A Search and Learning Model of Export Dynamics*

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Abstract

We develop a model of firm-level export dynamics that features costly customer search, endogenous terminations of buyer-seller relationships, and learning on both sides of the market. Specifically, firms update their beliefs about their products' popularity with each potential customer they meet (”learning effects”), and potential customers update their awareness of a firm's products with each successful buyer-seller match the firm makes (”visibility effects”).

We fit this model to customs records describing U.S. manufactured imports from Colombia. Identification comes from several key assumptions: buyer-seller meetings that generate only one shipment reflect unsuccessful encounters (rejected samples), multi-shipment pairings reflect successful buyer-seller relationships, and for a given firm, the time gap between its new meetings with potential buyers reflects its search intensity (up to a random shock).

Qualitatively, the model replicates a large set of patterns in the customs records regarding exporter maturation, transitions across states, and the distribution of partner counts across firms. It also provides a basis for assessing the value of business relationships, the importance of both types of learning, the role of luck in market penetration, and the key drivers of aggregate export dynamics. Inter alia, we find that learning and visibility effects are both statistically significant, but the former are more important than the latter, and while they both affect the maturation processes of new exporters, neither matters as much as search frictions in shaping aggregate export dynamics.

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1 Introduction

What drives firm-level export dynamics? In recent years, various papers have explored the effects of market entry costs, learning, search frictions, and reputation-building. We take this literature one step further by developing a dynamic model that simultaneously incorporates all of these elements and quantifies their individual effects on exporters' behavior. Further, we do so by focussing on the formation and maturation of exporters' relationships with individual foreign buyers. Fit to customs records, our model allows us to quantify the various exporting costs faced by different types of firms, as well as the value of the intangible capital they have amassed by building their portfolios of foreign clients. It also allows us to characterize the effects of informational and matching frictions on aggregate export dynamics.

We recognize and separately quantify three types of trade frictions in our analysis. The first type is standard: exporters must engage in costly search to identify potential clients abroad. Since search costs are convex in match rates, this forces firms to gradually build their portfolio of foreign buyers. The second type of friction arises from sellers' limited knowledge of foreign buyers' tastes. Potential exporters are unsure about the appeal of their products in foreign markets, but they gradually learn about this as they meet potential customers. Therefore, exporters with appealing products intensify their marketing efforts as they receive positive reinforcement, and they expand relatively quickly. The final type of friction has to do with buyers learning about sellers. Exporters that have already established a large number of business relationships are relatively visible to other buyers, so for a given level of spending on search, they meet relatively more potential customers. Thus, other things equal, new exporters add clients relatively slowly, and the per unit cost of replacing failing business relationships are relatively small at well-established firms.

We base our analysis on the cross-sectional and temporal variation in shipment-level customs records describing U.S. manufactured imports from Colombia. We begin by summarizing the main patterns in these data that we want our model to explain, including the dynamics of seller matching patterns and the life-cycle revenue trajectories generated by indiviudal matches. Then we develop a dynamic search and matching model in which firms expand by adding to their client base at home and in foreign markets. Finally, we fit this model to our customs data and explore its implications.

When bringing our model to data, we rely on a simple identification strategy. Specifically, we define a buyer-seller match to have occurred whenever we observe an initial shipment between a particular exporting firm and a particular importing firm. But if the exporter makes no further shipments to the importer, we say the match was a failure. That is, we view the first shipment as a sample, and we define *successful* matches to be those that result in at least one additional shipment. These assumptions are consistent with the large number of one-off shipments we observe between sellers and buyers, and they render several key variables in our model observable for each exporter at each point in time, namely, its match arrival rate, its cumulative number of matches, and its success rate.

The model qualitatively replicates a large set of patterns in the customs regarding exporter maturation, transitions across states, and the distribution of partner counts across firms. It also delivers a variety of implications regarding exporter behavior. First, the aggregate value of foreign business connections amounts to roughly US$28 billion dollars for Colombian manufacturers, with firm-specific values heavily dependent on productivity, product appeal, and portfolio of clients. Second, there is some scope for luck in the establishment of a new exporting firm, since a few successful matches early in its market exploration can encourage it to search more intensely and thereby to improve its visibility. Third, as new exporters meet potential customers, the knowledge they acquire about their product's appeal is more valuable to them than the heightened visibility they acheive by expanding their client base. Fourth, because of the various trade frictions in our model, aggregate export responses to permanent exchange rate shocks are more muted and gradual in our model than in other treatments. Finally, both learning effects and market visibility effects are statistically significant, but their effect on aggregate export dynamics is modest compared to the effects of search frictions. This is because they mainly affect the maturation processes of new exporters, which account for a small share of total export volumes.

1.1 Relation to literature

Calibrated GE models: Broadly speaking, our paper concerns firm dynamics in open economies. As such, it connects to a number of strands of the trade literature. The first uses calibrated general equilibrium models with productivity shocks that move firms through the size distribution. These include Alessandria and Choi (2007, 2014), Ruhl (2008), Atkeson and Burstein (2010), Burstein and Melitz (2013), Impullitti et al. (2013), Drozd and Nozal (2012), and Arkolakis (2015). We depart from all of these papers by focussing on the market knowledge and visibility that firms reap by investing in business relationships with new clients.

Our model does, however, share some features with these papers. Among them, Arkolakis (2015) is perhaps closest to ours, in that he uses convex market penetration costs to generate a number of stylized facts, including the age-dependence of export growth rates. However, since the exporting decision is static in his model and learning is absent, it does not explain the irreversibilities observed in firms' exporting behavior, nor does it speak to the duration of matches. Drozd and Nozal (2012) are also relatively close to us in the sense that they treat firms as building their foreign market shares gradually through a costly search and matching process. However, they do this using a representative agent RBC model that abstracts from firm heterogeneity, thereby missing most of the patterns in the firm-level data we focus upon.

Single-agent models with a customer margin: Another strand of the trade literature uses single-agent models to explore foreign customer accumulation in more detail. Like ours, the models in these papers are econometrically estimated using customs records. Fitzgerald et al. (2019) exploit Irish records to investigate whether firms build their customer base through non-price marketing efforts versus price discounts to establish a market presence. They find little role for discounting, but an important role for the customer base. Similarly, Pivetau (forthcoming) uses French data to investigate the importance of discounts versus non-price efforts as mechansims for building a foreign customer base. Unlike Fitzgerald et al. (2019), he finds a tendency for firms to discount their goods when they are new to a market, but he confirms that a large existing customer base improves product awareness in the remainder of the population and conveys an important advantage. Our model is perhaps closest to these papers, in that it is also a single-agent model fit to customs records. But we differ from them in that we study the joint evolution of home and foreign sales for each firm, allowing for endogenous entry and exit in both markets, learning effects, and search frictions. Also we characterize the life-cycle of successful matches, including their endogenous dissolution.[1]

Market equilibria models with matching Addional papers with a foreign consumer margin shut down dynamics in order to analyze assortative matching patterns between exporters and importers in particular markets. Blum et al. (2010, 2013), Bernard et al. (2018) and Sugita et al. (2019) explore the buyer-seller matching patterns that emerge in a full-information world with productivity heterogeneity on both sides of the market, fixed matching costs, and (in some cases) production complementarities.[2]

Eaton et al. (2016) also model market equilibria with 2-sided matching, but in a dynamic context. Their formulation lacks a mechanism for assortative matching, but it treats both buyers and sellers as building client portfolios by searching for each other, subject to matching frictions.[3] Fit to data on U.S. apparel trade, this model confirms an important role for existing business relationships as determinants of market visibility. In this respect, and because it is a search model with a customer margin, it resembles the present paper. But like the static models, it does not allow for learning effects, nor does it characterize the life

[1]From a very different perspective, Chaney (2014) uses customs records to model international customer accumulation as a contagion penomhenon, with exporters tending to break into markets that are geographically close to those that are they already service. The contagion processes are not based on optimizing behavior, however.

[2]Bernard and Moxnes (2018) provide a useful summary of the literature on networks and trade.

[3]An appendix shows how assortative matching can be added.

cycle of matches once they have been formed.

Search and trade Matching and/or screening frictions appear in a number of other trade models that are too abstract to bring directly to data, but are supported by reduced-form evidence. In Rauch and Watson (2003), importers experiment with foreign suppliers by placing trial orders with them, and they gain access to a supplier network if they establish a successful business relationship. We take the assumption that importers evaluate a sample shipment before forming business relationships from this formulation. In Albornoz et al. (2012), firms choose to experiment in markets with low entry costs in order to learn about their product's appeal elsewhere. Similarly, in Nguyen (2012), firms learn about idiosyncratic demands for their products by "testing" a subset of markets, each of which generates a correlated signal about unexplored markets. Although we focus on a single destination market, our model resembles these in that exporters learn about their products' through early shipments, and when new exporters survive, their sales tend to grow rapidly. Aeberhardt et al. (2014) and Araujo et al. (2016) explain the small scale and high exit rate of new exporters (inter alia) by assuming that exporters are initially uncertain about the reliability of their new buyers, whom they find through a random matching process. The learning mechanisms built into these models are different from ours, but they too are designed to capture some of the same patterns that we target.

Learning models: Other learning models of export dynamics are designed for structural estimation. Timoshenko (2015) adds age as a profit shifter to a standard sunk-cost model of exporter behavior. She then demonstrates that this variable helps explain exporter persistence in differentiated (but not homogeneous) product industries and interprets this to imply that learning effects are important.[4] Drawing from Jovanovic (1982), Arkolakis, et al. (2018) characterize firms as learning their types by observing their sales histories. This allows them to explain the relatively rapid expansion of new exporters. This paper resembles ours in that firms learn about their products' appeal in export markets from their match histories. It differs in that it is a calibrated equilibrium model with a representative consumer and no matching frictions or network effects. Li (2018) adds time variation to the idiosyncratic productivity shocks in Eaton et al (2014), replaces endogenous search efforts with a Markov process on the number of foreign orders firms receive, and treats firms as learning about their demand per order in each destination market. With these adjustments, he then quantifies the effects of priors, productivity, and learning on export market participation. Finally, without developing a complete model, Berman et al. (2019) use a simple decomposition to isolate residual fluctuation in foreign demand that they attribute to learning. Like the other

[4]This paper is not strictly structural in the sense that it uses a reduced-form approximation to exporters' value functions.

studies mentioned here, they find learning plays an important role.[5]

Micro-founded models of aggregate trade fluctuations: Finally, our model allows us to explore the microfoundations of aggregate export dynamics in the presence of several types of frictions. Alessandria and Choi (2014), Alessandria et al. (2014), and Alessandria et al. (2018) use foreign market entry costs to induce forward-looking behavior, with the last paper adding endogenous variable exporting costs. Ruhl and Willis (2017) extend the standard sunk cost model by assuming that firm-specific foreign demand grows with years of export market experience. This allows them to explain the fact that exporters typically start small, reduces estimates of sunk entry costs, and dampens simulated export responses to an exchange rate shock over medium-term horizons. Piveteau (forthoming) also explores these relationships using his model of export dynamics (discussed above). Our model can be thought of as providing a particular micro foundation for the dependence of exporting volumes on years in the market.

2 Firm-Level Trade: Transaction Level Evidence

Over the past fifteen years, a robust set of stylized facts has emerged regarding the foreign market entry and evolution of exporting firms.[6] The model we develop is designed to explain these facts as they manifest themselves in Colombian shipments to the United States. So we begin by summarizing the patterns of interest in these transactions.

2.1 Data

We base our analysis on a comprehensive data set that describes all imports by buyers in the United States from Colombian exporters during the period 1992-2009. It is an extract from the U.S. Census Bureau's Longitudinal Foreign Trade Transactions Database (LFTTD), which covers all commercial shipments into and out of the United States. Each record includes a date, the US dollar value of the product shipped, a 6-digit harmonized system product code, a quantity index, and, critically, ID codes for both sellers and buyers. These IDs allow us to identify the formation and dissolution of business relationships between

[5]Ruhl and Willis (2017) also note this pattern in plant-level export data and show that market entry costs are insufficient to explain it.

[6]Early contributions include Brooks (2006), Besedes (2008), and Eaton et al. (2008). The interested reader can find many of the more recent studies mentioned in Bernard et al. (2017).

individual buyers in the U.S. and sellers in Colombia, hereafter referred to as "matches."[7]

To identify foreign exporters, the U.S. import transactions records include a manufacturer's identification code.[8] This field is an amalgamation of the manufacturer's country, company name, street address, and city. Anecdotal information from customs brokers indicates that commonly used software constructs it automatically as the name and address information that is entered in other fields. So this variable is sensitive to differences in the way exporters' names and addresses are recorded as they pass through customs, and shipments from the same exporter can appear to originate from distinct Colombian firms. To gauge the importance of this problem, we have conducted various checks on the matches that are based on this variable; these are explained in Appendix B.

We limit our analysis to transactions between non-affiliated trade partners, and we consider only imports of manufactured goods. The latter restriction notably excludes oil and coffee exports, which constitute the bulk of trade between the two countries and are dominated by a few Colombian sellers.[9] Our final data set of manufacturing transactions spans the years 1992-2009. It contains 26,625 unique Colombian exporters, 12,921 unique U.S. importers, and 42,767 unique trading pairs. Value data have been deflated to 1992 prices using the U.S. CPI. Since we exclude a number of large HS codes from our data, as well as affiliated trade, and because we also lose information due to disclosure restrictions, the total value covered by our data is not comparable to total Colombian exports to the U.S. Table 18 in Appendix B compares patterns in our sample to patterns in official aggregates from both the U.S. and Colombia.

In addition to U.S. customs records, we use establishment level survey data from Colombia's national statistics agency (Departmento Administrativo Nacional de Estadistica, or DANE). These data provide annual information on the sales volumes, exports, and other characteristics of all Colombian manufacturing plants with at least 10 workers. Because they have been widely analyzed, we do not discuss summary statistics for this data set herein. Later, however, when estimating our search and learning model, we use such statistics to characterize the size distribution of Colombian firms, the fraction of Colombian plants that export and, among these firms, the relationship between exports and domestic sales.

[7]There are two ways to track U.S. importers in the LFTTD: Employment Identification Numbers (EINs) and the firm identifiers in the Longitudinal Business Database ("alphas"). Though an EIN does not necessarily identify a complete firm, it is unique to a firm, and there is one associated with every import transaction. Alphas map to entire firms, but the match rate between trade transactions and alphas is only about 80 percent (Bernard, Jensen, and Schott, 2009). To maximize the coverage of our sample, we use Employment Identification Numbers (EIN) to identify U.S. buyers.

[8]This variable is based on Block 13 of CBP form 7501, the import declaration form and customs brokers are required to input the data.

[9]Colombian commercialization of coffee is centralized to an important degree by the National Federation of Coffee Growers. A few players also dominate oil exports.

Table 1: Average aggregates by cohort age

Cohort age	Exporters	Total Exports	Average Exports
1 year	1	1	1
2 years	0.29	1.11	3.77
3 years	0.18	0.93	5.03
4 years	0.14	0.67	4.66
5 years	0.12	0.63	5.18
6 years	0.10	0.51	4.99
7 years	0.08	0.50	5.72
8 years	0.08	0.45	5.91
9 years	0.07	0.39	5.58
10 years	0.06	0.40	6.58

Notes: Based on LFTTD customs records, U.S. imports of manufactured goods from Colombia, 1992-2009. Figures for cohorts aged 2-10 are expressed relative to corresponding figures for one-year-old cohorts.

2.2 Exporter cohort maturation

Following Brooks (2006), we begin with Table 1, which summarizes the typical cohort maturation process for Colombian exporters of manufactured goods to the United States. It is based on observed evolution patterns among cohorts of firms that entered the market each year between 1993 and 1999, and it exploits U.S. customs records from 1992 through 2009.

To interpret the figures in this table, imagine for a moment that they describe a particular cohort, say, those Colombian firms that first entered the U.S. market in 1993. Then the second row of the Table would imply that only 29 percent of these firms continued exporting though 1994 (column 1), yet these survivors generated 11 percent more export revenue in 1994 than the entire cohort did in 1993 (column 2) because sales per surviving cohort member were 3.77 times as large in 1994 as sales per cohort member in 1993 (column 3). Other rows would have analogous interpretations, each normalized relative to the cohort's entry year.

The actual interpretation for Table 1 differs from this one only in that it is an average of all of the cohort-specific tables we can construct using cohorts observed for at least 10 years.[10] Taking averages saves space but does not affect the basic message, since maturation patterns vary little across cohorts (Appendix tables A.1-A.3).

Column 1 of Table 1 shows the rate of decline in cohort membership is especially high between the first and second year, with more than 70 percent of firms dropping out. But conditional on making it to the second year, the survival probability is much higher, with an attrition rate around 40 percent the second year, and further declines occur thereafter.

[10]Similar tables for Colombian exports of all goods and to all destinations appear in Eaton, et al (2008).

Thus, in terms of numbers, the most recent cohort is always larger than any previous one, and exporters with more than 15 years of market tenure are rare. For example, firms that were exporting to the United States in 1992 account for less than five percent of the firms exporting to the United States towards the end of the sample.

Column 2 shows that the rapid initial decline in its membership is not accompanied by a similar collapse in total cohort sales. The relative stability of total sales means that sales per firm are growing substantially. Indeed, as can been seen in column 3, sales per surviving exporter more than triple from the first to the second year, increase again in the cohort's third year, and show no strong tendency to grow further or shrink thereafter.

2.3 Patterns of buyer-seller matches

We next use the LFTTD data to characterize the buyer-seller matches that took place during 1992-2009. For compatibility with earlier papers, we do not distinguish here between matches that generated more than one shipment ("successful" matches, by our definition) and those that did not.

