ON THE PRACTICAL PERFORMANCE OF RATELESS CODES*

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Keywords: Coding, error-correction codes, rateless codes, online codes

Abstract: We propose a new parameter for optimizing the performance of rateless codes by minimizing latency and data overhead. We call it Latency-Overhead Product (LOP). It is based on ideas used in the analysis of digital circuits. We demonstrate the effectiveness of the LOP parameter for a specific class of rateless codes called Online codes (OC). We give results from experiments in ideal channel and simulated wireless channel with losses. In the second part of our experiments we analyze the relationship between the message block size and the performance of rateless codes. With the results from these experiments we extend the results from the performance of Luby Transform (LT) codes published in (Vukobratovic and Despotovic, 2005) by adding figures for Online codes.

1 INTRODUCTION

Rateless codes (RC) (Luby et al., 2001; Luby, 2002; Maymounkov, 2002; Shokrollahi, 2006; MacKay, 2005) are a class of error correction codes, used for encoding data transmitted over channels with variable packet error rate. An example of such a channel is a wireless link. Due to the *fading* effect present in wireless channels, the capacity of the latter is not constant. Non-constant channel capacity results in varying packet error rate. Although rateless codes are well suited for channels with varying error rate, their use often causes data overhead and delays. Minimizing both data overhead and delay is critical for achieving optimal performance of rateless codes. In the presented research, we address the mentioned problem for the case of rateless codes. We support our conclusions with experimental results for a specific class of rateless codes called Online codes (Maymounkov, 2002).

*THIS WORK IS FUNDED PARTIALLY BY IBBT, KATHOLIEKE UNIVERSITEIT LEUVEN (OT/06/40) AND FWO PROJECTS (G.0300.07 AND G.0450.04). THIS WORK WAS SUPPORTED IN PART BY THE IAP PROGRAMME P6/26 BCRYPT OF THE BELGIAN STATE (BELGIAN SCIENCE POLICY), BY THE EU IST FP6 PROJECTS (ECRYPT) AND BY THE IBBT-QOE PROJECT OF THE IBBT.

2 OUR CONTRIBUTION

We propose a new parameter - Latency-Overhead Product (LOP) for evaluating and optimizing the performance of rateless codes. We use the proposed parameter to experimentally find the optimal number of the blocks into which a message should be divided prior to being encoded with rateless codes. We evaluate the optimal block size in terms of latency and data overhead.

3 LATENCY-OVERHEAD PRODUCT

The proposed metric - Latency-Overhead Product (LOP) is inspired by ideas used in the analysis of digital circuits. In digital circuits theory there exists the so called parameter Area-Delay Product (ADP). ADP is widely used as a measure for evaluating the optimal performance of the integrated circuit in terms of minimum delay and consumed area. Analogously to ADP, we propose the Latency-Overhead Product (LOP) as a measure for evaluating the optimal number of blocks into which a message should be broken prior to being encoded with rateless codes. LOP evaluates optimality in terms of minimum latency and minimum overhead.

4 LOP AND RATELESS CODES

Let us consider an encoder for a file of size n blocks. Let δ be the average overhead and t_d total decoding time of the algorithm. We define Latency-Overhead Product as:

$$\mathrm{LOP} = \mathrm{LOP}(n) = \delta^{\alpha} t_d^{\beta} \tag{1}$$

where α and β are weight parameters of the LOP function. By fixing the weights α and β to specific values we are choosing whether the LOP minimization will be done with a stronger preference for small overhead ($\alpha > \beta$) or small delay ($\alpha < \beta$). For example if we are more interested in having small overhead rather than small delay (i.e. we want the delay also to be small, but this is not as important as having a small overhead) then we can indicate this by giving a larger weight value for α and smaller weight value for β e.g. $\alpha = 2, \beta = 1$. The resultant value for the LOP function is $\mathrm{LOP} = \delta^2 t_d$.

In general, the overhead δ and latency t_d are functions of several parameters. In realistic environments they depend on the state of the channel. For a very general case we can derive a theoretical condition that needs to be fulfilled in order to obtain optimal value for the block size n for a given rateless code:

$$\frac{\partial \mathrm{LOP}}{\partial n} = \alpha \delta^{\alpha-1} t_d^{\beta} \frac{\partial \delta}{\partial n} + \beta t_d^{\beta-1} \delta^{\alpha} \frac{\partial t_d}{\partial n} = 0 \ . \tag{2}$$

In other words, given the parameters α and β, using LOP we can find an optimal n such that Eq. (3) holds.

$$\frac{\alpha}{\delta} \frac{\partial \delta}{\partial n} = -\frac{\beta}{t_d} \frac{\partial t_d}{\partial n} \ . \tag{3}$$

Here δ and t_d depend on the type of rateless code that is used and on the state of the channel. If Eq. (3) has a real solution we can achieve an optimal value for n.

We show the effectiveness of the LOP in practice for a specific class of rateless codes called Online codes. Before presenting our experimental results, we give a short description of Online codes.

