Chapter - 3: Current Electricity

- Rate of flow of charges - Current

  SI unit : 1 Ampere = 1 coulomb / 1 second

- If the current is not steady, $I = \frac{dQ}{dt}$.

- **Ohm's Law:**

At constant temperature, the potential difference across the ends of wire is directly proportional to the current in the wire

$$V \propto I$$

$$V = IR$$

R = Resistance

$$R = \frac{V}{I}$$

SI unit = Ω

$$1\,\Omega = \frac{1V}{1A}$$

Resistance: Opposition offered by the conductor to the flow of current.

Factors affecting Resistance:

$$R \propto l$$

$$R \propto \frac{1}{A}$$

$$R \propto \frac{l}{A}$$

$$R = \frac{\rho l}{A}$$

$\rho \rightarrow$ Resistivity of a Specific resistance.

$$\rho = \frac{RA}{l}$$

Resistivity: Resistance offered by a wire of length 1 m and Area of cross section 1 m^2.

SI unit = Ωm.

Conductance: Reciprocal of Resistance

$$G = \frac{1}{R} = \frac{A}{\rho l}$$

SI unit = mho, Ω^{-1}, siemens

Conductivity: Reciprocal of Resistivity

$$\sigma = \frac{1}{\rho} = \frac{l}{RA}$$

SI unit = $\Omega^{-1} m^{-1}$, mho m^{-1}, siemens m^{-1}.

Current density: $\vec{J} = \frac{I}{\vec{A}}$

SI unit = Ampere / m^2
(It is a vector).

Relation between E, σ, J.

$$V = IR$$

$$V = \frac{I \rho l}{A}$$

$$V = J \rho l$$

$$\boxed{E = J\rho}$$

$$E = \frac{J}{\sigma} \quad \text{or} \quad \boxed{\sigma E = J}$$

~~The~~ **Drift Velocity**: The average velocity of all the electrons after the application of electric field.

- Consider a ~~dc~~ conductor with N no. of free electrons
- Let $u_1, u_2, u_3, \ldots u_N$ be the initial velocities of all the electrons before applying electric field.
- As the electrons are moving in random possible directions, the average velocity of all the electrons before applying electric field is zero.

$$\frac{u_1+u_2+u_3+\cdots+u_N}{N} = 0.$$

- Let $v_1, v_2, v_3, \ldots, v_N$ be the velocities of electrons after applying electric field
- On applying $\vec{E}$, e^- are accelerated but immediately in the next collision they ~~loose~~ lose their acceleration. The velocities of electrons after applying $\vec{E}$ is given as:

$$v_1 = u_1 + a t_1$$

$$v_2 = u_2 + a t_2$$

$$\vdots$$

$$v_N = u_N + a t_N$$

adding all equations and dividing by N.

$$\frac{v_1+v_2+v_3+\cdots+v_N}{N} = \cancel{\frac{u_1+u_2+u_3+\cdots+u_N}{N}} + a\left(\frac{t_1+t_2+t_3+\cdots+t_N}{N}\right)$$

$$V_d = 0 + a\tau$$

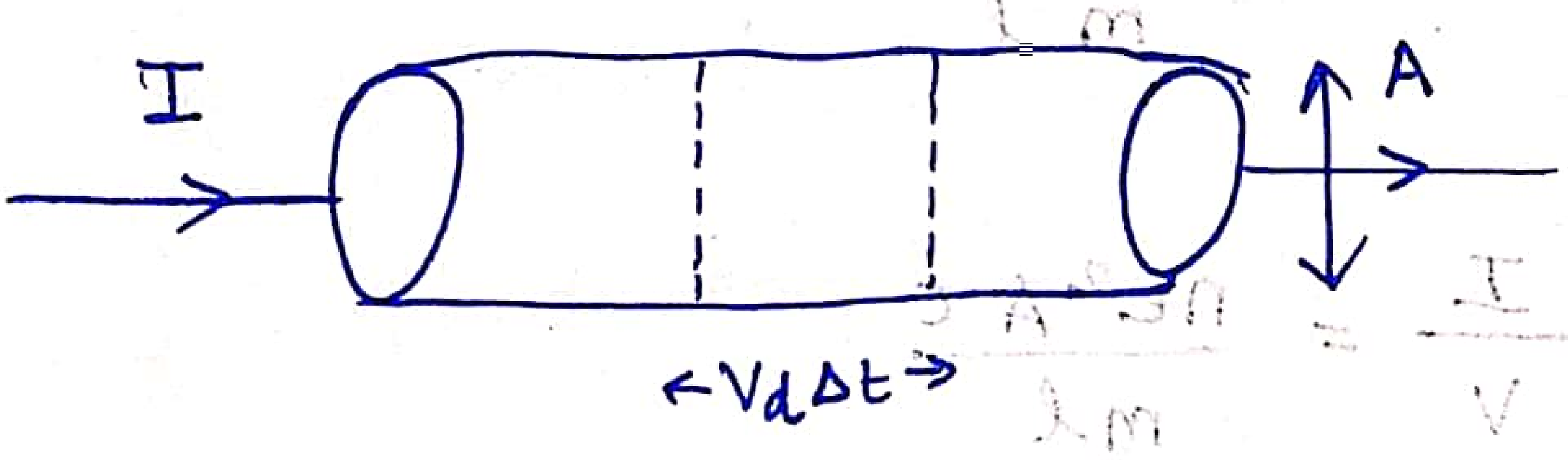
$$\left(\tau = \frac{t_1 + t_2 + t_3 + \cdots t_N}{N}\right)$$

→ **relaxation time** : Average time elapsed in between successive collisions

$$\left(\begin{aligned} \vec{F} &= -e\vec{E} \\ ma &= -eE \\ a &= \frac{-eE}{m} \end{aligned}\right)$$

$$\boxed{\therefore\ V_d = \frac{-eE\tau}{m}}$$

Relation between current and drift velocity



A - Area of cross section of the conductor

V_d - Drift velocity of the electrons

Δt - Time interval (For which current is determined)

e - charge of an electron

n - number density of electrons (no. of e^-s / volume)

Distance moved = $V_d \Delta t$

Volume covered = $A V_d \Delta t$

Total no. of e^-s moving across this vol $\Big\}$ = $n A V_d \Delta t$

Total charge of these e^-s = $neAV_d \Delta t$.

$$\text{current} = I = \frac{\Delta Q}{\Delta t} = \frac{neAV_d\Delta t}{\Delta t} = \boxed{neAV_d.}$$

Ohm's law and Resistivity:

Current through the conductor is given as

$$I = neAV_d$$

$$V_d = \frac{eE\tau}{m}$$

$$I = neA\left(\frac{eE\tau}{m}\right)$$

$$I = \frac{ne^2AE\tau}{m}$$

$$E = \frac{V}{l}$$

$$I = \frac{ne^2AV\tau}{ml}$$

$$\frac{I}{V} = \frac{ne^2A\tau}{ml}$$

$$\frac{V}{I} = \frac{ml}{ne^2A\tau}$$

$$R = \left(\frac{m}{ne^2\tau}\right)\frac{l}{A}$$

$R = \rho\frac{l}{A}$ (Comparing with this eq.)

$$\boxed{\rho = \frac{m}{ne^2\tau}}$$

$\rho \propto \frac{1}{n}$ and $\rho \propto \frac{1}{\tau}$

Variation of Resistance or resistivity with temperature.

$$R_t = R_0(1+\alpha t)$$

$$\rho_t = \rho_0(1+\alpha t)$$

$$\alpha = \frac{R_t - R_0}{R_0 t} = \frac{\rho_t - \rho_0}{\rho_0 t}$$

Temp. coefficient of Resistance and Resistivity $= \alpha$.

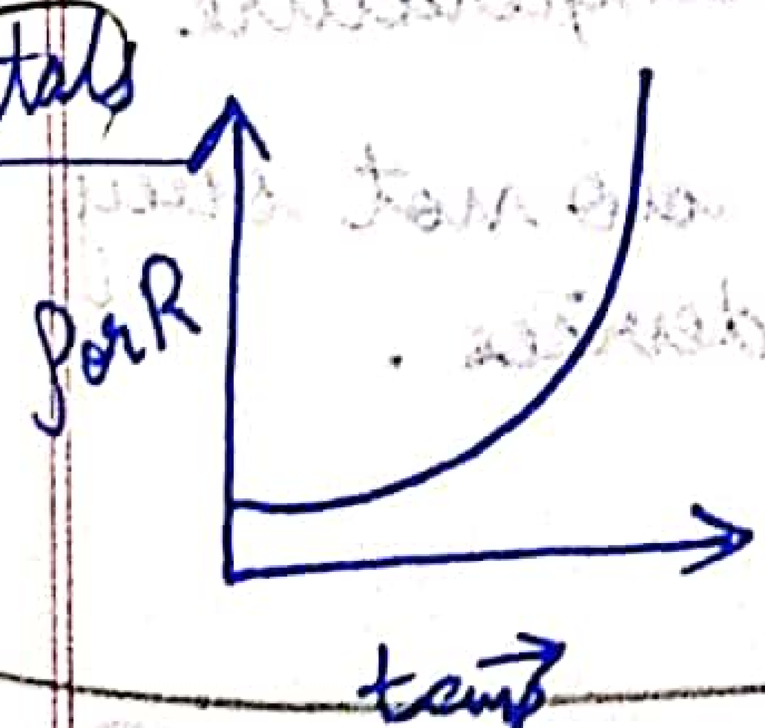
R_t/ρ_t = Resistance/ Resistivity at $t°C$.

