WSU Vancouver Parking Permit Price Optimization to Maximize Revenue

Math 464 Capstone Project

Skyler Vertner- Santos

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Overview

This project's purpose is to record the process and findings of implementing the abstract concepts taught in Math 464, Linear Optimization, and applying the modeling and solution finding methods to a decision making process in business. The document describes the process I went through to find an area where I could apply linear optimization methods to. This includes successful and failed attempts to find a contact to work with, and finally, the mathematical model of the constraints and variables involved in the decision making process and the solution I found to the goal that I set.

The final model I went with describes the planning and design that the WSU Vancouver Parking Services undergoes to decide on parking permit prices. I define the constraints that limit certain variables of consideration such as parking spot limitations, purchasing behavior of the students, and demand constraints such as the need to meet the parking demand for the projected amount of student drivers.

Contents

1 Contact List

The following is a list of the organizations I seek to contact. Below each is a bulleted list of possible optimization problems that I might expect to see.

1. Washington State University Vancouver Parking Services
   - Pricing of permits based on lots
   - Lot size constraints
   - Maximizing profit
2. Sportsman's Warehouse
   - Ordering stock based on needs
   - Budget and shipping size constraints
   - Maximizing quarterly profit based on product ordering
3. Michaels Framing
   - Ordering bulk parts based on demand and budget
   - Minimizing glass waste by maximizing glass cutting efficiency
   - Maximizing profit based on ordering lites and minimizing waste due to demand

Feasibility and Inclination WSU Parking services strikes my interest because it is directly relevant to me as a student who drives and potentially interesting. For example, if WSU-V made the grey lot free or extremely cheap, the passes would sell out according to lot size which may hurt sales in the other lots closer to campus building. Finding out the right pricing that would maximize profit or revenue based on their data of previously sold tickets could in fact benefit their financial model. Sportsman's Warehouse has a variety of departments such as fishing with tons of small jigs and hooks at low price vs more expensive products such as firearms in hunting. Figuring out how many products from a select few departments based on truck size and weight constraints and budgeting would be interesting to find. As for Michael's Framing, when a frame shop receives a frame order, it has to cut glass to fit the frame size. There are certain standard size lites/panes it can be cut from. These different standard sizes have different costs. If many orders are taken, it may be more cost effective to cut several small pieces for multiple orders from a larger lite. Maximizing profit based on these constraints is an optimization problem that would illuminate the relation of waste of glass to the shop's profit.

2 Contact Plan

WSU Parking Services To begin contacting WSU parking services I will email them at van.parkingservices@wsu.edu with the following:
To whomever it may concern,
I am a student in Dr. Asaki's Mathematical Optimization course and am located here on the Vancouver campus. As part of a class project, I am researching how to construct mathematical models that can be used to assist in decision making processes. I am interested in applying what I have learned to the parking services here at Vancouver. It would be helpful we could set up a short meeting of half hour or less to discuss lot permits and costs and perhaps other decisions that pertain to your services. I could meet any day of the week between 11:00 a.m. until 3:00.
Please Let me know. If you have any questions,please feel free to conact my professor at tasaki@wsu.edu.
Sincerely,
Skyler Vertner- Santos

Michaels Framing I used to work at Michaels and was a friend of the framing manager, Jason. I have his phone number, so reaching out to him might look like a text that reads as:
Jason,
I am currently taking a Math course in Dr. Asaki's Mathematical Optimization Course. As a part of a project, I am researching how mathematics can be used to aid in decision making processes. I would be very interested in applying the concepts I am learning to something of the nature of optimizing glass cutting to minimize waste and save money or other aspects of the frame shop. I can meet in accordance to your schedule, possibly Wednesday or Fridays. It would be helpful to meet for a half hour or less.
Please Let me know. If you have any questions, please feel free to contact my professor at tasaki@wsu.edu.
Sincerely,
Skyler Vertner- Santos

Sportsman's Warehouse I am currently employed at Sportsman's Warehouse and could gain email information to our Receiving manager, Hannah. Contacting her may look like:
Hannah,
I am currently in Dr. Asaki's Mathematical Optimization course at WSU. As part of a class project, I am researching how to construct mathematical models that can be used to assist in decision making processes. I am interested in applying what I have learned to the decision making process in optimizing truck freight. It would be helpful we could set up a short meeting of half hour or less to discuss the process of product ordering and truck delivery schedules. I could meet any time during, before, or after work.
Please Let me know. If you have any questions, please feel free to contact my professor at tasaki@wsu.edu.
Sincerely,
Skyler Vertner- Santos

3 Meeting Synopsis

After contacting each business, I have been successful in talking to a representative for each one. I joined Jason Craft at Michael's Framing to discuss the demand of glass cutting orders and the procedure and decision making in completing the task. I sat down with Anne O'Neill from WSU Vancouver Parking Services. She discussed pricing, lot sizes, and decisions that are involved in permit pricing. Lastly, I contacted Hannah at Sportsman's Warehouse about the procedure for stock ordering. Unfortunately, all the decision making involved at Sportsman's is done through corporate who places orders based on stock counts that are updated in store as well as seasonal sales. Scheduling for freight is done by each department manager who makes sure there are enough people to put stock out through the week. Because of this, I could not find anything worth trying to optimize here.

