7. ALTERNATING CURRENT

Average value of current over one cycle:

$$F(t) = \frac{1}{T}\int_0^T F(t)\,(dt)$$

$$I_{avg} = \frac{1}{T}\int_0^T I\,dt.$$

$$(I = I_0 \sin \omega t)$$

$$I_{avg} = \frac{1}{T}\int_0^T I_0 \sin \omega t\,dt.$$

$$I_{avg} = \frac{I_0}{T}\int_0^T \sin \omega t\,dt. = \frac{I_0}{T}\left[\frac{-\cos \omega t}{\omega}\right]_0^T$$

$$\left(\omega = 2\pi\nu = \frac{2\pi}{T}\right)$$

$$= \frac{-I_0}{T\omega}\left[\cos \omega t\right]_0^T$$

$$= \frac{-I_0}{\cancel{T}\times\frac{2\pi}{\cancel{T}}}\left[\cos \frac{2\pi}{\cancel{T}}\times\cancel{T} - \cos\frac{2\pi}{T}\times 0\right]$$

$$= -\frac{I_0}{2\pi}\left[\cos 2\pi - 1\right] = 0.$$

$$\therefore \langle \cos \omega t \rangle = 0$$

For half a cycle:

$$\frac{2I_0}{\pi}$$

$$F(t) = \frac{2}{T}\int_0^{T/2} F(t)\,dt$$

$$= \frac{2}{T}\int_0^{T/2} I\,dt$$

$$= \frac{2}{T}\int_0^{T/2} I_0 \sin \omega t \, dt$$

$$= \frac{2I_0}{T}\left[\frac{-\cos \omega t}{\omega}\right]_0^{T/2}$$

$$= -\frac{2I_0}{T\omega}\left[\cos \omega t\right]_0^{T/2}$$

$$= -\frac{2I_0}{T\omega}\left[\cos \frac{2\pi}{T} \times \frac{T}{2} - \cos 0\right]$$

$$= -\frac{2I_0}{T\omega}\left[\cos \pi - \cos 0\right]$$

$$= -\frac{2I_0}{T\omega}\left[-1-1\right]$$

$$= -\frac{2I_0 \times T}{T \times 2\pi}\left[-2\right] = \frac{2I_0}{\pi}$$

Phasors: Vector representation of current and potential

RMS value of current:

It is that value of alternating current which when flowing through a resistance for the same time produces the same heating effect as that produced by direct current under the same conditions.

$$I_{rms}^2 = \frac{1}{T}\int_0^T I^2 dt.$$

$(I = I_0 \sin \omega t)$

$$I_{rms}^2 = \frac{1}{T}\int_0^T I_0^2 \sin^2 \omega t \, dt$$

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$$= \frac{I_0^2}{T}\int_0^T \sin^2 \omega t \, dt$$

$$= \frac{I_0^2}{T}\int_0^T \left(\frac{1-\cos 2\omega t}{2}\right) dt$$

$\int_0^T \cos 2\omega t \, dt$

$= \left[\frac{\sin 2\omega t}{2\omega}\right]_0^T$

$$= \frac{I_0^2}{2T}\int_0^T dt - \underbrace{\int_0^T \cos 2\omega t}_{0}$$

$= \dfrac{\left[\sin 2 \cdot \frac{2\pi}{T} \cdot T - \sin 2\omega(0)\right]}{2\omega}$

$$= \frac{I_0^2}{2T}\int_0^T dt - 0$$

$$= \frac{I_0^2}{2T} \times [T - 0]$$

$= \dfrac{(\sin 4\pi - \sin 0)}{2\omega}$

$$= \frac{I_0^2}{2T} \times T = \frac{I_0^2}{2}.$$

$= \dfrac{0-0}{2\omega}$

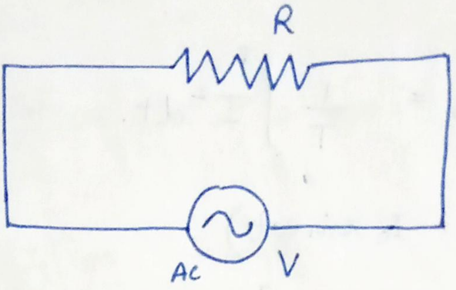
$= 0.$

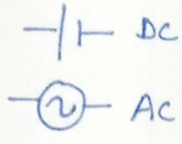
$$\therefore I_{rms}^2 = \frac{I_0^2}{2} \Rightarrow \boxed{I_{rms} = \frac{I_0}{\sqrt{2}}}$$

