A Search and Learning Model of Export Dynamics

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Two sets of relevant issues

- Aggregate/industry level export dynamics
  - What determines short and long-run responses to macroeconomic shocks?
  - Why are export responses to trade liberalization unpredictable?
  - What are the underlying causes of export booms?

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- Trade at the level of individual firms
  - What are the firm-level trade frictions?
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  - How reconcile the cross section and dynamic patterns?
- **This paper**: Approach these issues by studying formation, evolution, and dissolution of international buyer-seller relationships.

What we do

1. Chararcterize buyer-seller relationships in decade's worth of data on individual merchandise shipments from Colombia to the United States
2. Develop a (partial equilibrium) dynamic search and learning model motivated by features of the data
3. Fit the model and quantify exporting frictions:
    - costs of finding new buyers
    - costs maintaining relationships with existing ones.
    - learning about product appeal in foreign markets
    - network effects
4. Use our estimated model to analyze the aggregate response to policy shocks such as trade liberalization

Related literature

- Melitz (2003), etc.
  - More efficient firms more likely to export
  - More efficient firms sell more in any market
- Beachhead exporting costs:
  - *Theory*: Dixit (1989), Baldwin and Krugman (1989), Impullitti, Irarrazabal, and Opromolla (2012)
  - *Quantitative*: Roberts and Tybout (1997), Bernard and Jensen (2004), Das, Roberts, and Tybout (2008)
- Marketing costs: Arkolakis (2009, 2010); Drozd and Nozal (2011)
- Networks: Rauch (1999, 2001), Chaney (2011)
- Learning: Rauch and Watson (2002); Albornoz, Calvo-Pardo, Corcos, and Ornelas (2012), Li (2014)

Structure of the talk

1. Stylized facts
2. Model
3. Estimates
4. Policy experiments

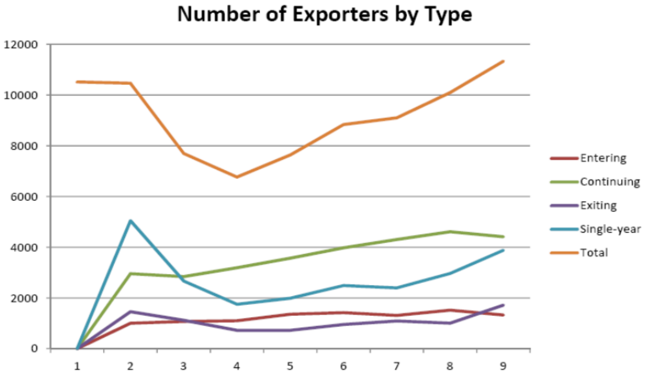
Stylized facts

- Evidence from Colombian customs data
    - Population of (legal) Colombian export transactions over the course of a decade (1996-2005).
    - Each transaction has a date, value, product code, firm ID, and destination country.
    - See also: Besedes (2006); Bernard et al (2007); Blum et al (2009); Albornoz, et al (2010)
- Evidence from U.S. customs records
    - Population of (legal) import transactions over the course of a decade (1996-2009).
    - Each transaction has a date, value, product code, affiliated trade indicator, exporter country *and* firm ID, and importer firm ID.
    - See also Blum et al, 2009a, 2009b; Albornoz et al, 2010; Carballo, Ottaviano and Martincus (2013).

The three main facts

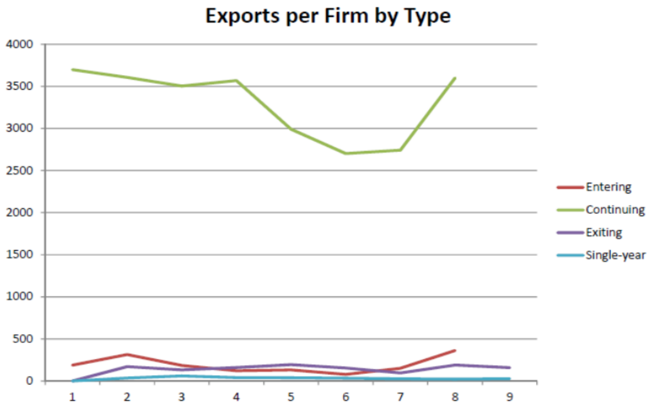
- The three key facts that motivate our model are:
  1. Churning: many firms export for only a short period
  2. Growth: firms that continue exporting grow quickly
  3. Fat tails: significant share of very large firms

Exporters by durability



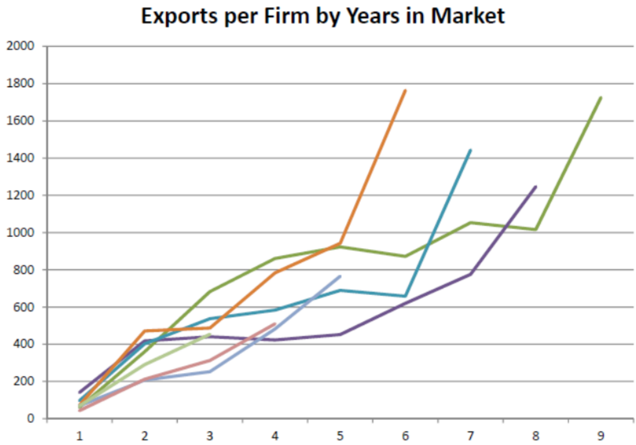
- As a fraction of total exporters, firms that enter a market and immediately exit are important.

