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Chapter - 4: Moving Charges and Magnetism

Force on a charged particle in a magnetic field

$$\vec{F}_m = q(\vec{v} \times \vec{B})$$

$$|\vec{F}_m| = qvB \sin\theta.$$

$q \rightarrow$ charge
$v \rightarrow$ velocity of the charge
$B \rightarrow$ strength of mag. field

direction of the ~~electric~~ magnetic field is given by right hand thumb rule or cross product rule.

Force on a (charge) in a magnetic field

$$\vec{F}_m = q(\vec{v} \times \vec{B})$$

$$|\vec{F}_m| = qvB \sin\theta.$$

Special cases:

1) $\vec{F}_m = 0$, $v = 0$, $\theta = 0°$ or $180°$

2) $|\vec{F}_m| = qVB$ (max) ; $\theta = 90°$.

$$F_m = qVB\sin\theta$$

$$\boxed{B = \frac{F_m}{qV\sin\theta}}$$

$q = 1C$, $V = 1 m/s$, $\sin\theta = 1$:

then $B = F_m$. SI unit of B = Tesla.

One Tesla : If a charge of one coulomb moving with a velocity of 1 m/s, experiences a force of 1N, the strength of the magnetic field is one Tesla.

If in a region, both electric and magnetic field are present, the total force acting on the charge is given by:

$$\vec{F} = \vec{F_E} + \vec{F_m}$$

$$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$$

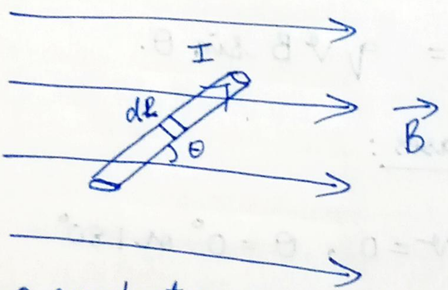
This force is called ~~la~~ <u>lorentz force</u>.

Electric part of lorentz force $\Rightarrow q\vec{E}$

Magnetic part of lorentz force $\Rightarrow q(\vec{v} \times \vec{B})$

Lorentz force $= q\vec{E} + q(\vec{v} \times \vec{B})$.

Force on a conductor carrying current placed in uniform magnetic field



Consider a conductor carrying current I placed in uniform magnetic field B.

n - no. density of electrons (per unit volume)

e - charge on an electron

dl - small length element of the conductor

A - Area of cross section of the conductor

V_d = Drift Velocity

l = length of the conductor

Force on each electron in the conductor is given by:

$\Rightarrow -e(\vec{V_d} \times \vec{B})$

Volume of length element $dl = + A dl$.

no. of electrons in this volume $= + n A dl$

Total charge of all these electrons present in that volume $= - ne A dl$.

Total force on the small length element dl

$$= d\vec{F} = - ne A dl \, (\vec{V_d} \times \vec{B})$$

$$d\vec{F} = - ne A (dl \, \vec{V_d} \times \vec{B})$$

(As drift Vel & current opp to each other) $[\, dl \, \vec{V_d} = - d\vec{l} \, V_d. \,]$

$$\therefore d\vec{F} = ne A \, (d\vec{l} \, V_d \times \vec{B})$$

$$d\vec{F} = ne A V_d \, (d\vec{l} \times \vec{B})$$

$$d\vec{F} = I \, (d\vec{l} \times \vec{B})$$

$$\int d\vec{F} = I \, (\vec{l} \times \vec{B})$$

$$\boxed{\vec{F} = I \, (\vec{l} \times \vec{B})}$$

↓

Total force on the entire conductor of length l.

<u>Special cases:</u>

(i) $\theta = 0^\circ$ or $180^\circ \Rightarrow F = 0$.

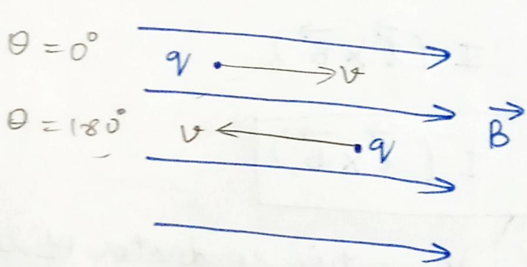
(ii) $\theta = 90^\circ \Rightarrow F = BIl$ (maximum)

Direction of force is given by right hand thumb rule or flemming's left hand rule

Stretch the first 3 fingers of your left hand mutually $\perp^r$ to each other. If the forefinger represents the field, centre finger represents the current, the thumb will show the direction of force on the conductor.