Similarly RMS value of voltage is given as:

$$V_{rms} = \frac{V_0}{\sqrt{2}}$$

Effect of a pure resistor in an AC circuit





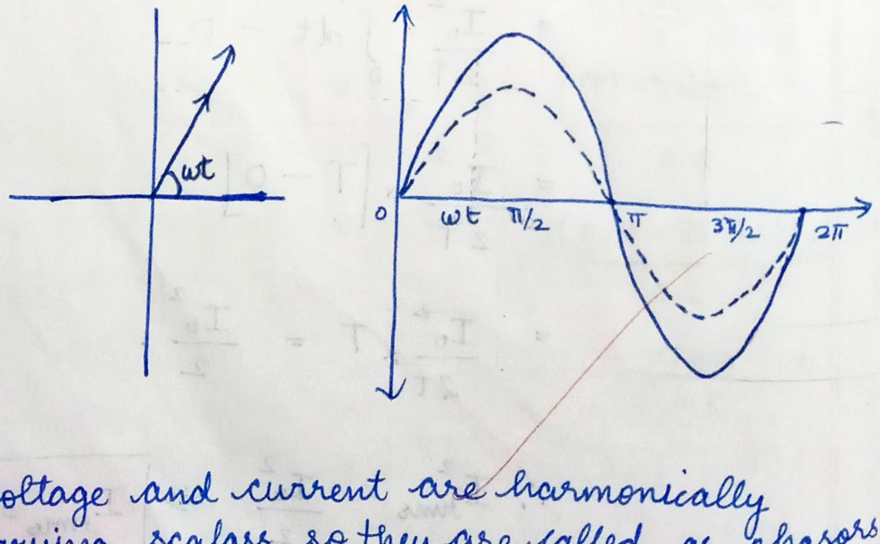
Potential diff. given by the supply is:

$$V = V_0 \sin \omega t. \quad \text{— (1)}$$

Current through the resistance is given by:

$$I = \frac{V}{R} = \frac{V_0 \sin \omega t}{R} \qquad I = I_0 \sin \omega t \quad \text{— (2)}$$

From equation (1) and (2) it is found that the voltage and the current are in phase with each other.

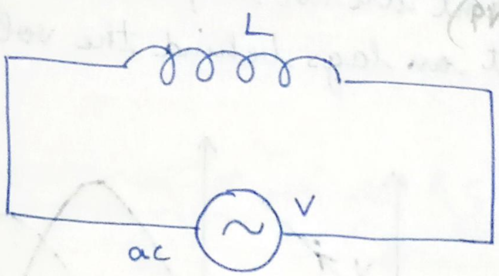


Voltage and current are harmonically varying scalars, so they are called as phasors or rotating vectors.

In the phasor diagram, the magnitude of the vector represents the peak value of voltage and current.

The vertical component represents the voltage and current at that instant of time.

Effect of a pure inductor in an AC circuit



(Phase diff 90)

V leading
I lagging
by

Consider a pure inductor of self inductance L connected to an AC circuit

By applying Kirchoff's rule,

Time const
$\tau = L/R$.

$$V - L\frac{dI}{dt} = 0$$

$$L\frac{dI}{dt} = V. \qquad [V = \sin\omega t\ (V_0)]$$

$$L\frac{dI}{dt} = V_0 \sin\omega t.$$

$$\therefore \quad dI = \frac{V_0}{L}\sin\omega t\, dt$$

$$\int dI = \int \frac{V_0}{L}\sin\omega t\, dt.$$

$$I = \frac{V_0}{L}\int \sin\omega t\, dt$$

$$= \frac{V_0}{L}\left[-\frac{\cos\omega t}{\omega}\right]$$

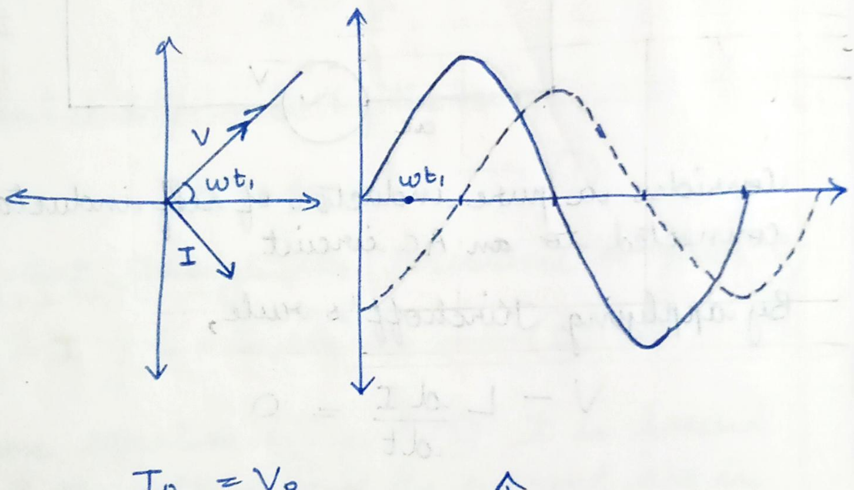
$$= \frac{V_0}{\omega L} \times (-\cos\omega t) = \frac{V_0}{L\omega}\left[-\sin(90 - \omega t)\right]$$