R_0/ρ_0 = Resistance /Resistivity at $0°C$

α = Temp. coefficient Resistance/ Resistivity

t = Temp. difference.

Metals



$$\rho = \frac{m}{ne^2\tau}$$

$\alpha = +ve$ and large

As temp. increases, KE increases, frequent collisions occur, τ decrease

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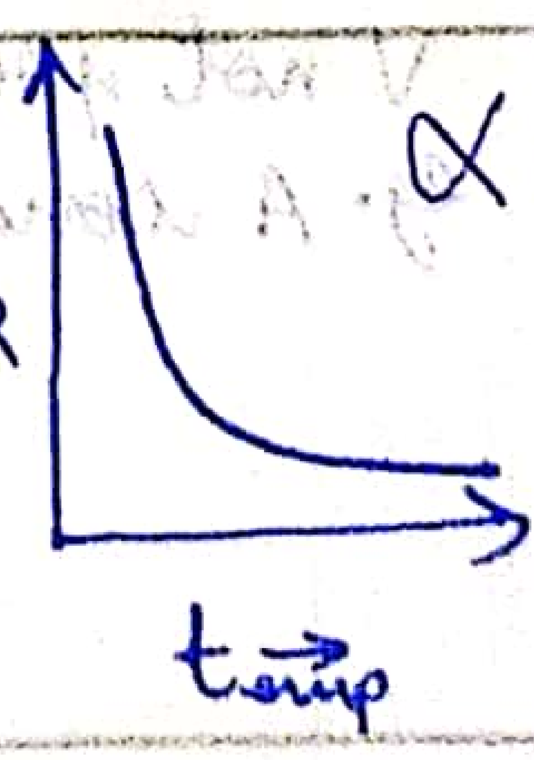
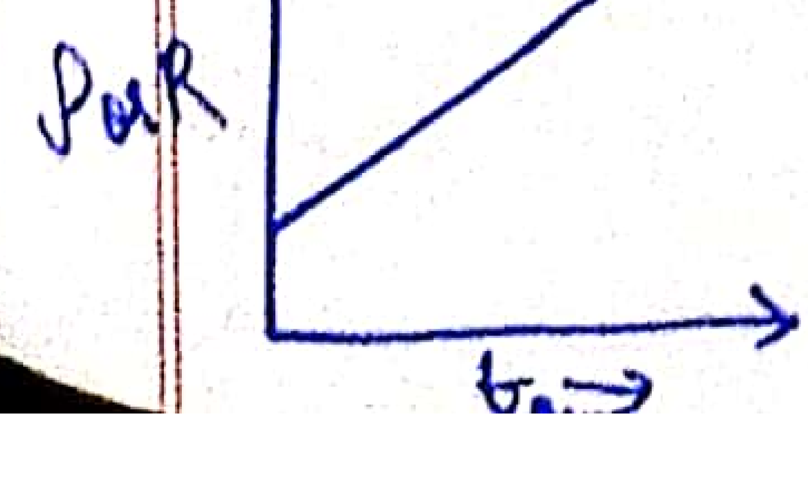
Semi Conductor

As temp increases, no. density of free electrons increase and relaxation time decreases. But increase in 'n' dominates decrease in τ $\therefore$ R decreases with increase in temp.



$\alpha = -ve$

Alloys

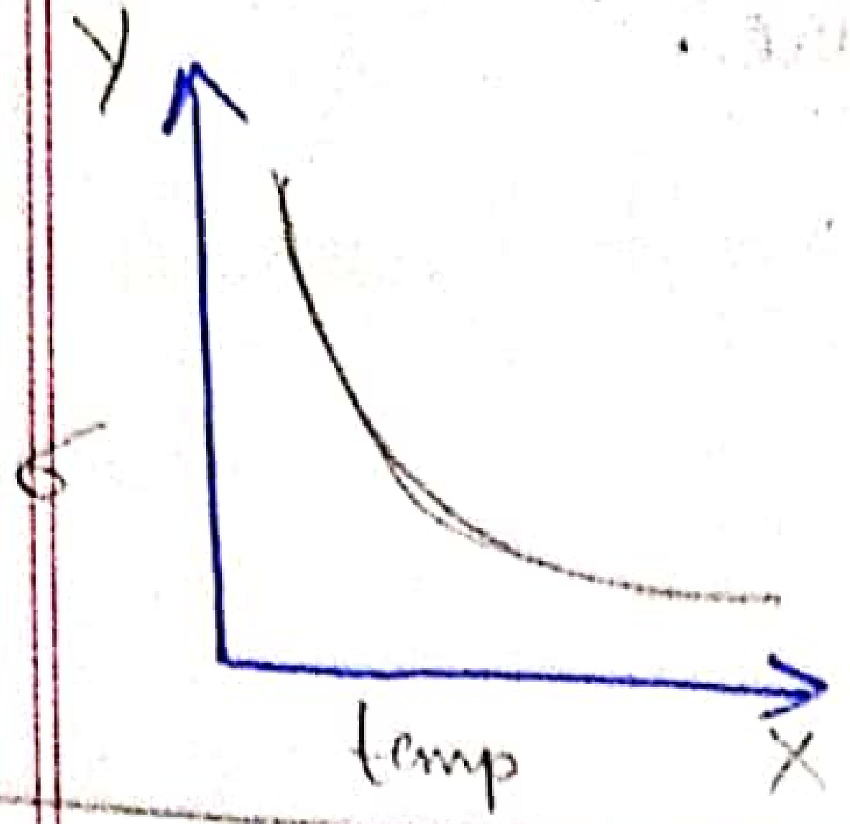


Nichrome, Constantan, Manganin

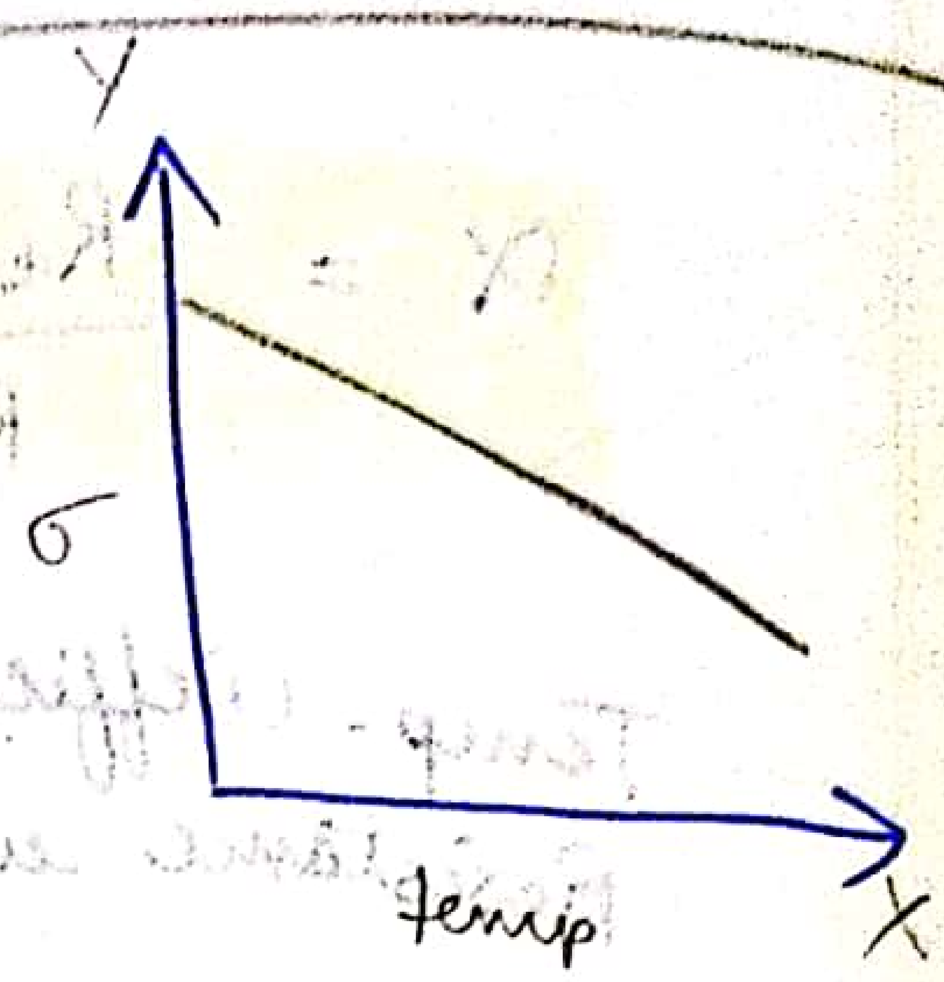
Alloys show negligible increase in resistance and resistivity with Temp. Hence they are used in standard resistance coil.

