University of Minnesota
School of Physics and Astronomy

2025 Fall Physics 8501
General Relativity I
Assignment Solution

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Assignment 8 due on Monday November 3 at 10PM

Question 1

In lecture we showed that $P^0 = M$ for the Schwarzschild solution in standard coordinates. Calculate P^z and show that it is zero. Since the metric is isotropic this shows that all components of the 3-momentum are zero.

Answer

Question 2

In lecture we were given the components of the affine connection $\Gamma^{\lambda}_{\mu\nu}$ for the Scharzschild solution in standard coordinates. Using these, calculate one component of the Riemann-Christoffel curvature tensor, namely R^{t}_{rtr}. This is nonvanishing everywhere and goes to zero as $r \to \infty$. This shows that space is curved even though both the Ricci tensor and curvature scalar vanish.

Answer

Question 3

A photon moves in the Schwarzschild metric in the equatorial plane $\theta = \pi/2$. Using standard coordinates, show that the shape of the orbit is given by the solution to the differential equation

$$\frac{d^2w}{d\phi^2} + w = 3w^2, \tag{1}$$

where $w = GM/r$. Assuming that $|w| \ll 1$, solve this equation iteratively to find the deflection angle $\Delta\phi$, and show that it agrees with the answer obtained in class by other means.

Question 4

The deflection of light by a spherical static body whose physical radius is smaller than its Schwarzschild radius should produce comet-like orbits suffering substantial deflection before returning to infinite radius if the distance of closest approach r_0 becomes comparable to the Schwarzschild radius. Using the differential equation in problem 1 show that there is a critical orbit $w_c(\phi)$, corresponding to a critical radius r_c. Explain what happens when $r_0 > r_c, r_0 = r_c$, and $r_0 < r_c$. Draw some orbits for illustration. This phenomenon was observed by the "Event Horizon Telescope".

Answer