EE5311 Tutorial 3 Report - EE22B070

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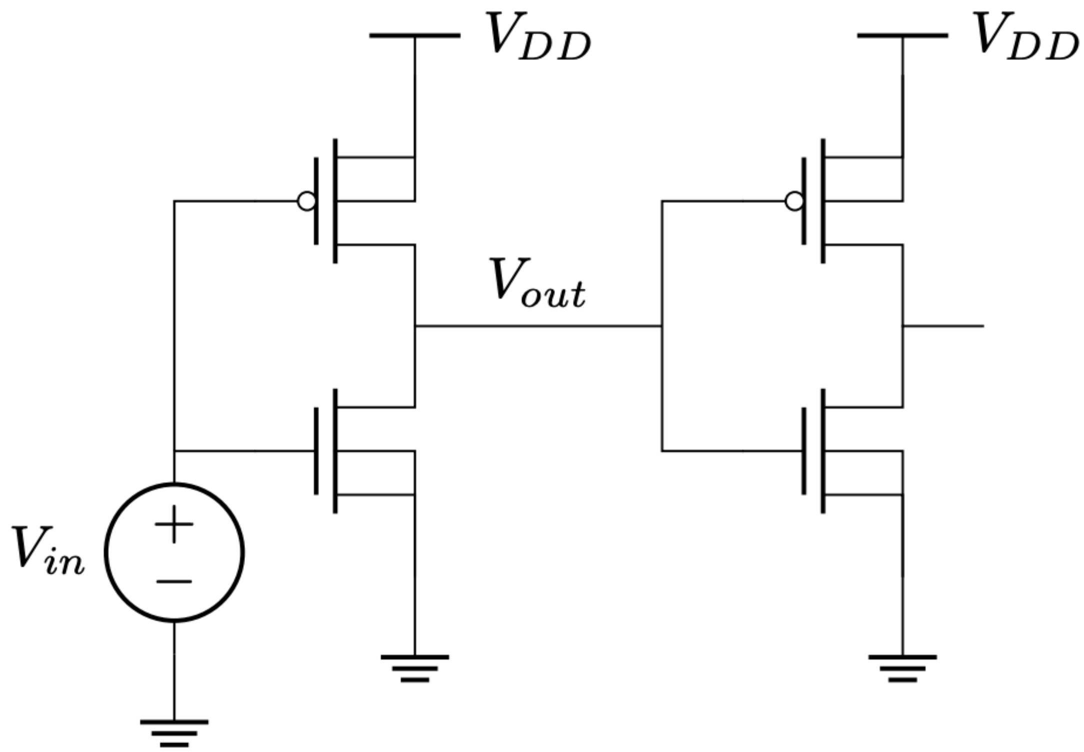
Problem Schematic:

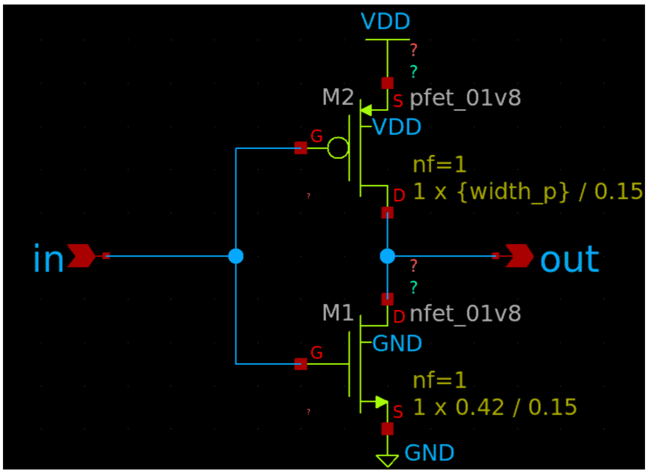


Pulse Specifications: 0 to V_{DD} with rise and fall time of 5 ps and pulse width of 250 ps.

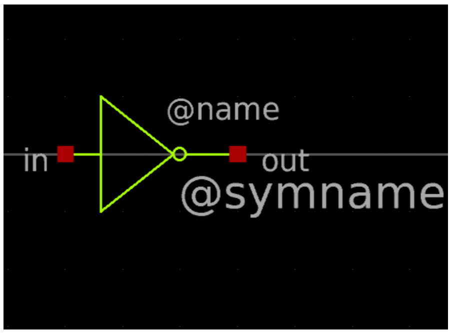
1.(a)

Set V_{DD} = 1.8V . Assume that L_n = L_p = 0.15 µm and W_n = 0.42 µm. Obtain the delay for W_p = 0.42 µm, 0.84 µm, 1.26 µm.

