Capítulo 7

Problema 01.

(a) $\int_0^{\infty} 2e^{-2x}dx = 2\times\left[\frac{e^{-2x}}{-2}\right]_0^{\infty} = 2\times\left[0+\frac{e^{-0}}{2}\right]=1$

(b) $P(X>10)=\int_{10}^{\infty} 2e^{-2x}dx = 2\times\left[\frac{e^{-2x}}{-2}\right]_{10}^{\infty} = e^{-20}$

Problema 02.

(a) $\frac{1}{2}\times\frac{C}{2}=1 \Rightarrow \text{C}=4$

(b)

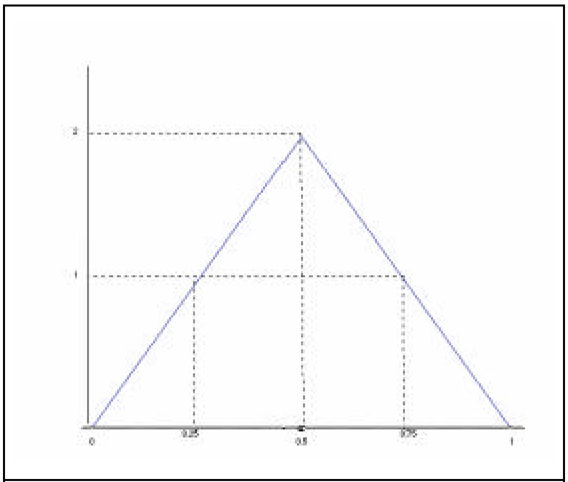

gráfico de f(x)

(c) $P\left(X\le\frac{1}{2}\right)=\frac{1}{2}=P\left(X>\frac{1}{2}\right)$

$P\left(\frac{1}{4}\le X\le\frac{3}{4}\right)=2\times P\left(\frac{1}{4}\le X\le\frac{1}{2}\right)=2\times\left(0{,}5-P\left(X\le\frac{1}{4}\right)\right)=$

$=2\times\left(\frac{1}{2}-\frac{1\times\frac{1}{4}}{2}\right)=2\times\left(\frac{1}{2}-\frac{1}{8}\right)=1-\frac{1}{4}=\frac{3}{4}$

Problema 03.

(a) Como $P(X\le 10)=1$ vem:

$\int_0^{10} kxdx=1$, ou seja, $\int_0^{10} kxdx=k\left[\frac{x^2}{2}\right]_0^{10}=50k=1 \Rightarrow \text{k}=0{,}02$

$$\therefore \quad \mathrm{F(x)} = \int_0^x 0{,}02\mathrm{x}\ \ \mathrm{dx} = 0{,}01x^2$$

Logo, $\mathrm{F}(1) = P(X < 1) = 0{,}01$

(b) $P(X < r) = 0{,}01r^2 = \dfrac{\pi r^2}{\pi(10)^2}$

Problema 04.

$$\int_{10}^{\infty} \frac{c}{x^2}dx = c \times \int_{10}^{\infty}\left[\frac{1}{x^2}\right]dx = c \times \left[-\frac{1}{x}\right]_{10}^{\infty} = c \times \frac{1}{10} = 1 \quad \rightarrow \ \mathrm{c} = 10$$

$$P(X > 15) = \int_{15}^{\infty}\frac{10}{x^2}dx = 10 \times \int_{15}^{\infty}\left[\frac{1}{x^2}\right]dx = 10 \times \left[-\frac{1}{x}\right]_{15}^{\infty} = 10 \times \frac{1}{15} = \frac{2}{3}$$

Problema 05.

$$E(X) = \int_0^{\frac{1}{2}} 4x^2 dx + \int_{1/2}^{1} x4(1-x) = 4 \times \left[\frac{x^3}{3}\right]_0^{1/2} + 4 \times \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_{1/2}^{1} = 4 \times \left\{\frac{1}{24} + \left(\frac{1}{2} - \frac{1}{3}\right) - \left(\frac{1}{8} - \frac{1}{24}\right)\right\} =$$

$$= 4 \times \left\{\frac{1}{24} + \frac{1}{6} - \frac{2}{24}\right\} = 4 \times \frac{3}{24} = \frac{1}{2}$$

$$E(X^2) = \int_0^{\frac{1}{2}} 4x^3 dx + \int_{1/2}^{1} x^2 4(1-x) = 4 \times \left[\frac{x^4}{4}\right]_0^{1/2} + 4 \times \left[\frac{x^3}{3} - \frac{x^4}{4}\right]_{1/2}^{1} = 4 \times \left\{\frac{1}{64} + \left(\frac{1}{3} - \frac{1}{4}\right) - \left(\frac{1}{24} - \frac{1}{64}\right)\right\} =$$

$$= 4 \times \left\{\frac{1}{32} + \frac{1}{3} - \frac{1}{4} - \frac{1}{24}\right\} = 4 \times 7 \times \frac{1}{96} = \frac{7}{24}$$

Logo,

$$\mathrm{Var(X)} = \frac{7}{24} - \frac{1}{4} = \frac{1}{24}$$

$$\mathrm{F(x)} = 4 \times \int_{\frac{1}{2}}^{x} (1 - \mathrm{t})dt = 4 \times \left[t - \frac{t^2}{2}\right]_{\frac{1}{2}}^{x} = 4 \times \left\{\left[x - \frac{x^2}{2}\right] - \left[\frac{1}{2} - \frac{1}{8}\right]\right\} = 4 \times \left\{x - \frac{x^2}{2} - \frac{3}{8}\right\} + \frac{1}{2} =$$

$$= 4x - 2x^2 - \frac{3}{2} + \frac{1}{2} = 4x - 2x^2 - 1$$

Logo,

$$\mathrm{F(x)} = \begin{cases} 0, \text{se } \ x < 0 \\ \dfrac{4\mathrm{x}^2}{2}, \ \text{se } \ 0 \le \mathrm{x} \le \dfrac{1}{2} \\ 4x - 2x^2 - 1, \ \text{se } \dfrac{1}{2} < x \le 1 \end{cases}$$

Problema 06.

$$E(X)=\int_0^{\frac{\pi}{2}}(x\,\text{sen}\,x)dx=\left[-x\cos x+\int\cos x dx\right]_0^{\frac{\pi}{2}}=$$

Tomando:

$u=x \quad\Rightarrow\quad du=1$

$dv=\text{sen}\,x \;\Rightarrow\; v=\text{-}\cos x$

$$=\left[-x\cos x+\text{sen}\,x\right]_0^{\frac{\pi}{2}}=\left[-\frac{\pi}{2}\cos\frac{\pi}{2}+\text{sen}\,\frac{\pi}{2}-0\cos 0-\text{sen}\,0\right]=\text{sen}\,\frac{\pi}{2}=1$$

$$E(X^2)=\int_0^{\frac{\pi}{2}}(x^2\,\text{sen}\,x)dx=$$

Tomando:

$u=x \quad\Rightarrow\quad du=1$

$dv=x\,\text{sen}\,x \;\Rightarrow\; v=\text{-}x\cos x+\text{sen}\,x$

$$=-x^2\cos x+x\,\text{sen}\,x+\int x\cos x+\text{sen}\,x=$$

$u=x \quad\Rightarrow\quad du=1$

$dv=\cos x \;\Rightarrow\; v=\text{sen}\,x$

$$=-x^2\cos x+x\,\text{sen}\,x+x\,\text{sen}\,x-\cos x+\cos x=\left[-x^2\cos x+2x\,\text{sen}\,x\right]_0^{\pi/2}=\pi$$

Logo,

$Var(X)=\pi-1$

Problema 07.

$$E(X)=\int_{10}^{\infty}x\frac{10}{x^2}dx=\int_{10}^{\infty}\frac{10}{x}dx=10\times\int_{10}^{\infty}\frac{1}{x}dx=10\times\left[\log x\right]_{10}^{\infty}=+\infty$$

Problema 08.

(a) $P\left(X>b\,\middle|\,X<b/2\right)=\dfrac{P\left(b<X<b/2\right)}{P\left(X<b/2\right)}$, onde

$$P\left(X<b/2\right)=\int_{-1}^{b/2}3x^2dx=\left(x^3\right)_{-1}^{b/2}=\frac{b^3}{8}+1$$

$$P\left(b<X<b/2\right)=\int_{b}^{b/2}3x^2dx=\left(x^3\right)_{b}^{b/2}=\frac{b^3}{8}-b^3$$

Logo,

$$P\left(X>b\,\middle|\,X<b/2\right)=\frac{P\left(b<X<b/2\right)}{P\left(X<b/2\right)}=\frac{\frac{b^3}{8}-b^3}{\frac{b^3}{8}+1}=\frac{-7b^3}{b^3+8}$$

(b) $E(X)=\int_{-1}^{0}3x^3dx=3\times\left[\frac{x^4}{4}\right]_{-1}^{0}=\frac{3}{4}\times[0-1]=-\frac{3}{4}$

$E(X^2)=\int_{-1}^{0}3x^4dx=3\times\left[\frac{x^5}{5}\right]_{-1}^{0}=\frac{3}{5}\times[0+1]=\frac{3}{5}$

Então,

$Var(X)=E(X^2)-[E(X)]^2=\frac{3}{5}-\left(-\frac{3}{4}\right)^2=\frac{3}{80}$

Problema 09.

$$E(X)=\frac{3}{5}\times10^{-5}\int_{0}^{100}x^2(1-x)dx=\frac{3}{5}\times10^{-5}\left[100\frac{x^3}{3}-\frac{x^4}{4}\right]_0^{100}=\frac{3}{5}\times10^{-5}\times100^3\left[\frac{100}{3}-\frac{100}{4}\right]=$$

$$=\frac{3}{5}\times10^3\times\left[\frac{1}{3}-\frac{1}{4}\right]=\frac{3}{5}\times10^3\times\frac{1}{12}=50$$

Logo,

$E(L)=C_1+50C_2$

Problema 10.

(a) $P(X>1{,}5)=\int_{1,5}^{3}\left(1-\frac{x}{3}\right)dx=\left[x-\frac{x^2}{6}\right]_{3/2}^{3}=\left(3-\frac{9}{6}\right)-\left(\frac{3}{2}-\frac{9}{24}\right)=\frac{3}{8}=0{,}375$

(a) $E(X)=\int_0^1\frac{2}{3}x^2dx+\int_1^3\left(x-\frac{x^2}{3}\right)dx=\frac{2}{3}\times\left(\frac{x^3}{3}\right)_0^1+\left(\frac{x^2}{2}-\frac{x^3}{9}\right)_1^3=\frac{2}{9}+\left[\left(\frac{9}{2}-\frac{27}{9}\right)-\left(\frac{1}{2}-\frac{1}{9}\right)\right]=$

$=\frac{4}{3}=1{,}33$ num dia $\Rightarrow 30$ dias $:\frac{4}{3}\times30=40\longrightarrow$ 4000 kg

(b) $P(X\le a)=0{,}95$

$P(\le 0X\le1)=\frac{1\times2}{2\times3}=\frac{1}{3}$

$\frac{1}{3}+\int_1^a\left(-\frac{x}{3}+1\right)dx=0{,}95$

$\int_1^a\left(-\frac{x}{3}+1\right)dx=0{,}95-0{,}33=0{,}62$

$\left(x-\frac{x^2}{6}\right)_1^a=a-\frac{a^2}{6}-\frac{5}{6}+\frac{1}{3}=0{,}62\longrightarrow -a^2+6a-3=5{,}7\longrightarrow a^2+6a+8{,}7=0$

Logo, resolvendo a equação de 2º grau acima, encontra-se que: $a=2{,}45\rightarrow$ 245 kg

Problema 11.

$$E(X)=2\times\int_0^\infty xe^{-2x}dx=2\times\left[\left(-\frac{xe^{-2x}}{-2}\right)_0^\infty-\frac{1}{2}\times\int_0^\infty e^{-2x}dx\right]=$$

Tomando:

$$v' = e^{-2x} \rightarrow v = \frac{e^{-2x}}{-2}$$

$$= \left[\left[\left(-xe^{-2x} \right)_0^{\infty} \right] + \int_0^{\infty} e^{-2x} dx \right] = \left[\frac{e^{-2x}}{-2} \right]_0^{\infty} = \frac{1}{2}$$

$$Var(X) = \frac{1}{4}$$

Problema 12.

Calculando o valor de c:

$$c\int_{-1}^{1}(1-x^2)dx = 1 \longrightarrow c\left[x - \frac{x^3}{3}\right]_{-1}^{1} = 1$$

$$c\left[\left(1 - \frac{1^3}{3}\right) - \left(-1 + \frac{1}{3}\right)\right] = 1 \longrightarrow \frac{4}{3} \times c = 1 \longrightarrow \mathrm{c} = \frac{3}{4}$$

$$E(X) = \int_{-1}^{1} \frac{3}{4} x(1-x^2)dx = \frac{3}{4} \times \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_{-1}^{1} = \frac{3}{4} \times \left[\left(\frac{1}{2} - \frac{1}{4}\right) - \left(\frac{1}{2} - \frac{1}{4}\right)\right] = 0$$

$$E(X^2) = \int_{-1}^{1} \frac{3}{4} x^2(1-x^2)dx = \frac{3}{4} \times \left[\frac{x^3}{3} - \frac{x^5}{5}\right]_{-1}^{1} = \frac{3}{4} \times \left[\left(\frac{1}{3} - \frac{1}{5}\right) - \left(-\frac{1}{3} + \frac{1}{5}\right)\right] = \frac{3}{4} \times \frac{4}{15} = \frac{1}{5}$$ Logo,

$$Var(X) = E(X^2) - [E(X)] = \frac{1}{5} - [0]^2 = \frac{1}{5}$$

Problema 13.