Exporters by durability



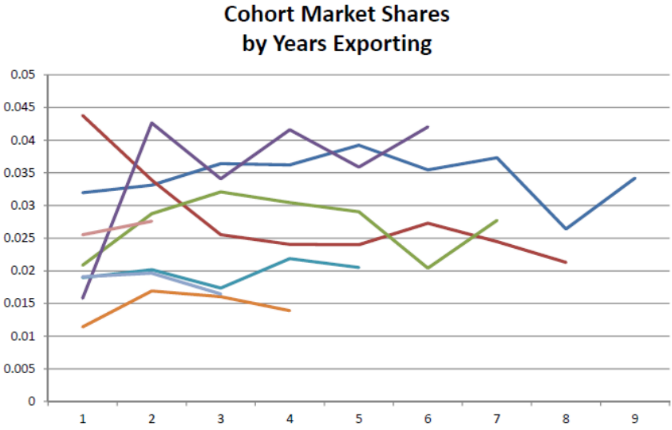
- But as a fraction of total export revenue, brand new exporters don't account for much.

Cohort maturation



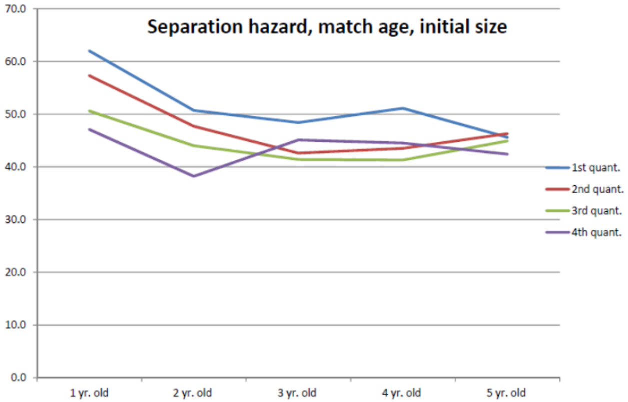
- The firms that survive their first year grow exceptionally rapidly (see also Ruhl and Willis, 2008).

Cohort maturation

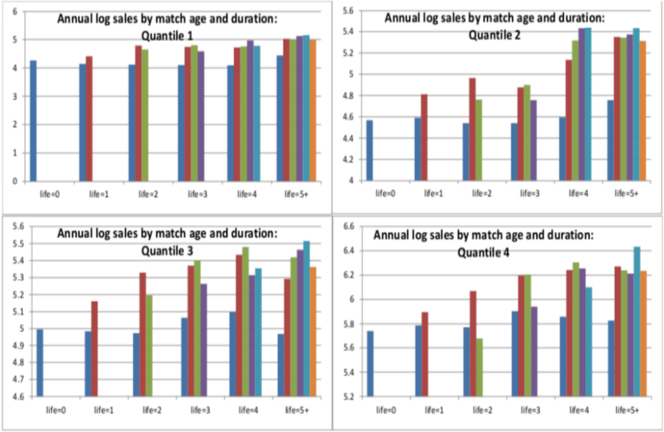


- Hence young cohorts typically gain market share despite rapid attrition.
- Post-1996 entrants account for about half of cumulative export expansion by 2005.

Cohort maturation



- Most new matches fail within a year, but
  - Chances of survival are higher for matches with large initial sales
  - Survival rates improve and converge for all matches after the first year.
  - To sustain or increase exports, firms must continually replenish their foreign clientele.



- Matches that start small tend to stay small.
- After a match's first year, there is no systematic tendency for its annual sales to grow.

Power-law distributions

- A distribution $G(x)$ is **power law** if its right-tail is distributed Pareto:

$$F_{\text{Pareto}}(x;\theta,x_{\min}) = 1 - \left(\frac{x}{x_{\min}}\right)^{-\theta}$$

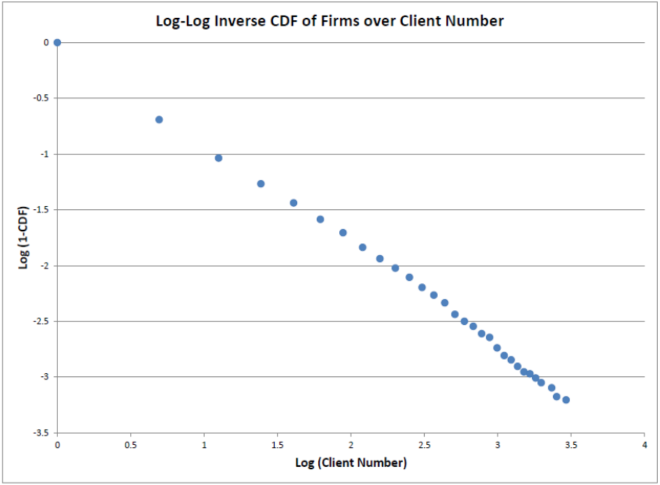
- The log of $1 - F_{\text{Pareto}}$ is a linear function of $\log x$:

$$\log\left(1 - F_{\text{Pareto}}(x;\theta,x_{\min})\right) = -\theta \log x + C$$

- If data are distributed power law, a scatter plot of the log empirical inverse CDF and log of the data will be linear in the tail ($\approx$ Gibrat's law)
- If data are distributed Pareto, a scatter plot of the log empirical inverse CDF and log of the data will be linear everywhere (Zipf's law)

A seriously Pareto client distribution

- Most firms have a single buyer, but the distribution of client counts across exporters is fat-tailed.



