

# 122 Final Project

Veronica Strempfer

2025-03-07

## Introduction

Airplanes are one of many forms of transportation that we use for travel and we often don't think about the factors that go into making it fly. Aerodynamics is one of the factors that go into airplane flight and is defined as follows: "Weight, lift, thrust, and drag are the four principles of aerodynamics. These physics of flight and aircraft structures forces cause an object to travel upwards and downwards, as well as faster and slower" (Shah, 2025). Furthermore, the weight of cargo is something taken into account in relation to the plane's flight. For example, "Amiouny et al. [1] presented an approach for the onedimensional loading problem where the constraint is to balance around the aircraft's midpoint" (FOK et al., 2004). Essentially, balancing the weight around a certain area of the plane influences its flight performance. Paper airplanes may seem like simple objects but we can actually use them to understand different parts of aerodynamics and other factors that go into making a well structured airplane. While we typically associate aerodynamics with large scale application, the same principles apply to smaller objects, such as paper airplanes. By studying paper airplanes, we can learn how factors such as weight distribution influence their flight.

Relating to previous studies on paper airplanes Ristroph states, "The key criterion of a successful glider is that the center of mass must be in the 'just right' place... Good paper airplanes achieve this with the front edge folded over several times or by an added paper clip, which requires a little trial and error" (Ristroph, 2022). This study indicates the importance of adding weight in specific areas of the plane to ensure proper flight dynamics, which is relevant to our study of placing paper clips in different areas. Furthermore, weight distribution is significant not just for paper airplanes, but also in actual airplane design, where shifts in weight can greatly affect flight performance.

The study I will be doing specifically focuses on answering the question how do different combinations of paper clip placements on a paper airplane influence its flight distance? Using a full factorial design, we will test the effects of placing paper clips on the nose, middle, and rear of the airplane, either individually or in combination. Unlike past studies that tested a single paper clip placement at a time, this experiment will allow for multiple paper clips in different positions, providing a broader understanding of their impact. The results of this study will help illustrate the role of weight distribution in flight design which is important to real world application. Understanding how multiple factors interact in flight design is important in fields like aerospace engineering, where weight is a major factor in the design of drones and different air crafts. Furthermore, this study serves as an example of how factorial design, can be used to test and understand specific outcomes that result from multiple contributing factors and interactions.

## Methods

A full factorial design was chosen for this experiment to evaluate the effects of different combinations of paper clip placements on flight distance. This is a good experiment design because "Factorial designs can reduce the potential problems associated with pure insertion and are argued to be a statistical, more powerful and generalizable alternative to cognitive subtraction" (Friston et al., 1996a). Unlike methods that assume

each factor acts independently, factorial designs allow for the analysis of both main effects and interactions between variables. This approach ensures a more in depth understanding of how weight distribution impacts flight distance. The factors of interest of this experiment were paper clip on nose, paper clip on the middle, and paper clip on the rear, each with two levels of yes or no to represent the presence of that condition in the given trial. With a total of eight possible combinations: nose, middle, rear, nose/middle, nose/rear, middle/rear, none, or all three conditions.

First a pilot study was conducted to be used for sample size calculation. Each paper airplane was made from the same type of printer paper to avoid variability from paper type. For the pilot study a total of 3 replications were performed, 24 trials. To determine the order of each combination condition for each trial the sample function was used to ensure as much randomization as possible. The trials were performed inside to eliminate external factors (ie. the wind). A measuring tape was used to measure the distance of the plane from the take off point to the point where the plane first hit the floor. I also performed each trial to ensure that each airplane was thrown with the same throwing technique and to maintain consistency. After sample size calculations were done another set of trials were run based on the recommended sample size calculation and the same process was done for conducting these new trials.

The statistical method that was used for this experiment was the linear model (lm) function which analyzed the effects of the factors and their interactions on the outcome variable, distance. This method ensures that all main effects, two way interactions, and three way interactions are taken into account in the analysis. The assumptions involved for the lm function are normality, structure to the data, and equality of variances. Before doing further testing since this is a factorial experiment that was done in a somewhat controlled environment, independence is expected to be met because each trial is separate, and factors are randomly assigned. Additionally, looking at my data there aren't any extreme outliers so the normality assumption should likely be met.

For technical issues the main issue was making sure each trial had the same consistent throwing technique; being able to maintain the same technique throughout was difficult to do or even identify a set technique. Additionally, the precision of measurement was difficult because seeing the exact spot that the plane landed was difficult. Moreover, the nose of some planes began to deteriorate as more trials were performed, in this case I would create new planes. Finally, due to the experiment being performed indoors space was an issue at times (ie. the plane would hit the wall or ceiling).

