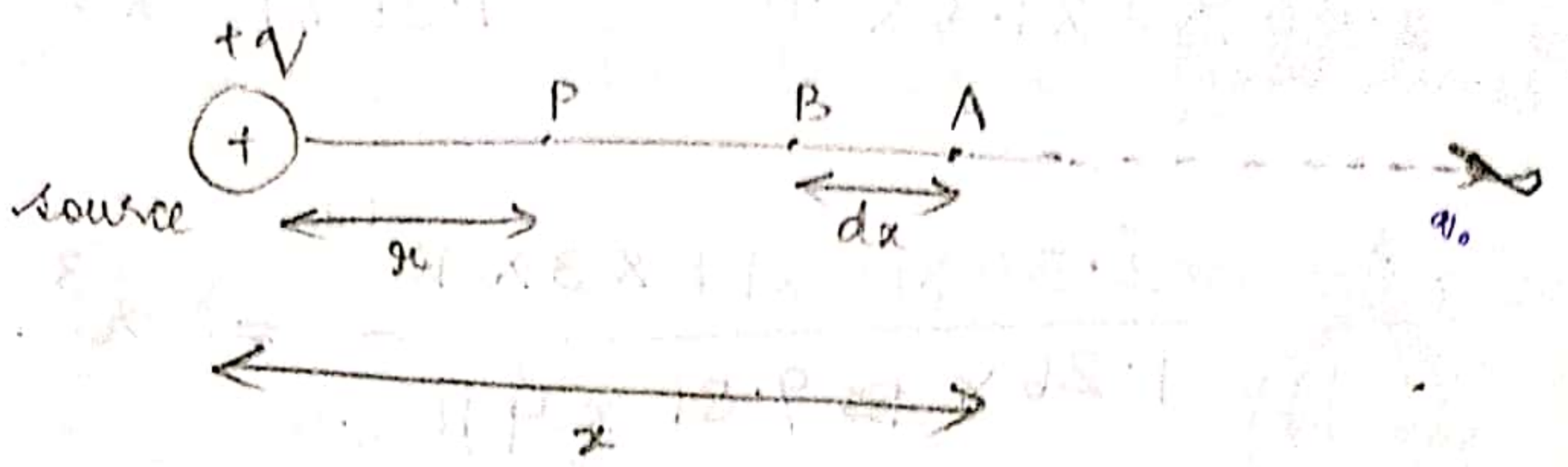
# Chapter 2: Electrostatics - II

**Electrostatic potential:** Work done per charge in bringing a positive test charge from infinity to that point in an electric field.

## ELECTROSTATIC POTENTIAL AT A POINT DUE TO A POINT CHARGE




Consider a positive point charge "+q" located as shown. Let P be a point at a distance "r" from the charge. Work done in moving a small test charge "$q_0$" from A to B through a distance dx is given as:

$$dW = \vec{F}_{ext.} \cdot dx.$$

$$= F_{ext}\, dx \cos\theta$$

$$= F_{ext}\, dx\,(-1) \qquad (\theta = 180^\circ)$$

$$= -\frac{kqq_0}{x^2}\, dx.$$

Total WD in moving the test charge from infinity to point P:

$$\int dW = \int_r^\infty -\frac{kqq_0}{x^2}\, dx.$$

$$W = -kqq_0 \int_r^\infty \frac{1}{x^2}\, dx = -kqq_0 \int_r^\infty x^{-2}\, dx.$$

$$= -kqq_0 \left[-x^{-1}\right]_r^\infty$$

$$= -kqq_0 \times \left[\frac{1}{x}\right]_r^\infty$$

$$= -kqq_0 \left[\frac{1}{\infty} - \frac{1}{r}\right]$$

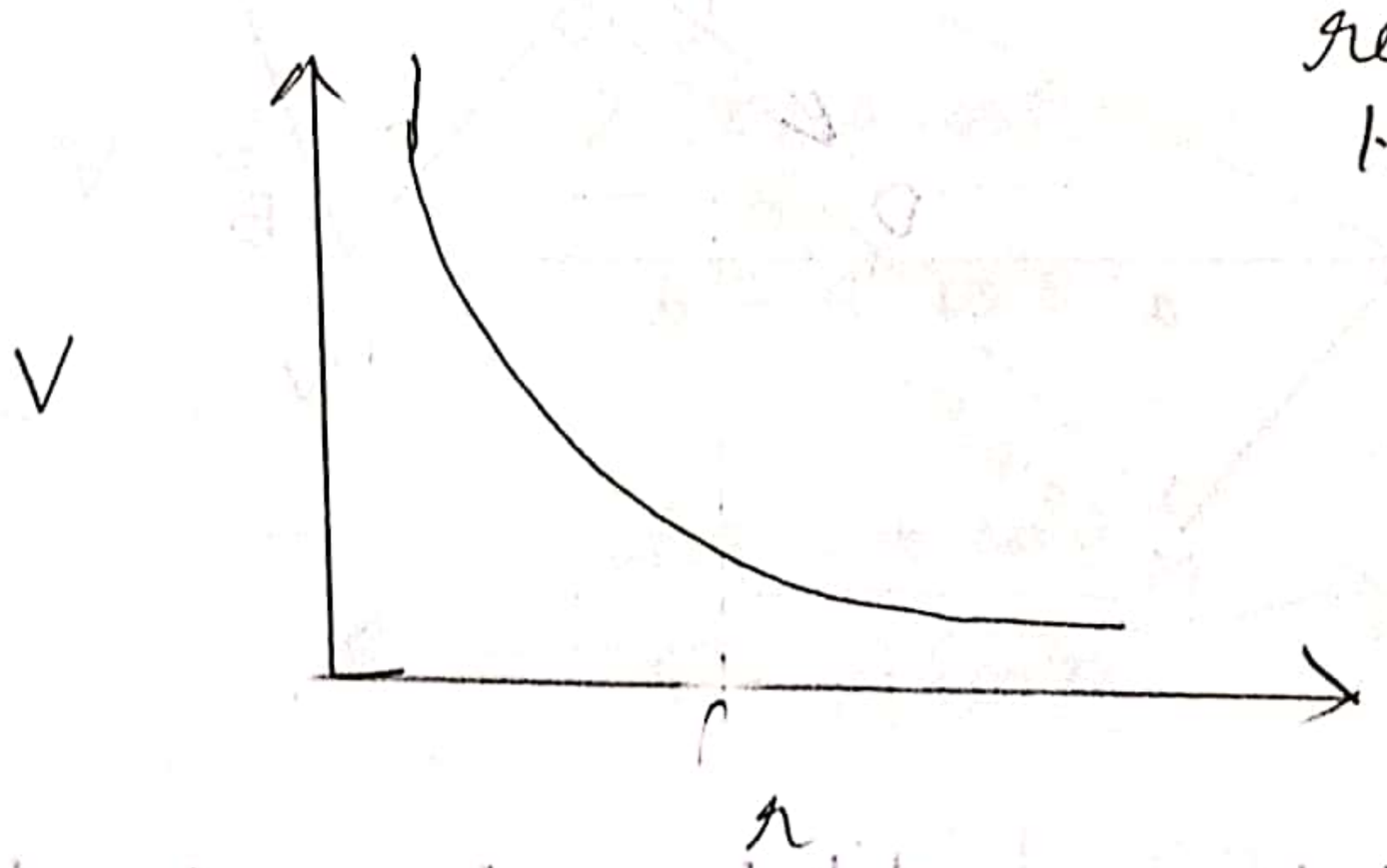
$$W = \frac{kqq_0}{r}$$

$$W = \frac{kqq_0}{r}.$$

$$V = \frac{W}{q_0} = \frac{kqq_0}{r} \times \frac{1}{q_0} = \frac{kq}{r}.$$

Potential due to "q" at dist. "r".

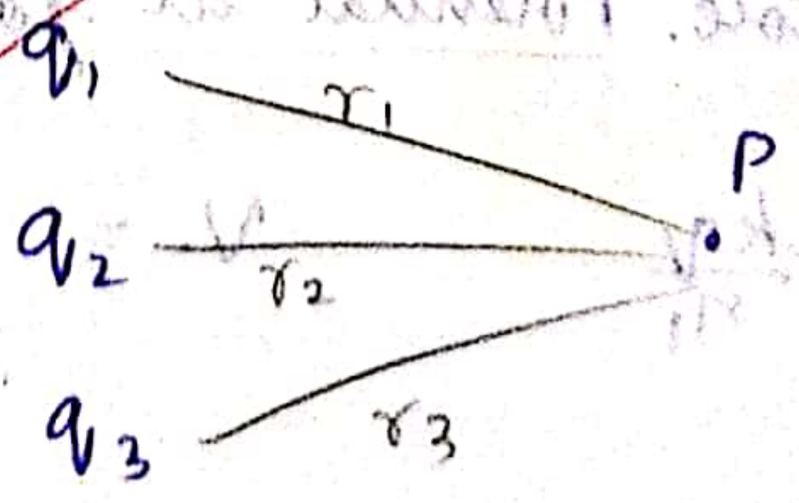
Units : Joules/Coulomb
= Volt.




rectangular Hyperbola.

Potential due to a point charge

## Potential at a point due to many charges



$$V_{q_1} = \frac{kq_1}{r_1} \qquad V_{q_2} = \frac{kq_2}{r_2} \qquad V_{q_3} = \frac{kq_3}{r_3}$$

$$V = V_{q_1} + V_{q_2} + V_{q_3}$$

$$= \frac{kq_1}{r_1} + \frac{kq_2}{r_2} + \frac{kq_3}{r_3}$$

Total potential at a point due to many charges is the algebraic sum of potentials due to individual charges.

