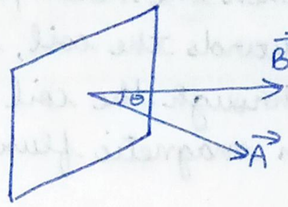
6. ELECTROMAGNETIC INDUCTION

Magnetic flux: Total number of magnetic field lines passing through unit area normal to it.

Mathematically it is defined as the scalar product of magnetic field and area.

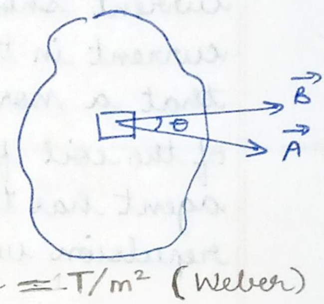
$$\phi_B = \vec{B} \cdot \vec{A}$$
$$= BA\cos\theta.$$



If A is not symmetrical, field not uniform

$$d\phi_B = \vec{B} \cdot d\vec{A}$$

$$\phi_B = \oint \vec{B} \cdot d\vec{A}$$



dimensions $= [ML^2T^{-2}A^{-1}]$ SI unit $= 1\,Wb = T/m^2$ (Weber)

FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION

Magnitude of induced emf in a closed circuit is equal to the time rate of change of magnetic flux taking place through the closed circuit

$$\mathcal{E} = \frac{-d\phi}{dt}.$$

If the coil has N no. of turns, $\mathcal{E} = -N\dfrac{d\phi}{dt}$

-ve sign indicates the direction of induced emf and hence the direction of induced current.

LENZ'S LAW

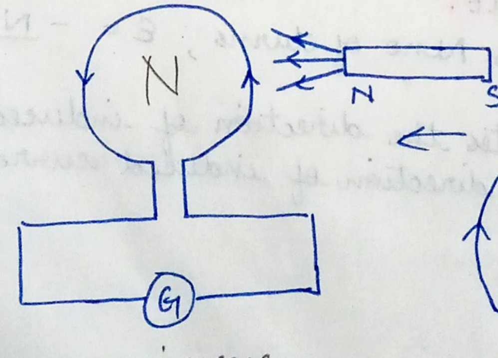
- The direction of induced emf is so as to oppose the cause that produced it.

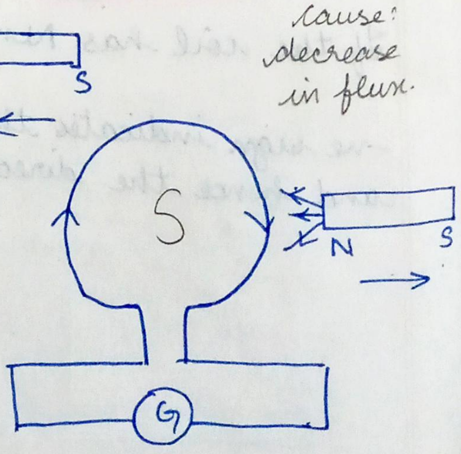
- It follows law of conservation of energy

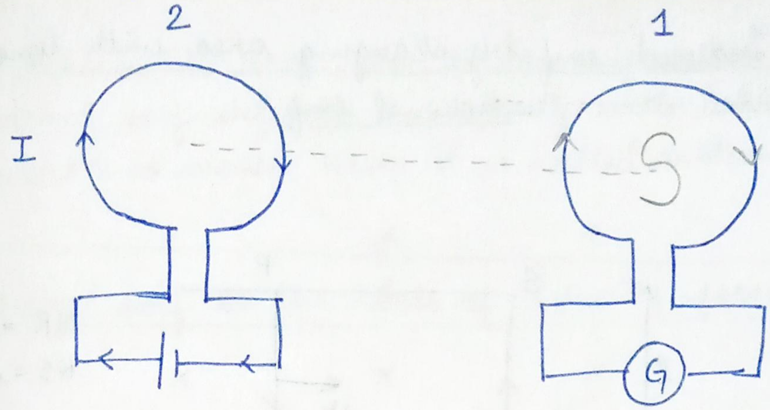
- When the north pole of the magnet is moved towards the coil, no. of field lines passing through the coil increases, there is an increase in magnetic flux through the coil.

