Chapter - 1 : Electrostatics - I

1) <u>Properties of charges:</u>

1. There are 2 kinds of charges: +ve and -ve.

2. By convention, the glass rod rubbed with silk cloth is said to acquire a +ve charge and the plastic rod rubbed with woolen cloth is said to have acquired a -ve charge.

3. Like charges repel and unlike charges attract each other.

4. A charged body can ~~or~~ charge an uncharged body.

5. <u>Additivity:</u>
Total charge of any system is an algebraic sum of individual charges present in the system.

$$Q = q_1 + q_2 + q_3 + \cdots$$

6. <u>Conservation of charge:</u>
Total charge of an isolated system is conserved. Charge can neither be created nor be destroyed, but can only be transferred from one body to the other.

7. <u>Quantisation of charge:</u>
Total charge of a system is quantised.
i.e; charge on any body can be expressed as an integral multiple of the most basic and fundamental charge → charge on an electron.

$$Q = ne$$

$n \rightarrow$ integer, $e = 1.6 \times 10^{-19} C$

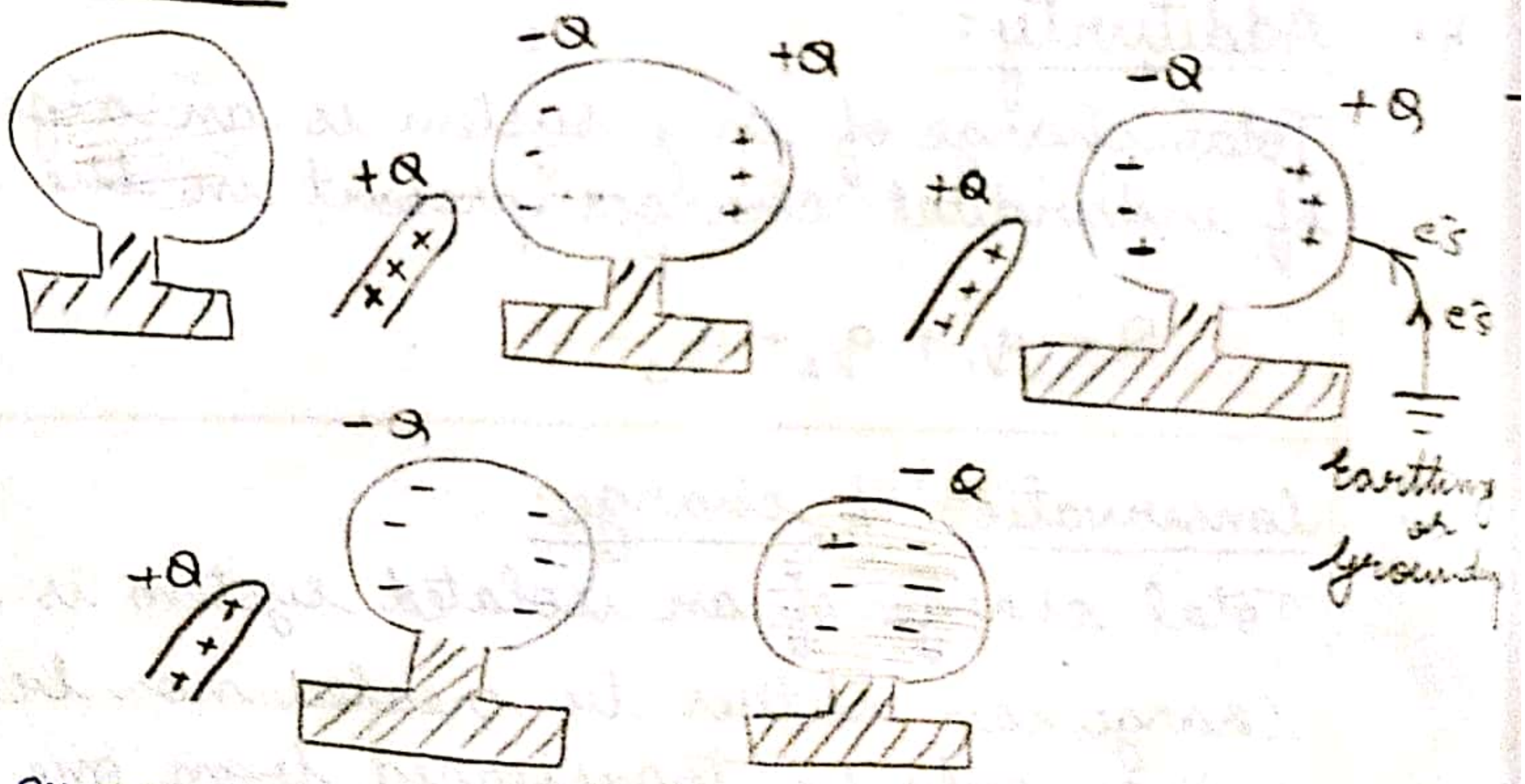
2) Methods of charging a body:

a) **By friction:** If 2 bodies are rubbed against each other, there is transfer of charge from one body to another.

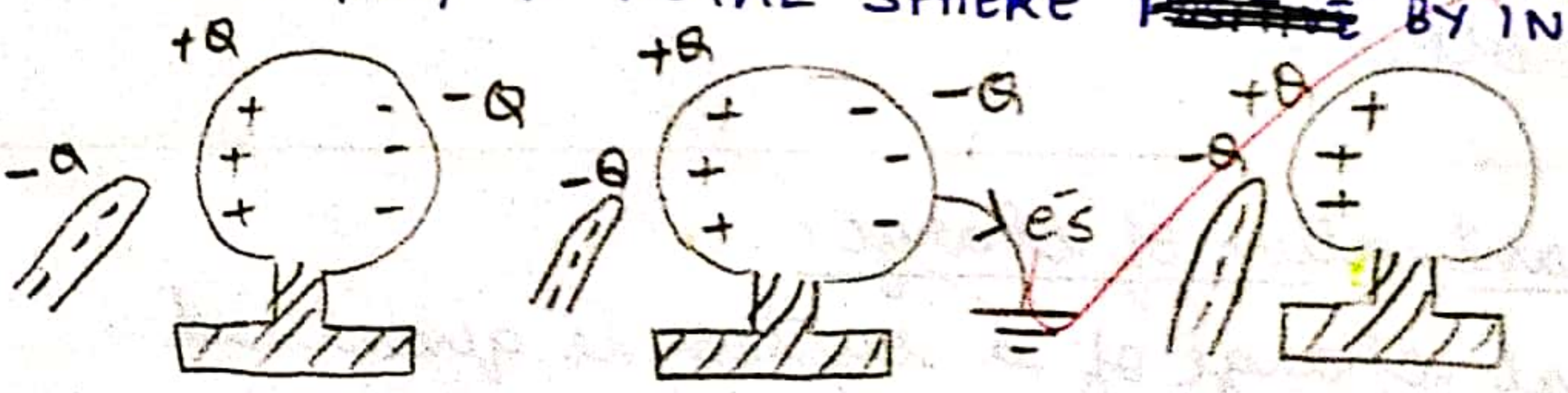
eg) A glass rod rubbed with silk cloth attains a positive charge while a plastic straw rubbed with wool attains a negative charge.

b) **By conduction:** Here an uncharged body is brought in contact with a charged body and it becomes charged. Both the bodies attain similar kind of charge and the total charge is equally distributed between the two bodies. (Both the bodies must be symmetrical)

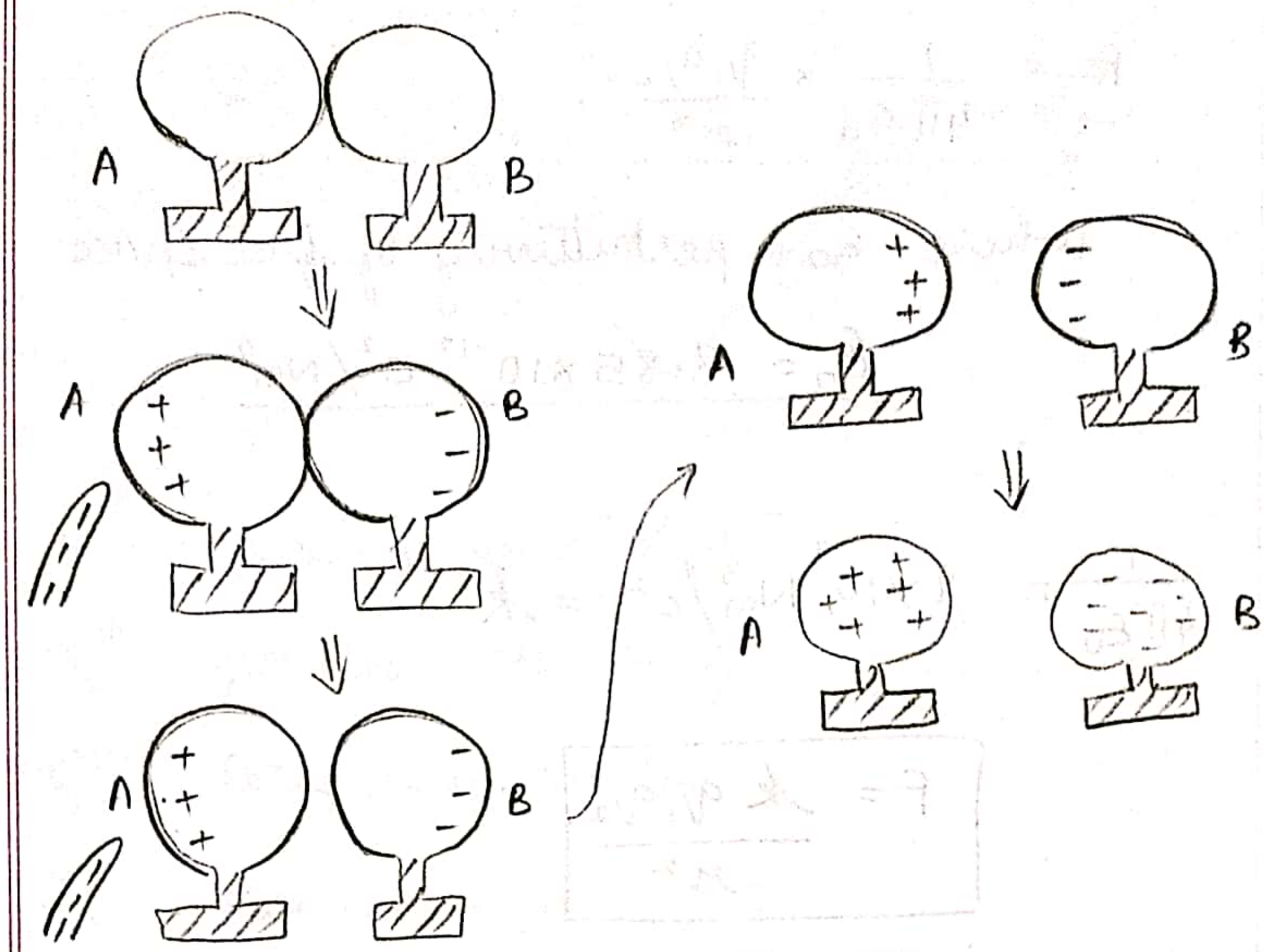
c) **By induction:**



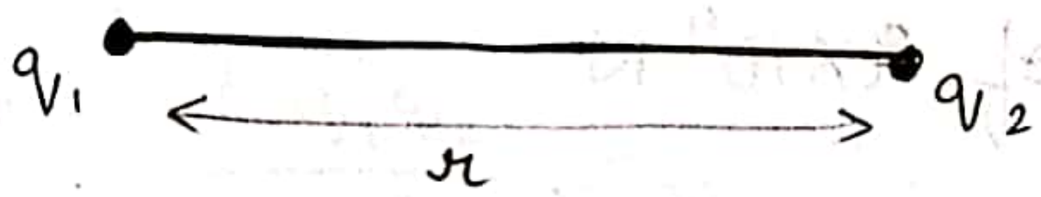
CHARGING A METAL SPHERE ~~NEGATIVE~~ ~~POSITIVE~~ BY INDUCTION



Charging two spheres by induction:



COULOMB'S LAW



$$F \propto q_1 q_2$$

$$F \propto \frac{1}{r^2}$$

$$\therefore F \propto \frac{q_1 q_2}{r^2}$$

$$F = \frac{1}{4\pi\varepsilon} \times \frac{q_1 q_2}{r^2}.$$

where ε = epsilon
= permittivity of the medium.

