A Search and Learning Model of Export Dynamics

Jonathan Eaton, a,b Marcela Eslava, c David Jinkins f, C.J. Krizan, d James Tybout a,e

^aBrown University, ^bNBER, ^cU. de los Andes, ^dCensus Bureau (CES), ^ePenn State, ^fCopenhagen Business School

November 7, 2014

Two sets of relevant issues

- Aggregate/industry level export dynamics
 - What determines short and long-run responses to macroeconomic shocks?
 - Why are export responses to trade liberalization unpredictable?
 - What are the underlying causes of export booms?

Two sets of relevant issues

- Aggregate/industry level export dynamics
 - What determines short and long-run responses to macroeconomic shocks?
 - Why are export responses to trade liberalization unpredictable?
 - What are the underlying causes of export booms?
- Trade at the level of individual firms
 - What are the firm-level trade frictions?
 - What determines the cross-firm distribution of export sales?
 - What determines firm-specific export growth patterns, once they start exporting?
 - How reconcile the cross section and dynamic patterns?

Two sets of relevant issues

- Aggregate/industry level export dynamics
 - What determines short and long-run responses to macroeconomic shocks?
 - Why are export responses to trade liberalization unpredictable?
 - What are the underlying causes of export booms?
- Trade at the level of individual firms
 - What are the firm-level trade frictions?
 - What determines the cross-firm distribution of export sales?
 - What determines firm-specific export growth patterns, once they start exporting?
 - How reconcile the cross section and dynamic patterns?
- **This paper**: Approach these issues by studying formation, evolution, and dissolution of international buyer-seller relationships.

What we do

- Characterize buyer-seller relationships in decade's worth of data on individual merchandise shipments from Colombia to the United States
- ② Develop a (partial equilibrium) dynamic search and learning model motivated by features of the data
- Sit the model and quantify exporting frictions:
 - costs of finding new buyers
 - costs maintaining relationships with existing ones.
 - learning about product appeal in foreign markets
 - network effects
- Use our estimated model to analyze the aggregate response to policy shocks such as trade liberalization

Related literature

- Melitz (2003), etc.
 - More efficient firms more likely to export
 - More efficient firms sell more in any market
- Beachhead exporting costs:
 - Theory: Dixit (1989), Baldwin and Krugman (1989), Impullitti, Irarrazabal, and Opromolla (2012)
 - Quantitative: Roberts and Tybout (1997), Bernard and Jensen (2004), Das, Roberts, and Tybout (2008)
- Marketing costs: Arkolakis (2009, 2010); Drozd and Nozal (2011)
- Networks: Rauch (1999, 2001), Chaney (2011)
- Learning: Rauch and Watson (2002); Albornoz, Calvo-Pardo, Corcos, and Ornelas (2012), Li (2014)

Structure of the talk

- Stylized facts
- Model
- Stimates
- Policy experiments

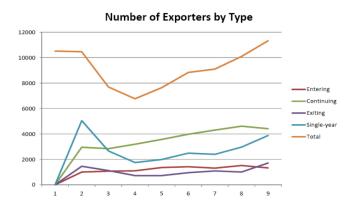
Stylized facts

- Evidence from Colombian customs data
 - Population of (legal) Colombian export transactions over the course of a decade (1996-2005).
 - Each transaction has a date, value, product code, firm ID, and destination country.
 - See also: Besedes (2006); Bernard et al (2007); Blum et al (2009);
 Albornoz, et al (2010)
- Evidence from U.S. customs records
 - Population of (legal) import transactions over the course of a decade (1996-2009).
 - Each transaction has a date, value, product code, affiliated trade indicator, exporter country and firm ID, and importer firm ID.
 - See also Blum et al, 2009a, 2009b; Albornoz et al, 2010; Carballo, Ottaviano and Martincus (2013).

The three main facts

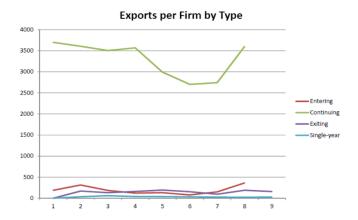
- The three key facts that motivate our model are:
 - Churning: many firms export for only a short period
 - @ Growth: firms that continue exporting grow quickly
 - Fat tails: significant share of very large firms

Exporters by durability



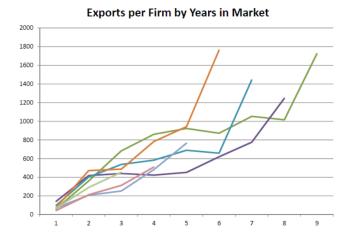
• As a fraction of total exporters, firms that enter a market and immediately exit are important.

Exporters by durability



 But as a fraction of total export revenue, brand new exporters don't account for much.

