UNIVERSITY OF WESTMINSTER

Predictive Analysis for Decision Making

# Investigating the Impact of Bitcoin Prices on the Volatility of the FTSE 100 Index: A time-series data analysis

Empirical Report

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# Abstract

This study examines the relationship between Bitcoin prices and the FTSE 100 stock market index using empirical analysis. The analysis is based on daily data covering the period from 01 January 2018 to 31 December 2023. Descriptive statistics reveal significant volatility in Bitcoin prices compared to the FTSE 100 index, along with some degree of co-movement between the two variables.

An OLS regression model confirms a statistically significant positive relationship between log Bitcoin prices and the log FTSE 100 index, but with a relatively small coefficient. An ARIMA (1,1,0) model is employed to capture underlying patterns and make forecasts, indicating that increases in Bitcoin prices tend to correspond with increases in the FTSE 100 index. Including the Bitcoin variable improves the model's explanatory power. Further analysis using a GARCH model demonstrates that Bitcoin price shocks, defined as deviations from the mean exceeding a certain threshold, significantly impact the volatility of the FTSE 100 index.

While the findings suggest a positive relationship between Bitcoin prices and the FTSE 100 index, the magnitude of the impact is relatively modest. The study contributes to understanding the interconnectedness between emerging cryptocurrency markets and established stock markets, with implications for investors, traders, and policymakers. Limitations and suggestions for future research are discussed.


**Keywords:** Bitcoin, FTSE 100 Index, ARIMA, GARCH, Volatility

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# 1. Introduction

Forecasting changes in the stock market index is crucial for investors, policymakers, and researchers to manage risk effectively and make informed decisions. While the volatility in the stock market is influenced by various factors, such as interest rate, exchange rate, inflation expectations and industry performances, cryptocurrency volatility has gained attention as a potential predictor of stock market movements.

Bitcoin, launched in year 2009 as the first cryptocurrency, operates with blockchain technology with a system of recording information in a way that makes it difficult to change, hack or cheat the system (Investopedia, 2024). While it is apparent that other factors beyond Bitcoin prices impact the stock market's volatility, exploring how the stock market reacts to Bitcoin price changes can offer valuable insights for investors and researchers, potentially improving their ability to forecast stock market movements. Dhingra, et al., (2023) mentioned in their review that researchers are becoming more interested in studying the association of stock markets with cryptocurrencies and Bitcoin during COVID-19.

Simultaneously, the daily chart shows that the FTSE index has experienced a strong bull run, consistently staying above the 50-day moving average (Investing.com, 2024). However, after consecutive interest rate rises in the UK, many investors worry about increased index volatility affecting possible recessions on their holdings (LSEG, 2023). To address this concern, some market analysts suggest for defensive techniques, such as target volatility index or setting the FTSE100 volatility index at 15%. However, this risk-aversive approach may limit the potential for higher returns on investments. The previous studies suggest that risk-tolerant investors prefer to allocate their investments in the Bitcoin market rather than in the stock market, thereby validating the interconnectedness between the stock market and Bitcoin market capitalisation (Mai, 2019). Therefore, considering the dynamic market sentiments, this paper aims to determine whether fluctuations in Bitcoin price movements have any evident and detectable impact on the volatility of the FTSE100 index.

While extensive literature exists on macroeconomic factors affecting stock market volatility or attempting to forecast cryptocurrency/Bitcoin volatility, this paper fills the gap by analysing the extent of the impact of Bitcoin volatility on the UK stock market, particularly the FTSE100 index. Ahmed (2021) notes a significant amount of research on the potential impact of stock returns on Bitcoin volatility, but few studies consider the reverse.

This study is motivated by a lack of empirical studies on whether Bitcoin prices contain useful information for the volatility of the UK stock market. Understanding the predictive power of Bitcoin prices for the UK stock volatility will aid relevant policymakers in developing policies to sustain economic growth. Additionally, stock market performance is an important indicator of macroeconomic stability and foreign investment attraction, necessitating policymakers to monitor risk factors that could destabilise it. Investors often seek information on market risk and its predictors to minimise risks and improve returns, necessitating the analysis of the interconnectedness between emerging cryptocurrency markets and established stock markets.

This analysis employs various econometric techniques to model the relationship between Bitcoin and the FTSE100 index to answer the research questions below:

(i) Does Bitcoin price volatility affect UK stock market performance?

(ii) Can Bitcoin be included as a significant predictor for the FTSE100 index?

(iii) Does the UK stock market exhibit asymmetric reactions to Bitcoin price shocks?

The remaining paper proceeds as follows. Section 2 reviews relevant literature on the nature of the relationship between cryptocurrencies and stock indices. Section 3 outlines the data and methodology used for this analysis. Section 4 presents the econometric models and empirical findings. Section 5 discusses the interpretation of results and model limitations. Finally, Section 6 provides concluding remarks and scope for future research.

# 2. Literature Review

Bouri, et al., (2023) contribute to the literature by investigating the predictive power of Bitcoin prices for US stock volatility by utilising a lag-distributed model on daily Bitcoin prices from 2017 to 2021, and 20-day annualised realised volatility of the US S&P 500 with sector-wise stock returns and identified an inverse relationship between Bitcoin prices and realised volatility of US stocks. Conversely, Ahmed (2021) investigated the volatility of Bitcoin prices on various equity markets, suggesting that developed-market returns are positively related to Bitcoin's realised variance, while emerging-market returns exhibit a mixed response depending on market conditions.

Empirical investigations show that Bitcoin behaves more like an asset rather than a currency (Kim, et al., 2021). If, however, Bitcoin is viewed as an alternative asset class, one must examine its risk-return profile and its ability to act as a hedge or a diversifier for other assets. Several market analysts and scholars consider Bitcoin as digital gold since Bitcoin and gold have similar characteristics, such as non-centrality, both have no identifiable cash flows, and both can be used as currency hedges. However, other economists argue that Bitcoin is primarily a speculative investment rather than a traditional monetary instrument. Notably, major financial exchanges like the Chicago Board Options Exchange (CBOE) and the Chicago Mercantile Exchange (CME) have been trading Bitcoin futures (Almansour, et al., 2021). Regardless of whether Bitcoin is considered a currency or an asset, understanding its volatility is crucial (Bergsli, et al., 2022).

The importance of assessing and predicting volatility in the cryptocurrency market is further evidenced by studies analysing Bitcoin prices' impact on expected inflation. By employing Granger causality tests, Blau et al. (2021) supported the argument that Bitcoin can be an effective hedge against inflation, as its unregulated nature contributes to its perceived ability to protect against rising prices. Bitcoin has the potential to be an inflation hedge due to its fixed supply and decentralised nature. However, the inherent volatility, regulatory uncertainties, and relatively short historical records compared to traditional hedges add a level of risk. An empirical assessment by Zeng and Ahmed (2023) on market integration of East Asian stock markets and the volatility spillover to the Bitcoin market suggests that Bitcoin offers diversification benefits. Just and Echaust (2024), while using data from five major cryptocurrencies from 2017 to 2022, explore whether cryptocurrencies added to a stock portfolio can decrease the overall portfolio's volatility. However, they concluded that conditional probabilities are not satisfactorily high to recommend cryptocurrencies as safe havens. Yet, investing in Bitcoin should not be based solely on historical performance but rather on thorough research and understanding of the asset and the broader cryptocurrency market (Nasdaq, 2023).

