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Prospect Theory Explains Newsvendor Behavior: The Role of Reference Points

Xiaoyang Long, Javad Nasiry

School of Business and Management, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong {xlongaa@ust.hk, nasiry@ust.hk}

Turrent understanding in operations management is that prospect theory, as a theory of decision making under uncertainty, cannot systematically explain the ordering behavior observed in experiments on the newsvendor problem. We suggest this is because the newsvendor's reference point is assumed to be the status quo, i.e., zero payoff. We propose an alternative based on newsvendor's salient payoffs and show that prospect theory can, in fact, account for experimental results.

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Introduction

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In the classic newsvendor problem, a newsvendor faces stochastic demand yet must make a single inventory ordering decision. The wholesale and retail prices are fixed at w and p, respectively. If the demand $\mathbf{x} \in [x, \bar{x}]$ follows a distribution with cumulative distribution function $F(\cdot)$, then the newsvendor's optimal order quantity q^* solves $F(q^*) = p - w/p$. The fraction p - w/p is called the *critical fractile*, and we refer to q^* as the optimal rational decision or normative order quantity. However, experiments have shown repeatedly that decision makers systematically deviate from the normative decision. More specifically, newsvendors consistently underorder in the high-profit regime (i.e., $w/p < \frac{1}{2}$) and overorder in the low-profit regime (i.e., $w/p > \frac{1}{2}$) (Schweitzer and Cachon 2000, Bolton and Katok 2008, Kremer et al. 2010). This phenomenon is known as the "pull-tocenter" effect because the newsvendor's decision \tilde{q} appears to lie between the normative decision q^* and the mean demand μ .

Schweitzer and Cachon (2000), who originally documented the overordering/underordering pattern, rule out prospect theory (PT) as an explanation. They study the high- and low-profit regimes under two demand scenarios. The design of their experiment in the *high-demand* scenario is such that the newsvendor never realizes a loss, so all prospects are assumed to be evaluated as gains. Because PT dictates that the newsvendor be risk averse in the domain of gains, he should therefore always underorder (Eeckhoudt et al. 1995). In the low-demand scenario, the newsvendor may realize losses as well as profits, depending on the order quantity and the realized demand. In this case, Schweitzer and Cachon explain the overordering/underordering pattern by relying on the risk attitudes toward gains and losses in PT. In particular, in the high-profit regime, the newsvendor is more likely to experience gains, and because the newsvendor is risk averse over gains, he is more likely to underorder. In the low-profit regime, the newsvendor is more likely to experience losses, and because the newsvendor is risk seeking over losses, he is more likely to overorder (Eeckhoudt et al. 1995).

More recently, Nagarajan and Shechter (2014) reconfirm that the newsvendor always underorders in the low-profit case by considering a more elaborate description of PT that incorporates probability weighting when the prospects are evaluated. These authors also show numerically that PT predicts overordering in the high-profit case. Because these predictions directly contradict existing experimental results, Nagarajan and Shechter conclude that PT does not explain the newsvendor's behavior, and they pose the question of "why PT, a well-known and widely accepted framework for decision making under uncertainty, may not apply to a fundamental operations management problem" (p. 1062). We shall elaborate on the reference point construct to show that PT can, in fact, explain newsvendor behavior.

Schweitzer and Cachon (2000) and also Nagarajan and Shechter (2014) implicitly assume that the decision maker compares the outcomes with the current level of wealth, which is assumed to be zero. Yet this need not be the case because "there are situations in which gains and losses are coded relative to an expectation or aspiration level that differs from the status quo" (Kahneman and Tversky 1979, p. 286).



In this paper we show that if a more plausible reference point is chosen, then PT can indeed explain observed ordering behavior without relying on risk attitudes or the distortion of probabilities. We suggest in particular that, given the order quantity q, the reference point is the weighted average of two salient payoffs: the lowest and highest possible profits. We interpret the weight assigned to the highest payoff as the degree of newsvendor optimism. Then we show, consistently with experimental results, that the newsvendor overorders if w/p is sufficiently large and underorders otherwise.

