Filters

Frequency selective circuits

How to make filters → Rs, Ls + Cs. Mostly Cs as Ls are bulky.

But we'll try implementing filters using active elements [semiconductor devices]. → Active filters.

Filters have a vital application in sampling.

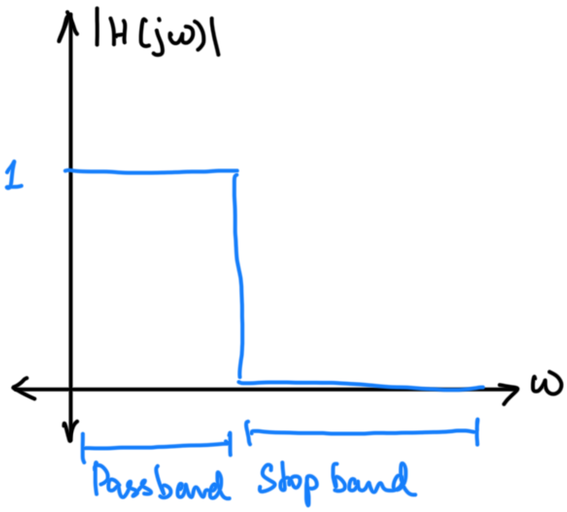
Synthesising first and second order filters

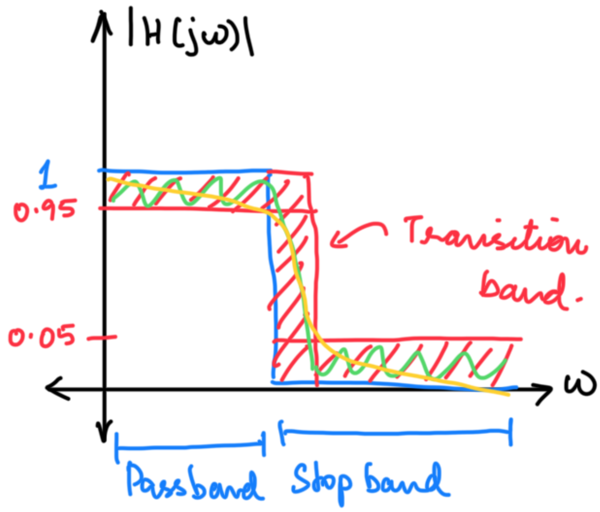
→ Channel selection
→ Noise removal
→ Anti-aliasing
→ Reconstruction

Filter:

$$H(s) = \frac{N(s)}{D(s)} = \frac{V_0}{V_i}$$

Filter synthesis – obtain N & D from specifications → Zeros and poles.





Let us assume $\frac{N(s)}{D(s)}$, # zeros ≤ # poles ⇒ Low Pass

N^{th} order polynomial (with Real coefficients)

$\downarrow$

Even N $\quad \frac{N}{2}$ 2^{nd} order polynomials w/ real coeffs.

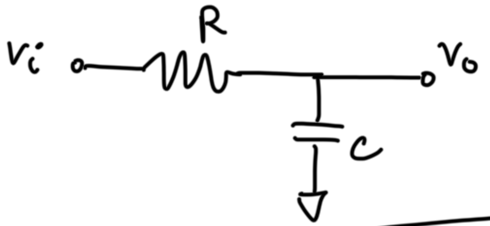
Odd N $\quad \left(\frac{N-1}{2}\right)$ 2^{nd} order $+$ 1 first order, all with real coeffs.

Let us consider Filters of this type : $H(s) = \frac{1}{D(s)}$

$$= \frac{1}{D_1(s) \cdot D_2(s) \cdots D_k(s)}$$ Product of 1^{st} and 2^{nd} order terms

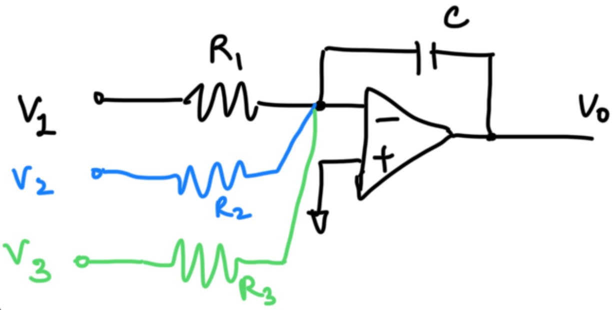
$$= \frac{1}{D_1(s)} \cdot \frac{1}{D_2(s)} \cdot \frac{1}{D_3(s)} \cdot \cdots \cdot \frac{1}{D_k(s)} \cdot$$ $\}\rightarrow$ Cascade!!

