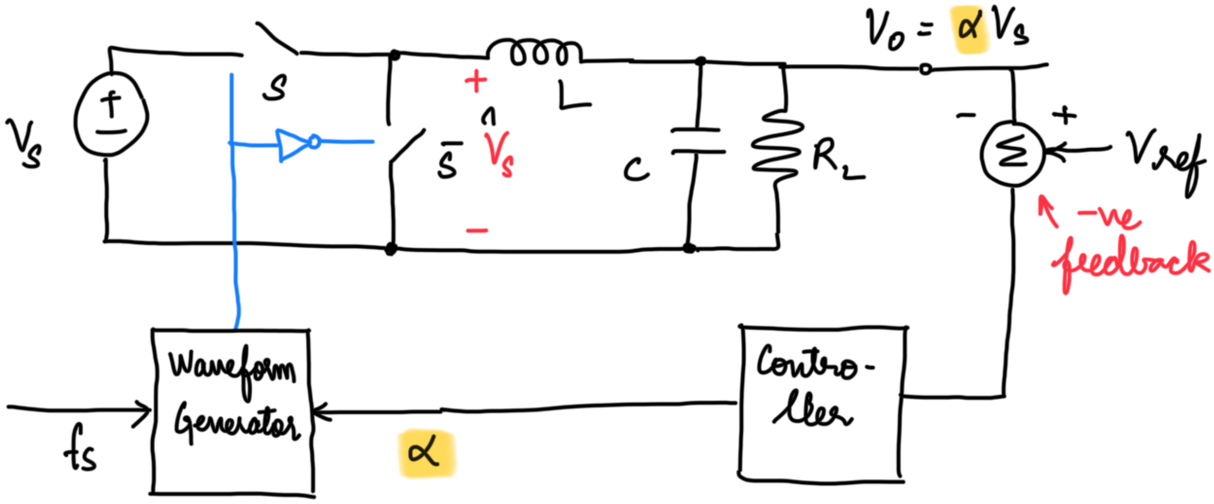
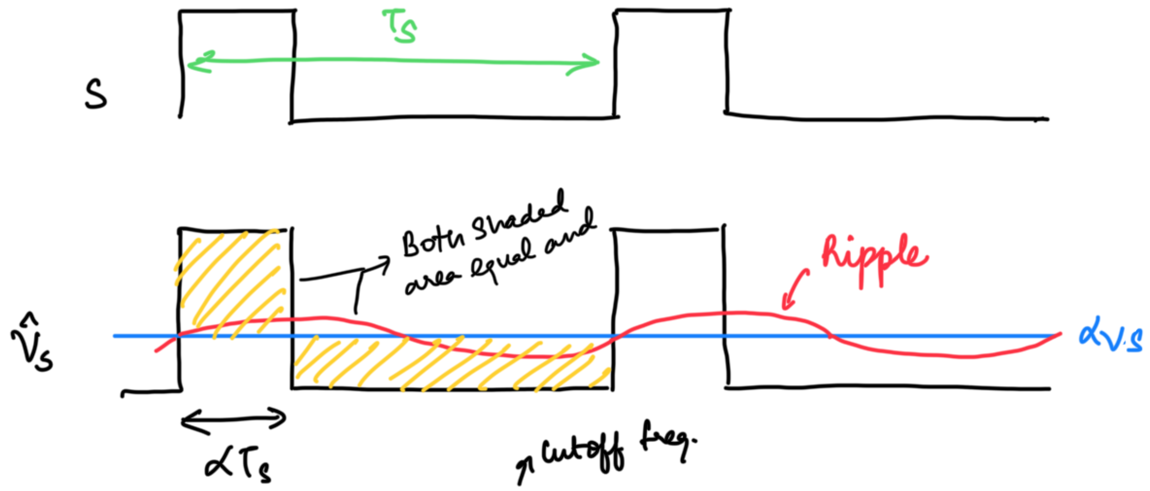
DC-DC Converters

Lossless ways of converting DC Voltages

We already have seen a topology to obtain average voltages lower than source voltage using switches and low pass filters (lossless LC)

