Electric Circuits & Networks

1) Voltage and Current Laws :

KVL and KCL

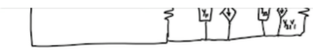
Tellegen's theorem : $\sum P_{absorbed} = \sum P_{supplied} \Rightarrow \sum P = 0$

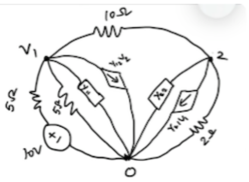
Impedances in series and Admittances in parallel = Algebraic Sum.

Resistances & Inductances - Impedances, Capacitances – Admittances.

2) Basic Nodal and Mesh Analysis :

Nodal analysis:





At node 1:

$$\frac{V_1 - \cancel{10}^{\,2}}{\cancel{5}} + \frac{V_1}{5} + y_{11}V_1 + \frac{V_1 - V_2}{10} + Y_{12}V_2 = 0$$

$$V_1\left(\frac{2}{5} + \frac{1}{10} + y_{11}\right) + \left(\frac{-1}{10} + y_{12}\right)V_2 = 2$$

$$\Rightarrow V_1\left(0.4 + 0.1 + 0.2\right) + \left(-0.1 - 0.05\right)V_2 = 2 \Rightarrow 0.7V_1 - 0.15V_2 = 2$$

At node 2:

$$\frac{V_2 - V_1}{10} + y_{22}V_2 + y_{21}V_1 + \frac{V_2}{2} = 0$$

$$\Rightarrow V_1\left(-\frac{1}{10} + y_{21}\right) + V_2\left(\frac{1}{10} + y_{22} + \frac{1}{2}\right) = 0$$

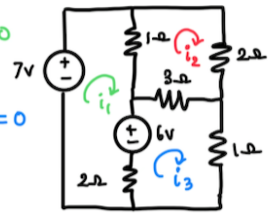
$(3.733 - 0.15)V_2 = 2$

$\Rightarrow V_2 = \frac{6}{11.2}$

$= 0.536V$

$\therefore V_1 = 2.86V$

Mesh Analysis :



$$-7 + (i_1 - i_2)1 + 6 + 2(i_1 - i_3) = 0$$

$$1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$$

$$-6 + 3(i_3 - i_2) + i_3 + 2(i_3 - i_1) = 0$$

Solve for i_1, i_2, i_3

$$6i_2 - i_1 - 3i_3 = 0$$

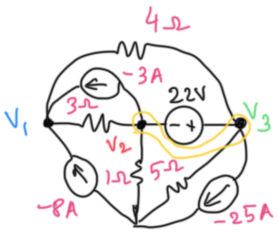
$$6i_3 - 2i_1 - 3i_2 = 6$$

$$3i_1 - i_2 - 2i_3 = 1$$

$$\begin{bmatrix} 3 & -1 & -2 \\ -1 & 6 & -3 \\ -2 & -3 & 6 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 9/13 & 4/13 & 5/13 \\ 4/13 & 14/39 & 11/39 \\ 5/13 & 11/39 & 17/39 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 6 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix} //$$

Supernode:



$\rightarrow$

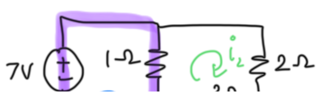
Node 1 : $(v_1 - v_2)/3 + (v_1 - v_3)/4 = -8 - 3$ and of course, $v_3 - v_2 = 22$

Super node : $\dfrac{(v_2 - v_1)}{3} + \dfrac{(v_3 - v_1)}{4} + \dfrac{v_3}{5} + \dfrac{v_2}{1} = 3 + 25$

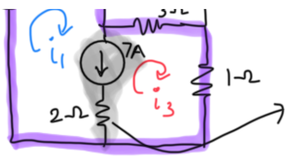
Basically, in super node & mesh, we reduce the no. of nodes / meshes.

Super mesh :

Super mesh eqn : $i_1 - i_3 = 7$



$$-7 + i_1 + 3(i_3 - i_2) + i_3 = 0 \Rightarrow i_1 - 3i_2 + 4i_3 = 7$$



$(i_2-i_1)+2i_2+3(i_2-i_3)=0 \Rightarrow -i_1+6i_2-3i_3=0$

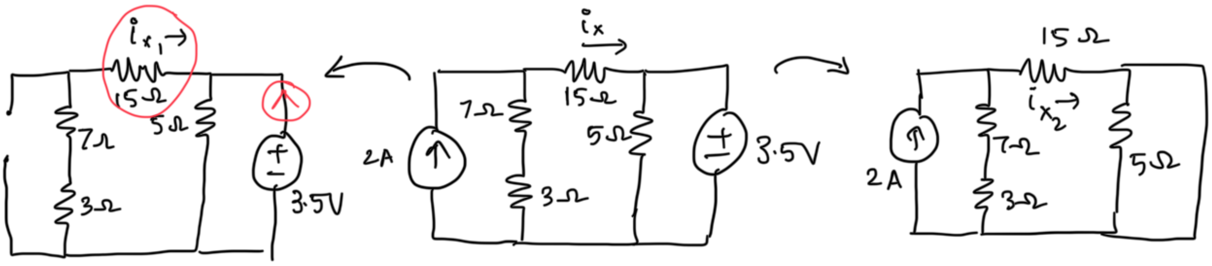
We don't consider the shaded (in black) region.
Instead, we take the purple region as one mesh

3. Handy Circuit Analysis Techniques:

Linearity: A circuit that has only linear Elements (Elements whose current / voltage depends on only 1st power of current / voltage variables or the like).

Superposition: Consequence of Linearity. The response in a linear circuit having more than one independent source can be obtained by adding the responses caused by the separate independent sources *acting alone*.

Current source – Open circuited; Voltage source – close circuited.



