

University of Minnesota  
School of Physics and Astronomy

**2026 Spring Physics 8502**  
**General Relativity II**  
Assignment Solution

Lecture Instructor: Professor Keith Olive

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## Problem Set 1 due on Due Feb 9 at 11:59pm

### Question 1

- (a) Consider the outer surface of the ergosphere for a Kerr black hole. Construct the normal vector to this surface and show that it is a 2-way surface.
- (b) Using the condition  $n_\alpha n^\alpha = 0$  for a null surface, derive the equation equation for the critical 1-way surface for a time-independent axial symmetric surface. That is start with some  $u(\rho, \theta)$  to derive the equation defining this surface. What is its physical interpretation.

### Answer

## Question 2

Show that

$$(L_X T)_{b_1 b_2 \dots b_s}^{a_1 a_2 \dots a_r} = (\nabla_X T)_{b_1 b_2 \dots b_s}^{a_1 a_2 \dots a_r} - \sum_i T_{b_1 b_2 \dots b_s}^{a_1 \dots j \dots a_r} X_{;j}^{a_i} + \sum_j T_{b_1 \dots j \dots b_s}^{a_1 a_2 \dots a_r} X_{;b_j}^j \quad (1)$$

**Answer**

### Question 3

- (a) Consider an arbitrary unit vector,  $X$  transported along a latitude line on the surface of the sphere. Use  $ds^2 = d\theta^2 + \sin^2 \theta d\phi^2$ , and recall that there are only 2 distinct  $\Gamma$ 's.
- (b) Do the same assuming the vector is Lie transported. That is instead of  $\nabla_T X = 0$ , assume that  $L_T X = 0$ .

### Answer