Michael's Framing Raw Data Collected:
42 orders going two weeks out (Feb 3- Feb 17)
29 Include some sort of glass involved
Glass panes come in 7 standard sizes which can be ordered in boxes: 8x10, 11x14, 18x24, 22x30, 24x36, 32x40, 42x60. These sizes come in boxes with varying amount of lites with restriction weight of 50lbs per box.
The prices for each vary from (40-300) dollars
The ordering process for shop supplies falls directly on the Manager , Jason. He can order supplies whenever he wants to (with permission from the store manager) as long as he stays within the shop spending budget that differs week from week depending on sale and season. The main strategy he uses is to always have a stock backup of at least two extra, unopened boxes for each size. For 24x36 and larger, he makes sure there are always at least four boxes in stock. This way they do not run out while waiting for shipments of more glass.
The costs on average remain the same for the boxes of glass. Rarely do they change largely in price, it mostly small fluctuations.
One variable discussed that would be hard to model is how many lites are gone to waste because they are shattered or miss-cut. For example, it has happened several times where a large box such as a 42x60 will come in with broken glass since the boxes were mishandled in transportation. I have two options for modeling this. One, I can leave this out and create a model where we live in a perfect world and assume there is no waste due to broken glass. Second, I can assign a probability of glass loss based on some arbitrary coefficient. This would be assigned by some arbitrary, estimated mean value.
The objective is to maximize profit by spending less on glass orders but by fulfilling the customer orders. This also means minimizing glass loss such as cutting a lite to 18x24 from a 22x30 lite when one could just use a pre cut 18x24 from its box since it is a standard size.
One important question I have to consider is how do I want to control the parameters. For example, there are really two different types of glasses that can be ordered. UV protective glass, or UV glass with a non-glare coating. This instantly doubles the amount of parameters I have to include. So I have to decide whether I want to simplify what is being ordered and used in order to achieve a feasible model.

WSU Vancouver Parking Services Raw Data Collected:
There are 5 lot types available to park at at WSU-V. There is the Orange lot, the Green lot, Red lot, Gray lot, and Blue lot.
The blue lot differs from the others since it is a daily lot. Anyone who parks at the blue lot pays 4 dollars for the day. This may be left out of my model but I do have some information on blue lot sales.
The Orange, Green, Red, and Gray lot require permits that are sold in three different date intervals. Students can buy an annual permit, a Fall/Spring permit, or per semester.
WSU-V is aloud to sell 170 percent of permits relative to how many parking spots they have per lot because of the inflow and outflow of traffic throughout the day. The permit rates are the following order: Orange, Green, Red, Gray.
Annual: 285, 243, 203, 161
Fall/Spring: 230, 192, 152, 112
Semester: 120, 101, 81, 61
On average, the Blue lot takes 450 dollars a day (weekday).
The amount of spots in each lot is:
458, 349, 169, 503, 300.
The parking services is only aloud to stretch the prices by 2.1 percent.
Re-striping and landscaping costs an average of 40,000 dollars a semester.

4 Problem Statement

WSU-V Parking Services There are five parking lots which WSU-V students can buy permits for. The orange, green, red, gray, and blue lot. WSU-V Parking services has to decide how to price the parking permits to maximize their revenue. Based on their data from the passed ten years, the Parking Services has two price models to offer that they think will make the most money based on this years projected student demand.
Option one entails purchasing option for the Fall and Spring, or per semester. One semester consists of 15 weeks and one year consists of two semesters. Previous years data suggests that the average student goes to campus 3 days a week. The prices are listed as:

Option 1: Permit Prices ($)

Lots	Orange	Green	Red	Gray	Blue
Fall/Spring	230	192	152	112	-
Semester	120	101	81	70	-
Daily	-	-	-	-	5

Option 2 does away with the semester purchases and has an increased price for all annual permits but a decreased price in daily parking. These prices are listed as:

Option 2: Permit Prices ($)