$\frac{1}{R} = \frac{1}{5} + \frac{1}{25} = \frac{6}{25}$

$R = 25/6\ \Omega$

$\therefore I = \frac{-3.5}{25} \times 6 = \frac{-3\times7}{25} = \frac{-21}{25}$ A

$\frac{21}{25} \begin{cases} 5 \\ 25 \end{cases} \Rightarrow i_{x_1} = -21/(25\times6)$ A

$2A \begin{cases} 15 & 3/5 \\ 10 & 2/5 \end{cases}$

$\Rightarrow i_{x_2} = \frac{2}{5}\times 2 = \frac{4}{5}.$

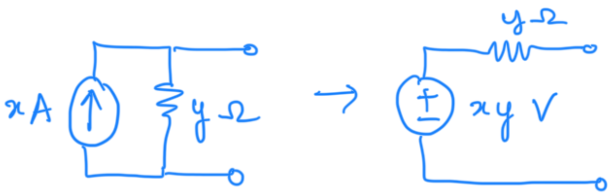
$$i_x = i_{x_1} + i_{x_2} = -\frac{21/30}{5} + \frac{4}{5} = \frac{99/30}{5}$$

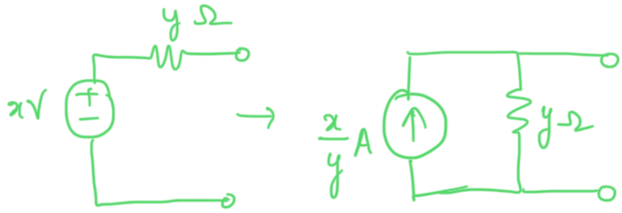
$$= \frac{33}{50} = 0.66 \text{ A}$$

The contribution of the 3.5 V source does not depend on the contribution of the 2 A source – Provided all of the elements used are linear.

Source Transformation

A practical voltage source can be electrically equivalent to a practical current source, and vice versa.





Head of the current source $\Leftrightarrow$ + Terminal of Voltage source

Thevenin and Norton Equivalent Circuits

Thevenin:

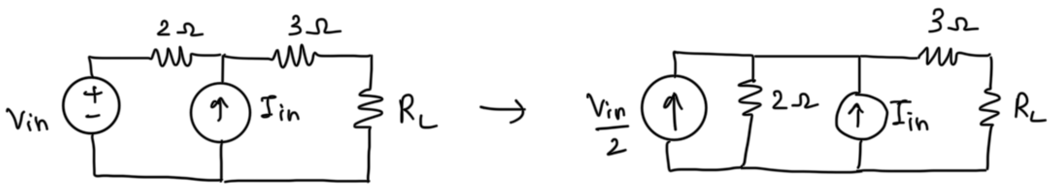
Replace everything except load resistor with an equivalent composed of an independent voltage source in series with a resistor

Norton:

Replace everything except load resistor with an equivalent composed of an independent current source in parallel with a resistor.

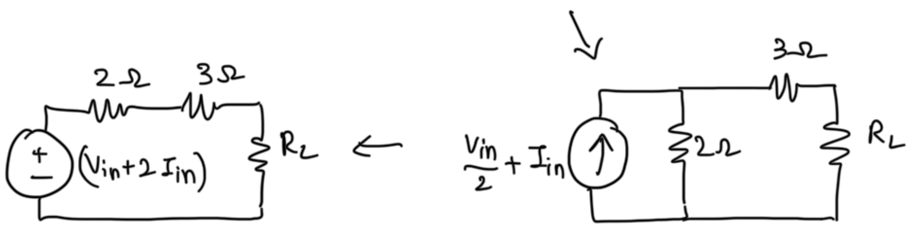
Can go from Thevenin to Norton and viceversa by source Transformations

How to get Thevenin and Norton Equivalents ? $\Rightarrow$ Repeated Source Transformation



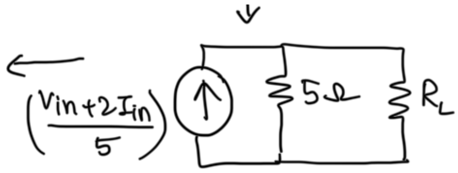
$R_{TH} = 5\,\Omega$

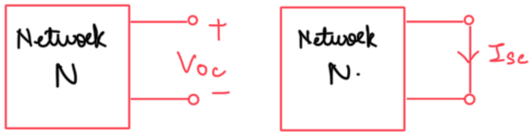
$V_{TH} = (V_{in} + 2I_{in})\,V$



$R_N = 5\Omega$

$V_N = \dfrac{V_{in} + 2I_{in}}{5}$





$V_{oc} = V_{\text{open circuit}}$, $I_{sc} = I_{\text{short circuit}}$

Driving point Impedance : Z_{TH}.

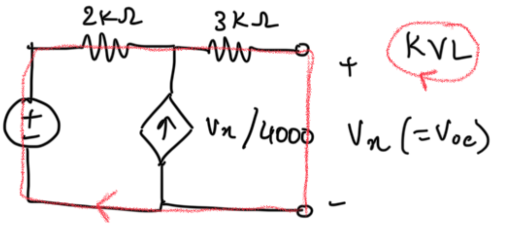
To find this, [According to Thevenin's Theorem] Zero out all independent sources and connect the part of the network to a source V_{oc} about which we have to find the equivalent circuit. Source is I_{sc} if Norton theorem is used.

$$\boxed{V_{oc} = R_{TH}\, i_{sc}}$$

When dependent sources are present :

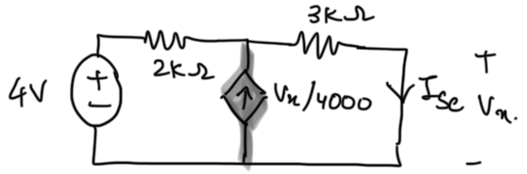
Treat them like other components. Write KVL and KCL. Simplify.
At the end, we must get only an Impedance if we don't have indep. sources, but only dependent sources.

eg.



KVL $\Rightarrow -4 + \cancel{2}\times 10^{3}\left(-\dfrac{V_x}{\cancel{4000}_{\,2}}\right) + 3K(0) + V_x = 0$

$\Rightarrow \dfrac{V_x}{2} = 4 \Rightarrow V_x = 8V.$



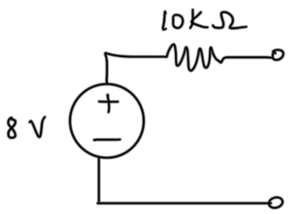
Since terminals are short circuited, $V_x = 0$

$\therefore$ Dependent source $= 0$

$\therefore I_{sc} = 4/5\times 10^{-3} = 0.8\times 10^{-3}$ A.

To get R_{TH}, we know, $R_{TH} = V_{oc}/I_{sc} = 8/0.8\times 10^{-3} = 10K\Omega$

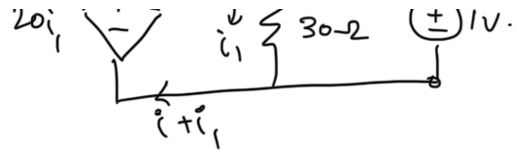
$\therefore$ Thevenin Equivalent =



Pro tip: Sometimes it might help to connect 1A [or 1V] source across the terminals and have V_{oc} [or I_{sc}] as $1\times R_{TH}$ [or $1/R_{TH}$], solve for V_{oc} [or I_{sc}] and then get R_{TH}.