As temperature increases $\sigma = \frac{ne^2\tau}{m}$

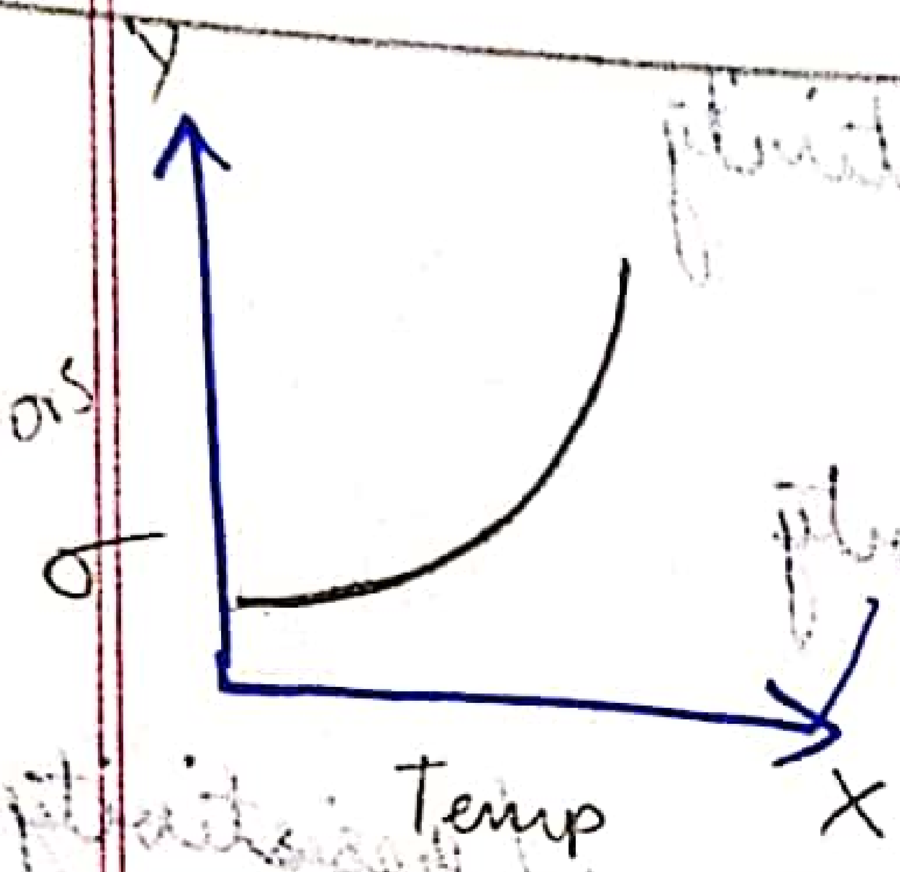
Metals



Alloys



semi conductors



Ohmic and non-ohmic devices

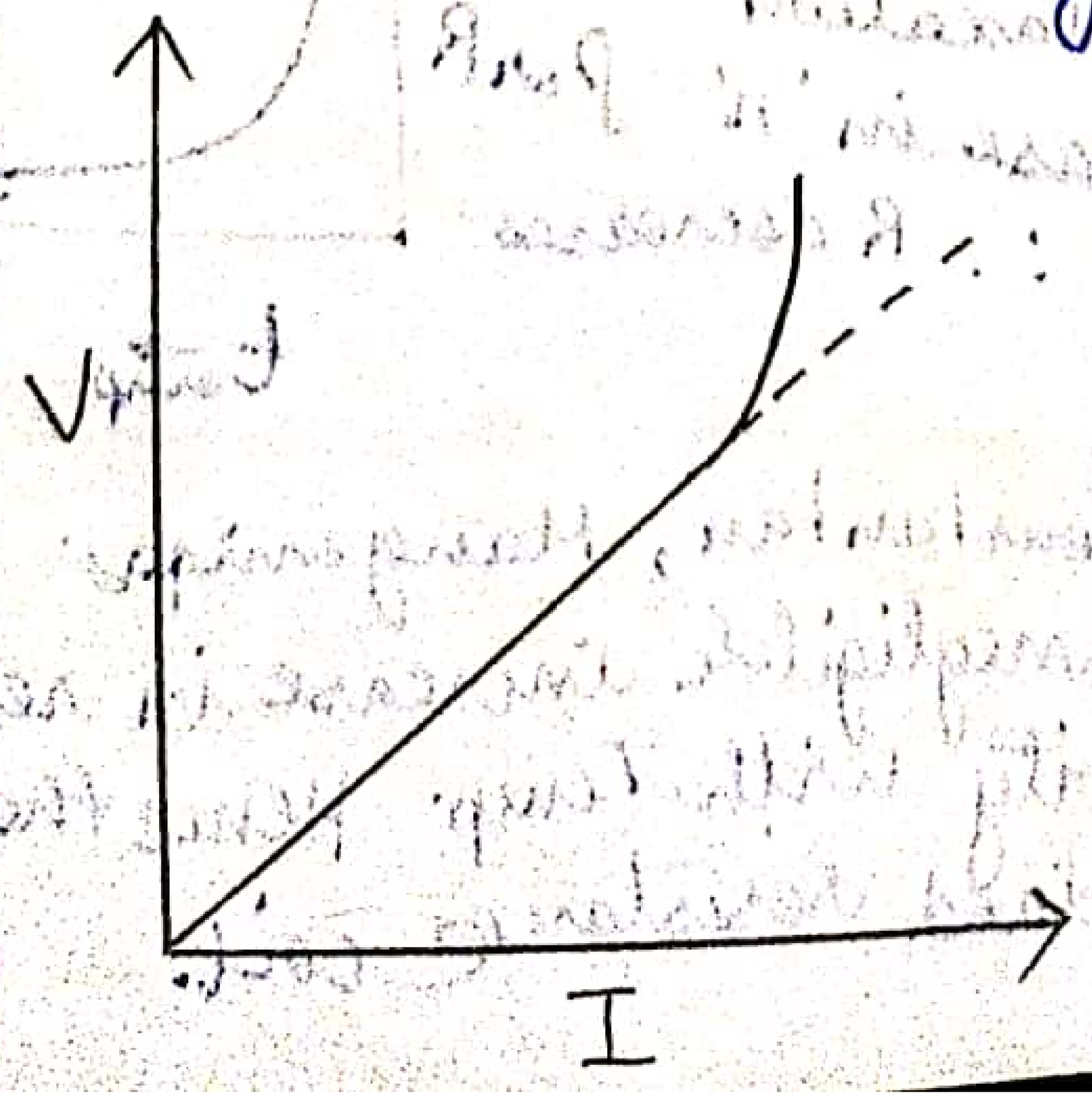
Ohmic devices: Devices which obey ohm's law are called ohmic devices.

- eg. Metallic conductor at constant temperature

Non ohmic devices: Devices which do not obey ohm's law are called non-ohmic devices.
If it satisfies any one of:

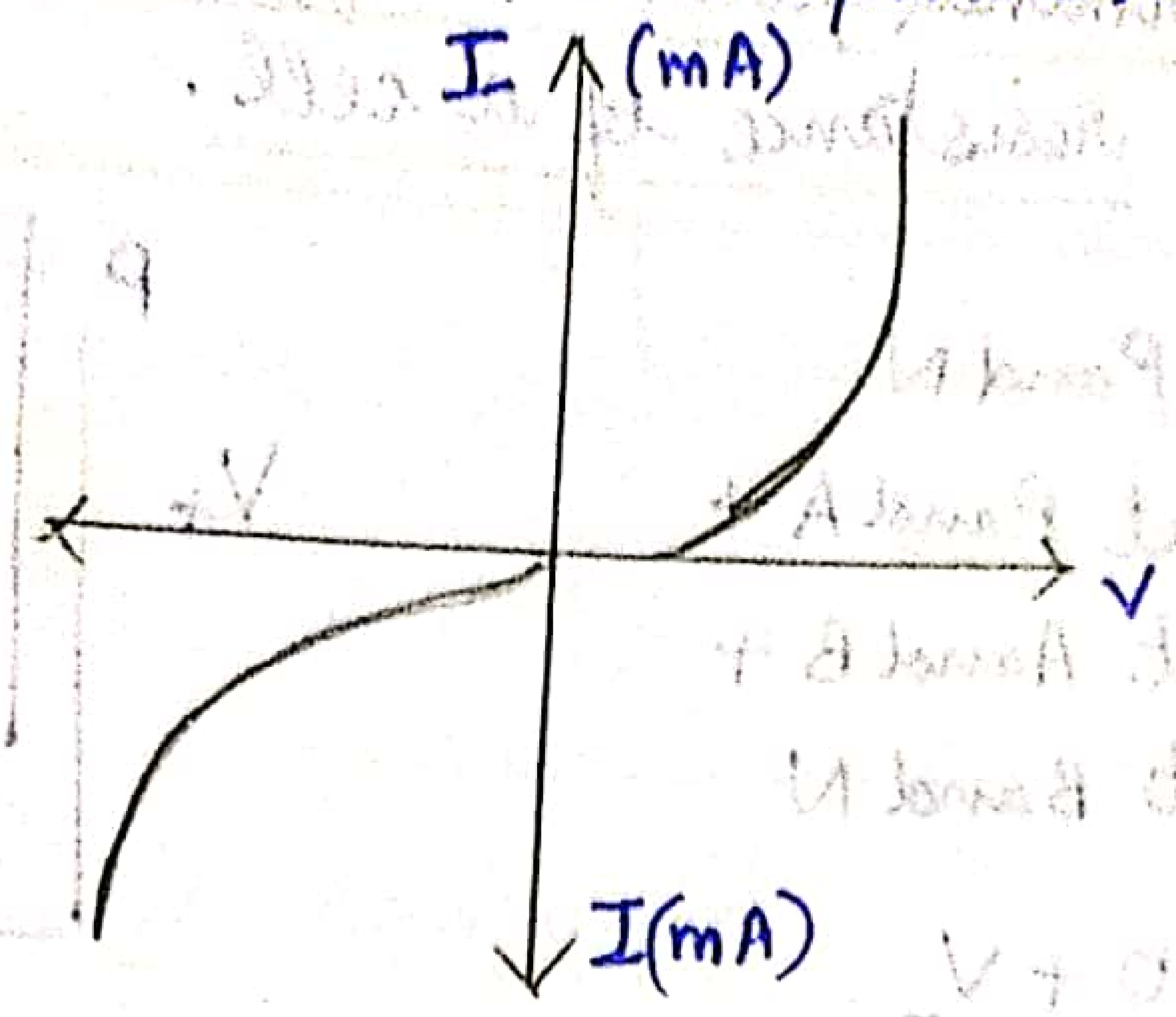
- V not proportional to I:

eg. A conductor at high temperature.



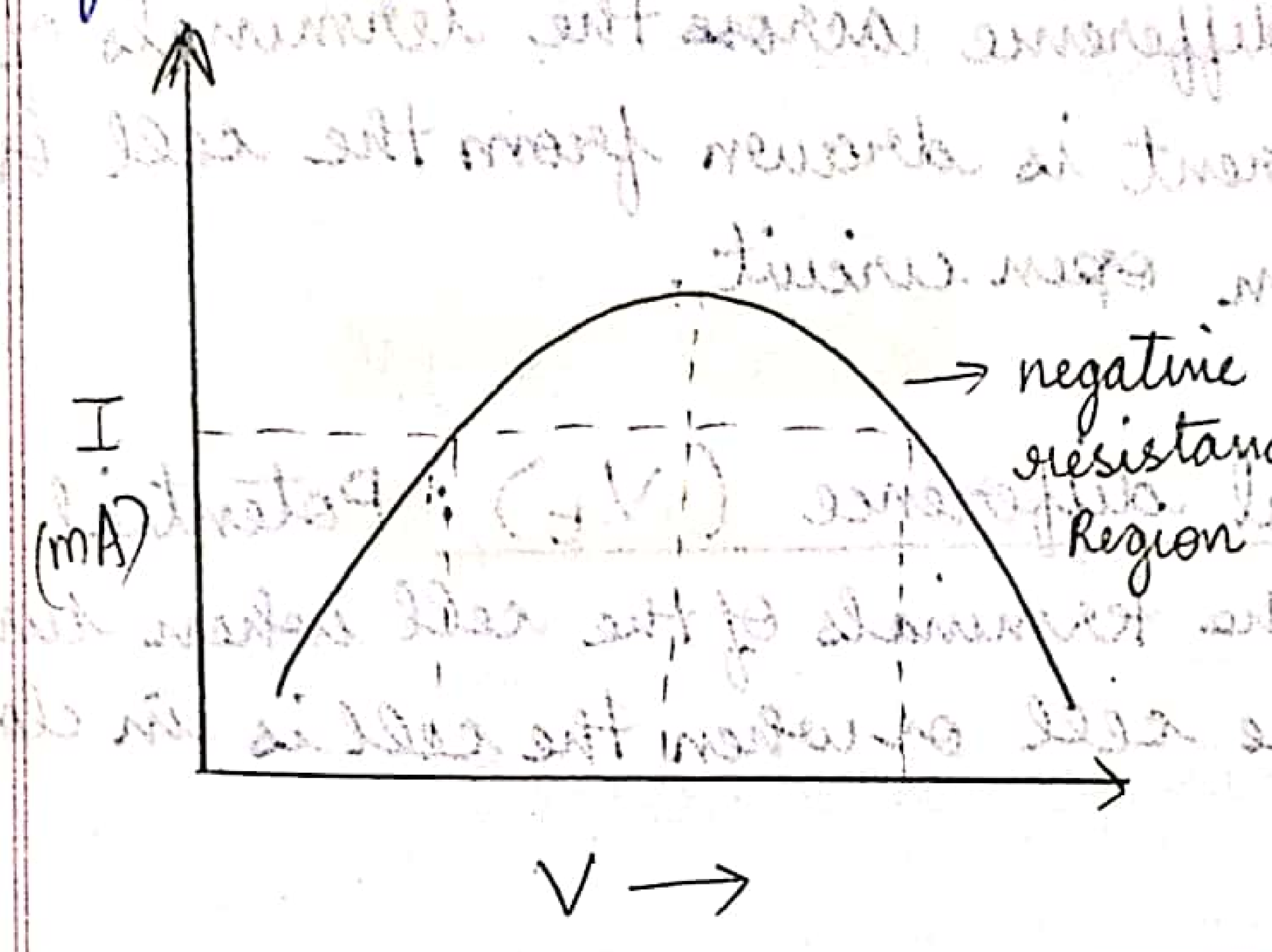
- Relation between V and I depends upon the sign of V.

eg: A diode



- Relation between V and I is not unique. i.e; for a given value of I, there may be more than one value of V.

eg. Gallium Arsenide (GaAs)



Mobility:

$$\mu = \frac{V_d}{E}$$

Drift Velocity per unit Electric field is called mobility.

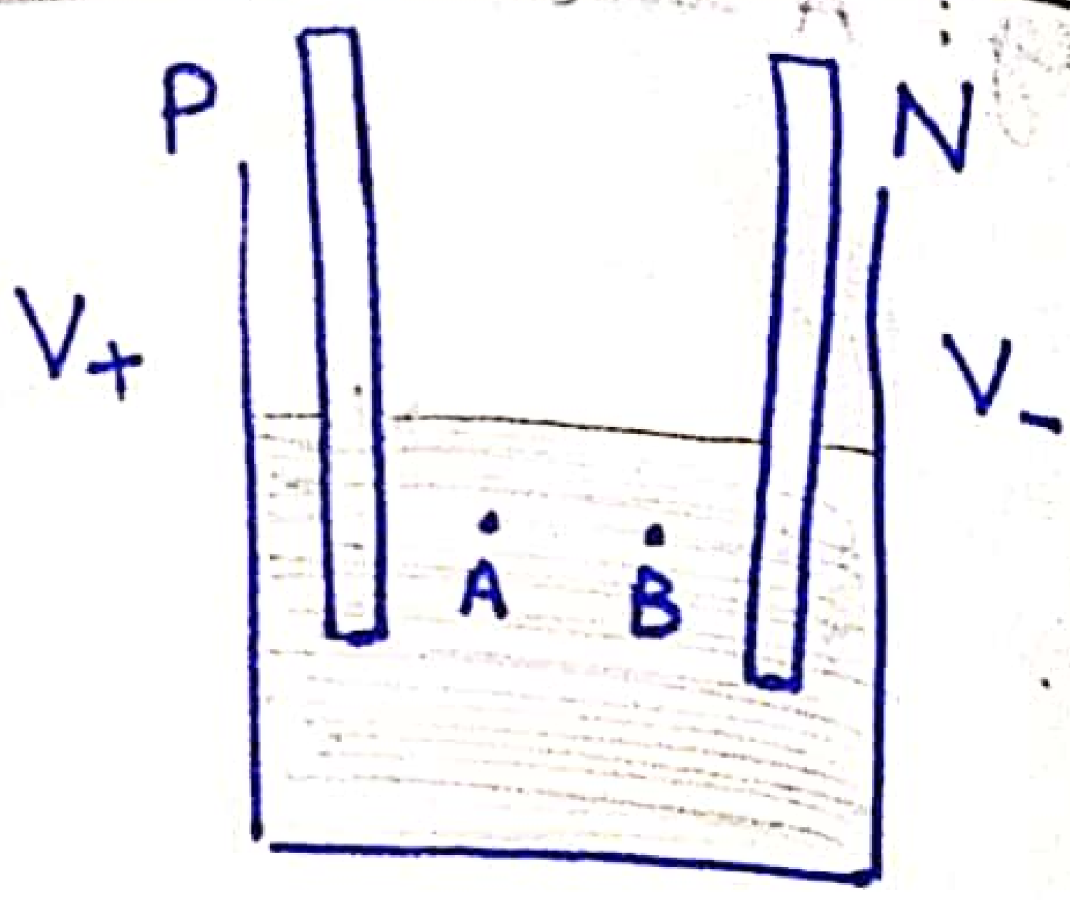
$$\mu = \frac{\frac{eE}{m}\tau}{E}$$

SI unit = C.s/kg

Cm/N.s

$$\mu = \frac{e\tau}{m}$$

EMF, terminal potential difference and internal resistance of a cell.



P.d bet P and N

= P.d bet P and A +
P.d bet A and B +
P.d bet B and N

$= V_+ + 0 + V_-$

$= V_+ - V_-$

→ not a force tho.