$$I = \frac{V_0}{L\omega} \sin(\omega t - \pi/2)$$

$$I = I_0 \sin(\omega t - \pi/2) \quad \text{---} (2)$$

$$\left[I_0 = \frac{V_0}{L\omega} \right]$$

From (1) & (2) it is seen that voltage and ~~a~~ current are not in phase with each other. Current ~~an~~ lags behind the voltage by $\pi/2$.

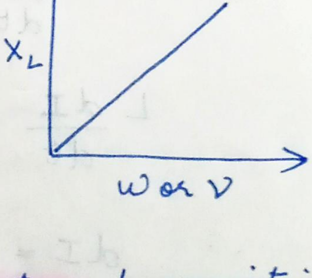


$$I_0 = \frac{V_0}{L\omega}$$

$$\frac{V_0}{I_0} = L\omega$$

$$X_L = L\omega \text{ ohm}$$

$$y = mx$$



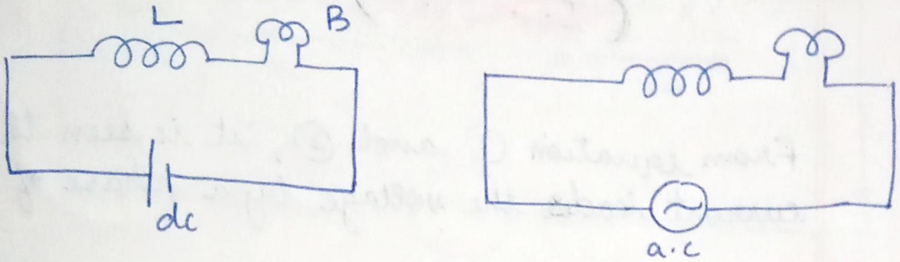
X_L is called inductive reactance / opposition offered by an inductor to the flow of current. (Resistance equivalence)

SI units = Ω

for DC, $\omega = 0$, $\nu = 0$ $\therefore X_L = 0$.

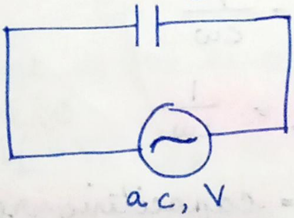
$\nu \propto X_L$ High ν = open circuit
Low ν = closed circuit.

2) Inductor coil; bulb in DC and AC:
When an ~~iron~~ iron core is introduced into the coil, what happens to the bulb's glow?



Effect of a capacitor in an AC circuit.

Consider a capacitor of capacitance C connected to an AC circuit. Let the charge on the plates of the capacitor at an instant of time be q and potential diff. bet. the plates of the capacitor be V.



$\tau = RC$
(Time const.)

Charging:
I = dec
V = inc
Q = inc

Discharging:
I = dec
V = dec
Q = dec.

