Moreover, it is verifiable that the SIL typically is charged at a positive price except for some fairly special cases, and we omit the details here to avoid redundancy.

4. Extensions

In the sequel, we consider several extensions and examine the validity of our qualitative implications. First, we investigate the role of SIL when the SIL is offered for free. Second, we discuss the scenario in which the seller does not impose any restriction on the availability of the products, and all the products are always available to all the consumers. Finally, we examine the case in which the SIL provides imprecise information, and the utility and cost functions take the general forms.

4.1. Free SIL

We now consider the case in which free SIL is offered ($p_t = 0$). We divide our analysis into two cases: uncertainty arises in the high segment and uncertainty arises in the low segment. To avoid redundancy, we focus on the regular case and summarize the results in the following proposition.

PROPOSITION 4. *When the seller provides free SIL and serves all the consumers, the corresponding quality levels are summarized as shown below:*

Scenarios	q_n	q_1	q_2
Uncertainty in the high segment		$q_n = q_1 = \theta_1 - \dfrac{(1-v)(1-\beta)(\theta_2 - \theta_1)}{v + (1-v)\beta}$	θ_2
Uncertainty in the low segment	θ_2	$q_1 = \theta_1 - \dfrac{v(1-\alpha)(\theta_2 - \theta_1) + (1-v)(\theta_2 - \theta_1)}{v\alpha}$	θ_2

Proposition 4 shows that when uncertainty arises in the high segment and the seller offers free SIL, the quality of low-valuation consumers in the high segment (q_1) is downward distorted. Compared to the scenarios in §3, the seller cannot counterbalance the loss caused by information asymmetry via charging for the SIL. Thus, he downward distorts the product quality intended for the low-valuation consumers after they experience the learning. Moreover, the seller provides identical products for all the low-valuation consumers (i.e., $q_n = q_1$ and $p_n = p_1$). This is because the free SIL cannot prevent the low-segment consumers from choosing to participate in SIL for a better offer. Similarly, when uncertainty arises in the low segment, the seller downward distorts the quality for the low-valuation consumers. Moreover, he provides identical and socially optimal qualities ($q_n = q_2 = \theta_2$) for all the high-valuation consumers.

4.2. Nonexclusive Offers

For some sellers (e.g., apparel and grocery stores), it may be difficult to monitor whether or not the consumers have participated in the SIL. Therefore, we consider the scenario with nonexclusive offers; that is, all the products are available to all the consumers. Similarly, we focus on the regular case and present our findings in the following proposition.

PROPOSITION 5. *When the seller does not impose any restriction on the availability of the products, the corresponding quality levels are summarized as shown below:*

Scenarios	q_n	q_1	q_2
Uncertainty in the high segment		$q_n = q_1 = \theta_1 - \dfrac{(1-v)(1-\beta)}{v + (1-v)\beta} \cdot (\theta_2 - \theta_1)$	θ_2
Uncertainty in the low segment	θ_2	$q_1 = \theta_1 - \dfrac{v(1-\alpha) + (1-v)}{v\alpha} \cdot (\theta_2 - \theta_1)$	θ_2

With nonexclusive offers, the seller should provide identical products for the consumers with identical willingness to pay to maintain the ex post incentive compatibility. Accordingly, when uncertainty arises in the high segment, the qualities for all the low-valuation (type θ_1) consumers are identical ($q_n = q_1$). Likewise, when the low-segment consumers are uncertain about their preferences, all the high-valuation consumers obtain identical qualities ($q_n = q_2 = \theta_2$). Thus, the seller inevitably distorts the product qualities for the low-valuation consumers to avoid cannibalization.

4.3. SIL Provides Imprecise Information

In this section, we extend our basic model to incorporate general utilities, costs, and the possibility of imprecise information updating from SIL. Specifically, we use $C(q)$ to denote the seller's production cost, where $C'(q) > 0$ (quality improvement is costly), $C''(q) > 0$ (increasing marginal cost), and $C(0) = 0$. We maintain our assumptions on the ex ante two consumer segments, but now their utilities are expressed by the following general functions: $\theta_i U(q) - p(q)$, where $\theta_i \in \{\theta_1, \theta_2\}$, $U'(q) > 0$, $U''(q) < 0$ and $U(0) = 0$. The seller and the consumers are assumed to be risk neutral, and the consumers' reservation utilities are normalized to zero.