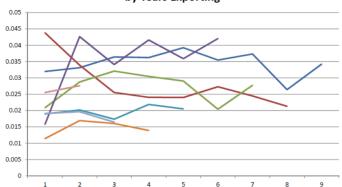
Cohort maturation



 The firms that survive their first year grow exceptionally rapidly (see also Ruhl and Willis, 2008).

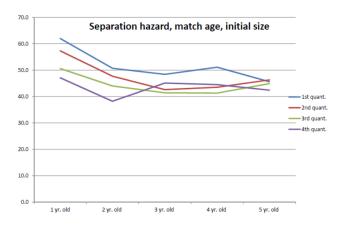
Cohort maturation

Cohort Market Shares by Years Exporting



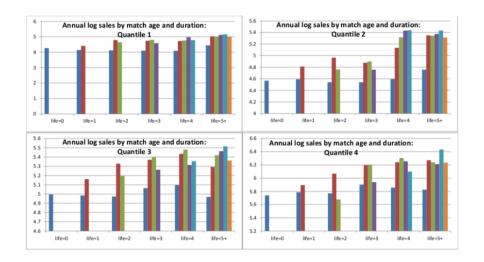
- Hence young cohorts typically gain market share despite rapid attrition.
- Post-1996 entrants account for about half of cumulative export expansion by 2005.

Match maturation



- Most new matches fail within a year, but
 - Chances of survival are higher for matches with large initial sales
 - Survival rates improve and converge for all matches after the first year.
 - To sustain or increase exports, firms must continually replenish their foreign clientele.

Match maturation



- Matches that start small tend to stay small.
- After a match's first year, there is no systematic tendency for its

Power-law distributions

• A distribution G(x) is **power law** if its right-tail is distributed Pareto:

$$F_{\mathsf{Pareto}}(x; heta, x_{\mathsf{min}}) = 1 - \left(rac{x}{x_{\mathsf{min}}}
ight)^{- heta}$$

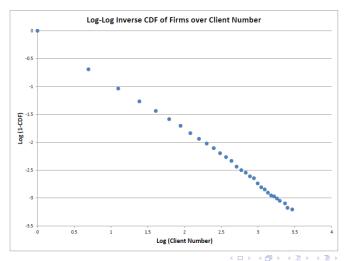
• The log of $1 - F_{Pareto}$ is a linear function of log x:

$$\log\left(1 - F_{\text{Pareto}}(x; \theta, x_{\min})\right) = -\theta \log x + C$$

- If data are distributed power law, a scatter plot of the log empirical inverse CDF and log of the data will be linear in the tail (\approx Gibrat's law)
- If data are distributed Pareto, a scatter plot of the log empirical inverse CDF and log of the data will be linear everywhere (Zipf's law)

A seriously Pareto client distribution

 Most firms have a single buyer, but the distribution of client counts across exporters is fat-tailed.



Year-to-year transitions in numbers of clients

Table 3: Transition Probabilities, Number of Clients

t t+1	exit	texit	1	2	3	4	5	6-10	11+
enter	0.000	0.000	0.947	0.044	0.007	0.002	0.001	0.001	0.000
texit	0.000		0.896	0.086	0.014	0.004			0.000
1	0.533	0.081	0.332	0.043	0.008	0.002	0.001		
2	0.180	0.081	0.375	0.249	0.077	0.026	0.007	0.005	0.000
3	0.074	0.043	0.225	0.282	0.206	0.093	0.047		
4	0.045		0.112	0.226	0.259	0.162	0.097	0.078	
5			0.103	0.184	0.197	0.184	0.094	0.197	
6-10				0.070	0.082	0.114	0.149	0.465	0.066
11+	0.000	0.000	0.000	0.000	0.000			0.440	0.460

Key model features

- Firms engage in costly search to meet potential buyers at home and (possibly) abroad.
- Firms new to the foreign market don't know what fraction of buyers there will be willing to do business with them.
- As they encounter potential buyers, firms gradually learn the scope of the market for their particular products, and they adjust their search intensities accordingly (learning).
- Search costs fall as firms accumulate successful business relationships (reputation effects).
- Maintaining a relationship with a buyer is costly, so a relationship that yields meager profits is dropped.
- Three types of shocks: marketwide, firm-specific, match-specific

Three model components

- A Seller-Buyer Relationship
- Learning About Product Appeal from Encounters with Potential Buyers
- Searching for Potential Buyers

Why continuous time?

