



МИНИСТЕРСТВО НАУКИ
И ВЫСШЕГО ОБРАЗОВАНИЯ
РОССИЙСКОЙ ФЕДЕРАЦИИ

Федеральное государственное бюджетное
образовательное учреждение высшего образования
«НОВОСИБИРСКИЙ ГОСУДАРСТВЕННЫЙ ТЕХНИЧЕСКИЙ УНИВЕРСИТЕТ»

НГТУ



НЭТИ

Кафедра прикладной математики

Лабораторная работа № 3

по дисциплине «Численное моделирование динамических систем, описываемых
обыкновенными дифференциальными уравнениями»

МЕТОДЫ РУНГЕ-КУТТЫ



Группа

ПМ-91

Бригада

ЗАТОЛОЦКАЯ ЮЛИЯ

КОНСТАНТИНОВА АНАСТАСИЯ

Преподаватель

ВАГИН ДЕНИС ВЛАДИМИРОВИЧ

Дата

20.02.2022

Новосибирск

1. Задание:

I часть

На трёх сетках $h=[0.1, 0.05, 0.025]$ решить задачу

$$y' = 4ty$$

$$t \in [0, 1]$$

$$y(0) = 1$$

с помощью классического явного четырёхэтапного метода Рунге-Кутты

$$y_{n+1} = y_n + hk_n, \quad k_n = \frac{1}{6} \left(k_n^{(1)} + 2k_n^{(2)} + 2k_n^{(3)} + k_n^{(4)} \right),$$

$$k_n^{(1)} = f(t_n, y_n), \quad k_n^{(2)} = f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_n^{(1)}\right),$$

$$k_n^{(3)} = f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_n^{(2)}\right), \quad k_n^{(4)} = f\left(t_n + h, y_n + hk_n^{(3)}\right).$$

II часть

Решить задачу

$$y' = -25y + \cos(t) + 25\sin(t)$$

$$t \in [0, 2]$$

$$y(0) = 1$$

Простые методы Эйлера (явный и неявный) $h=[0.1, 0.05, 0.025]$

Модифицированный Эйлер и Трапеция $h=[0.2, 0.1, 0.05]$

Р-К 4-го порядка (из учебного пособия) $h=[0.5, 0.25, 0.125, 0.1]$

2. Результаты:

2.1 Первая часть (метод Рунге-Кутты)

$$h = 0.1$$

tn	yn	y(tn)	yn-y(tn)
0	1	1	0
0.1	1.02020133	1.02020134	6.69342248e-09
0.2	1.08328699	1.08328707	7.49969853e-08
0.3	1.19721701	1.19721736	3.55232566e-07
0.4	1.37712642	1.37712776	1.34905728e-06
0.5	1.64871668	1.64872127	4.59400698e-06
0.6	2.05441881	2.05443321	1.44031991e-05
0.7	2.66441399	2.66445624	4.22509722e-05
0.8	3.59652192	3.59663973	0.000117801672
0.9	5.0527736	5.05309032	0.000316716972
1	7.38822484	7.3890561	0.00083125573

$$h = 0.05$$

tn	yn	y(tn)	yn-y(tn)
0	1	1	0
0.05	1.00501252	1.00501252	2.60675925e-11
0.1	1.02020134	1.02020134	2.68387312e-10
0.15	1.04602786	1.04602786	1.02274655e-09
0.2	1.08328706	1.08328707	2.85446355e-09

0.25	1.13314845	1.13314845	6.94928914e-09
0.3	1.19721735	1.19721736	1.57091791e-08
0.35	1.27762128	1.27762131	3.37357768e-08
0.4	1.37712769	1.37712776	6.94340154e-08
0.45	1.49930236	1.4993025	1.37608443e-07
0.5	1.64872101	1.64872127	2.63646732e-07
0.55	1.83125172	1.83125221	4.90237752e-07
0.6	2.05443232	2.05443321	8.88134995e-07
0.65	2.32797624	2.32797781	1.5733782e-06
0.7	2.66445351	2.66445624	2.73483456e-06
0.75	3.08021217	3.08021685	4.67825412e-06
0.8	3.59663183	3.59663973	7.89680547e-06
0.85	4.24183896	4.24185214	1.31841742e-05
0.9	5.0530685	5.05309032	2.18162665e-05
0.95	6.0799356	6.07997145	3.58438405e-05
1	7.38899753	7.3890561	5.85651118e-05