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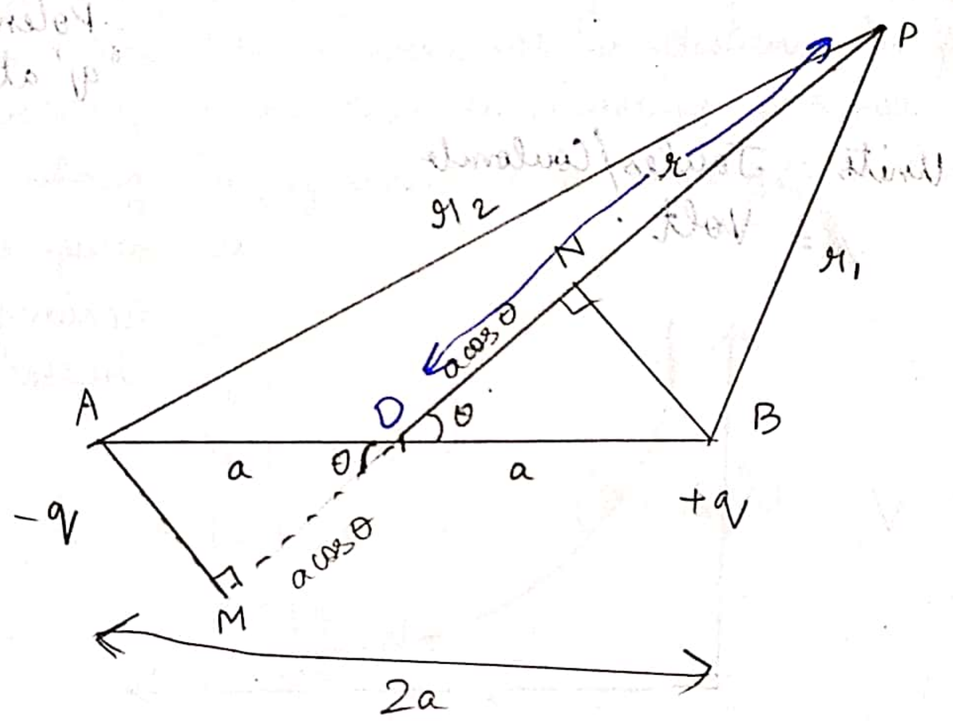
## Potential at a far off point due to dipole:-




~~to~~ Let $+q$ & $-q$ constitute 2 charges of a dipole as shown. Let P be a far off point at distance $r_1$ from $+q$ and distance $r_2$ from $-q$. Let $\theta$ be the angle subtended by point P at the centre O of the dipole. Potential at point P due to $+q$:

due to $-q$:

$$V_+ = \frac{kq}{r_1} \qquad V_- = -\frac{kq}{r_2}$$

$BP = NP = OP - ON$

In $\Delta OBN$

$$\frac{ON}{OB} = \cos\theta$$

$$ON = OB\cos\theta$$
$$= a\cos\theta$$

$$BP = OP - ON$$
$$= r - a\cos\theta$$

$AP = MP = OP + OM$

$\Delta OAM$,

$$\frac{OM}{OA} = \cos\theta$$

$$OM = a\cos\theta.$$

$$AP = OP + OM$$
$$= r + a\cos\theta.$$

$$V = \frac{kq}{r - a\cos\theta} - \frac{kq}{r + a\cos\theta}$$

$$= \frac{\cancel{kqr} + kqa\cos\theta - \cancel{kqr} + kqa\cos\theta}{r^2 - a^2\cos^2\theta}$$

$$V = \frac{2kqa\cos\theta}{r^2 - a^2\cos^2\theta}$$

$$= \frac{kq\,2a\cos\theta}{r^2 - a^2\cos^2\theta}$$

$$\boxed{V_{\text{arbitrary pt.}} = \frac{kp\cos\theta}{r^2 - a^2\cos^2\theta}.}$$

For short dipole, $r >>> a$.

$$\therefore \boxed{V_{\text{arbitrary pt}} = \frac{kp\cos\theta}{r^2}}$$

1) $\theta = 0^\circ$ (angle bet. OP & towards +q)

$$V = \frac{kq}{r^2}$$

$\theta = 180^\circ$

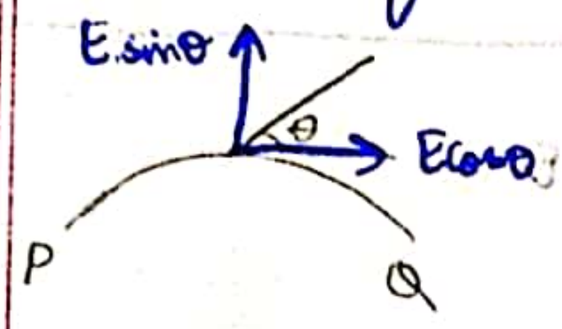
$$V = -\frac{kq}{r^2}$$

$\theta = 90^\circ$

$V = 0.$

## Equipotential Surfaces

- Imaginary surfaces drawn around a charge in an electric field where potential is the same.
- Electric field is always $\perp^r$ to equipotential surface

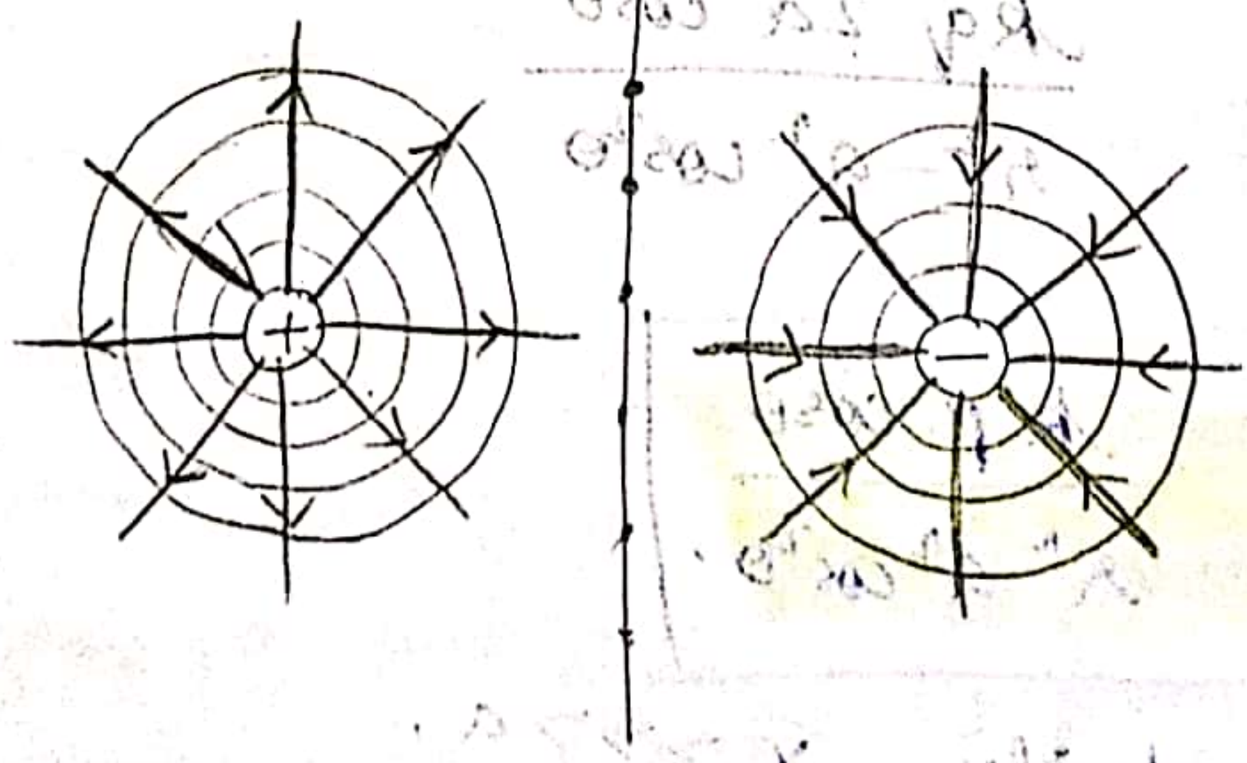



PQ - equipotential surface.

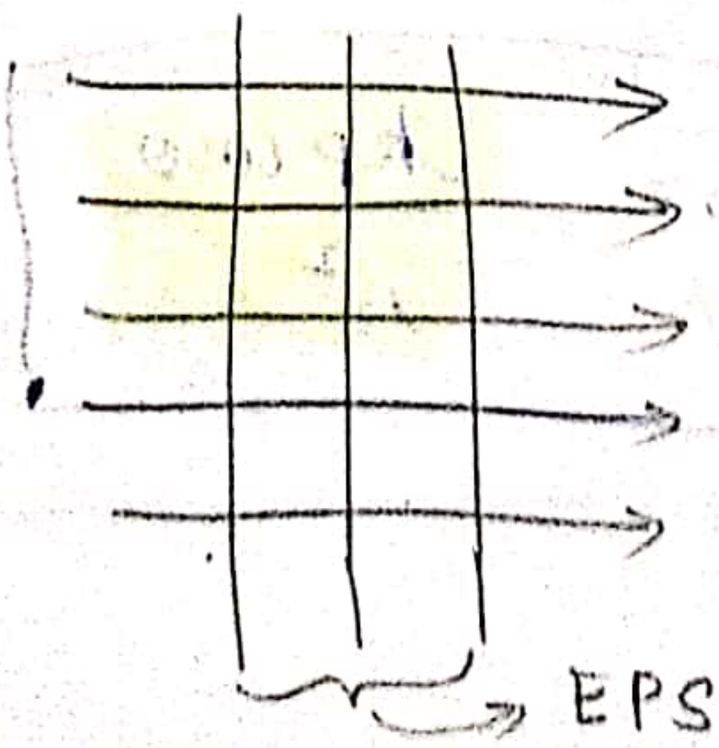
Ecosθ must be zero.