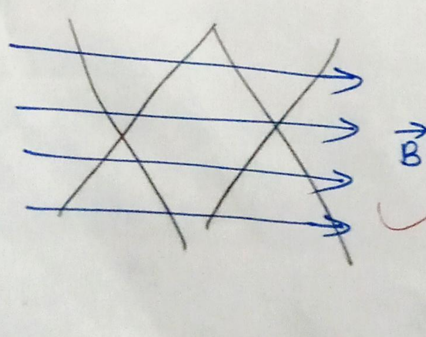
MOTION OF A CHARGED PARTICLE IN MAG. FIELD

a) Charged particle is moving parallel or anti-parallel to the field ($\theta = 0°$ or $180°$)



$F = 0$ so the charged particle continues to moves along the same path without any deviation, exhibits uniform motion.

b) Charged particle is moving perpendicular to the field ($\theta = 90°$)



When a charged particle moves perpendicular to the magnetic field, maximum force acts on it, which is perpendicular to the direction of motion. This force does not change the magnitude of velocity, it only changes the direction of Velocity. Hence the charged particle describes <u>uniform circular motion</u>.

Instantaneous power = $\vec{F} \cdot \vec{v}$

$$P = Fv\cos\theta$$

$$P = 0$$

$$\therefore W = 0$$

$\therefore$ Change in $KE = 0$
(Work-energy theorem)

$\therefore v^2$ is constant
or motion is uniform.

The centripetal force required for circular motion is provided by the magnetic field: $F = qvB$.

$$\therefore \frac{mv^2}{R} = qvB$$

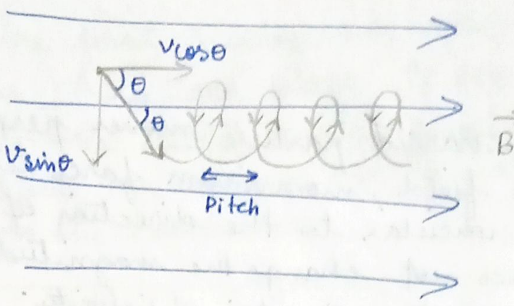
$$R = \frac{mv^2}{qvB} = \boxed{\frac{mv}{qB} = R}$$ $P =$

$$T = \frac{2\pi R}{v} = \frac{2\pi m\cancel{v}}{\cancel{v}qB} = \boxed{\frac{2\pi m}{qB} = T}$$

$$\boxed{\nu = \frac{qB}{2\pi m}} \quad 2\pi\nu = \frac{qB}{m}\ ; \ \omega = \frac{qB}{m}$$

If the charged particle moves at an angle other than 0°, 180°, 90° :

$\theta \neq 180^\circ, 0^\circ, 90^\circ$.



$v\cos\theta \rightarrow$ uniform motion
$v\sin\theta \rightarrow$ uniform circular motion } Produces a Helix.

Pitch = Velocity × T

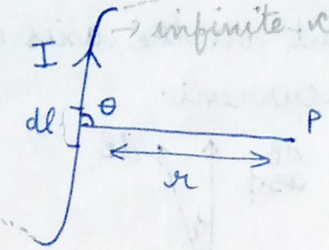
$= v\cos\theta \times T$

Radius $= \dfrac{mv_{\perp r}}{qB} = \dfrac{mv\sin\theta}{qB}$.

When a charged particle moves at an angle other than $0^\circ, 90^\circ, 180^\circ$ in a magnetic field, the velocity can be resolved into components – $v\cos\theta$ and $v\sin\theta$ as shown. Due to $v\cos\theta$, it describes linear motion, and due to $v\sin\theta$, it describes uniform circular motion. Combination of both is called a helix, and the path is called a helical path.

Pitch of the helix: Linear distance moved by the charge in one complete rotation.

Biot Savart Law:



$dB \propto I$

$dB \propto dl$

$dB \propto \sin\theta$

$dB \propto \frac{1}{r^2}$

$\Rightarrow dB \propto \frac{I\,dl \sin\theta}{r^2}$

$$\Rightarrow \boxed{dB = \frac{\mu_0}{4\pi} \frac{I\,dl\sin\theta}{r^2}}$$

μ_0 = permeability of free space

$\mu_0 = 4\pi \times 10^{-7}$ Tm/A...

Magnetic field produced at a point due to a current carrying conductor is directly proportional to current (I), length of the current element (dl), ~~sine~~ sine of the ~~cur~~ angle between the current and the position vector of the point and inversely proportional to the square of the distance between the conductor and the point.

