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We consider a problem faced by a contract assembler that both assembles finished goods and procures the associated component parts for one of its major customers. Because of rapid changes in technology and ongoing engineering changes, all parts subject to obsolescence are purchased only for the current customer order. The procurement lead times of the components are random. Moreover, although the order for the finished product has a defined due date, the contract allows the customer to change the order quantity. Consequently, the assembler also faces a random demand. The assembler must determine how much to order and when to order each component part. The objective is to minimize the total expected cost, including the cost of holding components prior to their assembly, penalties for tardiness vis-a-vis the assembly due date, and overage and underage costs in satisfying the demand quantity. We present some structural results and discuss insights regarding optimal policies. We also present several simple heuristic policies and compare them to optimal policies. Computational results indicate that ignoring lead time variability can be costly, but relatively simple heuristics that consider lead time variability perform quite well.
(*Inventory/Production; Multiitem; Optimal Policies; Stochastic Models*)

1. Introduction

We consider a problem faced by a contract assembler which not only assembles finished goods for one of its major customers, but is also responsible for procuring the associated component parts. The use of contract assemblers is growing rapidly, especially in the electronics industry. Some such contract assemblers specialize in printed circuit card assembly for computers and other similar products, while others specialize in the assembly of finished products including computers and industrial electromechanical products. Our focus is on contract assemblers whose primary responsibility is the assembly (versus fabrication) of products, and the procurement of some or all of the associated parts.

Typically, these contract assemblers have long-term (e.g., annual) arrangements with their customers. As such, a customer may order the same product several times in different quantities. However, because of rapid changes in the underlying technology and frequent engineering changes, key component parts have a high probability of becoming obsolete or significantly less valuable between one order and the next. For this reason, it is practical to think of each order as a custom order in terms of the physical product. Due to the rapid rate of technical obsolescence and corresponding changes in the value of the components, contract assemblers try to avoid ordering (or holding) excess components. It has been estimated that the value of

key electronic components declines about 40% per quarter.

Due to the nature of the contract between the manufacturer of the product and the customer, the customer may be allowed to adjust the order quantity (within limits) between placement of the order (or, in some cases, the statement of a forecast) and the due date. The assembler's uncertainty about the final order quantity is compounded by significant uncertainty in the delivery lead times for component parts. In some cases, this uncertainty is attributable to fluctuations in supply and demand for fairly generic components and in other cases it is attributable to fluctuating workloads and/or insufficient capacity at the manufacturers of more specialized component parts. In either case, from the point of view of the purchasing agents, the delivery lead times are random and little can be done to influence them in the short run.
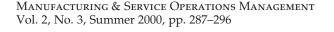
It should be pointed out, however, that purchasing agents typically have some information about each supplier's workload and general market conditions that affect each supplier's response time. Moreover, they may have been quoted delivery dates for recent orders, and some historical information may be available about the delivery performance of various suppliers. From this, the purchasing agents can construct lead time distributions that reflect their knowledge and uncertainty about what the actual delivery lead times will be. Indeed, in both of the industrial examples that we describe later in this section, major vendors do quote delivery lead times, but the bias and variability of the actual lead time vis-a-vis the quoted lead time differs by vendor. When historical lead times, or the deviations between quoted and actual lead times, are indicative of future performance, standard methods for fitting distributions may be used (for example, see Clemen 1996 and McClave and Benson 1991). In cases where the future is not expected to mirror the past, Bayesian techniques (e.g., Clemen 1996 and Sherbrooke 1992) may be used for representing lead time distributions.

The assembly process itself is fairly predictable in terms of both the time required and the final yields (after testing and rework). Thus, the uncertainty in the assembly lead time and output are small relative to the other sources of uncertainty. The contract assembler, therefore, views its main problem to be that of determining how much to order and when to order each of the component parts. The objective is to minimize the total expected cost, including the cost of holding components prior to their assembly, penalties for tardiness vis-a-vis the assembly due date, and overage and underage costs in satisfying the demand quantity.

