Management Science

Publication details, including instructions for authors and subscription information:
http://pubsonline.informs.org

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MANAGEMENT SCIENCE
Vol. 60, No. 2, February 2014, pp. 508–520
ISSN 0025-1909 (print) | ISSN 1526-5501 (online)
http://dx.doi.org/10.1287/mnsc.2013.1779

Financial Product Differentiation over the State Space in the Mutual Fund Industry

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By distancing themselves from others in risk factor loadings, mutual funds yield distinct returns and become better-performing funds in different market situations. This enables mutual funds to obtain stochastic market power and charge higher fees than they could otherwise. This strategy fundamentally differs from the conventional market segmentation strategy that targets investors with heterogeneous preferences. We present a model to study this novel form of financial product differentiation over the states of nature. Empirically, we find that the return attributable to risk factor loadings has a significant impact on a fund's market share. Fund fees are related to the positions of their factor loadings in the industry and funds with more extreme risk factor loadings charge higher fees.

Keywords: mutual fund styles; product differentiation; market share; fee; stochastic
History: Received March 14, 2012; accepted May 20, 2013, by Wei Jiang, finance. Published online in *Articles in Advance* October 24, 2013.

1. Introduction

Whether mutual fund fees are too high has long been a controversial topic among researchers, the media, and policy makers. Given the evidence that active managers cannot outperform passively managed portfolios in most asset classes, the answer tends to be that high fund fees for actively managed portfolios are unjustified (e.g., Jensen 1968, Gruber 1996, Carhart 1997). According to the annual statistics reported by the Investment Company Institute, in 2012 more than 7,500 mutual funds competed in the market. Yet mutual fund profits seem robust and their rents are not dissipated despite the enormous number of competitors in the industry. It is even more puzzling given the "smart money" effect documented by Zheng (1999) that investors are able to select funds by moving away from the poor performers and toward the good performers. One might expect that "performance chasing" by fund investors would force funds to lower their fees in order to improve performance and attract cash flow.

This paper proposes a novel explanation on how mutual funds are able to sustain economic rent when investors are chasing past fund performance. The idea is that, if investors invest in funds with good past performance, funds can avoid head-to-head competition by strategically holding portfolios with different risk factor loadings. Depending on the stochastic outcomes of risk factors in different states, some funds will appear to be winning funds, whereas others will be losing funds. Because winning funds attract cash inflows and gain market powers, mutual funds with different risk factor loadings will rotate market powers in different states of nature. The benefit of such a "rotation," as we will demonstrate, is that the stochastic market power makes it possible that funds charge higher fees even *before* the realization of the states of nature. This is an important finding because mutual fund fees are generally stable in practice. Our result shows that funds do not need to adjust their fees in response to the variation in their performance and still are able to extract economic rent. If we view a fund's portfolio as a "product" in the mutual fund industry and its return as a measure of the quality, the difference in risk factor loadings allows mutual funds to differentiate the quality of their products over the states of nature, even if ex ante they have the same likelihood of being the winning fund.

The logic of such product differentiation is not confined to the mutual fund industry. A similar example of mutual fund differentiation through the state space is the professional economic forecaster industry analyzed in Laster et al. (1999). Like mutual fund managers, macroeconomic forecasters who make accurate forecasts in the previous period have attracted great attention from media, gaining clients and market share. However, when they have common information about future economy, forecasters will not be able to differentiate themselves if they all make the mean ("consensus") prediction based on the common information. Therefore, to maximize their expected market shares, forecasters will distance themselves from others by deliberately biasing their predictions from the consensus prediction, resulting in a greater dispersion of forecasts. Depending on the realized outcome of macroeconomy, forecasters will alternately become the "super stars" and enjoy large market share. Laster et al. (1999) term such a bias in analyst forecasts as a "rational bias" because it is a rational response to the incentive provided by the market demand function.

To demonstrate the fee effect of product differentiation through the state space, we develop a model considering the strategic competition between two mutual funds and the demand for a mutual fund being a function of its past performance. We show that the expected price elasticity of demand is lower when fund returns are stochastically differentiated in different states of nature. Intuitively, if investors chase funds with good past returns, a fund will gain market power in its winning states. However, the degree of market power that it gains depends on the number of winning funds. If funds differentiate in their risk factor loadings, they will alternate their winning states, resulting in one winning fund in each state to enjoy the market power. Such a rotation of being the winner allows funds on average to enjoy greater market powers and charge higher fees. We provide some empirical evidence conforming to the theoretical analysis. We show that a fund's returns attributable to its risk factor loadings have a significantly positive impact on its market share, suggesting that investors chase funds with good past returns even if they are due to risk factor loadings. We also find that the deviation of a fund's factor loadings from the industry medians is significantly and positively associated with its fee, consistent with our analytical result that greater differentiation in risk factor loadings allows funds to charge higher fees.

Our study is part of the increasing literature on the strategic interaction in the financial industry. Several papers discuss fee setting in advisory contracts in the mutual fund industry from a principal–agent perspective (e.g., Golec 1992, Tufano and Sevick 1997, Deli 2002). However, considering the large number of investors and mutual funds, it is very costly to write and enforce any contract. Instead, the cost is low to transfer money from one fund to another. In essence, open-end funds provide a strong form of "voting with the feet." It is reasonable to believe that market competition is the most important means of disciplining fund managers. Wahal and Wang (2011) conclude that the mutual fund market has evolved into one that displays the hallmark of a competitive market after showing that incumbent funds that have a higher overlap in their portfolio holdings with entrants tend to reduce management fees. Hortaçsu and Syverson (2004) develop a structural model to show that search costs prevent the index fund offering the highest utility to capture the whole market, permitting seemingly homogeneous S&P 500 index funds to charge diffuse fees. Massa (2000) argues that brand proliferation in the mutual fund industry is the marketing strategy that management companies use to exploit investors' heterogeneity and the reputation of the top-performing fund. A recent paper by Carlin (2009) argues that price complexity is an important determinant of price formation in retail financial markets. Carlin (2009) shows that price complexity in an industry could be endogeneously determined through strategic interaction among firms and, in equilibrium, all firms enjoy a positive rent from having some degree of price complexity. Our study extends the literature and proposes a new explanation on how mutual funds, even if ex ante they are identical, can generate economic rents. The product differentiation we propose is fundamentally different from the conventional production differentiation. In our model, mutual funds achieve a stochastic product differentiation with the product quality (i.e., performance) differentiated in different states of nature. We call this type of product differentiation "financial product differentiation over the states of nature." To our knowledge, this special form of product differentiation and its effect on price competition have not been identified and analyzed in either the economics or finance literature.

The remainder of this paper is organized as follows. Section 2 constructs the model to analyze financial product differentiation and its implications. Section 3 presents empirical evidence on the relationship among market shares, fees, and risk factor loadings in the mutual fund industry. Section 4 concludes.

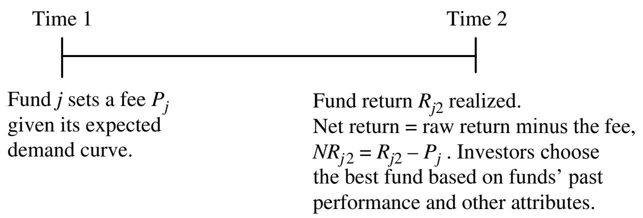
2. Theoretical Analysis

2.1. Setup

We consider two funds that compete for market shares. Their decision process is a two-stage game as shown in Figure 1. In the first stage (time 1), both funds set fee levels given their expected demand functions; in the second stage (time 2), returns of mutual funds are realized and investors invest their money in the best mutual fund based on a fund's realized performance and other fund attributes.

We solve the model in the reverse direction from the decision process. We first study investors' demand function at time 2. Then we show, at time 1, how funds decide optimal fees given the expected demand functions at time 2.

Figure 1 Timing and Nature of the Decision Process



2.1.1. Mutual Fund Return. The raw (before fee) return of mutual fund j is modeled as being generated by a factor return-generating process of the following form:

$$R_{j2} = \alpha_j + \beta_{jM} F_{M2} + \varepsilon_{j2}, \tag{1}$$

where R_{j2} is mutual fund j's excess raw return over risk-free return at time 2, β_{jM} is mutual fund j's loading on the factor, F_{M2} is the excess return of risk factor over risk-free return at time 2 with mean zero and variance σ_M^2,[1] α_j is manager j's ability after adjusted for a fund's exposure to the risk factor, and ε_{j2} is the idiosyncratic risk of mutual fund j's portfolio with mean zero and variance $\sigma_{\varepsilon_j}^2$. The net return of mutual fund j, NR_{j2}, at time 2 is equal to the raw return minus the fee, $NR_{j2} = R_{j2} - P_j$.