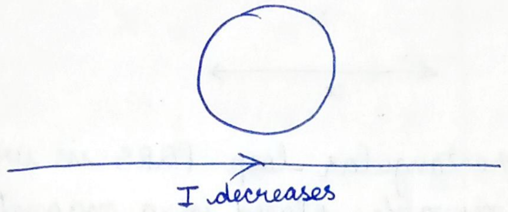
- According to lenz's law, the direction of the current should oppose the cause. Therefore current in the coil flow in such a direction that a north pole is produced on the side of the coil facing the magnet. The external agent has to do work against the force of repulsion which is converted to electrical energy.

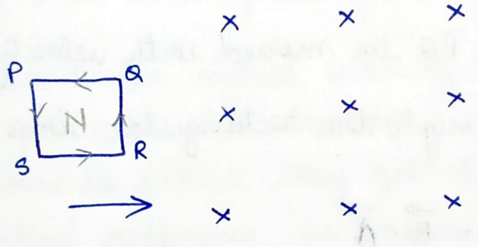
- When the north pole of the magnet is moved away from the coil, the current in the coil flows in such a way that the side of the coil facing the magnet has a south pole, the external agent has to do work against the force of attraction, which is converted to elec. energy.











<u>Methods of producing induced emf:</u>

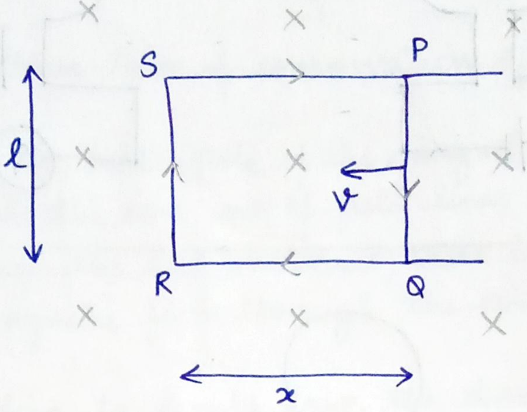
$$\phi = BA\cos\theta$$

$$\varepsilon = \frac{-d\phi}{dt} = \frac{-d}{dt}(BA\cos\theta)$$

1. By changing magnetic field with time (Pg: 205, 206)
2. By changing area with time
3. By changing the orientation of the coil with time.

1) ## Induced emf by changing area with time

– Motional emf



$QR = x$
$RS = l$

Consider a rectangular loop PQRS in which the arm PQ is movable, placed in a magnetic field as shown.

let $QR = x$, $RS = l$.

let the arm PQ be moved with velocity v.

The flux through the rectangular loop PQRS is given as:

$$\phi = \vec{B} \cdot \vec{A}$$

$$= BA\cos\theta.$$

$$\phi = BA \quad (\cos\theta = 1)$$

$$\boxed{\phi = B(lx)}$$

According to Faraday's law,

$$\mathcal{E} = -\frac{d\phi}{dt}$$

$$= -\frac{d}{dt}(Blx)$$

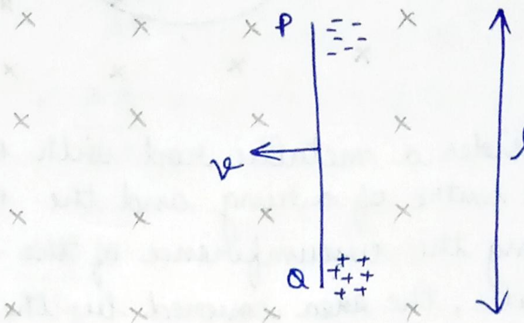
$$\mathcal{E} = -Bl\frac{dx}{dt} = -Bl(-v)$$

$$\boxed{\mathcal{E} = Blv}$$

As there is no area term in the expression, induced emf depends on velocity with which the arm PQ is moved. Hence it is called motional emf.

2 marks
2)

Motional emf on the basis of Lorentz force



Consider a conductor PQ of length l moving with velocity v in a magnetic field as shown.

