Week 10 Thursday
L-18
Convection

$$\frac{dM_r(r)}{dr} = 4\pi r^2 \rho(r)$$

$$\frac{dP(r)}{dr} = -\rho(r)\frac{GM_r(r)}{r^2}$$

$$\frac{dT(r)}{dr} = -\frac{3\kappa_R(r)\rho(r)}{64\pi\sigma}\frac{L_r(r)}{r^2 T(r)^3}$$

$$\frac{dL_r(r)}{dr} = 4\pi r^2 \rho(r)\epsilon(r)$$

$$M_r(r) \quad P(r) \quad L_r(r) \quad T(r)$$

$$\rho(r) \quad \mu(r) \quad \epsilon_{\rm nuc}(r) \quad \kappa_R(r)$$

$$P(r) = \frac{\rho(r)}{\mu(r)} kT(r)$$

$$\mu(r) = f(\text{comp}, T(r), P(r))$$

$$\kappa_R(r) = f(\text{comp}, T(r), P(r))$$

$$\epsilon_{\rm nuc}(r) = f(\text{comp}, T(r), P(r))$$

Other energy transport?

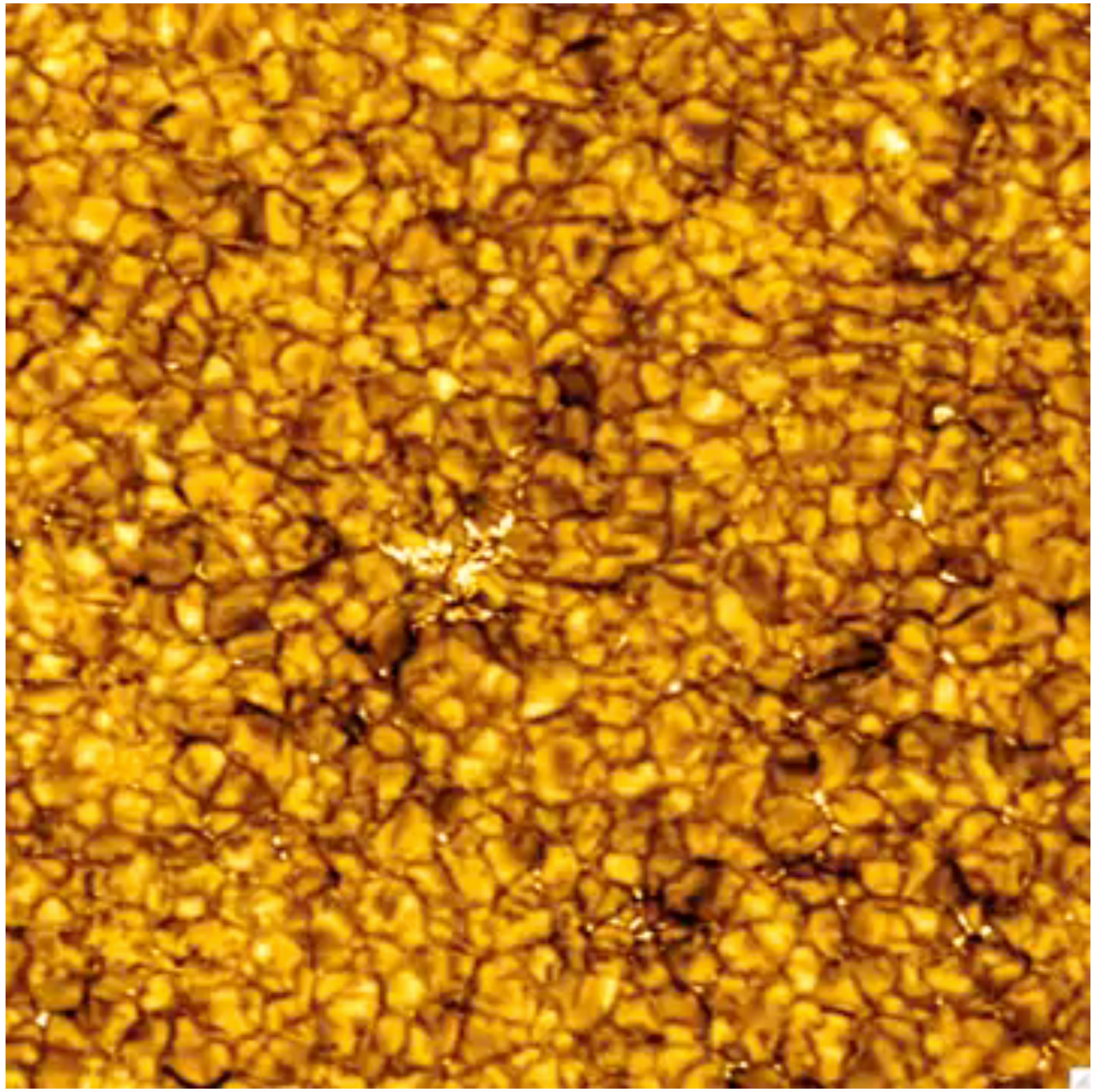
Ideal gas always valid?

Nuclear mechanism?

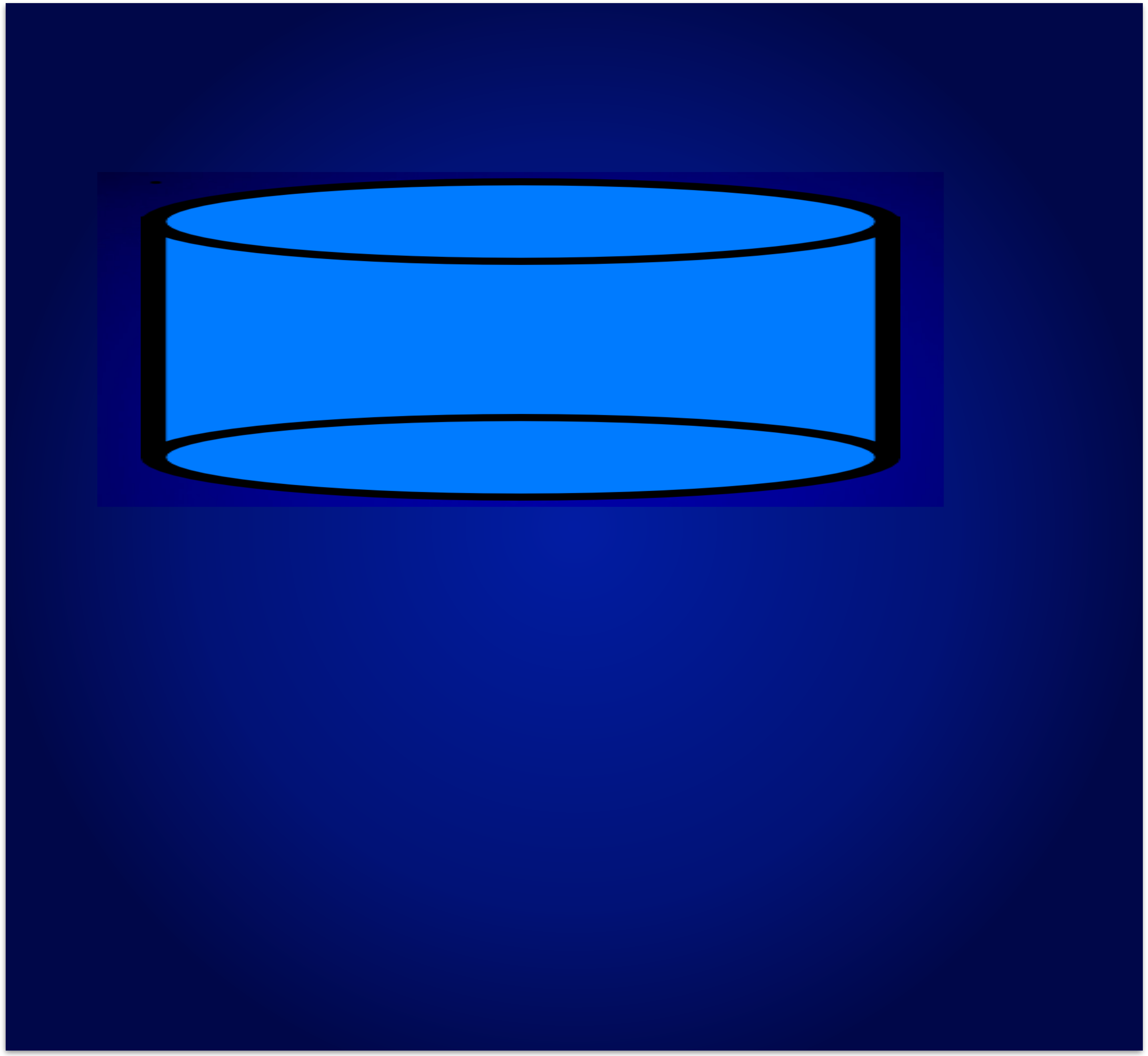
How to solve?