$h = 0,025$

tn	yn	y(tn)	yn-y(tn)
0	1	1	0
0.025	1.00125078	1.00125078	1.01696429e-13
0.05	1.00501252	1.00501252	1.02473585e-12
0.075	1.01131352	1.01131352	3.66706665e-12
0.1	1.02020134	1.02020134	9.17310672e-12
0.125	1.03174341	1.03174341	1.92181826e-11
0.15	1.04602786	1.04602786	3.641909e-11
0.175	1.06316467	1.06316467	6.49025278e-11
0.2	1.08328707	1.08328707	1.11060938e-10
0.225	1.10655325	1.10655325	1.84548599e-10
0.25	1.13314845	1.13314845	2.99576808e-10
0.275	1.16328744	1.16328744	4.76589213e-10
0.3	1.19721736	1.19721736	7.44417861e-10
0.325	1.23522112	1.23522112	1.14305299e-09
0.35	1.27762131	1.27762131	1.72718484e-09
0.375	1.32478476	1.32478476	2.57073274e-09
0.4	1.37712776	1.37712776	3.77262155e-09
0.425	1.43512219	1.4351222	5.46413803e-09
0.45	1.49930249	1.4993025	7.81829446e-09
0.475	1.57027379	1.5702738	1.10617271e-08
0.5	1.64872126	1.64872127	1.54898103e-08
0.525	1.73542092	1.73542094	2.14858402e-08
0.55	1.83125218	1.83125221	2.95453679e-08
0.575	1.9372123	1.93721234	4.03070568e-08
0.6	2.05443316	2.05443321	5.45918017e-08
0.625	2.18420074	2.18420081	7.34523087e-08
0.65	2.32797772	2.32797781	9.82359363e-08
0.675	2.48742975	2.48742988	1.30664338e-07
0.7	2.66445607	2.66445624	1.72934421e-07

0.725	2.86122519	2.86122542	2.27846341e-07
0.75	3.08021655	3.08021685	2.9896583e-07
0.775	3.32426927	3.32426966	3.90830126e-07
0.8	3.59663922	3.59663973	5.09209327e-07
0.825	3.90106593	3.90106659	6.61438252e-07
0.85	4.24185129	4.24185214	8.56838062e-07
0.875	4.62395205	4.62395315	1.1072522e-06
0.9	5.05308889	5.05309032	1.42772815e-06
0.925	5.53587516	5.535877	1.83738519e-06
0.95	6.07996909	6.07997145	2.36051979e-06
0.975	6.69425401	6.69425704	3.02801487e-06
1	7.38905222	7.3890561	3.87913771e-06

h	ln-1 max	ln-1(h)/ln-1(h/2)
0,100	0,00083125573	
0,05	5,85651118e-05	14,19
0,025	3,87913771e-06	15,10

Из последней таблицы видно, что при уменьшении шага погрешность уменьшается примерно в 2^4 раз, следовательно, первая схема имеет 4 порядок аппроксимации.

2.2 Вторая часть (явный метод Эйлера)

$$h = 0,1$$

tn	yn	y(tn)	yn-y(tn)
0	1,00	1,00	0,00
0.1	-1.4	0.181918415	1.58191842
0.2	2.44908396	0.205407278	-2.24367668
0.3	-3.07894595	0.296073291	3.37501924
0.4	5.45275309	0.389463742	-5.06328935
0.5	-7.11347769	0.479429265	7.59290695
0.6	11.9565386	0.564642779	-11.3918959
0.7	-16.4406682	0.644217712	17.0848859
0.8	26.3480307	0.717356093	-25.6306746
0.9	-37.6589852	0.78332691	38.4423121
1	58.5089561	0.841470985	-57.6674851
1.1	-85.6057264	0.89120736	86.4969338
1.2	130.681968	0.932039086	-129.749929
1.3	-193.656618	0.963558185	194.620176
1.4	292.920572	0.98544973	-291.935123
1.5	-436.900238	0.997494987	437.897733
1.6	657.851168	0.999573603	-656.851594
1.7	-984.280737	0.99166481	985.272402
1.8	1478.88738	0.973847631	-1477.91354
1.9	-2215.91918	0.946300088	2216.86548
2	3326.21219	0.909297427	-3325.30289

