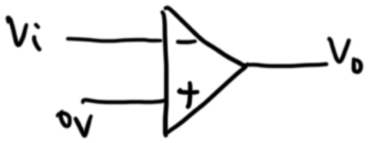
Nagendra Krishnapura Analog - 3.

Schmitt Trigger.

Realising a comparator using OpAmp.



$\Rightarrow$ $V_i < 0$, $V_0 = +V_{sat}$
$V_i > 0$, $V_0 = -V_{sat}$.

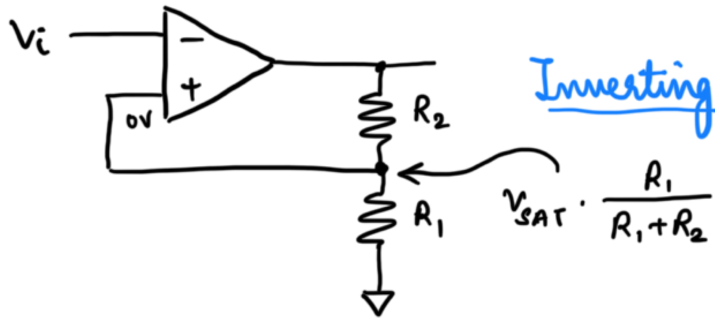
If V_i is small and it fluctuates a lot, then the output will also fluctuate, which we don't want.

So, instead of having the limit as (here, zero) one value, we will have 2 more thresholds:

For upper saturation, V_i must be larger than a threshold that's slightly higher than the current limit (some value > 0 here). |||ly for lower saturation, V_i must be smaller than a threshold that's slightly less (some value < 0 here in this case).

This thing - having upper and lower thresholds - is called "Hysteresis".

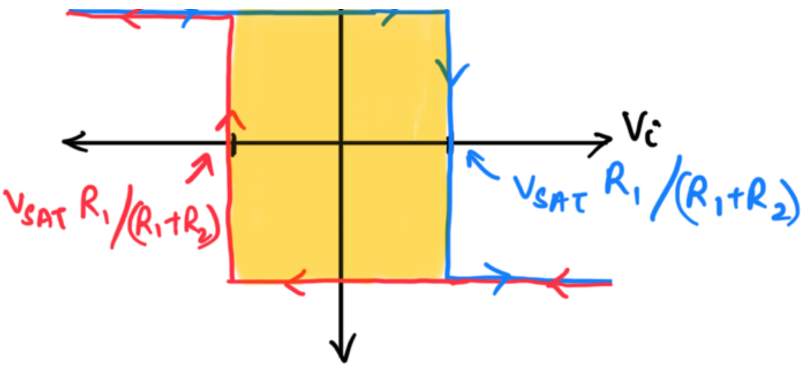
And the OpAmp that (we are going to realise) performs this "comparator" operation is called a Schmitt Trigger.



Inverting Schmitt Trigger

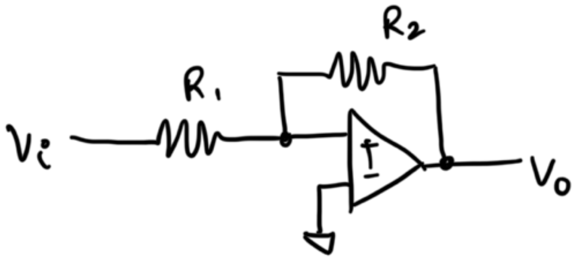
Now, how would the Vo vs V_i curve look like?

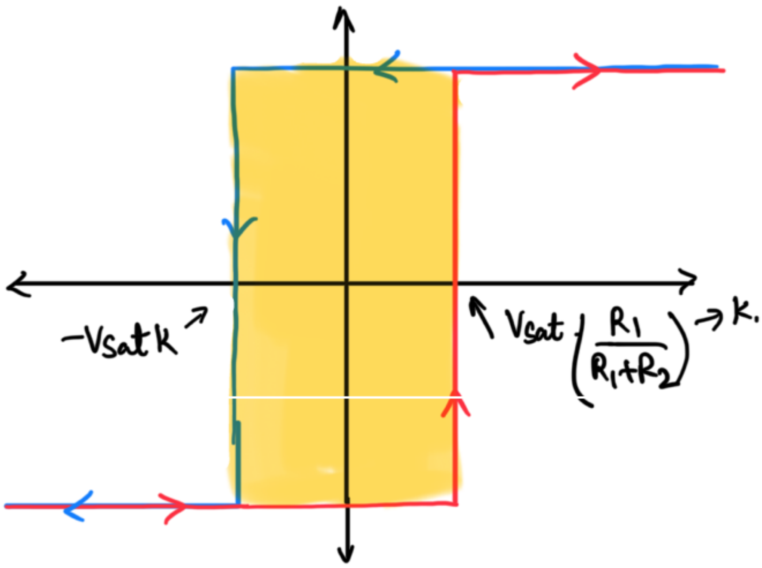
The satuation value for



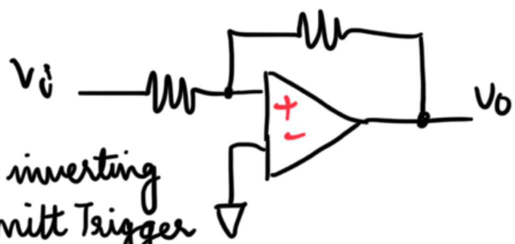
voltages inside the yellow shaded region depends on whether we came from low values (Red) or high values (Blue).

Non-inverting Schmitt Trigger:

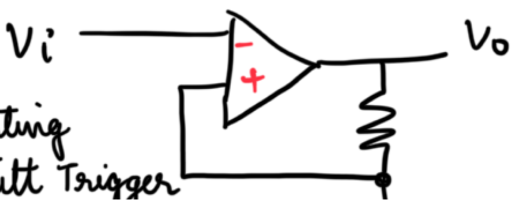




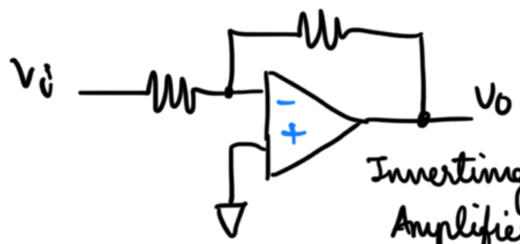
Schmitt Triggers | Amplifiers



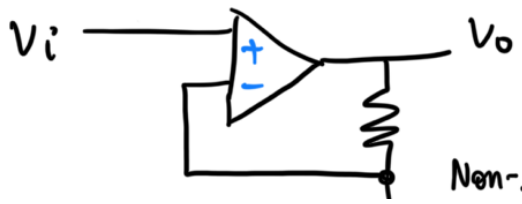
Non-inverting Schmitt Trigger



Inverting Schmitt Trigger



Inverting Amplifier

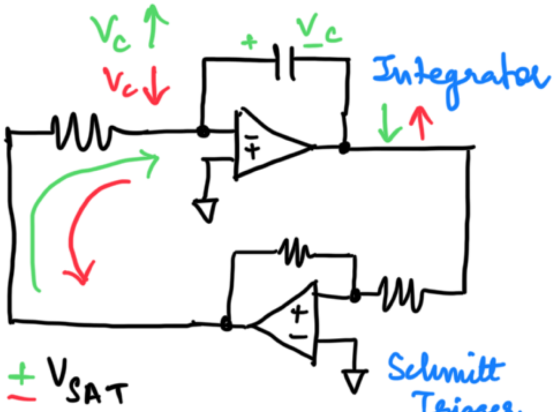


Non-inverting

Nagendra's Version

Saurabh's Version

Schmitt Trigger Oscillator – Ramp Generator



Will be used in Lab!