(a) $T \sim U[150, 300]$

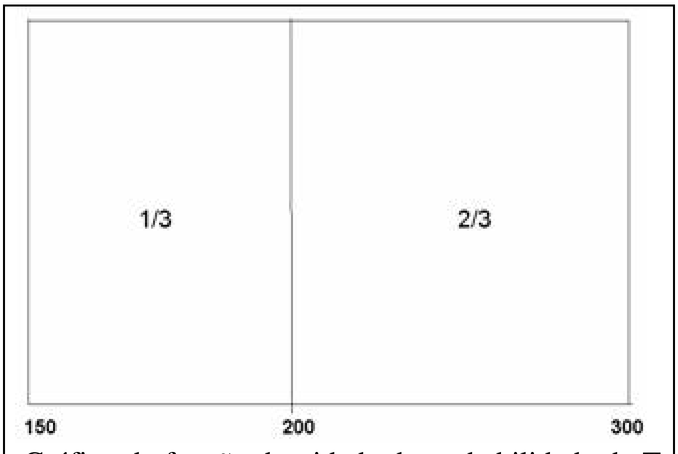

Gráfico da função densidade de probabilidade de T

$$C = C_1$$

$$V = \begin{cases} C_2, \mathrm{T} < 200 \\ \mathrm{C}_3, \mathrm{T} > 200 \end{cases}$$

(b) $L = V - C_1 = \begin{cases} C_2 - C_1, 150 < \mathrm{T} < 200 \\ C_2 - C_1, 200 < \mathrm{T} < 300 \end{cases}$

Logo,

$$E(L) = (C_2 - C_1) \times \frac{1}{3} + (C_3 - C_1) \times \frac{2}{3} = \frac{2}{3} C_3 + \frac{1}{3} C_2 - C_1$$

Problema 14.

$X \sim N(10;4)$

(a) $P(8 < X < 10) = P(-1 < Z < 0) = 0{,}34$

(b) $P(9 \le X \le 12) = P\left(-\frac{1}{2} < Z < 1\right) = 0{,}34 + 0{,}19 = 0{,}53$

(c) $P(X > 10) = P(Z > 0) = 0{,}5$

(d) $P(X < 8 \text{ ou } \mathrm{X} > 11) = P(Z < -1) + P(Z > 0{,}5) = 0{,}16 + 0{,}31 = 0{,}47$

Problema 15.

$X \sim N(100;100)$

(a) $P(X < 115) = P(Z < 1{,}5) = 0{,}933$

(b) $P(X \ge 80) = P(Z \ge -2) = 0{,}977$

(c) $P(|X - 100| \le 10) = P(-10 \le X - 100 \le 10) = P\left(-1 \le \frac{X - 100}{10} \le 1\right) = P(-1 \le Z \le 1) = 0{,}6827$

(d) $P(100 - a \le X \le 100 + a) = P(-a \le X - 100 \le a) = P\left(-\frac{a}{10} \le X \le \frac{a}{10}\right) = 0{,}95$

$$\Rightarrow \frac{a}{10} = 1{,}96 \to a = 19{,}6$$

Problema 16.

$X \sim N(\mu, \sigma^2)$

(a) $P(X \le \mu + 2\sigma) = P\left(\frac{X - \mu}{\sigma} \le 2\right) = P(Z \le 2) = 0{,}977$

(b) $P(|X - \mu| \le \sigma) = P(|Z| \le 1) = 0{,}68$

(c) $P(-a\sigma \le X - \mu \le a\sigma) = P(-a \le Z \le a) = 0{,}99 \longrightarrow a = 2{,}58$

(d) $P(X > b) = 0{,}90 \longrightarrow P\left(Z > \frac{b - \mu}{\sigma}\right) = 0{,}90$

Logo,

$$\left(\frac{b - \mu}{\sigma}\right) = -1{,}28 \longrightarrow b = \mu - 1{,}28\sigma$$

Problema 17.

$X \sim N(170;5^2)$

(a) $P(X > 165) = P(Z > -1) = 0{,}94134$

$\therefore$ Nº esperado $= 10000 \times 0{,}94134 = 9413$

(b) $P(\mu - a < X < \mu + a) = 0{,}75$

$P(170 - a < X < 170 + a) = 0{,}75$

$P\left(-\frac{a}{5} < Z < \frac{a}{5}\right) = 0{,}75$

$\frac{a}{5} = 1{,}15 \longrightarrow a = 5{,}75$

Logo o intervalo simétrico é:

$Intervalo = (164{,}25;175{,}75)$

Problema 18.

$V \sim N(500;50^2)$

$P(V > 600) = P(Z > 2) = 0{,}023$

Problema 19.

$D_1 \sim N(42;36)$

$D_2 \sim N(45;9)$

Para um período de 45 horas, tem-se:

$P(D_1 > 45) = P(Z > 0{,}5) = 0{,}31$

$P(D_2 > 45) = P(Z > 0) = 0{,}50$

Neste caso, D_2 deve ser preferido.

Para um período de 49 horas, tem-se:

$P(D_1 > 49) = P(Z > 1{,}17) = 0{,}121$

$P(D_2 > 49) = P(Z > 1{,}33) = 0{,}092$

E neste caso, D_1 deve ser preferido.

Problema 20.

$X \sim N\left(0{,}6140 ; (0{,}0025)^2\right)$

(a) $P(0{,}61 < X < 0{,}618) = 0{,}8904$ BOM

$P(0{,}608 < X < 0{,}610) + P(0{,}618 < X < 0{,}620) =$

$= P(-2{,}4 < X < -1{,}6) + P(1{,}6 < X < 2{,}4) = 0{,}0466 + 0{,}0466 = 0{,}0932$ RECUPERÁVEL

$P(X < 0{,}608) + P(X > 0{,}62) = P(Z < -2{,}4) + P(Z > 2{,}4) = 2 \times 0{,}0082 = 0{,}0164$ DEFEITUOSAS

(b) $E(T) = 0{,}10 \times 0{,}8904 + 0{,}05 \times 0{,}0932 - 0{,}10 \times 0{,}0164 = 0{,}09206$

Problema 21.