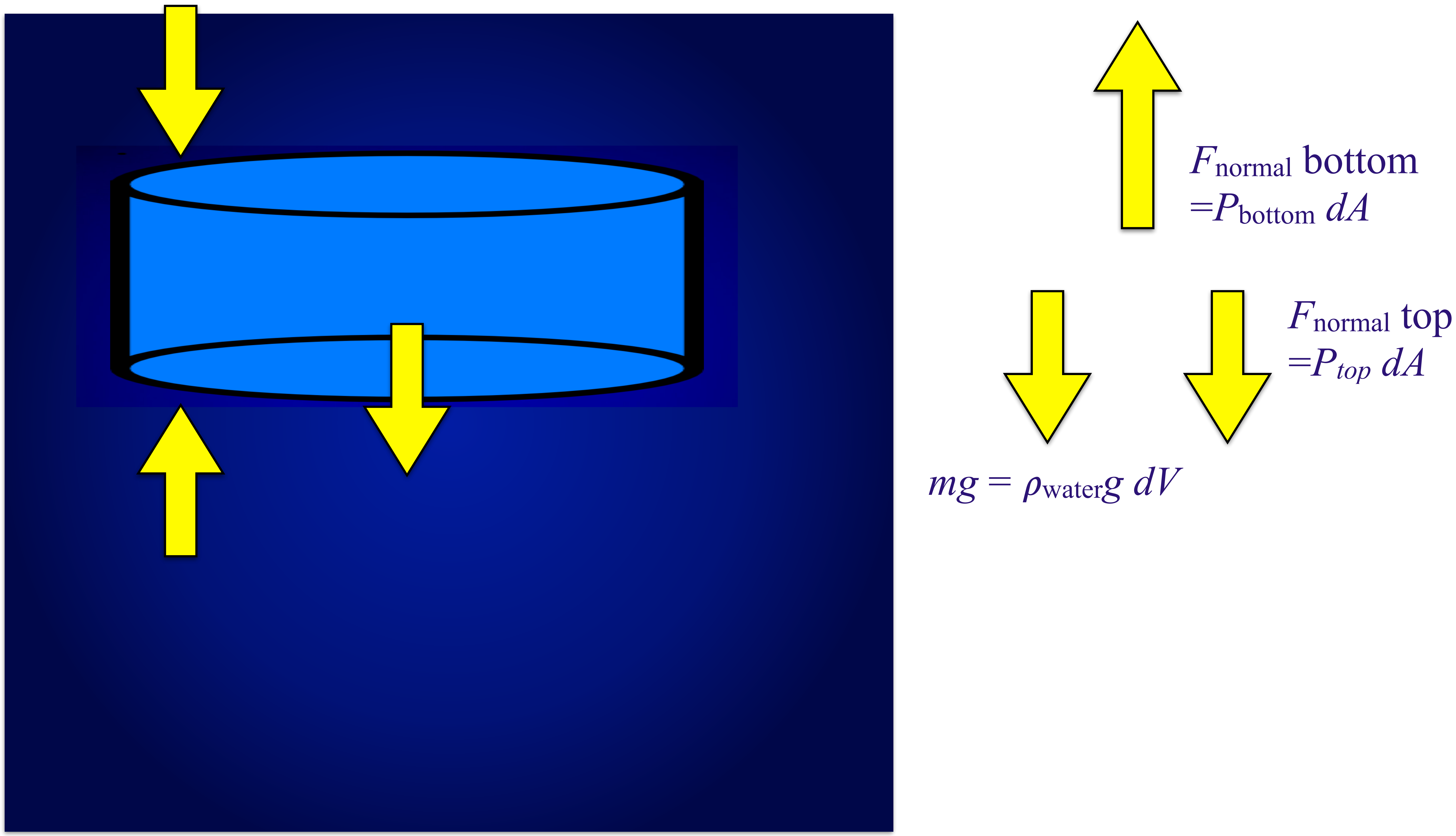
What changes with time?

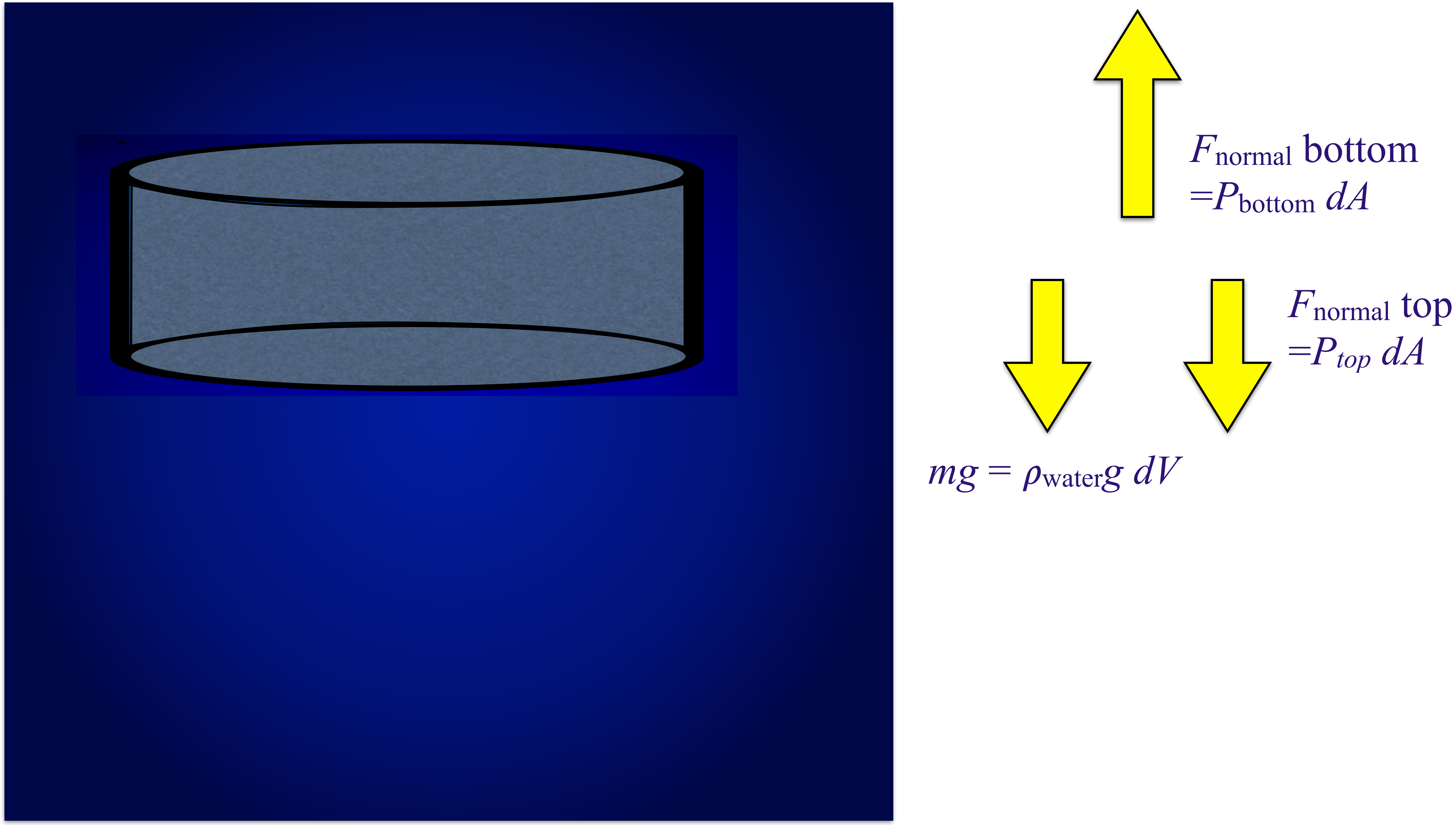
Other energy transport?