Lots	Orange	Green	Red	Gray	Blue
Fall/Spring	280	210	190	156	-
Daily	-	-	-	-	3

The amount of permits that can be sold is dependent on the lot sizes. The amount of permits that can be sold are represented by:

Amount of lot permits that can be sold

Lots	Orange	Green	Red	Gray	Blue
Permits	778	593	287	855	510

This year, WSU-V has 2650 vehicles from students who commute that are in need of a parking spot. From previous years, they can conclude that 50 percent of students buy annual permits, and 50 percent buy semester permits when there is an option for both. They can also forecast that in option one, students will compete to buy semester or annual permits before they take the option of daily parking in the blue lot. In option two, they forecast that students will compete for spots in the blue lot before they consider buying annual passes.
WSU-V plans to change class times around so that they can maximize their revenue from parking permits. They need to figure out the best combination of selling permits so that they can organize a schedule incentive to achieve this maximum revenue. What combination of selling parking permits maximizes their earnings? What is the maximum revenue they can take in from parking permits?

5 Optimization Model

To begin the modeling of the optimization problem, we shall begin by defining some variables:

- Let $N =$ (The # of parking spaces needed to meet student demand).
  In this case, we have $N = 2650$

- Let $W =$ (The amount of class days in a year).
  In this case we have 5 class days a week at 15 weeks a semester for 2 semesters. This amounts to 150 class days in a year.

- Let $D =$ (The number of days the average student goes to campus in a year).
  The average student only goes to campus three times a week so $D = 90$.
  Let us consider each option as a separate optimization program.

5.1 Option One Model

Let $x_k =$ (The amount of permits sold to the k^{th} lot type. We have $x = \{x_1, x_2, ..., x_{13}\}$. This is organized in the table below.

Lot color and type indicating k

Lots	Orange	Green	Red	Gray	Blue
Fall/Spring	1	2	3	4	-
Semester 1	5	6	7	8	-
Semester 2	9	10	11	12	-
Daily	-	-	-	-	13

The goal of the optimization model is to maximize revenue. This we define as the sum of each lot type revenues which is the price of a permit of lot type k multiplied by the amount of those permits sold, x_k. For option 1, we have the revenue, denoted as R_1, defined to be:

$$\begin{aligned} R_1 = 230x_1 + 192x_2 + 152x_3 + 112x_4 + 120(x_5 + x_9) \\ + 101(x_6 + x_{10}) + 81(x_7 + x_{11}) + 70(x_8 + x_{12}) \end{aligned}$$

The amount of permits that can be sold are limited by the amount of parking spaces.

The amount of parking spaces are:

Lots	Orange	Green	Red	Gray	Blue
Permits	778	593	287	855	510

Each lot can be filled per semester. The constraint for the orange lot would be:

$$x_1 + \frac{1}{2}(x_5 + x_9) \leq 778$$

To avoid the fraction, we can double each side of the equality. The same is done for the following lots. This gives the lot size constraints for the Orange, Green, Red, and Gray lots:

$$\begin{aligned} 2x_1 + x_5 + x_9 &\leq 1556 \\ 2x_2 + x_6 + x_{10} &\leq 1186 \\ 2x_3 + x_7 + x_{11} &\leq 574 \\ 2x_4 + x_8 + x_{12} &\leq 1710 \end{aligned}$$

The Blue lot is restricted to 510 spaces a day. Since there are W class days in a year, the most amount of blue lot permits that can be sold due to lot size is given by:

$$x_{13} \leq 510W$$

This is because the Blue lot permits are sold per each day.
Given that there are N students who are in need of a parking space, there are demand constraints that need to be met. The annual permits and the semester permits are accounted for once since each are sold once per year. The daily parking permits are sold per each day that a student parks. The amount of permits that are sold to the Blue lot can be expressed by the amount of students who aren't parking in the other lots times the amount of days that they will park in the blue lot.

$$x_{13} = (N - \sum_{i=1}^{12} x_i)D$$

The demand constraints are given by:

$$\begin{aligned} \textstyle\sum_{i=1}^{13} x_i = &\textstyle\sum_{i=1}^{12} x_i + x_{13} \\ = &\textstyle\sum_{i=1}^{12} x_i + (N - \sum_{i=1}^{12} x_i)D \end{aligned}$$

Additionally, Parking Services estimates that half the students will purchase annual permits and the other half will pay per each semester. This may be because half the students cannot afford to pay for annual pass all at once or other varying factors.
This means for each lot,

$$2x_{\text{annual}} = x_{semester1} + x_{semester2}$$

These are given by equality constraints:

$$\begin{aligned} 2x_1 - x_5 - x_9 &= 0 \\ 2x_2 - x_6 + x_{10} &= 0 \\ 2x_3 - x_7 + x_{11} &= 0 \\ 2x_4 - x_8 + x_{12} &= 0 \end{aligned}$$

