AI110 Assignment 1

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Question:

10.13.3.25:A coin is tossed 3 times. List the possible outcomes.
Find the probability of getting (i) all heads (ii) at least 2 heads

Solution:

The possible outcomes may be listed as follows:
$$\begin{pmatrix} H,H,T & H,H,H \\ H,T,H & H,T,T \\ T,H,T & T,H,H \\ T,T,H & T,T,T \end{pmatrix}$$
The sample space S has a total of eight cases.

(i).

- In order to solve this questions let us take a random variable x to represent the number of heads we get in three coin tosses and to explore all the events let us create a table to display the probabilities for varying values of x.

x	Pr(x)	Cases
0	$\frac{1}{8}$	$\{T,T,T\}$
1	$\frac{3}{8}$	$\{H,T,T\},\{T,H,T\},\{T,T,H\}$
2	$\frac{3}{8}$	$\{H,H,T\},\{H,T,H\},\{T,H,H\}$
3	$\frac{1}{8}$	$\{H,H,H\}$

- The probability of getting all heads is clearly $\frac{1}{8}$.

- In the next approach we will be using binomial distribution, but let us first define the random variables to be used. By definition, a binomial random variable is the total number of "successes" in a fixed number of independent and identical trials, each of which has only two possible outcomes (usually called success and failure, but heads and tails in this particular case). The "independent and identical" implies that every trial has the same probability of success (and therefore all of the probabilities of failure at the same, too).

- We may state this as a fixed number of iid (independent and identically distributed) Bernoulli trials, which means the same thing. So a binomial random variable is a member of a two parameters family of discrete random variables, the parameters being the number of trials and the probability of success.

Random variable	Physical representation
n	number of tosses
x	number of heads

- A further table to describe and summarize the parameters is given below:

Parameter	Value	Description
number of trials	3	number of tosses
probability of success in each trial	$\frac{1}{2}$	probability of head in each toss

- Using binomial probability distribution we can say that $\Pr(x) = \binom{n}{x}(\Pr(H))^x(\Pr(T))^{n-x}$. Substituting n as 3 and x as 3 for the first part of the question we get $\Pr(0) = (\frac{1}{2})^3 = \frac{1}{8}$.

(ii).

- For this question we may use the concept of binomial cdf which evaluates the distribution function of a binomial random variable with parameters n and p by summing probabilities of the random variable taking on the specific values in its range. These probabilities are computed by the following recursive relationship:

  $\Pr(X = j) = \frac{(n+1-j)p}{j(1-p)} \Pr(X = j - 1)$

- An input displaying the input parameters is given below:

Input Parameter	Value	Descripti
k	$\Pr(x) \geq 2$	Argument for which the binomial distribution function is to be evaluat
n	3	Number of Bernoulli tri
p	$\frac{1}{2}$	Probability of success on each tr

- The probability of getting at least 2 heads or $\Pr(x) \geq 2$ is equal to $\sum_{x=2}^{n} \Pr(x)$.

- Clearly $\Pr(2) + \Pr(3) = \frac{3}{8} + \frac{1}{8} = \frac{4}{8}$.

- Using binomial probability distribution to confirm probabilites we can say that $\Pr(x) = \binom{n}{x}(\Pr(H))^x(\Pr(T))^{n-x}$.

- Substituting n as 3 and x as 2 and n as 3 and x as 3 for the second case we get $\Pr(2) + \Pr(3) = \binom{3}{2}(\frac{1}{2})^2(\frac{1}{2})^1 + (\frac{1}{2})^3 = \frac{1}{2}$.

- The probability of getting at least 2 heads is therefore $\dfrac{1}{2}$.