University of Minnesota
School of Physics and Astronomy

2025 Fall Physics 8501
General Relativity I

Assignment Solution

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Assignment 4 due on Monday September 29th at 5PM

Question 1

Calculate the metric g_{ij} and its inverse g^{ij} , the affine connection Γ^i_{jk}, and the Laplacian ∇^2 in two dimensions for a polar coordinate system with $\xi^1 = x$ and $\xi^2 = y$ being Cartesian coordinates and $x^1 = r$ and $x^2 = \theta$ being polar coordinates.

Answer

Question 2

Calculate the compact expressions for the components of the affine connection when the metric g_{ij} is diagonal. See problem 3 in chapter 3 in chapter 3 of Carroll's book.

Answer

Question 3

Prove that if the equation for a geodesic has the form

$$\frac{d^2x^\alpha}{dp_i^2} + \Gamma^\alpha_{\beta\gamma}\frac{dx^\beta}{dp_i}\frac{dx^\gamma}{dp_i} = 0, \tag{1}$$

for two different parameters p_1 and p_2 defined along the geodesic then the most general relation between them is $p_2 = Ap_1 + B$ where A and B are constants.

Answer