## Results

```
set.seed(372025)
trials <- c("none", "nose_only", "middle_only", "rear_only", "nose_middle", "nose_rear", "middle_rear",
num_reps <- 3
replicated_trials <- rep(trials, each = num_reps)
ordered_trials <- sample(replicated_trials)
print(ordered_trials)

## [1] "rear_only"    "nose_only"     "middle_rear"   "nose_rear"    "nose_only"
## [6] "rear_only"    "all"          "nose_middle"  "none"        "all"
## [11] "middle_only"  "nose_rear"    "nose_middle"  "none"        "middle_only"
## [16] "nose_rear"    "nose_middle"  "none"        "rear_only"   "nose_only"
## [21] "all"          "middle_rear"  "middle_only"  "middle_rear"

# Pilot data
pilot_data <- data.frame(
  distance = c(218, 132, 118, 119, 130, 139, 107, 142, 218, 116, 110, 224, 138, 158, 137, 136, 150, 178),
  nose = c(0, 1, 0, 1, 1, 0, 1, 1, 0, 1, 0, 1, 1, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 1, 1, 0, 0, 0),
```

```

    middle = c(0, 0, 1, 0, 0, 0, 1, 1, 0, 1, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 0, 0, 1, 1, 1, 1),
    rear = c(1, 0, 1, 1, 0, 1, 1, 0, 0, 1, 0, 1, 0, 0, 0, 1, 0, 1, 0, 1, 1, 0, 1, 1)
)
print(pilot_data)

```

```

##      distance nose middle rear
## 1        218    0     0     1
## 2        132    1     0     0
## 3        118    0     1     1
## 4        119    1     0     1
## 5        130    1     0     0
## 6        139    0     0     1
## 7        107    1     1     1
## 8        142    1     1     0
## 9        218    0     0     0
## 10       116    1     1     1
## 11       110    0     1     0
## 12       224    1     0     1
## 13       138    1     1     0
## 14       158    0     0     0
## 15       137    0     1     0
## 16       136    1     0     1
## 17       150    1     1     0
## 18       178    0     0     0
## 19       115    0     0     1
## 20       131    1     0     0
## 21       123    1     1     1
## 22       125    0     1     1
## 23       124    0     1     0
## 24       126    0     1     1

```

```

library(knitr)
pilot_data_model <- lm(distance ~ nose*middle*rear, data=pilot_data)
pilot_data_model_summary <- summary(pilot_data_model)
output1 <- signif(summary(pilot_data_model)$coefficients, 4)
output1 <- as.data.frame(output1)
output1$`Pr(>|t|)`[1] <- formatC(output1$`Pr(>|t|)`[1], format = "e", digits = 3)
pf(pilot_data_model_summary$fstatistic[1], df1=pilot_data_model_summary$fstatistic[2], df2=pilot_data_m

```

```

##      value
## 0.1507106

```

```

knitr::kable(output1)

```

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	184.70	17.46	10.5800	1.254e-08
nose	-53.67	24.69	-2.1730	0.04513
middle	-61.00	24.69	-2.4700	0.02514
rear	-27.33	24.69	-1.1070	0.2847
nose:middle	73.33	34.92	2.1000	0.05196

	Estimate	Std. Error	t value	Pr(> t )
nose:rear	56.00	34.92	1.6030	0.1284
middle:rear	26.67	34.92	0.7636	0.4562
nose:middle:rear	-83.33	49.39	-1.6870	0.1109

```

power_factorial_23 <- function(beta_mean, beta_se, replicates,
                                iter=1000, alpha=0.05){

  # Make sure inputs are proper
  if(length(beta_mean) != 8){
    stop("beta_mean must be a vector with 8 values")
  }
  if(length(beta_se) != 8) {
    stop("beta_se must be a vector with 8 values")
  }

  # Generate data frame with NA outcome values
  sim_data <- data.frame(
    outcome = rep(NA, (8*replicates)),
    x1 = rep(c(0,0,0,0,1,1,1,1), replicates),
    x2 = rep(c(0,0,1,1,0,0,1,1), replicates),
    x3 = rep(c(0,1,0,1,0,1,0,1), replicates)
  )

  pvals <- NA

  for(j in 1:iter){
    for(i in 1:dim(sim_data)[1]){

      # Generate random values of beta based on mean and se
      beta <- rnorm(n=8, mean=beta_mean, sd=beta_se)

      # Generate simulated outcome variable values
      sim_data$outcome[i] <- beta[1] +
        beta[2]*sim_data$x1[i] +
        beta[3]*sim_data$x2[i] +
        beta[4]*sim_data$x3[i] +
        beta[5]*sim_data$x1[i]*sim_data$x2[i] +
        beta[6]*sim_data$x1[i]*sim_data$x3[i] +
        beta[7]*sim_data$x2[i]*sim_data$x3[i] +
        beta[8]*sim_data$x1[i]*sim_data$x2[i]*sim_data$x3[i]
    }

    # Run linear model for 2^3 data with all interaction terms
    model1 <- lm(outcome ~ x1+x2+x3+x1*x2+x1*x3+x2*x3+x1*x2*x3, data=sim_data)

    # Extract p-value and store it
    f <- summary(model1)$fstatistic
    pvals[j] <- pf(f[1], f[2], f[3], lower.tail=FALSE)
  }

  # Calculate estimate of power
}