Model

The newsvendor's material payoff for order quantity q and realized demand x is

$$\pi(q, x) = \begin{cases} px - wq & \text{if } x < q, \\ (p - w)q & \text{if } x \ge q. \end{cases}$$

We assume a piecewise-linear value function for the reference-dependent newsvendor. That is,

$$v(y) = \begin{cases} \eta y & \text{if } y \ge 0, \\ \lambda \eta y & \text{if } y < 0; \end{cases}$$

here, η captures the strength of reference effects, and λ is the coefficient of loss aversion. A higher value of η implies more sensitivity to deviations from the reference point; a higher λ indicates more sensitivity to losses than to gains. This operationalization of PT abstracts from the newsvendor's risk attitudes and is sufficient to explain the ordering behavior. In Appendix B we demonstrate that the results are robust to more general forms of the value function.

The newsvendor's ex post utility is $U(q, x) = \pi(q, x) + v(y)$. It consists of the realized profit and a psychological component that captures how the realized payoff compares with the reference payoff (see Kőszegi and Rabin 2006 and Herweg 2013 for similar model setups).

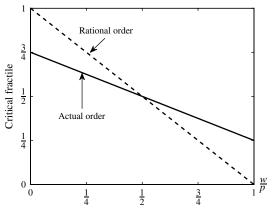
We hypothesize that the reference payoff for an order quantity q is a convex combination of the maximum possible payoff (p-w)q and the minimum possible payoff $p\underline{x} - wq$. That is, we assume

$$r(q) = \beta(p - w)q + (1 - \beta)(p\underline{x} - wq). \tag{1}$$

The parameter $\beta \in [0,1]$ can be interpreted as the decision maker's level of optimism. A high value of β means that the newsvendor holds high expectations for the final outcome, whereas a low β implies that he anchors more on the worst-case scenario. The following proposition summarizes our major findings. Proofs are in Appendix A.

PROPOSITION 1. Assume that demand x is uniformly distributed on $[\underline{x}, \bar{x}]$ and that the newsvendor's reference

Figure 1 The Newsvendor's Order Quantity Compared with the Rational Order Quantity



Note. $\eta = 1$, $\lambda = 1$, and $\beta = \frac{1}{2}$.

point is as in Equation (1). Then the following statements hold. (i) The newsvendor's optimal order quantity \tilde{q} satisfies $F(\tilde{q}) = (p - w + \eta(1 - \beta)p)/(p + \eta(\lambda - 1)\beta^2p + \eta p)$. (ii) The newsvendor overorders if $w/p > t(\lambda, \beta) = ((\lambda - 1)\beta^2 + \beta)/((\lambda - 1)\beta^2 + 1)$ and underorders otherwise.

Hence, the newsvendor overorders if the margin is sufficiently low, $w/p > t(\lambda, \beta)$, and underorders if the margin is sufficiently high, $w/p < t(\lambda, \beta)$. This predicted behavior is in line with the results from experimental research. Figure 1 depicts a scenario where we set $\lambda = 1$ and $\beta = \frac{1}{2}$ so that $t(\lambda, \beta) = \frac{1}{2}$. In this case, the newsvendor underorders $(F(\tilde{q}) < F(q^*))$ if the optimal rational order q^* exceeds mean demand and overorders $(F(\tilde{q}) > F(q^*))$ otherwise. This shows that even without loss aversion $(\lambda = 1)$, reference dependence in PT could predict the newsvendor's underordering/overordering pattern.

Moreover, because $0 \le t(\lambda, \beta) \le 1$ for $0 \le \beta \le 1$, our model accommodates ordering behavior from always overordering ($\beta = 0$) to always underordering ($\beta = 1$). Our model then helps explain the findings of Lau et al. (2014), who report that individual behavior varies widely in the newsvendor experiment and that the pull-to-center effect is not evident in the decisions of all subjects.

2.1. Asymmetries in Ordering Behavior

Our model also explains the "asymmetries" observed in ordering behavior between the high-profit and low-profit regimes. Schweitzer and Cachon (2000) find that subjects exhibit greater bias in the low-profit than in the high-profit case. That is, the actual order is farther away from the normative order in the low-profit case. Bostian et al. (2008) observe a like asymmetry, albeit to a lesser extent. In contrast, Ho et al. (2010) find a greater bias in the high-profit case. Still other works



¹ However, they use a Normal distribution for demand.

on the newsvendor problem do not report any asymmetry (Bolton and Katok 2008, Kremer et al. 2010).