This problem differs from others in the literature in that there is a single customer order with a due date and *both* the supply timing and the demand quantity are stochastic. Although such situations occur widely in practice, there are no known results regarding the structure of the optimal component ordering policy when there are multiple components.

Our assumptions are strongly motivated by what we have observed in several different settings. We provide two specific examples below. The first setting is a company that designs controllers used by the electric utility industry, and the primary contract assembler of these products. The design firm offers several families of semi-custom products. Products within a family differ in their capabilities, and the displays and housing are usually customized for each customer's needs. Often these differences in the displays and housings necessitate changes in the primary printed circuit boards. Although a given utility may place multiple orders for the same finished product, there is no guarantee that an additional order will occur, and a product designed for one utility generally is not suited to another utility. If the customer order is for a new or redesigned product, the design of the printed circuit board, displays and housing may entail a lead time of a month or more, and the manufacture of the boards may take several additional weeks. Meanwhile, the utility may seek to change its order quantity as the budget for equipment expenditures is periodically revised. Because of the importance of a continuing relationship with each utility, the design firm would like to accommodate customer needs, particularly upward changes in the order quantity, which allow the design firm to amortize the product design costs more quickly. The design firm orders the required components and arranges for them to be shipped to the contract assembler. At the same time, it reserves capacity at the contract assembler for the associated order. In this case, the design firm makes procurement quantity and timing decisions on behalf of

the contract assembler, and faces the same tradeoffs that we have described above.

The second setting is a contract assembler of motherboards for desktop computers. The contract assembler has a long-term contract with a major computer manufacturer, and is fully responsible for purchasing component parts. Some components are commonly-used inexpensive electronic parts, and we do not consider these in our model. Many of the expensive components (e.g., bare printed circuit boards) are specific to a particular product or product family and are subject to frequent engineering changes. Because of these engineering changes and because the contract assembler cannot use or sell excess product-specific components, or may have to wait a long time for a subsequent order, it tends to be fairly risk averse in the purchase of these components. In effect, the contract assembler behaves almost as if each customer order were independent of one another. Such a strategy may not be optimal except near the end of the product's life cycle. However, very rapidly declining component prices can make a myopic policy very prudent from a risk management perspective.

We acknowledge that the single-customer-order situation does not fully capture the additional decisions and factors that would arise in the case of multiple customer orders (such as the possibility of batching multiple orders). In these instances, combined customer orders and/or safety stock can provide some buffer stock that can be used in response to lead time variability. Our model does not account for such interactions, but we hope that our model can serve as a first step toward the development of more realistic models. Nevertheless, as pointed out above, our model closely resembles a number of practical situations.

Many articles, too numerous to mention here, have been written on continuous-review and periodic-review *single-item* inventory models with stochastic lead times and either deterministic or stochastic demand (a list is available from the authors upon request), but little has been done for assembly systems. Moreover, the vast majority of the inventory literature on stochastic assembly systems has considered only one primary source of uncertainty at a time.
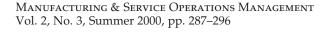
In particular, one body of literature assumes that the demand timing and quantity are known but the procurement lead times are stochastic. The primary concern is to determine the optimal planned lead times (i.e., how far in advance of the due date to order) for the components to minimize the expected cost. Related research includes Yano (1987), who considers two components with stochastic procurement and assembly times; Kumar (1989), Hopp and Spearman (1993) and Shore (1995), who consider n components with random procurement lead times and instantaneous assembly, and Chu, Proth and Xie (1993) who consider n components with random procurement times and a deterministic assembly time. Some of these papers seek to minimize expected costs, including time-weighted shortages, while others impose a constraint on the probability of achieving on-time delivery.

Gurnani et al. (1996) consider a finished product with two components and a single random demand. There is an independent supplier for each component and a joint supplier that can supply the components in pairs. Each component is assumed to arrive in the current period with some probability or in the subsequent period otherwise. The key decisions are how much to purchase from each supplier.