```

```

power <- sum(pvals<0.05)/iter
return(power)
}

library(ggplot2)
set.seed(123)
source("power_factorial_23.R")

beta_mean <- c(184.67, -53.67, -61.00, -27.33, 73.33, 56.00, 26.67, -83.33)
beta_se <- rep(24,8)

replicates <- 2:50

power1 <- NA
for(i in 1:length(replicates)){
  power1[i] <- power_factorial_23(beta_mean, beta_se, replicates[i])
}

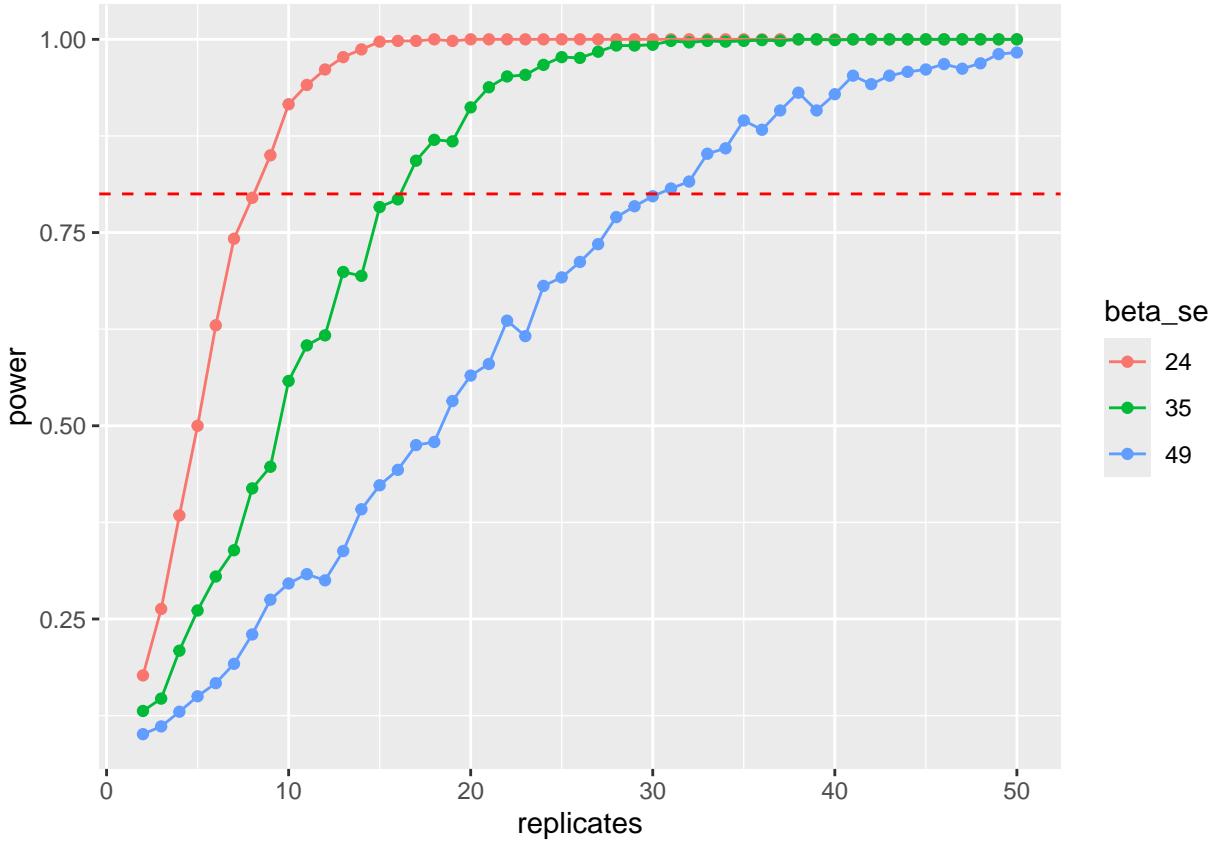
beta_se <- rep(35, 8)
power2 <- NA
for(i in 1:length(replicates)){
  power2[i] <- power_factorial_23(beta_mean, beta_se, replicates[i])
}

beta_se <- rep(49, 8)
power3 <- NA
for(i in 1:length(replicates)){
  power3[i] <- power_factorial_23(beta_mean, beta_se, replicates[i])
}

all_power <- data.frame(
  power = c(power1, power2, power3),
  beta_se = c(rep("24", length(power1)),
             rep("35", length(power2)),
             rep("49", length(power3))),
  replicates = rep(replicates, 3)
)

ggplot(data=all_power, mapping=aes(x=replicates, y=power, group=beta_se, color=beta_se)) +
  geom_point() + geom_line() + geom_hline(yintercept = 0.8, linetype = "dashed", color = "red")