First and second order active filters? : (low pass)



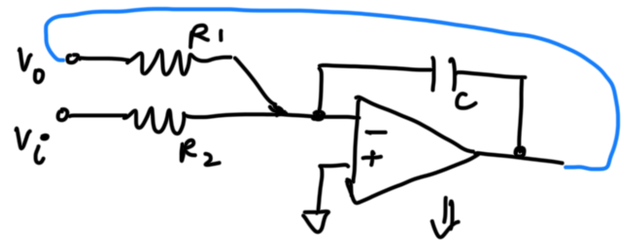
$$\frac{V_o}{V_i} = \frac{1}{1+sCR} = \frac{1}{1+(s/p_1)}$$

Building block of filters $\longrightarrow$ Integrators.



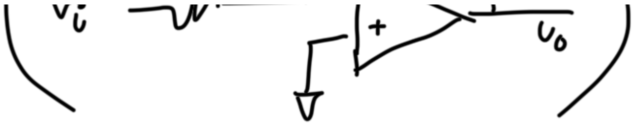
$$V_o = \left(-\frac{1}{sCR_1} \cdot V_1\right)\left(-\frac{1}{sCR_2} V_2\right) \cdot \left(\frac{-1}{sCR_3} V_3\right)$$

$$(V_o - V_i)\left(\frac{-p_1}{s}\right) = V_o \ ;$$

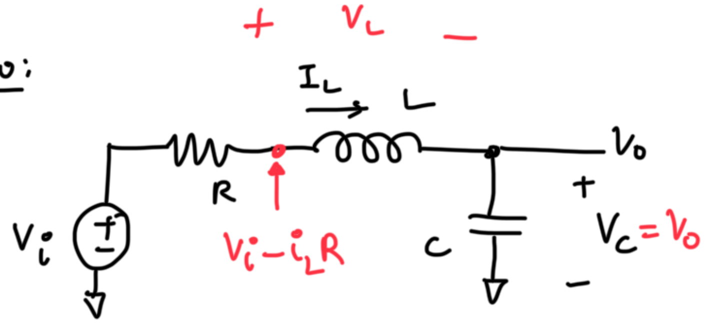


$\Downarrow$

!!!



Now:



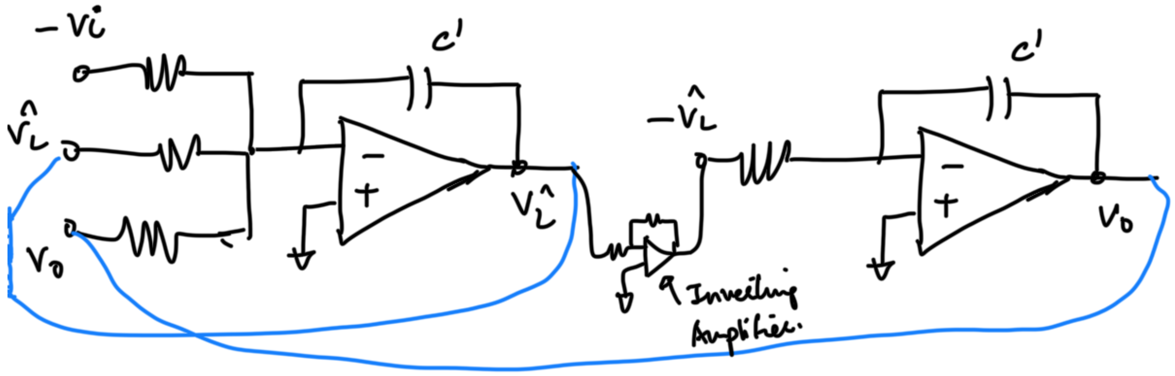
$$I_L = \frac{1}{sL} \cdot V_L$$

$$V_c = \frac{1}{sC} \cdot I_c .$$

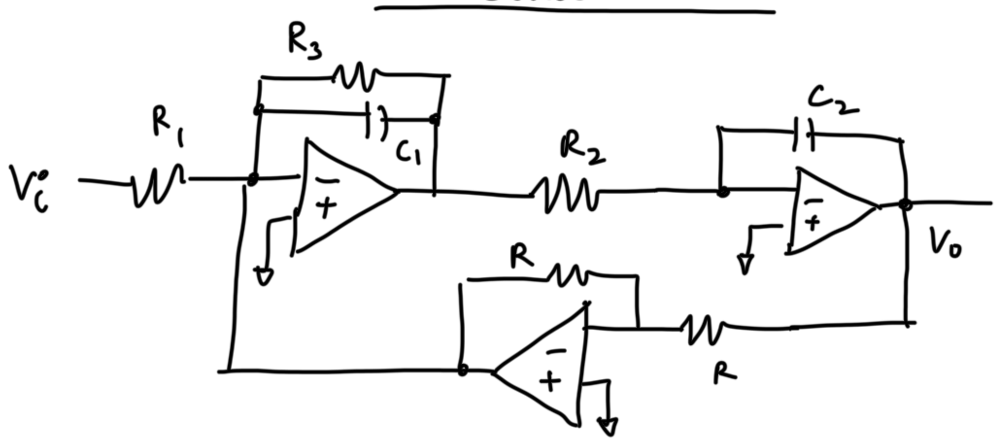
$$V_L = (V_i - i_L R - V_o) \qquad \therefore R\, I_L = \frac{(V_i - i_L R - V_o)}{sL} R$$

