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Optimal Energy Procurement in Spot and Forward Markets

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Spot and forward purchases for delivery on the usage date play an important role in matching the supply and the uncertain demand of energy because storage capacity for energy, such as electricity, natural gas, and oil, is limited. Transaction costs tend to be larger in spot than forward energy markets near maturity. Partially procuring supply in the forward market, rather than entirely in the spot market, is thus a potentially valuable real option, which we call the *forward procurement option*. We investigate the optimal value and management of this real option as well as their sensitivities to parameters of interest. Our research quantifies the value of the forward procurement option on realistic natural gas instances, also suggesting that procuring the demand forecast in the forward market is nearly optimal. This policy greatly simplifies the management of this real option without an appreciable loss of value. We provide some theoretical support for this numerical finding. Beyond energy, our research has potential relevance for the procurement of other commodities, such as metals and agricultural products.

Keywords: correlated price and demand uncertainty; energy and commodities; newsvendor model; OM–finance interface; procurement; real options; spot and forward markets; transaction costs; valuation
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1. Introduction

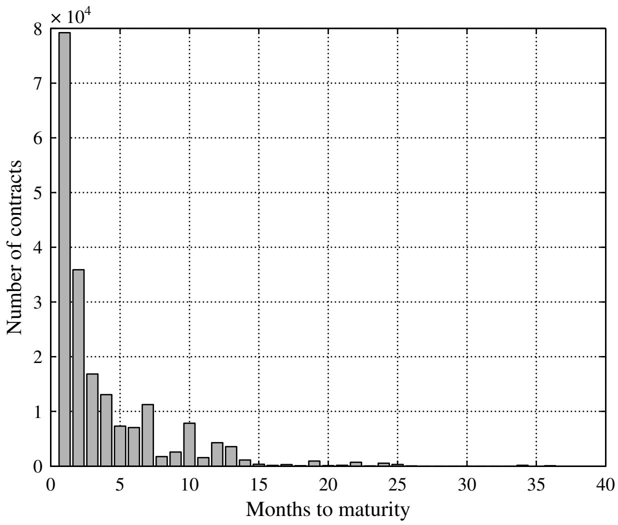
Energy, such as electricity, natural gas, and oil, plays a key economic role in supply chains because it constitutes a primary input to most industrial and commercial activities (Geman 2005). Spot and forward markets trade energy for immediate and future delivery, respectively. Storage capacity for energy is limited. For example, in 2009 the U.S. natural gas usable storage capacity was about 20% of annual demand (based on data from the U.S. Energy Information Administration; EIA 2011). Thus, procurement of supply in spot and forward markets for delivery on the usage date is important in matching a firm's uncertain demand for energy. Indeed, this research was motivated by our collaboration with an energy reselling company that operates without access to storage capacity.

Transaction costs in energy, and more generally commodity, markets tend to be higher for spot than forward trades near maturity. Specifically, these costs appear to be inversely related to trading volume, a feature for which there is both theoretical support and empirical evidence (Thompson and Waller 1988, Thompson et al. 1993, Bryant and Haigh 2004, Linnainmaa and Roşu 2009, Roşu 2009). Trading volume typically is thinner far away from the maturity of a futures (forward) contract and during its delivery period, that is, in the spot market, than for dates closer to this maturity—the reduced trading volume in the spot market reflects the reduced availability of the commodity for discretionary purposes in this market. Average transaction costs thus tend to be roughly U shaped in time to maturity (see, e.g., Figure 1 in Bryant and Haigh 2004). To partially illustrate this phenomenon, Figure 1 plots the average trading volume for the first 36 maturities of the NYMEX natural gas futures contract during the week of March 8, 2010. (This figure does not include data for spot trades.) This figure suggests that trading volume in NYMEX natural gas futures is higher for contracts with maturities that are closer to the trading date. Consistent with these observations, in their study of oil derivatives, Trolle and Schwartz (2009, p. 4434) state that "open interest for futures contracts tends to peak when expiration is a couple of weeks away, after which open interest declines sharply."

This feature of transaction costs in energy markets gives firms engaged in the procurement of energy the option to partially satisfy their uncertain commodity requirement (demand) in the forward market rather than entirely in the spot market. We call this real option the *forward procurement option*. The literature on this topic from the perspective of differential transaction costs is scant (see §2).

We formulate and analyze a parsimonious model of energy procurement, as well as a richer and more

Figure 1 Average Trading Volume for the First 36 Maturities of the NYMEX Natural Gas Futures Contract During the Week of March 8, 2010



realistic version of this model, to study the optimal valuation and management of this real option. Our models generalize the classical newsvendor model (Porteus 2002, §1.2) by including both demand and price uncertainty. We apply the valuation approach of Jouini and Kallal (1995), which extends the risk-neutral-valuation approach (Luenberger 1998, Chap. 16; Seppi 2002; Birge 2000; Smith 2005) to the case of transaction costs in security trading. We use our models to investigate, both structurally and numerically, the optimal forward procurement quantity and the value of the forward procurement option as well as their sensitivities to parameters of interest.

Our analysis provides insights into the management of an important business process in energy supply chains. In particular, by applying our model to realistic natural gas distribution instances, we *quantify* the value of the forward procurement option to be in the \$0.4–\$2.4 million per month range, which amounts to reducing the cost of procuring only in the spot market by about 0.61%–3.52% per month. Moreover, our research suggests the near optimality of a forward procurement policy based only on *demand forecasting*, which avoids (i) estimating the transaction costs, (ii) modeling the joint distribution of the spot demand and price, and (iii) optimizing the forward procurement decision. This policy thus substantially simplifies the management of the forward procurement option without a considerable loss of its value. We provide some theoretical justification for this numerical observation. Furthermore, this demand forecast forward procurement policy performs near optimally even when the spot and forward transaction costs have similar magnitudes. We observe this behavior despite the optimal forward procurement quantity being considerably below the demand forecast in this case. We attribute this finding to the initial flatness of the objective function for values of the forward procurement quantity above the optimal quantity.

Our analysis relies on assuming equality between the forward price and the expected spot price in the absence of transaction costs. We investigate numerically the effect of relaxing this assumption on our main insights, when arbitrage opportunities from trading in the forward and spot markets are excluded in the presence of transaction costs.

Our insights should be relevant to energy resellers, local distribution companies, and industrial and commercial users that purchase large amounts of energy, such as food processors, metal and chemical manufacturers, and large restaurant and hotel chains. Potentially, our insights also have broader applicability for the procurement of other commodities, such as metals and agricultural products.

We proceed by reviewing the related literature in §2. We present our base model in §3. We conduct our structural and numerical analyses of this model in §§4 and 5, respectively. We conclude in §6. Online Appendix A summarizes the notation used in the main text. All the relevant proofs of the results stated in the main text are in Online Appendix B. In Online Appendix C, we perform a numerical study with a variant of our base model that relaxes the assumption that the forward price and the expected spot price are equal when there are no transaction costs but precludes arbitrage opportunities from trading in the forward and spot markets when the transaction costs are positive. We formulate and analyze our extended model in Online Appendix D. (The online appendix is available as supplemental material at http://dx.doi.org/10.1287/msom.2013.0473.)