2.1.2. Investor Preference. When investors are faced with the decision of choosing a mutual fund, they evaluate a fund by its fee, returns, and other attributes. To capture investors' preference over different aspects of mutual funds, investor i's subjective utility function of choosing fund j at time 2 is defined as

$$U_{ij2} = Perf_j(\mathbf{\Phi}, P_j, R_{j2}) + \mathbf{X}'_j \mathbf{B} + \xi_j + e_{ij2}, \tag{2}$$

where U_{ij2} is the utility for investor i if she chooses fund j at time 2; $Perf_j(\mathbf{\Phi}, P_j, R_{j2})$ is an evaluation function constructed by investors to evaluate fund j's performance; $\mathbf{\Phi}$ is a vector of parameters in investors' performance evaluation function; $\mathbf{X}_j$ is an L-dimensional column vector where the kth row is mutual fund j's attribute k (e.g., fund age); $\mathbf{B}$ is an L-dimensional vector where the kth row measures investors' sensitivity to a fund's attribute k; ξ_j is a scalar, which denotes the quality of other service attributes of fund j that is observable to investors and funds but not to econometricians; and the term e_{ij2} captures investor i's idiosyncratic and unobservable taste for fund j. As in the classic logit demand model (e.g., Berry 1994, Berry et al. 1995), it is assumed to follow a type I extreme distribution.

[1] Assuming that the mean of excess return of risk factor equals zero is for the convenience of illustration. All conclusions do not change if the mean is positive.

Empirical evidence from household brokerage accounts (Barber et al. 2005) or mutual fund flows (e.g., Ippolito 1992, Gruber 1996, Chevalier and Ellison 1997, Brown et al. 1996) shows that investment decisions of retail mutual fund investors are affected by mutual funds' total returns, even after controlling for risk-adjusted returns. These results suggest that investors chase both risk-adjusted returns and the returns that come from risk factor loadings.[2] To accommodate the differential impacts of the risk-adjusted return and the returns attributed to risk factor loadings on investors' choices of funds, the performance evaluation function is specified as the following:

$$Perf_j(\mathbf{\Phi}, P_j, R_{j2}) = \phi_0 \alpha_j - b_0 P_j + \phi_1 \beta_{jM} F_{M2} - \tfrac{1}{2}\phi_2 \operatorname{Var}(R_{j2}), \tag{3}$$

where ϕ_0 measures investors' sensitivity to risk-adjusted return α_j; b_0 measures investors' sensitivity to fee; ϕ_1 is the sensitivity to a fund's return attributable to its risk factor loading $\beta_{jM} F_{M2}$; and ϕ_2 reflects investors' risk aversion to the volatility of mutual fund returns, $\operatorname{Var}(R_{j2})$. This performance evaluation equation is quite general. If $\phi_0 > 0$ and $\phi_1 = 0$, investors are sophisticated and evaluate fund past performance using only the risk-adjusted return α_j. In other words, investors know how to "purge" the performance of a fund that is attributable to its risk factor loading. If $\phi_1 \neq 0$, investors respond not only to a fund's past risk-adjusted return but also to its return attributable to the factor loading.

2.2. The Ex Post Market Share

We first derive the ex post demand function s_{j2} for each fund j at time 2 when the risk factor return is realized and $F_{M2} = f_{M2}$ (i.e., the state of nature is revealed). Note that we use lowercase letters s_{j2} and f_{M2} to denote realized values at time 2 and capital letters S_{j2} and F_{M2} to denote random variables. At time 2, after observing the realized returns of all funds, each investor allocates all her mutual fund investment to the fund that provides her with the highest utility.[3] The optimization problem of investor i is

$$\max_j U_{ij2} \quad \Leftrightarrow \quad \max_j \big(\phi_0 \alpha_j - b_0 P_j + \phi_1 \beta_{jM} f_{M2} - \tfrac{1}{2}\phi_2 \operatorname{Var}(R_{j2}) + \mathbf{X}'_j \mathbf{B} + \xi_j + e_{ij2}\big).$$

[2] Barber et al. (2005) argue that the representative heuristic leads investors to believe that recent performance is overly representative of a fund's future prospects and to chase past winners.

[3] The assumption that investors invest in only one fund that provides the highest utility allows us to focus on investors' return-chasing behavior rather than the diversification purpose. Barber et al. (2005) show that the median household holds two mutual funds, including bond funds, international equity funds, and specialized sector funds. Because our model studies the demand for active equity funds, it is reasonable to assume that an investor chooses one active equity fund.

Following the standard approach in empirical industrial organization literature (e.g., Berry 1994), we can integrate over individual investors' idiosyncratic tastes e_{ij2} to derive the total demand for each fund at time 2. That is, the market share for fund j s_{j2} at time 2 is equal to

$$s_{j2} = \frac{\exp(A_j - \frac{1}{2}\phi_2(\beta_{jM} - \beta^*_{M2})^2\sigma^2_M)}{\sum_{l=1}^2 \exp(A_l - \frac{1}{2}\phi_2(\beta_{lM} - \beta^*_{M2})^2\sigma^2_M)}, \quad j = 1, 2, \quad (4)$$

where

$$A_j = \phi_0\alpha_j - b_0 P_j - \frac{1}{2}\phi_2\sigma^2_{\varepsilon_j} + \mathbf{X}'_j\mathbf{B} + \xi_j, \quad \beta^*_{M2} = \frac{\phi_1 f_{M2}}{\phi_2\sigma^2_M}.$$

Proof. See Appendix A.1 for the proof of Equation 4.

All variables, except for A_j and β^*_{M2}, were defined earlier. A_j captures the total impact of a fund's attributes, excluding returns attributable to the risk factor loading, on its market share. β^*_{M2} needs a detailed explanation. In the proof, we show that β^*_{M2} is the ex post optimal factor loading at time 2 that provides investors with the highest evaluation of a fund's performance. β^*_{M2} is a function of the realized risk factor return f_{M2} whose value is state dependent.[4] Intuitively, if the realized risk factor return turns out be positive, it would be optimal for a fund to have positive risk factor loading. In contrast, if the realized risk factor return is negative, it is optimal for a fund to have a negative risk factor loading. β^*_{M2}, however, is unknown at time 1 because the realized value of risk factor return f_{M2} is unknown at time 1.[5]

The term $\beta_{jM} - \beta^*_{M2}$ measures the distance between fund j's risk factor loading, β_{jM}, chosen at time 1, and the ex post optimal risk factor loading β^*_{M2}, known at time 2. The smaller the distance, the closer is the risk factor loading chosen by a fund to be the optimal at time 2, resulting in a better return and a higher market share at time 2. Therefore, mutual funds with different risk factor loadings at time 1 will yield distinct returns at time 2 depending on distances between their risk factor loadings β_{jM} and the realized ex post optimal risk factor loading β^*_{M2}.

In the market share Equation (4), we can also see that mutual funds factor loadings β_{jM} will not affect their ex post market shares if either investors do not respond to the return attributable to risk factors loadings, i.e., $\phi_1 = 0$, or mutual funds' factor loadings β_{jM} are all equal, $\beta_{1M} = \beta_{2M}$. If $\phi_1 > 0$, there are opportunities for fund managers to differentiate their products by holding different factor loadings. Mutual funds with different risk factor loadings β_{jM}s will become better-performing funds alternately in different states as their distances with the ex post optimal risk factor loading β^*_{M2} vary with the value of risk factor return F_{M2}. Therefore, funds differentiate their performance across different states of nature. We call this special type of product differentiation "financial product differentiation over the states of nature." We would emphasize that mutual funds can horizontally differentiate their products even though investors are homogeneous. In this sense, financial product differentiation over states of nature differs from the conventional differentiation in a critical way.

2.3. Price Elasticity and Fee Setting

To isolate and demonstrate the effect of the differentiation through risk factor loadings, we consider two funds that are identical in other attributes: $A_j = A$, $j = 1, 2$. Because mutual funds set fees at time 1 before the realization of risk factor returns (i.e., the states of nature), we can derive funds' expected market shares S^e_j conditional on the information at time 1 by integrating all possible states in time 2:

$$S^e_j = E_1(S_{j2}) = \int \frac{\exp(A - \frac{1}{2}\phi_2(\beta_{jM} - \beta^*_{M2})^2\sigma^2_M)}{\sum_{l=1}^2 \exp(A - \frac{1}{2}\phi_2(\beta_{lM} - \beta^*_{M2})^2\sigma^2_M)}\, dF(f_{M2}), \quad (5)$$

where E_1 is the expectation operator conditional on the information at time 1, and $F(f_{M2})$ is the cumulative distribution function of the risk factor return F_{M2}.

