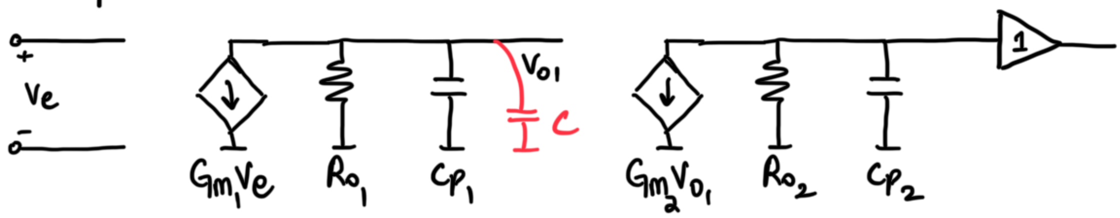
Compensation Methods

Now that we know a system is stable or not using phase margin, how to make an unstable system stable?

To move the pole in the right half plane to the left half of the plane, hopefully by not introducing any new poles.



Poles of this opamp →
[Poles of loop gain A(s)]

: $\dfrac{1}{R_{o_1}C_{p_1}}$, $\dfrac{1}{R_{o_2}C_{p_2}}$

If the phase margin is less , how will we stabilise it now? — By lowering one of its poles' frequencies.

By doing so, the unity gain frequency ω_{ugf} falls well below the other pole. If the poles are closer, the phase margin would be lesser, so we try shifting a pole to lower frequency & thus decrease ω_{ugf} and increase phase margin.

How do we lower a pole frequency: By adding a capacitance in parallel.

Now for the above opamp. let us say that the phase margin is 1°. How to increase it? → Move one of the poles to the lower frequency region by adding C., such that $\frac{A_0}{K}(P_1)$ is lower than the other pole ↖ ω_{ugf} Makes more sense if you go back to the bode plots we drew earlier.

This kind of stabilisation is ...

Let us say, $P_1 = 10$ K rad/s, $P_2 = 25$ k rad/s, $\frac{A_0}{K} = 1000$.

Now, which pole should I move to the left? (for $\zeta = 1$)

Recall $\zeta = \left(\sqrt{\frac{P_1}{P_2}} + \sqrt{\frac{P_2}{P_1}}\right) \frac{1}{2}\sqrt{\frac{K}{A_0}}$ and $\zeta = 1$, $\frac{P_2}{P_1} = \frac{4 A_0}{K}$

(i) $\left(\frac{A_0}{K} \cdot P_1\right) = \frac{P_2}{4} \Rightarrow 1000 \cdot P_1 = \frac{25 \times 10^3}{4} = 6.25$ rad/s.

10 k rad/s to ?

$\therefore$ $\boxed{\omega_{ugf} = 6.25 \text{ krad/s}}$ $\therefore$ $P_1 = 6.25$ rad/s.

(ii) $\frac{A_0}{K} P_2 = \frac{P_1}{4} \Rightarrow 1000 \cdot P_2 = \frac{10 \times 10^3}{4} = 2.5$ rad/s.

25 k rad/s to ?

$\therefore$ $\boxed{\omega_{ugf} = 2.5 \text{ krad/s}}$, $P_2 = 2.5$ rad/s.

What should I choose? Both of these are valid.

Remember, ω_{ugf} = Bandwidth, and we need as higher bandwidth as possible?! → That is why we'd Choose $\omega_{ugf} = 6.25$ krad/s

Usually, if you lower the pole that's the least, we'd get a higher bandwidth.

In the example we saw in Phase margin of higher order systems, that system is called a dominant pole system since it has one dominant pole (which is P_1) which governs the roll off of the loop gain at the ω_{ugf}. [Refer Bodeplot]

And the thing we did — Moving one of the poles to very low frequencies — to make the phase margin higher, thus making the system stable is called : **Dominant pole frequency compensation.**

Dominant pole system:

dominant pole

Non-dominant poles and zeros

M

M = no. of zeros

$$A(s) = \frac{A_0}{k} \cdot \frac{1}{1+s/p_1} \cdot \frac{\prod_{k=1} (1+ s/z_k)}{\prod_{k=2}^{N} (1+s/p_k)}$$

N = no. of poles.

$$p_1 << p_k, z_k \quad ; \quad \frac{A_0}{k} \cdot p_1 < \underset{(k\geq 2)}{p_k}, \underset{(k \geq 1)}{z_k}$$

