University of Minnesota
School of Physics and Astronomy

2025 Fall Physics 8901
Elementary Particle Physics I

Assignment Solution

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Question 1

The $\tau - \theta$ Puzzle

In the 1950's, two particles τ, θ were discovered with the same mass and lifetime that decayed differently. At the time, physicists believed that parity was conserved in all interactions.

(a) Consider the decay $\theta \to \pi^+\pi^0$. Assuming parity invariance and zero for the spin of θ, find the parity of θ.

(b) Now consider the decay process $\tau \to \pi^+\pi^+\pi^-$. (This is an old symbol for the K meson.) Let l be the orbital angular momentum of $\pi^+\pi^+$ and l' the orbital angular momentum of π^- relative to the center-of-mass of $\pi^+\pi^+$. Assuming parity invariance and the spin of τ equal to zero, find its parity.

(c) What resolved the $\tau - \theta$ puzzle?

Answer

(a)

First, we note that the intrinsic parity of a pion is -1. The parity of a two-particle system is given by

$$P_\theta = P_1 P_2 (-1)^l, \tag{1}$$

where P_1 and P_2 are the intrinsic parities of the two particles, and l is their relative orbital angular momentum. Since the θ and pions have spin 0, the system of two pions must have total angular momentum $j = s + l = 0$, in order to satisfy the conservation of total angular momentum. This means that l must be 0, too. Therefore, we have $P_\theta = 1$. It is even parity.

(b)

The parity of a three-particle system is given by

$$P_\tau = P_1 P_2 P_3 (-1)^{l+l'}, \tag{2}$$

where P_1, P_2, and P_3 are the intrinsic parities of the three particles, and l and l' are their relative orbital angular momenta. Since the τ and pions have spin 0, the system of three pions must have total angular momentum $j = s + l + l' = 0$, in order to satisfy the conservation of total angular momentum. This means that $l + l'$ must be 0, too. Therefore, we have $P_\tau = -1$. It is odd parity.

(c)

Since the τ and θ have the same mass and lifetime, they are actually the same particle, now known as the K meson. The resolution of the $\tau - \theta$ puzzle was the discovery that parity is not conserved in weak interactions, which is how the K meson decays. $\square$

Question 2

List all applicable conservation laws that are or would be violated in the following decays:

1. $\rho^0 \to \pi^0\pi^0$

2. $\rho \to \gamma\gamma$

3. $K^+ \to \pi^+\pi^0$

4. $\pi^0 \to 5\gamma$

(Look up the corresponding parities from the Particle Data Group at http://pdg.lbl.gov.)

Answer

Before we analyze each decay, we list the conservation laws that we will check for each decay:

- Conservation of electric charge
- Conservation of angular momentum (total angular momentum J).
- Conservation of isospin
- Conservation of parity
- Conservation of C-parity
- Conservation of G-parity

1. The ρ^0 has quantum numbers $I^G(J^{PC}) = 1^+(1^{--})$, the π^0 has quantum numbers $I^G(J^{PC}) = 1^-(0^{-+})$.

   - Electric charge: The ρ^0 has charge 0, and the two π^0's have charge $0 + 0 = 0$. Electric charge is conserved.
   - Angular momentum: The ρ^0 has spin 1, and the two π^0's have spin $0 \otimes 0 = 0$. To conserve total angular momentum, the two-pion system must have orbital angular momentum $l = 1$. Total angular momentum is conserved.
   - Isospin: The ρ^0 has isospin $I = 1$, and the two π^0's can form isospin $I = 1 \otimes 1 = 0, 1, 2$. Therefore, the decay can proceed through the $I = 1$ channel. Isospin is conserved.
   - Parity: The parity of the two-pion system is given by

     $$P_\rho = P_\pi P_\pi (-1)^l = (-1)(-1)(-1)^l = (-1)^l. \tag{3}$$

     Since the ρ^0 has spin 1 and the pions have spin 0, the two-pion system must have orbital angular momentum $l = 1$ to conserve total angular momentum. Therefore, the parity of the two-pion system is $P = -1$, which massches the parity of the ρ^0. Parity is conserved.

- C-parity: The C-parity of the two-pion system is given by

$$C_\rho = C_\pi C_\pi (-1)^l = (+1)(+1)(-1)^l = (-1)^l. \tag{4}$$

  Since the ρ^0 has spin 1 and the pions have spin 0, the two-pion system must have orbital angular momentum $l = 1$ to conserve total angular momentum. Therefore, the C-parity of the two-pion system is $C = -1$, which matches the C-parity of the ρ^0. C-parity is conserved.