If mutual funds do not engage in financial product differentiation through risk factor loadings and hold the same betas, $\beta_{1M} = \beta_{2M} = \beta_M$, it can be easily seen that, in this case, the expected market share of mutual fund j $\bar{S}_j$ at time 2 is constant and equal to

$$\bar{S}_j = E_1(S_{j2}) = \int \frac{\exp(A - \frac{1}{2}\phi_2(\beta_M - \beta^*_{M2})^2\sigma^2_M)}{2\exp(A - \frac{1}{2}\phi_2(\beta_M - \beta^*_{M2})^2\sigma^2_M)}\, dF(f_M) = \int \frac{1}{2}\, dF(f_M) = \frac{1}{2}. \quad (6)$$

This result is not surprising. Two funds that are identical in all aspects are expected to share the market. However, the following proposition shows that, by holding opposite risk factor loadings, mutual funds could have the same market shares as those if they hold the same risk factor loadings.

Proposition 1. *If two mutual funds differentiate themselves by holding opposite risk factor loadings $\beta_{1M} = -\beta_{2M}$, their expected market shares are the same as those if they hold the same risk factor loadings, i.e., $S^e_j = \bar{S}_j$, $j = 1, 2$.*

[4] Note that $\mathbf{\Phi}$ and σ^2_M are not state dependent.

[5] At time 1, given that F_{M2} has a distribution of mean zero and a variance σ^2_M, β^*_{M2} has a distribution with the mean equal to zero and the variance equal to $\mathrm{Var}(\beta^*_{M2}) = \mathrm{Var}((\phi_1 F_{M2})/(\phi_2\sigma^2_M)) = (\phi_1/\phi_2 \cdot 1/\sigma_M)^2$.

Proof. See Appendix A.2.

The above result indicates that by holding opposite risk factor loadings, both funds have a 50% chance of being the winning fund, but their expected market shares remain the same. The benefit of the differentiation, however, is that both funds' expected price elasticity of demand is reduced even though their expected market shares remain unchanged. This could be seen by comparing the expected price elasticity of demand with the risk-factor-loading differentiation versus the one without differentiation.

The expected price elasticity of demand for fund j at time 1 with the risk-factor-loading differentiation is

$$|\eta_j^e| = \left|\frac{\partial S_j^e/S_j^e}{\partial P_j/P_j}\right| = \left|\int -\frac{s_{j2}}{S_j^e} b_0 P_j (1-s_{j2})\, dF(f_{M2})\right| = \frac{b_0 P_j}{S_j^e}\int s_{j2}(1-s_{j2})\, dF(f_{M2}). \tag{7}$$

The expected price elasticity of demand for fund j at time 1 without the differentiation is

$$|\bar{\eta}_j| = \left|\frac{\partial \bar{S}_j/\bar{S}_j}{\partial P_j/P_j}\right| = \left|\int -\frac{s_{j2}}{\bar{S}_j} b_0 P_j (1-s_{j2})\, dF(f_{M2})\right| = b_0 P_j (1-\bar{S}_j) = \frac{1}{2} b_0 P_j. \tag{8}$$

The impact of the product differentiation over the state space on the expected price elasticity of demand is then shown in the following proposition.

Proposition 2. *The expected price elasticity of demand becomes lower if two funds engage in financial product differentiation over the states of nature by holding different risk factor loadings, i.e.,* $|\eta_j^e| < |\bar{\eta}_j|$.

Proof.

$$|\eta_j^e| = \left|-b_0 \frac{P_j}{S_j^e}\int s_{j2}(1-s_{j2})\, dF(f_{M2})\right| = |b_0 P_j(1-S_j^e - \mathrm{Var}(S_{j2})/S_j^e)| < b_0 P_j (1-S_j^e) = b_0 P_j(1-\bar{S}_j) = |\bar{\eta}_j|. \tag{9}$$

To better understand the mechanism behind the proof, we can rewrite the expected price elasticity when funds differentiate their risk factor loading η_{j2}^e into

$$|\eta_j^e| = \int \frac{s_{j2}}{S_j^e}[b_0 P_j (1-s_{j2})]\, dF(f_{M2}) = \int w_{j2} |\eta_{j2}|\, dF(f_{M2}), \tag{10}$$

where $|\eta_{j2}| = b_0 P_j (1 - s_{j2}) = |(\partial s_{j2}/s_{j2})/(\partial P_j/P_j)|$ is fund j's price elasticity of demand in the state when its market share is equal to s_{j2} at time 2, and $w_{j2} = s_{j2}/S_j^e$ is the weight associated with the state and is equal to fund j's market share in that state divided by its expected market share.

Similarly, the expected price elasticity of demand if funds do not differentiate their risk factor loading $\bar{\eta}_j$ can be written as

$$|\bar{\eta}_j| = \int \frac{s_{j2}}{\bar{S}_j} b_0 P_j (1-s_{j2})\, dF(f_{M2}) = \int b_0 P_j (1-\bar{S}_j)\, dF(f_{M2}) = \int |\bar{\eta}_j|\, dF(f_{M2}). \tag{11}$$

A comparison of Equations (10) and (11) reveals the reason that $|\eta_j^e| = \int w_{j2}|\eta_{j2}|\, dF(f_{M2}) < \int |\bar{\eta}_j|\, dF(f_{M2}) = \bar{\eta}_j$ is the following. If two funds hold different risk factor loadings, they differentiate states when they will become the winning fund and reduce the number of winning funds in each state. Consequently, in the state when a fund becomes the winning fund, it enjoys a higher market share s_{j2} and greater market power (i.e., lower price elasticity $|\eta_{j2}| = b_0 P(1-s_{j2})$. Therefore, the low price elasticity in the winning state is associated with a greater weight $w_{j2} = s_{j2}/S_j^e$. Conversely, in the state when a fund becomes the losing fund, it will incur a smaller market share s_{j2} and lower market power (i.e., high price elasticity $|\eta_{j2}|$). Hence, the higher price elasticity in a losing state is associated with a lower weight. A higher weight associated with a lower price elasticity, together with a lower weight associated with a higher price elasticity, reduces the expected price elasticity of demand. In essence, product differentiation over the state space "changes" the probability measure of expected price elasticity, resulting in lower expected price elasticity.[6]

To summarize, the differentiation in risk factor loadings allows two funds to avoid state-to-state competition and enjoys a larger market share and lower price elasticity of being the winner fund. Indeed, the price elasticity of demand could be further reduced if both funds engage in a greater degree of financial product differentiation. That is, if fund 1 increases its beta to $\beta_M^{1'} > \beta_M^1$ and fund 2 decreases its beta to $\beta_M^{2'} < \beta_M^2$, the new price elasticity $|\eta_j^{e'}|$ will be even smaller.

Proposition 3. *If two funds engage in a greater degree of product differentiation by holding risk factor loadings that are more disperse, their demand curves become even less price elastic, i.e.,* $|\eta_j^{e'}| < |\eta_j^e|$.

Proof. See Appendix A.3.

The key consequence of product differentiation over the states of nature is to change the slope of the

[6] To see this, $\int w_{j2}\, dF(f_{M2}) = \int (s_{j2}/S_j^e)\, dF(f_{M2}) = (1/S_j^e)\int s_{j2}\, dF(f_{M2}) = S_j^e/S_j^e = 1$. w_{j2} is a radon nikodym derivative in the change of probability measure.

expected demand curve. This will affect the fee setting by mutual funds who decide on the optimal fee level at time 1. Specifically, fund j chooses the fee to maximize profit given its expected market share function.[7] That is,

$$\max_{P_j} E_1[\Pi_{j2}] = E_1[TA \times S_{j2}(P_j)(P_j - MC_j) - FC_j]$$
$$= TA \times S_j^e(P_j)(P_j - MC_j) - FC_j, \qquad (12)$$

where Π_{j2} is the profit of mutual fund j at time 2; TA is the total asset size of the mutual fund industry; $S_{j2}(P_j)$ is the market share of mutual fund j at time 2 conditional on its fee, P_j; and MC_j and FC_j are the marginal and fixed cost of production for mutual fund j, respectively.