Furthermore, we introduce the informativeness of the SIL as follows. After learning, some consumers will receive a "good" signal (g), whereas others receive a "bad" signal (b). We can then express the *information precision*, $r \in (\frac{1}{2}, 1)$, via the following conditional probabilities: $P(b \mid \theta_1) = P(g \mid \theta_2) = r$. Given the conditional probabilities, a consumer updates her belief regarding her true type based on the Bayes' rule and behaves as if her expected willingness to pay is

$$\theta_g^H = \theta_2 - (\theta_2 - \theta_1)\frac{(1-r)\beta}{(1-r)\beta + r(1-\beta)} \quad \text{or}$$

$$\theta_b^H = \theta_1 + (\theta_2 - \theta_1)\frac{(1-r)(1-\beta)}{r\beta + (1-r)(1-\beta)}$$

in scenario H. Similarly, in scenario L, the consumer behaves as if her expected willingness to pay is

$$\theta_g^L = \theta_2 - (\theta_2 - \theta_1)\frac{\alpha(1-r)}{\alpha(1-r) + (1-\alpha)r} \quad \text{or}$$

$$\theta_b^L = \theta_1 + (\theta_2 - \theta_1)\frac{(1-\alpha)(1-r)}{\alpha r + (1-\alpha)(1-r)}.$$

Thus, if a consumer is uncertain regarding her true willingness to pay, in the consumer's utility $\theta_i U(q) - p(q)$, θ_i should be replaced by her expected willingness to pay.

We consider the case in which valuation uncertainty arises only in the high segment, and all the analysis about scenario L can be obtained similarly. In the following, we present the analytical results to illustrate the optimal product line design. We use the "*generalized*" setting to indicate the scenarios with the general seller's cost function, consumers' utility function, and an imprecise information revelation.

Because the seller's problem formulation is a straightforward extension of that in §3, we relegate it to the appendix and herein summarize our findings directly.

Proposition 6. *In the generalized setting with consumer uncertainty in the high segment, the quality allocations and the feasibility conditions of the four possible strategies are summarized as shown in the following table:*

	Regular solution	*Non-SIL*
Quality Allocation	q_n: *downward distortion*; q_b: *efficient*; q_g: *efficient*	q_l: *downward distortion*; q_h: *efficient*
Feasibility	$p_t \geq 0$ *and* $\theta_1 - (1-v)(\theta_\beta - \theta_1)/v \geq 0$	$\theta_1 - (1-v)(\theta_\beta - \theta_1)/v \geq 0$

	Shutdown 1	*Shutdown 2*
Quality Allocation	$q_n = 0$; q_b: *efficient*; q_g: *efficient*	q_n: *downward distortion*; $q_b = 0$; q_g: *efficient*
Feasibility	*Always*	$p_t \geq 0$ *and* $\theta_1 - (1-v)(\theta_\beta - \theta_1)/v \geq 0$

Proposition 6 shows that there are four possible optimal strategies from the seller's perspective, and the seller may abandon the SIL when it only partially resolves consumers' valuation uncertainty. This occurs when the information is so obscure that the SIL is unable to offer sufficient value to the unsure consumers. In such a scenario, to induce the unsure consumers to learn ex ante, the seller must largely compensate them ex post. Nevertheless, this generous offer also entices the consumers who do not exhibit valuation uncertainty to learn and walk away. Intuitively, SIL enables more efficient product line design, as it sharpens the distribution of consumers' valuations from two segments. Given this, it is natural to find that SIL would be abandoned if it does not sufficiently resolve uncertainties, since the distance between consumers' valuations from two segments is not sufficiently large.

Next, we articulate when the nondegenerate two-stage screening procedure is implemented (via the regular solution). We find the following corollary.

Corollary 3. *In the generalized setting with uncertainty in the high segment, the regular solution is the optimal one as long as it is feasible; moreover, the seller's profit under the regular solution is increasing in the information precision.*

Since the SIL uncovers the unsure consumers' preferences, despite the inaccurate information, the seller still promotes better matches in the high segment and creates the extra surplus due to the finer segmentation. This implies that the obstacles from facilitating this two-stage screening procedure are primarily attributed to two sources. First, the ex ante and ex post cannibalization problems may force the seller to exclude some consumers because maintaining the incentive compatibility along the product line is too costly. Second, the potential exacerbation of ex post adverse selection may result in a net loss even in the presence of efficiency gain. In this case, the seller is better off leaving consumers in a quandary. Furthermore, since more precise information leads to a more efficient allocation, the social surplus increases, as we verify rigorously in the appendix.