- Two types of discrete events occur at random intervals, sometimes with high frequency
 - Sellers meet buyers
 - Once business relationships are established, orders are placed
- With continuous time formulation we can:
 - allow for an arbitrarily large number of events during any discrete interval
 - allow agents to update their behavior each time an event occurs

1. Relationship dynamics

profits from a shipment

- Define exogenous state variables:
 - φ_i productivity of seller j
 - x_t^m size of market $m \in \{h, f\}$ (Markov jump process)
 - y_{ijt}^m idiosyncratic shock to operating profits from shipment to buyer i by seller j in market m (Markov jump process)
 - Π^m profit function scalar (so that all exogenous state variables can be normalized to mean log zero)
- When buyer *i* places an order with seller *j* in market *m* it generates operating profits:

$$\pi(x_t^m, \varphi_j, y_{ijt}^m) = \Pi^m x_t^m \varphi_j^{\sigma - 1} y_{ijt}^m.$$

Superscripts and subcripts mostly suppressed hereafter:

$$\pi_{\varphi}(x,y) = \Pi x \varphi^{\eta-1} y$$



1. Relationship dynamics

value of a business relationship

- In active business relationships, buyers place orders with exogenous hazard λ^b . Details
- After each order, sellers must pay fixed cost *F* to keep a business relationship active.
- Value to a type- φ seller of a relationship in state $\{x, y\}$:

$$\widetilde{\pi}_{\varphi}(\mathbf{x}, \mathbf{y}) = \pi_{\varphi}(\mathbf{x}, \mathbf{y}) + \max \left\{ \widehat{\pi}_{\varphi}(\mathbf{x}, \mathbf{y}) - F, \mathbf{0} \right\}$$

- $\widehat{\pi}_{\varphi}(x,y)$ is continuation value to a type- φ seller of a relationship in state $\{x,y\}$ Details .
- Continuation values depend negatively on
 - \bullet δ : exogenous hazard of relationship death.
 - ullet ho : seller's discount rate (including exogenous seller death probability)

<ロ > < 回 > < 回 > < 回 > < 互 > < 互 > の Q O

1. Relationship dynamics

expected value of a new relationship

- Sellers don't know what y value their next business relationship will begin from.
- Let $Pr(y^s)$ be the probability of initial shock y^s determined by the ergodic distribution of y.
- Expected value of a successful new encounter:

$$\widetilde{\pi}_{\varphi}(\mathbf{x}) = \sum_{\mathbf{y}^{\mathbf{s}}} \Pr(\mathbf{y}^{\mathbf{s}}) \widetilde{\pi}_{\varphi}(\mathbf{x}, \mathbf{y})$$

2. Learning about product appeal

the "true" scope of the market

- Fraction of potential buyers in market m who are interested in seller j's product: $\theta_j^m \in [0,1]$ of total potential buyers.
- Assume $\theta_j^{m'}$ s are time-invariant, mutually independent draws from a beta distribution:

$$r(\theta|\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} (\theta)^{\alpha-1} (1-\theta)^{\beta-1},$$

• Expected value:

$$E(\theta|\alpha,\beta) = \frac{\alpha}{\alpha + \beta}$$
.

• Posterior beliefs, after meeting n^m potential clients in market m, a^m of whom want to do business: • Details

$$\overline{\theta}^m(a^m, n^m) = E\left[\theta^m|a^m, n^m\right] = \frac{a^m + \alpha}{n^m + \alpha + \beta}$$

3. Searching for buyers

the cost of search

- Seller continuously chooses the hazard s with which she encounters a potential buyer at a flow cost c(s,a)
 - Maintain web site
 - Pay to be near top of web search listings
 - Attend trade fairs
 - Research foreign buyers
 - Send sales reps. to foreign markets
 - Maintain foreign sales office
- The number of successful encounters, a, allows for network effects (NYT 2/27/12: Panjiva, ImportGenius).
- Functional form used for estimation (Arkolakis, 2010):

$$c(s,a) = \kappa_0 \frac{(1+s)^{(1+1/\kappa_1)} - 1}{(1+a)^{\gamma \cdot (1+1/\kappa_1)} (1+1/\kappa_1)}$$



3. Searching for buyers

the value of search abroad

- Let $V_{\varphi}(a, n, x)$ be the value of continued search for a type- φ firm with a successes in n meetings.
- The first-order for optimal search abroad is: Details

$$\begin{array}{lcl} c_s(s^*,\mathbf{a}) & = & \overline{\theta}_{\mathbf{a},\mathbf{n}}(\widetilde{\pi}_{\varphi}(\mathbf{x}) + V_{\varphi}(\mathbf{a}+1,\mathbf{n}+1,\mathbf{x})) \\ & & + (1-\overline{\theta}_{\mathbf{a},\mathbf{n}})V_{\varphi}(\mathbf{a},\mathbf{n}+1,\mathbf{x}) - V_{\varphi}(\mathbf{a},\mathbf{n},\mathbf{x}). \end{array}$$