5 ONLINE CODES

Online codes (OC) are a type of rateless codes suitable for channels with an unknown or variable loss rate. OC were introduced by Petar Maymounkov in (Maymounkov, 2002). They are efficient error-correction codes, requiring $O(1)$ time to generate each encoding block and $O(n)$ time to decode an original message of length n. Detailed treatment of Online codes is given in (Maymounkov, 2002) and (Maymounkov and Mazieres, 2003).

In our experiments we use LOP to measure the optimal number of message blocks when message is encoded with Online codes in two cases: ideal channel and simulated wireless channel with losses. We simulate a channel with losses with a Gilbert-Elliot model. Next we describe our experimental settings and provide the obtained results.

Table 1: Calculating the Optimal Block Size using LOP parameter - Ideal Channel

Message size n	**Overhead [%]**	**Latency [s]**	**LOP [ms]**
667	31.23	0.22	21.46
833	28.36	0.23	18.50
1,333	21.38	0.26	11.88
2,000	16.93	0.30	8.60
4,000	8.21	0.55	3.71
10,000	**5.32**	**1.26**	**3.57**
12,000	5.00	1.82	4.59
20,000	4.76	3.92	8.88
40,000	4.05	12.52	20.54

6 EXPERIMENTS AND RESULTS

In order to show the usefulness of the proposed parameter LOP we have implemented Online codes (Maymounkov, 2002).[2]. With Online codes we conduct two sets of experiments: (1) finding an optimal block size with LOP; (2) measuring the general performance of Online codes in respect to packet size vs. total overhead. The first experiment is conducted for two cases: (a) ideal channel (b) channel with losses, simulated according to a Gilbert-Elliot model.

6.1 Optimal Block Size with LOP

In this set of experiments we use LOP in order to evaluate the optimal block size of a message encoded with OC. We run $10,000$ simulations per measurement.

[2]The source code for our Online codes implementation is licensed under GPL and is available at http://sourceforge.net/projects/onlinecodes

Table 2: Calculating the Optimal Block Size using LOP parameter - Wireless Channel

Message size n	**Overhead [%]**	**Latency [*s*]**	**LOP [*ms*]**
667	49.55	1.53	375.65
833	42.10	1.50	256.86
1,333	30.37	0.99	91.31
2,000	22.85	0.91	47.51
4,000	10.61	1.17	13.17
10,000	**6.23**	**2.11**	**8.19**
12,000	5.88	2.47	8.54
20,000	5.32	5.63	15.95
40,000	4.79	14.55	33.38

One simulation consists of encoding and decoding of a test file, using Online codes. In all simulations we use a test file of size 1 MB. During experiments we change the size of the blocks into which the test file is broken, resp. we change the *number* of blocks composing the file. In each simulation we measure the two parameters of the LOP function: (1) Overhead δ - the difference between the number of original message blocks n and the total number of sent message blocks necessary for successful decoding and (2) Latency t_d - the total time measured from start of the encoding until the end of decoding.

In Tables 1 and 2 are shown results from calculating LOP for the cases of ideal channel and channel with losses respectively. In each simulation we calculate the value of the LOP function, using the measured values for δ and t_d. For our test settings, we give preference to minimizing the packet overhead over minimizing the computational costs. That results in the values $\alpha = 2$, $\beta = 1$ for the weight parameters of the LOP function.

From Tables 1 and 2 we can see that for ideal channel the minimum value of the LOP parameter is 3.57 *ms* and for channel with losses this value is equal to 8.19 *ms*. Both values are obtained for the case when the original message is broken down to $10,000$ blocks. The encoding and decoding of this message, using Online codes results in 1.26 *s* latency and 5.32 % overhead for ideal and 2.11 *s* latency and 6.23 % overhead for channel with losses respectively. These values are optimal under the setup conditions of the experiment (global parameters of the Online codes algorithm and LOP constants - α and β).

The data from Tables 1 and 2 is used to construct the graphs in Figs. 1 and 2 respectively. The two graphs represent the values of Latency-Overhead Product as a function of the message size n.

6.2 Packet Size vs. Total Overhead

In this set of experiments we investigate the relationship between the packet size and the total overhead when transmitting a test file encoded with Online codes. We transmit a file of size 1M Byte using blocks of different size varying from 25 Bytes to 500 Bytes. This causes the number of blocks in which the original file is broken to vary from $40,000$ to $2,000$ blocks respectively. The test file is transmitted over the simulated wireless channel. The channel is simulated according to the Gilbert-Elliot model where packet error rate is packet length dependent.

The results from this test are presented in four graphs in Fig. 3. We have one graph for each block size. Each graph represents a histogram showing the total number of experiments (ordinate) where the overhead has a specific value (abscissa). White bars represent the overhead on the transmitting side and are more interesting for us. They show the total overhead of the transmitting file. The black bars give just an overhead at the receiving side. Observing the Fig. 3 we can notice that with increasing the packet size black and white bars are getting more separated. This behavior is natural as increasing the packet size causes larger packet error rate.