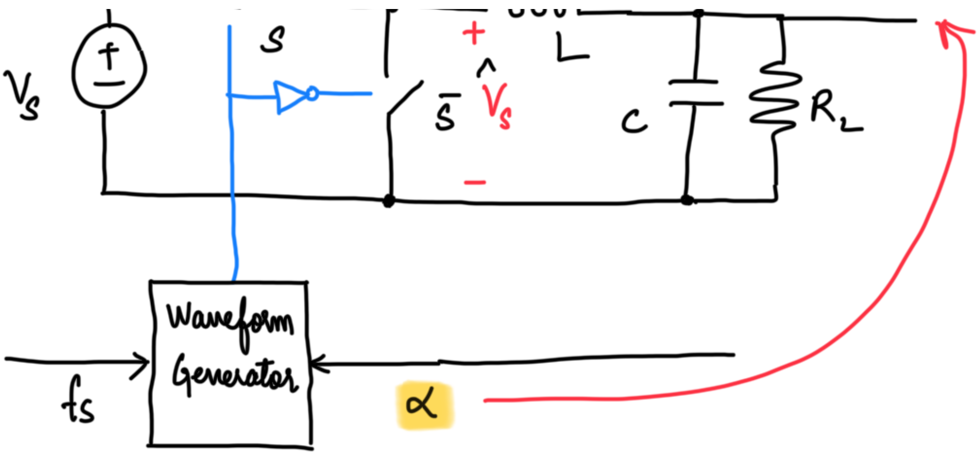




$$f_s \gg \frac{1}{2\pi\sqrt{LC}}$$ ↗ cutoff freq. → Ripple Amplitude drops and we get Nearly DC voltage!

Now, since we have a negative feedback loop, we need to make sure it is stable.

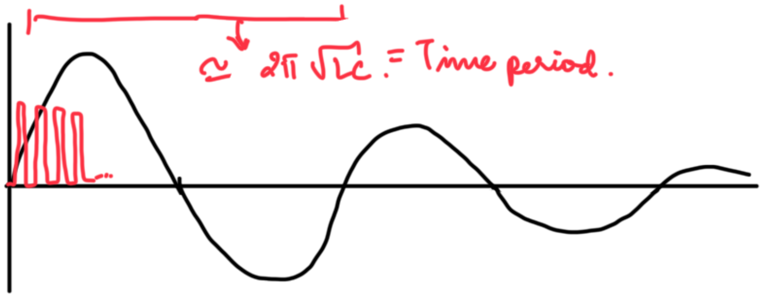
And for that, we need the loop gain – And for that we need the model of this circuit.



Transfer function − ? → we'll need impulse response.

How to determine the impulse response? → We'll try to evaluate the step response.

For ease of calculation, let α : α_0 in the beginning.



$T_s << 2\pi\sqrt{LC}$

We now are basically convolving the α [which is now the input voltage] and the system's $x(t) \rightarrow$ RLC circuit!

$$\therefore \int h(\tau)\,x(t-\tau)\,d\tau = h_0 \int x(t-\tau)\,d\tau .$$

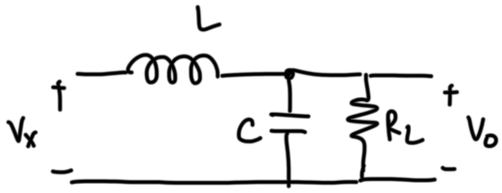
Impulse response of the LC filter hardly changes within a switching period.
← Approximation!

* The input can be approximately represented by its average in each period.

Response of the circuit to a step α_0 in the duty cycle
$\simeq$ Response of the LTI (LPF) to a step $\alpha_0 V_s$

Now: The transfer function: What is it?

$$\frac{V_o(s)}{\alpha(s)} = \;?$$



$$\rightarrow \quad \frac{V_o(s)}{V_x(s)} = \frac{1}{s^2 LC + \frac{L}{R_L} s + 1}.$$

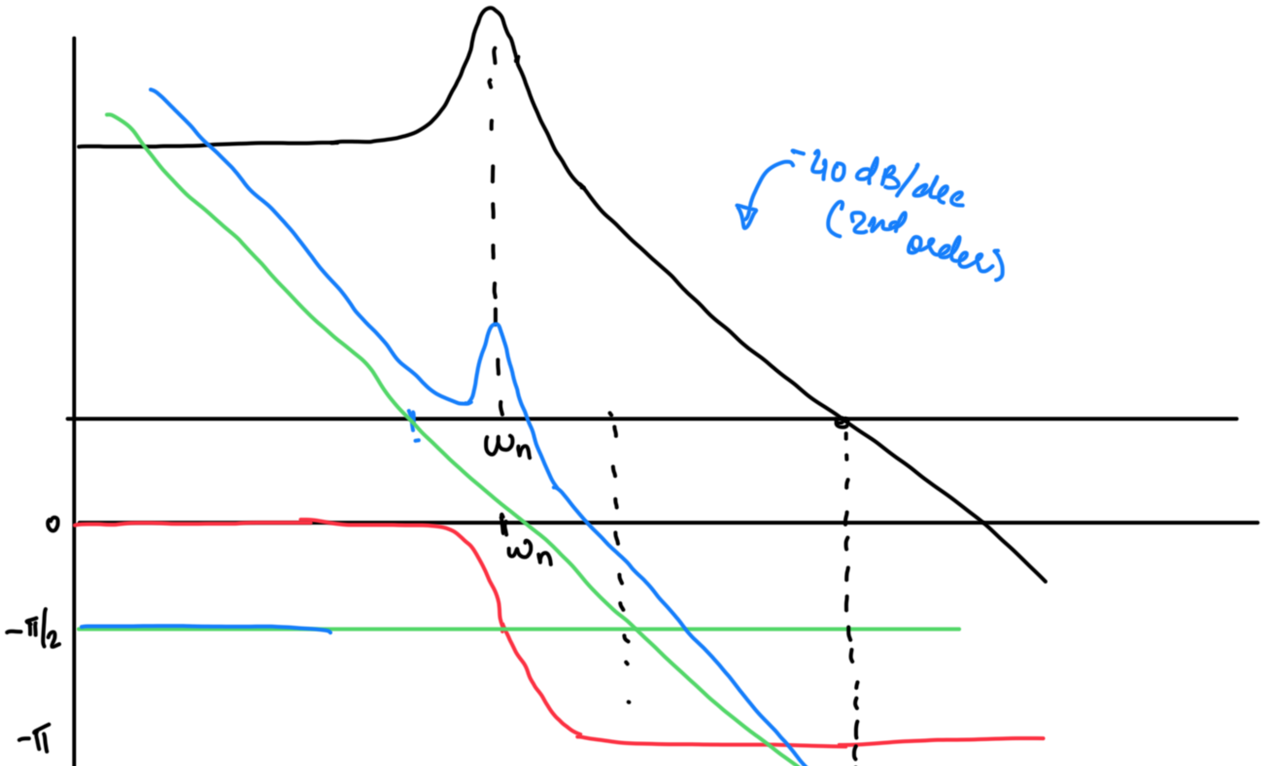
But $V_x(s) = \alpha(s) V_s$.