Vector form:

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I(\vec{dl} \times \vec{r})}{r^3}$$

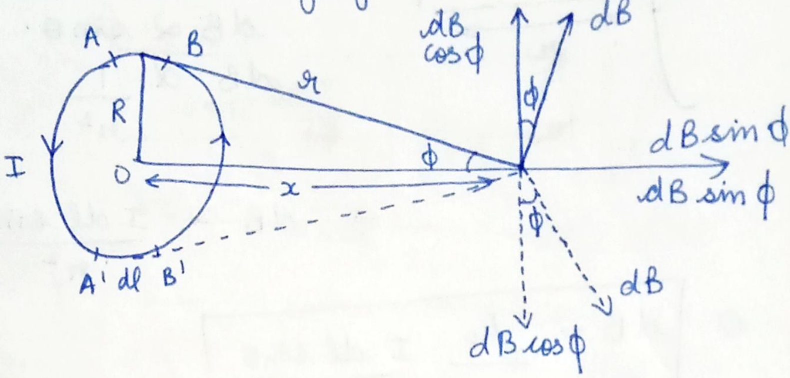
or

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi} \frac{I(\vec{dl} \times \hat{r})}{r^2}}$$

direction of mag. field $\Rightarrow$ given by right hand thumb rule.

Applications of Biot Savart law:

Magnetic field at a point on the axis of a circular coil carrying current.



Consider a circular coil of radius R carrying current I in anticlockwise direction.

dl - A small current element of the coil at AB

r - distance of point P from the current element dl

x - distance of point P from the centre O of the circular coil.

dB - Magnetic field at point P due to the current element dl.

According to Biot savart law, magnetic field at point P due to current element dl is given as:

$$dB = \frac{\mu_0}{4\pi} \frac{I\,dl \sin\theta}{r^2} \qquad \sin\theta = 1 \; ; (\theta = 90^\circ)$$

$$dB = \frac{\mu_0}{4\pi} \times \frac{I\,dl}{r^2}.$$

The direction of dB shown in the diagram as given by cross product rule.

resolving dB into components, we get:

dB cos ϕ ~~along~~ perpendicular to the axis and
dB sin ϕ along the axis.

We find that component $dB\cos\phi$ gets cancelled due to contribution from current element dl at $A'B'$ diametrically opposite to AB. Hence total magnetic field at point P is given as :

$$B = \Sigma\, dB \sin\theta.$$

$$= \Sigma \frac{\mu_0}{4\pi} \frac{I\,dl}{r^2} \sin\phi$$

$$B = \frac{\mu_0}{4\pi} \frac{I}{r^2} \Sigma dl \sin\phi$$

$$\sin\phi = \frac{R}{r}$$

$$\therefore B = \frac{\mu_0}{4\pi} \frac{I \cdot 2\pi R \cdot R}{r^3}$$

$$r^2 = R^2 + x^2$$

$$r = \sqrt{R^2 + x^2}$$

$$B = \frac{\mu_0}{4\pi} \times \frac{I \cdot 2\pi R^2}{(R^2+x^2)^{3/2}}$$

$$\boxed{B = \frac{\mu_0 I R^2}{2(R^2+x^2)^{3/2}}\ \hat{i}}$$

For n turns

$$\vec{B} = \frac{\mu_0 n I R^2}{2(R^2+x^2)^{3/2}}\ \hat{i}$$

At the centre of the coil : $x = 0$.

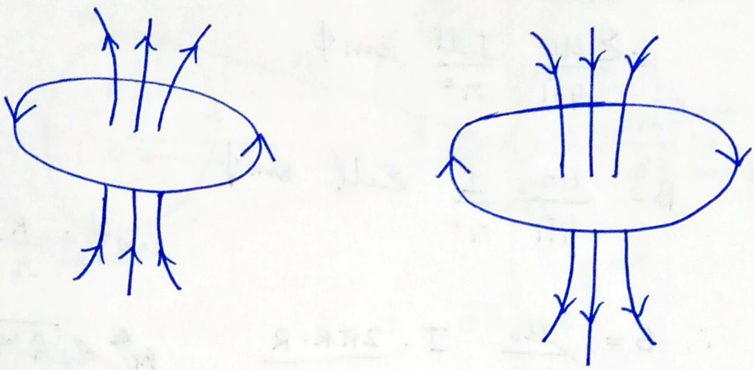
$$\therefore \vec{B} = \frac{n\mu_0 I R^2}{2(R^2)^{3/2}} = \frac{n\mu_0 I R^2}{2R^3} = \frac{\mu_0 n I}{2R}$$

$$\boxed{\therefore \vec{B} = \frac{\mu_0 n I}{2R}\ \hat{i}}$$

To determine the direction of magnetic field:

Right hand thumb rule :

Statement: Curl the fingers of your right hand along the direction of current, the thumb will show the direction of field at the centre.