$$i = i_0 e^{-t/\tau}$$

$$q = CV$$

$$I = \frac{dI}{dt} = \frac{d}{dt}(CV)$$

$$V = V_0 \sin \omega t \quad \text{—(1)}$$

$$I = \frac{d}{dt}(CV_0 \sin \omega t)$$

$$I = CV_0 \frac{d}{dt}(\sin \omega t)$$

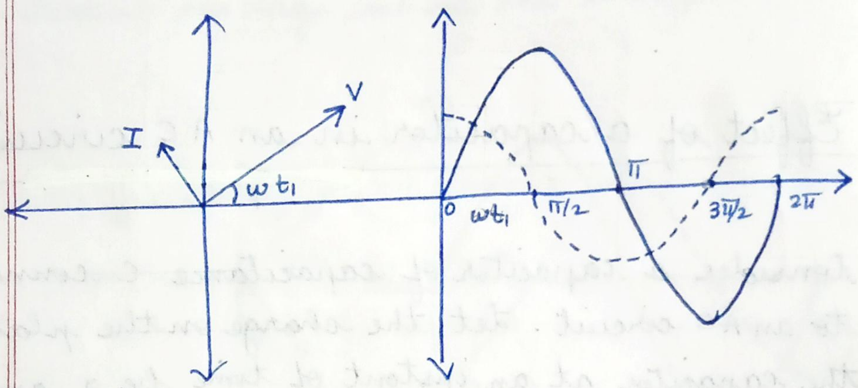
$$I = CV_0 \omega \cos \omega t$$

$I = \underbrace{CV_0\omega}\sin(\omega t + \pi/2)$. $\frac{V_0}{R} = \frac{V_0}{X_c} =$

$I = I_0 \sin(\omega t + \pi/2)$

$(I_0 = CV_0\omega)$ $\left(X_c = \frac{1}{C\omega}\right) \approx R$

From equation ① and ②, it is seen that current leads the voltage by a phase of $\pi/2$.



$I_0 = CV_0\omega$

$\frac{V_0}{I_0} = \frac{1}{C\omega}$

$X_c = \frac{1}{C\omega}$ ohm

X_c = capacitive reactance

It is the opposition offerred by the capacitance to the flow of current. (Resistance equivalent)

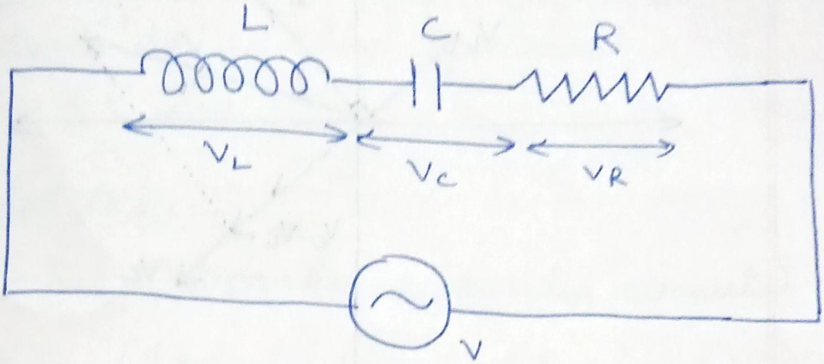
$\nu \propto \frac{1}{X_c}$

High Frequency — Short circuit

Low frequency – Open circuit

LCR circuit

5 marks



Consider an inductor of inductance L, capacitor of capacitance C and resist of resistance R connected in series to an alternating supply.

V_{OR} – Peak value of voltage across the resistor.
V_{OL} – Peak value of voltage across the inductor
V_{OC} – Peak value of voltage across the capacitor
I_0 – Peak Value of current in the circuit

The net voltage in the circuit is given as:

$$V_0 = V_L + V_C + V_R$$ as

Voltage across the capacitor lags behind the current and voltage across the inductor leads the current.

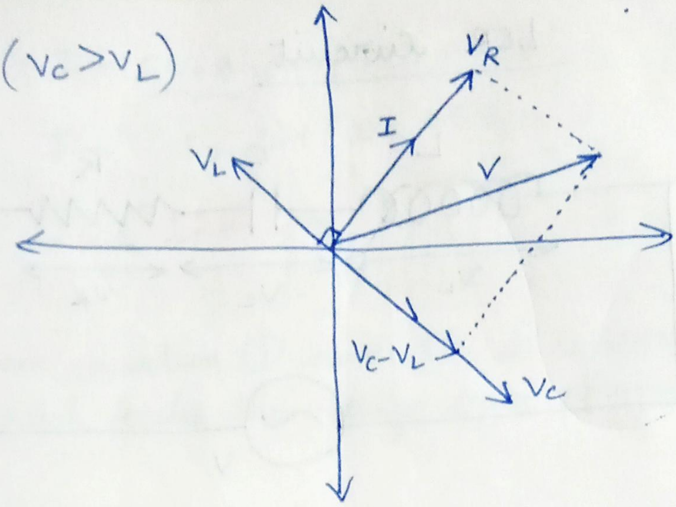
Therefore net voltage can be ~~done~~ found by using parallelogram law of vector addition in the phasor diagram.

$$V_0 = \sqrt{V_{OR}^2 + (V_{OC} - V_{OL})^2}$$

$$V_0 = \sqrt{I_0^2 R^2 + (I_0 X_C - I_0 X_L)^2}$$

$$\begin{bmatrix} V_{OR} = I_0 R \\ V_{OL} = I_0 X_L \\ V_{OC} = I_0 X_C \end{bmatrix}$$