This model assumes that each student attends each semester. This means that if a student buys a semester permit, they are guaranteed to buy the second semester permit as well.

$$x_{semester1} = n \ , \ x_{semester2} = n \rightarrow x_{semester1} - x_{semester2} = 0$$

These constraints are modeled by:

$$\begin{aligned} x_5 - x_9 &= 0 \\ x_6 - x_{10} &= 0 \\ x_7 - x_{11} &= 0 \\ x_8 - x_{12} &= 0 \end{aligned}$$

The general Integer Program becomes:

$$\begin{array}{ll} \max\limits_{x} & R_1 = 230x_1 + 192x_2 + 152x_3 + 112x_4 + 120(x_5 + x_9) \\ & +101(x_6 + x_{10}) + 81(x_7 + x_{11}) + 70(x_8 + x_{12}) + 5x_{13} \\ \text{s.t.} & 2x_1 + x_5 + x_9 \leq 1556 \\ & 2x_2 + x_6 + x_{10} \leq 1186 \\ & 2x_3 + x_7 + x_{11} \leq 574 \\ & 2x_4 + x_8 + x_{12} \leq 1710 \\ & x_{13} \leq 510W \\ & 2\sum_{i=1}^{4} x_i - \sum_{i=5}^{12} x_i = 0 \\ & \sum_{i=5}^{8} x_i - x_{i+4} = 0 \\ & (N - \sum_{i=1}^{12} x_i)D \leq 510W \\ & 0 \leq D \leq W \\ & N \geq 0 \\ & x \geq 0 \\ & x \in \mathbb{Z}^{13} \end{array}$$

The purchasing behavior for option one dictates that students will buy out permits for every lot but the Blue lot before they settle for daily parking. The total parking spots in the Orange, Green, Red, and Gray lots are:

$$778 + 593 + 287 + 855 = 2513$$

The purchasing behavior constraint can be given by:

$$\sum_{i=1}^{12} x_i = 2513$$

This indicates that the remaining students in need of spots have to take daily parking.

$$x_{13} = (N - 2513)W$$

There are 87 students who are forced to park daily in the Blue lot. The blue lot constraint becomes:

$$x_{13} = 87W$$

Using this and filling out for the given values of N, W, D we get the final Integer Program for option one:

$$\begin{aligned}
\max_{x} \quad & R_1 = 230x_1 + 192x_2 + 152x_3 + 112x_4 + 120(x_5 + x_9) \\
& +101(x_6 + x_{10}) + 81(x_7 + x_{11}) + 70(x_8 + x_{12}) + 5x_{13} \\
\text{s.t.} \quad & 2x_1 + x_5 + x_9 \leq 1556 \\
& 2x_2 + x_6 + x_{10} \leq 1186 \\
& 2x_3 + x_7 + x_{11} \leq 574 \\
& 2x_4 + x_8 + x_{12} \leq 1710 \\
& x_{13} = 7830 \\
& 2\sum_{i=1}^{4} x_i - \sum_{i=5}^{12} x_i = 0 \\
& \sum_{i=5}^{8} x_i - x_{i+4} = 0 \\
& x \geq 0 \\
& x \in \mathbb{Z}^{13}
\end{aligned}$$

5.2 Option Two Model

Let $x_k =$ (The amount of permits sold to the k^{th} lot type). We have $x = \{x_1, x_2, ..., x_5\}$ This is organized in the table:

Lot color and type indicating k

Lots	Orange	Green	Red	Gray	Blue
Fall/Spring	1	2	3	4	-
Daily	-	-	-	-	5

For option 2, the revenue denoted by R_2 is defined as:

$$R_2 = 280x_2 + 210x_2 + 190x_5 + 156x_4 + 3x_5$$

The lot restriction for option two remain the same as option one.