5. Conclusion

In this paper, we demonstrate that the SIL can serve as an effective tool for discriminating the consumers that face valuation uncertainty. We find that inducing the consumer learning is profitable as long as it completely resolves the consumers' uncertainty. In contrast, the seller may abandon the SIL due to the exacerbated adverse selection problem if the SIL does not provide adequate information. Moreover, we also discuss the scenarios in which the SIL is offered for free and the seller provides an identical menu to all the consumers, respectively. Our findings demonstrate that the SIL can no longer counterbalance the disincentive, and it only performs a function of exposing information to the uncertain consumers.

Our work certainly has some limitations. First, to highlight the role of SIL, we focus exclusively on the scenario with two possible values. To apply our idea to the practical situations, it may be essential to investigate other more complicated scenarios. The uncertain valuation may be a common theme for various segments of consumers; furthermore, the consumers may end up with more than two types (as the SIL provides several possible signals). Second, our monopoly setting is a bit restrictive. In reality, most sellers face competition from other firms in the same category. In such a scenario, whether the SIL can still serve as an effective tool for price discrimination becomes less obvious. Third, we assume that consumers do not obtain consumption utilities from the learning process. However, incorporating consumer utilities from the SIL is necessary for some scenarios, especially when the process of SIL is lengthy and functional. These constitute important directions for future work.

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Appendix

A.1 Major Steps of the Technical Proofs

This appendix covers only the major steps of the proofs. More details are given in the online supplement.

Proof of Proposition 1. In the sequel, we find the solution to problem (**P1**) assuming that all the consumers are included and all the uncertain consumers participate in the SIL. The shutdown cases and the case without SIL are omitted here.

We analyze problem (**P1**) based on the following procedure: (1) ignore some redundant constraints; (2) regroup the remaining constraints based on the price variables in the left-hand sides; (3) identify possible sets of binding constraints; (4) find the candidate solutions; (5) check the feasibility of the candidate solutions.

First, we identify some redundant constraints from problem (**P1**). When (IR_1) and (IC_{21}) hold, we obtain that $\theta_2 q_2 - p_2 \geq \theta_2 q_1 - p_1 \geq \theta_1 q_1 - p_1 \geq 0$, which implies that ($\mathrm{IR}_2$) is redundant. Moreover, we observe that $\beta(\theta_1 q_1 - p_1) + (1-\beta)(\theta_2 q_2 - p_2) - p_t \geq \theta_\beta q_n - p_n \geq \theta_1 q_n - p_n \geq 0$, where the first inequality follows from (IC_{tn}), and the third inequality follows from (IR_n). Thus, (IR_t) is redundant as well. Second, removing the two redundant constraints, we regroup the remaining constraints based on the price variables in the left-hand sides as follows:

$$p_n\colon (\mathrm{IR}_n) \text{ and } (\mathrm{IC}_{nt}); \quad p_1\colon (\mathrm{IR}_1),\ (\mathrm{IC}_{tn}),\ \text{and } (\mathrm{IC}_{12});$$
$$p_2\colon (\mathrm{IC}_{tn}) \text{ and } (\mathrm{IC}_{21}); \quad p_t\colon (\mathrm{IC}_{tn}).$$

From the structure of the optimization problem (**P1**), we can identify possible sets of binding constraints. Essentially, at least one constraint has to be binding in each group. Otherwise, since the seller's objective is strictly increasing in these price variables, the seller can achieve a higher expected profit; this can be done by increasing the corresponding price variable without sacrificing the feasibility (i.e., without violating the incentive compatibility and individual rationality). Therefore, there are two possible sets of binding constraints: (1) $[(\mathrm{IR}_n), (\mathrm{IR}_1), (\mathrm{IC}_{tn}), (\mathrm{IC}_{21})]$, and (2) $[(\mathrm{IR}_1), (\mathrm{IC}_{tn}), (\mathrm{IC}_{nt}), (\mathrm{IC}_{21})]$. Furthermore, there is no interior solution in case $[(\mathrm{IR}_1), (\mathrm{IC}_{tn}), (\mathrm{IC}_{nt}), (\mathrm{IC}_{21})]$, and we omit the details here. Now, we focus on *the regular case* $[(\mathrm{IR}_n), (\mathrm{IR}_1), (\mathrm{IC}_{tn}), (\mathrm{IC}_{21})]$ to obtain the candidate solution. Note that (IR_1) is binding. Thus, we can easily eliminate the nonlinear term $\max\{\theta_1 q_1 - p_1, \theta_1 q_2 - p_2, 0\}$ in the constraint (IC_{nt}). If (IC_{12}) is slack, we obtain that $\theta_1 q_2 - p_2 \leq \theta_1 q_1 - p_1 = 0$. Thus, $\max\{\theta_1 q_1 - p_1, \theta_1 q_2 - p_2, 0\} = 0$.

