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WHEN IS A CORRELATION MATRIX APPROPRIATE FOR FACTOR ANALYSIS?

SOME DECISION RULES

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Three techniques for the assessment of the psychometric adequacy of correlation matrices are discussed. These procedures are: (*a*) computation of Bartlett's test of sphericity, (*b*) inspection of the off-diagonal elements of the anti-image covariance matrix, and (*c*) computation of the Kaiser-Meyer-Olkin measure of sampling adequacy. The advantages and disadvantages of each are compared with respect to assessment of correlation matrices prior to factor analysis.

It has become common practice for researchers to report in professional journals the results of factor and component analyses of various correlation matrices. Undoubtedly this is, at least in part, due to the accessibility of computer programs that perform these tasks. One need only casually review the educational and psychological literature to find factor analysis applied to questionnaires, attitude scales, test batteries, and demographic data. Seldom, however, is evidence provided that the sample correlation matrices at hand are appropriate for factor-analytic methods. The necessity of determining the adequacy of one's data prior to the application of factor analysis is exemplified by the problems encountered in some recent attempts to interpret factor or component analysis of random data.

Armstrong and Soelberg (1968) demonstrated that an ostensibly acceptable pattern matrix might be obtained through application of a principal component solution to a correlation matrix based on random normal deviates ($N = 50$). Shaycoft [2] demonstrated that she was forced to attempt an interpretation of random numbers as the basis of meaningful components when she analyzed a matrix of 10 variables of interest (from PROJECT TALENT) and four random deviates. Unfortunately, the reader, if not provided with some evidence of the psychometric adequacy of the sample correlation matrix under consideration, has little effective way to assess the adequacy of the solution. There are, however, techniques specifically designed to be applied to correlation matrices prior to any factoring procedures. It is the purpose of this article to outline those procedures and to encourage their use prior to analysis.

One available practice is the application of Bartlett's test of sphericity (Bartlett, 1950). Explanations of the test may be found in Kendall (1957), Anderson (1958), and Cooley and Lohnes (1971). Maxwell (1959) and Tobias and Carlson (1969) recommended that the test be used prior to the application of factor analysis. It is computed by the formula:

$$-\left[(N-1)\ 1/6\ (2P+5)\right] \mathrm{Log}_e |R|,$$

where N is the sample size, P is the number of variables, and $|R|$ is the determinant of the correlation matrix. For large N, the statistic is approximately distributed as chi-square with $\frac{1}{2}P\ (P-1)$ degrees of freedom and has the associated hypothesis that the sample correlation matrix came from a multivariate normal population in which the variables of interest are independent. Rejection of the hypothesis is taken as an indication that the data are appropriate for analysis.

Knapp and Swoyer (1967) conducted a study of the power of the test and found it to be quite substantial. They determined

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[2] M. F. Shaycoft. The Eigenvalue Myth and the Data Reduction Fallacy. Paper presented at the meeting of the American Educational Research Association, Minneapolis, Minnesota, March 1970.

that for sample size equal to 200, number of variables equal to 10, and alpha equal to .05 one would be virtually certain to reject the independence hypothesis when the intercorrelations among the variables are as low as .09. Tobias and Carlson (1969) used the Bartlett test to compute the probability associated with the Armstrong and Soelberg (1968) matrix and found it to be .55, thus failing to reject the hypothesis of independence. Obviously the prior use of the Bartlett test would have precluded the application of component analysis to that matrix.

A second procedure involves the inspection of the off-diagonal elements of the anti-image covariance matrix $S^2R^{-1}S^2$, where R^{-1} is the inverse of the correlation matrix and the diagonal matrix S^2 is defined as $(\text{diag}R^{-1})^{-1}$. Psychometric theory suggests that if the factor-analytic model is appropriate for a set of data, R^{-1} should be near diagonal (Kaiser, 1963). Accordingly $S^2R^{-1}S^2$, the matrix of covariances of the unique parts of the tests, should also approach a diagonal.[3] If this matrix does not exhibit many zero off-diagonal elements, the investigator has evidence that the correlation matrix is *not* appropriate for factor analysis. Dziuban and Shirkey[4] (1973) applied this procedure to the Armstrong and Soelberg (1968) matrix. They found that 142 (or 37%) of the off-diagonal elements of $S^2R^{-1}S^2$ were greater than zero (> .09). Thus one may arbitrarily decide that the number of nonzero off-diagonal elements is sufficiently large to make the data undesirable for factor analysis.