2.3.1 Monogamous and polygamous matches

The number of Colombian exporters appearing in our sample grew at roughly 2 percent per annum, from 2,232 in 1992 to 3,300 in 2009, while the number of U.S. importing firms grew by 3 percent per annum, from 1,190 to 2,079 (Appendix A, Table 17). The number of Colombian exporter-U.S. importer pairs (representing at least one transaction between them in a year) also grew at an annual rate of 2 percent. Roughly 80 percent of matches are monogamous in the sense that the buyer deals with only one Colombian exporter and the exporter ships to only one buyer in the United States. However, since the remainder of the matches are polygamous, the average Colombian exporter was involved in relationships with around 1.3 U.S. firms while the average U.S. buyer was involved with around 2.3 Colombian firms. Both figures declined slightly over the period.

2.3.2 Transition Probabilities

Like firms' exporting stints (Table 1), most of their buyer-seller matches are short-lived. Even among those matches involving more than one shipment, the overall year-to-year death rate is roughly 40 percent, as we will show later. Not surprisingly, therefore, there is a great deal of flux in exporters' portfolio of clients.

Table 2 reports the probability with which a Colombian exporter transitions to a different number of clients the following year. Therein, a firm that stops exporting but re-appears as an exporter sometime later in our sample period is considered to have gone "dormant", while those exporters that drop to zero foreign sales for the extent of our sample are considered to have gone "out" of exporting. Those that have ever been observed to export constitute the pool of potential entrants.

Among first-time exporters, roughly 93 percent sell to only one firm.[11] Of these, 62 percent don't export the next year, and only 6 percent go on to establish a larger number of relationships. For firms with 3 relationships in a year, 12 percent enter into a larger number of relationships the next year, but 67 percent lose clients. Similar patterns obtain for firms starting with other client counts. Hence, in addition to an enormous amount of churning among smaller exporters, we see a general tendency for firms to lose clients on net from one year to the next.

2.3.3 Ergodic degree distribution

We can ask what this pattern of entry and growth implies about the ergodic distribution of relationships. If we assume that entrants in a year replace exiting firms, the ergodic distribution implied by this transition matrix is given by Table 3.

For purposes of comparison, the year-specific average share of Colombian firms in each group is reported as well. Note that the ergodic distribution implied by the transition matrix is very close to the cross-sectional distribution in the data, suggesting that over the period we observe the process has been quite stationary. Interestingly, both distributions are very nearly Pareto, reflecting the coexistence of many small scale exporters with a few "super-exporters."

2.3.4 Match maturation

The survival probability of new matches increases with initial sales volume. Table 4 sorts observations on matches according to their size in their first year of existence and reports year-to-year separation rates. In addition to the very low survival rates, two patterns stand out. First, those matches that begin with sales in the top quartile among all new matches are more likely to survive than matches that begin with smaller sales volumes. Second, survival probabilities improve after the initial year.

Further features of the match maturation process are evident in Figure 1, which shows average annual sales per match, broken down by initial size quartile. For each size quartile, matches are further distinguished according to their total life span: less than one year (life=0), 1 to 2 years (life=1), and so forth. And for each cluster of bars, the left-most bar corresponds to sales in the initial year of the match's existence, the next bar corresponds to sales during the second year of the match's existence, and so forth.

[11]Many of these matches involve a single shipment. As we will show later, the overall match success rate (i.e., multi-shipment rate) is roughly 41 percent.

Table 2: Transition Probabilities, Number of Clients

t \t+1	Out	Dormant	1	2	3	4	5	6-10	11+
Out	.	.	0.932	0.055	0.009	0.002	0.001	0.001	0.000
Dormant	.	.	0.876	0.100	0.015	0.008	.	.	0.000
1	0.539	0.080	0.321	0.048	0.010	0.002	.	0.001	.
2	0.194	0.077	0.375	0.241	.	0.024	0.009	0.004	.
3	0.090	0.042	0.220	0.271	0.210	0.092	.	0.027	.
4	0.059	.	0.129	0.216	0.215	0.184	0.083	0.095	.
5	.	.	0.095	0.184	0.181	0.181	0.126	0.178	.
6-10	.	.	0.039	0.073	0.089	0.123	0.157	0.419	0.073
11+	.	0.000	0.000	0.000	.	.	.	0.432	0.526

Notes: Based on LFTTD customs records, U.S. imports of manufactured goods from Colombia, 1992-2009. Figures are cross-year averages of annual transition rates. Confidentiality restrictions prevent us from reporting numbers for cells that are too sparsely populated.

Table 3: Ergodic Client Distribution Implied by Transitions

	1	2	3	4	5	6-10	11+
Erg Distribution	0.792	0.112	0.031	0.016	0.009	0.022	0.016
Data	0.778	0.116	0.043	0.021	0.011	.	.

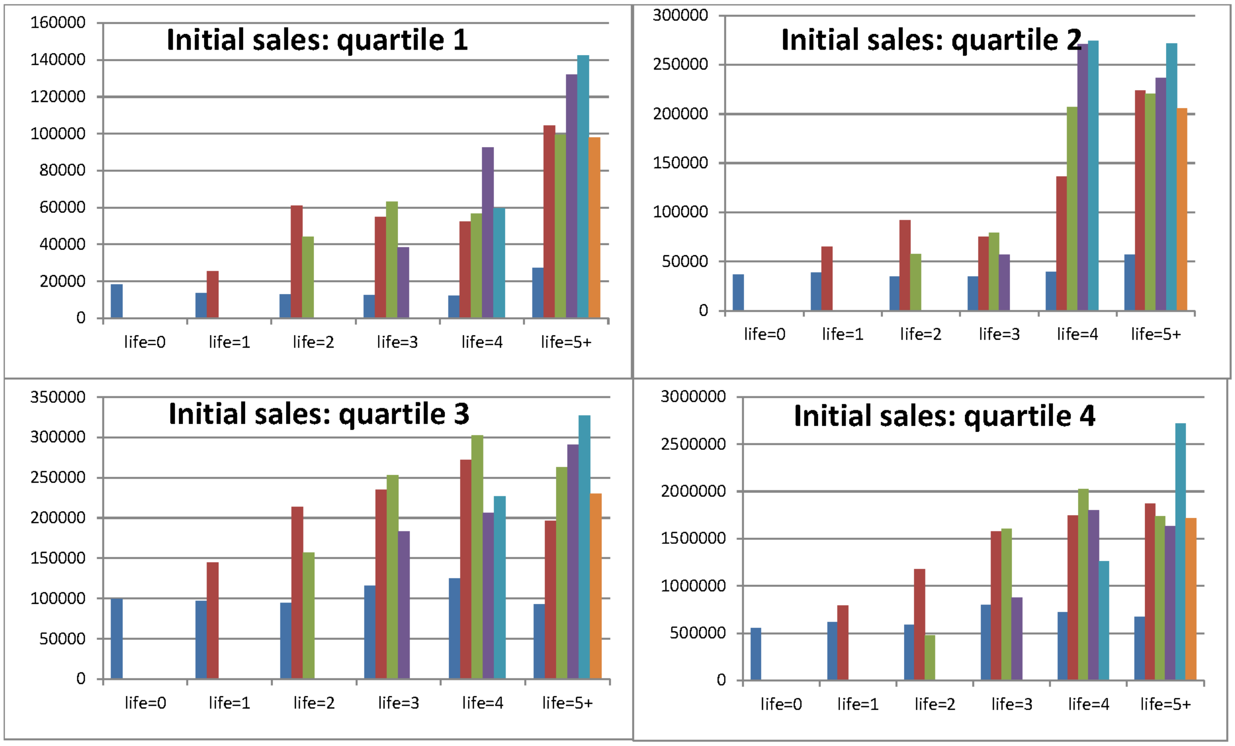
Notes: Based on transition probabilities reported in Table 2

Table 4: Separation Rates, by Age of Match and Initial Sales

	1 year	2 years	3 years	4 years	5+ years
Quartile 1	82.9	63.2	57.3	55.0	49.7
Quartile 2	75.6	58.4	49.4	46.8	43.7
Quartile 3	67.7	52.1	44.6	40.8	37.6
Quartile 4	52.1	44.5	40.3	39.2	36.7

Notes: Based on LFTTD customs records, U.S. imports of manufactured goods from Colombia, 1992-2009.

Figure 1: Average annual sales per match, by initial size quartile



Notes: Based on LFTTD customs records for manufactured goods imported from Colombia, 1992-2009.

The first message of these graphs is that initial sales are a good predictor of sales in subsequent years, conditioning on survival. Those matches with first-year sales in the smallest quartile systematically generated the lowest annual sales in subsequent years, and more generally, first-year sales are monotonically related to annual sales in subsequent years. (Note the different scales of the vertical axes in different panels of Figure 1.) Second, sales tend to jump from the first to the second year, in large part because observations on a match's first year correspond to less than a full calendar year. (There is an analogous effect at work in the final year of a match's life.) Looking at complete-year observations reveals a tendency for annual sales to grow among matches that start small and survive, but no such tendency among matches that start in the largest quartile. Finally, looking across matches with different life spans, those that survive more years tend to have higher sales in all (full) years than matches that fail relatively quickly. This pattern is robust across matches in the different quartiles for initial sales.

3 A Model of Exporting at the Transactions Level

We now develop a model of exporter behavior consistent with the patterns reviewed above. Buyer-seller relationships form and disband at irregular intervals. Similarly, export shipments are discrete events distributed unevenly through time. To capture these features of the data, and to allow agents to update their behavior each time their circumstances change, we formulate our model in continuous time, treating all of the exogenous random variables in our model as Markov jump processes.

Explaining the evolution of a firm's exports and domestic sales requires modeling both its sales to existing buyers and the evolution of its portfolio of clients. We can treat these two components sequentially. We first consider the relationship between a seller and an individual buyer. Having characterized the seller's profits from a relationship with an individual buyer, we then turn to her learning about the popularity of her product, i.e., the chance that a potential buyers likes her product. Finally, we characterize her search for buyers.

3.1 A Seller-Buyer Relationship

This section characterizes the profit streams that sellers generate from successful business relationships. The expressions we develop here describe relationships between domestic firms and foreign buyers, but with appropriate relabelling of market-wide variables they apply equally to relationships between domestic firms and domestic buyers.

3.1.1 Profits from a single shipment

Several features of our model are standard. First, at any time t seller j can hire workers at a wage w_t in real local currency units, each of whom can produce $\varphi_j \in \{\varphi^1, .., \varphi^{N_\varphi}\}$ units of output.[12] Hence seller j's unit cost in local currency is w_t/φ_j. If she sells at price p_{jt} in foreign currency her unit profit in local currency is

$$p_{jt}/e_t - w_t/\varphi_j, \tag{1}$$

where e_t is the exchange rate. Second, goods markets are monopolistically competitive and each producer supplies a unique differentiated product.

Once buyer i has agreed to form a business relationship with seller j, he periodically places sales orders with j. For j, an order from i that arrives at time t generates revenue:

$$X_{ijt} = \left(\frac{p_{jt}}{P_t}\right)^{1-\eta} y_{ijt}\overline{X}_t, \tag{2}$$

[12] We treat φ as time-invariant to facilitate model identification. Other sources of idiosyncratic temporal variation in sales will be discussed shortly.

where $\eta > 1$ is buyers' elasticity of demand, p_{jt} is the price of seller j's product, $\overline{X}_t$ is the average spending level among all potential foreign buyers, P_t is the relevant price index for all competing products in the foreign market, and $y_{ijt} \in \{y^1, .., y^{N_y}\}$ is a time-varying demand shifter idiosyncratic to the ij relationship.[13]

For simplicity, and to keep the analysis as close as possible to other heterogenous firm models, we assume that the seller posts a non-negotiable price, charging the optimal markup over unit cost:[14]

$$p_{jt} = \frac{\eta}{\eta - 1} \frac{e_t w_t}{\varphi_j} \tag{3}$$

By (1), (2), and (3), an order from buyer i at time t therefore generates the following profits for seller j:

$$\pi_{ijt} = \frac{1}{\eta} \frac{\overline{X}_t}{e_t} \left(\frac{e_t w_t \eta/(\eta - 1)}{\varphi_j P_t} \right)^{1-\eta} y_{ijt}.$$

We can combine all the macroeconomic variables affecting the profit of any seller from this source selling in this destination, along with constants, as:

$$x_t = \frac{1}{\eta} \frac{\overline{X}_t}{e} \left(\frac{e_t w_t \eta/(\eta - 1)}{P_t} \right)^{1-\eta},$$

where $x \in \{x^1, .., x^{N_x}\}$ is general to all potential buyers in the foreign market. Suppressing subscripts on state variables, this allows us to write the profits from a sale as:

$$\pi_\varphi(x, y) = x\varphi^{\eta-1}y, \tag{4}$$

In what follows, equation (4) is all we take from our specification of preferences and pricing behavior into the dynamic analysis. Any set of assumptions that deliver this simple multiplicative expression for a firm's profit from a sale would serve us equally well.

[13] Not all buyers necessarily face the same range of goods and hence the same aggregate price index P. We treat idiosyncratic components of the price index as P as reflected in y_{ijt}.

[14] Alternative specifications include bilateral bargaining between buyer and seller, as in Eaton et al. (2016), and pricing rules that recognize a link between current sales volume and futrure growth in customer base, as in Fitzgerald et al. (2019) and PIveteau (forthcoming). To keep our model tractable, and in view of Fitzgerald et al.'s (2019) finding that exporters' prices don't covary with market tenure, we opt for constant mark-up pricing.

3.1.2 Relationship dynamics

At any point in time, each seller maintains business relationships with an endogenous number of buyers. These relationships form as a consequence of a search process that will be characterized in the following section, and they dissolve for several reasons. First, there is a constant exogenous hazard δ that any particular relationship will terminate, which could be due to the demise of the buyer or the buyer no longer finding the seller's product useful. Second, after each sale to a particular buyer, the seller evaluates whether it is worth sustaining her relationship with him. Doing so keeps the possibility of future sales to him alive, but it also means paying the fixed costs F of maintaining the account, providing technical support, and maintaining client-specific product adjustments.[15]

When deciding whether to maintain a particular business relationship, the seller knows her own type, φ, the macro state, x and profits from the current sale, $\pi_\varphi(x, y)$ to the buyer in question. She can therefore infer this buyer's current y value and calculate the value of her relationship with him to be:

$$\widetilde{\pi}_\varphi(x, y) = \pi_\varphi(x, y) + \max\left\{\widehat{\pi}_\varphi(x, y) - F, 0\right\}.$$

Here $\widehat{\pi}_\varphi(x, y)$ is the expected value of continuing a relationship that is currently in state (x, y). Clearly the seller terminates this relationship if $\widehat{\pi}_\varphi(x, y) < F$.

If a seller pays F to keep a relationship active, and if the relationship does not end anyway for exogenous reasons, one of several events will next affect it: with hazard λ^b the buyer will place another order, with hazard $q^X_{xx'}$ x will jump to some new marketwide state $x' \neq x$, or with hazard $q^Y_{yy'}$ y will jump to some new buyer-specific shock $y' \neq y$.[16] Let τ_b be the random time that elapses until one of these events occurs. Given that x and y are Markov jump processes, τ_b is distributed exponentially with parameter $\lambda^b + \lambda^X_x + \lambda^Y_y$, where

$$\lambda^X_x = \sum_{x' \neq x} q^X_{xx'} \tag{5}$$

and

$$\lambda^Y_y = \sum_{y' \neq y} q^Y_{yy'}, \tag{6}$$

[15] For instance, Colombian producers of construction materials interviewed for a related project (Domínguez et al, 2013) mentioned that it is common for foreign buyers to request adjustments in the specifications of products or packages. In turn, these require adjustments in the production process that are costly to maintain.

[16] Since sales in the data are discrete events rather than flows, we model the buyer's purchases accordingly. We think of the buyer not as making use of the products continually but in discrete spurts. For example, the buyer might be a producer of a product that it makes in batches. At the completion of each batch it buys inputs for the next batch.

are the hazards of transiting from x to any $x' \neq x$, and from y to any $y' \neq y$, respectively. Then assuming the seller has a discount factor ρ, the continuation value $\widehat{\pi}_\varphi(x, y)$ solves the Bellman equation:

$$\begin{aligned}
\widehat{\pi}_\varphi(x, y) &= \mathbf{E}_{\tau_b}\left[e^{-(\rho+\delta)\tau_b}\frac{1}{\lambda^b + \lambda_x^X + \lambda_y^Y}\left(\sum_{x'\neq x} q_{xx'}^X \widehat{\pi}_\varphi(x', y) + \sum_{y'\neq y} q_{yy'}^Y \widehat{\pi}_\varphi(x, y') + \lambda^b \widetilde{\pi}_\varphi(x, y)\right)\right] \\
&= \frac{1}{\rho + \delta + \lambda^b + \lambda_x^X + \lambda_y^Y}\left(\sum_{x'\neq x} q_{xx'}^X \widehat{\pi}_\varphi(x', y) + \sum_{y'\neq y} q_{yy'}^Y \widehat{\pi}_\varphi(x, y') + \lambda^b \widetilde{\pi}_\varphi(x, y)\right)
\end{aligned}$$

Before a seller has met her next buyer, she does not know what state y this buyer will happen to be in. So when choosing her search intensity for new business relationships, she must base her decisions on the ex ante expected pay-off to forming a new business relationship. Given the market state x, a type-φ seller calculates this expected value as:

$$\widetilde{\pi}_\varphi(x) = \sum_s \Pr(y^s)\widetilde{\pi}_\varphi(x, y).$$

where $\Pr(y^s)$ is the probability that a randomly selected buyer is currently in state $y^s \in \{y^1, .., y^{N_y}\}$.[17]

For the purposes of the search model that follows, all that matters about an individual relationship is $\widetilde{\pi}_\varphi(x)$, and this object can be estimated directly from data on the revenue streams generated by matches. Nonetheless, the idiosyncratic state of each buyer-seller match affects whether it will be endogenously terminated and hence matters for our characterization of aggregate export dynamics.