$h = 0,05$

tn	yn	y(tn)	yn-y(tn)
0	1	1	0
0.05	-0.2	0.336483966	0.536483966
0.1	0.162411475	0.181918415	0.0195069407
0.15	0.13393911	0.172955878	0.0390167679
0.2	0.202751442	0.205407278	0.00265583591
0.25	0.246652132	0.249334413	0.00268228148
0.3	0.296037537	0.296073291	3.57538565e-05
0.35	0.343157698	0.343056269	-0.000101429709
0.4	0.38980147	0.389463742	-0.000337728101
0.45	0.43537561	0.434978541	-0.000397068592
0.5	0.47988537	0.479429265	-0.000456104999
0.55	0.523189709	0.522688297	-0.000501412151
0.6	0.565187835	0.564642779	-0.000545055773
0.65	0.605772914	0.605186493	-0.000586420343
0.7	0.644843969	0.644217712	-0.000626256319
0.75	0.682303226	0.681638767	-0.000664459027
0.8	0.718057087	0.717356093	-0.000700993951
0.85	0.752016177	0.751280406	-0.000735771633
0.9	0.784095619	0.78332691	-0.000768709582
0.95	0.814215231	0.813415505	-0.000799725765
1	0.842299728	0.841470985	-0.000828742987
1.05	0.868278914	0.867423226	-0.000855688753
1.1	0.892087856	0.89120736	-0.000880495737
1.15	0.913667042	0.91276394	-0.000903101937
1.2	0.932962537	0.932039086	-0.000923450853
1.25	0.949926111	0.948984619	-0.000941491622
1.3	0.964515365	0.963558185	-0.000957179153
1.35	0.976693832	0.975723358	-0.000970474234
1.4	0.986431074	0.98544973	-0.000981343634
1.45	0.993702751	0.992712991	-0.000989760187
1.5	0.998490689	0.997494987	-0.000995702855
1.55	1.00078292	0.999783764	-0.000999156784
1.6	1.00057372	0.999573603	-0.00100011334
1.65	0.997863599	0.996865028	-0.000998570137
1.7	0.992659341	0.99166481	-0.000994531027
1.75	0.984973953	0.983985947	-0.000988006107
1.8	0.974826643	0.973847631	-0.000979011687
1.85	0.962242773	0.961275203	-0.000967570247
1.9	0.947253798	0.946300088	-0.000953710385
1.95	0.929897182	0.928959715	-0.000937466744
2	0.910216307	0.909297427	-0.000918879925

$h = 0,025$

tn	yn	y(tn)	yn-y(tn)
0	1	1	0
0.025	0.4	0.560258824	0.160258824
0.05	0.19061556	0.336483966	0.145868406
0.075	0.127686572	0.228284674	0.100598102
0.1	0.119643252	0.181918415	0.0622751631
0.125	0.132137209	0.168611667	0.0364744579
0.15	0.152278103	0.172955878	0.0206777749
0.175	0.175222399	0.18669628	0.0114738813
0.2	0.199144149	0.205407278	0.00626312887
0.225	0.223349052	0.226712925	0.00336387323
0.25	0.247567224	0.249334413	0.00176718987
0.275	0.271687994	0.272580235	0.000892240697
0.3	0.295660463	0.296073291	0.000412827781
0.325	0.319456215	0.319604831	0.000148615477
0.35	0.34305534	0.343056269	9.28793224e-07
0.375	0.3664412	0.366357347	-8.38526548e-05
0.4	0.389598471	0.389463742	-0.000134728979
0.425	0.412512415	0.412345083	-0.000167332925
0.45	0.435168613	0.434978541	-0.000190071317
0.475	0.457552866	0.457345409	-0.000207456667
0.5	0.479651172	0.479429265	-0.000221907076
0.525	0.501449715	0.501214999	-0.000234715893
0.55	0.52293487	0.522688297	-0.000246573065
0.575	0.544093207	0.543835362	-0.000257845086
0.6	0.564911504	0.564642779	-0.000268725167
0.625	0.58537675	0.585097437	-0.000279313741
0.65	0.605476155	0.605186493	-0.000289661604
0.675	0.625197157	0.624897364	-0.000299793028
0.7	0.64452743	0.644217712	-0.000309718136
0.725	0.663454896	0.663135456	-0.000319439533
0.75	0.681967723	0.681638767	-0.00032895586
0.775	0.700054343	0.699716079	-0.000338263693
0.8	0.717703452	0.717356093	-0.000347358566
0.825	0.734904019	0.734547783	-0.000356235519
0.85	0.751645295	0.751280406	-0.00036488939
0.875	0.767916818	0.767543503	-0.000373314978
0.9	0.783708417	0.78332691	-0.000381507129
0.925	0.799010224	0.798620763	-0.000389460782
0.95	0.813812676	0.813415505	-0.000397170998
0.975	0.828106521	0.827701888	-0.000404632975
1	0.841882827	0.841470985	-0.00041184206
1.025	0.855132983	0.854714189	-0.000418793751
1.05	0.867848709	0.867423226	-0.000425483706
1.075	0.880022058	0.87959015	-0.000431907747
1.1	0.891645422	0.89120736	-0.000438061858
1.125	0.902711536	0.902267594	-0.000443942195