Additionally, where the efficient market hypothesis proposes that prices reflect all available information in the market, the cryptocurrency market is considered inefficient, characterised by its high volatility. Sodiq and Oluwasegun (2020) analyse the volatility of exchange rates and stock market prices in Nigeria in response to fluctuations in cryptocurrency prices (Bitcoin and Ethereum) using monthly data from August 2015 to December 2019. By employing statistical techniques including GARCH (1,1) and EGARCH (1,1), findings reveal that the stock market price in Nigeria is more influenced by Bitcoin and Ethereum price instability than by exchange rates, suggesting cryptocurrency volatility influences stock market changes. While others argue that Bitcoin is correlated with the VIX (volatility index) but not with the stock market (totalcoin.io, n.d.).

Evaluating the relation between cryptocurrencies and stock market dynamics is complex, as indicated by the contrary results observed in studies analysing the impact of stock market volatility on cryptocurrency price volatility. Such as, Zeng and Ahmed (2023), while utilising daily data from 2014 to 2020, find no intuitive effect of Bitcoin spillovers on the East Asian market, but note that the Bitcoin market itself is at significant risk of spillovers, particularly from the Chinese stock market. Conversely, Bhullar and Bhatnagar (2020) report, based on data from 2015 to 2019, that while the Indian (Sensex) stock market influences Bitcoin volatility, fluctuations in the Chinese (SSIC) stock index do not affect Bitcoin price movements.

This literature collectively contributes valuable insights into various aspects of the relationship between Bitcoin and traditional financial markets. However, the findings might be constrained by the chosen time frame and regional focus, addressing the scope for further research to capture a broader spectrum of Bitcoin dynamics. This paper intends to aid value to the literature by isolating and specifically monitoring the impact of Bitcoin price movements on FTSE100 index volatility to evaluate whether Bitcoin can be used as an effective determinant of one of a major stock index. Additionally, utilising recent data for monitoring Bitcoin dynamics under this paper can enhance forecasting accuracy and lead to higher economic gains for investors and portfolio managers.

# 3. Data and Methodology

The dataset utilised in this study comprises daily observations of BTC-GBP prices and FTSE100 index values spanning from January 1, 2018, to December 31, 2023. BTC data is sourced from (Yahoo Finance, n.d.), while FTSE100 index data is obtained from (investing.com, n.d.). Both variables are taken in GBP denomination to ensure an accurate comparison, and Python is the primary tool for analysis.

To conduct a comprehensive analysis, 'Price' values from the FTSE100 data and 'Adj Close' values from the BTC-GBP data are utilised. However, due to the differing trading schedules — the FTSE100 market being closed on weekends and the cryptocurrency market operating daily — gaps in time-series data occur. To address this, missing values in the FTSE100 data are interpolated with the last known value from the previous trading day using the 'Forward Fill' method, aligning timestamps across both datasets. The final dataset encompasses a total of 2,192 observations. Additionally, variables have been converted into logarithmic form to stabilise the variance and make the data more stationary. Incorporating the natural logarithm of variables allows to capture the percentage changes in the variables.

Subsequently, a series of econometric models are estimated to examine the relationship between BTC volatility and FTSE100 volatility, including autoregressive models like ARIMA (AutoRegressive Integrated Moving Average) and GARCH (Generalized Autoregressive Conditional Heteroskedasticity).

Past studies have commonly utilised ARIMA and GARCH models and their variants for estimating volatility in Bitcoin and financial markets or instruments. The study by (Almansour, et al., 2021) on nine major cryptocurrencies found that ARCH and GARCH models are effective predictors of cryptocurrency volatility. Bergsli et al. (2022) also utilised several GARCH models (GARCH, EGARCH, GJR-GARCH, IGARCH, MSGARCH, and APARCH) models with various error distributions and two HAR models (regular and logarithmic HAR models) on data spanning from January 2014 to September 2018 to forecast the volatility of Bitcoin. Kyriazis (2020) examined the data from March 2012 to March 2020 and by employing ARCH and GARCH models, found that Bitcoin returns and volatility are positively influenced by gold returns and the S&P500 volatility index (VIX), whereas the Geopolitical Risk Index exerts negative impacts on Bitcoin markets, further suggesting that the Simple Asymmetric ARCH methodology provides the best fit for the purposes of estimations.

Therefore, the choice of methodology is justified by the literature, the characteristics of the data exhibiting time-series patterns, and the research objective of investigating the impact of Bitcoin prices on FTSE 100 volatility. The ARIMA model is employed to forecast future volatility in the FTSE100 index, while the GARCH model is used to model time-varying volatility in financial data, with the inclusion of BTC prices as an exogenous variable in both models.

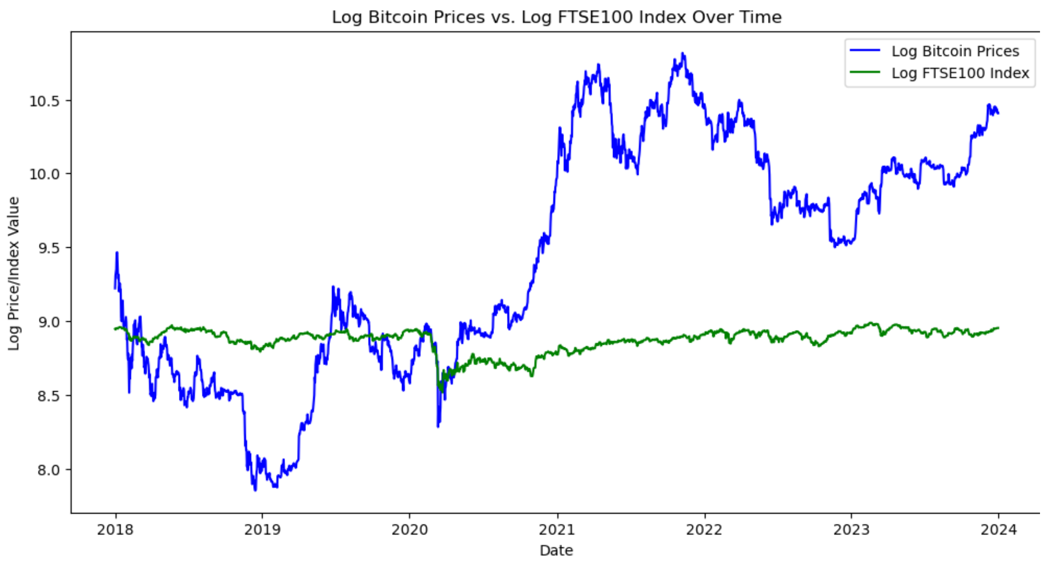
# 4. Econometric / Statistical Model

## Descriptive Analysis

The visual analysis shows significant volatility in both time series. Bitcoin prices exhibit more extreme fluctuations compared to the FTSE 100 index, with the COVID period in 2020 causing a notable downturn in both variables. However, Bitcoin's recovery appears quicker and more pronounced, displaying a random walk pattern. In contrast, the FTSE 100 index demonstrates a relatively stable long-term trend, with fluctuations around a consistent mean. Despite these differences, both variables exhibit some degree of co-movement, as depicted.

The correlation coefficient is derived as 0.187, which suggests a weak positive correlation between log Bitcoin prices and the log FTSE100 index. This indicates that while there is some degree of co-movement between the two variables, the relationship is not particularly strong.