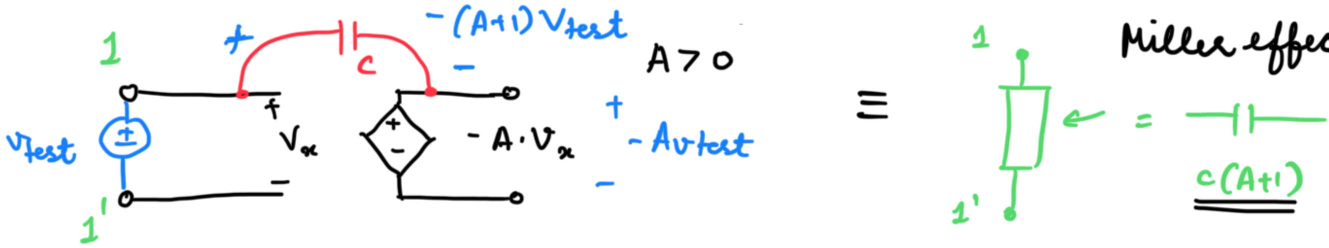
Unity gain Compensated OpAmps

As far as stability is concerned, $k=1$ (gain $= 1$) is the worst case.

$$\downarrow$$

$$\omega_{ugf} = \frac{A_0}{k} p_1 < p_2 \quad \text{for stability.}$$

Bandwidth → Higher frequency non dominant pole.



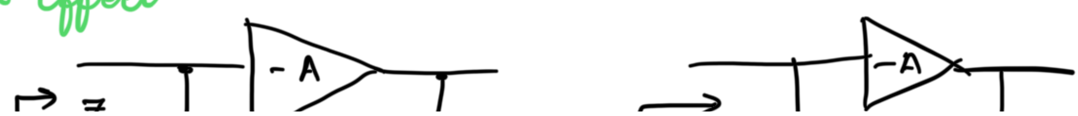
Miller effect.

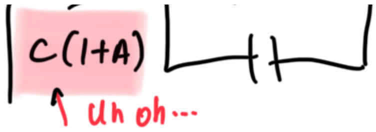
Miller effect = Phenomenon that occurs when an impedance is connected between the Cause + effect circuit of a Controlled source inside an opamp of gain A, the equivalent circuit between the input terminals of the opamp can become $Z \cdot \frac{1}{(A+1)}$

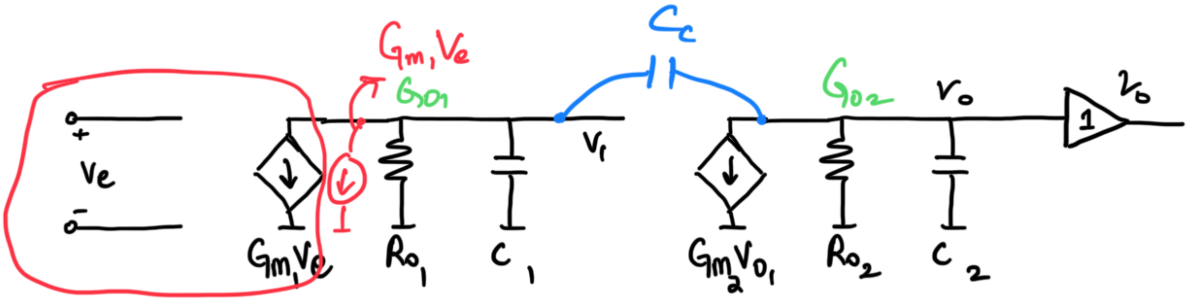
Dominant pole compensation – Making 1 pole to dominate the roll off of the magnetic response

Usually the opamps are 2 staged → Why? One stage gives us very less gain as we saw and also, the saturation limits would be too far from the supply levels.

Miller Effect:







Admittance Matrix

$$\underset{(2\times2)}{\begin{bmatrix} G_{o_1}+s(C_1+C_c) & -sC_c \\ -sC_c+G_{m_2} & G_{o_2}+s(C_2+C_c) \end{bmatrix}} \underset{(2\times1)}{\begin{bmatrix} V_1 \\ V_o \end{bmatrix}} = \underset{2\times1}{\begin{bmatrix} -G_{m_1}V_e \\ 0 \end{bmatrix}}$$

Cramer's rule :

$$\underline{\underline{V_o}} = \frac{\left| \begin{bmatrix} G_{o1} + s(C_1+C_2) & -G_{m_1}V_e \\ -sC_c + G_{m_2} & 0 \end{bmatrix} \right|}{\left| \begin{bmatrix} G_{o_1} + s(C_1+C_2) & -sC_c \\ -sC_c + G_{m_2} & G_{o_2} + s(C_2+C_c) \end{bmatrix} \right|}$$

$$\frac{V_o}{V_e} = \frac{G_{m_1}(G_{m_2} - sC_c)}{s^2(C_1C_2 + C_c(C_1+C_2)) + s(C_c(G_{m_2}+G_{o2}+G_{o_1}) + G_{o2}C_1 + G_{o_1}C_2) + G_{o1}G_{o2}}$$

Sanity Check ⇒ at $s=0$, Gain =
at $C_c=0$, Gain =

Now, $s = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$ → Not possible ...