As the conductor moves, each of the charge present in the conductor will experience a magnetic lorentz force, due to which they get separated, causing a potential difference as shown. Hence an induced emf is produced across the ends of the conductor.

This induced emf is work done per charge in moving positive charges from P to Q.

$$W = \vec{F} \cdot \vec{s}$$

$$= Fs\cos\theta$$

$$= Fs \qquad (\cos\theta = 1) \quad \theta = 0^\circ$$

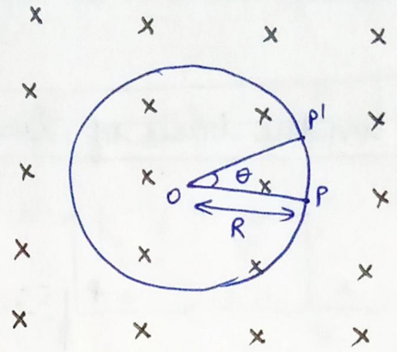
$$= q(\vec{v} \times \vec{B}) \cdot l$$

$$= qvB\sin\theta\, l \qquad (\sin\theta = 1) \quad \theta = 90^\circ$$

$$W = qvBl$$

3) **Induced emf by rotating a metallic rod along the circumference of a ring.**

(2/3 marks)



Consider a metallic rod with one end fixed at the centre of a ring and the other end moving along the circumference of the ring. As the rod rotates, the area covered by the rod changes, so there is a change in flux and hence an induced emf is produced at its ends.

let OP = R = initial position of rod.

OP' = position of the rod after 't' seconds

θ = angle covered by the rod when it moves from OP to OP'.

Flux through the conducting rod is given as:

$$\phi = \vec{B} \cdot \vec{A}$$

$$= BA\cos\theta$$

$$\phi = BA$$

$$\phi = B \cdot \frac{1}{2} R^2 \theta.$$

From faraday's law,

$$\varepsilon = \frac{-d\phi}{dt} = -\frac{d}{dt}\left(B \cdot \frac{1}{2} R^2 \theta\right)$$

$$= \frac{-d\theta}{dt} \times \frac{B \cdot R^2}{2}$$

$$= \frac{-1}{2} BR^2 \omega$$

$$\boxed{\mathcal{E} = -\frac{1}{2} BR^2 \omega}$$

Relation between charge and change in flux:

$$\mathcal{E} = -\frac{d\phi}{dt}$$

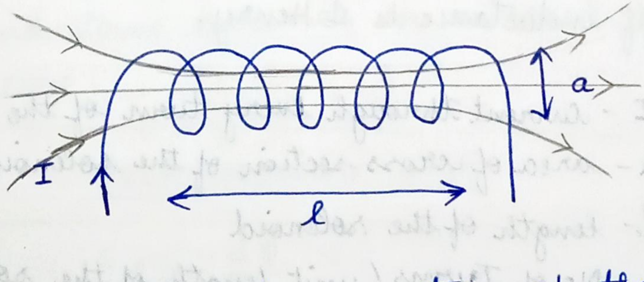
$$\mathcal{E} = \frac{\Delta \phi}{\Delta t}$$

$$IR = \frac{\Delta \phi}{\Delta t}$$

$$R \cdot \frac{\Delta Q}{\Delta t} = \frac{\Delta \phi}{\Delta t}$$

$$\boxed{\Delta Q = \frac{\Delta \phi}{R}}$$

SELF INDUCTION



When current through every turn of the solenoid changes, the magnetic field through every turn changes. Hence there is a flux change through every turn and an induced emf is produced across the ends of the solenoid. This phenomena is called self induction.

It is found that flux through every turn is directly proportional to the current through the solenoid.

$$B = \mu_0 n I.$$

$$\phi \propto I \qquad (B \propto I)$$

$$\phi = LI$$

L : self inductance or
coefficient of self induction

$$L = \frac{\phi}{I}$$

$$1\,H = \frac{1\,Wb}{1\,A} \qquad (I = 1A,\ L = \phi)$$

Self inductance: - electrical inertia

- Flux linked with every turn of the solenoid when current through every turn is 1 A.

SI unit = 1 Henry:

- When 1 Ampere of current flowing through a solenoid causes a flux change of 1 Weber, the self inductance is 1 Henry.