In air / free space / vacuum :

$$F = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r^2}$$

where ε_0 = permittivity of free space

$$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/Nm^2$$

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ Nm^2/C^2 = k$$

$$\boxed{F = \frac{k\, q_1 q_2}{r^2}}$$

Define one Coulomb :

One coulomb is that charge which when separated from ~~by~~ another equivalent charge ~~from~~ by a distance of one meter in vacuum such that there is a force of ~~attraction~~ repulsion of 9×10^9 N

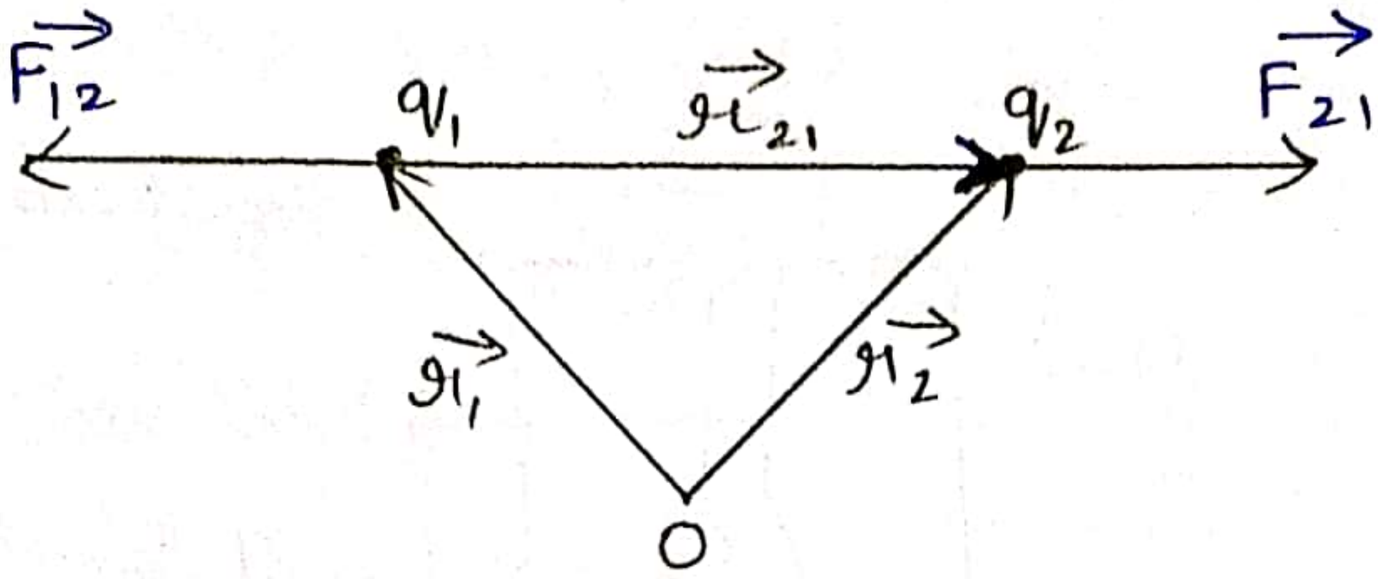
Dielectric constant or relative permittivity :

It is defined as the ratio of permittivity of medium to permittivity of free space

or

Force between 2 charges in vacuum to force between the same two charges separated by the same distance in ~~that~~ a medium.

Coulomb's law in vector form:



$\vec{r_1}$: position vector of q_1

$\vec{r_2}$: position vector of q_2

$\vec{F_{12}}$: Force on q_1 due to q_2

$\vec{F_{21}}$: Force on q_2 due to q_1

$$\vec{r_{21}} = \vec{r_2} - \vec{r_1}$$ (From triangle law of vector addition)

$$\vec{r_{12}} = \vec{r_1} - \vec{r_2}$$

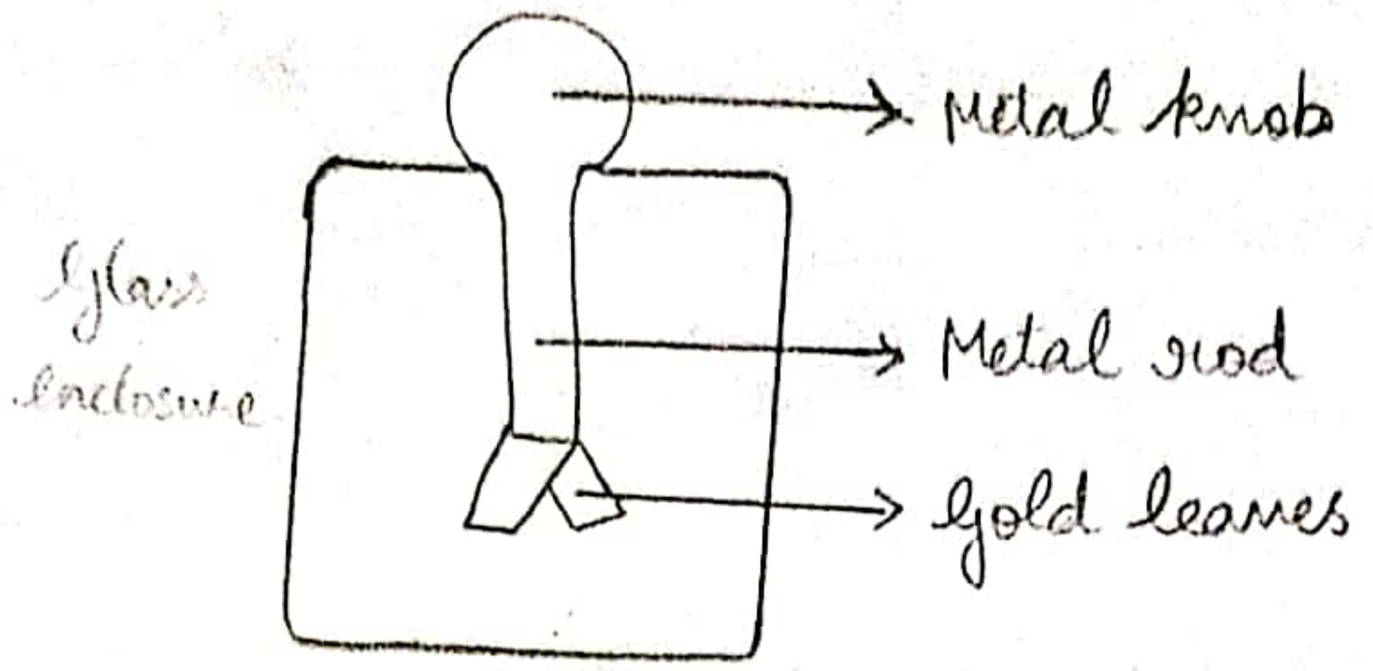
$$\vec{F_{21}} = \frac{k q_1 q_2}{r_{21}^2} \hat{r}_{21}$$

$$\vec{F_{12}} = \frac{k q_1 q_2}{r_{12}^2} \hat{r}_{12}$$

$$\vec{F_{12}} = -\vec{F_{21}}$$

∴ Coulomb's law follows newton's Third law.

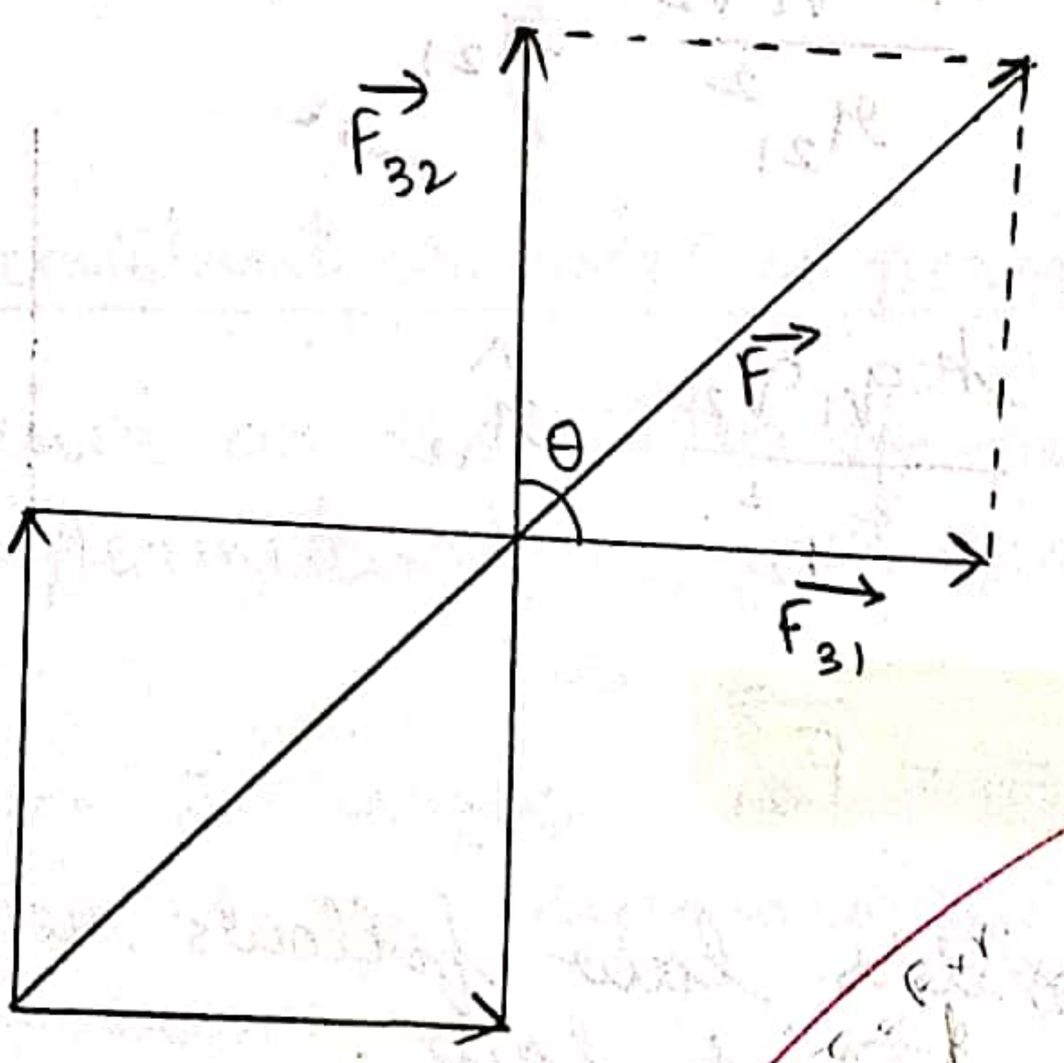
Gold leaf Electroscope



It is used to determine the amount of charge on a charged body. Degree of divergence of the leaves tells us about the amount of charge present on a body.

Principle of superposition of forces:

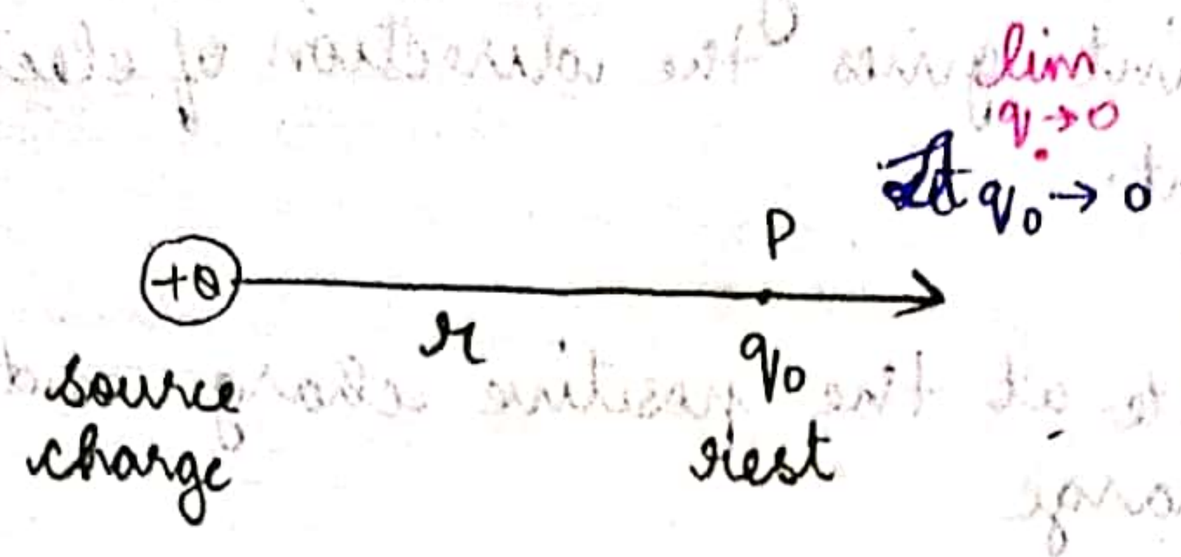
When many charges are present in a system, the total force on a charge is the vector sum of forces on the charge due to individual charges.