$\cos\theta = 0$ $\theta = 90^\circ$ — Normal

- 2 equipotential surfaces do not intersect
- No work is done in moving a charge on an equipotential surface

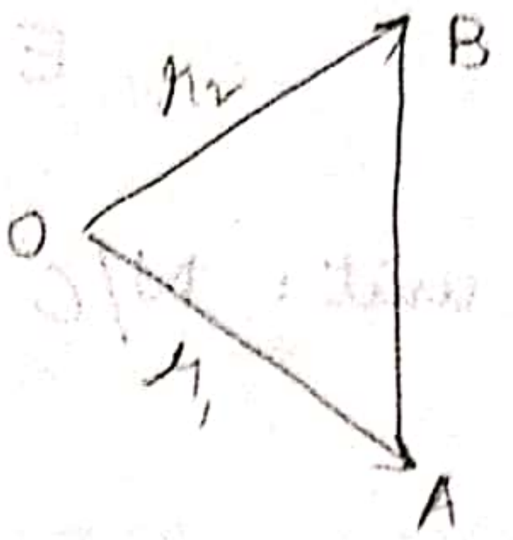


Ref. Pg-60



Potential difference between 2 points in an electric field

$$W = -\int_{r_1}^{r_2} \frac{kqq_0}{x^2}\,dx.$$




$$V = \frac{W}{q_0} = -\int_{r_1}^{r_2} \frac{kq}{x^2}\,dx.$$

$$V = -kq\left[-\frac{1}{x}\right]_{r_1}^{r_2}$$

$$= kq\left[\frac{1}{x}\right]_{r_1}^{r_2}$$

$$= \frac{kq}{r_2} - \frac{kq}{r_1}$$

P.d = V = Final potential - Initial potential.

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## Relation between Electric field and potential

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$$W = -\int \frac{kqq_0}{x^2}\,dx$$

$$V = \frac{W}{q_0} = -\int \frac{kq}{x^2}\,dx.$$

$$V = -\int \frac{kq}{x^2}\,dx.$$

$$V = -\int \vec{E}\cdot d\vec{x}.$$

$$dV = -\vec{E}\cdot d\vec{x}$$

Electric field = negative space rate of change of potential

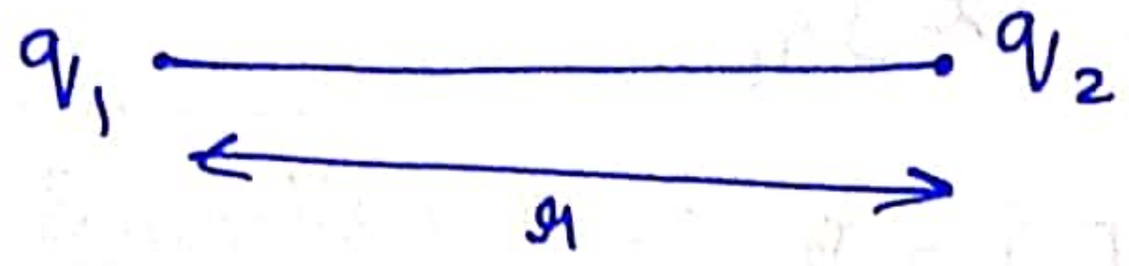
$$E = -\frac{dV}{dx}.$$ (Change in potential over a distance)

## For uniform electric field

$$E = \frac{V}{x}$$

unit : N/C or V/m.

## PE of a system of 2 charges




WD in bringing $q_1 = W_1 = 0$

WD in bringing $q_2 = W_2 = Vq_2$

$$= \frac{kq_1q_2}{r}$$

$$P.E = \text{Total } \cancel{\text{work}}\ W = W_1 + W_2$$

$$= 0 + \frac{kq_1q_2}{r}$$

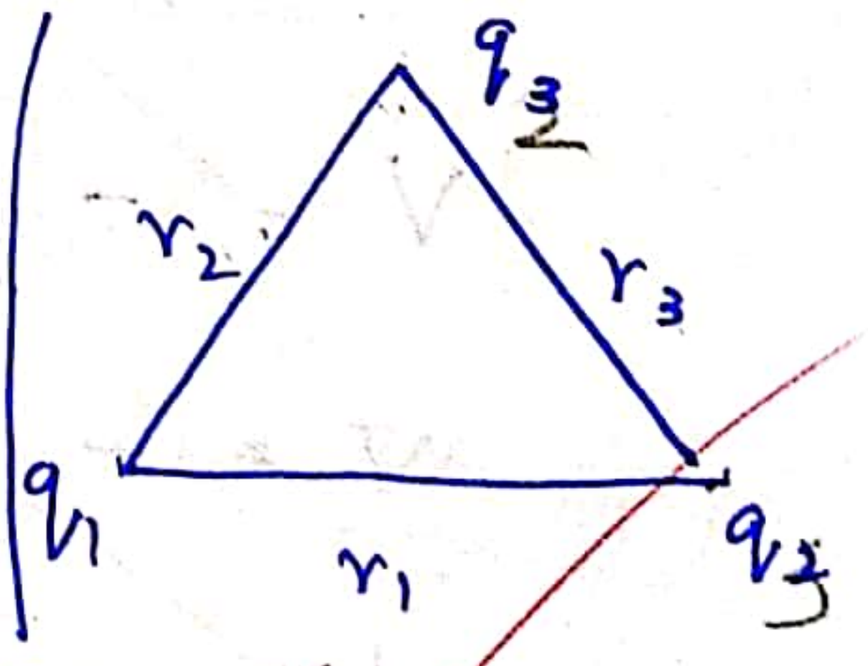
$$\boxed{PE = \frac{kq_1q_2}{r}}$$

## PE of a system of 3 charges

$W_1 = 0$

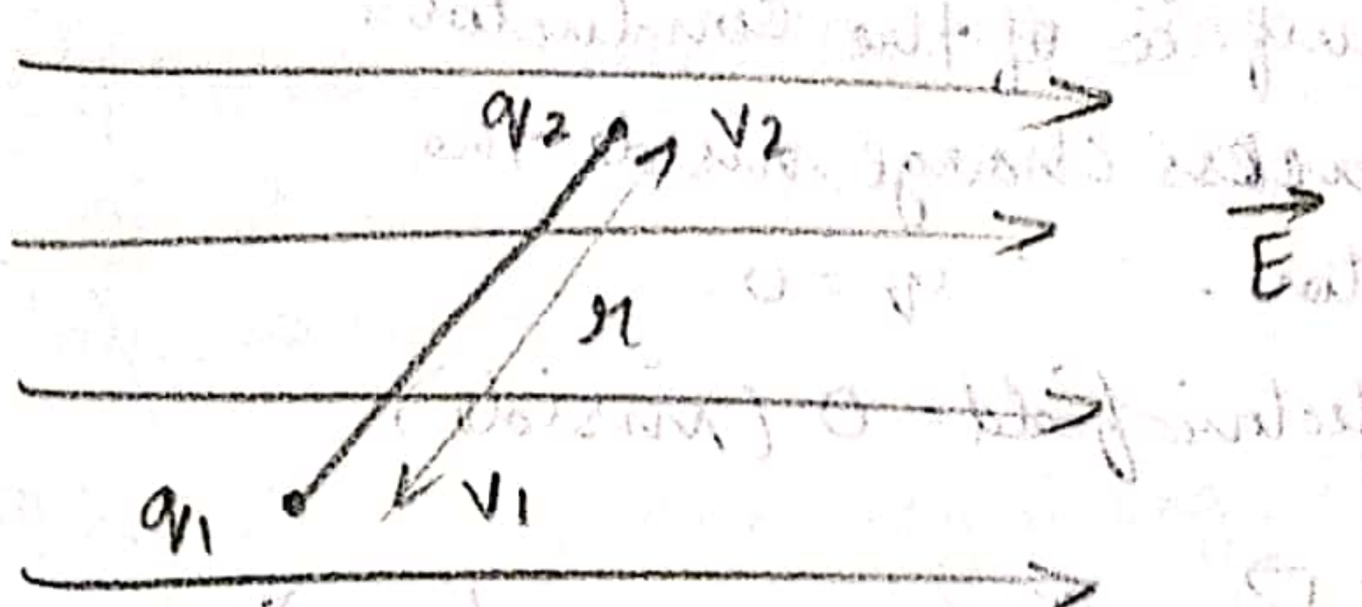
$W_2 = \frac{kq_1}{r_1} \times q_2$

$W_3 = \frac{kq_1}{r_2} \times q_3 + \frac{kq_2}{r_3} q_3$




$$\text{Total WD} = \frac{kq_1q_2}{r_1} + \frac{kq_1q_3}{r_2} + \frac{kq_2q_3}{r_3}$$