3. Searching for buyers

when the truth is known: the domestic market

- As n increases $\overline{\theta}_{a,n}$ converges to the true θ . There is no more learning and the reward to search depends on a and n only through network effects.
- We assume this characterizes the domestic market, so the first-order condition for optimal search at home is:

$$c_s(s^*, a) = \theta_j \widetilde{\pi}_{\varphi}(x).$$

The exogenous state variables

- Assume x^f , x^h , and y follow independent Ehrenfest diffusion processes.
 - Any variable z that obeys this process is discretized into 2e+1 possible values, $e \in I^+$: $z \in \{-e\Delta, -(e-1)\Delta, ..., 0, ..., (e-1)\Delta, e\Delta\}$.
 - Process jumps with hazard λ_z , and when it does so:

$$z' = \left\{ \begin{array}{l} z + \Delta \\ z - \Delta \\ \text{other} \end{array} \right. \text{ with probability } \left\{ \begin{array}{l} \frac{1}{2} \left(1 - \frac{z}{e \triangle}\right) \\ \frac{1}{2} \left(1 + \frac{z}{e \triangle}\right) \\ 0 \end{array} \right. .$$

 As the grid becomes finer, this type of random variable asymptotes to an Ornstein-Uhlenbeck processes:

$$dz = -\mu z dt + \sigma dW$$



The exogenous state variables

- \bullet If z observed at regular intervals, can estimate μ and σ by regressing z on lagged z
- For x^f , x^h , obtain maximum likelihood estimates of μ and σ using logged and de-meaned time series on total real consumption of manufactured goods in each country.
- Recover λ_z and Δ using Shimer's result that asymptotically, $\mu = \lambda_z/e, \ \sigma = \sqrt{\lambda_z}\Delta.$
- This gives us the $q_{xx'}^X$ values needed to construct $q_{yy'}^Y$'s for home and foreign markets.
- Since *y* is unobservable, recover the parameters of its jump processes using the structure of the dynamic model.

The exogenous state variables

Market-wide Shock Processes (x^f, x^h)						
Orstein-Uhlenbeck Parameters	Colombia	United States				
μ Mean Reversion	0.171	0.174				
σ Dispersion	0.003	0.058				
Ehrenfest Process Parameters						
λ Jump Hazard	1.200	1.215				
Δ Jump Size	0.003	0.053				
grid points	15	15				

remaining parameters

- Unidentified preference parameters taken from literature: Discount rate (including 0.03 exogenous death) $\rho=$ 0.05, Demand elasticity $\sigma=$ 5
- Remaining parameters identified using indirect inference

$$\Lambda = \left(\Pi^h, \Pi^f, \delta, F, \alpha, \beta, \sigma_{\varphi}, \lambda_y, \Delta_y, \lambda_b, \gamma, \kappa_0, \kappa_1\right)$$

Indirect inference (Gouriéroux and Monfort, 1996)

- Using reduced-form auxillary regressions and/or moments, summarize key relationships in the data using a vector of statistics $(\widehat{\mathbf{M}})$
- Similar to Simulated Method of Moments difference is a bit philosophical
- For a candidate set of parameter values (Λ) , simulate same statistics using the model $\widehat{\mathbf{M}}^{s}(\Lambda)$.
- Construct the loss function:

$$Q(\Lambda) = \left(\widehat{\mathbf{M}} - \widehat{\mathbf{M}}^{\mathfrak{s}}(\Lambda)\right)' \Omega \left(\widehat{\mathbf{M}} - \widehat{\mathbf{M}}^{\mathfrak{s}}(\Lambda)\right)$$

where Ω is a positive definite weighting matrix.

• Use a robust algorithm to search parameter space for $\widehat{\Lambda} = \arg\min Q(\Lambda)$.

basic idea

Indirect inference

identification

- Profit scaling constants, (Π^h, Π^f)
 - means of log home and foreign sales
- Shipment hazards (λ^b)
 - average annual shipment rates per match
- Product appeal parameters (α, β)
 - distribution of home and foreign sales
- \bullet Firm productivity dispersion (σ_{φ})
 - · covariance of home and foreign sales
- Search cost parameters $(\kappa_0, \kappa_1, \gamma)$
 - match rates
 - client frequency distribution (especially fatness of tail)
 - client transition probabilites
 - fraction of firms that export



Indirect inference

identification

- Idioysncratic shocks to importers (λ^y, Δ^y)
 - cross-plant variances in home and foreign sales
 - covariation of home and foreign sales
 - autocorrelation, match-specific sales
 - client frequency distribution, client transition probabilites
- Match maintenance costs (F)
 - client frequency distribution, client transition probabilites
 - sales among new versus established matches
 - age-specific match failure rates
- ullet Exogenous match separation hazard (δ)
 - separation rates after first year
 - age-specific match failure rates
 - client frequency distribution