$i = 1$.

A Quick Example!



R_{TH}

$$-20i_1 + 10(i+i_1) + 30i_1 = 0 \Rightarrow 20i_1 + 10i = 0 \Rightarrow 2i_1 = -i$$

$$\left(i_1 = -\frac{1}{2}i\right)$$

$$-30i_1 + 5i + 1 = 0 \Rightarrow -30\left(-\frac{1}{2}i\right) + 5i + 1 = 0$$

$$\Rightarrow 20i + 1 = 0$$

$$i = -1/20 \text{ A.}$$

$\therefore R_{TH} = 20\,\Omega \longrightarrow$

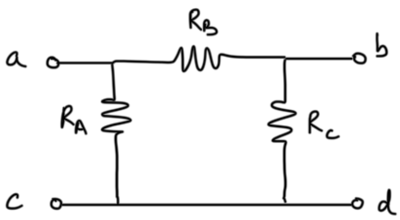
Maximum Power Transfer

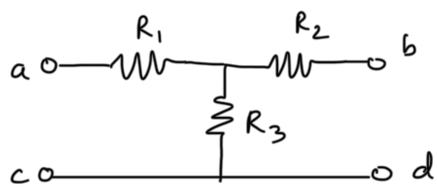
A network delivers maximum power to a load resistance when it is equal to the Thevenin equivalent resistance of a network.

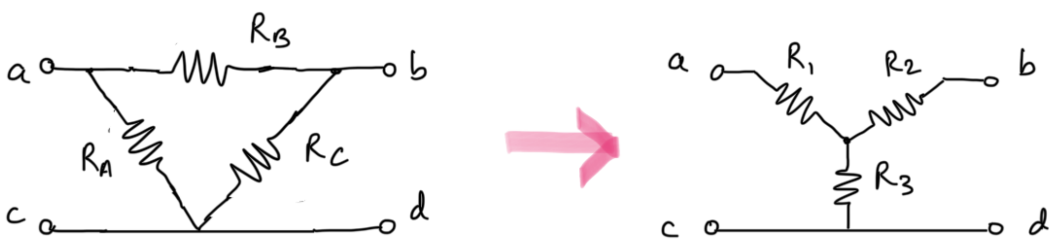
There is a distinct difference between "drawing" maximum power from a "source" and "delivering" maximum power to a "load."

(Drawing max power from a (voltage source) = Shorting the terminals)

Delta-Wye Conversion







$R_A = (R_1R_2 + R_2R_3 + R_3R_1)/R_2$

$R_B = (R_1R_2 + R_2R_3 + R_3R_1)/R_3$

$R_C = (R_1R_2 + R_2R_3 + R_3R_1)/R_1$

$R_1 = R_A R_B / (R_A + R_B + R_C)$

$R_2 = R_B R_C / (R_A + R_B + R_C)$

$R_3 = R_A R_C / (R_A + R_B + R_C)$

4) Laplace Domain

$$F(s) = \int_{-\infty}^{\infty} e^{-st} f(t)\,dt \quad , \quad f(t) = \frac{1}{2\pi j} \int_{\sigma_0 - j\infty}^{\sigma_0 + j\infty} e^{st} F(s)\,ds$$

Initial value theorem: $\lim_{t \to 0^+} f(t) = \lim_{s \to \infty} [s F(s)]$

Final value theorem: $\lim_{t \to \infty} f(t) = \lim_{s \to 0} [s F(s)]$.

Every analysis, except specified here, is carried out in the same manner in the laplace domain, as in time domain.

Capacitor: $i(t) = C \frac{dv}{dt} \longrightarrow I_C(s) = CS\,V_C(s) - C v_C(0^-)$

$V_C(s) = \frac{I_C(s)}{sC} + \frac{v_C(0^-)}{s}$.

Impedance = $1/sC$

Inductor: $v(t) = L \frac{di}{dt} \longrightarrow V_L(s) = Ls\,I_L(s) - L\,i_L(0^-)$

$I_L(s) = \frac{V_L(s)}{sL} + \frac{i_L(0^-)}{s}$

Impedance = sL

Analysis would be done just like time domain, but inductances and capacitances would be written in Laplace domain like mentioned above, and all other elements would also undergo Laplace transform. [$v(t) = u(t)$, $V(s) = 1/s$]

Poles, zeroes, transfer functions: $Z(s) = \frac{N(s)}{D(s)}$

$H(s)$ = transfer function = $\frac{V_{out}}{V_{in}}$, Poles $\Rightarrow D(s) = 0$; Zeros $\Rightarrow N(s) = 0$.

4) Bode Plots

Quick method of obtaining an approximate picture of the amplitude and phase variation of a given transfer function as functions of ω.

We will use logarithmic scales for frequency and magnitude, providing an improved snapshot of frequency over a wide range of values.

This approximate response curve : Asymptotic / Bode plot.

The Decibel Scale

We will be using logarithmic scales for frequency and magnitude for improved snapshot of frequency response over a wide range of values.

$$|H(j\omega)| = 10^{H_{dB}/20} \; ; \quad H_{dB} = 20 \log |H(j\omega)|$$

An increase of $|H(j\omega)|$ by a factor of 10 corresponds to an increase in H_{dB} by 20 dB.

$|H(j\omega)| = 1 \Leftrightarrow H_{dB} = 0$

$|H(j\omega)| = 2 \Leftrightarrow H_{dB} = 6dB$

$|H(j\omega)| = 10 \Leftrightarrow H_{dB} = 20dB$

$H(j\omega) \longleftrightarrow H(s)$ as well.

Decade : Range of frequencies defined by a factor of 10 [3→30]

Octave : Range of frequencies defined by a factor of 2 [7→14].

