Chapter - 10 : Wave Optics

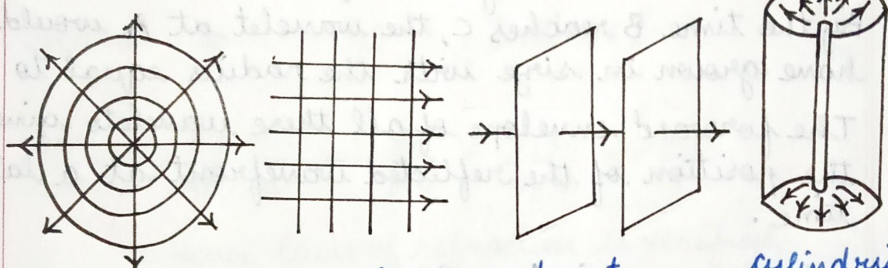
- **Wave front:** A surface of constant phase is called as a wave front.

(i) If the source is a point source, the wave front is spherical.

(ii) If the source is far away, the wave front is plane.

(iii) If the source is extended, the wave front is cylindrical.

- **Ray:** A line drawn $\perp^r$ to the wavefront.



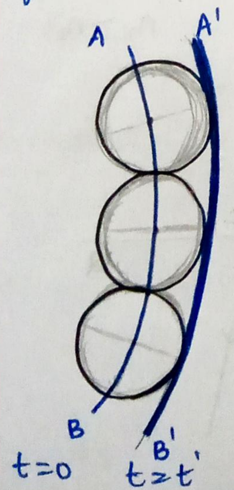
spherical wavefront

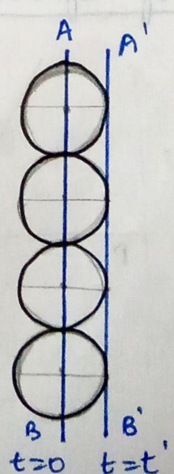
Plane wavefront

cylindrical wavefront

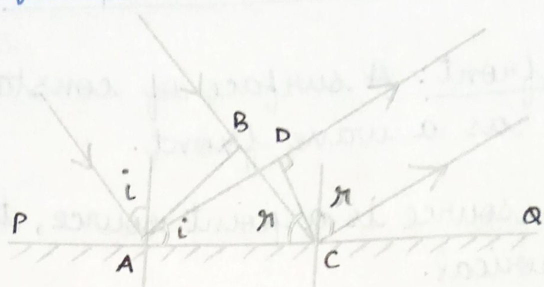
Huygen's Principle:

- Every point on the wavefront acts as a source of secondary disturbance giving out wavelets in all directions which move with the speed of wave in that medium.
- The forward envelope of all these wavelets is the position of new wavefront at a later time.





Laws of Reflection :



- AB is a plane wavefront incident on the surface PQ.
- According to Huygen's principle as the wavefront strikes the surface wavelets are produced.
- Every point on AB as it strikes the surface produces wavelets successively one after the other.
- By the time B reaches C, the wavelet at A would have grown in size with the radius equal to ct.
- The forward envelope of all these wavelets gives the position of the reflected wavefront at a later time.

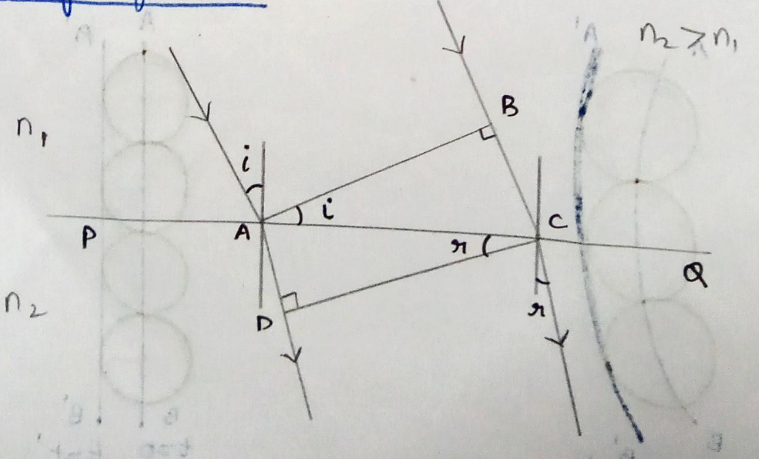
In ΔABC, $\sin i = \frac{BC}{AC}$.