PORTIONS FOR MID-TERM - I

1) Electrostatics I
2) Electrostatics II
3) Current Electricity.

A
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Ampere's circuital law (like Gauss's law)

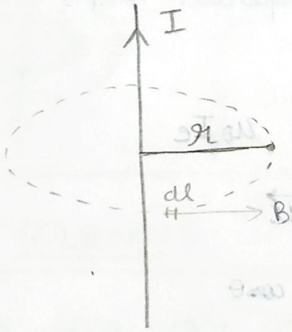
The line integral of magnetic field over a closed path or loop is equal to μ_0 times the current enclosed by the path. (I_e)

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_e$$

Applications of ampere's circuital law

Magnetic field at a point due to a straight conductor

a) At a point outside the conductor ($r > a$).



Consider a straight conductor carry current I as shown. Let "a" be the radius of the ~~da~~ conductor. Let P be a point at a distance "r" from the conductor. To determine Magnetic field at point P, Construct an amperian loop, a circle passing through point P, with the conductor at its centre.

According to ampere's circuital law,

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_e$$

($\cos\theta = 1$), $\theta = 0°$ angle between dl & B is same.

LHS: $\oint \vec{B} \cdot \vec{dl} = \oint B \cdot dl \cdot \cos\theta = \oint B dl$

$B \oint dl = B(2\pi r)$

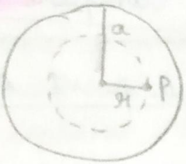
$$B\, 2\pi r = \mu_0 I \qquad (I_e = I)$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

$$B \propto \frac{1}{r}, \qquad B \propto I.$$

~~Consider a point P inside the surface of a conductor~~

b) At a point inside the conductor $(r < a)$



Consider a point P inside the surface of a conductor. To determine magnetic field at point P, construct an amperean loop, a circle passing through point P.

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I_e$$

$$\begin{aligned} \text{LHS} &= \oint \vec{B}\cdot\vec{dl} \\ &= \oint B\, dl \cos\theta \\ &= \oint B\, dl \qquad (\cos\theta = 1) \\ &= B \oint dl \\ &= B \cdot 2\pi r. \end{aligned}$$

$$B\cdot 2\pi r = \mu_0 I_e.$$

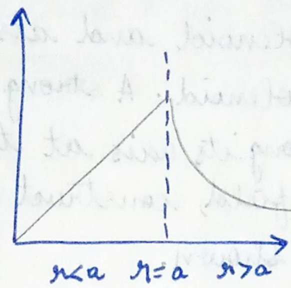
$$\left[I_e = \frac{I r^2}{a^2} \right]$$

$$\left(\begin{aligned} \pi a^2 &= I. \\ \pi r^2 &= \,?\, I_e \\ I_e &= \frac{I}{\cancel{\pi} a^2} \times \cancel{\pi} r^2 \end{aligned} \right)$$