- I - current through every turn of the solenoid
  a - area of cross section of the solenoid
  l - length of the solenoid
  n - No. of turns / unit length of the solenoid
  N - Total no. of turns

- Magnetic field along the axis of the solenoid is given as:

$$B = \mu_0 n I$$

- Flux through every turn of the solenoid:

$$\phi = \vec{B} \cdot \vec{a}.$$
$$= Ba\cos\theta$$
$$= Ba\,(1) \qquad (\cos 0^\circ = 1)$$
$$= \mu_0 n I a.$$

- Total flux through the solenoid:

$$\Phi = N \mu_0 n I a \qquad (N = nl)$$
$$\Phi = nl\mu_0 n I a$$
$$\Phi = (\mu_0 n^2 l a)\, I$$

- Comparing this with $\Phi = LI$,

air core
$$\boxed{L = \mu_0 n^2 l a} \qquad L \propto l$$
$$\boxed{L = \frac{\mu_0 N^2 a}{l}} \qquad L \propto \frac{1}{l}$$

$$L = \mu_r \mu_0 n^2 l a \quad \text{— iron core.}$$

- Self inductance increases on introducing an iron core.

SELF INDUCED EMF

$$\varepsilon = \frac{-d\phi}{dt}$$
$$= \frac{-d(LI)}{dt}$$
$$= -L\frac{dI}{dt}.$$

- This emf opposes the change in current, so it is called <u>back emf</u>.

ENERGY STORED IN AN INDUCTOR:

Due to self induced emf, work has to be done to build up current in a circuit, having an inductor.

This workdone gets stored in the inductor as its energy.

The rate at which work is done is given by:

$$\frac{dW}{dt} = |\varepsilon| \cdot I$$

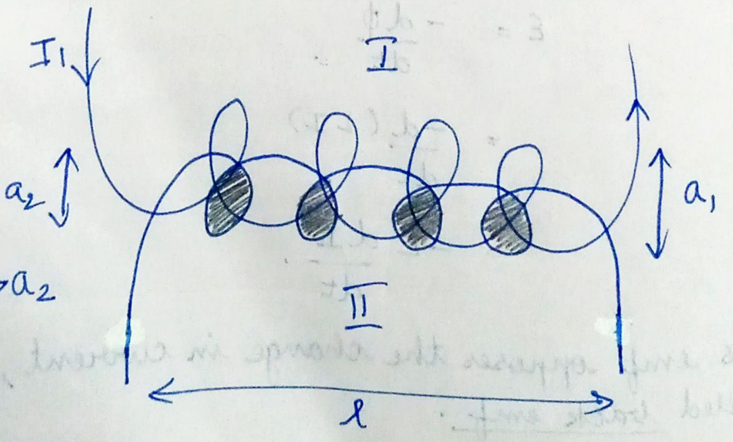
$$\frac{dW}{dt} = \left(L \cdot \frac{dI}{dt}\right) I.$$

$$\int dW = L \int_0^I I\, dI.$$

$$W = L\frac{I^2}{2}.$$

$$\boxed{\text{Energy} = \frac{1}{2} L I^2}$$

MUTUAL INDUCTION



$a > a_2$

Consider 2 solenoids or 2 coils wound over one another or kept near each other.

When current through one solenoid changes, magnetic field through every turn of the second solenoid changes. Hence there is a flux change through every turn of the second solenoid and an induced emf is produced across the ends of the second solenoid.

Such a phenomenon is called mutual induction.

It is found that flux through every turn of the second solenoid is directly proportional to the current through the first solenoid.

$$\phi_2 \propto I_1$$

$$\phi_2 = M_{21} I$$

M_{21} : Mutual inductance of 2^{nd} solenoid w.r.t. first solenoid (or)
Coefficient of mutual induction

$$M_{21} = \frac{\phi_2}{I_1}$$

$$\left(\begin{array}{c} I_1 = 1A \\ M_{21} = \phi_2 \end{array} \right)$$

<u>Mutual inductance</u> is defined as total flux linked with second solenoid when current through first solenoid is 1 A.