## PE of 2 charges on an external uniform electric field :




WD in bringing $q_1 = W_1 = V_1 q_1$

WD in bringing $q_2 = W_2 = V_2 q_2 + \frac{k q_1 q_2}{r}$

Total WD $= V_1 q_1 + V_2 q_2 + \frac{k q_1 q_2}{r}$

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Suppose the charges are replaced by a dipole

$q_1 = +q \qquad q_2 = -q \qquad r = 2a.$

Total WD $= +V_1 q - V_2 q - \frac{k q^2}{2a}$

$= q(V_1 - V_2) - \frac{k q^2}{2a}$

$= q(-\vec{E} \cdot \vec{dx}) - \frac{k q^2}{2a}$

$= q(-\vec{E} \cdot \vec{2a}) - \frac{k q^2}{2a}$

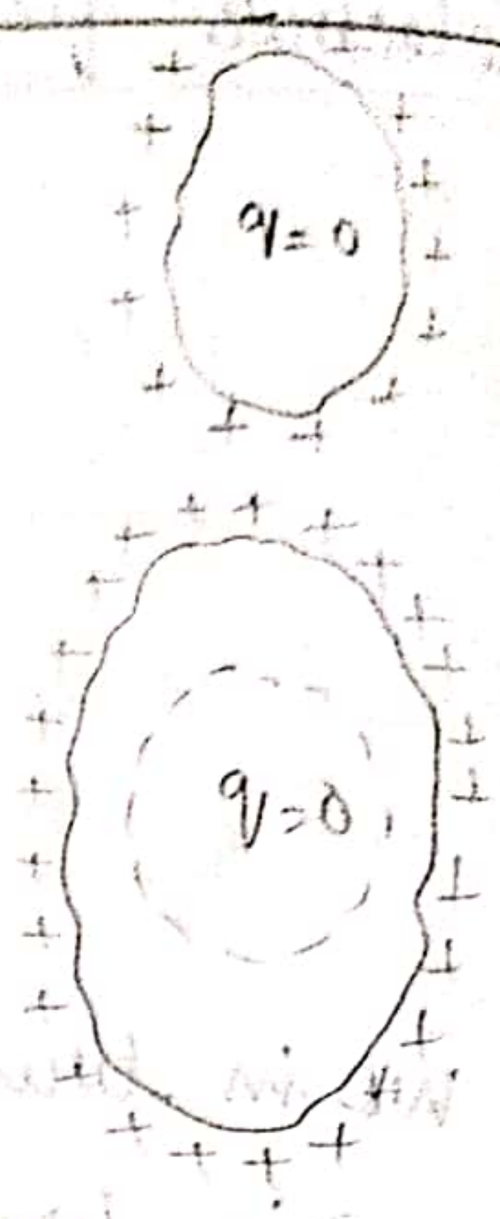
$= -qE\,2a\cos\theta - \frac{k q^2}{2a}$

$$\boxed{\text{Potential Energy} = -pE\cos\theta - \frac{k q^2}{2a}}$$

of a dipole

# Properties of a conductor given a charge

1) All the charges ~~beside~~ reside on the outer surface of the conductor.
∴ No excess charge inside the conductor. ∴ $q = 0$.




2) ∴ Electric field = 0 (inside)

$$\left( \oint \vec{E} \cdot d\vec{s} = \frac{q}{\varepsilon_0} = 0. \quad \therefore \vec{E} = 0. \right)$$

3) As $\vec{E}$ inside the conductor is zero, the entire volume of the conductor is an equipotential volume

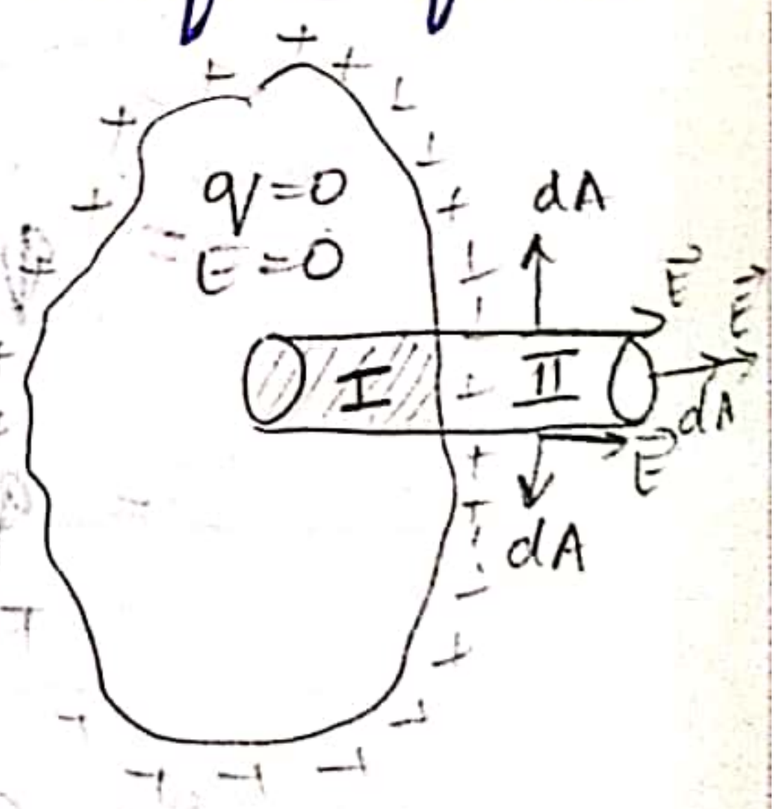
$$E = -\frac{dV}{dx} \qquad E = 0$$

$$0 = -\frac{dV}{dx} \qquad \therefore V = \text{const}.$$

4) $\vec{E}$ is always along the normal to the surface of the conductor

5) $\vec{E}$ at a point just outside the surface of the conductor is given as:

$$\vec{E} = \frac{\sigma}{\varepsilon_0} \hat{i}.$$




I:

$$\oint \vec{E} \cdot d\vec{x} = 0.$$

$$E = 0$$

II:

$$\oint \vec{E} \cdot d\vec{s} = \int_{CS} E \cdot ds + \int_{A} E \cdot ds$$

$$\oint \vec{E} \cdot d\vec{s} = 0 + \int_{A} E\, ds = EA = \frac{q}{\varepsilon_0}.$$

$$q = \sigma A.$$

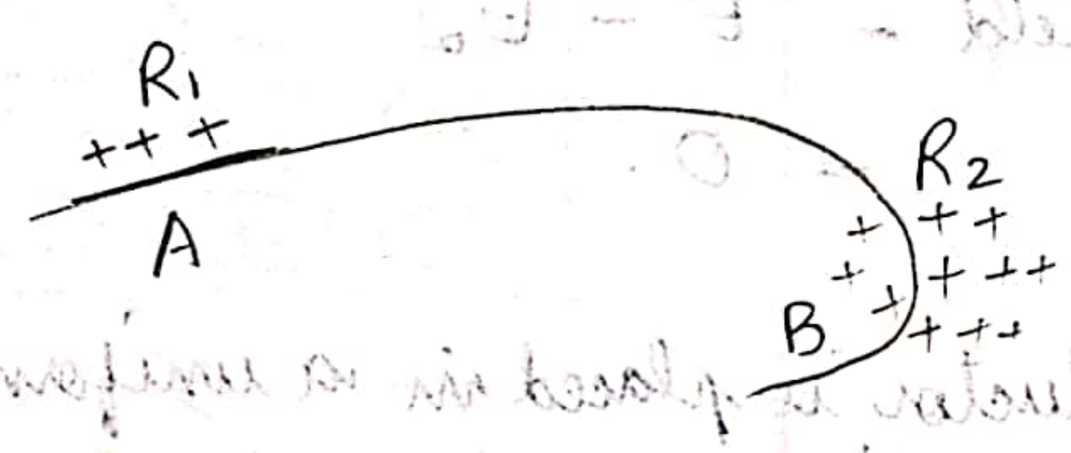
$$\boxed{\vec{E} = \frac{\sigma}{\varepsilon_0} \hat{i}}$$

6) Electrostatic shielding

- As charge inside a conductor is zero and there is no electric field, sensitive instruments can be protected by keeping them inside the hollow of a charged conductor. This is called electrostatic shielding.

- During thunderstorm and lightning, it is better to sit inside a car than standing outside.

