Management Science

Publication details, including instructions for authors and subscription information:
http://pubsonline.informs.org

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MANAGEMENT SCIENCE
Vol. 61, No. 3, March 2015, pp. 542–559
ISSN 0025-1909 (print) | ISSN 1526-5501 (online)

http://dx.doi.org/10.1287/mnsc.2013.1854

Bargaining for an Assortment

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A retailer's assortment decision results from a process of give-and-take, during which the retailer may bid manufacturers against one another, and the terms of trade offer plenty of flexibility for allocating the profit among the retailer and manufacturers. We adopt a bargaining framework to capture such an assortment selection process. We investigate the properties of the profit allocations that could emerge in a decentralized supply chain. In our model, the retailer engages in simultaneous bilateral negotiations with all manufacturers. Our model and analysis produce managerial insights that could not be obtained in the absence of a bargaining perspective on assortment planning. For example, we find that when a manufacturer improves its product, such improvements not only benefit the retailer but might even benefit competing manufacturers. In fact, even improvements to out-of-assortment products can increase the profits of the retailer and certain in-assortment manufacturers. Hence, our results suggest that a manufacturer can benefit from collaborating with judiciously chosen competitors.

Keywords: assortment planning; bargaining; game theory
History: Received March 22, 2012; accepted September 13, 2013, by Serguei Netessine, operations management. Published online in *Articles in Advance* March 3, 2014.

1. Introduction

One of the most important merchandising decisions a retailer needs to make is assortment selection, that is, choosing a subset from several products offered by different manufacturers. When composing an assortment or revising an existing one, the terms of trade between the retailer and a manufacturer may take forms that go beyond the simple wholesale-price contract. For example, it is not uncommon for the retailer to demand a slotting fee in exchange for carrying the manufacturer's product. In addition, according to a report by the Federal Trade Commission, the slotting fee is just one of many promotional allowances over which the retail buyer and the manufacturer's representative negotiate. Other allowances include advertising allowances and marketing funds, introductory allowances applied on a per-unit basis, buyback guarantees, and failure fees (Federal Trade Commission 2003). The negotiation between the retail buyer and the manufacturer's representative is often contentious, with the manufacturer facing the ever-present threat of exclusion from the assortment. In fact, according to an article about the negotiation tactics used by the retail buyers of big supermarket chains in the United Kingdom, "the main threat buyers use is that they will de-list a supplier, a tactic commonly used if a producer is refusing to reduce prices or accede to other requests" (Sibun and Hall 2008).

In short, a retailer's assortment decision results from a process of give-and-take, during which the retailer may potentially bid manufacturers against one another and the terms of trade offer substantial flexibility for allocating the system profit among the retailer and several manufacturers. In this paper, we adopt a bargaining framework to capture such an assortment selection process. We investigate the properties of the assortment and the resulting profit allocation that will emerge in a decentralized supply chain under such a bargaining framework.

Most research on assortment planning has considered the single-stage problem, where the retailer composes an assortment from a number of products, each with given characteristics and costs. We refer the reader to Kök et al. (2009) for an extensive survey of this literature. In contrast, we focus on a supply chain setting where the retailer negotiates with several manufacturers to choose an assortment. As such, our paper belongs to a small group of papers that consider assortment-related interactions between a retailer and its suppliers (Villas-Boas 1998, Aydin and Hausman 2009, Kurtuluş and Nakkaş 2011, Kurtuluş and Toktay 2011). Papers in this group assume a certain type of contract governing the trade between the retailer and manufacturers, and go on to study how the contract influences the eventual assortment. One common thread across these papers is that the assortment chosen in a decentralized supply chain differs from the assortment offered by a central decision maker. These papers show that the efficiency of the supply chain can be improved through contractual arrangements such as slotting fees or category captainship. In contrast, we avoid modeling any specific contractual terms that may be used to allocate the profit from the eventual assortment. Instead, to capture the flexibility of trade terms available to the retailer and manufacturers, we model a situation where the parties negotiate directly over the profit allocation from the eventual assortment.

Adopting a bargaining perspective on the retailer's assortment decision leads to novel modeling challenges. When the retailer and a manufacturer are negotiating over the allocation of profit from a given assortment, their negotiation is affected by the eventual outcomes of the retailer's negotiations with other manufacturers included in the same assortment. Each of these bilateral negotiations is conducted with the knowledge that, were the negotiation to fail, the retailer would default to its next best assortment. The difficulty is that the retailer's profit from the next best assortment is itself determined through a set of similar negotiations. In fact, one does not even know what the "next best assortment" is unless one can predict the outcomes of the retailer's negotiations with each and every manufacturer and for each and every assortment.

To capture the interdependence among the retailer's negotiations with different manufacturers, our modeling approach supposes that the retailer engages in simultaneous bilateral negotiations with each individual manufacturer and the retailer's profit in the event that the negotiation breaks down (i.e., the retailer's disagreement payoff) is the retailer's payoff from the best assortment that can be composed without that manufacturer and thus depends on the outcomes of all other negotiations. More specifically, we use the generalized Nash bargaining framework (see Nash 1950, Osborne and Rubinstein 1990) to model the bilateral negotiations between the retailer and each manufacturer, and we then require the outcomes of all these bilateral negotiations to be in equilibrium. This equilibrium concept, which requires interdependent Nash bargaining solutions to be in equilibrium with one another, is often referred to as the Nash–Nash solution.

The generalized Nash bargaining solution (GNBS), which we use to model the bilateral negotiation between the retailer and each manufacturer, has been used increasingly frequently to analyze supply chain interactions (see, e.g., Gurnani and Shi 2006, Nagarajan and Bassok 2008, Nagarajan and Sosic 2008). The Nash–Nash solution, on the other hand, has been widely used by researchers in economics (Davidson 1988, Horn and Wolinsky 1988, Dobson and Waterson 1997, O'Brien and Shaffer 2005, Bjoenerstedt and Stennek 2007) and marketing (Dukes and Gal-Or 2003, Shaffer and Zettelmeyer 2004, Dukes et al. 2006). In a recent paper, Feng and Lu (2012) use the Nash–Nash solution concept to study a supply chain where two competing manufacturers bargain with their supplier(s) to set the transfer price for an outsourced component. Their main finding, namely that the presence of a low-cost supplier may hurt the manufacturers' profits by diminishing their bargaining positions, attests to the nontrivial effects of negotiation on supply chains.

Another recent inquiry into interdependent negotiations in a supply chain is that of Lovejoy (2010a), who considers a multitier supply chain in which a customer-facing firm wishes to deliver a single product and bargains with multiple candidates to select a tier 1 supplier. Each tier 1 candidate must do the same to select the tier 2 supplier it would work with in the event that it is chosen by the downstream firm. The negotiations thus ascend the echelons of the supply chain. Lovejoy (2010a) proposes an axiomatic solution, which is proven to be consistent with the Nash bargaining outcome. Lovejoy (2010b) suggests that a different model is needed when a retailer offers multiple products from different suppliers (p. 54). In particular, instead of a retailer choosing one supplier as the tier 1 supplier, the retailer could work with multiple tier 1 suppliers, and the outcome would depend on the extent to which each of the several suppliers contributes to the supply chain. This is precisely the type of scenario that is captured in our assortment bargaining model.

To our knowledge, the only paper that considers manufacturer–retailer negotiations in the assortment planning context is that of Draganska et al. (2010), who empirically investigate how the channel constituents' relative bargaining powers affect pricing decisions, total channel profitability, and profit allocation. Their objective when positing a negotiation model is to develop a basis for empirical estimation, whereas we take a purely theoretical perspective and derive structural insights. For example, although a firm's bargaining power is an exogenously fixed input, (Draganska et al. 2010, p. 60) state: "This [negotiation] process gives rise to an extremely complicated game of interrelated negotiations, which the theory literature has yet to solve. The problem lies in the fact that the decision-relevant variables for one particular negotiation between a retailer and a manufacturer—namely, the profits that would be realized if the negotiation is successful and the profits in case the negotiation breaks down—depend on the outcomes of all other negotiations." In keeping with their empirical focus and motivated by the further challenges imposed by their setting with multiple retailers and manufacturers with multiple products, Draganska et al. (2010) resolve this difficulty by adopting a simpler model of what happens when two parties disagree. Specifically, in the bilateral negotiation between a retailer and a manufacturer for a given assortment, they relate the retailer's disagreement payoff (i.e., the retailer's profit in the event that the negotiation breaks down) to the retailer's profit from *the same assortment* minus the specific product under negotiation. Differing from this approach, we define the retailer's disagreement payoff as the retailer's profit from *the best possible assortment* it could compose without the specific product under negotiation.

Our model is sophisticated enough to capture multiple interdependent negotiations taking place over assortments and their associated profit allocations, yet tractable enough to allow us to characterize the payoffs in equilibrium. In fact, we derive a closed-form expression for equilibrium payoffs, which can be evaluated once the system profit for each and every assortment is known. We take advantage of this tractability to apply our equilibrium concept to the assortment selection of a price-setting newsvendor. We obtain insights that could not be obtained in the absence of a bargaining perspective on assortment selection.

2. The Model

We consider a market served by a single retailer R, who has access to N different manufacturers, each of which offers a single product. Let $S = \{1, 2, \dots, N\}$ be the set of products (equivalently, manufacturers). Every subset of S corresponds to a possible assortment for the retailer. We use i to index the assortments. Let $A_i \subseteq S$ be the set of products in assortment $i = 1, \dots, 2^N$. ($A_i = \varnothing$ corresponds to offering no products.)

Let π^i denote the *system profit* from assortment i, which is the surplus that remains after all parties have covered their costs. We do not impose any explicit restrictions on the demand model or the revenue and cost structure underlying the system profit from an assortment, and we treat the system profits as exogenously fixed input parameters. We further assume that no two assortments yield the same system profit. This assumption is made for expositional convenience, and it is not difficult to relax. We ignore the trivial assortments that lead to a net loss for the system, so we assume that all assortments yield nonnegative system profit.

When splitting the system profit from a given assortment, the retailer engages in simultaneous bilateral negotiations with all manufacturers included in that assortment. Let u_a^i denote the payoff to agent $a \in \{R, 1, 2, \dots, N\}$ from assortment $i = 1, \dots, 2^N$. If manufacturer j is not in assortment i, then its payoff from that assortment is zero; that is, $u_j^i = 0$ for $j \notin A_i$. We use the GNBS to model the outcome of the bilateral negotiation between the retailer and a manufacturer. According to the GNBS, the bargaining outcome must maximize a weighted product of the two parties' surpluses, where each party's surplus is weighted by the respective party's bargaining power. Let $\alpha_j \in (0, 1)$ denote manufacturer j's bargaining power in its bilateral negotiation with the retailer. This parameter is traditionally interpreted as a proxy for the patience and bargaining skill of the negotiating party (Muthoo 1999). Draganska et al. (2010), who empirically analyze manufacturer and retailer negotiations over assortments, focus on estimating the bargaining powers of several manufacturers and retailers. Their analysis of the German coffee market finds that manufacturers have larger bargaining power. Their analysis also reveals the effect of three important determinants of bargaining power: the size of the firm, store-brand introductions, and the size of the retailer's merchandise, that is, the number of products a retailer carries across all product categories. Instead of modeling the external constituents of bargaining power (such as firm size), we take it to be an exogenously fixed parameter.