Hereafter, we will denote the expected value of a relationship with a foreign buyer by $\widetilde{\pi}_\varphi^f(x)$ and the expected value of a relationship with a home market buyer by $\widetilde{\pi}_\varphi^h(x)$. These two objects are calculated in the same way, but since expenditure levels $(\overline{X}_t)$ and price indices (P_t) differ across markets, and no exchange rate factor e is necessary for domestic profit calculations, each has its own process for the market-wide state variable, x. These market-wide demand shifters are denoted x^f and x^h below.

3.2 Learning about Product Appeal

In each market, sellers conduct market-specific searches for buyers. When searching in market $m \in \{h, f\}$, each recognizes that some fraction $\theta^m \in [0, 1]$ of the potential buyers she meets there will be willing to do business with her. An encounter with one of these willing

[17]Here we take the probabilities $\Pr(y^m)$ to be the ergodic distribution of y implied by the transition hazards $q_{yy'}^Y$. We could assume that the distribution at the time of the first purchase is different from the ergodic one.

buyers generates an expected profit stream worth $\widetilde{\pi}^m_{\varphi,x}$, while an encounter with any of the remaining potential buyers does not generate a sale then or subsequently.

Each seller's θ^h and θ^f values are drawn before she has met any clients. These draws remain fixed through time, inducing permanent cross-market differences in her product's popularity. All θ^m draws are independently beta-distributed across sellers and markets:

$$b(\theta^m|\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\,(\theta^m)^{\alpha-1}\,(1-\theta^m)^{\beta-1},\ m \in \{h,f\},$$

where $\Gamma(\phi) = \int_0^\infty z^{\phi-1}e^{-z}dz$ is the gamma function (needed to ensure that the distribution has the proper limits). However, the independence of θ^h and θ^f does not mean sellers' domestic and foreign sales are likewise independent. Rather, cross-market correlation in sales will be induced by the firm type φ, which can be viewed as capturing aspects of product appeal that are common to both markets.[18]

Benchmark model: Sellers are presumed to have already met many potential customers in the domestic market, and thus to have learned their θ^h draws. But sellers typically have far less experience abroad, so in the benchmark version of our model, we allow them to still be learning about their θ^f draws. Specifically, each seller recognizes that for any given θ^f, the probability a random sample of n potential foreign buyers will yield a customers is binomially distributed:

$$q\left[a|n,\theta^f\right] = \binom{n}{a}\left[\theta^f\right]^a\left[1-\theta^f\right]^{n-a}.$$

So after she has met n^f potential buyers abroad, a^f of whom were willing to buy her product, a seller's posterior beliefs about her θ^f draw are distributed:

$$p(\theta^f|a^f,n^f) \propto q\left[a^f|n^f,\theta^f\right]\cdot b(\theta^f|\alpha,\beta)$$

where the factor of proportionality is the inverse of the integral of the right-hand side over the support of θ^f. Since the beta distribution is the conjugate prior for the binomial, a firm's expected success rate after a successes in n trials has a convenient closed-form representation:

$$\overline{\theta}^f_{a,n} = E\left[\theta^f|a^f,n^f\right] = \int_0^1 \theta p(\theta|a^f,n^f)d\theta = \frac{a^f+\alpha}{n^f+\alpha+\beta}. \tag{7}$$

This posterior mean converges to $p\lim\left(\frac{a^f}{n^f}\right) = \theta^f$ as n gets large.

[18]The firm effect is similarly interpreted to reflect both productive efficiency and product appeal in Melitz (2003) and many other papers based on CES demand systems. However in the present context, the global aspects of product appeal captured by φ are qualitatively distinct from the market-specific product appeal effects captured by θ. The former determines the amount of a product each buyer purchases, given that he is interested, while the latter determines what fraction of potential buyers are willing to place orders with the seller, should they happen to meet her.

Note that exporters learn something about their foreign demand with each new match—successful or otherwise. In this regard we depart from other models with learning effects, which have generally presumed that the number of signals per period is either zero or one, depending upon firms' foreign market participation (Fitzgerald et al., 2019; Arkolakis et al., 2018; Timoshenko, 2015). Does it matter? Our formulation creates an extra incentive for inexperienced exporters to search intensively, which we will quantify in Section 6 below.

known-θ^f model: As an alternative to our benchmark model, we consider the possibility that sellers already know their product's popularity in *both* markets, so that $p(\theta^f|a^f, n^f)$ is a degenerate distribution and $\overline{\theta}^f_{a,n} = \theta^f$. In this version of the model, sellers' matching histories only affect their search intensities by affecting their visibility in each market, as we will discuss shortly. Our known-θ^f model is not nested by the benchmark model, it is simply a different characterization of the role of information in driving search policies.[19]

3.3 Searching for Buyers

To complete our characterization of firms' behavior, we now consider sellers' search intensities in each market. Each seller continuously chooses the market-specific hazard s^m, $m \in \{h, f\}$, with which she encounters a potential buyer, recognizing that this involves the instantaneous flow cost $c^m(s^m, a^m)$, where $c^m(s^m, a^m)$ is increasing and convex in s^m.[20] Whether $c^m(s^m, a^m)$ increases or decreases in the number of successful matches, a^m, depends upon the relative strength of several forces and will be left for the data to determine. Costs might fall with a^m because encounters with interested buyers increase the seller's visibility and enhance her opportunities to meet additional potential buyers. Alternatively, costs might rise if the pool of easy-to-reach buyers becomes "fished out," as in Arkolakis (2010).

We can now describe optimal search behavior, beginning with the foreign market. Recall that when the foreign market state is x^f, a type-φ seller expects the value of a new business relationship will be $\widetilde{\pi}^f_\varphi(x^f)$. Further, she believes the next match will yield such a relationship with probability $\overline{\theta}^f_{a,n}$. Combined with search cost function $c^f(s^f, a^f)$ and the jump process for x^f, these objects imply sellers' optimal search policy abroad.

[19]In order for the learning model to nest the no learning model, each firm would have to have its own Beta distribution parameters, α and β.

[20]Interviews conducted with Colombian exporters revealed a variety of activities firms pursue to meet potential buyers abroad (Domínguez, et al, 2013). Ranked roughly in terms of decreasing cost, these included maintaining a foreign sales office; paying the exports promotion office to organize visits with prospective clients abroad, and sending their sales representatives to those visits; sending sales representatives abroad to visit potential clients on their own; attending trade fairs; paying a researcher to search the web for foreign firms that purchase products similar to their own; paying browsers to ensure that their site appear near the top of a search for their product type; maintaining a web site in English. Interviewees also reported that relatively low-cost activities, such as traveling to trade fairs, or translating their websites to English, led to relationships with one or two clients every few years. Establishing a larger network of clients required much more costly activities.

To characterize this policy, let τ_s^f be the random time interval until the next foreign search event, which could be either a change in the marketwide state x^f or an encounter with a potential buyer. Then, suppressing market superscripts, the optimal search intensity s for a type-φ firm with foreign market search history (a, n) solves the following the Bellman equation:

$$V_\varphi(a,n,x) = \max_s \mathbf{E}_{\tau_s}\left[-c(s,a)\int_0^{\tau_s} e^{-\rho t}dt + \frac{e^{-\rho\tau_s}}{s+\lambda_x^X}\cdot\left(\sum_{x'\neq x} q_{xx'}^X V_{\varphi,}(a,n,x')\right.\right.$$
$$\left.\left.+\, s\left[\overline{\theta}_{a,n}(\widetilde{\pi}_\varphi(x) + V_\varphi(a+1,n+1,x) + (1-\overline{\theta}_{a,n})V_\varphi(a,n+1,x)\right]\right)\right]$$

(Recall that λ_x^X is given by (5).) Taking expectations over τ_s yields:

$$\begin{aligned} V_\varphi(a,n,x) \quad &= \quad \max_s \frac{1}{\rho+s+\lambda_x^X}\left[-c(s,a) + \sum_{x'\neq x} q_{xx'}^X V_{\varphi,}(a,n,x')\right. \qquad (8) \\ &\qquad \left. +\, s\left\{\overline{\theta}_{a,n}\left[\widetilde{\pi}_\varphi(x) + V_\varphi(a+1,n+1,x)\right] + (1-\overline{\theta}_{a,n})V_\varphi(a,n+1,x)\right\}\right] \end{aligned}$$

Applying the multiplication rule for differentiation and using expression (8) for $V_\varphi(a, n, x)$, the optimal search intensity s^* satisfies:

$$\frac{\partial c(s^*,a)}{\partial s} = \overline{\theta}_{a,n}\left[\widetilde{\pi}_\varphi(x) + V_\varphi(a+1,n+1,x)\right] + (1-\overline{\theta}_{a,n})V_\varphi(a,n+1,x) - V_\varphi(a,n,x) \qquad (9)$$

That is, the marginal cost of search must equal the expected marginal benefit of a match, which includes the expected value of the associated profit stream, $\overline{\theta}_{a,n}\widetilde{\pi}_\varphi(x)$, and the expected value of the information generated.

Now consider the home market. Since we assume sellers have already learned their true success rates at home, θ_j^h, new encounters do not influence expectations, and we need not condition the value function or the expected success rate on search histories. Again suppressing market superscripts, the Bellman equation collapses to:

$$\begin{aligned} V_\varphi(x,a) \quad &= \quad \max_s \frac{1}{\rho+\lambda_x^X}\left[-c(s,a) + \sum_{x'\neq x} q_{xx'}^X V_\varphi(x',a)\right. \\ &\qquad \left. +\, s\left\{\theta_j^h\left[\widetilde{\pi}_\varphi(x) + V_\varphi(a+1,x)\right] + (1-\theta_j^h)V_\varphi(a,x)\right\}\right] \qquad (10) \end{aligned}$$

and the first-order condition is simply:

$$\frac{\partial c(s^*,a)}{\partial s} = \theta_j^h\left[\widetilde{\pi}_\varphi(x) + V_\varphi(a+1,x) - V_\varphi(a,x)\right].$$

The marginal cost of search equals the expected profit from a successful relationship times the probability of success. Of course, this condition also describes foreign market search in the known-θ^f version of the model.

4 An empirical version of the model

4.1 The search cost function

To implement our model empirically, we impose additional structure in several respects. First, we specify a functional form for our search cost function. Generalizing Arkolakis (2010) to allow for reputation/network effects, we write these costs as:

$$c^m(s^m, a^m) = \kappa_0^m \frac{[(1+s^m)]^{\kappa_1} - 1}{\kappa_1 [1 + \ln(1+a^m)]^{\gamma}}. \tag{11}$$

where $m \in \{h, f\}$. Several properties of this function merit note. First, marginal costs fall at a rate determined by γ with the number of successful matches a seller has already made, so $\gamma > 0$ implies that positive reputation/network effects dominate negative congestion effects.[21] Second, a seller who is not searching in a particular market incurs no search cost: $c^m(0, a^m) = 0$. Third, given the cumulative number of successful matches, a, the marginal cost of search increases with s at a rate determined by κ_1 : $\frac{\partial c^m(s^m,a^m)}{\partial s^m} = \kappa_0^m (1+s^m)^{\kappa_1 - 1} / [1 + \ln(1+a)]^{\gamma}$. Fourth, we allow the cost function scalar to vary across markets, since the cost of maintaining any given level of visibility is likely to be higher in foreign markets. Finally, since a^m is the cumulative number of successes in market m, reputation/network effects endure, even if a firm is not actively searching.

4.2 Processes for exogenous state variables

Next we impose more structure on the exogenous state variables, φ, x^h, x^f, y^h and y^f. All are assumed to have zero means in logs, and the net effect of these normalizations is undone by introducing scalars Π^h and Π^f into the home and foreign profit functions, respectively:

$$\pi_\varphi^f(x^f, y^f) = \Pi^f x^f \varphi^{\eta-1} y^f, \tag{12}$$
$$\pi_\varphi^h(x^h, y^h) = \Pi^h x^f \varphi^{\eta-1} y^f \tag{13}$$

More substantively, we impose that the cross-firm distribution of φ is log normal with variance parameter σ_φ, and we treat all of the Markov jump processes (x^h, y^h, x^f, y^f) as

[21] To contain the dimensionality of the computational problem we solve, we assume that firms with more than a^* buyers have (i) exhausted their learning effects, and (ii) reap no additional network effects at the margin from further matches. We choose a^* to exeed the observed maximum a for 99 percent of sellers in the foreign (United States) market. Also, we set $a = a^*$ for all sellers in their home (Colombian) market.

independent Ehrenfest diffusion processes. The idiosyncratic match shocks, y^f and y^h, are assumed to share the same distribution, but we allow the x^f and x^h processes to differ. Among other things, the latter accommodates the fact that the exchange rate affects aggregate demand and price indices in the two markets differently.

Any variable z generated by an Ehrenfest process can be discretized into $2g+1$ possible values, $g \in I^+ : z \in \{-g\Delta, -(g-1)\Delta, .., 0, .., (g-1)\Delta, g\Delta\}$. Further, it jumps to a new value with hazard λ_z, and given that a jump occurs, it goes to z' according to:

$$z' = \begin{cases} z+\Delta \\ z-\Delta \\ \text{other} \end{cases} \text{with probability} \begin{cases} \frac{1}{2}\left(1-\frac{z}{g\triangle}\right) \\ \frac{1}{2}\left(1+\frac{z}{g\triangle}\right) \\ 0 \end{cases} .$$

Thus, given a grid size g, the intensity matrices $Q^X = \left\{q_{ij}^X\right\}_{i,j=1,N^X}$ and $Q^Y = \left\{q_{ij}^Y\right\}_{i,j=1,N^Y}$ that were introduced in section 3.1 are each block-diagonal and characterized by a single parameter, Δ.

5 Estimation

5.1 Stage 1: estimating observable jump processes

Shimer (2005) shows that if z follows a continuous time Ehrenfest diffusion process, it asymptotes to an Ornstein-Uhlenbeck process with mean zero as the fineness of the grid increases:[22]

$$dz = -\mu z dt + \sigma dW.$$

Here $\mu = \lambda_z/g$, $\sigma = \sqrt{\lambda_z}\Delta$, and W follows a Weiner process. Accordingly, since it is possible to observe proxies for x^f and x^h, these can be viewed as discrete time observations on underlying Ornstein-Uhlenbeck processes, and the parameters of these processes can be econometrically estimated. Then, given μ and σ, estimates of Δ and λ for these processes can be inferred.

Measuring x^f as real expenditures on manufacturing goods in the U.S., and measuring x^h as real expenditures on manufacturing goods in Colombia, we obtain the results reported in Table 5. They imply that x^f and x^h both jump 1.2 times per year, on average. However, jumps in the U.S. market tend to be much larger, essentially because they reflect movements in the real exchange rate as well as movement in dollar-denominated expenditures.

[22]Specifically, replacing the parameter vector (λ, g, Δ) with $(\lambda/\epsilon, g/\epsilon, \Delta\sqrt{\epsilon})$, $\epsilon > 0$, leaves the autocorrelation parameter μ and the instantaneous variance parameter σ unchanged. But as $\epsilon \to 0$, the innovation dW approaches normal.

Table 5: Market-wide Demand Shifters

	Parameter	*value*
home macro state jump hazard	λ^{x_h}	1.200
foreign macro state jump hazard	λ^{x_f}	1.215
home macro state jump size	Δ^{x_h}	0.003
foreign macro state jump size	Δ^{x_f}	0.053

Notes: Our foreign market size mesaure is the OECD time series on American GDP in 'Industry, including energy' adding imports and subtracting net exports of manufactures. Our home market size measure is real Colombian expenditures on manufacturing goods, taken from DANE. We converted all of the data used for the estimation into real 1992 US dollars, deflating nominal US dollars with the consumer price index available on the US Bureau of Labor Statistic website. We used an official Colombian Peso - US Dollar exchange rate time series downloaded from the Central Bank of Colombia to convert Pesos to nominal US Dollars.

5.2 Stage 2: Indirect inference

Our data are relatively uninformative about the rate of time discount ρ and the demand elasticity η, so we do not attempt to estimate either one. For the former we follow convention and assume $\rho = 0.05$. For the latter, following many previous trade papers, we fix the demand elasticity at $\eta = 5$. Also, to limit the size of the estimated parameter vector, we take the exogenous match failure rate to be the observed match failure rate among matches at least 3 years old ($\delta = 0.326$), we take the search cost function to be quadratic in search intensity ($\kappa_1 = 2$), and we assume that the hazard rate for the buyer is once per quarter ($\lambda_y = 4$).[23]

All of the remaining parameters we estimate jointly using the transactions data summarized in Section 2 above. These parameters include the market size scalars ($\Pi^h, \Pi^{f\cdot}$), the fixed costs of maintaining a match (F^h, F^f), the parameters of the product appeal distributions (α,β), the dispersion of the productivity distribution (σ_φ), the jump size for the idiosyncratic buyer shocks (Δ_y), the hazard rate for shipments (λ_b), the network/congestion parameter (γ), and the market-specific cost function scaling parameters (κ_0^h, κ_0^f). For notational convenience we collect these parameters in the vector Λ :

$$\Lambda = \left(\Pi^h, \Pi^{f\cdot}, F^h, F^f, \alpha, \beta, \sigma_\varphi, \Delta_y, \lambda_b, \gamma, \kappa_0^h, \kappa_0^f\right)$$

We construct our estimator for Λ using the method of indirect inference (Gouriéroux and Monfort, 1996). That is, for each candidate Λ vector, we use the model to simulate the foreign and domestic transactions of an artificial sample of producers. Then, using these simulated

[23]These last three parameters could in principle be estimated, and in earlier drafts we have done so. However, they have not appeared to be well-identified.

data, we estimate a set of reduced-form regressions that summarize the relationships we want our model to capture. Finally, searching the support of Λ vectors, we choose the one that makes the regression coefficients from simulated data correspond as closely as possible to the corresponding regression coefficients based on sample data. Algebraically, our estimator is

$$\hat{\Lambda} = \min_{\Lambda} \left[\bar{m} - m(\Lambda)\right]' W \left[\bar{m} - m(\Lambda)\right],$$

where $\bar{m}$ is a column vector of regression coefficients obtained from sample data, $m(\Lambda)$ is the analogous vector of regression coefficients based on data simulated at Λ, and W is a compatible non-singular weighting matrix. Setting $W^{-1} = var(\bar{m} - m(\Lambda))$ maximizes the efficiency of this estimator, but any non-singular W will yield consistent estimates. We use a block-diagonal version of $var(\bar{m} - m(\Lambda))$, with each block corresponding to the moments from a particular regression.