7) $\sigma \propto \frac{1}{R}$ (R = Radius of curvature)




$R_1 > R_2$

$V_A = V_B$

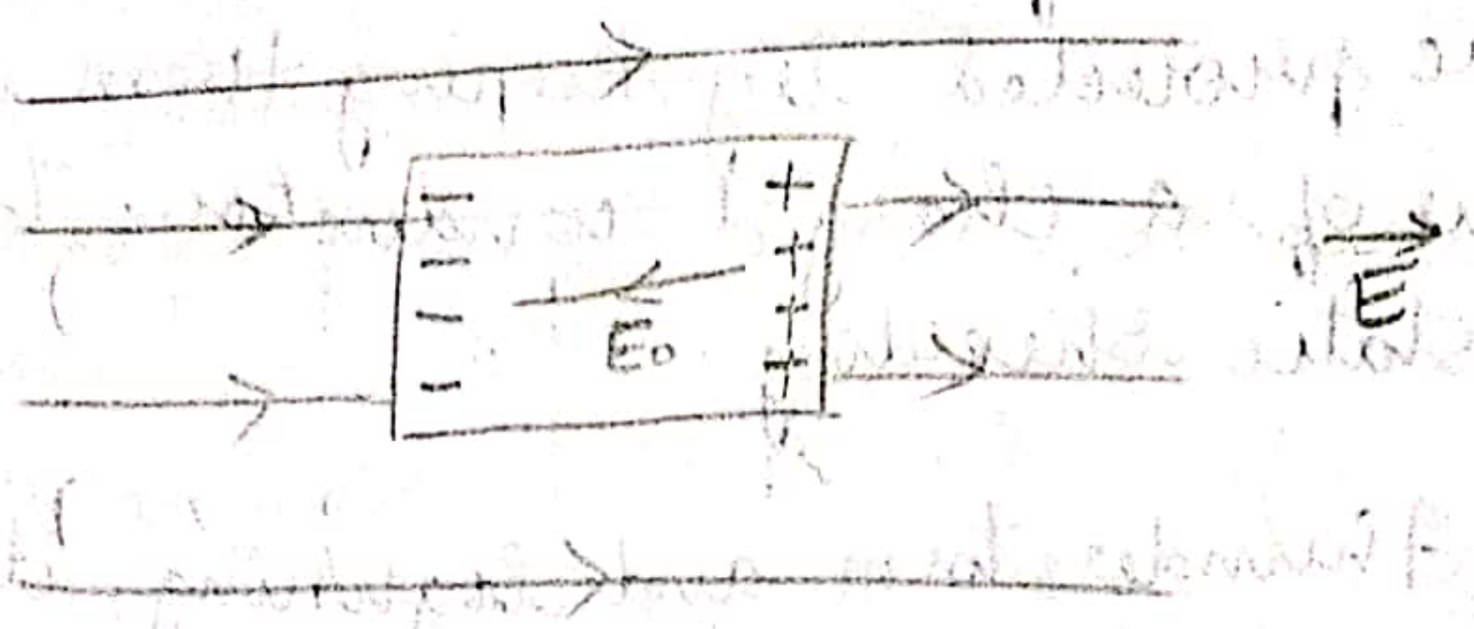
$$\frac{kq_1}{R_1} = \frac{kq_2}{R_2}$$

$$\frac{k\sigma_1 4\pi R_1^2}{R_1} = \frac{k\sigma_2 4\pi R_2^2}{R_2}$$

$$\sigma_1 R_1 = \sigma_2 R_2.$$

$$\frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1} \Rightarrow \sigma \propto \frac{1}{R}.$$

## Behaviour of a conductor in an uniform external elec. field




$$\vec{F} = q\vec{E}$$

$$\vec{F} = -q\vec{E}$$

$$\text{Net field} = \vec{E} - \vec{E_0}$$

$$= 0.$$

When a conductor is placed in a uniform electric field, there is displacement of charges.

+ve charges move along the field and -ve charges move opposite to the field. Hence they accumulate on either side of the conductor as shown.

Hence an internal field is created which is equal and opposite to the external field.

Therefore, Net field $= \vec{E} - \vec{E_0} = 0.$

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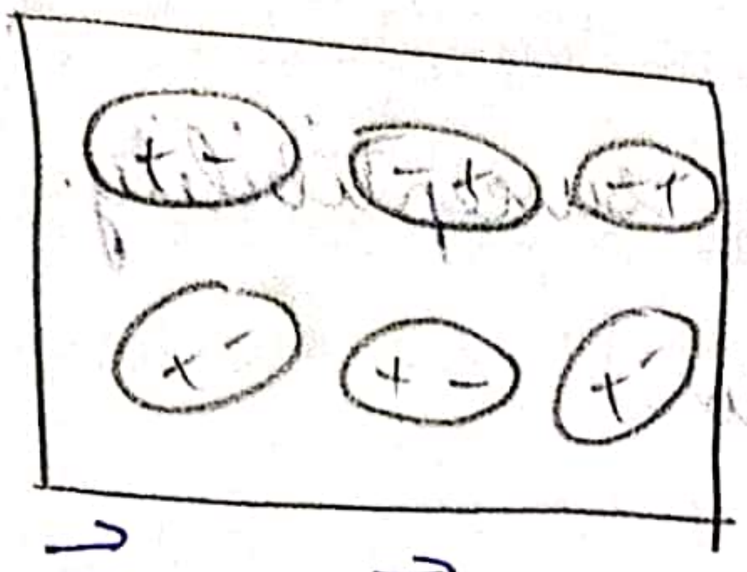
## Behaviour of a dielectric in an uniform elec. field

Dielectrics – Insulators that can be made of either polar or non polar molecules.

Polar molecules – Molecules with permanent dipole moment ($H_2O$, HCl)

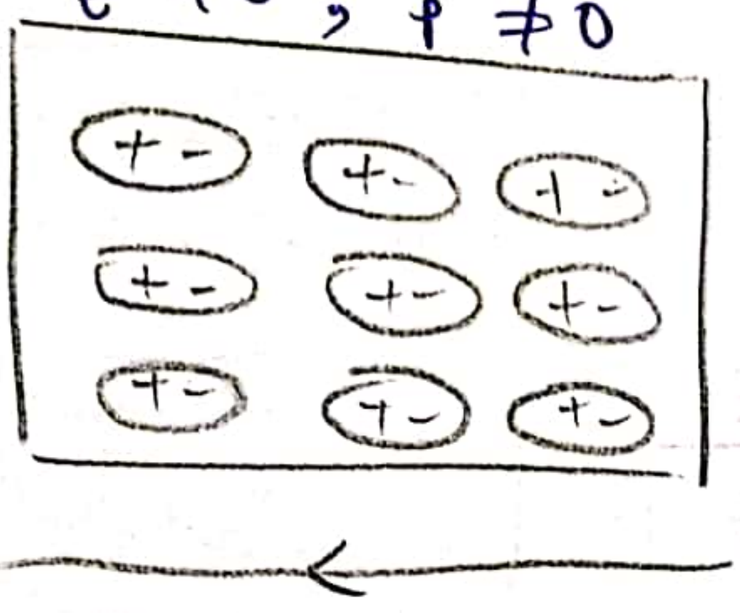
Non-polar molecules – Molecules without permanent dipole moment ($H_2$, $N_2$)

## POLAR DIELECTRIC



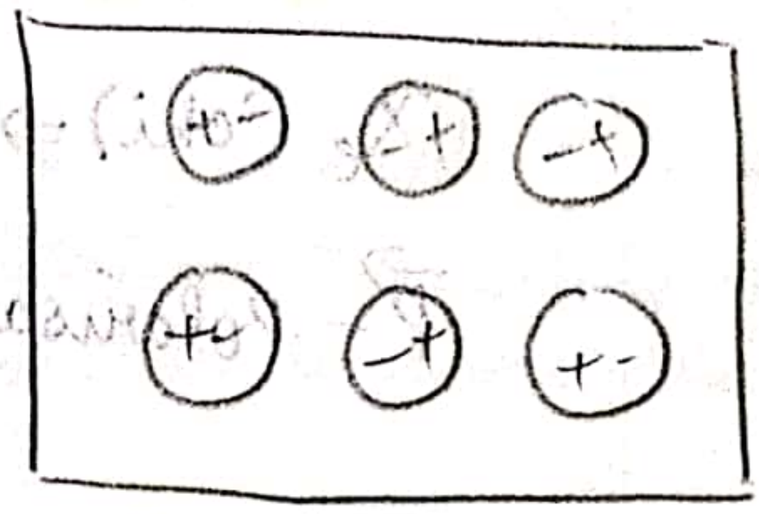
$\vec{P} = 0 \quad \vec{E} = 0$

$\vec{E} \neq 0, \vec{P} \neq 0$



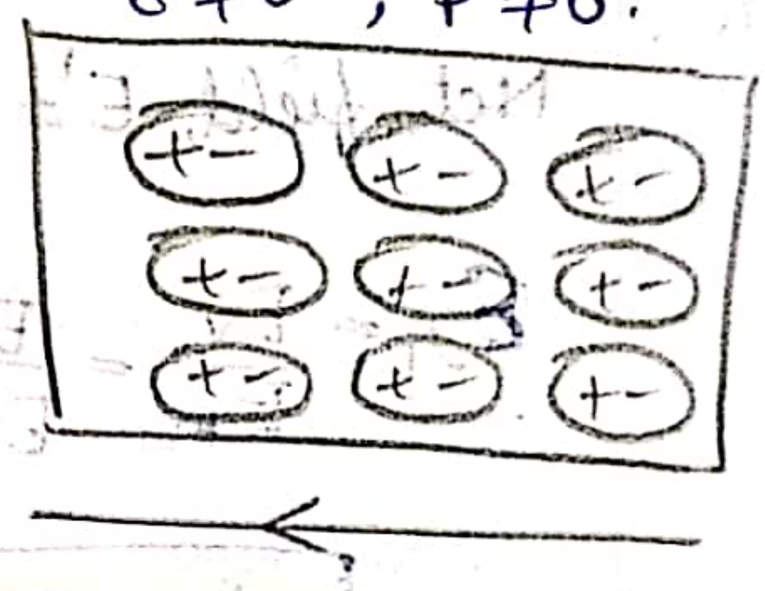
- In the absence of the external field, the dipole moment of all the molecules are oriented in random possible directions. Hence net dipole moment is 0.
- When it is placed in an external field, there is displacement of charges and ~~st~~ every molecule aligns its dipole moment along the field direction. Hence, there is a net dipole moment developed along the external field

## NON POLAR DIELECTRIC



$\vec{P} = 0, \vec{E} = 0$

$\vec{E} \neq 0, \vec{P} \neq 0.$



There is no dipole moment in any of the molecules. Hence in the absence of External field, dipole moment is 0.