The pure-strategy Nash equilibrium in prices satisfies the following first-order conditions:

$$S_j^e(p_j) + (P_j - MC_j)\frac{\partial S_j^e(P_j)}{\partial P_j} = 0. \qquad (13)$$

The fee charged by mutual fund j in equilibrium at time 1 if funds differentiate in their risk factor loadings is thus equal to

$$P_j^{\text{diff}} = MC_j \frac{|\eta_j^e|}{|\eta_j^e| - 1}. \qquad (14)$$

Similarly, if funds hold the same risk factor loadings, the equilibrium price charged by mutual fund j is equal to

$$P_j^{\text{no diff}} = MC_j \frac{|\bar{\eta}_j|}{|\bar{\eta}_j| - 1}.$$

$P_j^{\text{diff}} > P_j^{\text{no diff}}$ given $|\eta_j^e| < |\bar{\eta}_j|$. That is, both funds charge higher fees if they differentiate in their risk factor loadings. The above result follows naturally from Proposition 2 that the differentiation in risk factor loadings lowers a fund's expected price elasticity of demand. Intuitively, the rotation of being the winning fund in different states results in fewer funds in each state to share the market power of being the winner. If a fund charges a higher fee at time 1, the profit it gains in winning states is large because its market share is large and the price elasticity is low. The profit it loses in losing states is small because its market share is low even though the price elasticity is high. On average, the gain in winning states from a higher price outweighs the loss in losing states. Therefore, funds are able to charge a higher price if they engage in product differentiation over the states of nature.

We have demonstrated in Proposition 3 that if both funds engage in a greater degree of financial product differentiation by holding factor loadings that are further apart, the price elasticities of their demand curves become lower. Equation (14) then leads to the following proposition:

Proposition 4. *Two funds will charge higher fees when their risk factor loadings are further away from others.*

It is worth noting that higher fees are set by both funds based on the expected market shares before the realization of the states of nature in Nash equilibrium. This is important because they do not need to adjust their fees according to the realized states of nature in order to capture the benefit of such a product differentiation. By maintaining different risk factor loadings and charging a higher fee, both funds can on average enjoy higher profit margins, although their ex post market shares do vary with the states of nature.

2.4. Discussion

Our model takes the differentiation in factor loading as given and investigates its impact on the price elasticity. In reality, thousands of funds differentiate their products in various dimensions (e.g., size, location and reputation) besides risk factor loadings, which make it difficult to know the equilibrium distribution of mutual funds' risk factor loadings. Nonetheless, our analysis has implications for the potential distribution of mutual funds' risk factor loadings. A fund's ex post optimal risk factor loading depends on risk factor return. Ex ante, mutual funds do not know the optimal risk factor loading because risk factor return is unknown, but they know that the distribution of optimal risk factor loading is determined by the distribution of risk factor return. If the distribution of risk factor return is bell shaped, the distribution of optimal risk factor loading is also bell shaped.[8] This means that if a mutual fund chooses the mode of the distribution, it has the highest probability of having the optimal risk factor loading and

[7] That funds are risk neutral and care about their expected profits is assumed in theoretical literature (e.g., Carlin 2009, Guerrieri and Kondor 2012). If funds are risk averse, the greater variation in profits resulting from the product differentiation over the states of nature might lower their utilities and weaken incentives to engage in such a differentiation. Nevertheless, empirical literature on strategic risk-taking behavior in mutual fund tournaments shows that funds are willing to take significantly higher risk to increase their expected profit (e.g., Brown et al. 1996, Chevalier and Ellison 1997, Elton et al. 2010, Schwarz 2012). For example, Brown et al. (1996) show that the midyear losing funds increase their funds' volatility in the latter part of the year more than did the midyear winning funds. These results suggest that the primary focus in a fund's objective is its expected profit.

[8] Indeed, the empirical distributions of monthly returns market factor, size factor, value factor, and momentum factor are all bell shaped. Moreover, the empirical distributions of mutual funds risk factor loadings on these risk factors are also bell shaped.

becoming the winning fund ex post. However, if all funds choose the same loading, they will all become winning funds in the same state ex post. The competition among winning funds will be high, resulting in lower market shares and higher price elasticities. Therefore, some funds will have incentives to deviate from the mode because there will be fewer winning funds to compete with if their risk factor loadings turn out to be the optimal. The cost of deviating from the mode is that the probability of becoming winning funds diminishes. Therefore, in choosing its risk factor loading, a fund needs to consider both the probability of being a winning fund as well as the number of winning funds to compete with even if it wins. For a fund to choose an extreme risk factor loading that has a lower likelihood of being the winning fund, it should anticipate that few funds will choose the extreme risk factor loading so that it can enjoy a great market power in its winning state. Therefore, very few funds will choose extreme risk loadings factor loadings. Ultimately, the location of a mutual fund's risk factor loading is determined by the tradeoff between the benefit of being a winner fund with fewer competitors and the cost of having a lower likelihood of winning.

Mutual funds typically are members of a fund family. A fund's risk factor loadings could also be influenced by the strategic consideration at the level of fund family. Nanda et al. (2004) show that a star fund attracts greater cash inflow not only to the fund but also to other funds in the family. Such a "spillover" effect can give incentives to a fund family to increase its chance of having a star fund. Goetzmann and Ibbotson (1993) argue that fund family can maximize the probability of having a star fund by lowering the cross-fund return correlation within the family. With differentiation in risk factor loadings, product differentiation over the states of nature indeed lowers the correlation of returns across funds in the family and increases its chance of having a star fund in a certain state. It is conceivable that, compared to the standalone funds, the additional benefit of the spillover effect from having a star fund in a fund family would give funds in the family more incentives to engage in product differentiation over the state space.

Our analysis considers the product differentiation over the state space through the differentiation in returns attributed to risk factor loadings. Differentiation over the state space could also be achieved through the differentiation in funds' idiosyncratic returns. However, differentiation through idiosyncratic returns might not be as effective as differentiation through returns attributed to risk factor loadings. The average R-square of the four-risk-factor model in explaining mutual fund returns is around 90% (e.g., Carhart 1997), suggesting that return attributed to risk factor loading explains a majority of the variation in a fund's total return, and there is less room for funds to differentiate through idiosyncratic returns.

Finally, it is important to point out that product differentiation over the state space is not mutually exclusive with extant explanations on the proliferation of risk factor loadings. For example, a motive for mutual funds' differentiating in their risk factor loadings is to serve investors who have different needs. If investors cannot manufacture risk exposures for themselves, or they cannot do it as cost-effectively as mutual funds can, mutual funds can charge a fee for providing service to investors with heterogeneous needs. Differentiation in risk factor loadings could also be a strategy for mutual funds to create product complexity to increase investors' search costs that allow funds to sustain economic rents (e.g., Carlin 2009, Carlin and Manso 2011). The importance of product complexity has been investigated in the mortgage (Amromin et al. 2013), municipal bond (Green et al. 2010), and mutual fund markets (Sensoy 2009). Carlin and Gervais (2012) show how laws could be designed to protect uninformed individual investors in retail financial markets. Our theory identifies a new motive for mutual funds to differentiate their risk factor loadings, which is to reduce mutual funds' price elasticities through stochastic market power. It is possible that differentiation in risk factor loadings allows funds to achieve both a product differentiation that targets heterogeneous investors and a financial product differentiation over the state space.

3. Some Empirical Evidence

In this section, we provide some empirical relationships among market shares, fees, and risk factor loadings in the mutual fund industry. We use the Center for Research in Security Prices (CRSP) Survivor-Bias-Free Mutual Fund database for the information on fund returns, fund fees, and other fund characteristics. Because our focus is on the strategic competition by equity funds, we include only domestic, nonsector, nonindex equity funds. We also exclude funds that held fewer than 10 stocks and had less than \$5 million assets under management. Some mutual funds are multiclass funds that have the same manager and same portfolio composition, but with different fee structures (i.e., classes of shares). We aggregate all the observations pertaining to different share classes into one fund level observation.[9] The sample period is from 1991 to 2008. Our final sample includes 2,944 distinct funds with similar characteristics as those in Kacperczyk et al. (2008).

[9] We use a net asset value-weighted average to aggregate portfolio-level information into a fund-level observation.

Table 1 Distributions of Mutual Fund Risk Factor Loadings

Risk factor loadings	Mean	Median	SD	5th	95th	Skewness	Kurtosis
			Panel A: 1991–2008				
β_{MKTRF}	0.993	0.983	0.278	0.555	1.476	0.364	4.646
β_{SMB}	0.215	0.123	0.443	−0.380	1.038	0.603	2.855
β_{HML}	−0.013	0.000	0.475	−0.830	0.739	−0.216	3.916
β_{UMD}	0.041	0.029	0.291	−0.416	0.547	0.263	4.411
			Panel B: 1991–2000				
β_{MKTRF}	0.990	0.979	0.298	0.518	1.525	0.549	4.213
β_{SMB}	0.219	0.120	0.458	−0.380	1.108	0.671	2.911
β_{HML}	0.010	0.023	0.506	−0.877	0.850	−0.221	3.832
β_{UMD}	0.024	0.007	0.317	−0.477	0.591	0.313	3.939
			Panel C: 2001–2008				
β_{MKTRF}	0.995	0.985	0.263	0.582	1.444	0.380	4.996
β_{SMB}	0.213	0.125	0.432	−0.381	0.991	0.542	2.781
β_{HML}	−0.028	−0.016	0.452	−0.801	0.672	−0.238	3.922
β_{UMD}	0.054	0.042	0.270	−0.365	0.510	0.256	4.839

Notes. This table reports summary statistics of estimated four-factor loadings for the sample equity funds. A fund's annual risk factor loadings are estimated using Carhart's (1997) four-factor model.