When Schmitt Trigger is in positive saturation, Output is high, so current is pushed into the integrator. This increases the Capacitor voltage, and since the Op Amp of Integrator is in –ve feedback, the output will decrease.

But when Schmitt Trigger is in negative saturation, this draws current from the integrator, which will decrease the capacitor voltage, and thus the output will be high.

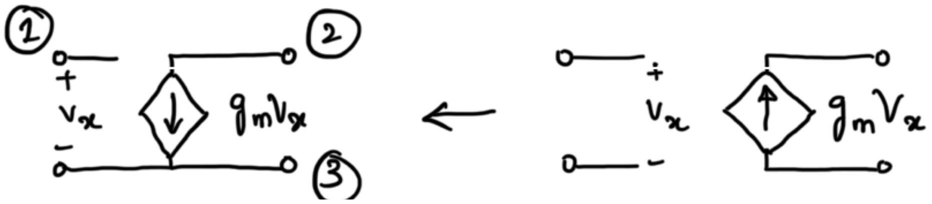
This way, the oscillator works. High to low, then back to High.

Reason why integrator is unstable: Has a pole in the imaginary axis → It'll never reach steady state.
↓
For stable systems, the poles of their transfer function must lie on the left half of the plane, excluding the imaginary axis.

Realising OpAmp with VCCS

OpAmps → Main job for it is to have very high gain
↓
Made of transistors, which are usually realised as VCCS.

VCCS with 3 terminals

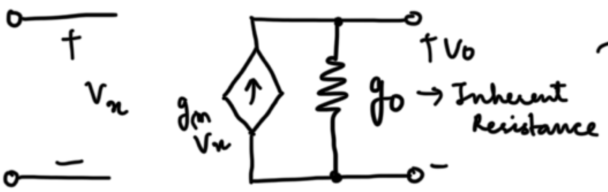


Now, we are trying to make OpAmp using VCCS?

But wait... Isn't an OpAmp, a VCCS? why are we making controlled sources out of controlled sources?

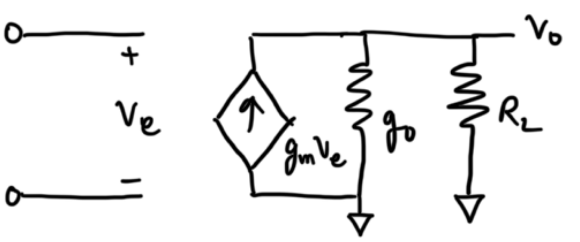
⇓

To get accurate gain. Transistors don't have accurate and convenient gains. For that we make use of Negative feedback loops and get the gain WE WANT, precisely too.



→ $\dfrac{V_o}{V_x} = \dfrac{g_m}{g_o}$.

Let us now try getting a gain of 10^6 from this:

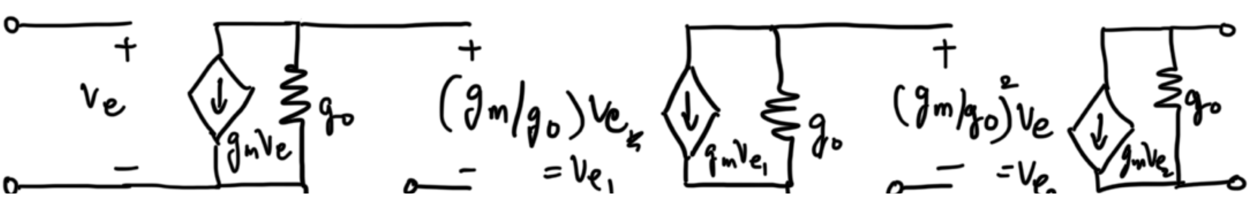


$\Rightarrow \dfrac{g_m}{g_o + G_L} = \dfrac{V_o}{V_e}$.

Why aren't we getting $\dfrac{g_m}{g_o} = \dfrac{V_o}{V_i}$ as the VCCS gain and going further? ⇒ The maximum value of $g_m/g_o \sim 10-100$.

∴ We need to do something. The problem of getting precise gain and not some random gain is solved. Now,

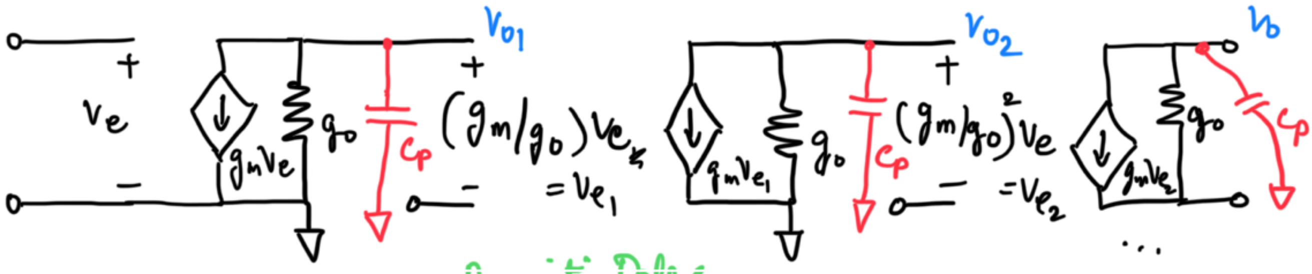
this is a device limitation ⇒ Small transistors have less g_m/g_o ($10 \sim 100$ gain). To get gain more than that, we cascade them!



But still ... We are missing something ⇒ This hypercascading: can it be done forever?

{ What we are missing is the capacitances and inductances.

There can be capacitances formed in the physical circuit, for the dimension / scale of the circuits we build.



Parasitic Poles.

The red capacitances are the ones which are the most dominant. Others are all insignificant. Basically, there will be capacitances between every node. But the V_o and the ground is the one having maximum capacitances due to ??

Inductances will also be there, but for the dimensions we are working on right now, it is not significant.