In ΔADC, $\sin r = \frac{AD}{AC}$.

$$\frac{\sin i}{\sin r} = \frac{BC}{AC} \times \frac{AC}{AD} = \frac{BC}{AD} = \frac{ct}{ct} = 1.$$

$$\frac{\sin i}{\sin r} = 1 \Rightarrow \sin i = \sin r \Rightarrow \angle i = \angle r.$$

Laws of Refraction :



- AB is a plane wavefront incident on the surface PQ separating the 2 media of refractive indices n_1 & n_2.
- According to Huygen's principle, as the wavefront strikes the surface, wavelets are produced.
- Every point on AB on reaching the surface produces wavelets successively one after the other.
- By the time B reaches C, the wavelet at **A** would have grown in size with radius equal to $v_2 t$.
- The forward envelope of all these wavelets is the position of new wavefront at a later time.

$BC = v_1 t \quad ; \quad AD = v_2 t.$

In ΔABC, $\sin i = \frac{BC}{AC}$; In ΔADC, $\sin r = \frac{AD}{BC}$.

$$\frac{\sin i}{\sin r} = \frac{BC/AC}{AD/AC} = \frac{BC}{AD} = \frac{v_1 t}{v_2 t} = \frac{v_1}{v_2}$$

$\Rightarrow$ Hence laws of refraction is verified

$BC = \lambda_1 \quad ; \quad AD = \lambda_2.$

$\lambda_1 = v_1 t \quad ; \quad \lambda_2 = v_2 t.$

$$\therefore \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} \Rightarrow \frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2}. \Rightarrow \nu_1 = \nu_2.$$

$\Rightarrow \nu = \text{constant}.$

$\therefore$ Frequency doesn't change when light undergoes refraction.

Interference

When 2 light waves of same amplitude, almost the same frequency moving along the same direction superimpose regions of maximum and minimum intensities are formed. This phenomenan is called interference.

Region of maximum intensity – Bright band.
Region of minimum intensity – Dark band.

eg. Colours in soap bubbles, colours in thin films of oil.

Analytical Treatment

3 marks

Let the displacement of particle of medium due to 2 light sources be given as:

constructive interference $\Rightarrow n\lambda$

destructive interference $\Rightarrow (n+1/2)\lambda$.

$$y_1 = a\cos\omega t$$
$$y_2 = a\cos(\omega t + \phi)$$
$$\phi = \text{phase difference}$$

According to principle of superposition, the net displacement is given as:

$$Y = y_1 + y_2$$

$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$$= a\cos\underbrace{\omega t}_{A} + a\cos(\underbrace{\omega t + \phi}_{B})$$

$$= 2a\cos\left(\frac{\omega t - \omega t - \phi}{2}\right)\cos\left(\frac{\omega t + \omega t + \phi}{2}\right)$$

$$Y = 2a\cos(-\phi/2)\cos(\omega t + \phi/2)$$

Comparing this with equation of standard form

$$Y = A\cos\omega t,$$

Amplitude of the resultant wave is given as:

$$A = 2a\cos(\phi/2)$$

I = Intensity $\propto A^2$

$$\therefore I = 4a^2\cos^2\phi/2$$

Let the intensities of 2 light sources be I_0.

$\Rightarrow$ $\therefore$ Resultant Intensity $= 4 I_0 \cos^2(\phi/2)$.

Conditions for maxima / bright band / constructive interference.

$I_{max} = 4 I_0$.

$\cos^2(\phi/2) = 1$.

$\therefore \phi = 2n\pi, n = 0, 1, 2, \ldots$

Conditions for minima / dark band / destructive interference.

