

University of Minnesota  
School of Physics and Astronomy

**2025 Fall Physics 8901**  
**Elementary Particle Physics I**  
Assignment Solution

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# Problem Set 2 due 9:30 AM, Monday, September 29th

## Question 1

### The $\tau - \theta$ Puzzle

In the 1950's, two particles  $\tau$ ,  $\theta$  were discovered with the same mass and lifetime that decayed differently. At the time, physicists believed that parity was conserved in all interactions.

- (a) Consider the decay  $\theta \rightarrow \pi^+\pi^0$ . Assuming parity invariance and zero for the spin of  $\theta$ , find the parity of  $\theta$ .
- (b) Now consider the decay process  $\tau \rightarrow \pi^+\pi^+\pi^-$ . (This is an old symbol for the  $K$  meson.) Let  $l$  be the orbital angular momentum of  $\pi^+\pi^+$  and  $l'$  the orbital angular momentum of  $\pi^-$  relative to the center-of-mass of  $\pi^+\pi^+$ . Assuming parity invariance and the spin of  $\tau$  equal to zero, find its parity.
- (c) What resolved the  $\tau - \theta$  puzzle?

## Answer

(a)

First, we note that the intrinsic parity of a pion is  $-1$ . The parity of a two-particle system is given by

$$P_\theta = P_1 P_2 (-1)^l, \quad (1)$$

where  $P_1$  and  $P_2$  are the intrinsic parities of the two particles, and  $l$  is their relative orbital angular momentum. Since the  $\theta$  and pions have spin 0, the system of two pions must have total angular momentum  $j = s + l = 0$ , in order to satisfy the conservation of total angular momentum. This means that  $l$  must be 0, too. Therefore, we have  $P_\theta = 1$ . It is even parity.

(b)

The parity of a three-particle system is given by

$$P_\tau = P_1 P_2 P_3 (-1)^{l+l'}, \quad (2)$$

where  $P_1$ ,  $P_2$ , and  $P_3$  are the intrinsic parities of the three particles, and  $l$  and  $l'$  are their relative orbital angular momenta. Since the  $\tau$  and pions have spin 0, the system of three pions must have total angular momentum  $j = s + l + l' = 0$ , in order to satisfy the conservation of total angular momentum. This means that  $l + l'$  must be 0, too. Therefore, we have  $P_\tau = -1$ . It is odd parity.

(c)

Since the  $\tau$  and  $\theta$  have the same mass and lifetime, they are actually the same particle, now known as the  $K$  meson. The resolution of the  $\tau - \theta$  puzzle was the discovery that parity is not conserved in weak interactions, which is how the  $K$  meson decays.  $\square$

## Question 2

List all applicable conservation laws that are or would be violated in the following decays:

1.  $\rho^0 \rightarrow \pi^0 \pi^0$
2.  $\rho \rightarrow \gamma \gamma$
3.  $K^+ \rightarrow \pi^+ \pi^0$
4.  $\pi^0 \rightarrow 5\gamma$

(Look up the corresponding parities from the Particle Data Group at <http://pdg.lbl.gov>.)

## Answer

Before we analyze each decay, we list the conservation laws that we will check for each decay:

- Conservation of electric charge
- Conservation of angular momentum (total angular momentum  $J$ ).
- Conservation of isospin
- Conservation of parity
- Conservation of C-parity
- Conservation of G-parity

1. The  $\rho^0$  has quantum numbers  $I^G(J^{PC}) = 1^+(1^{--})$ , the  $\pi^0$  has quantum numbers  $I^G(J^{PC}) = 1^-(0^{-+})$ .

- Electric charge: The  $\rho^0$  has charge 0, and the two  $\pi^0$ 's have charge  $0 + 0 = 0$ . Electric charge is conserved.
- Angular momentum: The  $\rho^0$  has spin 1, and the two  $\pi^0$ 's have spin  $0 \otimes 0 = 0$ . To conserve total angular momentum, the two-pion system must have orbital angular momentum  $l = 1$ . Total angular momentum is conserved.
- Isospin: The  $\rho^0$  has isospin  $I = 1$ , and the two  $\pi^0$ 's can form isospin  $I = 1 \otimes 1 = 0, 1, 2$ . Therefore, the decay can proceed through the  $I = 1$  channel. Isospin is conserved.
- Parity: The parity of the two-pion system is given by

$$P_\rho = P_\pi P_\pi (-1)^l = (-1)(-1)(-1)^l = (-1)^l. \quad (3)$$

Since the  $\rho^0$  has spin 1 and the pions have spin 0, the two-pion system must have orbital angular momentum  $l = 1$  to conserve total angular momentum. Therefore, the parity of the two-pion system is  $P = -1$ , which matches the parity of the  $\rho^0$ . Parity is conserved.