$$B\cdot 2\pi r = \mu_0 \frac{I r^2}{a^2}$$

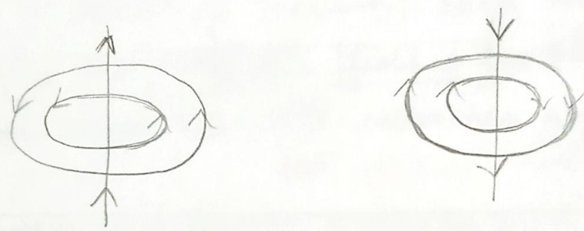
$$B = \frac{\mu_0 I r}{2\pi a^2}$$

$$B \propto r$$



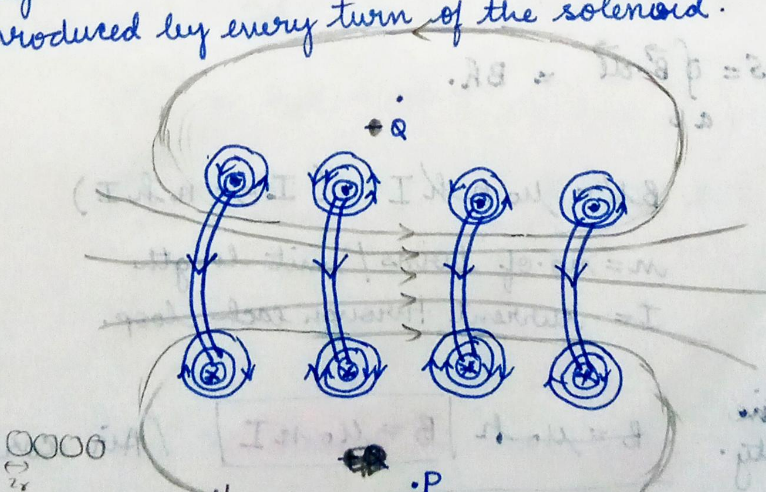
Direction to determine the magnetic field – Right hand thumb rule:

Hold the conductor in your right hand. If the thumb points along the direction of current, curling of fingers shows the direction of magnetic field.



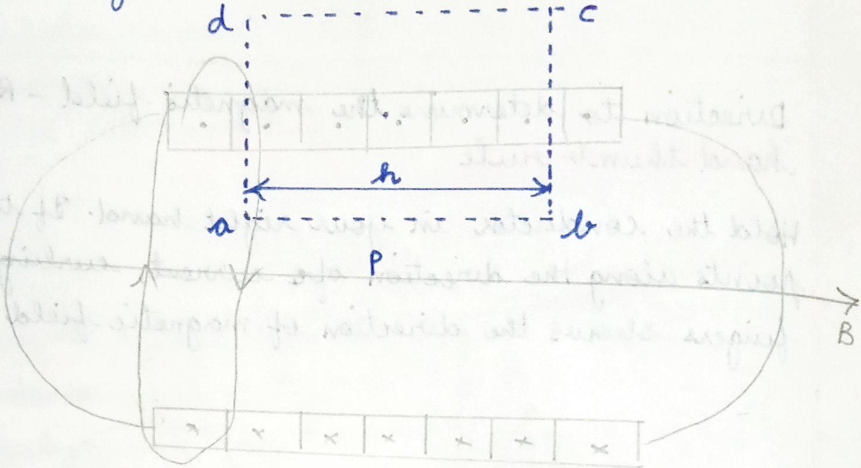
SOLENOID

Solenoid is a long wire wound into many circular turns in the form of an ellipse on a cylinder. The turns are insulated from each other by enamelled wires. Total magnetic field produced by the solenoid is the vector sum of magnetic fields produced by every turn of the solenoid.



$N \cdot 2r$. $n = \frac{N}{L} = \frac{N}{N2r} = \frac{1}{2r}$

It is found that there is no magnetic field outside the solenoid and also inbetween the turns of the solenoid. A strong uniform magnetic field exists along its axis at the centre. To determine this magnetic field, construct an amperean loop, a rectangle, as shown.



$$\oint_{abcda} \vec{B}\cdot\vec{dl} = \mu_0 I_e.$$

$$\oint_{cd} \vec{B}\cdot\vec{dl} = \oint 0 \quad \text{(outside the solenoid)}.$$

$$\oint_{ad} \vec{B}\cdot\vec{dl} = \oint_{bc} \vec{B}\cdot\vec{dl} = 0 \quad (\cos 90^\circ = 0^\circ)$$

$$LHS = \oint_{ab} \vec{B}\cdot\vec{dl} = Bh.$$

$$B\cancel{h} = \mu_0 n \cancel{h} I \quad (I_e = nhI)$$

n = no. of turns / unit length

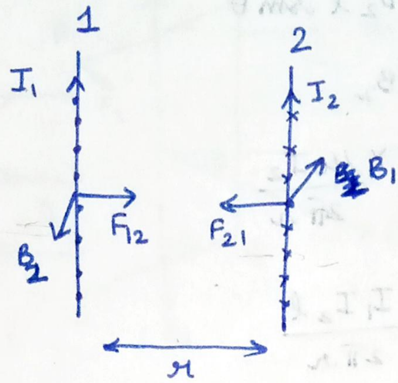
I = current through each loop.

μ_r = relative permeability.

~~$B = \mu_0 h$~~ $\boxed{B = \mu_0 n I}$ (Air core)

$\boxed{B = \mu_0 \mu_r n I}$ (Ironcore)

Force between two long straight parallel current carrying conductors.



