University of Minnesota
School of Physics and Astronomy

2026 Spring Physics 8902
Elementary Particle Physics II

Assignment Solution

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Problem Set 3 Due 11am, Monday, March 2, 2026

Question

Neutral kaons: oscillations, CP violation and CPT invariance

Neutral kaon mixing is described by the effective Hamiltonian $H = M - \frac{i}{2}\Gamma$, acting on the $(K^0, \bar{K}^0)$ basis, where $M(\Gamma)$ is the neutral kaon mass (decay) matrix.

(a)

Consider the following CP-conserving parameterization of the mass matrix as defined in the $(K^0, \bar{K}^0)$ basis:

$$M_0 = \begin{pmatrix} m & \Delta \\ \Delta & m \end{pmatrix}, \tag{1}$$

where Δ is real-valued. Determine the mass eigenvalues, $m_\pm$ and basis states $(K_-,\ K_+)$ in which M_0 becomes diagonal. Using the experimental inputs m_{K^0} and $\Delta m = m_L - m_S$, obtain numerical values for m and Δ.

(b)

Working in the (K_-, K_+) basis, basis, extend the model of (a) to allow for CP violation by introducing the real-valued parameter δ:

$$M_{\mathrm{CP}} = \begin{pmatrix} m_- & -i\delta \\ i\delta & m_+ \end{pmatrix}, \tag{2}$$

and assume there is no direct CP violation. This mass matrix corresponds to the *superweak* model. Assume Γ is diagonal in the (K_-, K_+) basis with eigenvalues Γ_L and Γ_S. By expressing M_{CP} in the $(K^0, \bar{K}^0)$ basis, predict the phase, φ_ϵ of ϵ defined by $|K_S\rangle \propto |K_+\rangle + \epsilon|K_-\rangle$ and $|K_L\rangle \propto |K_-\rangle + \epsilon|K_+\rangle$, and determine δ from the measured value of $|\epsilon|$.

(c)

Assume the $\Delta S = \Delta Q$ rule so that, to an excellent approximation, $K^0 \to \pi^- l^+ \nu$ and $\bar{K}^0 \to \pi^+ l^- \bar{\nu}$, with equal magnitudes of the corresponding decay amplitudes (no direct CP violation in semileptonic decays). Starting from an initially pure $|K^0\rangle$ beam at $t = 0$, use the time evolution in terms of $|K_S\rangle$ and $|K_L\rangle$ (including the mixing parameter ϵ defined in (b)) o derive the time-dependent semileptonic charge asymmetry

$$A(t) \equiv \frac{N_+(t) - N_-(t)}{N_+(t) + N_-(t)}, \tag{3}$$

where $N_+(t)$ and $N_-(t)$ are the number of semileptonic decays in a small time bin around proper time t that produce l^+ and l^-, respectively. Express $A(t)$ in terms of ϵ, Δm and $\Gamma_{L,S}$ (working to first order in ϵ). Show that at late times (when the beam is K_L dominated), $A(t) \to \delta_L \approx 2\mathrm{Re}(\epsilon)$. Using PDG values for Δm, $\Gamma_{L,S}$, make a plot of $A(t)$ versus t/τ_S over the range $0 \leq t/\tau_S \leq 10$.

(d)

Finally, extend the model in (b) to

$$M_{\mathrm{CPT}} = \begin{pmatrix} m_- & \chi \\ \chi^* & m_+ \end{pmatrix}, \tag{4}$$

where χ is complex and $Re(\chi)$ is a T-conserving, CP-violating and CPT-violating parameter. Show that, to first order in χ, the states which diagonalize the full Hamiltonian H are

$$|K_S\rangle \approx |K_+\rangle - \frac{\chi}{\xi}|K_-\rangle, \tag{5}$$

$$|K_L\rangle \approx |K_-\rangle + \frac{\chi^*}{\xi}|K_+\rangle, \tag{6}$$

where $\xi = (\Delta m)/2 - i(\Delta\Gamma)/4 \approx (m_L - m_S)/2 + i(\Gamma_S)4$, (assuming $\Delta\Gamma \equiv \Gamma_L - \Gamma_S$ and $\Gamma_S \gg \Gamma_L$). Then include direct CP violation by writing $\eta_{+-} \approx \epsilon + \epsilon'$ and $\eta_{00} \approx \epsilon - 2\epsilon'$ (to first order in small parameters), and determine how the CPT-violating mixing term χ/ξ modifies the phases of η_{+-} and η_{00}. Derive the following relation between phases

$$\frac{2}{3}\varphi_{+-} + \frac{1}{3}\varphi_{00} - \varphi_\epsilon \approx \frac{|m_{\bar{K}^0} - m_{K^0}|}{2|\epsilon|(m_L - m_S)} \sin\varphi_\epsilon. \tag{7}$$

The result $|m_{\bar{K}^0} - m_{K^0}|/m_{K^0} < 8.0 \times 10^{-19}$, which follows from this relation, provides one of the best limits on CPT invariance.