- C-parity: The C-parity of the two-pion system is given by

$$C_\rho = C_\pi C_\pi (-1)^l = (+1)(+1)(-1)^l = (-1)^l. \quad (4)$$

Since the  $\rho^0$  has spin 1 and the pions have spin 0, the two-pion system must have orbital angular momentum  $l = 1$  to conserve total angular momentum. Therefore, the C-parity of the two-pion system is  $C = -1$ , which matches the C-parity of the  $\rho^0$ . C-parity is conserved.

- G-parity: The G-parity of the two-pion system is given by

$$G_\rho = G_\pi G_\pi = (-1)(-1) = 1. \quad (5)$$

Hence, the G parity is  $G = +1$ , which matches the G-parity of the  $\rho^0$ . G-parity is conserved.

All conservation laws are satisfied.

**Remark:** If we check the decay mode of  $\rho \rightarrow \pi\pi$ , we find that the branch ratio is close to 100%. This is consistent with our analysis that the decay can occur.

2. The  $\rho$  has quantum numbers  $I^G(J^{PC}) = 1^+(1^{--})$ , the photon has quantum numbers  $I^G(J^{PC}) = 0^-(1^{--})$ .

**Remark:** Actually, PDG show that the photon has isospin  $I = 0, 1$ . Here, I choose  $I = 0$  because the photon do not involve in strong interactions. In other words, the photon is a singlet under the strong interaction. I think  $I = 1$  case is for the weak interaction, but I am not sure.

- Electric charge: The  $\rho$  has charge 0, and the two photons have charge  $0 + 0 = 0$ . Electric charge is conserved.
- Angular momentum: The  $\rho$  has spin 1, and the two photons have spin  $s = 1 \otimes 1 = 0, 1, 2$ . To conserve total angular momentum, the two-photon system must have orbital angular momentum  $l = 0$ (when  $s = 1$ ),  $l = 1$ (when  $s = 0, 1, 2$ ),  $l = 2$ (when  $s = 1$ ). Hence, the total angular momentum might be conserved. However, by the **Landau-Yang theorem**, a massive spin-1 particle cannot decay into two photons. Therefore, the decay cannot occur. **Angular momentum is not conserved.**
- Isospin: The  $\rho$  has isospin  $I = 1$ , and the two photons can couple to isospin  $I = 0$ . Therefore, the decay cannot proceed through any isospin channel. **Isospin is not conserved.**
- Parity: The parity of the two-photon system is given by

$$P_\rho = P_\gamma P_\gamma (-1)^l = (-1)(-1)(-1)^l = (-1)^l. \quad (6)$$

By checking the possible values of  $l$  above, we know the  $l$  should be 1 to conserve the parity.

- C-parity: The C-parity of the two-photon system is given by

$$C_\gamma C_\gamma = (-1)(-1) = 1 \neq -1 = C_\rho. \quad (7)$$

Hence, the C parity is  $C = +1$ , which does not match the C-parity of the  $\rho$ . **C-parity is not conserved.**

The angular momentum, isospin, and C-parity are not conserved.

**Remark:** If we check the decay mode of  $\rho \rightarrow \gamma\gamma$ , we find that the branch ratio is 0%.

3. The  $K^+$  has quantum numbers  $I(J^P) = \frac{1}{2}(0^-)$ , the  $\pi^+$  has quantum numbers  $I(J^P) = 1(0^-)$ , and the  $\pi^0$  has quantum numbers  $I(J^P) = 1(0^-)$ .

- Electric charge: The  $K^+$  has charge  $+1$ , and the two pions have charge  $1 + 0 = +1$ . Electric charge is conserved.
- Angular momentum: The  $K^+$  has spin 0, and the two pions have spin  $0 \otimes 0 = 0$ . To conserve total angular momentum, the two-pion system must have orbital angular momentum  $l = 0$ . Total angular momentum is conserved.
- Isospin: The  $K^+$  has isospin  $I = \frac{1}{2}$ , and the two pions can form isospin  $I = 0, 1, 2$ . **Therefore, the decay cannot proceed through any isospin channel.**
- Parity: The parity of the two-pion system is given by

$$-1 = P_K = P_\pi P_\pi (-1)^l = (-1)(-1)(-1)^l = (-1)^l. \quad (8)$$

Since the  $K^+$  has spin 0 and the pions have spin 0, the two-pion system must have orbital angular momentum  $l = 0$  to conserve total angular momentum. Therefore, the parity of the two-pion system is  $P = +1$ , which does not match the parity of the  $K^+$ . **Parity is not conserved.**

- C-parity: Not applicable, since the particles are not neutral.