The regressions themselves are reported in Tables 6, 7 and 8. In each table, the data-based regression estimates are reported, and their standard errors are reported below them in parentheses. To facilitate comparisons between the sample and the simulated data, and with no loss of information, we have replaced the intercept of each regression with the mean value of the dependent variable in cases where that was possible.[24] We now briefly describe these regressions, our reasoning in choosing them, and the parameters they help most to identify. These observations regarding identification are based partly on our calculation of Andrews et al.'s (2017) sensitivity matrix, which we report in full in Appendix C.

Search policy. The first regression in Table 6 summarizes the effects of firms' market experiences on their search intensity (s). Roughly speaking, this equation can be viewed as a second order approximation to the foreign market policy function (9).There is no simple mapping from its coefficients to particular elements of Λ. Nonetheless, our sensitivity matrix suggests that this equation is most helpful in identifying the fixed costs of maintaining a relationship, F^h and F^f, the parameters of the success rate (θ) distribution, α and β, and the network/congestion parameter, γ.

The dependent variable in equation (i) is a proxy for a firm's foreign market search intensity after n successful matches, namely, the inverse of the time interval between firm j's n^{th} and $n + 1^{st}$ match. And the right-hand side is a second-order translog function of this firm's cumulative number of successes (a) and cumuluative success rate ($\frac{a}{n}$). To deal with firms that have had no successes, we add 1 to a and to $\frac{a}{n}$ before taking logs.

The unit of observation here is an exporter-specific new match, and we define a new match to occur whenever an exporter makes a shipment to a buyer it has not dealt with

[24]Several regressions were done in real pesos within the Colombian national statistical agency (DANE). We are not confident that they can be expressed in units that are strictly comparable to the real dollar units in which U.S. customs records were expressed.

before. We view this first shipment as a sample of the exporter's merchandise, so we only consider this match to be successful if it results in at least one additional shipment. This interpetation of the data means we can use customs records to directly infer the cumulative number of successes for each firm j (a_{nj}) after each of its $n \in \{1, ..., N_j\}$ matches, and the associated cumulative success rates $(\frac{a}{n})_{nj}$.

Interpreting the coefficient estimates for this regression is problematic, both because it includes second order terms and because we have not controlled for the highly nonlinear firm effects generated by φ and θ^f. But evaluation of this equation on a grid of success rates and cumulative successes gives us a crude sense for the relationships implied by our estimates. The results (available upon request) show that search intensity is only mildly sensitive to success rates, but it strongly increases with cumulative successes.

Separation policy. Equation (*ii*) captures a second basic feature of firms' exporting behavior: match termination policies. Here the unit of observation is seller j's i^{th} match in year t, and the dependent variable, $D^{exit\ match}$, takes a value of unity when this match is in its final year.[25] Since matches endogenously terminate when $\widehat{\pi}_{\varphi_j}(x_t, y_{ijt}) < F^f$, the explanatory variables in this regression would ideally be the idiosyncratic demand shock y_{ijt} and/or the firm's productivity level φ_j. But neither variable is directly observable, so we use several of their correlates as predictors: current match sales, X^f_{ijt}, age of the match, A_{ijt}, and export market tenure, Δ_{ijt}. All variables are expressed in logs and, given the patterns revealed by Table 4, we allow firms in their first year of exporting ($D^{new\ to\ mkt} = 1$) to experience particularly high failure rates. [26]

The sensitivity matrix (Appendix C) implies that equation (*ii*) helps most to identify the fixed costs of maintaining an established foreign match, F^f. That is, conditioning on sales, X^f_{ijt}, matches are more likely to survive when fixed costs are low. It also proves to be helpful in identifying the jump size, Δ_y, since failure rates are also affected by the volatility of z_{ijt}.

Not surprisingly, estimates of equation (*ii*) reflect the same patterns that we noted in connection with Table 4. Matches in their first year are relatively likely to fail, as are matches that start with relatively small sales volumes. The results also show that exporters with more experience in foreign markets tend to have longer-lived relationships, a feature of the data that our model captures with cross-firm variation in productivity levels, φ.

Match success rates The remaining regressions in Table 6 concern the distribution of success rates, θ. Equation (*iii*) summarizes the average success rate among active exporters and its relation to the cumulative number of meetings an exporter has had (n). Accordingly

[25] Only active matches are included in the sample.

[26] Note, however, that in Table 7, matches that die after a single shipment are treated as having existed for less than one year, while our model-based estimates treat these cases of single shipments as unsuccessful meetings that did not lead to business relationships.

it is informative about $\alpha/(\alpha+\beta)$ and selection due to learning. Equation (iv) describes dispersion in success rates–i.e., the squared residuals from equation (iii)–among exporters with different experience (n) levels. Both regressions suggest that selection takes place as firms acquire market tenure, since success rates are higher among experienced (high-n) firms, and the dispersion in success rates among such firms is lower.

Client distributions and shipment frequencies. The next set of regressions appears in Table 7. Equation (v) summarizes the information on client distributions in Table 3. Specifically, letting $\Phi(\ell)$ be the fraction of exporters with no more than ℓ active clients, column (v) reports the regression of $\ln(1-\Phi(\ell))$ on $\ln \ell$ and $(\ln \ell)^2$.[27] We choose this functional form because earlier studies have found that exporters' foreign client distributions are approximately Pareto, implying that the relationship between $\ln(1-\Phi(\ell))$ and $\ln \ell$ is approximately linear. Note that our data confirm a nearly-Pareto client distribution, as the coefficient on the quadratic term is quite small (-0.055).

Equation (v) helps to identify the cost function parameters $(\kappa_0^h, \kappa_0^f, \gamma)$ because the client distribution largely reflects firms' search intensities. In particular, the network effects captured by the parameter γ determine how much of a search cost discount large (big a) firms enjoy, and thus the "fatness" of the right-hand tail of the client distribution $\Phi(\cdot)$.

Equation (vi), the other regression in Table 7, simply establishes the mean log number of shipments per year per continuing match. It serves as a target for the shipment arrival hazard and obviously helps to identify λ_b.

Match- and firm-level sales Regressions that characterize the time series properties of firms' exports, cross-firm dispersion in exports, and patterns of correlation between exports and domestic sales are collected in Table 8. These equations are particularly informative about the parameters $\left(\Pi^h, \Pi^{f\cdot}, F^h, F^f, \sigma_\varphi, \Delta_y\right)$. Equation (vii) is an AR1 in log match revenues, conditioned on match age and a dummy to control for first-year effects. By the logic reviewed in section 5.1 above, the root (0.826) and root mean square error (1.208) in this AR1 identify the jump size, Δ_y and the cross-firm variance in productivity, σ_φ, up to selection effects. Also, together with equation (ii), the mean log annual revenue per match (10.67) essentially pins down the profit function scalar and the fixed cost of maintaining a foreign match (Π_f, F^f).

The last four equations in Table 8 involve domestic sales. Since we don't observe firms' individual matches in the domestic market, these regressions describe establishment-level

[27]By construction, the intercept of the (non-parametric version of) this regression line must be zero.

panel data merged with Colombian customs records.[28] Equations (*viii*) is an AR1 for home sales, and is thus informative about the extent which firms adjust their domestic connections and their associated match specific sales in response to idiosyncratic shocks. Accordingly, the coefficients in this equation are particularly helpful in identifying κ_0^h and F^h, and the mean squared error helps identify σ_φ and $\alpha/(\alpha+\beta)$. Equation (*ix*) is a simple projection of firm level exports on firm-level domestic sales. It serves to distinguish market-specific variation in revenues from variation in revenues that is common to both markets. Thus the estimated parameters of this equation, including its mean squared error, are informative about the variance of productivity shocks (σ_φ^2), which are common to both markets, relative to the variance of market-specific appeal draws, θ^h and θ^f.[29]

Finally, equations (*x*) and (*xi*) describe the relative importance of home versus foreign sales. The former gives the share of firms that participate in the foreign market and thereby speaks to the relative return to maintaining foreign versus domestic business connections, that is (Π^f, F^f, κ_0^f) versus (Π^h, F^h, κ_0^h). The latter gives the average share of exports to the U.S. in total sales of exporting firms. Accordingly, it largely reflects the number of clients in each market, and thus responds especially to differences between κ_0^f and κ_0^h.

Sensitivity analysis As suggested by Andrews et al. (2017), we check which moments are important using the sample analog to the matrix $(G'WG)^{-1}G'W$ where $G = \frac{-\partial[m(\Lambda)]}{\partial\Lambda'}$ is the Jacobian for the vector of simulated moments. "Intuitively, this matrix is a local approximation to the mapping from moments to estimated parameters." (Andrews, et al., 2017, p. 1555) Evaluated at our benchmark estimates (to be discussed), we obtain the results reported in detail in Appendix C. Here we summarize the patterns that emerge.

First, most parameters respond to many moments rather than one or several. Limiting our attention to elasticities with absolute value greater than 0.1, most parameters show significant responses to at least 5 moments, and several (F^f, F^h, γ) respond to more than 15. All parameters respond to at least 2. The moments that affect the most parameters are those generated by the match sales autogression (equation *vii*), the shipping rate regression (equation *vi*), the domestic sales autoregression (equation *viii*), the regression explaining the variance in success rates (equation *iv*), and the fraction of firms that export (equation *x*).

[28]More precisely, regressions (*viii*) through (*x*) in Table 8 are done using a combination of the Colombian Annual Manufacturing Survey (AMS) and Colombian administrative records of exports transactions. The data used cover 1993-2007. Exports are merged into the AMS using firm identifiers. This is done because the AMS has no export information for 1993-1999, and because the dynamics of aggregate exports reported in the EAM starting in 2004 differ substantially from aggregate reports from other sources.

[29]Given the average success rate, $\alpha/(\alpha+\beta)$, the variances of θ^h and θ^f depend only on $\alpha+\beta$.

Table 6: Match hazards, success rates, and endurance

	(i) $\ln(s_{ij})$	*(ii)* $D_{ijt}^{exit\ match}$	*(iii)* $\frac{a_{ij}}{n_{ij}}$	*(iv)* $u^2_{a_{ij}/n_{ij}}$
mean, dep. variable	-0.719	0.395	0.413	0.091
	(0.621e-2)	(0.319e-2)	(0.153e-2)	(0.26e-3)
$\ln(1+n_{ij})$	–	–	0.093	-0.056
			(0.003)	(0.000)
$\ln(1+a_{ij})$	-0.818	–		
	(0.113)			
$\ln(1+a_{ij})^2$	0.312	–	–	–
	(0.017)			
$\ln(1+\frac{a_{ij}}{n_{ij}})$	-1.132	–	–	–
	(0.296)			
$\left[\ln(1+\frac{a}{n})\right]^2$	2.451	–	–	–
	(0.396)			
$\ln(1+a_{ij})\cdot\ln(1+\frac{a_{ij}}{n_{ij}})$	-0.708	–	–	–
	(0.134)			
$D_{ijt}^{new\ to\ mkt}$	–	0.034	–	–
		(0.011)		
$\ln X_{ijt}^{f}$	–	-0.031	–	–
		(0.002)		
$\ln A_{ijt}$	–	-0.054	–	–
		(0.009)		
$\ln \Delta_{jt}$	–	-0.028	–	–
		(0.007)		
observations (rounded)	38,500	23,500	35,800	35,800

Notes: Unit of observation, columns *i*, *iii* and *iv*: seller j's i^{th} match. Unit of observation, column *ii*: seller j's i^{th} match in its t^{th} year. s_{ij} = inverse of time interval between commencement of match i and commencement of the next one for exporter j $D_{ijt}^{exitmatch} = 1$ if exporter $j's$ i^{th} match dies in year t. a_{ij} = cumulative number of successes for exporter j at time of match i. $D_{ijt}^{newtomkt} = 1$ if exporter $j's$ i^{th} match is in its first year. $\ln A_{ijt}$ = log age of exporter $j's$ i^{th} match. $\ln \Delta_{jt}$ = log age of exporter j in year t. X_{ijt}^{f} = foreign sales volume generated by exporter $j's$ i^{th} match.

Table 7: **Client distribution and shipment frequency**

	(*v*) $\ln(1-\Phi(\ell))$	(*vi*) $\ln(s_{ijt})$
mean, dep. variable	-5.973 (2.173)	0.971 (0.004)
$\ln(\ell)$	-1.8813 (0.1123)	-
$(\ln \ell)^2$	-0.0545 (0.0211)	-
sample restrictions	$\ell > 0$	$s_{ijt} > 0$
observations	43	87,000

Notes: ℓ: number of active clients; $\Phi()$ = cumulative distribution of exporters in terms of ℓ; s_{ijt} = number of shipments per year to client i by exporter j in year t

Table 8: **Home and foreign sales regressions**

	(*vii*) $\ln X^f_{ijt}$	(*viii*) $\ln X^h_{jt}$	(*ix*) $\ln X^f_{jt}$	(*x*) D^f_{jt}	(*xi*) $\frac{X^f_{jt}}{X^f_{jt}+X^h_{jt}}$
mean, dep. variable	10.665 (0.002)	–	–	0.102 (0.003)	0.127 (0.002)
R_{ijt-1}	0.328 (0.018)	-	-	-	-
$\ln X^f_{ijt-1}$	0.826 (0.004)	-	-	-	-
$\ln X^h_{jt-1}$	-	0.976 (0.029)	-	-	-
$\ln X^h_{jt}$	-	-	0.323 (0.110)	-	-
$\ln \Delta_t$	0.063 (0.014)	-	-	-	-
root mse	1.2079	0.4621	2.1665	0.303	0.243
sample restrictions	$X^f_{ijt}, X^f_{ijt-1} > 0$	$X^h_{jt}, X^h_{jt-1} > 0$	$X^f_{jt}, X^h_{jt} > 0$	$X^h_{jt} > 0$	$X^f_{jt}, X^h_{jt} > 0$
observations	25,400	99,300	11,600	119,800	12,500

Notes: $R_{ijt} = 1$ if exporter $j's$ i^{th} match is in its first year. $\ln \Delta jt$ = log age of exporter j. X^f_{ijt} = foreign sales volume generated by exporter $j's$ i^{th} match. X^f_{jt} = total foreign sales volume generated by firm j. X^h_{jt} = total home sales volume generated by firm j. $D^f_{jt} = 1$ if firm j is an exporter.

5.3 Parameter estimates

Table 9 reports estimates of the structural parameter vector Λ for both the benchmark and the known-θ^f model. Although our estimator exploits month-to-month transitions in the customs records, all hazards are normalized so that the unit of time is one year. Thus, for example, our estimate of δ implies that on average, matches last roughly 4 months (one-third of a year) before separating for exogenous reasons. Most parameter estimates are similar for both models, though, as we'll argue below, the benchmark model fits the data better. We therefore focus our discussion on the results for this model, turning later to the main distinguishing features of the known-θ^f results.

Benchmark parameter estimates Active matches generate an average of $\lambda_b = 15.43$ shipments per year, and the profits associated with these shipments vary widely across firms and macro conditions. Evaluating the gross profit-per-shipment functions (12) and (13) at our estimated values for Π^h, Π^f and the parameters governing realizations for φ, x, and y, we find that gross profits per shipment (before fixed costs) for a firm at the median productivity level matched to a median buyer are essentially zero. Accordingly, these firms are not active. On the other hand, a firm with productivity 1.9 standard deviations above the mean earns gross profits per shipment ranging from $US 4 to $US 42, depending upon what state its buyer is in. In the domestic market, the analogous figures range from $US 45 to $US 405. Further, a firm with the highest productivity matched to the best possible buyer in the most favorable macro state earns $US 31,512 in gross profits per export shipment and $US 281,570 in profit per domestic shipment. Of course, firms almost never attain these maxima, and when they do they are very unlikely to repeat their performance. This is consequence of the short expected life span of matches, and the fact that buyers' demands change an average of $\lambda_y = 4$ times per year.

These seemingly small magnitude of these figures reflects several factors. First, the productivity distribution for exporting firms come from the right-hand tail of the unconditional productivity distribution. Thus those firms with productivity 1.9 standard deviations above the mean unconditional mean of φ are actually the smaller exporters. Second, since revenues per shipment are $\eta = 5$ times profits per shipment, and since an average of $\lambda_b = 15.43$ shipment occur per year, expected annual revenues from a match that survives the entire year are $\eta \cdot \lambda_b = 77.15$ times as large as profits per shipment for that match.

Turning to the fixed cost estimates, note that both are quite small (F^f=$US 0.30**,** F^h=$US 0.03). These costs thus have no affect on major exporters. Nonetheless, they affect the fraction of exporting firms by keeping fringe players that would otherwise sell tiny amounts out of foreign markets.