2. Literature Review

Our work is related to the operations management literature on long- and short-term contracting in business-to-business settings (Kleindorfer and Wu 2003, Kleindorfer 2008) and the real-option literature on energy and commodity applications (Dixit and Pindyck 1994, Sick 1995, Trigeorgis 1996, Smith and McCardle 1999, Seppi 2002). We add to these literatures by defining the forward procurement option and investigating its optimal exercise and valuation.

Real-option models of energy and commodity procurement contracts, such as the swing option and other contracts with varying amounts of sourcing flexibility (Li and Kouvelis 1999, Jaillet et al. 2004), address the uncertainty in the purchase price evolution, but they neglect demand uncertainty. In contrast, our models capture the joint uncertainty in the spot demand and price. Our research provides novel insights into the effect of demand uncertainty on the optimal forward procurement quantity and the forward procurement option value.

The operations management literature includes various models of the uncertain evolution of the demand forecast in procurement (see, e.g., Hausman 1969, Hausman and Peterson 1972, Graves et al. 1986, Heath and Jackson 1994). We integrate a model of demand uncertainty consistent with these demand forecast evolution models and a model of the uncertain evolution of the spot purchase price in a newsvendor setting (Porteus 2002, §1.2), thus developing and analyzing a novel variant of the newsvendor model.

Various authors have considered the procurement of a commodity in forward and/or spot markets, including Kalymon (1971), Ritchken and Tapiero (1986), Williams (1986, p. 146; 1987), Gurnani and Tang (1999), Gavirneni (2004), Seifert et al. (2004), Berling and Rosling (2005), Gaur and Seshadri (2005), Gaur et al. (2007), Goel and Gutierrez (2009), Nascimento and Powell (2009), Oum and Oren (2010), Berling and Martínez-de-Albéniz (2011), and Boyabatli et al. (2011). Unlike these authors, we analyze the impact of the correlation between the spot demand and price on the optimal forward procurement quantity and the value of the forward procurement option, and we bring to light the near optimality of the demand forecast forward procurement policy.

Ritchken and Tapiero (1986), Seifert et al. (2004), and Oum and Oren (2010) maximize the expected utility of a risk-averse procurement manager, whereas Gaur et al. (2007) and Goel and Gutierrez (2009) model the risk aversion of economic agents via the risk-neutral-valuation approach. Gaur and Seshadri (2005) consider both approaches. Risk-neutral valuation does not admit the presence of transaction costs in security trading. In particular, the model in Goel and Gutierrez (2009) includes differential spot and forward transportation costs, whereas the transaction costs in our models originate from bid-ask spreads rather than transportation costs. Thus, although we also consider risk-averse economic agents, this difference requires us to adopt a valuation framework that admits transaction costs in security trading, specifically the one of Jouini and Kallal (1995). Applying this valuation framework involves some discussion of the choice of an equivalent probability measure.

3. Base Model

There is a finite horizon that starts at time 0 and ends at time T. The firm satisfies an energy requirement at time T. We refer to this requirement as the spot demand, denoted by d. The spot demand is a random variable at any time before time T, and it becomes known at this time. In our base model, the time T may correspond to a single date, e.g., a day, or an aggregation of several dates, e.g., a month. Thus, the spot demand is a requirement at a single date (e.g., a daily requirement) in the former case and an aggregate requirement (e.g., a monthly requirement) in the latter case. Our extended model, discussed in Online Appendix D, refines the aggregate case by splitting the aggregate requirement over several dates.

The firm does not hold any inventory because of its lack of access to storage facilities and can satisfy this requirement by procuring energy in the spot market at time T. The firm can also procure supply in advance at time 0 by entering into a forward contract with physical delivery at time T. Time 0 is a time near time T when, as pointed out in §1, the forward transaction costs, introduced below and further discussed in §4.1, are at their lowest level, e.g., two weeks away from time T in our numerical study presented in §5. In practice, some energy forward contracts can be negotiated to entail delivery on a single date, e.g., a specific day of the month (Belak 2011). When time T is a single date, our assumption of delivery at time T of the supply procured in the forward market is realistic for such forward contracts. However, most energy forward contracts entail delivery at a constant rate during a given period, e.g., a month. When time T is interpreted as an aggregation of multiple dates, in our base model we do not distinguish when the supply procured in the forward market is actually delivered during the delivery period. In contrast, our extended model, presented in Online Appendix D, captures the constant and rateable delivery of typical energy forward contracts.

We denote by $F(0)$ the time 0 *nominal* price of a forward contract with time T delivery and simplify it to F. (We refer to a price in the absence of transaction costs as a nominal price.) This price evolves during the time interval $[0, T]$ as a known stochastic process $F(t) \in \Re_+$, $t \in [0, T]$. We focus on the time T forward nominal price, which is the spot nominal price f; that is, $F(T) \equiv f$.

Trading in spot and forward markets incurs transaction costs—that is, bid-ask spreads. Consistent with models studied in the finance literature (see, e.g., Constantinides et al. 2007), we model these costs as proportional. Let A and $B \in (0, 1)$. If the firm spot purchases one unit of energy at time T, it pays the spot ask price $(1 + A)f$; if the firm spot sells one unit of energy at this time, it receives the spot bid price $(1 - A)f$. At time 0 the firm can forward purchase one unit of energy at the forward ask price $(1 + B)F$. We do not allow the firm to short sell energy forward because this is suboptimal in the no arbitrage valuation framework that we use, which is explained in §4.1. However, a forward sale at time 0 of one unit of energy can be made at the forward bid price $(1-B)F$. (This is useful for the discussion in §4.1.) Consistent with theoretical work and empirical evidence on the structure of transaction costs in energy and commodity markets (see §1), the spot transaction costs are larger than the forward transaction costs: $A > B$. That is, we model the observed behavior of transaction costs *on average*. (Also recall that we assume that time 0 is near time T.)

The firm's time 0 forecast for its time T demand is $D(0)$, which we simplify to D. As the forward price, this forecast may evolve as a stochastic process, denoted by $D(t) \in \Re_+$, $t \in [0, T]$, which is correlated with the forward-nominal-price stochastic process. We focus on the spot demand d at time T; that is, $D(T) \equiv d$. Because the demand forecast and forward-nominal-price stochastic processes are correlated, the spot demand and nominal price are as well.

For notational convenience, we define $\mathbb{E}_t[\cdot] := \mathbb{E}[\cdot \mid D(t), F(t)]$, $\forall (t, D(t), F(t)) \in [0, T] \times \Re_+^2$, expectation given $D(t)$ and $F(t)$ with respect to a probability distribution discussed in §4.1. Our notation does not distinguish between random variables and their realizations; which is which should be clear from the context. We simplify $\mathbb{E}_0$ to $\mathbb{E}$.