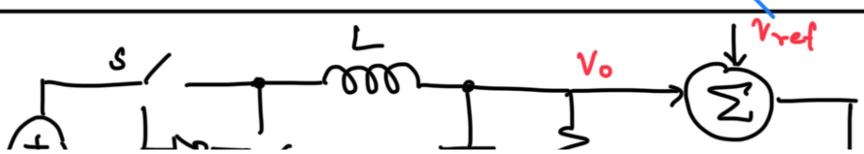
$$\therefore \quad \frac{V_o(s)}{\alpha(s)} = V_s \cdot \frac{1}{s^2 LC + s\frac{L}{R_L} + 1}$$

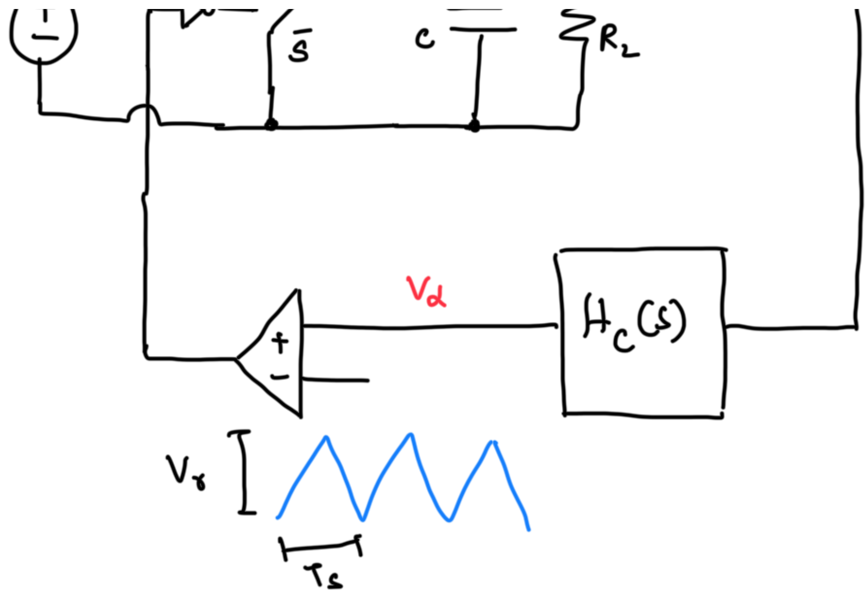
Now that we have the transfer function, we will now look at the loop gain, find the phase margin, and study the stability of the negative feedback that we encountered...

α cannot vary rapidly ... NOTE.

Let's bode plot $V_o(s)/\alpha(s)$.