Approximations for solving Quadratic eq^n : $\underline{ax^2+bx+c=0}$.

let's assume that x_1 & x_2 are the solutions, and

x_1 is such that $ax^2+bx \gg c \Rightarrow ax^2+bx=0$ $\quad x_1 \simeq -b/a$

x_2 is such that $ax^2 \ll bx+c \Rightarrow bx+c=0.$ $\quad x_2 \simeq -c/b.$

Now here, the poles are at $-P_1$ and $-P_2$.

$$\therefore \quad (s+P_1)(s+P_2)$$

$$\therefore P_1 = \frac{-c}{b} = \frac{G_{01}G_{02}}{C_c(G_{m_2}+G_{02}+G_{01})+C_1 \cdot G_{02}+C_2 \cdot G_{01}}$$

$$P_2 = -\frac{b}{a} = \frac{C_c(G_{m_2}+G_{02}+G_{01})+C_1 \cdot G_{02}+C_2 \cdot G_{01}}{C_1C_2+C_2C_c+C_cC_1.}$$

We can see that P_1 has decreased when we add C_c to it. and P_2 has been increased ...

This is what we were trying to do $\Rightarrow$ To figure out that we both decrease the smaller pole and increase the larger pole's frequencies in an attempt to make it stable.

This type of compensation is called the miller compensation.

Why is Nodal analysis preferred more than Mesh analysis?

Most of the cases, it is easier to identify the nodes than the meshes.

$$Z_1 = \frac{G_{m_2}}{C_c}$$

$$\frac{V_o}{V_e} = \frac{G_{m_1}(G_{m_2}-sC_c)}{s^2(C_1C_2+C_2C_c+C_cC_1)+s(C_c(G_{m_2}+G_{02})+C_1 \cdot G_{02}+C_2 \cdot G_{01})+G_{01}G_{02}}$$

Now, to interpret the equation, we approximated.

Before adding C_c, $P_1 = \frac{G_{01}}{C_1}$; $P_2 = \frac{G_{02}}{C_2}$

After adding C_c,

Conductance @ first stage o/p

capacitor @ first stage o/p

$$P_1 = \frac{G_{o1}}{C_c\left(\frac{G_{m2}}{G_{o2}} + 1 + \frac{G_{o1}}{G_{o2}}\right) + C_1 + C_2\,\frac{G_{o1}}{G_{o2}}} = \frac{G_{o1}G_{o2}}{G_{m2}C_c}\ ?$$

Miller Multiplied C_c @ first stage o/p THE DOMINANT ONE

Second stage approximations, not a VCVS.

$$P_2 = \frac{C_c\left(G_{m2} + G_{o2} + G_{o1}\right) + C_1 G_{o2} + C_2 G_{o1}}{C_c C_1 + C_1 C_2 + C_2 C_c}$$

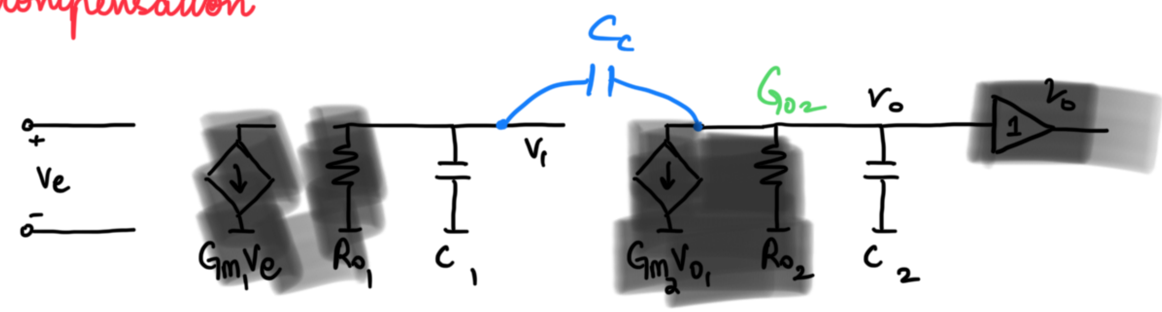
Before it was $\left(\frac{G_{o2}}{C_2}\right)$

$$= \frac{G_{m2}\,\frac{C_c}{C_1 + C_c} + G_{o2} + G_{o1}\,\frac{C_2 + C_c}{C_1 + C_c}}{C_2 + \frac{C_1 C_c}{C_1 + C_c}}$$