Y : *Lucro esperado por item*

$$P(X \le 0{,}9) = \int_0^{0{,}9} e^{-x}\,dx = 1 - e^{-0{,}9} = 0{,}5934$$

$$P(X > 0{,}9) = e^{-0{,}9} = 0{,}4066$$

$Y: \quad 2 \;\; ; \;\; 3$

$P(Y = y): 0{,}5934 \;\; ; 0{,}4066$

$E(Y) = -1{,}1868 + 1{,}2198 = 0{,}033$

Problema 22.

$Y \sim b(10;0{,}4) \qquad X \sim N(4\,;2{,}4)$

(a) $P(3 < Y < 8) = P(4 \le Y \le 7) \cong P(3{,}5 \le X \le 7{,}5) = P(-0{,}32 \le Z \le 2{,}26) = 0{,}4881 + 0{,}1255$
$= 0{,}6136$

(b) $P(Y \ge 7) \cong P(X \ge 6{,}5) = P(Z \ge 1{,}61) = 0{,}0537$

(c) $P(Y < 5) = P(Y \le 4) \cong P(X \le 4{,}5) = P(Z \le 0{,}32) = 0{,}6255$

Problema 23.

$X \sim b(100\,;0{,}1)$

$$P(X = 12) = \binom{100}{12} \times (0{,}1)^{12} \times (0{,}9)^{88}$$

$Y \sim N(10\,;9)$

$$P(X = 12) = P(11{,}5 \le Y \le 12{,}5) = P(0{,}5 \le Z \le 0{,}83) = 0{,}1043$$

Problema 24.

X : *número de defeitos*

$$P(X \ge 30) = \sum_{j=30}^{1000} \binom{1000}{j} \times (0{,}05)^j \times (0{,}95)^{1000-j}$$

$Y \sim N(50\,;47{,}5)$

$$P(X \ge 30) \cong P(Y \ge 29{,}5) = P\left(Z \ge \frac{29{,}5 - 50}{6{,}89}\right) = P(Z \ge -2{,}975) = 0{,}9986$$

Problema 25.

(a) $P(Y \le 5{,}5) = P(X + 5 \le 5{,}5) = P(X \le 0{,}5) = 0{,}50$

(b) $G(y) = P(Y \le y) = P(X + 5 \le y) = P(X \le y - 5) = F(y - 5)$

Então:

$$g(y) = f(y-5) = \begin{cases} 0, \; y < 5 \\ 4(\text{y - 5}), \; 0 \le y - 5 \le \dfrac{1}{2} \rightarrow 5 \le y \le 5{,}5 \\ 4(1 - y + 5) = 4(6 - y)\,, \dfrac{1}{2} \le y - 5 \le 1 \rightarrow 5{,}5 \le y \le 6{,}0 \\ 0, \;\; y > 6 \end{cases}$$

(c) $G(z) = P(Z \le z) = P(2X \le z) = P\left(X \le \frac{z}{2}\right) = F\left(\frac{z}{2}\right)$

Então:

$$g(z) = \frac{1}{2} \times f\left(\frac{z}{2}\right) = \begin{cases} 0, z < 0 \\ z,\ 0 \le \frac{z}{2} \le 1 \rightarrow 0 \le z \le 1 \\ 2 \times \left(1 - \frac{z}{2}\right), \frac{1}{2} \le \frac{z}{2} \le 1 \rightarrow 1 \le z \le 2 \\ 0, z > 2 \end{cases}$$

(d) Problema 26.

$$G(y) = P(Y \le y) = P(2X - 0{,}6 \le y) = P(2X \le y + 0{,}6) = P\left(X \le \frac{y+0{,}6}{2}\right) = F\left(\frac{y+0{,}6}{2}\right)$$

Logo,

$$g(y) = f\left(\frac{y+0{,}6}{2}\right) \times \frac{1}{2} = \frac{3}{2} \times \left(\frac{y+0{,}6}{2}\right)^2, -1 \le \frac{y}{2} + 0{,}3 \le 0 \rightarrow -2{,}6 \le y \le -0{,}6$$

$$E(Y) = \int_{-2,6}^{-0,6} \frac{3}{2} y \left(\frac{y+0{,}6}{2}\right)^2 dy = \frac{3}{8} \times \int_{-2,6}^{-0,6} y(y^2 + 0{,}36 + 1{,}2y)dy = \int_{-2,6}^{-0,6} (y^3 + 0{,}36y + 1{,}2y^2)dy =$$

$$= \frac{3}{8} \times \left[\frac{y^4}{4} + 0{,}36\frac{y^2}{2} + 1{,}2\frac{y^3}{3}\right]_{-2,6}^{-0,6} = -2{,}10$$

$$E(Y^2) = \int_{-2,6}^{-0,6} \frac{3}{2} y^2 \left(\frac{y+0{,}6}{2}\right)^2 dy = \frac{3}{8} \times \int_{-2,6}^{-0,6} y^2(y^2 + 0{,}36 + 1{,}2y)dy = \int_{-2,6}^{-0,6} (y^4 + 0{,}36y^2 + 1{,}2y^3)dy =$$

$$= \frac{3}{8} \times \left[\frac{y^5}{5} + 0{,}36\frac{y^3}{3} + 1{,}2\frac{y^4}{4}\right]_{-2,6}^{-0,6} = \ldots.$$

$$Var(Y) = E(Y^2) - [E(X)]^2 = E(Y^2) - 4{,}41$$

Problema 27.

$X \sim U[-1;1]$

Tomando $Y = X^2$:

$$G(y) = P(Y \le y) = P(X^2 \le y) = P(-\sqrt{y} \le X \le \sqrt{y}) = F(\sqrt{y}) - F(-\sqrt{y})$$

Logo:

$$g(y) = \frac{1}{2\sqrt{y}}\left[f(\sqrt{y}) + f(-\sqrt{y})\right]$$

$$f(x) = \begin{cases} \frac{1}{2}, -1 < x < 1 \\ 0,\ \text{caso contrário} \end{cases}$$

$$\therefore\ g(y) = \frac{1}{2\sqrt{y}}\left[\frac{1}{2} + \frac{1}{2}\right] = \frac{1}{2\sqrt{y}},\ 0 < y < 1$$

Tomando $W = |X|$:

$$G(w) = P(W \le w) = P(|X| \le w) = P(-w \le X \le w) = F(w) - F(-w)$$

Logo:

$$g(w) = f(w) - f(-w) = \frac{1}{2} + \frac{1}{2} = 1, \quad 0 < w < 1$$

Problema 28.