Determination of Asymptotes

$$H(s) = 1 + \frac{s}{a}$$ (pole at $s = -a$).

$$|H(j\omega)| = \left|1 + \frac{j\omega}{a}\right| = \sqrt{1 + \frac{\omega^2}{a^2}} \Rightarrow H_{dB} = 20 \log \sqrt{1 + \frac{\omega^2}{a^2}}$$

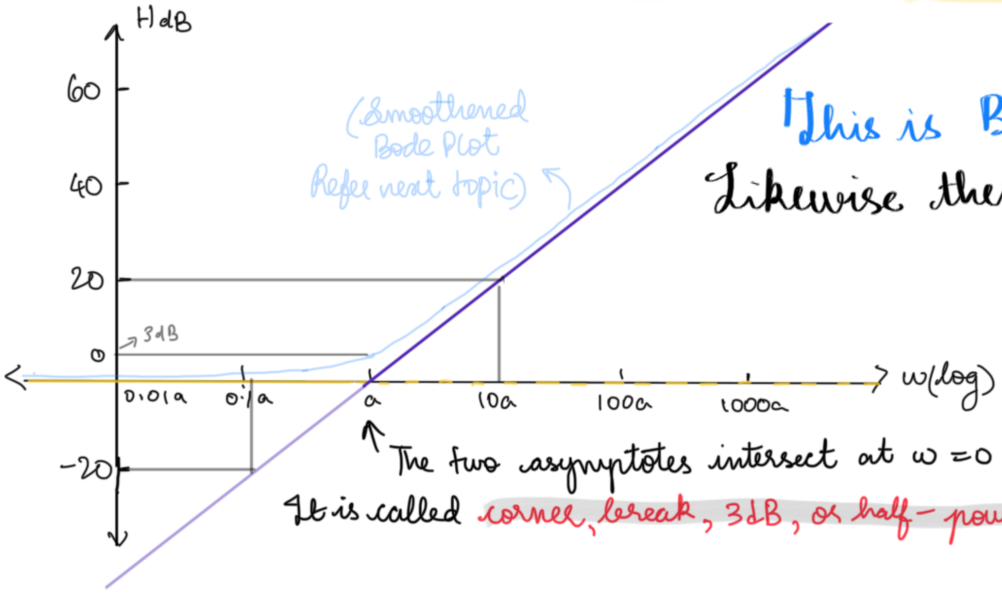
for $\omega \ll a$,

$H_{dB} = 20 \log (1) = 0$

for $\omega \gg a$,

$H_{dB} = 20 \log \left(\frac{\omega^2}{a^2}\right)^{1/2} = 20 \log \frac{\omega}{a}$

or, for $\omega = a$, $H_{dB} = 0$, ↙ For $\omega = a$, $H_{dB} = 0$; $\omega = 10a$, $H_{dB} = 20\,dB$.
$\omega = 2a$, $H_{dB} = 6\,dB, \ldots \rightarrow$ 6dB/octave! ↳ 20 dB / decade Both are same!



This is Bode Magnitude plot.
Likewise there is Bode Phase plot!

↑ The two asymptotes intersect at $\omega = 0$ → The frequency of the zero.
It is called corner, break, 3dB, or half-power frequency!

Smoothing Bode Plots

We have seen for $\omega \gg a$ and $\omega \ll a$ only ⇒ approximations. Now, for $\omega = a$,

$$H_{dB} = 20 \log \sqrt{1 + \frac{\omega^2}{a^2}} = 20 \log \sqrt{2} = \frac{1}{2}\, 20 \log 2 = \frac{1}{2} \times 6\,dB = 3\,dB.$$

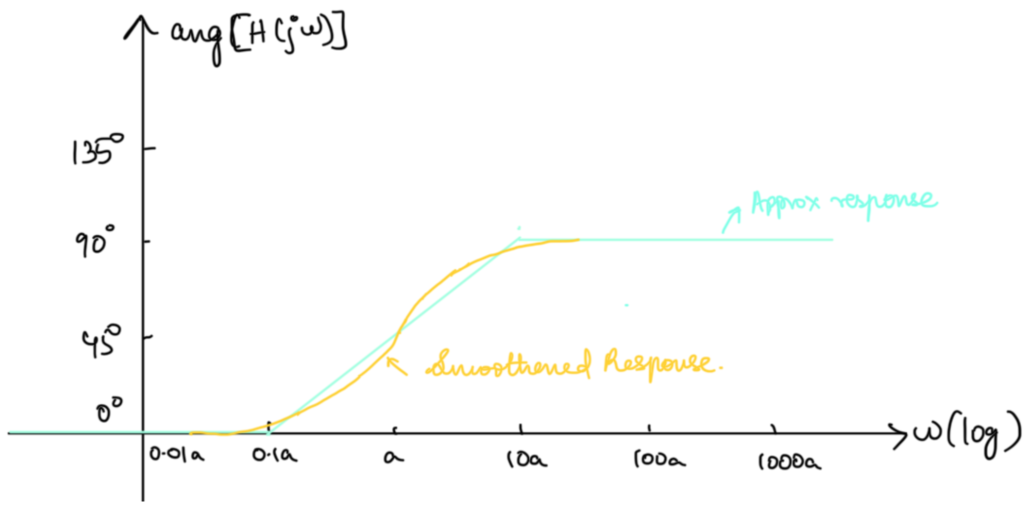
Now, for $\omega = 0.5a$, $H_{dB} = 20 \log \sqrt{1.25} \approx 1\,dB$. $|H(j\omega)| = \sqrt{2}$, $H_{dB} = 3\,dB$

Phase Response

$$\text{ang}\; H(j\omega) = \text{ang}\left(1 + j\frac{\omega}{a}\right) = \tan^{-1} \frac{\omega}{a}$$

For $\omega \ll a$, ang $H(j\omega) = 0°$ → for $\omega < 0.1a$ ||
For $\omega \gg a$, ang $H(j\omega) = 90°$ → for $\omega > 10a$ ||

For $\omega \in [0.1a, 10a]$, slope = 45°/decade.
[as $\omega = a \Rightarrow$ ang $H(j\omega) = 45°$]



Bode plots with multiple terms

For transfer functions with more than a simple pole → Bode method

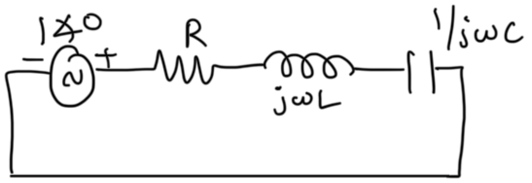
$H(s) = K\left(1+\frac{s}{s_1}\right)\left(1+\frac{s}{s_2}\right) \longleftrightarrow H_{dB} = 20\log K + 20\log\sqrt{1+\left(\frac{\omega}{s_1}\right)^2} + 20\log\sqrt{1+\left(\frac{\omega}{s_2}\right)^2}$

Plot everything and add all the asymptotes. Refer to this for each pole:

6) Resonance (Parallel and Series)

Frequency Response of circuits:

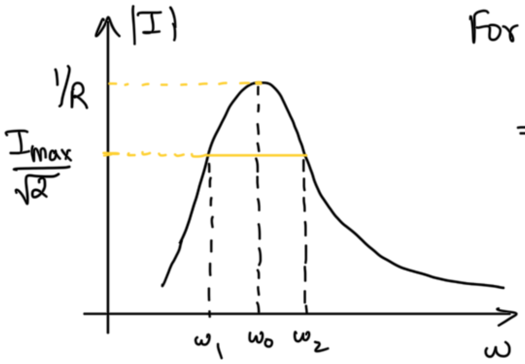
$Z(j\omega) = R + j(\omega L - 1/\omega C)$



$$I = \frac{1}{R + j(\omega L - 1/\omega C)} = \frac{e^{-j\angle Z(j\omega)}}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

$$|I| = \frac{1}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

For $\omega L = \frac{1}{\omega C}$, or $\omega_0 = 1/\sqrt{LC}$, $|I| = 1/R$ Purely Resistive.