The isospin and parity are not conserved.

**Remark:** If we check the decay mode of  $K^+ \rightarrow \pi^+\pi^0$ , we find that the branch ratio is 21.13%. This is a **weak decay**, in which isospin and parity are not conserved.

4. The  $\pi^0$  has quantum numbers  $I^G(J^{PC}) = 1^-(0^{-+})$ , the photon has quantum numbers  $I^G(J^{PC}) = 0^-(1^{--})$ .

- Electric charge: The  $\pi^0$  has charge 0, and the five photons have charge  $0 + 0 + 0 + 0 + 0 = 0$ . Electric charge is conserved.
- Angular momentum: The  $\pi^0$  has spin 0, and the five photons can have total spin 1, 2, 3, 4, 5. To conserve total angular momentum, the five-photon system must have orbital angular momentum  $l = 1, 2, 3, 4, 5$  to form the correct combinations. Hence, the total angular momentum might be conserved.
- Isospin: The  $\pi^0$  has isospin  $I = 1$ , and the five photons can form isospin  $I = 0$ . **Therefore, the decay cannot proceed through any isospin channel.**

- Parity: The parity of the five-photon system is given by

$$-1 = P_\pi = P_\gamma P_\gamma P_\gamma P_\gamma P_\gamma (-1)^l = -1 \times (-1)^l. \quad (9)$$

By checking the possible values of  $l$  above, we know the  $l$  should be 2, 4 to conserve the parity.

- C-parity: The C-parity of the five-photon system is given by

$$+1 = C_\pi \neq C_\gamma C_\gamma C_\gamma C_\gamma C_\gamma = (-1)^5 = -1, \quad (10)$$

Hence, the C parity is  $C = -1$ , which does not match the C-parity of the  $\pi^0$ . **C-parity is not conserved.**

The isospin and C-parity are not conserved.

**Remark:** If we check the decay mode of  $\pi^0 \rightarrow 5\gamma$ , we find that the branch ratio is 0%. The dominant decay mode is  $\pi^0 \rightarrow 2\gamma$ , which has a branch ratio of 98.823%. This is consistent with our analysis that the decay cannot occur.

□

## Question 3

List all states ( $J^{PC}$ ) with total spin  $J = 0, 1, 2$  and  $P, C$  parities that cannot be realized as a fermion-antifermion system (i.e., as  $e^+e^-$  or quark-antiquark). (Hypothetical particles with such combinations of quantum numbers are called exotic, and are being sought for in experiments, so far unsuccessfully.)

## Answer

First we note that a fermion-antifermion system has the following properties:

- The intrinsic parity of a fermion is  $+1$ , and the intrinsic parity of an antifermion is  $-1$ . Therefore, the intrinsic parity of a fermion-antifermion system is  $-1$ . Hence the parity of a fermion-antifermion system is given by

$$P = P_f P_{\bar{f}} (-1)^l = -(-1)^l = (-1)^{l+1}, \quad (11)$$

- The C-parity of a fermion-antifermion system is given by

$$C = (-1)^{l+s}, \quad (12)$$

where  $s$  is the total spin of the fermion-antifermion system, which can be 0 or 1.

Based on the above properties, we can list all possible states ( $J^{PC}$ ) with total spin  $J = 0, 1, 2$  for a fermion-antifermion system:

- For  $J = 0$ :
  - When  $l = 0, s = 0$ :  $P = (-1)^{0+1} = -1, C = (-1)^{0+0} = +1$ , so  $J^{PC} = 0^{-+}$ .
  - When  $l = 1, s = 1$ :  $P = (-1)^{1+1} = +1, C = (-1)^{1+1} = +1$ , so  $J^{PC} = 0^{++}$ .
- For  $J = 1$ :
  - When  $l = 0, s = 1$ :  $P = (-1)^{0+1} = -1, C = (-1)^{0+1} = -1$ , so  $J^{PC} = 1^{--}$ .
  - When  $l = 1, s = 0$ :  $P = (-1)^{1+1} = +1, C = (-1)^{1+0} = -1$ , so  $J^{PC} = 1^{+-}$ .
  - When  $l = 1, s = 1$ :  $P = (-1)^{1+1} = +1, C = (-1)^{1+1} = +1$ , so  $J^{PC} = 1^{++}$ .
  - When  $l = 2, s = 1$ :  $P = (-1)^{2+1} = -1, C = (-1)^{2+1} = -1$ , so  $J^{PC} = 1^{--}$ .
- For  $J = 2$ :
  - When  $l = 1, s = 1$ :  $P = (-1)^{1+1} = +1, C = (-1)^{1+1} = +1$ , so  $J^{PC} = 2^{++}$ .
  - When  $l = 2, s = 0$ :  $P = (-1)^{2+1} = -1, C = (-1)^{2+0} = +1$ , so  $J^{PC} = 2^{-+}$ .
  - When  $l = 2, s = 1$ :  $P = (-1)^{2+1} = -1, C = (-1)^{2+1} = -1$ , so  $J^{PC} = 2^{--}$ .