SI unit = H, $1H = \frac{1 Wb}{1 A}$.

<u>1 Henry</u>: When current of 1A flowing through first solenoid causes a flux change of 1 Weber in the second solenoid, mutual inductance is 1 Henry.

Expression for mutual inductance of a pair of solenoid.

N_1/N_2 - no. of turns in solenoid I/II.

n_1/n_2 - no. of turns/unit length in solenoid I/II

a_1/a_2 - area of cross section of solenoid I/II

I_1 - current in solenoid I.

Magnetic field along the axis of solenoid I:

$$B_1 = \mu_0 n_1 I_1$$

Flux through every turn of solenoid II:

$$\phi_2 = \vec{B_1} \cdot \vec{a_2}$$

$$\phi_2 = B_1 a_2 \cos\theta$$

$$\phi_2 = B_1 a_2 \quad (\cos 0^\circ = 1)$$

$$\phi_2 = \mu_0 n_1 I_1 a_2$$

Total flux through second solenoid:

$$\Phi_2 = N_2 \phi_2$$

$$= N_2 \mu_0 n_1 I_1 a_2$$

$$(N_2 = n_2 l)$$

$$\Phi_2 = n_2 l \mu_0 n_1 I_1 a_2$$

$$= (n_1 n_2 \mu_0 l a_2) I_1$$

$$(\Phi_2 = M_{21} I_1)$$

$$\boxed{M_{21} = \mu_0 n_1 n_2 l a_2}$$

Suppose current I_2 flows through the second solenoid, then flux through ~~the first~~ every turn of the first solenoid:

$$\phi_1 = \vec{B_2} \cdot \vec{a_2}$$

$$= B_2 a_2 \cos\theta$$

$$= B_2 a_2$$

$$= \mu_0 n_2 I_2 a_2$$

Total flux through first solenoid:

$$\bar{\Phi}_1 = N \phi$$

$$(N_1 = n_1 l)$$

$$\bar{\Phi}_1 = (\mu_0 n_1 n_2 l a_2) I_2$$

$$\bar{\Phi}_1 = M_{12} I_2 .$$

$$\boxed{M_{12} = \mu_0 n_1 n_2 l a_2 = M_{21} .}$$

Mutual inductance of a pair of solenoid is constant.

If an iron core is introduced:

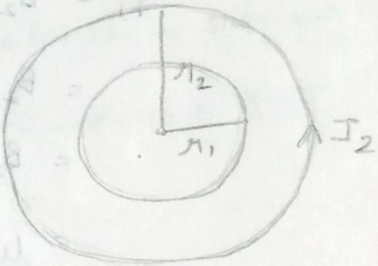
$$M_{12} = M_{21} = \mu_0 \mu_r n_1 n_2 l a_2 .$$

Factors on which self inductance and mutual inductance depend — ⊗

If 2 solenoids are placed apart —

Q. 2 concentric circular coils, one of small radius r_1 and other of large radius r_2 are placed coaxially with the centres coinciding. obtain the mutual inductance of the arrangement.

1-5 HW



$$B_2 = \frac{\mu_0 I_2}{2r_2}$$

$$\phi_1 = \vec{B_2} \cdot \vec{a_1}$$

$$= B_2 a_1 \cos\theta$$

$r_2 >>> r_1$

$$= \frac{\mu_0 I_2}{2r_2} \cdot a_1 \ (1)$$

$$\phi_1 = \frac{\mu_0 I_1}{2r_2} \times \pi r_1^2 = \left(\frac{\mu_0 \pi r_1^2}{2r_2}\right) I_1$$

$$\phi_1 = M_{12} \cdot I_1$$

$$\therefore M_{12} = \frac{\mu_0 \pi r_1^2}{2r_2} = M_{21}$$

<u>Induced emf by changing the orientation of the coil with time:</u>

When a coil is rotated in a magnetic field, the orientation of the coil changes continuously with time w.r.t. to the field.

So there is a flux change in the coil and hence an induced emf is produced at the ends of the coil. This phenomenon is called <u>electromagnetic induction</u>.