(a) $$E(X) = \int_0^2 \left(\frac{x^2}{2} + \frac{x}{10}\right)dx + \int_2^6 \left(\frac{-3x^2}{40} + \frac{9x}{20}\right)dx = \left[\frac{x^3}{30} + \frac{x^2}{20}\right]_0^2 + \left[\frac{-3x^3}{120} + \frac{9x^2}{20}\right]_2^6 =$$

$$= \left[\frac{8}{30} + \frac{4}{20}\right] + \left[\left(-\frac{648}{120} + \frac{324}{40}\right) - \left(-\frac{24}{120} + \frac{36}{40}\right)\right] = 2{,}47$$

(b) $$P(X > 3) = \int_3^6 \left(-\frac{3x}{40} + \frac{9}{20}\right)dx = \left[\frac{-3x^2}{80} + \frac{9x}{20}\right]_3^6 = \left(-\frac{108}{80} + \frac{54}{20}\right) - \left(-\frac{27}{80} + \frac{27}{20}\right) = 0{,}338$$

(c) $$\int_{Q_2}^6 f(x)dx = 0{,}5 \rightarrow \int_{Q_2}^6 \left(-\frac{3x}{40} + \frac{9}{20}\right)dx = \left[-\frac{3x^2}{80} + \frac{9x}{20}\right]_{Q_2}^6 = -\frac{108}{80} + \frac{54}{20} + \frac{3Q_2}{80} - \frac{9Q_2}{20} = 0{,}5$$

Portanto, $Q_2 = 2{,}06$.

Problema 29.

$$f(x) = \frac{1}{\beta - \alpha}, \ \alpha < x < \beta$$

(a) $$E(X) = \int_\alpha^\beta \frac{x}{\beta - \alpha}dx = \frac{1}{\beta - \alpha} \times \left[\frac{x^2}{2}\right]_\alpha^\beta = \frac{1}{\beta - \alpha} \times \frac{\beta^2 - \alpha^2}{2} = \frac{\alpha + \beta}{2}$$

$$E(X^2) = \int_\alpha^\beta \frac{x^2}{\beta - \alpha}dx = \frac{1}{\beta - \alpha} \times \left[\frac{x^3}{3}\right]_\alpha^\beta = \frac{1}{\beta - \alpha} \times \frac{\beta^3 - \alpha^3}{3} = \frac{(\beta - \alpha)(\beta^2 + \alpha^2 + \alpha\beta)}{3(\beta - \alpha)}$$

$$= \frac{\beta^2 + \alpha^2 + \alpha\beta}{3}$$

$$Var(X) = \frac{\beta^2 + \alpha^2 + \alpha\beta}{3} - \frac{\alpha^2 + \beta^2 + 2\alpha\beta}{4} = \frac{(\beta - \alpha)^2}{12}$$

(b) $$F(x) = \begin{cases} 0, x < \alpha \\ \int_\alpha^x \frac{1}{\beta - \alpha}dt = \frac{x - \alpha}{\beta - \alpha}, \alpha \le x < \beta \\ 1, x > \beta \end{cases}$$

Problema 30.

$$f(x) = \frac{1}{\beta - \alpha}, \alpha < x < \beta$$

$$U = \frac{X - \frac{\alpha+\beta}{2}}{\beta - \alpha} \longrightarrow \left[-\frac{1}{2}, \frac{1}{2}\right]$$

$$P(c < X < d) = F_U\left(\frac{d - \frac{\alpha+\beta}{2}}{\beta-\alpha}\right) - F_U\left(\frac{c - \frac{\alpha+\beta}{2}}{\beta-\alpha}\right)$$

Então:

$$G(u) = P(0 \le U \le u) = u,\ 0 \le u \le 1$$

$$u = 0{,}00 \longrightarrow G(0) = 0$$

$$u = 0{,}01 \longrightarrow G(0{,}01) = 0{,}01$$

$$u = 0{,}00 \longrightarrow G(0{,}02) = 0{,}02$$

e assim por diante.

Problema 31.

$$P(c < X < d) = F_U\left(\frac{d - 7{,}5}{5}\right) - F_U\left(\frac{c - 7{,}5}{5}\right)$$

(a) $P(X < 7) = P(5 < X < 7) = F_U(-0{,}1) - F_U(-0{,}5) = 0{,}4$

(b) $P(8 < X < 9) = F_U(0{,}3) - F_U(0{,}1) = 0{,}2$

(c) $P(X > 8{,}5) = P(8{,}5 < X < 10) = F_U(0{,}5) - F_U(0{,}2) = 0{,}3$

(d) $P(|X - 7{,}5| > 2) = 1 - P(-2 < X - 7{,}5 < 2) = 1 - P(5{,}5 < X < 9{,}5) = 1 - [F_U(0{,}4) - F_U(0{,}4)] =$
$= 1 - 0{,}8 = 0{,}2$

Problema 32.

$X \sim N(\mu; \sigma^2)$

$$E(X) = \int_{-\infty}^{\infty} x f(x) dx = \int_{-\infty}^{\infty} x \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

Fazendo $y = \dfrac{x-\mu}{\sigma} \longrightarrow x = \sigma y + \mu \longrightarrow dx = \sigma dy$, tem-se:

$$E(X) = \int_{-\infty}^{\infty} (\sigma y + \mu) \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2} dy = \mu \times \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2} dy + \sigma \times \int_{-\infty}^{\infty} y \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2} dy$$

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2} dy = 1,\ y \sim N(0,1)$$

$$\int_{-\infty}^{\infty} y \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2} dy = \int_{0}^{\infty} y \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2} dy + \int_{-\infty}^{0} y \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2} dy$$

Considerando:

$$\int_{-\infty}^{0} y\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}y^2}dy$$

Tomando $z=-y \longrightarrow y=-z \rightarrow dy=-dz$, tem-se:

$$\int_{-\infty}^{0} y\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}y^2}dy=-\int_{0}^{\infty} z\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2}dz$$

Logo,

$$\int_{-\infty}^{\infty} y\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}y^2}dy=\int_{0}^{\infty} y\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}y^2}dy-\int_{0}^{\infty} z\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2}dz=0$$

Logo,

$$E(X)=\mu$$

Sabe-se que:

$$Var(X)=E(X^2)-[E(X)]^2$$

$$E(X^2)=\int_{-\infty}^{\infty} x^2 f(x)dx=\int_{-\infty}^{\infty} x^2\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}dx$$