- When  $l = 3, s = 1$ :  $P = (-1)^{3+1} = +1$ ,  $C = (-1)^{3+1} = +1$ , so  $J^{PC} = 2^{++}$ .

Therefore, the states ( $J^{PC}$ ) with total spin  $J = 0, 1, 2$  and  $P, C$  parities that cannot be realized as a fermion-antifermion system are:

- $0^{+-}, 0^{--}$
- $1^{-+}$
- $2^{+-}$

□

## Question 4

State which of the following combinations can or cannot exist in a state of isospin  $I = 1$ , and give the reasons:

1.  $\pi^0\pi^0$
2.  $\pi^+\pi^-$
3.  $\pi^+\pi^+$
4.  $\Sigma^0\pi^0$
5.  $\Lambda\pi^0$

## Answer

First, we note the isospin quantum numbers of the particles involved:

- The  $\pi^0$  has isospin  $I = 1$ ,  $I_3 = 0$ .
- The  $\pi^+$  has isospin  $I = 1$ ,  $I_3 = +1$ .
- The  $\pi^-$  has isospin  $I = 1$ ,  $I_3 = -1$ .
- The  $\Sigma^0$  has isospin  $I = 1$ ,  $I_3 = 0$ .
- The  $\Lambda$  has isospin  $I = 0$ ,  $I_3 = 0$ .

Now we analyze each combination:

1.  $\pi^0\pi^0$ : The two  $\pi^0$ 's can form isospin  $I = 0, 1, 2$ . Therefore, the combination can exist in a state of isospin  $I = 1$ . But since the two pions are identical bosons, their total wavefunction must be symmetric under exchange. When the isospin state is  $I = 1$  (which is antisymmetric), the spatial part must be antisymmetric (odd orbital angular momentum) to make the total wavefunction symmetric. Hence, the combination can exist in a state of isospin  $I = 1$  **with odd orbital angular momentum**  $L = 1, 3, 5$  and so on.

**Remark:** I also check the C-G coefficients, and find that the state  $|I = 1, I_3 = 0\rangle$  is given by

$$|1, 0\rangle = \frac{1}{\sqrt{2}}(|\pi^+\pi^-\rangle - |\pi^-\pi^+\rangle), \quad (13)$$

$$|\pi^0\pi^0\rangle = \sqrt{\frac{2}{3}}|2, 0\rangle - \sqrt{\frac{1}{3}}|0, 0\rangle. \quad (14)$$

The  $|1, 0\rangle$  state does not contain the  $|\pi^0\pi^0\rangle$  component. This means that the two  $\pi^0$ 's cannot form isospin  $I = 1$  state. This is consistent with our analysis above that the two  $\pi^0$ 's can exist in a

state of isospin  $I = 1$ . In my opinion, I think we cannot consider orbital angular momentum when we analyze the isospin state. Therefore, I think the two  $\pi^0$ 's cannot form isospin  $I = 1$  state. But I am not sure about this point.

2.  $\pi^+\pi^-$ : The  $\pi^+$  and  $\pi^-$  can form isospin  $I = 0, 1, 2$ . Therefore, the combination can exist in a state of isospin  $I = 1$ . Since the two pions are not identical particles, there is no symmetry requirement on their total wavefunction. Hence, the combination can exist in a state of isospin  $I = 1$ .
3.  $\pi^+\pi^+$ : The two  $\pi^+$ 's can only form isospin  $I = 2$  since  $I_3 = +2$ . Therefore, the combination cannot exist in a state of isospin  $I = 1$ .
4.  $\Sigma^0\pi^0$ : The  $\Sigma^0$  has isospin  $I = 1$ , and the  $\pi^0$  has isospin  $I = 1$ . The combination can form isospin  $I = 0, 1, 2$ . Therefore, the combination can exist in a state of isospin  $I = 1$ .
5.  $\Lambda\pi^0$ : The  $\Lambda$  has isospin  $I = 0$ , and the  $\pi^0$  has isospin  $I = 1$ . The combination can only form isospin  $I = 1$ . Therefore, the combination can exist in a state of isospin  $I = 1$ .

□