1.15	0.913213485	0.91276394	-0.000449545081
1.175	0.923144706	0.922689839	-0.000454867017
1.2	0.932498991	0.932039086	-0.000459904676
1.225	0.941270494	0.940805839	-0.000464654909
1.25	0.949453734	0.948984619	-0.000469114747
1.275	0.957043596	0.956570315	-0.000473281405
1.3	0.964035338	0.963558185	-0.000477152277
1.325	0.970424588	0.969943863	-0.000480724944
1.35	0.976207355	0.975723358	-0.000483997174
1.375	0.981380024	0.980893057	-0.000486966921
1.4	0.985939362	0.98544973	-0.00048963233
1.425	0.989882521	0.989390529	-0.000491991734
1.45	0.993207035	0.992712991	-0.00049404366
1.475	0.995910827	0.99541504	-0.000495786825
1.5	0.997992207	0.997494987	-0.000497220139
1.525	0.999449874	0.998951531	-0.000498342706
1.55	1.00028292	0.999783764	-0.000499153826
1.575	1.00049082	0.999991165	-0.00049965299
1.6	1.00007344	0.999573603	-0.000499839888
1.625	0.999031055	0.998531341	-0.000499714402
1.65	0.997364305	0.996865028	-0.000499276611
1.675	0.995074235	0.994575708	-0.000498526788
1.7	0.992162276	0.99166481	-0.000497465403
1.725	0.988630248	0.988134155	-0.000496093117
1.75	0.984480358	0.983985947	-0.00049441079
1.775	0.9797152	0.97922278	-0.000492419472
1.8	0.974337751	0.973847631	-0.000490120407
1.825	0.968351374	0.967863859	-0.000487515034
1.85	0.961759808	0.961275203	-0.000484604979
1.875	0.954567174	0.954085782	-0.000481392062
1.9	0.946777966	0.946300088	-0.000477878291
1.925	0.938397053	0.937922987	-0.000474065861
1.95	0.929429672	0.928959715	-0.000469957156
1.975	0.919881428	0.919415873	-0.000465554742
2	0.909758288	0.909297427	-0.000460861373

Неявный метод Эйлера

$h = 0,1$

tn	yn	y(tn)	yn-y(tn)
0	1	1	0
0.1	0.385453	0.181918	-0.203534
0.2	0.280038	0.205407	-0.0746306
0.3	0.318392	0.296073	-0.0223187
0.4	0.395441	0.389464	-0.00597739
0.5	0.480504	0.479429	-0.00107452
0.6	0.564184	0.564643	0.000458915
0.7	0.643204	0.644218	0.0010142
0.8	0.716076	0.717356	0.00128055

0.9	0.781873	0.783327	0.00145439
1	0.83988	0.841471	0.00159092
1.1	0.889502	0.891207	0.00170505
1.2	0.930239	0.932039	0.00180029
1.3	0.961681	0.963558	0.001877
1.4	0.983515	0.98545	0.00193481
1.5	0.995522	0.997495	0.00197324
1.6	0.997582	0.999574	0.00199195
1.7	0.989674	0.991665	0.00199074
1.8	0.971878	0.973848	0.00196965
1.9	0.944371	0.9463	0.00192888
2	0.907429	0.909297	0.00186883

