University of Minnesota
School of Physics and Astronomy

2026 Spring Physics 8502
General Relativity II

Assignment Solution

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Problem Set 1 due on Due Feb 9 at 11:59pm

Question 1

(a) Consider the outer surface of the ergosphere for a Kerr black hole. Construct the normal vector to this surface and show that it is a 2-way surface.

(b) Using the condition $n_\alpha n^\alpha = 0$ for a null surface, derive the equation equation for the critical 1-way surface for a time-independent axial symmetric surface. That is start with some $u(\rho, \theta)$ to derive the equation defining this surface. What is its physical interpretation.

Answer

(a)

Starting with the metric for a Kerr black hole in Boyer-Lindquist coordinates,

$$ds^2 = -\left(1 - \frac{2Mr}{\Sigma}\right) dt^2 - \frac{4Mar\sin^2\theta}{\Sigma} dt d\phi + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + \left(r^2 + a^2 + \frac{2Ma^2 r \sin^2\theta}{\Sigma}\right) \sin^2\theta d\phi^2, \tag{1}$$

where $\Sigma = r^2 + a^2\cos^2\theta$ and $\Delta = r^2 - 2Mr + a^2$. The outer surface of the ergosphere is defined by the condition $g_{tt} = 0$, which gives

Question 2

Show that

$$(L_X T)^{a_1 a_2 \ldots a_r}_{b_1 b_2 \ldots b_s} = (\nabla_X T)^{a_1 a_2 \ldots a_r}_{b_1 b_2 \ldots b_s} - \sum_i T^{a_1 \ldots j \ldots a_r}_{b_1 b_2 \ldots b_s} X^{a_i}_{;j} + \sum_j T^{a_1 a_2 \ldots a_r}_{b_1 \ldots j \ldots b_s} X^{j}_{;b_j} \tag{2}$$

Answer

Question 3

(a) Consider an arbitrary unit vector, X transported along a latitude line on the surface of the sphere. Use $ds^2 = d\theta^2 + \sin^2\theta d\phi^2$, and ecall that there are only 2 distinct Γ's.

(b) Do the same assuming the vector is Lie transported. That is instead of $\nabla_T X = 0$, assume that $L_T X = 0$.

Answer