We next show that our model can predict the asymmetry in either direction. Define $B: (0,1) \rightarrow [0,1]$ as a function that maps the fraction w/p to the degree of bias in the ordering decision; i.e., $B(w/p) = |F(\tilde{q}) - F(q^*)|$.

PROPOSITION 2. Assume that demand \mathbf{x} is uniformly distributed on $[\underline{x}, \bar{x}]$ and that the newsvendor's reference point is as in Equation (1). Let $a \in (0, \frac{1}{2}]$. (i) If $t(\lambda, \beta) > \frac{1}{2}$, then $B(\frac{1}{2} - a) > B(\frac{1}{2} + a)$; i.e., there is a greater bias in the high-profit regime. (ii) If $t(\lambda, \beta) < \frac{1}{2}$, then $B(\frac{1}{2} - a) < B(\frac{1}{2} + a)$; i.e., there is a greater bias in the low-profit regime.

In other words, if $t(\lambda, \beta) > \frac{1}{2}$, the order quantity is farther away from the normative order quantity in the high-profit regime. On the other hand, if $t(\lambda, \beta) < \frac{1}{2}$, the order quantity is farther away from the normative order quantity in the low-profit regime.

Most experimental literature in support of the pull-to-center effect follows Schweitzer and Cachon (2000) in testing only two cases (e.g., $w/p=\frac{1}{4}$ for the high-profit case and $w/p=\frac{3}{4}$ for the low-profit case). Yet by anchoring on μ when interpreting the results, researchers implicitly assume that the newsvendor makes the rational decision (i.e., $\tilde{q}=q^*=\mu$) at $w/p=\frac{1}{2}$. To the best of our knowledge, Bostian et al. (2008) are unique in testing this assumption. They find that the newsvendor slightly overorders when $w/p=\frac{1}{2}$. In our model this would imply that $t(\lambda,\beta)<\frac{1}{2}$, and hence there should be a larger bias in the low-profit regime. Bostian et al. observe such an asymmetry.

In our model, underordering for $w/p = \frac{1}{4}$ implies that $F(\tilde{q}) - F(q^*) = ((1-\beta) - \frac{3}{4}[(\lambda-1)\beta^2+1])/((\lambda-1) \cdot \beta^2+1/\eta+1) < 0$ and overordering for $w/p = \frac{3}{4}$ implies that $F(\tilde{q}) - F(q^*) = ((1-\beta) - \frac{1}{4}[(\lambda-1)\beta^2+1])/((\lambda-1) \cdot \beta^2+1/\eta+1) > 0$. If, as claimed by Schweitzer and Cachon (2000), the degree of bias is higher in the low-profit case, then $\frac{3}{4}[(\lambda-1)\beta^2+1] - (1-\beta) \leq (1-\beta) - \frac{1}{4}[(\lambda-1)\beta^2+1]$; this expression simplifies to $(1-\beta) - \frac{1}{2}[(\lambda-1)\beta^2+1] \geq 0$. If we assume that $\lambda=2$ on average, 2 then these inequalities imply that the typical subject studied by Schweitzer and Cachon is not very optimistic $(0.22 < \beta < 0.41)$.

3. Discussion and Conclusion

In this paper, we show that a simple version of prospect theory can explain the behavioral deviations in the newsvendor problem without relying on risk preferences or distortion of probabilities. This demonstration is made possible by a novel interpretation of the reference point, which is a key concept and "open parameter" (Rabin 2013) in PT. We find

that PT explains the ordering behavior observed in newsvendor experiments, provided the newsvendor anchors on the most and least favorable outcomes. We also establish that individual characteristics of the newsvendor, such as his degree of optimism, significantly affect the ordering decision and may explain the heterogeneous behavior of the subjects in newsvendor experiments.

We conclude by highlighting two critical issues. First, our results are not driven by the assumption of a piecewise-linear value function. We show in Appendix B that more general value functions that also account for risk attitudes over gains and losses yield the same qualitative insights.