**Figure 1: Visual Analysis of Relation between BTC prices and FTSE100 Index**



## Regression Analysis

Further, an OLS regression model was fitted that revealed a statistically significant positive relationship between log Bitcoin prices and the log FTSE 100 index as implied by the low p-value of the t-statistic ($p < 0.05$). However, the coefficient of log_BTC is relatively small, indicating that a percentage change in the price of BTC by one unit increases the percentage change in the FTSE100 index by 0.0189 units only, ceteris paribus. The R-squared of 0.035 also indicates that log_BTC explains only a small portion of the variation, approximately 3.5%, in the log FTSE_100 index.

Given the results, the linear model can be estimated as:

$$\ln\left(\widehat{\mathrm{FTSE100}}\right)_t = 8.6925 + 0.0189 * \ln(BTC)_t + \varepsilon_t \qquad \text{----------- (Model 1)}$$

(t = 8.914, p=0.00) (t = 431.833, p=0.00)

This equation indicates that the predicted value of the log FTSE100 index is equal to the intercept term (8.6925) plus the coefficient for the log Bitcoin prices (0.0189) multiplied by the log Bitcoin prices at time t, where the error term $\varepsilon_t$ represents all other factors affecting the FTSE100 index but are not accounted for in the model.

**Figure 2: Output for OLS Regression (Model 1)**

```
                            OLS Regression Results
==============================================================================
Dep. Variable:             log_FTSE100   R-squared:                       0.035
Model:                             OLS   Adj. R-squared:                  0.035
Method:                  Least Squares   F-statistic:                     79.45
Date:                 Mon, 06 May 2024   Prob (F-statistic):           1.02e-18
Time:                         13:32:36   Log-Likelihood:                 2461.0
No. Observations:                 2191   AIC:                            -4918.
Df Residuals:                     2189   BIC:                            -4907.
Df Model:                            1
Covariance Type:             nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          8.6925      0.020    431.833      0.000       8.653       8.732
log_BTC        0.0189      0.002      8.914      0.000       0.015       0.023
==============================================================================
Omnibus:                       460.350   Durbin-Watson:                   0.013
Prob(Omnibus):                   0.000   Jarque-Bera (JB):              818.584
Skew:                           -1.331   Prob(JB):                    1.77e-178
Kurtosis:                        4.369   Cond. No.                         115.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

## ARIMA Modelling

To further analyse the data, ARIMA modelling is employed to capture underlying patterns and make forecasts in the FTSE100 index based on BTC price fluctuations.

To prepare the data for ARIMA modelling, the Augmented Dickey-Fuller (ADF) test was used to identify the presence of unit roots in time series data of log_FTSE100 index and log_BTC variables and determine the appropriate level of differencing needed to make the data stationary. The initial test results indicated that both time series are non-stationary (p-values > 0.05 for both variables), failing to reject the null hypothesis of a unit root in the data (Appendix 6).

To address non-stationarity, the first difference of both variables was applied. The first difference of a variable is calculated as $\Delta X_t = X_t - X_{t-1}$, where $X_t$ represent the value of a variable at time t and $X_{t-1}$ represent the value of a variable at the previous time. Using the first difference helps to capture the changes in the variables from one period to the next.

The results of the Augmented Dickey-Fuller (ADF) test on the differenced series are as follows:

- For the first difference of FTSE100:

  ADF Statistic: -13.5022

  p-value: 2.9623e-25

  Critical Values at 1%, 5%, and 10%: -3.4334, -2.8629, -2.5675

- For the first difference of BTC:

  ADF Statistic: -32.2573

  p-value: 0.0

  Critical Values at 1%, 5%, and 10%: -3.4333, -2.8629, -2.5675

Since the ADF test on the differenced series yields significantly lower ADF statistics compared to critical values at all common significance levels (1%, 5%, and 10%), with p-values close to zero, therefore, the null hypothesis of unit root is rejected, and it is concluded that both series are stationary at the first difference. The revised model is formulated as:

$$\Delta \ln\left(\widehat{\mathrm{FTSE100}}\right)_t = \beta_0 + \beta_1 * \Delta\ln(BTC)_t + \varepsilon_t$$

Where $\beta_0$ is the intercept term, $\beta_1$ is coefficient for the first difference of log BTC prices, $\Delta\ln(BTC)_t$ and $\Delta\ln(FTSE100)_t$ represent the first difference of log BTC prices and log FTSE100 index, respectively, and $\varepsilon_t$ is error term. By utilising the first difference of log BTC prices and log FTSE100 index, the model effectively isolates the alterations in both variables from one period to the next. This approach examines how changes in Bitcoin prices relate to changes in the FTSE100 index, providing insights into their short-term dynamic relationship.

OLS regression yields coefficients for the stationary model (output is in Appendix 9):

$$\Delta\ln\left(\widehat{\text{FTSE100}}\right)_t = -2.194e^{-05} + 0.0433 * \Delta\ln(BTC)_t + \varepsilon_t$$ ----------- (Model 2)

(t = -0.115, p=0.908) (t = 8.139, p=0.00)

It is observed that the coefficient estimates for log BTC with the first difference (0.0433) in model 2 are higher than the coefficient for log BTC (0.0189) in model 1, indicating that the impact of changes in BTC prices on FTSE100 returns is larger when considering the first difference of BTC prices compared to simply using the log of BTC prices. However, the insignificant constant term and lower R squared value of Model 3 indicate that Model 2 was a better-fit model. Nevertheless, taking the first difference and making the data stationary is crucial to proceed with ARIMA modelling.

Once the data is stationary, ARIMA modelling can be performed by defining its parameters: ‘p’ for the autoregressive (AR) order and ‘q’ for the moving average (MA) order to capture the underlying patterns in the data. The 'I' in ARIMA stands for "integrated," indicating the differencing order ‘d’ required to make the time series stationary, which is ‘1’ as defined by the ADF test. For defining the AR and MA orders, a grid search was performed that involved fitting ARIMA models with different combinations of $p$ and $q$ values and evaluating each model's performance using a chosen criterion (AIC or BIC).

**Figure 3: Output from Grid search to identify the best parameters for ARIMA Modelling**

```
Best AIC: -14235.594175064825
Best Parameters: (1, 1, 0)
```

The grid search revealed the best-fitting model as ARIMA (1,1,0) according to the AIC criterion, indicating an autoregressive component of order ($p$ = 1), a differencing order of ($d$ = 1), and no moving average component ($q$ = 0), provides a reliable framework for forecasting and analysing the FTSE100 index data.

Subsequently, ARIMA (1,1,0) models are fitted on log FTSE100 index data with and without including the differenced log BTC variable:

1. **ARIMA (1,1,0) Model without including log_BTC:**

$$\Delta\ln\left(\widehat{\text{FTSE100}}\right)_t = -0.0335 * \ln(\text{FTSE100})_{t-1} + \varepsilon_t$$ ----------- (Model 3)

(z = -3.183, p=0.001)

Figure 4: Output for ARIMA Model, without BTC - (Model 3)

```
                               SARIMAX Results
==============================================================================
Dep. Variable:            log_FTSE100   No. Observations:                 2161
Model:                 ARIMA(1, 1, 0)   Log Likelihood                7119.797
Date:                Mon, 06 May 2024   AIC                         -14235.594
Time:                        13:51:44   BIC                         -14224.238
Sample:                             0   HQIC                        -14231.441
                               - 2161
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ar.L1         -0.0335      0.011     -3.183      0.001      -0.054      -0.013
sigma2      8.026e-05   6.88e-07    116.617      0.000    7.89e-05    8.16e-05
===================================================================================
Ljung-Box (L1) (Q):                   0.00   Jarque-Bera (JB):             49069.66
Prob(Q):                              0.99   Prob(JB):                         0.00
Heteroskedasticity (H):               1.33   Skew:                            -1.35
Prob(H) (two-sided):                  0.00   Kurtosis:                        26.19
===================================================================================