$I_{max} = 0$.

$\cos^2(\phi/2) = 0$.

$\phi = n\pi, n = 0, 1, 2, \ldots$

To obtain stable and sustained interference pattern, position of bright and dark band should not change with time. Hence we require coherent sources of light.

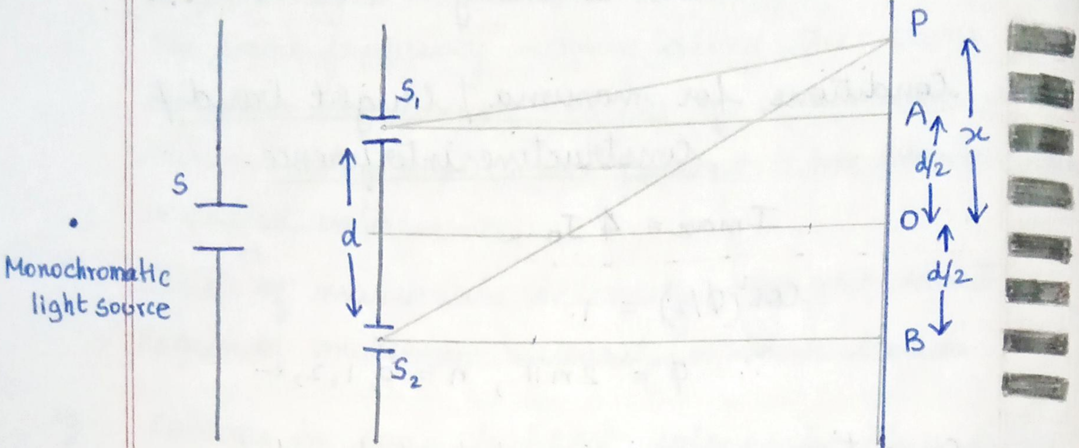
Coherent sources: Light sources which emit out light continuously, of same frequency, almost the same magnitude, and a constant phase difference are called coherent sources of light.

2 independent sources of light cannot be coherent because there will be abrupt or rapid phase changes in a time gap of 10^{-10}s.

If two individual light sources are used, then we will observe an average intensity given by: $I = 4 I_0 \cos^2(\phi/2)$, $\left[\langle \cos^2\phi/2 \rangle = \frac{1}{2} \right]$

$I = \frac{4 I_0}{2} = 2 I_0$

Young's double slit Experiment



- It consists of a monochromatic light source (Sodium vapour lamp)
- A slit S of width 1 mm is placed in front of the source.
- 2 more slits S_1 and S_2 of width 0.2 mm are kept in front of slit S.
- Light coming out of S_1 and S_2 act as coherent sources of light.
- Alternate bright and dark bands are formed on a screen kept 1 m away from the 2 slits.

Let D - distance of the screen from the 2 slits S_1 & S_2.

d - distance between the 2 slits S_1 and S_2.

x - distance of observation point P on the screen from the centre of the screen.

ΔS_1AP : $S_1P^2 = S_1A^2 + AP^2 = D^2 + (x - d/2)^2$

ΔS_2BP : $S_2P^2 = S_2B^2 + BP^2 = D^2 + (x + d/2)^2$.

$S_2P^2 - S_1P^2 = \cancel{D^2} + (x + d/2)^2 - \cancel{D^2} - (x - d/2)^2 = 2xd.$

$(S_2P - S_1P)(S_2P + S_1P) = 2xd.$

$$S_2P - S_1P = \frac{2xd}{S_2P + S_1P}$$

$S_2P \simeq D$, $S_1P \simeq D$.