Year-to-year transitions in numbers of clients

Table 3: Transition Probabilities, Number of Clients

t\t+1	exit	texit	1	2	3	4	5	6-10	11+
enter	0.000	0.000	0.947	0.044	0.007	0.002	0.001	0.001	0.000
texit	0.000	.	0.896	0.086	0.014	0.004	.	.	0.000
1	0.533	0.081	0.332	0.043	0.008	0.002	0.001	.	.
2	0.180	0.081	0.375	0.249	0.077	0.026	0.007	0.005	0.000
3	0.074	0.043	0.225	0.282	0.206	0.093	0.047	.	.
4	0.045	.	0.112	0.226	0.259	0.162	0.097	0.078	.
5	.	.	0.103	0.184	0.197	0.184	0.094	0.197	.
6-10	.	.	.	0.070	0.082	0.114	0.149	0.465	0.066
11+	0.000	0.000	0.000	0.000	0.000	.	.	0.440	0.460

Key model features

- Firms engage in costly **search** to meet potential buyers at home and (possibly) abroad.
- Firms new to the foreign market don't know what fraction of buyers there will be willing to do business with them.
- As they encounter potential buyers, firms gradually learn the scope of the market for their particular products, and they adjust their search intensities accordingly (**learning**).
- Search costs fall as firms accumulate successful business relationships **(reputation effects**).
- Maintaining a relationship with a buyer is costly, so a relationship that yields meager profits is dropped.
- Three types of shocks: marketwide, firm-specific, match-specific

Three model components

1. A Seller-Buyer Relationship
2. Learning About Product Appeal from Encounters with Potential Buyers
3. Searching for Potential Buyers

Why continuous time?

- Two types of discrete events occur at random intervals, sometimes with high frequency
  - Sellers meet buyers
  - Once business relationships are established, orders are placed
- With continuous time formulation we can:
  - allow for an arbitrarily large number of events during any discrete interval
  - allow agents to update their behavior each time an event occurs

- Define exogenous state variables:
  - φ_j productivity of seller j
  - x_t^m size of market $m \in \{h, f\}$ (Markov jump process)
  - y_{ijt}^m idiosyncratic shock to operating profits from shipment to buyer i by seller j in market m (Markov jump process)
  - Π^m profit function scalar (so that all exogenous state variables can be normalized to mean log zero)
- When buyer i places an order with seller j in market m it generates operating profits:

$$\pi(x_t^m, \varphi_j, y_{ijt}^m) = \Pi^m x_t^m \varphi_j^{\sigma-1} y_{ijt}^m.$$

Superscripts and subcripts mostly suppressed hereafter:

$$\pi_\varphi(x, y) = \Pi x \varphi^{\eta-1} y$$

- In active business relationships, buyers place orders with exogenous hazard λ^b. ▸ Details
- After each order, sellers must pay fixed cost F to keep a business relationship active.
- Value to a type-φ seller of a relationship in state $\{x, y\}$:

$$\widetilde{\pi}_\varphi(x, y) = \pi_\varphi(x, y) + \max\left\{\widehat{\pi}_\varphi(x, y) - F, 0\right\}$$

- $\widehat{\pi}_\varphi(x, y)$ is continuation value to a type-φ seller of a relationship in state $\{x, y\}$ ▸ Details.
- Continuation values depend negatively on
  - δ : exogenous hazard of relationship death.
  - ρ : seller's discount rate (including exogenous seller death probability)

- Sellers don't know what y value their next business relationship will begin from.
- Let $\Pr(y^s)$ be the probability of initial shock y^s determined by the ergodic distribution of y.
- Expected value of a successful new encounter:

$$\widetilde{\pi}_{\varphi}(x) = \sum_{y^s} \Pr(y^s)\widetilde{\pi}_{\varphi}(x, y)$$

- Fraction of potential buyers in market m who are interested in seller j's product: $\theta_j^m \in [0,1]$ of total potential buyers.
- Assume $\theta_j^{m\prime} s$ are time-invariant, mutually independent draws from a beta distribution:

$$r(\theta|\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} (\theta)^{\alpha-1} (1-\theta)^{\beta-1},$$

- Expected value:

$$E(\theta|\alpha,\beta) = \frac{\alpha}{\alpha+\beta.}$$

- Posterior beliefs, after meeting n^m potential clients in market m, a^m of whom want to do business: ▸ Details

$$\overline{\theta}^m(a^m,n^m) = E\left[\theta^m|a^m,n^m\right] = \frac{a^m+\alpha}{n^m+\alpha+\beta}$$

3. Searching for buyers

the cost of search

- Seller continuously chooses the hazard s with which she encounters a potential buyer at a flow cost $c(s, a)$
  - Maintain web site
  - Pay to be near top of web search listings
  - Attend trade fairs
  - Research foreign buyers
  - Send sales reps. to foreign markets
  - Maintain foreign sales office
- The number of successful encounters, a, allows for network effects (NYT 2/27/12: Panjiva, ImportGenius).
- Functional form used for estimation (Arkolakis, 2010):

$$c(s, a) = \kappa_0 \frac{(1+s)^{(1+1/\kappa_1)} - 1}{(1+a)^{\gamma\cdot(1+1/\kappa_1)}\,(1+1/\kappa_1)}$$

- Let $V_\varphi(a, n, x)$ be the value of continued search for a type-φ firm with a successes in n meetings.
- The first-order for optimal search abroad is: Details

$$
\begin{aligned}
c_s(s^*, a) \quad = \quad & \overline{\theta}_{a,n}(\widetilde{\pi}_\varphi(x) + V_\varphi(a+1, n+1, x)) \\
& + (1 - \overline{\theta}_{a,n}) V_\varphi(a, n+1, x) - V_\varphi(a, n, x).
\end{aligned}
$$

3. Searching for buyers
when the truth is known: the domestic market

- As n increases $\overline{\theta}_{a,n}$ converges to the true θ. There is no more learning and the reward to search depends on a and n only through network effects.
- We assume this characterizes the domestic market, so the first-order condition for optimal search at home is:

$$c_s(s^*, a) = \theta_j \widetilde{\pi}_\varphi(x).$$

- Assume x^f, x^h, and y follow independent Ehrenfest diffusion processes.
  - Any variable z that obeys this process is discretized into $2e+1$ possible values, $e \in I^+ : z \in \{-e\Delta, -(e-1)\Delta, .., 0, .., (e-1)\Delta, e\Delta\}$.
  - Process jumps with hazard λ_z, and when it does so:

$$z' = \begin{cases} z+\Delta \\ z-\Delta \\ \text{other} \end{cases} \text{with probability} \begin{cases} \frac{1}{2}\left(1-\frac{z}{e\triangle}\right) \\ \frac{1}{2}\left(1+\frac{z}{e\triangle}\right) \\ 0 \end{cases} .$$