A third procedure involves the computation of the Kaiser-Meyer-Olkin measure of sampling adequacy (Kaiser, 1970). The modified overall index is defined as:

$$\frac{\sum\sum_{j\neq k} r_{jk}^2}{\sum\sum_{j\neq k} r_{jk}^2 + \sum\sum_{j\neq k} q_{jk}^2},$$

where the q^2s are the squares of the off-diagonal elements of the anti-image correlation matrix $SR^{-1}S$ and r^2s are the squares of the off-diagonal elements of the original correlations. The index yields an assessment of whether the variables belong together psychometrically and thus whether the correlation matrix is appropriate for factor analysis. A similar measure may be defined for each variable separately. For j, the measure of sampling adequcy is:

$$\frac{\sum_{\substack{k \\ k\neq j}} r_{jk}^2}{\sum_{\substack{k \\ k\neq j}} r_{jk}^2 + \sum_{\substack{k \\ k\neq j}} q_{jk}^2}$$

It gives an indication of whether a particular variable "belongs to the family" psychometrically. The measure of sampling adequacy lies between zero and 1 and appears to be a function of four "main effects." Holding the others constant, it appears to improve as (*a*) the number of variables increases, (*b*) the (effective) number of factors decreases, (*c*) the number of subjects increases, and (*d*) the general level of correlation increases. Kaiser's (1974) present calibration of the index is as follows:

In the .90s—marvelous
In the .80s—meritorious
In the .70s—middling
In the .60s—mediocre
In the .50s—miserable
Below .50 —unacceptable

The overall measure of sampling adequacy for the Armstrong and Soelberg (1968) matrix was .42, which is a clear indication that it was inappropriate for factor analysis.

A Real Data Example

For illustrative purposes, results of the three procedures when applied to Harman's (1967) eight political variables ($N = 147$) are presented.

Bartlett's Test of Sphericity

The determinant of the correlation matrix was $.17 \times 10^{-3}$. This leads to a clear rejection of the hypothesis of independence, thereby providing evidence that the matrix may be appropriate for analysis.

[3] It is also true that $SR^{-1}S$, the anti-image correlation matrix, should be near diagonal.

[4] C. D. Dziuban and E. C. Shirkey. On the Psychometric Adequacy in Correlation Matrices. Paper presented at the annual meeting of The American Educational Research Association, New Orleans, Louisiana, February 25–March 1, 1973.

TABLE 1

ANTI-IMAGE COVARIANCE MATRIX ($S^2R^{-1}S^2$) AND MEASURES OF SAMPLING ADEQUACY FOR THE POLITICAL VARIABLES

Variable	Variable 1	2	3	4	5	6	7	8	Measure of sampling adequacy
1	—								.73
2	<u>−.11</u>								.76
3	.06	−.07							.84
4	−.03	.01	.00						.87
5	.00	.03	.06	−.06					.53
6	.04	−.05	−.02	.03	−.03				.93
7	.02	−.02	.05	−.05	<u>.19</u>	.01			.78
8	.04	.00	.04	−.09	.05	.04	.00	—	.86

Note. Overall measure of sampling adequacy = .81. Underscored elements are not zero to the first place.

Inspection of $S^2R^{-1}S^2$

The anti-image covariance matrix $S^2R^{-1}S^2$ and the measures of sampling adequacy are presented in Table 1. It may be observed that only four elements of $S^2R^{-1}S^2$ were not zero to the first decimal place (approximately 7%). Thus one may interpret this as sufficient evidence that the matrix is close enough to diagonal, thereby deciding that the data are good for factoring.

Measure of Sampling Adequacy

The overall measure of sampling adequacy for the correlation matrix was .81, which puts it into the "meritorious" range. Examining the individual measures of sampling adequacy, it may be observed that only Variable 5 (measure of sampling adequacy = .53) was located in the "miserable" range.

In this case, the results of each procedure give evidence that the matrix is generally appropriate for factor analysis.

DISCUSSION

There seem to be at least three procedures available to investigators for assessing the psychometric adequacy of their sample correlation matrices prior to factor analysis. The Bartlett test, which is a function of the sample size, number of variables, and $\log_e$ of the determinant of the correlation matrix, may be used as a lower bound to the quality of the matrix. This is, if one fails to reject the independence hypothesis, the matrix need be subjected to no further analysis. On the other hand, rejection of the independence hypothesis on the Bartlett test is not a clear indication that the matrix is psychometrically sound. Because for large N the statistic is approximately distributed as chi-square, the sample size has a substantial effect on the probability of rejection. Dziuban and Harris (1973) also demonstrated that not all contingencies may be guarded against by prior application of the Bartlett test.

Inspection of the off-diagonal elements of the anti-image covariance matrix $S^2R^{-1}S^2$ may also aid the investigator in judging the psychometric quality of his sample correlation matrix. If $S^2R^{-1}S^2$ turns out to be very close to a diagonal, then one may have confidence that factor analysis is indeed appropriate in that case. The problem here lies in determining how diagonal is diagonal. When Dziuban and Shirkey (see Footnote 4) examined matrices of known psychometric quality, they found between 7% and 14% nonzero elements for $j \neq k$. The matrix of random variables analyzed by Armstrong and Soelberg (1968) exhibited 37% nonzero off-diagonal elements.

Perhaps it seems reasonable that if say 25% of those elements are nonzero, one has reason to suspect the psychometric quality of his correlation matrix.

The use of the measure of sampling adequacy allows one to make decisions regarding individual variables as well as the overall quality of the correlation matrix. By the use of this index, the investigator may identify individual variables that might lead to erroneous interpretation. Because factor-analytic investigations require that a priori judgments be made concerning which variables should be included, the use of the measure of sampling adequacy might be the logical intermediate step to assess the effiicacy of those judgments. It seems desirable, however, that prior to the application of factor-analytic methods, the sample correlation matrices should be assessed for psychometric quality using at least one, or possibly all, of the procedures outlined in this article.

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