The firm needs to decide how much supply q to procure in the forward market at time 0. Such an optimal procurement decision can be obtained by solving the following optimization problem:

$$V := \max_{q \geq 0} \mathbb{E}[(1-A)f(q-d)^+ - (1+A)f(q-d)^-] - (1+B)Fq, \tag{1}$$

where $(\cdot)^+ := \max\{\cdot, 0\}$ and $(\cdot)^- := -\min\{\cdot, 0\}$. The first and second terms inside the expectation in (1) are the revenue collected from selling excess contracted supply on the spot market and the cost of any supply shortfall, respectively. The third term in (1) is the forward procurement cost. The objective function in (1) and its optimal value V are both expressed in time T money because money is exchanged only at this time.

Because of the presence of the spot market, the firm always achieves 100% service level. Indeed, $q = 0$ is a feasible solution to (1). This solution corresponds to simply waiting until time T, observing the realized demand and procuring this amount on the spot market. We denote the value of this spot procurement policy as $V^S := -\mathbb{E}[(1+A)fd]$. We define the value of the forward procurement option, V^P, as the additional value obtained by optimally procuring both in the forward and spot markets rather than only in the spot market:

$$V^P := V - V^S. \tag{2}$$

In other words, V^P is the value of the *cost savings* accrued by optimally trading both in the spot and the forward markets rather than only in the spot market.

4. Structural Analysis of the Base Model

In this section, we analyze the exercise and valuation of the forward procurement option by imposing more structure on our base model—in particular, the demand forecast and forward-nominal-price processes $D(\cdot)$ and $F(\cdot)$ and, hence, the joint probability distribution of the spot demand random variable d and the spot-nominal-price random variable f. We discuss in §4.1 the valuation framework that we use. We provide a general result in §4.2 and specific results in §4.3, assuming that the spot demand and nominal price are jointly lognormally distributed and satisfy a martingale condition: their expected values, with expectation taken with respect to an equivalent measure in the valuation framework of Jouini and Kallal (1995), are the demand forecast and the forward nominal price, respectively, at the forward trading time. We label this model the martingale lognormal (MLN) model.

4.1. Valuation Framework

If trading futures contracts for a commodity commands transaction costs, as we assume, standard risk-neutral valuation (Luenberger 1998, Chap. 16; Seppi 2002) of the cash flows from dealing this commodity does not apply. We thus apply the valuation framework with transaction costs of Jouini and Kallal (1995) to value these cash flows. These authors show that the absence of arbitrage in securities markets in the presence of transaction costs is equivalent to the existence of at least one probability measure that converts some process between the bid and the ask price processes of traded securities into a martingale. Any such measure, which Jouini and Kallal (1995) refer to as an equivalent measure, or a martingale measure, can be used for valuation purposes in a manner analogous to risk-neutral valuation.

To apply this theory here, suppose that the forward transaction costs do not decrease in the time interval $[0, T]$. That is, let $B(\cdot)$ be a time dependent function such that $B(0) \equiv B$, $B(t) \leq B(t')$, $\forall t, t' \in [0, T]$, $t \leq t'$, and $B(T) \equiv A$. Refer to the process $F(t)$, $t \in [0, T]$, as the process for the *nominal* price of a futures contract for time T delivery—that is, when the transaction costs are absent. (Here we assume equivalence between futures and forward nominal prices; this condition holds when the risk-free interest rate is deterministic (Cox et al. 1981), which we also assume to be the case here.) Then, a probability measure under which the process $F(\cdot)$ is a martingale is an equivalent measure in the valuation framework of Jouini and Kallal (1995). That is, under this measure it holds that $F(t) = \mathbb{E}_t[F(t')]$, $\forall t, t' \in [0, T]$, $t \leq t'$, and $F(t) \in \Re_+$.

We use this martingale measure to determine the equivalent probability distribution of the spot-nominal-price random variable. This is a natural choice of equivalent measure. For example, in the absence of transaction costs, it is the only choice consistent with risk-neutral valuation. That is, this equivalent measure is a risk-adjusted measure for the futures nominal price process. Under our chosen equivalent measure, the forward nominal price F is thus equal to the expected spot nominal price f (Shreve 2004, p. 244):

$$F = \mathbb{E}[f]. \tag{3}$$

In Online Appendix C, we conduct a numerical analysis with a variant of our model that relaxes condition (3).

Condition (3) and $F \in \Re_+$ impose the restriction $\mathbb{E}[f] \in \Re_+$ for all $(D, F) \in \Re_+^2$. To avoid trivial cases, we make the following assumption: $\mathbb{E}[d]$ and $\mathbb{E}[fd] \in \Re_+$ for all $(D, F) \in \Re_+^2$.

We assume that changes in the demand forecast process $D(\cdot)$ are uncorrelated with changes in the price of the market portfolio. Hence, we do not risk adjust the evolution of this stochastic process (Smith 2005). (This does not imply that the demand forecast and forward-nominal-price processes are uncorrelated.) This assumption is realistic for commercial and residential energy demand because it is largely determined by the weather. (See the related discussion in Hull 2012, Chap. 33.)

We focus on the uncertainty in the spot demand and nominal price given the information available at the beginning of the time horizon, that is, D and F. All probabilistic statements related to these random variables are under their joint distribution as determined by the previous assumptions. For simplicity, we assume that spot demand is a continuous random variable.

4.2. A General Result

Proposition 1 characterizes the optimality condition for problem (1) without assuming a specific (equivalent) joint probability distribution for the spot demand and nominal price. We let $\mathbb{P}$ denote probability.

Proposition 1 (Optimality Condition). *Assume that $F > 0$. Consider a positive feasible solution to problem* (1). *Such a solution is optimal if and only if it satisfies the following condition*:

$$\mathbb{E}\left[\frac{f}{F}\mathbb{P}\{d \le q \mid f\}\right] = \frac{1}{2}\left(1 - \frac{B}{A}\right). \tag{4}$$

If no positive solution to problem (1) *satisfies this condition, then zero is the optimal solution to this problem.*

As shown in the proof of Proposition 1, the objective function of problem (1) can be expressed as

$$(A-B)Fq - 2A\mathbb{E}[f(q-d)^+] - \mathbb{E}[(1+A)fd]. \tag{5}$$

The third term in (5) is the value of procuring only in the spot market. The first term in (5) is the value saved by the firm by purchasing an amount of supply q at time 0, irrespective of whether this amount is needed to satisfy the demand at time T. The second term in (5) is the reduction in this value if q exceeds this demand. Expression (5) thus shows that problem (1) can be thought of as maximizing the value of the net savings from operating both in the forward and spot markets relative to only operating in the spot market. In other words, the central concern in problem (1) is balancing the savings from procuring in the forward market, rather than only in the spot market, against the cost of buying too much in advance.