Consider 2 long straight parallel conductors 1 & 2 carrying currents I_1 and I_2 as shown, kept distance 'r' apart in vacuum. Each conductor is placed in the magnetic field of the other, and hence each conductor will experience a force.

F_{12} = Force on conductor 1 due to 2.

F_{21} = Force on conductor 2 due to 1.

Magnetic field at distance r due conductor 1 is given as:

$$B_1 = \frac{\mu_0 I_1}{2\pi r}$$

$$\vec{F_{21}} = I_2(\vec{l} \times \vec{B_1})$$ (l = length of conductor).

$$|\vec{F_{21}}| = I_2 B_1 l \sin\theta.$$

$$|\vec{F_{21}}| = I_2 l B_1 = \left(\frac{\mu_0 I_1}{2\pi r}\right) I_2 l.$$

$$\Rightarrow \frac{\mu_0 I_1 I_2 l}{2\pi r} = F_{21}$$

$$\vec{F_{12}} = I_1(\vec{l} \times \vec{B_2}) \qquad B_2 = \frac{\mu_0 I_2}{2\pi r}.$$

$$|\vec{F_{12}}| = I_1 B_2 l \sin\theta$$

$$= I_1 l B_2$$

$$= I_1 l \times \frac{\mu_0 I_2}{2\pi r}$$

$$F_{12} = \frac{\mu_0 I_1 I_2 l}{2\pi r}$$

$$F_{21} = F_{12}.$$

$$\frac{F_{12}}{l} = \boxed{\frac{F_{21}}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}}$$ Force acting per unit length

Define 1 Ampere:

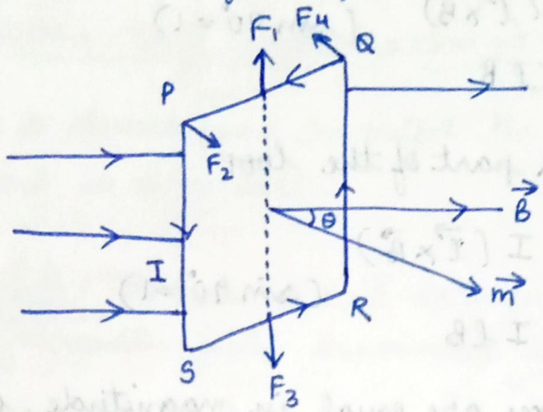
One Ampere is that current flowing through 2 long straight parallel conductors kept 1 meter apart in vacuum, such that they experience an attractive force per unit length of 2×10^{-7} N/m.

$$\frac{F_{12}}{l} = \frac{F_{21}}{l} = \frac{4\pi\times10^{-7}\times1\times1}{2\pi\times1} = 2\times10^{-7} \text{ N/m}$$

By using Flemming's left hand rule or cross product rule, the direction of force is found to be towards each other. i.e,

Parallel currents attracts and
Anti-parallel currents repel.

→ Torque on a rectangular current loop placed in a uniform magnetic field.



Consider a rectangular current loop PQRS placed in a uniform magnetic field as shown.

PQ = SR = b (breadth of the loop)

PS = QR = l (length of the loop)

θ = angle between magnetic field and the normal to the loop.

Every part of the loop ~~with~~ will experience a force.

Force on SR part of the loop:

$$\vec{F_3} = I\,(\vec{b} \times \vec{B})$$

$$|\vec{F_3}| = I\,b\,B \sin(90-\theta)$$

$$|\vec{F_3}| = I\,b\,B \cos\theta.$$

Force on PQ part of the loop:

$$\vec{F_1} = I(\vec{b} \times B)$$

$$|\vec{F_1}| = I\,b\,B \sin(90+\theta)$$

$$|\vec{F_1}| = I\,b\,B \cos\theta.$$

$\vec{F_1}$ and $\vec{F_3}$ are collinear, equal in magnitude, but opposite to each other, hence they cancel out each other. Hence net force in vertical direction is zero.

Force on PS part of the loop

$$\vec{F_2} = I(\vec{l} \times \vec{B}) \quad (\sin 90^\circ = 1)$$
$$|\vec{F_2}| = IlB$$

Force on QR part of the loop

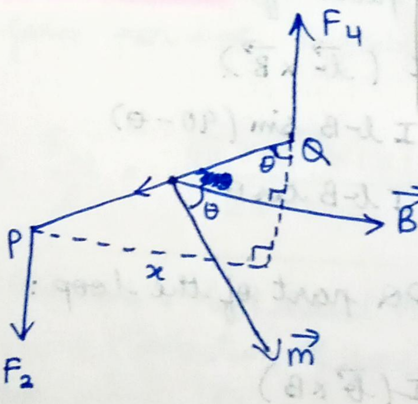
$$\vec{F_4} = I(\vec{l} \times \vec{B}) \quad (\sin 90^\circ = 1)$$
$$= IlB$$

These 2 forces are equal in magnitude, opposite in direction, but their line of action does not coincide. Hence they constitute a couple and the couple, and the couple gives rise to a torque.