Electric field:

Region around a charge where its influence can be felt is called electric field.

Electric field strength / intensity



$$\boxed{\vec{E} = \frac{\vec{F}}{q_0}}$$

It is defined as force per charge experienced by a small positive test charge kept at that point in the electric field.

$$\vec{F} = \frac{kQq_0}{r^2}$$

$$\vec{E} = \frac{kQ\cancel{q_0}}{r^2} \times \frac{1}{\cancel{q_0}} = \frac{kQ}{r^2}$$

$$\vec{E} = \frac{kQ}{\vec{r}^2} \text{ or } \frac{kQ}{r^2}\hat{r}$$

SI unit = N/C

dimensions = $[MLT^{-3}A^{-1}]$

Electric field lines :

1. These are imaginary lines drawn in an electric field which shows the direction of force on a positive test charge kept at that point in the electric field.

2. Tangent drawn through the electric field line at a given point gives the direction of electric field at that point.

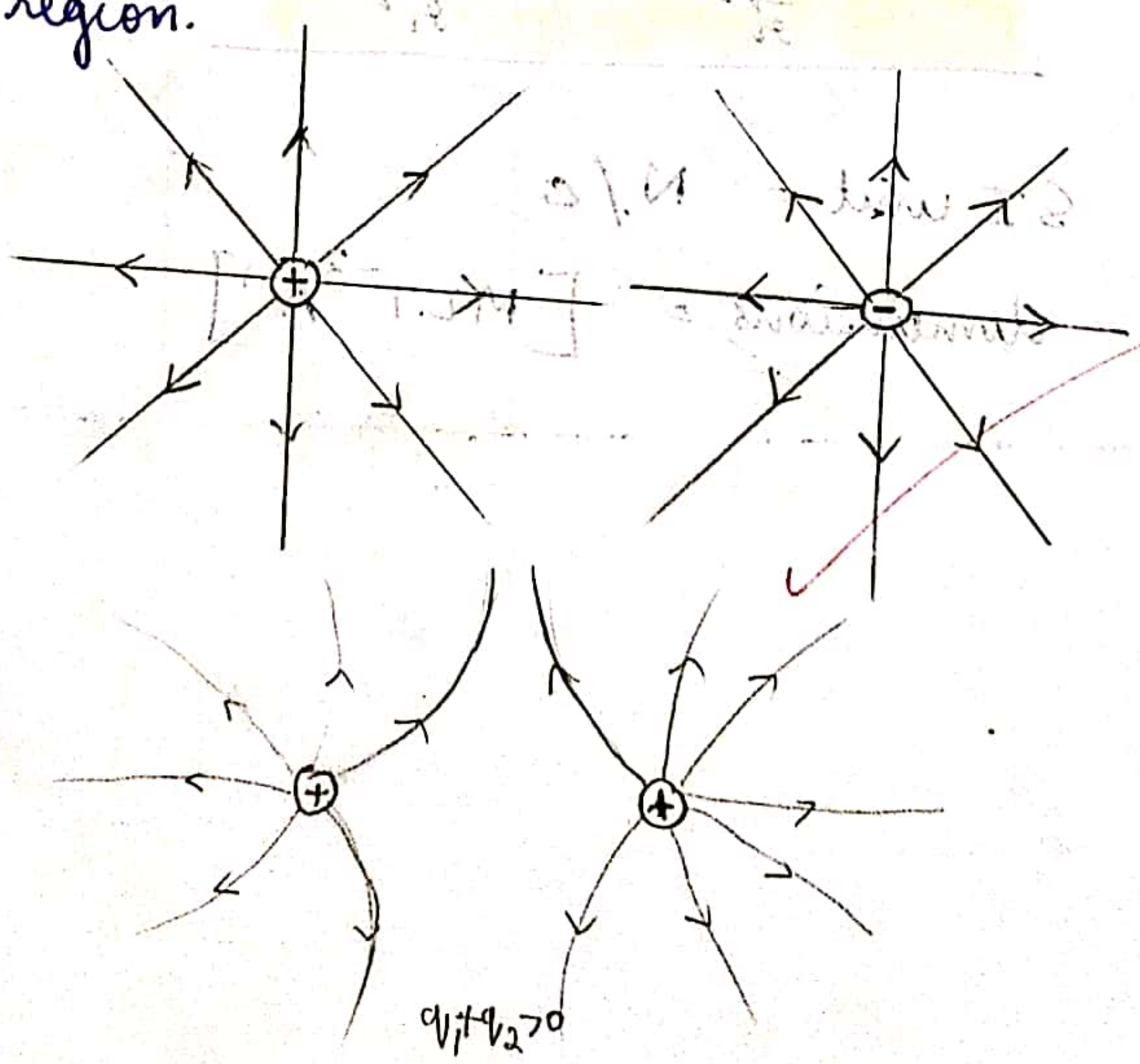
3. They originate at the positive charge and end at the negative charge

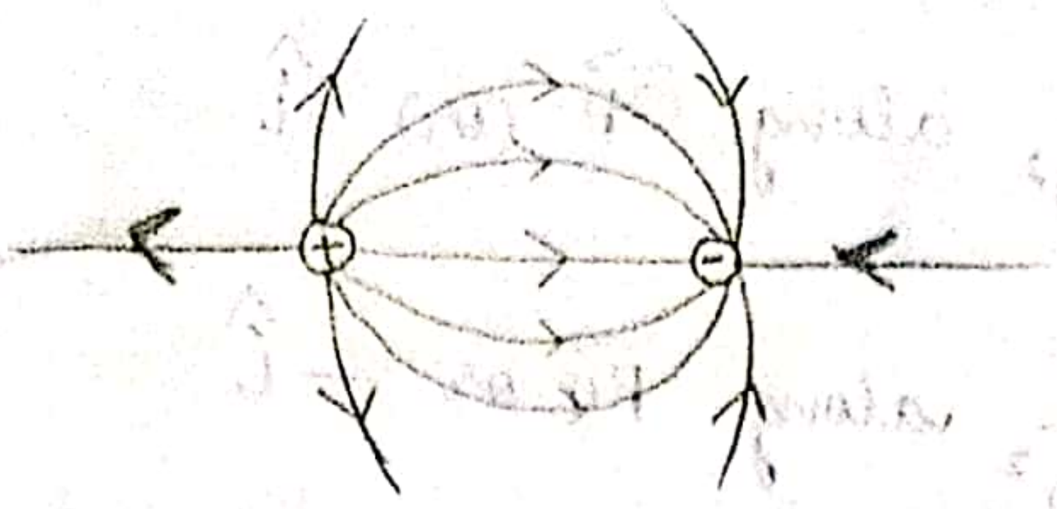
4. They do not intersect (at the point of intersection 2 tangents can be drawn showing 2 directions of electric field which is not possible.)

5. They are continuous without break (at the point of break infinite no. of tangents can be drawn showing infinite directions of electric field, which is not possible.)

6. They do not form closed loops (~~because these are~~

7. No. of field lines passing through unit area is directly proportional to electric field strength in that region.





$q_1 + q_2 = 0$

ELECTRIC DIPOLE :

Pair of equal and opposite charges separated by a small distance is called an electric dipole.

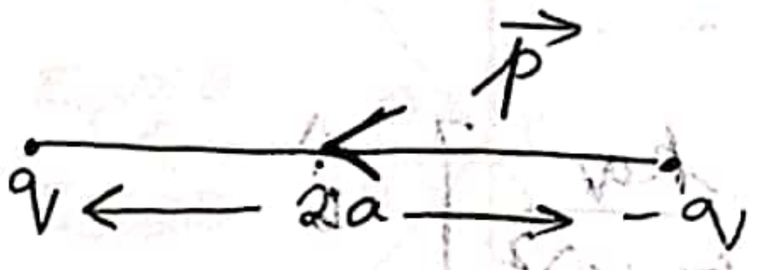
Electric dipole moment is defined as the product of one of the charges and the distance of separation between the two charges.

$$\vec{p} = q \cdot 2\vec{a}$$

It is a vector quantity, direction (-ve to +ve) from negative charge to positive charge

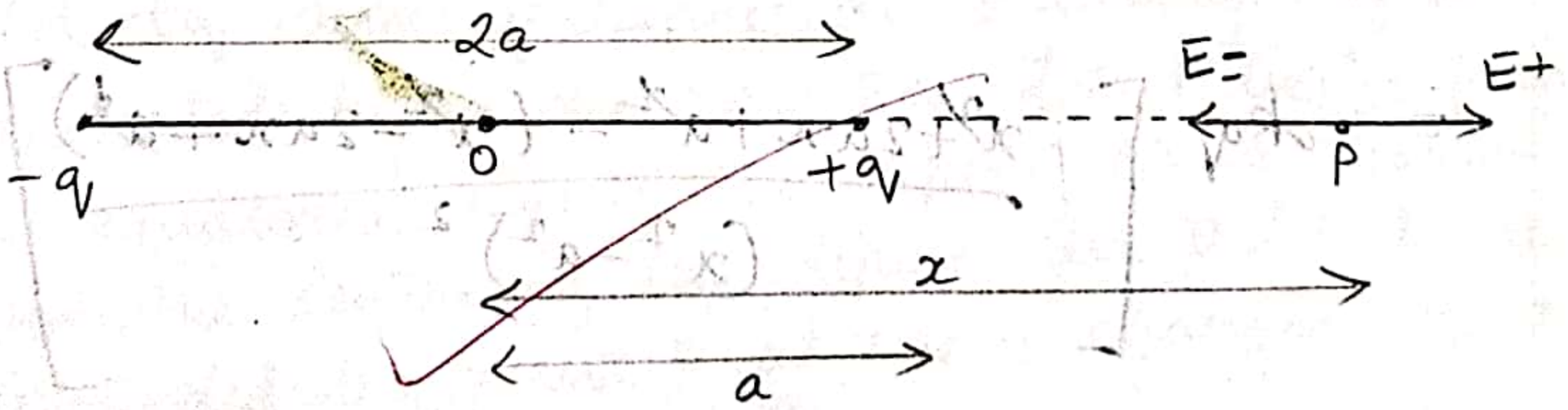
SI unit = C m

Dimensions = [LTA]



Axial field :

Electric field at a point on the ~~axe~~ axis of a electric field.



Let +q and -q constitute the 2 charges of a dipole separated by a distance 2a. Let P be a point along the axis of the dipole at a distance x from the centre. Electric field at point P due to +q is

$$\vec{E}_+ = \frac{kq}{(x-a)^2} \text{ along } \vec{OP} \text{ (or) } \hat{i}$$

$$\vec{E}_- = \frac{kq}{(x+a)^2} \text{ along } \vec{PO} \text{ or } -\hat{i}$$

$$\vec{E} = \vec{E}_+ + \vec{E}_-$$

$$= |\vec{E}_+| - |\vec{E}_-|$$

$$= \frac{kq}{(x-a)^2} - \frac{kq}{(x+a)^2}$$

$$= kq\left(\frac{1}{(x-a)^2} - \frac{1}{(x+a)^2}\right)$$

$$= kq$$

$$= \frac{kq}{(x-a)^2}\hat{i} + \frac{kq}{(x+a)^2}(-\hat{i})$$

$$= \left[\frac{kq}{(x-a)^2} - \frac{kq}{(x+a)^2}\right]\hat{i}$$

$$= kq\left[\frac{(x+a)^2 - (x-a)^2}{(x^2-a^2)^2}\right]\hat{i}$$

$$= kq\left[\frac{x^2+2ax+a^2-(x^2-2ax+a^2)}{(x^2-a^2)^2}\right]\hat{i}$$

$$= \frac{kq\,(+4ax)}{(x^2-a^2)^2}\hat{i} = \frac{kq\cdot 2a\cdot 2x}{(x^2-a^2)^2}\hat{i}$$

$$\vec{E} = \frac{k\vec{p}\cdot 2x}{(x^2-a^2)^2}$$

Axial field and dipole moment are parallel to each other.