$h = 0,05$

tn	yn	y(tn)	yn-y(tn)
0	1	1	0
0.05	0.494405	0.336484	-0.157921
0.1	0.29731	0.181918	-0.115391
0.15	0.237132	0.172956	-0.0641757
0.2	0.237543	0.205407	-0.0321356
0.25	0.264553	0.249334	-0.0152182
0.3	0.302987	0.296073	-0.00691326
0.35	0.346034	0.343056	-0.00297815
0.4	0.390605	0.389464	-0.00114088
0.45	0.43526	0.434979	-0.000280967
0.5	0.479298	0.479429	0.000131239
0.55	0.522348	0.522688	0.000340168
0.6	0.564186	0.564643	0.000457001
0.65	0.604655	0.605186	0.000531838
0.7	0.643631	0.644218	0.000587102
0.75	0.681006	0.681639	0.000632765
0.8	0.716683	0.717356	0.000673226
0.85	0.75057	0.75128	0.000710392
0.9	0.782582	0.783327	0.000745066
0.95	0.812638	0.813416	0.000777558
1	0.840663	0.841471	0.000807966
1.05	0.866587	0.867423	0.000836291
1.1	0.890345	0.891207	0.000862497
1.15	0.911877	0.912764	0.000886536
1.2	0.931131	0.932039	0.000908353
1.25	0.948057	0.948985	0.000927897
1.3	0.962613	0.963558	0.000945121
1.35	0.974763	0.975723	0.000959982
1.4	0.984477	0.98545	0.000972443
1.45	0.991731	0.992713	0.000982474
1.5	0.996505	0.997495	0.000990048
1.55	0.998789	0.999784	0.000995149

1.6	0.998576	0.999574	0.000997762
1.65	0.995867	0.996865	0.000997881
1.7	0.990669	0.991665	0.000995506
1.75	0.982995	0.983986	0.000990642
1.8	0.972864	0.973848	0.000983303
1.85	0.960302	0.961275	0.000973505
1.9	0.945339	0.9463	0.000961275
1.95	0.928013	0.92896	0.000946642
2	0.908368	0.909297	0.000929642

$h = 0,025$

tn	yn	y(tn)	yn-y(tn)
0	1	1	0
0.025	0.640379	0.560259	-0.08012
0.05	0.428667	0.336484	-0.0921834
0.075	0.307956	0.228285	-0.0796711
0.1	0.243216	0.181918	-0.0612981
0.125	0.212888	0.168612	-0.0442764
0.15	0.203696	0.172956	-0.0307402
0.175	0.207466	0.186696	-0.0207695
0.2	0.21916	0.205407	-0.0137532
0.225	0.235675	0.226713	-0.00896203
0.25	0.255092	0.249334	-0.00575804
0.275	0.276228	0.27258	-0.00364741
0.3	0.298345	0.296073	-0.00227205
0.325	0.320987	0.319605	-0.00138262
0.35	0.343866	0.343056	-0.000810129
0.375	0.3668	0.366357	-0.000442295
0.4	0.389669	0.389464	-0.000205569
0.425	0.412397	0.412345	-5.23133e-05
0.45	0.43493	0.434979	4.806e-05
0.475	0.45723	0.457345	0.000115053
0.5	0.479268	0.479429	0.00016103
0.525	0.501021	0.501215	0.000193792
0.55	0.52247	0.522688	0.000218242
0.575	0.543598	0.543835	0.000237451
0.6	0.564389	0.564643	0.000253335
0.625	0.58483	0.585097	0.000267087
0.65	0.604907	0.605186	0.000279448
0.675	0.624606	0.624897	0.000290876
0.7	0.643916	0.644218	0.000301653
0.725	0.662824	0.663135	0.000311951
0.75	0.681317	0.681639	0.000321873
0.775	0.699385	0.699716	0.000331484
0.8	0.717015	0.717356	0.000340819
0.825	0.734198	0.734548	0.000349899
0.85	0.750922	0.75128	0.000358734
0.875	0.767176	0.767544	0.000367329