- As the grid becomes finer, this type of random variable asymptotes to an Ornstein-Uhlenbeck processes:

$$dz = -\mu z dt + \sigma dW$$

- If z observed at regular intervals, can estimate μ and σ by regressing z on lagged z
- For x^f, x^h, obtain maximum likelihood estimates of μ and σ using logged and de-meaned time series on total real consumption of manufactured goods in each country.
- Recover λ_z and Δ using Shimer's result that asymptotically, $\mu = \lambda_z / e$, $\sigma = \sqrt{\lambda_z}\Delta$.
- This gives us the $q^X_{xx'}$ values needed to construct $q^Y_{yy'}$'s for home and foreign markets.
- Since y is unobservable, recover the parameters of its jump processes using the structure of the dynamic model.

Market-wide Shock Processes (x^f, x^h)		
Orstein-Uhlenbeck Parameters	**Colombia**	**United States**
μ Mean Reversion	0.171	0.174
σ Dispersion	0.003	0.058
Ehrenfest Process Parameters		
λ Jump Hazard	1.200	1.215
Δ Jump Size	0.003	0.053
grid points	15	15

Estimation
remaining parameters

- Unidentified preference parameters taken from literature: Discount rate (including 0.03 exogenous death) $\rho = 0.05$, Demand elasticity $\sigma = 5$
- Remaining parameters identified using indirect inference

$$\Lambda = \left(\Pi^h, \Pi^f, \delta, F, \alpha, \beta, \sigma_\varphi, \lambda_y, \Delta_y, \lambda_b, \gamma, \kappa_0, \kappa_1\right)$$

Indirect inference (Gouriéroux and Monfort, 1996)

basic idea

- Using reduced-form auxillary regressions and/or moments, summarize key relationships in the data using a vector of statistics ($\widehat{\mathbf{M}}$)
- Similar to Simulated Method of Moments – difference is a bit philosophical
- For a candidate set of parameter values (Λ), simulate same statistics using the model $\widehat{\mathbf{M}}^s(\Lambda)$.
- Construct the loss function:

$$Q(\Lambda) = \left(\widehat{\mathbf{M}} - \widehat{\mathbf{M}}^s(\Lambda)\right)' \Omega \left(\widehat{\mathbf{M}} - \widehat{\mathbf{M}}^s(\Lambda)\right)$$

  where Ω is a positive definite weighting matrix.
- Use a robust algorithm to search parameter space for $\widehat{\Lambda} = \arg\min Q(\Lambda)$.

Indirect inference
identification

- **Profit scaling constants**, (Π^h, Π^f)
  - means of log home and foreign sales
- **Shipment hazards** (λ^b)
  - average annual shipment rates per match
- **Product appeal parameters** (α, β)
  - distribution of home and foreign sales
- **Firm productivity dispersion** (σ_φ)
  - covariance of home and foreign sales
- **Search cost parameters** $(\kappa_0, \kappa_1, \gamma)$
  - match rates
  - client frequency distribution (especially fatness of tail)
  - client transition probabilites
  - fraction of firms that export

- **Idioysncratic shocks to importers (λ^y, Δ^y)**
  - cross-plant variances in home and foreign sales
  - covariation of home and foreign sales
  - autocorrelation, match-specific sales
  - client frequency distribution, client transition probabilites
- **Match maintenance costs** (F)
  - client frequency distribution, client transition probabilites
  - sales among new versus established matches
  - age-specific match failure rates
- **Exogenous match separation hazard** (δ)
  - separation rates after first year
  - age-specific match failure rates
  - client frequency distribution

Data versus simulated statistics

Transition probs., no. clients (n^c)	Data	Model	Share of firms exporting	Data	Model
$\widehat{P}[n^c_{jt+1} = 0 \mid n^c_{jt} = 1]$	0.618	0.650	$\widehat{E}(1_{X^f_{jt}>0})$	0.299	0.359
$\widehat{P}[n^c_{jt+1} = 1 \mid n^c_{jt} = 1]$	0.321	0.320			
$\widehat{P}[n^c_{jt+1} = 2 \mid n^c_{jt} = 1]$	0.048	0.027	**Log foreign sales on**		
$\widehat{P}[n^c_{jt+1} \geq 3 \mid n^c_{jt} = 1]$	0.013	0.002	**log domestic sales**	Data	Model
$\widehat{P}[n^c_{jt+1} = 0 \mid n^c_{jt} = 2]$	0.271	0.443			
$\widehat{P}[n^c_{jt+1} = 1 \mid n^c_{jt} = 2]$	0.375	0.339	$\widehat{\beta}^{hf}_1$	0.727	0.877
$\widehat{P}[n^c_{jt+1} = 2 \mid n^c_{jt} = 2]$	0.241	0.165	$\widehat{se}(\epsilon^{hf})$	2.167	0.640
$\widehat{P}[n^c_{jt+1} \geq 3 \mid n^c_{jt} = 2]$	0.113	0.052			