- G-parity: The G-parity of the two-pion system is given by

$$G_\rho = G_\pi G_\pi = (-1)(-1) = 1. \tag{5}$$

  Hence, the G parity is $G = +1$, which matches the G-parity of the ρ^0. G-parity is conserved.

All conservation laws are satisfied.
Remark: If we check the decay mode of $\rho \to \pi\pi$, we find that the branch ratio is close to 100%. This is consistent with our analysis that the decay can occur.

2. The ρ has quantum numbers $I^G(J^{PC}) = 1^+(1^{--})$, the photon has quantum numbers $I^G(J^{PC}) = 0^-(1^{--})$.
Remark: Actually, PDG show that the photon has isospin $I = 0, 1$. Here, I choose $I = 0$ because the photon do not involve in strong interactions. In other words, the photon is a singlet under the strong interaction. I think $I = 1$ case is for the weak interaction, but I am not sure.

- Electric charge: The ρ has charge 0, and the two photons have charge $0 + 0 = 0$. Electric charge is conserved.

- Angular momentum: The ρ has spin 1, and the two photons have spin $s = 1 \otimes 1 = 0, 1, 2$. To conserve total angular momentum, the two-photon system must have orbital angular momentum $l = 0(\text{when } s = 1), l = 1(\text{when } s = 0, 1, 2), l = 2(\text{when } s = 1)$. Hence, the total angular momentum might be conserved. However, by the **Landau-Yang theorem**, a massive spin-1 particle cannot decay into two photons. Therefore, the decay cannot occur. **Angular momentum is not conserved**.

- Isospin: The ρ has isospin $I = 1$, and the two photons can couple to isospin $I = 0$. Therefore, the decay cannot proceed through any isospin channel. **Isospin is not conserved**.

- Parity: The parity of the two-photon system is given by

$$P_\rho = P_\gamma P_\gamma (-1)^l = (-1)(-1)(-1)^l = (-1)^l. \tag{6}$$

  By checking the possible values of l above, we know the l should be 1 to conserve the parity.

- C-parity: The C-parity of the two-photon system is given by

$$C_\gamma C_\gamma = (-1)(-1) = 1 \neq -1 = C_\rho. \tag{7}$$

Hence, the C parity is $C = +1$, which does not match the C-parity of the ρ. **C-parity is not conserved**.

The angular momentum, isospin, and C-parity are not conserved.
Remark: If we check the decay mode of $\rho \to \gamma\gamma$, we find that the branch ratio is 0%.

3. The K^+ has quantum numbers $I(J^P) = \frac{1}{2}(0^-)$, the π^+ has quantum numbers $I(J^P) = 1(0^-)$, and the π^0 has quantum numbers $I(J^P) = 1(0^-)$.

   - Electric charge: The K^+ has charge $+1$, and the two pions have charge $1 + 0 = +1$. Electric charge is conserved.
   - Angular momentum: The K^+ has spin 0, and the two pions have spin $0 \otimes 0 = 0$. To conserve total angular momentum, the two-pion system must have orbital angular momentum $l = 0$. Total angular momentum is conserved.
   - Isospin: The K^+ has isospin $I = \frac{1}{2}$, and the two pions can form isospin $I = 0, 1, 2$. **Therefore, the decay cannot proceed through any isospin channel**.
   - Parity: The parity of the two-pion system is given by

     $$-1 = P_K = P_\pi P_\pi (-1)^l = (-1)(-1)(-1)^l = (-1)^l. \tag{8}$$

     Since the K^+ has spin 0 and the pions have spin 0, the two-pion system must have orbital angular momentum $l = 0$ to conserve total angular momentum. Therefore, the parity of the two-pion system is $P = +1$, which does not match the parity of the K^+. **Parity is not conserved**.
   - C-parity: Not applicable, since the particles are not neutral.

   The isospin and parity are not conserved.
   Remark: If we check the decay mode of $K^+ \to \pi^+\pi^0$, we find that the branch ratio is 21.13%. This is a **weak decay**, in which isospin and parity are not conserved.

4. The π^0 has quantum numbers $I^G(J^{PC}) = 1^-(0^{-+})$, the photon has quantum numbers $I^G(J^{PC}) = 0^-(1^{--})$.

   - Electric charge: The π^0 has charge 0, and the five photons have charge $0 + 0 + 0 + 0 + 0 = 0$. Electric charge is conserved.
   - Angular momentum: The π^0 has spin 0, and the five photons can have total spin $1, 2, 3, 4, 5$. To conserve total angular momentum, the five-photon system must have orbital angular momentum $l = 1, 2, 3, 4, 5$ to form the correct combinations. Hence, the total angular momentum might be conserved.
   - Isospin: The π^0 has isospin $I = 1$, and the five photons can form isospin $I = 0$. **Therefore, the decay cannot proceed through any isospin channel**.