In our model, when the retailer is negotiating with manufacturer j over the split of profits from assortment i, the "pie" that is being split by these two parties is the system profit from assortment i minus what is eventually allocated to all other manufacturers in assortment i. In the event that the retailer and manufacturer j cannot agree how to split the profit from assortment i, the negotiation will break down. The retailer could then choose the best assortment from the remaining $N-1$ manufacturers, so the retailer's disagreement payoff is its profit from the best assortment that does not include manufacturer j. Let $b(j)$ denote the best assortment the retailer can compose without including the product of manufacturer j; that is,

$$b(j) = \arg\max_{i:\, j \notin A_i} \{u_R^i\}.$$

Note that the lower the profit from assortment $b(j)$, the more the retailer is hurt by leaving out manufacturer j. Hence, the retailer's profit from assortment $b(j)$ can be thought of as an inverse metric for the attractiveness of manufacturer j's product.

As for manufacturers, we assume that their disagreement payoffs are zero. In fact, as long as manufacturers' disagreement payoffs are exogenously fixed, we can always normalize them to zero. Such a normalization must also reset the system profit from any assortment to the surplus that remains after each manufacturer in that assortment claims its nonzero disagreement payoff. Intuitively, in the normalized bargaining problem, the retailer and manufacturers negotiate over how to split the surplus that remains after each manufacturer claims its disagreement payoff. By assuming that a manufacturer's disagreement payoff is exogenously fixed (and normalized to zero), we are implicitly assuming that if the bilateral negotiation breaks down, then the manufacturer loses its incremental profit from selling through the retailer, but what the manufacturer derives from other sources (e.g., by selling through other retailers) is unaffected. This assumption is reasonable, for example, when the retailer in question is a manufacturer's only means to reach a certain market (so that the manufacturer does not have the option of giving up on this retailer and tapping all or part of the same market through an alternative retailer) or when each retailer makes up only a small portion of the manufacturer's business (so that the manufacturer's deal with a particular retailer has negligible effects on the rest of the manufacturer's business).

Although our solution concept is based on many bilateral negotiations, these negotiations take place at one point in time. Our model thus cannot capture the strategic incentives that might arise in settings with repeated interactions; for example, a retailer might omit a powerful manufacturer from the assortment to lower that manufacturer's bargaining power (which we assume to be exogenous and constant) and gain concessions in future periods.

2.1. An Illustration with Two Manufacturers

Before we formally define what constitutes an equilibrium in our model, we illustrate the equilibrium concept using an example with $N=2$ manufacturers. Label the assortments so that assortment 1 includes only manufacturer 1's product, assortment 2 includes only manufacturer 2's product, and assortment 3 includes both products. In addition, for the sake of this illustrative example, let us assume that any one of these three assortments will yield a positive system profit, whereas the system profit from the null assortment is zero.

Using this small example, we first highlight an important assumption. Given our model, the retailer and a manufacturer conduct a separate bilateral negotiation for each assortment that contains the manufacturer. For example, the retailer and manufacturer 1 conduct one bilateral negotiation to split the profit from assortment 1 and another bilateral negotiation to split the profit from assortment 3. If these two parties fail to reach a deal in any one of these bilateral negotiations, then we assume that the retailer reverts to the best assortment it can compose without manufacturer 1. Note the implicit assumption: If any one of the bilateral negotiations between the retailer and manufacturer 1 breaks down, then the retailer gives up on all possible assortments containing manufacturer 1.

Consider now the bilateral negotiation between the retailer and manufacturer 1 for the split of profits from assortment 3. Recall that, according to our model, these two parties are splitting the total profit from assortment 3 (π^3) minus manufacturer 2's payoff from assortment 3 (u_2^3). In addition, the retailer's disagreement payoff is its profit from the best assortment without manufacturer 1 ($u_R^{b(1)}$), and manufacturer 1's disagreement payoff is normalized to zero. Given these specifications, the GNBS stipulates that the retailer's payoff, u_R^3, and manufacturer 1's payoff, u_1^3, must solve the following maximization problem:

$$\begin{aligned}\max_{u_R^3,\,u_1^3}\quad & (u_R^3-u_R^{b(1)})^{1-\alpha_1}(u_1^3)^{\alpha_1}\\ \text{subject to}\quad & u_R^3+u_1^3+u_2^3=\pi^3,\\ & u_R^3,\,u_1^3\geq 0.\end{aligned}\tag{1}$$

Similarly, in the bilateral negotiation between the retailer and manufacturer 2 for assortment 3, the GNBS stipulates that the retailer's payoff, u_R^3, and manufacturer 2's payoff, u_2^3, must solve the following maximization problem, which is symmetric to (1):

$$\begin{aligned}\max_{u_R^3,\,u_2^3}\quad & (u_R^3-u_R^{b(2)})^{1-\alpha_2}(u_2^3)^{\alpha_2}\\ \text{subject to}\quad & u_R^3+u_2^3+u_1^3=\pi^3,\\ & u_R^3,\,u_2^3\geq 0.\end{aligned}\tag{2}$$

We have not yet specified what the retailer's disagreement payoffs, $u_R^{b(1)}$ in (1) and $u_R^{b(2)}$ in (2), will be. In this small example, the best assortment the retailer can compose without manufacturer 1 is assortment 2, which includes manufacturer 2's product only; that is, $b(1)=2$. Hence, if the retailer's negotiation with manufacturer 1 breaks down, the retailer's disagreement payoff ($u_R^{b(1)}$) is in fact the retailer's profit from assortment 2 (u_R^2), which is determined through a negotiation between the retailer and manufacturer 2 over the split of profit from assortment 2. In particular, according to the GNBS, the payoffs to the retailer and manufacturer 2 from assortment 2, u_R^2 and u_2^2, must solve the following maximization problem:

$$\begin{aligned}\max_{u_R^2,\,u_2^2}\quad & (u_R^2-u_R^{b(2)})^{1-\alpha_2}(u_2^2)^{\alpha_2}\\ \text{subject to}\quad & u_R^2+u_2^2=\pi^2,\\ & u_R^2,\,u_2^2\geq 0.\end{aligned}\tag{3}$$

Similarly, the retailer's payoff when the retailer disagrees with manufacturer 2 ($u_R^{b(2)}$) is the retailer's profit from assortment 1 (u_R^1), which is determined through a negotiation between the retailer and manufacturer 1 over the split of profit from assortment 1. According to the GNBS, the payoffs to the retailer and manufacturer 1 from assortment 1, u_R^1 and u_1^1, must solve the following maximization problem, which is symmetric to (3):

$$\begin{aligned} \max_{u_R^1,\, u_1^1} \quad & (u_R^1 - u_R^{b(1)})^{1-\alpha_1}(u_1^1)^{\alpha_1} \\ \text{subject to} \quad & u_R^1 + u_1^1 = \pi^1, \\ & u_R^1,\, u_1^1 \geq 0. \end{aligned} \tag{4}$$

Note that the optimal value of u_R^2, which solves (3), appears in problems (1) and (4) as well, because $u_R^{b(1)}$ is given by the optimal value of u_R^2. Similarly, the optimal value of u_R^1, which solves (4), appears in problems (2) and (3) in the form of $u_R^{b(2)}$. Hence, maximization problems (1)–(4) are all linked. In this simple two-manufacturer scenario, a bargaining equilibrium should specify the agents' payoffs from assortments 1–3 (u_R^i, u_1^i, and u_2^i for assortment $i = 1, 2, 3$) that simultaneously solve maximization problems (1)–(4).

Let us use the ongoing example to highlight another assumption of our model. Suppose that assortment 2 (which contains only manufacturer 2) is so poor that the system profit from assortment 2 is less than what the retailer would obtain if it disagreed with manufacturer 2 (i.e., $\pi^2 < u_R^{b(2)}$). In such a case, what happens when the retailer and manufacturer 2 are negotiating over the profit from assortment 2? In a one-shot Nash bargaining scenario, if the pie were less than one agent's disagreement payoff, the two agents would not negotiate in the first place. However, in our setting, one needs to make an assumption for the outcome of such scenarios, because how the agents split the profit from better assortments is contingent on how they would split the profit from such unattractive assortments. Our resolution is to assume that, in such an assortment, the manufacturer in question will obtain zero profit from the assortment. This is a reasonable assumption because in such instances the retailer could always improve its profit by dropping the manufacturer in question. Hence, it makes sense that the retailer would yield no profit to the manufacturer.

In short, the equilibrium we seek to characterize should specify each and every agent's payoff from all possible assortments so that those payoffs jointly solve problems (1)–(4). When there are $N > 2$ manufacturers, the characterization of the equilibrium becomes more challenging in two respects. First, the number of bilateral negotiations increases exponentially. Second, unlike the case with two manufacturers, the best assortment without manufacturer j is no longer obvious but must be determined through the equilibrium itself.

2.2. Bargaining Equilibrium

As illustrated in §2.1, a *bargaining equilibrium* must specify the profits allocated to the retailer and to each manufacturer for all possible assortments. These allocations must be such that the profits allocated to the retailer and a manufacturer j in assortment i must satisfy the GNBS with the manufacturer's disagreement payoff being zero and the retailer's disagreement payoff being its profit from assortment $b(j)$, which itself must satisfy the GNBS for each of the bilateral negotiations between the retailer and the manufacturers in assortment $b(j)$. More formally, we define a *bargaining equilibrium* as follows.

Definition 1. Consider the bilateral negotiation between the retailer and manufacturer j in assortment i over the retailer's payoff from assortment i (denoted by u_R^i) and manufacturer j's payoff from assortment i (denoted by u_j^i). Let $\mathbf{U}_{-j}^i$ denote the vector of payoffs from assortment i for all manufacturers other than j. Then, given $\mathbf{U}_{-j}^i$, we define the feasible set of payoffs from assortment i to the retailer and manufacturer j as follows:

$$\mathcal{A}(\mathbf{U}_{-j}^i) := \left\{ (u_R^i, u_j^i) \,\middle|\, u_R^i \geq 0,\, u_j^i \geq 0,\, u_R^i + u_j^i + \sum_{k \neq j} u_k^i = \pi^i,\, u_j^i = 0 \text{ for } j \notin A_i \right\}.$$

Given the feasible set $\mathcal{A}(\mathbf{U}_{-j}^i)$, the retailer's and manufacturer j's payoffs from assortment i are given by the GNBS; i.e., u_R^i and u_j^i solve

$$\max_{(u_R^i,\, u_j^i) \in \mathcal{A}(\mathbf{U}_{-j}^i)} (u_R^i - u_R^{b(j)})^{1-\alpha_j}(u_j^i)^{\alpha_j}, \tag{5}$$

where $u_R^{b(j)}$ is the retailer's disagreement payoff in the event that its negotiation with manufacturer j fails. In particular, we define the retailer's disagreement payoff as the retailer's payoff from the best possible assortment the retailer can compose without manufacturer j; i.e.,

$$b(j) = \arg\max_{i:\, j \notin A_i} \{u_R^i\}. \tag{6}$$

A bargaining equilibrium is a set of payoffs, u_R^i and u_j^i for all manufacturers $j \in \{1, 2, \ldots, N\}$, that simultaneously solve maximization problem (5) for each assortment $i = 1, \ldots, 2^N$ subject to the constraint (6).

The equilibrium concept described above requires that the outcomes of several bilateral negotiations, each of which is given by a GNBS, are in equilibrium. This concept, known as the Nash–Nash solution, has been adopted by many papers as discussed in the literature review. The Nash–Nash solution is an extension of the GNBS to a setting with multiple interdependent negotiations. The GNBS does not explicitly model the bargaining process and is cooperative in nature. The Nash–Nash solution also inherits these properties to a certain degree. Although the Nash–Nash solution avoids modeling the negotiation process, the literature contains evidence that the Nash–Nash solution captures the outcomes of certain bargaining processes. In particular, the Nash–Nash solution captures the outcomes of multiple simultaneous Rubinstein games, in which two parties make alternating offers in rapid succession until an agreement is reached or negotiation breaks down (e.g., as Feng and Lu 2012 also point out, Davidson 1988 and Bjoenerstedt and Stennek 2007 show such an equivalence in two separate applications of the Nash–Nash concept).