From the binding constraints (IR_n) and (IR_1), we can obtain that $p_n = \theta_1 q_n$ and $p_1 = \theta_1 q_1$. Moreover, from (IC_{21}), we obtain an expression of p_2 as follows:

$$\theta_2 q_2 - p_2 = \theta_2 q_1 - p_1 = \theta_2 q_1 - \theta_1 q_1 = (\theta_2 - \theta_1) q_1$$
$$\Leftrightarrow \quad p_2 = \theta_2 q_2 - (\theta_2 - \theta_1) q_1.$$

The binding constraint (IC_{tn}) leads to a relation between those prices:

$$\beta(\theta_1 q_1 - p_1) + (1-\beta)(\theta_2 q_2 - p_2) - p_t = \theta_\beta q_n - p_n$$
$$\Leftrightarrow \quad p_t = (1-\beta)(\theta_2 - \theta_1)q_1 - (\theta_\beta - \theta_1)q_n.$$

Having characterized these price variables, we can then substitute them into the objective function of problem (**P1**) as follows:

$$\pi = v\left\{\left[\theta_1 - \frac{(1-v)(\theta_\beta - \theta_1)}{v}\right]q_n - \frac{1}{2}q_n^2\right\} + (1-v)\beta\left(\theta_1 q_1 - \frac{1}{2}q_1^2\right) + (1-v)(1-\beta)\left(\theta_2 q_2 - \frac{1}{2}q_2^2\right).$$

The remaining decision variables are the quality levels q_n, q_2, and q_1. The candidate solution is given by the first-order conditions as follows:

$$\frac{\partial \pi}{\partial q_n} = v\left\{\left[\theta_1 - \frac{(1-v)(\theta_\beta - \theta_1)}{v}\right] - q_n\right\} = 0$$
$$\Leftrightarrow \quad q_n = \theta_1 - \frac{(1-v)(\theta_\beta - \theta_1)}{v},$$
$$\frac{\partial \pi}{\partial q_1} = (1-v)\beta(\theta_1 - q_1) = 0 \quad \Leftrightarrow \quad q_1 = \theta_1,$$
$$\frac{\partial \pi}{\partial q_2} = (1-v)(1-\beta)(\theta_2 - q_2) = 0 \quad \Leftrightarrow \quad q_2 = \theta_2.$$

The second-order conditions are easily verified. Given the quality levels, the corresponding prices can be determined accordingly. We now characterize the conditions under which the ignored constraints (IR_2), (IR_t), (IC_{nt}), and (IC_{12}) are satisfied. First, since (IC_{21}) and (IR_1) are binding, we obtain that $\theta_2 q_2 - p_2 = \theta_2 q_1 - p_1 \geq \theta_1 q_1 - p_1 = 0$, and thus ($\text{IR}_2$) holds. Furthermore,

$$\beta(\theta_1 q_1 - p_1) + (1-\beta)(\theta_2 q_2 - p_2) - p_t$$
$$= \theta_\beta q_n - p_n \geq \theta_1 q_n - p_n = 0,$$

where the first equality follows from the binding constraint (IC_{tn}). Thus, (IR_t) holds as well. Now we consider (IC_{12}). The left-hand side of (IC_{12}) minus the right-hand side is $\theta_1 q_1 - p_1 - \theta_1 q_2 + p_2 = (\theta_2 - \theta_1)(q_2 - q_1)$, where we have used the fact that (IC_{21}) is binding. Obviously, (IC_{12}) holds since $q_2 = \theta_2 \geq q_1 = \theta_1$.

Finally, because $p_t = (1-\beta)(\theta_2 - \theta_1)q_1 - (\theta_\beta - \theta_1)q_n = (1-\beta)(\theta_2 - \theta_1)(q_1 - q_n) \geq 0$, then $\max\{\theta_1 q_1 - p_1, \theta_1 q_2 - p_2, 0\} - p_t = -p_t \leq \theta_1 q_n - p_n$. Hence, ($\text{IC}_{nt}$) holds. In summary, the regular solution satisfies all the constraints. □