- Parity: The parity of the five-photon system is given by

$$-1 = P_\pi = P_\gamma P_\gamma P_\gamma P_\gamma P_\gamma (-1)^l = -1 \times (-1)^l. \tag{9}$$

  By checking the possible values of l above, we know the l should be $2, 4$ to conserve the parity.

- C-parity: The C-parity of the five-photon system is given by

$$+1 = C_\pi \neq C_\gamma C_\gamma C_\gamma C_\gamma C_\gamma = (-1)^5 = -1, \tag{10}$$

  Hence, the C parity is $C = -1$, which does not match the C-parity of the π^0. **C-parity is not conserved**.

The isospin and C-parity are not conserved.
Remark: If we check the decay mode of $\pi^0 \to 5\gamma$, we find that the branch ratio is 0%. The dominant decay mode is $\pi^0 \to 2\gamma$, which has a branch ratio of 98.823%. This is consistent with our analysis that the decay cannot occur.

□

Question 3

List all states (J^{PC}) with total spin $J = 0, 1, 2$ and P, C parities that cannot be realized as a fermion-antifermion system (i.e., as e^+e^- or quark-antiquark). (Hypothetical particles with such combinations of quantum numbers are called exotic, and are being sought for in experiments, so far unsuccessfully.)

Answer

First we note that a fermion-antifermion system has the following properties:

- The intrinsic parity of a fermion is $+1$, and the intrinsic parity of an antifermion is -1. Therefore, the intrinsic parity of a fermion-antifermion system is -1. Hence the parity of a fermion-antifermion system is given by

$$P = P_f P_{\bar{f}} (-1)^l = -(-1)^l = (-1)^{l+1}, \tag{11}$$

- The C-parity of a fermion-antifermion system is given by

$$C = (-1)^{l+s}, \tag{12}$$

  where s is the total spin of the fermion-antifermion system, which can be 0 or 1.

Based on the above properties, we can list all possible states (J^{PC}) with total spin $J = 0, 1, 2$ for a fermion-antifermion system:

- For $J = 0$:
  - When $l = 0$, $s = 0$: $P = (-1)^{0+1} = -1$, $C = (-1)^{0+0} = +1$, so $J^{PC} = 0^{-+}$.
  - When $l = 1$, $s = 1$: $P = (-1)^{1+1} = +1$, $C = (-1)^{1+1} = +1$, so $J^{PC} = 0^{++}$.
- For $J = 1$:
  - When $l = 0$, $s = 1$: $P = (-1)^{0+1} = -1$, $C = (-1)^{0+1} = -1$, so $J^{PC} = 1^{--}$.
  - When $l = 1$, $s = 0$: $P = (-1)^{1+1} = +1$, $C = (-1)^{1+0} = -1$, so $J^{PC} = 1^{+-}$.
  - When $l = 1$, $s = 1$: $P = (-1)^{1+1} = +1$, $C = (-1)^{1+1} = +1$, so $J^{PC} = 1^{++}$.
  - When $l = 2$, $s = 1$: $P = (-1)^{2+1} = -1$, $C = (-1)^{2+1} = -1$, so $J^{PC} = 1^{--}$.
- For $J = 2$:
  - When $l = 1$, $s = 1$: $P = (-1)^{1+1} = +1$, $C = (-1)^{1+1} = +1$, so $J^{PC} = 2^{++}$.
  - When $l = 2$, $s = 0$: $P = (-1)^{2+1} = -1$, $C = (-1)^{2+0} = +1$, so $J^{PC} = 2^{-+}$.
  - When $l = 2$, $s = 1$: $P = (-1)^{2+1} = -1$, $C = (-1)^{2+1} = -1$, so $J^{PC} = 2^{--}$.

- When $l = 3$, $s = 1$: $P = (-1)^{3+1} = +1$, $C = (-1)^{3+1} = +1$, so $J^{PC} = 2^{++}$.

Therefore, the states (J^{PC}) with total spin $J = 0, 1, 2$ and P, C parities that cannot be realized as a fermion-antifermion system are:

- 0^{+-}, 0^{--}
- 1^{-+}
- 2^{+-}

$\square$

Question 4

State which of the following combinations can or cannot exist in a state of isospin $I = 1$, and give the reasons:

1. $\pi^0\pi^0$
2. $\pi^+\pi^-$
3. $\pi^+\pi^+$
4. $\Sigma^0\pi^0$
5. $\Lambda\pi^0$

Answer

First, we note the isospin quantum numbers of the particles involved:

- The π^0 has isospin $I = 1$, $I_3 = 0$.
- The π^+ has isospin $I = 1$, $I_3 = +1$.
- The π^- has isospin $I = 1$, $I_3 = -1$.
- The Σ^0 has isospin $I = 1$, $I_3 = 0$.
- The Λ has isospin $I = 0$, $I_3 = 0$.