Fazendo $y=\dfrac{x-\mu}{\sigma} \longrightarrow x=\sigma y+\mu \longrightarrow dx=\sigma dy$, tem-se:

$$E(X)=\int_{-\infty}^{\infty}(\sigma y+\mu)^2\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}y^2}dy=\mu^2\times\int_{-\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}y^2}dy+\sigma^2\times\int_{-\infty} y^2\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}y^2}dy+$$

$$+2\mu\sigma\times\int_{-\infty}^{\infty} y\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}y^2}dy$$

Já vimos anteriormente que:

$$\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}y^2}dy=1$$

$$\int_{-\infty}^{\infty} y\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}y^2}dy=0$$

Queremos então calcular:

$$\int_{-\infty}^{\infty}\underbrace{y}_{u}\underbrace{y\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}y^2}dy}_{v'}=\underbrace{\left[y\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}y^2}\right]_{-\infty}^{\infty}}_{1}+\underbrace{\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}y^2}}_{0}=1$$

Logo,

$$E(X^2)=\sigma^2+\mu^2$$

$$Var(X)=\sigma^2+\mu^2-\mu^2=\sigma^2$$

Problema 33.

$X\sim N(6{,}4;0{,}8^2)$

$A \longrightarrow P(7{,}5\le X\le 10)=P(1{,}38\le X\le 4{,}5)=0{,}49997-0{,}41621=0{,}0837$

$N^o\,esperado=0{,}0837\times 80=6{,}696\cong 7$

$B \longrightarrow P(5\le X\le 7{,}5)=P(-1{,}75\le X\le 1{,}38)=0{,}41621+0{,}45994=0{,}876$

$N^o\,esperado=0{,}876\times 80\cong 70$

$C \longrightarrow P(X < 5) = P(X < -1{,}75) = 0{,}040$
$N^{o}\, esperado = 0{,}040 \times 80 \cong 3$

Problema 34.

$X = Peso\, Bruto \sim N(1000;20^2)$

(a) $P(X < 980) = P(Z < -1) = 0{,}15866$

(b) $P(X > 1010) = P\left(Z > \frac{1}{2}\right) = 0{,}30854$

Problema 35.

(a) $X = Peso \sim N(5;0{,}8^2)$ $\quad \mathrm{n} = 5000$

Então:

$$\frac{X-5}{0{,}8} = Z \longrightarrow X = 0{,}8Z + 5$$

$$z_1 = 0{,}84 \to x_1 = 4{,}33$$
$$z_2 = 0{,}68 \to x_2 = 5{,}54$$
$$z_3 = 1{,}28 \to x_3 = 6{,}02$$

Logo,

$$\text{se} \begin{cases} \mathrm{x} \le 4{,}33, \text{ então classifica como pequeno} \\ 4{,}33 < \mathrm{x} \le 5{,}54, \text{ então classifica como médio} \\ 5{,}54 < \mathrm{x} \le 6{,}02, \text{ então classifica como grande} \\ \mathrm{x} > 6{,}02, \text{ então classifica como extra} \end{cases}$$

Problema 36.

$VL \sim N(1000;10^2)$

(a) $\mathrm{P}(\mathrm{VL} < 990) = \mathrm{P}(\mathrm{Z} < -1) = 0{,}16 \longrightarrow 16\%$

(b) $\mathrm{P}(|\mathrm{VL} - 1000| < 20) = \mathrm{P}(-20 < \mathrm{VL} - 1000 < 20) = P(-2 < Z < 2) = 0{,}9545 \longrightarrow 95{,}5\%$

(c) $\mathrm{P}(|\mathrm{VL} - 1200| < 2 \times 20) = P(-2 < Z < 2) = 0{,}9545 \longrightarrow 95{,}5\% \quad \therefore não\, muda$

Problema 37.

$D \sim \mathrm{N}(0{,}10;\ (0{,}02)^2)$

$$\mathrm{V} = \begin{cases} 5, & |\mathrm{D} - 0{,}10| > 0{,}03 \\ 10, & |\mathrm{D} - 0{,}10| \le 0{,}03 \end{cases}$$

$$E(\mathrm{V}) = 5 \times \mathrm{P}(|\mathrm{D} - 0{,}10| > 0{,}03) + 10 \times \mathrm{P}(|\mathrm{D} - 0{,}10| \le 0{,}03)$$

$$\mathrm{P}(|\mathrm{D} - 0{,}10| \le 0{,}03) = \mathrm{P}(-0{,}03 < \mathrm{D} - 0{,}10 < 0{,}03) = \mathrm{P}(-1{,}5 < \mathrm{Z} < 1{,}5) = 0{,}867$$

Logo,

$$E(\mathrm{V}) = 5 \times 0{,}133 + 10 \times 0{,}867 = 9{,}34$$

Problema 38.

Aparelho A $\longrightarrow \begin{cases} Lucro = 1000, \text{ sem restituição.} \\ Prejuízo = 3000, \text{com restituição.} \end{cases}$

Aparelho B $\longrightarrow \begin{cases} Lucro = 2000, \text{ sem restituição.} \\ Prejuízo = 8000, \text{com restituição.} \end{cases}$

X : tempo para a ocorrência de algum defeito grave.

$X \mid A \sim N(9;4)$.

$X \mid B \sim N(12;9)$.

Então:

$$P(X \leq 6 \mid A) = P(Z \leq -1{,}5) = 0{,}066$$
$$P(X \leq 6 \mid B) = P(Z \leq -2) = 0{,}023$$

Portanto os lucros esperados para os dois produtos são:

$$A \longrightarrow 1000 \times 0{,}934 - 3000 \times 0{,}066 \cong 736$$
$$B \longrightarrow 2000 \times 0{,}977 - 8000 \times 0{,}023 \cong 1770$$

Portanto, incentivaria as vendas do aparelho do tipo B.

Problema 39.

(a) $X \sim U(1;3)$ então $E(X) = 2$

$Y = 3X + 4$ então $E(Y) = 10$

$$E(Z) = \int_1^3 e^x \times \frac{1}{2} dx = \frac{1}{2} \times \left[e^x\right]_1^3 = \frac{1}{2} \times \left(e^3 - e\right)$$

(b) $X \sim f(x) = e^{-x}, x > 0$ então $E(X) = 1$

$$E(Y) = \int_0^\infty x^2 e^{-x} dx$$

$$E(Z) = \int_0^\infty \frac{3}{x+1} e^{-x} dx$$

Problema 40.

$X \sim U(-a;3a)$ então $E(X) = a$

$$Var(X) = \frac{(3a+a)^2}{12} = \frac{16a^2}{12} = \frac{4}{3}a^2$$

Problema 41.

