University of Minnesota
School of Physics and Astronomy

2025 Fall Physics 8501
General Relativity I

Assignment Solution

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Assignment 6 due on Monday October 13th at 10PM

Question 1

Show that

$$g_{\mu\nu,\gamma} = \Gamma_{\mu\nu\gamma} + \Gamma_{\nu\mu\gamma}, \tag{1}$$

where

$$\Gamma_{\mu\nu\lambda} \equiv g_{\mu\sigma}\Gamma^{\sigma}_{\nu\lambda}, \tag{2}$$

and that

$$g_{,\gamma} = g g^{\mu\nu} g_{\mu\nu,\gamma}. \tag{3}$$

Answer

By definition of the covariant derivative, we have

Question 2

The metric for the surface of a sphere of radius a is determined by

$$ds^2 = a^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right) \tag{4}$$

(i) Calculate the components of the affine connection. It is conventional to denote the components as $\Gamma^{\phi}_{\theta\phi}$ and similarly for the other ones. In other words, $x^1 = \theta$ and $x^2 = \phi$.

(ii) Referring to figure 3.2 of the textbook, parallel transport a vector along two different paths, starting from a point on the equator $\theta = \pi/2, \phi = 0$, and ending at the north pole. Path A is along a line of fixed longitude $\phi = 0$. Path B first goes along the equator to $\phi = \pi/2$ and then goes north along a line of fixed longitude. Initially the vector you are parallel transporting points in the $-\hat{\theta}$ direction. What is the angle between the two vectors at the north pole? (You must solve the parallel transport equation, just drawing a picture is not sufficient.)

Answer

Question 3

Starting with the energy-momentum tensor of a massless scalar field ϕ

$$T_{\mu\nu} = \phi_{,\mu}\phi_{,\nu} - \frac{1}{2}g_{\mu\nu}\phi_{,\sigma}\phi^{,\sigma}, \tag{5}$$

derive the equation of motion for ϕ in the presence of a gravitational field.