Now, lets calculate V_o/v_e, V_{o1}/v_e, V_{o2}/v_e.

$$\frac{V_{o1}(s)}{V_e(s)} = \frac{g_m}{g_o + sC_p} = \frac{g_m/g_o}{1 + sC_p/g_o} \; ; \; \therefore \frac{V_o}{V_e} = \left(\frac{g_m/g_o}{1 + sC_p/g_o}\right)^3$$

Low pass Filter

Has 3 Poles.

Now... Why are we doing all this, finding Transfer function and all ?? → The Poles!

Remember? For stability of a system, all the poles need to lie in the left half plane, excluding the Im axis.

Now let us go to loop gain :

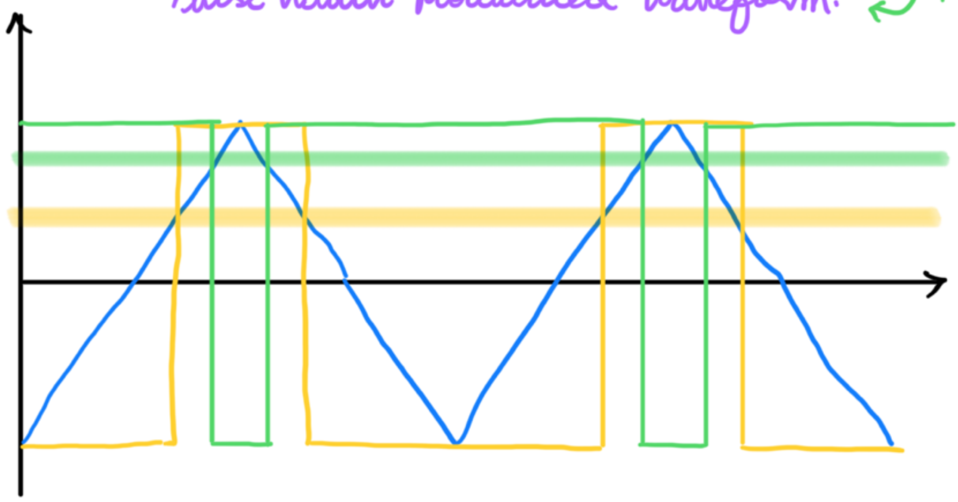
$$\frac{V_0}{V_i} = \frac{K}{1+(K/A_0)}$$, and if $A_0/K > 8$, system is unstable.

Hence, to get good loop gain, we usually cascade (typically more than 2) Transistors. But this poses a problem of having multiple poles.

Now, we need to find a way to make these stable. And we will be seeing this as a important part of this course.

Need to Recap 1st order 2nd order responses (DSP!!)

Pulse Width Modulated Waveform. Saurabh's Version



Last Lecture :

Those Capacitances (Parasitic Poles) are a problem when we try to Cascade a lot of transistors in order to get large gain.

Refer the Above Cascade of transistors diagram

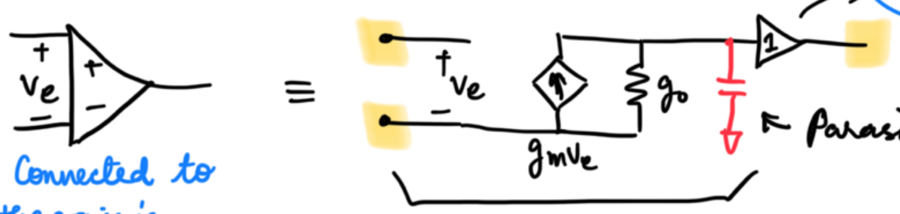
$$\frac{V_{o1}}{V_e} = \frac{g_{m1}/g_{o1}}{1+SC_1/g_{o1}} \; ; \; \frac{V_{o2}}{V_e} = \frac{g_{m1}g_{m2}/g_{o1}g_{o2}}{(1+SC_1/g_{o1})(1+SC_2 g_{o2})}$$ ← 2 poles

$$\frac{V_{o3}}{V_e} = \frac{g_{m1}g_{m2}g_{m3}/g_{o1}g_{o2}g_{o3}}{(1+SC_1/g_{o1})(1+SC_2/g_{o2})(1+SC_3/g_{o3})}$$ ← 3 Poles! → If just one of them somehow fall in the Right-half of

the s-plane, then the system becomes unstable.

Now – Our goal is to "Quantify" or basically analyse this unstability and find a way to make it stable – Because, we do see a lot of places where thousands of Transistors are stacked / Cascaded, giving rise to a possible Right side pole from the thousands of poles – Yet the devices work fine!

Single Stage Op-Amp – I^{st} order $H(s)$ ...



Buffer (VCVS of gain 1)

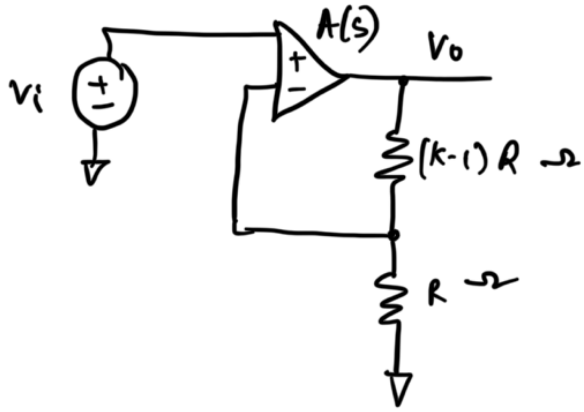
← Parasitic Capacitor.

Buffer is Connected to ensure the gain is g_m/g_o, regardless of the load we connect.

$$A(s) = \frac{A_o}{1+s/p_1}, \quad A_o = (g_m/g_o) \leq 100$$

Open loop transfer function / Gain of Op Amp.

Now, let us connect this Op Amp to a Non-inverting amplifier:



$$\frac{V_o}{V_i} = \frac{k}{1+\frac{k}{A(s)}} = \frac{k}{1+\frac{k(1+s/p_1)}{A_o}}$$

$$= \frac{k}{1+\frac{k}{A_o}+\frac{k}{A_o}\cdot\frac{s}{p_1}}$$

Closed loop transfer function

DC Gain of closed loop Transfer fn

$$\frac{k}{1+k/A_o}$$

Sanity check? $s=0$; this simplifies to $\frac{k}{1+k/A_o}$

Now, pole(s) of $\frac{V_o}{V_i}$? $\Rightarrow \frac{k}{\left(1+\frac{k}{A_o}+\frac{k}{A_o}\cdot\frac{s}{p_1}\right)}$ → $D(s)=0$

$\Rightarrow$ Pole: $1+\frac{k}{A_o} = \frac{ks}{A_o p_1}$; $s = \left(1+\frac{k}{A_o}\right)\frac{A_o}{k}\cdot p_1$

I seriously need to review the Routh ...

$S = -p_1\left(\frac{A_o}{k}+1\right)$ → Unconditionally stable

The Prerequisites ! ... stable...

→ RLC, 1^{st}, 2^{nd} order, Responses, Bode Plots.

