University of Minnesota
School of Physics and Astronomy

2025 Fall Physics 8901
Elementary Particle Physics I

Assignment Solution

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Question 1

The $\tau - \theta$ Puzzle

In the 1950's, two particles τ, θ were discovered with the same mass and lifetime that decayed differently. At the time, physicists believed that parity was conserved in all interactions.

(a) Consider the decay $\theta \to \pi^+\pi^0$. Assuming parity invariance and zero for the spin of θ, find the parity of θ.

(b) Now consider the decay process $\tau \to \pi^+\pi^+\pi^-$. (This is an old symbol for the K meson.) Let l be the orbital angular momentum of $\pi^+\pi^+$ and l' the orbital angular momentum of π^- relative to the center-of-mass of $\pi^+\pi^+$. Assuming parity invariance and the spin of τ equal to zero, find its parity.

(c) What resolved the $\tau - \theta$ puzzle?

Answer

(a)

First, we note that the intrinsic parity of a pion is -1. The parity of a two-particle system is given by

$$P_\theta = P_1 P_2 (-1)^l, \tag{1}$$

where P_1 and P_2 are the intrinsic parities of the two particles, and l is their relative orbital angular momentum. Since the θ and pions have spin 0, the system of two pions must have total angular momentum $j = s + l = 0$, in order to satisfy the conservation of total angular momentum. This means that l must be 0, too. Therefore, we have $P_\theta = 1$. It is even parity.

(b)

The parity of a three-particle system is given by

$$P_\tau = P_1 P_2 P_3 (-1)^{l+l'}, \tag{2}$$

where P_1, P_2, and P_3 are the intrinsic parities of the three particles, and l and l' are their relative orbital angular momenta. Since the τ and pions have spin 0, the system of three pions must have total angular momentum $j = s + l + l' = 0$, in order to satisfy the conservation of total angular momentum. This means that $l + l'$ must be 0, too. Therefore, we have $P_\tau = -1$. It is odd parity.

(c)

Since the τ and θ have the same mass and lifetime, they are actually the same particle, now known as the K meson. The resolution of the $\tau - \theta$ puzzle was the discovery that parity is not conserved in weak interactions, which is how the K meson decays. $\square$

Question 2

List all applicable conservation laws that are or would be violated in the following decays:

1. $\rho^0 \to \pi^0\pi^0$
2. $\rho \to \gamma\gamma$
3. $K^+ \to \pi^+\pi^0$
4. $\pi^0 \to 5\gamma$

(Look up the corresponding parities from the Particle Data Group at http://pdg.lbl.gov.)

Answer

Before we analyze each decay, we list the conservation laws that we will check for each decay:

- Conservation of electric charge
- Conservation of angular momentum (total angular momentum J).
- Conservation of isospin
- Conservation of parity
- Conservation of C-parity
- Conservation of G-parity

1. The ρ^0 has quantum numbers $I^G(J^{PC}) = 1^+(1^{--})$, the π^0 has quantum numbers $I^G(J^{PC}) = 1^-(0^{-+})$.

   - Electric charge: The ρ^0 has charge 0, and the two π^0's have charge $0+0=0$. Electric charge is conserved.
   - Angular momentum: The ρ^0 has spin 1, and the two π^0's have spin $0+0=0$. To conserve total angular momentum, the two-pion system must have orbital angular momentum $l=1$. Total angular momentum is conserved.
   - Isospin: The ρ^0 has isospin $I=1$, and the two π^0's can couple to isospin $I=0,1,2$. Therefore, the decay can proceed through the $I=1$ channel. Isospin is conserved.
   - Parity: The parity of the two-pion system is given by

     $$P_\rho = P_\pi P_\pi(-1)^l = (-1)(-1)(-1)^l = (-1)^l. \quad (3)$$

     Since the ρ^0 has spin 1 and the pions have spin 0, the two-pion system must have orbital angular momentum $l=1$ to conserve total angular momentum. Therefore, the parity of the two-pion system is $P=-1$, which massches the parity of the ρ^0. Parity is conserved.

- C-parity: The C-parity of the two-pion system is given by

$$C_\rho = C_\pi C_\pi (-1)^l = (+1)(+1)(-1)^l = (-1)^l. \tag{4}$$

  Since the ρ^0 has spin 1 and the pions have spin 0, the two-pion system must have orbital angular momentum $l = 1$ to conserve total angular momentum. Therefore, the C-parity of the two-pion system is $C = -1$, which matches the C-parity of the ρ^0. C-parity is conserved.

- G-parity: The G-parity of the two-pion system is given by

$$G_\rho = G_\pi G_\pi = (-1)(-1) = 1. \tag{5}$$

  Hence, the G parity is $G = +1$, which matches the G-parity of the ρ^0. G-parity is conserved.