The papers described above deal with a single customer order (as in our model). Several other papers treat the situation with ongoing (constant or stationary) demand for a single finished product. Fujiwara, Adikari and Perera (1994) and Fujiwara and Sedarage (1997) study a system in which each part i is controlled by a continuous-review (r_i, Q) policy. Chu et al. (1994), Mauroy and Wardi (1995) and Proth et al. (1997) propose different approaches for solving a discrete time problem in which multiple finished products are assembled from multiple components. Components may be shared among the products and both component delivery lead times and finished product demands are random. The problem is to decide finished product assembly quantities and component purchase quantities at the beginning of each period so as to minimize expected inventory and backorder costs. The emphasis of these papers is on developing a computational procedure to search for a good solution to an approximate version of the problem.

In our problem, we face uncertainty in supply and demand, with regard to both timing and quantity. So,

a natural question is: What is the optimal replenishment policy in these circumstances? It turns out there is no simple answer to this question. Whybark and Williams (1976) develop a simulation model to study the advantages of safety stock vs. safety lead time under different types of uncertainty. Their conclusion is that, under conditions of uncertainty in timing, safety lead time is preferred, while safety stock is preferred under conditions of quantity uncertainty. Other researchers (e.g., Vinson 1972, Bagchi et al. 1986, Song 1994) have pointed out the importance of lead time uncertainty in setting inventory levels. However, to our knowledge, there is no definitive answer to this question. Aside from the paper by Gurnani et al. which considers a very special case, we are not aware of any literature that simultaneously considers a make-to-order environment with demand uncertainty and a fixed due date in circumstances where multiple components with random lead times must be assembled.

The remainder of this paper is organized as follows. We first provide a formal statement of the problem. We then present a variety of structural results for the problem and a method to compute the optimal solution. We also propose several simpler heuristic policies and through computational studies, compare their performance with the optimal policy.

2. Problem Statement and Formulation

We consider a two-level assembly problem in which n distinct components are assembled into one finished product. Without loss of generality, we assume that exactly one unit of each component is required to produce the finished product. There is a one-time demand of random quantity D that will occur at a (known) future time t_0. We consider the static version of the problem where ordering (timing and quantity) decisions must be made simultaneously at the beginning of the horizon, say time 0. We must decide how far in advance of time t_0 to place the order for each component and what the order quantity should be. We are assuming that, although the customer may update the order quantity prior to the shipment date, the manufacturer must determine the component order quantity when the order for the first component is placed. At that

point in time, the manufacturer uses the initial customer order quantity, history about the customer's past behavior in updating order quantities, and any available information or forecasts on the customer's demand for the product in forming the demand distribution to be employed in making the decision.

Each component is supplied by a vendor whose manufacturing process is independent of both the demand and the component orders, but its operation determines the lead times of the component orders. Let random variable L_i be the lead time of component i which has a cumulative distribution function F_i, $i = 1, 2, \ldots, n$. Consistent with the static nature of our problem, we assume that no information updates on the status of orders from the supplier are available. Although this assumption is somewhat restrictive, in practice, relatively little information is available about the delivery dates of the components until close to the time of their delivery. At that point, it may be possible to request a delay in the delivery of a component that is likely to be otherwise early, but it is more difficult to request expediting of a component that is likely to be late. We do not consider these options here, because the possibility of making such changes is very situation-specific.

Our objective is to find an optimal component procurement policy that minimizes the total expected cost. We consider unit purchase costs, inventory holding costs, and per unit per unit time backlogging costs. We also consider losses due to salvaging, and penalties for demand that remains unsatisfied when the order is shipped.

We assume the assembly time is deterministic, which parallels the situation in our motivating application. Thus, the assembly process is started just in time to meet the due date if all components are available; otherwise, it is started as soon as the tardiest component arrives. We also assume that there is one order of each component for each assembly batch. This is common practice, and is also a realistic representation for instances where the finished product is custom or semi-custom, or where the batch identity of component parts and/or the finished product must be maintained for safety or administrative reasons.