Data versus simulated statistics

Match death hazards	Data	Model	**Exporter exit rate**	Data	Model
Death rate, $A^m_{ijt-1} = 0$	0.694	0.649	*Exit rate*, $A^m_{ijt-1} = 0$	0.709	0.725
Death rate, $A^m_{ijt-1} = 1$	0.515	0.484	*Exit rate*, $A^m_{ijt-1} = 1$	0.383	0.312
Death rate, $A^m_{ijt-1} = 2$	0.450	0.531	*Exit rate*, $A^m_{ijt-1} = 2$	0.300	0.462
Death rate, $A^m_{ijt-1} = 3$	0.424	0.546	*Exit rate*, $A^m_{ijt-1} = 3$	0.263	0.102
Death rate, $A^m_{ijt-1} = 4$	0.389	0.485	*Exit rate*, $A^m_{ijt-1} = 4$	0.293	0.200

Data versus simulated statistics

Log sales per client vs. no. clients	Data	Model	Ave. log sales by cohort age	Data	Model
$\widehat{\beta}_1^m$	2.677	0.422	$\widehat{E}(\ln X_{jt}^f \mid A_{jt}^c = 0)$	8.960	10.181
$\widehat{\beta}_2^m$	-0.143	0.317	$\widehat{E}(\ln X_{jt}^f \mid A_{jt}^c = 1)$	10.018	11.124
$\widehat{se}(\epsilon^m)$	2.180	1.449	$\widehat{E}(\ln X_{jt}^f \mid A_{jt}^c = 2)$	10.231	11.030
No. clients, inverse			$\widehat{E}(\ln X_{jt}^f \mid A_{jt}^c = 3)$	10.369	11.021
CDF regression	Data	Model	$\widehat{E}(\ln X_{jt}^f \mid A_{jt}^c \geq 4)$	10.473	11.178
$\widehat{\beta_1^c}$	-1.667	-2.501			
$\widehat{se}(\epsilon^{n^c})$	0.066	0.048			

Data versus simulated statistics

Match death prob regression	Data	Model	Log match sale autoreg.	Data	Model
$\widehat{\beta}_0^d$	1.174	1.277	$\widehat{\beta}_1^f$	0.811	0.849
$\widehat{\beta}_{\text{1st year}}^d$	0.166	0.061	$\beta_{\text{1st year}}^f$	0.233	0.150
$\widehat{\beta}_{\text{lsales}}^d$	-0.070	-0.055	$s\widehat{e}(\epsilon^f)$	1.124	0.330
$s\widehat{e}(\epsilon^d)$	0.453	0.441	**Log dom. sales autoregression**	Data	Model
Match shipments per year	Data	Model	$\widehat{\beta}_1^h$	0.976	0.907
$\widehat{E}(n^s)$	4.824	1.416	$s\widehat{e}(\epsilon^h)$	0.462	0.656

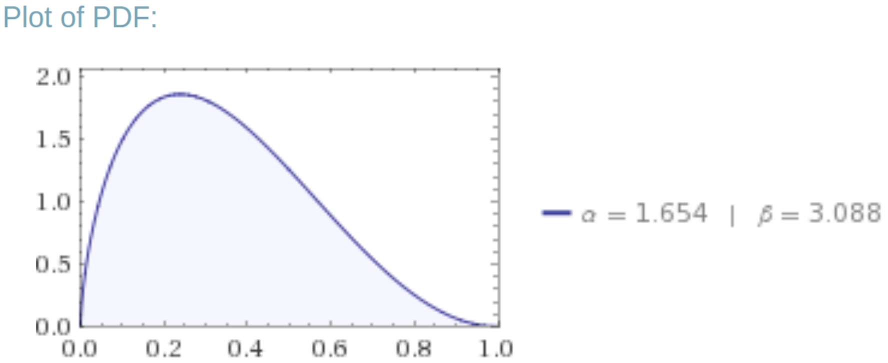
Parameters

Parameters Estimated using indirect inference (Λ)			
	Parameter	*value*	*std. error*
rate of exogenous separation	δ	0.65	
domestic market size	Π^h	7.914	
foreign market size	Π^f	7.363	
log fixed cost	$\ln F$	7.317	
First θ distribution parameter	α	1.654	
Second θ distribution parameter	β	3.088	

- fixed cost of maintaining a relationship: $\exp(7.317) = \$1{,}506$.
- about $\alpha/(\alpha+\beta) = 0.35$ of the potential buyers a typical exporter meets are interested in doing business
- success rates vary across exporters with standard deviation $\sqrt{\alpha\beta/\left[(\alpha+\beta)^2(\alpha+\beta+1)\right]} = 0.199$

Parameters

Estimated product appeal distribution:



Parameters

Parameters Estimated using indirect inference (Λ)

	Parameter	*value*	*std. error*
demand shock jump hazard	λ_y	0.501	
demand shock jump size	Δ^y	0.037	
shipment order arrival hazard	λ_b	1.216	
std. deviation, log firm type	σ_φ	0.628	
network effect parameter	γ	0.011	
search cost function curvature parameter	κ_1	0.041	
search cost function scale parameter	κ_0	132.533	

- convexity of search cost function is important
- cost of search at hazard $s = 0.6$: $\$6,829$ when $a = 0$; $\$5614$ when $a = 1$; $\$2,888$ when $a = 20$.
- "lock-in" effect

Parameters

restricted models

restricted models

		benchmark (Λ)	no learning (Λ^{NL})	no network (Λ^{NN})
rate of exogenous separation	δ	0.267	0.516	0.119
fixed cost	F	7.957	10.238	8.539
First θ distribution parameter	α	0.716	0.512	1.807
Second θ distribution parameter	β	3.161	0.351	0.963
demand shock jump hazard	λ_y	0.532	0.713	1.581
demand shock jump size	Δ^y	0.087	0.060	0.087
shipment order arrival hazard	λ_b	8.836	10.028	10.347
std. deviation, log firm type	σ_φ	0.650	1.268	1.355
network effect parameter	γ	0.298	0.112	0
fit metric	D	9.97 e+04	2.155 e+05	1.17 e+05
fit metric, no weighting	$\widetilde{D}$	0.117	0.182	0.143

- no-learning model, treats firms as knowing their exact θ^f draws
- no-network model shuts down reputation effects by imposing $\gamma = 0$

Parameters
The no-learning model

- Rapid turnover of novice exporters less likely:
  - discourages inexperienced low-θ^f firms from exploring foreign markets
  - eliminates learning-based exit.
- High-θ^f firms do not intensify their search efforts as they receive positive feedback.
- Lower productivity firms induced to participate in export markets by a
  - rightward shift in θ^f distribution and
  - higher values for Π^f and λ_b
- Match failure rates and market exit rates are sustained by
  - higher values for F, δ, and λ_y.
- Model badly overstates the share of firms that export, overstates the relationship between sales per client and number of clients.