```



The sample size calculation was done using the above graph for a  $2^3$  factorial experiment. Based on the graph we can see that for the lower `beta_se` of 24, 10 replicates are necessary for 80% power, for 35 about 15 reps, and for 49 around 30 reps. To satisfy the sample size calculation with high variability a sample size of 240 trials or 30 reps would be necessary. I decided to do 10 reps or 80 trials following the red curve that represents `beta_se` = 24. Given that I am not accounting for much variability in my data by using the lower `beta_se` and smaller sample size my results for my statistical tests will likely not have enough statistical power to show any significance in conditions and their interactions with other conditions on airplane distance.

```
set.seed(3122025)
trials <- c("none", "nose_only", "middle_only", "rear_only", "nose_middle", "nose_rear", "middle_rear",
num_reps <- 10
replicated_trials <- rep(trials, each = num_reps)
ordered_trials <- sample(replicated_trials)
print(ordered_trials)
```

```
## [1] "none"      "rear_only"   "all"        "nose_middle" "nose_rear"
## [6] "none"      "rear_only"   "none"       "nose_rear"   "nose_middle"
## [11] "rear_only"  "all"        "nose_middle" "none"       "nose_rear"
## [16] "all"        "nose_middle" "nose_middle" "nose_rear"   "nose_middle"
## [21] "rear_only"  "rear_only"   "nose_only"   "middle_rear" "middle_only"
## [26] "middle_only" "nose_middle" "middle_rear" "rear_only"   "nose_only"
## [31] "middle_rear" "middle_only" "middle_only"  "none"       "nose_only"
## [36] "all"        "nose_rear"   "middle_only" "nose_only"   "nose_only"
## [41] "rear_only"  "nose_rear"   "all"        "middle_rear" "nose_only"
## [46] "middle_rear" "nose_rear"   "rear_only"   "none"       "nose_rear"
## [51] "middle_only" "middle_rear" "middle_rear" "rear_only"   "middle_only"
```

```

## [56] "nose_rear"    "rear_only"     "nose_middle"   "nose_only"      "nose_rear"
## [61] "all"          "middle_only"   "middle_only"   "all"           "none"
## [66] "middle_only"  "nose_only"    "none"         "all"           "middle_rear"
## [71] "nose_middle"  "nose_only"    "none"         "nose_only"     "middle_rear"
## [76] "middle_rear"  "none"        "nose_middle"  "all"           "all"

# Full Data
experiment_data <- data.frame(
  distance = c(131, 178, 158, 124, 122, 125, 142, 176, 153, 134,
               198, 122, 125, 158, 112, 123, 197, 211, 158, 141,
               107, 102, 198, 131, 87, 152, 163, 157, 122, 178,
               173, 147, 163, 217, 173, 149, 195, 215, 218, 215,
               202, 171, 217, 150, 196, 160, 176, 219, 159, 212,
               152, 163, 157, 122, 178, 168, 172, 176, 153, 166,
               174, 168, 190, 185, 200, 155, 163, 177, 169, 182,
               132, 128, 140, 133, 150, 144, 135, 155, 147, 160),
  condition = c("none", "rear_only", "all", "nose_middle", "nose_rear",
                "none", "rear_only", "none", "nose_rear", "nose_middle",
                "rear_only", "all", "nose_middle", "none", "nose_rear",
                "all", "nose_middle", "nose_middle", "nose_rear",
                "nose_middle", "rear_only", "rear_only", "nose_only",
                "middle_rear", "middle_only", "middle_only", "nose_middle",
                "middle_rear", "rear_only", "nose_only",
                "middle_rear", "middle_only", "middle_only", "none",
                "nose_only", "all", "nose_rear", "middle_only", "nose_only",
                "nose_only", "rear_only", "nose_rear", "all", "middle_rear",
                "nose_only", "middle_rear", "nose_rear", "rear_only", "none",
                "nose_rear", "middle_only", "middle_rear", "middle_rear",
                "rear_only", "middle_only", "nose_rear", "rear_only",
                "nose_middle", "nose_only", "nose_rear",
                "all", "middle_only", "middle_only", "all", "none",
                "middle_only", "nose_only", "none", "all", "middle_rear",
                "nose_middle", "nose_only", "none", "nose_only", "middle_rear",
                "middle_rear", "none", "nose_middle", "all", "all")
)

experiment_data$nose <- ifelse(experiment_data$condition %in% c("all", "nose_middle", "nose_rear", "no
experiment_data$middle <- ifelse(experiment_data$condition %in% c("all", "middle_rear", "middle_only", "no
experiment_data$rear <- ifelse(experiment_data$condition %in% c("all", "rear_only", "middle_rear", "no

experiment_data$condition <- NULL

experiment_data
```

```

##      distance nose middle rear
## 1          131     0      0     0
## 2          178     0      0     1
## 3          158     1      1     1
## 4          124     1      1     0
## 5          122     1      0     1
## 6          125     0      0     0
## 7          142     0      0     1
```