Recall that exactly one unit of each component is needed for each unit of finished product, that there is

a single customer order, and that we are addressing a static version of the problem. As such, the order quantities of the components should be the same for all the components. (This is intuitive and easy to prove for the static case, but this property may not hold when ordering decisions can be made dynamically.)

Let y be the order quantity and x_i be the planned lead time for component i. Without loss of generality, we assume that y and x_i, $i = 1, \ldots, n$ are all nonnegative. Let

h_i = unit holding cost rate of component i,

p = unit tardiness cost rate of the end product.

In general, we use a bold faced letter to indicate an n-vector. In particular, $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ and $\mathbf{h} = (h_1, h_2, \ldots, h_n)$. For any two vectors $\mathbf{a}$ and $\mathbf{b}$, $\mathbf{a} \cdot \mathbf{b} = \Sigma_{i=1}^{n} a_i b_i$.

Clearly, $(x_i - L_i)^+$ is the earliness of component i, and $(L_i - x_i)^+$ is the "tardiness" of component i. Here, $z^+ = \max\{0, z\}$ for any real number z. The tardiness of the finished product is then

$$\tilde{T}(\mathbf{x}) = \max_{1 \le i \le n} (L_i - x_i)^+.$$

Since

$$\Pr\{\tilde{T}(\mathbf{x}) \le t\} = \Pr\{(L_i - x_i)^+ \le t \text{ for all } i\}$$

$$= \prod_{i=1}^{n} \Pr\{(L_i - x_i)^+ \le t\}$$

$$= \prod_{i=1}^{n} F_i(x_i + t),$$

the expected tardiness is

$$T(\mathbf{x}) = E[\tilde{T}(\mathbf{x})] = \int_0^{\infty} \left(1 - \prod_{i=1}^{n} F_i(x_i + t)\right) dt.$$

Also, the expected holding cost is

$$y \sum_{i=1}^{n} h_i E\{(x_i - L_i)^+ + [\tilde{T}(\mathbf{x}) - (L_i - x_i)^+]\}. \quad (1)$$

Observe that

$$E[(x_i - L_i)^+ - (L_i - x_i)^+] = x_i - E[L_i].$$

Let

$$l_i = E[L_i] \quad \text{and} \quad h_0 = \sum_{i=1}^{n} h_i.$$

Then, (1) becomes

$$y[\mathbf{h} \cdot (\mathbf{x} - \mathbf{l}) + h_0 T(\mathbf{x})]. \quad (2)$$

Obviously, the expected tardiness cost is $pE[D]T(\mathbf{x})$. So, applying (2), the expected earliness and tardiness cost can be written as

$$y[\mathbf{h} \cdot (\mathbf{x} - \mathbf{l}) + h_0 T(\mathbf{x})] + pE[D]T(\mathbf{x}). \quad (3)$$

Let c, π, and s be the unit product cost (sum of component procurement costs and assembly cost), the unit revenue, and the unit salvage value of the finished product, respectively. It is reasonable to assume that $\pi > c > s$. Otherwise, it might be unprofitable to produce at all, or it might be profitable to produce a unit in order to salvage it. The following expression is the variable procurement costs less expected revenue of the finished product:

$$cy - sE[(y - D)^+] - \pi E[\min\{y, D\}]$$

$$= -(\pi - c)y + (\pi - s)E[(y - D)^+]. \quad (4)$$

Combining (3) and (4), we obtain the total cost function:

$$C(\mathbf{x}, y) = [\mathbf{h} \cdot (\mathbf{x} - \mathbf{l}) + h_0 T(\mathbf{x}) - (\pi - c)]y$$