For $|I| = I_{max}/\sqrt{2} \Rightarrow 1/\sqrt{2}R = |I|$.

$$\Rightarrow R^2 + (\omega L - 1/\omega C)^2 = 2R^2.$$
$$(\omega L - 1/\omega C)^2 = R^2$$
$$\Rightarrow (\omega L - 1/\omega C) = \pm R$$

$$\Rightarrow \omega^2 LC - 1 = \omega RC, \qquad \omega^2 LC - 1 = -\omega RC$$
$$\omega^2 - \frac{\omega R}{L} - \frac{1}{LC} = 0, \qquad \omega^2 + \frac{\omega R}{L} - \frac{1}{LC} = 0$$

$$\Rightarrow \omega_{11} = \frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}} \; ; \quad \omega_{12} = \frac{R}{2L} - \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}$$

$$\omega_{21} = -\frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}} \quad ; \quad \omega_{22} = -\frac{R}{2L} - \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}$$

$\left(\alpha = R/2L \;;\; \omega_0 = 1/\sqrt{LC}\right)$: $\omega_1 = \omega_{21} = -\alpha + \sqrt{\alpha^2 + \omega_0^2}$

$\omega_2 = \omega_{11} = \alpha + \sqrt{\alpha^2 + \omega_0^2}$

Bandwidth: $B = \omega_2 - \omega_1 = 2\alpha$

Quality factor: $Q = \omega_0 / B = \omega_0 / 2\alpha = \frac{1}{R}\sqrt{\frac{L}{C}}$

7) Two port networks

Impedance parameters (Z) $V_1 V_2$ $I_1 I_2$

$V_1 = z_{11} I_1 + z_{12} I_2$; $V_2 = z_{21} I_1 + z_{22} I_2$

Reciprocity : $z_{12} = z_{21}$, Symmetry : $z_{11} = z_{22}$

Series - series connection : $Z = Z_1 + Z_2$

Admittance parameters $(Y) = 1/Z$ $I_1 I_2$ $V_1 V_2$

$I_1 = y_{11} V_1 + y_{22} V_2$; $I_2 = y_{21} V_1 + y_{22} V_2$

Reciprocity : $y_{12} = y_{21}$, Symmetry : $y_{11} = y_{22}$

Parallel - parallel connection : $Y = Y_1 + Y_2$

Hybrid Parameters (h) $V_1 I_2$ $I_1 V_2$

$V_1 = h_{11} I_1 + h_{12} V_2$; $I_2 = h_{21} I_1 + h_{22} V_2$

Reciprocity : $h_{12} = -h_{21}$, Symmetry : $|H| = 1$

Series - parallel connection : $H = H_1 + H_2$

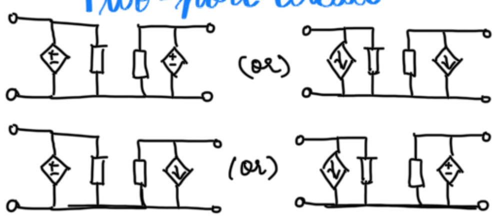
Inverse hybrid Parameters $(g) = 1/H$ $V_2 I_1$ $V_1 I_2$

$I_1 = g_{11} V_1 + g_{12} I_2$; $V_2 = g_{12} V_1 + g_{22} I_2$

Reciprocity : $g_{12} = -g_{21}$, Symmetry $= |G| = 1$

Parallel Series connection : $G = G_1 + G_2$

Two-port Circuit



Interconnection of Networks

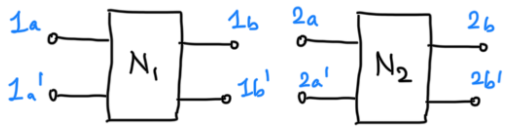
Series-Series : $1a' \to 2a$, $1b' \to 2b$.

Series - Parallel : $1a' \to 2a$, $1b \to 2b$, $1b' \to 2b'$

Parallel - Series : $1b' \to 2b$, $1a \to 2a$, $1a' \to 2a'$

Parallel - Parallel $1ab \to 2ab$, $1ab' \to 2ab'$

Cascade $1b \to 2a$, $1b' \to 2a'$.



$$H = \frac{1}{z_{22}} \begin{bmatrix} |Z| & z_{12} \\ -z_{21} & 1 \end{bmatrix}$$

$$Z = \frac{1}{h_{22}} \begin{bmatrix} |H| & h_{12} \\ -h_{21} & 1 \end{bmatrix}$$

$$G = \frac{1}{z_{11}} \begin{bmatrix} 1 & -z_{12} \\ z_{21} & |Z| \end{bmatrix}$$

$$Z = \frac{1}{g_{11}} \begin{bmatrix} 1 & -g_{12} \\ g_{21} & |G| \end{bmatrix}$$

Transmission Parameters (t) $V_1\ I_1 \quad V_2\ I_2$

$$T = \frac{1}{z_{21}}\begin{bmatrix} z_{11} & |Z| \\ 1 & z_{22}\end{bmatrix}$$

$V_1 = t_{11}V_2 + t_{12}I_2$; $I_1 = t_{21}V_2 + t_{22}I_2$

Reciprocity : $|T| = 1$, Symmetry = $t_{11} = t_{22}$

Cascade interconnection : $T = T_1 \times T_2$.

$$Z = \frac{1}{t_{21}}\begin{bmatrix} t_{11} & |T| \\ 1 & t_{22}\end{bmatrix}$$

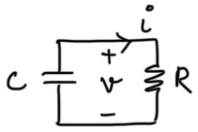
8) RL, RC, RLC Circuits

Natural Response - In the absence of all sources

Forced Response - describes the contribution from external sources.