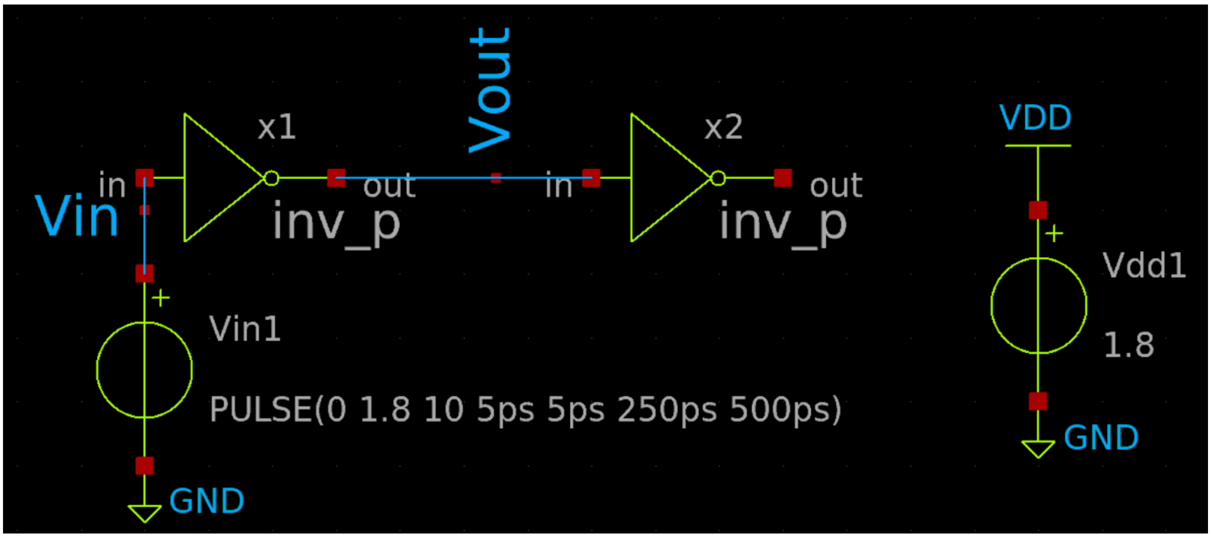
Schematic:



Schematic: inverter with parameterized pMOS width



Symbol for inverter given on the left



Schematic for 1(a)

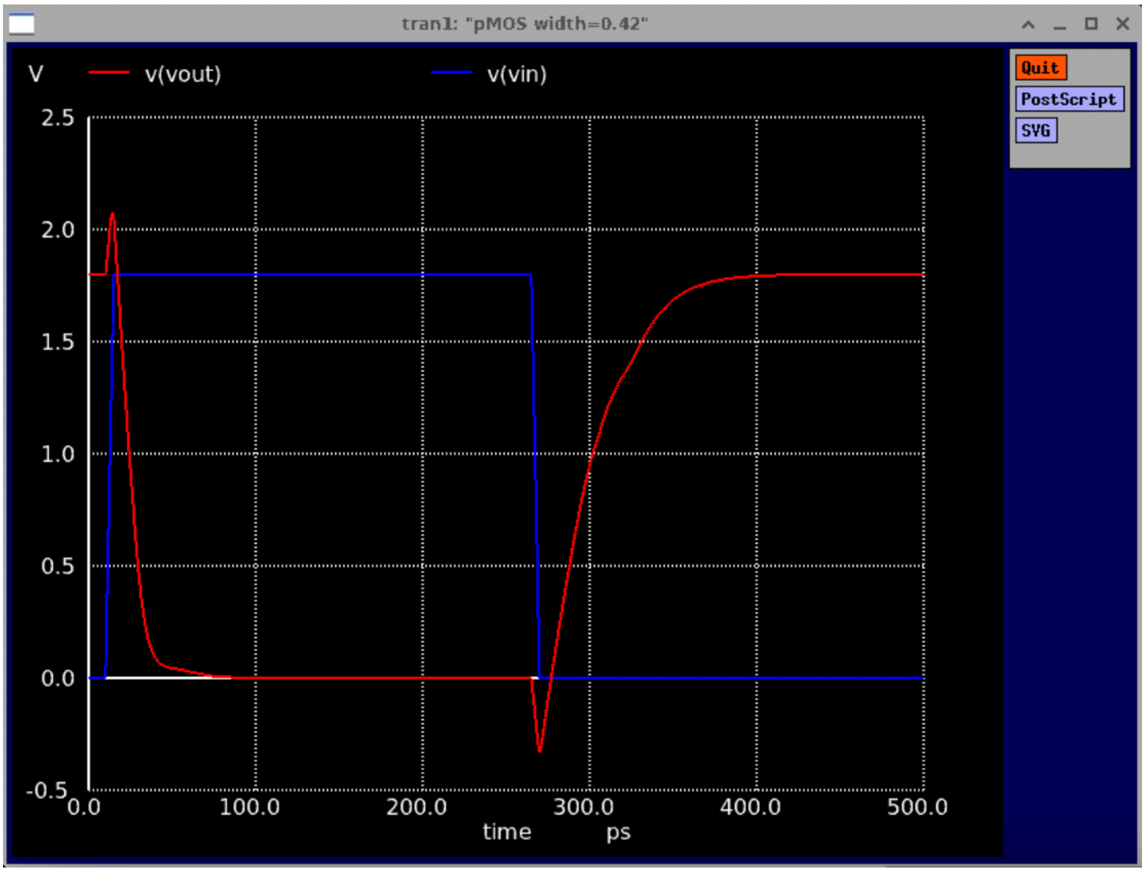
.