$H_C \frac{V_o}{V_d}$ = Loop Gain → Must have appropriate phase margin

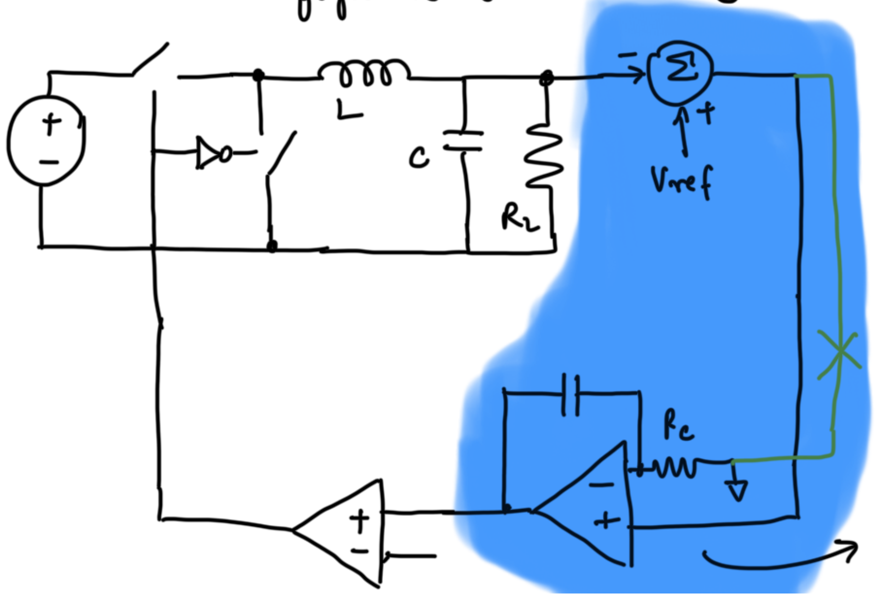
We want H_c to have high gain in DC. $\omega_c/s. = H_c$ [for now].

Basically an integrator: $H_c = \frac{\omega_c}{s}$. What/ How should ω_c be?

@ $\frac{1}{\sqrt{LC}}$: $\frac{V_s}{V_d}$ has its peak ; Peak : $\frac{V_s}{V_r} Q$.

$$H_c\left(j\frac{1}{\sqrt{LC}}\right) < \frac{1}{\frac{V_s}{V_r} Q} \quad ; \quad \omega_c < \frac{\omega_u}{\frac{V_s}{V_r} \cdot Q}$$

Now that we've figured out that H_c must be an integrator:



$H_c = -\frac{1}{sC_cR_c}$??

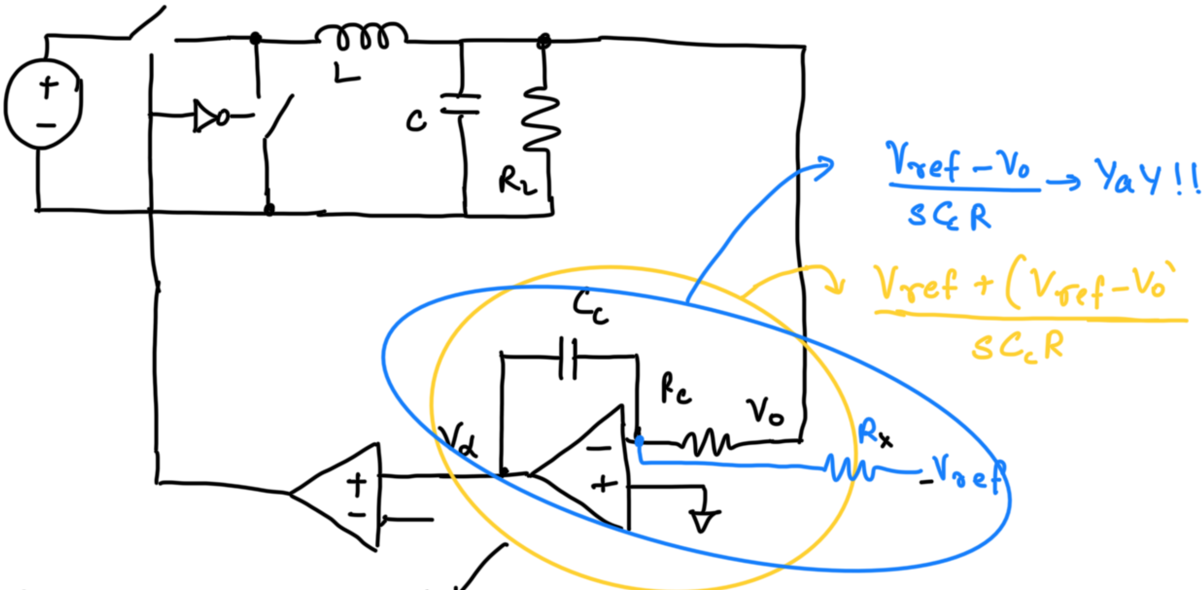
$1 + \frac{1}{sC_cR_c}$

$$W_{ugf} = \omega_c \cdot \frac{V_s}{V_r} = \frac{1}{C_c R_c} \cdot \frac{V_s}{V_r}$$

$CLB = W_{ugf}$ if the system is ideal.

We will now try to implement this part of the circuit...