Increases the numerator

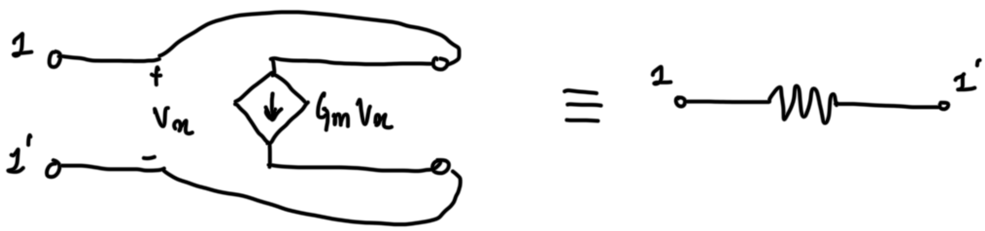
Series combination of C_1 and C_c ??

Since the poles are being split, We call this a pole split compensation

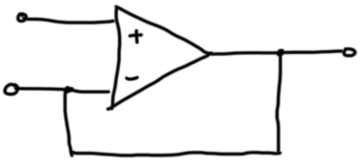


The equations suggest us that C_1 series with C_c and both parallel to C_2. If we remove all of the components from the circuit except capacitors, we see exactly the same combination of capacitors that equations gave us. Sweet!

Fun fact: A VCCS in feedback behaves as a Resistor



Now let us try finding the phase margin of the miller compensated opamp.



$\rightarrow L(s) = A(s)$ $\qquad A_0 = G_{m_1} R_1 G_{m_2} R_2$

$$\omega_{ugf} = A_0 \cdot p_1 =$$

$$= \frac{G_{m_1} G_{m_2}}{G_{o_1} G_{o_2}} \cdot \frac{G_{o_1}}{C_c\left(\frac{G_{m_2}}{G_{o_2}} + 1 + \frac{G_{o_1}}{G_{o_2}}\right) + C_1 + C_c \frac{G_{o_1}}{G_{o_2}}} = \frac{G_{m_1}}{C_c}.$$

Pole always gives a phase lag. If a zero is in the LHP, then it gives us a phase lead. But a zero in RHP will give a phase lag.

Phase lead leads to more stability.

Now, Phase margin:

$\pi/2$ DC loop gain → Very large no.!

$$\phi\Big|_{\omega_{ugf}} = \pi - \tan^{-1}\frac{\omega_u}{z_1} - \cancel{\tan^{-1}}\frac{\omega_u}{p_1} - \tan^{-1}\frac{\omega_u}{p_2}$$

$$= \pi/2 - \tan^{-1}\frac{G_{m_1}/C_c}{G_{m_2}/C_c} - \tan^{-1}\frac{\omega_u}{p_2}$$

$$\therefore \tan^{-1}\left(\frac{G_{m_1}}{G_{m_2}}\right) + \tan^{-1}\frac{\omega_u}{p_2} < 30^\circ.$$

This is known. It is fixed.

$$\tan^{-1}\frac{\omega_u}{p_2} < 30 - \tan^{-1}\frac{G_{m_1}}{G_{m_2}}$$

To choose the Appropriate C_c, $\tan^{-1}\frac{\omega_u}{p_2}$ would have it, solve it for C_c.

Equation to determine C_c: This dominates

We can neglect these terms

$$\omega_u = \frac{G_{m_1}}{C_c} \; ; \qquad p_2 = \frac{G_{m_2}\cdot\frac{C_c}{C_c + C_1} + \underbrace{G_{o_2} + G_{o_2}\cdot\frac{C_c + C_2}{C_c + C_1}}}{C_2 + \frac{C_c C_1}{C_c + C_1}}$$

$$\therefore \frac{\omega_u}{p_2} = \tan\left(\frac{\pi}{2} - \tan^{-1}\frac{G_{m_1}}{G_{m_2}} - \phi_M\right)$$

Phase shift due to p_2 @ ω_u.