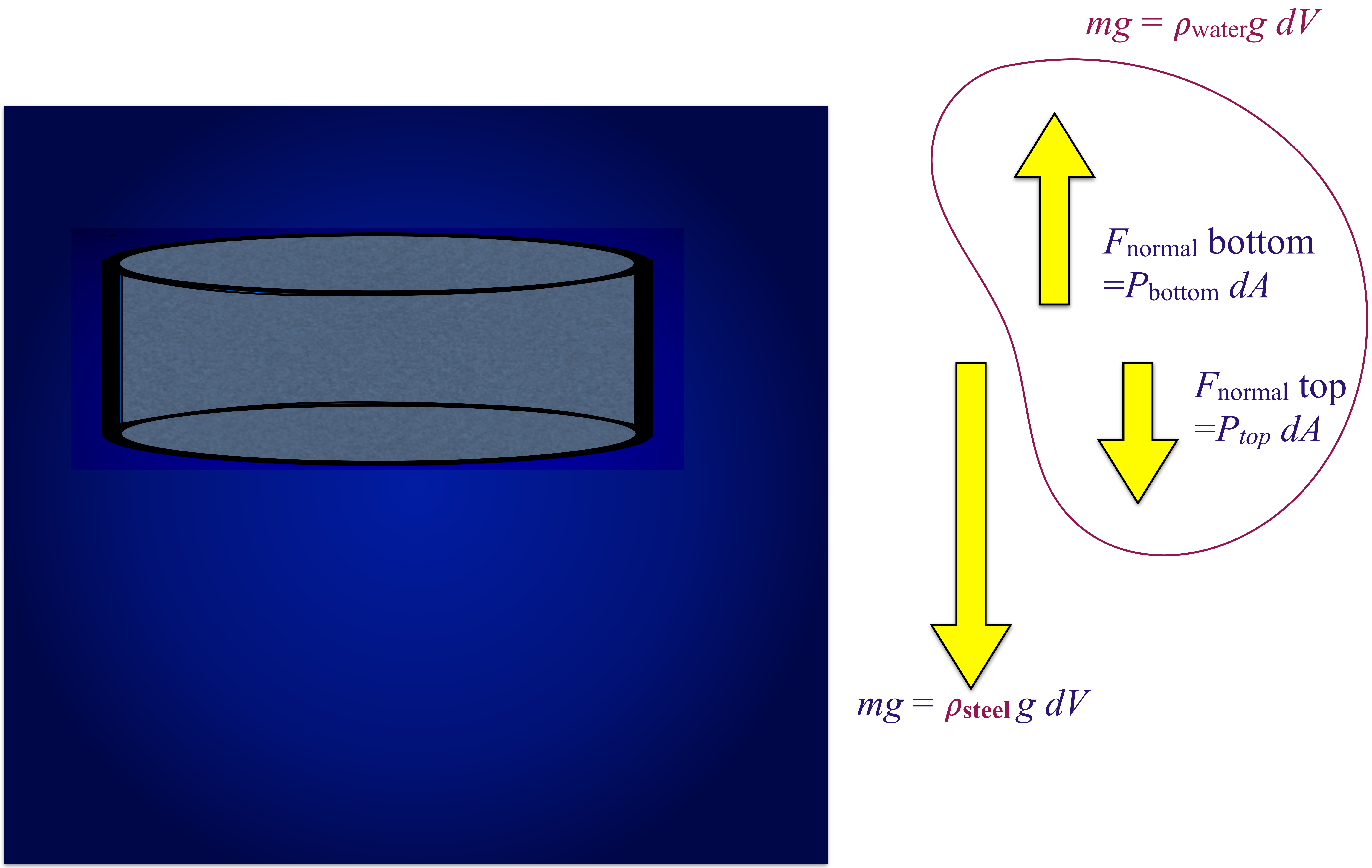


Reminder



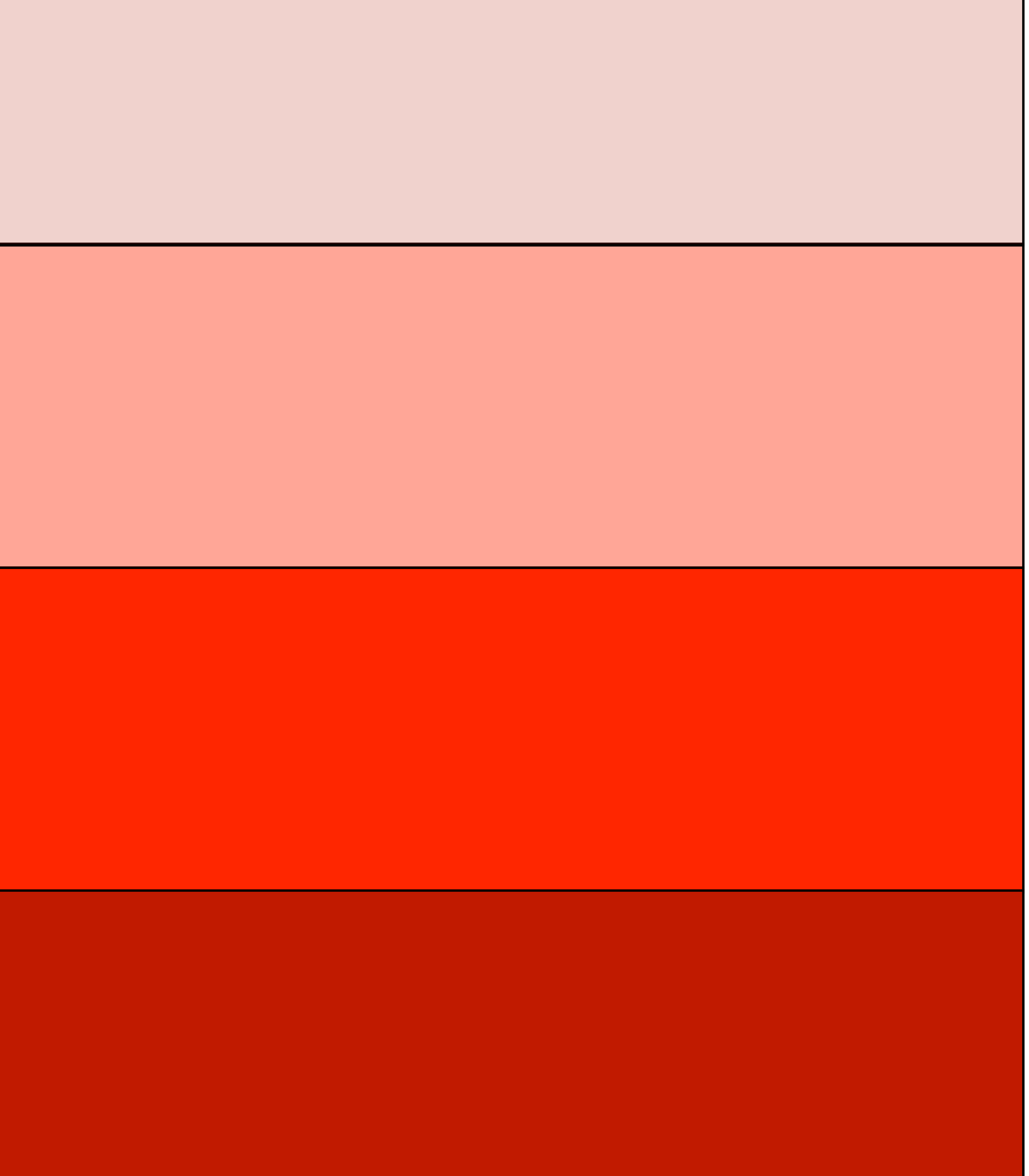

F_{normal} bottom
$=P_{\text{bottom}}\, dA$
F_{normal} top
$=P_{top}\, dA$
$mg = \rho_{\text{water}} g\, dV$


F_{normal} bottom
$=P_{\text{bottom}}\, dA$
F_{normal} top
$=P_{top}\, dA$
$mg = \rho_{\text{water}} g\, dV$



$mg = \rho_{\text{water}} g \, dV$
F_{normal} bottom
$= P_{\text{bottom}} \, dA$
F_{normal} top
$= P_{top} \, dA$
$mg = \rho_{\text{steel}} \, g \, dV$

Density



The "bubble test":

The "medium" is the structure you'll get ($T(r)$, $\rho(r)$, $P(r)$, etc) that you will get if indeed the photon are transporting all of the energy.

Continuity equation

$$\frac{dM_r(r)}{dr} = 4\pi r^2 \rho(r)$$

Hydrostatic equilibrium

$$\frac{dP(r)}{dr} = -\rho(r)\frac{GM_r(r)}{r^2}$$

Radiative energy transport

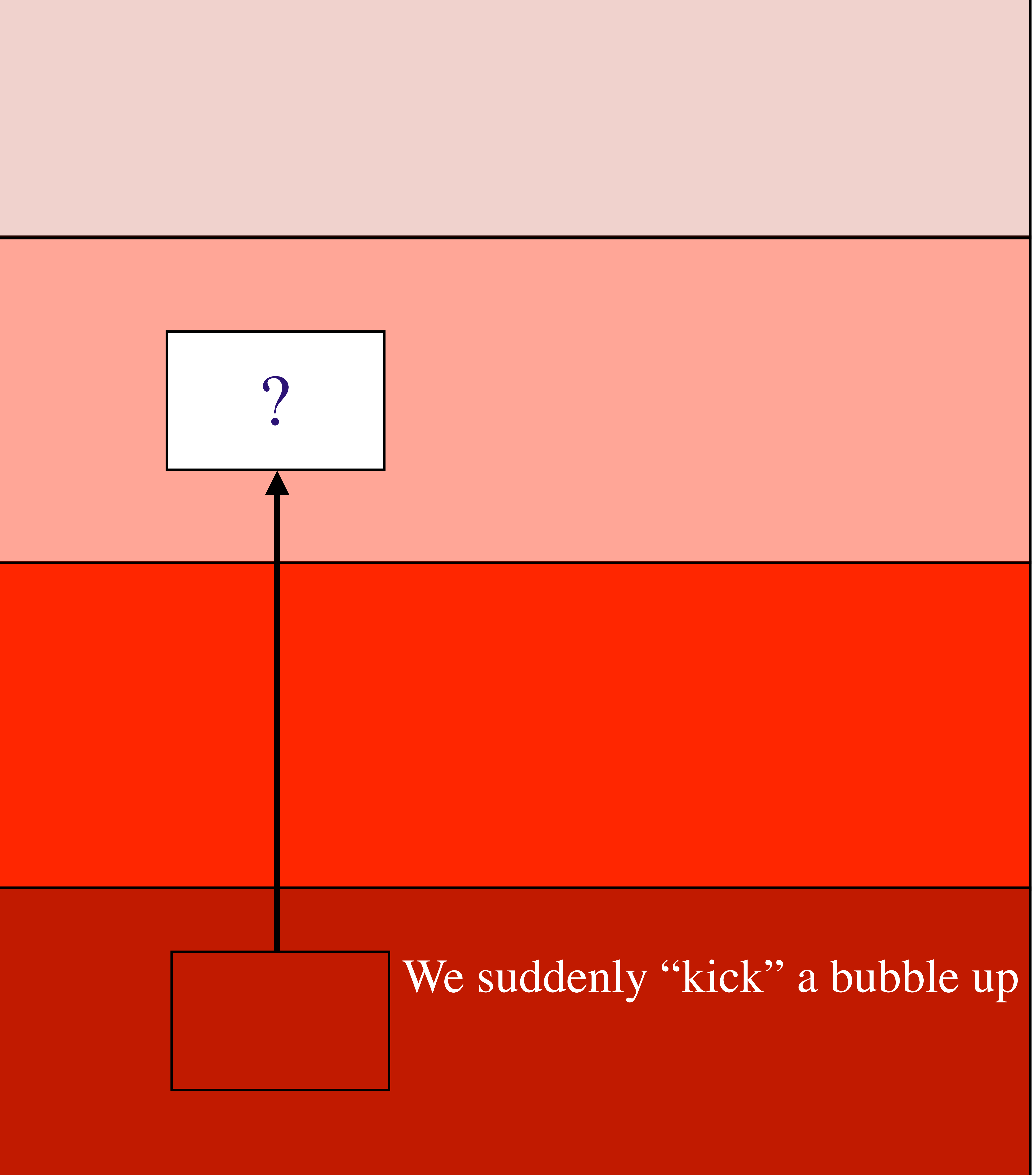
$$\frac{dT(r)}{dr} = -\frac{3\kappa_R(r)\rho(r)}{64\pi\sigma}\frac{L_r(r)}{r^2T^3(r)}$$

(Or = to convection transport)

Energy source

$$\frac{dL_r(r)}{dr} = 4\pi r^2 \rho(r)\epsilon(r)$$

Density



The "bubble test":