let $\hat{V}_L = I_L R$ $\qquad R \dfrac{I_L}{sC} = V_o \cdot R$

$$\therefore (V_i - \hat{V}_L - V_o) \cdot \frac{R}{sL} = \hat{V}_L \; ; \quad \hat{V}_L \cdot \frac{1}{sCR} = V_o .$$

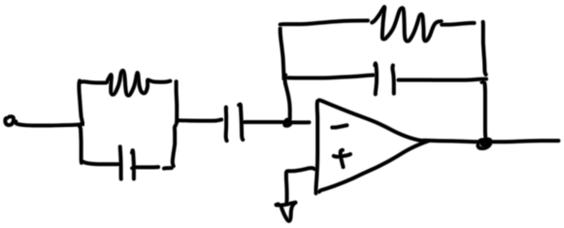


A Cleaner circuit:

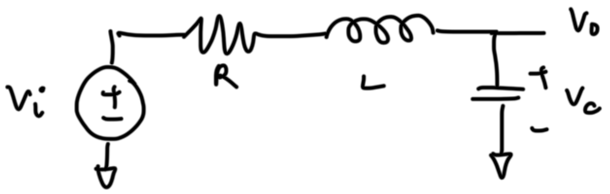


Most general form of a 1^{st} order Transfer function:

$$H(s) = \frac{b_0 + b_1 s}{a_0 + a_1 s}$$ Bilinear



2nd order filter:

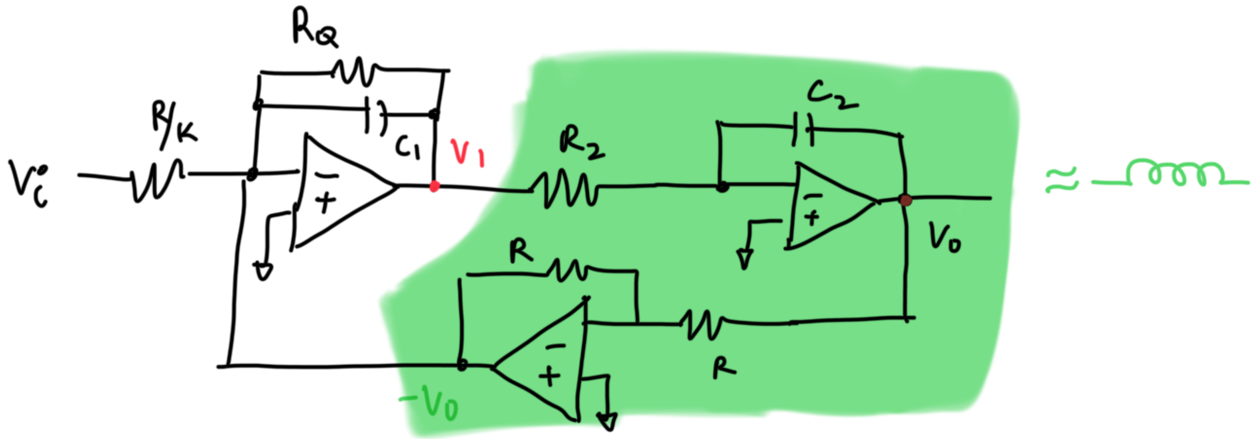


State Variables : I_L, V_c

Active implementation : Integrator outputs.

N^{th} order : N integrators...

$$R I_L = \frac{-R}{sL}\left(-V_i + I_L R + V_o\right) \quad ; \quad -V_o = \frac{-1}{sCR}\left(I_L R\right)$$

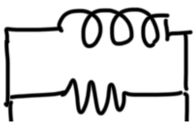


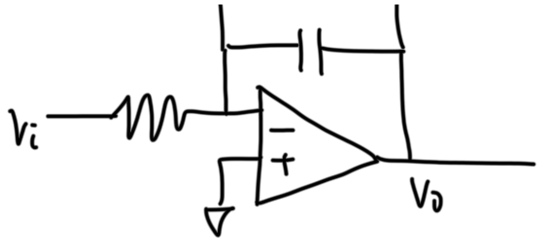
$$\left(\frac{s}{\omega_n}\right)^2 + \frac{s}{\omega_n}\frac{1}{Q} + 1 \quad ; \quad \omega_n = \frac{1}{CR} ; \quad Q = R_Q/R.$$

$$V_o = \frac{-1}{sCR} \cdot V_1 \quad ; \quad V_1 = -\left(\frac{R_Q}{1 + sCR_Q}\right)\left(-\frac{V_o}{R} + \frac{kV_i}{R}\right)$$

$$\boxed{\frac{V_o}{V_i} = \frac{k}{(sCR)^2 + sCR \cdot \frac{R}{R_Q} + 1}}$$ → Lowpass ; $\frac{R_Q}{R}$

$$\frac{V_1}{V_i} = \frac{-sCR}{(sCR)^2 + sCR \cdot \frac{R}{R_Q} + 1}$$ → Bandpass ; R_Q/R ; $\frac{1}{CR}$