If $A_0 \ggg k$, then $s = -\frac{P_1 A_0}{k}$.

Gain Bandwidth Product $\approx A_0 P_1$

Unity Gain frequency

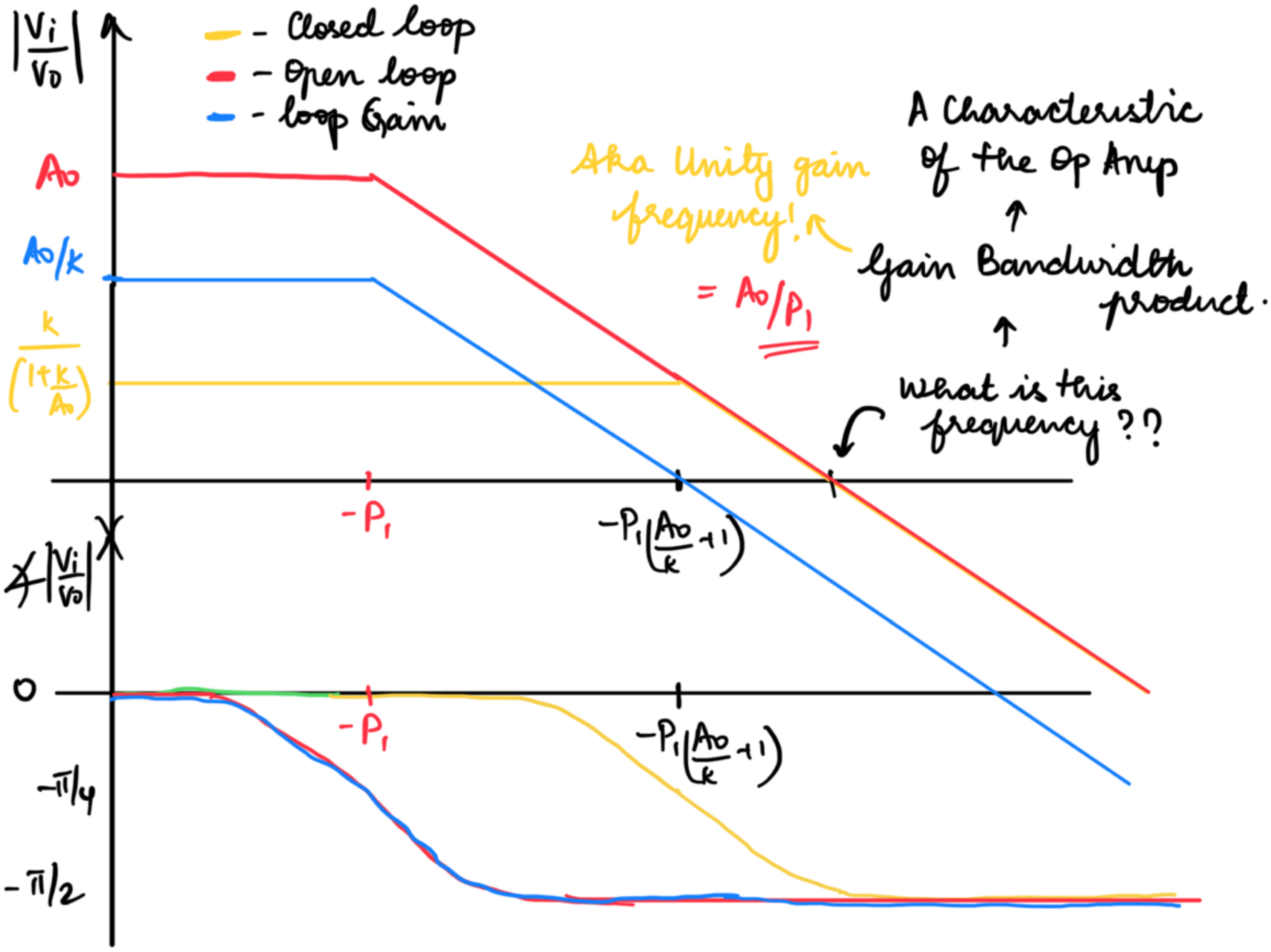
Basically, there is a Quantity called "Gain Bandwidth" Product, and this is the thing that Quantifies Stability...

See Lec (17) [15-Feb 2017], Around 15min, from when he draws Bode Plot.

Plot the Bode plot and all of that from that lecture.

P = pole frequency, $-P$ = pole. → Need to properly listen.

Unity Gain frequency $\omega_u = A_0 \omega_p$.
↳ Gain bandwidth product.



Why is GBP or UGF is more important ?

When we need to change an OpAmp, but retain its behaviour, stability and all that, it is quite not possible to get an OpAmp with the same gain A_0.

But if GBP or UGF is same, the behaviour will be very similar, so that we needn't worry about its gain anymore:

A change in gain will shift the low frequency behaviour up or down, that's it. (see Red & Yellow above)

For stability, GBP matters.

Step Response of Close loop: $\frac{k}{1+k/A_0}\left(1-\exp\left(-P_1\left(1+\frac{A_0}{k}\right)t\right)\right)$

Most important one for -ve feedback : Loop Gain.

Loop Gain for the above non-inverting amplifier: $A(s)/k$.

Refer the Bode Plot ...

	Open loop	Closed loop
DC Gain	A_0	$k/(1+k/A_0)$
Pole	$-P_1$	$-P_1\left(\frac{A_0}{k}+1\right)$
BandWidth	P_1	$P_1\left(\frac{A_0}{k}+1\right)$

1st order transient response: Exponential decay.

Second Order: 2-stage OpAmp (2 poles)

2 OpAmps Cascaded.

$$A(s) = \frac{A_0}{(1+s/p_1)(1+s/p_2)}$$

$$\frac{V_o}{V_i} = \frac{K}{1+\frac{K}{A(s)}} \rightarrow \text{2nd order Transfer function.}$$

$$= \frac{K}{1+\frac{K}{A_0}(1+s/p_1)(1+s/p_2)} \rightarrow \frac{\omega_n^2}{s^2+2\delta\omega_n s+\omega_n^2}$$

ω_n = Natural frequency.