The profit and cost function scalars are much more important. The model assigns lower search costs to the home market ($\kappa_0^h = 859.0$ versus $\kappa_0^f = 3,079.7$) and much larger profits per sale ($\Pi^h/\Pi^f = \exp(-3.88 + 6.14) = 9.77$). Both patterns help explain the small amount of output exported to the U.S. among these firms (Table 8, regression *xi*). And the two sets of scalars are separately identified by their different effects on match arrival rates (Table 4, regression *i*) and revenues from ongoing matches (Table 8, regressions *vii* and *viii*). The benchmark model also implies that search costs fall significantly as firms acquire market visibility through successful matches ($\gamma = 0.383$). As mentioned earlier, identification of this visibility effect comes largely from the shape of the client distribution (Table 7, regression *v*).

So what are the costs of making new matches? For a firm with no prior successful matches in the foreign market, a search intensity sufficient to yield an average of one new match per year costs $c^f(1,0)$ = \$US $1,539$, but an expected yield of four new matches—about one successful match for a firm with average product appeal—costs $c^f(4,0)$ = \$US $24,637$. The analogous figures in the home market are $c^h(1,0)$ = \$US 428 and $c^h(4,0)$ = \$US $6,848$. But having an established reputation is helpful. A firm that has already made 2 successful foreign matches could expect to pay only $c^f(4,2)$ = \$US $20,142$ for the next one—roughly 20 percent less than the cost of the first one. Similarly, a firm that has already made two successful home market matches could expect to pay $c^h(4,2)$ = \$US $5,598$ for the third. These reputation effects are nontrivial, and other things equal, they create a cost advantage for well-established firms.

Given match payoffs and search costs, firms' search intensity is determined by their expected success rates. Their (unobserved) actual rates are drawn from a beta distribution, which we estimate to have mean $\alpha/(\alpha+\beta) = 0.23$ and variance $\alpha\beta/\left[(\alpha+\beta)^2(\alpha+\beta+1)\right] = 0.23^2$. Hence, before they acquire export market experience, firms expect that roughly 1 in 4 new encounters with potential buyers will lead to business relationships. And since new exporters are uncertain about their θ^f draws, they expect to learn a good deal from the outcomes of their early matches.

Known-θ^f parameter estimates Recall that our known θ^f model differs from the benchmark model in that it presumes each firm j already knows the fraction of the foreign population of buyers that is willing to do business with it, θ_j^f. This assumption implies that low-appeal firms never bother to invest much in foreign market searches. Further, compared to firms that learn their θ_j^f draws through experience, fully-informed firms have less incentive to search intensively when they are new to export markets. That is, for these firms their is no information value to matches.

The last two columns of Table 9 present parameter estimates based on this version of the model. Most parameters are similar, but for the known-θ^f model the estimate of the network effect is larger ($\gamma = 0.50$ versus $\gamma = 0.38$) and the estimates of search costs are

Table 9: Structural parameter estimates

		Benchmark model		Known-θ^f model	
	Parameter	*value*	*std. error*	*value*	*std. error*
log of domestic profit scalar	$\ln \Pi^h$	-3.879	(0.1364)	-3.460	(0.0725)
log of foreign profit scalar	$\ln \Pi^f$	-6.135	(0.1993)	-6.273	(0.0759)
fixed cost, domestic	F^h	0.027	(0.0047)	0.037	(0.0064)
fixed cost, foreign	F^f	0.296	(0.0428)	0.301	(0.0359)
First θ distribution parameter	α	0.571	(0.0454)	0.581	(0.0703)
Second θ distribution parameter	β	1.894	(0.2320)	4.661	(0.2107)
demand shock jump size	Δ^y	1.882	(0.2222)	1.951	(0.1810)
shipment order arrival hazard	λ_b	15.426	(0.1991)	15.431	(0.1428)
std. deviation, log firm type	σ_φ	1.386	(0.0095)	1.401	(0.0051)
network effect parameter	γ	0.383	(0.0485)	0.508	(0.0479)
log of home search cost scalar	$\ln \kappa_0^h$	11.722	(0.1486)	12.480	(0.0850)
log of foreign search cost scalar	$\ln \kappa_0^f$	13.002	(0.0095)	13.666	(0.1373)
log of fit metric	$\ln(\Lambda)$	10.806		11.346	

higher ($\kappa_0^h = 859$ and $\kappa_0^f = 3,079$ versus $\kappa_0^h = 1,826$ and $\kappa_0^f = 5,982$). Higher search cost scalars and larger network effects appear to help the known θ^f model explain the observed pattern of small entry, gradual growth, and eventual dominance by high-θ entrants without relying on learning effects. However, the known-θ^f version of the model does substantially worse than the benchmark version according to Rivers and Vuong's (2002) test statistic for non-nested comparisons.[30]

6 Analysis of results

6.1 Model fit

Appendix D juxtaposes the data-based moments, $\bar{m}$, with their simulated counterparts, $m(\Lambda)$, from the benchmark model. Generally, the patterns in the data are replicated by our model, though not all of the model-based equation estimates correspond closely to their

[30]The Rivers and Voung (2002) statistic takes the form $T_n = \frac{\sqrt{n}}{\hat{\sigma}_n}\left[\hat{\Lambda}^1 - \hat{\Lambda}^2\right]$, where $\hat{\Lambda}^1$ and $\hat{\Lambda}^2$ are the MSM fit metrics for the two models, and $\hat{\sigma}_n^2$ approximates $var\left[\hat{\Lambda}^1 - \hat{\Lambda}^2\right]$. This statistic has a standard normal distribution under the null $E(\hat{\Lambda}^1) = E(\hat{\Lambda}^2)$. Applying it to our context, and treating the weighting matrix W as non-stochastic when calculating $\hat{\sigma}_n^2$, we get T_n = -1,583.2. Two caveats apply. First, it is not obvious what the right sample size n is in our context, given that some of our moments are constructed using firm-year level data, some are constructed using shipment-level, and some are constructed using match level. We used a very conservative approximation to the number of firms we base our inferences on ($n = 1000$), but clearly, the test statistic would have been highly significant at much smaller values. Second, this test statistic does not recognize randomness in the fit statistics due to the simulation draws we use. Time and hardware limitations prevented us from using samples so large that this was negligible, though we used common seeds for both sets of results.

data-based analogs. In particular, average exporting rates, match-specific sales dynamics, and the client distribution are well-captured by the model, as are most mean values of dependent variables. However the model fails to generate the association between success rates and firms' search intensities that we observe in the data. This relationship is relatively weak—note the large standard errors for the coefficients on $\ln(1+\frac{a}{n})$ and $\left[\ln(1+\frac{a}{n})\right]^2$ in column 1 of Table 6—so it doesn't receive much weight in the fit metric. A more detailed summary of the fit can be found in Appendix D.

Since we have not targeted the patterns described in Section 2 when estimating, it is instructive to ask how well they are replicated by our model. Tables 10, 11 and 12 below provide answers. In the top panel of table 10, the information in Tables 1-3 is collapsed by averaging across exporting cohorts for which we observe at least 10 years of data. (These cohorts were born in the years 1997 through 2002.) Recall that values of each aggregate for 2-year olds, 3-year olds, and so on are expressed as fractions of the corresponding values for 1-year olds. For example, the data tell us that, on average, only 29 percent of the exporters who began exporting in year t were still exporting in year t+1, and only 5 percent of those that began exporting in year t were present in year t+9. Likewise, among Colombian exporters that survive in the U.S. market for 10 years, average exports per firm are 6.58 times as large as they are among exporters that are in their first year of exporting.

The bottom panel of Table 10 shows corresponding figures based on model-simulated data. Qualitatively, the patterns in the actual and the simulated data match up. For both data sets, the largest drops in the number of exporters occur during the first two years, thereafter cohort size drops gradually. Likewise, total exports rise early in cohort's life, and decline thereafter. Finally, exports per surviving firm grow rapidly over time, reflecting both the exit of small-scale firms and client accumulation among survivors. It should be noted, however, that the "average exports" and "total exports" series based on actual data vary much less dramatically with cohort age than the simulated data. Also, in the data-based figures, the drop in cohort membership is more dramatic during the first year. In sigifnicant part, these discrepancies reflect the fact that the data-based figures were constructed by treating the first shipment between a buyer and a seller as establishing a match, while the model does not.[31]

Table 11 compares the match exit rates observed in the actual data with those observed in the simulated data. These are broken down by match age, and by the size of the match's first-year sales. As with the figures in Table 6, this comparison is imperfect because of the

[31] Specifically, since the data-based series count single-shipment buyer-seller encounters as matches, these series inflate the one-year-old firm and total export counts, while they depress mean exports among one-year olds. Restrictions on data access have temporarily prevented us from re-doing these tables in a fully compatible way. The issue will be addressed in a future draft.

Table 10: Cohort evolutions: data vs. model

	Actual data		
Cohort age	Exporters	Total Exports	Average Exports
1 year	1	1	1
2 years	0.29	1.11	3.77
3 years	0.18	0.93	5.03
4 years	0.14	0.67	4.66
5 years	0.12	0.63	5.18
6 years	0.10	0.51	4.99
7 years	0.08	0.50	5.72
8 years	0.08	0.45	5.91
9 years	0.07	0.39	5.58
10 years	0.06	0.40	6.58

	Simulated data		
Cohort age	Exporters	Total Exports	Average Exports
1 year	1.00	1.00	1.00
2 year	0.61	1.73	2.84
3 years	0.35	1.34	3.81
4 years	0.19	1.81	9.50
5 years	0.10	2.29	22.74
6 years	0.06	2.12	34.43
7 years	0.05	1.89	39.69
8 years	0.04	1.69	43.23
9 years	0.03	1.89	63.69
10 years	0.02	1.46	65.17

Notes: Figures for cohorts aged 2-10 are expressed relative to corresponding figures for one-year-old cohorts.

Table 11: Match separation rates

Match age	Quartile 1	Quartile 2	Quartile 3	Quartile 4
	Actual data			
1 year	82.9	75.6	67.7	52.1
2 years	63.2	58.4	52.1	44.5
3 years	57.3	49.4	44.6	40.3
4 years	55	46.8	40.8	39.2
5+ years	49.7	43.7	37.6	36.7
	Simulated data			
1 year	59.5	88.4	89.7	62.7
2 years	26.6	28.6	31.0	27.4
3 years	29.7	32.2	32.6	29.9
4 years	31.0	27.6	20.4	31.9
5+ years	28.4	29.5	36.3	36.1

Notes: Figrures are percentages of the exporters in each age-initial size category that do not export during the following year.

Table 12: Exporter distribution by number of buyers

	share of exporters	
Number of buyers	actual data	simulated data
1	0.79	0.77
2	0.11	0.10
3	0.03	0.05
4	0.02	0.03
5	0.01	0.02
6-10	0.02	0.03
11+	0.02	0.01

Notes: Figures give the ergodic distribution of current buyer counts across exporting firms.

differences in the way matches are defined in the two data sets. Nonetheless, the relatively high failure rates among first-year matches are replicated by the model, as is the tendency for matches that begin from the largest sales quartile to fail less frequently than others. However, the high failure rates are concentrated among one-year-old matches in the simulated data, while they decline more more gradually with age in the actual data. Also, unlike the actual data, the exporters that begin in the smallest size quartile exhibit failure rates as low as those of the largest exporters.

Finally, Table 12 reports the distribution of client counts across exporters in the actual versus simulated data. Overall the two distributions match up very well, though the actual data contain more exporters with two clients (and fewer with more than two clients) than the model predicts.

6.2 The value of relationships

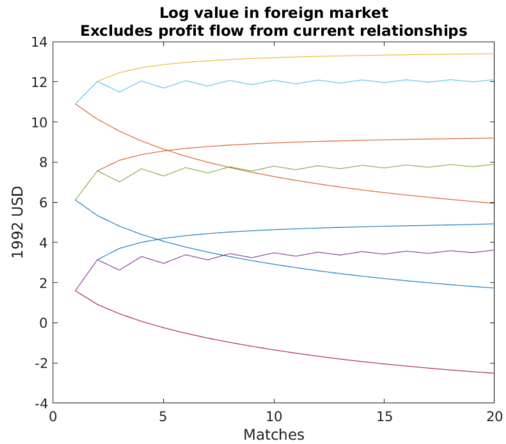
6.2.1 The value of clients

In addition to convex search costs, two forces in our model make exporting decisions forward looking. First, each successful business relationship improves an exporter's visibility and reduces its cost of finding additional potential clients. We call this the "network effect." Second, each match—successful or unsuccessful—conveys information about the scope of the market for the exporter's product. We call this the "learning effect." With Bayesian updating (equation 7), it means that early matches generate particularly valuable signals and may be worth pursuing even if they are not expected to generate significant earnings. It also means that two firms, *ex ante* identical, may have very different long term experiences in export markets, depending upon whether their early matches happened to yield successful business relationships.

To give some sense for the combined importance of the network effect and the learning effect, Figure 2 shows the perceived change in the firm's value with each additional meeting. These changes are exclusive of the profits generated by the new matches, so they describe the impact of each new match on continuation values solely through these two effects. We plot the continuation values for firms of three productivity types, taken from the 10th, 50th, and 90th productivity percentiles among simulated exporters.[32] These values depend upon firms' priors concerning their popularity ($\bar{\theta}^f$), which in turn depend upon the number of meetings (n) they have already experienced at the of time each increment to a. (They do not depend upon firms' *true* success rates, θ^f, as these are unobservable.) We demonstrate this dependence of perceived continuation values on match histories by considering several extreme cases: an unbroken string of successful matches ($n = a$) and an unbroken string of

[32] Of course, low-productivity firms do not export, so these are percentiles of a truncated distribution.

Figure 2: Log continuation values conditioned on match history



Notes: Continuation value trajectories for firms with productivity in the 10th, 50th, and 90th percentiles of the simulated productivity distribution of exporters. For each productivity type, we plot values for all successful matches, alternating success and failure, and all failures.

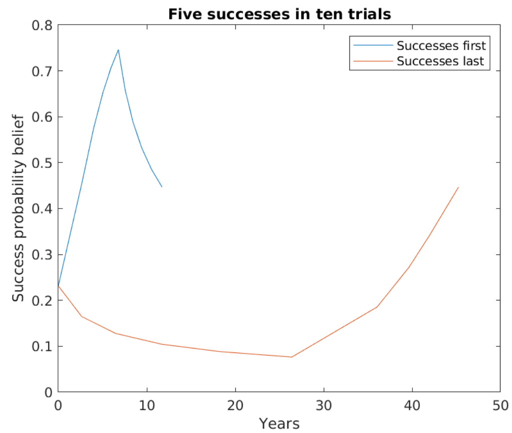
failures ($n = 0$). To provide a benchmark, we also graph the evolution of firms' values when they experience a strictly alternating succession of successes and failures ($n \approx 2a$).

Initial continuation values of the three productivity types of exporters vary widely. The foreign operations of the highest productivity type of firm are valued at US$ 53,800 before its first foreign match, while the median productivity firm's foreign operations are valued at only US$ 452 before its first match, and the foreign operations of the lowest productivity firm are initially worth only US$ 5.

The first match has the biggest impact on continuation values, and most of the impact of additional information has dissipated by the twentieth match. For example, if its first match is a success, the highest productivity firm's value jumps to US$ 165,000. On the other hand, failures quickly erase firm value. The continuation value of the median productivity firm with four successful matches is almost the same as the value of the high productivity firm with four failed matches, at US$ 5,669.

Because match histories affect continuation values, they also affect the intensity with which firms search for new clients. We explore this dependence in Figure 3, which plots beliefs regarding θ^f over time for a firm in the 90^{th} percentile of productivity. Here we assume that if a firm is searching with intensity λ, it meets its next match at exactly the mean waiting time $1/\lambda$. There are two lines on the plot, both containing five successes and five failures. The only difference is that in the top line, the successes come first, while in the bottom line the failures come first. Before any meetings, the beliefs are the same, and after all 10 meetings they are the same as well because at this point both histories contain

Figure 3: Evolution of success probability belief



Notes: Beliefs of a firm with productivity in the 90th percentile of exporters over success probability. Top line is five success followed by five failures. Bottom line is five failures followed by five successes.

5 successes.

The key message of Figure 3 is that if the successes come first, it takes 10.5 years to get 10 matches. But if the failures come first, it takes more than 43 years. Thus, simply because of luck, it takes four times longer for the failure-first firm to get to 10 meetings because it searches far less intensively along the way.

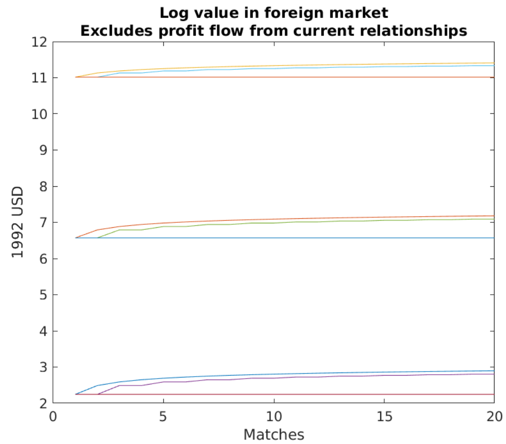
6.2.2 Value dynamics when θ^f is known

The patterns we have depicted thus far reflect both the network effect and the learning effect. To gauge their relative importance, we now redo Figure 2 under the assumption that firms know their true θ^f realizations from the start. More precisely, using our estimates of the "known-θ^f" policy function (see Table 9), we simulate the continuation values of firms at the 10th, 50th, and 90th productivity percentiles. And as in Figure 2 we consider three alternative match histories: only successes, only failures, and alternating successes and failures. Also, since true success rates now affect behavior, we give all firms a success probability of $\theta^f = 0.43$. This number corresponds to the 65th percentile of success probabilities among active exporters in our simulated data.[33]

The results appear in Figure 4. Overall, continuation values move much less as firms acquire experience, implying that the new exporter dynamics in Figure 2 were mainly due to firms learning their types. Continuation values do still rise a bit when firms make successful

[33]We chose this particular value because it is close to 50%, and it is one of the discretized points on the grid we use for estimation.