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).
```

## 2. ARIMA (1,1,0) Model with differenced log_BTC:

$$\Delta \ln\left(\widehat{\mathrm{FTSE100}}\right)_t = 0.0156 * \Delta\ln(BTC)_t - 0.0296 * \ln(\mathrm{FTSE100})_{t-1} + \varepsilon_t \qquad \text{--- (Model 4)}$$

(z = 9.085, p=0.000)  (z = -2.714, p=0.007)

Figure 5: Output for ARIMA Model, including BTC - (Model 4)

```
                               SARIMAX Results
==============================================================================
Dep. Variable:            log_FTSE100   No. Observations:                 2161
Model:                 ARIMA(1, 1, 0)   Log Likelihood                7128.418
Date:                Tue, 07 May 2024   AIC                         -14250.836
Time:                        10:48:49   BIC                         -14233.803
Sample:                             0   HQIC                        -14244.606
                               - 2161
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
diff_BTC       0.0156      0.002      9.085      0.000       0.012       0.019
ar.L1         -0.0296      0.011     -2.714      0.007      -0.051      -0.008
sigma2      7.962e-05   8.37e-07     95.112      0.000     7.8e-05    8.13e-05
===================================================================================
Ljung-Box (L1) (Q):                   0.00   Jarque-Bera (JB):             40265.95
Prob(Q):                              1.00   Prob(JB):                         0.00
Heteroskedasticity (H):               1.31   Skew:                            -1.18
Prob(H) (two-sided):                  0.00   Kurtosis:                        24.02
===================================================================================

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).
```

Models 4 and 5 describe how the current value of log_FTSE100 is related to its lagged value and the first difference of log BTC (if included as an exogenous variable), along with an error term capturing unexplained variability.

The autoregressive term (AR1) coefficient is -0.0335 for model 3 and -0.0296 for model 4. This represents the impact of the previous period's value of the log FTSE100 index on the current period's value, with a negative coefficient indicating a tendency for the series to revert towards its mean. The AR1 values provide insights into the short-term autocorrelation behaviour of the dependent variables, indicating how past values influence present values, suggesting a slightly weaker impact of past effects on the FTSE100 index in the presence of the log_BTC variable.

By including the log differenced BTC variable in the ARIMA model, we can account for the influence of Bitcoin price changes on the FTSE100 index. The positive coefficient for diff_BTC (0.0156) suggests that increases in Bitcoin prices tend to correspond with increases in the FTSE100 index. This finding provides valuable insights for investors and analysts, highlighting the interconnectedness between cryptocurrency and traditional financial markets such as stock indices. Additionally, the inclusion of the BTC variable improves the explanatory power of the model, as evidenced by the slightly higher log-likelihood. However, the AIC is slightly lower for the second model.

Moreover, the parameter sigma2 ($\sigma^2$) represents the estimated variance of the model's error term. For model 3, $\sigma^2$ is reported as $8.026e^{-05}$, while in model 4, with the inclusion of the BTC variable (diff_BTC), $\sigma^2$ is $7.962e^{-05}$. Therefore, a slightly lower $\sigma^2$ for model 4 indicates a reduced variability in the errors when BTC difference is included as an exogenous variable in the ARIMA model.

Therefore, in the context of this model, changes in Bitcoin prices, as captured by the log-differenced BTC variable, have a small but positive and statistically significant impact on the fluctuations of the FTSE100 index.

The Heteroskedasticity (H) test value of 1.31 indicates that there is some evidence of heteroskedasticity in the data, which is further diagnosed with white test as below:

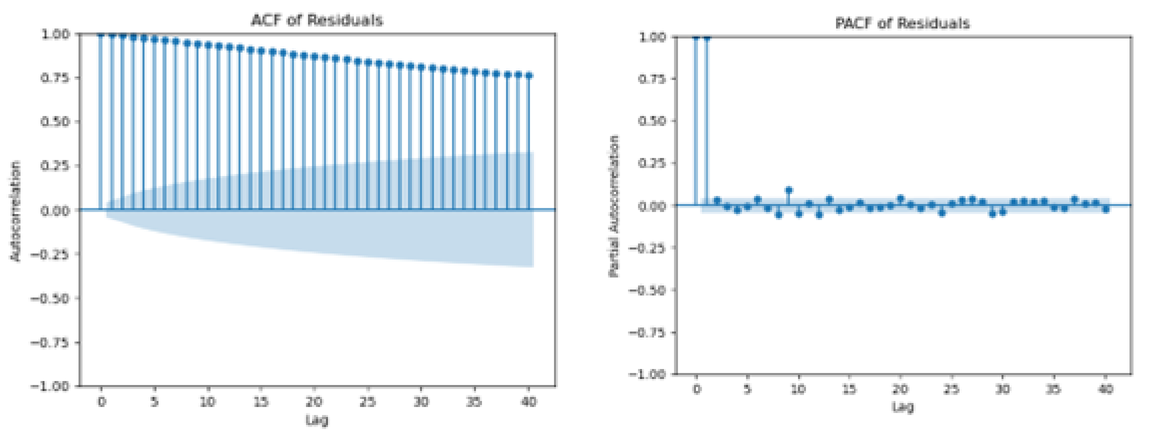
**Figure 6: White's Test Result for Heteroskedasticity**

```
White Test Results:
(207.91442690082312, 7.111412068882866e-46, 114.69922736315782, 4.253934403113874e-48)
```

Where 207.914 is the test statistic for the null hypothesis of no heteroskedasticity with a p-value of $7.111e^{-46}$, and 114.699 is the test statistic for the null hypothesis of no autocorrelation with a p-value of $4.254e^{-48}$ in the residuals of the model. Since the p-values are lower than 0.05 significance level, suggesting residuals are heteroskedastic and autocorrelated.

The presence of autocorrelation in residuals is further investigated by plotting the ACF and PACF functions. The Autocorrelation Function (ACF) measures the correlation between the residuals at different lags, however, the Partial Autocorrelation Function (PACF) measures the correlation between the residuals at a specific lag. In the plot, the ACF values remained consistently high and positive for all lags greater than 0, suggesting positive autocorrelation in the residuals between different lags. However, most of the PACF values lie within the confidence interval, indicating that after accounting for the correlations at intermediate lags, there is no strong evidence of a significant partial autocorrelation structure in the residuals. The ACF can help identify the MA order (q), while the PACF can help identify the AR order (p).

**Figure 7: Plots - ACF and PACF of Residuals**




## Volatility Modelling with GARCH

A GARCH model is used to assess the relationship between BTC and the volatility of FTSE100 while accounting for conditional heteroskedasticity in the data. For identifying the magnitude and direction of BTC price shocks that affect the volatility of FTSE100, a dummy variable for BTC prices is created based on its mean standard deviation as a threshold and assigns a value of 1 if it exceeds the threshold (indicating a shock) or 0 otherwise.

$$D_t = \begin{cases} 1 & if\ \Delta BTC_t > mean\ std.deviation \\ 0 & Otherwise \end{cases}$$

Here, $\Delta BTC_t$ represents the change in BTC prices at time *t*.

Subsequently, a GARCH model is fitted, and the conditional mean equation is derived as:

$$\mu_t = 8.9030$$

Indicating the estimated average value of the log FTSE100 index. The standard error of the mean coefficient is $1.973e^{-03}$, indicating the precision of the estimate, and the t-statistic is significant at $p<0.05$.

Additionally, the conditional variance of the volatility equation is:

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \gamma_1 D_t \varepsilon_{t-1}^2$$

Where:

$omega\ (\omega) = 1.2859e^{-04}$ is the constant term of the volatility equation, indicating average variance,
$alpha\ (\alpha_1) = 0.200$ is the coefficient of the ARCH term, suggesting 20% of the variance in the FTSE 100 index is explained by lagged squared residuals, indicating volatility clustering,
$beta\ (\beta_1) = 0.6800$ is the coefficient of lagged conditional variance (GARCH term), indicating about 68% of the variance contributes to volatility persistence, and
$gamma\ (\gamma_1) = 0.200$ is the coefficient of lagged squared residuals during periods of shock (leverage effect), suggesting a positive relationship between BTC price shock and associated risk in the FTSE100 index.

**Figure 8: Output for GARCH Model (Model 5)**