Our equilibrium concept is based on the assumption that the retailer conducts simultaneous bilateral negotiations with each manufacturer over every assortment that includes the manufacturer. We avoid modeling the detailed negotiation process, and instead we focus on modeling the outcome of these negotiations so that we can establish the bargaining equilibrium. One could envision the agents possibly converging on this equilibrium through infinite rounds of offers and counteroffers. Of course, such a process with infinite rounds of negotiations is infeasible in practice. While we acknowledge this discrepancy between our model and negotiation in practice, the equilibrium we establish still serves as a focal point on which negotiations in practice might converge, even if such convergence were eventually truncated.

To help with the exposition in the remainder of the paper, here we introduce an indexing of the assortments and manufacturers. Let us index the assortments so that $\pi^1 > \pi^2 > \cdots > \pi^{2^N}$. With this indexing, lower-indexed assortments are better for the system. In addition, let $B(j)$ denote the system-best assortment without manufacturer j; i.e., $B(j) = \arg\max_{i:\, j\notin A_i}\{\pi^i\}$. (Contrast this with $b(j)$, which denotes the retailer's best assortment without manufacturer j.) Let us label manufacturers so that the manufacturers in the system-optimal assortment (given by set A_1) are manufacturers 1 through $|A_1|$, and the remaining manufacturers carry the indices $|A_1|+1$ through N. Furthermore, we index manufacturers 1 through $|A_1|$ so that $B(|A_1|) \le \cdots \le B(2) \le B(1)$. For the remaining manufacturers, we have $B(|A_1|+1) = B(|A_1|+2) = \cdots = B(N) = 1$. With this indexing of the manufacturers, the best assortment without manufacturer 1 [$B(1)$] is worse than the best assortment without manufacturer 2 [$B(2)$], which is worse than the best assortment without manufacturer 3 [$B(3)$], and so on. Hence, with this indexing, lower-indexed manufacturers are more attractive for the system.

Despite the apparent complexity of the bargaining equilibrium specified in Definition 1, our model is tractable enough to allow a characterization of the payoffs in equilibrium, as we show in the next section.

3. Analysis and Findings

In this section, we first derive certain intuitive properties that characterize any bargaining equilibrium. Building on these results, we propose an intuitive allocation, which we then show to be the unique bargaining equilibrium. Finally, we characterize the equilibrium by obtaining closed-form expressions for the profit allocations from all possible assortments.

3.1. Equilibrium Properties

Consider first the negotiation between the retailer and manufacturer $j \in A_i$ regarding the distribution of profits derived from assortment i. In the classic version of the GNBS, where the disagreement payoffs are exogenously fixed parameters, the two parties would negotiate only if the "pie" is larger than the sum of their disagreement payoffs. In our version, where the retailer's disagreement payoff is endogenous, we need to consider what happens when the pie happens to be smaller than the retailer's disagreement payoff. Accordingly, two cases need to be distinguished.

If the pie under negotiation is smaller than or equal to the retailer's disagreement payoff (i.e., if $\pi^i - \sum_{k\in A_i,\, k\neq j} u_k^i \le u_R^{b(j)}$), then the solution to (5) is such that manufacturer j receives zero payoff, and the retailer receives the entire surplus from this bilateral negotiation:

$$u_R^i = \pi^i - \sum_{k\in A_i,\, k\neq j} u_k^i.$$

This solution reflects the intuitive expectation that the retailer has no reason to yield this particular manufacturer any profit from assortment i; otherwise, the retailer could do better simply by dropping the manufacturer and offering a different assortment.

If, on the other hand, the pie is larger than the retailer's disagreement payoff (i.e., if $\pi^i - \sum_{k\in A_i,\, k\neq j} u_k^i > u_R^{b(j)}$), then manufacturer $j \in A_i$ extracts a positive payoff. In this case, a standard result of the GNBS states that each party gets its disagreement payoff plus a fraction of the net surplus, which is defined to be the size of the pie minus the sum of all the parties' disagreement payoffs. The fraction collected by each party is equal to its bargaining power. More precisely, the payoffs to the retailer and the manufacturer that solve (5) are given by

$$u_j^i = \alpha_j\left(\pi^i - \sum_{k\in A_i,\, k\neq j} u_k^i - u_R^{b(j)}\right), \qquad (7)$$

$$u_R^i = (1-\alpha_j)\left(\pi^i - \sum_{k\in A_i,\, k\neq j} u_k^i - u_R^{b(j)}\right) + u_R^{b(j)}. \quad (8)$$

Using x^+ as a shorthand for $\max(x, 0)$ and combining the two cases discussed above, it follows from (7) and (8) that

$$u_j^i = \frac{\alpha_j}{1-\alpha_j}(u_R^i - u_R^{b(j)})^+, \quad (9)$$

which is well defined since $\alpha \in (0,1)$. Notice from (9) that for every dollar the retailer claims in excess of its disagreement payoff, manufacturer j claims β_j dollars, where $\beta_j := \alpha_j/(1-\alpha_j)$. Hereafter, when we refer to manufacturer j's bargaining power, we will be referring to the parameter β_j instead of α_j.

The following lemma, which compares the payoffs to two different manufacturers from the same assortment, follows directly from (9).

Lemma 1. *Consider assortment i and manufacturers $j, k \in A_i$. In any equilibrium,*

$$\frac{u_j^i}{\beta_j} > \frac{u_k^i}{\beta_k} \quad \Leftrightarrow \quad u_R^{b(j)} < u_R^{b(k)}.$$

Lemma 1 shows that a manufacturer's payoff from any given assortment is driven by two characteristics of the manufacturer. The first of these is the manufacturer's bargaining power (β_j for manufacturer j), which is exogenous to our model. The second characteristic is the attractiveness of the manufacturer's product, as captured by the best possible profit the retailer can make if it excludes the manufacturer (given by $u_R^{b(j)}$ for manufacturer j). This measure of "attractiveness" is endogenous to our model and it is an outcome of the equilibrium.

If a manufacturer's product is very attractive, in the sense that excluding that manufacturer leaves the retailer with a much less profitable assortment, then that manufacturer essentially brings a larger pie to split through the bilateral negotiation with the retailer. It then depends on the manufacturer's bargaining power (i.e., β_j), to what extent such a larger pie translates into a higher payoff for the manufacturer.

The next lemma compares the retailer's and manufacturer's payoffs from two different assortments. The proof of the lemma is provided in the appendix.

Lemma 2. *Consider any two assortments h and i. In any equilibrium,*
(a) $\pi^i > \pi^h \Leftrightarrow u_R^i > u_R^h$;
(b) *either* $u_j^i = u_j^h = 0$ *or* $\pi^i > \pi^h \Leftrightarrow u_j^i > u_j^h$.

Lemma 2 shows that if an assortment is better for the system, then it is better for the retailer and it is better for the manufacturers as well. Conversely, any assortment that makes either the retailer or any manufacturer strictly better off also makes the system strictly better off. In other words, the retailer's, manufacturers' and system's incentives are aligned (weakly in the case of manufacturers who receive zero profit from both assortments).

Given that in equilibrium the retailer's incentives are aligned with the system's incentives and given our earlier indexing of the assortments ($\pi^1 > \pi^2 > \cdots > \pi^{2^N}$), it now follows that $u_R^1 > u_R^2 > \cdots > u_R^{2^N}$ in equilibrium. That is, in equilibrium, lower-indexed assortments are better for both the system and the retailer. In addition, given the alignment of incentives, in equilibrium the retailer's best assortment without manufacturer j [$b(j)$] must be the same as the system-best assortment without manufacturer j [$B(j)$]. Figure 1 depicts how the assortments are ordered and the implications of this ordering.

The following lemma specifies the set of manufacturers that will receive a positive payoff from assortment i. In particular, the lemma states the intuitive property that, for manufacturer j to receive a positive payoff from assortment i, the manufacturer must be part of each and every assortment the retailer prefers to assortment i. In other words, if the retailer wants to do better than assortment i, then it must have no choice but to carry the product of manufacturer j. Otherwise, the retailer will not yield any profit from assortment i to manufacturer j, because the retailer could do better than assortment i if it simply ignored manufacturer j and chose an assortment from the remaining manufacturers.

Lemma 3. *In any equilibrium, for any assortment i, the set of manufacturers with positive payoff is*

$$P_i = \{j \in S\colon\ j \in A_h \text{ for all } h \leq i\}.^{1}$$

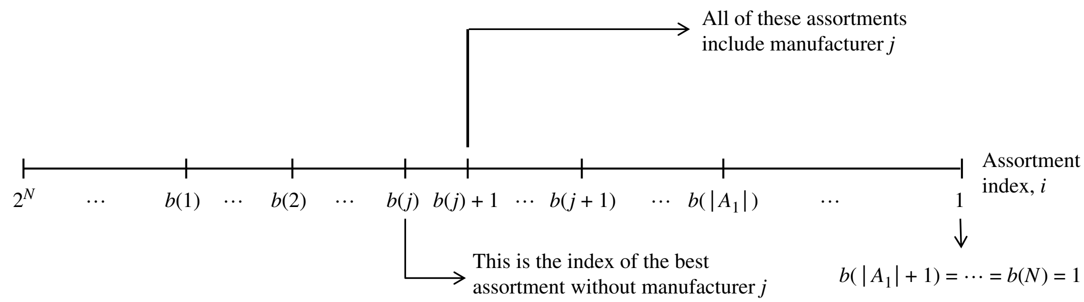
Given the manufacturer and assortment indexing we adopted, Corollaries 1–4, which we state next, follow from Lemma 3. Together these corollaries characterize which manufacturers receive positive payoff in assortment $i = 1, \ldots, 2^N$. Figure 2 provides a convenient way to visualize the results formally stated in these corollaries.

Corollary 1. *If $j \in A_1$, then $u_j^1 > 0$ in any equilibrium.*

Corollary 1 states that any manufacturer that is part of the system-optimal assortment (assortment 1) will receive a positive payoff from that assortment. As the retailer's incentives are aligned with the system's, we would expect the retailer to eventually select assortment 1, assuming that it is the retailer who ultimately

[1] Note that a manufacturer may be in assortment i and may still get zero equilibrium payoff from assortment i; this happens when the retailer could do better than assortment i by composing an assortment that does not include the manufacturer in question.

Figure 1 Illustration of Assortment Indexing



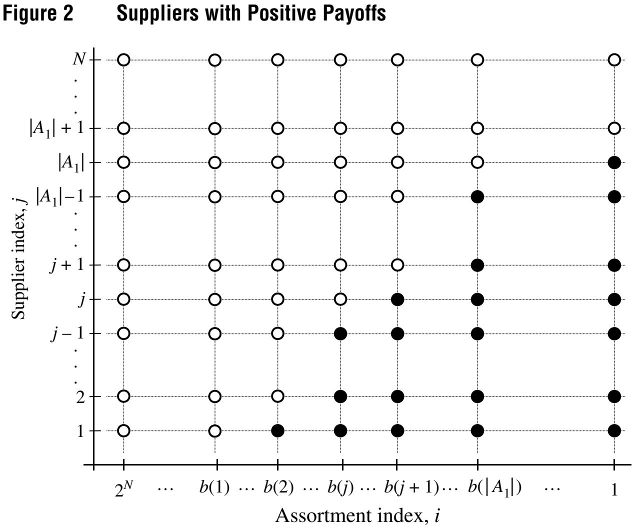
Notes. As we move from assortment 2^N to assortment 1, both system and retailer profits improve. Manufacturers are indexed so that the best assortment (assortment 1) consists of manufacturers 1 through $|A_1|$. Assortment $b(j)$ is the best assortment without manufacturer j, so all assortments with smaller indices include manufacturer j. For the manufacturers that are not in the system-best assortment (manufacturers $|A_1|+1$ through N), the best assortment without them is assortment 1.

decides which assortment to implement. All manufacturers that form part of that assortment will receive positive payoffs, as we would expect intuitively.