Complete Response (V or I) = Natural Response + Forced Response

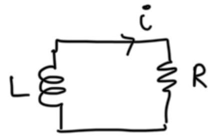
RC : $c\dfrac{dv}{dt} + \dfrac{v}{R} = 0$



; Energy $= \frac{1}{2}CV_0^2$, $\tau = RC \rightarrow e^{-t/\tau}$

$$\Rightarrow \frac{dv}{v} = -\frac{1}{RC}dt \Rightarrow v(t) = V_0 e^{-t/RC}.$$

RL : $L\dfrac{di}{dt} + Ri = 0$



; Energy $= \frac{1}{2}LI_0^2$, $\tau = L/R$.

$$\Rightarrow \frac{di}{i} = \frac{-R}{L}dt \Rightarrow i(t) = I_0 e^{-t/(L/R)}$$

General Perspective : $\tau = R_{eq}C$ (or) $\tau = L/R_{eq}$.

Source free RC and RL circuit with single Energy Storage Element (L or C)

1. Determine R_{eq} using Thevenin equivalent, then τ.
2. Find initial conditions of $v(t)$ or $i(t)$.
3. Solution = $v(t) = v(0^+)e^{-t/\tau}$ or $i(t) = i(0^+)e^{-t/\tau}$.

Initial Conditions

imposed when source is unit step or [$u(t)$]

Capacitor : $i(0^-) = i(\infty) = 0$; $v(0^-) = v(0^+)$ [Resists change in V]

open circuit $\quad i(0^+) = c(dv/dt)$ $\quad$ Capacitor becomes voltage source!

Inductor : $v(0^-) = v(\infty) = 0$; $i(0^-) = i(0^+)$ [Resists Change in I]

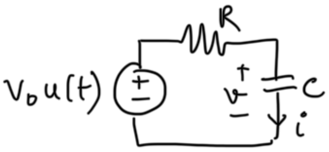
Short circuit $\quad v(0^+) = L(di/dt)$ $\quad$ Inductor becomes current source!

(Resistor — Reacts Instantaneously)

Forced Response of RC

↳ Let's assume that the forced response is some const.

$V_f = K$ (let's assume) $\Rightarrow$ In RC, $RC\dfrac{dv}{dt} + v = V_0$



$\rightarrow RC\cancel{\dfrac{d(K)}{dt}}^{\,0} + K = V_0 \Rightarrow K = V_0.$

Complete Responce of RC

$v_{total} = v_n + v_f$; $\quad v_n = A e^{-t/\tau}$; $\quad v_f = V_0$; $\quad v_{total} = A e^{-t/\tau} - V_0$.

↳ $t<0$, $u(t) = 0$ no?

Initial Conditions : $v(0^+) = 0 \Rightarrow v_{total}(0) = 0 = A + V_0 \Rightarrow A = -V_0$

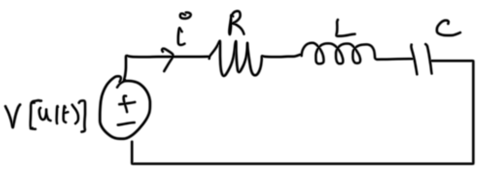
$\therefore V_{total}(t) = (1 - e^{-t/RC})V_0$ and $|||^{ly}$ for RL, $i_{total}(t) = \dfrac{V_0}{R}(1 - e^{-Rt/L})$

Procedure for Complete Response (For RL and RC)

1. Find τ by Finding Thevenin equivalent [$R_{eq} C_{eq}$ or L_{eq}/R_{eq}]
2. Determine initial conditions [Capacitor − V, Inductor I]
3. Determine final conditions [Cap − oc, Ind. − sc]
4. Final Response = $v(t) = v(\infty) + [v(0^+) - v(\infty)]e^{-t/\tau}$

   $i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-t/\tau}$

RLC Circuits (series)



$V = V_R + V_L + V_C$; $i = i_C = C\dfrac{dV_c}{dt}$.

$V_R = iR = RC\dfrac{dV_c}{dt}$, $\quad V_L = L\dfrac{di}{dt} = L\dfrac{d}{dt}\left[C\dfrac{dV_c}{dt}\right] = LC\dfrac{d^2V_c}{dt}$

$$\therefore V = LC\frac{d^2V_c}{dt} + RC\frac{dV_c}{dt} + V_c \Rightarrow \frac{V}{LC} = \frac{d^2V_c}{dt} + \frac{R}{L}\frac{dV_c}{dt} + \frac{V_c}{LC}.$$

Solving for V_c in the S domain for natural response, we get :

$$s^2 V_c + \frac{R}{L} s V_c + \frac{1}{LC} V_c = 0 \Rightarrow s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$S = -\alpha \pm \sqrt{\alpha^2 - w_0^2} \quad [\alpha = R/2L, \; w_0 = 1/\sqrt{LC}].$$

Damping Factor $= \dfrac{\alpha}{w_0} = \zeta = 1/2Q$

Case 1: $\alpha^2 > w_0^2$: Overdamped, $s = -\alpha \pm \sqrt{\alpha^2 - w_0^2}$, Real, -ve roots

$V_c(t) = A_1 e^{-s_1 t} + A_2 e^{-s_2 t}$

Case 2: $\alpha^2 = w_0^2$: Critically damped, $s = -\alpha$, Real, -ve & equal roots.

$V_c(t) = (A_1 t + A_2) e^{-\alpha t}$

Case 3: $\alpha^2 < w_0^2$: Underdamped, $s = -\alpha \pm i\sqrt{\alpha^2 - w_0^2}$, Imaginary, Roots are complex conjugates of each other

$V_c(t) = [A_1(\cos\beta t + i\sin\beta t) + A_2(\sin\beta t + i\cos\beta t)] e^{-\alpha t}$

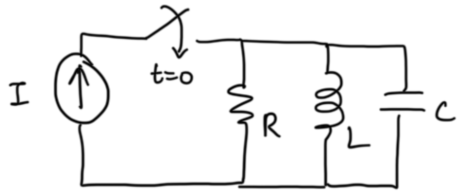
Re part only considered in Real circuits

$Re[V_c(t)] = e^{-\alpha t}[A_1 \cos\beta t + A_2 \sin\beta t]$, $\beta = \sqrt{w_0^2 - \alpha^2}$

Taking only the real parts

Case 4: $R = 0 \Rightarrow s = \pm i w$, Purely imaginary, $V_c(t) = A_1 \cos wt + A_2 \sin wt$