Figure 4: Log continuation values conditioned on match history, no learning



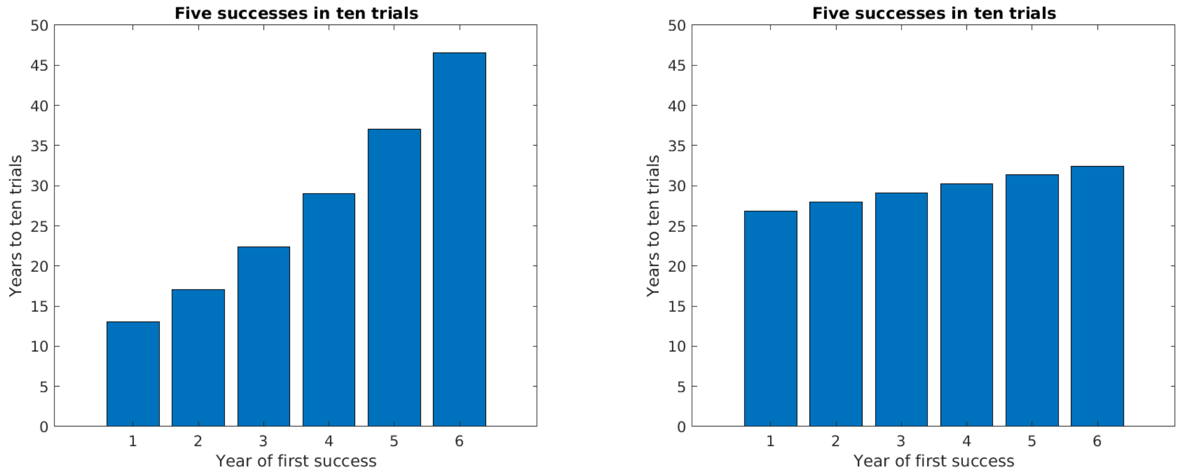
Notes: Continuation value trajectories for firms with productivity in the 10th, 50th, and 90th percentiles of the simulated productivity distribution of exporters in the learning version of the model. For each productivity type, we plot values for all successful matches, alternating success and failure, and all failures.

matches because these matches make is easier to meet additional buyers through the network effect. However, unsuccessful matches now have no effect on these values.

We can also examine how match arrival times depend on successes and failures in the known-θ^f model. Since firms know their success probabilities in this version of the model, the known-θ^f version of Figure 3 (not pictured) is simply two horizontal lines with height θ^f. But the lengths of these lines still depends upon match histories because of the network effect. To demonstrate this dependence, Figure 5 plots the expected time to ten meetings, when five consecutive meetings are successful and the others failures. The x-axis is the number of meetings that take place before the first success occurs. If it is one, then the first five meetings are successes, and the next five failures. If it is 6, then the first five meetings are failures, and the last five meetings successes.

To facilitate comparison, Figure 5 presents results for both the baseline model (panel a) and the known-θ^f model (panel b). All plots are for a firm in the 90^{th} percentile of productivity among exporters, and, in the case of the known-θ^f model, the success probability is set to $\theta^f = 0.43$. As we saw in Figure 3, the time it takes a learning firm to reach ten meetings depends heavily on the placement of the successes (panel a). If the successes come at the beginning of the ten meetings, it takes 12 years for this type of firm to reach ten meetings. If the failures come first, it takes 45 years. But for known-θ^f firms (panel b), time to ten meetings depends much less on the placement of successes. If the successes come

Figure 5: Time to ten meetings by placement of five consecutive successes



(a) Baseline

(b) known-θ^f

first, it takes 27 years to reach ten meetings. If they come last, it takes 32 years.[34] So here again we see that learning effects and network effects both matter, but the former are more important.

6.2.3 Exchange rate shocks and relationship values

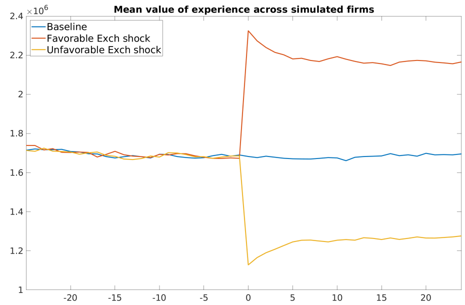
Finally, to inform our discussion of aggregate export dynamics below, we characterize the responses of firms' values to permanent but unanticipated switches in the exchange rate regime. Specifically, we generate 2000 different 100-year long foreign demand trajectories using the process reported in Table 5. Then we scale all realizations after the 50^{th} year by 1.20 (for our devaluation experiment) or 0.8 (for our revaluation experiment), and for each trajectory, we simulate all firms' exporting patterns using our estimated model.

In Figure 6 we plot the average and median value of firms' exporting experience during the 25 years immediately before the shock and the 25 years immediately thereafter.[35] At each point in time, these series indicate how much a typical exporter would lose if we returned it to the state it was in before it began meeting foreign clients. Thus they represent the value of the information it has learned about its success rate θ^f, plus the value of the market

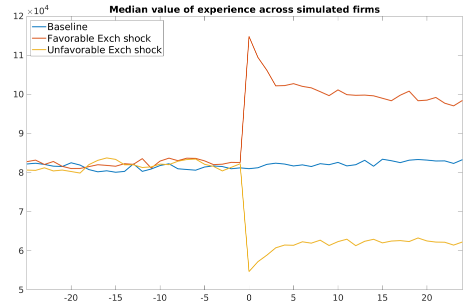
[34]The reason it takes so long is that this type of firm expects that only around half of its meetings will be successes. This corresponds to the ultimate 50% success rate we are simulating, but it also means that the firms are not searching very hard. If we were to make the firm believe that it has a close to 100% success rate, it would only take a few years to reach 10 meetings.

[35]Both the means (panel a) and the medians (panel b) are averages across our 2000 simulations.

Figure 6: Exchange rate shocks and exporter value



(a) Mean value of experience



(b) Median value of experience

visibility it has built through previous matches, plus the expected profits yet to be generated by its current portfolio of clients. These series were calculated under the presumption that firms are free to re-enter export markets after their exporting histories are wiped out, so they substantially understate the capital losses that would result from a permanent move to autarky.

Because the productivity distribution is right-skewed, the means are significantly higher than the medians. Before the exchange rate shock, the mean is around US$ 1.7 million, while the median is only US$ 80,000. After the shock, the values ultimately converge to around 20 percent higher or lower than their pre-shock values, but there is some overshooting in each case. For the positive shock, this pattern reflects the role of matching frictions, which delay the entry of marginally profitable firms. For the negative shock, the pattern reflects the combined effects of fixed costs (F^f) and idiosyncratiic shocks to y_{ijt}, which cause the eventual termination of matches at some marginally profitable firms.

The transition patterns for the means are nearly mirror images of each other in both the positive shock and the negative shock cases (panel a), but the medians exhibit more overshooting for the positive shock than for the negative shock, with a lower long run effect (panel b). This asymmetry implies that positive shocks tend to gradually increase the right-skewness of the value distribution, reflecting the gradual growth of large firms' client bases and the gradual entry of many marginal exporters.The adjustment to exchange rate appreciation is more rapid because search frictions don't inhibit downsizing. By not paying the fixed match maintenance costs, all exporting firms can quickly shed their low-value exporting relationships, and in the case of marginal exporters this amounts to exiting the market. We will explore the extent to which this asymmetry in responses creates hysteresis in aggregate exports in Section 6.3 below.

For a back-of-the-envelope calculation of the total (pre-shock) value of Colombian manufacturers' exporting experience, we multiply 1.7 million dollars by the approximately 3000 Colombian exporters we observe each year, obtaining a figure of roughly 5.1 billion dollars. This is about 18 percent of the total value of being able to export, 28.3 billion dollars.

6.3 Macroeconomic adjustment dynamics

6.3.1 Margins of adjustment

How do these changes in value translate into aggregate export dynamics? We conclude our counterfactual analysis by simulating the aggregate export trajectories associated with the exchange rate shocks described above.[36] Figure 7 summarizes the results. The panels in the left-hand column correspond to a permanent 20 percent peso devaluation and the panels in the right-hand column correspond to a 20 percent peso appreciation, each occurring at the end of the 50^{th} year (period 0). Each panel breaks down the total value of a particular export aggregate—sales, matches, or exporters—into contributions from matches that existed in period 0 (yellow area; hereafter "incumbent matches"), matches created after period 0 with exporters that existed in period 0 (orange area; hereafter "new matches with incumbent exporters"), and matches formed after period 0 with exporters that entered after period 0 (blue area; hereafter "new matches with new exporters"). Also, each panel includes a set of thin lines that show how the boundaries between the shaded areas would have been different if there had been no permanent shock. These lines are common to panels in the same row.[37]

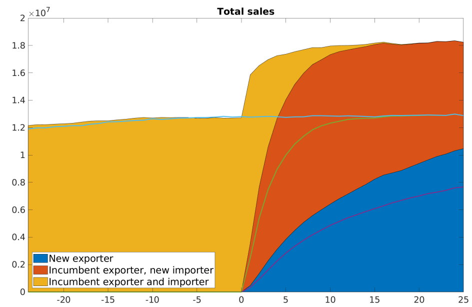
Consider first Panels (a) and (b), which describe total export sales. Perhaps the most striking pattern here is the rapid turnover in matches. Within several years, incumbent matches have lost about three-quarters of their market share, regardless of whether the exchange rate depreciates (figure a), appreciates (figure b), or fluctuates around a startionary mean (thin lines in both figures). Nonetheless, given their persistent productivity (φ) and product appeal (θ), incumbent exporters retain more than 50 percent of the market after 25 years by continuously replenishing their client portfolio.

Despite rapid match turnover, adjustments to a new exchange rate regime take time to play out. During the first post-shock year, valuation effects account for almost all of the movement in aggregate export sales (panels a and b). However, as time progresses the total

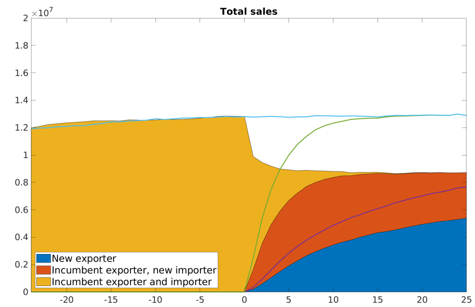
[36]Since we are using a single-agent model, we caution that these simulations miss interactions between exporters in the foreign market. However, they may be a reasonable approximation to the aggregate exports responses of a small country shipping to a large one, where its products constitute a small fraction of total supply.

[37]These figures are inspired by similar graphs in Piveteau (forthcoming), but to highlight the role of learning and endogenous match separations, we use a decompostion that distinguishes matches to new exporters from others. (Piveteau (2020) distinguishes the consumer margin, the extensive margin, and an aggregative valuation effect.)

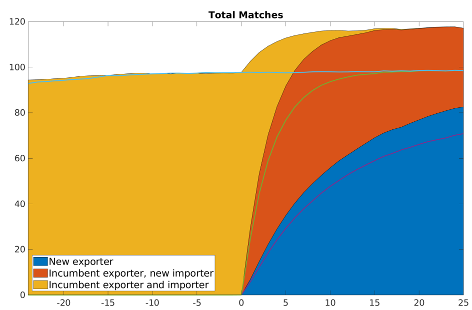
Figure 7: Baseline response to a permanent shock: export aggregates



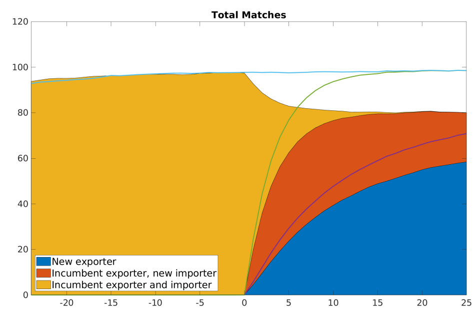
(a) Total sales: Devaluation



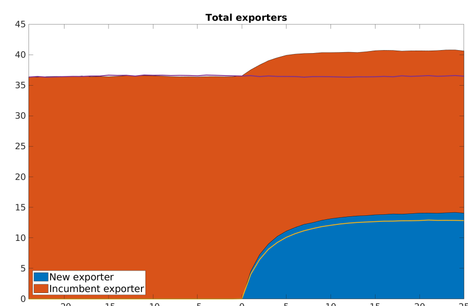
(b) Total sales: Revaluation



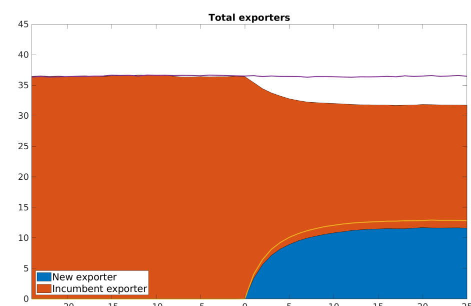
(c) Total active matches: Devaluation



(d) Total active matches: Revaluation



(e) Total active exporters: Devaluation



(f) Total active exporters: Revaluation

Notes: Figures in the left-hand (right-hand) column depict aggregate responses to a permanent 20 percent real devaluation (revaluation) at time 0. Shaded areas in panels a-d reflect contributions of matches that existed at time 0 (yellow), matches formed after time 0 by exporters that were active at time 0 (red), and matches formed after time 0 by exporters that entered the foreign market after time 0 (blue). Thin lines show patterns that would have obtained in the absence of the shock. All series are averages across 2000 simulations of the exchange rate process.

effect of the exchange rate shock grows, both because the total number of exporters adjusts, and because the number of matches per exporter moves sympathetically (refer to panels c through f). These processes take place over a period of roughly 15 years, adding an extra 40-50 percent to the initial valuation response.

Figure 8 describes the same experiment as Figure 7, but instead of showing the levels of each aggregate through time, it plots the percentage contribution of each type of match to the corresponding aggregate. Remarkably, the post shock transition patterns in all figures coincide very closely with the no-shock (thin line) patterns, implying that the cohort composition of active matches is nearly invariant to aggregate shocks. Hence, if we were only interested in the rate at which new exporters displace incumbent exporters, or the rate at which new matches displace incumbent matches, it would matter very little whether we were analyzing the aftermath of a permanant devaluation, a permanent revaluation, or a period without any regime switching.

6.3.2 Learning, networks and aggregate export dynamics

Nonetheless, the relative contributions of the different types of matches *do* depend upon the learning effects and visibility effects captured by our model. To demonstrate how, we next contrast the post-shock evolution pattern implied by our benchmark model (column 1 of Table 9) with the pattern implied by the known-θ^f version of the model (column 3 of Table 9), normalizing the predicted values of each aggregate to unity in the initial shock period. And we contrast the dynamic implications of the known-θ^f version of the model with those of a model in which θ^f's are known *and* all firms have maximum visibility, regardless of their exporting history.[38]

Our findings are summarized by Figure 9. As in figure 8, each panel describes the market share of icumbent matches (yellow region), new matches with incumbent exporters (orange region) and new matches with new exporters (blue region) after an exchange regime shock. But in this figure, we have superimposed lines that show how the shaded areas would have shifted if firms knew their θ^f draws with certainty (dashed red lines), and if they not only knew their θ^f values, but also had maximum visibility (dashed green lines).

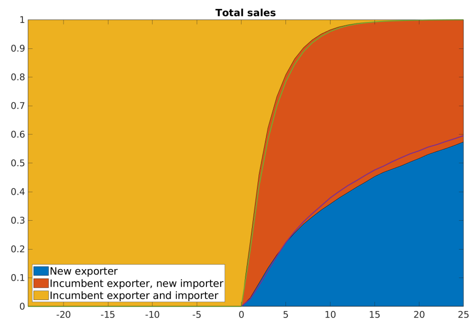
Several messages emerge from this exercise. Most notably, when firms always know their true θ^f values, the share of new matches that goes to new exporters is substantially higher. Why? Entering cohorts are dominanted by firms with high θ^f's, and these firms search more

[38] In our simulations of the benchmark model, the maximum number of successful matches is approximately $m = 40$ successful matches. So to characterize full visibility, we replace equation (11) with

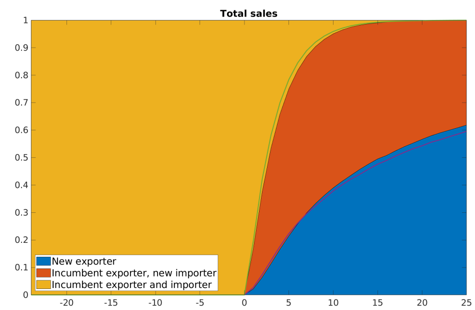
$$c^m(s^m, a^m) = \kappa_0^m \frac{[(1+s^m)]^{\kappa_1} - 1}{\kappa_1 [1 + \ln(1 + a^{40})]^\gamma}. \tag{14}$$

.