To obtain more insight into this trade-off, it is useful to derive condition (4) using marginal analysis. If a positive quantity is procured at time 0, then its associated marginal underage and overage costs are $(1+A)f - (1+B)F$ and $(1+B)F - (1-A)f$, respectively. Intuitively, such a decision is optimal, in which case we denote it by q^*, if and only if it balances its expected marginal underage and overage costs. Let $1\{\cdot\}$ be the indicator function of event $\{\cdot\}$. Exploiting condition (3), the optimal expected marginal underage cost is

$$\begin{aligned}
&\mathbb{E}[((1+A)f - (1+B)F)1\{d > q^*\}] \\
&\quad = \mathbb{E}[((1+A)f - (1+B)F)(1 - 1\{d \le q^*\})] \\
&\quad = (A-B)F - \mathbb{E}[((1+A)f - (1+B)F)1\{d \le q^*\}].
\end{aligned}$$

The optimal expected marginal overage cost is $\mathbb{E}[((1+B)F - (1-A)f)1\{d \le q^*\}]$. We obtain (4) by equating the expressions for these costs, rearranging, and noticing that $\mathbb{E}[f1\{d \le q^*\}] = \mathbb{E}[f\mathbb{E}[1\{d \le q^* \mid f\}]] = \mathbb{E}[f\mathbb{P}\{d \le q^* \mid f\}]$.

We can further interpret condition (4) as a generalization of the classical newsvendor critical fractile optimality condition (Porteus 2002, §1.2), which relates the overage probability and the critical ratio. Consider the right-hand side of (4). The expectations of the marginal underage and overage costs are $(A-B)F$ and $(A+B)F$, respectively. Thus, the term $(1 - B/A)/2 \in (0, 1/2)$ is the ratio of the expected marginal underage cost and the sum of the expected marginal underage and overage costs, and it can be interpreted as a critical ratio. The term f/F, with $F > 0$, is nonnegative and such that $\mathbb{E}[f/F] = 1$, so it acts as a weighting random variable for the term $\mathbb{P}\{d \le q^* \mid f\}$. Hence, the left-hand side of (4) assumes the interpretation of weighted overage probability. Condition (4) relates the weighted overage probability and the critical ratio at optimality. Given this interpretation, the fact that the right-hand side of (4) is positive and less than one-half emphasizes that the key concern in problem (1) is managing the cost component associated with purchasing too much supply in advance.

4.3. Results with the MLN Model

In general, condition (4) can be used as the basis for developing numerical methods to compute an optimal forward procurement decision at time 0 and, hence, the value of the optimal procurement policy. By making specific assumptions on the joint probability distribution of the spot demand and nominal price, we can obtain closed form expressions for these quantities.

We proceed by assuming a lognormal model with unbiased demand forecast. Specifically, we assume that the random variables spot demand d and spot nominal price f are jointly lognormally distributed, and $D = \mathbb{E}[d]$. That is, we assume that the natural logarithms of the random variables d and f are jointly normally distributed with standard deviations s_d and s_f and means $\ln D - s_d^2/2$ and $\ln F - s_f^2/2$, respectively, and correlation coefficient c. This is our MLN model. It satisfies condition (3).

The parameters s_d and s_f are related to the standard deviations $D\sqrt{\exp(s_d^2) - 1}$ and $F\sqrt{\exp(s_f^2) - 1}$ of the spot demand and the spot nominal price, respectively, and hence the coefficients of variation (CVs) $\sqrt{\exp(s_d^2) - 1}$ and $\sqrt{\exp(s_f^2) - 1}$, respectively, of these random variables. We thus interpret changes in the values of these parameters as analogous directional changes in these standard deviations and variabilities (CVs). The parameter c is related to the correlation between the spot demand and nominal price, which is

$$\frac{e^{cs_d s_f} - 1}{\sqrt{(e^{s_d^2} - 1)(e^{s_f^2} - 1)}}. \tag{6}$$

Changing the value of c can thus be interpreted as changing (6) in the same direction.

The MLN model is consistent with a version of the single-product multiplicative martingale model of the demand forecast evolution of Heath and Jackson (1994), which in our case essentially reduces to the model of Hausman (1969). It is also consistent with the futures price evolution model of Black (1976), as well as the one implied by assuming that the spot nominal price follows a mean reverting process (see, e.g., Jaillet et al. 2004).

Optimal Forward Procurement Quantity. Proposition 2 gives a closed-form expression for the optimal forward procurement quantity under the MLN model. This proposition uses the covariance, demand variance, and demand riskiness terms defined in Table 1. The covariance term depends on the covariance between the random variables $\ln d$ and $\ln f$—that is, $cs_d s_f$. The demand variance term depends on the variance of the random variable $\ln d$—that is, s_d^2. We denote by z the $(1 - B/A)/2$th percentile of the standard normal distribution. The demand riskiness term depends on the product of z, which is negative because the critical ratio $(1 - B/A)/2$ is positive and less than one-half, and the standard deviation of $\ln d$. (The riskiness label is motivated by the negative sign of the exponent of this term and contrasts the safety label that would be appropriate if this sign were positive.)

Table 1 Time Variance, Covariance, and Riskiness Terms

Term	Notation	Definition
Covariance	K^C	$\exp(cs_d s_f)$
Demand variance	K^V	$\exp(-s_d^2/2)$
Demand riskiness	K^R	$\exp(zs_d)$

Proposition 2 (Optimal Forward Procurement Quantity with the MLN Model). *Under the MLN model, the optimal amount of supply to procure forward in problem* (1) *is*

$$q^* = K^C K^V K^R D. \tag{7}$$

This proposition shows that under the MLN model the optimal quantity to procure forward is the current demand forecast D scaled by the product of the covariance, demand variance, and demand riskiness terms. In isolation, the demand variance and riskiness terms make the optimal amount of supply to procure in the forward market decrease below the demand forecast. The effect of the covariance term depends on the sign of the correlation coefficient c. By itself, this effect is to make q^* decrease below D if $c < 0$ and increase above D if $c > 0$. It is easy to verify that q^* is at least D if and only if $c \geq (s_d/2 - z)/s_f > 0$. A sufficiently large positive value of c is thus needed for the optimal procurement quantity to exceed D.

Comparative Statics of the Optimal Forward Procurement Quantity. Under the MLN model, the optimal forward procurement quantity does not depend on the forward nominal price. It instead depends on the standard deviation of the natural logarithm (log) of the spot nominal price and the correlation coefficient between the log spot demand and nominal price. Corollary 1, which follows easily from Proposition 2, states the comparative statics of the optimal forward procurement quantity with respect to these and other quantities of interest. We define $\bar{s}_d^q := (z + cs_f)^+$. Table 2 summarizes these comparative statics.

Corollary 1 (Comparative Statics of q^* with the MLN Model). *Consider the MLN model. Ceteris paribus, the optimal forward procurement quantity* (1) *increases in D;* (2) *increases in c;* (3) *decreases in s_f if $c < 0$, does not depend on s_f if $c = 0$, and increases in s_f if $c > 0$;* (4) *decreases in s_d if $c \leq 0$, and increases in s_d when s_d increases up to $\bar{s}_d^q$ and decreases in s_d for values of this parameter that exceed $\bar{s}_d^q$ if $c > 0$; and* (5) *decreases in B/A.*

Table 2 **Comparative Statics of the Optimal Forward Procurement Quantity Under the MLN Model**

Parameter	Sign of the effect
D	$+$
c	$+$
s_f	$c<0$: $-$; $c=0$: 0; $c>0$: $+$
s_d	$c\le 0$: $-$; $c>0$: $+$ for $s_d \in (0, \bar{s}_d^q)$, $-$ for $s_d \in (\bar{s}_d^q, \infty)$
B/A	$-$

Note. The minus sign ($-$), 0, and the plus sign ($+$) denote a negative, null, and positive effect, respectively.