```
                   Constant Mean - GJR-GARCH Model Results
==============================================================================
Dep. Variable:               log_FTSE100   R-squared:                       0.000
Mean Model:                Constant Mean   Adj. R-squared:                  0.000
Vol Model:                     GJR-GARCH   Log-Likelihood:                3878.80
Distribution:                     Normal   AIC:                          -7747.60
Method:               Maximum Likelihood   BIC:                          -7719.21
                                           No. Observations:                 2161
Date:                   Wed, May 08 2024   Df Residuals:                     2160
Time:                           09:33:21   Df Model:                            1
                                Mean Model
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
mu             8.9030  1.973e-03   4513.528      0.000 [  8.899,  8.907]
                              Volatility Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
omega      1.2859e-04  5.799e-06     22.174 6.057e-109 [1.172e-04,1.400e-04]
alpha[1]       0.2000  1.309e-02     15.274  1.146e-52     [  0.174,  0.226]
gamma[1]       0.2000  3.304e-02      6.053  1.422e-09     [  0.135,  0.265]
beta[1]        0.6800  1.711e-02     39.742      0.000     [  0.646,  0.714]
============================================================================

Covariance estimator: robust
```

Therefore,

$$\sigma_t^2 = 1.2859e^{-04} + 0.200 * \varepsilon_{t-1}^2 + 0.6800 * \sigma_{t-1}^2 + 0.200 * D_t \varepsilon_{t-1}^2 \quad \text{--- (Model 5)}$$

(t = 22.174) (t = 15.274) (t = 39.742) (t = 6.053)

($p<0.05$) ($p<0.05$) ($p<0.05$) ($p<0.05$)

All the coefficients are estimated using the maximum likelihood estimation method and are statistically significant, as indicated by their respective t-statistics and p-values, suggesting that this model plays a significant role in capturing the volatility in the FSTE100 index. The significant coefficient value of $\gamma$ captures the asymmetric effect, implying that when BTC prices experience higher than mean deviation (shocks), then the percentage change in volatility of the FTSE100 tends to increase by approximately 20%.

# 5. Interpretation

## Key Findings

The empirical analysis of the Bitcoin price data and FTSE100 index data spans from January 1, 2018, to December 31, 2023. The findings from the GARCH and ARIMA models are interpreted considering the research questions.

This study investigated the relationship between Bitcoin prices and the FTSE 100 stock market index using various econometric models. The descriptive analysis revealed significant volatility in Bitcoin prices compared to the FTSE 100 index, with some degree of co-movement between the two variables. The correlation analysis showed a weak positive correlation of 0.187, indicating a limited linear association.

The regression analysis demonstrated a statistically significant positive relationship between log Bitcoin prices and the log FTSE 100 index. However, the coefficient was relatively small (0.0189), indicating that a 1% change in Bitcoin prices is associated with only a 0.0189% change in the FTSE 100 index, ceteris paribus. The low R-squared value (0.035) suggests that log_BTC explains only a small portion (3.5%) of the variation in the log FTSE_100 index, indicating that changes in Bitcoin prices have a modest impact on the FTSE 100 index movements.

The ADF test revealed that the data was stationary after applying the first difference. To capture the underlying dynamics and make forecasts, an ARIMA (1,1,0) model was employed. The positive coefficient for the first difference of Bitcoin prices (0.0156) in the ARIMA model indicated that increases in Bitcoin prices tend to correspond with increases in the FTSE 100 index. Including the Bitcoin variable improved the explanatory power of the model, as evidenced by the slightly higher log-likelihood and lower error variance. This suggests that incorporating the BTC difference variable might have helped capture additional information or patterns in the data, leading to a slightly improved model fit. However, the presence of heteroskedasticity and autocorrelation in the residuals suggests that the ARIMA model may not fully capture the underlying dynamics in the data.

Further analysis using a GARCH model revealed that Bitcoin price shocks, defined as deviations from the mean exceeding a certain threshold, had a significant impact on the volatility of the FTSE100 index. Specifically, when Bitcoin prices experienced higher-than-mean deviations, the volatility of the log_FTSE100 tended to increase by approximately 20%. It appears that price changes in BTC, particularly the asymmetric shocks, do affect the volatility of FTSE100 in the model.

While the findings suggest a positive relationship between Bitcoin prices and the FTSE 100 index, the magnitude of the impact is relatively small. However, Bitcoin price shocks appear to have a more substantial effect on the volatility of the FTSE 100 index, highlighting the potential spillover effects between cryptocurrency markets and traditional financial markets.

## Evaluation of the Econometric Models

The research model employed in this study uses various econometric models, including OLS regression, ARIMA, and GARCH models, and provides a comprehensive approach to analysing the relationship between Bitcoin prices and the FTSE 100 index volatility. The log transformation and differencing techniques ensure stationarity of the time series data, which is a crucial assumption for reliable modelling. Including volatility modelling with GARCH models allows for capturing volatility clustering and persistence, which are commonly observed in financial time series.

Uzonwanne (2021) conducted a similar study by incorporating a Multivariate Vector Autoregressive Moving Average - Asymmetric Generalized Autoregressive Conditional Heteroscedasticity (VARMA-AGARCH) model to investigate the return and volatility spillovers between bitcoin price and five major stock index markets (FTSE 100, S&P 500, CAC 40, DAX 30, and Nikkei 225). The empirical findings highlighted the presence of both uni-directional and bi-directional spillovers across different market pairs, providing valuable insights into the interconnectedness of cryptocurrency and traditional financial markets. However, the empirical findings for the FTSE100-Bitcoin pair presented contrasting results to this study, showing there was no evidence of shock spillovers in the short-run and in the long-run, there was uni-directional shock spillover from FTSE 100 to bitcoin significant at 1%, implying that in the long-run, negative shocks to the FTSE 100 transmit to the bitcoin market. However, the study's reliance on data from March 2013 to March 2018 might have limited its ability to capture recent developments and fluctuations in cryptocurrency markets.

Overall, this analysis suggests that while Bitcoin prices exhibit a statistically significant positive relationship with the FTSE 100 index, the magnitude of the impact is relatively small. However, higher than mean Bitcoin price shocks appear to have a more substantial effect on the volatility of the FTSE

100 index, as captured by the GARCH model. These findings highlight the potential spillover effects between cryptocurrency markets and traditional financial markets.

## Limitations

- The low R-squared value in the OLS regression analysis suggests that Bitcoin prices alone may not strongly predict the FTSE 100 index movements.
- The limitations of heteroskedasticity and autocorrelation could be partially attributed to the economical nature of the current model, which includes only one exogenous variable. The estimated effect of BTC volatility on FTSE100 volatility may be biased if other relevant variables are not included in the model.
- The study assumes a linear relationship between Bitcoin prices and the FTSE 100 index. However, the relationship between these variables may be non-linear or exhibit regime shifts, which could affect the validity of the linear models used. As suggested by Kim, et al., (2021) that the Stochastic Volatility (SV) models are favoured over GARCH models for handling asymmetric effects in financial markets, due to their ability to handle fat-tailed and skewed returns, non-Gaussian correlations, and complex dependencies among assets.
- The dynamics between Bitcoin prices and the FTSE 100 index could change over time, and the study's findings might not be generalisable to other time frames.
- Bitcoin is a relatively new asset class, and traditional financial models or assumptions may not fully capture its behaviour.

# 6. Conclusion

The analysis employs various empirical models to explore potential patterns or connections between Bitcoin's value and the performance of the UK's top 100 listed companies represented by the FTSE100 index by analysing daily data from January 2018 to December 2023, to address the research questions as below:

(i) Does Bitcoin price volatility affect UK stock market performance?

The results from the GARCH model indicate that Bitcoin price shocks, defined as deviations from the mean exceeding a certain threshold, significantly impact the volatility of the FTSE 100 index. Specifically, when Bitcoin prices experience higher-than-mean deviations, the volatility of the log_FTSE100 tends to increase by approximately 20%. This finding suggests that elevated fluctuations in Bitcoin prices can have spillover effects on the UK stock market, potentially influencing its performance and increasing uncertainty for investors.

(ii) Can Bitcoin be included as a significant predictor for the FTSE 100 index?

While the OLS regression and ARIMA models reveal a statistically significant positive relationship between Bitcoin prices and the FTSE 100 index, the magnitude of the impact is relatively small. Therefore, while Bitcoin prices can be considered a predictor for the FTSE 100 index, its predictive power appears limited, and other factors likely play a more significant role in explaining the index's movements.

(iii) Does the UK stock market exhibit asymmetric reactions to Bitcoin price shocks?

The inclusion of the dummy variable capturing Bitcoin price shocks in the model suggests that the UK stock market, as represented by the FTSE 100 index, responds differently to significant deviations in Bitcoin prices compared to smaller fluctuations. This finding implies that the UK stock market may exhibit heightened sensitivity or asymmetric reactions to significant Bitcoin price shocks, potentially due to increased investor attention, risk perception, or market uncertainty during such events.

These findings underscore the dynamic interdependence between cryptocurrency markets, epitomised by Bitcoin, and traditional financial markets, such as the FTSE100. Consequently, stakeholders, including investors, policymakers, and researchers, gain valuable insights into the intricate dynamics between these markets, enabling them to make more informed decisions and develop robust risk management strategies in the evolving digital financial landscape.

However, it is essential to note that the study has certain limitations, including the sample period, data frequency, variable selection, and model assumptions. Future research could explore different time periods, incorporate additional variables, and consider non-linear models to provide a more comprehensive understanding of the relationships between cryptocurrency and stock markets.

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# Appendix

The Python file for the codes can be accessed at the following link:

https://github.com/vershasandesh/PADM/blob/main/PADM-%20codes.ipynb

1. Extracting the data for BTC prices and the FTSE100 index into a separate file and filling in the missing values in FTSE100 data.

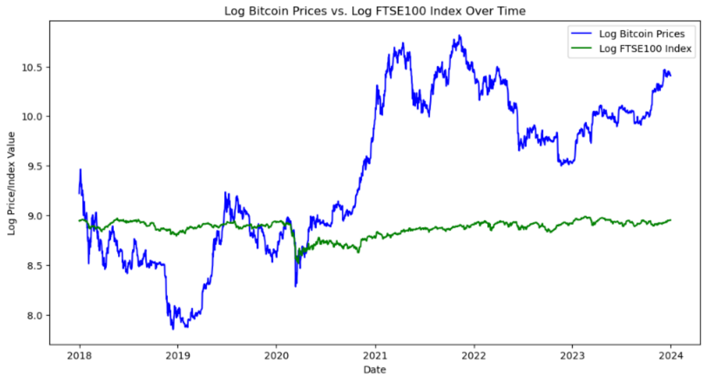
```
import pandas as pd