Data versus simulated statistics

Transition probs.,			Share of firms		
no. clients (n^c)	Data	Model	exporting	Data	Model
$\widehat{P}[n_{jt+1}^c = 0 n_{jt}^c = 1]$	0.618	0.650	$\widehat{E}(1_{X_{it}^f>0})$	0.299	0.359
$\widehat{P}[n_{it+1}^c = 1 n_{it}^c = 1]$	0.321	0.320	•		
$\widehat{P}[n_{jt+1}^c = 2 n_{jt}^c = 1]$	0.048	0.027	Log foreign sales on		
$\widehat{P}[n_{it+1}^c \ge 3 n_{it}^c = 1]$	0.013	0.002	log domestic sales	Data	Model
$\widehat{P}[n_{jt+1}^c = 0 n_{jt}^c = 2]$	0.271	0.443			
$\widehat{P}[n_{it+1}^c = 1 n_{it}^c = 2]$	0.375	0.339	\widehat{eta}_1^{hf}	0.727	0.877
$\widehat{P}[n_{it+1}^c = 2 n_{it}^c = 2]$	0.241	0.165	$s\widehat{e}(\epsilon^{hf})$	2.167	0.640
$\widehat{P}[n_{jt+1}^c \ge 3 n_{jt}^c = 2]$	0.113	0.052			

Data versus simulated statistics

Match death hazards	Data	Model	Exporter exit rate	Data	Model
Death rate, $A_{iit-1}^m = 0$	0.694	0.649	Exit rate, $A_{iit-1}^m = 0$	0.709	0.725
Death rate, $A_{iit-1}^{m} = 1$	0.515	0.484	Exit rate, $A_{iit-1}^{m} = 1$	0.383	0.312
Death rate, $A_{iit-1}^m = 2$					
Death rate, $A_{iit-1}^m = 3$					
Death rate, $A_{ijt-1}^{m} = 4$					

Data versus simulated statistics

Log sales per client	Ave. log sales						
vs. no. clients	Data	Model	by cohort age	Data	Model		
$\widehat{\beta}_1^m$	2.677	0.422	$\widehat{E}(\ln X_{jt}^f A_{jt}^c=0)$	8.960	10.181		
$\widehat{\beta}_2^m$	-0.143	0.317	$\widehat{E}(\ln X_{jt}^f A_{jt}^c=1)$	10.018	11.124		
$s\widehat{e}(\epsilon^{m})$	2.180	1.449	$\widehat{E}(\ln X_{it}^f A_{it}^c = 2)$	10.231	11.030		
No. clients, inverse			$\widehat{E}(\ln X_{it}^f A_{it}^c = 3)$	10.369	11.021		
CDF regression	Data	Model	$\widehat{E}(\ln X_{jt}^f A_{jt}^c \ge 4)$	10.473	11.178		
$\widehat{\beta_1}^c$	-1.667	-2.501					
$s\hat{e}(\epsilon^{n^c})$	0.066	0.048					

Data versus simulated statistics

Match death			Log match			
prob regression	Data	Model	sale autoreg.	Data	Model	
$\widehat{\beta}_0^d$	1.174	1.277	\widehat{eta}_1^f	0.811	0.849	
\widehat{eta}_0^d \widehat{eta}_1^d 1st year	0.166	0.061	eta_{1st}^f year	0.233	0.150	
$\widehat{\beta}_{Isales}^d$	-0.070	-0.055	$\widehat{se}(\epsilon^f)$	1.124	0.330	
$s\widehat{e}(\epsilon^d)$	0.453	0.441	Log dom. sales			
Match shipments			autoregression	Data	Model	
per year	Data	Model	$\widehat{\beta}_1^h$	0.976	0.907	
$\widehat{E}(n^s)$	4.824	1.416	$s\hat{e}(\epsilon^h)$	0.462	0.656	

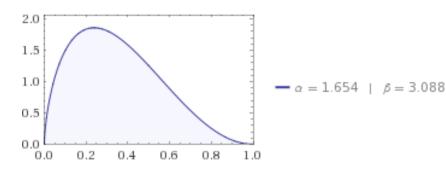
Parameters Estimated using indirect inference (\Lambda)						
	Parameter	value	std. error			
rate of exogenous separation	δ	0.65				
domestic market size	Π^h	7.914				
foreign market size	Π^f	7.363				
log fixed cost	In <i>F</i>	7.317				
First $ heta$ distribution parameter	α	1.654				
Second θ distribution parameter	β	3.088				

- fixed cost of maintaining a relationship: exp(7.317) = \$1,506.
- about $\alpha/(\alpha+\beta)=0.35$ of the potential buyers a typical exporter meets are interested in doing business
- success rates vary across exporters with standard deviation $\sqrt{\alpha\beta/\left[(\alpha+\beta)^2(\alpha+\beta+1)\right]}=0.199$



Estimated product appeal distribution:

Plot of PDF:



Parameters Estimated using indirect inference (Λ)

