University of Minnesota
School of Physics and Astronomy

2025 Fall Physics 8901
Elementary Particle Physics I

Assignment Solution

Lecture Instructor: Professor Tony Gherghetta

Zong-En Chen
chen9613@umn.edu

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Problem Set 3 due 9:30 AM, Monday, October 13th

Question 1

p-d reactions

Consider the reactions

$$p + d \to \pi^+ + {}^3\mathrm{H}, \quad p + d \to \pi^0 + {}^3\mathrm{He}. \tag{1}$$

Since the deuteron is in a ${}^3S_1$ state, it must be an isospin singlet. Therefore, the initial state $p + d$ is a pure $I = \frac{1}{2}$ state. Given that ${}^3\mathrm{H}$ and ${}^3\mathrm{He}$ form an isodoublet, write down the isospin decomposition of the final states, and from this, the ratio of the two cross sections.

Question 2

Particle production by strong interactions

Explain why the processes $\pi^- + p \to \pi^+ + \Sigma^-$, $\pi^- + p \to K^0 + n$, $\pi^- + p \to \Sigma^+ + K^-$ cannot be observed.

Question 3

SU(2) invariants and pseudoreal representations

(a) Show that $\delta^a{}_b$ and ϵ_{ab} are invariant tensors under SU(2) transformations.

(b) The nucleon doublet $N^a = \binom{p}{n}, a = 1, 2$ transforms as the fundamental 2 of SU(2), while its conjugate $\bar{N}_a \equiv (N^a)^\dagger = (\bar{p}, \bar{n})$ transforms as $\overline{\mathbf{2}}$. Use $\delta^a{}_b$ to form an SU(2) invariant with $N, \bar{N}$ and write it explicitly in terms of the proton and neutron fields.

(c) Define $\tilde{N}^b = \epsilon^{bc} \bar{N}_c^T$ which maps the $\overline{\mathbf{2}}$ representation (lower index) into $\mathbf{2}$ (upper index). Construct an SU(2) invariant with $N, \tilde{N}$ using ϵ_{ab}, and write it in terms of the components. Verify that the result is identical to part (b), demonstrating that the $\mathbf{2}$ and $\overline{\mathbf{2}}$ representations are equivalent (or pseudoreal) in SU(2) and that any invariant constructed with $\delta^a{}_b$ can be rewritten using ϵ_{ab}.

(d) Consider SU(3), with the quark triplet $q^a (a = 1, 2, 3)$ transforming as $\mathbf{3}$ and its conjugate $\bar{q}_a \equiv (q^a)^\dagger$ transforming as $\overline{\mathbf{3}}$. Discuss why a similar mapping using the SU(3) invariant ϵ_{abc} does not make $\mathbf{3}$ and $\overline{\mathbf{3}}$ equivalent. Write down the possible SU(3) invariants involving $q, \bar{q}$.

Question 4

Applications of U-spin

(a) Show that $U_\pm = t_6 \pm i t_7$ and $U_3 = \left(\sqrt{3} t_8 - t_3\right)/2$ satisfy the SU(2) algebra

$$[U_3, U_\pm] = \pm U_\pm, \quad [U_+, U_-] = 2U_3.$$

(b) Show that the charge operator $Q = t_3 + t_8/\sqrt{3}$ is a U-scalar i.e. it has U-spin $U = 0$ or $[Q, U_i] = 0$ for $i = \pm 3$. Write the electromagnetic current operator in terms of quark fields.

(c) Show that for the meson octet, the ($U_3 = 0$) component of the U-triplet is $\pi_U^0 = \left(-\pi^0 + \sqrt{3}\eta\right)/2$, and the U-singlet is $\eta_U^0 = \left(\sqrt{3}\pi^0 + \eta\right)/2$. Since π_U^0 is a U -spin vector component it cannot couple to the electromagnetic current. Show that for the 2γ decay mode, $\langle \pi^0 \mid 2\gamma \rangle = \sqrt{3} \langle \eta \mid 2\gamma \rangle$. How does this U-spin prediction compare with the experimental decay widths?