All conservation laws are satisfied.

Remark: If we check the decay mode of $\rho \to \pi\pi$, we find that the branch ratio is close to 100%. This is consistent with our analysis that the decay can occur.

2. The ρ has quantum numbers $I^G(J^{PC}) = 1^+(1^{--})$, the photon has quantum numbers $I^G(J^{PC}) = 0^-(1^{--})$.

Remark: Actually, PDG show that the photon has isospin $I = 0, 1$. Here, I choose $I = 0$ because the photon do not involve in strong interactions. In other words, the photon is a singlet under the strong interaction. I think $I = 1$ case is for the weak interaction, but I am not sure.

- Electric charge: The ρ has charge 0, and the two photons have charge $0 + 0 = 0$. Electric charge is conserved.
- Angular momentum: The ρ has spin 1, and the two photons have spin $1 + 1 = 0, 1, 2$. To conserve total angular momentum, the two-photon system must have orbital angular momentum $l = 0(\text{when } s = 1), l = 1(\text{when } s = 0, 1, 2), l = 2(\text{when } s = 1)$. Hence, the total angular momentum might be conserved. However, by the **Landau-Yang theorem**, a massive spin-1 particle cannot decay into two photons. Therefore, the decay cannot occur. **Angular momentum is not conserved**.
- Isospin: The ρ has isospin $I = 1$, and the two photons can couple to isospin $I = 0$. Therefore, the decay cannot proceed through any isospin channel. **Isospin is not conserved**.
- Parity: The parity of the two-photon system is given by

$$P_\rho = P_\gamma P_\gamma (-1)^l = (-1)(-1)(-1)^l = (-1)^l. \tag{6}$$

  By checking the possible values of l above, we know the l should be 1 to conserve the parity.

- C-parity: The C-parity of the two-photon system is given by

$$C_\gamma C_\gamma = (-1)(-1) = 1 \neq -1 = C_\rho. \tag{7}$$

Hence, the C parity is $C = +1$, which does not match the C-parity of the ρ. **C-parity is not conserved**.

The angular momentum, isospin, and C-parity are not conserved.
Remark: If we check the decay mode of $\rho \to \gamma\gamma$, we find that the branch ratio is 0%.

3. The K^+ has quantum numbers $I(J^P) = \frac{1}{2}(0^-)$, the π^+ has quantum numbers $I(J^P) = 1(0^-)$, and the π^0 has quantum numbers $I(J^P) = 1(0^-)$.

   - Electric charge: The K^+ has charge $+1$, and the two pions have charge $+1+0 = +1$. Electric charge is conserved.
   - Angular momentum: The K^+ has spin 0, and the two pions have spin $0+0 = 0$. To conserve total angular momentum, the two-pion system must have orbital angular momentum $l = 0$. Total angular momentum is conserved.
   - Isospin: The K^+ has isospin $I = \frac{1}{2}$, and the two pions can couple to isospin $I = 0, 1, 2$. **Therefore, the decay cannot proceed through any isospin channel**.
   - Parity: The parity of the two-pion system is given by

   $$-1 = P_K = P_\pi P_\pi (-1)^l = (-1)(-1)(-1)^l = (-1)^l. \tag{8}$$

     Since the K^+ has spin 0 and the pions have spin 0, the two-pion system must have orbital angular momentum $l = 0$ to conserve total angular momentum. Therefore, the parity of the two-pion system is $P = +1$, which does not match the parity of the K^+. **Parity is not conserved**.
   - C-parity: Not applicable, since the particles are not neutral.

   The isospin and parity are not conserved.
   Remark: If we check the decay mode of $K^+ \to \pi^+\pi^0$, we find that the branch ratio is 21.13%. This is a **weak decay**, in which isospin and parity are not conserved.

4. The π^0 has quantum numbers $I^G(J^{PC}) = 1^-(0^{-+})$, the photon has quantum numbers $I^G(J^{PC}) = 0^-(1^{--})$.

Question 3

List all states ((J^{PC})) with total spin $J = 0, 1, 2$ and P, C parities that cannot be realized as a fermion-antifermion system (i.e., as e^+e^- or quark-antiquark). (Hypothetical particles with such combinations of quantum numbers are called exotic, and are being sought for in experiments, so far unsuccessfully.)

Answer

Question 4

State which of the following combinations can or cannot exist in a state of isospin $I = 1$, and give the reasons:

1. $\pi^0\pi^0$
2. $\pi^+\pi^-$
3. $\Sigma^0\pi^0$
4. $\Lambda\pi^0$

Answer