(a) $E(T) = \int_0^\infty t \frac{1}{\beta} e^{\frac{-t}{\beta}} dt = \frac{1}{\beta} \times \int_0^\infty t e^{\frac{-t}{\beta}} dt = \beta$, usando integração por parte.

(b) $E(T^2) = \int_0^\infty t^2 \frac{1}{\beta} e^{\frac{-t}{\beta}} dt = \frac{1}{\beta} \times \int_0^\infty t^2 e^{\frac{-t}{\beta}} dt = 2\beta^2$

Logo,

$$Var(X) = E(T^2) - [E(T)]^2 = 2\beta^2 - \beta^2 = \beta^2$$

Problema 43.

(a) $F_y(y) = P(Y \le y) = P(X^2 \le y) = P\left(-\sqrt{y} \le X \le \sqrt{y}\right) = F_x\left(\sqrt{y}\right) - F_x\left(-\sqrt{y}\right)$

(b) $f_y(y) = \dfrac{f_x\left(\sqrt{y}\right)}{2\sqrt{y}} - \dfrac{f_x\left(-\sqrt{y}\right)}{2\sqrt{y}} = \dfrac{1}{2\sqrt{y}}\left[f_x\left(\sqrt{y}\right) - f_x\left(-\sqrt{y}\right)\right] = \dfrac{1}{2\sqrt{y}}$, 0< y <1

(c) $E(X^2) = \int_0^1 x^2 dx = \left[\dfrac{x^3}{3}\right]_0^1 = \dfrac{1}{3}$

(d) $E(Y) = \int_0^1 y \dfrac{1}{2\sqrt{y}} dy = \dfrac{1}{2} \times \left[\dfrac{y^{\frac{3}{2}}}{\frac{3}{2}}\right] = \dfrac{1}{2} \times \dfrac{2}{3} \times \left[y^{\frac{3}{2}}\right]_0^1 = \dfrac{1}{3}$

Problema 44.

$X \sim f(x) = e^{-x}, x > 0$ então $E(X) = 1 = Var(X)$

$$Z = \frac{X - \mu_x}{\sigma_x}$$

$$E(Z) = E\left(\frac{X - \mu_x}{\sigma_x}\right) = \frac{E(X-1)}{1} = 0$$

$$Var(Z) = Var\left(\frac{X - \mu_x}{\sigma_x}\right) = \frac{Var(X)}{\sigma_x^2} = 1$$

Problema 45.

(a) $\alpha = 1 \longrightarrow \int_0^\infty e^{-x} dx = 1 = 0!$

Vale para $\alpha = n$:

$$\Gamma(n+1) = \int_0^\infty e^{-x} x^n dx = \left[-x^n e^{-x}\right]_0^\infty + \int_0^\infty x^{n-1} e^{-x} dx = n \times \Gamma(n) = n \times (n-1) = n!$$

(b) O raciocínio em (a) vale para qualquer $n \in \Re_+$.

(c) $\Gamma(1) = \int_0^\infty e^{-x} dx = 1$

$\Gamma\left(\dfrac{1}{2}\right) = \int_0^\infty e^{-x} x^{-\frac{1}{2}} dx = \sqrt{\pi}$; faça $x = \dfrac{u^2}{2} \longrightarrow dx = udu$

$$\int_{-\infty}^{\infty} e^{-\frac{u^2}{2}} \left(\frac{u^2}{2}\right)^{-\frac{1}{2}} udu = \sqrt{2} \times \int_{-\infty}^{\infty} e^{-\frac{u^2}{2}} du = \sqrt{\pi}$$

(d) $E(X)=\int_{-\infty}^{\infty} x\frac{1}{\Gamma(\alpha)\beta^{\alpha}}x^{\alpha-1}e^{-x/\beta}dx=\int_{-\infty}^{\infty}\frac{1}{\Gamma(\alpha)\beta^{\alpha}}x^{\alpha}e^{-x/\beta}dx=\alpha\beta\int_{-\infty}^{\infty}\frac{1}{\Gamma(\alpha+1)\beta^{\alpha+1}}x^{(\alpha+1)-1}e^{-x/\beta}dx$

$\int_{-\infty}^{\infty}\frac{1}{\Gamma(\alpha+1)\beta^{\alpha+1}}x^{(\alpha+1)-1}e^{-x/\beta}dx=1$ pois é a f.d.p. de $X\sim Gama(\alpha+1,\beta)$.

Logo,

$E(X)=\alpha\beta$.

$$E(X^2)=\int_{-\infty}^{\infty}x^2\frac{1}{\Gamma(\alpha)\beta^{\alpha}}x^{\alpha-1}e^{-x/\beta}dx=\frac{(\alpha+1)\alpha\beta^2}{(\alpha+1)\alpha\beta^2}\int_{-\infty}^{\infty}\frac{1}{\Gamma(\alpha)\beta^{\alpha}}x^{\alpha}e^{-x/\beta}dx=$$

$$=\frac{(\alpha+1)\alpha\beta^2}{}\int_{-\infty}^{\infty}\frac{1}{\Gamma(\alpha+2)\beta^{\alpha+2}}x^{(\alpha+2)-1}e^{-x/\beta}dx=\alpha^2\beta^2+\alpha\beta^2$$

Então,

$Var(X)=\alpha^2\beta^2+\alpha\beta^2-\alpha^2\beta^2=\alpha\beta^2$

Problema 46.

(a) $\int_{-\infty}^{\infty}f(x)dx=\int_{b}^{\infty}\frac{\alpha}{b}b^{\alpha+1}x^{-\alpha-1}dx=\frac{\alpha}{b^{-\alpha}}\times\left[\frac{x^{-\alpha}}{-\alpha}\right]_b^{\infty}=b^{\alpha}\times b^{-\alpha}=1$

(b) $\alpha>1$

$$E(X)=\int_b^{\infty}\alpha b^{\alpha}x^{-\alpha}dx=\alpha b^{\alpha}\left.\frac{x^{-\alpha+1}}{1-\alpha}\right|_b^{\infty}=\frac{\alpha b}{\alpha-1}$$

$\alpha>2$

$$E(X^2)=\int_b^{\infty}\alpha b^{\alpha}x^{-\alpha+1}dx=\alpha b^{\alpha}\left.\frac{x^{-\alpha+2}}{2-\alpha}\right|_b^{\infty}=\frac{\alpha b^2}{\alpha-2}$$

Então,

$$Var(X)=E(X^2)-[E(X)]^2=\frac{\alpha b^2}{\alpha-2}-\left[\frac{\alpha b}{\alpha-1}\right]^2=\frac{\alpha b^2}{(\alpha-1)^2(\alpha-2)}$$

Problema 47.