Proof of Corollary 1. Let us prove that $\pi_R^H \geq \pi_N^H$; that is, the seller will obtain more profit in the regular solution than in the non-SIL solution. To this end, we express the difference between the optimal profits π_R^H and π_N^H as follows: $\pi_R^H - \pi_N^H = \frac{1}{2}(1-v)\beta(1-\beta)(\theta_2 - \theta_1)^2 \geq 0$. Moreover, when the regular solution is infeasible as $q_n = \theta_1 - (1-v)(\theta_\beta - \theta_1)/v < 0$, the non-SIL solution is also infeasible. As shown above, there are two possible optimal solutions when inducing learning: the regular solution and the shutdown solution. In the regular solution, the SIL is never offered for free as $p_t = (1-\beta)(\theta_2 - \theta_1)(q_1 - q_n) > 0$. Moreover, in the shutdown solution, $p_t = (1-\beta)(\theta_2 q_2 - p_2) > 0$; consequently, the price for the SIL is always strictly positive. □

Proof of Proposition 2. We divide our analysis into three cases: (1) problem (**P2**), (2) non-SIL case, (3) the shutdown cases. Similar to the approach we use to tackle problem (**P1**), we characterize the equilibria and present them in Proposition 2. In the following, we verify that the seller improves his revenue by inducing consumer learning.

Let π_R^L and π_N^L represent the profit in the regular solution and the non-SIL solution, respectively. We can express $\pi_R^L - \pi_N^L$ in the following:

$$\pi_R^L - \pi_N^L = v\alpha\left\{\left[\theta_1 - \frac{(1-v)(\theta_2 - \theta_1)}{v}\right]q_{1R} - \frac{1}{2}q_{1R}^2\right\} + v(1-\alpha)\left\{\theta_2 q_{2R} - \frac{1}{2}q_{2R}^2\right\} - v\left\{\left[\theta_\alpha - \frac{(1-v)(\theta_2 - \theta_\alpha)}{v}\right]q_{lN} - \frac{1}{2}q_{lN}^2\right\},$$

where the subscripts R and N of the quality levels are used to distinguish whether the quality levels are in the regular solution or in the non-SIL solution. Define

$$K(q_i, q_j) = v\alpha\left\{\left[\theta_1 - \frac{(1-v)(\theta_2 - \theta_1)}{v}\right]q_i - \frac{1}{2}q_i^2\right\} + v(1-\alpha)\left\{\theta_2 q_j - \frac{1}{2}q_j^2\right\}.$$

Note that $\max_{q_{lN}} K(q_{lN}, q_{lN})$ equals $\max_{q_{1R}, q_{2R}} K(q_{1R}, q_{2R})$ with an additional constraint $\{q_{1R} = q_{2R}\}$. Therefore, $\max_{q_{1R}, q_{2R}} K(q_{1R}, q_{2R}) \geq \max_{q_{lN}} K(q_{lN}, q_{lN})$. Recollecting that

$$(q_{1R}, q_{2R}) = \underset{q_{1R},\, q_{2R}}{\arg\max}\, K(q_{1R}, q_{2R}) \quad \text{and}$$
$$q_{lN} = \underset{q_{lN}}{\arg\max}\, K(q_{lN}, q_{lN}),$$

we obtain

$$\pi_R^L - \pi_N^L = \max_{q_{1R}, q_{2R}} K(q_{1R}, q_{2R}) - \max_{q_{lN}} K(q_{lN}, q_{lN}) \geq 0;$$

thus, $\pi_R^L \geq \pi_N^L$. Furthermore, it is easy to verify the regular solution dominates the shutdown solution, and we omit the proof here.

When the regular solution is not feasible, $q_1 = \theta_1 - (1-v)(\theta_2 - \theta_1)/v \leq 0$. Meanwhile, in the non-SIL solution, $q_{lN} = (1-\alpha)\theta_2 + \alpha q_1 \leq (1-\alpha)\theta_2$. Consequently, the shutdown solution dominates the non-SIL solution because $\pi_S^L - \pi_N^L \geq \frac{1}{2}v[(1-\alpha)\theta_2^2 - (1-\alpha)^2\theta_2^2] \geq 0$. □

Proof of Corollary 2. This comes directly from the proof of Proposition 2 in the online supplement, as $p_t = 0$ occurs only in the case $q_1 = 0$. □

Proof of Proposition 3. The problem formulation and solving procedure are similar to that in the proof of Proposition 1. □

Proof of Proposition 4. To emphasize the role of SIL, we analyze the cases in which the seller induces learning for free, and characterize the equilibrium when uncertainty arises in the high segment. Likewise, the equilibrium when uncertainty arises in the low segment can be obtained.