# Convert 'Date' column to datetime type with custom format
bitcoin_data['Date'] = pd.to_datetime(bitcoin_data['Date'])
ftse100_data['Date'] = pd.to_datetime(ftse100_data['Date'], format='%d/%m/%Y')

# Merge datasets on 'Date' with an outer join to include all dates
data = pd.merge(bitcoin_data[['Date', 'Adj Close']], ftse100_data[['Date', 'Price']], on='Date', how='outer')

# Sort by date
data = data.sort_values(by='Date')

# Forward fill missing values in FTSE100 data
data['Price'] = data['Price'].ffill()

# Rename columns
data = data.rename(columns={'Adj Close': 'BTC', 'Price': 'FTSE100'})

# Filter data for the specified date range
data = data[(data['Date'] >= '2018-01-01') & (data['Date'] <= '2023-12-31')]

# Save the merged data to a new file named 'data.csv'
data.to_csv('data.csv', index=False)
```

2. Calculating the log of both variables:

```
import numpy as np

# Calculate the logarithm of Bitcoin prices and FTSE100 index
data['log_BTC'] = np.log(data['BTC'])
data['log_FTSE100'] = np.log(data['FTSE100'])

# Descriptive statistics for log-transformed variables
log_btc = data['log_BTC'].describe()
log_ftse100 = data['log_FTSE100'].describe()

# Print descriptive statistics
print("Log Bitcoin Descriptive Statistics:")
print(log_btc)
print("\nLog FTSE100 Descriptive Statistics:")
print(log_ftse100)
```

3. Exhibiting the relation between BTC price movement and the FTSE100 index by plotting log variables:

```
# Line plots for Bitcoin prices and FTSE100 index
plt.figure(figsize=(12, 6))
plt.plot(data['Date'], data['log_BTC'], label='Log Bitcoin Prices', color='blue')
plt.plot(data['Date'], data['log_FTSE100'], label='Log FTSE100 Index', color='green')
plt.title('Log Bitcoin Prices vs. Log FTSE100 Index Over Time')
plt.xlabel('Date')
plt.ylabel('Log Price/Index Value')
plt.legend()
plt.show()
```




4. Calculating the correlation coefficient:

```
# Calculate correlation coefficient
correlation_coefficient = data['log_BTC'].corr(data['log_FTSE100'])
print("Correlation Coefficient between Log Bitcoin Prices and Log FTSE100 Index:", correlation_coefficient)
```

```
Correlation Coefficient between Log Bitcoin Prices and Log FTSE100 Index: 0.18715088686993894
```

5. Output of Model 1:

```
import statsmodels.api as sm

# Add a constant term to the independent variable
X = sm.add_constant(data['log_BTC'])

# Specify the dependent variable
y = data['log_FTSE100']

# Fit the linear regression model
model = sm.OLS(y, X).fit()

# Print the summary of the model
print(model.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:            log_FTSE100   R-squared:                       0.035
Model:                            OLS   Adj. R-squared:                  0.035
Method:                 Least Squares   F-statistic:                     79.45
Date:                Mon, 06 May 2024   Prob (F-statistic):           1.02e-18
Time:                        13:32:36   Log-Likelihood:                 2461.0
No. Observations:                2191   AIC:                            -4918.
Df Residuals:                    2189   BIC:                            -4907.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          8.6925      0.020    431.833      0.000       8.653       8.732
log_BTC        0.0189      0.002      8.914      0.000       0.015       0.023
==============================================================================
Omnibus:                      460.350   Durbin-Watson:                   0.013
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              818.584
Skew:                          -1.331   Prob(JB):                    1.77e-178
Kurtosis:                       4.369   Cond. No.                         115.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