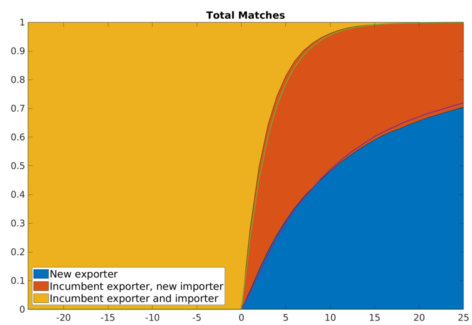
Figure 8: Baseline responses to a permanent shock: export composition



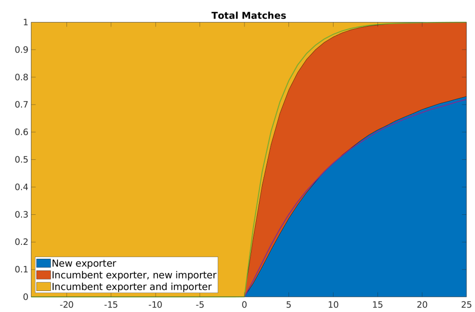
(a) Total sales: Devaluation



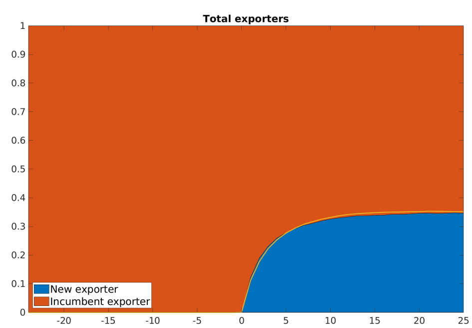
(b) Total sales: Revaluation



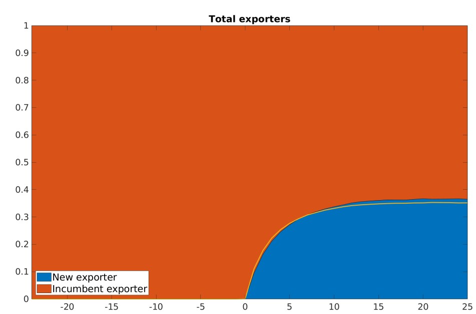
(c) Total active matches: Devaluation



(d) Total active matches: Revaluation



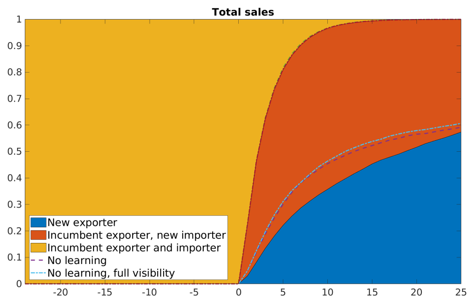
(e) Total active exporters: Devaluation



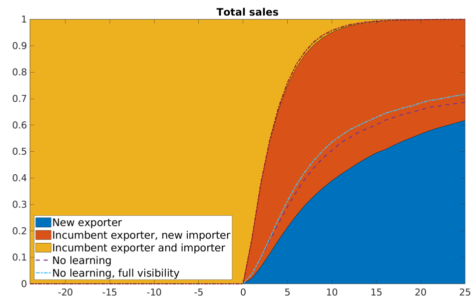
(f) Total active exporters: Revaluation

Notes: Figures in the left-hand (right-hand) column depict aggregate responses to a permanent 20 percent real devaluation (revaluation) at time 0. Shaded areas in panels a-d reflect contributions of matches that existed at time 0 (yellow), matches formed after time 0 by exporters that were active at time 0 (red), and matches formed after time 0 by exporters that entered the foreign market after time 0 (blue). Thin lines show patterns that would have obtained in the absence of the shock. All series are averages across 2000 simulations of the exchange rate process.

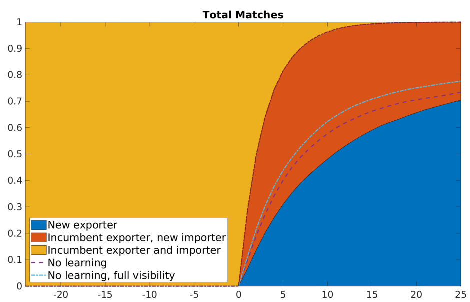
Figure 9: Responses to a permanent shock: baseline vs. other specifications



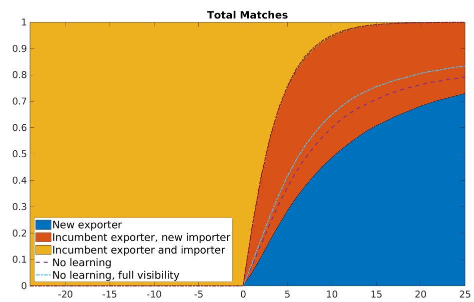
(a) Total sales: Devaluation



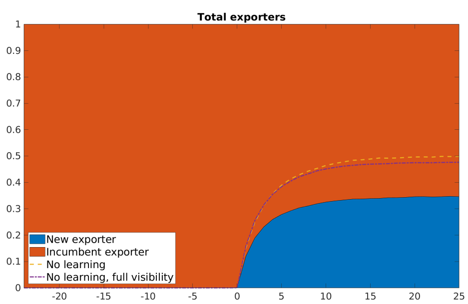
(b) Total sales: Revaluation



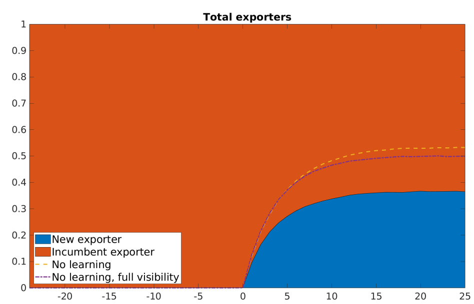
(c) Total active matches: Devaluation



(d) Total active matches: Revaluation



(e) Total active exporters: Devaluation



(f) Total active exporters: Revaluation

Notes: Figures in the left-hand (right-hand) column depict aggregate responses to a permanent 20 percent real devaluation (revaluation) at time 0. Shaded areas in panels a-d reflect contributions of matches that existed at time 0 (yellow), matches formed after time 0 by exporters that were active at time 0 (red), and matches formed after time 0 by exporters that entered the foreign market after time 0 (blue). Thin dashed lines show patterns that would have obtained if learning effects (red) or both learning and network effects (green) had been absent. All series are averages across 2000 simulations of the exchange rate process.

intensely. Of course, firms that were incumbent in period 0 also tend to have high θ^f's and invest heavily in search, but since many of these firms were already well established, revealing their θ^f draws to them doesn't change their behavior as much.

Second, when firms always know their θ^f draws, giving everyone maximum visibility tends to increase the relative importance of new matches with new exporters. (Note that the green lies above the red dotted line in panels a through d of Figure 8.) The simple reason is that new exporters typically have less visibility than incumbent exporters, and this experiment eliminates their associated cost disadvantage. Note, however, just as we found in Section 6.2 above, visibility effects are not as important as learning effects.

Finally, regardless of whether all firms have maximum visibility and/or full knowledge of their θ^f draws, the matches of entering cohorts displace incumbent matches at almost exactly the same pace. That is, the border between the orange and yellow regions nearly coincides with the dashed red and green lines in all panels. This reflects the fact that incumbent matches are unaffected by either θ^f beliefs or search costs.

6.3.3 Trade elasticities

To conclude our analysis of export aggregates, we present short and long-run simulated trade elasticities in Table 13, with cross-simulation standard deviations in parentheses. The first three columns report results for the benchmark model, the middle three for the known-θ^f model, and the last three for the known-θ^f, full-visibility model.

The results do not appear to be very sensitive to our assumptions regarding learning or visibility, though the known-θ^f model generates a bit *less* long run responsiveness, mainly because of smaller match elasticities. Apparently, therefore, when potential firms are ignorant of their potential, their search intensities are relatively relatively responsive to permanent exchange rate shocks.. But these effects are concentrated among new, small-scale exporters and therefore quantitatively small.

While our long run sales elasticities resemble Piveteau's (forthcoming) and Boehm et al.'s (2020), we find a much longer transition period. This is likely due to the frictions in our model that derive from convex search costs and learning effects. For similar reasons, and because ours is a single agent model, our figures are substantially lower than the long run elasticities typically generated by calibrated general equilibrium models (e.g., Alessandria and Choi, 2014; Alessandria, et al., 2018).[39]

[39] Alessandria and Choi (2014) use a symmetric 2-country dynamic model with endogenous firm creation, capital accumulation, fixed exporting costs, and iceberg costs but no other trade frictions or factor adjustment costs. Analyzing movement from a global 8 percent tariff to free trade, they find the trade elasticity rises from about 5 in the short run to 8 in the long run, which is reached in 5-8 years. In a similar model, but with firms' exporting costs depending upon their incumbency, Alessandria et al. (2018) estimate a short-run trade elasticity of 4 and a long-run elasticity of 11.55. This model generates transition dynamics over a period of 10-15 years.

Table 13: Simulated Trade Elasticities

Favorable	**Baseline**			**No learning**			**No learning/Full visibility**		
Time since shock	1 year	5 years	25 years	1 year	5 years	25 years	1 year	5 years	25 years
Sales	1.20	1.69	1.88	1.18	1.66	1.65	1.22	1.63	1.67
	(0.05)	(0.05)	(0.05)	(0.04)	(0.04)	(0.04)	(0.03)	(0.03)	(0.03)
Matches	0.26	0.80	0.94	0.27	0.76	0.85	0.29	0.72	0.76
	(0.03)	(0.03)	(0.03)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Exporters	0.14	0.50	0.60	0.19	0.51	0.61	0.16	0.49	0.55
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Unfavorable									
Sales	1.15	1.62	1.74	1.20	1.70	1.81	1.13	1.65	1.63
	(0.04)	(0.04)	(0.04)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)
Matches	0.24	0.73	0.93	0.25	0.75	0.88	0.20	0.70	0.77
	(0.03)	(0.03)	(0.03)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Exporters	0.14	0.48	0.63	0.16	0.49	0.65	0.14	0.47	0.59
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)

Notes: All elasticities are based on 2000 simulations of favorable and unfavorable 20 percent changes in the mean exchange rate. Starndard errors based on cross-simulation standard deviations are in parentheses.

Total sales react a bit more to the favorable exchange rate shock than to the unfavorable shock: the 25-year elasticity of sales with respect to the former is 1.88, and it is 1.74 with respect to the latter. Interestingly, this difference between expansion and contraction elasticities does not derive from differences in match elasticities, which are nearly symmetric in both dimentions, regardless of what time horizon we consider. Rather, it traces mainly to sales per match, implying that selection effects are in play. Specifically, ast we noted above in connection with figure 6, the smallest, least profitable matches are dropped in a downturn. But in an upturn, there is no selection on the productivities of new matches, since these are unseen until the match is consummated.[40]

[40] An early literature noted the phenomenon of export hysteresis, and attributed it to sunk market entry costs (Dixit, 1989; Baldwin and Krugman, 1989). While search costs are sunk, that explanation does not explain the differences in long run elasticities our model generates. The reason is that matches are short-lived, and search costs must be continuously incurred.

7 Summary

A robust set of stylized facts regarding firm-to-firm trade dynamics has emerged from more than a decade of research on customs records. First, most exporters are inexperienced, ship small quantities, and have few foreign clients. Second, the typical buyer-seller relationship lasts only year or two, so business connections evolve rapidly, and it is common to see firms with only a few clients cease exporting entirely, giving way to the next entering cohort of inexperienced exporters. Third, however, each new cohort contains a small number of firms that survive and grow many times faster than aggregate exports. They do so not by selling increasing amounts to the same clients, but by expanding their customer base abroad.

After confirming these patterns for Colombian manufacturers shipping to the United States, we develop a continuous time model that explains them. Firms wishing to export must engage in costly search to identify potential buyers abroad. The buyers they encounter either reject their products or form finite-lived business relationships with them. Buyer who form business relationships with exporters send them favorable signals about the appeal of their products, and in doing so, encourage them to search more intensively for additional buyers. Successful business relationships also reduce search costs by improving sellers' visibility (network effects). Finally, sellers' search intensities depend upon their permanent idiosyncratic characteristics and marketwide conditions.

Fit using the method of simulated moments, the model replicates the patterns in customs records described above and allows us quantify several types of trade costs, including the search costs of identifying potential clients and the costs of maintaining business relationships with existing clients. It also allows us to estimate the network effect of previous exporting successes on the costs of meeting new clients, and to characterize the cumulative effects of learning on firms' search intensities and intangible capital stocks.

Both the learning effect and the network effect prove to be statistically significant, and combined, they serve to increase aggregate export responsiveness to permanent exchange rate shocks. Their effect is modest, however, mainly because they are most important among newer exporters, which account for a small share of total export volume. Finally, the effects of permanent exchange rate shocks are larger over longer time horizons, mostly because of search frictions, but also because of learning effects, which make exporters with appealing products gradually increase their search intensities.

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A data tables

year	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	total
1992	2,232																		2,232
1993	823	1,235																	2,058
1994	583	330	1,160																2,073
1995	440	213	339	953															1,945
1996	372	163	178	255	899														1,867
1997	321	128	133	170	248	877													1,877
1998	268	104	124	132	153	256	893												1,930
1999	232	85	87	114	117	187	262	1,026											2,110
2000	203	85	79	91	103	136	170	344	1,372										2,583
2001	187	70	65	79	85	109	145	229	389	1,251									2,609
2002	173	64	62	72	68	88	112	171	242	399	1,373								2,824
2003	165	51	58	62	62	77	86	140	185	301	440	1,719							3,346
2004	150	52	41	53	63	76	80	132	164	223	327	616	1,768						3,745
2005	140	52	47	39	54	77	69	115	145	196	235	398	661	1,902					4,130
2006	122	46	44	39	44	71	65	110	131	157	168	308	410	564	1,896				4,175
2007	113	37	39	31	42	55	48	91	101	132	156	240	305	365	548	1,681			3,984
2008	93	29	30	24	38	50	45	74	90	117	130	184	198	230	331	447	1,455		3,565
2009	80	25	28	24	28	40	39	60	72	88	97	145	175	157	230	248	386	1,378	3,300

Table 14: Number of Exporting Firms, by Entry Cohort

year	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	total
1992	469																		469
1993	352	83																	435
1994	336	83	92																510
1995	313	75	102	58															549
1996	256	67	62	40	60														484
1997	247	84	43	41	48	119													581
1998	225	49	42	36	45	131	63												590
1999	207	51	49	41	39	197	74	81											739
2000	180	53	55	37	51	102	53	158	109										799
2001	150	22	51	41	28	57	36	80	101	111									677
2002	124	23	47	34	27	28	23	45	65	83	40								538
2003	147	42	51	31	42	24	22	37	71	107	50	78							702
2004	156	43	53	19	57	21	23	42	78	106	60	107	90						855
2005	150	22	75	17	52	18	23	43	78	80	58	81	75	84					855
2006	117	31	52	14	64	43	17	38	61	79	32	51	52	112	78				838
2007	103	7	18	11	67	58	19	30	28	64	22	35	33	66	67	62			689
2008	95	6	9	8	33	37	17	33	26	34	20	31	37	54	42	53	57		591
2009	68	22	7	6	13	24	10	23	16	16	14	22	41	25	39	37	36	64	485

Table 15: Value of Exports, by Entry Cohort (millions of $US)

year	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	pooled
1992	210																		210
1993	428	67																	211
1994	576	251	79																246
1995	712	353	300	61															282
1996	687	411	346	158	67														259
1997	771	652	321	241	192	136													310
1998	839	468	339	269	297	510	71												306
1999	893	601	561	361	336	1,054	281	79											350
2000	885	623	697	407	496	750	313	460	80										309
2001	801	316	783	519	329	521	251	350	259	89									260
2002	716	353	757	473	399	318	207	260	268	207	29								191
2003	891	827	870	493	677	315	257	260	385	355	114	46							210
2004	1,039	828	1,281	358	900	281	291	318	478	476	183	174	51						228
2005	1,071	413	1,593	444	967	231	326	375	535	408	248	204	113	44					207
2006	958	675	1,177	356	1,448	605	256	341	464	505	188	165	126	198	41				201
2007	915	175	466	357	1,606	1,048	391	327	278	481	140	145	108	181	123	37			173
2008	1,023	208	283	341	860	747	379	443	289	287	153	166	186	236	125	120	39		166
2009	855	864	262	266	478	607	255	389	221	176	143	152	235	162	169	151	93	47	147

Table 16: Exports per Firm, by Entry Cohort (thousands of $US)

Year	Colombian Sellers	U.S. Importers	Pairs
1992	2,232	1,190	3,087
1993	2,058	1,183	2,824
1994	2,073	1,212	2,810
1995	1,945	1,173	2,588
1996	1,867	1,191	2,490
1997	1,877	1,208	2,480
1998	1,930	1,191	2,495
1999	2,110	1,386	2,793
2000	2,583	1,661	3,411
2001	2,609	1,698	3,483
2002	2,824	1,826	3,733
2003	3,346	2,110	4,483
2004	3,745	2,296	5,071
2005	4,130	2,457	5,552
2006	4,175	2,471	5,607
2007	3,984	2,343	5,307
2008	3,565	2,221	4,751
2009	3,300	2,079	4,467

Table 17: Exporters and importers by year

B Data checks

To investigate the quality of the exporter id (manuf_id) in the U.S. import records, we ran a series of robustness checks. The Colombian and U.S. data overlap for the years 2000-2008 and both contain measures of the value of exports as well as the number of exporting firms. If the manuf_id variable is error-prone and noisy, we would expect the U.S. data to over-report the number of Colombian firms exporting to the U.S. That is, each time a customs broker wrongly enters the data in the field, a new firm would be created. Table 18 below summarizes the total value of exports to the U.S. and the number of Colombian firms, by year, for each data set.

The datasets align much more closely on value than they do on firm counts. The difference in value is never more than 10% while the firm count difference ranges from 18% to 74%. The differences are stable over time.