$$\textbf{(F1)}\qquad \max_{p_n,q_n,p_1,q_1,p_2,q_2} \pi, \quad \text{where}$$

$$\pi = v\big(p_n-\tfrac{1}{2}q_n^2\big)+(1-v)\beta\big(p_1-\tfrac{1}{2}q_1^2\big)+(1-v)(1-\beta)\big(p_2-\tfrac{1}{2}q_2^2\big)$$

$$\begin{aligned}
\text{s.t.}\ &(\mathrm{IR}_n)\quad \theta_1q_n-p_n\ge 0,\quad (\mathrm{IR}_1)\quad \theta_1q_1-p_1\ge 0,\\
&(\mathrm{IR}_2)\quad \theta_2q_2-p_2\ge 0,\\
&(\mathrm{IR}_t)\quad \beta(\theta_1q_1-p_1)+(1-\beta)(\theta_2q_2-p_2)\ge 0,\\
&(\mathrm{IC}_{tn})\quad \beta(\theta_1q_1-p_1)+(1-\beta)(\theta_2q_2-p_2)\ge \theta_\beta q_n-p_n,\\
&(\mathrm{IC}_{nt})\quad \theta_1q_n-p_n\ge \max\{\theta_1q_1-p_1,\theta_1q_2-p_2,0\},\\
&(\mathrm{IC}_{12})\quad \theta_1q_1-p_1\ge\theta_1q_2-p_2,\\
&(\mathrm{IC}_{21})\quad \theta_2q_2-p_2\ge\theta_2q_1-p_1.
\end{aligned}$$

As the analysis in the previous sections, we obtain the candidate solution: $q_n=q_1=\theta_1-((1-v)(1-\beta)(\theta_2-\theta_1))/(v+(1-v)\beta)$; $q_2=\theta_2$. Furthermore, the candidate solution satisfies all the ignored constraints. □

Proof of Proposition 5. From now on, we consider the scenario in which all the products are available to all the consumers in both stages, and we assume that the price of SIL is nonnegative. Similarly, the analysis is presented when the uncertainty arises in the high segment, and we relegate the analysis when the uncertainty arises in the low segment to the online supplement.

$$\textbf{(A1)}\qquad \max_{p_n,q_n,p_1,q_1,p_2,q_2} \pi, \quad \text{where}$$

$$\begin{aligned}
\pi = {}& v\big(p_n-\tfrac{1}{2}q_n^2\big)\\
&+(1-v)\beta\big(p_1-\tfrac{1}{2}q_1^2\big)\\
&+(1-v)(1-\beta)\big(p_2-\tfrac{1}{2}q_2^2\big)+(1-v)p_t
\end{aligned}$$

$$\begin{aligned}
\text{s.t.}\ &(\mathrm{IR}_n)\quad \theta_1q_n-p_n\ge 0,\quad (\mathrm{IR}_1)\quad \theta_1q_1-p_1\ge 0,\\
&(\mathrm{IR}_2)\quad \theta_2q_2-p_2\ge 0,\\
&(\mathrm{IR}_t)\quad \beta(\theta_1q_1-p_1)+(1-\beta)(\theta_2q_2-p_2)-p_t\ge 0,\\
&(\mathrm{IC}_{tn})\quad \beta(\theta_1q_1-p_1)+(1-\beta)(\theta_2q_2-p_2)-p_t\\
&\qquad\qquad \ge \max\{\theta_\beta q_n-p_n,\theta_\beta q_1-p_1,\theta_\beta q_2-p_2\},\\
&(\mathrm{IC}_{12})\quad \theta_1q_1-p_1\ge\theta_1q_2-p_2,\\
&(\mathrm{IC}_{1n})\quad \theta_1q_1-p_1\ge\theta_1q_n-p_n,\\
&(\mathrm{IC}_{21})\quad \theta_2q_2-p_2\ge\theta_2q_1-p_1,\\
&(\mathrm{IC}_{2n})\quad \theta_2q_2-p_2\ge\theta_2q_n-p_n,\\
&(\mathrm{IC}_{n1})\quad \theta_1q_n-p_n\ge\theta_1q_1-p_1,\\
&(\mathrm{IC}_{n2})\quad \theta_1q_n-p_n\ge\theta_1q_2-p_2.
\end{aligned}$$

We find the candidate solution as follows: $q_1=q_n=\theta_1-((1-v)(1-\beta)/(v+(1-v)\beta))(\theta_2-\theta_1)$; $q_2=\theta_2$. Moreover, the feasibility constraint is $p_t\le\beta(1-\beta)\cdot(\theta_2-\theta_1)^2(1/(v+(1-v)\beta))$. □