0.9	0.782951	0.783327	0.000375685
0.925	0.798237	0.798621	0.000383799
0.95	0.813024	0.813416	0.00039167
0.975	0.827303	0.827702	0.000399293
1	0.841064	0.841471	0.000406666
1.025	0.8543	0.854714	0.000413784
1.05	0.867003	0.867423	0.000420642
1.075	0.879163	0.87959	0.000427238
1.1	0.890774	0.891207	0.000433566
1.125	0.901828	0.902268	0.000439623
1.15	0.912319	0.912764	0.000445405
1.175	0.922239	0.92269	0.000450909
1.2	0.931583	0.932039	0.00045613
1.225	0.940345	0.940806	0.000461067
1.25	0.948519	0.948985	0.000465716
1.275	0.9561	0.95657	0.000470074
1.3	0.963084	0.963558	0.000474137
1.325	0.969466	0.969944	0.000477905
1.35	0.975242	0.975723	0.000481374
1.375	0.980409	0.980893	0.000484542
1.4	0.984962	0.98545	0.000487407
1.425	0.988901	0.989391	0.000489967
1.45	0.992221	0.992713	0.000492222
1.475	0.994921	0.995415	0.000494168
1.5	0.996999	0.997495	0.000495806
1.525	0.998454	0.998952	0.000497134
1.55	0.999286	0.999784	0.000498152
1.575	0.999492	0.999991	0.000498857
1.6	0.999074	0.999574	0.000499252
1.625	0.998032	0.998531	0.000499334
1.65	0.996366	0.996865	0.000499104
1.675	0.994077	0.994576	0.000498562
1.7	0.991167	0.991665	0.000497709
1.725	0.987638	0.988134	0.000496544
1.75	0.983491	0.983986	0.00049507
1.775	0.978729	0.979223	0.000493285
1.8	0.973356	0.973848	0.000491193
1.825	0.967375	0.967864	0.000488794
1.85	0.960789	0.961275	0.000486089
1.875	0.953603	0.954086	0.00048308
1.9	0.94582	0.9463	0.000479769
1.925	0.937447	0.937923	0.000476159
1.95	0.928487	0.92896	0.000472251
1.975	0.918948	0.919416	0.000468048
2	0.908834	0.909297	0.000463552

Модифицированный Эйлер

$h=0,2$

tn	yn	y(tn)	yn-y(tn)
0	1	1	0
0.2	10.3316289	0.205407278	-10.1262216
0.4	87.6717545	0.389463742	-87.2822907
0.6	743.178934	0.564642779	-742.614291
0.8	6313.27601	0.717356093	-6312.55865
1	53657.6255	0.841470985	-53656.784
1.2	456083.417	0.932039086	-456082.485
1.4	3876701.81	0.98544973	-3876700.83
1.6	32951957.7	0.999573603	-32951956.7
1.8	280091633	0.973847631	-280091632
2	2.38077887e+09	0.909297427	-2.38077887e+09

$h=0,1$

tn	yn	y(tn)	yn-y(tn)
0	1	1	0
0.1	2.66581444	0.181918415	-2.48389603
0.2	5.18594994	0.205407278	-4.98054266
0.3	9.09697311	0.296073291	-8.80089982
0.4	15.2726451	0.389463742	-14.8831814
0.5	25.1346238	0.479429265	-24.6551945
0.6	40.9948365	0.564642779	-40.4301937
0.7	66.611819	0.644217712	-65.9676013
0.8	108.094749	0.717356093	-107.377393
0.9	175.372486	0.78332691	-174.589159
1	284.580745	0.841470985	-283.739274
1.1	461.941245	0.89120736	-461.050037
1.2	750.065303	0.932039086	-749.133264
1.3	1218.19718	0.963558185	-1217.23362
1.4	1978.8595	0.98544973	-1977.87405
1.5	3214.90204	0.997494987	-3213.90454
1.6	5223.45602	0.999573603	-5222.45645
1.7	8487.35984	0.99166481	-8486.36817
1.8	13791.2258	0.973847631	-13790.252
1.9	22410.0489	0.946300088	-22409.1026
2	36415.6952	0.909297427	-36414.7859

$h=0,05$

tn	yn	y(tn)	yn-y(tn)
0	1	1	0
0.05	1.08290722	0.336483966	-0.746423255
0.1	1.11915025	0.181918415	-0.937231832
0.15	1.13063804	0.172955878	-0.957682164
0.2	1.12902915	0.205407278	-0.923621876
0.25	1.12053646	0.249334413	-0.871202046

