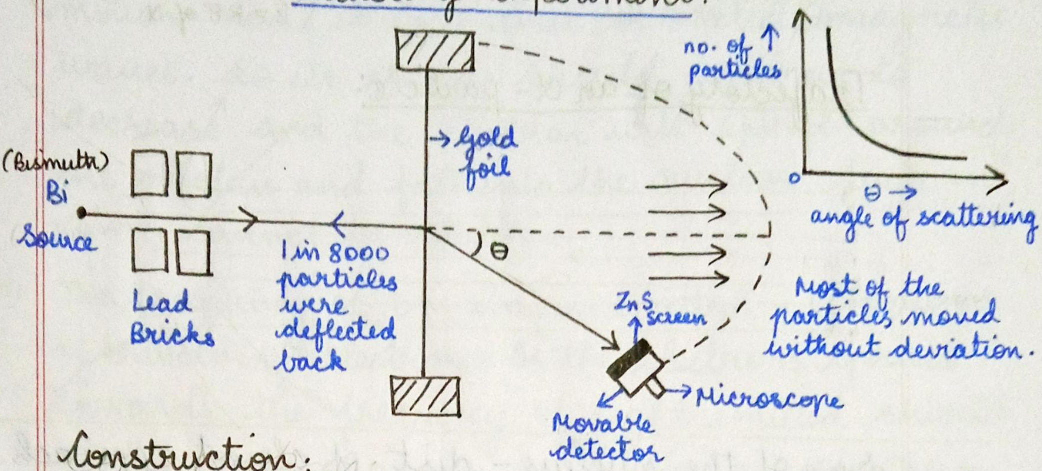
Chapter-12: Atoms

Rutherford's, Geiger and Marssden's α particle scattering experiment.



Construction:

(i) It consists of a source of α-particles - Bismuth (Bi)

(ii) Two sets of Pb bricks are used to produce a collimated beam of α-particles.

(iii) A thin gold foil (10^{-7}m thickness) is kept in the path of α particles.

(iv) A movable detector with ZnS screen and a microscope are used to detect the α-particles.

Observation:

(i) Most of the α-particles move without any deviation.

(ii) 1 in 8000 α-particles were deflected back in their own path.

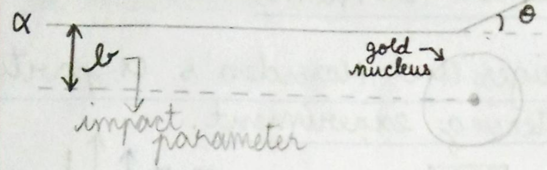
(iii) Few of the α-particles (less than 1%) are scattered through small angles.

Conclusion:

(i) Most of the space in the atom is empty.

(ii) 1 in 8000 α-particles were deflected back along their path as they must have encountered a very large repulsive force from another +ve charge.
Hence the entire +ve charge and the entire mass of the atom is conc. in a small region at the centre - nucleus.

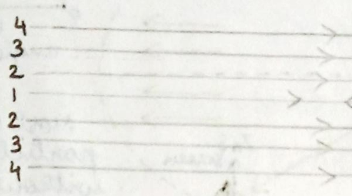
(iii) Few of the α particles were scattered through small angles as they must have approached the nucleus with different values of impact parameter - The $\perp^r$ dist. bet. the inital velocity of the α particle and the line joining the centre of the gold nucleus when it is far away from the nucleus.



$$b = \frac{Z e^2 \cot \theta/2}{4\pi \varepsilon_0 E}$$

(Z = at. no. of gold
E = KE of α)

Trajectory of an α-particle:



Size of the nucleus - dist. of closest approach r_0

As the α particle starts moving towards the nucleus, it possess kinetic energy as it approaches the nucleus, its kinetic energy gets converted to PE of a system of 2 charges (α particles and gold nucleus). At the distance of closest approach r_0, it has only PE.

Hence, $KE = PE$

$$\frac{1}{2} m v^2 = \frac{k q_1 q_2}{r_0}$$

$$q_1 = +2e \qquad q_2 = +79e.$$

$$r_0 = \frac{k q_1 q_2}{\frac{1}{2} m v^2}$$

$$r_0 = 10^{-14} \text{ to } 10^{-15} \text{ m.}$$

Drawbacks of Rutherford's model of an atom.

(i) Electrons revolving in the orbit is in accelerated motion. Hence it will emit out electromagnetic waves. So its energy should continue to decrease and the electron will spiral around the nucleus and fall into the nucleus. Such an atom cannot be stable.

(ii) The frequency of EM waves emitted = frequency of revolving electron. As the electrons spirals inwards, the frequency of light emitted should continuously change. So continuous spectrum should be observed, but what is seen is line spectrum.

Bohr's model of an atom

(i) Electrons revolve in a certain stable or stationary orbits without emission of energy.