$\therefore S_2P - S_1P =$ Path difference $= \frac{2xd}{2D}$.

$$\boxed{\text{Path difference} = \frac{xd}{D}}$$

Condition for bright band / Maxima / constructive interference:

Path difference $= \lambda . n$.

$$\frac{xd}{D} = n\lambda.$$

$$\boxed{x = \frac{n\lambda D}{d}} \quad (n = 0, 1, 2, \dots)$$

Condition for dark band / Minima / Destructive interference:

$$\frac{xd}{D} = (n + 1/2)\lambda.$$

$$x = \frac{(n + 1/2)\lambda D}{d} \quad (n = 0, 1, 2, \dots)$$

<u>Bandwidth / Fringewidth</u>: (β)

Distance between 2 consecutive maxima or minima

$x = \frac{n\lambda D}{d}$, $x_1 = \frac{\lambda D}{d}$, $x_2 = \frac{2\lambda D}{d}$

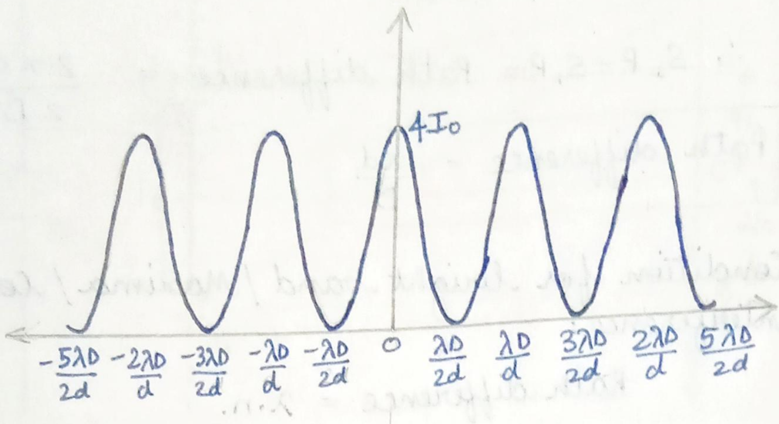
$\therefore \beta = x_2 - x_1 = \frac{2\lambda D}{d} - \frac{\lambda D}{d} = \boxed{\frac{\lambda D}{d} = \beta}$

$x = (n + 1/2)\frac{\lambda D}{d}$, $x_1 = \frac{\lambda D}{2d}$, $x_2 = \frac{3\lambda D}{2d}$.

$\therefore \beta = x_2 - x_1 = \frac{3\lambda D - \lambda D}{2d} = \boxed{\frac{\lambda D}{d} = \beta}$

All the bands have equal width in Interference.

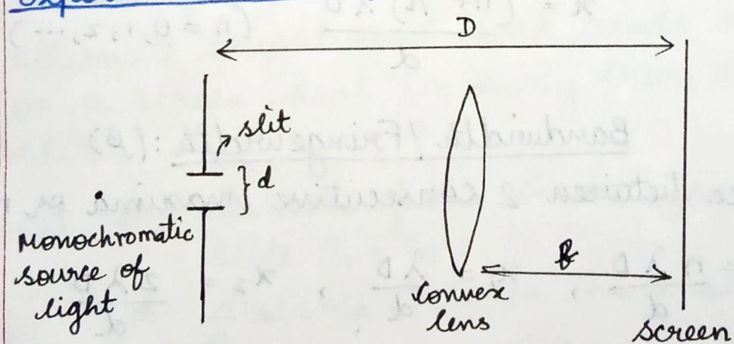
Variation of intensity with position of the point or Path difference



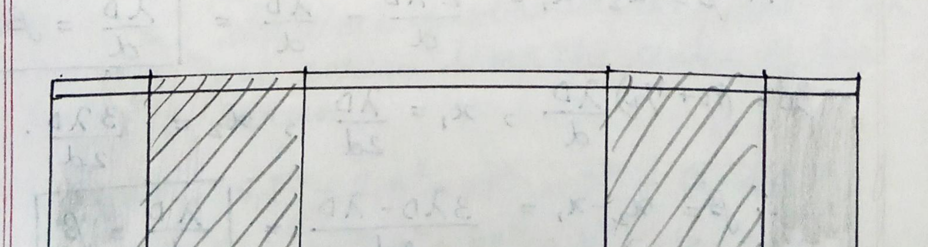
Diffraction:

- Bending of light around obstacles is called diffraction.
- It can be observed only when the size of the slit is comparable to the wavelength of light.
- It can be explained on the basis of Huygen's principle.

