

AI110 Assignment 1

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Question:

10.13.3.25: A coin is tossed 3 times.

1. List the possible outcomes.
2. Find the probability of getting (i) all heads (ii) at least 2 heads

Solution:

The possible outcomes may be listed as follows:

$$\begin{pmatrix} H, H, T & H, H, H \\ H, T, H & H, T, T \\ T, H, T & T, H, H \\ T, T, H & T, T, T \end{pmatrix}$$

The sample space S has a total of eight cases.

(i).

1. In order to solve this questions let us take a random variable x to represent the event of getting a head on a coin toss.
2. In this approach we will be using binomial distribution which we may state this as a fixed number of independent and identically distributed Bernoulli trials or binomial cdf which evaluates the distribution function of a binomial random variable with parameters n and p by summing probabilities of the random variable taking on the specific values in its range. These probabilities may be computed by the following recursive relationship:

$$\Pr(X = j) = \frac{(n+1-j)p}{j(1-p)} \Pr(X = j - 1)$$

3. An table displaying the input parameters is given below:

Input Parameter	Value	Description
k	$\Pr(x) \geq 2$	Argument for which the binomial distribution function is to be evaluated
n	3	Number of Bernoulli trials
p	$\frac{1}{2}$	Probability of success on each trial

- Using the formula for binomial probability distribution we can say that $\Pr(x) = \binom{n}{x}(\Pr(H))^x(\Pr(T))^{n-x}$. Substituting n as 3 and x as 3 for the first part of the question we get $\Pr(0) = (\frac{1}{2})^3 = \frac{1}{8}$.

(ii).

- The probability of getting at least 2 heads or $\Pr(x) \geq 2$ is equal to $\sum_{x=2}^n \Pr(x)$.
- Substituting n as 3 and x as 2 and n as 3 and x as 3 for the second case we get $\Pr(2) + \Pr(3) = \binom{3}{2}(\frac{1}{2})^2(\frac{1}{2})^1 + (\frac{1}{2})^3 = \frac{1}{2}$.
- The probability of getting at least 2 heads is therefore $\frac{1}{2}$.