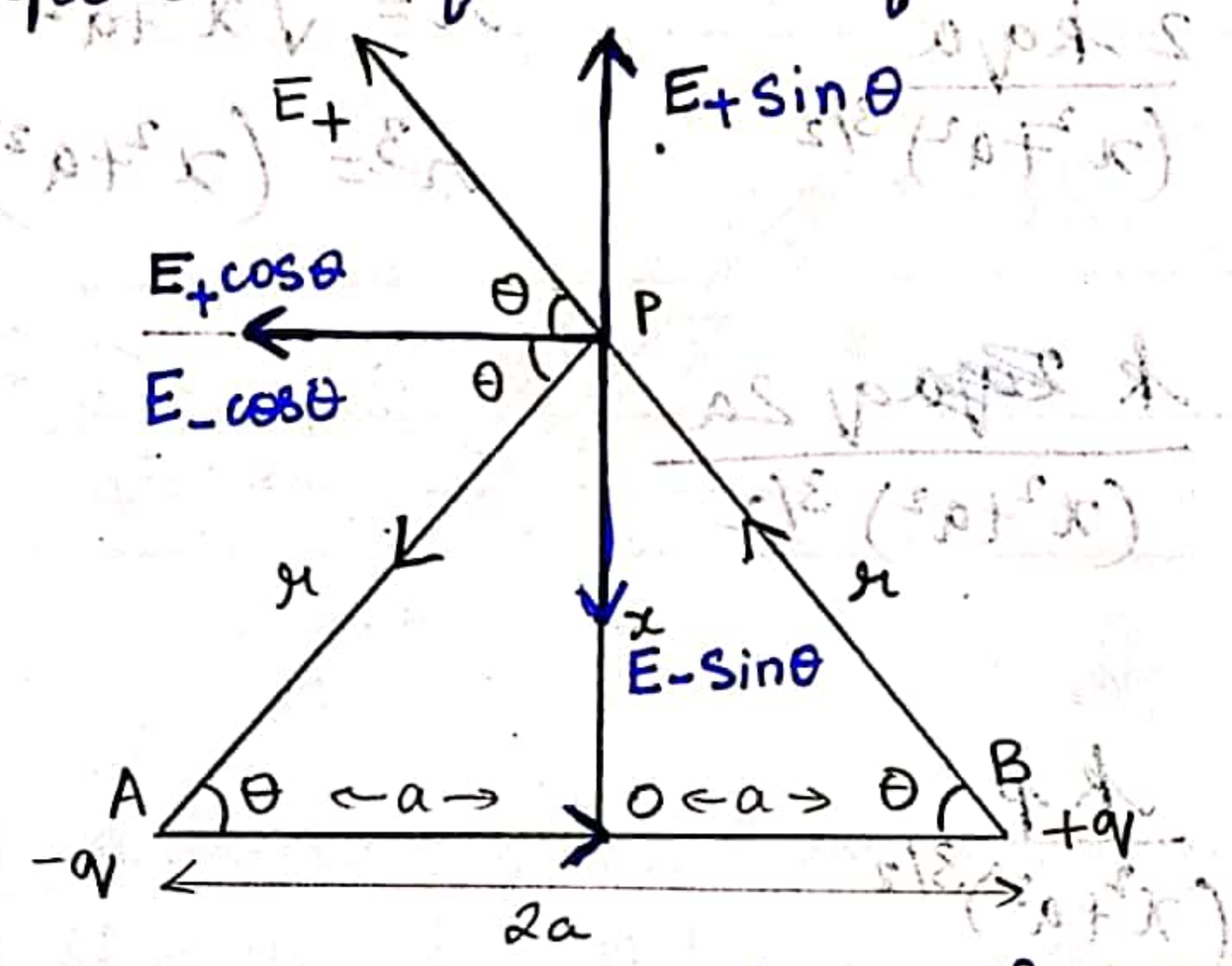
for a short dipole $x >>> a$

$\therefore a \approx 0$ (negligible)

$$\vec{E} = \frac{k\vec{p}\cdot 2x}{(x^2)^2}$$

$$\vec{E} = \frac{2k\vec{p}}{x^3} \rightarrow \boxed{\vec{E} \propto \frac{1}{x^3}}$$

Electric field at a point on the equatorial line of the dipole — Equatorial field.



Let +q and -q represent 2 charges of a dipole separated by a distance 2a. Let P be a point on the equatorial line of the dipole at a distance x from the centre of the dipole. Let θ be the angle subtended by point P at the 2 charges -q and +q

$$|\vec{E_+}| = \frac{kq}{r^2} \qquad |\vec{E_-}| = \frac{kq}{r^2}$$

$$E = E_+ \cos\theta + E_- \cos\theta$$

$$= 2E_+ \cos\theta$$

$$= \frac{2kq}{r^2}\cos\theta \qquad \left(\cos\theta = \frac{a}{r};\right)$$

$$E = \frac{2kq}{r^2} \times \frac{a}{r}$$

$$E = \frac{2kqa}{r^3}$$

$$r^2 = x^2 + a^2$$

$$r = \sqrt{x^2+a^2}$$

$$r^3 = (x^2+a^2)^{3/2}$$

$$E = \frac{2kqa}{(x^2+a^2)^{3/2}}$$

$$= \frac{k\,2qa}{(x^2+a^2)^{3/2}}$$

$$= \frac{kp}{(x^2+a^2)^{3/2}}$$

$$\vec{E} = \frac{-k\vec{p}}{(x^2+a^2)^{3/2}}$$

Equatorial field and dipole moment are opp. to each other.

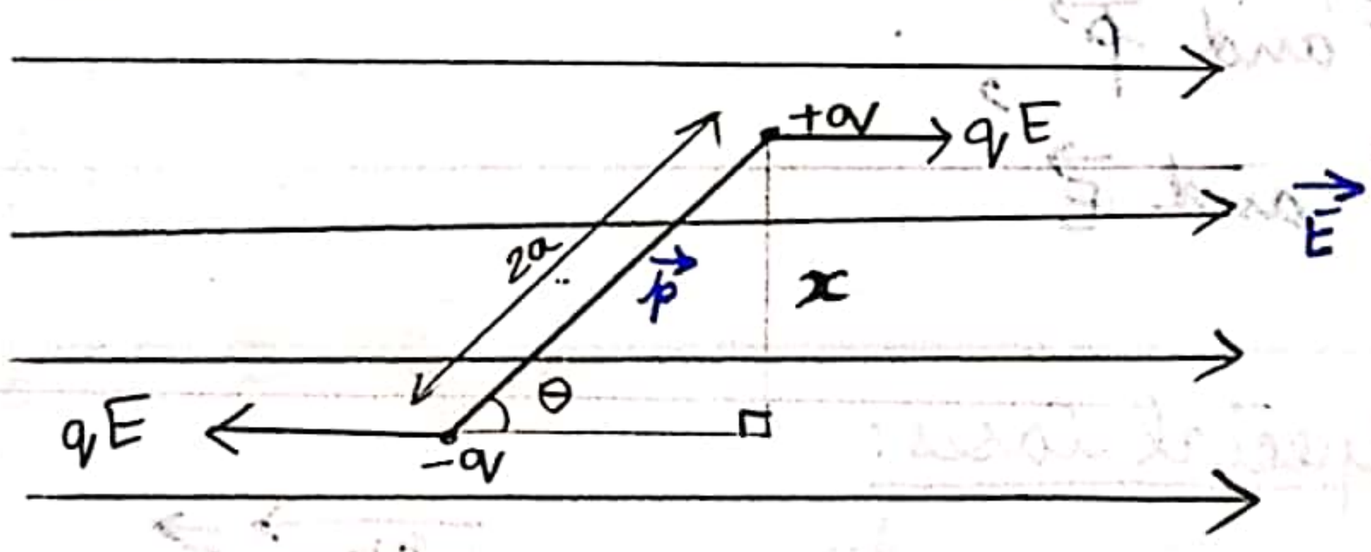
For a short dipole : $x >>> a$.

$$\vec{E} = -\frac{k\vec{p}}{x^3}, \quad E \propto \frac{1}{x^3}.$$

AXIAL FIELD	EQUATORIAL FIELD
• Parallel to dipole moment	Antiparallel to dipole moment
• Chosen point is not equidistant from O	Chosen point is equidistant
• $\vec{E}_{axial} = 2\vec{E}_{equatorial}$ (when it is a short dipole) $\Rightarrow x >> a$.	$\vec{E}_{equatorial} = \frac{1}{2}\vec{E}_{axial}$

Similarity : Both are inversely proportional to r^3.

Torque on a dipole placed in a uniform external electric field



Consider a dipole placed in a uniform external field as shown. Let θ be the angle between the dipole moment 'p' and the electric field 'E'.

Force on +q : $\vec{F}_+ = q\vec{E}$

Force on −q : $\vec{F}_- = -q\vec{E}$

As net force is zero, there is no translatory motion in the dipole.

But the line of action of 2 forces do not coincide, hence they constitute a couple.
Couple gives rise to ~~rota~~ torque
Rotational motion

Torque = one of the forces × ⊥^r dist.

$$= qE \times x$$

$$= qE \times 2a \sin\theta$$

$$= q \cdot 2a \cdot E \sin\theta$$

$$\tau = pE \sin\theta$$

$$\vec{\tau} = \vec{p} \times \vec{E}.$$

Direction of torque is given by right hand thumb rule or cross product rule. It is perpendicular to both $\vec{p}$ and $\vec{E}$.

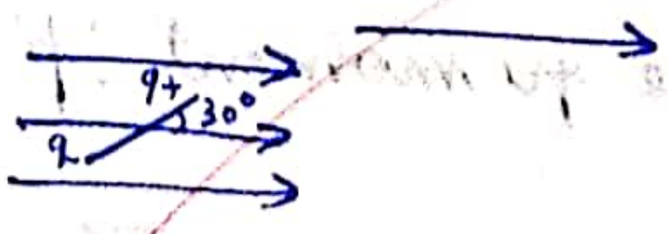
2 pairs of perpendicular vectors are:
$\vec{\tau}$ and $\vec{p}$,
$\vec{\tau}$ and $\vec{E}$

Special Cases:

1) $\theta = 90^\circ$, $\tau = pE$

2) $\theta = 0^\circ$, $\theta = 180^\circ$, $\tau = 0$

3) $\theta = 30^\circ$, $\tau = \frac{pE}{2}$



Work done on rotating a dipole in a uniform external electric field

The work done in rotating a dipole through a small angle $d\theta$ is

$$dW = \tau \, d\theta.$$

If the dipole is rotated from an initial angle θ_1 to a final angle θ_2, Work done is given as,

$$W = \int_{\theta_1}^{\theta_2} \tau \, d\theta$$

$$W = \int_{\theta_1}^{\theta_2} PE \sin\theta \, d\theta$$

$$= PE \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta$$

$$W = -PE \left(\cos\theta\right)_{\theta_1}^{\theta_2}$$

$$= -PE \left[\cos\theta_2 - \cos\theta_1\right]$$

$$W = PE \left[\cos\theta_1 - \cos\theta_2\right]$$ — WD by field

potential energy is calculated assuming that initially the dipole is $\perp^r$ to the field and finally at some angle θ.

$$\therefore \theta_1 = 90^\circ, \quad \theta_2 = \theta.$$

$$W = PE\left(\cos\theta_1 - \cos\theta_2\right)$$

$$= PE\left(\cos 90^\circ - \cos\theta\right)$$

$$U = -PE\cos\theta \quad \text{or} \quad U = -\vec{P}\cdot\vec{E}$$

Special Cases

1) $\theta = 0^\circ$, $U = -PE$, $\tau = 0$ $\Rightarrow$ stable equilibrium
(P.E least, $\tau = 0$)

2) $\theta = 180^\circ$, $U = +PE$, $\tau = 0$ $\Rightarrow$ Unstable equilibrium.