## 8	176	0	0	0
## 9	153	1	0	1
## 10	134	1	1	0
## 11	198	0	0	1
## 12	122	1	1	1
## 13	125	1	1	0
## 14	158	0	0	0
## 15	112	1	0	1
## 16	123	1	1	1
## 17	197	1	1	0
## 18	211	1	1	0
## 19	158	1	0	1
## 20	141	1	1	0
## 21	107	0	0	1
## 22	102	0	0	1
## 23	198	1	0	0
## 24	131	0	1	1
## 25	87	0	1	0
## 26	152	0	1	0
## 27	163	1	1	0
## 28	157	0	1	1
## 29	122	0	0	1
## 30	178	1	0	0
## 31	173	0	1	1
## 32	147	0	1	0
## 33	163	0	1	0
## 34	217	0	0	0
## 35	173	1	0	0
## 36	149	1	1	1
## 37	195	1	0	1
## 38	215	0	1	0
## 39	218	1	0	0
## 40	215	1	0	0
## 41	202	0	0	1
## 42	171	1	0	1
## 43	217	1	1	1
## 44	150	0	1	1
## 45	196	1	0	0
## 46	160	0	1	1
## 47	176	1	0	1
## 48	219	0	0	1
## 49	159	0	0	0
## 50	212	1	0	1
## 51	152	0	1	0
## 52	163	0	1	1
## 53	157	0	1	1
## 54	122	0	0	1
## 55	178	0	1	0
## 56	168	1	0	1
## 57	172	0	0	1
## 58	176	1	1	0
## 59	153	1	0	0
## 60	166	1	0	1
## 61	174	1	1	1

```

## 62      168    0     1     0
## 63      190    0     1     0
## 64      185    1     1     1
## 65      200    0     0     0
## 66      155    0     1     0
## 67      163    1     0     0
## 68      177    0     0     0
## 69      169    1     1     1
## 70      182    0     1     1
## 71      132    1     1     0
## 72      128    1     0     0
## 73      140    0     0     0
## 74      133    1     0     0
## 75      150    0     1     1
## 76      144    0     1     1
## 77      135    0     0     0
## 78      155    1     1     0
## 79      147    1     1     1
## 80      160    1     1     1

library(knitr)
real_data_model <- lm(distance ~ nose*middle*rear, data=experiment_data)
real_data_model_summary <- summary(real_data_model)
output <- signif(summary(real_data_model)$coefficients, 4)
output<-as.data.frame(output)
output$`Pr(>|t|)`[1]<-formatC(output$`Pr(>|t|)`[1],format = "e",digits = 3)
pf(real_data_model_summary$fstatistic[1], df1=real_data_model_summary$fstatistic[2], df2=real_data_model_summary$fstatistic[3])
##      value
## 0.888128

knitr::kable(output)

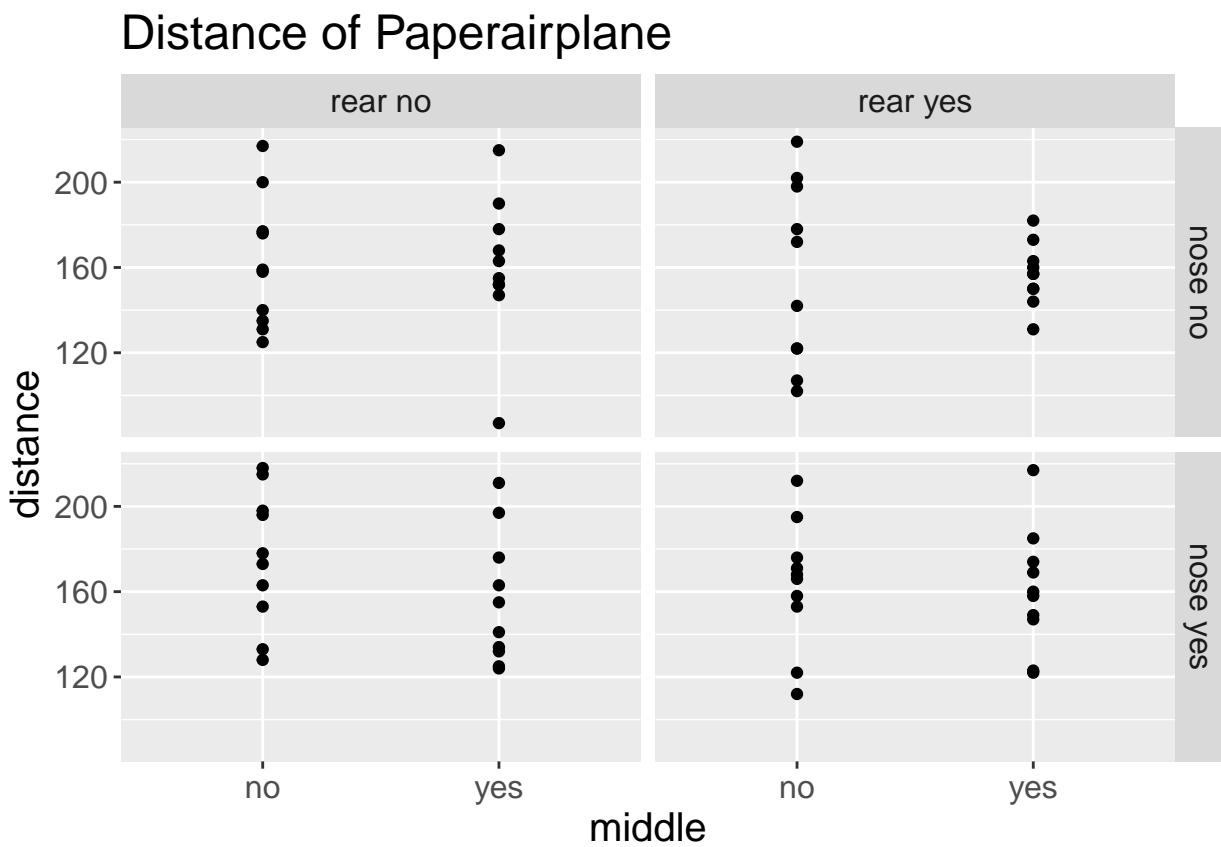
```

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	161.8	9.826	16.47000	4.057e-26
nose	13.7	13.900	0.98590	0.3275
middle	-1.1	13.900	-0.07916	0.9371
rear	-5.4	13.900	-0.38860	0.6987
nose:middle	-18.6	19.650	-0.94650	0.3471
nose:rear	-6.8	19.650	-0.34600	0.7303
middle:rear	1.4	19.650	0.07124	0.9434
nose:middle:rear	15.4	27.790	0.55410	0.5812