The increase of the optimal forward procurement quantity in the demand forecast, D, is obvious because this amounts to an increase in the expected spot demand.

The behavior of this optimal quantity in the correlation coefficient, c, can be explained by noticing that increasing the value of c decreases the weighted overage probability. Intuitively, procuring in the forward market thus becomes more appealing.

The effect of changing the log spot-nominal-price standard deviation, s_f, depends on the sign of the correlation coefficient, c, and modulates its effect. An increase in s_f weakens the effect of c if $c<0$, strengthens it if $c>0$, and does not affect it if $c=0$.

Changing the log spot demand standard deviation, s_d, affects the demand variance, demand riskiness, and covariance terms. If $c\le 0$, an increase in s_d affects all these terms in the same direction, so the optimal forward procurement quantity decreases. If $c>0$, the increase in the covariance term due to an increase in s_d dominates the corresponding decreases in the demand variance and riskiness terms, and the optimal forward procurement quantity also increases; but when s_d grows sufficiently large, the decreases in the latter two terms dominate the increase in the covariance term, and the optimal forward procurement quantity also decreases.

The decrease of the optimal forward procurement quantity in the transaction cost ratio, B/A, is intuitive.

Valuations. Proposition 3 provides closed-form expressions for the values of the spot procurement policy, the optimal forward procurement policy, and the forward procurement option as well as a bound on the relative value of this option. Two of these expressions are closed form in the sense that they depend, through the term $\alpha := 2A\Phi(z-s_d)$, on the cumulative distribution function of the standard normal distribution, $\Phi(\cdot)$, which can be readily evaluated in a spreadsheet.

Proposition 3 (Valuations with the MLN Model). *Consider the MLN model. The values of the spot procurement policy, the optimal procurement policy, the forward procurement option, and a bound on the relative value of this option, respectively, are*

$$V^S = -(1+A)K^C FD, \tag{8}$$

$$V = V^S + \alpha K^C FD, \tag{9}$$

$$V^P = \alpha K^C FD, \tag{10}$$

$$\frac{V^P}{|V^S|} < \frac{A}{1+A}. \tag{11}$$

The value of the spot procurement policy is the product of the transaction cost term $-(1+A)$, the covariance term, the forward nominal price, and the demand forecast. The value of the optimal procurement policy is the value of the spot procurement policy plus the value of the net savings that accrue to the firm by optimally procuring in the forward market–that is, the term $\alpha K^C FD$ (see the discussion following expression (5)). By definition (2), the value of these net savings is the value of the forward procurement option. Obviously, the value of this option cannot be negative. Moreover, this value relative to the absolute value of the spot procurement option is bounded above by less than half the spot bid-ask *spread*. (This spread is $2A$.) That is, this amount is the most that can be gained from optimally procuring both in the forward and spot markets rather than only in the spot market.

Comparative Statics of the Value of the Forward Procurement Option. Corollary 2 gives some of the comparative statics of the value of the forward procurement option. Table 3 summarizes them. We denote by $\phi(\cdot)$ the standard normal density function. We label with C the condition $s_f c \le \phi(z)/\Phi(z)$ and with $\bar{C}$ its complement. If $\bar{C}$ is true, then we define $\bar{s}_d^V$ as the value of s_d that satisfies the equality $s_f c = \phi(z-s_d)\Phi(z-s_d)$. (The quantity $\bar{s}_d^V$ exists and is positive, as shown in the proof of Corollary 2.)

Corollary 2 (Comparative Statics of V^P with the MLN Model). *Consider the MLN model. Ceteris paribus, the value of the forward procurement option, V^P, (1) increases in D; (2) increases in F; (3) increases in c; (4) decreases in s_f if $c<0$, is not affected by s_f if $c=0$, and increases in s_f if $c>0$; (5) decreases in s_d if either $c\le 0$ or $c>0$ and C holds, and increases up to the threshold $\bar{s}_d^V$ and decreases thereafter in s_d if $c>0$ and $\bar{C}$ holds; and (6) decreases in B/A.*

Table 3 **Comparative Statics of the Value of the Forward Procurement Option with the MLN Model**

Parameter	Sign of the effect
D	$+$
F	$+$
c	$+$
s_f	$c<0$: $-$; $c=0$: 0; $c>0$: $+$
s_d	Either $c\le 0$ or $c>0$ and C holds: $-$ $c>0$ and $\bar{C}$ holds: $+$ for $s_d \in (0, \bar{s}_d^V)$, $-$ for $s_d \in (\bar{s}_d^V, \infty)$
B/A	$-$

Note. The minus sign ($-$), 0, and the plus sign ($+$) denote a negative, null, and positive effect, respectively.

Consider part (1). From expression (5), the value of the forward procurement option is the difference between the value of the savings and the value of the overprocurement cost when acting optimally in the forward market. These values are both linear in the forward nominal price, F. (This is obvious for the savings; see (28) in Online Appendix B for the overprocurement cost.) The value of these savings is obviously greater than the value of this overprocurement cost for every given such price. The difference between the former and the latter values thus increases in this price. A similar argument holds for part (2) with respect to the demand forecast, D. Parts (3) and (4) mirror how the correlation coefficient and the log spot-nominal-price standard deviation affect the optimal forward procurement quantity. Part (5) is related to the behavior of the optimal forward procurement quantity in the log spot demand standard deviation. Part (6) is intuitive.

Demand Forecast Procurement Policy. Determining the optimal forward procurement quantity requires knowledge of the parameters of the joint spot demand and nominal price distribution as well as the spot and forward transaction costs. It is thus of interest to consider a forward procurement policy that does not require this information and investigate the suboptimality of such a policy.

We take this policy to be the one that procures forward an amount equal to the demand forecast; that is, $q = D$. We choose this policy because it does not require knowledge of the transaction costs and ignores all but one of the parameters of the MLN model—that is, D.

We denote the value of the demand forecast procurement policy by V^D. It follows from (5) and Lemma 2 in Online Appendix B that this value with the MLN model is

$$V^D = V^S + \left\{A - B - 2A\left[\Phi\left(\frac{s_d}{2} - cs_f\right) - K^C\Phi\left(-\left(\frac{s_d}{2} + cs_f\right)\right)\right]\right\}FD. \quad (12)$$

The demand forecast procurement policy is optimal when there is no *demand* uncertainty (irrespective of the specific spot-nominal-price distribution used). This is because the choice in this case is between procuring an amount D in the forward market if $(1+B)F \le (1+A)\mathbb{E}[f] = (1+A)F$ and procuring this amount in the spot market otherwise, and $B < A$.