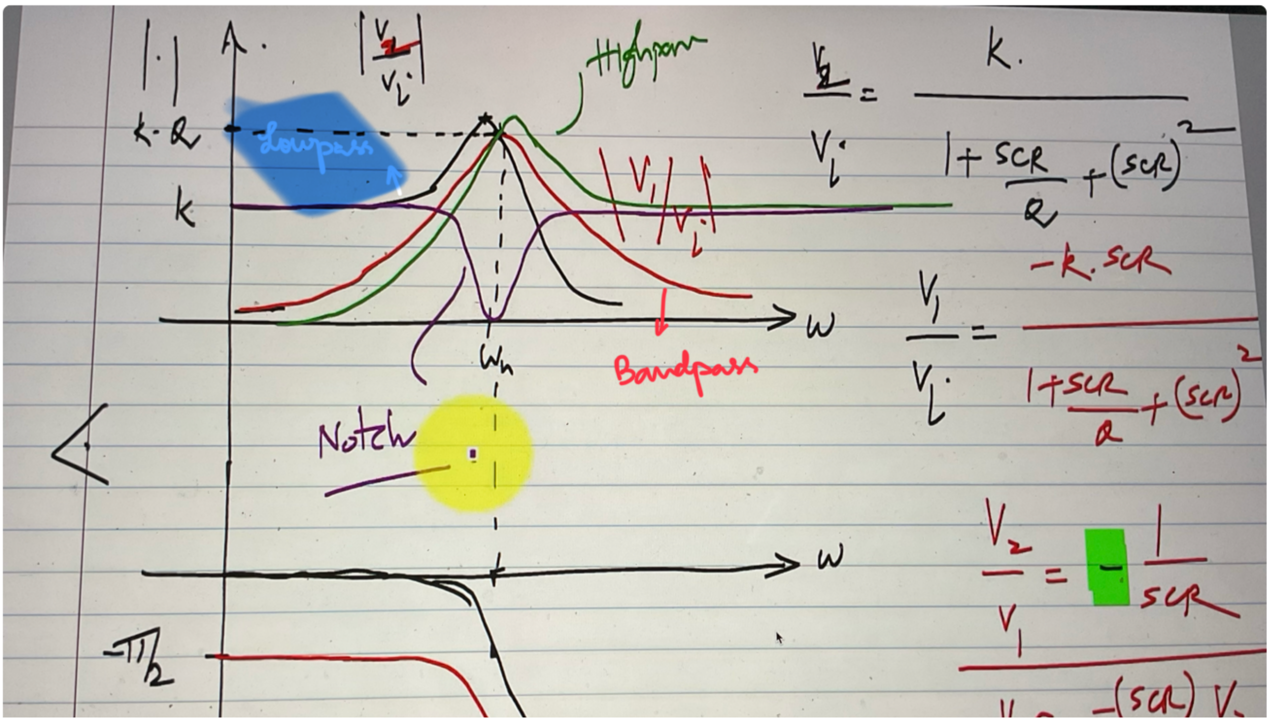
$\Rightarrow$ 

Low Pass : $\dfrac{K}{\left(\frac{s}{\omega_n}\right)^2 + \frac{s}{\omega_n R} + 1}$

Bandpass: $\dfrac{k' \, s/\omega_n}{\left(\frac{s}{\omega_n}\right)^2 + \frac{s}{\omega_n R} + 1}$

High pass : $\dfrac{k'' \,(s/\omega_n)^2}{\left(\frac{s}{\omega_n}\right)^2 + \frac{s}{\omega_n R} + 1.}$

Notch / Bandstop : $\dfrac{k\left(1 + \left(\frac{s}{\omega_n}\right)^2\right)}{\left(\frac{s}{\omega_n}\right)^2 + \frac{s}{\omega_n R} + 1.}$



