University of Minnesota
School of Physics and Astronomy

2026 Spring Physics 8902
Elementary Particle Physics II

Assignment Solution

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February 4, 2026

Problem Set 2 Due 11am, Monday, February 16

Question 1

Weak decay of pions

(a) Find the electron energy spectrum $d\Gamma/dE_e$ for the decay $\pi^- \to \pi^0 + e^- + \overline{\nu}_e$ in the π^- rest frame keeping $m_e \neq 0$ (take $m_\nu = 0$). Assume the hadronic current is dominated by $f_+(0)$ and neglect radiative corrections. Perform the phase-space integration by integrating over the π^0 and $\overline{\nu}_e$ momenta (i.e. treat E_e as the only observed variable). Give the kinematic endpoints and verify the $m_e \to 0$ limit.

(b) Using the electron energy spectrum obtained in part (a), integrate over E_e to extract the leading correction of order m_e^2/Δ^2 to the total decay rate. Write the result in the form

$$\Gamma(\pi^- \to \pi^0 + e^- + \overline{\nu}_e) = |V_{ud}|^2 \frac{G_F^2 \Delta^5}{30\pi^3} \left(1 - a\frac{\Delta}{m_\pi} - b\frac{m_e^2}{\Delta^2}\right), \tag{1}$$

where $\Delta = m_{\pi^-} - m_{\pi^0}$, and neglecting higher-order terms in Δ/m_π and m_e^2/Δ^2. In the lectures it was shown that $a = 3/2$. Determine the coefficient b.

Answer

(a)

(b)

Question 2

Tau decays

(a) Find the decay rate for the two-body decay $\tau^- \to \pi^- + \nu_\tau$, neglecting neutrino masses and using $\langle 0|\overline{d}\gamma^\mu\gamma^5 u|\pi^-\rangle = if_\pi p_\pi^\mu$. Determine the ratio

$$R_\pi = \frac{\Gamma(\tau^- \to \pi^- + \nu_\tau)}{\Gamma(\tau^- \to e^- + \overline{\nu}_e + \nu_\tau)}, \tag{2}$$

using the tree-level leptonic rate with $m_e = 0$, and compare with the corresponding PDG branching-fraction ratio.

(b) Now consider $\tau^- \to \rho^- + \nu_\tau$ with $\langle 0|\overline{d}\gamma^\mu u|\rho^-(q,\epsilon)\rangle = f_\rho m_\rho \epsilon^\mu$, and derive the decay rate $\Gamma(\tau^- \to \rho^- + \nu_\tau)$ (neglect the neutrino mass). Form the ratio

$$R = \frac{\Gamma(\tau^- \to \rho^- + \nu_\tau)}{\Gamma(\tau^- \to e^- + \overline{\nu}_e + \nu_\tau)}, \tag{3}$$

and nd compare with the PDG data to extract f_ρ (or the ratio f_ρ/f_π).
Hint: Use the polarization sum

$$\sum_\lambda \epsilon_\mu^{(\lambda)}(q)\epsilon_\nu^{(\lambda)*}(q) = -g_{\mu\nu} + \frac{q_\mu q_\nu}{m_\rho^2}. \tag{4}$$