0.3	1.10847964	0.296073291	-0.812406353
0.35	1.09464115	0.343056269	-0.751584877
0.4	1.07998647	0.389463742	-0.690522726
0.45	1.06504678	0.434978541	-0.630068243
0.5	1.05012217	0.479429265	-0.570692901
0.55	1.03538949	0.522688297	-0.512701194
0.6	1.02095973	0.564642779	-0.45631695
0.65	1.00690834	0.605186493	-0.40172185
0.7	0.993291379	0.644217712	-0.349073666
0.75	0.980153988	0.681638767	-0.298515221
0.8	0.967534916	0.717356093	-0.250178823
0.85	0.955468839	0.751280406	-0.204188433
0.9	0.943987583	0.78332691	-0.160660674
0.95	0.933120733	0.813415505	-0.119705228
1	0.922895918	0.841470985	-
1.05	0.913338947	0.867423226	0.0814249333
1.1	0.904473838	0.89120736	-
1.15	0.896322822	0.91276394	0.0132664781
1.2	0.888906308	0.932039086	0.0164411187
1.25	0.882242854	0.948984619	0.0431327783
1.3	0.876349125	0.963558185	0.0667417657
1.35	0.87123986	0.975723358	0.0872090601
1.4	0.86692783	0.98544973	0.104483498
1.45	0.863423815	0.992712991	0.1185219
1.5	0.860736576	0.997494987	0.129289176
1.55	0.858872828	0.999783764	0.136758411
1.6	0.85783723	0.999573603	0.140910937
1.65	0.857632371	0.999573603	0.141736373
1.7	0.858258763	0.996865028	0.139232658
1.75	0.85971484	0.99166481	0.133406048
1.8	0.861996963	0.983985947	0.124271107
1.85	0.865099428	0.973847631	0.111850668
1.9	0.869014481	0.961275203	0.0961757746
1.95	0.873732335	0.946300088	0.077285607
2	0.879241198	0.928959715	0.0552273802
		0.909297427	0.0300562284

Модифицированная Трапеция
 $h=0,2$

tn	yn	y(tn)	yn-y(tn)
0	1	1	0
0.2	-	-	-
0.2	0.230091	0.205407	0.435499
0.4	0.572991	0.389464	-0.183527
0.6	0.485801	0.564643	0.0788414
0.8	0.751	0.717356	-0.0336435
1	0.826934	0.841471	0.0145369
1.2	0.938183	0.932039	-0.0061438

1.4	0.982766	0.98545	0.00268396
1.6	1.00071	0.999574	-0.00113681
1.8	0.973385	0.973848	0.000462686
2	0.909557	0.909297	-0.000259811

$h=0,1$

tn	yn	y(tn)	yn-y(tn)
0	1	1	0
	-		
0.1	0.0113147	0.181918	0.193233
0.2	0.210983	0.205407	-0.00557523
0.3	0.294116	0.296073	0.00195709
0.4	0.38954	0.389464	-7.5818e-05
0.5	0.479379	0.479429	5.05369e-05
0.6	0.564616	0.564643	2.66719e-05
0.7	0.644191	0.644218	2.65728e-05
0.8	0.717332	0.717356	2.41451e-05
0.9	0.783305	0.783327	2.17553e-05
1	0.841452	0.841471	1.91212e-05
1.1	0.891191	0.891207	1.62994e-05
1.2	0.932026	0.932039	1.33143e-05
1.3	0.963548	0.963558	1.01963e-05
1.4	0.985443	0.98545	6.97641e-06
1.5	0.997491	0.997495	3.68679e-06
1.6	0.999573	0.999574	3.60343e-07
1.7	0.991668	0.991665	-2.96971e-06
1.8	0.973854	0.973848	-6.27009e-06
1.9	0.94631	0.9463	-9.50782e-06
2	0.90931	0.909297	-1.26505e-05

$h=0,05$

tn	yn	y(tn)	yn-y(tn)
0	1	1	0
0.05	0.280742	0.336484	0.055742
0.1	0.15308	0.181918	0.0288384
0.15	0.161719	0.172956	0.0112364
0.2	0.201497	0.205407	0.00391011
0.25	0.24805	0.249334	0.00128412
0.3	0.295663	0.296073	0.0004101
0.35	0.342925	0.343056	0.000131539
0.4	0.389419	0.389464	4.51516e-05
0.45	0.43496	0.434979	1.87896e-05
0.5	0.479419	0.479429	1.07613e-05
0.55	0.52268	0.522688	8.23768e-06
0.6	0.564635	0.564643	7.33961e-06
0.65	0.60518	0.605186	6.90896e-06
0.7	0.644211	0.644218	6.60348e-06