3) $\theta = 90^\circ$, $U = 0$, $\tau = max$ $\Rightarrow$ not in equilibrium

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Continuous charge distribution

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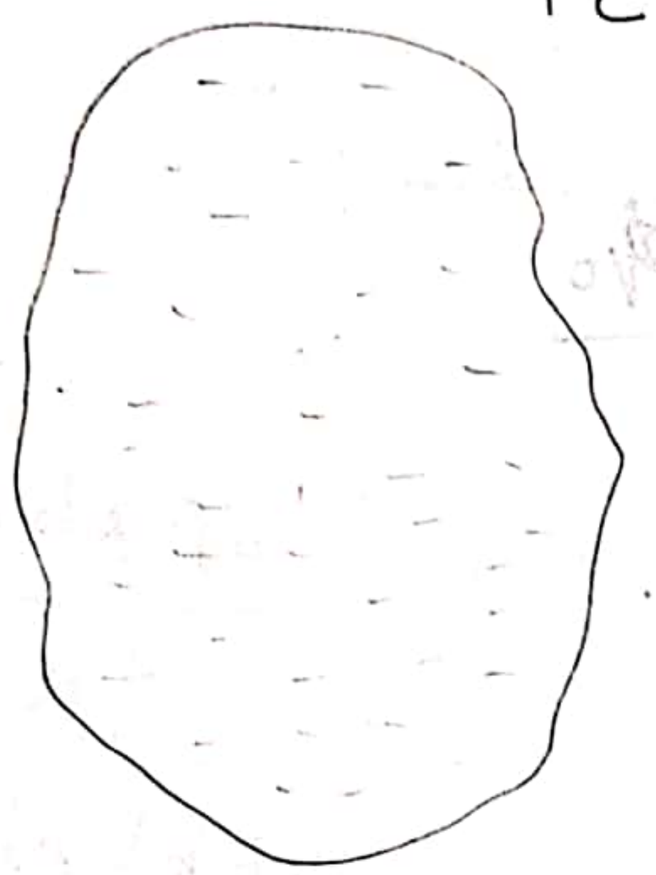
1 C



$Q = ne$

$$n = \frac{Q}{e} = \frac{1}{1.6\times10^{-19}} \times 1C = 6\times10^{18} \text{ electrons}$$

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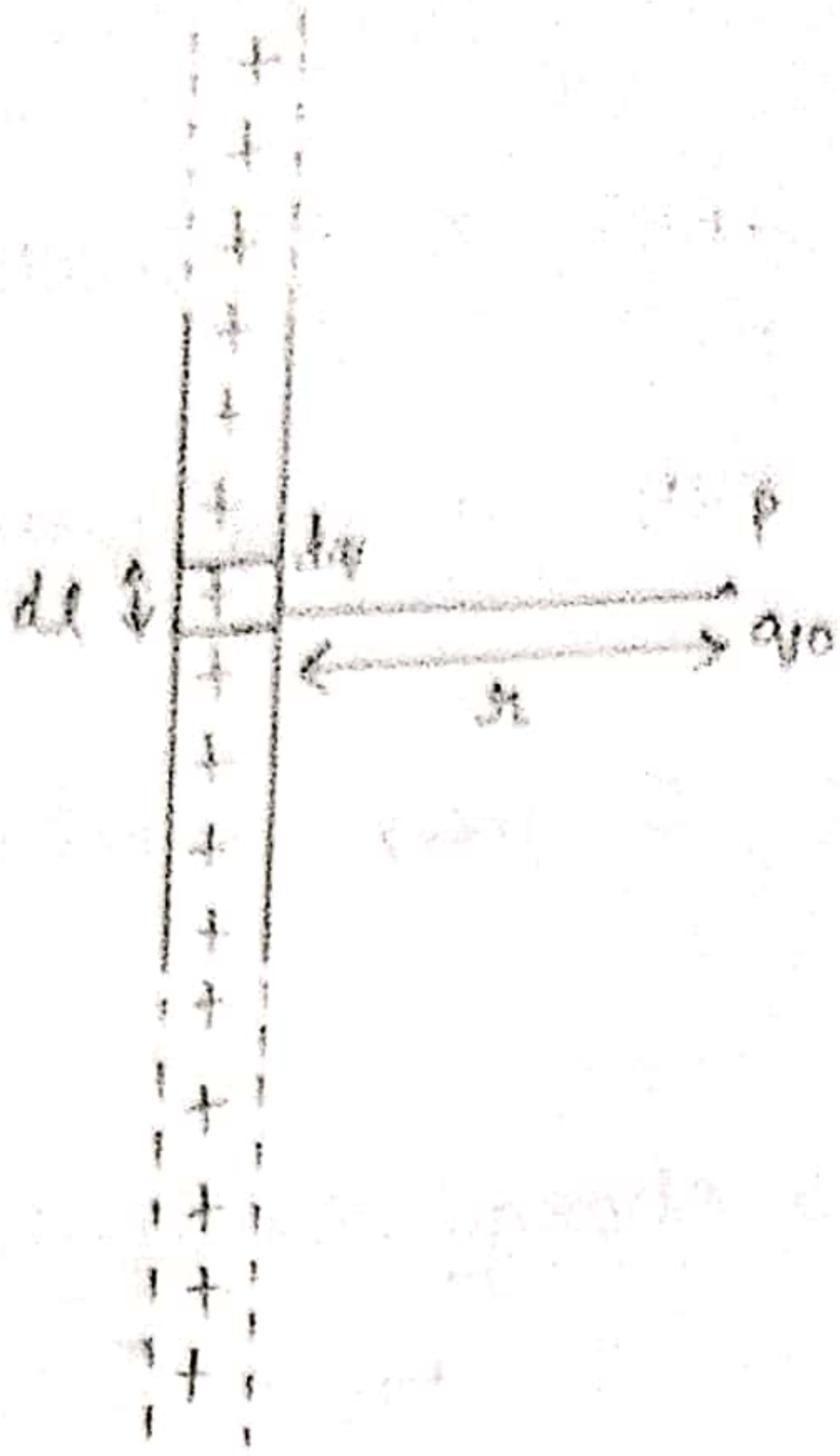
a) Linear charge distribution (λ)

If the charges are distributed over a thin line or a thin wire, it is called as linear charge distribution.

* Linear charge density: It is defined as charge per unit length. $\lambda = Q/l$

SI unit = C/m dimensions = $[M^{-1}TA]$



$$dF = \frac{k\, dq\, q_0}{r^2}$$

$$\lambda = dq/dl.$$

$$dq = \lambda dl$$

$$dF = \frac{k \lambda dl\, q_0}{r^2}$$

$$\vec{F} = \int_0^L \frac{k \lambda dl\, q_0}{r^2}\, \hat{r}$$

$$\boxed{\vec{E} = \frac{\vec{F}}{q_0} = \int_0^L \frac{k \lambda dl}{r^2}\, \hat{r}}$$

b) Surface charge distribution (σ)

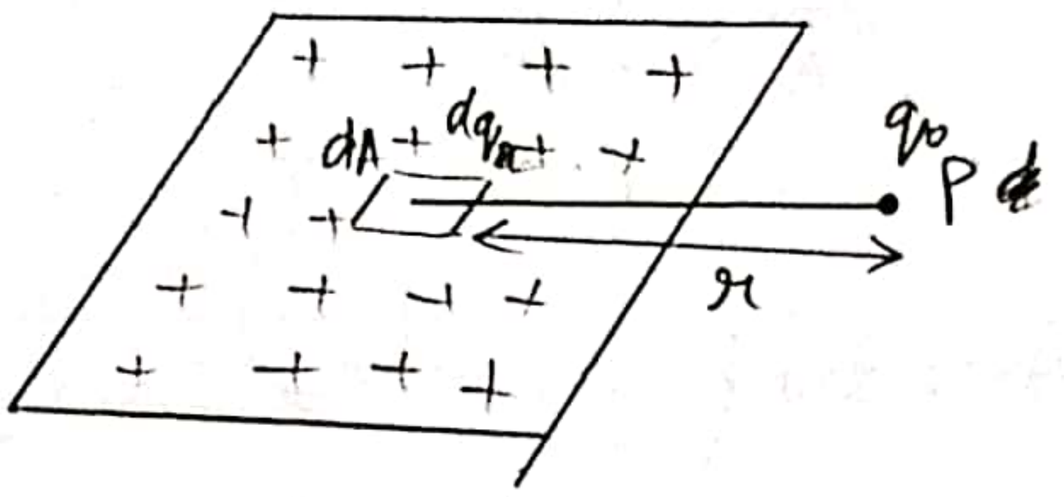
If the charges are distributed over a surface, it is called surface charge distribution

Surface charge density : Charge per unit area.

$\sigma = \frac{Q}{A}$, SI unit = C/m^2

Dimensions : $[M^{-2}TA]$

~~$dF = k\,dq$~~



$$dF = \frac{k\,dq\,q_0}{r^2}$$

$$\sigma = dq/dA.$$

$$dq = \sigma\,dA.$$

$$dF = \frac{k\sigma\,dA\,q_0}{r^2}$$

$$\vec{F} = \int_0^L \frac{k\sigma\,dA\,q_0}{r^2}\hat{r}$$

$$\boxed{\vec{E} = \frac{\vec{F}}{q_0} = \int_0^L \frac{k\sigma\,dA}{r^2}\hat{r}}$$

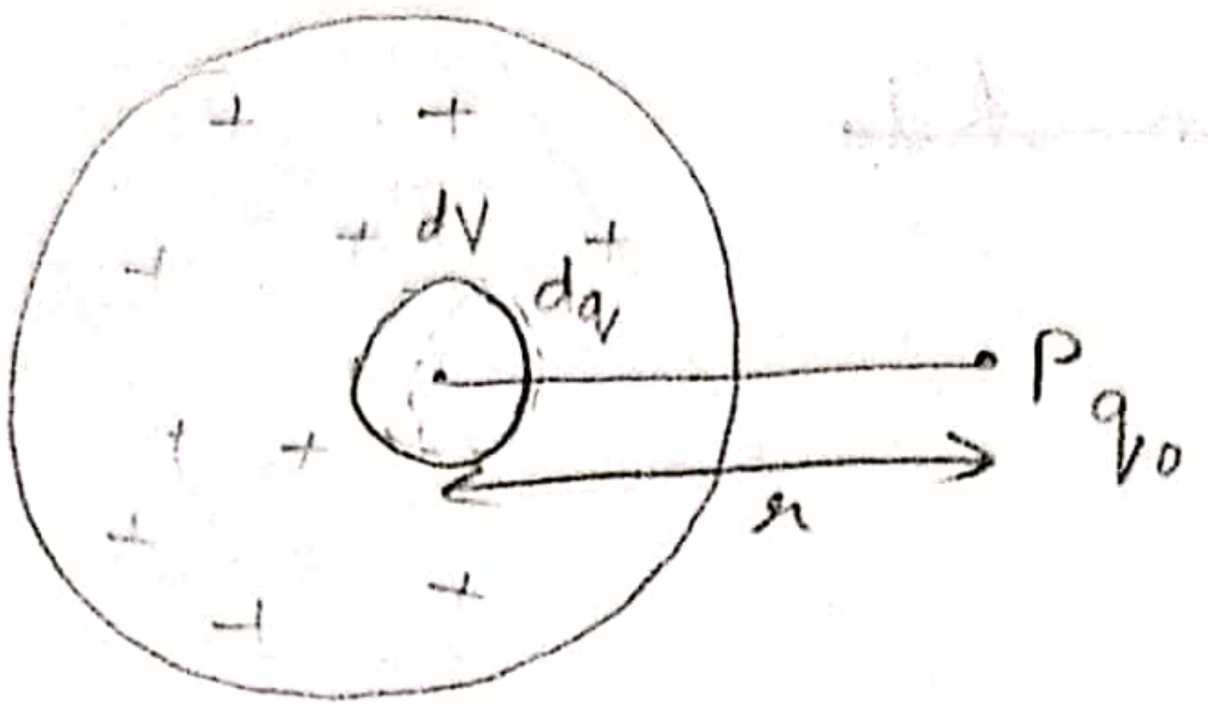
c) Volume charge distribution (ρ)

If the charges are distributed over some volume space, it is called volume charge distribution.