Corollary 2. *If $j \notin A_1$, then $u_j^i = 0$ for all i in any equilibrium.*

Corollary 2 states that any manufacturer that is *not* part of the system-optimal assortment (assortment 1) will receive zero payoff from *any* assortment. Intuitively, the retailer could do better by dropping any of these manufacturers and reverting to the system-optimal assortment. Hence, in equilibrium, the retailer will not yield any profit to these manufacturers from any assortment. On a related note, when modeling the profit allocation from an assortment, the Shapley value allocation (Shapley 1953) would yield an alternative to the Nash–Nash solution concept. However, as Lovejoy (2010b) also points out, the concept of the Shapley value seems ill-fitted to a setting such as ours because it determines each firm's allocation based on its contribution to *any* possible coalition (or assortment) and thus often allocates profits to firms that would obtain zero profits in practice. We specifically establish each firm's actual contribution in comparison to the best alternative assortment. Hence, manufacturers that are not part of the best possible assortment do not derive any profits.

Figure 2 Suppliers with Positive Payoffs



Note. A solid (blank) circle at the intersection of a manufacturer and assortment indicates that the manufacturer receives positive (zero) payoff in that assortment.

Corollary 3. *For assortment $i \geq b(1)$, we have $u_j^i = 0$ for all $j \in A_i$ and $u_R^i = \pi^i$ in any equilibrium.*

Corollary 3 summarizes an observation that will simplify our analysis in the rest of the section. Specifically, Corollary 3 focuses on the very poor assortments, indexed from 2^N to $b(1)$. In these assortments, the retailer could drop any one manufacturer from consideration, say manufacturer j, and the best assortment the retailer can compose without manufacturer j will still be at least as good as assortments 2^N through $b(1)$ (i.e., $b(j) \leq b(1) \leq 2^N$ for $j = 1, \ldots, N$). Intuitively, if the retailer offered one of assortments 2^N through $b(1)$, the retailer would have no reason to yield any profit to any manufacturer (because the retailer would be settling for an assortment that can be readily dominated after dropping any one manufacturer). Thus, according to Corollary 3, the retailer claims the entire system profit from assortments 2^N through $b(1)$ in equilibrium.

Corollary 4. *For any assortment i such that $b(j) > i \geq b(j+1)$, where $j = 1, \ldots, |A_1|$, the set of manufacturers with positive payoffs is $P_i = \{1, \ldots, j\}$ in any equilibrium.*

In contrast to Corollary 3, Corollary 4 focuses on better assortments. Specifically, Corollary 4 focuses on assortment i, which is better than assortments $b(1)$ through $b(j)$, that is, better than the best the retailer could do if it left out any one of manufacturers 1 through j. Hence, in assortment i, manufacturers 1 through j make an indispensable contribution and each one of them must receive positive payoff.

Equipped with these properties, which must be satisfied by any bargaining equilibrium, in the rest of the section, we establish the existence and uniqueness of a bargaining equilibrium.

3.2. Equilibrium Payoffs

In the previous subsection, we established a set of properties that must be satisfied by any equilibrium, but we have not yet shown that an equilibrium exists. In this subsection, we propose an allocation of profits motivated by the equilibrium properties. In §3.4 we will show that this particular allocation is indeed an equilibrium (i.e., the payoffs meet the conditions laid out in Definition 1) and it is unique under our assumptions.

Consider the following allocation of profits from assortment 2^N (the system-worst assortment) through assortment 1 (the system-best assortment); we discuss the rationale for it immediately after the statement of the allocation.

Profit Allocation 1. *Consider the following allocation of profits:*

(a) *For assortment i such that $2^N \geq i \geq B(1)$, $u_R^i = \pi^i$ and $u_j^i = 0$ for $j = 1, \ldots, N$.*

(b) *For assortment i such that $B(k) > i \geq B(k+1)$, where $k = 1, \ldots, |A_1|$,*[2]

$$u_R^i - u_R^{B(k)} = \frac{\pi^i - \pi^{B(k)}}{1 + \sum_{l=1}^k \beta_l},$$

$$u_j^i - u_j^{B(k)} = \beta_j \frac{\pi^i - \pi^{B(k)}}{1 + \sum_{l=1}^k \beta_l} \quad \text{for } j = 1, \ldots, k,$$

$$u_j^i = 0 \quad \text{for } j = k+1, \ldots, N.$$

First and foremost, this allocation is intended to make sure that if an assortment is strictly better for the system, then it is strictly better for the retailer—we know from the previous subsection that an equilibrium must satisfy this property. Indeed, if one follows the proposed allocation, then the retailer's profit strictly improves as one moves from assortment 2^N toward assortment 1. Hence, given the proposed allocation, "the retailer's best assortment without manufacturer j" (denoted by $b(j)$) is the same as "the system's best assortment without manufacturer j" (denoted by $B(j)$). In the rest of the subsection, we use $b(j)$ to refer to both.

Part (a) describes the allocation of profit from the poor assortments, namely assortments 2^N through $b(1)$, and it is motivated by the property we already noted in Corollary 3. Namely, those assortments are poor enough that the retailer could drop any one of the manufacturers and compose a better assortment with the other manufacturers. Hence, the retailer yields zero profit to the manufacturers and claims the entire system profit from the assortment.

[2] For notational convenience, we define $B(N+1) = 1$.

Part (b) describes the allocation of profit from the better assortments. In particular, consider an assortment i, which is more profitable than the best assortment without manufacturer k, i.e., $i < b(k)$, but not more profitable than the best assortment without manufacturer $k+1$, i.e., $i \geq b(k+1)$. First, observe from part (b) that the allocation assigns positive profits to manufacturers 1 through k. This is motivated by the property noted in Corollary 4. Namely, if the retailer dropped one of manufacturers 1 through k, it could not have composed as good an assortment as assortment i and, hence, those manufacturers must be rewarded with a positive profit for the indispensable contribution they make to assortment i. Second, the specific profit allocations in part (b) have an intuitive interpretation, which is easier to see for the case where the retailer and manufacturers have the same bargaining power, that is, when $\beta_j = 1$ for all $j = 1, \ldots, N$. For that special case, the allocation of profit from assortment i reduces to the following:

$$u_R^i - u_R^{b(k)} = \frac{\pi^i - \pi^{b(k)}}{1+k},$$

$$u_j^i - u_j^{b(k)} = \frac{\pi^i - \pi^{b(k)}}{1+k} \quad \text{for } j = 1, \ldots, k,$$

$$u_j^i = 0 \quad \text{for } j = k+1, \ldots, N.$$

According to the above allocation, when splitting the profit from assortment i, the retailer and manufacturers 1 through k are essentially splitting assortment i's incremental profit over assortment $b(k)$ (i.e., $\pi^i - \pi^{b(k)}$). In particular, each of these $k+1$ agents claims its payoff from assortment $b(k)$ plus an even share of assortment i's incremental profit over assortment $b(k)$. The even-split of the incremental profit is intuitive: All of manufacturers 1 through k are making an indispensable contribution to assortment i; dropping any one of them would lead to a worse assortment. Thus, if the retailer wishes to offer assortment i instead of assortment $b(k)$, it must successfully conclude its bilateral negotiation with each of manufacturers 1 through k. In those negotiations, for every dollar of incremental profit the retailer extracts, each of these manufacturers will also demand a dollar. Thus, the incremental profit must be shared evenly among those $k+1$ players.

Even though the retailer is obtaining an even share of assortment i's *incremental* profit over assortment $b(k)$, it will obtain a larger share of the *total* profit from assortment i. If one pictures the supply chain progressing from the worst possible assortment to the best possible assortment, the retailer claims a share from each and every improvement. In comparison, a manufacturer starts to claim a share of the improvements only after the supply chain reaches an assortment for which the manufacturer is indispensable.

Thus, the retailer's *total* profit will be larger than that of an individual manufacturer, owing to the retailer's ability to bid manufacturers against one another in its bilateral negotiations.

In essence, part (b) of Profit Allocation 1 follows the same intuition described above, but for the case where manufacturer j's bargaining power relative to the retailer is β_j—in that case manufacturer j receives β_j dollars for every dollar received by the retailer.

3.3. An Example

In this subsection, we illustrate Profit Allocation 1 using a numerical example with two manufacturers, and we show that the resulting allocation is a bargaining equilibrium.

Suppose that the retailer is negotiating with two manufacturers, whose bargaining powers are $\beta_1 = \beta_2 = 1$ (equivalently, $\alpha_1 = \alpha_2 = 1/2$). The four possible assortments, their system profits (which have been chosen arbitrarily and for expositional convenience), and the indexing of the assortments (which is consistent with the convention laid out earlier) are shown in Table 1.

For this example, we next compute the payoffs prescribed by Profit Allocation 1. Consider how different agents' payoffs change as we move from the system-worst assortment (assortment 4) to the system-best assortment (assortment 1).

In the system-worst assortment, which in this example happens to be the empty assortment $i = 4$, all agents receive zero payoff.

For assortment $i = 3$, which consists of manufacturer 2 only, part (a) of the profit allocation applies (because $3 = B(1)$), and we obtain $u_R^3 = \pi^3 = 10$ and $u_1^3 = u_2^3 = 0$. Intuitively, assortment 3 is poor enough that the retailer could drop any manufacturer in the assortment (in this case, manufacturer 2) and compose a better assortment from remaining manufacturers (in this case, manufacturer 1). Thus, the retailer yields no profit to manufacturer 2 from assortment 3 and claims the entire system profit.

For assortment $i = 2$, which consists of manufacturer 1 only, part (b) of the profit allocation applies (because $3 = B(1) > 2 = B(2)$). Intuitively, in assortment 2, the retailer must yield some profit to manufacturer 1, because the retailer could not compose as good an assortment without this manufacturer. Specifically, the retailer and manufacturer 1 receive equal shares of the incremental profit obtained by moving from assortment 3 to assortment 2. More precisely, applying part (b) with $i = 2$ and $k = 1$, and recalling that $B(1) = 3$, the retailer receives $u_R^2 = u_R^3 + (\pi^2 - \pi^3)/2 = 10 + (20 - 10)/2 = 15$, and manufacturer 1 receives $u_1^2 = u_1^3 + (\pi^2 - \pi^3)/2 = 0 + (20-10)/2 = 5$.

Table 1 **Assortments and Exogenously Fixed System Profits**

Assortment	Assortment index, i	System profit, π^i
{1, 2}	1	50
{1}	$2 = B(2)$	20
{2}	$3 = B(1)$	10
∅	4	0

Table 2 **Assortments, System Profits, and Payoffs Prescribed by Profit Allocation 1**

Assortment	Assortment index, i	System profit, π^i	u_R^i	u_1^i	u_2^i
{1, 2}	1	50	25	15	10
{1}	$2 = B(2) = b(2)$	20	15	5	0
{2}	$3 = B(1) = b(1)$	10	10	0	0
∅	4	0	0	0	0

For assortment $i = 1$, which consists of manufacturers 1 and 2, part (b) of the profit allocation applies (because $2 = B(2) > 1$). If the retailer dropped any one of these two manufacturers, it would be worse off than under this assortment, so the retailer must yield some profit to both manufacturers. Specifically, each of the retailer, manufacturer 1 and manufacturer 2 receives an equal share of the incremental profit obtained by moving from assortment 2 to assortment 1. More precisely, applying part (b) with $i = 1$ and $k = 2$, and recalling that $B(2) = 2$, the retailer receives $u_R^1 = u_R^2 + (\pi^1 - \pi^2)/3 = 15 + (50 - 20)/3 = 25$, manufacturer 1 receives $u_1^1 = u_1^2 + (\pi^1 - \pi^2)/3 = 5 + (50 - 20)/3 = 15$, and manufacturer 2 receives $u_2^1 = u_2^2 + (\pi^1 - \pi^2)/3 = 0 + (50 - 20)/3 = 10$. Table 2 summarizes the payoffs we computed above.