The bubble **does**

- Adjusts its pressure to the pressure of the medium

The bubble **does not**

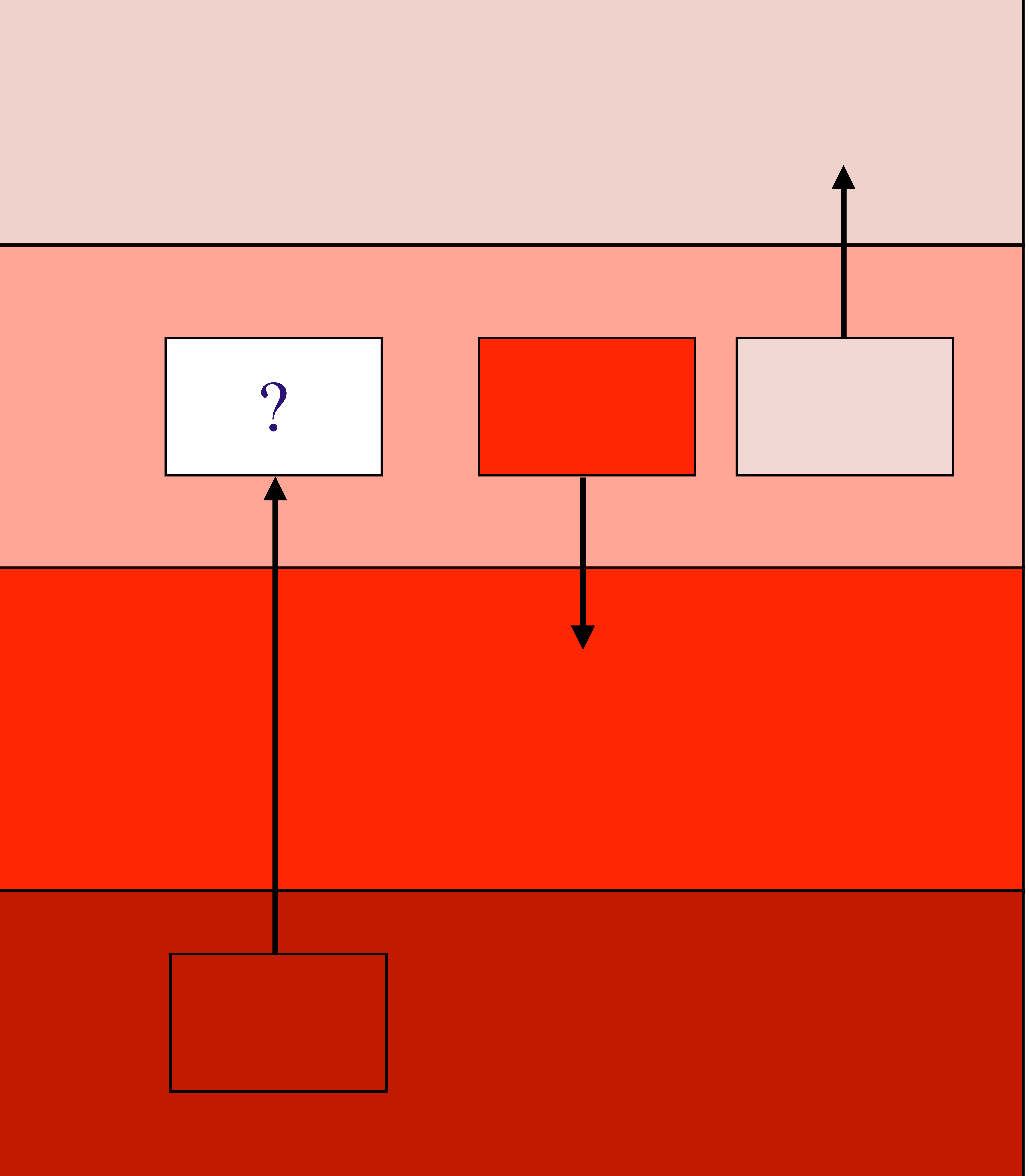
- Adjusts its mean molecular weight to that of the medium
- Adjusts its ρ and T to that of the medium

=> The change in T and ρ for the bubble is an adiabatic process

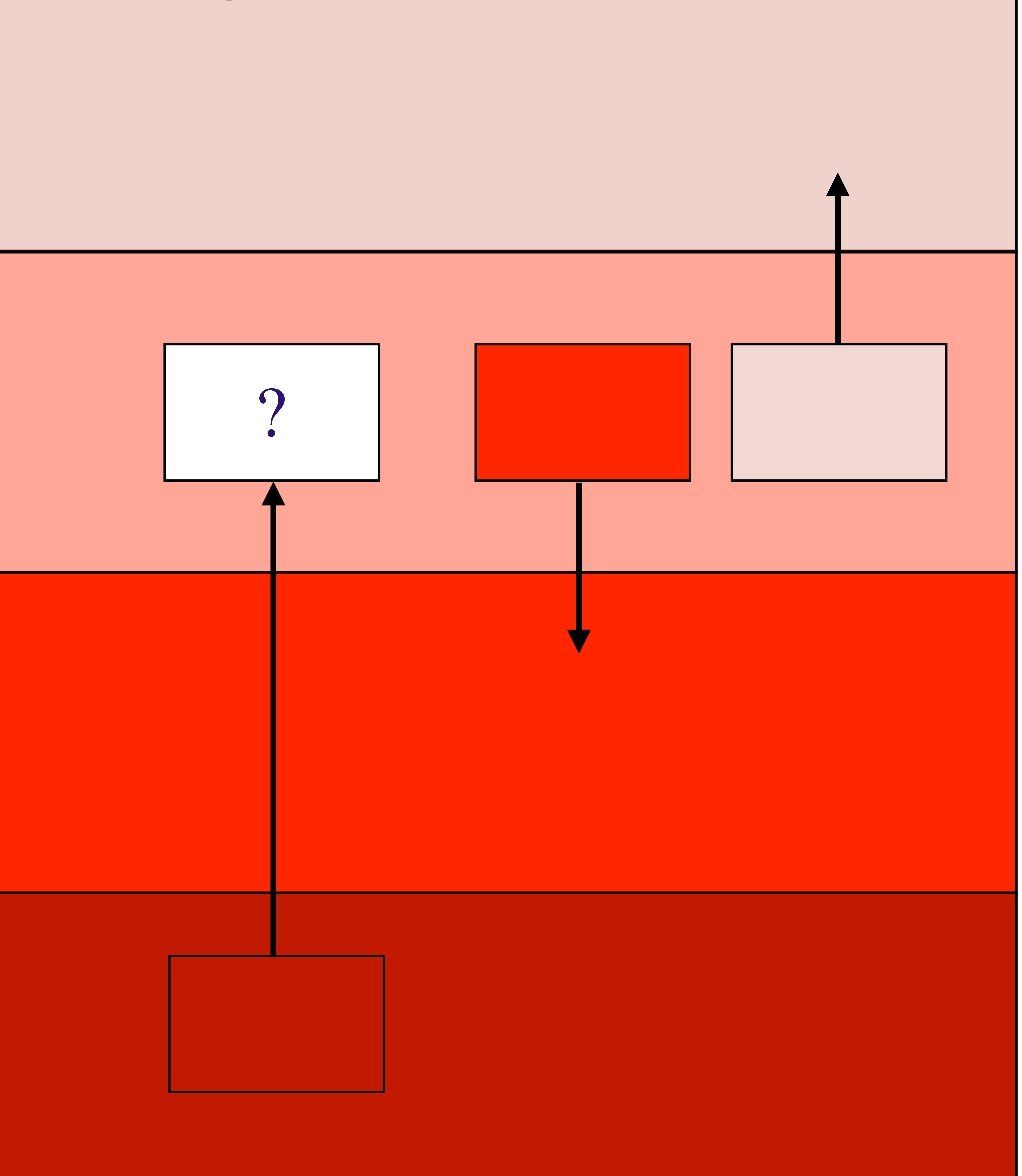
$$\gamma = \frac{C_P}{C_V} = \frac{5}{3} \text{ for a mono-atomic ideal gas}$$