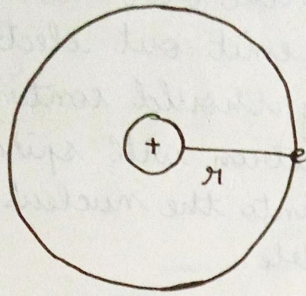
(ii) Electrons revolve in only those orbits for which the angular momentum of the electron is an integral multiple of $h/2\pi$.

$$L = \frac{nh}{2\pi} \quad (n = 1, 2, 3, \ldots)$$

(iii) When an electron makes a transition from higher energy level to a lower energy level, a photon is emitted whose energy is equal to the energy difference between the 2 levels.

$$E = h\nu = E_i - E_f$$

Total energy of an electron in its orbit.



The centripetal force required for the e^- to revolve is provided by the coulomb force of attraction between the proton and electron.

$$F_c = \frac{mv^2}{r} \text{ (centripetal force)}$$

$$F_e = \frac{ke^2}{r^2} \text{ (coulomb force)}$$

$$F_c = F_e.$$

$$\frac{mv^2}{r} = \frac{ke^2}{r^2}$$

$$v^2 = \frac{ke^2}{rm} \quad \text{— (1)}.$$

$$\text{KE of the } e^- = \frac{1}{2}mv^2 = \frac{1}{2}\cancel{m}\frac{ke^2}{\cancel{m}r}$$

$$KE = \frac{ke^2}{2r}. \quad \text{— (2)}$$

$$\text{PE of the } e^- = -\frac{ke^2}{r} \quad \text{— (3)}$$

$$\text{Total energy} = KE + PE = \frac{ke^2}{2r} - \frac{ke^2}{r}.$$

$$TE = -\frac{ke^2}{2r} \quad \text{— (4)} \quad \left[\begin{array}{l} \therefore KE = -TE \\ PE = 2\,TE. \end{array}\right]$$

-ve sign indicates that e^- are bound to the nucleus by attractive force and energy must be supplied to displace 'em.

Velocity of the e^- in the n^{th} orbit.

Acc. to Bohr's 2^{nd} postulate, angular momentum of the e^- is given as:

$$L = \frac{nh}{2\pi}$$

$$mv_n r_n = \frac{nh}{2\pi}$$

$$mr_n = \frac{nh}{2\pi v_n} \quad \text{— } 5$$

Sub (5) in (1)

$$v_n^2 = \frac{ke^2}{mr_n} = ke^2 \times \frac{2\pi v_n}{nh}$$

$$v_n^2 = \frac{ke^2 \, 2\pi v_n}{nh}$$

$$v_n = \frac{e^2 \, 2\cancel{\pi}}{\cancel{4\pi}_{2} \varepsilon_0 nh}$$

$$v_n = \frac{e^2}{2\varepsilon_0 nh} \quad \text{— } (6) \qquad \therefore v_n \propto \frac{1}{n}.$$

Radius of the orbit.

Sub 6 in 5 we get:

$$mr_n = \frac{nh}{\cancel{2}\pi} \times \frac{\cancel{2}\varepsilon_0 nh}{e^2}$$

$$r_n = \frac{n^2h^2\varepsilon_0}{\pi e^2 m} \quad \text{— } (7) \qquad \therefore r \propto n^2$$

$n=1$, $r_1 = 0.53 \times 10^{-10}$ m or $0.53\,\text{Å}$ = Bohr radius

Total energy of e^- in orbit.

sub (7) in (4)

$$TE = -\frac{ke^2}{2r_n} = \frac{-ke^2}{2\,\frac{\varepsilon_0 n^2 h^2}{\pi m e^2}}$$

$$E_n = -me^4 \cdot \cancel{\pi} \cdot \frac{1}{4\cancel{\pi}\varepsilon_0} \cdot \frac{1}{2} \cdot \frac{1}{\varepsilon_0 n^2 h^2}.$$

$$\boxed{E_n = \frac{-me^4}{8\varepsilon_0^2 h^2 n^2}} \quad \left(E_n = -\frac{13.6}{n^2}\ eV\right)$$

$$\therefore E_n \propto \frac{1}{n^2}.$$

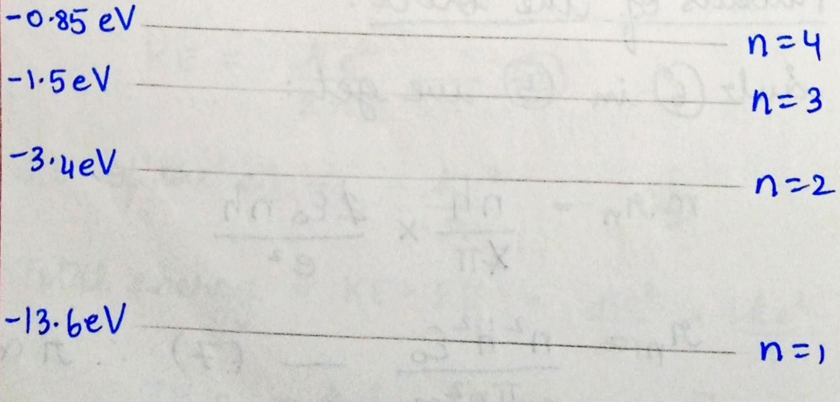
Energy levels of H atom

$n=1$, $E_1 = -13.6\ eV$

$n=2$, $E_2 = -3.4\ eV$

$n=3$, $E_3 = -1.5\ eV$

$n=4$, $E_4 = -0.85\ eV$



Spectral series of hydrogen atom

When an e^- makes a transition from higher energy to a lower energy level photon is emitted whose energy is equal to the energy differente between the 2 levels.