	Parameter	value	std. error
demand shock jump hazard	λ_y	0.501	
demand shock jump size	Δ^{y}	0.037	
shipment order arrival hazard	λ_b	1.216	
std. deviation, log firm type	$\sigma_{m{arphi}}$	0.628	
network effect parameter	$\gamma^{'}$	0.011	
search cost function curvature parameter	κ_1	0.041	
search cost function scale parameter	κ_0	132.533	

- convexity of search cost function is important
- cost of search at hazard s = 0.6: \$6,829 when a = 0; \$5614 when a = 1; \$2,888 when a = 20.
- "lock-in" effect



restricted models

restricted models

		benchmark	no learning	no network
		(Λ)	(Λ^{NL})	(Λ^{NN})
rate of exogenous separation	δ	0.267	0.516	0.119
fixed cost	F	7.957	10.238	8.539
First $ heta$ distribution parameter	α	0.716	0.512	1.807
Second $ heta$ distribution parameter	β	3.161	0.351	0.963
demand shock jump hazard	λ_y	0.532	0.713	1.581
demand shock jump size	Δ^{y}	0.087	0.060	0.087
shipment order arrival hazard	λ_b	8.836	10.028	10.347
std. deviation, log firm type	σ_{φ}	0.650	1.268	1.355
network effect parameter	γ^{\cdot}	0.298	0.112	0
fit metric	D	9.97 e+04	2.155 e+05	1.17 e+05
fit metric, no weighting	\widetilde{D}	0.117	0.182	0.143

- ullet no-learning model, treats firms as knowing their exact eta^f draws
- ullet no-network model shuts down reputation effects by imposing $\gamma=0$

The no-learning model

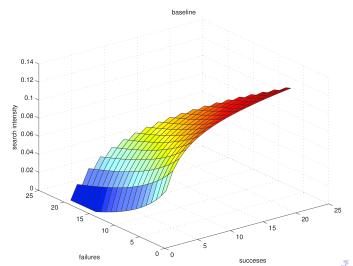
- Rapid turnover of novice exporters less likely:
 - \bullet discourages inexperienced low- θ^f firms from exploring foreign markets
 - eliminates learning-based exit.
- High- θ^f firms do not intensify their search efforts as they receive positive feedback.
- Lower productivity firms induced to participate in export markets by a
 - ullet rightward shift in eta^f distribution and
 - \bullet higher values for Π^f and λ_b
- Match failure rates and market exit rates are sustained by
 - higher values for F, δ , and λ_y .
- Model badly overstates the share of firms that export, overstates the relationship between sales per client and number of clients.

The no-network model

- Model moves part way toward matching the Pareto shape by reducing the convexity of the search cost function, κ_1 .
- This is an imperfect fix because all exporters are equally affected by κ_1 , not just the larger ones.
- Various other adjustments occur, including:
 - modest increase in F,
 - ullet rightward shift in the heta distribution, an
 - ullet increase in the variance of φ ,
 - ullet increase in the jump hazard for buyer shocks, λ_y
- Client distribution is far from Pareto: model is unable to explain the existence of very large exporters; overstates the fraction of firms that export.

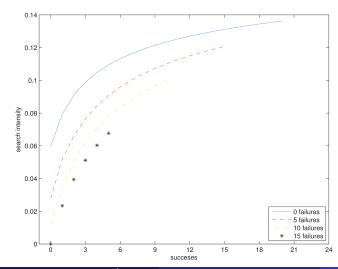
Learning and the policy function

 Fix productivity: search intensity as a function of past successes and failures



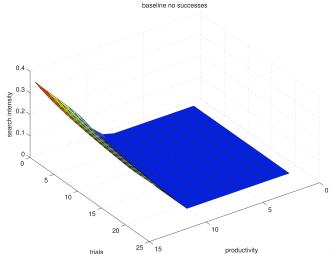
Learning and the policy function: 2D

 Fix productivity: search intensity as a function of past successes and failures



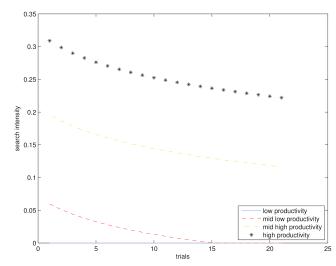
Productivity and search

• Fix successes at zero: search intensity as a function of productivity and failures



Learning and the policy function: 2D

 Fix successes at zero: search intensity as a function of productivity and failures



Incorrect permanent exits from export market

- A firm with high product appeal and bad luck will believe product appeal is low
- If belief is low enough, firm may permanently exit export market
- After exit, no more learning, mistake is never corrected

Incorrect permanent exits from export market

Full information search intensities

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0.005
0	0	0.014	0.027	0.037	0.045	0.052
0.012	0.042	0.060	0.073	0.084	0.092	0.099
0.084	0.116	0.135	0.149	0.160	0.169	0.177
0.144	0.177	0.197	0.212	0.223	0.233	0.241
0.198	0.233	0.254	0.269	0.281	0.291	0.299
0.253	0.289	0.311	0.327	0.340	0.350	0.358
0.309	0 347	0.370	0 387	0.400	0.410	0 419