$$P \propto \rho T \propto \rho^{\gamma} \propto T^{\frac{\gamma}{\gamma - 1}}$$

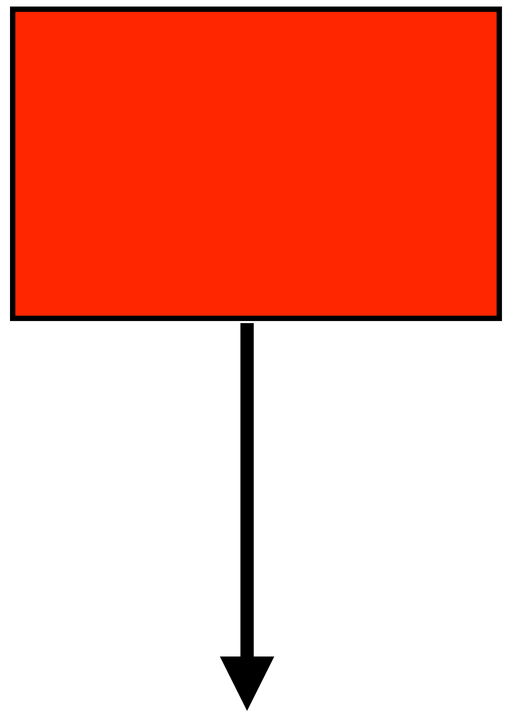
Density



Density



The density in the medium decreased quicker than the density in the bubble



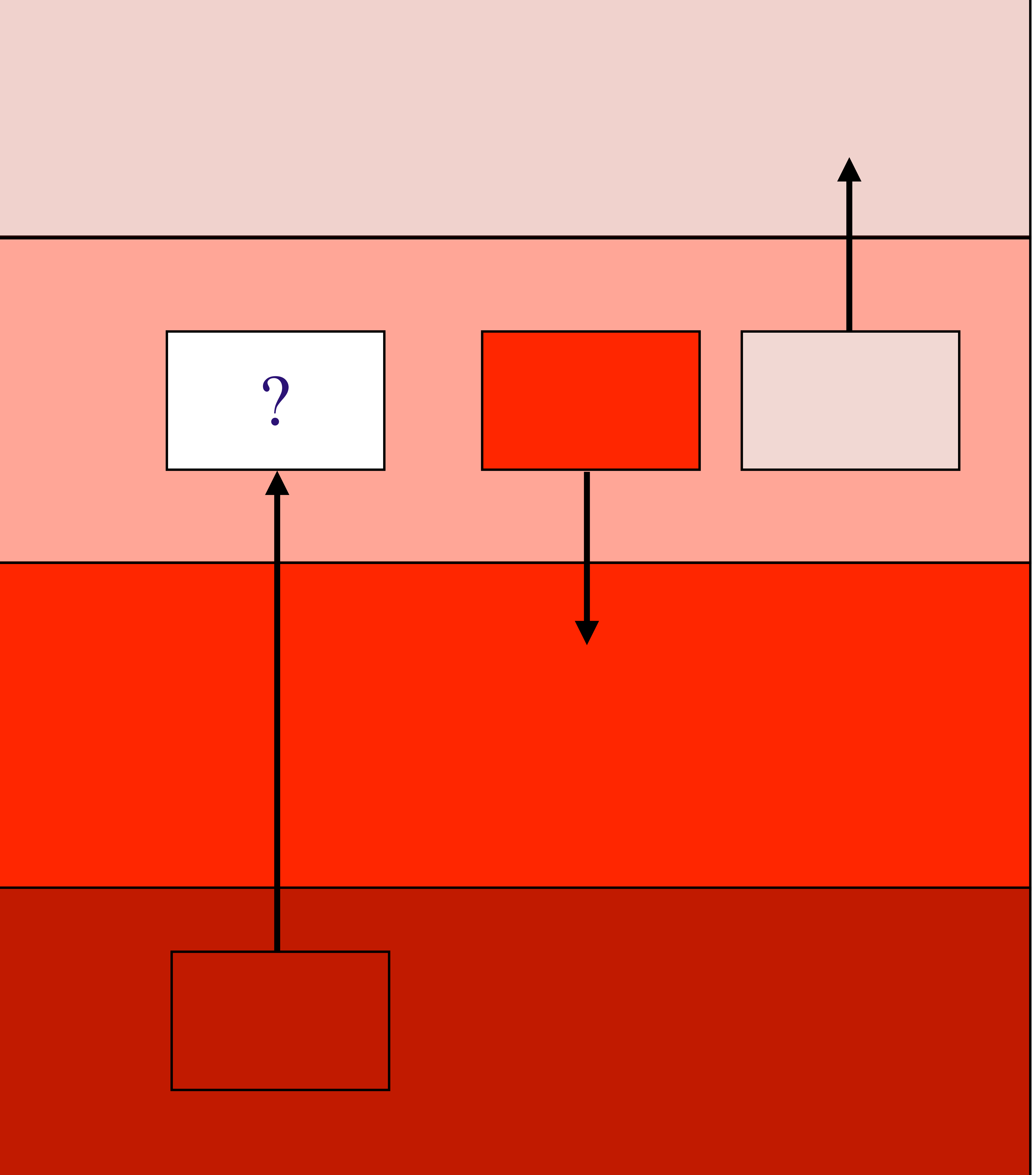
Stable if $\left(\frac{\partial\rho}{\partial r}\right)_{\text{bubble}} > \left(\frac{\partial\rho}{\partial r}\right)_{\text{medium}}$

Negative number

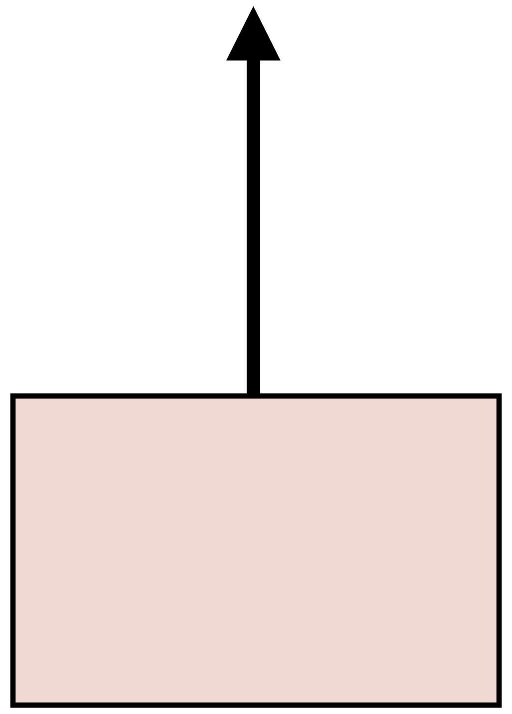
More negative number

Graph on the board

Density



The density in the bubble decreased quicker than the density in the medium



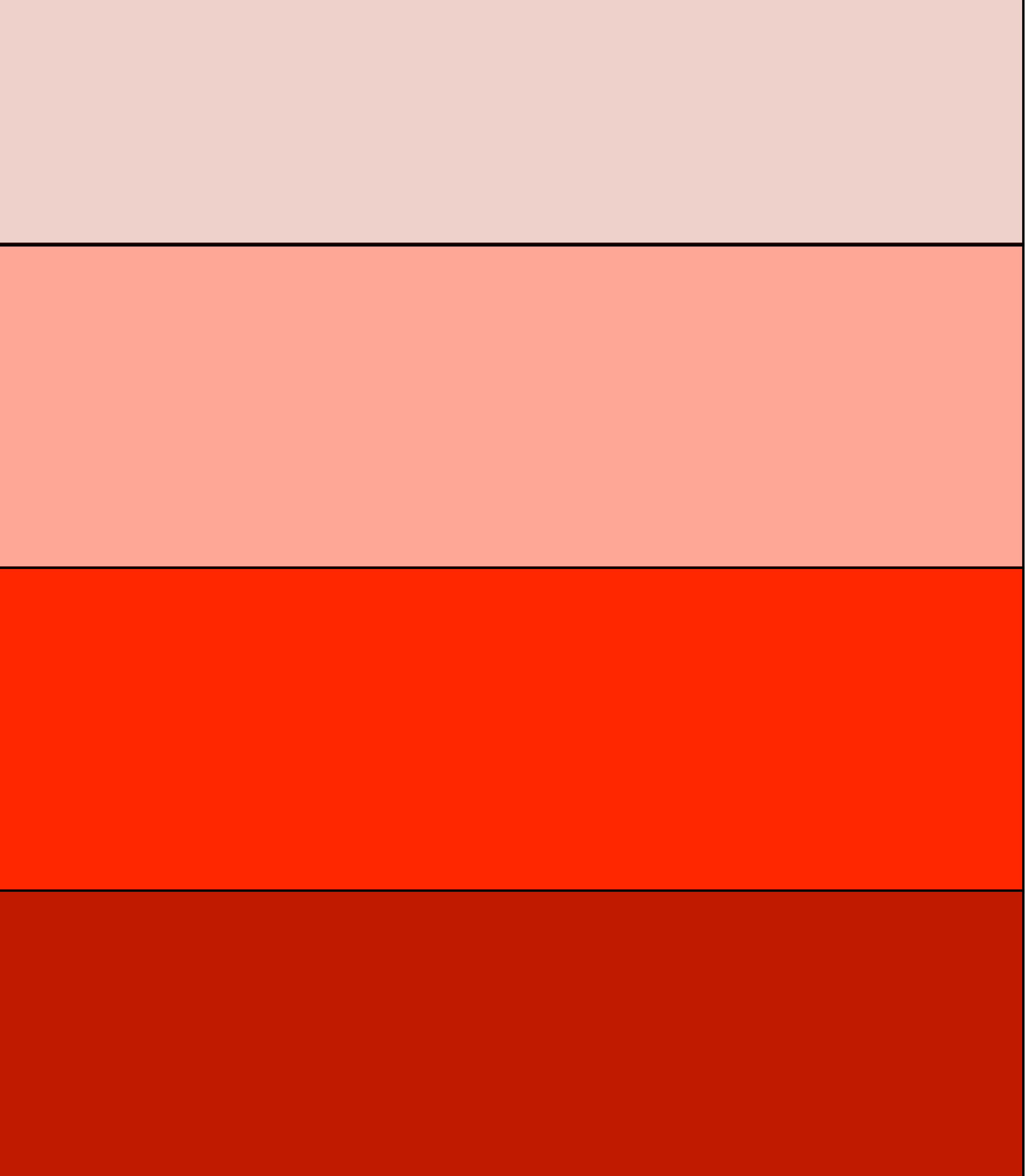
$$\text{Unstable if} \left(\frac{\partial \rho}{\partial r}\right)_{\text{bubble}} < \left(\frac{\partial \rho}{\partial r}\right)_{\text{medium}}$$

More negative number

Negative number

Graph on the board

Density



The “bubble test”:

The “medium” is the structure you’ll get $(T(r), \rho(r), P(r)$, etc) that you will get if indeed the photon are transporting all of the energy.