Next, we verify that the above profit allocation is indeed a bargaining equilibrium. As described in Definition 1, a bargaining equilibrium "is a set of payoffs, u_R^i and u_j^i for all manufacturers $j \in \{1, 2, \ldots, N\}$, that simultaneously solve maximization problem (5) for each assortment $i = 1, \ldots, 2^N$ subject to the constraint (6)." In what follows, we take the payoffs listed in Table 2 as given, and we confirm that they solve the maximization problems posed in Definition 1.

For assortment $i = 3$, Definition 1 requires that the payoffs solve the following maximization problem:

$$\max_{(u_R^3,\, u_2^3) \in \mathcal{A}(\mathbf{U}_{-2}^3)} (u_R^3 - u_R^{b(2)})^{1/2} (u_2^3)^{1/2},$$

where $b(2)$ denotes the retailer's best assortment without manufacturer 2. For the payoffs to be feasible (i.e., for the condition $(u_R^3, u_2^3) \in \mathcal{A}(\mathbf{U}_{-2}^3)$ to hold), we must have $u_R^3 \geq 0$, $u_2^3 \geq 0$, $u_1^3 = 0$ and $u_R^3 + u_2^3 = \pi^3 = 10$. In addition, if one follows the allocation summarized in Table 2, then $b(2)$ is given by assortment 2, and the retailer's payoff from that assortment is $u_R^{b(2)} = u_R^2 = 15$. Thus, the above maximization problem reduces to

$$\max_{u_R^3,\, u_2^3:\, u_R^3 \geq 0,\, u_2^3 \geq 0,\, u_R^3 + u_2^3 = 10} (u_R^3 - 15)^{1/2} (u_2^3)^{1/2}.$$

Observe that the objective function's maximum value is zero, which is attained by setting $u_2^3 = 0$ and thus

$u_R^3 = 10$. This is exactly what the allocation in Table 2 does.

Likewise, for assortment $i = 2$, the payoffs must solve the following maximization problem:

$$\max_{(u_R^2,\, u_1^2) \in \mathcal{A}(\mathbf{U}_{-1}^2)} (u_R^2 - u_R^{b(1)})^{1/2} (u_1^2)^{1/2},$$

where $b(1)$ is the retailer's best assortment without manufacturer 1, and the feasibility condition $(u_R^2, u_1^2) \in \mathcal{A}(\mathbf{U}_{-1}^2)$ translates into $u_R^2 \geq 0$, $u_1^2 \geq 0$, $u_2^2 = 0$ and $u_R^2 + u_1^2 = \pi^2 = 20$. Following the allocation in Table 2, the retailer's payoff from assortment $b(1)$ is $u_R^{b(1)} = u_R^3 = 10$. Thus, the maximization problem reduces to

$$\max_{u_R^2,\, u_1^2:\, u_R^2 \geq 0,\, u_1^2 \geq 0,\, u_R^2 + u_1^2 = 20} (u_R^2 - 10)^{1/2} (u_1^2)^{1/2}.$$

The objective function is the familiar "Nash product," which is maximized when the two agents evenly split what remains from the "pie" after each agent claims its disagreement payoff. In this case, the size of the pie is 20, the retailer's disagreement payoff is 10, and manufacturer 1's disagreement payoff is 0. Thus, the maximum value of the objective function is attained by setting $u_R^2 = 15$ and $u_1^2 = 5$, which is consistent with the allocation in Table 2.

Finally, for assortment $i = 1$, the payoffs (u_R^1, u_1^1, and u_2^1) must solve the following maximization problems:

$$\max_{(u_R^1,\, u_1^1) \in \mathcal{A}(\mathbf{U}_{-1}^1)} (u_R^1 - u_R^{b(1)})^{1/2} (u_1^1)^{1/2},$$

$$\max_{(u_R^1,\, u_2^1) \in \mathcal{A}(\mathbf{U}_{-2}^1)} (u_R^1 - u_R^{b(2)})^{1/2} (u_2^1)^{1/2}.$$

Consider the first of these two maximization problems, which captures the bilateral negotiation between the retailer and manufacturer 1 over assortment 1. The feasibility condition $(u_R^1, u_1^1) \in \mathcal{A}(\mathbf{U}_{-1}^1)$ translates into $u_R^1 \geq 0$, $u_1^1 \geq 0$, $u_2^1 \geq 0$ and $u_R^1 + u_1^1 + u_2^1 = \pi^1 = 50$. If we follow the allocation in Table 2, manufacturer 2 receives a payoff of 10 from assortment 1, so the "pie" that the retailer and manufacturer 1 are sharing in this bilateral negotiation over assortment 1 is $\pi^1 - u_2^1 = 50 - 10 = 40$. In addition, following the allocation in Table 2, we have $u_R^{b(1)} = 10$. Therefore, the first maximization problem reduces to

$$\max_{u_R^1,\, u_1^1:\, u_R^1 \geq 0,\, u_1^1 \geq 0,\, u_R^1 + u_1^1 = 40} (u_R^1 - 10)^{1/2} (u_1^1)^{1/2}.$$

Once again, the objective function is a Nash product, which is maximized when the retailer and manufacturer 1 evenly split what remains from the pie after each agent claims its disagreement payoff. In this case, the pie is 40, the retailer's disagreement payoff is 10, and manufacturer 1's disagreement payoff is 0. Thus, the maximum value of the objective function is attained by setting $u_R^1 = 25$ and $u_1^1 = 15$, as prescribed by the allocation in Table 2. Similarly, one can verify that the allocation in Table 2 solves the second maximization problem as well.

3.4. Characterizing the Equilibrium

In the preceding subsection, we used a small numerical example to illustrate that the proposed profit allocation is a bargaining equilibrium. The following theorem establishes that the proposed profit allocation always satisfies the conditions of Definition 1 and, hence, yields an equilibrium, which is shown to be unique under our assumptions. Furthermore, the theorem specifies an expression for the retailer's profit for any given assortment.[3]

Theorem 1. *Profit Allocation* 1 *yields the unique bargaining equilibrium, where the retailer's profit from assortment i is given by*

$$u_R^i = \sum_{j=0}^{|P_i|-1} \frac{\pi^{b(j+1)} - \pi^{b(j)}}{1 + \sum_{k=1}^{j} \beta_k} + \frac{\pi^i - \pi^{b(|P_i|)}}{1 + \sum_{k=1}^{|P_i|} \beta_k}. \qquad (10)$$

A couple remarks are in order regarding the computation of the equilibrium characterized in Theorem 1. It is worth noting that the above is a closed-form expression for the retailer's profit. While these expressions involve the $b(j)$ and $|P_i|$ constructs, note that their derivation requires only the knowledge of the system profits π^i. By definition, assortment $B(j)$ is simply the system-best assortment without manufacturer j, and the result in Lemma 2 implies that $b(j) = B(j)$. Given the $b(j)$, Corollary 4 directly establishes the $|P_i|$ (the number of suppliers who receive positive profit from assortment i) for any given assortment i.

Even with system profits from different assortments given, the complexity of determining the payoffs under the bargaining equilibrium is at least $O(N \cdot 2^N)$, as it is necessary to specify payoffs for $N+1$ agents from 2^N possible assortments. While the complexity of determining the outcomes for each of the agents is not polynomial, we expect that in practice one could often utilize the structure of the specific cost, revenue, and demand models to render computations more efficient. Even in the worst-case, the computational burden might not be insurmountable, if the number of products considered in the assortment is not too large. For example, Draganska et al. (2010) included seven brands of coffee in their empirical study of where the bargaining power lies in the supply chain.

Given that our characterization of the equilibrium allocation requires only the knowledge of system profits, it is fairly easy to obtain comparative statics, which highlight the effect of important parameters on the equilibrium. In particular, the next corollary states that any increase in a manufacturer's bargaining power (weakly) improves that manufacturer's equilibrium profits and (weakly) worsens the profits of all other parties.

[3] For notational convenience, we define $\pi^{b(0)} = 0$ in the statement of the theorem. This expression is derived by using an inductive approach.

Corollary 5. *If β_j increases, u_j^i (weakly) increases and u_a^i, $a \in \{R, 1, \dots, N\}/\{j\}$ (weakly) decreases for any given assortment i.*

4. Applications and Implications

The framework we described so far is general enough to accommodate many demand/supply models frequently adopted in the operations management literature on assortment planning. In this section, we focus on a specific model of demand and supply to illustrate the application of our framework. To that end, we first describe the model, followed by a description of the equilibrium results obtained under that model. Throughout this section, we use the term "product j" to refer to manufacturer j's product.

4.1. A Retailer Facing Logit Demand with Uncertainty

Consider a retailer who is a price-setting newsvendor. More specifically, given assortment i, the retailer chooses the inventory level and the price of each product $j \in A_i$. Let y_j^i and p_j^i denote, respectively, the inventory level and the unit retail price of product j the retailer chooses if it offers assortment i. Let $\mathbf{y}^i$ and $\mathbf{p}^i$ denote the corresponding vectors. Once the retailer makes its inventory and pricing decisions, the manufacturers produce the quantities ordered by the retailer. The manufacturer of product j incurs a unit production cost of c_j. In addition, we assume that whenever product j is included in the assortment, the system incurs a fixed cost K_j, which may be incurred by the retailer, the manufacturer, or the two of them combined. After the retailer receives the orders it placed with the manufacturers, the stochastic demand for product j, denoted by $D_j(\mathbf{p}^i, A_i)$, is realized. Suppose that $D_j(\mathbf{p}^i, A_i)$ is a multiplicative perturbation of an expected demand, which is a function of all the products included in the assortment and their prices. More precisely,

$$D_j(\mathbf{p}^i, A_i) = \lambda q_j(\mathbf{p}^i, A_i)\epsilon_j,$$

where λ is the market size, $q_j(\mathbf{p}^i, A_i)$ is the market share of product j given assortment i and its vector of prices $\mathbf{p}^i$, and the ϵ_j's are independent normal random variables with a mean of one and standard deviation of σ_j. We use the logit model to capture the market share as is frequently done in the operations management literature on assortment planning. The logit model is motivated by the multinomial logit consumer choice model and posits that product j's market share is given by

$$q_j(\mathbf{p}^i, A_i) = \frac{\exp(v_j - p_j^i)}{1 + \sum_{k \in A_i} \exp(v_k - p_k^i)},$$

where v_j can be interpreted as a metric for the attractiveness of product j.

After the random demands are realized, the excess demand for a product is backordered and the retailer incurs a cost of b_j per unit of backordered demand for product j. The leftover inventory is disposed, and the system incurs a net cost of h_j per unit of leftover inventory of product j (which may be higher or lower than c_j, depending on whether the disposal incurs additional cost or accrues salvage value). The system profit from assortment i is then given by

$$\begin{aligned}\pi^i(\mathbf{y}^i, \mathbf{p}^i) = \sum_{j \in A_i} \{ &(p_j^i - c_j)E[D_j(\mathbf{p}^i, A_i)] \\ &- b_j E[D_j(\mathbf{p}^i, A_i) - y_j^i]^+ \\ &- h_j E[y_j^i - D_j(\mathbf{p}^i, A_i)]^+ - K_j \}.\end{aligned}$$

For any assortment i the retailer has no incentive to deviate from the price vector $\mathbf{p}^i$ and the inventory vector $\mathbf{y}^i$ that maximize the system profit from assortment i (see Lemma 2). Hence, in this example, we fix the prices and inventory levels at the system-optimal level.