$$\Rightarrow \frac{(G_{m1}/C_c)\cdot(C_2 + C_cC_1/C_c+C_1)}{(G_{m2}\, C_c/\, C_c+C_1)}$$

Required Phase margin (We can set it to how much ever we want)

$$= \frac{G_{m1}}{C_c} \cdot \frac{C_2(C_c+C_1) + C_cC_1}{G_{m2}\, C_c}$$

$$= \frac{G_{m1}}{G_{m2}} \cdot \frac{C_c(C_1+C_2) + C_1C_2}{C_c^2} = \tan\left(\frac{\pi}{2} - \tan^{-1}\frac{G_{m1}}{G_{m2}} - \phi_M\right)$$

This is nothing but a Quadratic equation in C_c.

let $\frac{G_{m1}}{G_{m2}} = G$.

ax^2+bx+c, where $a = \tan\left(\frac{\pi}{2} - \tan^{-1}G - \phi_M\right)$

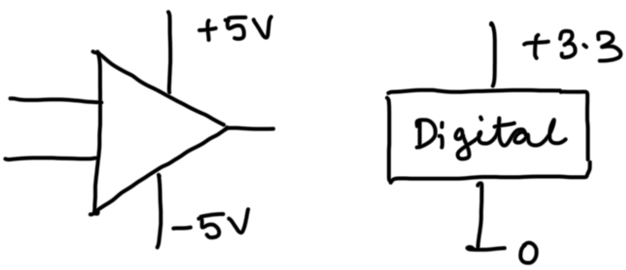
$b = G(C_1+C_2)$; $c = G\,C_1C_2$.

We can make another approximation : $C_c \gg C_1$, Meaning C_c is a short.

$$P_2 = \frac{G_{m2}}{C_2+C_1}$$, and this approximation makes sense when $C_2 \gg C_1$.

The third Miller compensation lecture was a pain.

Introduction to DC-DC Converters



Now, using just a battery, I need to power my

Opamp → which is dual supply, and a digital circuit with single supply.

How will I get precisely +5 & −5V for Opamp, and +3.3V for Digital from one single battery source??

→ To keep something accurate, we need negative feedback

→ To convert from one voltage to another:

- AC – Transformers which are magnetically coupled and electrically isolated.
- DC – Resistor dividers.

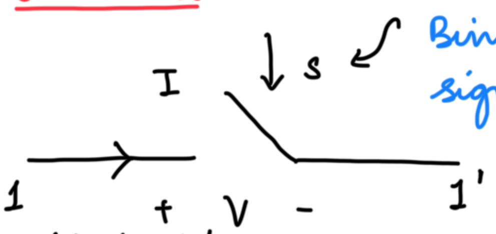
But again, resistor dividers will introduce errors and is not considered ideal in practical applications:

If we need an increased voltage, or negative voltage, we won't be able to achieve this. But with a transformer, this operation is trivial

Hence we need to have some mechanism to convert DC to AC

AC to DC – Rectifiers! Then DC to AC?

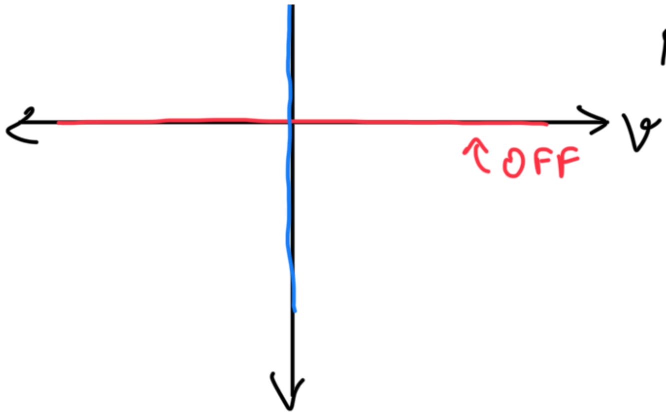
Switch:



Binary logic signal → 0 – off, 1 – on.