Our equations for the medium does not give us $\dfrac{\partial \rho}{\partial r}$ directly

Next step: find $\dfrac{\partial \rho}{\partial r}$

Continuity equation

$$\frac{dM_r(r)}{dr} = 4\pi r^2 \rho(r)$$

Hydrostatic equilibrium

$$\frac{dP(r)}{dr} = -\rho(r)\frac{GM_r(r)}{r^2}$$

Radiative energy transport

$$\frac{dT(r)}{dr} = -\frac{3\kappa_R(r)\rho(r)}{64\pi\sigma}\frac{L_r(r)}{r^2T^3(r)}$$

(Or = to convection transport)

Energy source

$$\frac{dL_r(r)}{dr} = 4\pi r^2 \rho(r)\epsilon(r)$$

Finding $\dfrac{\partial \rho}{\partial r}$

We will start with the ideal gas law: $P = \dfrac{\rho}{\mu[m_H]} kT$ (I will wrap up the m_H inside of the μ to make it simpler)

Isolate ρ: $\rho = \dfrac{1}{k} \dfrac{P\mu}{T}$

Or in order words: $\rho = \text{constant} \dfrac{P\mu}{T}$

We could also be general, to accommodate for different equation of state: $\rho = C \dfrac{P^{\alpha}\mu^{\varphi}}{T^{\delta}}$

For later: $\ln \rho = \ln C + \alpha \ln P - \delta \ln T + \varphi \ln \mu$

Finding $\dfrac{\partial \rho}{\partial r}$

General gas law $\rho = C \dfrac{P^{\alpha}\mu^{\varphi}}{T^{\delta}}$ or $\ln\rho = \ln C + \alpha \ln P - \delta \ln T + \varphi \ln \mu$

1. Partial derivative
2. ρ divide
3. Add X/X

$$\frac{d\rho}{\rho} = \boxed{\frac{1}{\rho}\left(\frac{d\rho}{dP}\right)_{T,\mu} P}\,\frac{dP}{P} + \boxed{\frac{1}{\rho}\left(\frac{d\rho}{dT}\right)_{P,\mu} T}\,\frac{dT}{T} + \boxed{\frac{1}{\rho}\left(\frac{d\rho}{d\mu}\right)_{T,P} \mu}\,\frac{d\mu}{\mu}$$

4. $\dfrac{dx}{x} = d[\ln x]$

$$\frac{d\rho}{\rho} = \boxed{\left(\frac{d\ln(\rho)}{d\ln(P)}\right)_{T,\mu}}\frac{dP}{P} + \boxed{\left(\frac{d\ln(\rho)}{d\ln(T)}\right)_{P,\mu}}\frac{dT}{T} + \boxed{\left(\frac{d\ln(\rho)}{d\ln(\mu)}\right)_{T,P}}\frac{d\mu}{\mu}$$

5. Replace with the α, δ, φ exponents

α $\qquad$ $-\delta$ $\qquad$ φ

$$\frac{d\rho}{\rho} = \alpha\frac{dP}{P} - \delta\frac{dT}{T} + \varphi\frac{d\mu}{\mu}$$

Finding $\dfrac{\partial\rho}{\partial r}$

General gas law $\rho = C\,\dfrac{P^{\alpha}\mu^{\varphi}}{T^{\delta}}$ or $\ln\rho = \ln C + \alpha\ln P - \delta\ln T + \varphi\ln\mu$

6. Divide by ∂r everywhere, and move ρ to the right.

$$\frac{d\rho}{\rho} = \alpha\,\frac{dP}{P} - \delta\,\frac{dT}{T} + \varphi\,\frac{d\mu}{\mu}$$

$$\boxed{\frac{d\rho}{dr} = \frac{\alpha\rho}{P}\,\frac{dP}{dr} - \frac{\delta\rho}{T}\,\frac{dT}{dr} + \frac{\varphi\rho}{\mu}\,\frac{d\mu}{dr}}$$

$$\boxed{\frac{d\rho}{dr} = \frac{\alpha\rho}{P}\frac{dP}{dr} - \frac{\delta\rho}{T}\frac{dT}{dr} + \frac{\varphi\rho}{\mu}\frac{d\mu}{dr}}$$

Unstable if: $\left(\frac{d\rho}{dr}\right)_{\text{medium}} - \left(\frac{d\rho}{dr}\right)_{\text{bubble}} > 0$

$$\left(\frac{\alpha\rho}{P}\cancel{\frac{dP}{dr}}\right)_{\text{med}} - \left(\frac{\delta\rho}{T}\frac{dT}{dr}\right)_{\text{med}} + \left(\frac{\varphi\rho}{\mu}\frac{d\mu}{dr}\right)_{\text{med}} - \left(\frac{\alpha\rho}{P}\cancel{\frac{dP}{dr}}\right)_{\text{bub}} + \left(\frac{\delta\rho}{T}\frac{dT}{dr}\right)_{\text{bub}} - \left(\frac{\varphi\rho}{\mu}\cancel{\frac{d\mu}{dr}}\right)_{\text{bub}} > 0$$

Same pressure, terms cancel out

No change of molecular weight inside the bubble

$$-\left(\frac{\delta\rho}{T}\frac{dT}{dr}\right)_{\text{med}} + \left(\frac{\varphi\rho}{\mu}\frac{d\mu}{dr}\right)_{\text{med}} + \left(\frac{\delta\rho}{T}\frac{dT}{dr}\right)_{\text{bub}} > 0$$

ρ term cancels

Rearrange

$$-\left(\frac{d\ln(T)}{dr}\right)_{\text{med}} + \left(\frac{d\ln(T)}{dr}\right)_{\text{bub}} + \left(\frac{\varphi}{\delta}\frac{d\ln(\mu)}{dr}\right)_{\text{med}} > 0$$

$$-\left(\frac{d\ln(T)}{dr}\right)_{\text{med}}+\left(\frac{d\ln(T)}{dr}\right)_{\text{bub}}+\left(\frac{\varphi}{\delta}\ \frac{d\ln(\mu)}{dr}\right)_{\text{med}}>0$$

Multiply everywhere by pressure scale height:

$$-\frac{dr}{d\ln(P)}$$

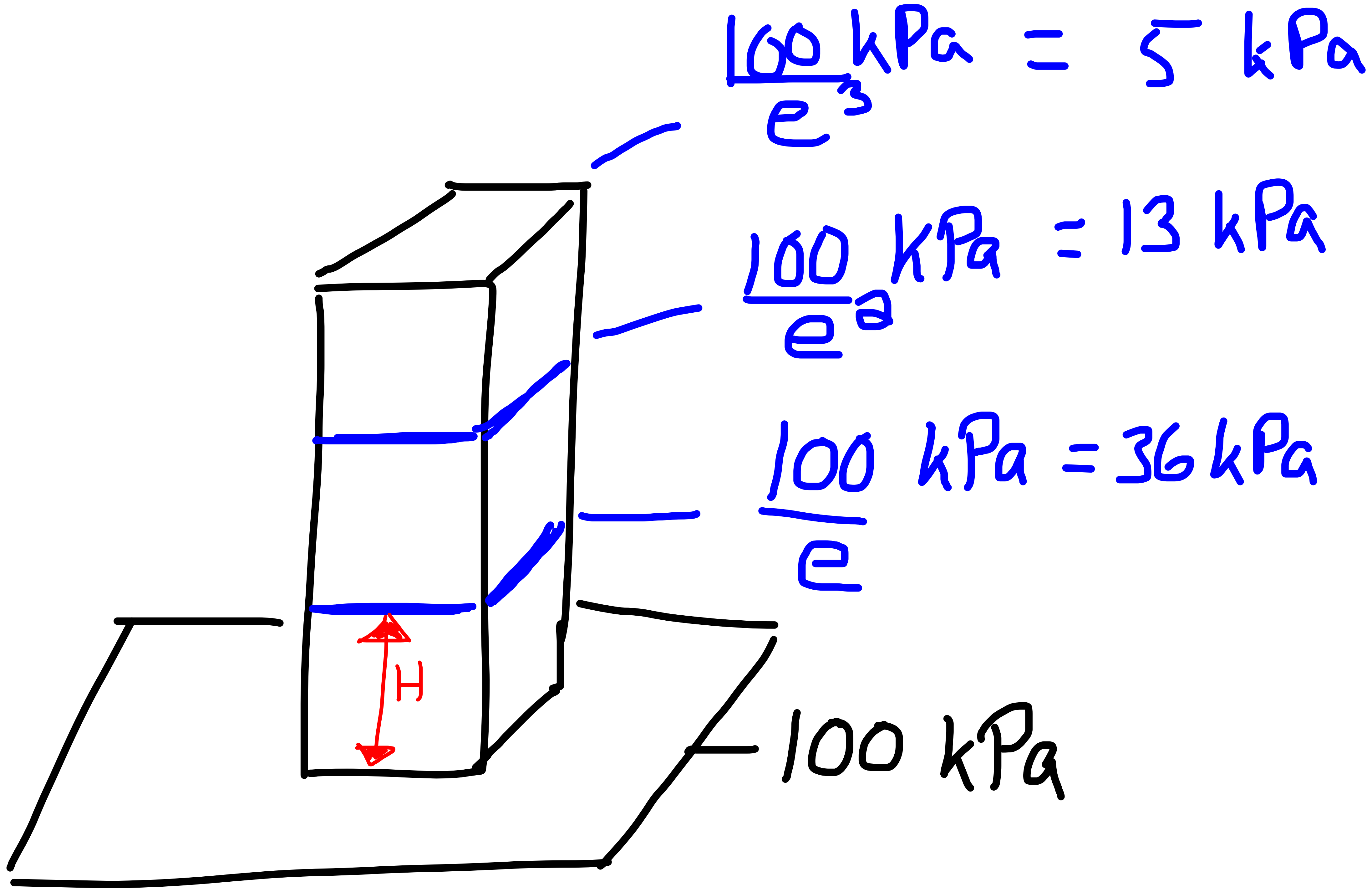
$$\left(\frac{d\ln(T)}{d\ln(P)}\right)_{\text{med}}-\left(\frac{d\ln(T)}{d\ln(P)}\right)_{\text{bub}}-\left(\frac{\varphi}{\delta}\ \frac{d\ln(\mu)}{d\ln(P)}\right)_{\text{med}}>0$$

Unstable if:

$$\nabla_{\text{med}}-\nabla_{\text{ad}}-\nabla_{\mu}>0$$

The pressure scale height: distance over which the pressure change by a factor of "e"