The null hypothesis for this experiment is none of the factors (nose, middle, rear) or their interactions has an effect on the distance of the paper airplane. The alternative hypothesis is that at least one of the factors or one of their interactions has an effect on the distance of the paper airplane. Looking at the above p-values of each individual factor and the two way and three way interactions we can see that each p-value is more than 0.05. These values are already larger when evaluated at alpha = 0.05 but for full completion, when comparing these values to the bonferroni-adjusted value which is  $0.05/7 = .007$  all independent p-values are more than this value. Finally, when examining the overall p-value at alpha = 0.05, 0.888128 is also greater than 0.05. Thus, based on these results we fail to reject the null hypothesis and can conclude that none

of the factors (nose, middle, rear) or their interactions has an effect on the distance of the paper airplane. However, a very important thing to note here is that my sample size calculation gave me a recommended sample size larger than the one I completed. I only did a total of 10 reps, 80 trials which didn't offer the full statistical power, the largest beta\_se value offered a sample size of 30 reps, 240 trials. Given that the full number of trials couldn't be completed the p-values aren't as precise as they would be with full statistical power. It is difficult to detect true significance because of the lack of statistical power/sample size.

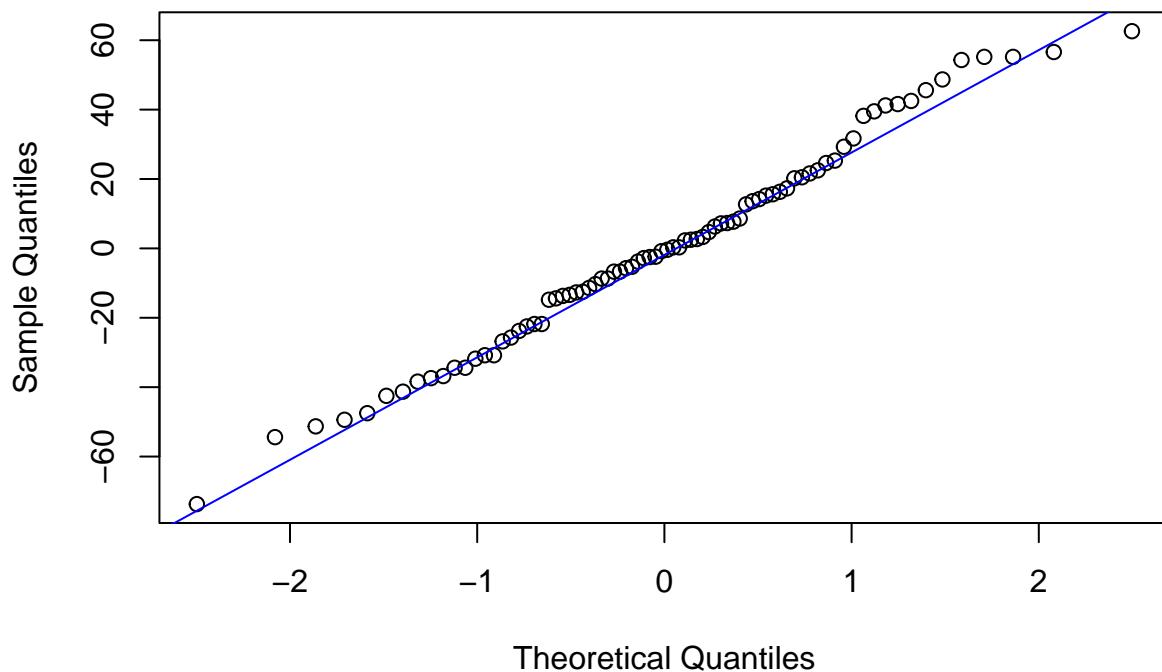
```
library(ggplot2)
experiment_data$middle <- as.factor(experiment_data$middle)
theme_update(text = element_text(size = 15))
ggplot(data = experiment_data, aes(x = middle, y = distance)) +
  geom_point() +
  facet_grid(nose ~ rear,
             labeller=labeller(nose = c("0" = "nose no",
                                         "1" = "nose yes"),
                                 rear = c("0" = "rear no",
                                         "1" = "rear yes")))) +
  scale_x_discrete(name = "middle", labels = c("no", "yes")) +
  ggtitle("Distance of Paperairplane")
```



Looking at this plot we can see that for the most part all eight combinations seem to range between the same values of about 120-250. One note is that the combination of middle yes, nose no, rear yes (middle\_rear) has a smaller range variance in comparison to the other combination groups. Furthermore, there is one outlier for middle yes, rear no, nose no (middle\_only) below the 120 range. Overall, from this graph we can see that there are no major differences between the different combinations and their effects on distance of the plane.

```
# Normality
qqnorm(real_data_model$residuals)
qqline(real_data_model$residuals, col = "blue")
```