δ = damping factor

$Q = 1/2\delta$

$$\delta = \frac{1}{2}\sqrt{\frac{K}{A_0}}\left(\sqrt{\frac{p_1}{p_2}}+\sqrt{\frac{p_2}{p_1}}\right)$$

To make a reasonable δ, $p_2 \gg p_1 \rightarrow \frac{p_2}{p_1} \sim \frac{A_0}{K}$.

for $\delta = 1$, $\frac{p_2}{p_1} = 4\frac{A_0}{K}$

$\delta = \frac{1}{2}$, $\frac{p_2}{p_1} = \frac{A_0}{K}$

$\delta = \frac{1}{\sqrt{2}}$, $\frac{p_2}{p_1} = 2\frac{A_0}{K}$

Significance of $\delta = \frac{1}{\sqrt{2}}$:

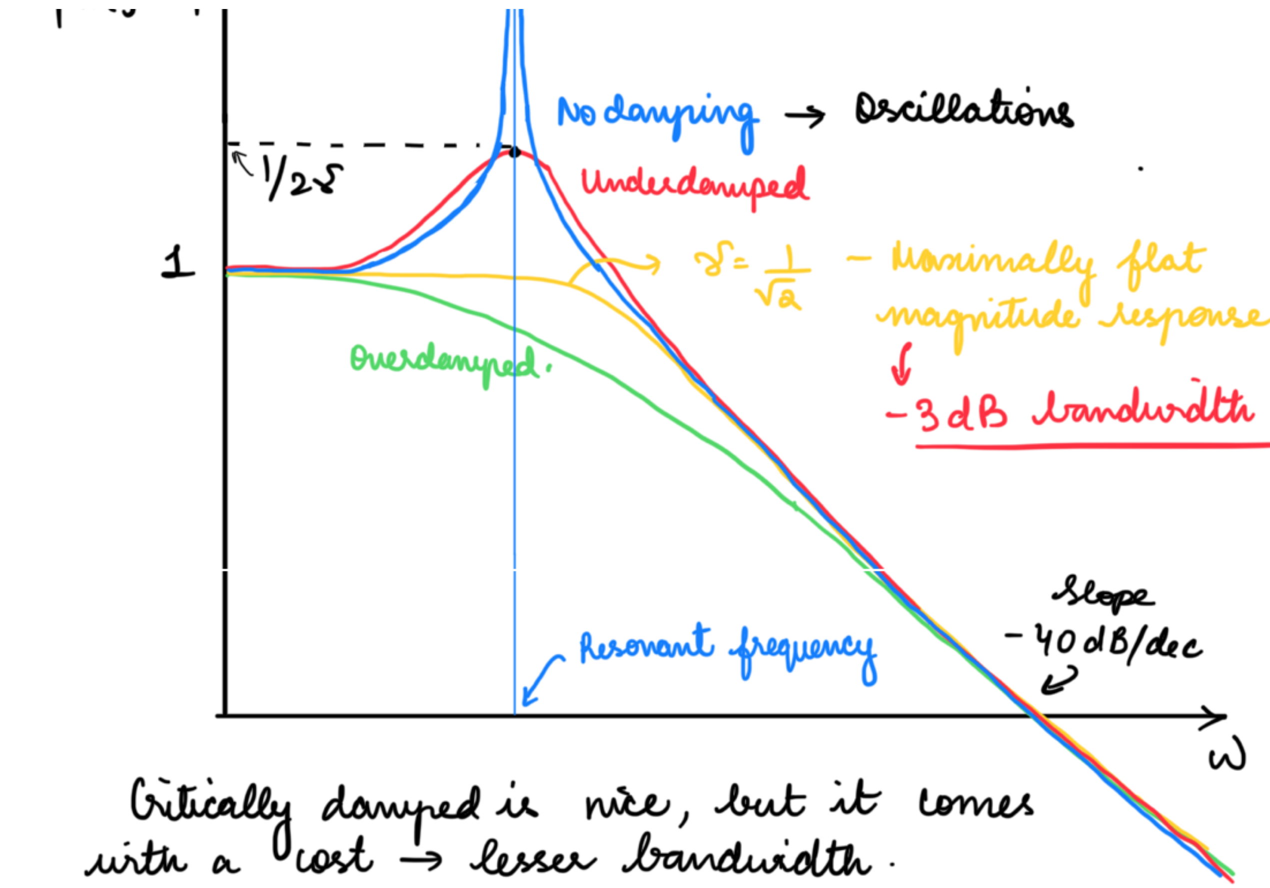
The fastest way to reach 50% of Saturation..., also gives a wider bandwidth without giving a peak.

Maximally flat magnitude response.

for $\delta = 1/\sqrt{2}$,

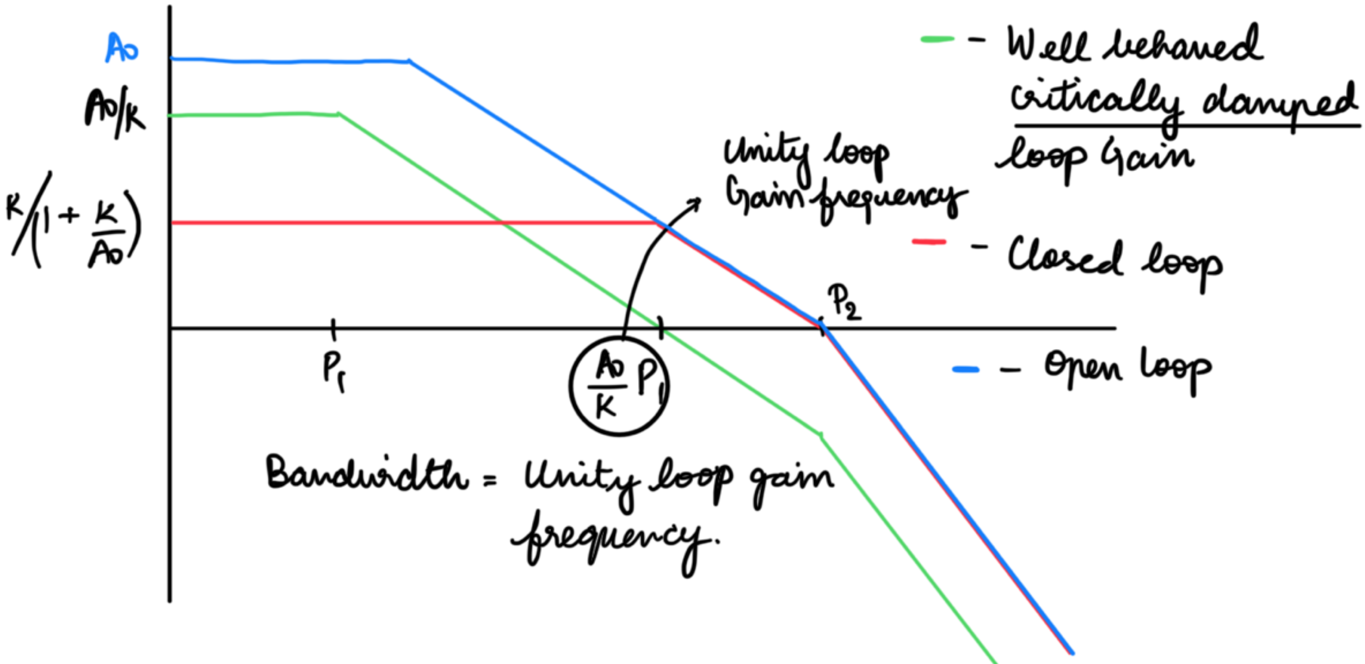
$$H(s) = \frac{1}{1+2\delta\frac{s}{\omega_n}+\frac{s^2}{\omega_n^2}} = \sqrt{\frac{1}{\left(\frac{\omega}{\omega_n}\right)^4+\left(\frac{\omega}{\omega_n}\right)^2\cdot(4\delta^2-2)+1}}$$