= EMF (Electromotive force) of the cell

EMF : Potential difference across the terminals of a cell when no current is drawn from the cell or when the cell is in open circuit.

Terminal potential difference (V_t) : Potential difference across the terminals of the cell when current is drawn from the cell or when the cell is in closed circuit.

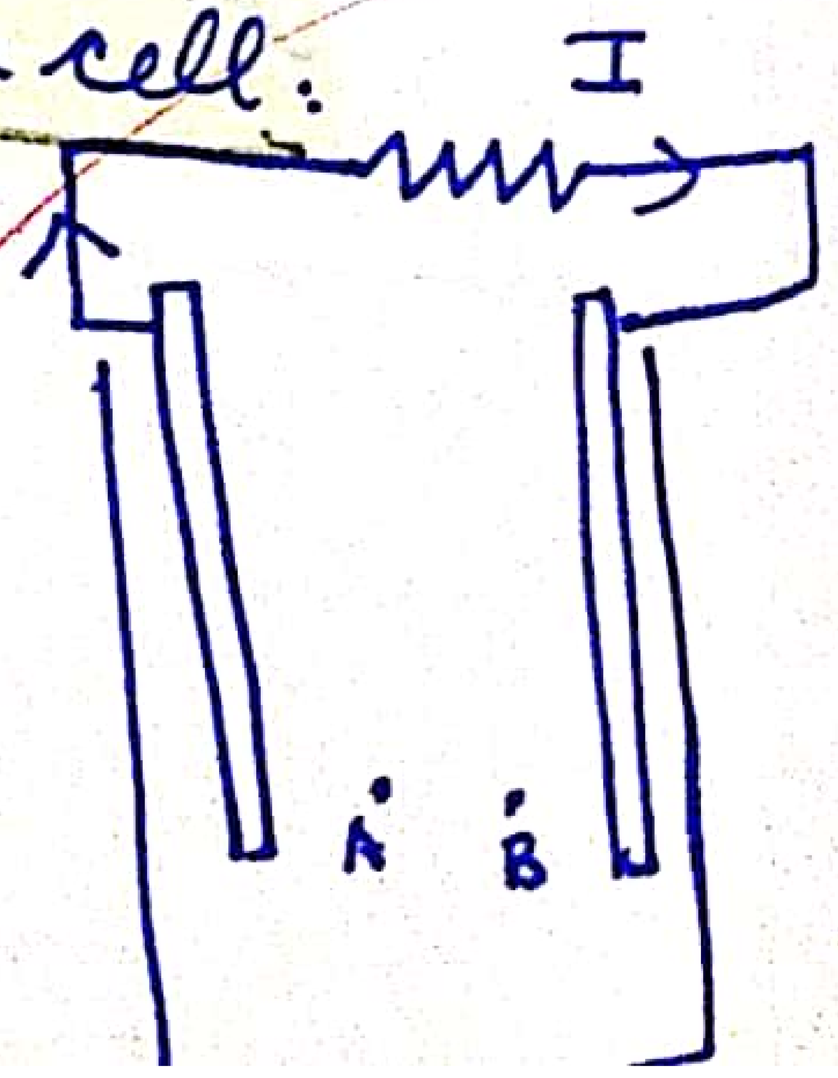
Internal resistance of the cell (r) : Opposition offered by the electrolyte of the cell is called internal resistance of the cell. It depends on :

- length of the electrodes
- Nature of the electrolyte
- Concentration of the electrolyte
- Area of cross section of the electrodes
- Temperature.

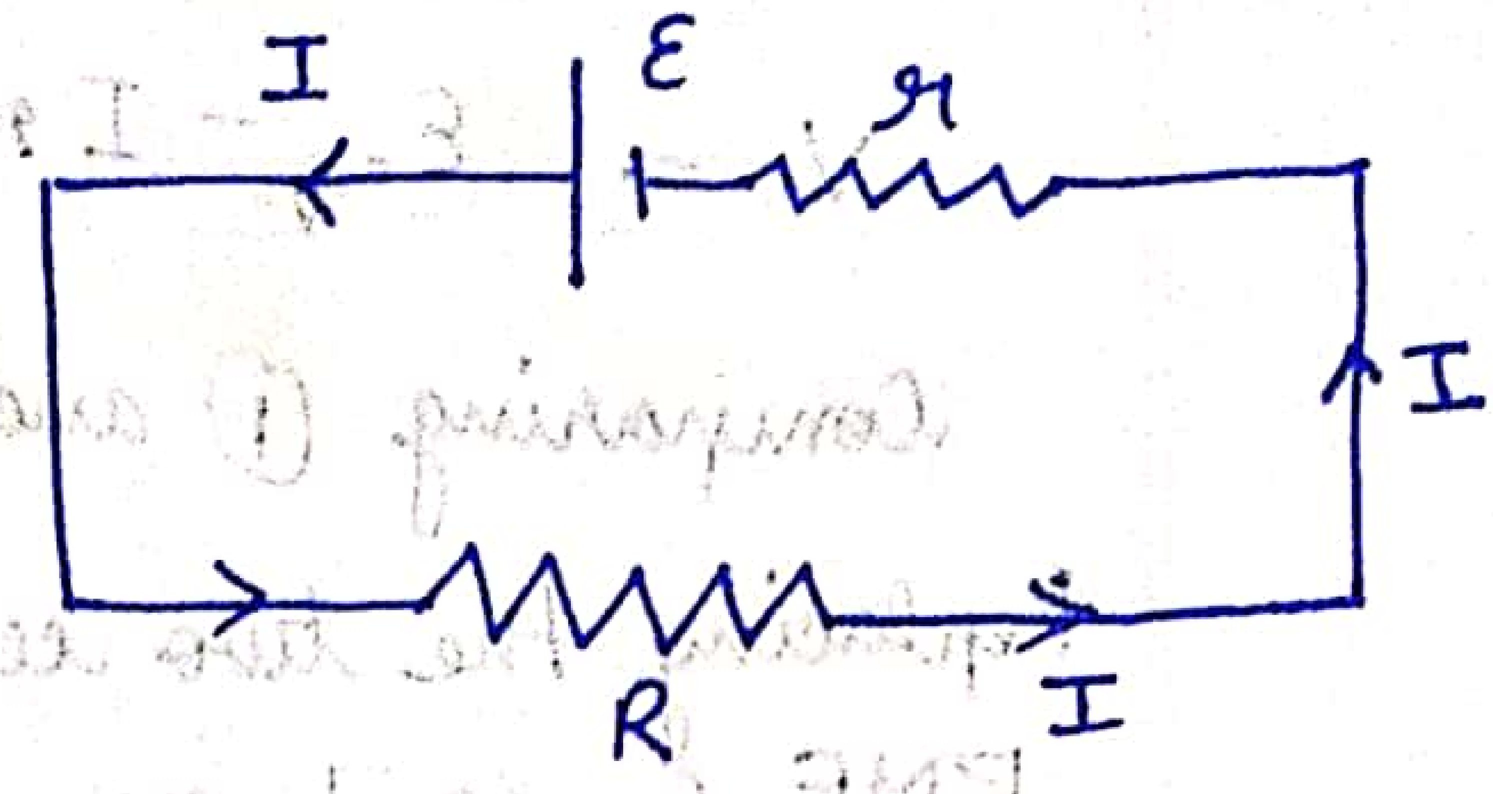
When current is drawn from a cell :