Second, our formalization of the reference point departs from the notion of reference point as a fixed benchmark, since the reference point in our model is a function of the order quantity. An interpretation of our approach is that the newsvendor forms an "expectation" about the unknown payoff that depends on the order quantity. We are not the first to propose this approach, however. Kőszegi and Rabin (2006) suggest that, in an uncertain environment, the reference point is stochastic and determined in a "personal equilibrium" by the decision maker's rational expectations, which depend on his actions. The advantage of the personal equilibrium framework is that it does not impose an exogenous structure on the reference point. Herweg (2013) applies this framework to the newsvendor's problem and finds that the newsvendor always underorders.

Overall, our results shed light on the importance of properly calibrating behavioral theories to the specific problems in operations management.

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Appendix A. Proofs

PROOF OF PROPOSITION 1. (i) We put $\bar{F}(q) = 1 - F(q)$. Then the newsvendor's expected utility (suppressing the argument in r(q)) is

$$EU(q) = \int_{\underline{x}}^{q} (px - wq) dF(x) + (p - w)q\bar{F}(q)$$

$$-\lambda \eta \int_{\underline{x}}^{\frac{r+wq}{p}} (r - px + wq) dF(x) +$$

$$\eta \int_{\frac{r+wq}{p}}^{q} (px - wq - r) dF(x)$$

$$+ \eta ((p - w)q - r)\bar{F}(q). \tag{A1}$$

Given r(q) as in (1), EU(q) is concave. This is because

$$\frac{\partial \text{EU}(q)}{\partial q} = (p - w) - pF(q) - \lambda \eta (r' + w) F\left(\frac{r + wq}{p}\right) - \eta (r' + w) \bar{F}\left(\frac{r + wq}{p}\right) + \eta p \bar{F}(q) = 0, \quad (A2)$$



² See Ho and Zhang (2004).

and

$$\begin{split} \frac{\partial^2}{\partial q^2} \mathrm{EU}(q) &= -pf(q) - \eta p f(q) \\ &- (\lambda - 1) \eta(r' + w) f\left(\frac{r + wq}{p}\right) \left(\frac{r' + w}{p}\right) \\ &- \lambda \eta r'' F\left(\frac{r + wq}{p}\right) - \eta r'' \bar{F}\left(\frac{r + wq}{p}\right) < 0. \end{split}$$

The inequality follows because $r'(q) + w = \beta p > 0$ and r''(q) = 0. Substituting r(q) into (A2) and recalling that demand is uniform, we obtain the desired result.

(ii) Observe that $F(\tilde{q}) - F(q^*) = (p - w + \eta p(1 - \beta))/(p + (\lambda - 1)\eta\beta^2p + \eta p) - (p - w/p) = (\eta(1 - \beta) - (1 - w/p) \cdot [(\lambda - 1)\eta\beta^2 + \eta])/((\lambda - 1)\eta\beta^2 + 1 + \eta) > 0$ if and only if $w/p > ((\lambda - 1)\beta^2 + \beta)/((\lambda - 1)\beta^2 + 1)$. \square

Proof of Proposition 2. By Proposition 1, $F(\tilde{q}) - F(q^*) = (1 - \beta - (1 - w/p)[(\lambda - 1)\beta^2 + 1])/((\lambda - 1)\beta^2 + 1/\eta + 1)$. Therefore, $B(1/2 - a) = |(1 - \beta - (1/2 + a)[(\lambda - 1)\beta^2 + 1])/((\lambda - 1)\beta^2 + 1/\eta + 1)|$ and $B(1/2 + a) = |(1 - \beta - (1/2 - a)[(\lambda - 1)\beta^2 + 1])/((\lambda - 1)\beta^2 + 1/\eta + 1)|$.

(i) If $t(\lambda,\beta) > \frac{1}{2}$, then by Proposition 1, $F(\tilde{q}) - F(q^*)|_{w/p=\frac{1}{2}} = (1-\beta-(1/2)[(\lambda-1)\beta^2+1])/((\lambda-1)\beta^2+1/\eta+1) < 0$. It follows that $B(1/2-a) = |(1-\beta-(1/2)[(\lambda-1)\beta^2+1])/((\lambda-1)\beta^2+1])/((\lambda-1)\beta^2+1]/((\lambda-1)\beta^2+1]/((\lambda-1)\beta^2+1]/((\lambda-1)\beta^2+1)/((\lambda-1)\beta^2+1)/((\lambda-1)\beta^2+1)/((\lambda-1)\beta^2+1)/((\lambda-1)\beta^2+1)/((\lambda-1)\beta^2+1)/((\lambda-1)\beta^2+1)/((\lambda-1)\beta^2+1)/((\lambda-1)\beta^2+1)/((\lambda-1)\beta^2+1)/((\lambda-1)\beta^2+1)/((\lambda-1)\beta^2+1/\eta+1) < 0$, $\lambda \ge 1$, and a > 0.