Under the MLN model, this policy is optimal when the log spot demand standard deviation is equal to $2\bar{s}_d^q > 0$, because in this case $q^* = D$. Proposition 4 provides additional conditions such that the demand forecast policy outperforms the spot procurement policy, and is hence able to capture a fraction of the value of the forward procurement option, and bounds on both the absolute and relative suboptimality of the demand forecast procurement policy in a specific case.

Proposition 4 (Performance of the Demand Forecast Procurement Policy with the MLN Model). *Consider the MLN model.* (1) *A value $\bar{s}_d^D > 2\bar{s}_d^q$ exists that depends on the problem parameters such that $V^D > V^S$ when and only when $s_d \in (0, \bar{s}_d^D)$ and $V^D = V^S$ when $s_d = \bar{s}_d^D$.* (2) *If $s_d \in (2\bar{s}_d^q, 2cs_f]$, then it holds that*

$$V - V^D < FDB, \quad (13)$$

$$\frac{V - V^D}{|V|} < \frac{B}{K^C}. \quad (14)$$

Part (1) of Proposition 4 indicates that the demand forecast policy outperforms the spot procurement policy when the variability in the spot demand is "not too large." The bounds (13) and (14) on the absolute and relative suboptimality of the demand forecast procurement policy require the log spot demand and nominal price to be positively correlated, which is realistic. These bounds can be small because realistic values of the forward transaction cost B can be small (see §5.1). In particular, the bound (13) is less than the suboptimality of the spot procurement policy, that is, the value of the forward procurement option, if the forward transaction cost is sufficiently small, that is, $B < 2A\Phi(z - s_d)\exp(cs_d s_f)$. There are realistic parameter values that satisfy this condition, as discussed in §5.2. Overall, Proposition 4 suggests that the demand forecast policy can capture a substantial amount of the value of the forward procurement option in realistic cases.

It is possible to establish bounds on the suboptimality of the forward procurement policy in more cases than the one specified in part (2) of Proposition 4. However, it is challenging to make these bounds as informative as the ones stated in this proposition.

5. Numerical Study with the Base Model

In this section, we report the results of a numerical study conducted with our base model to assess the value of the forward procurement option and the performance of the demand forecast procurement policy as well as illustrate some of the comparative statics of this option and its optimal exercise. We present the instances used as test beds in §5.1. We discuss our results in §5.2.

5.1. Instances

In our study, the firm is a hypothetical natural gas distributor operating in the United States. We let the length of the time horizon be two weeks and thus set T equal to 14/365. The end of the time horizon corresponds to February 2011; that is, we use the aggregate interpretation of our base model. We take the firm's February 2011 demand forecast as of time 0 to be 14,593,766 MMBtu, which corresponds to the February 2001 demand faced by the utility considered in the study of Muthuraman et al. (2008, Table 1, p. 1143). Here, we make the convenient assumption that this figure is indicative of the demand forecast of this firm for February 2011. We use \$4.4315/MMBtu as the forward nominal price at the beginning of the time horizon; this is the average of the low and high prices of the NYMEX natural gas futures contract for delivery in February 2011 observed on January 14, 2011.

We specify the parameters of the MLN model by employing the single-product multiplicative martingale model of the demand forecast evolution of Heath and Jackson (1994) and the seasonal mean reverting model for energy spot prices of Jaillet et al. (2004). Specifically, the demand forecast for time T evolves as a geometric Brownian motion with volatility σ_D and zero drift so that each demand forecast is unbiased; that is, the expected value of the spot demand is equal to the current demand forecast. The log deseasonalized natural gas spot nominal price is $\chi(t) := \ln(f(t)/S(t))$, where $f(t)$ is the spot nominal price at time t and $S(t)$ is the deterministic seasonality factor for this price. This log price evolves as a mean reverting process with speed of mean reversion κ, *risk-adjusted* mean reversion level ξ, and volatility σ_χ. The dynamics of these processes in the time interval $[0, T]$ are

$$dD(t) = \sigma_D D(t)dZ_D(t), \tag{15}$$

$$d\chi(t) = \kappa[\xi - \chi(t)]dt + \sigma_\chi dZ_\chi(t), \tag{16}$$

$$dZ_D(t)dZ_\chi(t) = \rho dt, \tag{17}$$

where $dZ_D(t)$ and $dZ_\chi(t)$ are increments to standard Brownian motions with instantaneous correlation coefficient ρ and dt is an infinitesimal time increment. Under model (15)–(17), the parameters of the MLN model are as follows:

$$s_d = \sigma_D\sqrt{T}, \quad s_f = \sigma_\chi\sqrt{\frac{1-e^{-2\kappa T}}{2\kappa}},$$
$$c = \frac{\rho(1-e^{-\kappa T})/\kappa}{\sqrt{T(1-e^{-2\kappa T})/(2\kappa)}}.$$

Given that F is known, the values of the parameters $\chi(0)$, ξ, and $S(T)$ are not needed because they are assumed to satisfy condition (3), which reduces to

$$F = S(T)\exp\left(\chi(0)e^{-\kappa T} + (1-e^{-\kappa T})\xi + \frac{\sigma_\chi^2}{4\kappa}(1-e^{-2\kappa T})\right).$$

However, the values of these parameters are needed when dealing with our extended model, as discussed in Online Appendix D, §D.3.

We set a base value for the log demand forecast volatility, σ_D, to make the CV of spot demand equal to 0.05. We have confirmed with practitioners that this is a reasonable figure for natural gas demand. In our specification of the MLN model, the CV of spot demand given the information available at time 0 is $\sqrt{\exp(\sigma_D^2 T) - 1}$. Thus, for given values of T and this CV, the corresponding value of σ_D is $\sqrt{\ln(1+\mathrm{CV}^2)/T}$. For T equal to 14/365 and the CV of spot demand equal to 0.05, we obtain a value for σ_D equal to 0.2551. To obtain insights into the effect of changes in the value of this parameter on the value of the forward procurement option, we also consider values for σ_D equal to 0.1276, 0.3824, 0.5093, 0.6358, 0.7616, 0.8868, 1.0112, 1.1347, 1.2572, 1.3786, and 1.4989, which roughly correspond to spot demand CV values in the 0.025–0.3 range in increments of 0.025, respectively. Moreover, our range of values for σ_D includes the 0.6458–1.1157 range of electricity log demand volatilities reported in Pilipovic (2007, Figure 11-28, p. 343). Although we deal with natural gas, rather than electricity, demand, the overlap between these ranges provides some support for our choice of values of the parameter σ_D.

For the log deseasonalized spot-nominal-price speed of mean reversion and volatility, κ and σ_χ, we consider base values of 1.0547 and 0.6696, which are the estimates of these parameters obtained by Lai et al. (2011, Table 2) using NYMEX data. Similar to σ_D, we consider additional values for σ_χ, namely 0.2696, 0.3696, 0.4696, 0.5696, and 0.7696, leaving the value of the parameter κ at its base level.

