University of Minnesota
School of Physics and Astronomy

2026 Spring Physics 8012
Quantum Field Theory II

Assignment Solution

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Homework 3 Due to February 12 9:00 AM

Question 1

(a) Write down the generators of the SU(3) group in the fundamental representation. Use the standard form of the Gell-Mann matrices. Commute them and find the set of the structure constants for SU(3).

(b) Calculate anti-commutators of the same generators. The constants in the right-hand-side of the anti-commutators are called d symbols.

(c) Compare the result to anti-commutators for SU(2) (in which the generators in fundamental representation are (1/2)×Pauli matrices). What is the qualitative difference?

(d) For $N = 3$ check the the following equation is valid:

$$f^{abc} f^{adg} = \frac{2}{N} \left(\delta^{bd} \delta^{cg} - \delta^{bg} \delta^{cd} \right) + d^{abd} d^{acg} - d^{acd} d^{abg} \tag{1}$$

Answer

(a)

The generators of the SU(3) group in the fundamental representation are given by the Gell-Mann matrices divided by 2:

$$T^1 = \frac{1}{2} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad T^2 = \frac{1}{2} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad T^3 = \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{2}$$

$$T^4 = \frac{1}{2} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad T^5 = \frac{1}{2} \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad T^6 = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \tag{3}$$

$$T^7 = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad T^8 = \frac{1}{2\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}. \tag{4}$$

The commutation relations for the generators can be expressed in terms of the structure constants f^{abc} as follows:

$$f^{abc} = \frac{2}{i} \mathrm{Tr}(T^a [T^b, T^c]) \tag{5}$$

Calculating the commutators and using the above formula, we find the non-zero structure constants for SU(3) (see the calculation details in the Mathematica notebook):

$$f^{123} = 1, \quad f^{147} = \frac{1}{2}, \quad f^{156} = -\frac{1}{2}, \quad f^{246} = \frac{1}{2}, \quad f^{257} = \frac{1}{2}, \quad f^{345} = \frac{1}{2}, \quad f^{367} = -\frac{1}{2}, \tag{6}$$

$$f^{458} = \frac{\sqrt{3}}{2}, \quad f^{678} = \frac{\sqrt{3}}{2}. \tag{7}$$

(b)

The anti-commutators of the generators can be expressed in terms of the d symbols as follows:

$$d^{abc} = 2\mathrm{Tr}(T^a\{T^b, T^c\}) \tag{8}$$

Calculating the anti-commutators and using the above formula, we find the non-zero d symbols for SU(3) (see the calculation details in the Mathematica notebook):

$$d^{118} = d^{228} = d^{338} = -d^{888} = \frac{1}{\sqrt{3}}, \quad d^{448} = d^{558} = d^{668} = d^{778} = -\frac{1}{2\sqrt{3}}, \tag{9}$$

$$d^{146} = d^{157} = -d^{247} = d^{256} = d^{344} = d^{355} = -d^{366} = -d^{377} = \frac{1}{2}. \tag{10}$$

Note that the anti-commutators yield a more complex structure involving the d symbols, which is

$$\{T^a, T^b\} = \frac{1}{3}\delta^{ab} I + d^{abc} T^c. \tag{11}$$

However, we can also derive d^{abc} using the relation:

$$d^{abc} = 2\mathrm{Tr}(T^a\{T^b, T^c\}), \tag{12}$$

since the generators are traceless.

(c)

For SU(2), the generators in the fundamental representation are given by (1/2) times the Pauli matrices:

$$T^1 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad T^2 = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad T^3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{13}$$

Calculating the anti-commutators for SU(2), we find that they are proportional to the identity matrix:

$$\{T^a, T^b\} = \frac{1}{2}\delta^{ab} I. \tag{14}$$

The qualitative difference between SU(2) and SU(3) is that for SU(2), the anti-commutators yield a simple result proportional to the identity matrix, while for SU(3), the anti-commutators yield a more complex structure involving the d symbols, which are not simply proportional to the identity matrix. This reflects the richer structure of the SU(3) group compared to SU(2).

(d)

Please refer to the Mathematica notebook for the detailed calculation of the equation, and the result is valid. $\square$