University of Minnesota
School of Physics and Astronomy

2025 Fall Physics 8501
General Relativity I
Assignment Solution

Lecture Instructor: Professor Joseph Kapusta

Zong-En Chen
chen9613@umn.edu

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Assignment 7 due on Monday October 20th at 10PM

Question 1

The metric for the surface of a sphere of radius a is given by

$$g_{\theta\theta} = a^2, \quad g_{\phi\phi} = a^2 \sin^2\theta, \quad g_{\theta\phi} = g_{\phi\theta} = 0. \tag{1}$$

Calculate the Gaussian curvature $K = -\frac{1}{2}R$ for this space. Then calculate the Gaussian curvature for a space with a metric given by

$$g_{xx} = \frac{a^2(1-y^2)}{(1-x^2-y^2)^2}, \quad g_{yy} = \frac{a^2(1-x^2)}{(1-x^2-y^2)^2}, \quad g_{xy} = g_{yx} = \frac{a^2 xy}{(1-x^2-y^2)^2}, \tag{2}$$

where $x^2 + y^2 < 1$. Notice that the distance between two points is unbounded because of the denominators. Can you imagine such a space?

Answer

Question 2

In class we were to led to Einstein's field equation with the inclusion of a cosmological constant.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = -8\pi G T_{\mu\nu}. \tag{3}$$

Find the nonrelativistic, static, weak field limit of this equation. The constant Λ has dimension of $1/\text{length}^2$. Calculate the numerical value of Λ based on the numerical value of the dark energy inferred from cosmological observations.