6. Initial ADF statistic, indicating unit-root in both variables:

```
from statsmodels.tsa.stattools import adfuller

# Perform ADF test on FTSE100 index (log_FTSE100)
result_ftse100 = adfuller(data['log_FTSE100'])
print('\nADF Statistic for log_FTSE100:', result_ftse100[0])
print('p-value:', result_ftse100[1])
print('Critical Values:', result_ftse100[4])

# Perform ADF test on Bitcoin prices (log_BTC)
result_btc = adfuller(data['log_BTC'])
print('ADF Statistic for log_BTC:', result_btc[0])
print('p-value:', result_btc[1])
print('Critical Values:', result_btc[4])
```

```
ADF Statistic for log_FTSE100: -2.6186235445823023
p-value: 0.08920892301374672
Critical Values: {'1%': -3.433355979713365, '5%': -2.8628679395895342, '10%': -2.5674769282439556}
ADF Statistic for log_BTC: -0.7281469706219738
p-value: 0.8393235202135165
Critical Values: {'1%': -3.4333422250634027, '5%': -2.8628618662999394, '10%': -2.567473694616472}
```

7. Taking the first difference of variables:

```python
import pandas as pd

# Calculate first differences for FTSE100 (dependent variable)
data['diff_FTSE100'] = data['log_FTSE100'].diff()

# Calculate first differences for BTC (independent variable)
data['diff_BTC'] = data['log_BTC'].diff()

# Drop NaN values resulting from differencing
data = data.dropna()

# Display the first few rows of the differenced data
print(data.head())
```

```
         Date           BTC  FTSE100   log_BTC  log_FTSE100  diff_FTSE100  \
1 2018-01-02  11021.922852  7648.10  9.307642     8.942213     -0.005174
2 2018-01-03  11252.498047  7671.11  9.328345     8.945217      0.003004
3 2018-01-04  11512.325195  7695.88  9.351173     8.948440      0.003224
4 2018-01-05  12846.035156  7724.22  9.460790     8.952116      0.003676
5 2018-01-06  12917.895508  7724.22  9.466369     8.952116      0.000000

   diff_BTC
1  0.086316
2  0.020704
3  0.022828
4  0.109617
5  0.005578
```

8. Performing the ADF test again after taking the first difference:

```python
from statsmodels.tsa.stattools import adfuller

# Perform ADF test on differenced FTSE100
result_diff_FTSE100 = adfuller(data['diff_FTSE100'])
print('ADF Statistic for differenced FTSE100:', result_diff_FTSE100[0])
print('p-value:', result_diff_FTSE100[1])
print('Critical Values:', result_diff_FTSE100[4])

# Perform ADF test on differenced BTC
result_diff_BTC = adfuller(data['diff_BTC'])
print('\nADF Statistic for differenced BTC:', result_diff_BTC[0])
print('p-value:', result_diff_BTC[1])
print('Critical Values:', result_diff_BTC[4])
```

```
ADF Statistic for differenced FTSE100: -13.502183861089488
p-value: 2.9622550189994806e-25
Critical Values: {'1%': -3.433355979713365, '5%': -2.8628679395895342, '10%': -2.5674769282439556}

ADF Statistic for differenced BTC: -32.2573025679446
p-value: 0.0
Critical Values: {'1%': -3.4333422250634027, '5%': -2.8628618662999394, '10%': -2.567473694616472}
```

9. Regressing OLS on differenced log_BTC and log_FTSE100 (Model 2):

```python
import statsmodels.api as sm

# Add a constant term to the independent variable (BTC) for the intercept
data['const'] = 1

# Define the dependent variable (first difference of log FTSE100 index)
y = data['diff_FTSE100']

# Define the independent variables (first difference of log BTC prices and the constant)
X = data[['diff_BTC', 'const']]

# Fit the linear regression model
model = sm.OLS(y, X)
results = model.fit()

# Print the regression results
print(results.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:           diff_FTSE100   R-squared:                       0.030
Model:                            OLS   Adj. R-squared:                  0.029
Method:                 Least Squares   F-statistic:                     66.25
Date:                Tue, 07 May 2024   Prob (F-statistic):           6.64e-16
Time:                        11:25:00   Log-Likelihood:                 7154.7
No. Observations:                2161   AIC:                        -1.431e+04
Df Residuals:                    2159   BIC:                        -1.429e+04
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
diff_BTC       0.0433      0.005      8.139      0.000       0.033       0.054
const      -2.194e-05      0.000     -0.115      0.908      -0.000       0.000
==============================================================================
Omnibus:                      756.391   Durbin-Watson:                   2.076
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            26931.649
Skew:                          -0.974   Prob(JB):                         0.00
Kurtosis:                      20.184   Cond. No.                         28.0
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

10. Grid search to identify optimal parameters for the ARIMA model:

```python
import itertools
import statsmodels.api as sm

# Define a range for p and q
p_range = range(0, 4)
q_range = range(0, 4)

# Perform grid search
best_aic = float("inf")
best_params = (0, 0, 0)

for p, q in itertools.product(p_range, q_range):
    try:
        # Fit ARIMA model
        model = sm.tsa.ARIMA(data['log_FTSE100'], order=(p, 1, q))
        results = model.fit()

        # Check AIC
        if results.aic < best_aic:
            best_aic = results.aic
            best_params = (p, 1, q)
    except:
        continue

print("Best AIC:", best_aic)
print("Best Parameters:", best_params)
```

```
Best AIC: -14235.594175064825
Best Parameters: (1, 1, 0)
```

11. Fitting ARIMA Model with identified parameters, without BTC variable (Model 3):

```python
import statsmodels.api as sm

# Fit the ARIMA model
p = 1  # Autoregressive order
d = 1  # Differencing order
q = 0  # Moving average order

# Create and fit the ARIMA model
model = sm.tsa.ARIMA(data['log_FTSE100'], order=(p, d, q))
results = model.fit()

# Print the summary of the model
print(results.summary())
```

```
                               SARIMAX Results
==============================================================================
Dep. Variable:            log_FTSE100   No. Observations:                 2161
Model:                 ARIMA(1, 1, 0)   Log Likelihood                7119.797
Date:                Mon, 06 May 2024   AIC                         -14235.594
Time:                        13:51:44   BIC                         -14224.238
Sample:                             0   HQIC                        -14231.441
                               - 2161
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ar.L1         -0.0335      0.011     -3.183      0.001      -0.054      -0.013
sigma2      8.026e-05   6.88e-07    116.617      0.000    7.89e-05    8.16e-05
===================================================================================
Ljung-Box (L1) (Q):                   0.00   Jarque-Bera (JB):             49069.66
Prob(Q):                              0.99   Prob(JB):                         0.00
Heteroskedasticity (H):               1.33   Skew:                            -1.35
Prob(H) (two-sided):                  0.00   Kurtosis:                        26.19
===================================================================================