Amount of lot permits that can be sold

Lots	Orange	Green	Red	Gray	Blue
Permits	778	593	287	855	510

We have the lot constraints:

$$\begin{aligned}
x_1 &\leq 778 \\
x_2 &\leq 593 \\
x_3 &\leq 287 \\
x_4 &\leq 855 \\
x_5 &\leq 510W
\end{aligned}$$

We have the forecast that students will fight for daily parking over purchasing the annual pass. If we imagine that the Blue lot permits can be sold in advance, then the Blue lot permits will be sold in reserve to the first students to buy permits. Since N is larger than the amount of parking spots in the Blue lot, we know that

$$x_5 \quad = 510D = 45900$$

Although $510D$ Blue lot permits are sold during the year, they are only sold to 510 students. This implies that the rest of the students will have to purchase annual passes.
We have a constraint on $x_1 ... x_4$

$$\sum_{i=1}^{4} x_i \geq N - 510 = 2140$$

Since the WSU-V parking Services has a student demand of N, we can adjust this to be an equality constraint.

$$\sum_{i=1}^{4} x_i = 2140$$

The Integer Program becomes:

$$\begin{array}{rl}
\max\limits_{x} & R_2 = 280x_1 + 210x_2 + 190x_3 + 156x_4 + 3x_5 \\
\text{s.t.} & x_1 \leq 778 \\
& x_2 \leq 593 \\
& x_3 \leq 287 \\
& x_4 \leq 855 \\
& x_5 = 45900 \\
& \sum_{i=1}^{4} x_i = 2140 \\
& x \geq 0 \\
& x \in \mathbb{Z}^5
\end{array}$$

6 Solution

The following is broken down into two solutions. The first, for Option One, and the second for Option Two. Each solution has an analysis of expected findings. They are then compared and related to the original problem statement.

6.1 Solution to Option One

The given Integer Program that models WSU Vancouver's Parking Services revenue based on the constraints laid out in Option One above:

$$\begin{aligned}
\max_{x} \quad & R_1 = 230x_1 + 192x_2 + 152x_3 + 112x_4 + 120(x_5 + x_9) \\
& +101(x_6 + x_{10}) + 81(x_7 + x_{11}) + 70(x_8 + x_{12}) + 5x_{13} \\
\text{s.t.} \quad & 2x_1 + x_5 + x_9 \le 1556 \\
& 2x_2 + x_6 + x_{10} \le 1186 \\
& 2x_3 + x_7 + x_{11} \le 574 \\
& 2x_4 + x_8 + x_{12} \le 1710 \\
& x_{13} = 7830 \\
& 2\textstyle\sum_{i=1}^{4} x_i - \sum_{i=5}^{12} x_i = 0 \\
& \textstyle\sum_{i=5}^{8} x_i - x_{i+4} = 0 \\
& x \ge 0 \\
& x \in \mathbb{Z}^{13}
\end{aligned}$$

Given that $x \in \mathbb{Z}^{13}$, the feasible region for the Program is discrete rather than a convex hull. The Program must be solved via Integer Programs solving methods. This model is easily solved by modeling the problem in MATLAB.

The IP can be put into the form:

$$\boxed{\begin{aligned}
\max_{x} \quad & z = c^{\mathsf{T}} x \\
\text{s.t.} \quad & Ax \le b \\
& A_e x = b_e \\
& x \ge 0 \\
& x \in \mathbb{R}^n
\end{aligned}}$$

Since we have a maximization problem, we minimize over $Q = -z$.

Let $x = [x_1\ x_2\ ...\ x_{13}]^T$. Converting the IP into equivalent matrix form:

$$
\begin{aligned}
\min_{x} \quad & Q = -[230\ 192\ 152\ 112\ 120\ 101\ 81\ 90\ 120\ 101\ 81\ 70\ 5]^T \\
\text{s.t.} \quad & [2\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0]x \leq 1556 \\
& [0\ 2\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 1\ 0\ 0]x \leq 1186 \\
& [0\ 0\ 2\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 1\ 0]x \leq 574 \\
& [0\ 0\ 0\ 2\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 1]x \leq 1710 \\
& [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1]x = 7830 \\
& -[-2\ -2\ -2\ -2\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 0]x = 0 \\
& [0\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ -1\ -1\ -1\ -1\ 0]x = 0 \\
& x \geq 0 \\
& x \in \mathbb{Z}^9
\end{aligned}
$$

From above, we can model:

$$Ax \leq b\,,\ A_e x = b_e$$

by:

$$A = \begin{bmatrix} 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$b = \begin{bmatrix} 1556 \\ 1186 \\ 574 \\ 1710 \end{bmatrix},$$

$$A_e = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 2 & 2 & 2 & 2 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 0 \end{bmatrix},$$

$$b_e = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Using the “Intlinprog” program in MATLAB, the solution is generated.
The Optimal solution point is

$$x^* = \begin{bmatrix} 389 & 296 & 143 & 427 & 389 & 296 & 143 & 427 & 389 & 296 & 143 & 427 & 7830 \end{bmatrix}^T$$

