

University of Minnesota
School of Physics and Astronomy

2025 Fall Physics 8901
Elementary Particle Physics I
Assignment Solution

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Question 1

Gell-Mann Okubo for the baryon octet

The generators t_a ($a = 1, \dots, 8$) of $SU(3)$ are normalized as

$$t_a = \frac{\lambda_a}{2}$$

with

$$\text{Tr}(t_a t_b) = \frac{1}{2} \delta_{ab},$$

where λ_a are the Gell-Mann matrices. They satisfy

$$[t_a, t_b] = i f_{abc} t_c$$

and

$$\{t_a, t_b\} = \frac{1}{3} \delta_{ab} \mathbf{1} + d_{abc} t_c,$$

where f_{abc} are totally antisymmetric and d_{abc} are totally symmetric structure constants.

Let B and \bar{B} be the baryon octet 3×3 traceless matrices, expanded in the generator basis as

$$B = B^i t_i, \quad \bar{B} = \bar{B}^i t_i$$

where B^i, \bar{B}^i are the adjoint components. Define the two bilinear combinations

$$O_A \equiv [\bar{B}, B] = \bar{B}B - B\bar{B}, \quad O_S \equiv \{\bar{B}, B\} - \frac{2}{3} \mathbf{1} \text{Tr}(\bar{B}B).$$

- (a) Show that both O_A and O_S are traceless and therefore transform in the adjoint (octet) representation.
- (b) Expand O_A and O_S in components using the generator basis and show that

$$O_A = i(\bar{B}^i B^j) f_{ijk} t_k, \quad O_S = (\bar{B}^i B^j) d_{ijk} t_k,$$

so that

$$(O_A)^k = i f_{ijk} \bar{B}^i B^j, \quad (O_S)^k = d_{ijk} \bar{B}^i B^j.$$

- (c) Introduce a flavor-breaking spurion $H_8 = H_8^i t_i$, with real components H_8^i . Construct the two independent $SU(3)$ -invariant mass terms:

$$S_f = (O_A)_b^a (H_8)_a^b, \quad S_d = (O_S)_b^a (H_8)_a^b.$$

Assuming H_8 points in the 8-direction (i.e. $H_8^i \propto \delta_{i8}$), argue that S_f and S_d correspond to the f -type and d -type symmetry breaking terms in the baryon mass operator, respectively.

- (d) Given that an adjoint operator O_8 acts on an octet state B as $O_8(B) = [O_8, B]$, show that the invariant scalars in (c) are equivalent to the matrix elements

$$S_f \propto \langle \bar{B} | t_8 | B \rangle \equiv \text{Tr}(\bar{B} [t_8, B]),$$

$$S_d \propto \langle \bar{B} | d_{8ij} t_i t_j | B \rangle \equiv \text{Tr}(\bar{B} [d_{8ij} t_i t_j, B]).$$

- (e) Hence, argue that

$$S_f \propto Y, \quad S_d \propto I(I+1) - \frac{Y^2}{4}$$

where I, Y are the isospin and hypercharge of the baryon B , respectively, thereby reproducing the Gell-Mann-Okubo mass formula for the baryon octet.

Hint: Verify that

$$\left[\frac{2}{\sqrt{3}} t_8, B \right] = Y_B B, \quad [d_{8ij} t_i t_j, B] = \left(I(I+1) - \frac{Y^2}{4} \right) B$$

where B is the baryon octet 3×3 matrix.

Answer

Question 2

ρ - ω mixing

The vector mesons $\rho(770)$ and $\omega(782)$ are very close in mass. For this reason the effects of isospin violation are somewhat enhanced in these mesons and can be parametrized in terms of ρ - ω mixing. Namely, the physical ρ^0 and ω mesons can be viewed as orthogonal mixed states of a pure isospin triplet and isospin singlet:

$$\begin{aligned}\rho^0 &= \cos\theta \frac{(u\bar{u} - d\bar{d})}{\sqrt{2}} + \sin\theta \frac{(u\bar{u} + d\bar{d})}{\sqrt{2}}, \\ \omega &= -\sin\theta \frac{(u\bar{u} - d\bar{d})}{\sqrt{2}} + \cos\theta \frac{(u\bar{u} + d\bar{d})}{\sqrt{2}},\end{aligned}$$

where θ is a (small) mixing angle.

- (a) Determine θ (up to a sign) using experimental data on the decay $\omega \rightarrow \pi^+\pi^-$. Estimate the error in the value of the mixing angle.
- (b) Using the value of θ predict the decay rates $\Gamma(\rho^0 \rightarrow e^+e^-)$ and $\Gamma(\omega \rightarrow e^+e^-)$, assuming the amplitude for a quark pair annihilation into an e^+e^- pair is proportional to the electric charge Q of the quark.
- (c) Assume that the transition amplitude between different spin states of a $q\bar{q}$ quark pair with emission of a photon: $(q\bar{q}) \rightarrow (q\bar{q}) + \gamma$ is proportional to the quark electric charge Q . Use the value of the ρ - ω mixing angle θ to determine the ratios of the decay rates:
 - (i) $\Gamma(\rho^0 \rightarrow \pi^0\gamma)/\Gamma(\omega^0 \rightarrow \pi^0\gamma)$,
 - (ii) $\Gamma(\rho^0 \rightarrow \eta\gamma)/\Gamma(\omega^0 \rightarrow \eta\gamma)$.

Compare with the PDG experimental data. How does the inclusion of ρ - ω mixing improve the agreement with the data?

Answer

Question 3

Baryon magnetic moments

The octet of spin- $\frac{1}{2}$ baryons has magnetic moments μ . The operator that describes the magnetic moment is an $SU(3)_f$ octet operator which is proportional to the quark charge Q . The charge

$$Q = t_3 + \frac{1}{\sqrt{3}}t_8$$

is traceless ($\text{Tr } Q = 0$) and can be promoted to a purely $SU(3)_f$ octet spurion $\mathbf{8}_Q$ (with no singlet piece, as in contrast to the GMO mass formula). Hence, when determining the baryon magnetic moment

$$\mu(B) = \langle \bar{B} | \mu | B \rangle \propto \mathbf{8}_{\bar{B}} \times \mathbf{8}_Q \times \mathbf{8}_B$$

there are two independent octet structures (the f - and d -type couplings, as for the baryon mass), given by

$$\mu(B) = c_f \text{Tr}(B^\dagger [Q, B]) + c_d \text{Tr}(B^\dagger \{Q, B\}) = \alpha_+ \text{Tr}(BB^\dagger Q) + \alpha_- \text{Tr}(B^\dagger BQ),$$

where $\alpha_+ \equiv c_d + c_f$, $\alpha_- \equiv c_d - c_f$ are arbitrary constants and

$$B = \begin{pmatrix} \frac{\Sigma_u^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma_u^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ -\Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}, \quad Q = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix}.$$

Determine all the spin- $\frac{1}{2}$ baryon magnetic moments in terms of $\mu(p)$ and $\mu(n)$ (by eliminating $c_{f,d}$ or α_{\pm}) and compare with the PDG experimental values. These predictions were first worked out by Coleman and Glashow in 1961. Note that imposing the full $SU(6)$ spin-flavor symmetry further predicts $\mu(p)/\mu(n) = -\frac{3}{2}$, which you can ignore in this problem.

Answer