$(V_C > V_L)$



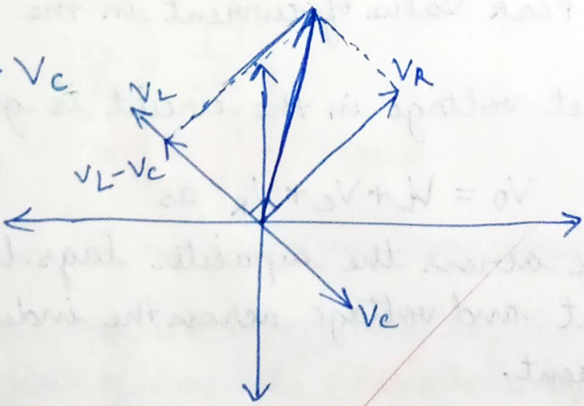
$$V_0 = I_0\sqrt{R^2 + (X_C - X_L)^2}$$

$$\frac{V_0}{I_0} = \sqrt{R^2 + (X_C - X_L)^2}$$

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$Z \Rightarrow$ Impedance.

$V_L > V_C$



Impedance : Opposition offered by all the 3 circuit elements, inductor, capacitor and resistance to the flow of current.

SI unit $= \Omega$.

Special Cases:

1) $X_C > X_L$ (Capacitor dominated circuit)
2) $X_L > X_C$ (Inductor dominated circuit)
3) $X_L = X_C$; $Z = R$;

$$L\omega = \frac{1}{C\omega} \Rightarrow \omega^2 = \frac{1}{LC} \Rightarrow \omega = \frac{1}{\sqrt{LC}}$$

$$2\pi\nu = \frac{1}{\sqrt{LC}} \; ; \; \nu_r = \frac{1}{2\pi\sqrt{LC}}$$ resonant frequency

When $X_L = X_C$, $Z = R$, impedance is minimum, current is maximum, power absorbed is maximum. This condition is called resonance condition of LCR circuit.

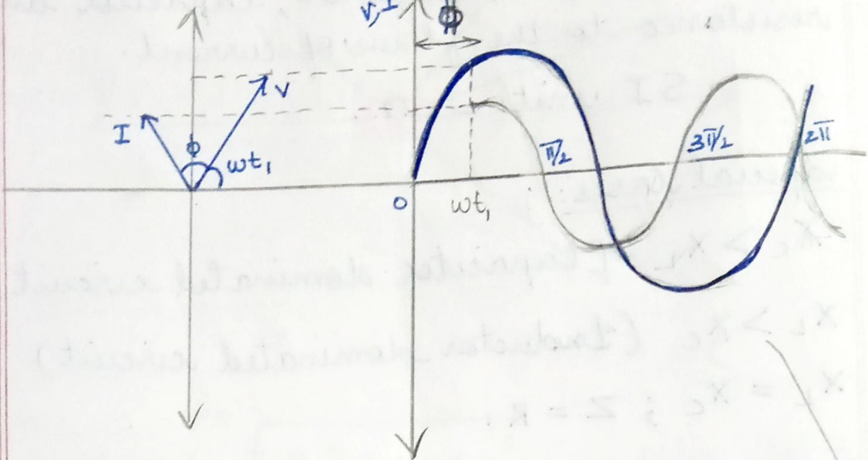
Applications of LCR circuit:

1) Tuner circuits in TV and Radio makes use of the condition of resonance in LCR circuit.

It is used ~~to~~ while tuning the frequency of LCR circuit is matched with the freq. of the incoming signal (i.e.,) the resonance condition is satisfied, power absorbed is max. and the signal is received with atmost clarity.

2) Metal Detector (Refer Eg. 7.10 Pg 225)

Impedance Triangle:

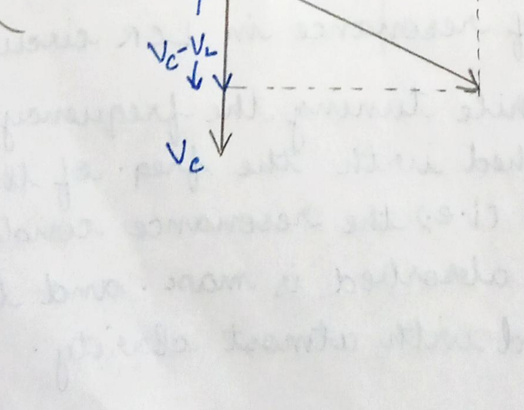


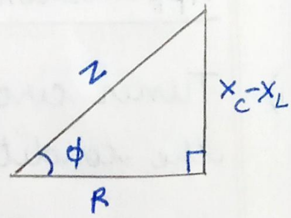
$$\cos\phi = \frac{R}{Z} \qquad Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$\cos\phi \Rightarrow$ power factor

$$\tan\phi = \frac{V_C - V_L}{V_R} = \frac{I_0 X_C - I_0 X_L}{I_0 R}$$

$$\tan\phi = \frac{X_C - X_L}{R}$$





Average power dissipated over one cycle

Resistors	Inductor	Capacitor
$P_{avg} = \frac{1}{T}\int_0^T P_{ins}\,dt$	$P_{avg} = \frac{1}{T}\int_0^T P_{ins}\,dt$	$P_{avg} = \frac{1}{T}\int_0^T P_{ins}\,dt$
$P_{ins} = V_{ins} I_{ins}$ $V_{ins} = V_0 \sin\omega t$ $I_{ins} = I_0 \sin\omega t$	$P_{ins} = V_{ins} I_{ins}$ $V_{ins} = V_0 \sin\omega t$ $I_{ins} = I_0 \cos\omega t$	$P_{ins} = V_{ins} I_{ins}$ $V_{ins} = V_0 \sin\omega t$ $I_{ins} = I_0 \cos\omega t.$
$P_{avg} = \frac{1}{T}\int_0^T V_{ins} I_{ins}\,dt$	$P_{avg} = \frac{1}{T}\int_0^T V_{ins} I_{ins}\,dt$	$P_{avg} = \frac{1}{T}\int_0^T V_0 \sin\omega t.\, I_0 \cos\omega t\,dt$
$= \frac{1}{T}\int_0^T V_0 \sin\omega t\, I_0 \sin\omega t\,dt$	$= \frac{1}{T}\int_0^T V_0 \sin\omega t(-I_0\cos\omega t)\,dt$	$= \frac{V_0 I_0}{2T}\int_0^T 2\sin\omega t\cos\omega t\,dt$
$= \frac{I_0 V_0}{T}\int_0^T \sin^2\omega t \cdot dt$	$= -\frac{V_0 I_0}{2T}\int_0^T 2\sin\omega t\cos\omega t\,dt$	$= \frac{V_0 I_0}{2T}\int_0^T \sin 2\omega t\,dt$
$= \frac{I_0 V_0}{2T}\int_0^T 1-\cos\omega t\,dt$	$= \frac{-V_0 I_0}{2T}\int_0^T \sin 2\omega t\,dt$	$= \frac{V_0 I_0}{2T}\left[\sin \cdot \frac{2\pi}{\cancel{T}}\cancel{T}\cdot 2\right]$
$= \frac{I_0 V_0}{2T}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T$	$= \frac{-V_0 I_0}{2T}\left[\sin 2\omega T\right]$	$= \frac{V_0 I_0}{2T} \cdot \sin 4\pi$
$= \frac{I_0 V_0}{2T}\left[T - 0\right]$	$= \frac{-V_0 I_0}{2T}\left[\sin 2 \cdot \frac{2\pi}{\cancel{T}}\cancel{T}\right]$	$\boxed{P_{avg} = 0}$
$= \frac{I_0 V_0 \cdot \cancel{T}}{2\cancel{T}}$	$= \frac{-V_0 I_0}{2T}\sin 4\pi$	
$P_{avg} = \frac{I_0 V_0}{2}$	$\boxed{P_{avg} = 0}$	
$P_{avg} = \frac{I_0}{\sqrt{2}} \cdot \frac{V_0}{\sqrt{2}}$		
$P_{avg} = I_{rms} V_{rms}$		
$\boxed{P_{avg} = \frac{V_0 I_0}{2}}$		