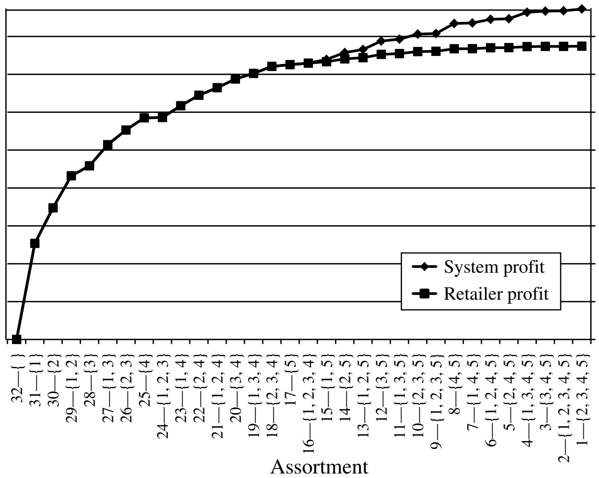
Of course, in the inventory control literature, there are more sophisticated models for managing the inventory of substitutable products. For example, see Nagarajan and Rajagopalan (2008) for a model that accommodates the random allocation of random aggregate demand, which also allows for substitutions upon stockouts.

4.2. Equilibrium Profit Allocation

Using the model described in §4.1, we now illustrate the profit allocations in equilibrium through a numerical example. Suppose that the retailer is bargaining with five manufacturers. For the numerical example, we use the following parameter values: $\lambda = 100$, $\alpha_1 = 0.8$, $\alpha_2 = 0.7$, $\alpha_3 = 0.7$, $\alpha_4 = 0.6$, $\alpha_5 = 0.6$, and for $j = 1, \dots, 5$: $v_j = j + 1$, $c_j = j/2$, $K_j = j + 4$, $\sigma_j = 0.3$, $b_j = 9$, $h_j = 1$.

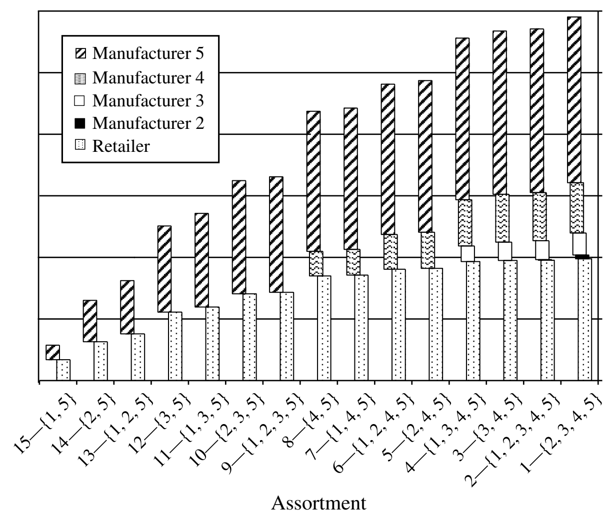
Given that there are five manufacturers to choose from, the equilibrium should specify the profit allocations in $2^5 = 32$ different assortments (one being the empty assortment). Figure 3 shows the system profit and the retailer's portion of it in each of assortments 1–32 (assortment 32 is the empty assortment, in which case there is no profit to be allocated)—the indexing of the assortments is such that lower-indexed assortments yield higher system profits. First, we note that in assortments 16–31, the retailer will claim the entire system profit. This happens because, in each of assortments 16 through 31, the retailer could drop any one of the manufacturers and compose a better assortment with the remaining manufacturers. Therefore, no manufacturer can claim any

Figure 3 System and Retailer Profits Across Assortments



Note. The *x*-axis shows the assortment index along with the set of products included in that assortment; for example, assortment 19 consists of manufacturers 1, 3, and 4.

Figure 4 Allocation of Profits in Assortments 1–15



Note. In assortments 16–31, the retailer claims the entire system profit.

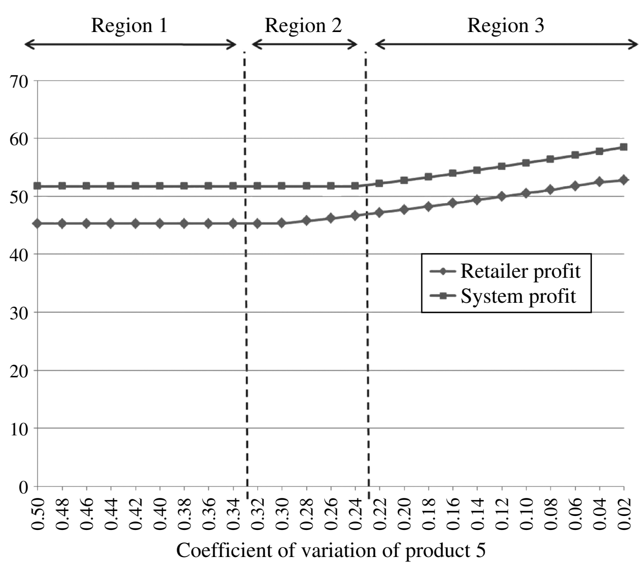
part of the system profit in assortments 16–31. Furthermore, in this example, the best assortment consists of products 2–5. Consequently, manufacturer 1 receives zero profit in any assortment because the retailer could always do better after leaving out that manufacturer's product. Observe from the figure that if the retailer wants to do better than assortment 16, then it must carry the product of manufacturer 5. Hence, the retailer must yield positive profit to manufacturer 5 in assortments 15 and better. Likewise, if the retailer wants to do better than assortment 9, then it must carry the product of manufacturer 4 and, hence, the retailer must yield positive profit to manufacturer 4 in assortments 8 and better. In fact, Figure 4 shows how the system profit is allocated among the retailer and the manufacturers in assortments 1–15.

Assuming that the retailer is the party that ultimately decides what assortment to offer, we expect that the assortment that will be eventually offered is the best assortment (assortment 1), which consists of manufacturers 2–5.

4.3. Effect of Demand Variability

In this subsection, we focus on the system's, retailer's, and manufacturers' equilibrium profits from the best assortment. To illustrate the nontrivial effect of model parameters, we focus on one of the most important parameters from an operational perspective—demand variability. In the model described in §4.1, the demand's coefficient of variation for product j is given by σ_j, which is the standard deviation of the random variable ϵ_j (recall that $E(\epsilon_j) = 1$). Hence, to examine the effect of demand variability, we use a numerical example in which we vary σ_j. In particular, we consider a retailer who is bargaining with five manufacturers, each of whom offers a single product. In Figure 5 we show the retailer's equilibrium profit from the best assortment as product 5's coefficient of variation, σ_5, changes from 0.5 to 0.02. The rest of the parameter values are as follows: $\lambda = 100$, $\alpha_1 = 0.8$, $\alpha_2 = 0.7$, $\alpha_3 = 0.7$, $\alpha_4 = 0.6$, $\alpha_5 = 0.6$, $v_j = 0.5(j+1)$, $c_j = 0.4j$, $K_j = 3j$, $b_j = 9$, $h_j = 1$ for $j = 1, \ldots, 5$ and $\sigma_j = 0.3$ for $j = 1, \ldots, 4$. As product 5's demand becomes less variable, we observe three distinct regions. In region 1, the retailer's best assortment includes products (manufacturers) 1–4. In this region, the demand for product 5 is so unpredictable (i.e., the coefficient of variation, σ_5, is so large) that product 5 does not enter the best assortment and influences neither the system profit from the best assortment nor the retailer's share of it.

Figure 5 Retailer and System Profits from the Best Assortment



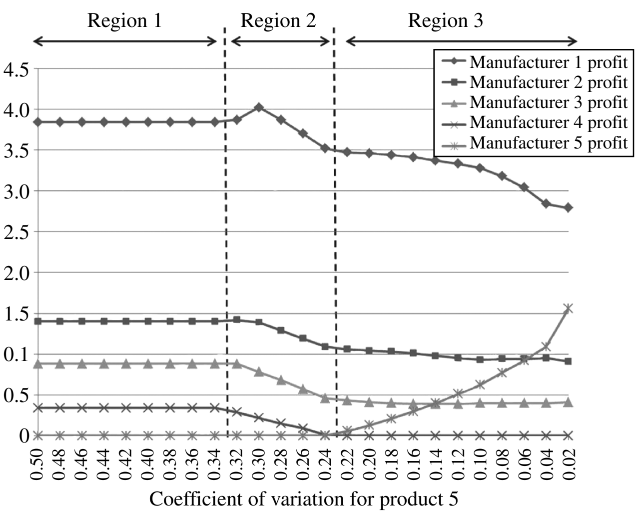
In region 2, σ_5 is still large enough that product 5 is not part of the best assortment. Hence, in region 2, the system profit from the best assortment remains constant as σ_5 continues to decrease. Unlike the system profit, however, the retailer's profit from the best assortment does increase as σ_5 further decreases in region 2. In other words, an improvement to an out-of assortment product (product 5) improves the retailer's profit from the assortment. This happens because, once we reach region 2, product 5 is attractive enough that it would enter the assortment if product 2, 3, or 4 were dropped. As σ_5 decreases further, product 5 becomes more attractive and provides a superior bargaining chip to the retailer in its negotiations with the manufacturers of these products.

In region 3, the demand for product 5 is so predictable that product 5 is attractive enough to enter the best assortment. In fact, in region 3, the best assortment consists of products 1–3 and 5 (product 4 drops out of the best assortment to save on the fixed costs). Of course, in this region, where product 5 is now part of the best assortment, any further reduction to σ_5 improves both the system and the retailer profit from the best assortment.

For the same example depicted in Figure 5, we show in Figure 6 the manufacturers' equilibrium profits from the best assortment as σ_5 changes from 0.5 to 0.02.

Consider first region 2. In region 2, an improvement to the demand variability of product 5 may hurt or help other manufacturers. Consider, for example, what happens when σ_5 is reduced from 0.32 to 0.3. Recall that, in region 2, the best assortment consists of all products except for product 5. Hence, reducing σ_5 from 0.32 to 0.3 has no effect on the system profit from the best assortment. Nevertheless, when σ_5 is reduced from 0.32 to 0.3, manufacturer 1 now claims a larger share of the system profit at the expense of manufacturers 2–4. In other words, an improvement to an out-of-assortment product (product 5) increases the profit of one manufacturer in the assortment (manufacturer 1) and decreases the profit of all other manufacturers in the assortment. How do we explain that σ_5's effect on manufacturer 1 is the opposite of its effect on manufacturers 2–4? The explanation lies in the best assortments the retailer could compose after dropping either of those manufacturers, which are shown in Table 3. Note that when σ_5 is 0.32 or 0.3, product 5 is in the best assortment the retailer could compose without products 2–4. Hence, product 5 can be used by the retailer to credibly threaten manufacturers 2–4, and the retailer can use any improvement to product 5 to extract more profit from those manufacturers. However, product 5 is not attractive enough to be in the best assortment the retailer could compose without product 1. Consequently, improving

Figure 6 **Manufacturers' Profits from the Best Assortment**



Note. The regions coincide with those in Figure 5.

product 5 has no effect on the retailer's bargaining position relative to manufacturer 1. Hence, when σ_5 changes from 0.32 to 0.3, the retailer squeezes additional profit out of manufacturers 2–4, and ends up sharing some of that additional profit with manufacturer 1.

