University of Minnesota
School of Physics and Astronomy

2026 Spring Physics 8012
Quantum Field Theory II

Assignment Solution

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Homework 2 Due to February 5 9:00 AM

Question 1

Consider a set of three differential operators

$$\left\{T^{+}=x^{2}\frac{d}{dx},\quad T^{0}=-x\frac{d}{dx},\quad T^{-}=\frac{d}{dx}\right\} \tag{1}$$

acting on an arbitrary smooth function $f(x)$. Find the algebra of these operators, i.e. all three commutators $[T^{i},T^{j}]$ where $i,j=+,-,0$. Show that this algebra is closed.

Answer

Question 2

Compare it with the algebra of three Pauli matrices:

$$\left\{\frac{1}{2}\sigma_1, \quad \frac{1}{2}\sigma_2, \quad \frac{1}{2}\sigma_3\right\} \tag{2}$$

How this algebra is called? Find the linear combinations of the Pauli matrices in this which form exactly the same algebra as the operators in question 1. What is the difference between the representations of the two algebras above?

Answer