Switches are linear? How to find if a component is linear or not.?
↓
Superposition, Homogeneity. ; Graphically straight line characteristics

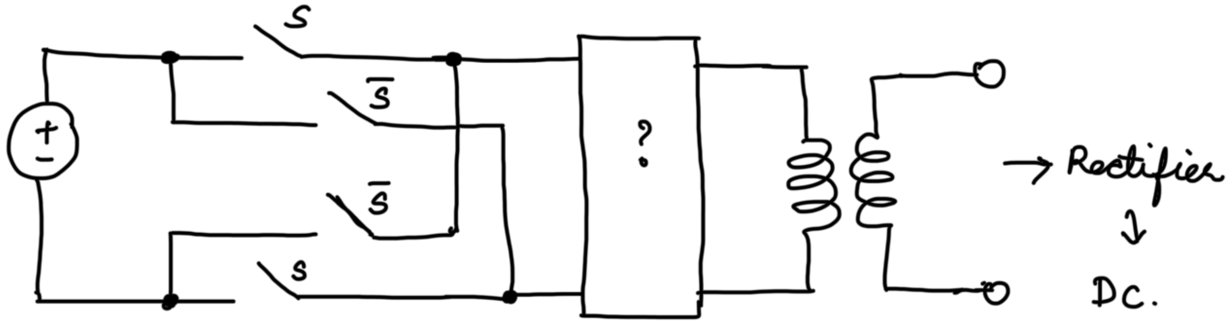
This is linear, but time variant.



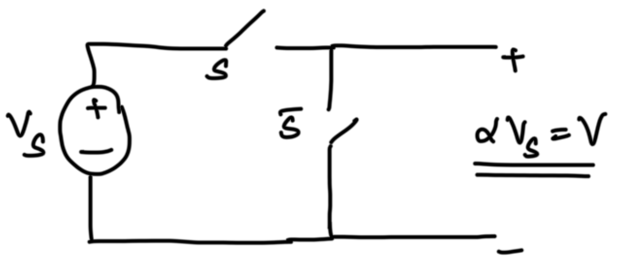
Power dissipation? Zero. How?
Area under the graph.

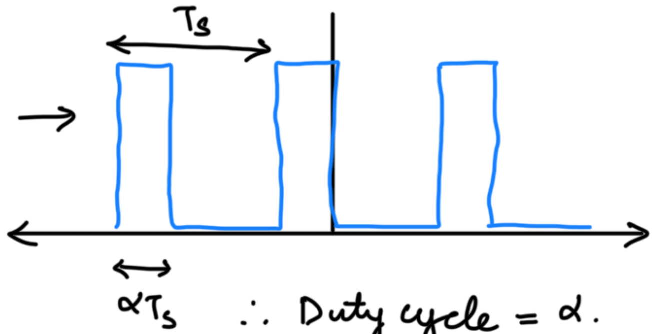
Now, how does a switch helps us? → can convert DC to AC! → By switching it ON and OFF periodically...

But how many switches?



Now to get, say a Voltage V, from V_s → Battery Voltage,





∴ Duty cycle = α.

∴ A pulse width modulator, which is periodic but with varying duty cycle can be used to get a required V, just by varying the Duty cycle, and αV_s is the Average Voltage value.

Problem now → αV_s is periodic?! But we need the constant DC value right? How to fix this?:

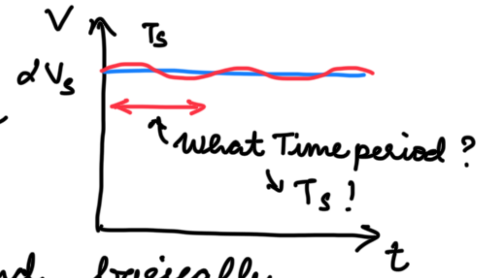
Remember? We can decompose any periodic signal as its fourier series? We need the constant a_0 term, i.e., the DC term. So, how to get rid of the higher freq. and just get freq. = 0 [DC]? → LPF obvious!

Cutoff frequency should be less than the first freq. term in the fourier series expansion.

And that is literally the frequency of the signal itself!

But, real circuits can't get rid of the harmonics, just like that [Ouch].

So, we'll get some amplitude, like 10 mV, for the periodic signal as a ripple — But hey! 10 mV is nothing when compared to some 5 or 10V, even 2 or 3 V. Even 1V. So, works? Yay.



Now, what if we want to decrease the amplitude further?

