

A Minimal Spectral Leray Projection for Enforcing Incompressibility in Periodic Three-Dimensional Flows

Author: Anthony Scott Hood

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Abstract

We present a minimal, exact implementation of the Leray projection for enforcing incompressibility in three-dimensional periodic flows using Fourier spectral methods. The method achieves divergence errors on the order of machine precision ($\sim 1e-16$).

1. Introduction

Projection methods are a foundational technique in incompressible computational fluid dynamics. In periodic domains, Fourier spectral methods allow incompressibility to be enforced exactly in spectral space.

2. Mathematical Formulation

Given an intermediate velocity field u^* , the pressure Poisson equation is solved and a Leray projection is applied to remove the irrotational component of the velocity.

3. Numerical Implementation

The method uses FFT-based differentiation and a spectral Poisson solve. All operators are applied in Fourier space.

4. Verification

Tests using both divergence-free and compressible initial conditions show that the resulting velocity field satisfies the incompressibility constraint to machine precision.

5. Conclusion

This work provides a compact, reference-grade kernel suitable for DNS and LES solvers in periodic domains.

References

Chorin (1968); Temam (1969); Canuto et al. (2006).