Experimental set up:

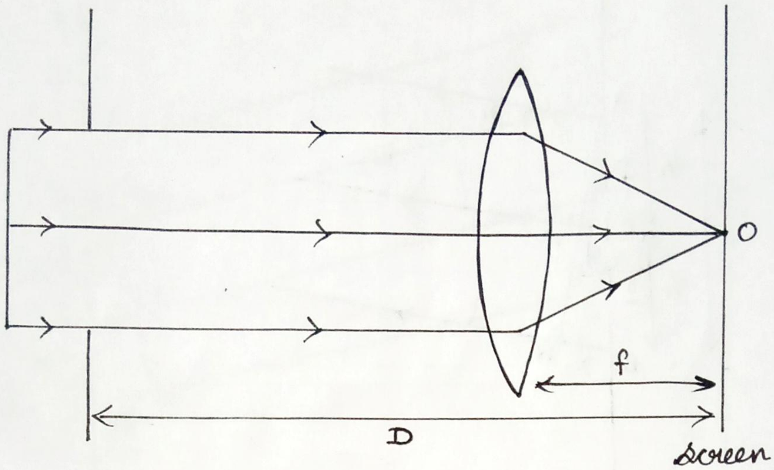


$(d \simeq \lambda)$



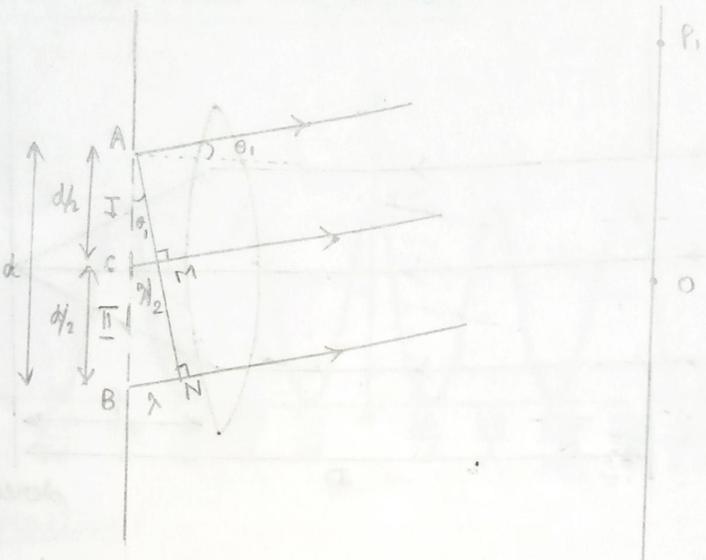
Diffraction Pattern

Formation of central maximum:



- Consider a plane wavefront incident on the slit AB.
- Every point on the wavefront acts as a source of secondary disturbance giving out wavelets in all directions.
- At the centre O of the screen bright band, a central maximum is formed as all the wavelets coming from every point on the source travels the same distance.
- Hence there is no path difference and no phase difference.
- So they superimpose constructively and form a central maximum at the centre of the screen.

Formation of secondary minimum :



- Consider wavelets moving at angle θ_1, with respect to incident direction reaching pt. P on the screen.
- These wavelets have a path difference.

$$BN = d \sin \theta_1$$

such that they superimpose destructively to form a dark band at point P_1

(assume the slit to be divided into 2 regions I and II), wavelets from region I differ in path of $\lambda/2$ from wavelets in region II.

- Hence they superimpose destructively to form a minima at point P_1 on the screen.