We set a base value for the instantaneous correlation coefficient, ρ, by making the correlation between the spot demand and nominal price with $T = 14/365$ equal to 0.2. (This value is used in the study of Seifert et al. 2004.) Specifically, in our specification of the MLN model, the correlation between the spot demand and nominal price is $\{\exp(\rho\sigma_D\sigma_\chi[1 - \exp(-\kappa T)]/\kappa) - 1\}/(G_D G_f)$, with $G_D := \sqrt{\exp(\sigma_D^2 T) - 1}$ and $G_f := \sqrt{\exp(\sigma_\chi^2[1-\exp(-2\kappa T)]/(2\kappa)) - 1}$. Thus, for given values of this correlation, denoted by CORR, and T, the corresponding value of ρ is $\ln(1 + G_D G_F \mathrm{CORR})/\{\sigma_D\sigma_\chi[1-\exp(-\kappa T)]/\kappa\}$. For $T = 14/365$, $\sigma_D = 0.26$, and $\sigma_\chi = 0.6696$, we obtain $\rho = 0.1968$. Based on the argument made by Seifert et al. (2004) that the correlation between a commodity price and the demand for this commodity faced by a firm should be positive, we also consider values for ρ equal to 0.0246, 0.0492, 0.0738, 0.0984, 0.1230, 0.1476, 0.1722, 0.2952, 0.3934, 0.4916, 0.5897, and 0.6878, each of which roughly corresponds to an

Table 4 **Parameter Values Used in Our Numerical Study with Our Base Model**

Parameter	Value(s)
T	14/365
D	14,593,766 MMBtu
F	\$4.475/MMBtu
σ_D	0.1276, **0.2551**, 0.3824, 0.5093, 0.6358, 0.7616, 0.8868, 1.0112, 1.1347, 1.2572, 1.3786, 1.4989
σ_χ	0.2696, 0.3696, 0.4696, 0.5696, **0.6696**, 0.7696
κ	1.0547
ρ	0.0246, 0.0492, 0.0738, 0.0984, 0.1230, 0.1476, 0.1722, **0.1968**, 0.2952, 0.3934, 0.4916, 0.5897, 0.6878
A (%)	3.75
B (%)	**0.025**, 0.25, 2.5

Note. When there are multiple values for a parameter, the boldface value corresponds to the base case.

equal CORR value. We have verified with practitioners that this range includes realistic values. In addition, this range includes the 0.0267–0.2134 range of correlations between the electricity price and demand reported in Pilipovic (2007, Table 11-2, p. 342). Despite the difference in commodity, the overlap between our and this range gives some support for our choice of values of the parameter ρ.

In terms of the transaction costs, we rely on indicative figures provided to us by practitioners at a major U.S. energy trading company and at a major U.S. natural gas broker. These figures are consistent with our assumed transaction cost structure. We set the forward bid-ask spread ($2B$) equal to 0.05% and the spot bid-ask spread ($2A$) to 7.5%. We also consider forward bid-ask spreads equal to 0.5% and 5% to quantify the impact of changes in the value of the forward and spot transaction cost differential on the forward procurement option value and optimal quantity.

Table 4 summarizes the parameters used to generate our 2,808 ($= 12 \cdot 6 \cdot 13 \cdot 3$) instances.

5.2. Results

In discussing our results, we focus on the value of the forward procurement option, the optimal forward procurement quantity, some of their comparative statics, and the performance of the demand forecast procurement policy. Table 5 summarizes our results.

Value of the Forward Procurement Option. The upper bound (11) on the relative value of the forward procurement option is 3.61%. In the base case, the relative value of this option is 3.45%, which is 95.57% of this bound. Thus, the bound (11) is informative in this case. Across all our instances, the relative value of the forward procurement option varies between 0.75% and 3.52%, which correspond to 20.78% and 91.73% of the value of the bound (11). In particular, the ratios of the relative value of the forward procurement option and this bound vary between 76.45% and 97.51%, 70.64% and 91.41%, and 20.78% and 32.13% when the forward transaction cost is 0.025%, 0.25%, and 2.5%, respectively. The bound (11) thus becomes looser when the forward transaction cost increases. This behavior is expected given that the value of the forward procurement option decreases when the value of B increases, as a result of the corresponding decrease in the value of z, but this bound is unaffected by such changes.

In absolute terms, the forward procurement option is worth \$2,315,354 per month in the base case and ranges from \$502,975 to \$2,366,699 per month across all of our instances. Because these are reductions in the cost of the spot procurement policy achieved by the optimal forward procurement policy, these findings suggest that a firm may derive substantial benefit from optimally exploiting the differential transaction costs between the forward and spot markets.

Comparative Statics of the Value of the Forward Procurement Option. Proposition 3 implies that the ratio of the forward procurement option and the absolute value of the spot procurement policy is equal to $\alpha/(1+A)$, which only depends on the transaction cost parameters, A and B, the log demand forecast volatility, σ_D, and the length of the time horizon, T (because α depends on σ_D and T, via $s_d = \sigma_D\sqrt{T}$, and z, which in turn depends on A and B). Increasing the log demand forecast volatility decreases these ratios by less than 1%. Specifically, these ratios vary between 2.76% and 3.52%, 2.55% and 3.30%, and 0.75% and 1.16% when the forward transaction cost is 0.025%, 0.25%, and 2.5%, respectively. As expected, each set of ratios decreases when the forward transaction cost increases. More interesting, each set of ratios decreases below 2% only when this cost increases by two orders of magnitude, that is, from 0.025% to 2.5%. Furthermore, even when the spot and forward transaction costs share the same order of magnitude, the forward procurement option retains some value.

Even though these ratios do not vary by more than 1% for a given value of the forward transaction cost, the monetary value of the forward procurement option varies in a more substantial manner. Specifically, it ranges between \$1,850,447 and \$2,366,699, \$1,712,886 and \$2,221,029, and \$502,975 and \$780,464 per month when the forward transaction cost is 0.025%, 0.25%, and 2.5%, respectively.

Table 5 **Summary of Our Numerical Results with the Base Model**

Quantity	Base case	Range
V^P (\$)	2,315,354	502,975–2,366,699
$V^P/\lvert V^S\rvert$ (%)	3.45	0.75–3.52
q^*/D (%)	99.96	72.13–100.43
$V - V^D$ (\$)	3	0–259,780
$(V - V^D)/\lvert V\rvert$ (%)	0.00	0.00–0.39

These variations in the option value are almost completely attributable to changes in the log demand forecast volatility, σ_D. (Increasing σ_D decreases the option value in our instances.) That is, the value of the forward procurement option is relatively insensitive to changes in the log deseasonalized spot-nominal-price volatility, σ_χ, and the instantaneous correlation coefficient, ρ, on our instances.