In the presence of an external field, charges displace themselves and there is a net dipole moment developed along the external field direction.

Hence whether its polar/non polar dielectric, there is a net dipole moment developed in the presence of external elec. field. This net dipole moment per unit volume is called as Polarisation.

## Polarisation :

$$\vec{P} \propto \vec{E}$$

$$\vec{P} = \chi_e \vec{E}$$

$\chi_e$ (chi) ⇒ Electric susceptibility.

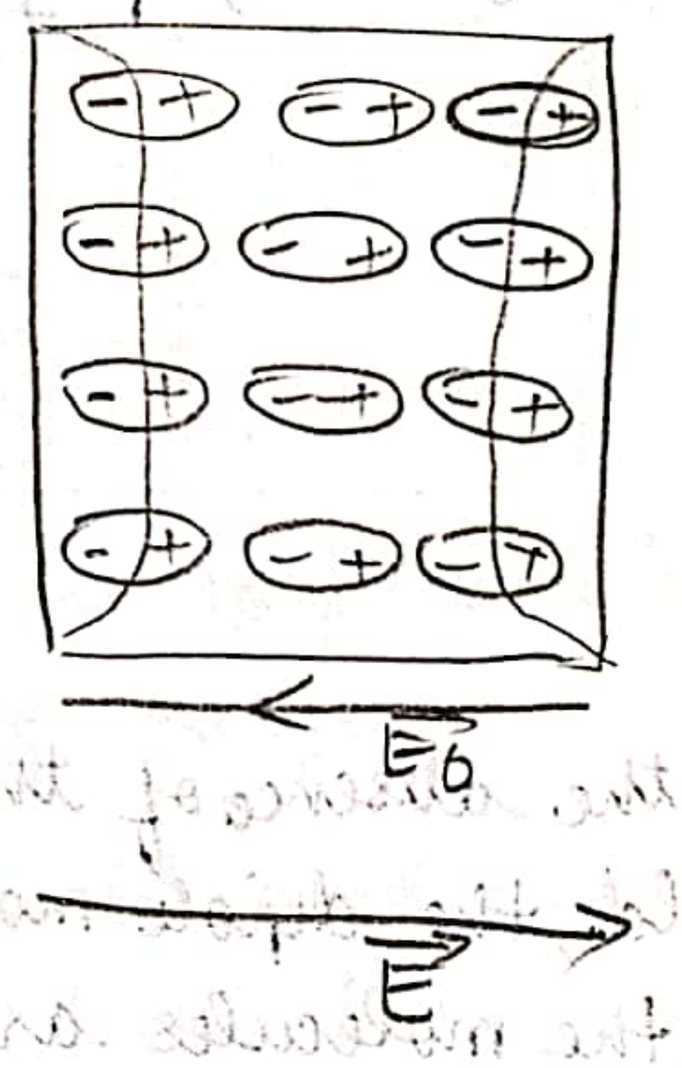
$\vec{P}$ – Polarisation Vector.

How does a polarised dielectric modify the elec. field within it?

Net field $E' = E - E_0$

$$\varepsilon_r = \frac{E_v}{E_m} = \frac{E}{E - E_0} = \frac{E}{E'}$$

$$\boxed{E' = \frac{E}{\varepsilon_r}}$$




Polar / Non-polar dielectric when kept in an external field develops a net dipole moment along the external field. Due to this there is accumulation of +ve charge on one side and -ve charge on the other side as shown. This results in an internal field $E_0$ in a direction opposite to the Electric field (external). Hence the net field inside the dielectric is: $\frac{E}{\varepsilon_r}$

# Capacitors

It is a device which is used to store charges and hence electrical energy in the form of electric field. It is found that

Charge stored by a capacitor $\propto$ Potential differente developed across it

[Capacitance : The ability of a conductor to store charge.]

$$Q \propto V$$

$$Q = CV$$

$$C = \text{capacitance} = \frac{Q}{V}.$$

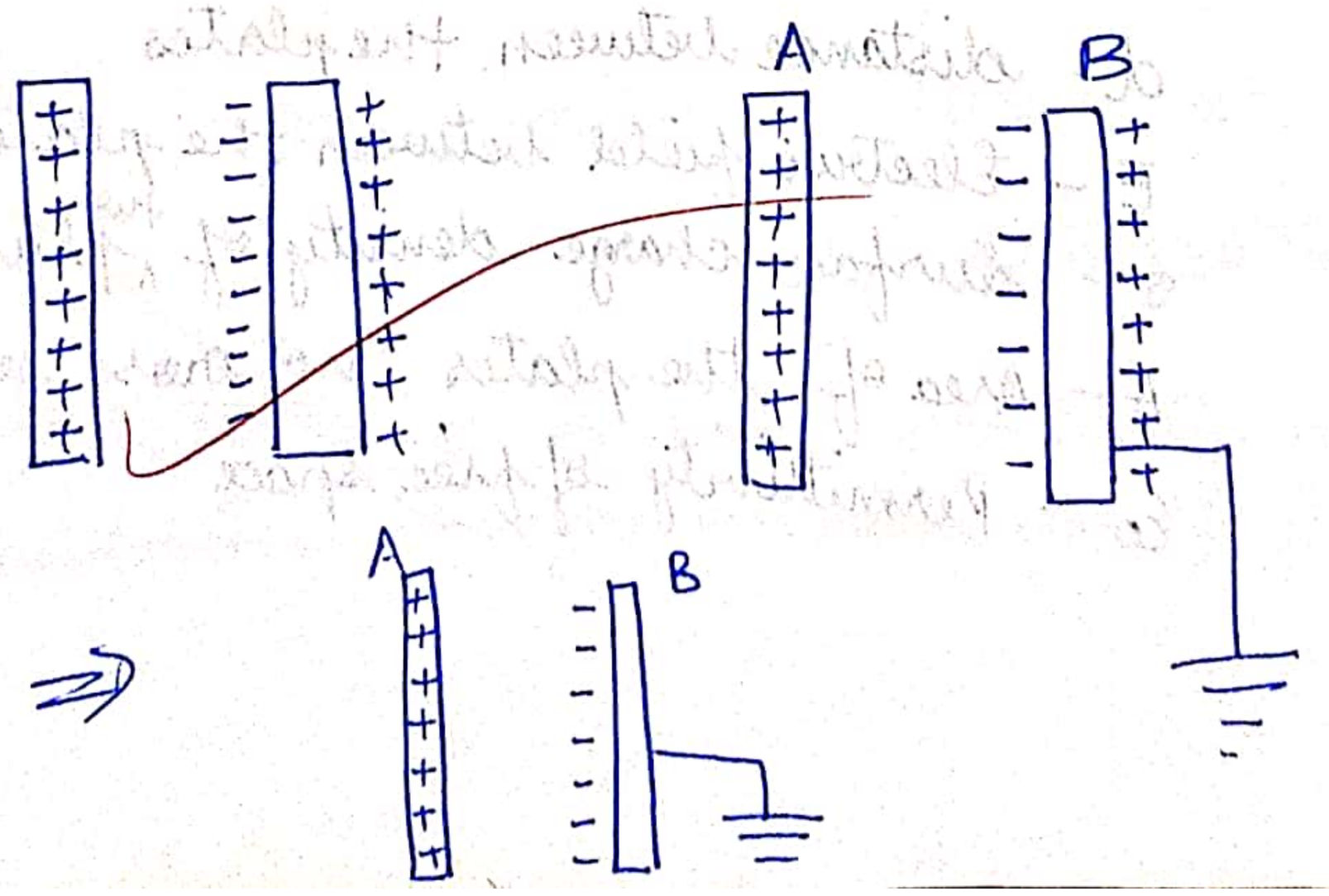
SI unit of capacitance = Farad.

1 Farad: The capacitance of a capacitor when 1 coulomb of charge on the plates causes a potential difference of one Volt across the plates.

## Parallel plate capacitor

consists of 2 long, thin metallic plates kept near each other, separated by a small distance, (one plate given a positive charge and the other plate given a negative charge).

## Principle of Parallel plate capacitor:



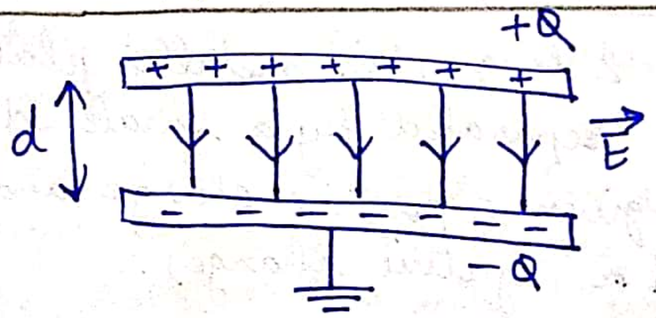
Consider a positively charged metal plate A as shown. Bring a similar neutral metal plate B near it. Due to induction, negative charges will be induced on the side of B near A and positive charges will be induced on the side of B farther from A.

Negative charges will reduce the potential of A and positive charges will increase the potential of A.