RLC Circuits (Parallel)



$$I = \frac{V_c}{R} + \frac{1}{L}\int v_c\,dt + C\frac{dV_c}{dt} \quad \text{(differentiate)}$$

$$\frac{1}{R}\frac{dV_c}{dt} + \frac{1}{L}V_c + C\frac{d^2V_c}{dt} = 0.$$

$$\Rightarrow \frac{d^2V_c}{dt} + \frac{1}{RC}\frac{dV_c}{dt} + \frac{1}{LC}V_c = 0 \Rightarrow s^2 V_c + \frac{s}{RC}V_c + \frac{1}{LC}V_c = 0$$

$$\Rightarrow s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0 \quad \Rightarrow s = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$(\alpha = 1/2RC, \; w_0 = 1/\sqrt{LC}) \leftarrow \quad = -\alpha \pm \sqrt{\alpha^2 - w_0^2}$$

The same 3 cases for parallel, except Case 4

Complete Response of RLC

1. Determine the initial conditions
2. Obtain a numerical value for the forced response
3. Write the form of natural response with unknown constants A_1, A_2, etc. [Find α, w_0, β] (which Case also)
4. Forced + Natural = Total

5. Find $i(0)$ or $v(0)$ and $\frac{di(0)}{dt}$ or $\frac{dv(0)}{dt}$ and put initial conditions to get A_1, A_2, etc.

To find $\frac{dv(0)}{dt}$ or $\frac{di(0)}{dt}$ might be tricky $\Rightarrow$ If $\frac{dv(0)}{dt}$, relate with capacitor current

$\frac{di(0)}{dt} \rightarrow$ inductor voltage.

9) Sinusoidal Steady State (All about Phasors)

Steady state Response = Forced Response.

Steady state ≠ not changing with time

Steady state = Reached after transient / natural response die out

Transient Response = Natural Response

The Phasor

$$I_m \cos(\omega t + \phi) = I_m \angle \phi$$

Phasor transformation : $I_m \cos(\omega t + \phi) \rightarrow I_m e^{j\omega t + j\phi}$

$Re\{i(t)\} = I_m e^{j\phi} \rightarrow I_m \angle \phi.$

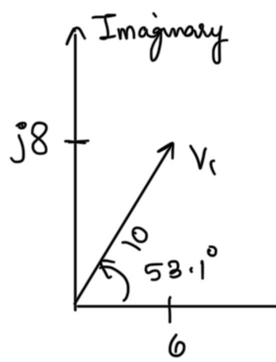
$$Re\left\{I_m e^{j(\omega t + \phi)}\right\} = I_m \cdot Re\left\{\cos(\omega t + \phi) + i \sin(\omega t + \phi)\right\} = I_m \cos(\omega t + \phi).$$

Resistor : $\bar{V} = R\bar{I}$, Inductor : $\bar{V} = j\omega L \bar{I}$, Capacitor : $\bar{V} = \frac{\bar{I}}{j\omega C} = \frac{-j\,I}{\omega C}$

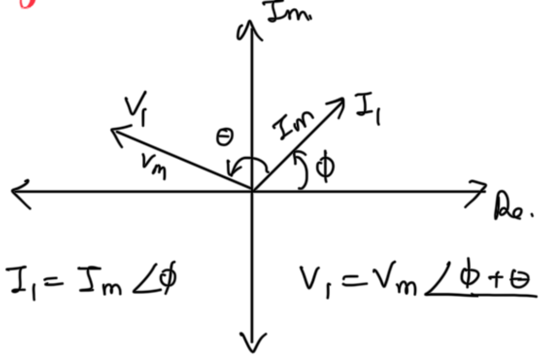
Impedance = Resistance + j Reactance

Admittance = Conductance + j Susceptance

Phasor Diagrams



$V_1 = 6 + 8j$
$= 10 \angle 53.1°$



$I_1 = I_m \angle \phi$

$V_1 = V_m \angle \phi + \theta$

10) Power Analysis

Instantaneous Power = $p(t) = v(t)\,i(t)$.

Power factor.

Average power $P = \lim_{\tau \to \infty} \frac{1}{\tau} \int_{-\tau/2}^{\tau/2} p(t)\,dt = \frac{1}{2} V_m I_m \cos(\theta - \phi)$

$$P = \frac{1}{2} \mathrm{Re}\{V I^*\}.$$

Average power absorbed by purely reactive elements = 0.

Maximum Power Transfer : $(Z_{TH} = R_{TH} + j X_{TH})$

An independent voltage source in series with Z_{TH} or an independent current source in parallel with Z_{TH}, delivers a maximum average power to a load Z_L, when $Z_L = Z_{TH}^*$

Impedance matching is done to make $Z_L = Z_{TH}^*$ for maximum power transfer by adding parallel capacitor and series inductor, which will be added to Z_{TH} and Z_L to make them complex conjugates!

Effective Values of I and V (rms)

$$I_{eff} = I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt} = I_m/\sqrt{2}, \quad V_{eff} = V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2\,dt} = V_m/\sqrt{2}$$

$$\Rightarrow \quad P = \frac{1}{2} V_m I_m = V_{eff} I_{eff}$$

$$= V_{eff}^2 / R = I_{eff}^2 R.$$

Apparent power and power Factor ✗

Apparent power = $V_{eff} I_{eff}$. ; Average Power = $V_{eff} I_{eff} \cos\phi$.

$$\therefore PF = \cos\phi = P / I_{eff} V_{eff}.$$

lagging PF = inductive. leading PF = capacitive.

Complex power $\bar{S} = \bar{V}_{eff} \bar{I}_{eff}^* \Rightarrow \bar{S} = V_{eff} I_{eff} \angle\phi$

$\bar{S} = P + jQ$, P = Average power, Q = Reactive power.

$P = V_{eff} I_{eff} \cos\phi$, $Q = V_{eff} I_{eff} \sin\phi$.

Sign of Q ⇒ Q = −ve, capacitive load, Q = +ve, inductive load.

Summary of diff. powers

Quantity	Symbol	Formula	Units
Average power	P	$V_{eff} I_{eff} \cos\phi$	Watt (W)
Reactive power	Q	$V_{eff} I_{eff} \sin\phi$	volt-amp-reactive (VAR)
Complex power	$\bar{S}$	$P + jQ$ $V_{eff} I_{eff} \angle\phi$ $\bar{V}_{eff} \bar{I}_{eff}^*$	volt-amp.
Apparent power	$\lvert\bar{S}\rvert$	$V_{eff} I_{eff}$	volt-amp.

$$\bar{S} = \bar{S}_1 + \bar{S}_2 = \underbrace{V I_1^* + V I_2^*}_{\text{(parallel)}} = \underbrace{(V_1 + V_2) I^*}_{\text{(series)}}$$

Power factor Compensation

To make PF close to 1 (making it close to 1 means fully resistive), we do this compensation where we add $S_2 = jQ_2$ to $S_1 = P + jQ_1$ such that $(Q_1 + Q_2)$ is less.