The objective value, or the maximum revenue that the program found was ($):

$$z^* = 491110$$

6.2 Solution One Analysis

There are a few expected patterns in the magnitudes of components in the solution vector due to what you would expect the constraints to do. Such are the equivalence of:

```
Optimal solution found.

Intlinprog stopped at the root node because the objective value is within a gap tolerance of the optimal value,
options.AbsoluteGapTolerance = 0 (the default value). The intcon variables are integer within tolerance,
options.IntegerTolerance = 1e-05 (the default value).


dv =

         389
         296
         143
         427
         389
         296
         143
         427
         389
         296
         143
         427
        7830


ov =

     -491110
```

Figure 1: Option One Solution

$$x_1 = x_5 = x_9,\ x_2 = x_6 = x_{10},\ \ldots,\ x_4 = x_8 = x_{12}$$

Since the semester permits are sold an equal amount each semester, the x_k terms that represent the sale of semester permits for the k^{th} lot will be equal. The same lot terms that represent the annual sales of permits will have a proportional doubling relation to semester permits for that lot. This proportionality is determined by the purchasing behavior of the students.

$$\begin{aligned} x_{semester1} &= x_{semester2} = C \\ 2x_{\text{annual}} &= x_{\text{semester1}} + x_{\text{semester2}} \\ &= 2C \\ x_{annual} &= C \end{aligned}$$

In general, this proportional relation is given by the distribution of how students buy lot permits in regard to annual or semester purchases. It can be generalized, letting P be the percentage of students buying annual passes. We define P to be $0 \leq P \leq 1$, such that $P \in \mathbb{R}$. This way the percentage of students who buy semester permits is $1 - P$. The general relation for this model is:

$$Px_{annual} = \frac{(1-P)}{2}(x_{\text{semester1}} + x_{\text{semester2}})$$

The same relation follows as in the particular case:

$$x_{semester1} = x_{semester2} = C$$

We can conclude:

$$\begin{aligned} Px_{annual} &= \tfrac{(1-P)}{2}(2C) \\ Px_{annual} &= (1-P)C \end{aligned}$$

Which can be written as either equivalent form:

$$\begin{aligned} \tfrac{P}{1-P}x_{annual} &= C \\ x_{annual} &= \tfrac{1-P}{P}C \end{aligned}$$

The following pattern to expect in x^* would then be:

$$x_1 = \tfrac{1-P}{P}x_5 = \tfrac{1-P}{P}x_9,\ x_2 = \tfrac{1-P}{P}x_6 = \tfrac{1-P}{P}x_{10},\ ...,\ x_4 = \tfrac{1-P}{P}x_8 = \tfrac{1-P}{P}x_{12}$$

To verify that this alters the solution in this way, we can modify our model to have different purchasing behavior constraints. For example, if we assume that 60 percent of students will buy annual permits and 40 percent will purchase two semester permits, we can modify the purchasing behavior to follow:

$$3x_{annual} = x_{semester1} + x_{semester2}$$

This is because the ratio of students buying annual to semester is 3 : 2.
The purchasing behavior constraints become:

$$\begin{aligned} 3x_1 - x_5 - x_9 &= 0 \\ 3x_2 - x_6 + x_{10} &= 0 \\ 3x_3 - x_7 + x_{11} &= 0 \\ 3x_4 - x_8 + x_{12} &= 0 \end{aligned}$$

We use the new A_e matrix to be:

$$A_e = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 3 & 3 & 3 & 3 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 0 \end{bmatrix}$$

We expect that in this case where $P = 0.6$ we will get:

$$\begin{aligned} x_{annual} &= (\tfrac{1-0.6}{0.6})C \\ &= (\tfrac{0.4}{0.6})C \\ x_{annual} &= \tfrac{2}{3}x_{semester1} = \tfrac{2}{3}x_{semester2} \end{aligned}$$

Solving the new Integer Program with new purchasing behavior constraints we find that:

```
Optimal solution found.

Intlinprog stopped at the root node because the objective value is within a gap tolerance of the optimal value,
options.AbsoluteGapTolerance = 0 (the default value). The intcon variables are integer within tolerance,
options.IntegerTolerance = 1e-05 (the default value).


dv =

   1.0e+03 *

    0.3100
    0.2360
    0.1140
    0.3400
    0.4650
    0.3540
    0.1710
    0.5100
    0.4650
    0.3540
    0.1710
    0.5100
    7.8300


ov =

     -493380
```

Figure 2: Option One Solution with Modified Purchasing Behavior Constraints

Taking a look at the Orange lot:

$$\begin{aligned} x_1 &= 310 \\ x_5 &= 465 \\ x_9 &= 465 \end{aligned}$$

We see that

$$310 = \frac{2}{3}(465)$$

as expected. The same pattern is true for the Green, Red, and Grey lot as well, that is

$$3x_{annual} = x_{semester1} + x_{semester2}$$

holds true.