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).
```

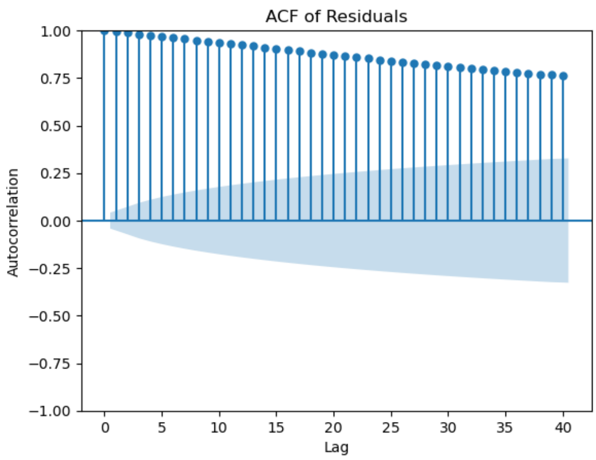
12. Fitting ARIMA Model with identified parameters, including log_BTC variable (Model 4):

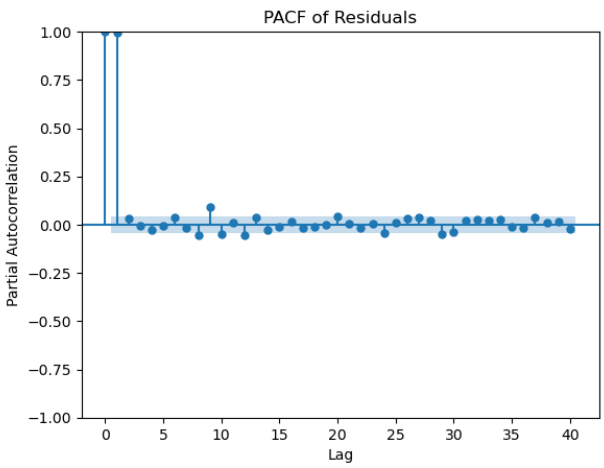
```python
# Fit the ARIMA model with log differenced BTC as exogenous variable
model_with_exog = sm.tsa.ARIMA(data['log_FTSE100'], order=(p, d, q), exog=data['diff_BTC'])
results_with_exog = model_with_exog.fit()

# Print the summary of the model with exogenous variable
print(results_with_exog.summary())
```

```
                               SARIMAX Results
==============================================================================
Dep. Variable:            log_FTSE100   No. Observations:                 2161
Model:                 ARIMA(1, 1, 0)   Log Likelihood                7128.418
Date:                Tue, 07 May 2024   AIC                         -14250.836
Time:                        10:48:49   BIC                         -14233.803
Sample:                             0   HQIC                        -14244.606
                               - 2161
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
diff_BTC       0.0156      0.002      9.085      0.000       0.012       0.019
ar.L1         -0.0296      0.011     -2.714      0.007      -0.051      -0.008
sigma2      7.962e-05   8.37e-07     95.112      0.000     7.8e-05    8.13e-05
===================================================================================
Ljung-Box (L1) (Q):                   0.00   Jarque-Bera (JB):             40265.95
Prob(Q):                              1.00   Prob(JB):                         0.00
Heteroskedasticity (H):               1.31   Skew:                            -1.18
Prob(H) (two-sided):                  0.00   Kurtosis:                        24.02
===================================================================================

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).
```

13. Plotting ACF and PACF:

```
import matplotlib.pyplot as plt
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf

# Obtain the residuals from the ARIMA model
residuals = results.resid

# Plot ACF of residuals
plot_acf(residuals, lags=40, alpha=0.05)
plt.title('ACF of Residuals')
plt.xlabel('Lag')
plt.ylabel('Autocorrelation')
plt.show()

# Plot PACF of residuals
plot_pacf(residuals, lags=40, alpha=0.05)
plt.title('PACF of Residuals')
plt.xlabel('Lag')
plt.ylabel('Partial Autocorrelation')
plt.show()
```




14. Computing white test for heteroskedasticity:

```
import statsmodels.api as sm
from statsmodels.stats.diagnostic import het_white

# Assuming residuals and exogenous variables are already defined
# Fit your model
model = sm.OLS(y, X)
results = model.fit()

# Obtain residuals
residuals = results.resid

# Perform White test for heteroskedasticity
white_test_results = het_white(residuals, X)
print("White Test Results:")
print(white_test_results)
```

```
White Test Results:
(207.91442690082312, 7.111412068882866e-46, 114.69922736315782, 4.253934403113874e-48)
```

15. Calculating the values for the dummy variable:

```
# Calculate the mean standard deviation of BTC prices
btc_mean_std = data['log_BTC'].std()

# Use the mean standard deviation as the threshold for identifying shocks
threshold = btc_mean_std

# Define a function to identify shocks based on the threshold
def identify_shocks(btc_price):
    if btc_price > threshold:
        return 1  # Shock
    else:
        return 0  # No shock

# Apply the function to the BTC prices to identify shocks
data['BTC_Shock'] = data['log_BTC'].apply(identify_shocks)

# Display the first few rows of the data
print(data.head())
```

```
          Date          BTC  FTSE100   log_BTC  log_FTSE100  diff_FTSE100  \
30  2018-01-31  7199.478516  7533.55  8.881764     8.927122     -0.007199
31  2018-02-01  6428.699707  7490.39  8.768528     8.921376     -0.005746
32  2018-02-02  6254.072266  7443.43  8.740988     8.915087     -0.006289
33  2018-02-03  6497.811523  7443.43  8.779221     8.915087      0.000000
34  2018-02-04  5866.059570  7443.43  8.676938     8.915087      0.000000

    diff_BTC  volatility_FTSE100  BTC_Shock  lagged_BTC  const
30  0.008262            0.007588          1         NaN      1
31 -0.113236            0.009076          1    8.881764      1
32 -0.027539            0.010948          1    8.768528      1
33  0.038233            0.012459          1    8.740988      1
34 -0.102282            0.013668          1    8.779221      1
```

16. Fitting the GARCH Model:

```
import pandas as pd
import arch

# Assuming 'log_FTSE100' contains the log returns of the FTSE100 index
returns = data['log_FTSE100']

# Define the exogenous variable (Bitcoin shock)
exog = data['BTC_Shock']

# Fit GARCH(1,1) model with Bitcoin shock dummy
garch_model = arch.arch_model(returns, vol='Garch', p=1, q=1, o=1, x=exog)
garch_results = garch_model.fit(disp='off')

# Print summary of the model
print(garch_results.summary())
```

```
                   Constant Mean - GJR-GARCH Model Results
==============================================================================
Dep. Variable:            log_FTSE100   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                  GJR-GARCH   Log-Likelihood:                3878.80
Distribution:                  Normal   AIC:                          -7747.60
Method:            Maximum Likelihood   BIC:                          -7719.21
                                        No. Observations:                 2161
Date:                Wed, May 08 2024   Df Residuals:                     2160
Time:                        09:33:21   Df Model:                            1
                               Mean Model
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
mu             8.9030  1.973e-03   4513.528      0.000 [  8.899,  8.907]
                              Volatility Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
omega      1.2859e-04  5.799e-06     22.174 6.057e-109 [1.172e-04,1.400e-04]
alpha[1]       0.2000  1.309e-02     15.274  1.146e-52     [  0.174,  0.226]
gamma[1]       0.2000  3.304e-02      6.053  1.422e-09     [  0.135,  0.265]
beta[1]        0.6800  1.711e-02     39.742      0.000     [  0.646,  0.714]
============================================================================

Covariance estimator: robust
```