$$+ (\pi - s)E[(y - D)^+] + pE[D]T(\mathbf{x}). \quad (5)$$

Our objective is then to solve the optimization problem

$$\textbf{(P1)} \qquad \min_{\mathbf{x}, y \ge 0} C(\mathbf{x}, y).$$

3. Problem Analysis

Notice that for any fixed $\mathbf{x}$, (5) is just a newsboy function of y (shifted by the last term). Therefore, it is convex in y, and for any fixed $\mathbf{x}$, there exists $y^*(\mathbf{x})$ such that

$$C(\mathbf{x}^*, y^*) = \min_{\mathbf{x}, y \ge 0} C(\mathbf{x}, y) = \min_{\mathbf{x} \ge 0} \left\{ \min_{y \ge 0} C(\mathbf{x}, y) \right\}$$

$$= \min_{\mathbf{x} \ge 0} C(\mathbf{x}, y^*(\mathbf{x})).$$

Next, we show that for any fixed y, (5) is convex in $\mathbf{x}$. To do so, we need the following obvious fact.

LEMMA 1. *If a real function $g(u, v)$ defined on $A \times B$ is convex in u for all v, then for any random variable V taking values in B, $E[g(u, V)]$ is still convex in u.*

Observe that the function $(u_i - x_i)^+$ is convex in x_i for all u_i. Recall that the maximum of several convex functions is still convex. So, the function

$$g(\mathbf{x}, \mathbf{u}) = \max_{1 \le i \le n} \{(u_i - x_i)^+\}$$

is convex in $\mathbf{x}$. Let $\mathbf{L} = (L_1, \ldots, L_n)$. From Lemma 1, $T(\mathbf{x}) = E[g(\mathbf{x}, \mathbf{L})]$ is convex in $\mathbf{x}$. Now, because $\mathbf{h} \cdot \mathbf{x}$ is also convex in $\mathbf{x}$, for any fixed $y \ge 0$, $C(\mathbf{x}, y)$ is a nonnegative linear combination of convex functions of $\mathbf{x}$ and therefore it is convex. Thus, we have shown:

PROPOSITION 1. (i) *For any fixed y, $C(\mathbf{x}, y)$ is convex in* $\mathbf{x}$. (ii) *For any fixed* $\mathbf{x}$, *$C(\mathbf{x}, y)$ is convex in y.*

Thus, by solving

$$\frac{\partial}{\partial y} C(\mathbf{x}, y) = 0,$$

and substituting appropriately into the original objective function, we have $y^*(\mathbf{x})$ satisfying

$$F_D(y^*(\mathbf{x})) = \frac{\pi - c - [\mathbf{h} \cdot (\mathbf{x} - \mathbf{l}) + h_0 T(\mathbf{x})]}{\pi - s}, \quad (6)$$

where F_D is the cumulative probability distribution of D.

Observe that the optimal order-up-to value y^* is generally *less* than in the standard newsvendor model. This occurs because $h_0 T(\mathbf{x})$ is nonnegative and $\mathbf{h} \cdot (\mathbf{x} - \mathbf{l})$ is typically (although not always) nonnegative. The latter term is strictly nonnegative if the planned lead times are at least as large as the respective mean lead times—that is, if the safety times are nonnegative. Typically, the safety times will be nonnegative if the lead time densities are not too skewed and the cost of tardiness is much higher than the component inventory holding costs. Under these common conditions, uncertainty in the lead times causes one to be more conservative in ordering than if the lead times were deterministic. This relationship has rarely been observed in single-item inventory models; indeed, under certain mild conditions that are qualitatively similar to those just stated, the order-up-to points increase as the lead time variance increases (see, for example, Gerchak and

Mossman 1992 and Song 1994). Intuitively, the reason for our result is that reducing the order quantity reduces the holding costs that accrue due to differences in the arrival times of the components. These holding costs do not occur in single-item models because no mating of components is necessary.

In general, the function C is not jointly convex in $(\mathbf{x}, y)$. Also, it is clear that the function is not separable in $\mathbf{x}$ and y. Therefore, the search for the optimal solution of (**P1**), namely, $(\mathbf{x}^*, y^*)$, will require a special solution procedure. One such technique is a parametric optimization procedure described below.