Reminder



$$P(r) = \frac{\rho(r)}{\mu(r) m_H} kT(r)$$

$$H = \frac{P}{dP/dr}$$

$$\frac{dP(r)}{dr} = -\rho(r) g(r)$$

$$= \frac{kT/\mu m_H}{g}$$

$$= \frac{r^2 P(r)}{\rho(r) G M_r(r)}$$

$$\left(\frac{d\ln(T)}{d\ln(P)}\right)_{\mathrm{med}} - \left(\frac{d\ln(T)}{d\ln(P)}\right)_{\mathrm{bub}} - \left(\frac{\varphi}{\delta}\,\frac{d\ln(\mu)}{d\ln(P)}\right)_{\mathrm{med}} > 0$$

Unstable if:

$$\nabla_{\mathrm{med}} - \boxed{\nabla_{\mathrm{ad}}} - \nabla_{\mu} > 0$$

$$P \propto \rho T \propto \rho^{\gamma} \propto T^{\frac{\gamma}{\gamma-1}}$$

$$\ln P \propto \frac{\gamma}{\gamma - 1} \ln T$$

$$\frac{d\ln T}{d\ln P} = \frac{\gamma - 1}{\gamma}$$

$$\left(\frac{d\ln(T)}{d\ln(P)}\right)_{\mathrm{med}} - \left(\frac{d\ln(T)}{d\ln(P)}\right)_{\mathrm{bub}} - \left(\frac{\varphi}{\delta}\frac{d\ln(\mu)}{d\ln(P)}\right)_{\mathrm{med}} > 0$$

Unstable if:

$$\boxed{\nabla_{\mathrm{med}}} - \nabla_{\mathrm{ad}} - \nabla_{\mu} > 0$$

Remember: The "medium" is the structure you'll get if indeed the photon are transporting all of the energy.

$$\nabla_{\mathrm{med}} = \left(\frac{d\ln(T)}{dr}\right)_{\mathrm{med}} * H$$

$$= \frac{1}{T}\left(\frac{3\kappa_R \rho L_r}{64\pi\sigma r^2 T^3}\right) * \left(\frac{Pr^2}{\rho G M_r}\right)$$

$$= \frac{3\kappa_R P L_r}{64\pi\sigma G M_r T^4}$$

Continuity equation

$$\frac{dM_r(r)}{dr} = 4\pi r^2 \rho(r)$$

Hydrostatic equilibrium

$$\frac{dP(r)}{dr} = -\rho(r)\frac{GM_r(r)}{r^2}$$

Radiative energy transport

$$\frac{dT(r)}{dr} = -\frac{3\kappa_R(r)\rho(r)}{64\pi\sigma}\frac{L_r(r)}{r^2T^3(r)}$$

(Or = to convection transport)

Energy source

$$\frac{dL_r(r)}{dr} = 4\pi r^2 \rho(r)\epsilon(r)$$

$$\left(\frac{d\ln(T)}{d\ln(P)}\right)_{\text{med}} - \left(\frac{d\ln(T)}{d\ln(P)}\right)_{\text{bub}} - \left(\frac{\varphi}{\delta}\frac{d\ln(\mu)}{d\ln(P)}\right)_{\text{med}} > 0$$

Unstable if:

$$\nabla_{\text{med}} - \nabla_{\text{ad}} - \nabla_{\mu} > 0$$

Let's ignore the gradient of μ for a moment, to simplify things

$$\nabla_{\text{med}} > \nabla_{\text{ad}}$$

$$\nabla_{\text{med}} = \frac{3\kappa_R P L_r}{64\pi\sigma G M_r T^4}$$

$$\nabla_{\text{ad}} = \frac{\gamma - 1}{\gamma}$$

$\gamma = \frac{5}{3}$ for a mono-atomic ideal gas

Q: When is ∇_{med} (∇_{rad}) large?

Continuity equation

$$\frac{dM_r(r)}{dr} = 4\pi r^2 \rho(r)$$

Hydrostatic equilibrium

$$\frac{dP(r)}{dr} = -\rho(r)\frac{GM_r(r)}{r^2}$$

Radiative energy transport

$$\frac{dT(r)}{dr} = -\frac{3\kappa_R(r)\rho(r)}{64\pi\sigma}\frac{L_r(r)}{r^2 T^3(r)}$$

(Or = to convection transport)

Energy source

$$\frac{dL_r(r)}{dr} = 4\pi r^2 \rho(r)\epsilon(r)$$

Unstable if:

$$\nabla_{\mathrm{med}} - \nabla_{\mathrm{ad}} - \nabla_{\mu} > 0$$

$$\nabla_{\mathrm{med}} = \frac{3\kappa_R P L_r}{64\pi\sigma G M_r T^4}$$

$$\nabla_{\mathrm{ad}} = \frac{\gamma - 1}{\gamma}$$

Ok, if there is indeed convection (it is unstable!), what should we use then?

Continuity equation

$$\frac{dM_r(r)}{dr} = 4\pi r^2 \rho(r)$$

Hydrostatic equilibrium

$$\frac{dP(r)}{dr} = -\rho(r)\frac{GM_r(r)}{r^2}$$

Radiative energy transport

$$\frac{dT(r)}{dr} = -\frac{3\kappa_R(r)\rho(r)}{64\pi\sigma}\frac{L_r(r)}{r^2 T^3(r)}$$

(Or = to convection transport)

Energy source

$$\frac{dL_r(r)}{dr} = 4\pi r^2 \rho(r)\epsilon(r)$$

It is not a "either or":

- When stable, there is no convection, so $\nabla_{\mathrm{real}} = \nabla_{\mathrm{med}}$
- But when non-stable, the photons are still there and still participate in energy transport!
  So $\nabla_{\mathrm{rad}} > \nabla_{\mathrm{real}} > \nabla_{\mathrm{ad}}$

Unstable if:

Let's ignore the gradient of μ for a moment, to simplify things

$$\nabla_{\rm med} > \nabla_{\rm ad}$$

$$\nabla_{\rm med} = \frac{3\kappa_R P L_r}{64\pi\sigma G M_r T^4}$$

$$\nabla_{\rm ad} = \frac{\gamma - 1}{\gamma}$$

$\gamma = \frac{5}{3}$ for a mono-atomic ideal gas

Q: When is $\nabla_{\rm med}$ ($\nabla_{\rm rad}$) large?

Notebook: 2 example stars (Sun, 10 times the mass of the Sun)

1. Where is κ_R large (hint, it will matter)
2. Where are these stars convective?

mesa.sourceforge.net

MESA
Modules for Experiments in Stellar Astrophysics

MESA home
code capabilities
prereqs & installation
getting started
using pgstar
using MESA output
beyond inlists (extending MESA)
troubleshooting
FAQ
star_job defaults
controls defaults
pgstar defaults
binary_controls defaults
news archive
documentation archive

MESA

*You may also want to visit **the MESA marketplace**, where users share the inlists from their published results, tools & utilities, and teaching materials.*

Why a new 1D stellar evolution code?

The MESA Manifesto discusses the motivation for the MESA project, outlines a MESA code of conduct, and describes the establishment of a MESA Council. Before using MESA, you should read the **manifesto document**. Here's a brief extract of some of the key points

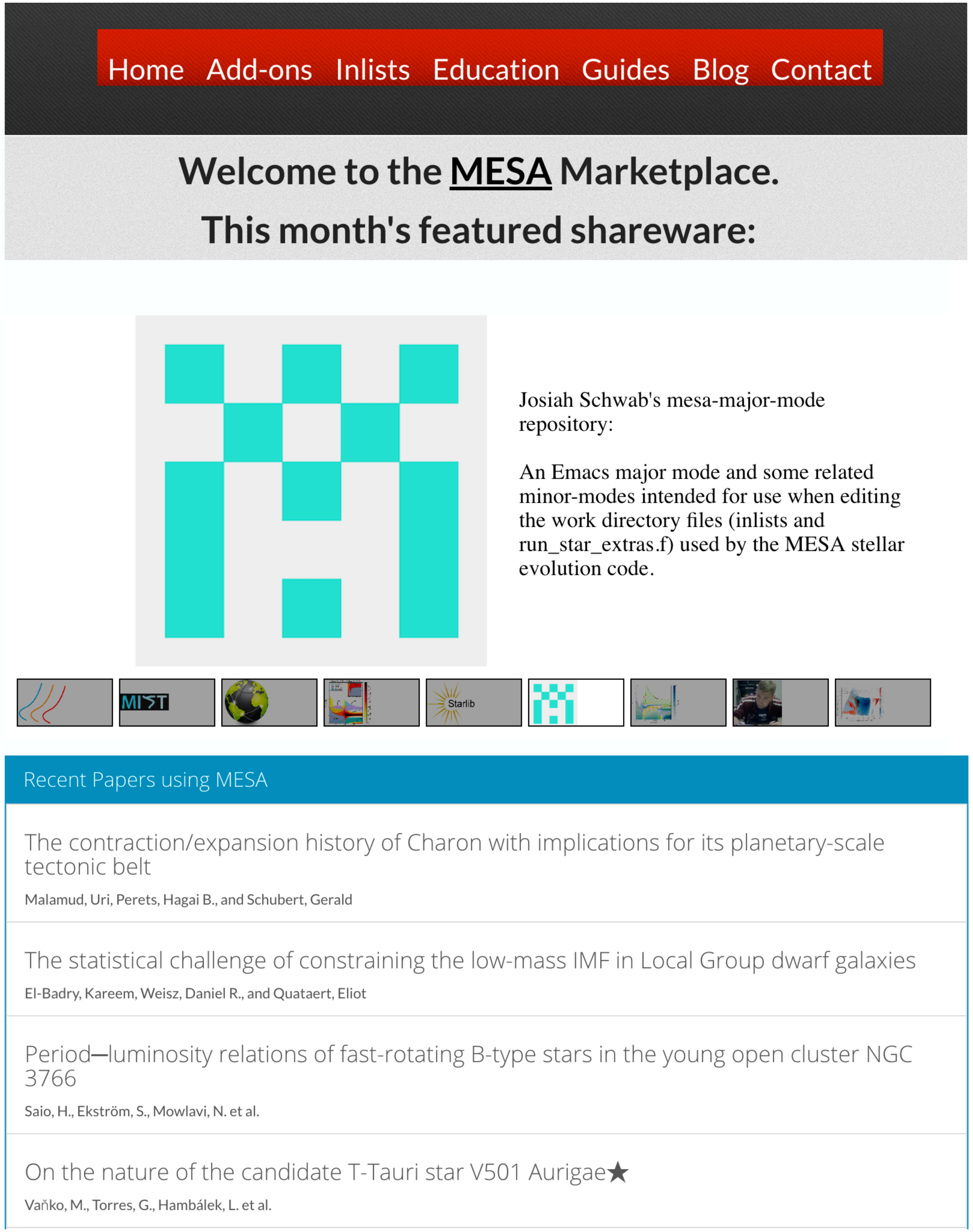
Stellar evolution calculations remain a basic tool of broad impact for astrophysics. New observations constantly test the models, even in 1D. The continued demand requires the construction of a general, modern stellar evolution code that combines the following advantages:

- **Openness**: anyone can download sources from the website.
- **Modularity**: independent modules for physics and for numerical algorithms; the parts can be used stand-alone.
- **Wide Applicability**: capable of calculating the evolution of stars in a wide range of environments.
- **Modern Techniques**: advanced AMR, fully coupled solution for composition and abundances, mass loss and gain, etc.
- **Comprehensive Microphysics**: up-to-date, wide-ranging, flexible, and independently useable microphysics modules.

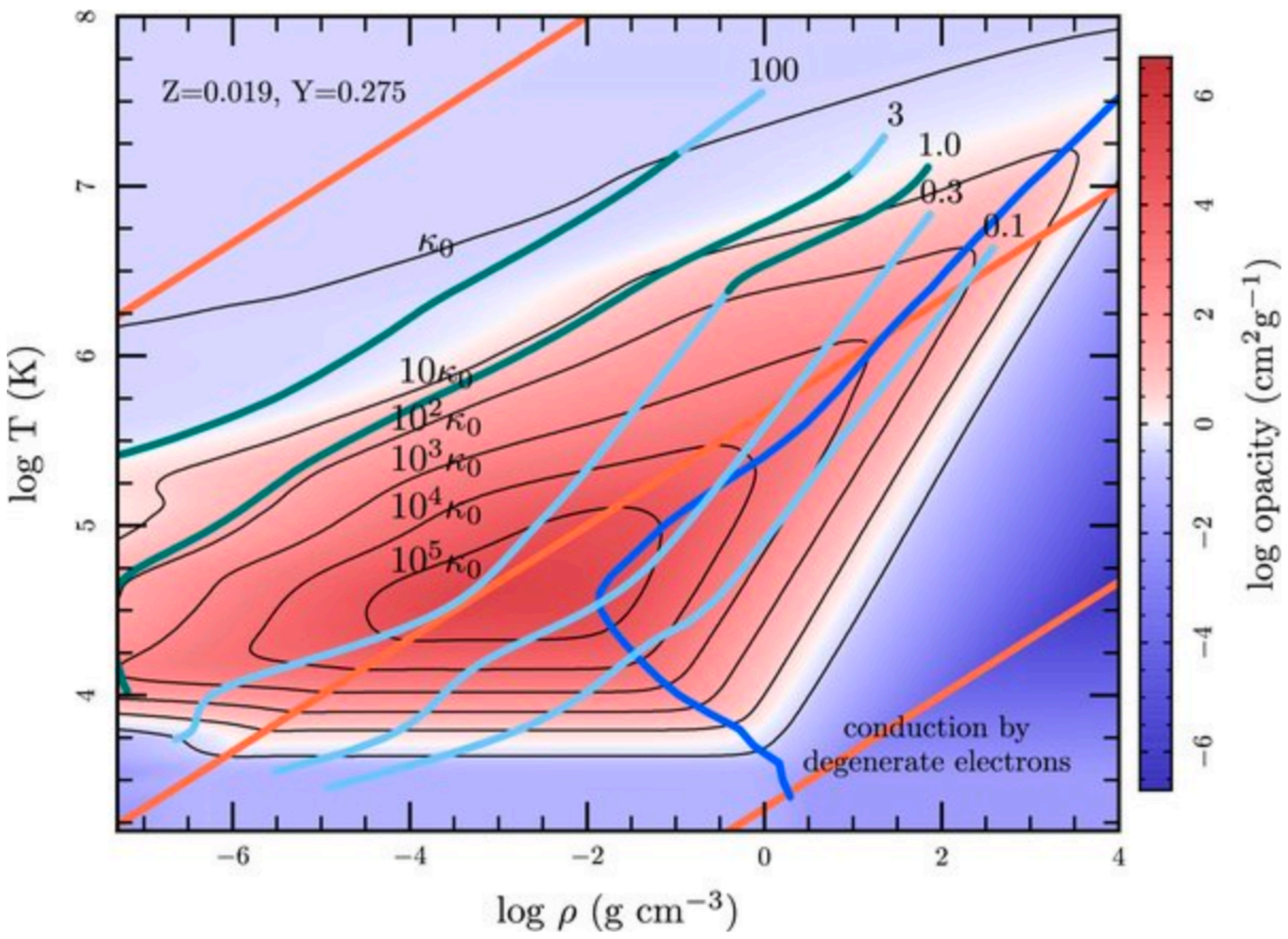
Latest News

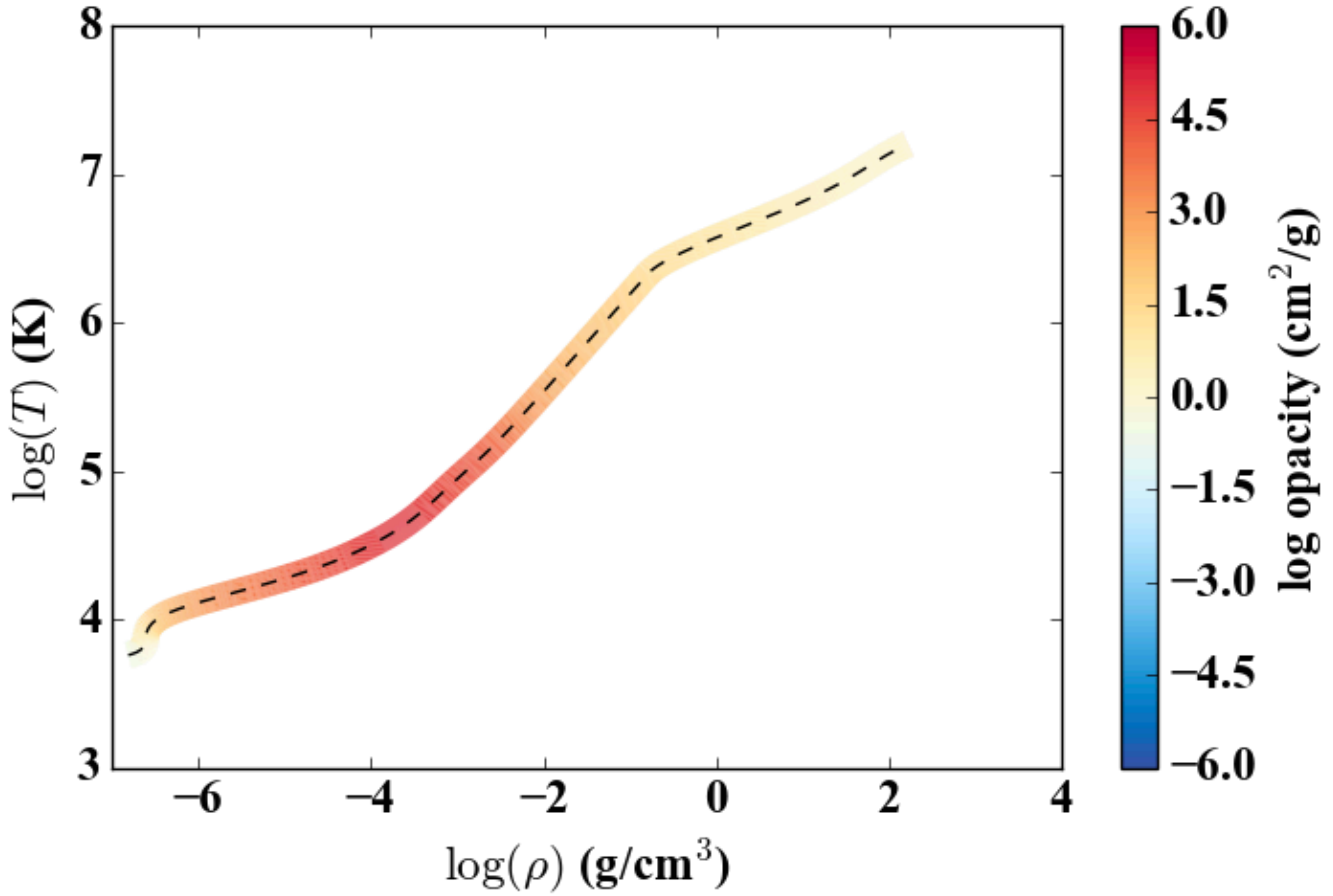
- 08 Apr 2017
  » **MESA Marketplace**
- 17 Feb 2017
  » **Release 9575**
- 25 Jan 2017
  » **Diffusion Updates**
- 08 Jan 2017
  » **Summer School 2017**
- 10 Aug 2016
  » **Documentation Archive**
- 19 Jun 2016
  » **Release 8845**
- 03 Feb 2016
  » **Release 8118**
- 29 Jan 2016
  » **New MESA SDK Version**
- 10 Jan 2016
  » **Summer School 2016**
- 27 Sep 2015
  » **Instrument Paper 3**

mesastar.org



Paxton et al. 2011, Fig. 3





Kip (first edition) fig 22.7
(hint for notebook interpretation: read the textbook!)