

University of Minnesota
School of Physics and Astronomy

2025 Fall Physics 8501

General Relativity I

Assignment Solution

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Assignment 3 due on Monday September 22th at 5PM

Question 1

Show explicitly that the 4-vector current density for a collection of point charges satisfies $\partial_\mu J^\mu = 0$

Answer

In the class, we defined the 4-vector current density for a collection of point charges as

$$J^0(t, \mathbf{x}) = \rho(t, \mathbf{x}) = \sum_a q_a \delta^{(3)}(\mathbf{x} - \mathbf{x}_a(t)) \quad (1)$$

$$\mathbf{J}(t, \mathbf{x}) = \sum_a q_a \mathbf{v}_a(t) \delta^{(3)}(\mathbf{x} - \mathbf{x}_a(t)), \quad \mathbf{v}_a(t) = \frac{d\mathbf{x}_a(t)}{dt}. \quad (2)$$

Question 2

Prove that the electromagnetic energy density squared minus the square of the Poynting vector is a Lorentz invariant for an electromagnetic field by expressing this quantity in terms of tensors. You might consider using the dual field strength tensor defined by $\tilde{F}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$.

Answer

Question 3

Calculate the scalar T^α_α associated with the electromagnetic stress tensor.

Answer