$|H(j\omega)|\uparrow$ $\uparrow\infty$



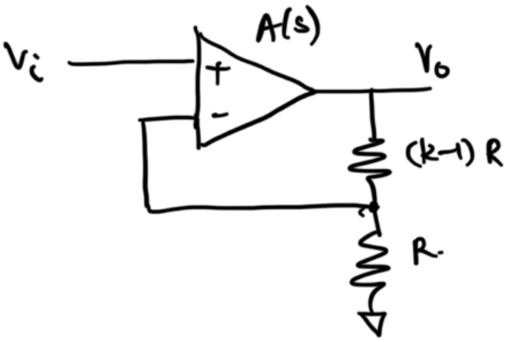
Critically damped is nice, but it comes with a cost → lesser bandwidth.

To get a higher bandwidth only, we are going for slightly underdamped, and the highest bandwidth we can ever get is for a specific underdamped case: $\delta = 1/\sqrt{2}$.



- The feedback system behaves in an ideal way until the unity loop gain frequency.

Higher Order Circuits!



Let's assume that the opamp has 3 identical poles:

$$A(s) = \frac{A_0}{\left(1+\frac{s}{P_1}\right)^3}$$

$$\frac{V_o}{V_i} = \frac{k}{1+\frac{k}{A_0}\left(1+\frac{s}{P_1}\right)^3}$$

$|H(j\omega)| = \infty$ for some ω if $H(s)$ has poles on the $j\omega$ axis

$$H(s) = \frac{\prod_{i=1}^{n}\left(1-\frac{s}{z_i}\right)}{\prod_{i=1}^{n}\left(1-\frac{s}{P_i}\right)}$$

n = order / no. of poles.

Now lets solve the higher order circuit...

$$1+\frac{k}{A_0}\left(1+\frac{s}{P_1}\right)^3 = 0$$

$$\Rightarrow \left(1+\frac{s}{P_1}\right)^3 = -\frac{A_0}{k} = \frac{A_0}{k} e^{-j\pi}$$

$$\therefore \quad 1+\frac{s}{P_1} = \sqrt[3]{\frac{A_0}{k}}\left[-1,\ e^{-j\frac{\pi}{3}},\ e^{j\frac{\pi}{3}}\right]$$

$\therefore$ Will be instable for some value of ω

$$\left(1+\frac{j\omega}{P_1}\right) = \left(\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)\sqrt[3]{\frac{A_0}{k}}$$

$$\frac{1}{2} + j\left(\frac{\omega}{P_1} - \frac{\sqrt{3}}{2}\sqrt[3]{\frac{A_0}{k}}\right) = \sqrt[3]{\frac{A_0}{k}}$$

Equating the Real and imaginary parts,

$\frac{p_0}{K} = 8$. (Real)

$\frac{\omega}{P_1} = \frac{\sqrt{3}}{2} \Rightarrow \omega = \frac{\sqrt{3}}{\cancel{2}} P_1 \times \sqrt[3]{8} = \sqrt{3} P_1$ (imaginary)

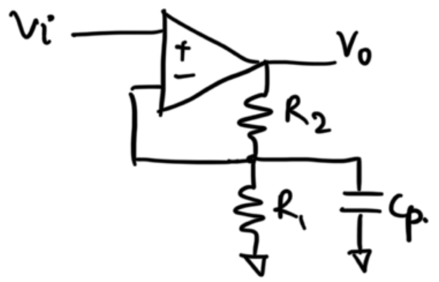
How to solve the ringing (oscillating) problem: Separate the poles much further!

Some of the worst positioning of the poles is to have them identical — Compromises stability.

Vaguely speaking, In second order, we saw that the damping factor is the least when the poles are identical.

If the damping factor is less, then the output of the system oscillates — posing problems.

We'll see why identical poles is the most stupid thing to do.



What is the loop gain of this circuit?

If Cp wasn't there, it'd be $\frac{A(s) R_1}{R_1 + R_2}$

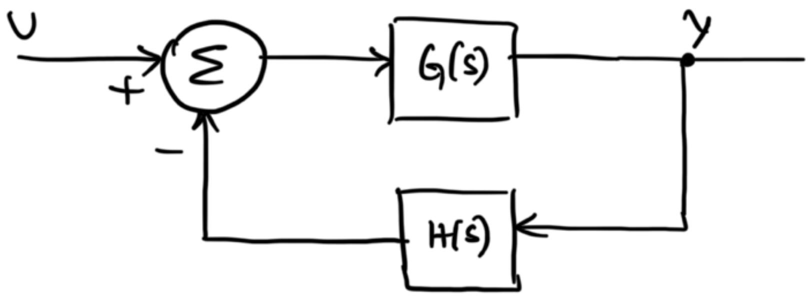
If Cp is there, $\Rightarrow \frac{A(s) G_2}{G_1 + G_2 + sC_p}$

Poles $\Rightarrow \frac{G_1 + G_2}{C_p} = \frac{1}{R_1 \| R_2 \cdot C_p}$.

$$D_N(s) = \left(1 + \frac{s}{P_1}\right)\left(1 + 2\delta_1 \frac{s}{\omega_1} + \frac{s^2}{\omega_1^2}\right) \cdots$$

If order is odd

We will figure out a way to quantitatively determine well behaved" in terms of the pole's position on the s plane.



$$Y = [U - Y(H(s)]\,G(s)$$

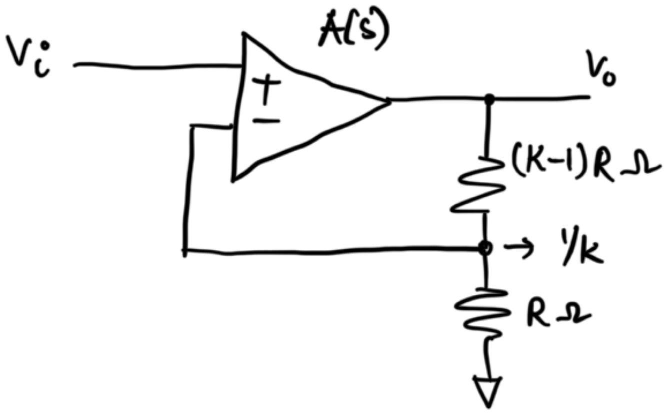
$$Y = U\,G(s) - Y\,H(s)\,G(s)$$

$$U\,G(s) = Y\,[1 + H(s)\,G(s)]$$

$$\Rightarrow \frac{Y}{U} = \boxed{\frac{G(s)}{1 + H(s)\,G(s)}}$$

→ Sanity Check: If $H(s) = 0$, $\frac{Y}{U}$ = just $G(s)$. ✓

Now, what is $G(s)$ and $H(s)$ for a non-inverting amplifier ?!