If Q_1 = +ve, (→ Inductor) Q_2 = leading PF (capacitor) in Parallel
If Q_1 = −ve, Q_2 = lagging PF (inductor) in Series.

11) Polyphase Circuits :

ab = line, AB (an) = phase

Y Connection : $V_{an} = V_p \angle 0°$ [Phase Voltage],

(V √3 + 30) $V_{AB} = \sqrt{3}\, V_{an} \angle 30°$ [line voltage]

Phase current = line current $= V_{an}/Z_Y = \dfrac{1}{\sqrt{3}} \dfrac{V_p \angle \phi - 30°}{Z_Y}$.

Δ Connection : $V_{AB} = V_{ab} = \sqrt{3}\, V_{an} \angle +30^\circ$ [Phase volt = line volt]
$(I \sqrt{3} - 30^\circ)$

$I_{AB} = V_{AB} / Z_\Delta$ (Phase current)

$I_a = \sqrt{3}\, I_{AB} \angle -30^\circ$ (line current)

$$\boxed{Z_Y = Z_\Delta / 3}$$

Total Power.

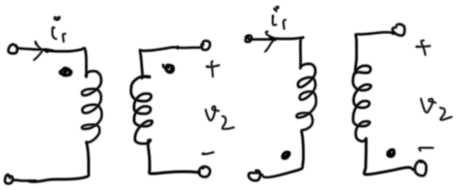
$P = 3 V_p I_p$, P_p = Power perphase = $P/3$

$= \sqrt{3} V_L I_L \times PF.$ [V_L and I_L = mag. of line current & voltage].

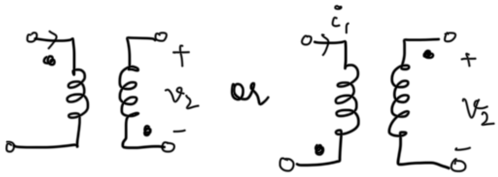
These Formulae are for both Δ & Y

12) Magnetically Coupled Circuits

Dot Convention :



M +ve



M −ve

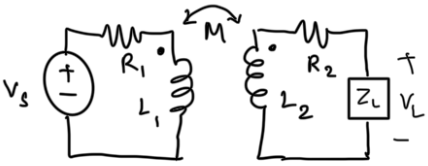
An upper limit for M and Coupling coefficient k.

$$M \le \sqrt{L_1 L_2} \Rightarrow k = \frac{M}{\sqrt{L_1 L_2}} \ ; \ k \in [0, 1].$$

loosely coupled. Tightly coupled

Linear Transformer: No magnetic material is involved.

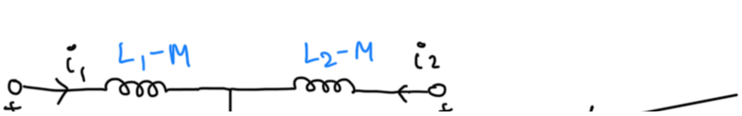
Reflected Impedance : $(R_{22} = R_2 + R_L,\ X_{22} = \omega L_2 + X_L)$

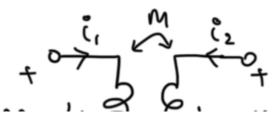


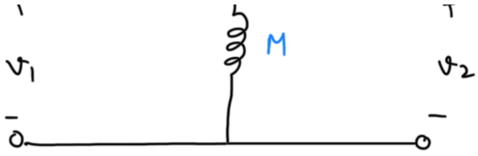
$Z_{11} = R_1 + j\omega L_1$, $Z_{22} = R_2 + j\omega L_2 + Z_L$
$= R_{22} + j X_{22}$

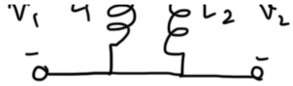
$$Z_{in} = Z_{11} + \frac{\omega^2 M^2}{Z_{22}} = Z_{11} + \frac{\omega^2 M^2 R_{22}}{R_{22}^2 + X_{22}^2} - j\frac{\omega^2 M^2 X_{22}}{R_{22}^2 + X_{22}^2}$$

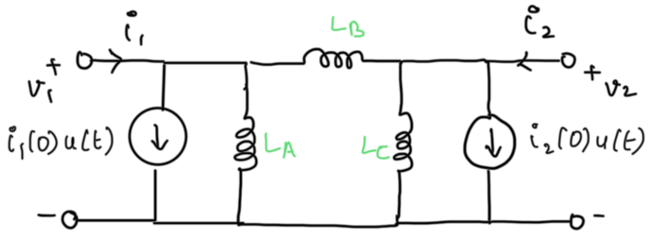
T and π Equivalent Circuits









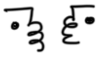


$$L_A = L_1L_2 - M^2 / L_2 - M$$
$$L_B = L_1L_2 - M^2 / M$$
$$L_C = L_1L_2 - M^2 / L_1 - M$$

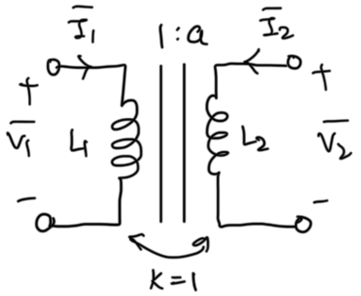
for  $\rightarrow \frac{I_2}{I_1} = \frac{-N_1}{N_2}$

Ideal Transformer:

(Turns Ratio)

$\rightarrow$ for 

$$a = \frac{N_2}{N_1} = \frac{\bar{V}_2}{\bar{V}_1} = \frac{\bar{I}_1}{\bar{I}_2} \qquad (\bar{V} = \text{Phasor})$$



$a > 1$: Step-up Transformer

$a < 1$: Step-down Transformer.

$$Z_{in} = Z_L / a^2$$

$$\bar{V}_2 \bar{I}_2 = \bar{V}_1 \bar{I}_1 \rightarrow |\bar{V}_2 \bar{I}_2| = |\bar{V}_1 \bar{I}_1| = \text{max allowable value on power transformers}$$

In time domain: $\frac{v_2}{v_1} = a$, $i_1 = -i_2$

(11)

Equivalent Circuits