We employ a four-factor model (Fama and French 1993, Carhart 1997) to capture the factor loadings of mutual funds. For each fund/year, we regress the monthly gross excess return of mutual fund j on monthly four-risk-factor returns as the following:

$$R_{js} = \alpha_j + \beta_{jMKTRF}MKTRF_s + \beta_{jSMB}SMB_s + \beta_{jHML}HML_s + \beta_{jUMD}UMD_s + e_{js}, \quad s = 1, 2, \ldots, 12, \tag{15}$$

where R_{js} is the raw return on the portfolio of fund j in excess of the one-month T-bill return in the month s; $MKTRF_s$ is the difference between the return on the CRSP value-weighted portfolio of all NYSE, Amex, and Nasdaq stocks and one-month T-bill yields in month s; and SMB_s, HML_s, and UMD_s are returns on factor-mimicking portfolios for size, book-to-market, and momentum factors, respectively.[10] To avoid extreme values that result from estimation error in the factor model (e.g., Berk 1995, Mamaysky et al. 2007, Shanken and Zhou 2007), we winsorize estimated risk factor loadings at the top and bottom 1%.

Table 1 presents the distributions of annual mutual fund factor loadings estimated from the four-factor model. One salient feature is that the distributions of all four-factor loadings are dispersed. For instance, β_{MKTRF} ranges from 0.555 to 1.476 between the 5th percentile and the 95th percentile, and β_{SMB} ranges from −0.380 to 1.038 between the 5th percentile and the 95th percentile. The skewnesses of the distributions of all factor loadings are close to zero, suggesting that the distributions of risk factor loadings are symmetric.

[10] The data is from Kenneth R. French's data library.

3.1. Determinants of Mutual Fund Market Shares

A critical condition for mutual funds to be able to achieve product differentiation over the states of nature is that returns attributable to risk factor loadings affect investors' demand for mutual funds. To test the validation of this condition, we estimate the impact of returns attributable to risk factor loadings on mutual fund market shares, i.e.,

$$\Delta \ln(s_{j,t}) = a + \phi_0 \alpha_{j,t-1} + \sum_{k=1}^{K} \phi_k (\beta_{jk} F_k)_{t-1} - b_0 P_{j,t-1} - \phi_{K+1} \mathrm{Var}(R_{j,t-1}) + \mathbf{X}'_{j,t-1}\mathbf{B} + \xi_{j,t}, \tag{16}$$

where $\Delta \ln(s_{j,t})$ is the change of the logarithm of fund j's market share between time $t-1$ and t. A fund's market share $s_{j,t}$ is defined as the total net asset value of fund j divided by the aggregated net asset value of all funds at time t. a is a constant, and $\alpha_{j,t-1}$ is fund j's risk-adjusted return at time $t-1$. The return attributable to the loading on the risk factor k, $\beta_{jk}F_k$, are calculated using the estimated annual factor loading $\hat{\beta}_{jk}$ times the annual risk factor return F_k. Specifically, the returns attributable to four risk factors at time $t-1$ are denoted as $(\hat{\beta}_{jMKTRF}MKTRF)_{t-1}$, $(\hat{\beta}_{jSMB}SMB)_{t-1}$, $(\hat{\beta}_{jHML}HML)_{t-1}$, and $(\hat{\beta}_{jUMD}UMD)_{t-1}$, respectively; $P_{j,t-1}$ is fund j's expense ratio at time $t-1$; $\mathrm{Var}(R_{j,t-1})$ is the volatility of a fund's return at time $t-1$; $\mathbf{X}$ includes fund characteristics such as ln(fund age) and turnover ratio; and $\xi_{j,t}$ is the error term.

Table 2 reports the results on the determinants of mutual fund market shares. Column (1) includes only alpha, the returns attributable to factor loadings, and year dummies as explanatory variables. The significance of variables is tested based on the heteroskedasticity robust t-statistics adjusted for clustering within funds. The result shows that the change of mutual fund's market share is significantly affected by both its alpha and returns attributable to factor loadings. The coefficients on alpha and four-factor returns are all positive and significant at 1% level. The results support the notion that investors chase funds with past high returns that could be due to either better alphas or higher returns attributed to risk factor loadings.

To further distinguish the effect of returns attributed to risk factor loadings $\beta_{jk}F_k$ from the effect of risk factor loadings β_{jk} itself on mutual fund market shares, column (2) adds risk factor loadings $\hat{\beta}_{jMKTRF,t-1}$, $\hat{\beta}_{jSMB,t-1}$, $\hat{\beta}_{jHML,t-1}$, and $\hat{\beta}_{jUMD,t-1}$ as explanatory variables for the change of a mutual fund's market share. The results show that including risk factor loadings has virtually no impact on

Table 2 Determinants of Mutual Fund Market Shares

	(1)	(2)	(3)
$Alpha_{jt-1}$	0.798***	0.949***	0.930***
	(14.19)	(13.88)	(18.28)
$(\beta_{MKTRF}MKTRF)_{jt-1}$	0.174***	0.176***	0.149***
	(4.85)	(4.95)	(3.84)
$(\beta_{SMB}SMB)_{jt-1}$	0.497***	0.428***	0.387***
	(7.14)	(6.23)	(5.31)
$(\beta_{HML}HML)_{jt-1}$	0.506**	0.499***	0.485***
	(12.18)	(12.22)	(11.69)
$(\beta_{UMD}UMD)_{jt-1}$	0.396***	0.261***	0.230***
	(6.70)	(4.41)	(3.64)
$\beta_{MKTRF,t-1}$		−0.028**	0.005
		(−2.29)	(0.38)
$\beta_{SMB,t-1}$		0.002	0.014
		(0.29)	(1.59)
$\beta_{HML,t-1}$		0.098***	0.089***
		(12.09)	(10.35)
$\beta_{UMD,t-1}$		0.081***	0.088***
		(6.09)	(6.52)
Fee_{jt-1}			−2.412***
			(−3.09)
$\mathrm{Var}(R_j)_{t-1}$			−0.668***
			(−3.85)
$Turnover\ ratio_{jt-1}$			−0.001
			(−0.11)
$\mathrm{Ln}(age)_{jt-1}$			−0.048***
			(−14.48)
Constant	−0.219***	−0.028	0.144
	(−9.34)	(−1.19)	(1.24)
Year dummies	Yes	Yes	Yes
N	21,372	21,372	20,364
Adj. R^2	0.07	0.07	0.10

** and *** denote significance at the 5% and 1% levels, respectively.

the estimated coefficients on returns attributed to risk factor loadings. This is not surprising; a fund's risk factor loadings β_{jk} generally are stable and persistent. The variation in returns attributed to risk factor loadings $\beta_{jk}F_k$ largely comes from the variation in risk factor returns F_k. The identification of the effect of returns attributed to risk factor loadings $\beta_{jk}F_k$ on mutual fund market shares comes from the time-series variation in risk factor returns F_k instead of the variation in risk factor loadings β_{jk}. This is consistent with our analysis that the differentiation in risk factor loadings generates stochastic market power through the randomness of risk factor returns, which in turn affects a fund's market share.

Column (3) controls for other fund attributes such as expense ratio, ln(fund age), volatility of return, and turnover ratio. The result indicates that higher expense ratio, older age, higher volatility, and greater turnover negatively affect a fund's market share. The inclusion of those fund attributes again has very little impact on the coefficients of returns attributed to risk factor loadings. Given that the dependent variable is a change of logarithm function of market share, the coefficient of 0.930 on alpha indicates that a 1% increase in a fund's annual risk-adjusted return will lead to about 0.930% percentage increase in the growth rate of a fund's market share. The coefficients on four-factor returns, $\hat{\beta}_{jMKTRF}MKTRF$, $\hat{\beta}_{jSMB}SMB$, $\hat{\beta}_{jHML}HML$, and $\hat{\beta}_{jUMD}UMD$, are also economically significant and equal to 0.149, 0.387, 0.485, and 0.230, respectively. The coefficients on factor loadings $\hat{\beta}_{jMKTRF}$ and $\hat{\beta}_{jSMB}$ are insignificant, suggesting that there is no significant difference in the growth of market share between funds with different loadings on market factor and size factor. However, $\hat{\beta}_{jHML}$ and $\hat{\beta}_{jUMD}$ are significantly positive, suggesting that funds with greater loading on value and momentum factors have a higher growth rate during our sample period. Interestingly, the coefficient on expense ratio is equal to −2.412, whose absolute value is greater than the one on the raw alpha. It suggests that investors are more sensitive to the change in expense ratio than to the change in a fund's alpha. This may be explained by the fact that the expense ratio generally is stable and an increase in expense ratio could be long-lived, whereas an increase of fund performance in alpha tends to be temporary.