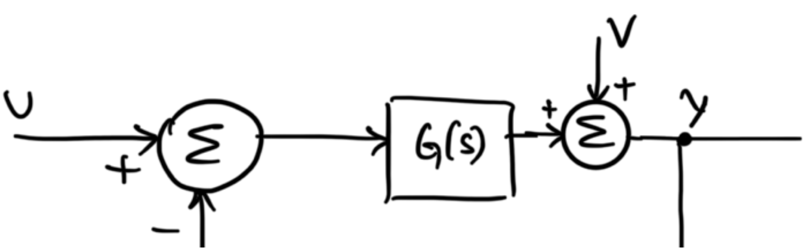
$\therefore G(s) = A(s)$

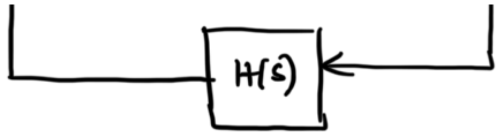
$H(s) = \frac{1}{K}$

$$\therefore \frac{V_i}{V_o} = \frac{U}{Y} = \frac{A(s)}{1 + \frac{A(s)}{K}}$$

and $G(s)\,H(s)$ = loop gain.

Now, lets add 'V' to the output of OpAmp ($G(s)$).



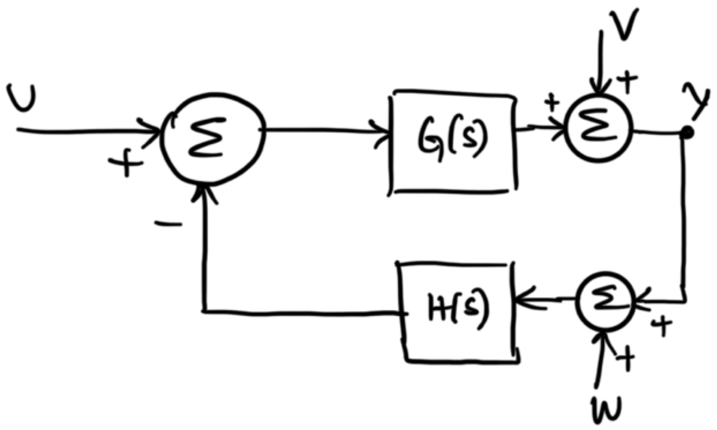


Now, $\left.\frac{V}{Y}\right|_{U=0} \Rightarrow ??$

$$Y = (-Y[H(s)])\, G(s) + V$$

$$Y\left[1 + H(s)\,G(s)\right] = V$$

$$\frac{Y}{V} = \frac{1}{1 + H(s)G(s)} = \boxed{\frac{1}{1+GH}}$$



Now, $\left.\frac{Y}{W}\right|_{U,V=0} = ?$

$$Y = (-\,H(s)\,[W+Y])\,G(s))$$

$$= -H[W+Y]\,G$$

$$Y = -GH(W+Y)$$

$$Y(1+GH) = -GHW$$

$$\boxed{\frac{Y}{W} = \frac{-GH}{1+GH}}$$

What do we infer from these results? The denominator is always $1+GH$. → This is a property of any negative feedback system.

Poles are the property of a network : Not where you apply the input and the output.

All good things happen when $|GH| \gg 1$;

$$\frac{Y}{U} \simeq \frac{1}{H} , \quad \frac{Y}{V} = \frac{1}{GH} , \quad \frac{Y}{W} = -1.$$

Now, what happens if $|GH| \ll 1$? [Not Having feedback]

$$\frac{Y}{U} \simeq G , \quad \frac{Y}{V} \simeq 1 , \quad \frac{Y}{W} \simeq -GH.$$

Note: (Nagendra OP)

- Forward path has to provide a large gain, but can be imprecise.

- Feedback path doesn't have to provide gain, but needs to be precise.

If these 2 things are taken care of, then the gain of the Amplifier can be precisely defined using -ve feedback.

loop Gain = GH

$1 + GH$ or $1 + L$ will appear in the denominator of every -ve feedback system transfer function.

$$\boxed{\frac{Y}{U} = \frac{G}{1+GH}}.$$

Now, we are trying to study stability.

Let's try figure out for what value (just one for now) of L the system becomes unstable:

Loop Gain → if −1, Transfer function goes to ∞

For an amplifier network to be stable, the Closed loop transfer function must be stable → i.e., the closed loop transfer function's poles must lie on the left half plane.

But not always it is easy to solve and get the poles of a Closed loop transfer function.

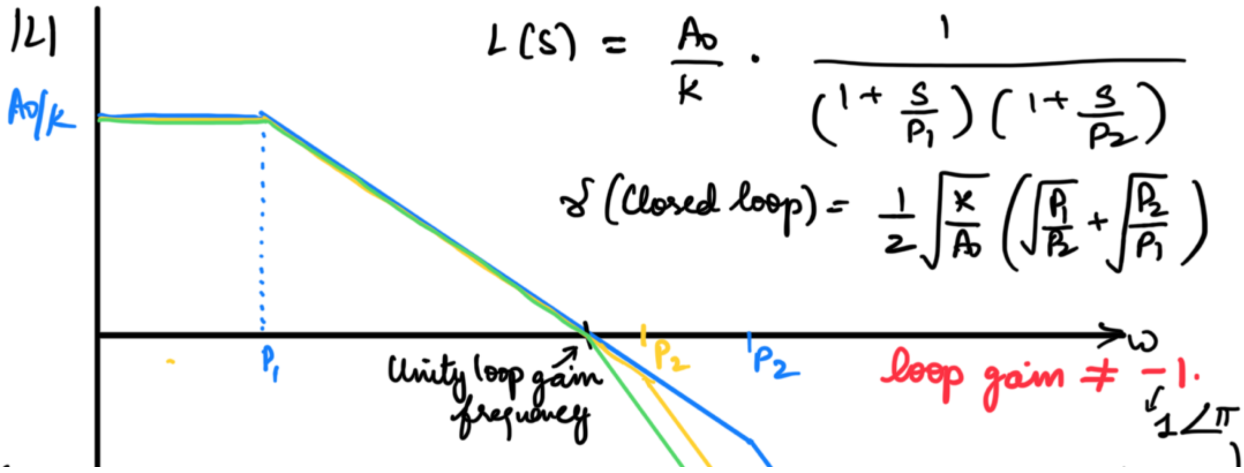
So, we'll figure out a way to find out (or get a rough idea of where they are) the poles — without having to solve for the polynomials of higher orders (and eventually fix them!)

We'll also figure out WHAT the value of loop gain should be for stable network at what frequency.