6.3 Solution to Option Two

$$\begin{aligned} \max_{x} \quad & R_2 = 280x_1 + 210x_2 + 190x_3 + 156x_4 + 3x_5 \\ \text{s.t.} \quad & x_1 \leq 778 \\ & x_2 \leq 593 \\ & x_3 \leq 287 \\ & x_4 \leq 855 \\ & x_5 = 45900 \\ & \textstyle\sum_{i=1}^{4} x_i = 2140 \\ & x \geq 0 \\ & x \in \mathbb{Z}^5 \end{aligned}$$

Minimizing in MATLAB we define $Q = -z$. Letting $x = [x_1\ x_2\ x_3\ x_4\ x_5]$
The program becomes:

$$\begin{array}{ll} \max\limits_{x} & Q = -[280\ 210\ 190\ 156\ 3]^T \\ \text{s.t.} & [1\ 0\ 0\ 0\ 0] \leq 778 \\ & [0\ 1\ 0\ 0\ 0] \leq 593 \\ & [0\ 0\ 1\ 0\ 0] \leq 287 \\ & [0\ 0\ 0\ 1\ 0] \leq 855 \\ & [0\ 0\ 0\ 0\ 1] = 45900 \\ & [1\ 1\ 1\ 1\ 0] = 2140 \\ & x \geq 0 \\ & x \in \mathbb{Z}^5 \end{array}$$

We can modify this Integer program to fit the same form,

$$\boxed{\begin{array}{ll} \max\limits_{x} & z = c^{\mathsf{T}}x \\ \text{s.t.} & Ax \leq b \\ & A_e x = b_e \\ & x \geq 0 \\ & x \in \mathbb{R}^n \end{array}}$$

Defining

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

$$b = \begin{bmatrix} 778 \\ 593 \\ 287 \\ 855 \end{bmatrix},$$

$$A_e = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix},$$

$$b_e = \begin{bmatrix} 45900 \\ 2140 \end{bmatrix}.$$

Entering the Integer Program in Matrix form, we find the solution.

$$x^* = \begin{bmatrix} 778 & 593 & 287 & 482 & 45900 \end{bmatrix}^T$$

The maximum revenue found is given by the objective value ($):

$$z^* = 609792$$

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Optimal solution found.

Intlinprog stopped at the root node because the objective value is within a gap tolerance of the optimal value,
options.AbsoluteGapTolerance = 0 (the default value). The intcon variables are integer within tolerance,
options.IntegerTolerance = 1e-05 (the default value).