To summarize, these results indicate that returns attributable to risk factor loadings significantly affect a mutual fund's market share after controlling for risk-adjusted returns, risk factor loadings, and other fund attributes. This could be because either that some investors are not sophisticated enough to know how to use risk-adjusted returns in evaluating mutual fund performance, or that they know how to but prefer to evaluate fund performance by the total return. Regardless of the reason, as long as investors are responsive to a fund's return attributable to risk factors loadings, mutual funds can load risk factors differently and achieve the product differentiation over the state space.

3.2. Risk Factor Loadings and Fees

Our theoretical analysis suggests that mutual funds with risk factor loadings that deviate further from the median will charge higher fees because they face less-elastic expected demand curves. To test this implication, for each year, we calculate the deviation by subtracting the industry median risk factor loading from a fund's risk factor loading. Since a negative deviation is the same as a positive deviation in terms of its impact on a fund's fee setting, we take the absolute value of the deviation. We add up four deviations for each risk factor loading to measure total deviation of risk factor loadings (*TDRFL*) for fund j in year t. That is,

$$\begin{aligned} TDRFL^{Industry}_{jt} &= |\beta_{MKTRF,\,jt} - Industry\ Median\ \beta_{MKTRF,\,t}| \\ &\quad + |\beta_{SMB,\,jt} - Industry\ Median\ \beta_{SMB,\,t}| \\ &\quad + |\beta_{HML,\,jt} - Industry\ Median\ \beta_{HML,\,t}| \\ &\quad + |\beta_{UMD,\,jt} - Industry\ Median\ \beta_{UMD,\,t}|. \end{aligned} \tag{17}$$

We then estimate the following fee determination equation:

$$\begin{aligned} Fee_{j,\,t} &= a + \beta_1 TDRFL^{Industry}_{j,\,t-1} + \beta_2 \ln(TNA)_{j,\,t-1} \\ &\quad + \beta_3 \ln(Stock)_{j,\,t-1} + \beta_4 \ln(age)_{j,\,t-1} \\ &\quad + \beta_5 Turnover_{j,\,t-1} + Objective_j + Year_t \\ &\quad + \varepsilon_{i,\,t}, \end{aligned} \tag{18}$$

where $Fee_{j,\,t}$ is the expense ratio charged by fund j at time t; $\ln(TNA)_{j,\,t-1}$ is the logarithm of fund j's total net asset at time $t-1$; $\ln(stock)_{i,\,t-1}$ is the total number of stock held fund j at time $t-1$; $\ln(age)_{j,\,t-1}$ is the logarithm of fund j's age at time $t-1$; $Turnover_{j,\,t-1}$ is fund's portfolio turnover ratio at time $t-1$; $Objective_j$ is a dummy variable indicating if a fund is an aggressive growth, a long-term growth, or a growth and income fund; $Year_t$ is year t's fixed effect; and $\varepsilon_{i,\,t}$ is the error term.

Column (1) of Table 3 reports the regression results of the impact of *TDRFL* on mutual fund fees. For the ease of discussion, fees are measured in basis points. Fund size is negatively related to fees, whereas the number of stocks held by a fund is also negatively related to fees. Younger funds with higher turnover ratios tend to charge higher fees. Both aggressive growth and long-term growth funds charge significantly higher fees than growth and income funds do. These results are consistent with previous findings on the determinants of mutual fund fees. Interestingly, $TDRFL^{Industry}_{j,\,t-1}$ is significantly and positively associated with fees. If a fund increases the deviation of each of its four-risk-factor loadings by one standard deviation, the $TDRFL^{Industry}_{j,\,t-1}$ will increase by 1.487 ($= 0.278 + 0.443 + 0.475 + 0.291$), which could lead to a 5.616 ($= 3.777 \times 1.487$) basis points higher fee. To put this into perspective, the expense ratio for the average fund in our sample is 126 basis points. An increase of 5.616 is equal to a 4.5% ($= 5.616/126$) increase in expense ratio for the average fund.

To check the robustness of the result, we calculate two alternative measures for the total deviation of risk factor loadings. Instead of using industry median risk factor loadings, $TDRFL^{IOC}_{jt}$ and $TDRFL^{Lipper}_{jt}$ use median factor loadings in each fund investment objective category classified by the Investment Objective Classifications and Lipper Objective Classifications to calculate the total deviation of risk factor loadings, respectively. Columns (2) and (3) of Table 3 report the results using these two alternative measures. One can see that estimated coefficients on $TDRFL^{IOC}_{jt-1}$ and $TDRFL^{Lipper}_{jt-1}$ are all significant at the 1% level with a similar magnitude to the one on $TDRFL^{Industry}_{j,\,t-1}$. In sum, the findings conform to the theoretical implication, suggesting that a fund's fee setting is determined not only by its own attributes but also by positions of its risk factor loadings within the industry.

A caveat in interpreting the results is that, as we discussed earlier, differentiation in risk factor might allow funds to charge higher fees by targeting heterogeneous investors with different needs or search costs. For example, Carlin (2009) shows that price complexity prevents some consumers from becoming informed and allows firms to charge a higher price. To the extent that funds with more extreme risk factor loading are associated with greater price complex, our results are also consistent with the price complexity explanation. Further identification of potentially coexisting plausible explanations will be an interesting direction for future empirical research. We view our current empirical evidence as suggestive.

Table 3 Mutual Fund Fees and Risk Factor Loadings

	(1)	(2)	(3)
$TDRFL^{Industry}_{j,\,t-1}$	3.777***		
	(4.96)		
$TDRFL^{IOC}_{j,\,t-1}$		3.440***	
		(4.42)	
$TDRFL^{Lipper}_{j,t-1}$			4.631***
			(5.03)
Ln(TNA)$_{j,\,t-1}$	−7.495***	−7.496***	−7.477***
	(−14.08)	(−14.06)	(−13.02)
Ln($nstock$)$_{j,\,t-1}$	−11.007***	−11.002***	−8.952***
	(−10.29)	(−8.37)	(−6.21)
Ln(age)$_{j,\,t-1}$	−3.450***	−3.463***	−3.627***
	(−5.46)	(−5.48)	(−4.76)
Turnover ratio$_{j,\,t-1}$	7.261***	7.328***	7.951***
	(4.96)	(4.99)	(5.04)
AG dummy$_j$	16.685***	17.198***	12.217***
	(5.18)	(5.33)	(3.36)
LG dummy$_j$	5.653***	5.472***	4.587***
	(2.77)	(2.67)	(2.06)
Year dummies	Yes	Yes	Yes
N	16,112	16,112	10,844
Adj. R^2	0.25	0.24	0.24

***Significance at the 1% level.

4. Conclusion

This study proposes that financial product differentiation over the state space through risk factor loading is a factor in driving style proliferation in the mutual fund industry. We develop a model of strategic competition between two mutual funds that accommodates stochastic fund characteristics to show how financial product differentiation affects mutual fund market shares, price elasticities, and fees. In particular, our model demonstrates that through product differentiation over the state space, two funds become the winning fund and obtain market power alternately in different states, thereby avoiding state-by-state competition. The novelty of this product differentiation is that both funds are able to lower price elasticities and charge higher fees even before the realization of market conditions, although they are identical in having the same likelihood of becoming the winning fund.

A unique feature of the product differentiation over the state space is that it does not require investors to be heterogeneous. As long as investors chase past fund performance, mutual funds are able to attract investors and generate market powers alternately in different states of nature by distancing themselves in risk factor loadings. It highlights the importance of interaction between consumer information and knowledge and funds' strategic considerations in determining the market structure of the mutual fund industry.

Acknowledgments

The authors thank department editor Wei Jiang, an associate editor, two anonymous referees, Takeshi Amemiya, Patric Bajari, Timothy Bresnahan, Andrew Carrothers, Xiaowei Li, Barr Rosenberg, John Shoven, Neng Wang, Frank Yu, Eric Zitzewitz, Huayue Zhang, and seminar participants at Stanford University for helpful comments. Jiaping Qiu thanks the Social Sciences and Humanities Research Council of Canada for financial support.