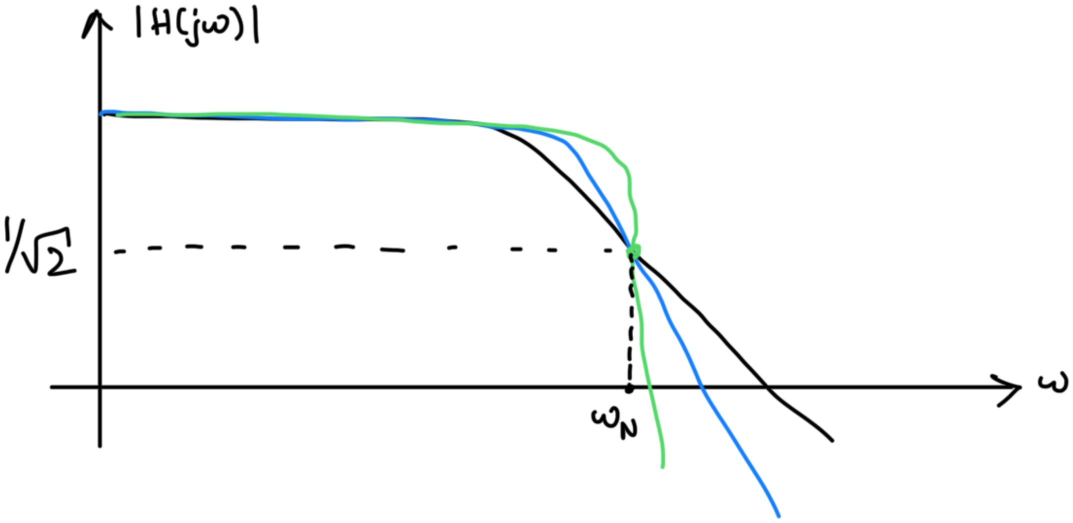
N^{th} order lowpass:

$$H(s) = \frac{1}{1 + a_1 s + \cdots + a_N s^N}$$

$$|H(j\omega)|^2 = \frac{1}{1 + \underbrace{c_2\left(\frac{\omega}{\omega_n}\right)^2 + \cdots + c_{2N-2}\left(\frac{\omega}{\omega_n}\right)^{2N-2}}_{0 \text{ for maximally flat magnitude}} + c_{2N}\left(\frac{\omega}{\omega_n}\right)^{2N}}$$

Only even order terms, order 2N.

N^{th} order max. flat magnitude filter $|H(j\omega)|^2 = \dfrac{1}{1+\left(\frac{\omega}{\omega_N}\right)^{2N}}$



$$H(s) = \frac{1}{D(s)} \qquad |H(j\omega)|^2 = \frac{1}{|D(j\omega)|^2} = \frac{1}{1+\left(\frac{\omega}{\omega_N}\right)^2}$$

Need $D(s)$ such that

$$|D(j\omega)|^2 = 1 + \left(\frac{\omega}{\omega_N}\right)^{2N}$$

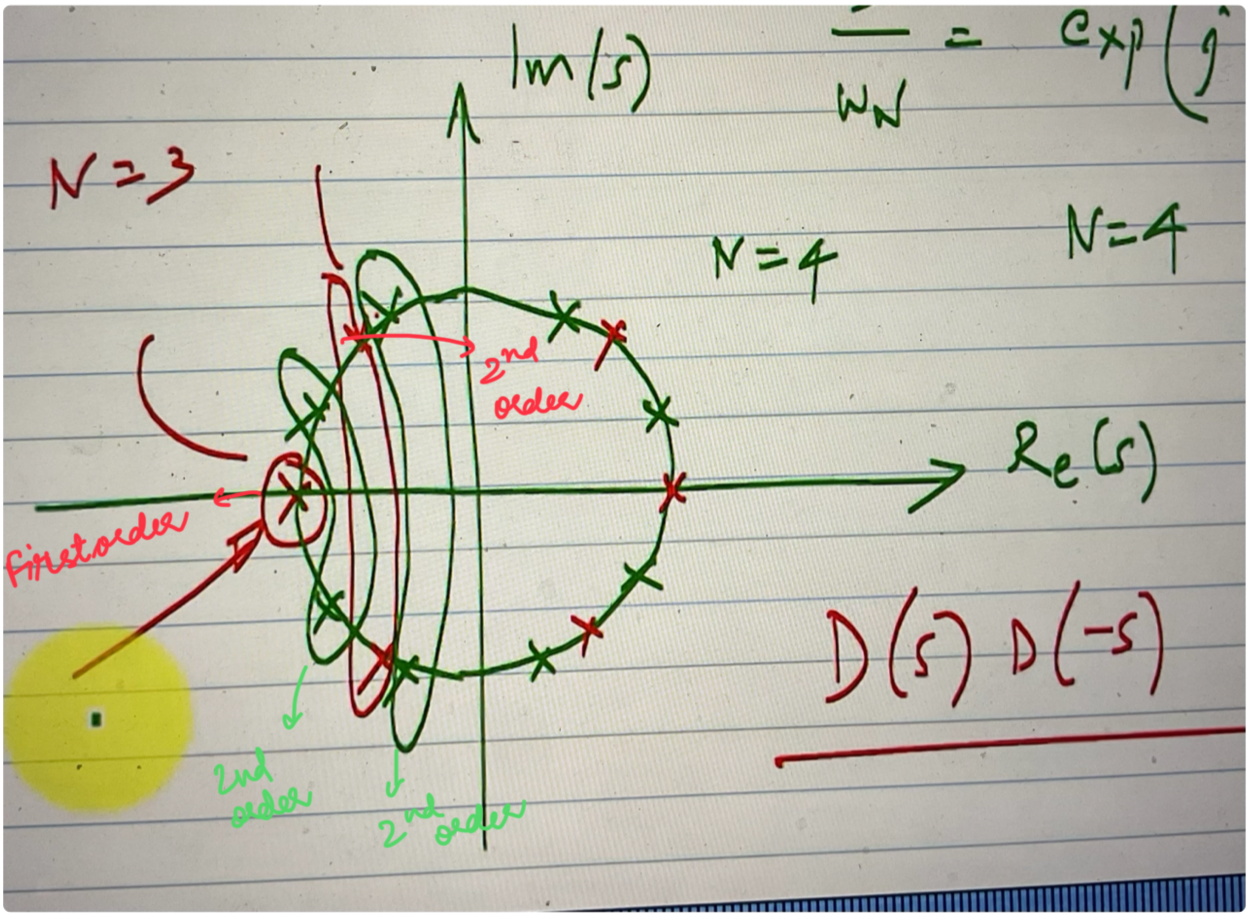
$$|D(j\omega)|^2 = D(j\omega)\cdot D^*(j\omega) = D(s)\cdot D(-s)\Big|_{s=j\omega}$$

