

**NUMERICAL METHODS LABORATORY( MA29202) &  
NUMERICAL TECHNIQUES LABORATORY( MA39110)**  
***Assignment-1 based on Time Dependent PDEs***<sup>1</sup>

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1. Find  $u(x, t)$  using Lax-Friedrichs Method for the following linear convection equation with  $f(u) = u$ , that is,

$$\frac{\partial u}{\partial t} + \frac{\partial f(u)}{\partial x} = 0,$$

and the initial conditions are given as,

$$u(x, 0) = \begin{cases} 1, & |x| < 1/3, \\ 0, & 1/3 < |x| \leq 1. \end{cases}$$

Use 201 evenly spaced grid points to divide the given domain  $[-1, 1]$  into 200 equally spaced intervals with each of length  $\Delta x$ . The final solutions are required to be obtained at  $t = 4$  units. Also, take  $\lambda = \frac{\Delta t}{\Delta x} = 0.8$  and periodic boundary conditions are imposed at the boundary, *i.e.*,  $u(x(0), t) = u(x(200), t)$ . Similarly, use  $u(x(201), t) = u(x(1), t)$ . Also, plot a reference solution using 5001 grid points and plot both the solutions together.

2. Find  $u(x, t)$  using Lax-Friedrichs Method for the following inviscid Burgers' equation with  $f(u) = \frac{u^2}{2}$ , that is,

$$\frac{\partial u}{\partial t} + \frac{\partial f(u)}{\partial x} = 0,$$

and initial conditions are given as,

$$u(x, 0) = \begin{cases} +1, & |x| < 1/3, \\ -1, & 1/3 < |x| \leq 1. \end{cases}$$

Use 201 evenly spaced grid points to obtained final solutions at  $t = 0.3$  units. Also, take  $\lambda = \frac{\Delta t}{\Delta x} = 0.8$  and periodic boundary conditions are imposed at each boundary. Also, plot a reference solution using 5001 grid points and plot both the solutions together.

**Lax-Friedrichs Method:** To derive the Lax-Friedrichs method, first consider Forward in Time and Centre in Space (FTCS) discretization:

$$u_i^{n+1} = u_i^n - \frac{\Delta t}{2\Delta x} (f(u_{i+1}^n) - f(u_{i-1}^n)).$$

Now, replace  $u_i^n$  with  $\frac{(u_{i+1}^n + u_{i-1}^n)}{2}$  in the first-term on the right hand side. With this modification, FTCS becomes the Lax-Friedrichs Method:

$$u_i^{n+1} = \frac{1}{2} (u_{i+1}^n + u_{i-1}^n) - \frac{\Delta t}{2\Delta x} (f(u_{i+1}^n) - f(u_{i-1}^n)).$$

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<sup>1</sup>Sent on: January 22, 2023.