Proof of Proposition 6. We formulate the seller's optimization problem as follows:

$$\textbf{(P4)}\qquad \max_{p_n,q_n,p_b,q_b,p_g,q_g} \pi, \quad \text{where}$$

$$\begin{aligned}
\pi = {}& v(p_n-C(q_n))+(1-v)P_b^H(p_b-C(q_b))\\
&+(1-v)P_g^H(p_g-C(q_g))+(1-v)p_t
\end{aligned}$$

$$\begin{aligned}
\text{s.t.}\ &(\mathrm{IR}_n)\quad \theta_1U(q_n)-p_n\ge 0,\\
&(\mathrm{IR}_b)\quad \theta_b^HU(q_b)-p_b\ge 0,\\
&(\mathrm{IR}_g)\quad \theta_g^HU(q_g)-p_g\ge 0,\\
&(\mathrm{IR}_t)\quad P_b^H(\theta_b^HU(q_b)-p_b)+P_g^H(\theta_g^HU(q_g)-p_g)-p_t\ge 0,\\
&(\mathrm{IC}_{tn})\quad P_b^H(\theta_b^HU(q_b)-p_b)+P_g^H(\theta_g^HU(q_g)-p_g)\\
&\qquad\qquad -p_t\ge\theta_\beta U(q_n)-p_n,\\
&(\mathrm{IC}_{nt})\quad \theta_1U(q_n)-p_n\\
&\qquad\qquad \ge\max\{\theta_1U(q_b)-p_b,\theta_1U(q_g)-p_g,0\}-p_t,\\
&(\mathrm{IC}_{bg})\quad \theta_b^HU(q_b)-p_b\ge\theta_b^HU(q_g)-p_g,\\
&(\mathrm{IC}_{gb})\quad \theta_g^HU(q_g)-p_g\ge\theta_g^HU(q_b)-p_b,
\end{aligned}$$

whereas $P_b^H=r\beta+(1-r)(1-\beta)$ and $P_g^H=(1-r)\beta+r(1-\beta)$. In the case [($\mathrm{IR}_n$), ($\mathrm{IR}_b$), ($\mathrm{IC}_{tn}$), ($\mathrm{IC}_{gb}$)], we obtain the candidate solution as follows:

$$C'(q_n)=\left[\theta_1-\frac{(1-v)(\theta_\beta-\theta_1)}{v}\right]U'(q_n),\quad C'(q_b)=\theta_b^HU'(q_b),$$
$$C'(q_g)=\theta_g^HU'(q_g).$$

We then characterize the conditions under which the ignored constraints (IR_g), (IR_t), (IC_{nt}), and (IC_{bg}) are satisfied. It is easy to verify that (IR_g), (IR_t), (IC_{bg}) hold. Furthermore, when $p_t\ge 0$ and consequently (IC_{nt}) holds, we obtain the regular solution in this case. For ease of presentation, let us introduce the following notation. Let q_{yz}^x denote the quality level for type y consumer in solution z in scenario x, where $x\in\{L,H\}$ and $y\in\{b,g,n,h,l\}$, and p_{yz}^x is the corresponding price. For example, we use q_{nR}^H, q_{bR}^H, and q_{gR}^H to substitute the quality level variables in the regular solution of scenario H. □

Proof of Corollary 3. First, we prove that $\pi_R^H\ge\pi_N^H$. The difference of the optimal profits π_R^H and π_N^H can be expressed as follows:

$$\begin{aligned}
\pi_R^H-\pi_N^H = {}&(1-v)P_b^H(\theta_b^HU(q_{bR}^H)-C(q_{bR}^H))\\
&+(1-v)P_g^H(\theta_g^HU(q_{gR}^H)-C(q_{gR}^H))\\
&-(1-v)(\theta_\beta U(q_{hN}^H)-C(q_{hN}^H)).
\end{aligned}$$

Note that q_{gR}^H and q_{bR}^H maximize $\theta_g^HU(q)-C(q)$ and $\theta_b^HU(q)-C(q)$, respectively (i.e., they are both efficient). Thus, following the approach in the proof of Proposition 2, we obtain $\pi_R^H>\pi_N^H$. The comparison between π_R^H and π_{S1}^H or π_{S2}^H follows directly since the latter two are simply solutions in which some quality levels are forced to zero.