$$\therefore \; D(s)\,D(-s)\Big|_{s=j\omega} = 1 + \left(\frac{s}{j\omega_N}\right)^{2N}.$$

Solving for the roots :

$$1 + \left(\frac{s}{j\omega_N}\right)^{2N} = 0 \;\Rightarrow\; \left(\frac{s}{j\omega_N}\right)^{2N} = e^{\,j\left(\frac{2\pi k+\pi}{2N}\right)}$$

$$\therefore \; \frac{s}{\omega_N} = e^{\,j\left(\frac{2\pi k+\pi}{2N} + \frac{\pi}{2}\right)}$$



This filter is called a Butterworth filter ...

Now, to implement a 3rd order filter, we have to cascade a 1st and a 2nd order filter.

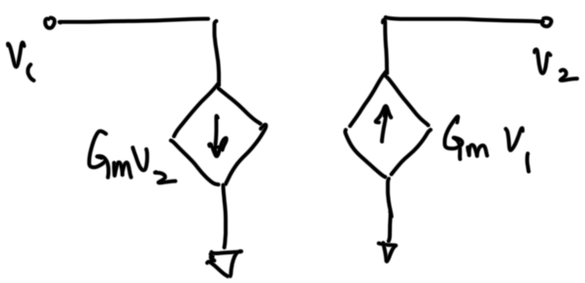
But, what should be the order? 1 then 2 ? or 2 then 1 ?

1st order, and 2nd order. Why? This will prevent V_{pp} from going to $\pm V_{SAT}$.

Ordering of sections:

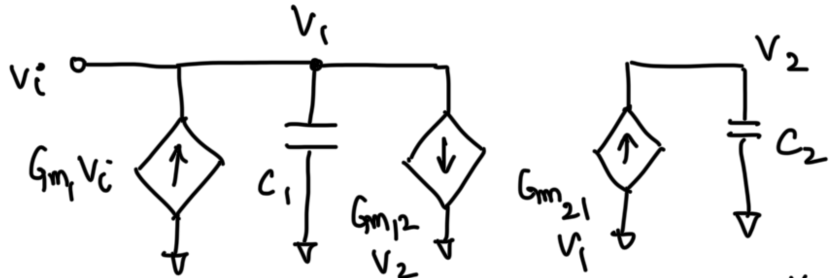
Increasing order of Q to maximize the input that can be applied (outputs limited to V_{SAT})

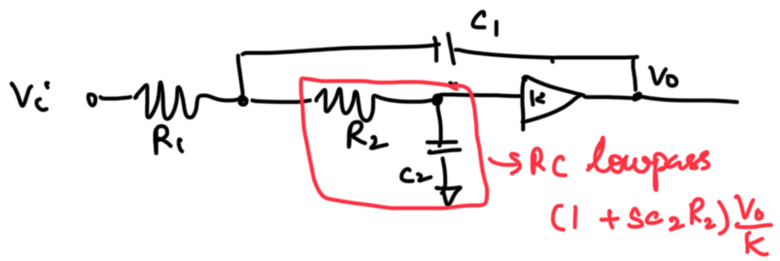
Gyrator:

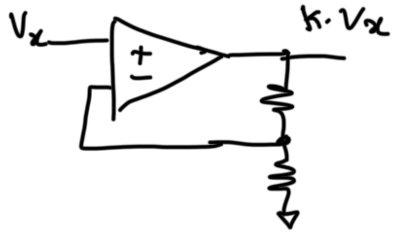


Capacitor	$\leftrightarrow$	Inductor
Resistor	$\leftrightarrow$	Resistor
$R \parallel C$	$\leftrightarrow$	R series C
Z	$\leftrightarrow$	$1/G_m^2 Z$.