Sim-Code:

```
sim
.param width_p = 0.42
.control
foreach wp 0.42 0.84 1.26
  alterparam width_p = $wp
  reset
  tran 0.1p 500p
  plot v(Vout) v(Vin) title "pMOS width = $wp"
  meas tran thl trig v(Vin) val = 0.9 rise = 1 targ v(Vout) val = 0.9 fall = 1
  meas tran tlh trig v(Vin) val = 0.9 fall = 1 targ v(Vout) val = 0.9 rise = 1
  let delay = ($&thl + $&tlh)/2
  echo w : $wp delay : $&delay
end
.endc
```

Measurements:

- For W_p = 0.42 µm



V_{in} (pulse) vs V_{out}

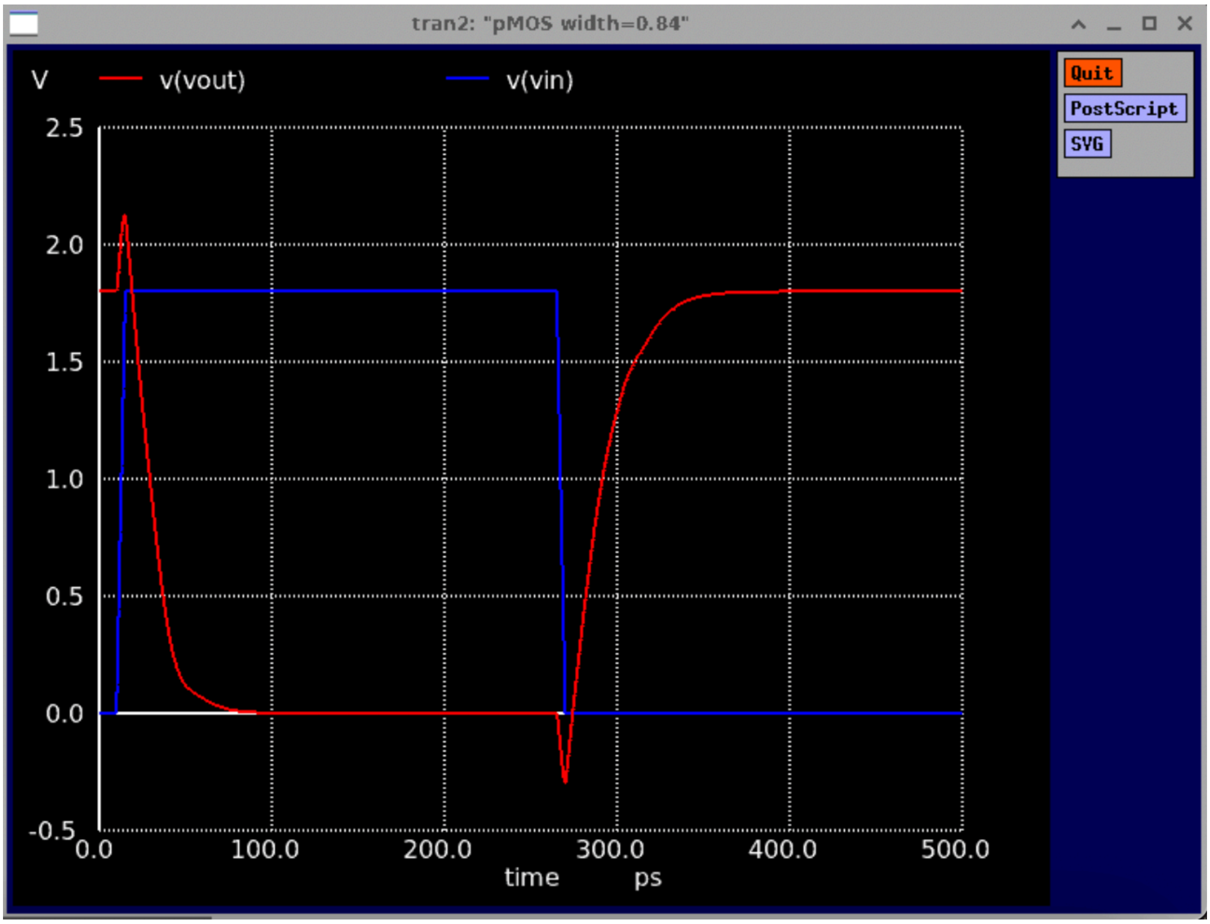
- There are noticeable spikes at the rising and falling edges due to the charging and discharging of the junction and sw cap between the drain and gate.

```
Initial Transient Solution
--------------------------

Node                                   Voltage
----                                   -------
vin                                          0
vdd                                        1.8
vout                                       1.8
net1                               4.53092e-08
vdd1#branch                       -1.81498e-11
vin1#branch                                  0

 Reference value :  3.55950e-10
No. of Data Rows : 5020
thl                 =  1.277011e-11 targ=  2.527011e-11 trig=  1.250000e-11
tlh                 =  3.088892e-11 targ=  2.983889e-10 trig=  2.675000e-10
w : 0.42 delay : 2.18295E-11
```