Volume charge density:

It is defined as charge per unit ~~long~~ volume

$$\rho = \frac{Q}{V} = C/m^3.$$



$$dF = \frac{k\, dq\, q_0}{r^2}$$

$$\rho = \frac{dq}{dv}$$

$$dq = \rho\, dv.$$

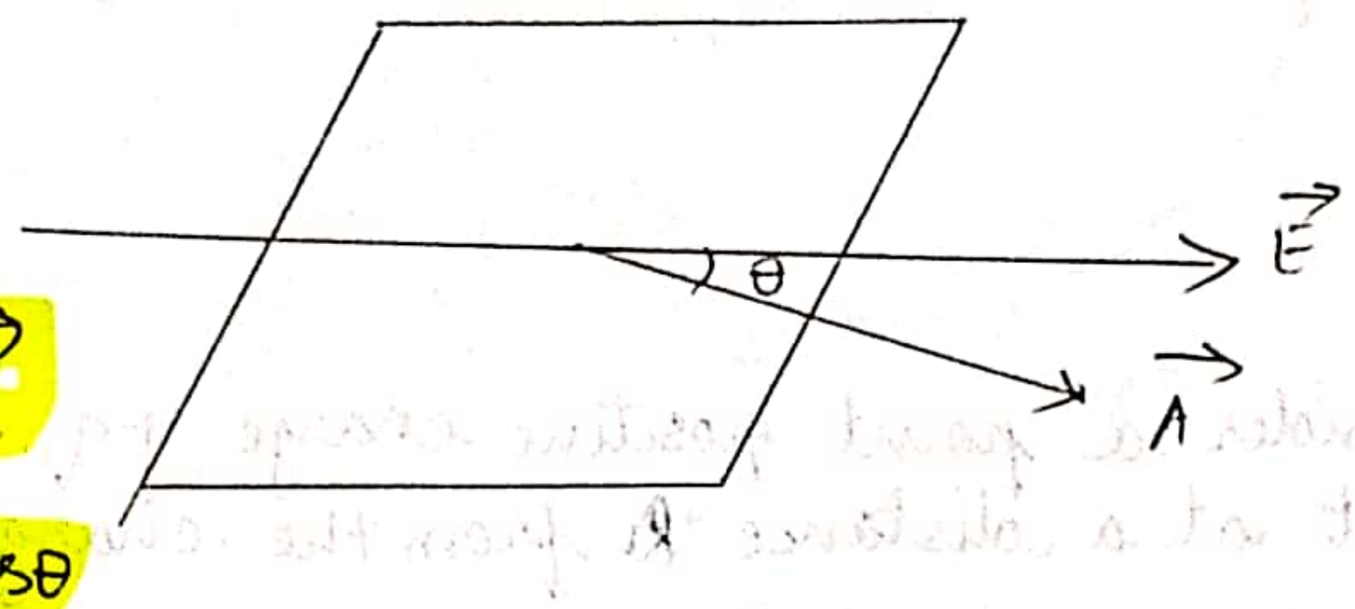
$$dF = \frac{k \rho\, dv\, q_0}{r^2}$$

$$\vec{F} = \int_0^2 \frac{k \rho\, dv\, q_0}{r^2}\, \hat{r}$$

$$\boxed{\vec{E} = \frac{\vec{F}}{q_0} = \int_0^4 \frac{k \rho\, dv}{r^2} \cdot \hat{r}}$$

ELECTRIC FLUX :

Total no. of electric field lines crossing unit Area normal to it is called electric flux.
Mathematically it is the scalar product of electric field and area vector.



$\phi = \vec{E} \cdot \vec{A}$

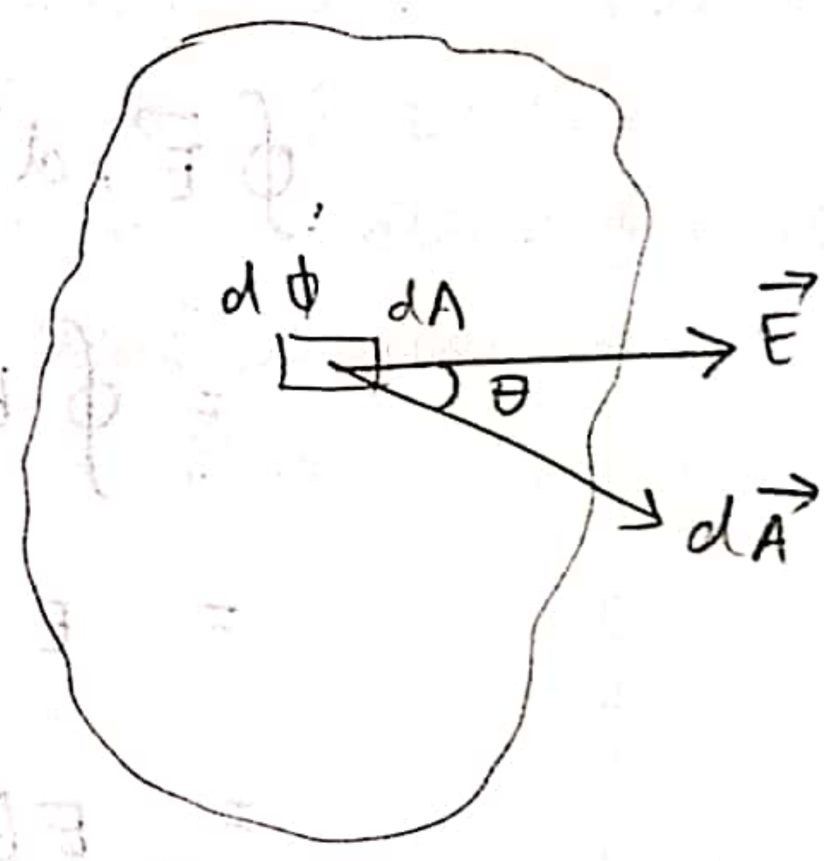
$\phi = EA\cos\theta$

SI unit = Nm^2/C , volt metre

If the area is not symmetrical and electric field is not uniform:

$$d\phi = \vec{E} \cdot d\vec{A}$$

$$\Phi = \int \vec{E} \cdot d\vec{A}$$



GAUSS's LAW:

$$\Phi = \frac{Q}{\varepsilon_0}$$

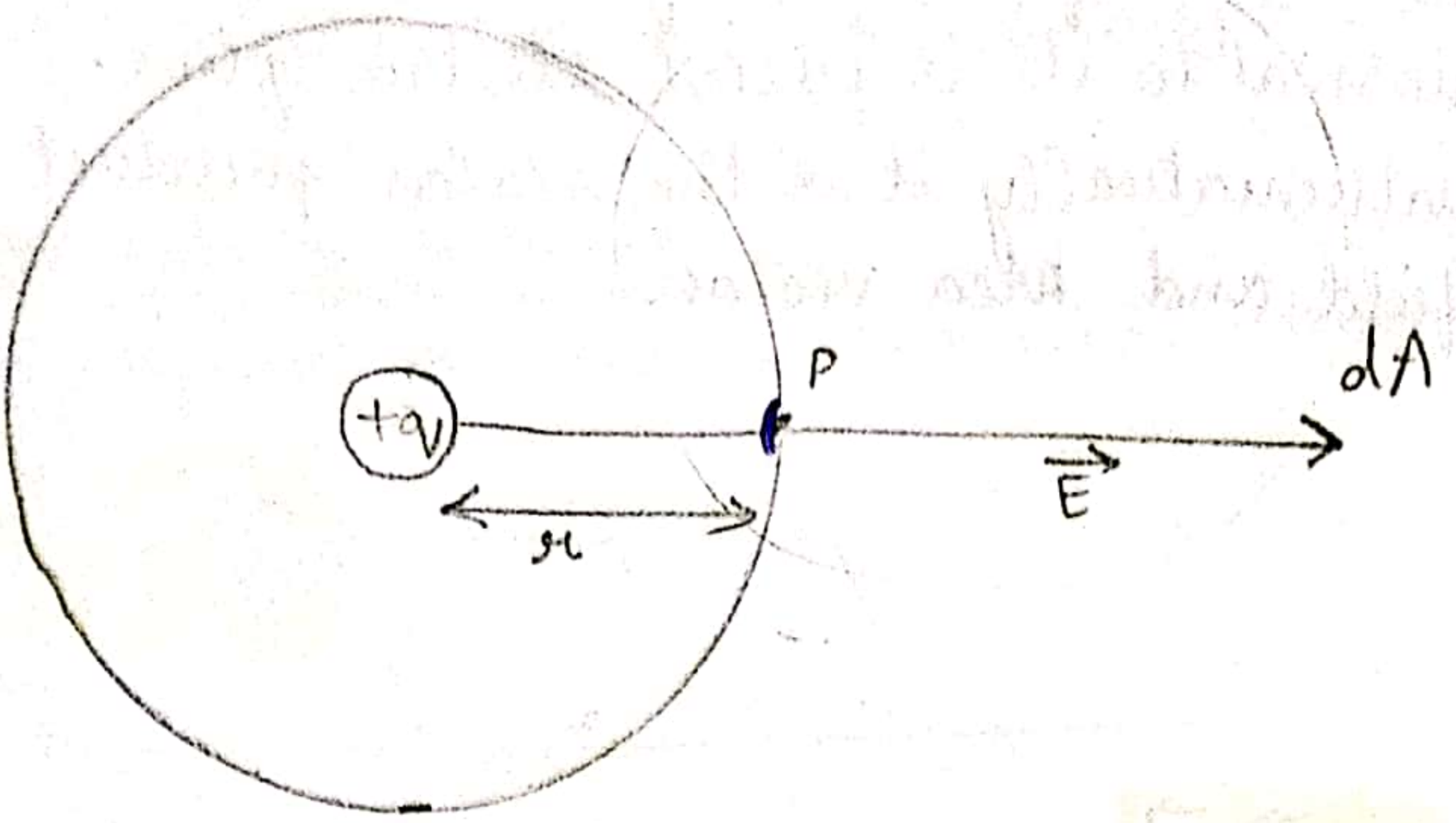
Summation of Area elements in that closed surface

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$ (Provided it is in vacuum)

In medium: $\boxed{\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon}}$ Permittivity in medium.

The total electric flux through a closed surface due to an electric field is equal to $\frac{1}{\varepsilon_0}$ times the total charge enclosed by the surface.

GAUSS'S LAW FROM COULOMB'S LAW



Consider a point positive charge +q. Let P be a point at a distance r from the charge +q. Consider a spherical surface passing through point P ~~passing through po~~ with the charge at centre. Consider a small surface area dA at pt P as shown

Total Flux through the surface is given as:

$$\oint \vec{E} \cdot d\vec{A} = \oint E\,dA \cos\theta$$

$$= \oint E\,dA$$

$$= E \oint dA$$

$$= E(A) = E \times 4\pi r^2$$

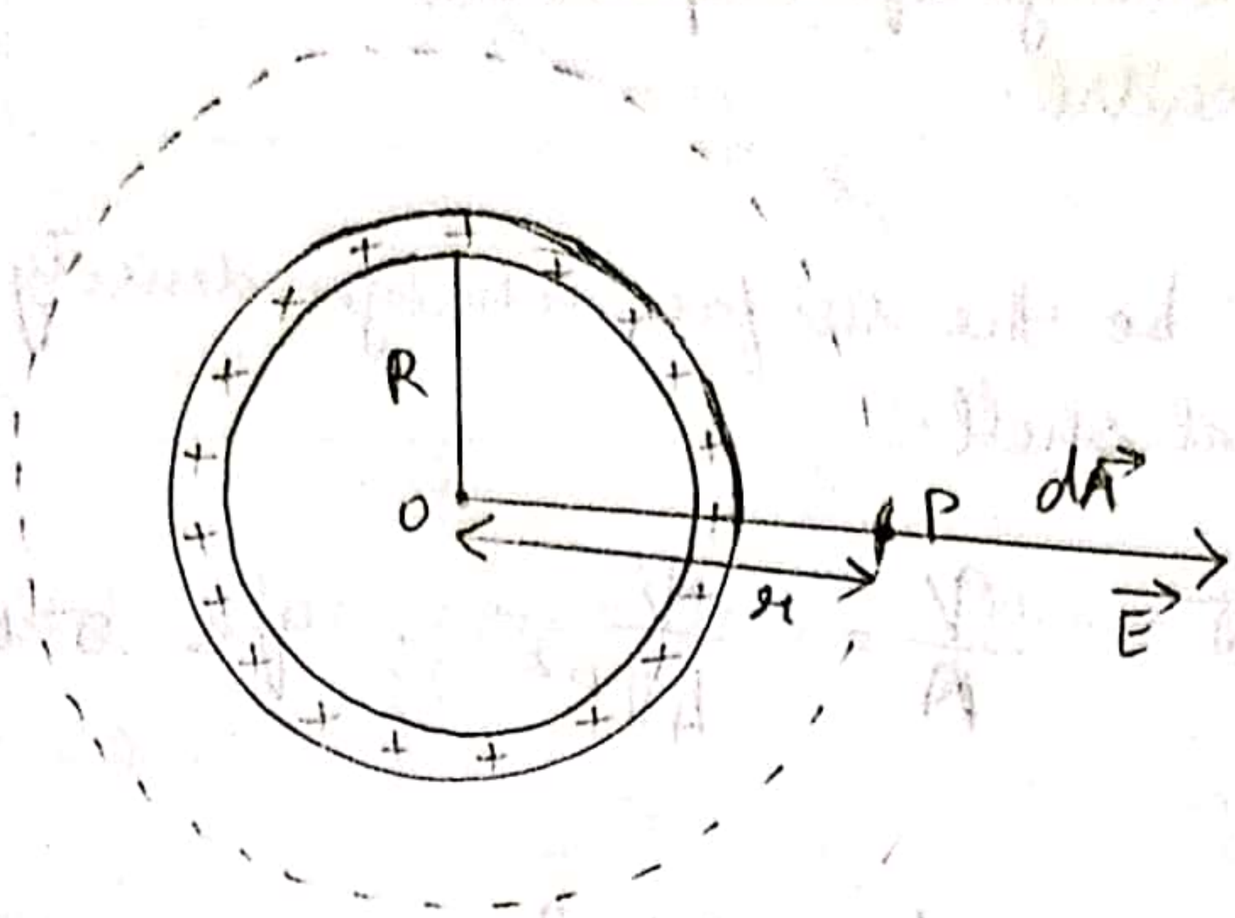
$$= \frac{kq}{\cancel{r^2}} \times 4\pi \cancel{r^2}$$

$$= \frac{1}{4\pi\varepsilon_0} \times q \times \cancel{4\pi} = \frac{q}{\varepsilon_0} = \text{Gauss law.}$$