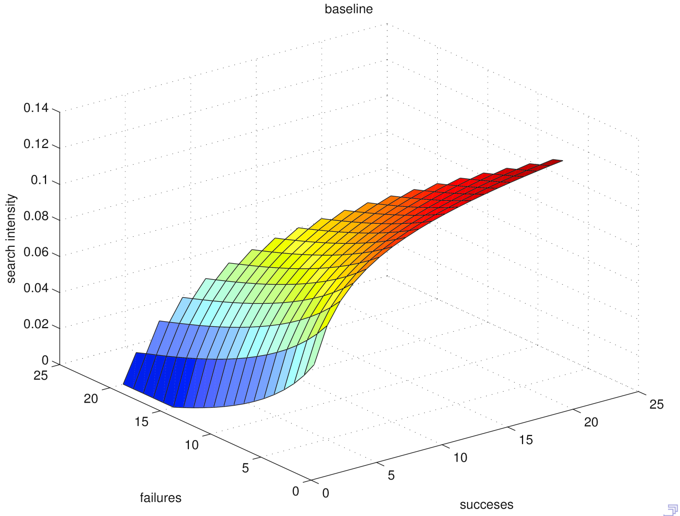
Parameters

The no-network model

- Model moves part way toward matching the Pareto shape by reducing the convexity of the search cost function, κ_1.
- This is an imperfect fix because all exporters are equally affected by κ_1, not just the larger ones.
- Various other adjustments occur, including:
    - modest increase in F,
    - rightward shift in the θ distribution, an
    - increase in the variance of φ,
    - increase in the jump hazard for buyer shocks, λ_y
- Client distribution is far from Pareto: model is unable to explain the existence of very large exporters; overstates the fraction of firms that export.

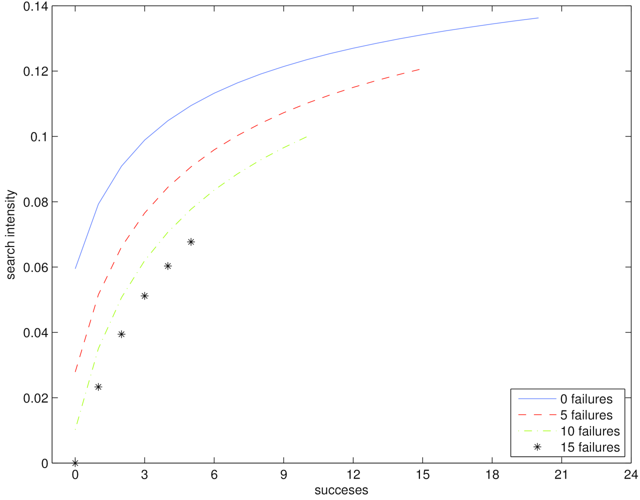
Learning and the policy function

- Fix productivity: search intensity as a function of past successes and failures



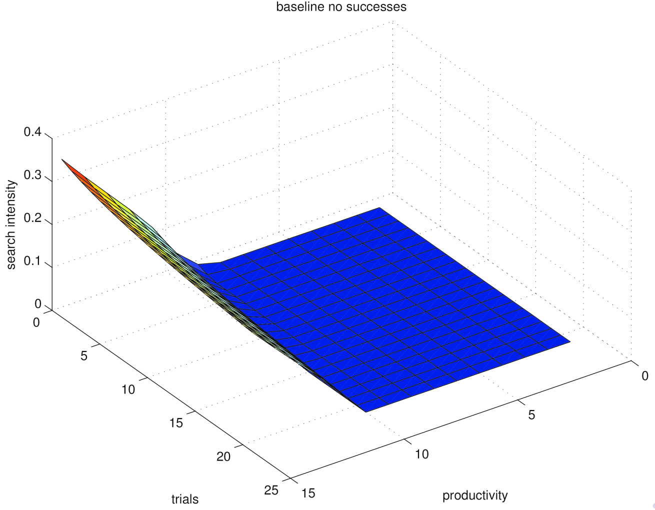
Learning and the policy function: 2D

- Fix productivity: search intensity as a function of past successes and failures



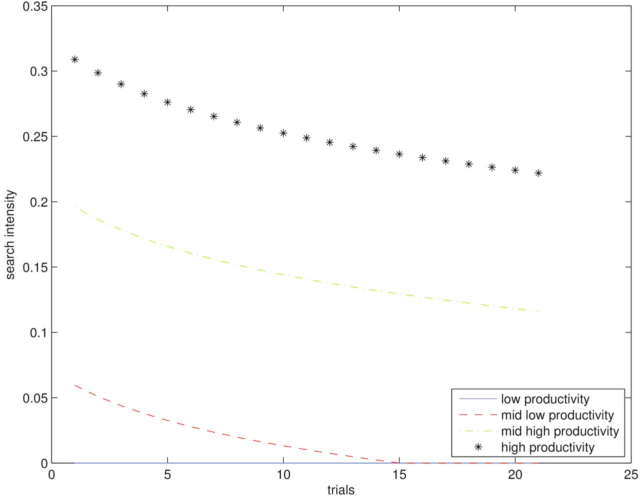
Productivity and search

- Fix successes at zero: search intensity as a function of productivity and failures