## Normal Q–Q Plot

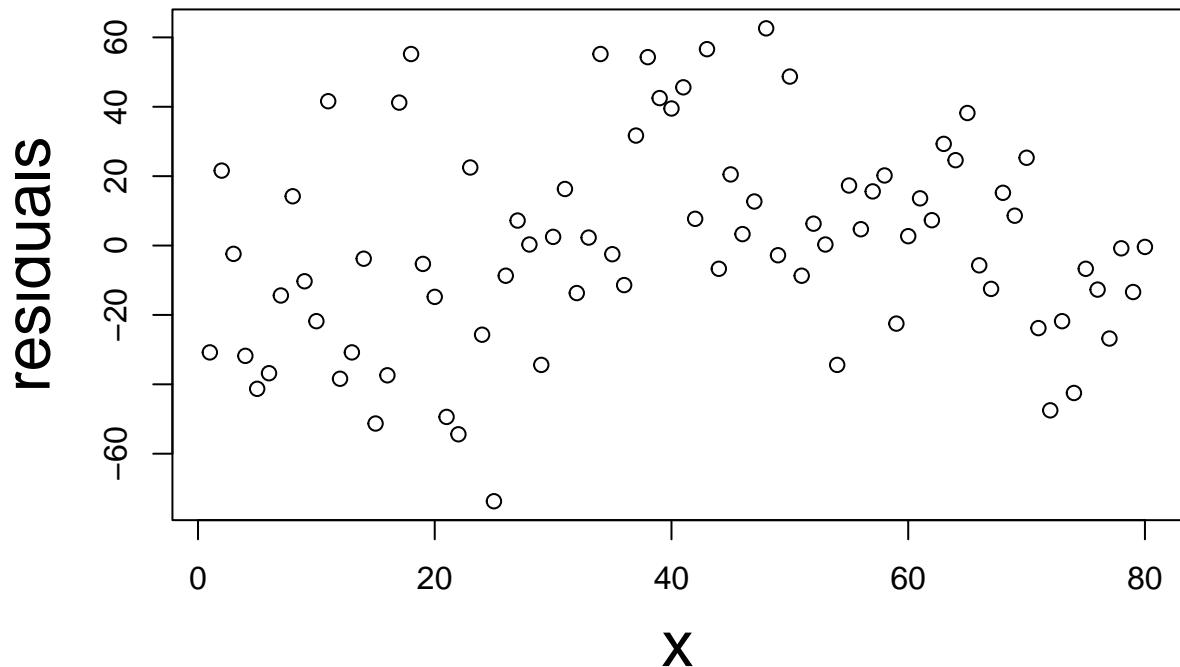


```
shapiro.test(real_data_model$residuals)
```

```
##
## Shapiro-Wilk normality test
##
## data: real_data_model$residuals
## W = 0.98818, p-value = 0.6769

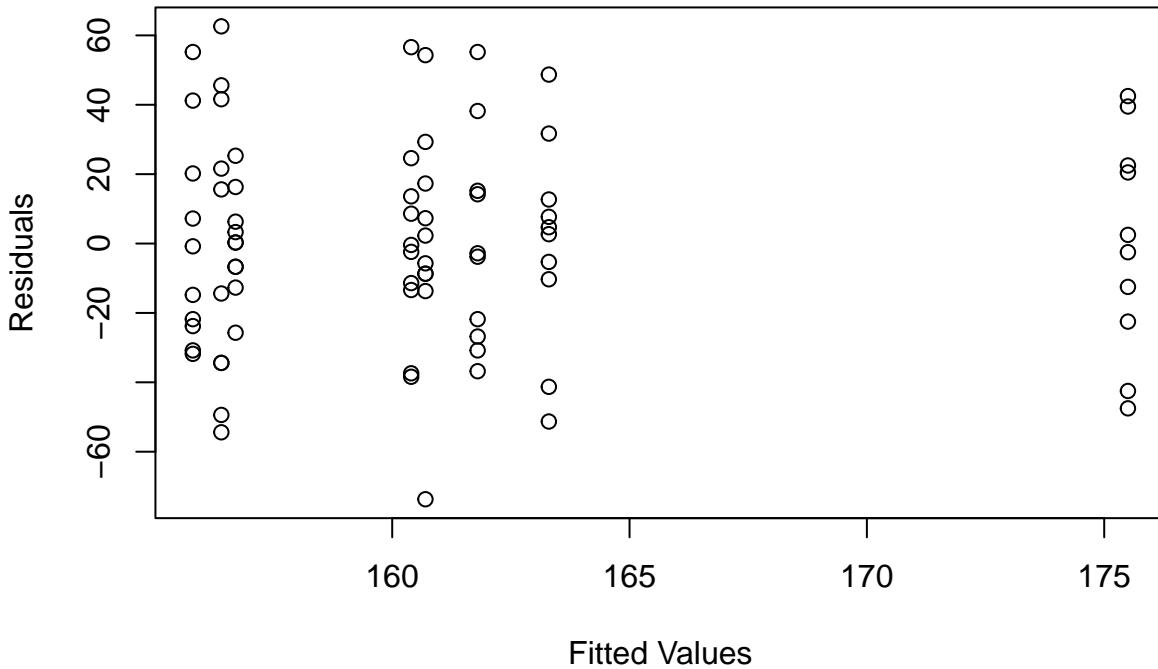
# Structure to the data
x <- 1:length(real_data_model$residuals)
plot(real_data_model$residuals ~ x, ylab="residuals", cex.lab=2,
     main="Residuals vs. Order of Data Collection", cex.main=2)
```

# Residuals vs. Order of Data Collection



```
# Equality of Variances
plot(real_data_model$residuals ~ real_data_model$fitted.values,
     main = "Residuals vs. Fitted Values",
     xlab = "Fitted Values",
     ylab = "Residuals")
```