APPLICATIONS OF GAUSS'S LAW

1) Electric field due to a hollow charged spherical shell.

a) At a point outside the shell $(r > R)$



Consider a hollow spherical shell given positive charge as shown. Let P be a point at a distance r from the centre of the shell. let R be the radius of the shell. To determine electric field at point P, construct a spherical gaussian surface passing through point P as shown.

According to gauss's law

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$

$$\oint \vec{E} \cdot d\vec{A} = \oint E\, dA \cos\theta$$

$$= \oint E\, dA$$

$$= E \oint dA$$

$$= E\, 4\pi r^2$$

$$\frac{Q}{\varepsilon_0} = E \cdot 4\pi r^2$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$\vec{E} = \frac{kq}{r^2}\hat{r}$$

↳ electric field due to a point charge.

As this expression is similar to the electric field due to a point charge, we can say that the entire charge of a spherical shell is concentrated at its centre.

Let σ be the surface charge density of the spherical shell.

$$\sigma = \frac{q}{A} = \frac{q}{4\pi R^2} \; ; \; q = \sigma 4\pi R^2$$

$$E = \frac{k\sigma 4\pi R^2}{r^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\sigma 4\pi R^2}{r^2}$$

$$\vec{E} = \frac{\sigma R^2}{\varepsilon_0 r^2}\hat{r}$$

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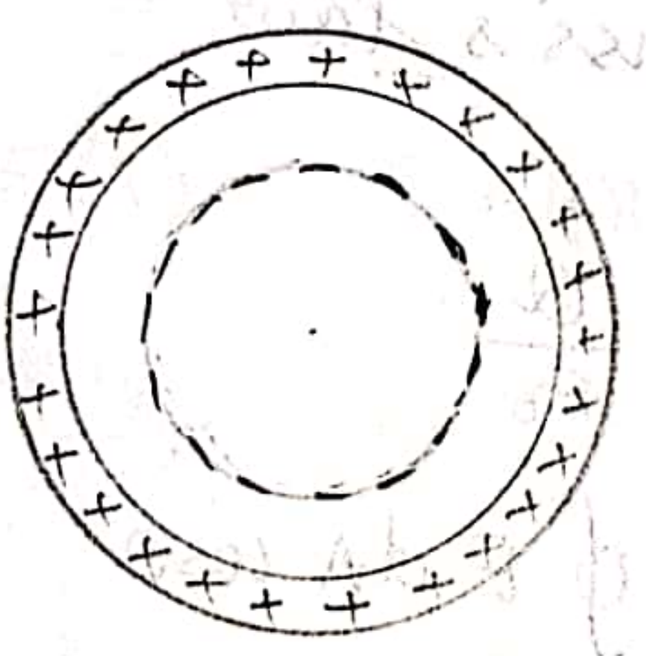
b) At a point inside the shell ($R > r$)



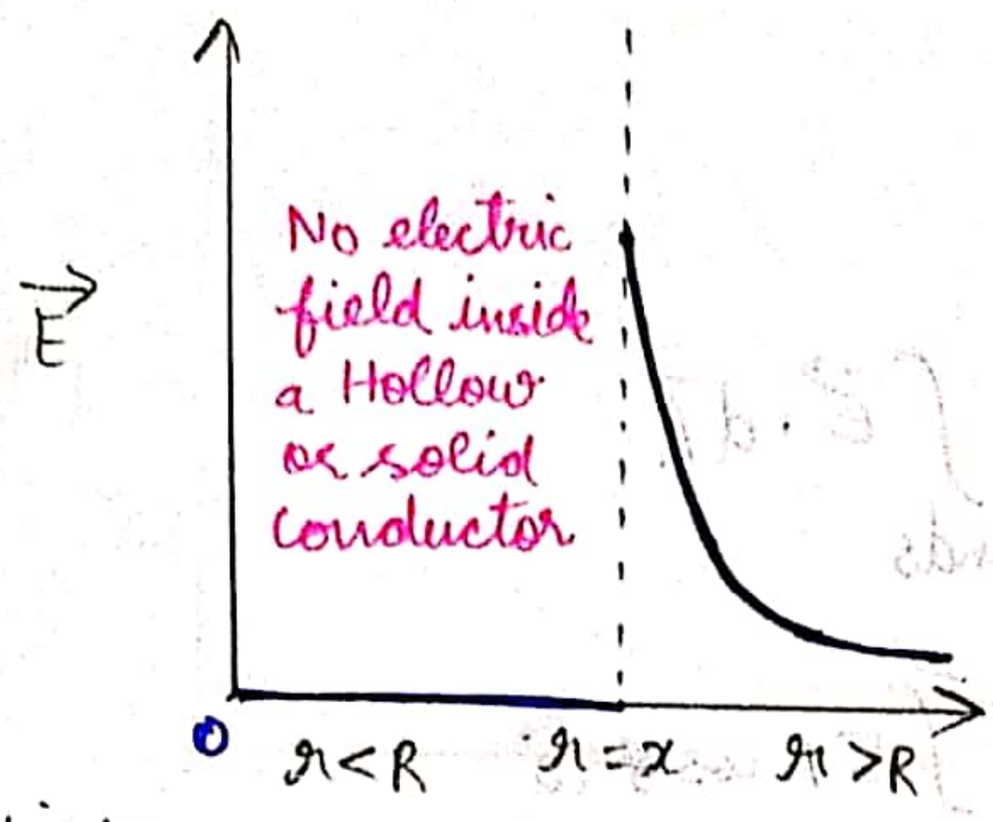
To determine ~~the~~ elec. field at point P, construct a spherical gaussian surface passing point P as shown.

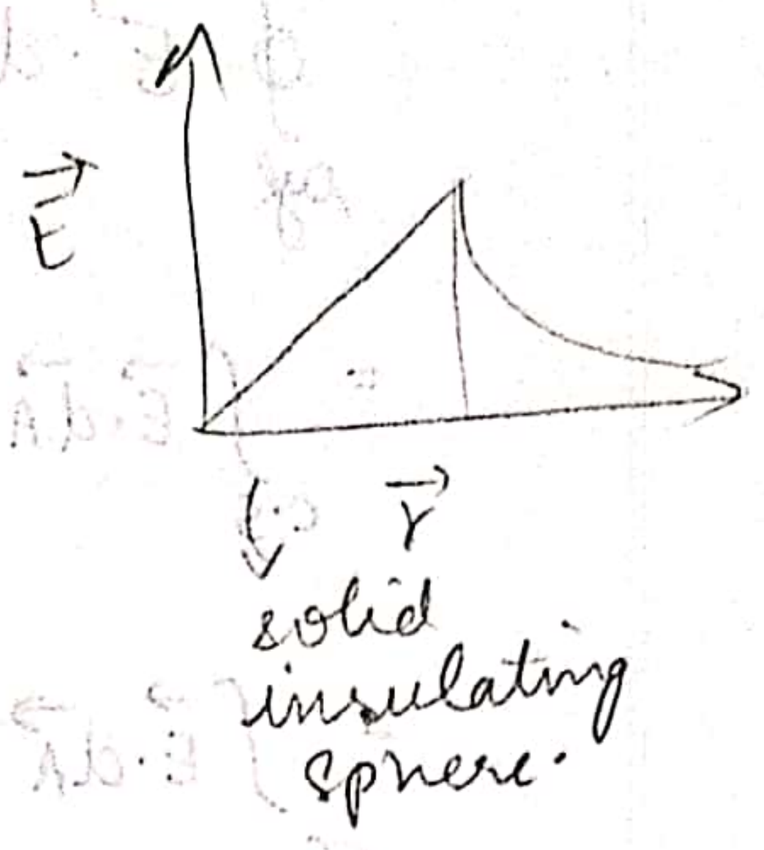
According to Gauss's law

$$\oint \vec{E}\, d\vec{A} = q/\varepsilon_0 . \quad q = 0 .$$

$$\oint \vec{E} \cdot d\vec{A} = 0 , \; d\vec{A} \neq 0 . \quad \therefore \vec{E} = 0 .$$

Variation of electric field with distance





Conducting shell/sphere - solid
hollow

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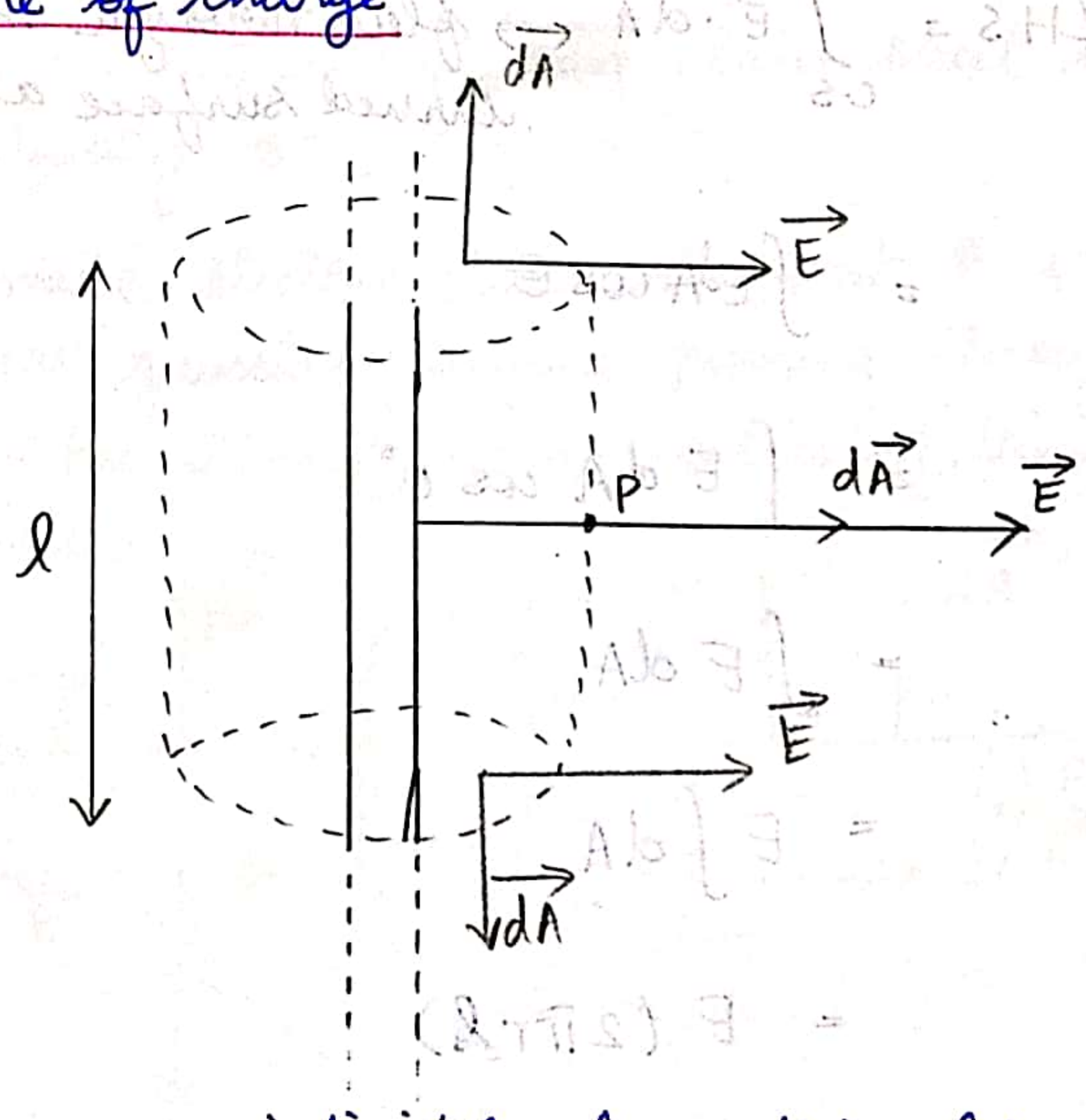
2) Electric field at a point due to an infinitely long ~~than~~ line of charge