To look more closely at the cause of the difference in firm counts, we compared the number of firms across sources by HS2 categories. The counts in the LFTTD were higher than the Colombian data in only 28 of the 82 codes and by far the biggest differences are in HS codes 61 and 62: textiles. In these two product classes the U.S. data identifies 4025 more firms than the Colombian data. If we remove these two sectors from the list, the difference in firm counts flips and the Colombian data contain 1001 more firms than the LFTTD.

Interestingly, Title 19 of U.S. code specifically requires that the manuf_id variable for textile products represent the manufacturer of the textile products, not an intermediary. That is, for this sector in particular the manufacturer, not an intermediary must be reported on the CBP 7501 form. By contrast, prior work by several authors of this paper has shown (Marcela's 8/9/13 e-mail referenced this) that the Colombian data reports the exporter, which may or may not be the manufacturer. Given that revious research (Tybout, 2000 JEL) has shown that developing countries tend to have a disproportionately large share of

	Colombia		**United States**		**% difference**	
Year	# exporters	value	# exporters	value	# exporters	value
2000	1775	1038	2721	1140	53%	10%
2001	2026	995	2744	1019	35%	2%
2002	2230	870	2986	855	34%	-2%
2003	2800	1113	3579	1119	28%	1%
2004	3035	1379	4002	1415	32%	3%
2005	2861	1554	4288	1438	50%	-7%
2006	2689	1665	4361	1552	62%	-7%
2007	2420	1540	4175	1496	73%	-3%
2008	2161	1570	3758	1474	74%	-6%

Table 18: Colombian versus U.S. Customs Records

small manufacturing firms, it is reasonable to assume that a large part of the reason why the U.S. data report so many more firms in the textile sector is that due to administrative reasons the U.S. data count many small manufacturers and the Colombian data are, in many cases, reporting aggregators and intermediaries.

As a final check of the integrity of the manuf_id variable - and the robustness of our main results - we experimented with a "fuzzy" version of the manuf_id variable that did not contain any street numbers in the string (a likely source of input errors). The effect of this is to reduce the number of Colombian firms in the data, an approximation of fixing any extraneous noise from data entry errors. Next we re-ran Table 4 with the fuzzy data and compared the results to the original version.

One of the key findings from Table 4 is the high match separation rates ranging from about 40% to 66%. Using the fuzzy version did not reduce the separation rates substantially and left the patterns intact. The fuzzy separation rates ranged from 26% to 62%, a drop of 6% on average. It does not appear that our results are sensitive to a modest reduction in data entry errors.

C Identification

Table 19: Sensitivity matrix

	$\ln \Pi^h$	F^h	F^f	$\ln \Pi^f$	α	β	Δ_y	λ_b	γ	$\ln \kappa_0^h$	$\ln \kappa_0^f$	σ_φ
avg. mat death	-0.112	0.007	0.011	-0.111	-0.019	0.127	0.006	0.068	-0.045	-0.097	-0.119	0.000
new to mkt	-0.841	-0.034	0.420	-0.656	-0.283	0.772	1.199	-0.325	-0.482	0.329	0.131	0.040
current sales	-2.164	0.294	2.080	2.905	0.003	0.454	11.016	4.386	-2.072	-12.437	-3.520	-0.326
exporter age	1.027	0.092	-0.745	1.029	0.411	-1.124	-1.216	1.074	0.586	-0.966	-0.726	-0.089
match age	-0.876	0.117	-0.216	-0.557	0.068	0.868	-0.389	1.422	-0.136	-2.108	-1.579	-0.045
avg. match sales	0.190	-0.012	-0.042	0.075	0.040	-0.045	-0.228	-0.176	0.072	0.256	0.214	0.011
1st yr dummy	1.616	-0.096	0.462	5.391	0.419	-4.141	6.635	1.058	0.501	-2.377	2.464	-0.175
match sales, t-1	-0.683	0.058	-0.097	-1.127	-0.183	1.293	0.170	0.742	-0.469	-0.082	-1.077	0.012
exporter age	0.428	-0.047	0.196	1.050	0.169	-1.178	-0.081	-0.493	0.410	-0.419	0.891	-0.018
MSE, match AR1	-0.349	0.015	-0.132	-0.122	-0.043	0.105	0.091	-0.045	-0.017	-0.019	-0.204	-0.003
degree dist. slope	0.033	-0.002	0.001	0.136	0.071	0.757	-0.020	-0.502	-0.038	0.237	-0.142	-0.016
degree dist. curv.	0.509	-0.016	0.059	0.830	0.287	1.327	0.246	-0.858	0.006	0.377	-0.403	-0.068
avg. ln #shipments	0.206	-0.020	0.237	0.177	0.046	-0.063	-0.803	10.713	-0.144	0.359	-0.001	-0.021
export/dom coef.	-0.442	0.021	0.069	0.714	0.009	-0.302	1.730	0.672	-0.067	-1.311	-0.135	-0.045
dom. sales AR1	2.835	0.051	-4.213	-2.908	-0.602	3.213	1.311	-1.069	-1.155	14.694	-0.998	0.174
avg. match hazard	-0.002	0.007	0.205	0.005	-0.006	0.037	0.055	-0.083	-0.223	-0.418	0.013	0.105
$\ln(1+a)$	-0.040	0.002	0.004	-0.071	-0.003	0.093	-0.044	0.003	-0.016	-0.033	-0.037	0.002
$\ln(1+a)^2$	-0.066	-0.003	0.212	0.770	0.035	-0.790	0.746	0.402	0.081	-0.890	0.169	-0.039
$\ln(1+\frac{1}{n})$	0.018	-0.001	-0.003	0.029	0.000	-0.036	0.024	-0.002	0.004	0.025	0.016	-0.001
$\ln(1+\frac{1}{n})^2$	-0.027	0.001	0.008	-0.027	-0.001	0.037	-0.013	0.011	-0.007	-0.043	-0.020	0.000
$\ln(1+\frac{1}{n}) \cdot \ln(1+a)$	0.089	-0.003	-0.035	0.041	0.006	-0.063	-0.003	-0.065	0.020	0.164	0.057	0.001
avg. succ. rate, $\frac{a}{n}$	-0.181	-0.035	-0.547	-2.688	1.697	-0.361	1.759	-2.682	-0.563	1.323	0.037	0.095
coef., $\ln n$	-1.121	0.020	2.076	-3.493	-0.655	-2.049	-7.981	-0.494	0.032	-0.423	-2.017	0.023
$var(\frac{a}{n}\|n)$	14.085	-0.180	5.673	33.822	8.725	-29.932	-12.829	-3.497	-5.688	-1.592	15.193	-1.119
coef., $\ln n$	11.610	-0.994	9.961	17.641	-2.014	-42.636	19.778	9.299	7.756	-4.401	9.615	-0.933
$\frac{foreign\ sales}{total\ sales}$	-9.785	-0.068	1.997	1.591	0.443	-2.183	-1.189	2.443	1.057	-10.317	1.247	-0.052
$\frac{\#exporters}{\#firms}$	-3.256	0.110	-3.178	-2.103	-1.139	0.187	3.049	2.308	-0.959	9.688	-2.247	0.034

Table 20: Sensitivity matrix, elasticity form

	$\ln \Pi^h$	F^h	F^f	$\ln \Pi^f$	α	β	Δ_y	λ_b	γ	$\ln \kappa_0^h$	$\ln \kappa_0^f$	σ_φ
avg. mat death	0.008	0.064	0.010	0.005	-0.009	0.018	0.001	0.001	-0.031	-0.002	-0.002	0.000
new to mkt	-0.029	0.167	-0.189	-0.014	0.066	-0.054	-0.085	0.003	0.168	-0.004	-0.001	-0.004
current sales	-0.019	-0.360	-0.234	0.016	0.000	-0.008	-0.195	-0.009	0.180	0.035	0.009	0.008
exporter age	0.020	-0.259	0.193	0.013	-0.055	0.046	0.050	-0.005	-0.118	0.006	0.004	0.005
match age	0.005	0.086	-0.015	0.002	0.002	0.009	-0.004	0.002	-0.007	-0.004	-0.002	-0.001
avg. match sales	-0.536	-4.896	-1.569	-0.134	0.762	-0.259	-1.329	-0.125	2.067	0.239	0.181	0.084
1st yr dummy	-0.353	-2.996	1.325	-0.745	0.622	-1.855	2.991	0.058	1.111	-0.172	0.161	-0.107
match sales, t-1	0.107	1.303	-0.200	0.112	-0.194	0.414	0.055	0.029	-0.745	-0.004	-0.050	0.005
exporter age	-0.007	-0.105	0.040	-0.010	0.018	-0.038	-0.003	-0.002	0.065	-0.002	0.004	-0.001
MSE, match AR1	0.065	0.392	-0.325	0.014	-0.054	0.040	0.035	-0.002	-0.033	-0.001	-0.011	-0.002
degree dist. slope	0.010	0.091	-0.004	0.027	-0.150	-0.479	0.013	0.039	0.120	-0.024	0.013	0.014
degree dist. curv.	0.020	0.094	-0.031	0.021	-0.078	-0.109	-0.020	0.009	-0.003	-0.005	0.005	0.008
avg. ln #shipments	-0.079	-1.086	1.195	-0.043	0.120	-0.050	-0.636	1.034	-0.560	0.046	0.000	-0.023
export/dom coef.	0.079	0.546	0.163	-0.081	0.011	-0.111	0.639	0.030	-0.122	-0.078	-0.007	-0.023
dom. sales AR1	-0.705	1.813	-13.736	0.457	-1.018	1.636	0.672	-0.067	-2.912	1.209	-0.074	0.121
avg. match hazard	0.000	-0.245	-0.663	0.001	0.010	-0.019	-0.028	0.005	0.558	0.034	-0.001	-0.073
$\ln(1+a)$	-0.004	-0.025	-0.005	-0.004	0.002	-0.018	0.009	0.000	0.015	0.001	0.001	-0.001
$\ln(1+a)^2$	0.000	-0.003	0.017	-0.003	0.001	-0.010	0.010	0.001	0.005	-0.002	0.000	-0.001
$\ln(1+\frac{1}{n})$	-0.018	-0.107	-0.037	-0.018	-0.002	-0.072	0.048	0.000	0.035	0.008	0.005	-0.002
$\ln(1+\frac{1}{n})^2$	-0.039	-0.216	-0.149	-0.024	0.009	-0.108	0.039	-0.004	0.097	0.020	0.009	-0.002
$\ln(1+\frac{1}{n}) \cdot \ln(1+a)$	-0.013	-0.069	-0.067	-0.004	0.006	-0.019	-0.001	-0.002	0.030	0.008	0.002	0.000
avg. $\frac{a}{n}$	0.022	-0.603	-0.870	0.206	1.399	-0.090	0.440	-0.082	-0.693	0.053	0.001	0.032
coef., $\ln n$	-0.003	-0.006	-0.061	-0.005	0.010	0.009	0.037	0.000	-0.001	0.000	0.001	0.000
$var(\frac{a}{n}\|n)$	-0.238	-0.435	1.257	-0.361	1.001	-1.035	-0.447	-0.015	-0.974	-0.009	0.077	-0.053
coef., $\ln n$	0.100	1.217	-1.121	0.096	0.117	0.749	-0.350	-0.020	-0.675	0.012	-0.025	0.022
$\frac{foreign\ sales}{total\ sales}$	0.157	-0.155	0.419	-0.016	0.048	-0.072	-0.039	0.010	0.172	-0.055	0.006	-0.002
$\frac{\#exporters}{\#firms}$	0.118	0.572	-1.517	0.048	-0.282	0.014	0.229	0.021	-0.354	0.117	-0.024	0.003

D Model Fit

Each table in this appendix reports model-based based moments below their data-based counterparts, which are repeated from Tables 6, 7 and 8. Standard errors for the data-based estimates appear in parentheses below each pair of figures; these too are repeated from Tables 6, 7 and 8.

Looking first at table 21, column 1, we see the model understates monthly log match hazards.The quadratic relationship between match hazards and cumulative successes in the data is also present in the model-based simulations, albeit somewhat dampened. And the relation between success rates and match hazards changes curvature. Column 2 shows that the model under-predicts match death rates a bit, though it picks up their negative relationship to match sales and age. (The first year effect seems to be entirely absorbed by this age variable.) As for success rates, the model comes reasonably close to the data. It misses the positive association between this variable and number of matches, but does replicate the reduction in success rate dispersion as the cumulative number of matches grows.

Turning to table 22, we see that model gets the nearly-Pareto distribution of client counts across firms, as the coefficient on $\ln(\ell)^2$ is negative but close to zero, just as in the data. However, the slope of regression v is less negative in the simulated data than in the actual data, implying that the model predicts relative more exporters have high-client counts. As for equation (vi), the estimated model generates more shipments per month among active matches than we find in the data.

Finally, table 23 shows that the model does a good job of explaining match-level sales dynamics (equation vii), including the dependence of sales on exporters' market tenure, Δ. It also gets the persistence in home market sales almost exactly right (equation $viii$). It is less successful at explaining the weak correlation between domestic and foreign sales, perhaps because the dependent variable is exports destined for the U.S. alone, and exports to other destinations—which are not in our model—are not really independently determined.

Table 21: Match hazards, success rates, and endurance: Model vs. Data

	(i) $\ln(s_{ij})$	*(ii)* $D_{ijt}^{exit\ match}$	*(iii)* $\frac{a_{ij}}{n_{ij}}$	*(iv)* $u^2_{a_{ij}/n_{ij}}$
mean, dep. variable	-0.719 1.527 (0.621 E-2)	0.395 0.267 (0.319 E-2)	0.413 0.470 (0.153 E-2)	0.091 0.066 (0.265 E-3)
$\ln(1 + a_{ij})$	–	–	0.093 -0.009 (0.003)	-0.060 -0.033 (0.000)
$\ln(1 + a_{ij})$	-0.818 -0.371 (0.113)	–	–	–
$[\ln(1 + a_{ij})]^2$	0.312 0.024 (0.017)	–	–	–
$\ln(1 + \frac{a_{ij}}{n_{ij}})$	-1.132 3.774 (0.296)	–	–	–
$\left[\ln(1 + \frac{a_{ij}}{n_{ij}})\right]^2$	2.451 -5.555 (0.396)	–	–	–
$\ln(1 + a_{ij}) \cdot \ln(1 + \frac{a_{ij}}{n_{ij}})$	-0.708 0.564 (0.134)	–	–	–
$D_{ijt}^{new\ to\ mkt}$	–	0.034 -0.133 (0.012)	–	–
$\ln X_{ijt}^f$	–	-0.032 -0.033 (0.002)	–	–
$\ln A_{ijt}$	–	-0.054 -0.077 (0.009)	–	–
$\ln \Delta_{jt}$	–	-0.028 0.020 (0.007)	–	–

Notes: Unit of observation, columns i, iii and iv: seller j's i^{th} match. Unit of observation, column ii: seller j's i^{th} match in its t^{th} year. s_{ij} = inverse of time interval between commencement of match i and commencement of the next one for exporter j $D_{ijt}^{exitmatch} = 1$ if exporter $j's$ i^{th} match dies in year t. a_{ij} = cumulative number of successes for exporter j at time of match i. $D_{ijt}^{newtomkt} = 1$ if exporter $j's$ i^{th} match is in its first year. $\ln A_{ijt}$ = log age of exporter $j's$ i^{th} match. $\ln \Delta_{jt}$ = log age of exporter j in year t. X_{ijt}^f = foreign sales volume generated by exporter $j's$ i^{th} match.

Table 22: Client distribution and shipment frequency, model vs. data

	(v) $\ln(1-\Phi(\ell))$	(vi) $\ln(\lambda_b)$
mean, dep. variable	–	0.971 1.489 ()
$\ln(\ell)$	-1.881 -1.199 (0.112)	-
$(\ln \ell)^2$	-0.056 -0.115 (0.021)	-
sample restrictions	$\ell > 0$	$\lambda_b > 0$
observations	43	87,000

Notes: ℓ: number of active clients; $\Phi()$ = cumulative distribution of exporters in terms of ℓ; s_{ijt} = number of shipments per year to client i by exporter j in year t

Table 23: Home and foreign sales regressions

	(*vii*)	(*viii*)	(*ix*)	(*x*)	(*xi*)
	$\ln X^f_{ijt}$	$\ln X^h_{jt}$	$\ln X^f_{jt}$	D^f_{jt}	$\frac{X^f_{jt}}{X^h_{jt}+X^f_{jt}}$
mean, dep. variable	10.665 10.957 (0.002)	-	-	0.102 0.141 (0.003)	0.127 0.062 (0.002)
R_{ijt-1}	0.328 0.607 (0.018)	-	-		
$\ln X^f_{ijt-1}$	0.826 0.848 (0.004)	-	-		
$\ln X^h_{jt-1}$	-	0.976 0.964 (0.001)	-		
$\ln X^h_{jt}$	-	-	0.323 0.811 (0.012)		
$\ln \Delta_{jt}$	0.063 0.060 (0.014)	-	-		
sample restrictions	$X^f_{ijt} > 0$ $X^f_{ijt-1} > 0$	$X^h_{jt} > 0$ $X^h_{jt-1} > 0$	$X^f_{jt} > 0$ $X^h_{jt} > 0$	$X^h_{jt} > 0$	$X^f_{jt}, X^h_{jt} > 0$
observatiaons	25,400	99,300	11,600	119,800	12,500

Notes: $R_{ijt} = 1$ if exporter $j's$ i^{th} match is in its first year. $\ln \Delta jt$ = log age of exporter j. X^f_{ijt} = foreign sales volume generated by exporter $j's$ i^{th} match. X^f_{jt} = total foreign sales volume generated by firm j. X^h_{jt} = total home sales volume generated by firm j. $D^f_{jt} = 1$ if firm j is an exporter.