[Loop Gain = $L(j\omega)$]

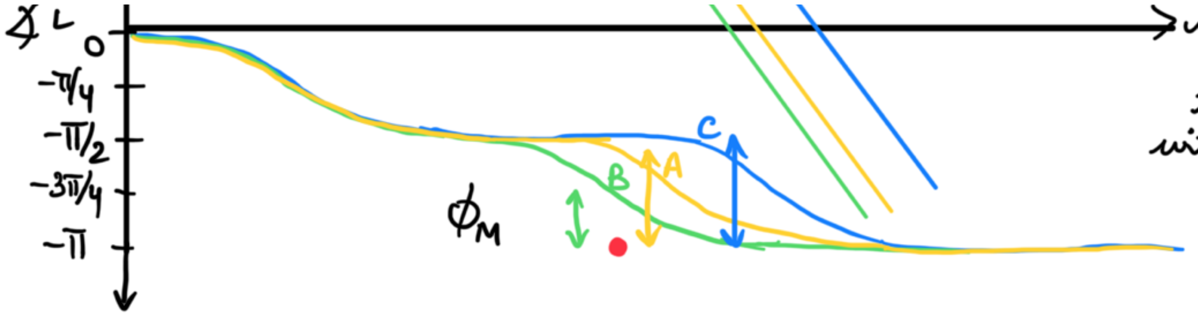
For now, **Loop Gain = −1 : That is Something we need to stay away from...**

Now let us see the Magnitude plot of a well behaved 2nd order system. (loop gain)



$$L(s) = \frac{A_0}{K} \cdot \frac{1}{\left(1+\frac{s}{P_1}\right)\left(1+\frac{s}{P_2}\right)}$$

$$\gamma\,(\text{Closed loop}) = \frac{1}{2}\sqrt{\frac{K}{A_0}}\left(\sqrt{\frac{P_1}{P_2}}+\sqrt{\frac{P_2}{P_1}}\right)$$



Marked the point with Red dot

C – We know, well behaved.
A – slightly lower damping factor → second pole comes closer
B – Even lesser damping.

As we can see, decrease in damping results in the phase plot to go closer to the unstable region.

So, is there a way to quantify how closer we are to the instability? – Yes.

The arrows, color coded accordingly, gives the distance between the phase of that transfer function in magnitude 1, and the point $1\measuredangle\pi$ for that transfer function

This distance between $\measuredangle L(j\omega_{ugf})$ and $-\pi$ = Phase margin.
Very widely used for stability.

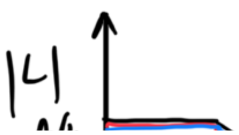
~~Generalising stability condition~~
even for systems of higher order,
without explicitly evaluating the polynomial for poles

The phase margin $\rightarrow \phi_M$ = $\min |1+L|$

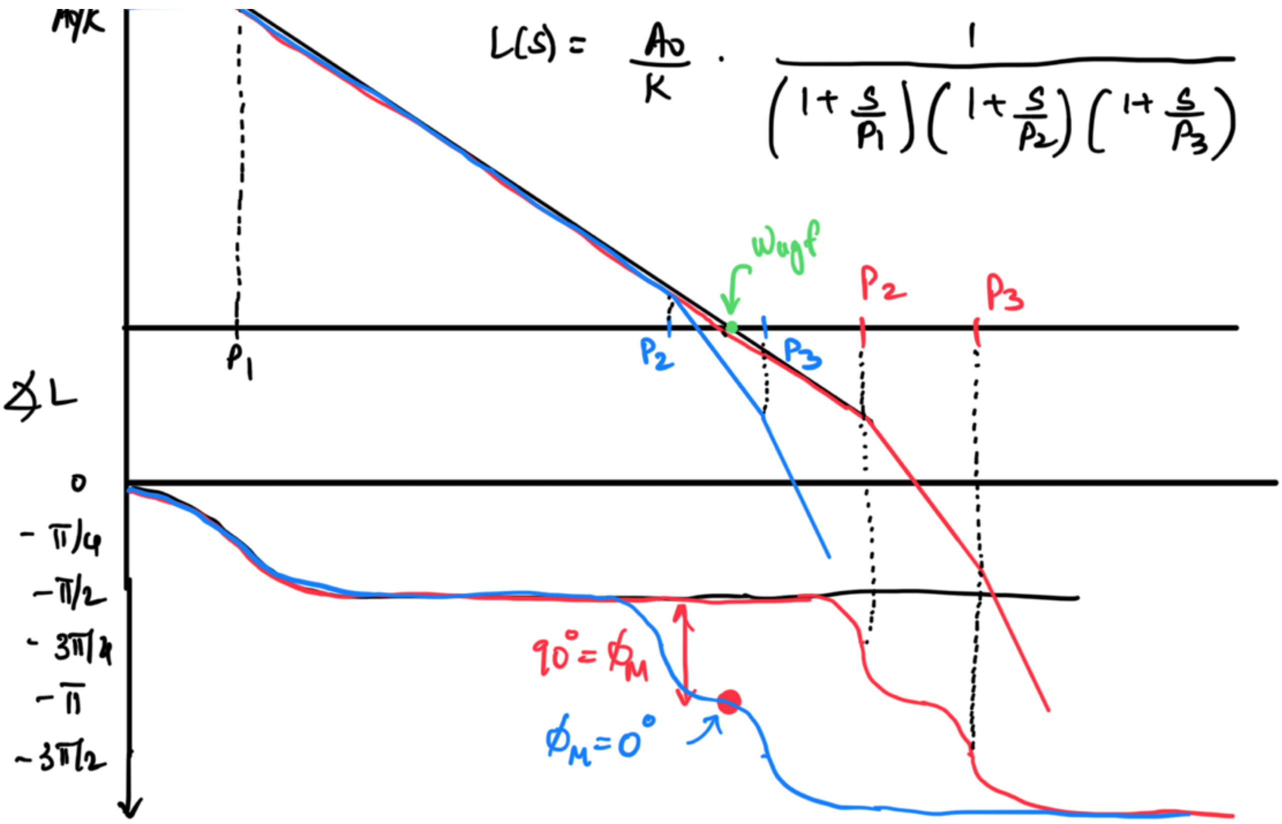
$$\phi_M = \tan^{-1}\left(\frac{\omega_{ugf}}{P_1}\right).$$

The best it can be $= 90° \rightarrow \delta =$
The worst it can be, yet be stable $= 45° \rightarrow \delta = 1/2$
$\delta = \frac{1}{\sqrt{2}} = 63°$; $\delta = 1 = 76°$.

Higher order systems



$$L(s) = \frac{A_0}{K} \cdot \frac{1}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)\left(1+\frac{s}{p_3}\right)}$$



When $L(s)$ has no zeroes (and how many ever poles)

if $\measuredangle L < -\pi$ at w_{ugf}, the system is unstable.

→ From Nyquist theorem, complex analysis.