Now we analyze each combination:

1. $\pi^0\pi^0$: The two π^0's can form isospin $I = 0, 1, 2$. Therefore, the combination can exist in a state of isospin $I = 1$. But since the two pions are identical bosons, their total wavefunction must be symmetric under exchange. When the isospin state is $I = 1$ (which is antisymmetric), the spatial part must be antisymmetric (odd orbital angular momentum) to make the total wavefunction symmetric. Hence, the combination can exist in a state of isospin $I = 1$ **with odd orbital angular momentum** $L = 1, 3, 5$ and so on.
   Remark: I also check the C-G coefficients, and find that the state $|I = 1, I_3 = 0\rangle$ is given by

$$|1, 0\rangle = \frac{1}{\sqrt{2}}(|\pi^+\pi^-\rangle - |\pi^-\pi^+\rangle), \tag{13}$$

$$|\pi^0\pi^0\rangle = \sqrt{\frac{2}{3}}|2, 0\rangle - \sqrt{\frac{1}{3}}|0, 0\rangle. \tag{14}$$

   The $|1, 0\rangle$ state does not contain the $|\pi^0\pi^0\rangle$ component. This means that the two π^0's cannot form isospin $I = 1$ state. This is consistent with our analysis above that the two π^0's can exist in a state of isospin $I = 1$. In my opinion, I think we cannot consider orbital angular momentum when we analyze the isospin state. Therefore, I think the two π^0's cannot form isospin $I = 1$ state. But I am not sure about this point.

2. $\pi^+\pi^-$: The π^+ and π^- can form isospin $I = 0, 1, 2$. Therefore, the combination can exist in a state of isospin $I = 1$. Since the two pions are not identical particles, there is no symmetry requirement on their total wavefunction. Hence, the combination can exist in a state of isospin $I = 1$.
Remark: I also check the C-G coefficients, and find that the state $|I = 1, I_3 = 0\rangle$ is given by
$$|1,0\rangle = \frac{1}{\sqrt{2}}(|\pi^+\pi^-\rangle - |\pi^-\pi^+\rangle). \tag{15}$$
The $|1,0\rangle$ state contains the $|\pi^+\pi^-\rangle$ component. This means that the π^+ and π^- can form isospin $I = 1$ state. This is consistent with our analysis above.

3. $\pi^+\pi^+$: The two π^+'s can only form isospin $I = 2$ since $I_3 = +2$. Therefore, the combination cannot exist in a state of isospin $I = 1$.
Remark: I also check the C-G coefficients, and find that the state $|I = 2, I_3 = 2\rangle$ is given by
$$|2,2\rangle = |\pi^+\pi^+\rangle. \tag{16}$$
The $|2,2\rangle$ state contains the $|\pi^+\pi^+\rangle$ component. This means that the two π^+'s can only form isospin $I = 2$ state. This is consistent with our analysis above.

4. $\Sigma^0\pi^0$: The Σ^0 has isospin $I = 1$, and the π^0 has isospin $I = 1$. For the same realized reason and the C-G coefficients in part (1), this cannot exist in a state of isospin $I = 1$.
Remark: I also check the C-G coefficients, and find that the state $|I = 1, I_3 = 0\rangle$ is given by
$$|1,0\rangle = \frac{1}{\sqrt{2}}(|\Sigma^+\pi^-\rangle - |\Sigma^-\pi^+\rangle), \tag{17}$$
$$|\Sigma^0\pi^0\rangle = \sqrt{\frac{2}{3}}|2,0\rangle - \sqrt{\frac{1}{3}}|0,0\rangle. \tag{18}$$
The $|1,0\rangle$ state does not contain the $|\Sigma^0\pi^0\rangle$ component. This means that the Σ^0 and π^0 cannot form isospin $I = 1$ state. This is consistent with our analysis above.

5. $\Lambda\pi^0$: The Λ has isospin $I = 0$, and the π^0 has isospin $I = 1$. The combination can only form isospin $I = 1$. Therefore, the combination can exist in a state of isospin $I = 1$.
Remark: I also check the C-G coefficients, and find that the state $|I = 1, I_3 = 0\rangle$ is given by
$$|1,0\rangle = |\Lambda\pi^0\rangle. \tag{19}$$
The $|1,0\rangle$ state contains the $|\Lambda\pi^0\rangle$ component. This means that the Λ and π^0 can form isospin $I = 1$ state. This is consistent with our analysis above. □