Torque = one of the forces $\times \perp^r$ dist.

$$= IlB \times x$$
$$= IlB \times b\sin\theta$$
$$= I\,lb\,B\sin\theta$$
$$\tau = IAB\sin\theta$$

$\vec{A}$'s direction: Direction of Mag. field caused by the current in the rectangular loop, (Not the external magnetic field $\vec{B}$)



If the loop has N no. of turns, $\tau = NIAB\sin\theta$

$\tau = mB\sin\theta \quad (m = NI\vec{A})$

$\tau = \vec{m} \times \vec{B}$ $\quad$ m = magnetic moment of the current loop.

$\boxed{\tau = \vec{m} \times \vec{B}}$ $\quad$ direction: along outward drawn normal

$\tau = \vec{p} \times \vec{E}$ $\quad$ (p = dipole moment, $\vec{E}$ = elec. field)

SI unit of $\vec{m}$ = Ampere metre^2 = $A\,m^2$.

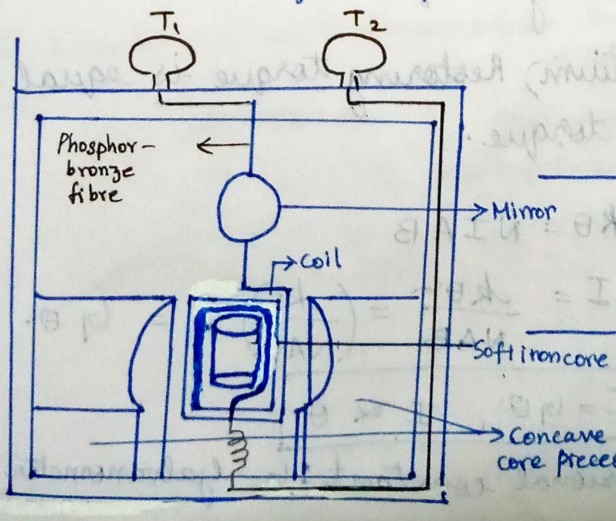
MOVING COIL GALVANOMETER

It is a device used to detect the presence of current in a circuit.

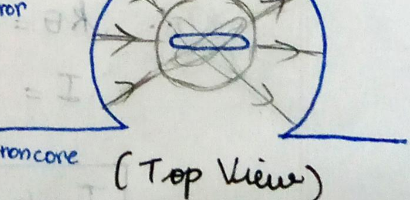
__Principle__: ~~It cons~~ A coil carrying current placed in a uniform magnetic field experiences a torque.

__Construction__:

- It consists of coil of many turns (copper) wound over a non-magnetic metallic frame.
- The coil is suspended by phosphor bronze fiber. A mirror is attached to the fibre.
- The coil is placed between concave pole pieces of a strong magnet.
- A soft iron cylinder is placed within the coil.
- The 2 ends of the coil are connected to 2 terminals T_1 and T_2 which provide contact with external circuit.
- A lamp and scale arrangement is provided to measure the deflection produced in the coil



Radial Field



(Top View)

Radial Field:
Field lines along the radius of curvature of concave pole pieces is called radial field. It makes the torque maximum.

Soft Iron Cylinder:
It increases the strength of the magnetic field.

THEORY:
When T_1 and T_2 are connected to the external circuit, current I flows through the coil and it experiences a deflecting torque on the coil.

$$\tau = NIAB\sin\theta$$
$$\tau = NIAB \quad (\sin\theta = 1)$$

As the coil rotates, the phosphor-bronze fiber will also rotate. So, a restoring torque comes into play within the fiber, which will try to bend bring the fiber back to its original position.

Let the angular twist in the fiber be θ.

Let the restoring torque/angular twist $= k$.

$\therefore$ Restoring torque $= k\theta$.