Solution Procedure

This solution procedure takes advantage of the fact that for each fixed y we can determine the best $\mathbf{x}$. For fixed y, the problem is to:

$$\min_{\mathbf{x}} y\mathbf{h} \cdot (\mathbf{x} - \mathbf{l}) + (h_0 y + pE[D])[T(\mathbf{x})] + G(y)$$

where $G(y)$ represents the expected holding and shortage cost for the newsvendor problem.

We can solve the problem of finding the x_is by a parametric approach in which we optimize $\mathbf{x}$ for a given expected tardiness level, t, in an inner optimization problem and optimize the expected tardiness level in an outer optimization problem. The problem we need to solve is:

$$\min_t \min_{\mathbf{x}} [y(\mathbf{h} \cdot \mathbf{x} + h_0 t) + pE[D]t \mid T(\mathbf{x}) = t]. \quad (7)$$

Let us first consider the problem of finding the optimal $\mathbf{x}$ for a given y and t. This is a highly structured optimization problem, if either the lead times are discrete, or if we are willing to use a discrete approximation of them. Discretization to time intervals of a day or a shift would be sufficient in practical applications. For fixed y and t, the problem reduces to

$$\begin{vmatrix} \min & \mathbf{h} \cdot \mathbf{x} \\ \text{s.t.} & T(\mathbf{x}) = t. \end{vmatrix}$$

This problem has a linear objective with a convex equality constraint and therefore can be solved by standard convex optimization procedures. Note that for $T(\mathbf{x}) = t$, the resulting solution is a (possibly nonunique) dominant solution; all others can be eliminated from consideration for that value of t. Furthermore, we know that each x_i has decreasing marginal

returns in decreasing $T(\mathbf{x})$ (i.e., $T(\mathbf{x})$ is convex decreasing in each x_i). Because the inventory costs are linear, the "bang for buck" (i.e., the decrease in $T(\mathbf{x})$ per dollar increase in the objective function) also exhibits decreasing marginal returns. So we can start with $\mathbf{x} = \mathbf{0}$ and iteratively increase the x_i with the greatest "bang for buck" until we achieve $T(\mathbf{x}) = t$.

Now the question is what happens to the inner minimization as t changes? Note that in the inner minimization, for fixed y, the terms yh_0t and $pE[D]t$ are linear in t. On the other hand, $\mathbf{h} \cdot \mathbf{x}$ must increase as t decreases, and due to the diminishing marginal returns, it must increase at an increasing rate. So we can conclude that with y fixed, $\min_{\mathbf{x}}(y\mathbf{h} \cdot \mathbf{x} \mid T(\mathbf{x}) = t)$ is convex decreasing in t, and therefore the entire expression in brackets in (7) is convex in t. This result also follows directly from the fact that the problem is a right-hand-side parametric convex optimization problem.

Let $f(t) = \min_{\mathbf{x}}(\mathbf{h} \cdot \mathbf{x} \mid T(\mathbf{x}) = t)$, which is also convex decreasing in t. We can rewrite the original objective function as:

$$\min_{y,t} y[f(t) + h_0t] + pE[D]t + G(y) - (\mathbf{h} \cdot \boldsymbol{l})y.$$

We now return to the problem of determining the optimal values of y and t. The objective function may not be jointly convex in these variables, so one must take care to consider all viable values of y and t. This is not difficult, however, as y is bounded above by the newsvendor solution (cf. (6)), and t will be bounded above by practical limits on the tardiness. For each t, however, we only need to retain one nondominated $\mathbf{x}$ (and the same nondominated $\mathbf{x}$ can be used for all y), which considerably reduces the computational effort by decoupling $\mathbf{x}$ from y in the search procedure.