Gm-C filter:







$$\frac{V_o}{V_i} = \frac{V_i - (1 + sC_2R_2)V_o/K}{R_1}$$

$$= \frac{K}{s^2 C_1 C_2 R_1 R_2 + s\left[R_1 C_2 + R_2 C_2 + R_1 C_1 (1-K)\right] + 1}$$

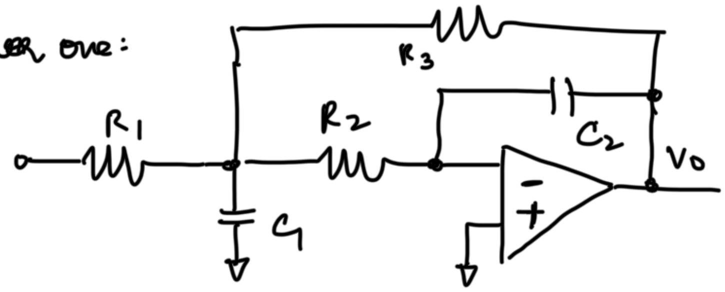
If $R_1 C_1 = R_2 C_2$, $\quad = \dfrac{K}{s^2 C^2 R^2 + sCR(3-K) + 1} \approx \dfrac{K}{(s/\omega_n)^2 + \dfrac{s}{Q\omega_n} + 1}$.

$R_1 = R_2 = R$

$C_1 = C_2 = C$

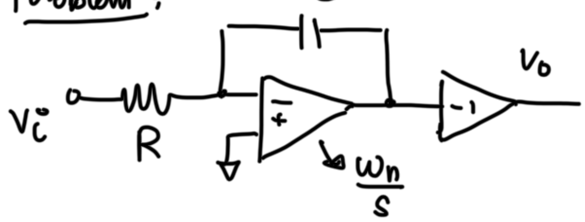
For Real poles Q not greater than 1/2 !

Another one:



