

University of Minnesota  
School of Physics and Astronomy

**2025 Fall Physics 8501**

**General Relativity I**

Assignment Solution

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# **Assignment 4 due on Monday September 29th at 5PM**

## **Question 1**

Calculate the metric  $g_{ij}$  and its inverse  $g^{ij}$ , the affine connection  $\Gamma^i_{jk}$ , and the Laplacian  $\nabla^2$  in two dimensions for a polar coordinate system with  $\xi^1 = x$  and  $\xi^2 = y$  being Cartesian coordinates and  $x^1 = r$  and  $x^2 = \theta$  being polar coordinates.

## **Answer**

## **Question 2**

Calculate the compact expressions for the components of the affine connection when the metric  $g_{ij}$  is diagonal. See problem 3 in chapter 3 in chapter 3 of Carroll's book.

## **Answer**

## Question 3

Prove that if the equation for a geodesic has the form

$$\frac{d^2x^\alpha}{dp_i^2} + \Gamma_{\beta\gamma}^\alpha \frac{dx^\beta}{dp_i} \frac{dx^\gamma}{dp_i} = 0, \quad (1)$$

for two different parameters  $p_1$  and  $p_2$  defined along the geodesic then the most general relation between them is  $p_2 = Ap_1 + B$  where  $A$  and  $B$  are constants.

## Answer