University of Minnesota
School of Physics and Astronomy

# 2025 Fall Physics 8011
# Quantum Field Theory I

Assignment Solution

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# HW2 Due to October 7 11:59 PM

## Question 1

Problem 5.1

Work out the LSZ reduction formula for the complex scalar field that was introduced in problem 3.5. Note that we must specify the type ($a$ or $b$) of each incoming and outgoing particle.

## Answer

# Question 1

Problem 6.1

(a) Find an explicit formula for $\mathcal{D}q$ in eq. (6.9). Your formula should be of the form $\mathcal{D}q = C\prod_{j=1}^{N} dq_j$, where $C$ is a constant that you should compute.

(b) For the case of a free particle, $V(Q) = 0$, evaluate the path integral of eq. (7.9) explicitly. Hint: integrate over $q_1$, then $q_2$, etc, and look for a pattern. Express you final answer in terms of $q', t', q'', t''$ and $m$. Restore $\hbar$ by dimensional analysis.

(c) Compute the $\langle q'', t''|q', t'\rangle = \langle q''|e^{-iH(t''-t')}|q'\rangle$ by inserting a complete set of momentum eigenstates, and performing the integral over the momentum. Compare your result in part (b).

# Answer

# Question 1

Problem 7.3

(a) Use the Heisenberg equations of motion, $\dot{A} = i[H, A]$, to find explicit expressions for $\dot{Q}$ and $\dot{P}$. Solve these to get the Heisenberg-picture operators $Q(t)$ and $P(t)$ in terms of the Schrödinger-picture operators $Q$ and $P$.

(b) Write the Schrödinger-picture operators $Q$ and $P$ in terms of the creation and annihilation operators $a$ and $a^\dagger$, where $H = \hbar\omega(a^\dagger a + \frac{1}{2})$. Then, using your result from part (a), write the Heisenberg-picture operator $Q(t)$ and $P(t)$ in terms of $a$ and $a^\dagger$.

(c) Using your result from part (b), and $a|0\rangle = \langle 0|a^\dagger = 0$,verify eqs. (7.16) and (7.17).

# Answer

# Question 1

Problem 7.4

Consider a harmonic oscillator in its ground state at $t = -\infty$. It is then subjected to an external force $f(t)$. Compute the probability $|\langle 0|0\rangle_f|^2$ that the oscillator is still in its ground state at $t = +\infty$. Write your answer as a manifestly real expression, and in terms of the Fourier transform $\tilde{f}(E) = \int_{-\infty}^{+\infty} e^{iEt} f(t)$. Your answer should not involve any other unevaluated integrals.

# Answer