Flux through the coil is given as:

$$\phi = BA\cos\theta \qquad \left(\omega = \frac{\theta}{t}\ ;\ \text{let } \theta = \omega t\right)$$

$$= BA\cos(\omega t)$$

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA\cos\omega t)$$

$$= -BA\frac{d(\cos\omega t)}{dt}$$

$$= -BA.\ \omega(-\sin\omega t)$$

$$= BA\omega\sin(\omega t)$$

coil has one turn.

for n turns

$$\varepsilon = N BA\omega\sin\omega t.$$

ωt	0	$\pi/2$	π	$3\pi/2$	2π
ε	0	$NAB\omega$	0	$-NAB\omega$	0

ε_0 = max value for ε

$= NAB\omega.$

Since ε varies periodically, ε produces alternating current.

This emf varies continuously with time and also changes its direction periodically. Such an emf is called alternating emf, and the current that it drives in the circuit is called alternating current.

AC GENERATOR

It is a device which converts mechanical energy to electrical energy.

Construction:

- Armature – It is a coil of many turns wound on a frame.
- Slip rings (S_1 + S_2)
- Carbon brushes (B_1 and B_2)
- Field Magnets – Powerful pole pieces of a strong magnet.

Principle:

Electromagnetic induction (define during exams)

Theory: Derivation of $\varepsilon = \varepsilon^{\circ} \sin \omega t$.

Working:

When the coil is rotated in a magnetic field, the orientation of the coil changes with respect to the field. Hence there is an induced emf produced across the ends of a coil.

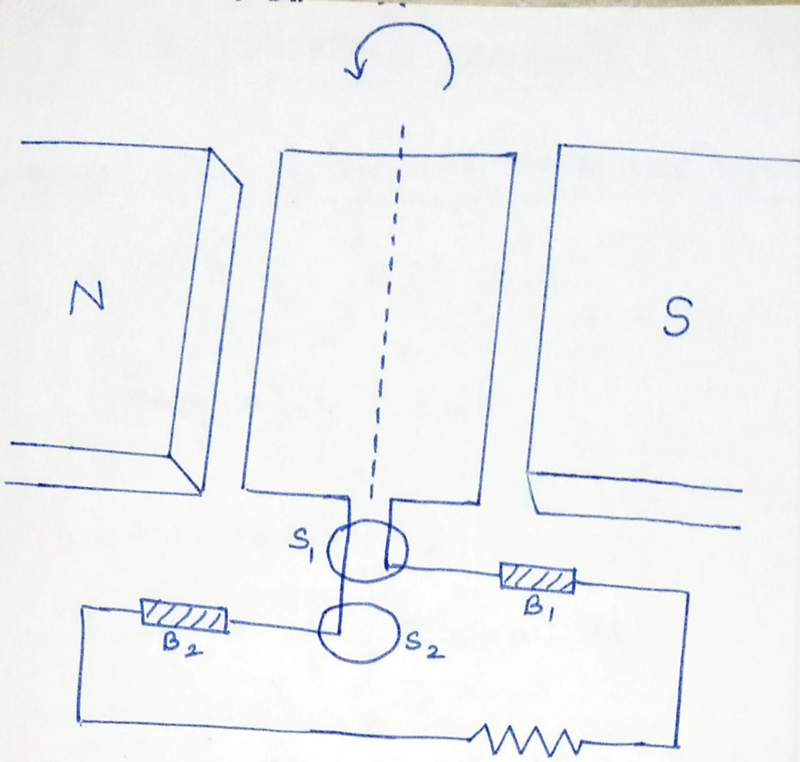
This emf is given as : $\varepsilon = \varepsilon_0 \sin \omega t$.

If the ~~coil~~ resistance of the coil and the external circuit is R, then current produced can be given as:

$$I = \frac{\varepsilon}{R} = \frac{\varepsilon_0 \sin \omega t}{R} \rightarrow I_0$$

$$\therefore \quad I = I_0 \sin \omega t$$

(I_0 = max value of current)



Direction of current is given by Flemming's right hand rule:

Stretch the first 3 fingers of your right hand mutually perpendicular to each other. If the forefinger shows the direction of field and the thumb points in the direction in which the conductor moves, the centre finger points in the direction of current through the conductor.