$V_t < \varepsilon$ $\quad I \neq 0, r \neq 0.$

$V_t = \varepsilon$ $\quad I = 0, r = 0.$



$$V_t = \mathcal{E} - Ir$$

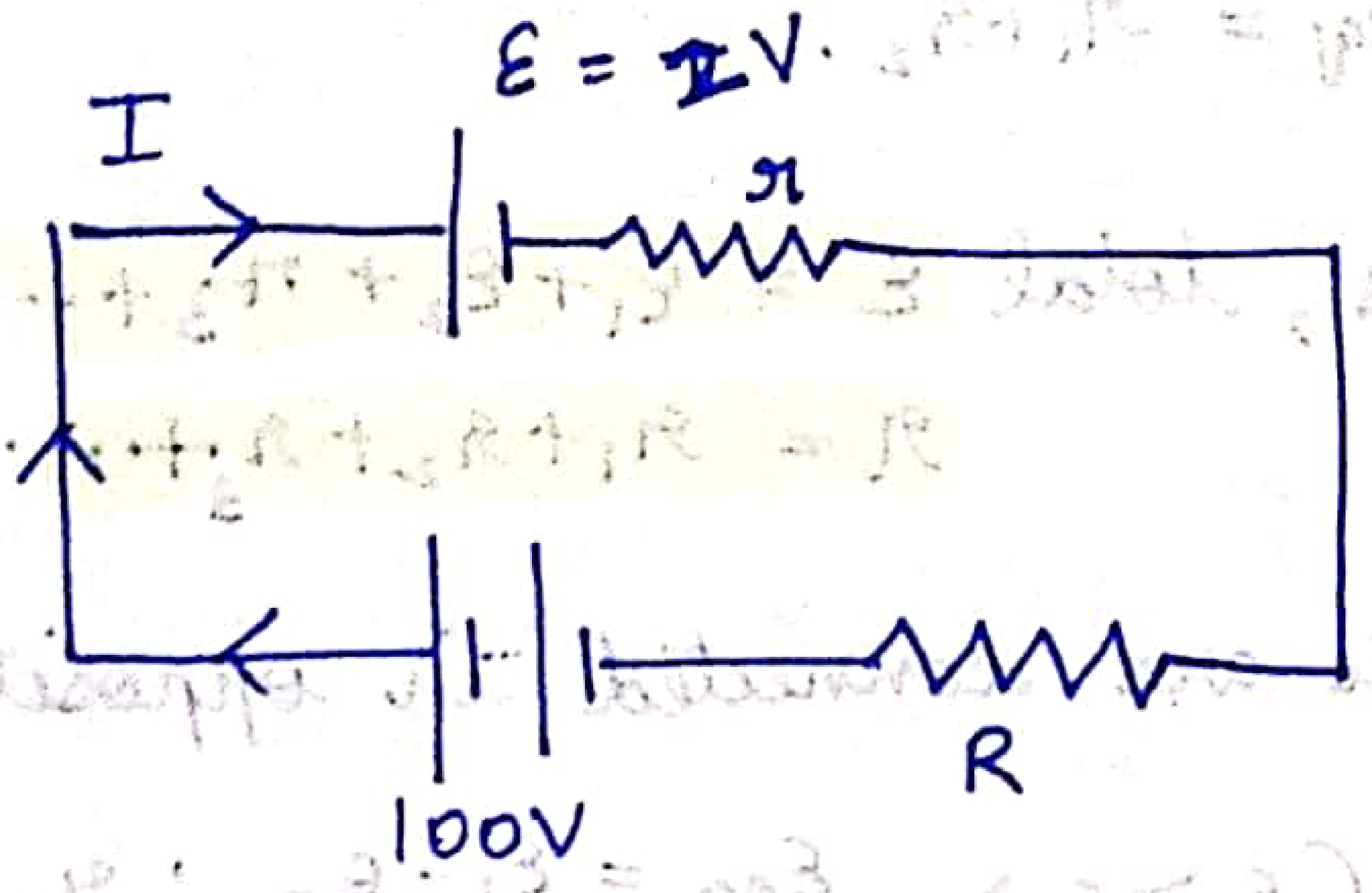


$$V_t = IR$$

$$IR = \mathcal{E} - Ir$$

$$\boxed{I = \frac{\mathcal{E}}{R+r}}$$

When a cell is recharged

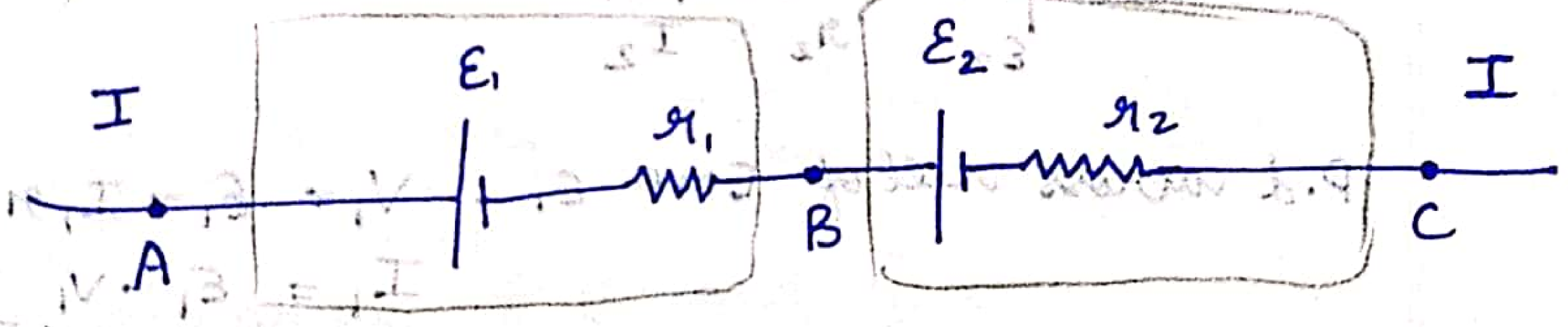


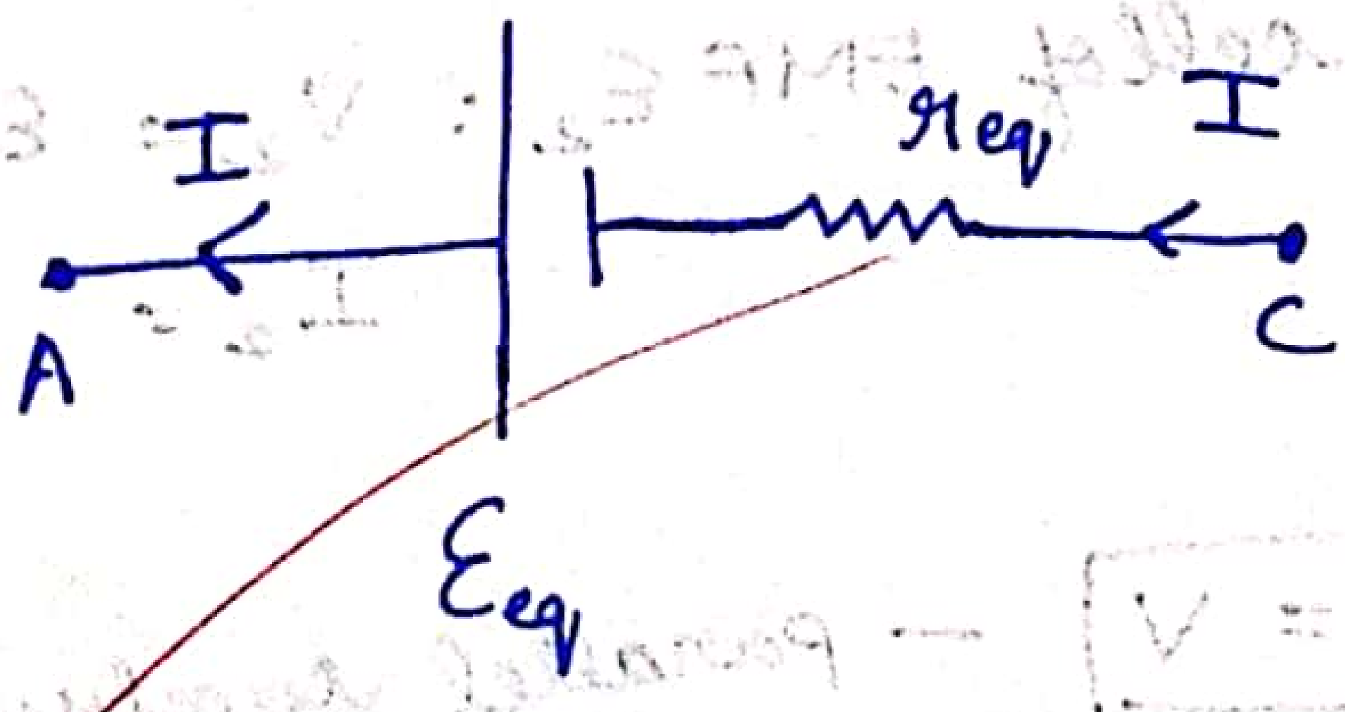
$$V_t = \mathcal{E} + Ir$$

$$V_t > \mathcal{E}.$$

Combination of cells

Series combination





$V_{AB} = \mathcal{E}_1 - Ir_1 \qquad V_{BC} = \mathcal{E}_2 - Ir_2 \qquad V_{AC} = V_{AB} + V_{BC}$

$$= \mathcal{E}_1 - Ir_2 + \mathcal{E}_2 - Ir_2 = (\mathcal{E}_1 + \mathcal{E}_2) - I(r_1 + r_2) \quad \text{—} (1)$$

$$\therefore V = \varepsilon_{eq} - I r_{eq} \quad — (2)$$

Comparing (1) and (2)

Replacing the two cells with another cell with EMF ε_{eq} and r_{eq}.

$$\therefore \varepsilon_{eq} = \varepsilon_1 + \varepsilon_2$$
$$r_{eq} = r_1 + r_2 .$$