$$E_i = \frac{-me^4}{8\varepsilon_0^2 n_i^2 h^2} \qquad E_f = \frac{-me^4}{8\varepsilon_0^2 h^2 n_f^2}$$

$$\Delta E = h\nu = E_i - E_f.$$

$$h\nu = -\frac{me^4}{8\varepsilon_0^2 h^2 n_i^2} - \left(\frac{-me^4}{8\varepsilon_0 h^2 n_f^2}\right)$$

$$h\nu = \frac{me^4}{8\varepsilon_0^2 h^2}\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

$$\frac{hc}{\lambda} = \frac{me^4}{8\varepsilon_0^2 h^2}\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

$$\frac{1}{\lambda} = \frac{me^4}{8\varepsilon_0^2 h^3 c}\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

$$\frac{1}{\lambda} = R\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

$$R = \frac{me^4}{8\varepsilon_0^2 h^3 c} = 1.03 \times 10^7/\text{m}.$$ (Rydberg's const)

This value was found by Balmer to be $1.09 \times 10^7/\text{m}$, experimentally, which further gave proof to Bohr's atomic model.

Q) 12.1, 12.3

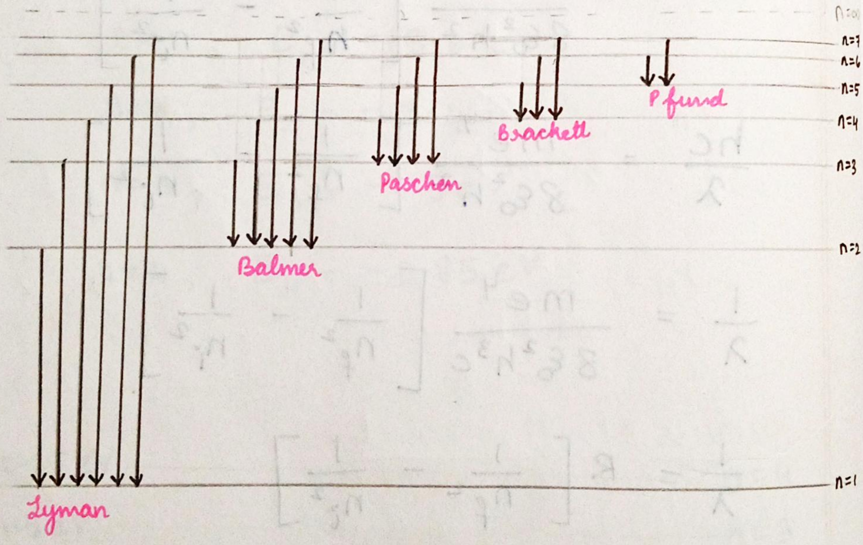
1) $n_f = 1, \; n_i = 2, 3, 4, \ldots$
→ Lymann series – UV

2) $n_f = 2, \; n_i = 3, 4, 5, \ldots$
→ Balmer series – VIS

3) $n_f = 3, \; n_i = 4, 5, 6 \ldots$
→ Paschen series – IR

4) $n_f = 4, \; n_i = 5, 6, 7, \ldots$
→ Brackett series – IR

5) $n_f = 5, \; n_i = 6, 7, 8, \ldots$
→ Pfund series – Far IR.



$H\alpha$ – $3 \rightarrow 2$

$H\beta$ – $4 \rightarrow 2$

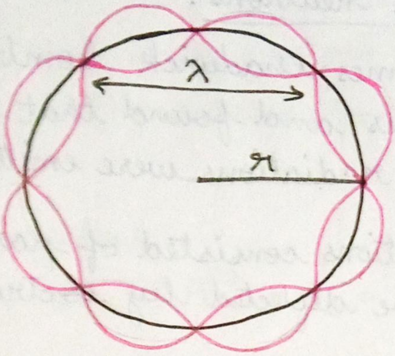
$H\gamma$ – $5 \rightarrow 2$

no. of transitions (to ground state)

$$\frac{n(n-1)}{2}$$

n = Highest level.

Debroglie's explanation of Bohr's 2nd Postulate



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According to Debroglie's hypothesis, a moving material particle has wave nature associated with it. So the electrons moving in orbit also have a wave nature. These waves will super impose and stationary waves are produced.

For an e^- travelling in n^{th} orbit (circular), the total distance travelled is the circumference of the orbit.

$$\text{Total dist.} = n\lambda.$$

$$n\lambda = 2\pi r_n$$

$$2\pi r_n = n\frac{h}{p}$$

$$2\pi r_n = \frac{nh}{mv_n}$$

$$mv_n r_n = \frac{nh}{2\pi}.$$

$$L = \frac{nh}{2\pi}.$$