Optimal Forward Procurement Quantity and Its Comparative Statics. The optimal forward procurement quantity, q^*, is 99.96% of the demand forecast, D, in the base case. It varies between 72.13% and 100.43% of this forecast across all our instances. More specifically, it ranges between 95.58% and 100.43%, 93.49% and 100.01%, and 72.13% and 97.83% of the demand forecast when the forward transaction cost is 0.025%, 0.25%, and 2.5%, respectively. The optimal forward procurement quantity thus drops substantially below the demand forecast in some of the instances in which the spot and forward transaction costs are of the same order of magnitude.

To explain this observation, recall from Proposition 2, specifically (7), that the optimal forward procurement quantity is the demand forecast scaled by the product of the demand variance, demand riskiness, and covariance terms. In isolation, the demand variance and riskiness terms make the optimal forward procurement quantity decrease below the demand forecast, whereas the covariance term has the opposite effect when the instantaneous correlation coefficient is positive, which is the case in our numerical study. Across all our instances, the covariance and demand variance terms range from 1.0000 to 1.0303 and from 0.9578 to 0.9997, respectively, and their product ranges from 0.9582 to 1.0052. Thus, the combined effect of these two terms is to keep the optimal forward procurement quantity close to the demand forecast. As opposed to these two terms, the demand riskiness term depends on the forward transaction cost. This term varies between 0.9976 and 0.9998, 0.9757 and 0.9979, and 0.7528 and 0.9761 when this cost is 0.025%, 0.25%, and 2.5%, respectively. Consequently, the optimal forward procurement quantity drops considerably below the demand forecast only on some instances when the spot and forward transaction costs share the same order of magnitude.

Similar to the value of the forward procurement option, the optimal forward procurement quantity is more sensitive to changes in the volatility of the log demand forecast than changes in the volatility of the log deseasonalized spot nominal price and their instantaneous correlation coefficient.

Performance of the Demand Forecast Procurement Policy. Proposition 4 suggests that the demand forecast procurement policy can perform well in some cases. Specifically, the value of the bound (13) on the absolute suboptimality of the demand forecast procurement policy is less than the value of the forward procurement option on 164 out of 291 instances that satisfy the condition for the bound validity. (These 164 instances correspond to all the 38 instances with $B = 0.025\%$ and all the 126 instances with $B = 0.25\%$ for which this bound is valid; that is, the value of this bound exceeds the value of the forward procurement option on all the 127 instances with $B = 2.5\%$ for which this bound is valid.) Across these 164 instances, their respective forward transaction costs, that is, 0.025% and 0.25%, turn out to be the largest values of the bound (14) on the relative suboptimality of this policy. More broadly, our numerical analysis of the optimal forward procurement quantity suggests that the demand forecast procurement policy should be near optimal on all the instances for which the spot and forward transaction costs have different orders of magnitude because, for these instances, the demand forecast is close to the optimal forward quantity. Indeed, the suboptimality of the demand forecast procurement policy is at most 0.01% and 0.02%, which translates to \$6,613 and \$12,564 per month, when the forward transaction costs are 0.025% and 0.25%, respectively.

It is, however, unclear if the demand forecast procurement policy would also perform well on the instances for which the forward and spot transaction costs have the same order of magnitude because, for these instances, the demand forecast can be substantially above the optimal forward procurement quantity, that is, by as much as 32.87% ($= (1/0.7526 - 1)\%$). Our numerical results reveal that the demand forecast policy is near optimal even on these instances, and its suboptimality is at most 0.39% (\$259,780 per month).

This finding indicates that when the spot and forward transaction costs have the same order of magnitude, so that the demand forecast can considerably exceed the optimal forward procurement quantity, the objective function of problem (1) is relatively flat to the right of its optimal solution. To gain some intuition on this behavior, let η be a positive number and consider the loss from procuring ηq^* more than is optimal in the forward market. This loss can be easily shown to be

$$2A\mathbb{E}[f((1+\eta)q^* - d)^+ - f(q^* - d)^+] - (A-B)Fq^*\eta. \quad (18)$$

Given the discussion following (5) in §4.2, the first term in this difference is the value of the *additional* reduction in savings from sourcing in the forward market, as a result of the forward supply exceeding the realized demand, when procuring $(1+\eta)q^*$ rather than q^*; the second term is the value of the savings obtained from overprocuring in the forward market the amount $q^*\eta$. Because (18) cannot be negative, the second term of this difference never exceeds the first term. Moreover, although both these terms become larger when η increases, it is easy to verify that their difference is an increasing and convex function of η. That is, the first term grows faster than the second term as η increases. Our finding indicates that initially the increments in these two terms tend to offset each other, and, hence, the initial growth in their difference is slow.

These results suggests that achieving near-optimal management of the forward procurement option requires only demand forecasting rather than estimating the transaction costs and the joint distribution of the spot demand and nominal price as well as optimizing the forward procurement quantity. Thus, these results suggest that the demand forecast procurement policy is appealing for practical implementation.

6. Conclusions

In this paper, we study the optimal valuation and management of the forward procurement option, a real option that arises from the differential transaction costs in spot and forward energy and, more broadly, commodity markets. We do this by formulating, analyzing, and applying to data a parsimonious procurement model with correlated spot demand and nominal price random variables as well as an extension thereof. In particular, we quantify the value of the forward procurement option on realistic natural gas distribution instances. Our numerical work suggests that procuring the demand forecast in the forward market is near optimal. We provide some theoretical support for this numerical observation. This policy thus substantially simplifies the management of this real option without a considerable loss of its value. We observe that the demand forecast forward procurement policy performs near optimally even when the magnitudes of the spot and forward transaction costs are similar. In this case, the optimal forward procurement quantity is substantially smaller than the demand forecast, but the near optimality of the demand forecast policy is due to the observed initial flatness of the objective function to the right of the optimal forward procurement quantity. Our findings are relevant to energy resellers and local distribution companies, but they retain potential significance in other industrial and commercial contexts.

Our work can be extended in several directions. We consider a single procurement date in the forward market. It would be of interest to consider a more dynamic model with additional procurement dates later on in the forward market. This model may yield some additional benefit relative to our models, but it would require estimating the dynamics of the transaction costs in a more granular fashion.

Firms may face limits to the amount of trading that they can perform in the forward market at a given date, e.g., because of limited working capital availability, counterparty risk, and supply chain disruptions (Tomlin and Wang 2012) that may affect the availability of contracted supply. Our models could be extended to consider these features.

The transaction costs are deterministic in our models. This simplification captures the average behavior of these costs over time. A more refined approach might model the stochastic evolution of transaction costs, possibly linking it to the one of trading volume, and the relationship between the evolution of these costs and those of demand and price. Relevant empirical research would be useful in this respect.

In this paper, we use the valuation approach of Jouini and Kallal (1995). Considering the spot and forward procurement choices of multiple firms within an equilibrium model would allow one to assess the limitations of this no-arbitrage approach applied to the valuation and management of the forward procurement option.

Despite the limited availability of energy storage capacity in practice, our models could be extended to include storage in a manner similar to the inventory models reviewed in §2. In particular, it would be of interest to assess the value of storage for energy users in practical settings (see, e.g., Butler and Dyer 1999).

Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/msom.2013.0473.

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