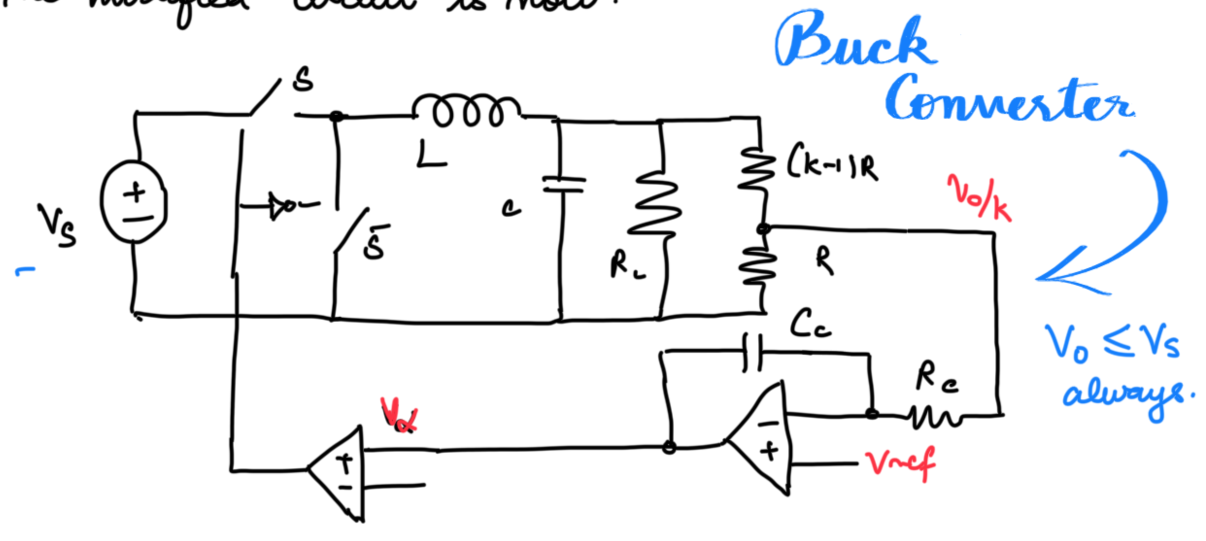
For addition - summing amplifier?

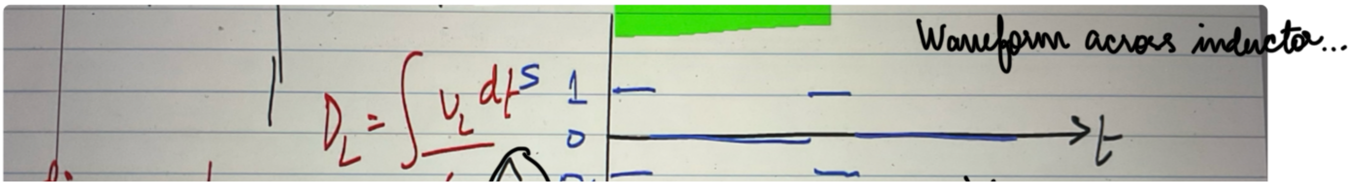


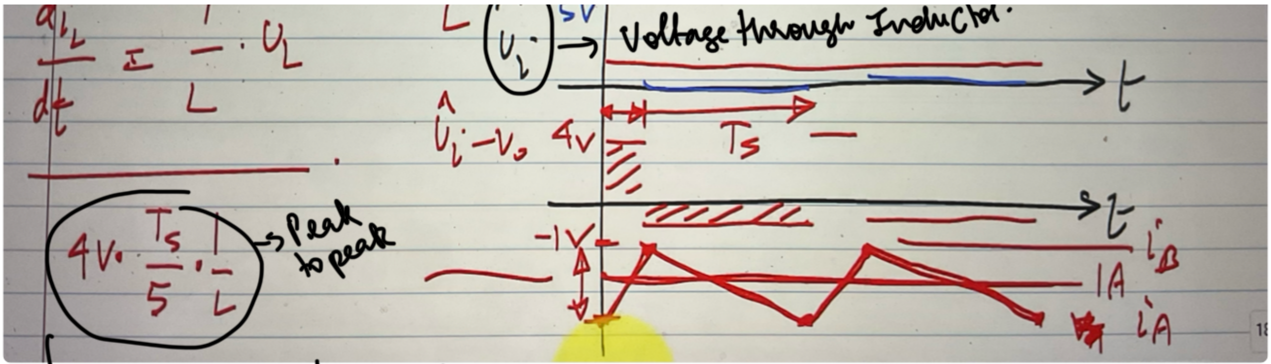
We now have a summing integrator, instead of a summing amplifier.

But we usually compare an attenuated version of V_o with V_{ref}.