(a) $E(X)=\int_0^{\infty}x\frac{1}{x\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\ln x-\mu}{\sigma}\right)^2}dx \quad (1)$

Tomando $\ln x=y\to x=e^y, dx=e^y dy$

$\frac{y-\mu}{\sigma}=z\longrightarrow y=\mu+\sigma z, dy=\sigma dz$

Voltando a (1):

$$E(X)=\int_0^{\infty}x\frac{1}{x\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\ln x-\mu}{\sigma}\right)^2}dx=\int_{-\infty}^{\infty}e^{\mu+\sigma z}\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}(z)^2}\sigma dz=e^{\mu+\frac{\sigma^2}{2}}\times\int_{-\infty}^{\infty}\frac{e^{(z^2-2\sigma z+\sigma^2)}}{\sqrt{2\pi}}dz=$$

$$= e^{\mu+\frac{\sigma^2}{2}} \times \int_{-\infty}^{\infty} \frac{e^{(z-\sigma)^2}}{\sqrt{2\pi}} dz = e^{\mu+\frac{\sigma^2}{2}} \times \underbrace{\int_{-\infty}^{\infty} \frac{e^{-\frac{1}{2}v^2}}{\sqrt{2\pi}} dz}_{1} = e^{\mu+\frac{\sigma^2}{2}}$$

(b) $E(X^2) = m^2 e^{\sigma^2}$, $m = E(X) = e^{\mu+\frac{\sigma^2}{2}}$

$Var(X) = m^2 e^{\sigma^2} - m^2 = m^2 (e^{\sigma^2} - 1)$

Problema 48.

$P(X > x) = e^{-x}$, $P(X > t + x) = e^{-(t+x)}$

$$\therefore \frac{P(X > t + x)}{P(X > x)} = \frac{e^{-(t+x)}}{e^{-x}} = e^{-t} = P(X > t)$$

Problema 49.

$$E(Y) = \int_{-\infty}^{\infty} |x| f(x) dx = \int_{-\infty}^{0} -x \frac{1}{2} e^{x} dx + \int_{0}^{\infty} x \frac{1}{2} e^{-x} dx = \frac{1}{2} \times \int_{-\infty}^{0} -xe^{x} dx + \frac{1}{2} \times \int_{0}^{\infty} xe^{-x} dx =$$

$$= -\frac{1}{2} \times (-1) + \frac{1}{2} \times 1 = 1$$

$$\int_{-\infty}^{0} -xe^{x} dx = \left[xe^{x}\right]_{-\infty}^{0} - \int_{-\infty}^{0} e^{x} dx = -1$$

$$\int_{-\infty}^{0} xe^{-x} dx = \left[xe^{-x}\right]_{0}^{\infty} - \int_{0}^{\infty} e^{x} dx = 1$$

Problema 50.

$X \sim U(0,1)$ $\qquad Y = \frac{1}{2} X^2$

Então:

$$E(Y) = \int_0^1 \frac{1}{2} x^2 dx = \frac{1}{2} \times \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{6}$$

Problema 51.

(a) $\beta = 1 \longrightarrow f(x) = \begin{cases} \beta e^{-\beta x}, x \geq 0 \\ 0, x < 0 \end{cases}$ logo f.d.p. de uma exponencial.

(b) $\beta = 2 \longrightarrow f(x) = 2xe^{-2x}, x \geq 0$

$E(X) = 2 \times \int_0^{\infty} x^2 e^{-2x} dx$ (integrar por partes!)

Problema 52.

$$f(x) = \frac{\Gamma(4)}{\Gamma(2)\Gamma(2)} x(1-x) = 6x(1-x), \quad 0 < x < 1$$

$$P(X \le 0{,}2) = 6\times\int_0^{0,2} x(1-x)dx = 6\times\left[\frac{x^2}{2}-\frac{x^3}{3}\right]_0^{0,2} = 0{,}104$$

$$E(X) = 6\times\int_0^1 x^2(1-x)dx = 6\times\left[\frac{x^3}{3}-\frac{x^4}{4}\right]_0^1 = \frac{1}{2}$$

$$E(X^2) = 6\times\int_0^1 x^3(1-x)dx = 6\times\left[\frac{x^4}{4}-\frac{x^5}{5}\right]_0^1 = \frac{3}{10}$$

$$Var(X) = E(X^2)-[E(X)]^2 = \frac{1}{20}$$

Problema 53.

$$E(X) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{x}{1+x^2}dx = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{2x}{1+x^2}dx \qquad (1)$$

Tomando:

$$1+x^2 = y, \quad dy = 2xdx$$

Voltando a (1):

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{2x}{1+x^2}dx = \frac{1}{2\pi}[\log y]_{-\infty}^{\infty} = \infty$$

, logo não existe.

Problema 56.

$X \sim N(10;16)$

Então:

$$Q_x = 10+4Q_z$$

$$Q_z(0{,}10) = -1{,}28 \longrightarrow Q_x(0{,}10) = 10+4\times(-1{,}28) = 4{,}88$$

$$Q_z(0{,}25) = -0{,}67 \longrightarrow Q_x(0{,}25) = 10+4\times(-0{,}67) = 7{,}32$$

$$Q_z(0{,}50) = 0 \longrightarrow Q_x(0{,}1) = 10+4\times(0) = 10$$

$$Q_z(0{,}75) = 0{,}67 \longrightarrow Q_x(0{,}75) = 10+4\times(0{,}67) = 12{,}68$$

$$Q_z(0{,}90) = 1{,}28 \longrightarrow Q_x(0{,}90) = 10+4\times(1{,}28) = 15{,}12$$

Problema 57.

Considerando agora $Y \sim \chi^2(5)$, tem-se:

$$Q(0{,}10) = 1{,}610$$

$$Q(0{,}25) = 2{,}672$$

$$Q(0{,}50) = 4{,}351$$

$$Q(0{,}75) = 6{,}676$$

$$Q(0{,}90) = 9{,}236$$

Problema 58.

(a) $P(\chi^2(4) > 9{,}488) = e^{-4,724}\sum_{j=0}^{1}\frac{(4{,}724)^j}{j!} = 0{,}0089\times[1+4{,}724] = 0{,}051$

Da tabela da ditribuição qui-quadrado vem que: $P\left(\chi^2(4) > 9{,}488\right) = 0{,}05$.

(b) $P\left(\chi^2(10) > 16\right) = e^{-8}\sum_{j=0}^{1}\frac{(8)^j}{j!} = 0{,}00034 \times [1 + 8 + 32 + 85{,}3 + 170{,}7] = 0{,}101$

Da tabela da ditribuição qui-quadrado vem que: $P\left(\chi^2(10) > 16\right) = 0{,}10$.