Consider now the behavior in region 3. As explained earlier, product 5 is part of the best assortment in that region. Hence, any improvement to product 5 (which here takes the form of reduced σ_5) improves manufacturer 5's profit. As for the other manufacturers, their equilibrium profits may increase or decrease, depending on which one of two opposing effects dominates. On one hand, an improvement to product 5 improves the system profit, thus increasing the size of the pie to be shared. This increase may translate into higher profits for other manufacturers as well. On the other hand, the improvement to product 5 improves the bargaining position of manufacturer 5. Thus, manufacturer 5 may claim larger profit at the expense of other manufacturers. In the example depicted in Figure 6, the latter effect dominates in much of region 3, thus reducing the profits of manufacturers 1–3 as σ_5 improves from 0.22 to 0.04. However, it is worth noting that the former effect dominates for manufacturer 3 at the tail end of region 3

Table 3 **Best Assortment That Could Be Composed Without Each of Manufacturers 1–4, When σ_5 Is 0.32 or 0.3**

j	$A^{b(j)}$
1	2, 3, 4
2	1, 3, 4, 5
3	1, 2, 4, 5
4	1, 2, 3, 5

and manufacturer 3's profit slightly improves as σ_5 improves from 0.04 to 0.02.

Note that, while our discussion here was on the effects of a decrease in a product's demand variability, similar observations apply if a product's attractiveness improves for other reasons, for example, due to cost reductions or if customers associate a higher expected utility with the product.

5. Conclusion and Discussion

Assortment selection decisions are a key driver of retailer profitability. Negotiations between retailers and manufacturers over the composition of assortments are very involved and of flexible form, allowing an almost arbitrary allocation of profits among different parties. Since retailers are in control of the final assortment decision, they can bid manufacturers against one another, threatening any given manufacturer with an alternative assortment composed solely of other manufacturers' products. In this paper, we adopt a bargaining framework to capture the complex interactions that arise in the negotiation of such an assortment selection process. We derive closed-form solutions that define the profit allocations for each party and for each possible assortment. We also illustrate how the outcomes of the negotiation process are affected by changes to the characteristics of an individual product, considering an application where the demands for the different products in an assortment are derived from a multinomial logit model and the retailer is a price-setting newsvendor.

Our model produces actionable insights for the retailer and manufacturers. The retailer should keep in mind that it always benefits from having a larger pie to share. Therefore, the retailer should actively seek low-cost opportunities to improve its manufacturers' products and operations. For example, if timely sharing of detailed advertising or point of sale information improves a manufacturer's demand forecasting, some of the additional profit will find its way to the retailer through the bargaining process. In addition, the retailer should be willing to help even those manufacturers who are currently not being carried by the retailer. For example, it would be a good idea for the retailer to engage out-of-assortment manufacturers and to provide guidance as to improvements that could qualify those manufacturers for the retailer's assortment. Improvements to such manufacturers provide the retailer with a more advantageous threat point in its negotiations with other manufacturers.

As for the manufacturer, interestingly, there might be gains from cooperating with competitors. An improved competitor has two effects on a manufacturer: (1) The competitor becomes a more attractive fallback option for the retailer, thus eroding the manufacturer's payoff; (2) the system profit becomes larger, thus improving the manufacturer's payoff. In sum, if the second effect dominates the first, the manufacturer may indeed benefit from an improved competitor. Similarly, an in-assortment manufacturer would improve its profit by collaborating with an out-of-assortment manufacturer, if the out-of-assortment manufacturer is a good alternative for an in-assortment competitor. Such a collaboration improves the out-of-assortment manufacturer, thus forcing the in-assortment competitor to yield more profit to the retailer, who would then have to share some of that improvement with the in-assortment manufacturer.

Acknowledgments

The authors thank Serguei Netessine (the department editor), the associate editor, and two reviewers for their valuable comments and suggestions, from which the paper benefited greatly.

Appendix

Proof of Lemma 2

Proof of (a). The proof is by contradiction. Suppose $\pi^i > \pi^h$ and assume that $u_R^i \leq u_R^h$. The total surplus to be split is larger for assortment i. In the following we show that the aggregate payments to manufacturers for assortment h are not smaller than those for assortment i, implying $u_R^i > u_R^h$, thus leading to a contradiction. For $u_R^i \leq u_R^h$, (9) implies that $u_j^i \leq u_j^h$ for all $j \in A_i \cap A_h$. Manufacturers $j \in A_i \backslash A_h$ receive a payoff $u_j^i = 0$, since $u_R^h \geq u_R^i$ implies $u_R^{b(j)} \geq u_R^h \geq u_R^i$. Manufacturers $j \in A_h \backslash A_i$ do not receive any payoff from assortment i ($u_j^i = 0$). Thus, $\pi^i > \pi^h \Rightarrow u_R^i > u_R^h$. The equivalency in Lemma 2 then follows from another simple proof by contradiction: assuming $u_R^i > u_R^h$, $\pi^i < \pi^h$ cannot be true as the latter would imply $u_R^i < u_R^h$. (Recall that $\pi^i \neq \pi^h$ by assumption.)

Proof of (b). Suppose $\pi^i > \pi^h$. Note that, by virtue of part (a), we also have $u_R^i > u_R^h$. Consider the case where $u_j^h > 0$. (The result for $u_j^h = 0$ holds trivially.) In this case, it must be that $j \in A_i$. (Otherwise, we would have $u_R^{b(j)} \geq u_R^i > u_R^h$, which along with (9) would imply that $u_j^h = 0$, leading to a contradiction.) Now, given that $j \in A_i$ and $u_R^i > u_R^h$, it follows from (9) that $u_j^i > u_j^h$. □

Proof of Lemma 3

Consider manufacturer j in assortment i. We first prove that if $j \in A_h$ for all $h < i$, then manufacturer j receives positive payoff from assortment i. To that end, first note that $b(j) > i$ (given that manufacturer $j \in A_h$ for all $h \leq i$). Hence, $\pi^{b(j)} < \pi^i$. By Lemma 2(a), it then follows that $u_R^{b(j)} < u_R^i$. This fact, along with (9), implies that $u_j^i > 0$, that is, manufacturer j receives positive payoff from assortment i.

We next prove that if $j \notin A_h$ for some $h < i$, then manufacturer j receives zero payoff from assortment i. In this case, note that $b(j) < i$ (given that there is an assortment $h < i$ such that assortment h does not include manufacturer j). Hence, $\pi^{b(j)} > \pi^i$. Once again invoking Lemma 2(a), it follows that $u_R^{b(j)} > u_R^i$. Hence, by (9), we must have $u_j^i = 0$, that is, manufacturer j receives positive payoff from assortment i.

Thus, we have shown the following: Manufacturer j in assortment i receives positive payoff from assortment i only if $j \in A_h$ for all $h < i$, concluding the proof. □

Proof of Theorem 1

We prove three different claims: (i) Profit Allocation 1 yields a bargaining equilibrium; i.e., it satisfies the conditions laid out in Definition 1. (ii) The bargaining equilibrium is unique. (iii) The expression in the statement of the theorem yields the retailer's profit from assortment i in equilibrium.

Proof of (i). First, given Profit Allocation 1 and the assumption that $\pi^1 > \pi^2 > \cdots > \pi^{2^N}$, note that $u_R^1 > u_R^2 > \cdots > u_R^{2^N}$. Hence, the retailer's best assortment without manufacturer j is given by the system's best assortment without manufacturer j, i.e., $b(j) = B(j)$. Throughout the proof, we use $b(j)$ to refer to both.

Next, we prove that for each assortment i and each manufacturer $j \in A_i$, u_R^i and u_j^i given by Profit Allocation 1 satisfy the conditions in Definition 1. To do so, we must show that

(a) u_R^i and u_j^i are in the feasible set $\mathcal{A}(\mathbf{U}_{-j}^i)$, where $\mathbf{U}_{-j}^i$ is the vector of payoffs from assortment i for all manufacturers other than j;

(b) u_R^i and u_j^i solve the optimization problem in (5).

We divide the proof into two cases, depending on the value of i:

Case 1: $i \geq b(1)$. For such an assortment, the proposed allocation sets $u_R^i = \pi^i$ and $u_j^i = 0$ for $j = 1, \ldots, N$. Hence, u_R^i and u_j^i, $j = 1, \ldots, N$ satisfy the conditions $u_R^i \geq 0$, $u_j^i \geq 0$, $u_R^i + u_j^i + \sum_{m \neq j} u_m^i = \pi^i$, and $u_j^i = 0$ for $j \notin A_i$. Therefore, for any manufacturer $j \in A_i$, we have $(u_R^i, u_j^i) \in \mathcal{A}(\mathbf{U}_{-j}^i)$. This proves (a).

To prove (b), first note that $i \geq b(1) \geq b(j)$ for $j = 1, \ldots, N$. Hence, we have $u_R^i \leq u_R^{b(1)} \leq u_R^{b(j)}$ for $j = 1, \ldots, N$. Because $u_R^i \leq u_R^{b(j)}$, it now follows that the maximum value one can feasibly attain for the objective function of (5) is zero. Setting $u_j^i = 0$ attains this maximum and, hence, solves the optimization problem (5).

Case 2: $b(k) > i \geq b(k+1)$ for $k = 1, \ldots, |A_1|$. For assortment i such that $b(k) > i \geq b(k+1)$, the proposed allocation of profits is as follows:

$$\begin{aligned} u_R^i - u_R^{b(k)} &= \frac{\pi^i - \pi^{b(k)}}{1 + \sum_{l=1}^k \beta_l}, \\ u_j^i - u_j^{b(k)} &= \beta_j \frac{\pi^i - \pi^{b(k)}}{1 + \sum_{l=1}^k \beta_l} \quad \text{for } j = 1, \ldots, k, \\ u_j^i &= 0 \quad \text{for } j = k+1, \ldots, N. \end{aligned} \tag{11}$$

Note that u_R^i and u_j^i, $j = 1, \ldots, N$ satisfy the conditions $u_R^i \geq 0$, $u_j^i \geq 0$, and $u_j^i = 0$ for $j \notin A_i$. In addition, the condition $u_R^i + u_j^i + \sum_{m \neq j} u_m^i = \pi^i$ is also satisfied because

$$\begin{aligned} &u_R^i + u_j^i + \sum_{m \neq j} u_m^i \\ &\quad = u_R^i + \sum_{m=1}^k u_m^i \quad [\text{by } u_m^i = 0 \text{ for } m = k+1, \ldots, N] \\ &\quad = u_R^{b(k)} + \frac{\pi^i - \pi^{b(k)}}{1 + \sum_{l=1}^k \beta_l} + \sum_{m=1}^k \left(u_m^{b(k)} + \beta_m \frac{\pi^i - \pi^{b(k)}}{1 + \sum_{l=1}^k \beta_l} \right) \\ &\qquad [\text{by substituting for } u_R^i \text{ and } u_m^i \text{ from (11)}] \\ &\quad = u_R^{b(k)} + \sum_{m=1}^k u_m^{b(k)} + (\pi^i - \pi^{b(k)}) \end{aligned} \tag{12}$$

Now, for assortment $i = b(k)$, the proposed allocation of profits yields

$$\begin{aligned} u_R^{b(k)} - u_R^{b(k-1)} &= \frac{\pi^{b(k)} - \pi^{b(k-1)}}{1 + \sum_{l=1}^{k-1} \beta_l}, \\ u_j^{b(k)} - u_j^{b(k-1)} &= \beta_j \frac{\pi^{b(k)} - \pi^{b(k-1)}}{1 + \sum_{l=1}^{k-1} \beta_l} \quad \text{for } j = 1, \ldots, k-1, \\ u_j^i &= 0 \quad \text{for } j = k, \ldots, N. \end{aligned} \tag{13}$$