If one side of B is earthed as shown in the diagram it is found that potential of A further reduces thereby increasing the capacitance of the capacitor.

As $C = \frac{Q}{V}$, $C \propto \frac{1}{V}$. $V \downarrow$, $C \uparrow$.

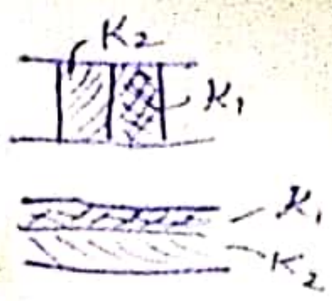
## Capacitance of a parallel plate capacitor




Q – Charge on the plates
V – Potential difference between the plates.
d – distance between the plates
E – Electric field between the plates
σ – Surface charge density of the plates
A – Area of the plates (not cross section)
$\varepsilon_0$ – Permittivity of free space

$$C = \frac{Q}{V}$$

Area - parallel
Thickness - series




$$C = \frac{\sigma A}{V} \qquad \left(\sigma = \frac{q}{A}\right)$$

$$C = \frac{\sigma A}{Ed} \qquad \left(E = \frac{V}{d}\right)$$

$$C = \frac{\cancel{\sigma} A \varepsilon_0}{\cancel{\sigma} d} \qquad \left(E = \frac{\sigma}{\varepsilon_0}\right)$$

$$\left(\frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}\right)$$

one plate  another plate

$$\boxed{C = \frac{\varepsilon_0 A}{d}}$$

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Q) If $C = 1F$, $d = 1cm$, $\varepsilon_0 = 8.85 \times 10^{-12}$.

$$1 = \frac{8.85 \times 10^{-12}}{1 \times 10^{-2}}$$

$$1 = 8.85 \times 10^{-10} \cdot A$$

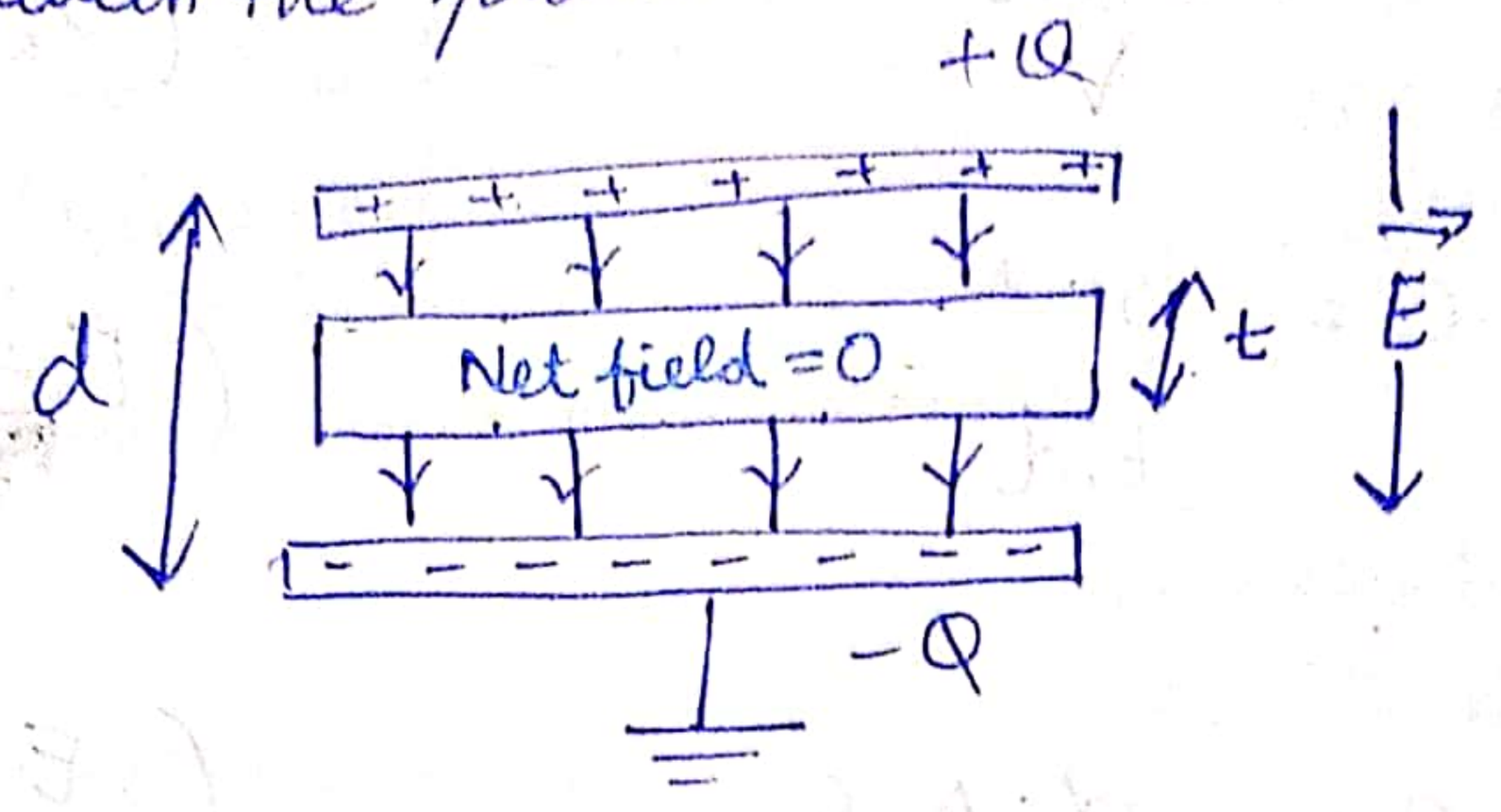
$$A = \frac{1}{8.85} \times 10^{10}$$

$$\approx 10^{9} m^2. \approx 10 \times 10^{8} m^2$$

~~30 m × 30 m~~

Such a large area is not practically possible to manufacture. Therefore we make use of $\mu F$, $nF$, $pF$

Capacitance of a parallel plate capacitor with a conducting slab ($t < d$) in between the plates.




Consider a conducting slab of thickness $t < d$ placed inbetween the plates of the capacitor.

$$C = \frac{Q}{V}$$

$$C = \frac{\sigma A}{V} \qquad (Q = \sigma A)$$

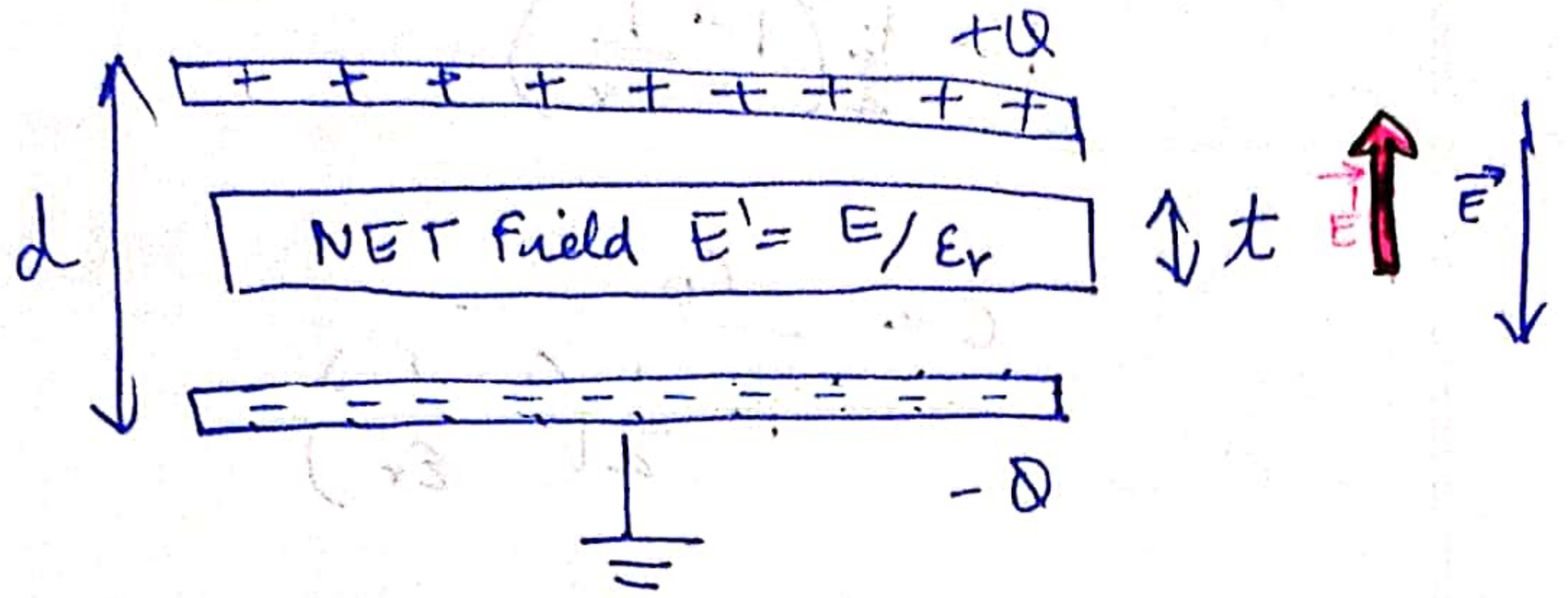
$$C = \frac{\sigma A}{E(d-t)} \qquad (V = Ed)$$

$$C = \frac{\sigma A \varepsilon_0}{\sigma (d-t)} \qquad \left[E = \frac{\sigma}{\varepsilon_0}\right]$$

$$\boxed{C = \frac{\varepsilon_0 A}{d-t}}$$

$\therefore$ Capacitance increases when a conductor is introduced.