- Final Ans: **The delay for W_p = 0.42 µm is *21.82 ps.***

  T_{rise} = 30.88 ps, T_{fall} = 12.77 ps

- For W_p = 0.84 µm:



V_{in} (pulse) vs V_{out}

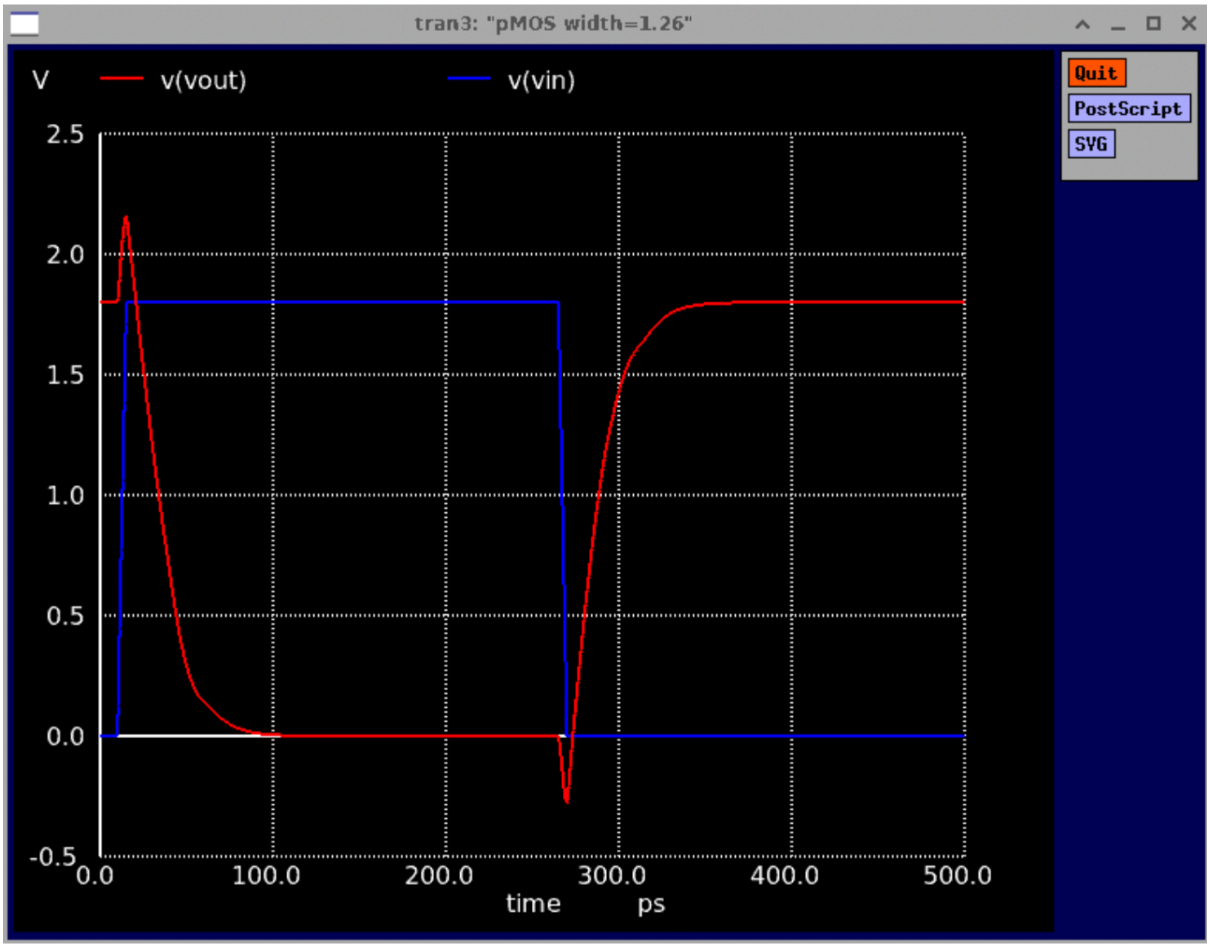
- Spikes are similar to the 0.42 µm case.

```
Initial Transient Solution
--------------------------

Node                                   Voltage
----                                   -------
vin                                          0
vdd                                        1.8
vout                                       1.8
net1                              6.71229e-07
vdd1#branch                      -2.43518e-10
vin1#branch                                  0

 Reference value :  1.49250e-10
No. of Data Rows : 5020
thl                 =  1.822964e-11 targ=  3.072964e-11 trig=  1.250000e-11
tlh                 =  2.168397e-11 targ=  2.891840e-10 trig=  2.675000e-10
w : 0.84 delay : 1.99568E-11
```

- Final Ans: **The delay for W_p = 0.84 µm is *19.95 ps.***

  T_{rise} = 21.68 ps, T_{fall} = 18.23 ps

- For W_p = 1.26 µm:



V_{in} (pulse) vs V_{out}

- Spikes are similar to the 0.42 µm and 0.84 µm cases.

```
Initial Transient Solution
--------------------------

Node                                   Voltage
----                                   -------
vin                                          0
vdd                                        1.8
vout                                       1.8
net1                               9.60947e-07
vdd1#branch                       -3.47833e-10
vin1#branch                                  0

 Reference value :  3.57050e-10
No. of Data Rows : 5020
thl                  =  2.327333e-11 targ=  3.577333e-11 trig=  1.250000e-11
tlh                  =  1.898207e-11 targ=  2.864821e-10 trig=  2.675000e-10
w : 1.26 delay : 2.11277E-11
```

- Final Ans: **The delay for W_p = 1.26 µm is *21.12 ps***

  T_{rise} = 18.98 ps, T_{fall} = 23.27 ps

Inference from Plots:

As we increase the W_p, the rise time decreases. This is explained by the formula:

$$t_{PLH} \propto C_{Total}/W_p$$

And C_{Total} is also dependent on W_p.

And in the case of fall time, there is no W_p in denominator, hence, the rise time increases.

1.(b) & (c)

b) *Set W_p so that the delay is minimized. Plot the delay as a function of V_{DD} for V_{DD} = 1V to 1.8V in steps of 0.1V. How does it compare with analytical estimates?*

c) *Plot the measured and estimated energy-delay product as a function of V_{DD}. What is the optimum V_{DD}?*

Calculations:

1.(b) from (a), we obtained the optimal W_p value to be 0.84 μm. Now we consider the variation of delay w.r.t time.

$$t_{PLH} = \frac{C_{total}(V_{DD}/2) \cdot (E_{cp}L + (V_{DD} - |V_{TP}|))}{k_p (\frac{W}{L})_p E_{cp}L (V_{DD} - |V_{TP}|)^2}$$

for $\boxed{W_p = 0.84}$

$$t_{PHL} = \frac{C_{total}(V_{DD}/2) \cdot (E_{cn}L + (V_{DD} - V_{TN}))}{k_n (\frac{W}{L})_n E_{cn}L (V_{DD} - V_{TN})^2}$$

$C_{total} = 3.2 fF$

$$t_{PLH} = \frac{C_{total}(V_{DD}/2) \cdot (E_{cp}L + (V_{DD} - |V_{TP}|))}{k_p (\frac{W}{L})_p E_{cp}L (V_{DD} - |V_{TP}|)^2}$$

$$\Rightarrow \quad t_{pLH} = C \frac{(E_{cp}L + V_{DD} - |V_{Tp}|)\ V_{DD}}{(V_{DD} - |V_{Tp}|)^2}$$

$$\Rightarrow \quad E_{cp}L = \frac{2 v_{SAT,p}}{\mu_p} L_p = \frac{2 \times 3\times10^4}{0.009} \times 0.15\,\mu$$

$$= 100 \times 10^4 \times 10^{-6} = 1\ V$$

$$\Rightarrow \quad t_{pLH} = C_1 \left(\frac{1 + 0.3}{0.3^2} \right)$$

$$C_1 = \frac{3.2 \times 10^{-\cancel{15}}\ 10^{-9}}{2\left(36.72 \times \left(\frac{0.84}{0.15}\right) \times 1\right)\cancel{\mu}}$$

$$= 0.00778 \times 10^{-9}$$

$$= 7.78 \times 10^{-12}$$

$$\frac{2\times 8\times10^4}{0.028} \times 0.15$$

$$\underline{\underline{0.96}}$$

$$\therefore \quad t_{pHL} = \frac{C_{total}(V_{DD}/2) \cdot (E_{cn}L + (V_{DD} - V_{Tn}))}{k'_n \left(\frac{W}{L}\right)_n E_{cn}L\ (V_{DD} - V_{Tn})^2}$$

for $V_{DD} = 1.8\ V$:

$$\Rightarrow \quad t_{pLH} = C_1 \frac{(E_{cp}L + V_{DD} - |V_{Tp}|)\ V_{DD}}{(V_{DD} - |V_{Tp}|)^2}$$

1/4

$$= \frac{7.78\times10^{-12}\ (1 + 1.8 - 0.7)\ (1.8)}{(1.8 - 0.7)^2}$$

$$= 24.3\ ps$$

$$t_{pHL} = \frac{5.71\times10^{-12} \times (0.96 + 1.8 - 0.7)\ (1.8)}{(1.8-0.7)^2}$$

$$= 17.49\ ps$$

$$\approx \boxed{20.1\ ps}$$

for $V_{DD} = 1.7$ V,

$$\Rightarrow \quad t_{PLH} = \frac{7.78 \times 10^{-12}(1 + 1.7 - 0.7)}{(1.7-0.7)^2}(1.7)$$

$$= 26.452 \text{ ps}$$

$$t_{PHL} = \frac{5.71 \times 10^{-12}(0.96 + 1.7 - 0.7)}{(1.7-0.7)^2}(1.7)$$

$$= 19.02 \text{ ps}$$

$$t_{delay} = 22.73 \text{ ps}$$

similarly we can do for other values of V_{DD} as well.