$\therefore$ The modified circuit is now:







$= 1V \cdot \frac{4T_s}{5} \cdot \frac{1}{L}$

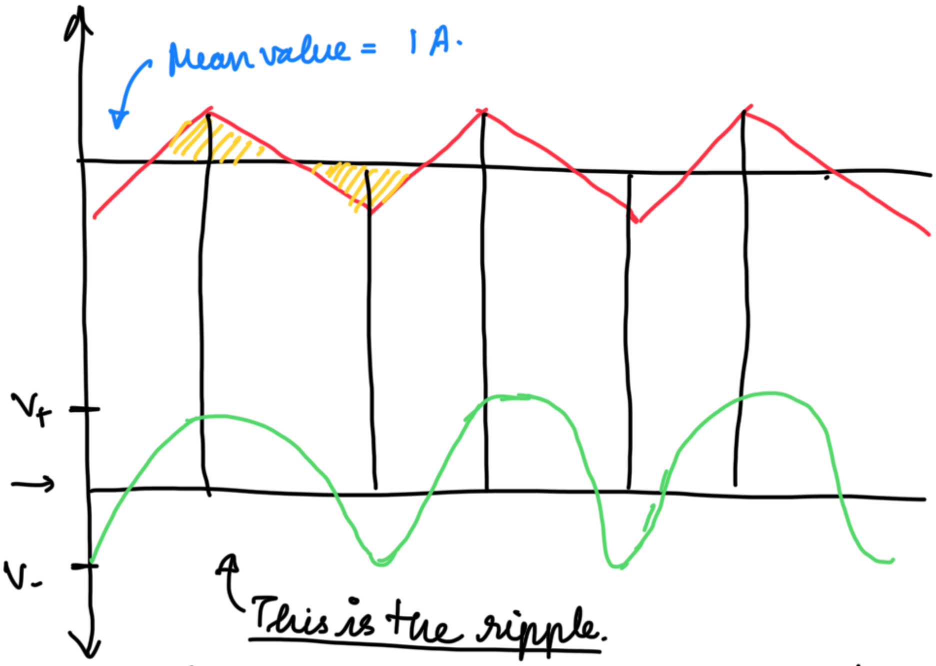
This is taking $V_s = 5V$, $\alpha = \frac{1}{5}$.

Now after the inductor, the ripple goes through the capacitor and the DC part will go through the resistor → Which is exactly what we want.

If this is the current flowing through the inductor, if R is high, then it flows into the capacitor.

So, what would be the capacitor voltage? – Parabolae?

$\left(i_c = C\frac{dV}{dt}\right)$



$V_+ - V_-$ = Peak to Peak voltage

= ?

1V

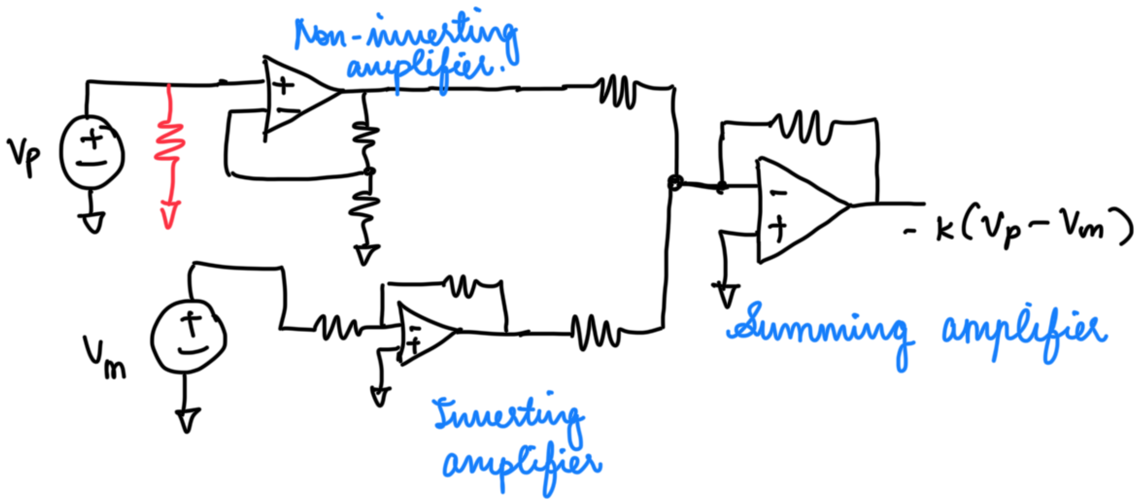
This is the ripple.

and we need the peak-peak voltage to be as less as possible!! ?!

We'll look at how to get KV_i from V_i.