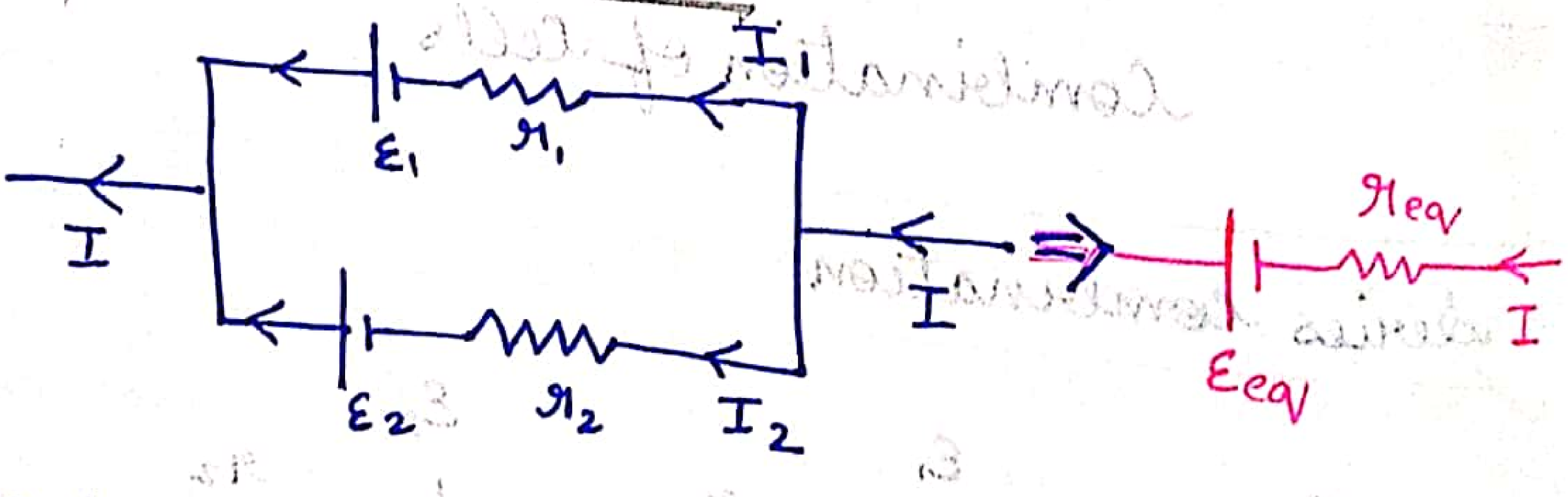
$\therefore$ In series, total $\varepsilon = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \ldots$

$r = r_1 + r_2 + r_3 + \ldots$

If the cells are connected in opposition

$(\varepsilon_1 > \varepsilon_2)$ $\varepsilon_{eq} = \varepsilon_1 - \varepsilon_2$; $r_{eq} = r_1 + r_2$.

Parallel Combination:



P.d across cell of EMF ε_1 : $V_1 = \varepsilon_1 - I_1 r_1$

$$I_1 = \frac{\varepsilon_1 - V_1}{r_1} \quad — (1)$$

P.d across cell of EMF ε_2 : $V_2 = \varepsilon_2 - I_2 r_2$

$$I_2 = \frac{\varepsilon_2 - V_2}{r_2} \quad — (2)$$

$\boxed{V_1 = V_2 = V}$ — parallel combination.

Total $I = I_1 + I_2$

$$I = \frac{\varepsilon_1 - V}{r_1} + \frac{\varepsilon_2 - V}{r_2} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} - V\left[\frac{1}{r_1} + \right.$$

$$V\left[\frac{1}{r_1}+\frac{1}{r_2}\right] = \frac{\varepsilon_1}{r_1}+\frac{\varepsilon_2}{r_2} - I$$

$$V\left[\frac{r_2+r_1}{r_1 r_2}\right] = \frac{\varepsilon_1}{r_1}+\frac{\varepsilon_2}{r_2} - I$$

$$V = \left[\frac{\varepsilon_1}{r_1}+\frac{\varepsilon_2}{r_2}\right]\left[\frac{r_1 r_2}{r_1+r_2}\right] - I\left[\frac{r_1 r_2}{r_1+r_2}\right]$$

$$= \left[\frac{\varepsilon_1 r_2+\varepsilon_2 r_1}{r_1 \cancel{r_2}} \times \frac{r_1 \cancel{r_2}}{r_1+r_2}\right] - I\left[\frac{r_1 r_2}{r_1+r_2}\right]$$

$$= \left[\frac{\varepsilon_1 r_2+\varepsilon_2 r_1}{r_1+r_2}\right] - I\left[\frac{r_1 r_2}{r_1+r_2}\right] \quad \text{— (3)}$$

If the 2 cells are replaced by a single cell of EMF ε_{eq} and internal resistance r_{eq}, then

$$V = \varepsilon_{eq} - I r_{eq} \longrightarrow (4)$$

Comparing (3) and (4)

$$\varepsilon_{eq} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1+r_2}$$

$$r_{eq} = \frac{r_1 r_2}{r_2+r_1}$$

For many no. of cells: $\varepsilon_{eq} = \frac{\varepsilon_1}{r_1}+\frac{\varepsilon_2}{r_2}+\frac{\varepsilon_3}{r_3}+\cdots$

$$\frac{1}{r_{eq}} = \frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}+\cdots$$

"n" cells

In **parallel**, if ~~all~~ have same $\mathcal{E}$ and r,

$\mathcal{E}_{eq} = \mathcal{E}$. $r_{eq} = r/n$.

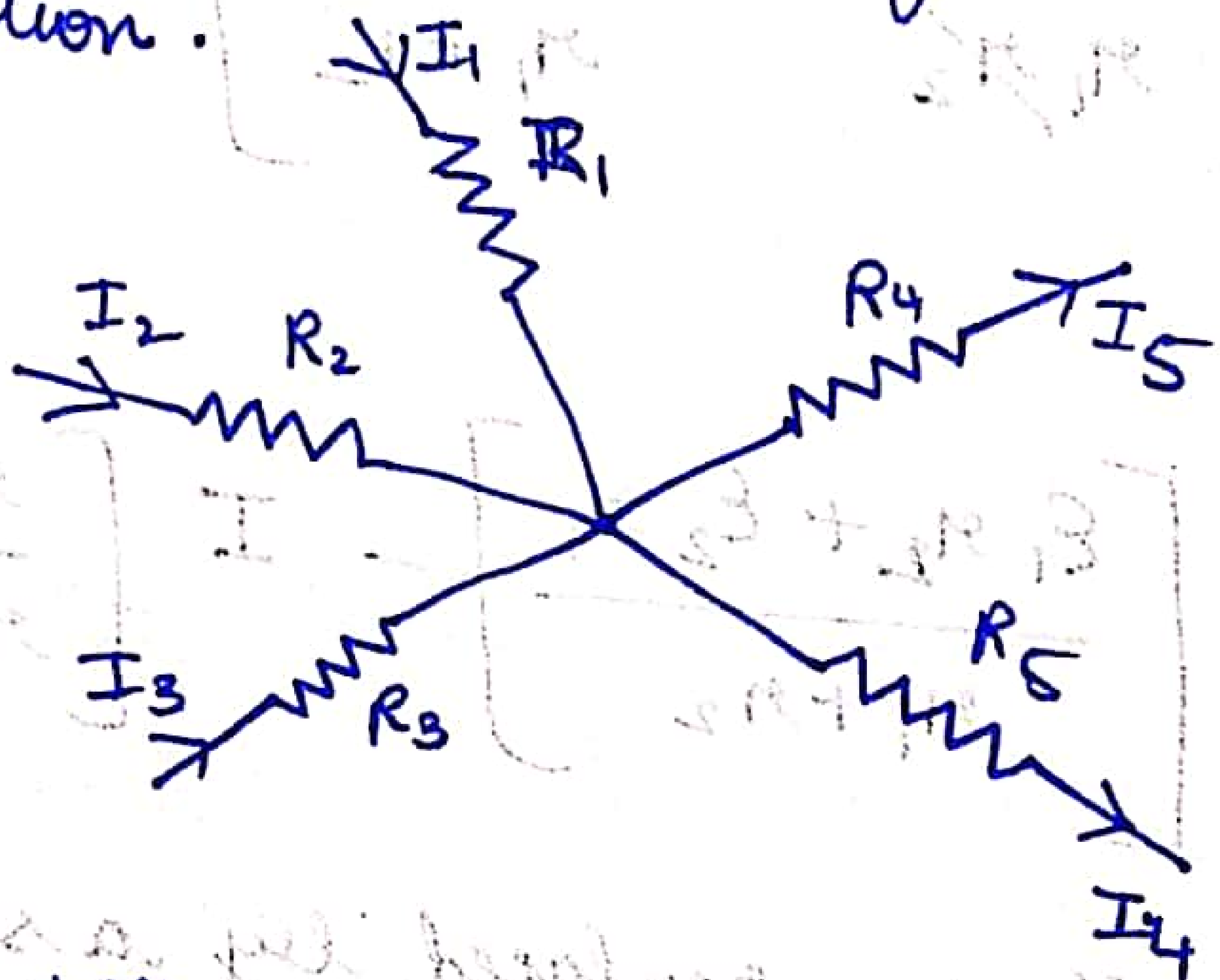
In **series**, if 'n' cells have same $\mathcal{E}$ and r,

$\mathcal{E}_{eq} = n\mathcal{E}$ $r_{eq} = nr$

KIRCHOFF'S RULES

a) Junction Rule:

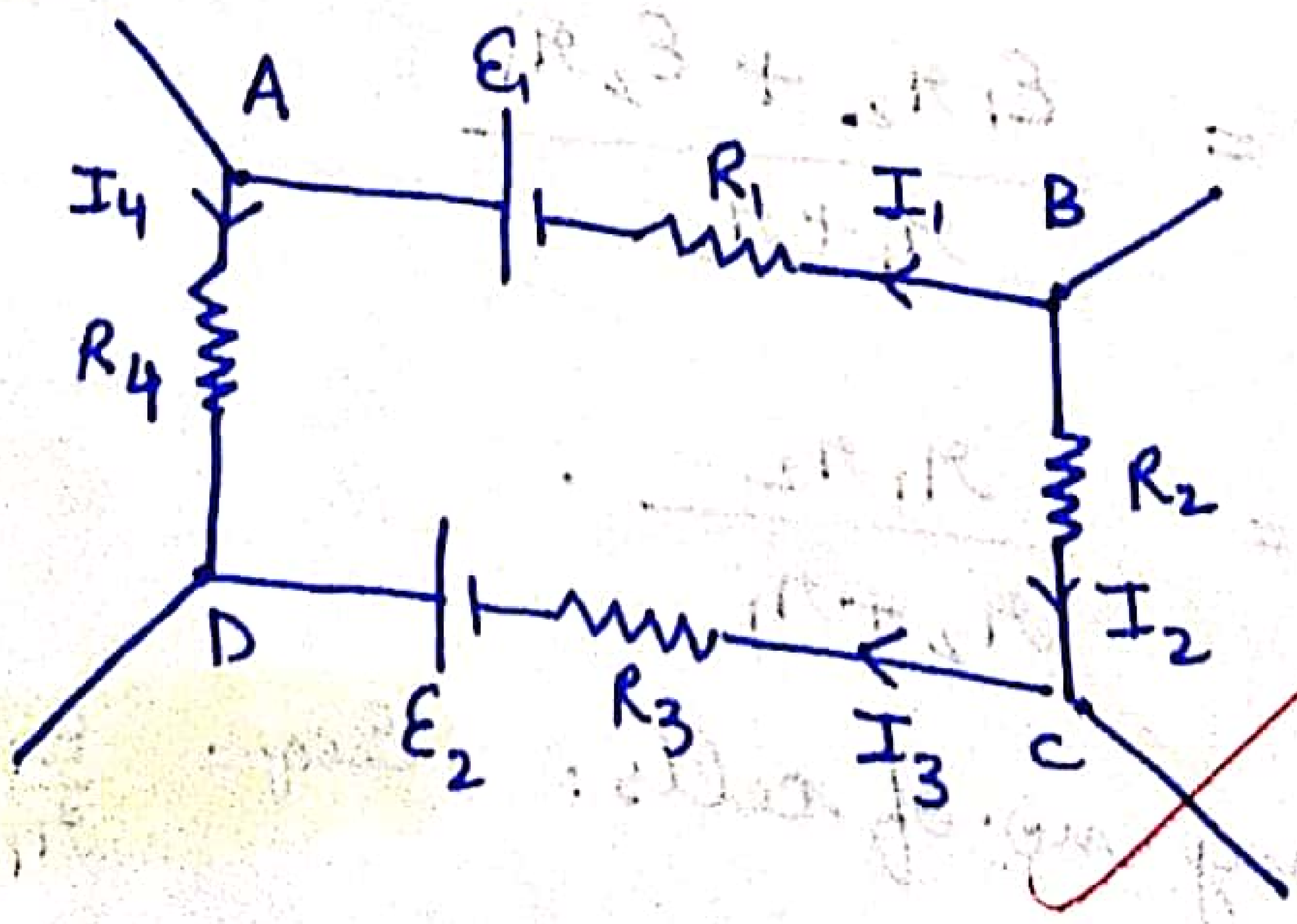
The total sum of current entering the junction is equal to the ~~total~~ sum of current leaving the junction.



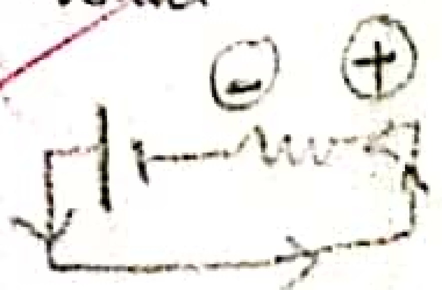
$$I_1 + I_2 + I_3 = I_4 + I_5.$$

b) Loop Rule:

The algebraic sum of potential difference across all the circuit elements in a closed loop is equal to zero.



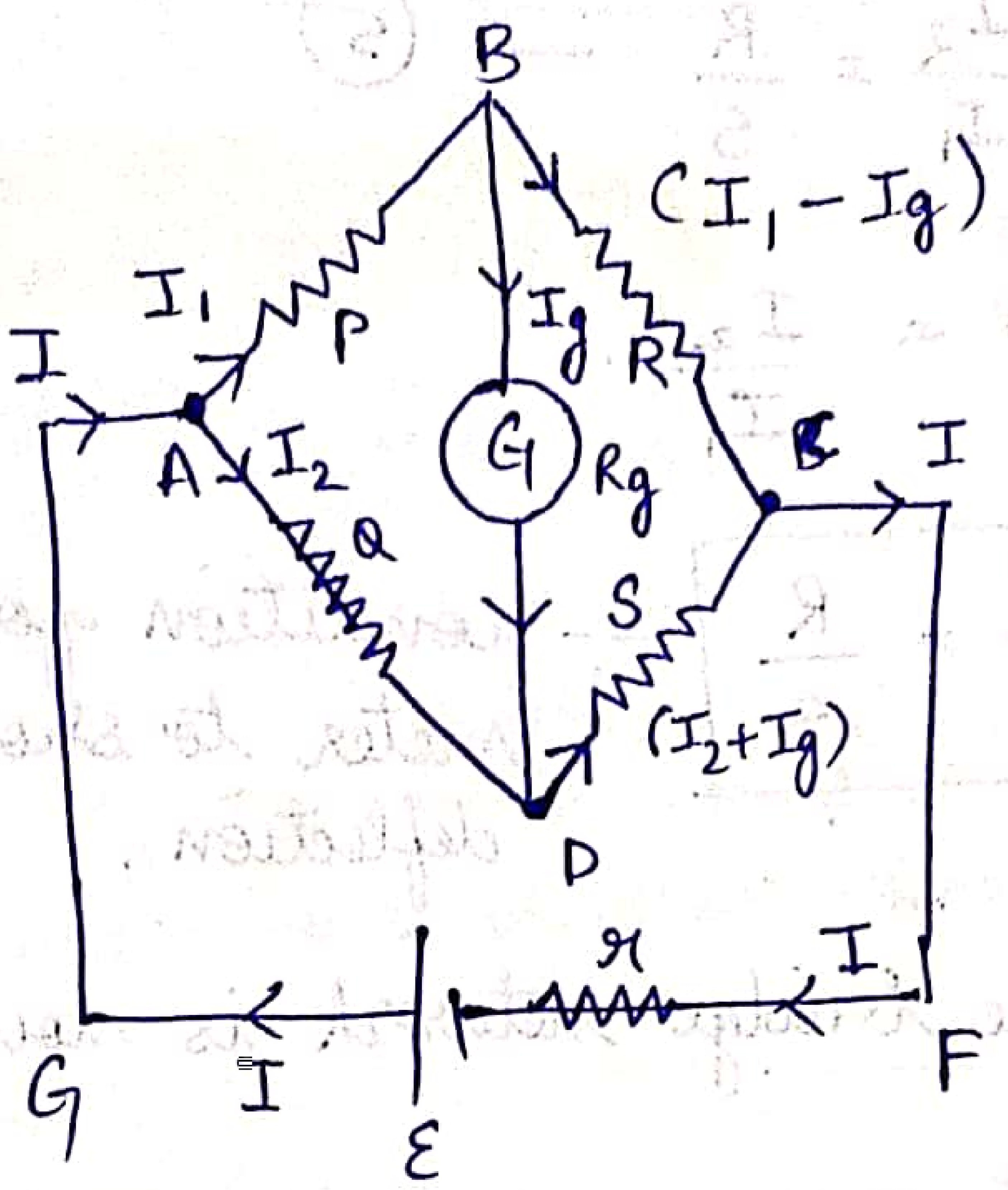
For a resistor, current always flows from high potential to low potential.



$$-\mathcal{E}_1 + I_1R_1 - I_2R_2 - I_3R_3 + \mathcal{E}_2 + I_4R_4 = 0.$$

Application of Kirchoff's rules

Wheatstone bridge network



It consists of 4 resistors PQRS connected to a galvanometer and a cell of EMF ε with internal resistance r. as shown.

Meter bridge is an example of wheatstone bridge network

Applying junction rule we get $I = I_1 + I_2$ — (1)

loop ABDA: $-I_1 P - I_g R_g + I_2 Q = 0$ — (2)

loop BCDB: $-(I_1 - I_g)R + (I_2 + I_g)S + I_g R_g = 0$ — (3)

If the bridge is balanced, the current through the galvanometer is zero.

$\therefore I_g = 0$, equations 2 and 3 becomes:

2 $\Rightarrow$ $-I_1 P + I_2 Q = 0$ $\Rightarrow$ $I_2 Q = I_1 P$.

$\therefore \frac{I_2}{I_1} = \frac{P}{Q}$ — 4

③ $\Rightarrow$ $-I_1 R + I_2 S = 0.$

$$I_1 R = I_2 S$$

$$\frac{I_2}{I_1} = \frac{R}{S} \quad \text{--- ⑤}$$

$$④ = ⑤ = \frac{I_2}{I_1}$$

$$\boxed{\therefore \quad \frac{P}{Q} = \frac{R}{S}}$$ — condition for galvanometer to show null deflection.

i.e., the bridge network is now balanced.