Productivity

Product Appeal

Incorrect permanent exits from export market

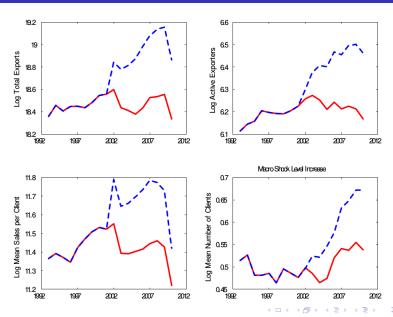
Fraction of incorrect exits after ten meetings

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	1.000
0	0	0.356	0.190	0.082	0.020	0.000
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

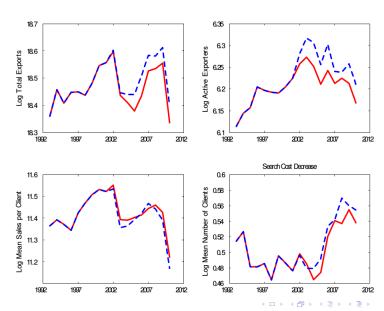
Productivity

Product Appeal

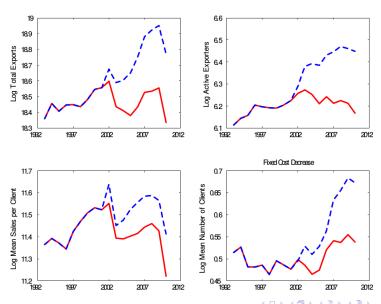
A 20% increase in foreign market size



A 20% reduction in search costs



A 20% reduction in fixed costs



Summary

- Micro patterns of transactions and buyer-seller relationships through the lens of the model:
 - Large volume of small scale exporters explained by large volume of inexperienced firms, searching at a low level.
 - High exit rate reflects short lifespan of typical match, combined with low-level search and learning about product appeal.
 - Small number of major exporters reflects combination of skewed distribution of product appeal and reputation effects.
- Search costs, multi-period matches, learning, and reputation effects combine to provide an explanation for hysteresis in trade.
 - Reputation effects appear to be particularly important.
 - Since learning is mainly relevant for new, marginal players, probably doesn't have a big effect on short-run export dynamics.

A Digression: hazards

• From the perspective of time 0, let the probability that an event will occur before time *t* be described by the exponential distribution:

$$F[t] = 1 - e^{-qt}$$

 The likelihood of the event happening exactly at t (the "hazard rate" at t) is then:

$$\frac{f(t)}{1 - F(t)} = \frac{qe^{-qt}}{e^{-qt}} = q$$

• This hazard rate doesn't depend upon t.

Hazards

• Suppose k independent events occur with hazard $q_1, q_2, ... q_k$. The probability that none occur before t is:

$$\prod_{j=1}^{k} \left(1 - F_j(t)\right) = e^{-t\Sigma_j q_j}$$

• So by time t, at least one event occurs with probability $1-e^{-t\Sigma_j q_j}$, and the likelihood that this happens exactly at t is

$$\frac{\sum_{j} q_{j} \left[e^{-t\sum_{j} q_{j}} \right]}{e^{-t\sum_{j} q_{j}}} = \sum_{j} q_{j}$$



Relationship dynamics

Markov jump processes

- x (market-wide) follows Markov jump process, hazard $q_{xx'}^X$ of transiting from state x to state x'.
- y (match-specific) follows Markov jump process, hazard $q_{yy'}^{Y}$ of transiting from state y to state y'.
- $\lambda_x^X = \sum_{x' \neq x} q_{xx'}^X$ is hazard of a change in market-wide state x
- $\lambda_y^Y = \sum_{y' \neq y} q_{yy'}^Y$ is hazard of a change in match-specific state y.
- ullet λ^b is hazard of a new purchase order from existing client.
- τ_b time until the next change in state, which occurs with hazard $\lambda^b + \lambda_x^X + \lambda_y^Y$

Relationship dynamics

the continuation value

- δ exogenous hazard of relationship death.
- ullet ρ seller's discount rate.