(ii) The proof is similar to that of part (i).

Appendix B. General Value Functions

In this appendix, we show that our results in Proposition 1 do not depend on the linearity of the value function. Assume, for example, that

$$v(y) = \begin{cases} \eta y^d & \text{if } y \ge 0, \\ -\lambda \eta (-y)^d & \text{if } y < 0, \end{cases}$$

where 0 < d < 1 and $\lambda > 1$ (see Tversky and Kahneman 1992). This value function incorporates the risk attitudes of the subjects as specified by PT. In short, decision makers are risk averse in the domain of gains but are risk seeking in the domain of losses. The resulting threshold pattern is similar to that described in Proposition 1.

PROPOSITION 3. Suppose that demand is uniformly distributed on $[\underline{x}, \overline{x}]$ and that the newsvendor's reference point is given by Equation (1). The newsvendor overorders if $w/p > (\lambda \beta^{d+1} + \beta (1-\beta)^d)/(d(1-\beta)^d + \lambda \beta^{d+1} + \beta (1-\beta)^d)$ and underorders otherwise.

PROOF. The newsvendor's expected utility is

$$EU(q) = \int_{\underline{x}}^{q} (px - wq) dF(x) + (p - w)q\overline{F}(q)$$
$$-\lambda \eta \int_{\underline{x}}^{\frac{r + wq}{p}} (r - px + wq)^{d} dF(x)$$
$$+ \eta \int_{\underline{r + wq}}^{q} (px - wq - r)^{d} dF(x) + \eta ((p - w)q - r)^{d} \overline{F}(q).$$

Differentiating twice with respect to q and considering that x is uniformly distributed, we have

$$\frac{\partial \text{EU}(q)}{\partial q} = p - w - pF(q) - \lambda \eta \beta \frac{(\beta p(q - \underline{x}))^d}{\bar{x} - \underline{x}}$$
$$- \eta \beta \frac{((1 - \beta)p(q - \underline{x}))^d}{\bar{x} - \underline{x}}$$
$$+ \eta d((1 - \beta)p(q - \underline{x}))^{d-1}(1 - \beta)p\bar{F}(q),$$

and

$$\begin{split} \frac{\partial^{2} \mathrm{EU}(q)}{\partial q^{2}} &= -pf(q) - \lambda \eta \beta^{2} p d \frac{(\beta p(q - \underline{x}))^{d-1}}{\bar{x} - \underline{x}} \\ &- \eta \beta (1 - \beta) d p \frac{((1 - \beta) p(q - \underline{x}))^{d-1}}{\bar{x} - \underline{x}} \\ &+ \eta d (d - 1) ((1 - \beta) p(q - \underline{x}))^{d-2} (1 - \beta)^{2} p^{2} \bar{F}(q) \\ &- \eta d ((1 - \beta) p(q - x))^{d-1} (1 - \beta) p f(q) < 0. \end{split}$$

The inequality follows because 0 < d < 1. Therefore, EU(q) is concave.

Substituting q^* into $\partial EU(q)/\partial q$ now yields

$$\frac{\partial \mathrm{EU}(q)}{\partial q}\Big|_{q=q^*} = (\bar{x} - \underline{x})^{d-1} \left(-\lambda \eta \beta^{d+1} (p-w)^d - \eta \beta (1-\beta)^d \cdot (p-w)^d + \eta d (1-\beta)^d w (p-w)^{d-1}\right),$$

which is positive if and only if $w/p > (\lambda \beta^{d+1} + \beta (1-\beta)^d)/(d(1-\beta)^d + \lambda \beta^{d+1} + \beta (1-\beta)^d)$. \square

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