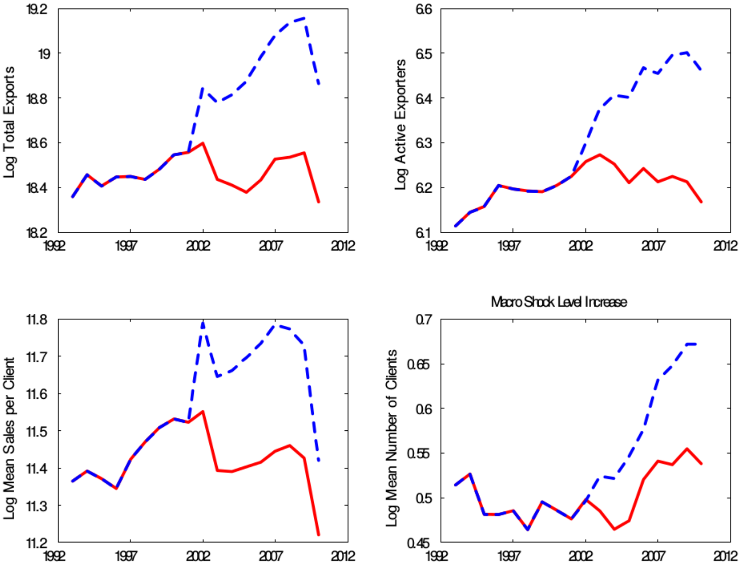


Learning and the policy function: 2D

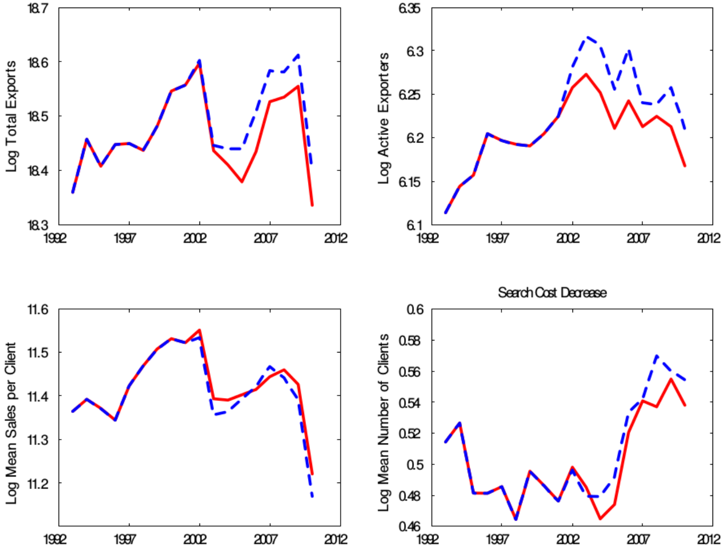
- Fix successes at zero: search intensity as a function of productivity and failures



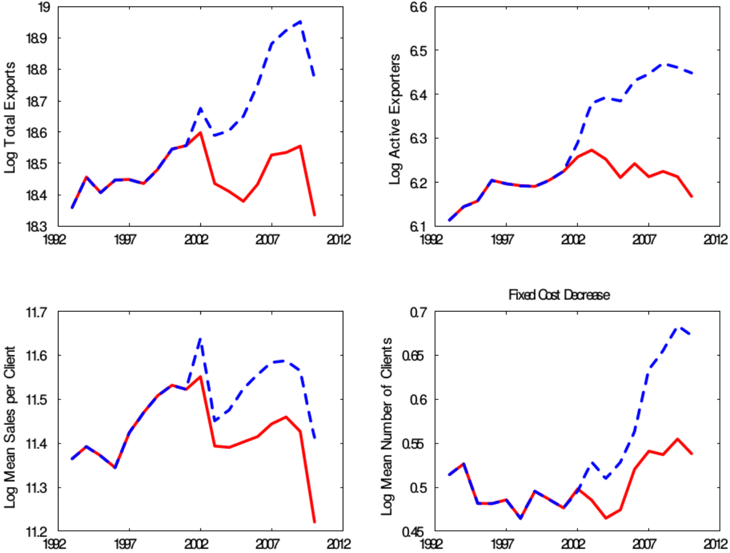
A 20% increase in foreign market size



A 20% reduction in search costs



A 20% reduction in fixed costs



Summary

- Micro patterns of transactions and buyer-seller relationships through the lens of the model:
    - Large volume of small scale exporters explained by large volume of inexperienced firms, searching at a low level.
    - High exit rate reflects short lifespan of typical match, combined with low-level search and learning about product appeal.
    - Small number of major exporters reflects combination of skewed distribution of product appeal and reputation effects.
- Search costs, multi-period matches, learning, and reputation effects combine to provide an explanation for hysteresis in trade.
    - Reputation effects appear to be particularly important.
    - Since learning is mainly relevant for new, marginal players, probably doesn't have a big effect on short-run export dynamics.

A Digression: hazards

- From the perspective of time 0, let the probability that an event will occur before time t be described by the exponential distribution:

$$F[t] = 1 - e^{-qt}$$

- The likelihood of the event happening exactly at t (the "hazard rate" at t) is then:

$$\frac{f(t)}{1 - F(t)} = \frac{qe^{-qt}}{e^{-qt}} = q$$

- This hazard rate doesn't depend upon t.