4. Heuristics

What is most interesting about our results is the nature of the relation between $\mathbf{x}$ and y. In the vast majority of the literature on inventory models, incorporating random lead times into a model with random demand tends to increase order quantities, order-up-to points, and/or reorder points. In our model, we have the opposite effect. That is, under common conditions in which the optimal planned lead times are at least as large as the respective mean lead times, the optimal

order quantity declines when lead time randomness is considered. (Similar phenomena have been observed in other instances. For example, Zipkin (2000) notes that the variance of lead time demand decreases when lead times are random, and demand is more variable than is the Poisson process.) This raises the question of the magnitude of the effect, and whether this effect can be modeled in an approximate fashion so that good solutions can be obtained quickly. Because the objective function may not be jointly convex in all variables, the optimal solution may be difficult to obtain for large problems, and a practical alternative may be needed. Below we suggest several heuristic procedures.

Newsvendor Heuristic
Solve the *standard* newsboy problem to obtain the order quantity, denoted y^N, then find $\mathbf{x}^*(y^N)$.

Mean Lead Time Heuristic
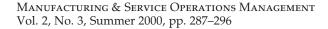Set $x_i = E[L_i]$. Then solve for y using (6).

Mean Demand Heuristic
Set $y = E[D]$. Then solve for $\mathbf{x}$ using this value of y.

Descent Procedure
Initially set y according to one of the heuristics above. (We used y^N.) Then find $\mathbf{x}^*$ given this y, then y^* given the most recent $\mathbf{x}$ and iterate until convergence is achieved. Because the cost function may not be jointly convex, this procedure may converge to a local minimum. It is an improvement procedure, however, so it is guaranteed to converge.

5. Computational Results and Conclusions

We generated 48 problems with identical component costs $c_i = 100$ for all i and 96 problems with nonidentical component costs. The latter problems are divided between two different scenarios: (a) $\mathbf{c} = (100, 500, 100, 100, 100)$ and (b) $\mathbf{c} = (100, 500, 100, 100, 500)$, where $\mathbf{c}$ is the vector of component costs. Demands are Poisson with $\lambda = 60$. For each component cost vector, there are six different lead time distribution vectors. In all cases, we used negative binomial distributions, which reflect the positive skewness of lead time distributions observed in practice. The parameters r_i and p_i, where r_i

is the integer parameter and p_i is the fractional parameter, for these lead time distributions are shown in Table 1.

The annual inventory holding cost rate was set to 20%. There are two different values for each of the other costs parameters. The value of π is either $1.5c$ or $2.5c$, where c is the total cost of the finished product, i.e., the sum of unit costs of the components and the assembly cost. The parameter f, which represents the assembly cost as a fraction of the total cost of the product, is either 0.1 or 0.5. The value of p is either 0.01π or 0.05π; these values were selected so that both the penalties for unsatisfied demand and the penalties for delayed delivery would have an influence on the decisions. The value of s is either $0.4c$ or $0.9c$ representing situations where the salvage value is low or high, respectively. By considering all combinations, we have 48 ($= 6 \times 2 \times 2 \times 2 \times 2$) problems in the case of identical components. Similarly, there are 48 problems for each of the two nonidentical component cost vectors.

For each problem, we applied the four heuristics described above and an optimal procedure which parallels that given above. Because we are using discrete demand distributions, we could enumerate values of y. Likewise, we used integer increments in solving for the planned lead times. Technically, it is possible for the optimal planned lead times to be nonintegral even when the lead time distribution is discrete. However, in practice, rounding to the nearest integer (or to the nearest practical increment) should be sufficient, and we compute solutions accordingly. For our problem parameters, the typically observed planned lead times are long enough (mostly low two-digit values) that a one time period increment is reasonable.

Table 2 presents summary statistics on the performance of the heuristic procedures. In all of the test problems, the descent heuristic identified the optimal solution, and the newsvendor heuristic, which does not require an iterative search, performed extremely well. The mean demand heuristic also performed well, but with the loss of a few percentage points from the newsvendor heuristic, which requires virtually the same computational effort. The mean lead time heuristic performed poorly in all problems.