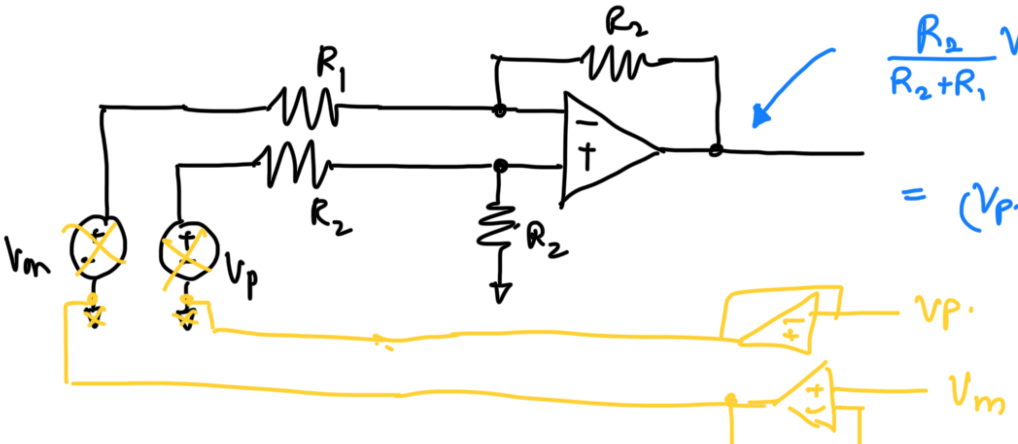
→ Trivial right? — Non inverting amplifier.

But what about getting $k(V_p - V_m)$ from V_p & V_m?



Is there a way to do all of this with lesser OpAmps? Yes!

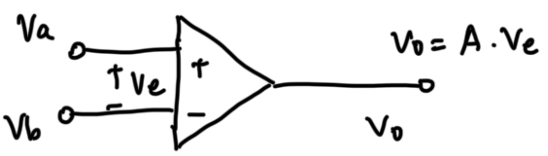
Infact, we can do it with just one OpAmp!! How? :

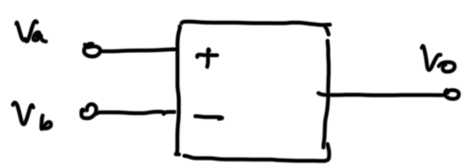


$$\frac{R_2}{R_2+R_1} V_p \left(1 + \frac{R_2}{R_1}\right) - V_m \frac{R_2}{R_1}$$

$$= (V_p - V_m)\frac{R_2}{R_1}.$$

} → Buffers connected to ensure no current is drawn by the major OpAmp.





$V_o = \alpha V_a + \beta V_b$.

For an op Amp, $\alpha = -\beta$.

[For any general 3-terminal device]

$V_d = (V_a - V_b)$ $\quad V_{cm} = \frac{V_a + V_b}{2}$ Common mode voltage

(differential
Component)

$$V_{cm} = \frac{V_a + V_b}{2} \quad \text{(average)}.$$

$$V_a = V_{cm} + V_d/2 \quad ; \quad V_b = V_{cm} - V_d/2.$$

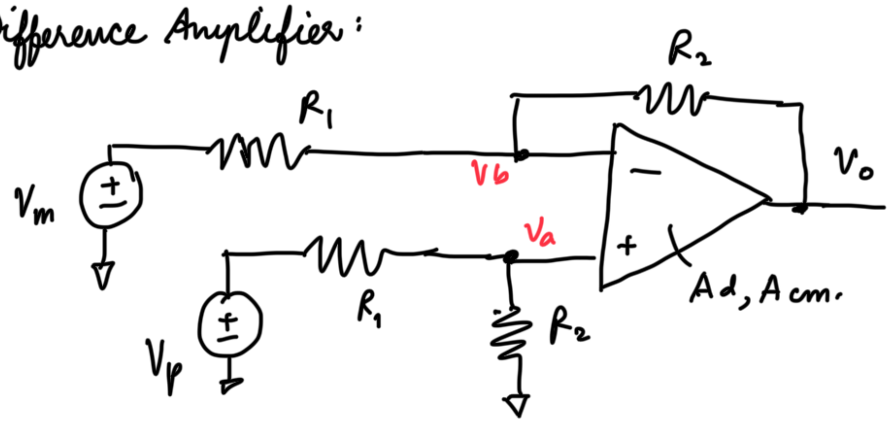
Ideally, $\alpha = -\beta$ for opAmp. But it is never perfect.

$$V_o = \alpha V_a + \beta V_b = \frac{(\alpha - \beta)}{2} V_d + \frac{(\alpha + \beta)}{2} V_{cm}.$$

$$= A_d V_d + A_{cm} V_{cm}$$

Differential gain. Common mode gain

Difference Amplifier:



$$V_a = V_m \frac{R_2}{R_2 + R_1} \quad ; \quad V_b = \frac{V_o R_1}{R_1 + R_2}.$$

$$V_o = A_d \cdot V_d + A_{cm} \cdot V_{cm}.$$

$$V_d = \frac{(V_p - V_m) R_2}{R_1 + R_2} - \frac{V_o R_1}{R_1 + R_2} ; \quad V_{cm} = \frac{V_p + V_m}{2} \frac{R_2}{R_1 + R_2} + \frac{V_o}{2} \frac{R_1}{R_1 + R_2}.$$

$$= A_d (V_p - V_m) \left[\frac{R_2}{R_1 + R_2}\right] - A_d V_o . \frac{R_1}{R_1 + R_2}$$