1.(c)
similarly for EDP:

@ $V_{DD} = 1.8$ V

$$EDP = \frac{1}{2} C_{total} V_{DD}^2 t_p$$

$$= \frac{1}{2} \times 3.2 \times 10^{-15} \times 1.8^2 \times 20.1 \times 10^{-12}$$

$$= 104.198 \times 10^{-27} = 1.04 \times 10^{-24}$$

for $V_{DD} = 1.7$ V:

$$EDP = \frac{1}{2} \times 3.2 \times 10^{-15} \times 1.7^2 \times 22.73 \times 10^{-12}$$

$$= 105.103 \times 10^{-27}$$

similarly, we can calculate EDP for other values of V_{DD}.

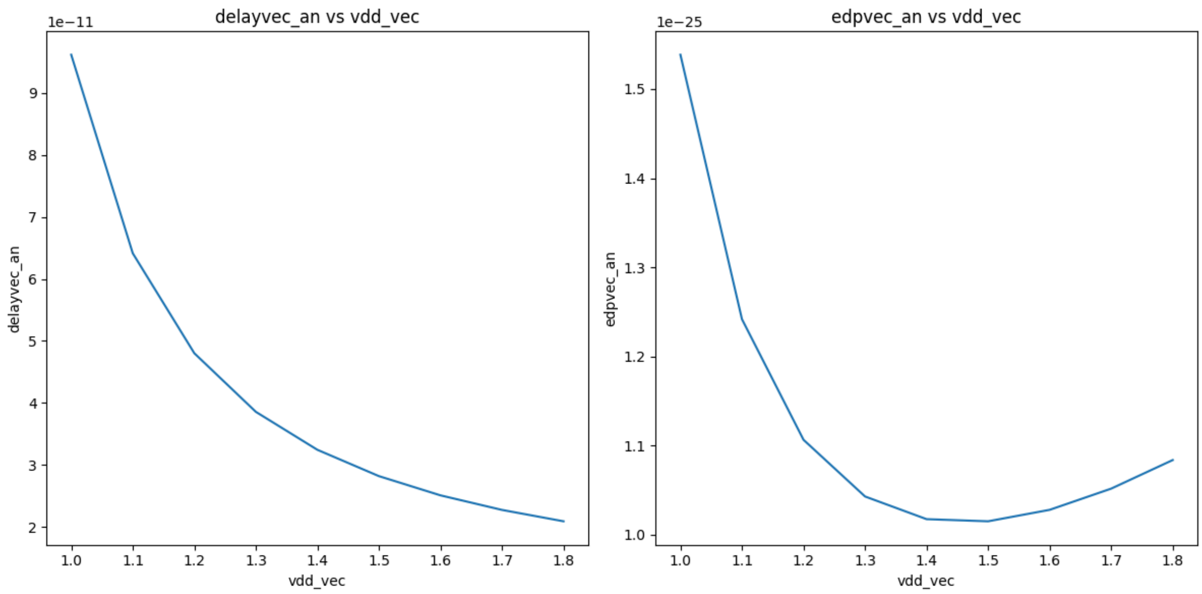
★ I have made a python script to plot the analytical values & compare it with the simulation values.

★ There is a huge difference in the two, maybe the simulation take the non idealities into account.

★ But the optimum value for V_{DD} (EDP optimised) is 1.4 V

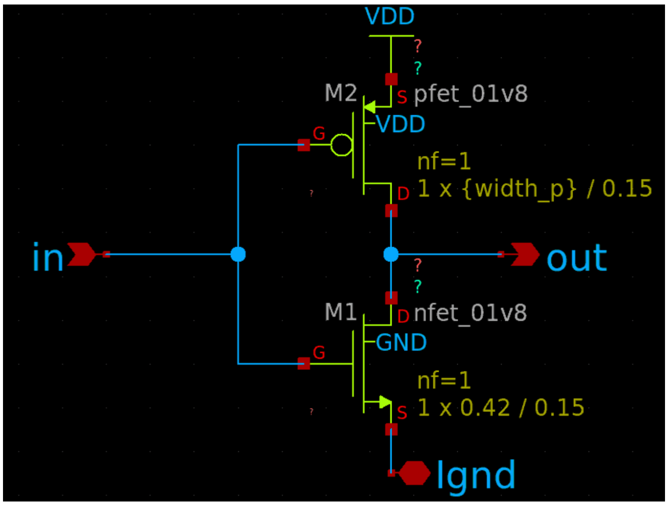
```python
import matplotlib.pyplot as plt

vdd_vec = [1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8]
delayvec_an = []
edpvec_an = []

for vddv in vdd_vec:
  delay_an = 0.5 * (((7.78e-12 * (0.3 + vddv) * vddv) / ((vddv - 0.7)**2)) + ((5.71e-12 * (0.26 + vddv) * vddv) / ((vddv - 0.7)**2)))
  edp_an = 0.5 * 3.2e-15 * vddv**2 * delay_an
  delayvec_an.append(delay_an)
  edpvec_an.append(edp_an)

plt.figure(figsize=(12, 6))  # Adjust figure size as needed

# Plot delayvec_an vs vdd_vec
plt.subplot(1, 2, 1)  # Create a subplot (1 row, 2 columns, plot 1)
plt.plot(vdd_vec, delayvec_an)
plt.xlabel('vdd_vec')
plt.ylabel('delayvec_an')
plt.title('delayvec_an vs vdd_vec')

# Plot edpvec_an vs vdd_vec
plt.subplot(1, 2, 2)  # Create a subplot (1 row, 2 columns, plot 2)
plt.plot(vdd_vec, edpvec_an)
plt.xlabel('vdd_vec')
plt.ylabel('edpvec_an')
plt.title('edpvec_an vs vdd_vec')

plt.tight_layout()  # Adjust spacing between subplots
plt.show()  # Display the plots
```