Consider an infinitely long, thin line of charge of linear charge density λ. To determine electric field at point P, ~~consider~~ construct a ~~a~~ cylindrical gaussian surface passing through point P. l will be the length of the cylinder

According to Gauss's law:

$$\oint \vec{E}\cdot d\vec{A} = q/\varepsilon_0$$

LHS =

$$\oint_{S} \vec{E}\cdot d\vec{A}$$

$$= \int_{C.S} \vec{E}\cdot d\vec{A} + \int_{2\,ends} \vec{E}\cdot d\vec{A}$$

$$= \int_{C.S} \vec{E}\cdot d\vec{A} + \int E A \cos 90^\circ$$

$$= \int_{C.S} \vec{E}\cdot d\vec{A} + 0$$

LHS = $\int_{C.S} \vec{E}\cdot d\vec{A}$ ⇒ flux through the curved surface alone.

$$= \int E dA \cos\theta$$

$$= \int E\, dA \cos 0^\circ$$

$$= \int E\, dA$$

$$= E \int dA$$

$$= E\,(2\pi r l)$$

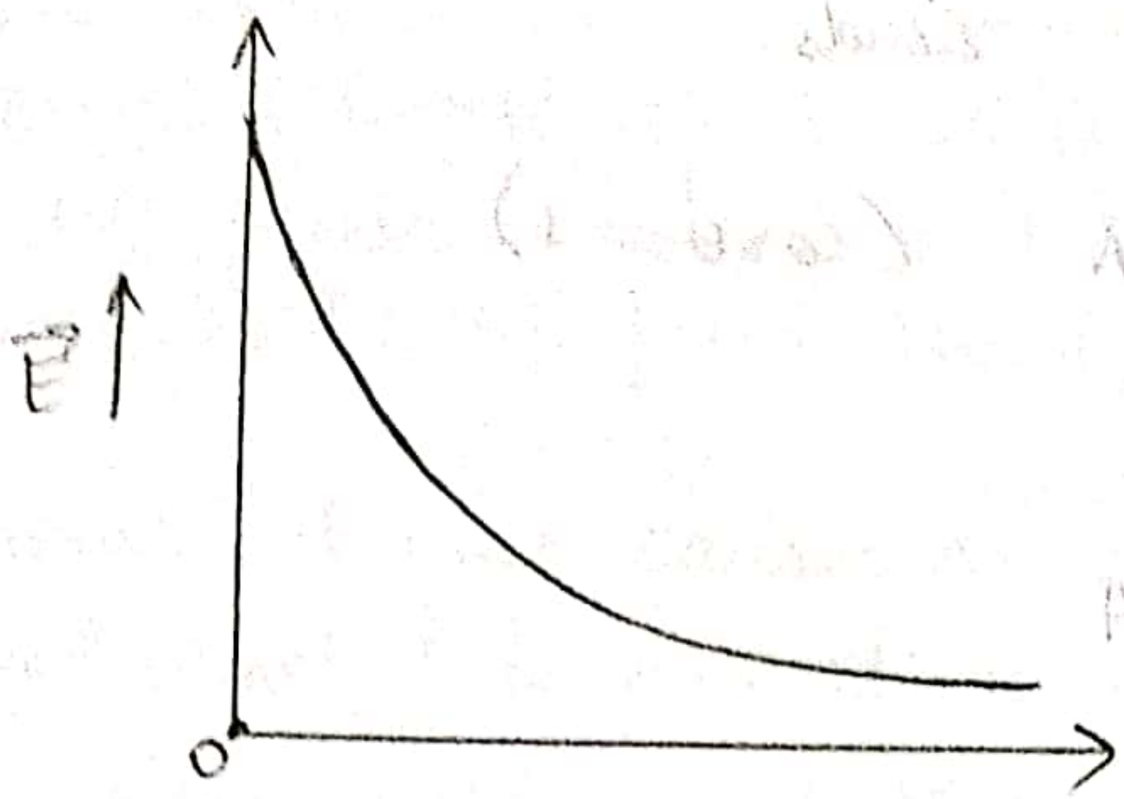
$$2E\pi r l = Q/\varepsilon_0.$$

$$\lambda = q/l.$$

$$2E\pi r l = \frac{\lambda l}{\varepsilon_0}$$

$$\boxed{\vec{E} = \frac{\lambda}{2\pi r \varepsilon_0}\,\hat{r}}$$

Variation of electric field with distance r.



$E \propto \frac{1}{r}$

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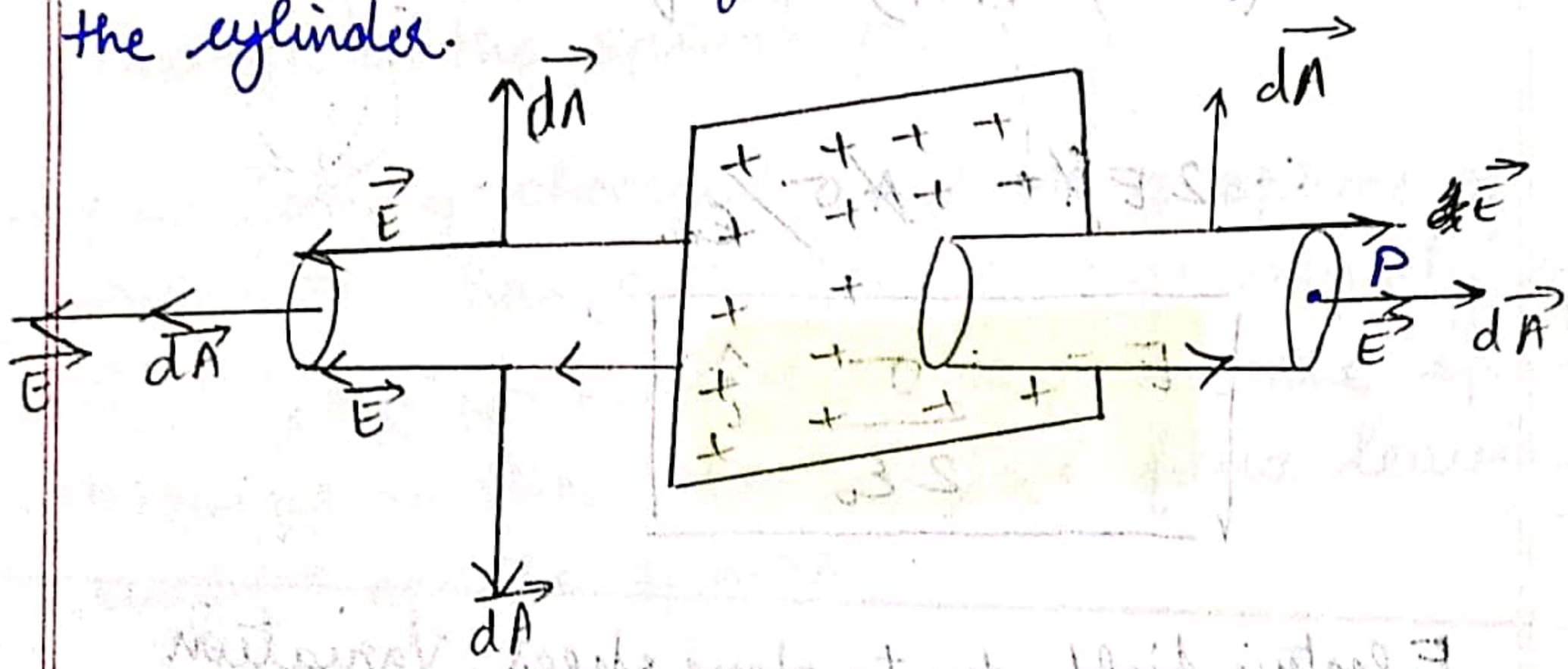
3) Electric field at a point due a plain sheet of charge (σ)

Consider an infinitely long plain sheet of surface charge density σ

To determine electric field at point P, construct a cylindrical gaussian surface passing through point P. Let A be the area of cross section of the two ends of the cylinder.



$$\oint_{cyl} \vec{E}\cdot\vec{dA} = q/\varepsilon_0$$

$$\oint \vec{E}\cdot\vec{dA} = \int_{C.S} \vec{E}\cdot\vec{dA} + \int_{2\,ends} \vec{E}\cdot\vec{dA}$$

$$\int_{C.S} \vec{E}\cdot\vec{dA} = 0 \quad [\vec{E} \perp^r \vec{dA}]$$

$$LHS = \int_{2ends} \vec{E} \cdot \vec{dA} = \int_{2ends} E\, dA \cos\theta$$

$$= \int_{2ends} E\, dA \qquad (\cos\theta = 1)$$

$$= E \int_{2ends} dA$$

$$= E \cdot 2A$$

$$= 2EA.$$

$$RHS = q/\varepsilon_0 = LHS = 2EA.$$

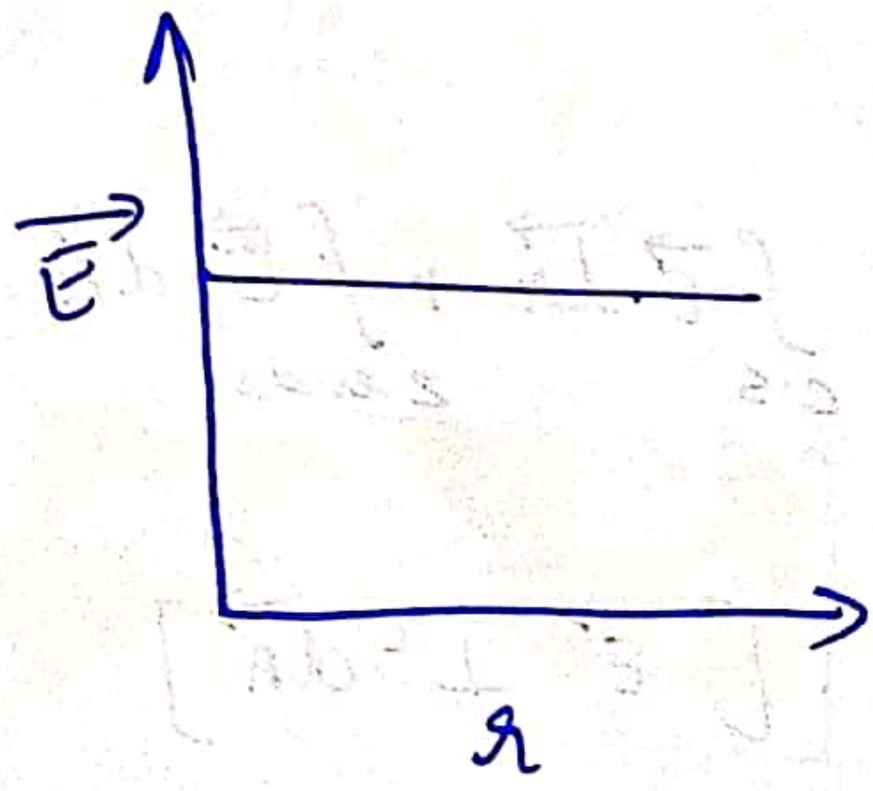
$$2EA = q/\varepsilon_0.$$

$$(\sigma = q/A) \qquad q = A\sigma.$$

$$\therefore 2E\cancel{A} = \cancel{A}\sigma/\varepsilon_0.$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}\hat{i}}$$

Electric field due to plane sheet: Variation with distance r:



(independent of r)