$$+ A_{cm} \left(\frac{V_p + V_m}{2}\right) \frac{R_2}{R_1 + R_2} + \frac{A_{cm}}{2} \left(\frac{V_o R_1}{R_2 + R_1}\right)$$

$$\therefore V_o = \left(A_d (V_p - V_m) \frac{R_2}{R_1 + R_2} + A_{cm} \left(\frac{V_p + V_m}{2}\right) \frac{R_2}{R_1 + R_2} \right)$$

$$\left(\frac{\cdots}{\left(1 + A_d - \frac{A_{cm}}{2}\right) \cdot \frac{R_1}{R_1+R_2}} \right) \div A_d \frac{R_1}{R_1+R_2}$$

$$= (V_p - V_m)\,\frac{R_2}{R_1} \times \frac{1}{1 - \frac{A_{cm}}{2A_d} + \frac{1}{A_d}\left(\frac{R_1+R_2}{R_1}\right)}$$

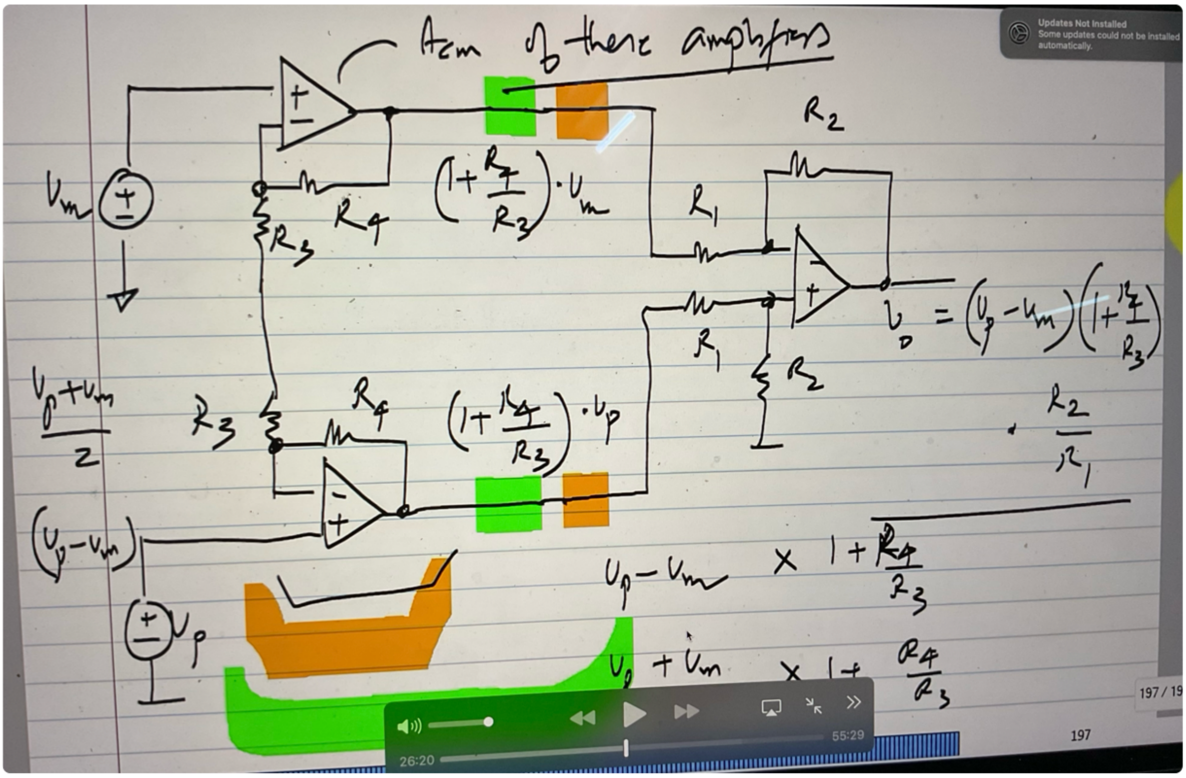
$$+ \left(\frac{V_p+V_m}{2}\right) \cdot \frac{R_2}{R_1} \cdot \frac{A_{cm}/A_d}{1 - \frac{A_{cm}}{2A_d} + \frac{1}{A_d}\left(\frac{R_1+R_2}{R_1}\right)}$$

Not good...

A_d/A_{cm} → Called as the Common mode Rejection Ratio.

Instrumentation Amplifier

Something similar to CMRR → PSRR (Power supply rejection Ratio)



What is the difference between the previous circuit and the current circuit?

$$V_o = (V_p - V_m)\left(1 + \frac{R_4}{R_3}\right) \cdot \frac{R_2}{R_1} \cdot \frac{1}{\cdots} + \left(\frac{V_p+V_m}{2}\right)\left(1 + \frac{R_4}{R_3}\right) \cdot \frac{R_2}{R_1} \cdot \frac{A_{cm}}{A_d} \cdot \frac{1}{\cdots}$$

3 opamp instrumentation Amplifier.