Code for Analytical plotting of delay and edp vs V_{DD}

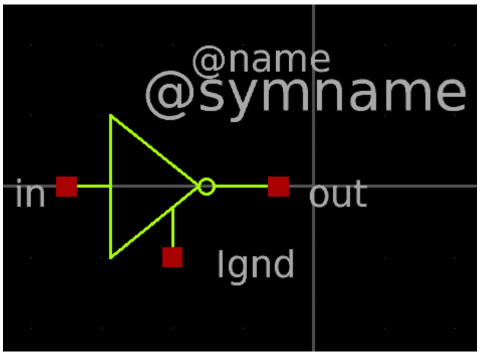


Analytical Plots for Delay and EDP vs V_{DD}

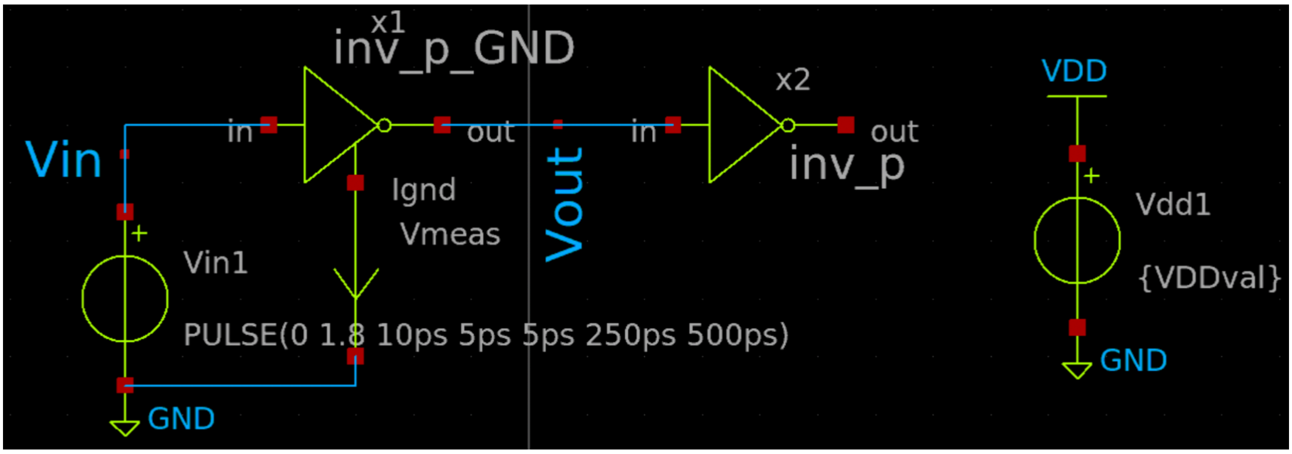
Schematic:



Schematic: inverter with parameterized pMOS width with GND input pin



Symbol for inverter given on the left

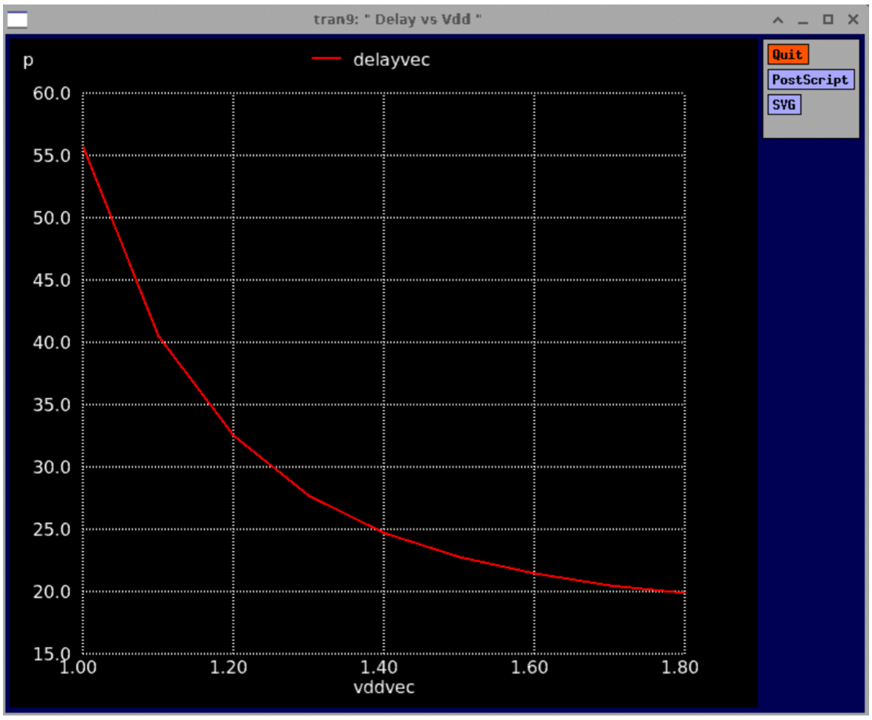


Schematic for 1(b) and 1(c)

