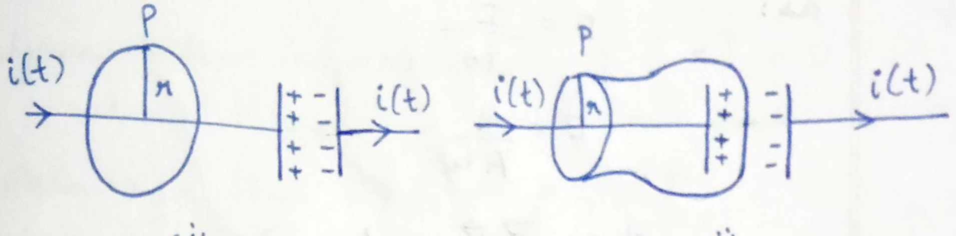
8. Electromagnetic Waves

Inconsistency in Ampere's circuital law and maxwell's correction



(i) (ii)

To determine magnetic field at point P, construct an amperean loop through a circle passing through point P with radius r, distance of point P from the wire. By applying ampere circuital law,

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I_0$$

$$\text{LHS}: \oint \vec{B}\cdot dl$$

$$= \oint B dl.$$

$$= B \oint dl = B(2\pi r) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r} \quad \text{— } ①$$

By using diagram 2, maxwell applied ampere circuital law to determine magnetic field at point P (pot like surface)

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I_e$$

$$I_e = 0$$

$$B = 0 \quad \text{— } ②$$

Eq. ① and ② shows that magnetic field at a given point found by using 2 different surfaces is different. This is the inconsistency found by maxwell while applying ampere circuital law.

To remove inconsistency he introduced the concept of displacement current.

$\vec{E}$ between the plates of capacitor is given as:

$$E = \frac{I}{\varepsilon_0}$$

$$E = \frac{Q}{A\varepsilon_0}$$

$$\phi E = \vec{E}\cdot\vec{A} = EA$$

$$\phi_E = \frac{Q}{A\varepsilon_0} \cdot A = \frac{Q}{\varepsilon_0}$$

$$\frac{d\phi_E}{dt} = \frac{1}{\varepsilon_0}\frac{dQ}{dt}$$

$$\frac{dQ}{dt} = \varepsilon_0 \frac{d\phi_E}{dt}$$

$$i_d = \frac{\varepsilon_0 d\phi_E}{dt}$$ = displacement current

It is defined as current which arises due to variation of electric field with time.

$\therefore$ 2 becomes $B = \frac{\mu_0 I_e}{2\pi r}$, $I_e = i_d$.

$$B = \frac{\mu_0 i}{2\pi r}$$

Maxwell modified ampere circuital law

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 (i_c + i_d)$$

$$= \mu_0 i_c + \mu_0 \frac{\varepsilon_0 d\phi}{dt}$$

Maxwell's 4 Equations

1. Gauss's law in electrostatics $\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$

2. Gauss's law in magnetism $\oint \vec{B} \cdot d\vec{A} = 0$

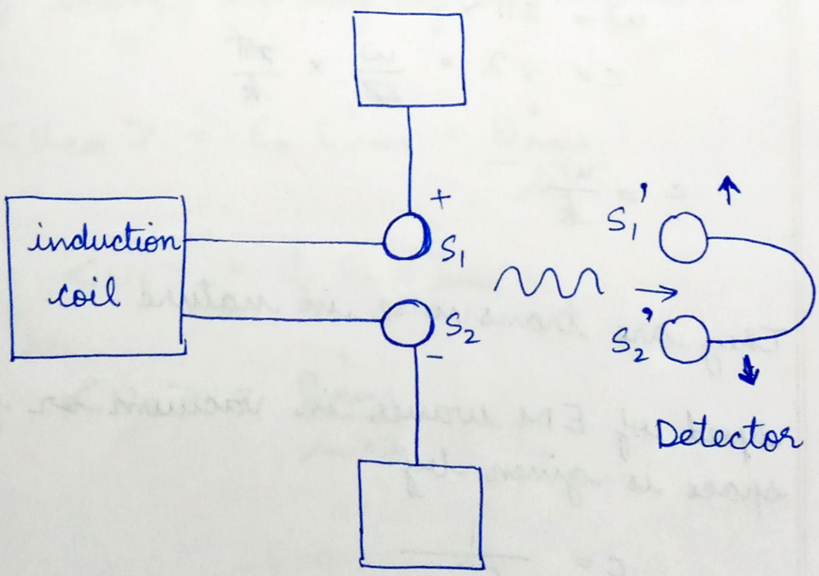
3. Faraday's law $\oint \vec{E} \cdot d\vec{l} = -\frac{d\phi}{dt}$.