0.75	0.681632	0.681639	6.32305e-06
0.8	0.71735	0.717356	6.0389e-06
0.85	0.751275	0.75128	5.74316e-06
0.9	0.783321	0.783327	5.43407e-06
0.95	0.81341	0.813416	5.1117e-06
1	0.841466	0.841471	4.77663e-06
1.05	0.867419	0.867423	4.42965e-06
1.1	0.891203	0.891207	4.0716e-06
1.15	0.91276	0.912764	3.70338e-06
1.2	0.932036	0.932039	3.3259e-06
1.25	0.948982	0.948985	2.94011e-06
1.3	0.963556	0.963558	2.54697e-06
1.35	0.975721	0.975723	2.14746e-06
1.4	0.985448	0.98545	1.74259e-06
1.45	0.992712	0.992713	1.33336e-06
1.5	0.997494	0.997495	9.20801e-07
1.55	0.999783	0.999784	5.05938e-07
1.6	0.999574	0.999574	8.981e-08
1.65	0.996865	0.996865	-3.26542e-07
1.7	0.991666	0.991665	-7.42078e-07
1.75	0.983987	0.983986	-1.15576e-06
1.8	0.973849	0.973848	-1.56655e-06
1.85	0.961277	0.961275	-1.97343e-06
1.9	0.946302	0.9463	-2.37537e-06
1.95	0.928962	0.92896	-2.77138e-06
2	0.909301	0.909297	-3.16046e-06

Р-К 4-го порядка

$h=0,5$

tn	yn	y(tn)	yn-y(tn)
0	1	1	0
0.5	758.34676	0.479429265	-757.867331
1	574731.815	0.841470985	-574730.974
1.5	435851122	0.997494987	-435851121
2	3.30530649e+11	0.909297427	-3.30530649e+11

$h=0,25$

tn	yn	y(tn)	yn-y(tn)
0	1	1	0
0.25	37.4081102	0.249334413	-37.1587758
0.5	1381.67414	0.479429265	-1381.19472
0.75	51338.8233	0.681638767	-51338.1417
1	1908210.38	0.841470985	-1908209.53
1.25	70927068.4	0.948984619	-70927067.4
1.5	2.63631892e+09	0.997494987	-2.63631892e+09
1.75	9.79904811e+10	0.983985947	-9.79904811e+10
2	3.64225068e+12	0.909297427	-3.64225068e+12

$h=0,125$

tn	yn	y(tn)	yn-y(tn)
0	1	1	0
0.125	1.76969763	0.168611667	-1.60108596
0.25	2.95299633	0.249334413	-2.70366192
0.375	4.81610079	0.366357347	-4.44974344
0.5	7.7982742	0.479429265	-7.31884494
0.625	12.6234842	0.585097437	-12.0383868
0.75	20.4840614	0.681638767	-19.8024226
0.875	33.3428336	0.767543503	-32.5752901
1	54.4301755	0.841470985	-53.5887046
1.125	89.0617335	0.902267594	-88.1594659
1.25	145.983723	0.948984619	-145.034739
1.375	239.58619	0.980893057	-238.605297
1.5	393.544231	0.997494987	-392.546736
1.625	646.808502	0.998531341	-645.80997
1.75	1063.46074	0.983985947	-1062.47675
1.875	1748.92798	0.954085782	-1747.97389
2	2876.65809	0.909297427	-2875.74879

$h=0,1$

tn	yn	y(tn)	yn-y(tn)
0	1	1	0
0.1	0.748222121	0.181918415	-0.566303706
0.2	0.618897971	0.205407278	-0.413490693
0.3	0.567641299	0.296073291	-0.271568008
0.4	0.565344645	0.389463742	-0.175880903
0.5	0.5928245	0.479429265	-0.113395235
0.6	0.637352105	0.564642779	-0.0727093262
0.7	0.690406794	0.644217712	-0.0461890821
0.8	0.746221835	0.717356093	-0.0288657418
0.9	0.800843916	0.78332691	-0.0175170058
1	0.85152542	0.841470985	-0.0100544355
1.1	0.896332193	0.89120736	-0.00512483338
1.2	0.933890726	0.932039086	-0.00185163966
1.3	0.963225411	0.963558185	0.000332774047
1.4	0.98365387	0.98544973	0.00179586026
1.5	0.994719564	0.997494987	0.00277542308
1.6	0.996148248	0.999573603	0.00342535541
1.7	0.987819511	0.99166481	0.00384529985
1.8	0.969747748	0.973847631	0.00409988334
1.9	0.942068898	0.946300088	0.00423118949
2	0.905030582	0.909297427	0.00426684439