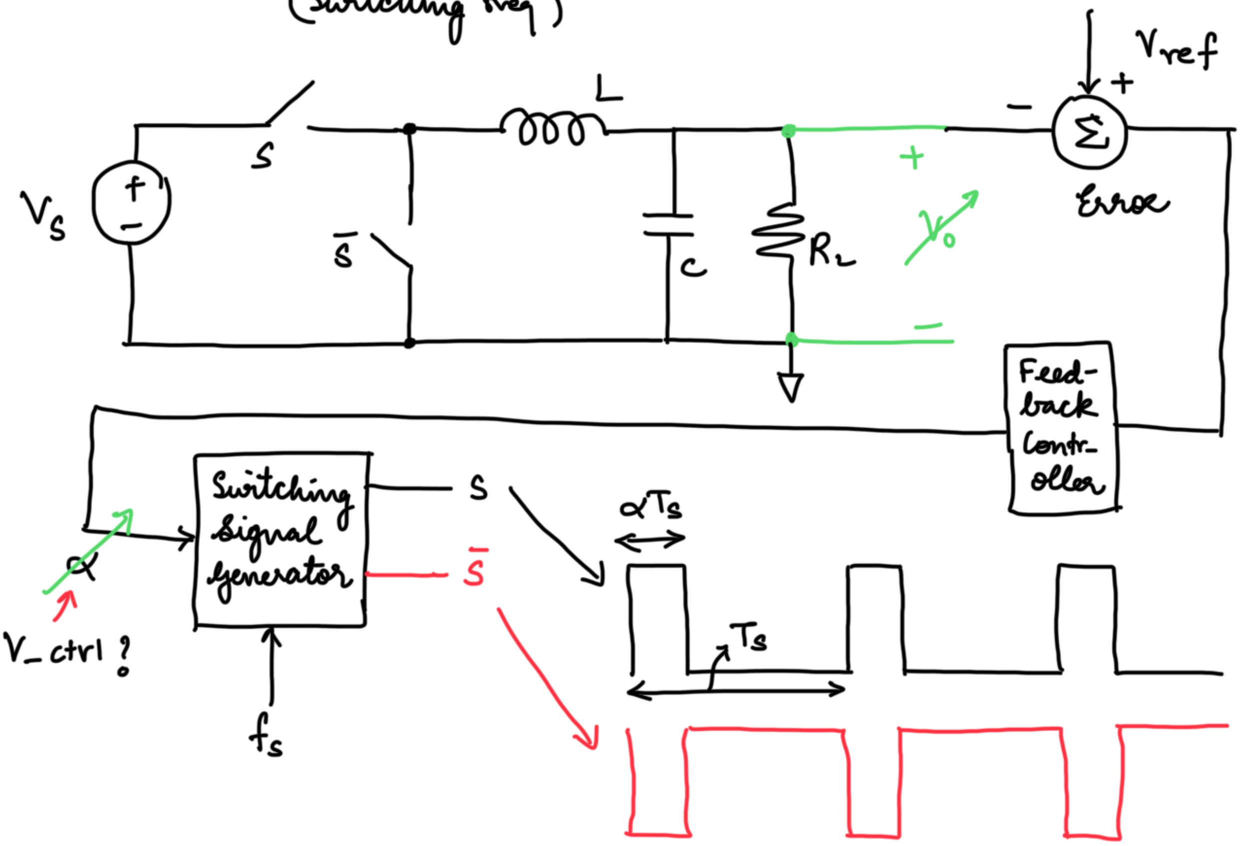
Can increase the value of L and/or C, and basically move the low pass filter to a shorter bandwidth - So it filters even more.

But L and C are bulky, and you need them to be sleek. Let's say we've used the maximum possible physically large L and C has been used, and we can't increase L and C further. Now how do we decrease the amplitude?

Increase the switching frequency. But this also has a cost → Switching faster dissipates more power in real case. In ideal case, we assume no losses. Actually negligible. But if you frequently switch ON and OFF, then the power dissipation is significant and we do not want that. [Power dissipation in changing the switch state, I mean]

Ripple $\propto$ |

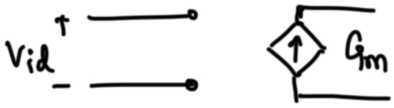
$\propto \dfrac{\quad}{(\text{Switching freq})^2}$



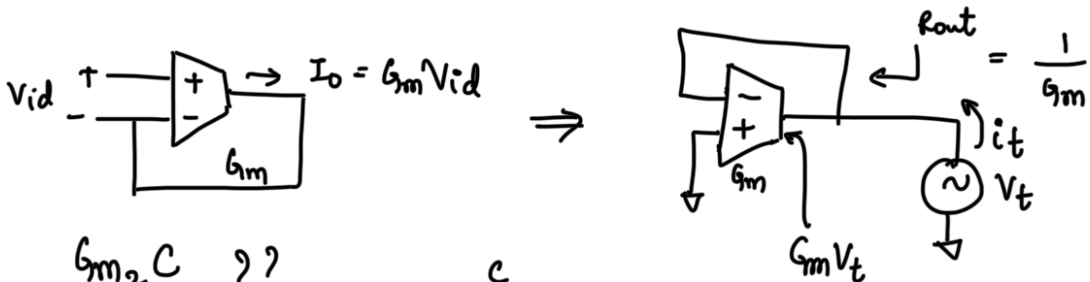
Now we'll try modelling the above circuit, find its stability conditions, and basically analyse.