dv =

         778
         593
         287
         482
       45900


ov =

     -609792
```

Figure 3: Option Two Solution

6.4 Solution Two Analysis

The solution for Option Two is rather straightforward. Since the annual lots fill up to satisfy student demand after the Blue lot spots are taken, the optimum revenue will result in the more expensive lots being filled before the lower lots. This is exactly what x^* depicts. The first three lots are filled to capacity and the remaining students settle with the grey lot. Although this is guaranteed to happen, understanding this answer incentivizes parking services to model class schedules so that students are incentivized to purchase lot permits reflecting x^*.

6.5 Comparison of Solutions

The main goal that WSU-V Parking Services has is to figure out the best combination of selling permits so that they can organize a schedule incentive to achieve a maximum revenue. By modeling the two options they proposed as possible selling models, we found that for Option One, we achieve a maximum revenue of $491,110. In Option Two we found that the maximum value the revenue can take is $609,792.

To maximize revenue based on the student demand and purchasing behavior projections that the Parking Services anticipates, WSU should model their class schedule to incentivize the parking permit sales as shown by x^* in Option One's solution. That is, the parking permit price layout that produces the largest revenue is:

Permit Prices ($)

Lots	Orange	Green	Red	Gray	Blue
Fall/Spring	230	192	152	112	-
Semester	120	101	81	70	-
Daily	-	-	-	-	5

This price layout produces the projected sales:

Permits sold (#)

Lots	Orange	Green	Red	Gray	Blue
Fall/Spring	389	296	143	427	-
Semester	778	592	286	854	-
Daily	-	-	-	-	7830

7 Reflection

This project served as an instructive application of pure mathematics to the practical world. In many ways, interior point methods, ellipse algorithms, and cycling from vertex to vertex of high dimensional polyhedra seems highly abstract- and they are. Despite seeming distant and purely conceptual, the solution finding methods of linear optimization, as well as the construction of linear models, are prevalent in their manifestation of natural and artificial phenomena. In other words, there exist many cases in the real world that can be modeled by Linear Programs and their solutions can be found by varying, above-mentioned methods. The process of working "hands on" with a real situation, sorting and modeling it into a mathematical model that I know enough about to solve, was the most valuable aspect that this project brought to me. In this case, the model happened to be an Integer program rather than the relaxed Linear Program. Many of the solution finding methods taught in the class pertain to solving Linear Programs which have continuous, convex feasible regions. Since Integer programs are discrete and not convex, I had to solve the model I made using an algorithm in MATLAB. In the modern world of computing, being able to use computer programs designed to solve problems is an essential skill required for all applied mathematicians. There may be cases where finding solutions by hand would take far too long, or is not a realistic approach given the steps necessary to do so. Instead, being able to understand the problem and model it, such that there are no errors due to misunderstanding, is imperative. This project happened to be an example where I had to understand the model to be able to define parameters in MATLAB to find the solution. And for the sake of saving time, learning to get rid of redundant constraints and unnecessary mathematical statements was helpful so I didn't have to complicate the program.

There were many factors in the process of discussing decision making processes in the businesses I reached out to that led to my final optimization model. Finding a model that seemed to be a feasible option made it difficult to begin the developing a program. There weren't many options for creating a model with Sportsman's Warehouse. After speaking to the receiving manager, it was apparent that many of the decision making was made higher up in the chain, meaning that corporate managed the ordering of products. Alternatively, modeling how departments scheduled workers seemed uninteresting and rather detached from any real quantitative constraints. The idea of modeling glass optimization for Michaels frame shop seemed very interesting to me on the other hand. The difficulty in doing so seemed to be very different from the models we had explored in lectures and the textbook. This made the modeling of the problem slightly difficult, as the only insight I had into modeling was looking at making constraints that came from partitions of a plane. I wasn't sure how to model this correctly. Modeling the WSU Vancouver Parking Services problem with pricing permits seemed the most achievable to me. All the constraints seemed linear such as the parking lot limit. The difficulty in modeling this problem was that there were so many factors that were in place that the job of incorporating them all would be too difficult a task to take on.

Many of the complications of producing a realistic model was due to the complex fluctuation of students dropping out, enrolling, and getting parking citations. Furthermore, Parking

Services offers summer permits, night class permits, metered parking, and daily orange lot parking. All of these to take into consideration of the model would be too difficult with the limited data I had received. The major benefit of choosing this was that it was doable and instructive. That is why I chose this route. The major issue I had was that the model I ended with isn't entirely realistic. There are far more constraints that need careful attention and inclusion. Also, the objective function only focuses on revenue. There may exist possibilities where lesser revenue cases have higher profit since they may have less expenses. This would be an optimal objective function for the parking services. Or alternatively, eating a smaller profit may be more beneficial to the institution as they may make a larger profit elsewhere that would supersede the need to achieve the largest parking service profit. This could include discounted or waived parking for certain partnerships that bring more money to the school in other ways. These possibilities were never discussed and I found that focusing on revenue would be the cleaner option. It was also hard to obtain information after my initial contact with the head of the Parking Services at WSU-V.

The initial meeting I had with was extremely helpful for obtaining the data necessary for the model above. Upon that, I never received emails with additional data that was offered to me and I received no responses to my follow up. This of course was understandable due to their high demand on a small staff, although perhaps I wouldn't be able to create a more detailed model unless I had been more persistent and visited more often. In retrospect, if I could do anything different, I would have included more constraints such as metered parking, night class parking, and orange lot daily parking. Unfortunately I didn't have any data as to how much WSU makes for each of these options.

The style I used in the document above was more instructive and drawn out than rigorous. I believe the notation I used and the clarifications I used made for proper mathematical grammar. On the other hand, the formatting of the Integer Programs did not satisfy my desire of alignment. They were still clear enough to understand and followed general conventions of Linear Programming. This includes keeping them in standard form as the problem naturally is formulated that way. The results I attained were not surprising and somewhat predictable. Because the problem was very simple, due to its unrealistic simplification, it was pretty straightforward to see what the solution would look like. This was the case more so for the solution to the second option since there were very few constraints. In fact, this problem could most likely be solved without using any of the methods discussed in class.

I understood the type of constraints that would result from the problem statement very well such that the process of elaborating on it was more of a process than deriving them. I did make some initial mistakes such as modeling the lot constraints where you couldn't have an odd number of semester permits. I also tried tying both option one and option two into one Binary Integer Program. This, unfortunately, was not linear. Once I had sketched out all the components to model option one and two, completing the assigned tasks was only difficult because of the time it took me write out my answer in an organized and explanatory fashion, and then typeset it in LaTeX.