Appendix. Proofs of Propositions

A.1 Proof of Equation (4)

The utility of investor i choosing fund j at time 2 is

$$\begin{aligned} U_{ij2} &= Perf_j(\mathbf{\Phi}, P_j, R_{j2}) + \mathbf{X}'_j\mathbf{B} + \xi_j + e_{ij2} \\ &= \phi_0\alpha_j - b_0P_j + \phi_1\beta_{jM}f_{M2} - \tfrac{1}{2}\phi_2\,\mathrm{Var}(R_{j2}) \\ &\quad + \mathbf{X}'_j\mathbf{B} + \xi_j + e_{ij2} \\ &= \phi_0\alpha_j - b_0P - \tfrac{1}{2}\phi_2\sigma^2_{\varepsilon_j} + \mathbf{X}'_j\mathbf{B} + \xi_j \\ &\quad + \{\phi_1(\beta_{jM}f_{M2}) - \tfrac{1}{2}\phi_2\beta^2_{jM}\sigma^2_M\} + e_{ij2} \\ &= A_j + u_{j2}(\beta_{jM}) + e_{ij2}, \end{aligned}$$

where $A_j = \phi_0\alpha_j - b_0P_j - \frac{1}{2}\phi_2\sigma^2_{\varepsilon_j} + \mathbf{X}'_j\mathbf{B} + \xi_j$ and $u_{j2}(\beta_{jM}) = \phi_1\beta_{jM}f_{M2} - \frac{1}{2}\phi_2\beta^2_{jM}\sigma^2_M$.

To find the ex post optimal risk factor loading that provides investor i with the highest utility at time 2, the first-order condition for β_j to maximize the utility is

$$U'_{j2}(\beta_{jM}) = 0 \quad\Leftrightarrow\quad \mu'_{j2}(\beta_{jM}) = 0 \quad\Leftrightarrow\quad \beta^*_{jM} \equiv \beta^*_{M2} = \left(\frac{\phi_1}{\phi_2}\right)\frac{f_{M2}}{\sigma^2_M}.$$

Thus,

$$u^*_{j2} = u_{j2}(\beta^*_{M2}) \equiv u^*_{M2} = \tfrac{1}{2}\phi_2\cdot(\beta^*_{M2})^2\sigma^2_M,$$

so that

$$U_{ij2} = U_{ij2} - u^*_{j2} + u^*_{j2} = A_j - \tfrac{1}{2}\phi_2(\beta_{jM} - \beta^*_{M2})^2\sigma^2_M + u^*_{M2} + e_{ij2}.$$

Thus, the set of unobserved stochastic shocks e_{ij2} that lead to the choice of fund j at time 2 is defined by

$$\psi_j = \{e_{ij2} \mid U_{ij2} \geq U_{il2}\}.$$

Then, mutual fund j's aggregate market share at time 2 is

$$S_{j2} = \int_{\psi_j} dP(e_{ij2}),$$

where $P(\cdot)$ is the probability distribution function of the stochastic shocks e_{ij2}. Following the standard assumption in the logit model that e_{ij2} has a type I extreme-value distribution, the ex post market share of mutual fund j, s_{j2}, at time 2 is equal to

$$s_{j2} = \frac{\exp(A_j - \frac{1}{2}\phi_2(\beta_{jM} - \beta^*_{M2})^2\sigma^2_M)}{\sum_{l=1}^2 \exp(A_l - \frac{1}{2}\phi_2(\beta_{lM} - \beta^*_{M2})^2\sigma^2_M)}.$$

A.2 Proof of Proposition 1

Fund 1 holds high beta $\beta_{1M} = \beta^H_M$, and fund 2 holds low beta $\beta_{2M} = \beta^L_M$, where $\beta^H_M = -\beta^L_M$. The ex post market shares of the two funds are equal to

$$s_{12} = \frac{\exp(A - \frac{1}{2}\phi_2(\beta^H_M - \beta^*_{M2})^2\sigma^2_M)}{\exp(A - \frac{1}{2}\phi_2(\beta^H_M - \beta^*_{M2})^2\sigma^2_M) + \exp(A - \frac{1}{2}\phi_2(\beta^L_M - \beta^*_{M2})^2\sigma^2_M)},$$

$$s_{22} = \frac{\exp(A - \frac{1}{2}\phi_2(\beta^L_M - \beta^*_{M2})^2\sigma^2_M)}{\exp(A - \frac{1}{2}\phi_2(\beta^H_M - \beta^*_{M2})^2\sigma^2_M) + \exp(A - \frac{1}{2}\phi_2(\beta^L_M - \beta^*_{M2})^2\sigma^2_M)}.$$

The expected market shares for funds 1 and 2 are equal to

$$\begin{aligned} S^e_1 &= E_1(S_{12}) \\ &= \int \big(\exp\big(A - \tfrac{1}{2}\phi_2(\beta^H_M - \beta^*_{M2})^2\sigma^2_M\big)\big) \\ &\qquad \cdot \big(\exp\big(A - \tfrac{1}{2}\phi_2(\beta^H_M - \beta^*_{M2})^2\sigma^2_M\big) \\ &\qquad\quad + \exp\big(A - \tfrac{1}{2}\phi_2(\beta^L_M - \beta^*_{M2})^2\sigma^2_M\big)\big)^{-1}\, dF(f_{M2}), \\ S^e_2 &= E_1(S_{22}) \\ &= \int \big(\exp\big(A - \tfrac{1}{2}\phi_2(\beta^L_M - \beta^*_{M2})^2\sigma^2_M\big)\big) \\ &\qquad \cdot \big(\exp\big(A - \tfrac{1}{2}\phi_2(\beta^H_M - \beta^*_{M2})^2\sigma^2_M\big) \\ &\qquad\quad + \exp\big(A - \tfrac{1}{2}\phi_2(\beta^L_M - \beta^*_{M2})^2\sigma^2_M\big)\big)^{-1}\, dF(f_{M2}). \end{aligned}$$

Given that $\beta^H_M = -\beta^L_M$ and the density function of $F(f_{M2})$ is symmetric,

$$\begin{aligned} S^e_1 = \int &\big(\exp\big(A - \tfrac{1}{2}\phi_2(\beta^H_M - \beta^*_{M2})^2\sigma^2_M\big)\big) \\ &\cdot \big(\exp\big(A - \tfrac{1}{2}\phi_2(\beta^H_M - \beta^*_{M2})^2\sigma^2_M\big) \end{aligned}$$

$$
\begin{aligned}
&\quad+\exp\big(A-\tfrac{1}{2}\phi_2(\beta_M^L-\beta_{M2}^*)^2\sigma_M^2\big)\big)^{-1}\,dF(f_{M2})\\
&=\int\big(\exp\big(A-\tfrac{1}{2}\phi_2(\beta_M^L-(-\beta_{M2}^*))^2\sigma_M^2\big)\big)\\
&\quad\cdot\big(\exp\big(A-\tfrac{1}{2}\phi_2(\beta_M^L-(-\beta_{M2}^*))^2\sigma_M^2\big)\\
&\quad+\exp\big(A-\tfrac{1}{2}\phi_2(\beta_M^H-(-\beta_{M2}^*))^2\sigma_M^2\big)\big)^{-1}\,dF(f_{M2})\\
&=\int\big(\exp\big(A-\tfrac{1}{2}\phi_2(\beta_M^L-(-\beta_{M2}^*))^2\sigma_M^2\big)\big)\\
&\quad\cdot\big(\exp\big(A-\tfrac{1}{2}\phi_2(\beta_M^L-(-\beta_{M2}^*))^2\sigma_M^2\big)\\
&\quad+\exp\big(A-\tfrac{1}{2}\phi_2(\beta_M^H-(-\beta_{M2}^*))^2\sigma_M^2\big)\big)^{-1}\,dF(-f_{M2})\\
&=\int\big(\exp\big(A-\tfrac{1}{2}\phi_2(\beta_M^L-\beta_{M2}^*)^2\sigma_M^2\big)\big)\\
&\quad\cdot\big(\exp\big(A-\tfrac{1}{2}\phi_2(\beta_M^L-\beta_{M2}^*)^2\sigma_M^2\big)\\
&\quad+\exp\big(A-\tfrac{1}{2}\phi_2(\beta_M^H-\beta_{M2}^*)^2\sigma_M^2\big)\big)^{-1}\,dF(f_{M2})\\
&=S_2^e.
\end{aligned}
$$

Therefore, $S_1^e=\bar{S}_1=\frac{1}{2}$ and $S_2^e=\bar{S}_2=\frac{1}{2}$.