Continuation value of a business relationship in state (x,y) for a type- φ exporter :

$$\begin{split} \widehat{\pi}_{\varphi}(x,y) &= \mathbf{E}_{\tau_{b}} \left[e^{-(\rho+\delta)\tau_{b}} \frac{1}{\lambda^{b} + \lambda_{x}^{X} + \lambda_{y}^{Y}} \right. \\ & \left. \cdot \left(\sum_{x' \neq x} q_{xx'}^{X} \widehat{\pi}_{\varphi}(x',y) + \sum_{y' \neq y} q_{yy'}^{Y} \widehat{\pi}_{\varphi}(x,y') + \lambda^{b} \widetilde{\pi}_{\varphi}(x,y) \right) \right] \\ &= \frac{1}{h} \left(\sum_{x' \neq x} q_{xx'}^{X} \widehat{\pi}_{\varphi}(x',y) + \sum_{y' \neq y} q_{yy'}^{Y} \widehat{\pi}_{\varphi}(x,y') + \lambda^{b} \widetilde{\pi}_{\varphi}(x,y) \right) \end{split}$$

where

$$h = \rho + \delta + \lambda^b + \lambda_x^X + \lambda_y^Y$$

- Suppress market superscripts to reduce clutter.
- The **prior distribution** is:

$$r(\theta|\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} (\theta)^{\alpha-1} (1-\theta)^{\beta-1},$$

• The likelihood: Given θ , and given that a seller has met n potential buyers, the probability that a of these buyers were willing to buy her product is binomially distributed:

$$q[a|n,\theta] = \binom{n}{a} [\theta]^a [1-\theta^m]^{n-a}.$$

• The posterior distribution for θ :

$$p(\theta|a, n) \propto q[a|n, \theta] \cdot r(\theta|\alpha, \beta)$$

• The expected success rate after a successes in n trials is thus:

$$\overline{\theta}(a, n) = E[\theta|a, n] = \frac{a + \alpha}{n + \alpha + \beta}$$

Sellers base their search intensity on this posterior mean.

Searching for buyers

the value of search

The value of continued search for a type- φ firm with a successes in n meetings is:

$$\begin{split} &V_{\varphi}(\textbf{\textit{a}},\textbf{\textit{n}},\textbf{\textit{x}}) = \\ &\max_{s} \mathbf{E}_{\tau_{s}} \left[-c(s,\textbf{\textit{a}}) \int_{0}^{\tau_{s}} e^{-\rho t} dt + \frac{e^{-\rho \tau_{s}}}{s + \lambda_{x}^{X}} \cdot \left(\sum_{\textbf{\textit{x}}' \neq \textbf{\textit{x}}} q_{\textbf{\textit{x}}\textbf{\textit{x}}'}^{X} V_{\varphi,}(\textbf{\textit{a}},\textbf{\textit{n}},\textbf{\textit{x}}') \right. \\ &+ s \left[\overline{\theta}_{\textbf{\textit{a}},\textbf{\textit{n}}}(\widetilde{\pi}_{\varphi}(\textbf{\textit{x}}) + V_{\varphi}(\textbf{\textit{a}}+1,\textbf{\textit{n}}+1,\textbf{\textit{x}}) + (1 - \overline{\theta}_{\textbf{\textit{a}},\textbf{\textit{n}}}) V_{\varphi}(\textbf{\textit{a}},\textbf{\textit{n}}+1,\textbf{\textit{x}}) \right] \right) \end{split}$$

where:

- $\lambda_x^X = \sum_{x' \neq x} q_{xx'}^X$ is the hazard of any change in the market-wide state Χ.
- \bullet τ_s is the random time until the next search event, which occurs with hazard $s + \lambda_{\times}^{X}$.

Taking expectations over τ_s yields:

$$V_{\varphi}(a, n, x)$$

$$= \max_{s} \frac{1}{\rho + s + \lambda_{x}^{X}} \left[-c(s, a) + \sum_{x' \neq x} q_{xx'}^{X} V_{\varphi,}(a, n, x') + s \left\{ \overline{\theta}_{a, n} \left[\widetilde{\pi}_{\varphi}(x) + V_{\varphi}(a + 1, n + 1, x) \right] + (1 - \overline{\theta}_{a, n}) V_{\varphi}(a, n + 1, x) \right\} \right]$$

The first-order condition is thus:

$$\begin{array}{lcl} c_s(s^*,a) & = & \overline{\theta}_{a,n}(\widetilde{\pi}_{\varphi}(x) + V_{\varphi}(a+1,n+1,x)) \\ & & + (1-\overline{\theta}_{a,n})V_{\varphi}(a,n+1,x) - V_{\varphi}(a,n,x). \end{array}$$

◆ロト ◆団 ト ◆ 豆 ト ◆ 豆 ・ からぐ

when the truth is known: the domestic market

- In the domestic market the reward to search depends on a and n only through network effects.
- The value of search at home is thus simply:

$$V_{\varphi}(x) = \max_{s} \frac{1}{\rho + s + \lambda_{x}^{X}} \left[-c(s, a) + \sum_{x' \neq x} q_{xx'}^{X} V_{\varphi}(x') + s\theta_{j} \widetilde{\pi}_{\varphi}(x) \right]$$

The associated first-order condition is:

$$c_s(s^*, a) = \theta_j \widetilde{\pi}_{\varphi}(x).$$