Substituting the expressions for $u_R^{b(k)}$ and $u_m^{b(k)}$ from (13) in (12), we obtain

$$\begin{aligned} &u_R^i + u_j^i + \sum_{m \neq j} u_m^i \\ &\quad = u_R^{b(k-1)} + \frac{\pi^{b(k)} - \pi^{b(k-1)}}{1 + \sum_{l=1}^{k-1} \beta_l} \\ &\qquad + \sum_{m=1}^{k-1} \left(u_m^{b(k-1)} + \beta_m \frac{\pi^{b(k)} - \pi^{b(k-1)}}{1 + \sum_{l=1}^{k-1} \beta_l} \right) + (\pi^i - \pi^{b(k)}) \\ &\quad = u_R^{b(k-1)} + \sum_{m=1}^{k-1} u_m^{b(k-1)} + (\pi^{b(k)} - \pi^{b(k-1)}) + (\pi^i - \pi^{b(k)}) \\ &\quad = \cdots = u_R^{b(1)} + u_1^{b(1)} + (\pi^{b(2)} - \pi^{b(1)}) + \cdots \\ &\qquad + (\pi^{b(k)} - \pi^{b(k-1)}) + (\pi^i - \pi^{b(k)}) \quad [\text{by repeated} \\ &\qquad \text{substitutions for } u_R^{b(k-1)}, \ldots, u_R^{b(2)} \text{ and } u_m^{b(k-1)}, \ldots, u_m^{b(2)}] \\ &\quad = \pi^i \quad [\text{because } u_R^{b(1)} = \pi^{b(1)} \text{ and } u_1^{b(1)} = 0]. \end{aligned}$$

Therefore, for any manufacturer $j \in A_i$, we have $(u_R^i, u_j^i) \in \mathcal{A}(\mathbf{U}_{-j}^i)$. This proves (a).

To prove (b), we will first show that the proposed values for u_R^i and u_j^i solve the optimization problem (5) for $j = 1, \ldots, k$. Given that the pair $(u_R^i, u_j^i) \in \mathcal{A}(\mathbf{U}_{-j}^i)$ (which makes sure that the profits allocated to all agents add up to the system profit π^i), we know from GNBS that the (u_R^i, u_j^i) pair that solves the optimization problem (5) is the unique pair that satisfies $(u_R^i - u_R^{b(j)})/(1 - \alpha_j) = u_j^i/\alpha_j$. Recalling that $\beta_j = \alpha_j/(1 - \alpha_j)$, the optimal (u_R^i, u_j^i) pair is the unique pair that satisfies $\beta_j(u_R^i - u_R^{b(j)}) = u_j^i$. Observe from (11) that

$$\begin{aligned} u_j^i - u_j^{b(k)} &= \beta_j (u_R^i - u_R^{b(k)}), \\ u_j^{b(k)} - u_j^{b(k-1)} &= \beta_j (u_R^{b(k)} - u_R^{b(k-1)}), \\ &\cdots \\ u_j^{b(j+1)} - u_j^{b(j)} &= \beta_j (u_R^{b(j+1)} - u_R^{b(j)}). \end{aligned}$$

Summing all these equalities together yields $u_j^i - u_j^{b(j)} = \beta_j(u_R^i - u_R^{b(j)})$, which is the equality that should be satisfied by the (u_R^i, u_j^i) pair that solves optimization problem (5).

It remains to be shown that the proposed values for u_R^i and u_j^i solve the optimization problem (5) for $j = k+1, \ldots, N$ as well. To that end, note that $i \geq b(k+1) \geq b(j)$ for $j = k+1, \ldots, N$. Hence, we have $u_R^i \leq u_R^{b(j)}$ for $j = k+1, \ldots, N$. Therefore, for $j = k+1, \ldots, N$, the maximum value one can feasibly attain for the objective function of (5) is zero. Setting $u_j^i = 0$ for $j = k+1, \ldots, N$ attains this maximum and, hence, solves the optimization problem (5).

Proof of (ii). In part (i), we showed that Profit Allocation 1 is a bargaining equilibrium. Here, we show that, in any equilibrium, the payoffs are given by Profit Allocation 1. Thus, we will conclude the equilibrium is unique. Once again, we divide the proof into two cases, depending on the value of i:

Case 1: $i \geq b(1)$. First, note that, in any equilibrium, u_R^i, u_j^i and $u_R^{b(j)}$ must satisfy (9) for each assortment i and manufacturer $j \in A_i$. For assortment $i = 2^N, \ldots, b(1)$, we have $u_R^i \leq u_R^{b(j)}$ for $j = 1, \ldots, N$. Hence, the only way to satisfy (9) is to set $u_j^i = 0$ for $j = 1, \ldots, N$. This coincides with the payoffs in Profit Allocation 1.

Case 2: $b(k) > i \geq b(k+1)$ for $k = 1, \ldots, |A_1|$. Consider assortment i such that $b(k) > i \geq b(k+1)$ for some $k \in \{1, \ldots, |A_1|\}$. In assortment i, in equilibrium, we have $u_j^i = 0$ for $j = k+1, \ldots, N$ (by Corollary 4). For manufacturer $j = 1, \ldots, k$, on the other hand, using (9) and recalling the definition $\beta_j := \alpha_j/(1-\alpha_j)$, we have $u_j^i = \beta_j(u_R^i - u_R^{b(j)})$. Hence, in equilibrium, we can write the retailer's payoff from assortment i as follows:

$$u_R^i = \pi^i - \sum_{j=1}^{k} \beta_j (u_R^i - u_R^{b(j)}).$$

It follows from the above equation that

$$u_R^i = \frac{\pi^i + \sum_{j=1}^k \beta_j u_R^{b(j)}}{1 + \sum_{j=1}^k \beta_j}.$$

By adding and subtracting $u_R^{b(k)}$ to the right-hand side and rearranging terms, we obtain the following:

$$u_R^i = u_R^{b(k)} + \frac{\pi^i - u_R^{b(k)} + \sum_{j=1}^{k-1} \beta_j (u_R^{b(j)} - u_R^{b(k)})}{1 + \sum_{j=1}^k \beta_j}.$$

Now note that $\beta_j(u_R^{b(j)} - u_R^{b(k)}) = -u_j^{b(k)}$ (this follows from (9)). Using this observation, we can rewrite the right-hand side of the above equality:

$$u_R^i = u_R^{b(k)} + \frac{\pi^i - u_R^{b(k)} - \sum_{j=1}^{k-1} u_j^{b(k)}}{1 + \sum_{j=1}^k \beta_j}.$$

Since $u_R^{b(k)} + \sum_{j=1}^{k-1} u_j^{b(k)} = \pi^{b(k)}$, the retailer's profit from assortment i in equilibrium can be expressed as follows:

$$u_R^i = u_R^{b(k)} + \frac{\pi^i - \pi^{b(k)}}{1 + \sum_{j=1}^k \beta_j}.$$

Note that this payoff coincides with the retailer's payoff in Profit Allocation 1. Given the retailer's payoff and using (9), one can obtain expressions for the payoffs of manufacturers 1 through k as well—we omit this step. Those expressions also coincide with the payoffs in Profit Allocation 1. Hence, any equilibrium must satisfy Profit Allocation 1, and thus Profit Allocation 1 yields the unique equilibrium.

Proof of (iii). We divide the proof into two cases, depending on the value of i.

Case 1: $i \geq b(1)$. For assortment $i \geq b(1)$, recall from Corollary 3 that the retailer claims all the system profit, i.e., $u_R^i = \pi^i$. For such i, (10) also yields $u_R^i = \pi^i$. To see why, note that $P_i = \varnothing$ given that all manufacturers in assortment i receive zero payoff. Hence, $|P_i| = 0$. Substituting $|P_i| = 0$ in (10) and recalling that $\pi^{b(0)} = 0$ by definition, we verify that (10) yields $u_R^i = \pi^i$.

Case 2: $b(j+1) \leq i < b(j)$ for $j = 1, \ldots, |A_1|$. The proof is by induction on j. For $j = 1$, the retailer's payoff from assortment i such that $b(2) \leq i < b(1)$ is given by

$$\begin{aligned} u_R^i &= u_R^{b(1)} + \frac{\pi^i - \pi^{b(1)}}{1+\beta_1} \quad \text{by Profit Allocation 1} \\ &= \pi^{b(1)} + \frac{\pi^i - \pi^{b(1)}}{1+\beta_1} \quad \text{since } u_R^{b(1)} = \pi^{b(1)} \text{ by Corollary 3} \end{aligned}$$

We next verify that (10) also yields the above expression for u_R^i when i is such that $b(2) \leq i < b(1)$. For such i, it follows from Corollary 4 that $P_i = \{1\}$ and, hence, $|P_i| = 1$. Substituting $|P_i| = 1$ in (10) and recalling that $\pi^{b(0)} = 0$ by definition, we verify that (10) also yields

$$u_R^i = \pi^{b(1)} + \frac{\pi^i - \pi^{b(1)}}{1+\beta_1}.$$

Hence, the result holds for $j = 1$.

We make the *induction assumption* that the result holds when $j = h$ for some $h = 1, \ldots, |A_1| - 2$. In other words, for assortment i such that $b(h+1) \leq i < b(h)$, (10) in Theorem 1 yields the retailer's payoff from assortment i. To complete the induction, we next show that the result holds for $j = h+1$ as well. Consider assortment i such that $b(h+2) \leq i < b(h+1)$. By Profit Allocation 1, the retailer's payoff from assortment i is

$$u_R^i = u_R^{b(h+1)} + \frac{\pi^i - \pi^{b(h+1)}}{1 + \sum_{k=1}^{h+1} \beta_k}. \tag{14}$$

By the induction assumption, we can use (10) in Theorem 1 to obtain the following expression for $u_R^{b(h+1)}$—in writing this expression, we use the fact that $P_{b(h+1)} = \{1, \ldots, h\}$ (by Corollary 4) and, hence, $|P_{b(h+1)}| = h$:

$$u_R^{b(h+1)} = \sum_{j=0}^{h-1} \frac{\pi^{b(j+1)} - \pi^{b(j)}}{1 + \sum_{k=1}^{j} \beta_k} + \frac{\pi^{b(h+1)} - \pi^{b(h)}}{1 + \sum_{k=1}^{h} \beta_k} = \sum_{j=0}^{h} \frac{\pi^{b(j+1)} - \pi^{b(j)}}{1 + \sum_{k=1}^{j} \beta_k}.$$

Substituting the above expression for $u_R^{b(h+1)}$ in (14) yields

$$u_R^i = \sum_{j=0}^{h} \frac{\pi^{b(j+1)} - \pi^{b(j)}}{1 + \sum_{k=1}^{j} \beta_k} + \frac{\pi^i - \pi^{b(h+1)}}{1 + \sum_{k=1}^{h+1} \beta_k}.$$

Noting that $P_i = \{1, \ldots, h+1\}$ for $b(h+2) \leq i < b(h+1)$ (by Corollary 4), we have $|P_i| = h+1$. Hence, the expression above can also be written as

$$u_R^i = \sum_{j=0}^{|P_i|-1} \frac{\pi^{b(j+1)} - \pi^{b(j)}}{1 + \sum_{k=1}^{j} \beta_k} + \frac{\pi^i - \pi^{b(|P_i|)}}{1 + \sum_{k=1}^{|P_i|} \beta_k},$$

which is identical to (10) in Theorem 1. $\square$

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