A.3 Proof of Proposition 3

We show that the price elasticity for mutual funds will be lower if their risk factor loadings deviate further away from the median. Consider two scenarios: In scenario A, two funds have factor loadings $\beta_{1M}=\beta_M^H$ and $\beta_{2M}=\beta_M^L$, where $\beta_M^H=-\beta_M^L$; in scenario B, $\beta_{1M}^{'}=\beta_M^{H'}$ and $\beta_{2M}^{B}=\beta_M^{L'}$, where $\beta_M^{H'}=-\beta_M^{L'}$. Funds' factor loadings are more distant from the median in scenario B, i.e., $\beta_M^{H'}>\beta_M^H$.

The ex post market share of the fund 1 in scenario A is equal to

$$
\begin{aligned}
s_{12}&=\left(\exp\left(A+\left(\phi_1(\beta_M^H f_{M2})-\frac{1}{2}\phi_2(\beta_M^H)^2\right)\sigma_M^2\right)\right)\\
&\quad\cdot\left(\exp\left(A+\left(\phi_1(\beta_M^H f_{M2})-\frac{1}{2}\phi_2(\beta_M^H)^2\right)\sigma_M^2\right)\right.\\
&\quad\left.+\exp\left(A+\left(\phi_1(\beta_M^L f_{M2})-\frac{1}{2}\phi_2(\beta_M^L)^2\right)\sigma_M^2\right)\right)^{-1}\\
&=\frac{\exp(\phi_1\beta_M^H f_{M2}\sigma_M^2)}{\exp(\phi_1\beta_M^H f_{M2}\sigma_M^2)+\exp(\phi_1\beta_M^L f_{M2}\sigma_M^2)}\\
&=\frac{1}{1+\exp(2\phi_1\beta_M^L f_{M2}\sigma_M^2)}.
\end{aligned}
$$

Similarly, the ex post market share of the fund 1 in scenario B is equal to

$$s_{12}'=\frac{1}{1+\exp(2\phi_1\beta_M^{L'} f_{M2}\sigma_M^2)}.$$

The expected market shares are the same in scenarios A and B. That is,

$$S_1^e=S_2^e=\tfrac{1}{2}\ \text{and}\ S_1^{e'}=S_2^{e'}=\tfrac{1}{2}.$$

The variances of market share in these two scenarios are equal to

$$
\begin{aligned}
\mathrm{Var}(S_{12})&=\int(s_{12}-S_1^e)^2\,dF(f_{M2})=\int_{-\infty}^{0}(s_{12}-S_1^e)^2\,dF(f_{M2})\\
&\quad+\int_0^{\infty}(s_{12}-S_1^e)^2\,dF(f_{M2})\quad\text{and}
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{Var}(S_{12}')&=\int(s_{12}'-S_1^{e'})^2\,dF(f_{M2})=\int_{-\infty}^{0}(s_{12}'-S_1^{e'})^2\,dF(f_{M2})\\
&\quad+\int_0^{\infty}(s_{12}'-S_1^{e'})^2\,dF(f_{M2}).
\end{aligned}
$$

Because

$$
\begin{cases}
s_{12}'-S_1^{e'}>s_{12}-S_1^e>0 & \text{if } f_{M2}>0,\\
s_{12}'-S_1^{e'}<s_{12}-S_1^e<0 & \text{if } f_{M2}<0,
\end{cases}
$$

we have $\mathrm{Var}(S_{12}')>\mathrm{Var}(S_{12})$. The expected price elasticity in scenario A $|\eta_1^e|$ is greater than that in scenario B $|\eta_1^{e'}|$ because

$$
\begin{aligned}
|\eta_1^e|&=\left|\frac{\partial S_1^e P_1}{\partial P_1 S_1^e}\right|=\left|-b_0\frac{P_1}{S_1^e}\int s_{12}(1-s_{12})\,dF(f_{M2})\right|\\
&=|b_0P_1(1-S_1^e-\mathrm{Var}(S_{12})/S_1^e|\\
&>|b_0P_1(1-S_1^{e'}-\mathrm{Var}(S_{12}')/S_1^{e'}|=|\eta_1^{e'}|.
\end{aligned}
$$

References

Amromin G, Huang J, Sialm C, Zhong E (2013) Complex mortgages. Working paper, University of Texas at Austin, Austin.

Barber B, Odean T, Zheng L (2005) Out of sight, out of mind: The effects of expenses on mutual fund flows. *J. Bus.* 78(6):2095–2119.

Berk J (1995) A critique of size-related anomalies. *Rev. Financial Stud.* 8(2):275–286.

Berry S (1994) Estimating discrete choice models of product differentiation. *RAND J. Econom.* 25(2):242–262.

Berry S, Levinsohn J, Pakes A (1995) Automobile prices in market equilibrium. *Econometrica* 63(4):841–890.

Brown K, Harlow V, Starks L (1996) Of tournaments and temptations: An analysis of managerial incentives in the mutual fund industry. *J. Finance* 51(1):85–110.

Carhart M (1997) On persistence in mutual fund performance. *J. Finance* 52(1):57–82.

Carlin IB (2009) Strategic price complexity in retail financial markets. *J. Financial Econom.* 91(3):278–287.

Carlin IB, Gervais S (2012) Legal protection in retail financial markets. *Rev. Corporate Finance Stud.* 1(1):68–108.

Carlin IB, Manso G (2011) Obfuscation, learning, and the evolution of investor sophistication. *Rev. Financial Stud.* 24(3):754–785.

Chevalier J, Ellison G (1997) Risk taking by mutual funds as a response to incentives. *J. Political Econom.* 105(6):1167–1200.

Deli D (2002) Mutual fund advisory contracts: An empirical investigation. *J. Finance* 57(1):109–133.

Elton E, Gruber M, Krasny Y, Ozelge S (2010) The effect of the frequency of holding data on conclusions about mutual fund management behavior. *J. Banking Finance* 34(5):912–922.

Fama E, French K (1993) Common risk factors in the returns on stocks and bonds. *J. Financial Econom.* 33(1):3–56.

Goetzmann WN, Ibbotson RG (1993) Games mutual fund companies play: Strategic response to investor beliefs in the mutual fund industry. Working paper, Yale School of Management, New Haven, CT.

Golec JH (1992) Empirical tests of a principal-agent model of the investor-investment dvisor relationship. *J. Financial Quant. Anal.* 27(1):81–95.

Green RC, Li D, Schurhoff N (2010) Price discovery in illiquid markets: Do financial asset prices rise faster than they fall? *J. Finance* 65(5):1669–1702.

Gruber MJ (1996) Another puzzle: The growth in actively managed mutual funds. *J. Finance* 51(3):783–810.

Guerrieri V, Kondor P (2012) Fund managers, career concerns, and asset price volatility. *Amer. Econom. Rev.* 102(5):1986–2017.

Hortaçsu A, Syverson C (2004) Product differentiation, search costs and competition in the mutual fund industry: A case study of S&P 500 index funds. *Quart. J. Econom.* 119(2):403–456.

Ippolito R (1992) Consumer reaction to measures of poor quality: Evidence from the mutual fund industry. *J. Law Econom.* 35(1):45–70.

Jensen MC (1968) The performance of mutual funds in the period 1945–1964. *J. Finance* 23(2):389–416.

Kacperczyk M, Sialm C, Zheng L (2008) Unobserved actions of mutual funds. *Rev. Financial Stud.* 21(6):2379–2416.

Laster D, Bennett P, Geoum I (1999) Rational bias in marcroeconomic forecasts. *Quart. J. Econom.* 114(1):293–318.

Mamaysky H, Spiegel M, Zhang H (2007) Improved forecasting of mutual fund alphas and betas. *Rev. Finance* 11(3):359–400.

Massa M (2000) Why so many mutual funds? Mutual fund families, market segmentation and financial performance. Working paper, INSEAD, Fontainebleau, France.

Nanda V, Wang ZJ, Zheng L (2004) Family values and the star phenomenon. *Rev. Financial Stud.* 17(3):667–698.

Schwarz CG (2012) Mutual fund tournaments: The sorting bias and new evidence. *Rev. Financial Stud.* 25(3):913–936.

Sensoy AB (2009) Performance evaluation and self-designated benchmark indexes in the mutual fund industry. *J. Financial Econom.* 92(1):25–39.

Shanken J, Zhou G (2007) Estimating and testing beta pricing models: Alternative methods and their performance in simulations *J. Financial Econom.* 84(1):40–86.

Tufano P, Sevick M (1997) Board structure and fee-setting in the U.S. mutual fund industry. *J. Financial Econom.* 46(3):321–355.

Wahal S, Wang YA (2011) Competition among mutual funds. *J. Financial Econom.* 99(1):40–59.

Zheng L (1999) Is money smart? A study of mutual fund investors' fund selection ability. *J. Finance* 54(3):901–933.