Hazards

- Suppose k independent events occur with hazard $q_1, q_2, ...q_k$. The probability that none occur before t is:

$$\prod_{j=1}^{k}(1 - F_j(t)) = e^{-t\Sigma_j q_j}$$

- So by time t, at least one event occurs with probability $1 - e^{-t\Sigma_j q_j}$, and the likelihood that this happens exactly at t is

$$\frac{\Sigma_j q_j \left[e^{-t\Sigma_j q_j}\right]}{e^{-t\Sigma_j q_j}} = \Sigma_j q_j$$

- x (market-wide) follows Markov jump process, hazard $q^X_{xx'}$ of transiting from state x to state x'.
- y (match-specific) follows Markov jump process, hazard $q^Y_{yy'}$ of transiting from state y to state y'.
- $\lambda^X_x = \sum_{x' \neq x} q^X_{xx'}$ is hazard of a change in market-wide state x
- $\lambda^Y_y = \sum_{y' \neq y} q^Y_{yy'}$ is hazard of a change in match-specific state y.
- λ^b is hazard of a new purchase order from existing client.
- τ_b time until the next change in state, which occurs with hazard $\lambda^b + \lambda^X_x + \lambda^Y_y$

Relationship dynamics

the continuation value

- δ exogenous hazard of relationship death.
- ρ seller's discount rate.

Continuation value of a business relationship in state (x, y) for a type-φ exporter :

$$\begin{aligned}
\widehat{\pi}_{\varphi}(x,y) &= \mathbf{E}_{\tau_b}\Bigg[e^{-(\rho+\delta)\tau_b}\frac{1}{\lambda^b+\lambda_x^X+\lambda_y^Y} \\
&\quad \cdot\left(\sum_{x'\neq x} q_{xx'}^X\widehat{\pi}_{\varphi}(x',y)+\sum_{y'\neq y} q_{yy'}^Y\widehat{\pi}_{\varphi}(x,y')+\lambda^b\widetilde{\pi}_{\varphi}(x,y)\right)\Bigg] \\
&= \frac{1}{h}\left(\sum_{x'\neq x} q_{xx'}^X\widehat{\pi}_{\varphi}(x',y)+\sum_{y'\neq y} q_{yy'}^Y\widehat{\pi}_{\varphi}(x,y')+\lambda^b\widetilde{\pi}_{\varphi}(x,y)\right)
\end{aligned}$$

where

$$h=\rho+\delta+\lambda^b+\lambda_x^X+\lambda_y^Y$$

Learning about product appeal

experience and expected success rates

- Suppress market superscripts to reduce clutter.
- The **prior distribution** is:

$$r(\theta|\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} (\theta)^{\alpha-1} (1-\theta)^{\beta-1},$$

- **The likelihood:** Given θ, and given that a seller has met n potential buyers, the probability that a of these buyers were willing to buy her product is binomially distributed:

$$q[a|n,\theta] = \binom{n}{a} [\theta]^{a} [1-\theta^{m}]^{n-a}.$$

Learning about product appeal

experience and expected success rates

- **The posterior** distribution for θ:

$$p(\theta|a, n) \propto q\left[a|n, \theta\right] \cdot r(\theta|\alpha, \beta)$$

- The expected success rate after a successes in n trials is thus:

$$\overline{\theta}(a, n) = E\left[\theta|a, n\right] = \frac{a + \alpha}{n + \alpha + \beta}$$

- Sellers base their search intensity on this posterior mean.

Searching for buyers
the value of search

The value of continued search for a type-φ firm with a successes in n meetings is:

$$V_\varphi(a,n,x) =$$
$$\max_s \mathbf{E}_{\tau_s}\left[-c(s,a)\int_0^{\tau_s} e^{-\rho t}dt + \frac{e^{-\rho\tau_s}}{s+\lambda_x^X}\cdot\left(\sum_{x'\neq x} q_{xx'}^X V_{\varphi,}(a,n,x')\right.\right.$$
$$\left.\left.+\, s\left[\bar{\theta}_{a,n}(\widetilde{\pi}_\varphi(x) + V_\varphi(a+1,n+1,x) + (1-\bar{\theta}_{a,n})V_\varphi(a,n+1,x)\right]\right)\right]$$

where:

- $\lambda_x^X = \sum_{x'\neq x} q_{xx'}^X$ is the hazard of any change in the market-wide state x.
- τ_s is the random time until the next search event, which occurs with hazard $s+\lambda_x^X$.

Taking expectations over τ_s yields:

$$
\begin{aligned}
& V_\varphi(a, n, x) \\
= \quad & \max_s \frac{1}{\rho + s + \lambda_x^X} \Bigg[-c(s, a) + \sum_{x' \neq x} q_{xx'}^X V_{\varphi,}(a, n, x') \\
& \quad + s \left\{ \overline{\theta}_{a,n} \left[\widetilde{\pi}_\varphi(x) + V_\varphi(a+1, n+1, x) \right] + (1 - \overline{\theta}_{a,n}) V_\varphi(a, n+1, x
\end{aligned}
$$

The first-order condition is thus:

$$
\begin{aligned}
c_s(s^*, a) \quad = \quad & \overline{\theta}_{a,n} (\widetilde{\pi}_\varphi(x) + V_\varphi(a+1, n+1, x)) \\
& + (1 - \overline{\theta}_{a,n}) V_\varphi(a, n+1, x) - V_\varphi(a, n, x).
\end{aligned}
$$

Searching for buyers

when the truth is known: the domestic market

- In the domestic market the reward to search depends on a and n only through network effects.
- The value of search at home is thus simply:

$$V_{\varphi}(x) = \max_{s} \frac{1}{\rho + s + \lambda_x^X} \left[-c(s, a) + \sum_{x' \neq x} q_{xx'}^X V_{\varphi}(x') + s\theta_j \widetilde{\pi}_{\varphi}(x) \right]$$

- The associated first-order condition is:

$$c_s(s^*, a) = \theta_j \widetilde{\pi}_{\varphi}(x).$$