4. Modified Version of ampere's circuital law $= \oint \vec{B} \cdot d\vec{l} = \mu_0 i_c + \mu_0 i_d$
$= \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\phi}{dt}$.

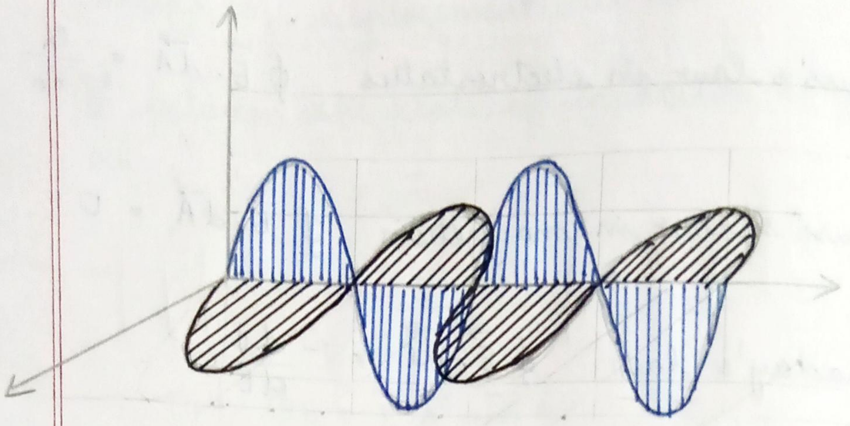
Production of EM waves

Source : accelerated charge (oscillating charge)

Hertz experiment : (Refer reader).



Properties of EM waves



(i) It consists of elec. and mag. field mutually $\perp^r$ to each other and $\perp^r$ in the direction of propagation of wave.

(ii) $\vec{E} = E_0 \sin(kx - \omega t)\hat{j}$

$\vec{B} = B_0 \sin(kx - \omega t)\hat{k}$

$k = \frac{2\pi}{\lambda} \Rightarrow$ propagation constant

$$\omega = 2\pi\nu$$

$$c = \nu\lambda = \frac{\omega}{\cancel{2\pi}} \times \frac{\cancel{2\pi}}{k}$$

$$c = \frac{\omega}{k}$$

(iii) They are transverse in nature

(iv) Speed of EM waves in vacuum or free space is given by:

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

In medium,

$$v = \frac{1}{\sqrt{\mu \varepsilon}}$$

$$= \frac{1}{\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}}$$

$$v = \frac{c}{\sqrt{\mu_r \varepsilon_r}}$$

(v) At any instant of time, it can be shown that,

$$c = \frac{E_0}{B_0} = \frac{E}{B}$$

(vi) EM waves carry energy and momentum

If total energy of EM waves incident on a surface

(vii) Avg. energy density of EM waves is given as:

$$\langle u_{em} \rangle = \varepsilon_0 E_{rms}^2 = \frac{B_{rms}^2}{\mu_0}$$

$$\langle U_E \rangle = \frac{1}{2} \varepsilon_0 E_{rms}^2$$

$$\langle U_B \rangle = \frac{B_{rms}^2}{\mu_0 \times 2}$$

Intensity of electromagnetic wave :-

Energy of the wave ~~indicient~~ incid per unit area, per second.

$$\text{Intensity} = \frac{\text{energy}}{\text{area} \times \text{time}}$$

$$= \frac{\text{energy}}{\text{area} \times \text{dist.}} \times \frac{\text{dist}}{\text{time}}$$

$$\text{Intensity} = u \times c$$

Pointing vector :-

Represents the direction of progation of wave

$$\vec{S} = \frac{1}{\mu_0} (\vec{E} \times \vec{B})$$