Average power dissipated over one cycle in LCR Circuit

$$P_{avg} = \frac{1}{T}\int_0^T P_{ins}\, dt$$

$$P_{ins} = V_{ins} I_{ins}$$

$$V_{ins} = V_0 \sin \omega t$$

$$I_{ins} = I_0 \sin(\omega t + \phi)$$

$$= \frac{1}{T}\int_0^T V_0 \sin\omega t \; I_0 \sin(\omega t + \phi)\, dt$$

$$= \frac{V_0 I_0}{T}\int_0^T \sin\omega t\,(\sin\omega t \cos\phi + \cos\omega t \sin\phi)\, dt$$

$$= \frac{V_0 I_0}{T}\left[\int_0^T \sin^2\omega t \cos\phi\, dt + \int_0^T \sin\omega t \cos\omega t \sin\phi\, dt.\right.$$

$$= \frac{V_0 I_0}{T}\cos\phi\left[\int_0^T \left(\frac{1-\cos 2\omega t}{2}\right) dt\right]$$

$$= \frac{V_0 I_0}{2\cancel{T}}\cos\phi \cdot \cancel{T} = \frac{V_0 I_0}{\sqrt{2}\cdot\sqrt{2}}$$

$$= V_{rms} I_{rms} \cos\phi.$$

$\cos\phi$ = power factor of LCR circuit

Special cases:

1) If $\phi = 0$, $Z = R$.

$$P_{avg} = V_{rms} I_{rms}$$

This corresponds to resonance condition of LCR circuit

It is a pure resistive circuit.

2) $\phi = \pi/2$,

$$P_{avg} = 0. \Rightarrow \text{Wattless currents}$$

This corresponds to purely inductive or purely capacitive circuit.

Such currents which do not dissipate any power are called as wattless currents.

Choke coil:

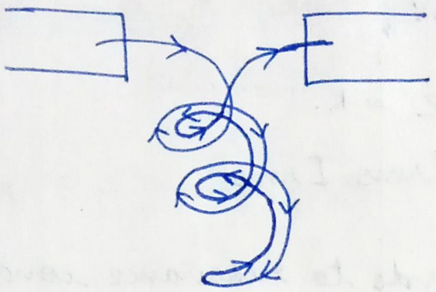
In AC circuits, resistors are used to restrict the flow of current, due to which there will be power loss. Therefore in AC circuits, capacitors or inductors are used to restrict the flow of current as they do not consume or dissipate power over one cycle.

Air core choke coil - high frequency

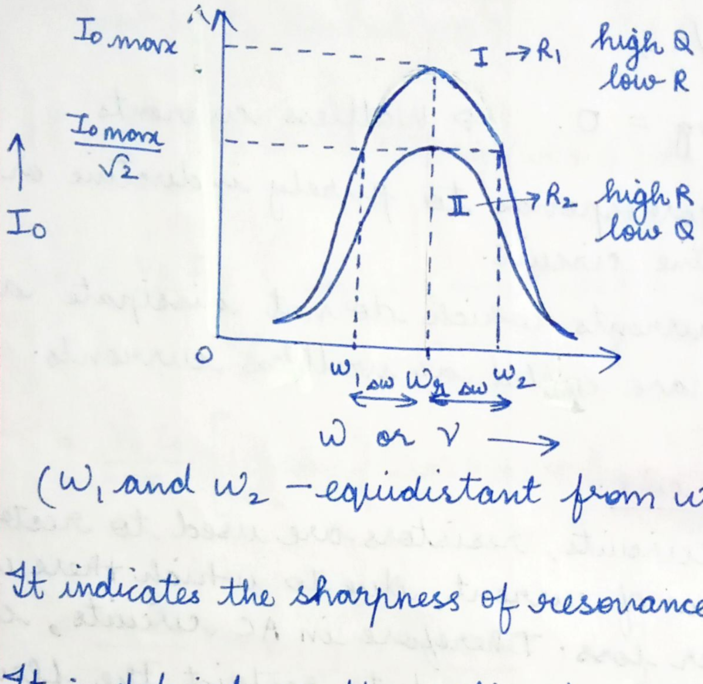
Iron core choke coil - low frequency.

Non inductive winding:

A coil doubly wound over itself provides no self induction as the current flowing in 2 turns being in opposite direction causes flux change in opposite direction. Hence they cancel each others' effect.



Quality Factor: (Q factor)



(ω_1 and ω_2 – equidistant from ω_r)

- It indicates the sharpness of resonance
- It is defined as the ratio of resonance frequency to bandwidth of frequency, i.e., difference between the frequencies for which the amplitude is $\frac{1}{\sqrt{2}}$ times the max value of current.

$$Q = \frac{\omega_r}{\omega_2 - \omega_1} = \frac{\omega_r}{2\Delta\omega}$$

$2\Delta\omega$ = Bandwidth

It is also defined as the ratio of voltage across the inductor or capacitor to voltage across the source at resonance.