Inference:

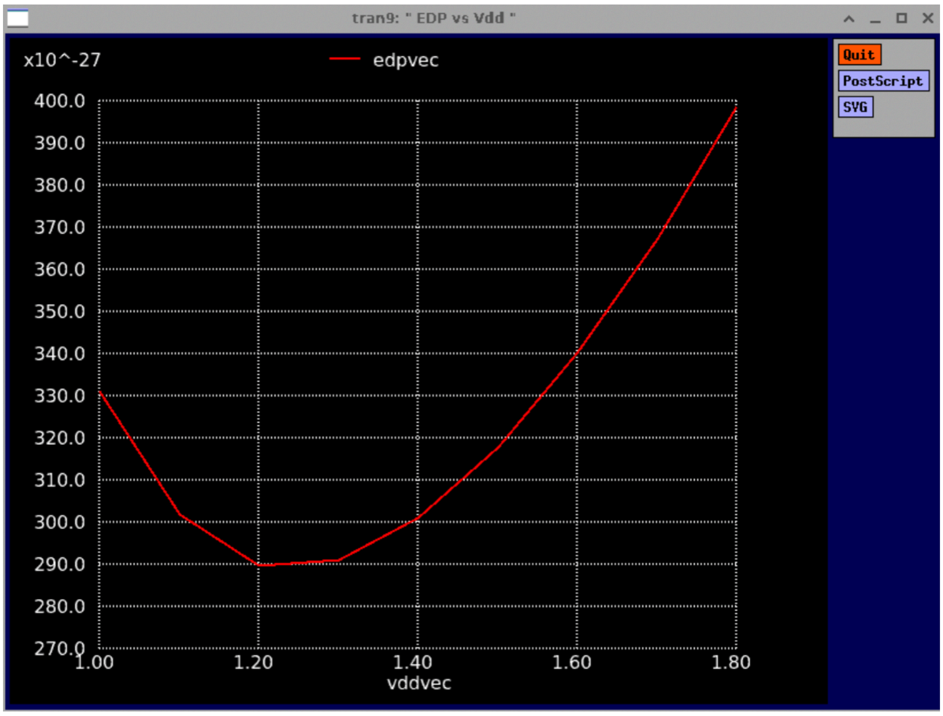
From the above observations, we see that the minimum delay is **W_p = 0.84 µm** (delay = 19.95 ps).

Measurements:



Delay vs V_{DD}

- The optimal delay for the given range is at V_{DD} = 1.8 V.



Energy Delay Product (EDP) vs V_{DD}

- The EDP presents a convex nature and the value for V_{DD} for optimal EDP is V_{DD} = between 1.2 and 1.3 V., which is very close to the analytical results given above.

Inference on Delay: 1(b)

so, from 1.0 V → 1.4 V, there is a steep decline in delay as in this region:

$$E_{cn}L \gg V_{DD} - V_{Tn}$$

$$\therefore \quad t_{pHL} \propto \frac{V_{DD}}{(V_{DD} - V_{Tn})^2}$$

$$\& \quad t_{pLH} \propto \frac{V_{DD}}{(V_{DD} - |V_{Tp}|)^2}$$

in 1.4 V → 1.8 V region, the decline is less steep due to:

$$E_{cn}L \ll V_{DD} - V_{Tn}$$

$$\therefore \quad t_{pHL} \propto \frac{V_{DD}}{V_{DD} - V_{Tn}}, \quad t_{pLH} \propto \frac{V_{DD}}{V_{DD} - |V_{Tp}|}.$$

Sim-Code:

```
.param VDDval = 1.8
.param width_p = 0.84
.control
let Nsim = 9
let delayvec = vector(Nsim)
let vddvec = vector(Nsim)
let edpvec = vector(Nsim)
let index = 0
while index < Nsim
  let vddv = 1.0 + (index * 0.1)
  let vby2 = vddv/2
  alterparam VDDval = $&vddv
  reset
  tran 1p 600p
  meas tran thl trig v(Vin) val = $&vby2 rise = 1 targ v(Vout) val = $&vby2 fall = 1
  meas tran tlh trig v(Vin) val = $&vby2 fall = 1 targ v(Vout) val = $&vby2 rise = 1
  meas tran iinteg integ i(vmeas)
  let delayvec[index] = ($&thl + $&tlh)/2
  let vddvec[index] = vddv
  let edpvec[index] = $&iinteg * vddv * delayvec[index]
  let index = index + 1
end
plot delayvec vs vddvec title " Delay vs Vdd "
plot edpvec vs vddvec title " EDP vs Vdd "
print delayvec
.endc
```