(Continued in Nagi Notes -5)

Aniruddhan sir Insights on Miller compensation:

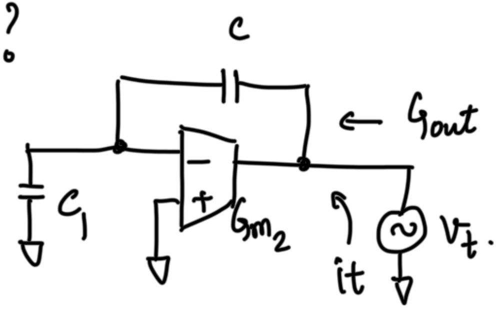


$G_m V_{id}$ → VCCS. ⇒ Put it in −ve feedback.



$\dfrac{G_{m_2} C}{C_1 + C}$??

Now,



$$i_t = G_{m2} \frac{C V_t}{C_1 + C} \Rightarrow \frac{V_t}{i_t} = \frac{C_1 + C}{G_{m2} C}$$

Now, $R_{eff} = R_2 \,||\, R_{out}$

$$P_2' = \sqrt{3}\, \omega_u \qquad p_1' = \frac{1}{R_1 G_{m2} R_2 C}$$

$$\omega_u = \frac{G_{m1}}{C} \qquad p_2' = \frac{1}{R_{eff} \, C_{eff}}.$$

$$1/R_{eff} = G_{out} + \frac{1}{R_1} + \frac{1}{R_1}\frac{C + C_2}{C + C_1}$$

$$= G_{out} + G_1 + \frac{G_1 C + C_2}{C + C_1}$$

$$C_{eff} = C_2 \,||\, C_{series} C_1 = C_2 + C \text{ series } C_1$$

$$= C_2 + \frac{C_1 C}{C_1 + C}$$

$$\therefore P_2' = \frac{G_{out} + G_1 + \frac{G_1(C + C_2)}{(C + C_1)}}{C_2 + \frac{C_1 C}{C_1 + C}}$$

$$P_2' = \frac{\frac{G_{m2} C}{C_1 + C} + G_1 + G_1 \frac{(C + C_2)}{(C + C_1)}}{C_2 + \frac{C_1 C}{C_1 + C}}$$

$$= (G_{m2} + 2G_1) C + G_1 C_1 + G_1 C_2$$

$$\frac{C_1 + C(C_1+C_2)}{(\cancel{C_1+C})}$$

$$= \frac{(G_{m_2} + 2G_1)C + G_1(C_1+C_2)}{C_1 + C(C_1+C_2)} = P_2'$$

$$W_u = \frac{P_2'}{\sqrt{3}} = \frac{G_{m_1}}{C} \Rightarrow P_2' C = \sqrt{3}\, G_{m_1}$$

$$\sqrt{3}\, G_{m_1} = \sqrt{3}\; 100 \times 10^{-6} = \sqrt{3} \times 10^{-4}$$

$$\frac{(G_{m_2} + 2G_1)C + G_1(C_1+C_2)}{\frac{C_1}{C} + C_1 + C_2} = \sqrt{3} \times 10^{-4}$$

$$(10^{-3} + 2\times 10^{-5})C + 10^{-5}(866+100)\times 10^{-12} = \sqrt{3}\times 10^{-4} \times \left(\frac{10 \times 10^{-12}}{C} + (866+100)\times 10^{-12}\right)$$

$$= 10^{-3} C = \sqrt{3}\left(\frac{10^{-7}}{C} + 966\times 10^{-8}\right)$$

$$C 10^{-3} - \frac{10^{-7}}{C}\sqrt{3} = \sqrt{3} \times 9.66 \times 10^{-8}$$

$$C^2 \times 10^{-3} - 10^{-7}\sqrt{3} = \sqrt{3} \times 9.66 \times 10^{-7} C$$

$$10^{-3} C^2 - 1.67 \times 10^{-6} C - 1.73 \times 10^{-7} = 0$$

$$10^4 C^2 - 16.7 C - 1.73 = 0$$

$$C = 1.4 \times 10^{-2} C \times 10^{-7} = \underline{1.4\,nF}$$