To summarize the results from our two sets of experiments: using LOP, given the weight parameters α and β, we experimentally obtain the value of $n = 10,000$ blocks as an optimal message size for both ideal channel and channel with losses. Observing the simulation results for the relationship between packet size vs. total overhead it is straightforward to conclude that the use of smaller packets results in smaller total overhead and hence - in better performance. On the other hand, by decreasing the packet size we increase the number of packets and as a result decoding becomes computationally more expensive.

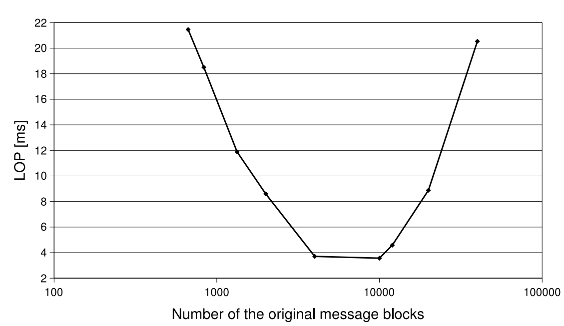

Figure 1: LOP as a function of the message size - Ideal Channel.

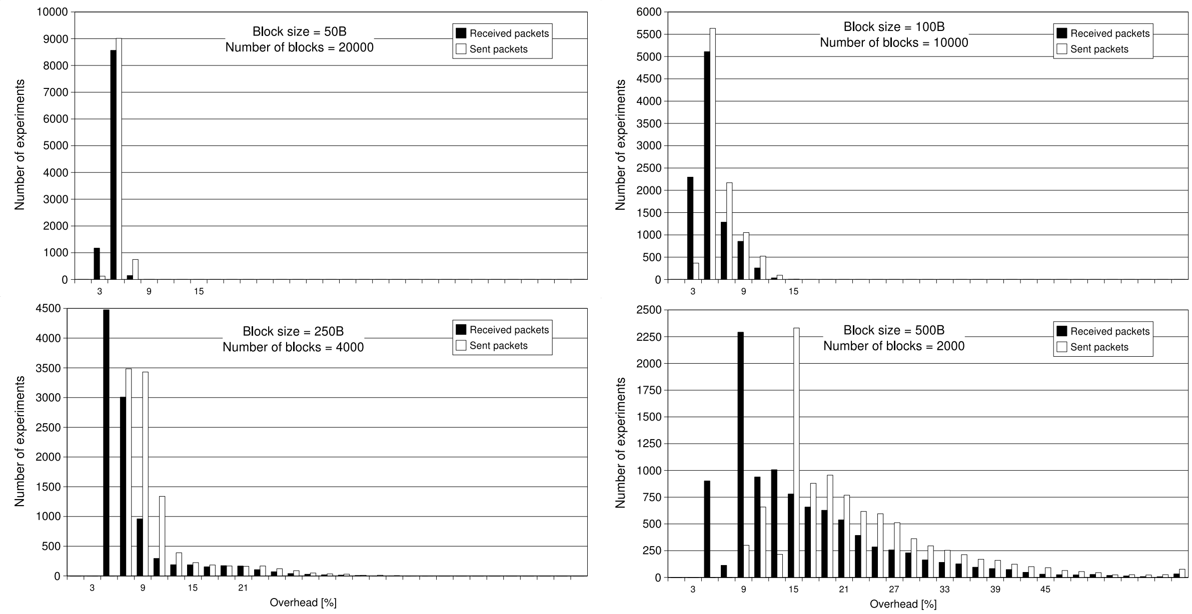

Figure 3: Packet size vs. total overhead for four different packet sizes - channel with losses.

7 CONCLUSION

In this paper we have proposed a new parameter - Latency-Overhead Product (LOP) - for finding the optimal block size for a message encoded with rateless codes. We implemented Online codes and showed how LOP can successfully be used to determine the optimal block size in terms of best compromise between latency and data overhead. Experimenting with the behavior of Online codes in simulated wireless environment, where the packet error rate is packet-length dependant, we showed that the use of smaller block sizes results in a smaller total overhead. Although we experimentally proved the effectiveness of LOP for evaluating and optimizing the performance of rateless codes, this parameter can be further investigated from theoretical point of view. This is a possible direction for the future work.

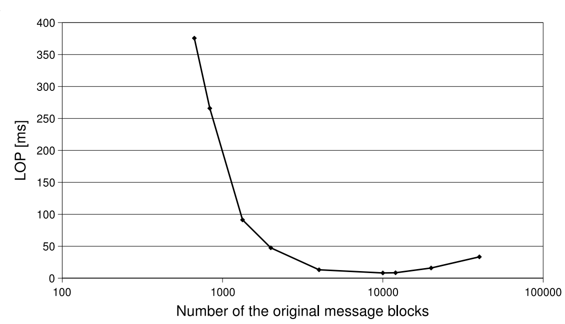

Figure 2: LOP as a function of the message size - Wireless Channel.

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