At equilibrium, Restoring torque is equal to deflecting torque.

$$\therefore k\theta = NIAB$$
$$I = \frac{k\theta}{NAB} = \left(\frac{k}{NAB}\right)\theta = G\theta.$$

$$\boxed{I = G\theta, \quad I \propto \theta}$$

$k \rightarrow$ torsional constant; G – Galvanometer constant.

Current sensitivity $= \frac{\theta}{I}$: defined as deflection per unit current

$= \frac{NAB}{k}$ (rad/Amp)

Voltage sensitivity $= \frac{\theta}{V}$: defined as deflection per unit voltage.

no effect on Voltage sensitivity due to no. of turns as $N\uparrow$, $R\uparrow$, ratio same.

$= \frac{NAB}{kR}$ (rad/Volt)

By increasing the no. of turns, Area bet. the loop, Magnetic field, and by decreasing 'k'.

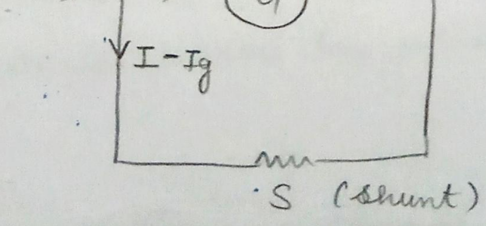
Increasing current sensitivity need not necessarily increase the Voltage sensitivity as:

$\left(\frac{\theta}{I}\right)' = \frac{2NAB}{k}$; $\left(\frac{\theta}{IR}\right)' = \frac{2NAB}{k \times 2R} = \frac{NAB}{kR}$ itself.

PROPERTIES OF THE FIBER

1. It should be a good conductor
2. It should be elastic in nature
3. It must have low k value.
4. It should be anti-corrosive in nature.

CONVERSION OF GALVANOMETER TO AMMETER



To convert a galvanometer into ammeter, a small resistance called shunt is connected in parallel to it.

$$-I_g R_g + (I - I_g) S = 0 \quad ; \quad I_g R_g = I \cdot S$$

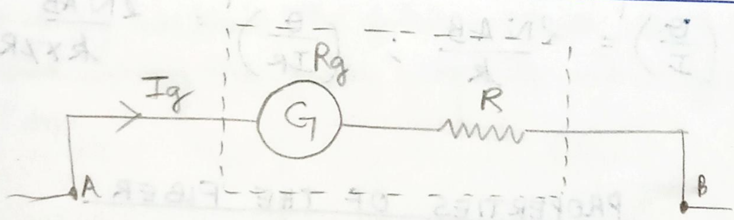
$$S = \frac{I_g R_g}{I - I_g}$$

$$\frac{1}{R_A} = \frac{1}{R_g} + \frac{1}{S}$$

$$R_A = \frac{R_g S}{R_g + S}$$

An ideal ammeter must have zero resistance.

CONVERSION OF GALVANOMETER to VOLTMETER



To convert galvanometer into voltmeter, a high resistance is connected in series.

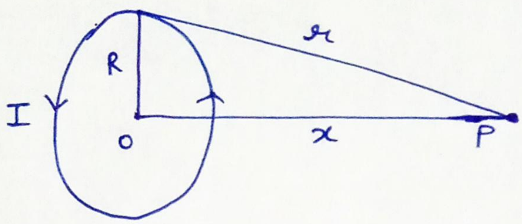
$$V = I_g R_g + I_g R.$$

$$R = \frac{V - I_g R_g}{I_g}$$

$$R_V = R_g + R.$$

An ideal voltmeter must have infinite resistance

CIRCULAR LOOP (carrying current) AS A MAGNETIC DIPOLE



$$B = \frac{\mu_0 I R^2}{2(R^2+x^2)^{3/2}}$$

$x >> R.$

$$\therefore B = \frac{\mu_0 I R^2}{2x^3}$$

$$= \frac{\mu_0 I \pi R^2}{2\pi x^3}$$

$$= \frac{2\mu_0 I \pi R^2}{4\pi x^3}$$ $I\pi R^2$ → magnetic moment. = (IA)

$$B = \frac{\mu_0 2m}{4\pi x^3} \quad \text{— (1)} \quad \Rightarrow \quad E = \frac{2p}{4\pi \varepsilon_0 r^3} \quad \text{— (2)}$$

$B \to E$
$\mu_0 \to 1/\varepsilon_0$
$m \to p$

This is the axial field of elec. dipole.

Eq. (1) and (2) are similar.

A circular loop in a magnetic field experiences a torque as is experienced by an electric dipole in a uniform elec. field.

Hence a circular current loop acts as a magnetic dipole.