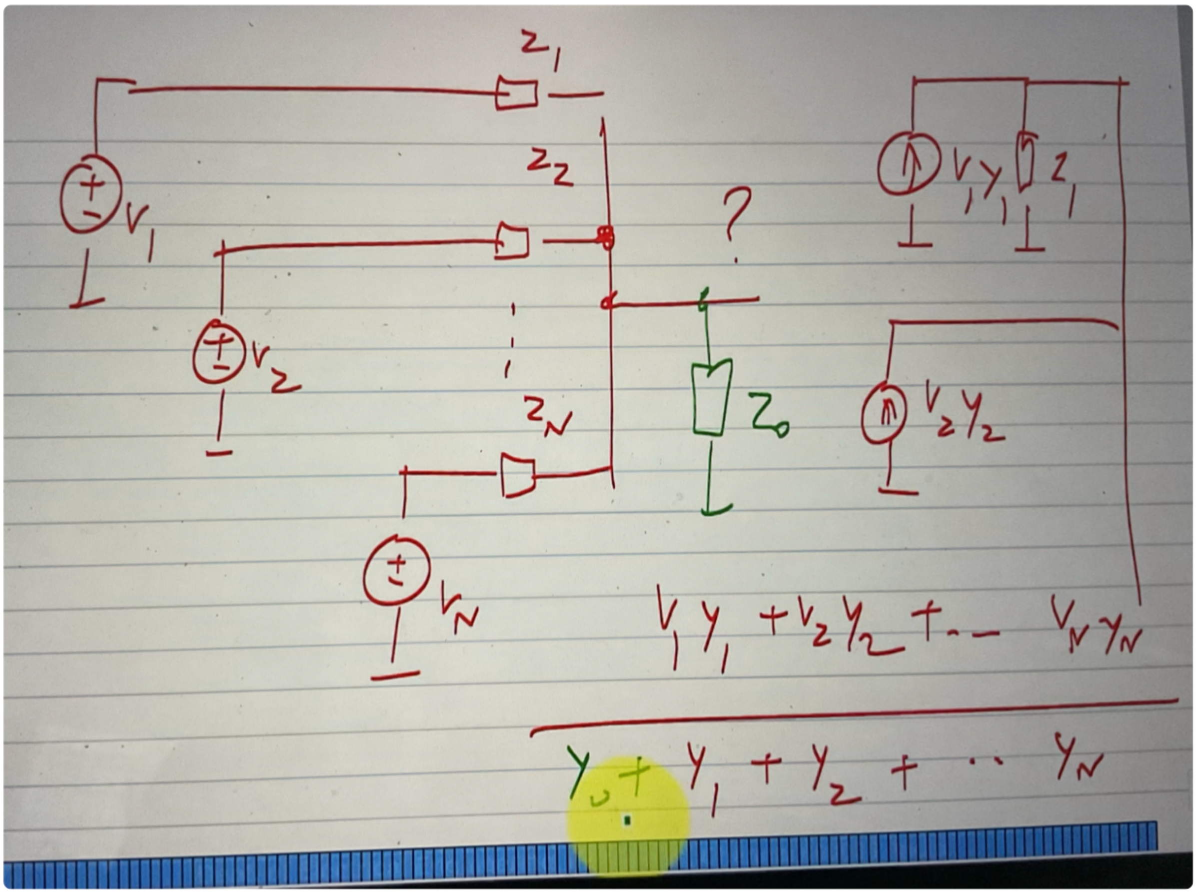
Rauch Filter

Problem:



$\frac{V_o}{V_i} = \frac{1}{sCR}$ with ideal opamp.

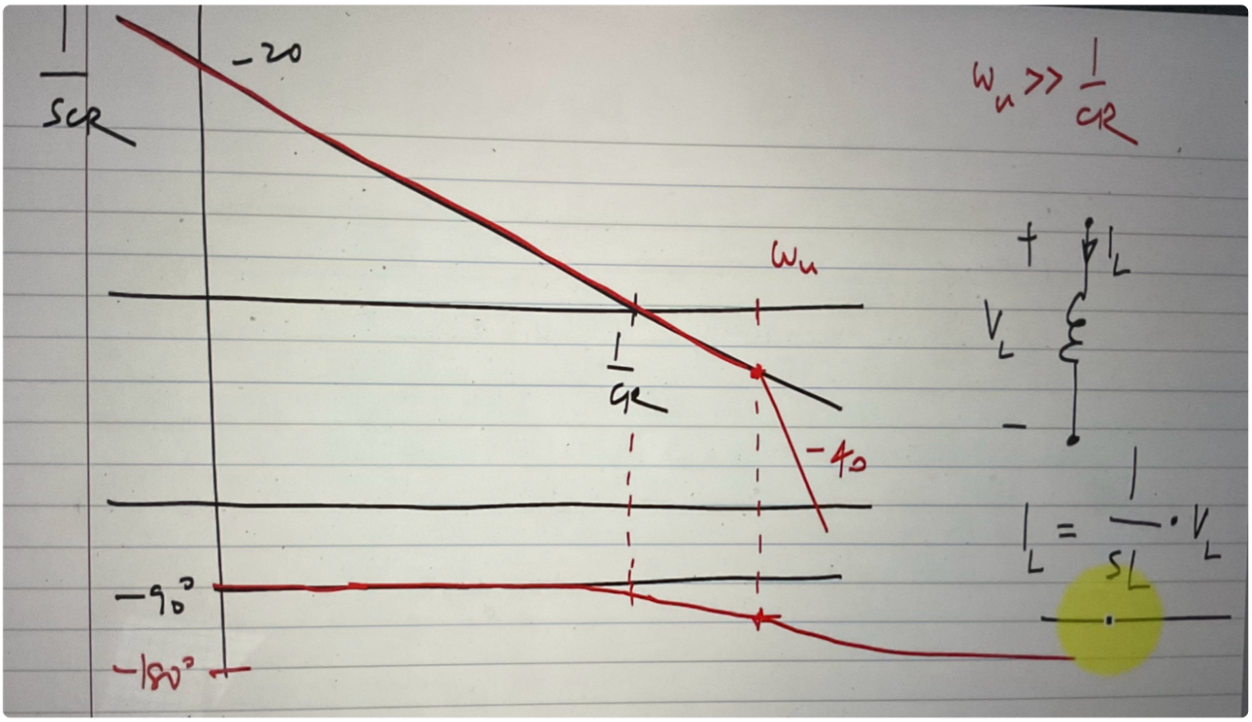
$$V_e = \frac{s}{\omega_n} V_o = \left(\frac{V_i}{R} - V_o sC\right)\frac{1}{R+sC}$$



$$\frac{V_o}{V_i} = \frac{1}{sCR\left(1+\frac{s}{\omega_n}+\cancel{\frac{1}{\omega_n CR}}^{\,0}\right)}$$

$\omega_n \gg \frac{1}{CR}$.

Lets bodeplot this Transfer function:



1/sCR
-20
ωu >> 1/CR
ωu
1/CR
-40
+
IL
VL
−
IL = 1/sL · VL
-90°
-180°