Second, we show the profit in the regular solution is increasing in r. The derivative can be expressed as follows:

$$\frac{\partial\pi}{\partial r} = (1-v)\{\beta[(\theta_1 U(q_b)-C(q_b))-(\theta_1 U(q_g)-C(q_g))] + (1-\beta)[(\theta_2 U(q_g)-C(q_g))-(\theta_2 U(q_b)-C(q_b))]\}\geq 0,$$

whereas the first-order conditions are used to cancel out some terms, and the last inequity can be easily verified as follows. For ease of exposition, define $Z_1(q_i)=\theta_1 U(q_i)-C(q_i)$. Apparently, $Z_1'(q_b)=\theta_1 U'(q_b)-C'(q_b)\leq \theta_b U'(q_b)-C'(q_b)=0$ and $Z_1''(q_i)\leq 0$. Therefore, $Z_1(q_b)\geq Z_1(q_g)$. Similarly, we obtain $(\theta_2 U(q_g)-C(q_g))\geq(\theta_2 U(q_b)-C(q_b))$. □

A.2 Heterogeneous Consumers in the Segment Without Uncertainty

In the sequel, we extend our discussion to the scenario with heterogeneous consumers in the segment without uncertainty. Specifically, in both segments, there are consumers with willingness θ_1 or θ_2, and the proportion of $\theta_1(\theta_2)$ is $\alpha(1-\alpha)$. Moreover, only one segment exhibits uncertainty. For those consumers with uncertainties, they realize their willingness is either θ_1 or θ_2 after learning. Then we can formulate the optimization problem as follows:

$$(\mathbf{M}) \qquad \max_{p_1,q_1,p_2,q_2,p_b,q_b,p_g,q_g} \pi, \quad \text{where}$$

$$\pi = v\alpha\left(p_1-\tfrac{1}{2}q_1^2\right)+v(1-\alpha)\left(p_2-\tfrac{1}{2}q_2^2\right) +(1-v)\alpha\left(p_b-\tfrac{1}{2}q_b^2\right) +(1-v)(1-\alpha)\left(p_g-\tfrac{1}{2}q_g^2\right)+(1-v)p_t,$$

s.t.

(IR_1) $\theta_1 q_1-p_1\geq 0$, (IR_2) $\theta_2 q_2-p_2\geq 0$,

(IR_b) $\theta_1 q_b-p_b\geq 0$, (IR_g) $\theta_2 q_g-p_g\geq 0$,

(IR_t) $\alpha(\theta_1 q_b-p_b)+(1-\alpha)(\theta_2 q_g-p_g)-p_t\geq 0$,

(IC_{t1}) $\alpha(\theta_1 q_b-p_b)+(1-\alpha)(\theta_2 q_g-p_g)-p_t\geq \theta_\alpha q_1-p_1$,

(IC_{t2}) $\alpha(\theta_1 q_b-p_b)+(1-\alpha)(\theta_2 q_g-p_g)-p_t\geq \theta_\alpha q_2-p_2$,

(IC_{1t}) $\theta_1 q_1-p_1\geq \max\{\theta_1 q_b-p_b,\theta_1 q_g-p_g,0\}-p_t=\theta_1 q_b-p_b-p_t$,

(IC_{2t}) $\theta_2 q_2-p_2\geq \max\{\theta_2 q_b-p_b,\theta_2 q_g-p_g,0\}-p_t=\theta_2 q_g-p_g-p_t$,

(IC_{12}) $\theta_1 q_1-p_1\geq \theta_1 q_2-p_2$,

(IC_{21}) $\theta_2 q_2-p_2\geq \theta_2 q_1-p_1$,

(IC_{bg}) $\theta_1 q_b-p_b\geq \theta_1 q_g-p_g$,

(IC_{gb}) $\theta_2 q_g-p_g\geq \theta_2 q_b-p_b$.

Following the aforementioned approach, the candidate solution is given by first-order conditions:

$$q_1=\theta_1-\frac{(1-\alpha)(\theta_\alpha-\theta_1)}{\alpha}-\frac{(1-v)(\theta_\alpha-\theta_1)}{v\alpha}, \quad q_2=\theta_2,$$

$$q_b=\theta_1-\frac{v(1-\alpha)(\theta_2-\theta_1)}{(1-v)}, \quad q_g=\theta_2.$$

Similarly, we characterize the conditions under which the ignored constraints (IR_2), (IR_g), (IR_t), (IC_{t2}), (IC_{1t}), (IC_{12}), (IC_{21}), and (IC_{bg}) are satisfied. After derivations, we find that as long as $q_1\leq q_b\leq q_g\leq q_2$ holds, all the ignored constraints hold. To avoid redundancy, the details are omitted here. Furthermore, it is easy to find that when $v/(1-v)\leq(1-v\alpha)/(v\alpha)$, $q_1\leq q_b$.

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