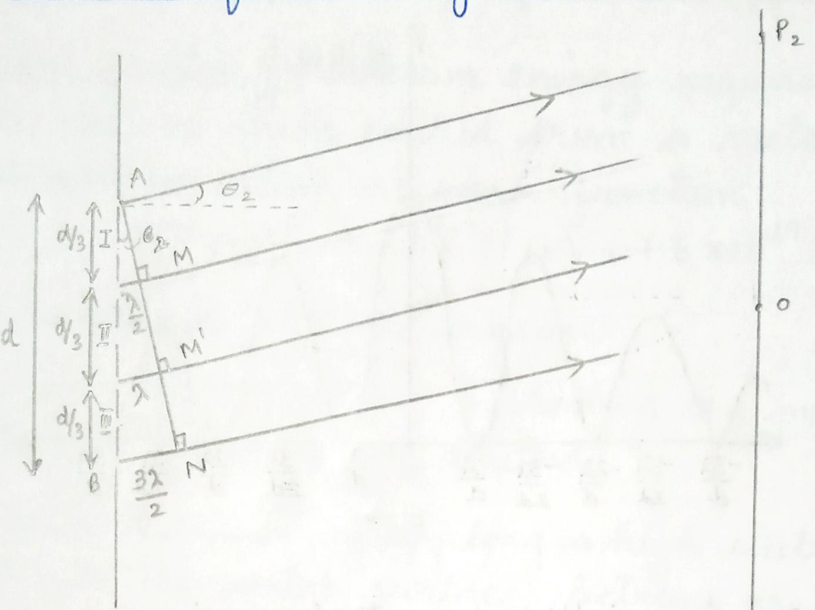
Condition for secondary minima:

$$d \sin \theta = n\lambda.$$

$$d\theta = n\lambda$$

$$\theta = \frac{n\lambda}{d}, (n = 1, 2, 3)$$

Formation of secondary maxima:



In ΔABN, $BN = d \sin\theta_1$; $d \sin\theta = \lambda$.

Condition for secondary maxima:

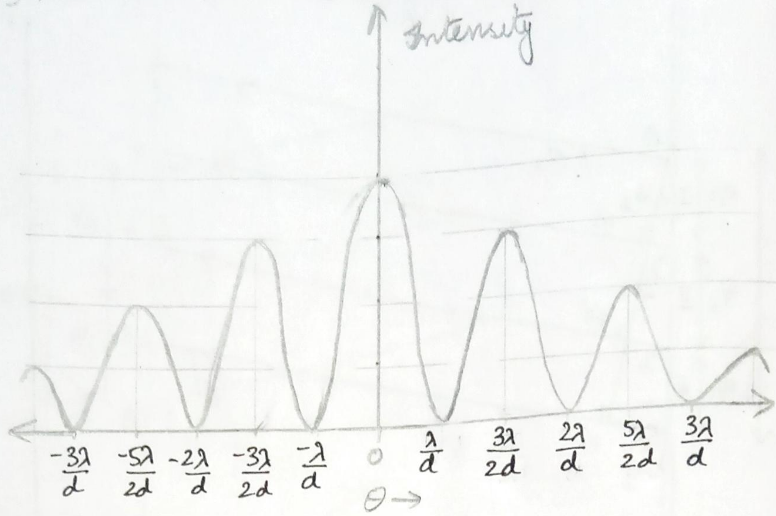
$$d \sin\theta = (n + 1/2)\lambda.$$

$$d\theta = (n + 1/2)\lambda.$$

$$\theta = (n + 1/2)\frac{\lambda}{d} \quad (n = 1, 2, 3 \ldots)$$

Consider wavelets moving at angle θ_2, with respect to incident direction reaching point P_2 on the screen. These wavelets have a path difference of $BN = d \sin\theta_2$, Such that they superimpose constructively to form a maxima at point P_2 (assume the slit to be divided into 3 equal parts) the wavelets from region 1 and the wavelets from region 2 differ in path of $\lambda/2$, hence they superimpose destructively but the wavelets from region 3 superimpose constructively and form a maxima.

372 to end x



Angular width of central maxima

$$= 2\frac{\lambda}{d}$$

Linear width of central maxima $= 2\frac{\lambda}{d} \cdot D$

Angular width of secondary maxima

$$= \lambda / d.$$

Linear width of secondary maxima $= \frac{\lambda D}{d}$

Angular width and linear width of central maxima is twice the angular width and linear width of secondary maxima.