## Residuals vs. Fitted Values



Performing model checking is done here in order to confirm that all assumptions for a factorial experiment are met. These assumptions are normality of the data, structure to the data, and equal variances. From the qqnorm plot we can see that the residual values follow closely to the normal line meaning that the residuals follow a normal distribution. Additionally, from the Shapiro-Wilk test we get a value of 0.6769 which is more than 0.05 so we can conclude that there is statistically significant evidence that the residuals follow a normal distribution. The result of the qqnorm plot and the Shaprio-Wilk test confirm that the normal distribution assumption of a factorial experiment is met. The next graph is titled "Residuals vs. Data Collection" and is used to examine the structure of the data. Based on the plot the residuals appear to be scattered both above and below zero across the range of the x-axis indicating that there is no consistent upward or downward trend satisfying the condition of no structure to the data. There is a slight u-shape to the data but the trend isn't distinct enough to violate the assumption and it is still valid to say that the structure of the data assumption is met. The last plot is titled "Residuals vs. Fitted Values" and this plot tests to see if the variances of the residuals are equal. Looking at the plot we can see that the residual values are evenly distributed around the same values, indicating that they have an equal variance, satisfying the assumption. After examining each of these forms of model checks we can conclude that the data passes the three assumptions of a factorial design and using the lm function of normality, structure to the data, and equal variances. Thus, no further methods are required, meaning no permutation test is necessary to support further findings and the p-values found in the lm function are accurate.

## Discussion

This experiment worked to answer the question: how do different combinations of paper clip placements on a paper airplane influence its flight distance? A full factorial design was used and I tested the effects of placing paper clips on the nose, middle, and rear of the airplane, either individually or in combination. After collecting a pilot study to determine sample size calculation, a new data set was collected based on

this sample size. The lm function was then used as one of the statistical methods to determine if any of the conditions or interactions were significant in affecting flight distance. The lm function showed that none of the individual factors or their interactions were significant. Leading me to fail to reject the null hypothesis and conclude that none of the factors or their interactions had a significant effect on flight distance.

However, a crucial aspect is that in my sample size calculation my range of potential sample sizes ranged from the low value of 10 reps - 30 reps. I chose to only do 10 reps resulting in lack of full statistical power of 80%. This limitation was highlighted in previous research, where it was stated: "Our findings suggest that randomized, controlled trials in clinical orthopaedic research utilise sample sizes which are too small to ensure statistical significance for what may be clinically important results." (Freedman et al., 2001). In this context, the small sample size in my experiment similarly prevented me from detecting true significance. Given that the full number of trials couldn't be completed, the p-values aren't as precise as they would be with full statistical power.

Despite the sample size limitations the assumptions for the lm function used for the factorial design were met. The normality assumption was validated through a qq plot and a Shapiro-Wilk test, with values following closely to the normal line and the p-value from the Shapiro-Wilk test being more than 0.05. Next, the residuals showed no distinct trends up or down besides a small u-shaped trend but nonetheless the data still passed the structure of the data assumption. Finally, the variance was approximately equal across values satisfying the equal variances assumption.

One of the issues that was involved in data collection was maintaining the same throwing technique for each trial. Not being able to control how to throw the plane from the same height and angle each time may have caused some variability across trials. Additionally, the precision of measurement was difficult because seeing the exact spot that the plane landed was difficult. The plane slid at some points and went certain distances that were difficult to see from afar. Moreover, the nose of some planes began to deteriorate as more trials were performed, in which case I would create new planes with new paper. Finally, due to the experiment being performed indoors space was an issue at times, where the plane would hit furniture or ceilings. When this occurred the trial would be performed again.

These findings can be applied to aspects beyond just paper airplanes. The lack of effect of weight distribution from the paperclip placements challenge the common idea that weight adjustments significantly influence performance in aerodynamics. In the field of engineering, designs often rely on the assumption that weight adjustments can significantly alter an object's trajectory and stability. For example, in aerospace engineering, the addition of weight in certain areas is sometimes assumed to improve control or balance. However, my findings from this paper airplane experiment show that other factors may play a role in flight distance/performance. This insight can be associated with engineering, emphasizing the importance of optimizing more types of design elements; such as wing shape and material type, rather than weight modifications. Similarly, in automotive and drone designs, engineers prioritize airflow dynamics and material type together rather than just one condition. Ultimately, this experiment demonstrated that many factors go into flight performance, not just weight distribution, and this illustrates the complexity of how planes and other types of machines work.

## References

- Fok K, Chun A. Optimizing air cargo load planning and analysis. InInternational Conference on Computing, Communications and Control Technologies 2004 Aug 14.
- Freedman KB, Back S, Bernstein J. Sample size and statistical power of randomised, controlled trials in orthopaedics. The Journal of Bone & Joint Surgery British Volume. 2001 Apr 1;83(3):397-402.
- Friston KJ, Price CJ, Fletcher P, Moore C, Frackowiak RS, Dolan RJ. The trouble with cognitive subtraction. Neuroimage. 1996 Oct 1;4(2):97-104.
- Scientists experiment with paper planes to study aerodynamics, flight stability. NSF - National Science Foundation. 2022. <https://www.nsf.gov/news/scientists-experiment-paper-planes-study>