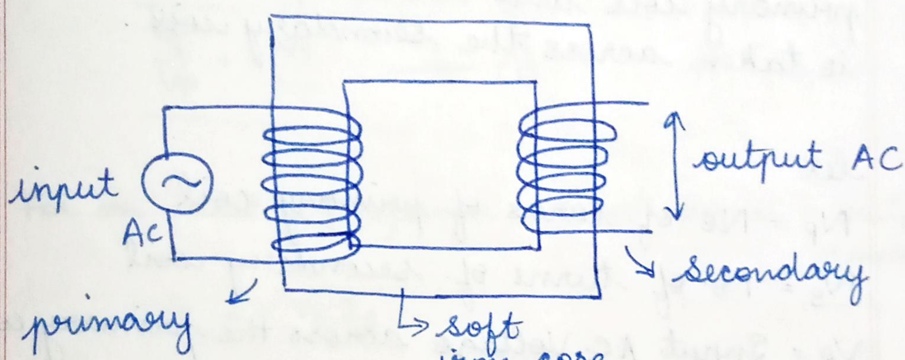
$$Q = \frac{V_L}{V_s} \text{ or } \frac{V_C}{V_s} \text{ at resonance.}$$

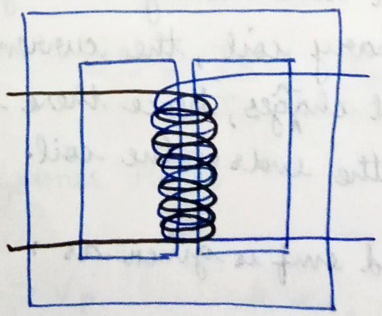
$$= \frac{I X_L}{I R} = \frac{L \omega_r}{R}$$

$$= \frac{L}{R}\sqrt{\frac{1}{LC}}$$

$$\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

Transformer





Step up: No of turns of Secondary coil is more

Step down: No. of turns of primary coil is more

It is a devices used to step up or step down the alternating voltage.

Principle: Mutual Induction (defn in exams)

Construction and Working:

It consists of 2 coils namely primary coil and secondary coil either wound on two separate limbs of a soft iron core or wound on the same limb of the soft iron core.

The input AC voltage is supplied to the primary coil and the output AC voltage is taken across the secondary coil.

let
N_P = No. of turns of primary coil
N_S = No of turns of secondary coil
V_P = Input AC Voltage across the primary coil
V_S = Output AC Voltage across the secondary coil.

As the input alternating voltage is supplied to the primary coil, the current through primary coil changes, hence there is an induced emf across the ends of the coil.

This induced emf is given as : $\varepsilon_P = -N_P \frac{d\phi}{dt}$

due to mutual induction, same flux change occurs through the secondary coil, hence induced emf across the secondary coil is :

$$\varepsilon_s = -N_s \frac{d\phi}{dt}$$

For an ideal transformer, the following 2 assumptio

- The primary coil has negligible resistance

  i.e; $V_p = \varepsilon_p$

- The secondary coil ~~has~~ is an open circuit.

  i.e; $V_s = \varepsilon_s$.

$$\therefore \quad V_p = -N_p \frac{d\phi}{dt}$$

$(\div)$

$$V_s = -N_s \frac{d\phi}{dt}$$

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} \quad \text{— (1)}$$

For an ideal transformer, input power = output power.

$$V_p I_p = V_s I_s.$$

$$\frac{V_s}{V_p} = \frac{I_p}{I_s} \quad \text{— (2)}$$

$$(1) = (2), \quad \frac{N_s}{N_p} = \frac{I_p}{I_s} = \frac{V_s}{V_p} = k$$

k- transformer ratio

$k > 1$, $V_s > V_p$	$k < 1$, $V_s < V_p$
$I_s < I_p$	$I_s > I_p$
$N_s > N_p$.	$N_s < N_p$
Step up transformer	Stepdown transformer

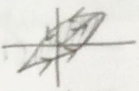
Efficiency of Transformer:

Ratio of output power to input power

$$\eta = \frac{\text{output power}}{\text{input power}} \times 100$$

$$= \frac{V_S I_S}{V_P I_P} \times 100.$$

No transformer has 100% efficiency due to energy loss by: (Refer pg - 261)

- Flux leakage - 2nd diagram
- Resistance of primary and secondary coil
- Eddy current loss. - lamination
- Hysteresis loss -  Area ∝ Energy loss

  (use substances in which magnetic retentivity is less (and less area under hysteresis loss)

ADVANTAGES OF TRANSFORMERS:

Pg 268 Large scale transmission of electrical energy over long distance with minimum energy loss.