From these results, we can draw several conclusions. First, it may be more important to consider lead time variability than demand variability. The mean lead time heuristic performed very poorly, whereas the mean demand heuristic, although suboptimal, never produced poor results. This result is surprising in view of the fact that the numerical results are based on Poisson demands, which have a higher coefficient of variation than many demand distributions observed in practice. Second, it is very important to consider uncertainty of the lead times, even if it is possible to do so only approximately or heuristically. Neither the newsvendor heuristic nor the mean demand heuristic is guaranteed to find optimal planned lead times for the original problem, but they, nevertheless, produce very good results because lead time uncertainty is considered explicitly. Finally, in all of our test problems, the descent procedure produced the optimal solution. Thus, although we were not able to prove joint convexity of the objective function with respect to all variables, the function appears to be sufficiently well behaved that standard procedures can be used with confidence.

The results underscore the importance of considering the effects of lead time uncertainty when multiple

Table 1 Parameters of the Six Lead Time Distribution Vectors

Vector.	r_1, p_1	r_2, p_2	r_3, p_3	r_4, p_4	r_5, p_5
1	9.0, 0.9	9.0, 0.9	9.0, 0.9	9.0, 0.9	9.0, 0.9
2	4.0, 0.4	4.0, 0.4	4.0, 0.4	4.0, 0.4	4.0, 0.4
3	28.0, 0.7	28.0, 0.7	28.0, 0.7	28.0, 0.7	28.0, 0.7
4	6.0, 0.15	6.0, 0.15	6.0, 0.15	6.0, 0.15	6.0, 0.15
5	4.0, 0.4	4.0, 0.4	4.0, 0.4	6.0, 0.15	6.0, 0.15
6	9.0, 0.9	4.0, 0.4	28.0, 0.7	6.0, 0.15	6.0, 0.15

Table 2 Evaluation of Heuristic Solutions: Percent Deviation from Optimality

Heuristic	Average	Std Dev	Max
Newsvendor	0.0011	0.0047	0.0407
Mean LT	71.18	81.66	338.67
Mean Demand	1.75	1.33	3.80
Descent	0	0	0

components need to be assembled, even in a single-period framework. Further research is needed to address the multi-period problem. In the multi-period problem, there is some opportunity for risk-pooling across time, particularly when orders can cross (which could occur if multiple suppliers are used for the same component), but also when orders do not cross. Moreover, unless transaction costs are high, it may even be optimal to place more component orders than there are distinct customer orders (analogous to order-splitting). Some work has been done on single-item (nonassembly) periodic review models with random lead times (e.g., Kaplan 1970, Liberatore 1977, Nahmias 1979, Ehrhardt 1984, Anupindi et al. 1996, and Song and Zipkin 1996), and work has been done on assembly systems with random demand and deterministic lead-times (see Clark and Scarf 1960, Schmidt and Nahmias 1985, Graves 1988, Yano and Carlson 1988 and Rosling 1989 and references therein). Various approaches and results in these papers may be useful in extending our model to multiple periods. We note, however, that these models do not incorporate the concept of a custom order that must be shipped as a batch. Instead, each unit of demand in a given period can be treated independently and satisfied in different periods. Thus, extending these results may pose considerable modeling and technical challenges.

The value of reducing lead time variability has been investigated by Gerchak and Parlar (1991) and Paknejad et al. (1992) for single-item inventory models. We expect the benefits of reducing lead time variability to be even greater in assembly systems, and research is needed to understand how to efficiently allocate improvement efforts.

Research is also needed to consider multiple customer orders with shared components. In these situations, questions also arise as to how to allocate the available inventory, further complicating the problem. Opportunities also exist to generalize all of the aforementioned problems to accommodate information updates, with perfect or imperfect information, and to more accurately quantify the value of information and expediting in such environments. We hope that our research motivates further work in these directions.[1]

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