## Capacitance of a parallel plate capacitor with a dielectric slab in between the plates.




Consider a dielectric slab of $t < d$ in between the plates of the capacitor

let $\varepsilon_r \rightarrow$ dielectric constant of slab

$$C = \frac{Q}{V}$$

$$Q = \sigma A$$

$$V = E(d-t) + E't$$

$$V = E(d-t) + \frac{E}{\varepsilon_r} t$$

$$C = \frac{\sigma A}{E(d-t) + \frac{E}{\varepsilon_r} t} = \frac{\sigma A}{E\left[(d-t) + \frac{1}{\varepsilon_r} t\right]}$$

$$= \frac{\sigma A}{\frac{\sigma}{\varepsilon_0}\left[(d-t) + \frac{t}{\varepsilon_r}\right]} = \frac{A\varepsilon_0}{d - t + \frac{t}{\varepsilon_r}}$$

$$C = \frac{A\varepsilon_0}{d\left(1 - \frac{t}{d} + \frac{t}{d\varepsilon_r}\right)}$$

$$C = \frac{A\varepsilon_0/d}{1 - \frac{t}{d}\left(1 - \frac{1}{\varepsilon_r}\right)}$$

$$C = \frac{C_0}{1 - \frac{t}{d}\left(1 - \frac{1}{\varepsilon_r}\right)}$$

$$C > C_0$$

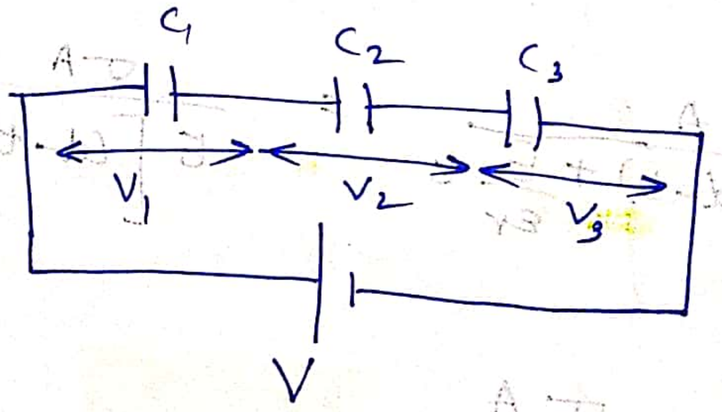
$$t = d$$

$$C = C_0 \varepsilon_r$$

$$C = C_0 \varepsilon_r$$

## Combination of Capacitors

### Series Combination:



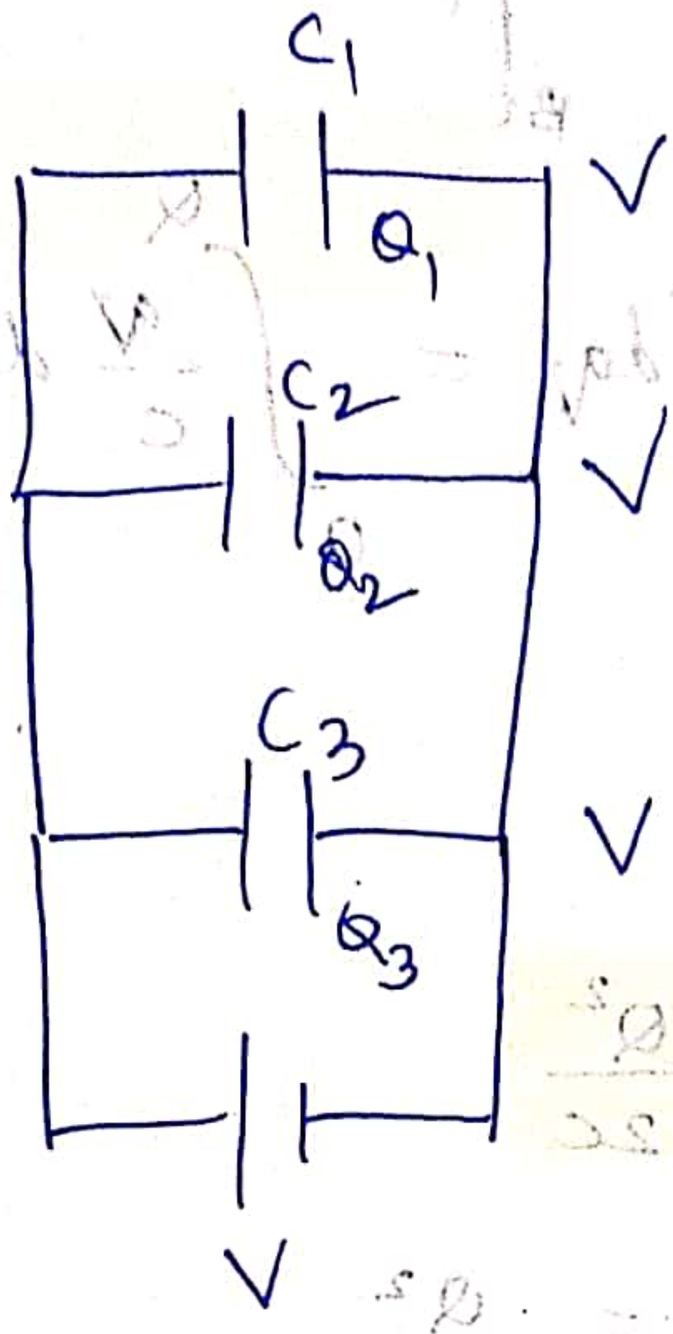

In series combination, charge on all the capacitors is the same, but potential difference is different

$$V = V_1 + V_2 + V_3$$

$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\therefore \boxed{\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

## Parallel combination :

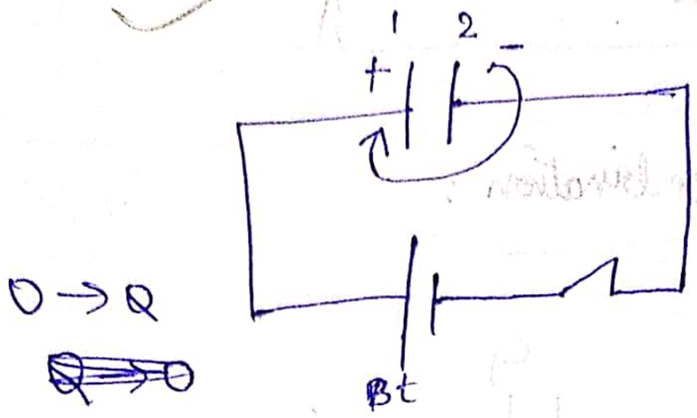



$$Q = Q_1 + Q_2 + Q_3$$

$$C_{\frac{p}{s}} V = C_1 V + C_2 V + C_3 V$$

$$\boxed{C_{parallel} = C_1 + C_2 + C_3}$$

# Energy stored in a Capacitor




$0 \rightarrow Q$

$\cancel{Q \rightarrow 0}$

$$\int dW = \int_0^Q V\,dq = \int_0^Q \frac{q}{C}\,dq$$

$$\left(V = \frac{q}{C}\right)$$

$$W = \frac{Q^2}{2C}$$

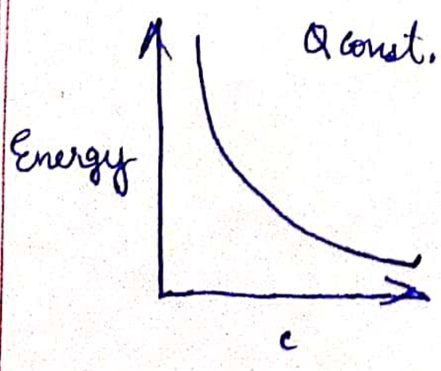
$$\text{Energy} = \frac{Q^2}{2C}$$

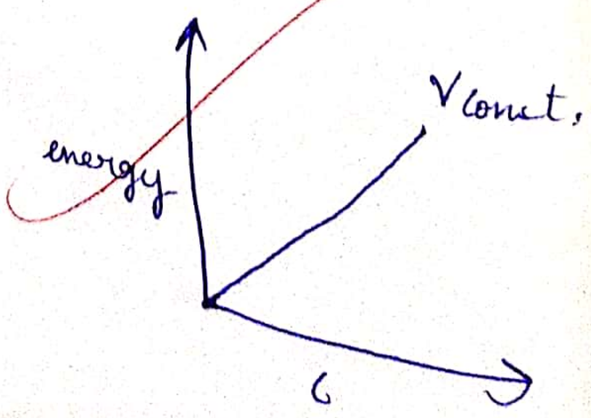
$$= \frac{(CV)^2}{2C}$$

$$= \frac{1}{2}CV^2$$

$$= \frac{1}{2}\frac{Q}{V}V^2$$

$$\text{energy} = \frac{1}{2}QV$$



Energy density = $\frac{\text{Energy}}{\text{Volume}}$

$$u = \frac{\frac{1}{2} C V^2}{A d} = \frac{1}{2} \frac{\varepsilon_0 A}{d} \frac{(E d)^2}{A \cdot d}$$

$$\boxed{u = \frac{1}{2} \varepsilon_0 E^2}$$